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DESIGN OF THE TRACKING COMPLEX FOR A TRACKING RADAR SYSTEM.

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ABSTRACT

This report deals with the design of the Tracking Loop and those functions involved with it, for a ship-borne Tracking Radar System.

The report begins by placing the problem in perspective from an historical point of view by surveying some of the literature available.

A realistic environment in which the radar could be expected to operate is postulated in some detail. Following that, the actual design of a tracker to suit that environment is conducted.

Simulations of proposed configurations were a major part of the design methodology, as they best indicated the performance of the loop. These were written in FORTRAN-77 on a VAX Computer and displayed using modified PLOT-10 and PLOT-21 software packages.

Since the design and simulation of the entire Tracking Radar System was not covered in this dissertation, no major conclusions could be drawn concerning the performance of the Tracking Loop in the overall Tracking Radar System. Such results will have to await a full system simulation such as that proposed later in this report.

DECLARATION

I declare that this report is my own unaided work. It is being submitted for the degree Master of Science in Electrical Engineering at the University of the Witwatersrand, Johannesburg, and has not been submitted before for any degree or examination at any other University.

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(Name of candidate)

2nd day of December, 1983.

DEDICATION

I would like to thank Dr John Howard for the useful advice (both technical and grammatical) which has made this report readable.

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1.0 INTRODUCTION

1.1 Tracking Loop Aspects Of A Radar

The primary function of the tracking loop in a Tracking Radar is to generate estimates of the angle to a ta get, the range to a target and target range rate in order to drive the servos and hence point the radar antenna in the correct direction, irrespective of the target or radar platform motion or change in motion.

This function must, if possible, be performed continuously, irrespective of whether or not measurement data is being supplied by the sensor, or whether the supplied measurements have been corrupted in any way.

Measurement data will be corrupted by a number of disturbances, the result of which will be degraded tracking performance. The major sources of these disturbances will include measurement noise (glint, thermal, multipath, etc), ship's motion (roll, pitch and heave) and Electronic Counter-measures (ECM) such as Seduction and Jamming.

The first two problems may be overcome to a large extent by good tracking loop design. However it may be expected that ECM will pose a more difficult problem, possible solutions to which will be discussed in the report.

1.2 Overview Of The Design Strategy

The design of the tracking loop follows a particular path, the development of which was governed for the most part by the following restraints:

1. Specified Radar Sensor Design
2. Specified initial Servo Design and Timing
3. Limited information on Target Dynamics
4. Limited information on Measurement Noise
5. Ship-Borne System
6. Possibility of losing measurement information
7. Limited Processing Speed

1.2.1 Fixed Sensor And Servo Designs -

As both the sensor and servo systems have been specified, the tracking loop configuration has to be tailored to fit in with their data types and timing constraints.

1.2.2 Information On Targets And Noise -
With regard to the design of the tracking filters, it was felt that since this is critically dependant on the target motion and measurement noise spectra to be catered for, detailed information on target dynamics and measurement noise spectra was required. Hence the inclusion in Chapters 3 and 4 of a survey of these topics.

1.2.3 Ship-Borne System -
Because the effects of ship motion on tracking accuracy are quite considerable, it was essential that this motion be considered during the design of the tracking loop (Chapter 5). To implement the above design it was necessary to conduct a study of ship motion prediction techniques. The results of the study are outlined in Chapter 8.

1.2.4 Lost Measurement Information -
The problem of estimation in the case where the target position is not completely defined due to a lack of range or angle information is dealt with in Chapter 7.

1.2.5 Processor Speed -
Implementation of the designed loop has not been considered in this report. However, results of a brief analysis show that an 8087 microprocessor will be fast enough to run the tracking loop at the specified rate if careful design procedures are followed.

1.3 The Design Report

1.3.1 Design Justification -
Each stage of the design procedure will be suitably justified using either some reasonable argument or a mathematical formulation.

The conclusion of the above-mentioned procedure will be a series of FORTRAN Program listings, the construction of which would allow for their direct translation to Micro Assembler and subsequent inclusion in any Radar System.

1.3.2 Scope -
The report will include all aspects of the design of the tracking loop except for the implementation of joystick control functions.

It must be stressed that the tracking loop is only a small part of the whole system, hence even functions such as target acquisition, which would be essential for the operation of the tracking loop, are not included in this report.

2.0 BACKGROUND TO THE DIGITAL TRACKING PROBLEM

2.1 Historical Development

The purpose of the Tracking Radar is to measure the coordinates of a target with the object of determining its trajectory, and thus determining its position at some future time. In general, the better the accuracy of the initial measurement, the better the overall accuracy of the prediction.

It is possible to perform the basic tracking function using analog error actuated controllers; however, for optimum performance, the controller should match its bandwidth to that of the target which it is tracking. The complexity required to implement this adaptive strategy using analog circuitry makes it prohibitively expensive and unreliable, hence an alternative solution is required.

This alternative solution has been achieved in the past twenty years. As the sophistication required from tracking systems has increased and the cost of computers has decreased, most new systems have been developed using digital control techniques.

2.2 Literature Survey

A large amount has been written on the subject of Tracking Radar. In general, however, papers available in the 'open' literature have been limited to those concerned with estimation and prediction techniques and have not dealt with the specific problems involved in their implementation in real systems.

Hence (for the design of this system), since the purpose of this report is to design a tracking loop to the point where implementation details are investigated, as it was not possible to obtain much information concerning the working of existing Fire Control Systems as a whole, it was necessary to determine what had been written concerning each of the following sections:

1. Tracking Radar Systems
2. Tracking Filters
3. Aircraft Motion Models and Process Noise
4. Sensor Errors and Measurement Nois
5. Ship Motion

It should be noted that all of the texts refered to in the following subsections are listed in the Bibliography.

2.2.1 Tracking Radar Systems -

The basic principles by which tracking radars operate are well explained by Dunn et al (1.2) and v.d.Merwe (1.8). In particular the latter 'paper' at least introduces most aspects of tracking that must be considered in the design of a Tracking System.

A subsystem of the above which is also dealt with by v.d.Merwe (1.6) and (1.7) concerns the implementation of a range tracking loop.

Richardson (1.4) presents a more systems orientated approach, in which the role of Fire Control Radars for ship defence is evaluated. This aspect has become particularly important since the Falklands crisis where FCR Systems reportedly did not prove to be as effective as expected.

Lastly, Boyell (1.1) outlines some of the techniques used in the defence of a moving ship against attack.

2.2.2 Tracking Filters -

It is virtually impossible to read all that has been written on tracking filters in the last twenty years. However a number of 'classic' papers marking milestones in the development of digital tracking filters must be read. These include Benedict and Bordner (2.1) on 'The Synthesis of an Optimal Set of Track While Scan Smoothing Equations' and Kalman (2.11) on 'A new Approach to Linear Filtering and Prediction'.

In recent years most papers on Tracking Filters have been concerned with the Kalman Filter and its ramifications. The small number which have been read are listed in the Bibliography.

Because of the computational overheads involved with the implementation of the Kalman Filters, there is still a certain amount of research being conducted into other techniques where fast real time operation is essential. In this field the Alpha Beta and Alpha Beta Gamma Filters are the most popular. V.d.Merwe (2.25) and van Zyl (2.24) have conducted fairly detailed studies into the performance of the former, while Cantrell (2.5) and Simpson (2.19) have studied the latter.

For fast applications, Finite Memory filters are also quite popular, these are dealt with by Nesline and Zarchan (2.14), by Brooker (2.3) and by Harris (2.10). It has been found that for certain radar applications, this type of filter performs better than its recursive counterparts.

Lastly, in the field of estimation and prediction, there are a number of papers which compare and evaluate filter performances for particular applications, and in particular environments. Of particular interest in this case are papers by Singer and Behuke (2.20) and Brown et al (2.4). More detailed comparisons are presented by Mayiatis (2.12), Rametti (2.16) and Mendez (2.13) all of which are very useful in making a choice of which filter to use.

2.2.3 Aircraft Motion Models And Process Noise -

With the development of the Extended Kalman Filter, accurate models of aircraft dynamics have become very important, hence a fair amount of literature is now available in this field. Singer (3.6) presents the case for a constant velocity target with manoeuvres and atmospheric turbulence viewed as correlated perturbations upon this velocity. Gholson and Moose (3.3) use a Semi-Markov model for target accelerations, which Moose et al (3.5) later combines with the Singer model.

Observations of real radar measurements made by Harris (3.4) have lead to another process noise model in which a correlated velocity function is used.

Some of the models have become quite complex, for example Bryson (3.2) uses fifth order systems to model lateral and longitudanal motions of an aeroplane, however this is the exception rather than the rule.

Ahlers (3.1) attempts to define a number of operati tack profiles which can be used to test a Tracking Radar simu.

The testing of Tracking Radar Systems can be a problem, especially where Sea Skimming Missiles are involved, hence the role of simulation plays a more important part in their evaluation every day. For this reason the development of target models should not be neglected.

2.2.4 Sensor Errors And Measurement Noise -

A detailed overview of the effects of Sensor errors on Gunfire Control is presented by Seifer (4.14). This paper is useful from an error budget point, however with regard to the actual measurement errors involved, both Dunn et al (4.5) and Howard (4.6) have produced comprehensive summaries of most of the forms of measurement noise that can be expected.

In a more specific vein, Barton (4.3), Mrstic and Smith (4.9) and Asseo (4.1) investigate multipath problems associated with low angle tracking, and propose some possible solutions. These papers are most important as they point out some of the difficulties involved with tracking low altitude targets such as Sea Skimming Missiles.

Target glint is a major source of measurement noise; it may however be reduced quite substantially by using frequency agility as explained by Lind (4.7) and Loomis and Graf (4.8).

Finally a report by Pryce (4.12) supplies details of a general purpose Radar Sensor Package which will simulate most of the measurement noise types found in the above references.

2.2.5 Ship Motion -

Ship motion spectral densities are supplied in papers by Gibson (5.1) and Harrison (5.3), while details of a navigation system for the measurement of the above can be found in the Technical Report (5.5).

Harris (5.2) derives a elegant method for the rotation of coordinate systems as required for the removal of ship motion.

2.2.6 General -

As well as the papers and reports mentioned above, a number of comprehensive texts exist on radar in general, these include; Skolnik (6.4), Skolnik (6.5), Barton (6.1) and Brookner (6.2).

With respect to signal processing techniques Schwartz and Shaw (6.3) is very useful, as is Stanley (6.6) from a digital point of view.

2.3 Conclusion

As such a vast amount of research is being undertaken in the Tracking Radar field, in general it should be possible to solve most problems encountered by finding and reading the correct paper on the subject.

This method has been followed to some extent. However, it is felt that by following existing developments too closely, the originality of the research may be lost.

The extent to which other people's work has been used is indicated by references included in the main body of the report.

3.0 DETAILS OF PROFILES OF TARGETS TO BE TRACKED

3.1 Introduction

The lack of real data records has necessitated the development of a computer package which would be able to generate a series of realistic Target Profiles by which the Tracking Filters could be designed and evaluated.

Because the profiles are to be used for both of the above purposes, it is necessary that they include all possible configurations that may be met in reality, and particularly that they be 'worst case' realistic in all possible respects.

3.2 Expected Profile Types

Ahlers (1981) breaks the profile types down into two basic categories; viz. Aircraft and Ships. As the reason for the development of the model is to test the dynamic response of the Tracking Filters, it was deemed necessary to include only profiles which would tax the filters to some extent, hence ships have been excluded.

Current day piloted aircraft have the following typical major characteristics:

1. True Air Speed
   During Approach < 400m/s
   During Search and Attack < 300m/s

2. Angular Acceleration
   6g Maximum
   Time constant to 4g > 1.0sec

3. Linear Acceleration
   1.5g Maximum

4. Dive Angle
   60 deg Maximum

5. Cruise Height
   Minimum > 10m
   Maximum (typical) < 6000m

6. Climb Angle
   25 deg Typical

7. Radar Cross Section
   1 sq metre minimum (fluctuating - Swerling 3)

The radar may be required to track cruise missiles whose launching platform may be below the radar horizon, or guided missiles whose launching platform is above the radar horizon. The characteristics postulated for such targets are as follows:

Cruise Missiles

1. Air speed up to 500m/s
2. Cruise Altitude. Between 15m and 9000m
3. Attack Altitude. Minimum 2m above average one-tenth of highest wave height above mean sea level.
4. Diving Angle 65 deg maximum
5. Radar cross section 0.1 sq metre

Guided Missile

1. Air speed maximum 800m/s
2. Guidance Altitude minimum 3m
3. Homing Altitude. Minimum 2m above average one-tenth of highest wave height above mean sea level.
4. Diving Angle 65 deg maximum
5. Radar cross section 0.1 sq metre

The following typical profiles are generated:

3.2.1 Piloted Aircraft -

3.2.1.1 Pitch-up Dive Attack -

Typically this profile would involve the aircraft approaching the ship at a low altitude (30 to 50 metres) to delay detection by the ships early warning system. Once the aircraft is within 10km it will climb to an acceptable altitude, turn toward its target and proceed with a dive attack. The trajectory of the attack is a function of the weapon type carried by the aircraft.

The following characteristics have been postulated for typical pitch-up attack:

Approach Cruise Altitude		> 30m
Approach Speed		< 400m/s
Off Angle Approach		typ 45 deg
Pull up Range (from ship)		min 5000m
Pull up Angular Acceleration		max 5g
Climb Angle		typ 30 deg
Apex Altitude	(bomb run)	typ 2000m
	(rocket run)	typ 2000m
	(gun run)	typ 1000m
Dive LOS Depression	(bomb run)	typ 8 deg
	(rocket run)	typ 45 min
	(gun run)	none
Speed at Weapon Release Point		max 300m/s
Release Point:	(bomb run)	1200,900m
coordinates (x,z)*	(rocket run)	1500,900m
	(gun run)	1000,400m
Dive Recovery Angular Acceleration		typ 6g

* refer to Fig.3.4 for a definition of coordinates

3.2.1.2 High Altitude Dive Attack -
The final approach in this form of attack is assumed to be similar to that in the pitch-up attack, the differences being in the method of approach.

In general the Aircraft approaches its target directly at an altitude of between 6000 and 9000 metres. Once the target is near enough, the aircraft loses altitude as fast as possible and then proceeds with a dive attack.

Characteristics of the attack type are tabulated as follows:

Approach Cruise Altitude	typ 6000m
Diving Turn angular acceleration	typ 3.5g
Initial Diving Angle	40 - 60 deg

3.2.2 Missile Attack Profiles -
Currently there are five methods employed of guiding a missile to its target:

1. LOS (line of sight guidance). The attacker tracks the target and then commands the missile to track the target line of sight.

2. Wire Guided or Fibre Optics are used by the attacker to communicate with his missile. Missile range is limited by the length of wire (typ max 5000 metres).

3. Semi Active Homing. The missile carries the receiving end of a tracking device. It therefore homes in on a target that has been illuminated by a non resident source.

4. Active Homing. The missile actively employs a tracking device to home in on the target.

5. Passive Homing. The missile carries a sensor (infra red) to track and home in on the target.

6. Pre-programmed Guiding. The Missile memory is programmed with a flight path and impact point, however it is unlikely that this type of guidance will be used against a moving target.

The following sections detail the profiles that will be generated to represent the above types of missile flight.

3.2.2.1 High Altitude Diving Profile -
The missile approaches at a high altitude (max 9000 metres) until it reaches the predetermined range, it then dives directly toward the target.

Air Speed	cruising	< 500m/s
	diving	< 800m/s
Cruise Altitude		< 9000m
Diving Angle		< 65 deg
Diving Range		< 25km
RCS		> 0.1 sqm

3.2.2.2 Sea Skimming Profile -
The missile cruises toward the target at a fairly safe altitude (15 metres) until it is within 10km, it then dives to an attack altitude which will be dependent on the sea state.

Air Speed	< 500m/s
Cruise Altitude	> 15m
Attack Altitude	min 2m above average 1/10 of highest wave height above mean sea
Diving Range	typ 10km
RCS	> 0.1 sqm

3.3 Computer Generation Of Target Profiles

Each profile consists of a number of independent sections which are linked together by matching their position and velocity vectors at the point of joining. It was decided that to match acceleration as well would make the model too complex, so each section was constructed with zero acceleration at its beginning and end.

An example of the profile sections for the pitch-up dive attack is given:

APPROACH	Constant Veloc.ty
PULLUP	Z acceleration, keep overall velocity constant. Allow acceleration to decay so that acceleration is zero when the correct climb angle is reached.
CLIMB	Deceleration calculated to give the correct velocity at the top of the climb.
TOP TURN	Acceleration in X,Y and Z directions calculated to result in the correct velocities and altitude at the end of the turn.
BUNT	Constant speed profile with a *fixed LOS depression* until correct altitude is reached.
PULLUP	Z acceleration to ensure that the aircraft does not fly into the ground.

To ensure that the profile is generated correctly with respect to the ship, the following occurs. A dummy run is made, and the end points (x,y) noted, then a real run is made with the outputs shifted by the measured amount, so that the profile will end at the origin.

3.3.1 *Angular Acceleration Model* -

It was decided that a simple step would not be a sufficiently accurate model of aircraft acceleration. Without trying to model the aircraft dynamics completely, a more accurate model was constructed using a pair of cascaded low pass filters with their gains set to give a rise time of 1.5 sec for a 4g turn, as per Ahlers (1981).

A plot of the final acceleration model is shown in Fig.3.1. It has been found that this is a fairly accurate representation of aircraft dynamics.

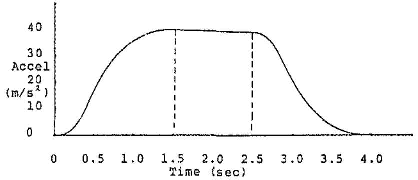

Fig.3.1 Plot of Angular Acceleration Profile

3.3.2 Process Noise Model -
Process noise can be thought of as variations of the target velocity vectors from those predicted kinematically, by such factors as local air turbulance and variations in aircraft thrust etc.

Measurements by Harris et al (1981b) indicate that this noise type may be broken down into a series of ramps and sinusoids. These can be differentiated to produce velocity steps of up to 4m/s with durations of up to 5sec.

A simple velocity model was constructed by passing white noise through a first order low pass filter with a cutoff frequency of 0.2Hz. A plot of the noise typically generated by this model is shown in Fig.3.2. The position is obtained by integrating this series, from which it can be seen that the noise on position does in fact resemble a construction made of ramps and sinusoids.

Ideally a certain amount of correlation should have been introduced as there is bound to be some cross coupling between the coordinates, however this was neglected in the interests of simplicity.

The process noise generated by the above model is added to the recorded velocity and position output of the profile generator, and these are recorded for use in the filter performance evaluation.

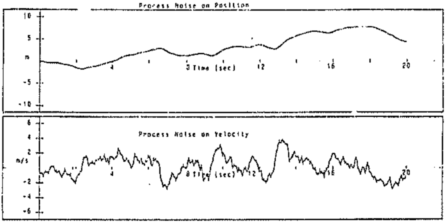

Fig.3.2 Plot of Typical Process Noise

3.3.3 Target Radar Cross-section (RCS) -

Skolnik (1970) shows that the RCS of an aircraft is directly related to its attitude with respect to the line of sight. In general this relation is only valid in the long term. In the short term the fluctuations of RCS have been catagorised in terms of Swerling Numbers as explained in Skolnik (1970).

In the interests of simplicity however, it was decided that only the long term RCS should be modelled. From Pryce (1982) this relationship is plotted for a jet aircraft in Fig.3.3, however in the case of missiles, the RCS is assumed to remain constant, as in general the missile is flying directly towards the Radar.

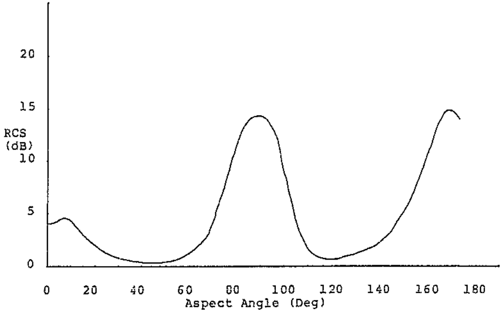

Fig.3.3 RCS of a Jet as a function of Attitude

3.3.4 Examples Of Profiles -
The figures on the following pages demonstrate the capabilities of the profile generator a listing of which may be found in Brooker (1983).

It is these seven profiles that are used to design and evaluate the tracking filters.

The Target position is plotted in the following coordinate frame:

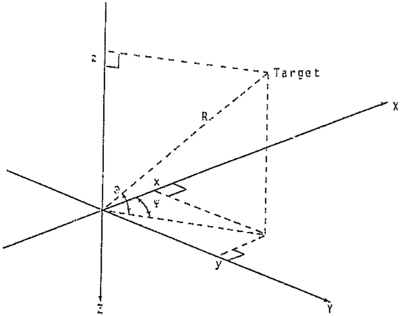

Fig.3.4 Target Coordinate Frame of Reference

The plot consists of four graphs which show the target position, velocity and RCS. It can be seen that the target position is displayed in both its polar and cartesian form. The former is included as it is what the Radar sees, while the latter is included as it is easier to visualise the target profile in 3-D from this representation.

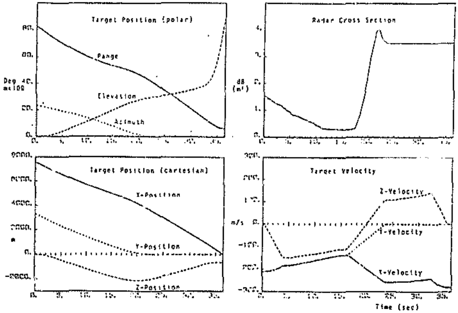

Fig.3.5 Pitch-up Dive Attack with Bomb Bunt

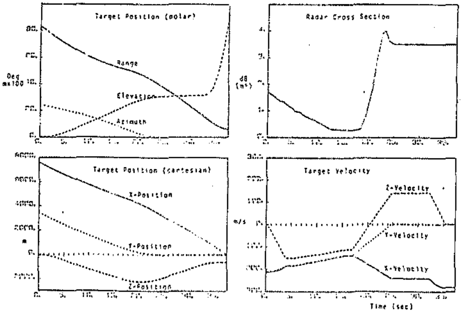

Fig.3.6 Pitch-up Dive Attack with Rocket Bunt

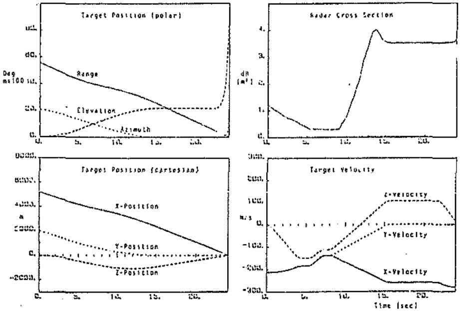

Fig.3.7 Pitch-up Dive Attack with Gun Run

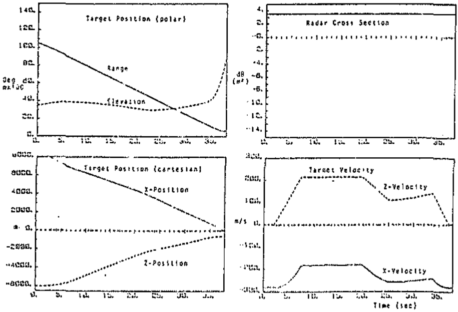

Fig.3.8 High Altitude Attack with Bomb Bunt

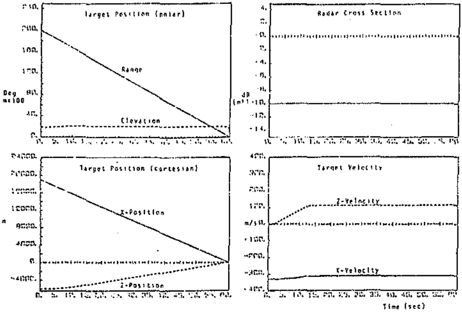

Fig.3.9 High Altitude Guided Missile Attack

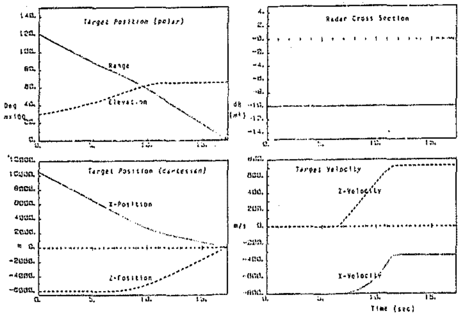

Fig.3.10 High Altitude Cruise Missile Attack

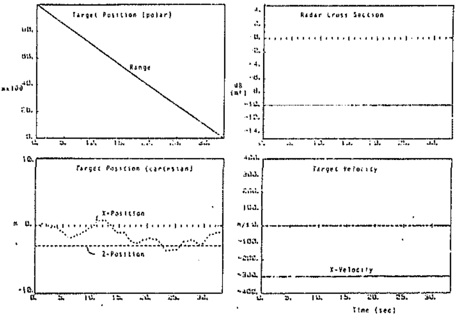

Fig.3.11 Sea Skimmer Attack

3.4 Determination Of The Frequency Content Of A Manoeuvre

3.4.1 Introduction -

Before the design of the tracking filters could be initiated, it was advantageous to investigate aircraft dynamics from a frequency domain point of view.

It would be possible to achieve this end by determining the frequency content of the target profiles illustrated in the previous section, however it was decided that a more simple analytical approach would be easier to modify or interpret. The method used in the end allowed the determination of the peak amplitude of a manoeuvre as a function of its fundamental frequency.

3.4.2 Frequency Content Of A Manoeuvre (position) -

An aircraft manoeuvre is bounded by the maximum acceleration which is acceptable to either the structure or the pilot. A profile which maintains the maximum acceleration would consist of a circle, or linked arcs of a circle, as shown in Appendix A. Hence to determine the frequency content of such a manoeuvre it is necessary only to determine its spectral content. The circle is defined by a maximum radial acceleration and a velocity. The repetition period and depth of the arc define both the major frequency component, and the magnitude of the contribution.

The relationship between the magnitude of the contribution and the frequency is defined by the following formula which is derived in Appendix A.

$$A = \frac{v^2}{a}\left[1 - \cos\left(\frac{a}{4vf}\right)\right] \qquad f > \frac{a}{4.pi.v}$$

A = Amplitude (m)
a = Acceleration (m/s^2)
v = Velocity (m/s)
f = Frequency

The above relationship is graphically displayed in the following figure.

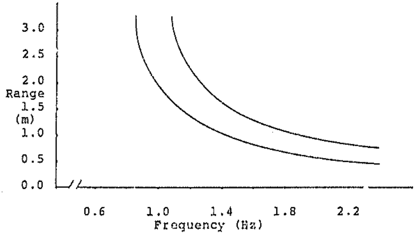

Fig.3.12 Frequency content of a Manoeuvre

3.4.3 Frequency Content Of A Manoeuvre (velocity) -
This relationship is derived in a similar way to that for the position. However, it has been found that the accuracy with which the velocity may be determined is a function of the initial velocity.

The relationship is described by the following formula derived in Appendix A, and illustrated in the following graph (Fig.3.13).

$$dR = v \left[1 - \cos\left(\frac{a}{4vf}\right) \right]$$

dR = Magnitude of velocity change (m/s)
Other variables have been defined

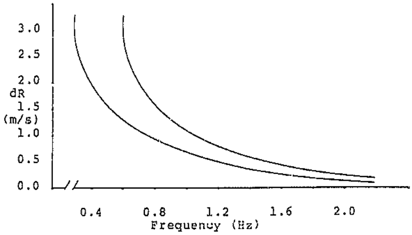

Fig.3.13 Velocity Frequency Content of Manoeuvre

3.4.4 Conclusions -
It can be seen from the graphs that the possible dynamics of a jet aircraft limit its bandwidth to below 1.0Hz for manoeuvres exceeding 2.0m or 1.5m/s. Hence if it is required that a target be tracked to within those limits by an 'ideal' t cker in a noise free environment, its bandwidth need not exceed the 1.0Hz specified.

4.0 MEASUREMENT NOISE

4.1 Introduction

Using information about Radar Sensors from Barnard et al (1981) and much that has been written on radar measurements from Dunn and Howard (1970a) and Barton (1974) amongst others, it has been possible to construct a highly simplified model of the measurement noise that can be expected from the complete system. The rest of the chapter outlines the the most important findings of the investigation, and ends with a table outlining the noise amplitudes and spectra used in the design of the tracking loop.

4.2 Signal Processing In The Sensor

With regard to tracking, the object of the sensor is to measure the range of a target and the angular error of the radar boresight to the line of sight angle to the target.

Reflected signals are coherently detected and sampled using fast sample and hold circuits known as range gates. Both the I and Q channels of both the sum and difference channel outputs are sampled. It is the contents of these gates that are used to generate the range and angular measurements supplied by the Sensor.

The signal processing involved with extracting the range and angles is explained in the following sections with the aid of Fig.4.1.

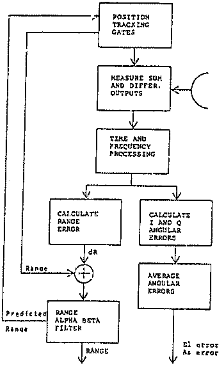

Fig.4.1 Sensor Signal Processing

4.2.1 Range Tracking -
Range measurement is performed in two parts. Initially an Alpha Beta filter is used to predict the target position for the next sample period. Measurements are made at that range, and the range error calculated using a variation of the Early/Late Gate technique explained in v.d.Merwe (1981c). The sum of the predicted range and the range error is taken as the measured range.

The following formula is used to determine the range error:

$$Re = \frac{[sL] - [sE]}{[sL] + [sC] + [sE]}$$

Re	= Range Error
[sE]	= Amp sq output of Early Gate
[sL]	= Amp sq output of Late Gate
[sC]	= Amp sq output of Central Gate

It can be seen from Fig.4.2 that the resultant Transfer Function is highly nonlinear, hence the advantages of operating in the linear region need no explanation. For this reason accurate prediction of target range is required, for this reason the Sensor will accept an external accurate estimate of range rate.

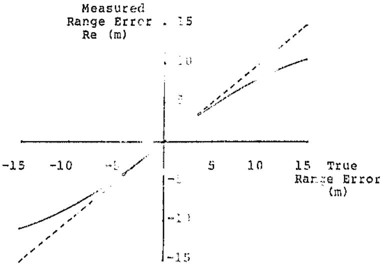

Fig.4.2 Range Transfer Function

4.2.2 Angular Error Estimation -
For a Two Axis Monopulse Tracking Radar if both the I and Q components of the tracking range gate sum and difference channel outputs are used, the I and Q components of the angular error are calculated for both Elevation and Azimuth.

The following formulae are used to perform the angular error calculations:

$$AI = \frac{dAI.sCI + dAQ.sCQ}{[sC]}$$

$$AQ = \frac{dAQ.sCI + dAI.sCQ}{[sC]}$$

AI	= Inphase angular error
AQ	= Quadrature angular error
dAI	= Angular difference Channel Inphase component
dAQ	= Angular Difference Channel Quadrature component
sCI	= Central Gate Sum Channel Inphase Component
sCQ	= Central Gate Sum Channel Quadrature Component

The angular measure output from the sensor is generally the I component only, because the Q component is usually zero unless multipath returns exist. It is assumed that this Q component is calculated in case complex target resolution is required for multiple target returns.

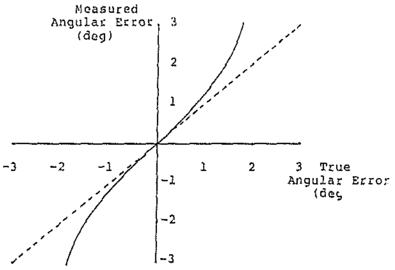

Fig.4.3 Azimuth and Elevation Transfer Functions

4.3 Multipath

In general, multipath effects are restricted to the elevation channel only, however in the case of a shipborne radar system, a certain amount will be coupled into the azimuth channel as the ship tilts.

Mrstik (1978) divides the multipath effects into three main regions:

1. Sidelobe Reflection Region, where the main beam is clear of the surface of the sea and reflections enter the sidelobes only.

2. Mainlobe Reflection Region, where surface reflections appear at one side of the main beam, but fading is moderate.

3. Horizon Region, where the target and the image are close enough that nearly full antenna gain applies to both, and specular reflections may cancel the direct signal almost completely.

4.3.1 Sidelobe Region -
Significant errors may arise from specular reflections entering the sidelobes of the beam, the RMS error can be approximated by the following formula:

$$Er = \frac{r.BWe}{\sqrt{8.Gse}}$$

Er = RMS Tracking Error (rad)
r = Reflectivity
BWe = Elevation Beamwidth (rad)
Gse = Sum Channel main lobe to difference channel sidelobe gain ratio.

4.3.2 Mainlobe Region -
For reflecting sources within 1.5 beamwidths of the beam axis, reflections will have entered the main lobe of the difference channel. Tracker response must be calculated from the vector relationships of the sum and difference channels arising from the target and reflection positions and their relative phases.

4.3.3 Horizon Region -
This is a region of unstable tracking in which the radar may track the centroid of the target and its reflection, or nod from one to the other as the signal strength of each echo varies. A point is reached at a target elevation of about one third beamwidth where cancellation due to strong specular returns may result in deep signal fades.

4.3.4 Multipath Model -
A simple model was constructed with reference to Mrstik (1978) and Barton (1974) which was used to illustrate some of the effects of multipath on the tracking of a target. A FORTRAN listing of the program is included in Brooker (1983).

The simulation used a pair of exponential functions to model the sum and difference patterns for the Four Horn Monopulse Antenna. These equations are listed below:

$$SUM(a) = e^{-1.386(a/BW)}$$

$$DIF(a) = 1.94(a/BW)e^{-0.83(a/BW)}$$

SUM(a) = Gain in sum Channel
DIF(a) = Gain in difference channel
a = Angle with respect to beam axis
BW = Elevation Beamwidth

The model assumes that the specular return will be shifted in phase from the direct return by the signal path difference and attenuated by the reflectivity of the sea.

The sum and difference channel contributions will be a function of the gains in the directions of the direct and specular signals, and their relative phases as illustrated by the following formulae:

$$S = Ae^{jwt} SUM(ad) + rAe^{j(wt+p)} SUM(ar)$$

$$D = Ae^{jwt} DIF(ad) + rAe^{j(wt+p)} DIF(ar)$$

S = Sum channel signal
D = Difference channel signal
A = Reflected signal amplitude
r = Reflectivity of the sea
ad = Angle of direct return wrt beam axis
ar = Angle of specular return wrt beam axis
p = Phase shift of Specular Signal

The real and imaginary components of the above signals are derived mathematically, and these are substituted into the formula for the angular error which is given at the beginning of the chapter.

Fig.4.4 and Fig.4.5 show the results of simulations for a Sea Skimmer at an altitude of 7m. The three curves in each plot show the true elevation, and the measured elevation for both normal and open loop offset tracking.

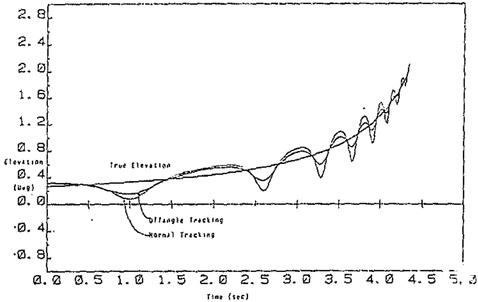

Fig.4.4 Specular Multipath for a Sea Skimmer (ref=0.5)

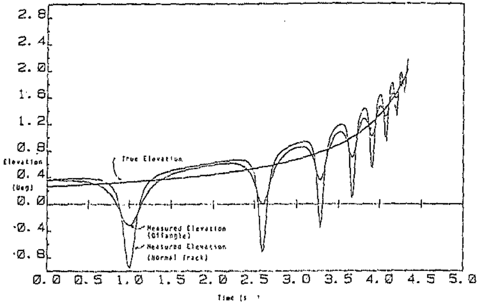

Fig.4.5 Spec lar Multipath for a Sea Skimmer (ref=0.9)

4.3.5 Offangle Tracking -
Mrstik (1978) states that it is possible to improve tracking during multipath conditions to some extent by tilting the radar antenna up from the horizon. This reduces the gain in the direction of the reflected signal, and hence reduces its effect of the tracking accuracy.

One disadvantage of this form of tracking is that the target signal return is in a very nonlinear portion of the angular error transfer curve, hence an accurate mapping is required between the measured angular error and the real error. A second disadvantage is that the antenna gain in the direction of the target is also reduced, so if the RCS of the target is small, signal strength problems may arise.

4.4 Glint

According to Dunn and Howard (1970b), glint noise is generated by the following mechanism; to a radar beam, a target such as an aircraft appears to be a large number of independent reflectors each of which moves relative to the others as the aircraft changes its attitude. The changes need only be of the order of the wavelength of the radar to cause a major change in phase between the reflected signals. Hence the vector sum of the return signals (which is generated by the radar) appears to wander all over the actual target, and even beyond its physical boundaries.

4.4.1 Glint Magnitude And Spectrum -
According to Dunn and Howard (1970a), the RMS magnitude of the glint noise is a function of the aircraft shape, especially such appendages as wing tanks, or other good reflectors on the wingtips.

Ar = 0.1 Lt	[No tanks]	Aircraft viewed from
Ar = 0.3 Lt	[Tanks]	the front
Ar = 0.3 Lt		Viewed from the side

Ar = RMS Glint (metres)
Lt = Target dimension perpendicular to LOS (metres)

Glint spectra are generally confined to fairly low frequencies with the actual bandwidth a function of the target velocity in the direction of the measurement. Hence an aircraft approaching radially with very little tangential motion can be expected to show wide bandwidth glint in range and narrow bandwidth glint in angles.

An empirical formula from Lind (1968) relating the Bandwidth of the glint to the angular velocity is as follows:

$$Bg = \frac{2.L.w}{0.03}$$

Bg = Glint Bandwidth (Hz)
L = Target angular dimension (m)
w = Angular Velocity (rad/s)

A formula which describes the spectral distribution of typical glint noise again from Dunn and Howard (1970b) is as follows:

$$N(f) = Var \cdot \frac{2.Bg}{Pi.(Bg^2 + f^2)}$$

N(f) = Noise power density (power/Hz)
Bg = Glint Bandwidth (Hz)
f = Frequency (Hz)
Pi = 3.14159....

4.4.2 Glint Model -
Glint Noise is modelled by passing white noise through a si first order low pass filter with the correct bandwidth. The fi is digital and was constructed using the Bilinear Transform as follows (Stanley 1975):

$$H(z) = \frac{b(1+z^{-1})}{1 + az^{-1}}$$

$$d = \frac{1}{Tan(Pi.fc/fs)}$$

$$b = \frac{1}{1+d} \qquad a = \frac{1-d}{1+d}$$

fc = Cutoff frequency (Hz)
fs = Sampling Frequency (Hz)

4.4.3 Glint Spectral Smearing And Frequency Agility -
The case of a radar sensor with true frequency agility (random selection of any frequency between two limits) has been studied by Lind (1968). He shows that for a jump in frequency less than the critical limit no noise decorrelation occurs, while if the jump is greater than critical, complete decorrelation occurs.

$$fcrit = \frac{c}{2Lt}$$

fcrit = Critical frequency
c = Speed of light
Lt = Depth of the target perpendicular to LOS

For a "pseudo agile" radar in which there are only a few frequencies from which to choose, assuming that the same frequency was never selected twice in a row, there will never be any subcritical jumps and so ideally the full glint spectrum should be spread to the sampling frequency. However, if the number of available frequencies is too small, the time between subsequent transmissions would be too short to ensure total decorrelation, hence the glint spectrum would not be spread by the full amount.

It is possible however, that if the available frequencies are selected at random that this will aid with the overall decorrelation of the glint.

A model was constructed to test this hypothesis. In the model ten independent glint time series were generated, and sampled on a random basis, The spectrum of the resultant time series was determined using a 1024 point FFT and the results plotted. The glint cutoff frequency for the test was chosen to be 2.5Hz and the sampling frequency 5msec

To test the random sampling hypothesis, a model was constructed as above, but the glint series were sampled in sequence.

The above tests are illustrated in Fig.4.6. The normal glint spectrum has a 6dB per decade roll-off which is a characteristic of a first order low pass filter. The pseudo agile mode with sequential sampling shows by its periodicity that the correlation still exists in the glint series after a period of ten samples, however, the plot of the spectrum of the pseudo agile mode with random sampling shows by its flatness that the above process does in fact aid with the decorrelation.

The above tests show that a pseudo agile radar with as few as ten frequencies separated by more than the "critical limit" can serve to give the full benefit of frequency agility.

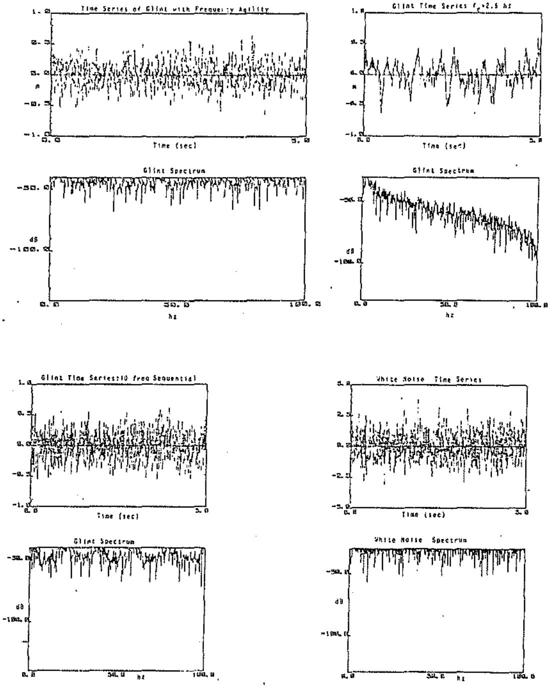

Fig.4.6 Glint Spectrum Spread using Pseudo Frequency Agility

4.5 Other Measurement Noise

There are a number of contributors which will be dealt with briefly.

4.5.1 Thermal Noise -

This contribution is assumed to be 'White' and will have a RMS magnitude given by the following formula: (Dunn and Howard 1970a):

$$Ea = \frac{BW}{km\sqrt{B.T.(S/N).(prf/Bs)}} \qquad \text{Angular Error (units of BW)}$$

$$Er = \frac{T}{\sqrt{(S/N).(prf/Bs)}} \qquad \text{Range Error (units of T)}$$

Ea = RMS Thermal Noise Error (Angles)
Er = RMS Thermal Noise Error (Range)
km = Angle-error detector slope
BW = Antenna 3dB Beamwidth
T = Pulse width
S/N = Signal to noise power Ratio at Servo
prf = Pulse repetition frequency
B = Receiver Bandwidth
Bs = Servo Bandwidth

4.5.2 Quantisation Noise -

This noise contribution is also assumed to be 'white' with a RMS magnitude given by the following formula (Barnard 1981):

$$Eq = \frac{6.6}{\sqrt{S/N}} \qquad \text{(mRad RMS)}$$

Eq = Angular Quantisation Error
S/N = Signal to Noise Ratio (>10dB)

There are a number of other errors whose contributions will be white but that are not quantified using a formula, among these will be errors in signal extraction and errors due to timing jitter.

4.6 Errors Caused By Sensor Nonlinearities

These are errors caused by the nonlinear Range and Angle Transfer Functions.

4.6.1 Range Nonlinearity Errors -

The Range Transfer Function for a modern tracking radar will be such that at the extremes of a range gate, the difference between the actual and measured target positions should not be greater than 1m. Hence if the central tracking gate is moved only when the target reaches its edge, for an aircraft approaching the radar at a constant velocity the resultant noise output will have the form of a sawtooth with a p-p amplitude of 2m.

If an aircraft was approaching at 300m/s the fundamental would have a frequency of about 20Hz and an RMS value of 0.6m. However, this noise is filtered before it is output. Assuming the filter bandwidth is 15Hz from Brooker (1981), the final noise contribution will be of the order of 0.2m RMS.

4.6.2 Angle Nonlinearity Errors -

In this case the error will be a bias, the magnitude of which will be a function of the slackness of the track. The track type is a function of the servo design which is not considered within the scope of this report, so no quantitative analysis will be made. However, the principles by which the error is generated will be the same as for the range error case.

4.7 Final Noise Model

In the interests of simplicity a number of assumptions have been made with respect to the measurement noise.

1. Targets are assumed to be approaching radially
2. Frequency Agility is not used
3. Multipath contributions are taken at the average height of the overall target profile

Using the above assumptions, the noise contributions listed on the previous pages can be summarised in the following table:

Table.4.1 Measurement Noise Contributions

Coordinate	Contributor	3dB Bwidth (Hz)	RMS Value (m,mrad)
RANGE	X-White	80	0.7
	Range Nolin	0 - 30	0.6 - 0.2
	Multipath	1.0	0.1
	Glint: A/C	25	5.0
	Missile	25	1.0
ELEVATION	X-White	80	0.002R
	Glint	4.0	5/R
	Multipath	1.0	20
AZIMUTH	X-White	80	0.002R
	Glint	4.0	5/R
	Multipath	1.0	5.0

R = Range in Km

5.0 DEVELOPMENT OF THE TRACKING LOOP

5.1 Introduction

The system which performs all the tracking functions will be called the Tracking Controller (TC).It is shown in the Tracking Radar context in th following block diagram.

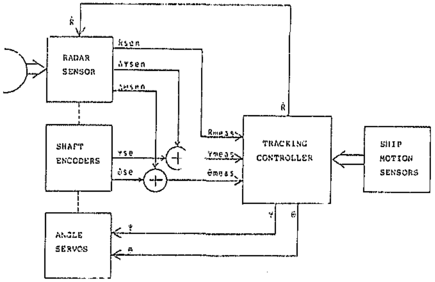

Fig.5.1 Block Diagram of a Tracking Radar

The Tracking Controller consists of the following components:

1. TRACKING FILTER GROUP. This includes the basic loop including all the coordinate transform blocks and the tracking filters proper.

2. SIGNAL FLOW CONTROLLER. Consists of all the logical components, software switches etc that are used to control the loop configuration.

3. SHIP MOTION EXTRAPOLATION. These are the routines which read the ships attitude (Roll, Pitch and Heading) and extrapolate it for use by the tracking filter group.

5.2 Function Of The Tracking Controller

The TC is the main link between the Radar's Measuring Functions and the Directive Functions. It takes measurement data supplied by the sensor, the shaft encoders and the ship motion sensors, and processes it in such a manner as to generate the 'best' information possible about the position and velocity of a target that is being tracked.

This in tion is used mainly to ensure that the Radar Sensor rema ng in the direction of the target. This is achieved by e angle servos with the direction (Elevation and Bea target.

The complex of the TC is due mainly to the fact that it must ensure that tracking continues regardless of any adverse conditions that may be encountered, and if track is lost, that the target be reacquired as quickly and efficiently as possible.

To ensure that the TC does perform its required functions, it must be able to track a target through all of the following distractions:

1. Range Gate Seduction
2. Predispensed Chaff
3. Selfdispensed Chaff
4. Angle deception (However it may be achieved)
5. Crossing Targets
6. Multipath Effects

The differentiation between the true target and those which may attempt to decieve the TC can be achieved by a number of methods:

Firstly, the adaptive tracking filters will be 'tuned' to the dynamics of the target being tracked, This fact should ensure that the filters memory will pull it through any fleeting disturbance.

Secondly, if it is seen that a disturbance is of such a magnitude that it is unlikely that the filters will pull through, then a memory track mode is selected, this will ensure that the filter's memory will not be further corrupted. Hopefully the predicted target position will then be accurate enough to allow tracking to continue once the disturbance has passed.

Other special features of the Tracking Controller that may help improve tracking are Imposed Altitude and Offset angle Tracking. These are explained fully in sections 5.7 and 5.8.

The following diagram illustrates how the Tracking Filter Group and Ship Motion Extrapolation Routines fit into the Tracking Controller structure. The representation shows the I/O configurations during both halves of the 40msec computer cycle.

LEGEND TO FIG.5.2

TRACKING FILTER GROUP:	Basic Tracking Loop including Filters
XTRAP xx	: Ship Motion Extrapolation Routines
SEC	: Routine to perform Secant Correction
TPD (xp,yp,zp)	: Target Position Data
SMS	: Ship Attitude (Roll, Pitch, Heading)
Datum	: Ship Position wrt Datum Point
V Ship	: Ship Velocity
$d_\psi$sen	: Sensor Azimuth Error
$d_\theta$sen	: Sensor Elevation Error
Rsen	: Sensor Range
ψsect	: Shaft Encoder Azimuth Reading
θsect	: Shaft Encoder Elevation Reading
Zimp	: Imposed Tracking Altitude
Xd,Yd,Zd	: Target Designation Inputs
$\psi r'$	: Estimated Azimuth (one sample ahead)
$\theta r'$	: Estimated Elevation (one sample ahead)
Rr'	: Estimated Range (one sample ahead)

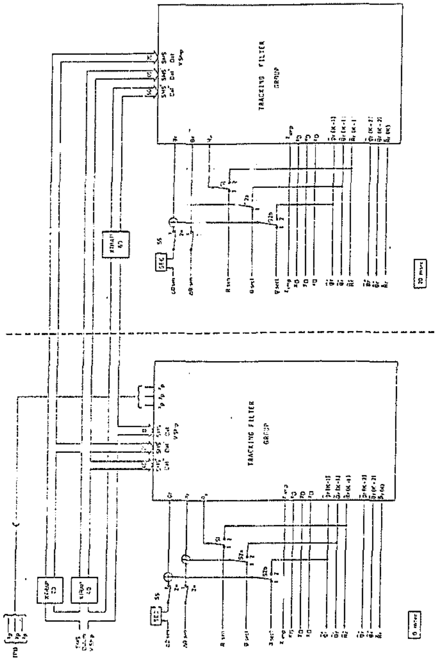

Fig.5.2 Tracking Controller: Tracking Filter Group in Context

5.3 Tracking Filter Group Configuration Development

The tracking filter group consists of a series of routines which shift the measurement frame of reference to a fixed stable frame for filtering, and then back to the original frame to supply the angle servos.

The development of this structure which is illustrated in Fig.5.3 is outlined below.

5.3.1 *Filtering Coordinate System* -

The first decision that needs to be made is whether to filter in a Polar or Cartesian System. From a point of view of simplicity the former is superior, as all the measurements and outputs must be in that coordinate system. However there are a number of major disadvantages that were explored before a decision was taken.

If the dynamics of a target flying at constant velocity are examined in terms of radial and tangential components (centered at the radar), it can be shown (Brooker 1981) that for normal target approach profiles, substantial radial and angular accelerations are generated. Hence if a simple second order (position, velocity) filter is used, its bandwidth will have to be fairly large to limit the dynamic lag errors. Alternatively, if it is adaptive, the adaptation strategy will have to consider the range of the target.

Further difficulties involve memory track configurations. In general an aircraft is more likely to follow a course of constant velocity in the cartesian sense, than one of constant range and angular rates. Hence if its position is to be predicted, the prediction is more likely to be correct in the former case.

In the defence of using a polar system for Memory Track, it is more simple to perform extrapolations if range and angular rates are known, than if the x,y and z velocities are known. This is because the outputs must be in the polar form, and so fewer transformations would be required.

Other considerations include the difficulties of translating the measurement origin and , particularly, rotating the coordinate frame of reference in polar space, both of which must be performed in the tracking loop.

It can be seen from the above, that the few advantages to filtering in a polar system are easily outweighed by the major difficulties and disadvantages. Hence a cartesian system will be used.

5.3.2 Choice Of Coordinate Reference System -

If the dynamics of ships motion (roll particularly) are translated to the target, as would happen if ship motion were not accounted for, any subtle changes in target motion would be swamped, and hence the tracking controller task of maintaining a filter finely tuned to target dynamics would be difficult, or even impossible.

From the point of view of Memory Track (Estimation of lost target measurements), where the target velocity is used to predict its future position, if the platform motion (both angular and linear) is not accounted for before the velocity is calculated, then any predictions would be corrupted by changes in the platform velocity.

From the above arguments it can be seen why it was decided that filtering should take place in a datum centered coordinate system which would be free of both the radar platform's rotational and translational motion.

5.3.3 Controlling The Servo -

In older systems, the angle servos were driven directly from the Sensor angular error signals, however this required that good measurements were available before tracking could be achieved. In the proposed system this will no longer be required as the target position can be predicted during memory track.

One problem which has not yet been dealt with is the conversion of the data supplied by the Tracking Filters to a format which is acceptable to the angle servos. Two aspects of this problem will be dealt with. The first is the frame of reference, while the second is a problem of loop speed.

Solution of the first is simple as it requires only a shift. Rotation and translation, however the second is more difficult.

A brief investigation has shown that using an 8087 based computer, the fastest update rate possible for the Tracking Filter Group (and support software) is every 20msec, this is only one quarter as fast as a typical angle servo loop.

To link the fast and slow loops it was decided that the TC would supply the servos with position data valid at the time of sending, and also 20msec ahead. The reason for the two data points is that it allows the servos to interpolate rather than to extrapolate to determine the instantaneous target position.

The main advantage of this format has again to do with platform motion. It is felt that as the TC has better estimates of the extrapolated ship attitude, it will generate more accurate estimates of the target position relative to the ship than could the angle servos.

Because the servo operates in the BDSF(R) frame (see Appendix B), ships motion must have been added to the predicted positions before they are sent. This motion data will have to be extrapolated so that it is valid at the same time as the target data is.

5.3.4 Range And Range Rate Estimation -

In general an external estimate of range is not required by the sensor. However provision is also made for its supply in those cases when the sensor does need it.

Provision is made to supply the Sensor with an external range rate estimate, as that generated internally by the Sensor alpha-beta filter is fairly inaccurate.

It is for the above reasons that estimates of range and range rate are generated.

5.3.5 Memory Track Considerations -

It is possible to use typical target dynamics to obtain better estimates of missing measurement data than is posible using linear extrapolation. However, it was found (see Chapter 7) that the improved performance was not consistent, and in general did not warrent the increased loop complexity.

Linear extrapolation is simply achieved by feeding the 1 sample ahead estimates back into the loop as pseudo measurement inputs.

5.3.6 Tracking Filter Group Block Diagram -

The block diagram (Fig.5.3) shows the results of the above configuration development.

LEGEND TO FIG.5.3

[P/C]	Polar to Cartesian Transformation
[Tr/Tm]	Translation from Radar to SMS centered
[-SM]	Rotation of frame to remove ship motion
[Tm/Tp]	Translation from SMS to Datum centered
[FILTER]	Tracking Filters
[MANU]	Manoeuvre quantisation algorithms
[XTRAP xx]	Position Extrapolation
[Tp/Tm]	Translation from Datum to SMS centered
[+SM]	Rotation of frame to add ship motion
[Tm/Tr]	Translation from SMS to Radar centered
[C/P]	Cartesian to Polar Transformation
[-S]	Removal of Ship Speed
[RDOT]	*Calculation of Range Rate*

The Inputs and Outputs correspond to those in Fig.5.2 which have already been defined.

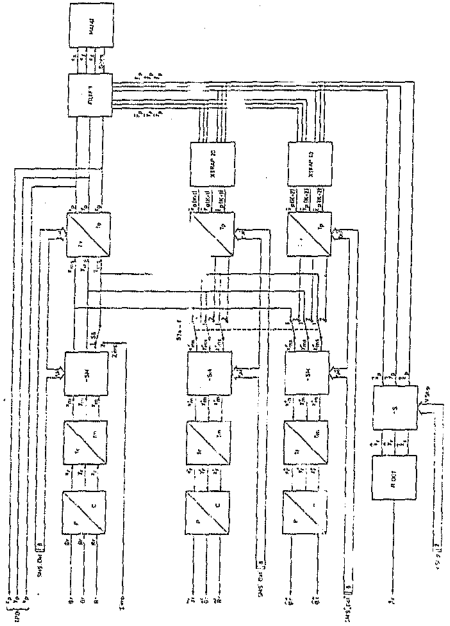

Fig.5.3 Block diagram of the Tracking Filter Group

5.4 Tracking Filter Group: Block Definition

Once the structure of the Tracking Filter Group had been defined, it was possible to derive a series of arithmetic algorithms to perform the required transformations and filtering. The following sections show these details, derivations for which can be found in Appendix C.

The spatial significance of the following translation and rotation algorithms can be better understood if the frames of reference defined in Appendix B are understood.

5.4.1 Secant Correction (SEC) -

This operation projects the azimuth error as generated by the sensor from the plane of the antenna to the plane of the deck for use by the Servos.

Psen = Sensor Azimuth Error
Psenp= Projected Azimuth Error
Tr = Elevation (wrt deck)

Psenp = Psen.Sec(Tr)

5.4.2 Polar To Cartesian Transform (P/C) -

This transforms the information from the polar format (Range, Elevation, Azimuth) to a cartesian format (X, Y, Z).

Rr=Range
Tr=Elevation
Pr=Azimuth

Xr = Rr.Cos(Tr).Cos(Pr)

Yr = Rr.Cos(Tr).Sin(Pr)

Zr = -Rr.Sin(Tr)

5.4.3 Radar To Ship Centered Translation [Tr/Tm] -
This shifts the origin of the coordinate system in which the target position is defined from Radar centered to Ship Motion Sensor Centered (SMS).
The reason for this shift is that ships motion is most simply removed from the target position data if it is in this frame.

Xr,Yr,Zr = Target Data Radar Centered
Xm,Ym,Zm = Target Data SMS Centered
Xrm,Yrm,Zrm = Position of SMS wrt Radar

$$Xm = Xr-Xrm$$

$$Ym = Yr-Yrm$$

$$Zm = Zr-Zrm$$

5.4.4 Removal Of Ship's Motion [~SM] -
This function rotates the frame of reference in which the target position is defined, from BDSF(M) to NSF(M) (see Appendix B for definition of coordinate frames of reference), it ensures that the target position is independent of the ships attitude (roll, pitch, heading).

Xm,Ym,Zm = Target Data BDSF(M) frame
Xms,Yms,Zms = Target Data NSF(M) frame

PH = Ship's Roll (+ve Port side up)
TH = Ship's Pitch (+ve Bows up)
PS = Ship's Heading (+ve Clockwise from True North)

$$Zi = Zm.Cos(PH)+Ym.Sin(PH)$$

$$Yi = Ym.Cos(PH)-Zm.Sin(PH)$$

$$Xi = Xm.Cos(TH)+Zi.Sin(TH)$$

$$Zms = Zi.Cos(TH)-Xm.Sin(TH)$$

$$Yms = Yi.Cos(PS)-Xi.Sin(PS)$$

$$Xms = Xi.Cos(PS)+Yi.Sin(PS)$$

5.4.5 Ship To Datum Centered Translation [Tm/Tp] -
This translation shifts the frame of reference of the target position from that centered on the SMS to one centered at a datum point which has been previously defined and remembered by the Ship Motion Sensors.
Filtering occurs in this frame as it ensures that the target velocity remains independent of that of the Ship, hence more accurate estimation is possible.

Xms,Yms,Zms = Target positions wrt the SMS
Xp,Yp,Zp = Target positions wrt a datum point
Xmp,Ymp,Zmp = Datum point,wrt the SMS

$$Xp = Xms - Xmp$$

$$Yp = Yms - Ymp$$

$$Zp = Zms - Zmp$$

5.4.6 Tracking Filters [FILTER] -
As filtering is performed in cartesian space, it is necessary to have three filters running simultaneously, each of which will deal with one dimension.

The filters perform two basic functions. The first is that they generate a smoothed estimate of the target position, and the second is that they estimate the target's velocity.

Most important from a tracking point of view is that the filters be optimised to the current target dynamics, thereby obtaining the best possible estimates. This is achieved by making them adaptive, with the Bandwidth dependent on the magnitude of the Target Manoeuvre in that direction. For this reason the magnitude of a manoeuvre must be estimated to some degree, without going to a higher order filter.

The results of tests conducted in the following chapter indicate that the Modified Centroid Beta filter is superior to the other two by a small margin.

The selection of the tracking filter is dealt with in detail in the next chapter. This will include the actual filter type as well as the adaptive strategy and the manoeuvre detection algorithms.

The filter function is described by the following equations:

$Xm(k)$ = Latest Measurement
$Xp^*(k)$ = Intermediate position estimate
$Xp(k)$ = Position Estimate

$\dot{X}p(k-1)$ = Last Velocity Estimate
$\dot{X}p(k)$ = Latest Velocity Estimate

N = Number of samples in moving Average (50)
B = Gain of Velocity Estimator

$$Xp^*(k) = \frac{1}{N}\sum_{j=k-N+1}^{k} Xm(j) + \frac{N-1}{2} . Ts . \dot{X}p(k-1)$$

$$\dot{X}p(k) = \dot{X}p(k-1) + \frac{B}{Ts}[Xm(k) - Xp^*(k)]$$

$$Xp(k) = Xp^*(k) + \frac{N-1}{2} . Ts .(\dot{X}p(k) - \dot{X}p(k-1))$$

The filter is adaptive, and again with reference to the following chapter five gain states are used. These are listed in Table.5.1 along with the lag transition threshold.

Table.5.1 Filter Gain Adaption Thresholds

BETA	THRESHUP	THRESHDN
0.0009	1.3	0.0
0.0021	1.8	0.486
0.0041	2.2	0.792
0.0057	2.4	1.35
0.0155	1000	0.576

5.4.7 *Manoeuvre Detection (MANU)* -
Manoeuvre quantisation performs the basic function of detecting whether a manoeuvre of sufficient magnitude to warrant increasing the bandwidth is occuring, or if the acceleration has diminished to such a small magnitude that the bandwidth may be decreased.

With reference to the following chapt..r it was decided that the manoeuvre quantisation routines would consist of alpha filters on the Position Residual with separate thresholds for each filter gain state.

The Alpha filter is described by the f.llowing equations:

Alpha = Gain of the Filter
E(k) = Position Residual
L(k) = Present Estimate of *Lag*
L(k-1) = Last Estimate of Lag

$$E(k) = Xm(k) - Xp(k)$$

$$L(k) = L(k-1) + Alpha \,.\, [\, E(k) - L(k-1) \,]$$

5.4.8 Filter Adaption Algorithm -
While the Tracking Filters are still settling (before the 50 point memory is filled for the first time), the manoeuvre algorithm is not operational. Instead, as the number of points in the memory increases from one to fifty, the velocity gain, Beta, is calculated to maintain a fairly constant bandwidth. *The following formula is* used:

$$Beta = \frac{0.77}{N}$$

Once the tracking filter memory is full the gain state (see Table.5.1) is set to the largest bandwidth for all the filters. Following that, on every period the absolute lag is examined to *determine whether it exceeds or is* less than the up or down thresholds respectively. If it exceeds 'Threshup' the gain is increased to the next higher value, while if it falls below 'Threshdn' then the gain is decreased to next value.

5.4.9 Position Extrapolation (XTRAP Xx) -
As the process of filtering takes a finite time, a current smoothed estimate of the target position taken directly from the measurements is too late. Hence, the filter outputs must be extrapolated to where the target will be once the filtering has been completed, or to whatever period in the future that the position is required.

In this case the target position is extrapolated to the end of the current sample period, and to the end of the following one. This allows that interpolation be used by the Angle Servos in determining the target position at times other than those predicted.

XP, YP, ZP = Current Target position
$\dot{X}p, \dot{Y}p, \dot{Z}p$ = Current Target Velocity
$Xp(k+1)$ = One sample ahead prediction
$Xp(k+2)$ = Two sample ahead prediction
Ts = Sample Period

$$Xp(k+1) = Xp+\dot{X}p.Ts$$

$$Xp(k+2) = Xp+2.\dot{X}p.Ts$$

5.4.10 Datum To Ship Centered Translation (Tp/Tm) -
Ships position with respect to the datum as supplied by the SMS gives the present position. However, at this stage the target position data has been extrapolated to one and two samples ahead, hence the position of the ship must be predicted for these times. This is simply achieved, as the ship velocity is also supplied by the SMS.

$Xp(k+.)$ = Target extrapolated position wrt datum
$Xms(k+.)$ = Target Extrapolated position wrt SMS
$Xpm(k+.)$ = Ship extrapolated position wrt datum

$$Xms(k+.) = Xp(k+.)-Xpm(k+.)$$

$$Yms(k+.) = Yp(k+.)-Ypm(k+.)$$

$$Zms(k+.) = Zp(k+.)-Zpm(k+.)$$

5.4.11 Addition Of Ship Motion [+SM] -
As the servos require to know the target position with respect to the ships deck, angular information presented to them must include ships motion. Hence, the filtered target position data must be rotated from NSF(M) to BDSF(M).

In this case, the ship attitude is predicted for the correct time in the future by using filters, the workings of which are explained in Chapter 8.

Xms(k+.) = Target position NSF(M) frame
Xm(k+.) = Target position BDSF(M) frame

PH' = Ship's Roll Extrapolated
TH' = Ship's Pitch Extrapolated
PS' = Ship's Heading Extrapolated

Xi = Xms.Cos(PS')-Yms.Sin(PS')

Yi = Yms.Cos(PS')+Xms.Sin(PS')

Zi = Zms.Cos(TH')+Xi.Sin(TH')

Xm = Xi.Cos(TH')-Zms.Sin(TH')

Ym = Yi.Cos(PH')+Zi.Sin(PH')

Zm = Zi.Cos(PH')-Yi.Sin(PH')

The above rotation is performed on both the data which has been extrapolated to one sample, and that extrapolated to two samples ahead.

5.4.12 Ship Centered To Radar Translation [Tm/Tr] -
This translation is identical for bot' of the extrapolated outputs, as the position of the Radar wrt the remains constant.

Xr = Xm+Xrm

Yr = Ym+Yrm

Zr = Zm+Zrm

Valid for Xr' and Xr"

5.4.13 Cartesian To Polar Transform (C/P) -
The Sensor requires the range which has been extrapolated to one sample ahead, while the Servos require *Elevation and Bearing* which have been extrapolated to both one and two samples ahead. Hence there are two transforms one of which requires range and the other of which does not.

$$Rr = \sqrt{Xr^2 + Yr^2 + Zr^2}$$

$$Tr = \text{Arcsin}(-Zr/Rr)$$

IF(Xr=0)then

$$Pr = Pi/2$$

ELSE

$$Pr = \text{Arctan}(Yr/Xr)$$

END IF

IF(Zr<0)then

$$Pr = Pr+Pi$$

5.4.14 Calculation *Of Range Rate (RDOT)* -
The sensor requires the target range rate relative to itself, hence the target velocity must be determined wrt the ship before the range rate is calculated.

$$\dot{X}r = \dot{X}p-\dot{X}ship$$

$$\dot{Y}r = \dot{Y}p-\dot{Y}ship$$

$$\dot{Z}r = \dot{Z}p-\dot{Z}ship$$

$$\dot{R}r = \frac{Xr.\dot{X}r+Yr.\dot{Y}r+Zr.\dot{Z}r}{Rr}$$

where $\dot{R}r$ is positive away from the origin

5.5 Signal Flow Controller

The Signal Flow Controller (SFC) controls the logical implementation of the different tracking configurations that must be accommodated by the TC. These Configurations consist of both the Normal and Memory Track Modes, and Imposed Altitude as well as the sub-modes that form minor parts of the major tracking configurations.

1. Normal Track I. Track Settling
   II. Track Running

2. Range Memory Track

3. Angle Memory Track

4. Full Memory Track

5. Imposed Altitude

6. Off angle Tracking

The last two constitute configurations that exist in parallel with the other Modes.

5.5.1 Mode Select Logic -
The SFC functions by controlling the settings of a number of Software switches (real or imaginary) which alter the signal flow structure. If Figs 5.2 and 5.3 are examined in conjunction with the following list of switch settings it should be possible to understand the basics of the flow.

Table.5.2 Signal Flow Controller Switch Positions

MODE	S1	S2a-b	S3a-f	S5	S6
Track Settle	1	1	1	1	n/a
Normal Track	1	1	2	1	n/a
Range Memory	2	1	2	1	n/a
Angle Memory	1	2	2	2	n/a
Full Memory	2	2	2	2	n/a
Imposed Alt	n/a	n/a	n/a	n/a	2
Off Angle	n/a	n/a	n/a	n/a	2

5.6 Ship Motion Extrapolation

As can be seen if Figs 5.1 and 5.3, estimates of ships attitude valid at the present time and both 1 and 2 samples ahead must be supplied. This is achieved using the following extrapolation algorithm the design of which is dealt with in Chapter 8.

Ap(k+xx) = Sin/Cos of Roll, Pitch or Heading Extrapolated to xx msec into the future
A(k) = Sin/Cos of Roll, Pitch or Heading Present Estimate
Alpha = Position Gain (0.7345)
Beta = Velocity Gain (0.4694)
Gamma = Acceleration Gain (0.15)

$$Ap(k) = A(k-1) + \dot{A}(k-1).Ts + \ddot{A}(k-1).Ts^2/2$$

$$A(k) = Ap(k) + Alpha.[\ Am(k) - Ap(k)\ ]$$

$$\dot{A}(k) = \dot{A}(k-1) + \ddot{A}(k-1).Ts + Beta/Ts.[\ Am(k) - Ap(k)\ ]$$

$$\ddot{A}(k) = \ddot{A}(k-1) + 2.Gamma/Ts^2.[\ Am(k) - Ap(k)\ ]$$

$$Ap(k+20) = A(k) + 20.\dot{A}(k) + 0.2.\ddot{A}(k)$$

$$Ap(k+40) = A(k) + 40.\dot{A}(k) + 0.8.\ddot{A}(k)$$

$$Ap(k+60) = A(k) + 60.\dot{A}(k) + 1.8.\ddot{A}(k)$$

Extrapolation of ship position with respect to the datum point is also performed. However, this uses linear prediction using ship velocity which is supplied by the Ship Motion Sensor.

5.7 Peripheral Functions

There are a number of major functions that are linked to the Tracking Controller, either to supply it with information or to be supplied by it.

5.7.1 Ships Motion Sensor [SMS] -
The SMS will supply the TC with estimates of the current ships attitude (roll, pitch and heading) as well as an estimate of the ships speed and position relative to a previously defined datum point.

5.7.2 The Sensor (SENSOR) -
The Sensor is the heart of the Tracking Radar System. It supplies range information, and angle error information for tracking purposes, as well as a host of other outputs to aid this function.

More details of typical Radar Sensors can be found in Chapter 4.

5.7.3 Servo Functions -

5.7.3.1 Tilt Correction (+/- TILT) -
It is important that the thc base of the antenna pedestal be on the same plane as the ships deck. In general this may be achieved using shims or by some sort of correction algorithm in the software. Fig.5.4 indicates that the latter method is being used.

5.7.3.2 Shaft Encoders (SE) -
The shaft encoders output the angular position of the sensor with respect to the ship's deck.

5.7.3.3 Offset Angle (OFANGLE) -
This function supplies the Servo inputs with a fixed Elevation bias of about 1 deg. This, in conjunction with an Imposed Altitude setting will ensure that the antenna elevation remains constant irrespective of the target altitude or the ships attitude.

The elevation error signal will have been determined from an inaccurate section of the transfer function, hence to correct for these errors it must be passed through the inverse function. It is expected that a polynomial function will be used to perform this correction.

5.7.3.4 Angular Interpolation (INT 5) -
The data which is supplied by the Tracking Controller has been extrapolated to the present (wrt to the time when data is received by the Servo Computer) and to 20msec in the future.

The first servo output occurs 5msec later, hence the linear interpolation performed between the two data samples in each Bearing and Elevation will start at 5msec after the present position estimate, and end at the 20msec after. This is four samples in all.

The above peripheral functions are illustrated in block diagram form in Fig.5.4.

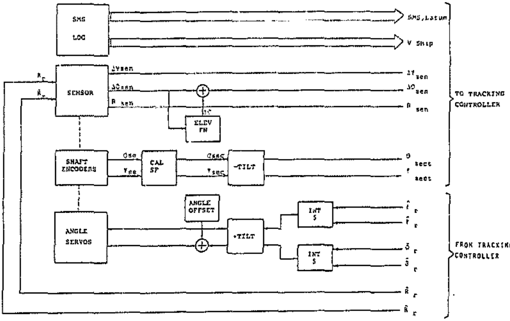

Fig.5.4 Peripheral Functions to the Tracking Controller

5.8 Operation Of The Tracking Controller

As soon as the Radar is switched on, the Ship Motion Prediction routines are actuated and given time to settle. This ensures that the data supplied will be accurate by the time any designation or tracking functions need to be *performed*.

After a designation has been supplied and a target acquired, the Tracking Controller is activated.

Initially the Sirnal Flow Controll selects the Track Settling Mode. This allows fairl tracking to continue using raw data while the *Tracking Filt* e.

To decrease the filter settling time still further, as the filter memory is filling for the first time, the velocity gain Beta is calculated as a function of the number of points in an attempt to maintain a fairly constant bandwidth.

Once the filters have settled the SFC selects the Normal Track Mode and the *full tracking* loop is utilised.

The adaptive tracking filters function automatically. Initially they are set to their widest bandwidth so that they will settle *as* quickly as possible. After that they are sensitive only to target manoeuvres.

If either range or angle information is deemed to be corrupt, or is unavailable, the SFC selects the correct Partial Memory Track Mode using S1 and S2.

Imposed altitude is used to simplify the tracking of a target flying at a constant altitude, or *for the tracking* of ships which are known to remain at a fixed altitude. Its use is mainly to minimise the *effects of multipath*.

The final logical function is offangle track which is used for the same reasons as imposed altitude. It is *implemented by selecting* imposed altitude and supplying an angle offset with the associated inverse elevation *transfer function*.

5.9 Simulation Of The Tracking Controller

A detailed FORTRAN model of the TC is included in Brooker (1983). This includes the *basic loop* structure as has been defined above, and such peripheral functions as are required for its correct operation.

Programs include the following:

1. *Tracking* Controller
2. Ship Motion Generator
3. Simplified Measurement Noise / Target Generator
4. Ship Motion Extrapolation Routines
5. Graphics Packages

A *simplified* Measurement Noise / Target model was used instead of the full Sensor Simulation to shorten run times. It is in fact a data file which was generated from the Sensor model developed by Pryce (1982) using the noise contributions outlined in Chapter 4.

Similarly the Ship Motion data *is taken from* a file which had been created by the roal Model.

An example of the overall simulation can be seen in Fig.5.5 and Fig.5.6. The former is a simulation with no measurement noise on the Target, while the latter is a full imulation.

The target being tracked is performing a pitchup/dive attack with a bomb bunt as can be seen from the profile in Graph 9.

Polar measurement noise inputs are shown in Graphs 1 to 3 from which it can be seen that the first simulation is indeed noise free. The next three graphs correspond to the 'smoothed' estimates supplied by the Tracking Controller.

As the first plot uses noise free data, all of the errors on the output must be the effects of dynamic lag. It can be seen from either plot that the tracking error seldom exceeds 4m in range and 1mrad in angles.

The error in range rate also seldom exceeds 5m/s as can be seen from Graph 7.

The following table lists the RMS errors calculated for the full simulation including noise:

Table.5.3 Simulation RMS Tracking Errors

Estimate	RMS Error
Range	1.75m
Range Rate	5.5m/s
Elevation	1.6mrad
Azimuth	0.4mrad

Graph 8 shows that the manoeuvre detection and gain adaptation functions do work. That they are effective is borne out by the fact that the lag is constrained within acceptable levels.

A comparison between the two figures seems to indicate that the lag is the major contributing factor to tracking error. However, as will be seen in a later chapter, the filter gains are calculated in such a way as to minimise the mean squared tracking error at each state.

Fig.5.5 Tracking Controller Simulation (Noise Free).

Fig.5.6 Full Tracking Controller Simulation.

Assuming a critically damped filter, this reduces the possible selection of the Alpha Beta Filter from any point on a plane (the Alpha-Beta Plane) to points on a line *defined* by the following relationship

$$ALPHA = 2.\sqrt{BETA} - BETA$$

For the Centroid Beta Filter, *Harris and Clarke* (1981b) have found that a moving average one second long is a good length to ensure good *Variance Reduction* for the correlated noise that can be expected (Note: Harris invented the original Centroid Beta Filter for use as a Tracking Filter) Hence it was decided that as the sample period is 20 msec, 50 samples would be used *in the average*.

This restriction limits *possible filters* from any point in the N-Beta Plane to those on a line. Details of filter performance on *the N-Beta plane may* be found in Brooker (1982).

One further restriction is applied, and that is that the position bandwidth of the filters should lie in the region *between 0.5Hz and* 4.0Hz, as that is where all major aircraft dynamics occurs.

The 0.5Hz minimum is again the result of investigations conducted by *Harris* (1981b). In their system which had a minimum Bandwidth of the order of 0.4Hz it was found that the minimum was never reached before a manoeuvre was detected and the bandwidth opened up again.

6.1.2 Parameters For Comparison -

Any number of parameters can be calculated for a filter. But, all of those will not be very useful in deciding how well it will fulfil the role as a tracking filter in a Tracking System. For this reason, and those dictated by the *overall* TC structure, the following are used in the comparison.

1 Sample Ahead Position Variance Reduction Ratio
2 Sample Ahead Position Variance Reduction Ratio
100 Sample Ahead *Position Variance* Reduction Ratio
200 Sample Ahead Position Variance Reduction Ratio

1 Sample Ahead Position Lag
2 Sample Ahead Position Lag
Velocity Lag

1% Response time to a Position Step
1% Response time to a Position Ramp (Rate Step)

Other characteristics are calculated which are not used in the filter selection routines. These are used to help define the filter as a complete unit.

Gains: Alpha, Beta, N
Position Frequency Response
Position Estimate Bandwidth

It can be seen that the comparison does not include computer time or storage. This is because the implementation time is very similar for the three filters.

All of the formulae used in the comparison are derived by v.d.Merwe (1981b) and Brooker (1982) or in Appendix D.

The following list of formulae are all used to calculate the variables which will be used in the comparison.

The Centroid Beta Filter

a. Position Variance Reduction

$$VRRP = \frac{1}{1-a^2}\left\{\frac{2\,(1-a^N)}{N^2} + (1-a)^2 + \frac{2\,(1-a)(a-a^{N+1})}{N}\right\}$$

b. Velocity Variance Reduction

$$VRRV = \frac{1}{1-a^2}\left\{\frac{4\,(1-a)^2}{N\,(N-1)} + \frac{8a\,(a^{N-1}-a^N)}{N^2\,(N-1)} - \frac{8a\,(1-a^{N-1})}{N^2(N-1)^2}\right\}$$

c. Covariance Reduction Ratio

$$COVR = \frac{1}{1+a}\left\{\frac{2\,(a^{N-1}-1)}{N\,(N-1)} + \frac{2\,(1-a^N)}{N^2} + \frac{2a\,(1-a)}{N}\right\}$$

d. M Sample Ahead Position Variance Reduction

$$VRRP.M = VRRP + 2M*COVR + M^2*VRRV$$

e. Position Lag

$$Lp = \ddot{x}\,\frac{Ts^2}{B}$$

f. Velocity Lag

$$Lv = \ddot{x}\,\frac{Ts}{1}\left\{\frac{2}{(N-1)B} + \frac{2N-7}{6}\right\}$$

g. 1 sample Ahead Position Lag

$$L1p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+1}{N-1} + \frac{N-2}{3}B\right\}$$

h. 2 Sample Ahead Position Lag

$$L2p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+3}{N-1} + \frac{2N-1}{3}B\right\}$$

i. Position Frequency Response

$$\left|H(w)\right|^2 = \frac{\frac{2}{N^2}(1-\mathrm{Cos}NwTs) + (1-a)^2 + \frac{2(1-a)}{N}(\mathrm{Cos}wTs-\mathrm{Cos}(N-1)wTs}{1 + a^2 - 2a\,\mathrm{Cos}wTs}$$

$$a = 1 - \frac{N-1}{2}B$$

j. 1% Step Settling Time (Samples)

$$n = \frac{\mathrm{Log}\,\dfrac{0.01\ (a-1)}{(1-a) + 1/N\ (a-a^{1-N})}}{\mathrm{Log}(a)}$$

k. Ramp Response Times (0.01 accuracy)

$$n = \frac{\mathrm{Log}\,\dfrac{0.01\ (1-a)^2}{a/N\ (a^{-N}-1) + (a-1)}}{\mathrm{Log}(a)}$$

l. Filter Algorithm

$$x(k) = \frac{1}{N}\sum_{j=k-N+1}^{k} xm(j) + \frac{N-1}{2}.Ts.\dot{x}(k-1)$$

$$\dot{x}(k) = \dot{x}(k-1) + B/Ts\ [\ xm(k)-x(k)\ ]$$

Mod No2 Centroid Beta Filter

a. Position Variance Reduction

$$VRRP = \frac{1}{1-a^2}\left\{\frac{(1-a)^2}{1} + \frac{2\,(1-a^N)(Na - (N-1)a^2)}{N^2}\right\}$$

b. The Velocity VRR is the same for the Modified filter as it is for the original version.

c. Covariance Reduction Ratio

$$COVR = \frac{1}{1+a}\left\{\frac{2a\,(a^{N-1}-1)}{N\,(N-1)} + \frac{2a\,(1-a^N)}{N^2} + \frac{2\,(1-a)}{N}\right\}$$

d. Position Lag

$$Lp = \ddot{x}\,\frac{Ts^2}{B}\left\{1 - \frac{N-1}{2}\,B\right\}$$

e. Velocity Lag is the same as that for the original Centriod Beta Filter

f. 1 Sample Ahead Position Lag

$$L1p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+1}{N-1} - \frac{N+1}{6}\,B\right\}$$

g. 2 Sample Ahead Position Lag

$$L2p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+3}{N-1} + \frac{N+1}{6}\,B\right\}$$

h. Position Frequency Response

$$\left|H(w)\right|^2 = \frac{\frac{4}{N^2}a^2.\mathrm{Sin}^2\frac{NwTs}{2} + (1-a)^2 + \frac{4(1-a)}{N}\,a.\mathrm{Sin}^2\frac{NwTs}{2}}{1 + a^2 - 2a\,\mathrm{Cos}wTs}$$

i. 1% Step Settling Time (Samples)

$$n = \frac{\mathrm{Log}\,\frac{0.01\ (a-1)}{a(1-a) + a/N(a-a^{1-N})}}{\mathrm{Log}(a)}$$

j. Ramp Response Time *(0.01 accuracy)*

$$n = \frac{\mathrm{Log}\,\frac{0.01\ (1-a)^2}{a^2/N(a^{-N}-1) + a(a-1)}}{\mathrm{Log}(a)}$$

k. Filter Algorithm

$$x^*(k) = \frac{1}{N}\sum_{j=k-N+1}^{k} xm(j) + \frac{N-1}{2}.Ts.\dot{x}(k-1)$$

$$\dot{x}(k) = \dot{x}(k-1) + B/Ts\ [\ xm(k)-x^*(k)\ ]$$

$$x(k) = x^*(k) + \frac{N-1}{2}.Ts.[\ \dot{x}(k)-\dot{x}(k-1)\ ]$$

The Alpha Beta Filter

a. Position Variance Reduction Ratio

$$VRRP = \frac{2g^2 - 3gh + 2h}{g(4 - 2g - h)}$$

b. Velocity Variance Reduction Ratio

$$VRRV = \frac{2h^2}{g(4 - 2g - h)}$$

c. Covariance Reduction Ratio

$$COVR = \frac{h(2g - h)}{g(4 - 2g - h)}$$

d. Position Lag

$$Lp = \ddot{x} \frac{Ts^2}{h} (1-g)$$

e. Velocity Lag

$$Lv = \ddot{x} \frac{Ts}{h} \left\{ \frac{2g - h}{2} \right\}$$

f. 1 Sample Ahead Position Lag

$$L1p = \ddot{x}\,\frac{Ts^2}{h}$$

g. 2 Sample Ahead Position Lag

$$L2p = \ddot{x}\,\frac{Ts^2}{h}(1 + g + h)$$

h. Position Frequency Response

$$|H(w)|^2 = \frac{g^2 + (h-g)^2 + 2g(h-g)\mathrm{Cos}\,wTs}{1+(h+g-2)^2+(1-g)^2+(2-g)(h+g-2)\,2\mathrm{Cos}\,wTs+2(1-g)\mathrm{Cos}\,2wTs}$$

i. 1% Step Settling Time (Samples)

$$n = \frac{\mathrm{Log}\,\dfrac{[4(1-g) - (2-g-h)^2]\,0.01^2}{4h(1-g)}}{\mathrm{Log}(1-g)}$$

This formula is valid for an underdamped filter only. It is based on the envelope decay.

For a critically damped filter, the settling time cannot be represented in a closed form.

j. Ramp Response Time (0.01 absolute accuracy)

$$n = \frac{\text{Log}\ \dfrac{[4(1-g) - (2-g-h)^2]\,0.01^2}{4(1-g)^2}}{\text{Log}(1-g)}$$

This formula is valid for an underdamped filter.

$$n = \frac{\text{Log}\ \dfrac{(1-\sqrt{h})\,0.01}{1 + h - 2\sqrt{h}}}{\text{Log}(1-\sqrt{h})}$$

This formula gives the 1% Ramp Settling Time for a Critically Damped Filter.

k. Filter Algorithm

$$x^*(k) = x(k-1) + Ts.\dot{x}(k-1)$$

$$\dot{x}(k) = \dot{x}(k-1) + h/Ts\ [\ xm(k)-x^*(k)\ ]$$

$$x(k) = x^*(k) + g\ [\ xm(k)-x^*(k)\ ]$$

It was felt that as some of the above formulae represent a compromise, and that as the correct formulae can only be obtained by recursive methods, it would be simpler to determine the step and ramp response times dynamically.

6.1.3 Filter Comparison Method -
A program was written which would calculate any of the above parameters while holding one particular one equal for the three filters (Listing is included in Brooker (1983)).

It was found that if one filter was superior in one aspect, it was inferior in others, as can be seen if the tables in Appendix E are examined. For this reason it was decided to rate each parameter on a scale of importance (0 to 1) and using that and its actual value (normalised wrt the average for the three filters) find an overall performance rating for the filter at that particular point. The filter with the lowest rating is deemed to be the best for this particular application.

$$\text{Param rating} = \frac{\text{Param Value}}{\text{Para Value Average}} \times \text{Weight}$$

Filter rating = Sum of Param r tings

The scales (weightings) chosen are listed in the following table:

Table.6.1 Filter Parameter Comparison Ratings

PARAMETER	WEIGHTING
VRR-1	1.0
VRR-2	1.0
VRR-100	0.6
VRR-200	0.6
LAG-1	0.3
LAG-2	0.3
LAG-VEL	0.6
STEP RESP	0.5
RAMP RESP	0.5

6.1.3.1 Reasons For Particular Ratings -
The 1 and 2 sample ahead VRRs are rated very highly as they define how well the filter estimates the actual target position for tracking purposes.

The 100 and 200 sample ahead VRRs are rated fairly highly as they dictate how accurate the prediction of the target position will be during memory track.

The position lags are not rated very highly as they will be limited to an acceptably small value by he filters adapting their gains during target accelerations.

The velocity lag has a very marked influence on the performance of the memory track predictor, hence it is rated equal to the memory track variance ratings.

The position and velocity step response times determine how fast the filter will react to a transient, in which not much importance is placed, hence they have been given ratings of 0.5 each.

6.1.4 Parameters To Hold Equal -

Initially bandwidth was going to be used as the control parameter; however, after due consideration it was decided that as the response of the Centroid Beta Filters is not a monotonic function of frequency it would not be an apt control.

The second control parameter considered was the variance reduction ratio. It was felt that this was just, as the VRR's are the most highly rated parameters in the comparison. There is one flaw to this choice, however, and that is the fact that the VRRs calculated are those for white noise, and hence are not a completely accurate reflection of what the filters will encounter.

It is shown in Brooker (1982) that the filter performance trends obtained using white noise inputs are similar to those using noise as defined in Chapter 4, even though the actual VRRs obtained will be different, hence using white noise VRR's is acceptable.

In each case the evaluation is performed at a number of points along the lines defined earlier (Crit damping or N=50). As explained in the last chapter, the filter will operate in any of five gain states. Hence, to check the overall filter performance it would be best to evaluate it at the states defined. In this case it is not possible, as at the time that the comparison was made the actual states had not been decided upon. A compromise was achieved by evaluating the filters at points representative of bandwidths between 0.5 and 4.0Hz.

6.1.5 Results Of The Comparison. -

6.1.5.1 Equal Bandwidth -
The following bandwidths were selected as being representative of the range 0.5 to 4.0Hz:

Table.6.2 Equal Bandwidth Filter Ratings

BANDWIDTH	BEST FILTER
0.5	C-BETA
0.75	ALPHA BETA
1.0	ALPHA BETA
1.5	C-BETA
2.0	NO2 C-BETA
3.0	NO2 C-BETA
4.0	NO2 C-BETA

This does not really give one filter any marked superiority over any of the others.

6.1.5.2 Equal 1 Sample Ahead VRR -
The following Variances are representative of bandwidths of the same order as those above:

Table.6.3 Equal VRR-1 Filter Ratings

VRR POS-1	BEST FILTER
0.0242	ALPHA BETA
0.0524	NO2 C-BETA
0.0784	NO2 C-BETA
0.1061	NO2 C-BETA
0.1514	NO2 C-BETA
0.2177	NO2 C-BETA
0.2630	NO2 C-BETA

The superiority of the Modified Centroid Beta filter is easy to see from this table.

6.1.5.3 Other Variance Comparisons -
The two-sample ahead VRR comparison again shows the marked superiority of the Mod No2 Centroid Beta Filter, while at 150 samples ahead the Alpha Beta and Mod No2 Centroid Beta filters gain about equal credit (Tables in Appendix E).

6.1.5.4 Comparisons With Fixed Lag -
Such comparisons are a problem as the lags often cannot be made equal over the range required, however it was decided that as the position lags will be limited by the filters adaptive strategy, they will not be used as a reference for comparison purposes.

6.1.6 Graphic Display -
As an aid to the understanding of the way that the parameters change, a number of plots have been made, (see Fig.6.1 to Fig.6.3) each with a different control parameter. From the plots it is difficult to find any major differences between the filter performances which could be used to select or disqualify any particular one.

From an interest point of view, the most noticable characteristic of any filter is the way that the bandwidth of the Centroid Beta filters increase with increasing beta. The reason for this is best explained if a plot of the actual frequency response is examined in Brooker (1982). The response consists of a number of bumps, the position of which is governed by the zeros of the filter. With changing beta, the bumps shift both in amplitude (vertically) and in frequency (horizontally). As the amplitude of each bump crosses the magic 0.707 threshold, there will be an abrupt change in the bandwidth of the filter.

This is one of the reasons why in this case a fixed bandwidth comparison is not used.

The other graphs have less of interest to offer. In general, the lags and step or ramp response times are very similar for the three filters, while the variance reduction ratio plots show that the Alpha Beta filter is superior in the short term ,while the Centroid Beta filters are superior in the long term.

In each case the horizontal scale represents a linear variation of the defined variable over the region plotted by that variable. For example in Fig.6.1 the horizontal axis represents filters with gain Beta varying from 0.0 to 4.0 as shown in Graph 1.

As the graphs are to be used for comparison purposes only, the dependent variables have been calculated for normalised Ts (filter sample period). This affects all the graphs except Beta and Bandwidth.

Legend to Figures 6.1 to 6.3

(1) - Centroid Beta Filter

(2) - Mod No-2 Centroid Beta Filter

(3) - Alpha Beta Filter

(X) - Equal Variable

B - Velocity Gain (Beta)

r - Ramp Response

s - Step Response

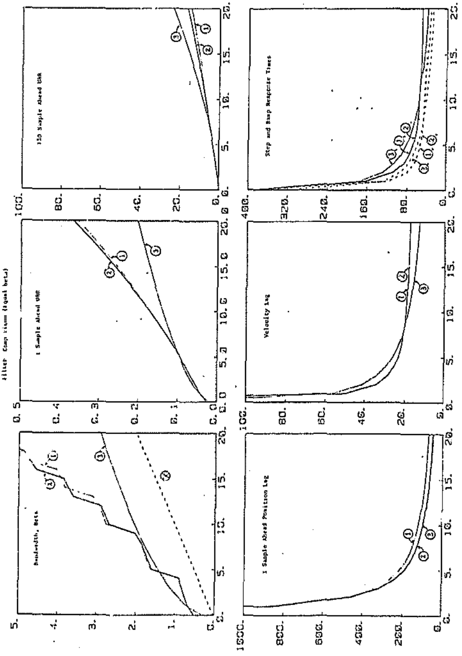

Fig.6.1 Graphical Filter Comparison (Equal Beta)

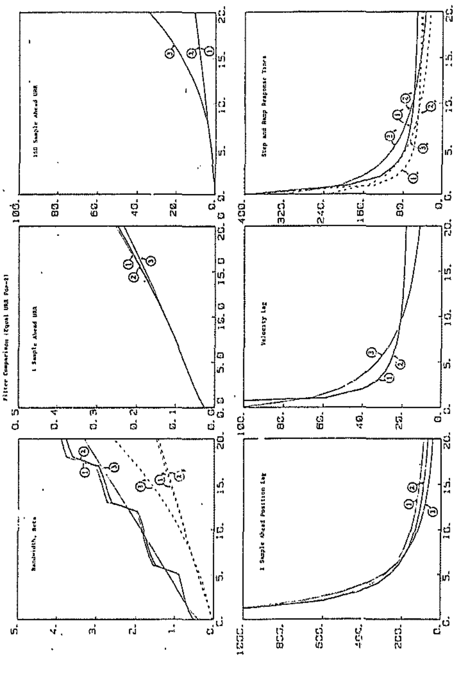

Fig.6.2 Graphical Filter Comparison (Equal VRRP-2)

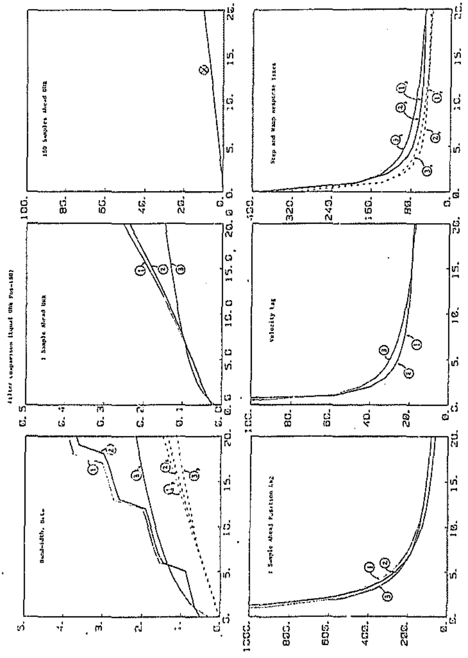

Fig.6.3 Graphical Filter Comparison (Equal VRRP-150)

6.2 Filter Optimisation Technique

6.2.1 Introduction -

The ultimate performance measure of a tracking filter is a function of how well it matches itself to the target state. This performance is measured not only with respect to the accuracy wit' which the Tracking Filter directs the antenna, but also by the filters rejection of any deceptive strategy.

If the former were the only criterion by which the filter performance were to be judged, then a fixed gain filter would be sufficiently good if it were optimised for the target profiles as a whole. But as the second criterion must be taken into account it is imperative that the filter have more states.

The main reason that the Tracking Filter has more than one state is that if a single state filter were used, its bandwidth would have to be sufficiently high to cater for real manoeuvres with magnitudes of 6g and more. This wide bandwidth would make it susceptible to being pulled off the real target quite rapidly. If however the filter had more states, its response would be slower in general, and hence the 'pulloff' time would hopefully be increased to such an extent that the deception would have been terminated before track was lost.

6.2.2 Optimisation Technique -

It is not really possible to find a quantitive function that will minimise the Tracking Filters susceptibility to deception, however it is fairly simple to find an optimisation method which will minimise the tracking error.

6.2.3 Optimisation Of Tracking Accuracy -

The two main functions which affect the tracking accuracy for an accelerating target are the measurement noise and the filter position lag for an accelerating target.

A fairly comprehensive study has been made of the Measurement Noise that is expected. Details of this may be found in Chapter 4.

With respect to the acceleration lag, the most common target profiles were examined and a distribution of accelerations made. Firstly the acceleration distribution is broken up into a series of quantum levels, each of which has a similar probability of an acceleration occuring. The exception is the no acceleration state which will have a higher probability. The quantum levels are listed in the following table.

It should be noted that this derived distribution is very different from that which is found in Schwartz and Shaw (1975). The main reason for this difference is that their analysis is theoretical and assumes that maximum or no acceleration is most common.

Table.6.4 Target Acceleration Probabilities

ACCELERATION m/s.s	PROBABILITY pu
0 - 3	0.733
3 - 10	0.089
10 - 25	0.083
25 - 40	0.081
> 40	0.014

The above have been chosen as the acceleration states for which the optimum filter gains will be calculated for each of the states of the adaptive tracker.

6.2.3.1 Optimum Filter Gain -
From the magnitude and spectral distribution of the measurement noise, and the Tracking Filter with a particular gain, can be obtained the filtered noise characteristics. Of particular interest is the variance, which will be used in conjunction with the acceleration lag to optimise the filter.

As the filter is being optimised for minimum mean squared error for each state, this should be derived as a function of the worst case filter lag and the variance for that state. It can be shown that the Mean Squared error is given by the sum of the variance of zero mean noise and the square of the bias (constant lag error). This derivation is found in Appendix F.

$$MSerr = Var + Lag^2$$

Now all that is required is to find a set of Gains for the Tracking Filter that will minimise the sum of the variance and the square of the worst expected lag (for that quantum level).

Because the measurement noise is not white, there is no simple analytical method of finding the output variance, hence it is determined by Impulse Methods as explained in Brooker (1982).

With respect to the bias, there are formulae which will give the steady state lag of the tracking filter, given the gains and the acceleration.

Because of the structure of the Tracking Controller (which supplies one and two sample ahead predicted positions), the filter gains must be optimised for the one and two samples ahead variances and acceleration lags. The two formulae used are given below:

One Sample Ahead Lag (Mod No2 C-Beta)

$$L1p = \ddot{X} \frac{Ts^2}{B} \left\{ \frac{N+1}{N-1} - \frac{N-1}{6} . B \right\}$$

Two Sample Ahead Lag

$$L2p = \ddot{X} \frac{Ts^2}{B} \left\{ \frac{N+3}{N-1} + \frac{N-1}{6} . B \right\}$$

A FORTRAN program which will perform this optimisation is listed in Brooker (1983). For the given noise and the worst case acceleration for each filter state, the program selected the following gains:

Table.6.5 Selection of Optimum Filter Gain

ACCELERATION m/s.s	BETA	BANDWIDTH Hz
0 - 3	0.0009	0.630
3 - 10	0.0021	0.745
10 - 25	0.0041	0.905
25 - 40	0.0057	1.650
> 40	0.0155	3.915

The above results are very sensitive to the noise spectrum used in the optimisation program. Hence, it is reasonable to assume, that, as better noise models are developed, the optimum filter gains will change.

The next procedure in the development of the filter is an algorithm which will detect whether the threshold for maximum acceleration has been exceeded.

6.3 Manoeuvre Quantisation Algorithm

This consists of a detection routine and an adaptive state selector. The former will detect the presence or absence of a manoeuvre, while the latter will select a new 'state' which has filter gains which are suitable for the detected acceleration.

6.3.1 Detection Of Lag -
The manoeuvre detector is a device which monitors the tracking filter residue (difference between measure and estimated position) and hence decides whether a bias exists or does not.

As with most Radar functions, a compromise must be reached between the probability of detection and probability of false alarm. A study conducted by Harris (1981b) indicates that if the manoeuvre detection threshold is 3RMS of the noise output by the detector, then the probability of false alarm is negligible.

Two methods of detection have been investigated, they are the Moving Sum and the Alpha Filters. The former is proposed by Harris (1981b) and the latter by v.d.Merwe (1981a).

6.3.2 The Moving Sum -
If the measurement noise is passed through a moving sum it will have an RMS value which increases in a logarithmic fashion with increasing N. If a Gaussian distribution is assumed for the noise, then the probability of the peak of the noise exceeding 3RMS is negligible.

The lag (once it has reached steady state) will increase in a linear fashion with N. Hence all that is required is a sum which is long enough to ensure the eventual detection of the minimum detectable magnitude lag required. Minimum detectable lag is given by the following formula:

$$LAGmin = 3RMS/N$$

N = Number of Points in the Sum

The 3RMS thresholds levels in the following table are derived in this case by passing noise of the correct variance and spectrum (see Chapter 4) through moving sum filters of different lengths. The RMS value is then calculated using a long time average (5000 points).

The results of this test are pessamistic, as, in the case where the lag detection filter uses the residual stream, most of the low frequency components will have already been extracted by the tracking filter, hence the RMS values generated by the moving sum will be smaller.

Table.6.6 Lag Detection limits for Moving Sum Filters

N	3RMS	3RMS/N
5	59.3	11.86
10	94.2	9.42
15	119.9	7.99
20	141.0	7.05
25	159.3	6.37
30	175.8	5.86
40	204.8	5.12
50	230.1	4.60
60	253	4.22
70	274	3.91
80	293	3.66
90	312	3.47
100	329	3.29
200	467	0.84

For an allowed lag of 3m at least 100 samples are required, hence it is possible that it will take 2sec to detect an acceleration (Ts = 20msec).

The main disadvantage of the moving sum is that it takes time for the sum to fill, and prior to that time detection of any manoeuvre is unlikely.

6.3.3 The Alpha Filter -
The Alpha Filter is basically a low pass filter, and hence will smooth out the noise so as to better [illegible] the steady state lag. Again there will be a minimum threshold for detection dependent on the 3RMS output of the filter. The following formula defines its operation:

$$L(k) = L(k-1) + ALPHA \cdot [\, E(k) - L(k-1) \,]$$

E(K) = Residual stream of the Tracking Filter

The following table determines the detection threshold, the 3RMS values were obtained in the same manner as those for the Moving Sum.

Table.6.7 Lag detection limits for the Alpha Filter

ALPHA	3RMS
0.01	2.32
0.015	2.84
0.02	3.27
0.025	3.64
0.03	3.96
0.04	4.59
0.05	5.07
0.06	5.52
0.07	5.92
0.08	6.29
0.1	6.94
0.15	8.23
0.2	9.24
0.3	10.73
0.5	12.63

Again for a 3m minimum detectable lag, the alpha required will be of the order of 0.02.

With respect to the time to detection, that is a function of the step response of the filter. It can be shown that the 90% step response time for the alpha filter is given by the following function.

$$N = \frac{LOG(\ 1-0.9\ )}{LOG(\ 1-alp\ )} - 1$$

For the Alpha chosen 113 samples would be required to ensure the detection of a manoeuvre, this puts the Alpha Filter on a par with the Moving Sum.

It should be noted that if faster lag detection is required, either the same threshold and a larger alpha can be used, or a lower threshold and the same alpha can be used. In either case the probability of false alarm will increase as the thershold will no longer be at the 3RMS level.

6.3.4 Effects Of Track Filter Bandwidth On Manoeuvre Detection - Because the manoeuvre detectors are basically ultra low pass filters, high frequency noise will have a minimal effect on their accuracy.

The residue frequency response is shown for tracking filter bandwidths between 0.5 to 4.0Hz. As can be seen from Fig.6.4, very little noise is passed into the lag detection filter, hence it will have only a small effect on the probability of false alarm.

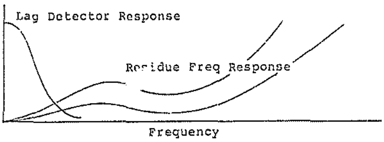

Fig.6.4 Lag Detector and Residual Frequency Responses

The bandwidth will have a major effect on the time to detection of a given manoeuvre, because, as the bandwidth decreases, the larger will be the lag per unit acceleration, hence the faster it will cross a given threshold.

6.3.5 Calculating The Transition Thresholds -

Each change of state requires an up threshold which will result in a step change to a wider bandwidth, and a down threshold which will result in the reverse.

6.3.5.1 Calculating The Up Thresholds -
Each filter state is defined by a minimum and a maximum acceleration. It is required that the state changes once the acceleration exceeds the the upper limit.

For the Tracking Filter the present estimate residual stream is monitored and its lag is extracted. As the relationship between the acceleration and lag is known, the lag for the maximum allowed acceleration is calculated and used as the up threshold.

This up threshold is calculated for all the filter states except for the highest, where it is set to a large number and thus ensures that the filter never tries to increase its bandwidth further.

6.3.5.2 Down Threshold -
This threshold is calculated *in the following manner* (derivation in Appendix F). It is known that when the bandwidth steps down, the resultant lag at the lower bandwidth must be lower than the maximum allowed. A 10% safety margin is allowed.

The Down Threshold is calculated from the following formula:

$$\text{Threshdn} = \text{Thresnup} \cdot \frac{\text{Acc Max}(N-1)}{\text{Acc Max}(N)} \cdot 0.9$$

Threshdn	=	Down threshold for state N
Threshup	=	Up threshold for state N
AccMax(N-1)	=	*Max acceleration allowed* in state N-1
AccMax(N)	=	Max acceleration allowed in state N

In the same manner that there is no up threshold for the top state, the down threshold for the bottom state is set to zero.

6.4 Filter Settling Algorithm

On startup, the filter moving sum must be *filled*. After each addition the correct average is calculated. If a constant velocity gain is maintained during this time, the filter bandwidth will fluctuate. To reduce this effect, Beta is made a function of the number of points in the memory.

The following equations were tried in an attempt to perform this smoothing:

$$\text{Beta} = 0.1 - \frac{N}{590} \qquad \text{Beta} = \frac{0.77}{N}$$

N = Number of Samples in Average

Graphically the Gains and Bandwidths of the settling strategies are illustrated in the following pair of graphs:

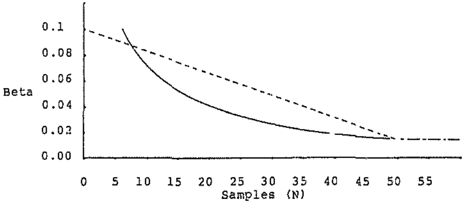

Fig.6.5 Velocity Gain as a function of Sample Number

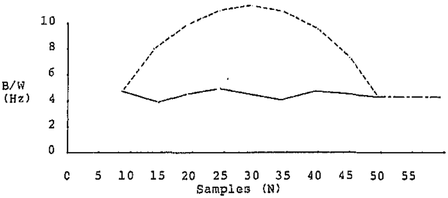

Fig.6.6 Filter Bandwidth as a function of Sample Number

6.5 *Final Adaptive Filter*

From the formulae and tables outlined in this chapter it is possible to select *an 'optimum'* adaptive tracking filter.

The Tracking Filter selected initially was the Mod No2 Centroid Beta Filter with a 50 point moving average.

It was decided that five states should be used for the filter, hence the chart of acceleration probabilities could be used as they are tabulated.

Table.6.8 Parameters of an optimum *adaptive Track Filter*

ACCEL m/s.s	BETA	THRESHUP m	THRESHDN m	WORST LAG m
0 - 3	0.0009	1.3	0.0	1.45
3 - 10	0.0021	1.8	0.486	2.09
10 - 25	0.0041	2.2	0.792	2.7[illegible]
25 - 40	0.0057	2.4	1.35	3.16
> 40	0.0155	1000	0.576	?

To ensure the detection of the smallest lag (0.486 m) it has been calculated that an Alpha = 0.001 would be required. However it was found that 2300 points would be required to ensure detection, hence it is not feasable to allow such a high confidence level.

It has been found by experiment that the minimum alpha that gives acceptable detection times is *0.01*.

Derivation of any of the formulae used in the section on manoeuvre detection can be *found in Appendix F*.

7.0 DEVELOPMENT OF MEMORY TRACK ALGORITHMS

7.1 Introduction

Three measurements are required to define a target position in space, which for a tracking radar are range elevation and bearing.

If some or all of the measurements are absent, the target position cannot be resolved exactly. Memory Tracking is thus only an attempt to estimate the missing measurements.

A number of techniques have been developed to perform this estimation. These vary from very simple linear predictions using available target velocity and position data to very complex methods which involve probable target behaviour patterns.

The three memory track configurations that will be considered are as follows:

1. Range Memory Track (Estimate Range)

2. Angle Memory Track (Estimate Elevation and Bearing)

3. Full Memory Track (Estimate both Range and Angles)

These will be dealt with in turn in the following sections.

7.2 Range Memory Track

In this case no range measurement is available, as would be the case if the target being tracked was using a *Self Screening Jammer*, or if some form of Range Gate Seduction was being attempted, and the operator was not happy about the range estimate.

With no range available, the target can be thought of as at any point along the angular measurement vector. The object of range memory track is thus to try to pinpoint the target position on this 'vector'.

7.2.1 Linear Extrapolation -

The most simple method of estimating the range will be by extrapolating the last measured range using the latest estimate of the range rate (polar extrapolation). A similar method would be to perform the linear extrapolation in cartesian space and to neglect the resultant angular estimates, and use only the resultant range.

The former method is ideal for a radially approachng target, while the latter would be superior in the case of a flypast where the range rate can be expected to change rather rapidly.

For typical attack profiles (pitchup/dive attacks with a bomb bunt, and a gun run), a comparison has been made assuming successive periods of memory track each 2 sec long, and repeated over the whole profile.

The results of the tests are as follows:

Table.7.1 Linear Range Extrapolation Comparison

EXTRAP TYPE	MEAN ABS ERROR
Polar	42.5m
Cartesian	39.2m

7.2.2 Other Methods -

A number of other estimation methods were tested. These are listed briefly below.

1. Shortest Distance Correction: This is a method for correcting the linearly extrapolated position by assuming that the correct target position is that on the angular measurement vector, at a point closest to the extrapolated estimate.

2. Constant Speed Correction: The assumption which is made in this case is that the traget is flying at a constant speed, and that this speed is known. The estimated target position will thus be at the intersection of the angular measurement vector, and the constant speed sphere. Such a correction will only be performed every 1/2 sec, as if it were performed more often, the effects of measurement noise on the accuracy of the correction would be substantial.

Results of the tests were promising, it was found that a full pitchup profile could be followed using either method with a final tracking error of less than 1000m. Standard linear extrapolation methods would have resulted in errors exceeding twice this amount.

More detail of the methods outlined above may be found in Appendix G.

7.3 Angle Memory Track

In this case, the angle measurement is unused or unavaila le, hence the target can be thought of as being at any point c a sphere centered at the radar. In reality however, for measuremen s to have taken place, the target must have been within the radar bea .

A number of techniques have been developed to estimate the unmeasured angles, some of which will be outlined in the f llowing paragraphs.

7.3.1 Linear Extrapolation -
This method uses the latest estimate of target velocity (carte ian) or angular rates, to linearly predict the target position. As ith the range method, if the extrapolation is performed in cartes an space, the calculated value of the measured variable is ignored.

There are two major disadvantages in extrapolating in a polar frame. Firstly the formulae for calculating the angular rates from the estimated cartesian velocities are complex, and secondly, prediction may only be performed in a ship stable (NSF) frame, hence all of the angular data must be converted to the correct frame of reference before it can be used.

The following formulae for the angular rate conversions are derived in Appendix G:

$$\dot{T} = \frac{z}{R^2\sqrt{R^2 - z^2}}(x\dot{x} + y\dot{y}) - \frac{\sqrt{R^2 - z^2}}{R^2}\dot{z}$$

$$\dot{P} = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2}$$

$\dot{T}$ = Rate of change of Elevation
$\dot{P}$ = Rate of change of Azimuth
R = Range
x,y,z = Cartesian position Estimates
$\dot{x},\dot{y},\dot{z}$ = Cartesian velocity Estimates

For the typical profiles as used for the range comparison, the results show that polar extrapolation is slightly superior to the cartesian method.

Table.7.2 Linear Angle Extrapolation Comparison

EXTRAP TYPE	MEAN ABS ERROR
Polar	37.0m
Cartesian	44.7m

7.3.2 Other Methods -

Again a number of other estimation methods were developed and tested, these are briefly outlined below.

1. Constant Direction Correction: This technique assumes that the measured target direction is correct, but that magnitude may be wrong, hence the estimated target position will the intersection of the above vector and the measurement sphere.

2. Constant Speed Correction: The assumption is made that the target speed is accurate, hence the position estimate will be on the intersection of the speed sphere and the measurement sphere (ie. a circle). The correct position on the circle is assumed to be the point which is closest to the linear extrapolated position.

Results of these techniques for estimation are not so promising as those for the range memory track type, however in some cases when the estimates were filtered, final errors of less than 500m were obtained. The tests conducted were not conclusive however, as radar beamwidth limitations were not considered.

Appendix G contains more detail of the above outlined estimation methods.

7.4 Full Memory Track

It is not possible to perform any type of correction on the full memory track estimates, hence they are simple linear extrapolation using the last valid velocity estimates and measured position.

7.5 Results

Because of the complexity of the memory track methods which use typical target behaviour, and because the improvements obtained were neither as good as expected, nor consistently obtainable, it was decided that simple linear extrapolation would be used.

Also from point of view of simplicity, it was decided that extrapolation will be performed in cartesian space in the NSF(p) frame.

Fig.7.1 to Fig.7.3 are plots of the three memory track types (range, angle and full). Each of the plots shows the effect of sequentially switching between normal and memory track.

Typical errors after 4sec range from 30m for a fairly constant velocity target, to 150m for the target accelerating at 4g. It should be noted also that the error contributions are not only caused by the targets accelerating away from the constant velocity prediction used, but also that a substantial velocity lag error already exists to corrupt the extrapolation.

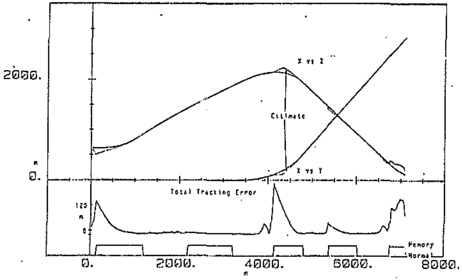

Fig.7.1 Range Memory Track: Linear Extrapolation

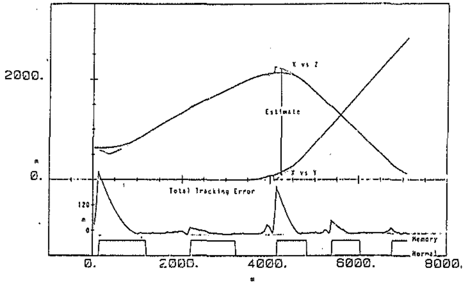

Fig.7.2 Angle Memory Track: Linear Extrapolation

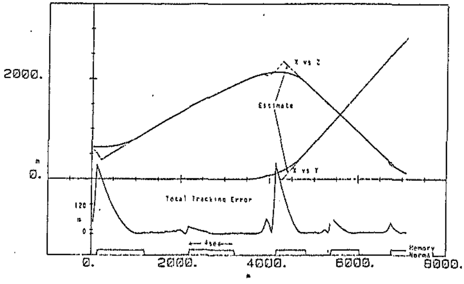

Fig.7.3 Full Memory Track

8.0 SHIP MOTION EXTRAPOLATION FILTERS

8.1 Introduction

The chapter on the Tracking Controller included the fact that the filtered data fed to the Servos was extrapolated one and two samples into the future, hence the addition of ships motion to that data must also use roll, pitch and heading that have been extrapolated to those times.

As well as the extrapolation of the ships attitude, the Filters must also predict the ships position with respect to the datum for the two times in the future.

To complicate matters, the SMS is read only every second tracking sample. For the first sample after the SMS has been read, the attitude is extrapolated to one and two samples ahead. However for the second sample period, the one sample ahead prediction is used as the present attitude, and the two sample prediction is used as a one sample prediction, so the SMS data must be extrapolated to three samples ahead for use as the two sample ahead prediction.

8.2 Extrapolation Filter Comparison

A technique of designing such Extrapolation Filters using FIR (finite impulse response) techniques has been developed by Kirsten (1981), and a filter optimised to perform a 55msec prediction designed. It is this filter that is used in the comparison.

Another method to perform the extrapolation would be to use an Alpha-Beta-Gamma filter to generate estimates of position, velocity and acceleration, then use them to extrapolate. This particular method allows for the prediction of 20, 40 and 60 msec ahead using one filter only, compared to the three filters that would be required if FIR filters were used.

Finally it may be possible to use a simple Alpha Beta filter if the magnitude of the motion is not too large, and the prediction time not too long.

The ships motion used in the comparison is generated using a formulation supplied by Gibson (1981) and expanded by Harrison (1981). The generator supplies roll, and pitch calculated to be worst case for South African waters.

The Alpha Beta Gamma Filter is optimised very roughly for minimum error variance for typical roll motion and the noise indicated in the error variance table. It would have been possible to perform the same optimisation for both pitch and heading, but as the main factor contributing to the error is the measurement noise, this would have been pointless.

Table.8.1 Ship Motion Prediction Filter Comparison

FILTER	GAINS	ERROR VARIANCE	
		NO NOISE	0.175mrad
FIR	see Kirsten	0.17094	0.612
AB	0.5 0.5	1.5240	1.535
ABG	.67 .37 .1	0.3350	0.4757
	.73 .47 .15	0.2320	0.4619
	.81 .64 .25		0.5323

Note: The Technical Manual (1978) on the Ship Motion Sensors gives a figure for the noise on the SM outputs of 0.175mrad RMS, however no bandwidth figures are given, so for the above measurements a B/W of 7.0Hz is assumed.

Table.8.1 and Figs 8.1 and 8.2 show the relative superiority of the Alpha Beta Gamma Filter for noisy measurement data.

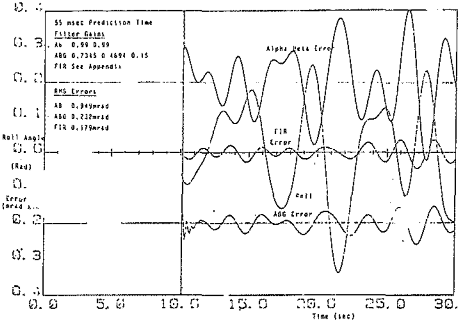

Fig.8.1 Ship Motion Prediction Comparison (Noise Free)

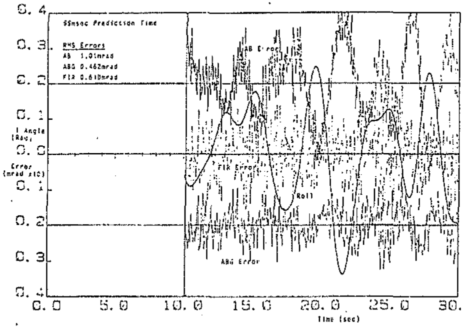

Fig.8.2 Ship Motion Prediction Comparison (Noise)

8.3 The Alpha Beta Gamma Filter

8.3.1 Configuration -

The filter may be represented by the following formulae:

Ap(20) = 20msec Ahead Extrapolation
Ap(40) = 40msec Ahead Extrapolation
Ap(60) = 60msec Ahead Extrapolation

$Ap(k)$ = One sample ahead predicted position
$A(k)$ = Present Position Estimate
$\dot{A}(k)$ = Present Velocity Estimate
$\ddot{A}(k)$ = Present Accelaration Estimate

Alpha = Position Gain
Beta = Velocity Gain
Gamma = Acceleration Gain
Ts = Sampling Frequency (40msec)

$Am(k)$ = Input: Cos/Sin of Roll, Pitch or Heading

$$Ap(k) = A(k-1) + \dot{A}(k-1).Ts + \ddot{A}(k-1).Ts^2/2$$

$$A(k) = Ap(k) + Alpha[\ Am(k) - Ap(k)\ ]$$

$$\dot{A}(k) = \dot{A}(k-1) + \ddot{A}(k-1).Ts + Beta/Ts[\ Am(k) - Ap(k)\ ]$$

$$\ddot{A}(k) = \ddot{A}(k-1) + 2.Gamma/Ts^2[\ Am(k) - Ap(k)\ ]$$

$$Ap(20) = A(k) + \dot{A}(k)20.0E-03 + \ddot{A}(k)2.0E-04$$

$$Ap(40) = A(k) + \dot{A}(k)40.0E-03 + \ddot{A}(k)8.0E-04$$

$$Ap(60) = A(k) + \dot{A}(k)60.0E-03 + \ddot{A}(k)1.8E-03$$

A brief analysis showed that it would be more efficient time wise to accept the Cosine and Sine values of the ship attitude from the SMS and use them to perform the predictions rather than to perform the predictions and then perform the conversions. Fig.8.3 which performs the extrapolation in this order indicates that the accuracy of the prediction is still good.

The selection of gains for the Alpha Beta Gamma filter is calculated from the Niel relationship out of Simpson (1963):

$$BETA^2 = 2.ALPHA.GAMMA$$

$$ALPHA(ALPHA + BETA + GAMMA/2) = 2.BETA$$

8.3.2 Performance -

A Program was written which, using the Ships Motion Model as before, determined the RMS prediction error for both Roll and Pitch for all the prediction times required, these errors are displayed in the following table.

The gains used in generating the following RMS errors, and the plot are as follows:

Alpha = 0.7345
Beta = 0.4694
Gamma = 0.1500

Table.8.2 RMS Prediction Errors for SM Extrapolation

XTRAP PERIOD msec	ROLL mrad	PITCH mrad
0.0 msec	0.1545	0.1507
20.0 msec	0.2287	0.2189
40.0 msec	0.3[illegible]48	0.3132
60.0 msec	0.4747	0.4[illegible]30

Note: The Noise on the measurements is assumed to be the same as that in the comparison. 0.175 mrad RMS with B/W of 7.0Hz

A plot which is displayed in Fig.8.3 shows the dynamic performance of the SMS prediction filters for the final choice of the gains, alpha, beta and gamma.

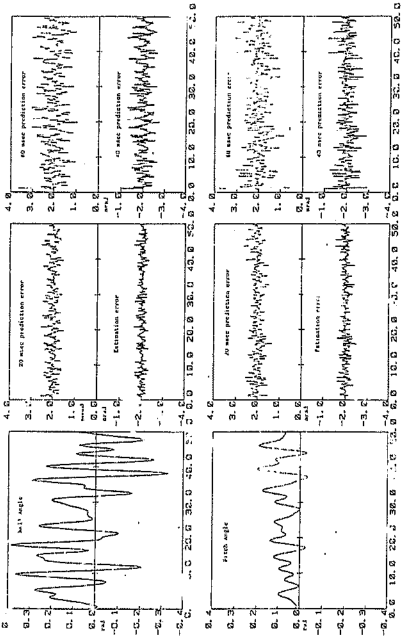

Fig.8.3 Performance of the Alpha Beta Gamma Filter.

9.0 TRACKING LOOP DEVELOPMENT EVALUATION

9.1 Summary

The Tracking Complex is one of the main sections of a Tracking Radar System, hence its performance dictates the overall performance of the radar to a large extent. For this reason, care must be taken in its development, both in terms of its operation and particularly its function in the overall system.

With regard to the latter, a loop structure was developed that would function best in the system. The main constraints in this case, were that the radar would operate from a moving platform, and that the angle servos operate at a different speed to the tracking loop.

In essence, the loop structure is as follows; The measurement data is converted to be relative to a fixed stable frame, filtering then takes place and the target position is predicted one and two samples ahead. The predictions are then converted back to the correct frame for the operation of the angle servos which generates its own faster estimates of the target position by linear interpolation between the two points.

With regard to the development of the tracking filters, a more analytical approach was taken, firstly a set of parameters by which a filter would be evaluated was drawn up. These were then used to evaluate a number of filters over a bandwidth ranging from 0.5Hz to 4.0Hz. The best filter was then used in the tracking loop.

To optimise the filter performance for a real tracking environment, it was necessary to develop realistic target and measurement noise models, which were then used as inputs when a gain adaptation strategy was developed for the tracking filters.

Finally, for the case where the measurement information was corrupt or incomplete, it was necessary to develop a number of algorithms that could be used to estimate this information.

9.2 Conclusions

With regard to the Tracking Loop development and performance a number of important conclusions can be drawn. These are dealt with in the following section.

9.2.1 Tracking Performance -

As far as can be determined from simulation, the Tracking Loop performs the required target position estimation adequately under normal conditions. As no performance specification was introduced, these results must be dealt with in absolute terms.

During normal conditions with standard noise inputs as defined in Chapter 4, for sea states leading to ship motion as defined in Chapter 8 and for a target whose acceleration does not exceed 6g as defined in Chapter 3; the following table summarises the RMS tracking errors that were obtained for the overall tracking loop:

Table.9.1 Simulation RMS Tracking Errors

Estimate	*RMS Error*
Range	1.75m
Range Rate	5.5m/s
Elevation	1.6mrad
Azimuth	0.4mrad

note: the difference between the azimuth and elevation errors is due to multipath effects and larger acceleration lags in the latter.

During memory tracking conditions the mean absolute tracking error for a series of two and four second periods was measured during simulation to be the following:

Table.9.2 Simulation Mean Absolute Prediction Error

Memory Track Type	Error over 2 sec	Error over 4 sec
Range	40.0m	100.0m
Angle	45.0m	150.0m
Full	60.0m	180.0m

The main implications of the above table are that almost instant reacquisition of a target is possible after as long as four seconds. This is because the target will still be within the radar beamwidth and the range acquisition-gate length after that time.

Possibly the most important aspect of these results is that this tracking performance is obtained on board a moving ship which is rolling as much as 25deg from the horizontal. This means that the performance of the Tracking Radar should be virtually independent of sea state.

9.2.2 Tracking Loop Development -

It is felt that the following two major considerations have lead to the above tracking performance:

1. Tracking Loop Configuration

2. Choice of Prediction Filters

With respect to the configuration, it is felt that decoupling the target measurement data to make it completely independent of the measurement platform is the main reason for the consistantly good estimates. This is particularly so for the memory track modes on whose accuracy any extraneous motion will have a catastrophic effect.

Another major aspect of the loop structure which is expected to have a considerable effect on the overall system tracking accuracy is the generation of a pair of position estimates. This innovation allows the angle servos to interpolate between two good target position estimates rather than to extrapolate from the current estimate. This will help minimise the non-linear effects of ship motion as explained in Chapter 5.

A substantial portion of this dissertation deals with the development of filters for estimation and prediction, these include the Tracking and Ship Motion Estimation filters.

It was shown that for the specified constraints and criteria that the Mod No-2 Centroid Beta Filter is superior to either the Centroid Beta or Alpha Beta filters. However, examination of the test results shows that its superiority is marginal.

It is believed that any one of the filters would have performed adequately in this tracking context, and that the main performance considerations are the gain optimisation and adaptive strategies employed.

A more important group of filters with regard to the overall tracking accuracy are the Alpha Beta Gamma filters used for the estimation and prediction of ship attitude. These are more critical than the tracking filters as the effect of a poor attitude estimate can have a more marked effect on tracking than can a single poor target position estimate.

In general, however, the design of such filters is more simple than those for tracking as the measurement data supplied by a Ship Motion Sensor will, in general, be more accurate and more reliable than that supplied by a Radar Sensor.

9.3 Future Work (Simulation)

Ideally the simulation would include the following functional blocks:

1. Target Model
2. Sensor Model
3. Tracking Filter Group
4. Servo Model
5. Ships Motion Model

The basic configuration of which is displayed in Fig.9.1.

9.3.1 Proposed System Performance Tests -

The main functions w[illegible]ld be improved to perform the overall system performance [illegible]re the target and sensor models. These would be altered [illegible] for multiple target returns, the importance of which c[illegible]e underestimated.

Multiple target model facilities will cater for the following Radar System tests:

1. Predispensed Chaff
2. Self dispensed Chaff
3. Crossing Targets [Birds, Shells, Other A/C]
4. Multipath
5. Range Gate *Seduction*

Some of the above have been conducted in 1-Dimension during the simulation of the Sensor Alpha-Beta Filter (Brooker 1981).

Another major effect on system performance which cannot be investigated to any degree without the overall system simulation is the effect on tracking of supplying the Sensor with an external estimate of range rate.

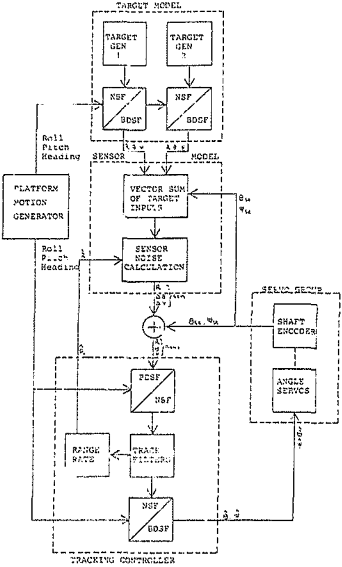

Fig.9.1 Tracking Radar Simulation Block Diagram

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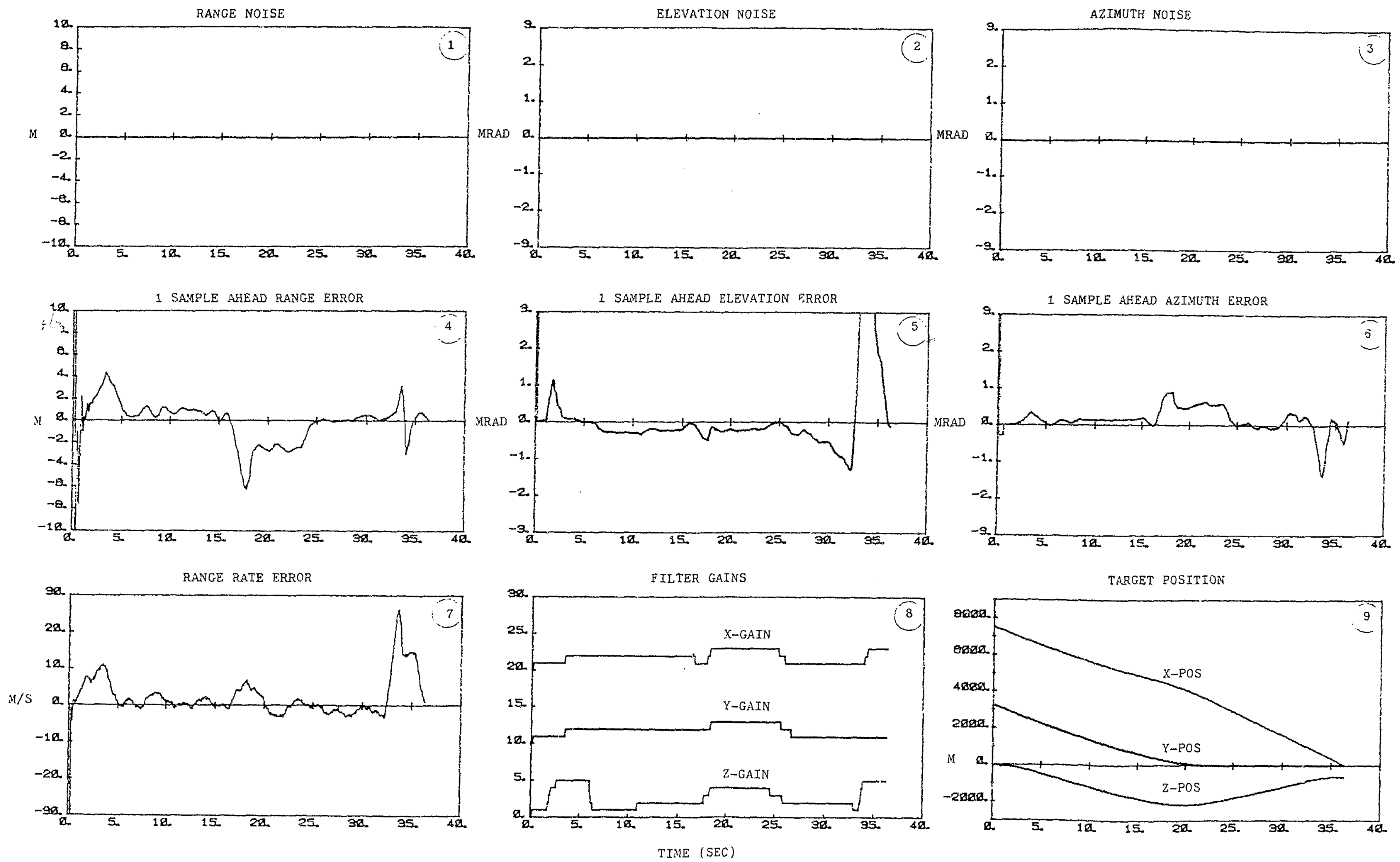
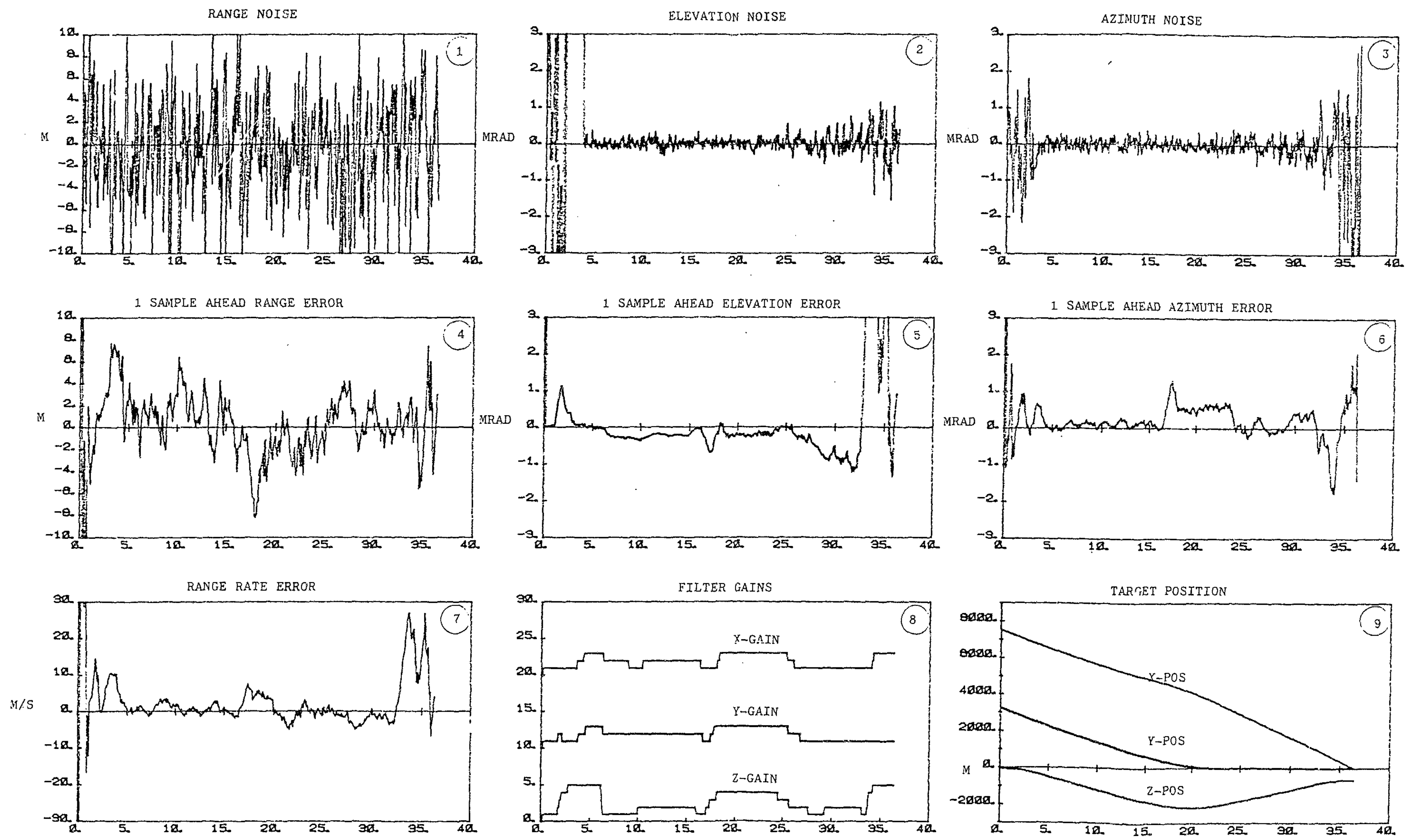
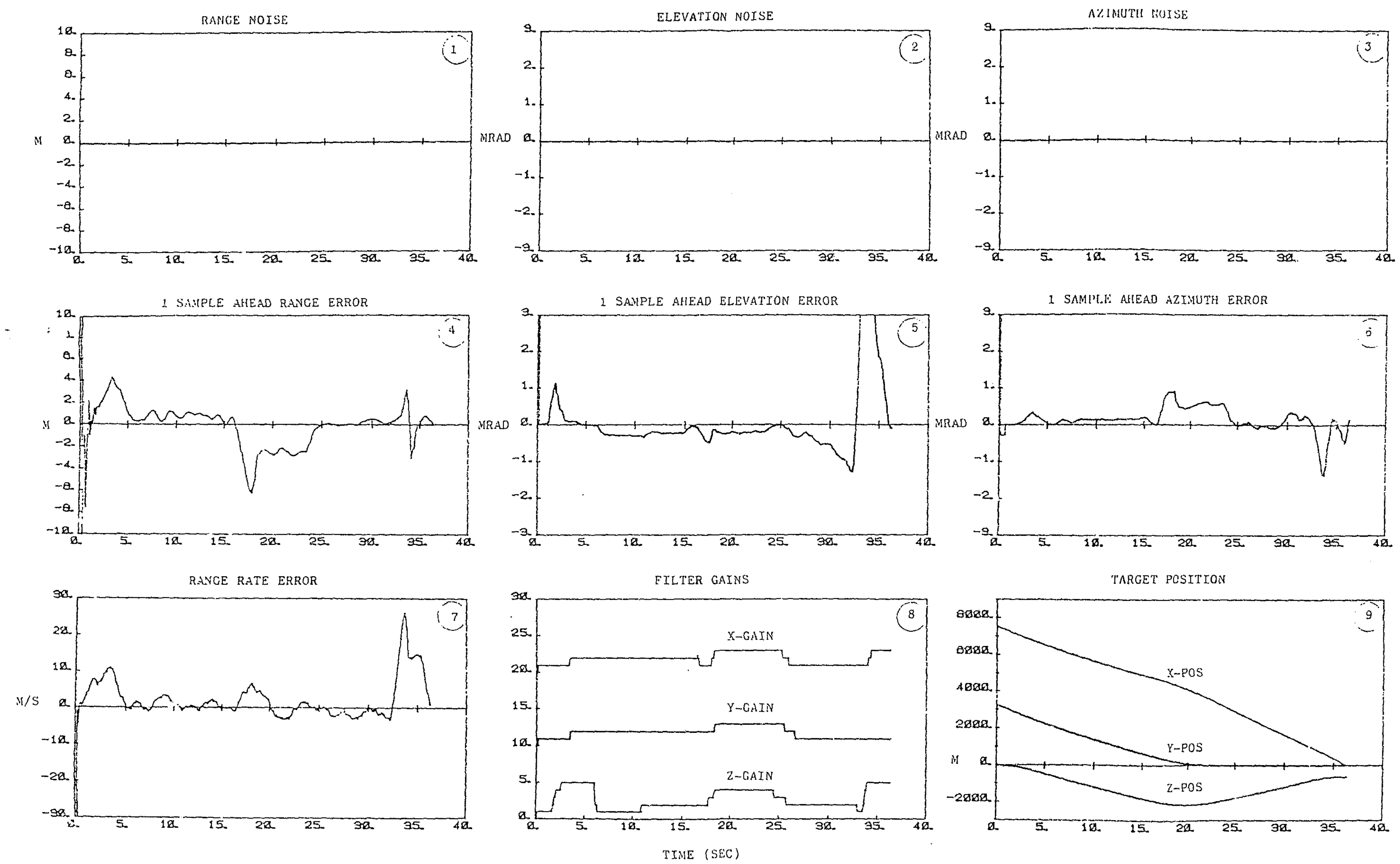
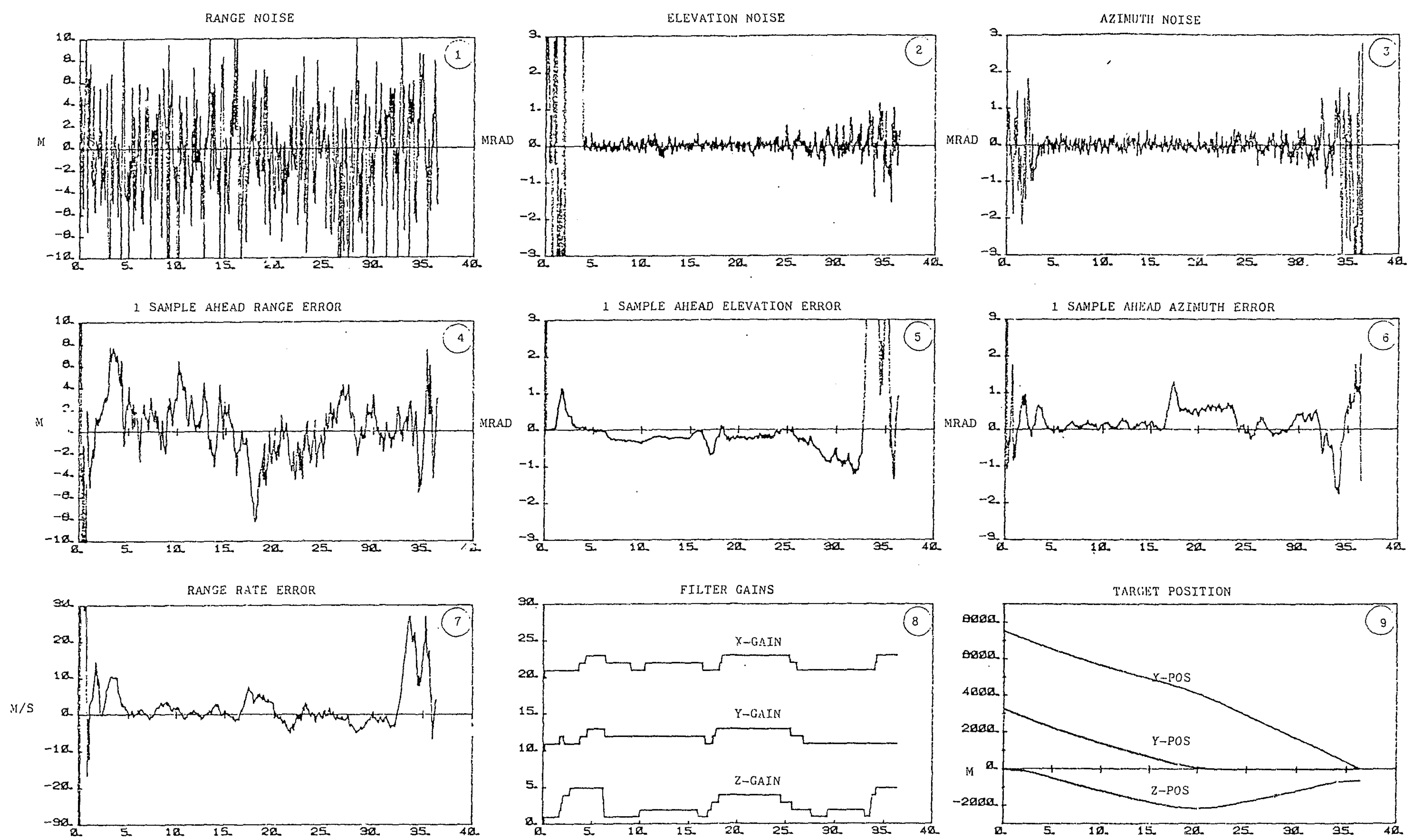
TRACKING CONTROLLER SIMULATION
(NOISE FREE)



FULL TRACKING CONTROLLER SIMULATION




RANGE NOISE
ELEVATION NOISE
AZIMUTH NOISE
1 SAMPLE AHEAD RANGE ERROR
1 SAMPLE AHEAD ELEVATION ERROR
1 SAMPLE AHEAD AZIMUTH ERROR
RANGE RATE ERROR
FILTER GAINS
TARGET POSITION
M
MRAD
M/S
X-GAIN
Y-GAIN
Z-GAIN
X-POS
Y-POS
Z-POS
TIME (SEC)



RANGE NOISE
ELEVATION NOISE
AZIMUTH NOISE
1 SAMPLE AHEAD RANGE ERROR
1 SAMPLE AHEAD ELEVATION ERROR
1 SAMPLE AHEAD AZIMUTH ERROR
RANGE RATE ERROR
FILTER GAINS
TARGET POSITION
M
MRAD
M/S
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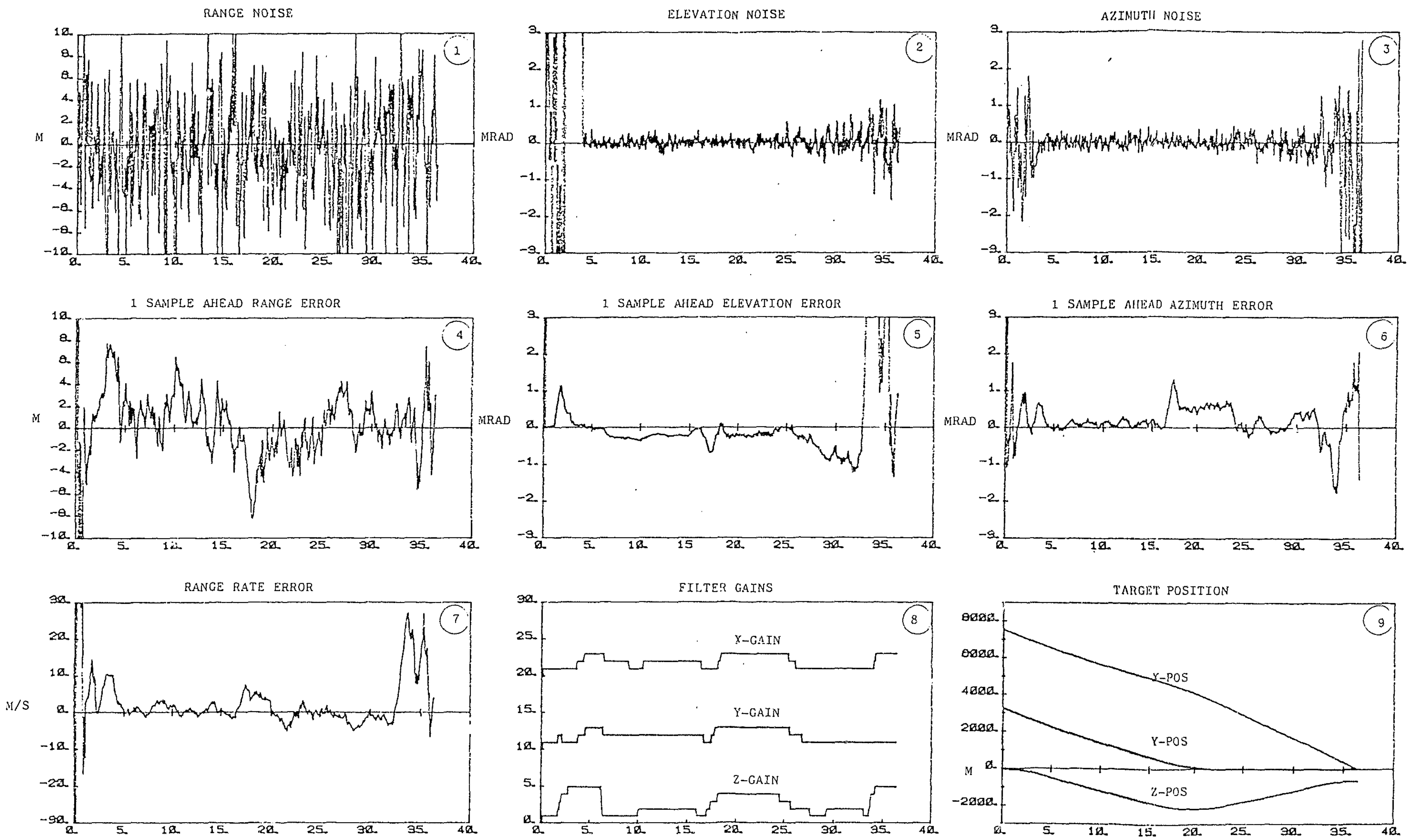
Z-GAIN
X-POS
Y-POS
Z-POS


RANGE NOISE
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1 SAMPLE AHEAD ELEVATION ERROR
1 SAMPLE AHEAD AZIMUTH ERROR
RANGE RATE ERROR
FILTER GAINS
TARGET POSITION
X-GAIN
Y-GAIN
Z-GAIN
X-POS
Y-POS
Z-POS
M
MRAD
M/S

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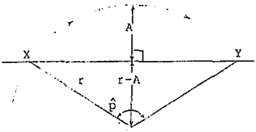
APPENDIX A

DERIVING THE FREQUENCY CONTENT OF A MANOEUVRE

Position Frequency Content



Length of arc XY subtended by angle $\hat{p}$

$$XY = r.\hat{p}$$

Time to fly the arc at speed v (m/s) is t

$$t = \frac{r.\hat{p}}{v} \qquad (1)$$

Hence the major frequency component is given by fm

$$fm = \frac{1}{2.t} = \frac{v}{2.r.\hat{p}} \qquad (2)$$

Angle $\hat{p}$ is given by the following formula:

$$\hat{p} = 2.\text{Arccos}\frac{r-A}{r} \qquad (3)$$

For a constant Acceleration $a=v.v/r$

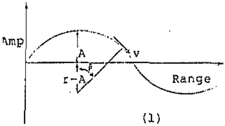
$$r = \frac{v^2}{a} \qquad (4)$$

Substituting (3) and (4) into (2)

$$fm = \frac{a}{4.v.\text{Arccos}\left\{1 - \frac{a.A}{v^2}\right\}} \qquad (5)$$

$$A = \frac{v^2}{a}\left[1 - \text{Cos}\left(\frac{a}{4.v.fm}\right)\right] \qquad (6)$$

Velocity Frequency Response



Range Rate $\dot{R}$

$$\dot{R} = v.\mathrm{Cos}(\hat{p}) \qquad (1)$$

Maximum Angle $\hat{p}$

$$\hat{p}max = \mathrm{Arccos}\ \frac{r-A}{r} \qquad (2)$$

hence $\dot{R}max = v$

$$Rmin = v.\frac{r-A}{r} \qquad (3)$$

Possible difference in range rate is:

$$d\dot{R} = v.[\ 1 - \frac{r-A}{r}\ ] = \frac{v.A}{r} \qquad (4)$$

For constant acceleration $r=v.v/a$

$$d\dot{R} = \frac{a.A}{v} \qquad (5)$$

hence.

$$A = \frac{d\dot{R}.v}{a} \qquad (6)$$

Substituting (6) into the position frequency response

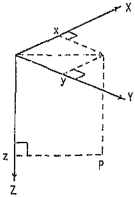
$$d\dot{R} = v\ [\ 1 - \mathrm{Cos}(\frac{a}{4.v.fm})\ ] \qquad (7)$$

APPENDIX B

DEFINITION OF COORDINATE SYSTEMS

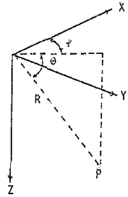
B.0.1 Types Of Coordinate Systems

B.0.1.1 Cartesian Coordinate System - Three orthogonal axes form a right handed system. Any point P in space can be expressed in terms of its three cartesian coordinates (x,y,z).



B.0.2 Polar Coordinate System

Using the above coordinate system as a base, the position of a point P is expressed in terms of its three polar coordinates (R,P,T) as shown in the following figure.



B.0.3 Definition Of Coordinate Systems

B.0.3.1 North Orientated Surface Fixed Frame [NSF] - This system is defined by the following:

1. X-Axis pointing true North
2. Y-Axis pointing East
3. Z-Axis pointing vertically downward (toward the center of the earth)

The mnemonic for the axis system NSF(x) where (x) denotes the origin of the system as defined in the next section.

B.0.4 Bow/deck Orientated Platform Fixed Frame [BDSF]

This system is defined by the following:

1. X-Axis parallel to the platform longitudinal centre line pointing toward the bows
2. Y-Axis pointing to starboard
3. Z-Axis pointing downward in the platforms vertical plane of symmetry

The mnemonic for this system is BDSF(x).

B.0.5 Specifications For The Origin Of The Axis

When using BDSF(x) and NSF(x) representations. the (x) represents the following:

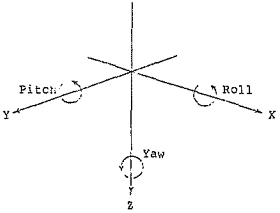
R (origin Radar)
M (origin Platform Motion Sensor)
P (origin Datum Point)
S (origin Mid-platform)

For example, a BDSF coordinate system with origin at the FCR will be called BDSF(R).

APPENDIX C

DERIVATION OF FORMULAE USED BY THE TRACKING CONTROLLER

Definition of Platform Attitude



Roll +ve port side up
Pitch +ve bows up
Yaw +ve clockwise from north

Removal of platform motion shall be in the reverse order to that which they are gymbalised in the SMS.

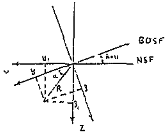
Removal of Platform Motion (BDSF to NSF Rotation)

Roll

yl = R.Cos(a+roll)   zl = R.Sin(a+roll)

Now: Cos(a+roll) = Cos(a).Cos(roll) - Sin(a).Sin(roll)
Sin(a+roll) = Sin(a).Cos(roll) + Cos(a).Sin(roll)

And: y = R.Cos(a)   z = R.Sin(a)



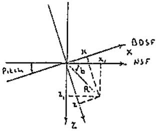
Hence: yl = y.Cos(roll) - z.Sin(roll)
zl = z.Cos(roll) + y.Sin(roll)

Pitch

xl = R.Cos(b-pitch)   zl = R.Sin(b-pitch)

Now: Cos(b-pitch) = Cos(b).Cos(pitch) + Sin(b).Sin(pitch)
Sin(b-pitch) = Sin(b).Cos(pitch) - Cos(b).Sin(pitch)

And: z = R.Sin(b)   x = R.Cos(b)



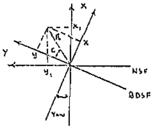
Hence: xl = x.Cos(pitch) + z.Sin(pitch)
zl = z.Cos(pitch) - x.Sin(pitch)

Yaw

yl = R.Cos(c+yaw)   xl = R.Sin(c+yaw)

Now: Cos(c+yaw) = Cos(c).Cos(yaw) - Sin(c).Sin(yaw)
Sin(c+yaw) = Sin(c).Sin(yaw) + Cos(c).Cos(yaw)

And: y = R.Cos(c)   x = R.Sin(c)



Hence: yl = y.Cos(yaw) - x.Sin(yaw)
xl = x.Cos(yaw) + y.Sin(yaw)

Addition of Platform Motion [NSF to BDSF Rotation]

This is performed in the reverse order to the removal.

Yaw

Given: $y1 = y.Cos(yaw) - x.Sin(yaw)$
$x1 = x.Cos(yaw) + y.Sin(yaw)$

Then: $y1.Cos(yaw) + x1.Sin(yaw) = y.Cos^2(yaw) + y.Sin^2(yaw)$

Hence: $y = y1.Cos(yaw) + x1.Sin(yaw)$
$x = x1.Cos(yaw) - y1.Sin(yaw)$

By a similar method the addition of pich and roll can be determined.

Pitch

$z = z1.Cos(pitch) + x1.Sin(pitch)$
$x = x1.Cos(pitch) - z1.Sin(pitch)$

Roll

$y = y1.Cos(roll) + z1.Sin(roll)$
$z = z1.Cos(roll) - y1.Sin(roll)$

The above formulae can be combined to form a series of equations which will perform either the removal or the addition of platform motion.

BDSF to NSF Rotation

```
z* = z.Cos(roll) + y.Sin(roll)
y* = y.Cos(roll) - z.Sin(roll)
x* = x.Cos(pitch) + z*.Sin(pitch)
z1 = z*.Cos(pitch) - x.Sin(pitch)
y1 = y*.Cos(yaw) - x*.Sin(yaw)
z1 = x*.Cos(yaw) + y*.Sin(yaw)
```

NSF to BDSF Rotation

```
x* = x1.Cos(yaw) [illegible].Sin(yaw)
y* = y1.Cos(yaw[illegible] Sin(yaw)
z* = z1.Cos(pit[illegible].Sin(pitch)
x = x*.Cos(pitc[illegible] Sin(pitch)
y = y*.Cos(roll) [illegible].Sin(roll)
z = z*.Cos(roll) - y*.Sin(roll)
```

Range Rate Calculation [Cartesian to Polar]

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\dot{R} = \frac{dR}{dt} = \frac{dR}{dx}.\frac{dx}{dt} + \frac{dR}{dy}.\frac{dy}{dt} + \frac{dR}{dz}.\frac{dz}{dt}$$

And:

$$\frac{dR}{dx} = \frac{1}{2}(x^2 + y^2 + z^2)^{-1/2}.2x$$

$$\frac{dR}{dx} = \frac{x}{R}$$

Hence:

$$\dot{R} = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{R}$$

APPENDIX D

DERIVATION OF THE FORMULAE USED IN THE FILTER COMPARISON

The Centroid Beta Filter

Velocity Lag

For unit step acceletration, the Z-Transform of the Velocity and of the Position are given by the following Formulae:

$$\dot{X}m(z) = \frac{Ts.z}{(z-1)^2}$$

$$Xm(z) = \frac{Ts^2.z.(z+1)}{2.(z-1)^3}$$

The Residual E(z) is found by taking the difference between the actual and the estimated velocities.

$$E(z) = \dot{X}m(z) - Xm(z).H(z) \qquad H(z) \text{ Velocity Function}$$

The steady state velocity lag is calculated using this residual as follows:

$$Lv = \lim_{z \to 1} \frac{z-1}{z}.E(z)$$

Substituting for H(z), Xm(z) and $\dot{X}m(z)$

$$Lv = \lim_{z \to 1} \frac{Ts}{z-1} - \frac{Ts(z+1)}{2(z-1)^2}.B.z.\frac{\left[1 - 1/N \sum_{j=0}^{N-1} z^{-j} \right]}{z - \frac{N-1}{2}.B - 1}$$

Substituting for $a = 1 - \frac{N-1}{2}.B$

$$Lv = \lim_{z \to 1} Ts.\frac{2(z-1)(z-a) - B.z(z+1) + B/N \sum_{j=1}^{N} z^{-j} (z+1)}{2(z-1)^2 (z-a)}$$

As this has the form of 0/0 it is possible to use L'Hospitals rule, which when applied results in the following:

$$Lv = \frac{4 - 2B + B.\dfrac{[2N^2 - 9N + 13]}{3}}{2(N-1)B}$$

Which can be rewritten in the following form:

$$Lv = \ddot{X}\,\frac{Tc}{1}\left\{\frac{2}{(N-1)B} + \frac{2N-7}{6}\right\}$$

One and Two Sample Ahead Position Lag

This could be derived from the Z-Domain, however it is more simple to use the position and velocity lags as follows:

$$L1p = Lp + Lv.Ts + 1/2.\ddot{x}.Ts^2$$

$$L2p = Lp + 2.Lv.Ts + 2.\ddot{x}.Ts^2$$

The following formulae result after substitution:

$$L1p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+1}{N-1} + \frac{N-2}{3}.B\right\}$$

$$L2p = \ddot{x}\,\frac{Ts^2}{B}\left\{\frac{N+3}{N-1} + \frac{2N-1}{3}.B\right\}$$

Step Response Time

A unit step in the Z-Domain is given by:

$$Xm(z) = \frac{z}{z-1}$$

The residual of the actual and estimated positions:

$$E(z) = Xm(z) - Xm(z).H(z) \qquad H(z) \text{ Position Function}$$

Substituting:

$$E(z) = \frac{z}{z-1} - \frac{z}{z-1} \cdot \frac{(z-1)\frac{1}{N}\sum_{j=0}^{N-1} z^{-j} + \frac{N-1}{2}.B}{z + \frac{N-1}{2}.B - 1}$$

Substituting for $a = 1 - \frac{N-1}{2}.B$

$$E(z) = \frac{z[1 - \frac{1}{N}\sum_{j=0}^{N-1} z^{-j}]}{z-a}$$

Expanding to a series:

$$E(z) = \frac{z}{z-a} - \frac{1}{N}\left\{ \frac{z}{z-a} + \frac{z^{-1}}{z-a} + \ldots \frac{z^{-N+2}}{z-a} \right\}$$

$$= \frac{1}{z^{N-1}(z-a)}\left\{ z^{N} - \frac{1}{N}.\{ z^{N} + z^{N-1} + \ldots z \} \right\}$$

By residual methods for n>N the error stream is as follows:

$$en = \mathrm{Res}\left[\frac{1}{z-a} \left[z^{N} - \frac{1}{N}.\left(z^{N} + z^{N-1} + \dots z \right) \right] z^{n-N} \right]_{z=a}$$

$$= \frac{\left[a^{N} - \frac{1}{N}.\left(a^{N} + a^{N-1} + \dots a \right) \right] a^{n}}{a^{N}}$$

This can be written in terms of the following Sum:

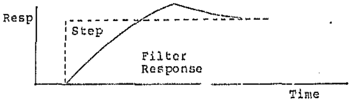
$$\frac{1}{N} \sum_{j=1}^{N} a^{j} = \frac{a}{N} \sum_{j=0}^{N-1} a^{j} = \frac{1}{N} \left\{ \frac{a - a^{N+1}}{1-a} \right\}$$

Resulting in the following:

$$en = \frac{(1-a) + \frac{1}{N}.\left(a - a^{1-N} \right)}{1-a} \; . \; a^{n}$$

Require that en < 0.01   that is 1% of a unit step
Hence by substituting and solving for n the following formula is obtained:

$$n = \frac{\mathrm{Log} \dfrac{0.01\ (a-1)}{(1-a) + 1/N\ (a-a^{1-N})}}{\mathrm{Log}(a)}$$



The Ramp Response Time

The ramp response is obtained in a similar manner, though instead of using a unit step in the Z-Domain, a unit ramp is used:

$$Xm(z) = \frac{Ts.z}{(z-1)^2}$$

The resultant error stream is as follows:

$$en = \frac{Ts\left\{\frac{a}{N}.(1-a^N) - a^N.(1-a)\right\}a^{n-N}}{(1-a)^2}$$

For the required 1% accuracy en < 0.01.n.Ts
After substituting into th above equation, it can be seen that n can only be ..ved using inerative means.

$$n.Log(a) - Log(n) = Log\frac{0.01\ (1-a)^2}{a/N\ (a^{-N}-1) - (1-a)}$$

If an absolute error is acceptable, then the following form of the equation can be used:

$$n \approx \frac{Log\dfrac{0.01\ (1-a)^2}{a/N\ (a^{-N}-1) + (a-1)}}{Log(a)}$$

Mod No2 Centroid Beta Filter

One and Two Sample Ahead Position Lags

The velocity lag of this filter is the same as that for the standard centroid beta filter, hence the differences in the prediction position lag will be a function of the present position lag only.

$$Lp = \ddot{x} \frac{Ts^2}{B} \left\{ 1 - \frac{N-1}{2}.B \right\}$$

$$L1p = \ddot{x} \frac{Ts^2}{B} \left\{ \frac{N+1}{N-1} - \frac{N+1}{6}.B \right\}$$

$$L2p = \ddot{x} \frac{Ts^2}{B} \left\{ \frac{N+3}{N-1} + \frac{N+1}{6}.B \right\}$$

Step Response Time

Substituting into the following equation and simplifying:

$$E(z) = Xm(z) - Xm(z).H(z)$$

$$E(z) = \frac{z}{z-1} - \frac{z}{z-1}.\frac{(1 - \frac{N-1}{2}.B)\ \frac{1}{N}\sum_{j=0}^{N-1} z^{-j} - z.\frac{N-1}{2}.B}{z + \frac{N-1}{2}.B - 1}$$

Replacing $1 - \frac{N-1}{2}.B$ by 'a'

$$E(z) = \frac{z.a.(1 - \frac{1}{N}\sum_{j=0}^{N-1} z^{-j})}{z-a}$$

It can be seen that this is the same as the step response for the *standard* Centroid Beta Filter, except that it is scaled by the factor 'a'.

Hence:

$$e_n = \frac{a\left\{a^N - \frac{a}{N}.\frac{(1-a^N)}{1-a}\right\}a^n}{a^N}$$

For a 1% accuracy the following equation results:

$$n = \frac{\mathrm{Log}\ \frac{0.01\ (a-1)}{a(1-a) + a/N(a-a^{1+N})}}{\mathrm{Log}(a)}$$

Ramp Response Time

Substituting the Z-Domain representation of the unit ramp and the position transfer function into the following formula:

$$E(z) = Xm(z) - Xm(z).H(z)$$

Where:

$$Xm(z) = \frac{Ts.z}{(z-1)^2}$$

Gives the following:

$$E(z) = \frac{Ts.a\left\{z^N - \frac{1}{N}\sum_{j=0}^{N-1} z^{-j}\right\}}{z^{N-1}\ (z-1).(z-a)}$$

Which is again similar to the standard Centroid Beta residue, hence the error stream will be as follows:

$$en = \frac{Ts.a\left\{\frac{a}{N}.(1-a^N) - a^N.(1-a)\right\}a^{n-N}}{(1-a)^2}$$

The solution for n will again be by an iterative process unless an absolute error is acceptable:

$$n = \frac{\text{Log}\ \dfrac{0.01\ (1-a)^2}{a^2/N\ (a^{-N}-1) + a(a-1)}}{\text{Log}(a)}$$

The Alpha Beta Filter

Velocity Lag

The velocity residual is found by comparing the actual target velocity with the velocity estimate supplied by the filter.

The Z-Domain representations of the position and velocity of a unit step acceleration are given below:

$$\dot{X}m(z) = \frac{Ts.z}{(z-1)^2}$$

$$Xm(z) = \frac{Ts^2.z.(z+1)}{2.(z-1)^3}$$

Substituting into the following formula:

$$E(z) = \dot{X}m(z) - Xm(z).H(z) \qquad H(z) \text{ Velocity Function}$$

$$E(z) = \frac{Ts.z}{(z-1)^2} - \frac{Ts^2.z.(z+1)}{2.(z-1)^3} . \frac{h}{Ts} . \frac{z(z-1)}{z^2 + (h+g-2)z + (1-g)}$$

Taking limit $\lim_{z \to 1} \frac{z}{z-1}.E(z)$

Using L'Hospitals rule:

$$Lv = \ddot{X}.Ts\left\{\frac{2g - h}{2h}\right\}$$

The One and Two Sample Ahead Position Lags

Substituting into the following formula:

$$L1p = Lp + Lv.Ts + 1/2.\ddot{x}.Ts^2$$

$$L2p = Lp + 2.Lv.Ts + 2.\ddot{x}.Ts^2$$

The following Lags result:

$$L1p = \ddot{x}.\frac{Ts^2}{h}$$

$$L2p = \ddot{x}.\frac{Ts^2}{h}.\{ 1 + g + h \}$$

Step Response Time

Substituting into the Following Formula:

$$E(z) = Xm(z) - Xm(z).H(z)$$

$$E(z) = \frac{z.(1-g).(z-1)}{z^2 + (h+g-2).z + (1-g)}$$

For an underdamped system, the response may be obtained in the following form:

$$en = 2.A.R^n \, Cos(nT+P)$$

For the step response, we are interested only in the envelope:

$$en = 2.A.R^n \qquad \text{where A is an absolute value}$$

It can be shown that:

$$2.A = \sqrt{\frac{4.h.(1-g)^2}{4.(1-g) - (2-g-h)^2}}$$

And that:

$$R = (1-g)$$

Hence the envelope is given by the following formula:

$$en = \sqrt{\frac{4.h.(1-g)^2}{4.(1-g) - (2-g-h)^2}} \, . \, (1-g)^{n/2}$$

For the 1% accuracy required en=0.01 hence:

$$n = \frac{\operatorname{Log} \dfrac{[4(1-g) - (2-g-h)^2]0.01^2}{4h(1-g)}}{\operatorname{Log}(1-g)}$$

For the critically damped filter, the roots are equal.

$$R_1 = R_2 = \frac{2-g-h}{2}$$

Resulting in the following Residual:

$$E(z) = \frac{z.(1.g).(z-1)}{\left\{z - \dfrac{2-g-h}{2}\right\}^2}$$

Expanding by partial fractions and using the following identity:

$$n^1 x(n) = (-z.\frac{d1}{dz})^1.X(z)$$

$$\frac{z}{(z-R)^2} = \frac{1}{R}.n.R^n = n.R^{n-1}$$

Hence:

$$en = (1-g).R^n - (1-g)(\frac{g+h}{2}).n.R^{n-1}$$

Substituting for R and $g = 2\sqrt{h} - h$:

$$en = \{ 1 + h - 2\sqrt{h} \}\{ 1 - (n-1)\sqrt{h} \}(1 - \sqrt{h})^{n-1}$$

Again to solve for n, some iterative method must be used.

Ramp Response Time

It can easily be shown that the relationship between the step and ramp response residues is:

$$E(z)r = \frac{E(z)s}{(z-1)}$$

$$E(z)r = \frac{z.(1-g)}{z^2 + (h+g-2).z + 1-g}$$

Expanding by partial fractions and calculating the envelope.

$$2.A.R^n = \frac{2.(1-g)}{\sqrt{4(1-g)-(2-g-h)^2}} \cdot (1-g)^{n/2}$$

Again, if an absolute accuracy of 0.01 is acceptable for the underdamped system, then the following form can be used:

$$n = \frac{\text{Log} \dfrac{[4(1-g) - (2-g-h)^2]0.01^2}{4(1-g)^2}}{\text{Log}(1-g)}$$

For the Critically Damped Filter:

$$E(z) = \frac{z.(1-g)}{\left(z - \frac{2-g-h}{2}\right)^2}$$

The solution can be found using the step response formulation:

$$n = (1 + h - 2\sqrt{h}).n.(1 - \sqrt{h})^{n-1}$$

A proper 1% accuracy is thus given by the following:

$$n = \frac{\text{Log}\,\frac{(1-\sqrt{h})0.01}{1 + h - 2\sqrt{h}}}{\text{Log}(1-\sqrt{h})}$$

APPENDIX E

TRACKING FILTER COMPARISON TABLES

EQUAL BANDWIDTH COMPARISON

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	4.0000	4.0050	4.0100
BETA	0.03687	0.01589	0.01589
ALPHA or N	0.3472	50.00	50.00
VRR POS	0.24356595	0.25814146	0.25734147
VRR VEL	0.00239631	0.00039331	0.00039331
COV RR	0.02136358	0.00589406	0.00966179
VRR POS-1	0.28868943	0.27032289	0.27705836
VRR POS-2	0.3386055	0.2832910	0.2975619
VRR POS-150	60.56971	10.87592	12.00544
LAG POS	17.704	62.933	38.433
LAG VEL	8.9152	18.0687	18.0687
LAG POS-1	27.119	81.501	57.001
LAG POS-2	37.534	101.070	76.570
STEP RESP	27.0	52.0	51.0
RAMP RESP	19.0	42.0	38.0
FILTER RATE	2.00452	1.72912	1.66636

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	3.0000	3.0050	3.0050
BETA	0.02143	0.01270	0.01270
ALPHA or N	0.2714	50.00	50.00
VRR POS	0.18522361	0.20207520	0.20127520
VRR VEL	0.00098538	0.00029905	0.00029905
COV RR	0.01198342	0.00506682	0.00736284
VRR POS-1	0.21017583	0.21250789	0.21629994
VRR POS-2	0.2370988	0.2235387	0.2319228
VRR POS-150	25.95130	8.45072	9.13873
LAG POS	33.996	78.740	54.240
LAG VEL	12.1612	18.7139	18.7139
LAG POS-1	46.657	97.954	73.454
LAG POS-2	60.318	118.168	93.668
STEP RESP	37.0	53.0	52.0
RAMP RESP	27.0	44.0	41.0
FILTER RATE	1.91514	1.76274	1.72212

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.0000	2.0050	2.0050
BETA	0.00978	0.00904	0.00904
ALPHA or N	0.1880	50.00	50.00
VRR POS	0.12467687	0.14407063	0.14327066
VRR VEL	0.00028132	0.00020068	0.00020068
COV RR	0.00526866	0.00386880	0.00497407
VRR POS-1	0.13549551	0.15200889	0.15341946
VRR POS-2	0.1466768	0.1603485	0.1639697
VRR POS-150	8.03495	5.82003	6.15081
LAG POS	83.069	110.620	86.120
LAG VEL	18.7284	20.0151	20.0151
LAG POS-1	102.297	131.135	106.635
LAG POS-2	122.526	152.650	128.150
STEP RESP	58.0	57.0	56.0
RAMP RESP	[illegible]2.0	47.0	45.0
FILTER RATE	1.79990	1.80980	1.79030

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.5050	1.5050	1.5050
BETA	0.00559	0.00439	0.00463
ALPHA or N	0.1440	50.00	50.00
VRR POS	0.09413928	0.07953422	0.08181071
VRR VEL	0.00011729	0.00008709	0.00009275
COV RR	0.00296024	0.00202042	0.00239758
VRR POS-1	0.10017704	0.08366[illegible]14	0.08669861
VRR POS-2	0.1064494	0.0879642	0.0917720
VRR POS-150	3.62120	2.64508	2.88786
LAG POS	153.005	227.790	191.483
LAG VEL	25.2390	24.7976	24.7976
LAG POS-1	178.744	253.088	216.298
LAG POS-2	205.483	279.386	242.114
STEP RESP	78.0	74.0	71.0
RAMP RESP	57.0	57.0	55.0
FILTER RATE	1.93036	1.72917	1.74047

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.0050	1.0050	1.0050
BETA	0.00251	0.00482	0.00482
ALPHA or N	0.0977	50.00	50.00
VRR POS	0.062952[illegible]0	0.08504312	0.08425462
VRR VEL	0.000033[illegible]	0.00009724	0.00009724
COV RR	0.00130462	0.00220505	0.00250246
VRR POS-1	0.06559542	0.08955045	0.08935677
VRR POS-2	0.0683066	0.0942523	0.0946534
VRR POS-150	[illegible].21674	2.93437	3.02281
LAG POS	359.126	207.469	182.969
LAG VEL	38.4012	23.9681	23.9681
LAG POS-1	398.027	231.937	207.437
LAG POS-2	437.928	257.405	232.905
STEP RESP	120.0	71.0	70.0
RAMP RESP	87.0	55.0	54.0
FILTER RATE	1.74047	1.83257	1.82696

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.7550	0.7550	0.7550
BETA	0.00142	0.00204	0.00220
ALPHA or N	0.0740	50.00	50.00
VRR POS	0.04729771	0.05102950	0.05[illegible]2346
VRR VEL	0.00001418	0.00003248	0.00003610
COV RR	0.00073118	0.00096790	0.00110183
VRR POS-1	0.04877427	0.05299778	0.05456322
VRR POS-2	0.0502792	0.0550310	0.0568752
VRR POS-150	0.58580	1.07231	1.19502
LAG POS	651.479	490.223	430.046
LAG VEL	51.5482	35.5091	35.5091
LAG POS-1	703.527	526.232	464.598
LAG POS-2	756.575	563.241	500.151
STEP RESP	161.0	114.0	108.0
RAMP RESP	117.0	85.0	80.0
FILTER RATE	1.78763	1.78869	1.82368

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.5050	0.5000	0.5050
BETA	0.00064	0.00018	0.00019
ALPHA or N	0.0498	50.00	50.00
VRR POS	0.03159123	0.02404823	0.02424013
VRR VEL	0.00000416	0.00000049	0.00000054
COV RR	0.00032388	0.00008784	0.00009313
VRR POS-1	0.03224316	0.02422439	0.02442693
VRR POS-2	0.0329034	0.0244015	0.0246148
VRR POS-150	0.22239	0.06132	0.06429
LAG POS	1494.992	5555.556	5238.658
LAG VEL	77.8302	242.2574	242.2574
LAG POS-1	1573.322	5798.313	5469.480
LAG POS-2	1652.652	6042.070	5701.304
STEP RESP	243.0	400.0	400.0
RAMP RESP	178.0	400.0	400.0
FILTER RATE	1.93138	1.73429	1.73432

EQUAL 1 SAMPLE AHEAD VARIANCE *REDUCTION RATIO*

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	3.6800	3.9550	3.8650
BETA	0.03152	0.01550	0.01518
ALPHA or N	0.3236	50.00	50.00
VRR POS	0.22504173	0.25099003	0.24431087
VRR VEL	0.00184945	0.00038131	0.00037144
COV RR	0.01805856	0.00580483	0.00912796
VRR POS-1	0.26300830	0.26298100	0.26293823
VRR POS-2	0.3046738	0.2757346	0.2823005
VRR POS-150	47.25519	10.57199	11.34019
LAG POS	21.457	64.516	41.394
LAG VEL	9.7643	18.1333	18.1333
LAG POS-1	31.721	83.149	60.084
LAG POS-2	42.985	102.783	79.773
STEP RESP	30.0	52.0	51.0
RAMP RESP	21.0	42.0	38.0
FILTER RATE	1.96196	1.75543	1.68261

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	3.1000	3.0650	3.0300
BETA	0.02280	0.01300	0.01277
ALPHA or N	0.2792	50.00	50.00
VRR POS	0.19108006	0.20711967	0.20250717
VRR VEL	0.00108898	0.00030755	0.00030113
COV RR	0.01279140	0.00515372	0.00741343
VRR POS-1	0.21775185	0.21773466	0.21763515
VRR POS-2	0.2466016	0.2289648	0.2333654
VRR POS-150	28.53045	8.67315	9.20187
LAG POS	31.620	76.923	53.787
LAG VEL	11.7463	18.6397	18.6397
LAG POS-1	43.866	96.063	72.982
LAG POS-2	57.112	116.203	93.178
STEP RESP	36.0	53.0	52.0
RAMP RESP	26.0	43.0	41.0
FILTER RATE	1.93472	1.75832	1.70696

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.2200	2.0000	1.9800
BETA	0.01196	0.00900	0.00892
ALPHA or N	0.2068	50.00	50.00
VRR POS	0.13803193	0.14347170	0.14141764
VRR VEL	0.00038723	0.00019966	0.00019751
COV RR	0.00649997	0.00385438	0.00489741
VRR POS-1	0.15141912	0.15138012	0.15140998
VRR POS-2	0.1655807	0.1596878	0.1617973
VRR POS-150	10.80079	5.79205	6.05462
LAG POS	66.306	111.111	87.657
LAG VEL	16.7856	20.0351	20.0351
LAG POS-1	83.591	131.646	108.235
LAG POS-2	101.877	153.181	129.812
STEP RESP	52.0	57.0	56.0
RAMP RESP	37.0	47.0	46.0
FILTER RATE	1.87957	1.77446	1.74596

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.5900	1.7200	1.6800
BETA	0.00624	0.00600	0.00600
ALPHA or N	0.1518	50.00	50.00
VRR POS	0.09946292	0.10067120	0.09982215
VRR VEL	0.00013910	0.00012536	0.00012527
COV RR	0.00331285	0.00269799	0.00316356
VRR POS-1	0.10622773	0.10619254	0.10627454
VRR POS-2	0.1132707	0.1119646	0.1129775
VRR POS-150	4.22304	3.73067	3.86743
LAG POS	135.916	166.667	142.273
LAG VEL	23.8166	22.3027	22.3027
LAG POS-1	160.232	189.469	165.080
LAG POS-2	185.549	213.272	188.887
STEP RESP	74.0	65.0	64.0
RAMP RESP	53.0	51.0	50.0
FILTER RATE	1.84821	1.78351	1.76828

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.1900	0.9100	0.9000
BETA	0.00352	0.00400	0.00402
ALPHA or N	0.1152	50.00	50.00
VRR POS	0.07462621	0.07462716	0.07414963
VRR VEL	0.00005727	0.00007791	0.00007846
COV RR	0.00184329	0.00185075	0.00206628
VRR POS-1	0.07837006	0.07840656	0.07836065
VRR POS-2	0.0822285	0.0823418	0.0827286
VRR POS-150	1.91616	2.38286	2.45947
LAG POS	251.011	250.000	224.038
LAG VEL	32.1867	25.7041	25.7041
LAG POS-1	283.698	276.204	250.183
LAG POS-2	317.384	303.408	277.327
STEP RESP	100.0	77.0	76.0
RAMP RESP	73.0	59.0	58.0
FILTER RATE	1.83786	1.78506	1.77708

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.8050	0.7500	0.7400
BETA	0.00163	0.00200	0.00203
ALPHA or N	0.0792	50.00	50.00
VRR POS	0.05070525	0.05055421	0.05035327
VRR VEL	0.00001754	0.00003159	0.00003229
COV RR	0.00084164	0.00094945	0.00101505
VRR POS-1	0.05240608	0.05248469	0.05241565
VRR POS-2	0.0541420	0.0544784	0.0545426
VRR POS-150	0.69776	1.04620	1.08147
LAG POS	563.953	500.000	467.765
LAG VEL	47.9954	35.9082	35.9082
LAG POS-1	612.449	536.408	503.878
LAG POS-2	661.944	573.816	540.971
STEP RESP	150.0	116.0	113.0
RAMP RESP	109.0	86.0	84.0
FILTER RATE	1.81784	1.79311	1.78905

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.3800	0.5000	0.5000
BETA	0.00036	0.00018	0.00018
ALPHA or N	0.0377	50.00	50.00
VRR POS	0.02384360	0.02404823	0.02403165
VRR VEL	0.00000178	0.00000049	0.00000049
COV RR	0.00018386	0.00008784	0.00008823
VRR POS-1	0.02421309	0.02422439	0.02420859
VRR POS-2	0.0245861	0.0244015	0.0243865
VRR POS-150	0.11895	0.06132	0.06142
LAG POS	2654.604	55[illegible].556	5531.055
LAG VEL	103.5457	242.[illegible]574	242.2574
LAG POS-1	2758.650	5798.3[illegible]	5773.813
LAG POS-2	2863.696	6042.070	6017.570
STEP RESP	324.0	400.0	400.0
RAMP RESP	237.0	400.0	400.0
FILTER RATE	1.73833	1.83130	[illegible] 83037

EQUAL 2 SAMPLES AHEAD VARIANCE REDUCTION RATIO

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	3.3950	3.9550	3.8200
BETA	0.02710	0.01550	0.01486
ALPHA or N	0.3021	50.00	50.00
VRR POS	0.20850258	0.25099003	0.23869826
VRR VEL	0.00144311	0.00038131	0.00036202
COV RR	0.01536794	0.00580483	0.00889795
VRR POS-1	0.24068156	0.26298100	0.25685617
VRR POS-2	0.2757468	0.2757346	0.2757381
VRR POS-150	37.28880	10.57199	11.05349
LAG POS	25.752	64.516	42.780
LAG VEL	10.6492	18.1333	18.1333
LAG POS-1	36.901	83.149	61.526
LAG POS-2	49.050	102.783	81.272
STEP RESP	33.0	52.0	51.0
RAMP RESP	23.0	42.0	39.0
FILTER RATE	1.90152	1.78912	1.70937

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.9200	3.0650	2.9650
BETA	0.02029	0.01300	0.01255
ALPHA or N	0.2646	50.00	50.00
VRR POS	0.18019150	0.20711967	0.19875199
VRR VEL	0.00090214	0.00030755	0.00029479
COV RR	0.01131216	0.00515372	0.00725922
VRR POS-1	0.20371796	0.21773466	0.21356522
VRR POS-2	0.2290487	0.2289648	0.2289681
VRR POS-150	23.87202	8.67315	9.00938
LAG POS	36.236	76.923	55.188
LAG VEL	12.5392	18.6397	18.6397
LAG POS-1	49.275	96.063	74.441
LAG POS-2	63.314	116.203	94.693
STEP RESP	39.0	53.0	53.0
RAMP RESP	27.0	43.0	41.0
FILTER RATE	1.88724	1.78583	1.72693

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.1500	2.0000	1.9550
BETA	0.01126	0.00900	0.00880
ALPHA or N	0.2010	50.00	50.00
VRR POS	0.13368912	0.14347170	0.13967149
VRR VEL	0.00035184	0.00019966	0.00019452
COV RR	0.00610328	0.00385438	0.00482515
VRR POS-1	0.14644752	0.15138012	0.14951630
VRR POS-2	0.1597096	0.1596878	0.1597501
VRR POS-150	9.88121	5.79205	5.96392
LAG POS	70.954	111.111	89.150
LAG VEL	17.3469	20.0351	20.0351
LAG POS-1	88.801	131.646	109.789
LAG POS-2	107.648	153.181	131.428
STEP RESP	54.0	57.0	57.0
RAMP RESP	39.0	47.0	46.0
FILTER RATE	1.86142	1.78668	1.75190

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.5750	1.7200	1.6750
BETA	0.00612	0.00600	0.00594
ALPHA or N	0.1505	50.00	50.00
VRR POS	0.09847914	0.10067120	0.09902947
VRR VEL	0.00013487	0.00012536	0.00012386
COV RR	0.00324613	0.00269799	0.00313014
VRR POS-1	0.10510627	0.10619254	0.10541361
VRR POS-2	0.1120032	0.1119646	0.1120455
VRR POS-150	4.10698	3.73067	3.82494
LAG POS	138.861	166.667	143.919
LAG VEL	24.0679	22.3027	22.3027
LAG POS-1	163.429	189.469	166.793
LAG POS-2	188.997	213.272	190.667
STEP RESP	75.0	65.0	64.0
RAMP RESP	54.0	51.0	50.0
FILTER RATE	1.84604	1.78834	1.76562

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.1950	0.9100	0.8950
BETA	0.00354	0.00400	0.00400
ALPHA or N	0.1155	50.00	50.00
VRR POS	0.07478839	0.07462716	0.07385680
VRR VEL	0.00005765	0.00007791	0.00007791
COV RR	0.00185145	0.00185075	0.00205353
VRR POS-1	0.07854895	0.07840656	0.07804178
VRR POS-2	0.0824248	0.0823418	0.0823826
VRR POS-150	1.92741	2.38286	2.44292
LAG POS	249.861	250.000	225.500
LAG VEL	32.1140	25.7041	25.7041
LAG POS-1	282.475	276.204	251.704
LAG POS-2	316.089	303.408	278.908
STEP RESP	100.0	77.0	76.0
RAMP RESP	73.0	59.0	58.0
FILTER RATE	1.84080	1.78589	1.77331

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.8100	0.7500	0.7400
BETA	0.00165	0.00200	0.00203
ALPHA or N	0.0795	50.00	50.00
VRR POS	0.05094250	0.05055421	0.05035327
VRR VEL	0.00001779	0.00003159	0.00003229
COV RR	0.00084963	0.00094945	0.00101505
VRR POS-1	0.05265955	0.05248469	0.05241565
VRR POS-2	0.0544122	0.0544784	0.0545426
VRR POS-150	0.70605	1.04620	1.08147
LAG POS	558.512	500.000	467.785
LAG VEL	47.7657	35.9082	35.9082
LAG POS-1	606.778	536.408	503.878
LAG POS-2	656.044	573.816	540.971
STEP RESP	149.0	116.0	113.0
RAMP RESP	109.0	86.0	84.0
FILTER RATE	1.81990	1.79213	1.78797

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.3800	0.5000	0.5000
BETA	0.00036	0.00018	0.00018
ALPHA or N	0.0375	50.00	50.00
VRR POS	0.02371758	0.02404823	0.02403165
VRR VEL	0.00000175	0.00000049	0.00000049
COV RR	0.00018191	0.00008784	0.00008823
VRR POS-1	0.02408315	0.02422439	0.02420859
VRR POS-2	0.0244522	0.0244015	0.0243865
VRR POS-150	0.11761	0.06132	0.06142
LAG POS	2683.386	5555.556	5531.055
LAG VEL	104.1028	242.2574	242.2574
LAG POS-1	2787.989	5798.313	5773.813
LAG POS-2	2893.592	6042.070	6017.570
STEP RESP	326.0	400.0	400.0
RAMP RESP	238.0	400.0	400.0
FILTER RATE	1.73496	1.83298	1.83206

EQUAL 150 SAMPLES AHEAD VARIANCE REDUCTION RATIO

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.2000	3.9550	3.7550
BETA	0.01179	0.01550	0.01433
ALPHA or N	0.2054	50.00	50.00
VRR POS	0.13702518	0.25099003	0.22927815
VRR VEL	0.00037841	0.00038131	0.00034619
COV RR	0.00640235	0.00580483	0.00851180
VRR POS-1	0.15020829	0.26298100	0.24664792
VRR POS-2	0.1641482	0.2757346	0.2647101
VRR POS-150	10.57204	10.57199	10.57207
LAG POS	67.395	64.516	45.274
LAG VEL	16.9189	18.1333	18.1333
LAG POS-1	84.814	83.149	64.122
LAG POS-2	103.233	102.783	83.970
STEP RESP	52.0	52.0	52.0
RAMP RESP	38.0	42.0	39.0
FILTER RATE	1.59424	1.95141	1.85435

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	2.0550	3.0650	2.9000
BETA	0.01030	0.01300	0.01215
ALPHA or N	0.1927	50.00	50.00
VRR POS	0.12801440	0.20711967	0.19219603
VRR VEL	0.00030560	0.00030755	0.00028373
COV RR	0.00556351	0.00515372	0.00698990
VRR POS-1	0.13944703	0.21773466	0.20645957
VRR POS-2	0.1514909	0.2289649	0.2212906
VRR POS-150	8.67316	8.67315	8.67310
LAG POS	78.366	76.923	57.784
LAG VEL	18.2050	18.6397	18.6397
LAG POS-1	97.071	96.063	77.142
LAG POS-2	116.776	116.203	97.501
STEP RESP	56.0	53.0	53.0
RAMP RESP	41.0	43.0	42.0
FILTER RATE	1.64531	1.91470	1.83999

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.7800	2.0000	1.9250
BETA	0.00779	0.00900	0.00858
ALPHA or N	0.1688	50.00	50.00
VRR POS	0.11122791	0.14347170	0.13636506
VRR VEL	0.00019693	0.00019966	0.00018885
COV RR	0.00416623	0.00385438	0.00468822
VRR POS-1	0.11975731	0.15138012	0.14593035
VRR POS-2	0.1286806	0.1596878	0.1558733
VRR POS-150	[illegible].79207	5.79205	5.79198
LAG POS	106.664	111.111	92.107
LAG VEL	21.1556	20.0351	20.0351
LAG POS-1	128.319	131.646	112.866
LAG POS-2	150.975	153.181	134.626
STEP RESP	66.0	57.0	57.0
RAMP RESP	47.0	47.0	46.0
FILTER RATE	1.74179	1.85562	1.80259

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.5200	1.7200	1.6600
BETA	0.00571	0.00600	0.00581
ALPHA or N	0.1455	50.00	50.00
VRR POS	0.09515104	0.10067120	0.09727282
VRR VEL	0.00012124	0.00012536	0.00012073
COV RR	0.00302566	0.00269799	0.00305599
VRR POS-1	0.10132359	0.10619254	0.10350554
VRR POS-2	0.1077386	0.1119646	0.1099797
VRR POS-150	3.73070	3.73067	3.73060
LAG POS	149.529	166.667	147.698
LAG VEL	24.9564	22.3027	22.3027
LAG POS-1	174.986	189.469	170.726
LAG POS-2	201.442	213.272	194.755
STEP RESP	77.0	65.0	65.0
RAMP RESP	56.0	51.0	51.0
FILTER RATE	1.82829	1.80300	1.76871

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	1.2900	0.9100	0.8900
BETA	0.00414	0.00400	0.00391
ALPHA or N	0.1245	50.00	50.00
VRR POS	0.08087536	0.07462716	0.072796[illegible]0
VRR VEL	0.00007336	0.00007791	0.00007591
COV RR	0.00217125	0.00185075	0.00200732
VRR POS-1	0.08529122	0.07840656	0.07688695
VRR POS-2	0 0898538	0.0823418	0.0811293
VRR POS-150	2.38287	2.38286	2.38287
LAG POS	211.658	250.000	230.955
LAG VEL	29.5969	25.7041	25.7041
LAG POS-1	241.755	276.204	257.382
LAG POS-2	272.852	303.408	284.809
STEP RESP	92.0	77.0	77.0
RAMP RESP	67.0	59.0	59.0
FILTER RATE	1.88511	1.76987	1.74501

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.9450	0.7500	0.7350
BETA	0.00224	0.00200	0.00198
ALPHA or N	0.0923	50.00	50.00
VRR POS	0.05937980	0.05055421	0.04973488
VRR VEL	0.00002841	0.00003159	0.00003111
COV RR	0.00115885	0.00094945	0.00098797
VRR POS-1	0.06172591	0.05248469	0.05174192
VRR POS-2	0.0641288	0.0544784	0.0538112
VRR POS-150	1.04624	1.04620	1.04616
LAG POS	405.843	500.000	480.921
LAG VEL	40.7911	35.9082	35.9082
LAG POS-1	447.134	536.408	517.550
LAG POS-2	489.425	573.816	555.179
STEP RESP	127.0	116.0	115.0
RAMP RESP	93.0	86.0	85.0
FILTER RATE	1.89484	1.76202	1.74314

VARIABLE	ALPHA-BETA	C-BETA	MOD2 C-BETA
BANDWIDTH	0.2750	0.5000	0.5000
BETA	0.00019	0.00018	0.00018
ALPHA or N	0.0274	50.00	50.00
VRR POS	0.01729139	0.02404823	0.02402224
VRR VEL	0.00000067	0.00000049	0.00000048
COV RR	0.00009641	0.00008784	0.00008800
VRR POS-1	0.01748489	0.02422439	0.02419872
VRR POS-2	0.0176797	0.0244015	0.0243762
VRR POS-150	0.06136	0.06132	0.06129
LAG POS	5096.356	5555.556	5545.199
LAG VEL	143.2775	242.2574	242.2574
LAG POS-1	5240.134	5798.313	5788.534
LAG POS-2	5384.912	6042.070	6032.868
STEP RESP	400.0	400.0	400.0
RAMP RESP	329.0	400.0	400.0
FILTER RATE	1.56679	1.91727	1.91595

APPENDIX F

DERIVATION OF FORMULAE USED IN MANOEUVRE DETECTION

The Alpha Filter

The filter equation is as follows:

$$L(k) = L(k-1) + g.[\ E(k) - L(k-1)\ ]$$

Which results in the following Transfer Function:

$$H(z) = \frac{g.z}{z + (g-1)}$$

From which can be obtained the residual error for a unit step input.

$$E(z) = \frac{z}{z-1} - \frac{z}{z-1} . \frac{g.z}{z + (g-1)}$$

$$= (1-g) \frac{z}{z - (1-g)}$$

The error stream will be given by the following:

$$en = (1-g)^{n+1}$$

Hence for an allowable error of 10%

$$n = \frac{Log(0.1)}{Log(1-g)} - 1$$

Calculation of Manoeuvre Detection Thresholds

Up Threshold

This is given by the present lag for the maximum allowable acceleration for each state. This will be the lag on the present estimate, as the residual stream operates on the present estimate.

Max Accel (m/s.s)	Gain B	Up Thres (Lp m)	Max Lag (L2p m)
3	0.0009	1.3	1.45
10	0.0021	1.8	2.09
25	0.0041	2.2	2.71
40	0.0057	2.4	3.16
>40	0.0155	1000	?

For a given maximum acceleration there will be lag given by the following formula:

$$Lp = \frac{Ts^2}{B}\left\{1 - \frac{N-1}{2}.B\right\}\ddot{x}$$

Substituting for B=0.0009 Ts=20E-03 and x=3:

$$Lp = 1.30m$$

Similar calculations are made for all the states except the top state which is set at a very large value which will never be exceeded.

Down Threshold

When the gain state switches down, the acceleration must be such that the resultant lag in the new state will be lower than the maximum allowed. It has been decided that an acceptable margin of safety will be 10%, hence, the lag will be 90% of the allowable maximum.

The required down threshold can thus be calculated:

For a given acceleration $\ddot{x}$, the lag will be:

$$L(n) = \ddot{x}.K(n)$$

Similarly:

$$L(n-1) = \ddot{x}.K(n-1)$$

Assuming that the lag is 90% of the maximum, then:

$$L(n-1) = 0.9\ TUP(n-1) = \ddot{x}.K(n-1)$$

And that it has just come from the next state up:

$$L(n) = TDN(n)$$

Substituting for the acceleration:

$$TDN(n) = \ddot{x}.K(n) = \frac{0.9\ TUP(n-1)}{K(n-1)}\ .\ K(n)$$

Now:

$$TUP(n-1) = \ddot{x}max(n-1).K(n-1)$$

and:

$$K(n) = \frac{TUP(n)}{\ddot{x}max(n)}$$

Thus:

$$TDN(n) = 0.9\ TUP(n).\frac{\ddot{x}max(n-1)}{\ddot{x}max(n)}$$

Calculation of RMS Error from Variance and Lag

For a particular fixed lag 'L' acting as a bias on noise, the RMS Error can be approximated by the following formula:

$$Erms = \left(1/N \sum_{j=1}^{N} (xj+Lj)^2\right)^{1/2}$$

$$= \left(1/N \sum_{j=1}^{N} xj^2 + 1/N \sum_{j=1}^{N} L^2 + 2/N \sum_{j=1}^{N} xj.L\right)^{1/2}$$

The Expectation Value:

$$E[2/N \sum_{j=1}^{N} xj.L] = 2/N \sum_{j=1}^{N} L.E[xj] = 0$$

for zero mean noise xj.

Now:

$$1/N \sum_{j=1}^{N} xj^2 = Var(x)$$

$$1/N \sum_{j=1}^{N} L^2 = L^2 \quad \text{(for constant lag.}$$

Hence:

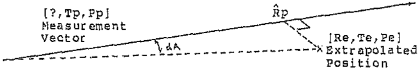
$$Erms = [\, Var(x) + L^2 \,]^{1/2}$$

APPENDIX G

DERIVATION OF FORMULAE USED IN MEMORY TRACK

Range Memory Track

Shortest Distance



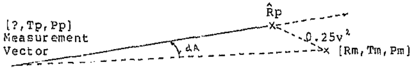
$$\hat{R}p = Re.Cos(dA)$$

$$dA = [(Tp-Te)^2 - (Pp-Pe)^2]^{1/2}$$

Hence the Estimated Range is given by:

$$\hat{R}p = Re.Cos[(Tp-Te)^2 - (Pp-Pe)^2]^{1/2}$$

Constant Speed



For a time interval between corrections T=0.5sec

Target speed $v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2}$

By the cos rule:

$$0.25v^2 = Rm^2 + Rp^2 - 2.Rm.Rp.Cos(dA)$$

Hence:

$$\hat{R}p = Rm.Cos(dA) \pm [\ Rm^2\{Cos^2(dA)-1\} - 0.25v^2]^{1/2}$$

Now:

$$Cos^2(dA)-1 = -Sin^2(dA) = -dA^2 \quad \text{for small dA}$$

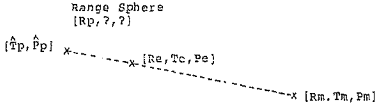
$$Cos(dA) = 1$$

Hence the predicted range is:

$$\hat{R}p = Rm \pm [\ 0.25v^2 - Rm^2.dA^2]^{1/2}$$

Angle Memory Track

Constant Direction

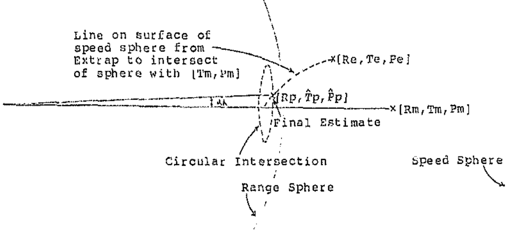


Assuming small angular changes, it is possible to use linear extrapolation as follows:

$$\hat{T}p = Tm + \frac{(Rp-Rm)(Te-Tm)}{Re-Rm}$$

$$\hat{P}p = Pm + \frac{(Rp-Rm)(Pe-Pm)}{Re-Rm}$$

Constant Speed



By the Cos Rule:

$$dA = \mathrm{Acos}\left\{\frac{Rm^2 + Rp^2 - 0.25v^2}{2.Rm.Rp}\right\}$$

By linear extrapolation assuming small angles:

$$\frac{dA}{[(Te-Tm)^2 + (Pe-Pm)^2]^{1/2}} = \frac{dT}{Te-Tm} = \frac{dP}{Pe-Pm}$$

Hence by substitution of the above:

$$\hat{T}p = Tm + \frac{(Te-Tm)dA}{[(Te-Tm)^2 + (Pe-Pm)^2]^{1/2}}$$

$$\hat{P}p = Pm + \frac{(Pe-Pm)dA}{[(Te-Tm)^2 + (Pe-Pm)^2]^{1/2}}$$

Note: For a target approaching radially, the function takes the form 0/0 which does not calculate well, hence some form of checking is required.

Derivation of Angular Rates

Elevation Rate

$$\frac{dT}{dt} = \frac{dT}{dx}.\frac{dx}{dt} + \frac{dT}{dy}.\frac{dy}{dt} + \frac{dT}{dz}.\frac{dz}{dt}$$

Where the elevation is given by the following formula:

$$T = \mathrm{Asin}\,\frac{z}{[x^2+ y^2+ z^2]^{1/2}} = \mathrm{Asin}(u)$$

Performing the partial differentiation:

$$\frac{dT}{dx} = \frac{1}{(1-u^2)^{1/2}}.\frac{du}{dx}$$

$$= \frac{-z.x}{R^2(R^2- z^2)^{1/2}}$$

Similarly:

$$\frac{dT}{dy} = \frac{-z.y}{R^2(R^2- z^2)^{1/2}}$$

While for z the differential changes:

$$\frac{dT}{dz} = \frac{(R^2- z^2)^{1/2}}{R^2}$$

Hence the elevation rate is:

$$\dot{T} = \frac{z}{R^2(R^2- z^2)^{1/2}}[x\dot{x} + y\dot{y}] - \frac{(R^2- z^2)^{1/2}}{R^2}.\dot{z}$$

Azimuth Rate

Given that the Azimuth P is related to x,y,z by the following:

$$P = \text{Atan}\,\frac{y}{x} = \text{Atan}(u)$$

Calculating the differentials:

$$\frac{dP}{dx} = \frac{1}{1 + u^2}\cdot\frac{du}{dx}$$

$$= \frac{-y}{x^2 + y^2}$$

Similarly:

$$\frac{dP}{dy} = \frac{x}{x^2 + y^2}$$

Hence:

$$\dot{P} = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2}$$