An Application of Goal Programming Model to a Bank Balance Sheet Management

Denise D M Chiwandire

Supervisor: Dr. Blessing Mudavanhu

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University of Witwatersrand

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Supervisor: Signature:

DECLARATION

I, Denise D M Chiwandire declare that the research work reported in this dissertation is my own, except where otherwise indicated and acknowledged. It is submitted for the degree of Masters in Management in Finance and Investment at the University of Witwatersrand, Johannesburg, South Africa. This thesis has not, either in whole or part, been submitted for a degree or diploma to any other universities.

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Abstract

The research presented the application of a goal programming model to the balance sheet management of South African Banks in segmented markets based on multiple goals such as asset accumulation, total liability, shareholders wealth, profitability, earnings and total goal achievement. The significance of the study can measure the effectiveness of policy decisions and plan future investments effectively. The model determined the optimal structure of the balance sheet based on the fulfilment of strategic objectives. Data collected from annual financial reports covered the period 2011 to 2019. The solutions showed the achievement of all goals and possible improvements in target values for optimization.

Key-words: Goal programming, assets, liability, balance sheet management

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1.0 Introduction

Financial intermediaries in particular commercial banks provide financial services to households and business entities through their role of channelling funds from savers to borrowers, Giokas and Vassiloglou (1991). The ease in which credit can be supplied to the economy is managed through the balance sheets as highlighted by Adrian and Shin (2014). Balance sheet management[1] efficiency requires determining the present and future changes to the structure of the balance sheet, Kumar et al (1988). The careful management of the balance sheet is key to the success of any financial intermediary.

Over time financial intermediaries, in particular banks have faced ambiguity of cash flows, costs of funds and return on investment, Kusy and Ziemba (1983). An example which was explained by Arewa et al (2013) was during the mid-nineteenth century banks had excess funds in the form of demand and savings deposits. Banks adopted asset management as a strategy to efficiently keep track of excess funds, Owoputi et al (2013). Over time the deposits began to decline and liabilities started to increase forcing banks to give more attention to liability management, Arewa et al (2013). The turn of the twentieth century then saw the publicized need for the assets and liabilities in the banks' balance sheet to be monitored efficiently, hereafter bank asset and liability (balance sheet) management concept was established, Torbira et al (2013).

Kumar et al (1988) explained how the raw materials of a bank are loanable funds which consist of money in the economy that willing to be saved and lent out as an investment rather than for consumption. The authors further explained how the balance sheet is composed of assets which include cash, bonds, securities against contractual obligations on the liabilities side, Kumar et al (1988).

Balance sheet management can increase efficiency and uncover unrealised potential in underutilised assets, CIPFA (2017). In the case of banks, funds need to be raised and deployed in such a way that keeps risks at an optimal level while maximising return and ensuring shareholders wealth is capitalized, CAFRAL (2012). The ever-changing external environment has heightened the intensity of unforeseen events which banks need to be

[1] Balance Sheet Management coordinates assets, liabilities and equity of an organization order to reduce the risk of loss.

vigilant of and continuously need to monitor to reduce exposure to liquidity risk and meeting other set goals, CAFRAL (2012).

Decision-makers can formulate, implement, monitor and evaluate strategies related to the bank's balance sheet to achieve set out goals and set risk acceptance levels, Corsaro et al, (2010). Such strategic goals included guaranteeing the bank remains a going concern, is competitive in the current and foreseeable future and maximising returns as explained by Guven and Persentili (1997).

Bank management constructs a number of objectives which may conflict with each other such as maximise profitability, minimise risk, maximise shareholders wealth and liquidity adequacy as suggested by Halim et al (2015). In order to accomplish these objectives, management is faced with a dynamic environment which is regarded as uncertain, potentially biased and information asymmetry rendering it difficult for decision makers to formulate reliable solutions, Karin et al (2013).

This facilitated the need for researches to study and find models which take into account simultaneous objectives and the dynamic environment to find the best outcome from viable alternatives, Aouni and Kettani (2001). This led to the formulation of multi-criteria decision making (MCDM[2]) framework. In the MCDM field, Goal Programming[3] (GP) is seen as a unique problem-solving methodology, Schniederjans (1995) on how paramount banks composition of assets and liabilities needed to be optimized to trade off risk, return and liquidity. Brodt (1978) highlighted that the model can also measure the cost/benefits of changing policies, losing deposits, changing equity and the maximum interest rate to pay for time deposits.

GP is the most widely used analytical technique which has great flexibility for decision-makers to provide solutions to problems incorporating multiple, pre-specified objectives and goals, Neelavathi (2015). The method is a technique to reduce deviations as well as priority deviations as much as possible from the set target, Dan E and Onuoha D (2013).

The framework is expressed in linear form to show the optimum solution to multi-dimensional objective function with a given set of limitations, Neelavathi (2015).

[2] A subset of operational research focused on solving multiple conflicting goals.

[3] Goal programming is an improved mathematical method used to solve multi objective problems.

Kosmidiu and Zopounidis (2004) highlighted that Pareto efficiency in goal programming is optimised when no other outcome can achieve better results for one or more objectives in the model.

In this research, the GP model will determine optimal structures of asset accumulation, total liability, shareholders wealth, profitability, earnings and total goal achievement for a wholesale market segment[4] bank, ABSA Bank and a retail market segment[5] bank, Capitec Bank in South Africa.

1.1 Research Problem

In today's competitive environment, management in banks is facing complex problems which require them to achieve various conflicting objectives such as maximizing profits, minimizing risk and maximising shareholders wealth. This highlighted the importance of understanding the motivation, mechanics and cost of financial intermediary balance sheet management in the financial sector, Tobias and Song (2011). The Multi-Criteria Decision Making Techniques (MCDM)[6] has a unique problem-solving methodology called GP which can be adopted to manage conflicting objectives set by bank management, Romero (1991).

Decision-makers are faced with multiple conflicting objectives being a mixture of operational and personal preferences. The ability to achieve these objectives is constrained by environmental conditions within which the decision maker operates. Therefore, the research problem is to find the optimal size and composition of assets and liabilities of a wholesale market segment bank and a retail market segment bank from a set of goals that assists decision-makers in strategic planning and combating the dynamic business environment.

[4] Whole sale market segment banks include corporate banking , foreign exchange and rates

[5] Retail market segment banks include electronic banking , traditional retail banking and personal banking

[6] Is a discipline which involves evaluating complex decision making scenarios with often conflicting objectives that relevant stakeholders value differently.

1.2 Research Objectives

The study was to prioritize and achieve the main set of objectives formulated by decision-makers in the two segmented banks in South Africa. The mathematical model was designed and formulated using Goal programming (GP) to optimize the assets and liabilities in the balance sheet in order for the set objectives to be accomplished.

Other objectives were abridged as follows:

a) To study the effect of internal and external determinates that affect the structure and composition of the balance sheet.
b) To use GP, a field in Multi-Criteria Decision Making Techniques (MCDM)
c) To make decision-maker formulate reliable solutions in the ever-changing business environment.

1.3 Research Importance

The below points highlight the importance of the research model:

a) Optimizing a bank's balance sheet using quantitative models.
b) Bank management can test and measure the effectiveness of policy decisions.
c) Plan future investments effectively.
d) Define the size and quality of assets and liabilities.

1.4 Methodology

Historical data was obtained from the annual financial reports of a wholesale bank balance sheet from 2011 until 2018 and from a retail bank from 2012 to 2019. This set of data was used to create a model that optimized the balance sheets for financial management efficiency. The different years used were due to the banks having different accounting periods but all covered a period of seven years. The analysis included using GP to solve multiple objectives and simultaneously using LINGO software to give a suitable balance sheet structure for management of assets and liabilities.

1.5 Outline of the Study

The research paper is structured as follows; the second chapter outlines the literature review, highlighting key researchers who developed and practically applied goal programming models for balance sheet management. The Goal programming process is outlined in chapter three, showing the basic concepts and formulation of the model. The methodology in chapter four shows the data collected and analysis done for goal programming using LINGO software. Results and analysis from the model are presented in the fifth chapter leading to conclusions in the final sixth chapter of how goal programming can be used to manage a banks' balance sheet.

2.0 Literature Review

In this chapter, we will look at research papers on the foundation and elucidation of a balance sheet in union with goal programming for banks. The mathematical deterministic and stochastic models formulated by researchers related to bank balance sheet management. Emphasis will be made on deterministic models with a glance at stochastic models. Examples will be shown of applications of goal programming in bank balance sheet management in different countries over the years.

2.1 Theoretical Framework

The use of models and tools to manage assets and liabilities enables management in banks to make informed business decisions in order to meet desired goals as explained by Chaturvedi (2014) in the paper, "Asset and Liability Management in Banks." The bank's management is guided by the board's risk philosophy to monitor the market risk profile and choose a suitable course of action given the impact of alternative business decisions that can affect the risk profile and profitability.

Chaturvedi (2014) explored how asset and liability management focuses on micro-level objectives to attain the macro-level objectives. Chaturvedi said, "Macro-level involves formulating critical business policies and efficient allocation of capital and designing products with appropriate pricing strategies." The author further explained, "Micro-level aims at profitability through price matching while ensuring liquidity by means of maturity matching." The immunization process of assets and liabilities aims to maintain the spread so the deployments of liabilities are at a higher rate than the costs. However, the process of matching prices and maturity levels is not easy for management.

Adam (2007) described the bank balance composition in the book titled, "Hand Book of Asset and Liability Management from Models to Optimal Return Strategies." as being defined by the accounting and banking industry definitions. From an accounting arrangement, the balance sheet of a bank should have assets comprising of treasury products being equal to liabilities such as commercial trades, debt and equity, Adam (2007).

Banking industry divides balance sheet into two parts: Trading Book[7] and Banking Book[8]. Hull (2007) explained that under Basel II and III, the trading book assets are marked to market daily. Value-at-risk (VAR) is measured on a ten day horizon. While banking book, assets are not marked to market unless there is a risk of default from the counter-party, they are held at the acquisition price or the book value. The banking book is measured on a one year horizon, Hull (2007).

Choudhry (2012) wrote that assets and liabilities of banks need to be managed to balance the interest rate and liquidity risk in the book titled, "An Introduction to Banking: Liability Risk and Asset-Liability Management." The banks operating strategy is guided by the four key concepts of Asset and Liability Management (ALM) according to Choudhry. The first key factor is liquidity which refers to how easily cash equivalent assets can be turned into money. Regulatory authorities require that the banking book holds a portion of assets in liquid instruments such as overnight funds, treasury bills (T-Bills) and certificate of deposits (CDs) to less liquid assets which include medium-term bonds.

The second factor is the term structure of interest rates in the money market. The interest rate yield curve shape can greatly impact the strategies employed by the bank. Specific instruments can highlight the significance of interest sensitivity. The second from last factor is the matching or mismatching of assets and liabilities maturity profiles to specific strategies employed. The final key factor is default risk exposure of defaulters on payments due to the bank.

Charnes and Cooper in 1961 coined the term goal programming (GP) in their published book titled, "Management Models and the Industrial Applications of Linear Programming," Caballero et al (2009). A number of publications followed from the Charnes and Cooper 1961 contribution notably by authors such as Ijiri (1965), Lee (1972) Ignizio (1976) as well as papers by Romero (1986, 1991), Schniederjans (1995), Tamiz et

[7] The portfolio of financial instruments help by the bank in the form of assets for trading such as derivatives, bonds, equities, foreign exchange, commodities and additional financial contracts

[8] The assets of a bank expected to be held to maturity such as loans to individuals , corporates , deposits , investments,

al (1995, 1998), Jones and Tamiz (2002) whose work appeared in scientific literature and is considered to be the most significant, Caballero et al (2009), Aouni and Kettani (2001).

Aouni and Kettani (2001) highlighted in their paper, "Goal programming model: A glorious history and a promising future," that GP has received criticism over time. In particular the area of aggregation procedures of deviations, objectives have no common standard of measurement. Satisfaction functions were introduced by Martel and Aouni (1990) to answer the incommensurability of the scales and integrate the decision maker's preferences. The authors conclude by saying GP models is a tool which allows the collective decision-making process to be adapted for networking and globalization.

In a recent research paper, Neelavathi (2015) explained how Goal programming (GP) is an improved mathematical tool used to solve multiple conflicting objectives to reach the best solution while satisfying a set of constraints. It is an extension of linear programming (LP) which uses the simplex method to find optimum solutions, Ekezie and Onuoha (2013). The process is to state satisfactory goals, ranked in order of importance which are computed in linear form to provide a set of solutions that satisfy identified constraints. The method is to reduce both the sum of deviations and priority deviations against set target values as well as realised results, Neelavathi (2015).

The objective being to minimize the deviations of ranking goals of main importance to the least important depending on the number of goals set, Neelavathi (2015). The shortfall in using goal programming is the assumption that constraints in the model are satisfied. In reality, not all constraints necessarily need to be met.

2.2 Deterministic Models

Deterministic models explained by Kosmidou and Zopounidis in 2004 in their book title, "Goal Programming Techniques for Bank Asset Liability Management," use linear programming where the outputs of the model are assumed to use random events whose parameter values are determined by the initial conditions and be computed for large problems.

According to Kosmidou and Zopounidis (2004), the pioneer of balance sheet management in banks was Chambers in 1961 for the paper titled, "Inter-Temporal Analysis and Optimization of Bank Portfolios." A mathematical model was derived to interpret after formulation the conditions of current operations as opposed to the former efforts, Chamber (1961). The authors sought how to optimise the balance sheet of a bank over several time periods given two restrictions which were the reserve requirements for liquid assets and "balanced" portfolio, Kosmidou and Zopounidis (2004),

They measured the level of risk a particular portfolio can carry using intertemporal linear programming (ILP). Chambers found a solution to the problem and incidentally found new information that could be used to study the implications of financial regulators policies. ILP models went on to be used by banks in the USA, Germany, Japan and Denmark. Cohen and Hammer (1967, 1972) applied and further extended the Chambers model in the paper titled "Linear programming and optimal bank asset management decisions." It included variables which author Brodt (1978) identified as, "Bankers' policy constraints, market constraints and feedback between banks' loans and deposits," while optimizing an objective function of profit.

Kosmidou and Zopounidis (2004) showed how linear programming models focused on one goal in the objective function and was not capable of analysing multi objectives such as market share retention or the size of deposit and loans being expanded. However, the model assumed that bankers knew beforehand the future levels of risk of types of deposits, rates of interest and the market value of the bank which was implausible, Kosmidou and Zopounidis (2004).

Bank management takes on a number of complex objectives which need solving by using multi-objective programming, Giokas and Vassiloglou (1991), Abid and Chakroun (2013). The authors Eatman and Sealey in 1979 research paper, "A Multi-Objective Linear Programming Model for Commercial Bank Balance Sheet Management," formulated a multi-objective linear programming model for banks which addressed the problem of dealing with an objective function with one goal while factoring in the constraints of policy and management. Their primary objectives were profitability, solvency and ratio of risky assets to capital, of which solvency was stated in terms of liquidity and risk.

Kosmidou and Zopounidis (2004) explained how solvency was measured by the ratios of capital-adequacy ratio (CA) and risk-asset to capital ratio (RA). When determining the ratios CA and RA, both asset and liability compositions were considered. As CA and RA ratio increase, liquidity diminishes and risk increases. Therefore banks can capitalize on liquidity and decrease risk by reducing both the CA and RA ratios. Kosmidou and Zopounidis (2004) highlighted that multi-objective models give management selection in the direction of the bank and allows the factoring in of dynamics to achieve a utility maximising solution. Bankers used this model to maximise interest rates to be paid out for deposits, costs and benefits of policy changes, increasing the banks equity and measuring uncertainty of future economic situations.

Tayi and Leonard (1988) wrote a paper on the "Bank Balance-Sheet Management: An Alternative Multi-Objective Model," which developed an alternative model to ensure that the solutions obtained overcame the decision-makers trade-off between conflicting goals and reduce their cognitive burden to maximise their utility. The changes made to the structure of the bank balance sheet would indicate efficiency in its management.

The research presented an example of a multi-objective balance sheet problem which served to demonstrate the formulation and the solution would vary between decision-makers depending on their preferences. The model attempted to minimize disutility function which had positive and negative deviations of objectives. The results showed the bank hold diverse asset and risk portfolios and utility functions affect the return on assets.

2.3 Stochastic Models

Stochastic models or mean-variance models, estimate the probability distribution of potential outcomes by allowing for random variations in one or more inputs over time. With this approach variance measures the risk and earnings are evenly distributed with the use of risk-averse utility functions to make decisions. However, Kosmidou K and Zopounidis C (2004) emphasized, "The use of stochastic models, dynamic programming, sequential decision theory and linear programming under uncertainty pose computation difficulties."

Harry Markowitz (1952) pioneered the roadmap for building models in financial economics and corporate finance with what is popularly known as the "Markowitz Portfolio Theory." Mangram (2013) described it as, "An investment framework for the selection and construction of investment portfolios based on the maximization of expected portfolio returns and simultaneous minimization of investment risk." The average investor has two objectives when it comes to making an investment decision for their portfolio, maximising expected return and minimising risk.

The problem noted on the portfolio theory by Giokas and Vassiloglou (1991) was the complex statistical-based mathematical modelling and formulas. The theory made underlying assumptions which were highlighted by Mangram (2013) as being not practical. Some of the key criticisms discussed by Mangram (2013) below were, assuming perfect information where investors have all the relevant information regarding their investment on hand. In reality, however, world markets have information asymmetry and insider trading is widespread. The model assumed unlimited access to capital allowing investors to borrow anytime, at risk-free interest rates. The government is the only investor who can borrow continuously at the interest-free rate whereas individuals are subject to credit limits set by regulators. The models did not factor in taxes and transaction costs. Investors are subject to broker fees, administrative costs which can change the optimal portfolio selection.

A bank seeks to maximise their future profits subject to a mix of constraints, Kusy and Ziemba (1983). The authors developed a stochastic linear programming model in their paper titled, "A Bank Asset and Liability Management Model," to determine the optimal trade-off between liquidity, risk and return to bring greater productivity to the organization of assets and liabilities in banks.

2.4 Application of GP

Giokas and Vassiloglou in 1991 applied the goal programming model as a case study to a Greek bank on their paper titled, "A Goal programming model for bank assets and liabilities management." The findings showed management could pursue the objective of maximising revenue in conjunction with retaining market share and increasing bank assets and liabilities. However, the goal model was a build-up of the linear programming model which had been in operation prior. The study maintained the key elements from the linear model but in the array of objectives.

Kosmidou and Zopoundi (2004) used goal programming for their paper on, "Goal programming techniques for Bank Asset and Liability Management." A large commercial Greek bank in 1999 was facing conflicting goals and the balance sheet investigated to find the optimal balance sheet combination for 2000 which presented multiple solutions. However, the various alternative solutions found had a major defect in that it allowed one goal to be improved without degrading the others.

A study was done in Nigeria by Arewa, Owopti and Tobira (2013) titled, "Financial Statement Management, Liability Reduction and Asset Accumulation: An Application of Goal programming model to a Nigerian bank." The study was based on six goals identified in the bank; a model was formulated and found that the bank's goals except for liability reduction could be attained. The bank thrived on liability accumulation yearly through deposit mobilization activities defeating the objective of liability minimization. The authors recommended the bank convert their liabilities to assets, the removal of equity and earnings as goals and recommending the application of the model to other locally-owned banks.

A multi-objective approach was formulated by Kruger (2011) for a hypothetical South African bank to manage the balance sheet. The model objective was to maximize earnings while incorporating other goals such as risk all within set guidelines from management and regulatory bodies. Although a trial and error approach was used for optimization, a single scenario was input. The model needed to be extended by applying stochastic programming.

Abid and Chakround (2013) applied goal programming technique to a commercial bank in Tunisia. The two aims of the study were to first apply goal programming on the balance sheet of a commercial bank within structural, regulatory and bank policy limits to obtain the optimal balance sheet structure for the year 2007 using 2006 balance sheet and then compare the computations with the actual financial statement values for 2007. The next aim was to develop the asset-liability model as an alternative, robust model to aid decision-maker when formulating strategic plans in the bank. The results obtained differed with the actual balance sheet statement of 2007, which showed the applicability of the model as a planning tool and informed policy choice. Further developments on the model could include forecasting assets and liabilities while taking into account different time horizons, interest rate risk and changes to the market parameters which decision-makers can monitor and analyse.

Jamshidinavid and Mehri (2016) designed a model for an Iranian bank to manage their assets and liabilities. The mathematical model incorporated the organisational and goal restrictions and the objective using the information from the financial statements for the fiscal years 2005-2014, analytic hierarchy process technique and legal requirements. The degree of importance of objectives was identified using the analytic hierarchy process technique through a questionnaire for top management of the bank. The financial statement information was used to form the equations within the structural, goal restrictions and objective functions. LINGO Software was used to solve for the bank's optimal allocation of resources within the balance sheet. The results calculated were compared to the actual results and showed that fixed assets with low return were allocated fewer resources and assets with a higher return, such as bonds should be invested more to increase income and ultimately retained earnings for the bank. This would lead to the shareholder's satisfaction being increased.

Two Malaysian banks had goal programming applied to manage their asset and liabilities by Lam J, Chen and Lam W in 2017. Their study was similar in the goal formulation of Arewa et al (2013), in which six objectives were observed. The pre-emptive programming method provides the fulfilment of each goal systematically. Each aim is given a priority level where the highest priority is achieved first before the second. Once all priorities have been achieved without making anyone worse off the model is said to have been optimised. The outcomes for both banks showed attainment of all goals and highlighted improvements in target values for some goals.

Lin and O'Leary (1993) highlighted that the literature presented GP is a relatively new operational research technique. In the studies published, there is very little evidence on implementation. There was a survey conducted by Zanakis in 1985 which showed the GP received limited usage and limited awareness of it. A limitation in the literature of GP is that there is little evidence that it is a useful tool to analyse behaviour descriptively. An example given was that linear regression is frequently used to investigate archival data to understand and describe behaviour. Whereas if in GP papers where people have multiple goals this behaviour should be more consistent with GP solutions than with single goal solutions. Literature to show this type of scenario is very few.

Though academics employ many methodologies to investigate theories in more detail. There is little evidence to show academics are using it to address issues of academic concern. GP has been used sparingly as a research methodology by academics.

3.0 Goal Programming Process

The basic concepts and outline of the steps taken to formulate a goal programming model will be covered in the chapter. Goal programming variants will be discussed and a mathematical derivation of the model will be derived.

3.1 Basic Concepts and Definitions

Goal programming is a popular technique in the field of multi-criteria decision making as explained by Jones and Tamiz (2010). Therefore is it necessary to understand the theories and methodologies of MCDM to build a GP model, Jones and Tamiz (2010)

3.1.1 Multi-Criteria Decision Making

MCDM is a field which deals with deciding on the selection of the best alternative from several possible options in making a decision, subject to several attributes, Romero (1991).

Zeleny (1982), Romero (1991) explained the basic concepts involved in MSDM as first defining attributes. These are captions or descriptions of an unbiased representation of values for the decision-makers (DM[9]). The quantifiable values can be written as a mathematical expression $f\ (x)$ of decision variables. Decision variables are factors the DM has control over. Liquidity, risk, profit and turnover are examples that banks factor into the decision process.

Romero (1991) further went on to explain the mathematical function of the corresponding attribute can be maximized or minimised to represent objectives, for example maximising profits, maximising turnover, minimizing risk, minimizing costs. The mathematical expression would be Max $f(x)$ or Min $f(x)$ where $f(x)$ is the attributes form.

A goal, explained by Romero (1991) is an attribute with an attribute and a target combined. When DM set a reasonable level of attainment for an attribute that is defined as a target. Thus, if a bank wants to set net profit margin for the year at 20% that is a goal.

[9] DM refers to various stakeholders who are choice makers who gather information and access the best alternative solution to overcome a problem.

Goals generally are expressed as $f(x) \leq b$ or $f(x) \geq b$, where b is the target value. In our example of net profit margin, the expression will be $f(x) \geq 20\%$.

Kumar (2015) defined a constraint as, "A restriction on the decision variable that must be fulfilled in order for the solution to be feasible." Constraints can be defined into two categories, structural or system constraints and goal constraints. The difference between a goal and constraint in mathematical programming lies in the inequalities right-hand side meaning of the value, Romero (1991). The right-hand side in goals, that is the b value, represents the desirable value decision-makers hope to achieve or not. While the right-hand side of a constraint must be attained to avoid non-viable outcomes.

The last concept explained by Romero (1991) which encompasses all the three aforementioned concepts is criterion. Andre et al (2010) showed that the attributes, objectives and goals set by DM are appropriate to a particular problem and form the measure of criterion. If a problem has more than one criterion this is referred to as MCDM. Hence MCDM is a framework to analyse decision making problems with diverse attributes, objectives and goals, Andre et al (2010).

The above meaning to basic concepts will form the foundation for understanding the building of a goal programming model. Different algebraic symbols for the concepts may be used but the approach involved is the same.

3.1.2 Goal programming

GP is a popular technique in the field of MCDM which is relatively easy to formulate, Jones and Tamiz (2010). It is a linear programming (LP)[10] mathematical tool which has been extended to find the best possible outcome from a given set of parameters expressed in a linear equation, Dan, Onuoha (2013) and Hassan (2015)

The purpose of GP is to satisfy multiple conflicting goals which are relevant to the decision making problem being considered, Romero (1991). There are three types of analysis GP is used to perform, Kumar (2015):

a) Identifying the necessary resources needed to realize the desired set of objectives.
b) Determining the degree to which goals can be realized with limited resources.
c) Finding the best possible solution under varying priorities of goals.

3.2 Formulation of GP

To formulate goal programming model Schniederjans (1984, 1995) highlighted Ignizios (1986) key basic steps as.

a) Define the decision variables.
b) State the constraints.
c) If necessary to determine the pre-emptive priorities.
d) If necessary to determine the relative weights.
e) State the objective function.
f) State the nonnegative requirement.

Romero (1991), Jones and Tamiz (2010) explained the steps in detail. The first step is to establish the number of Q goals, where the index $q = 1, \dots, Q$ and define the n decision variables with term $\underline{x} = x_1, x_2, \dots, x_n$. Decision-makers have control over these variables and define the decision to be made. Once the goals have been established, their target value b_q need to be determined. The target values being the desired level DM aim to attain for

[10] LP is used to determine the best possible outcome from asset of parameters.

each goal. The target level should be specified as to whether it will be equal to, greater than or less than the value set, Jones and Tamiz (2010).

Romero (1991) went on to explain how the term deviation variables[11] was introduced into GP models. Deviation variables can be negative or positive and represented as n_q and p_q respectively. The negative deviation variable (n_q) represents the number of units by which the q^{th} goal was not achieved against the target set. The positive deviation variable (p_q) represents the number of units that were surpassed by the q^{th} goal against the target, Romero (1991). The deviation variables are constrained from taking non zero value and non-negative values at the same time, Jones and Tamiz (2010).

The q^{th} goal expressed by Romero (1991) and Jones and Tamiz (2010) arithmetically as

$$f_q(\underline{x}) + n_q - p_q = b_q \quad (1)$$

If $f_q(\underline{x}) \geq b_q$ that means the q^{th} goal is greater than or equal to the set target level. The negative variable n_q will take the smallest possible value and be minimized. Romero (1991) explained that if $f_q(\underline{x}) \leq b_q$ the positive variable p_q must take the smallest possible value and be minimized. However, when $f_q(\underline{x}) = b_q$ both the negative n_q and positive variables p_q must take the smallest values and the sum of minimization being $n_q + p_q$.

DM can decide which deviation variables to remove based on the three basic penalisation rules formulated by Jones and Tamiz (2010) in Table 3.1

[11] Deviation variable is the aberration measurement between specified goals in a GP.

Goal type	Significance	Deviation Variables	Examples
1	Achieve at most the target level	p_q	Maintain costs within a budget of R 1.5 m
2	Achieve at least the target level	n_q	Aim for a net profit margin of at least 20%
3	Achieve the target level exactly	$n_q + p_q$	Aim to employ exactly 15 workers

Table 3 1: Three principal types of goals[12]

Any positive deviation above the goal would result in goal type 1 being penalised. A negative deviation below the goal level would penalize goal type 2. For goal type 3 any positive or negative deviation from the target level would result in the goal being penalised, Jones and Tamiz (2010).

Decision-makers desire to meet all goals set, however, if the goal is not achieved this does not imply that the solution is not feasible. Such a goal is termed soft constraint. But hard constraints are those goals whose violation will result in the solution being infeasible, Jones and Tamiz (2010). Such constraints are modelled by the condition

$$\underline{x} \in F$$

Jones and Tamiz (2010) explained F as, "The feasible region made up of points in the decision space that satisfy all of the constraints and sign restrictions."

The purpose of GP is to minimize the deviations between the fulfilment of the goals and their desired targets through the formulation of an achievement function. The minimization process can be achieved using different methods which lead to different GP variants, Romero (1991).

[12] Jones and Tamiz (2010)

After considering the above factors the GP model can take the algebraic form, Jones and Tamiz (2010).

$$Min\ a = h(\underline{n}, \underline{p}) \tag{2}$$

Subject to

Goal (soft) constraints	$f_q(\underline{x}) + n_q - p_q = b_q$	$q = 1, \ldots, Q$
Hard constraints	$\underline{x} \in F$	
	$\underline{x}, n_q, p_q \geq 0$	$q = 1, \ldots Q$

Where $\underline{n}$ is the vector of q negative deviational variables, $\underline{p}$ is the vector of q positive deviational variable. Due to the nature of GP minimisation n_q x $p_q = 0$ will be satisfied. Jones and Tamiz (2010).

The nature of the achievement function is depended on the GP variants used, Jones and Tamiz (2010), Kumar (2015). The main variants are:

a) Weighted goal programming (WGP)
b) Lexicographic goal programming (LGP)
c) Chebyshev goal programming

3.2.1 Weighted Goal Programming (WGP)

Romero (1991) together with Jones and Tamiz (2010) explained that WGP considers all goals together as the DM is interested in direct comparison. The goals are represented in a composite objective function. The function tries to minimize the sum of the deviations (positive p_i, negative n_i) between their goals and desired targets. The DM will rank based on weights the importance of the deviations of each goal. The linear WGP models geometric structure is:

$$Min\ a = \sum_{q=1}^{Q}\left(\frac{u_q n_q}{k_q} + \frac{v_q n_q}{k_q}\right) \qquad (3)$$

Subject to linear constraints

Goal constraint: $f_q(\underline{x}) + n_q - p_q = b_q \qquad q = 1, \dots, Q$

Hard Constraint: $x \in F$

With $\underline{x} \geq 0,\ n_q \geq 0,\ p_q \geq 0 \qquad q = 1, \dots, Q$

Where the definitions of the variables introduced for the formulation of GP, except u_q and v_q are the weights for the negative and positive deviations from the q^{th} goal. k_q is the normalisation constant necessary to scale all the goals onto the same units of measurement, Jones and Tamiz (2010). F is the feasible or constraint set, Romero (1991) made up of points that satisfy the decision space. Equation (3) corresponds to a traditional linear programming model (LP) problem and can be solved using simplex algorithm, Romero (1991).

Jones and Tamiz (2010) highlighted that WGP affords greater flexibility and allows DM makers to do more 'trade off' analysis and direct comparison between goals. The advancement in computer software and decreased solution time to compute large scale goal programmes has seen a trend from using lexicographic goal programming to WGP.

3.2.2 Lexicographic Goal Programming (LGP)

LGP is also known as pre-emptive or non-Archimedean, ranks different goals into pre-emptive priorities or priority levels. The unwanted deviations need to be minimised for each priority level. If priority L_i is favoured more to another priority L_k , L_i will have a higher priority level and be more infinitely important than L_k,($L_i >>> L_j >>> L$), where >>> means very much greater than, Iserman (1982), Sherali (1982), Ignizio (1983a), Jones and Tamiz (2010) Kumar (2015). The goals with higher priority are fulfilled first before the lower priorities according to their lexicographic order, Romero (1991).

Romero (1991), Jones and Tamiz (2010) presented the LGP model where each priority is a function of a subset of unwanted deviational variables, $h_l(\underline{n}, \underline{p})$ as

$$Lex\ Min\ a = \left[h_1\left(\underline{n}, \underline{p}\right), h_2\left(\underline{n}, \underline{p}\right), \dots, h_l\left(\underline{n}, \underline{p}\right)\right] \tag{4}$$

Subject to linear constraints

Goal constraint: $f_q(\underline{x}) + n_q - p_q = b_q \qquad q = 1, \dots, Q$

Hard Constraint: $x \in F$

With $\underline{x} \geq 0,\ n_q \geq 0,\ p_q \geq 0 \qquad q = 1, \dots, Q$

The authors Romero (1991), Jones and Tamiz (2010) criticised LGP for not being compatible with utility function theory and not appropriate for every multi-objective situations.

3.2.3 Chebyshev Goal Programming

Flavell (1976) introduced this variant of goal programming. Chebyshev (L_∞) underlying philosophy is that of balance when using the distance metric. The DM is trying to achieve an ideal balance between the achievement of the set of goals rather than extreme optimization, Jones and Tamiz (2010), Kumar (2015). The model is sometimes referred to as Minmax goal programming because it minimizes the maximum unwanted deviation variables, rather than the sum of deviations, Kumar (2015). The variant was thought to be of little practical use according to Jones and Tamiz in 2010. However, the authors pointed out that greater awareness and development of methodologies of extended GP which encompass Chebyshev goal programming will increase the usage in practice of the variant.

4.0 Methodology

In this chapter, the goal programming formulation was applied to manage two balance sheets of the South African bank's segmented markets, wholesale and retail. The methodology used in the research started with data collection, criteria definition, and ended with the application of the GP model to determine suitable balance sheet item structures.

4.1 Data Collection

Secondary data was used which had been published and not sourced directly by the researcher. In order to design the GP model the balance sheet items are identified from published financial annual reports of 7 years for ABSA bank (2011-2018) in Appendix A and Capitec bank from 2012 to 2019 in Appendix C. The financial year for ABSA bank runs from January to December whereas for Capitec bank it's from March to February, however, both are twelve month periods. The information gathered will generate the structural, goal restrictions and objective function. LINGO software will be used to solve the equations and find the banks optimal allocation of the decision variables. The balance sheet information for 2018 for ABSA and 2019 for Capitec will be used to compare to those derived from the model and analysed for forecasting.

4.2 GP Model design

Goal programming models are suited to solve such problems where a bank has to achieve goals simultaneously that contradict each other such as maximising assets while minimizing liabilities, Siew et al (2017).

The GP variant that will be used to solve the GP model is WGP. This variant is more popular as reported by Jones and Tamiz (2002, 2010) survey report. The variant also offers greater flexibility and for DM to do more, "Trade of analysis and direct comparison between goals," Jones and Tamiz (2010)

The GP formulation will take equation (3) from Jones and Tamiz and change the algebraic symbols and follow the Arewa et al (2013) and Chen et al (2017) methodology as below,

$$Min\ z = w_1 G_1 + w_2 G_2 + \cdots + w_i G_i \tag{5}$$

Where $i = 1,2,3, \dots, n$

Subject to linear constraints

Goal constraint:

$$\sum_{j=1}^{m} (a_{ij} x_j + d_i^- - d_i^+) = g_i \tag{6}$$

$$x_j, d_i^-, d_j^+ \geq 0$$

Whereas described by Chen et al (2017) as,

z = objective function;

w_i= weight for $i = 1,2,3, \dots, n$;

d_i^-= negative deviation variable (underachievement) for $i = 1,2,3, \dots, n$;

d_i^+= positive deviation variable (overachievement) for $i = 1,2,3, \dots, n$;

x_j=decision variables for $j = 1,2,3, \dots, m$;

a_{ij}=parameter for decision variables;

g_i=aspiration level for $i = 1,2,3, \dots , ni = 1,2,3, \dots , n$;

Table 4.1 shows the 6 important goals for bank financial management for the GP model, Chen et al (2017).

Goal	Objective
1	Maximize total assets
2	Minimize total liabilities
3	Maximize total equity
4	Maximize profitability
5	Maximize earnings
6	Maximize total goal achievements

Table 4 1: The goals of the banks[13]

[13] Chen et al (2017)

4.2.1 ABSA Bank GP Model

The financial statement in the annual report of ABSA bank from the year 2011 to 2017 is investigated. A GP model is developed to optimize the financial management of the bank to achieve multiple goals. The WGP method allows the decision-maker to satisfy one goal at a time. The first goal of maximising total assets (1) will be satisfied first then followed by the second goal of minimizing total liabilities (2) and so on.

The financial data relating to ABSA banks total assets, liabilities, equity, profit and earnings for the period 2011 to 2017 is presented in Table 4.2. The figures comprise of totals presented in the balance sheet for the respective years. The Assets for 2011 amounted to R 703 324 000 and so on. The detailed line items that make up the figure of each respective goal is found in Appendix B. For example total assets for 2011 comprised of loans, loans and advances, derivatives, other securities, and remaining earning assets to give the sum total presented in Table 4.2.

YEARS	2011	2012	2013	2014	2015	2016	2017	
GOALS	000'' Rands							Total
Assets	703,324	722,036	750,543	772,476	877,653	857,320	925,446	5,608,798
Liabilities	685,044	703,008	735,376	755,135	875,644	849,012	903,029	5,506,248
Equity	52,748	56,268	52,615	54,282	55,853	64,655	80,685	417,106
Profit	8,193	7,482	8,735	9,300	10,047	9,934	8,477	62,168
Earnings	37,217	38,011	34,506	33,713	32,033	36,099	37,855	249,434
Total	1,486,526	1,526,805	1,581,775	1,624,906	1,851,230	1,817,020	1,955,492	11,843,754

Table 4 2: Financial Data of ABSA Bank

The decision variables are the totals for the individual goals balance line items in the financial statement for the respective years, that is $x_1 = 2011, x_2 = 2012, \ldots\ x_7 = 2017$. Table 4.2 values will be coded[14] and reduced to more manageable units to be processed by the computer program, summarized as below in Table 4.3

[14] To code is to reduce large quantities of data into a form more manageable especially by computer programs.

YEARS	2011	2012	2013	2014	2015	2016	2017	
GOALS	000'' Rands							Total
Assets	70.3324	72.2036	75.0543	77.2476	87.7653	85.7320	92.5446	560.8798
Liabilities	68.5044	70.3008	73.5376	75.5135	87.5644	84.9012	90.3029	550.6248
Equity	5.2748	5.6268	5.2615	5.4282	5.5853	6.4655	8.0685	41.7106
Profit	0.8193	0.7482	0.8735	0.9300	1.0047	0.9934	0.8477	6.2168
Earnings	3.7217	3.8011	3.4506	3.3713	3.2033	3.6099	3.7855	24.9434
Total	148.6526	152.6805	158.1775	162.4906	185.1230	181.7020	195.5492	1,184.3754

Table 4 3: Coded Financial Data of ABSA Bank

Table 4.3 summarizes the weight in relation to the value of the goal based on Table 4.2. For decision-makers at ABSA bank to formulate a plan for year 2018, the 6 goals can be achieved by assessing the performance from the previous year (2011-2017) and coming up with an ideal scenario from 2018.

The goal constraints for derived from each goal are as follows

Assets:

$$70.3324x_1 + 72.2036x_2 + 75.0543x_3 + 77.2476x_4 + 87.7653x_5 + 85.7320x_6 + 92.5446x_7 \geq 560{,}8798 \quad (7)$$

Liabilities:

$$68.5044x_1 + 70.3008x_2 + 73.5376x_3 + 75.5135x_4 + 87.5644x_5 + 84.9012x_6 + 90.3029x_7 \leq 550.6248 \quad (8)$$

Equity:

$$5.2748x_1 + 5.6268x_2 + 5.2615x_3 + 5.4282x_4 + 5.5853x_5 + 6.4655x_6 + 8.0685x_7 \geq 41.7106 \quad (9)$$

Profit:

$$0.8193x_1 + 0.7482x_2 + 0.8735x_3 + 0.9300x_4 + 1.0047x_5 + 0.9934x_6 + 0.8477x_7 \geq 6.2168 \quad (10)$$

Earnings:

$$3.7217x_1 + 3.8011x_2 + 3.4506x_3 + 3.3713x_4 + 3.2033x_5 + 3.6099x_6 + 3.7855x_7 \geq 24.9434 \quad (11)$$

Total goal achievement:

$$148.6526x_1 + 152.6805x_2 + 158.1775x_3 + 162.4906x_4 + 185.1230x_5 + 181.7020x_6 + 195.5492x_7 \geq 1184.375 \quad (12)$$

Positive and negative variables are added to into the constraint to determine the increase or decrease of the goals, Arewa et al (2013) and Chen et al (2017). The goal programming model is to be developed like Chen et al (2017) as follows,

Objective Function:

$$Min = d_1^- + d_2^+ + d_3^- + d_4^- + d_5^- + d_6^- \qquad (13)$$

Subject to

$$70.3324x_1 + 72.2036x_2 + 75.0543x_3 + 77.2476x_4 + 87.7653x_5 + 85.7320x_6 + 92.5446x_7 + d_1^- - d_1^+ = 560{,}8798$$

$$68.5044x_1 + 70.3008x_2 + 73.5376x_3 + 75.5135x_4 + 87.5644x_5 + 84.9012x_6 + 90.3029x_7 + d_2^- - d_2^+ = 550.6248$$

$$5.2748x_1 + 5.6268x_2 + 5.2615x_3 + 5.4282x_4 + 5.5853x_5 + 6.4655x_6 + 8.0685x_7 + d_3^- - d_3^+ = 41.7106$$

$$0.8193x_1 + 0.7482x_2 + 0.8735x_3 + 0.9300x_4 + 1.0047x_5 + 0.9934x_6 + 0.8477x_7 + d_4^- - d_4^+ = 6.2168$$

$$3.7217x_1 + 3.8011x_2 + 3.4506x_3 + 3.3713x_4 + 3.2033x_5 + 3.6099x_6 + 3.7855x_7 + d_5^- - d_5^+ = 24.9434$$

$$148.6526x_1 + 152.6805x_2 + 158.1775x_3 + 162.4906x_4 + 185.1230x_5 + 181.7020x_6 + 195.5492x_7 + d_6^- - d_6^+ = 1184.375$$

$$x_1, x_2, x_3, x_4, x_5, x_6,\ d_1^-, d_2^-, d_3^-, d_4^-, d_5^-, d_6^-, d_1^+, d_2^+, d_3^+, d_4^+, d_5^+, d_6^+ \geq 0$$

4.2.2 Capitec Bank GP Model

The financial data for Capitec banks total assets, liabilities, equity, profit and earnings for the financial years 2012 to 2018 are presented in Table 4.4. The detailed line items for the respective goals are found in Appendix D.

YEARS	2012	2013	2014	2015	2016	2017	2018	Total
GOALS	000" Rands							
Assets	18 620	29 961	35 015	41 466	47 614	52 048	61 125	285 849
Liabilities	18 436	29 834	36 209	42 353	49 286	57 240	66 066	299 424
Equity	5 185	8 513	9 982	11 564	13 659	16 118	18 892	83 913
Profit	1 094	1 605	2 038	2 564	3 228	4 387	4 471	19 386
Earnings	2 002	2 939	4 130	5 701	7 772	10 330	13 153	46 027
Total	45 338	72 852	87 373	103 647	121 560	140 122	163 706	734 599

Table 4 4: Financial Data of Capitec Bank

The decision variables are $x_1 = 2012, x_2 = 2013, \ldots\ x_7 = 2018$ the respective financials years represented will have the data coded and summarized as below in Table 4.5

YEARS	2012	2013	2014	2015	2016	2017	2018	Total
GOALS	000" Rands							
Assets	1,8620	2,9961	3,5015	4,1466	4,7614	5,2048	6,1125	28,5849
Liabilities	1,8436	2,9834	3,6209	4,2353	4,9286	5,7240	6,6066	29,9424
Equity	0,5185	0,8513	0,9982	1,1564	1,3659	1,6118	1,8892	8,3913
Profit	0,1094	0,1605	0,2038	0,2564	0,3228	0,4387	0,4471	1,9386
Earnings	0,2002	0,2939	0,4130	0,5701	0,7772	1,0330	1,3153	4,6027
Total	4,5338	7,2852	8,7373	10,3647	12,1560	14,0122	16,3706	73,4599

Table 4 5: Coded Financial Data of Capitec Bank

The goal constraints for derived from each goal are as follows

Assets:

$$1.8620x_1 + 2.9961x_2 + 3.5015x_3 + 4.1466 + 4.7614x_5 + 5.2048x_6 + 6.1125x_7 \geq 28.5849 \quad (14)$$

Liabilities:

$$1.8436x_1 + 2.9834x_2 + 3.6209x_3 + 4.2353x_4 + 4.9286x_5 + 5.7240x_6 + 6.6066x_7 \leq 29.9424 \quad (15)$$

Equity:

$$0.5185x_1 + 0.8513x_2 + 0.9982x_3 + 1.1564x_4 + 1.3659x_5 + 1.6118x_6 + 1.8892x_7 \geq 8.3913 \quad (16)$$

Profit:

$$0.1094x_1 + 01605x_2 + 0.2038x_3 + 0.2564x_4 + 0.3228x_5 + 0.4387x_6 + 0.4471x_7 \geq 1.9386 \quad (17)$$

Earnings:

$$0.2002x_1 + 0.2939x_2 + 0.4130x_3 + 0.5701x_4 + 0.7772x_5 + 1.0330x_6 + 1.3153x_7 \geq 4.6027 \quad (18)$$

Total goal achievement:

$$4.5338x_1 + 7.2852x_2 + 8.7373x_3 + 10.3647x_4 + 12.1560x_5 + 14.0122x_6 + 16.3706x_7 \geq 73.4599 \quad (19)$$

The goal programming model is to be developed as follows

Objective Function:

$$Min = d_1^- + d_2^+ + d_3^- + d_4^- + d_5^- + d_6^- \quad (13)$$

Subject to

$$1.8620x_1 + 2.9961x_2 + 3.5015x_3 + 4.1466 + 4.7614x_5 + 5.2048x_6 + 6.1125x_7 + d_1^- - d_1^+ = 28.5849$$

$$1.8436x_1 + 2.9834x_2 + 3.6209x_3 + 4.2353x_4 + 4.9286x_5 + 5.7240x_6 + 6.6066x_7 + d_2^- - d_2^+ = 29.9424$$

$$0.5185x_1 + 0.8513x_2 + 0.9982x_3 + 1.1564x_4 + 1.3659x_5 + 1.6118x_6 + 1.8892x_7 + d_3^- - d_3^+ = 8.3913$$

$$0.1094x_1 + 01605x_2 + 0.2038x_3 + 0.2564x_4 + 0.3228x_5 + 0.4387x_6 + 0.4471x_7 + d_4^- - d_4^+ = 1.9386$$

$$0.2002x_1 + 0.2939x_2 + 0.4130x_3 + 0.5701x_4 + 0.7772x_5 + 1.0330x_6 + 1.3153x_7 + d_5^- - d_5^+ = 4.6027$$

$$4.5338x_1 + 7.2852x_2 + 8.7373x_3 + 10.3647x_4 + 12.1560x_5 + 14.0122x_6 + 16.3706x_7 + d_6^- - d_6^+ = 73.4599$$

$$x_1, x_2, x_3, x_4, x_5, x_6,\ d_1^-, d_2^-, d_3^-, d_4^-, d_5^-, d_6^-, d_1^+, d_2^+, d_3^+, d_4^+, d_5^+, d_6^+ \geq 0$$

The data used in the formulation of the goal programming model will be solved using a mathematical software called LINGO. This is a software used in MDCM theories for optimization models. Chen et al (2017).

5.0 Results and Analysis

In this chapter, the aim is to answer the problem statement of finding the optimal size and composition of assets and liabilities for a bank in the wholesale and retail market segments in South Africa.

5.1 ABSA Bank

Table 5.1 shows the LINGO optimal solution key output for ABSA bank a wholesale market segment. The LINGO output results in detail can be found in Appendix E.

Variable	Value	Reduced Cost
D1-D6	0,000000	1,00000
X2	2,032621	0,00000
X4	2,408777	0,00000
X6	2,659723	0,00000
D4-	0,186563	0,00000
D5-	0,505979	0,00000
D6-	0,692941	0,00000

Table 5 1: The ABSA Bank solution output from LINGO 17.0

The deviation variables for all goals have been achieved throughout the seven years as shown by the zero values of $d_1^-, d_2^+, d_3^-, d_4^-, d_5^-, d_6^-$ respectively. The reduced cost of 1 for the variables represents the opportunity cost that an objective functions coefficient would have to improve. For a minimization function, this would translate to an increase of 1 and for a maximization function a decrease of 1. Therefore the reduced cost of 1 for the variables translates to the optimal value of the MIN problem decreasing by 1 unit when 1 additional unit of the variable is included.

The zero values on the deviation variables show that ABSA bank overall financial performance for the years 2011 to 2017 is respectable and stable. There are potential improvements in the target value based on the positive value of deviation variables of $d_4^+(0.1865626)$, $d_5^+(0.5059788)$ and $d_6^+(0.6929414)$ shown in Table 5.2.

The three goals that can be improved are profitability (Goal 4), earnings (Goal) and total goal achievement (Goal 6) as Table 5.2 shows.

Goals	d_i^-	d_i^+	Total	New Total (d_i^+ + Total)
Maximize Total Assets	0	0.0	560.8798	560.8798
Minimize Total Liabilities	0	0.0	550.6248	550.6248
Maximize Total Equity	0	0.0	41.4106	41.4106
Maximize Profit	0	0.1865626	6.2168	6.4033626
Maximize Earnings	0	0.5059788	24.9434	25.4493788
Maximise Total Goal Achievement	0	0.6929414	1184.3754	1185.0683414

Table 5 2: Potential Improvement of Goals for ABSA bank

ABSA's assets (goal 1), liabilities (goal 2) and equity (goal 3) have negative deviation variables (d_i^-) being equal to zero which means these goals are fully achieved. For profitability, goal 4 can be increased by 0.1865626 or R1.865 million. Cumulative earnings can also be increased by R5.0599 million and total goal achievement can be increased by R6.9294 million for continuous improvement.

In the Lingo software solution report, the objective function is represented by row one and the constraints by rows two to seven. The slack or surplus column shows a value of zero which shows that the constraint is satisfied as an equality. No additional units of the variable can be included in the optimal solution and the constraint has not been violated. The dual price column of negative one defines how the constraining value is decreased by one unit in order for the objective function to improve.

5.2 Capitec Bank

Table 5.3 shows the LINGO optimal solution output for Capitec bank a retail market segment. The detailed LINGO output can be found in Appendix F.

Variable	Value	Reduced Cost
D1-D6	0,000000	1,00000
X2	1,929574	0,00000
X5	3,516145	0,00000
X6	0,5827271	0,00000
X7	0,5328767	0,00000
D1-	0,2282598	0,00000
D6-	0,2284998	0,00000

Table 5 3: The Capitec Bank solution output from LINGO 17.0

The output values show that the zero value for the negative deviations for Capitec Banks' 6 goals have been accomplished, shown in Table 5.4.

Goals	Output Value	Goals Achievement
Maximize Total Assets	$d_1^- = 0$	Accomplished
Minimize Total Liabilities	$d_2^+ = 0$	Accomplished
Maximize Total Equity	$d_3^- = 0$	Accomplished
Maximize Profit	$d_4^- = 0$	Accomplished
Maximize Earning	$d_5^- = 0$	Accomplished
Maximise Total Goal Achievements	$d_6^- = 0$	Accomplished

Table 5 4: Goals Accomplished for Capitec Bank

The results of the positive deviation variables for all the goals can be shown in Table 5.5

Goals	d_i^-	d_i^+	Total	New Total (d_i^+ + Total)
Maximize Total Assets	0	0,2282598	28,5848828	28,8131426
Minimize Total Liabilities	0	0	29,942369	29,942369
Maximize Total Equity	0	0	8,3912952	8,3912952
Maximize Profit	0	0	1,9385958	1,9385958
Maximize Earning	0	0	4,6027106	4,6027106
Maximise Total Goal Achievements	0	0,2284998	73,4598534	73,6883532

Table 5 5: Potential Improvement of Goals for Capitec Bank

Based on Table 5.5, Goal 1 of maximizing total assets for the bank can further be increased from the current level by R2.283 million to give a new optimum total of R28.813 million. Goals 2 to 5 which are to minimize total liabilities, maximize total equity, maximize total profits and maximize earning respectively indicated the goals are fully achieved. The positive deviation $d_i^+ = 0$ for the goals shows that the totals should remain the same. Goal 6 of total goal achievement shows that the overall sum of selected balance sheet lines can be increased by R2.284 million to achieve a new level of R73.688 million.

5.3 Sensitivity Analysis

An important part of the GP model solution is the sensitivity analysis of a model. If subtle changes to certain parameters in the model can affect overall optimization then attention should be on approximating future values, Kumar (2015). If the optimal solution is not sensitive to changes then there is no need to estimate the values of the parameters, Holzman (1981) and Kumar (2015).

In LINGO the effect of changes on the generated solutions objective function can be investigated using range analysis. Table 5.6 shows the range analysis for ABSA Bank.

Right-hand Side Ranges:			
Row	**Current RHS**	**Allowable Increase**	**Allowable Decrease**
2	560,8798	3,460897	0,4800009
3	550,6248	1,026359	1,257913
4	41,41060	0,9490901	0,4011671
5	6,216800	0,1865626	INFINITY
6	24,94340	0,5059788	INFINITY
7	1184,375	0,6929414	INFINITY

Table 5 6: Objective Coefficient Ranges ABSA Bank

Table 5.7 shows the range of analysis for Capitec Bank.

Right-hand Side Ranges:			
Row	**Current RHS**	**Allowable Increase**	**Allowable Decrease**
2	28,58490	0,2282598	INFINITY
3	29,94240	0,2070178	0,7828663E-01
4	8,391300	9,376422E-02	0,7545660E-01
5	1,938600	6,436190E-02	0,3262650E-01
6	4,602700	0,2322523	0,2067316
7	73,459900	0,2284998	INFINITY

Table 5 7: Objective Coefficient Ranges Capitec Bank

For both banks there is little sensitivity, as the allowable increase is infinity, meaning it can take any positive value with no limit. Allowable decrease for the $d_1^-, d_2^+, d_3^-, d_4^-, d_5^-, d_6^-$ variables is 1. Any further decrease will make the solution not optimal.

For ABSA bank the model solution figures can predict the 2018 model figures and be compared to the actual 2018 figures to test if the future value parameters should be focused on by management as shown in Table 5.8.

YEARS	Model	Actual 2018	Deviation	Allowable Decrease	Allowable Increase
GOALS					
Assets	96.0055	101.1840	5.1785	1.0000	INFINITY
Liabilities	91.3293	99.5993	8.2700	1.0000	INFINITY
Equity	9.0176	7.9042	-1.1134	1.0000	INFINITY
Profit	1.0343	0.8022	-0.2321	1.0000	INFINITY
Earnings	4.2915	3.5209	-0.7706	1.0000	INFINITY
Total	196.2421	213.0106	16.7685	1.0000	INFINITY

Table 5 8: Findings and Comparison with ABSA's real bank balance sheet for 2018

All parameters except for Equity are within the allowable increases and decreases ranges. Equity fell by more than one which is above the allowable decrease.

Capitec bank parameters are all within the allowable increase ranges shown in Table 5.9.

YEARS	Model	Actual 2019	Deviation	Allowable Decrease	Allowable Increase
GOALS					
Assets	6,3407	7,6103	1,2696	1,0000	INFINITY
Liabilities	6,8136	7,8752	1,0616	1,0000	INFINITY
Equity	1,9829	2,1676	0,1846	1,0000	INFINITY
Profit	0,5114	0,5295	0,0181	1,0000	INFINITY
Earnings	1,5476	1,5950	0,0474	1,0000	INFINITY
Total	16,5991	19,7776	3,1785	1,0000	INFINITY

Table 5 9: Findings and Comparison with Capitec's real bank balance sheet for 2019

Capitec Bank financial performance is stable as the 6 goals of assets, liability, equity, profit, earnings and total achievements can be increased without making the model less optimum.

6.0 Conclusion

The application of goal programming model to two segmented banks in South Africa showed that the balance sheets were optimized and improvements could be made to certain line items. The findings of the model showed that ABSA Bank and Capitec Bank had good financial performances as all six goals examined were achieved.

For ABSA bank potential improvements for some goals were identified namely maximization of total profitability, total earning and total goal achievements. However, the sensitivity analysis on equity highlighted that any further decrease of total equity will mean the bank cannot absorb more losses on its assets. Further decrease in equity will affect debt holders and result in the bank being insolvent. Management needs to strike a balance with increases and decreases to equity to protect investors' interests. Capitec bank has two goals, namely maximizing total assets and total goal achievements whose proportions in the balance sheet can be amended to levels beyond the current level.

Arewa et al (2013) applied GP to a Nigerian bank and their results showed that the optimum solution satisfied goals of asset accumulation, shareholder wealth, increasing earning and maximizing profit. But the goal of minimizing liability failed to be satisfied. The target of liability goal was overstated and hence violated the output value of not being equal to zero. The bank needed to source capital from within or other sources apart from liability in order to fulfil the liability reduction goal. The results form ABSA and Capitec are divergent to the Nigerian results. The Nigerian case study has five years under observation whereas the study presented in the paper has eight years under observation. The weights used in the Nigerian study are different from the ABSA and Capitec case study.

Besides GP being used in financial management for both practitioners in industry and academics it has been used in real world applications for various research methodologies as shown by Lin and O'Leary (1993). These included an analysis of the Bell system breakup by Charnes et al in 1988. Olve in 1981 used GP to analyse a problem faced by the Swedish National Telecommunications Administration, involving multiple success measures for local telephone service. A model for developing countries and the marine industry was developed by Taguchi in 1983. In Finland, Wallenius et all in 1978 developed an approach to analyse macroeconomic problems for the country. However, Lin and O'Leary (1993) noted that the papers had two elements that stood out from other studies. Firstly the studies were associated with a government decision problem where the authors were involved in the project beyond the development of the GP model and data was publicly available. In other studies the authors had to conceal the model, data and results due to due to corporate constraints leading to an underestimate of the extent of use of GP models.

Secondly, the decision problems have established areas representing the need for different goals associated with them. In many corporate settings, top management establishes the different goals however the nature of the business organization may diminish the need for a tool like GP. It may not be appropriate for a business to disclose the different goals their different departments aim to achieve.

Many of the papers listed addressed problems of direct concern (practical issues) for decision-makers in organizations. However in reality there are two types of practitioners, decision-makers and academics. There are limited papers in the financial GP to address academic issues and highlighted by Lin and O'Leary (1993). The authors noted that it was difficult to determine what an academic issue is and what a practical issue is.

Goal programming may be a decision support tool that helps a bank or financial institution adopt a blueprint plan for benchmarking goal levels to be achieved in the future. The model can simply be included as a guidance tool in decision making processes such as yearly budget formulations. Further developments of goal programming can be explored to target additional goals, interest risk consideration, focusing on optimizing the asset portfolio and monitor the effectiveness of policy decisions over different time horizons.

7.0 References

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8.0 Appendix

Appendix A: Statement of Financial Position: ABSA Bank 2011-2018[15]

ABSA BANK LTD	Years							
Consolidated data	Cons 31/12/2011 th ZAR	Cons 31/12/2012 th ZAR	Cons 31/12/2013 th ZAR	Cons 31/12/2014 th ZAR	Cons 31/12/2015 th ZAR	Cons 31/12/2016 th ZAR	Cons 31/12/2017 th ZAR	Cons 31/12/2018 th ZAR
	"000	"000	"000	"000	"000	"000	"000	"000
Balance sheet								
Assets								
Loans	486 719	501 682	526 884	540 848	578 599	614 530	641 176	705 786
Gross Loans	498 107	514 680	539 640	553 171	590 594	628 740	654 799	728 253
Less: Reserves for Impaired Loans/ NPLs	11 388	12 998	12 756	12 323	11 995	14 210	13 623	22 467
Other Earning Assets	216 605	220 354	223 659	231 628	299 054	242 790	284 270	306 054
Loans and Advances to Banks	57 486	44 440	46 249	47 038	76 643	55 412	62 533	69 947
Derivatives (Assets)	49 773	52 249	48 616	42 630	79 753	46 751	59 140	46 130
Other Securities	107 506	123 334	128 554	141 708	142 140	140 405	162 597	189 797
Remaining earning assets	1 840	331	240	252	518	222	n.a.	180
Total Earning Assets	703 324	722 036	750 543	772 476	877 653	857 320	925 446	1 011 840
Fixed Assets	7 268	7 653	8 504	9 137	10 955	12 726	13 519	13 609
Non-Earning Assets	31 844	34 231	33 588	32 448	47 533	48 265	49 393	54 230

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	-	-	-	-	-	-	-	-
Total Assets	742 436	763 920	792 635	814 061	936 141	918 311	988 358	1 079 679
Consolidated data	Cons 31/12/2011 th ZAR	Cons 31/12/2012 th ZAR	Cons 31/12/2013 th ZAR	Cons 31/12/2014 th ZAR	Cons 31/12/2015 th ZAR	Cons 31/12/2016 th ZAR	Cons 31/12/2017 th ZAR	Cons 31/12/2018 th ZAR
	"000	"000	"000	"000	"000	"000	"000	"000
Liabilities & Equity								
Deposits & Short Term Funding	556 239	555 300	610 135	631 545	685 422	701 389	729 201	733 606
Total Customer Deposits	423 028	466 241	488 049	519 491	556 030	560 842	578 825	592 854
Deposits from Banks	53 436	44 588	67 035	56 269	65 646	64 118	79 110	140 752
Other Deposits and Short-term Borrowings	79 775	44 471	55 051	55 785	63 746	76 429	71 266	n.a.
Other interest bearing liabilities	114 015	130 103	110 991	106 922	169 759	123 380	143 493	227 717
Derivatives (Liabilities)	51 159	50 618	50 629	44 362	90 856	42 770	52 810	36 551
Trading Liabilities	529	1 126	2 472	2 795	1 242	1 787	8 141	26 960
Long term funding	62 327	78 359	57 890	59 765	77 661	78 823	82 542	164 206
Other (Non-Interest bearing)	13 333	16 211	12 888	14 811	18 493	22 183	28 262	31 988
Reserves	1 457	1 394	1 362	1 857	1 970	2 060	2 073	2 682
Equity	57 392	60 912	57 259	58 926	60 497	69 299	85 329	83 686
	-	-	-	-	-	-	-	-
Total Liabilities & Equity	742 436	763 920	792 635	814 061	936 141	918 311	988 358	1 079 679
Consolidated data	Cons 31/12/2011 th ZAR	Cons 31/12/2012 th ZAR	Cons 31/12/2013 th ZAR	Cons 31/12/2014 th ZAR	Cons 31/12/2015 th ZAR	Cons 31/12/2016 th ZAR	Cons 31/12/2017 th ZAR	Cons 31/12/2018 th ZAR

	"000	"000	"000	"000	"000	"000	"000	"000
Notes								
Impaired Loans (Memo)	42 879	30 007	23 959	20 675	19 462	22 918	23 164	38 455
Loan Loss Reserves (Memo)	11 388	12 998	12 756	12 323	11 995	14 210	13 623	22 467
Liquid Assets (Memo)	119 134	93 932	85 923	102 440	143 956	120 994	146 860	191 392
Intangibles (Memo)	700	1 160	1 303	1 422	2 029	2 339	3 861	7 246
Off Balance Sheet Items	63 210	68 174	137 888	146 453	176 413	158 207	178 032	212 666
Hybrid Capital (Memo)	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Subordinated Debts (Memo)	14 051	17 907	15 762	10 535	12 954	15 679	15 866	20 052

Appendix B Statement of Financial Position: Capitec Bank 2012-2019

CAPITEC BANK HOLDINGS LIMITED	Years							
Consolidated data	Cons 29/02/2012 th ZAR	Cons 28/02/2013 th ZAR	Cons 28/02/2014 th ZAR	Cons 28/02/2015 th ZAR	Cons 29/02/2016 th ZAR	Cons 28/02/2017 th ZAR	Cons 28/02/2018 th ZAR	Cons 28/02/2019 th ZAR
	"000	"000	"000	"000	"000	"000	"000	"000
Balance sheet								
Assets								
Loans	16 863	27 935	30 053	32 484	35 760	39 205	41 814	44 515
Gross Loans	18 408	30 658	33 690	36 341	40 891	45 135	47 642	54 879
Less: Reserves for Impaired Loans/ NPLs	1 545	2 723	3 637	3 857	5 132	5 930	5 828	10 364
Other Earning Assets	1 757	2 026	4 962	8 982	11 854	12 843	19 310	31 588
Loans and Advances to Banks	558	0	0	504	806	757	4 767	11 107
Derivatives (Assets)	0	3	203	36	225	58	0	0
Other Securities	1 199	2 023	4 759	8 442	10 823	12 027	14 544	20 481
Remaining earning assets	0	0	0	0	0	0	0	0
Total Earning Assets	18 620	29 961	35 015	41 466	47 614	52 048	61 125	76 103
Fixed Assets	543	698	855	849	1 111	1 523	1 754	2 210
Non-Earning Assets	4 458	7 688	10 321	11 602	14 220	19 787	22 078	22 115
Total Assets	23 622	38 347	46 191	53 916	62 946	73 358	84 957	100 428
Consolidated data	Cons 29/02/2012 th ZAR	Cons 28/02/2013 th ZAR	Cons 28/02/2014 th ZAR	Cons 28/02/2015 th ZAR	Cons 29/02/2016 th ZAR	Cons 28/02/2017 th ZAR	Cons 28/02/2018 th ZAR	Cons 28/02/2019 th ZAR

	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR
Liabilities & Equity								
Deposits & Short Term Funding	11 827	19 013	24 965	31 330	39 355	48 863	57 925	71 861
Total Customer Deposits	10 363	17 179	23 601	30 029	37 787	48 039	57 824	71 365
Deposits from Banks	167	143	0	0	156	0	0	0
Other Deposits and Short-term Borrowings	1 297	1 692	1 364	1 302	1 412	824	100	495
Other interest bearing liabilities	3 870	7 180	8 421	7 851	7 289	5 901	5 274	4 598
Derivatives (Liabilities)	3	26	1	22	0	46	55	15
Trading Liabilities	0	0	0	0	0	0	0	0
Long term funding	3 867	7 154	8 421	7 829	7 289	5 855	5 220	4 583
Other (Non-Interest bearing)	2 715	3 612	2 812	3 107	2 535	2 395	2 800	2 203
Reserves	25	28	11	64	108	81	67	91
Equity	5 185	8 513	9 982	11 564	13 659	16 118	18 892	21 676
Total Liabilities & Equity	23 622	38 347	46 191	53 916	62 946	73 358	84 957	100 428
Consolidated data	Cons 29/02/2012 th ZAR	Cons 28/02/2013 th ZAR	Cons 28/02/2014 th ZAR	Cons 28/02/2015 th ZAR	Cons 29/02/2016 th ZAR	Cons 28/02/2017 th ZAR	Cons 28/02/2018 th ZAR	Cons 28/02/2019 th ZAR
	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Qual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR	12 months Unqual IFRS AR
Notes								
Impaired Loans (Memo)	724	1 407	2 417	2 674	3 314	4 239	3 863	8 440
Loan Loss Reserves (Memo)	1 545	2 723	3 637	3 857	5 132	5 930	5 828	10 364

Liquid Assets (Memo)	5 365	8 512	13 707	13 215	13 349	17 756	23 953	28 081
Intangibles (Memo)	69	136	201	239	243	280	283	316
Off Balance Sheet Items	1 382	1 678	1 727	1 981	3 157	3 399	3 333	3 874
Hybrid Capital (Memo)	0	0	0	0	0	0	0	0
Subordinated Debts (Memo)	1 091	2 932	2 932	2 937	2 937	2 685	2 482	1 846

Appendix C Detailed Financial Data of ABSA Bank

YEARS	2011	2012	2013	2014	2015	2016	2017	2018
GOALS	000" Rands	000" Rands	000" Rands	000" Rands	000" Rands	000" Rands	000" Rands	000" Rands
Assets								
Loans	486,719	501,682	526,884	540,848	578,599	614,530	641,176	705,786
Loans and Advances to Banks	57,486	44,440	46,249	47,038	76,643	55,412	62,533	69,947
Derivatives (Assets)	49,773	52,249	48,616	42,630	79,753	46,751	59,140	46,130
Other Securities	107,506	123,334	128,554	141,708	142,140	140,405	162,597	189,797
Remaining earning assets	1,840	331	240	252	518	222	0	180
Total Earning Assets	**703,324**	**722,036**	**750,543**	**772,476**	**877,653**	**857,320**	**925,446**	**1,011,840**
Liabilities								
Total Customer Deposits	423,028	466,241	488,049	519,491	556,030	560,842	578,825	592,854
Deposits from Banks	53,436	44,588	67,035	56,269	65,646	64,118	79,110	140,752
Other Deposits and Short-term Borrowings	79,775	44,471	55,051	55,785	63,746	76,429	71,266	0
Derivatives (Liabilities)	51,159	50,618	50,629	44,362	90,856	42,770	52,810	36,551
Trading Liabilities	529	1,126	2,472	2,795	1,242	1,787	8,141	26,960
Long term funding	62,327	78,359	57,890	59,765	77,661	78,823	82,542	164,206
Other (Non-Interest bearing)	13,333	16,211	12,888	14,811	18,493	22,183	28,262	31,988
Reserves	1,457	1,394	1,362	1,857	1,970	2,060	2,073	2,682
Total Liabilities	**685,044**	**703,008**	**735,376**	**755,135**	**875,644**	**849,012**	**903,029**	**995,993**
Equity								
Shareholders equity	52,590	56,220	52,565	54,280	55,842	64,629	80,683	79,051
Non-controlling interests	158	48	50	2	11	26	2	-9
Total equity	**52,748**	**56,268**	**52,615**	**54,282**	**55,853**	**64,655**	**80,685**	**79,042**
Profit								
Net income	**8,193**	**7,482**	**8,735**	**9,300**	**10,047**	**9,934**	**8,477**	**8,022**

Earnings								
Retained earnings	**37,217**	**38,011**	**34,506**	**33,713**	**32,033**	**36,099**	**37,855**	**35,209**
Total	**1,486,526**	**1,526,805**	**1,581,775**	**1,624,906**	**1,851,230**	**1,817,020**	**1,955,492**	**2,130,106**

Appendix D: Detailed Financial Data of Capitec Bank

CAPITEC BANK HOLDINGS LIMITED

YEARS	Cons 29/02/2012 th ZAR	Cons 28/02/2013 th ZAR	Cons 28/02/2014 th ZAR	Cons 28/02/2015 th ZAR	Cons 29/02/2016 th ZAR	Cons 28/02/2017 th ZAR	Cons 28/02/2018 th ZAR	Cons 28/02/2019 th ZAR
GOALS	"000	"000	"000	"000	"000	"000	"000	"000
Assets								
Loans	16 863	27 935	30 053	32 484	35 760	39 205	41 814	44 515
Loans and Advances to Banks	558	0	0	504	806	757	4 767	11 107
Derivatives (Assets)	0	3	203	36	225	58	0	0
Other Securities	1 199	2 023	4 759	8 442	10 823	12 027	14 544	20 481
Remaining earning assets	0	0	0	0	0	0	0	0
Total Earning Assets	**18 620**	**29 961**	**35 015**	**41 466**	**47 614**	**52 048**	**61 125**	**76 103**
Liabilities								
Total Customer Deposits	10 363	17 179	23 601	30 029	37 787	48 039	57 824	71 365
Deposits from Banks	167	143	0	0	156	0	0	0
Other Deposits and Short-term Borrowings	1 297	1 692	1 364	1 302	1 412	824	100	495
Derivatives (Liabilities)	3	26	1	22	0	46	55	15
Trading Liabilities	0	0	0	0	0	0	0	0
Long term funding	3 867	7 154	8 421	7 829	7 289	5 855	5 220	4 583
Other (Non-Interest bearing)	2 715	3 612	2 812	3 107	2 535	2 395	2 800	2 203
Reserves	25	28	11	64	108	81	67	91
Total Liabilities	**18 436**	**29 834**	**36 209**	**42 353**	**49 286**	**57 240**	**66 066**	**78 752**
Equity								

Equity	5 185	8 513	9 982	11 564	13 659	16 118	18 892	21 676
Profit								
Net Income	1 094	1 605	2 038	2 564	3 228	4 387	4 471	5 295
Earnings								
Retained earnings	2 002	2 939	4 130	5 701	7 772	10 330	13 153	15 950
Total	**45 338**	**72 852**	**87 373**	**103 647**	**121 560**	**140 122**	**163 706**	**197 776**

Appendix E: The ABSA Bank solution output from LINGO 17.0

Variable	Value	Reduced Cost
D1MINUS	0.000000	1.000000
D2PLUS	0.000000	1.000000
D3MINUS	0.000000	1.000000
D4MINUS	0.000000	1.000000
D5MINUS	0.000000	1.000000
D6MINUS	0.000000	1.000000
X1	0.000000	0.000000
X2	2.032921	0.000000
X3	0.000000	0.000000
X4	2.408777	0.000000
X5	0.000000	0.000000
X6	2.659723	0.000000
X7	0.000000	0.000000
D1PLUS	0.000000	0.000000
D2MINUS	0.000000	0.000000
D3PLUS	0.000000	0.000000
D4PLUS	0.1865626	0.000000
D5PLUS	0.5059788	0.000000
D6PLUS	0.6929414	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000

Appendix F: The Capitec Bank solution output from LINGO 17.0

Variable	Value	Reduced Cost
D1MINUS	0.000000	1.000000
D2PLUS	0.000000	1.000000
D3MINUS	0.000000	1.000000
D4MINUS	0.000000	1.000000
D5MINUS	0.000000	1.000000
D6MINUS	0.000000	1.000000
X1	0.000000	0.000000
X2	1.929574	0.000000
X3	0.000000	0.000000
X4	0.000000	0.000000
X5	3.516145	0.000000
X6	0.5827271	0.000000
X7	0.5328767	0.000000
D1PLUS	0.2282598	0.000000
D2MINUS	0.000000	0.000000
D3PLUS	0.000000	0.000000
D4PLUS	0.000000	0.000000
D5PLUS	0.000000	0.000000
D6PLUS	0.2284998	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000

Appendix G: The Range Analysis for ABSA Bank

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
D1MINUS	1.000000	INFINITY	1.000000
D2PLUS	1.000000	INFINITY	1.000000
D3MINUS	1.000000	INFINITY	1.000000
D4MINUS	1.000000	INFINITY	1.000000
D5MINUS	1.000000	INFINITY	1.000000
D6MINUS	1.000000	INFINITY	1.000000
X1	0.000000	INFINITY	0.000000
X2	0.000000	0.000000	0.000000
X3	0.000000	INFINITY	0.000000
X4	0.000000	0.000000	0.000000
X5	0.000000	INFINITY	0.000000
X6	0.000000	0.000000	0.000000
X7	0.000000	INFINITY	0.000000
D1PLUS	0.000000	INFINITY	0.000000
D2MINUS	0.000000	INFINITY	0.000000
D3PLUS	0.000000	INFINITY	0.000000
D4PLUS	0.000000	0.000000	0.000000
D5PLUS	0.000000	0.000000	0.000000
D6PLUS	0.000000	0.000000	0.000000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
2	560.8798	3.460897	0.4800009
3	550.6248	1.026359	1.257913
4	41.71060	0.9490901	0.4011671
5	6.216800	0.1865626	INFINITY
6	24.94340	0.5059788	INFINITY
7	1184.375	0.6929414	INFINITY

Appendix H: The Range Analysis for Capitec Bank

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
D1MINUS	1.000000	INFINITY	1.000000
D2PLUS	1.000000	INFINITY	1.000000
D3MINUS	1.000000	INFINITY	1.000000
D4MINUS	1.000000	INFINITY	1.000000
D5MINUS	1.000000	INFINITY	1.000000
D6MINUS	1.000000	INFINITY	1.000000
X1	0.000000	INFINITY	0.000000
X2	0.000000	0.000000	0.000000
X3	0.000000	INFINITY	0.000000
X4	0.000000	INFINITY	0.000000
X5	0.000000	0.000000	0.000000
X6	0.000000	0.000000	0.000000
X7	0.000000	0.000000	0.000000
D1PLUS	0.000000	0.000000	0.000000
D2MINUS	0.000000	INFINITY	0.000000
D3PLUS	0.000000	INFINITY	0.000000
D4PLUS	0.000000	INFINITY	0.000000
D5PLUS	0.000000	INFINITY	0.000000
D6PLUS	0.000000	0.000000	0.000000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
2	28.58490	0.2282598	INFINITY
3	29.94240	0.2070178	0.7828663E-01
4	8.391300	0.9376422E-01	0.7545660E-01
5	1.938600	0.6436190E-01	0.3262650E-01
6	4.602700	0.2322523	0.2067316
7	73.45990	0.2284998	INFINITY