Teaching Problem Solving in Foundation Phase Mathematics

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Abstract

The South African Curriculum for mathematics in the Foundation Phase promotes problem solving as both a curriculum outcome and as a means to develop the mathematics proficiency of young pupils. A three-way approach is recommended that includes developing a strong sense of number; using meaningful problems; and discussing ideas and approaches. This qualitative research, which is grounded in a situated approach to learning, investigated how a small group of Foundation Phase educators in two independent schools in Johannesburg, taught mathematical proficiency and how they integrated problem solving into their pedagogy. Data was gathered through lesson observations, initial and reflective interviews and a focus group discussions with the Foundation Phase educators in each school. Drawing on a selected set of concepts from Sfard's (2008) operationalisation of what it means to *do* mathematics, and Bernstein's (2000) language of description for pedagogic practices the study described and analysed the 'what' and 'how' of their practices, focusing on how they *do* mathematics in their classrooms. The aim was to describe what the educators *actually* do, not to focus on absences. The study found that although the educators promote *some* form of mathematical proficiency, there were variations in the ways and extent to which they integrated problem solving into their practices and afford their pupils opportunities to *do* mathematics and acquire mathematical discourse. Given that teacher development programmes must work in the gap between educators' actual practices and desired practices, the descriptions of variations in the form and content of educators' *actual* practices may inform the design of teacher development- programmes intended to strengthen their competencies to work with problem solving in their classrooms.

Key words: problem solving, Foundation Phase, mathematising, problematising, situated learning.

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree of examination at any other university.

Karen Wynne Davies Stein

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Chapter 1: Introduction

This research report is positioned in the field of Curriculum Studies and investigated whether and how Foundation Phase educators in two independent schools, promoted the development of mathematical proficiency in their classrooms. In particular, it explored whether they included problem solving and, if so, the ways in which they integrated it into their mathematical teaching practices.

In the past few decades there have been developments in thinking, with respect to: the purpose of mathematics education; the nature of the mathematical knowledge taught in schools; what educators should be teaching; and how it should be taught. Much time has been spent on reforming and developing the mathematics curriculum with the focus on identifying strong standards and a quality curriculum for what students should achieve (Ball, Hill, & Bass, 2005). Questions have also been asked about the purpose of mathematics, what it is educators ought to be teaching and there has been much research into new and innovative ways of ensuring that students become mathematically proficient.

In South Africa, the *Curriculum and Assessment Policy Statement* (CAPS) document (DBE, 2011), states as one of its aims, the development of students who are able to "identify and solve problems and make decisions using critical and creative thinking" (2011, p. 5). It also places emphasis, at all levels of mathematics, on teaching students to think mathematically and this is expanded in the South African *Numeracy Handbook for Foundation Phase Teachers Grade R-3* (DBE, 2012) in which mathematical proficiency is described as:

- Understanding what you are doing (*conceptual understanding*);
- Being able to apply what you have learnt *(strategic competence*);
- Being able to reason about what you have done (*adaptive reasoning*);
- Recognising that you need to engage (*productive disposition*) with a problem to solve it; and, of course,
- Being able to calculate/compute (*procedural fluency*) with confidence. (2012, p. 11)

Both *CAPS* (DBE, 2011) and the *Numeracy Handbook* (DBE, 2012) highlight understanding, applying and reasoning are fundamental skills in doing mathematics. A three-way approach is recommended that includes developing a strong sense of number; using meaningful problems; and discussing ideas and approaches. The Handbook, in particular, addresses the significance of problem solving as a teaching tool through which mathematical knowledge and skills can be developed. If understanding is the goal of mathematics teaching, then the curriculum and mathematics instruction should begin with problems, dilemmas and questions for students to solve (Hiebert et al., 1996).

During the 1990s in South Africa, the concept of "Learning through Problem Solving" (Murray, Olivier, & Human, 1998) was introduced into the Foundation Phase of a few state schools in the Cape Province. The aim was to move educators away from a strictly transmission-type approach to one where students were required to construct their own knowledge. Problem solving was used as the tool of learning. With extensive support and inservice training of educators, it was successfully introduced, and by 1993, it had been adopted by the Foundation Phase of state schools in other provinces and some independent schools. However, fears and objections arose from both educators and parents because of its innovative approach. However, while it persisted in some schools, lack of ongong support saw it being abandoned in others (Murray et al., 1998).

The introduction of *CAPS* in 2011, reaffirmed the importance of problem solving as a mathematical tool. However research indicates that there may be a greater, or lesser gap between what the official curriculum prescribes and what educators actually do (McCutcheon, 1997). Even if educators accept a curriculum in principle, along with a school's ethos and its mathematics philosophies, curriculum implementation is contingent on effective input from educators (Ball, Hill & Bass, 2005) who have the power to determine what it is their students have the opportunity to learn. The ways in which educators' interpret and make decisions about what and how they teach it (how they recontextualise the official curriculum for their students in their context of practice) are shaped by their professional knowledge base and the contexts in which they are teaching (Cornbleth, 1988; McCutcheon, 1997; Stenhouse, 1975).

According to Shulman (1986), an educator's knowledge base includes: their knowledge of the disciplinary practice to be taught (content knowledge); their knowledge of subject specific pedagogies (pedagogical content knowledge); their beliefs about learning and their students; and their own general experiences, as students and educators, of the particular subject to be taught. From this complex background of curriculum, personal knowledge and experiences, and the contexts and conditions in which they are teaching, educators create their own pedagogy in their contexts of practice (Shulman, 1986).

The concept of context is complex. Cornbleth (1988) suggests that it is important to distinguish between nominal context and relevant contexts. For this study, the relevant context included: the subject matter and social organization in the classroom, which provide a setting for academic activities that can extend or constrain students' learning; educators' biographies, beliefs and knowledge; as well as the institutional ethos and demography of the school. A key consideration was that educators might be significantly influenced by the school's ethos and orientations to mathematics, and perhaps the extent to which they follow a prescribed textbook.

In addition to the fact that there may be a gap between what the official curriculum prescribes and what educators actually do, there may also be a gap between what educators *say* they do and what they actually do. Educators may invoke, or use, language and terminology inscribed in the official curriculum and in the orientation to maths teaching favoured in their schools, however their actual practices in their mathematics classes may be at odds with what they say they do. It is not what educators say they do but what they actually do that has implications for students' access to mathematical knowledge and the development of mathematical proficiency. Despite potential gaps between 'saying' and 'doing', educators' reflections on practices they have enacted, may illuminate the underlying logic of their approach (Schoenfeld, 2013a).

If we consider these constraints which influence how educators recontextualise the curriculum - how they think and implement it; their knowledge of and experience in mathematics; and the school and classroom context in which they teach - then what educators are saying and what they are doing could differ from school to school and between educators in the same school.

In the dynamic and changing educational context of South African education, there is a growing body of research on primary mathematical education, much of which has been done in under-resourced State schools where there is often no strong culture of learning or teaching (Ensor & Hoadley, 2004; Venkat & Spaull, 2015). Until now, the focus has been, predominately, on educator knowledge, identity and practices – particularly pedagogic content knowledge but little attention had been given to specific mathematical topics (Adler, Alshwaikh, Essack, & Gcsamba, 2017).

While it is understood that most schools in the country are struggling with more elementary educational problems, the fact that the curriculum has raised the importance of problem solving as a tool, through which mathematics knowledge and skills can be developed, means that research into local educators' views on mathematical thinking and problem solving is pertinent and many independent schools are endeavouring to integrate thinking skills into all curriculum areas. Exploring the ways in which these schools are integrating problem solving into their mathematics practices makes it relevant.

Preschool children enter the Foundation Phase with some informal, intuitive mathematical knowledge and understanding learned in the course of their development (Piaget, 1964) and through experiential interaction in everyday life practices (Vygotsky, 1978). However, students at this age generally do not have *conscious awareness or volitional control* of knowledge they have learned tacitly in the course of day to day socio-cultural practices (Vygotsky, 1978). Furthermore, mathematics is a specialised discourse that is not reducible to every discourse (Sfard, 2008; Vygotsky, 1978). Foundation Phase students' intuitive or tacit knowledge of mathematics may serve as foundations for the development of mathematics proficiency but *whether* they develop conscious awareness and mastery of mathematical knowledge, as well as the procedures and forms of reflection entailed in these five strands, is contingent on, what and how they are taught mathematics. In other words, it is contingent on 'what' and 'how' the educator teaches (NRC, 1989; Duncan et al., 2008; Green & Gallagher, 2014; Mulligan, Mitchelmore, English, & Crevensten, 2013).

It is the role of Foundation Phase educators to advance their students' mathematical thinking and promote the development of a sound foundation for mathematical proficiency. This is especially significant as current research indicates that mathematical competence has a profound effect on further academic achievement (NRC, 1989; Duncan et al., 2008; Green & Gallagher, 2014; Mulligan et al., 2013). In South Africa, most Foundation Phase educators are not necessarily mathematics specialists and are responsible for teaching all areas of the curriculum. Developing and using mathematical knowledge for teaching is often more beneficial than mathematical content knowledge (Ball et al., 2005). With problem solving and critical thinking being aims of the South African curriculum, many educators may already be integrating these practices into their everyday teaching. Thinking skills have been included in the educational philosophy at many independent schools, in particular, and problem solving is also receiving more attention. It is what educators do purposefully during mathematics lessons which was the aim of this research.

As a mathematics and Foundation Phase educator who has taught in well-resourced independent schools for many years, I have always valued the importance of solving problems in the development of mathematical practice. Therefore, the focus of this research is, the ways in which Foundation Phase educators are taking up the challenge of recontextualising the mathematics curriculum and promoting problem solving and mathematical reasoning in the development of their mathematical practice.

Accordingly, the aim of this study was to *describe* and *analyse* how the Foundation Phase educators in two independent schools promote mathematical proficiency in their teaching practices, in order to explore whether they included problem solving in their work, and if so, how. I was also curious about how the ethos of the school and its particular orientation to mathematic education, might influence and shape the educators' practices, however, this was not central to my research.

The two schools were purposively selected because of their explicit, well-defined educational ethos and the different mathematical philosophies and approaches they have elected to implement. As independent schools, both are well-resourced, cater for a middle-class student body and share a strong culture of teaching. The schools have two or three highly-qualified Foundation Phase educators in each grade. Although this well-resourced (arguably 'optimal') educational environment increases the likelihood that they will promote the strands of mathematical proficiency which includes problem solving, it does not follow that they actually implement the prescribed curriculum or enact the school's approach to it.

In order to investigate how the educators enacted their pedagogy in the classroom with particular emphasis on whether and how they teach problem solving, this study collected data, through interviews and classroom observations, to address the following questions:

- What are the educators' conceptions of teaching mathematics and problem solving in the Foundation Phase and how do they engage with the Foundation Phase mathematics curriculum?
- How do the educators teach for mathematical proficiency, and do their practices actually follow the orientation to mathematics teaching favoured in their particular school?
- Do the educators integrate problem solving into their mathematics lessons? If so, how?
- Are there differences in the individual educators' practices, within the same school, and at the different schools?

At the outset of this research report it is imperative to emphasise two important points. First, the focus of this research was on what educators do. In order to do this, it was obviously necessary to look specifically at what educators say and do and how they engage with their students. Accordingly, I have endeavoured to reflect and represent the students' interactions in the classroom activities. However, a deeper analysis of student learning and development, and whether they incorporate problem solving in other subjects and spheres of their lives overtime, is beyond the scope of this research report. A deeper focus on student learning and development would have required a very different approach to sampling, data collection methods and interpretation of findings.

Second, the aim of this research is not to judge the educators or to focus on the absences in their practices relative to normative ideals and standards outlined in the literature. Instead I chose to approach teacher development through a Vygotskian lens. The first step was to describe what educators are actually doing. This could be the basis for designing teacher development programmes that build on from what educators *are* already doing successfully.

In the next section of the report, I review literature that relates directly to the teaching of mathematical proficiency. The focus is on mathematics knowledge, mathematics education and mathematical pedagogy. I explore contemporary thinking on mathematics which promotes thinking mathematically and making sense of mathematics. This is then followed by the conceptual framework for the research.

Chapter 2: Literature Review

2.1 Introduction

This literature review examines contemporary thinking in literature on mathematics education to provide my research with an overview of important aspects of the field, to establish a sense of what I should be looking for, to describe and interpret educators' teaching practices. First, I discuss how mathematical knowledge has been reconceptualised and what is meant by mathematical proficiency. Then I look at mathematics education, the ways in which it is recontextualised in the school environment and how theories of learning and teaching influence the way in which it is learned. Next, I address the role of problem solving and reasoning in mathematics practices and their significance in the Foundation Phase. Finally, I consider mathematical thinking and the different pedagogic practices that promote mathematically proficiency.

2.2 Mathematics Knowledge

In the past few decades, influenced by the work of constructivists, social constructivist theory and situated cognition, there have been new developments in how we think about mathematics. Together with our understanding of cognition, and learning and teaching, so too have the practices of knowledge and education been informed and shaped. (Greeno, Collins, & Resnick, 1996). Traditionally mathematics knowledge was perceived as a cognitive activity in which facts and procedures were mastered, and the depth of mathematical knowledge was determined by the extent to which this was done effectively. However, this emphasis on computational skills and performance, with little concern for the meaning and significance of mathematics, proved rather narrow in outlook. Contemporary research into how mathematics is learned, shows that greater value should be placed on the process of 'doing' mathematics rather than just learning content or skills-based knowledge (Boaler, 2009). Mastering the subject involves more than acquiring a set of mathematical facts. It requires proficiency in a range of mathematical practices.

In examining concepts of 'knowing' and 'doing', important ideas about mathematical knowledge have evolved. In the social context, mathematics is viewed as a specialised practice with its own, specific conceptual tools, artefacts, procedures and discourse that each need to be meaningfully interpreted. 'Doing' mathematics (Boaler, 2009) entails being able to: recognise and understand the logic of thinking and performing mathematical tasks, in relation to these tools; understand and implement the mathematical conventions; identify and grasp the particular references inferred; make connections as to how they are structurally related; and situate them in the context where they are going to be used and experienced (Thompson, 1990). Boaler defines it more succinctly, with her description of mathematics as "a human activity, a social phenomenon, a set of methods used to help illuminate the world, and it is part of our culture" (2007, p 20).

Mathematics is essentially a process of thinking, mathematising, sense-making and developing competence with mathematical tools. Along with the publication of *Everyone Counts* (NRC, 1989), research informed by constructivist, socio-cultural and situated learning started to stress the importance of mathematics as a process that involves observing, testing, estimating, experimenting, and discovering. By seeking solutions, exploring patterns and formulating conjectures, students are involved in using and interpreting mathematics, empowering them to think flexibly (NRC, 1989; Schoenfeld, 1992).

In *Adding it up: Helping children learn mathematics,* Council and Committee (2001) conceptualises mathematics proficiency as having five strands: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. All five need to be integrated and balanced in order to achieve mathematical proficiency. Ball (2002) and her group of mathematicians go further and describe mathematical proficiency as focusing: "on the mathematical know-how, beyond content knowledge, that characterises expertise in learning and using mathematics" (Boaler, 2002, p. 8). Both Council and Committee's conceptualisation of mathematics proficiency and Ball's work on mathematics education have informed the development of the South Africa's *CAPS* curriculum and the *Numeracy Handbook*.

Most researchers agree that different forms of knowledge can be characterised as conceptual or procedural and there is consensus in the belief that mathematics, as an activity, is of value. Engaging in the 'doing' of mathematics leads to meaningful understandings of concepts; develops a sense of how the mathematical system works and generates new ideas (Noss, Healy & Hoyles, 1997; Simon, 1996).

This broadening of conceptions of mathematical knowledge has brought about a lively debate on the characteristics of mathematics practice and much has been written about the interrelationship between the two (Ball, 2003; Ball & Bass, 2002; Boaler, 2002; Noss et al., 1997; Putnam, Lampert, & Peterson, 1990; Schoenfeld, 1992). While mathematics is something students have to do, it is not defined by the knowledge students own. In undergoing this reconceptualisation, important changes have occurred in how mathematics is practised in schools. With a larger focus on processes, this 'doing' of mathematics is an increasingly social and collaborative act. Mathematical practices refer to the specific things that successful mathematicians should do, such as: drawing connections, observing relationships, identifying patterns, justifying claims, using symbolic notation efficiently and making generalisations.

In line with the reconceptualisation of mathematics knowledge, thinking in mathematics education has undergone a *renaissance* too, as everyday mathematical knowledge experiences from outside the classroom are now linked to the mathematics curriculum. By way of pedagogic discourse, this specialised knowledge is integrated and mediated into the classroom environment. If educators are to provide students with the opportunity to participate in the specialised practice of 'doing' mathematics, then they need to be able to interpret the curriculum, and to adopt appropriate theories of teaching and learning, so their students are able to acquire, produce and assess this knowledge. In the next section, I discuss the ways in which mathematics knowledge from the curriculum can be mediated by educators through theories of teaching and learning.

2.3 Mathematics Education

Underpinned by educational policy, which has been influenced by cultural and social contexts, the curriculum consists of the selection and organisation of knowledge to be learned in schools. Mathematics, as a specialised practice, is different from both everyday and academic knowledge and what students experience in school is a discourse from which mathematical content is transmitted and acquired through social interaction. As a result, this mathematics must undergo a process of recontexualisation before it can be transformed into a curriculum suitable for use in the classroom.

The concept of recontextualisation is drawn from Bernstein and refers to how schools go about selecting and organising mathematics, and interacting with students, to promote learning (Lerman & Tsatsaroni, 1998; Lilliedahl, 2015). Bernstein describes recontextualisation as a pedagogic process through which mathematics is reconstructed and reproduced ready for learning. It is about which knowledge is selected as most important and how it is transmitted to students. It is important to note that recontextualisation can also be seen a bi-level process in which, first, the official curriculum regulates the selection and transformation of knowledge and, second, the teacher recontextualises the curriculum into her classroom (Lilliedahl, 2015).

Recognising the importance of 'doing' mathematics, emulating the work of mathematicians and integrating mathematical knowledge and practices, also requires a recontextualisation of mathematics knowledge. Contemporary Literature in mathematics education is increasing acknowledging the need for mathematics to be a purposeful and meaningful activity where students are offered opportunities to engage with a broad range of problems and problem situations, and apply a variety of approaches and techniques. This 'doing' of mathematics is reliant on a socially meaningful context, as well as communication and collaboration with each other, and the availability of a wide range of resources (Boaler, 2002; Long & Dunne, 2014; Putnam et al., 1990; Schoenfeld, 1992).

Knowing and doing mathematics are intricately connected to theories of learning and teaching. How and what students learn in class, and the set of practices they develop in pursuit of mathematical proficiency, is strongly influenced by the work of constructivists, social constructivist theory and situated cognition.

Constructivists were the first to recognise that students are creators of their own learning and that understanding comes when knowledge is constructed and then reconstructed. Students' learning and thinking is governed by what they already know and this influences how they construct new knowledge. Working from their pre-existing concepts, they need to actively engage in opportunities where they are able to apply previous ideas and formulate new knowledge by restructuring, reorganising and transforming them, to create a richer, more powerful understanding. Knowledge is modified when students reflectively sort through existing information and relate them to current thoughts and ideas. As this knowledge is constructed, new meaning is given to the way in which things are perceived. Social interaction is fundamental to this process, creating access for individual learning. To construct and understand a new idea, students have to actively think and reflect, both independently and with others. Constructivists also raised the importance of problem solving as a process through which students, using their own knowledge, could interact with others, create new experiences and construct new concepts (Boaler, 2010, 2016; Brodie, 2009; Schoenfeld, 1985; Van de Walle, 1998).

Vygotsky (1978), and subsequent proponents of his theory, maintain that while spontaneous concepts are formed through everyday interactions in the environment, the development of scientific concepts requires semiotic mediation (Hasan, 1995; Schmittau, 2004a). Students accept mathematical concepts that are culturally constructed but, to go beyond mere conceptual understanding and cognitive development, the curriculum and pedagogy are needed to mediate them further. An awareness of the importance of conceptual understanding and reorientating it, allows for this cognitive development of concepts which Vygotsky, in his research, believes occurs when students are involved with tasks that are, initially, not easy to solve. When these tasks lie outside the students' Zone of Proximal Development interaction with fellow students, along with mediation by the educator can help in the process of developing new concepts. Vygotskian problem solving is a sequenced series of interesting and accessible tasks (Schmittau, 2004a). As each solution is mastered, the students progress and develop more powerful mathematical insights and methods to find solutions. With the support and help of educators, students are challenged to work together through the problem-solving and inquiry activities, productively expanding their Zone of Proximal Development (Lerman & Tsatsaroni, 1998; Schmittau, 2003, 2004a; Swanson, 2013).

More recently, situated theories of learning have advocated that a student's knowledge is constructed within, and linked to, the activity, context and culture in which it is learned. This interconnectedness means that what is learned cannot be separated from how it is learned and used. Learning is social and occurs when students interact with each other through shared activities and shared knowledge to solve problem situations. For learning to be authentic, the environment plays an important role. It needs to be situated in a context where students are able to incorporate the knowledge they already possess and instinctively use intuitive reasoning and "negotiate meaning" through everyday activities. Often this social interaction is within a community of practice, where participation moves from cursory to significant through shared knowledge, skills, and experiences (Boaler, 2001; Brodie, 2009; Brown, Collins, & Duguid, 1989).

All three perspectives champion learning that occurs when students have opportunities to engage in the work of mathematicians and can interact to communicate their ideas and use and apply their mathematical knowledge. Learning happens in an environment that supports mathematising, mathematical modelling and encourages different practices so the acquired knowledge can be used and applied in other contexts. This suggests that, within the five strands of mathematic proficiency, strategic competence and adaptive reasoning go beyond pure mathematical knowledge and are valued in the greater *milieu* of learning.

If complex problem solving and critical reasoning now top the list of essential skills of the future (Gray, 2016), then their role in the process of mathematical proficiency is more significant than ever. Problems have always played an important role in school mathematics, however problem solving has not (Boaler, 2016; Schoenfeld, 1992; Stanic & Kilpatrick, 1992). Initially problem solving was rather superficial in nature but, as more detailed understandings of thinking and learning developed, and conceptions of mathematics knowledge evolved, the role of problem-solving strategies in mathematical practices took on greater importance. Likewise, the impact of mathematical reasoning on mathematical understanding has been overlooked. Now considered an essential skill, and fundamental to constructing knowledge, problem solving enables students to use mathematics in useful and doable ways (Ball & Bass, 2003). What follows is an overview of the role that problem solving and reasoning play in students becoming mathematically proficient.

2.3.1 Problem Solving

One of the strands of mathematical proficiency is strategic competence, which involves the "ability to formulate, represent and solve mathematical problems" (Council & Committee, 2001, p. 5). In the South African curriculum, it includes problems, open-ended investigations and mathematical models and is explained in the *Numeracy Handbook* (2012) as being able to apply conceptual understanding and procedural fluency to non-routine problems.

Mathematicians like Pólya (1954) and Halmos (1980) were among the first to realise the importance of problem solving in mathematical practices. Pólya's work conceptualises problem solving as actively engaging in challenging problems through guessing, insight and discovery. He believes that experience in mathematics should be consistent with the way mathematics is done. Halmos claims that solving problems is at "the heart of mathematics" (Schoenfeld, 1992, p. 15) and it is his belief that mathematicians exist to solve problems and that mathematics is about problems and solutions. He argues that schools should offer students mathematical experiences that prepare them for challenges of significant difficulty and complexity, encouraging better problem posers and problem solvers (Schoenfeld, 1992).

In mathematics education, proponents of problem solving suggest that it provides a background not only for practicing learned skills but also for developing new concepts. It integrates all parts of mathematical proficiency, expanding conceptual understanding and provides opportunities to apply mathematical knowledge. The essence of problem solving refers to the whole process of attending to the problem and working towards a solution. It involves mathematical thinking, problem-solving processes and different ways in which they can be taught and learnt in the classroom. Working towards a solution starts with responding to the problem with understanding; suggesting conjectures and hypotheses that build understanding and move closer to a solution; and then examining and questioning possible solutions. Hence, Heddens and Speer (2006) define problem solving as "the (interdisciplinary) process an individual uses to respond to, and overcome, obstacles or barriers when a solution, or method of solution, to a problem is not immediately obvious" (Petersen, 2017, p. 82). They argue that this deepens students' conceptual understanding and allows them to engage in the process of sense making which allows for mathematical skills and knowledge to have more meaning and purpose (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016; Petersen, McAuliffe, & Vermeulen, 2017; Schoenfeld, 2013b).

Applied mathematical problem solving is distinguished by acquiring knowledge and applying it. In other words, conceptual knowledge should be learnt first and then applied to solve problems. The underlying idea being that this school knowledge should be useful and applicable to situations beyond the school context, with problems being the medium through which students can apply their conceptual knowledge. Consequently, the mathematical problem-solving process starts with a *real problem situation,* which is interpreted into a *mathematical model, then mathematised* until *mathematical results* are obtained and finally applied to the *real situation solution*. Mathematising is the work of applying and generating mathematical methods, drawing conclusions, calculating and interpreting results (Blum & Niss, 1991).

Problem solving can also provide a starting point for developing new concepts. When "mathematical ideas are embedded in situations that provide the possibilities of making connections to previous experiences, knowledge and needs" (Murray, 2012), students can learn mathematics with understanding. By introducing an activity, which is familiar to the students, and problematising it, students are more likely to construct their own understanding. Understanding comes from the culture within the classroom, where educators and students collaborate and a collective activity is created. The educator is active in selecting and presenting the task and constructing a community in which the students problematise the situation. Critical to this, is the process through which students explain and justify their thinking and methods, then examine and discuss other ideas and learn from one another. Understanding also develops from what students take away with them after the activity is complete. Problems should involve analysing patterns and relationships, developing strategies for solving other problems and constructing their own strategies, and forming dispositions towards mathematics which recognise their own agency (Hiebert et al., 1996; Murray, 2012; Murray et al., 1998).

Problematising starts with students making the activity their own by constructing their understanding. Using prior knowledge, they make personal and collective sense of what they are learning and assign meaning to it. Reflecting on the problem engages both the educator and the students in practicing the mathematics. The organisation of a lesson around a contextual problem often gives meaning to the concepts, transforming implicit knowledge into explicit and generalised knowledge that can be applied to other situations (Hiebert et al., 1996; Long & Dunne, 2014; Murray et al., 1998).

An important part of problem solving is being able to see the mathematics in a non-mathematical situation. When students are introduced to situations that are meaningful and real to them, they have to interpret the context and make sense of what is being asked. Mathematising is being able to organise their thinking so that they can make the link to mathematics. Freudenthal (Gravemeijer & Terwel, 2000) described it has being able to take "matter from reality" and make it "mathematical matter" (2000, p. 781). Students engage in the inquiry, elaborate on how the problem is posed and ask questions. Further they have to bring the mathematics in by organising their thoughts, applying their knowledge and strategies with an understanding of the possibilities and the limitations. (Kazemi & Stipek, 2009; Franke & Kazemi, 2001; Bos, 2017). In other words, mathematising is the work of thinking about different strategies, constructing the correct knowledge, and being able to reflect on and clarify thinking (Yackel, 2012). More succinctly put, it is about students performing a problem-solving process, acquiring knowledge of mathematics and critically analysing and assessing their process (Blum & Niss, 1991; Putnam et al., 1990).

Mastering mathematical proficiency is a set of processes in which problem solving occupies a predominant position. Once engaged in solving problems, students have to navigate through facts, concepts, procedures, methods and solutions, simultaneously making sense of their ideas and checking that everything fits together. If problem solving is important in building mathematical proficiency then, in analysing and assessing their thinking, students also need reasoning, and the skills it teaches, to build mathematical understanding. In the next section, I look at reasoning and the role it plays in mathematical proficiency.

2.3.2 Mathematical Reasoning

In order to engage in mathematical practices, and especially in solving problems, students need to be able to express, clarify, and define their thinking. Ideas should be scrutinised and reflected upon to deepen and extend conceptual understanding. Reasoning allows students to foster ways of thinking or develop arguments to solve problems and integrate ideas into a more coherent whole (Brodie, 2009). Council and Committee (2001) includes adaptive reasoning as one of the five strands of mathematical proficiency, defining it as the "capacity for logical thought, reflection, explanation and justification" (2001, p.5.). The *Numeracy Handbook* (2012) describes it as reflecting on and thinking about ways in which problems can be solved and explaining and justifying solutions. When integrated with the other strands of mathematical proficiency, reasoning offers students the chance to reflect on their thinking, relate it to conceptual understanding and competencies that they already own, and to make sense of the mathematics.

Reasoning provides the pathway to conceptual understanding. Adaptive reasoning is a basic skill that builds understanding, is essential to the use of mathematics, and is crucial to reconstructing previous knowledge when it is required. Accepted mathematical knowledge and understanding is established when students explore problems, formulate and examine claims, and prove their cogency by engaging in extensive procedures of adaptive reasoning (Ball & Bass, 2003). Reasoning also assists with the understanding of concepts because it allows for different ways of thinking about mathematical situations and could raise further questions. Reasoning assists with the understanding of concepts by mentally, or physically, envisioning the transformation of a mathematical situation and its results. Simon (1996) postulates that it is "a natural inclination of the human learner who seeks to understand and to validate mathematical ideas" (1996, p. 207). Reasoning also gives students access to establishing a deeper understanding of mathematical conceptual ideas and competencies (Boaler, 2009, 2016; Brodie, 2009; Simon, 1996).

Intuitive and inductive reasoning go further than informal clarifications and justifications. Tools such as the use of analogies and metaphors, and mental and physical images, encourage thinking and serve as problem-solving strategies (Council & Committee, 2001). There are two ways in which mathematical reasoning can be used: as a source of inquiry and justification of claims. By creating a demand for adaptive reasoning through the solving of problems, students are exposed to, and engage in, well established mathematical ideas and language (Ball & Bass, 2003).

Both problem solving and reasoning need to be at the heart of mathematics. Encouraging rich conversations between students, where they are able to defend their thinking, justify their claims and explain their choice of methods, is powerful in the sense-making of real and living mathematics. The use of intuition, creativity, discovery, imagination and communication are all valuable and important tools when justifying claims and developing arguments (Boaler, 2009, 2016; Brodie, 2005).

Piaget (1964) suggests that pre-schoolers enter Kindergarten with some informal, intuitive mathematical knowledge and understanding learned in the course of their development. Furthermore, he suggests that this experiential interaction in everyday life practices serves as a sound foundation for the development of mathematics knowledge and mathematics achievement at school, which are strong predictors of mathematical competence later (Ripple & Rockcastle, 1972). Young children are ready to be introduced as early as kindergarten to the practices of mathematical proficiency. The five strands of mathematical proficiency are based on the premise that Foundation Phase students are cognitively ready and able to be empowered by mathematics. In the following section, I look at why opportunities for students to experience, and observe, problem-solving and reasoning situations in the early grades are fundamental to knowing and understanding mathematics.

2.4 Mathematics in the Foundation Phase

Bruner's hypothesis that "any subject can be taught effectively in some *intellectually honest* form to any child at any stage of development" (Ball, 1993, p. 3), suggests that young children have the power to learn to think mathematically, and to think mathematically to learn (Ball, 1993). Mathematical development in young children is not fixed. Current research posits that young children are more capable of complex mathematical thinking and abstract reasoning than previously thought . The intuitive and informal mathematical knowledge, with which young children start school, provides a basis from which educators can develop students' mathematical thinking and mathematical understanding. Mathematical proficiency, therefore, can be enhanced when students use this power to make sense of experiences and in particular, make sense of mathematical opportunities (Carpenter & Fennema, 1996;

Council and Committee, 2001; Mason, 2008; Stacey, 2010). Furthermore, both the NRC (U.S.) and DBE (S.A.) recommend that the five strands of mathematic proficiency are introduced from the first grade.

In the Foundation Phase mathematics curriculum, the focus of the content area is on the conceptual knowledge of number, operations and relationships. However, unless the students develop a deep conceptual understanding and make sense of mathematics, they will be unable to be mathematically proficient. Researchers suggest that mathematics instruction should extend beyond basic numeracy skills (DBE, 2011; Gamble, 2014; Mason, 2008; Mulligan et al., 2013). Introducing mathematical pattern and structure, and informal pre-algebraic skills, are both critical and significant to mathematical understanding. Mathematical patterning looks for regularity and predictability in numbers, while structure refers to the way numbers can be organised. Informal pre-algebraic skills, like comparing, constrasting, generalising and justifying, can be introduced by asking students to think about what they do and make sense of number. Exposure from the beginning, to the most general and abstract level of understanding, orientates students towards understanding and powerful insights.

Mathematical understanding is developed when students build connections with new concepts and ideas and all students will develop their understandings differently. Defining understanding is being able to measure the quality and quantity of connections a concept or idea has with existing knowledge (Van de Walle, 1998). Understanding differs from student to student and is dependent on opportunites to create ideas and make connections. Skemp (1976) defines understanding along a continuum – moving from instrumental understanding (‘doing’ without understanding) to relational understanding (knowing what to do and why). Relational understanding comes from working with pre-existing knowledge, building connections, interrelating concepts, creating various representations and solving problems. It is generally agreed that students need to build a deep interconnected understanding of the five strands of mathematical proficiency and, for this to be achieved, students should be introduced to opportunities which foster understanding and effective and productive thinking in their first encounters with mathematics in the classroom (Green & Gallagher, 2014; Mason, 2008; Skemp, 1976).

It is the cognitive processes of solving problems and reasoning mathematically that provide an understanding of how these concepts and skills all fit together and make sense (Stacey, 2010). Vygotsky (1978) asserts that cognitive development occurs when students encounter problems to which they have no immediate solution. He argues that most mathematical concepts are already culturally constructed and it is when students are given opportunities to think and mediate these concepts within the classroom that they are able to go beyond conceptual understanding (Schmittau, 2004a, 2004b). Solving problems challenges students to go beyond their everyday experiences, to refine their intuitive understanding and to find new ways to express it (Carraher & Schliemann, 2002).

In the Foundation Phase, the educators' role is particularly significant: they should lay the groundwork for the practice of 'doing' mathematics. By extending mathematical knowledge beyond just number, introducing mathematical thinking skills and problem solving, students are in a position to investigate tasks creatively, logically and critically. Deliberate and intentional teaching of problem-solving and reasoning skills can then occur at appropriate moments (Ball & Bass, 2003; Green & Gallagher, 2014).

In accepting that mathematical knowledge has undergone reconceptualisation, that mathematical education has to be recontextualised, and that strategic competence and adaptive reasoning have a significant place in mathematical proficiency (especially in the Foundation Phase), then how do educators implement the curriculum? Through their knowledge, beliefs and actions, educators provide their students with access to the transmission and acquisition of mathematical knowledge. Next, I address the role of educators in developing a pedagogy that endorses the mathematical practices of conceptual understanding, strategic competence and adaptive reasoning.

2.5 Mathematical Pedagogy

Teaching and learning mathematics is a multi-faceted process: it is about developing concepts, guiding practices, discussing student thinking, attending to relationships between problems and solutions and justifying the mathematical argument (Council & Committee, 2001). Consequently, through their pedagogical practices, educators are at the centre of curriculum delivery and student learning. Educators are the "gatekeeper[s] to the meaning" (Moore, 2013, p. 70) of the world of mathematics and are responsible for ensuring their students become mathematically proficient. The type of interactions that develop between educators, students and the content will determine what is taught and what is learned and, in the school and classroom context, this is affected by the knowledge, beliefs, decisions and actions of educators as well as the expectations, knowledge, interests and responses of the students (Council & Committee, 2001). Mathematical pedagogic practices can be wide and varied but, ultimately, effective teaching is determined by the quality of instruction, that which ensures higher order knowledge is constructed, mastered, systematised, internalised, has a broad sense of transfer and is consciously used (Karpov & Bransford, 1995; Moore, 2013).

Research by Council and Committee (2001) made three key findings on the instructional practices that lead to mathematical proficiency: first, that instructional practice and content cannot be considered independently of the context in which the learning takes place or the ways in which the educators and students interpret and value the curriculum, time, tasks and other such issues; second, that the curriculum can only be effective if educators implement it, in their classrooms, lesson-by-lesson and third, that the ways in which educators and students interact shapes the effectiveness of instruction that develops mathematical proficiency and is "neither simple, common nor well understood. It comes in many forms and can follow a variety of paths" (2001, p. 359).

According to Bernstein, there are three important message systems of schooling: curriculum, pedagogy and evaluation. Curriculum defines what counts as valid knowledge; pedagogy defines what counts as valid transmission of knowledge; and evaluation defines what counts as valid realisation of knowledge. The relationship between the three is hierarchical, with the curriculum influencing the pedagogy and the pedagogy shaping the evaluation (Moore, 2013; Morais, 2002; Pires, Neves, & Morais, 2004; D. Scott, 2007).

Curriculum refers to the content: what is transmitted or acquired through the pedagogic discourse. How specialised the knowledge remains is dependent on how it is structured and how it is interrelated to everyday knowledge. With strong boundaries, knowledge remains more specialised, giving educators and students limited control over the selection and organisation. With weaker boundaries, knowledge is integrated, and educators and students have greater control over the selection, organisation and pace (Scott, 2007) .

Pedagogy is the relationship between educators and students: how the discourse regulates the principles of the transmission and acquisition of knowledge. Pedagogic discourse describes the social interaction that characterises the teaching-learning context in a classroom (Morais, 2002). Distinguished by both instructional and regulative contexts, it is a relay between the pedagogical relationship of educators and students and their control over the selection, organisation and pacing of the knowledge being transmitted and acquired. The instructional context is embedded in regulative context (Hoadley, 2006). It provides the discourse of competence and is concerned with the knowledge and skill being transmitted. While the regulative context provides the discourse of the underlying theory of pedagogy and the expectations of character, conduct and manner (Hoadley, 2006). Both contexts are defined by the relationship between the educator and the student, with classification (power) and framing (control) determining how pedagogic practices are enacted in the classroom. In line with control over the structure of knowledge, strong framing in the instructional context empowers the transmitter, while weaker framing enables the acquirer. Rules of communication, in the regulative space, and relations within the classroom space are determined by the way they are framed. Framing supported by classification, sets the rules of interaction, opening or closing boundaries, and defining, maintaining and changing the pedagogic discourses. These characteristics which describe the relationship between the educator and students offered a useful, descriptive framework for my conceptual framework (Moore, 2013; Morais, 2002; Morais, Neves & Pires, 2004; Scott, 2007).

Evaluation is established in pedagogic practice but is influenced by how the curriculum, the school and educators structure it. The outcome of the instructional context is performance, while the regulative context emphasises competence. Demonstrating an understanding of both performance and competence is governed by recognition and realisation rules. Students recognise, through the classification of pedagogic discourse, how to distinguish between contexts and orientate themselves to what is expected and legitimate within that context. Framing, on the other hand, controls what is apt within the context and allows students to produce contextually specific practices. Hence, in evaluating the transmission of valid knowledge, students are required to recognise the specialised context and realise the appropriate practices that apply (Moore, 2013).

In her research on the teaching and learning of mathematics, Sfard (2007) defines mathematics as a specialised form of discourse. Mathematical thinking is a distinctive discourse characterised by its own vocabulary, visual mediators, narratives and routines. Vygotsky claims it is communication that drives cognition and, before learning can take place, students have to engage collectively in activities. Commognition, a combination of communication and cognition, is the thinking form of communication and learning mathematics is about modifying and extending that discourse (Sfard, 2007).

In school, effective learning is driven by an active educator, experienced in mathematical discourse, and a learning-teaching agreement between the educator and the students. Students learn about mathematical objects out of the need to communicate and visual mediation is crucial to the success of that communication. During the discourse, consideration is given to: the use of mathematical words; how visual mediators (the object of learning) co-ordinate the communication; the way in which spoken or written narratives (texts of the object of learning) frame descriptions, relations or activities; and the application of routines (well-defined repetitive actions). Learning occurs when there is a transformation in the students' discourse, which they have to understand and execute competently on either an object (explicit) level or meta (implicit) level. This is, then, evident in their refined use of mathematical words, visual mediators, narratives and routines. Learning can also occur when students are confronted by commognitive conflict, such as when they encounter meta-level learning, misconceptions and contradictory claims. With the educators taking the lead and mediating the process, students have to make sense and understand the new discourse. Sfard's ability to make sense of the mathematical processes in the classroom will guide my Conceptual Framework (Sfard, 2000, 2007, 2008).

Successful instruction is largely dependent on how educators provide students with opportunities and experiences. Believing that all strands of mathematical proficiency can converge in problem solving (Council & Committee, 2001), it is the opportunities provided to students to integrate those strands that are critical and significant. Particularly important is how educators view task selection and their mediation of the discourse. The tasks should have depth, providing opportunities for different solution strategies, various kinds of representations, discourse and be cognitively demanding (Stein, Grover, & Henningsen, 1996). Alternatively, students can do the problematising themselves by framing a meaningful question to be explored by the class (Engel, cited in Schoenfeld, 2013). When mediating the discourse, educators take deliberate actions to participate or influence discourse in the mathematics lesson. Their actions have purpose (an intended outcome), take place in a specific setting, have a particular form (verbal and non-verbal) and result in consequences for both students and educators (Krussel, Edwards, & Springer, 2004; Schoenfeld, 2013b).

Pedagogic practices are complex and often uncertain. If educators are aspiring to create pedagogic practices that are intellectually honest, to both their students and mathematics, then the uncertainty and complexity intensifies. Educators should be empowered by pedagogical dilemmas, enabling them to determine their own direction and how they work with their students (Ball et al., 2005; Ball, Thames, & Phelps, 2008).

2.6 Mathematical Thinking

From the literature review so far, certain insights can be highlighted. Acquiring mathematical knowledge is about constructing conceptual understanding through the doing of mathematical processes. Thinking, mathematising, sense-making and competence are the skills students need to cultivate. The curriculum is recontextualised by the schools, in line with their educational ethos; and by the educators, consistent with their knowledge and beliefs about mathematics. Foundation Phase students enter school with intuitive and informal knowledge that provides the basis from which educators can build more formal mathematical knowledge. The curriculum encourages the development of students who are successful, skilled problem solvers. If all this is taken into consideration, then there should be a shift in what, and how, educators practice in the classroom.

In many countries, problem solving, combining mathematical knowledge, reasoning ability and strategies, cooperative communication skills and thinking skills, are all recognised as fundamental components of the school mathematics curricula. As in South Africa, curriculum developers in countries like Australia, Singapore, Hong Kong, England and the Netherlands have recognised that if students are going to be able to use and apply mathematical knowledge in a meaningful way, educators have to be encouraged to provide more problem-solving experiences. However, as research has found, its implementation can be problematic (Anderson, 2009).

The assumption that students construct their knowledge forms the basis of the CAPS document and therefore implies there should be a shift in how educators adapt their pedagogic practices in order, to offer opportunities for mathematical knowledge to be constructed (Council & Committee, 2001). Recent research has focused on students' mathematical thinking being important in developing conceptual understanding and problem-solving strategy. In understanding student thinking, educators are able to build coherence between their mathematical knowledge and their pedagogic practices. Carpenter and Fennema (1996) argue that if educators recognise how "students' typical understanding and its evolution" (1996, p. 4) develop, this then provides a framework for educators' knowledge of mathematics and a context in which educators can interpret and apply their pedagogic practices. Franke and Kazemi (2001) posit that student thinking is core to educator and student learning. In focusing on student's mathematical thinking, educators integrate their pedagogy, mathematics and students' understanding. Further research by Franke et al. (2009), found that through questions, educators could support their students' thinking allowing them to develop more explicit and detailed explanations. To engage students in mathematical inquiry, Kazemi and Stipek (2009) propose that educators examine the nature and degree of conceptual thinking and create a "press" (2009, p 61.) for conceptual learning.

Cognitive Guided Instruction, values the importance of problem*s and the ways in which they are solved. Students are encouraged to spend most of their time working on problems and discussing alternative solutions. Educators move away from finding ways to represent the mathematical concepts, allowing the students to construct their own representations based on what they intuitively think. To begin with students use concrete apparatus or pictures to model the problem, moving on to abstract it using more efficient strategies. Students effectively model the structure of the problem while educators, learn to understand what their students are thinking and construct a coherent, organised base to use when teaching mathematics (Carpenter & Fennema, 1996).

The key to Cognitive Guided Instruction is the development of students' mathematical thinking and by observing the process educators, are able to make sense of it, evaluate their understanding, adapt and build their knowledge and, finally, use it in the context of their pedagogical practice. Focusing on making sense of students' thinking, empowers educators to elaborate on how they can pose problems, ask questions, nurture interactions, and allow learning to develop (Franke & Kazemi, 2001).

The development of students' mathematical thinking comes from opportunities for them to participate in classroom discourse, by listen and talking to each other, as well as responding to their educator. In this way, not only does the educator monitor the thinking but so do the students. When talking to each other, students can help each other, learn different strategies, and improve understanding. They need time to describe, explain and justify their own thinking which, in turn, helps them to internalise their knowledge which leads them to developing new perspectives and understanding. It goes beyond just finding the answers, allowing students to be precise and explicit in their explanation ensuring that the educator and fellow students understand what they are proposing (Franke et al., 2009).

Educators' questions, particularly those that follow students initial responses can support the development of more explicit and detailed explanations. A probing sequencing of specific questions, a single specific question, or even a general question, results in students elaborating on their initial explanations; helps students correct their thinking; and makes sense of their ideas. There is a potential benefit for all participants in the classroom: the educators are able to more fully understand their students thinking; the students clarify and solidify their own thinking; and for other students, they connect to their own thinking. Hence, educators' questions can position their students' thinking to support mathematical understanding (Franke et al., 2009)

Classroom practices can press students for conceptual mathematical thinking. Socio-mathematical norms provide a set of expectations that indicates what constitutes mathematical thinking. In understanding these norms, educators can construct pedagogic strategies that can be applied in a variety of contexts. The difference between "high- and low-press" exchanges suggest how important that it is:

(a) an explanation consists of a mathematical argument, not simply a procedural description;

(b) mathematical thinking involves understanding relations among multiple strategies;

(c) errors provide opportunities to reconceptualise a problem, explore contradictions, and pursue alternative strategies;

(d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation. (Kazemi, 2009, p. 78)

In supporting student understanding, a high-press for conceptual thinking in mathematics is guided by the students' engagement in activities, educators valuing their students' ideas, and students participating in the discourse (Kazemi & Stipek, 2009).

For many educators providing a classroom culture which facilitates the learning of problem solving, includes opportunities for collaborative participation, and results in conceptual learning and understanding of mathematical knowledge – among other expectations - is a complex undertaking. It also requires them to develop a new culture of mathematics, which not only requires a mindset change in their understanding of the process of mathematics but has them reconceptualising their role and position in the classroom. These are challenging tasks, requiring an intentional shift in the ways they teach and offer opportunities for doing mathematics.

2.7 Problem Solving in South African Schools

The introduction of *CAPS* in 2011, mathematics education in South Africa paid cognisance to the reconceptualisation of mathematics and the importance of problem solving. One of the aims of mathematics education, as stated in CAPS (DBE, 2011) requires students to be able to "identify and solve problems and make decisions using critical and creative thinking" (2011, p. 5). It subsequently points out that one of the strands of mathematical proficiency is a productive disposition which is "recognising that you need to engage with a problem to solve it" (DBE, 2012, p. 11). The *Numeracy Handbook for Foundation Phase Teachers Grade R-3* (DBE, 2012) which accompanies *CAPS* (DBE, 2011), encourages educators to teach mathematics more effectively by using a child-centred approach and provides educators with practical ideas on the teaching of concepts and skills required in *CAPS* (DBE, 2011).

Successful teaching of problem solving requires many educators to make a paradigm shift in their approach to mathematics pedagogy and their belief system. Many educators in South Africa have not been afforded the right to a sound mathematical education themselves, nor effective teacher training in the recent developments in mathematical thinking and, as Foundation Phase educators, they are not mathematics specialists. They are responsible for teaching all parts of the curriculum and do not necessarily have specialised knowledge of mathematics. To implement recommendations from research and the curriculum is a complex and challenging task and while professional development can be successful, it is often insufficient or absent in many schools, and even where it exists many educators are reluctant to part with their strongly held beliefs.

Integrating problem solving in mathematics in South African schools is largely dependent two major factors: the ethos of the school and the mathematical knowledge of the educators. Schools, coping with large classes and lack of resources, are not always able to reconceptualise their approach to mathematics. Research in South African schools (Askew, Venkat & Matthews, 2012; Venkat & Naidoo, 2012; Ensor et al., 2009) shows that mathematics is still taught as an isolated set of facts and procedures which need to be mastered; tasks in mathematics are not built on what students have previously learned; nor are aspects of mathematics connected together; and tasks are answered through simple, practical methods (Askew, 2016).

Where problem solving is undertaken, sense making of problem situations is not addressed. Educators are directly modelling the situations using concrete resources to produce the answer. Students often lack the understanding, when the familiar problem situations are explained in everyday language. Opportunities for informal modelling to create a basis for more formal methods are limited. Further, educators in explaining the problem situations, are not making the mathematical concepts explicit (Venkat & Askew, 2016).

Tasks involving word problems, should through task design and pedagogic approaches, provide narratives which can support understanding (Venkat & Askew, 2016). These tasks encourage students to act out and use informal methods, include discussions on the advantages and disadvantages of different models, and lead to calculation strategies. However, whole class discussions can lead to “standardised, generalisable and efficient models and strategies” (Venkat & Askew, 2016, p. 264) which distracts from more sophisticated strategies. Projects undertaken by Wits Mathematics Connect-Primary (WMC-P)[1] have developed interventions which addresses gaps in mathematical knowledge and improving the mathematical pedagogy. Using word problems, with particular emphasis on multiplicative reasoning, lessons are structured so that students work in pairs first, followed by a whole class discussion led by the educator, and then students work independently.

The quality of the tasks used by educators is significant to the success of sense making in problem solving. Part of that sense making is for students to be able to clarify and express their thinking. By modelling writing tasks to support strategies in problem solving, educators can encourage their students to describe the thinking behind the process. Petersen, McAuliffe & Vermeulen (2017) in exploring writing strategies in mathematical problem solving in Grade 3 students noted a development in problem-solving strategies as students applied their conceptual knowledge.

While there are significant challenges regarding the integration of problem solving in Foundation Phase mathematics in South Africa, interventions which model an enriched view of both the teaching and learning of mathematics, can extend educators' understanding, students opportunities and lead to more effective teaching (Long & Dunne, 2014; Deacon 2016).

However, there are well resourced independent schools striving to be accredited as "Thinking School".[2] They are actively promoting an ethos that subscribes and supports a wide range of thinking skills. The importance of problem solving in mathematics has grown with this explicit development. Educators in these schools have been privileged with professional development in these areas and a supportive school environment.

As a Grade 4 educator, I was fortunate to undergo professional development in Hanlie Murray's (1996) Learning through Problem Solving approach. Then later as a mathematics educator in the Senior Prep, I was able to use it to develop conceptual understanding from Grade 5 to Grade 7. Strangely, the greatest opposition to this approach came from my colleagues who either disagreed with this view of teaching mathematics or feared its implementation (Murray et al., 1998).

[1] https://www.wits.ac.za/wits-maths-connect/wits-maths-connect-primary/

[2] Thinking Schools @Exeter. Retrieved from https://socialsciences.exeter.ac.uk/education/thinkingschools/

2.8 Conclusion

Many of the changes to the mathematics curriculum have happened in the last ten years or so, which means that many experienced educators are expected to teach mathematics to which they have not been exposed as either a student or student-teacher. To implement recommendations from research and the curriculum is a complex and challenging task and while professional development can be successful, it is often insufficient or absent in many schools, and even where it exists many educators are reluctant to part with their strongly held beliefs.

Chapter 3: Conceptual Framework

3.1 Introduction

In recent times many schools, especially independent ones, have developed their own educational ethos and, a consequence, educators' perceptions have been shifted and their new pedagogic mindsets are being reflected in their classroom practices. It was, therefore, my intention to investigate what educators are currently doing in their classrooms: first; by looking at their beliefs and practices, then by; describing the mathematics they teach; and, finally by establishing whether and how they were integrating problem solving into their practices. This research was not concerned with evaluating educators and their practices, nor with identifying what educators are not doing. The focus was on developing a descriptive, interpretative account of *what* and *how* educators teach mathematics and, in particular, whether they teach problem solving.

Researchers, like Ball and Schoenfeld, promote the idea that educators should be helping students to become active participants in the 'doing' of mathematics (Sfard, 1998). What is learned cannot be separated from how it is learned and used, hence, mathematical knowledge is a product of activities and situations in which they are produced (Brown et al., 1989). Given the importance of 'doing' mathematics, I drew on insights from a range of theorists who have adopted a situated approach to researching educators' practices, in particular Bernstein and Sfard.

In developing a construct that would enable me to describe and interpret what educators do in their classrooms, I started with Bernstein's theory of pedagogic practice and communication and Sfard's theory of commognition (Brown et al., 1989; Moore, 2013; Sfard, 2007). Both Bernstein and Sfard posit interpretive frameworks for making sense of classroom processes. Drawing on their work, I also included characteristics of situated learning and problem solving (Blum & Niss, 1991; Gravemeijer & Terwel, 2000). Shulman (1986) and Schoenfeld (2015) offered further insights.

Conceptual Framework			
Situated Learning	Culture of Mathematics	Interaction between educators and students	Authentic activities
Bernstein	Spaces Communication Sequencing Pacing	Relationship Instructional Regulative	Selection Organisation Pacing Conceptual Demand Investigative proficiency
Sfard	Object of Learning Process Outcome Resources	Discourse Words. Visual mediators, narratives and routines	renegotiate, reframe and restructure tasks Mediation
Problem Solving	Real-life problem situations Resources	Problematising Agency and authority Disciplinary accountability	Multi-dimensional, quality tasks.

Table 1: Overview of Conceptual Framework

3.2 Bernstein

Bernstein provided a means for describing the relationship between educators and students – the forms of pedagogic discourse that allow for the valid transmission of knowledge. Characteristised by both instructional and regulative contexts, it is their control over the selection, organisation and pacing of the knowledge, which distinguishes the educator and student relationship.

In describing the teaching-learning context, the educators' particular pedagogical practices are characterised by: the structure of the educators' and students' spaces; the communication relations between the educators and students and the students with each other; the sequencing and pacing of the learning; the intra-disciplinary relations; the level of conceptual demand and the level of investigative proficiency. The *how* of pedagogic practices is a consequences of classification and framing in a regulative and instructional context. Strong or weak classification and framing determines whether the boundaries between the charcteristics are bounded or blurred (Moore, 2013; Pires, 2004). Consequently by using Bernstein and the characteristics listed above, I was able to describe the educator-student relationship in the classroom.

3.3 Sfard

Sfard (2007) offered a generative framework for operationalising the mathematics processes in the classroom. She identifies three broad areas with respect to what it means to do mathematics: the focus on the object of learning, the focus on the process and the focus on the outcome. Sfard describes mathematics as a specialised form of discourse that is characterised by: the use of mathematical words; coordinating visual mediators; allowing narratives to frame descriptions or activities; and applying well defined and repetitive routines. Sfard stands by the view that all thinking, mathematical thinking included, is a discourse because the activities are social and because the more aware educators are of the discourse processes, the more able they are to effect student learning. By using Sfard's framework to identify these characteristics in the lessons, it allowed for a deeper understanding of the educators' use of opportunities and the 'doing' of mathematics. With respect to mathematics, I looked for a rich, clear sense of mathematical discourse, which included the use of mathematical words, visual mediators, mathematical narratives and routine, as well as the use of varied and purposeful resources (Sfard, 2007). I also noted student communication skills, and how they interpreted and expressed their thinking (Sfard, Nesher, Streefland, Cobb, & Mason, 1998).

3.4 Situated Learning

Creating a culture of mathematics, learning by participation and the use of authentic activities are also important aspects of situated learning and valuable to the 'doing' of mathematics (Brown et al., 1989). Situated learning points to a culture of mathematics which is recognised and identified and related to conceptual knowledge. For an activity to be authentic, it should be productive and useful; framed by a culture of mathematics; and should put conceptual knowledge to use. The meaning and purpose of the activity is socially constructed and, if it is authentic, it will be deeply informative, and be set in a life-world context (Brown et al., 1989). Situated theories favour a form of learning that prioritises participation over acquisition. Learning is visualised as a shifting, ongoing activity that is characterised by the context in which it occurs. This context, which is rich in discourse and communication, is the interaction between the students and the activities that creates learning. Not only do students interact with knowledge, they interact with each other and with objects of learning and such participation means that conceptual understanding is always being constructed. Each new opportunity, situation, negotiation and activity allows for the concepts to be renegotiated, reframed and restructured (Greeno, 1997; Sfard, 1998). Of importance here is the activity, or practice, the educators are undertaking and the transformation in learning that takes place (Sfard, 2007).

3.5 Problem Solving

Problem solving, as outlined in the literature review, is thinking mathematically. It is a process of identifying a real problem situation, interpreting it into a mathematical model, then mathematising and seeking a result. The nature of the classroom environment is essential to supporting students' thinking and engagement (Schoenfeld, 2013b) and Engle (in Schoenfeld, 2013b) has developed a framework for powerful learning environments. It includes four aspects: problematising; agency and authority, disciplinary accountablity and resources. The educator develops the environment where students have opportunities to propose and explore meaningful questions. They are empowered to seek information, extract ideas, shape arguments, and explain them. They learn to make claims and arguments and justify their reasons. The environment encourages the use of a wide range of different resources (Schoenfeld, 2013b).

The choice of task is central to problem solving. In order for students to be able to work through the problem-solving process, the task should be multi-dimensional. It should provide opportunities for the representation and drawing of mathematical ideas, including the use of properties of numbers and operations; students to explain their thinking mathematically, then follow the thinking of others and evaluate it; prompting different ways to solve the problem; connecting the mathematics to the students' environment; and the educators to take up or set aside students' ideas (Delaney, 2010).

3.4 Conclusion

By combining the characteristics (see Table 1) of situated learning, Sfard's and Bernstein's interpretive frameworks and problem solving, I was able to develop a construct which offered a complementary analysis of what educators are doing in their classrooms as they teach mathematics. Using the conceptual framework as a guide I established a research methodology and proceeded to data collection.

Chapter 4: Research Methodology

4.1 Introduction

Informed by the insights from the literature review and the conceptual framework, this research report explored how the educators enacted their pedagogy in the classroom with particular emphasis on whether and, if so, how they taught problem solving.

My report followed a qualitative research methodology. Qualitative research is a contextualised, interpretive process, which allows for the data to be collected at source. The emphasis is on qualitatively describing and interpreting events through sense making and understanding (Atieno, 2009; Creswell, Hanson, Clark Plano, & Morales, 2007; McMillan & Schumacher, 2010; Merriam, 2002; Ponterotto, 2006). In order to describe educators' practices and make sense of them required extensive data collection. This extensive data collection included: the undertaking of interviews, lesson observations, focus groups interviews and the collection of artefacts (relevant documents, drawings or other responses).

As the sole researcher, it was necessary to limit the scope and scale of the research. Using a purposively selected sample of educators, collecting data in a school setting, and using multimethod strategies, I gathered information through initial, reflective and focus group interviews in order to understand educators' beliefs about and experiences in the learning of mathematics, and their understanding of their relationship with mathematics. This was followed by observing lessons with each educator, to see how they enacted their practices. The interviews were audio-taped and the lessons video-taped and then transcribed.

Finally, the interpretation of the research was "richly descriptive" (Merriam, 2002, p. 5). Looking beyond what educators say and do, I developed a detailed reflection of the context, the educator and students, and their social relationships by looking for meanings, intentions, strategies and motivations (Atieno, 2009; Creswell et al., 2007; McMillan & Schumacher, 2010; Merriam, 2002; Ponterotto, 2006).

4.2 Research Method

In line with a qualitative research methodology and an interpretative approach, the research methods plan was flexible, exploratory and observational with an emphasis on developing rich descriptions of practice.

4.2.1 Selection of Schools

For this study, I worked with a purposive sample. I chose two independent schools in the broader Johannesburg area. The schools were selected because of differences in their general educational ethos and their distinct orientations to mathematics education. Pseudonyms have been used to protect the identity and maintain confidentiality of both the schools and the educators.

- **Hoopoe School** is a well-established independent monastic school. The educators are all highly qualified and have extensive teaching experience. The classes range from 20 to 24 students of mixed races and abilities. There are three classes in each Grade. The school subscribes to the Singapore mathematics philosophy, "Pr1me Mathematics" which follows two principles: the first being "(T)eaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving" and the second being "(T)eaching should build on students' knowledge; take cognisance of students' interests and experiences; and engage them in active and reflective learning."[3]. "Pr1me Mathematics" supports problem solving and concept development and its approach to the former is for the educators to explicitly model examples using a combination of processes and strategies and think through to the solution using appropriate mathematical language. "Pr1me Mathematics" also carefully scaffolds conceptual development by building with each concept, on pre-existing conceptual understanding.

[3]Principles of Teaching and PR1ME Mathematics. p. 1, Retrieved from http://www.scholastic.co.nz/media/1335/prime_teaching_principals_document.pdf

Beyond its “Pr1me Mathematics” philosophy, Hoopoe School, also has an explicit “Thinking School” ethos. This is based on the Growth Mindset (Anderson, 2009; Anderson, Krathwohl & Bloom, 2001; Dweck, 2008) approach to learning and includes the use of multidimensional thinking tools. It is guided by Costa and Kallick’s (2005; 2013) ‘16 Habits of Mind’, which is focused on dispositions towards learning; Hyerle’s (1996) ‘Thinking Maps’, which is composed of eight visual organisers; and Anderson’s (2001) ‘Taxonomy’ which is a cognitive dimension of thinking skills that empowers students with mindful attitudes and tools for lifelong learning.[4] It is noted that a whole-school approach has been adopted, where educators, students and parents have been included in training programmes and have been encouraged to use an explicit approach to thinking skills. The school has recently been awarded Advanced Thinking School Status by the University of Exeter’s Cognitive Education Development Unit.

- **Weaver School** is a newly established, independent, co-educational school. Again, the educators are all highly qualified and have varied teaching experience. Student numbers range from 14 to 22 in each class and there are two classes of mixed ability and race in each Grade.

  They have recently introduced the “NumberSense” programme, which is aligned with the South African curriculum and promotes a sense of number, sophisticated strategies and a deep understanding of mathematics. “NumberSense” is founded on what it means to do mathematics and the five strands of mathematical proficiency, as defined in *Adding it Up* (Council & Committee, 2001), as well as how students learn mathematics. It is developmental and builds on concepts and skills using problem solving from the beginning, to introduce new concepts[5].

[4] Taken from the School’s website
[5] Taken from the NumberSense website: www.numbersense.co.za

> Weaver School has a strong constructivist education philosophy and promotes an ethos of students learning naturally, through inquiry-based and project-based learning, which encourages collaboration, exploration and curiosity[6].

Of interest was the extent to which the schools' educational ethos would influence the teaching and learning of mathematics. In following the "Thinking Skills" approach and integrating it into all parts of the curriculum, would educators from Hoopoe School explicitly reflect it in the mathematics classroom? Likewise, would the constructivist approach be manifested in the classroom practices of educators at Weaver School?

As mentioned in the introduction to this research, beyond the considerations under review in this study, both schools are well-functioning, well-resourced and serve middle class students. These factors were considered significant because of the likelihood that their educators would be given more opportunity to work with problem solving, and to promote the development of mathematical proficiencies, than their contemporaries working in under-resourced schools where no strong culture of learning or teaching exists.

4.2.2 Selection of Participants

Selecting the educators for my study, was a three phase process (see Table 2).

First Phase

After obtaining Ethics clearance, and permission from the principals of both schools, I was able to complete informal observations of mathematics lessons in their Foundation Phases. Altogether I spent time with 12 educators. I observed a mathematics lesson of about an hour's duration in each class. I shared the purpose of my research with the educators but they were not specifically asked to plan a lesson for my benefit. My intention was to choose an educator from each grade.

[6] Taken from the school website.

Phases of Selection	Type of Selection	Number of Educators	Grade level	Lessons
First Phase	Informal observations to identify potential participants	12 Educators	Two or three from Grades 1, 2 and 3	One lesson of about an hour's duration with each educator.
Second Phase	Selection of participants	6 educators	Two each from Grades 1, 2 and 3	With each educator: Initial interview. 3 consecutive lessons of about an hour each. Reflective Interview. Focus group interview.
Third Phase	After data collection and detailed analysis of practices observed.	4 Educators	One from Grade 1, one from Grade 2 and two from Grade 3	Detailed explanation follows.

Table 2: Selection of Participants

During these informal observations I focused on the way in which the educators: worked with mathematical ideas and properties of number and operations; followed and evaluated students' explanations; explored different ways to solve a problem; anticipated difficulties students might be having; selected tasks; and connected mathematics to the environment (Delaney, 2010). While I did not include these informal observations as part of my analysis, they did influence my final purposive selection.

Phase 2

Then, reflecting on the mathematics observed, the interaction between the educators and the students, and the discourse in the classroom, I used these informal criteria to select a cohort of six, that included one Grade 1, 2 and 3 educator each from Hoopoe and Weaver schools.

Phase Three

Following the initial interviews, the classroom observations, reflective interviews, and the focus group interviews with the six educators, all the data was transcribed. I used my research question as the starting point to read through them and gain a sense of what mathematics and problem solving was evident. After tracking how each educator progressed and developed the mathematics topics in all of their lessons and developing a big picture overview that this data this provided, I decided to focus on only four of the educators (See Table 3).

These educators were purposively selected because they each offered something different in their approach that would enrich the quality of my data.

- **Teacher Taryn**: Of all the educators I found her lessons to be the most exciting from a mathematics perspective: the way in which her lessons unfolded; her understanding of the mathematical concept; and her use of discourse with her students.
- **Teacher Emma**: In the informal observation she gave her class a problem situation and using their mathematics skills of measuring, they had to complete the task. In the formal lesson observations she chose a skills-based approach. This I believe is a dilemma many teachers have: they vacillate between a classroom culture which facilitates learning where concepts and skills are constructed, to that of a structured lesson where competency is valued.
- **Teacher Sam**: The students in these lessons embraced the idea of discourse not only with each other but also with Teacher Sam. She was also explicit in connecting the mathematics to life-world activities.
- **Teacher Alice**: In my experience, her lessons reflect those of many educators. Her lessons are primarily skills based and the focus is on competency. Word problems are included, with solutions taking precedence over the thinking and reasoning process.

Schools	**Hoopoe School**	**Weaver School**

Grade	Grade 1	Grade 2	Grade 3	Grade 1	Grade 2	Grade 3
Teacher	Taryn	Emma	Sam	Sophie	Lucy	Alice
Final Selection	Yes	No	Yes	No	Yes	Yes

Table 3: Final Selection of Educators

4.3 Data Collection

4.3.1 Interviews

Initial Interviews

The initial interviews (see Appendix 1) with the educators took place first. They were semi-structured in nature and the aim was to explore, and gain an in-depth understanding of, their thinking on mathematical knowledge, the curriculum and their approach to teaching mathematics. Moreover, the interviews offered me an opportunity to probe deeper into the educators' perceptions, thoughts, beliefs and concerns (Creswell et al., 2007; McMillan & Schumacher, 2010; Merriam, 2002). I used a selection of probing, open-ended questions and, for transcribing purposes, these interviews were audio-taped. Where specific information was required, the questions were more structured but the rest acted as a guide to the issues to be explored. The exact wording, and order in which the questions were asked, was flexible.

To start the initial interview, I showed the educators three definitions of mathematics (see Table 4) and asked them which one best described their beliefs and attitudes. Then, I established their backgrounds in teaching: their teaching qualifications, experience and mathematical professional development. Lastly, I addressed their mathematical knowledge with reference to their mathematical identity, the curriculum and their mathematics practices. Here I asked the educators to expand on what they understood as the valid knowledge and the valid transmission of knowledge.

1. Mathematics is doing the work of mathematicians. It is about patterns: noticing them, describing them, and generalising them. Mathematicians are curious, they make connections, they love being challenged to find creative solutions to real life problems. They 'do' mathematics.

2. Mathematics is a toolbox that is filled with mathematical facts, skills and procedures. It is a hierarchical discipline, where educators teach students the relevant facts, skills and procedures. Students are required to master the mathematics, develop precision and get the correct answers.

3. Mathematics is about empowering students to think, communicate, reason and connect with everyday reality. It is about providing opportunities to explore problem situations, experiment with modelling, ask questions, make decisions, investigate, and discuss outcomes in a rich learning environment.

(These definitions are adapted from Boaler (2016), Schoenfeld (1985, 2013), Hugo (2014) and my literature review.)

Table 4: Definitions of Mathematics

Reflective Interviews

The reflective interviews (see Appendix 3) followed the lesson observations. They were also semi-structured, afforded the educators the chance to evaluate the lessons they taught, by reflecting on the learning that took place and their teaching practice. Using questions specific to each educator, I also asked them to reconsider the definition of mathematics they had chosen and took specific moments from the lessons to examine, clarify and further understand their teaching practices.

Focus Group Interviews

Finally, I conducted focus groups interviews (see Appendix 4). All three educators from each school were invited and the objective was to gain a broader understanding of mathematics in their schools and, more particularly, in the Foundation Phase. Altogether with the educators' understanding, experiences, reasoning and thinking, this provided a context for the culture of mathematics in the school. This more relaxed environment allowed the educators to influence, and be influenced by the others, hence adding to the quality and structure of the data collected (Creswell et al., 2007; Dilshad & Latif, 2013; Merriam, 2002). The focus group interviews were audio-taped and transcribed for analysis.

4.3.2 Lesson Observations

The aim of the lesson observations was to be a non-participant observer, gain first-hand experience of the educators' teaching practices and witness how the mathematics unfolded in the lesson. This was to facilitate a deeper understanding of the context and the behaviour of the individual educators and the interaction with their students.

Following the initial interviews, I observed three lessons over three days in the educator's classrooms. The educators were asked to teach consecutive lesson in which a mathematical concept could be developed. Each lesson was about an hour long. These lessons were video-taped by a third party. When possible I took informal notes but mostly I focused on the interaction between the educator and the students and the mathematics being in the lesson. I also took photographs of the students' work, worksheets and workcards used in the lessons (Creswell et al., 2007; McMillan & Schumacher, 2010; Merriam, 2002).

4.4 Analysis of Data

The interviews and lessons observations were all transcribed. Based on my conceptual framework, the general observations (see Appendix 2) I looked first at general pedagogic approaches (Bernstein's theory of pedagogic practice). From the mathematics perspective, I was looking for evidence of mathematics (Sfard's framework of mathematic processes), situated learning and what problem solving had taken place. It was then that I decided to focus on only four of the educators. As mentioned earlier, each offered something in their approach that would enable me to be selective in the quality of my data.

I developed tables to capture the integrity of the educators' practices. I identified characteristics, patterns and themes that, provided insights, possible motivations and developed clearer ideas as to the *what* and *how* of educators' practices. I completed tables which tracked the progression of the mathematics topic they developed in all of the lessons linking it to the development of mathematics concepts and the discourse between the educators and their students (See Appendices 5,6,7,8).

I referred back to the learning ethos of the respective schools and approach to mathematics, as well as the initial and reflective interviews, for links to understand the educators' experiences, relationships, and practices. By looking at the educators' similarities I was able to developed further categories which helped connect the pedagogies: personal experience of learning mathematics; their definition of mathematics; their use of CAPS; and textbooks. Although the focus was on the educator, it was important to capture all the nuances and authentically transcribe the teaching practice. Classroom analysis is not only about the educator and the students but is influenced by a myriad of visible and invisible factors. In the search for understanding, empirical observations can capture the superficial but not always the deeper contextual value of the moment (Schoenfeld, 2013a).

Within the mathematics category, I was specifically interested in the kind of mathematical knowledge being transmitted and acquired. In examining the tasks used by the educators in their lessons, I classified them according to how meaningful and purposeful they were, the cognitive demand they created, and whether they were skills-based or offered opportunities in which connections between concepts, procedures and contexts could be made. Here the quality of the mathematics and opportunities for reasoning were important.

When I explored the role of the educators, in particular, the 'how' of transmitting mathematical knowledge, most important was the use of mathematics as a discourse. This involved recognising how the educators: make use of mathematical vocabulary, visual mediators, narrative and routines in the discourse; develop mathematical concepts from horizontal to vertical; vary the kinds of questions asked and feedback given; interact with their students, to provided more detail to their practice.

Further examining of evidence of problem solving, I considered how the educators had problematised the situation; if the students had worked through a process of interpreting, mathematising and reasoning; and the opportunities given different approaches and accurate mathematical thinking.

4.5 Ethical Considerations

In collecting data for the study, ethics was a critical consideration. I was responsible for protecting the rights and welfare of the educators, the students in their class and the schools that participated in this study. The Principals gave me permission to conduct my research at their schools and ethics approval was granted (Protocol Number: 2018ECE038M) before any research was undertaken. Then I approached the educators I had chosen, invited them to take part and once I had been granted the necessary approvals to proceed, I introduced myself to the students and explained to them the purpose of the research. Although my focus was on the educators' practices, rather than on the students themselves, it was necessary for the educator and I to clarify this in the classroom. They were also given letters and asked to sign Assent Forms and, because they are minors, letters and Consent Forms were sent home for their parents to read and complete too.

In hindsight, the letters and Consent Forms were too detailed and many parents were tardy in returning them. A more user-friendly version would have made the process less tedious for the schools and the educators. Some parents wanted their students to be included but were overly concerned about the lessons being video-taped despite reassurances of confidentiality and anonymity which was maintained at all levels. To ensure that the parents' wishes were respected, teacher interns were asked to supervise those students in the classroom so they did not miss out on the lessons. These students were not video-taped.

4.6 Validity and Limitations

As with all qualitative research, the collection and analysis of data must have validity. With this in mind, I chose to use a multi-method strategy that included: semi-structured interviews, observations and artefacts. Questions for the interviews, and observation criteria, were guided by the conceptual framework and constructs extracted from the literature. By employing multiple methods, it was possible to gain a fuller understanding of the educators' interpretations, perspectives and perceptions. The evidence collected was intended to be credible and to effectively argue the position of the educators. The variety of different methods ensured that, when analysing the data, triangulation was possible.

Understanding that qualitative research in primary research cannot be neutral, objective or disconnected, I endeavoured to be rigorous in hearing the meanings behind what the educators said and did. I was also mindful to describe educators' practices and not evaluate them.

The key limiting factors in this research were its small size, the focus on the educators' orientation to mathematics and the complexities and challenges of conducting research in busy, active classrooms. Furthermore, it was restricted to well-resourced independent schools, where classes were small and relatively heterogenous. As a consequence, the schools and educators selected do not constitute a representative sample and, as such, findings from the research cannot be regarded either as conclusive or generally reflective of South African educators or schools (McMillan & Schumacher, 2010).

On reflection, had more time been available to conduct the research, it would have been preferable to observe the educators on more than one occasion and at different points during the school year. Being able to capture the dimensions of an educator's practice in a managed observation situation, working with such young students, is highly complex and three lessons is but a snapshot of the educators' teaching practice. So, although every detail of each lesson was examined meticulously, the small sample size does limit the quality of the research.

More depth could have been provided to the data by adding a control group that followed the CAPS curriculum, as long as it was a similarly well-functioning, well-resourced school in the same area, serving the same middle-class demographic.

Chapter 5: Descriptive Interpretative Analysis

5.1 Introduction

As this is an interpretive and descriptive account, I have endeavoured to understand the educators' contexts, experiences, relationships and practices, giving attention to the schools' learning ethos and mathematical approach. By using inductive reasoning, the categories constructed from the data are linked and informed by the literature and my conceptual framework, and the interpretation of the research is "richly descriptive" (Merriam, 2002). Looking beyond what the educators say and do, I have presented a detailed reflection of the context, the educator and students, and their social relationships by identifying meanings, intentions, strategies and motivations. My focus has not been evaluative, instead I have looked at the *what* and *how* of the mathematics, and what educators are doing in in their classrooms. (Atieno, 2009; Creswell et al., 2007; McMillan & Schumacher, 2010; Merriam, 2002; Ponterotto, 2006).

In presenting my descriptive analysis about teaching problem solving in the Foundation Phase, I started with a portrait of each educators background and teaching experience and I identified the similarities between them. Drawing on both the interviews and the lesson observations, I then describe the significance of each educator's teaching practices. As indicated in the previous chapter, I focused on four educators: Teacher Taryn, Teacher Lucy, Teacher Sam and Teacher Alice, because together their teaching practices offered something different and noteworthy. The descriptions of their practices are shown in sequence, from Grade 1 through to Grade 3.

Teacher Taryn has 12 years of teaching experience with eight of those years being in Grade 1. She has an Honours Degree in inclusion as well as her Junior Primary qualification. She has been at the school for 8 years and is presently a Grade 1 class teacher with 24 pupils in her class.

Teacher Lucy is in her second year at the school teaching, Grade 2. She has had varied experience throughout the Foundation Phase, having taught for more than 20 years. After obtaining her Senior Primary qualification at the Johannesburg College of Education, she studied further for a Diploma in Psycho-Neurological Learning Disabilities. Of all the classes video-taped, this class of 14 students were a little overwhelmed and reluctant to participate without encouragement from their teacher.

Teacher Sam has a B.Prim. Ed from Rhodes University. She has been teaching for 26 years and has experience in all the Foundation Phase classes. This is her third year of teaching Grade 3 at the school. There were 22 pupils in her class.

Teacher Alice is in her first year of Grade 3 at the school. She qualified as a Psychometrist with a B.Ed. Honours in Psychology after obtaining her Junior Prep qualification at Johannesburg College of Education. She has taught mostly in Grade 3 and has 20 years of experience. There were 19 students in this class.

Schools	**Hoopoe**		**Weaver**	
Grade	**Grade 1**	**Grade 3**	**Grade 2**	**Grade 3**
Teacher	Taryn	Sam	Lucy	Alice
Qualifications	B. Ed Honours Inclusion	B. Prim. Ed	HDE (SP) Diploma in Learning Disabilities	B. Ed Honours Psychometrist
Experience	12 years	26 years	20 years	20 years
Time spent at school	8 years	3 years	2 years	1 year
Class size	24	22	14	22

Table 5: Educators' Background in Teaching

5.1 Analysis of Descriptions

In promoting mathematical proficiency, educators are expected to help their students develop: a strong sense of number, meaningful problem-solving skills and the ability to discuss their ideas and approaches (Council & Committee, 2001). Understandably most Foundation Phase educators are not mathematics specialists, however it is the 'what' and 'how' of teaching and learning that influences the quality and mathematical honesty of educators' pedagogic practices. In my descriptions, I have started by first discussing the similarity between the educators' personal experiences in learning mathematics at school, their definition of mathematics and their use of CAPS and their chosen textbook. Following that, I give an interpretive and descriptive account of the lessons I observed.

5.2.1 Similarities between Educators

Personal Experience of Learning Mathematics

During the initial interviews, the four educators were asked about their own experiences of learning mathematics at school and of being an educator. Ironically, all had unfavourable experiences when learning mathematics at school and this had affected the way in which they approached their teaching of mathematics.

Teacher Taryn said the reason she taught was because she struggled at school.

> *I hated mathematics and I really struggled with it at school.*

She liked to *"help her students who are struggling to understand mathematics."* Instead of the *"chalk and talk"* methods used by her teachers, Teacher Taryn made sure she linked everything together.

> *So, if I teach one concept, I'll keep revisiting it or linking it across so when the children are doing problem solving, they have to [think and apply all] that they have learnt.*

Teacher Lucy clearly remembered how anxious she felt when asked her Times Tables. Being told she was *"not good at mathematics"* led her to believe that she was not. For her *"confidence"* and *"believing in one's self"* were essential for learning mathematics.

Consequently, her methods were now very different and by teaching it, she had learnt how to do mathematics.

> *We are not all going to learn it the same way and not all at the same time.*

Teacher Sam found that when she was learning mathematics, too much emphasis was placed on getting the answer correct and that *"there was only one way to do it whether you understood it or not."* She has embraced the thinking that:

> *If they can explain their method and their way of thinking and it makes sense to somebody else then they have achieved.*

Similarly, Teacher Alice also had a terrible experience learning mathematics and developed a *"mathematics block."* Knowing how hard it was for her has helped her in her teaching. Instead of standing at the blackboard and rote teaching, her methods are completely different.

> *My method has changed into hands on, interactive and engage.*
> *[P]hysically show them what you are talking about [so] they can visualise it.*

For all four educators, their experiences of learning mathematics at school had personally affected their sense of their own mathematics ability and had a direct effect on the way they each teach mathematics today. Acknowledging that their teaching practices were informed by the negative experience they had at school, they each recognised that there was more to mathematics than just computational skills and competence and that ideas of teaching mathematics by "*chalk and talk*", "*drill, drill and drill*" and "*rote*", were no longer part of their pedagogic practices. Instead they recognised the value of students being able to understand and implement the concepts.

Definition of Mathematics

In the initial interviews all four educators were asked to select one of three definitions which best reflected their position on teaching mathematics. They all chose the following:

Mathematics is about empowering students to think, communicate, reason and connect with everyday reality. It is about providing opportunities to explore problem situations, experiment with modelling, ask questions, make decisions, investigate, and discuss outcomes in a rich learning environment. (See Table 3)

This definition suggested that the educators were accepting of an inquiry-based method that was in keeping with the educational philosophies of both schools in the study. Moreover it implied that the educators had bought into problem solving as a major component of teaching and learning mathematics.

Teacher Taryn reasoned that:

We problem solve every day, all the time.

Teacher Lucy indicated it was about:

[E]mpower[ing] the students to think. I am not thinking for them. They are learning. They are discovering.

Teacher Sam believed that mathematic could not be *"a separate thing"* and had value in everyday life. She further explained that problem solving enabled a teacher to tell it in a story:

by telling your children a story it is more real for them.

Teacher Alice explained that for her:

[M]athematics is about everyday reality, problem solving, exploring problem situations [using] what makes sense to [the students].

In juxtaposition to those comments, later in the interviews, Teacher Alice and Teacher Lucy indicated that they found the teaching of problem solving challenging as the students' conceptual understanding was not well consolidated.

Curriculum and Assessment Policy Statement (CAPS)

Both schools consulted the CAPS document (DBE, 2011) as a guide to what mathematics they needed to teach in their Grade and in combination with the specific philosophy chosen by the school, recontextualised the mathematical content for use in their classroom. When asked about their use of CAPS, and its emphasis on the five strands of mathematical proficiency, none of the educators could specifically recall having read about it.

Teachers Taryn and Sam both indicated that their interpretation of CAPS was significantly influenced by being a 'Thinking School'. Teacher Taryn felt they *"would be doing [mathematical proficiency] anyway."*

Because the focus of CAPS is primarily on formal assessment which Teacher Lucy said she is *"not prepared to do"*, she noted the expectations of her Grade and *"adds in other stuff"* to create her own year plan. She also acknowledged that there were *". . . some good ideas, teaching ideas, not all of them though . . ."* in the document.

Teacher Alice used CAPS *"to do her year planner"* to *"meet the criteria by the end of the year."* As a curriculum, it was her view that CAPS is *"just a guideline"* and *"not advanced enough for [her grade]."*

The educators used CAPS as a guide to determine which criteria should be covered in their grade but, further than that they were not predisposed to its teaching guidelines or assessment policy. Worthy of note, was that not one of them had consulted the *Numeracy Handbook* (2012).

Textbooks

In addition to CAPS, the schools are guided by the textbook they choose to endorse. Problem solving is promoted in the curriculum, as well as in "Pr1me Mathematics" and "NumberSense", albeit in different ways. The "Prime Mathematics'" approach to problem solving was in the view of the educators to explicitly model examples using a combination of processes and strategies and think through to the solution using appropriate mathematical language. On the other hand "NumberSense" uses problem solving from the beginning to build concepts.

At Hoopoe School the Foundation Phase mathematics curriculum had been shaped in parallel with CAPS and "Pr1me Mathematics"[7]. In the focus group interview, the educators explained

[7] http://emea.scholastic.com/en/scholastic-prime-mathematics

that they followed the order and sequence of 'Pr1me Mathematics' and embraced the language and some methods (bar-model, whole-part-part, UPAC) to ensure continuity throughout the school. Rather than be bound to either, the educators had recontextualised the mathematics to ensure they taught *"quality"* not just *"quantity"*. Teacher Alice described it as a *"conscious decision"* to ensure the students know *"some things well"* rather than *"knowing a whole lots of stuff . . . not even half well."* Teacher Taryn said it was about *"doing it properly and making it relevant"* to her students. The students at Hoopoe School did not physically use the "Pr1me Mathematics" textbook but it guided and influenced the educators' practices.

In a similar manner, the Foundation Phase mathematics curriculum at Weaver School had been developed in tandem with CAPS and 'NumberSense". Neither dominated and the educators were free to teach the mathematics using a variety of different methods and approaches. They perceived the problem solving as word problems and not as the medium through which a concept is built and understood. Each educator has their own way of using the textbooks, placing the students on different levels, so it is used mostly for independent work.

Even though she had worked with the book for years, Teacher Alice felt that the different ways in which the textbook was being used meant that *"we have not come to grips with it."* She did not *"like the NumberSense book"* as she found it repetitive and *"not stimulating enough"* but admitted that her students were *"excited"* by it.

Teacher Lucy explained that she did not agree with the NumberSense method that *"every child be on the same page"* as *"you have some children that cannot be on that page"* and others *"who have to be three books ahead"*. Her view was that the textbook gave her *"that opportunity to work individually"* with her students and allowed her to *"see what areas they are struggling in"* but she did not see it as "*being her core focus*."

5.2.2 Teaching Practices

Using the initial and reflective interviews and the lesson observations, I have attempted to describe the essence of each educator's teaching practice. In every case I have looked at their mathematics beliefs and teaching practices; general pedagogic practices; the mathematics in the lesson; and evidence of problem solving. Table 6 gives a broad overview of the educators lessons.

Lesson Observations				
Educator	**Teacher Taryn**	**Teacher Lucy**	**Teacher Sam**	**Teacher Alice**
Lesson	Grouping	Addition	Fractions	Multiplication
General Pedagogic Practices				
Educators and Students Spaces	Flexible boundaries	Flexible boundaries	Flexible boundaries	Flexible boundaries
Communication Relations	Open between educator-student Opportunities for student-student	Open between educator-student Opportunities for student-student.	Open between educator-student Opportunities for student-student	Mixed between educator-student No opportunities for student-student
Pacing of Learning	Varied	Varied	Varied	Varied
Intra-disciplinary Relations	Strong – combination of every day and mathematics	Weak – closed boundaries around mathematics	Strong – combination of every day and mathematics	Weak – closed boundaries around mathematics
Cognitive Demand	High	Low	Medium	Low
Investigative Proficiency	Some opportunities	No opportunities	Some opportunities	No opportunities
Mathematics Process				
Vocabulary	Integrated use of words, connections made	Use of addition words	Integrated use of words, connections made	Use of multiplication words
Visual Mediators	Varied use. Apparatus, visual, written and verbal	Apparatus, visual, written and verbal	Varied use. Apparatus, visual, written and verbal	Apparatus, visual, written and verbal
Narratives	Students encouraged to use descriptions	Some descriptions used	Students encouraged to use descriptions	Some descriptions used
Routines	Developing through the lessons	Developing through the lessons	Developing through the lessons	Some narratives developing.
Discourse	Varied	Stilted	Students keen to offer ideas,	Educator controlled,

	Some students eager, others reticent	Encouraged by educator	thoughts and opinions.	helped students when needed
		Situated Learning		
Culture	Visible mathematics in the classroom. Ideas integrated	Visible mathematics in the classroom.	Visible mathematics in the classroom. Ideas link to other situations	Visible mathematics in the classroom
Participation	Engaged interaction between educator and class - pairs and mathematics	Interaction between educator and class, pairs	Engaged interaction between educator and class - pairs, groups	Whole group interaction, some sharing but no real discussions
Authentic Activities	Productive, meaningful Connections made from between concepts, procedures and context	Useful Connections made between concepts and procedures	Meaningful Connections made from between concepts and context	Productive Connections made from between concepts and procedures
		Problem Solving		
	Use of life-world contexts Problem situations Possible solutions Word problems	Incidental moment	Use of life-world contexts Problem situations Possible solutions	Incidental moment Word problems

Table 6: Overview of Educators' Lessons

Teacher Taryn

Teacher Taryn saw the building of number concepts as important to establish a good foundation. She valued conceptual understanding and emphasised the importance of teaching a concept in a variety of different ways, then revisiting it and connecting it to other areas in the curriculum like language and handwriting. In order to understand the basics she believed that the students should do lots of practical work to build on the concepts they have already learnt. She also indicated that the use of the correct language, which is part of the "Pr1me Mathematics" approach, had changed her teaching practice. When questioned about problem solving, Teacher Taryn, said she believed that:

> *Every day we problem solve all the time. So I think it is good for them to think out of the box and to understand why we have problems and why we have to solve them. I like doing problem solving.*

In Grade 1, Teacher Taryn taught three lessons on grouping and sharing leading to division (see Appendix 5). Each lesson was well-structured so that the activities built on one another. The lessons started with revision of halving and then, Teacher Taryn introduced the idea of sharing with more groups. On each occasion, there was lots of practical work to start with and the students moved on to more formalised recording of the concept. In the third lesson the students were asked to solve problems; first using drawings and then recording the solutions.

General Pedagogic Practices

During the lessons, Teacher Taryn consciously connected the mathematics to spelling and reading of the words 'half' and 'halving'. She also referred to everyday knowledge and contexts familiar to the Grade 1s when discussing the concepts. The boundaries around the everyday and specialised knowledge of mathematics being taught were a combination of integrated and separate depending on the context.

The topic of mathematics was taken from the curriculum and recontextualised by Teacher Taryn, in collaboration with her colleagues, through a "Pr1me Mathematics" lens. The sequencing of the mathematics in the lesson was organised and explicitly controlled by Teacher Taryn with each new concept built on prior knowledge. As the class became more proficient with the mathematics, she withdrew the practical apparatus and introduced more abstract methods.

Although there was a hierarchical relationship between Teacher Taryn and her students, there was evidence of a relaxed and open dynamic which allowed her students to engage in discussions and to feel their opinions were valued. The lessons took place in the front of the classroom where the students sat in a circle with their teacher. When doing paired work on the carpet or book work at their desks, Teacher Taryn moved around checking on how each student was doing and offering help where necessary. She created flexible boundaries both in her relationships with her students and in how the space in the classroom was utilised.

Mathematics – Grouping and sharing, leading to division.

Teacher Taryn structured the tasks carefully, introducing her students to grouping and sharing through halving. Each activity was meaningful and purposeful. She started with a life-world context (sharing a choc-chip cookie between two people) and linked it to the mathematics concept of halving (two equal pieces). Not only did they group and share numbers, they folded paper shapes in half and drew lines to halve their white boards.

The students had all been folding the shapes in different ways:

> ***Teacher Taryn:*** *Who has a circle?*
> ***Child 2:*** *(Child holds up circle.) You can fold it in half like that, and then in half like that.*
> ***Teacher Taryn:*** *Good.*
> ***Child 2:*** *I also did it like this and then like this.*
> ***Teacher Taryn****: Look here. (She holds a circle in one hand, biscuit in the other.)*
>
> ***Child 1:*** *I did it like this.*
> ***Teacher Taryn:*** *Okay, is that half?*
> ***Child 1:*** *Quarters.*
> ***Teacher Taryn****: Yes, that is quarters.*
>
> ***Teacher Taryn :*** *Who has a star? Show me your star.*
> *(Child shows star folded in half, another child shows star.)*
> ***Teacher Taryn:*** *Can you fold it in half any other way?*
> ***Child 6:*** *No.*
> ***Teacher Taryn:*** *Why?*
> ***Child 6:*** *You could fold it like this.*
> ***Teacher Taryn:*** *But why can't you?*
> ***Child 6:*** *Because it has five pieces.*

In the above examples Teacher Taryn provided opportunities for the students to make different connections: some shapes you could halve in more than one way, others you could not, and with the star, the halving of an odd number was introduced.

Throughout the three lessons, Teacher Taryn ensured that each activity built on the one before, creating cognitive demand. The number range from 2 to 26 enabled the students to work with easier number through to more difficult ones. Likewise, when sharing, the groups ranged from 2 to 6. The grouping and sharing were situated in word problems, providing the transition from everyday to specialised knowledge.

Teacher Taryn consciously mathematised different situations. She asked what shape the cookie was and what the names of lines were which divided the white boards in half. When the students offered answers, she mathematised their answers to further their understanding. Here the students were sharing 9 beans between 3 students:

> ***Child 1:*** *Teacher Taryn, I knew the answer all along. I just counted in 3s.*
> ***Teacher Taryn:*** *Good. So you know how to count in 3s. 3, 6, 9. So what is your answer?*
> ***Child 1:*** *9.*
> ***Teacher Taryn:*** *Oooh. That is your whole. What is your answer?*
> ***Child 1:*** *Your answer is 3.*

In the above example, Teacher Taryn made the connection between counting in groups and sharing. By asking for the answer, she was checking whether the child had understood the difference between 'whole' and 'part'.

Here Teacher Taryn had asked the students to move from using beans to drawing instead. They are sharing 16 sweets between 2 boys.

> ***Teacher Taryn:*** *How would you do that example on your board? Have you figured it out?*
> ***Child:*** *You could draw circles.*
> ***Teacher Taryn:*** *You could draw circles. Okay. Let's hear you example.*
> *(Child takes worksheet and places one dot in each circle counting up to 16.)*
> ***Teacher Taryn:*** *Look what you did? Instead of taking the counters and going one for you and one for you . . . all the way up to 16. You did dots.*
> ***Child:*** *That is what I did.*
> ***Teacher Taryn:*** *Okay tell me what you did.*
> ***Child:*** *I did the same.*
> ***Teacher Taryn:*** *Okay so tell me how you did it even if it is the same.*
> ***Child:*** *I started at 16 and I drew 2 circles and then I drew 1, 2, 3, till 16. And then I counted how many I had.*
> ***Teacher Taryn*** *Good.*

Child: *Teacher Taryn, you know what I did? Because it is 10, I put 5 in each and then 3 because you are counting in 3s and 10s. (Child shakes head.) 3s and 5s. So you out 5 there and 5 there and 3 there and 3 there.*
Teacher 1A: *So you did a whole-part-part with your 16. You halved your 10 into 5, 5 and you 6 into 3, 3. Nice!*

In this example, Teacher Taryn, invited the students to engage in the discourse by giving explanations and mathematising their thinking. In using a mixture of general and specific questions, she was able to ensure that the responses from her students ranged from answers, to explanations, to strategies and different ideas.

Teacher Taryn used the correct vocabulary throughout, reminding the students that the key concepts of halving and grouping was dividing the whole into equal groups. In using Unifix blocks, then beans, then their own drawings, the students were exposed to different visual mediators, which helped to reinforce the mathematising needed. Teacher Taryn modelled narratives with her students to develop routines. At different times, when examples were completed, she would ask the students:

Half of ______ is ______.
What is your whole? And how many are you sharing between? So what is your answer?
*Share 9 rocks with 3 friends. How many rocks will **each** friend get?*

The feedback Teacher Taryn gave to her students indicated that she had listened to their ideas and on occasion used the opportunity to build on it.

Teacher Taryn: *So, let's do the next one. It starts with a 12.*
Students: *12 suckers, 2 friends. How many suckers will **each** friend get?*
Child 3: *6.*
Teacher Taryn: *How do you know that?*
Child 3: *Because I know what 12 + 6 is.*
Teacher Taryn: *6 + 6 is*
Child 3: *12.*
Teacher Taryn: *And half of 12 is*
Child 3: *6.*
Teacher Taryn: *Good.*

In this case the child was not clear about her thinking and Teacher Taryn assisted her in refining and correcting it.

Problem Solving

Throughout the lessons, Teacher Taryn used opportunities to problematise situations. She started off the topic of halving and sharing with everyday problem situations, such as sharing a choc-chip cookie between two people, and sharing 9 choc-chip cookies between Bean, Bird and Mouse. Ideas were discussed, a solution was found, the mathematics recognised and the concept discussed. She guided the mathematising of the concepts asking for reasons where appropriate.

> ***Teacher Taryn:*** *But why can't you?*
> ***Child 6:*** *Because it has five pieces*
> ***Teacher Taryn*** *Why can't you fold something that has 5 points?*
> ***Child 6:*** *Because it is uneven. Because it is an unequal number.*
> ***Teacher Taryn:*** *Ok, we are going to get to that as well. Keep that in your brain.*

Teacher Taryn probed the reasons for this child's answer, allowing time for thinking and for them to clarify their thinking.

Two students worked together, sharing 16 sweets between 2 boys. Teacher Taryn did not correct the students but instead provided the opportunity for them to re-evaluate their answers, check their method and get the correct answer.

> ***Teacher Taryn:*** *Oh dear, that is not fair? She has 7 and she's got . ?*
> ***Child 5:*** *8. Take one away. (Child takes one bean away so it is 7 and 7 in each group.)*
> ***Teacher Taryn:*** *That 7 + 7 doesn't make 16. Start again with your whole.*

On reflection, Teacher Taryn was satisfied with the lessons. She reiterated that she liked to develop new concepts slowly, with lots of examples so her students understand the language. She was satisfied that they now knew different ways of doing things and the progression of it and could now see the difference between plus, minus and grouping and sharing.

> *We thought about it, we discussed it, we did it in different ways. We looked at the progression of it. We moved from the concrete to the abstract.*

I discuss Teacher Taryn further in Chapter 6.

Teacher Lucy

As a Grade 2 educator, Teacher Lucy spoke about the importance of empowering students to think and to learn and discover things for themselves. She described her pedagogic practice as:

> *I am not setting down and saying - this is how we do it; it is how we could do it. How could you do it? Can you show us your way? The kids are actually learning from each other. I am not teaching, I am facilitating.*

She valued mathematical concepts and understanding of what it is that is being done, above methods, and explained that students learn things differently and not all at the same time. She believed that concrete apparatus was essential in developing conceptual understanding and encouraged her students to visualise ideas in their heads.

Teacher Lucy admitted that problem solving was a challenge to teach and therefore she did not spend enough time on it. She felt her young students did not understand it and were unable to apply even basic concepts. However, she went on to say that when her students did problem solving from their textbooks, they figured it out together, drawing, or using concrete apparatus to develop understanding. She distinguished between word problems and problem solving, saying that she focused a lot on problem solving but not necessarily in mathematics.

> *We figure things out. They figure it all by themselves. It is the conscious thing and I do it all the time.*

Teacher Lucy introduced her Grade 2s to vertical addition using decomposing (see Appendix 6). Each lesson started with mental mathematics practice, in the form of games working in groups or as a class. Teacher Lucy had her students decomposing numbers using Base 10 blocks and a place value chart, and then adding them together. To begin with they merely had to write the answers. In the second lesson, Teacher Lucy wrote the numbers on the board and introduced them to the format of vertical addition. Then, in the third lesson they were able to write the sum on their white boards and use the Base 10 block to work it out, if they needed to. The sums given to the students involved bonds up to ten with no re-grouping required.

General Pedagogic Practices

In teaching vertical addition with decomposing (see Appendix 6), Teacher Lucy kept the focus on the mathematical knowledge. Her intention was to introduce vertical addition in a mathematical situation and ensure her students were proficient in the method before situating it and applying it to relevant contexts. She kept the boundaries around the specialised knowledge of mathematics fixed, concentrating on developing the skill.

The selection of mathematics was taken from the curriculum and recontextualised by Teacher Lucy. Although she said the selection of mathematics for the lesson came from the CAPS document, there was no specific reference to vertical addition. The sequencing and logic of the mathematics developed in the lessons was not always explicit and although the lesson was structured to build on from the prior knowledge of decomposing, the class did not recognise or realise the connection at first. Once she had reminded them that they had done decomposing before, there was more clarity. The students all used the Base 10 apparatus to begin with but as they became more competent, she introduced the written method. Timing was flexible dependent on the activity.

There was a hierarchical relationship between Teacher Lucy and her students. She was visibly instructional with the students, who followed the procedures. She sat with the students in the front of the class and checked on what they were doing, offering feedback and assistance where necessary. When the students were seated at their desks, she again moved around ensuring that each student was involved in their task. At times she relaxed the boundaries in her relationship with her students while keeping explicit control of order in the classroom.

Mathematics – Vertical Addition

Teacher Lucy chose a closed, routine skill to teach her class. Everyone did the same tasks, following the procedures she set out. No connections were made to everyday contexts, nor to the purpose or value of the skill. Neither were opportunities provided to make connections between concepts, procedures and contexts. The numbers in the sums were straight forward and the students simply had to apply their bonds of 10 knowledge. Except for introducing a subtraction sum at the end of the third lesson the cognitive demand was routine.

The tasks for the mental mathematics offered opportunities for the students to mathematise their thinking. The students were multiplying the numbers on two dice together: a six and a four.

> ***Teacher Lucy:*** *Can you tell us how you worked that out?*
> ***Child Lucy:*** *I counted in 6. I did 6 then 12. Then I counted from 12, 13, 14, 15, 16, 17, 18. (Child demonstrates fingers.) At 18 I counted 6 more. And it was 24.*
> ***Teacher Lucy:*** *Okay. Did someone else do it differently? What did you do lovey?*
> ***Child 8:*** *I flipped it around. I did 4 times 6.*
> ***Teacher Lucy:*** *4 times 6. So, 4 groups of 6, so then what did you do?*
> ***Child 8:*** *I counted in 4s 6 times.*

In the above example, two different methods were explained. Teacher Lucy mathematised the method. However, Child 8 could not differentiate between "4 times 6, 4 groups of 6 and counting in 4s six times".

Here the students were multiplying the numbers on two dice together: a three and a five. Child 9 chose not to draw circles on his white board but rather use his fingers. Neither he, nor Teacher Lucy mathematised the situation.

> ***Teacher Lucy:*** *What you do there? You could count in what?*
> ***Child 9:*** *I had 5 circles (uses fingers) and then I counted 1,2,3 (on each finger) until 15.*
> ***Teacher Lucy:*** *But you didn't draw the circles so where did you see them?*
> ***Child 9:*** *My hands were the circles.*
> ***Teacher Lucy:*** *So, your fingers were the groups. Fabulous.*

Because the lessons were skills based, everyone was doing the same sums and procedures and there were no opportunities to share thinking. Two students did use different methods and Teacher Lucy took time to listen to the strategies they used, but these were not shared with the class. Those who took time to understand the procedure were supported and assisted by Teacher Lucy. She identified their mistakes and worked with them to solve them correctly. For those students who persevered, their efforts were recognised, as she encouraged them to trust themselves. Lots of praise was given to the students as Teacher Lucy checked their work.

Teacher Lucy's approach to the discourse in her class was authoritative but interactive as her class was reticent in their responses. Opportunities for student-class talk were offered and with reassurance from the teacher, student interaction did improve. Mathematical vocabulary was used, especially in connection with the understanding of addition. However, its clarity in connection with the mathematics was not always explicit. Some of the Grade 2s did not immediately recognise that Base 10 blocks and place value chart as the objects of learning. They were given ten green sticks (10 tens) and 15 yellow cubes (15 ones) and first asked to place the 15 'ones' on the place value chart and then make 15 in another way. Many of the students rearranged the 15 cubes into different patterns.

Towards the end of lesson one, Teacher Lucy, developed a narrative for the skill of vertical addition.

> *We have started with a 23 and we have started with a 16 . . . you are going to put them together, so you are going to add them. . . You start with the ones. Then you are going to write the answer for me on your white board.*

Teacher Lucy's questions were mostly specific, requiring recall, rules and procedural answers from the students. Some of the questions tended to be leading which encouraged the students to respond and she did ask some students for explanations and strategies. Her feedback was evaluative and she praised her students when their work was correct. Classroom talk was on a mainly question-response-feedback approach as illustrated in these episodes:

> **Teacher Lucy:** Is this a 9? *(Teacher Lucy points to the 9 in 97.)*
> ***Students:*** *No. No.*
> ***Teacher Lucy:*** *What is it?*
> ***Child:*** *90.*
> ***Teacher Lucy:*** *Why is it a 90?*
> ***Child:*** *Because you hear the 90 in 97.*
> ***Teacher Lucy:*** *Very good. Look at the column the 9 is in.*
> ***Child:*** *It is in the 10s.*

In this episode the class have added 32 and 44 together. Teacher Lucy has written the sum on the board. She probes a bit more, getting the students to reason out an answer.

> ***Teacher Lucy:*** *You added your ones together and you had . . .*
> ***Students:*** *6*
> ***Teacher Lucy:*** *6 of them. (Writes 6 on board under the ones.) Then you added your tens together and you had. . .*
> ***Students:*** *7*
> ***Teacher Lucy:*** *7. (Writes 7 on the board underneath the tens.) So, this is 7 and 6?*
> ***Students:*** *76.*
> ***Teacher Lucy:*** *Why is it 76? Why is this not a 7?*
> ***Child 1:*** *Because there are two numbers together.*
> ***Teacher Lucy:*** *What are the two numbers?*

Child 1: *7 and 6*
Teacher Lucy: *Is it 7 and 6?*
Child 1: *70. 6.*
Teacher Lucy: *70 and 6.*

Problem Solving

Teacher Lucy consciously chose to do vertical addition from an abstract number approach only and then she intended to introduce the concept in an everyday context to her students. So she made no provision for problem solving to be included in her lesson. Nonetheless, during the first lesson, an incidental problem situation arose. Using the Base 10 blocks of which each student had 10 tens and 15 ones, Teacher Lucy asked her class to find another way of making 15.

Teacher Lucy: *Right, is there another way you can make the number 15 for me?*
That is the same way. You still have 15 ones. Use any of your base 10 blocks and give me 15 in another way. Let me see.
(Child 5 puts 1 ten and 5 ones.)
Teacher Lucy*: Trust yourselves. Don't look at someone else's and copy them you may be right and they may be wrong. As long as you can explain to me. Okay you still have got 15 blocks even though you've changed the pattern?*
Child 2*: (Nods.) Ja.*
Teacher Lucy: *Have you got 15 ones even though you have changed the pattern?*
Child 9: *(Nods.) Yes.*
Teacher Lucy: *Have you got 15 ones even though you've changed the pattern?*
Child 8: *Yes.*
Teacher Lucy: *I have asked you to use another way not another pattern of showing me 15. All you have done is rearranged your 15 ones.*
Using any of your Base 10 blocks show me another way of making 15?
(Child 4 spread them out further).
Okay, you have rearranged them again. We've still got 5, 5 and 5 ones.
Can you explain to us what you have done? (Child 7)
Child 7: *I have used one of these (Child 7 holds a green block up) and 5.*
Teacher Lucy: *Why did you use one of those green ones?*
Child 7: *The greens ones are 10 and five of these yellow ones are 5 and I added them together to make 15.*
Teacher Lucy: *Very good. Can you explain why you (Child 5) have done it that way? Why did you use a green one?*
Child 5: *Because 10 plus 5 makes 15.*

The students found a variety of ways to represent 15 most of them mathematically correct and they could have been mathematised (5 x 3 = 15). What was unclear to them, was that Teacher Lucy was looking for them to decompose the number (10 + 5 = 15).

Reflecting on her lessons afterwards, Teacher Lucy was happy with the outcome. She explained that she had started with the basics and knew it had been successful because except for a few expected ones, her class could do vertical addition and explain what they were doing. She confirmed that it was a conscious choice not to integrate the everyday and mathematically contexts.

> *I did the abstract first. I was focused on using Base 10 blocks. I wanted them to use the Base 10 blocks to have the visual in front of them. And I don't want them to now try and visualize that the 10s or the 1s were also a cake. It is too much for this age. I wanted it to be concrete, concrete. But I specifically want them to see that so that they can have a concept with them and have a picture because maths is a pictures.*

Teacher Sam

Teacher Sam described a broad view of mathematics. She saw it as being part of everyday life and something that is useful and purposeful. She believed that seeing patterns and relationships, thinking skills and solving problems are all part of mathematics. She described her lessons as being hands on:

> *It is about physically doing it. Working in pairs, applying the skills, really getting into the discussion and working together to solve problem.*

When questioned further about problem solving she said:

> *Problem solving is great because you can tell it in a story and by telling young students a story is more real for them. Story sums and word problems are really great because you can make it real for the group.*

Teacher Sam valued taking time to consolidate each concept before moving on to a new one so that her students could build on what they already know.

In Grade 3, Teacher Sam introduced fractions: the notation and name of unitary and non-unitary fractions (see Appendix 7). The three lessons were structured and the activities built on each other referring back to their prior knowledge of "bar model" and "whole-part" and sharing. Teacher Sam used lots of practical apparatus making the mathematics very visual for her class.

General Pedagogy Practices

When planning her lessons, Teacher Sam consciously chose meaningful contexts to which the Grade 3s could relate. She referenced fractions in different contexts such as quantities in recipes, groups of sweets, pizza slices, a netball court, and a clock. Throughout the lessons she made connections between the context and the mathematics of fractions. Only the worksheets completed by the students were focused purely on fractions. Teacher Sam was successfully able to integrate the boundaries between the everyday and specialised knowledge of mathematics and other curriculum areas and then combine the fraction knowledge into her lessons.

The selection of mathematics was taken from the curriculum and recontextualised by Teacher Sam, in collaboration with her colleagues, through a "Pr1me Mathematics" lens. The sequencing of the activities in the lesson was organised and explicitly controlled by Teacher Sam. Each activity built on the concept before moving from concrete to the more abstract. Timing was flexible and varied according to the needs of the students.

The relationship between Teacher Sam and her class was hierarchical. Her approach to the discourse was interactive, moving between authoritative and dialogical. The boundaries were flexible and her students readily contributed to the discourse and offered their opinions. The lessons took place in the front of the classroom with Teacher Sam sitting in a circle with her students. She noted what the students were writing on their white boards, asking some to share or commenting on the ideas. While doing group work with some of her students, Teacher Sam encouraged each child to contribute. Regulative rules where implicitly controlled, providing for the space in the classroom to vary depending on use and allowing her to open and close the relationship boundaries when necessary.

Mathematics - Fractions: notation and name of unitary and non-unitary fractions.

Each task that Teacher Sam introduced to the Grade 3s had been carefully thought out and planned ensuring they were meaningful and purposeful. By moving between unitary and non-unitary fractions (fractions of a whole and fractions of groups), the students had to recognise and realise the different rules which applied to the notation, comparison and identity of different fractions, and she created cognitive demand to engage the students.

In this episode Teacher Sam chose six students to stand in front:

> ***Teacher Sam:*** *How many do not have a blazer on?*
> ***Child 1:*** *4.*
> ***Teacher Sam:*** *How would I represent that now?*
> ***Children:*** *(No response.)*
> ***Teacher Sam:*** *So earlier on you told me that it has a part. So there is a part of the group that is not wearing blazers. Right? How many?*
> ***Children:*** *4. 4. 4.*
> ***Teacher Sam:*** *4. But the whole group is 6. Do I want to write it like this 4/6 or 6/4?*
> ***Child 14:*** *The first one. I mean the second one. 6 out of 4.*
> ***Teacher Sam:*** *6 out of 4. So 6 out of 4 are not wearing blazers.*
> ***Child 14:*** *Yes. There are 6 altogether and 4 are wearing blazers.*
> ***Teacher Sam:*** *But you said 6 are not wearing blazers.*
> ***Child 6:*** *4 out of 6 are not wearing blazers.*
> ***Teacher Sam:*** *What could we say about those wearing blazers? How could we represent that?*
> ***Child 6:*** *2 out of 6 are wearing blazers.*
> ***Teacher Sam:*** *2 out of 6 are wearing blazers. How could we represent those with their hair not tied up?*
> ***Child ?:*** *1 out of 6.*
> ***Teacher Sam:*** *Tied up?*
> ***Child 7:*** *5 out of 6 do have their hair tied up.*
> ***Teacher Sam:*** *How could we represent those wearing earrings?*
> ***Child 11:*** *2 out of 6*

The situation was mathematised and she provided opportunities for the students to connect concepts (whole-part) and procedures (notation) in the different contexts.

The correct mathematical vocabulary was used by both Teacher Sam and her students. By using a variety of visual mediators, such as jelly beans, pizza slices, fraction pictures and written fractions, she was able to mathematise the contexts. The students were encouraged to “recall and apply past knowledge” (division, bar-models, whole-part) when interpreting and applying the new concepts and developing narratives. The combination of the everyday context and the symbolic notation enabled the students to shift from the horizontal to the vertical building understanding of the concepts.

In this episode the class had counted out a packet of jelly beans and were deciding how they could express that in a packet of 18 jelly beans there is only one blue jelly bean.

> ***Child 8:*** *Everyone is writing 18/1 or 1/18 but it is actually 17 and 1.*
> ***Child ?:*** *I don’t think it is.*
> ***Child 8:*** *Because we started with 18. And took away 1. So now it is 17.*
> ***Teacher Sam*** *Aaah. So let’s listen to what my question could be. And that might help you to determine the strategy you want to use to write this representation. Listen to the question*

with meaning and understanding. My question is . . . I want to know how many of my favourite colour blue jelly beans there are out of the whole packet.
Child ?: *So you put 1 over 18.*

Teacher Sam did not agree or disagree with either student. Instead she used the interaction to create a learning opportunity. By focusing their attention on the question they could think again and make more sense of the situation.

All three lessons provided moments for great discourse. Despite the fact that Teacher Sam tended to control the discourse, her class was comfortable in engaging in discussions, offering their opinions, suggesting ideas, asking questions and verbalising their thinking. She encouraged participation from all and included those who were not always as forthcoming. Her questions varied from general to specific. Most were recall, procedural and leading but she did include a few probing ones.

Written on the board was ½ > ¼ and Teacher Sam asked the class what it said:

Child 9: *Half is greater than a quarter.*
Teacher Sam: *But 4 the number 4 is bigger than the number 2.*
Child 9: *But it is 1 of 4 pieces.*
Teacher Sam: *Can anybody explain a bit more? I am confused because 4 is bigger than 2.*
Child 14: *Umm 4 means just means quarters because 4 means 4 pieces but it is smaller. And half is if you cut a pizza in half there are going to be two pieces. And if you cut a pizza into quarter there are going to be 4 pieces.*
Teacher Sam: *Okay. Tell us a bit more.*
Child 2: *Because the one ½ tells you, you have 1 whole and you have to share it between two people. But if you have 1 whole and you share it between 4 people you are going to have to make the pieces smaller because everyone has to have an even amount.*
Teacher Sam: *Right.*
Child 8: *Umm. Because it has a 1 on top which means it was a whole piece then you went and cut it into 2 pieces which is two halves. But then more people wanted pizza so you had to cut it into another half which made the quarters and there'd be 4 quarters. So that is why it is bigger.*

Responses from her students went beyond just answers, included explanations and some thinking and reasoning. Teacher Sam listened carefully to her students' responses implicitly getting them to discuss and agree on answers and think more about their explanations.

The discourse in class allowed for two different conflicting ideas to be analysed and a new understanding created. One child explained that in a fraction like ½ and ¼:

Child 8: *1 represents that it has one group. But you split it in to 2 or you split it into 4. So the second number is how many groups you split it into?*

After further discussion using different examples, the child was heard to say:

> ***Child 8:*** *So it would depend on the story on how you say it and how you would write it down?*

On another occasion, a student became confused when comparing two fractions:

> ***Child 12:*** *Umm. 4/20 is equal to 1/5.*

Despite Teacher Sam pointing out the size of 1/5 and 1/20 and the student agreeing that 1/5 was bigger, she still said 4/20 was equal to 1/5. When another student entered the discourse, the misunderstanding was cleared up.

> ***Child 11:*** *I think what she is saying is umm. . . 5 + 5 + 5 + 5 is 20*
> ***Child 12:*** *Oh.*
> ***Child 11:*** *If you were doing a plus sum.*
> ***Teacher Sam:*** *That is going back to our repeated addition and multiplication. But if we are comparing the size of the fraction, if I joined all of these together in a bar model and one of those underneath. (Teacher indicates 4/5 and 1/20.) Are 4/5 equal to 1/20?*
> ***Child 12:*** *No*

Problem Solving

Using moments during the lessons, Teacher Sam did create problem situations like this one:

> *I am confused. This doesn't make sense to me. These are parts (she holds two halves of pizza in her hand and points to the whole pizza) and that is a whole. So if you ate two parts out of the 2, you are telling me that you ate the whole pizza. Can anyone explain?*

Once the correct explanation was given the problem-solving process ended.

At the end of third lesson the class was given a group activity to complete. The students were put into groups and given a task in which they had to make either a poster, a game, a booklet, a song or a story to teach the class a certain fraction concepts. The process involved understanding what was required of them, collaborating and agreeing on an idea, using the material given to them, and then producing an that which would be used as a teaching aid.

When looking back on her lessons Teacher Sam, said the groups were right on target with their final tasks. In presenting, she could see that they understood the concept. Some students still needed help with comparing of fractions and she had to go back and reteach that concept. They had now moved on to time and could connect their fraction knowledge with: 'o'clock', 'quarter past', 'half past', and 'quarter to'.

Teacher Sam was conscious that:

for every lesson that I plan and every lesson I evaluate, I keep reminding myself what is the task, what is the purpose and what am I trying to achieve? And I often say to them [the students] why do you think we are learning this, why do we learn fractions and time?

Teacher Alice

Teacher Alice emphasised that a sound knowledge of number was essential; explaining that being able to manipulate and make sense of number concepts was important. When planning her lessons she considered it important that there were opportunities for:

the students to problem solve, critically analyse and come up with their own reasoning as to why they've or how they've got a certain answer.

Teacher Alice also made mention of the importance of counting skills, manipulation of skills and allowing the students to be interactive and collaborate with each other.

The most challenging part of the curriculum for Teacher Alice to teach was problem solving because students do not know how to apply the different operations. She suggested that it should rather be done incidentally and related to the students so they would have a better understanding. When asked to define problem solving she stated that:

I think we use it in everything. It is how to critically analyse, be able to come up with a solution. So it being able to critically analyse, find an answer and all steps involved in that process.

These Grade 3 lessons were about multiplication: its meaning and calculation strategies (see Appendix 8). Each lesson started with mental mathematical practice, either counting or games. The class revisited the work they had done on multiplication already, including vocabulary, groups of, repeated addition and an array. Under the direction of Teacher Alice, the students then practised a method of multiplication on their white boards, using flard cards to decompose the numbers before multiplying. After more practice during the third lesson on their white boards, the students completed examples in their books.

General Pedagogic Practices

To start the lessons, Teacher Alice emphasised the importance of the mathematical vocabulary associated with multiplication. Even though the students then had a chance to integrate this knowledge by completing word problems, she initially kept it separate. When she moved on to the calculation method, the skill dominated. The boundaries around the mathematics were fairly strong and separate from everyday contexts and other curriculum areas.

The selection of mathematics was taken from the curriculum and recontextualised by Teacher Alice. The sequencing of the mathematics in the lesson was predominately skills-based and was organised and controlled by her. Word problems were included briefly but the emphasis was on working through the calculation method and gaining competence. Timing was flexible and varied according to the activities.

The relationship between Teacher Alice and her students was hierarchical. Much of the lesson took place in the front of the classroom where Teacher Alice sat with her students. Bookwork was completed at the students' desks which were arranged in groups of three or four. She moved around the classroom and worked in groups with some of her students. Using a visible instructional style, there was an explicit regulative order which was recognised and realised. She controlled the boundaries around the relationship with her students and in how the space in the classroom was utilised, opening and closing them when necessary.

Mathematics – Multiplication Method

In teaching multiplication (see Appendix 8), Teacher Alice was mainly focused on the skill, so the activities were closed and routine. The students all did the same tasks and followed procedures they had already learnt. The focus of the lessons was therefore to recall the procedures and rules and competently complete the method. Connections were made to prior knowledge (words associated with multiplication), other mathematical concepts (repetitive addition and multiplication) and the students completed some word problems. Opportunities to make broader connections between concepts, procedures and contexts were not explicit. The sums given to the class focused on the application of tables and did not allow for greater cognitive demand.

Teacher Alice controlled the discourse and classroom talk tended to be question-response-feedback interaction. Her questions were mainly specific, requiring recall, rules and procedural answers from her students. When doing work on their white boards as a class, Teacher Alice took time to include everyone, asking different students to answer questions. Often the questions were leading and only on some occasions were explanations expected.

In this episode, she took time to probe further and explore a mistake with a student. The class were answering this question: Nine bicycles, how many wheels? Child 9 was working on his white board.

> ***Teacher Alice****: Why are you timesing by 4?*
> ***Child 9:*** *I did 4 - 9 times.*
> ***Teacher Alice:*** *But why 4?*
> ***Child 9:*** *I was counting . . 4s.*
> ***Teacher Alice:*** *So your bicycle has 4 wheels.*
> ***Child 9:*** *No.*
> ***Teacher Alice:*** *But you are timesing by 4.*
> ***Child 9:*** *Ohhhh.*

Teacher Alice had introduced the students to the vocabulary associated to multiplication and encouraged them to use it. She used 'product', 'times', repeated addition' and 'groups of' when asking questions or talking about multiplication. In explaining multiplication by a hundreds or tens number, the narrative she had created was not mathematically accurate. In this episode the class is multiplying 70 by 2:

> ***Teacher Alice:*** *70 x 2. What have I told you about the 0 there? We are going to* ***add*** *our . . .*
> ***Child 2:*** *0 (Teacher Alice points to 7 and 2.)*
> ***Child 2:*** *70 x 2. 7 x 2. 140.*

Other narratives evident in the lessons, had been taught previously by Teacher Alice and she reinforced them with her students. The focus of the lessons was therefore to recall the procedures and rules and competently complete the method. Using 3-D shapes as visual mediator confused her students when she asked them to "multiply the cone by the cylinder". Teacher Alice had placed numbers on the shapes expecting the students to multiply the numbers. This had to be clarified with the students, before they understood what they were expected to do.

The feedback given by Teacher Alice was evaluative - mistakes were acknowledged, identified and assistance given. Praise was given to students who completed the tasks accurately and gave correct answers.

Problem Solving

An unplanned problem situation occurred in the first lesson. Initially Teacher Alice was going to show her students how to fold the paper into four equal parts and then asked for ideas.

> ***Teacher Alice:*** *Don't start it until I have shown you. Any ideas how I can fold this into four equal pieces? Without using my ruler and measuring.*
> ***Child 7:*** *You fold it in half like that. (Child makes a fold vertically across the paper)*
> ***Teacher Alice:*** *So I fold it in half like that.*
> ***Child 7:*** *Also. You fold it in half like that. (Child opens the paper and then folds it horizontally)*
> ***Teacher Alice:*** *I do that. Is that what you are telling me to do?*
> ***Child 7:*** *Yes*
> ***Teacher Alice:*** *But that is only two equal pieces. Oh, I see. First you want me to fold it this way, then open it up and fold it that way. That is very good. I didn't even think of doing it that way.*
> ***Child 8:*** *Or Teacher there is this way.*
> ***Teacher Alice:*** *Aah. Let's see another way. How would you do it, love?*
> ***Child 8:*** *You fold it in half like this and then in half again. (Child folds it in half horizontally and then vertically.)*
> ***Teacher Alice:*** *Excellent. So there are two different ways. Does everybody know? You can choose whichever way you want, as long as you fold it into four equal pieces. Then open up your piece of paper.*

The two students solved the problem using two different methods and they were able to explain their thinking to their teacher.

After Teacher Alice had taught the lessons she considered she had successfully achieved her outcomes. The students were able to define multiplication in four different ways and do the calculations with decomposing.

> *Now I am doing the reverse with the division and they are struggling and we have gone back to [multiplication]. We are doing it both ways and I think it is consolidated now and they got the concept and I think they enjoyed working with it because we did so many diverse activities.*

Table 6 to be deleted and placed on p 61 & 62

Chapter 6: Discussion

In this chapter I discuss how educators develop mathematical proficiency in their classrooms, with particular emphasis on, whether they teach problem solving. In order to promote mathematical proficiency, it follows that educators should be skilled in the teaching of mathematics. The four educators, whose lessons I observed, are all highly qualified, experienced and engaged in very different teaching practices. Not only did their approach reflect their personalities but their educational beliefs were also evident.

All four promoted mathematical proficiency in varying degrees and their teaching practices were directly influenced by their mathematical knowledge and how they believed conceptual understanding is created. They each had distinctive approaches to the teaching of problem solving.

In my conceptual framework, I argued there were important aspects to situated learning that were valuable to the 'doing' of mathematics namely: a culture of mathematics, participation and authentic activities. There should be a rich culture of mathematics visible in the classroom and the lessons and this was the case in Teacher Taryn's classroom, where a series of mathematical posters are predominantly displayed for her to refer to during her lessons. She made use of a wide variety of apparatus, from Unifix blocks to beans to white boards, for sharing and grouping. Along with her students, there was good use of the correct mathematical vocabulary. Teacher Lucy's classroom also had appropriate mathematics posters and, in addition, she and her students made use of a wide variety of apparatus, from games to white boards and Base 10 blocks. In her classroom, Teacher Sam had a board dedicated to mathematics concepts, and other posters in the front of the class. During the lessons, the students who were familiar with the correct mathematics vocabulary, were able to interact with different mathematical representations of fractions, posters and real pizza. Teacher Alice's classroom had relevant mathematical posters on display too, in addition, she made use of games, flard cards and white boards and had introduced her students to the correct multiplication terminology.

From a situated perspective, learning happens when conceptual knowledge is used and understanding of concepts occurs when these are tested in different situations (Brown et al., 1989). Teacher Alice preferred to focus a being competent in the use of concepts before paying attention to its application to life-world contexts. Teacher Lucy separated the learning of concepts from authentic activities believing the first priority was to become competent and then apply the understanding to a different situations. Both these educators found problem solving difficult to teach, believing that the students did not have conceptual understanding. Teacher Sam integrated the use of concepts into different situations which were relevant to her students and allowed them to transfer the ideas across. Teacher Taryn encouraged her students to build their own understanding, by interacting with the concepts in different situations, moving from concrete to abstract using a variety of apparatus, different modalities and forms of representation.

The literature reviewed posits that, when students participate in new opportunities, situations and activities they are able to renegotiate, reframe and restructure mathematical concepts (Greeno, 1997; Sfard, 2007). Teacher Sam provided plenty of opportunities for her students to discuss and offer ideas about the concepts they were learning. She allowed time for her students to think ideas through and offer their own solutions to problem situations. Teacher Lucy, when working with different mental mathematics games and activities, encouraged her students to offer different methods and thinking to find solutions. Teacher Taryn, in her careful selection of activities provided opportunities for different connections to be made and discussed stretching beyond just the present concept. Teacher Alice's focus was on being fluent and accurate in completing the calculations.

Learning mathematics is about conversing mathematically with oneself and with other students. All four educators controlled the discourse in their lessons restricting the time given to their students to talk about mathematics. The discourse in Teacher Sam's class was the most interesting. Her approach, though interactive, was both authoritative and dialogical and her students were comfortable in sharing their ideas and thoughts and helping each other out. She also made time for paired and group work. Teacher Alice preferred a fully authoritative non-interactive discourse and helped her students out when they were unsure of what to say. Teacher Lucy used an authoritative and dialogical approach, and while the discourse in her class was stilted to begin with, she strongly encouraged interaction and, by the third lesson, her students were more willing to talk. The discourse in Teacher Taryn's class varied. She encouraged all to participate, and offered opportunities for paired work, and some students were eager to share while others were reticent.

Being able to apply mathematical concepts to a problem context or link it to a real-life situation ensures conceptual understanding and a sense that mathematics is useful and worthwhile. Teacher Taryn began each of her lessons by connecting the concept to a context her students could relate to, along with using visual apparatus and the beginnings of mathematical representations. Word problems were used to situate all concepts then mathematised and solutions found. Teacher Lucy used visual apparatus to introduce the concepts and then moved on to numerical images. Teacher Alice introduced a few word problems but concentrated on the use of visual images and numerical representations. Teacher Sam found a variety of problem contexts which illustrated fractions in different situations. Her students were able to link the visual apparatus to mathematical representations.

It has never been my intention to judge or compare the educators or the schools. However, I would like to highlight two factors which I believe are significant. First, is how Teacher Taryn carefully sequenced and built on the mathematical concepts in her lessons. She sequenced the concept of sharing from halving, a concept with which her class was already familiar. Building on this foundation she introduced sharing into larger groups. By contextualising it with problem situations she allowed for opportunities where links and connections could be made with previous knowledge. Next she introduced the mathematics part of grouping changing the modality from verbal to written by making reference to the correct vocabulary and notation. Conceptual understanding and strategic competence were being constructed in parallel before introducing productive disposition, adaptive reasoning and procedural fluency. The sequencing of the mathematic knowledge in Teacher Sam's lessons was not as clear. She introduced wide range of fraction concepts by contextualising them within problem situations. She then returned to particular concepts and made reference to the mathematics so that links and connections could be made. In the Teacher Lucy's and Teacher Alice's the sequence of the mathematical knowledge was static as their main focus was on procedural fluency and they did not make explicit reference to conceptual understanding or strategic competence. This has serious implications for how students acquire mathematical proficiency (Council & Committee, 2001)

The second factor relates to the difference between the two schools. Hoopoe School, over the last four years, has been successful in developing a whole school approach, which includes educators, students and parents, to the explicit use of thinking skills. Their programmes of professional development have been particular successful with the educators and students buying into the skills and routines which are purposefully used in all curriculum areas. Likewise while the Foundation Phase do not use the Pr1me Mathematics textbooks, they are committed to using its terminology and concept development. Weaver School is a young school which is still in the process of developing its ethos. All the educators are new to the concepts of constructivism and their understanding and ability to implement it explicitly is growing. Not all the educators have had the opportunity for professional development in NumberSense and its method of concept development. Therefore professional development programmes which offer a framework to a constructivist way of learning, would lead to shared understanding, strengthen of teaching practices, without forfeiting the integrity of each educators' style. Consensus in this way would lead to a more cohesive approach.

Students acquire mathematical proficiency more readily when a balance between situated learning and problem solving is created. They are mutually interdependent and they require deliberate, intentional teaching. All the educators offered different practices in which they promote mathematical proficiency and engaged in some form of problem solving. Getting the balance right is challenging for all educators and certainly not always evident in just a few lessons.

I learnt a great deal from observing all of the educators' lessons, reflecting on their orientations and approaches to teaching mathematics and problem solving, relative to my own. Teacher Taryn's lessons, in particular, extended my thinking about a range of new ways to work beyond my current approaches and develop an imagination for further ideas.

Exemplar

Teacher Taryn - Grade 1 – Sharing and Grouping leading to Division

In the initial interview, Teacher Taryn clearly articulated her orientation to mathematics and mathematics teaching. She spoke about:

understanding mathematics better; linking everything together; mathematics is all around you; I like doing problem solving and *lots of practical, hands on, concrete, lots of repetition.*

When reflecting on her lessons, and because she found sharing and grouping difficult to teach, she questioned whether she should be doing them: "separately" or "together" but, in her final analysis decided that *"[they] need to flow because they all work together."* She noted that addition, subtraction and the language associated with them gave her students a foundation from which to work.

When planning her lessons, Teacher Taryn, said she thought about:

how much they know already, what you can apply to what they already know or have learnt in the previous year and how much detail they have done.

Then she *"plans lots of concrete activities before we do anything written or expand on it."* Her favourite part of teaching mathematics was *"pulling it all together. [So] teaching all the concepts and applying it all across."*

Teacher Taryn's lessons indicated careful thought and planning (see Annexure 5). The first lesson started with a problem context as she guided her students to the mathematics. First, she began with a relevant problem situation – sharing a choc-chip cookie in half equally - and

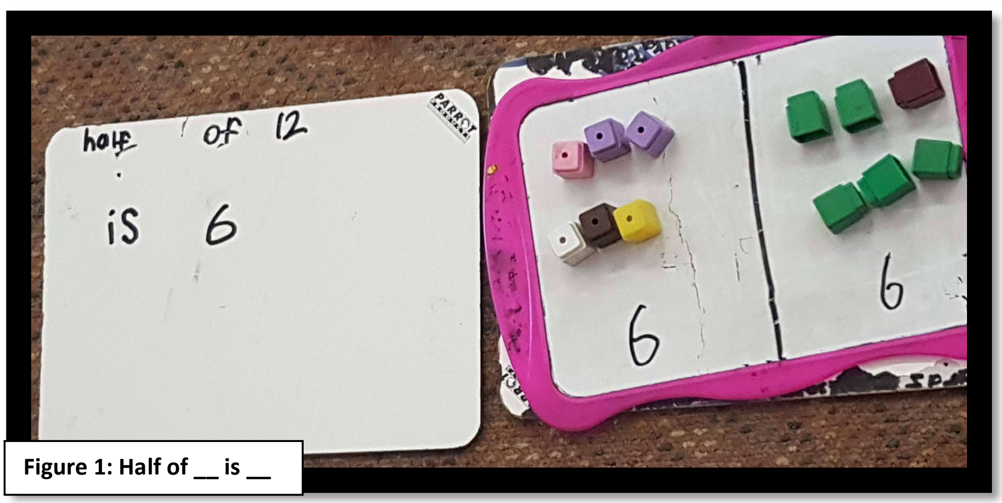

Figure 1: Half of __ is __

involved her students in the discourse from the beginning. Then she gave her class paper shapes which included: squares, rectangles, triangles, stars and circles, to fold in half. The selection of shapes created problem situations (circle and star) and the students had to explain if it could be halved, or not, and why not. Taking advantage of the fact her students had white boards with them, she asked them to use lines to halve their boards and revised the names of the lines. She introduced the words "half" and "halving" and linked them to reading and spelling. Using Unifix blocks, Teacher Taryn then revised halving with the class. She worked with them, modelling the method and then allowing them to work in pairs monitoring and engaging them in talking mathematics. Her students used numbers from 2 to 26 and were encouraged to use *"Half of __ is __ "* (see Figure 1) as they halved the numbers. Before the class moved on to do the independent work, she revisited what they had been doing and linked it to the worksheet.

To begin lesson two, Teacher Taryn recapped what they had done the day before, discussing the halving of the cookie and the vocabulary. Moving on from halving to sharing, she again created a problem situation – sharing nine choc-chip cookies between Bean, Bird, and Mouse. Ideas were discussed, a solution was found and the mathematics recognised. Next the class were given workcards (see Figure 2). Using beans, they had to share them into groups and write the answer. To start with, Teacher Taryn modelled the process and slowly withdrew, allowing her students to work in pairs. She reinforced the vocabulary: *"What is your whole? How many are you sharing between? So what is your answer?"* Once most of the students had completed the workcards, Teacher Taryn changed the activity. This time the students had to draw the circles, share out the beans and write the answer (see Figure 3). Once again she modelled the process and had the class check her answer. The students had to

Figure 2: Sharing workcards

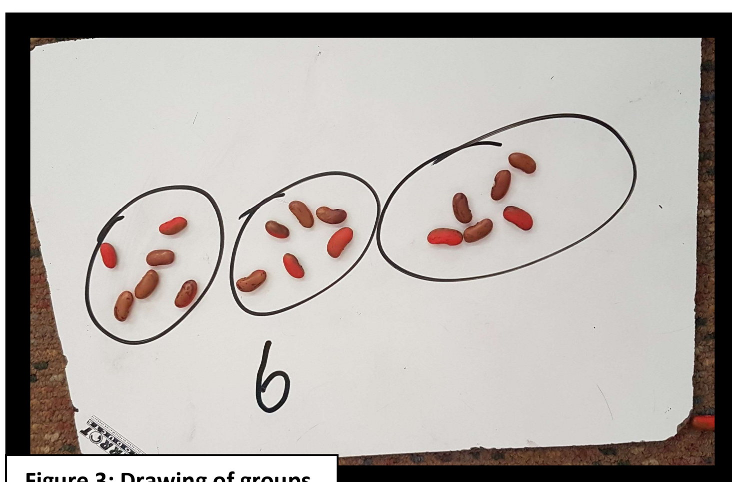

Figure 3: Drawing of groups

complete two word sums in their books independently. Teacher Taryn read the two problems together with her students, and she asked them to identify the whole and the number of groups.

Teacher Taryn started lesson three by revisiting halving and sharing and the vocabulary associated with it. She referred back to how they had drawn when doing addition and subtraction and asked them how they could draw sharing. Taking ideas from the different students, she had them explain their different methods. Working from those students' ideas, Teacher Taryn modelled the sharing process. They drew their whole as little circles, then drew large circle for their groups (see Figure 4). They shared out their whole by crossing out and redrawing them in the large circles. They counted each group and wrote down the answer. Teacher Taryn worked step by step with them reinforcing the process and then withdrew allowing them to work on their own. The final activity was to complete a selection of workcards in their books (see Figure 5). They read the cards together highlighted the important word *"each"* and explained what they had to do.

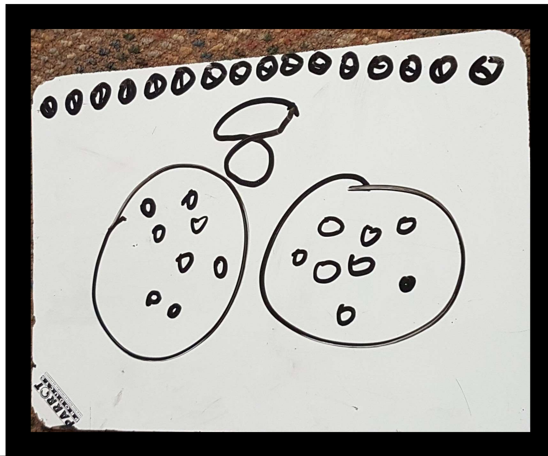

Figure 4: Sharing and grouping

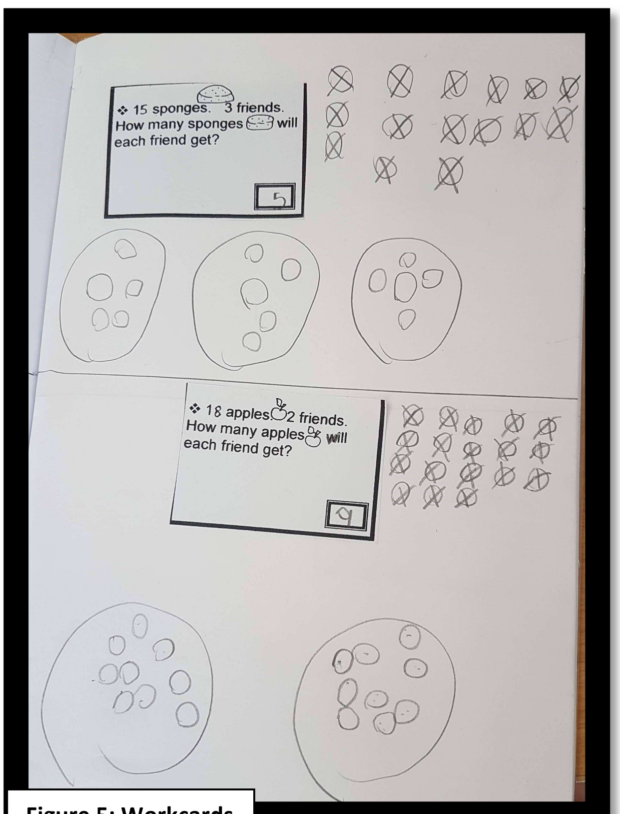

Figure 5: Workcards

By creating a problem situation each time, Teacher Taryn was able to make the mathematical concept of halving and sharing meaningful and relevant to her students. She created a logical continuum along which she developed the concepts, from: halving using Unifix blocks; sharing into groups using beans; to sharing by drawing circles and using beans; then, to drawing the whole, group circles and sharing out by drawing in each circle. Each activity was more cognitively demanding for the children and, at each stage Teacher Taryn used language to promote understanding 'half', 'whole', 'groups', 'answer', 'share between', and 'each' and created opportunities for them to interact and take part in the mathematical discourse.

In encouraging the mathematical discourse, Teacher Taryn invited her students to share their thinking. Using recall, procedural or probing questions, she listened carefully to her students responses: acknowledging, refining, connecting, or exploring further. Rather than evaluating their answers, she probed further, creating a learning moment. She was also implicit in her praise.

In summary, Teacher Taryn's success came from using problem solving as a foundation from which to build conceptual understanding. The problem situations provided the background for mathematising and a "press for learning" (Kazemi & Stipek, 2009). Her tasks increased in cognitive demand, moving from concrete to abstract, changing modalities of communication and representation. Her language and orientation to language reflected precision and care for correct mathematical conventions, vocabulary and discourse. She valued the importance of communicating mathematically, providing opportunities for her students to talk mathematics.

Chapter 7: Conclusions

The aim of my research study was to investigate how educators enact their mathematics pedagogy in the classroom with particular emphasis on whether they teach problem solving. My research was conducted in two well-resourced, independent schools where the educators were all highly qualified and experienced and had attended recent professional development sessions. While discussions from this purposive sample cannot be generalised, I do believe that other educators could identify with some of the issues raised.

From the interviews, the snapshot of mathematical teaching I observed, and textbooks and book work completed by the students, it was possible to see that the educators enacted their beliefs in the classroom. Teacher Taryn and Teacher Sam, in particular, reflected not only their own beliefs but the inquiry-based approach, that was in keeping with their school's educational philosophy. There was, however, a dissonance between what Teacher Lucy and Teacher Alice defined as mathematics and how they enacted it. Even though they identified with the definition of mathematics aligned to an inquiry-based approach, the pedagogic practices of Teachers Lucy and Alice indicated a toolbox approach where students are taught facts, skills and procedures.

Using their own distinct pedagogic practices, all the educators promoted aspects of mathematical proficiency in their classrooms. Nevertheless, whether, and how they integrated and balanced situated learning, mathematical discourse and problem solving, depended on the individual educator's beliefs. Teacher Taryn's ability to successfully incorporate all the factors, while teaching a complex concept like sharing and grouping, in a Grade 1 class, suggests that it is possible to establish an integrated balance.

In reviewing my descriptions and analysis, there are three observations which made a such a significant impact on me that they have caused me to reflect on and re-evaluate my own practices: first is understanding that, everyday life-world activities provide ideas for how problem solving connects with mathematics; that, as educators, we tend to control the discourse in the classroom, and third, that it is the quality of the task that is of utmost importance.

Opportunities for problem solving can be sourced from the students in the class, are present in the classroom and around the school and as educators, we need to be attentive to them

~~a~~nd use them. Problem solving should empower all of us: educators and students - to seek information, extract ideas, shape arguments and explain them. The more powerful the learning environment, the greater the extent to which we can problematise and build identities as doers of mathematics (Schoenfeld, 1985, 2013b). Problem solving can be seen as applying conceptual knowledge; the solving of word problems once students have conceptual understanding and procedural fluency. This is reinforced by CAPS and some textbooks. Developing strategic competence and adaptive reasoning, goes beyond students being able to solve just word problems. More importantly, if problem solving is used as a starting point for developing new concepts, students can learn mathematics with understanding (Hiebert et al., 1996; Murray et al., 1998). Problematising starts with students making the activity their own and having an opportunity to construct their own understanding.

After observing 18 lessons in Grade 1, 2 and 3, the most striking feature was that, in most instances, the educators tended to control the discourse. Scott, Mortimer & Aguiar (2006) identify two approaches to communication in the classroom: authoritative/dialogic and interactive/non-interactive. The authoritative approach focuses on one meaning, while the dialogic recognises a range of meanings. Interactive communication opens the boundaries to more people while noninteractive restricts the boundaries, excluding others. Most educators followed an authoritative/interactive approach, leading their students through questions and responses to establish understanding. The literature highlighted the importance of students participating in collective activities and engaging in mathematical discourse. As educators, it is our responsibility to create more interactive/dialogical opportunities, opening and closing the interaction to structure and frame mathematics and mathematising. In doing so, we also have to ensure that our students have time to think, time to develop the correct mathematical discourse and the time to experience commognitive conflict within that same discourse (Scott, Mortimer, & Aguiar, 2006; Sfard, 2007).

In their research, Kazemi and Stipek (2009), identified four norms that provides a framework for mathematical discourse in the classroom. Not only do they have a positive effect on the social environment but they ensure a “press for learning” (2009, p. 61). Mathematical inquiry is established when students: describe their thinking; find multiple ways to solve their problems and explain their strategies to their class; use their mistakes as learning opportunities and collaborate to find solutions. As educators, we shape the environment by giving priority to mathematics, sense making and giving all students a chance to participate in meaningful classroom activities (Schoenfeld, 2013b).

While we do not exclude our students from the discourse, their participation can be limited. The quality of the tasks opens and closes opportunities for them to take part and the more meaningful and purposeful the task, the greater the chances are that they will become involved in the collective activity. Such tasks should be presented in rich contexts, with fairly open problem situations. The mathematising and mathematics vocabulary, are discussed first enabling our students to think informally before moving on to the abstract, thus experiencing both horizontal and vertical mathematics. Reflecting on and drawing conclusions, followed by the sharing of findings makes the mathematics explicit and integrates the ideas (Schoenfeld, 1992; Bos, 2017). The quality of the tasks impacts directly on what our students can offer the discourse and the cognitive demands the tasks place on them are conducive to stimulating and creating productive discourse through which they can all engage purposefully.

All three of these observations are closely connected. Each one impacts on and creates opportunities for the other. The environment created by us as educators is contingent on how we recontextualise the mathematics curriculum in our classrooms. In other words, it is depends on whether and how, we create rich tasks with open problem situations, and, under our careful guidance, encourage mathematical discourse between our students. It was evident in all four classes that there were students who had ideas and solutions to problem situations.

Finally, it is far easier to describe omissions in educational practice than to describe and interpret what is present and possible. However, if one is concerned with opportunities for teacher development and committed to zones of potential development, then it is important to build on what educators *are* already doing successfully, and then offer resources and an imagination for what is possible.

This descriptive interpretive analysis of four educators' practices may enable teacher educators and developers to think about the diverse ways in which educators strive to balance the development of mathematical proficiency and problem solving, and to identify ways and directions that could take them further in this quest.

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Appendix 1 – Initial Interview

Questions for Educators

(Qualitative Instrument)

The purpose here is for me to understand more about your views on mathematics knowledge, the curriculum and your approach to teaching mathematics. Because this is an empirical study, everything you tell me is important and relevant and will provide a deeper understanding of your knowledge and practices.

Part One – Educator's Background

- Name and School
- Job description
- Teaching qualification
- Teaching experience, Grade experience
- Mathematics professional development (Which course/s made the most impact on you? Give reasons why.)

From these three perspectives of mathematics, which do you think would best describe your beliefs and attitudes relating to the teaching and learning of mathematics.

1. Mathematics is doing the work of mathematicians. It is about patterns: noticing them, describing them, and generalising them. Mathematicians are curious, they make connections, they love being challenged to find creative solutions to real life problems. They 'do' mathematics.

2. Mathematics is a toolbox that is filled with mathematical facts, skills and procedures. It is a hierarchical discipline, where educators teach students the relevant facts, skills and procedures. Students are required to master the mathematics, develop precision and get the correct answers.

3. Mathematics is about empowering students to think, communicate, reason and connect with everyday reality. It is about providing opportunities to explore problem situations, experiment with modelling, ask questions, make decisions, investigate, and discuss outcomes in a rich learning environment.

(These definitions are adapted from Boaler (2016), Schoenfeld (1985, 2013), Hugo (2014) and my literature review.)

Part Two – Mathematical Knowledge (valid knowledge)

Mathematics Identity

- Why do you teach?
- As far as you can recall, what have been your own experiences of learning mathematics – both as a student at school and as an educator?
- How were you taught mathematics?
- How do you teach? Tell me more about how you teach your own class and why you use this approach?
- Would you say you are a "maths" person? If so, why?
- What do you like about teaching mathematics in your Grade?

Mathematics Curriculum

- How would you define mathematics in the Foundation Phase?
- What is the value of mathematics? Why is it important to include in curriculum?
- How have CAPS or the ECD: Curriculum Guidelines informed your teaching?
- Mathematical proficiency refers to conceptual understanding, strategic competence, adaptive reasoning, productive disposition and procedural fluency. How do these strands inform your approach to teaching mathematics? How would you rate them in relation to each other?
- If you could change the curriculum, would you change anything?

Part Three - Mathematics Practice (valid transmission of knowledge)

- In your view, what are the most important considerations to bear in mind when planning and teaching mathematics to your students?
- What are the challenges of teaching in your Grade?
- In the years you have been teaching mathematics, have your practices changed? If so, in what ways?

- Which aspects of the mathematics curriculum do you enjoy teaching?
- Which aspects of the mathematics curriculum are more challenging to teach?
- Your school follows “Pr1me Mathematics” / “NumberSense” (to insert as appropriate). Tell me more about this approach. Does it differ from approaches you’ve used before? If so, how?
- Has “Pr1me Mathematics”/“NumberSense” had an impact on your approach to teaching? If so, how?
- What do you see as the major benefits and challenges of teaching mathematics using this approach?
- What are your thoughts on problem solving in mathematics?
- Are there any guidelines you would like to share with newcomers to Foundation Phase mathematics teaching?

Teachable Moments

- Can you recall and describe an occasion (during a lesson or any other time) when you experienced a “teachable moment”? What made it so special?

Appendix 2: Lesson Observation Guidelines

(These procedures are adapted from Bernstein (Moore, 2013), Sfard (2007, 2008) and Hugo (2014).)

General Pedagogic Observations		
Educators and Students Spaces	Open and closed boundaries Separate, integrated, both	
Communication Relations between Educator-Student, Student-Student.	Solid or weak boundaries Integrated Educator-Student, Student-Student	
Pacing of Learning	Timing – fast or slow Making meaning	
Intra-disciplinary Relations	Separate – Strong Integrated – Open	
Cognitive Demand	High, mixed, low	
Investigative Proficiency	Thinking, reasoning, modelling and making connections	

Mathematics Process	
Mathematical words – use Visual mediators – how they coordinate communication Mathematical narratives – frame descriptions Mathematical routines -well-defined repetitive actions. Mathematical discourse - calculational discourse - reasons behind procedures, conceptual discourse – analyse comparative and contrasting task interpretations, mathematical thinking, educators role in discourse – questions, feedback	

Situated Learning	
Culture – interaction with concepts Participation - educators and students	

Interaction – discourse and communication Interaction – knowledge, objects of learning, conceptual understanding Authentic Activities – productive, useful, meaningful, cognitive demand Opportunities to make connections between concepts, procedures, contexts	

Problem Solving	
Problematising, process, solution Meaningful questions, information, explain ideas, justify reasons represent, explain, solve, connect mathematical ideas	

Appendix 3: Reflective Interviews

Questions for Educators

Part Four – Evaluation (The valid realisation of knowledge)

This interview is for you to evaluate your thoughts on the lessons, evaluate the learning that took place and looking at your teaching practice. I am looking for your opinion.

- Looking back, how do you feel the lessons went?
- How does this compare with what you expected would happen? Did you teach what you intended to teach?
- Are there any factors, that impacted directly on your teaching practice in these lessons?
- You chose No 3 as your definition of mathematics. How do you think your lessons allowed for: "thinking, communicating, reasoning and connecting with everyday reality"?
- Do you think your children talk enough about mathematics in your lessons? (To you, as the teacher, to their friends/partners or to the class?)

Taking moments from the lessons (mathematics, participation, discourse, problem solving)

Teacher Taryn

- I noticed you make mathematics real for the students. You situate the concept in a real-life situation, you use lots and lots of visual representations, and you use the language of mathematics throughout your lesson. You are connecting the visual and the language all the time.
- I am interested in the way you integrate spelling, reading and other aspect of mathematics. How effective do you find it? Do you do connect to the mathematics in other lessons?
- When the students offer explanations, using the mathematics language you have taught them, you extend their thinking by offering alternatives. Do you ask them to share their different strategies and reason with each other?

- Your students obviously vary in ability. When doing the sums, some can do them mentally, others need to use methods. How do you balance the differing cognitive demand?
- I noticed that when you taught this new concept, progression was important. Is this a “Pr1me Mathematics” thing or something you have developed? Halving, sharing, practical, visual . . . what happens next?
- Are the worksheets and work-cards you use from “Pr1me Mathematics” or your own?
- You situate all your number work in real-life problems. How often do you focus just on problem solving? (i.e. not necessarily word sums but giving the students a situation to examine and letting them take it from there?)
- How do you evaluate how much the students have learned?

Teacher Lucy

- I noticed you are able to read the children, and change the flow of the lesson when necessary? How does this impact on the pace of learning?
- I also noticed that you use the language of mathematics throughout your lesson, and the children are starting to use the language back to you in their explanations. Is that your intention?
- You do lots of mental mathematics, reinforcing it with games, which the children clearly enjoy. How effective is this and do you need to reinforce in other ways too?
- With the mental work you talk a lot about strategies. Does this extend to calculations? In the lessons I observed, I only had the opportunity to see you practicing vertical addition.
- I was interested to see that, when you did the sums, it was all about the numbers. Was it a conscience choice not to situate them in real-life?
- How often do you focus just on problem solving? In other words, not necessarily word sums, letting the children examine a situation and take it from there?

- Your students obviously vary in ability. When doing the sums, some can do it in their mentally, other need to use the method and others need scaffolding. How do you balance the differing cognitive demands?

Teacher Sam

- How did the teach backs by the students go?
- Fractions are difficult to understand. Too difficult for Grade 3s? How far do you go?
- It was so interesting to see you encourage purposefully discussions and some lovely conflicting thoughts arose. You do not openly negate an answer. How does this work for you? Does it ever give rise to misconceptions?
- I notice you focus a lot on thinking skills: listening with meaning and understanding, reflecting, previous knowledge. How has that impacted on the learning of maths in your class?
- You situate all your number work in real-life problems. How often do you focus just on problem solving not necessary word sums but letting the students examine a situation and letting them take it from there?
- How do you evaluate how much the students have learned?

Teacher Alice

- You do lots of mental mathematics, reinforcing with songs and games, and the children clearly enjoy themselves. How effective is it and do you need to reinforce in other ways too?
- I noticed your idea to use shapes to help with the calculations – cube x cone – seemed to confuse the children. Would you use the same example again or would you do it differently next time ?
- In the lessons I observed, I only had the opportunity to see you practicing one method in multiplication. Do you encourage the use of different strategies in both mental work and calculations?

- I noticed that you did to word problems with the children, but when you did the calculations you didn't situate them in real-life. Was this a conscious choice?
- I noticed that you use the language of mathematics throughout your lesson. How important is for the children to use the language back to you in their explanations?
- Your students obviously vary in ability. When doing the sums, some can do it in their mentally, other need to use the method and others need scaffolding. How do you balance the differing cognitive demands?
- How often do you focus just on problem solving? In other words, not necessarily word sums, letting the children examine a situation and take it from there?

Appendix 4: Focus Group Interviews

Proposed Questions

In this focus group interview, I am looking for your perspective on the way mathematics is perceived and practiced in the Foundation Phase and across you school.

Interestingly, all three of you gave me three completely different mathematics lessons but when you looked at the descriptions of mathematics I gave you all chose the same one: No 3.

> *Mathematics is about empowering students to think, communicate, reason and connect with everyday reality. It is about providing opportunities to explore problem situations, experiment with modelling, ask questions, make decisions, investigate, and discuss outcomes in a rich learning environment.*

Would you say that it describes what mathematics is all about at your school?

Part One – Curriculum

- How has CAPS or the ECD: Curriculum Guidelines informed your school's mathematics policy in the Foundation Phase?
- How has being a Thinking School/Constructivist School informed the teaching of mathematics in the Foundation Phase?
- Hoopoe School - In each lesson there was a focus on using the correct mathematical language and strategies like whole-part-part, bar model, and UPAC. Is this motivated by school's philosophy on mathematics and thinking? Or does it come from the "Pr1me Mathematics" approach?
- Hoopoe School - Not one of you used the Pr1me mathematics textbook? Is this a personal choice or a Foundation Phase decision?
- Weaver School - How has being a Constructivist thinking school informed the teaching of mathematics in the foundation phase?
- Weaver School - Not one of you used the "NumberSense" textbook? Is this a personal choice or a Foundation Phase decision?
- How do the students connect with mathematics in the school?

Part Two - Pedagogy

- How do you decide what to teach each week?
- How does the "textbook" inform your teaching?
- Effective teaching practices can be wide and varied. Do you agree or disagree with this statement? Please explain your position.
- At your school, how is feedback/evaluation/assessment valued in the teaching of mathematics?