SOME RECENT CONTRIBUTIONS TO FLUID FLOW MEASUREMENT AND INSTRUMENTATION

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I hereby declare that the subject matter contained in this thesis is my own work and that it has not previously been submitted to another University for degree purposes.

14th December, 1964.

TO MILLA

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S Y N O P S I S

Advance in technology over various fields of science and engineering during the last decade, has made its impact felt also in the sphere of fluid mechanics and Hydraulic engineering. Here progress has set a [illegible] for ever increasing accuracies in measurement of velocity, pressure and discharge in an increasing variety of fluids.

The very nature of fluid flow mechanisms is such that it cannot be pinned down to a rigorous mathematical treatment - consequently resort must still be had to instrument application for measuring basic flow parameters.

However, the existing accepted accuracies of two to three per cent in flow measurement are simply not acceptable any longer - workers are searching for measuring techniques to yield accuracies of $\pm \frac{1}{2}$ to $\pm \frac{1}{4}$ per cent when dealing with the more refined aspects of, to mention only a few, water and air flow, jet engine research, the chemical industry and hydraulic machine design.

This thesis comprises a comprehensive theoretical analysis of all the factors influencing the conventional methods of measuring total and static pressure, velocity and discharge, with the ultimate goal of improved designs, methods of instrument calibration and application and methods of evaluating or correcting errors involved. The author is motivated by the conviction that mere review and accumulation of experimental data does not lead to the required higher accuracies and feels that a new approach is necessary in the form of basic analysis and, where suitable, mathematical treatment of the various flow measuring methods. Resulting hypotheses can then be substantiated or refuted in the light or reliable experimental evidence.

Two total pressure probes have been developed which combine excellent performance characteristics with simplicity of manufacture. Each incorporates a small cylinder having a single orifice inside a spherical shield which is vented at the rear. At air speeds up to ~ 275 ft/sec probe No. I with a shield diameter of 0.344 inch had a coefficient of unity and recorded total pressure to within one per cent of the dynamic pressure for angles of yaw up to ~ ± 31 degrees and angles of pitch up to ± 30 degrees. In the speed range ~ 70 ft/sec to ~ 275 ft/sec, the coefficient of probe No. II with the same overall dimensions was 0.991 to within ± 0.2 per cent and the range of insensitivity to angle of flow effects was extended to ~ 48 degrees angle of yaw and ~ ± 44 degrees angle of pitch. The response time of these probes under a suddenly applied air pressure of 500 mm water gauge was ~ 54 seconds when linked to a Betz projection manometer. Theory and experiment were in good agreement.

Development of a single-purpose static pressure probe which showed reasonable performance characteristics is described. This instrument had a coefficient of 0.655 being to within ± 1 per cent of the measured values over the speed range 70 ft/sec to 280 ft/sec. The average angular insensitivity over the range was very encouraging, giving ± 33 degrees in yaw and -15 degrees to +30 degrees in pitch for a coefficient value to within 1 per cent.

Other methods for measuring static pressure were analysed theoretically. The wall static tapping method proved to be the most reliable and accurate, provided it was applied with the necessary 'know-how' and related corrections. It is pointed out that there are still many complex factors demanding further investigation and clarification, whatever the method of measurement adopted. These factors are identified and analysed and some correction factors are suggested.

Furthermore, basic principles and factors affecting the operation and performance of current meters are dealt with theoretically. The basic meter equations were derived and results from calibration, laboratory and field experiments are analysed in the light of the factor analysis. Suggestions are made for the possible development of more accurate and refined techniques for meter calibration and application, in the hope that these would be of some assistance to research workers and would-be users of current meters.

A comprehensive analysis was made of all the factors likely to influence the functioning of venturi meters. Theoretical expressions for calculating discharge coefficients for conical-type meters were derived and the values thus calculated were compared with experimental results from N.E.L. and Liège. The author's theoretical values for Reynolds numbers 2×10^5 and higher are in very close agreement with results from the final tests at N.E.L. and Liège, the maximum discrepancies being of the order of -0.4 per cent at the 'hump' of the meter characteristics, and ranging from +0.03 to zero per cent for Reynolds numbers 4×10^5 and upwards. In the laminar range, Hall's part of a proposed theoretical curve shows maximum deviations of $\sim \pm 0.3$ per cent up to a Reynolds number of 1.5×10^5. Laminar, transitional and turbulent zones in the characteristic curve have been suggested on experimental and theoretical evidence and there seems to be theoretical verification of the hypothesis relating meter characteristics to pipe friction curves. Numerical application of theoretical considerations will continue to meet with difficulties until venturi forms and designs, allowing better and more controlled flow characteristics, are specified by International Codes. In this respect certain suggestions are submitted for consideration. It is possible that more reliable relationships for predicting discharge coefficient within specified limits of accuracy would result thus rendering meter calibration superfluous.

The author trusts that the subject matter dealt with, and the general approach in this thesis, would eventually contribute to the ultimate aim of higher accuracies in flow measurement.

CHAPTER I

Many centuries ago a poor man asked a sage : 'Why am I in such need?' The sage broke off a willow twig and cut notches on it at equal intervals. Then he gave it to the poor man, saying: 'I give thee this sceptre of success, a stick of measuring. Remember, all things are made to measure : sandals for thy feet, the sheath for thy sword. Drive the stick straight into the ground and according to its shadow, which follows the sun, thou shalt measure time and arrange thy life. In the spring, when the shadow shortens, sow thy grain; in autumn when it lengthens, gather in the crops. Measure thy share and thy neighbour's share; do it honestly and thou shalt fare well.'

To measure as faithfully and accurately as possible is the theme of this thesis which deals with fluid-flow measurement.

1. INTRODUCTION

Hydraulic measurements date back to the beginnings of civilisation when water was, as it still is today, essential to man's very existence. A log timed in its progress with the current, a marked stick dipped into a fluid .... these probably were the first crude methods of measuring or defining the state of a fluid in motion or at rest.

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C H A P T E R   I

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As human needs and progress became more and more sophisticated, measurements of velocity, pressure and discharge, in widening varieties of fluids, grew in importance. Further progress demands that measurements of the basic fluid flow parameters be accomplished with ever higher accuracy.

In modern science and technology, the mechanisms of fluid flow are so complex that they cannot as yet be fully explained and defined by rigorous mathematical treatment, and therefore resort must still be had to instrument application for the measurement of basic flow parameters. Accuracy in measurement is thus of vital importance not only to design and development but also to the advancement of the science of fluid mechanics.

The concept of 'accurate measurement' can perhaps be clarified by the following thoughts. Although under ideal conditions and with a suitable instrument, a flow parameter may be measured so accurately that the differences between the measured and true values are negligibly small, this no longer holds once conditions become those normally encountered in practice. The characteristics inherent both in the instrument and in the flow are such that a measurement will always be subject to some degree of error. Accurate measurement is thus possible only if the error involved can be minimised - by instrument design or method of measurement - or if the error can be estimated and due allowance made. How can this ideal of accurate measurement be achieved?

2. THE PROBLEM

Errors in measurement stem from complex interrelated influences of different flow factors on the operation of a particular instrument. Elimination and/or correction of errors becomes feasible only if the effects of these factors can be analysed basically and assessed quantitatively. Examination of the associated theoretical and experimental treatment of these influences could lead to improved designs and methods of instrument calibration and application, and of error estimation.

Experts in this field in the research laboratories of various countries still cannot reach agreement, however, on the influences - direct and indirect - of the multitude of factors affecting the characteristics and operation of flow measuring instruments.

In many instances workers do not measure what they believe they are measuring, often because of thoughtless and faulty application of instruments in accordance with International Code specifications which are largely based on experimental work conducted in the late 1930's. In many electronic measuring devices, instrumentation may be near-perfect, from the transducer to the indicating or recording device, but what the transducer actually senses cannot always be defined in terms of the parameter that is supposed to be measured.

A colleague, recently returned from an extensive study tour in the United States of America, expressed the opinion that the fundamental factors affecting the measurement of basic flow parameters are at the present time often ignored provided the sensing and recording are done electronically. Many investigators seem to be 'orbiting in outer space' and refuse to 'come down to earth' on unsolved fundamental problems related to instrument development and application.

The accuracy with which the basic flow parameters can be measured dictates the reliability of conclusions drawn from experimental results; accuracy of measurement determines the weight to be assigned to quantitative work and often forms the basis upon which a hypothesis can be confirmed or refuted. particularly so when mathematical analysis proves impossible or inconclusive.

Haughton, in a recent contribution [(1)] stressed the urgent necessity for more accurate flow measurement by pointing out that various aspects of liquid, air, and gas flow measurement were for diverse reasons being reappraised in the United States. 'Accuracies of the order of ± ½ per cent on an absolute basis', wrote Haughton, 'had required that factors in the basic flow equations and those associated with the testing and calibrating techniques, be meticulously examined for effects that could disturb the overall measurement by a 1/10 of 1 per cent, or less'.

He pointed out further that 'existing national and international flow codes had been devised by consolidating the work of many individuals and organisations that, over the past fifty years, had endeavoured, either to solve their own local flow problems - or had very definitely tried to cover or fill in gaps of flow knowledge through investigations commissioned by voluntary organisations interested in preparing a code. However, [illegible]ming that there was a wide interest in, and a definite need for, higher flow accuracies, it would seem that the entire body of flow measurement knowledge should be re-examined with the idea of re-checking it, filling in gaps, cleari[illegible] uncertainties, and extending existing ranges of information to new conditions of current interest, but which had been up till now termed extreme'.

In research and design in the fields of aeronautical, civil, chemical, marine and mechanical engineering, a knowledge of velocity and pressure distribution in fluids, in motion or along boundaries, is often indispensable.

Static pressures and velocities at points in free streams, in turbulent flow regimes, and in flows encountered in hydraulic machinery are often required to be measured with extreme accuracy. Accurate measurement of total pressure distributions in free streams, inside turbo-machines, and in the wakes of totally submerged bodies or near solid boundaries, especially in three-dimensional and turbulent flow fields, are often vital in research and design. Pressure and velocity gradients in boundary-layers characterise the very nature of fluid flow and need to be determined accurately in investigations related to heat exchange, mixing processes, and energy transfer, while the quantity of a fluid flowing is important in modern industrial plant, especially in the chemical industry and in water supply.

In medical research and diagnosis, particularly in circulatory and heart diseases, further progress is heavily dependent upon improved accuracy in the measurement of blood pressure, flow rates, and pressure differentials. This particular aspect is dealt with in some detail in the next chapter.

Many trusting investigators solemnly believe that accuracies of ± 1 per cent and better can be expected in ordinary routine measurements of pressure, velocity and discharge of water, gas or air. Little do they appreciate how difficult, if not impossible, it is to achieve accuracies of this order. At the present time many groups - particularly those dealing with the more refined aspects of water and air flow, in jet engine research and development, and in chemical technology - are clamouring for flow measurement techniques that will yield accuracies of ± 1/4 to 1/2 per cent. [(1), (2), (18)] As pointed out in the discussion on Ref. 3, we cannot 'go on living with our two or three per cent accuracies in flow measurement'.

This demand for greater accuracy can best be met by developing new instruments, applying new techniques or by improving existing methods and the author submits that the most fruitful approach for ensuring success is through intensive basic study and theoretical analysis - supported by experimental verification - rather than through continued accumulation and review of experimental data.

Engel, [(3)] an internationally recognised hydraulician, said in 1960 - several years after the author had commenced his researches - 'Any further research should not be started without a very thorough theoretical investigation, the simple accumulation of empirical values being stopped immediately' .... He concluded his remarks with the following statement : 'An analysis of our knowledge on flow through contractions in closed conduits at the present time discloses many controversial points. The main task in the near future would be to advance certain hypotheses and to confirm or refute them, systematically narrowing down the wide scope of contradictory arguments, and thus establishing a better insight into the mechanism of flow through contractions'

The author submits that the above statement is applicable not only to instruments associated with 'contractions', but to most instruments and techniques currently used in flow measurement.

Fundamental analysis of all flow factors affecting the characteristics and operation of flow measuring instruments, therefore, would seem to be the logical foundation for future research programmes.

3. SCOPE OF INVESTIGATIONS

Having regard to the immense diversity of measurement both in closed conduits and open channels, only the methods and instruments most commonly employed have been selected for detailed study. In fluid mechanics the fundamental parameters are : length, surface, volume, angle, pressure, velocity and mass flow-rate; in this study the physical properties of fluids are not considered. Particular attention is given to the measurement of total pressure, static pressure, velocity of flow and flow-rate.

As a first step, an intensive literature survey covering the instruments, techniques, accuracies and problems associated with the measurement of the above flow parameters under different conditions of flow was undertaken. The instruments selected for detailed study were total and static pressure probes, wall static tappings for static pressure measurement, current meters and the venturi meter.

The survey identified those factors which, directly and indirectly, affect the operation of the instruments in question. A preliminary analysis confirmed the importance of a deep understanding of the different instrument characteristics and flow mechanisms involved.

In the first instance therefore, an attempt was made to analyse, in the light of available experimental evidence and contradictory theories, all the factors which could possibly influence the functioning of these instruments in flow measurement, with a view to differentiating among the 'known', the 'uncertain' and the 'unknown'. In the second place, the 'uncertain' and 'unknown' influences were analysed critically and where possible, theoretically in order to establish a comprehensive factor analysis for use in possible instrument development and application. Finally, the above analysis was presented in such a form as to provide a guide for attaining improved accuracies from use of these instruments in laboratory and field.

To delineate the theme, the problem and the scope of this investigation there follows a brief review of each of the measurement processes and each of the instruments to be dealt with; attention is drawn to faulty practices in laboratory and in industry and to the many aspects about which there is lack of agreement among experts in this field.

3.1 The measurement of total pressure

Total pressure at a point in a fluid in motion may be defined as the sum of velocity head and static pressure head at that point or, neglecting the effect of turbulence at this stage :

$$p_t = p_s + \tfrac{1}{2}\rho V_c^2 \quad * \qquad (1.1)$$

* A list of symbols with meanings appears in Appendix I.

where p_t = total pressure;

p_s = static pressure;

ρ = mass density of fluid;

V_o = free stream velocity.

Instruments used for measuring total pressure are : Pitot tubes, Pitot cylinders, Pitot spheres and claw-tube devices.

Under normal conditions it is not difficult to measure total pressure accurately; a great variety of geometrical shape is permissible and is to be found. Any suitable opening in a probe of reasonable geometric shape can be arranged to sense a pressure closely approaching the total pressure. The simplest form might be a tube bent at right angles, the leading leg of which is cut square and located in the flow with the axis parallel to the direction of flow. Alternatively, a hole in a sphere or other symmetrical body, such as a small disc, connected to a tube, will sense total pressure when the hole is normal to the direction of flow. If the positioning and geometry of the hole in a simple hook type, cylindrical, or cantilever Pitot tube are correctly chosen, these instruments will be insensitive to yaw and pitch over a range ± 12 degrees.[4]

The instrument constant or coefficient of these instruments is usually taken as unity and depending on the geometrical shape selected, errors result mainly from such factors as : incorrect location, size and shape of the total pressure opening; improper alignment of the instrument to the direction of flow, especially in two-dimensional and strongly three-dimensional flow fields; incorrect instrument dimensions relative to duct size; and vibrations caused by shedding of vortices behind the transverse leg of the probe. Errors can also arise when Pitot tubes are used at low Reynolds numbers, that is in fluids of high viscosity flowing at low velocities, when viscous forces become predominant. Departure of the instrument constant from unity in these circumstances may be considerable.

Other factors having a marked influence on the performance of these instruments are turbulence, swirl, centre line displacement in zones of steep total pressure gradient, and proximity to solid boundaries.

How are these instruments generally employed? In practice it is not uncommon to find pressure probes inserted into strongly turbulent fields; the total pressure measured is incorrect because the angle of attack is at times well beyond the yaw insensitivity range. Total pressure probes are often to be found near solid boundaries where .racteristics, as determined in free-stream calibration tests, do not apply; the resulting error is unknown.

When cantilevered probes are used in pipes of large diameter, the probe often vibrates in the flow as the instrument traverses the pipe diameter. This factor is sometimes neglected in measurements, especially in industry, but can give rise to serious errors. (5)

Engineers in the field are often obliged to employ the 'instrument available', whereas a reasonable result is possible only if the various factors affecting probe performance are given due consideration. In two large industrial undertakings in the Republic the structural lay-out is such that suitable locations for flow gauging are simply not accessible. Pitot tubes are inserted, often on one diameter only, in positions immediately downstream of a 90 degree bend or valve or other disturbing feature.

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In one instance, the only practicable position for gauging the quantity of flue gasses discharged through a 200-ft. high chimney was at the base of the chimney immediately following a 90 degree vertical bend. A traverse was possible on one diameter only of the circular section. Different traverses on this diameter were completed, and the resultant distorted velocity profile was plotted. In accordance with the initial interpretation of the data available the quantity discharged was estimated and compared with the value computed from measurements of quantity of combustion materials involved. The discrepancy was more than 100 per cent. The author's investigations revealed that the traverse had been located in the separation zone immediately downstream of the inside of the 90 degree bend. Upon adjustment of the velocity diagram, recomputation of the discharge through the chimney gave a result within 5 per cent of that calculated thermochemically. This example illustrates the importance of correctly evaluating a particular flow effect.

In steady flow the Pitot cylinder type of instrument should yield results having a standard deviation less than $\pm$ 0.3 per cent. On the other hand, accurate measurement of total pressure in strongly three-dimensional turbulent flows is cumbersome, difficult, and often impossible. The mechanical alignment of directional standard-type probes with the spatial velocity is not always possible because of the bulkiness of probe and traverse gear. In general, a probe having adequate stiffness, rapid response, and a wide range of insensitivity to angle of attack, is the ideal.

Kiel [6] was the first to develop a 'shielded' total pressure probe. The range of insensitivity to yaw and pitch is about ± 55 degrees. The instrument has size limitations, however, whereas the smaller or miniature Kiel probe is difficult to manufacture and identical instruments suffer discrepancies in characteristics.

The author's objective was the design of a probe of smaller dimensions than the Kiel probe, easy to manufacture, having rapid response and comparable insensitivity range. The success with which this aim was achieved, is discussed in Chapter III.

3.2 <u>The measurement of static pressure</u>

The static or piezometric pressure in a

fluid is the pressure that would be registered by a pressure gauge if it were moving along with the stream so as to be at rest or 'static' with respect to the fluid. To design a pressure gauge to move with the fluid is impracticable. The customary way of measuring static pressure is to connect a pressure gauge or manometer to a small hole drilled normal to the wall of the duct in which the fluid is flowing. The location, geometry and edge finish of this hole are all-important factors.

If the hole is too large the piezometric pressure will be overestimated. A protruding burr, on the other hand, causes separation and therefore an underestimate of the static pressure. If the entrance edges are too generously rounded, impact occurs, leading again to a pressure rise above that of the true static pressure. If the tapping diameter is much greater than the pipe wall thickness, stream filaments are split, giving rise to secondary flow inside the hole and over-registering of the pressure reading.

The best arrangement is a perpendicular hole not more than 3 to 8 mm in diameter but never greater in diameter than the pipe wall thickness. In aerodynamic work diameters vary from 0.5 to 1 mm. Burrs must be removed and the internal edges of the tapping should be slightly rounded. It is important that there should be no change of hole diameter for a distance, measured from the flow boundary, at least 2.5 times the hole diameter. One or more of these holes, with or without interconnecting pressure chambers, may be used.

A probe consisting of a flat circular disc with a piezometer opening at the centre, through which the static pressure is transmitted, may also be used. It is self-evident that such an instrument will give erroneous readings if the disc is not perfectly aligned with the flow direction. In turbulent flow there are fluctuating components of velocity in all directions which complicate accurate static pressure measurement.

To provide an ideal piezometer hole is seldom possible in practice because of the difficulty of removing burrs and rounding the inside entrance edges. The possibility of introducing slight suction or impact pressures is therefore always present.

In one industrial concern in South Africa the author witnessed measurements of flow in a large water main; the static pressure was determined by means of a single piezometer tapping. The hole was 3/4 inch in diameter and 1/2 inch deep at the inner boundary increasing in diameter to accommodate the external tapping plug. The entrance edges were not rounded and, since drilling was from the outside, internal burrs had almost certainly been left. Measurements of piezometric pressure at this hole were used in conjunction with total pressure readings to determine the velocity profile. The results obtained were so inaccurate that they had to be discarded.

An instrument which may also be used for measuring static pressure consists of a cylindrical probe aligned with axis in the direction of flow and having suitably located side openings. Here, however swirl, turbulence, piezometer hole geometry, and boundary-layer effects on the probe itself can lead to inaccurate measurement.

Piezometer tappings in a duct wall or static pressure orifices located on a cylindrical probe, as described above, are the methods most commonly employed for measuring static pressure.

Recent laboratory work by Preston and Norbury (7) showed, in measurements of flow at a Reynolds number of 4×10^4, employing a total pressure probe associated with wall tappings, an error of 0.0035 compared with 0.0054 discharge units in a Pitot-static traverse thus demonstrating the superiority of the wall tapping-method for static pressure measurement.

In an attempt to measure mass air flow to an accuracy of better than 0.25 per cent by means of a Pitot-static traverse, Stratford and Ascough (8) found that, using high quality static hole tappings, the static pressure could be measured in steady flow to an accuracy of 0.1 per cent, but that to achieve this the surface finish on the inside of the tube, in the vicinity of the hole, as well as the tapping hole itself required 'meticulous attention'. Outside of direct gravimetric or volumetric measurement, however, an accuracy of this order is open to question.

One may well ask whether it is practicable to develop a single-purpose static pressure probe or whether one should concentrate on the measurement of total and static pressure with one probe as, for example, with the Pitot-static tube.

The latter method, as has been indicated, is not as accurate as that given by static tapping holes in a duct wall.(8) This accuracy is however only realised when using very high quality tappings which, in turn, poses a prerequisite often very difficult and even impossible to satisfy outside the laboratory.

Whatever method is employed it seems imperative to concentrate future researches on the possible development of a single-purpose probe, as well as on the derivation of reliable correction factors for the inherent wall static hole error, turbulence effects and errors caused by static pressure distribution over a duct cross-section.

The author has developed a spherical single-purpose static pressure probe which shows promise, and has also derived correction factors to take account of the abovementioned errors in wall static and Pitot-static pressure measurements. These are dealt with in Chapter IV.

3.3 The measurement of velocity

The term 'velocity' means 'point' or local velocity and can be defined as the derivative of the displacement, s, of a fluid particle with respect to time t, or

$$\bar{v} = \frac{ds}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.2)$$

where $\bar{v}$ = time average of 'point' velocity.

If, as [illegible]ted in the previous section, the total and static pressures at a point in a fluid can be measured accurately, the velocity at that point could be determined indirectly from the expression :

$$\bar{v} = \sqrt{\frac{2\Delta p}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots (1.3)$$

In equation (1.3), Δp is the difference between total and static pressures, the instrument constants being taken as unity. This method will again be referred to in Chapter V.

For direct 'point' velocity determination, however, the current meter, which dates back to 1790, remains the instrument most commonly employed. There are today more than 300 papers dealing with current meter performance, but there is still speculation and doubt about the accuracy of this instrument and about factors affecting its functioning.

The operation of the current meter is based on the principle that the local flow velocity in the fluid is proportional to the speed of rotation of a vane when the axis of rotation is in the direction of flow, or to the speed of rotation of a 'cup' when the rotor axis is perpendicular to the direction of flow.

It is usual to develop an expression for the velocity of flow which, for a given range of flow, approximates the true calibration curve. For the screw-type meter the curve is a flat hyperbola and a typical expression is :

$$V = K_a n + A \quad \ldots\ldots\ldots\ldots (1.4)$$

where n represents the number of rotor revolutions per second and K_a and A are constants for the particular meter over the specified velocity range.

Current meters do not register the momentary velocity but actually average out the pulsating flow variations at a point in the fluid; this has a distinct advantage. When correctly applied under ideal conditions, current meters can be relied on to register average point velocity to an accuracy of about $\pm$ 0.5 per cent.(9) In oblique or strongly pulsating flows, however, current meters might overestimate velocity by as much as 2 to 4 per cent.[10]

Uncertainties and doubts about the exact functioning of current meters may be summarised thus :

What accuracies can reasonably be expected near solid boundaries? How is the application of the meter affected when calibration has been by towing through still water? Do the waiting times between towing runs affect the calibration? What is the influence of temperature changes during rating or between rating and application? How do viscosity changes in the lubricating oil (due to temperature or pressure changes) affect the rating equation?

Relative importance and effect of supporting structure, turbulence, strongly oblique flow, and velocity gradients across the meter circle are additional sources of speculation among leading authorities.

Kolupaila[9] at a recent symposium on flow measurement at the National Engineering Laboratory, East Kilbride, Scotland, (N.E.L.) stated that current meters cannot accurately register velocities at positions closer than 0.75D to a solid boundary, D being the runner diameter. (This proximity requirement derives from the German test code). On the other hand, Professor Benini [11] reported at the same symposium that the error caused by wall proximity cannot be distinguished from ordinary experimental errors which eclipse possible proximity effects and that these do not exceed 0.2 to 0.3 per cent.

Specialists such as Professor Benini, R.G. West, Dr. Böhm-Raffray, and Professor Gerber [12] are attempting to link wall proximity effects to those associated with rating by towing through still water, and to waiting periods between runs, in order to determine a logical interdependence of these factors.

Coffin [13] reports a marked influence of temperature on rating characteristics. In contradiction, Dr. Böhm-Raffray, in Vienna, could not establish any temperature effects at all. In this respect it is important to note that, due to temperature changes, and under pressure changes when the instruments are employed in closed conduits under high pressures, viscosity alters in different types of lubricating oil, thus complicating the logical interpretation of research results.

The influence on flow measurement of the frame supporting the current meters is yet another controversial factor. Serpand and Coffin (14) found that errors in current meter measurements of large discharges were mainly caused by the change in velocity profile at the measuring section due to the presence of the current meter frame. They also reported that model tests allowed this phenomenon to be evaluated and offered a correction factor which could be applied to subsequent field tests.

In a discussion on the Serpand - Coffin paper, West disagreed with the above findings because, inter alia, the Reynolds numbers were different; Chaix wanted to know to what extent reduction of flow speeds, due to the grid effect, could be attributed to the supports. (15) Professor Gerber stressed the importance of this unknown factor in rating experiments; Professor Bogena was sceptical about the magnitude of the correction factor based on the model tests and Dr. Winternitz argued 'that a fairly wide uncertainty interval must be applied to the results from (some of) these tests' [15] .

Disagreement seems to centre on the altered velocity profile caused by the supporting frame. Flow retardation, accompanied by acceleration due to venturi-effect, in the vicinity of the frame at the gauging section require further investigation.

In South Africa, current meters are widely used in industry and in the field without adequate laboratory facilities for calibration. The International Code [16] specifies that meters be calibrated before and immediately after a test series to determine the limits of accuracy of the test results. In the Republic this specification is very seldom, if ever, met.

For acceptance tests for pumps and turbines, high class current meters, properly calibrated, should be employed. The limits of error of these instruments should be accurately known.

Clearly, there is no adequate guide to the procedure to be adopted for the use of current meters and the precautions to be taken to achieve desired accuracies; many uncertainties still exist. It was therefore the author's from (some of) these tests'[15].

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Clearly, there is no adequate guide to the procedure to be adopted for the use of current meters and the precautions to be taken to achieve desired accuracies; many uncertainties still exist. It was therefore the author's aim to examine critically, with reference to tests carried out in the laboratory and in the field, all the factors influencing the functioning of current meters. This work is dealt with in Chapter V.

3.4 <u>The measurement of discharge in closed conduits</u>

Indirect measurement of discharge by means of the venturi meter is considered in this section.

Clemens Herschell named the venturi met after the French engineer Venturi, who in 1797 was first to formulate the basic principle of conversion of pressure head to kinetic energy in a pipe constriction.

The venturi meter incorporates a constriction located between two tapered sections and the device is usually inserted between two flanges in a pipeline.

Acceleration of the fluid in the constriction results in local lowering of average static pressure. Suitable pressure tappings allow the difference in pressure between the 'inlet' and 'throat' sections to be observed, and the discharge can then be deduced from the fundamental relationship, viz :

$$Q = C_d \frac{A_2}{\sqrt{1 - m^2}} \cdot \sqrt{\frac{2 \Delta p}{\rho}} \quad .... (1.5)$$

where, Q = discharge;

C_d = discharge coefficient;

$\frac{A_2}{\sqrt{1 - m^2}}$ = K = instrument constant;

Δp = differential pressure as measured;

ρ = mass density of the fluid;

A_2 = throat cross-sectional area, and

m = $\frac{A_2}{A_1}$, where A_1 is the area at inlet to the meter.

The calibration characteristic of a venturi meter is given by an experimentally determined graph representing C_d vs R_e. For different meters these characteristics show inherent properties which have, as yet, not been fully explained. Theoretical progress has been hampered by the many complex flow factors associated with flow through constrictions. These effects could not in the past, and perhaps cannot even now, be analysed mathematically, with the result that to-day, after almost four decades of concentrated experimental research, no acceptable method of predicting C_d exists.

One of the main problems associated with venturi meters is the assessment of reliability and constancy of margins in the higher accuracy ranges. A meter having high internal roughness will remain reasonably stable and register discharge within ± 1 per cent to ± 2 per cent, whereas a meter having 'smooth' internal finish may, after having become soiled, suffer discharge errors upwards of 4 per cent.[(2), (17)]

At the present time, many workers, especially those associated with jet propulsion research and modern chemical technology, are clamouring for flow measurement to accuracies of ± 1/4 per cent to ± 1/2 per cent.[(1), (2), (18)] The crux of the problem is that a standard venturi meter operating to such accuracies does not exist and, although individual meters might achieve this accuracy margin, the duration or consistency of such performance cannot be guaranteed or predicted.

In an attempt to distinguish between the 'known' and the 'unknown', the author has listed more than thirty different but interrelated factors that have a direct influence on the operation of a venturi meter; there are others that have an indirect influence. It is astonishing to report that there seems to be very little agreement among leading authorities about the spheres of influence of these factors on venturi meter behaviour. The whole topic is indeed surrounded by disagreement and contradiction.

At a recent symposium on flow measurement held at N.E.L., the fact was stressed that 'flow measurement was at the cross-roads' and that it has become necessary to separate the 'known from the unknown'. Haughton [19] advocated the preparation of a suitable guide to workers in the field of meter improvement - 'as a means of explaining to venturi meter users just why it was that high accuracies of better than ± 1/2 per cent were so very hard to obtain'. A changed approach to methods of research and modus operandi would have to be investigated.

In South African industrial plants venturi meters are often placed immediately downstream of pipe bends. A typical case is that of a fuel injection plant in which air and oil must be supplied in correct proportion. The air supply is to be injected from a ring main at various elevations. Complete combustion is essential because a deficiency of air would cause freezing of the charge near the injection point (as a result of endothermic decomposition of oil). Venturi meters or orifice plates are to be installed on pipes feeding from the ring main, the only positions available being immediately downstream of 5-foot long bends on the 10-inch diameter pipe. If special precautions are not taken grave errors in proportioning of the air and oil can result.

Quantities of oxygen of up to 100 tons a day are metered by means of orifice plates having long thin leads to the recording devices; the readings are interpreted in terms of the original C_d values and little regard has been given to the fact that coefficients of discharge change with time. Many examples like the foregoing could be cited. It is in circumstances such as these that accuracies of ± 2 per cent and even ± 1 per cent are often claimed.

To stress the need for further research in this field, a few of the 'factors in dispute', referred to earlier, are now dealt with.

Neale [17] reports that difference in surface texture of upstream pipe surfaces can alter C_d by as much as 4 per cent in large meters. West [20] argued that the marked effect of upstream pipe roughness was caused by incomplete development of the entrance velocity profile. Schlag[21] found that a rough upstream pipe in meters having large area ratio caused the 'hump' in the characteristic curve to disappear, while Hall [1] explained that the increase in C_d, which causes the hump, was due more to thinning of the throat boundary-layer at transition than to upstream pipe conditions.

The question here seems to be : How do upstream pipe roughness and length affect the throat boundary-layer and what effect has the throat boundary-layer, in turn, on the discharge coefficient at, say, fully developed entry conditions?

Concerning radii at the entrance and throat sections, employed to prevent separation at high Reynolds numbers, Rouse [22] states that separation causes a lowering of the local boundary pressure, while Engel [3] maintains that, according to Newton's second law, a marked increase in pressure could result. Why rounded corners should affect the discharge coefficient, and to what degree, is still uncertain.

Concerning other factors governing the flow in a venturi throat, Hall [20] submits that Hutton's [23] analysis, based on assumption of fully developed flow in the throat section, 'does not lead to a better understanding of the problem'. Hutton, in turn, argued that, although Hall's boundary-layer approach seems promising, factors such as velocity distribution corrections, losses over the venturi, and upstream pipe and internal meter conditions cannot be ignored. [1]
Ziegler [24] seems to be in agreement with Hall on the question of boundary-layer effects but then disagreed with those authorities who advocate the use of meters with 'rough' internal finish.
Spencer [24] pointed out that if Hall's theory applied quantitatively to rounded entrance nozzles then why did he use the N.E.L. experimental results in association with a classical venturi to substantiate his theory? Spencer pleaded for controlled experiments and subsequent theoretical

treatment.

Sanker [24] suggested the introduction of trip wires to control the velocity profile and possibly the boundary-layer, while Engel [3] was of the opinion that there still existed considerable confusion regarding the influence of the boundary-layer and separation; until more experimental information on boundary-layer effects became available, it remained doubtful whether artificial control by a trip wire would be helpful.

Disagreement and uncertainty also range over factors such as : Static tapping geometry, radii of proposed roundings, angles of convergence and divergence, throat length, whether C_d increases with Reynolds number or becomes constant at higher Reynolds numbers, effects of installation conditions, methods of defining and standardising 'roughness' or surface finish, effects of different area ratios, influence of the so-called 'impact pressure', conditions of fluid to be metered, etc. Factors which, until now, have been completely neglected are : the effects of turbulence, the static hole error, and the error caused by the fact that even the exact wall static pressure differs from the average static pressure over the gauging section.

The author has therefore attempted to analyse each of the controversial factors theoretically or critically. A theoretically derived relationship for C_d has yielded values which agreed closely with those determined experimentally. A second relationship for C_d der[illegible]tension of this theory is proposed as a [illegible] future experimental research. This work [illegible] described in Chapter VI.

4. <u>CONCLUSION</u>

Engel, [3] said in 1960 :

'One of the most serious shortcomings of the present state of the art is that the various National Standards are based on experimental investigations which were conducted in the thirties before the work of Prandtl and Schlichting were available' .... These standards therefore cannot yield the desired accuracies. He stated emphatically : 'Any further research should not be started without a very thorough theoretical investigation, the simple accumulation of empirical values being stopped immediately .....'

What Engel said in 1960 was in almost perfect agreement with the author's ideas at the time this research project was commenced in 1956. His words enunciate the theme of this thesis which, it is hoped, will prove a positive contribution in the desired direction of higher and more reliable accuracies in fluid flow measurement.

CHAPTER II

THE MEASUREMENT OF BLOOD-FLOW PARAMETERS ASSOCIATED WITH MEDICAL RESEARCH, AND CARDIAC DIAGNOSIS AND SURGERY

1. INTRODUCTION

It is not only in engineering and pure science that accuracy in the measurement of fluid pressure, flow velocity, and volume flowing constitute a problem. In the human body the heart pumps blood through a complex network of elastic ducts. The problem of accurate measurement of flow parameters in this field is aggravated not only by changes in pressure and flow velocity, resulting from distensibility of veins and arteries, but also by the inaccessibility of 'suitable gauging points'.

It is not at present possible to compute, from volumetric or suchlike measurements, the flow of blood either through the heart or peripheral circulatory system. The volume of internal organs can be estimated from X-ray silhouettes, but no recognised technique is available for accurately recording absolute blood flow rate directly. Of the abovementioned three parameters, 'blood pressure' alone can be measured directly. By inserting hypodermic needles or catheters, connected to manometers, pressures in virtually all parts of the cardiovascular system can be studied.

Measurement of these pressures is, however, widely misinterpreted. The hypodermic needle or catheter, if oriented against the direction of flow, is in effect a blunt-nosed total pressure probe sensing total, not static, pressure. When the blood velocity at the point of measurement is relatively low, the percentage error associated with incorporation of the velocity head in a pressure measurement is small. On the other hand, if one is interested in static pressure wave forms in regions where the blood velocity is of the order of 10 ft/sec and higher, neglect to deduct velocity head would introduce a percentage error of 50 per cent or even as much as 100 per cent. Increases in venous total pressures have been interpreted as increased volume of blood in the veins, increased effective filling pressure (which is actually static pressure), and increased diastolic volume of the ventricles. To postulate a change in volume solely on the basis of a change in pressure, tacitly overlooks possible variations in the distensibility of veins, cardiac walls, or arteries and could lead to misleading conclusions.

Differences in pressures (i.e. total pressures) at two points in the circulatory system measured at different times has often been interpreted to mean a change in blood flow, whereas it should be appreciated that changes in the pressure gradient along a series of vessels will produce corresponding changes in flow only if the resistance to flow between the two poinss of measurement remains constant. The problem here, however, is that the resistance to flow can be determined only by simultaneous measurement of both the pressure gradient and the flow velocity of the blood.

It is obvious, therefore, that attempts to estimate the flow velocities, volumes, and static pressures in the circulatory system by measuring total pressures only and observing related clinical phenomena can result in faulty diagnosis. Direct total pressure measurement has, however, instrinsic value in enabling certain of the conditions under which the circulatory system functions to be defined.

2. THE MEASUREMENT OF PERIPHERAL VENOUS PRESSURE

Venous pressure is usually measured by means of a hypodermic needle; the pressure at the needle point is balanced by an equivalent height of saline solution in a manometer. Alternatively, the phlebomanometer, which has a small compression chamber between the needle and the manometer, can be used for pressure measurement in both large and small peripheral veins. [25]

It is important to note here that with both these methods it is a form of total and not static pressure which is being measured. Depending on the orientation of the needle in the vein and the flow velocity of the blood, a pressure is measured which is erroneously referred to in medical terminology as static pressure. If this measurement is treated as static pressure it is subject to considerable error because of the velocity-head component of unknown magnitude.

3. EXTRAVASCULAR PRESSURES

For the measurement of tissue pressures, for example, in the skin, in subcutaneous tissue, and in the muscles, water manometers are generally used and the transducers are similar to those described above. Cerebrospinal fluid pressure too is usually measured with vertical manometers. Since the pressures in veins and extravascular spaces rarely change abruptly,they can be measured fairly accurately with simple fluid manometers. Such instruments, however, are entirely inadequate for recording pressures which fluctuate rapidly, and this is especially so when it is the extremes of pressure that are significant.

If a mercury manometer is connected directly to an artery through a hypodermic needle, the mercury column will oscillate with small amplitude and low frequency above and below the mean pressure value. These fluctuations do not reliably indicate the magnitude of either the systolic or the diastolic pressure. The same problem arises in measuring the widely fluctuating pressures within ventricular cavities.

MEASUREMENT OF ARTERIAL BLOOD PRESSURE

Measurement of arterial blood pressure entails determination of both the systolic and the diastolic pressures. These two pressure levels actually represent the amplitude of the arterial pressure wave at the points of measurement.

The arterial blood pressure is generally measured by means of a sphygmomanometer. This is the familiar instrument used by the general practitioner to measure the so-called 'blood pressure' of patients. The apparatus consists of an inelastic cuff containing an inflatable rubber bag. The bag is connected by rubber tubing to a rubber bulb and a mercury manometer.

Significant errors in this type of pressure measurement result from improper selection or application of sphygmomanometer cuffs. The pressure in the rubber bag is transmitted to the greatest depth at the centre of the cuff. If the cuff is sufficiently wide and properly adjusted, the pressure indicated by the manometer is that exerted on the tissues immediately surrounding the artery. If, however, the limb is too thick in relation to the width of the cuff, the pressure around the artery may be significantly less than that recorded from the rubber bag.

Under these conditions the cuff pressure required to collapse the artery must exceed the pressure in the artery at the point, and consequently both systolic and diastolic pressure readings will be too high. It should be clear that this method of pressure measurement is not direct. It is, in fact, the measurement of a certain pressure at two conditions of flow, viz. at a highly turbulent level (turbulence being caused by the constriction in the artery at high cuff pressure) which yields the systolic pressure, and at a condition of laminar flow when the artery is almost 'open' again as the cuff pressure is reduced, giving the diastolic pressure. Thus these pressures are measured under certain physiological and anatomical conditions and, simply defined as systolic and diastolic 'pressures', are of clinical meaning to the medical practitioner.

5. CONTINUOUS RECORDING OF BLOOD PRESSURE

Measurement of arterial blood pressure plays an important role in cardiovascular research. Recent developments in cardiac catheterisation (to be discussed presently) have created widespread interest among medical doctors, hydraulic engineers, and scientists in their searches for accurate ways of recording both pulmonary and systematic arterial pressures.

Rapidly fluctuating pressures can be accurately recorded only by means of apparatus having adequate frequency response. Pressure transducers that will respond to the rapidly fluctuating arterial and intraventricular pressures, have become imperative. Mechanical pressure transducers in association with optical manometers have been employed extensively,[26] and recent developments have almost brought the instrumentation aspect of medical research in this particular field into line with that of research in hydraulics and fluid mechanics. The emphasis is now shifting to electronic pressure transducers.

For any particular application the transducer, amplifier, and recorder must be suitably matched for optimum performance. This process invariably involves a compromise among factors such as sensitivity, convenience and stability or frequency response, and it is clear from the literature that the nature and significance of frequency response are widely misunderstood.

6. BULK FLOW MEASUREMENT OF BLOOD

The principles employed for engineering measurements of bulk flow have been adopted in medical science.

Determination of the total volume of fluid in a container of complex shape by sample measurement of the dilution of a known quantity of added dye after thorough mixing, is a familiar and straightforward engineering technique. The circulatory system of the higher animal body is so complex, however, that injected dye short-circuits close to the point of injection before commencing to circulate, with the result that the concentration at a remote sampling point reaches a peak, then subsides and subsequently rises to a second peak as re-circulation occurs. If the once-circulated dye could be distinguished from that which has re-circulated, blood volume and cardiac bulk throughput could be computed with reasonable accuracy. Stewart (27) has attempted to refine the method and as a result of his work the following procedure is now suggested :

First a control sample of blood is withdrawn at the point of sampling. Hypertonic salt solution is then injected. The arrival of salt in a peripheral artery is signalled by a change in the tone emitted by a telephone connected to a bridge circuit, one leg of

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The flow rate can then be determined from the following relationship :

$$Q = \frac{q}{Ct} \quad \text{.............................. (2.1)}$$

where

Q = the blood flow rate;

q = the volume of injected dye;

C = the average sample concentration over a time interval t, and

t = the time of sampling.

This method is similar, in principle, to the sodium dichromate method applied in engineering for direct bulk flow determinations.

Possible sources of error in the application of this method to blood flow measurement are discussed in the following paragraphs.

Errors must result if the foreign substance, injected into a vein at constant rate, begins to re-circulate before a concentration plateau is established at the sampling point. Errors result also from : incomplete mixture of the dye or salt with the blood; poor velocity distribution over the artery cross-section due to pathological conditions; loss of blood from sampling, and dilution by overdose injections.

It follows that diffusible substances likely to be absorbed during passage through the heart and lungs should be avoided. Stable dyes are 'Evans blue' or cardio-green, which tend to remain in the blood stream for a considerable period. When applied with care, the method gives fairly good results.

Heavy water, which is quickly absorbed into the blood stream, is a valuable aid to the determination of blood flow rate. Administered orally the radiation intensity in an internal organ, vein or artery, is measured and the foregoing principles applied. Physiological effects and the reliability of this method are at present under scrutiny by leading specialists.

7. THE CATHETERISATION TECHNIQUE

A modern technique for measuring pressure wave forms in cardiac research is that of catheterisation.(29)

A thin, inelastic plastic tube, or catheter, of 2 to 3 mm diameter and 100 or 130 cm long is threaded through a vein at the elbow into the caridiac chambers. The exact position of the catheter tip is determined from X-ray silhouettes.

In this position the catheter fills with blood so that it is primed automatically by the pressure in the cardiac chamber. The external end of the tube communicates with a variable inductance type pressure transducer. Usually the pressure vessel of the transducer is primed with water before the catheter tip is connected, care being taken to ensure that no air bubbles are trapped in the transmitting fluid. A multi-channel cardiograph is usually connected to enable the surgeon to watch the heart performance during the measurement and the pressure wave form is viewed on an oscilloscope.

An application of cardiac pressure measurement by catheter is in the treatment of so-called 'blue blood' cases, where the 'dirty' or 'blue blood' mixes with the purified or red blood. This usually happens when an opening exists between the right and left ventricles. A catheter is inserted first into the one, then into the other of these two chambers. If the total pressures recorded are almost equal, it can be presumed that there is an opening in the partition between them.

The pressure in the right ventrical should be about one-fifth of that in the left ventrical; any marked increase in this ratio points to the presence of a communication between the two chambers. This can be remedied by open-heart surgery. Here it should be noted that two pressure determinations are compared, the difference aiding diagnosis of a certain clinical condition.

In other applications, however, the 'pressure' determined by a catheter could be misleading. The catheter tube operates as a flexible-blunt-nosed total pressure probe. When it is used to evaluate a cardiac pulse or pressure in vein or artery, the position of the tip relative to the direction of flow is of utmost importance. It can readily be appreciated that there is an unknown velocity-head component incorporated in the pressure reading. However, the distensibility of cardiac and 'duct' walls also complicates accurate measurement.

Electronic pressure transducers and recording equipment are usually employed and pressure fluctuations are displayed on an oscilloscope. Since it is merely the changes in shape of the pressure wave and the differences in mean or peak values - not the absolute values - that are considered adequate for the purposes of clinical diagnosis, the scale is never calibrated.

8. BLOOD AS A FLUID

Blood is a suspension of small cells of approximately neutral buoyancy in a liquid known as plasma. The cells consist of red cells, white cells and platelets, the red cells being in the majority by far and consequently the most important in relation to effect on fluid properties. Liquid from the plasma can permeate into these red cells through osmosis; permeabilities are affected by temperature changes which in turn change the physical flow characteristics of the blood. Thus, the flow of blood differs from that of simple fluids, the principal difference being that blood is a 'living liquid' which changes its physical properties when removed from the blood vessels; coagulation is the most important change that takes place.

If, therefore, blood is pumped through an artificial circulation system, these changes must be prevented. An anti-coagulant used is the snake venom, heparin; the effect can be reversed by administration of the chemical protamine sulphate. Damage to cells when blood is passed through mechanical devices alters the physical character of the fluid and this affects pressure measurement.

It becomes evident that when a pressure probe is to be used for the measurement of pressure inside blood vessels or in the heart, emolysis of the blood in the artificial calibrating circuit must be prevented or appropriate corrections made.

9. CONCLUSIONS

In the light of the factors discussed, it may be stated that accurate determination of absolute pressures and velocities in the circulatory system of the human body are very seldom achieved. The problems associated with pressure and velocity measurements in this field are, in some respects, analogous to those encountered in fluid mechanics and hydraulics. Techniques of improved reliability are essential and must be developed.

The author is at present assisting Drs. J.M. Combrink and Wil. Davies, two heart specialists attached to the heart unit of the University of Pretoria, in an attempt to devise a method of measurement by which the total pressure can be broken down into the two components - static pressure and velocity-head.

In the following chapter the development of a small total pressure probe, insensitive to angle-of-attack effects, is described and the experimental method to be employed at the heart unit in an attempt to determine these parameters is outlined.

CHAPTER III

THE MEASUREMENT OF TOTAL PRESSURE WITH SPECIAL REFERENCE TO THREE-DIMENSIONAL FLOW

1. INTRODUCTION

The principle embodied in conventional total pressure probes is that an orifice facing the flow will record total pressure to a high degree of accuracy, provided the probe geometry is correct - as pointed out in Chapter I. In practice, however, it must be accepted that a total pressure orifice 'faces the flow' only under the most ideal conditions, and if the flow is inclined at an angle to the centre line of the orifice bore, probe geometry will affect the resulting pressure distribution at the point of measurement and consequently the total pressure reading.

In three-dimensional flows inside turbo-machines, or in situations where it is not possible to insert a probe in such a way that its opening faces the flow, the angle of attack at the gauging position is liable to vary widely. Under these conditions accurate point measurement of total pressure must be preceded by improvement in the design of pressure probes.

Pressure energy is a scalar quantity and conse quently instruments are to be preferred which, irrespective of the angle of flow at the measuring point, will register total pressure to the desired degree of accuracy. These probes must be small, easy to manufacture, and should respond quickly to pressure changes.

2. NECESSITY FOR DEVELOPMENT OF A NEW TYPE OF TOTAL PRESSURE PROBE

To improve the accuracy of total pressure measurement especially in three-dimensional flows, either conventional instruments have to be modified or improved designs must be developed. The analysis which follows illustrates existing shortcomings and defines the problem.

2.1 Modification of conventional instruments

The hook-type pitot tube and the pitot cylinder are the two basic forms of conventional instrument in general use.

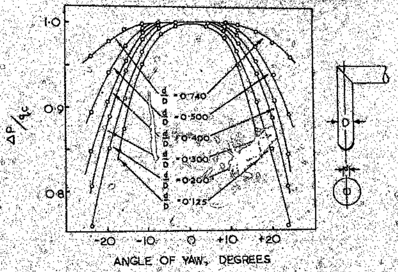
In an attempt to improve the performance of conventional total pressure probes, designers aimed at deflecting the flow to the centre line of the orifice bore.

In the hook-type probe, with hemispherical tip, sensitivity to flow inclination can be reduced by increasing the ratio of orifice-to-probe diameter.[30],[31] From Fig. 3.1(a) it may be seen that an increase in the conventional value of $\frac{d}{D} \approx 0.25$, for standard designs, to $\frac{d}{D} \approx 0.$[illegible] will eliminate an error of 2 per cent of the dynamic pressure at an angle of attack of $\pm$ 10 degrees. With a sharp-edged orifice further improvement is possible, but an instrument of this kind would be liable to damage. A simple arrangement features an internal taper having an included cone angle of 30 to 40 degrees extending about one to one and a half probe diameters from the square face of the tip.(32) At the end of such a taper, pressure rises to within 1 per cent of the true stagnation pressure for flow inclinations up to $\pm$ 25 degrees.

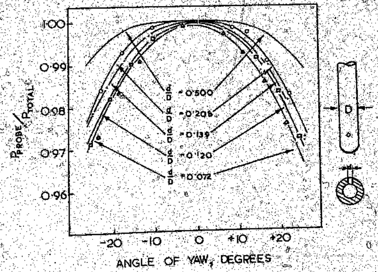
The effect of angle of attack on total pressure recovery is more pronounced with instruments of cylindrical form than with those of other geometrical shapes. In two-dimensional flow of an ideal fluid the pressure coefficient K, for a cylinder of infinite length, is given by :

$$K = 1 - 4 \sin^2 \alpha,$$

where α is the angle of flow. A variation in α of 30 degrees from the centre line of the orifice bore results in a zero value of K, i.e. a 100 per cent variation. The corresponding experimentally determined values of α range from 38 to 42 degrees. It is important, however,



3.1(a) HOOK TYPE PITOT WITH HEMISPHERICAL HEAD



3.1 (b) CANTILEVERED PITOT CYLINDER WITH HEMISPHERICAL HEAD

FIG. 3.1 TOTAL PRESSURE RECOVERY AS A FUNCTION OF ANGLE OF YAW AND DIFFERENT RATIOS OF ORIFICE TO PROBE DIAMETER to note here that complete recovery of total pressure occurs only within the narrow range of ± 3 degrees from the flow direction. (33)

Modification of conventional probes along the lines described above have proved effective in reducing sensitivity to flow inclination. As shown in Fig. 3.1(b) it is possible to obtain complete total pressure recovery over the range $\alpha \approx \pm 12$ degrees by making the orifice diameter equal to one half the probe diameter. (34) This was achieved at a Mach number of 0.37 and the indications are that at lower speeds the range of insensitivity would be greater. It has been claimed that spherical countersinking of the orifice had proved particularly effective, the range of insensitivity being $\alpha = \pm 30$ degrees for an error of the order of 0.3 per cent at Mach numbers below 0.31. (35) This range of accuracy claimed, however, was not confirmed in tests carried out at N.E.L.

Other factors affecting the operation of conventional total pressure probes are overall probe dimensions, complexity of traversing gear and response times. Because of the dimensions of a probe, the size and complexity of traversing gear, and the possibility of mechanical failure, directional probes of the standard type are rarely aligned mechanically with the velocity vector in the flow. Because the presence of a probe produces a change in momentum and thereby modifies the prevailing flow conditions, it is essential that the probe be kept as small as possible. On the other hand, particularly when automatic traverse gear is employed, quick response to pressure changes is necessary. This requirement tends to increase instrument size, because adequate cross-sections are needed to house internal leads. Clearly, therefore, hook-type as well as cylindrical probes can be modified, relatively simply, to register total pressure over ranges of flow inclinations far wider than those of their conventional versions. (Approximately ± 25 degrees for 1 per cent error margin). There are, however, applications that call for directional insensitivity in excess of ± 40 degrees and demand smaller probe dimensions, simpler traversing techniques and reasonable response times. This is particularly so in flow surveys in turbo-machines and jet engines; load changes may produce variations in flow directions as great as 45 degrees;[36] very small overall probe dimensions are imperative and often a probe has to be inserted obliquely up to angles exceeding 25 degrees to the direction of flow. Special total pressure probes must be developed to meet these requirements.

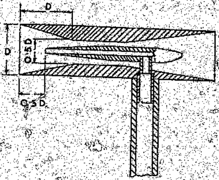
2.2 Development of improved designs

It has been established experimentally that, in order to obtain further improvement of the range of insensitivity, the 'point' of measurement must be shielded by means of a device which will artificially create the conditions required for registration of the true total pressure over as wide a range of flow angles as possible.

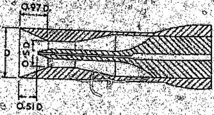
The principle embodied in shielded instruments is that of conservation of total pressure energy along a streamline in loss-free flow. The actual pressure probe is thus encased in an outer shield, generally of cylindrical shape, the function of which is to deflect the stream along the axis of the orifice bore. The total pressure can then be determined correctly, irrespective of flow inclination, provided the streamlines are deflected into the shield and the actual measurement is carried out in a region of loss-free flow.

It has long been known that a total-pressure tube placed inside the throat of a venturi-type shield constitutes a probe which exhibits complete regain of total pressure for angles of attack up to ± 40 degrees. (37)

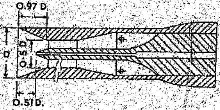
Probably the first detailed study of such a design was carried out by Kiel, (6) who specified the essential probe dimensions. (Fig. 3.2(a)) Tests showed that for an angle-of-flow range extending to ± 47 degrees the



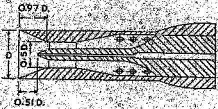
3.2 (a) ORIGINAL KIEL PROBE



3.2 (b) MODIFIED VERSION WITHOUT VENTING ORIFICES.



3.2 (c) MODIFIED VERSION WITH ONE ROW OF VENTING ORIFICES.



3.2 (d) MODIFIED VERSION WITH THREE ROWS OF VENTING ORIFICES.

FIG. 3.2. FORMS OF SHIELDED PRESSURE PROBES

error was less than 1 per cent of dynamic pressure.

The Kiel probe is suitable for aircraft use and is adaptable to some flow survey work. For rake mounting it must, however, be replaced by modified versions. See Figs. 3.2(b), (c) and (d). The characteristics of these probes are summarised in Reference 4. Inner contour dimensions are critical and must be accurately reproduced in order to attain the performance characteristics of the prototype. This, in turn, places a limitation on size. The diameter of the original Kiel probe was 20 mm, the length 80 mm, whereas diameters of the modified versions were 19 mm. Although this instrument has a range of insensitivity of ± 63 degrees, there still remains a need for much smaller shielded total-pressure probes, easier to manufacture and having comparable insensitivity range and response time for flow surveys in turbo machines, jet engines and modern hydraulic plant.

In general therefore it can be stated that the necessity exists for a probe having the following requisites : Smallest size commensurate with adequate structural stiffness, short response time, insensitivity to angle-of-attack over as wide a range as possible, and ease of manufacture. These can best be realised in a single-purpose instrument, i.e. a probe which measures only one parameter of flow, in this instance total pressure. The development of such an instrument (4) is dealt with in the following sections.

3. THE DEVELOPMENT OF A NEW TYPE OF SHIELDED TOTAL-PRESSURE PROBES

Markowski and Moffat (38) called attention to a simplified miniature Kiel probe of only 0.3 inch length, equivalent by their specification to 2.5 times the outer shield diameter. At low Mach numbers and for a version with sharp-edged entry, the range of insensitivity claimed was ± 40 degrees, increasing to approximately ± 55 degrees if the entry is well rounded. This performance was comparable to that of the original Kiel probe.

Neither shield nor 'sting' dimensions of this probe were stated, however, in reference 38; the information given is restricted to a specification of overall length and location of the total-pressure orifice with respect to the entry face of the probe, which is identical to that of the original Kiel probe.

It was therefore decided to have an experimental probe made along the general lines suggested by Markowski and Moffat, and to check its performance. The aim of the investigation was to accumulate basic design data and to explore the possibilities of developing miniature shielded pressure probes which would have the desired characteristics. The steps in the development of such an instrument are briefly as follows :

3.1 Testing of probes having cylindrical shields constructed on the general lines suggested by Markowski and Moffat.[38]

3.2 Testing of modified probes, having a 'through' instead of 'forward-bent' central sting.[1] See Fig. 3-3.

3.3 Introducing spherical shields - theoretical analysis of flow around and through a spherical shield as a guide for further experimental investigations.

3.4 Testing of different probes having spherical instead of cylindrical shields.

3.5 Testing of the versions conforming most closely to the desired characteristics.

3.6 Comparison between theoretically derived flow characteristics and those observed during actual probe performance.

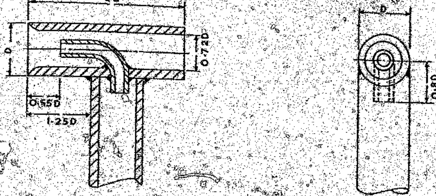
3.1 Early tests on probes with cylindrical shields

3.1.1 Test facilities and method of testing

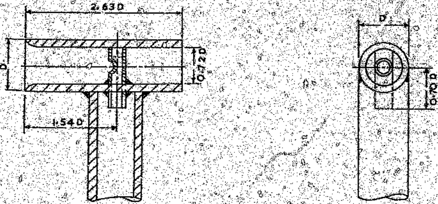
Details of test procedure and apparatus used are given in reference 39. The following is a short summary :

The tests were performed in a wind tunnel having interchangeable free jets of 3 inch and 6 inch. diameter and a closed duct of 3 inch diameter as well as a closed test section of 7.5 inch diameter through which air was sucked by means of a centrifugal fan. The distribution of velocities in the free jets and closed test sections had previously been examined and found suitable for the testing of small pressure probes.

Readings were taken at constant air speeds ranging from about 65 to 275 ft/sec, corresponding to dynamic pressures of approximately 24 to 460 mm water gauge. All probe readings were referred to those for zero angle of attack and results were presented as percentage errors in centre-line dynamic pressure, 'q_c'. The instrument on test was itself used to register the reference pressure, for no metering plane was large enough to accommodate a separate total-pressure reference tube in an interference-free position; during tests, repeated check readings were taken on the centre-line position. In addition to these tests, the response times of each probe were determined.



3.3(a) VERSION WITH CENTRAL STING BENT FORWARD



3.3 (b) MODIFIED VERSION WITH CENTRAL STING SPANNING CYLINDRICAL SHIELD

FIG. 3.3. SIMPLIFIED SHIELDED PRESSURE PROBES

Response characteristics are important only where probes of small size such as those tested are contemplated. In single-orifice instruments of this type a relatively small increase in size would alleviate or eliminate the time-response problem.

3.1.2 Test results

Full details of results are given in reference 39. In all, seven simplified versions of the probe were tested.

Fig. 3.3(a) shows instrument No. 1, which was the first probe tested. The central sting was of hypodermic tubing, bent forward as shown. The final form of this particular version, namely instrument No. 4, was arrived at in four steps; none of the intermediate designs was entirely satisfactory. For this instrument the error was within 0.5 per cent of the dynamic pressure for angles of flow ranging from 0 to 45 degrees. The time response under 500 mm water gauge was approximately 100 seconds.

The probe characteristics are given in Fig. 3.4; while showing satisfactory performance and being much simpler to construct than the original Kiel probe, it is not easy to manufacture in small sizes owing to difficulties encountered in bending forward the central sting. Unless the bent sting is truly central, with respect to

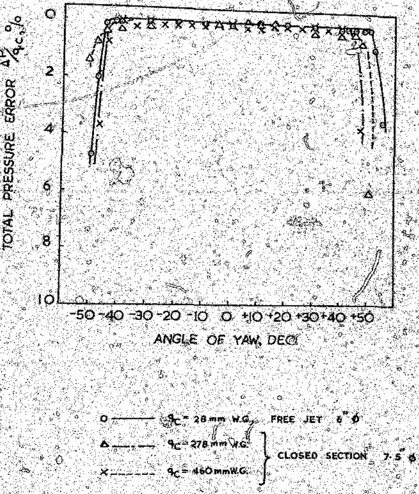


FIG. 3·4. VARIATION OF TOTAL PRESSURE ERROR WITH ANGLE OF ATTACK (INSTRUMENT NO. 4) the cylindrical shield, the characteristics will reflect the asymmetry and change accordingly. Further, the response time depends to a great extent on the quality of workmanship; the bore of the sting can inadvertently be crimped during bending causing partial blockage. To illustrate the difficulty of manufacture, it may be stated that one of the most able instrument mechanics in the workshops at N.E.L. could not make two probes having identical or even reasonably similar hydrodynamic characteristics.

The search for a simpler and more satisfactory design was continued.

3.2 Tests on modified versions of cylinder-shielded probes

The instrument described in Section 3.1 was modified by altering the design of the central sting and the entrance geometry.

Since the forward-bent sting and its correct location constituted the main difficulties, a design was next considered in which this part was replaced by a small central sting located on a vertical diameter inside the tubular shield instead of being bent forward into the flow. See Fig. 3.3(b). The impact orifice was drilled centrally into this sting. Three of these designs

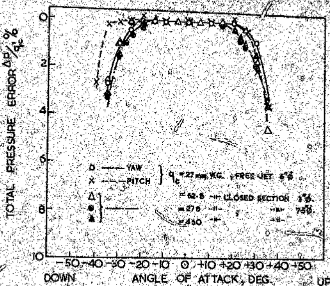


FIG. 3·5. (a) VARIATION OF TOTAL PRESSURE ERROR WITH ANGLE OF ATTACK (INSTRUMENT NO. 7)

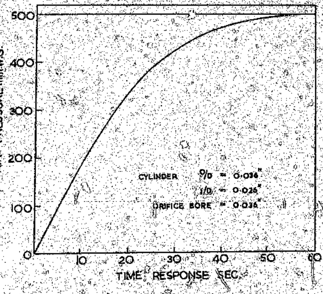


FIG. 3·5. (b) TIME RESPONSE OF INSTRUMENT NO. 7

were tested by altering central sting diameter, impact orifice configuration, and entrance edges of the tubular shield.

The test rig was similar to that used for the tests in the previous section.

3.2.1 Test results

Details of results are given in reference 39.

Reasonably satisfactory characteristics were reflected by instrument No. 7, which records total pressure to within 0.5 per cent of the dynamic pressure in the range ± 22 degrees; if a pressure error of up to 1 per cent can be tolerated, the flow angle may be increased to ± 28 degrees. The response time was less than 60 seconds under an air pressure of 500 mm water gauge. Probe characteristics are shown in Figs. 3.5 (a) and (b).

The forming of the impact orifice required meticulous care. Denting of the hypodermic tube or leaving burrs on the edges of the orifice would have marked effects on total pressure registration, probably due to secondary flow in or near the impact orifice.

Evidently a probe of this design could be used where small overall dimensions, quick response, and adequate pressure recovery are required in an angle-of-attack range of ± 28 degrees or less. It is very much simpler to manufacture than the version with the forward-bent central sting and there is little doubt that in slightly larger versions than were tested, the insensitivity range could possibly be pushed up to ± 30 degrees or more. Within the present context, however, interest was restricted to probes of small dimensions.

At this stage of development the author started thinking in terms of spherical instead of cylindrical shields.

3.3 Introduction of spherical shields

Studies of flow round cylinders and spheres revealed that the normal flow pattern was distorted more severely by a cylinder than by a sphere. Compared with a cylinder, a sphere, if used as a probe shield, would moreover have more stable pressure conditions in its wake and therefore inside the shield itself particularly when the probe is yawed or pitched in relation to the flow. The manufacturing process, too, could be facilitated if the stem and central sting could be simply press-fitted into a spherical shield of which the desired internal contours had been drilled out beforehand.

It was therefore decided to adopt spherical in

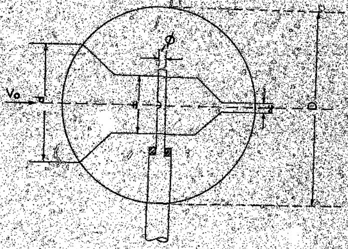


FIG. 3·6. THEORETICAL PROBE CONTOURS

place of cylindrical shielding and to undertake prior to further testings, a theoretical analysis of probe geometry and of flow round and through a spherically-shielded probe with the object of optimizing the internal configuration and critical parameters.

3.3.1 Theoretical analysis of proposed spherical probe

3.3.1.1 Internal probe geometry

As a guide to the shaping of the interior of the spherical shield, wind tunnel experiments were conducted on a small araldite sphere, through which a cylindrical passage had been drilled, and into which a central sting with impact orifice had been fitted normal to the axis of the passage. On the basis of these experiments it was decided to adopt an internal form having the following features : diameter of shield sphere D, diameter of frontal entrance d, diameter of central tubular passage x, diameter of exhaust or venting orifice at rear, y, and diameter of central sting which embodies the impact orifice ϕ. The probe contours are shown in Fig. 3.6.

With the aid of Buckingham's Π-theorem [40] it can be shown that the inter-relationship of the above features with flow parameters can be expressed by the function :

$$P_t = \xi \left[\left(\frac{x}{D}\right)^2, \left(\frac{x}{D}\right), \left(\frac{y}{D}\right), (R_e), (e), \left(\frac{\phi}{D}\right), \left(\frac{d}{D}\right) \right] \cdot (\rho V_o^2) \quad \ldots (3.1)$$

where R_e = Reynolds number;

e = $\frac{e_v}{D}$, the relative roughness;

ρ = mass density of fluid, and

V_o = free stream velocity.

The other symbols are defined above.

An analysis of equation (3.1) shows that :

(i) P_t is directly related to ρV_o^2 and hence this element may be neglected in a discussion of geometric features.

(ii) From a study of the pressure distribution on the surface of a sphere in a flow, it would seem that the best value of d would be dictated by the extent of the zone of positive pressure on the frontal face. This would be related to the sphere diameter D;

thus $d = f_1(D)$ ....(3.2)

(iii) For smooth entry, x would be controled by d; thus

$x = f_2(d)$, whence from equation (3.2)

$x = f_3(D)$ ....(3.3)

(iv) If the venting orifice diameter y is less than x, a smooth contraction would define, within experimental limits, values of y in relation to x; thus

$$y = f_4(x)\text{, whence from equation (3.3),}$$

$$y = f_5(D)$$

(v) The relative roughness e is important and for reproducibility, the internal finish should be of high standard and free from burrs.

(vi) The central sting diameter ϕ is related to x to ensure conservation of momentum and hence

$$\phi = f_6(x)$$

From the above equations one can write :

$$D = f(x)$$

$$D = F(y)$$

$$\phi = f'(y)\text{, and}$$

$$D = f''(d)$$ which on substitution in equation (3.1) yield * :

$$P_t = \xi\left[\left(\frac{x}{F(y)}\right)^2, \left(\frac{x}{F(y)}\right), \left(\frac{y}{f(x)}\right), (R_e), (e), \left(\frac{f'(y)}{f(x)}\right), \left(\frac{d}{f''(d)}\right)\right] \quad \ldots\ldots(3.4)$$

$$\text{or } P_t = \chi\left[\left(\frac{x}{y}\right)^n, \left(\frac{x}{y}\right), \left(\frac{y}{x}\right), (R_e), (e), \left(\frac{y}{x}\right)\right] \quad \ldots\ldots\ldots(3.5)$$

* With an eye to experimental convenience in examining the effects of parameter changes on probe performance, the author decided to consider x and y independent variables.

where χ represents a different functional relationship between the parameter groups, and n is a postive integer. It is also important to note that equation (3.4) can be written in the form given by equation (3.5) only if the functions F(y), f(x), f'(y) etc. are polynomials free from exponential terms - a condition which will be accepted at this stage as true. If, for a particular sphere of diameter D, d is fixed in terms of the frontal pressure distribution curve, a smooth internal finish is assured, and ϕ and x are chosen in relation to d such that no momentum change takes place at the measuring section, equation (3.5) reduces to :

$$P_t = \chi_1 \left[\left(\frac{1}{\sigma}\right)^n, \left(\frac{1}{\sigma}\right), (\sigma), (R_e) \right] \qquad \ldots\ldots\ldots\ldots\ldots (3.6)$$

where σ is defined as $\frac{y}{x}$, and n again is a positive integer.

Equation (3.6) is written in this form in order to illustrate the relative importance of the parameters concerned. It shows that for given sphere - entrance - and central sting-diameter, the probe performance will be controlled by parameters σ and R_e. The dominating influence of σ is clearly reflected by equation (3.6).

3.3.1.2 Analysis of flow through and round spherical probe

Early efforts to derive an approximate theoretical

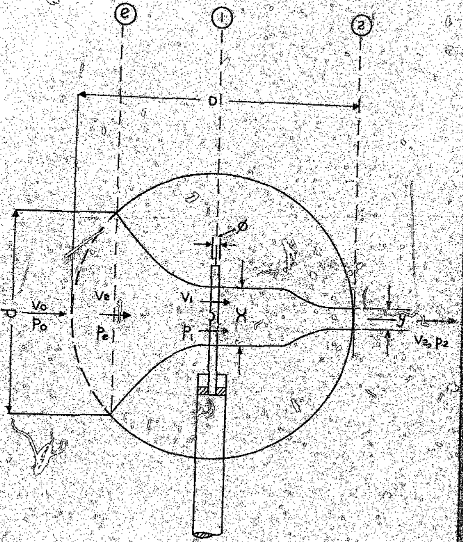


FIG. 3·7.

expression for the probe total pressure proved fruitless. The potential flow theory for ideal flow round a sphere was applied and disturbance equations introduced to correct for the effects of entrance opening, internal flow passage, central sting, and supporting stem. Approximate solutions of the resulting 'forest' of equations were not possible; in some instances attempts gave results not even remotely comparable with experimental trends.

In search of a reasonable mathematical model to represent the flow through and round a spherical shield, the author considered the following hypothesis which was suggested by the clearly dominating role of $c = \frac{y}{x}$.

Referring to Fig. 3.7 it seems logical to consider the venting orifice at the rear of the sphere to act as a control valve which regulates the flow through the sphere. If y is made considerably smaller than x, throttling results. Should y be made equal to x, the flow round the sphere would simply be partially diverted through the shield. Depending on the degree of throttling, therefore, the free-stream velocity, V_o on the probe axis would be decreased gradually up to the entrance face at section e. If, in the limit, y is chosen equal to the entrance diameter d, the velocity

V_e would be nearly equal to the free stream velocity V_o; putting y = o would prevent flow through the shield and hence V_e would be zero.

When throttling takes place, a larger volume of fluid arrives at the sphere entrance than can actually be discharged through the venting orifice. It is submitted that, under these conditions, a 'pneumatic cushion' or 'stagnant eddy reservoir' is formed immediately within the entrance. From this 'reservoir' fluid flows through the probe past the pressure orifice in the central sting before discharging through the venting orifice.

In accordance with the above hypothesis a stagnation point or near-stagnation point could be formed within the 'reservoir', yielding a value of V_e on a centre line stream tube of approximately zero, depending on the degree of throttling or the value of σ. Mathematically this concept can be expressed as :

$$V_e = K'V_o \qquad (3.7)$$

K' being fractional and small. Thus, if a stagnation point is formed, K' would tend to zero.

As stated earlier, the principle embodied in shielded instruments is that of conservation of total energy along a streamline in loss-free flow. Total pressure can be determined correctly, irrespective of flow inclination, provided the streamlines are deflected into the shield and the actual measurement is carried out in a region of loss-free flow. A streamline in loss-free flow would theoretically be considered as being on the shield axis, and hence the total pressure orifice is located in the central sting at this position. It is however clear that the ideal conditions pertaining to loss-free flow through a probe shield passage would not be realised.

In the light of the above theoretical considerations the author decided to apply the total energy equations for flow through and round a spherical shield, assuming the flow to be incompressible - Mach number not exceeding about 0.3 - and allowing for energy losses through the shield.

The following expression for the total pressure P_t was developed, viz :

$$P_t = V_o^2 \left\{ C_1 + (K')^2 \cdot C_2 \left[\Delta_1 + \frac{\Delta_2}{K'} + \frac{\Delta_3}{K'} + \Delta_4 \right] \right\} \quad ..(3.8)$$

(Details of the mathematical analysis are given in Appendix II).

K' is defined by equation (3.7) ;

$$C_1 = \rho \frac{C_p - 1}{2(1 - K_o)} ;$$

$$C_2 = \frac{1}{1 - K_o} ;$$

$$\Delta_1 = \frac{\rho d^4}{2y^4} ;$$

$$\Delta_2 = \frac{64 \mu \ell_1 d^2}{V_o x^4} ;$$

$$\Delta_3 = \frac{32 \mu \ell_2 d^2}{V_o y^4} \quad \text{and}$$

$$\Delta_4 = \frac{4.8 \rho \phi d^4}{\pi x^5}$$

Also : C_p = Base pressure coefficient;

K_o = Instrument coefficient, and

V_o = Free-stream velocity.

Thus, for a particular probe with known dimensions, if K_o, K' and C_p can be estimated, the theoretical value of total pressure is given by equation (3.8) at any known free-stream velocity V_o.

At this stage equation (3.8) yields one important result, viz. : Equation (3.4) can be expressed in the form given by equation (3.5) (See sub-section 3.3.1.1.) on account of the fact that equation (3.8), and hence