MATHEMATICAL KNOWLEDGE FOR TEACHING GEOMETRY TO GRADE 10 LEARNERS

A Mini – Dissertation

by

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DECLARATION

I declare that this research report is my own work. It has being submitted for the Degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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(Signature of Candidate)

20th day of December 2007

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DEDICATION

Gratitude to my parents and family.

Whose unconditional love and tireless devotion inspired me through my years of studying.

I am truly blessed.

Thank – you.

ABSTRACT

The purpose of this project is to investigate the Mathematical work that educators goes through when teaching Geometry to grade 10 learners using the National Curriculum Statement. It is important to establish if educators understand the concepts of the new curriculum.

This case study was used to gain insight into the mathematical problems and the mathematical problem solving that this teacher uses in his teaching practices.

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Introduction

*1.1 **Aim:***

The implementation of the new National Curriculum Statement for Grade 10 – 12 (DoE, 2002), in South Africa is currently under way. The National Curriculum Statement is a product of reform in education in South Africa that tends to equip all stakeholders, especially teachers, to deal with and encompass the process of change. One of the aims of this new curriculum is for teachers to provide unique opportunities of enriched learning processes to improve the education learners actually receive. The statement: "A suitable range of mathematical process skills and knowledge enables an appreciation of the discipline itself" (DoE, 2001:9) amplifies the purpose of the curriculum.

Thus this study makes it possible for us, as researchers, to investigate classroom practices to see the influences of teachers and their teaching on and between the curriculum and the learners. The construction and the implementation of a new curriculum go hand in hand thus making it necessary to investigate whether teachers and learners at school are equipped to deal with the challenges of the new curriculum and the demands within the confines of the learners' mathematical learning institutions. This study is located in the midst of the sea of educational changes in South Africa and its challenges, specifically focussing on mathematics teaching. In particular, the purpose of this study is to investigate the mathematical work[1] that the teacher does or needs to do when teaching a specific section in geometry to grade 10 learners, in the context of the new curriculum demands. I have briefly introduced the problem area in which this study is located. In the next section I will make explicit the research questions that this study focuses on and the critical questions that underpin this study.

*1.2 **Research Problem:***

This is an in-depth study of how a selected teacher works with the mathematical problem solving and the challenges this teacher encounters when implementing the complex tasks[2] of teaching and mathematical demands on the teacher related to the selected section of

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1 Mathematical work refers to the problem solving teacher do as they teach. Refer to Ball. Bass and Hill (2004) and Hill, Rowan and Ball (2005). This will be further unpacked later on in chapter 2.

2 Task refers to the mathematical work that one is assigned to do.

mathematics in the teacher's grade 10 classroom, in the context of the new curriculum. The intention of this study is to investigate the mathematical work the teacher does as the teacher figures out what the learners know about this specific geometry section (polygons, in particular quadrilaterals) in the curriculum of mathematics and to determine how to move the learners on. I want to observe what mathematical work the teacher does (or needs to do) which is only observable over time:

- To know the learners level of understanding;
- To understand what there is to know about polygons, in particular quadrilaterals, over and above being able to 'do' specific problems or exercises;
- To know multiple perspectives, representations and arguments to make relevant decisions in his teaching;
- To turn his content knowledge into pedagogical action.

The above points assisted in looking at the mathematical knowledge for teaching geometry to grade 10 learners, which would assist in answering the following critical question.

1.3 ***<u>Critical Questions:</u>***

1) What mathematical work does the teacher do as he teaches geometry (polygons, in particular quadrilaterals) to his Grade 10 learners?
   a) What mathematical problems or challenges does he encounter?
   b) How does he engage with these mathematical problems of teaching?

2) What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?

3) How does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?

1.4 ***<u>Rationale:</u>***

The goals of the QUANTUM[3] research project that investigates mathematics for teaching gave a sense to why this study focuses on the pedagogic practices of a mathematics teacher when teaching geometry that is informed by the new curriculum. Thus I am trying to get an

[3] The QUANTUM project is a larger study on mathematics for teaching in teacher education and school classroom practices. See projects by: Adler & Davis (2005), Adler and Pillay (2006), Kazima and Adler (2006) and Pillay (2006).

understanding of the mathematical work of teaching in this case which is aligning itself to one of the aims of the QUANTUM project.

1.4.1 Geometry is difficult to teach:

It is well known that some teachers and learners encounter difficulties in secondary school geometry. From my experience as a secondary school teacher, I found that most mathematics learners either find tasks too simple or too difficult. In providing tasks that are at the appropriate level for all learners, it places teaching demands on the teacher, one in particular being the need to restructure or rescale tasks (Ball and Bass, 2000) so that they challenge a diverse range of learners. Rescaling of tasks poses problems and challenges for a teacher: scaling down can enable mathematical development for some learners, but at the same time it can make the task too easy for others. On the other hand scaling up a task could be what is needed for some learners, but this then makes it inaccessible to others. Hence the importance of a study that investigates how a teacher works with geometric tasks to meet varying learners' needs, interests and developmental levels in their classrooms, and the mathematics demands of this work.

In relation to development, the Van Hiele's (in Feza and Webb, 2005) state that one of the reasons for difficulties with learning and teaching geometry is because secondary school geometry requires the learners to think at higher levels, while the lower levels of geometric thinking have not been adequately developed. The hierarchical, levels of geometric thinking makes it interesting for this study to see how the teacher moves the learners from one level to the next level. Through my personal experience, I found that learners often learn the definitions of various quadrilaterals through rote learning without understanding them. Ball and Bass (2000) contend that when learners are confined by *bounded tasks* [4] they can often produce the correct answers and so appear successful at mathematics despite the lack of understanding. This mismatch between what learners might be expected to do, and what understanding they are presumed to have could also create problems or challenges for teachers. Success in mathematics with understanding is important for this study and it will be interesting to see how this teacher works with tasks in relation to the Van Hiele levels of geometric thinking.

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[4] Bounded tasks are opposite to opened tasks. Bounded tasks have definite solutions which are obtained by using set procedures and operations.

1.4.2 ***<u>Changes to teaching geometry in the new curriculum:</u>***

A study of geometry teaching has also become very important given the current curriculum changes in South Africa. Through redesigning the interim syllabus (National Curriculum Statement – General Policy, 2003) there emerged a new integrated and responsive curriculum statement, known as the National Curriculum Statement for Grade 10-12. The National Curriculum Statement is the formal intended curriculum in South African schools. The implementation of this changed curriculum is not only the responsibility of specialists in curriculum development but of all stakeholders in the education fraternity, especially schools and teachers. The new curriculum encompasses everything (ongoing curriculum development and delivery) that happens in the school as a result of what teachers do, and promotes a view of education as learner-centred rather than teacher–centred: the teacher is seen as a facilitator and resource for learners who learn by interacting, problem solving, discovering and investigating. This new curriculum also contains and refers to the official policy on what is to be taught and it includes statements about the importance of the integration of knowledge and learning being relevant and connected to real life situations (DOE, 1997). Taking parallelograms as an example, the 'old' curriculum emphasised properties, definitions and proof. For *example*, one can prove that a quadrilateral PQRS is a parallelogram by definition and by general proof, working with properties that constitute sufficient to ascertain a parallelogram such as:

- i) Opposite sides are parallel.
- ii) Opposite angles are equal in size.
- iii) Opposite sides are equal in length.
- iv) Diagonals bisect one another.
- v) Each diagonal bisects the parallelogram.
- vi) One pair of opposite sides is equal and parallel.

This is the kind of problem we would have found in previous grade 10 textbooks. In the new National Curriculum Statement (DoE, 2002), the teaching and learning of space, shape and measurement compels us to rethink Euclid's definitions from a spatial point of view, to establish knowledge about the plane and space which are more relevant to learners' lives and their experiences. From an epistemological perspective, the study of space, shape and measurement started long before school mathematics (geometry) but the mathematics tasks and approach used to teach geometry in schools constantly changes. Geometry is about the orientation of space so geometric objects would appear in 1 dimensional, 2 dimensional and 3 dimensional spaces. This means an orientation to the exploration of space. It is not easy to show this spatiality, although through education we can enquire about what mediations passes spatiality into space, i.e. when pragmatic space becomes systematic space. Although it is not simple to move from pragmatic space to systematic space this is what the new curriculum intends of the learner, and so has implications for teaching. There is emphasis in the new geometry syllabus (both Euclidean geometry and Analytical geometry) on proof and proving, conjectures and conjecturing about properties of quadrilaterals. For *example*, in contrast to the kinds of problems that dominated geometry textbooks and exemplified above, here is a task one is likely to find in current textbooks:

The vertices of quadrilateral ABCD are A (2; 1), B (5; 2), C (6; 5), and D (3; 4).

i) Sketch and show that ABCD is a parallelogram.

ii) Write down the relationship, as investigated by construction and measurement between opposite sides, opposite angles, consecutive angles, a rectangle, square, rhombus, and trapezium.

This approach is used to build on the learners' previous experiences from the General Education and Training band [5] to make formal and extended levels of knowledge assessable, thus deepening the learners experience and proficiency in the processes of geometric proof. The Further Education and Training band[6] requires the learners to acquire functioning knowledge and skills of mathematics such as investigating, generalising and proving together with content subject knowledge. This is aptly captured in the curriculum statement for mathematics, which includes inter alia:

> "**Use various logical processes to formulate, test and justify conjectures:**
> Reasoning is fundamental to mathematical activity. Active learner's question, examine, conjecture and experiment. Mathematics programmes should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct convincing arguments of others."
>
> [Government Gazette, June 6, 1997:115, my emphasis]

In addition, the National Curriculum Statement emphasizes the following principles: social transformations, high knowledge and high skills, integration and applied competence,

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5 General Education and Training band refers to the beginning phase of formal schooling education, from grade 0 to Grade 9.

6 Further Education and Training band refers to the last phase of formal schooling education, from grade 10 to grade 12.

progression, articulation and portability, social and environmental justice, human rights and inclusivity, valuing indigenous knowledge systems, credibility, quality and efficiency. In Bernstein's (1996) terms, we can understand the shift from the old "intended curriculum" which had a rigid design, sequence, pacing and was highly insulated and classified against interferences from other subjects (what Bernstein referred to as a 'collection code'), to a more integrated code. The integrated code where there is weaken classification and weaken framing between contents leads to an integrated curriculum that is non- specialized with weak boundaries and linked more to everyday life. According to Bernstein's (1996) terms the integrated curriculum is defined by the fact that the contents instead of going their own separate ways stand in open relation to each other. This interactive curriculum requires a more spatial orientation to geometry. While this is not a fully integrated curriculum code in Bernstein's (1996) terms, there is an intention to weaken insulation. This thus has implications for both the teacher and learners. In particular, the teacher must be able to transfer additional skills: to think logically, analytically, holistically and laterally because problem solving is not a set of learnt techniques but it is a tool for analysing, understanding and engaging with the world. It is in this intended context that teachers might face new problems or challenges as learners work with geometric tasks in line with the new curriculum goals and so with new demands on how they need to work with mathematics in their classroom.

At present the outcomes - based education system is under scrutiny, learners are now required to read, describe, represent, analyse and interpret, explain properties of shape in 2-dimensional and 3-dimensional spaces with support and justification (DoE, 2002). The study of space, shape and measurement requires learners to explore relationships, make and test conjectures, solve problems and disprove false conjectures involving polygons, in particular quadrilaterals (DoE, 2002). This in-turn leads to investigating geometric properties of polygons, in particular quadrilaterals to establish, justify, prove conjectures and give alternative definitions of polygons. These make new and different demands on teachers and their teaching, including new orientations to mathematical activity. This study is aimed at providing insights into some of the difficulties and challenges faced by the grade 10 mathematics teachers teaching the grade 10 learners with the new curriculum.

Hence the questions:

1) What mathematical work does the teacher do as he teaches geometry to his Grade 10 learners?

i. What mathematical problems or challenges does he encounter?
ii. How does he engage with these mathematical problems of teaching?

2) What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?
3) How does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?

1.5 <u>Plan of Study:</u>

In this Chapter, the problem and aims of the study have been outlined. In particular aspects and demands of the new curriculum as they are to be used in the rest of the work have also been clarified. For example, investigate; justify; clarify; conjecture; test and derive.

In Chapter Two, a review of literature dealing with demands of the new curriculum and the changes for teaching geometry is done together with an in–depth review of research on the mathematical work of teaching and what teachers do or need to do when teaching geometry. In addition, this chapter reports on how Bernstein's theory is used as a wider lens for the study – as its broad theoretical framework; and how Ball and Bass (2000) work on the mathematical problem solving teachers do contributes further to the development of the analytic framework for the study.

In Chapter Three, the qualitative method of investigation that has been used will be discussed and the empirical investigation will be outlined.

Chapter Four provides an analysis of the data (Interviews and Lesson Observation).

In Chapter Five, the results and conclusions of the investigation will be dealt with and recommendations will be made.

Chapter Two

Literature Review and Theoretical Framework

2.1. Introduction:

> "A teacher must interpret students' written work, analyse their reasoning, and respond to the different methods they might use in solving a problem. Teaching requires the ability to see mathematical possibilities in a task, sizing it up and adapting it for a specific group of students. Familiarity with the trajectories along which fundamental mathematical ideas develop is crucial if a teacher is to promote students' movement along those trajectories. In short, teachers need to master and deploy a wide range of resources to support the acquisition of mathematical proficiency."
>
> [Kilpatrick, Swafford and Findell, 2001:370]

The above caption aptly illustrates the aim of how a specific teacher engages in the 'mathematical work' and the knowledge resources that the teacher calls upon as he goes about this mathematical work to fix meaning for the learners. These are not immediately visible in teaching practice, and so require an orientation to an analytical framework. I see mathematical work as a mental process that is also social. The teaching and learning of mathematics is envisaged as a social activity – occurring through interaction between the teacher and the learner. Learners' mental activity, however, is not visible, and in mathematics much of the activity involves abstract processes. Hence, one of the ways a teacher can gauge what the learners have acquired is by encouraging and enabling learners to verbalize their understanding. Thus when one considers the role of mathematics education in a developing country like South Africa, one needs to keep in mind the statement from the National Curriculum Statement (DoE 2002) about the nature of mathematics:

> "The study of Mathematics contributes to personal development through a deeper understanding and successful application of its knowledge and skills, while maintaining appropriate values and attitudes. … Competence in mathematical process skills such as investigating, generalising and proving is more important than the acquisition of content knowledge for its own sake."
>
> [National Curriculum Statement Grade 10 – 12. 2002:9]

From this it is evident that change is not merely new content, or new method, but a mind shift. It necessitates that reform in mathematics education brings the need to reason critically, logically and mathematically in order to explain and justify mathematical thinking. These reforms require the teacher to not only know the rules in the mathematical content and context, but also why these rules exist and mathematical processes that produce these rules and how these rules come about. My review of relevant literature that follows has been selected about the teaching of geometry in focus and this discussion of mathematical work and the new curriculum's intentions forms the backdrop against which this study is conducted.

I have reviewed a selection of literature in mathematics education in order to be well informed about prevailing debates on the teaching and learning of geometry that inform my study. I have also focused on literature related to pedagogical and mathematical knowledge for teaching. The following research is of central importance: the Van Hiele and Van Hiele-Geldof (1958) level of geometric thought as the frame through which to interpret research on the teaching and learning of geometry; a range of research on teachers' knowledge stemming from Shulman's (1986 and 1987) distinctions between mathematical content knowledge, pedagogical content knowledge and curricular knowledge including Ball and Bass (2000) who describes and develop Shulman's (1986 and 1987) notion of pedagogic content knowledge (PCK), Kazima and Adler (2006) and Adler and Pillay (2006) who researched teaching practice and how teachers wrestle with various tasks of teaching. Also Kilpatrick, Swafford and Findell's (2001) postulation of five interrelated strands that constitute mathematical proficiency and Stein, Schwan-Smith, Henningson, and Silver (2000) analysis of the range of cognitive demands that mathematical instructional tasks place on learners contribute to understanding the tasks that teachers offer learners. This selected array of research literature on teaching and learning mathematics enlighten my study about mathematics for teaching in practice.

The Van Hiele (1984) levels of geometric thought and Kilpatrick et al. (2001) strands of proficiency forms the backdrop for me to interpret the teaching and learning of geometry; and the notion 'mathematical work' as mathematical problem-solving in mathematics teaching of Ball, Bass and Hill (2004) and Hill, Rowan and Bass (2005) is anticipate to assist me in answering the first critical. While Kazima and Adler (2006), Pillay (2006) and Alder and

Pillay (2006) casts light to answer the second and third critical questions as discussed in 2.2 below.

*2.2 **Panorama on the teaching and learning of geometry:***

The teaching and learning of geometry in South Africa requires specific attention because many schools have been educationally and economically disadvantaged, with effects on the quality of instruction in geometry and so too learners' understanding of geometry. Thus, for this study, pedagogic practice is important since it focuses on how and what a teacher does to develop the learners' understanding of high school geometry. Ball, Lubienski and Mewborn (2001) argue that conceptual understanding is crucial if learners are to become proficient in mathematics, i.e. the teacher must know how to make the mathematics accessible to the learners. This is also evident from the extensive research of Bennie (1998), Burger & Shaughnessy (1986), De Villiers (1987), Feza & Webb (2005), Fuys et al. (1988), Mason (in press), Mayberry (1981) and Mayberry (1993) that uses Van Hiele's theory of geometric thought to describe the way in which the teaching and learning of geometry links to the school curriculum.

To probe difficulties encountered in school geometry, the Van Hiele et al.'s theory (1958) postulates a model of five hierarchical levels for geometric thinking. In view of this, Van Hiele's theory is going to be used in two ways in this study: a) to look at the geometric requirements (polygons, in particular quadrilaterals) stipulated by the National Curriculum Statement because it is informed by the Van Hiele levels, and b) to see where in the National Curriculum the learners in grade 10 are expected to be, thus relating to the five phases of learning. Given the research, I looked at both South African and international research and given that the National Curriculum Statement is new, the question arises as to whether it is likely that the teacher's class will or will not be at the appropriate Van Hiele level for grade 10. If not, the teacher is going to have difficulty in managing where the learners are coming from and where the teacher needs to assist the learners to get to. In addition, the teacher will be working with his interpretation of the orientation to geometry expected by the new curriculum and this increases the demands on him. As it shall be seen, the teacher in this study does indeed attempt to work with his interpretation of the goals for mathematics, which is aligned with the principles inherent in the new curriculum.

In the 1960's, the former Soviet Union changed its geometry curriculum to accommodate the Van Hiele's levels of development. In the 1980's the United States also showed an interest in the Van Hiele's model of geometric thought levels; and in South Africa changes in 2006 at school level, implemented through the National Curriculum Statement, infers the Van Hiele levels. With reference to the National Curriculum Statement, looking at Van Hiele's level[7] (1984) of thinking the General Education and Training (GET) learner must have acquired level 0, 1 and 2. That is, the GET) learners in alignment with the Van Hiele levels should be able to:

At level 0 – Visualise. Here the teacher must expose learners to more sensory experiences to create the objects of thought and then structure thinking by perceptions for example, *recognising rectangles and squares as quadrilaterals and polygons* in order for learners to move to level 1;

At level 1 - Analyse. Where teachers must provide more exercises on recognition of properties so that learners can analyse component parts that should be used. For example, *opposite angles of parallelograms are congruent*;

At level 2 – Informally deduce. Where teachers must provide more examples and ask learners to formulate and explain informally the interrelationships of properties within and among figures, for example, *in a quadrilateral, if the opposite side are equal then the opposite angles are congruent, or a square is a rectangle that is a quadrilateral because of the same properties.*

In relation to this study, from this point on when learners enter the Further Education and Training (FET) band they should be at *level 3 moving on to level 4. At level 3* learners must be able to deductively prove theorems and form interrelationships among axioms, definitions, theorems and formal proofs. For example, *the learners must operate on the objects from the ordering of relations, i.e. every defined term may be traced to undefined terms for example, if the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram;*

At *level 4, teachers* must teach with more rigor so that, learners are able to postulate theorems in different axiom systems by comparison and analysis. By the time learners reach their senior years in the Further education and training band, learners should be at level 3 and 4 seeing geometry as abstract, with a high degree of rigor, which is prescribed for the grade 10's as the

[7] Van Hiele's numbering scheme: This project uses the original numbers of level 0 - Visualisation, level 1 – Analysis, level 2 – Informal Deduction, level 3 – Formal Deduction, and level 4 - Rigor.

ability to describe, represent, analyse, explain and justify the properties of shape and space by the National Curriculum Statement.

[Teppo, Anne. 1991:211-215]

Thus the above explanations on the Van Hiele levels are aptly stated in the National Curriculum Statement:

> "The **investigation of polygons, using any logical method** (Euclidean, co–ordinate and/or transformation) and **alternative definitions of polygons**"
>
> [National Curriculum Statement Grade 10 – 12. 2002:54, my emphasis]

As depicted in the table below the five Van Hiele levels in relation to polygons in particular, quadrilaterals show that there is going to be mathematical work that the teacher is going to have to do, since this is an old topic that requires new demands in a new curriculum. The two bold columns indicate the two levels that need to be developed at grade 10. In working with the tasks and the mathematical demands from geometry a Van Hiele point of view, the learners will require much more than an empirical or practical skill. Thus more demands will be placed on the teacher to develop more mathematical and problem solving skills.

Level 0 ***Visualisation***	***Level 1*** ***Analysis***	***Level 2*** ***Informal deduction***	***Level 3*** ***Formal deduction***	***Level 4*** ***Rigor***
Recognize different polygons	Sees properties of polygons	Recognises interrelationships between polygons	**Uses polygon information to deduce more information**	**Recognizes unjustified assumptions**
Name polygons	Describe various polygon properties	Defines & formulates words & sentences accurately & concisely	**Know what is given and what is required to distinguish definitions, postulates & theorems**	**Formulates and describes various deductive systems**
Sketch polygons	Translates verbal information into drawings	Construct other polygons from related ones.	**Recognizes when & how to use & deduce given information to construct specific polygons**	**Understands limitations and capabilities of drawing tools**
Realises similarities & differences	Understands different types of polygon classification.	Understands qualities and properties contained in different classes	**Uses logical rule to develop proofs**	**Know when postulates are independent, consistent & categorical**
Identifies polygons in physical objects.	Recognises geometric properties of physical objects.	Understand maths concept models that represent relationships between objects	**Deduces properties & solve properties of polygons from given or deduced information.**	**Uses & develops maths models to represent abstract systems.**

Table 1 - *2.1 A representation of Van Hiele's five levels in relation to five skills*

The table below represents the sequence of Van Hiele’s model as well as the proposed five hierarchical phases to help learners to move through each of the five levels from understanding the objects of thought to the structure of thinking specific to polygons in particular, quadrilaterals. In a classroom situation it is often that the learners might be on the same level but not on the same phase. Therefore it is the teacher’s duty (work) to bring all learners not only to the required level but also the required phase, by moving the learners through the five phases of each level.

Level 0 Visualisation	**Level 1** Analysis Exploration	**Level 2** Informal deduction Ordering	**Level 3** Formal deduction	**Level 4** Rigor
Phase 1: Inquiry/ Information	***Phase 1:*** Inquiry/ Information	***Phase 1:*** Inquiry/ Information	***Phase 1:*** Inquiry/ Information	***Phase 1:*** Inquiry/ Information
Phase 2: Directed Orientation	***Phase 2:*** Directed Orientation	***Phase 2:*** Directed Orientation	***Phase 2:*** Directed Orientation	***Phase 2:*** Directed Orientation
Phase 3: Explication	***Phase 3:*** Explication	***Phase 3:*** Explication	***Phase 3:*** Explication	***Phase 3:*** Explication
Phase 4: Free Orientation	***Phase 4:*** Free Orientation	***Phase 4:*** Free Orientation	***Phase 4:*** Free Orientation	***Phase 4:*** Free Orientation
Phase 5: Integration	***Phase 5:*** Integration	***Phase 5:*** Integration	***Phase 5:*** Integration	***Phase 5:*** Integration

Table 2 - *2.2 A representation of Van Hiele's five level and the five phases of development*

According to the Van Hieles, learners move through each level of thought, through organised instruction of five phases of learning as described below by Mason (in press). According to Teppo (1991) these five phases can be applied to move the learners to the next level of geometric development.

Phase 1: Inquiry or Information: Through discussion the teacher identifies what learners already know about quadrilaterals and polygons and the learners become oriented to this.

Phase 2: Direct or Guided Orientation: Learners explore the objects of instruction from structured quadrilateral and polygon tasks to explore specific concepts.

Phase 3: Explication: Learners describe in their own words what they have learnt from the teacher's explanation of mathematical terms and concepts.

Phase 4: Free Orientation: Learners solve problems and investigate more open – ended tasks by applying the relationships learnt.

Phase 5: Integration: Learners develop a network of objects and relations by summarising and integrating what they learnt.

[Teppo, Anne. 1991:213]

According to the five learning phases, the development of mathematical geometric proficiency largely depends on the approach used to teach geometry and the tasks that make up the lessons. In terms of geometry teaching, research by Cabassut (2005) and Nordström (2003 and 2004), has shown that European countries like France, Germany and Sweden are using textbooks that approach geometric arguments and proofs from a realistic or practical perspective. For example, Cabassut (2005) describes how the French based the proof of Pythagoras' theorem on a puzzle technique (pragmatic argument) where the main function of the proof is explanation of implicit properties. While in comparison, for the same proof, the Germans used more visual arguments of explicit properties. The Asian countries like China and Japan used a more theoretical approach of short tasks and questions as reported by Jones, Fujita, and Ding (2004).

In prior studies done by De Villiers, (1987 and 1990), it is revealed that learners have a problem when moving from the concrete level to the abstract level of geometric thinking, especially second language speakers. In relation to the Van Hiele levels, a study conducted by De Villiers (1994) demonstrates that learners only acquire the first three levels of the Van Hiele model because teachers and textbooks encourage development at these three levels and they neglect or ignore development opportunities at the formal deduction and rigor levels. Another reason might be that teachers are not skilled enough and do not have sufficient geometry knowledge to teach geometry although they may have an understanding of it, and be able to solve geometric problems. De Villiers' study involves conducting interviews with learners (individually and in a class context) regarding their understanding of conceptual relationships. While learners were unable to understand the role, function or value of specific mathematical content e.g. polygons in particular quadrilaterals, they possessed an understanding of formulating definitions and following logic. These results showed that learners are able to understand content and underlying logic of these conceptual relationships that exist in geometry curriculum.

According to Malan (1986), De Villiers & Njisane (1987) and Govender (1995), learners from grade 7 to grade 12 are able to, but tend not to draw conclusions from definitions and hierarchical classification of quadrilaterals. Thus the learners show problems with both the interpretation of the language and the functional understanding (understanding the role, function or value of hierarchical classification of quadrilaterals and processes within mathematics) of the meaning of the activity. Similarly, as citied in De Villiers (1994), Battista and Clements (1993) conclude that learners are able to follow the logic of a hierarchical classification of squares and rectangles but they have difficulties in accepting the rational or logical understanding i.e. they lack functional understanding. Many other studies for example, Mayberry (1981), Usiskin (1982), Burger & Shaughnessy (1986) and Fuys, Geddes & Trischler (1988), on the Van Hiele theory, also show that learners have difficulty with the hierarchical classification of quadrilaterals. Thus there has been considerable research based on the Van Hiele theory and all concur on two things:

i) there is a mismatch between the geometry curriculum at a specific grade and the learners' actual level at that specific grade; ii) and between the teachers instructions with the level of the learners understanding of geometry and the learners home language. And this contributes to complexities of teaching and learning of polygons in particular, quadrilaterals.

According to Feza and Webb's (2005) study, 30 grade seven learners (whose first language was not English) from six previously disadvantaged primary schools were interviewed to elicit the learners understanding of geometry in relation to the assessment standards set by the Revised National Curriculum Statement (RNCS) and the implied Van Hiele levels. Feza and Webb (2005) also found that grade 7 learners did not reach Van Hiele level 2 thus not satisfying the geometry assessment criteria. They argued that the learner's cultural background has to be considered when developing geometric mathematics curriculum and assessment standard has to also consider where the learner is coming from i.e. the learner's cultural background. These research findings by Feza and Webb (2005) show that most learners move on without attaining the required Van Hiele levels, although research by Mason (in press) and Mayberry (1983) argues that primary school learners should pass through Van Hiele level[8] 0, 1 and 2 at the end of grade 7. Thus, most learners enter high school without having reached level 2, so high school teachers should be aware that they should expect to teach geometry from the level that the learners are on instead of the intended levels 3 and 4 (Feza and Webb 2005).

The new curriculum i.e. The Revised National Curriculum and the National Curriculum Statement does encourage tasks that accommodates the Van Hiele demands. Thus this study takes cognisance that teachers should also be aware that the content and mode of instruction

[8] Van Hiele's numbering scheme: This project uses the original numbers of level 0 - Visualisation, level 1 - Analysis, level 2 – Informal Deduction, level 3 – Formal Deduction, and level 4 - Rigor.

must be at the level and understanding of the learners. It must be remembered that each level of geometric understanding has its own vocabulary and its own system of relations that will affect the teacher's pedagogy of geometry. Indeed teachers are bound to face challenges and problems (because learners experiences can either facilitate or impede progress) as teachers rescale tasks that they take into their diverse classrooms to suit learner's development levels and geometry demands of the curriculum.

Van Hiele's (1984) geometric thought model will be used to answer questions like: What mathematical problems or challenges does the teacher encounter as he teaches a section in geometry on polygons in particular, quadrilaterals in grade 10 to meet the requirements of the National Curriculum Statement? What could teachers do to manage the challenges or problems that they encounter as they teach? What mathematical work and pedagogical work does the teacher do as he teaches in the classroom to enhance the learners' geometric development? What kind of mathematical work does the teacher do to develop a specific level and to move the learners to the next level? What mathematical knowledge resources does the teacher call on to ensure that the learners acquire mathematical proficiency? What happens in teaching when the teacher presents the learning materials, i.e. above or below the learners' level of geometric thought? These questions sheds light to answer the critical questions of what mathematical knowledge is required for teaching geometry to grade 10 learners.

At the moment South Africa is trying to combine deductive and inductive approaches in Geometry, like appealing to specific examples, perceiving patterns for validation of conjectures and using logical deductions to validate conjectures. Thus in this study the teacher uses open tasks (tasks that can not be solved just by looking at it, open tasks have many different approaches that can lead to the solution) that the learners must solve, using any approach. These open tasks are also facilitated by the teacher's explanations. In papers by Herbst (2002a and 2003) he stresses how the didactical contract between the teacher and learner regulates mathematics in the classroom. Later research by Herbst (2006) discusses "what kinds of negotiations are required to engage learners in geometric problem solving and how these negotiations impact on the mathematical activity". This is important for my study since the mathematical work that is needed to teach geometry specifically, to teach polygons in particular, quadrilaterals requires continuous interaction especially for the teacher, who has to build skills, break down concepts and pose questions and answers.

The National Curriculum Statement, geometry requirement for school Grade 10 learners, requires that the learners develop skills in conjecturing and proving geometry problems. It is interesting to note that mathematicians do not support Herbst (2006) notion that conjecturing and proving in geometry in a classroom situation are different in logical form and in creating mathematical substance. This study is on geometry and the teacher's intention is to teach conjecturing and proving, so the statement by Herbst (2006) is therefore appropriate that learners first work inductively to produce conjectures and then deductively to produce proofs, however, he also stresses the argument on what makes a conjecture true and not just perceiving the conjecture as a fact. At this point there will be a shift in the focus from the development of geometry to cognitive demands on tasks for proficiency in geometry. So what I am saying now is, "*What is the issue about tasks set up (possibly for inductive or deductive approaches) and the different cognitive demands and proficiencies that relates to tasks?"*

The next section contains a discussion of some research on tasks, and related cognitive demands. Tasks analysis, while not the focus of the study, is important because of three main issues for this study i.e. the scaling up and down of the tasks, the gap between the learner's levels and the new curriculum demands. This is because, of course, what happens in the classroom is a function of what the teacher does, and one of the things that the teacher does is to set up such tasks. So, in order to be able to study what this teacher does (mathematical work) and what mathematical demands these make on teachers, I need to look at how this teacher sets up the tasks and how geometry is carried throughout these tasks. So I am going to move on to a discussion about the mathematical classroom tasks.

2.3 Tasks, Cognitive demands and Proficiencies:

What are teaching tasks? To start the learning process in the classroom one of the first things that a teacher could possibly engage with is to provide his/her learners with a task to work on. The selection of an appropriate task by the teacher is just one example of the mathematical work of teaching that a teacher could engage in. Since, one of the central aims of this study is to investigate the mathematical work of teaching that the teacher engages in, the selection of the tasks becomes an important aspect for me to look at. Selecting a task is also then the first form of scaling[9] (i.e. the first attempt to meet learners' needs/levels). Particularly, having to

[9] Scaling entails moving the tasks in an upward or downward direction to meet the learners' needs/level so that they can engage with the tasks.

see what the learner's need and where the learners are coming from could be a problem in terms of getting the learners to where they need to be (at a grade 10 level).

The National Curriculum Statement's outcomes for Outcome Based Education (OBE) creates opportunities for making connections across the geometries (Euclidean, Analytical and Transformational) by solving routine and non-routine problems. This is in line with Stein, Grover and Henningsen's (1996) explanation of tasks. They distinguish between *tasks features* (number of solutions and strategies, number of representations and communication requirements) and the *cognitive demand of the task* (memorisation, procedures and doing mathematics), which together cast light on how to analyse the tasks to answer the first critical question in this study. This literature points to the view that learners should be exposed to tasks that facilitate procedural and conceptual learning if they are to achieve mathematically proficiency. Stein et al. (1996), informs our understanding that some tasks make high cognitive demands[10] on the learner. But not all tasks do so since there are no guarantees in the enactment of the tasks. Sometimes, tasks can start off with high cognitive demands but then get watered down by the teacher, or tasks start off with high cognitive demands and remain this way during the implementation phase. Thus, it is important for the teacher to be well informed about the level of development of the learner so as to control the destiny of the tasks (the degree of cognitive effort) and to handle the challenges or problems that the task can bring to the classroom. The focus in this study points to an element of the mathematical work that a teacher does. The teachers hold the key to what geometric knowledge the learners learn: in Stein et al. (2000) terms, different mathematical instructional tasks placed different cognitive demands on learners, and so will have different efforts on their mathematical learning.

The instructions during the set up and planning stages ensure that the task caters for the geometric needs of the learners in terms of Van Hiele's levels and Stein et al's high and low cognitive demands levels. As Stein et al. (1996) argue that it is the teacher's goals, the teacher's subject knowledge and the teacher's pedagogic knowledge that are in the forefront during the implementation phase of task work in the classroom; and this involves actual

[10] High cognitive demands which is conceptual refers to procedures with connections and doing mathematics, while low cognitive demands are procedural referring to memorization and procedures without connections.

learners' engagement (classroom norms, task conditions, teacher's instructional habits and disposition and learners' learning habits and disposition).

Thus teachers form an important link between the teaching of geometry and the learning of geometry, i.e. forming the link between the tasks and the situation. As stated by Stein et al. (1996), teachers do influence learners' cognitive development, which in turn influences the learning process. I agree, since teachers provide learners with opportunities, language and tasks that if appropriate to learners' thinking in geometry (polygons in particular, quadrilaterals), eventually lead to learning of all phases of each of the Van Hiele levels. The interaction between the teacher and the learners - what Herbst (2002a and 2003), calls the didactical contract (between the teacher and learner) - must occur to address misconceptions and rescaling (to a higher or lower cognitive demand) of tasks to achieve the relevant outcomes specified in the National Curriculum Statement. This requires that the teacher have to look beyond the surface of the task to ensure that tasks are set up and implemented as intended to promote the understanding of geometry. Specific for this study, could be tasks according to the specific curriculum requirements and appropriate Van Hiele levels and how the teacher uses and develops the tasks to accommodate the learners' present Van Hiele levels in the classroom, which is not very visible or identifiable.

It is well known that some mathematics teachers emphasise skills performance while others emphasise learning procedures with conceptual understanding. Kilpatrick et al (2001) agree with the 1950's and 1960's "maths movements" that emphasises understanding and unifying ideas and computational skills, as well as, the 1980's and 1990's movement of "maths power". Kilpatrick et al (2001) introduces us to what they call mathematical proficiency. They state that in order to *recognise* whether learners are mathematically proficient they must display the following five stands of proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Thus in order to do the analysis of the lessons I am going to draw on the following key ideas of Kilpatrick's five strands because these strands provides categories for mathematics proficiency to look for in the lessons and interviews. Mathematical proficiency also consists of: knowledge, skills, abilities and beliefs and these are rephrased in the new curriculum statement to attitudes, beliefs and values.

According to Kilpatrick et al (2001: 116):

1) *Conceptual understanding* is the 'comprehension of mathematical concepts, operation, and relations.' They believe that when a learner has acquired this strand, they are able to verbalise relationships with different concepts and are not only restricted to isolated facts and methods. Learners who have developed this strand are logical and are able to organise their knowledge and thoughts to represent mathematical situations in different ways, e.g. pictures, diagrams, stories and renaming facts, and how these can be used in different and new situations. Because of this, they are able to make connections with their existing knowledge. Being able to retain this knowledge is an important aspect of conceptual understanding. Since a learner is able to understand and not merely memorize a method or solution to a problem, they are better equipped to retain what they've learnt. It can be concluded that learners' understanding of concepts, affects how and why they respond to certain tasks, thus promoting mathematical proficiency.

2) *Procedural fluency*. Kilpatrick et al. (2001:121) describe procedural fluency as 'knowledge of procedures, knowledge of when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently'. They say that in order for learners to be proficient at mathematics, they need to develop this strand. How do teachers develop this? Firstly Kilpatrick et al. (2001) suggested appropriate skills that learners should be allowed to think and reason critically by knowing when, where and how to use procedures and not just merely follow procedures. The danger of this as mentioned by Kilpatrick et al. (2001) is that when learners do not understand what they are doing, they will more than likely practise incorrect procedures.

3) *Strategic competence* i.e. when learners are able to apply different strategies in formulating, representing and solving mathematical problems. To develop strategic competence the teacher needs to provide learners with experiences, opportunities and practices in problem formulation and problem solving which includes both routine and non-routine problems, which require flexibility. Once the learner has learnt how to form mental representations of problems, detect mathematical relationships and devise novel solution methods, then only can the teacher claim that the learner is a proficient problem solver.

4) *Adaptive reasoning* i.e. when learners are able to adapt and relate concepts and procedures to new circumstances, by reflecting, thinking, explaining and justifying logically. When learners are able to justify and explain their work in order to clarify their understanding, procedures and competence, then the learner is applying adaptive reasoning skills.

5) *Productive disposition* i.e. learners are able to make sense, apply and see the usefulness of mathematics. The learners have to develop self-confidence in knowledge, ability as well as in mathematics. Thus the teacher has to make it possible for the learners to adopt a positive attitude towards mathematics and by focusing on the learners own abilities.

Thus it is imperative to look at the framework that has an influence on the teacher's ability to work with the mathematical knowledge for schooling. The use of Kilpatrick et al. (2001) five intertwining and independent strands describe, cognitive changes that enables young learners to be experts, competent and knowledgeable in mathematics learning, i.e. to master and to be proficient in mathematics. Kilpatrick et al. (2001) also mention that instructional programs are required for the learner to acquire mathematical proficiency to cope with daily challenges in life and to continue to study mathematics. It is the teacher's mathematical work to develop these strands to ensure mathematical proficiency.

As part of the teacher's work, the teacher has to ensure that all five strands, work inter-connectedly and together to achieve successful learning. What would also help the teacher is time, i.e. if learners are given enough time to work on mathematical problems they will achieve proficiency and if the curriculum allows enough time for the teacher to actually develop the skill of solving geometric problems. Thus successful learning refers to the fulfilment of the didactical contract (Herbst, 2006) and how learners represent and connect pieces of knowledge in solving problems i.e. different kinds of connections or flexible approaches are required to achieve mathematical proficiency, which depicts the work of the teacher. This is also stressed in the National Curriculum Statement as alternative proofs.

Kilpatrick, et al. (2001: 118) state that "learning with understanding is more powerful then simply memorising because the organization improves retention, promotes fluency and facilitates learning related materials". This then leads to adaptive expertise and meta-cognition, i.e. knowledge about one's own thinking and understanding and problem solving and the ability to monitor one's self. Other factors that affect mathematical proficiency are differences between gender, race, ethnicity, poverty and socio-economic situations but this is not the focus of this study. Kilpatrick et al., (2001) also state that there is need for mathematics instructions and programs to improve the quality of teaching and to help learners become mathematically proficient. This is significant because the new curriculum expects teachers to use this theory as a way for teaching and developing the learners' mathematical knowledge of quadrilaterals and polygons.

2.4 Geometric Tasks:

Since teachers need to interact with learners to determine what learners know and still need to learn, Ainley, Pratt and Hansen's (2006) offer a curriculum design analysis and the set of heuristics that are helpful in teasing out the challenge between the intentions of the teachers and the experiences of the learners. Looking at the appropriate knowledge that teachers need to have, Freudenthal's philosophy leads to reality and guidance principles for progressive mathematization involving vertical[11] and horizontal[12] mathematization. Ainley et al., (2006) believes that the inclusion of purpose and utility of tasks can help teachers resolve the challenges or problems in the set up and implementation phase of geometry tasks which provides answers to the first critical question.

It is important to review the research as a backdrop to this study, as cited in Bennie (1998), of the interpretation by Fuys et al. (1988) of the Van Hiele's model of thinking describes how learners learn geometry by developing their own models when working with geometric tasks, to see how the teacher's mathematical work is carried out. In view of this, Ainley et al. (2006) investigates how illustrative tasks designed with utility and purpose helps learners engage with mathematics in a more meaningful way. They describe two tasks i.e. the spinner and mending gargets, which were found to be challenging tasks that lead to identifying another dimension of learning called *utility focused learning* that enables a rich understanding of mathematics. Although task is not a focus of this study, it nevertheless has particular significance in the context of this study, which will add knowledge of teaching and learning of geometry tasks at school level.

Although this project is not focused only on tasks, it is important that teachers take cognisance of the task features and whether the task fits the requirements of the curriculum and assessment standards of the National Curriculum Statement, as well as whether it encourages learners to think, reason, justify, conjecture, hypothesis and make sense. Stein et al. (1996) states that the teacher should ensure that the solution of the task starts from a social point and becomes more individual as the learner solves it, by using mathematical explanations and justifications. Ainley et al. (2006) addresses the challenges or problems of

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11 Vertical mathematization: refers to mathematics that is taught both in preceding and later years in school.

12 Horizontal mathematization: relates to the mathematical content or lesson to a specific topic in various learning area of the same grade.

whether the teachers plan from the lesson's objectives (resulting in unrewarding mathematics for the learners) or plan from the tasks that increases the learners' involvement (resulting in unforced learning which then is in turn difficult to assess). Facilitating learners' development of geometric understanding and skill, the tasks that support this, and related cognitive demands all have implications for what the teacher needs to know and know how to use in their teaching. I have discussed the studies of Van Hiele (1984), Stein et al. (1996) and Kilpatrick et al. (2001) related to geometry and what the teacher will do, but what I am also interested in is how the teacher manages this. For this I need to look at the problem solving for teaching geometry, actually what this work is and how I am going to analyse the data. Hence the next section focuses on mathematics for teaching.

2.5 Mathematics for Teaching:

While Outcomes Based Education is evidently shaping our education system, the National Curriculum Statement (NCS 2002: 9) stresses that teaching practice and learners' education must be in the forefront. The curriculum mentions that proficiency in geometry needs will be a function of improvements in teaching. Thus teachers' knowledge of subject matter draws increasing attention from policymakers. In order to examine the rationale behind this focus on teachers' subject knowledge and related practices, the relevant sections of the National Curriculum Statement (NCS 2002) and the assessment standards will be discussed and kept in mind when understanding the knowledge that the teachers need to have to promote geometric thinking (proficiency). Thus the literature on Stein et al. (1996) and Kilpatrick et al. (2001) is pertinent since it points to the fact that it is needed to gauge proficiency in geometry.

When examining teachers' knowledge, I used Shulman's (1986) three distinct categories of: content knowledge, pedagogical content knowledge and curriculum knowledge. Shulman (1986) infers that there is a fine line between content knowledge and curriculum knowledge, as he discusses how the teacher understands and transmits content knowledge, pedagogical knowledge and curriculum knowledge, which are inherently inter-related. I agree with Shulman (1986) when he refers to content/subject knowledge for teaching as going deep into conceptual understanding of facts and concepts as opposed to a mere procedural understanding. Shulman (1986) adds to this by stating that teachers must understand *why* in relation to the subject they are teaching, and so too on what grounds assertions and their justifications are warranted.

When Shulman (1986) mentions content knowledge, this refers to the knowledge the teacher needs to structure subject matter for the learner, in this case the notions of polygons in general quadrilaterals in particular. This structuring will include various ways in which concepts, syntax and procedures are organised to validate solutions of how polygons in particular, quadrilateral tasks are taught. Thus the teacher must know why it is so; on what grounds and under what conditions and circumstances the solutions are justified in order to mediate between the learners understanding and the objectives of the tasks i.e. as Skemp (1987) states, the teacher needs to be able to guide learners through a transition from intuitive to reflective thinking.

Recent emphases on alternative definitions of concepts or equivalent statement require teachers to provide learners' with rich opportunities and experience with proof in school at all levels. Researchers define proof in various ways but Schoenfeld's (1994: 76) definition seems most appropriate "proof is not a thing separable from mathematics as it appears to be in our old curricula, it is an essential component of doing, communicating and recording mathematics". Similarly Wheeler (1990: 3) states that the "mathematics community views proof as a form of discourse". Thus proof involves mathematical practices that form relationships between problem solving and conjuring, resulting in a deductive system of definitions, axioms and theorems. In school geometry content knowledge of reasoning and proof includes mathematical discourse of reasoning such as deducing, inducing, conjecturing, justifying, proving, testing examples and counter examples, geometric arguments that take the learner to what Van Hiele calls level four of rigor or formal logic. Learners work inductively to produce conjectures and then deductively to produce proofs as stated by Herbst (2006).

Pedagogical content knowledge goes beyond just quadrilateral and polygon subject matter. It refers to the knowledge for teaching used when doing their work and what makes understanding of quadrilaterals and polygons easy or difficult and how to enhance or rectify learners' existing knowledge. Shulman (1986: 9) defines "pedagogical content knowledge for teaching as ways of representing and formulating the subject matter so as to make it comprehensible to others, together with an understanding of what makes learning of specific topics easy or difficult from the learner's perspective".
Shulman (1986) describes curriculum knowledge as a range of designed programmes for teaching particular subjects and topics at specific levels and grades. Under curriculum knowledge, Shulman (1986) distinguishes between lateral curriculum knowledge that displays

the teacher's ability to relate the contents which is simultaneously discussed in other learning fields and vertical curriculum knowledge, which refers to the teacher's familiarity with the topics that are taught in the same subject areas in the past, at the moment and later on in his/her standards/grades. It is apparent that teachers need more than just subject knowledge to teach geometry (polygons in particular, quadrilaterals), since the teacher has to impart this subject knowledge in a form that the learners can understand and apply. Since this study investigate how the teacher teaches and manages mathematical teaching, it will be interesting to see how the teacher manages these three components of knowledge to promote geometric development. I then turned to problem solving as part of teaching.

The idea of the teachers' mathematical work being a particular kind of mathematical problem-solving comes from Ball and Bass (2000), and is a function of the teacher's interaction with learners, listening to learners' ideas and working with the learners' ideas. Ball and Bass (2000) talk about what kind of knowledge teachers require in order to manage the real situation, describing this deeply detailed knowledge of mathematics and the ability to use it in real contexts of practices. Ball, Bass and Hill (2004) describe eight categories of problem - solving based on their study of teaching practices. These eight types are:

1) Design mathematically accurate explanations that are comprehensible and useful for students;
2) Use mathematically appropriate and comprehensible definitions;
3) Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process;
4) Interpret and make mathematical and pedagogical judgements about students' questions, solutions, problems, and insights (both predictable and unusual);
5) Be able to respond productively to students' mathematical questions and curiosities;
6) Make judgements about the mathematical quality of instructional materials and modify as necessary;
7) Be able to pose good questions and problems that are productive for students' learning;
8) Assess students' mathematics learning and take next steps.

[Ball et al. 2004:59]

Kazima and Adler (2006) reduced these eight types of problem-solving categories to six categories (i) Defining; (ii) Explaining; (iii) Representing; (iv) Questioning; (v) Scaling (vi) Learners Production, with a slight adaptation in the labelling. They found in their study that the categories that referred to learners' productions were not easily distinguished. In a similar study, Pillay (2006) also works with the reduced six categories of problem–solving that teachers do when they are teaching mathematics. Following these studies, I plan to use the same six categories (once again with a slight adaptation in the labelling) in this study to analyse the teachers problem-solving practices. The same framework is used to contribute and elaborate Mathematics for Teaching (MfT) that used in the QUANTUM Project. Teaching practice is filled with problems and challenges that the teacher has to solve and these six problem-solving categories seem the most appropriate to depict the teachers enactment.

Of all the research related to my study and discussed above, Feza and Webb's (2005) research is the one that relates most directly to this study. Their focus, however, was on learner's understanding of geometry using tasks. In contrast, this study concentrates on what is taught, what work the teacher does as he teaches and what resource the teacher calls on when scaling tasks intended to promote mathematical development (proficiency) in geometry at a grade 10 level. Their study will nevertheless, help to examine the learners' responses to relevant tasks as well as relate these to observations of what and how teachers handle these challenges or problems that arise from the mediation that takes place during the various phases of teaching (i.e. what work the teacher does). At the same time this will show how the teacher moves learners on to the next stage of development.

For this study the question arises as to what kind problem solving is enacted in teaching and what knowledge resources the teacher calls on when "unpacking" the mathematical work for mathematical proficiency and so a review of research that has begun to address the first and second critical questions.

2.6 *Teacher's Mathematical Work:*

The literature that relates to the teacher's mathematical work suggests that whatever the teacher is going to do, he is going to deal with pedagogic and mathematical problems and the teacher is likely to confront some challenges. From my experience as a teacher, these kinds of mathematical problems that Adler (2001), Ball (1993) and Lampert (1985) see as dilemmas are likely to crop up in any teaching situation, so I refer to them as challenges or problems. This study's focus is on how the teacher manages these challenges or problems while implementing the tasks (the new mathematical ideas and the mathematical language) and how the teacher teaches and handles the challenges or problems during the different phases of teaching. Thus this teacher has to find ways of managing these kinds of mathematical problems.

Ball (1993) and Ball and Bass (2000) elaborate further. The research by Ball (1993), on reforming school mathematics, discusses the dilemmas of content *i.e. representing the content* so that grade 10 learners engage in exploring space, shape and measurement and discourse i.e. *respecting mathematical thinkers* and community by creating a mathematical community. Ball (1993) suggests that teachers must have an adequate understanding and knowledge of geometric concepts to be taught. Ball and Bass (2000) describe the mathematical practices as challenges or problems that teachers handle during teaching and learning. They refer to the work that teachers do as "unpacking" or "decompressing" mathematical knowledge. Unpacking is a compelling notion, but as Adler and Davis (2005) argue, it is not well defined. In this study, I thus turn to the work done in the QUANTUM[13] project where unpacking in the context of teaching can be interpreted through the resources the teacher calls on to manage the challenges and problems of teaching that might ensue.

Although operation of pedagogic judgement is central in teaching, it is not the central focus of this study. The focus of this study is on how events and sub–events condenses into evaluative events over time and the work that the teacher does and the evaluative appeals the teacher uses during teaching quadrilaterals and polygons, which is the focus of this study. The key focus will be on how the teacher's knowledge is mediated from pedagogic content knowledge to what is expected by and from the teacher. The Van Hiele's five geometric thought levels and Bernstein's (1990) pedagogic device will be used as the theoretical framework and Kazima and Adler and Pillay's problem solving categories and appeals will be used as an analytical lens to investigate geometric development of the teaching geometry.

So as one looks at Kazima and Adler (2006) and Adler and Pillay (2006) research, we see that the outcome changes from topic to topic and that it depends on how the teacher does the mathematical work. What is seen is that, what teachers choose to teach and how they choose to teach it affects what resources teachers call on. Thus it is going to be interesting to see in this study if this actually supports the above mentioned research. The work of mathematical

13 Quantum Project - This is a larger study on mathematics for teaching, headed by Professor J. Adler. Thus drawing the framework from the QUANTUM Project, allows me to see that the teachers' work is situated in their pedagogical practice.

problem solving that the teacher does require the teacher to use different knowledge resources, when fixing meaning of the mathematics for the learners. Since the content is conceptual in nature it is expected that all or most appeals[14] will be mathematical in nature. Thus the analytic and theoretical framework therefore follows.

2.7 Analytical Framework and Theoretical Framework:

In relation to the principles of the QUANTUM project, I have drawn on a theory of pedagogy in order to able to 'see' what the teacher is doing (teachers work) over time in the pedagogic practice. Recapping first the literature reviewed up to this point, the following are important. It is likely that the teacher might encounter problems. Given the discussion about the Van Hiele levels of the development of geometric thinking, this together with Stein et al's (1996) tasks analysis provides us with tools to see and interpret differences between the tasks selected and the implementation of the tasks.

I analysed the teacher's lesson plans in relation to the grade 10, geometry syllabus, and it revealed information about the learners' levels and backgrounds that the teacher was expecting the learners to be at. The pre-interview sheds light on what the teacher anticipates during the lessons whilst the post-interview sheds light on what the teacher encountered when teaching. Together with the observations of the lessons in progress, it gave good idea/s of whether there is a gap between the development levels and the geometry demands of the curriculum or a gap between teacher's knowledge and skills to teach. That is, if the teacher is talking and doing mathematics that the learners can understand to promote successful mathematics learning.

Taking the Quantum Project and the above discussions of mathematical work of teaching practice into consideration, I have drawn literally from aspects of Bernstein's theory of pedagogic discourse as a lens for this study. He describes a pedagogic device as a "symbolic ruler of consciousness" (Bernstein, 1990: 180). In other words, it acts to mediate specialised consciousness to be formed through pedagogical practices. In simpler terms and in relation to the study the pedagogic device will be acting in this teachers' pedagogic practice towards a particular experience of geometry. According to Bernstein (1990) pedagogy condenses into evaluation and, following the QUANTUM project, I have interpreted this to indicate that

[14] Appeals to mathematics, the curriculum and different experiences.

pedagogic practice is driven by evaluation. Evaluation in this sense is observed particularly in the work of the teacher during his interaction with his learners in his attempts to legitimate what is to count as appropriate geometric thinking and reasoning that takes place in the classroom.

In more practical terms, I will need to look at what the teacher is doing and what resources the teacher calls on as he legitimates (shows, gives or affirms) geometric meanings in his class. But how are we going to see what the teacher taught, what problems he faced and what resources he called on? We are going to actually study pedagogic discourse, thus we have to turn to Bernstein but more specifically to how Adler and Davis and other researchers interpret pedagogic discourse. The following diagram depicts the analytical framework that will be used to analysis the collected data.
The diagram that follows is a representation that summaries the framework.

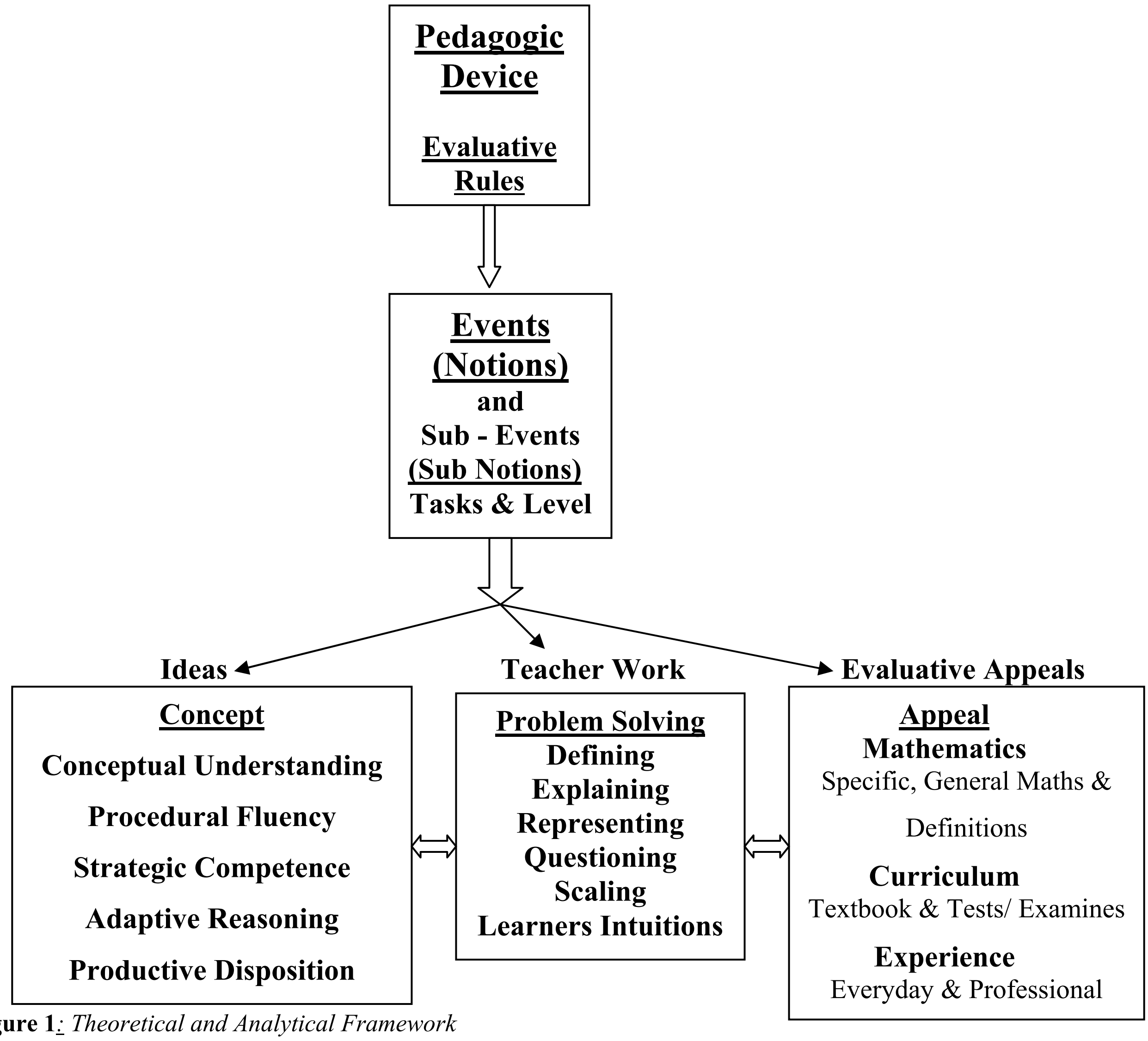

Figure 1: *Theoretical and Analytical Framework*

The framework for this study is thus divided into three sections:

1) How the teacher advances or attempts to advance learners' mathematical proficiency (Kilpatrick et al. 2001) by looking at the prescribed geometric requirements (DoE 2002), the geometric levels of thought (Van Hiele, 1986) and the cognitive demands of tasks (Stein et al. 1996).
2) What and how the teacher calls on and handles the teaching of geometry? (Shulman, 1986 and Ball& Bass, 2000).

What the teacher appeals too and how the teacher legitimates his appeals? (Adler and Pillay, 2006; Kazima and Adler, 2006 and Pillay, 2006).

The above diagram involves each of the following to a greater and lesser extent: teacher knowledge, teacher practice, teacher reasoning, unpacking mathematics, policy, curriculum, learning perspectives, for example, conjectures and proofs, tasks, learners' misconceptions and mathematics proficiency. All of the above is produced in pedagogic discourse that is practiced at a school level. I therefore allude to Singh (2002) when she states:

> "I explicate the dimensions and complexity of the pedagogic device as a model for analysing the processes by which discipline – specific or domain – specific expert knowledge is converted or pedagogised to constitute school knowledge (classroom curricula, teacher – student talk, online learning)."
>
> [Singh 2002:572]

The above extract amplifies the pedagogic device through pedagogical practices at a school level. Thus Bernstein's theory of the pedagogic device forms a theoretical foundation for this study, since Bernstein (1990) refers to the recontextualisation of knowledge into pedagogic communication. Bernstein's pedagogic device condenses into evaluative events over time. Thus I looked at what evaluation the teacher is doing and where is it coming from and how is it legitimated.

For this study it is important, to discuss Ball and Bass (2000) because they talk about what kind of knowledge teachers require in order, to manage the real situation in the classroom i.e. in–depth knowledge of mathematics and the ability to use it in real contexts of practice. Bernstein talks about a rule from a social point of view that is at work. Mathematical work cannot be done without also looking at the teacher's work that takes place between the initial planning of the lessons and the set up and the implementation of the tasks in the mathematics classroom. That leads to Bernstein's (1990) recontextualisation rule that provides an explanation of how knowledge is mediated between the geometry task, the teacher and the learner. Bernstein (1990) refers to this work as the "recontextualising rule that regulates the formation of pedagogic discourse" or "the conversion of knowledge into pedagogic communication" (Bernstein, 1990, p. 184). In contrast Ball et al. (2004) called it "unpacking[15]" and this study focuses on problem solving and rescaling of tasks. Likewise the teacher has to take these tasks and "delocate it so as to relocate it in the set up to make sense so that it can be delocated" (Bernstein, 1996, p. 47) and relocated to reach the focus in the implementation phase as. Thus there are two processes of recontextualisation or unpacking that takes places in the mathematics classroom: 1) between the initial development of the lessons and the set up and 2) between the set up and implementation. These two phases are link by cognition, i.e. thinking, reasoning, perception, recognition and understanding.

The call to make mathematics more real is not new in itself and most schools in Gauteng have a multicultural milieu therefore it is imperative that the social and cultural aspects enhances and enriches the development of the teaching and learning of mathematics (geometry). It is of importance for the teacher to know at what level of interaction and interpretation the learners are because learners try to interpret and understand information in term of what they already know. In this case it is apt because the teacher uses and provides the learners with appropriate geometric examples of quadrilaterals and polygons to boost the learner's cognitive development and conceptual understanding of geometry. From birth to adulthood, we interact with our parents, siblings, peers, teachers, society, contexts and environment, thus the child does not learn in isolation but in a physical, social and historical environment. The child develops culturally, socially and psychologically. Social interaction affects the nature of knowledge like prior knowledge affects new knowledge. This is evident when the teacher prompts the learners to use the correct mathematical language. In different social contexts there are shared values and attitudes, which is seen when the teacher reminds the learners to respect each other.

The following questions will be answered:

[15] Unpacking means, that the teacher is elaborating and keeping it as mathematical as possible.

1) What mathematical work does the teacher do as he teaches geometry to his Grade 10 learners?
    i. What mathematical problems or challenges does he encounter?
    ii. How does he engage with these mathematical problems of teaching?

2) What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?
3) How does this work do and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?

2.8 Conclusion:

This chapter reviews the literature that clearly shows the importance of teaching geometry in the curriculum at the level of school geometry as well as what, how and why the teacher teaches the way he does. In conjunction there is also a model of the theoretical and analytical framework. In conclusion this study presents the teacher knowledge when working with learners intuitions to acquire mathematical proficiency while working with specialised content knowledge that is located in appropriate tasks. The term 'unpacking' will be used to capture the ways the teacher structures and provides experiences for the learners' participation in teaching-learning situation. The next chapter will highlight the methodological approach used in this research.

Chapter Three

Research Design

This chapter describes the research approaches and methods adopted for this research study and how the data collection techniques were employed. I will also engage in a discussion by using the actual data and table of analysis to describe how the analysis was done – the concepts derived from the analytic framework and the indicators used of code these. This chapter also includes a discussion of ethical issues that were considered.

3.1 ***Methodology:***

This research study investigates mathematics for teaching geometry in a particular teacher's practice. This investigation therefore lends itself to a qualitative case study within an interpretive paradigm, since I attempt to understand this in terms of the teacher's own descriptions and circumstances. The study aims to extend our understanding of a particular teacher's teaching practices rather than generalise the results. Relative to this particular high school it is a subjective perspective of this grade 10 teacher, but the description of what goes on in the mathematics classroom during geometry lessons is broader because of the theoretically informed interpretative paradigm which Cohen, Manion and Morrison (2002: 22) argue "characterises as a concern for the individual." Qualitative research is concerned with human behaviour and how individuals derive meaning from their interaction, concerning the quality of the teaching and learning of geometry.

The qualitative research methodology allows for a combination of different strategies that can be used together to understand teaching practices. Conducting pre-interviews and post-interviews with the teacher allowed me to understand the teachers' intentions and reflections. The pre-interviews and post-interviews conducted with the teacher and the observations of the teachers and learner's actions and reactions in the classroom also provided opportunities for various methods of data collection. According to Maxwell (1996) collecting data from various sorts of individuals and settings allows for the principle of triangulation, which increases the validity of the data received. Maxwell (1996) also amplifies another advantage of using the qualitative research method as it allows for formative evaluations intended to improve existing processes or programs rather than to simply assess the value of the curriculum.

Thus by doing a qualitative case study with one grade 10 mathematics teacher and his grade 10 class, from a particular secondary school, I was able to gain meaning and perspective from the teacher and his class within this social setting. Opie (2004: 74) states that, "… a case study can be viewed as an in-depth study of interactions of a single instance in an enclosed system". Thus this research needs to understand teachers in their own context and the influence that this context has on their actions. The full detail of the design elements of this study is worked out during the course of the study since the qualitative approach can supplement and orientate the understanding of teaching practice. Of course, the results of this case study are not used to generalise or change a situation and this may be a limitation. According to Opie (2004) these results can be used to influence practice or make recommendations for change.

The aim of this study is to get closer to the teachers' understanding and use of polygons in particular quadrilateral tasks in the grade 10 geometry syllabus and what mathematical work the teacher does to promote development in geometry. This is done through investigating what work the grade 10 teacher does as he teaches polygons in particular quadrilateral tasks. The focus is on what problem solving happens between the teacher's work and the learner's understanding and what knowledge resources the teacher call on when dealing with the challenges or problems that he experiences, and when he works to legitimate meanings in his class.

This qualitative research approach allows me to able to move back and forth for gaining different meanings, gathering diverse data and identifying various perspectives on teaching practices. The case study shows the extent to which the teacher engages learners with polygons, in particular quadrilateral tasks, as informed by the new curriculum.

3.2 ***<u>Empirical setting: (Selected case: Sample):</u>***

School's at present need to cope with the new National Curriculum Statement (NCS). A case study was chosen as a research strategy to examine the teaching practices in mathematics (geometry) teaching. By using a case study, I was able to use a variety of evidence such as research literature, interviews, and observations, thus lending itself as an empirical inquiry. The sample case for this study is aptly described as a theoretical sample and one of opportunity and purpose in a high school in the south of Johannesburg. A qualified experienced grade 10 teacher (Mr. Ken)[16] from a public Gauteng Department of Education high school was selected. This teacher was a suitable sample, because of his expertise in mathematics and his willingness and eagerness to participate in this project. Mr. Ken has completed a BSc. Honours in Mathematics Education and he is concurrently doing his Master in Mathematics Education, and so, he is well informed about the current curriculum changes. The language of teaching and learning at this school is English and this teacher was teaching learners from a range of socio-economic backgrounds whose home language is not necessarily English. The teacher having taught mathematics from grade 8 to grade 12 is dedicated towards wanting to make a difference to educating the youth of Gauteng to be critical thinkers. As is evident from the pre-interview - Question 3b

Line 1: Mr. Ken: *Although it is important to cover the syllabus, I like the task to make the* ***learner want to do it****, not just because the learners have too. I want to teach learners* ***to think mathematically and to do mathematics on their own.***

Although the teacher had eleven years of experience in the mathematical teaching field, he was teaching the new National Curriculum Statement (NCS) for the first time. He had access to a range of resources for example: National Curriculum Statement, textbooks, chalkboard, overhead projector, inter-net etc. This teacher was one of three teachers who taught the grade 10's. Mr. Ken was the only teacher who was willing to be interviewed and video recorded during his teaching for the research. This research was carried out at the secondary school during the third term[17]. I had relatively easy access to the school and direct contact with the teacher at this school. This study took place in one of the three grade 10 - mathematics classes taught by the teacher that was chosen by this teacher. The basis for his choice is unknown. This class consists of 37 learners, 20 males and 17 females.

3.3 Data Collection and Instruments:

To answer the first critical question of "What mathematical problems or challenges does the teacher encounter and how the teacher engages with these problems and challenges as he implements a section in geometry". What is of great concern is how the teacher teaches the section on polygons in particular, quadrilaterals in grade 10 to meet the requirements of the National Curriculum Statement? This was ascertained via pre and post semi-structured interviews, classroom observations and field notes. A general discussion follows about the

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[16] Mr. Ken is a pseudonym that refers to the teacher and all names used for the learners are also pseudonyms.

[17] Third term: according to the public school calendar the school year is divided into four terms.

data collection techniques employed and the reasons for the range of techniques that were employed.

3.3.1. ***Interviews:***

An initial and post semi-structured interviews (see appendix A, B) were conducted once to establish, from the perspective of the teacher, why certain tasks were chosen, the purposes they intended to serve and how the teacher intended to use the tasks in class to teach geometry. According to Maykut and Morehouse (1994), the interview is a form of discourse that is shaped and organized by asking and answering questions.

A semi-structured interview was most appropriate as there are clear areas of focus and concern for me, which allowed for a rich discussion of thoughts and feelings. At the same time, I needed to be open to interpretations and comments from the teacher that might not have been anticipated. The teacher was an active participant in the interviews, that is, the teacher was given a voice, since the teacher has his own ideas, feeling, insights, expectations and attitudes.

The interviews consisted of broad, open questions that allowed me the opportunity to investigate, explore, probe and develop a conversation with the teacher. That is, I was able to probe the teacher's verbal responses to the approaches he used in choosing mathematical tasks and the mathematical work he did when teaching polygons in particular, quadrilateral related tasks. The interview was also aimed at investigating how the teacher worked with learners' errors (common errors) and misconceptions that surface during teaching; how the teacher was able to explain the concepts he intends learners to understand and what he hoped learners would be able to do in order to complete and learn from the set tasks. The interviews also explored what the teacher knew about his learners in relation to the tasks prepared for the lessons, which illuminated the second research question "what knowledge resources (mathematical and other) does the teacher call on as he goes about this pedagogical work to enhance geometric development?" The teacher could say what he was thinking and why he chose these particular tasks that contributes to the richness and spontaneity (Oppenheim, 1966) of the data. Each response generated more information, particularly as the teacher was encouraged to elaborate on his ability to cope with the demands from the learners, his positive and negative feelings towards the new curriculum and his expectations and fears.

Thus before the implementation of the first lesson and after the implementation of the last lesson, reflective semi–structured interviews were conducted with the teacher, to ascertain his experience of teaching, the challenges he faced and so the problems he might have confronted. The reflective pre-interview schedule (Appendix A) was designed before the observation of the lessons took place and the types of questions that are included in this interview schedule are informed by the first two critical questions. The post–interview schedule was designed (Appendix B) after the data was collected from the lessons that contributed to the answers of the third critical question " how does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?"

3.3.2. ***Lesson Observation:***

In addition to interviews with the teacher, a period of one week was spent with the teacher to observe the lessons in which mathematical work took place. All classroom observations were video taped to capture uninterrupted raw data and the videos were transcribed into full lesson excerpts. The analytical and theoretical framework then informed the choice of which excerpt to use in the analysis and it also had an impact on me wanting to see what happens in the classroom over time. In conjunction field notes and tape recordings was used as back up during the observation phase.

Since the purpose of this study is to find answers to the burning question, the literature that possibly would provide insight into what might be observed for example, Ball et al. (2004), Ball & Bass (2000), Shulman (1986, 1987) and Adler (2005) as well as from observing the teacher when he attempts to fix meaning for his learners. The lesson observation was thus used to illuminate the intended and implemented task so as to be able to relate this to the problem–solving the teacher was doing, and the resources called on for this.

In conclusion, the focus of the interviews was on how the teacher reflected on his teaching, both prior to the first lessons and after observing the last lessons. From the observation I identified, teaching work in action and (at a theoretical level) the recontextualisation of mathematical knowledge into pedagogic communication (Bernstein, 1996). The analysed data was compared and contrasted to previous research evidence. Thus triangulation took place by cross – sectioning (cross checking) the interview data, field notes and the observation data to qualify the results. Three established researches verified the analysis of the data tables by listening to the lesson transcripts and checking the coding of the data tables. This preliminary analysis informed me how to redesign the critical questions to focus on the central topic. After the post interview a more refined analysis of all the observed data of the lessons took place.

3.4. Data Analysis:

3.4.1. Tasks / Lesson Analysis:

The typological and inductive data analysis method was used to make sense of the data collected and to use the results to answer the research questions (Hatch, 2002). This analysis involved breaking up the raw data from the interviews and observation of lessons into manageable themes, patterns, trends and relationships, through an inspection of the relationships between concepts, constructs or variables. This process of typological analysis helped to analyse the tasks used in the lessons according to the five Van Hiele levels to suit the critical questions asked about mathematics, as well as the problem – solving the teacher enacted.

After analysing the levels of the data, there was a further investigation of the tasks as high-level tasks or low-level tasks according to Stein et al (2000). I also used this data later for some interpretative work in the final analysis when comparing the findings with other research and contesting or confirming theory.

An interesting feature of the lessons observed, is that in Mr. Ken's classroom, the work of teaching is distributed between the learners and the teacher. What I mean by this is that development of notions and meanings throughout the lessons is a function of the interactions between him and his learners, where explanations of ideas are distributed between the learner's and himself. This is attributed to the fact that Mr. Ken's pedagogic practice is more learner–centered. He is constantly providing learners with opportunities to think and explore, and works to have them explain their thinking and justify their conjectures. Since the work of teaching is distributed in Mr. Ken's classroom, it is not only through his utterances but also through what he is doing to solicit responses from his learners that it is possible to observe what notions are being engaged, and the mathematical work he does or needs to do, and the resources called in for this.

3.4.2. ***Interview and Lesson Analysis:***

From the above tasks / lesson analysis the data was fine-combed to interpret what work the teacher does to manage the mathematical challenges and problems he faced during the teaching processes. A set of categories (typology) was drawn up to capture the choice and explanations of the teacher from the pre and post interviews (Hatch 2002), as well as in the lessons, as the first step of the analysis. I then look at the analysis and reflect back on the interviews and lessons, to see if the typology captured everything that happened. As is typical in qualitative research, I had to adjust and induce additional categories under the appeal section because the whole data set was mathematical which was not very helpful.

Thereafter I worked inductively to go through the transcripts of the lessons again to develop categories like these provisional typologies, from the data and the already created typologies. The category of specific, general and definition/rule were thus used as further divisions to define the mathematic appeals as discussed later. These categories were drawn up in relation to the literature that was covered in the literature review of research works of Kilpatrick et al. (2001), Ball et al (2004), Kazima and Adler (2006), Adler and Pillay (2006) and Pillay (2006). Looking at the framework in the previous chapter, the table below is a translation of the analytical framework. This re–representation of the analytical framework is to represent the concepts and problem solving categories from the typology developed, as well as the appeals that evolved inductively. This table also includes the timing of the events, the recognition of a notion and sub – notion and how they come into being.

		Concept							Problem Solving						Appeal					
															Maths					
Events	Time	Notion	Sub-Notion	Conceptual Understanding	Procedural Fluency	Strategic Competence	Adaptive Reasoning	Productive Disposition	Defining	Explaining	Representing	Questioning	Scaling	Working with Learner Intuition	Specific	General	Definition/Rule	Curriculum	Experience	Comments

Table 3 - 3.4.1: *Framework to Analysis*

3.5 Recognition Rules of observed lessons:

An appropriate point of departure is to briefly review the analytical framework that underpins this study. In essence the unit of analysis is an event, which is marked at the beginning by the announcement of a mathematical notion. Typically with the introduction of an object to engage viz, a task, a mathematical statement, an exercise, an example etc. and at the end, by the teachers attempt to fix meaning related to the object for and with learners. In a pedagogic practice that is task based and more learner–centred as in the case in Mr. Ken's class, one task is the object of attention over three lessons, hence, fixing meaning happens in stages, and may not be explicitly observable like it is in a more traditional mathematics classroom (such as Nash's[18] teaching of linear functions). Therefore, as discussed previously, the fixing of meaning by the teacher is extended to also include a move that is made to shift the notion or sub–notion to another level (i.e. shifting so that the notion in question tends toward a more 'fuller' notion for the learners).

A new sub–notion begins with bringing in an additional object of attention related to the problem or task (another representation, a question that focuses on a particular example etc.). In each of the events I have identified the notions and sub–notions and their nature. By which I mean their potential to promote what Kilpatrick et al. (2001:116) list as the interwoven strands of mathematical proficiency:

Conceptual understanding – the comprehension of mathematical concepts and skills as well as the teacher's ability recognise the learners ability to verbalise and represent mathematical ideas in different ways.

Procedural fluency – the teacher recognise the learners ability to carry out mathematical procedures accurately and efficiently by modifying or adapting procedures by making them easier while using them.

Strategic Competence – the teacher provide the learners with tasks that the learners can form mental representation and relationships of the examples and non–examples.

Adaptive reasoning – the teacher's ability to follow the learners' logically explanation and justification as the learner relate and adapt concepts and procedures to new conditions.

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[18] See Pillay (2006) and Adler and Pillay (2006) – Nash's teaching was described as 'traditional' where the teacher stood in front and did most of the talking whilst learners were expected to 'listen'. Nash taught with the examinations in mind and hence his teaching emphasised the importance of the procedure for doing the mathematics that would ultimately result in obtaining the correct solution.

Productive disposition – the teacher had to judge whether the learner's is convincing in making sense of the mathematical concepts and if the learners are confident and if they see the usefulness of the mathematics at their own level of development.

The purpose of this section is to provide the reader with a set of 'recognition rules' as to how I chunked and categorized the data. The video recordings of the lessons were transcribed so that it could be read and re-read and broken up into appropriate units or evaluative events. After chunking the data into events these were timed according to the sub notions to help to answer the critical questions. The chunks were then grouped according to notions like conjecturing, justifying, define, proof and hypothesis which were divided further into sub-notions. The relevant data was then coded so as to identify relationships and themes between the notions and sub–notions.

The notions and sub–notions were then also put through categories to see (if at all) the data fits Kazima and Adler's 2006 and Adler and Pillay's 2006 problem solving categories that depicts what mathematical work the teacher is doing. In this case study the teacher works extensively with learner intuitions. My analysis focused on how the teacher represents mathematical concepts and ideas, how he poses questions and respond to questions from the learners, how the teacher defines and explains mathematical concepts and how the teacher scales (upwards and/or downwards) the tasks to ensure that the learning process takes place so that learners can represent their understanding mathematically. These notions together with the sub–notions were looked at to see what problem solving the teacher uses. The notions and sub–notions of data were then also examined in term of the categories of appeals to see how I see the teacher legitimate the learners' mathematical knowledge as specific, general and defining or using rules in mathematics. I also noticed that the teacher legitimates mathematics through the curriculum and everyday experience, though this was far less frequent.

This brief discussion of the first three lessons will locate the extract below. In the first lesson the teacher introduces a non-routine tasks by merely presenting the learners with the problem of "how many diagonals are there in a 700 – sided polygon". The learners were then expected to find ways of understanding and solving the problem. The teacher guided the learners after the learners came up with possible solutions to develop concepts like deduce, test, justify and conjecture. The teacher also steered the learners to start from simple polygons to show and explain why? The second lesson continued from the previous lesson with teacher developing concepts like: testing, investigating and justifying. The teacher encouraged learners to work co–operatively within their groups with the number of diagonals from one vertex of various polygons. The third lesson leads to the conclusion of this task, where learners moved to more complex polygons and eventually to the solution of the task. To demonstrate how I categorised the data I will be making use of the following extract of event 2 from lesson three. This extract enables me to demonstrate classification of the data under most of the categories mentioned in the above table. Nevertheless there are limitations to these categories since all the categories are not observable in each notion and sub–notion. For example, it was difficult to observe 'Productive Disposition' in only a few lessons.

Transcript of Lesson Three

07:07 – 08:00

Line 1: **Lynita:** I think we should divide the sides.

Line 2: **Nicole:** Are we coming to a conclusion or are we back on our page?

Line 3: **Tebogo:** I know, I know, I think its best we do this. (*Showing Lynita)* It's going to give us a pattern, we are going to see it better.

Line 4: **Mr. Ken:** Let's get back to what we were doing. Ok! We had that group in the middle explaining why three? Thokozani you wanted to add something. Remember you wanted to say something Durrell.

Line 5 and line 6: An inaudible discussion between Mr. Ken and Durrell taking place. Mr. Ken also disciplined the learners by warning them that they will not leave until they arrive at a solution.

Line 7: **Lauren:** Since we realised that we've, we got now like this sort of base set.

08:00 – 09:00

Line 8: **Lauren:** We've got now based it on facts.

Line 9: **Mr. Ken:** Ok, I want you to come back and explain from scratch.

Line 10: ***Lauren:*** But mustn't they explain the part first. *(Pointing to group 5)*

Line 11: ***Learners:*** Yes, sir.

Line 12: ***Mr. Ken:*** What part?

Line 13: **Lauren:** Where they came about the three diagonals?

Line 14: ***Mr. Ken:*** They did explain it, but you dealt with it.

Line 15: **Lauren:** Ya, but. O, this is my other thing, my other pattern that I found about my. *(Showing the transparency)*

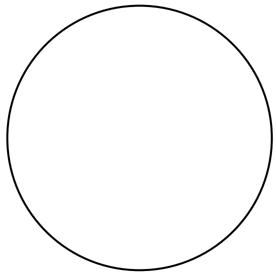
Line 16: ***Mr. Ken*****:** Yes. Now I want you to come and maybe work on the exercise.

Line 17 to line 19: A discussion between Mr. Ken and a learner takes place, but the discussion is inaudible.

Line 20: **Mr. Ken:** You ready, ok let Lauren explain, and after Lauren you can go.

Line 21: ***Lauren:*** Ok, what I did was take a 24 - sided polygon and I just made a circle and I just made the points. And from one vertex I got 21 diagonals.

09:00 – 10:00



Line 22: **Lauren:** I got 21 diagonals. Ya, 21 times 24, count each vertex gives you 504, but we realised that 504 isn't correct we must divide by 2, which is going to give me 252 diagonals, and then Sir asked why do you divide it by 2, so I quickly had to come up with why, so I thought maybe because we have two vertex points that I only count two, we don't really use them cause they on that what that words again.

21 diagonals from one vertex. 24 sides 504 diagonals 504/2 = 252

Line 23: **Mr. Ken:** What word?

Line 24: **Learner:** Consecutive.

Line 25: **Lauren:** Consecutive thing so I thought maybe that's why you divide by 2 because I count two points that are on…*(inaudible)*

Line 26: **Mr. Ken:** That's why you divided by 2, somebody else here said.

10:00 – 11:00

Line 27: **Mr. Ken:** I'm not convinced about that.
Mr. Ken: Because I mean, why, how does the "2" known consecutives vertices that you don't draw diagonals to, relate to the 2 that you divide the total number of diagonals by?
Mr. Ken: I don't get it.

Line28: **Durrell:** I know sir.

Line 29: **Mr. Ken:** Because ultimately you will be forming diagonals from those two vertices as well, you will be forming diagonals from all 24 vertices. *(Demonstrating with his hands)*

Line 30: **Lauren:** okay, from each vertex. But from that one point you do not use two vertices *(Showing with hands).*

Line 31: **Mr. Ken:** Yes, you will.

Line 32: **Lauren:** Inaudible. From that the one point you will not have two diagonals.

Line 33: **Mr. Ken:** Okay Lauren, think about what does it do to divide by 2, what does that mean, to 504? What did you do to it?

Line 34: **Lauren:** You halve it.

Line 35: **Mr. Ken:** Do you half it, so why would you halve this number of diagonals?

11:00 – 11:11

Line 36 and Line 37: A learner and Mr. Ken, engages in a discussion which is inaudible. Mr. Ken finally insists that the learner needs to think about their discussion.

11:30 – 12:00

Line 38: **Lauren:** Sir you know the other page that you had. *(Educator goes to the front to give Lauren the transparency)*

Line 39: **Mr. Ken:** Ok! Tebogo I am going to ask you, come and try this. Oh! No. Thokozani first, then Tebogo.

Line 40: ***Lauren:*** Okay, what I first realised. When we pattern between each number, like from 14 to 20 there's 6 so we think that…(Interrupted by educator)

Sides	***Diagonals***	***Vertex***
3	*0*	*0*
4	*2*	*1*
5	*5*	*2*
6	*9*	*3*
7	*14*	*4*
8	*20*	*5*

Line 41: **Mr. Ken:** So you saying that the numbers in between, what is that called again what did we say that. The numbers in between the number of diagonals per the number of diagonals per vertex increases by one each time.
Mr. Ken: Do you understand what I am saying? From 0 to 2 there's 2 and then from 2 to 5 there will be 2 plus 1, increases by 1.

12:00 – 13:00

Line 42: **Learners:** That what we saying.

Line 43: **Mr. Ken:** That's exactly what she's saying. So you guys also came up with that.
Mr. Ken: Ok now you come explain it nicely to us. *(Pointing to Thokozani)*

Line 44: **Thokozani:** *(Moves to the front of class).* Ok, sir. We came up with. *(Class is very noisy)*

Line 45: **Mr. Ken:** Ok, now guys.
Mr. Ken: Thokozani is up front, so that means that we give him the same courtesy, we listen.

Line 46: **Thokozani:** *(Works from his page).* Ok Sir, after doing the several of these polygons, this is the method that we came up with, so we need to invent this because no diagram could solve it.

13:00 – 14:00

Line 47: **Thokozani:** So 1 to 3 has no diagonals so we stared working from number 4 a 4 sided polygon, so from the 4 sided polygon you get 2 and then to the next one to 5 you add 3 cause from the 2 from the 1 the previous one you add just 1 more and then to get the sixth one you add 4 more, from the 5 one you add 4 more.

Line 48: **Mr. Ken:** Thokozani use the transparency that's there. You got the same numbers there, no, no, you can't use that, use those numbers there.
Line 49: **Mr. Ken:** Because we not following what you saying.

Line 50: **Thokozani:** From the 4 Sir you begin with 2 right 'cause it's an easy one to work out you can easily work out how many diagonals you can get from a 4 sided shape. From a 4 sided shape we figured that if you add 3 more to your.

14:00 – 15:00

Line 51: **Thokozani:** From your 3 from the one that you got first you get your diagonals for number 5. From the 5 you add 4 then you get 9, which is diagonals for 6 - sided polygon. For 7 you add 5 to that 9 then you'll get 14 diagonals in a 7 one and that's how you carry on throughout it, you just add one more.

Line 52: **Mr. Ken:** Ok, So what you guys are basically saying is that you can increase the number of diagonals by one each time and you can work out by adding and adding and adding and adding you can work out to how many there will be a 700 sided polygon?
Mr. Ken: So I need to know, first of all, how many diagonals there'll be in how many? 699 (Learners Chorus 699) sided polygon before I can work out how many is in 700.

15:00 – 15:41

Line 53: **Learners:** Ya.

Line 54: **Mr. Ken:** That's what you saying. So, you're going to start by 3 sides and work all your way up to 700.

Line 55: **Thokozani:** That's why sir, I said, work with smaller numbers sir.

Line 56: **Mr. Ken:** But we, the question here is "How many diagonals in a 700 - sided polygon"?
Mr. Ken: Your method will obviously work based on the pattern that we see. If that, pattern continues, now once again we call that a conjecture. It's not true yet.

Line 57: **Learner:** But sir.

Line 58: **Learner:** We proved.

Line 59: **Mr. Ken:** We proved how many diagonals from that particular vertex.

15:41 – 16:17

Line 60: **Lerato:** We proved 700

Line 61: **Mr. Ken:** She can show 700. Can you, explain why that is so?

Line 62: **Learners:** Discussion inaudible.

16:17 – 17:00

Line 63: **Mr. Ken:** Okay, you can come and tell me.
Mr. Ken: Thokozani, I want you to think about in that group.
Mr. Ken: Ok guys, shhh listen. I need to short cut. I do not have to draw all that polygons to get to 700. and that is what your method is going to lean, lead me to do, okay.
Mr. Ken: And that is the point of mathematics, to come up with a way, from using those patterns to come up with a certain way we can find the number of diagonals without actually sketching all 700. Or at least 699.

Line 64 to line 66: Inaudible discussion between the learner and Mr. Ken takes place.

Line 67: **Mr. Ken:** Okay, let's hear what you've come up with then.
Mr. Ken: Let's see.
Mr. Ken: You said you have a solution.
Mr. Ken: We would like to leave.

Line 68: **Learner:** Let's leave.

Line 69: **Mr. Ken:** So let's make us leave. We all would like to leave.

17:00 – 18:00

Line 70: **Mr. Ken:** You know why you leave for break.
Mr. Ken: That's besides everything.(Class very noisy).
Mr. Ken: Ok, listen, let's settle down.
Mr. Ken: Lauren has a particular way of doing it.
Mr. Ken: Can you tell me how many in 700?

Line 71: ***Lauren:*** I never have the time but I can.

Line 72: ***Mr. Ken:*** You can. Show me how using your conjecture. Because it's not proved as yet.

(Nombulelo and Kinesh goes up to the board to explain the method. Nombulelo first works on transparency).

18:00 – 19:00

Line 73: **Kinesh*:*** Because of our pattern, we decided to take 700-3, which gives you, 697. So that the number of diagonals from one vertex but then we said that the number of 697 x 700, because there's 700 sides. And that how much of diagonals we got. So we think that's the number of diagonals in a whole polygon.

Line 74: **Nombulelo:** And there seven and one vertex. Did we divide it?

Line 75: **Kinesh:** How many diagonals from one vertex?

Line 76: **Nombulelo:** 697.

Line 77: **Kinesh:** So we going to say that times 700. Too get the number of diagonals in the whole diagram.

Line 78: **Nombulelo:** In the whole figure. But then we not sure, not sure if 700 is the total or we divide by two. *(Discussion between Kinesh and Nombulelo is inaudible because they not sure of the answer although they worked it out).*

19:00 – 20:00

Line 79: **Mr. Ken:** *(Walks to the front of the class, to check Nombulelo's and Kinesh's work).* That's how many diagonals from a vertex.

Line 80: **Nombulelo:** And here 700/2 is 350 and 700 x 350 is that. *(Showing the calculation of 245000)* And you just say 700.

Line 81: ***Mr. Ken:*** Why divide 700 by 2?

Line 82: **Kinesh:** Explanation inaudible.

Line 83: **Nombulelo:** *(Instructs Kinesh to calculate).* Do that *(Kinesh does the calculation while Nombulelo writes it down).*

20:00 – 21:00

700 – 3 = 697 700 x 697 = 487900

Line 84: **Kinesh:** From our pattern that we found here.
We decided to take 700 and minus 3 and we got 697.
That's the number of diagonals from one vertex.
Kinesh: So then we said that 697 times 700, because that's the number of sides and we came up with this figure. *(Pointing to the 487900 on the calculator)* And so we think that's the number of diagonals in the whole polygon.

Line 85: **Nombulelo:** The 700 sided polygon.

Line 86: **Tebogo:** That's what Lauren said first.

Line 87: **Mr. Ken:** That's what Lauren said first. What that flaw in that, Lauren? Let hear.
Line 88: **Lauren:** When ya, I also worked it out like they did. I did that by 600. I did by 700 equal that as well. But according to what I discovered is that you must divided the answer by two, which gives you 243 950

Line 89: **Mr. Ken:** So you have to divide that by two.

Line 90: **Lauren:** Yes.

Line 91: **Mr. Ken:** So you are saying that, their number of 4 is 48, then if this 400 then it gives you 7000.

21:00 – 21:35

Line 92: **Mr. Ken:** So we can see that's reasonable. Why was it reasonable?
Mr. Ken: Because it worked for a triangle, it worked for a quadrilateral, it worked for a pentagon, it worked for a hexagon, so it seems to be a reasonable conjecture, it seems to be working.
Mr. Ken: The question was however 'Why divide by two'?

Line 93: **Lauren:** Sir.

Line 94: **Learners:** Sir. *(Students have a discussion).*

Line 95: **Mr. Ken:** *(Everyone wants to answer, so educator counts 4 people to answer the question).* Okay, I get one, two, three, four. Durrell, Tebogo, Lauren and then Peane *(Dikgabiso)*

Line 96: ***Durrell:*** Sir, I think I know why, sir, because it starts on the one side sir, like a six, a hexagon sir. It goes to the 3 sides. Once you gone with the three sides you can't go on. I don't know how to explain this so *properly.*

(Lesson 3, Event 2, Time Interval 7:07 to 20:35)

Lesson Three

Events	Time	Notion	Sub-Notion	Conceptual Understanding	Procedural Fluency	Strategic Competence	Adaptive Reasoning		Defining	Explaining	Representing	Questioning	Scaling	Working with Learner Intuition	Specific	General	Definition / Rule	Curriculum	Experience	Comments
			Concept						Problem Solving						Appeal: Maths					Comments
Event 2	07:07 to 11:11	Conjecturing and justifying	Conjecturing - Empirical Case	√		√	√			√	√	√	√	√	√					Feedback: Various polygons. Number of diagonals from 1 vertex. E.G. 24 - sided polygon.
	11:30 to 15:41		Conjecturing - Proving patterns	√	√		√				√		√	√	√					Algebraic pattern
	16:17 to 21:35		Conjecturing - Systematically	√		√				√	√	√		√		√				Providing a 700 – sided polygon

***Table 4** - 3.4.2: Event 2 from Lesson Three*

The overall **notion** in this extract is conjecturing and justifying with the following **sub–notions** that have been timed accordingly: *Conjecturing - Empirical Case (11:30 to 15:41), Conjecturing - Proving patterns (11:30 to 15:41) and Conjecturing – Systematically (16:17 to 21:35).*

Between the time interval 07:07 to 11:11, the nature of the sub–notion of Conjecturing – Empirical Case is **conceptual** since the teacher pursues for justification by using words like: *give facts, convince, what is your understanding, tell me and show me.* This sub–notion also involves strategic competence and adaptive reasoning, since the teacher uses statements like: *explain why, explain from scratch, what part, come back and work on the exercise, why you divided by 2, I am not convinced.* The teacher engages in five of the six mentioned problem solving categories during his mathematical work that is explaining, representing, questioning, scaling and working with learner intuition. In order to legitimate meaning the teacher appeals to specific mathematics. My reason for suggesting that the teacher appeals to specific mathematics is because the teacher probes the learners for specific mathematical reasoning, for example, *the teacher asks Lauren to explain*. In this sub–notion the teacher never appeals to the curriculum and experience.

The kind of problems that these learners are grappling with is different from the routine type of questions that one would find in a typical textbook. After defining, naming and proving the theorems on quadrilaterals, the learners are required to use algebraic methods for proving riders for example:

a) Calculate area ABFE

b) Calculate area ABCD if AB = 82mm, BC = 32mm and BF = 20mm

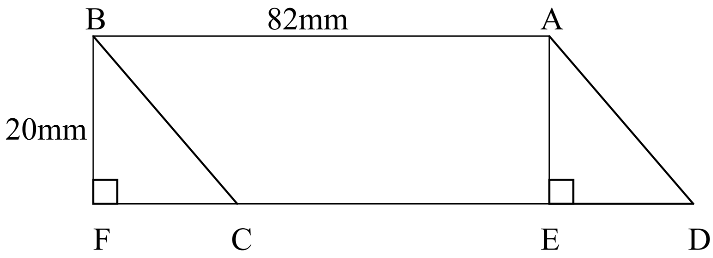


Figure 2*: Quadrilateral ABFD*

The above example is a combination of algebra with geometry. Thus, Mr. Ken now, needs to engage with his learners in a different fashion, Mr. Ken then also provides them with opportunities to work with non–routine problems, which is essential for developing strategic competence. The teacher chooses this task because it requires the learner to understand the problem so as to represent it mathematically either algebraically or geometrically in order solve the problem.

Strategic competence deals with the ability to formulate, represent and solve mathematical problems. Remember, that the work of the teaching is distributed in Mr. Ken's classroom, it is not only through his utterances but also the work that he is doing to solicit the kinds of responses from his learners that I will need to consider (i.e. this form of negotiated work needs to be looked at). In view of this and the extract above it is explicit that the mathematical problems of determining the number of diagonals in a 700 sided polygon are constantly being reformulated and thus also re– represented. These are the activities that characterises what Kilpatrick et al. (2001) refer to as strategic competence. The problem of determining the number of diagonals is located within a specific case and to move to the general case is difficult – hence the appeal to the specific.

With respect to the above extract Line 27 illustrates that Mr. Ken's responses does not require learners to engage procedurally with the concept. Mr. Ken's statement seduces learners to reflect critically on their thinking (solutions) – they need to think logically about their reasoning so as to find a solution that would convince someone else (in this case Mr. Ken). This demonstrates Mr. Ken's attempts to foster some form of **adaptive reasoning** amongst his learners.

In respect to similar studies conducted by Kazima and Adler (2006) and Adler and Pillay (2006) and Pillay (2006) shows that, in teaching practice learners are encouraged to grapple with various tasks that involve the six condensed problem solving techniques. What is quite different in this study is that because of the chosen tasks, the teacher is forced to work mathematically with *representing and working with learner's intuitions* to a larger extent in comparison to *defining*, *explaining, questioning and scaling.* Thus I used an adapted vision of Kazima and Adler's six types of problem solving categories to fit with what I identified as defining, explaining, representing, questioning, scaling and working with learner's intuitions

Thus referring to the above table and transcript, it is interesting to notice that the teacher does not use **defining** when problem solving in the third lesson at all. Although defining did not take place in this event but it did occur in the other lessons in lesson 4 and lesson 5. The teacher used the textbook and the mathematics dictionary to legitimate the mathematical definitions and rules according to the curriculum. In this event the teacher often uses: *explaining, questioning and scaling* (in two of the three sub-notion) as a technique of problem solving to fix meaning to the conjecturing process by using specific empirical cases and general systematic mathematical problems.

I had to re–define the category of explaining to suit Mr. Ken's lessons where he encourages the learners to explain why by saying what and showing what they understand, which is quite different from how Kazima and Adler (2006) used explanations. I also see that **explanations** are a central aspect of the work of teaching that Mr. Ken engages with. Here again, the teacher does not necessarily provide explicit explanations but he rather prompts learners or probes learner's responses so that they can give explanations for their responses. Thus in order to engage learners in this fashion the teacher himself needs to have an idea of what a convincing explanation is. The explanation need not be mathematically robust or correct and I am not looking at explanations being uttered by the teacher alone. This is how I have defined explanations as a category of the mathematical work of teaching. For example the teacher illuminates the explanation aspect by saying “Ok, I want you to come back and explain from scratch”. This then links to the idea that the teacher is working with learner’s intuitions as well.

The problem solving techniques of *representing and working with learner’s intuition* are used most frequently in this event which is legitimated through specific mathematical cases which is eventually generalised in the third sub-notion. Various learner **representations** and re–representations of the problem for example “*the number of diagonals in a 24 – sided polygon*” has to be understood and recognized by the teacher (Mr. Ken) in order for the teacher to work mathematically to fix meaning of this specific representation for the learners to understand this representation.

Both the teacher and the learners engage in **questioning** often**.** The teacher poses most of the thought provoking questions to move the learners on instead of explaining to the learners. Thus the teacher uses questions to encourage the learners to justify their mathematical thinking by using this specific representation.

Scaling occurred in two of the three sub–notions. Showing that, the teacher **scaled** the task for the learners at first by asking the learners to start with simple polygons and to move to more complex polygons.

Working with learner’s intuition is seen to be the most prominent problem solving technique in the above notion. Throughout this event the teacher works with the learner’s mathematical intuitions. It is interesting to note that during this teachers’ problem solving (mathematical work) he only make appeals to specific and general mathematic.

The categories that are not covered thus far, are between the time interval 11:30 to 15:41 of the sub–notion Conjecturing - Proving patterns. Here, the teacher is testing **procedural fluency** when he states *“So you saying that the numbers in between, what is that called again what did we say that. The numbers in between the number of diagonals per the number of diagonals per vertex increases by one each time”* and *“do you understand what I am saying? From 0 to 2 there’s 2 and then from 2 to 5 there will be 2 plus 1, increases by 1”.* This is evident that the teacher is testing the accuracy of the learner's knowledge of the procedure followed to move to the final solution.

In the sub–notion Conjecturing – Systematically (16:17 to 21:35) the teacher appeals once again to mathematics of a **general** nature to fix meaning. For example: when the teacher *said, "So we can see that's reasonable. Why was it reasonable? Because it worked for a triangle, it worked for a quadrilateral, it worked for a pentagon, it worked for a hexagon, so it seems to be a reasonable conjecture, it seems to be working"*. The appeal that the teacher makes to legitimate is meaning requires the learners to generalise mathematics.

3.6 <u>Ethical Considerations:</u>

I applied for permission from the Gauteng Department of Education's (GDE) research unit and for ethical clearance from the University of Witwatersrand, School of Education to conduct this study. I also explained the aim and purpose of the research to the Institutional Development Support Officer (IDSO) of district 6, the principal, the Head of Department of Mathematics (HOD) and the Mathematics teachers of the grade 10's. I obtained written consent from the above-mentioned people to do the research at XXX Secondary School. In collaboration with the mathematics head of department and the principal of XXX Secondary School, the teacher (Mr. Ken) granted permission for the interviews to be tape–recorded. The teacher and principal granted permission for the observations of the lessons to be video recorded, tape-recorded and written field notes to be made as well as permission was received to make copies of learners work done during this period.

I explained to the grade 10 learners that their real names will not be used in the write up. The learner's were also given the assurance that their responses will be treated with the utmost confidentiality. Learners had an option of not participating if they so wished not to, which Cohen et al. (2002: 279) refers to as "informed consent, guarantees of confidentiality, beneficence and non– maleficence". Parents of the chosen grade 10 learners had to sign consent forms before commencing with the research, since most of the learners are not of age to participate in the research. The teacher also had to sign a similar consent form before the research commenced. The consent form contained information about: an outline of the research topic and aims, an assurance that the findings of this research will not be a reflection of the learners, their families or school, a guarantee of autonomy of the participants and that participation is voluntary. It also included an explanation of how and by whom the findings are to be used, how and why there were interviews with the teacher and lesson observations, an explanation for the need for video recording and tape – recording of the lessons, since learners will appear in the video. I also informed learners' that the learners' written responses will be selected to assist in researching the teacher's work of enhancing the learners' understanding of polygons in particular, quadrilaterals at the end of the research process. Draft letters seeking permission are attached in Appendix C - (Principal, Teacher, Learner, Parent / Guardian).

As an ethical consideration I considered the integrity of the school and promised to acknowledge all those who helped make this study possible by using pseudonyms, so not to reveal the true identity of the participants. Care was taken not to contaminate the data by putting words into the teacher and learners' mouths. The data was used as evidence and a way of persuading readers that reports and conclusions are believable. Care was also taken not to let personal prior knowledge; interests and personal preferences influence the process of data collection. As part of the ethical considerations I worked to minimise disruption as far as possible to the normal functioning of the school and I will ensure that the school and interested parties gets written and verbal feedback of the results of this research.

*3.7 **Rigour in this research:***

As Opie (2004) states that reliability refers to the properties of the whole data collection process and not just the results of the research. Qualitative reliability refers to trustworthiness (relatability) of what extent this research can be conducted again under similar circumstances in another place and time. This situation is unique therefore it will be difficult to replicate this study. The fact that this study is to explore teaching practice it would be possible to discover the same or similar results if they conduct the same or similar study using the same or similar participants. Reliability refers to the consistency and accuracy of the instruments of the pre and post interviews and the classroom observations of the lessons. Thus I made explicit, all aspect of the research design (as above) to look like another similar study. To ensure increased reliability a combination of data collection strategies was used like; extracts from the transcripts, detailed descriptors, and mechanical methods of recording and constant checking, reviewing and analysis.

To obtain reliability in the design, I had to develop a social relationship with the grade 10 teacher first which included explaining and informing this teacher about the criteria, rationale, decision making process and the data analysis processes that were used, especially the codes used and how the selection for this case study was made. To add to the trustworthiness, the information collected will then be checked informally with the grade 10 - teacher for accuracy during the process. And to maintain reliability in the data collection, verbatim accounts of conversations, transcripts and direct quotes from documents were used together with concrete, precise descriptions. Using recording from tape recorder and videotape will make precise detailed records and descriptions of situations. Thus, affirming confirmability during the interpretation process of the data that contributes to the triangulation of the data.

Qualitative research has high *internal validity* because of the long data collection period and subjectivity (Opie 2004). Continual data analysis, collaboration and refinement of ideas will ensure internal validity over a minimum period of a week to collection data. Classroom observations and interviews were conducted in a natural setting and monitored by myself, thus maintaining internal validity.

Threats to *external validity* are the effects that limit this study's usefulness, (Opie 2004). Thus this study was comparable and translatable. This case study design was adequately described so that its' extended finding can be used as a comparison and a contrast for further studies but not to be generalized. Thus this study aims at extending the educational understanding of the teachers teaching practice rather than to make generalizations, thus making this study more credible. The credibility of the interpretations of the observed lessons was verified through the interviews with the teacher. The teacher was given the freedom in the interviews to say what he wanted too, while I was empathic. I also took the non – verbal and verbal communication into consideration while being sensitive to the teacher.

Due to the limitations of a case study, this study cannot be generalised but this study can be more profitable since it can be transferable. Thus this transferability leads to being more trustworthy with increased relatability that could be applicable to other further studies.

3.8 ***Conclusion:***

This research study employed the strategies of participation, observation and interviewing in the process of data gathering thus warranting a qualitative approach to data gathering. I managed to confirm the data that covered the same areas of concern by using two semi–structured interviews. This chapter also highlighted the ethical issues that were considered for this study. I ensured the rigour of trustworthiness; confirmability; credibility and transferability of the data received from the observation lessons and then crossed checked the data during the interviews that allow for triangulation of the data.

Chapter Four

Analysis and Interpretation of Data

4.1 ***Introduction:***

The purpose of this chapter is to report on the investigation carried out in relation to the questions set out in chapter one. This chapter begins the analyses of the findings of this case study by discussing the possible solutions and demands of the tasks and by giving a background and an overview of the lessons observed. I then provide a largely qualitative analysis of the observed lessons according to the framework and the indicator descriptors in the previous two chapters. Finally, attempts are made to answer the following research questions:

1) What mathematical work does the teacher do as he teaches geometry to his Grade 10 learners?
    a. What mathematical problems or challenges does he encounter?
    b. How does he engage with these mathematical problems of teaching?
2) What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?
3) How does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?

4.2 ***The tasks, learner's possible approaches and demands on the teacher:***

I solved the two tasks that the teacher chose to administer to the learners during the observation period to see what demands and requirements the tasks could place on the teacher and learners. If the teacher is going to teach these tasks, what level (in Van Hiele's terms) is he expecting the learner's to work at? If the learners are not at the appropriate level, what might the teacher do to bridge the gap? Considering the geometric phenomena in these tasks, what do the teacher and learners need to know of polygons in relation to their specific properties.

4.2.1 ***<u>T ask 1:</u>***

The first task: **"How many diagonals are there in a 700 – sided polygon"?**

How does an informal[19] proof as a solution to **this** task relate to Van Hiele levels of geometric thought development? One has to look at the requirements in relation to the Van Hiele levels to see what mathematical knowledge the teacher and learners need to answer the above question. As mentioned in chapter 2, here is a table that shows how the task fits in the Van Hiele Levels, especially in the early learning experiences when we want the learners to recognise simple regular polygons.

Looking at the table below the learners must be able to use the following skills: visual, verbal, drawing, logical and application at levels zero, one and two and possibly level three. The learner needs to see the difference between various polygons, and that a diagonal is a line segment joining two vertices that are not adjacent to each other. The learner must also realise that once a polygon has more than four sides, more than one diagonal can be drawn from a single vertex; and further that a diagonal from point A to point C is the same diagonal as that from point C to point A.
Teppo (1991) re-describes the five levels in relation to the five phases:

Level 0 - To recognize the above diagrams as well as name and sketch various polygons as similar and different physical objects.

Level 1 - To describe the differences verbally of what they see of the properties of the various polygons by recognising the geometric properties.

Level 2 - To recognise and understand interrelationships of mathematical concepts using mathematical language accurately and concisely. It is also the starting point for induction and conjecturing.

Level 3 - *To use the above information of what is given and what is required to proof, to deduce definitions, postulates and theorems.*

Level 4 - To recognise examples and non – examples of concrete and abstract systems.

[Teppo, Anne. 1991:211]

[19] Informal proof is use because it refers to the development of conjecturing, testing and justifying which is the road to formal proof. Working towards a proof.

As will be seen below Van Hiele level three and four is required at least to solve this task.

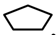
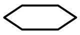
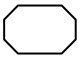
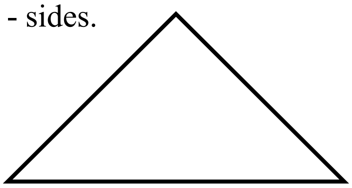
Level 0 ***Visualisation***	***Level 1*** ***Analysis***	***Level 2*** ***Informal deduction***	***Level 3*** ***Formal deduction***	*Level 4* *Rigor*
E.g. Triangle - Δ, Quadrilateral-⌂, ◊, pentagon - **, hexagon -**  **, Octagon**  **etc.**	**E.g. 4 - sides Parallelogram – = opposite sides = opposite angles diagonals bisct**	**E.G. Quadrilaterals = trapezium, square rectangle, rhombus, parallelogram because all have 4 sides.**	Regular polygons have equal sides and interior angles. No. of diagonals in various polygons.	E.g. Diagonals for 1 vertex. Triangles – no diag., Quadrilaterals – 1 diag., Pentagons – 2 diag.

Table 5 *- 4.2.1: A representation of Van Hiele's five levels in relation to five skills*

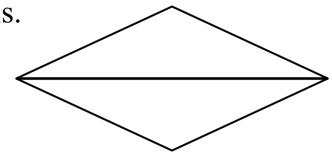
4.2.2 Approach 1 – Practical and Visual:

One has to look at various ways learners might attempt to solve this task practically or empirically. Thus in order to generate a pattern, the task level may possibly be altered to a low - level task or high - level task according to Stein et al. (1996). Learners will then have to physically draw in the diagonals in various figures; starting from a 3 – sided figure and move sequentially up to the 699 – sided polygon and finally to a 700 – sided polygon, thus processing from simple known polygons to more complex polygons which tends to create a problem, for example:

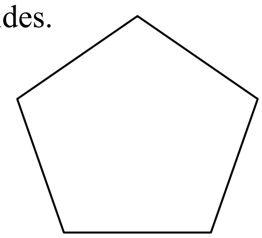
Triangle has three - sides.
Zero diagonals.



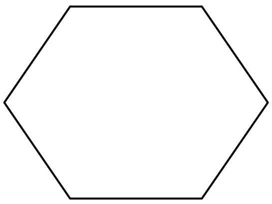
Quadrilateral has four – sided.
Two diagonals.



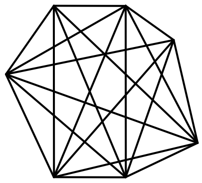
Pentagon has five –sides.
Five diagonals.



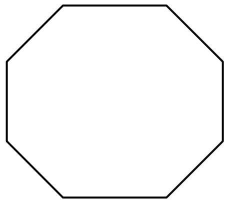
Hexagon has six – sides.
Nine diagonals.



Heptagon has seven-sides.
14 diagonals.



Octagon has eight – sides.
20 diagonals.



But as we see that from here on the drawing becomes too cumbersome, tedious and messy which makes finding the solution unlikely. So this is as far as (8 – sided or 9 – sided figure) the learners are likely to get by drawing in the diagonals empirically. The learners thus have to develop some sort of system. They might use a table in order to determine a pattern as the number of sides plus the number of diagonals of the next polygon equals the number of diagonals of the following polygon as presented below.

SIDES	***DIAGONALS***
3	0
4	2
5	5
6	9
7	14
8	20
:	:

Table 6 - *4.2.2: Possible solution to a seven hundred - sided polygon*

For this attempt the learner does not have to necessarily see relationships. So if the learners are working at Van Hiele levels 0, 1 and 2, this is how far they will get. They have to just work with the numbers to see a pattern. The pattern does not relate to spatiality, other than in the ability to draw in the diagonals. But once you get to an 8 - sided figure you can see the pattern. Although this method might also be cumbersome and tedious it is possible to reach the solution eventually. By starting with known simple polygons and moving to the more complex polygons, and because the learners do not have much time on their hands, they might look for a shorter method by looking at more specific properties about diagonals.

4.2.2.1 <u>Teacher's Work:</u>

The teacher has to consider the constraints of time and learners not being on the appropriate levels to find a solution to this task. So for this study, the question is; what challenges does the teacher confront here and how does the teacher deal with these challenges. The teacher is faced with the challenge that the learners are required to understand polygons and the relationship between diagonals i.e. the learner must have some spatial knowledge and geometry knowledge. In Van Hiele terms the teacher is going to have to move the learners from whatever level or phase they are at to the required level and phase by working mathematically.

4.2.3 <u>Approach 2 - Geometric and Algebraic:</u>

If the learners use an approach that requires a higher level of reasoning as compared to the first attempt, what is require then in Van Hiele terms is level 3 and 4.

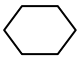
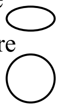
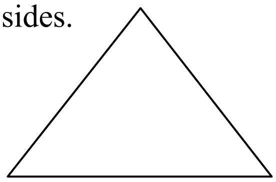
<u>Level 0</u> ***<u>Visualisation</u>***	***<u>Level 1</u>*** ***<u>Analysis</u>***	***<u>Level 2</u>*** ***<u>Informal deduction</u>***	***<u>Level 3</u>*** ***<u>Formal deduction</u>***	***<u>Level 4</u>*** ***<u>Rigor</u>***
E.g.  Hexagon Nonagon 24 –sided figure 700 –sided figure 	E.g. 3 – sided has No diagonals. Definition of Diagonal – line joining opposite sided	E.G. A triangle is formed by a vertex and its two consecutive points (sides). This is informal deducing and conjecturing.	**Formula: Number of sides x (number of sided – three) divided by two equals number of diagonals**	**E.g. Use formula two solve other polygons, including examples and non – examples.**

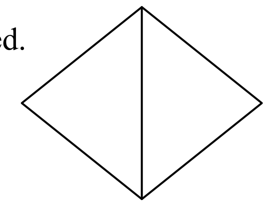
Table 7 *- 4.2.3a: A representation of Van Hiele's five levels in relation to five skills*

For example: looking at the number of diagonals from one vertex. The learners have to realise that there is no diagonal from any vertex in a triangle because the vertices are adjacent to each other.

Triangle has three - sides.
Zero diagonals.



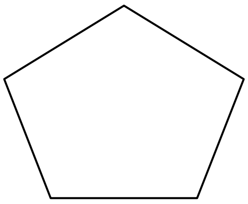
Quadrilateral has four – sided.
Two diagonals.
One diagonal from 1 vertex.



Pentagon has five –sides.

Five diagonals.

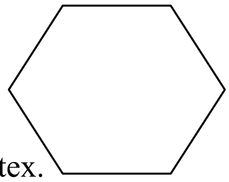
Two diagonals from 1 vertex.



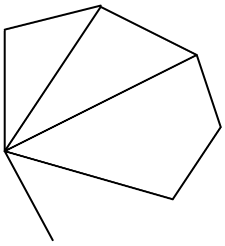
Hexagon has six – sides.

Nine diagonals.

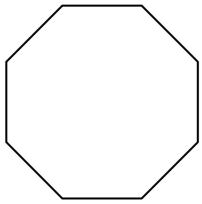
Three diagonals from 1 vertex.



Heptagon has seven-sides.
14 diagonals.
Four diagonals from 1 vertex



Octagon has eight – sides.
20 diagonals.
Five diagonals from 1 vertex



SIDES	***DIAGONALS***	***NO. of DIAGONALS from 1 vertex***
3	0	0
4	2	1
5	5	2
6	9	3
7	14	4
8	20	5
:	:	:
24	252	21
:	:	:
699	243 252	696
700	243 950	697

Table 8 *- 4.2.3b: Solution to a seven hundred - sided polygon*

Some learners will reason as follows for example: In a 4–sided polygon there is one diagonal from one vertex. There are four vertices so there are four diagonals, but of course one diagonal is doubled each time, so there are two diagonals.

In a 5–sided polygon there are two diagonals from one vertex. There are five vertices so there are ten diagonals, but of course one diagonal is doubled each time, so there are five diagonals.

In a 6–sided polygon there are three diagonals from one vertex. There are six vertices so there are eighteen diagonals, but of course one diagonal is doubled each time, so there are nine diagonals.

In a 7–sided polygon there are four diagonals from one vertex. There are seven vertices so there are twenty-eight diagonals, but of course one diagonal is doubled each time, so there are fourteen diagonals.
In an 8–sided polygon there are five diagonals from one vertex. There are eight vertices so there are forty diagonals, but of course one diagonal is doubled each time, so there are twenty diagonals.

In order for the learners to see some properties, they have to see how diagonals came into play. In seeing the relationships of diagonals to diagonals from a vertex, it requires both spatiality and mathematical reasoning. Thus, learners must start to realize that a triangle has no diagonals and this triangle is formed by the vertex and its two consecutive adjacent points therefore you subtract three from the number of sides of any polygon and divide by two because a diagonal extends from one vertex to the other opposite vertex.
Other learners might reason as follows, the number of sides of the polygon multiplied by the number of diagonals from one vertex of the polygon divided by the two consecutive sides (i.e. the number of sides minus three) equals the number of diagonals in the polygon. For example:

A. <u>Seven – sided polygon</u>
(7 x 4) / 2
= 28 / 2
= 14 diagonals

B. <u>Nine – sided polygonal</u>
(9 x 6)/2
= 54 / 2
= 27 diagonals

C. <u>Twenty four – sided polygon</u>
(24 x (24 - 3)) / 2
= (24 x 21) / 2
= 504 / 2
= 252 diagonals

D. <u>700 – sided polygon</u>
(700 x (700 – 3)) / 2
= (700 x 697) /2
= 487 900 / 2
= 243 950 diagonals

4.2.3.1 <u>Teacher's Work:</u>
The teacher started with a very complex first task which learners will be able to solve if they are at Van Hiele level 3 and 4. If the learners are not at level 3 or 4 then the teacher has to get learners to work with an understanding of a diagonal from a vertex. From the possible task solutions it can be seen that the task places high–level demands on the teacher's mathematical work since it, cannot just be solved by looking at it. So in a sense the object here is not only

empirical work but also more geometric work. The teacher needs pedagogic strategies to engage learners in conversation to clarify and justify their ideas and methods. The teacher also needs to know what to do to actually understand the relationship between polygons and diagonals. This requires more then just spatiality, it also requires mathematical reasoning and generalisations. This discussion has illuminated how the task demands relate to the Van Hiele levels, the proficiency for mathematics, problem-solving and knowledge demands on the teacher.

4.3 Second Task – Application of Proof:

There is a real leap in this second task (see box below). The task integrates with the real world.

> **A scenario: There are these developers. Real estate developers and they come up with the idea that, you have got a nice area with a dam on the one side and nice roads linking this area with all the other areas. The Real Estate wants to develop property, so that each of the four rectangular properties must maintain their original surface area. What's important is that all four rectangular properties should have excess to the dam and at the same time they must have excess to the road. The Real Estate wants this, by spending the minimum amount of money.**

The first thing to notice in this task, are the words **"four rectangular properties".** It is confusing because the shapes do not remain the same. To solve the problem, the properties must change shape – to **parallelograms**. In addition the learners must know the specific geometry theorem entailed in the solution to this task. In the old curriculum, this problem would require the learners to prove the theorem of "the area of a rectangle and the area of a parallelogram between the same parallel lines and on the same bases are equal".

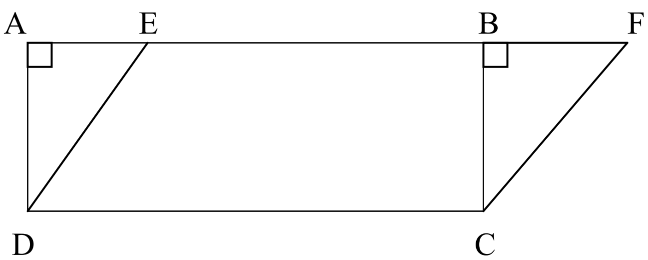


Figure 3: Quadrilateral ADCF

The learners are clearly going to have to go through the Van Hiele levels, the demands of the task and what do they have to do. And what will be interesting is to see if and how the teacher and learners reduce the task, but they do not necessarily do that. What would be even more interesting then what the teaching is doing is how the teacher manages this problem.

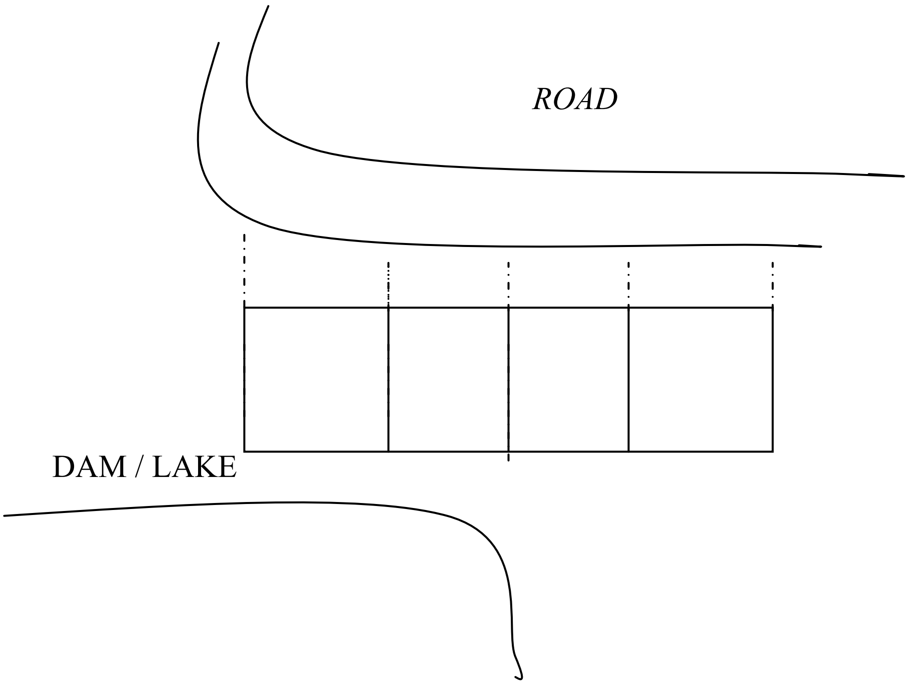


Figure 4: *Task 2*

Level 1 - To recognize, name and sketch the similarities and differences between rectangles and parallelogram as physical objects.

Level 2 - To describe the differences verbally of what they see of the properties of the rectangle and parallelogram by recognising the geometric properties.

Level 3 - To recognise and understand interrelationships of mathematical concepts for example, area using mathematical language accurately and concisely.

Level 4 - To use the above information of what is given and what is require to proof, to deduce definitions, postulates and theorems.

Level 5 - To recognise two conditions must apply for the theorem to be true.

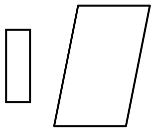
Level 0 ***Visualisation***	***Level 1*** ***Analysis***	***Level 2*** ***Informal deduction***	***Level 3*** ***Formal deduction***	***Level 4*** ***Rigor***
Rectangle and Parallelogram: 4 – sided, hence 4 vertices. **E.g.** 	**Parallelogram and Rectangle –** **= opposite sides** **= opposite angles** **= diagonals bisct** **Rectangle each angle = 90º**	**All rectangles are parms.** **But all parms. are not rectangles.**	**Rect. and parm. between same parallel lines and the same base are equal in area.**	**Two conditions must hold. E.g.** **Rect: L = 8, B = 3** **LxB = 8x3 = 24** **Parm: H = 8 x B = 3** **HxB = 8x3 = 24**

Table 9 - *4.3: A representation of Van Hiele's five levels in relation to five skills*

By looking at the table above, the solutions demands Van Hiele level 3 and 4. A possible solution could be to change the rectangular shaped properties to parallelogram shaped properties. Remember that the base must be the same (the base will be the side next to the road) and the rectangle and the parallelogram must be between the same two parallel lines. Through solving this task the learners are taught to know and understand the theorem in order to use it "*If a rectangle and a parallelogram that are on the same base and between the same two parallel lines, then they will have equal areas"*.

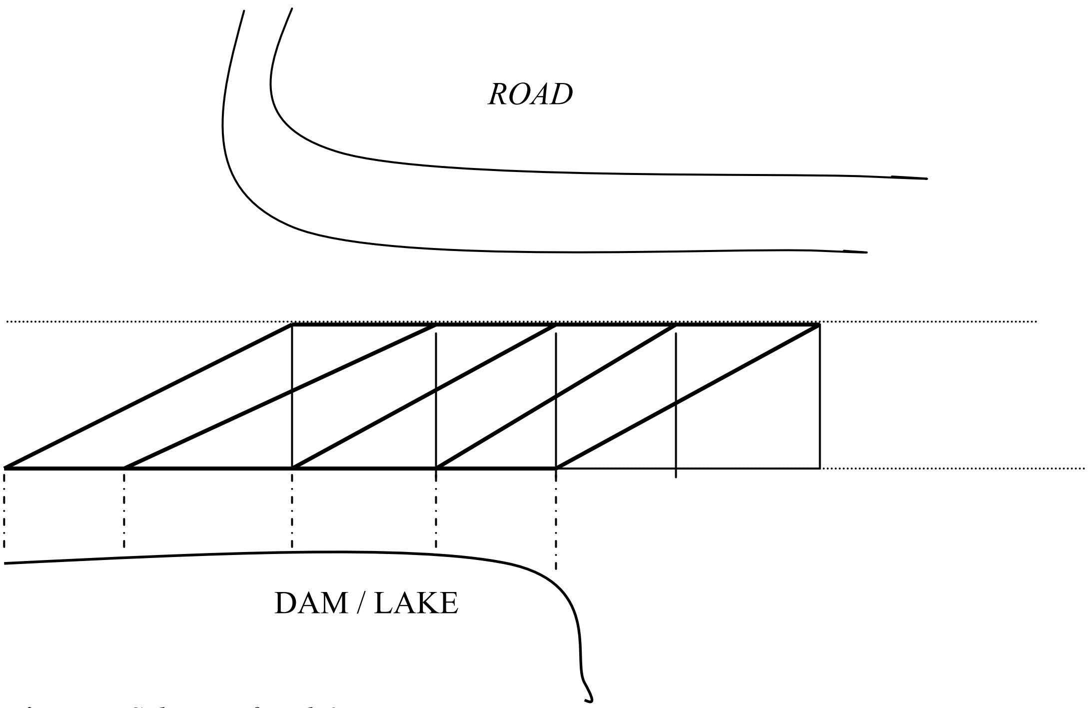


Figure 5: *Solution of Task 2*

This task is difficult and demanding for the learners. In Van Hiele terms it is not just on the level of formal deduction and rigor, it is more than that. So in fact the Van Hiele levels do not really fit neatly here but it is necessary to use them because it requires an application of a proof of a theorem.

From these two tasks it is seen that the teacher chose a spread of tasks with content that requires low and high levels of mathematical thinking, in order to be of value and to provide challenges to develop the learner's mathematical thinking. Stein et al. (2000) helps us to understand that even if the teacher's chosen tasks deals with everyday life, it can still make high level demands on the learner.

So what an analysis of the tasks in terms of Van Hiele levels shows is that there is going to be mathematical work that the teacher is going to have to do. Will the teacher rescale these high–level tasks, and if so how? How will he work to enable learners to engage with the problem and so develop proficiency in geometry? This highlights the notion of scaling that the teacher needs to do.

4.4 <u>Background of the Lessons:</u>

In this section I describe and analyse the data that I collected from two interviews with Mr. Ken and the five lessons observed during the data collection process. I support the results by discussing these in relation to the relevant literature I also use tables to present pertinent results of the analysis of this case study. An analysis of the data is presented by first giving an overview of all five lessons observed. This is then followed by an analysis of the teacher's interviews and the teacher's responses to learner's questions regarding the task.

The tasks were administered to 37 of Mr. Ken's learners from a grade 10 mathematics class. The five lessons totalled to 2½ hours of viewing, each period's duration was 30 minutes long. What is of interest is the content of these lessons and how Mr. Ken explained why he chose these tasks. Since he has taught this geometry section before, the teacher now wants to do something different that would push the learners thinking (i.e. what is going on in these lessons is not what he would normally teach). Therefore it took the teacher 5 lessons to reach the aims of these two tasks so as to complete these two tasks with his class. By and large from the summary below, we see that the teacher works largely with conjectures to reach the solutions that I discussed earlier in this chapter. I demonstrated that these lie inside Van Hiele level 3 and level 4.

4.4.1. Overview of the lessons:

This summary and the table below shows an overview of the five lessons observed:

Lesson **One**: During the first lesson the teacher gave the first task of "**How many diagonals are there in a 700 –sided polygon?"** to the learners without any particular instruction besides to use any method to solve the problem. Groups of various learners found it difficult to get started. After some time and with the teacher refusing to give suggestions or directions i.e. Mr. Ken insisted that they make a start on their own, the groups then got started by attempting various methods and approaches. Mr. Ken observed for 5 - 8 minutes and then he started to interact, prompt and build on what the learners had done. The learners then started deducing, testing and justifying solving the problem. They used examples and non–examples that involved mathematical concepts like: sides, diagonals, vertex, pentagon and polygons.

Lesson **Two**: The same task continued with more testing, investigating and again with prompting, questioning etc from the teacher. New mathematical concepts were dealt with like: odd number of sides and even number of sides, hexagon and the number of diagonals from a vertex of a polygon.

Lesson **Three**: The conclusion of the first task was arrived at after more justifying and conjecturing with the number of diagonals in a 21 – sided and 24 –sided polygon and eventually a 700 – sided polygon. The teacher prompted the learners to look for a pattern by using the smaller polygon that lead to the solving of the 700 – sided polygon.

Lesson **Four**: The second task of "**Where 4 rectangular properties has access to the road but not to the lake must retaining their surface area and have access to both the road and lake."** was given to the learners. The learners found it difficult so the teacher sketched the scenario and eventually demonstrated the solution, while still encouraging learners to work towards their own solutions. The teacher then moved the learners to prove the required theorem which forced the learners to start to define, justify, hypothesis, investigate and conjecture, so that they could see that the rectangles had to be changed to parallelograms.

Lesson **Five**: In the concluding lesson, still working on the second task, learners displayed misconceptions about the vertical height, horizontal height and diagonal length. The teacher dealt with these misconceptions as they occurred so that the learners could move to proving the theorem. Learners nevertheless, concluded that the two conditions for this theorem must hold for the theorem to be true. This was consolidated after more defining, justifying, investigating, testing and conjecturing.

Date	***Duration***	***Topic***	***Concepts Discussed***	***Comments***
Tues 19/09	**5th period.** 30 minutes	**How many diagonals are there in a 700 – sided polygon?**	Conceptual with some strategic competence. Deduce, test, justify, counter example, conjecture. *Sides, diagonals, vertex, pentagon, polygon.*	Introduction – open task – Application of proof. Learners start with difficulty from simple polygons. Used dictionary
Tues 19/09	**7th period.** 30 minutes	**Continuation of lesson:** How many diagonals are there in a 700 –sided polygon?	Conceptual and a bit of procedural. Test, investigate, justify. *Odd & even No. of sides, hexagon, no. of diagonals from one vertex.*	Work in groups. Tests various polygons. Diagonals from 1 vertex. The teacher is aware and assists learners with difficulties.
Wed 20/09	**3rd period.** 30 minutes	**Conclusion of lesson:** How many diagonals are there in a 700 –sided polygon?	Conceptual and a bit of strategic competence. Justify and conjecture.*21-sided polygon, 700-sided polygon, misconception - 100x7-sided polygon = 700- sided polygon*	By using a 24 – sided polygon. Concluded with an algebraic solution for a 700 – sided polygon. The teacher highlights misconceptions.
Thur 21/09	**1st period.** 30 minutes	**4 – Rectangular properties to have access to the road and lake.**	Definition, Formulae. Justify, hypothesis, investigate, conjecture, counter examples. *Parallelogram and rectangle. Parm. Area. = Rectangle Area.*	For the Proof – Two conditions must hold. Scenario - Practical situation change shape. The teacher eventually demonstrates the answer.
Thur 21/09	**2nd period.** 30 minutes	Area of rectangle and parm. between the same two // lines and on the same base are equal.	Definition, Formulae. Justify, hypothesis, test, and conjecture. *Area of parm. = area of rectangle* *Two conditions must hold.*	Teacher deals with misconception – vertical, horizontal and diagonal length to move learners on.

Table 10 - *4.4: An overview of the five lessons' observed*

The above table provides an overview of the five lessons, where the first three lessons focus on one task on polygons and diagonals that deals informally with conjecturing and proofs. The fourth and fifth lessons focus on the second task dealing with the formal proof of quadrilaterals.

I began by making an assessment of what I consider as the teacher's mathematical work, by referring to specific sections involving the teacher's input and the learners' responses, which the teacher guides throughout the lessons. Although the teacher's practice is in the foreground for this study, the learners' responses are important because engaging these is what constitutes the mathematical work that the teacher has to do.

4.4.2. <u>Constitution of the five lessons:</u>

What I am going to present now is a table that summarises the analysis of the lesson of event by event *(see Appendix D for the full table).* I identified: 3 events (notions) and 8 sub-events (sub-notions) in lesson **one.** Most of these entailed working with learners on *justifying* their mathematical thinking, to the teacher and the class as a whole. In lesson **two***,* there were 2 main events (notions) and 10 sub-events (sub-notions), and most of them were focused on *testing* conjectures that were being made. There were 4 events (notions) and 6 sub-events (sub-notions) in lesson **three** and most of them were *conjecturing.* Similarly there were 3 events (notions) and 7 sub-events (sub-notions) in lesson **four** and most of them were *justifying,* 4 events (notions) and 6 sub-events (sub-notions) in lesson **five** and most of them were *conjecturing*.

So what then is different across the lessons and the mathematical work the teacher did? What geometry work did the teacher try to do, in terms of the above explanation of the five strands in relation to the condensed six problem solving types and what kinds of appeals did the teacher make as he worked to have his learners come to know and be able to do the tasks he presented to them.

4.5 <u>Teacher's Intentions:</u>

It has been extensively argued that prior to the 1950's the dominant mode of mathematics teaching was skills performance based or what Kilpatrick et al. (2001) called procedures without understanding. It is also well known and well accepted that the 1950's and 1960's math's movements and the 1980's and 1990's movement of "maths power", emphasises understanding and unifying ideas and computational skills or what Kilpatrick et al. (2001) called procedures with understanding. It is thus interesting that during the pre-interview and the post–interview with the teacher (Mr. Ken) explained that his intention was to develop the learners' mathematical understanding. In the interview extract below it is evident that Mr. Ken wants to develop Kilpatrick et al. (2001) five strands by providing the learner's with a supportive environment.

[Pre - Interview - Question 2]

Line 1 Mr. Ken: *I'm going to try and focus on the understanding of proofs. Why do we have proofs in geometry and what is the purpose? So what I want to get them to do is. I don't want them to see the proof as another problem solving activity or application of solving a problem. Rather that it's a way of doing maths and getting a deeper understanding and communicating that maths to others. What you understand from it. I would like to go into* **conjecturing** *and from having a series of activities that leads them up to* ***why and how to actually prove the theorem of parallelograms****?*

Therefore the teacher's explanation is evidence of what he wants to do with the learners and what the learners have to do. The teacher also makes his intentions clear, about what the tasks must focus on and why, i.e. the tasks must focus on the geometry requirements and the mathematical work of why proofs and what is the purpose of proof? It also shows that the teacher's intentions are aligned to Kilpatrick's et al. (2001) five strands of proficiency. Thus the tasks must make mathematical demands on the learners' level of competence.

[Pre - Interview - Question 3a and 3b]

Line 1: Mr. Ken: *Although it is important to cover the syllabus, I like the task to make the* ***learner want to do it****, not just because the learners have too. I want to teach learners* ***to think mathematically and to do mathematics on their own.***

It is clear that Mr. Ken intends to develop **productive disposition** in order to enhance his learner's mathematical proficiency. Mr. Ken wants to make mathematics worthwhile for his learners so that they enjoy and see the sense of mathematics.

Line 2: Mr. Ken: *I use the materials that I received with the file that I spoke about earlier. I also try to be innovative by using other textbooks and inter-net. I have access to a lot of in-service materials that I sometimes use. Sometimes there is just not enough time.*

Line 3: Mr. Ken: *I'm going to try and focus on the* ***understanding of proofs. Why do we have proofs in geometry and what is the purpose?***

Mr. Ken wants his learners to gain a deeper understanding of mathematics by using innovatively a variety of resources to develop c**onceptual understanding.**

Line 4:* Mr. Ken:** *So what I want to get them to do is. I don't want them to see the proof as another problem solving activity or application of solving a problem. Rather that* ***it's a way of doing maths and getting a deeper understanding and communicating that maths to others.

This shows that Mr. Ken wants to enhance **adaptive reasoning** by nurturing logical thinking in his learners so that they can explain and justify their mathematical reasoning, by applying learned procedures involving a number of steps.

***Line 5:* Mr. Ken:** *What you understand from it. I would like to go into conjecturing and from having a series of activities that leads them up to why and how to actually prove the theorem of parallelograms?*

[Post - Interview - Question 1 and 1.1]

Line 1:* Mr.Ken:** *Basically, I knew that it would be a real challenge for learners to actually do that and in order to solve the problem there would be a lot of maths, before we could solve the problem and the learners would* ***get into the maths and start learning as they try and solve the problem.

Although the teacher is talking about conjecturing, what Mr. Ken is getting at, is that in a similar way learners need to have some kind of procedure to follow in order to do problems of this nature, otherwise it does not make sense. That is what I mean by **procedural fluency** here, in other words learners need to have a set of procedures that is also part of the problem solving. That is, "what do I know, how do I get started, what is a similar case", that is the set of procedures needed to move forward in solving the problem. This was a real challenge for Mr. Ken, because he wanted his learners to carry out appropriate procedures accurately. Because of this blurring, the procedure is not given in these kinds of problems where strategies need to be used and followed, i.e. procedural with or without understanding.

Line 2:* Mr.Ken:** *First of all,* ***they need to know what a polygon is *so* ***we*** *would have* ***to investigate that they get a better understanding of what it is? What makes a polygon?***

Line 2 and line 3 is evidence of how Mr. Ken attempts to develop **strategic competence**. Mr. Ken interchanges the words "they" and "we" throughout the lessons accentuates just how the work of teaching is distributed.

Line 3:* Mr. Ken:** *What is the relationship between diagonals and the sides?* ***So they needed to establish some kind of understanding about how the two relate to each other and then

maybe use that to come up with some possible conjecture *of how to calculate that. I chose 700 sides because I knew that no one would be able to draw sides and then be able to draw in 700 diagonals. That is why I made it difficult I didn't want to choose a smaller number.*

From the above extract Mr. Ken is convinced that the learners will be able to solve the mathematical tasks of this nature by using different representations. Thus, it is evident from Mr. Ken's response that, he is determined to develop his learners to be 'proficient' and critical thinkers of geometry, by developing them holistically (knowledge, skills, abilities, and beliefs) to connect pieces of knowledge by representing it in different ways.

In constructing a suitable framework for analysing data, it is imperative to look at why the teacher chose a particular task to enhance his teaching/learning style during the planning and set up of the tasks and the questioning during the lessons, and what factors were involved in the decisions that the teacher made during the lesson. Mr. Ken, actively involved learners rather than the learners passively listening whenever possible, for example the learners were invited to work at the overhead projector and they were encouraged to teach and learn from each other. During the lesson the teacher monitored the effectiveness of his teaching by finding out about learners' thinking by listening carefully to what they say and by reading what they write or watch what they do in their explanation and drawings. Mr. Ken also gave immediate feedback, which was often in the form of questions rather than explanations, because he looked for presentations that evoke mathematics from his learners, thus enhancing their mathematical proficiency by legitimating meaning for the learners through mathematics. Mr. Ken uses Kilpatrick's et al. (2001) five strands to assess what the learners know and what they need to learn mathematical for example:

[Lesson 1, 08:00 to 09:00]

Conceptual understanding for example ***Thokozani:*** *Sir what I've done sir is? First 700 are too many sides to draw so if there are 4 - sides how will I do that sir? Then I figure that the 4 - sides must be divided by 2. 4/2 = 2 diagonals. So take 700/2 will give you the answer. So that's the answer I got.*

This learner verbalises why he divides by two because he realises that if it works for a smaller polygon (quadrilateral) then it will work for any polygon, thus concluding that there is 350 diagonals in a 700 – sided polygon.

[Lesson 1, 09:00 to 10:00]

Mr.Ken reply of "*Two". So you deduced from that one example that you should divide the 700 by two as well? So you only went as far as a 4 - sided shape? You didn't test anything else"*

The teacher promotes conceptual understanding and pushes the learner to practice **procedural fluency**. But it is also clear that what is happening here is not only fluency but also **adaptive reasoning.** The teacher notices that the learners are relating mathematical concepts by integrating their knowledge to an unfamiliar problem by interpreting and extrapolating the new situation. This shows that Kilpatrick et al. (2001) stands are interrelated in interlinked.

[Lesson 2, 09:00 to 10:00]

Procedural fluency for example*:* ***Mr. Ken:*** *Ok! I see no triangle here guys.*

Lawrence: *Because a triangle doesn't have one. (A diagonal)*

[Lesson 2, 10:00 to 11:00]

Mr. Ken: *Ok! What I suggest before we go that complicated.*

Mr. Ken: *Ok! Start with the simple ones and try and find a solution from that. Ok! What are we saying about that?*

[Lesson 2, 11:00 to 12:00]

Mr. Ken: *How many sides are there?*

Sanele: *Three.*

Mr. Ken: *So what you are doing is, you are investigating it, and you record the data so that you can look back to find solutions.*

Mr. Ken: *So what are you doing is finding patterns. And from those patterns you might come up with the solution.*

Mr. Ken: *What do you get from a 4 – sides.*

Lawrence: *Two.*

[Lesson 2, 12:00 to 13:00]

Mr. Ken: *There might be a pattern. But how do we discover the pattern.*

Mr Ken: *So it's also important to go systematically, from one to the other.*

Mr. Ken: *If you go systematically you can see a pattern. If you 18, 17, 16 it is going to be hard to see a pattern. (Teacher move to the* **ninth group** *and listens to learner's explanation)*

The teacher is leading the learners into using strategies and approaches that would lead to them finding a solution. The learners thus used similar procedures to get the number of sides and diagonals in various polygons that eventually lead them developing a pattern. Mr. Ken is forcing the learners to use well–known procedures that are not obvious from the way the task is set–up. The learners have to decide on the most appropriate procedure to solve for the solution performing one or more procedures.

[Lesson 2, 17:00 to 18:00]

Strategic competence for example ***Esli:*** *Explanation is inaudible. (He uses the diagram that they drew and the pattern that they founded).*

4 – sided has 2 diagonals
5 – sided has 5 diagonals
6 – sided has 9 diagonals
7 – sided has 14 diagonals
8 – sided has 20 diagonals

5 – 2 = 3
9 –5 = 4
14 – 9 = 5
20 – 14 = 6

Mr. Ken: *Ok! Find a pattern that works for you. That works for all the diagrams. How will that help you predict? For example: if I said to you now, in a 10 – sided.*

Mr. Ken: *In a 10 –sided penta, polygon, how many diagonals will there be?*

Some learners develop the skill by demonstrating the solution algebraically while others use geometric diagrams of the various polygons.

Productive disposition is one of Kilpatrick's strands that is not possible to be observed in the lessons presented. Although this was one of the strands that Mr. Ken proposed to develop as seen in the interview. During the lessons the teacher realises that the learner's make sense when they give their reasons how they would determine the number of corners in a 700 – sided polygon but it is not enough evidence for me to say that this strand was developed. Therefore in conclusion, in the context of this research it is not easy to show productive disposition, so I removed this strand completely from the table, although it was my intention to include it.

This analysis of how the teacher uses the five strands is of interest because it shows how the teacher's teaching practices move across almost all five strands when developing mathematical concepts in various polygons – unlike other studies (Kazima and Adler, 2006 and Pillay, 2006), where there appears to be a focus on a few of the strands. This analysis also shows that these conceptual understanding also calls' for the reform in mathematical education.

4.5. Composite Table of Event Chunking

Event Chucking												
	Lessons	L1	L2	L3	L4	L5	L1	L2	L3	L4	L5	
	Total number of events & sub-events	8	10	6	7	6	%	%	%	%	%	Average
Concept (Notions and Sub – Notions)												
	Conceptual	6	10	6	7	6	75	100	100	100	100	95.0
	Procedural	0	4	1	1	2	0	40	17	14	33	20.9
	Strategic Competence	5	2	3	5	2	63	20	50	71	33	47.5
	Adaptive Reasoning	2	6	4	2	5	25	60	67	29	83	52.7
Problem – Solving												
	Defining	2	1	0	5	4	25	10	0	71	67	34.6
	Explaining	1	6	4	7	4	13	60	67	100	67	61.2
	Representing	0	5	5	4	4	0	50	83	57	67	51.4
	Questioning	6	8	4	7	6	75	80	67	100	100	84.3
	Scaling	6	8	4	4	4	75	80	67	57	67	69.1
	Working with Learner Intuitions	6	9	4	4	6	75	90	67	57	100	77.8
Appeals												
Mathematics	Specific	6	4	4	4	5	75	40	67	57	83	64.4
Mathematics	General	2	6	2	3	1	25	60	33	43	17	35.6
Mathematics	Definition	2	1	0	2	3	25	10	0	29	50	22.7
	Curriculum - Exam / Tests	1	1	0	5	4	13	10	0	71	67	32.1
	Experience	1	0	0	1	0	13	0	0	14	0	5.4

***Table 11** - 4.6: Composite Results per Lesson*

This composite quantified table of the five lessons that were observed reveals the mathematical work that this teacher did during his teaching. In relation to the different elements of mathematical proficiency (Kilpatrick et al. 2001), and the different kinds of problem–solving (Ball et al. 2001) a mathematics teacher does, we can see from the table, that the major demands on learners are conceptual, and these correlate with the dominance of questioning by the teacher. These in turn are correlated with a dominance of appeals to mathematics.

It is evident that the concept of *conceptual understanding (95%)* is fixed through problem solving i.e. *questioning (84.3%)* which is used most often by the teacher to legitimate meaning of the mathematical proof, when justifying, testing and conjecturing of polygons, in particular quadrilaterals through *specific mathematical* (64.4%) appeals. The rest of the events and sub– events are very spread, so this teacher actually does work with and across all of these notions, but if we look at what is similar and dominant after conceptual understanding, it is adaptive reasoning at 52.7% that is dominated by the teacher working with the learner's intuitions at 77.8%. The following two concepts of strategic competence at 47.5% and followed by procedural fluency ranking the lowest at 20.9% although both ranked below fifty percentages, it still demands a lot of mathematical work from the teacher.

Looking at the six types of problem solving work that the teacher engagers in there are three types that rank highly that is, questioning at 84.3%, working with learners' intuitions at 77.8% and scaling at 69.1% while explaining, representing and defining followed with 61.2%, 51.7% and 34.6%. Together with all this work, it was important to see how the teacher legitimates this mathematical work. It is evident that the teacher uses a mathematical resource that is more specific (64.4%) in nature followed by general mathematics (35.6%) and the curriculum (32.1%). There is limited appeal to experience (5.4%). Each type of problem–solving is elaborated below, followed by a discussion of the appeals made.

4.7 ***<u>Analysis of the problem solving (mathematical work) by the teacher:</u>***

I will answer the first question of this study: What mathematical work does the teacher do as he teaches geometry to his Grade 10 learners?

a) What mathematical problems or challenges does he encounter?
b) How does he engage with these mathematical problems of teaching?

Mr. Ken's engages all six problem solving tasks as defined in Kazima and Adler, though to different degrees: *questioning* (84.3%), *working with learner intuitions* (77.8%), *scaling* (69.1%), *explaining* (61.2%), *representing* (51.4%) and *defining* (34.6) during the tasks. This is interesting when contrasted with the teacher in Kazima and Adler's study, as well as the teacher in Pillay's study. In each of these latter studies, the teacher only engaged with some of these problem-solving tasks, and this appeared linked to the way mathematics was treated in those pedagogical practices.

From the analysis of Mr. Ken's teaching, is evident that it is not just new topics in teaching practice that increase the teacher's mathematical work, but also the new curriculum and the type of activity or task that is presented to the learners. The teacher's mathematical problems and challenges are identified in the following extracts from different lesson throughout the observation. Different extracts are used because Mr. Ken does not engage in defining, explaining, representing, questioning, scaling and working with learner intuitions at the same time. Therefore each category will be discussed separately.

4.7.1. ***<u>Questioning:</u>***

The most conspicuous category of the teacher's problem solving in all five lessons is questioning. Mr. Ken responds to many of the inputs from his learners with different questions. For example,

[Lesson 1, 09:00 to 10:00]

Line 1: Mr. Ken: *Two. So you deduced from that one example that you should divide the 700 by two as well? So you only went as far as a 4 - sided shape? You didn't test anything else?*

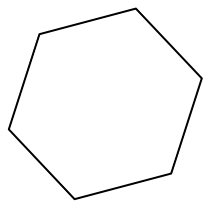
Implicit in this last statement is the idea that one test is insufficient, and through this question he nudges learners to reconsider their response.

[Lesson 2, 03:00 to 04:00]

Line 2: Mr. Ken: *Can you see how many diagonals will this have?*

(Pointing to the hexagon on the transparency)…

Line 3: Mr. Ken: *How many?*



From Mr. Ken's questioning, he urges the learner to move on to solve the problem by looking at a polygon with more sides, but still a small enough number of sides to make sense.

[Lesson 3, 25:00 to 26:00]

Line 4: Mr. Ken: *Okay, just listen to me. What does this answer mean?*

Line 5: Mr. Ken: *What does it actually give you?*

Line 6: Mr. Ken: *What does the quantity for 400 give you?*

Line 7: Mr. Ken: *Does it give you the number of diagonals in a 700 – sided polygon?*

These examples reflect how the teacher is working mathematically with what the learners are producing. He formulates relevant questions according to what specific learners are working with, and so we can see that he is managing a complex and difficult practice. He is constantly engaging with learner's intuitions in such a way that they are encouraged to think further or differently

4.7.2. ***Working with learners intuitions:***

One predominant feature of Mr. Ken's mathematical work was his engagement with what learners produced in the first three lessons. This provides further contrast with the teacher's work as seen in Kazima and Adler (2006), where working with learners' probability intuitions also featured, but less so than here. In order to illuminate the mathematical problem solving (work) that the teacher does during this process of teaching, it is important to look at the three different intuitions of three groups of learners that the teacher had to deal and reflect on what he would need to do, as he worked to move their thinking on towards solving the task.

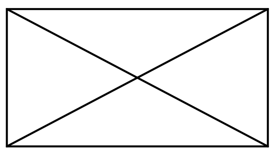
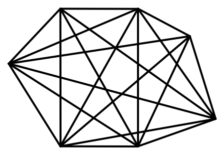
DURRELL's Group	**THOKOZANI's Group**	**LEESHAN's Group**
700 – sided polygon 700 / 2 = 350 diagonals	4 –sided polygon 4 / 2 = 2 diagonals	7 – sided polygon 14 diagonals 14 x 100 = 1400 diagonals
Possible Answer: Verbal response	**Possible Answer:** 	**Possible Answer:** 
Reasoning: Because of sides – corners. 700 / 2 = 350 corners and 175 diagonals	**Reasoning:** Too big a number therefore use s quadrilateral. 4 / 2 = 2 diagonals therefore 700/2	**Reasoning:** 7 – sided polygon has 14 diagonals therefore multiply by 100 which equals 1400
Teacher's Work: Is representation mathematically correct? Reasoning correct? In each case? What questions do I ask?		

Table 12 *- 4.7.2: Three Group Responses*

The above table represents Durrell's, Thokozani's and Leeshan's groups constructed their own initial understanding of and orientation to the problem. Each of the groups explained their interpretation of what is meant, either by verbal or geometrical representations. Durrell's group works with the general problem.

[Lesson 1, 07:00 to 08:00]

Line 1: Durrell: *(Learner continues but in part inaudible) Sir, one of the side's have, like a corner. Yes, explanation inaudible, because of the diagonals. Therefore two of the sides makes like a corner. So I just divided by two.*

Line 2: Mr. Ken: *So you just divide the 700 by 2. And how do you base that on? So how do you base that on because there's 700 – sides? So how many corners will there be if there's, 700 –sides?*

Thokozani's group starts with a specific quadrilateral and moves to generalise the actual problem.

[Lesson 1, 08:00 to 10:00]

Line 3: Thokozani: *Sir what I've done sir is? First 700 is too many sides to draw so if there is 4 - sides how will I do that sir. Then I figure that the 4 - sides must be divided by 2. 4 / 2 = 2 diagonals. So take 700/2 will give you the answer. So that's the answer I got.*

Line 4: Mr. Ken: *So you say that, there's too many sides to draw. If I can just hear you clearly? That 700 sides are too many sides, too big a polygon to draw. Let me get it clear.*

Line 5: Mr. Ken: *So you took a smaller polygon of four sides and draw the diagonals in there. So how many diagonals you get?*

Although Leeshan's group started with a specific 7 – sided polygon, and the assumption therefore that 100 polygons with 7 sides each will have the same number of diagonals as a 700 sided figure.

[Lesson 2, 22:00 to 23:00]

Line 6: Leeshan: *Sir because for seven sides, we did this. So for 700 – sides we multiply by 100.*

Line 7: Mr. Ken: *So you just multiplied by 100.*

Line 8: Mr. Ken: *Okay, that's interesting.*

Line 9: Leeshan: *Is that right.*

Line 10: Mr. Ken: *I don't know. But it's interesting. Why am I multiplying that by 700? So 7 give you 14 and 700 should give you 1400.*

Mr. Ken's response here indicates that he has not considered this as a possible solution and needs to think about it – it is not immediately obvious what might be right about this approach to the problem. Here, quite clearly, we see the mathematical work the teacher has to do as he, on his feet, has to work out whether this approach is a mathematically valid one, and then what next question he could ask learners to reorient their underlying interesting but nevertheless mathematical thinking.

These three different learners' responses reveal that Mr. Ken has to substantial work to do when dealing with the learner's different intuitions, be it verbal, a calculation, a visual demonstration, a discussion and a formal or informal proof.

4.7.3. ***Scaling:***

What is interesting in the above three examples, is that in two groups, learners engaged the problem by starting with something they could grasp (a 4 – sided or 7 – sided polygon). The problem they faced was in their attempt to generalise from that instance. Hence the mathematical work that Mr. Ken has to do also entails scaling the task to the learners' levels. We can see him doing this work below.

A brief period of individual working follows while the teacher monitors the groups working.

[Lesson 2, 04:00 to 05:00]

Line 1: Mr. Ken: *Ok! The problem doesn't seem as simple as it initially appears to be.*

Line 2: Mr. Ken: *Ok, what, I want you to do is, in your groups, investigate. Starting with the triangle and moving up as far as you possible can.*

Line 3: Learners: *Inaudible.*

Line 4: Mr. Ken: *Listen. I want you to move up from a triangle. Drawing the number of diagonals and come up with some understanding of how you can possibly calculate the number of diagonals in a 700 – sided polygon.*

Line 5: Mr. Ken: *And I want you to work as a group.*

Line 6: Mr. Ken: *Ok, let's go ahead.*

…

[Lesson 2, 05:00 to 06:00]

…

Line 3: Mr. Ken: *(Works with* **group six***). A 700 – sided polygon, how many diagonal will that have. In a square there is, two diagonals. Now starting for a triangle draw in the diagonals and start to think and come with a way to work out the problem.*

…

[Lesson 2, 06:00 to 07:00]

Line 1: Mr. Ken: *Will you say that a 700 – sided polygon …? (Inaudible) Take your ideas and put them together.*

Line 2: Mr. Ken: *I want you to take a 5–sided polygon and investigate how many diagonals there are? I suggest that you take the first polygon and progress in that way. And find a way to predict the number of polygons for a 700 –sided polygon.*

Line 3: Mr. Ken: *Many polygons. We need to look at all possible ones and make sure that it works for all polygons.*

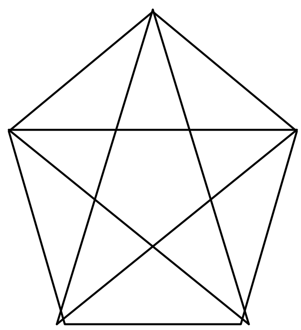
The work continues in this way, short interludes of individual working interlacing with whole-class discussion to give all learners opportunities to contribute their ideas. The teacher is continuously active, using the individual responses to prepare for the next whole–class discussion. The above transcript shows how the teacher is scaling down to smaller polygons and how he works his questions to get learners to think about predicting, and then ensuring how the general solution holds in all cases. The teacher uses scaling as a scaffolding mechanism to move the learners to the next step of conjecturing and testing.

4.7.4. ***Explaining:***

An interesting feature of Mr. Ken's mathematical work is how he works with mathematical explanations as a kind of problem solving. In contrast to the teacher in Pillay's study (2006) where the teacher explains what he wants learners to know and do all the time, Mr. Ken calls on learners to explain their rationale or thinking. Mr. Ken in turn must know and understand these explanations in order to move learner thinking on. He has to gauge whether learner explanations are valid, as well as provide explanations that will lead the discussion on.

[Lesson 1, 23:00 to 24:00]

Line 1: Mr. Ken: *For example (Uses the diagram to show learners) those two vertices will be conservative, they follow each other, either clock wise or anti-clock wise. Depending on how you want to look at it. Okay, so in other words, from there, (pointing to the pentagon that Tebogo drew) how many vertices, I mean how many diagonals can we draw?*



Line 2: Learner: *(Counts quickly.) We can draw five.*

Line 3: Mr. Ken: *You can draw five.*

Line 4: Learners: *Yes.*

The explanation that Mr. Ken offers to the learner does not provide implicit explanation but it takes the lesson a step further, which eventually leads to the explanation of the concept of a counter– example and by asking the learners to compare and form relationships between a pentagon and a hexagon. Although there are procedural explanations by the learners, Mr. Ken's explanations are twofold: first it is to recap what the learners have explained in order to see if they understood the mathematical concepts and ideas and secondly to take the learner a step further (moves the learners on). Mr. Ken's explanations have to be mathematically robust and correct. Therefore the teacher even uses a diagram to illustrate the concept dealing with the notion of diagonals in a pentagon. This is evident that a reform approach was used and not the traditional "talk and chalk" approach because of the continuous scaffolding that takes place.

[Lesson 2, 04:00 to 05:00]

Line 5: ***Mr. Ken:*** *Listen. I want you to move up from a triangle. Drawing the number of diagonals and come up with some understanding of how you can possibly calculate the number of diagonals in a 700 – sided polygon.*

There is further evidence, when most learners ask questions; the teacher's response helps mathematically to move the learners on. The teacher encourages learners to help each other by explaining answers in different ways. This is evident that the teacher's experience promoted the teacher's mathematical work when the homework or worksheet was used in these lessons.

4.7.5. ***Representing:***

This is the second lowest frequently occurring category. It is very interesting to note that during the introduction of both the tasks no or very little representing took place. Towards the end of both tasks a bit more representing takes place. This probably happened because the teacher wanted to give the learners opportunities to approach the solution in ways that they will understand it.

For example:

[Lesson 3, 11:00 to 12:00]

Line 1:Lauren: *Okay, what I first realised. When we Find a pattern between each number, like from 14 to 20 there's 6 (20 – 14) so we think that… (Interrupted by teacher)*

Sides	Diagonals	Vertex
3	*0*	*0*
4	*2*	*1*
5	*5*	*2*
6	*9*	*3*
7	*14*	*4*
8	*20*	*5*

This extract illustrates a learner's representation that Mr. Ken has to work with. Mr. Ken moves the learner from the geometric representation to the numeric representation. Mr. Ken then steers the learners to look for and grapple with a numeric pattern to predict and justify "How many diagonals are there in a 700 – sided polygon".

Although at first the transition of the various groups' understanding is not clear because Mr. Ken seems concerned with concepts and rules but later he moves the groups in the correct direction to proving the 700 – sided problem. This is evident of the problem - solving that this practice requires of the teacher. Thus, Mr. Ken puts Kilpatrick et al. (2001) five strands into practice by moving from the simple to the complex structures according to the learner's developmental levels as the Van Hiele levels show that there is going to be mathematical work that the teacher is going to have to do. Thus when Mr. Ken intervenes and the intervention is modified to the learner's actual needs rather than to the assumed needs of the learners in general in that particular group. From Van Hiele the teacher can see that the task is on level 3 going on to level 4.

4.7.6 ***Defining:***

This category occurs least frequently. Defining mainly occurred at the beginning of the first task and throughout the second task. It is noticed that when a task is introduced and when it relates directly to the curriculum then it requires defining by the teacher.

[Lesson 3, 15:00 to 16:00]

Line 1: Mr. Ken: *Your method will obviously work based on the pattern that we see. If that, pattern continues, now once again we call that a conjecture. It's not true yet.*

Mr. Ken spends some time to drive the concept of conjecturing home. Mr. Ken engages with each group of learners in a discussion to explain in a step-by-step fashion to clarify the groups understanding explicitly to the group before going on to the next task. Mr. Ken also reminded the learners about minor things and previous knowledge like the definition of pentagons and hexagons.

4.8. ***Knowledge Resources to nourish appeals:***

Looking at table 4.6 above, we see that Mr. Ken's problem-solving includes all six types of problem-solving identified by others in the field, though his predominant problem-solving work is questioning, scaling and working with the learners' intuitions. How does the problem - solving he does relate to the kinds of resources he calls in to do this work? As he asks questions, engages with learners' ideas, scales tasks and so on, what resources does he draw on? This is of central interest to this study, and its focus in the second research question: "What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?"

As mentioned before in my initial analysis, it was apparent that Mr. Ken chiefly appealed to mathematical resources as he went about his work, but that suggested this to be too broad category for this study. So the category of mathematics was then divided into sub–categories of: *specific mathematics* that occurred most frequently, *general mathematics* occurred when concluding the tasks and *definitions and rules* was used by to a lesser extent unlike, Pillay (2006) where Nash appealed mostly to *rules and the empirical mathematics*. These sub-categories are discussed in more detail below.

Mr. Ken obtains knowledge about the learner's conceptual learning through analysing and observing what the learners do, as well as by checking the learner's written work. The teacher (Mr. Ken) was able to draw from the functional relationships and mathematical connections by focusing on the learner's actions. Mr. Ken also helped learners to see how mathematics is integrated by applying familiar methods to unfamiliar tasks that learners were required to explain and discuss the meaning of notation and comment on possible derived solutions. The extracts below are the same extracts from the section of the problem–solving (4.8) that is used to show how the teacher appeals to mathematics that is specific and general.

4.8.1 <u>Specific Mathematics:</u>

Looking at the same extracts used under the problem - solving category of questioning in paragraph 4.7.1, these extracts are referring to specific polygons like quadrilaterals, pentagons, hexagons etc. This indicates that the teacher is using specific mathematical case's that requires the learner to select and use that mathematical content that will eventual lead to the general case.

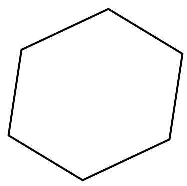
[Lesson 1, 09:00 to 10:00]

Line 1: Mr. Ken: *Two. So you deduced from that one example that you should divide the 700 by two as well? So you only went as far as a <u>4 - sided shape</u>? You didn't test anything else?*

[Lesson 2, 03:00 to 04:00]

Line 2: Mr. Ken: *Can you see how many diagonals will this have?*
(<u>Pointing to the hexagon</u> on the transparency)…

Line 3: Mr. Ken: *How many?*



[Lesson 3, 25:00 to 26:00]

Line 4: Mr. Ken: *Okay, just listen to me. What does this answer mean?*

Line 5: Mr. Ken: *What does it actually give you?*

Line 6: Mr. Ken: *What does <u>the quantity for 400</u> give you?*

4.8.2 <u>General Mathematics:</u>

After the teacher persuaded the learners to find a pattern from the specific methods, he then encouraged the learners to apply the pattern, thus to generalize the pattern in order to make predictions based on the pattern and / or other evidence that will lead to a general solution.

For example:

[Lesson 3, 15:00 to 16:00]

Line 1: Mr. Ken: *Your method will obviously work based on the pattern that we see. If that, pattern continues, now once again we call that a conjecture. It's not true yet.*

4.8.3 Definition and Rules:

The teacher deals with definitions and rules as previous knowledge that has been acquired. Thus the learners know when and how to use appropriate vocabulary, definitions and rules. He does not give the learners the required definitions, but instead insists that they use valid resources like dictionaries, previous notes from their geometry books etc. For example: He thus encourages the learners to read information directly from the dictionary.

[Lesson 1, 15:00 to 19:00]

00: 15 – 00: 16

Line 1: Mr. Ken: *Page 91 everybody*

Line 2: Lauren: *Page 90.*

Line 3: Mr. Ken: *Page 90. (Addressing Kinesh). Why do you think that a polygon and a pentagon is not the same thing?*

…

Line 4: Mr. Ken: *Okay, lets then just go back to defining a polygon. What is a polygon?*

…

00: 17 – 00: 18

…

Line 5: Mr. Ken: *Ok! What does it say in the dictionary? Lets hear*

Line 6: Lawrence: *It says that there is less then five edge inaudible.*

Line 7: Mr. Ken: *Can you go under, look under the definition of polygon? What does it say?*

Line 8: Lawrence: *Reads definition. A polygon is a plane shape completely enclosed by 3 or more, straight lines or edges. Usually edges are not allowed to cross one another. And the word is not often used (Inaudible).*

00:18 – 00: 19

…

Line 9: Mr. Ken: *Does it say it is "not" used or that it is not "often" used?*

The definitions are recalled and read from the dictionary, to indicate to learners what the definitions are so that they can move on to solving the task which is the most important part of the lesson. That is, the teacher did not want to make the appeal to defining as the focus of the actual lesson.

4.8.4. Curriculum:

The teacher's intention is not only to teach for examinations and tests but for the learners to gain mathematical knowledge that will enhance their mathematical proficiency. Mr. Ken uses the curriculum to teach basic knowing and routine mathematical applications that is backed by reasoning and reflecting.

4.8.5. Experience:

Although this teacher, has a wealth of experience in mathematics teaching, as is evident in this case study. Mr. Ken did admit that he does not always teach mathematics in this fashion, although he would like to. Mr. Ken is fully aware that he has to pull together integrated mathematical knowledge from various areas of mathematics to solve these tasks.

4.9. Relations to the Curriculum:

Having discussed the problem-solving the teacher does, and the appeals called on, I can move on and attempt to answer the final question: " How does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?" As mentioned in earlier chapters, it is important to highlight the five interwoven strands of mathematical proficiency that needs to be developed in unison, as described by Kilpatrick et al. (2001). The strands of: c*onceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition* develop over time as the teacher (Mr. Ken) alluded that it is necessary for every learner to learn mathematics. Mr. Ken is fully aware that this section of geometry on polygons, in particular quadrilaterals is important as it is required again in grade 11 and grade 12 curriculum in a different and more complex way.

Even (1990) discusses the relations and inclusion of both procedural knowledge and conceptual knowledge for the nature of mathematics. Shulman (1986) also alluded that teachers need to have a deeper conceptual understanding of the mathematics being taught as opposed to a mere procedural understanding of the mathematical knowledge. It is evident from these lessons that this pedagogic knowledge will ensure that the teacher does not overlook misconceptions, which in turn will not impact negatively but positively on the learning process by rectifying these misconceptions. Mr. Ken then needed to be flexible so as to provide accurate and correct responses to the learners' claims so as to steer them to arriving at the correct conjecture.

Mr. Ken needed to know the possible directions that learners might pursue in future, as well as which directions are worth pursuing (which directions are most likely dead ends) and what would lead to the development of mathematical concepts. In view of this how much of mathematics must a teacher understand or what level of mathematics must the teacher have as to ensure that he/she does not pass wrong judgements. Because of Mr. Ken's experience, he applies his vertical curriculum knowledge he has an overview of the whole curriculum. He knows with what mathematical knowledge his learners came with from previous grades, to what mathematical knowledge they need to acquire at this stage and what they need to know for grade 12 and later. Thus Mr. Ken works with the sense of where the learners fit in and where they coming from and where they have to go. Although it is not possible to discern lateral curriculum knowledge as described by Shulman (1986 and 1987). From the pre–interview I learnt that Ken was one of the three teachers that taught grade 10 mathematics and in this school one teacher designs the prep. Mr. Ken was not responsible for the year 2006 grade 10 preparations that is, why he found the revision lessons as an opportunity to do something different from what was prescribed.

For the teacher to work in this way he had to know mathematics for teaching according to researchers (Ball and Bass, 2000; Ball, Lubienski and Mewborn, 2001; Ball, Bass and Hill 2004; Even, 1990 and Shulman, 1986) who stress the need for teacher's to know in detail the section in mathematics, the topic and the curriculum requirements. This mathematical knowledge for teaching is needed by the teacher to have a sound knowledge on how to locate and address learners' errors. It is evident in the five lessons, that Mr. Ken constantly corrects the learners' errors and assists learners to move forward to understanding mathematics in order to enhance and promoting their mathematics proficiency. Mr. Ken's experience and involvement in mathematics education is used as his main resource to promote mathematical proficiency amongst his learners.

[Pre - Interview]

Line 1: Mr. Ken: *Ok! We got a system at this school, where one teacher, designs for a particular grade so you don't always agree with what the prep is. But because of time constraints and assessments, you follow the prep and do it.*

Line 2: Interviewer: *What you are saying to me is that prior to this year you did not set the grade10 Prep for the previous years.*

At various points during the lessons, I noticed that the learners responded by using rules, but there is no evidence that they understand why these works. The teacher makes long-term decision, as well as immediate decisions during the course of the lessons. It is evident when Mr. Ken allows learners time to deduce a pattern by using a 3–sided polygon and than moving to other polygons so that a pattern can be seen.

Line 3:Mr. Ken: *No, I did not set it. No not at all but what I do is take the prep and edit it, by trying to be innovative.*

In the implementation of the new National Curriculum Statement (NCS), Mr. Ken is aware of the challenges of changing his old ways of teaching to meet the needs of the new curriculum, which he did successfully in these five lessons. Although Mr. Ken has received intervention in mathematics education and he had access to inter-net sources in term of appropriate pedagogic methodology, he nevertheless still uses his old methods of pedagogy during his normal teaching. In contrast to the interview it is interesting to notice that Mr. Ken did not anticipate that he has to go through the different levels and this is evident when he expects the learners to start with the task. In fact the learners are not all at the required level so the teacher thus has to do a lot of mathematical work that may prevent the learners from completing the task in the prescribed time.

If we consider the curriculum demands in the sense that teachers are required to complete the syllabus so that the learners can sit for their examinations. Perhaps the teacher would grapple with this mode of teaching just to consolidate a topic and to give the learners' different perspectives of a topic, but then some times revert to the expository, transmission model of teaching. The methods that Mr. Ken used to manage the lessons shows that he depended on his development of content/subject knowledge and pedagogic knowledge, which Ball and

Bass (2000) sees as a prerequisite for teaching. This content/subject knowledge that Mr. Ken has, allows him to be an innovative manager/facilitator in creating opportunities for learning and to take learners' experiences, interests and needs into account Ball and Bass (2000) thus creating diversity. Thus Mr. Ken extensive experience allowed him to engage learners in the teaching and learning process with ease.

So the mathematical demands and the teacher's mathematical work in support of the new curriculum goals are extensive. In this study, Mr. Ken steps out during these lessons to unfreeze the dynamics and interactive dimensions and transforms the teaching of polygons, in particular quadrilaterals. Mr. Ken takes up the responsibility to set up learning opportunities that provides encouragement and promotes cognitive conflict and draw out mathematical structures that are essential for conceptual development. He certainly engages with the learners' meaning and provides guidance and steers mathematical interactions in fruitful directions. In comparison to Kazima and Adler (2006) and Pillay (2006) to this study, Mr. Ken can do what the new curriculum demands but he does not, do all this mathematical work in his normal teaching. So, all that is needed now is for this type of teaching to filter through the institution.

Chapter 5

Findings and Conclusions: Insights, Discussion and Review

5.1 ***Introduction:***

My aims in this research have been twofold. My first interest was to raise issues about polygons, in particular the teaching on diagonals of polygons at school level and the other on respective areas of parallelograms and rectangles on the same base. There is considerable research on teaching and learning of geometry at school level, with emphasis on task development and learners' thinking. I have focused on the other side of the coin – on the mathematical work that the teacher does. This is necessary to see if objectives for tasks on polygons, in particular diagonals of polygons teaching are being met. Secondly, I want to contribute to the transferable theory that emerges from this research, since research cannot be prescriptive but the explanatory power of a theory is useful. In this regard I have tried to apply and extend some of the important and far reaching ideas of the Van Hiele's (1986) and Kilpatrick et al (2001). In this chapter I show how these aims work in tandem by offering insights on teaching practice by concentrating on the mathematics work that the teacher does in his practice.

5.2 ***Synthesising the study:***

Many important results were found in investigating the critical research questions on mathematics for teaching. That is, the problem solving and resources that the teacher used and how this teacher legitimated mathematics for the learners were guided by:

1) What mathematical work does the teacher do as he teaches polygons, in particular quadrilaterals to his Grade 10 s?
    a. What mathematical problems or challenges does he encounter?
    b. How does he engage with these mathematical problems of teaching?
2) What knowledge resources (mathematical and other) does the teacher call on as he goes about this work?
3) How does this work and the resources called on in this class, relate to new curriculum goals for mathematical proficiency and how can this relationship be explained?

The above research questions were answered in chapters 4. I analysed the ways in which orientations to polygons, in particular quadrilaterals are underpinned by the different level the learners are at and argued that it is likely that the teacher is going to have to do mathematical work, as most learners are likely to be at Van Hiele level 2. In which case the teacher is going to have to shift the learners to level 3 and than to level 4, to meet the demands of the new curriculum and the tasks he set. In fact if I look at Teppo's description of the different phases, what I would be able to see is whether or not the teacher is able to identify the phases the learner are on and to move them on to the next phase, and the work that he did to accomplish this.

In this study, I summarise and illustrate the theoretical ideas that I based on Bernstien's theory. It was in the light of: Van Hiele's (1986) discussion on the level of mathematical development; Kilpatrick's et al. (2001) discussion on mathematics proficiency; the discussion on Stein et al. (1996) and Ainley et al. (2006) task development and Ball et al. (2004) discussion on Mathematics for Teaching (MfT). It was very interesting to see what this teacher did to get the learners to work at the appropriate level. Given the literature I was pleasantly surprised to see how the teacher worked with learners to get them to manage the tasks at level 3. What work did he do to accomplish this, and how does this relate to other studies on teachers' mathematical work such as Kazima and Adler (2006), Pillay (2006) and Davis (2006)? I view the teaching and learning of polygons, in particular quadrilaterals in the light of my theoretical framework. Thus I discuss challenges inherent in this (institutional) pedagogic system.

The section there after was a discussion of the methodology and an exploration of issues arising from them. In the next section on the results, I first summarise some important findings that emerged from my analysis and I review the insights provided by my research methods and explore the limitations of these methods and suggest directions for further research. I looked at some important and broad issues and implications concerning polygons, in particular quadrilaterals in mathematics education in general. How is teaching of polygons, in particular quadrilaterals, constituted?

My findings in this investigation have shown that the teachers' orientations to teaching polygons, in particular quadrilaterals, including their findings of conceptions of polygons and approaches to teaching it, are complex, inter-related and diverse. How teachers position themselves to teach polygons, in particular quadrilaterals are critical to the ways in which they engage with the learners and the learning task and hence affecting the quality of the learner's mathematical knowledge.

This summarises the ideas I have raised throughout this investigation. The tasks are shaped by how this teacher orients himself with respect to the teaching of the task, his goals and the tools and constraints accompanying the tasks. It also involves indirectly the learner actions by them ensuing that outcomes unfold. On one hand, the teacher must be aware of the learner actions relate to the purposes and to the resources available, as well as the constraints of the task. On another hand, though the tasks goals are linked to specific conditions that are connected to outcomes. This teacher achieved success by overcoming mathematical challenges and the demands of the new curriculum. The teacher sees that the task's unfolds through interactions, including verbal communication with other people, and through objects or resources, such as books and internet. Contextual elements are relatively stable aspects of the schooling system, compared to individual actions. Therefore institutionalised practices, such as the writing of examinations at schools, seem to reproduce similar actions and outcomes in an unchanging tradition.

The teaching of polygons, in particular quadrilaterals is complex and a fluid formation that may be transformed by the actions of individual teacher's or by groups of teachers in a classroom situation. It is transformed by new resources, such as the availability of technology, or by changes in departmental or institutional practices or by new government policies. Furthermore this transformation influences other school systems for example, the curriculum, teaching methods and assessment for examination purposes. Depending on the aspect of mathematics that is being taught for example polygons, in particular quadrilaterals, I feel that to introduce such a topic via whole-class discussion per say would not be most suitable to allow the learners to explore and experiment to observe the features of the various polygons. However, for the skill to be developed, so that the learners can apply it successfully in a formal examination, I feel that a drill and practice approach, together with the above mentioned approach would have being more suitable.

*5.3 **Theoretical Perspectives On Teaching***

The theoretical framework I have used in this study, originates from a larger study, the QUANTUM project that emphasises the mathematical work the teacher does, in shaping of the learners proficiency and the teaching practice. Teachers are an important part of this organisational network because they influence the perceptions and actions of the learners. In turn, teachers' perceptions and behaviour are formed by their own experiences, both their past experiences and their current awareness about the learner's needs and the teaching practice. Teachers' actions are constrained by the conditions surrounding them. Hence part of my understanding of teachers teaching relates to interpreting the mathematic tasks (geometry) so that the learners understand as they manage their classrooms.

In teaching polygons, in particular quadrilaterals Mr. Ken used traditional methods at first where he took for granted that learners' should learn certain topics at a specified level using prescribed techniques without questioning the framework within which this learning takes place. The attention, it is assumed, is focused on the content to be learned rather than the learning situation. What was interesting is that when Mr. Ken suggested that he wanted to re–teach this section using reformed methods I was excited to see how the lessons unfold and how the learners adapted. My project draws on the teaching situation, showing that contrary to these suppositions, learners' orientations to learning are mediated by the teacher's findings of the context in the teaching practices in which they take part. Further, the outcome of these findings may not concur with the objective of school education which is, surely, high quality learning. Geometry (Polygons, in particular quadrilaterals) is a compulsory component of the Mathematics curriculum at school level. I as a researcher saw Mr. Ken use polygons, in particular quadrilaterals as a tool for understanding the research of others and to help teachers, to make sense, if they carried out their own research.

Through this study I have a better understanding of the dynamics of the teaching practice of polygons, in particular quadrilaterals and hence this provides a guide for the improvement of pedagogy in mathematics. The teacher creates an appropriate environment to manage and guide a history of learning experiences of individual learners that will influence how the teacher responds to a given learning experience and the learning task that is situated in a school setting, which has its own traditions and practices. For the teacher's guidance to be effective, underlying the content, presentation and evaluation of the resource need to be examined. In particular, the evaluation procedure is a powerful signal from the teacher to the learners. The intention of the teacher is to ensure that deeper interpretations of the resource material is achieved by most learners, but careful scrutiny of the evaluation from the perspective of the teacher is needed to achieve this difficult aim.

The evidence from my observations suggests this teacher's do not value some learners behave as practices in the classroom. The teacher ignored some learner's moaning and groaning, when in fact they were not following the explanations. Learners have diverse needs that form and are transformed by the teacher's perceptions of the learning task and the learners' actions in carrying out these tasks. Furthermore, for some learners, grappling with the task of learning polygons, in particular quadrilaterals led to increased confidence, enhanced insight into mathematical thinking and personal empowerment. While other learners, appeared to be overwhelmingly concerned with mastering concepts, techniques and skills, in order to solve the tasks. Since mathematical tasks are imposed on the learners with little or no room for negotiation, some learner grappled to make new information meaningful and to relate it to their lives. Thus this knowledge furthermore, may not reflect a learner's ability to communicate the knowledge, apply it in different situations or even remember it the following year.

At times some learners seemed disinterested, I am not sure if this happened because of the presence of the video camera in the class. The fact that one is eating, another is sleeping and the other is posing for the camera, makes me wonder if they perceive that, what they are required to do is not mathematical? I am of the opinion that once learners perceive that what they are doing as irrelevant it would result in classroom management problems such as idle talk, high noise levels and unnecessary wandering about to mention but a few. In order to avoid the ethos of classroom from becoming chaotic, I feel that teachers must acquire the knowledge of all three of Shulman's (1986) forms of knowledge (PCK, SMK and CK). From my observation and view it is clearly evident from the video that this teacher has acquired all forms of knowledge, since he has excellent classroom control.

My data has provided many examples showing challenges between the tasks of polygons, in particular quadrilaterals and the teacher's actions as a function of a single institutional setting. On the other side the effect of the historical and institutional context is equally potent. Those currently organising and reforming, or teaching, the polygons, in particular quadrilaterals curriculum have inherited practices from their predecessors. Such practices are not lightly tossed aside, even if they do not fit with personal educational aims. My research shows that these findings are still with us. Teachers may also act in accordance with an academic system that does not always match their personal views, as is evident in the interviews.

There are pertinent challenges. The first challenge is between what teacher perceives as the learners real needs - their current concerns or their goals for their future lives - and how the mathematical knowledge is presented to them in the school context. This challenge lies partly in the gap between educational aims and educational practices. The second challenge is between the high standards of learning that is expected at school level and the current conditions surrounding school education in South Africa. These conditions of dwindling resources and increasing demands on both teachers and learners can lead to expectations of education. Thus these conditions can lead to demands for the economical transmission of information, efficiently delivered and in a form that is easy to absorb. The third challenge is the gap between the complexity and changeable nature of geometry itself in our information rich and technologically advanced era, and mainly traditional tools and resources available for its teaching. In my experience, it is still common for school geometry to rely on the established modes of teaching and tutoring, centred on the teachers and depending heavily on textbooks and technology.

5.4 Discussion

The discussion will focus on three aspects:

5.4.1 Overview Of The Key Results

In this project, my findings suggest that the teacher's motivational levels are associated with more desirable approaches to teaching polygons, in particular quadrilaterals and are reflected in better results of mathematical proficiency. My observation of some learners behaviour shows that it is reasons for their willingness or reluctance to learn were dominated by perceptions of polygons, in particular quadrilaterals as uninteresting, unappealing or difficult. This is consistent with research on polygons, in particular quadrilaterals education showing that many learners are averse to studying polygons, in particular quadrilaterals and have a poor understanding of geometric concepts. Since learners learnt procedures (without connections) or mastering decontextualised concepts by using routine skills and operations to solving problems. For example, the response from Leeshan's group indicates that thinking in "that way" was rote learning.

The patterns that emerged from my studies are consistent with previous research except here Mr. Ken meets the demands of this new curriculum. They may also match the intuition of some teachers. However, caution should be exercised in generalising the results from this particular study. Research is needed to describe the teaching of polygons, in particular quadrilaterals in different contexts. The methods and research tools I used provide wider perspectives on the data. These are discussed in section 5.4.2, below. I will also consider some general issues suggested by the results, in section 5.4.3.

5.4.2 ***Outcomes deduced By the Research Strategies***

In this study, I used an innovative package of research strategies to try and reveal the worlds of the learners. By combining qualitative description, and a bit of quantitative summaries, I had different lenses with which to view the findings. My aims are described below:

- I proposed to explore patterns in the data as a whole as well as to investigate how this teacher manages his mathematical knowledge and experience.
- I intended to view the data systemically, as is consistent with my theoretical framework building on the work of Bernstein (1996) and the Van Hiele’s (1986). For this reason, I interviewed the teacher. This also involved paying attention to the dynamics of the relationship teaching and teaching practice in context. In much research on polygons, in particular quadrilaterals, education in the context is taken as the background - unchanging and uniform - is ignored.
- I aimed to use research methods acceptable to the research community whilst acknowledging the role of my own beliefs, interpretations and values. Hence, my findings represent my interpretations of the data. The analyses of the, research objectives permeate the entire investigation. They motivate the enterprise and effectively shape the evaluation of observations and explanation of facts as perceived. No method of analysis has an internal validity of its own. Its worth is justified by its appropriateness for exploring the data and by the value of the results brought out. The different methods and research tools I used led me to discover different aspects of the data and suggested further hypotheses and analyses.

My findings showed a relationship between the categories of problem solving. This suggested a link between conceptions of polygons, in particular quadrilaterals and approaches to teaching it as a powerful result available through direct observation. Since this result was entirely unexpected, it was unlikely to have been revealed by imposed categories of description or through usual methods of content analysis that are based on the personal knowledge and perspectives of the researcher. Such anomalies challenged my inference of superficial or transparent conclusions, leading me to propose alternative and deeper explanations. Opportunities for finding and validating alternative explanations and for rejecting superficial conclusions, as is important in good research were also provided by the qualitative analyses. Important quantitative data have also been reported in this project, for example, composite table of events and their problem solving categories. Analyses of these data are tools for suggestion and discovery. They have added to my understanding of what mathematical work the teacher does to teach polygons, in particular quadrilaterals and the ways of experiencing it. That is, I consider that the quantitative data complement the qualitative data in forming the hypotheses on which my interpretations and explanations were founded.

I conducted a pre and post interview with the teacher in this research. I tried to establish a rapport with the teacher being interviewed, to interpret correctly what he was saying to me. This could be termed personal conversations. A number of questions arise from my methods. How should one interpret interview data, in view of the above suggestion that not all interviews take place on the same footing? What are reliable data? Is this valid? In short, do we need improved criteria for evaluating how data was chunked and analysed?

The research strategies described in chapter three were useful in exploring the data. The open-ended questions used in interviews allowed the teacher his own awareness of the meaning of teaching polygons, in particular quadrilaterals to be explored. However, limitations of my methods were also discovered. As for any qualitative interpretation, analysis of data was extremely painstaking and time consuming; I sometimes felt I was not only swimming, but also drowning in the data. The teacher was confident about his ability to articulate his thoughts but he had time constraints. These limitations were unavoidable under the conditions in which the research was carried out and with the resources available to me. The tasks with their accompanying goals unfold continuously over time. The lessons observed, however, could only capture perceptions and actions during that time. I could not capture the movement and transformations that undoubtedly took place when the learners completed part of the tasks as homework. I am therefore unable to assess the directions of these transformations.

This project sets the scene for further investigations. What is needed now is a longitudinal study to investigate the mathematical work the teacher does as the teacher figures out what the learners know about this specific geometry section (polygons, in particular quadrilaterals) in the curriculum of mathematics and to determine how to move the learners on, that is, what mathematical work the teacher does (or needs to do). This will impinge on improving education and the teaching practice in fields of mathematics. Research which uses technology, such as video recorders and interactive computer applications to record actions at a large number of different times would have been more effective in capturing the dynamic nature of teaching practice than my studies did. Such methods could also compare what learners say or write about learning polygons, in particular quadrilaterals with what they actually do.

My quantification of data allowed overall patterns to be discerned and provides a basis for comparative studies. This can be more difficult if an entirely narrative style is used to describe data. Also, my quantitative analyses provided support for qualitatively logical relationships. However, qualitative data is essential for conveying the richness of pedagogic practices. My selection of research tools and methods depended on what questions I wanted to answer.

5.4.3 <u>Concerns and Implications</u>

The first issue raised by my results is how policy makers can take account of teachers' personal views. The problem of curriculum design is complex, balancing the needs and expectations of society with the various needs and purposes of individuals. In my experience, school geometry concentrates mainly on content, with little attention to the energising function of a learner activity and its goals. If teachers of mathematics assume that "rational" assessments of their courses will prevail, then they will endeavour to help learners appreciate polygons, in particular quadrilaterals by stressing the usefulness of the subject to their fields. It appears from this study that teachers who do that are preaching to the converted. My findings suggest that those who teach polygons, in particular quadrilaterals as a compulsory component of a mathematical course need support to explicitly address personal evaluations of their teaching.

The second issue concerns the aims of teaching polygons, in particular quadrilaterals in terms of standard techniques for analysing data and/or applications of these, with few interpreting it as a guide to thinking and "mastering oneself". One central value of school education is its power to enlarge the understanding and imagination, to produce a perspective on the particular facts and skills that are learned. Further, the power of a tool lies in its effective use.

The third issue suggested by my investigation pertains to the nature of polygons, in particular quadrilaterals as taught and learned today. It is well known in mathematics education that high - level thinking requires some automation of low level processes.

Finally, an important issue arising from my data concerns the challenge between teachers' responsibilities as managers and teachers. Due to the abstract and difficult nature of polygons, in particular quadrilaterals concepts, the teacher has some hard work to do. The teacher makes sure that he has researched and understands every aspect of the topic. He analyses the underlying concepts and checks that learners can be expected to have the prerequisite knowledge. He ensures that the topic is broken down into logical and sequential portions that the explanations are clear and succinct, that applications of the concepts are illustrated by means of lucid examples and their relations to other concepts are clarified. He indicates where the topic is leading and works at conveying the information in a way that is interesting, manageable and fits into the 35 minute periods.

The learner role in this setting is to write everything down, to nod wisely, and to voice answers to the teacher's questions in a way that corresponds to what is in the teacher's mind. The learner is encouraged to ask appropriate questions, that is, questions indicating to the teacher how well the learner is following the teacher's train of thought. Questions indicating the learner awareness of total incomprehension are not appropriate. They waste time, which could be used to convey more content.

Job satisfaction in this context is high. The teacher is satisfied with the careful, explicit and thorough way he has taught the topic, and relieved to have fitted it into the period. All these activities of the teacher are fitting and understandable in the context of time management. If, however, we look at those same activities, regarding the situation as an educational one, it seems evident that the teacher has gained far more from his activities than the learners from theirs. It is our aim is to educate learners in polygons, in particular quadrilaterals, rather than train them to master techniques

An important implication of my theoretical perspective is that the teacher goals cannot be viewed as isolated and individual but must be understood in the context of institutional or societal forces. Institutions, in turn, are part of communities and cultures that can suffer from: an overloaded curriculum, imposing too much detail at too advanced a level, failing to connect learning with the world of practice and using forms of teaching and assessment that encourage rote learning.

Problem solving in the school setting, explains, why so many learners find the hardest part of a polygons, in particular quadrilateral problem is "trying to understand the question". Learners may be trying to match the question with one encountered before. The teacher must perceive context of tasks and determines how it is construed. While there is no simple answer to the dilemma, an initial step is for teachers to recognise that the learning of polygons, in particular quadrilaterals as a meaningful activity must be negotiated, not assumed.

Mathematics education could equip teachers to apply their polygons, in particular quadrilateral knowledge to new and unfamiliar situations. It could enable them to assess critically the ways in which this knowledge influences how decisions are made in society. It could enable teachers to develop confidence in using information technology appropriately and it could also imbue some teachers with the desire to continue teaching mathematics.

Intellectual development is characterised by an improved capacity for abstract thinking, better methods of learning, and conscious meta-cognitive control. Mathematical activities in this context would be made up not only of those actions aimed at the mastery of knowledge, skills and technical abilities, but also of those directed at enhancing mathematical proficiency such as the ability to reflect, to understand the connections between polygons, in particular quadrilaterals concepts, to see ahead by conjecturing and to generalise. It could also be expected that mathematical activities would result in learners developing their conceptions of what polygons, in particular quadrilaterals are. My data shows that some learners who succeed in overcoming their reluctance to tackle mathematical problems, who conquer long standing difficulties with mathematics or a severe lack of confidence in their abilities to do mathematics, report feelings of achievement.

To make a change the level of knowledge and the level of educational and pedagogical consciousness of the teacher are important factors. For example there is a need for enthusiastic teachers, competent teachers and sufficiently trained teacher. A central assumption of this project is that an analysis of particular actions must take into account issues and dimensions neither apparent in the immediate context nor pertaining to the participants of the studies alone. I have tried to explicate facets of the institutional, cultural and social settings integral to the way each individual's learning developed and progresses.

5.5 <u>CONCLUSION</u>

My investigation raises questions about the central premises behind teaching and learning polygons, in particular quadrilaterals at schools in South Africa. That is, what polygons, in particular quadrilaterals should be taught and how is it, experienced by learners? Mathematics as an exact and elite discipline has led to enormous technological and scientific advances. It is the basis of scientific thinking, which underpins geometry and other methods of interpreting our world. The very objectivity and abstraction of mathematics, however, may be its downfall in mathematics education in an era when the rapid advance of technology is outstripping progress in solving escalating social and environmental problems.

The time has come for us to think about which polygons, in particular quadrilaterals that are appropriate and useful for humans in the information era, rather than continuing teaching the sort of polygons, in particular quadrilaterals which could be done by machines, in environments which constitute the teacher as the expert and regulator of knowledge.

My project has shown that an orientation to learning polygons, in particular quadrilaterals are integral to their activities. In this project I have tried to understand what the teacher's mathematical work is and how the teacher handle the problems that is encountered by drawing on a theoretical perspective which are based on the work of Bernstein (1986), the Van Hiele's (1986), Ball and Bass (2000) and Shulman (2001). Further, polygons, in particular quadrilaterals tasks or activities evolve from a dynamic process of thinking, acting and interacting in contexts that form and reform geometry awareness. The challenge for mathematics teachers is to find a way to communicate to enable learners to view geometry as meaningful and useful knowledge that promotes their development and helps them tackle the complex issues of modern society. In these tasks there is much work to be done.

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Appendix A

Mathematics Education Research Project

Preliminary Interview Schedule

These interviews will be semi – structured. Although there is many question with clear ideas not all the question will be posed, it will depend on the circumstances that prevail.

Educator's Tasks Information: Initial interview (after planning, prior to teaching).

1. Why do you feel that these revision lessons are necessary?
   a. What are you hoping to achieve?

2. Can you describe to me what you are going to do/cover in these revision geometry lessons you have planned.

3. How do you plan to teach these tasks? I see you have selected a range of revision tasks/exercises – I would like to discuss these with you.
   a. How do you choose tasks?
      i. What resources do you use? Textbooks? Mathematics books? Internet? In-service materials?
   b. How and why did you choose these specific tasks as compared to the tasks used earlier? Lets discuss them one by one …
      i. What makes a good task for you for this/these lessons
      ii. What about the mathematics in the task? Has it changed as compared to previous tasks?
      iii. Activity for learners in the task?
   c. Are all tasks at the same level of difficulty? How do you plan for this?

4. How do you plan to teach these tasks?
   a. Will you teach the same tasks to other grade 10 classes?

b. Will you teach using any other teaching approach?

5. What mathematical knowledge do you hope your learners will learn through these lessons and tasks?
   a. Are you able to explain the benefits of the tasks from the learners' point of view?

6. What problems or misconception are you trying to address when planning to teach
   a. How will you overcome them if there are any problems or misconceptions?
   b. What indicators will you use to move to the next concepts?

<u>Educator's Biography Information:</u>

1. What qualifications do you have?

2. How many years have you been teaching mathematics for?

3. What grade have you taught mathematics to?

4. Have you attended any in – service training? Explain?

5. How would you describe your purpose in the mathematics department?

6. How does the mathematics department function in terms of planning, meetings etc.?

7. How does the mathematics department integrate with the other learning areas?

8. What is the ethos of the grade 10 mathematics classes?

Appendix A

Mathematical Education Research Project

Interview Schedule

I would like to thank you for agreeing to be interviewed. This interview will take about 20 to 25 minutes of your time. There are no right or wrong answers. I would like you to relax and feel free to ask me to repeat or explain if you do not understand the questions.

The italics print, are possible answers and each question will lead to possibility more probing questions.

1. Why do you (educator) select specific quadrilateral tasks?
    - o *To find out what learners have learned.*
    - o *To plan learning programmes*
    - o *To make formative decision* (Mention a few decision)
    - o *To make summative decision* (Mention a few decision)

2. Can you (educator) tell me briefly, how you select specific tasks to teach quadrilaterals?
    - *By selecting outcomes*
    - *By collecting reliable and valuable evidence*
    - *Possible answers*
    - *Continuously*

3. How do you know at what level of geometry development the learners are at?

4. When you teach quadrilaterals, what difficulties do you think learners would have?

5. How would you teach to avoid some of these difficulties?

6. **What assessment methods do you use to assess what the learners have learnt from these tasks?**
    - ❑ *Text – based*
    - ❑ *Observation – based*
    - ❑ *Task – based*
    - ❑ *Portfolio*
    - ❑ *Self, peer, and group*

7. What aspects do you look for in a specific task?
    - ➢ *Appropriateness and accuracy*
    - ➢ *Flawless solution*
    - ➢ *Relationships*
    - ➢ *Interpretation of mathematics*
    - ➢ *Insightfulness*
    - ➢ *Pertinent communication using geometric models and comments*

Appendix B

Mathematics Education Research Project

Post Interview Schedule

1. Why did you choose this problem (how many diagonals in a figure with 700 sides) as an introductory problem to the geometry lesson I have observed?

   1.1 What did you hope learners would learn from this particular task?

   1.2 Have you used a task like this before? For what purpose?

   1.3 How did you see this problem linking with the way quadrilaterals are done in the curriculum in Grade 10, and so the geometry grade 10 learners need to know?

2. You spent three lessons on this task. Was this expected?

3. Were you pleased/ satisfied with what you accomplished with the learners in these lessons and on this task? Elaborate?

   3.1 What in your view was positive about the task and lessons?

   3.2 What in your view were the problems you faced as you worked with learners on this task?

   3.3 What was unexpected?

4. In lesson 4 you moved onto a different task – on area – why did you choose this quadrilateral problem in lesson four?

   4.1 What did you hope learners would learn from this particular task / problem?

   4.2 Have you used a problem / task like this before? Foe what purposes?

   4.3 How did you see this problem linking with the way quadrilaterals are done in the curriculum in Grade 10, and so the geometry grade 10 learners need to know?

5. If you think back over all the other lessons and these two major tasks, it would be interesting for me if you could reflect on whether you faced any problems or dilemmas when you were:

   5.1 Planning these revision lessons and the tasks to go into them.

   5.2 Starting the lessons and setting up the tasks for learners to do.

   5.3 As the lessons progressed.

6. What problems/ dilemmas did you think learners confronted as they worked on:

   6.1 The '700' task (remember some of these: 700/2 etc…)

   6.2 How did you try to offer learners assistance with these problems?

   6.3 Again, how did you try to assist learners with their difficulties.

7. If I was a new or inexperienced educator in your school, and asked for assistance with teaching the geometry curriculum for grade 10, what advice would you give me?

8. I know we discussed this a little in the first interview, but could you talk some more, reflecting on these lessons, about what was different from what you normally do in class when you teach geometry.

Appendix C
Letter to Principal

153 Village Green
Denton Street
Ridgeway
2091
31 June 2006

Dear Mrs Nicholia
Mondeor High School
[School's Address]

As a follow up to our telephonic conversation early this year, I write to formally request your consent to participate in a research project. This research project is a necessity to complete my Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. Professor Jill Adler supervises this project. It is hoped that the findings of this research will help educators find ways of dealing with the problems / dilemmas that educators and learners encounter with the concepts and proofs of quadrilaterals.

The aim of project is to investigate what mathematical work done or needs to be done by educators when implementing tasks on quadrilaterals to grade 10 learners to enhance their geometric development. To accomplish this research I will need to interview the grade 10 educator about teaching practices and also observe the educator in the teaching practice. The focus will be on the implementation of tasks on quadrilaterals in geometry.

The lessons will continue as normal and as scheduled according to the educator's timetable. I have identified Mr. Coetzee who has already given me a verbal consent that he is prepared to be a subject in this project. I therefore, humbly request your permission to allow me the opportunity to observe one grade 10 mathematics class as they are being taught this section on quadrilaterals for one or two weeks at most. I also request that I tape –record and make field notes of the educator's interviews and videotape and tape –record the lessons. I will also need to have access to copies of any material/s produced by the educator for teaching this section and the learner's works that is produced through this process. The interviews will be conducted at the educator convenience before and after the implementation of the tasks so as not to interrupt the smooth functioning of the school.

The data collected will be used only for research purposes. The research could be used for reports at conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in al written and verbal reports. After the completion of the project, all data collected will be securely stored at the University of the Witwatersrand for a maximum of five years. The results if this project will be communicated with you upon completion of this project. You may withdraw your consent at any time during the project, without any penalties or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. I f there is anything, within reason, that I can do in return please do not hesitate to inform me.

Looking forward to hearing from you with a favorable response.

Yours sincerely.
Ms. S Naidoo
Researcher
Cell No. 0827049018

Mathematics Education Research Project
Researcher: Ms. S. Naidoo

<u>Consent form for Principal of Mondeor High School in a research project</u>

I, ____ (please print)

Principal of Mondeor High School give consent to the following:

- **Research related to mathematical knowledge for teaching geometry (quadrilaterals) can be conducted at Mondeor High School**
  Yes / No

________________________________ ________________

Signature of Principal **Date**

________________________________ ________________

Signature of Researcher **Date**

Appendix C

Letter to the Head of Department of Mathematics

153 Village Green
Denton Street
Ridgeway
2091
31 June 2006

Dear Mrs K. Naidoo
Mondeor High School
[School's Address]

As a follow up to our telephonic conversation early this year, I write to formally request your consent to participate in a research project. This research project is a necessity to complete my Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. Professor Jill Adler supervises this project. It is hoped that the findings of this research will help educators find ways of dealing with the problems / dilemmas that educators and learners encounter with the concepts and proofs of quadrilaterals.

The aim of project is to investigate what mathematical work done or needs to be done by educators when implementing tasks on quadrilaterals to grade 10 learners to enhance their geometric development. To accomplish this research I will need to interview the grade 10 educator about teaching practices and also observe the educator in the teaching practice. The focus will be on the implementation of tasks on quadrilaterals in geometry.

The lessons will continue as normal and as scheduled according to the educator's timetable. I have identified Mr. Coetzee who has already given me a verbal consent that he is prepared to be a subject in this project. I therefore, humbly request your permission to allow me the opportunity to observe one grade 10 mathematics class as they are being taught this section on quadrilaterals for one or two weeks at most. I also request that I tape –record and make field notes of the educator's interviews and videotape and tape –record the lessons. I will also need to have access to copies of any material/s produced by the educator for teaching this section and the learner's works that is produced through this process. The interviews will be conducted at the educator convenience before and after the implementation of the tasks so as not to interrupt the smooth functioning of the school.

The data collected will be used only for research purposes. The research could be used for reports at conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in al written and verbal reports. After the completion of the project, all data collected will be securely stored at the University of the Witwatersrand for a maximum of five years. The results if this project will be communicated with you upon completion of this project. You may withdraw your consent at any time during the project, without any penalties or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. I f there is anything, within reason, that I can do in return please do not hesitate to inform me.

Looking forward to hearing from you with a favorable response.

Yours sincerely.
Ms. S Naidoo
Researcher
Cell No. 0827049018

Mathematics Education Research Project

Researcher: Ms. S. Naidoo

<u>Consent form for Head of Department in a research project</u>

I, ____ (please print)

Head of Department of Mathematics at Mondeor High School give consent to the following:

- **Research related to mathematical knowledge for teaching geometry (quadrilaterals) can be conducted at Mondeor High School**
  Yes / No

______________________________ ______________

Signature Head of Department **Date**

______________________________ ______________

Signature of Researcher **Date**

Appendix C
Letter to School Governing Body of Mondeor High School

153 Village Green
Denton Street
Ridgeway
2091
31 June 2006

Dear Chairman
Mondeor High School
[School's Address]

As a follow up to our telephonic conversation early this year, I write to formally request your consent to participate in a research project. This research project is a necessity to complete my Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. Professor Jill Adler supervises this project. It is hoped that the findings of this research will help educators find ways of dealing with the problems / dilemmas that educators and learners encounter with the concepts and proofs of quadrilaterals.

The aim of project is to investigate what mathematical work done or needs to be done by educators when implementing tasks on quadrilaterals to grade 10 learners to enhance their geometric development. To accomplish this research I will need to interview the grade 10 educator about teaching practices and also observe the educator in the teaching practice. The focus will be on the implementation of tasks on quadrilaterals in geometry.

The lessons will continue as normal and as scheduled according to the educator's timetable. I have identified Mr. Coetzee who has already given me a verbal consent that he is prepared to be a subject in this project. I therefore, humbly request your permission to allow me the opportunity to observe one grade 10 mathematics class as they are being taught this section on quadrilaterals for one or two weeks at most. I also request that I tape –record and make field notes of the educator's interviews and videotape and tape –record the lessons. I will also need to have access to copies of any material/s produced by the educator for teaching this section and the learner's works that is produced through this process. The interviews will be conducted at the educator convenience before and after the implementation of the tasks so as not to interrupt the smooth functioning of the school.

The data collected will be used only for research purposes. The research could be used for reports at conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in al written and verbal reports. After the completion of the project, all data collected will be securely stored at the University of the Witwatersrand for a maximum of five years. The results if this project will be communicated with you upon completion of this project. You may withdraw your consent at any time during the project, without any penalties or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. I f there is anything, within reason, that I can do in return please do not hesitate to inform me.

Looking forward to hearing from you with a favorable response.

Yours sincerely.
Ms. S Naidoo
Researcher
Cell No. 0827049018

Mathematics Education Research Project

Researcher: Ms. S. Naidoo

<u>Consent form for School Governing Body in a research project</u>

I, ____ (please print)

School Governing Body of Mondeor High School give consent to the following:

- **Research related to mathematical knowledge for teaching geometry (quadrilaterals) can be conducted at Mondeor High School**
  Yes / No

________________________________ ________________

Signature of School Governing Body **Date**

________________________________ ________________

Signature of Researcher **Date**

Appendix C
Letter to Teacher

153 Village Green
Denton Street
Ridgeway
2091
31 June 2006

Dear Mr. K Coetzee
Mondeor High School
[School address]

As a follow up to our telephonic conversation early this year, I write to formally request your consent to participate in a research project. This research project is a necessity to complete my Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. Professor Jill Adler supervises this project. It is hoped that the findings of this research will help educators find ways of dealing with the problems or dilemmas that educators and learners encounter with the concepts and proofs of quadrilaterals.

The aim of project is to investigate what mathematical work done or needs to be done by educators when implementing tasks on quadrilaterals to grade 10 learners to enhance their geometric development. To accomplish this research I will need to interview you as the grade 10 educators, about teaching practices and also observe you in the teaching practice. The focus will be on the implementing on quadrilaterals tasks in geometry.

The lessons will continue as normal and as scheduled according to your timetable. I therefore, humbly request your permission to allow me the opportunity to:

- Observe one grade 10 mathematics class as they are being taught this section on quadrilaterals for one or two weeks at most.
- I request that I tape –record and make field notes of the interviews
- I also request to videotape and tape –record the lessons.
- I will also need to have access to copies of any material/s produced by you for teaching this section and the learner's works that is produced through this process.

The interviews will be conducted at your convenience before and after the implementation of the tasks so as not to interrupt the smooth functioning of the school.

The data collected will be used only for research purposes. The research could be used for reports at conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports. After the completion of the project, all data collected will be securely stored at the University of the Witwatersrand for a maximum of five years. The results if this project will be communicated with you upon completion of this project. You may withdraw your consent at any time during the project, without any penalties or prejudice.

Kindly complete the attached form and return it to me at your earliest convenience. I will be happy to answer any questions or queries that you might have. If there is anything, within reason, that I can do in return please do not hesitate to inform me. Looking forward to hearing from you with a favorable response.

Yours sincerely.
Ms. S Naidoo (Cell No. 0827049018)
Researcher

Mathematics Education Research Project

Researcher: Ms. S. Naidoo

<u>Consent form for educator to participate in a research project</u>

I, ____ (please print)

Educator at Mondeor High School gives consent to the following:

- **Research related to mathematical knowledge for teaching quadrilaterals can be conducted at Mondeor High School**
  Yes / No

- **The possible future use of the written tasks for research purposes.**
  Yes / No

- **Copies made of class work or homework of tasks that my child might produce as part of these observed lessons.**
  Yes / No

- **Being video tape-recorded and taped-recorded during the observation lessons.**
  Yes / No

______________________________ ______________

Signature of Educator **Date**

______________________________ ______________

Signature of Researcher **Date**

Appendix C

Mathematics Education Research Project

Letter to Parent/Guardian and Learner

31 May 2006

Dear Parent/Guardian, Grade 10 Learner
Mondeor High School

Greetings from Ms. S. Naidoo. I am currently studying for a Masters of Science degree in Mathematics Education at the University of the Witwatersrand in Johannesburg. Professor Jill Adler supervises this project. As part of my thesis, I am doing research to investigate what mathematical work is done or needs to be done by educators when teaching quadrilaterals in grade 10 to enhance learners geometric thinking / development. It is hoped that the findings of this research will help educators find ways of dealing with the problems / dilemmas that educators and learners encounter with the concepts and proofs of quadrilaterals.

The grade 10 Mathematics educator and the mathematics Head of Department, the Principal and School Governing Body have given me permission to send you, the parents/guardian this letter to invite your child to participate in this research project. Once you have read this letter you can decide whether you want your child to participate in this research project or not. The parent/guardian must give their child permission to participate or not, because your child is not of age to make this decision on their own. Remember that your child does not have to take part unless you want them to or if you want your child to take part. Please complete the attached form and sign in the space provided and make your child sign as well and return it to the mathematics educator. If you have any questions about this research project you could contact me at the school.

If you agree for your child to participate in this research project, your child will need to be present during the lesson observations when the educators will administer the class tasks. The purpose of these tasks is not for marks but for research. The date for this will be negotiated with your child's educator. With your permission the research will involve:

- Observations of a set of lessons will be video tape-recorded and tape –recorded.
- Learners' work will be collected.

Your child's anonymity and the confidentiality will be protected.

Looking forward to hearing from you.

Yours sincerely.
Ms. S Naidoo
Researcher
Cell No. 0827049018

Mathematics Education Research Project

Researcher: Ms. S. Naidoo

<u>Consent form for Parents/ Guardian and learners participating in a research project</u>

Name of learner: ____

Name of Parent/guardian: ____

I give consent to the following:

(Please circle your responses)

- **Copies made of class work or homework of tasks that your child might produce as part of these observed lessons.**
  Yes / No

- **Being video recorded and taped-recorded during the observation lessons.**
  Yes / No

_____________________________ _______________

Signature of learner **Date**

_____________________________ _______________

Signature of Parent/Guardian **Date**

_____________________________ _______________

Signature of Researcher **Date**

Appendix D

LESSON ONE

Events	***Time***	***Concept***						***Problem Solving***						***Appeal***					***Comment***
														Maths					
		Notion	**Sub-Notion**	***Conceptual***	***Procedural***	***Strategic***	***Adaptive***	***Defining***	***Explaining***	***Representing***	***Questioning***	***Scaling***	***Learner Intuit.***	***Specific***	***General***	***Definition/ Rule***	***Curriculum***	***Experience***	
Event 1	00:00 to 06:50	Proof in relation to polygons & diagonals	Interpretation			√							√		√			√	Open instruction. Solves problem on own
	07:08 to 08:38		Justify - specific	√		√					√		√	√					1st Attempt (700/2 = 350 diagonals)
	08:40 to 09:48		Justify -Deduce and Test	√		√	√				√	√	√	√					2nd Attempt (4/2 = 2 diagonals)
	09:52 to 10:40		Justify					√				√	√	√		√			3rd Attempt (7 sided polygon x 100 = 700) 14x 100 = 1400 diag.
	10:40 to 14:00		Justify	√							√	√	√		√				Because of justifying - agrees.
Event 2	14:00 to 15:20	Define polygon & Pentagon	Justify - Explaining	√							√	√		√					Confusion between polygon & pentagon
	15:23 to 22:00		Define polygon & pentagon	√		√		√			√	√		√		√	√		Help from dictionary
Event 3	22:32 to 25:00	Conjecture of diagonals of pentagon	Justify - counter example	√		√	√		√		√	√	√	√					Eg. Pentagon - demonstration of diagonals from 1 vertex and all vertices
			Total	6	0	5	2	2	1	0	6	6	6	6	2	2	1	1	

LESSON TWO

Events	Time	Notion	Sub-Notion	Concept: Conceptual	Concept: Procedural	Concept: Strategic	Problem Solving: Adaptive	Problem Solving: Defining	Problem Solving: Explaining	Problem Solving: Representing	Problem Solving: Questioning	Problem Solving: Scaling	Problem Solving: Learner Intuit.	Appeal: Maths: Specific	Appeal: Maths: General	Appeal: Maths: Definition/Rule	Appeal: Curriculum	Appeal: Experience	Comment
Event 1	00:00 to 04:00	Con - jecture		√						√	√		√	√		√	√		Odd & even number of sides / 2. Want to test
Event 2	05:00 to 07:54	Conjecture of 700 - sided polygon	Investigate Pentagon & Hexagon	√		√		√	√	√	√	√	√	√					Group 6: $(\text{Sides} / 2)^2$
	07:56 to 09:44		Testing various polygons	√					√			√			√				Group 4: Finding patterns and record data
	09:48 to 12:30		Testing various polygons	√	√		√		√	√	√	√	√		√				Group 8: Not working systematically
	12:35 to 14:22		Testing Algebraically	√	√		√		√		√	√	√		√				Group 9: Solves algebraically
	14:32 to 16:59		Testing diagonals from 1 vertex	√	√	√	√			√	√	√	√	√					Group 2: Testing various poly - gons: Diagonals from 1 vertex
	17:05 to 18:49		Testing Algebraically	√	√		√		√		√	√	√		√				Group 3: Solves algebraically
	18:54 to 20:58		Testing various polygons	√			√			√	√	√	√		√				Group 5: Not working systematically
	21:56 to 23:15		Justifying	√					√				√	√					Group 7: Testing not very successful
	23:20 to 25:00		Justify Method	√			√				√	√	√		√				Group 1: Testing not very successful
			Total	10	4	2	6	1	6	5	8	8	9	4	6	1	1	0	

LESSON THREE

Events	Time	Notion	Sub-Notion	Concept				Problem Solving						Appeal					Comments
														Maths					
				Conceptual	Procedural	Strategic	Adaptive	Defining	Explaining	Representing	Questioning	Scaling	Learner Intuit.	Specific	General	Definition/Rule	Curriculum	Experience	
Event 1	03:00 to 04:50	Justifying		√							√				√				Looking for a pattern and why always two. Learners reasoning
Event 2	07:07 to 11:11	Conjecturing and justifying	Conjecturing: Empirical Case	√		√	√		√	√	√	√	√	√					Feedback: Various polygons. No.of diagonals from 1 vertex. E.G. 24 - sided polygon.
	11:30 to 15:41		Conjecturing: Proving patterns	√	√		√			√		√	√	√					Algebraic pattern
	16:17 to 20:35		Conjecturing systematically	√		√			√	√	√		√		√				Providing 700 - sided polygon
Event 3	21:41 to 23:00	Justifying. Half the diagonals	Justifying. Dividing by 2	√			√		√	√		√	√	√					Number of diagonals from 1 vertex. Non consecutive
Event 4	24:00 to 29:00	Justifying. Using 7 & 9 - sided.		√		√	√		√	√	√			√					Educator convinces 2 learners - they wrong.
			Total	6	1	3	4	0	4	5	4	4	4	4	2	0	0	0	

LESSON FOUR

Events	***Time***	***Concept***						***Problem Solving***								***Appeal***			***Comment***
		Notion	***Sub-Notion***	***Conceptual***	***Procedural***	***Strategic***	***Adaptive***	***Defining***	***Explaining***	***Representing***	***Questioning***	***Scaling***	***Learner Intuit.***	***Specific***	***General***	***Definition/Rule***	***Curriculum***	***Experience***	
Event 1	00:58 to 01:58	Informal: Proof, justify, hypothesis, investigate and conjecture.	Justifying and hypothesis	√				√	√		√				√		√		Tracing procedure by using correct maths terms. Tries to define hypothesis.
	02:04 to 03:25		Justifying + hypothesis	√		√		√	√		√	√	√	√					Attempts to define conjecture.
	03:30 to 05:28		Justifying and conjecture as proof	√		√		√	√		√	√	√		√	R √			Attempts to define conjecture as a proof.
	05:40 to 07:00		Justifying and proving	√		√	√		√	√	√		√		√		√		Forming a maths community.
Event 2	07:20	Justifying and conjecturing: Rect. & Parm	Hypothesising:	√					√	√	√	√	√	√			√	√	Practical situation to investigate formal
	10:40 to 18:55		Conjecturing	√		√	√	√	√	√	√			√			√		Algebraic pattern
Event 3	19:20 to 28::00	Justifying and conjecturing: Rect. & Parm	Two conditions must hold: same base & bet. Same // lines	√	√	√		√	√	√	√	√		√		√	√		Two conditions must hold: same base & between Same // lines
			Total	7	1	5	2	5	7	4	7	4	4	4	3	2	5	0	

LESSON FIVE

Events	Time	Notion	Sub-Notion	Conceptual	Procedural	Strategic	Adaptive	Defining	Explaining	Representing	Questioning	Scaling	Learner Intuit.	Specific	General	Definition/Rule	Curriculum	Experience	Comment
		Concept						***Problem Solving***						***Appeal***					
														Mathematics					
Event 1	03:30 to 06:49	Justifying by testing em - pirical case	Conjecturing: Empirical Case	√	√				√	√	√		√	√					Find a counter example and then move on.
	06:52 to 12:00		Conjecturing: Empir. Case.	√			√	√	√	√	√	√	√	√			√		Trying different examples: different length, base and slant.
Event 2	14:00 to 20:00	Hypo - thesis	Vertical, horizontal & diagonals lengths	√		√	√	√	√	√	√	√	√	√		R √	√		Use square to rectify length concept
Event 3	20:02 to 22:22	Conjecturing - using two conditions	Apply both conditions	√			√	√			√	√	√	√		R √	√		Justification that both conditions hold to be true. Both conditions was used.
	22:24 to 24:52		Apply one condition to	√	√		√	√			√	√	√	√		R √			Highlights that only one conditions was used
Event 4	25:00 to 30:00	Justifying & Con - jecturing	Equal area	√		√	√		√	√	√		√		√		√		Conclusion of property for rect. & parm.
			Total	6	2	2	5	4	4	4	6	4	6	5	1	3	4	0	

Appendix D

Composite table of Event Chunking

Lesson Number		**Total Occurences**					**% of Occurences**					
		1	**2**	**3**	**4**	**5**						
Notions and Sub - Notions		8	10	6	7	6	**1**	**2**	**3**	**4**	**5**	**Averages**
Notions and Sub - Notions Concept												
Conceptual		6	10	6	7	6	75	100	100	100	100	95.0
Procedural		0	4	1	1	2	0	40	17	14	33	20.9
Strategic Componence		5	2	3	5	2	63	20	50	71	33	47.5
Adaptive Reasoning		2	6	4	2	5	25	60	67	29	83	52.7
Problem - Solving												
Defining		2	1	0	5	4	25	10	0	71	67	34.6
Explaining		1	6	4	7	4	13	60	67	100	67	61.2
Representing		0	5	5	4	4	0	50	83	57	67	51.4
Questioning		6	8	4	7	6	75	80	67	100	100	84.3
Scaling		6	8	4	4	4	75	80	67	57	67	69.1
Working with Learner Intuition		6	9	4	4	6	75	90	67	57	100	77.8
Appeals												
Mathematics	Specific	6	4	4	4	5	75	40	67	57	83	64.4
	General	2	6	2	3	1	25	60	33	43	17	35.6
	Definition	2	1	0	2	3	25	10	0	29	50	22.7
Curriculum - Exam / Tests		1	1	0	5	4	13	10	0	71	67	32.1
Experience		1	0	0	1	0	13	0	0	14	0	5.4

Appendix E

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Date:	08 August 2006
Name of Researcher:	Naidoo Shiela
Address of Researcher:	153 Village Green
	Denton Street
	Ridgeway 2091
Telephone Number:	(011) 9072656
Fax Number:	(011) 9078364
Research Topic:	Mathematical knowledge for teaching geometry to grade 10 learners
Number and type of schools:	1 Secondary School
District/s/HO	Johannesburg South

Re: Approval In Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Senior Manager – Strategic Policy Research & Development
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ALBERT CHANEE
ACTING DIVISIONAL MANAGER: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	28/08/2006

Appendix F

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 • Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3007 • Fax: +27 11 717-3009 • E-mail: enquiries@educ.wits.ac.za • Website: www.wits.ac.za

STUDENT NUMBER: 0316764X
Protocol number: 2006ECE23

Mrs Shiela Naidoo
153 Village Green
Denton Street
RIDGEWAY X 4
2091

19 December 2007

Dear Mrs Naidoo

Application for Ethics Clearance: Master of Science

I have pleasure of advising you that the Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the senate has agreed to approve your application for ethics clearance submitted for your proposal entitled:

Mathematical knowledge for Teaching Geometry to Grade ten Learners

Recommendation:

Ethics clearance is granted

Yours sincerely

Matsie Mabeta
Wits School of Education

Cc: Supervisor Prof. J Adler (via email)