UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

EXPLORING GRADE 2 LEARNERS' MULTIPLICATIVE REASONING SKILLS

A research report submitted to the Wits School of Education, in partial fulfillment of the requirements for the Masters degree in Mathematics Education

By

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ABSTRACT

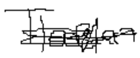
Mathematics performance of learners in South Africa is well below expectations. Studies such as the Third International Mathematics and Science Study Repeat (TIMSS-R) Survey and Southern and Eastern Africa Consortium for Monitoring Educational Quality (SECMEQ) have reported that South African learners’ performance in mathematics is lower than the average mathematics performance achieved by learners in other countries. Multiplicative reasoning which is essential for learners’ development and understanding of mathematics was a key area of concern in learners’ performance. The concern regarding the teaching and learning of multiplicative reasoning in South African schools was the catalyst for the Wits Maths Connect - Primary project conducting a series of short interventions with the aim of supporting effective learning and teaching of multiplicative reasoning at foundation and intermediate phase. This study in particular, is located within the Wits Maths Connect - Primary multiplicative reasoning project aimed at exploring grade 2 learners’ multiplicative reasoning skills in the context of the North West province of South Africa.

The broad focus of this study was to explore shifts in grade 2 learners multiplicative reasoning skills after the implementation of an intervention conducted in the North West province. The intervention ran for a period of 6 weeks and consisted of four carefully designed lessons which were taught over four weeks: one lesson per week. The areas focused on in the lessons included multiplicative arrays, linking arrays to division, making equal groups, and linking making equal groups to division. As a form of assessment to understand the effect of the intervention, a pre-test and post-test which was composed of bald number (context free) calculations and word problems (context-based) focused on multiplicative reasoning were administered to the learners. The study contains two research questions. The first one focuses on the shift in learner performance in the pre-test compared to the post test and the second focuses on learners’ performance in the multiplicative word problems compared to related bald calculation problems.

There were two key findings which emerged through a quantitative analysis of the collected data. It was found that learners performed significantly better in the post-test compared to the pre-test, and learners performed significantly better in word problem questions compared to bald calculations in both the pre-test and post-test. This therefore provides evidence that the intervention was indeed successful in helping to improve learners’ performance and enhance their multiplicative reasoning skills, and also that multiplicative reasoning context-based problems support learner understanding.

DECLARATION

I, Tasmiyah Hoosen, hereby declare that this research report is my own work. It is being submitted for the Degree of Master of Education at the University of the Witwatersrand, Johannesburg. This report has not been submitted for any other degree or examination at any other university.



Tasmiyah Hoosen

DEDICATION

To my parents. Thank you for your constant support, guidance and encouragement.

ACKNOWLEDGEMENTS

First and foremost, I would like to thank the Almighty Allah for granting me the knowledge and ability to complete this research project – indeed it is thanks to Him that I have made it this far in my academic career.

I would also like to offer my deepest thanks to Professor Hamsa Venkatakrishnan and Dr Lawan Abdulhamid for providing the funding for my studies this year. I will forever be grateful to you for awarding me the opportunity to achieve my Masters in Education; it is because of your kindness and generosity that this research project was made possible.

Furthermore, I wish to offer my utmost thanks to my supervisors; Dr Lawan Abdulhamid and Dr Samantha Morrison. Words cannot express how grateful I am for the patience, guidance, encouragement and invaluable insights you have granted me throughout this M Ed journey. I truly appreciate all your intellectual contributions, not to mention all the time you spent helping me to complete this research project. This achievement would not have been possible if it were not for all your support and effort – you are truly two outstanding academics and I am eternally grateful to you.

Lastly, I would like to thank my family for their constant support and encouragement; your motivation always inspires me and drives me to do the best that I can.

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LIST OF ABBREVIATIONS

WMC-P: Wits Maths Connect – Primary

MR: Multiplicative Reasoning

SACMEQ: Southern and Eastern Africa Consortium for Monitoring Educational Quality

TIMSS-R: Third International Mathematics and Science Study Repeat Survey

ANA: Annual National Assessment

SES: Socio-Economic Status

UK: United Kingdom

DBE: Department of Basic Education

CHAPTER 1: INTRODUCTION

1.1 Background to the study

It has been noted in the literature that there is a frequent occurrence of low achievement with regard to mathematics in South Africa (Askew, Takane, Ramsingh, Mathews, Venkat & Roberts, 2019). The Third International Mathematics and Science Study Repeat Survey (TIMSS-R) of worldwide trends with regard to physical science and mathematics performance proved that "South African learners' performance in mathematics was well below that of other participating countries in the tests that measured basic mathematical skills" (Howie, 2001, p. 18). Results from the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) III study additionally highlighted the issue surrounding learner performance in mathematics in South Africa by declaring it being below average (Spaull, 2011). Thus, the poor performance of learners in South Africa is alarming at international and regional levels.

At the national level, the results from the 'Annual National Assessment' (ANA) also confirmed poor mathematics achievement in primary grades in South Africa. The average results showed a drastic decline across grades from 68% to 43% from grade 1 to grade 6 and a decrease of 68% to 62% from grade 2 to grade 3 (Department of Basic Education, 2014). Schollar (2008) points to the shortage of move from more concrete to more abstract methods of dealing with number concepts as possible reasons for the poor performance in primary grades. He found that 60% of Grade 7 and 80% of Grade 5 learners resort to counting-by-ones solution strategies. Figure 1 provides evidence of a MR task completed by a grade 5 learner which displays the use of an inefficient strategy of unit-counting or counting-by-ones.

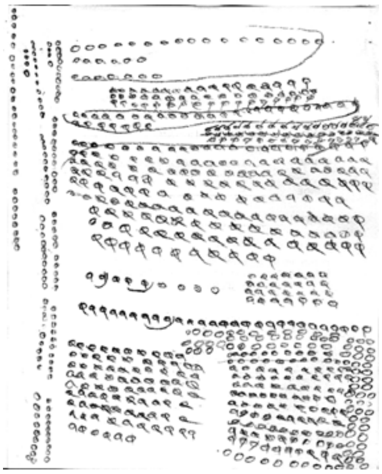

Figure 1: strategy used by grade 5 learner (Schollar, 2006, p.8)

As a result, Schollar (2008) stated that "learners at all primary grade levels in all provinces in South Africa routinely reduced all addition, subtraction, multiplication and division tasks to counting forwards and backwards, usually in single units" (p. 5). It is believed that this poor learner performance in South Africa stems from an over-reliance on counting-by-ones and a scarce understanding about the base-ten numeration system and the related understanding of place value at early grades (Schollar, 2008).

Comparative results from international, regional and local studies show that senior, intermediate and foundation phase learners are finding mathematics challenging and are not performing as well as they are required to. Documented in the context of South Africa was the fact that learners' poor mathematics performance stems from struggles with multiplicative reasoning (MR) concepts like multiplication and division (DBE, 2011). MR is defined as multiple ways of representing and thinking about situations which include ratio and proportion, rates and scale factor, and also multiplication and division (Brown, Hodgen, & Küchemann, 2014). MR is essential when it comes to the development of mathematics processes and concepts such as data analysis, probability, volume and area, and proportion and ratio. It has been noted that a lack of multiplicative structure development in early grades limits the development of learning in later years (secondary school) in areas such as using graphs, algebra, and functions (Mulligan & Watson, 1998). MR is considered as one of the common challenging domains in mathematics education (Venkat & Mathews, 2019).

Whilst MR is largely accepted as a crucial element of the mathematics curriculum, the teaching of it has also been a huge challenge for many South African primary mathematics teachers (Venkat &

Mathews, 2019). Askew (2018) notes that difficulty with developing MR might not stem from learners' lack of development, but rather from teaching approaches which limit opportunities for functional thinking. Struggles with division and multiplication are not exclusive to the context of South African, but have also been noted in a large variety of international literature (Anghileri, 2006; Nunes & Bryant, 1996; Hansen, 2011; Greer, 1992), and these difficulties are one of the components that result in poor learner performance in mathematics (DBE, 2011). Hence, this study is set to explore young learners' MR skills by means of a tightly focused intervention program conducted by Wits Maths Connect-Primary (WMC-P) project.

This concern regarding the learning and teaching of MR in South African schools was the catalyst for the WMC-P project conducting a series of short interventions with the aim of supporting effective learning and teaching of MR at foundation and intermediate phase. The intervention ran for a period of 6 weeks and consisted of four carefully designed lessons which were taught over four weeks: one lesson per week. The lessons had four steps; firstly, lesson related mental starter activities; second, practicing multiplication facts; third, two MR word problems completed in pairs as a whole class activity; and lastly, a set of MR tasks to be tried out by individual learners. A teacher guide and learner activity booklet were provided to the teachers. The areas focused on in the lessons included multiplicative arrays, linking arrays to division, making equal groups, and linking making equal groups to division. As a form of assessment to understand the effect of the intervention, a pre-test and post-test which was composed of bald number calculations and word problems based on MR were administered to the learners. One part of this study focuses on learners' performance in the multiplicative word problems compared to related bald calculations. There have been mixed findings related to learner's performance in word problems and bald number calculations, hence the findings of this study could lead to further insights being provided to this area. The broad focus of this study was to explore shifts in grade 2 learners MR skills in a context of one such intervention conducted in the North West province of South Africa.

1.2 Problem statement

There are widespread concerns on the state of South African learners' mathematical skills, with many learners performing below curriculum expectations at the end of primary schooling. MR is at the center of the development of mathematical learning (Venkat & Mathews, 2019); however, research has shown that struggles to teach MR concepts well are experienced by many primary school teachers

(Venkat & Mathews, 2019). This situation calls for teaching interventions to support learners' development of MR skills at early grades.

The poor performance in mathematics across South Africa calls for a pressing need to promote learners' mathematical skills and understanding. A poor grasp of MR in foundation phase grades leads to low performance in later grades (Brown et al., 2010; Schollar, 2008). There has been a limited amount of large-scale intervention studies conducted in the Northern parts of South Africa that focus on the development of young learners' MR skills. It is because of this gap that this study aimed to focus on the effect of a carefully designed teaching intervention on grade 2 learners' development of MR skills through the analysis of pre-test and post-test results.

This study, therefore, has the potential to provide valuable information with regard to strengthening learners' MR skills in early grades in the South African context. The findings could possibly: provide insights into the development of MR skills of learners in a province where the intervention was conducted in learners' mother tongue; show whether a short, structured intervention that has proven to raise the level of young learners MR skills in another province with a higher socio-economic status (SES) can also be effective in a lower SES context like the North West Province; and provide answers about a conceptual field which is not well developed in South African learners, but that research shows is important for learners' later success in mathematics and for accessing topics such as proportion, ratio, fractions, etc. (Mulligan & Watson, 1998).

1.3 Purpose statement

The purpose of this study was to explore shifts in grade 2 learners MR skills in a context of a short, structured teaching intervention conducted by the WMC-P project.

1.4 Research questions

The main question that guided my study was:

> To what extent does a short-term intervention, that focused on making connections between representations and the multiplicative field, improve grade 2 learners multiplicative reasoning skills?

The following sub-questions were interrogated in order to answer the main question:

1. What shift (if any) is evident between learners' performance in the multiplicative tasks contained in the pre- and post-test?
2. What is evident in learners' performance in the multiplicative word problems compared to the multiplicative bald number problems?

1.5 Research hypotheses

The following hypotheses were formulated to guide the study.

1. There is no significant difference between learners' performance in the pre-test and post-test in questions involving multiplicative reasoning.
2. There is no significant difference between the learners' mean scores in the pre-test and post-test in questions involving word problem problems and bald calculation problems.
3. There is no significant difference between learners' performance in bald calculation questions compared to the word problem questions involving multiplicative reasoning in the pre-test and post-test.

1.6 Significance of the study

The beneficiaries of this study include Foundation Phase teachers and learners in South Africa in general and in the North West province in particular. It could be beneficial for teachers in a way that it could improve their teaching styles and practices, and for learners as it may improve their MR skills and may lead to their overall better performance in mathematics. If the findings show that the intervention was successful in improving learners' MR skills, the intervention could be implemented in other provinces to promote other learners' MR skills and the overall mathematics performance in South Africa.

1.7 Conclusion

In this chapter, a background to the study was provided in which key aspects were introduced. A justification for the need of a study to investigate the effects of an intervention focused on MR concepts in the early grades was also provided. Lastly, I provided a purpose statement and finally research questions that guided the investigation.

Chapter 2 provides a discussion about the theoretical framework used in this study and, in addition, provides a literature review which focuses on MR, learner performance and problem format with regard to MR, problem solving, and types of multiplication and division problems.

Chapter 3 concentrates on the methodology and research design used in this study. It also includes the context of the study, information about sampling, validity, and ethical considerations.

Chapter 4 presents an analysis of the data and findings and discussion of the findings. A t-test analysis was used to test whether there were significant differences between learner performance in the pre- and post-test and in word problems and bald calculations. It was found that learners performed significantly better in the post-test compared to the pre-test, and also performed significantly better in word problem questions compared to bald calculations.

Chapter 5 concludes this study by discussing the implications that emanated from the findings, highlights the limitations that were faced, and lastly provides recommendations for future research that emerge from the findings of this research.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

This section provides a discussion about the theoretical framework that framed this study and provides reviews of current literature related to the development of learners MR skills. In doing so, the literature review is organized into two parts. The first part deals with the multiplicative conceptual field, and the MR types of problems and models that are advocated in the international literature. The second part deals with different types of early grade mathematics interventions done internationally and in the South African context, the outcomes of these interventions, as well as a literature-based discussion on word problems and bald calculations.

2.2 Theoretical framework

This study was based on Vergnaud's (1994) conception of the *multiplicative conceptual field* and is complemented by Haylock and Cockburn's (2008) work that provided me with tools to unpack how making significant connections aids secured mathematical understanding amongst learners in the early grades.

2.2.1 The multiplicative conceptual field

Vergnaud (1994) groups different types of MR into three categories; simple proportions, Cartesian product of two measures and multiple proportions (p. 1). The type of problems that are normally introduced in the context of schools are the simple proportions type and these are therefore the most appropriate to focus on for an intervention taking place in the foundation phase. Two variables in a fixed ratio are involved in simple proportion problems. An example of this type of problem that was used in this study is:

Mongezi has 3 bags of apples. There are 8 apples in each bag. How many apples does Mongezi have?

This is a multiplication problem which contains two variables: bags and apples, with a fixed ratio of 8:1. Two other forms of problems can also be set in the context of simple proportion problems, these being:

Mongezi buys 16 doughnuts in 4 equal sized boxes. How many donuts are in each box?

Hamsa is taking 30 eggs to the shop and puts them in boxes that hold 6 eggs each. How many boxes does she need?

The two problem types above can be represented and solved by means of division; the first problem is an example of a sharing (partitive) situation which is initially solved by learners' using one-by-one counting of concrete objects. After a while these one-by-one counting methods can be changed to composite sharing actions, and later on to replications of the number of shares as a composite unit (p. 3). The second problem type represents a grouping (quotative) situation where learners initially solve the problem by forming a group and duplicating it as a combined unit. Even though these "simple proportion" situations can be solved by means of repeated subtraction or addition, Vergnaud states that "multiplicative structures have their own intrinsic organization which is not reducible to additive aspects" (Vergnaud, 1983, p. 128). If learners' understandings of multiplicative situations are restricted to repeated subtraction and addition it hides the fact that simple proportion problems include four numbers, where a phrase like 'each box' shows that there is a 'hidden' number of 1. Hence, this proposes that MR problems are different from additive reasoning problems since MR problems include four numbers (they are quaternary), not three. Presenting the problem using a T-table is useful in revealing this hidden relationship. Figure 2 shows that, for example, the first problem (about Mongezi and the apples in bags) consists of two variables – the number of apples and the number of bags – and the constant ratio between them (Askew et al., 2019).

Bags	Apples
1 3	8 ?

Figure 2: T-table representing a simple proportion problem

Bags	Apples
1	4
?	12

Bags	Apples
1	?
3	12

Figure 3: T-tables of the problems related to division

Changing the unknown variable in Figure 2 allows for an extension of two division problems related to the first problem as show in Figure 3. Therefore, it represents a link between multiplication and division and displays the ratio nature included in the problem.

The T-table method, which Vergnaud (1994) argues to be useful for specifically highlighting multiplicative relationships, was strongly advocated in the intervention lesson. “For simple proportions, the T-table’s spatial arrangement helps to make explicit the scalar relationship within quantities (reading down the columns of the table) and the functional relationship between quantities (reading across the rows)” (Vergnaud, 1994, p. 6). This encourages learners to make connections between multiplication and division instead of looking at them as separate operations. The lessons used in the intervention included the use of T-tables and were taught by first introducing a concept in the context of multiplication (example introducing arrays) and was followed by a lesson that would link the concept to division (example linking arrays to division).

2.2.2 The connections framework

Vergnaud’s (1994) idea of a multiplicative field is complemented by the concept of significant connections in the teaching of simple proportion. I draw on the work of Haylock and Cockburn (2008) that recognise the involvement of the manipulation of symbols in mathematics. They believe that simply just memorising processes for the manipulation of those symbols for answering a variety of questions is not how the basis for understanding mathematics is built. They found that learning with understanding is more beneficial and meaningful than the rehearsal of routines and recipes. “For a teacher committed to promoting understanding in their children’s learning of mathematics, the challenge is to identify the most significant ways of thinking mathematically that are characteristic of understanding in this subject.” (Haylock & Cockburn, 2008, p. 6). This refers to the essential cognitive processes used by learners to internalise information they received externally and construct meaning. This consists of exploring the relationship between mathematical symbols and the other factors of learners’ experiences of mathematics, such as various kinds of pictures, every day and formal mathematical language, and concrete or real-life situations. They created the “*connections framework”* for discussing learner’s understanding of number and number operations. It is based on the idea that building up connections in learners’ minds develops understanding.

The arrows represented in Figure 4 show the connections that underpin learners’ understanding in mathematics.

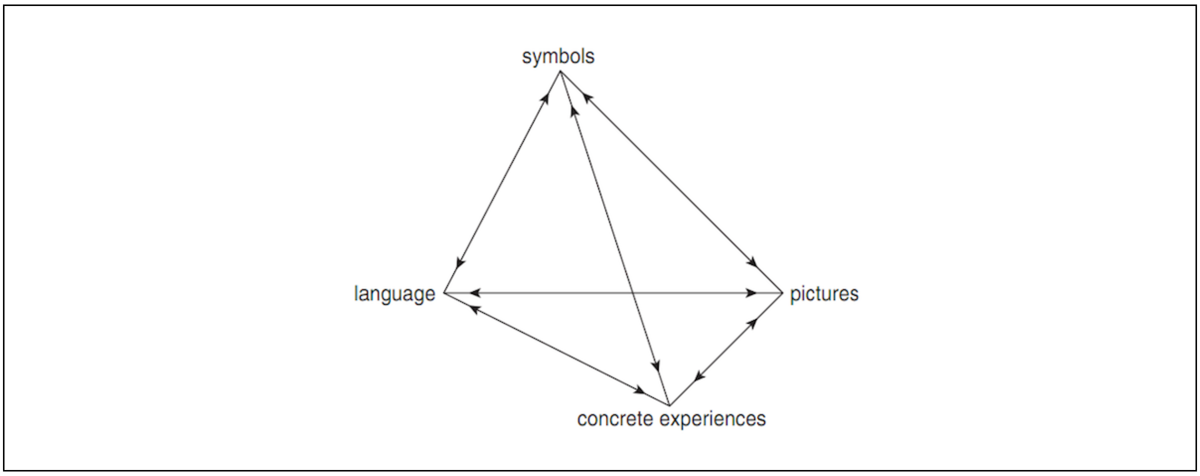

Figure 4: Connections model (Haylock & Cockburn, 2008, p. 10)

One or more of these four key components of mathematical experience are actively employed when learners are engaged in a meaningful mathematical activity: “concrete materials, symbols, language and pictures” (Haylock & Cockburn, 2008, p. 7).

The connections between the four components in Haylock and Cockburn’s framework guided the framing of the enactment of the MR interventions, where learners were invited to model the MR word problems by acting out the situations, using a range of diagrams to represent them, and to generate mathematical equations using symbols. A further important connection made is that between multiplication and division. Figure 5 presents the representational model of the conceptual framework for my study.

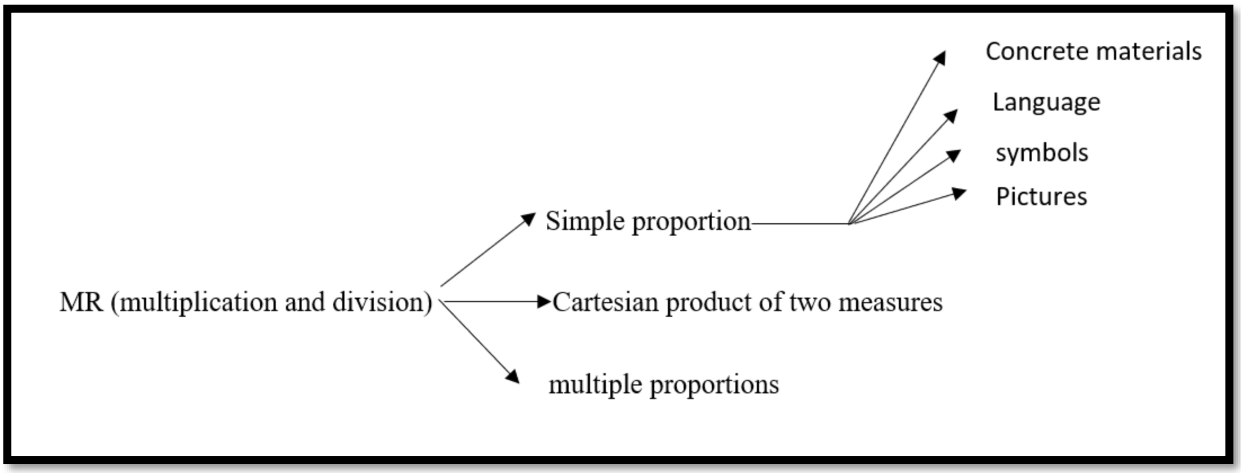

Figure 5: Conceptual framework for the study

Figure 5 represents Vergnaud's (1994) multiplicative conceptual field by showing that MR consists of a combination of both multiplication and division concepts. His work is further elaborated on by showing the three different types of MR i.e., simple proportions, cartesian product of two measures and multiple proportions (the type used in foundation phase is simple proportions). The link in relation to this study is made where the simple proportion type is enacted in such a way that it promotes the connections between pictures, language, symbols, and concrete materials to promote learning with understanding.

2.3 Multiplicative reasoning concepts, types of problems, and models

In this section I will focus on the concepts, types of problems and models involved in MR. To do this, I will begin by introducing the concepts of multiplication and division, then move on to concepts, types of problems and models in MR. The concepts include the definition and key ideas in MR; the types of problems include different types of MR (simple proportion) problems; and the different models in MR focus on those models that can be used to promote learners' understanding.

2.3.1 Multiplicative reasoning concepts

MR is defined as different ways of representing and thinking about situations which include ratio and proportion, scale factor and rates, and multiplication and division (Brown et al., 2014). These situations could involve whole numbers, real and rational numbers or integers. Unlike subtraction and addition which include the action of separating and joining, multiplicative operations cannot merely be solved by separating and joining actions (Nunes & Bryant, 1996).

The skill of MR consists of the capability of working adaptively with an extensive range of numbers and also to solve and recognise an assortment of division and multiplication problems (including indirect and direct proportion) and to effectively communicate this in diverse ways i.e., by means of diagrams, words, written algorithms and symbolic expressions (Siemon, Breed & Virgona, 2005). It is important for learners to adopt this sort of thinking as it concentrates mainly on problem structures and on how learners make sense of the multiplication of variables instead of arithmetic operations.

MR is essential when it comes to changing the ways in which mathematics is learnt and thought about by learners. It includes being able to work better with representations, concepts and strategies of division and multiplication as they arise in an expansive variety of contexts (Siemon et al., 2005). Nunes & Bryant (2007) provide three key considerations about MR:

1. Learners need to understand the inverse relationship between multiplication and division in order to wholly understand the concepts.

2. There are conceptual connections between multiplication and division and they relate to connections within the realms of reasoning

3. Multiplication and addition are sufficiently different, despite procedural connections between the two concepts.

These statements made by Nunes and Bryant show that division and multiplication cannot only be understood as repeated subtraction and addition since they are more complex than that. Opportunities should be given to students to investigate the connections between multiplication and division. If learners understand multiplication as simply being an extension of addition, and understand division as an extension of subtraction, it will cause them to have a limited view about what multiplication and division mean, therefore taking meaning away from the multiplicative structure (Barmby, Bilsborough, Harries, & Higgins, 2009).

In addition, these statements highlight the necessity to avoid teaching multiplication and division as two unconnected ideas but to rather show learners the relationships and connections that are shared by these concepts. Learners must acquire the skill that allows them to understand the interrelatedness of these concepts and the calculations which stem from them (Anghileri, 2006). Understanding the connections between division and multiplication arise from situations that can be sorted into a range of categories: (1) Cartesian products, (2) multiplicative comparison – scale factor, (3) equivalent groups – groups of, (4) rectangular arrays, and (5) partitioning (Siemon et al., 2005). All of the situations can be connected with certain ways of asking a question, with each represented in ways that show "repeated sets, many-to-one correspondence, arrays of rows and columns and many-to-many correspondences" (Anghileri, 2006, p. 85) (see Figure 6).

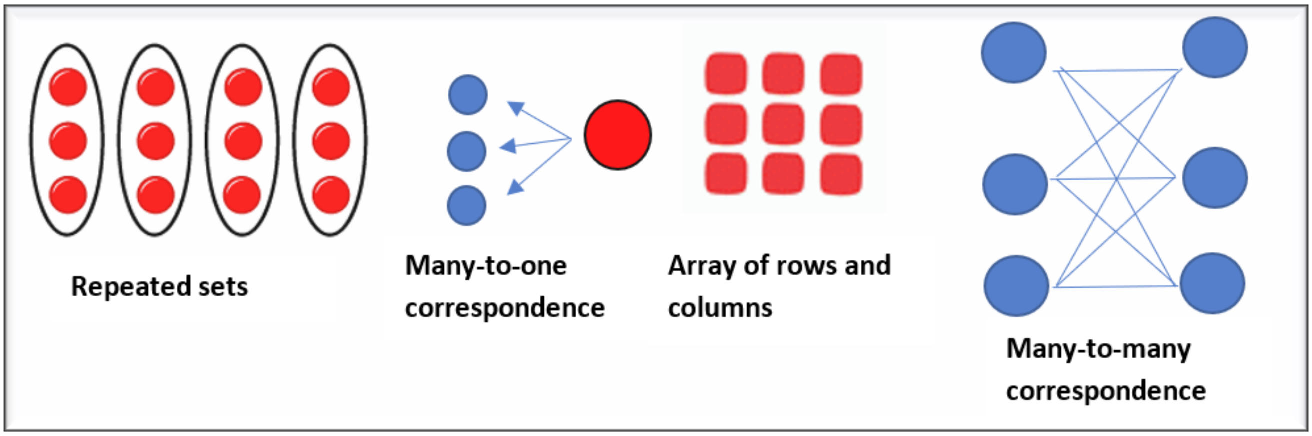

Figure 6: Repeated sets, many-to-one correspondence, an array of rows and columns, and a many-to-many correspondence

Anghileri (2006) shows that all situations comprise of three numbers, namely the total amount of numbers, the number of sets and the number of objects that each set contains. This variety of numbers can be seen both in terms of division and multiplication. When it comes to multiplication the aim is to find the total amount and with division the aim is to calculate either the number of sets or the number in each set (Anghileri, 2006). When it comes to division operations, two types are evident. The first being measurement or grouping situations and the second being sharing situations (i.e., quotative and partitive). In sharing situations, the number known as the dividend is distributed equally among a specific number of recipients and the quotient depends on the divisor. In measuring/grouping situations the quotient or number of recipients depends on the divisor or size of the portion and the dividend is divided into fixed portions (Squire & Bryant, 2002).

All the information provided above offer a base upon which an intervention that aims to develop multiplicative thinking can be conducted. Another noteworthy point is that moving from additive reasoning (the understanding of the inverse relationship between subtraction and addition) to MR is not an easy shift to make and therefore makes developing multiplicative thinking a difficult process. There is a dominance of using repeated addition to solve multiplication operations and this leads to many learners failing to make the shift from additive reasoning to MR (Hurst, 2015). It is commonly mentioned in literature that MR develops from deliberate instruction instead of naturally (Siemon et al., 2005; Moss & Case, 1999; Lamon, 2007).

2.3.2 Types of multiplicative reasoning problems

In this section, I intend to discuss each of the division and multiplication problems separately for the purpose of creating a clear picture of each of the concepts involved. The different types of multiplication and division word problems found in the literature consist of:

- Multiplication as repeated aggregation/addition
- Multiplication as Rate
- Multiplication as Scaling
- Multiplication as Array
- Division as Ratio
- Division as Sharing
- Division as Grouping

2.3.2.1 Multiplication as Repeated Aggregation/Addition

Haylock and Manning (2014) refer to repeated aggregation as being the fundamental idea that multiplication means "so many sets of" or "so many lots of" (p. 135). An example of a type of question that would require repeated addition would be: *Susan has 10 sets of 3 marbles. How many marbles are there altogether?* The question "how many marbles are there altogether?" is associated with multiplication (3 × 10). In this number sentence, the number 10 represents the multiplier and the number 3 represents the multiplicand is. In order to solve this problem, we should note that since there are 10 sets of 3 marbles, the total number of marbles can be calculated by adding 3 ten times (Haylock & Manning, 2014). The sum will therefore be: 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 30. Wallace and Gurganus (2005) state that in repeated addition one number represents the number of groups, the other represents the number of objects in each group, and the sum is the total amount of objects. A representation of this type of problem is shown in figure 7.

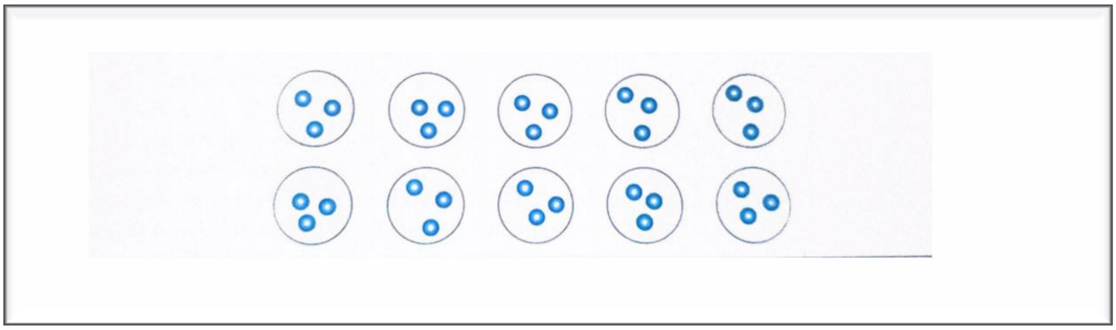

Figure 7: Multiplication as repeated aggregation (3 × 10) (Haylock & Manning, 2014, p. 135)

2.3.2.2 Multiplication as Rate

This problem type includes situations where an implicit ratio is involved, and the word ‘per’ applies in the context either explicitly or implicitly (Askew, 2004). Words such as ‘every’, ‘each’ and ‘per’ are usually found in these problems. Often, we either work with a constant relation of one-to-many correspondence between two sets or comparing two quantities (Nunes & Bryant, 1996). A connection can be seen between these types of problems and repeated addition since equal groups are evident, but they are not the same. This is due to the invariance of the one-to-many correspondence situations which is typically not seen in additive reasoning. For instance, if a chair has 4 legs, each time we add more chairs the number of legs is increased by 4. This differs from additive situations since in order to keep the difference between the sets constant, we need to add the same amount of legs to each set (Nunes & Bryant, 1996). Hence, in this situation, the idea of replicating arises when maintaining the ratio invariant.

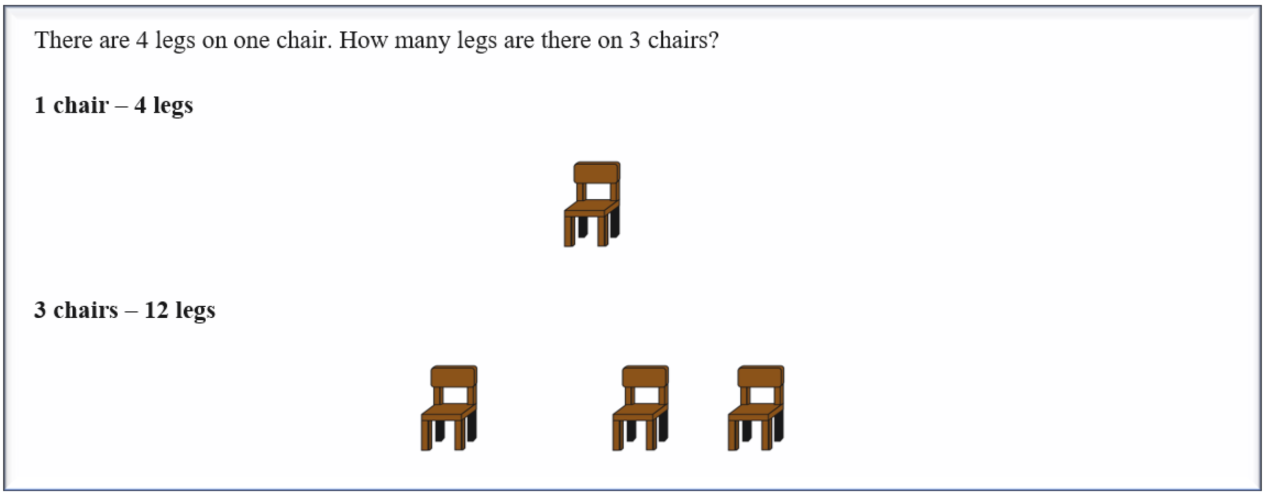

Figure 8: Multiplication as rate (3 × 4 = 12)

2.3.2.3 Multiplication as Scaling

Scaling types of multiplication problems involve increasing a quantity by a certain amount or a "scale factor" (Haylock & Manning, 2014, p. 103). This is noted to be a more challenging idea. An example would be: *Betty and Thabo have rulers. Betty's ruler is 3 units long and Thabo's ruler is 10 times as long as Betty's ruler. How long is Thabo's ruler?* In this case, the relationship between the two sets involved in the problem is shown by the phrase "times as many". Hence, in this case, multiplication is seen as "a process of enlargement, of scaling up by a factor; a process of making a quantity *so many times bigger* (Haylock & Cockburn, 2013, p.103). Figure 9 represents a multiplication as scaling type of problem.

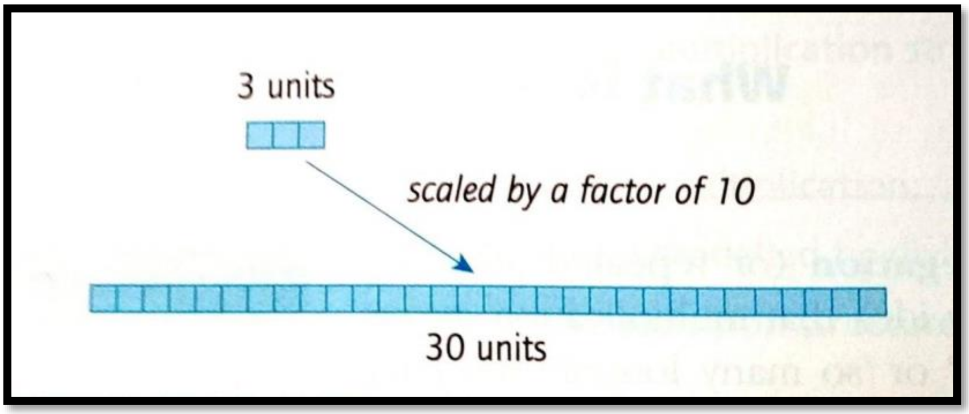

Figure 9: Multiplication as scaling (3 × 10) (Haylock & Manning, 2014, p. 136)

2.3.2.4 Multiplication as Array

Commutativity in multiplication needs to be established by learners. This can be done by making connections between one picture of multiplication and different experiences found in this context (Anghileri, 2006; Haylock & Cockburn, 2013). Arrays are a type of multiplication representation and is considered to be powerful. It shows clearly the commutative property since the multiplicand and multiplier are interchangeable in the rectangular array representation (Hurst & Hurrell, 2014) while the product remains the same.

In Figure 10 below, the array 3 × 5 could be calculated by viewing the problem as three rows of five doughnuts or five rows of three doughnuts; this shows that 3 × 5 is the same as 5 × 3. Visualisation of the properties of distributivity, associativity, and commutativity can be developed when working with arrays. They can be used with the help of other representations to promote a more flexible and deeper understanding of division and/or multiplication (Young-Loveridge, 2005).

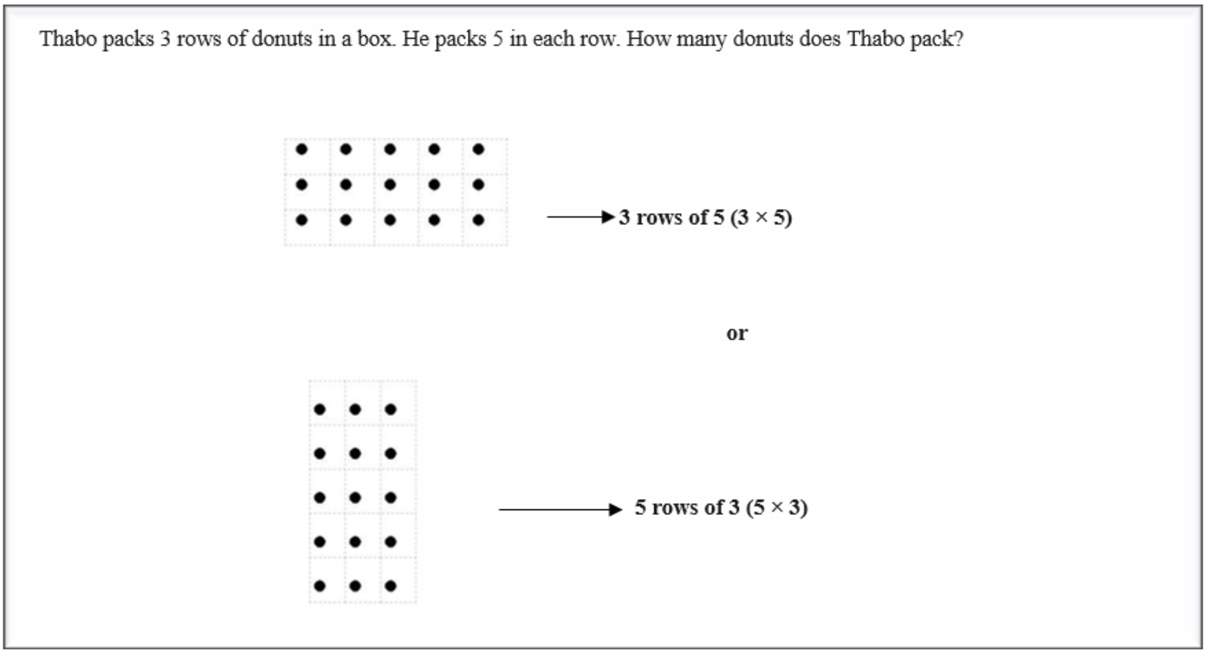

Figure 10: Multiplication as Array (3 × 5 = 15)

2.3.2.5 Division as Ratio

This type of problem involves situations where two quantities are being compared through the use of division (Haylock & Manning, 2014). An example would be: *If Ben earns R300 a week and Thembi* *earns R900 a week, how much more does Thembi earn per week compared to Ben?* This could be solved by showing that R900 – R300 = R600, or the earnings could be compared by looking at the ratio. For example, Thembi earns three times more than Ben (since 900 ÷ 300 = 3) (Haylock & Manning, 2014).

2.3.2.6 Division as Sharing

Types of problems that involve grouping objects into equal amounts where the portion or number of groups is known or given is called "division as sharing" (Haylock & Cockburn, 2013; Nunes & Bryant, 1996). For instance, *5 friends have a total of 25 marbles and wanted to share the marbles equally among themselves. How many marbles will each friend get?* We can see that 25 marbles need to be shared equally among the 5 friends. Learners are familiar with sharing as they experience it in their everyday lives i.e., sharing with their friends. Anghileri (2006) states that this concept is different from subtraction and addition as it involves determining multiplicative connections between two sets or more. Sharing situations are defined by three elements; the number of parts, the amount of the total, and the amount of the parts that should be equal amounts. One-to-many correspondence situations can be similar to sharing situations; however, the difference is that the number of children is inversely related to the number of marbles per child. Another difference noted by Anghileri (2006) is that one-to-one correspondence involves mainly whole numbers whereas division may result in solutions that are fractions. Young learners usually carry out actions when dealing with sharing type problems (i.e. giving one at a time to each person) and this is very different compared to grouping. The figure 11 is an example of division as sharing.

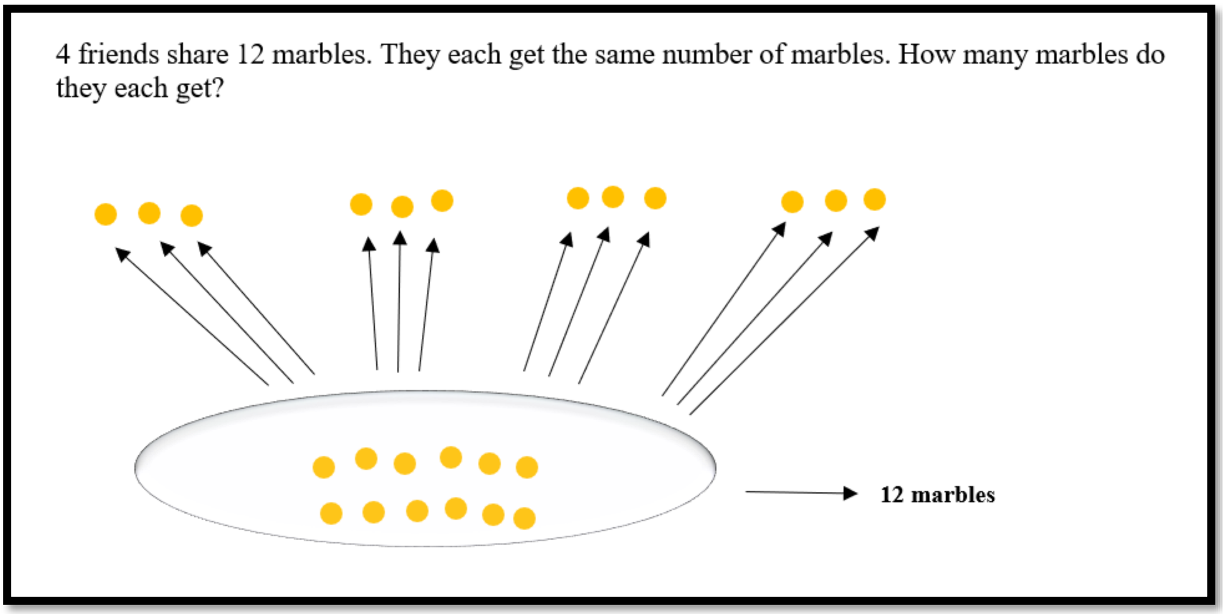

Figure 11: Division as sharing (12 ÷ 4 = 3)

2.3.2.7 Division as Grouping

In these types of problems, the total amount has to be separated into equal sized groups and the size of the group is known. Learners are required to think about and develop understanding about the numbers involved (Anghileri, 2006). For instance, *Hamsa is taking 30 eggs to the shop and puts them in boxes that hold 6 eggs each. How many boxes does she need?* In this situation, learners can work out the number of boxes by drawing dots to represent the 30 eggs then draw circles around or group 6 dots at a time. There will be 5 circles or groups, therefore, they can see that she will need 5 boxes.

In the section that follows, I will look at different models' learners may use to solve these types of simple proportion problems.

2.3.3 Multiplicative reasoning models

In this section I focus on the different models that can be used when solving multiplicative ratio type of problems. An important aspect of mathematics stems from the fact that concrete or real-life situations can be represented using pictures, language and mathematical symbols (Haylock & Cockburn, 2013). These mathematical pictures can also be called models. The models used for ratio type problems help learners to fully understand the relationships between numbers instead of following procedures where no understanding occurs. The models for ratio type problems consist of the following:

- Ratio table
- Double number line
- T – table

2.3.3.1 Ratio table

One of the ways in which teachers can assist learners when it comes to developing strategies for solving problems involving proportions is by using a ratio table (Middleton & van den Heuvel-Panhuizen, 1995). The relationship between two quantities can be seen by looking at a ratio table. It is assembled in a way that shows the values of both quantities. “Progressive and simultaneous operation on the given numbers shows how the relationship (ratio) is preserved proportionately.” (Dole, 2008, p. 19). Take this simple proportion problem for instance: *One enclosure can hold 12 rabbits. How many rabbits can fit in 14 enclosures?* Figure 12 provides a representation of a ratio table that could be used to solve this problem.

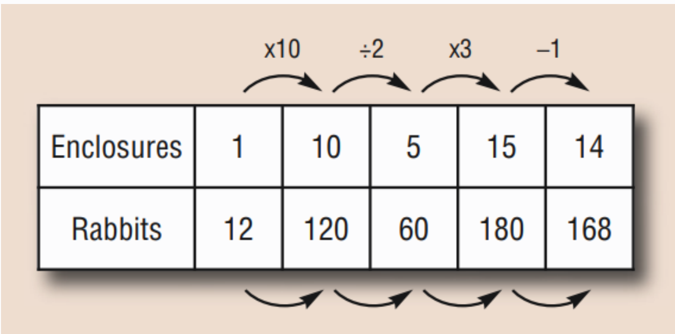

Figure 12: Ratio table representing word problem (Dole, 2008, p. 19)

In this example, in order to get to the solution, one could multiply by 10, then divide by 2, then multiply by 3 to get to 15. Since 15 is one more than the required number, then just subtract 1 group of rabbits (12) to achieve the answer of 168. An important characteristic of a ratio table is that involves showing the sequence of calculations, not just showing ordered calculations from smallest to largest (Dole, 2008).

2.3.3.2 Double number line

The double number line can be seen as precursor to the ratio table, where the double number line holds all possible ratio pairs (Küchemann, Hodgen & Brown, 2011). An example of using a double number line can be showed by considering the following simple proportions problem: *Sam is packing eggs in 5 baskets. She packs 2 eggs in each basket. How many eggs does she pack altogether?* By using the double line all the ratio pairs involved in the problem are represented. Figure 13 represents how the double number line can be used to solve the problem.

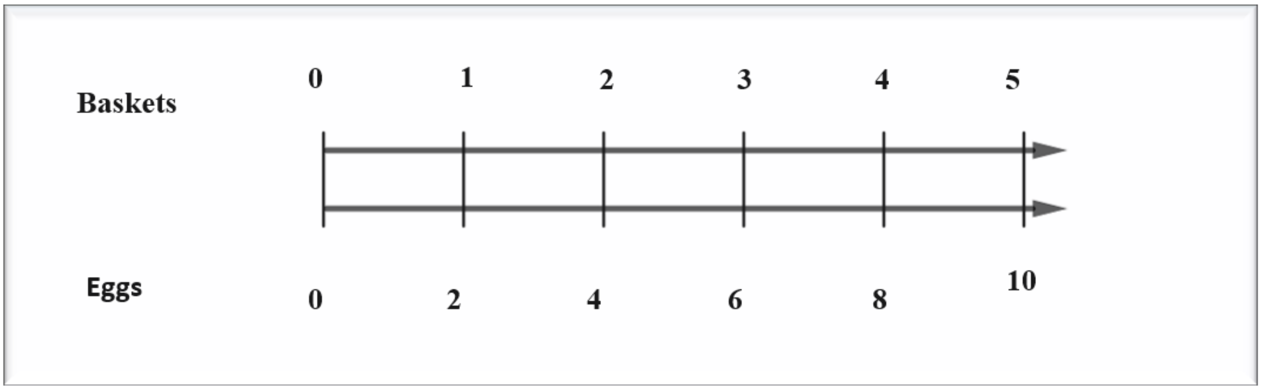

Figure 13: Double number line for 1:2 ratio

In figure 13 the numbers above the line represent the number of baskets, and the numbers below the line represent the number of eggs. If we extend the problem to eight baskets, we could add six, seven and eight. "The key conceptual move in MR is realising the ratio, functional, relationship that exists when reading the figures above and below on the double number line." (Askew, et al, 2019, p. 3).

2.3.3.3 T – table

Streefland (1985) and Vergnaud (1994) have argued that using a T-table is particularly helpful when it comes to focusing on multiplicative relationships. "For simple proportions, the T-table's spatial arrangement helps to make explicit the scalar relationship within quantities (reading down the columns of the table) and the functional relationship between quantities (reading across the rows)" (Vergnaud, 1994, p. 6). Take the following simple proportion problem for instance: *Neo bought 5 boxes of donuts. Each box contains 3 donuts. How many donuts did Neo buy?* Learners may use a T-table to solve the problem, as shown in figure 14.

Boxes	Donuts
1	3
2	6
5	15

Figure 14: T-table representing simple proportion problem

Using this t-table where numbers and relationships are vertically represented brings opportunities that encourage learners to recognise and understand the relationships between the variables (Askew et al, 2019), for example learners may see, in the T-table, the relationship between the boxes and doughnuts and are able to find the solution. The T-table also allows for truncated working rather than having to show each ratio pair as with the double number line.

2.4 Intervention studies

There have been several intervention studies conducted internationally and in South Africa that focus on the development of learners' MR skills. One of such interventions carried out by Hodgen, Coe, Brown and Küchemann (2014) aimed to increase learners' confidence and competence when dealing with multiplicative structures by enabling teachers to adapt their teaching practices according to learners' needs. The study was conducted in the context of 11 schools in London and focused on learners in year 8 (13–14-year-olds). The intervention included twenty-two participating teachers and occurred over the academic year 2010 to 2011. This idea of teaching learner's multiplicative structures in a particular way was a focus of another study conducted in the UK (e.g. Park & Nunes, 2001).

Park and Nunes (2001) carried out an intervention in England which was based on the starting point of the concept of multiplication, and included 42 learners from 2 primary schools. The findings suggested that "the concept of multiplication is in the schema of correspondence rather than in the idea of repeated addition" since learners performed better when they were taught it in that way. A number of intervention studies conducted in South Africa also focus on getting learners to perform better in MR tasks.

A local intervention study conducted by Venkat & Mathews (2014) within the broader WMC - P project aimed at improving grade 7 learners' MR skills in a context of low performance in the Gauteng Province. The intervention included a chain of four lessons which focused on MR, which were taught over a period of 6 weeks. Data was collected through the use of pre- and post-tests which included 14 MR problems. The outcomes of the intervention showed that learners' MR performance can improve through the use of key models and connections for sense making. Another intervention study that formed part of the WMC – P project aimed to "improve learners' understanding of and attainment in MR when solving context-based problems" (Askew et al, 2019, p.1). The research took place in a suburban school and included teachers and learners from three classes in each of Grades 1–3 in a predominantly historically disadvantaged learner population. The intervention was conducted over a period of four weeks and included promoting the use of array images to solve context-based problems. The findings showed evidence of gains which point to the success of using models to improve learners' MR skills.

The studies discussed above prove that there have been a number of interventions locally and abroad aimed at improving learners' MR skills. However, there have not been many large-scale intervention studies based specifically in the North West Province that focus on exploring grade 2 learners' MR skills. Therefore, the findings of this study can potentially provide valuable insights with regard to learners' MR skills in the foundation phase.

2.5 Word problems and bald calculations

Part of this research project focuses on the difference in learner performance between multiplicative word problems (context-based) and bald calculations (context-free). These two types of problems (word problems and bald calculations) can be connected to Treffer's (1978) notion of horizontal and vertical mathematisation. Horizontal mathematization involves moving from real life situations to the world of symbols (i.e., word problems) whereas vertical mathematisation involves the manipulation of symbols (i.e., bald calculations) (van den Heuvel-Panhuizen, 2001).

There have been countless studies which point to the difficulty of solving word problems (Daroczy Wolska, Meurers and Nuerk, 2015; Koedinger & Nathan, 2004; Cummins, Kintsch, Reusser and Weimer, 1988). In their study, Cummins et al. (1988) found that a class of learners in grade 1 managed to solve all problems when represented as bald calculations but only 29% were able to solve them in the format of word problems. In addition, when learner performance in word problems and matching numeric problems were measured, it was found that 27% of learners were correct when answering the word problems and 100% of learners answered the matching bald calculation correctly (Cummins et al., 1988). Koedinger and Nathan (2004) also point to the fact that learners find symbolic representations more accessible than word problems.

The difficulty of word problems has also been pointed out in South African literature. Askew and Venkat (2018) mention that learners across grades find word problems to be challenging. One argument regarding the difficultly of word problems is that the challenge stems from language components such as semantic and syntactic structures; learners find word problems difficult to understand if the word problem is not given in their first language (Setati & Barwell, 2006). Sepeng (2014) argues that "computational errors made by learners, in particular with regard to number skills, appear to stem from the inability to use language(s) (home and/or language of learning and teaching) effectively in order to resolve problems in realistic situations" (p. 22).

Contrary to the common view that word problems are more challenging than numeric calculations, some literature suggest that word problems can be easier to solve than numeric calculations. Koedinger, Alibali and Nathan (2008) argue that learners are less prone to errors and find word problems easier if problem contexts are familiar to them. Moreover, Carraher, Carraher and Schliemann (1987) found that third graders in Brazil were considerably more successful when solving word problems than numeric calculations.

2.6 Conclusion

The literature mentioned above has shown that mathematics performance in South Africa is quite poor, and learners struggle with the concepts of multiplication and division. This is problematic since MR is important for mathematics development in secondary grades. Essentially, division and multiplication are meant to be taught as being inverses of each other and not as separate concepts. The key concepts introduced in this chapter include discussions of the concepts of MR, the types of MR problems, and the models that can be used to solve ratio type problems. A look at the intervention studies conducted internationally and in South Africa show that this study may be useful and important for providing valuable insights about learners' MR skills in the foundation phase.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

Educational research is important as it involves determining credible answers to questions that are asked, by collecting and analysing data (Creswell, 2012). By doing so, we are able to gain knowledge through research that can allow us to improve educational practices since it leads to a more in-depth understanding of particular issues and problems. Opie (2004) sees research in education as a methodological process that adds not only to the knowledge in the field of one's study, but also to one's own knowledge. Hence, the aims of educational research consist of the growth of knowledge by means of building ideas in a field of study and providing a framework for research to be conducted. Educational research, therefore, can be seen as a science that uses scientific approaches such as observing, testing, explaining, and making predictions (Opie, 2004). In order to explore grade 2 learners' MR skills, I adopted a quantitative research design.

This chapter aims to describe the methodology that was used to conduct this study. It focuses on the research design, a discussion about the WMC-P project, context of the study, participants, description of the intervention, instruments for data collection, data source, data analysis, validity of the research and ethical considerations.

3.2 Research design

Research design is a set of procedures and methods used for the collection and analysis of data. Commonly there are three main types of research methods, namely qualitative, quantitative, and mixed methods. The research design adopted in this study is mainly quantitative. Aliaga and Gunderson (2000) define quantitative research as: "Explaining phenomena by collecting numerical data that are analysed using mathematically based methods (in particular statistics)" (p. 1). A quantitative research design was found to be more useful for this study due to the large scale of the collected data and was also in helping to provide a clear an accurate conclusion concerning learner performance.

The type of quantitative design adopted in this study is known as an experimental approach. This involves intervening in a natural setting and controlling several variables in order to determine a relationship between two or more properties of an individual unit (Scott & Morrison, 2005). This study assumes specifically a single-group experimental design which consists of the process of administering pre and post-tests, an intervention, and comparing the differences noted over time. I hold the constructivist belief that all individuals construct knowledge in their own unique ways using the tools, experiences, and information available to them at a particular time. The study was located within the WMC – P project; a brief description of this project is provided in the next section.

3.3 Wits Maths Connect – Primary (WMC-P) project

The WMC-P project was initiated in 2011 with a starting five year-remit to implement, develop, and design research-based interventions in ten partner state primary schools and study their outcomes. "The WMC-P project has four interrelated objectives: (i) to improve the quality of teaching of in-service teachers at the primary school level; (ii) to improve learner performance in primary school mathematics; (iii) to research sustainable and practical solutions to the challenges of improving numeracy in primary schools; and (iv) to provide leadership in numeracy education and increase dialogue[s] around solutions for the mathematics education crisis in South Africa" (Abdulhamid, 2016, p. 53)

One of the key initiatives within the broader WMC-P project was piloting short-term interventions with a focus on MR at both intermediate and foundation phase. This initiative was first carried out in Gauteng schools, and has recorded substantial success. An intervention study conducted within the WMC – P project found that the use of models and materials can help to support primary teachers' work with learners on multiplicative word problems (Venkat & Askew, 2016). Insights about the nature of teachers' mathematical content and pedagogic content knowledge, practice-based expertise, and the possibilities for development of these aspects with regard to MR were gained through another intervention piloted by the WMC – P project (Venkat & Askew, 2020). The intervention, like this study, consisted of a chain of four lessons concerning MR carried out in grade 2. This study, on the other hand, aims to explore grade 2 learners' MR skills and is based on data collected in the North West province. Hence, it may provide new and insightful gains in addition to the findings recorded previously within the WMC – P project conducted in Gauteng.

3.4 Context of the study and Participants

This study used data that was collected by WMC-P MR project located in the North West province. The data was gathered from 35 classrooms across 15 schools which serves previously disadvantaged populations. The focus was on grade 2 learners (approximately 7 to 9 years of age); being in their second year of schooling, the students already have one year's experience of school and are getting familiar with the school norms. Carrying out the lessons and the pre- and post-tests in the context of these North West schools was part of the WMC - P project expansion of its activities beyond Gauteng. The schools are located in underprivileged settings and the classrooms consist of predominantly black learners whose home language is Setswana.

The participants in this study included 1454 grade 2 learners from 35 classes across 15 schools in the North West Province as well as 35 foundation phase teachers and 15 Subject Advisors. The subject advisors involved in this study had undergone one and a half hours of training by members of the WMC-P team. Subject advisors then trained the 2 or 3 participating grade 2 teachers at their selected school prior to the intervention. The subject advisors held a mid-intervention reflection session with teachers after lesson 2 and intended to do a post-intervention reflection session after the post-test but this was interrupted by the COVID-19 pandemic.

3.5 Description of the intervention

The six-week intervention consisted of a pre-test, four intervention lessons (1 hour each) run once per week, followed by a post-test. The intervention consisted of four lessons (See Appendix B) that aimed to improve learners' MR skills. Each lesson started off with mental mathematics relevant to the focus of the intended lesson, then went onto a recall practice activity, then moved onto working with word problems, and ended with giving the learners practice questions. The aim of lesson 1 was to introduce the concept of arrays in multiplication. It mainly involved getting learners to act out and work with multiplication as array type of word problems and find ways to represent and solve the problem. A key point that was focused on was allowing learners to make a connection between different types of arrays (for example, one problem represents four rows of five, or four groups of five, the other represents five rows of four or five groups of four). The aim of lesson 2 was to make a link between arrays and division. This was done by implementing the same practices used in lesson 1 but with division word problems. Lesson 3 aimed to introduce the concept of equal groups in multiplication. It mainly involved getting learners to act out and work with multiplication as equal groups type of word problems and find ways to represent and solve the problem. T-tables were used to get learners to visualize and better understand the problem. Lesson 4 involved making connections between equal groups and division. This was done by implementing the same practices used in lesson 3 but with division word problems.

3.6 Instrument for data collection

The data collection instrument used in this study was pre and post-tests (See Appendix A). The test included addition and subtraction word problems (one each) and addition and subtraction bald number problems (one each); this was included so that learners were not cued into using multiplication for every question. It is further composed of ten MR questions: eight word problems and two bald number calculations. Four of the word problems are solved through multiplication and the other four are solved through division. The last two multiplicative questions are bald number calculations. The test was designed in such a way that there is one word problem that matches with each of the bald number calculations. For example, the word problem: *There are 3 legs on one pot. How many legs are on 5 pots?* matches the bald number problem: $5 \times 3 = ?$

3.7 Data source

The data used in this research report was pre-existing; it was already collected by the WMC-P team. The data was part of a MR 'Coaching for Development' project that focused on Subject Advisors' pedagogical content knowledge related to MR. This project was jointly undertaken by the WMC-P project and the North West Department of Education conducted during the first term of the 2020 school year. The data collected included learners' pre- and post-test scripts. This study involved a pre-test, teaching a set of four lessons, and then a post test. After receiving consent from all the participants and the relevant authorities, the subject advisors began administering the pre-tests. The pre-tests and post-tests consisted of the exact same questions and contained a total of fourteen questions. After administrating the pre-test, the four lesson plans, compiled by the Wits Maths Connect team, was used by the class teachers to teach the four lessons, one lesson per week. The aims of these lessons included: introducing the concept of arrays in mathematics, linking arrays to division, making equal groups, and linking the concept of making equal groups to division. Finally, the subject advisors administered the post-test and the results of both tests (pre and post) were compiled in an excel spreadsheet.

3.8 Data Analysis

This study focused on two main variables – performance and item format:

- **performance** across pre- and post-tests for the whole group
- **item format** looking across all 10 multiplicative items in the pre and post-test and also looking at the two 'matched' word and bare number problems

The pre and post-test consists of 14 questions in total. My focus was specifically on the 10 multiplicative questions in the test. Descriptive statistics (mean, bar charts) were used to analyze the data by looking at the learners' performance in the pre-test compared to the post-test. Another focus of the analysis is to look at the two 'matched' word and bare number problems. This refers to 4 questions in the test; two pairs which consists of a word problem and a bald number problem which requires using the same multiplication or division calculation to solve it. A t-test at 95% confidence level was used to test the significance of the mean differences between the pre- and post-test, and between the word problem and bald calculations.

Miller (2005) describes the t-test as being the most powerful of the related samples tests. A t-test is used in order to establish whether or not there is a significant difference between variables. Macdonald and Headlam (2008) proposed that a t-test measures whether there is a statistical difference between the mean of two groups (Macdonald & Headlam, 2008). Through the use of a t-test the mean of two sets data can be found and whether or not the difference between the means is significant. Borden (2016) suggested that a t-test analysis is a common mechanism to investigate the differences between two means since it allows the researcher to distinguish between chance and an actual difference. This study used the t-test analysis approach in order to confirm the differences in learner performance, and whether or not those differences are significant.

3.9 Validity of research

There are two different types of validity: "internal validity is a measure of accuracy and whether it matches reality; external validity, on the other hand, is a measure of generalizability" (Scott & Morrison, 2005, p. 253). Data on its own is not a guarantee of validity. Instead, it depends on the connection between the claim and the data gathering process which measures what it is really meant to measure (Opie, 2004).

There are two types of collected data: primary and secondary. The data used in this study was initially collected for a different purpose. It was collected as part of the WMC-P project, hence making it secondary data. Secondary data refers to data that was collected for a difference purpose than it was used for, and primary data is data that the research investigator collected for a particular research goal (Hox & Boeije, 2005). I had no inclusion in the data collection process, therefore the data employed in this study is secondary data. It is "data collected earlier by other researchers or for other purposes than research, such as official statistics, administrative records, or other accounts kept routinely by organisations." (Hox & Boeijie, 2005, p. 596).

There are a number of risk factors linked to an experimental research design (Scott and Morrison, 2005). To ensure validity the pre- and post-tests that were administered to learners were first translated from English to the learners' home language (Sestwana) to make it clearer and more understandable. Class teachers read every question out loud to avoid learners' possible lack of reading skills from impeding their sense making process.

In addition to that, the pre and post-tests consisted of the exact same questions; this allowed me to see clearly if learners' performances improved since the results display a precise measurement of the effectiveness of the intervention lessons. A threat to validity stems from the fact that the same questions were used in the pre- and post-test. The same questions were used in order to see more accurately the shift in performance from pre-test to post test. To ensure that the marks did not increase due to practicing, the teachers and learners were not informed that the tests were in fact the same. The pre- and post-tests were also printed out on different colour paper so as to not make learners aware that they were repeating the pre-test.

3.10 Ethical consideration

Ethics clearance is a crucial part of research as it ensures the safety of individuals participating in a study (Bell, 2005). It is the job of the researcher to ensure the safety of any participating individuals, hence no research may be carried out without receiving ethical clearance (McMillan & Schumacher, 2010).

Due to the fact that the data was not collected by me personally but rather previously collected by the WMC – P project, the matter of requesting and receiving ethical clearance was completed by the project. In addition, I obtained ethical clearance from the Wits University ethical committee for using the data for my study (see appendix I).

In addition to gaining ethical clearance, a form which contained information about the nature of this study was submitted to the Department of Education. Information letters explaining what the study was about and the role the teachers and learners would play were given to the principles of the participating schools as well as the participating teachers, learners and their parents. Consent forms were also given to learners and their parents; only parents and learners who gave their consent to being included in the study were included as participants. All learners and their parents were also made aware that the learners' participation was completely voluntary and there would be no consequences if they were not willing to be included.

CHAPTER 4: ANALYSIS AND FINDINGS

4.1 Introduction

The aim of this study was to explore the extent to which a short-term intervention, that focused on making connections between representations and the multiplicative field, improved grade 2 learners MR skills. This chapter provides an analysis of the data and presents the findings and discussion of results. The findings are presented according to the two research questions stated in Chapter 1. The first research question focused on whether there was any evidence of shifts between learners' performance on the multiplicative tasks contained in the pre- and post-test and the second research question focuses on what is evident in learners' performance on the two multiplicative word problems compared to the matching bald calculations. In order to test for significance of difference in means, a t-test analysis was conducted at 95% confidence level and the findings that arise from the analysis are discussed here.

4.2 Differences in performance between pre-test and post-test

Research Question 1: What shift (if any) is evident between learners' performance in the multiplicative tasks contained in the pre- and post-test?

To answer research question 1, the mean value (presented in percentages) was computed for the pre- and post-test and the difference between them was calculated. This is presented in Table 1

Table 1: Percentages of the mean performance of pre- and post-test

	Pre-test	**Post-test**	**Difference**
Mean	24.79%	42.57%	17.78%

One of the important outcomes of the analysis can be seen by looking at the mean values of the pre- and post-tests. The mean values in this case represent the learners' average mark in the pre- and post-tests. The percentage value for the pre-test is 24.79%. The mean percentage in the post-test stands at 42.57%. The difference between the mean in the pre- and post-test is 17.78%. Figure 15 presents a line graph and shows the pictorial differences between the pre- and post-test.

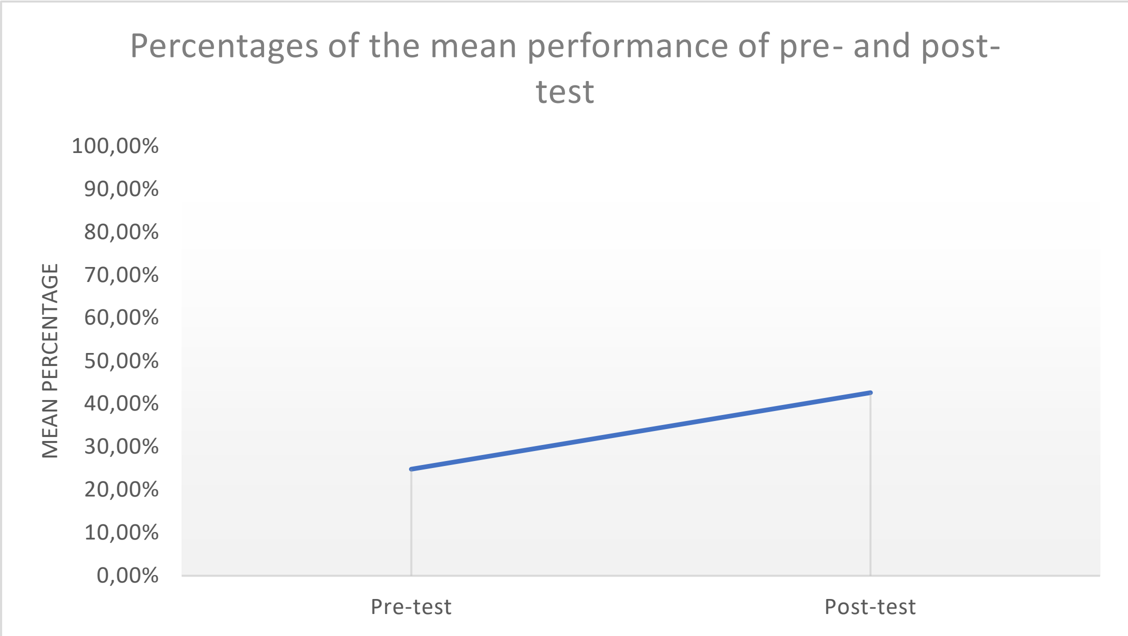

Figure 15: Percentages of the mean performance in the pre- and post-test

In Figure 15 it is evident that the mean value increased from the pre-test to the post-test which means that the average learner performance was higher in the post-test. However, it is important to note that even though the mean value was higher in the post-test, the overall performance on both the pre- and post-tests was low. The learners achieved an average of 24.79% in the pre-test and 42.57% in the post-test; the averages of both tests were below 50%.

To test for the significance of this difference, I formulated the following hypothesis and conducted a t-test analysis at 0.05 level of significance.

Hypothesis 1: There is no significant difference between learners' performance in pre-test and post-test in questions involving multiplicative reasoning

The essence of the t-test was to establish whether or not there was a significant difference between learner performance in the pre- and post-test. Whether or not there is a significant difference can be seen by looking at a *t value* after running the t-test analysis. The critical value of t for the desired level of significance should be any value above 1.64 for two-tailed test (Miller, 2005). This means that if the t value is greater than 1.64 a significant difference exists between the two scores, and if it is below 1.64 there is no significant difference. Table 2 presents the results of the t-test analysis when comparing learners' performance across all 10 multiplicative questions in the pre-test and post-test.

Table 2: T-test summary of results of learners' pre- and post-test performance

Categories	N	Mean	df	t-value	p-value	Conclusion
Pre-test	1454	24.79%	1453	27.41	0.0000	Significant
Post-test	1454	42.57%				

Table 2 shows the matching pre- and post-test data for 1454 learners. The t-value is 27.41 and p-value of 0.000 with degree of freedom (df) of 1453. The t value (27.41) is considerably higher than the critical t value of 1.64, hence there is a significant difference between the learners' performance in the pre- and post-test. This analysis indicates that learners performed significantly better after the teaching intervention. The null Hypothesis 1 is therefore rejected.

4.2.1 Comparison between multiplicative word problems and bald calculation problems

Research Question 2: What is evident in learners' performance in the multiplicative word problems compared to the multiplicative bald calculation problems?

The matching word problems and bald calculations in the pre- and post-test are shown in Table 3.

Table 3: Two multiplicative word problems and the matching bald calculations

Multiplicative word problem	Matched bald calculation
There are 3 legs on one pot. How many legs are on 5 pots?	5 × 3 = ?
Sonny plants rows of potatoes. Each row has 4 potatoes. Sonny plants 24 potatoes altogether. How many rows does Sonny plant?	24 ÷ 4 = ?

To answer research question 2, I computed the percentage mean performance of the two matching word problems and bald calculation problems in both pre- and post-test. Table 4 provides the summary of these percentages.

Table 4: Mean percentages of multiplicative word problems and bald calculations in pre- and post-test

Categories	N	Pre-test	Post-test	Difference
Word problems	1454	32,0%	48,5%	16,5%
Bald calculations	1454	21,0%	40,0%	19,0%
Difference		11%	8,5%	

Table 4 shows that the percentage for average learner performance across the multiplicative word problems in the pre-test was 32.0%. Through the analysis of the post-test results, it can be seen that the percentage of the mean or average learner performance in the post-test was 48.5%. Therefore, the average achieved by learners for the word problem questions in the pre-test increased from 32.0% to 48.5% in the post-test (a difference of 16.5%).

With regard to learner performance of bald calculations in the pre-test and post-test – looking at Table 4, it can be observed that the percentage of average learner performance across the bald calculation questions in the pre-test was 21.0%. This means that the average mark received for the bald number tasks in the pre-test was 0.42 out of a total of 2. Through the analysis of the post-test results, it is clear that the percentage of the mean or average learner performance in the post-test was 40.0%. Therefore, it is apparent that the average achieved by learners in the bald number calculations increased by 19% from the pre-test to the post-test.

In addition, looking at Table 4, it is clear that the percentage of average learner performance on the matched multiplicative word problems in the pre-test was 32.0%. Since the pre-test included two matching multiplicative word problems and bald calculations, the average mark received by learners on these multiplicative word problems was 0.64 out of a total 2. The percentage of average performance on the matched bald calculations in the pre-test was 21.0%, meaning that the average mark was 0.42 out of 2. There is a difference of 11% between the mean percentage of the matched word problems and bald calculations in the pre-test.

In relation to learner performance in word problems and bald calculations in the post-test, Table 4 shows that the percentage of average learner performance on the matched multiplicative word problems in the post-test was 48.5%. Similar to the pre-test, this indicates that the average mark received by learners on the two multiplicative word problems in the post-test was 0.97 out of a total of 2. The average learner performance on the bald calculation problems in the post-test was 40.0%, meaning that the average mark achieved by learners was 0.80 out of 2. Hence, there is a difference of

8.5% between the mean of the two multiplicative word problems and matching bald calculations in the post-test.

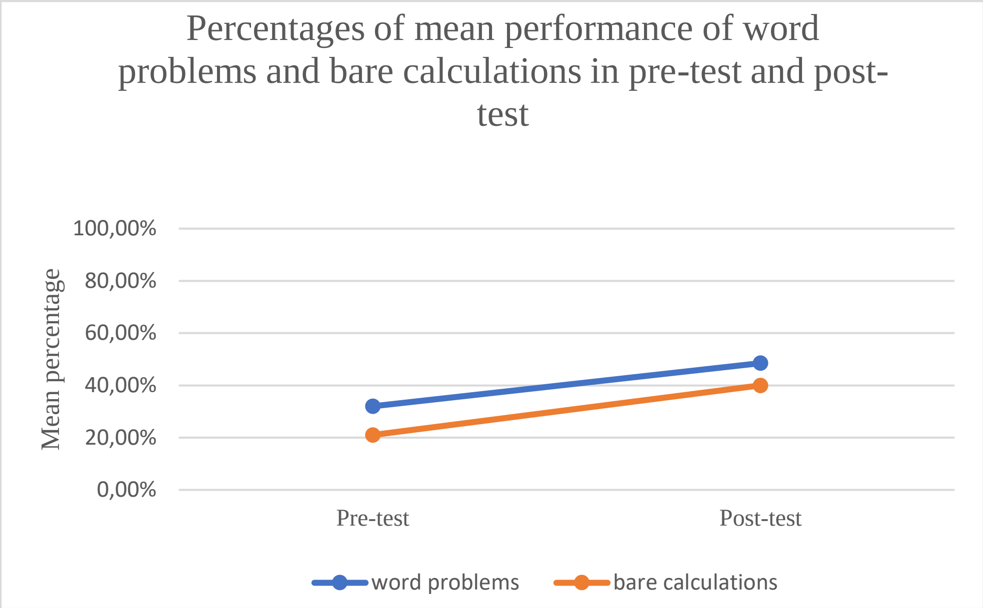

Figure 16: Percentages of mean performance of word problems and bald calculations in pre-test and post-test

Figure 16 shows that the learners' performance in both the word problems and bald calculations improved from the pre-test to the post-test. However, the learners' general performance in the multiplicative word problems and bald calculations in both the tests was relatively low. The percentages of average learner performance achieved by learners in the multiplicative word problem questions and bald calculations in the pre-test and post-test were all below 50%. It is also clear that the learners performed better in word problems compared to bald calculations in both the post-test and in the pre-test.

To test the significance of the mean differences, I formulated the following two null hypotheses

1. There is no significant difference between the learners' mean scores in pre-test and post-test in questions involving word problem problems and bald calculation problems.
2. There is no significant difference between learners' performance in bald calculation questions compared to the word problem questions involving multiplicative reasoning in the pre-test and post-test.

A t-test analysis carried out at 95% confidence level was used to test the hypotheses to establish whether the differences presented in Table 4 are significant or not. Table 5 presents the summary of results concerning the differences in learner performance of word problems and bald calculations in the pre-test and post-test.

Table 5: T-test summary of results between pre- and post-test multiplicative word problems performance and bald calculation problems

Categories	**Pre-test**	**Post-test**	**t-value**	**p-value**	**Conclusion**
Word problems	32,0%	48,5%	16,35	0,000	Significant
Bald calculations	21,0%	40,0%	17,22	0,000	Significant

In order to confirm if there indeed was a significant difference in the learners' performance, the t value in Table 5 is considered. If the t value is greater than 1.64, the difference in learner performance is significant. It is evident that the t value when comparing the difference in learner performance of the word problems in the pre- and post-test is 16.35 which is considerably higher than 1.64. Hence, there is a significant difference between learners' performance on the multiplicative word problems in the pre- and post-test. Therefore, learners performed significantly better in multiplicative word problems in the post-test than in the pre-test.

In addition, by looking at Table 5 it is evident that the t value when comparing the difference in learner performance of the bald calculations in the pre- and post-test is 17.22 which is much higher than 1.64. This means that the difference in performance on the bald calculations in the pre-test and post-test is significantly different; learners performed significantly better in the bald calculations in the post-test than in the pre-test. This further proves that learners performed better in the post-test compared to the pre-test, and therefore, hypothesis 1 was rejected. In order to conclude whether the second hypothesis is true or false, the data in Table 6 is considered.

Table 6: T-test summary of learner results for multiplicative word problems and bald calculation problems in pre- and post-test

Categories	**Word problem**	**Bald calculation**	**t-value**	**p-value**	**Conclusion**
Pre-test	32.0%	21.0%	10.93	0,000	Significant
Post-test	48.5%	40.0%	8.35	0,000	Significant

Table 6 shows that the t value, when comparing the difference in learner performance of the word problems and bald calculations in the pre-test, is 10.93 which is significantly higher than 1.64. This means that the difference in learner performance on the two multiplicative word problems and the matching bald calculations in the pre-test is significant; learners performed significantly better on the two multiplicative word problems than the matching bald calculations in the pre-test. The same can be said with regard to learner performance in the post-test. Since the t value is higher than 1.64, the difference in learner performance on the two multiplicative word problems was significantly better than learner performance on the two matching bald calculations in the post-test. Hence, the same outcome was found in both the pre-test and the post-test: learners performed better on the multiplicative word problems. Therefore, the second hypothesis was also rejected. The section that follows provides a summary of the findings emanating from the analysis done above.

4.3 Summary of findings

This section aims to provide a summary of the findings that emerged from this study. Through a quantitative analysis and interpretation of the data above, two key findings emerged:

1. *Intervention improved learner performance in multiplicative reasoning.*

   Looking at the analysis in the previous sections, it is clearly proven that learners performed significantly better in the post-test compared to the pre-test. The overall performance in the post-test improved by 17.78% after the intervention was carried out. Hence, the null hypotheses which stated that there would be no significant difference in learner performance on the multiplicative word problems, bald calculations, and the overall performance in the pre- and post-test, were rejected. It can therefore be concluded that the intervention was successful in improving learners MR skills.

2. *Multiplicative reasoning context-based problems support learner understanding*

It was clearly proven that learner performance on MR problems, where context is involved, was significantly better than learner performance on the matching context-free calculation problems in both the pre-test and post-test. In the pre-test, learners' average score for the two multiplicative word problems was 11% higher than their average score for the two matching bald calculations. Similarly, in the post-test, learners' average performance for the multiplicative word problems was 8.5% higher than their average for the matching bald calculations. Therefore, the third null hypothesis was rejected: there was a significant difference in learners' performance on the multiplicative word problems and bald calculations in the pre- and post-test.

4.4 Discussion of findings

One of the key findings stated that learners performed better in the post-test compared to the pre-test. This provides evidence that the intervention was indeed successful in helping to improve learners' performance and enhance their MR skills. The lessons in the intervention involved allowing learners to make connections between symbols, language, pictures, and concrete experience (Haylock & Cockburn, 2008). This strategy is therefore helpful in getting learners to understand MR concepts. This finding corresponds with previous research conducted in the South African context; there have been a number of studies which involved conducting interventions to improve learners MR skills (Venkat & Mathews, 2014; Askew et al., 2019). Within the broader WMC - P project, Venkat & Mathews (2014) aimed at improving grade 7 learners' MR skills in a context of low performance through an intervention carried out in the Gauteng Province. The outcomes of the intervention showed that learners' MR performance can improve through the use of key models and connections for sense making. Another intervention study part of the WMC – P project aimed to "improve learners' understanding of and attainment in MR when solving context-based problems" (Askew et al, 2019, p.1). The findings showed evidence of gains which point to the success of using models to improve learners' MR skills. This study further shows that an intervention, which teaches multiplication and division as inverse operations and not as separate operations, can be successful in improving learners' MR skills.

The second key finding, on the other hand, contradicts the majority of research findings around word problems and bald calculations. This study proves that learners performed better in the context-based problems than in context-free calculation problems. This finding contests many studies conducted locally and internationally. There have been a number of international studies which point to the difficulty of solving word problems (Daroczy et al., 2015; Koedinger & Nathan, 2004; Cummins, 1988). In their study, Cummins et al. (1988) found that a class of learners in grade 1 managed to solve all problems when represented as context-free but only 29% were able to solve the same problems when presented in a context-based. In addition, when learner performance in word problems and matching numeric problems were measured, it was found that 27% of learners were correct when answering the word problems and 100% of learners answered the matching bald calculation correctly (Cummins et al., 1988). Koedinger and Nathan (2004) also point to the fact that learners find symbolic representations more accessible than word problems. Studies conducted in South Africa have also pointed to the difficulty of word problems compared to bald calculations. Askew and Venkat (2018) mention that learners across grades find word problems to be challenging. Studies show that learners find word problems more difficult due to language; word problems are given in languages that are not learners' home languages (Setati & Barwell, 2006; Sepeng, 2014).

There have been some research findings which correspond to the second key finding in this study. Contrary to the common view that word problems are more challenging than numeric calculations, some literature suggest that word problems can be easier to solve than numeric calculations. Koedinger et al. (2008) argue that learners are less prone to errors and find word problems easier if problem contexts are familiar to them. Due to word problems involving horizontal mathematisation, learners could find them to be familiar, relatable and easier to understand. Moreover, Carraher et al. (1987) found that third graders in Brazil were considerably more successful when solving word problems than numeric calculations.

4.5 Conclusion

This section aimed to provide answers to the research questions which drove this study and test the three null hypotheses. The first research question, which I used to answer the main research question, focused on whether there were any shifts evident between learners' performance in the multiplicative tasks contained in the pre- and post-test. The first two null hypotheses stated that there would be no significant difference between learner performance in the pre-and post-test. Through quantitative analysis and interpretation of the data the hypotheses were rejected. There were shifts in learners' performance: learners performed significantly better in the post-test. The second research question aimed to find out what is evident in learners' performance on the multiplicative word problems compared to the matching bald calculations. The third null hypothesis that was made in response to this question was that there would be no significant difference in learner performance on the word problems and matching bald calculations in the pre- and post-test. This hypothesis was rejected: there was a significant difference. Learners performed significantly better in the multiplicative word problems compared to the matching bald calculations. Hence, the intervention was successful in improving learners' MR skills and MR context-based problems support learner understanding.

CHAPTER 5: CONCLUSION

5.1 Introduction

In this chapter I conclude this study by discussing the implications that emanate from the findings, highlighting the limitations that were faced, and lastly providing recommendations for future research that emerge from the findings of this research.

5.2 Summary of the study

The focus of my study involved exploring shifts in grade 2 learners MR skills after the implementation of an intervention conducted in the North West province. The two research questions that drove this study were:

(1) What shift (if any) is evident between learners' performance in the multiplicative tasks contained in the pre- and post-test?

(2) What is evident in learners' performance in the multiplicative word problems compared to the multiplicative bald calculation problems?

This study involved a total of 35 classrooms across 15 schools which serves previously disadvantaged populations. The focus was on grade 2 learners. The participants included a total of 1454 grade 2 learners as well as 35 foundation phase teachers and 15 Subject Advisors. It involved a six-week intervention which consisted of a pre-test, four intervention lessons run once per week, followed by a post-test. The instruments for data collection therefore included a pre- and post-test.

This study employed a quantitative research design. The results of the pre- and post-test were analysed using a t-test analysis at 95% confidence level. The t-test was used to prove whether there was a significant difference in learner performance in the pre-test compared to the post-test, and whether there was a significant difference in learner performance in word problems compared to bald calculations.

In the section that follows, the two key findings which emerged from this study are mentioned, and implications of those findings are discussed.

5.3 Findings and Implications

Two key findings emerged through analysis of the collected data. Firstly, it was found that an intervention based on teaching multiplication and division as inverses of each other instead of separate operations, while also allowing learners to make connections between symbols, language, pictures and concrete experiences, improves learner performance in MR skills at grade 2 level. Secondly, it was seen that learners performed better in context-based problems than in context-free problems.

The findings emerging from this study can be used to contribute to teacher practice in schools. This study proves that teaching multiplication and division as inverses of each other and also in a way that allows learners to make connections between symbols, language, pictures and concrete experience (Haylock & Cockburn, 2008), while also using strategies such as t-tables, ratio tables and double number lines to help learners understand the concepts can be helpful in improving learners' MR skills. Previous research has also pointed to the success in improving learners' MR skills when using models and connections for sense making (Venkat & Mathews, 2014). This generates a rational for implementing this kind of teaching in practice.

5.4 Limitations

Like any research, there were a number of limitations attached to this study. This section aims to point out and discuss the three limitations that were encountered when conducting this research report.

The first limitation involved the limited amount of time that the intervention was held over. Due to there being only 6 weeks for the intervention, only four lessons could be taught. If the intervention continued for a longer period, there may have been more gains; learners may have understood the concepts better and the average learner performance might have been above 50%.

The second limitation stemmed from the fact that there were only 2 pairs of matched word problems and bald calculation problems. More matched word problem and bald calculation questions would have provided a more in-depth insight into learners' performance across the two strands.

The third limitation was that I was unable to conduct interviews with the learners involved in this study. Since the data used in this research report was secondary data, I was not the one to collect the data and therefore could not conduct interviews with the learners. If this was possible, I would have had the opportunity to ask learners why they think they performed better in the word problem questions compared to the bald calculations, and in addition, question whether they find word problems less challenging, and if they do then why.

5.5 Recommendations for future research

The findings of this study provide opportunities for a few research areas to be further explored.

One recommendation for a future study is to carry out the same intervention but altering the intervention to include more matched word problem questions and bald calculation questions in the pre- and post-test. This would allow the researcher to get a clearer picture concerning learners' performance in the two types of questions.

The second recommendation is to conduct the intervention in the different provinces in South Africa. This would allow researchers to investigate whether or not the intervention is successful in promoting learners' MR skills even when implemented in different contexts. The participating schools could also be chosen in such a way that they vary with regard to the learners that attend i.e., not only select schools which serve previously disadvantaged populations and schools that are currently disadvantaged by government's inability to improve their contexts.

A third recommendation would be to conduct the intervention over a longer period. This would allow participating learners more time to work with and try to better understand MR concepts. In turn, the researcher would be able to notice whether there are greater gains when there are more lessons taught to learners.

The fourth and final recommendation is to implement the intervention and include interviews of learners after the post-test is administered. The researcher could find out more about learners' views on word problems and bald calculations by asking which they find more challenging and why. This may be able to provide valuable insight regarding why learners find one more challenging than the other, and also how to teach the concept of word problems or bald calculations in a more suitable, beneficial way – depending on the findings.

5.6 Conclusion and self-reflection

Opie (2004) sees educational research as a methodological process that adds not only to the knowledge in the field of one's study, but also to one's own knowledge. The findings of this study have contributed to my growth as a mathematics teacher. Not only have the findings provided valuable strategies regarding improving learners' MR skills, they have also allowed me to reflect on my own teaching of the concepts of multiplication and division. In all my previous observations in mathematics classrooms, the teaching of multiplication and division always included teaching the concepts as separate operations and mainly required learners to simply learn routines and recipes to do calculations instead of teaching them in a way that allows for deeper understanding. This study has shown me that by teaching these concepts as inverses of each other and also persuading learners to make connections between language, pictures, symbols, and concrete experiences, it could help to improve their MR skills and allow for a higher level of sense making. This has encouraged me as a teacher to adopt this strategy when teaching these operations in future. In addition, context-based problems in this situation will allow for a better understanding which is why learners may find word problems easier to solve. This brought my attention to the benefits of relating mathematics to learners' everyday lives, and is definitely another aspect I have learnt that will potentially help improve my teaching.

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APPENDIX A – PRE AND POST TEST

MR Assessment **Grade 2**

First name:____________ **Surname:______________**

Boy or girl:______________ **Class:__________________**

1.

On Thursday Lerato picked 17 apples. On Friday Lerato picked 13 apples. How many apples did Lerato pick altogether?

Lerato picked ______ apples altogether.

2.

Look at this pot. There are 3 legs on one pot.

How many legs are on 5 pots?

There are ____ legs on 5 pots.

3.

Sam plants 4 rows of cabbages. She plants 6 cabbages in each row. How many cabbages does Sam plant?

Sam plants __________ cabbages.

4.

Three friends share 30 sweets. They each get the same number of sweets How many sweets do they each get?

They each get ____ sweets.

5.

Busi packs out 5 rows of counters. Each row has the same number of counters. Busi packs out 40 counters altogether. How many counters are in each row?

There are ____ counters in each row.

6.

Thami had 25 marbles in his pocket. He lost some on the way home and was left with 18 marbles.

How many marbles did he lose?

Thami lost _____ marbles.

7.

Mongezi has 3 bags of apples. There are 8 apples in each bag. How many apples does Mongezi have?

Mongezi has __________ apples.

8.

Sonny plants rows of potatoes. Each row has 4 potatoes. Sonny plants 24 potatoes altogether. How many rows does Sonny plant?

Sonny plants ____ rows.

9.

Lawan puts pies in rows on a tray. Each row has 8 pies. He puts out 5 rows. How many pies does Lawan put out on the tray?

Lawan puts out _____ pies.

10.

Hamsa is taking 30 eggs to the shop. She puts the eggs into egg boxes. Each box can hold 6 eggs. How many boxes does Hamsa need?

Hamsa needs ____ boxes.

11.

17 + 13 =

12.

25 – 18 =

13.

5 x 3 =

14.

24 ÷ 4 =

APPENDIX B – LESSONS

LESSON 1 **Aim:** Introduce arrays

Oral skip counting	**Approx. 3 mins**
Count on in 5s – 5, 10, 15 … Repeat starting at 20 and then 35. Play with this in different ways. For example: Take it in turns for the teacher and learners to count, saying alternate numbers in the sequence. Split the class in two – each half takes it in turns to say the next number.	
Recall practice	**Approx. 5 mins**
Direct the children to the first page of the practice booklet. They are going to multiply each number in the top half of the page by 5. They write 5 in the box at the top. Tell them they only have 3 minutes to answer as many questions as possible (they just write the answers by each number, they do not write x 5) After three minutes, children swap booklets and mark each other's work as you go through the answers. Talk about quick ways to answer lots, for example, doing all the boxes with 1 in, or all the 10s. Tell the children to practice for doing the second set later in the week – can they beat their first score?	
Problem solving	**Approx. 30 mins**
Problem 1: Act out and involve children in setting up Samir's blueberry cupcake problem. After acting out the problem, draw an array sketch of the situation on the board. Mark in 1st row with 5 cupcakes, 2nd row with 10 cupcakes. Leave this on the board. Leave this on the board What are quick ways to count the total number of cupcakes altogether on the tray – without counting in 1s? Ask children to copy down the array image and the answer. **Problem 2**: Again, act out and involve children in setting up Samir's chocolate cupcake problem. After acting out the problem, draw an array sketch of the situation on the board. Again, leave this on the board. Ask for quick ways to count the total number of cupcakes on the tray listen for learners who count in either 4s or 5s and communicate this to the whole class, marking these two options on the array. Ask children to copy down the array image and the answer. **Talk about:** What is the same about the two problems? How are the problems different? One is four rows of five, or four groups of five, the other is five rows of four or five groups of four. **Problem 3**: Set up the story of Corin putting out rows of chairs. Invite children to the front to act out the story, using real chairs. Ask learners to solve the problem using a quick diagram . Look out for learners who: Make a clear representation of an array See the connection with the first two problems See the connection with the arrays that are on the board. Choose 2 or 3 children to share their solution with the class. **Talk about:** What is the same about the three problems? How are the problems different? Talk about how all three problems are about arrays	
Practice	
Direct learners to the page of practice problems in their booklets. Tell them to look out for the problem that is not an arrays problem.	

LESSON 2 **Aim:** Linking arrays to division

Oral skip counting	**Approx. 3 mins**
Count on in 5s – 5, 10, 15 … Count back in 5s starting at 50 and then 45. Count on in 2s – 2, 4, 6, 8 … Count back in 2s starting at 20 and then 16 Play with this in different ways. For example: Take it in turns for the teacher and learners to count, saying alternate numbers in the sequence. Split the class in two – each half takes it in turns to say the next number.	
Recall practice	**Approx. 5 mins**
rect the children to page 5 of the practice booklet. They are going to multiply each number in the top half of the page by 2. They write 2 in the box at the top. ll them they only have 3 minutes to answer as many questions as possible (they just write the answers by each number, they do not write x 2) After three minutes, children swap booklets and mark each other's work as you go through the answers. Talk about quick ways to answer lots, for example, doing all the boxes with 1 in, or all the 10s. Tell the children to practice for doing the second set later in the week – can they beat their first score?	
Problem solving	**Approx. 30 mins**
Problem 1: Act out and involve children in setting up Corin's chairs problem. After acting out the problem, draw a sketch of the situation on the board. Leave this on the board Work on helping children be clear about the difference between talking about the total number of chairs and the number of chairs in each row. Mark the first row with 5 chairs, the second with 10 chairs and so on. Ask children to copy down the array image and the answer. **Problem 2**: Again, act out and involve children in setting up Nomonde's bricks problem. After acting out the problem, draw a sketch of the situation on the board. Mark and number the rows of bricks. Again, leave this on the board. Ask children to copy down the array image and the answer. **Talk about:** What is the same about the two problems? w are the problems different? One made four rows of five, or four groups of five, the other is five rows of five, or five groups of five. Mark each of these on arrays. Could the answer to the first problem help in finding the answer to the second? **oblem 3**: Set up the story that Russell is packing out counters. Invite children to the front to act out the story, using real counters. Set everyone off to solve the problem. Look out for learners who: Make a clear representation of an array See the connection with the first two problems See the connection with the arrays that are on the board. Choose 2 or 3 children to share their solution with the class. **Talk about:** What is the same about the three problems? How are the problems different? Talk about how all three problems are about arrays.	
Practice	
Direct learners to the page of practice problems in their booklets. Tell them to look out for the problem that is not an arrays problem.	

LESSON 3 **Aim:** Making equal groups

Oral skip counting	**Approx. 3 mins**
Count on in 4s – 4, 8, 12 … Repeat starting at 20 and then 12. Count on in 3s – 3, 9, 12 … Repeat starting at 18 and then 30. Play with this in different ways. For example: Take it in turns for the teacher and learners to count, saying alternate numbers in the sequence. Split the class in two – each half takes it in turns to say the next number.	
Recall practice	**Approx. 5 mins**
Direct the children to the page 9 of the practice booklet. They are going to multiply each number in the top half of the page by 4. They write 4 in the box at the top. Tell them they only have 3 minutes to answer as many questions as possible (they just write the answers by each number, they do not write x 4) After three minutes, children swap booklets and mark each other's work as you go through the answers. Talk about quick ways to answer lots, for example, doing all the boxes with 1 in, or all the 10s. Tell the children to practice for doing the second set later in the week – can they beat their first score?	
Problem solving	**Approx. 30 mins**
Problem 1: Act out and involve children in setting up the situation of children getting into equal teams. After acting out the problem, record the solutions systematically in a table. Leave this on the board. Teams Players 1 5 2 10 3 15 Ask children to copy down the table and to find the answer. **Problem 2**: Again, act out and involve children in setting up the situation of packing mangoes into bags. After acting out the problem, record the solutions systematically in a table. Leave this on the board. Bags Mangoes 1 3 2 6 3 9 Ask children to copy down the table and to find the answer. **Talk about:** What is the same about the two problems? How are the problems different? One was making groups of five, the other was making groups of three. **Problem 3**: Set up the context that Mother is giving out sweets. Invite children to the front to act out the story, using real counters. Set everyone off to solve the problem. Look out for learners who: Make a clear table to help them. See the connection with the first two problems. Choose 2 or 3 children to share their solution with the class. **Talk about:** What is the same about the three problems? How are the problems different? Talk about how all three problems are about making equal groups of things.	
Practice	
Direct learners to the page of practice problems in their booklets. Tell them to look out for the problem that is not an equal groups problem.	

LESSON 4 **Aim:** Linking making equal groups to division

Oral skip counting **Approx. 3 mins**

Count on in 4s – 4, 8, 12 …
Count back in 4s starting at 40 and then 28.
Count on in 3s – 3, 6, 9 …
Count back in 3s starting at 30 and then 21.
Play with this in different ways. For example:

- Take it in turns for the teacher and learners to count, saying alternate numbers in the sequence.
- Split the class in two – each half takes it in turns to say the next number.

Recall practice **Approx. 5 mins**

- Direct the children to the page 13 of the practice booklet. They are going to multiply each number in the top half of the page by 3. They write 3 in the box at the top.
- Tell them they only have 3 minutes to answer as many questions as possible (they just write the answers by each number, they do not write x 3)

After three minutes, children swap booklets and mark each other's work as you go through the answers.
Talk about quick ways to answer lots, for example, doing all the boxes with 1 in, or all the 10s.
Tell the children to practice for doing the second set later in the week – can they beat their first score?

Problem solving **Approx. 30 mins**

Problem 1: Act out and involve children in setting up the situation of packing bananas into bags.
After acting out the problem, record the solutions systematically in a T-table and adding rows adding rows until they get to a total of 25. Leave the table on the board

Bags	Bananas
1	5
2	10
3	15
4	20
5	25

Problem 2: Again, act out and involve children in setting up the situation of putting legs on pots.
After acting out the problem, again record the solutions systematically in a table. Leave this on the board.
Ask children to copy down the table and to find the answer.

Pots	Legs
1	3
2	6
3	9
4	12
5	15
6	18

Talk about: What is the same about the two problems?
How are the problems different? One was making groups of five, the other was making groups of three.

- **Problem 3**: Set up the context that Thulelah has baked cupcakes. Invite children to the front to act out the story, using real counters.

Set everyone off to solve the problem. Look out for learners who:
Make a clear table to help them.
See the connection with the first two problems.
Choose 2 or 3 children to share their solution with the class.
Talk about: What is the same about the three problems? How are the problems different?
Talk about how all three problems are about making equal groups of things.

Practice Direct learners to the page of practice problems in their booklets.
Tell them to look out for the problem that is not an equal groups problem.

APPENDIX C – POSTERS

Lesson 1, problem 1: Blueberry cupcakes

Samir puts blueberry
cupcakes in rows on a tray.
Each row has 5 cupcakes.
Samir puts out 4 rows.
How many cupcakes does
Samir put out on the tray?

Lesson 1, problem 2: Chocolate cupcakes

Samir puts chocolate
cupcakes in rows on a tray.
Each row has 4 cupcakes.
Samir puts out 5 rows.
How many cupcakes does
Samir put out on the tray?

Lesson 1, problem 3: Rows of chairs

Corin puts out rows of chairs.
Each row has 4 chairs.
Corin puts out 5 rows.
How many chairs does Corin
put out altogether?

Lesson 2, problem 1: Chairs in rows

Corin puts out 20 chairs in rows.
He puts 5 chairs in each row.
How many rows of chairs does Corin put out?

Lesson 2, problem 2: Laying bricks

Nomonde lays out 25 bricks in rows.
She lays 5 bricks in each row.
How many rows of bricks does Nomonde put out?

Lesson 2, problem 3: Packing out counters

Russell packs out 30 counters.
He packs 10 counters in each row.
How many rows of counters does Russell make?

Lesson 3, problem 1: Teams

Children are playing games in teams.
There are 5 players in each team.
How many children in 3 teams altogether?

Lesson 3, problem 2: Packing mangoes

Constance is packing bags of mangoes.
She packs 6 bags of mangoes.
She puts 3 mangoes in each bag.
How many mangoes does Constance pack?

Lesson 3, problem 3: Giving out sweets

Mother gives 10 children some sweets.
She gives each child 3 sweets.
How many sweets does mother give out?

Lesson 4, problem 1: Packing bananas

Hamsa is packing bananas into bags.
She puts 5 bananas into each bag.
How many bags does Hamsa need to pack 25 bananas?

Lesson 4, problem 2: Making pots

Buyelwa makes pots.

He sticks 3 legs to each pot.

Buyelwa has 18 legs.

How many pots can Buyelwa make?

Lesson 4, problem 3: Packing cupcakes

Thulelah bakes 20 cupcakes.
She puts 4 cupcakes into each box.
How many boxes can Thulelah fill?

APPENDIX D – SAMPLE OF PRACTICE BOOK

Practice book **Grade 2**

Name: ______________________ Class: ___________________

Recall practice 1a X ☐

1	10	5	2	4	8	6	7	9	3
3	1	9	8	6	7	2	4	10	5
4	8	2	1	3	10	5	6	7	9
6	7	2	4	8	9	1	10	5	3
7	9	10	5	2	3	4	8	1	6

Total ☐

Recall practice 1b X ☐

7	2	4	8	9	1	10	5	6	3
10	5	2	4	8	6	7	9	3	1
9	10	5	2	3	4	8	1	7	6
1	9	8	6	7	2	4	10	3	5
8	2	1	3	10	5	6	7	4	9

Total ☐

Multiplicative Reasoning in Foundation Phase: Grade 2

Problems 1

1: Blueberry cupcakes

Samir puts blueberry cupcakes in rows on a tray.
Each row has 5 cupcakes.
Samir puts out 4 rows.
How many cupcakes does Samir put out on the tray?

Samir puts out _______ blueberry cupcakes.

2: Chocolate cupcakes

Samir puts chocolate cupcakes in rows on a tray.
Each row has 4 cupcakes.
Samir puts out 5 rows.
How many cupcakes does Samir put out on the tray?

Samir puts out _______ chocolate cupcakes.

3: Rows of chairs

Corin puts out rows of chairs. Each row has 4 chairs.
Corin puts out 5 rows.
How many chairs does Corin put out altogether?

Corin puts out _______ chairs.

APPENDIX E - PRINCIPAL INFORMATION LETTER

Dear Principal,

I write this letter to you on behalf of a research group employed at the University of the Witwatersrand.Thank you for agreeing to participate in the NWDE – Wits Maths Connect-Primary partnership Multiplicative Reasoning project. We are working with the NWDE Subject Advisers on this project. Each Subject Adviser will be working with two teachers and their classes in your school to run the intervention. The six-week intervention consists of a pre-test, four intervention lessons run once per week, followed by a post-test. The Subject Advisers will be working with the two teachers on marking the pre- and post-tests, and supporting their implementation of the four lessons. The proposed dates for the work in schools are attached on the next page.

On our side, for research purposes, we would like to collect and analyze the data on learners' pre- and post-test responses. This will help us to understand whether a model like this, where Subject Advisers work alongside teachers on classroom teaching, is helpful for improving learning outcomes in primary mathematics. In all of our collation and writing of this data, we will ensure that learner, teacher and school names are kept confidential and anonymized. Learners' names will not be included on the tests and their names and identities won't be included in the research project. Any data collected will be stored in a password protected computer and will be destroyed after 5 years of completion of the project.

At the end of the project, we will work with the Subject Advisers to prepare a short report detailing the project outcomes for learners in your school. This report will be sent out to you early in term 3, following the intervention's conclusion in term 2.

We ask you to confirm that you are happy for this project to proceed with the two teachers and their class learners in your school by signing and returning the tear-off slip below. Please contact us if you require any further details.

Dr. Samantha Morrison

Samantha.Morrison@wits.ac.za

011 717 3106

Ms. Tasmiyah Hoosen

1101913@students.wits.ac.za

I am happy for the GDE - Wits Maths Connect Primary Multiplicative Reasoning Scale Up project to be run in my school, and for the learner data to be collated for research purposes alongside the development purposes detailed above.

Principal name: _________________________ School: ___________________________

Date: _______

Intervention timetable

AM		Week 1 Run pre-test with 2/3 G3 classes Mon 15th April	Week 2 Observe/coach **Lesson 1** with 2/3 G3 teachers Thurs 25th April	Week 3 2/3 G3 teachers teach **Lesson 2** Thurs 2nd May	Week 4 Observe/coach **Lesson 3** with 2/3 G3 teachers Thurs 9th May	Week 5 2/3 G3 teachers teach **Lesson 4** Thurs 16th May	Week 6 Run post-test with 2/3 G3 classes Thursday 23rd May	
PM	Wits pre-intervention session (1,5 hours) Mon 8th April	Mark scripts with teachers Capture marks Reflect & Plan Meeting 1 (1.5 hours) Email spreadsheet marklists to Hamsa & Mike by Fri 19th April	Reflect & Plan Meeting 2 (1 hour) Thurs 25th April	Wits mid-intervention session (1,5 hours) Thurs 2nd May	Reflect & Plan Meeting 3 (1 hour)		Mark scripts with teachers Capture marks Reflect Meeting 4 (1 hour) Email spreadsheet marklists to Hamsa & Mike By Fri 24th April	Wits post-intervention session (1,5 hours) Mon 27th May

Wits based dates (red)

School based dates (blue)

APPENDIX F – PARENT INFORMATION SHEET AND CONSENT FORM

Dear Parent/Guardian,

I write this letter to you on behalf of a research group employed at the University of the Witwatersrand. We are working with the NWDE Subject Advisers on a NWDE – Wits Maths Connect-Primary partnership project focused on multiplication and division. The project consists of a six-week intervention involving a written pre-test that your child will take, four intervention lessons run one per week, followed by a post-test that is like the pre-test. The Subject Adviser will work with your child's teacher on marking the pre- and post-tests, and support the implementation of the four lessons and the tests.

We have implemented this model previously with Primary Mathematics Subject Advisors in Gauteng (2017/2018) and have achieved good success in terms of improved learner attainment on our side, we would like to collect and analyse the data on the class's pre- and post-test responses – your child's work on the tests would be part of this. This data will help us to understand whether a model like this where Subject Advisers work alongside teachers on classroom teaching is helpful for improving learning outcomes in primary mathematics.

In our research work, we will ensure that learner, teacher and school names are kept confidential and anonymized in all of our collation and writing up of this data, so your child will not be identifiable in any way. Learners' names will not be included on the tests and their names and identities won't be included in the research project. Any data collected will be stored in a password protected computer and will be destroyed after 5 years of completion of the project. If you are happy for your child's pre- and post-test responses to be used within our research work, we ask you to confirm this by signing and returning the tear-off slip below. If you do not wish for your child to be part of this project then he/she will be allowed to join another Grade 2 class when her/his class either writes a test or does the lesson (one lesson per week for 4 weeks) linked to this project. Please contact us if you require any further details.

Dr. Samantha Morrison

samantha.morrison@wits.ac.za

011 717 3106

Ms. Tasmiyah Hoosen

1101913@students.wits.ac.za

I am happy for my child's pre- and post-test work in the NWDE - Wits Maths Connect Primary Multiplicative Reasoning Scale Up project to be collated and used for research purposes alongside the development purposes detailed above.

Parent/Guardian name: ________________________

Learner name: ________________________

School: ________________________

Date: ________________________

APPENDIX G – LEARNER INFORMATION SHEET AND ASSENT FORM

Dear Learner,

I write this letter to you on behalf of a research group working at the University of the Witwatersrand. We are working on a project looking at multiplication and division. Your teacher will be working with a Subject Adviser on this project. As part of the work, your teacher will be giving you a short test that he or she will mark, and will then run four lessons on these topics, one a week. At the end of this work, she will set you another short test to see if your work has improved. Your work on these tests does not count for marks, so you don't have to worry about how you do on them. We would just like you to try them and do as many of the questions as you can. A Subject Advisor from the GDE will be in two of these lessons and helping out in the test sessions.

As part of our work at Wits University, we would like to collect in and look at your pre- and post-test answers. This will help us to understand whether these lessons are helpful for improving your mathematics learning.

In our research work, we don't put in your real name, so nobody reading our work will know that it is your answers we are talking about. All your answers will be kept on a computer with a password and will be deleted after 5 years. If you are happy for your work to be used in our research, tick 'Yes' below. If you do not want to partake in this project then you will visit another Grade 2 class when your class is doing a test or lesson linked to this project.

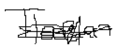
Dr. Samantha Morrison

samantha.morrison@wits.ac.za

011 717 3106

Ms. Tasmiyah Hoosen

1101913@students.wits.ac.za



____

I am happy for my pre- and post-test work to be used in the Wits research project.

YES ☐ NO ☐

Learner name: ____

School: ____

Date: ______________________________________

APPENDIX H – T-TEST ANALYSIS

```
T-TEST PAIRS=Pretest WITH Posttest (PAIRED)

  /CRITERIA=CI(.9500)

  /MISSING=ANALYSIS.
```

T-Test

Notes

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Comments		
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	Active Dataset	DataSet1
	Filter	<none>
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Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
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Paired Samples Statistics

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Pair 1	Pretest	3.47	1454	3.225	.085
	Posttest	5.96	1454	3.997	.105

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Pretest & Posttest	1454	.558	.000

Paired Samples Test

	Paired Differences							
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pair 1 Pretest - Posttest	-2.488	3.462	.091	-2.666	-2.310			

Paired Samples Test

		t	df	Sig. (2-tailed)
Pair 1	Pretest - Posttest	-27.405	1453	.000

```
DATASET ACTIVATE DataSet2.

T-TEST PAIRS=TW_Pre WITH TS_Pre (PAIRED)

  /CRITERIA=CI(.9500)
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/MISSING=ANALYSIS.

T-Test

Notes

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Comments		
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	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
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[DataSet2] C:\Users\Dr U Inuwa\Desktop\Dr Lawan 2.sav

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TW_Pre	.64	1454	.687	.018
	TS_Pre	.42	1454	.677	.018

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	TW_Pre & TS_Pre	1454	.351	.000

Paired Samples Test

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1 TW_Pre - TS_Pre	.223	.777	.020	.183	.263			

Paired Samples Test

		t	df	Sig. (2-tailed)
Pair 1	TW_Pre - TS_Pre	10.932	1453	.000

```
T-TEST PAIRS=TW_Post WITH TS_Post (PAIRED)

  /CRITERIA=CI(.9500)

  /MISSING=ANALYSIS.
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T-Test

Notes

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	Split File	<none>
	N of Rows in Working Data File	1454
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST PAIRS=TW_Post WITH TS_Post (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.
Resources	Processor Time	00:00:00.02
	Elapsed Time	00:00:00.06

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TW_Post	.97	1454	.727	.019
	TS_Post	.80	1454	.791	.021

Paired Samples Correlations

	N	Correlation	Sig.
Pair 1 TW_Post & TS_Post	1454	.480	.000

Paired Samples Test

	Paired Differences							
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower				
Pair 1 TW_Post - TS_Post	.170	.776	.020	.130				

Paired Samples Test

	Paired Differences	t	df	Sig. (2-tailed)
	95% Confidence Interval of the Difference			
	Upper			
Pair 1 TW_Post - TS_Post	.210	8.347	1453	.000

```
T-TEST PAIRS=TW_Pre WITH TW_Post (PAIRED)
  /CRITERIA=CI(.9500)
  /MISSING=ANALYSIS.
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T-Test

Notes

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	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	1454
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST PAIRS=TW_Pre WITH TW_Post (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.11

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TW_Pre	.64	1454	.687	.018
	TW_Post	.97	1454	.727	.019

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	TW_Pre & TW_Post	1454	.420	.000

Paired Samples Test

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower				
Pair 1 TW_Pre - TW_Post	-.327	.762	.020	-.366				

Paired Samples Test

		Paired Differences			
		95% Confidence Interval of the Difference			
		Upper	t	df	Sig. (2-tailed)
Pair 1	TW_Pre - TW_Post	-.287	-16.346	1453	.000

```
T-TEST PAIRS=TS_Pre WITH TS_Post (PAIRED)

  /CRITERIA=CI(.9500)
```

/MISSING=ANALYSIS.

T-Test

Notes

Output Created		02-FEB-2021 15:11:54
Comments		
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	Active Dataset	DataSet2
	Filter	<none>
	Weight	<none>
	Split File	<none>
	N of Rows in Working Data File	1454
Missing Value Handling	Definition of Missing	User defined missing values are treated as missing.
	Cases Used	Statistics for each analysis are based on the cases with no missing or out-of-range data for any variable in the analysis.
Syntax		T-TEST PAIRS=TS_Pre WITH TS_Post (PAIRED) /CRITERIA=CI(.9500) /MISSING=ANALYSIS.
Resources	Processor Time	00:00:00.00
	Elapsed Time	00:00:00.30

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	TS_Pre	.42	1454	.677	.018
	TS_Post	.80	1454	.791	.021

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	TS_Pre & TS_Post	1454	.353	.000

Paired Samples Test

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1 TS_Pre - TS_Post	-.380	.840	.022	-.423	-.336			

Paired Samples Test

		t	df	Sig. (2-tailed)
Pair 1	TS_Pre - TS_Post	-17.224	1453	.000

```
T-TEST PAIRS=WM_Pre WITH WD_Pre (PAIRED)
  /CRITERIA=CI(.9500)
  /MISSING=ANALYSIS.
```

APPENDIX I – ETHICS CLEARANCE CERTIFICATE

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2021ECE010M**
PROJECT TITLE	**Exploring Grade 2 leaners' multiplicative reasoning skills**
INVESTIGATOR	**TASMIYAH HOOSEN**
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	12 April 2021
DECISION OF THE COMMITTEE	Approved unconditionally
EXPIRY DATE	Date of submission of the project report

ISSUE DATE OF CERTIFICATE 16 April 2021 **CHAIRPERSON** (Dr Paul Goldschagg)

cc: Supervisors: Dr Lawan Abdulhamid and Dr Samantha Morrison

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DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

Signature **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

1101913:1101913_Research_Report_Masters.docx

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