2. Background

2.1 Introduction

The estimation of resistant coefficient and hence discharge capacity in a channel or river is one of the fundamental problems facing river engineers.

When applying Manning's equation to natural channels, its n value represents the total resistance to flow. Rouse (1965) pointed out that Manning's n can be affected by many factors from the fluid properties, flow characteristics, cross-sectional geometry and the geomorphology of the reach, to the sediment content. The coefficient determined by local roughness is not sufficiently representative of the overall resistance. Therefore in practice, n values are obtained from those presented in texts such as (Chow, 1956), (Barnes, 1967), and (Acrement and Schneide, 1989). As many practitioners have experienced, the selected n value may not give reasonable flow estimates, and hence have to be modified by some trial-and-error approach. Soong and Depue (1996) indicated that such an approach employs no physical reasoning, but is accepted as the only way to determine n because tabulations and illustrations cannot be covered for all types of channels. On many occasions, experience alone has led to the determination of n values (Barnes, 1967).

Manning's equation can also be notated as:

$$Q = \frac{AR^{\frac{2}{3}}S^{\frac{1}{2}}}{n} \qquad 2.1$$

V = Velocity, R = Hydraulic radius, S = slope of the bed,
A = Cross-sectional area and n = Manning's roughness coefficient.

Investigations of a fixed-bed, open channel resistance was first attempted by Antoine Chézy in his unpublished work which was reported by Herschel (1897) where Chézy inferred that channel resistance is directly proportional to wetted perimeter and the square of the velocity, and inversely proportional to the cross-sectional area and the hydraulic slope i.e.

Resistance

$$C = \frac{V^2 P}{AS} \quad 2.2$$

Or normally expressed as in the Chézy equation as;

$$V = C\sqrt{RS} \quad 2.3$$

C = resistance coefficient (Chézy coefficient).

Other experiments by Darcy (1854) and Bazin (1865) on artificial roughened surfaces showed that

$$C = \frac{1}{\sqrt{a+\frac{b}{R}}} \text{ (in metric units)} \quad 2.4$$

Where a and b are constant dependent on boundary roughness.

Additional experiments by Bazin (1897) resulted in a similar relation which for English units of measurement gives,

$$C = \frac{157.6}{1+\frac{m}{\sqrt{R}}} \quad 2.5$$

Where m = roughness factor, such that $0.109 < m < 3.17$.

Manning determined by experiment that the Chézy coefficient varied as the sixth root of the hydraulic radius, and in metric units is,

$$C = \frac{R^{\frac{1}{6}}}{n} \quad 2.6$$

Rouse and Ince (1957) made similar determinations of the Chézy coefficient.

Substitution of the value for C from equation 2.6 into the Chézy equation 2.1 results in the widely used equation, ascribed to Manning,

$$V = \frac{R^{\frac{2}{3}} S^{\frac{1}{2}}}{n} \quad 2.7$$

2.2 Composite resistance coefficient

Different methods have been presented for computing a composite resistance coefficient (i.e. combined resultant resistance due to different components).Where only surface roughness contributes to resistance, but the roughness size varies across the channel cross section (but not longitudinally), a number of formulations for calculating the overall, effective value of Manning's n (n_e) have been proposed. These can be expressed as weighted averages of functions of the local n values, but this accounts only for variation across a section assumed to be longitudinally continuous:

$$n = \left(\frac{(\sum_{i=1}^{N}(K_i n_i^a))}{K}\right)^{\frac{1}{a}} \qquad 2.8$$

where the subscript i refers to the subsection associated with the local value n_i, $N =$ the number of subsections specified, $K =$ the weighting variable (normally the wetted perimeter), and $a =$ an exponent which depends on the nature of the relationship assumed between subsection flow conditions. A few commonly used expressions of this form are the ones of Horton (1933) who assumed $a = 3/2$, Pavlovskii (1931), and Einstein and Banks (1950) who assumed $a = 2$. Other similar ones are those of Felkel (1960), Lotter (1933). Most of these formulas however do not account for interactions between subsection flows through transverse momentum exchange, which is considerable for overbank flows but less influential for most low, inbank flows (James and Jordanova, 2010).

The effects of these interactions can, however, be accounted for (Wallingford 2004) using a lateral distribution model, such as the Conveyance Estimation system (CES). For local resistance due to different influences such as vegetation, irregularities and surface roughness equation 2.9 provides a way of combining these effects (James and Jordanova, 2010). HR Wallingford (2004) equation was also corroborated by James and Jordanova (2010) below

$$n_l = \left(n_{sur}^2 + n_{veg}^2 + n_{irr}^2\right)^{\frac{1}{2}} \qquad 2.9$$

n_{sur} = surface roughness, n_{irr} for irregularity and n_{veg} for vegetation

n_l is similar to the Manning's n.

Equation 2.9 defines local composite unit roughness and is intended for use at the local sub-cross section scale.

Based on the concept introduced by Cowan (1956) for roughness variations not restricted to the transverse direction, the United States Soil Conservation Service (1963) (SCS) proposed an equation for the overall Manning's coefficient

$$n_e = (n_b + n_1 + n_2 + n_3 + n_4)m \qquad 2.10$$

where $n_b =$ a base value of n for a straight uniform section of channel in natural materials,

$n_1 =$ a correction factor for the effect of surface irregularities

$n_2 =$ a correction value for variation in shape and size of channel cross-section,

$n_3 =$ a correction value for obstructions,

$n_4 =$ a correction value for vegetation and flow conditions on the flood plain,

$m =$ a correctional factor for the meandering of the channel cross-section.

It is clear that equation 2.9 and 2.10 both have terms accounting for resistance accruing from the combined influence of form drag and surface drag. According to James et al. (2008) these phenomena are different in nature and produce effects that are different as flow depth changes therefore they should be described in different terms. James et al. (2008) therefore proposed an equation for combining contributions to resistance from local form and bed shear. The equation which they developed for combined vegetation stem and bed resistance has been applied at the channel reach scale to account for large emergent elements such as boulders.

$$V = \sqrt{\frac{1}{C_s + C_d} 2gls} \qquad 2.11$$

In this equation $l =$ a roughness length related to spacing (s) and width (d) of the form roughness elements by

$$l = \frac{s^2}{d} \qquad 2.12$$

Therefore the terms are now accounted for separately where form resistance is accounted for by a drag coefficient (C_D) and the bed shear resistance by the term C_s, which can be expressed in terms of the local surface Manning's n or Darcy-weisbach friction factor f by

$$C_s = \frac{fl}{4D} = \frac{2gln^2}{D^{\frac{4}{3}}} \qquad 2.13$$

in which $D =$ the flow depth and g = acceleration due to gravity (James et al. 2008).

Quantifying flow resistance is essential to understand the actual hydraulics of streams. Interactions between stream flow and channel boundaries lead to dissipation of energy as water moves within and between bed irregularities. Flow resistance is created by viscous skin friction around objects as well as form or pressure drag created from differential pressures around objects (Ferguson, 2007). According to David et al. (2011) the total value of the frictional losses can be represented with the dimensionless Darcy-Weisbach friction factor f

$$f = \frac{8\,\mathrm{g}\,\mathrm{R_h S_f}}{\mathrm{V.^2}} \qquad 2.14$$

where $f =$ Darcy-Weisbach friction factor, $g =$ acceleration due to gravity (m/s^2),

$R_h =$ hydraulic radius (m), $S_f =$ friction slope (m/m), and V = mean velocity (m/s).

Each parameter (V, S_f, R_h) has error associated with the measurement method (David et al., 2011). The use of *f*, along with Manning's n, nonetheless remains the most common approach to quantifying resistance in steep streams despite indications that Manning's equation in particular is poorly suited to steep streams with shallow flows (Ferguson, 2010). Einstein and Barbarossa (1952) proposed that, despite interactions among different components of resistance, the individual components could be quantified and summed in terms of f.

Manning and Darcy-Weisbach hydraulic roughness coefficient can then be related using the following equation:

$$n = \left[\frac{fR^{\frac{1}{3}}}{8g}\right]^{\frac{1}{2}} \qquad 2.15$$

Therefore once we have f we can determine n.

The f_{total} according to Morris and Wigget (1972) is a combination of bed, bank and skin roughness, however for relatively low flows as we are considering bank roughness is of secondary importance. Skin friction consists of material and form drag. This skin friction is described by the concept of bed shear stress while the form resistance was described by the balance between hydrodynamic and resistance forces (Morris and Wigget, 1972).

$$f_{total} = f_{grain} + f_{form} \quad 2.16$$

Where f_{grain} = viscous friction and form drag around grains in the absence of bedforms, f_{form} = form drag around bedforms, which includes the individual component of form drag around other objects such as boulders.

2.3. The combined resistance of bed shear and form roughness

James (2012) stated that considering the downward slope weight component of a volume of water in a uniform flow to be balanced, the relationship between the sum of the forces arising from bed shear and form drag, becomes

$$F = F' + F'' \quad 2.17$$

Where F = the downslope weight component of the water, F' = the bed shear force and F'' = the form drag (James, 2012). The weight component is

$$F = \rho g v S \quad 2.18$$

Where ρ = water density, g = gravitational acceleration, S = channel slope and v = volume of water.

The bed shear force is given by

$$F' = \tau_o A_{bf} \quad 2.19$$

in which A_{bf} = the surface area on which the shear stress acts and τ_o = bed shear, which can be expressed in terms of the surface friction factor, f', as

$$\tau_o = \frac{\rho f' V^2}{8} \quad 2.20$$

where V = the average velocity (James, 2012).

The form drag can be quantified through the drag equation,

$$F'' = \frac{1}{2} C_D A_p \rho \quad 2.21$$

Where C_D = drag coefficient and A_p = projected area of the form roughness elements in the flow direction (James, 2012).

Substituting the force expressions in the force balance equation and rearranging gives an equation for the velocity in terms of f', C_D and the channel characteristics, i.e.

$$V = \sqrt{\frac{1}{\frac{f'}{4}\frac{A_{bf}}{v} + C_D\frac{A_p}{v}}}\sqrt{2gS} \qquad 2.22$$

Further manipulation of equation (2.22) by James (2012) leads to

$$V = \sqrt{\frac{8g}{f' + 4C_D\frac{A_p}{A_{bf}}}}\sqrt{\frac{v}{A_{bf}}S} \qquad 2.23$$

which can be expressed as

$$V = \sqrt{\frac{8g}{f}}\sqrt{RS} \qquad 2.24$$

i.e. the conventional form of the Darcy-Weisbach equation, with

$$f = f' + f'' \qquad 2.25$$

Where f = the total friction factor and f'' = effective friction factor associated with form resistance, given by

$$f'' = 4C_D\frac{A_p}{A_{bf}} \qquad 2.26$$

and the hydraulic radius

$$R = \frac{v}{A_{bf}} \qquad 2.27$$

Expressing the bed shear in terms of the Manning resistance coefficient for the surface (n') rather than the friction factor, i.e.

$$\tau_o = \frac{\rho g n'^2}{R^{\frac{1}{3}}}V^2 \qquad 2.28$$

leads to the conventional Manning's equation

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad 2.29$$

with

$$n = \sqrt{n'^2 + n''^2} \qquad 2.30$$

Where n'' = resistance coefficient accounting for the form resistance component, given by

$$n'' = \sqrt{\frac{R^{\frac{1}{3}}}{8g} 4C_D \frac{A_p}{A_{bf}}} \qquad 2.31$$

James (2012) used Friction factor relationships presented by the ASCE Task Force on Friction Factors in Open Channels (1963) for hydraulically smooth channels

$$\frac{1}{\sqrt{f}} = c\log\left(\frac{R_e\sqrt{f}}{b}\right) \qquad 2.32$$

where $b = 9.55$ $c = 2$

And for hydraulically rough channels

$$\frac{1}{\sqrt{f}} = c\log\left(\frac{aR}{k_s}\right) \qquad 2.33$$

where $a = 12$ and $c = 2$

2.4 Resistance components of river with low flow

Over the years it has become obvious that the different elements present in a water body contribute to the total resistance to flow within the channel reach. Extensive effort has been devoted to quantifying the relative importance of different components of f during the past few decades, yet no consensus has been reached regarding the most important components or the most appropriate method to calculate individual components. Additive approaches have been used to investigate the contribution of grains by Millar (1999), Millar and Quick (1994), Einstein and Barbarossa (1952), Parker and Peterson (1980), wood and spill resistance by, Shields and Gippel (1995), Curran and Wohl (2003), and bar resistance in gravel bed rivers by Parker and Peterson (1980), Prestegaard (1983). Wilcox et al. (2006) demonstrated, however, that the immeasurable component was always the largest contributor to total resistance, so that an additive approach inflates the leftover component. Thus, quantifying the relative contribution of different sources of resistance remains a great challenge to understanding flow resistance in streams.

Grain resistance is most often defined as the viscous friction around grains, but in high-gradient channels, where boulders are on the same order of magnitude as flow depth, the grains can contribute significantly to form drag and spill resistance (Zimmerman, 2010).

Due to the above statement grain resistance is defined here as the combined flow resistance (i.e., form drag, skin friction, spill resistance) that results from the presence of the grains in the flow. In this work it will be referred to as f_{form}.

2.5 Investigations on Emergent Bank Vegetation.

Emergent bank vegetation is that vegetation that is normally found along the banks of a river. Over time these vegetation strips have been found to contribute to the total composite resistance to flow in any river they are found. James and Mokoa (2006) conducted experiments with longitudinal strips of artificial vegetation in a 12.3m long, 1.0m wide, plaster-lined channel on a slope of 0.001. They represented the vegetation stems by 5mm diameter rods mounted in 1.0m by 0.125m frames in a staggered arrangement with centre spacing of 25mm in both directions.(longitudinal and transverse).Also arranging the frames in longitudinal strips with seven different widths and locations, each covering 50% of the channel area. Patterns producing realistic width to depth ratios (W/D) greater than 2 were selected and analysed. “The value of n for the flume surface was found from a test with no stems to be 0.0102 and a value of 0.0432 was found for the stem-water interface through application of a side wall correction procedure using discharges determined from integrated velocity measurements across the clear sections. Each strip pattern was tested with four or five different discharges” (James and Mokoa, 2006).

Hirschowitz and James (2009), used the Darcy-Weisbach friction factor rather than Manning n with $a = 1$ to establish the friction factor for the vegetation-water interface and recommended Kaiser's (1984), equation

$$f_v = f_{to} + 0.18log\left(0.0135\frac{V_{inf}^2}{h_t V_{veg}^2}\right) \qquad 2.34$$

Where v_{inf} = the depth-averaged velocity in the channel as unaffected by vegetation, v_{veg} = the depth-averaged velocity within the vegetated zone, h_t = the flow depth and f_{to} = a constant to be equal to zero (Hirschowitz and James, 2009) for W/D greater than about 5 and between 0.06 and 0.1 for narrow channels.

Hirschowitz and James (2009) suggested a formula for calculating V_{Veg}

$$V_{veg} = \sqrt{\frac{1}{C_D}}\sqrt{2gls} \qquad 2.35$$

In which s = the channel slope, C_d is the plant drag coefficient, and

l = concentration length (i.e. the ratio of projected plant area to water volume), which for the experiments described here may be calculated as:

$$l = \frac{\pi d_{p/4N}^2}{A_c} \qquad 2.36$$

A_c = Longitudinal stem spacing

N = Number of stems

d_p = the stem diameter.

Finally Hirschowitz and James (2009) proposed a formula for the total friction in the channel with vegetation which is similar to that of Pavloski (1931).

$$f = \frac{f_b B + 2 f_v h_t}{B + 2h_t} \qquad 2.37$$

wheref_b = bed resistance value of the channel. B = width of the channel.h_t = depth of flow in the channel. f = total Darcy-Weisbach friction factor in the channel. Equation 2.37 is used when the vegetation is arranged on the two sides of the channel length

According to Hirschowitz and James (2009) when the vegetation is on one side of the channel length equation 2.37 will be modified by replacing $2f_v$ with $f_v + f_{side}$ to become

$$f = \frac{f_b B + (f_v + f_{side}) h_t}{B + 2h_t} \qquad 2.38$$

where f_{side} = the resistance coefficient of the solid boundary.

2.6 Effects of bank irregularities to total resistance

The flow resistance and thus water levels in channels are influenced by both the roughness of the bed and the roughness of the banks. For relatively small flow depths, the roughness of the banks is of secondary importance. This is generally the case for wide rivers at low flows (Meile et al. 2011).

According to the concept of Morris (1955) and Jiménez (2004), in principle, three different flow types in the large-scale depressions at the banks can be distinguished: the reattachment flow type, the normal recirculating flow type, and the square-grooved flow type.

The aspect ratio of the large-scale depression $\frac{\Delta B}{Lb}$ is an important parameter in order to classify the flow into these three. The smaller the $\frac{\Delta B}{Lb}$ is, the more easily the flow attaches to the sidewall of the cavity. When approaching $\frac{\Delta B}{Lb} = 1$, a circular gyre is revealed inside the cavity. If $\frac{\Delta B}{Lb} = 0.1$, the low-discharge flow does not reattach, but an elongated primary gyre and a very small secondary gyre can be observed. The increase of the discharge results in an increase of the rotating velocity of the gyre (Meile et al. 2011). For cavity aspect ratios $\frac{\Delta B}{Lb}$ between 0.15 and 0.6, the observations showed a recirculation of the flow with a primary and secondary gyre (fig.2.42b).Their smallest value of $\frac{\Delta B}{Lb}$ that has been tested is 0.05. In this case, the flow clearly reattaches to the cavity sidewall for all investigated discharges (Fig.2.42a).Their highest tested value $\frac{B}{Lb} = 0{:}8$. In this case, a single and almost circular gyre is located in the cavity (Fig.2.42c). In all cases, the secondary gyre is rotating with a much smaller velocity than the primary gyre, as stated by Uijttewaal et al. (2001); Uijttewaal (2005); Weitbrecht (2004).

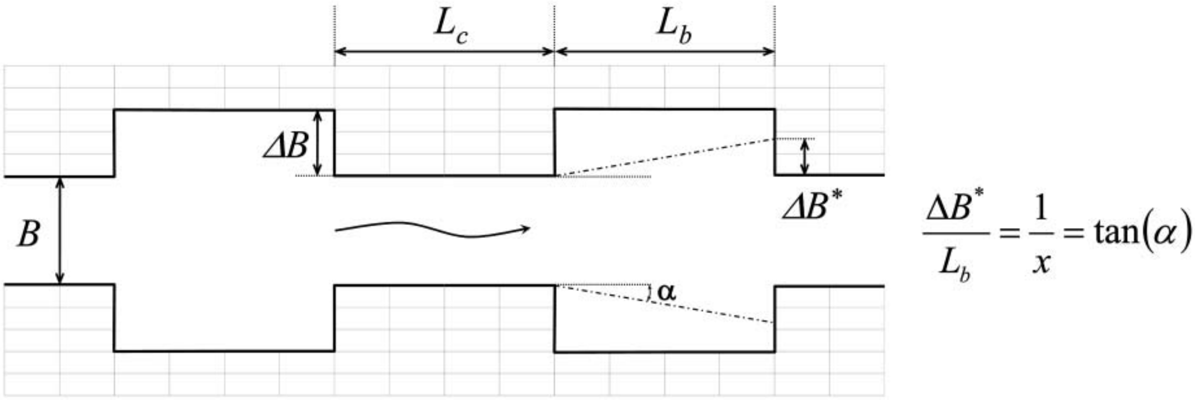

Fig.2.41. (a) Plan view of the test flume with (b) the definition of the parameters of the macro rough configurationsL_b, L_c, and ΔB(Meile et al. 2011).

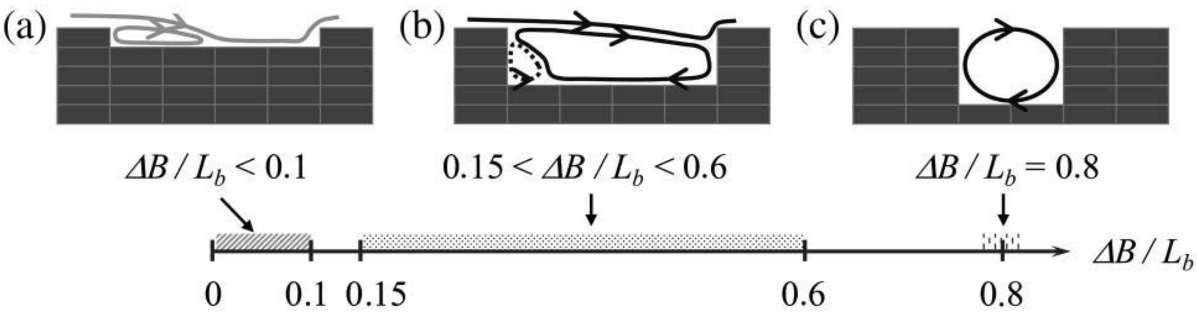

Fig.2.42. Observed basic flow types as a function of the aspect ratio of the large-scale depressions: (a) reattachment flow type; (b) normal recirculating flow type; and (c) square-grooved flow type; grey zones indicate the range of performed experiments (Meile et al. 2011).

2.6.1 Semi-empirical Drag-Coefficient Model

Meile et al. (2011) suggested a formula

$$f_{total} = f_{prism} + f_{Mr}. \qquad 2.39$$

which is analogous to equation 2.16 by Morris and Wigget (1972) where

$f_{Mr} =$ resistance due to micro-roughness of the sidewalls of the elements causing the irregularities,f_{prism} = the resistance due to the bed.

The power-law optimization suggested by Meile et al. (2011) is a formula for calculating or predicting the friction factor due to bank irregularities alone and can be expressed as

$$f_m = \frac{h_r}{\Delta l}\frac{2g}{u^2}4R_h \qquad 2.40$$

Where $\frac{h_r}{\Delta l}$ = the slope of the water surface. R_h = Hydraulic radius, u^2 = Velocity of flow

f_m = total friction factor for bed with irregularities.

Furthermore, Meile et al. (2011) arrived at a general formula for micro-roughness friction coefficient (f_{Mr}) for a channel

$$f_{MR} = C_d \frac{8R_h \Delta B^*}{B(L_b+L_c)} \qquad 2.41$$

They ascertained by experiment that $C_d = 0.475$ but for this experiment C_d is observed to be 0.5

With $\Delta B^* = \min\left(\Delta B, \frac{l_b}{x}\right)$ 2.42

Where $\Delta B^* =$ minimum of the geometric cavity depth and the effective cavity depth considering a certain expansion of the flow $(1/x)$ inside of the cavity (fig.2.41)

$$x = \left(\frac{R_{lim}}{R_m} + x_o\right)\left(\frac{l_b}{\Delta B}\right) \qquad 2.43$$

$x_o = 4.5$ and $R_{lim} = 150000$.

Basically so many other publications related the determination of total roughness coefficients exist and were reviewed but three basic literatures of which will be revisited later in chapter five are those of James (2012), Meile et al. (2011) and Hirschowitz and James (2009).