UNIVERSITY OF THE WITWATERSRAND

A research report to satisfy the requirements of a Master's Degree in Education (MEd) at the University of the Witwatersrand.

Exploring learner errors and misconceptions in algebraic expressions with Grade 9 learners

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Declaration

I, Bulelwa Stemele, hereby declare that this research report is my own work. It is being submitted for the Degree of Master's Degree in Education at the University of the Witwatersrand, Johannesburg. This report has not been submitted for any other degree or examination at any other university.

_______________________ 15 March 2023

Bulelwa Stemele Date

Abstract

Mathematics is a crucial skill for the acquisition of relevant skills in society and is required for admission to South African universities and other higher education institutions. However, South African learners' performance in mathematics on local and international educational achievement tests has been a major source of concern. Algebra is one area of mathematics that learners struggle with. Algebra is challenging because it is a more abstract form of mathematics and learners are unable to relate it to their daily lives. Algebra is essential because it serves as the foundation for further study in mathematics and other disciplines. Learners in Grade 9 struggle with the variables, equations, and abstract concepts found in algebra. Most errors and misconceptions that learners commit in mathematics stem from a lack of algebra background knowledge. Therefore, the purpose of this study was to explore algebraic expression errors and misconceptions in Grade 9. The study was based on the Vygotskian sociocultural theory of learning. According to the Vygotskian sociocultural theory, teachers and mathematics manipulative plays an essential role in facilitating learning in their learners Zone of Proximal Development (ZPD). A mixed-methods study was used to explore the errors and misconceptions committed by Grade 9 learners when solving algebraic expressions. The data was collected through tests and learner interviews from a class of 22 Grade9s. After analysing the pre-test data and identifying common errors and misconceptions, an intervention involving the use of algebra tiles to teach algebraic expressions was implemented. Firstly, my study supports the error types identified in the literature. Secondly, my study demonstrates an improvement in performance on the post-test following an intervention using algebra tiles. According to research, the use of manipulatives helps learner construct a conceptual understanding by consolidating abstract ideas. This study adds to previous research indicating the usefulness of manipulatives in mathematics classrooms.

Dedication

This study report is dedicated to my parents, Ntlupheko Moris Stemele and Nombulelo Olpa Stemele, who are incredibly proud of me and were so supportive throughout this journey. It is also dedicated to my amazing daughter, Fezeka Alwande Stemele, who has been my greatest supporter and has been patient with my late-night study sessions. I would also like to thank my siblings for their assistance during the process.

Acknowledgements

I cannot adequately express my gratitude to Dr Zaheera Jina Asvat, my supervisor, for her commitment, patience, and invaluable feedback. I had no experience with academic writing prior to beginning this journey. Without her assistance and knowledge, I would not have been able to embark on this journey. I am grateful to you!

In addition, this effort would not have been possible without the generous funding for my research studies provided by the Nurturing Emerging Scholars Programme (NESP).

I am also grateful to the school and the principal for granting me permission to conduct research in the school. In addition, I would like to thank my colleagues for their moral support. Finally, I would be remiss if I did not acknowledge my family, especially my parents and daughter: their belief in me kept me going even when I felt like surrendering.

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Chapter 1

1.1 Introduction: Background of the Study

Mathematics is a crucial skill for the acquisition of relevant skills in society and is required for admission to South African universities and other higher education institutions. However, South African learners' performance in mathematics on local and international educational achievement tests has been a major source of concern (Chirinda & Barmby, 2017). In 2012, the government began administering Annual National Assessments (ANA) in mathematics to learners in Grade 9. The main goal was to implement effective evaluation and allow learners to exhibit their relevant arithmetic skills and their inadequacies. The national average for these mathematics examinations in 2013 and 2014 was 13% and 14% respectively. After the release of the Grade 9 Annual National Assessments (ANA) 2013 results, it had been reported that learner achievements in mathematics were still at an unacceptable level (Department of Basic Education [DBE], 2014). The Annual National Assessments have been discontinued since 2015. A more recent Trends in International Mathematics and Science Study (TIMMS) in 2019 reported that 59% of Grade 9 learners did not have basic mathematics skills, and 64% did not possess basic science skills. As a result of their dismal performance, South Africa's Grade 9 learners are rated last worldwide in terms of academic achievement in mathematics and science (TIMSS 2019). The reasons for these results within a South African context have been attributed to socio-economic conditions, which include overcrowding in classes, a lack of resources such as textbooks, a rapid change in the curriculum without effective implementation, gender inequality, child-headed families, poverty, a lack of parental participation, and the list is endless.

Further research argues that questions have been raised as to why South Africa performs worse than its neighbouring countries, which are less wealthy (Graven & Heyd-Metzuyanim, 2014). Therefore, learner mathematical success cannot be solely a socio-economic issue. The errors that learners make when solving mathematics require further exploration because only then we will better understand the cause for their incorrect answers. From my experience teaching Grade 9 learners, I realise that algebra is a critical area in which learners struggle. Therefore, I consider it necessary to consider the slips, errors and misconceptions that learners have in this area and how these errors may be resolved.

Grade 9 is an important year in high school. The results of Grade 9 mathematics determine whether a learner continues with mathematics or moves on to mathematical literacy in Grade 10. Algebra is a foundational skill that prepares learners for other areas of mathematics. MacIntyre (2005) asserts that success in mathematics is highly contingent upon understanding algebraic concepts. Learners in Grade 9 should be able to simplify, multiply, divide, and factorise algebraic expressions among other computations (National Department of Basic Education, 2011). Algebraic expressions include both numerical and variable operation symbols. Algebra application-based questions remain evident throughout the high school years. Learners need to understand how algebraic expressions have developed from an operational to a structural point of view through time (Sfard, 1995).

Learners have trouble when transitioning from numbers into symbols, and these difficulties may be conceptual. However, when they get an incorrect answer, the cause of this incorrect answer could be a slip, an error or a misconception. Learners make errors all the time, and while this is normal, it is also critical to learning mathematics. Slips occur irregularly. Errors, however, occur on a regular basis and are often constant (Gardee & Brodie, 2015). Errors have a more conceptual underpinning than slips. This conceptual framework is called a misconception (Nesher, 1987).

In this study, I have investigated the types of errors Grade 9 learners make when solving algebraic expressions. My study focused on exploring the errors and misconceptions learners made when solving algebraic expressions, to gain a better understanding of why they make them and how they can be resolved. Algebra tiles was employed in intervention lessons to improve learners' understanding of algebraic expressions.

1.2 Research Questions

My study is guided by the following research questions:

1. What are the errors and misconceptions that Grade 9 learners make when solving algebraic expressions prior to the intervention process?

2. Are there any changes in the Grade 9 pre-test and post-test results?

3. How do the results of the post-test relate to the content in the intervention?

1.3 Rationale

It has always been problematic for me when learners make mistakes solving algebraic expressions. As I continued my work as a teacher, my main interest became understanding the mistakes that learners make. The literature speaks to three categories of mistakes that learners make when solving algebraic expressions: slips, errors, and misconceptions. Slips constitute a subset of errors and are defined as minor errors made by learners, which they should not repeat, and which they are aware of and can correct when called out (Luneta and Makonye, 2010). Slips do not indicate conceptual misunderstanding. They are sporadic. Errors are systematic and are grounded at a deeper conceptual level. Learners make an error when their answers are inaccurate, and Brodie (2014) explains that when they make multiple errors and gain confidence as a result, they develop misconceptions. Misconceptions in turn lead to multiple errors. The errors that learners make in tests are important to understand because they reveal misconceptions. The way teachers address these errors is important as it can develop learners' conceptual understanding or create deeper misconceptions. These errors can be used by teachers to provide learners with epistemological access to mathematics and contribute to developing learners' conceptual understanding. Therefore, the way a teacher deals with learner errors are crucial, as it can either enhance or limit learners' understanding of mathematics. In this study, I focus on algebra tiles as a manipulative in intervention lessons: how they may help learners correct the mistakes they make when solving algebraic expressions.

It is very easy for teachers to give learners algebraic expressions to solve, mark them and write the correct answers on the board and then move on. However, it is something else for teachers to understand why learners make the errors that they do and design ways in which to better address it. According to Webb (2009) considerable research focuses on how teachers work using whole class interaction and questioning to elicit learners thinking to correct misconceptions. Other research focuses on tasks and questions that teachers can use to understand learners' thinking (Webb, 2009). The selection and implementation of suitable questions/tasks can create a way to determine the errors learners make. I usually give questions that are from textbooks or previous question papers. I took this study as an opportunity to develop questions to draw out the mistakes that learners might make when solving algebraic expressions. In this study, I designed both a pre-test and post-test with questions selected from Chapter 8 of the *Mind Bourne textbook* used in Grade 9.

Employing manipulatives like algebra tiles is important to me; this is mainly because when used effectively they can benefit learners. Algebra tiles are a form of mathematical manipulative that facilitates learners' comprehension of algebraic topics (Salifu, 2022). Learners can visualise expressions using the algebra tiles. As the tiles are tangible, they can be touched, felt, and manipulated by learners as they solve problems. In my experience, learners struggle to visualise problems, but algebra tiles can assist in this regard. In this study, I used algebra tiles in the intervention lessons. In Chapter 3 I provide details on how learners worked with algebra tiles and in Chapter 4, I report on analysis from the pre- and post-tests.

This research will be useful for me, teachers in my school, as well as other teachers. While assessing the learners, I categorised the mistakes they made, these categories are supported by the literature and this will, firstly, inform teachers in their understanding of learners' mistakes when solving algebraic expressions. Second, this study will help teachers to understand how learners address their own mistakes when working with algebra tiles. Finally, the literature reports on studies where ZPD is created between teachers and learners. Here I elaborate on how the ZPD developed between the learners working with the algebra tiles. This contributes to both teacher knowledge and researcher knowledge on the potential of employing manipulatives with learners.

1.4 Structure of research report

This chapter serves as an overview of the entire report, discussing the background, rationale, literature review, and primary research questions of the study. Chapter 2 reviews the broad scope of literature pertaining to errors and misconceptions, algebra tile as well as the theoretical framework, whereas Chapter 3 explains the study's methodology. Chapter 4 presents and discusses the study's findings in light of the research questions and literature review, while Chapter 5 provides the study's conclusion, limitations, and suggestions for future research.

Chapter Two: Theoretical Framework and Literature review

2.1 Introduction

In this chapter, I discuss the theoretical framework and supporting literature that served as the basis for my research. Vygotsky's theory of social development serves as the foundation for the framework. I will examine the literature that discusses and differentiates the types of errors and misconceptions that frequently occur in mathematics classrooms when teaching algebra. I reviewed the literature on how errors and misconceptions are typically addressed. The discussion then shifts to the error coding tool, which was adapted from Ncube (2016). The reader is reminded that I employed algebra tiles to assist learners in addressing their errors and misconceptions during intervention classes; subsequently, this chapter will also discuss algebra tiles and review the existing literature to determine how algebra tiles have been used to address errors and misconceptions by other researchers.

2.2 Vygotsky's sociocultural theory

My study was prompted by my curiosity about how interaction with manipulatives could assist learners in overcoming challenges with algebraic expressions. Using manipulatives improves mathematical learning and comprehension (Furner and Worrell, 2017). I choose to explore this relationship through the sociocultural theory of Lev Vygotsky. My research draws on Vygotsky's sociocultural theory since its concepts of zone of proximal development and scaffolding support ways that errors and misconceptions were addressed in my study.

2.3 Zone of Proximal development

Vygotsky's (1978) social constructivism argues that knowledge is formed in a social environment and subsequently appropriated by individuals. The zone of proximal development (ZPD) is cruciaall notion in sociocultural theory. Vygotsky described the zone of proximal development as "the area between actual and potential development, as judged by problem-solving with adult guidance or more skilled peers" (1978, p. 86).

The ZPD represents the gap between the independent and assisted skills of a learner. Algebra tiles can be seen as the manipulative to close this gap. Vygotsky emphasised that an individual possesses both actual and potential skills that can be developed with assistance, in my case with algebra tiles. It should be noted here that even though the teacher provided instruction regarding the use of the tiles, the learners worked independently with the tiles.

Learners can build their mathematical comprehension using algebra tiles, which are tactile objects that they can manipulate. The use of algebra tiles was crucial to the process of acquiring an understanding of algebraic expressions. The solving of algebraic expressions was accomplished by the utilization of the algebra tiles' inherent characteristics. The learning of learners is improved when they use mathematical representations because these representations offer learners practical approaches to mathematical concepts and linkages to symbolic mathematics.

According to Vygotsky (1978), for development to take place, learner must feel that learning exercise that are relevant to them. The exercise must involve applying acquired knowledge and skills to a real-world exercise in a meaningful cultural context. The algebra tiles that learners used in the lessons were relatable because they were tangible tools that could be explored and manipulated as learners completed the exercises. Hall (1999) demonstrated that algebra tiles are a useful tool for conceptual understanding.

Thornton (1995) revealed that a programme of instruction using manipulative materials –such as algebra tiles –was effective and that learners who received instruction using these concrete materials appeared to have a better understanding of the concepts covered. The visual displays of the tiles transformed abstract concepts of algebraic expressions, and the tiles have the primary advantage of strengthening connections between algebraic expressions topics covered in Grade 9. Learners can use algebra tiles to solve solutions to algebraic expressions and explain their answers. The advantage of using a mathematical manipulative is that it facilitates both moderate and weak learners' understanding of algebraic concepts by allowing them to visualise the terms or unknowns in a concrete manner (Farah and Kim, 2017).

Learners had the chance to experiment with the algebra tiles under the supervision of the teacher to acquire deeper learning. While solving algebraic expressions, algebra tiles provide learners of various comprehension levels with an entry point. The teacher inspired learners to generate their own depiction of the problem and then allows them to make connections between the various representations. Gabina (2019) discovered that when learners use algebra tiles during lessons on algebraic expressions taught with manipulatives, their participation and comprehension of operations on algebraic expressions are enhanced.

2.4 Algebraic Expressions

An algebraic expression is a combination of numeric and symbolic notation with a variety of variables and signs (Sfard, 1995). As an example, consideration can be made to $5x - 2y + 10$ which as it stands is recognised as an algebraic expression. The idea behind algebraic expressions is to express numbers by means of letters, without indicating the values that those letters represent (McNeil et al, 2010). Variables are first misunderstood by learners as soon as they are introduced in the classroom. I agree with research that assert that learners have misconceptions about the meaning of variables (Cholily, et al, 2020; Booth et al, 2017; Sakow and Karma, 2015). Learners believe that a variable represents a single value rather than a range of possible values. Learners have historically struggled to solve algebraic expressions and research indicates that the variable and the algebraic expression are the two most difficult concepts for learners to grasp (Schoenfeld, 1988; Carraher et al 2008; Gabina, 2019; Marpa, 2019).

The concept of solving algebraic expressions is grounded within the curriculum documents that serve as a guide for teachers. Below I present a table from the (CAPS, 2011) document that specifies the algebraic expressions knowledge that learners in Grades 7 to 9 should acquire.

TOPICS	GRADE 7	GRADE 8	GRADE 9
2.3 **Algebraic expressions**	**Algebraic language** • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and/or equations	**Algebraic language** • Revise the following done in Grade 7: - recognize and interpret rules or relationships represented in symbolic form - identify variables and constants in given formulae and/or equations • Recognize and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coefficients and exponents in algebraic expressions	**Algebraic language** • Revise the following done in Grade 8: - recognize and identify conventions for writing algebraic expressions - identify and classify like and unlike terms in algebraic expressions - recognize and identify coefficients and exponents in algebraic expressions • Recognize and differentiate between monomials, binomials and trinomials
		Expand and simplify algebraic expressions Use commutative, associative and distributive laws for rational numbers and laws of exponents to: • add and subtract like terms in algebraic expressions • multiply integers and monomials by: - monomials - binomials - trinomials • divide the following by integers or monomials: - Monomials - Binomials - trinomials • simplify algebraic expressions involving the above operations	**Expand and simplify algebraic expressions** • Revise the following done in Grade 8, using the commutative, associative and distributive laws for rational numbers and laws of exponents to: - add and subtract like terms in algebraic expressions - multiply integers and monomials by: ♦ monomials ♦ binomials ♦ trinomials - divide the following by integers or monomials: ♦ monomials ♦ binomials ♦ trinomials - simplify algebraic expressions involving the above operations

TOPICS	GRADE 7	GRADE 8	GRADE 9
2.3 Algebraic expressions		• Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms • Determine the numerical value of algebraic expressions by substitution	- Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms - Determine the numerical value of algebraic expressions by substitution • Extend the above algebraic manipulations to include: - Multiply integers and monomials by polynomials - Divide polynomials by integers or monomials - The product of two binomials - The square of a binomial **Factorize algebraic expressions** • Factorize algebraic expressions that involve: - common factors - difference of two squares - trinomials of the form: ♦ $x^2 + bx + c$ ♦ $ax^2 + bx + c$, where a is a common factor. • Simplify algebraic expressions that involve the above factorisation processes. • Simplify algebraic fractions using factorisation.

Table 2.1 displaying Grade 7-9 algebraic expressions topics adapted from the CAPS document.

From the table, it can be noted that algebraic expressions are introduced to learners in Grade 7. At this level they are required to be able to identify the components of an algebraic expression. Learners are expected to know the number of terms in a particular algebraic expression. In addition, learners are expected to transform words into algebraic expressions. Lastly, in Grade 7 learners are introduced to algebraic number sentence, also known as equations.

Addition and subtraction of like terms must be mastered by learners in Grade 8, as the concept was introduced in Grade 7. Learners are introduced to polynomials such as monomial, binomial, and trinomial at the Grade 8 level. Learners are expected to recognise numerous algebraic expression types. Learners must be able to determine the value of algebraic expressions through use of substitution. Like the Grade 7 curriculum, Grade 8 learners must be able to convert words to algebraic expressions and simplify expressions using like and unlike terms. In Grade 8, learners learn to simplify algebraic expressions by calculating the square root and cube root. Learners are also taught on how to multiply and divide polynomials by monomials to further simplify algebraic expressions.

At Grade 9 level, learners should have mastered all the algebraic expression content from Grade 7 and Grade 8, including simplifying algebraic expressions through multiplication, division, calculating square and cube roots of expressions, as well as through subtraction and addition by collecting like terms. Learners must also use substitution to determine the precise value of an algebraic expression. The multiplication of binomials is introduced for the first time to Grade 9 learners. In Grade 9 factorisation is the most important topic. All factorisation types are introduced to Grade 9 learners. Learners are expected to factorise by identifying the greatest common factor, common bracket, difference between two squares, grouping, and trinomials.

The content that must be covered in algebraic expressions from Grades 7-9 has been described in detail above. As stated, learners are expected to comprehend algebraic language and be able to expand and simplify algebraic expressions. The dynamics of the algebraic language, the simplification of algebraic expressions by addition and subtraction, and the simplification of algebraic expressions by multiplication and division must be elaborated on for the purposes of this study. Subsequently, the factorisation types covered in Grade 9 are discussed. The topics that should be covered in algebraic expressions for Grade 7 to Grade 9 are discussed below in further detail with relevant examples.

2.4.1 Algebra language

Algebra is a mathematical language used to represent mathematical models of real-world situations and to solve problems that cannot be solved using arithmetic alone (Stillman and Galbraith, 1998). The ability of learners in Grades 7 to 9 to convert common language into algebraic expressions is a necessary skill that will later aid learners in forming algebraic equations to solve for unknowns. The table below shows some examples of algebraic language forms.

Sentence	Algebraic expressions
Multiply a number by 2 and add 7	$2x + 7$
Add 6 to a number and then multiply the results by 4	$4(x + 6)$
The square of the product of x and y	$(x \times y)^2$

Table 2.2 showing algebraic language from words to expressions.

Learners in Grades 7-9 will compose algebraic expressions and equations, and it is customary to represent unknown numbers using variables. When translating words into algebraic expressions, it is expected that learners know what a sum, total, difference, product, and quotient imply.

Moss, Czocher, and Lamberg (2018) explain that middle school learners may find the shift from mathematics to algebra challenging. As learners combine new algebraic concepts with their previous knowledge, misconceptions occur naturally in their learning process. For example, the belief that a variable is merely a letter that abbreviates a phrase or represents a constant number is one such misconception. The formulation of expressions and equations using variables and symbols to solve for the unknown in a problem-solving question is one of the most important applications of algebraic expressions (Moss, Czocher, and Lamberg, 2018).

Cholily, Kamil, and Kusgiarohmah (2020) indicate that learners display challenges in presenting the correct expressions for algebraic problems. In their study they investigate whether Grade 7 learners properly understand and correctly communicate algebraic forms. Cholily et al (2020) establish that learners had trouble communicating using the correct algebraic language as they are incapable of formulating statements in algebraic form.

Furthermore, the Cholily et al (2020) investigation reveals that learners hold a conceptual error in the solution of algebraic expressions. It is observed that learners do not comprehend why the coefficient of an a is 1, and that they provide a solution of a^2 for $a + a$ instead of $2a$. It makes sense therefore to conclude that learners lack a fundamental understanding of algebraic expressions (Cholily et al, 2020).

2.4.2 Simplify by addition and subtraction.

The fundamental operations taught in primary school are addition and subtraction. Multiplication and division are likely to cause considerable challenges for learners, although addition and subtraction should not provide any concerns, as they are encountered throughout the primary years. The addition and subtraction of like terms in algebra are substantially identical to the addition and subtraction of operations in mathematics, with the exception that variables are now associated with the terms. Given the amount of time learners spend on addition and subtraction in primary school, one might assume they would have no struggle with this, but from experience, I note that this is not the case. Here I remind the reader that, in my study, I focus on the errors and misconceptions that learners make when adding and subtracting like terms in algebraic expressions.

Some researchers propose introducing algebraic concepts in early primary school to better prepare learners for future algebraic studies (National Council of Teachers of Mathematics, 2000). Algebra is present in a variety of forms throughout the primary curriculum, but interestingly it is not explicitly taught; therefore, I agree with NCTM (2000) that teaching algebra in very early grades could help learners better comprehend the forms of algebraic expressions. According to research, the deficit in mathematical and algebraic comprehension emerges as learners make the transition from concrete representation of numbers to abstract conceptualisation (Hulse, Daigle, Manzo, Braith, Harrison and Ottmar, 2019).

Addition and subtraction are crucial concepts that are introduced in the early grades using concrete objects (Van Zanten and Van den Heuvel-Panhuizen, 2021). During early algebra instruction, learners and teachers use physical objects to represent specific mathematical forms. Learners must determine the effect of actions and operations on objects. Van Zanten and Van den Heuvel-Panhuizen (2021) introduce an effective method for dealing with addition and subtraction of integers by using a primary school bus ride as an illustration. A description of the illustration includes: one learner is a bus driver, and the other learners are passengers. At demarcated bus stops in the classroom, learners exit, and others enter. All the learners in the bus are required to keep track of the total number of passengers. In this way the learners are simultaneously taught addition and subtraction.

In other research, Du Plessis (2018) investigated foundation phase teachers' curriculum and teaching resources for repeating patterns. The study included six Grade 2 teachers from three schools in the same district. The study focuses on curriculum, supporting materials, and classroom resource texts, not teachers or teaching. The research was part of a larger study on Foundation Phase sequencing in South Africa. Du Plessis (2018) explains that young learners' foundation phase numeracy includes repeating patterns that can lead to relational thinking and the advancement of early algebra. At the foundational level, learners are expected to be able to solve problems involving patterns, such as finding the next shape in a pattern, drawing given item patterns, and creating their own patterns without teacher guidance. Du Plessis (2018) discovered that the curriculum statement (CAPS 2010) and the supporting document *Numeracy Handbook for Foundation Phase Teachers* (NHFPT) (2012) merely states and does not fully support the logic of dealing with repeating patterns in foundation phase. This lack of good guidance for teaching topics that could lead to an early introduction of algebra closes the door on connecting algebra concepts at a young age, which research suggests is the optimal foundation.

Learners who struggle with fundamental mathematics in primary school often continue to struggle in high school unless the challenges are effectively addressed. Learners have trouble when adding and subtracting integers in high school. Utilising qualitative research, Makonye and Fakude (2016) investigated the errors and misconceptions that learners exhibit in the addition and subtraction of directed numbers. Errors and misconceptions in the work of Grade 8 learners were analysed, and then in-depth interviews were conducted. Makonye and Fakude (2016) show that the most challenging aspect of learning integers is when addition and subtraction involve different signs. They assert that learners do not use the number line presentations to demonstrate their understanding of the relationships between addition and subtraction of directed numbers (Makonye and Fakude, 2016). Integer addition and subtraction knowledge is crucial for understanding the concept of adding and subtracting like terms.

At the beginning of Grade 7, learners differentiate between like and unlike terms. Learners must be proficient in addition and subtraction of like terms per CAPS requirements.

Below are examples of addition and subtraction in performed in Grade 7.

1. $a + 2a = 3a$
2. $-4ab + 6ab - 2a = 2ab - 2a$

In Grade 7, it is emphasised that learners must examine the coefficient of the variable and may only add or subtract the coefficient if the variables are identical and, thus, are considered to be like terms. Learners are prohibited from adding or subtracting coefficients when variables are not identical, as in example two above. Learners in Grade 8 are expected to simplify algebraic expressions by adding and subtracting like terms, some of the examples from Grades 7 and 8 are listed below.

1. $7a - 2a = 5a$
2. $-xy^2 - 5xy^2 + 10xy = -6xy^2 + 10xy$
3. $\sqrt{64a^2 + 36a^2} = \sqrt{100a^2} = 10a$

In Grade 9, learners are expected to have mastered addition and subtraction of like terms from Grade 7 and Grade 8, as well as subtraction of one algebraic expression from another and simplification of polynomials such as:

1. $(-4x^3)(-2x^3) - (5x)(2x^5) = 8x^6 - 10x^6 = -2x^6$
2. Subtract $t^4 - 3t^2 + 7$ from $5t^4 + 7t^2 - 1$

$$5t^4 + 7t^2 - 1 - (t^4 - 3t^2 + 7)$$
$$5t^4 + 7t^2 - 1 - t^4 + 3t^2 - 7$$
$$4t^4 + 10t^2 - 8$$

3. $\sqrt[3]{19x^3 + (2x)^3} = \sqrt[3]{19x^3 + 8x^3} = \sqrt[3]{27x^3} = 3x$

Addition and subtraction of like terms resembles addition and subtraction of integers in many ways. Variables and powers are the only differences between integers and an algebraic expression. The learner must maintain the same variables and exponents for each variable when adding and subtracting like terms.

Figure 2.1 illustrates the relationship between addition and subtraction of integers and algebraic expressions. The examples of like and unlike terms are also provided.

Addition and subtraction of integers: $-7 + 3 = -4$

Addition and subtraction of algebraic expressions: $-7xy + 3xy = -4xy$ since all the variables are same, the two terms are considered like terms and may be added.

Additionally, it is essential to be able to recognize unlike terms; these terms may not be added for an example $-10xy + 4xyz = -10xy + 4xyz$. Since the variables are not identical, these terms are unlike terms and cannot be added.

Figure 2.1 addition and subtraction of integers

Teachers and researchers have used a variety of methods for teaching and presenting the concept of integer addition and subtraction to alleviate the challenges encountered by learners while adding and subtracting integers. Verzosa et al (2018) investigated the use of Alge Ops, a mobile app. This was used in a study to teach integer addition and subtraction, with 11 to 12-year-olds participating in the study. The app also helped with addition and subtraction of algebraic expressions. The study found that learners could work flexibly with written picture representations for integer addition and subtraction but struggled with symbolic representations. Learners struggled to connect and represent symbols, especially for subtraction. As Verzosa et al (2018) concluded that secondary mathematics requires integer operations, making it difficult for teachers to teach advanced topics such as algebraic expressions to learners without basic addition and subtraction skills.

Taban and Cadorna (2018) explicate the challenge that Grade 8, 9, and 10 learners experience when performing addition and subtraction of like terms. In their study the structural aspects of learners' algebraic solutions were analysed and evaluated. The algebra test contained ten problems. Testing was restricted to addition and subtraction of like terms, expanding brackets, simplifying expressions, distribution, solving linear equations, factorisation, solving rational equations, evaluating algebraic expressions, rational expressions, formula derivation, and solving radical equations. Figure 2.2 shows some examples of common errors made by learners.

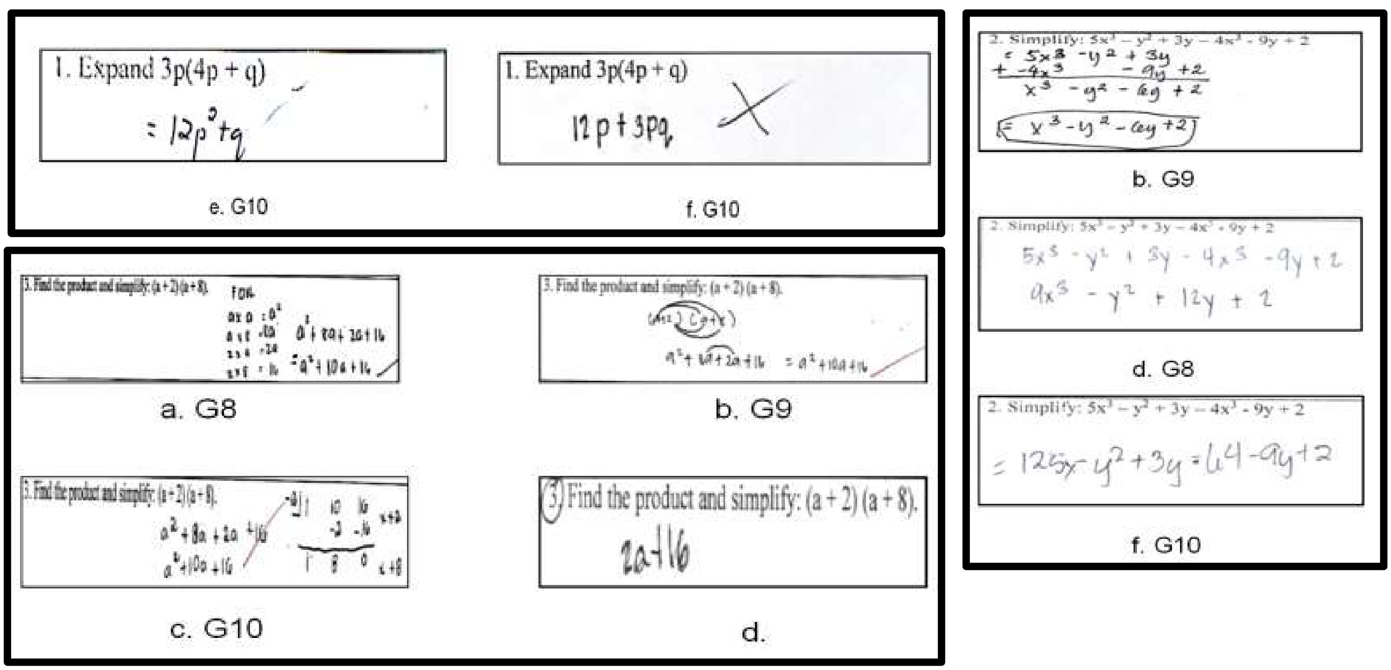


Figure 2.2 displaying some of the identified errors of Grade 8 to Grade 10 learners.

I have found the errors identified by Taban and Cadorna (2018) to also be present in the work of the learners I teach. They make the same errors: they ignore the negative sign and combine unlike terms. The authors found that the observed errors and misconceptions stem from learners' inability, across the three grade levels, to conduct addition and subtraction of like terms correctly (Taban and Cadorna, 2018). They attribute some of the errors to learners' conceptual understanding. They further establish that in addition to a lack of conceptual comprehension and expression manipulation skills, learners struggle with a conceptual understanding of the presented situation. The study suggests that mathematics teachers give learners ample learning chances that would help them gain confidence when completing mathematical exercises and improve their algebraic proficiency.

Muchoko, Jupri and Prabawanto (2019) show that learners struggle to visualise algebraic forms and apply the associativity and distributive rules of algebraic expressions when factorising or simplifying algebraic expressions. Muchoko et al (2019) examined algebraic visualisation in middle school. Some 30 learners were evaluated on their proficiency with algebraic operations. The challenges encountered by learners in factoring and simplifying algebraic expressions were discovered by analysing their work. The learners failed to combine like terms and re-arrange expressions in a way that was easy for them to recognise. In addition, learners disregarded the order of operations by performing exercises from left to right. Muchoko et al (2019) also propose that teachers should routinely examine classroom complications, such as identifying learners' learning difficulties and assisting them in overcoming these obstacles. Teachers are also expected to emphasize and analyse their learners' past knowledge to identify misconceptions, which is vital for the acquisition of new concepts (Muchoko et al, 2019).

It should be noted here that while researchers like Taban and Cadorna (2018) and Muchoko et al (2019) provide ways to assist teachers in effectively addressing these errors, my study employed a manipulative to assist learners in correcting these errors for themselves. I agree that teachers must know why learners carry errors and misconceptions to help them; by effectively doing so, teachers need to hold a strong understanding of the mathematical knowledge. However, the reader is reminded that the teacher is not the focus of my study. I am interested in understanding the errors that the learners made in the pre-test and whether the algebraic tiles as a manipulative helped them correct theses errors and misconceptions in the post test.

2.4.3 Simplify by multiplication and division.

In Grade 8 and 9, algebraic expressions can be simplified by multiplying or dividing by a monomial or integer using the distribute law. After learners have fully expanded and simplified an expression using the distributive law, it may be necessary to collect like terms and add or subtract them as appropriate.

Listed below are examples of simplifying algebraic expressions by multiplying or dividing by an integer or monomial.

1. $2(x + y) = 2x + 2y$
2. $4m(m - 1) - 5m(2m + 2) = 4m^2 - 4m - 10m^2 - 10m = -6m^2 - 14m$
3. $\frac{3p^5+15p^4-9p^3}{3p^2} = p^3 + 5p^2 - 3p$

The following two examples discovered by the researchers demonstrate that learners' misconceptions and errors stem from a concept they understand; however, they incorrectly apply previously learned rules (Brodie, 2014). Ncube, (2016) discovered a pattern of simply cross multiplication when learners were required to add the algebraic fractions: $\frac{x}{y} + \frac{w}{z} = xz + yw$. Zubainur and Ali (2018) also found a similar pattern that most learners are unable to solve algebraic fractions $\frac{x}{2} + \frac{x}{3} = \frac{x^2}{6}$ as learners multiplied the two fractions rather than finding a (lowest common denominator) LCD to add them.

2.4.4 Factorise algebraic expressions.

In Grade 9, a number of factorisation types are covered: highest common factor (HCF), grouping (a form of HCF), difference of two squares, and trinomial. To perform factorisation, the opposite of distribution is applied, one must identify the expression's factors. For an example $2x$ and $(x + 3)$ are the factors of $2x^2 + 6x$ because $2x\ (x + 3)$ must be multiplied to obtain $2x^2 + 6x$.

The following are examples of the various types of factorisation covered in Grade 9:

HCF (highest common factor)	Grouping
$2x + 4y = 2(x + 2y)$	$4x + 8y - 2bx - 4by$ $4(x + 2y) - 2b(x + 2y)$ $(x + 2y)(4 - 2b)$ $2(x + 2y)(2 - b)$

Difference of two squares	Trinomial
$16x^8 - y^4$ $(4x^4 + y^2)\,(4x^4 - y^2)$ $(4x^4 + y^2)\,(2x^2 - y)\,(2x^2 + \text{y})$	$x^2 + 8x - 12$ $(x + 6)(x - 2)$
Grouping $4x + 8y - 2bx - 4by$ $4(x + 2y) - 2b(x + 2y)$ $(x + 2y)(4 - 2b)$ $2(x + 2y)(2 - b)$	

Table 2.3 factorisation types covered in Grade 9.

Factorisation and simplification of algebraic expressions depend primarily on the addition and subtraction of like terms, as well as the distinction between like and unlike terms. Even though learners in Grade 7 were taught algebraic expressions, the same topic is taught at an advanced level in Grades 8 and 9, so they must understand the content of each grade to perform well in algebraic expressions in the next grade. This means that algebraic expressions as a concept requires prior knowledge from the previous grades. As stated in the CAPs document and from my own experience and understanding, I note that it is essential for learners to understand the addition and subtraction of like terms, as all other types of algebraic expression questions will ultimately require learners to add and subtract like terms. Mastering the addition and subtraction of like terms is a fundamental skill in algebraic expressions (Zubainur and Ali, 2018).

Error analysis of Grade 7 learners in simplifying algebraic expressions was the foundation of Lim's (2010) investigation. Learners were given 32 algebraic expression items adapted from previous research on algebraic expression errors and misconceptions. Lim (2010) found 12 typical common types of errors made by 265 male Grade 7 learners when simplifying algebraic expressions. Answer scripts and interview transcripts revealed algebraic expression simplification errors and misconceptions among learners. Learners made the most errors in order of operations, symbolic notation, negative pre-multiplier, and integer addition. Several factors could account for these errors and misconceptions, including failure to apply the order of operations with an emphasis on first dealing with brackets, orders, division, multiplication, addition and subtraction (BODMAS), rule of arithmetic in algebra, a lack of arithmetic background, interference from new learning, an inability to deal with direction and operation signs, difficulties with algebraic notation, and improper rule application (Lim, 2010). Some of these errors are displayed on table:

Common errors	Examples
Addition and subtraction of unlike terms.	$5xy + 4x - 7y = 2xy$
Incorrectly applies the laws of exponents when multiplying and dividing algebraic expressions.	$\frac{(4x^2y^4)^2}{\sqrt{64x^{16}}} = \frac{8x^4y^6}{8x^4} = y^6$ $2x(x^2 - 3x + 4) - 3x^2(x + 8)$ $= -2x^2 - 2x+12$
Conjoin error - wanting the final answer to contain only one term.	$\frac{2x + 4}{2} = x + 2 = 2x$
Confuses the various forms of factorisation.	$4x^2 - 8xy = (2x - 4)(2x + 4y)$
Illegal cancellation.	$\frac{5x + 10}{x + 2} = 5 + 5 = 10$

Table 2.4 illustrating some common errors and misconceptions in algebraic expressions.

The errors listed in the table above are among the most common errors made by Grade 9 learners when solving algebraic expressions. Lim (2010) emphasises that addition and subtraction of like terms forms the fundamentals in algebra and if learners cannot master these concepts, then their performance will impact other areas in the mathematics curriculum. Lim's (2010) research therefore emphasises the importance of addressing errors and misconceptions at Grade 7 level to prevent learners from carrying them onto further grades.

This section highlighted algebraic expressions content from Grades 7 to 9. It also discussed common algebraic expression errors from Grades 7 to 9. The section also investigates the relationship between integer operations and the concept of like and unlike terms in algebra. To accomplish my goals, it was necessary to review the existing literature on errors and misconceptions. The following section discusses various types of errors.

2.5 Slips, errors and misconceptions

Earlier I explained that there are three categories of mistakes. Here I remind the reader that slips can be corrected and are sporadic. However, multiple errors can result in misconceptions which indicate conceptual misunderstanding. The literature also explains that misconceptions lead to multiple errors.

2.5.1 Slips and Errors

Research indicates that slips are not a problem when executing mathematics exercises because learners are able to swiftly correct them when encouraged to do so (Luneta and Makonye, 2010). Even the most competent mathematicians can make slips, as has been noticed by (Luneta and Makonye, 2010). I occasionally make slips while writing on the whiteboard and am often corrected by the learners. Making these mistakes, however, does not hinder the learning process. Gardee (2015) asserts that slips are common; we all make them and as learners and doers of mathematics, they are sporadic. Research indicates that when learners are made aware of their slips, they quickly correct themselves or each other, and the likelihood of them repeating the slip is drastically reduced (Luneta and Makonye, 2010; Gardee, 2015). Gardee (2015) notes that probing or correcting slips would be a more suitable method of dealing with the mistake. An error is a mistake, slip, blunder or inaccuracy and a deviation from accuracy (Luneta and Makonye, 2010). Gardee and Brodie (2015) state that errors occur on a regular basis and are often constant. Errors are critical and play an important role in learning mathematics. Errors have the potential to develop into significant learning milestones (Nesher, 1987). Addressing the errors in learners' work helps them to develop and learn more effectively (Luneta and Makonye, 2010).

2.5.2 Misconceptions

Nesher (1987) defines misconception as a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected, and non-systematic errors. Luneta and Makonye (2010) assert that errors and misconceptions demonstrate that there is a structure to the misconceptions learners have and that these misconceptions are the result of earlier recognition as learners attempt to establish mathematical meanings. Brodie (2014) asserts that misconceptions result from learners applying what they have previously learnt in an incorrect manner. Gardee (2015) concurs with the assertions made by Brodie (2014) that that misconceptions are the outcome of correct prior knowledge interfering with new information.

Misconceptions do not only affect Grade 9 learners but addressing them is a global challenge. Using 20 years of data from the Trends in International Mathematics and Science Study (TIMSS) and TIMSS Advanced assessments, Neidorf, Arora, Erberber, Tsokodayi, and Mai (2020) investigated patterns of learner misconceptions, errors, and misunderstandings across education systems, grade levels, gender, and time (1995–2015). They selected 29 mathematics items from the TIMSS and TIMSS Advanced tests administered between 1995 and 2015 that examined learners' linear equation knowledge and errors. The incidence of certain types of learner errors and misconceptions at each grade level varied among the five nations studied. In each country and at each grade level, at least 50% of the learners displayed errors and misconceptions. This demonstrated that algebra misconceptions and errors are a global problem requiring immediate attention.

A persistence of errors and misconceptions that are not addressed in the lower grades continue to negatively impact performance in other areas of mathematics (Luneta and Makonye, 2010). In the Luneta and Makonye (2010) study, a test was administered to 45 Grade 12 learners in a rural school in the province of Limpopo who had used specially developed calculus teaching materials to determine any remaining errors and misconceptions on the topic. Frequent errors and misconceptions in algebra were a considerable impediment to learning calculus at Grade 12 level and therefore the focus of the study was to assess the errors and misconceptions that the learners had developed. The findings indicate that some learners could not correctly add or multiply elementary algebraic expressions. Learners demonstrated confusion when applying rules for indices, particularly for exponents with negative or fractional powers. Luneta and Makonye, (2010) found that most errors and misconceptions stem from lack of elementary algebra, which should have been mastered in the lower grades.

Ncube (2016) investigated the errors and misconceptions that Grade 9 learners hold in simplifying algebraic expressions. It should be noted that Ncube's (2016) study is important since it supports the coding tool of my study. Some 82 Grade 9 learners completed a 20-item test in elementary algebra. The test consisted of addition and subtraction of like terms, addition and subtraction of fractions, distribution of terms, multiplication of a binomial by a binomial, substitution, and an application question on algebraic expressions. Ncube was particularly interested in finding the common errors and misconceptions that learners make when solving algebraic expressions and then employing interviews to understand why they made these errors. Ncube (2016) identified common errors made by the learners and categorised them into six different types of errors, namely:

1. *Misapplication of rules*
2. *Misuse of the distributive property*
3. *Conjoin error.*
4. *Substituting letters by numbers*
5. *Sign errors*
6. *Misinterpretation of symbolic notation*

I now elaborate on each of the identified errors.

1. Misapplication of rules

Ncube, (2016) observed that learners create new rules for their own gain. Ncube explained that learners who learn algebraic rules with little or no conceptual understanding make more errors. Learners also misapply rules resulting in getting incorrect solutions. In some cases when simplifying the expression $2p - 4p(2p^2 - 1)$, learners inserted the bracket for $2p - 4p$ and change the question into multiplying two binomial $(2p - 4p)(2p^2 - 1)$ then they multiplied the binomial correctly. This procedure is incorrect and illustrates that the learner has changed the question by inventing new rules.

Learners are also confusing addition and subtraction rules with multiplication and division. In some cases, they are adding instead of multiplying. Due to the numerous mathematical rules and the interference of other concepts, it has been observed that learners tend to confuse the rules. In addition, they do not comprehend the rules, making it more difficult for them to keep up with the work being covered (Ncube, 2016).

Gardee (2015) concurs with some of Ncube's findings. Gardee (2015) examined how a Grade 9 mathematics teacher who participated in a professional development programme centered on learners' errors dealt with mathematical errors in her classroom. Videotapes were used to collect data over a two-year period that was then analysed qualitatively. Gardee (2015) examines in detail the types of errors that the mathematics teacher dealt with in her classroom and how the teacher dealt with them. Through analysis of the videotapes, Gardee (2015) identified four categories of errors that occurred in the teacher's classroom. These errors included the slips, errors resulting from misconceptions, language-related errors, and calculator-related errors. The video analysis revealed that the teacher handled more errors in a variety of ways. The teacher dealt with the errors through interrogating the learner about their responses; at some point the teacher also embraced the errors. The teacher encouraged learners to learn from their own errors by requesting additional responses to a question rather than correcting the error and providing the correct response.

Gardee and Ncube both identified the misapplication of rules error. Gardee (2015) indicated that misapplication of rules could also be found in the multiplication of variables. Most of the Grade 9 learners concluded that $a.a = 2a$, learners added rather than multiplying and applying law 1 of exponents (Gardee, 2015). In a similar manner, learners multiplied 2 to 15 in $2.a + a + 15 = 30a$ because they considered that the numbers were like terms in this question, they did not consider the use of BODMAS (Bracket, Order, Division, Multiplication, Addition, and Subtraction).

2. Misuse of the distributive property

Ncube (2016) found that the misapplication of the distributive property was the second most frequent error, accounting for 22% of learners' mistakes. Learners assigned the outside number to only one of the terms within the brackets, ignoring the second and third terms. The learners continued to multiply brackets even in cases where there was a plus or minus sign between the brackets. This means that the learners overgeneralised the distributive law by employing known rules in appropriate situations and then incorrectly adapting the known rules. Learners did not correctly perform expansion of a binomial. Ncube (2016) found that the learners held several misconceptions about expansion, which led them to commit numerous errors.

It should be noted that Ncube (2016) study supports findings from Egodawatte's (2011) that argues that in the application of the distributive property, learners can apply the rule to a simple, single statement without any attached terms, but have trouble applying it to complex expressions that include multiple terms. A learner may be able to provide the following correct solution $3(n + 7) = 3n + 21$ but will struggle to obtain a correct solution for $2p - 4p(2p^2 - 1)$.

Multiplication of algebraic expressions involving the square of a binomial presents the most difficulty for pre-service teachers (Marpa, 2019). Teachers' responses about the simplification of $(a + b)^2$ were one of the primary findings of a study conducted by Marpa (2019) on common errors in algebraic expressions. This study was conducted to identify frequent algebraic expression errors made by pre-service teachers. The majority of participants (61.26%) provided the solution of simply squaring the first and last terms of the binomial: $(a + b)^2 = a^2b^2$. This error demonstrates that many pre-service teachers have difficulty squaring binomials. If pre-service teachers struggle with the concept, then teaching it to learners is a challenging exercise.

Egodawatte (2011) identified the same error of simply squaring the first and last terms and labelled it invalid distribution. In this study, this error was made by the learner, not the teacher. Egodawatte (2011), additionally revealed that learners continue and oversimplify the solution of $(a+b)^2 = a^2b^2$ to ab or a^2+b^2.

3. Conjoin error.

This was the third-most frequent major error on the test written by Grade 9 learners. This error is the result of a misunderstanding of the concept underlying algebraic expressions. The learners paid no attention to the letters and concentrated solely on the numbers. They incorrectly added coefficients and constants, which is unacceptable.

Learners were able to remove brackets, but they were unable to distinguish between like and unlike terms when required to simplify further such as $2(3a+4)$ to $6a + 8 = 14a$. Ncube also observed that whenever learners saw the "+" sign, they simply added unlike terms together. In algebra, it is possible to have solutions with three terms, contrary to the rule in arithmetic that only answers with a single digit can be solutions. The learners then apply this arithmetic rule and add unlike terms to arrive at a single solution. According to Ncube (2016), failure to recognise like terms was the primary cause of the conjoin error.

4. Substituting letters by numbers

Ncube (2016) discovered that approximately 10% of the learners in her study made the substitution error, and, based on their responses Ncube concluded that there was a lack of knowledge regarding substitution. The learners in the study were given a value of one expression letter, and they were supposed to express the other number in terms of the unknown. However, the learners ended up presuming that the two expressions were equal, and they assigned numbers to the expressions they had been given. By assigning the same numeric value to the unknown variable, the learners did not respond to the question correctly.

5. Sign errors

Ncube found that 9% of learners committed errors involving signs. Learners made more errors when the coefficients held negative integers. Learners frequently commit errors when subtracting integers because they struggle with working with negative integer numbers. The study revealed that learners struggled with integers and operation symbols.

Ncube (2016) discovered that learners struggled with numbers and operation signs, since they were unable to provide the correct solution for $(8x^2 + 3x + 4) - (5x^2 - 7x + 2)$ and could not distribute the negative sign correctly. Lim's (2010) study supports this perception: he observed that learners routinely omit the negative sign while simplifying complex algebraic expressions, as well as when performing addition and subtraction operations on algebraic expressions. For example, in the expression $-6a + 3a = 9a$, learners ignore the fact that the 6 is negative and they simply add like terms.

Lim (2010) and Booth, McGinn, Barbieri and Young (2017) assert that learners of all ages are subject to sign error and that the sequencing of operations and use of brackets create a challenge. Learners solve algebraic expressions from left to right regardless of operation order (Booth et al, 2017). The learners struggle with integer addition and subtraction, which hinders their ability to simplify algebraic expressions. Learners were unable to simplify the algebraic expression such as $(2x^2 + 3x - 4) - (x^2 - 2x - 6)$, as they only collect like terms and added without distributing the negative sign in the second bracket Booth et al, (2017). The removal of the negative sign prevented learners in this study from subtracting the second expression from the first, resulting in incorrect results. Faramarzpoor (2020) conducted a study that investigated the causes of learners' errors in simplifying algebraic expressions. He also found that improper integer addition and subtraction calculations are the fundamental cause of so many errors and misconceptions among learners in algebra.

6. Misinterpretation of symbolic notation

In the Ncube (2016) study about 6% of learners made the mistake of misinterpreting symbolic notation. Learners misinterpreted terms containing invisible coefficients. Learners assumed that 0 represented terms with invisible coefficients because there were no numbers preceding the letters. Learners also failed to apply arithmetic meaning, as when asked to divide 2 by 2, they provided the correct answer of 1, but when asked to divide m by m, they provided the incorrect answer of 0.

It should be noted here that the six identified errors from Ncube's (2016) study supported the analysis of my data. The reader is advised that I provide more detail in the methodology chapter. In this section I provided thorough explanation of slips, errors, and misconceptions. I have provided a comprehensive literature review on algebraic expression slips, errors, and misconceptions in this section. Additionally, I must review the relevant literature to determine how to address the identified errors. The section that follows will discuss different strategies for addressing errors and misconceptions in algebra.

2.6 Addressing errors and misconceptions

To prevent learners from repeating errors and misconceptions, it is imperative to address them during the learning process. The literature highlights the role of the teacher in addressing the errors and misconceptions that learners hold when solving algebraic expressions. I must emphasise here that my study tried to investigate whether the intervention of employing a manipulative to address the learners' errors and misconceptions enabled them to make the necessary shifts. My focus is not on understanding the teacher's practices even though I am aware that my teaching promoted or constrained the learners' understanding of solving algebraic expressions. As a result, I will discuss the literature that focuses on the two most important considerations when dealing with misconceptions and errors. The teacher must first understand the learning styles of the learners. The teacher must, secondly, also understand the origins of mistakes and misunderstandings. I will elaborate on these two strategies for addressing errors and misunderstandings in the section that follows.

2.6.1 Teachers must understand learners' learning styles.

Teachers must have a thorough understanding of their learners' style of thinking to direct their lessons in a manner that tackles these misconceptions and errors (Enu and Ngcobo, 2020). Enu and Ngcobo (2020) recommend that teachers take note of the frequent occurrence of misconceptions in the classroom. They provide teachers with guidance on how to use formative assessment strategies as error-correcting tactics in the classroom to better assist learners who are struggling with mathematics. They discovered that errors in mathematics not only occur on quizzes, assignments, and end-of-term examinations, but virtually every day in the mathematics classroom. Errors are normal part of the learning process and should not be viewed as failures (Enu and Ngcobo, 2020). Learners do not commit errors out of stupidity, mathematical errors are valuable and could help learners in learning and gaining a deeper understanding of the idea (Enu and Ngcobo, 2020). Enu and Ngcobo (2020) show that errors are an integral part of the learning process, and as teachers we must avoid simply correcting them and instead embrace and accept them as a component of the learning process.

Egodawatte (2011) discuss the details of a collaborative teacher inquiry project that aimed to improve the quality of learning in Grade 9 applied mathematics while also enhancing professional development opportunities for teachers. Eleven schools participated in this project over the course of three semesters. Teachers, department heads, curriculum leaders, and administrators were among the participants. The aim of the study was to ensure that teachers gain a deeper understanding of their learners' thought processes and learners have more opportunities to rectify their own errors and misconceptions (Egodawatte, 2011). The findings suggest that if the follow-up questions are designed to allow learners to explain their reasoning and struggle with inconsistencies, then learners will be able to demonstrate their understanding of the material. Based on the findings of the study it was observed that engaging learners individually is more effective than doing so in large groups (Egodawatte, 2011).

2.6.2 Teachers must understand the sources of errors and misconceptions.

Researchers propose probing and discussing errors and misconceptions as the most common solution (Luneta and Makonye, 2010; Brodie, 2014). Luneta and Makonye (2010) showed that learners were able to reconsider their misconceptions after being questioned about the errors and misconceptions they committed during the calculus activity in Grade 12. Instead of re-teaching concepts, Brodie (2014) explains that the most effective way for teachers to address misconceptions is to comprehend their origins.

Booth et al (2017) also proposed conducting probing, however their probing method differs since they recommend that instead of beginning probing at the secondary level, the method should be implemented at the elementary level. In the early grades, they explained that teachers can encourage learners to explain correct and incorrect worked examples to help them focus on developing a solid conceptual foundation as well as the necessary procedural skills (Booth et al, 2017).

The Cholily, Kamil, and Kusgiarohmah (2020) study identify errors in learners' ability to comprehend and communicate the terminology of algebraic forms in written and spoken form. Using the case study research method, the researcher analyses Grade 7 term algebraic forms exercises and problems (learners aged 13 to 14). Cholily et al (2020) emphasise that algebra teachers should make every effort to ensure that their learners understand the prerequisite concepts correctly and without confusion. This supports learners in committing fewer errors when solving algebraic problems, boosting their self-confidence, and resulting in improved performance (Cholily et al, 2020).

Marpa (2019) proposed using error pattern analysis to address common errors in algebraic expressions. The error pattern analysis is a method of determining whether learners make consistent errors when performing basic computations online. Mathematics teachers can directly teach the correct procedure for solving and doing mathematics after identifying the pattern of learners' errors. Marpa (2019) also suggested that mathematics teachers should review previous topics before beginning a new lesson, especially if the previous topic is a prerequisite to the new one.

From the literature, it can be concluded that a summary of the strategies that researchers have employed to address mathematical misconceptions and errors that learners hold include:

1. Using mathematics manipulatives can assist learners with understanding abstract mathematical concepts.
2. Revising the concept, particularly if it is a prerequisite for the subsequent section.
3. Taking note of learners who continue to struggle with basic algebra concepts and offer them additional support.
4. Questioning learners about their errors and misconceptions so that teachers can determine where these errors and misconceptions originate from.
5. Acknowledging that errors and misconceptions are part of the learning and teaching process and should be embraced and used to enhance the learning process.
6. Using online software that alerts the learner immediately when they are attempting a question and making an error; error analysis helps in identifying the typical type of error that learners make, and teachers are aware of the common errors and patterns of errors.

The reader is advised that even though my research pertains to each of the aforementioned strategies for addressing errors and misconceptions, my study also lends itself to the employment of mathematical manipulatives and probing to address the errors and misconceptions that learners hold. Numerous studies have demonstrated the benefits of using concrete manipulatives in mathematics with learners (Graham, 2013).

In the following section, I elaborate on the various research-based strategies for addressing algebraic errors and misconceptions. The use of manipulatives to address errors and misconceptions was discussed in great detail. I concluded the section by summarising all the recommendations supported by research. My study employed algebra tiles, and I review literature in this area in the section below.

2.7 Algebra Tiles

Algebra tiles are two-dimensional shapes used to represent variables and constants (Salifu, 2022). The four colours that make up algebra tiles are yellow, blue, green, and red. yellow, blue, and green tiles represent positives, while only red tiles represent negatives (Salifu, 2022). The colours and shapes of the different algebra tiles are represented in Figure 2.3

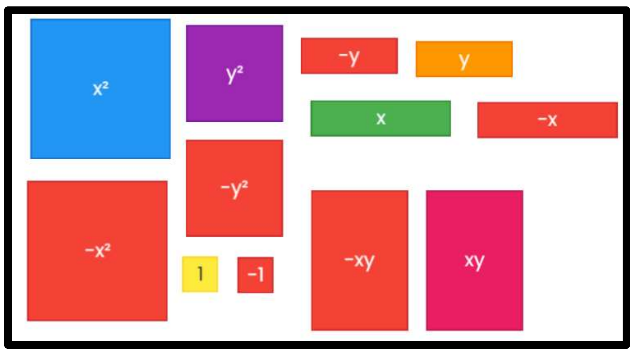


Figure 2.3: A representation of algebra tiles.

Thorton (1995) argued that teachers must be trained in the effective use of manipulative materials, and the curriculum and textbooks must be restructured away from more traditional methods. Pranada, Dizon and Sabalza (2019) assert that using algebra tiles as a teaching device in enhancing skills will help learners overcome their mathematics anxiety and will give them an easier way to solve or understand the concepts of algebra lessons like addition, subtraction, multiplication, and division of polynomials as well as factoring. Salifu (2022) asserts that introducing and continuing to use algebra tiles improves learners' algebra skills. Additionally, it is stressed that the tiles facilitate mental visualisation by incorporating concrete objects.

Learners can add, subtract, multiply, divide, simplify, and factorise algebraic expressions with the aid of algebra tiles, which prove to be a highly effective tool for assisting learners in manipulating algebraic expressions (Salifu, 2022). The tiles also provide an opportunity for learners to improve their performance with algebraic expressions (Ergene and Haser, 2021). Algebra tiles, facilitate the transition between manipulating algebraic expressions and manipulating concrete examples by allowing the creation of transitional situations (Sharp, 1995). Sharp (1995) asserts that the greatest influence of the tiles is not in improving test scores, but in providing alternative representation systems that are internalised by individual learners in a way that memorised facts and rote manipulations do not.

Hall (1999) demonstrated that algebra tiles are an effective tool for conceptual comprehension. Hall (1999) offered a workshop on the effective use of algebra tiles. The workshop presentation contained enough exercises for use with middle school or high school teachers and for also teachers new to using algebra tiles. The purpose of the workshop was to assist participants in effectively implementing algebra tiles in their classrooms. The participants were middle and high school teachers.

All participants were given algebra tiles, and a presentation explaining how to use them was shown using an overhead projector. In addition to having their own set of algebra tiles, teachers collaborated while learning the new skills. Below are examples of presentations from the workshop that utilised algebra tile to solve algebraic expressions. The following photos were extracted from Hall (1999) workshop, and they concisely demonstrate addition and subtraction of integers, simplification of algebraic expressions, multiplication of a binomial, and factorisation of a trinomial using algebra tiles.

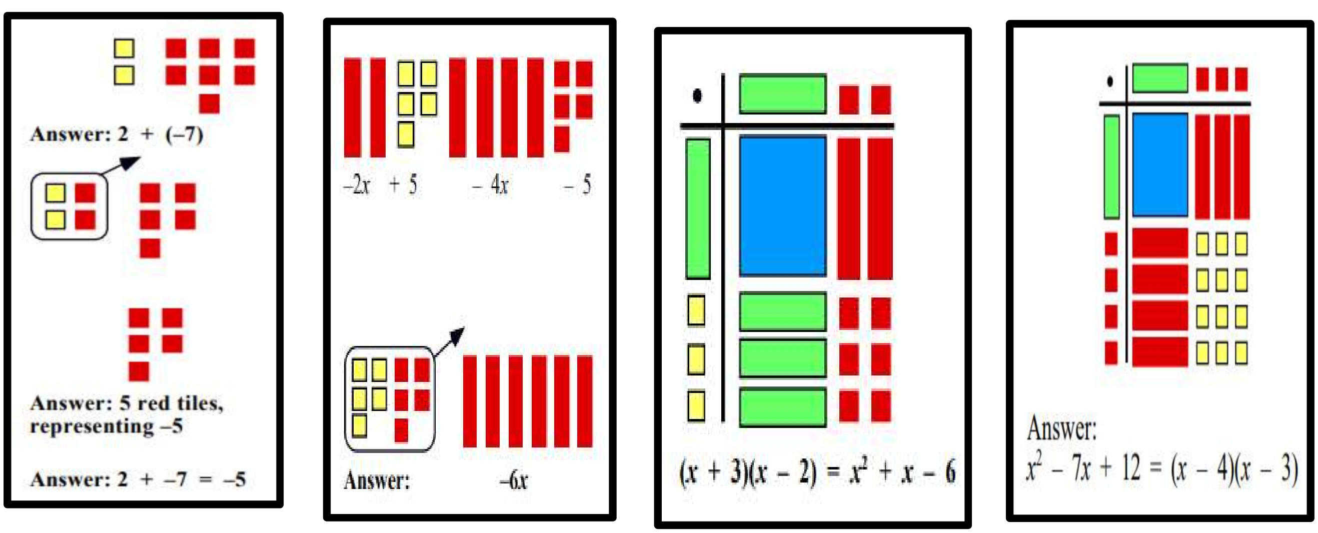


Figure 2.4 depicts the manipulation of algebra tiles.

The initial image depicts the addition of two integers. Adding 2 to -7. As shown in the image, the positive 2 is represented by two yellow ties, while the negative 7 is represented by red tiles.

A pair of yellow tiles are paired with a pair of red tiles (the cancellation procedure), and there are only five red tiles remaining, so the answer is -5. Hence $2 - 7 = -5$.

The second image depicts addition and subtraction expressions in algebra. The positive 5 illustrated by yellow squares is added to the negative 5 illustrated by red squares. The negative $2x$ red rectangle tiles are like the negative $4x$ red rectangle tiles. The positive 5 and negative 5 will pair using the procedure for cancellation. With negative $2x$ and negative $4x$, the tiles do not pair because they are all negative like terms, so they are added together to obtain $-6x$.

The final two images illustrate how to multiply a binomial by a binomial using tiles, as well as how to factorise a trinomial, which is the opposite of multiplying a binomial using the tiles. To obtain the solution, one must multiply each tile in the first column by each tile in the top row.

Thornton (1995) examined the effectiveness of a programme of instruction in polynomials and factoring which made extensive use of the algebra tiles. There were 132 participants. Every participant followed a six-week programme of instruction utilising the algebra tiles. The teacher-created polynomial and factoring tests and attitude questionnaires were used to evaluate learner achievement. The research showed that the programme of instruction using manipulative materials, such as algebra tiles, was effective, and that learners who received instruction using these concrete materials appeared to have a better understanding of the covered concepts. However, the study also showed that the six weeks assigned was limited and did not allow learners sufficient time to consolidate the concepts and skills learned using algebra tiles.

Farah and Kim (2017) have conclusively demonstrated that the use of algebra tiles is advantageous when adding and subtracting like terms. Farah and Kim (2017) conducted a study that tested the Colour Triciare Model for algebra addition and subtraction. The Colour Triciare Model is a manipulative that is very similar to the algebra tiles. This model consists of the three fundamental geometric shapes of triangle, circle, and square. All geometric shapes are comprised of two distinct colours, yellow and red. The red geometric shapes represent positive algebraic terms, whereas the yellow geometric shapes represent negative algebraic terms. The sample included 60 Grade 8 learners in Raub, Pahang. The samples were divided into two groups: a control group that used conventional methods and an experimental group that used the Colour Triciare Model. The learners took 26question pre- and post-tests. The Colour Triciare Model improved low-achieving learners' addition and subtraction skills in algebra. The benefit of using a mathematical manipulative such as algebra tiles and the Colour Triciare

Model is that it facilitates moderate and weak learners' comprehension of algebraic concepts by allowing them to visualise the terms or unknowns in a concrete manner (Farah and Kim, 2017).

The Ergene and Haser (2021) study supports the findings of Farah and Kim (2017). Ergene and Haser (2021) also showed that algebra tiles enhance performance. In their research they analysed the efficacy of the tiles for sixth-grade learners. In an experiment involving classes taught by the same mathematics teacher, one class was taught algebra without the use of algebra tiles, whereas the other class utilised algebra tiles throughout their seven-hour algebra lesson. It was concluded that learners in the experimental group performed better in the following areas: writing algebraic expressions for the given questions; determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions; performing operations with the given models of algebraic expressions; performing addition and subtraction with the given models of algebraic expressions; and writing given algebraic expressions as multiplicative expressions (Ergene and Haser, 2021).

Chaurasia (2019) worked with secondary school learners and showed that algebra tiles can help uninterested learners in the classroom since these tiles accommodate all types of learners. The study revealed that by manipulating mathematics tiles, learners were able to interact and engage with one another in groups. The research comprised of nine exercises performed by the participants. These exercises include integer addition and subtraction, addition and subtraction of like terms, simplification of algebraic expressions, linear equation, simplification of polynomials, and factoring.

Chaurasia (2019) found that algebra tiles motivate learners to investigate models. In every activity, learners modelled problems using their own algebra tiles, and they collaborated and worked in groups. The use of concrete materials improved learners' confidence and understanding, and this was true for all learning style groups. Learners were able to adapt knowledge obtained from concrete experiences to abstract situations (Chaurasia, 2019).

Wingett (2019) evaluated algebra tiles for Grade 9 learners in the United States who failed Algebra One for at least one semester. Two learner groups were given a pre- and post-test of a specific ability (binomial multiplication). The second group, but not the first, was instructed using algebra tiles. The second groups used algebra tiles for manipulating concrete, abstract, and representational models. The learners were expected to have drawn the tiles and written the algebraic terms indicated by the tiles in response to the exercise during the intervention lessons. Wingett (2019) showed that the algebra tile manipulators enhanced test scores. The experimental group learners grew more at ease with the manipulatives once they recognised that the tiles represented a concrete depiction of an abstract ability.

Gyampoh, Opoku-Mensah and Sefah (2020) employed a game board resembling algebraic tiles to address the algebraic expressions errors and misconceptions that learners may hold. Gyampoh et al (2020) explained that algebraic expressions posed many difficulties for learners, such as comprehending or manipulating them in accordance with accepted rules, procedures, or algorithms, assisting the learners in adding algebraic expressions using an improvised game board. Using the game board to add algebraic expressions, a colour is assigned to a given variable. Different colours cannot be added, signifying that different variables cannot be added (Gyampoh et al, 2020).

Gabina (2019) observed that when learners utilissed algebra tiles during algebraic expression lessons taught with manipulatives, their level of participation and comprehension of operations on algebraic expressions increased. Salifu, (2022)also assert that the use of algebra tiles is extremely beneficial in solving linear equations with one variable, the experimental groups (algebra tiles and balance model) outperformed their counterparts in the control group, who did not use any manipulatives, according to a post-test achievement comparison conducted by (Salifu, 2022).

The above research informed my study as I employed algebra tiles as an intervention manipulative to help learners address the errors that they made in the pre-test of the study. The use of algebra tiles serves to excite and motivate learners to create their own solutions; consequently, learners are actively engaged and responsible for their own learning. The reader is advised that the methods for manipulating algebraic expressions using algebra tiles is discussed in greater detail in the methodology section.

2.8 Conclusion

In this chapter, I propose a theory for understanding mathematics instruction and learning in the classroom. My analysis of algebraic errors and misconceptions is based on the zone of proximal development (ZPD). The ZPD theory is essential for the development of skills that facilitate the acquisition of mathematical information. Scaffolding with the use of manipulatives to represent answers through interaction with the goal of learning has proven to be an effective and much-needed theory in the teaching and learning of mathematics concepts.

I have argued in my discussion that re-teaching and simply providing the correct answer to a question are ineffective methods. The teacher is responsible for mediating and assisting learners in expressing meaning in a manner that addresses errors and misconceptions. I have also argued that mathematics teachers must observe learner work and identify errors and misconceptions in the classroom. In addition, I have argued that teachers must manage classroom interaction as a means of addressing errors and misconceptions. Most importantly, I have emphasised the significance of addressing errors and misconceptions using algebra tiles. I also argued that errors and misconceptions in algebraic expressions must be addressed with algebra tiles. In the following chapter, I will describe the research methodology to answer my research questions. The methodology chapter elaborates on how algebra tiles were utilised in the intervention lessons to assist learners with algebraic expressions exercises.

Chapter 3: Methodology

3.1 Introduction

In this chapter, I will describe the methods used to collect data, including the pre-test and post-test, algebra tile-based intervention lessons, and interviews conducted following the pre-test and the post-test. The issues of the validity and dependability of the findings are also addressed.

The purpose of education research is to diagnose various problems prevalent in our society and education system and to analyse them critically and logically (Basu, 2020). Research help teachers to evaluate their practice and make decisions. Research can support teachers in building new skills. In order to implement new educational strategies and evaluation techniques, the education system requires 21^{st} century skills. Teachers should be able to use educational research and scientific theories into their professional actions and decisions (Diery, Vogel, Knogler and Seidel, 2020). The reader is reminded that the primary objective of the study is to address the misconceptions and errors made by Grade 9 learners when solving algebraic expressions.

I am aware, based on my own experience, that Grade 9 learners hold several misconceptions, and introduce errors when solving algebraic expressions questions. I utilised both quantitative and qualitative data when conducting research. The pre-test and post-test was given before and after the intervention, respectively. The tests will measure the effectiveness of the intervention classes. In this context, tests conducted on learners will possess a quantitative nature, while qualitative data included interviews. I employed this combination of research methodologies to gain a better knowledge of the research problem and contribute to the formation of a more comprehensive view of the problem. I collected data by administering written pre- and post-tests, teaching lessons during the intervention programme and conducting interviews after the post-tests. In this chapter I will describe the changes that learners demonstrated on the post-test.

I will also provide an overview of the procedures that were used to gather data. This chapter also discusses the design of the study, the population and sample that was evaluated, and the data collection procedures. In addition, this chapter discusses the techniques that were used in order to analyse the data, the measures that were taken to guarantee the validity of the study, and the ethical concerns that were taken into consideration.

3.2 Research questions

The reader is reminded that the purpose of this study was to explore learner errors and misconceptions in algebraic expressions with Grade 9 learners. The following research questions acted as a guide throughout the data collection process; in this section, I will briefly describe how each question was addressed.

1. What are the errors and misconceptions that Grade 9 learners make in solving algebraic expressions prior to the intervention process?

2. Are there any changes in the Grade 9 pre-test and post-test results?

3. How do the results of the post-test relate to the content in the intervention?

To address the first question, the administered pre-test was recorded and evaluated, and errors and misconceptions were analysed. To address the second question, the learners' pre- and post-test scores were compared to determine whether they had improved. Lastly to address the third question the shifts were then carefully examined to determine how the intervention influenced the post-test results.

3.3 Research design

Harris and Brown (2010) assert that mixed methods research focuses on the potential capabilities of both qualitative and quantitative methods, enabling researchers to investigate multiple perspectives and discover correlations between the complex layers of numerous research questions. Abowitz and Toole (2010) assert that mixed method approaches are effective and that utilising two or more data collection methods whose validity and reliability problems counterbalance each other, enables us to triangulate in on the "true" result. Therefore, the use of a mixed-methods strategy resulted in considerable improvements to the findings of this research. Shorten and Smith (2017) assert that the use of a mixed research approach considerably increased the overall quality of the research study because it permits both the analysis of the contents and the interviewing of learners to gain their explanations.

Ivankova, Creswell and Stick (2006) explain that the mixed-methods sequential explanatory design implies collecting and analysing quantitative and then qualitative data in two consecutive phases within one study. The alternative approach is a sequential exploratory design; but this approach was not chosen because it would be difficult to mutually corroborate the findings from the quantitative method through triangulation, which is one of the main reasons for choosing a mixed method research design (Saunders, Lewis, and Thornhill, 2019).

The merging of quantitative and qualitative research methodologies has been employed since at least the 1960s and is known as triangulation (Jick, 1979). According to Jick (1979) triangulation offers researchers several major opportunities, and the employment of both qualitative and quantitative methodologies should be considered as complimentary rather than as rival camps. Therefore, explanatory-sequence design is the form of mixed method research that I employed in my study to facilitate more effective triangulation to answer the research questions. Creswell and Plano Clark (2011) assert that this technique entails collecting quantitative and qualitative data concurrently or sequentially, but with one type of data supporting the other.

The research design behind the collection of the quantitative data was a quasi-experiment, consisting of a one-group pre- and post-test. Harris, McGregor, Perencevich, Furuno, Zhu, Peterson and Finkelstein (2006) assert that quasi-experimental research includes non-random intervention trials and that the pre-test-post-test quasi-experimental study designs are used to evaluate the benefits of specific interventions. This study design is a quasi-experiment, consisting of a one-group pre- and post-test. Bryant, Gersten, Scammacca, Funk, Winter and Pool (2008) concur that small group interventions that emphasise arithmetic operations and quantitative reasoning exercise will benefit struggling learners. The Grade 9 participants were given a pre-test in order to investigate and analyse the errors and misconceptions that they commit; immediately following the pre-test, an intervention programme with numerous lessons was implemented with the same group of learners. They then completed a post-test to determine whether the misconceptions and errors identified in the pre-test had been addressed in the intervention lessons.

The first stage of the research consisted of analysing and identifying prevalent errors and misconceptions on the pre-tests. Berry (2008) states that at the outset of a course pre-tests can be administered to establish a topic knowledge baseline, which can then be correlated to an end-of-course test to measure knowledge additions. The pre-tests in the study served as the foundation for several of the intervention lessons taken. In the intervention programmes using algebra tiles, a sort of mathematical manipulative, was utilised. Algebra tiles are two-dimensional shapes used to represent variables and constants (Salifu, 2022).

Yellow, blue, green, and red are the four colours that make up algebra tiles. Positives are represented by yellow, blue, and green tiles, while negatives are represented by red tiles only. Figure 3.1 represents algebra tiles that learners utilised during intervention lessons.

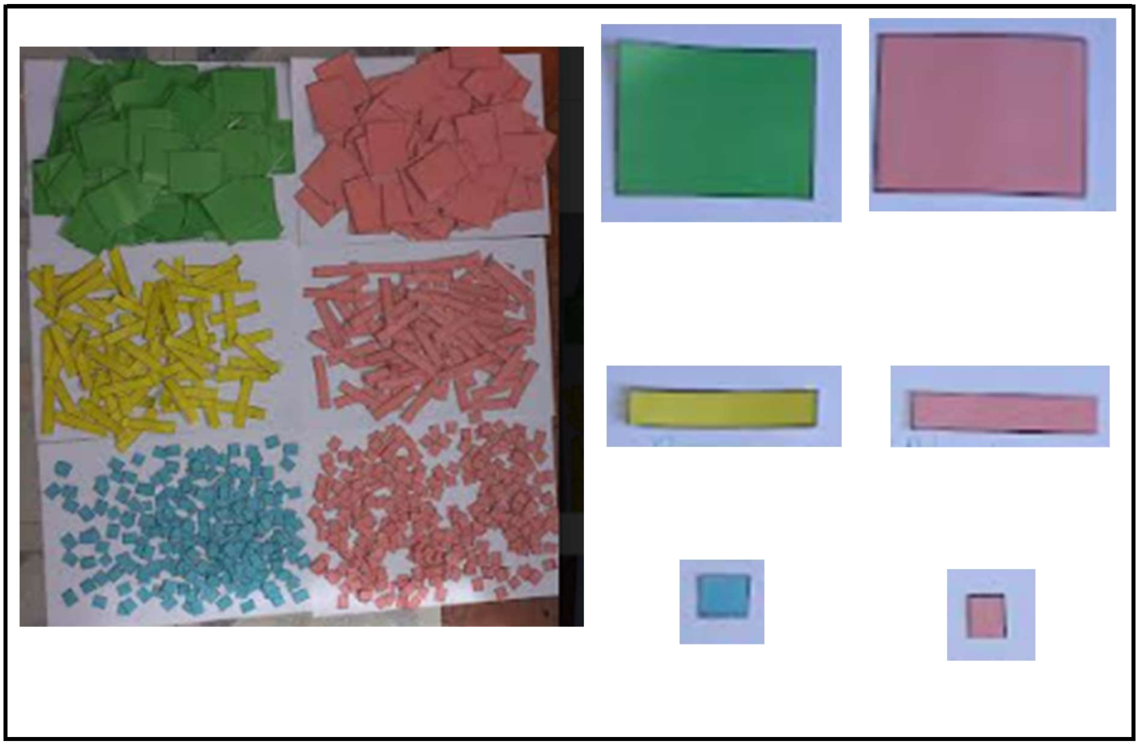


Figure 3.1: The algebra tiles used by the learners during the intervention lessons.

As seen on the table above the tiles represent algebraic expressions and I have explained the tiles further below.

The green large square tile represents an x^2 that is positive.

The red large square tile symbolises a negative x^2 value.

The small yellow rectangle represents an x that is positive.

The small red rectangle denotes negative x.

The small blue square represents a positive 1.

The small red square represents a negative 1.

Learners used the tiles to simplify algebraic expressions, divide and multiply polynomials, and factorise a variety of expressions. Sharp (1995) asserts that algebra tiles do not improve test scores, but they do provide learners with alternate representation systems that are internalised in ways that memorised information and rote manipulations do not. The tiles are intended to aid learners in the process of addressing algebraic expression-based problems.

The research design behind the collection of the qualitative data consisted of interviews with learners. I chose this strategy because I had planned to acquire a full explanation of the learners' responses to the pre-test and post-test. In my research, I used both the pre and post-tests; however, relying solely on the tests was insufficient, so in addition to that, I also used interviews to provide a more in-depth explanation of the research problem.

The second stage of the research was thus conducted after the post-tests that were given to the learners; the results of which were analysed, and extra interviews were conducted in order to verify and better comprehend their manner of thought. Throughout the duration of the research, my role was both researcher and teacher. As a teacher, it was my mission to assist learners in developing a better understanding of algebraic expressions and to address the errors and misconceptions that learners make when attempting to solve algebraic problems. I encouraged learners to make use of algebra tiles to manipulate them to complete the necessary algebraic exercise. As a researcher, it was my duty to determine whether the learners benefited from the intervention lessons through analysis of the data; and this was achieved through the implementation of a mixed-method research design to triangulate my quantitative and qualitative findings gather through my joint role in the process as a teacher and researcher.

3.4 Target population and sample

The target sample for this study consisted of Grade 9 learners aged 14 to 16 from a Pretoria East school. Since I am a teacher at this school, it was convenient for me to conduct the necessary research and provide a better-informed perspective. The study used algebra tiles, a manipulative mathematical tool, to address the misconceptions and errors that learners make when solving algebraic expressions.

The Grade 9 level was chosen because it is a deciding year in high school. This is the year when learners can choose whether to continue studying mathematics in the following grade or to study mathematical literacy. Since this decision is based on their performance in mathematics, this is a crucial year for guiding their career decisions. I was unable to introduce the tiles to all the classes simultaneously, nor could I request that the other teachers utilise these manipulatives due to time constraints. A rigid curriculum must be adhered to during school hours; consequently, I was required to conduct my research with a class I teach in the afternoon rather than during school hours. I teach 15 Grade 9 learners, but the study required a much larger sample size; seven learners were from a different class. All learners were able to attend the afternoon classes, and all participants gave their informed consent to participate.

In mixed method studies, semi-structured interviews are frequently employed to yield confirmatory results despite disparities in data collection, analysis, and interpretation techniques. In the post-test, the learners who scored the greatest improvement and those whose scores did not show any improvement were interviewed. Learners whose post-test scores increased by 18% were interviewed, as were those whose scores increased by 30% and one whose score decreased by 28%. The learner who showed only a 3% increase was also interviewed.

3.5 Data collection

Pre- and post-testing, as well as interviews, proven to be the most efficient approach for collecting data for my research. Utilising all three of these strategies enabled me to accumulate the most pertinent data necessary for the conclusion of my data collection.

I now detail each of the pertinent research instruments I employed.

3.6 Research instruments

3.6.1 Testing

Pre- and post-tests

The pre- and post-test was the primary study instrument that was used. The pre-test and post-test were analysed in detail in Chapter 4. The content in the tests regarding algebraic expressions originated from the Chapter 8 of the *Mind Bourne* textbook used in Grade 9. The reader is advised that the pre-test and post-test can be found in the appendix.

On each test, the following algebraic expressions-related topics were assessed:

a) Fundamental algebraic concepts including addition and subtraction of like terms.
b) Multiplying and dividing polynomials.
c) Products of binomials.
d) Factorisation types of algebraic expressions: highest common factor, grouping in pairs, common bracket, sign change rule, difference of two squares and trinomials.

I had selected these areas to be assessed because they form part of the curriculum for Grade 9 mathematics. The reader is reminded that the Curriculum Assessment Policy Statements

(CAPS) documents state that learners should:

"Know the language of algebra, be able to simplify and expand algebraic expressions and factorize algebraic expressions."

With the South African Curriculum Grade 9 learners are expected to be able to solve problems involving algebraic terminology, including extending, simplifying, and factoring algebraic expressions. The questions on the pre- and post-tests are like the questions Grade 9 learners will be asked on their school formal assessments; therefore, this exercise was consistent with what they normally do in school and was designed to improve their understanding. There was no statistically significant difference between the required mathematical cognitive levels and the percentages of levels on the pre- and post-test. The tests were within the required cognitive level for Grade 9 learners.

I utilised the pre-experimental one-shot case study in my research. A pre-experimental one-shot, according to Komala (2018), is a one-group study in which an experimental treatment (e.g., team teaching) is performed, followed by observation of the experimental participants (e.g., learners). The table below shows in detail how the pre-experimental one-shot case study was utilised in this study.

Group	Pre-test	Interviews	Treatment	Post-test	Interviews
A group of 22 Grade 9 learne rs	A 40 marks algebrai c expressions test. Polynomials, multiplication, division, and factorization were tested.	Based on their test scores, five learners were interviewed to learn more about the solutions they had presented.	Over the course of two weeks, algebraic tile-based intervention classes were administered.	A 40 marks algebrai c expressions test. Polynomials, multiplication, division, and factorization were tested.	Based on their test scores, another set of five learners were interviewed to learn more about the solutions they presented.

Table 3.1: One-group pre-test-post-test design.

The use of the pre-test allowed for an analysis of the learners' common errors and misconceptions. The goal was also to make learners aware of common errors and misconceptions when solving algebraic expressions. These misconceptions and errors were reviewed and studied prior to the intervention sessions, and they aided in the development of some of the intervention lessons. The post-test findings help establish whether the intervention lessons were beneficial. Table 3.2 provides overviews of the questions that were asked during the tests. The test questions are also provided in appendix A.

Topics covered	Content	Pre-test (40 marks)	Post-test (40 marks)
Revision concepts	Expression manipulation in algebra.	Q1 and Q2	Q1
Polynomials	Using addition and subtraction to simplify algebraic expressions.	Q3 (a) and (b) and (f)	Q2 (a) (1-4)
Products of binomials	Expanding and simplifying Distribute law. Square a binomial.	Q3 (c) and (d) and Question 4 all questions	Q2 (a) (5) and Q3 all questions
Division of algebraic expression	Use of exponential laws to divide and simplify. Dividing polynomials by monomials	Q3 (e)	Q2 (a) (7)
Algebraic expressions can be simplified by factoring.	Highest common factor Grouping Common bracket Difference of two squares Trinomials	Question 5-all questions	Question 4-all questions

Table 3.2: Content covered on both tests.

3.6.2 Interviews

Interviews, according to Lee, Capraro and Capraro (2018) have become one of the most important research procedures in mathematics education because researchers in this field ask questions, collect responses, and then attempt to interpret the responses. Rohid and Rusmawati (2019) state that interviews are vital for understanding learner cognition by scrutinising the words and actions of interviewers to analyse the mind of a learner.

I conducted interviews with 10 learners because I was more interested in their reasons for their written test responses. The purpose of the interviews was to collect as many explanations of the learners' work as possible, and a range of explanations based on test scores were necessary for gaining a deeper understanding and enriching the acquired data.

The purpose of the interviews conducted was to identify all the common algebraic expression errors and misconceptions made by learners and to obtain insight into what they are thinking when they make these errors. The purpose of the interviews was also to get insight into how the participants perceived the intervention lesson and to assess the effectiveness of the intervention lessons.

I developed at least five interview questions based on the learners' responses. The viewpoints were amended and adjusted properly in response to the learners' answers.

The following is a list of semi-structured interview questions.

1. Please explain how you arrived at the offered answer.
2. Why did you arrive at this conclusion?
3. What methods did you use to get your conclusions?
4. How do you differentiate the many types of factorisation?
5. Please mention any additional test-taking tactics you employed.

I attempted to be as objective as possible during the interviewing process, because as teacher and the researcher, I was expected to pay close attention to what the learners had to say without attempting to influence their responses.

3.6.3 Intervention lessons

The intervention lesson emphasises teaching learners how to utilise algebra tiles and to solve algebraic expression exercise and present their own models with guidance. It was primarily with the intention of targeting the learners' zone of proximal development (ZPD) that manipulatives were incorporated into mathematics instruction. The reader is reminded that the zone of proximal development was thoroughly discussed in Chapter 2. Marita and Hord (2017) defined visual representation interventions as any intervention that primarily uses a visual to scaffold learning. Thornton (1995) asserts that manipulative algebra tiles have potential as a tool to aid instruction in algebra courses and indications are that learners of all learning styles preference are well served by using these concrete materials. Dekker and Elshout-Mohr (2004), in their research on teacher interventions meant to improve mathematical levels, agreed that interactions-based interventions are more effective and helpful for learners. My research approach was structured around an intervention programme based on the concept of altering algebraic expressions by employing algebra tiles.

The learners in my study used an exploratory method to model the problems and build mutual understanding through group and peer conversations. The learners modelled algebraic expressions and completed the exercises using algebra tiles. The learners reported that algebra tiles facilitated meaningful learning, enhanced their conceptual understanding, and made lessons enjoyable.

The intervention lesson comprised of 10 lessons taught over a two-week period. Each lesson lasted for an hour. Each day, the intervention took a different form, which was fully determined by the algebraic expressions that we had to model and solve for on that particular day. The most significant aspect of the initial courses was recognising frequent errors and misconceptions and determining how to address them. The remaining lessons focused on familiarising learners with the use of algebraic tiles in solving algebraic expressions. Table 3.3 provides a brief description of all the intervention classes that occurred over the course of two weeks.

Intervention lessons	**Lesson objectives**
1	Examining pre-test misconceptions and errors. I assessed all the pre-tests, and the learners made a variety of errors, so in the first session, we analysed all the errors and misconceptions so the learners could identify their errors. This first session was similar to the corrections, but it also included a discussion of pre-test errors. This helped learners identify algebraic expression errors and misconceptions. This facilitated an in-depth discussion on errors and misconceptions; making learners aware of them.
2	Modelling addition and subtraction of like terms using algebra tiles by shifting from an abstract manipulative to concrete algebra tiles.
3-4	Instead of FOIL (distributive law), learners used algebra tiles to find products and square binomials in lessons three and four.
5-6	Dividing polynomials by integers and monomials and modelling this problem with algebra tiles. This exercise took place over two days.

	Learners also modelled various problems and completed the selected exercise.
7	Factorising utilising algebraic tiles to get the highest common factor and enforce the rectangular solution method.
8	This is the discovery of the distributive property and factorisation by the greatest common factor. Learners made this discovery independently and quickly realised the direct connection between the distributive law and the highest common factor.
9	Using algebraic concepts to factorise the difference of two squares, the learner understands that to construct a perfect rectangle, additional tiles that were not part of the original calculation must be added; however, they can be cancelled out by adding additional tiles of the opposite sign.
10	Trinomial factorisation: learners were comfortable arranging algebra tiles to get solutions. The idea is that each time the solution must be in a rectangle. The length and width of the rectangle will be the solution. The learners are now aware of the interlinked between all types of factorisation; they could do backwards by finding the factors from the solution or the solution from the factors.

Table 3.3 Intervention lessons

In each lesson, an expression-related concept was covered. The modelling of the problem using algebraic tiles was explained, and then additional examples for learners to model were displayed on the whiteboard; learners modelled at least four of the problems prior to completing a specific textbook exercise using algebra tiles individually, this was done in groups and with peers. The final step was to correct the exercises as a class discussion, focusing on only selected questions, and verify that the modelling was done correctly. The objective was to correct recognised errors and misconceptions. This required learners to use both their concrete algebra tiles and critical reasoning to get solutions. As the session progressed and the learners were familiar with arranging the algebra tiles to provide solutions, their approaches for obtaining the rectangular solution with factors diverged.

3.7 **Data analysis**

Data analysis is the process of developing answers to questions through the examination and interpretation of data (Sharma, (2018). This section examines the data collection methodology. This data was used to determine how algebra tiles enhanced the test performance of learners. In Chapter 4, a comprehensive analysis of the performance will be provided.

Finding the proper analytical framework for the data analysis or developing the best categories for the data was challenging. I discovered that the errors and misconceptions identified by Ncube (2016) provided the best lens for describing errors and misconceptions in algebraic expressions. I analysed my data using these categories.

The six errors identified by Ncube (2016) were used to create a coding tool for the errors identified in my own study. All categories identified by Ncube are included in the coding system, all the errors were also identified during the study besides the conjoining error because it was less frequent in my research. The reader is reminded that my study further employed an intervention using algebra tiles to assist learners in addressing the errors that they made in the pre-test.

The following table provides categories of errors identified in the Ncube (2016) study.

Categories	**Description for errors**	**Examples of common errors**
Conjoin error	Lacked understanding of like terms and unlike terms.	1) $3c + 4d \ = 7cd$ 2) $8a + \ 6 \ = 14a$
Misapplication of rules	Adding instead of multiplying, incorrect application of previously learned procedures.	1) $4m \times m = 5\text{m}$ 2) $\frac{x}{y} + \frac{w}{z} = xz + yw$

Misinterpretation of symbolic notation	Learners showed partial misunderstanding of factorisation.	1) $\frac{ma+mb}{m+md} = \frac{a+b}{d}$
Invalid distribution of brackets	Incomplete expansion of brackets and learners overgeneralised the distributive law.	1) $2(3a + 4) = 6a + 4$ 2) $(2m - n) + n = 2mn - n^2$
Sign errors	Problems working with integers and misunderstanding operation signs.	1) $(8x^2 + 3x + 4) - (5x^2 - 7x + 2)$ $= 3x^2 - 4x + 6$
Substituting letters by numeric values	Substituting letters with random numbers.	1) if $b + d = 6$ *then* $b + d + e = ?$ Learner solution $b + d + e = 9$ Learner assuming b, d, and e are identical

Table: 3.4 Errors identified by Ncube (2016)

The reader needs to be advised that I created the table in accordance with Ncube's findings (2016). I adapted the error types identified by Ncube (2016) by further categorising and describing them. The following is a summary of the six error categories identified by Ncube (2016) as utilised to code my data. The reader is reminded that I discussed these categories in more detail in the literature review.

3.7.1 Conjoin error.

Ncube (2016) suggested that failure to recognise like terms was the primary cause of this error. Most learners simply added unlike terms. Learners frequently desire a one term solution. In the example where 3c and 4d are added to get 7cd, they have incorrectly combined everything, including unlike terms. Learners added variable to constants to produce a single solution term. Based on my personal experience, regardless of unlike terms, learners are convinced that only a single term can be a solution.

3.7.2 Misapplication of rules

Ncube suggested that improper rule application is the primary cause of errors in Grade 9 algebraic expression simplification. This error resulted from previously acquired knowledge, according to Ncube. Learners often apply rules in inappropriate situations. Learners here misapplying a concept. This includes exponential rules like multiplying the coefficient instead of the powers. Learners add the numerator and denominator instead of using an LCD when adding or subtracting fractions with different denominators. This error usually indicates that the learner already has a foundation upon which the teacher must build, so teachers must verify that both the prerequisite knowledge and, most importantly, the application questions are understood.

3.7.3 Misinterpretation of symbolic notation

The reason for this error, according to Ncube, is that learners failed to make the connection between algebra and arithmetic. This error is most common among Grade 9 learners. Learners consider a coefficient to be zero when it is one. Learners can confidently determine that $2 \div 2 = 1$ but struggle to divide $m \div m = 0$, instead of 1. As teachers, we must stress the similarities between the concepts of integers and algebraic expressions. If learners comprehend how to divide integers with the same value, they should have no trouble dividing expressions with the same value, as the underlying concepts are identical.

3.7.4 Invalid distribution of brackets

Learners frequently commit three distribution errors when expanding an algebraic expression: incomplete distribution, failure to distribute the negative sign and squaring only the first and last term when expanding a binomial. Like the preceding illustrative example, learners would only perform distribution for the first term and not complete the distribution procedure for the remaining terms, resulting in incorrect solutions.

In algebraic expressions, it is extremely common for learners to disregard the negative sign outside the bracket and simply collect like terms. Another common error made by learners when expanding binomials is to square the first and last terms without considering the middle term. This demonstrates that learners do not realise that expanding a binomial always results in a trinomial. The errors cannot be ignored and must be addressed appropriately for the benefit of the learners.

3.7.5 Sign errors

According to Ncube, the cause of this error is a lack of understanding of what a variable represents and a lack of experience operating with directed numbers. In solving algebraic expressions questions, it is extremely common for learners to ignore the negative sign outside the brackets and simply group like terms together. When expanding binomials, another common mistake made by learners is to square the first and last terms without considering the middle term. This indicates that learners are unaware that expanding a binomial always yields a trinomial. The errors cannot be ignored and must be appropriately addressed for the learners' benefit.

3.7.6 Substituting letters by numeric values

Learners frequently make the error of arbitrarily assuming values for unknown variables. Learners are only given the value of one expression and failed to express the other variable in terms of the Unknown. As demonstrated in the last example in the table, the learners assumed that all letters are equal to six and replaced all letters with six, even though the question case stated that only $b + d = 6$.

Throughout my study I encountered the six errors listed above. The conjoin error was not as common in my study so I disregarded it. In my analysis I discussed in detail the five common errors committed and the frequency of each error.

3.8 Analysis of tests

I marked and assessed both the pre- and post-tests. I used a Microsoft excel spreadsheet as the primary tool to develop tables and graphs in analysing the tests. The learners' pre-test and post-test scores, as well as the difference between the two, were recorded. To establish whether there were shifts because of the intervention lessons we offered; a weighted average of both tests was entered in the spreadsheet. A double bar graph was produced to compare all pre- and post-test scores for each participant. Each table and graph should be self-explanatory; they should be understandable without the need to consult the accompanying text, as stated by Duquia, Bonamigo, González-Chica, and Martínez-Mesa (2014). I used the tables and graphs because it is easier to spot trends when data is displayed graphically as opposed to numerically, graphs are employed to represent data of the study. The reader is advised that a comprehensive study of the tables and graphs can be found in Chapter 4.

Coding enables researchers to conduct analyses of their data and uncover previously unrevealed insights. Furthermore, I coded my data by identifying common errors and misconceptions on both the pre- and post-tests. The frequency of the errors and misconceptions was recorded. This study's primary objective is to address these errors and misconceptions. The fourth chapter provides a full explanation of the common misconceptions and errors, as well as how algebra tiles attempted to address them.

The codes were written out in excel as a table to simplify comparison of the pre-test and post-test results. The test scores for each question were recorded on both the pre-tests and the post-tests, and these values were averaged across all the categories that were discovered. It is critical to have a full grasp of a learner's performance on each section of each test to determine whether the intervention lessons caused any changes in a learner's performance.

3.9 Analysis of Interviews

All 10 interviews were audio recorded to have an exact record of the dialogue between the researcher and each learner. The interviews were very helpful in supporting the test answers that the learners submitted. The interviews were not meticulously transcribed; rather, key statements from the interviews were chosen to support the quantitative data.

3.10 Rigour

Cypress (2017) states that researchers consider the rigour of qualitative research to be equivalent to the principles of reliability and validity, and that all three are essential attributes. My research design, data collecting, analysis, and reporting is of the required standard. This study design shows in detail how I answered my research questions, as well as how the data was collected and analysed.

3.10.1 Validity

The application of algebra tiles as a strategy for addressing errors and misconceptions and overcoming algebraic challenges was the primary focus of my research. The pre- and post-tests that the learners wrote lacked bias, were age-appropriate, did not discriminate, were neither excessively brief nor excessively lengthy, and nevertheless met the minimum quality standards. Test questions were written down and numbered in the correct sequence. The mathematics cognitive level supplied by the South African Government Grade 9 curriculum is called the National Curriculum and Assessment Policy Statement (CAPS). It was also utilised to confirm the grade-appropriateness of the questions; the content of both assessments was identical, but the tests themselves were not identical. I was able to determine the intervention's effectiveness by measuring whether learner performance improved.

In the study, I was the only researcher, which meant that the quality of my study was largely reliant on my own talents and ability to identify areas of potential subjectivity and prejudice. I am also a teacher at the institution where the research was conducted and worked with my supervisor and research colleague to analyse and verify the findings.

3.10.2 Reliability

A similar test was employed in the post-test however the structure and number of questions changed. Maintaining the identical nature of the pre-test and post-test would have compromised the reliability of the data, as learners had become familiar with the pre-test questions due to the in-depth discussion that occurred during the first intervention class session. I intended to determine if the learners 's improvement could be attributed to the competencies they acquired during the intervention session, as opposed to being solely attributable to the pre-test corrections.

The questions on the two tests were not the same. Each question on the test had clear instructions on what was expected of the learners and all questions were clearly worded to avoid any misinterpretation. Anxiety over the test was avoided by reassuring learners that the test was not for marks and the tests were only administered by me in both cases under similar circumstances with the same class.

It would be misleading to claim that my study was 100% reliable because it would have been difficult to replicate all the parameters of a small-scale study. Classroom-based study can be difficult at times because conditions are never constant: learners were absent from the intervention lessons in some cases owing to sporting events or school absences, but all learners wrote both tests. Also, the intervention lesson model research suggests that such investigations are promising Bryant et al (2020).

That the tests were valid and reliable, making it sufficient to claim that this study and its results meet the requirement for validity, which enabled me to relate a story about how Grade 9 learners addressed errors and misconceptions by using algebra tiles. In addition, the study also presents the numerous approaches that learners use to solve algebraic problems both before and after the intervention.

3.10.3 Generalisability

The purpose of my study was not to make generalisations or build theory, but rather to gain a thorough grasp of how Grade 9 learners might address the errors and misconceptions that they make when solving algebraic expressions: using a mathematical manipulative. This research will therefore inform how I teach algebraic expressions. I will be able to modify my own teaching and incorporate additional mathematical manipulatives to guarantee that the learners derive maximum benefit and fully understand the concept.

3.11 Ethical considerations

Before beginning this research, I applied to the Ethical Committee for Human Research and received approval to proceed. The application was processed on 17 February 2022, and the study was given the protocol number **2022ECE011M**. Appendix B contains the letter granting clearance on ethical concerns.

In addition to the university committee and application, the Gauteng Department of Education (GDE) was also informed about the nature of my study. The school principal, parents, and learners were also informed about the study, and permission to conduct my research was sought from all relevant parties. Official letters describing the nature of my study were provided to these parties, and learners in the school were informed about what their participation would entail.

The parties involved were informed that the teachings and test were in line with the CAPS curriculum. Before completing and submitting the consent form, learners and their parents were provided with all important study information for evaluation at home. To protect their identities learners' actual names were kept a secret, learners were given numbers ranging from learner one to learner 22. The classes were held after school, allowing learners to gain a deeper knowledge of algebraic expressions that had previously been taught using traditional methods by the school's three distinct mathematics teachers.

All parents and learners were informed that their participation was fully voluntary and that they could withdraw at any time without repercussions. In addition, participants were advised that they would not receive monetary remuneration, but that the information acquired throughout the study would be kept confidential and destroyed after five years. Appendices C and D contain copies of the letters and consent forms. Initially, the group's results revealed positive gains. The lesson plans used for the intervention group were shared with other teachers at my

Grade 9 school, along with explanations and guidance on how to conduct lessons if they wished to use such an intervention in the future. This was done to ensure that all learners would benefit from the intervention.

3.12 Conclusion

This chapter is dedicated to discussing the methods utilized in the report. The chapter explained the techniques employed for data gathering and analysis, encompassing both quantitative and qualitative approaches. Additionally, the merits of employing a mixed methodology were expounded upon. The questions that were addressed on the pre- and post-tests were also discussed. The chapter also provided a more comprehensive explanation of the algebra tiles that were utilized during the intervention sessions.

The chapter included a detailed discussion of the target population, sample and the methodology employed for data analysis. The discussion also included the analysis of the instruments and tactics employed to ensure the adherence to ethical standards, as well as the establishment of reliability and validity in the study.

Chapter 4: Analysis and findings

4.1 Introduction

As discussed in the previous chapter, the data for this study were derived from the errors and misconceptions committed by 22 Grade 9 learners while responding to algebraic expressions-based tests and interviews. In this section, I will present the findings of my study on the intervention lessons implemented based on the pre- and post-test scores of my learners. In addition, I will reveal the findings of the interviews done following the pre-test and post-test.

The study began with the learners completing a pre-test that I briefly did a first analysis on. I then conducted an intervention which involved me teaching the concept of algebraic expressions using algebra tiles to address the errors that the learners had made in the pre-test. The reader is reminded that algebra tiles are square and rectangular tiles that represent numbers and variables and can be used as manipulatives to solve algebraic expression questions. Algebra tiles are a form of mathematical manipulative that facilitates learners' comprehension of algebraic topics (Salifu, 2022)

The learners then wrote a post-test to determine whether the intervention was successful. I then selected a sample of five learners to interview. Details of these forms of data collection can be found in the methodology chapter. My focus in the intervention lessons was to address errors and misconceptions in algebraic expressions, with a particular emphasis on addition and subtraction of like terms. The reader is reminded that in these lessons learners had completed exercises from the prescribed Grade 9 textbook called *Mindbourne*. The topics covered during the lessons included polynomials, addition and subtraction of like terms, multiplication and division of expressions and factorisation. The lessons on factorisation included factorising a trinomial, a difference of two squares, determining the highest common factor, eliminating a common bracket, and grouping. Through conversations with the learners, the intervention lessons tried to address the errors and misconceptions. Learners in these classes worked through problems using algebra tiles and then explained their thought process and methodology.

The purpose of using algebra tiles was to teach the topic so that learners have a better grasp and do not confuse the rules of algebraic expressions. The usage of algebra tiles was intended to assist learners in identifying and addressing any errors and misconceptions they may have had. All 10 lessons were designed to assist in addressing these misconceptions. In addition, I conducted interviews to gain a deeper knowledge of the learners' work. Learners used these tiles to better visualise algebraic relationships. The usage of the algebra tiles was to assist learners to address errors in subtraction and addition, multiplication, division, and factorisation of algebraic expressions. Algebra tiles were intended to aid learners in developing skills and alternative approaches for solving algebraic expressions.

This chapter begins with a discussion of the overall test performance of the learners. Graphs and tables are used to compare the pre- and post-test results as a discussion structure. I then performed a comprehensive error analysis explanation of the actual causes of errors and misconceptions; a framework comprising these causes is subsequently explored in detail. I briefly describe how the lessons addressed the errors and misconceptions. I also provided brief commentary on the interview sessions. I additionally offered an analysis on the general findings of my study. I then provide a comprehensive review and analysis of the effect of the intervention lesson on the overall performance of the learners.

4.2 Pre- and post-test performance of learners

All learners were required to take a pre- and post-test, each worth 45 points and based on the content of algebraic expressions. The contents of both tests were similar, but not identical. After the intervention lessons had taken place, the pre-test results were compared to the post-test results. Table 4.1 summarises the performance of learners on their pre- and post-tests. Pre-test results indicated that learners initially performed poorly, with an average score of 48%; however, post-test results indicate remarkable progress, with a 18%-point gain resulting in an average score of 66%. The gains indicate that the algebra tile-based 10-lesson intervention strategy was successful. Considering this, it is vital to analyse the magnitude of the disparities in greater depth, as well as the various methods by which learners acquired the capacity to solve algebraic expressions. The table below displays the performance of all 22 learners on both the pre-test and post-test.

22 Learners	**Pre-test %**	**Post-test %**	**Gains %**
Mean%	48	66	18

Table 4.1: The average overall performance on the pre- and post-tests.

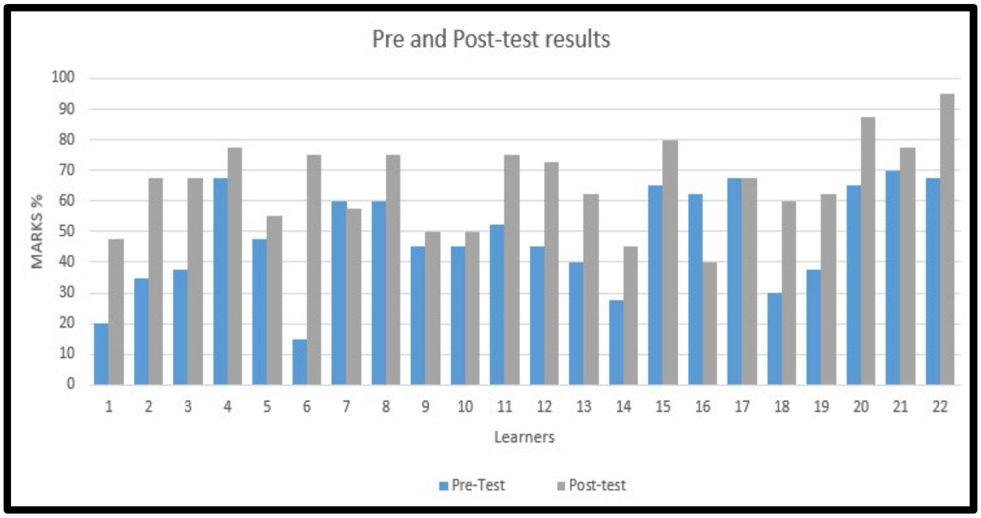


Figure 4.1: Learners' performance on each test

The results of each learner's performance on both tests are displayed in the figure located above.

ALGEBRAIC EXPRESSIONS					
Learner	Pre-Test		Post-Test		Increase/Decrease
	40	%	40	%	%
1	8	20	19	47,5	27,5
2	14	35	27	67,5	32,5
3	15	37,5	27	67,5	30
4	27	67,5	31	77,5	10
5	19	47,5	22	55	7,5
6	6	15	30	75	60
7	24	60	23	57,5	-2,5
8	24	60	30	75	15
9	18	45	20	50	5
10	18	45	20	50	5
11	21	52,5	30	75	22,5
12	18	45	29	72,5	27,5
13	16	40	25	62,5	22,5
14	11	27,5	18	45	17,5
15	26	65	32	80	15
16	25	62,5	16	40	-22,5
17	27	67,5	27	67,5	0
18	12	30	24	60	30
19	15	37,5	25	62,5	25
20	26	65	35	87,5	22,5
21	28	70	31	77,5	7,5
22	27	67,5	38	95	27,5
AVERAGE	19,31818	48,29545	26,31818	65,79545	17,5

Table 4.2: A summary of the results from both the pre- and post-tests that demonstrate learner achievement.

In the final column of Table 4.2 the percentage of growth for each learner is displayed. The table revealed that there were only two negative gains and one learner who neither gained nor lost. At least seven learners' post-test improvements exceeded 20%. The average increase of 18% for all learners implies that the learners benefited from the intervention classes.

I used the dependent sample test because I was comparing the results of the same learners before and after the intervention. The following results were obtained using a significance level of 005 and a two-tailed hypothesis:

Pre-test	Post-test	Difference(Post-test - Pre-test)	Standard Deviation (Diff - M)	Sq. Dev
20	48	28	10.55	111.21
35	68	33	15.55	241.66
38	68	30	12.55	157.39
68	78	10	-7.45	55.57
48	55	7	-10.45	109.3
15	75	60	42.55	1810.12
60	58	-2	-19.45	378.48
60	75	15	-2.45	6.02
45	50	5	-12.45	155.12
45	50	5	-12.45	155.12
53	75	22	4.55	20.66
45	73	28	10.55	111.21
40	63	23	5.55	30.75
28	45	17	-0.45	0.21
65	80	15	-2.45	6.02
63	40	-23	-40.45	1636.57
68	68	0	-17.45	304.66
30	60	30	12.55	157.39
38	63	25	7.55	56.93
65	88	23	5.55	30.75
70	76	6	-11.45	131.21
68	95	27	9.55	91.12
		M: 17.45		**S: 5757.45**

Difference Scores Calculations

Difference Scores Calculations
Mean: 17.45
$\mu = 0$
$S^2 = SS/df = 5757.45/(22-1) = 274.16$
$S^2_M = S^2/N = 274.16/22 = 12.46$
$S_M = \sqrt{S^2_M} = \sqrt{12.46} = 3.53$

T-value Calculation
$t = (M - \mu)/S_M = (17.45 - 0)/3.53 = 4.94$

Table 4.3: The dependent sample test results.

The value of t is 4.944408. The value of p is .00007. The result is significant at $p < 0.05$. The errors and misconceptions committed throughout the tests had an impact on the results attained by the learners. I used algebra tiles to address the errors and misconceptions highlighted on the pre-test in each of the preceding lessons. I will discuss all identified common errors in the section that follows.

4.3 Description of identified frequent errors.

The test results were rigorously evaluated to identify common errors and misconceptions made by learners when simplifying algebraic expressions. The frequency of each type of error was also recorded. The reader is reminded that the categories of this study were adapted from Ncube (2016). Except for the conjoin and the misinterpretation of symbolic notation, the errors identified by Ncube were common in my study. In contrast to the Ncube study, the conjoin and misinterpretation of symbolic notation were minimal and were not analysed in my study. Slips were common in my study but not in any of the Ncube categories.

There was a total of 366 errors committed on the pre-test, however there were only 230 errors committed on the post-test. To further organize my data, I divided common errors into five main categories. These errors consist of slips, sign errors, misconceptions, substitution errors, and multiplication errors. The following bar graph illustrates the identified common errors on the pre- and post-tests.

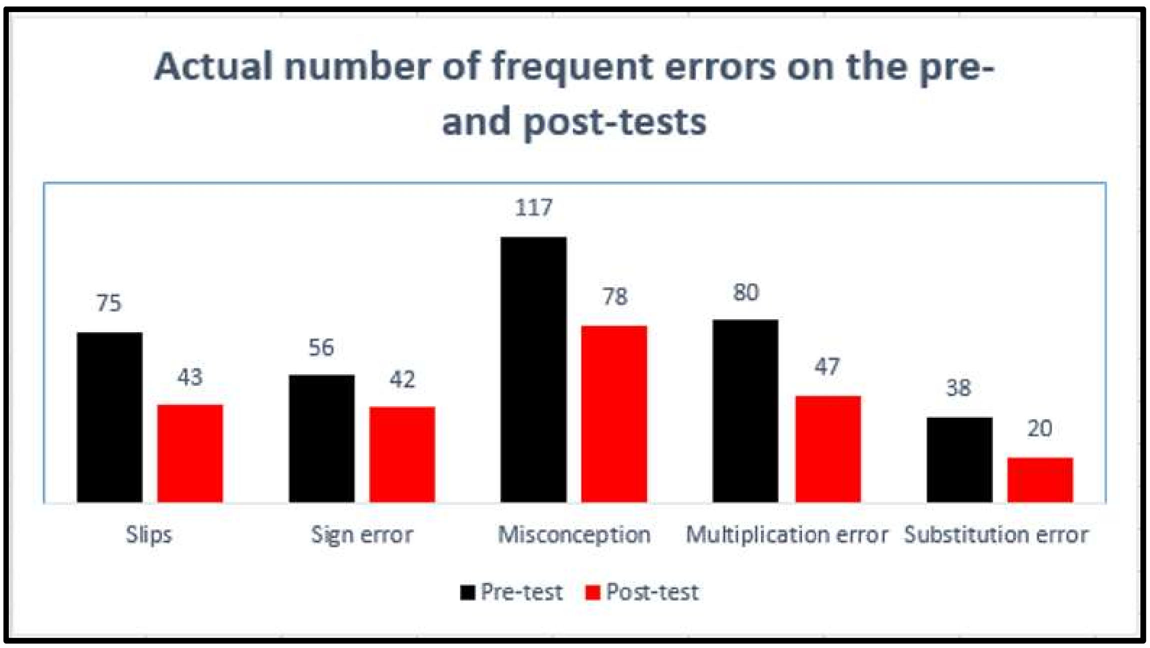


Figure 4.2: Learners' common errors on the tests

Here is an elaboration of each type:

Slips are incorrect responses that originate from carelessness, whereas errors are incorrect responses that reflect a lack of conceptual understanding (Moru et al, 2014). Luneta and Makonye (2010) explain that slips are unintended mistakes. This means that the learner made a little error in the solution steps, which affected their final answer; if this error is rectified, the learner will be able to complete the exercise without further problems.

Sign errors are possible outcomes of performing operations such as subtracting numbers, adding integers in the wrong order (Seng, 2010). This error is caused by improper sign usage while simplifying algebraic expressions.

I identify mixing the rules as a misconception because the learners are applying their prior knowledge to the new information in an overly generic manner (Brodie ,2014). Misconceptions can be described because of the overgeneralisation of existing knowledge, and they are pervasive, therefore they will continue to arise because of inadequate conceptual knowledge (Egodawatte, 2011; Moyo, 2020).

Learners make substitution errors when they substitute correctly but are unable to determine the numerical value of the expression, showing a lack of fundamental knowledge on the correct use of signs while doing addition and subtraction. In the substitution error, letters are substituted for numbers in an algebraic expression to find the expression's value.

Multiplication errors are caused by learners' misunderstanding of the BODMAS rules. From my experience, I notice that learners often tend to add and subtract before performing multiplication. Learners demonstrate difficulty with multiplication of algebraic expressions involving many terms, especially when stated in a different format with brackets.

4.3.1 Slips

Slips are minor errors that can be rectified by the learner. All learners committed 75 slip errors on the pre-test, compared to 43 slip errors on the post-test. In some instances, learners failed to fully factorise, while in others, their answers were inadequate. Learners were not writing exponential answers with positive powers. Moreover, it was common for learners to omit signs while arranging polynomials. Figure 4.3 demonstrates some slip errors on the tests.

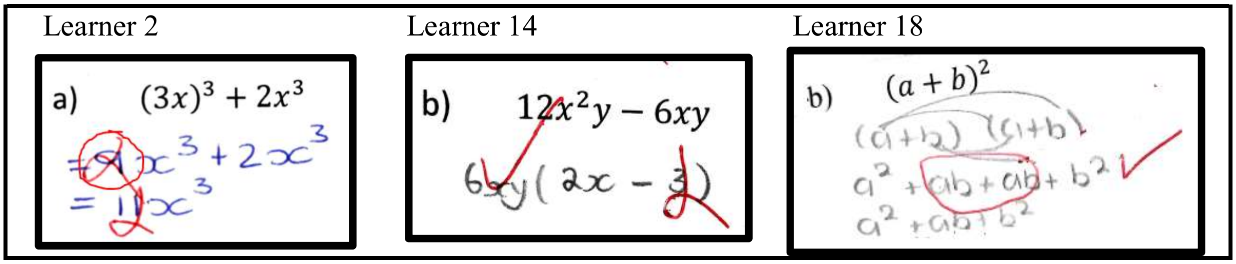


Figure 4.3: Some common slips on the tests.

Figure 4.3 illustrates some of the common slips made by Learner 2, Learner 14, and Learner 18 respectively. As depicted in figure 4.3, I will elaborate on the most frequent errors on the tests. The initial error proposed raising an expression to a power. If an expression within the brackets is raised to an outer power, learners can raise the variable to the power using law 3 of exponents, the learners multiply the power and integer instead of raising the power to the integer.

In this second picture, the learner made the slip of dividing $6\,x\,y \div 6xy = 3$ instead of 1, despite being able to recognise a common factor and factorize. In the final image, learners were expected to add the like terms; they properly expanded the binomial; all they had to do was add $ab + ab$; nevertheless, they left it as ab instead of $2ab$.The number of slips made by the learners was significantly higher; these slips cost them points. It was discovered that when these slips were brought to their attention, they were promptly rectified.

4.3.2 Sign errors

On the pre-test, the sign error was more prevalent than on the post-test. This sign error was committed 56 times during the pre-test, but only 42 times during the post-test. Working with numbers and operation signs presented challenges to learners. Learners frequently ignore negative signs to subtract or interpreted negative signs as minuses in interesting ways (Ncube, 2016). When learners factorised trinomials, they changed the signs of the terms. Then they added like terms, and they get incorrect answers. In other cases, learners correctly substituted the value of a given variable, but their misunderstanding of addition and subtraction of integers hindered accurate solutions. Learners struggled with the addition and subtraction of like terms. I illustrate some of these errors below.

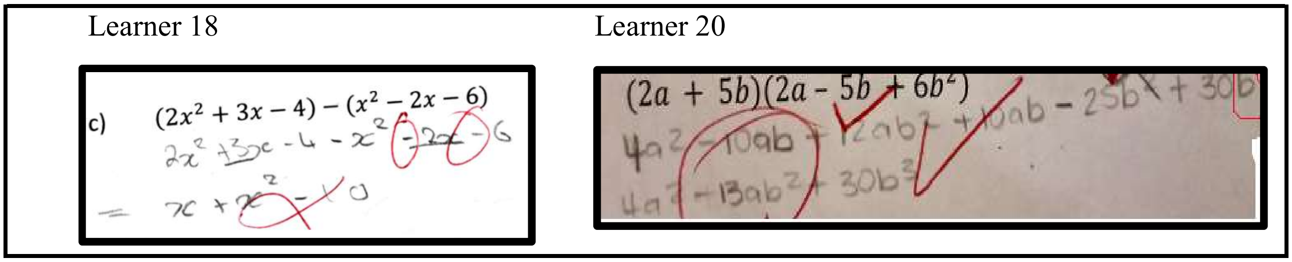


Figure 4.4: Some sign errors that were recorded.

Like learner 18, most learners disregarded the negative sign outside the brackets and did not distribute it. Learners were expected to distribute the negative sign prior to gathering like terms. Some learners correctly distributed the negative sign but were unable to add or subtract like terms. In the second image, learners were expected to multiply the binomial by the trinomial and again collect like terms. Due to a lack of knowledge, the addition and subtraction of like terms posed a barrier for this question. Learner 20 demonstrated accurate distribution, but they did not appropriately combine like terms. Learners are required to simplify their responses in most algebraic expressions questions by adding and subtracting like terms. It is noted that the ability to add and subtract like terms is crucial when working with algebraic expressions (Zubainur and Ali, 2018).

4.3.3 Misconceptions

Nesher (1987) defines misconceptions as arising from learners' prior learning, either in the classroom (especially for mathematics) or their interaction with the physical and social world. Consequently, misconceptions stem from a deeper conceptual misunderstanding (Neidorf et al, 2020). In the majority of test questions learners often confused rules, which could have arisen from their prior knowledge of these concepts. There were 117 misconceptions on the pre-test, compared to 78 misconceptions on the post-test. Figure 4.5 below shows an example of misconceptions.

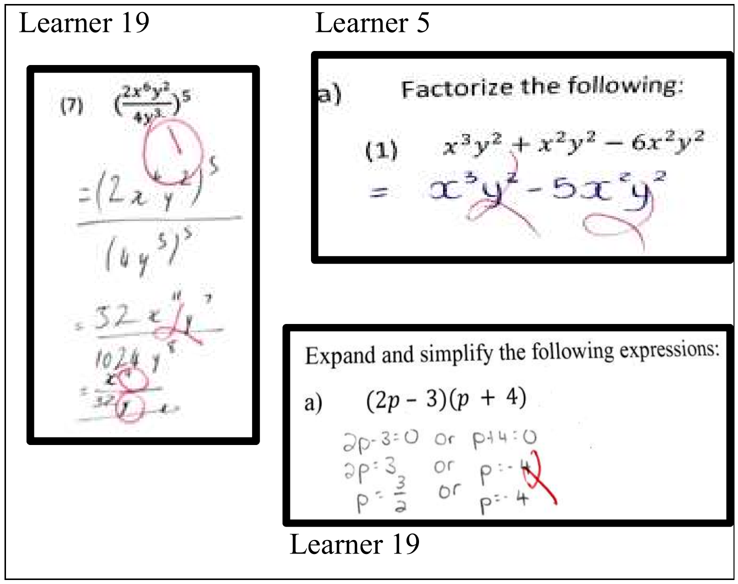


Figure 4.5: Typical misconceptions that were encountered on the tests.

The work of learners 19, 5, and 18 is shown in Figure 4.5. In the first image, learner 19 applies law 1 of exponents rather than law 3. The learner should multiply the powers rather than add them. The second image illustrates how learner 5 added rather than factorised. The learners were instructed to factorise. The learner added the terms without considering the instructions of the question. Essentially, the learner's response is correct, but it does not apply to this question.

In the last extract, learner 18 is instructed to expand and simplify the two binomials. The learner applies their knowledge of solving equations by setting each bracket equal to zero and solving for the unknown. If the question required the learner to solve for x, then the solution provided by the learner is correct. The learner does not appear to distinguish between an equation and an expression. The learner is not attentively following and reading the instructions in the question.

The above extracts are characterised as misconceptions because the learners overgeneralise their correct knowledge but apply it incorrectly. Gardee (2015) explains that misconceptions are the outcome of correct prior knowledge interfering with new information.

4.3.4 Error in multiplication

The pre-test contained 80 errors in multiplication, while the post-test contained 47 errors. The multiplication errors manifested in various ways. The most frequent error was when some learners distributed only one term and left the other, as in Question 4 c of the pre-test. The multiplication errors were widespread on both the pre- and post-test. In some instances, prior to distributing the numbers on the brackets, the learners added or subtracted the numbers not attached to the brackets.

Significantly (75%) of the learners lacked the ability to multiply expressions with fractions. This question was badly answered $\frac{-4a^2b\times6ab^4}{12a^4b^2}$. To simplify the expression, the learners were expected to multiply and divide. This question required the use of exponent laws 1 and 2. Multiplying the numerator expression was challenging for the learners. Others omitted the numbers, multiplied the powers, and assumed the first b's power was 0 instead of 1. Figure 4.6 shows some of the multiplication errors made on the tests.

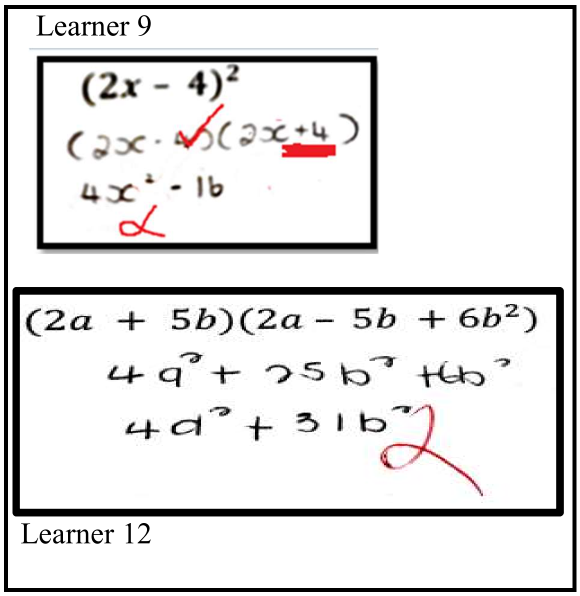


Figure 4.6: Some of the most common multiplication errors discovered on the tests.

In the first image, the learner 9 can expand the binomial, but then they incorrectly alter the sign. It is evident in the final step that just the first and last terms are squared to obtain the result, while the middle term is ignored. This demonstrates that the learner is unaware that when multiplying squared binomials, a trinomial is always the result. Most learners committed this error when solving for a square binomial; they just squared the first and last term. Egodawatte (2011) found similar results, demonstrating that learners persist in oversimplifying the solution to the expression $(a + b)^2 = a^2b^2$ or ab or $a^2 + b^2$.

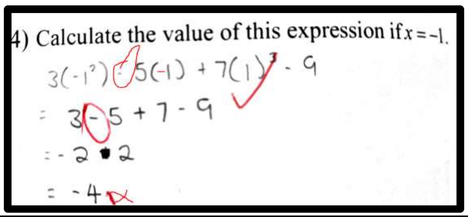
Learner 12 was able to distribute 2a from the binomial to the trinomial in the second figure but had trouble applying the same method to 5b. Also, the 5b was to be distributed prior to simplifying the answers. This method of distribution employed by the learner is incorrect.

4.3.5 Substitution error

Learners were able to perform substitutions, but they were unable to determine the numerical value of the expression when given the x and y values. This could be owing to a lack of conceptual understanding of the addition and subtraction of integers. The substitution errors were committed 38 times on the pre-test, compared to 20 times on the post-test.

The results for this sort of error did not change significantly. Figure 4.7 illustrates some examples of these substitution errors.

Learner 13



Learner 17

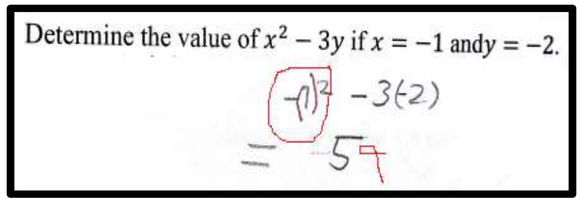


Figure 4.7: Example of Substitution errors observed on the tests.

In the first extract, learner 13 substituted correctly; however, it was evident that the learner did not get the correct solution because of not fully understanding addition and subtraction of integers. The same is true for the second figure, where learner 17 does the correct substitution; however, once again because of a lack of understanding of addition and subtraction of integers the value of the expression is incorrect.

In this section detailed the identified errors in some depth. In addition, examples of the errors on the learner's test were analysed in detail. The next section provides an overview of how the identified errors were addressed in the intervention lessons.

4.4 A concise summary of the lessons

In Chapter 3, I offered a full overview of each lesson, including the topic of algebraic expressions accomplished on each lesson and how algebra tiles were used to support the numerous exercises assigned during each session. This section now provides a concise description of how the lessons addressing the errors and misconceptions were conducted.

Here I present Table 4.4 as an overview of each lesson that served as a foundation for a more in-depth investigation of the common errors and misconceptions addressed in each lesson. Algebraic tiles were used to teach learners an algebraic concept, and they were required to complete textbook problems to apply the concepts they had learned using algebraic tiles.

Lesson	The focus of the lesson	Summary of each lesson
1	Examining pre-test errors.	To make learners aware of their errors and use them as a teaching point, the pre-test assessed learners work and revealed the errors.
2	Algebra tiles add and subtract like terms.	Learners s combine tiles to produce 0s by pairing like terms. Exercise 8.1 question (a) focuses on adding and subtracting like terms.
3-4	Multiplying binomial products and squares.	Learners often multiply binomials by squaring the first and last term, e.g. $(x-2)^2 \neq x^2+4$ I believed this activity would clarify for learners that all terms must be multiplied. Learners solved exercise 8.2 (a) and (c) by multiplying binomials with algebra tiles.
5-6	Polynomial division by integers and monomials	Model the divisor vertically. Learners s must utilize dividend tiles to form a rectangle whose height equals the divisor's length and whose quotient is the horizontal width. In addition, learners conducted exercise 8.1 question (d) ,
7	Factorisation: highest common factor	Learners build a rectangle with the tiles. Rectangle length is solution; width is HCF. Exercise 8.5, part (a) was solved using the factorization method.
8	Factorisation: Common bracket:	This lesson showed the learners that distributing a number outside the bracket is like finding the highest common factor, but in expanded form. (b) of Exercise 8.5 required factoring, hence the common bracket must be removed. In certain instances, learners must alter the symbol to fit the bracket. Learners s did not mind the obligation to change signs.
9	Factorisation: Difference of two squares	Learners were told to first assemble the squares and then add the 'missing' pieces to complete the rectangle to find the difference of perfect squares. As part of their daily work, learners completed exercise 8.8 (a)–(b).
10	Factorisation-Trinomial	The solutions for trinomial factorization were illustrated with algebra tiles. Learners solved exercises 8.9 (a) questions (b). Learners that use algebra tiles effectively appear to understand the relationship between multiplication, division, and factorization of algebraic expressions.

Table 4.4 Lesson outline overview

Using the required algebra tiles, each lesson was devoted to addressing errors and misconceptions. In addition, several opportunities for practicing and demonstrating with the tiles were provided. Learners were also encouraged to use algebra tiles for the exercises.

4.5 An overview of efforts to address the common errors using an intervention lesson.

It is argued that that it is critical to address errors and misconceptions during teacher and learner contact time so that the misconceptions that learners have developed do not inhibit further learning (Makonye, 2012). Makonye and Khanyile (2015) assert that questioning learners about their errors would assist them in reducing these errors.

In the intervention lessons, I addressed the errors in the following ways: In the first session of the intervention programme, I presented the commonly made errors and misconceptions that were discovered in the pre-test to learners to make them aware of these errors.

I requested that learners verify the accuracy of their answers. I then instructed learners, using algebra tiles, and encouraged them to break down their procedures into steps.

I looked for opportunities to get learners to explain their presentation so I could understand their thinking and reasoning. The objective of the 10 lessons was to make use of the algebra tiles and assist learners in addressing the errors and misconceptions identified on their pre-tests. Not every lesson was dedicated to addressing specific errors and misconceptions, but, throughout the lessons errors and misconceptions were addressed by using algebraic tiles to solve a variety of algebraic expression problems.

4.5.1 Sign error

In the second lesson of the intervention lessons, the incorrect sign was addressed. It was of the utmost importance to have a conversation with the learners about the 0 notion, which states that a negative tile and a positive tile will cancel each other out. Learners collaborated in groups and worked in pairs to formulate an algebraic expression that described how to combine the tiles. I did various examples of adding and subtracting tiles using an overhead projector in the second lesson. The learners were given the opportunity to practise using questions from the exercise. Learners addressed the sign error by using algebra tiles during the intervention lessons, which were relatively like the one shown in the figure below. Figure 4.8: demonstrates in detail, using the tiles, how to add and subtract unlike terms.

$$-2x^2 + 2x - 4 + (3x^2 - x + 1) = x^2 + x - 3$$

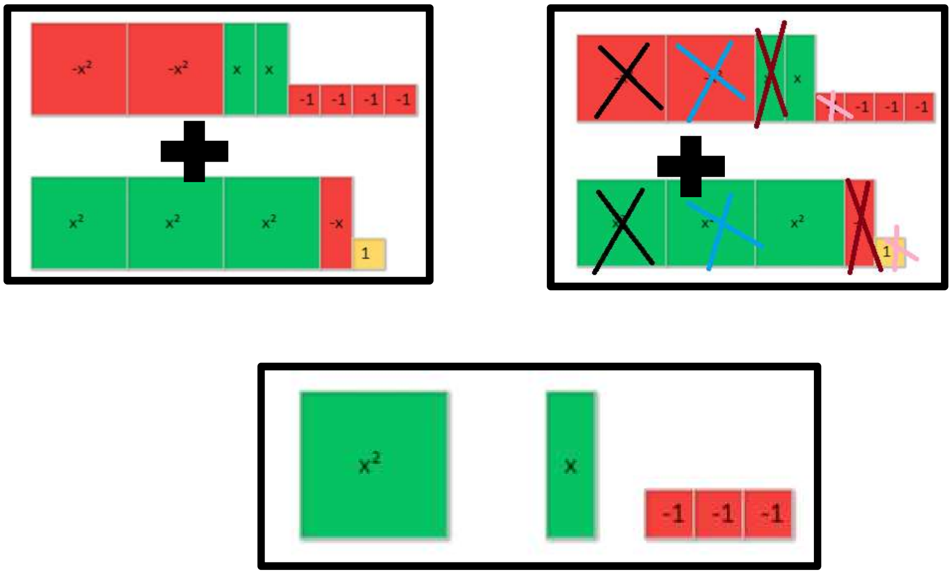


Figure 4.8: A representation of addition and subtraction of like terms using the algebra tiles.

The two expressions $-2x^2 + 2x - 4 + (3x^2 - x + 1) = x^2 + x - 3$ are added. The first image depicts the expressions as tiles, the second image demonstrates how the zero-pairs cancel, and the third image depicts the remaining solution.

4.5.2 Substitution error

According to the learners, evaluating expressions using algebraic tiles was among the simplest exercises. This session was conducted in conjunction with the addition and subtraction of algebraic expressions. To simplify algebraic expressions learners were asked to perform the substitution using algebra tiles. This activity ensured that learners avoided inaccurate addition and subtraction of integers due to signs by forcing them to generate a representation of the phrase using tiles and then replace each rectangle with the correct tile value prior to combining like terms.

In class, learners solved many problems like the one below:

What will the value of $3x - 2$ be if $x = 3$? To solve the problem each green x tile must be replaced with three yellow positive one-unit tiles. The two negative one-unit green tiles will combine with the two positive one-unit yellow tiles to produce zeros, which will cancel out, resulting in a solution of seven.

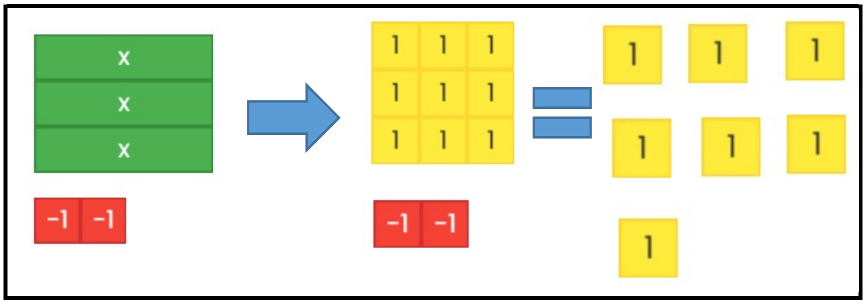


Figure 4.9: Substitution using algebra tiles $3(3) - 2 = 7$

4.5.3 Misconceptions

Learners frequently respond to the following question as follows: $(x - 3)^2 = x^2 + 9$. This misconception may result from the fact that $(4x)^2 = 16x^2$.

Learners use their understanding of the laws of exponents to factorise, but this understanding is applied incorrectly. To correct this misconception, it was first explained to the learners that $(x - 3)^2$ means you have two of the brackets. Learners will need to arrange $x - 3$ on the far left-hand side, on top learners will have the other $x - 3$ as depicted below:

$x - 3$

$x - 3$

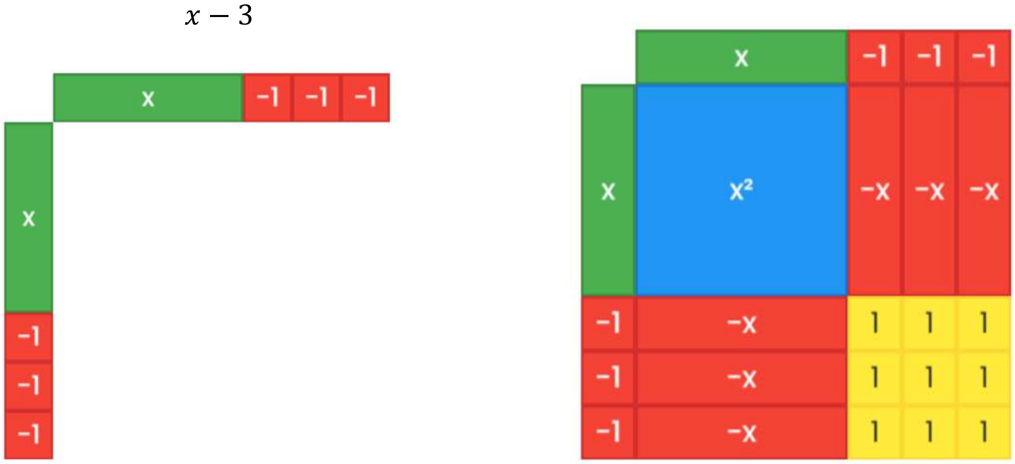


Figure 4.10: squaring a binomial using algebra tiles

$(x - 3)^2 = x^2 - 3x - 3x + 9$

The aim now is to fill in the inside in such a way that the square is formed. Learners accomplish this by multiplying the x on the left by the x on the right to obtain x^2. This answer is then placed inside the empty shell, the next step is to multiply the x on the left with the remaining terms at the top. The same procedure was applied for the three negatives ones on the left. Once steps of demonstrating the tiles are complete then the solution will be as below

The above figure shows exactly how the answer is obtained as a trinomial instead of the two terms. This was explained to the learners as to address the misconception that $(x - 3)^2 = x^2 + 9$. To address this misconception, learners also created similar solutions to the one shown above.

The following Figure 4.11 depicts how one learner solved for the product of two binomials before and after the intervention classes.

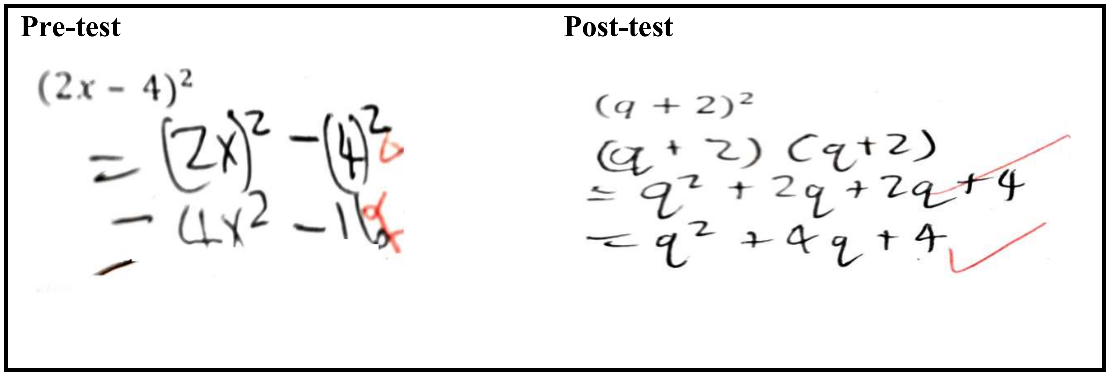


Figure 4.11: An illustration of how learner 11 gains better insight on multiplying a binomial by making use of algebra tiles.

Learner 11 is doing precisely what was taught in the intervention lessons, as described above. First, they expanded the two brackets, as instructed in the intervention lesson, and then they must multiply. The same learner 11 in the pre-test was simply multiplying the first and the last term.

4.5.4 Error in multiplication

I demonstrated how to multiply a monomial by a binomial using the tiles. I also explained to the learners how to multiply algebraic expressions by determining the product by filling in the area of a rectangle. I explained several of these examples and how learners were supposed to construct a framework and then fill it in. In addition, learners were taught how to multiply a binomial by another binomial and a binomial by a trinomial. Learners discussed the exercise in small groups and with their peers. We completed the multiplication questions as a class, and learners regularly came up with new ideas for how to correctly fill in the rectangles for the multiplication problems.

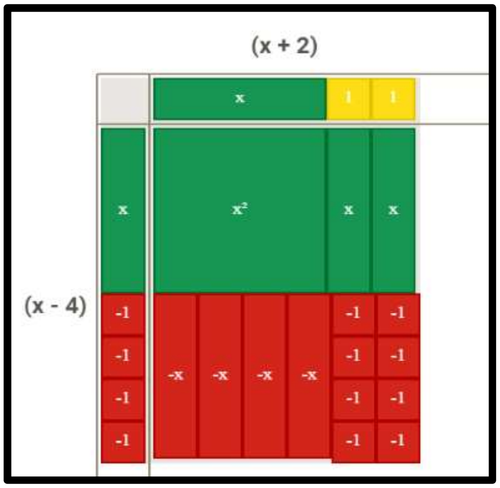


Figure 4.12: the tiles illustrate that $(x-4)(x+2)=x^2-2x-8$

The learners recognised from the solutions that this procedure for multiplying a binomial is similar to factoring a trinomial.

Evidently, some learner has gained the ability to apply or manipulate algebraic tiles to multiply binomials, based on the extract below.

4.6 The interviews

The reader is reminded that five learners were interviewed after the pre-test and post-test to determine where the errors and misconceptions originated from.

4.6.1 The pre-test interviews revealed the following key points:

The following learners were interviewed during the pre-tests: Learner 1,6, 10,14, and 18.

The lack of understanding addition and subtraction of like terms.

For question 3 item (a) $(3x)^3+2x^3=5x^3$ learner 1 provided this response, and in the interview the learner explained as follows: "I added the coefficients and the powers for the same reasons, as they are like terms." The learner was expected to first expand $(3x)^3$before adding $2x^3$, and they could not add the powers because this is addition, not multiplication, of algebraic expressions.

The learners' background knowledge about integers and algebraic expressions appears to be the most significant obstacles to learners' mastery of algebraic expressions. In some cases, the steps were correct, but learners ended up adding terms that were not like terms. When requested to explain the answer to question 3(c)

$(2x^2 + 3x - 4) - (x^2 - 2x - 6)$ learner 6 exclaimed in excitement: "This was simple, ma'am; I simply multiplied by the minus sign outside the brackets". " For $2x^2 + 3x - 4 - x^2 + 2x + 6$, I then gathered all the like terms to get $7x$ and the numbers I got 10!" This example demonstrates that, once again, learners' inability to identify like terms affects their answers. This question was a multiplication problem, which the learner answered correctly. However, when simplifying the problem, the learner made errors, indicating that identifying like terms is a problem. This also shows that learners overgeneralise their understanding of integer addition and subtraction to addition and subtraction of like terms. When asked to explain their solution for question 3(f) $-2x + 3 - \frac{5}{x}$ learner 10 and 14 interviewed indicated that they did not know how to answer the question. The other two simply indicated that they added $-2x$ and $-\frac{5}{x}$ and left 3 as it is, $-2x + 3 - \frac{5}{x} = -7x + 3$. Learner 18 was aware that he was supposed to take out an (lowest common denominator) LCD but admitted he did not know how to. He explained: "Ma'am whenever the denominators are the same in a set of fractions, you add the top numbers together; whenever the bottom numbers are different, you take out and LCD; but I don't know how to take out and LCD when there is only x in the bottom.". The learner altered the question by making up new rules. Ncube (2016) observed the same pattern, learners create new rules for their own gain.

Learners also respond to questions without considering the instructions. The inability of learners to understand instructions was also problematic, when asked to explain their solution for question 4(c) $(2a + 5b)(2a - 5b + 6b^2)$, learner 6 said: "Ma'am, I recognised the combination of the terms like and added, my answer was $4a + 0 + 6b^{2}$." The learner did not pay attention to the instructions, disregarded the existence of brackets, and did not multiply the binomial by the trinomial.

Although learners had already been introduced to algebra for two years, at the beginning of Grade 7, it was evident from the interview that a lack of background knowledge is one of the reasons for the errors and misconceptions. Learners reply to questions without conceptual knowledge. In addition, the interviews showed that learners were getting the wrong responses because of their tendency to improperly apply mathematical rules.

4.6.2 The post-test interviews revealed the following key points:

Another group of five learners was selected for an interview based on the results of their tests, to gain further insight into the solutions they had proposed. Learners 2, 5, 16, 17, and 22 were interviewed following the post-tests. The goal was to gain a deeper understanding of the errors made and determine whether the algebra tiles helped the learners address the errors and misconceptions that they had previously held.

Interestingly, when asked about extra test-taking strategies, only three out of five learners acknowledged utilising algebra tiles to assist them on the post-test, even though the tiles were available to all during the post-test. The three learners that employed the tiles were learners 2, 5, and 22. Learners 16 and 17 reported forgetting how to use the tiles, they admitted that they knew that the end results were a square but did not remember how to solve the solutions using the algebra tiles. Leaner 16 said: “I find the tiles confusing, and I prefer using the formal way of calculations.” Learner 2 and 22 exhibited a good attitude regarding the use of algebra tiles; one of the comments given during the interview was: “I find that the algebra tiles make things easier, and I find them enjoyable.”

When asked to explain their solution to question 3a, $(q+2)^2$, learner 17, who did not use algebra tiles said, "I squared the first and last term to obtain q^2+4.” Learner 16, who did not utilise the algebra tiles explained that he first expanded the bracket and then used the method of expanding two binomials as previously taught in class, obtaining the value q^2+4q+4. The learner’s response was correct, and he emphasised that he found the algebra tiles were confusing. The purpose of the algebra tiles was to address the errors and misconceptions. If the learner already understands the subject using a means other than algebra tiles, this is acceptable. Yet, it is challenging if a learner is struggling with a concept and is unwilling to explore alternative approaches to solving the problem.

For the Question 2(a) number 8. $2x-4y+3xy+2y-x$ for adding and subtracting like terms learner 2 said: "I expressed the expression using the tiles, I paired the zeros, and my answer was $x-2y+3xy$; I did not have to recall signs because my answers are normally incorrect because I got my signs wrong," was the response that one learner gave to the question 2a item number (8) $2x-4y+3xy+2y-x$. In response to the question of what other approaches had been taken to respond to the questions, learner 22 made the comment that “using the tiles makes it straightforward for me to answer questions that involve numerous forms of factorisation”.

It should be noted that the use of the algebra tiles was to help learners with conceptual understanding. Most of the learners during the lessons seemed certain that the use of the tiles improved their understanding.

The performance of learners on the pre-test and post-test was analysed in depth, along with the findings of the dependent sample test. There was a thorough discussion of the identified errors adopted from Ncube (2016) as well as samples of learners' actual work. In addition, a detailed explanation of how the algebra tiles were utilised in each session was provided. Interviews were conducted to establish the origins of the errors and misconceptions. In addition, the responses to interview questions were briefly discussed. The following section discusses the research findings.

4.7 Discussions and findings

With regards to the use of mathematical manipulatives in Grade 9, the analysis of the data reveals the essence of learner errors and misconceptions. In South Africa it is important to note that addressing learner errors and misconceptions in algebraic expressions using manipulatives has not been the subject of extensive research. However, studies on learner errors and misconceptions have shown that they are related to learners' prior experiences Nesher (1987). In analysing the data, five error categories were identified. This section will detail the findings about the five identified error categories.

The reader is reminded that the five identified categories were adopted from the Ncube (2016). Analysing the pre- and post-tests, the researcher identified five error categories. These error categories are slips, sign error, misconception, multiplication error and substitution error. The frequency of errors in each category was recorded. As shown in Figure 4.5, the percentage decrease in errors on the post-test compared to the pre-test was also recorded.

Errors	**Pre-test**	**Post-test**	**Percentage decrease**
	Actual number of errors		
Slips	75	43	43%
Sign error	56	42	25%
Misconception	117	78	33%
Multiplication error	80	47	41%
Substitution error	38	20	47%
TOTAL	**366**	**230**	**37%**

Table 4.5 The average percentage decrease between the pre- and post-tests for common errors.

The pattern of the results indicates a slight decline in sign errors and misconceptions, while slips, multiplication errors, and substitution errors decreased by at least 40% on average. The intervention lessons using algebra tiles were effective in addressing errors and misconceptions, according to the findings. Since the gains were modest, the results indicated that algebra tiles should be utilised more often in algebraic expressions exercises. A discussion on the findings pertaining to each of the five categories of errors that were discovered can be found in the following section.

4.7.1 Slips

The number of slip errors decreased by 43%, as displayed in the table. Most of these slips were due to carelessness on the part of the learner. The most frequent slips made by learners was insufficient simplification of solutions. In addition, the slip included omitting variables from the submitted answers. Some learners factorise the expression $x^2 - 16$ incorrectly as $(x-4)(x-4)$, when it should be factorized as $(x-4)(x+4)$. Another common slip with the exponents was writing $(3x)^3 = 3x^3$ when the correct form is $27x^3$. Another common slip committed by the learners was arranging an algebraic expression with increasing rather than decreasing powers of y, as requested by the question. The slips as suggested by (Luneta and Makonye, 2010) can be reduced easily if they are called out.

4.7.2 Sign errors

According to the results, there was not a significant difference between the pre-test and post-test for sign error. There were only 25% fewer errors on the post-test than on the pre-test. The primary reason for this error, according to the findings, is that learners struggle with integer addition and subtraction. There is a direct relationship between the concepts of integers and addition and subtraction of algebraic terms. The findings indicate that since the learners struggle with integers, they also tend to struggle with addition and subtraction of like terms. Lim (2010) also found comparable results. The primary focus of Lim's (2010) investigation was an error analysis of learners in Grade 7 as they attempted to simplify algebraic expressions. According to Lim (2010), the addition and subtraction of like terms make up the fundamentals of algebra, and if learners are unable to master these concepts, it will impact their performance in other areas of the mathematics curriculum.

It was a challenge for learners to collect like terms in $2x - 4y + 3xy + 2y - x$, the main challenge was that the signs of the terms were different. In another expression learners were able to distribute the expression $(2a + 5b)(2a - 5b + 6b^2)$ but could not simplify their answers by collecting like terms or added the like terms incorrectly. The expression $(2p-3)(p+4)$ was correctly expanded to $2p^2+8p-3p-12$, but because 8p and -3p contain different signs, a variety of answers were provided, including $11p$, $-5p$, and $5p^2$. Some learners added all terms containing variable p and obtained the answer $7p$. Additionally, some learners added all three terms in the expression, obtaining $-5p$.

Various types of algebraic expressions must be simplified by addition and subtraction of like terms. Since this knowledge is applicable to the addition and subtraction of like terms, it is crucial that learners have a solid foundation in the addition and subtraction of integers. The results indicate that learners have difficulty on identifying like and unlike terms. In some instances, learners added algebraic expressions to constants. The results also indicate that the learners were incapable of adding and subtracting algebraic expressions. The outcomes of the study are like those of Faramarzpoor (2020). Faramarzpoor (2020) conducted a study to determine the causes of learners' errors in simplifying algebraic expressions. Faramarzpoor (2020) discovered that improper integer addition and subtraction calculations are the root cause of numerous algebraic errors and misconceptions.

Muchoko et al (2019) researched factoring and simplifying algebraic expressions in middle school. The learners' work was evaluated. Analyses revealed that the learners were unable to combine like terms and rearrange expressions in a manner that made simplification easier for them. Similar to the study leaners work from left to right without considering BODMAS. The findings show that Grade 9 learners struggle with addition and subtraction of like terms and suggests that the concept was not fully understood in Grade 8. In all likelihood, they will continue to struggle with the same concept in Grade 10. Taban and Cadorna (2018) explain how learners in Grades 8 to 10 struggle with addition and subtraction involving like-terms. They examined the structure of learners' algebraic solutions. The authors found that the errors and misconceptions were caused by learners' inability to add and subtract like terms across three grade levels.

4.7.3 Misconceptions

On the post-test, misconceptions decreased by only 33% compared to the pre-test. The findings of the study indicate that ninth graders frequently commit misconceptions. Learners apply irrelevant rules or overgeneralise a concept. According to research by Brodie (2014) when learners make numerous errors and gain confidence as a result, they have developed a misconception. The learners are simply memorising the rules without a solid understanding of them; as a result, they will commit misconceptions. According to Luneta and Makonye (2010), most algebra errors and misconceptions can be traced back to a lack of proficiency in elementary algebra, which is a topic that should have been mastered in earlier grades.

The findings also indicate that learners create their own rules to solve questions, which is consistent with the findings from Ncube (2016) that if learners cannot solve a problem, they create their own rules to solve the problem. Learners identified $-2x$ and $-\frac{5}{x}$ as like terms and added them for the pre-test question $-2x + 3 - \frac{5}{x}$. The results show that learners change $-\frac{5}{x}$ to $-5x$; learners were unable to take the LCD and simplify appropriately, so they altered the question and the original form. This is consistent with Zubainur and Ali's (2018) and Ncube's (2016) findings that learners cannot add algebraic fractions. Learners do not understand the concepts of LCD (lowest common denominator) and instead treat fractions as normal numbers. Zubainur and Ali (2018) found that most learners multiplied $\frac{x}{2} + \frac{x}{3} = \frac{x^2}{6}$ instead of finding an LCD and adding. Ncube (2016) found a similar pattern of simply cross multiplication when learners had to add algebraic fractions: $\frac{x}{y} + \frac{w}{z} = xz + yw$. The researchers' two examples show that learners misapply rules they understand (Brodie, 2014).

The findings also revealed that learners frequently misapplied the rules of exponents. For example, for the post-test question $(\frac{2x^6y^2}{4y^3})^5$, learners would add the powers rather than multiply. Gardee (2015) identified the misapplication of rules error as well. According to Gardee (2015) most learners in Grade 9 concluded that $a.a = 2a$ by using addition rather than multiplication. In a similar manner, the findings show that learners multiplied 2 to 15 in the expression $2a + a + 15 = 30a$ because they assumed the numbers in the question are like terms. Based on the findings, learners continue to struggle with identifying like terms and overgeneralise the rules for integer addition and subtraction to apply to the addition and subtraction of like terms.

The findings also indicate that, despite being taught a new method for solving algebraic questions using tiles, learners continue to hold the misconception discovered in the pre-test. The findings also revealed that these misconceptions were not addressed during the first two years of algebraic expressions teaching. This is consistent with Luneta and Makonye's (2010) assertion that errors and misconceptions that are not addressed in the early grades continue to have a negative impact on mathematics performance in other areas. Using specially designed instructional materials, Luneta and Makonye (2010) analysed the remaining calculus misconceptions and errors among grade12 learners. They found that the misconceptions of Grade 12 learners were due to a lack of basic algebra knowledge and provided evidence that these misconceptions were carried over from earlier grades.

4.7.4 Multiplication errors

In a comparison of pre-test and post-test scores, the multiplication error ranked third among those that decreased the most. On post-tests, learners committed 41% fewer multiplication errors. In accordance with what Ncube referred to as invalid multiplication, the findings indicate that when multiplying, learners omitted negative signs or fail to fully distribute. Instances such as $(2a + 5b)(2a - 5b + 6b^2)$ learners would collect the like terms $2a$ and $2a$ along with $5b$ and $-5b$ as like terms. They disregarded the brackets completely. In cases, learners exclude the $6b^2$ and multiply the two binomials instead. In addition to multiplying the variable's powers instead of adding them, this was a common expansion error. Ncube (2016) reports that learners can apply the distributive property rule to a simple statement without terms but struggle with complex expressions with multiple terms. A learner can answer $3(n + 7) = 3n + 21$, but $2p - 4p(2p^2 - 1)$.

The findings also indicate that learners are proficient at multiplying less complex terms but struggle with complex ones, for this question $(t + 3)^2 - 2(t + 1)(t - 10)$, learners would multiply and simplify, but they would not correctly distribute the 2. In some cases, learners would distribute the 2 to the bracket prior to simplifying the binomial. Multiplying variable powers when expanding instead of adding them is another common multiplication error. It should be noted that Egodawatte (2011)'s research supports findings that learners can apply the distributive property rule to a simple, single statement without any attached terms, but struggle to apply it to complex expressions with multiple terms.

4.7.5 Substitution errors

There was a 47% decrease in substitution errors on the post-test. The findings show that learners can substitute numbers in algebraic expressions; the problem is performing the correct calculations and obtaining the correct value. Again, the findings indicate that understanding addition and subtraction of integers is the key challenge. The use of algebra tiles in the intervention made it significantly simpler for learners to cancel zero pairs.

This substitution question on the pre-test to determine the value of $x^2 - 3y$ if $x = -1$ and $y = -2$ was presumably straightforward; however, learners struggled greatly due to the substitution of negative numbers. The post-test question on substitution for calculating the value of $8y \times 7y^3 - \frac{2y^2}{3} - 4$ if $y = -2$ was not as poorly answered, but a few learners omitted the negative sign again, while others correctly substituted but were unable to obtain the correct expression value. Comparing the pre-test and post-test, the substitution error was the most improved. The results of struggling with addition and subtraction of integers align precisely with the findings of Ncube (2016). Ncube (2016) concluded that there was a lack of knowledge regarding substitution, and when asked to solve an application question based on substitution, learners simply assign random values.

4.8 Conclusion

The statistics indicate that learners' work contained a high number of errors prior to their exposure to the intervention lessons. Learners struggle with sign errors, multiplication errors, misconceptions, substitution error and multiplication error. According to the analysis of the tests, the most frequent error made by learners was misconceptions.

Comparing the pre-test to the post-test, it was discovered that the learners' overall performance had increased by 18% after receiving the intervention sessions. The results of the post-test indicate that the intervention lessons addressed the errors and misconceptions effectively. The algebra tiles as manipulatives employed in the intervention classes led to significantly higher post-test scores for the learners.

After the intervention lessons, learners were willing to use algebra tiles that had not been used previously. All the collected and analysed data for this study indicate that teaching algebra expressions with mathematical manipulatives is advantageous and beneficial for learners. The literature reviewed also supported the finding of the study. Following a comprehensive discussion of the findings in this chapter, the following chapter will provide conclusions and recommendations based on these findings.

Chapter 5: Conclusion

5.1 Introduction

In this final chapter, I summarise the findings presented in Chapter 4. I have discussed the significance of implementing the pre-test, post-test, and intervention as a means of enhancing learners' comprehension of algebraic expressions. I now discuss the limitations of the study. I will also consider the implications of these findings and make recommendations based on them. In conclusion, I consider the implications of the research for teaching.

My research supports the error types identified by Ncube and provides a new approach for teachers to address errors and misconceptions in algebraic expressions by employing algebra tiles. The reader is reminded that not all Ncube's categories were utilised in the study. In my research, the conjoin error and misinterpretation of symbolic notation were minimal and not common. Slips were prevalent in my study, although they were not one of the Ncube categories.

The fact that learners in South Africa find algebra to be the most challenging topic to study in the mathematics curriculum served as the motivation for me to start this research. South Africa's mathematical achievement has been dismal for decades (Spaull and Kotze, 2015). As a result of failing the subject on their Grade 9 November exam, a number of Grade 9 learners choose mathematical literacy over mathematics in Grade 10. Learners in Grade 9 are expected to be able to simplify a wide range of algebraic expressions questions. Knowledge of algebraic expressions is required for other related topics within algebra and serves as a foundation for other topics in mathematics (Lim, 2010).

Algebra frequently causes problems for learners because it is taught through rules in a very traditional classroom setting. It is because of these concerns; I chose to use algebra tiles to explore errors and misconceptions with Grade 9 learners. The key focus of the study was the role of the algebra tiles in the learner's Zone of Proximal Development (ZPD). Algebra tiles were used to help learners better comprehend algebraic expressions.

The theoretical foundation of my research's was the Zone of Proximal Development. The ZPD refers to a learner's developmental potential. The ZPD is where learning occurs after the identification of present knowledge. It should be noted that while I as the teacher supported and assisted the learners in using the algebra tiles, my practice is not the focus of this study. The contribution of this study is in the ZPD that had been created between the learner in using algebra tiles to solve algebraic expressions.

The study demonstrates that using algebra tiles can help learners understand the abstract concepts of algebraic expressions. Using algebra tiles, learners were able to demonstrate their own solutions and justify supplied solutions. I could determine, based on the displayed solution, where the learner's needed assistance. By introducing algebra tiles and guiding learners through the process of solving algebraic expression problems, I was engaging within the individual learners' ZPD.

Surprisingly little academic literature in South Africa addresses common learner errors and misconceptions in algebraic expressions incorporating algebra tiles.

The following research questions guided the study:

1. What are the errors and misconceptions that Grade 9 learners make in solving algebraic expressions prior to the intervention process?
2. Are there any changes in the Grade 9 pre-test and post-test results?
3. How do the results of the post-test relate to the content in the intervention?

5.2 Key findings

The results are presented in accordance with the research questions that guided this study. The results are listed below.

The errors and misconceptions in algebraic expressions are reviewed in a study's data, which reveals some intriguing findings. The pre-test data revealed five common algebraic expression errors made by Grade 9 learners. The reader is reminded that these common errors were adapted from Ncube (2016). After a series of algebra tile-based intervention lessons, the post-test data revealed a significant reduction in the five errors. Additionally, interviews were conducted to determine the origin of the errors and misconceptions.

5.2.1 First research question

The analysis of the pre-test written by Grade 9 learners revealed five common errors. The five most common errors identified were:

1. Slips
2. Sign errors
3. Misconceptions
4. Multiplication error
5. Substitution error

1. Slips

Many slips discovered on the test were due to learners leaving their answers incompletely simplified. The primary cause of the errors was carelessness, which can be easily remedied by requiring learners to double-check their calculations Gardee (2015) show that probing or correcting slips would be a more appropriate method of dealing with the error.

2. Sign errors

Inadequate conceptual understanding of integer addition and subtraction was the leading cause of the observed sign errors on the test. It was discovered that learners lack the ability to differentiate between like and unlike terms. Regardless of whether the variables and exponents were not identical, learners were adding or subtracting coefficients of the algebraic expressions. My findings align with Lim's (2010), as he discovered that learners routinely omit the negative sign while simplifying complex algebraic expressions, as well as when performing addition and subtraction operations on algebraic expressions. The sign error was most noticeable when adding or subtracting a negative term; the incorrect application of signs influenced learners' responses (Ncube 2016). The results are consistent with Booth, McGinn, Barbieri, and Young's (2017) assertions that learners of all ages are vulnerable to sign error and that the sequencing of operations and usage of brackets present a challenge.

3. Misconceptions

On the pre-test, misconceptions made up most errors. It was discovered that most of the misconceptions were due to learners applying their correct knowledge incorrectly when answering questions (Brodie, 2014). As a result of the discovered misconceptions, it was determined that learners have no understanding of the fundamental concept of algebra because they incorrectly apply it when required to do so in a different context.

It was observed that learners developed their own rules for algebraic expressions to accommodate their answers as suggested by Egodawatte (2011). The learners also mixed up the fundamental exponential rules when performing multiplication and division, which is another misconception. In addition, it was discovered that learners added the numerator and denominator separately rather than using the LCD when adding and subtracting algebraic fractions. Finally, it was discovered that learners were solving algebraic expressions from left to right without considering BODMAS. My findings acknowledge Ncube's (2016) findings as she exposed a similar pattern: when learners solved for algebraic fractions, BODMAS was disregarded.

4. Multiplication error

It was revealed that when expanding a binomial, learners only squared the first and last term. Learners also did not distribute all terms as instructed when multiplying binomials and trinomials. While multiplying algebraic expressions with exponents, learners were unsure whether the exponents should be added or multiplied. Ncube (2016) also discovered that the learners held several misconceptions about expansion, which led them to make a variety of multiplication errors. Egodawatte (2011) discovered the same error of just squaring the first and last terms and labelled it incorrect distribution. Ncube (2016) also found that learners did not remember the correct expansion of a binomial; they squared only the first and last terms.

5. Substitution error

Learners were able to substitute the given letter values, but the correct expression value was not obtained. It was discovered that the learners did not use brackets when performing substitution, and if they did use brackets correctly, they failed to apply BODMAS, demonstrating once again that their knowledge of integers was extremely limited and that they did not have a decent understanding of addition and subtraction of integers. According to Makonye and Fakude (2016), learners do not find it easy to accommodate negative numbers or the subtraction operation involving negative integers.

5.2.2 Second research question

What are the errors and misconceptions that Grade 9 learners make in solving algebraic expressions prior to the intervention process?

Slip errors decreased from the pre-test to the post-test; when learners correctly demonstrated the question using the algebra tile, they were less likely to write incorrect solutions for most questions on the post-test. The number of slips decreased by at least 43% when comparing the two tests.

The sign error was an area of concern in most questions. To arrive at the final solution for most algebraic expression questions, learners were required to further simplify their answers by adding and subtracting like terms. Sign error improved slightly between the pre and post-tests, as described in Chapter 4. Ncube (2016) suggests that the primary cause of the sign error is the learners' inability to comprehend addition and subtraction of like terms, which stemmed from a lack of understanding of addition and subtraction of integers. My results about the primary cause of the sign error coincide with Ncube's (2016) suggestions. The algebra tiles required learners to represent expressions with tiles and to pair zeros. By cancelling the zeros, addition and subtraction of like terms were simplified. Comparing the errors committed on both tests, there was a slight reduction of 25% in sign errors.

On both the pre- and post-tests, misconceptions were the most frequent type of error committed by Grade 9 learners. The misconceptions stem from prior knowledge acquired by learners in previous grades or from a misunderstanding of the mathematics rules taught in class. It was remarkable to observe a 33% reduction in misconceptions between the two tests. In other cases, as noted by Egodawatte (2011), learners invent their own rules that apply to them only and suit only their interests. It was evident from the test analysis that some of the misconceptions stemmed from prior knowledge. Luneta and Makonye (2010) further highlighted that the persistence of errors and misconceptions that are not addressed in the early grades continue to have a negative effect on performance in subsequent areas of mathematics.

There was also a substantial improvement in multiplication, and the difference between the pre- and post-test scores decreased by nearly 41%. The multiplication errors as highlighted was primarily since learners were only multiplying the first and last terms when expanding a binomial. Learners were also not distributing the outside number to all the numbers inside the bracket.

The substitution error was the most frequently committed error in both tests. The main issue with the substitution error, according to the findings, was that learners demonstrated a lack of conceptual understanding in integer addition and subtraction. Although they were able to correctly substitute the numbers into the expression, they were unable to determine the expression's exact value. The most improved errors were substitution errors, which were reduced by nearly 50%.

5.2.3 Third research question

How do the results of the post-test relate to the content in the intervention?

The post-test scores are higher and the difference between the pre- and post-test scores is statistically significant, indicating that the use of algebra tiles had a positive effect. The work done with learners in intervention lessons using algebra tiles may have influenced how they answered questions in the post test, resulting in improved performance and gains. It was critical to use manipulatives when addressing learner errors and misconceptions in algebraic expressions. The manipulative aided in making the abstract concept of algebraic expressions concrete. The post-test analysis revealed that errors and misconceptions can be remedied by probing learners' written work and teaching the concept using algebra tiles.

5.3 Limitations

Regarding my data set, the number of learners used for the study was relatively small; only one class with a maximum of 22 learners was used, more generalised claims could not be made. The study spanned two weeks with a one-hour lesson each day in the afternoon; however, this time was insufficient to work rigorously with learners. As a result, due to time constraints, only one error was covered in each day, and the next day was devoted to a different type of error. There were only 10 lessons conducted by me.

Due to the time constraints, it is possible that some of the errors and misconceptions were not fully remedied. A longer time frame could permit learners to be exposed to algebra tiles more frequently, thereby boosting their confidence when solving algebraic expressions. I could have broadened the data collection process by including all the Grade 9 learners at the school. The sample size of interviewing ten learners was insufficient, as their responses did not accurately represent the entire group.

Furthermore, after completing my data collection, I pondered the impact of using another class. Although it would have been preferable for another teacher to instruct the class without the use of algebra tiles. I thought if I had included another class to increase the sample size. There would be a treatment group and a control group. A larger sample would have been a more accurate representation of the population, yielding more accurate results for the study.

Even though definitive claims cannot be drawn from this one-time, relatively small-scale study, it does provide a useful lens and sufficient evidence to indicate the important role of algebra tiles in addressing errors and misconceptions in algebraic expressions.

5.4 Recommendations for future research

The findings of this study provide excellent motivation for future research into the use of algebra tiles to address algebra errors and misconceptions in South Africa. Algebra tiles may help learners comprehend algebraic thought processes, according to the findings. The results also suggest that algebra tiles support a variety of skills, including expression simplification, factorisation, distribution, and integer rules.

The findings have demonstrated that when learners are engaged in their learning, they attribute greater importance to abstract concepts, resulting in greater comprehension and the ability to conceptualise abstract concepts. My research confirms the findings of previous studies that teachers do not adequately address errors and misconceptions in the classroom. According to the research conducted by Brodie (2014), re-teaching the topic is not the most effective method for resolving errors. Another challenge is that secondary school teachers do not use manipulatives to aid conceptual comprehension because they are viewed as elementary school teaching techniques. Chaurasia (2019) worked with secondary school learners and shown that algebra tiles can assist disinterested learners in the classroom since they are adaptable to all types of learners. Following the use of algebra tiles learners were able to apply knowledge gained from concrete experiences to abstract conditions (Chaurasia, 2019).

It is suggested that mathematics teachers explore the use of manipulatives in their lessons. The study emphasises how manipulatives help learners learn by allowing them to transition from concrete experiences to abstract reasoning. According to research, when manipulatives are used, learner’s achievement improves over time.

5.5 Conclusion

The study conducted here focuses on manipulatives like algebra tiles, how these manipulatives can be employed to help learners solve algebraic expressions and how they can assist learners in correcting the errors they make. From the findings, I have shown the different errors and misconceptions that were identified in the learners' pre-tests. I then show that the learners performed better in the post-test. I also show how the algebra tiles can initiate a ZPD to assist learners in solving the algebraic expressions, first with the assistance of the tiles, and then independently. When misconceptions surface, instead of re-teaching, teachers can provide algebra tiles that will enable learners to solve the algebraic expressions for themselves rather than having procedures explained repeatedly. Conducting this study helped me to find better ways of addressing the errors and misconceptions that learners make when solving algebraic expressions. Consequently, learners will have better opportunities of making sense of mathematics, and hence develop independent learning.

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Appendices

APPENDIX A: Pre-test and Post-test

GRADE 9

PRE-TEST ALGEBRAIC EXPRESSIONS [40 MARKS]

QUESTION 1 [5 marks]

Choose the correct answer from the options provided. Please show all your working.

a) Simplify: $\sqrt{64x^{64}}$ (1)

(1) $32x^{32}$

(2) $8x^{32}$

(3) $32x^{8}$

(4) $8x^{8}$

b) $\left(\frac{a}{b}\right)^{-1} =$ (1)

(1) $\frac{a}{b}$

(2) 0

(3) 1

(4) $\frac{b}{a}$

c) Determine the value of $x^2 - 3y$ if $x = -1$ and $y = -2$. (1)

(1) 5

(2) -6

(3) -7

(4) 7

d) How many terms are in the expression $\frac{(x+3)(x-2)}{4x}$? (1)

(1) 1

(2) 2

(3) 3

(4) 4

f) $(x-3)^2 =$ (1)

(1) $x^2 - 9$

(2) $x^2 + 9$

(3) $x^2 - 6x + 9$

(4) $x^2 - 6x - 9$

QUESTION 2 [5 marks]

a) Given: $3x^2 - 5x + 7x^3 - 9$

1) What is the degree of the polynomial? (1)

2) Write this polynomial in descending powers of x. (1)

3) How many terms are in this expression? (1)

4) Calculate the value of this expression if $x = -1$. (2)

QUESTION 3[13 marks]

Simplify the following expressions:

a) $(3x)^3 + 2x^3$ (2)

b) $(a + b)^2$ (2)

c) $(2x^2 + 3x - 4) - (x^2 - 2x - 6)$ (2)

d) $\frac{2}{3}(2a^2 - 3a - 6)$ (2)

e) $\frac{-4a^2b \times 6ab^4}{12a^4b^2}$ (3)

f) $-2x + 3 - \frac{5}{x}$ (2)

QUESTION 4 [7 marks]

Expand and simplify the following expressions:

a) $(2p - 3)(p + 4)$ (2)

b) $(2x - 4)^2$ (2)

c) $(2a + 5b)(2a - 5b + 6b^2)$ (3)

QUESTION 5 [10 marks]

Factorise the following expressions completely.

a) $x^2 + 64$ (1)

b) $12x^2y - 6xy$ (2)

c) $t(x - y) - 2(x - y)$ (2)

d) $x^2 + 8x + 15$ (2)

e) $x^4 - 16$ (3)

GRADE 9

POST-TEST ALGEBRAIC EXPRESSIONS [40 MARKS]

QUESTION 1[8 marks]

a) Study the following expression and answer the questions that follow.

$$8y \times 7y^3 - \frac{2y^2}{3} - 4$$

(1) How many terms are there in the expression? (1)

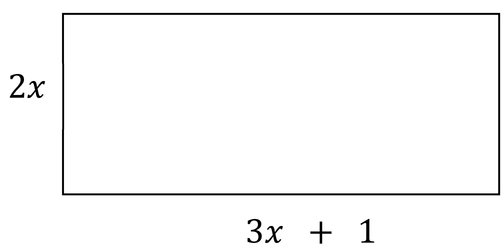
(2) What is the value of the constant term? (1)

(3) Write down the co-efficient of y^2 . (1)

(4) Arrange the expression in decreasing powers of y. (1)

(5) Calculate the value of this expression if $y = -2$. (2)

b) Write down and simplify an expression for the perimeter of the given rectangle, in terms of x: (2)



QUESTION 2 [16 marks]

a) Simplify the following:

(5) $5 - 3(x - y)$ (1)

(6) $3x + (x + 2)$ (2)

(7) $5 - 2(y + 3)2$ (2)

(8) $2x - 4y + 3xy + 2y - x$ (2)

(9) $(t + 3)^2 - 2(t + 1)(t - 10)$ (4)

(10) $3x^4 \times 2x^3 \times 2x^2$ (2)

(11) $(\frac{2x^6y^2}{4y^3})^5$ (3)

QUESTION 3 [7 marks]

a) Expand and simplify:

(1) $(q+2)^2$ (2)

(2) $2a\ (a - 3b)\ (a + 3b)$ (2)

(3) $(x + 2y)\ (2x + 3xy - 5y2)$ (3)

QUESTION 4 [9 marks]

a) Factorize the following:

(1) $x^3y^2 + x^2y^2 - 6x^2y^2$ (2)

(2) $m^2 - 16$ (2)

(3) $k^2 - 5k + 6$ (2)

(4) $2(x - y) - 2x(y - x)$ (3)

Appendix B: Exercises completed during the intervention lessons.

Exercises adapted from Mindbourne. Grade 9 textbook.

EXERCISE 8.1 *Revision of basic concepts.*

(a) Consider the expression $2x^2 - 8xy - 9y^2 + 7$.

(1) How many terms are there?

(2) What is the constant term?

(3) State the coefficient of the term in x^2, y^2 and x.

(4) Determine the value of the expression if $x = -\frac{1}{2}$ and $y = -2$.

(b) Simplify:

(1) $2m^2 + 2m + 3m^2 + 3m$ (2) $-4x^3 + 4x^2 - 2x^3 - 6x^2$

(3) $4m^2n - 7m^2n^2 + 3(mn)^2$ (4) $(-4x^3)(-2x^3) - (5x)(2x^5)$

(5) $(2x^7)^2(x^2)^7$ (6) $(2x^7)^2 - (x^2)^7$

(7) $3(a^2)^3 + (3a^2)^3$ (8) $(-2x^2)^3 - (-2x)^4$

(c) Simplify:

(1) $2(x-3)$ (2) $-3(x+6)$

(3) $-x(x+7)$ (4) $4x(2x-3y)$

(5) $-(m^2 - 3m)$ (6) $3(a-b^2) - 2(2a-b^2)$

(7) $-(x^2 - 2x - 8)(-3x)$ (8) $-(x^2 - 2x - 8) - 3x$

(9) $(2p-4p)(2p^2-1)$ (10) $2p - 4p(2p^2-1)$

(11) $-3x(x^3 - 2x^2 - 4x) - x^2(2x^2 - 3x + 1)$

(d) Simplify:

(1) $\frac{8x+16}{8}$ (2) $\frac{12x-18y}{3}$

(3) $\frac{3x^2-12x}{3x}$ (4) $\frac{4a^3b^2+8ab^2}{2ab}$

(5) $\frac{6x^3-12x^2}{-3x}$ (6) $\frac{9a^2b-18ab^2}{-3ab}$

(7) $\frac{5x^4-10x^3-15x^2}{5x^2}$ (8) $\frac{2a^5b^4+8a^5b^4-12a^5(-b^2)^2}{2(-ab)^3}$

(9) $\sqrt{9x^{16}+(4x^8)^2}$ (10) $\sqrt[3]{25m^9+100m^9}$

(11) $\sqrt[3]{(4x^3+4x^3)(-2x^5)^3}$ (12) $\frac{(-a^4-a^4)^4}{\sqrt{(-8a^8)(-8a^8)}}$

EXERCISE 8.2

(a) Expand and simplify:

(1) $(x+2)(x+1)$ (2) $(x+3)(x+2)$ (3) $(a+4)(a+6)$

(4) $(b-1)(b+5)$ (5) $(y-3)(y-2)$ (6) $(p-7)(p+4)$

(7) $(2x+1)(x+3)$ (8) $(3y+2)(y+2)$ (9) $(4w+1)(2w-3)$

(10) $(5n-2)(n+5)$ (11) $(8m-3)(2m+7)$ (12) $(x+2y)(x+3y)$

(13) $(m-2n)(m-4n)$ (14) $(4-6h)(3+4h)$ (15) $(6+2p)(3-4p)$

(16) $(-3x-3)(-4x+3)$ (17) $(3-7x)(9x+1)$ (18) $(a^3-2b)(a^3+3b)$

(b) Expand and simplify:

(1) $(x+1)(x-1)$ (2) $(x+2)(x-2)$ (3) $(a-4)(a+4)$

(4) $(m+6)(m-6)$ (5) $(p+5)(p-5)$ (6) $(2x+1)(2x-1)$

(7) $(3x-2)(3x+2)$ (8) $(7p-6)(7p+6)$ (9) $(12-8p)(12+8p)$

In the next part of the exercise, you need to be aware that $(a+b)^2=(a+b)(a+b)$.

(c) Expand and simplify:

(1) $(x+1)^2$ (2) $(x+2)^2$ (3) $(x+3)^2$

(4) $(x+4)^2$ (5) $(2y+3)^2$ (6) $(4m+7)^2$

(7) $(8p+6)^2$ (8) $(9n+4)^2$ (9) $(x-1)^2$

(10) $(x-2)^2$ (11) $(x-3)^2$ (12) $(x-4)^2$

(13) $(2y-3)^2$ (14) $(4m-7)^2$ (15) $(8p-6)^2$

(16) $(9n-4)^2$ (17) $(12x+5)^2$ (18) $(12x-5)^2$

(19) $(x^2-2)^2$ (20) $(3x^3+4y^2)^2$ (21) $(3x^3-4y^2)^2$

EXERCISE 8.5

(a) Factorise the following expressions:

(1)	$3x+6$	(2)	$4x-8$	(3)	$2a+12$
(4)	$8b-16$	(5)	$5ab+5ac$	(6)	$9ab+18a$
(7)	$15ab+3a$	(8)	$6mn+12n$	(9)	$7p-14pq$
(10)	$2xy-3y$	(11)	$15x^2+10x^2y$	(12)	$5x^2+10x^2y$
(13)	x^2+2x	(14)	$2y^2+4y$	(15)	$2a^2-2a$
(16)	$6x-x^2$	(17)	$5d^2-15d$	(18)	$9m^2-15m$
(19)	mn^2-m^2n	(20)	$\pi r^2+2\pi r$	(21)	x^3+2x
(22)	$3a^3+12a^2$	(23)	$16y^3+12y$	(24)	$6n^2-18n^3$
(25)	x^4-2x	(26)	$4p^4-8p$	(27)	$9m^4-9m^3$
(28)	$19x^5-38x^2$	(29)	$27g^6-18g^3$	(30)	$26c^7-13c^6d$
(31)	$3gh^3-33h^5$	(32)	$17a^5b+34b^2$	(33)	$3a^2b+9ab^3$
(34)	$18x^6y^4+12x^4y^7$	(35)	$36m^5n^3-15mn^6$	(36)	$16pq^7+32p^2qr$
(37)	$45x^3y^{10}-5x^2y^8$	(38)	$27a^3b^2c+81a^4b^3c^2$	(39)	$25x^9y^{10}-35x^7y^5$

(40)* $3ab^2+6a^2b^3-9a^3b^4$ (41)* $7m^3n^7-14m^4n^8+63m^5n^9$

(42)* $56\pi r^3+8\pi r^4+16\pi r$ (43)* $17\pi r^2-10\pi r^2+14\pi^2 r$

(b) Factorise the following expressions:

(1)	$a(b+c)+d(b+c)$	(2)	$2x(x-y)-y(x-y)$
(3)	$p(q+r)+q(q+r)$	(4)	$m(a-2b)+n(a-2b)$
(5)	$3x(x+1)-2(x+1)$	(6)	$m(n-2)-2n(n-2)$
(7)	$3a(b-4c)-6(b-4c)$	(8)	$4x(x-2)-8(x-2)$
(9)*	$(a-b)^2-2(a-b)$	(10)*	$(x-1)^2-3(x-1)$
(11)*	$(y-5)^3-(y-5)^2$	(12)*	$(x-2y)^4-3(x-2y)^3$

EXERCISE 8.8

(a) Factorise:

(1)	x^2-1	(2)	x^2-4	(3)	y^2-9
(4)	a^2-16	(5)	p^2-25	(6)	m^2-36
(7)	n^2-81	(8)	$4x^2-1$	(9)	$16x^2-9$
(10)	$81y^2-16$	(11)	$49n^2-121$	(12)	$100d^2-169$
(13)	x^2-4y^2	(14)	$16b^2-25c^2$	(15)	x^4-9
(16)	x^4-16	(17)	a^4-81	(18)	$a^2b^2-4c^4$

(b) Factorise fully:

(1)	$4x^2-4$	(2)	$8y^2-2$	(3)	$2x^2-50$
(4)	$3p^2-27$	(5)	$5n^2-20$	(6)	$100m^2-25$
(7)	$4x^2-16$	(8)	$4x^2-64$	(9)	$12x^2y-27y^3$
(10)	$75a^2-12b^2$	(11)	x^3-36x	(12)	x^2-4x^4
(13)	$3p^2q-48q^3$	(14)	$2ax^4-50a$	(15)	$3x^2y-243y$
(16)	$-x^2+4$	(17)	$-x^2+16$	(18)	$-9x^2+1$
(19)	$-25x^2+4$	(20)	$-2p^2+98$	(21)	$-3n^3+3n$
(22)	$-8b^5+32b$	(23)	$-7a^2b^2+28$	(24)	$-2a^5b^2+18a$
(25)*	$(x-y)^2-1$	(26)*	$(a-b)^2-9$	(27)*	$4(m-2n)^2-25$

EXERCISE 8.9

(a) Factorise:

(1)	x^2+3x+2	(2)	a^2+6a+5	(3)	$p^2+7p+12$
(4)	$y^2+7y+12$	(5)	$x^2+11x+18$	(6)	$k^2+8k+15$
(7)	$m^2+14m+24$	(8)	$d^2+14d+40$	(9)	$y^2+13y+40$
(10)	$x^2+12x+35$	(11)	x^2+5x+6	(12)	$x^2-7x+12$
(13)	$w^2-8w+15$	(14)	$x^2-11x+28$	(15)	$x^2-10x+9$
(16)	$x^2-10x+16$	(17)	n^2+n-20	(18)	$x^2+7x-18$
(19)	$t^2+4t-60$	(20)	$x^2+3x-88$	(21)	$r^2+5r-50$
(22)	x^2-x-6	(23)	$x^2-5x-24$	(24)	$k^2-9k-36$
(25)	x^2+2x+1	(26)	a^2-6a+9	(27)	$y^2-10y+25$
(28)	$x^2+13x-30$	(29)	$x^2+3x-54$	(30)	$x^2-17x-38$
(31)	$a^2-20a+64$	(32)	$x^2+22x+72$	(33)	$x^2-25x+100$
(34)*	$27x+x^2-90$	(35)*	$-64+30y+y^2$	(36)*	$k(k-9)-52$

(b) Factorise fully:

(1)	$2x^2-12x+16$	(2)	$2y^2+10y+8$	(3)	$3a^2-15a-18$
(4)	$5d^2-45d+100$	(5)	x^3-12x^2+27x	(6)	$4p^2-48p+80$
(7)*	$-x^2-2x+8$	(8)*	$-a^2-22a+75$	(9)*	$-4x^2+20x+24$
(10)*	$-2k^2+22k+52$	(11)*	$-a^2b+19ab-84b$	(12)*	$126+36x-2x^2$

APPENDIX C : Ethics clearance letter

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	PROTOCOL NUMBER: 2022ECE011M
PROJECT TITLE	Exploring errors and misconceptions in solving algebraic expressions with Grade 9 learners
INVESTIGATOR	BULELWA PENELOPE STEMELE
SCHOOL/DEPARTMENT OF INVESTIGATOR	WSoE
DATE CONSIDERED	18 MARCH 2022
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	MINIMAL
EXPIRY DATE	Date of submission of the Research Report
ISSUE DATE OF CERTIFICATE	CHAIRPERSON [signature] Dr. Batseba Mofolo-Mbokane

cc: Dr. Zaheera Jina Asvat

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

[signature] Signature

Date 24 / 03 / 2022

27 St Andrews Road, Parktown, Johannesburg, 2193 | Pvt Bag 3, WITS, 2050 | T +27 11 717 3007 | www.wits.ac.za/education

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APPENDIX D : Information letters

LETTER TO THE PRINCIPAL

February 2022

Bulelwa Stemele

Silverton, Pretoria 0084

0717101078

Email:552425@learners.wits.ac.za

bstemele@cbcpretoria.co.za

Dear Mr. Langton

Request for permission into school as a research site

I am a University of the Witwatersrand learner and a mathematics teacher at your school. I am currently pursuing a masters in mathematics education. I'm proposing a research project on teachers' classroom methods as part of the program's requirements. The study's main goal is to explore learner errors and misconceptions in algebraic expressions with Grade 9 learners. I am interested in how I interact with my learners and the regular patterns in mathematics classes, as well as helping them address slips, errors and misconceptions in algebraic expression. Afternoon lessons and a series of tests and interviews (to be audiotaped) was part of the study processes. I am aware of the ethical issues surrounding human subject research, and I have put in place a variety of safeguards to protect study participants. For example,

- I do not plan to complete the activities during school hours; instead, the lessons will take place after school.
- Other ethical considerations include: the videos will be used solely for research and teacher training; written consent will be obtained from parents and learners (see attached); and I will make the final report available to all participating learners after it has been examined by the University of the Witwatersrand.

If you have any questions or concerns concerning the research, please do not hesitate to contact me at any time.

Yours sincerely,

Bulelwa Stemele

INFORMATION LETTER TO PARENTS

Bulelwa Stemele

Silverton, Pretoria

0084

0717101078

Email:552425@learners.wits.ac.za

bstemele@cbcpretoria.co.za

Dear Parent of Learner

I work at CBC Mount Edmund High School as a mathematics teacher. I would like to propose a study on algebraic expressions. The study's main goal is to educate grade 9 learners about the common errors they make when solving algebraic expressions.

Your child is kindly invited to participate in the study. The study will take place during March and April for at least three weeks. The lesson will last thirty minutes. As part of the study, the teachings from your child's algebraic expressions class will be recorded. Your child will take a pre- and post-test, as well as an interview, as part of the course requirements. It is permissible to photocopy portions of your child's solutions.

I will not ask for your child's name or any other identifying information, and the tests and interview replies will be completely confidential and anonymous. Any information supplied to me will be kept secure and will not be shared with anyone else. I will employ a pseudonym to reflect your child's participation in my research articles (fake name).

If your child joins in this initiative, there will be no personal charges to you or your child. If you opt not to allow your child to participate or if you withdraw your child from the research, there are no disadvantages or fines. If you do not want your child to, she or he can withdraw at any time or refuse to answer any questions. The study will have no bearing on your child's mathematics report card.

If you have any questions about this research at any time during or after it is completed, please contact me using the information provided above. This research will be published as scholarly articles, which will be accessible via the university library's website. The information gathered for this study will be saved on a hard disk and preserved for five years. Other researchers may utilize the data collected from this study project with your approval.

Yours sincerely,

Bulelwa Stemele

INFORMATION LETTER TO LEARNERS

Bulelwa Stemele

Silverton, Pretoria

0084

0717101078

Email:552425@learners.wits.ac.za

bstemele@cbcpretoria.co.za

Dear Learner

I work at CBC Mount Edmund High School as a mathematics teacher. I'd like to propose a study on algebraic expressions. The study's main goal is to educate grade 9 learners about the common errors they make when solving algebraic expressions.

You are cordially invited to take part in the research. The research will take place between March and April 2022. The course is three weeks long. I'd like to request thirty minutes of your time each day. I'd like you to interact with me as the teacher, who will teach you algebraic expressions in a pleasant and informal setting. I ask for your permission to fully participate in these sessions. I'm also asking for your permission to record the lessons during the algebraic expression course that you'll be doing as part of the study.

I'd want to ask for your permission to write the pre- and post-tests, as they are required for the study. I might photocopy a portion of your workout. The test results will be kept fully private because I will not ask for your name or any other identifying information, and the information you provide during the interviews and tests will be kept secure and not shared with anyone else. To represent your participation in my academic articles, I will use a pseudonym (fake identity).

You will not be charged or compensated for your participation in this study. You have the option to leave the study at any time. Your Mathematics performance report will have nothing to do with the research.

If you have any questions about this research at any time during or after it is completed, please contact me using the information provided above. This research will be published as scholarly articles, which will be accessible via the university library's website. The information gathered for this study will be saved on a hard disk and preserved for five years. Other researchers may utilize the data collected from this study project with your approval.

Yours sincerely,

Bulelwa Stemele

APPENDIX E: Consent forms

PARENT'S CONSENT FORM

I, ………………………………………………….. (parent), agree for my child, ………………………………. to participate in this research project. The research has been explained to me and I understand what my child's participation will involve. I agree to the following:

(Please circle the relevant options below).

I agree that my child's participation will remain anonymous.

YES NO

I agree that the researcher may use anonymous quotes of my child in her academic articles.

YES NO

I give consent to classroom sessions being recorded with my child in them.

YES NO

I give consent for part(s) of my child's test solutions to be photocopied.

YES NO

I agree that the information my child provides may be used anonymously after this project has ended, for academic purposes by other researchers, subject to their own ethics clearance being obtained.

YES NO

…………………………………… (signature)

…………………………………… (name of participant)

…………………………………… (date)

…………………………………… (signature)

Bulelwa Stemele (name of person seeking consent)

…………………………………… (date)

LEARNERS' CONSENT FORM

I, ……………………………………………………, agree to participate in this research project. The research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below).

I agree that my participation will remain anonymous.

YES NO

I agree that the researcher may use anonymous quotes in her academic articles.

YES NO

I give consent to video-record by a camera man.

YES NO

I give consent for part(s) of my journal notes to be photocopied.

YES NO

I agree that the information I provide may be used anonymously after this project has ended, for academic purposes by other researchers, subject to their own ethics clearance being obtained.

YES NO

…………………………………… (signature)

…………………………………… (name of participant)

…………………………………… (date)

………………… (signature)

Bulelwa Stemele (name of person seeking consent)

…………………………………… (date)