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Segmented multifit of polynomial function for mass attenuation and energy-absorption coefficient values

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Abstract

The photon energy region 1–20 keV cannot be ignored for any theoretical beam-path attenuation and/or dosimetry calculations for low energy (mammography, dental units, etc.) bremsstrahlung spectra. The polynomial function presented by Tucker et al. for fitting the mass attenuation coefficient data for dry air (assumed at temperature of 20°C and pressure of 1.013×10^5 Pa) has therefore been adapted to the photon energy region below 20 keV. This region was excluded by Tucker et al. It is found that only by dividing the 1–20 keV region up into smaller sub-regions to obtain values for the polynomial parameters could a good fit be obtained over the whole of the region from 1 to 200 keV.

Keywords: Bremsstrahlung; Attenuation coefficients; X-rays

1. Introduction

Accurate values of mass and mass-energy attenuation coefficients are essential when quantifying the interactions of an X-ray beam with matter. X-ray spectra measurements, exposure or air kerma and dose calculations, are areas of both diagnostic and therapeutic radiology where these attenuation coefficients are employed. It is well known that the coefficients are functions of both the photon energy and the physical properties of the irradiated material. Hubbell (1969, 1982), Cullen et al. (1989), Seltzer (1993), Seltzer and Hubbell (1995), and Hubbell and Seltzer (1996) and many other authors have compiled mass attenuation data. These compilations have been used in bremsstrahlung spectra model, exposure and dose calculations. The mass attenuation compilations do not however cover every energy value in the energy ranges normally presented. To interpolate values for the mass attenuation coefficients for energies of interest, other than those tabulated, the compiled data are often fitted utilising least-square techniques and an appropriate mathematical expression. A mathematical expression given by Tucker et al. (1991) for modelling tungsten target spectra is recommended as being suitable for fitting mass attenuation data. The Tucker et al. expression is

$$\mu(x) = a + \frac{b}{x^{1.6}} + \frac{c}{x^{2.7}} + \frac{d}{x^{3.5}} + \frac{e}{x^{4.5}} \quad (1)$$

where $x = E/100$, E = X-ray photon energy (keV) and a, b, c, d, and e are derived parameters. Of particular interest for exposure and dose calculations for low energy spectra e.g. mammographic exposures, is the region below 20 keV; a region not addressed by Tucker et al. They address the region from 20 to 200 keV. For this lower end of bremsstrahlung energies, air attenuation cannot be ignored for theoretical computations of absorbed dose and/or exposure values. The Tucker et al. expression has been used therefore to fit mass attenuation coefficient data as well as mass energy-absorption coefficient data, for dry air in the region below 20 keV using a least-square technique. The mass energy-absorption and mass attenuation coefficients data used

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for this exercise were those compiled by Hubbell and Seltzer (1996) now very conveniently assembled by Gerward in a Windows platform programme called WinXCom.

2. Methodology

For the total region looked at in this work, data points from 1 to 200 keV were used. Three types of approximation approaches were adopted for fitting the data.

(1) Fitting the entire suite of data values once for one set of polynomial parameter values.
(2) Fitting the data values in two regions. From 1 to 20 keV and from 20 to 200 keV to obtain two sets of parameter values, one for each region.
(3) Dividing the 1–200 keV regions into four separate regions and fitting the data points in each sub-region separately for four sets of parameter values. The mass energy-absorption and mass attenuation data were fitted together with the *K* edge. (It is important that data points below and above the *K* edge are fitted separately and that the values of the edges be used as boundaries of contiguous regions.) The derived parameters from our fitting procedures as well as those given by Tucker et al. for mass energy-absorption and mass attenuation coefficients for air were used to compute attenuation

Table 1
Derived constants *a*, *b*, *c*, *d*, and *e* including Tucker et al. in cm^2/g for air mass energy-absorption coefficients

Energy range (keV)	*a*	*b*	*c*	*d*	*e*
Single fit					
1–200	11.599	−3.38E−01	2.24E−02	−2.34E−04	8.22E−07
Double fit					
1–20	−1.7431	0.064174	6.97E−03	3.33E−04	−1.58E−06
20–200	3.10E−02	−1.88E−02	1.29E−02	−1.93E−03	2.12E−04
Multiple fit					
1–3.203	343.78	−4.3313	1.26E−01	−3.33E−03	1.18E−05
3.203–8	−9.7059	6.92E−01	−6.02E−02	5.94E−03	−7.14E−05
5–20	1.03E−01	−2.66E−02	8.38E−03	5.25E−04	−5.11E−06
20–200	3.10E−02	−1.88E−02	1.29E−02	−1.93E−03	2.12E−04
Tucker et al. constants					
20–200	2.892E−02	−1.170E−02	5.836E−03	7.588E−04	−7.892E−06

Table 2
Derived constants *a*, *b*, *c*, *d*, and *e* including Tucker et al. in cm^2/g for air mass attenuation coefficients

Energy range (keV)	*a*	*b*	*c*	*d*	*e*
Single fit					
1–200	−2.35E−01	4.17E−02	7.96E−03	2.96E−04	−1.42E−06
Double fit					
1–20	−1.9125	8.66E−02	6.30E−03	3.56E−04	−1.67E−06
20–200	1.04E−01	8.34E−02	−5.28E−02	2.01E−02	−1.40E−03
Multiple fit					
1–3.203	3.7989	−4.762	1.37E−01	−3.67E−03	1.30E−05
3.203–8	−11.13	8.10E−01	−7.07E−02	6.76E−03	−8.10E−05
5–20	2.24E−01	−1.13E−02	6.67E−03	7.17E−04	8.36E−06
20–200	1.04E−01	8.34E−02	−5.28E−02	2.01E−02	−1.40E−03
Tucker et al. constants					
20–200	1.088E−01	6.004E−02	−2.581E−02	8.473E−03	−3.613E−04

coefficient values. The results from the computation have been compared to the original mass energy-absorption and mass attenuation coefficients data compiled by Hubbell and Seltzer (1996).

3. Results and discussion

The parameters *a*, *b*, *c*, *d*, and *e* derived from our fitting exercise and those presented by Tucker et al. for air, in units of cm^2/g, are presented in Tables 1 and 2 for the mass energy-absorption coefficients and mass attenuation coefficients data, respectively. Figs. 1–4 illustrate the fits graphically, for the regions 1–20 and 20–200 keV which also includes the results of the fit using the Tucker et al. parameters.

The four sub-region energy range fit of the mass energy-absorption coefficients data gave the lower deviations and are in much better agreement with the original data than the single or two energy range fits. Fitting data points of 1–20 keV as one-region, results in

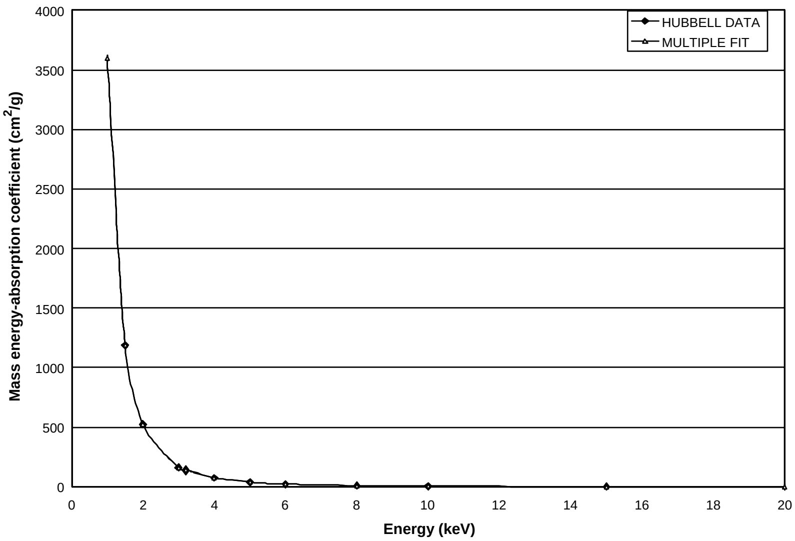

Fig. 1. Comparison of mass energy-absorption coefficient data by Hubbell and Seltzer with values obtained from multiple energy fit, for the energy range 1–20 keV.

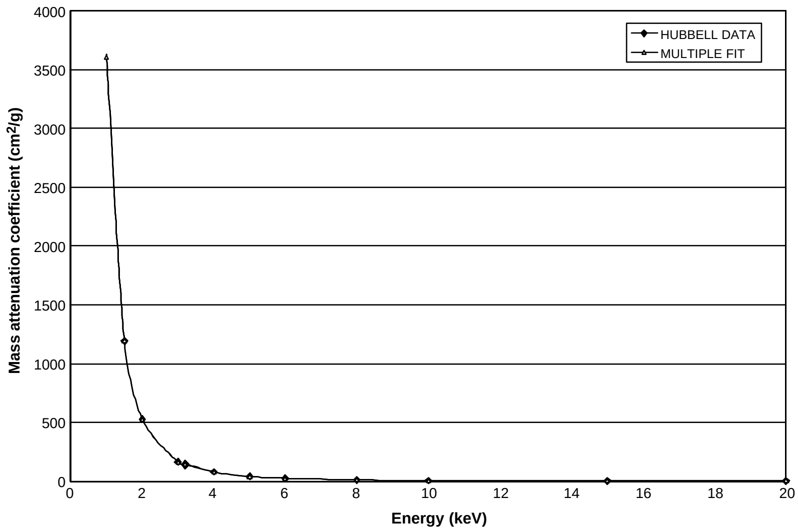

Fig. 2. Comparison of mass attenuation data by Hubbell and Seltzer with values obtained from multiple energy fit, for the energy range 1–20 keV.

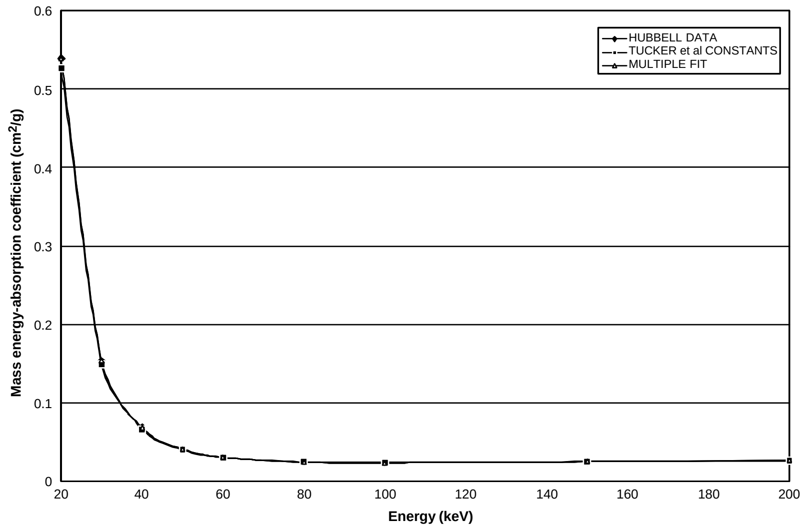

Fig. 3. Comparison of mass energy-absorption coefficient data by Hubbell and Seltzer with values obtained from multiple energy fit and Tucker et al. constants, for the energy range 20–200 keV.

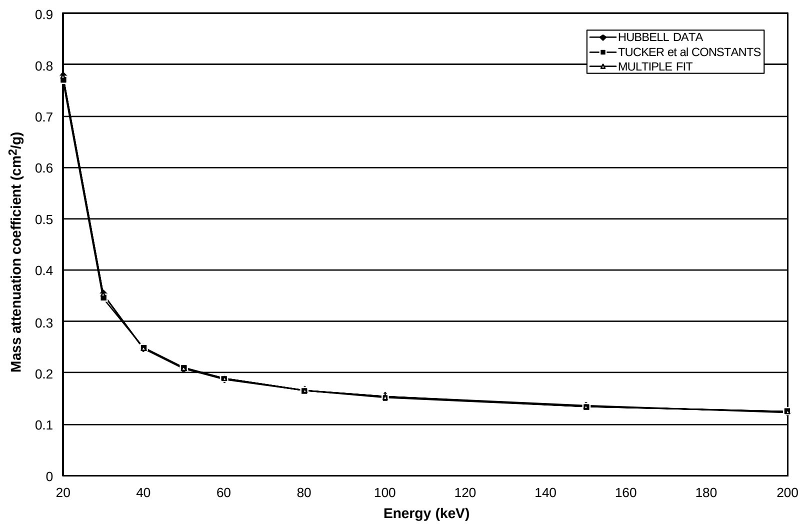

Fig. 4. Comparison of mass attenuation data by Hubbell and Seltzer with values obtained from multiple energy fit and Tucker et al. constants, for the energy range 20–200 keV.

an overestimation of the data by between 10% and 30% at energies 8–20 keV. The single, double and multiple energy range fits showed comparable deviation at the K-absorption edge, cf. Figs. 5 and 7. Using the parameters given by Tucker et al. for air mass energy-absorption coefficients to recalculate the mass energy-absorption coefficients data by Hubbell and Seltzer, deviations of less than 4% from the original data were observed between 20 and 200 keV overall, cf. Fig. 6. The mass attenuation coefficients data fit showed a similar pattern to that of the mass energy-absorption coefficients data fit cf. Fig. 8. Generally, the deviations were greater for the mass attenuation coefficients data than for the mass energy-absorption coefficients data for both the two and four energy region fits. However, the deviations were significantly lower than those from the mass energy-absorption coefficients data in the case of the single region (1–200 keV) fit. It was also observed that for data points 30–200 keV, the single energy region fit of the mass energy-absorption coefficients data produced

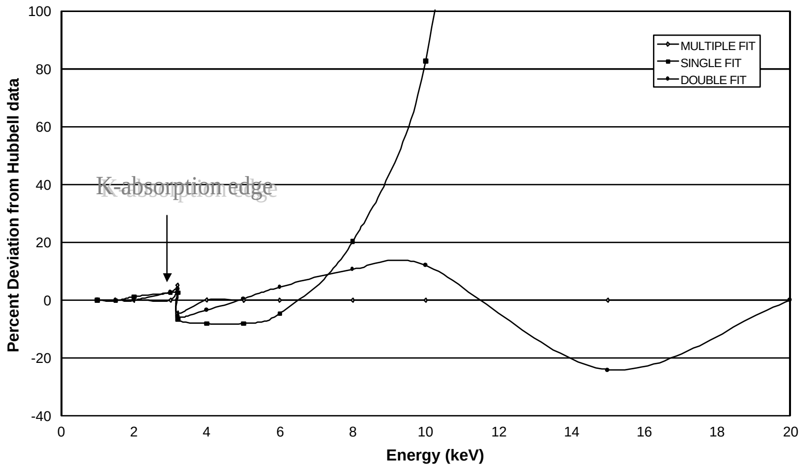

Fig. 5. Deviation of the mass energy-absorption coefficient data derived from the single, double and multiple energy fit, from the original values by Hubbell and Seltzer for the energy range 1–20 keV.

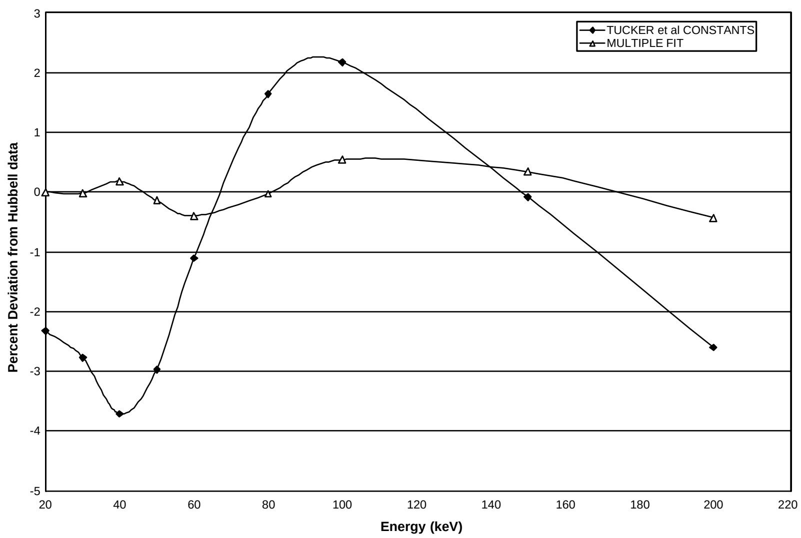

Fig. 6. Comparison of deviation of the mass energy-absorption coefficient data derived from 20 to 200 keV multiple fit parameters and Tucker et al. parameters from the original values by Hubbell and Seltzer for energy range 20–200 keV.

overestimations of the data whilst the mass attenuation data was underestimated for the same energy range. The deviations obtained with the Tucker et al. parameters from the mass attenuation coefficients data were comparable to those obtained when compared to the mass energy-absorption coefficients data.

4. Conclusion

The results from the fitting procedures confirm the suitability of Eq. (1) for interpolating mass attenuation and mass energy-absorption coefficients data. The exercise has shown, however, that when data points containing high and low energies (1–200 keV) are fit together with a single set of parameters, an overestimation of about 20% at the lower energies with far greater deviations at higher energies can result. Thus, to avoid over or under estimation of mass attenuation or mass energy-absorption data, it is important to group the data into smaller energy regions when fitting. This is especially important if the data were to be used in low energy photon calculations such as would be the case for mammography beams for example.

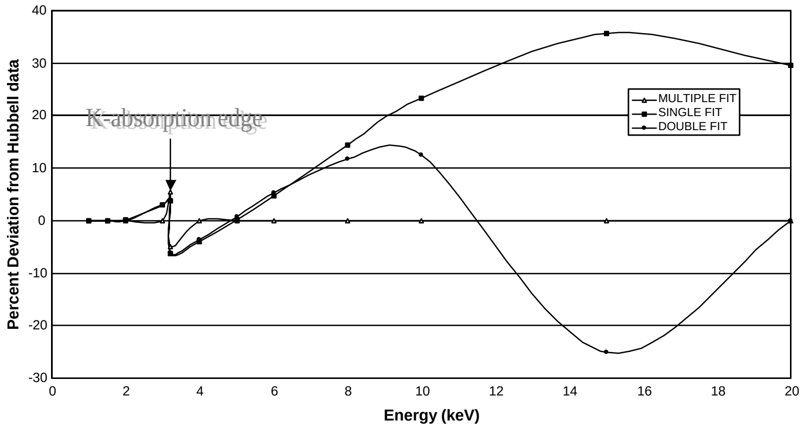

Fig. 7. Deviation of the mass attenuation coefficient data derived from the single, double and multiple energy fit, from the original values by Hubbell and Seltzer for the energy range 1–20 keV.

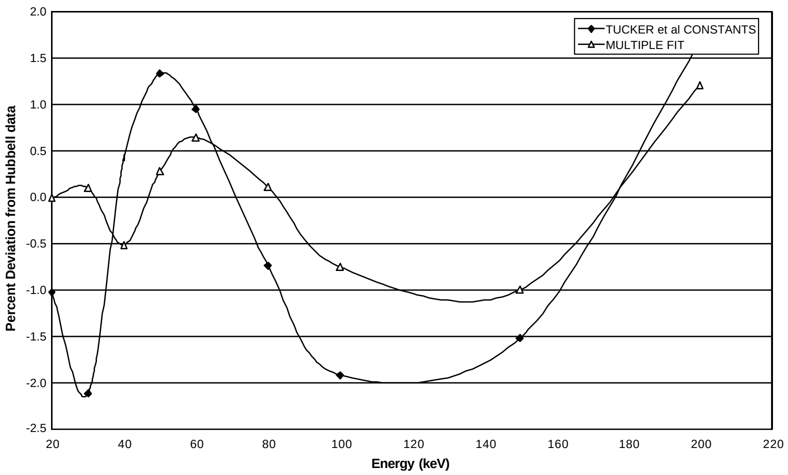

Fig. 8. Comparison of deviation of the mass attenuation data derived using the 20–200 keV multiple fit parameters and Tucker et al. parameters from the original values by Hubbell and Seltzer for energy range 20–200 keV.

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