STIRLING CYCLE ENGINE DESIGN AND OPTIMISATION

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Declaration

I, David Montague Berchowitz, hereby declare that this thesis is my own work and that the material herein has not been submitted by me for degree purposes at any other University. Some of the experimental data contained herein was obtained by me while in the employment of Sunpower Inc. During the course of this work, some sections have been published. Where this is so, it is clearly indicated within the text

David M. Berchowitz 9th day of August 1986

Abstract

This thesis describes means and analyses for application to the preliminary design and optimisation of Stirling cycle engines. To this end, analytical models for engine thermodynamics, heat exchanger performance, engine dynamics and various parasitic losses are developed. The engine thermodynamics are based upon a linearised version of the ideal adiabatic cycle. An iterative method is used to solve the thermodynamic equations which, for similar accuracy, is shown to be nearly two orders of magnitude faster than the typically used time stepping routines (explicit methods). Parasitic losses are subtracted from the ideal performance in order to obtain a more realistic measure of a real engine's performance (referred to as the second order method). By this method the degradation of the ideal performance due to the various losses is clearly identified and offers insight as to how the actual performance might be improved. Comparison with experimental data taken from three fairly well documented engines shows good agreement.

A simplified analysis of the heat exchangers is also included. This analysis simultaneously evaluates the irreversibilities due to viscous effects and temperature gradients thus allowing for simple optimisation for minimum temperature gradients and flow losses. A regenerator effectiveness-NTU model is developed in closed-form which is shown to give similar results to a previous method which required numerical integration.

Since free-piston configurations are considered to offer substantial advantages to the practical implementation of the Stirling cycle, an analysis of these machines is also presented. Previous analytical work is extended by more accurately accounting for pressure drop effects and by including the casing mass in the dynamic equations. Approximate closed-form solutions for the damping on the moving parts are included.

In the final chapter, the analytical methods and models are brought together in order to outline an approach to preliminary design. The selection of source temperature, charge pressure and working gas is outlined and the value of scaling in the preliminary layout is discussed. Optimisation and sensitivity studies are then used to refine the design. Simulation is used to obtain thermodynamic and gas dynamic details not addressed by the second order approach and is also useful in the final stages of design refinement.

To my mother and in memory of my father
Marie and Lukie Berchowitz

Acknowledgements

The work presented here spans some eight years of working in the research and design of Stirling engines. During this time I have had the honour and pleasure of working with many people in this field. Firstly, Professor Costa Rallis of the University of the Witwatersrand, under whose guidance much of my orginal thinking and approaches were formulated. Particularly the importance of design [Ral73] and the value of the simple calculation in order to achieve that end. Also while at the University of the Witwatersrand, I worked with Dr Israel Urieli on the first really significant numerical simulation of Stirling engine thermodynamics [UR77]. Later Dr Urieli moved to Israel where through correspondence, we were able to improve the simulation model to a point where it still remains the most complete physical model of Stirling engine thermodynamic performance [BU79]. The initial work on the analysis of free-piston Stirling engines was also done at the University of the Witwatersrand. Here Gavin Wyatt-Mair was particularly helpful, teaching me how to use the analog computer and later working with me to develop a linear analysis of free-piston engines. In 1979 I took up employment at Mechanical Technology Inc (MTI) and was able to benefit from working with many talented engineers engaged in Stirling engine research. In particular I would like to thank Mike Walsh, the senior design engineer, who helped me through my first experiences in the design of a Stirling cycle machine. After two years at MTI I moved to Sunpower Inc to work with William Beale, the inventor of the free-piston Stirling engine. Sunpower had already established itself as the major innovator in Stirling engine technology and I was and continue to draw great benefit from my work there. The people at Sunpower have always been willing to share their knowledge and experiences in an environment which is both challenging and supportive. In particular, William Beale has always been a source of encouragement and inspiration. He has encouraged me to seek the limits of this technology and to appreciate that in order to succeed one should not be intimidated by failure. At Sunpower I have been given the opportunity to design, build, test and experiment with Stirling engines. For this rare privilege, I will always be grateful. Also while at Sunpower I have been fortunate to work with Dr Robert Redlich who suggested fundamental improvements to the linear analysis of free-piston machines. With his help, I was able to gain a far more complete understanding of free-piston machines, particularly issues surrounding stability, starting and tuning.

Finally, this thesis would never have seen the light of day had it not been for my family: Gillian, Lukie and Andrea who have provided constant love and encouragement.

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List of Symbols

Wherever possible, the symbols for physical quantities are those in common use. Stirling engine design covers a wide range of engineering disciplines, and in some cases the common symbol for a quantity could not be used since it had already been used elsewhere. For example velocity is generally denoted by v, but this symbol also denotes specific volume. Thus in this case velocity has been denoted by U and in vector form by **U**. Other examples will be noted in the nomenclature list.

Subscripts are mainly used to differentiate between components to which a particular variable has been prescribed. Thus V_h refers to heater volume and V_k refers to cooler volume. Where a subscripted variable is used uniquely it is listed in the main list, eg. a_v.

Roman

a	dimension [m]; grouping of terms (Equation (3.28) and (5.55)); tube radius; gap between fins
a	solid particle surface area per unit bed volume [m^{-1}]
a_n	Fourier coefficient
$a_{1,2}$	amplitude defined by Figures C.5 and C.6
a_v	total particle surface area per unit total particle volume
A	area, free-flow area [m^2]; cylinder area (unsubscripted); constant
A_c	canister free-flow area [m^2]

A_m	amplitude defined by Figure C.4
A_n	free flow area per flow passage [m^2]; amplitude defined by Figure C.5
A_R	rod area [m^2]
A_s	contact area of ring seal
A_w	wetted area [m^2]
b	dimension [m]; height of fin
b_n	Fourier coefficient
B	constant
c	grouping of terms (Equation (5.71)); clearance seal gap [m]
c_p	specific heat capacity at constant pressure [J/kgK]
c_v	specific heat capacity at constant volume [J/kgK]
clc	clearance between piston and displacer at mean positions [m]
cle	clearance between displacer and cylinder head at displacer mean position [m]
C	value of Reynolds number at transition
C	constant
C_L	laminar flow friction coefficient
C_{HL}	laminar flow head loss coefficient
C_{pb}	bulk specific heat capacity at constant pressure [J/kgK]
d	derivative with respect to an independent variable
d	diameter [m]
d	clearance seal half-gap [m]

d_h	hydraulic diameter (Equation (4.36)) [m]
d_m	wire diameter
d_{pk}	diameter of gas spring puck
D	derivative with respect to time (d/dt)
D	damping coefficient [Ns/m]; cylinder diameter; casing diameter
D_p	defined by Equation (4.73)
D_s	gas spring damping coefficient
e	eccentricity of clearance seal
f	Fanning friction factor
f_o	steady flow friction factor
f_r	Reynolds friction factor ($f_r = f\,\mathrm{Re}$)
f	frequency [Hz]
F	force [N]
$F_{s\ell}$	sliding frictional force [N]
g	rate of entropy generation per unit volume [W/Km^3]
G	rate of entropy generation [W/K]
G_0	mass flux density [kg/s m^2] (Equation (4.73))
h	heat transfer coefficient [W/m^{2}K]
h_b	bulk heat transfer coefficient [W/m^{2}K]
h_g	hysteresis heat transfer coefficient [W/m^{2}K]
h	distance from gas spring wall to gas spring centre line [m]; appendix gap [m]

h_{lim}	appendix gap at which shuttle loss reaches a constant limiting value
H	enthalpy [J/K]
$\hat{H}$	internal energy per unit length [J/m]
$\dot{H}_{\ell}$	enthalpy transported along clearance seal [W]
H_{sh}	shuttle heat transfer [J]
$H_{\mathrm{L\,gas}}$	gas enthalpy transport in appendix gap [J]
$H_{\mathrm{sh\,lim}}$	limiting value of shuttle loss
imep	indicated mean effective pressure [Pa]
j	$\sqrt{-1}$
j_{H}	Colburn factor (Equations (4.78) and (4.79))
k	thermal conductivity [W/mK]
k_s	thermal conductivity of wall
k_{wd}	thermal conductivity of displacer wall
k_{wr}	thermal conductivity of regenerator wall
K	spring rate [N/m]
L	length [m]
L_0	length of zero leakage seal [m]
LHP	left half plane
m	mass [kg]
m_e	effective mass [kg]
$\mathfrak{m}$	component mass [kg]

m_a	mass of gas in appendix volume
$\dot{m}_\ell$	seal leakage [kg/s]
n	number of parallel paths
$\mathbf{N}$	unit normal vector
p	pressure [Pa]
p_d	gas spring pressure
P	power [W]
q	specific heat transferred [J/kg]
$\dot{q}$	heat flux [W/m^2]
$\mathbf{q}$	vector of heat flux components [W/m^2]
Q	heat transfer [J]; stored energy/energy dissipated per cycle
$\langle \dot{Q}_a \rangle$	appendix loss [W]
$\dot{Q}_c$	conduction heat transfer [W]
$\lvert Q_r \rvert$	regenerator unidirectional heat transfer [J]
r	stroke ratio; grouping of terms (Equation (4.25) or Table 4.4
r	volume ratio (Equation (2.2))
R	gas constant [J/kgK]
$\mathbb{R}$	real value
r_p	pressure ratio (Equation (6.82))
r_T	temperature ratio (Equation (6.82))
r_x	stroke to length ratio (Equation (6.82))
RHP	right half plane
s	specific entropy [J/kgK]; sinusoidal quantity

s	Laplace variable [s^{-1}]
S	entropy [J/K]; grouping of terms (Equations (3.28) and (5.50))
t	time [s]; wall thickness [m]
T	temperature [K]
T'	conditional temperature
$T(s)$	grouping of dynamic terms (Equations (5.9) and (5.13d))
T_g	gas temperature
T_ℓ	lower fixed temperature in appendix gap
$\bar{T}_L$	spatial mean gas temperature in appendix gap
T_s	local mean surface temperature; external wall temperature
T_w	wall temperature
u	specific internal energy [J/kg]; instantaneous velocity [m/s]
U	internal energy [J]; velocity [m/s]
$\mathbf{U}$	velocity vector [m/s]
U_p	piston velocity [m/s]
v	specific volume [m^3/kg]
V	volume [m^3]
$\mathbf{V}$	phasor
V_a	appendix volume
V_b	bounce volume
V_d	gas spring mean volume
V_D	reduced dead volume (Equation (3.54))
V_f	grouping of terms (Equation (4.12) or Table 4.4) [m^3]

V_{gsi}	volume towards hot end of double acting gas spring (*in* spring)
V_{gso}	volume towards cold end of double acting gas spring (*out* spring)
V_m	mean volume
V_r'	grouping of volume terms (Equation (3.68))
V_{rD}	grouping of volume terms (Equation (4.11))
V_{sw}	swept volume [m^3]
V_{swp}	swept volume of piston
w	mass flow rate [kg/s]
W_a	mass flow at entrance to appendix gap
w	specific work [J/kg]; inverse of thermal wavelength in solid [m^{-1}]
W	work [J]
W_s	work lost against seal friction
W_{sc}	work lost against constant frictional force
$\langle \dot{W} \rangle_g$	working gas hysteresis loss [W]
$\langle \dot{W} \rangle_{gs}$	gas spring hysteresis loss
$\langle \dot{W} \rangle_{seal}$	work delivered to displacer step (at seal)
$W_{Schmidt}$	work given by Schmidt analysis (§3.4) [J]
x	length, displacement [m]
y	length, coordinate [m]
z	coordinate [m]
z_h	wetted perimeter per flow passage [m]

Greek

α	phase angle between expansion and compression space volume variations; thermal diffusivity [m^2/s]
$\bar{\alpha}$	thermal diffusivity at mean wall temperature and mean pressure
α_c	thermal diffusivity of wall material
$\bar{\alpha}_h$	thermal diffusivity at hot wall temperature and mean pressure
α_0	thermal diffusivity at mean wall temperature
α_p	pressure coupling between piston motion and displacer force [N/m]
α_T	thermal coupling between displacer motion and piston force [N/m]
A	lagging phase angle of compression space volume to piston motion
β	angle, pressure phase angle; inverse of thermal wavelength [m^{-1}]; angle defined by Figure C.5 (pressure phase angle for adiabatic analysis)
β'	phase angle between pressure and piston motions
β_T	phase angle between regenerator heat transfer and compression space volume
γ	ratio of specific heat capacities
δ	collection of terms (§3.7); radial offset between cylinder and piston [m]
δp	pressure fraction ($\lvert p \rvert / \langle p \rangle$)
δT	temperature fraction ($\Delta T / \langle T \rangle$)
Δ	differential quantity
ϵ	regenerator effectiveness (Equation (2.8) and Equation (4.14))

ζ	dimensionless work (Equation (2.6)); angle defined by Figure C.2
η	indicated or thermal efficiency
$\mathcal{K}$	head loss coefficient (Equation (4.57))
θ	crank angle or angle
θ_1	angle obtained from Figure C.4
Θ	temperature difference [K]
θ_r	phase angle defined by Equation (4.25)
θ_w	phase angle between regenerator temperatures on either side of regenerator
κ	swept volume ratio (Equation (3.53b))
λ	thermal wavelength [m]
μ	dynamic viscosity [Pa s]; ring friction coefficient
ν	kinematic viscosity [m^2/s]
ξ	angle defined by Figure C.3
π	3.14159...
ρ	density [kg/m^3]; dead volume ratio (Equation (3.53c))
σ	stress tensor [Pa]
σ_h	hoop stress [Pa]
τ	temperature ratio (Equation (2.3) and Equation (C.15) for the adiabatic analysis)
υ	volume ratio (Equation (C.24))
Υ	dT/dx
ϕ	phase angle between displacer and piston motions
ϕ_m	phase angle defined by Equation (6.85)

Φ	lost work dissipated against viscous friction [W]
$\langle\Phi\rangle_S$	viscous dissipation in clearance seal [W]
φ	angle defined by Figure C.5
ψ	porosity (void volume/total volume)
ω	angular frequency [s^{-1}]; component resonance [s^{-1}]

Subscripts

act	actual value
av	average value
b	bulk averaged value
c	compression space
clc	compression space clearance volume
cle	expansion space clearance volume
d	displacer
dc	displacer to casing
dp	displacer to piston
e	expansion space
exc	heat exchanger
ext	external
f	evaluated at film temperature
g	gas
gs	gas spring
h	heater
hl	head loss
hp	half peak value
i	ideal value; inlet conditions

k	cooler
kr	cooler-regenerator interface
loc	local
L	laminar
max	maximum or peak value
n	per flow passage
o	outlet conditions; oscillatory conditions; reference conditions
p	peak value
p	piston
r	regenerator
R	rod
rh	regenerator-heater interface
s	steady flow conditions
sw	swept volume
t	total, transition value
v	volumetric mean
w	wall or wetted, mass flow
wg	wall to gas interface
wh	heater wall
wk	cooler wall

Non-dimensional Groups

N	number of transfer units (Equation (4.30))
Nu	Nusselt number (Equation (4.2))
Pe	Peclet number ($Pe \equiv Re \cdot Pr$)
Pr	Prandtl number
Re	Reynolds number