Does an intervention focused on discussing problem solving show potential for improving individual learning outcomes?

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Declaration

I declare that this Research report is my own unaided work. It is being submitted for the degree of Masters in Science Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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Abstract

Attainment in problem solving in mathematics is one of the cognitive levels that is used to guide the assessments in South Africa. Unfortunately, it is one of the levels in which most students do not perform well. This research investigates whether the approach taken in an intervention focused on discussing problem solving shows potential for improving individual learning outcomes? A pre-test and post-test on problem solving questions was administered to two Grade 8 classes from a private school in a Northern suburb of Johannesburg. The intervention 1 class wrote a pre-test and thereafter the class was taught how to deal with mathematical problems and later the class wrote a post-test. The intervention 2 class only wrote the pre-test and the post-test, with the usual kind of teaching. Not only did the intervention 1 class average improve by 10% from 38% on the pre-test to 48% on the post-test but also the class improved in terms of using models to solve problems. On the other hand, the intervention 2 class improved by 14% from 24% to 38%. However, while the intervention 2 class had an improvement in terms of using models to solve problems, the improvement was not substantial. The results also show that mathematical problem solving involving ratios remain a challenge for the grade 8 students in my school and that more work needs to be done to ensure success in mathematical problems involving ratios.

Keywords: Problem-solving; intervention 2 class; intervention 1 class, cognitive levels

Glossary of Terms or Abbreviations

- IEB (Independent Examination Board)
- TIMSS (Trends in international mathematics and science study)
- ANA (Annual National Assessment)
- CAPS (Curriculum and Assessment Policy)
- SP (Senior Phase)
- FET (Further Education and Training)
- DBE (Department of Basic Education)
- ZPD (Zone of Proximal development)
- H_0 (Null Hypothesis)
- H_a (Alternative Hypothesis)
- ES (Effect size)
- MD (Mean Difference)
- SD (Standard Deviation)
- ZPD (Zone of Proximal Development)
- M (Mean)

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1. Introduction

Mathematical problem solving research has received attention all over the World. One of the greatest contributions that came out of the research into mathematical problem solving in the 20th century was George Polya's model of problem solving. However, as the research in mathematical problem solving intensified, Polya's model received criticism for being too linear and too broad. My study will therefore focus more on problem solving models developed after that of Polya. For example, Brown and Walter's framework of problem posing, an illustration of Polya's problem solving model cited in Silver (1994).

In South Africa, there are issues around the Education system, which is so weak that the country continues to perform poorly in the Trends in International Mathematics and Science study (TIMSS). For example, South Africa was one of the bottom three countries (Botswana and Hondurus being the other two) that administered the TIMSS assessment for grade 9 in 2011 and showed low performances in Mathematics and Science (TIMSS, 2011).

Judging from the interactions with parents of the students I teach, there is an increased eagerness that children must have done mathematics and science during high school. Some of the reasons for this, include the fact that there is a shortage of skills in most fields that require mathematics and science. It is believed that having done these subjects at school will increase the individual's chances of being employable. However, students continue to perform poorly in mathematics and this poor performance in this subject is a serious concern for parents.

One of the elements of mathematics that emerges as a particular challenge is problem solving. According to the Department of Basic Education (2011), 15% of any assessment task should be on problem solving (see figure 1) below.

CAPS

4.4 Programme of Assessment

The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999. Descriptors for each level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge **20%**	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle ....
Routine Procedures **35%**	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1. Solve for $x : x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2\sin(x - 30°) + 1 = 0$ (Grade 11) 2. Prove that the angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle. (Grade 11)
Complex Procedures **30%**	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is $100km/h$ and the average speed for the return journey is $80km/h$? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x. (Grade 12)
Problem Solving **15%**	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

The Programme of Assessment is designed to set formal assessment tasks in all subjects in a school throughout the year.

53 MATHEMATICS GRADES 10-12

Figure 1: The programme of assessment to guide all assessment tasks taken from (*Department of Basic Education, 2011, p.53)*

However, there seems to be an emerging trend that students do not cope with the problem-solving questions, especially in my school. The Independent Examination Board (IEB) also confirms this notion. For example, the Independent Examination Board (IEB) which administers independent school examinations has recently found that out of all mathematics first paper questions in the examination written in 2015, the only two problem-solving questions that were on the paper were poorly answered with the mean percentage of these questions (Question 9 and Question 10) being 48% and 23% respectively (see figure 2 and figure 3) below. The full questions are provided in appendix I and appendix J.

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Question	Degree of difficulty of the question									
	MARKS				Algebra and Equations and Inequalities	Patterns and Sequences	Finance, Growth and decay	Functions and Graphs	Calculus	Probability
	K	RP	CP	PS						
1 (a) (1)	3				3					
1 (a) (2)	3				3					
1 (a) (3)		3			3					
8 (b) (2)			7			7				
9 (a)				4						4
9 (b)			6							6
9 (c)				5						5
10 (a)				7					7	
10 (b)				7	7					
Total	29	48	50	23	28	26	14	29	38	15

Figure 2. The analysis grid for national senior certificate: Mathematics: Paper 1. IEB. 2015

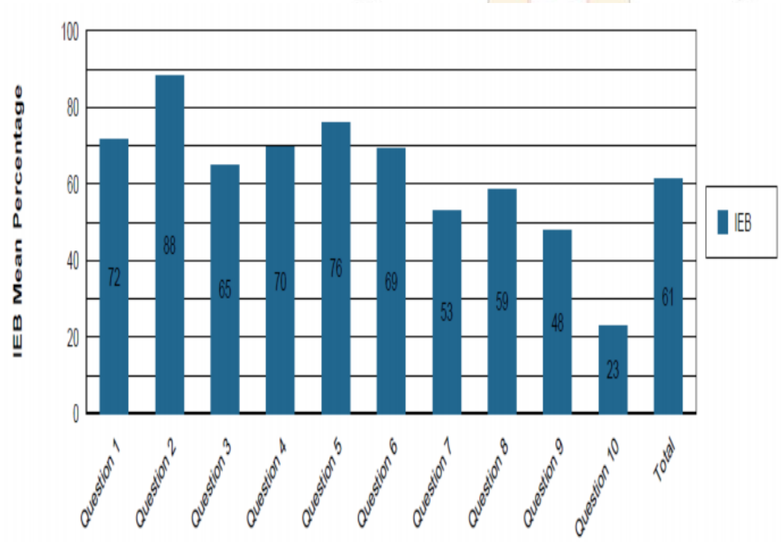


Figure 3: Bar graph showing mean percentage per question code in IEB paper 1. 2015

Furthermore, the performance of Grade 9 students in the Annual National Assessments (ANAs) for mathematics has been poor with the average percentages of 13% in 2012, 14% in 2013 and 11% in 2014 (Annual National Assessment, 2014). Given these low scores, success on the problem solving items is unlikely to be good.

Many researchers have argued in favour of problem solving as a primary goal for Education. For example, in my school, only 15% of the assessment task is on problem solving, following the recommendation by the Department of Education. This means that although, there are other levels that students need to be assessed on, such as knowledge, routine procedures, and complex procedures, problem solving is at the bottom of the list (at 15%) it is less than other assessment levels. Students find it extremely difficult. The implication of this is that, although the Department of Education recognises problem solving as an important skill that it advocates for, a conflict exits between students' performance in tasks involving this and what the Department of Education envisages. However, Jonassen (2004) has argued that "the only legitimate goal of education and training should be that of solving problems" (p.2). Therefore, problem solving should be a primary goal of mathematics education. Teachers should be inculcating a culture of problem solving in mathematics and preparing students to be good problem solvers. Unfortunately, as it is now, students are only getting a taste of what it means to solve problems because of the policy directives and the limited time spent on problem solving.

The current mathematical problem solving state in my school has many limitations. Among these is the fact that students are solving mathematical problems pre-determined by the textbooks and/or by me (the teacher). At high school level, there is usually a prescribed textbook that students use for mathematics. If the text book is not clear in a particular section, teachers are required to supplement it with activities from other sources. Therefore, solving mathematical problem from one textbook can be a limitation in the sense that students are not exposed to a variety of mathematics problems. Furthermore, students are constrained in terms of time in which to finish solving the problem. Problem solving requires time. Unfortunately, the pressure that comes with finishing the syllabus does not do justice to teaching of mathematical problem solving. At high school level, spending the lessons teaching problem solving only, might lead to inability to finish the syllabus. Therefore, teachers at my school tend to rush through problem solving activities which is not beneficial to students. With such constraints in mind, I understand as well that schools have the syllabus to finish which is pre-determined by the Department of Education and that teachers have to absorb the pressure coming from all directions (parents, the school and the Department of Education) to finish the syllabus. All this pressure is to ensure that students are not disadvantaged due to inability to finish a syllabus, especially when they have to write their final examinations that are set externally. I feel, however, that the state of mathematical problem solving in my school should not stay this way.

This exploratory study therefore looks at ways to improve students' performance in mathematical problem-solving through a small-scale intervention focussed on developing problems-solving skills in my school. In particular, the study focuses on Grade 8 students who received two teaching interventions, one class received an intervention that comprised a series of problems and a particular style of teaching, the other class received the same tasks only. A pre-test was administered to the two classes that received either intervention. A post-test was administered after the intervention to ascertain if students made a shift in terms of solving mathematical problems. The first intervention focused on encouraging students to discuss various solutions to problems and consider which solutions are the most effective, the second intervention provided students with the same problems but they worked on them individually and without discussion. Therefore, the research question is: Does the approach taken in the intervention 1 class, which focused on discussing problem solving show potential for improving individual learning outcome in comparison to the intervention 2 class where only the problems were given?

The study is likely to benefit any teacher of mathematics whose interest is in the teaching of problem solving in mathematics. The study can also assist the Department of Education, especially policy makers, to make informed decisions about the teaching and assessing of problem solving in mathematics. More importantly the study could help students from my school to become better problem solvers and help me change the way problem solving is taught at my school.

2. Literature Review

2.1. Defining problem solving

Problem solving in mathematics has many interrelated definitions. Isaacs (1987) defines problem solving as “explaining mathematical concepts, terms, principles and processes in one[s] own words; checking or verifying results; choosing appropriate techniques to solve problem” (p.178). This definition ties well with Polya’s model which states that “students need to understand the problem, make a plan, carry out a plan, look back when solving mathematics problems” (Polya, 1988). On the other hand, Mayer & Wittrock (1996) define problem solving as “a cognitive process directed at achieving a goal in which the solution method is not immediately obvious to the solver” (p.47). (Carson, 2007) cites Krulik & Rudnick (1980)) who define problem solving as the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The definition provided by Callejo & Vila (2009), cited in Nieuwoudt (2015), defines problem-solving as “a mathematical situation that poses a mathematical question to which the solution is not immediately accessible to the solver” (p.2).

What is common in definitions by Callejo & Vila and by Mayer and Wittrock is that the solution should not be immediately known to the solver. This is what makes problem solving both important and interesting. The mystery of the solution is what motivates good problem solvers to continue to look for a solution to a problem. The definition by Krulik and Rudnick, on the other hand, encapsulates what problem solving in mathematics should be about. Prior knowledge is key to problem solving and the use of previously acquired knowledge, skills and understanding to unfamiliar situations means positive transfer of knowledge. Silver (1981) takes a similar position to Krulik and Rudnick in arguing that “good problem solvers are able to recall structural features of the problem that they had previously solved and are able to use the knowledge they acquired solving that specific problem to solve a new problem” (p.54).

These definitions suggest that there are no set or prescribed algorithms to be followed to get to a solution. This immediately rules out the use of any kind of formula, which my students often expect to get whenever a question is posed, which they follow like a recipe, often without understanding the underlying mathematical concepts. Hence the first intervention focused on the meaning made by students when engaging with problems.

2.2 Why teach mathematical problem-solving?

2.2.1 Policy directives

The Curriculum and Assessment Policy Statement (CAPS) for the Senior Phase (SP) Grade 7-9 stipulates that one of the specific skills required to be developed in students is that of being able "to pose and solve problems" (Department of Education, 2011). CAPS for Further Education and Training (FET) Grades 10-12 also supports the teaching of mathematical problem solving. One of the specific aims of the policy is "to develop problem-solving and cognitive skills" (Department of Education 2011, p.8). It further argues that teaching should not be limited to "how" but should rather feature the "when" and "why" of problem types" (ibid, p.8). Hence, teaching problem solving is important across both SP and FET phases.

However, as a practising teacher, my experience and what I have seen happening with teachers in my school, leads me to think that for many different reasons problem-solving questions are not adequately dealt with when teaching mathematics. Among these, is the challenging nature of problem-solving questions and the time-consuming nature of problem-solving questions. Teachers fear that they might end up not finishing the syllabus if they spend too much time on problem solving questions. Although the curriculum advocates for problem solving, there is an issue that students need to have completed the syllabus for them to be ready for their examinations. So I try to finish the syllabus. Therefore, problem solving is given less teaching time despite it being an important part of the curriculum. Students often encounter problems solving questions for the first time in examinations or tests.

2.2.2. Teaching mathematical problem-solving benefits

Teaching problem solving has many benefits for students. For example Pehkonen (1997) sets out four categories on the benefit of teaching problem solving, arguing that:

(1) " Problem solving develops general cognitive skills.

(2) Problem solving fosters creativity.

(3) Problem solving is a part of mathematical application process.

(4) Problem solving motivates pupils to learn mathematics.” (p. 64).

Regarding (1), mathematical problems involve higher order thinking where students are ‘forced’ to think about the problem and their actions as they go about solving the problem. Therefore, when one is involved in more problem solving activities, one’s cognitive skills develops and consequently, one gets better at problem solving.

Sometimes, the answer to a mathematics problem is not immediately known to the solver when solving mathematics problems. Sometimes one’s strategies and approaches in problem solving fail. That is when one’s creativity becomes a useful tool in solving the problem. Problem solving offers the opportunity for one to be creative.

Application of acquired knowledge in mathematics is one of the most important processes towards the understanding of mathematics. Problems that are solved in mathematics are in context. Therefore, when one solves contextualized problems, one applies knowledge acquired previously and hence problem solving is referred to as part of mathematical application process.

Motivation is defined in multiples ways. However, Irvine (2015) defines it as “the student’s willingness or desire to engage in their learning” (p.109). Problem solving activity presents the opportunity to the students to engage in learning and to have a sense of fulfilment when they arrive at the solution. That enthusiasm and the quest to know what the solution will be to a problem becomes a motivation for students to continue to solve the problem. Wilson et al. (1993) argue that problem-solving can be fun especially because of the satisfaction when the solution has been arrived at and students get a sense of achievement. I can certainly attest to this notion of students getting a sense of achievement. I have seen in my class a boost in some students’ confidence when they feel that they have arrived at the solution of the problem. Even if they did not arrive at the solution, the fact that they were making progress made them feel confident about themselves and evokes the quest of wanting to know where they might have made a mistake which resulted in them not arriving at the solution. While I agree that problem solving can be fun, I have also seen the frustration in my students when they do not arrive at the solution. Some give up due to this frustration while others become even more enthusiastic about the problem, wanting to change the approach to a problem for a better one. I do think, however, that sometimes fun does not translate to good performance as far as my students are concerned.

Problem solving is not just about a set of exercises to be solved. As I argue below when I discuss a ‘drill and practice’ way of teaching mathematics, problem solving is about quality and not about quantity. As Wilson et al (1993) have argued “problem-solving forms a major part of mathematics, therefore to reduce it to a set of exercises and skills devoid of problem solving is misrepresenting mathematics as a discipline and short-changing the students” (ibid. p.66).

Teaching problem-solving in South African classrooms should prepare students for disciplines such as engineering or architecture where high level of thinking is involved. Complex mathematical calculations are often dealt with in fields such as engineering since they inculcate the culture of thinking and ability to solve problems.

Successful problem solvers generally use information and procedures that they have gained from previous experiences and training (Silver, 1981). Silver (1981) warns that prior experiences may, however, have a negative effect in new problem-solving situations and therefore, there is a need to focus on identifying situations that are conducive to positive transfer. Discussing a study conducted by (Krutetskii, 1976), Silver (1981) argues that according to Krutetskii (1976), those students who tend to recall the structural features of a problem could be regarded as good problem solvers whereas at the other end of the spectrum are those students who tend to recall, if anything, only the specific details of a problem statement. Furthermore, Jonassen (2004), agrees with Silver (1981) and argues that “classifying problem type is essential for students’ understanding and transfer of problem solving. Novice students tend to classify problems based on surface content rather than the relationships in principles which in turn results in errors” (p.21). Frank & Lester (1994), also in agreement with Silver (1981), argue that good problem solvers are aware of their strengths and weaknesses as problem solvers. Furthermore, Schoenfeld (2013), argues that “effective problem solvers monitor how well they are making progress and persevere or change direction accordingly when solving mathematical problems whereas on the other hand, unsuccessful problem solvers tend to choose a solution path quickly and then persevere at it even if they making little or no progress” (p.11). In this study, students from both intervention 1 and intervention 2 were given sufficient time to look for solutions to the problem, enough time to change their strategy if they felt that it was not leading to the solution to the problem. Some students did not need to change their strategies but the majority of them started and failed the first time. Only when they were doing the problem for the second time did some of them succeed, particularly students from intervention 1.

The success or failure of one's problem-solving attempts is based on four theoretical categories of problem solving (Schoenfeld, 2013), with the following categories as a guide of the analysis of someone's problem solving attempt

a) "The individual's knowledge
b) The individual's use of problem solving strategies, known as heuristic strategies
c) The individual's monitoring and self-regulation
d) The individual's belief system and their origins in the students' mathematical experiences" (p.11).

Regarding individual knowledge, one's success or failure could be determined mainly by one's knowledge. For example, if dealing with mathematical problem solving activity and one's knowledge is limited, one is likely not to succeed in that activity.

Polya's heuristic strategies and other strategies in dealing with mathematical problem solving activities (see for example, Krulik and Rudnick, 1980 or even Dewey, 1933) believe that with sufficient guidance and the right kind of assistance, students could succeed in problem solving.

Concerning the individual's monitoring and self-regulation, one's success or failure in problem solving could lie on how one monitors progress in problem solving and perseveres at it and change the approach accordingly if the progress made is not assisting in arriving at a solution.

How one comes to understand mathematics and how to solve mathematical problems could influence how one perceives mathematical problem solving. For example, if one was introduced to mathematical problems that are usually solved in less than 10 minutes, when faced with a problem that might take long to solve, one is likely to give up and could end up believing that that specific problem has no solution. However, if that student persevered, he or she might have arrived at the solution and perhaps have his/her belief about mathematical problem solving changed.

In summary, despite good problem solvers' awareness of their knowledge and beliefs, mathematics teachers also have a role to play in assisting students to become good mathematical problems solvers. This role ranges from how teachers introduce students to mathematical problem solving as well as the guidance and support offered to students when they solve problems.

2.3 How to teach mathematical problem solving?

As a practicing teacher, I have tried the 'drill and practice' way of teaching mathematics because I was taught this way during my school days. This is a method where a teacher starts by introducing a concept or topic, explains what it is about, provides definitions where necessary, shows an example and thereafter gives students examples to try on their own. Homework after the lesson is believed to allow students to practice what they had learned in class and help them retain it for a long time. My experience of paying my colleagues' class visits is that some of them are still using 'drill and practice'.

This traditional method of teaching in general has been criticized as ineffective in teaching students to be problem-solvers. For example, Buschman (2004) has argued that "teachers should encourage students to solve problems in ways that make sense to them instead of using traditional ways of solving mathematical problems" (p.304). When teaching using 'drill and practice, the focus is on quantity. Students do as many problem as they possible can, with a view that by doing many problems, their understanding will be developed. In so doing, there is danger of solving problems mindlessly or without understanding. Therefore, asking students to explain their solution or their approach to a problem enables them to reflect on their own understanding. During the teaching in intervention 1 students were asked to explain their solutions. They were asked questions like, "Would you have done the problem differently?" "What made you successful in solving the problem?" These questions probed for understanding and of making sense when solving problems. Intervention 2 did not include such discussions.

There is no one way of teaching or solving mathematics problems. Therefore, 'drill and practice' is likely to give students a perception that when solving mathematical problems, one has to follow a certain path. Problem solving in mathematics should be exploratory and students should have the opportunity to explore different ways of solving the problem. The quantity of the problems to solve should not compromise the quality of the problems. One could learn to be a good problem solver having solved just one ‘good’ mathematics problem.

Teaching problem solving is not the easy process that some of the models for problem solving seem to indicate. If teaching problem solving was that easy, students would have been able to solve mathematical problems. For example, as discussed earlier, Polya’s model of teaching problem-solving suggests these steps to be followed by students: “understand the problem, make a plan, carry out a plan, look back” (Polya, 1988). Even when students understand the problem, they still may not know how to do it. This means that more is required than providing steps to be followed in order for students to be able to solve mathematical problems. The heuristic nature of Polya’s model and others, can be interpreted as suggesting that problem solving is about following step-by-step algorithms or procedures which is not necessarily the case, hence such heuristic models have received heavy criticism. For example, Wilson et al. (1993), argued that such models have, among others, the following defects:

1. “They depict problem solving as linear process.
2. They present problem solving as a series of steps.
3. They imply that solving mathematics problems is a procedure to be memorised, practiced and habituated” (p.61).

Therefore, just to say students must understand the problem, make a plan, carry out a plan, look back is merely not good enough and therefore, Polya’s model fails to assist students to become good problem solvers.

Brown and Walter (1976) (cited in Silver, 1994) , came up with a framework that illustrates a dynamic and cyclic interpretation of Poyla’s model (see Figure 4). They argue that a student may start with a diagram in trying to understand the problem or may start with a plan and try to carry it out only to find that it does not work and then change it in trying to understand the problem. The illustration is shown in figure 4 below. This framework has been used in the University of Georgia for a course on problem solving for many years. It starts with problem posing followed by understanding the problem, making a plan, carrying out a plan and then looking back. The process starts all over again in a cyclic manner. The arrows indicate the direction each step could take in the process.

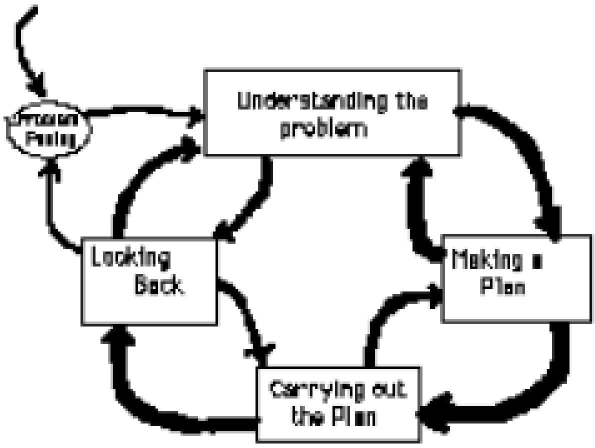


Figure 4: Brown and Walter's framework of problem posing, an illustration of Polya's problem solving model cited in Wilson et al (1993)

They go on to argue that any of the arrows in Figure 4 could describe the student's activity or thought process in mathematical problem solving. This illustration rules out the linearity of many of the models, among them, Polya's model. The framework by Brown and Walter was used during intervention 1.

Frank & Lester (1994) discussing five results that emerge when looking at the literature on problem-solving, have also argued that "teaching students about problem-solving strategies and heuristics and phases of problem solving such as Polya's four-phase problem-solving model does little to improve students' ability to solve mathematics problems in general" (p.666). They continue to argue that "teaching students to be more aware of their cognitions and better monitors of their problem-solving actions should take place in the context of learning specific mathematics concepts and techniques otherwise, general metacognition is likely to be less effective" (p.667).

Teaching problem solving can be a long, daunting, and at times, boring process. This is evident when I give my students a problem to solve: after 10-15 minutes of inability to arrive at the solution, they tend to just ask if I could tell them the answer so that they can continue to the next question. So it is of high significance that students remain engaged in the process. There are factors that support students' engagement in high-level tasks such as problem solving task. Henningsen & Stein (1997) have isolated some of the factors that support engagement of students in high-level tasks: "Tasks that builds on students' prior knowledge, scaffolding, appropriate amount of time, high-level performance, sustained pressure for explanation and meaning, student self-monitoring" (p. 534). These are the factors that are likely to improve success in mathematical problem-solving and were applied during the intervention which is discussed later in this paper.

2.4 How to assess mathematical problem solving?

A rubric is defined as a set of scoring guidelines for evaluating student's work (Montgomery, 2000). Montgomery continues to argue that assessing the students "work through the use of a rubric helps to clarify the critical learning that should be taking place and increase the likelihood that student will produce quality work". (p.325). According to Jonassen (2004), the rubric should be such that it should emphasise the aspects of the performance that are deemed most important. A rubric was used for the assessment of students' pre-test and post-test. Specifically, this rubric was designed to assess whether students understood the problem, were able to use some kind of a model to help solve the problem and finally whether the students were able to select information appropriate to solve the problem.

According to the Independent Examination Board (IEB) that set examinations for the independent schools, 15% of the assessment task should be on problem-solving. The Department of Education also requires 15% of the assessment task to be on problem solving. However, although in my school we strive to ensure that the IEB standards in terms of assessment are followed, and although only 15% of the assessment should be on problem solving and forms part of the whole assessment, 15% is far less than might be expected for a skill that is so important in mathematics. Problems that are meant to assess problem solving in my school are referred to as 'unseen' problems and when students come across these problems in the examination, they struggle and end up performing poorly in this problem solving section because of less attention given to this section during teaching time. As a teacher, it makes sense to focus more on the section where the bulk of the marks come from, consequently, less time is dedicated on problem solving.

This research involved the evaluation of problem solving by diagnosing the students' cognitive processes through evaluating the amount and type of help needed by an individual as well as small groups during problem solving activities, in the first intervention. How this will be done is discussed later when describing each intervention.

Assessment is an important part of learning. What needs to be assessed as well is equally important. Rubrics are one of the tools that play a critical role in assessment of open-ended problem-solving questions where critical thinking is involved. The allocation of 15% to problem solving could indicate a lack of recognition of the importance of problem solving in mathematics. Teaching problem solving needs time. The time spent on problem solving is not wasted since problem solving has many benefits, as discussed earlier in this chapter. One of the goals that should be sought is to produce good problem solvers that are able to think critically, select effective strategies and persevere in solving the problem.

The definitions of problem solving have one thing in common, that the path towards the solution to the problem is not immediately known to the problem solver. Due to the fact that the path is not immediately known to the solver, prior knowledge and exposure to problem solving questions that have been solved before play a significant role in the success of solving the problem.

There are many benefits of mathematical problem solving, among others, the development of cognitive skills of the problem solver and becoming an 'expert' or good problem solver due to a variety of problems that one has been exposed to as well as the ability to recall and use strategies used in solving previous problems .

Good problem solvers are aware of their strengths and weaknesses in mathematical problem solving. They know when they are making progress and when they are not and are able to change strategies (heuristics) accordingly.

3. Research Methodology

3.1 Theoretical framework

3.1.1 Socio-cultural theory

The intervention 1 is informed by a Vygotskian socio-cultural view of learning whereby the activity between students and the teacher leads the individual learning. The essence of this is the use of the Zone of Proximal development (ZPD) to help students through the process of problem-solving. ZPD is defined as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978). Daniels (2009) argues that "in a problem involving scientific concepts (defined as concepts introduced by a teacher in school), a student in collaboration with the teacher should be able to do what he has never done on his own". Hence the intervention 1 involves me, the teacher collaborating with students and students collaborating with their peers to help develop understanding in problem-solving.

Stein et al. (2008) came up with a model that deals with five practices that facilitate mathematical discussion around cognitively demanding tasks. The intervention will be based more on this model. These five practices are discussed in detail under intervention.

3.1.2 My constructivist position

It is a worrisome reality that my students find it difficult to deal with mathematical problem-solving questions and consequently perform poorly in tasks that involve mathematical problem-solving. However, I do believe that this reality is constructed by a number of factors. Among other factors, may be the kind of schooling that students are receiving as well as the kinds of mathematical problem-solving strategies that have been implemented and that they have or have not been exposed to prior to grade 8. It is befitting to mention that I am the researcher and the participant trying to construct understanding of whether the approach taken in the intervention 1 shows potential for improving learning outcomes. As I proceeded to look for the answer to this research question through this case study I was particularly interested in finding out whether the teaching that I provided to students using Stein et al.'s problem-solving strategy has had an impact or not.

3.2 Research design

Case study is a "systematic and in-depth study of one specific case in its context" (Bertram & Christiansen, 2014, p.42); (Opie, 2004, p.74). This case study involved two teaching interventions with two grade 8 classes, one with 18 students and the other with 19 students. One class (intervention 1) received an intervention that comprised a series of problems and a particular style of teaching (see below) and the other class (intervention 2) got the tasks only. Both classes wrote a pre-test and post-test which were carried out with the view to try and understand if the approach to teaching mathematical problem-solving with a particular style of teaching had a greater impact on students' performance compared to the intervention that only involved tasks. This design was used because it makes it easier to track if there is any shift in strategies used by students from pre-test to post-test. Dlamini et al. (2014) have argued that this kind of design allows the researcher "to track shifts with regard to previous (pre-test) strategies used to solve mathematical problems to the kind of strategies that students will be able to produce after participating in the intervention lessons" (p.7).

This case study also involves quantitative data in that it involved analysis of pre-test and post-test performance before and after intervention on mathematical problem solving.

3.2.1 Participants

The study took place in a private college in a Northern suburb of Johannesburg in South Africa. The College is divided into three schools. Junior preparatory (Grade 0-3), Senior Preparatory (Grade 4-7) and High School (Grade 8-12). There are four Grade 8 classes. I (the researcher) teach two of those Grade 8 classes and chose those two classes to be the focus of the study. This was convenient for me as a researcher as I already had time allocated in the timetable to see and teach these two classes. It would have been an inconvenience for the other two teachers who teach the other two classes for me to conduct my research in their classes. Therefore, the two Grade 8 classes involved in this study were chosen as a convenience sample. The poor performance from students in problem solving in mathematics might emanate from the

Foundation phase and manifests itself throughout Senior phase. In other words, the damage might have already been done when the students get to Grade 8. However, this does not mean that one should stop trying to rectify the situation. Therefore, grade 8 is also an appropriate grade to try and remedy the situation since the students still have 5 years of schooling left at this point.

Two classes, a total of 37 Grade 8 students, were involved. One class (19 students) received intervention 2 comprising of tasks only, wrote the pre-test and post-test (the repeat of pre-test) and the other (18 students) received intervention 1 comprising of tasks and teaching in a particular style, wrote pre-test and post-test after the lessons which were given by me (the researcher). The parents of the students involved signed the consent form, which gave permission for students to take part in this study. Students were also informed that participation in this study was voluntary and should they wish not to continue taking part in this study they could withdraw at any point. Fortunately, no student pulled out of this study throughout the research process. Permission was also granted by the college in which this study is taking place.

4. Data collection

4.1 Lessons

4.1.1 The intervention 1 class

The teaching took place during school hours. Specific lessons on the timetable were used for this purpose and were carefully chosen so that these lessons were at the times when students could still function at the optimum level. The lessons took place twice a week for three consecutive weeks and were each one-hour long. Although an hour was not long enough to solve many problems, it was the maximum amount of time the school timetable could accommodate. The lessons covered problem solving in three areas that were examined in the pre and post-test. The areas were, Numbers, Ratios and Algebra. These problems examined the same content in the pre-test and post-test but the context for the questions was different.

Students from the intervention 1 class would report for a mathematics lesson as normal on the day and already knew that they would be solving mathematics problems as it had been agreed upon prior to the teaching taking place. They sat in pairs and waited for the problem to be given. When the problem was projected on a screen via the data projector, students had, at most, 5 minutes to think about the problem and try to unpack what the problem was about and what would be required in order to successfully solve the problem. Some students took longer than others in this phase due to mathematical arguments that started as soon as the problem was projected. After the brainstorming session, students began to solve the problem.

Students were introduced to the‘bar model’, a way of representing problems in terms of bars. This way of representing a problem was meant to help show students how numbers or variables are related in a problem. For example, one student represented the light question as shown in figure 5 :

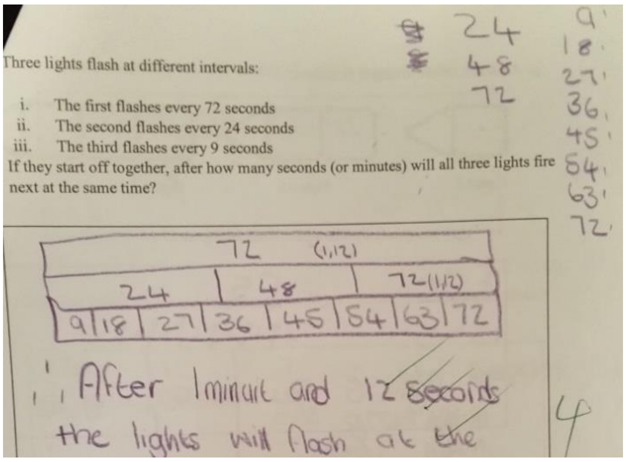


Figure 5: Student 1's representation of a light question using 'bar model'

The process of solving the problem would begin with students trying to model the situation. At this stage, some of the models that emerged did not make sense, however, students would modify these models until they began to make sense in terms of the situation. Often the process of solving the problem took about 20 to 25 minutes.

The process of solving the problem was subsequently followed by a report back where specific student pairs carefully chosen by me (the researcher) would be asked to put their solutions up on the whiteboard. They would then explain their reasoning. The reason for choosing specific pairs was to expose other students to the different ways that these pairs approached a problem. The decision about whose solution was to be written on the board was based on how much others could potentially learn from the approach taken. The methods used to solve the problem, which usually differed from pair to pair, offered the opportunity for others to learn.

A whole class discussion followed after the student pairs had reported back where a comparison of methods used took place. Students tried to answer questions such as: What is common between the methods used? What is different? Which method seems better than the other? And lastly, what could be learned from solving this specific type of mathematical problem? This process would take about 20-25 minutes.

The process was repeated for the second problem similar to the one previously dealt with. The process was quicker the second time around since students already knew the method they had to follow. This happened in all six lessons over the 3-week period with different types of problems.

During the intervention I closely monitored the progress students were making. For example, according to Schoenfeld (2013), "unsuccessful problem solvers tend to choose a solution path quickly and then persevere at it even if they are making little or no progress" (p.11). This means that unsuccessful problem solvers do not take time to attempt to understand the problem and select the best possible strategy in solving the problem. During this study, the reason for students taking some 3 to 5 minutes to discuss the problem in pairs was to ensure that before they started solving the problem they would have had a chance to think about the problem and share some ideas around the problem. During this time students were allowed to jot down notes of important information given about the problem.

When students had chosen a strategy to use to solve a problem, I monitored whether the strategy chosen was leading to the solution or not. Where the strategy chosen was not leading to the solution, I asked students to walk around to see how other groups / pairs were approaching the problem. This process took place before the whole class discussion / report back. Steering the group(s) in the right direction was important so that students did not spend too much time making little or no progress. Therefore, I sometimes asked students if the strategy that they were using was the only one to use to solve the problem. In so doing I was trying to guide students to think of other strategies if the one they were using was not leading to the answer.

As a teacher my duty was to introduce the task to students and to answer any questions that students might have, for clarity. I then facilitated the process of solving the problem by engaging students more on mathematical conversations that emerged around the class. Some of the questions I asked when I visited the pairs were: Could that method work? Why would it not work? What would you need to make it work? I also steered the whole class discussion in the direction that would assist students in the discovery process.

This intervention incorporated Stein et al.'s (2008) five practices for facilitating mathematical discussions around cognitive demanding tasks. The five practices are: "1) anticipating likely students' responses to cognitively demanding mathematical tasks; 2) monitoring students' responses to the tasks during the explore phase; 3) selecting particular students to present their mathematical responses during the discuss-and-summarise phase; 4) purposefully sequencing the student responses that were to be displayed; and 5) helping the class make mathematical connections between different students' responses and the key ideas" (p.321). The schematic diagram for this model is shown below in figure 6.

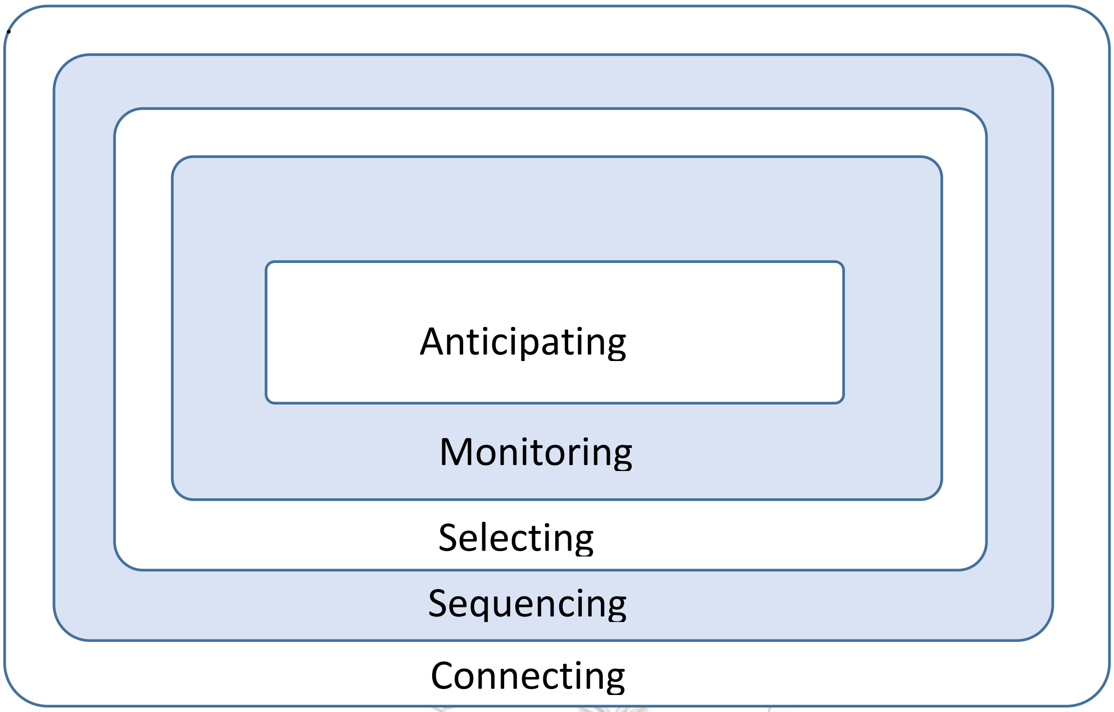


Figure 6: The schematic diagram of five practices for facilitating Mathematical discussions around cognitive demanding tasks.

Below is how each stage of the five practices relates to what was happening during the intervention.

Students were given a mathematics problem to solve in small groups or pairs. In this stage (the anticipating phase), learners are bouncing ideas off one another. Goos et al. (2002) have argued that a "ZPD could be created when peers of comparable expertise interact with one another" (p.193). At this stage the ZPD was likely to be developed when knowledgeable students interact with less knowledgeable peers.

While they were solving the problem in pairs or small groups, as the teacher I was walking around looking and listening to what students were doing and saying. This phase is called monitoring and Stein et al. (2008) have argued that the teacher in this phase needs to "pay attention to the mathematical thinking in which students engage as they work on a problem during the explore phase" (P. 326). During this process I provided guidance where necessary. Wilson et al. (1993) argue that "the amount and type of help needed can provide good insight into learners' problem solving abilities, as well as their ability to learn and apply new principles" (p.73).

After the process of solving the problem I asked one or two students to put their solutions up on the whiteboard in order for others to see what they had done in trying to solve the problem (Selecting phase). This process could also be referred to as whole class discussion. Stein et al. (2008) have argued that during this process the role of a teacher is "to develop and then build on the personal and collective sense-making of students rather than to simply sanction particular approaches as being correct or demonstrate procedures for solving predictable tasks" (p. 315).

The class reflected on their solutions to see if there was anything that they thought was useful, or whether one pair's solution was better than theirs (connecting phase). We then had the whole class discussion which took about 20-25 minutes. Wilson et al (1993) argue that through the use of this method "students were able to discuss and reflect on their approaches by tracing their joint work" (p.73).

A second problem-solving question, similar to the one that had been done, was given to students to attempt to solve where better strategies discovered from the previous problem solving process were applied and tested to see if they worked.

During the teaching students needed assistance from time to time either to clarify the question or use hints when they got stuck while solving the problem. This is the point where ZPD (the difference between what students could do on their own and what they could do with the help from the knowledgeable others) was developed. Students would do everything they could to solve the problem but when they got stuck they needed assistance from a knowledgeable being (me or peers) in order to progress further.

The lessons took the approach of teaching mentioned by Henningsen & Stein (1997) where factors that are perceived to support high-level students' engagement in 'doing mathematics' were applied.

During the intervention students were encouraged to use models to solve problems. Students were also introduced to a model called 'bar model' where students used bars to represent a problem.

According to Stephen & Mourat (2001), "representations refer to internal abstractions of mathematical ideas developed by a student through experience" (p.119). They continue to argue that representations such as tables, diagrams and graphs are external manifestations of mathematical concepts that act as stimuli and help in understanding of these concepts. The representations play a significant role in solving mathematical problems. For example, one of the process standards of the NCTM (2000) calls for all students to be able to select, apply, and translate among mathematical representations to solve problems. Hence, representations were encouraged during the teaching.

When the teaching was completed, both intervention 1 and intervention 2 classes wrote a post-test which was the same as the pre-test.

4.1.2 Intervention 2 class

The intervention 2 class, in contrast, only did the tasks. It had the same problems to solve except that they did not get to have a whole class discussion as the intervention 1 class did. Nor was the intervention 2 class taught to model the problem. In some cases they did not know if their solution was correct or not. They had to deal with the problem as if they encountered it in the test or examination. Another difference is that they did not have to work in pairs but some did voluntarily.

This class often asked, after solving the problem, whether their solution was correct or not. I often told them to compare their solutions with one another. Their excitement at the realization that their solutions were the same was an indication that they managed to get the correct solution. The input from me, the teacher, was however minimal.

The intervention 2 class also had the hour to solve two problems during the lesson. There was no whole class discussion as I specifically wanted this process to be as passive as possible or as close to how some of my colleagues at my school teach as it possibly could be.

The completion of lessons by both classes was followed by the post-test which was written during the same lesson time by both classes. The test was administered under strict test conditions. Students were not allowed to use a calculator. The process that subsequently followed was the marking of the test.

4.2. Test design

See appendix F for the complete test

According to Department of Education (2011), there are five content areas the Grade 8 syllabus should focus on. Among those content areas is; number operations and relationships as well as patterns, functions and algebra. In the number, operations and relationship content area, one of the focus areas involves "solving a variety of problems, using the increased range of numbers and the ability to perform multiple operations" (p.10). On the other hand, the foci of the pattern, functions and algebra as a content area includes "representation and description of situations in algebraic language, formulae, expressions, equations and graphs" (p.10).

Since this test was designed to assess Grade 8 content, it was appropriate to choose the content areas above as the areas of focus. Therefore, the questions in the pre/post-test involved the above mentioned content focus. The numbers at the end of each question represent the mark allocation. This was to make it easier to capture the data on excel and analyse it.

a) There are 4 red pens in Anna's desk drawer. There are 3 more black pens than red pens. There are also 7 more blue pens than red pens. How many pens are there in total? (Anna's desk drawer) (3)

b) At a sale, Mrs Math spent R6360 on a table, a chair and an iron. The chair cost R720 more than the iron. The table cost R960 more than the chair. How much did the chair cost? (Mrs Math) (4)

c) Mike is 180 cm tall and makes a shadow of 150cm. If Hamsa's shadow is 100 cm, how tall is she? (Hamsa's shadow) (2)

d) A stick of length a makes a shadow of length b. At the same time a tree has a shadow of length c. How tall is the tree in terms of a, b and c. (Stick length).

(4)

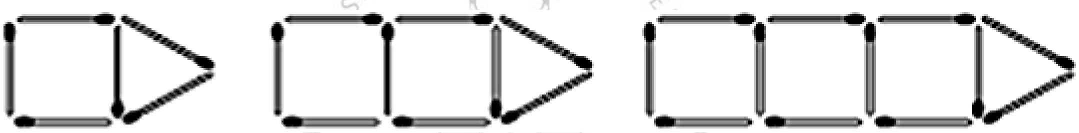
e) Three lights flash at different intervals:

i. The first flashes every 72 seconds
ii. The second flashes every 24 seconds
iii. The third flashes every 9 seconds

If they start off together, after how many seconds (or minutes) will
(The light question) (4)

f) A unit fraction has 1 as the numerator and a denominator greater than 1. The ancient Egyptians used only unit fractions. For $\frac{5}{8}$ they wrote the sum as $\frac{1}{2}+\frac{1}{8}$. Give two ways the Egyptians could write $\frac{10}{21}$ as a sum of unit fractions?
(unit fraction) (4)

g) Matchsticks are arranged as shown: (match stick)



Pattern 1 Pattern 2 Pattern 3

i. How many matchsticks are required for pattern 5? (2)

ii. Which pattern would need exactly 51 matches? Explain how you got your answer. (2)

iii. Give an expression for the number of matches required for the n^{th} pattern. (2)

h) Helen has 24 red apples and 12 green apples. What fraction of her apples are green?
(Helen's question) (2)

i) In a theatre between each row the number of chairs in a row increases by a constant amount. That is, the increase from row 1 to row 2 is the same as the increase from row 2 to row 3 and so on. There are 23 chairs in row 1. Row 10 has 50 chairs and the last row has 353 chairs. How many rows are there in the auditorium? (Auditorium) (3)

j) Molly likes to call her friend every Thursday night. Sometimes they have a lot to talk about, other times not so much. There are two phone companies Molly can use: TCC Cellular charges R0.10 per minute. UFC Network charges R0.50 for the first minute plus R0.08 for every minute thereafter. If Molly uses TCC Cellular for 8 minutes:

i) How much will her charge be? (2)

ii) Using your answer from i), how much time could Molly have spoken for using the UFC network? (Molly's question) (2)

The pre-test was exactly the same as post-test and designed to cover mathematical problem solving questions in three topics covered in Grade 8, namely numbers, algebra and ratios. Number topics specifically covered number relationships. Some of the examples of the items in the pre-test are provided below.

Some of the questions came from the textbook by Cai & Knuth (2011), Early algebraization. Some of the questions came from my own teaching. I had hoped that the questions were going to reveal where my students were in terms of different mathematical sections that were being assessed in a form of problem solving which, to a certain extent they did.

Examples:

Numbers

An example below required students to remember multiples of a number.

(The light question) (**Algebra)**

Students were expected to introduce a variable to represent the relationship in the prices of the items bought and equate the prices to the total amount paid by Mrs Math.

Mrs Math question (**Ratios)**

The first ratio problem given to students was to ascertain whether students were able to work with problem solving questions involving ratios and the second one that involved variables attempted to elicit whether students' knowledge was transferable to an unfamiliar situation.

For example, Hamsa's question and the stick length question.

4.3. Rationale for questions in the pre/post-test

There is a strong belief in algebra education that problem-solving contexts are foundational to algebraic activity (Cai & Knuth, 2011). Anna's desk drawer's question and the Mrs Math question were there for the students to think representationally about the relations in the problem. One of the ways in which students could solve this problem was by drawing a sketch representing a situation. The focus here was to have a pictorial representation that could assist students in visualization which is, to a certain extent, one of the skills that is difficult to acquire. The second way might be to start introducing variables for the unknowns and start looking at the relations between the variables.

Hamsa's shadow question and stick length question are looking at integrating algebraic thinking through ratios and proportion. Hamsa's shadow question, required students to find Hamsa's height through ratios if her shadow is 100cm. Students may have introduced a variable for Hamsa's height since it is an unknown and set up an equation that they could solve. However, the stick length question required students to generalise ratios using variables. This question also introduced students to algebraic expression of rules through ratios.

The question about the matchsticks (question g) was specifically looking at Algebraic generalisation involving figural patterns. Cai & Knuth (2011) argue that this kind of algebraic generalisation could be classified as constructive generalisation since it involves "direct or closed polynomial formulas that learners construct from known stages in a figural pattern as a result of cognitively perceiving figures that structurally consists of non-overlapping constituent gestalts or parts" (p.330). In this question students are repeatedly adding three matchsticks to form an extra square. This yields a formula of linear form

$y = mx + c.$

The light question was getting students to think about multiples of a number in an unfamiliar context. The lights flashing might be a simple context for some students, but to find out that the question was about multiples may not have been that simple. I anticipated that those who did not find out that the question was about multiples, might draw some kind of a diagram that was going to help them solve the problem.

Fractions is one of the concepts with which my students struggle. The unit fraction question was about fractions, specifically Egyptian unit fractions. Students had to attempt to write the given fraction in two different ways Egyptians would write this fraction. Helen's question was also developing the fraction concept.

The auditorium question (question *i*), assessed numbers that form patterns through sequences but students may have not thought about it in terms of sequences. However, students would still be able to use some kind of a diagram where they would have tried to work out what was the constant number that was repeatedly added to a previous row until the 10^{th} and last row, to be able to find the number of rows in the theatre, which was 111.

Question *j* (Molly's question), involved equations and costs through a familiar context where one uses a phone to call using different networks. Some students who could not derive the equation to calculate the cost could have rather used other ways of solving this problem, such as partitioning.

4.4 Trustworthiness of the study

4.4.1. Validity in data collection

4.4.1.1 Construct validity

According to Bertram & Christiansen (2014), validity refers to how believable or sound or justifiable the study is. In this study, the pre-test and post-test on problem solving administered to Grade 8 students in trying to ascertain whether the intervention focused on problem solving has potential to improve the outcomes. The statistical significance of the study means that the improvement in the results from pre-test to post-test could be due to teaching, although 'chance' cannot be ruled out as one of the possibilities. However, other factors such as the time of the day in which the tests were written could have had a negative impact on the validity of the study. It is easy for one to think that the study indeed measured what it was meant to measure meanwhile it ended up measuring something else, in this case whether students were good in guessing or not. On the other hand, problem solving is not like 'true or false' questions and therefore it would not have been possible to guess answers.

4.4.1.2 Reliability

Reliability refers to the extent to which the instrument can be repeated with the same or similar group and still produce the same results (Bertram & Christiansen, 2014). The results of the students from intervention 1 class improved after writing the pre-test and after the teaching. Since the intervention 1 class was taught, it is possible for any student to produce the similar result if he or she had to take the test again since they had been taught. However, the intervention 2 class, although their t-test showed statistical significance, I doubt that the students from intervention 2 class will be able to produce the similar results when taking a test again as they were not taught and their results of pre and post-test vary greatly thereby suggesting inconsistency. Therefore, the test, to a certain extent, is reliable perhaps not with the intervention 2 class. To test the reliability of the intervention 2 class' results would mean giving them the test again in the beginning of the following year when they are in Grade 9.

4.4.2. Validity in data analysis

The two classes that were involved in the study were taught by me (the researcher). As much as I tried to distance myself from the research, there was always the possibility of being subjective. This subjectivity ranges from test marking to analysis of data. For example, during test marking, I was always subjective since I knew the students and had an idea of their abilities. This could have been avoided had I given the scripts to a teacher from a different school to mark. Unfortunately, I could not find one available to assist. Furthermore, when interpreting the marks from the scripts, there was always the possibility of researcher bias. Therefore, validity could have been compromised in that regard.

4.5. ETHICAL CONSIDERATIONS

The University of the Witwatersrand ethics committee had to give permission to conduct this study following my application and the permission was granted. I also needed apply for permission from the College in which the study was taking place, and the College approved. Unfortunately, I cannot include the permission letter since it is on a school letterhead and I would like the school to remain anonymous. The indemnity forms were signed by both the parents of the students involved in the study and the students themselves. The consent form stipulated that students may withdraw from the study at any point should they wish to do so.

Examples of consent forms for parents and students are provided in appendix G and appendix H.

4.6 Data analysis

4.6.1 Test administration and marking

Both pre-test and post-test were written in class during the time for lessons with the two classes that were chosen to be part of this research. These tests were written in the presence of me (the researcher) in order to provide the students with clarity when required. Unfortunately, the two classes did not write all at the same time. The lessons in which the two classes wrote the test were consecutive. Therefore, students' time to interact and discuss the test was minimal. Although the influence of one class by the other would have been possible, it would not have been to a great extent.

Both tests were marked by me (the researcher) which enabled me to immediately find out the areas of focus when giving lessons. The marks attained by students for these tests were recorded on an excel spreadsheet and the scripts kept for further qualitative analysis.

Three types of scoring were used. There is a scoring out of 36, scoring using the rubric as well as coding the answers in terms of correct, incorrect, partially correct or omitted.

4.6.2 Scoring out of 36

This is the maximum possible mark that a student could attain both in a pre-test or post-test which is discussed further below.

4.6.3. Scoring using rubric

The rubric was also used to score the students' work. The rubric had three process dimensions to assess: understanding the problem, using appropriate information and representation and solving the problem and were listed vertically. Horizontally is scale from 1 to 4. There is further discussion on the rubric in 4.6.7.

4.6.4 Scoring out of 13

The scoring out of 13 arose from overall answers being coded as correct, partially correct, incorrect or omitted. This is also discussed in detail below under coding.

4.6.5 Raw marks

Table 2: Pre-test averages per question for the intervention 1 class

Pre-test (intervention 1)														
	Question number													
	j1	a	g1	j2	h	g2	g3	e	c	b	f	i	d	
Student no.	2	3	2	2	2	2	2	4	2	4	4	3	4	36
WXH020	2	3	2	0	2	2	2	4	2	4	2	3	0	28
WId015	2	3	2	1	2	0	2	0	2	0	2	3	0	19
WPM026	2	3	2	2	2	2	0	0	0	4	2	0	0	19
WEB003	2	3	2	2	2	2	0	4	0	0	0	0	0	17
WJB008	2	3	2	2	2	2	0	4	0	0	0	0	0	17
WSC011	2	3	2	1	2	2	1	4	0	0	0	0	0	17
WOv040	2	3	2	2	0	2	1	2	2	0	0	0	0	16
WBB006	2	3	2	1	2	2	2	0	0	0	0	0	0	14
WJR035	2	3	2	1	2	0	1	0	0	0	2	0	0	13
WDB010	2	3	0	2	0	0	1	2	0	0	2	0	0	12
WTD013	2	3	2	1	0	0	2	0	0	2	0	0	0	12
WAM030	2	3	2	2	2	0	1	0	0	0	0	0	0	12
WBS038	2	3	2	0	0	0	0	4	0	0	0	0	0	11
WJG017	2	1	2	2	2	0	0	1	0	0	0	0	0	10
WJF016	2	3	1	2	0	0	0	0	0	0	0	0	0	8
WGK022	2	3	2	1	0	0	0	0	0	0	0	0	0	8
WSM029	2	3	2	0	0	0	1	0	0	0	0	0	0	8
WMA001	2	2	2	1	0	0	0	0	0	0	0	0	0	7
Total for the class	36	51	33	23	20	14	14	25	6	10	10	6	0	14
% for each question	100	94	92	64	56	39	39	35	17	14	14	11	0	38

Table 2 is used as an example to discuss how scoring out of 36 looked after it had been captured on excel an spreadsheet. The table above is arranged vertically from the most successful student to the least successful one. Horizontally, it is arranged according to the question in which the class was most successful to the question in which the class was least successful.

4.6.5.1 Class performance.

The raw marks for each question were captured on an excel spreadsheet (see table 2 above as an example). The total and the percentage for each question were calculated. For example, question a) as seen above under test design, was out of 3 marks, therefore the maximum mark that the intervention 1 class could attain in this question is 3 x 18 = 54, since there were 18 students in the intervention 1 class. Similarly for the intervention 2 class, the maximum mark for question a) would be 3 x 19 = 57, since there were 19 students in the intervention 2 class. Therefore, the percentage would be the total mark attained by the class as a whole in question a) out of the total possible mark multiplied by 100.

The percentages at the bottom in a row titled '% for each question' were used to decide whether a class as whole was successful in a question or not. If the class attained 50% or higher in a question, it was considered successful in that question and if the class attained less than 50%, the class was considered unsuccessful in that question. Although individually some students may have done well in that specific question these averages were looking at the class as a whole.

The column on the right gives the overall average for the class which is the value in blue. This single value is representative of the whole class performance.

The class performance can also be discussed in terms of the coding discussed below. For example, looking at the number of correct answers at the bottom of Table 5, one can tell if the class performance on an item was above 50% or not. If there are many answers coded 3 that means the class performance was 50% or higher in that specific item.

4.6.5.2 Item performance

The averages at the bottom of the table in a row titled '% for each question' were also used to discuss performance in each item. The success or failure in the item was determined by whether the class attained 50% or higher or less than 50% in each item.

The choice of 50% was taken as a single percentage value to which one could draw a conclusion about a success or failure in a question. For instance, in my school, the pass mark for mathematics is 50% and hence 50% was chosen as a cut-off value.

The item performance can also be analysed in terms of Table 5. If there are more answers coded 3 in an item, that means that the item was well answered.

4.6.5.3 Student performance

Table 2 also shows the captured marks per question for each student. The overall total of the mark attained by a student is recorded in the last column. This table can be used to look at the performance of each student on each item.

The student performance is also examined in terms of the coding discussed below. For example, the student with most correct answers would be regarded as the student that performed well in the pre/post-test.

4.6.6 Coding

Table 8: Post-test coded answers per question for the intervention 1 class

	Intervention 1 (Post-test)													
					Question no.									
Student no.	j1	e	g1	a	g2	h	g3	j2	c	i	d	b	f	
WXH020	3	3	3	3	3	3	3	1	3	3	3	2	2	**10**
WJB008	3	3	3	3	1	3	3	3	3	1	1	1	1	**8**
WSC011	3	3	3	3	3	3	3	3	1	1	1	1	2	**8**
WOv040	3	3	3	2	3	1	3	3	3	1	3	1	2	**8**
WEB003	3	3	3	2	3	3	3	3	1	1	1	1	1	**7**
WBB006	3	3	3	3	3	1	3	1	1	3	2	1	1	**7**
WDB010	3	3	3	3	1	1	3	3	1	3	1	1	1	**7**
WId015	3	1	3	3	3	3	3	1	3	1	2	1	2	**7**
WPM026	3	3	3	3	3	3	3	1	1	0	0	1	2	**7**
WMA001	3	3	3	3	3	3	2	0	0	0	0	1	0	**6**
WJG017	3	3	3	1	3	3	2	3	1	0	0	1	2	**6**
WTD013	3	3	1	3	0	3	0	1	3	1	1	1	2	**5**
WJF016	3	3	3	3	3	1	1	1	1	0	0	1	0	**5**
WSM029	3	1	3	3	1	1	3	3	1	0	0	1	0	**5**
WJR035	3	3	1	3	3	3	2	1	1	1	1	1	1	**5**
WBS038	3	3	3	2	3	1	3	2	1	0	2	1	1	**5**
WGK022	1	3	3	3	1	3	2	1	0	1	0	1	0	**4**
WAM030	3	3	1	3	0	3	0	1	1	0	0	1	2	**4**
	17	**16**	**15**	**14**	**12**	**12**	**11**	**7**	**5**	**3**	**2**	**0**	**0**	

Table 8 above is used as an example to show how coding of the answers was captured on the excel spreadsheet. It is ordered vertically according to the students with the most correct answers to the students with the least. Horizontally, the table is ordered according to the question students were most successful at to the question in which they were least successful. The right-hand column is a summary of the number of correct questions the students attained.

The scripts of the students were marked and the following coding was used to code the answers: 0 was used if the question was omitted, 1 was used for incorrect answer, (that is the student scored no mark in the question), 2 was used for partially correct. (meansing that the student would have attained some marks in the question), lastly, 3 was for correct (that is a student would have scored full marks in the question). This coding resulted in a mark out of 13 since there were 13 items in the test. It is worth emphasizing that these are merely codes. These codes are not the marks and therefore need to be read as such.

This coding enabled me to note which question(s) students could cope with in the test and which questions were omitted. Therefore, this coding was used to refer to student's performance in the items.

4.6.7 Rubric

A rubric is defined as "scoring guidelines for evaluating students' work" (Montgomery, 2000). According to Mertler (2001), there are two types of rubrics, holistic rubric and analytic rubrics. Depending on the nature of the task, one can decide on the type of rubric to use. "Holistic rubrics are mainly used when the focus of the score reported is an overall proficiency or understanding of a specific content and or skills" (Mertler, 2001, p.2). On the other hand, "an analytic rubric is used when a fairly focused response is sought" (Mertler, 2001). The analytic rubric tends to focus on a task where there may be one or two acceptable solutions and creativity is not a key feature of students' response. However, the scoring using analytic rubrics tends to take longer because since the teacher examines work several times. This ensures substantial feedback to students due to rigour.

An analytic rubric was used to assess students' tests (see Figure 7 the rubric) below.

Mathematical Problem solving scoring guide

Process Dimensions	**4**	**3**	**2**	**1**
Making sense of the problem Interpret the concepts of the task and translate them into Mathematics	The interpretation and /or translation of the task are • thoroughly developed and /or • enhanced through connections and /or extensions to other contexts	The interpretation and translation of task are • adequately developed and • adequately displayed	The interpretation and /or translation of the task are • partially developed, and/or • partially displayed	The interpretation and/or translation of the task are • underdeveloped, • sketchy, • using inappropriate concepts, • minimal, and/or • not evident
Using information appropriately Takes the given information and uses it in suitably in trying to find an answer	Shows an understanding of why certain information is essential in solving the problem	Uses all information correctly	Uses some appropriate information correctly	Uses inappropriate information
Representation and solving the task Use models , pictures, diagrams and or symbols to represent and solve the task situation and select the effective strategy to solve the problem	The strategy and representation used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy that has been selected and applied and the representations used are • partially effective and/or • partially complete.	The strategy selected and representations used are • underdeveloped, • sketchy, • not useful, • minimal, • not evident, and/or • in conflict with the solution/outcome.

Figure 7 : The rubric used to assess students' pre-test and post-test adapted from Oregon Department of Education mathssp scoring guide combined with a rubric from Northwest Regional Education Laboratory Mathematics Center.

The rubric shown in Figure 7 was an adaptation of the rubric from Oregon Department of Education mathssp scoring guide combined with a rubric from Northwest Regional Education Laboratory Mathematics Center. The rubric from Oregon Department of Education mathssp was chosen because it was a rubric that was simple to work with and was easy to adapt. It had five process dimensions, namely, making sense of the task, representation and solving the task, communication and reasoning, accuracy as well as reflecting and evaluation and six performance indicators. I removed three of the process dimensions, namely communicating and reasoning, accuracy as well as reflecting and evaluation. These were irrelevant for the kind of task I had given my students to do. I replaced them with one process dimension from the rubric from the Northwest Regional Education Laboratory Mathematics Center, that is using appropriate information to create a new rubric that was suitable for assessing students' answers. Since the task I had given students did not require them to reflect on a solution and their approaches to the task therefore, it was not necessary to assess them on the above mentioned process dimensions which is reflection and evaluation. Although accuracy as well as communicating form part of students' solution, no mark was allocated for these dimensions but as a marker I paid attention as I marked to how students were doing in terms of these two dimensions. I also scaled the performance indicators down to a scale from 1 to 4 instead of 1 to 6. This enabled me to be thorough with the marking and made the marks much more manageable instead of having large numbers.

4.6.8 Distinguishing Categories

4.6.8.1 Making sense of the problem

Making sense of the problem is closely related to the representation category. In this category students needed to take a question and translate it into mathematics. For example, the Anna's desk drawer question, introducing a variable to show how the number of different coloured pens relate to each other could show that a student is translating the concept of the task into mathematics and in terms of mathematical language. However, even if a student was not using variables, he still needed to show that he understood the concept of the task by not confusing the given information. For example, Mrs Math question, a student that said a table costs R960 and that a chair costs R720 clearly shows no understanding.

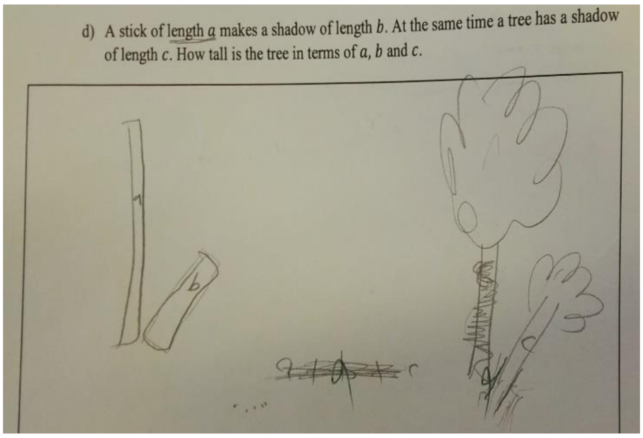
4.6.8.2 Using information appropriately

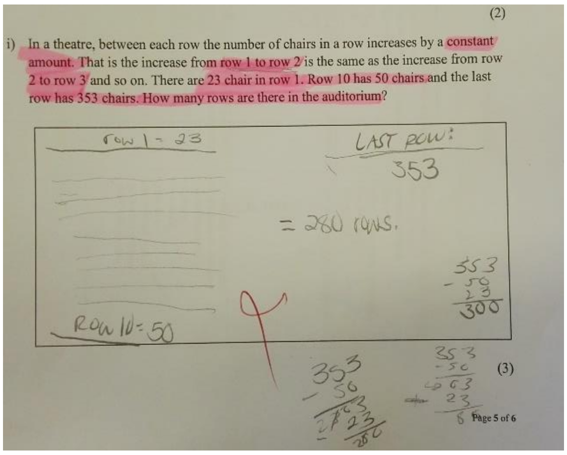
The given information is meant to be used to solve the problem. In this category, students needed to take the given information and use it appropriately in trying to solve the problem. I was particularly interested in whether a student showed understanding of working with a specific concept that a question was about. For example, in the Hamsa shadow question, a student that gave the answer of 130cm clearly did not understand the concept of ratios because he would have subtracted 50 cm. A student also needed to use all the given information to solve the problem.

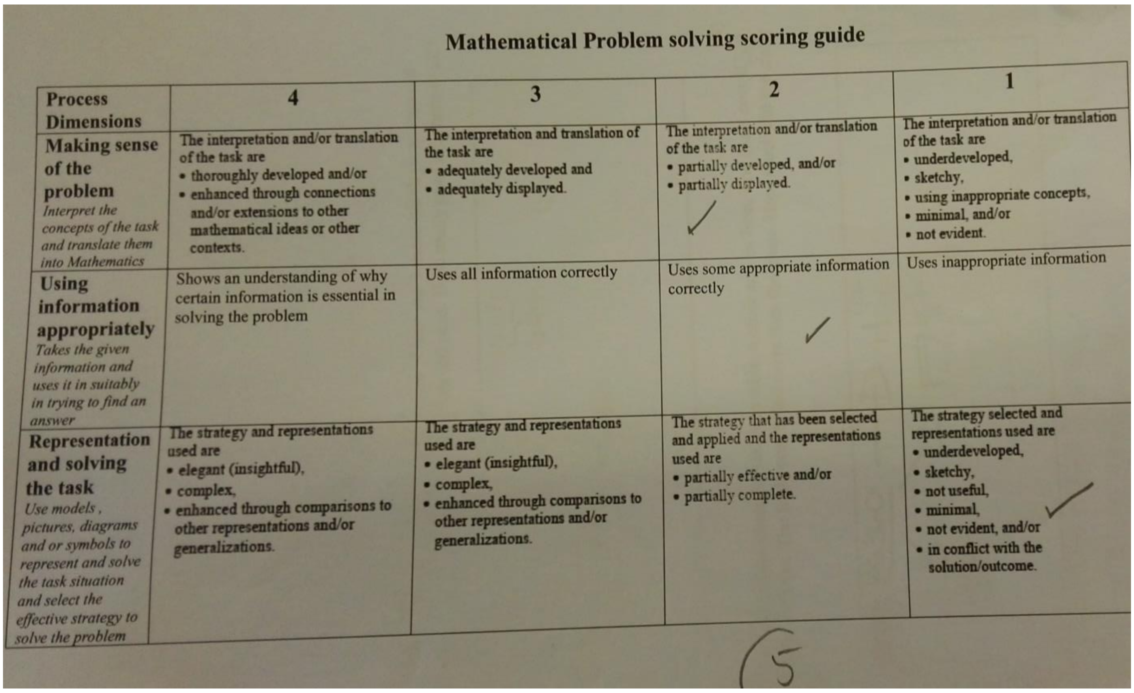
4.6.8.3 Representation and solving of the task

This category looks at the diagrams used and any form of model used to represent the situation. Over and above this, the model used had to be the model that could help the student solve the problem. For example, figure 8 below shows the model used by a student in the Hamsa shadow question. These models would have not helped the student solve the problem. So it is not just any model that was marked correct but the one that enabled a student to solve the problem. This category also looked at how well the model capture the given information.

I specifically wanted to assess whether the students understood the problem or the question. This was measured through investigating whether a student was able to translate or interpret the concept of the task into mathematics. I particularly looked at whether this was thoroughly developed or not. Below is some work from the student that was given a score of 5. Marked rubric is also provided below in figure 8.






Mathematical Problem solving scoring guide

Process Dimensions	4	3	2	1
Making sense of the problem *Interpret the concepts of the task and translate them into Mathematics*	The interpretation and/or translation of the task are • thoroughly developed and/or • enhanced through connections and/or extensions to other mathematical ideas or other contexts.	The interpretation and translation of the task are • adequately developed and • adequately displayed.	The interpretation and/or translation of the task are • partially developed, and/or • partially displayed. ✓	The interpretation and/or translation of the task are • underdeveloped, • sketchy, • using inappropriate concepts, • minimal, and/or • not evident.
Using information appropriately *Takes the given information and uses it in suitably in trying to find an answer*	Shows an understanding of why certain information is essential in solving the problem	Uses all information correctly	Uses some appropriate information correctly ✓	Uses inappropriate information
Representation and solving the task *Use models, pictures, diagrams and or symbols to represent and solve the task situation and select the effective strategy to solve the problem*	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy that has been selected and applied and the representations used are • partially effective and/or • partially complete.	The strategy selected and representations used are • underdeveloped, • sketchy, • not useful, • minimal, • not evident, and/or • in conflict with the solution/outcome. ✓

(5

Figure 8: Student 2 representation of stick length question and marked rubric

The dimension on the understanding of the problem was given a score of 2 because although the student did not get the above questions correct, there are questions that he managed to get right such as Anna's drawer question, Light question, Hamsa's shadow question and Helen's question. This meant that there is some kind of understanding and therefore this dimension was partially developed. The dimension on the appropriate use of information was given a score of 2 because even though the student was unable to solve the task, there is some evidence that through the sketches drawn the information given was going to be appropriately used to solve the problem. For example, the diagram drawn above about a stick length question shows that the student understood that the height of a tree is given as *a* and shadow as *b*. On the other hand, the second tree's shadow is given but not the length. Had the student known how the variables relate to one another, he would have easily solved this problem. Representation was given a score of 1 because the models were not useful since the student was unable to solve some of the problems with the aid of the models drawn.

The third process dimension was whether there was any form of representation or model used as an aid to solve the problem. I looked at whether the student was able to use models. This was measured by looking at strategies and representations used and how effective they were in assisting the student to arrive at the solution. Pictures, diagrams and or symbols can be an effective way to summarize and represent and solve the task. If selected effectively, students may be able to solve the problem much faster. This is what I noticed as I was offering intervention to my students. Those that were able to come up with a 'good' model, were successful in solving the problems. For example, in figure 9 is an example from the same student that used some models to work out solutions:

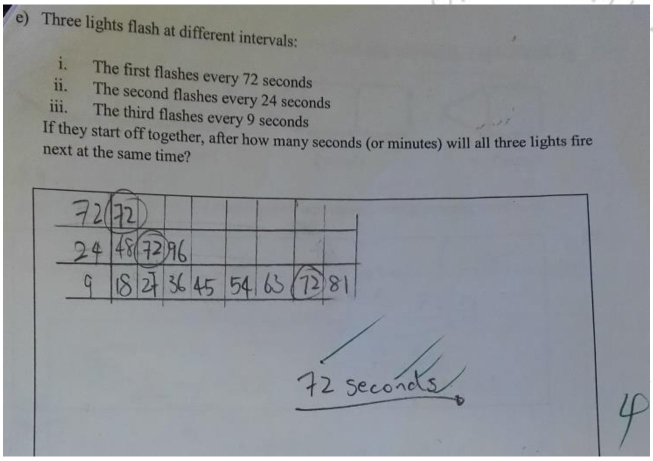
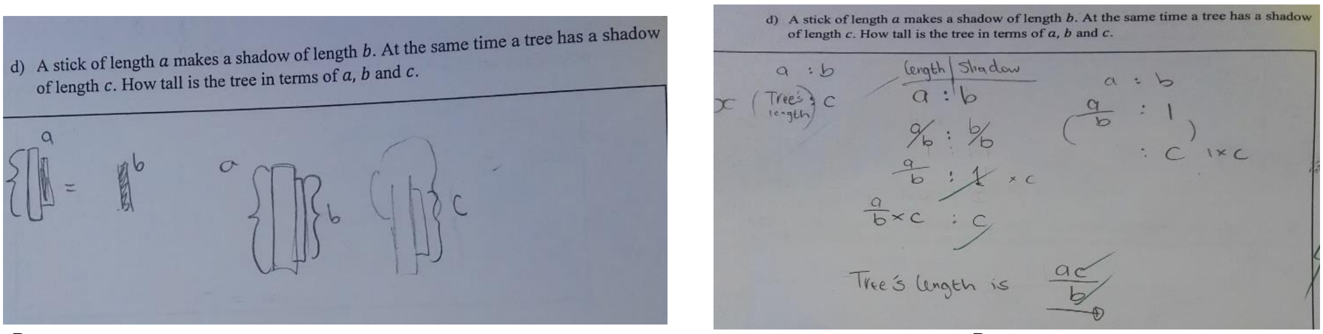


Figure 9: Student 3, solution of Light question

This model clearly shows the number of flashes of each light at the same time showing the multiples of the three lights.

There was also a big transition between pre and post tests in terms of coming up with these models from the model that seemed like it could not yield a positive result to a model that worked well.

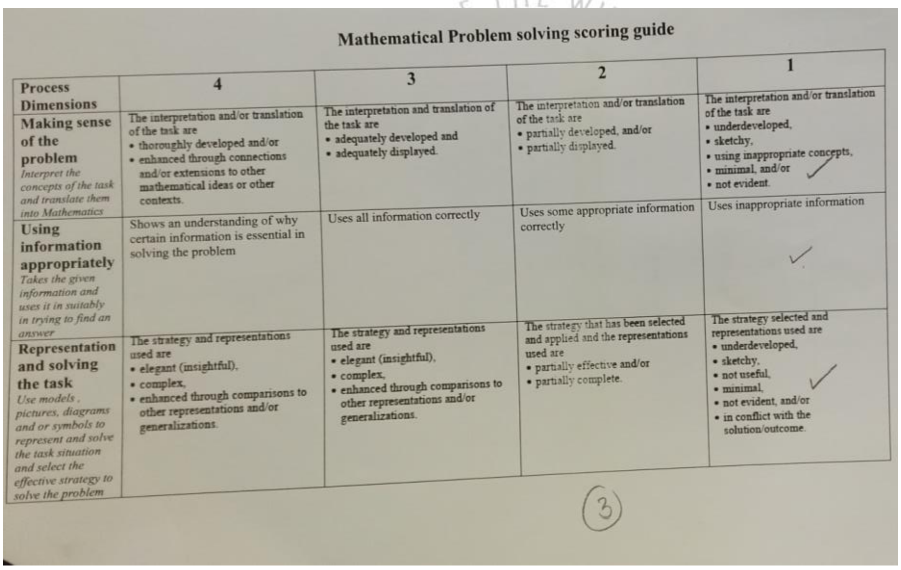


Pre-test Post-test

Figure 10: Student 4 pre and post-test solution of stick length question

Lastly, students were assessed on whether they were able to use given information appropriately. In this process dimension, students had to show why certain information was required in solving the problem. Therefore, taking given information and using it suitably in solving the problem was key.

For example, some of the work below comes from a student whose marked rubric is shown below in figure 11.



Mathematical Problem solving scoring guide

Process Dimensions	4	3	2	1
Making sense of the problem *Interpret the concepts of the task and translate them into Mathematics*	The interpretation and/or translation of the task are • thoroughly developed and/or • enhanced through connections and/or extensions to other mathematical ideas or other contexts.	The interpretation and translation of the task are • adequately developed and • adequately displayed.	The interpretation and/or translation of the task are • partially developed, and/or • partially displayed.	The interpretation and/or translation of the task are • underdeveloped, • sketchy, • using inappropriate concepts, • minimal, and/or • not evident.
Using information appropriately *Takes the given information and uses it in suitably in trying to find an answer*	Shows an understanding of why certain information is essential in solving the problem	Uses all information correctly	Uses some appropriate information correctly	Uses inappropriate information
Representation and solving the task *Use models, pictures, diagrams and or symbols to represent and solve the task situation and select the effective strategy to solve the problem*	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy that has been selected and applied and the representations used are • partially effective and/or • partially complete.	The strategy selected and representations used are • underdeveloped, • sketchy, • not useful, • minimal, • not evident, and/or • in conflict with the solution/outcome.

Figure 11: Student 5, marked rubric for student 5 work.

This student attained a score of 3 overall. Some of her work that was scored is shown below.

In terms of making sense of the problem, figure 12 below indicates lack of understanding from a student. The student, after working with the given information, came to the conclusion that the lights will flash at the same time again after 376 minutes which clearly shows lack of understanding. This also indicates that the information given was not used properly.

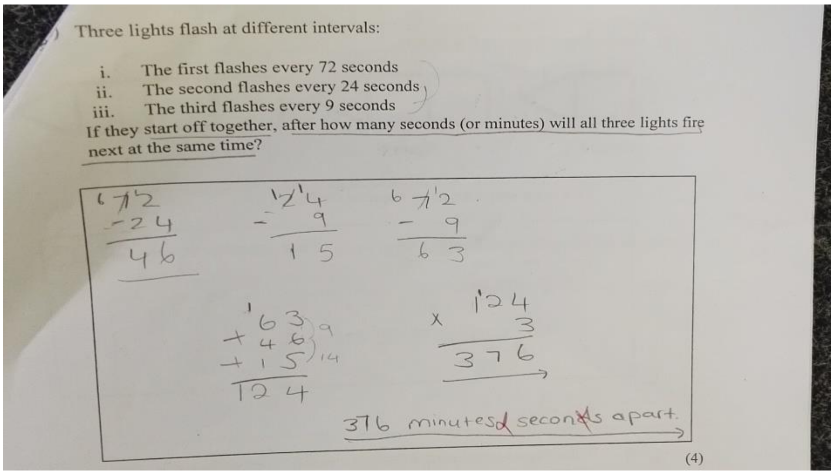


Figure 12: Student 6 solution of a light question

The next example in figure 13 also shows, not only lack of understanding and the use of appropriate information, but it also shows wrong strategy selection.

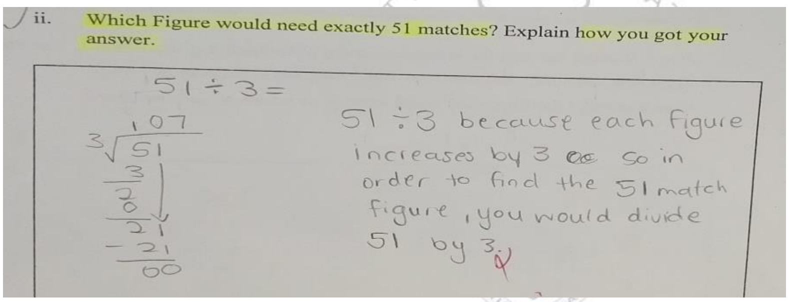


Figure 13: Student 7, solution of match sticks question

First of all, the student believes that 51 divided by 3 is equal to 107 as it is indicated as a quotient in figure 13. The explanation states that the figure that would require 51 matches according to her should be 17 because 51 divided by 3 is 17. However, that is not true as the figure that requires 51 matches is figure 16. Therefore, this was given a score of 1 as there was no score of 0 in the scale on the rubric.

There are other rubrics that could have been used such as a classic 5-level math rubric which is based on revised NTCM standards. This NCTM rubric had good performance indicators represented horizontally. Vertically, it had terms such as novice, practitioner apprentice and expert used. I would not like to use such terms to describe the work for my students or students themselves. These terms have the potential of demotivating students especially if a student is called a novice. Although, students were not going to get feedback on their scores using rubrics, if there were to demand to see their scores, I would be obliged to show them and their realization that they were called novice would not be pleasant. I therefore avoided these terms completely.

The focus of the assessment was on the overall gains in terms of the process dimensions in which students were assessed. The holistic rubric was limiting in terms of the scoring as it makes scoring quicker but the marking might not be rigorous and one is likely to miss some key information if less focus is paid to students' work. The analytic rubric was appropriate for this task because students could come up with different ways of solving the same problem and the rubric catered for that. If one is looking at whether students were able to devise other ways of arriving at the solution, that is creativity, the analytic rubric would allow that. Hence an analytic rubric was preferred for this specific pre-test and post-test. Therefore, it was appropriate to use an analytic rubric in a sense that in problem solving process there are many ways in which students could arrive at the desired solution. To give a quick scoring of students' work would not do justice to the assessment of what students could do when solving mathematics problems. It is argued that no rubric is better than the other, one has to choose the rubric appropriate for the purpose of the task. Although the feedback was not going to be given to the students in the end, I still needed to be thorough in the assessment of the task.

According to Polya's model and other models such as Stephen Krulik and Jesse Rudnick (1980), understanding the problem and coming up with a plan are amongst steps that are important and hence these are included in the process dimensions in the rubric used. One of the reasons why representation is important in problem solving is that it enables students to visualise and summarise information given about that specific problem. (Jonassen, 2004) argues that "students' construction of conceptual models indicates understanding of relationships between variables in that specific problem". Polya's model third step is to carry out a plan which ties with using a model to solve the problem which the intervention 1 class was encouraged to do.

The score of the whole class was recorded as a percentage. This score was interpreted holistically, looking at the performance of the class in a specific skill. The total possible mark that the intervention 1 class could attain in each skill was 72 which is (4 x 18 students). The maximum mark that a intervention 2 class could score in each skill assessed is 76 since there were 19 students in this class.

This form of assessing students' work enabled me to compare two classes, intervention 2 class and intervention 1 class in terms of which class might have done better in terms of three skills that were being assessed.

Exploratory statistics was used, specifically a t-test to investigate the likelihood that the improvement in averages of the pre-test and post-test was by chance. Therefore, after the pre-test and the post-test had been marked and the difference in the averages of the pre-test and the post-test had been established, the exploratory statistics was used.

4.6.9 The null hypothesis (H_o)

The H_o for the marking is that the approach taken in the teaching shows no potential for improving the learning outcomes.

4.6.10 The alternative hypothesis (H_a)

The H_a for the marking is that the approach taken in the teaching shows potential for improving the learning outcomes.

4.6.11 The alpha value

"The independent sample t-test compares the means of the two unrelated groups on the same continuous, dependent variable" (Lund & Lund, 2013, p.1). The alpha value is a single value that is representative of a number of t-test values. The alpha value for this research was taken as 0,05 and was used to compare two t-test values. One for the intervention 2 class and one for the intervention 1 class to ascertain if the H_o could be rejected or not.

4.6.12 Decision rule

If p > 0,05, the null hypothesis cannot be rejected. This means that the change in the marks of the students from pre-test to post-test is highly unlikely to have been caused by the teaching.

If p < 0,05, the null hypothesis can be rejected. This means that the change in the marks is highly likely to have been caused by the teaching provided to students.

4.6.13 Effect size (ES)

Effect size is simple a way of quantifying the size of the difference between two groups (Coe, 2002) .

ES = 0,2 means that there is small effect.

ES = 0,5 means there is medium effect

ES = 0,8 means there is large effect.

5. Findings

The first question that is going to be answered through the findings is, 1) How did the performance compare across the two classes? Then, 2) What do the findings reveal about the different topic areas? And finally 3) What do the findings reveal about the individual students? And then lastly, the research question, 4) Does the approach taken in the intervention 1, focused on discussing problem solving show potential for improving individual learning outcome in comparison to that of intervention 2 where only the problems were provided?

5.1 OVERALL SUCCESS

5.1.1 Whole class overall average

Table 1: Pre-test and post-test means, standard deviations for the marks for the intervention 1 class and intervention 2 class.

	Pre-test			**Post-test**				**T.test**	**Effect size**
	n	***M***	***SD***	***n***	***M***	***SD***	***MD***	***p***	
Intervention 1 class	18	38	34	18	48	33	10	0,0165	0,261
intervention 2 class	19	24	30	19	38	33	14	0,0037	0,4486

Notes: M = mean, SD = standard deviation, MD = difference between pre-test and post-test mean scores.

Table 1 above shows the number of students in the intervention 1 class as well as the intervention 2 class and the class level mean, standard deviation and the mean differences for pre-test and post-test. The t-test values as well as the effect size for both classes are also recorded.

As Table 1 shows, the intervention 1 class had a 10% difference improvement in average percentage from 38% pre-test average with a standard deviation of 34 to a mean of 48% post-test average with the standard deviation of 33. On the other hand, the intervention 2 class had a

14% difference in average percentage from 24% average in the pre-test with a standard deviation of 30 to 38% average in the post-test with a standard deviation of 33 which is a relatively greater improvement compared to that of the intervention 1 class.

A two tailed t-test was conducted comparing pre-test and post-test for both intervention 1 class and intervention 2 class (see table 1 above). As discussed under methodology, the p-value for this study is 0,05. This is the value to which the p-values for both intervention class and intervention 2 class were compared and a decision made about whether the difference in the mean marks is statistically significant or not is discussed below.

The intervention 1 class p-value of 0,0165 < 0,05 means that the mean mark is statistically significant. In contrast, the p-value of the control class of 0,0037 < 0,05 implies that the mean mark is also statistically significant. This means that the change in the mean mark of the two classes is highly likely to have been caused by some factors, for the intervention 1 class, it might be the teaching and for the intervention 2 class, it might be the way they were taught which was conventional in nature.

Interestingly, the standard deviation for both classes in the post-test is the same. This means that the marks deviate by 33% from the mean. In other words 66% of the students from the intervention 1 class had scores between 15% and 81%. Contrary to this, the 66% of the students from the intervention 2 class had scores between 5% and 71%. The huge spread in the marks for both classes means that some students scored high marks and some had low scores. The question that arises from this discrepancy in the improvement would then be, why did the intervention 2 class seem to have done better in terms of gains than the intervention 1 class?

5.2 Pre-test averages per question

5.2.1. Intervention 1 class

Table 2 below shows the pre-test results for the intervention 1 class. All the scores are for individual students who took both pre-test and post-test. The table shows the pre-test results for intervention 1 class ordered in two directions. Vertically, it is ordered from the student achieving the highest mark to the student achieving the lowest mark. Horizontally the table shows the test item in which the students were successful, to the item in which the students were least successful.

Table 2: Pre-test averages per question for the intervention 1 class

Pre-test (intervention 1)														
	Question number													
	j1	a	g1	j2	h	g2	g3	e	c	b	f	i	d	
Student no.	2	3	2	2	2	2	2	4	2	4	4	3	4	36
WXH020	2	3	2	0	2	2	2	4	2	4	2	3	0	28
WId015	2	3	2	1	2	0	2	0	2	0	2	3	0	19
WPM026	2	3	2	2	2	2	0	0	0	4	2	0	0	19
WEB003	2	3	2	2	2	2	0	4	0	0	0	0	0	17
WJB008	2	3	2	2	2	2	0	4	0	0	0	0	0	17
WSC011	2	3	2	1	2	2	1	4	0	0	0	0	0	17
WOv040	2	3	2	2	0	2	1	2	2	0	0	0	0	16
WBB006	2	3	2	1	2	2	2	0	0	0	0	0	0	14
WJR035	2	3	2	1	2	0	1	0	0	0	2	0	0	13
WDB010	2	3	0	2	0	0	1	2	0	0	2	0	0	12
WTD013	2	3	2	1	0	0	2	0	0	2	0	0	0	12
WAM030	2	3	2	2	2	0	1	0	0	0	0	0	0	12
WBS038	2	3	2	0	0	0	0	4	0	0	0	0	0	11
WJG017	2	1	2	2	2	0	0	1	0	0	0	0	0	10
WJF016	2	3	1	2	0	0	0	0	0	0	0	0	0	8
WGK022	2	3	2	1	0	0	0	0	0	0	0	0	0	8
WSM029	2	3	2	0	0	0	1	0	0	0	0	0	0	8
WMA001	2	2	2	1	0	0	0	0	0	0	0	0	0	7
Total for the class	36	51	33	23	20	14	14	25	6	10	10	6	0	14
% for each question	100	94	92	64	56	39	39	35	17	14	14	11	0	38

Ratio
Number
Algebra

The word ‘successful’ means that the class achieved 50% or more in a question. The intervention 1 class was only successful in five out of thirteen problem solving questions. This could be seen in the row titled ‘% for each question’. The questions in which the intervention 1 class was successful in the pre-test were: question a) which was on algebra (Anna’s desk drawer question; question g1) (matchstick question part one) where students had to extend the pattern, question h) (Helen’s question on fractions); question j1 and j2) which is part one and two of Molly’s question) on numbers.

5.2.2 Intervention 2 class

Tables 3 below shows pre-test results for the intervention 2 class. It is organized exactly the same as table 2 above.

Table 3: Pre-test averages per question for the intervention 2 class

Pre-test (intervention 2)														
	Question number													
	g1	a	j1	g2	h	e	j2	f	b	g3	c	d	i	
	2	3	2	2	2	4	2	4	4	2	2	4	3	36
Student no.														
YLB004	2	2	2	2	0	4	1	0	0	0	0	0	0	13
YMB005	2	0	2	2	2	4	0	0	0	0	0	0	0	12
YMH018	2	2	2	2	2	0	2	0	0	0	0	0	0	12
YTA002	2	3	2	2	2	0	0	0	0	0	0	0	0	11
YSM027	2	3	0	0	0	0	0	2	4	0	0	0	0	11
YSB007	2	0	0	2	2	4	0	0	0	0	0	0	0	10
YGO034	2	2	0	0	0	4	0	2	0	0	0	0	0	10
YAS037	2	3	0	2	0	0	0	2	0	1	0	0	0	10
YLM028	2	3	2	0	2	0	0	0	0	0	0	0	0	9
YKN032	2	3	2	2	0	0	0	0	0	0	0	0	0	9
YER036	2	3	0	2	2	0	0	0	0	0	0	0	0	9
YTT039	2	3	0	0	0	4	0	0	0	0	0	0	0	9
YAC012	2	2	1	0	0	2	1	0	0	0	0	0	0	8
YAM024	2	2	2	0	0	2	0	0	0	0	0	0	0	8
YIH019	0	3	2	0	2	0	0	0	0	0	0	0	0	7
YEB009	2	3	0	0	0	0	0	0	0	0	0	0	0	5
YLN031	2	2	0	0	0	0	0	0	0	0	0	0	0	4
YPM025	2	1	0	0	0	0	0	0	0	0	0	0	0	3
YNN033	2	1	0	0	0	0	0	0	0	0	0	0	0	3
Total for the class	**36**	**41**	**17**	**16**	**14**	**24**	**4**	**6**	**4**	**1**	**0**	**0**	**0**	**9**
% for each question	95	72	45	42	37	32	11	8	5	3	0	0	0	24

Ratio
Number
Algebra

The intervention 2 class was successful in only two questions in a pre-test. Question a) which is Anna's desk drawer question on algebra and question g1) which is part one of the matchstick problem on numbers. This can be seen in the above table looking at the percentages for each question. The class attained less than 50% in the rest of the questions

In summary, both classes were not as successful in the pre-test but the intervention 1 class performed better in 5 out of 13 questions in the pre-test whereas the intervention 2 class only managed to do better in 2 out of 13 questions. Both classes performed better in Anna's desk drawer's question algebra and part one of the matchstick question on numbers.

5.3 Post-test averages per question

5.3.1 Intervention 1 class

Table 4 below shows post-test results for the intervention 1 class. All the scores are for individual students who took both pre-test and post-test. Once again, the table shows the pre-test results for intervention 1 class ordered from the student achieving the highest mark to the student achieving the lowest mark vertically and horizontally, the table shows the test item in which students were successful to the item in which students were least successful.

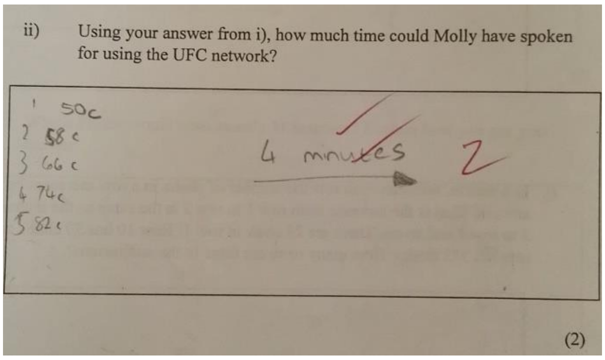
Table 4: Post-test averages per question for the intervention 1 class

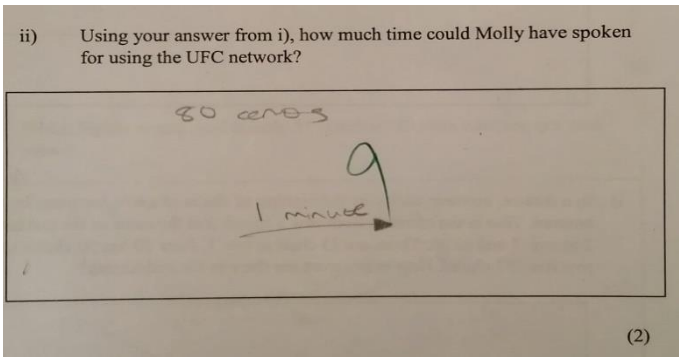
	Post-test (intervention 1)													
	Question number													
	j1	a	e	g1	g3	g2	h	j2	c	f	i	d	b	
Student no.	2	3	4	2	2	2	2	2	2	4	3	4	4	36
WXH020	2	3	4	2	2	2	2	0	2	2	3	4	2	30
WOv040	2	2	4	2	2	2	0	2	2	2	0	4	0	24
WSC011	2	3	4	2	2	2	2	2	0	2	0	0	0	21
WBB006	2	3	4	2	2	2	0	0	0	0	3	1	0	19
WJB008	2	3	4	2	2	0	2	2	2	0	0	0	0	19
WPM026	2	3	4	2	2	2	2	0	0	2	0	0	0	19
WEB003	2	2	4	2	2	2	2	2	0	0	0	0	0	18
WDB010	2	3	4	2	2	0	0	2	0	0	3	0	0	18
WId015	2	3	0	2	2	2	2	0	2	2	0	1	0	18
WJG017	2	0	4	2	1	2	2	2	0	2	0	0	0	17
WMA001	2	3	4	2	1	2	2	0	0	0	0	0	0	16
WBS038	2	2	4	2	2	2	0	1	0	0	0	1	0	16
WTD013	2	3	4	0	0	0	2	0	2	2	0	0	0	15
WJR035	2	3	4	0	1	2	2	0	0	0	0	0	0	14
WJF016	2	3	4	2	0	2	0	0	0	0	0	0	0	13
WAM030	2	3	4	0	0	0	2	0	0	2	0	0	0	13
WGK022	0	3	4	2	1	0	2	0	0	0	0	0	0	12
WSM029	2	3	0	2	2	0	0	2	0	0	0	0	0	11
Total for the class	34	48	64	30	26	24	24	15	10	16	9	11	2	17
% for each question	94	89	89	83	72	67	67	42	28	22	17	15	3	48

The intervention 1 class was successful in seven out of thirteen questions in the post-test. The intervention 1 class was successful in questions a) (Anna's desk drawer question) on algebra, question j1) (Molly's question), question g1) and g2) and g3) which is part 1, 2 and 3 of the matchsticks question and question e) (the light question) on numbers as well as and question h) (Helen's question). This can be seen in the above table looking at percentages for each question.

It is however, worth pointing out that the class was initially successful in question j2) which is Molly's question part 2 in the pre-test and unsuccessful in the same question in the post test which was rather strange.

It would appear that some students were confused by this question. Some were able to get the first part of the question correct but did not know how the first part of the question relates to the final answer. This could be because different students would interpret the question differently the first time. On a different day, the student might start interpreting the same question differently depending on the individual understanding of what the problem requires. Below in figure 14 is some student's work that shows being confused by the question in the post-test





Pre-test work Post-test work

Figure 14: Student 8 pre and post-test solution of Molly's question

5.3.2 Intervention 2 class

Table 5 below shows post-test results for the intervention 2 class combining all the scores scored by each student. All the scores are for individual students who took both pre-test and post-test. It is ordered the same as table 4 above from the most successful student to the least and from the item in which the class was successful to the item in which the class was successful the least.

Table 5: Post-test averages per question for the Intervention 2 class

Post-test (intervention 2)													
	Question number												
	g1	a	j1	e	g2	j2	h	g3	b	f	c	i	d	
	2	3	2	4	2	2	2	2	4	4	2	3	4	36
Student no.														
YMH018	2	3	2	4	2	2	2	0	0	2	0	3	0	22
YMB005	2	3	2	4	2	1	0	2	4	1	0	0	0	21
YSM027	2	3	2	4	0	1	2	1	4	0	0	0	0	19
YER036	2	3	2	4	0	2	2	1	0	2	0	0	0	18
YSB007	2	3	2	4	2	0	2	0	0	0	0	0	0	15
YIH019	2	3	2	4	2	2	0	0	0	0	0	0	0	15
YTA002	2	2	2	4	2	0	2	0	0	0	0	0	0	14
YLB004	0	2	2	4	2	2	0	0	0	0	2	0	0	14
YAS037	2	3	2	3	2	2	0	0	0	0	0	0	0	14
YTT039	2	3	0	4	0	0	2	1	0	2	0	0	0	14
YAC012	2	3	2	4	2	0	0	0	0	0	0	0	0	13
YEB009	2	3	0	4	0	0	0	1	1	0	0	0	0	11
YPM025	2	0	2	0	2	2	2	0	0	1	0	0	0	11
YKN032	2	3	2	0	2	1	0	0	0	1	0	0	0	11
YGO034	0	3	2	4	0	2	0	0	0	0	0	0	0	11
YAM024	2	0	2	4	2	0	0	0	0	0	0	0	0	10
YLM028	0	3	2	0	0	2	2	1	0	0	0	0	0	10
YNN033	2	1	0	4	0	2	0	0	0	0	0	0	0	9
YLN031	2	2	0	0	2	0	0	0	0	0	0	0	0	6
Total for the class	32	46	30	59	24	21	16	7	9	9	2	3	0	14
% for each question	84	81	79	78	63	55	42	18	12	12	5	5	0	38

The intervention 2 class was successful in six out of thirteen questions. Again, 'successful' means the class was able to achieve a score of 50% or higher in that question. The class improved in four more questions. This means that the class as a whole managed to improve its average in four questions to 50% or higher. The questions that the class improved on were question e) (the light question); question g1 and g2) which is part 1 and 2 of the matchstick question; question j1) and j2 (Molly's question) and question a) which is Anna's desk drawer question on algebra and question h) (Helen's question). The improvement in the average in four questions had a positive impact on the overall average for the class in the post-test, which is 38%.

Both classes managed to improve in at least two more questions from pre/ to post-test. While the intervention 2 class improved in question g 2) and g3) which is part 1 and 2 of matchstick question, the intervention 1 class improved in g2) part 2 of matchstick question, question h) (Helen's question), question e) (light question) and question j1 (Molly's question).

5.4 Pre-test results from coded answers

Table 6 below shows answers from the intervention 1 class coded according to 0, if the answer was omitted, 1 if the answer was incorrect, 2 if the answer was partially correct and 3 if the answer was completely correct. The table is ordered vertically according to the students with the most correct answers to the students with the least. Horizontally, the table is ordered according to the question students were most successful at to the question in which they were least successful. The last column vertically is a summary of the number of correct questions the students attained.

5.4.1 Intervention 1 class

Table 6: Pre-test coded answers per question for the intervention 1 class

	Intervention 1 (Pre-test)													
					Question no.									
Student no.	j1	a	g1	h	j2	g2	e	g3	c	b	i	d	f	
WXH020	3	3	3	3	1	3	3	3	3	3	3	1	2	**10**
WEB003	3	3	3	3	3	3	3	0	1	1	1	0	1	**7**
WJB008	3	3	3	3	3	3	3	0	1	1	1	0	1	**7**
WId015	3	3	3	3	2	1	1	3	3	1	3	0	2	**7**
WPM026	3	3	3	3	3	3	1	0	1	3	0	0	2	**7**
WBB006	3	3	3	3	2	3	1	3	1	1	1	1	1	**6**
WSC011	3	3	3	3	2	3	3	2	1	0	1	0	0	**6**
WOv040	3	3	3	1	3	3	2	2	3	1	1	1	1	**6**
WAM030	3	3	3	3	3	1	1	2	1	1	0	0	0	**5**
WTD013	3	3	3	1	2	1	1	3	1	2	1	0	1	**4**
WJG017	3	2	3	3	3	0	2	0	1	0	1	0	0	**4**
WJR035	3	3	3	3	2	1	1	2	1	0	1	1	2	**4**
WBS038	3	3	3	1	1	1	3	0	1	1	1	0	1	**4**
WDB010	3	3	1	1	3	1	2	2	1	1	1	0	2	**3**
WJF016	3	3	2	1	3	1	1	1	1	1	0	0	0	**3**
WGK022	3	3	3	1	2	1	1	0	1	1	0	1	1	**3**
WSM029	3	3	3	1	0	1	1	2	1	1	1	0	0	**3**
WMA001	3	2	3	1	2	1	1	0	1	1	0	1	1	**2**
	18	**16**	**16**	**10**	**8**	**7**	**5**	**4**	**3**	**2**	**2**	**0**	**0**	

Shifting the focus to the questions that were most omitted from the above table, question d) (the stick length question) which is a problem solving question involving ratios was the most omitted question by the intervention 1 class in the pre-test. In addition to this, the students that attempted the question did not obtain the correct answer. However, two students managed to arrive at the solution for question d) in the post-test. The work for these two individual students is discussed under the heading ‘students making more gains’ and a sample of their work is provided.

5.4.2 Intervention 2 class

Table 7 below shows answers for the Intervention 2 class coded according to 0, if the question was omitted, 1 if the answer was incorrect, 2 if the answer was partially correct and 3 if the answer was completely correct. The table is ordered exactly the same as table 6 above.

Table 7: Pre-test coded answers per question for the intervention 2 class

	Pre-test (intervention 2)													
				Question number										
	g1	g2	j1	a	h	e	b	j2	c	d	f	g3	i	
YMB005	3	3	3	1	3	3	0	1	1	0	0	1	1	**5**
YMH018	3	3	3	2	3	1	1	3	1	0	0	0	0	**5**
YTA002	3	3	3	2	3	1	1	0	1	0	0	0	0	**4**
YSB007	3	3	0	1	3	3	1	0	1	1	1	1	0	**4**
YLM028	3	1	3	3	3	1	1	0	1	0	0	1	0	**4**
YKN032	3	3	3	3	1	1	1	1	1	0	1	1	1	**4**
YER036	3	3	0	3	3	1	1	0	1	1	1	0	1	**4**
YLB004	3	3	3	2	1	1	1	2	1	1	1	0	1	**3**
YIH019	1	1	3	3	3	1	1	1	1	1	1	1	1	**3**
YAS037	3	3	1	3	1	1	1	1	1	1	2	2	1	**3**
YTT039	3	0	0	3	0	3	1	0	1	0	1	0	0	**3**
YEB009	3	1	1	3	1	1	1	1	1	1	1	0	1	**2**
YAM024	3	1	3	2	1	2	1	1	1	0	0	0	0	**2**
YSM027	3	1	0	2	1	1	3	0	1	0	2	0	0	**2**
YGO034	3	0	0	2	0	3	1	0	1	0	2	0	0	**2**
YAC012	3	1	2	2	1	2	1	2	1	1	1	1	1	**1**
YPM025	3	0	0	2	0	1	1	0	1	1	1	0	0	**1**
YLN031	3	1	1	2	1	1	1	1	1	0	1	1	1	**1**
YNN033	3	1	1	2	1	1	1	1	1	0	0	0	1	**1**
	18	**8**	**8**	**7**	**7**	**4**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	

Shifting the focus to the question most omitted for the intervention 2 class, this was question d) (the stick length) in the pre-test. Although students tried question c) (Hamsa’s shadow question) on ratios, all students could not arrive at the solution. Other questions that were a challenge for the students from the Intervention 2 class in the pre-test were question e) (light question) on numbers. This question had only four students able to arrive at the solution. Question b) (Mrs Math question) on numbers, was also a challenge with only one student able to do the question correctly. Question j2) (Molly’s question part 2) also had one student able to complete successfully in the pre-test. Questions f) (unit fraction) and question i) (auditorium question) had none of the students arriving at the solution.

It would appear that while the intervention 1 class struggled with question d) (stick length question) on ratios in the pre-test, the Intervention 2 class struggled with question c) (Hamsa's shadow question) on ratios, Mrs Math question on numbers, unit fraction question, auditorium question and Molly's question part 2. The common question that seems to have been a problem for both classes, is question d) on ratios.

In addition, algebraic problems could be solved using variables. However, when the pre-test was administered, it was alarming how students avoided using variables or completely forgot about the use of them in solving these problems.

5.5. Post-test results from coded answers

5.5.1 Intervention 1 class

Table 8 below shows post-test coded answers. Once again, the answers are coded according to 0 if the solution was omitted, 1 if the solution is incorrect, 2 if the solution is partially correct and 3 if the solution is completely correct.

Table 8: Post-test coded answers per question for the intervention 1 class

	Intervention 1 (Post-test)													
				Question no.										
Student no.	j1	e	g1	a	g2	h	g3	j2	c	i	d	b	f	
WXH020	3	3	3	3	3	3	3	1	3	3	3	2	2	10
WJB008	3	3	3	3	1	3	3	3	3	1	1	1	1	8
WSC011	3	3	3	3	3	3	3	3	1	1	1	1	2	8
WOv040	3	3	3	2	3	1	3	3	3	1	3	1	2	8
WEB003	3	3	3	2	3	3	3	3	1	1	1	1	1	7
WBB006	3	3	3	3	3	1	3	1	1	3	2	1	1	7
WDB010	3	3	3	3	1	1	3	3	1	3	1	1	1	7
WId015	3	1	3	3	3	3	3	1	3	1	2	1	2	7
WPM026	3	3	3	3	3	3	3	1	1	0	0	1	2	7
WMA001	3	3	3	3	3	3	2	0	0	0	0	1	0	6
WJG017	3	3	3	1	3	3	2	3	1	0	0	1	2	6
WTD013	3	3	1	3	0	3	0	1	3	1	1	1	2	5
WJF016	3	3	3	3	3	1	1	1	1	0	0	1	0	5
WSM029	3	1	3	3	1	1	3	3	1	0	0	1	0	5
WJR035	3	3	1	3	3	3	2	1	1	1	1	1	1	5
WBS038	3	3	3	2	3	1	3	2	1	0	2	1	1	5
WGK022	1	3	3	3	1	3	2	1	0	1	0	1	0	4
WAM030	3	3	1	3	0	3	0	1	1	0	0	1	2	4
	17	16	15	14	12	12	11	7	5	3	2	0	0	

While question d) (stick length question) remained the most omitted question in the pre-test, two students from the intervention 1 class managed to successfully complete this question in the post-test. While question c) (Hamsa’s shadow question) was successfully completed by three students in the pre-test, five students managed to do this question in the post-test. Both these questions were on ratios. Two students were able to do question i) (auditorium question) in the pre-test and three were able to do the same question in the post-test.

None of the students were able to successfully complete question f) (the unit fraction question) in the pre-test as well as the post test.

5.5.2 Intervention 2 class

Table 9 below represents the post-test coded answers for the intervention 2 class. The answers were coded according to 0 if the question is omitted, 1 if the solution is incorrect, 2 if the solution is partially correct and 3 if the solution is completely correct.

Table 9: Post-test coded answers per question for the Intervention 2 class

	Post-test (intervention 2)													
	Question number													
	g1	j1	e	a	g2	j2	h	b	c	g3	i	d	f	
YTA002	3	3	3	3	3	3	3	1	1	1	3	1	2	**8**
YLB004	3	3	3	3	3	2	1	3	1	3	0	1	2	**7**
YMB005	3	3	3	3	3	1	3	0	1	1	0	1	1	**6**
YSB007	3	3	3	3	3	3	1	1	1	1	1	1	1	**6**
YEB009	3	3	3	3	1	2	3	3	1	2	1	1	1	**6**
YAC012	3	3	3	3	1	3	3	1	1	2	1	1	2	**6**
YMH018	3	3	3	2	3	1	3	1	1	0	0	0	0	**5**
YIH019	1	3	3	2	3	3	1	0	3	1	1	1	0	**5**
YAM024	3	3	3	3	3	1	1	0	1	1	1	1	1	**5**
YPM025	3	3	1	1	3	3	3	1	0	0	0	0	2	**5**
YSM027	3	3	2	3	3	3	1	1	1	1	1	1	1	**5**
YLM028	3	3	3	1	3	1	1	0	1	0	0	0	0	**4**
YLN031	1	3	1	3	1	3	3	0	1	2	0	0	0	**4**
YKN032	3	3	1	3	3	2	1	1	1	1	1	1	2	**4**
YNN033	1	3	3	3	1	3	1	1	1	0	0	1	1	**4**
YGO034	3	0	3	3	1	0	3	1	1	2	1	0	2	**4**
YER036	3	1	3	3	1	1	1	2	1	2	1	1	1	**3**
YAS037	3	1	3	2	1	3	1	0	1	0	0	0	1	**3**
YTT039	3	1	1	2	3	1	1	1	1	1	1	0	1	**2**
	16	**15**	**14**	**13**	**12**	**9**	**8**	**2**	**1**	**1**	**1**	**0**	**0**	

There were fewer omissions for question d) (stick length question) on ratios in the post-test and more students attempted this question. However, the students did not manage to arrive at the solution for this question. 14 students were able to do question e) (light question) in the post-test, 2 students managed to do question b) (Mrs Math question) in the post-test. Students successfully completed question j2 (Molly’s question part 2) in the post-test with only one student able to do question c) (Hamsa’s shadow question). Only one student was able to do question i) (auditorium question) and none of the students managed question d) (stick length) and question f) (unit fraction) in the post-test.

Both intervention 1 and intervention 2 classes struggled with the auditorium and unit fraction question, which were testing them on numbers. They also struggled with stick length question and Hamsa's shadow questions which were on ratios.

5.6 The results of pre-test and post-test in a bar graph

5.6.1 Intervention 1 class

Figure 15 below is the bar graph of the intervention 1 class. It compares the percentages of each question in the pre-test and post-test. Vertically, the bar graph represents percentages attained in each question by the whole class and horizontally, the bar graph shows the test items.

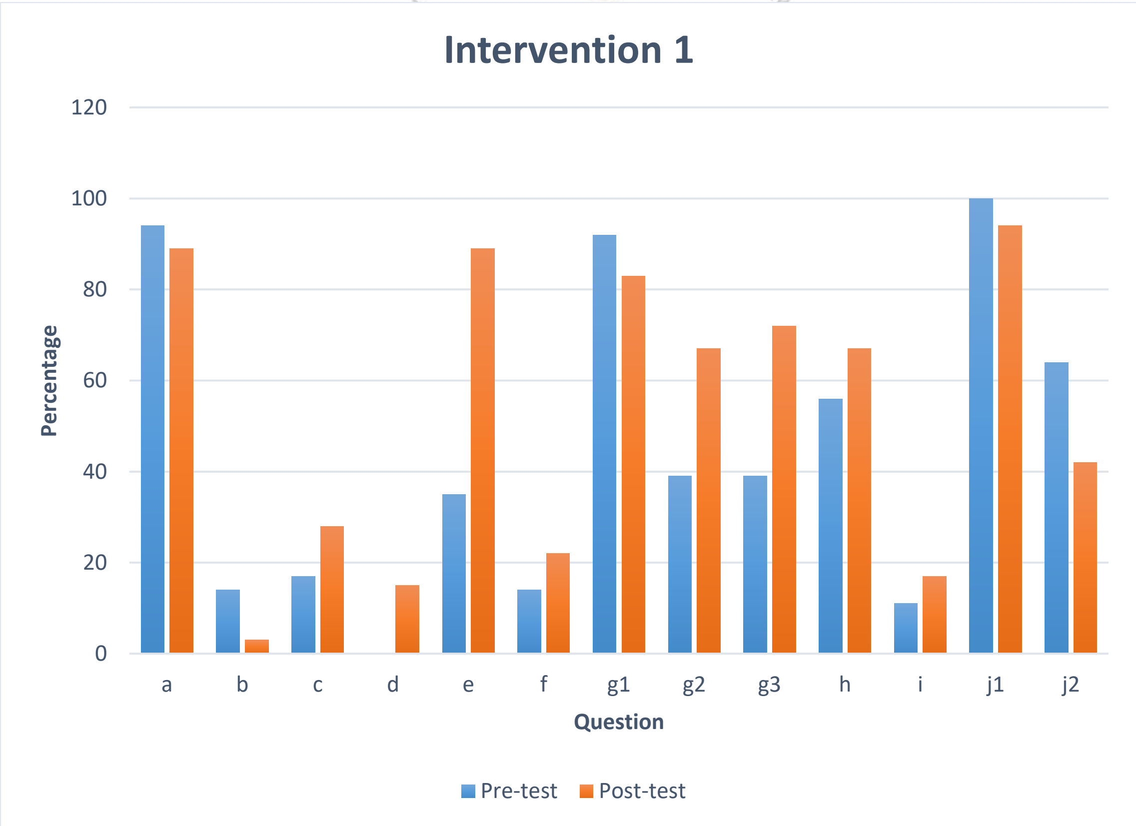
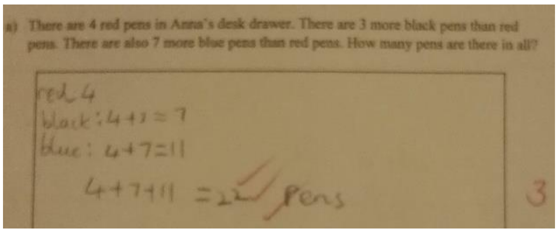
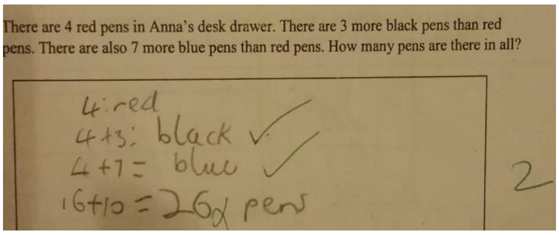


Figure 15 : Bar graph comparing the averages per question of the pre-test and post-test of the intervention 1 class

The bar graph in Figure 15 shows that the intervention 1 class dropped in some of the questions in which they were successful in the pre-test. The class dropped in question a); b); g1) j1) and j2. This could be seen in the height of the columns of the post-test. Consequently, although the overall average for the class improved by 10%, the average could have been better looking at the students' scripts, it would appear that some drop in the marks is due to careless mistakes, such as paying less attention on computation. The students' work shown in Figure 16 supports this notion. Student 9 (error in the post-test working out. 4+ 4 +4 is not equal to 16)



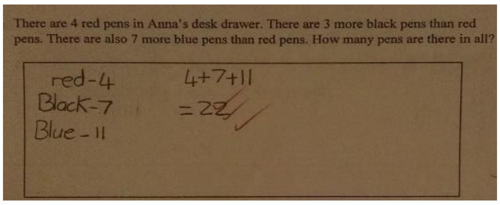


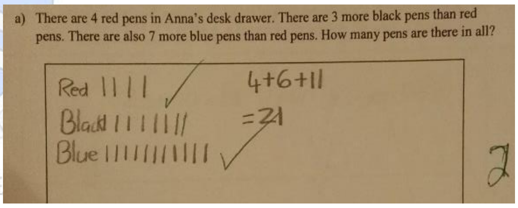
Pre-test

Post-test

Figure 16: Student 9 pre and post-test solution of Anna desk drawer's question

Student 10 (error in the post-test working out. Did not count blue pens properly. He has 7 tallies but wrote 6 as the total number of tallies)



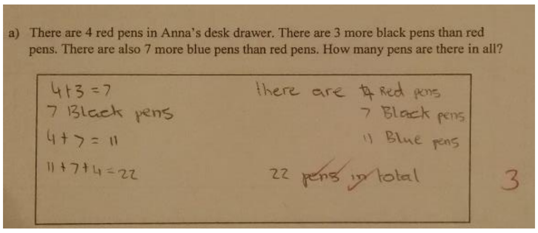


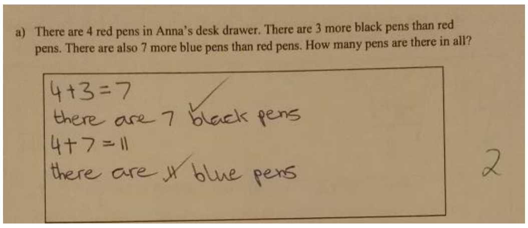
Pre-test

Post-test

Figure 17: Student 10, pre and post-test solution of Anna desk drawer's question

Student 11(is not answering the question asked. The question is how many pens are there in all but the student thought the question is asking for the number of blue pens)





Pre-test

Post-test

Figure 18: Student 11, pre and post-test solution of Anna desk drawer's question

One other reasons that might be attributed to the drop in the marks by some of the students from the intervention 1 class, is not reading the question properly. For example, the student's work in figure 19 below shows that the question was not read properly and consequently, the question was incorrectly answered.

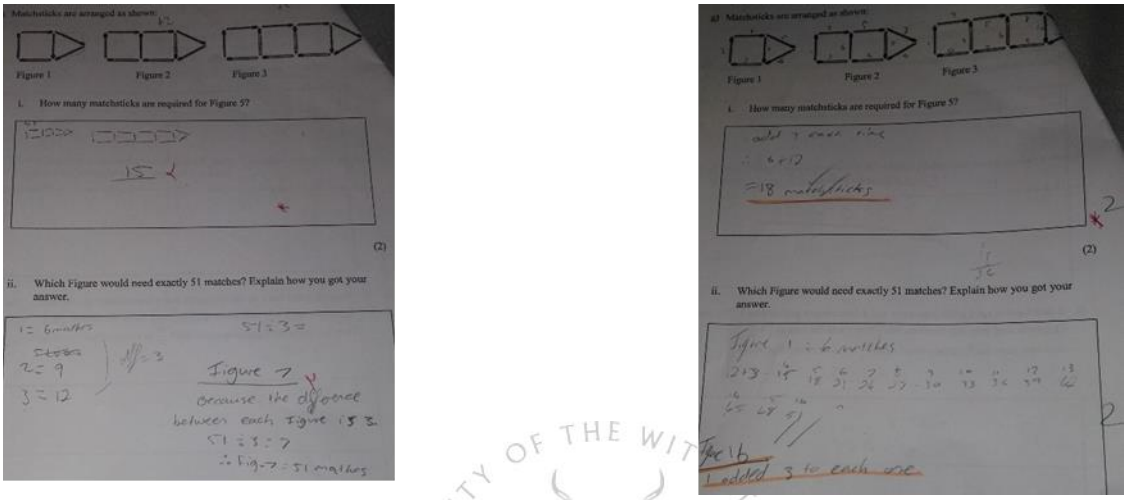


Figure 19: Student 12, pre and post-test solution of match stick question

The question was clearly asking for the number of matchsticks required to build pattern 5. The student gave the number of matchsticks required to build pattern 4.

Lastly, some students were able to do the question in the pre-test and unable to do the same question in the post-test. This is rather strange and raises questions about the understanding of the question. For example, the student 13's work in figure 20 below shows that the student was able to do Mrs Math question in the pre-test but unable to in the post-test.

Student 13(although the student was able to do this question in the pre-test, it seems like the same question completely confused the student in the post test)

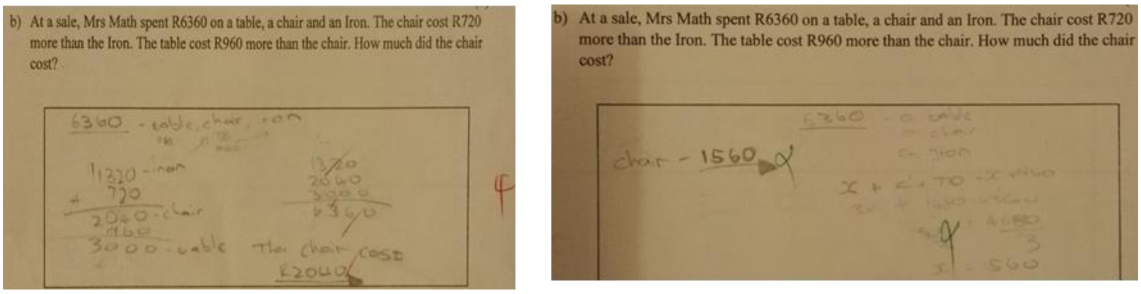
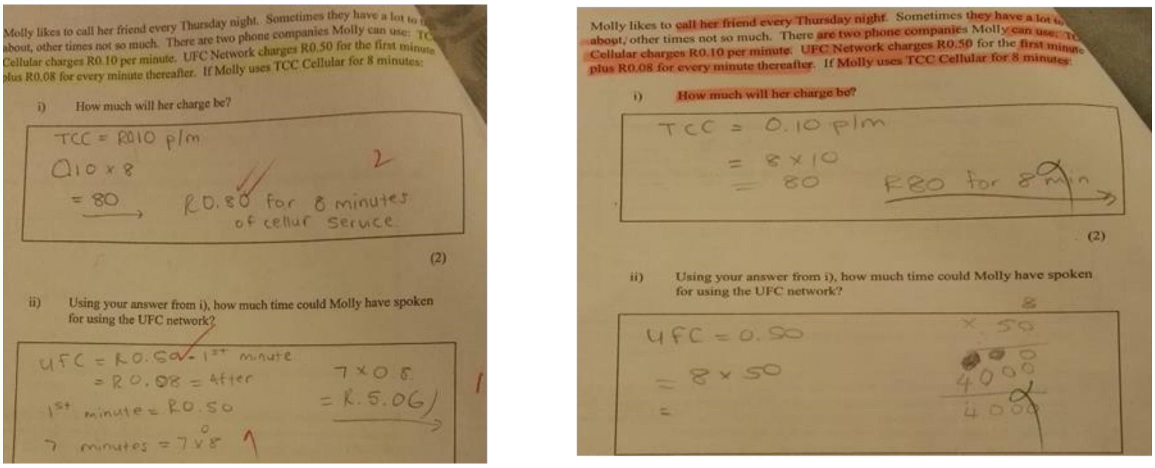


Figure 20: Student 13 pre and post-test solution of Mrs Math question

Student 14 (Student 14 was completely confused by these two question which she was able to do in the pre-test). See figure 21 below.



Pre-test Post-test

Figure 21: Student 14, pre and post-test solution of Molly's question

5.6.2 Intervention 2 class

Figure 22 below is the bar graph of the intervention 2 class. It compares the percentages of each question in the pre-test and post-test. Vertically, the bar graph represents percentages attained in each question by the whole class as a whole and horizontally, the bar graph shows the items.

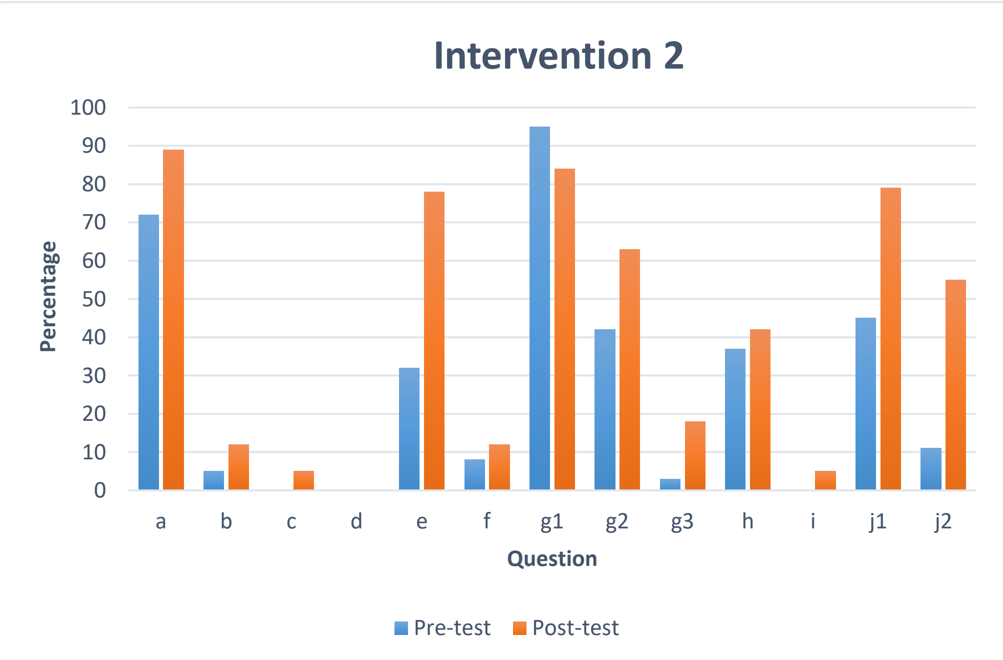
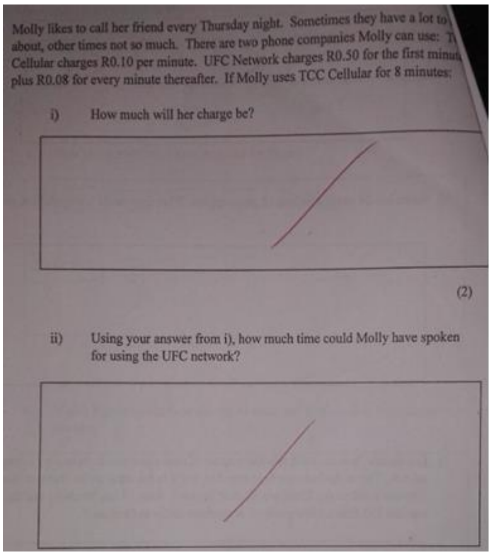
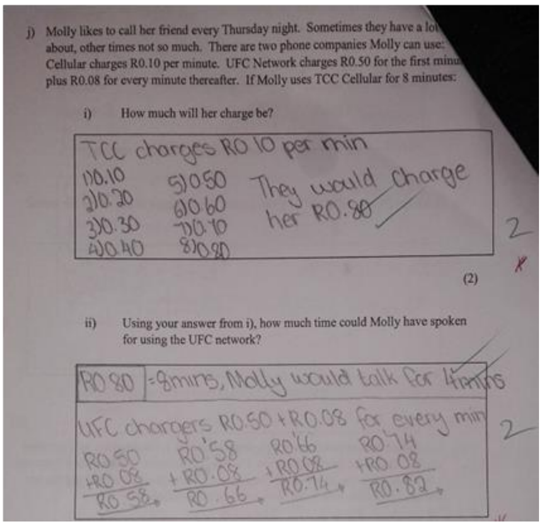


Figure 22: Bar graph comparing the averages per question of pre-test and post-test for the Intervention 2 class

The bar graph (Figure 22) shows that the intervention 2 class improved across most of the items from pre-test to post-test. The columns representing post-test averages are generally higher than the columns representing pre-test averages except for question g1) where a drop in the average is observed. Six students improved by at least 22% which impacted positively to the overall average improvement by 14%.

Looking at the scripts again of the students that improved by at least 22%, it appears that some of the improvement is due to trying out more problems in the post-test that were otherwise left un-attempted in the pre-test. See the student's work in figure 23 below.





Pre-test　　　　Post-test

Figure 23: Student 15 pre and post-test solution of Molly's question

While the intervention 1 class dropped in 5 questions from pre to post-test, the Intervention 2 class improved in all except one item.

5.7 Pre-test and post-test scores using a rubric.

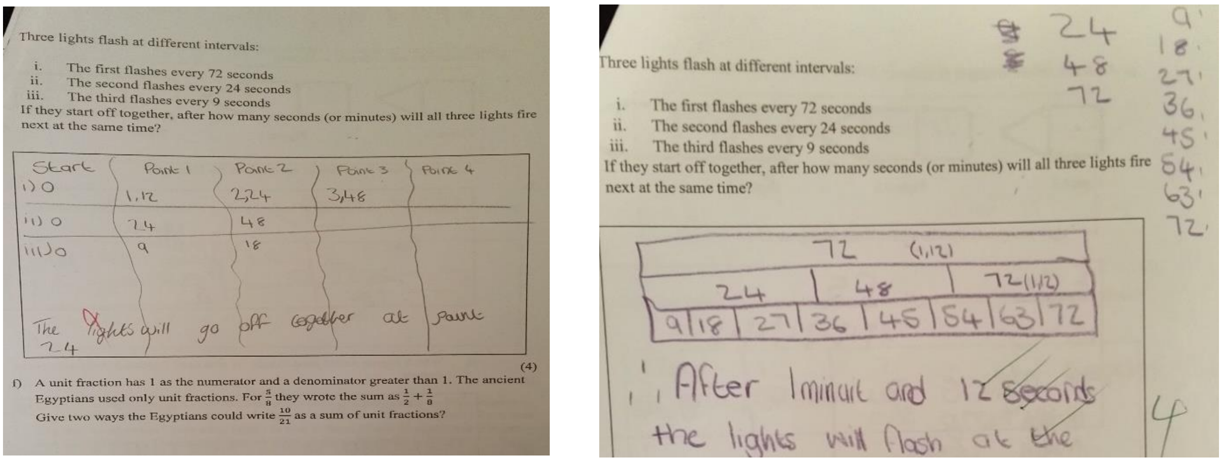
5.7.1 Intervention 1 class

Table 10 below shows the results of the Intervention 1 class when the pre-test and the post-test were scored using the rubric. Three process dimensions were assessed, these are, understanding of the problem, using appropriate information as well as representation and solving of the problem. The scoring was on a scale from 1 to 4. The table combines the scores achieved by each student on each process dimension and the total score for the whole class is written at the bottom of the table. For example, the highest score that the whole class could attain in each process dimension is (4 x 18) which equals 72 since there were 18 students in the intervention 1 class and were assessed in a scale 1 to 4. Once again, the reason why these three process dimensions were the focus, was to measure whether there were any gains by the intervention class in terms of understanding, using appropriate information and using models to solve the problem.

Table 10: Pre-test and post-test scores using a rubric

	Intervention 1 (Pre-test)								Intervention 1 (Post-test)		
	Making sense of the problem	Using information appropriately	Representation and solving the task						Making sense of the problem	Using information appropriately	Representation and solving the task
Student no.	**4**	**4**	**4**					Student no.	**4**	**4**	**4**
WMA001	1	1	1					WMA001	2	2	1
WEB003	3	2	2					WEB003	3	2	2
WBB006	1	2	1					WBB006	3	2	1
WJB008	3	2	1					WJB008	3	2	1
WDB010	2	1	1					WDB010	3	2	1
WSC011	3	2	1					WSC011	3	3	2
WTD013	2	1	1					WTD013	2	2	1
WId015	3	2	1					WId015	3	2	1
WJF016	2	1	1					WJF016	2	2	2
WJG017	2	1	1					WJG017	3	2	1
WXH020	4	4	2					WXH020	4	4	4
WGK022	1	1	1					WGK022	2	2	2
WPM026	3	2	1					WPM026	3	2	1
WSM029	1	1	1					WSM029	2	1	1
WAM030	1	1	1					WAM030	2	1	1
WJR035	2	1	1					WJR035	2	1	1
WBS038	1	1	1					WBS038	2	2	2
WOv040	2	1	1					WOv040	3	3	1
Total %	**51**	**38**	**28**					**Total %**	**65**	**51**	**36**

The intervention 1 class improved in all three process dimensions. The class had noticeable improvement especially in the process dimension of making sense of the task. However, it is worth pointing out that the representation and solving of the task, although there was an improvement, this improvement was not substantial. This correlates well with what could be seen in their scripts when looking at how they tried to solve the task in the pre-test and the post-test. Although there were very few diagrams used to assist with the solving of the task in the pre-test, it was good to note that some of the students tried to use models as an aid in solving the task in the post-test. Again, some of the models used were not 'perfect', but it was pleasing to note that students might have learnt that a diagram can be a useful aid in the understanding and ultimately in solving the task. See figure 24 below of the pre-test and post-test work of one student from the intervention class.



Pre-test diagram

Post-test diagram

Figure 24: Student 16, pre and post-test solution of the light question

The diagrams used in the pre-test started to improve in the post-test as it can be seen in the example in Figure 24.

5.7.2 Intervention 2 class

Table 11 below shows the results of the Intervention 2 class when the pre-test and the post test is scored using the rubric. Three process dimensions were assessed, that is, understanding of the problem, using information appropriately as well as representation and solving of the problem. The scoring was on a scale from 1 to 4. The table combines the scores for the individual student and tally the scores at the bottom for the class as a whole. For example, the highest possible score the intervention 2 class could attain in each process dimension is (4 x 19) which is equal to 76 since there were 19 students in the intervention 2 class . The reason why these three process dimensions were the focal point, was to measure whether there was any difference in gains between the intervention 1 class and the intervention 2 class in terms of understanding the problem, using information appropriately as well as using models to solve the problem.

Table 11: Pre-test and post-test scores using rubric for the intervention 2 class.

	Intervention 2 (Pre-test)				Intervention 2 (Post-test)		
	Making sense of the problem	Using information appropriately	Representation and solving the task		Making sense of the problem	Using information appropriately	Representation and solving the task
Student no.	4	4	4	Student no.	4	4	4
YTA002	1	1	1	YTA002	1	1	2
YLB004	2	1	1	YLB004	2	2	2
YMB005	2	1	1	YMB005	3	2	2
YSB007	2	1	1	YSB007	2	1	2
YEB009	1	1	1	YEB009	2	2	2
YAC012	1	1	1	YAC012	2	1	1
YMH018	1	2	1	YMH018	3	4	2
YIH019	1	1	1	YIH019	2	2	1
YAM024	1	2	1	YAM024	1	1	1
YPM025	1	1	1	YPM025	1	2	2
YSM027	2	1	1	YSM027	3	2	1
YLM028	1	1	1	YLM028	1	1	1
YLN031	1	1	1	YLN031	1	1	1
YKN032	1	1	1	YKN032	2	2	2
YNN033	1	1	1	YNN033	1	1	1
YGO034	1	1	1	YGO034	1	1	1
YER036	1	1	1	YER036	2	2	1
YAS037	2	1	1	YAS037	2	1	1
YTT039	1	1	1	YTT039	1	1	1
Total %	**30**	**26**	**24**	**Total %**	**42**	**38**	**33**

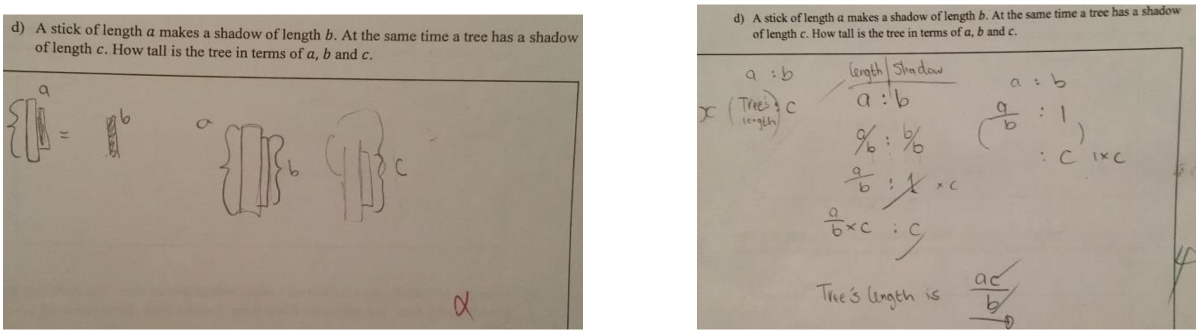
The intervention 2 class showed improvement across the three process dimensions. I had not expected the intervention 2 class to improve merely because they received a different kind of teaching. Their teaching was not as interactive as it was with the intervention 1 class, especially the interaction with me as a teacher. This clearly shows that the class had potential to do better. The understanding of the task was not where it should be as expected and hence the averages in the pre-test and the post-test show this notion. Although both classes improved on the process dimensions assessed but the improvement was not substantial. The scores of the intervention 1 class in the process dimensions assessed are higher than those of the control class. The improvement in the scores of the intervention 1 class could be due to them being introduced for example to 'bar model' one way of representing a problem diagrammatically.

5.8 Students making most gains

5.8.1 Intervention 1 class

This section is about those students that demonstrated most gains during teaching. Arguably, this is not easy to prove that a student has gained a lot out of the teaching. How does one measure gains and how does one account for that claim? In this section I looked at the marks of the students. In particular, those students with higher marks and with a significant difference between pre-test and post-test and discuss what they did and present some of their work as examples. I also looked at those students who were able to do some questions that the majority of the students were unable to do. For example, auditorium question, stick length question and Hamsa’s shadow question.

Two students from the intervention 1 class managed to arrive at the solution for question d) (stick length question) that was the most omitted question in the pre-test. These individual students seem to have made most gains. The fact that they were able to do this question in the post-test when other students could not do it, differentiates them from the rest of the class. Their work is shown in figure 25 below.



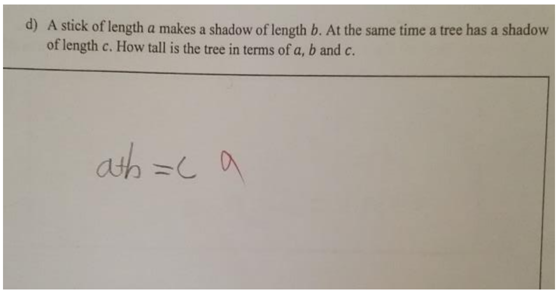
Pre-test response

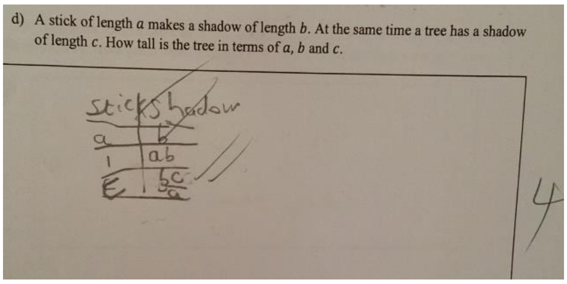
Post-test response

Figure 25: Student 17, pre and post-test solution of stick length question

Student 17 made most the gains looking at his work samples above and comparing pre-test and post-test responses. Initially, this student’s model could not help him solve the problem. Although in the post-test response, the student did not use ‘bar model’ but was successful in

terms of getting the solution. This student was the most successful student in the intervention 1 class overall. The very same student was one of the two students who were able to work out a solution to an auditorium question. The following example in figure 26 is how she worked out an auditorium question.





Pre-test response

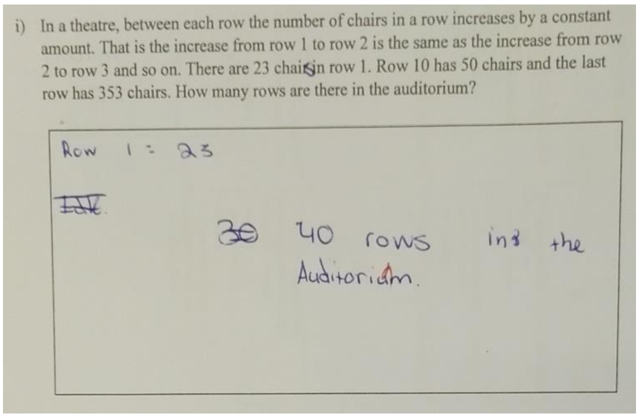
Post-test response

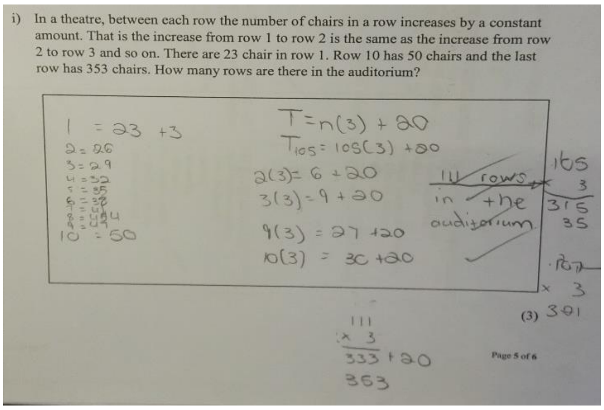
Figure 26: Student 18, pre and post-test solution of stick length question

Student 18 also made most gains. This particular student, even though he confused the variables in the post-test, it is evident that he had an idea of the concept of ratios and how to work with them. Looking at the pre-test response, it is noticeable that the students merely added the variables and equated them to c whereas in the post-test, the same student did better in the same problem.

5.8.2 Intervention 2 class

Only one student from the intervention 2 class was able to do the auditorium question in the post-test. The same student was unable to do this question in the pre-test. Her work is shown in figure 27 below.





Pre-test

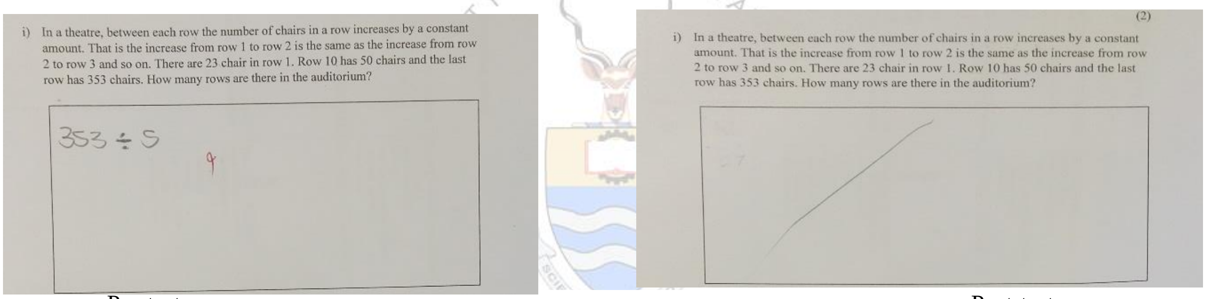
Post-test

Figure 27: Student 19: pre and post-test solution of the auditorium question

By looking at her work it is evident that initially in the pre-test this student did not understand the question. In the post-test, the student started to apply what she had learned on how to deal with patterns. There is an introduction of some kind of a formula that helped her in arriving at the solution. Therefore, it would appear that by dealing with problem solving in this study, this student was able to transfer what she had learned (previous knowledge) to a new situation and seems to have gained more.

5.9 Students making least gains

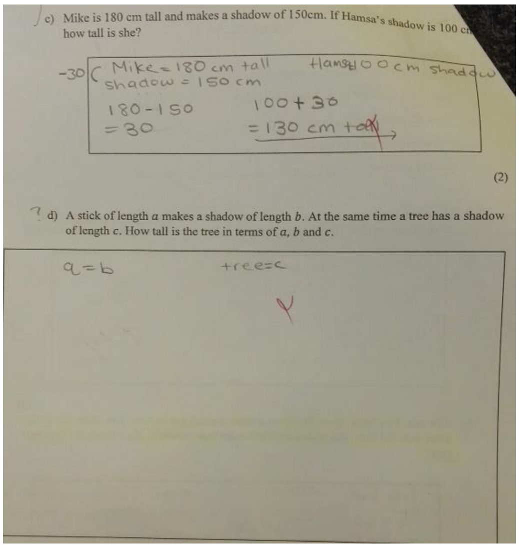
There are two students from the intervention 1 class that seem to have made least gains. These two students are at the bottom of the intervention 1 class (Table 4). Looking at their scripts, these students seem to have given up trying. Although some of the pre-test answers were wrong, they were attempting questions. In the post-test, some of the questions they had tried to do in the pre-test were omitted. See examples of students' work in figure 28 below.



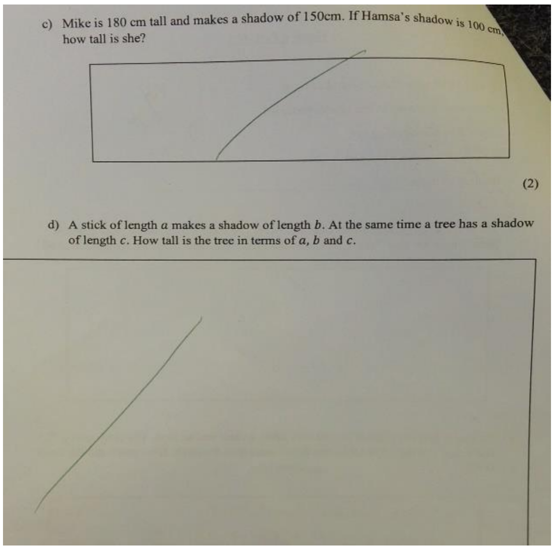
Pre-test Post-test

Figure 28: Student 20: pre and post-test solution of the auditorium question

Student 20 above tried the problem in the pre-test and in the post-test, student 20 did not try the question at all.



Pre-test

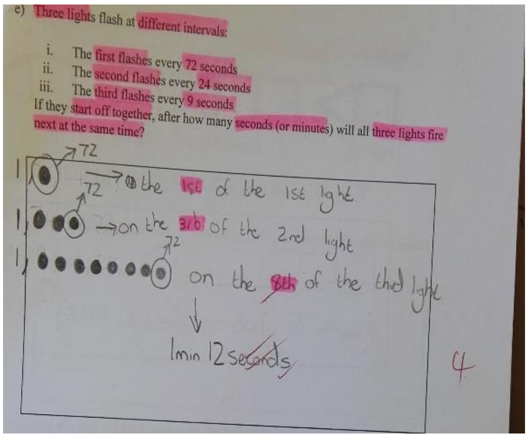


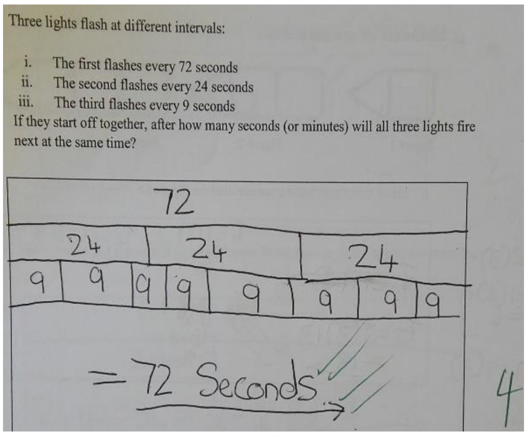
Post-test

Figure 29: Student 21: pre and post-test solution of Hamsa's shadow question

Student 21 was trying the questions on ratios in the pre-test. It would appear that in the post-test, the student gave up trying as shown in figure 29.

In solving the problem, it was found that students did not follow the heuristics suggested by Polya. That is, understand the problem, make a plan, carry out a plan and look back. Students often started with the model and tried to use the model to solve the problem. Sometimes the model did not fit the problem; they had to go back to the question to try to understand it better and thereafter refine the model or develop a new model. For example, the model in figure 30 below was used by one of the students to solve the stick length question in the pre-test.





Pre-test Post-test

Figure 30: Student 22: pre and post-test solution of light question

The model helped the student solve the problem but a modified model looks better and resembles that of a bar model that student had been exposed to. As the student continued to try to understand the problem, he then modified the model to better suit the context.

This is what I would call the cyclic nature of Brown and Walter's framework of problem solving because some students did not sometimes strictly follow the heuristics suggested by Polya.

6. Discussion

The following questions will be answered through the discussion of the results.

1) Why the intervention 2 class seems to have done better in terms of gains than the intervention 1 class?

2) What do findings reveal about the different topic areas?

3) What do the findings reveal about individual students?

6.1 Why did the class seem to have done better in terms of gains than the intervention class?

The overall improvement of 14% for the control class seems to suggest that the control class performed better in items than the intervention 1 class which had 10% improvement. While both classes managed to improve in at least two more questions from pre- to post-test, the intervention 1 class dropped in other questions which consequently impacted negatively on the overall average for the class in the post-test.

Scoring using the rubric suggests that the intervention 1 class performed better in all the process dimensions in which both classes were assessed. Students were assessed on the understanding of the problem, using appropriate information and on representation and solving of the task. This suggests that the intervention 1 class had more gains in terms of all these process dimensions making it the class that gained more in terms of these.

Depending on how one looks at gains, one might arrive at different conclusions. If looking at the overall average as the determiner of gains, then one might say the intervention 2 class gained more. If one looks at process dimensions and use the performance on them to determine gains, then the intervention 1 class gained more.

The illusion that the intervention 2 class gained more was brought about by the drop in the marks of the intervention 1 class in some of the items they had initially performed better initially. However, the overall picture is that the intervention 1 class still gained more because the class improved in terms of being able to use diagrams to solve problems and also in terms of understanding of the problem.

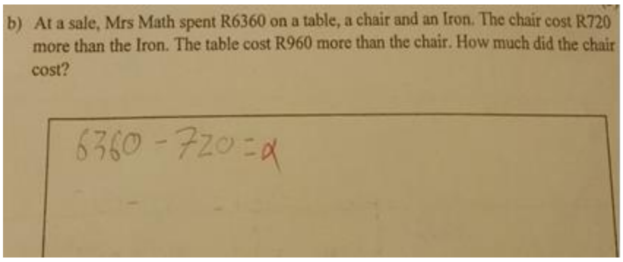
The intervention 2 class always wrote their test after the intervention 1 class had written. This was due to the timetable at the school. Having said that, there was always a possibility that the students might interact and talk about what transpired during the intervention class lesson. This may have included students telling each other how they approached different problems. This could be one of the reasons that 6 students from the intervention 2 class improved by at least 22% giving an impression that the intervention 2 class had more gains.

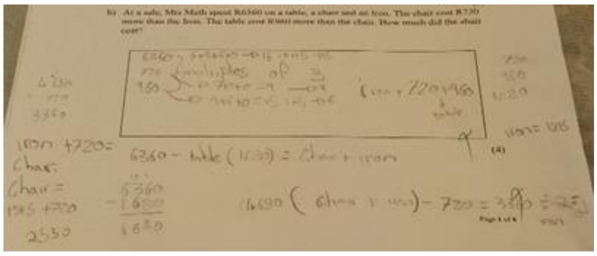
6.2 What do findings reveal about topic areas?

Both intervention 1 and intervention 2 class struggled with the auditorium and unit fraction questions, which were testing them on numbers. They also struggled with stick length question and Hamsa's shadow questions which tested ratios. In addition to the above questions, these questions were either omitted or students performed poorly on them.

This notion suggests that students find ratios challenging. It could also be argued that the question such as the auditorium question and unit fraction question, were the two questions that were particularly long in terms of the context and the wording. Hence, it could be that the reason why students performed poorly in them might be due to inability to recall all the information given and also selection of the appropriate information to use in order to solve the problem.

The last reason that could be the cause of poor performance in these questions might be that of too many figures given to a problem. Students may have found this confusing as they try to find out how the figures given relate to each other. For an example, Mrs Math question, some students just did not know what to do and there is evidence that due to the number of figures and wording, they got confused. See the work of a student in figure 31 below as an example.



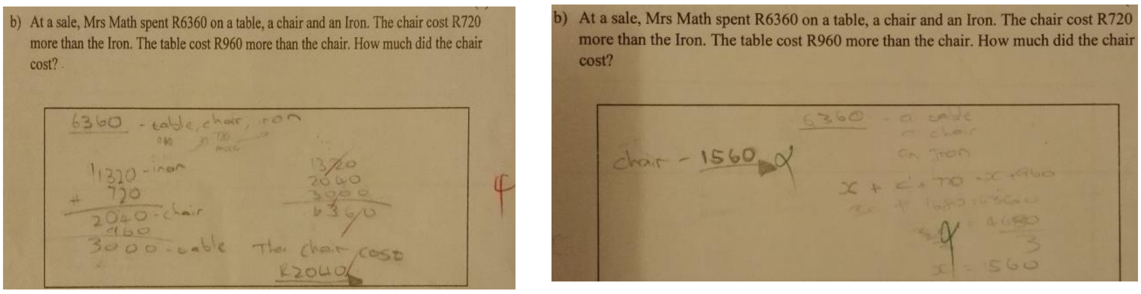


Pre-test

Post-test

Figure 31: Student 23: pre and post-test solution of Mrs Math question

Some students, as argued before in 5.6.1under the bar graph of the intervention 1 class (Figure 15), strangely, were able to do a question in pre-test and unable to in the post-test. For example, Figure 32 shows the work of a student that was able to do Mrs Math question in the pre-test and was unable to in the post-test.

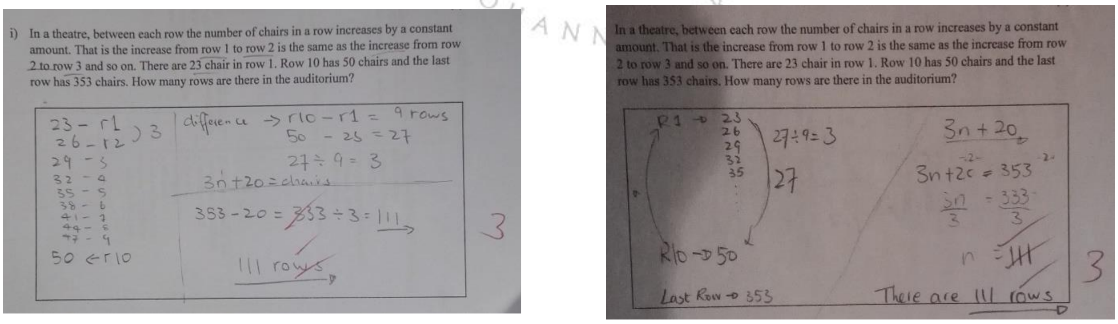


Pre-test Post-test

Figure 32: Student 24: pre and post-test of Mrs Math question

6.3 What do the findings reveal about individual students?

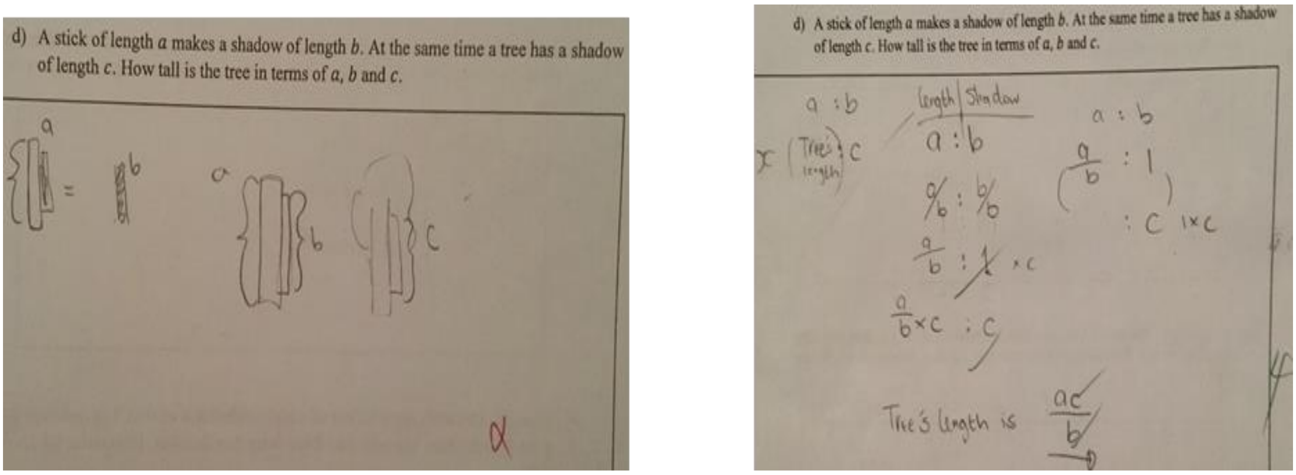
Different students might have made different gains as far as the teaching is concerned. Some students performed well in the pre-test and performed poorly in the post-test. However, there is a student in particular who was able to solve the auditorium question in the pre-test and the post-test. Her work is shown in Figure 33.



Pre-test Post-test

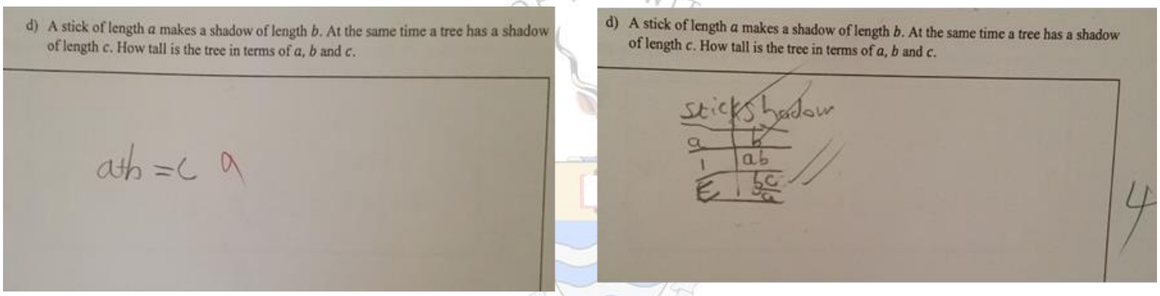
Figure 33: Student 25: pre and post-test of the auditorium question

Some students managed to do questions on ratios when the class struggled with these questions. For examples, the work of the students shown in Figures 34 and 35 indicates that there were more gains from pre-test to post-test. Their work for the post-test indicates the understanding of the concept, ratios in particular.



Pre-test response Post-test response

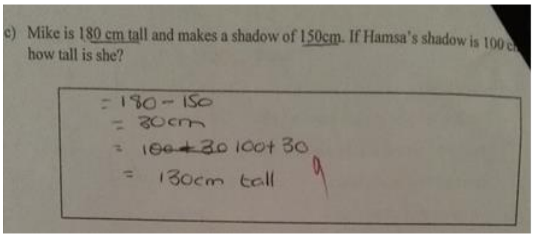
Figure 34: Student 26: pre and post-test solution of the stick length question



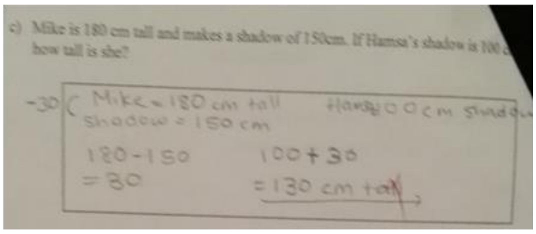
Pre-test response Post-test response

Figure 35: Student 27: pre and post-test solution of the stick length question

Apart from the students who made more gains, there are students from the intervention class that, although there was a slight improvement in the marks, there are still issues with understanding of some concepts, such as ratios. For example, the work of the student shown in Figure 36 shows some challenges with understanding. It would appear that the student's understanding of ratios is adding the same amount instead of multiplying by the same amount.



1st Student



2nd Student

Figure 36: Student 28: pre and post-test solution of Hamsa's shadow question

Solving mathematical problems can be daunting at times. The lapse of concentration and not reading the question properly are some of the challenges that also contributed to the drop in the marks.

The intervention 1 class p-value of 0,0165 and the intervention 2 class p-value of 0.0037 mean that the results are statistically significant. This means that the change in averages is highly unlikely to have happened by chance. One of the factors that might have had an impact on the change in averages for the intervention 1 class is the teaching itself. However, the 'chance' factor cannot be ruled out. It is possible that the change in averages happened by chance. On the other hand, the intervention 2 class p-value as well suggests that the 'chance' factor cannot be ruled out and that there are factors that are likely to have influenced the averages. In case of the intervention 2 class, it might be that when I asked the class to get on with solving the problems given to them, I may have indirectly given them heads up to do the problems in an exploratory manner which may have resulted in positive influence on their post-test average.

The effect size of the intervention 1 class of 0.26 means that the change in the averages is not large which support the 10% improvement in the average from the pre-test to a post-test.

In summary, it is widely known that there are issues around mathematical problem solving, as is the case in my school. The question is, what is the extent of this problem? This research has shed some light on this with problem solving questions involving different topics. In this case, problem-solving questions involving ratios is an issue in my school. This necessitates that teachers in my school spend more time guiding and assisting students to become good problem solvers. Silver (1981) has argued that good problem solvers use information and procedures that they have gained from previous experiences and training. If students have not been educated before to solve problems, they are likely to be unsuccessful in solving problems in future. The guidance was significant during teaching in this study. Hence there were shifts and gains in some items in terms of strategies of solving different problem. Depending on how students have been exposed to problem solving, some students struggled to solve problems because they had not been exposed to problem solving. Some students struggled with using models to solve problems and reverted to the ways that they had been exposed to, such as trying to find a formula, which at times did not succeed. Therefore, some individual students showed more gains as they were able to apply previous knowledge.

The situation at the school did not allow both classes to write at the same time. This means that there are factors that I could not control, such as students talking about the test with other students that had not written the test, consequently giving students that had not written the upper hand over those that had written already. I believe that with different conditions, this study could yield different results.

7 Recommendations and conclusion

In conclusion, some students were able to do problems that they would not have been able to do on their own. Vygotsky (1978) would argue that the students were working in ZPD, the difference between what students could do on their own and what they could do with the assistance from the knowledgeable being.

Firstly, a period of six weeks and six lessons was never going to be enough to deal with many different types of mathematical problems. There is a variety of mathematical problems. It is simply not possible to have dealt with all of them thoroughly in six lessons. Secondly, two classes of 18 students in a class are just not a big enough sample to start making generalisations about whether the approach taken in teaching is likely to improve learning outcomes. Thirdly, it is worth pointing out that the same study with a different group of students, in a different environment might yield different results. I would have liked to give the same group of students a delayed post-test to see if the skills acquired from the teaching are transferable. Therefore, giving these students a delayed post-test in their current grade, which is grade 9, might perhaps assist in finding out which class between intervention and control class made long term gains.

Given the opportunity to conduct the study again, I would investigate the intervention 2 class's improvement further by giving students a test to determine their attitude after writing the pre-test and the post-test. The test would have had questions such as, how did you feel before and after writing the test? I would then look at the marks attained by the student and compare it to their responses for the test. Using the available technology, I would find the regression line and work out if there is any correlation in the marks attained and attitude towards the test. A correlation co-efficient could be calculated to determine how strong or weak the correlation is.

The teaching offered to this selected few students does shed some light that perhaps with a larger sample and with the teaching over a longer period of time and also with a different class, the approach taken in teaching might improve or is likely to improve learning outcomes. Further research with different students, a bigger sample and with more time will have to investigate this topic further.

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Appendices

Appendix A Mathematical Problem solving scoring guide

Process Dimensions	**4**	**3**	**2**	**1**
Making sense of the problem Interpret the concepts of the task and translate them into Mathematics	The interpretation and /or translation of the task are • thoroughly developed and /or • enhanced through connections and /or extensions to other contexts	The interpretation and translation of task are • adequately developed and • adequately displayed	The interpretation and /or translation of the task are • partially developed, and/or • partially displayed	The interpretation and/or translation of the task are • underdeveloped, • sketchy, • using inappropriate concepts, • minimal, and/or • not evident
Using information appropriately Takes the given information and uses it in suitably in trying to find an answer	Shows an understanding of why certain information is essential in solving the problem	Uses all information correctly	Uses some appropriate information correctly	Uses inappropriate information
Representation and solving the task Use models , pictures, diagrams and or symbols to represent and solve the task situation and select the effective strategy to solve the problem	The strategy and representation used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy that has been selected and applied and the representations used are • partially effective and/or • partially complete.	The strategy selected and representations used are • underdeveloped, • sketchy, • not useful, • minimal, • not evident, and/or • in conflict with the solution/outcome.

Figure 7: The rubric used to assess students' pre-test and post-testadapted from Oregon Department of Education mathssp scoring guide combined with a rubric from Northwest Regional Education Laboratory Mathematics Center.

Appendix B:

CAPS

4.4 Programme of Assessment

The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999. Descriptors for each

level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge 20%	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle ....
Routine Procedures 35%	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1. Solve for $x : x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2\sin(x - 30^\circ) + 1 = 0$ (Grade 11) 2. Prove that the angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle. (Grade 11)
Complex Procedures 30%	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is $100km/h$ and the average speed for the return journey is $80km/h$? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x. (Grade 12)
Problem Solving 15%	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

The Programme of Assessment is designed to set formal assessment tasks in all subjects in a school throughout the year.

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MATHEMATICS GRADES 10-12

Figure 1: The programme of assessment to guide all assessment tasks

Appendix C

NATIONAL SENIOR CERTIFICATE: MATHEMATICS: PAPER I – ANALYSIS GRID Page 1 of 1

Question	Degree of difficulty of the question									
	MARKS				Algebra and Equations and Inequalities	Patterns and Sequences	Finance, Growth and decay	Functions and Graphs	Calculus	Probability
	K	RP	CP	PS						
1 (a) (1)	3				3					
1 (a) (2)	3				3					
1 (a) (3)		3			3					
8 (b) (2)			7			7				
9 (a)				4						4
9 (b)			6							6
9 (c)				5						5
10 (a)				7					7	
10 (b)				7	7					
Total	**29**	**48**	**50**	**23**	**28**	**26**	**14**	**29**	**38**	**15**

Figure 2. The analysis grid for national senior certificate: Mathematics: Paper 1. IEB. 2015

Appendix D

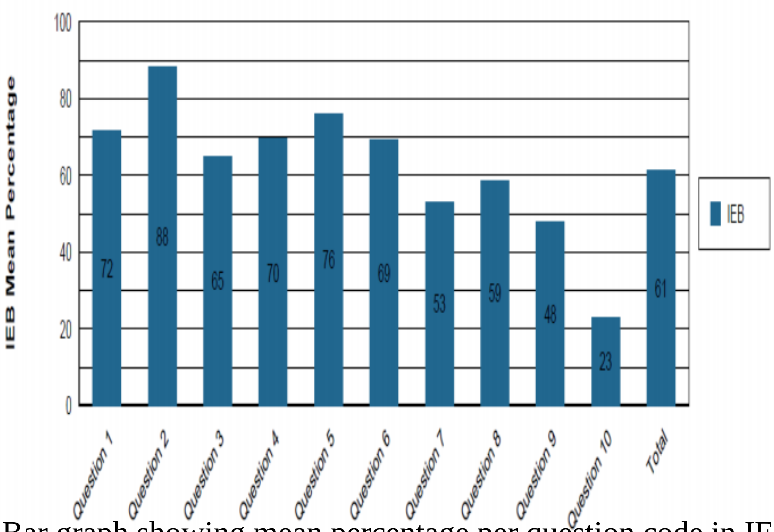


Figure 3: Bar graph showing mean percentage per question code in IEB paper 1. 2015

Appendix E

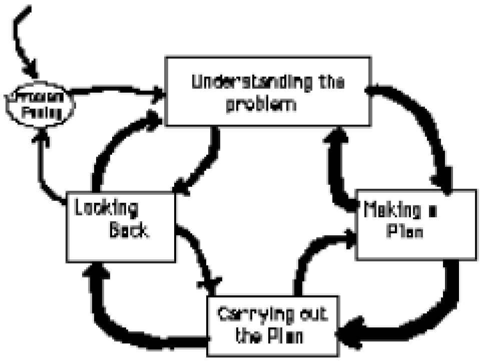


Figure 4: Brown and Walter's framework of problem posing, an illustration of Polya's problem solving model.

Appendix F

<u>Grade 8 Pre/ Post- test</u>

a) There are 4 red pens in Anna's desk drawer. There are 3 more black pens than red pens. There are also 7 more blue pens than red pens. How many pens are there in all? (3)

b) At a sale, Mrs Math spent R6360 on a table, a chair and an Iron. The chair cost R720 more than the Iron. The table cost R960 more than the chair. How much did the chair cost? (4)

c) Mike is 180 cm tall and makes a shadow of 150cm. If Hamsa's shadow is 100 cm, how tall is she? (2)

d) A stick of length a makes a shadow of length b. At the same time a tree has a shadow of length c. How tall is the tree in terms of a, b and c. (4)

e) Three lights flash at different intervals:

i) The first flashes every 72 seconds
ii) The second flashes every 24 seconds
iii) The third flashes every 9 seconds

If they start off together, after how many seconds (or minutes) will the lights flash all at the same time? (4)

f) A unit fraction has 1 as the numerator and a denominator greater than 1. The ancient Egyptians used only unit fractions. For $\frac{5}{8}$ they wrote the sum as $\frac{1}{2}+\frac{1}{8}$

Give two ways the Egyptians could write $\frac{10}{21}$ as a sum of unit fractions? (4)

g) Matchsticks are arranged as shown:

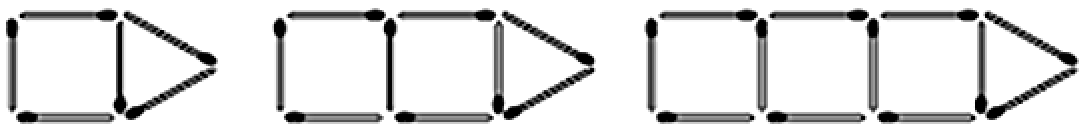


Figure 1 Figure 2 Figure 3

i. How many matchsticks are required for Figure 5? (2)

ii. Which Figure would need exactly 51 matches? Explain how you got your answer. (2)

iii. Give an expression for the number of matches required for the n^{th} figure. (2)

h) Helen has 24 red apples and 12 green apples. What fraction of her apples are green? (2)

i) In a theatre, between each row the number of chairs in a row increases by a constant amount. That is the increase from row 1 to row 2 is the same as the increase from row 2 to row 3 and so on. There are 23 chair in row 1. Row 10 has 50 chairs and the last row has 353 chairs. How many rows are there in the auditorium? (3)

j) Molly likes to call her friend every Thursday night. Sometimes they have a lot to talk about, other times not so much. There are two phone companies Molly can use: TCC Cellular charges R0.10 per minute. UFC Network charges R0.50 for the first minute plus R0.08 for every minute thereafter. If Molly uses TCC Cellular for 8 minutes:

i) How much will her charge be? (2)

ii) Using your answer from i), how much time could Molly have spoken for using the UFC network? (2)

TOTAL: 36

Appendix G

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called: an intervention to improve Grade 8 learners mathematical problem solving.

My name is: ________________________

Permission to review/collect documents/artifacts	**Circle one**
I agree that my test scripts can be used for this study only.	YES/NO
Permission to be audiotaped	
I agree to be audiotaped during the intervention lessons	YES/NO
I know that the audiotapes will be used for this project only	YES/NO
Permission for test	
I agree to write a test for this study.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotape
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign______________________________ Date____________________________

Appendix H

Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called an intervention to improve Grade 8 learners mathematical problem solving.

I, ________________________ the parent of ______________________

Permission to review/collect documents/artifacts	**Circle one**
I agree that my child's test scripts can be used for this study only.	YES/NO
Permission to be audiotaped	
I agree that my child may be audiotaped during intervention lessons.	YES/NO
I know that the audiotapes will be used for this project only	YES/NO
Permission for test	
I agree that my child may write a test for this study.	YES/NO

Informed Consent

I understand that:

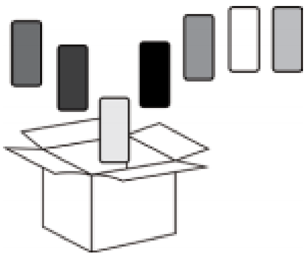
- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audiotaped, photographed and/or videotape
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

Appendix I

QUESTION 9

(a) A box contains 7 cards numbered 1 to 7. Two cards are drawn at random without replacement. Find the probability that the numbers on the two cards drawn out of the box give an odd product.



(4)

(b) For two events A and B, in the sample space S, it is given that $P(A) = 0,55$; $P(B) = 0,6$ and $P(A \text{ and } B) = 0,25$.

(1) Draw a Venn diagram to represent the information. (3)
Determine:

(2) $P(A \text{ and } B')$ (2)

(3) $P(A \text{ or } B')$ (1)

(c) A group of friends decide to plan a trip to Europe with the intention of visiting Rome, Madrid, Florence, Milan, Geneva and Paris. They choose the order of their visits randomly.

(1) Determine the possible number of different orders of their visits. (1)

(2) If Rome, Madrid and Florence are grouped together in that order, determine the number of different orders of their visits. (1)

(3) What is the probability that they will visit Rome, Madrid and Florence one after the other in any order. Give your answer correct to one decimal place. (3)

Appendix J

QUESTION 10

(a) The diagram below shows a solid right circular cone that is set centrally within a hemispherical container.

The radius of the hemisphere is $5\sqrt{3}$ cm. Of all possible right circular cones that can fit into the hemisphere, calculate, showing all working, the value of x (height of the cone in cm) for which the cone will have a maximum volume. Let radius of cone be p cm.

Useful formulae:

Volume of a Sphere $= \frac{4}{3}\pi r^3$

Volume of a right circular cone $= \frac{1}{3}\pi r^2 H$ (7)

(b) Busi would like to become an Olympic sprinter.

Her younger sister Khanya helps by racing against her.

When they tried the 100 metre sprint, Busi crossed the winning line when Khanya was still 25 metres short of it.

Busi wanted something more challenging, so it was agreed that Busi would start 25 metres behind the starting line.

They both ran at exactly the same speeds as in the first race.

Where was Busi and Khanya when the winning line was crossed by whoever arrived at it first? Explain your answer. (7)

[14]