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Optimal sizing of hybrid renewable energy systems using quasi-optimal control

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ABSTRACT

This paper addresses the critical challenge of sizing hybrid renewable energy systems over their lifespan while accounting for uncertainties in energy sources and load demands. By leveraging stochastic programming, we introduce a novel modeling technique that ensures robust optimization without compromising numerical tractability. Unlike conventional methods, which determine system parameters for the entire project duration upfront, our approach enables annual adjustments to the renewable components' sizing. This dynamic strategy alleviates early-stage under-utilization, aligning system capacity with evolving demand patterns. Central to our approach is the reformulation of the two-stage stochastic program as a quasi-optimal control. This reduces the number of optimization variables significantly, simplifying the solution process. Moreover, thousands of constraints are replaced with a system of differential equations, enhancing computational efficiency. By minimizing capital costs and dynamic operating expenses, we achieve an optimal system size. Through a real-world application in a rural area of South Africa, we demonstrate the effectiveness of our approach. The results showed a remarkable improvement in problem-solving efficiency, reduced under-utilization, and heightened adaptability to unpredictable weather conditions. This innovative framework represents a substantial leap forward in hybrid renewable energy systems design, offering a more agile and efficient solution for sustainable energy generation.

1. Introduction

Conventional energy generation suffers from significant drawbacks, including its reliance on finite fossil fuels that are unevenly distributed and depleting worldwide [1]. This dependence not only increases the risk of price volatility and supply interruptions but also contributes to the emission of greenhouse gases, worsening environmental concerns, and climate change. In contrast, renewable energy harnesses natural environmental resources such as the sun, wind, water, and hydrogen energy to generate electricity. With its capacity to reduce reliance on finite fossil fuels and mitigate greenhouse gas emissions, renewable energy is gaining popularity as a sustainable solution for power generation [2].

While renewable energy offers the advantage of utilizing widely distributed renewable resources, it also presents limitations. The availability of these resources may be contingent on local weather conditions, impacting the predictability of electricity generation. For this reason hybrid systems consisting of more than one renewable plants are designed. However, the installation and maintenance costs associated with renewable energy sources can be higher than those of conventional alternatives. Therefore, hybrid renewable energy systems, while promising, come with a higher price tag, necessitating a careful design process to achieve optimal component sizing [3].

In the evolving landscape of hybrid renewable energy systems (HRES), there have been notable advancements in sizing methods in the last decade. Prominent of these are the mathematical modeling techniques and the optimization methods. Optimization methods are essential for determining efficient and cost-effective combinations of renewable energy resources within a certain reliability level. Both the classical [4], heuristic and meta-heuristic type finite dimensional optimization methods [5,6] have been used. Pattern search, particle swarm optimization and genetic algorithm have emerged as significant tools in this domain, with researchers utilizing it for optimizing various aspects of HRES, from solar cell parameters to automatic generation control systems [7,8]. Despite these advancements, these sizing methods face unique challenges with respect to constraint handling and dimensionality, highlighting the need for continuous research and development in this field [9,10].

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Nomenclature

Abbreviations

HRES	Hybrid Renewable Energy Systems
SP	Stochastic Programming
ADMM	Alternating Direction Method of Multipliers
PH	Progressive Hedging
kW	Kilowatt

Set and Indices

D	Set of Positive Spanning Direction

Constants

μ_t	Penalty parameter at year t

Variables

Δ_k	Step size at iteration k
p_i	Trial point
T_ω, W_ω	Matrices ensuring the capacity and demand constraints per scenario ω
$q_\omega(t)$	Operating cost at time t per scenario ω (\$/kWh)
$y_\omega(t)$	Capacity used to produce electricity by power plants at time t per scenario ω (kW)
$x(t)$	Power plants capacities to be constructed at time t (kW)
$d_\omega(t)$	Load demand at time t per scenario ω (kW)
c	Capital cost per kW (\$/kW)
b^t	Capital cost at time t (\$)
$z(t)$	Operating cost at time t (\$)

Reviews of hybrid renewable energy systems are reported in [11, 12]. The design of hybrid renewable energy systems integrating solar and wind has been reported in the literature [13–16]. A comparative study of these systems has been given in [17]. Roughly speaking, two main tasks involved in designing any hybrid system are mathematical modeling of the problem [18,19] and the optimization technique employed [3,12]. By formulating a stochastic programming model and subsequently transforming it into a deterministic version, a finite-dimensional optimization problem is obtained. Once the problem is in its deterministic form, a range of traditional optimization methods can be used to solve the sizing problem [20]. However, it is important to note that the deterministic version results in a significant number of variables and constraints, particularly due to second-stage variables. Mathematical optimization techniques based on decomposition such as Brender's decomposition and progressive hedging have been suggested [21] for dealing with large problem sizes. Such techniques generally require multiple solutions of subproblems and they may also depend on the optimization problem at hand. There are many heuristics and meta-heuristics that have also been suggested in the literature [3,22–24]. However, solutions obtained by such heuristics cannot be verified mathematically. Moreover, these methods cannot be applied to problems with a large number of constraints. Hence, the numerical intractability of the problem with many variables and constraints remains as a drawback.

The second drawback is related to the rolling out of the renewable components during a project's lifespan. The renewable energy systems are designed for the entire lifetime, considering population growth and the corresponding increase in demand. However, approaches reported in the literature mandate the installation of all renewable components at the beginning of the project. Therefore, these approaches accommodate the projected increase in demand by building more power plants than initially required. This can lead to resource underutilization in the initial period because some of the system's components may remain unused. This is because the initial load demand can be met without utilizing all the constructed power plants, resulting in higher operational and maintenance costs and a waste of resources during the initial years of a project's lifespan.

In this paper, we propose a novel approach to the sizing of hybrid renewable energy systems, marked by several key advancements:

1. **Overcoming Drawbacks**: The noble approach introduced successfully addresses two major limitations discussed above.
2. **The Dynamic System**: The approach transforms the mathematical problem into a dynamical system over continuous time, giving rise to a quasi-optimal control problem.
3. **Components and Installation**: The method optimizes the dynamic operating cost, together with the components of the hybrid system and their installation rolling out over time.
4. **Reduction of Optimization Variables** : The approach includes a novel use of gradient flow in the dynamical system to manage constraints related to second-stage variables, and thereby effectively reducing the curse of dimensionality.
5. **Case Study Validation**: We apply our model to a case study in rural South Africa, aiming to find the optimal sizing for hybrid renewable energy systems and thereby validating the approach.
6. **Comparative Study**: A numerical comparison of our approach with the classical three-block Alternating Direction Method of Multipliers (ADMM) and Progressive Hedging (PH) algorithms is presented, illustrating the efficacy and innovation of our approach.

The rest of this paper is organized as follows. Section 2 presents the general modeling approach to the problem formulation. Section 3 presents a detailed description of the optimization problem in the case study. Section 4 introduces the pattern search optimization algorithm. Section 5 delves into the numerical results, comparison thereof, and discussion of the presented method. Finally, Section 6 provides the concluding remarks on the optimal sizing of hybrid renewable energy system.

2. The modeling approach to problem formulation

The two-stage SP approach has been used in the literature to determine the optimal sizing of a hybrid energy system [4,18,19]. The two-stage SP is defined as follows

$$\begin{aligned} \min_x \quad & c^T x + \mathbf{E}_\omega \mathcal{Q}(x,\omega), \quad \text{subject to} \\ & Ax = b, \quad x \geq 0, \quad x \in \mathbb{R}^{n_1}, \end{aligned} \tag{1}$$

where $c \in \mathbb{R}^{n_1}$, c^T denotes the transpose of the vector c, $A \in \mathbb{R}^{m_1 \times n_1}$, $b \in \mathbb{R}^{m_1}$, are constants, and $\mathbf{E}_\omega$ is the expected value over the random variable ω that follows a known distribution [25]. The term $\mathcal{Q}(x,\omega)$ outlines the second-stage optimum value and is defined as

$$\begin{aligned} \mathcal{Q}(x,\omega) = \min_{y_\omega} \quad & q_\omega^T y_\omega, \quad \text{subject to} \\ & T_\omega x + W_\omega y_\omega + \xi = h_\omega, \\ & y_\omega \geq 0, \quad y_\omega \in \mathbb{R}^{n_2}, \end{aligned} \tag{2}$$

where $q_\omega \in \mathbb{R}^{n_2}$, $T_\omega \in \mathbb{R}^{m_2 \times n_1}$, $W_\omega \in \mathbb{R}^{m_2 \times n_2}$, $h_\omega \in \mathbb{R}^{m_2}$ encode the random variable data [26], and ξ is some tolerance value.

The value of the first-stage variable, x, needs to be determined before any future revealing of the random variable ω; y_ω is known as the second-stage decision variable, which corresponds to the decisions after the realizations of the random variable revealed. It is also known as the recourse decision variable since it compensates for any bad decisions that may occur at the first stage. When all scenarios are considered in problem (2), then problem (1) becomes a deterministic problem.

In order to present our quasi-optimal control model, we need to define both types of variables in (1)–(2) in relation to our problem. We now motivate and introduce our continuous time problem. We consider the second-stage variable as a function of time and denote it by $y_\omega(t)$, where t is used as a year. The time dependent variable $y_\omega(t)$ depends on $x(t)$ and also on the demand, which varies with time. This allow us to use a dynamical system using $y_\omega(t)$. The first-stage variable $x(t)$ depends on time in a discrete sense. For the energy problem at hand, the second-stage variable $y_\omega(t)$ (capacity used) is dependent (influenced by) on the first-stage variable $x(t)$ (capacity proposed). Hence, for the dynamical system to be presented, we treat $y_\omega(t)$ as the state variable and $x(t)$ as the control variable. We assume a finite number of scenarios, $\omega \in \{1, 2, \ldots, s\}$. We can construct a performance index, over the planning horizon, using the continuous operating cost $q_\omega(t)$ due to $y_\omega(t)$. The expected operating cost over the entire horizon, say N years, can now be written as

$$\int_0^N \sum_{\omega=1}^{s} p_\omega q_\omega^T(t) y_\omega(t) dt \tag{3}$$

which we minimize subject to

$$T_\omega x(t) + W_\omega y_\omega(t) + \xi = h_\omega(t), \tag{4}$$

where $y_\omega(t)$ and $h_\omega(t)$ are vectors of appropriate dimensions under scenario ω. Here, $x(t) \in \mathbb{R}^{n_1}$ is the capacity in kW to be built for the power plants at time t; p_ω the probability associated with scenario ω; $q_\omega(t) \in \mathbb{R}^{n_2}$ is the operating cost of the power plants in \$/kWh at time t under scenario ω; $y_\omega(t) \in \mathbb{R}^{n_2}$ the amount of electricity capacity used to produce electricity by the power plants at time t measured in kW under scenario ω. The constraint (4) results from the capacity and demand constraints such as $y_\omega(t) \le x(t)$ and $-y_\omega(t) \le \tilde{d}_\omega(t)$, $\tilde{d}_\omega(t) = (\alpha - 1) d_\omega(t)$, respectively, using some tolerance values, e.g $\xi_i = 10^{-3}$. In particular, $h_\omega(t) = (\mathbf{0}, \tilde{d}_\omega(t))^T \in \mathbb{R}^{m_2}$ where $\mathbf{0} \in \mathbb{R}^{n_1}$ and $d_\omega(t) \in \mathbb{R}^{m_2 - n_1}$ is the load demand at time t in kW under scenario ω, and α being the reliability level; $T_\omega \in \mathbb{R}^{m_2 \times n_1}$ and $W_\omega \in \mathbb{R}^{m_2 \times n_2}$ are matrices ensuring the capacity and demand constraints under scenario ω. The reason for the operating cost $q_\omega(t)$ is also a function of time is that it increases annually. The same arguments apply for the demand $d_\omega(t)$, and hence $h_\omega(t)$ is treated as a function of time. The value of the control variable $x(t)$ is taken so that $T_\omega x(t) + W_\omega y_\omega(t) + \xi = h_\omega(t)$ holds, $\forall t$. Further explanation of the variables are given in the next section where we have considered a case study. Notice that the cost due to the first-stage variable $x(t)$ and the constraints related to it in (1) are not considered now but will be given shortly.

Treating $x(t)$ as the control variable, we now construct the following functional

$$F(y_\omega(t)) = \frac{1}{2} \left\| T_\omega x(t) + W_\omega y_\omega(t) + \xi - h_\omega(t) \right\|^2 . \tag{5}$$

We would like to achieve the minimum value zero of $F(y_\omega(t))$ by the following trajectory of $y_\omega(t)$ through the gradient flow

$$\begin{aligned} \dot{y}_\omega(t) &= -\nabla_{y_\omega(t)} F(y_\omega(t)) \\ &= -W_\omega^T W_\omega y_\omega(t) + W_\omega^T (h_\omega(t) - T_\omega x(t) - \xi). \end{aligned} \tag{6}$$

We define the following optimal control problem

$$\begin{aligned} &\min_{x(t)} \int_0^N \sum_{\omega=1}^{s} p_\omega q_\omega^T(t) y_\omega(t) dt, \qquad \text{such that} \\ &\dot{y}_\omega(t) = -W_\omega^T W_\omega y_\omega(t) + W_\omega^T (h_\omega(t) - T_\omega x(t) - \xi). \end{aligned} \tag{7}$$

The value $t = 0$ is the beginning of the first year, and hence $y_\omega(0) = \mathbf{0} \in \mathbb{R}^{n_2}, \forall \omega$.

In order to optimize problem (7), we introduced the new variable $z(t)$ such that

$$z(t) = \int_0^t \sum_{\omega=1}^{s} p_\omega q_\omega^T(\tau) y_\omega(\tau) d\tau, \quad 0 \le t \le N.$$

Table 1
Probability distribution of PV and wind turbine operating costs.

PV operating costs			Wind turbine operating costs	
Scenario	Cost \$/kWh	Probability	Cost \$/kWh	Probability
1	0.0583	10%	0.0141	10%
2	0.0592	20%	0.0148	20%
3	0.0603	40%	0.0149	40%
4	0.0614	20%	0.0153	20%
5	0.0615	10%	0.0154	10%

Clearly, $z(0) = 0$.

Using the Fundamental Theorem of Calculus, differentiating the objective function $z(t)$ with respect to time, we get the following differential equation

$$\dot{z}(t) = \sum_{\omega=1}^{s} p_\omega q_\omega^T(t) y_\omega(t), \quad \text{with} \quad z(0) = 0. \tag{8}$$

The optimization problem (7) now becomes

$$\begin{aligned} &\min_{x(t)} \quad z(N) \qquad \text{subject to} \\ &\dot{y}_\omega(t) = -W_\omega^T W_\omega y_\omega(t) + W_\omega^T (h_\omega(t) - T_\omega x(t) - \xi), \\ &\dot{z}(t) = \sum_{\omega=1}^{s} p_\omega q_\omega^T(t) y_\omega(t). \\ &\quad z(0) = 0, \quad y_\omega(0) = \mathbf{0}, \forall \omega. \end{aligned} \tag{9}$$

We now present the final problem by considering the cost due to the first stage decision as

$$\begin{aligned} &\min_{x(t)} \quad C_1(x(t)) + C_2(x(t)) + z(N) \qquad \text{subject to} \\ &\dot{y}_\omega(t) = -W_\omega^T W_\omega y_\omega(t) + W_\omega^T (h_\omega(t) - T_\omega x(t) - \xi), \\ &\dot{z}(t) = \sum_{\omega=1}^{s} p_\omega q_\omega^T(t) y_\omega(t). \\ &\quad z(0) = 0, \quad y_\omega(0) = \mathbf{0}, \forall \omega, \end{aligned} \tag{10}$$

where $C_1(x(t))$ and $C_2(x(t))$ are costs due to first stage decision $x(t)$, and the cost for budget violation due to $x(t)$, respectively. Full details of these costs together with the description of the control variable $x(t)$ are given in the section below.

3. The case study problem

In this study, we optimally build a hybrid renewable energy system in Ga-Nkoana to meet the current and future load demands of electrical power. Ga-Nkoana is a suburb located in Limpopo province, South Africa, at a latitude of −24.416673° and a longitude of 29.783335°. The suburb has 623 households in 2023. The renewable system is to be constructed and used for the whole 20 years duration of the project starting from the year 2023. The purpose of this case study is to find the ideal mix of these renewable plants for Ga-Nkoana that minimizes the total capital cost and the expected operating cost over 20 years while maintaining a certain level of reliability, α, of demand satisfaction. There are budget restrictions on yearly basis although we have used a single budget constrained in the two-stage SP presented below. We have chosen four renewable resources for this study, solar power (x_1), wind power (x_2), hydrogen power or fuel cell (x_3), and backup batteries (x_4).

For this problem, there are five possibilities of operating costs for PV and five possible operating costs for wind turbines (see Table 1), giving rise to $\omega = 5 \times 5 = 25$ scenarios. The reason is that the probabilities of the operating costs are independent. There are 5 blocks yearly. Taking into account the above specification the discrete-time version of two-stage

Table 2
Operation cost of power generation.

Plant	Cost $/kWh ($q$)
Solar[a]	0.060
Wind turbine[a]	0.015
Fuel cell & Electrolyzer	0.025
Backup batteries	0.150

[a] Solar plant and wind turbine operating costs are expected values.

Table 3
Expected power demand data for Ga-Nkoane.

Demand block	Demand (kW) (d)
1	23.1380
2	18.0440
3	12.9510
4	7.8570
5	2.7640

SP is given by

$$\begin{aligned}
&\min_{x,y} \sum_{k=1}^{4} c_k x_k + \sum_{\omega=1}^{25} p_\omega \sum_{k=1}^{4}\sum_{j=1}^{5}\sum_{l=1}^{20} q_{k\omega}\tau_j y_{kjl\omega}, \quad \text{subject to} \\
&\textstyle\sum_{k=1}^{4} c_k x_k \le b, && \text{budget constraint,} \\
&y_{kjl\omega} \le x_k, \quad k = 1,\dots,4, \forall j,l,\omega && \text{capacity constraint,} \\
&\textstyle\sum_{k=1}^{4} y_{kjl\omega} \ge (1-\alpha) d_{jl\omega}, \quad \forall j,l,\omega, && \text{demand constraint.}
\end{aligned} \tag{11}$$

The sub-indices k, j, l, and ω are used in variables and parameters to denote plant, yearly block, year, and scenario, respectively. For instance, τ_j is the duration of block j in hours; p_ω is the probability of scenario ω; $y_{kjl\omega}$ is the capacity fully used to produce electricity by power plant k for demand block j in year l under scenario ω measured in kW; $q_{k\omega}$ is the operating cost of plant category k under scenario ω in \$/kWh. The budget constraint guarantees that the capital cost does not surpass the available budget b. The capacity constraint ensures that the electricity generated by plant unit k does not exceed the total capacity of that unit. The demand constraint guarantees that the hybrid system meets the demand with a minimum level of reliability given by $\alpha = 0.01$.

The expected operating costs for $q_{k\omega}$ are given in Table 2. We employed the *randsample* function in MATLAB to generate 25 scenarios representing the operating costs for both PV and wind turbine systems using the probability distribution in Table 1. This function allowed us to generate random data by defining three essential parameters: *values, sample size*, and *probabilities*. The parameter *values* will be the operating costs $q_{k\omega}, \forall k$ in this context. By specifying the desired *sample size*, which is $\omega = 25$, as the second argument, we generated the requisite number of random data points. It is worth mentioning that the operating cost for fuel cell & electrolyzer, and batteries will remain the same for all 25 scenarios.

The demands $d_{jl\omega}$ are found in Table 3. The expected load demand for 20-year duration is presented in Fig. 1. Demands for electric power are described using the load duration curve, which illustrates the relationship between generation capacity requirements and capacity utilization [27]. The load duration curve is ordered in descending order by magnitude. The electric load for Ga-Nkoana using the continuous load duration curve is shown in Fig. 2, which displays the load duration curve for a total of 8760 hours (one year). For example, in Fig. 2, the load required was approximately above 5 kW for 5256 h and above 9 kW for 1752 h. We obtained the quantized demand set in Table 3 from the continuous curve, presented in Fig. 2, using the Mid-riser and Mid-tread uniform quantizer [28].

The size of problem (11) is enormous. There are four first-stage variables and $4 \times 5 \times 20 \times 25$ second-stage variables in this problem, giving a total of 10 004 variables. There is also one budget constraint, $4{\times}5{\times}20{\times}25$ capacity constraints, and $5 \times 20 \times 25$ demand constraints,

Table 4
Capital cost per kW capacity [29–32].

Plant	Cost per kW in \$ (c)
Solar	3000
Wind turbine	1300
Fuel cell & Electrolyzer	1700
Batteries	1500

totaling 12 501 constraints. Such a problem size presents difficulties to modern LP solvers. Increasing the number of scenarios will lead to an explosion in the problem size, making the use of the LP solver unrealistic.

It is remarkable to point out how the dynamical system in our optimal control problem (10) engulfs all these enormous numbers of constraints where only the first-stage optimization variables are needed, thus reducing the problem size dramatically. The fundamentals of our dynamic optimization problem (10) in relation to problem (11) are described below.

Juxtaposition of the discrete-time problem (11) and the continuous time problem (10) shows that $C_1(x(t))$ corresponds to $\sum_{k=1}^{4} c_k x_k$ (see Table 4), and $C_2(x(t))$ overtakes the budget constraint in (11)-more of which will follow shortly. The remaining two constraints in (11) can be converted to Eq. (4) by using tolerance values from which the first system of differential equations in (10) follows. Notice also that τ_j has no relevance in (10) due to the use of continuous time. The continuous form of Problem (11), e.g. Problem (10), therefore shows that it has gotten rid of 12 501 constraints. Moreover, the optimization problem involving 10 004 variables becomes the problem of only 12 variables, which we have shown below.

Before optimization, we need to define how the system of differential equations is solved, and the cost is calculated in (10). Under our assumption that the hybrid system needs to be built within, say the initial three years, we begin elaborating on the time dependent variable $x(t)$ followed by yearly budget restrictions. Each of the four components/plants of $x(t)$ is decomposed into a variable at the beginning of each year, such as $x(i)_j$ denoting the jth variable at the beginning of ith year, $j = 1, 2, \dots, 4$. If we assume that due to the budget restriction, the entire hybrid system will have to be built over three years, then there will be 12 variables $x(i)_j$ with $i = 1, 2, 3$ and $j = 1, 2, \dots, 4$.

At the beginning of the first year, there are four decision variables $x(1)_1$, $x(1)_2$, $x(1)_3$, $x(1)_4$, respectively, for four plants; at the beginning of the second year there are another four decision variables, e.g., $x(2)_1, x(2)_2, x(2)_3, x(2)_4$; at the beginning of the third year the variables are $x(3)_1, x(3)_2, x(3)_3, x(3)_4$. The 12 variables $x(i)_j$ are such that $x = (x_1, x_2, x_3, x_4)^T$ with $x_j = x(1)_j + x(2)_j + x(3)_j$ for $j = 1, 2, \dots, 4$. The operating cost will be incurred due to the generation capacity of a particular plant at any given time. The cost function is evaluated after the integration of the system of differential equations in (10). We have solved (10) by the Euler method using time discretization by taking the step length $L = 0.2$. The reason for this choice of step size is that we have divided the year into five block demands. Hence, $t = 0$ is the beginning of the first year; $t = 1$ is the beginning of the second year, and so on.

The cost due to the first-stage variable x is calculated by aligning the variables in time line as follows:

At $t = 0$ until $t = 0.8$,

$$x(0) = \bar{x}^0 = (x(1)_1, x(1)_2, x(1)_3, x(1)_4)^T \in \mathbb{R}^4.$$

At $t = 1$ until $t = 1.8$,

$$x(1) = x(0) + \bar{x}^1 = (x(1)_1 + x(2)_1, x(1)_2 + x(2)_2, x(1)_3 + x(2)_3, x(1)_4 + x(2)_4)^T \in \mathbb{R}^4.$$

At $t = 2$ until $t = 20$,

$$x(2) = \bar{x}^0 + \bar{x}^1 + \bar{x}^2 = (x(1)_1 + x(2)_1 + x(3)_1, x(1)_2 + x(2)_2 + x(3)_2, x(1)_3 + x(2)_3 + x(3)_3, x(1)_4 + x(2)_4 + x(3)_4)^T \in \mathbb{R}^4, \tag{12}$$

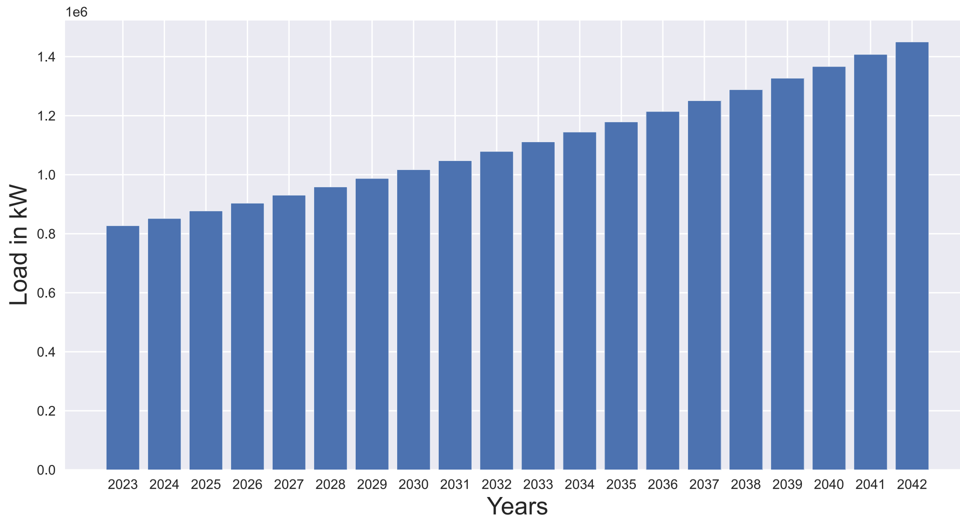

Fig. 1. Total expected load demand over 20-year duration.

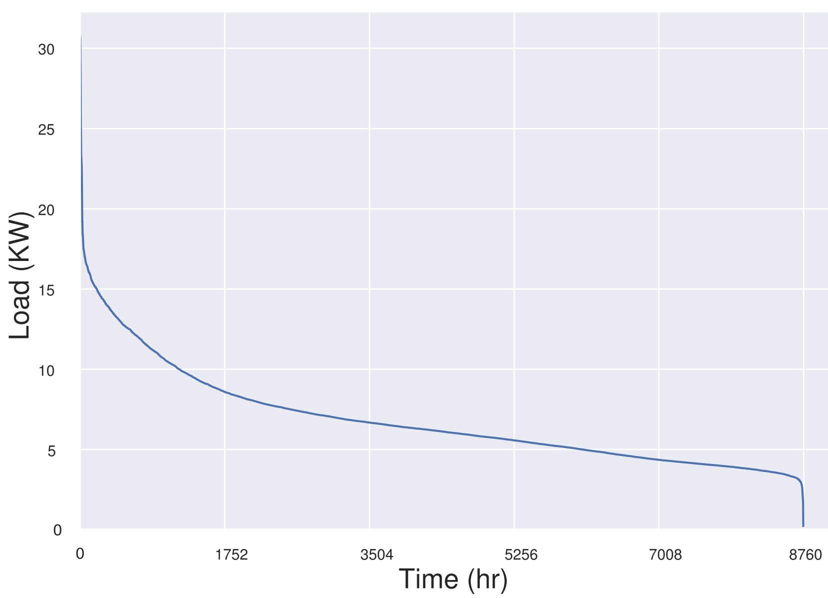

Fig. 2. Load duration curve of the demand.

where
$\bar{x}^0 = (x(1)_1, x(1)_2, x(1)_3, x(1)_4)^T \in \mathbb{R}^4$,
$\bar{x}^1 = (x(2)_1, x(2)_2, x(2)_3, x(2)_4)^T \in \mathbb{R}^4$,
$\bar{x}^2 = (x(3)_1, x(3)_2, x(3)_3, x(3)_4)^T \in \mathbb{R}^4$.

Implementing yearly budget constraints over a number of years is central to the successful construction of power plants. The budget serves as a limit for constructing new power plants each year, ensuring that the financial resources are allocated effectively and sustainably. If the construction costs for the power plants exceed the budget, we penalize the objective function with a penalty parameter. The objective function of the combined cost (operating, capital, and constraint violation) is defined as

$$\min_x \left\{ f(x) = c^T(\bar{x}^0 + \bar{x}^1 + \bar{x}^2) + \sum_{t=0}^{2} \mu_t \max\{c^T\bar{x}^t - b^t, 0\} + z(20) \right\}, \tag{13}$$

where b^t is the budget allocation for year $t = 0, 1, 2$, and μ_t is the penalty parameter for year t. The values $(b^0, b^1, b^2)^T = (\$80\,000, \$25\,000, \$15\,000)^T$ have been used. As mentioned earlier in (10), $C_2(x(t)) = \sum_{t=0}^{2} \mu_t \max\{c^T\bar{x}^t - b^t, 0\}$ is used to prevent violating the budget constraints. We have used $\mu_t = 10\,000, \forall t$. Notice that we have three budget constraints instead of one in (11). The roll out period can be increased by extending the summation in the last term of (13).

We have used derivative-free pattern search as our optimization technique. The calculation of $f(x)$ in pattern search requires $z(20)$, which is found by solving (10) with the initial conditions $z(0) = 0$, and $y_\omega(0)$ and $x \in \mathbb{R}^{12}$. We explain how x is fed into the solution process of (10) for the initial x^0 for pattern search, described in the next section. The same process is applied to the other iteration of the pattern search. The initial value is given as $x^0 = (\bar{x}^0, \bar{x}^1, \bar{x}^2)^T$. From $t = 0$ to $t = 0.8$, we assign $x(0)$ as our initial value, from $t = 1$ to $t = 1.8$, we assign $x(1)$, and from $t = 2$ to $t = 20$, we assign $x(2)$ as our initial values as presented in (12).

The last remaining issue to consider for our problem is the growth rate. Because we are assuming a yearly growth rate of $r = 3\%$, the operating costs and demands over time can be expressed as a function of time through the following equations:

$$
\begin{aligned}
q_{\omega}(t) &= q_{\omega}(t-1)\,(1+r)^{t} \\
d_{\omega}(t) &= d_{\omega}(t-1)\,(1+r)^{t}\,.
\end{aligned}
\tag{14}
$$

The value of $q_{\omega}(t-1)$ and $d_{\omega}(t-1)$ represents the operating costs and demands, respectively, from the previous year, and $(1+r)^{t}$ calculates the growth factor.

Clearly, our optimal control model formulation makes sense in that it allows incremental decisions on x. This approach involves constructing only the power plant components that are required for the first year, followed by building other components in the second and third year. However, since the variable x is scalar-valued, the problem (10) is a quasi optimal control problem and as such Pontryagin's Maximum Principle [33] is not applicable. Moreover, the objective function (13) is not differentiable. Therefore, we have decided to use the pattern search approach for our 12 variable problem. The Algorithm 1 used to optimize our quasi optimal control problem which calls the pattern search technique presented below.

Algorithm 1 Quasi-optimal control using Pattern Search Optimization

Initialization for pattern search: feasible solution x^0, positive spanning set D, initial step size $\Delta_0 = 1$, stopping tolerance $\Delta_{tol} = 10^{-7}$, $n = 12$(size of x)
Initialize the dynamical differential equations-based model: $N = 20, L = 0.2, z(0) = 0, y(0) = \mathbf{0} \in \mathbb{R}^4$
Set the function at (13) to be optimized as the total cost for 20 years
Evaluate the function $f(x)$ at the starting point x^0
Set counter $k \leftarrow 0$ and $i \leftarrow 1$
Solve the differential Equations in (10) using the starting point, and the initial conditions
while $\Delta_k \geq \Delta_{tol}$ **do**
  Evaluate the objective function f in (13) at the trial point $p_i = (x^k + \Delta_k d_i) \in P_k, \quad d_i \in D$
  Set the solution of Equation (6) and Equation (8) with the new trial point p_i
  if $f(p_i) < f(x^k)$ **then**
    Update the current best point $x^{k+1} \leftarrow p_i$
    Increase $k \leftarrow k+1$
    Set $i \leftarrow 1$
    Expand the mesh by increasing $\Delta_{k+1} \leftarrow 2\Delta_k$
  else
    Increase $i \leftarrow i+1$
    if $i > 2n$ **then**
      Set $x^{k+1} \leftarrow x^k$
      Increase $k \leftarrow k+1$
      Set $i \leftarrow 1$
      Reduce the mesh by decreasing $\Delta_{k+1} \leftarrow \frac{1}{2}\Delta_k$
    end if
  end if
end while
Return the optimal solution $x^* = x^k$

4. Pattern search optimization

Pattern search optimization [34] operates on the idea of searching for a pattern of points that converge to the optimal solution. The algorithm is beneficial for derivatives-free optimization which is ideal for the problem considered.

The basic idea of pattern search is to start with an initial guess of the optimal solution and then iteratively explore the search space. The algorithm uses a set of pattern points, which are chosen based on the current search direction, and then evaluates the function at these points to determine the next search direction. This process is repeated until the algorithm stops or we find a satisfactory solution [35].

The method generates a sequence of iterates $\{x^1, x^2, \ldots, x^k, \ldots\}$ where the objective function values are non-increasing. The method consists of two steps: the SEARCH step and the POLL step. The POLL step has been implemented here. The POLL process starts by selecting a trial point, denoted as p_i, from the poll set P_k. The trial point is calculated as follows [34,36]:

$$p_i = \{x^k + \Delta_k d_i \,:\, d_i \in D, i = 1, \ldots, 2n\},$$

where

$$D = \{e_1, \ldots, e_n, -e_1, \ldots, -e_n\},$$

$e_i \in \mathbb{R}^n$ is the ith unit coordinate vector.

The current iterate x^k is used as the starting point for this calculation. The trial point, p_i, is then evaluated to determine if it is a better solution than the current iterate x^k. If a better solution is found ($f(p_i) < f(x^k)$), the next iterate, x^{k+1}, is updated to be the better solution $x^{k+1} = p_i$. If no better solution is found for all $d_i \in D$, the current iterate is retained $x^{k+1} = x^k$. In the case of a successful POLL step, the step size parameter, Δ_{k+1}, is increased to $2\Delta_k$ to enhance exploration. On the other hand, if the POLL step is unsuccessful, the step size parameter is decreased to $\frac{1}{2}\Delta_k$. Thus, the iterates are updated as follows [34,36]:

$$
x^{k+1} = \begin{cases} p_i & \text{if } f(p_i) < f(x^k) \\ x^k & \text{if } f(p_i) \geq f(x^k). \end{cases}
\tag{15}
$$

Also, the updates for the step size is summarized as follows [34,36]:

$$
\Delta_{k+1} = \begin{cases} 2\Delta_k & \text{if } f(p_i) < f(x^k) \\ \frac{1}{2}\Delta_k & \text{if } f(p_i) \geq f(x^k). \end{cases}
\tag{16}
$$

Therefore, the update of the iteration k links to the update of Δ_k.

The pattern search method continues to iterate through the POLL step until the objective function values stop decreasing or meet the stopping criterion, i.e. $\Delta_k < \Delta_{tol}$.

5. Numerical results and discussion

Numerical results were obtained by implementing Algorithm 1 in MATLAB 2022b on a computer with a Core i7 processor and 16 GB of RAM. Results obtained by the quasi-control dynamic approach were presented first, followed by the comparison with the three-block ADMM and PH approaches, which allow parallel computing.

Table 5 summarizes the optimal sizes for 12 optimization variables and the cost of four types of renewable energy plants over a period of three years. The first column presents the optimal rollout of four plants during the first year, similarly for columns 2 & 3. The last column in Table 5 presents the total size in kW for each plant. It can be seen in the table that the size of each plant gradually decreases over the years. For instance, the first-year solar optimal size of 14 kW ($x(1)_1$) decreases in the second year to 3.75 kW ($x(2)_1$) to 1.87 kW ($x(3)_1$) for the third year. The last data 19.63 kW in column 1 denotes $x_1 = x(1)_1 + x(2)_1 + x(3)_1$.

Due to the pattern of the rollout of plants for each year, the capital cost decreases: \$70 900 in year 1 to \$21 100 in year 2 to \$10 550 in year 3. This shows that the hybrid system is gradually built over three years instead of building the entire renewable energy system at once which has been reported in the literature. Because of the rollout of plant capacities, the expansion of new power plants in a year is based on the existing system of the previous years, and therefore the operating cost over time must be monotonically increasing. This can be noticed in Table 5, and Fig. 3, given for the first three years. The decision to expand the system from one year to the next is meaningful due to the fact that the demand increases annually, which we have assumed in our case to be 3%.

Table 5 shows that the total cost \$108 516 consisting of 3 capital and operating costs for the first three years. Fig. 4 depicts the process

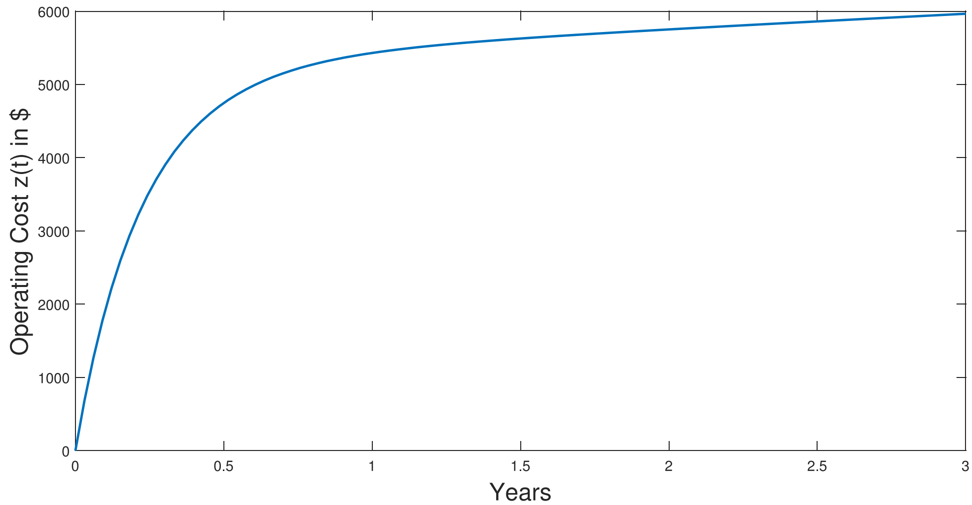

Fig. 3. Operating cost over three years using quasi-optimal control.

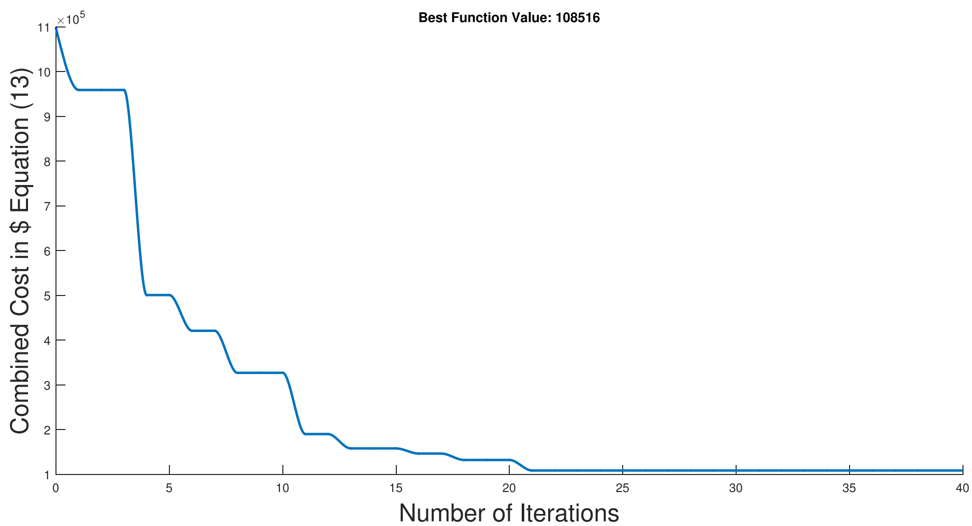

Fig. 4. Convergence to the optimal value (13) using pattern search.

by which the objective function reaches its optimal value using pattern search. It is important to note that the choice of the starting point in optimization can also impact the rate of convergence and the final solution obtained. However, we have obtained the same optimal value after running the algorithm several times. The only difference noticed was the number of iterations incurred by pattern search is different for each run. The initialization was carried out by randomly starting the 12 variables in $[0, 10]^{12}$.

Due to the fact that the larger sizes of plants are built at the beginning of the first year than the subsequent two years that followed, there is a jump in operating costs from 0 to 1 year in Fig. 3. This stabilizes as small plant sizes are rolled out in year 2 and even lesser sizes in year 3.

Next, we compare the performance of our dynamic approach materialized by Algorithm 1 with the three-block ADMM and the PH algorithm for the same problem where the entire system has to be built at the beginning of 20 years. In other word, we have solved the problem (11) with $b = b^0+b^1+b^2 = \$120\,000$ using three-block ADMM [4] and PH with all other input values remaining the same. In this comparison, we must emphasize that problem (11) is a large-size problem with 10 004 optimization variables and 12 501 constraints. However, ADMM and PH allow parallel computations. On the other hand, our quasi-control dynamic problem (10) is a 12-variable problem, but it involves the solution of a large-size system of ODE (ordinary differential equation). The comparison in terms of CPU time shows that our approach is 1.47 times faster than ADMM, and 7.62 times faster than PH, with execution times of 0.1350 s, 0.1985 s, and 1.0289 s, respectively. The optimal control approach does not use parallel computing, yet it is much faster than the other two algorithms. This superiority is because the optimal control approach addresses the feasibility via the dynamical system and optimization together. Conversely, the other two algorithms, though use parallel computing, spend much time to stay feasible due to the number of constraints of the problem. In terms of operating cost,

Table 5
Optimal power plants decision in kW in yearly-basis for 3 years with costs using quasi-optimal control.

Plant	1st year $x(1)_j, j \downarrow$	2nd year $x(2)_j, j \downarrow$	3rd year $x(3)_j, j \downarrow$	Total size
Solar ($x(i)_1, i \rightarrow$)	14	3.75	1.87	19.63
Wind ($x(i)_2, i \rightarrow$)	6	2.0	1.0	9.0
Batteries ($x(i)_3, i \rightarrow$)	5	2.0	1.0	8.0
Fuel cell ($x(i)_4, i \rightarrow$)	8	2.5	1.25	11.75
Capital cost ($)	70 900	21 100	10 550	102 550
Accumulated operating cost ($)	5431	5752	5966	5966
Total cost ($)	76 331	26 852	16 516	108 516

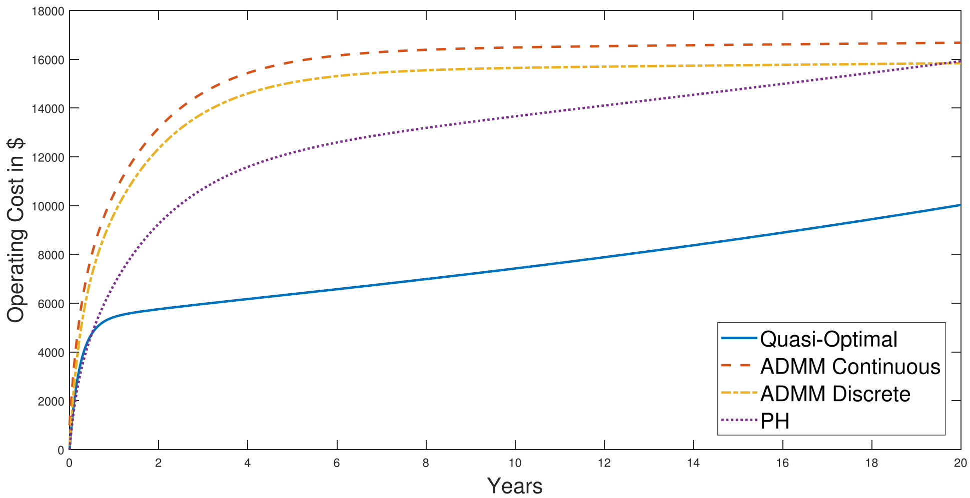

Fig. 5. Total operating cost for 20 years using quasi-optimal control, ADMM, and PH algorithms.

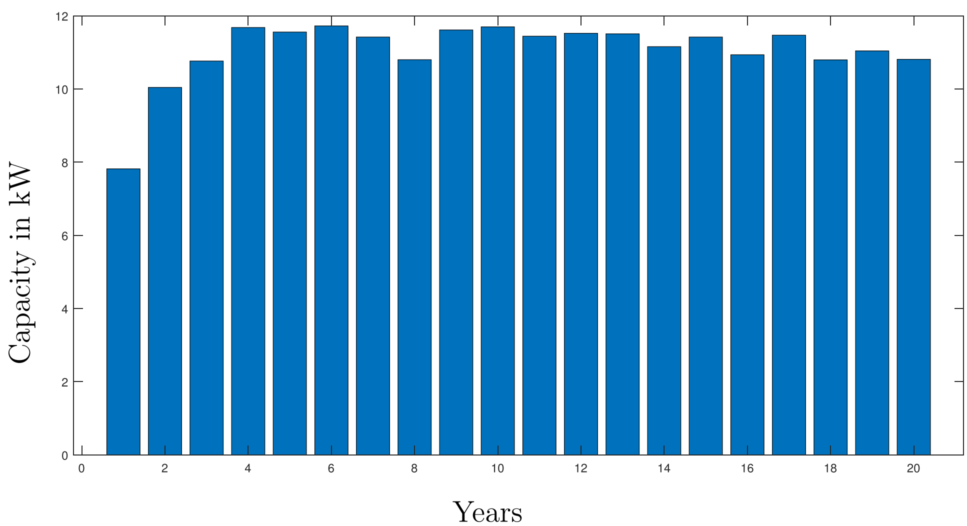

Fig. 6. Fuel cell capacity over 20 years.

Table 6
Optimal power plant that supply the load of Ga-Nkoane using ADMM and PH.

Plant	Optimal decision using ADMM	Optimal decision using PH
Solar PV	20.3829 kW	21.2450 kW
Wind Turbine	9.1330 kW	8.7217 kW
Fuel cell	12.1059 kW	11.5284 kW
Batteries	10.4619 kWh	10.2205 kWh
Capital cost $	109 950	110 000
Operating cost $	16 679	15 921
Total cost $	126 629	125 921

ADMM and PH are always higher because they install all plants at the very beginning, and obtain a higher size of plants in total. The total size of plants in the last column on Table 5 is 48.38 for quasi-optimal control, as opposed to 52.08 for ADMM and 51.72 for PH in Table 6.

In terms of capital cost, the three-block ADMM and PH methods calculate the capital cost at the beginning of the project, and therefore we can consider this as the present value. The total capital cost obtained by ADMM and PH was \$109 950, and \$110 000, respectively; see Table 6. To make the comparison fair, we calculate the present value for the three capital costs each for a year obtained by Algorithm 1. We calculate the present value of the capital cost as follows:

$$
\begin{aligned}
PV &= \sum_{t=0}^{2} \frac{cost_t}{(1+r)^t} \\
&= \$70\,900 + \frac{\$21\,100}{(1+0.03)^1} + \frac{\$10\,550}{(1+0.03)^2} \\
&= \$101\,329,
\end{aligned}
\tag{17}
$$

where the data 70 900, 21 900, and 10 550 denote the capital cost the beginning of year 1, year 2, and year 3, respectively, and are given in Table 5.

The formula (17) discounts the future cash flows at an interest rate of 3%. A comparison between the three costs of 109 950, \$110 000, and 101 329 show that the optimal control-based model decreases the total capital cost by 7.8%. This is a significant cost saving and could be a deciding factor in choosing which method to use for the project.

Fig. 5 is a graphical representation of the operating cost $z(t)$ associated with a hybrid renewable energy system using the quasi-optimal control and ADMM algorithms over a period of 20 years. The graph shows that the operating cost started at zero at $t = 0$ and increased with time. The operating cost itself increases annually, as shown in Eq. (14). The comparison of accumulating operating cost as t increases to 20 years is shown in Fig. 5 both in discrete (block-wise) and continuous time. In continuous time, after 20 years, the operating costs of ADMM and PH are higher than that of quasi-optimal control by approximately 64% and 62%, respectively. At the beginning of the time, the operating costs of ADMM and PH were also higher. It is because their solution constructs all the renewable components upfront; it also selects more components that are expensive, e.g., solar PV. For example, ADMM uses 20.38 kW of PV as opposed to 14 kW of PV using the quasi-optimal control approach in year 1.

From the comparison, it is now clear that although ADMM and PH allow parallel computing, our quasi-control dynamic approach is superior to ADMM and PH in all respects.

In order to demonstrate the adherence of our solution to the suggested capacities, we present Fig. 6, which shows the utilization of fuel cell capacity as an example. The figure illustrates that at time $t = 0, t = 1$, and $t = 2$, the fuel cell capacity employed remains below the corresponding limits of $x_3(0) = 8$, $x_3(1) = 10.5$, and $x_3(2) = 11.75$, respectively. This graphical representation solidifies our solution's compliance with the prescribed capacities.

6. Conclusion

We have presented a model for a hybrid renewable energy system with four components, which replaces the current two-stage stochastic programming approach with a quasi-optimal control involving a system of differential equations. The approach minimizes both the operating cost and capital cost. It allows the optimal decision on capital cost over a rollout period and uses a bare minimum constraint set and decision variables. Thus the sizes of the renewable components are found annually based on an annual budget and the current demand, ensuring that the financial resources are allocated effectively and sustainably.

We have compared the new approach with the three-block ADMM and PH approaches numerically using a case study. Due to the fact that the new approach uses a reduced number of variables and constraints, it is 1.47 times faster than the ADMM and 7.62 times faster than PH, which even both use parallel computing. We have also shown that our quasi-optimal control dynamic approach uses less total of hybrid components, thus making a saving of 7.8% in the budget when compared to ADMM and PH. The largest benefit is, however, the saving on the operating cost, which we have demonstrated graphically.

The mathematical modeling of the problem using the standard optimal control problem instead of the quasi-optimal control will be our future research.

CRediT authorship contribution statement

M. Montaz Ali: Conceptualization, Formal analysis, Methodology, Writing – original draft. **Nouralden Mohammed:** Data curation, Resources, Software, Validation, Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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