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FACULTY OF COMMERCE, LAW AND MANAGEMENT

AN ARCH/GARCH ARBITRAGE PRICING THEORY APPROACH TO MODELLING THE RETURN GENERATING PROCESS OF SOUTH AFRICAN STOCK RETURNS

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DECLARATION

I, Jan Jakub Szczygielski, declare that this dissertation is my own, unaided work, the substance of or any part of which has not been submitted in the past for any degree or examination in this or any other university or will be submitted in the future for a degree in this or any other university. The information contained herein has not been obtained during my employment or working under the aegis of, any other person or organisation other than this university. It is submitted in fulfilment of the requirements for the degree of a Masters of Commerce at the University of the Witwatersrand, Johannesburg.

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(Name of candidate) Signature

Signed this_________________ day of ___________________________ 2012 at Johannesburg.

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“Battle not with monsters, lest ye become a monster, and if you gaze into the abyss, the abyss gazes also into you.” - Friedrich Nietzsche

An ARCH/GARCH Arbitrage Pricing Theory Approach to Modelling the Return Generating Process of South African Stock Returns

ABSTRACT

This study investigates the return generating process underlying the South African stock market. The investigation of the return generating process is framed within the Arbitrage Pricing Theory (APT) framework with the APT reinterpreted so as to provide a conceptual framework within which the return generating process can be investigated. In modelling the return generating process, the properties of South African stock returns are taken into consideration and an appropriate econometric framework in the form of Autoregressive Conditional Heteroscedastic (ARCH) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models is applied. Results indicate that the return generating process of South African stock returns is described by innovations in multiple risk factors representative of several risk categories. The multifactor model of the return generating process explains a substantial amount of variation in South African stock returns and the ARCH/GARCH methodology is an appropriate econometric framework for the estimation of models of the return generating process. The APT framework is successfully applied to model and investigate the return generating process of South African stock returns.

Keywords: Arbitrage Pricing Theory, Autoregressive Conditional Heteroscedastic, Generalized Autoregressive Conditional Heteroscedastic, ARCH, GARCH, time series, risk factors, multifactor model, return generating process.

JEL Classification: C01, C32, C51, G10, G12

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
AMEX	American Stock Exchange
A-PARCH	Asymmetric Power ARCH
APT	Arbitrage Pricing Theory
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedastic
ARCH-M	ARCH-in-Mean
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BP_t	Building Plans Passed
CAC	Compagnie des Agents de Change
CAPM	Capital Asset Pricing Model
CI_t	Composite Coincident Index
CGARCH	Component GARCH
COM_t	All Commodity Index
CPI_t	Consumer Price Index
CREF	College Retirement Equity Funds
CRSP	Centre for Research in Security Prices
CTT_t	Composite Coinc. Index of Trad. Partners
DAX	Deutscher Aktien IndeX
DJ_t	Dow Jones Industrial Average (denotes the DJIA in empirical analysis)
DJIA	Dow Jones Industrial Average
DTS_t	Changes in the Term Structure
EGARCH	Exponential GARCH
EIV	Errors-in-variables
EWMA	Exponentially Weighted Moving Average
FIGARCH	Fractionally Integrated GARCH
FT 30	Financial Times Ordinary Share 30
FTSE	Financial Times Stock Exchange
$FTSE_t$	FTSE 100 Index (denotes the FTSE 100 Index in empirical analysis)
FTW_t	FTSE World Index
G7	Group of Seven
GARCH	Generalized Autoregressive Conditional Heteroscedastic
GARCH-M	GARCH-in-Mean
GDP	Gross Domestic Product
GED	Generalized Error Distribution
GLS	Generalized Least Squares
GNP	Gross National Product
$GOLR_t$	Rand Gold Price
HIS	Historical Average

HKI	Hong Kong Index
HSE	Helsinki Stock Exchange
IGARCH	Integrated GARCH
ISD	Implied Standard Deviation
JB	Jarque-Bera
JSE	Johannesburg Stock Exchange
LI_t	Composite Leading Index
LM	Lagrange Multiplier
LS	Least Squares
LSE	London Stock Exchange
LSPD	London Share Price Database
LTT_t	Composite Lead. Index of Trad. Partners
M_t	JSE All-Share Index (Total returns)
$M1A_t$	M1A (Narrow) Money Supply
$M3_t$	M3 (Broad) Money Supply
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
ME	Mean Error
MET_t	Metal Index
MP_t	Industrial Production
MSCI	Morgan Stanley Capital International
$MSCI_t$	MSCI World Index (denotes the MSCI World Index in empirical analysis)
$MSCIR_t$	MSCI World Index (Local Currency)
MSE	Malaysian Stock Exchange
NARCH	Non-linear ARCH
NASDAQ	National Association of Securities Dealers Automated Quotation
NBF	Net Business Formation
$NFCI_t$	Non-Fuel Commodity Index
N.I.I.D	Normally, Independently and Identically Distributed
NK_t	Nikkei 225
NLSUR	Non-linear Seemingly Unrelated Regression
NRI	Nomura Research Institute
NYSE	New York Stock Exchange
OIL_t	Rand Brent Crude Price
OECD	Organization for Economic Co-operation and Development
PPI_t	Producer Price Index
$\bar{R}^2$	Adjusted Coefficient of Determination
$RBAS_t$	Inflation Expectations
REIT	Real Estate Investment Trust
RMSE	Root Mean Square Error
S&P	Standard & Poor's
$SAGB10_t$	10 Year Government Bond Yield
$SAGB30_t$	30 Year Government Bond Yield

SARB	South African Reserve Bank
SIC	Schwarz Information Criterion
SLS_t	Retail Sales
SSE	Sydney Stock Exchange
SV	Stochastic Volatility
SWARCH	Switching ARCH
$TBT3_t$	Three Month Treasury Bill Rate
TGARCH	Threshold GARCH
TSE	Tokyo Stock Exchange *or* Toronto Stock Exchange (*contextual*)
TT_t	Terms of Trade
U	Prefix used to denote that a factor is in terms of innovations
UK	United Kingdom
US	United States
$ZARBA_t$	Rand/Currency Basket Exchange Rate
$ZARUS_t$	Rand-Dollar Exchange Rate

GLOSSARY OF TERMS

The abbreviations above are applicable, unless otherwise stated.

APT framework: Conceptual framework consisting of the linear factor model representative of the return generating process and the cross-sectional APT model relating expected returns to estimated factor loadings.

APT model: Model that relates expected (equilibrium) returns to factor loadings estimated within the linear factor model. Unlike the linear factor model which explains the time series behaviour of returns, the APT model explains cross-sectional differences in expected returns (D. Bower, R. Bower & Logue, 1984).

ARCH effect: Varying amplitude of returns or residuals over time indicative of unequal variance (Engle, 2001).

ARCH/GARCH framework: Econometric methodology based upon ARCH and GARCH models and their extensions. Used in the modelling of time-varying variance and the estimation of models of the return generating process.

ARCH and GARCH parameters: Terms in ARCH and GARCH models (e.g. lagged residual and conditional variance terms).

Assets: Financial instruments with a market value. Throughout this study, assets are also referred to as securities in generic terms, and bonds and stocks where appropriate.

Factor loading: Reaction of the dependent factor to movements of factors in the linear factor model (Reinganum, 1981). Also referred to in this study as a coefficient, factor beta, exposure or sensitivity.

Generalization/Generalizability (of model): Extension of a model specification/applicability of a model specification to an extended number of financial series which are not used in initial model building and estimation.

Heteroscedasticity: The unequal variance of random observations over time. In econometrics, this term is used to describe the non-constant variance of the residual terms obtained from regression (Gujarati, 2003; Brooks, 2008).

Idiosyncratic risk: Risk that is specific to an asset. Factors representative of idiosyncratic risk are factors that are unrelated to overall economic conditions (Roll & Ross, 1995). Also referred to in this study as unique, firm specific, diversifiable or unsystematic risk.

Innovations: Unexpected changes in factor series which are representative of the unexpected components of factors (Priestley, 1996).

Linear factor model: Model of the return generating process characterized by systematic risk factors under the APT framework. Used in the estimation of factor loadings within the APT framework. The linear factor model relates returns to movements in the factors describing the return generating process over time (Roll & Ross, 1980).

Priced factor: A factor for which the risk premium associated with a factor loading is statistically significant in the cross-sectional APT model (Roll & Ross, 1980).

Residual market factor: The unexpected change in the market index that is not attributable to factors in the base specification. The residual market factor is a *catch-all* proxy for omitted and unidentified risk factors in the return generating process specification (Burmeister & Wall, 1986; Van Rensburg 1996).

Returns: Changes in asset prices over a period of time.

Serial correlation: Statistical relationship between the time series of observations (Gujarati, 2003).

South African stock returns: Returns on the JSE All-Share Index *or* returns on the industrial sector and economic group indices that represent the economic groups and industrial sectors that compromise the JSE All-Share Index (contextual).

Systematic risk factor: Risk factor that has a pervasive influence on stock returns and reflects economy-wide risk (Roll & Ross, 1995). Systematic risk factors are those that influence returns on aggregates such as market indices and large portfolios.

1. INTRODUCTION

1.1. Background

The Arbitrage Pricing Theory (APT) proposed by Ross (1976) postulates a model which establishes the relationship between multiple sources of risk and asset returns (Gibbons, 1986). A number of studies, starting with those of Roll and Ross (1980) and Chen (1983), test the propositions of the APT by investigating the number of factors in the return generating process and the number of priced factors in expected (equilibrium) returns. The APT permits such inquiry as it consists of two equations. The first equation assumes that asset returns are generated by a *k*-factor model which describes the return generating process (Brealey & Meyers, 2003). The second equation relates factor loadings estimated in the *k*-factor model to expected returns (Roll & Ross, 1980). Early studies[1] of the APT rely upon factor analytic approaches to estimate loadings on factors and in doing so identify the structure of the return generating process (Reinganum, 1981). The use of factor analytic approaches however poses a limitation in that only the number and not the identity of factors in the return generating process is established. The first widely cited study to address this limitation is that of Chen, Roll and Ross (1986) who identify a set of macroeconomic factors that are considered to be proxies for systematic risk. A number of these pre-specified factors are found to be "priced" – able to explain the time series characteristics of returns *and* expected returns (Elton & Gruber, 1988). It is the latter aspect of the APT – the modelling of equilibrium relationships - that is studied widely.

Although investigations of the APT focusing upon asset pricing have been undertaken, there is another aspect of the APT framework that is not afforded the same consideration in the empirical literature, but is of immense interest; the theory underpinning the APT model that proposes that the return generating process can be described by a multifactor model. Whereas the APT may lead to useful predictions regarding expected returns even if the underlying multifactor model of the return generating process is not correctly specified, the time series behaviour of returns is still worthy of consideration if efficient estimates of factor loadings or sensitivities are desired (Gibbons, 1986). Elton, Gruber and Blake (1998) argue that the return generating process is an important building block in asset pricing models. However,

[1] See Roll and Ross (1980), Chen (1983), Kryzanowski and To (1983), Hughes (1984), Beenstock and Chan (1986), and Elton and Gruber (1988) amongst others.

according to Gibbons (1986), research on asset pricing models focuses upon testing the equilibrium implications of asset pricing models and assumes that the model of the return generating process is adequately specified. This suggests that in this line of research, less consideration is given to the structure of the return generating process.

Perhaps, the reason for this is the complexity of the return generating process. Researchers must specify the structure of the return generating process, identify factors that feature in the return generating process and define how these factors enter the return generating process. Fortunately, the APT provides a *conceptual framework* ("the APT framework") which permits the investigation of the return generating process. As a conceptual framework, the APT proposes that the return generating process is characterized by multiple factors entering as innovations and adapts existent theory so as to permit the identification of systematic risk factors that feature in the return generating process. The development of this framework can be traced through the literature. Early studies such as those of Roll and Ross (1980) focus upon identifying the structure of the return generating process and the pricing of assets. While the focus of these early studies is primarily asset pricing, it is acknowledged that returns are driven by multiple factors. Studies such as those of Chen *et al*. (1986) and Hamao (1988) employ pre-specified factors to explain expected returns and acknowledge an underlying return generating process characterized by (assumed) innovations in macroeconomic factors measuring the impact unspecified systematic risk factors. Studies such as those of Burmeister and Wall (1986) apply the APT *directly* as a framework to model the return generating process of asset returns.

1.2. Purpose and contribution of the study

The purpose of this study is to model and investigate the return generating process of South African stock market returns within the confines of the APT framework. To do so, it is first shown that the APT does indeed provide a framework for not only modelling the cross-section of expected returns but also for investigating the return generating process. In the first part of this study, the APT is reinterpreted as a conceptual framework with a focus upon explaining the time series behaviour of asset returns. It is within the APT framework that the model of the return generating process of South African stock returns is assumed to follow a *k*-factor structure. Furthermore, the category and identity of risk factors featuring in the return generating process is motivated by APT literature. Chen *et al*. (1986), Beenstock and Chan (1988), Elton and Gruber (1988), and Clare and Thomas (1994) acknowledge that the number of factors that are priced in cross-sectional studies and are assumed to have a pervasive influence on the time series behaviour of stock returns is potentially large. Notably, Berry, Burmeister and McElroy (1988: 30) state that "there is no 'correct' set of factors; there are many equivalent sets of correct factors," which give rise to similar empirical results. Fortunately, the APT framework sets out criteria for the identification of these factors, their derivation, statistical properties and identifies the genus of these factors. In investigating the return generating process of South African stock returns within the APT framework, a number of hypotheses are *indirectly* investigated:

Hypothesis 1: *South African stock returns are characterized by a multifactor return generating process.*

Hypothesis 2: *A multifactor model of the return generating process provides a better description of the time series behaviour of South African stock returns relative to a single-factor alternative.*

Hypothesis 3: *International risk plays an important role in explaining South African stock returns.*

Prior to building, modelling and investigating the return generating process of South African stock returns, a detailed consideration of the time series properties of stock returns and volatility is undertaken. This constitutes the second part of the study. The distribution of returns is described by both the mean and variance, and therefore a comprehensive investigation of the return generating process must take into consideration the properties of these two moments. Having considered the properties of the first two moments of the return distribution, an appropriate econometric framework that captures the observed properties is considered. Therefore, the return generating process of South African stock returns is modelled within an *appropriate* econometric framework.

The contribution of this study is therefore threefold. First, this study is not concerned with pricing implications; it is concerned with identifying and describing the return generating process of South African stock returns. In accordance with the APT framework, innovations in factors are used to model the return generating process (Berry *et al*., 1988). Second, this study models the return generating process as a multifactor model within an appropriate econometric framework, which takes into consideration the properties of stock returns and variance. This presents an improvement in terms of econometric methodology over a number of similar studies which rely upon the Least Squares (LS) methodology (see Burmeister & Wall, 1986; Berry *et al*., 1988; Bilson, Brailsford & Hooper, 2001). Finally, and perhaps most importantly, this study addresses a gap within the literature. Many studies employing multifactor models either directly or indirectly acknowledge the role of the APT framework in motivating for multifactor specifications (see Burmeister & Wall, 1986; Berry *et al*., 1988; Chen, Hsieh, Vines & Chiou, 1998; Cheung & Ng, 1998; Liow, 2004; Sadorsky, 2001; Sadorsky & Henriques, 2001; Sadorsky, 2008). These studies rely upon the APT framework to motivate for the factor structure of the model and to select risk factors used to explain returns. Whereas these studies serve as examples of how the APT is applied as a framework and reference is made to APT literature to motivate for multifactor cross-sectional and time series specifications, the link between the APT as an asset pricing theory and the APT as a framework is not adequately explored. This study addresses this gap; the role of the APT in informing the factor structure and identity of risk factors is investigated and the propositions of the APT framework are applied within a time series context.

1.3. Methodology

Aside from relying upon the APT framework as a conceptual framework, Autoregressive Conditional Heteroscedastic (ARCH) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models proposed by Engle (1982) and extended by Bollerslev (1986), Engle, Lilien and Robins (1987), Nelson (1991) and others are employed in modelling the return generating process of South African stock returns. This econometric framework ("the ARCH/GARCH framework") is well-suited for the modelling of stock returns as it discards the assumptions of normality, independence and a constant variance and provides insight into the second moment of the distribution (Elyasiani & Mansur, 1998; Engle, 2004). This framework is particularly appealing as it can be applied in a wide variety of contexts and is more robust to the presence of heteroscedasticity and ARCH effects in the residuals relative to the LS framework. The ARCH/GARCH framework provides a more statistically adequate and robust description of the return generating process (Sadorsky & Henriques, 2001).

1.4. Outline of the study

Chapter 2 outlines the APT framework and discusses some of its central propositions. Evidence concerning the validity of the framework is presented; if the APT is to serve as a conceptual basis for the investigation of the return generating process, its propositions must be valid. Important and relevant propositions of the APT framework are considered and it is shown that there is a linkage between the cross-sectional APT model describing expected returns and the underlying linear factor model describing the return generating process. Chapter 3 introduces the macroeconomic APT model and applications of the APT framework in examining the return generating process of stock returns are demonstrated. A theoretical model and criteria for selecting and identifying factors within the APT framework are outlined and discussed in Chapter 4, and a number of systematic risk factors are identified. A note of caution is in order here; the list of systematic risk factors is potentially large and therefore the purpose of this discussion is *not* to provide an exhaustive list of these factors, but rather to introduce this category of risk factors. Chapter 5 discusses the assumptions underlying the first two moments of the return distribution, namely the mean and the variance. Studies that challenge these assumptions are reviewed. Chapter 6 introduces the ARCH and GARCH (ARCH/GARCH) econometric framework and discusses its applicability not only to the modelling of the conditional variance, but also to the modelling of returns in various contexts. Chapter 7 outlines the methodology employed and presents empirical evidence on the properties of South African stock returns. The empirical analysis is undertaken in Chapter 8. The return generating process – motivated by the APT framework – is modelled and analyzed within the ARCH/GARCH econometric framework. Chapter 9 summarizes the primary empirical results, emphasizes the role of the APT framework as a conceptual basis, notes the role of the ARCH/GARCH framework and identifies areas for further research.

1.5. Delimitations

APT literature mainly focuses upon testing the APT model and implications arising from the equilibrium relationships between risk and returns. A number of studies, such as those of Hamao (1988) and Van Rensburg (1996, 2000) seek to establish which factors are priced in specific markets. Other studies, such as those of Poon and Taylor (1991) and Clare and Thomas (1994), challenge the findings of prior studies and point out deficiencies in prior work. This study is of an exploratory nature and focuses upon investigating the return generating process *within* the APT framework. As in Burmeister and Wall (1986) and Berry *et al*. (1988), estimates of risk premia are *not* of interest and the pricing of factors is not investigated. This study is a starting point for further research, be it of an exploratory nature or be it research seeking to establish equilibrium relationships in the South African stock market.

2. THE ARBITRAGE PRICING THEORY AS A CONCEPTUAL FRAMEWORK

2.1. An outline of the Arbitrage Pricing Theory

The Arbitrage Pricing Theory (APT) is a response to the acknowledgement that a number of factors other than market returns drive asset returns (Kandir, 2008). Chen *et al*. (1998) state that the APT is a natural successor to the Capital Asset Pricing Model (CAPM), its single-factor predecessor which relies upon a single factor, the market beta (β_M), to explain the cross-section of expected returns (see Sharpe, 1964). Similarly, Roll and Ross (1980) argue that the APT is a response to King's (1966) and Meyers' (1973) findings challenging the validity of a single-factor model and also a response to the findings of Basu (1977) and Banz (1981) which suggest that the CAPM β_M is not the only factor affecting expected returns. In fact, it is King (1966) who prior to the seminal work of Ross (1976) proposed a multifactor return generating process characterized by a number of "random change functions (factors)" and showed that returns can be decomposed into market and industry components (King 1966: 141). Similarly to the CAPM, the APT seeks to describe expected returns but in doing so, more than one source of risk is considered in explaining expected returns *and* in the underlying return generating process.

The APT was introduced by Ross (1976) around the same time studies challenging the CAPM began emerging and the role of multiple factors in the return generating process and equilibrium pricing relationships begun to be acknowledged. For example, in noting the role of multiple factors in the return generating process, Roll and Ross (1980: 1074) argue that the major difference between the CAPM and APT is that the APT permits a number of factors in the return generating process whereas the single-factor model - "the intuitive grey eminence behind the CAPM" - relies upon a single factor. The implication of permitting a number of factors to feature in the return generating process is that systematic risk may not necessarily be measured by a single factor as is the assumption underlying the CAPM, which postulates that systematic risk is fully captured by the market (Burmeister, Roll & Ross, 1994). In light of this, the APT can be considered as an alternative to the CAPM and as an alternative to a single-factor framework (Reinganum, 1981; Chen, 1983). However, unlike the CAPM which relies upon a broad stock market index as a proxy for systematic risk, the APT does not identify particular factors that proxy for systematic risks and further, does not indicate how many such factors exist (Burmeister *et al.*, 1994). Despite these limitations, the APT framework still qualifies as an attractive proposition from a theoretical perspective by proposing that the return generating process can be decomposed into a number of factors and that the compensation for bearing risk is reflected by several risk premia, as opposed to a single risk premium as assumed under the single-factor CAPM framework.

The APT framework is described by two equations. The first equation, often referred to as the *linear factor model,* is based upon the assumption that returns are generated by a *k*-factor return generating process. The discrepancy between *realized* returns on an asset and expected returns is postulated to equal the sum of the different quantities of risks relevant to an asset multiplied by realizations of corresponding risk factors and an asset specific shock (Ross, 1976; Roll & Ross, 1980; Berry *et al.*, 1988):

$$R_{it} - E(R_i) = \sum_{k=1}^{K} b_k F_{kt} + \varepsilon_{it} \tag{2.1}$$

where R_{it} is the realized return on asset *i* at time *t*, $E(R_i)$ is the expected return on asset i and $R_{it} - E(R_i)$ is the discrepancy between actual and expected returns on asset *i*. The sensitivity of asset *i* to realizations of systematic (common) risk factor *k* is denoted by b_k, which is the coefficient (beta, exposure or loading) on risk factor *k* denoted by F_{kt}. Each risk factor is assumed to be unpredictable at the beginning of every period and has an expected value of zero, which suggests that realized returns are equal to their expected returns $E(R_i)$ at the beginning of every period. Factor loadings can be estimated using factor analysis when factors are not specified or by ordinary LS regressions when factors are pre-specified (Reinganum, 1981; Chen *et al.*, 1986). In both instances, a time series model of the return generating process of asset returns is derived (Connor, 1995). The term “systematic” carries a particularly important implication within the framework; risk factors are assumed to have a pervasive influence on stock returns and reflect *systematic* or economy-wide risks common to all assets (Berry *et al.*, 1988; Roll & Ross, 1995). Dhrymes, Friend, M. Gultekin and N. Gultekin (1985) suggest that it is this assumption relating to the nature of the risk factors that makes models derived within the APT framework simple and parsimonious. As the assumption that returns are influenced by systematic risk factors is central to the APT framework, the factors that matter are those that move aggregates (market indices, large portfolios).

The APT framework also makes specific assumptions relating to the residuals (stochastic error terms) of the linear factor model. Residuals, denoted by ε_{it} in equation (2.1), are assumed to be asset specific where *i* represents asset *i* (Van Rensburg, 1996; Burmeister *et al*., 1994). The importance of the residuals stems from the assumption that idiosyncratic risk may be diversified away, and therefore, returns on aggregates are driven by systematic risk only (Roll & Ross, 1995). As a result, firm specific factors or events are relegated to the residuals, ε_{it}, which capture risk specific to asset *i* (Roll & Ross, 1980, 1995). The second assumption relating to the residuals of assets *i* and *j* as denoted by ε_{it} and ε_{jt} respectively, is that the covariance between the residuals is equal to zero:

$$\text{cov}[\varepsilon_{it}, \varepsilon_{jt}] = 0 \tag{2.2}$$

Strong dependence between ε_{it} and ε_{jt} implies that there are other systematic risk factors in the return generating process aside from the k hypothesized factors (Roll & Ross, 1980; Elton & Gruber, 1988). Van Rensburg (2000) cautions that this assumption is likely to be violated for specifications of the return generating process which employ pre-specified systematic risk factors. Finally, it is assumed that residuals are uncorrelated with factor realizations (Burmeister *et al*., 1994):

$$\text{cov}[\varepsilon_{it}, F_{kt}] = 0 \tag{2.3}$$

This assumption suggests that after controlling for multiple risk factors, the residuals no longer reflect the impact of these and other risk factors not directly incorporated into the return generating process. Equations (2.2) and (2.3) can be used to identify omitted risk factors and equation (2.1) is the motivation for studying the return generating process of South African stock returns within a multifactor framework. Whereas numerous studies have investigated the equilibrium implications and the theoretical underpinnings of the APT, the implication of equation (2.1) that returns can be described by a *k*-factor model within the

APT framework has not been considered as extensively. In other words, fewer studies have applied the APT as a framework to describe the time series behaviour of asset returns.

Aside from assuming a multifactor return generating process and making assumptions relating to the residuals, the APT framework retains a number of neoclassical assumptions:

1) Markets are assumed to be perfectly competitive and frictionless (Fama, 1995; Van Rensburg, 1996).
2) Investors prefer more wealth to less wealth and are therefore, risk averse wealth maximizers (Reinganum, 1981; Van Rensburg, 1996).
3) Individuals are assumed to have homogenous beliefs regarding the form of the return generating process (Van Rensburg, 1996).

Burmeister *et al*., (1994) and Brown and Reilly (2009) state that the APT framework is free of some of the restrictive assumptions of the CAPM, and this is evident from the limited set of assumptions above. Unlike the CAPM, returns are not required to be normally distributed, no assumptions are made regarding investors' utility functions and a market portfolio that contains all risky assets and is mean-variance efficient need not exist.

The second equation which completes the APT framework, referred to as the APT model or the APT relation[2] (see Elton, Gruber & Blake, 1995; Van Rensburg, 1996), establishes the equilibrium relationship between expected returns and risk premia by determining whether factors in the return generating process are "priced" (Roll & Ross, 1980; Amenc & Le Sourd, 2005). For a factor to be priced, estimates of risk premia, usually denoted by λ_k, must be statistically different from zero for an associated factor sensitivity to explain expected returns These risk premia, estimated using methods such as those of Fama and MacBeth (1973), represent compensation for exposure to systematic risk in the return generating process, which is rewarded by the market with increased expected returns (Van Rensburg, 1996; Burmeister *et al*., 1994). Exposure to systematic risk is rewarded because of the assumption

[2] Henceforth, this equation will be referred to as the "APT model" to distinguish it from the linear factor model representative of the return generating process specification in equation (2.1). This terminology is borrowed from Elton *et al*. (1995) who formerly define the cross-sectional model of expected returns as the "APT model." Although, this terminology is often used interchangeably to describe multifactor specifications of the return generating process and the cross-sectional relationship linking expected returns to factor sensitivities, a distinction is maintained throughout this study.

that although firm specific factors can influence returns on individual assets, these factors can be cancelled out by holding well-diversified portfolios. Therefore, any remaining risk is systematic in nature. Finally, central to the framework is the assumption that because risk-free arbitrage profits are impossible, a positive expected return can only be earned by taking on exposure to risk, which requires undertaking a net investment of funds (Burmeister *et al*., 1994). This implies that in equilibrium, the expected return on a zero investment and a zero systematic risk portfolio is zero assuming that firm specific risk is eliminated through diversification (Reinganum, 1981). Assuming that there exists a riskless asset - an asset with a zero beta - the expected return, $E(R_i)$, on asset *i* in equilibrium is denoted by the APT model as (Roll & Ross, 1980; Amenc & Le Sourd, 2005):

$$E(R_i) = \lambda_0 + \sum_{k=1}^{K} \lambda_k b_k \tag{2.4}$$

where λ_0 is the expected return on an asset with zero systematic risk (a riskless asset) and λ_k denotes the risk premium or the price of risk corresponding to the exposure (b_{ik}) to risk factor F_{kt} estimated in equation (2.1) (Roll & Ross, 1980; Chen, 1983; Burmeister *et al*., 1994). If the risk premium on the exposure to a given risk factor is statistically significant, then this factor is said to be priced. Such a finding leads to the main implication of equation (2.4); the APT model explains *cross-sectional* differences in expected returns. Assuming that factor realizations have an expected value of zero at the beginning of each period, the APT model states that expected returns are equal to λ_0, and the sum of different types of risk *quantified* by the respective b_k s multiplied by the corresponding risk premium λ_k (Berry *et al*., 1988; Burmeister *et al*., 1994). A divergence between the number of factors in equation (2.1) and the number of priced factors in equation (2.4) may arise; although *k*-factors may characterize the return generating process, not all may be priced with the number of priced factors ranging between 1 and *k* (Chen, 1983). Importantly, Chen (1983) emphasizes that although certain factors may not be priced, this does not mean that they are irrelevant. Factors that are not priced are not relevant in explaining expected returns, but still play a role in informing investment decisions and in explaining the return generating process. Elton and Gruber (1988) define factors that are not priced as those that only explain the time series behaviour of returns and factors that are priced as those that explain both the time series behaviour of returns *and* expected returns. Risk factors that explain expected returns can however only be identified through empirical research, which requires returns to exhibit sensitivity to realizations of the risk factors. This, in essence, motivates for a time series analysis of the relationships between systematic risk factors and asset returns (Berry *et al*., 1988). The arguments of Chen (1983) and Elton and Gruber (1988) are important as they suggest that APT studies, which focus on explaining expected returns, also provide insight into the identity of factors that explain the time series behaviour of returns. Moreover, such studies *acknowledge* a multifactor return generating process. In this spirit, seminal studies such as Chan, Chen and Hsieh (1985), Chen *et al*. (1986) and Hamao (1988) provide insight into factors that explain returns over time and tacitly identify the structure of the return generating process.

Essentially, the APT is a two-stage framework. In the first stage, factor loadings are estimated using time series techniques and the return generating process is modelled. In the second stage, the equilibrium relationships between expected returns and the risk premia associated with exposures to risk factors in the return generating process are estimated in cross-sectional regressions. Factor loadings estimated in the first stage are used as "data" to obtain estimates of λ_k in the second stage (McElroy & Burmeister, 1988). This is the conventional approach to estimating risk premia. Equations (2.1) and (2.4) can be reinterpreted as a single equation, reflecting the role of the APT as a unified framework. Burmeister and Wall (1986) substitute the APT model in equation (2.4) into the model of the return generating process in equation (2.1). This translates into what Berry *et al*. (1988: 31) call the "full APT":

$$R_{it} = \lambda_0 + \sum_{k=1}^{K} \lambda_k b_k + \sum_{k=1}^{K} b_k F_{kt} + \varepsilon_{it} \tag{2.5}$$

The importance of the APT framework is threefold. First, it suggests that returns are described by a multifactor return generating process. Second, it suggests that there is more than a single source of systematic risk. Finally, it provides a framework for investigating the return generating process; Roll and Ross (1980) state that it is the formalism of the APT that suggests both the theoretical and empirical structure of the framework needs to be explored to understand which economic forces affect returns. According to the authors, the APT provides a solid theoretical framework for ascertaining whether multiple factors feature in the return generating process and whether these factors are priced.

Chapter 2 proceeds by outlining and investigating relevant and specific propositions that make the APT framework a suitable framework for modelling and investigating the return generating process. A motivation for a conceptual framework for the modelling of the return generating process is provided and the relevance of the stated propositions is contextualized (section 2.2). These propositions are then investigated with reference to the literature (section 2.2.1 - 2.2.4) and the limitations of the APT framework are noted (section 2.2.5). A summary of this chapter is provided in the conclusion (section 2.3).

2.2. Motivation for a conceptual framework

By postulating that multiple factors drive returns and that these factors are systematic in nature, the APT framework provides motivation and direction for investigating and modelling the return generating process within a multifactor context. In particular, there are four propositions stemming from the APT that warrant further consideration and are indicative of the APT's role as a conceptual framework within which the return generating process may be modelled and investigated (Roll & Ross, 1980). These propositions are as follows:

1) The return generating process can be described by a linear *k*-factor model.
2) Expected returns are explained by factors that feature in the return generating process – expected returns depend upon *k*-factors and *k*-factors are priced reflecting the structure of the return generating process (Beenstock & Chan, 1986).
3) Only factors indicative of systematic risk explain expected returns.
4) A multifactor model of returns is superior in terms of explanatory power relative to a single-factor alternative.

Proposition (1) suggests that the return generating process can be described by a multifactor model (as opposed to a single-factor model) and as such, the return generating process should be modelled within a multifactor framework (section 2.2.1). Early studies, such as those of Roll and Ross (1980), Hughes (1984) and Beenstock and Chan (1986), rely upon factor analytic techniques[3] and various selection criteria to derive the optimal or sufficient number

[3] For a brief review of classical factor analytic techniques see Kryzanowski and To (1983).

of factors required to describe the return generating process. The advantage of a factor analytic approach is that the number of relevant factors in the return generating process is derived as opposed to imposed as is the case with models employing pre-specified factors (Kryzanowski & To, 1983). Proposition (2) suggests that priced factors – factors that explain expected returns - are also those that explain the time series behaviour of returns (Elton & Gruber, 1988). This proposition is usually not considered in detail as most APT studies employing pre-specified factors, such as those of Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988), focus upon explaining the cross-section of expected returns and give little consideration to the underlying return generating process and the role of pre-specified risk factors in the return generating process. Nevertheless, if it can be shown that *some* factors in the return generating process are reflected in the cross-section of expected returns, then there is a basis for drawing inferences from APT literature which focuses on explaining equilibrium returns and applying these inferences in investigating the return generating process (section 2.2.2). Indeed, this line of reasoning is accepted by Elton, Gruber, Brown and Goetzmann (2003) who state that although the work of Chen *et al*. (1986) – which is directly based upon the APT framework – focuses upon explaining expected returns, it provides a framework for multifactor models of the return generating process. Many studies that directly draw upon the APT model and the framework in general to motivate for multifactor time series specifications of the return generating process fail to establish the link between the APT model and the underlying multifactor return generating process (see Bower *et al*., 1984; Liow, 2004). For example, Liow (2004: 51) argues that the multifactor time series model of the return generating process of returns on commercial real estate in Singapore is “governed conceptually by the multiple-factor model implied under the APT.” It is assumed in Liow’s (2004) study that the APT can be applied to explain the time series variation in returns without consideration being given to the linkage between the APT model and the underlying linear factor model. Together, these two models form the APT framework although they are often considered in isolation. Showing that factors that feature in the return generating process are also those that explain the cross-section of expected returns lends credence to the application of inferences drawn from cross-sectional literature to time series analysis and to the argument that the APT is a conceptual framework within which the return generating process can be investigated. By showing that expected returns reflect the nature of the return generating process, an often ignored linkage between the two components of the framework is established and the case for using the APT as a framework is strengthened. If markets compensate exposure to multiple risk factors in the return generating process, then multiple risk factors must explain stock returns over time. The argument that inferences can be drawn from APT literature if such a linkage is shown to exist is especially pertinent given Elton and Gruber's (1988) assumption that priced factors also explain the time series behaviour of returns.

If propositions (1) and (2) hold, then the APT provides impetus for a detailed examination of the return generating process by suggesting that multiple factors explain returns over time. For proposition (2) to be accepted, the number of priced factors must reflect the number of factors in the return generating process (Chen, 1983). Proving proposition (2) suggests that factors identified as priced also explain the time series variation in returns. This proposition is relatively easy to confirm; all that is required is that when the return generating process is shown to incorporate a number of factors, some of these factors are priced. Additionally, testing proposition (2) also translates into a test of the APT model by showing that systematic risk is not measured in only one way as suggested by the CAPM (Burmeister *et al.*, 1994). Moreover, showing that multiple factors are priced also suggests that a multifactor return generating process is indeed the basis for a relationship between expected returns and sensitivities to multiple risk factors (Bower *et al*. 1984).

An important question relating to the return generating process and the APT model is the identity of factors. While the APT framework does not specifically identify factors, some guidance is provided in this regard. The APT suggests that only systematic risk factors explain expected returns and therefore it is systematic risk that is of interest in the return generating process. Proposition (3) can be confirmed by showing that only systematic risk factors are priced (section 2.2.3). This proposition originates from the assumption that within the APT framework, risk exposures to systematic risk factors determine the volatility and performance of well-diversified portfolios (Burmeister *et al*., 1994). Confirming this proposition requires for it to be shown that once diversification has taken place, the only sources of risk that remain – and that are priced - are systematic risks not eliminated through diversification. In the context of APT literature which focuses upon cross-sectional relationships, this can be investigated by showing that expected returns on well-diversified portfolios are not explained by factors representative of idiosyncratic risk (Reinganum, 1981; Burmeister *et al*., 1994). Proposition (4) does not follow directly from the assumptions underlying the APT framework, but is based upon the reasoning that there is no justification in relying upon a more complex framework if it does not convey more information relative to a simpler model (Reinganum, 1981). A simpler model and an alternative theoretical framework are offered by the CAPM which is based upon the assumption that market betas completely explain the cross-section of expected returns (Campbell, Lo & MacKinlay, 1997; Beenstock & Chan, 1986). Underlying the CAPM is Sharpe's (1963: 281) single-factor model[4] which is considered by Roll and Ross (1980) to be a simpler model of the return generating process; an alternative to the multifactor linear factor model assumed to underpin the APT model. Proposition (4) can be confirmed by showing that a multifactor model within the context of the APT framework provides a superior description of the return generating process and the cross-section of expected returns relative to a single-factor model (addressed in section 2.2.4; discussed within the South African context in Chapter 8). If this is the case, then adopting the multifactor APT framework is justified. If this is not the case, the single-factor CAPM framework should be adopted. As the APT model is central to the framework and is widely studied, establishing that the APT model is superior to an alternative in explaining expected returns provides a basis for *inferring* that a multifactor model of the return generating process is superior in explaining the time series behaviour of returns. Notwithstanding this, the explanatory power of the underlying linear factor model is also of direct interest.

2.2.1. Number of factors in the return generating process

The literature recognizes that the APT framework can be used to establish the number of factors in the return generating process (see Barr, 1989). Roll and Ross (1980) undertake the first extensive empirical study of the APT. Time series and cross-sectional analysis is conducted using a two-step approach utilizing returns on stocks listed on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) in the United States (US) over the July 1962 to December 1972 period. In the first step, time series data is used to estimate factor loadings on groups of individual stocks[5] and in the second step, factor loadings are used to explain the cross-section of expected returns. The results of the time series analysis relating to the structure of the return generating process indicate that at most five factors are required to explain the return generating process. Although this is not a direct test of the APT model, these results support the proposition of a multifactor return generating process.

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[4] The original formulation of Sharpe's (1963) "single-index" model is as follows:
$R_i = A_i + B_i I + C_i$ where A_i and B_i are parameters of the model and C_i is a random factor. Sharpe (1963) postulates that I is the level of any factor that is deemed to be the single most important factor influencing returns, such as the level of the stock market as a whole or the Gross National Product (GNP).

[5] To estimate factor loadings, 42 groups of 30 stocks each are used.

Kryzanowski and To (1983) state that the number of factors in the return generating process is of particular interest; if the number of factors is equal or similar to the number of assets in the economy, or if the number of factors is excessively large, then the APT framework is unsatisfactory and not a viable conceptual framework for the modelling of the return generating process and asset pricing. While recognizing that there are a number of assumptions underlying the APT framework, the authors seek to validate the assumption that the return generating process underlying the NYSE and Toronto Stock Exchange (TSE)[6] is characterized by at least one systematic risk factor. In contrast to the findings of Roll and Ross (1980), returns on US securities are characterized by at least ten factors over the entire sample period, although factors beyond the fourth and fifth factor are seemingly trivial in terms of explanatory power (Kryzanowski and To, 1983: Table 4). Having noted that Roll and Ross' (1980) samples of 30 securities require no more than five factors, in contrast to their samples of 50 securities which require ten factors, Kryzanowski and To (1983) investigate the relationship between sample size and the minimum number of factors. Results suggest that although the number of factors is positively related to sample size, this does not mean that the number of *non-trivial* factors is dependent upon sample size; the number of *relevant* factors ranges between four and five regardless of sample size. This proposition is supported by the finding that factors beyond the fourth or fifth factor lie on a flat gradient according to the scree test[7] (see Kryzanowski & To, 1983: Figure 1). This finding is interpreted as implying that factors beyond the fifth factor are either trivial or non-trivial but not general in that they are important for specific security subsets or over specific time periods. In the latter case, where factors are specific to certain security subsets, these factors are not systematic and therefore do not qualify as legitimate APT factors. Kryzanowski and To (1983) conclude that a five-factor structure should be sufficient from an economic perspective to describe the return generating process – a conclusion similar to that of Roll and Ross (1980). Results of the analysis conducted on Canadian securities yield similar conclusions (see Kryzanowski & To, 1983).

Hughes (1984) suggests that the theoretical formulation of the APT formalizes the notion of a multifactor return generating process and given this proposition, seeks to test the hypothesis that Canadian security returns are described by multiple factors. Unlike Roll and Ross (1980),

[6] The sample period for US securities covers the January 1948 to December 1977 period and for Canadian securities, the sample period is from January 1962 to December 1971.
[7] Kryzanowski and To (1983) state that the scree test is similar to tests used in stepwise regression to examine changes in the coefficient of determination.

Hughes (1984) not only considers the number of factors required to explain returns on two groups of Canadian securities listed on the TSE over the January 1971 to December 1980 period, but also investigates the amount of time series variation explained by systematic risk factors. Initially, a model with twelve factors is estimated using factor analysis for each group. Together, these factors explain over 50 percent of the variation in returns. The first factor accounts for a third of the variation explained and therefore, plays the most important role in explaining the variation in returns.[8] The remaining eleven factors account for approximately a fifth of the variation in returns. In light of these results, Hughes (1984) states that although these additional eleven factors explain less variation relative to the first factor, a multifactor model nevertheless has greater explanatory power than a single-factor model. Moreover, results indicate that the four factors subsequent to the first factor individually explain between 2 percent and 4 percent of the variation in returns, whereas the last seven factors individually explained between 1 percent and 2 percent of the variation in returns. Together, the four factors subsequent to the first factor explain an additional 10.8 percent of the variation in Group A returns and an additional 12.3 percent of the variation in Group B returns.[9] Similarly to the findings of Roll and Ross (1980) and Kryzanowski and To (1983), Hughes' (1984) findings suggest that five factors explain *most* of the variation in returns and factors beyond the fifth factor contribute marginally to the explanation of the return generating process.

Beenstock and Chan (1986) investigate numerous aspects of the APT framework using data for the December 1961 to December 1981 period from the London Share Price Database (LSPD). In an initial set of tests, the structure of the return generating process is explored using three samples of 80 firms each. Linear factor models with a minimum number of factors are fitted initially and the number of factors is increased by one at a time until likelihood ratio tests of the goodness-of-fit of the proposed model are no longer statistically significant. The stationarity of the return generating process is also tested by fitting models over two sub-periods (Beenstock & Chan, 1986: Table 1). Over the entire sample period of twenty years, the number of factors identified for the three sample groups is sixteen, nineteen

[8] A possible reason for the disproportionate explanatory power of the first factor is that this factor is the market index (Chen, 1983).
[9] If one considers the percentage increase in the proportion of variation explained by the additional four factors, then the percentage increase in the proportion of variation explained is 36.49 percent for Group A and 38.43 percent for Group B. These values are arrived at by subtracting the proportion of total variation explained by the first factor from the proportion of total variation explained by the first five factors and dividing by the proportion of total variation explained by the first factor.

and twenty-two respectively. Over the sub-periods, the number of factors for each sample remains more or less constant, suggesting that the return generating process is of a stationary nature. Given the extensive number of factors in the return generating process, Beenstock and Chan (1986) acknowledge that the number of factors may increase with sample size. The authors show that when samples consist of approximately 45 stocks, the number of factors is over ten, but when a sample consists of 80 stocks, the number of factors in the return generating process is approximately twenty. This is attributed to the emergence of sector specific factors which *should* be diversifiable and are therefore not relevant within the APT framework - a proposition similar to that put forward by Kryzanowski and To (1983). Beenstock and Chan's (1986) observation regarding the positive relationship between the number of factors and sample size is in line with Roll and Ross' (1984) argument that it is always possible to induce factors that are idiosyncratic and not systematic in nature with increases in sample size. By this line of reasoning, Beenstock and Chan (1986) argue that although the number of factors may increase with sample size, this does not invalidate the appropriateness of the APT framework as by definition, these factors are diversifiable. Similarly to Kryzanowski and To (1983) and Hughes (1984), Beenstock and Chan (1986) find that most of the explanatory power is concentrated in the first four factors and factors beyond the fourth factor contribute small and comparable amounts of explanatory power.

Elton and Gruber (1988) state that identifying factors which influence stock returns and modelling the return generating process is important for a number of applications such as estimating the covariance structure of returns for use in asset pricing and event studies. To identify the factor structure of the return generating process, the authors consider four groups of Japanese stocks compromising the Nomura Research Institute (NRI) 400 stock index over the April 1971 to March 1986 period. This index accounts for over half of the total capitalization on the Tokyo Stock Exchange (TSE) (as of April 1987). Based upon the Schwarz Information Criterion (SIC), three groups are found to be characterized by a four-factor return generating process whereas a single group is characterized by a three-factor return generating process (Elton & Gruber, 1988: Figure 1). Further tests conducted by Elton and Gruber (1988) confirm that the most likely number of *common* (as opposed to sample specific) factors in the return generating process is four. To confirm the multifactor structure of the return generating process, securities are re-sorted into size based portfolios and returns on these portfolios are regressed onto alternative two, three, four, five and six-factor solutions. The reported adjusted coefficient of determination ($\bar{R}^2$) indicates that gains in explanatory power beyond the fourth factor are negligible relative to the explanatory power of the first four factors. For example, the $\bar{R}^2$ increases from 0.686 when two factors derived from the first sample are used to explain returns to 0.771 for four factors and to 0.772 for six factors (Elton & Gruber, 1988: Table 3). These results demonstrate that the gain in explanatory power is negligible when additional factors beyond the first four factors are used to explain returns. Elton and Gruber (1988) go onto state that this finding supports the preliminary evidence that a four-factor model is sufficient to describe the return generating process of stocks on the TSE.

Roll and Ross (1980), Kryzanowski and To (1983), Hughes (1984), Beenstock and Chan (1986) and Elton and Gruber (1988) all arrive at a similar conclusion; returns across markets are described by a multifactor return generating process with the number of factors ranging between four and five. Regardless of what the exact number of factors is, what is certain and common across these studies is that the number of factors in the return generating process is *always* greater than one. In their entirety, these findings - made within the APT framework - support the proposition of a multifactor return generating process. It does, however, remain to be seen whether factors in the return generating process are priced.

2.2.2. Number of priced factors

In a formal test of the APT model, Roll and Ross (1980) find that in cross-sectional analysis with an assumed zero beta (risk-free) return of 6 percent, at least one factor is priced in 90 percent of groups of securities and two or more factors are priced in 60 percent of groups of securities considered in the study. Furthermore, a third of the groups exhibit at least three statistically significant risk premia. Roll and Ross (1980) conclude that at least three factors are priced and it is unlikely that more than four factors are reflected in expected returns. These findings suggest that a description of returns in equilibrium also relies upon a multifactor model and therefore, the APT model reflects the multifactor structure of the return generating process. The authors recognize that with APT factors being unidentified, it is possible that factors that generate returns differ from group to group and therefore, are not systematic as required by the APT framework. To determine whether the same factors generate returns in each sample, Roll and Ross (1980) test whether the intercept, λ_0, of the cross-sectional regressions used to estimate risk premia on factor exposures differs statistically across groups. As the intercept is assumed to be the expected return on a riskless rate and is unrelated to systematic factors, it should be identical across groups. The authors find that the intercepts do not differ across groups.[10] Although this implies a consistent underlying factor structure, Roll and Ross (1980) caution that this test is weak. Nevertheless, these findings suggest that the APT model reflects the multifactor structure of the underlying return generating process and point towards the existence of a set of *priced* factors that have a systematic effect upon returns.

Chen (1983) does not employ a factor analytic approach to derive the number of factors in the return generating process and instead *imposes* a five-factor model upon return series obtained from the Center for Research in Security Prices (CRSP) database. The five-factor model of the return generating process is, according to Chen (1983), motivated by the findings of Roll and Ross (1980) and therefore the number of relevant factors in the return generating process is not considered directly. Results of the cross-sectional regressions reveal that in the different sub-periods considered, the number of priced factors is *always* greater than one. For example, during the first sub-period (1963 to 1966) four factors are priced whereas during the third sub-period (1971 to 1974), two factors are priced. As in Roll and Ross (1980), the multiple priced factors reflect the multifactor structure of the (imposed) return generating process. Chen (1983) further tests whether the APT model has cross-sectional explanatory power by testing whether risk premia are jointly equal to zero. The null hypothesis is rejected confirming the presence of multiple priced factors in expected returns. The finding that at least two factors are priced, depending upon the sub-period considered, suggests that at least two factors explain the time series behaviour of returns even if the structure of the return generating process is not directly investigated. The finding that multiple priced factors are reflected in expected returns supports the proposition of an underlying multifactor return generating process.

Hughes (1984), having shown that up to twelve factors describe the return generating process, finds that between the two groups of Canadian securities considered, three to four factors are priced. As in Roll and Ross (1980) and Chen (1983), the presence of multiple priced factors imperfectly reflects the multifactor structure of the return generating process suggested by factor analysis. To ascertain the consistency of the factor structure, Hughes

[10] To obtain the time series of intercepts, cross-sectional regressions are estimated for *each* group using factor loadings for each asset in the 42 groups in the sample. For an outline of this procedure see Roll and Ross (1980).

(1984) investigates whether the intercept across the two groups is constant. Results suggest that this is indeed the case; the factors that describe expected returns on the first group are the same factors that explain expected returns on the second group. In an extension of the tests relating to the consistency of the factor structure, Hughes (1984) demonstrates that the intercept for both groups is equal to proxies of the risk-free interest rate in the form of rates on treasury bills and the bankers' acceptance rate. These findings suggest that not only does the intercept term have an economic interpretation, the intercept term in the APT model is equal to the risk-free rate of return as assumed by the APT framework. In a further test of the consistency of the factor structure, Hughes (1984) uses risk premia estimated for one group to explain returns on the alternate group.[11] Results show that the risk premia for a given group are able to explain expected returns for the alternate group suggesting that pricing relationships are not unique for each group.[12] According to the author, this points towards a consistency in the factor structure between sets of economic factors which drive returns for the two groups. Moreover, Hughes' (1984) findings demonstrate that multiple factors are priced and therefore, the cross-sectional APT model reflects the underlying multifactor return generating process.

Beenstock and Chan (1986) test the APT model by establishing how many of the factors in the return generating process are priced in three sub-samples spanning two sub-periods. In contrast to the findings of Roll and Ross (1980), in the first sub-period (1962 to 1971), the number of factors that are priced ranges between zero and two across sub-samples when a two-tailed *t*-test is applied and between two and three when a one-tailed *t*-test is applied. The finding of no priced factors is limited to a single sub-sample (Sample 3) when a two-tailed *t*-test is applied and it is common to observe between two and three priced factors for all sub-samples regardless of the type of *t*-test applied. For the second sub-period (1972 to 1981), the number of priced factors ranges between zero and one when a two-tailed *t*-test is applied and between one and two when a one-tailed *t*-test is applied. The correlation between the number of factors in the return generating process derived by Beenstock and Chan (1986) and the number of priced factors is weak. This is especially evident in the second sub-period, where only one factor is priced for most sub-samples. These findings suggest that the number of priced factors does not fully reflect the number of factors in the return generating process.

[11] For example, returns for each firm in Group A are regressed on estimates of *risk premia* for Group B.
[12] Hughes (1984) reports that when Group A returns are regressed on Group B risk premia, between 2 and 4 coefficients are statistically significant in 80 percent of regressions. When Group B returns are regressed on Group A risk premia, between 2 and 5 coefficients are statistically significant in 80 percent of regressions.

This inference is especially applicable to the second sub-period where the number of priced is centred on one, depending upon the type of *t*-test applied. In their entirety, Beenstock and Chan's (1986) findings provide somewhat ambiguous evidence relating to the linkage between the number of factors in the return generating process and the number of priced factors in the cross-sectional APT model.

Elton and Gruber's (1988) findings are similar to those of Beenstock and Chan (1986). After having found that four factors are sufficient to explain the return generating process underlying the returns on groups of securities compromising the NRI 400 stock index, factor sensitivities estimated in time series regressions are used to explain expected returns within the APT model. Results indicate that only a single factor is priced suggesting the multifactor structure of the return generating process is not fully reflected by the APT model. This suggests that an overt reliance upon the APT model to infer the number of factors in the return generating process in this instance will point towards a single factor where it has been shown by Elton and Gruber (1988) that four factors are more appropriate. Nevertheless, these findings suggest the APT model does reflect the structure of the return generating process, albeit weakly; a factor that features in the return generating process *is* priced in expected returns. However, these findings also suggest that the correlation between the number of priced factors in the APT model and the number of factors in the underlying return generating process is weak.

The findings of Roll and Ross (1980), Chen (1983) and Hughes (1984), Beenstock and Chan (1986) and Elton and Gruber (1988) suggest that relying upon the number of priced factors in the APT model to draw inferences regarding the structure of the underlying return generating process will result in an understatement of the number of factors in the return generating process. However, as in each of these studies one or more factors that feature in the return generating process are priced, it can be argued that the APT model indicates the *minimum* number of factors that feature in the return generating process. Most importantly, there *is* a linkage between the two components of the APT framework. Literature concerned with multifactor explanations of the return generating process often cites cross-sectional APT studies to motivate for multifactor specifications. As these cross-sectional APT studies usually acknowledge the multifactor structure of the return generating process which is (imperfectly) reflected by the APT model, these studies *can* serve as a motivation for multifactor specifications of the return generating process and *can* inform the identity of factors (Elton & Gruber, 1988; Elton *et al*., 2003). The presence of a linkage between the APT model and the return generating process (even if weak at times) suggests that inferences drawn from APT literature dealing with asset pricing form part of the APT framework in its entirety as a conceptual framework.

2.2.3. Role of undiversifiable risk

Roll and Ross (1980) investigate whether factors unrelated to systematic risk are priced. Prior results suggest that as many as four factors are priced, implying that investors are compensated for variation in the return generating process that arises from systematic (undiversifiable) risk. As the APT framework postulates that only systematic risk will be priced, a finding of priced factors that are unrelated to undiversifiable risk will lead to a rejection of the APT (Roll & Ross, 1980). Such a finding will be exceptionally disconcerting in the context of this study as the APT framework will no longer provide a definitive conceptual framework for the selection and identification of factors characterizing the return generating process.

Roll and Ross (1980) hypothesize that the total variance of individual returns should not explain expected returns as the diversifiable component of the total variance is eliminated through diversification and the non-diversifiable part is captured by factor loadings. To test whether the total variance plays a significant role in explaining expected returns, expected returns are regressed onto factor loadings *and* the standard deviation of individual returns within each group. Preliminary results indicate that in almost half of the groups considered, the standard deviation *is* statistically significant. However, Roll and Ross (1980) argue that this initial result arises due to positive skewness in individual returns which is believed to create a positive dependence between the sample mean and sample standard deviation. To address this problem, differing observations for each asset in each group are used to estimate factor loadings, the own standard deviation for each asset and to conduct the cross-sectional regressions.[13] Results indicate that the own standard deviation is priced in less than 10 percent of the sample groups considered. On the basis of these results, Roll and Ross (1980) argue that there is little reason to reject that the hypothesis that expected returns are

[13] Cross-sectional regressions are estimated using daily observations for days 1, 7 and 13…etc, the factor loadings are estimated using days 3, 9 and 15…etc, and own standard deviations are estimated using days 5, 11 and 17…etc. The own standard deviation is the standard deviation of returns on individual assets in Roll and Ross' (1980) sample. The time series of the risk premium on the own standard deviation is tested for statistical significance (see Roll & Ross, 1980: 1098).

unaffected by the own standard deviation. The finding of a statistically insignificant risk premium on the standard deviation for most groups supports the notion that it is only systematic risk that is priced and therefore, diversifiable risk does not play a role. This supports the proposition that only factors that have a systematic impact upon returns are of interest.

Similarly to Roll and Ross (1980), Chen (1983) tests the APT model against its own variance. Own variance is estimated using all even days in each sub-period and all securities are sorted into two portfolios characterized by high or low variance but the same factor loadings. It is hypothesized that if the returns on each portfolio do not differ significantly, then the APT model holds as own variance does not have an effect upon returns and all risk is reflected in factor loadings. If own variance does have an effect, then the portfolio with higher own variance should have returns that are higher than those on the portfolio with lower own variance. Results indicate that the difference between portfolio returns is statistically insignificant and therefore, the APT framework is validated. As in Roll and Ross (1980), factors representative of firm specific (idiosyncratic) risk are irrelevant after controlling for systematic risk (Chen, 1983: Table 5, Panel A). Chen (1983) however goes one step further than Roll and Ross (1980) and considers the impact of another firm specific risk factor - size. The size effect refers to the observation that small firms have higher returns relative to large firms after controlling for risk. In the context of the APT model, the presence of a size effect suggests that factor loadings do not fully capture risk and that some of the risk is reflected in a firm specific factor in the form of size (Banz, 1981; Reinganum, 1981). The rationale behind Chen's (1983) test is the same as before; firm size should not have an impact upon the returns on portfolios consisting of similarly sized firms after controlling for systematic risk. This hypothesis is tested by separating firms into two portfolios of assets with the same factor loadings but of different sizes; namely, small and large, and estimating the differences between average returns. Only for one out of the four sub-periods under consideration (1985-1978), a statistically significant difference between returns on the portfolios of small and large firms is observed after adjusting for systematic risk. However, this difference becomes statistically insignificant after adjusting for serial correlation. Based upon these results, Chen (1983) concludes that the null hypothesis of firm size having no explanatory power cannot be rejected after adjusting for risk captured by the factor loadings. Similarly to the results of Roll and Ross (1980), these findings suggest that idiosyncratic risk as measured by firm specific factors has no impact upon expected returns and therefore, only systematic risk is relevant.

Beenstock and Chan (1986) state that the APT framework implies that idiosyncratic risk should not be priced and in stating so, again echo this widely recognized tenet of the APT framework. To test whether this assumption holds for the stock market in the United Kingdom (UK), expected returns are estimated using odd months and own variance is estimated from even months covering the sample period. Cross-sectional regressions are run incorporating the estimated factor loadings and own variance as explanatory factors. The process is repeated using even months to obtain expected returns and odd months to obtain own variance. In two out of six instances – as Beenstock and Chan (1986) use three sub-samples – own variance is priced. However, Beenstock and Chan (1986) warn that because there is an overlap in the samples (220 securities divided into three samples of 80 securities each), even and odd observations are unlikely to be completely independent and thus, these mixed results should not be seen as a rejection of the null hypothesis of own variance not being priced. Given the recognized limitation of the test and the balance of probabilities, the null hypothesis of own variance not being priced cannot be rejected outright. These findings, although ambiguous, continue to point towards the relevance of systematic risk as opposed to idiosyncratic risk. Beenstock and Chan (1986) also consider the effect of firm size on UK firms. A test similar in rationale to that of Chen (1983) is conducted whereby firm size is used as an explanatory factor alongside factor loadings. Results indicate that firm size is statistically insignificant over the entire sample period and over the second sub-period (1972 to 1981).[14] This result is to be expected if only systematic risk is relevant.[15] In light of the latter finding, Beenstock and Chan (1986) state that the APT cannot be rejected upon the basis of a firm size effect although, the results of the test relying upon own variance to explain expected returns are somewhat ambiguous.

Yli-Olli and Virtanen (1992) investigate the applicability of the APT framework to Finnish stocks listed on the Helsinki Stock Exchange (HSE) over the February 1970 to December 1986 period. It is argued that because idiosyncratic risk can be diversified away, it should not be priced. To test whether this assumption holds within the APT framework, the authors employ a three-step procedure. After estimating factor loadings within a four-factor model, and using these factor loadings to explain expected returns, the resulting residuals are regressed onto the own variance and firm size. Results indicate that whereas own variance is

[14] Beenstock and Chan (1986) do not consider the first sub-period separately and therefore, no results are reported (see Beenstock & Chan, 1986: Table 11).
[15] It must however be noted that it is by no means certain that there is a firm size effect present within the UK market in the first instance.

statistically insignificant in all three sub-periods considered; the impact of firm size is statistically significant for a single sub-period (January 1981 to December 1986).[16] Yli-Olli and Virtanen (1992: 520) conclude that own variance and firm size have only "slight" explanatory power for expected returns suggesting that idiosyncratic risk is accounted for by the factor loadings.

The preceding discussion mostly supports the assumption that only systematic risk is priced; Roll and Ross (1980) find no evidence of an own variance effect, a finding confirmed by Chen (1983). Beenstock and Chan's (1986) results cast some doubt upon the findings of Roll and Ross (1980) and Chen (1983) with regard to the role of own variance in the UK market. However, on a balance of probabilities, their results are supportive of the assumption that only systematic risk is priced. Yli-Olli and Virtanen (1992) show by employing a three-step procedure that idiosyncratic risk is irrelevant for expected returns on Finnish stocks after controlling for systematic risk. These findings are generally congruent with Roll and Ross' (1980) argument that the absence of priced idiosyncratic risk factors implies that it is systematic risk present in the return generating process that is compensated. It is therefore this category of risk which should be considered when investigating the return generating process within the APT framework.

2.2.4. The APT framework versus a single-factor alternative

Reinganum (1981) states that although the APT framework is a plausible alternative to the single-factor CAPM framework, the reliance upon a more complicated model is justified only if it conveys more information relative to a simpler model. By this reasoning, the APT framework must provide a better description of returns relative to the CAPM to be considered a replacement and viable alternative. It is within this context that Chen (1983) investigates how the APT model fares against the CAPM. To investigate the cross-sectional explanatory power of the APT and CAPM, factor loadings are used to explain expected returns within the APT model and betas estimated using the Standard & Poor's (S&P) 500 Index, CRSP value-weighted and equally-weighted stock indices are used to explain expected returns within the CAPM. The resultant $\bar{R}^2$s are considered as indicators of explanatory power for the respective models. The $\bar{R}^2$ for the APT model employing factor loadings is found to be

[16] The other sub-periods span the February 1970 to December 1975 and the January 1976 to December 1980 periods.

almost double that of the CAPM employing market betas,[17] suggesting that the multifactor APT model is superior in explaining the cross-section of expected stock returns relative to the single-factor CAPM. To establish which model best describes actual returns, (r_i), actual returns are regressed on expected returns ($\hat{r}_i$) generated by the APT model and the CAPM:[18]

$$r_i = \alpha\hat{r}_{i,APT} + (1-\alpha)\hat{r}_{i,CAPM} + e_i \tag{2.6}$$

where $\hat{r}_{i,APT}$ and $\hat{r}_{i,CAPM}$ are returns generated cross-sectionally by the APT model and CAPM respectively. If the APT model is the correct model describing returns, then α, the coefficient on returns generated by the APT model, should be close to 1 if cross-sectional variation in r_i is fully explained by the APT model. Results for each of the sub-periods and indices used to estimated market betas are reported below:

Table 2.1: Estimated weights of the expected return from APT and CAPM

Period	S&P 500	α Value-Weighted Stock Index	α Equally-Weighted Stock Index
1963-1966	0.968	0.970	0.992
	(0.014)	(0.014)	(0.010)
1967-1970	1.006	0.994	0.952
	(0.014)	(0.011)	(0.010)
1971-1974	0.938	0.945	0.951
	(0.021)	(0.019)	(0.025)
1975-1978	0.953	0.970	0.994
	(0.014)	(0.010)	(0.020)

Notes:
1. Standard errors in parenthesis.

Source: Chen (1983)

Although, Chen (1983) notes that in a number of instances the estimated α differs from 1, it is clearly evident that the α s are all very close to one in all sub-periods and regardless of the market proxy used. These results are further supported by posterior odds analysis which overwhelmingly favours the APT model over the CAPM. This confirms that the multifactor APT model provides a more adequate description of expected returns relative to the single-factor CAPM. Furthermore, this implies that a multifactor model of the return generating process is also superior in describing returns; if the APT model is superior in describing

[17] The average $\bar{R}^2$ for the APT is 0.12 and 0.076 for the CAPM over all sub-periods and market proxies. The average $\bar{R}^2$ for the APT model is defined as the sum of $\bar{R}^2$ over the sub-periods divided by the number of sub-periods. For the CAPM, the average $\bar{R}^2$ is defined as the sum of $\bar{R}^2$ over the sub-periods *and* market proxies divided by the number of sub-periods multiplied by the number of market proxies used.

[18] See Chen (1983: 1398). APT ($r_i = \lambda_0 + \lambda_1\hat{b}_{i1} + ...\lambda\hat{b}_{ik} + \varepsilon_{it}$) against the CAPM alternative of $r_i = \lambda_0 + \lambda_1\hat{\beta}_i + \eta_i$ (Notation unchanged).

expected returns and the APT model reflects the underlying return generating process, it may be inferred that the underlying return generating process is superior in describing the time series behaviour of stock returns.

Bower *et al.* (1984) argue that it is undesirable to adopt a single-factor approach if it can be shown that a multifactor approach provides a better indication of asset risk. While the single-factor framework proposed by the CAPM has furthered the understanding of expected returns on assets, the APT model not only contributes to the understanding of expected returns but also offers a systematic link between expected returns and the return generating process. Unlike Chen (1983) who only considers the performance of the APT model against an alternative, Bower *et al.* (1984) consider both aspects of the APT framework. In the first test, the authors use returns on the CRSP value-weighted index to estimate a single-factor model of the return generating process underlying the CAPM. A four-factor model of the return generating process underlying the APT model is also estimated using factor scores.[19] The returns to be explained are returns on stock and bond portfolios over the January 1971 to December 1979 period.[20]

Returns on each portfolio are regressed on the CRSP value-weighted index and the factor scores. The results indicate that the average $\overline{R}^2$ for the multifactor return generating process underlying the APT model is 0.869 whereas the average $\overline{R}^2$ for the single-factor model of the return generating process underlying the CAPM is approximately 0.605. The implications of these results are best summarized by Bower *et al.* (1984: 1046) who state that "these findings are consistent with a conclusion that the APT provides a better description of the return generating process than does CAPM." A second test is conducted whereby factor scores are used to explain returns on a holdout sample consisting of the securities of electric utilities, gas companies, telecommunication providers and industrials.[21] As before, results indicate that the $\overline{R}^2$ is greater for 80 percent of individual stocks when the four APT factors are used to

[19] Factor scores are the values of a given factor at time *t* derived through factor analysis (Blume, M. Gultekin & N. Gultekin, 1986). Bower *et al.* (1984) refer to the return generating processes underlying the APT model and CAPM as the "characteristic line." This is another name for the return generating process (Ruppert, 2011).

[20] Twenty-six portfolios consisting of stocks and four bond portfolios. Another four stock portfolios, consisting of electric utilities, gas companies, telecommunication utilities and industrials are excluded from the estimation procedure and are treated as a holdout sample to test the predictive ability of each model in further tests.

[21] The factor scores are derived from a sample which excludes securities in the holdout sample. This addresses the criticism that the high level of explanatory power observed for the APT framework is the result of using factor scores derived from the very same return series that these factor scores are used to explain (see Bower *et al.*, 1984).

model the return generating process relative to the single-factor model characterizing the CAPM. The average $\bar{R}^2$ for the multifactor model is 0.323 whereas for the single-factor model, the average $\bar{R}^2$ is 0.263. Based upon these findings, Bower *et al*. (1984) again acknowledge that the multifactor APT framework is better at explaining the return generating process relative to the single-factor CAPM framework. Although, the authors warn that this superior explanatory power may be attributed to the use of factor scores derived from the returns that they are used to explain, this criticism is addressed by the use of a holdout sample.

It is desirable to extend the finding of superior explanatory power to the cross-sectional APT model, as this will indicate that both aspects of the APT framework - the return generating process and the APT model - are superior relative to a simpler framework. If the APT model reflects the superior explanatory power of the underlying return generating process, then it may be concluded that the superior cross-sectional explanatory power observed in studies focusing upon modelling returns in equilibrium is indicative of the explanatory power of the multifactor return generating process underlying the APT model. This indeed appears to be the case; Bower *et al*. (1984) find that the four-factor APT model explains over 40 percent of cross-sectional variation in expected returns whereas the CAPM explains just under 30 percent. The authors conclude that although the case for the APT is not absolute, the APT framework appears to be better at explaining both the time series and cross-sectional variation in returns. Moreover, it is demonstrated that both aspects of the APT framework - the return generating process and the APT model - are superior relative to a single-factor model in terms of explanatory power. A linkage between the superior explanatory power of the APT model and the underlying multifactor return generating process is demonstrated.

Beenstock and Chan (1986) compare the adequacy of the CAPM and APT models *within-sample* and *out-of-sample* by comparing the $\bar{R}^2$ of the two models. The APT model significantly outperforms the CAPM model in-sample; the average $\bar{R}^2$ for the APT model over the two sub-periods and three sub-samples is 0.263 whereas the average $\bar{R}^2$ for the CAPM is negligible.[22] Out-of-sample tests are conducted by estimating market betas and factor loadings over a ten-year period and running cross-sectional regressions over each month in the subsequent year (subsequent to estimation sample) starting in 1973. As before,

[22] 0.009 to be precise.

the multifactor APT model outperforms the CAPM; the average $\bar{R}^2$ for the CAPM is 0.023 whereas the average $\bar{R}^2$ for the APT model is substantially greater at 0.18. Expected returns are then regressed on returns predicted (fitted) by the APT model and the CAPM (equation (2.6)). As in Chen (1983), the data favours the multifactor APT model over the CAPM both in-sample and out-of-sample (see Beenstock & Chan, 1986: Table 8). Both Chen's (1993) and Beenstock and Chan's (1986) studies suggest that the APT model is superior and better suited to explaining the cross-sectional variation in expected returns relative to the CAPM.

Similarly to Bower *et al*. (1984), Elton and Gruber (1988) consider both aspects of the APT framework, although the emphasis is on explaining the time series variation in returns and comparing the performance of a four-factor model against the performance of a single-factor model of the return generating process. A four-factor return generating process specification is estimated for returns on size sorted portfolios together with a single-factor model utilizing returns on the NRI 400 stock index as the only explanatory factor. The average $\bar{R}^2$ reveals that while the single-factor model explains 55 percent of the time series variation in returns, the four-factor model explains 78 percent of the time series variation in returns. These results suggest that a multifactor model is superior relative to a single-factor model in explaining the return generating process of securities. Elton and Gruber (1988) also investigate the consistency of the explanatory power of the single-factor and the four-factor models and find that it differs across the differently sized portfolios. Whereas the single-factor model incorporating returns on the NRI 400 stock index explains between 14 percent and 90 percent of the variation in returns on the smallest and largest portfolios, the explanatory power of the four-factor model lies within a *narrower* range of between 66 percent and 82 percent respectively. This implies that a multifactor model is far more consistent and uniform in its ability to explain the time series variation in returns. Although Elton and Gruber (1988) also conduct cross-sectional tests of the APT model, unlike Chen (1983), Bower *et al*. (1984) and Beenstock and Chan (1986), the cross-sectional explanatory power of the APT model is not compared against that of a single-factor alternative. In conclusion, the authors suggest that a four-factor model is sufficient in explaining the return generating process. Notably, the study introduces another criteria upon which to judge the appropriateness of a model; consistency in explanatory power. In this regard, the APT framework is more consistent relative to a single-factor alternative.

The above studies suggest that the APT framework is a superior framework for modelling the return generating process and expected returns. Chen (1983) and Beenstock and Chan (1986) show that the APT model is more adequate and superior in explaining expected returns relative to the CAPM. Bower *et al*. (1984) show that the APT framework is better at explaining the time series variation in returns and the cross-section of expected returns. Elton and Gruber (1988) find that although only a single-factor is priced in cross-sectional analysis, the multifactor return generating process specification underlying the APT model is superior in a number of respects relative to a single-factor model relying upon an aggregate index. These findings suggest that the more complex APT framework is superior in explaining return behaviour and therefore, this multifactor framework should serve as a conceptual basis for investigating the return generating process or explaining equilibrium returns.

2.2.5. Limitations of the APT framework

The APT framework fulfils the role of an informative conceptual framework within which equilibrium relationships can be modelled and within which the return generating process can be explored. Propositions stemming from the APT find support; the return generating process is characterized by more than one factor and expected returns reflect the structure of the return generating process to some extent. Idiosyncratic risk factors are not priced suggesting that they are of no concern when modelling the return generating process. Systematic risk is of primary importance within the APT framework. The APT framework outperforms a single-factor alternative in explaining the time series behaviour of returns and expected returns. However, a comprehensive assessment and interpretation of the framework requires that limitations of the framework are acknowledged and criticisms noted.

Dhrymes, Friend and Gultekin (1984) re-examine the results obtained by Roll and Ross (1980) arguing that any empirical investigation of a new theory or concept should be subjected to replication to confirm the findings. The authors point out that more than five factors may be necessary to describe the return generating process of returns on NYSE and AMEX securities. Using an almost identical dataset[23] to that of Roll and Ross (1980), Dhrymes *et al*. (1984) show that that the number of groups for which a five-factor structure is inadequate is greater than that suggested by Roll and Ross (1980). Whereas Roll and Ross (1980) arrive at the conclusion that it is unlikely that more than five factors are necessary to

[23] Dhrymes *et al*. (1984) replace 13 securities for which data is unavailable and a further 11 securities characterized by a large number of missing observations.

describe the return generating process based upon a finding that in only 2.3 percent (1 out of 42) of groups a five-factor decomposition is inadequate, Dhrymes *et al*. (1984) find that a five-factor decomposition is inadequate for 16.6 percent (7 out of 42) of groups. This discrepancy is attributed to Roll and Ross' (1980) use of groups with missing observations or the greater precision of the statistical software used by Dhrymes *et al*. (1984), or both. Regardless of the source of error, this limitation potentially translates into erroneous conclusions regarding the complexity of the return generating process.

Dhrymes *et al*. (1984) also investigate how the number of factors in the return generating process varies with group size. Results indicate that returns on the smallest group (15 securities) are characterized by at most two factors whereas returns for groups consisting of 90 and 240 securities are described by at most nine and six factors respectively. This suggests that the number of factors is positively related to the number of securities in a group. Whereas Beenstock and Chan (1986) attempt to rationalize the positive relationship between the number of factors and group size, and Hughes (1984) shows that the first five factors are the most important in explaining the variation in returns, the findings of Dhrymes *et al*. (1984) should not be ignored. What is of concern is the large discrepancy in the number of factors in the return generating process of group of varying sizes. Furthermore, Dhrymes *et al*. (1984) do not investigate the group size at which the number of factors stabilizes.[24] The question that arises is whether *in practice* a two-factor model is sufficient and does not omit any relevant factors or whether a nine-factor model should be adopted on the basis of statistical and practical significance. With such a large discrepancy in the number of factors, the problem is not easily solved. Moreover, although an indication of an upper bound is desirable, it is not provided by Dhrymes *et al*. (1984). The consequences of a failure to identify a stable or upper bound to the number of factors in the return generating process is best articulated by Bodie, Kane and Marcus (2005) who state that if a model requires hundreds of explanatory factors, the consequence is a failure to simplify the return generating process. These findings indicate a major limitation; the APT framework fails to unambiguously identify the number of factors that are sufficient to describe the return generating process. Notwithstanding this limitation, what is certain is that more than one factor characterizes the return generating process.

[24] Only five factors are estimated for groups consisting of 240 securities as a result of "accelerating computer costs."

Central to the APT framework is the APT model which postulates that expected returns reflect compensation for exposures to multiple systematic risk factors in the return generating process. Roll and Ross (1980) find that at least three factors are priced, Chen (1983) finds that between two and four factors are priced, Hughes (1984) finds that between three and six factors are priced and Beenstock and Chan (1986) show that up to three factors are priced. Dhrymes *et al*. (1984) investigate whether a five-factor model explains the cross-section of expected returns by testing the null hypothesis of risk premia jointly equalling zero (see Chen, 1983). The null hypothesis is rejected in only 14.29 percent (6 out of 42) of the groups considered suggesting that a central component of the APT framework, the APT model, fails to explain expected returns. Furthermore, when factors representative of idiosyncratic risk such as the standard deviation and the skewness of individual returns are incorporated into the model, the null hypothesis is rejected for only 4.76 percent (2 out of 42) of groups. These findings contrast with those of Roll and Ross (1980) who find that at least one or more factors are priced in 28.57 percent (12 out of 42) of groups when idiosyncratic risk factors are considered. Dhrymes *et al*.'s (1984) findings suggest that APT factor loadings fail to explain expected returns and that APT factor loadings fare even worse in the presence of idiosyncratic risk factors. Moreover, when securities are sorted into groups by mean returns, no statistically significant risk premia vectors are found. Dhrymes *et al*. (1984) attribute this discrepancy in results to Roll and Ross' (1980) reliance upon the validity of the assumption regarding the normality of cross-sectional Generalized Least Squares (GLS) risk premia estimates and the use of tests that fail to consider departures from these assumptions. These findings cast some doubt upon the validity of the APT framework; the APT model is a critical component of the framework and if it does not hold, then why should an investigation of the return generating process be based upon a framework of which a central proposition is deficient. Dhrymes *et al*. (1984) state that the finding that few risk premia vectors differ statistically from zero reduces the APT to an explanation of the return generating process only. If this is the case, the APT can no longer be considered as a *comprehensive* conceptual framework which can be used to model *both* the return generating process of returns and to study equilibrium relationships.

Another set of tests of the APT considers whether the model is able to explain anomalies not explained by other models (Brown & Reilly, 2009). In essence, such tests are a comparison of alternate models on criteria other than explanatory power. One such anomaly is the so called January effect, which may bias asset pricing tests in favour of finding significant relationships between risk and expected returns. For example, Tinic and West (1984) show that when January is excluded from the analysis of the CAPM, there is a breakdown in the relationship between expected returns and risk. M.Gultekin and N.Gultekin (1987) investigate whether the January effect plays a role in tests of the APT model using CRSP return data for the July 1962 to December 1981 period. Preliminary analysis reveals that there is a strong seasonal effect in returns; mean returns are almost 30 percent for January whereas the month with the second highest mean returns is November with 11 percent. This seasonality is reflected in cross-sectional regressions estimated using only January returns; the null hypothesis of risk-premia being jointly equal to zero is *always* rejected in each of the sample groups of 30 and 90 securities. However, when returns for all other months excluding January (average of eleven months) are considered, risk premia are priced in just under a tenth (9 percent) and under a third (27 percent) of the groups respectively (see M.Gultekin & N. Gultekin, 1987: Table III, Panel A&B). While the seasonality results are robust to group size in that the January effect is observed in groups of 30 *and* 90 securities, the presence of a strong January effect is likely to bias results in favour of the APT model if January is considered together with the other eleven months. This is confirmed by M. Gultekin and N. Gultekin's (1987) findings that the statistical significance of the risk premia is greatly diminished when the sample excludes January and returns on the remaining eleven months are used. These findings suggest that the APT framework is only able to explain the risk-return relationship in January and is therefore, not applicable during other months. If this is the case, then drawing inferences from APT literature may not be appropriate as results reflect the presence of seasonal effects and are not a true reflection of equilibrium relationships.

Reinganum (1981) argues that the APT framework is a plausible alternative if it can explain differences in returns on firms of different sizes. The author's test is based upon the assumption that securities with similar factor loadings should have similar returns. To test this hypothesis, a two-stage procedure using CRSP data for securities traded on the NYSE and AMEX since July 1962 is employed. In the first stage, control portfolios are created by grouping securities with similar factor loadings into control groups where factor loadings are estimated over a sixteen year period. Excess returns on securities are then estimated by subtracting control portfolio returns from security returns. In the second stage, having obtained excess returns, securities are grouped into portfolios according to size. The rationale is that because all securities within the control groups have similar exposure to factors, excess returns are risk-adjusted and therefore, should be near zero. The validity of the APT model can then be investigated by testing the null hypothesis of average excess returns jointly equalling zero (Reinganum, 1981). Reinganum (1981) however finds that this is not the case; average excess returns are not equal to zero regardless of whether three, four or five-factor models are used to model APT risk. Returns on the smallest portfolio are positive and statistically significant whereas returns on the largest portfolio are negative and statistically significant. Returns on smaller portfolios are higher than returns on larger portfolios. A formal test of the equality of means confirms that there is evidence of a size effect after controlling for APT risk. This suggests that the APT framework fails to account for all risk in returns and fails to explain the size effect. In failing to do so, the APT framework fails to explain a phenomenon not explained by a simpler model. Thus, the indictment against the APT framework comes from the finding that it fails to account for anomalies that arise within the CAPM; the APT does not perform better relative to a simpler framework on criteria *other* than explanatory power. If the APT model does not present an improvement over the CAPM in the cross-sectional context, then the argument for adopting the more complicated APT framework is somewhat weakened.[25]

At the core of the APT framework lies the assumption that only systematic factors drive returns and as idiosyncratic risk can be mitigated through diversification, only systematic risk factors are relevant to security returns (Reinganum, 1981; Burmeister *et al*., 1994). Roll and Ross (1980), Chen (1983), Beenstock and Chan (1986) and Yli-Olli and Virtanen (1993) either find no evidence or limited evidence of priced idiosyncratic risk factors. Dhrymes *et al*. (1985) examine this proposition using CRSP return data for the July 1962 to December 1981 period. It is hypothesized that measures of idiosyncratic risk, namely the total and residual standard deviation, should be irrelevant when considered together with systematic measures of risk. Results for three group sizes (30, 60 and 90 securities) indicate that both systematic and idiosyncratic risk factors derived over the first sub-period (July 1962 to March 1972) provide little insight into the behaviour of expected returns although, idiosyncratic risk appears to be more important relative to systematic risk. For groups of the 30 securities, the null hypothesis that the risk premia vector is zero is rejected in 3.33 percent (1 out of 30) of

[25] Reinganum (1981) does however note that a number of hypotheses are tested jointly; for example, the ability to explain anomalies and also indirectly that only undiversifiable factors are relevant in explaining the cross-sectional characteristics of expected returns. For this reason, tests cannot reveal with absolute certainty which hypotheses are supported. Potential sources of error cited are the (potentially incorrect) assumption of a linear return generating process, the (in)ability to completely diversify away idiosyncratic variance and the existence of arbitrage on the NYSE and AMEX.

groups when factor loadings are used exclusively as explanatory factors. However, when own standard deviation and residual standard deviation are used as explanatory factors in addition to factor loadings, the vector of risk premia is found to be statistically insignificant for all groups suggesting that measures of idiosyncratic risk subsume any explanatory power that factor loadings may have. Furthermore, measures of total and residual standard deviation by themselves are statistically significant for 17 percent (5 out of 30) of groups suggesting that idiosyncratic risk is more important than systematic risk in explaining expected returns. These results are generalizable to groups of 60 and 90 securities (Dhrymes *et al*., 1985: Table III).

In the second sub-period (March 1972 to December 1981), factor loadings by themselves appear to play somewhat more important role. For groups of 30 securities, the risk premia vector is statistically significant for 20 percent (6 out of 30) of groups. However, the risk premia vector becomes statistically insignificant for all groups when idiosyncratic risk factors are considered in addition to factor loadings. This again suggests that any explanatory power attributable to factor loadings is subsumed by measures of idiosyncratic risk. By themselves, the own and residual standard deviation are statistically significant for 20 percent (6 out of 30) and 13.333 percent (4 out of 30) of groups respectively. These findings are generalizable to groups of 60 and 90 securities; fewer risk premia vectors are statistically significant when idiosyncratic risk measures are considered in addition to factor loadings and idiosyncratic risk measures appear to be marginally more important than factor loadings in explaining expected returns. If systematic risk factors explain expected returns and account for risk, then own variance and residual standard deviation should not play a role. Yet, Dhrymes *et al*.'s (1985) results suggest that these idiosyncratic risk factors may be just as or even more important than factor loadings. Such findings are concerning as they are incompatible with central propositions of the APT framework.

M.Gultekin and N.Gultekin (1987) present evidence supporting the findings of Dhrymes *et al*. (1985). The authors report that when residual standard deviation is introduced after excluding January returns, the number of priced risk premia decreases even further. However, the most pronounced result is observed when January returns are considered in isolation. Whereas risk premia are *always* statistically significant in January for groups of 30 and 90 securities, factor loadings are priced in 60 percent (18 out of 30) and 90 percent (9 out of 10) of groups respectively when residual standard deviation is incorporated as an additional explanatory factor. Reported *t*-statistics indicate that the residual standard deviation by itself is statistically significant in more than half of the groups, regardless of size. Such results are concerning; if loadings on APT factors, which are assumed to capture systematic risk fail to explain expected returns and if factors that proxy for idiosyncratic risk are priced, then this suggests that the APT framework's assumption that systematic risk is the *only* risk category that is important for explaining returns may not be valid (Dhrymes *et al.*, 1985).

Perhaps the most notable criticism is that the APT is a complete generalization and does not identify the risk factors that characterize the return generating process (Burmeister *et al.*, 1994). This limitation is widely recognized in the literature. Kandir (2008) states that a major criticism of the APT framework is that the framework derives factors statistically and does not identify them. Priestley (1996) suggests that factor analytic and principal component techniques make estimated risk premia uninterpretable. Brown and Reilly (2009) state that the APT framework does not identify the factors that describe returns and note that this is considered to be the primary shortcoming of the APT. Without knowing the identity of factors, it is impossible to determine which factors drive returns and to meaningfully interpret risk premia (Kandir, 2008). Dhrymes *et al.* (1984) are more forthcoming in their criticism of the APT and argue that without knowing the economic meaning of the factors, it is difficult to determine how the empirical application of the APT framework is useful for predictive and explanatory purposes. This contrasts with the CAPM framework, which identifies the market proxy as the single risk factor and therefore, makes the CAPM and the underlying single-factor model easier to apply once a suitable market proxy has been identified (Brown & Reilly, 2009). Roll and Ross (1980) note this shortcoming in their empirical investigation of the APT and argue that further research should be undertaken to determine the identity of the underlying factors. It is suggested that if there are a few systematic sources of risk, these are likely to be related to economic aggregates such as the GNP and interest rates. It is further argued that factors that have explanatory power for returns should be considered as substitutes for the unidentified factors. The authors go onto state that the formulation of the linear factor model within the APT framework motivates for further research into the theoretical and empirical structure of the model so as to better understand which systematic factors drive returns. The need to interpret unidentified systematic risk factors is recognized by Chen (1983) as the most important direction of further research.

2.3. Conclusion

The APT framework begins with the assumption that returns are generated by a *k*-factor model incorporating multiple factors and that sensitivities to unspecified factors explain the cross-sectional variation in expected returns (Ross, 1976; Roll & Ross, 1980). Together, these two aspects constitute the APT framework, a framework within which the return generating process can be modelled and equilibrium relationships established (section 2.2, proposition 1) & 2)).

Early studies conducted on foreign markets within the context of the APT framework, such as those of Roll and Ross (1980) and Chen (1983), suggest that there are multiple factors that feature in the return generating process and that exposure to these factors is rewarded by markets (section 2.2.1 & 2.2.2). These propositions are supported by empirical evidence and there is a linkage between the APT model and the underlying return generating process. Therefore, the APT framework is a comprehensive framework for investigating the return generating process *and* asset pricing. It is further hypothesized that only systematic risk factors are priced and therefore, relevant as firm-specific risk is diversifiable (see section 2.2, proposition 3); Reinganum, 1981; Berry *et al*., 1988; Roll & Ross, 1995). Amongst those that find evidence supporting this important proposition are Beenstock and Chan (1986) and Yli-Olli and Virtanen (1992) suggesting that when describing stock returns, the search for risk factors with explanatory power should focus upon systematic risk factors (see section 2.2.3). Moreover, the APT framework is a viable alternative to a single-factor framework based upon the CAPM (section 2.2, proposition 4)). Bower *et al*. (1984) show that the APT framework is better at explaining the time series and cross-sectional variation in returns relative to a single-factor alternative (section 2.2.4). Other studies support the superiority of the multifactor APT framework and suggest that the framework is a credible and viable alternative.

An important criticism of the framework arises from the use of statistically derived factors, which do not lend themselves to interpretation (see section 2.2.5; Dhrymes *et al*., 1984; Priestley, 1996; Brown & Reilly, 2009). Fortunately, this criticism is addressed by macroeconomic APT studies such as those of Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) who employ pre-specified macroeconomic factors as proxies for unidentified systematic risk factors. It is the macroeconomic APT model – together with its numerous extensions and applications - that completes a multifactor conceptual framework within which the return generating process can be modelled and investigated. The macroeconomic APT, its extensions, applications and its role in providing a conceptual framework for the modelling of the return generating process are discussed in the chapter that follows, Chapter 3.

3. THE APT MODEL AND THE RETURN GENERATING PROCESS

3.1. The macroeconomic APT model

Notwithstanding the criticisms and evidence contradicting the central propositions of the APT (section 2.2: 12), the APT as a framework has been readily accepted and widely applied. To address the limitation of unidentified factors (section 2.2.5: 37), Chen (1983) suggests that the time series of statistically derived factors be correlated with the time series of (identified) macroeconomic factors. This approach is applied by Elton and Gruber (1988) who consider the correlation between a set of statistically derived factors and an extensive set of macroeconomic factors. A strong statistical relationship between the two sets of factors is found suggesting that factors that are macroeconomic in nature are representative of systematic risk. Barr (1990) follows a similar procedure by extracting two factors from returns on non-gold Johannesburg Stock Exchange (JSE) indices and investigating the correlation between a set of macroeconomic factors and the two extracted factors. Another solution proposed by Chen (1983) involves correlating returns with the behaviour of risk factors identified through theory as important in the pricing of stocks. It is this latter approach that has prevailed with most APT studies employing pre-specified macroeconomic factors as proxies for unidentified systematic risk factors, giving rise to what Cauchie, Hoesli and Isakov (2004: 181) term the "macroeconomic APT." Although the focus of these studies is on explaining returns in equilibrium, these studies lay the foundation for multifactor models of the return generating process by assuming that the linear factor model underlying the APT model is characterized by multiple *pre-specified* macroeconomic factors representative of systematic risk (Elton *et al*., 2003). To explain expected returns, factor loadings on pre-specified factors are employed and by implication, these factor loadings are derived from a multifactor model of the return generating process. Therefore, underlying each macroeconomic APT model is a time series model of the return generating process (Connor, 1995).

Having introduced the macroeconomic APT model, this chapter proceeds by outlining the macroeconomic APT model and presenting numerous extensions (section 3.1.1 - 3.1.6). It is demonstrated that the APT model and the return generating process are linked (section 3.2). It then follows that if the APT model can be used to describe the cross-section of expected returns, it can also be used to describe the return generating process. Studies demonstrating this are presented in section 3.2. It is then shown that the APT framework goes beyond providing a description of expected returns; the APT framework acts as a conceptual framework for the modelling of the return generating process (section 3.3, 3.3.1 & 3.3.2). It is further demonstrated that APT framework can be extended to applications that require the modelling of the return generating process but are not directly concerned with the return generating process (section 3.3.3 & 3.3.4). A summary is provided in the conclusion (section 3.4).

3.1.1. Introducing the macroeconomic APT model

While Chen *et al*. (1986) are credited with being the first to employ proxies for unidentified risk factors to explain expected returns, it was Chan *et al*. (1985) who laid the groundwork for this approach citing the APT as their motivation in an investigation of the firm size effect for firms listed on the NYSE over the January 1953 to December 1977 period. Chan *et al*. (1985) postulate that returns are sensitive to changes in the economic environment and these changes are indicative of the risks that investors can hedge against. A theory linking stock prices to pre-specified factors is proposed whereby prices are assumed to be determined by expected cash flows and the discount rate - a pricing equation informing the choice of factors. Factors identified by Chan *et al*. (1985) in this manner are the monthly growth rate in industrial production, the unanticipated inflation rate, changes in expected inflation, changes in the term structure of interest rates, the default spread and changes in the business cycle as measured by the growth rate of Net Business Formation (NBF). The equally-weighted NYSE Index serves as a proxy for returns on the market. While it is recognized that *innovations* (*unexpected* changes) in these factors should be used in empirical tests, the authors choose not to use innovations and warn that the generation of innovations through pre-whitening[26] may lead to a loss of information. Time series relationships between returns and the candidate risk factors are established by examining the correlation between aggregate returns, as measured by returns on the equally-weighted NYSE Index and the set of risk factors. Table 3.1 reproduces the correlation matrix in Chan *et al*. (1985) to demonstrate the approach undertaken in identifying factors that are correlated with returns over time and to provide insight into potential time series relationships. Although this approach reveals the presence of time series relationships between returns and factors, macroeconomic APT studies give little direct consideration to these relationships.

[26] A process whereby the unexpected components of a series are extracted.

Table 3.1: Correlation among factors

	EWNY	*IPISA*	*UITB*	*DEI*	*UTS*	*BUSF*
EWNY						
IPISA	0.0738					
UITB	-0.1937	-0.1367				
DEI	-0.2120	0.0474	0.4333			
UTS	0.1399	-0.0898	-0.0251	-0.2267		
BUSF	0.2100	0.2640	-0.1436	-0.0927	-0.1260	
PREM	0.3325	0.0885	-0.0962	0.0357	-0.5813	0.2019

EWNY – Equally-Weighted NYSE Stock Index.
IPISA – Growth Rate of industrial production from month *t* to *t+1* (seasonally adjusted)
UITB – Unanticipated inflation, defined as CPI less expected inflation.
DEI – Change in expected inflation.
UTS – Difference in return on long-term government bond portfolio and the one-month T-bill.
BUSF – Growth rate of the Net Business Formation series from *t* to *t+1*. Seasonally adjusted.
PREM – Difference in return of 'under BAA' (rated by Moody) bond portfolio and long-term government bond portfolio.

Source: Chan, Chen & Hsieh (1985)

As evident from Table 3.1, all factors show some level of correlation with each other *and* with returns on the equally-weighted NYSE Index over time. In macroeconomic APT studies, the presence of correlation between returns on a market aggregate and candidate risk factors, is often cited as a justification for the inclusion of specific factors in the return generating process and consideration in cross-sectional tests (see Van Rensburg, 2000). Chan *et al*. (1985) proceed to estimate factor loadings for use in cross-sectional tests by first regressing 60 months of returns on size sorted portfolios on the candidate risk factors and then performing month-by-month cross-sectional regressions in the second stage over each month in the subsequent year.[27] The first stage is important. Regressing returns on macroeconomic factors to obtain factors loadings requires a formulation of a multifactor model of the return generating process. However, as asset pricing is of primary concern in the study, this aspect of the APT framework is not considered further by Chan *et al*. (1985).[28] The focus upon the implications of the cross-sectional APT model in this study is indicative of the focus of most macroeconomic APT studies. In these studies, the only (and limited) insight into the structure of the return generating process is provided by the correlation matrix.

Chan *et al*. (1985) find that three factors are priced over the entire sample period, namely the default spread, the growth rate in industrial production and unanticipated inflation. Together, these factors explain 35 percent of the cross-sectional variation in expected returns. Other factors that are priced over the sub-periods considered, aside from these three factors, are the

[27] This two-stage procedure is consistent with the Fama-Macbeth approach and represents the thrust of early macroeconomic APT literature (see Fama & Macbeth, 1973; Chen *et al*., 1986; Hamao, 1988).
[28] A two-step procedure need not be followed to establish which factors are priced. However, regardless of the approach undertaken in estimating factor loadings and risk premia, the APT framework consists of two components; a time series model and the APT model.

changes in expected inflation (1968-1977) and the term structure of interest rates (1958-1972). In a first test of the validity of the model, the authors consider whether a firm size effect is reflected in the residuals of the APT model. The differences between the residuals of the largest and smallest portfolio, and the top and bottom quintiles are statistically insignificant suggesting that macroeconomic factors used in place of *unidentified* APT factors account for systematic risk and risk that is associated with firm size. It is suggested that the firm size effect is related to risk associated with a changing default spread (see Chan *et al*., 1985: Table 4). In a second test of the model, the business cycle indicator is substituted for the default spread factor. Results show that the indicator is priced implying that it can replace the default spread as an indicator of business conditions. In a third test, Chan *et al*. (1985) formally consider whether a size proxy has explanatory power in the cross-sectional context; it is postulated that size is a proxy for unspecified risks. When size is the only factor in cross-sectional analysis, the risk premium on size is statistically significant. However, it is statistically insignificant when considered in combination with a set of macroeconomic factors which excludes market indices,[29] but includes the growth rate of industrial production, unexpected and expected inflation, the default spread and the term structure. In this version of the APT model, the default spread is statistically insignificant implying that that size and the other risk factors proxy for risk associated with the changing default spread. Chan *et al*. (1985) conclude that the size effect is explained by a multifactor pricing model. The authors' contribution is important in that pre-specified macroeconomic factors are used as proxies for systematic risk. Notably, these factors are correlated with returns over time suggesting that they explain the time series behaviour of returns. As a number of these risk factors are priced, it can be inferred that the APT framework can be used to not only explain expected returns, but to also model the time series variation in returns (Elton & Gruber, 1988).

Notwithstanding Chan *et al*.'s (1985) important contribution, Chen *et al*. (1986) are widely credited in the literature as being the first to utilize macroeconomic factors as proxies for unidentified APT risk factors. The influence of the APT framework on Chen *et al*. (1986) is evident; the authors, with reference to the work of Roll (1976), acknowledge that modern financial theory has focused upon the pervasive and systematic influences that affect stock prices. It is further argued that while it is accepted that individual stock prices are influenced by unexpected events, little is known about the identity of systematic factors that influence

[29] See equation (vi) in Table 5 in Chan *et al*. (1985).

prices; although, the co-movement of stock prices points towards their existence. It is with this argument in mind and within the APT framework, that Chen *et al*. (1986: 384) refer to an "embarrassing gap" between systematic factors and their identity. It is this gap that the authors seek to close by investigating the identity and nature of systematic risk factors.

The existence of systematic risk factors is suggested by observed co-movements of stock prices and implicit in the assumption that investors diversify. This suggests that only factors that are associated with the economic state have an impact upon the pricing of stock market aggregates. Although, Chan *et al*. (1985) allude to a theory upon which the identification and selection of factors can be based, Chen *et al*. (1986) formally identify and elaborate upon a theoretical model - the dividend discount model[30] - that aids the identification and selection of risk factors within the APT framework. It is hypothesized that any systematic factor that influences the expected stream of dividends, cash flows or/and the discount rate will impact stock prices. As current beliefs regarding potential and identified factors are assumed to be already incorporated into stock prices, it is only innovations in these factors that impact returns. While it is recognized that a failure to remove expected movements in the explanatory factors may introduce an errors-in-variables (EIV) problem, Chen *et al*. (1986) employ a simple rate of change methodology to represent (assumed) innovations[31] in factors. Their set of candidate risk factors incorporates the monthly and annual industrial production growth rates, the change in expected and unanticipated inflation, changes in the default spread, the term structure, consumption growth and changes in the oil price. Returns on the equally-weighted and value-weighted NYSE indices for the January 1953 to November 1983 period are used as a proxy for the market index. Factors considered, but not utilized, in the preliminary specification, are changes in real consumption and oil prices. As in Chan *et al*. (1985), time series relationships between returns on the two market aggregates and the macroeconomic factors are examined using a correlation matrix. Each of these factors is correlated with returns on the value and equally-weighted NYSE indices over the sample period (see Chen *et al*., 1986: Table 2, Panel A). The factors identified by Chen *et al*. (1986) are what Amenc and Le Sourd (2005: 153) term as "classic" factors suggesting wide usage in multifactor models employing pre-specified factors as proxies for systematic risk. The basic

[30] Discussed in greater detail in Chapter 4.
[31] As will become evident later, the rate of change methodology fails to generate true innovations.

multifactor specification[32] describing the return generating process of individual stock returns proposed by Chen *et al*. (1986) is given by:

$$R = \alpha + b_{MP} MP + b_{DEI} DEI + b_{UI} UI + b_{UPR} UPR + b_{UTS} UTS + \varepsilon \tag{3.1}$$

where R is the return on an individual security, α is the constant and the factor loadings, b, are the sensitivities of returns to changes in macroeconomic factors. The residual term is denoted by ε. *MP* is the monthly industrial production growth rate, *DEI* is the change in expected inflation, *UI* is unexpected inflation, *UPR* is the change in the default spread and *UTS* is the change in the term structure. Equation (3.1), as stated in Chen *et al*. (1986: 394), represents an important acknowledgement that returns can be described by innovations in multiple macroeconomic factors representative of unspecified APT risk factors.[33] Most importantly, equation (3.1) represents the return generating process underlying the macroeconomic APT model which relates returns to innovations in macroeconomic factors over time. However, the authors do not use equation (3.1) to study the return generating process of US returns but rather to estimate factor loadings for use in the corresponding cross-sectional macroeconomic APT model. Factor loadings are estimated by regressing returns on size sorted portfolios onto innovations in macroeconomic factors.[34] Risk premia are estimated in the second stage by employing Fama-Macbeth regressions relating expected returns to factor exposures (Chen *et al*., 1986):

$$R = \alpha + \lambda_{MP} b_{MP} + \lambda_{DEI} b_{DEI} + \lambda_{UI} b_{UI} + \lambda_{UPR} b_{UPR} + \lambda b_{UTS} \tag{3.2}$$

where R is the asset return for a given month and the bs estimated in equation (3.1) are explanatory factors in equation (3.2). The coefficients on the betas, λs, are interpreted as the risk premia – the price of risk - associated with a given macroeconomic factor. By taking

[32] Chen *et al*. (1986) vary the model specification to investigate various aspects. The notation used by Chen *et al*. (1986) is retained for demonstrative purposes.

[33] The separation of the APT into a time series model *and* a cross-sectional model employing factor loadings estimated in the time series model is explicitly acknowledged by Hamao (1988). Hamao (1988) specifies the time series model alongside its cross-sectional counterpart. Chen *et al*. (1986) specify the time series model but not its cross-sectional counterpart. The estimation of the cross-sectional regression using factor loadings estimated in the time series model can be seen as an implicit acknowledgement of a link between the two models.

[34] Equation (3.1) is estimated as a time series model using size sorted portfolios to control for the EIV problem and to achieve a spread of expected returns required for cross-sectional tests.

equations (3.1) and (3.2) together, it then follows by implication that priced factors are risk factors that feature in the return generating process.

Results of the cross-sectional analysis over the entire sample period (1958-1984) indicate that the monthly industrial production growth rate, the unexpected inflation rate, the default spread and the term structure of interest rates are priced suggesting that these factors play an important role in explaining both the cross-section of expected returns and the time series behaviour of returns. This argument is strengthened by a finding that these factors that are also correlated with return aggregates over time (see Chen *et al*., 1986: Table 2, Panel A). Risk premia on industrial production and the default spread are positive whereas the risk premia on unexpected inflation and the term structure are negative. Chen *et al*. (1986) hypothesize that the positive risk premium on industrial production reflects the benefit of insuring against real production risks whereas the positive risk premium on the default spread suggests that investors seek to hedge against unexpected increases in uncertainty. The negative risk premium on unexpected inflation is hypothesized to imply that assets are hedges against adverse influences on assets that are fixed in nominal terms. The sign of the risk premium on the term structure factor implies that stocks for which returns are negatively related to changes in the term structure are more valuable (Chen *et al*., 1986). In using pre-specified factors and ascribing meaning to estimated risk premia, Chen *et al*. (1986) address criticisms of the APT framework whereby it is not possible to ascribe meaning to the risk premia on statistically derived factors. In a second set of results, returns on equally and value-weighted indices compromising securities listed on the NYSE are incorporated into the model to test the pricing influence of the market indices and to gauge how the set of macroeconomic factors fares in comparison to market indices. Although Chen *et al*. (1986) note that the indices are the most statistically significant factors in unreported time series regressions, the equally and value-weighted market indices are not priced over the entire sample period and during any of the sub-periods. The macroeconomic factors however retain significance in the APT model suggesting that factors *aside* from market returns (extra-market factors) are priced in stock returns and therefore, are important in the return generating process (Chen *et al*., 1986).

Chen *et al*. (1986) acknowledge that the set of factors employed in the study is not exhaustive and that the identification of other potential factors should not be abandoned. Based upon the preceding findings, the authors state that stock returns are responsive to systematic news and priced according to their exposure to factors describing macroeconomic conditions. Similarly to Chan *et al*. (1985), little consideration is given to the time series relationships between returns and macroeconomic factors, and the validity of the underlying return generating process. However, explicit recognition is given to the form and composition of the return generating process. Furthermore, the results of the macroeconomic APT model suggest that macroeconomic factors are also important in the time series context. This, together with a more detailed exposition of a theory dealing with the identification of factors, points towards the APT's role as a framework for identifying and employing pre-specified systematic risk factors to explain the return generating process.

Hamao (1988) seeks to confirm the robustness of Chen *et al*.'s (1986) results by performing a parallel analysis on the Japanese market. The macroeconomic factors identified by Chen *et al*. (1986) are interpreted as proxies for underlying risk factors that drive stock returns. It is acknowledged that the advantage of using pre-specified macroeconomic factors lies in that economic meaning can be ascribed to these factors. The specification of the return generating process and the cross-sectional macroeconomic APT model is identical to that of Chen *et al.* (1986) in that the same factors are incorporated into the base model (equation (3.1)). Similarly to Chen *et al.* (1986), Hamao (1988) directly acknowledges the multifactor return generating process underlying the macroeconomic APT model although, its role is again limited to that of a time series regression run to estimate inputs for the cross-sectional APT model. The growth in oil prices is also considered in addition to two factors hypothesized to be relevant in the Japanese context; namely, unexpected changes in the foreign exchange rate and changes in the terms of trade.[35] The inclusion and consideration of these factors represents an early acknowledgement that there may be other relevant risk factors aside from those suggested by Chen *et al*. (1986) and that these factors may be specific to a given market. Moreover, this also suggests that a given set of factors should not be considered as "fixed" and is in line with Chen *et al*.'s (1986) argument that there are other influential systematic risk factors. Equally and value-weighted market indices are constructed using returns on the Tokyo Stock Exchange (section I) (TSE) Index.

To gain preliminary insight into the time series relationships between the macroeconomic factors and the return aggregates, the correlation between returns on the market indices and

[35] Hamao (1988: 52) refers to the constructed exchange rate factor as an "innovation variable for the exchange rate change." This reflects the APT framework's emphasis on the use of innovations.

the macroeconomic factors is investigated. To obtain factor loadings, size sorted portfolios are formed and time series regressions of returns on these portfolios on the five Chen *et al*. (1986) factors are conducted. Estimated factor loadings are used as independent factors in cross-sectional regressions over the same period that is used to estimate factor loadings in time series regressions (January 1975 - December 1984). Results are similar to those of Chen *et al*. (1986); over the entire sample period, changes in industrial production, changes in unanticipated inflation, the default spread and the term structure explain expected returns. In contrast to Chen *et al*. (1986) and in addition to these four factors, changes in expected inflation are also priced (Hamao, 1988: Table 5, Part 1). Changes in the terms of trade are not priced and the reason cited for this is the presence of serial correlation in the time series related to this factor (Hamao, 1988: Table 5, Part 3). A second set of tests is conducted by using factor loadings estimated over the first part of the sample (January 1975-December 1979) to explain expected returns in the second part of the sample (January 1980-December 1984). Hamao (1988) finds that factors with consistent explanatory power for expected returns are changes in expected inflation, changes in the default spread and to a (much) lesser extent changes in the term structure. These factors are also notably correlated with returns on the value and equally-weighted TSE indices over time. The foreign exchange rate, terms of trade and the oil price have no impact upon expected returns and the value and equally-weighted market indices are not associated with statistically significant risk premia in the same APT model specification. This implies that market indices are not associated with missing factors and confirms the cross-sectional explanatory power of extra-market factors (Hamao, 1988). The estimation of market betas together with factor loadings requires a specification of the return generating process which combines market factors and extra-market factors representative of systematic risk. This yields a multifactor model of the return generating process.

In a final test, Hamao (1988) estimates the market beta using the value-weighted market index *separately* from other factor loadings and combines the market beta with remaining factor loadings to explain expected returns. The risk premium on the market beta is found to be statistically insignificant – a finding that confirms the results of Chen *et al*. (1986) with regard to the explanatory power of factors aside from the market beta in the APT model. Risk premia on the inflation factors are positive, contrasting with the findings of Chen *et al*. (1986) implying that stocks for which the price increases with inflation are more valuable (Hamao, 1988). Expected returns are positively related to the default spread. Hamao (1988) concludes that the most important factors that explain expected returns are changes in expected inflation, changes in the default spread and the term structure. It is also stated that the evidence of statistically significant risk premia on a number of factors supports the approach of Chen *et al*. (1986), which proposes that multiple systematic risk factors are important in the pricing of stock returns and by implication, for the description of the return generating process (Elton & Gruber, 1988).

The studies of Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) are notable for a number of reasons. Although the aim of these studies is to explain returns in equilibrium, they suggest that systematic risk, as measured by pre-specified macroeconomic factors, features in the return generating process regardless of the role of the market portfolio. This is further implied by evidence of time series correlation between returns on market aggregates and the macroeconomic factors. The role of a multifactor return generating process is recognized and the macroeconomic APT models considered in these studies reflect the underlying return generating process. While early APT studies recognize the presence of multiple factors in the return generating process, Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) replace statistically derived APT factors with pre-specified macroeconomic factors. Finally, whereas Chan *et al*. (1985) and Chen *et al*. (1986) introduce and expound the macroeconomic APT approach to asset pricing, Hamao (1988) confirms the validity of this approach using non-US data. In light of the successful application of this approach, Hamao (1988: 60) states that “the evidence presented here is encouraging and it is certainly worth exploring further as new data become available.”

3.1.2. Expanding the search for risk factors

Poon and Taylor (1991) investigate whether the findings of Chen *et al*. (1986) are applicable to UK stocks by re-examining the methodology and the set of factors employed by Chen *et al*. (1986). The authors note that a failure to use innovations has the potential to result in spurious relationships and give rise to the EIV problem. Each series is pre-whitened using univariate Autoregressive Integrated Moving Average (ARIMA) models and the residuals of these models are treated as the unexpected series of macroeconomic factors. Factor loadings estimated using returns on size sorted portfolios are used to explain the cross-section of expected returns for each month subsequent to the five year period used to estimate factor loadings in time series regressions.[36] In contrast to Chen *et al*. (1986) who update factor loadings on an annual basis, Poon and Taylor (1991) update loadings monthly implying that an underlying return generating process is estimated for each five year period. The authors report that none of the original Chen *et al*. (1986) factors show a statistically significant contemporaneous pricing relationship. Noting that the relationship between returns and macroeconomic factors may not be contemporaneous, factor loadings are re-estimated using lags and leads. As before, no important pricing relationships between expected returns and the factors emerge. When isolated statistically significant pricing relationships are observed for individual factors, these are either of the wrong sign or inconsistent, suggesting that these relationships are unimportant.

In suggesting reasons for the poor performance of these factors in explaining expected returns, Poon and Taylor (1991) find that the Fama-Macbeth two-step procedure is sensitive to the number of factors included in regressions; certain factors are priced in given instances but not in others when the model specification is changed or the impact of factors is analyzed within a univariate context. This is attributed to the potentially narrow range of factor loadings arising from the use of size based portfolios. The authors further argue that the return-risk relationships may not be contemporaneous because of the announcements of macroeconomic factors being subject to lags and subsequent revisions. The pre-whitening process, which can potentially remove pricing information, is also cited as a reason. This limitation is recognized by Chan *et al*. (1985). However, when unfiltered series are used, Poon and Taylor (1991) find that the only statistically significant contemporaneous relationship is between changes in inflation expectations and expected returns. It is therefore unlikely that the lack of significant relationships arises due to the loss of pricing information as a result of the pre-whitening process.

At the very least, these results suggest that there may be other macroeconomic factors that are relevant to the UK market (Poon & Taylor, 1991). Importantly, and in contrast to Hamao (1988), it may be inferred that the Chen *et al*. (1986) factors may *not* be applicable across markets and therefore, factors that drive returns differ from market to market. It is however impossible to definitively answer this question as Poon and Taylor (1991) provide no insight

[36] Poon and Taylor's (1991) sample spans the January 1965 to December 1984 period. For example, factor loadings estimated over the January 1965 to December 1969 period are used to explain expected returns in January 1970.

into the time series relationships between returns and macroeconomic factors. This finding nevertheless motivates for a more extensive search for macroeconomic factors that can explain returns. It also suggests that the set of factors that is important for the South African stock market is likely to differ from the set of factors that is important for foreign markets (see Seneque, 1987).

3.1.3. Impact of portfolio formation criteria upon pricing

Clare and Thomas (1994) investigate the effect of alternative portfolio formation criteria upon asset pricing over the January 1978 to December 1990 period and consider an extended set of factors chosen on the basis of their perceived relevance to the UK economy (see Clare & Thomas, 1994: Table 1). The authors employ Autoregressive (AR) models to generate innovations in macroeconomic factors in the form of uncorrelated residuals. Results from a restricted model based upon *beta* sorted portfolios suggest that out of the extensive set of factors considered; the oil price, redemption yields on UK debentures, the default spread, the comfort index, the retail price index, private sector bank lending and the current account balance are priced. All risk premia are positive with the exception of the risk premium associated with the oil price. This proposed specification is further tested by incorporating excess market returns (excess returns on the Financial Times Stock Exchange (FTSE) All-Share Index) into the model. This serves to determine whether a factor has been omitted and whether the CAPM is preferable to the APT model. Results show that the market is not priced suggesting that no relevant factors have been omitted. All factors with the exception of the current account balance remain priced in returns. This implies that the APT model is correct in attributing the variation in expected returns to multiple sources of risk. Furthermore, this also implies that as in Chen *et al*. (1986) and Hamao (1988), extra-market sources of risk are important in the underlying return generating process.

Clare and Thomas' (1994) results for the restricted model employing factor loadings estimated using *size* sorted portfolios differ substantially from results of the model employing factor loadings estimated using beta sorted portfolios. Only two factors are priced, the comfort index and the retail price index – this contrasting with prior results where seven factors (excluding excess market returns) are priced. When excess market returns are incorporated into the model, the market factor is priced. This suggests that factors *are* omitted and that the CAPM may indeed be a more suitable model. However, as two other factors remain priced, the market factor *still* appears to omit information relevant to pricing. Clare and Thomas (1994) conclude that results are dependent upon the criteria used for sorting stocks into portfolios; beta sorted portfolios reveal more factors than size sorted portfolios and therefore, the search for macroeconomic factors is dependent upon the sorting methodology used. If this is indeed the case, the findings of Chan *et al*. (1985) and Chen *et al*. (1986) and Hamao (1988) should be approached with caution. It may be that there are other factors that explain the cross-section of expected returns and therefore feature in the return generating process. However, as results are dependent upon the sorting procedure used, these are not revealed in APT literature seeking to establish equilibrium relationships. At the very least, aside from pointing out limitations relating to the two-step procedure, these findings suggest that caution must be exercised when generalizing inferences from APT literature to multifactor models of the return generating process derived within the APT framework. Similarly to Poon and Taylor (1991), Clare and Thomas (1994) also suggest that risk factors vary across markets and therefore, it may be incorrect to expect the same set of factors to be priced across markets and to feature in the return generating process.

3.1.4. Expectations generating process and the role of innovations

Chan *et al*. (1985) and Chen *et al*. (1986) recognize that the APT framework requires *innovations* in factors but make no attempt to obtain or estimate uncorrelated series. Poon and Taylor (1991) and Clare and Thomas (1994) model innovations using ARIMA and AR models respectively. However, no further consideration is given to the methodology used to estimate innovations or to the consequences of a failure to use innovations. Priestley (1996) investigates the expectations generating process in greater detail and considers how alternative methodologies of modelling innovations affect inferences and the application of the APT framework. It is argued that tests of the APT rely upon the assumption that asset prices react to news regarding innovations in macroeconomic and financial factors. Following from this assumption, is the premise that investors *form expectations* of factors that are rewarded by markets with a premium. Although, the APT framework sheds no light upon how expectations are formed, the formation of these expectations requires an expectations generating process that specifies how innovations enter the framework. What is required of the time series of innovations under the APT framework is that they have an expected (mean) value of zero and are serially uncorrelated (Priestley, 1996).

Two methodologies have been widely used to generate (assumed) innovations. The first methodology, the rate of change methodology, employed by Chan *et al*. (1985) and Chen *et* *al*. (1986), assumes that first differences in factors represent innovations which follow a random walk. The second methodology relies upon an autoregressive time series technique whereby expectations are generated by AR models and the residuals are taken to be the innovations in factors (Priestley, 1996). However, Priestley (1996) argues that the former methodology fails to generate serially uncorrelated series, thus violating the basic requirement that unexpected components are serially uncorrelated. This methodology also fails to take into account prior information. While the latter methodology permits the use of past information, it assumes that the parameters of the model are stable and therefore, fails to account for changes in the parameters. An alternative, that Priestley (1996) argues is superior, is to use Kalman filter techniques to generate innovations. This approach also avoids the possibility that investors may make systematic errors in their predictions. Under this approach, expectations are updated recursively in each period as more information becomes available and the expectations generating process is described by an unobserved components model. The residuals of the model represent innovations.

To compare and assess the three methodologies, Priestley (1996) generates innovations for ten candidate risk factors[37] and tests for serial correlation to establish whether each technique *does* indeed generate innovations. Results for the rate of change methodology indicate that only for one factor, industrial production, is the series uncorrelated. This suggests that the rate of change methodology fails to generate the uncorrelated series required by the APT framework. The residual series generated by the autoregressive time series methodology are serially uncorrelated suggesting that the autoregressive methodology satisfies the requirement of true innovations. However, Priestley (1996) finds that the parameters of the AR models used to generate the innovations in factors are unstable. Series generated by the Kalman technique are uncorrelated for all factors with the exception of unexpected inflation. Unlike the autoregressive time series methodology, the technique permits time-varying parameters and therefore allows a learning process while avoiding parameter instability. To determine how the different specifications of the expectations generating process affect the results of the APT model, Priestley (1996) employs the Non-Linear Seemingly Unrelated Regression (NLSUR) framework to jointly estimate factor loadings and risk premia. The NLSUR framework, by permitting the joint estimation of the parameters of the return generating

[37] Candidate factors that Priestley (1996) hypothesizes carry a risk premium in the UK stock market are the default spread, industrial production, exchange rate, retail sales, money supply, unexpected inflation, changes in expected inflation, the term structure of interest rates, commodity prices and the market portfolio.

process and the APT model, eliminates the EIV problem and the limitations arising from the use of different portfolio formation criteria as portfolio formation is not longer necessary (Priestley, 1996; Antoniou, Garret & Priestley, 1998). Hence, Priestley's (1996) sample consists of returns on *individual* British firms over the December 1979 to August 1993 period.

Results indicate that when the (assumed) innovations generated by the rate of change methodology are employed, *seven* factors are priced; the default spread, unexpected inflation, real industrial production, commodity prices, changes in expected inflation, the money supply and returns on the market portfolio. For factors generated using the time series autoregressive methodology, *five* factors are priced; industrial production, unexpected inflation, retail sales, commodity prices and returns on the market portfolio. *Five* factors are priced in the APT model when factors are generated using Kalman filer techniques. These factors are the default spread, the exchange rate, the money supply, unexpected inflation and returns on the market portfolio. What is concerning is that these results indicate that there is no consistency across methodologies in the number of factors that are priced, the sign of the risk premia and the identity of the priced factors (see Priestley, 1996: Table 6). Furthermore, Priestley (1996) finds that innovations estimated using the Kalman filter technique provide the best performance in-sample and out-of-sample in terms of predicting returns within the APT model. While in-sample, the time series methodology outperforms the rate of change methodology, it underperforms the Kalman filter technique. Out-of-sample, the APT model employing the Kalman filter technique does not yield vastly superior results relative to the APT model employing the time series methodology.[38] However, both the autoregressive and Kalman filter methodologies lead to superior results relative to the rate of change methodology. This suggests that the methodology used to estimated innovations also has an impact upon the descriptive accuracy of the APT model.

Priestley's (1996) findings suggest that the method of generating innovations may result in misleading inferences when investigating which factors are priced. Furthermore, these results imply that the consequences of a failure to generate innovations or the consequences of the use of an inappropriate methodology extend into the return generating process. Importantly,

[38] In-sample tests are conducted by regressing actual returns on expected returns where expected returns are generated by models employing different methodologies to generate innovations. To assess out-of-sample performance, mean-squared errors are used to quantify the discrepancy between predicted and actual returns.

Priestley's (1996) study sheds light upon the role of innovations within the APT framework and details *how* innovations are estimated. This aspect of the APT framework is often ignored in asset pricing literature and in literature which investigates the return generating process within the APT framework. Priestley (1996) concludes by stating that caution must be exercised when constructing factors as this has important implications for the number of significant factors, their sign and the performance of the APT model.

3.1.5. Generalizability of the APT model

Antoniou *et al*. (1998) argue that any evaluation of the empirical performance and the validity of the APT model must consider its ability to explain expected returns *outside* of the estimation sample. This requires that the same factors are priced in different subsets of securities and the respective risk premia on these factors are the same across subsets of securities. The authors investigate whether this is the case using return data for stocks traded on the London Stock Exchange (LSE) over the January 1980 to August 1993 period. Similarly to Priestley (1996), the NLSUR econometric framework is employed owing to the limitations of the Fama-Macbeth two-step procedure. After testing various combinations of factors, Antoniou *et al*. (1998) arrive at a model with six priced factors; namely unexpected inflation, changes in expected inflation, money supply, the default spread, the exchange rate and returns on the market portfolio (FTSE All-Share Index). These six factors are then used to explain expected returns on a second sample of stocks. Results indicate that all factors are priced in the second sample with the exception of expected inflation suggesting that the *exact* same set of factors does not explain returns out-of-sample. Furthermore, the signs of the risk premia on the default spread and exchange rate are found to be positive and not negative as in the first sample. The hypothesis that the risk factors jointly carry the same risk premia (in magnitude) as those in the first sample is also rejected. Antoniou *et al*. (1998) state that these results show that the proposition that a model with an identical factor structure can be used to explain expected returns in both samples does not hold. In other words, the factor structure is not *fully* generalizable out-of-sample. However, after re-estimating the model for the second sample without expected inflation, the risk premia on unexpected inflation, the money supply and the market portfolio are similar in magnitude and of the same sign as those in the first sample. The null hypothesis that these three factors have risk premia of a similar magnitude in both samples is not rejected suggesting that although, the results are somewhat ambiguous at first, the same factors *can* be used to price assets across different samples and out-of-sample. Antoniou *et al*.(1998) go onto show that when a three-factor model incorporating factors with similar risk premia in both groups is estimated for both samples and outliers are excluded, the model is capable of explaining a significant amount of variation in expected returns for *both* samples. In light of these findings, the authors state that this supports the hypothesis that a specific APT model specification can describe the cross-sectional variation in returns in-sample and out-of-sample.

Importantly, Antoniou *et al*.'s (1998) findings suggest that an APT model specification estimated or based upon a certain set or subset of securities can be *generalized* to other subsets of securities that are not used in the estimation of the initial model specification. This property is especially important if one attempts to derive a generalizable APT model *and/or* the underlying linear factor model of the return generating process that is applicable to subsets of securities other than the securities employed in deriving the model. However, the ambiguity noted earlier in Antoniou *et al*.'s (1998) study must not be ignored; factors that explain returns in one sample may not *always* explain returns in other samples. A specific factor structure may be *partially* (as opposed to fully) generalizable. In this regard, Antoniou *et al*.'s (1998) findings provide evidence suggesting that within the APT framework, the APT model and by implication the return generating process specification, is (at the very least) partially generalizable across samples.

3.1.6. Role of international risk

Clare and Priestley (1998) state that the growth in non-US stock markets, especially those in South East Asia, has focused attention upon emerging markets. However, although the Chen *et al*. (1986) factors have been shown to be sources of systematic risk in developed markets, little research has gone into establishing whether these factors are also sources of systematic risk in developing economies. Moreover, studies drawing upon the APT framework have failed to take into account the removal of legislative barriers and increased capital mobility. It is argued that as a result of these developments in financial markets, domestic *and* international risk factors should be considered when examining risk-return relationships.

Clare and Priestley (1998) address this gap in the literature by considering the risk-return relationships in the Malaysian stock market using an APT model that incorporates domestic factors *and* an international equity index in the form of the Morgan Stanley Capital International (MSCI) World Index. Four of the Chen *et al*. (1986) factors are initially considered; namely, the unexpected changes in the term structure of interest rates, growth in industrial production, changes in expected inflation and unexpected inflation. An additional factor, the unexpected change in the risk-free rate, is also considered. As in Antoniou *et al*. (1998) the NLSUR framework is applied and a "domestic" APT model is estimated using returns on stocks traded on the Malaysian Stock Exchange (MSE) over the January 1986 to August 1994 period. Results indicate that the unexpected changes in the risk-free rate, the term structure of interest rates, unexpected inflation and changes in expected inflation are priced. This suggests that the Chen *et al*. (1986) factors explain expected returns on emerging markets. The model is then augmented with a domestic market index in the form of the Kuala Lumpur Composite Index which is found to be priced alongside all the other factors. Finally, the "international" APT model is estimated by incorporating returns on the MSCI World Index into the model. Clare and Priestley (1998) argue that this index fulfils the role of a *catch-all* proxy for international risk, similar to that of the residual market factor proposed by Burmeister and Wall (1986), by accounting for unobserved international factors that drive national markets. The authors find that while the size of the estimated risk premia on the domestic factors remains approximately the same, the risk premium on the MSCI World Index is statistically significant, suggesting that international risk plays a role in explaining expected returns on the Malaysian stock market. Moreover, a comparison of the explanatory power of the CAPM, the APT and the "international" APT model indicates that the augmented APT model is superior to both the CAPM and domestic APT model which rely exclusively upon domestic factors to explain expected returns. These results imply that an international risk factor contributes positively to the description of expected returns on the Malaysian stock market.

Clare and Priestley (1998) conclude by stating that although domestic factors are important for the pricing of Malaysian stocks, an international risk factor such as a market index, which proxies for unobserved international risk factors, plays an important role. These findings serve as an extension of the framework and an important acknowledgement that it is not only domestic risk that is relevant in asset pricing. International risk factors or proxies thereof should be considered in asset pricing studies and the role of international risk factors in the return generating process must be investigated accordingly.

Whereas Clare and Priestley (1998) employ a proxy for international risk, Cauchie *et al*. (2004) consider the role of specific international risk factors in returns on industrial sectors compromising the Swiss market over the January 1986 to November 2002 period. The authors state that identifying factors that drive returns is of major interest for practical and academic research, and the identity of factors is informed by the assumptions upon which a model is built. Implicit in these assumptions is that investors are limited to domestic stocks. Whereas this assumption is valid for closed or unintegrated markets, it is reasonable to assume that developed markets are at the very least partially integrated in an era of increasing globalization. Cauchie *et al*. (2004) seek to establish the role of domestic and international risk factors in the Swiss stock market, which is assumed to have international exposure accompanied by a perception of imperfect integration. The authors use specific international risk factors as opposed to a general proxy for international risk with the choice of risk factors guided by Chen *et al*.'s (1986) hypothesis that any factor which affects future cash flows and/or the discount rate will impact prices.

The model is chosen on the basis of explanatory power maximizing criteria and the final specification incorporates four factors, two of which are domestic and two of which are of an international nature. The domestic risk factors are returns on the Swiss stock market and the Swiss term structure whereas the international risk factors are the changes in industrial production and expected inflation for the group of seven (G7) countries. Although none of the factors are priced, the model explains approximately 10 percent of the variation in expected returns suggesting that *both* domestic and international risk factors play a role in the pricing of Swiss stocks.[39] Cauchie *et al*. (2004) argue that not only do these results confirm the identity of the factors used by Chen *et al*. (1986), each factor – domestic and international - is also representative of a risk category identified by Chen *et al*. (1986). These results, although somewhat ambiguous given the lack of statistically significant pricing relationships, motivate for the consideration of the impact of specific international factors on stock returns. Moreover, *regardless* of which approach is undertaken – whether a proxy for international risk is utilized as in Clare and Priestley (1998) or specific international risk factors are considered as in Cauchie *et al*. (2004) - it is evident that the macroeconomic APT model and the APT as a conceptual framework should consider sources of international risk.

3.2. The APT model and the return generating process: A linkage

The findings of studies dealing with the macroeconomic APT model lead to a number of inferences. Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) show that changes in

[39] Cauchie *et al*.'s (2004) assessment of this model is primarily based upon its explanatory power ($\bar{R}^2$).

macroeconomic factors assumed to represent changing systematic risk explain expected returns. As these factors are priced, this also suggests that macroeconomic factors are able explain time series variation in the return generating process (Elton & Gruber, 1988). This is supported by the presence of correlation between market aggregates and the macroeconomic factors. Furthermore, Chen *et al*. (1986) and Hamao (1988) explicitly acknowledge that the return generating process can be modelled as a function of multiple macroeconomic factors. Studies that follow highlight some of the limitations of the approach, suggest that the set of factors important to returns may differ across markets, emphasize the role of innovations in the APT framework, suggest that APT specifications can be generalized across assets and draw attention to the role of international risk factors.

Whereas APT studies employing pre-specified factors allude to the structure of the return generating process, inferences based upon these studies have mainly been drawn from results pertaining to the APT model, which establishes equilibrium relationships. The lack of direct consideration given to the time series relationships between returns and risk factors is indicative of the gap within the literature regarding the relationship between the APT model and the linear factor model underlying it. Fortunately, there exists a strand of literature which considers the time series relationships between returns and macroeconomic factors, and links the return generating process to the APT model by considering both aspects of the APT framework simultaneously. These studies suggest that the APT framework is not only a framework within which pricing relationships are established, but also a framework within which the return generating process can be modelled and investigated

3.2.1. Pricing and the return generating process

Beenstock and Chan (1988) state that the main limitation of the factor analytic approaches employed in the APT framework is that factors cannot be interpreted and therefore, it is impossible to determine whether factors derived using factor analysis represent systematic risk or idiosyncratic risk. Instead of focusing upon the cross-sectional implications of the APT model, the authors consider both the APT model and the return generating process by first establishing whether returns are linearly related to innovations in macroeconomics factors over time and then by establishing whether expected returns are linked to the estimated factor loadings. Two approaches to estimating factor loadings are applied, and in doing so, models of the relationship between macroeconomic factors and returns over time are estimated. In the first approach, innovations are generated and used to estimate factor loadings over the first T_1 observations whereas the remaining T_2 observations are used to estimate the APT model. In the second approach, factor loadings are estimated from odd observations whereas the APT model is estimated using even observations. A set of eleven candidate risk factors is considered.[40] Using UK return data for the October 1977 to December 1983 period, Beenstock and Chan (1988) find that according to the first approach, returns on seventy-six portfolios are described by four factors over time. These are interest rates as measured by treasury bill rates, the broad measure of the money supply, fuel and material costs and retail prices. Under the second approach, results are almost identical in that returns on the portfolios are significantly related to innovations in the same four factors. However, whereas under the first approach the relationship between the money supply and returns is negative, under the second approach it is positive.

Beenstock and Chan (1988) suggest that these findings imply a four-factor model to describe returns – this in essence representing a four-factor model of the return generating process derived within the APT framework. The number of positive and negative factor loading estimates is noted, and the authors ascribe economic meaning to these results in terms of the expected cash flow model employed by Chen *et al*. (1986). Factor loadings estimated under the two approaches are then used to estimate the cross-sectional APT model. Treasury bill rates, the money supply, fuel and material costs, and retail prices are priced under the first approach whereas treasury bill rates, the money supply and retail prices are priced under the second approach. The APT model explains over a third of cross-sectional variation in expected stock returns under both approaches. Unlike Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) who focus only upon the cross-sectional implications of the APT model, Beenstock and Chan (1988) directly consider the return generating process underlying the APT model and show that multiple factors drive returns. Furthermore, while it has been argued by Bower *et al*. (1984) and implied by Chen *et al*. (1986) and Hamao (1988) that the APT framework offers a systematic link between the APT model and the return generating process, none of the macroeconomic APT studies discussed so far have considered both the return generating process and the APT model together. Beenstock and Chan (1988) on the other hand show that factors that explain the time series behaviour of returns are also priced.

[40] The UK treasury bill rate, broad money supply, fuel and material cost index, general index of retail prices, general index of wages, industrial stoppages, export and import volume indices, relative export prices, Gross Domestic Product (GDP), and total production in countries belonging to the Organization for Economic Co-operation and Development (OECD).

In doing so, a linkage between the macroeconomic APT model and the return generating process underlying the APT model is demonstrated. Notably, the authors identify a return generating process *and* APT model specification for UK stocks *within* the APT framework.

McElroy and Burmeister (1988) re-examine the APT framework as a multifactor non-linear regression with pre-specified factors and across-equation restrictions. Whereas the approach of pre-specifying factors is consistent with Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988), the authors estimate the return generating process and APT model using NLSUR methods. As this approach yields joint estimates of factor sensitivities and risk premia, it permits insight into both aspects of the APT framework simultaneously. McElroy and Burmeister (1988) specify a five-factor model to describe returns on a sample of individual firms in the CRSP database over the January 1972 to December 1982 period. The factors incorporated into the model are the term structure of interest rates, the default spread, unexpected deflation, real final sales and the residual market factor where the residual market factor is a *catch-all* proxy representing variation in the return generating process not explained by the macroeconomic factors employed in the model (Burmeister & Wall, 1986). In selecting these factors, reference is made to the expected cash flow model as discussed in Chan *et al*. (1985) and Chen *et al*. (1986). The residual market factor is constructed by regressing returns on the S&P Composite Index on the four remaining factors. This yields initial insight into the structure of the return generating process underlying returns as an aggregate. Returns on the S&P Composite Index are negatively and significantly related to changes in the default spread and final retail sales and positively and significantly related to changes in the term structure and unexpected deflation. Together, these four factors explain almost a quarter of the time series variation in S&P Composite Index returns.

The second set of results reported by McElroy and Burmeister (1988) shows the factor sensitivities of returns on individual stocks to the five factors. These results, in essence, provide insight into the return generating process underlying the returns on individual stocks in the sample. Together, these five factors explain between 30 percent and 50 percent of the variation in returns. The relationship between returns on individual stock and the default spread is found to be mostly significant and predominantly negative whereas the relationship between returns and changes in the term structure is mostly significant and predominantly positive.[41] The relationship between unexpected deflation and retail sales differs in direction and statistical significance across stocks. McElroy and Burmeister (1988) report that over 60 percent of the estimated factor sensitivities are statistically significant suggesting that macroeconomic factors that are widely used in asset pricing and are assumed to measure systematic risk, also feature in and explain the return generating process. In turn, all risk premia in the corresponding APT model are priced suggesting that the very factors that characterize the return generating process are also those that explain the cross-section of expected returns. In conclusion, the authors state that the proposed set of factors is not unique and suggest that further research should be undertaken into whether there are other factors that explain returns. Similarly to Beenstock and Chan (1988), McElroy and Burmeister's (1988) results point towards a relationship between the APT model and the underlying return generating process. The role of the APT framework as a valid conceptual framework is demonstrated; it is successfully adapted for the purposes of identifying and modelling the return generating process as well as for asset pricing.

3.2.2. Pricing and the return generating process of South African stock returns

Van Rensburg (1996) argues that part of the reason for the emergence of the APT framework is the failure of the single-factor model underlying the CAPM to capture the numerous influences that drive stock returns. Although, the APT framework addresses this shortcoming by permitting a number of factors to influence returns, the main limitation of the model is the lack of clarity regarding the identity and number of factors in the return generating process. To investigate the pricing and return generating process of South African stock returns and to address these limitations, the author regresses returns on the JSE All-Share Index over the January 1980 to December 1989 period on unexpected movements in ten candidate risk factors.[42] Out of this set of risk factors, four factors have a statistically significant impact upon aggregate returns; namely, the Rand gold price, returns on the Dow Jones Industrial Average (DJIA), inflation expectations and the term structure of interest rates. Van Rensburg (1996) states that *a posteriori*, these factors should have a pervasive impact upon stock returns – an important assumption suggesting that a multifactor model derived upon the basis of a market aggregate should be generalizable to various securities compromising the JSE

[41] The words "mostly" and "predominantly" are used purposefully within this context. The relationship between returns on the securities in the sample and these two factors is not always of the same direction nor is always statistically significant. This can be seen in the context of Antoniou *et al*.'s (1998) findings which suggest that specifications derived within the APT framework are not always fully generalizable.

[42] Inflation, growth rates of manufacturing production, retail sales, the money supply, building plans passed, the Rand-Dollar exchange rate, the Rand gold price and returns on the Dow Jones Industrial Average (DJIA).

All-Share Index. The four-factor model of the return generating process explains 30 percent of the variation in returns. Changes in the gold price and returns on the DJIA have a positive and statistically significant impact upon returns and changes in inflation expectations and the term structure of interest rates have a negative and statistically significant impact upon returns. The residual term of this four-factor model is treated as the residual market factor.

Van Rensburg (1996) then turns to the identification of priced factors in a sample of South African firms using the NLSUR approach. Returns on the DJIA, changes in inflation expectations and changes in the term structure of interest rates are found to be priced. The residual market factor is not priced and neither is gold. It is suggested that despite gold's importance to the South African economy, it is not priced because of the sample's composition. The sample includes industrial and financial firms which are *unaffected* by movements in the gold price. This finding implies that gold may have an industry wide effect as opposed to a systematic effect and it is this industry wide effect that is reflected in the four factor-model of the return generating process. When gold is excluded from the estimation of the residual market factor and the APT model, the residual market factor is priced alongside returns on the DJIA, changes in inflation expectations and changes in the term structure of interest rates. Van Rensburg (1996) concludes that further research should be undertaken into the APT specification proposed in the study. A noteworthy finding is that the South African stock market is integrated with global markets. This inference is based upon the finding that returns on the DJIA feature in the return generating process specification and explain expected returns on South African stocks.

In a subsequent article, Van Rensburg (2000) again investigates the pricing of South African stocks over the January 1985 to January 1995 period and conducts a more extensive investigation of the return generating process within the APT framework.[43] Innovations, as required by the APT framework, are generated utilising Vector Autoregressive models. To identify risk factors, the correlation between the candidate risk factors and returns on the JSE All-Share Index is investigated. Factors that are not significantly correlated with JSE All-Share Index returns are omitted from further analysis. Implicit in this approach, as in Van Rensburg (1996), is the assumption that factors that drive returns on the JSE All-Share Index

[43] The expanded set of risk factors includes returns on the JSE All-Share, the Industrial and All-Gold Indices, the respective earnings on these sectors, returns on the DJIA, the Rand gold price, rates on a 10-year government bond, the three month banker's acceptance rate, the level of gold and foreign exchange reserves, and the money market shortage.

have a pervasive influence on securities throughout the South African stock market. However, Van Rensburg (2000) goes further in the exposition of the return generating process. An initial model of JSE All-Share Index returns is estimated incorporating returns on the DJIA, the 10-year government bond rate, the Rand gold price, growth in All-Share Indexed Earnings and changes in gold and foreign reserves. Jointly, these factors explain just under 30 percent of the variation in returns on the JSE All-Share Index with the former three factors having a statistically significant impact upon returns. Van Rensburg (2000) then augments this model with *two* residual market factors in the form of (uncorrelated) returns on the JSE Industrial and All-Gold Indices to correct for an upward bias in the variance of the estimated model parameters arising from potential model under-specification. It is argued that this bias translates into an (incorrect) acceptance of the null hypothesis of no relationship between returns and the factors describing the return generating process. *All* factors in the augmented model now have a statistically significant impact upon returns and the model explains over 90 percent of the variation in returns. This substantial improvement in explanatory power over the unaugmented model is attributed to the incorporation of a second residual market factor. Van Rensburg (2000) refers to this set of models as models of the return generating process which explain the time series of equity returns. As in Van Rensburg's (1996) study, the linear factor model underlying the APT model is the motivation for these multifactor specifications of the return generating process.

The results of the APT model indicate that all factors that appear in the underlying return generating process, with the exception of indexed earnings, are priced (Van Rensburg, 2000: Table 6, Panel B&G). Risk premia on the DJIA and changes in the level of gold and foreign exchange reserves are negative whereas risk premia associated with the remaining factors are positive. All factors are also priced individually. As with the findings of McElroy and Burmeister (1988), Beenstock and Chan (1986) and Van Rensburg (1996), these findings indicate that there is a relationship between the APT model and the return generating process. Furthermore, Van Rensburg's (2000) use of the DJIA as a risk factor again points towards the significance of international risk in the return generating process and the pricing of South African stocks. Last but not least, the use of a second residual market factor represents an important extension of the APT framework. The use of a second residual market factor is motivated by the potential failure of a limited set of pre-specified factors to sufficiently capture all risk in returns and to fully account for variation in the return generating process (Van Rensburg, 2000).

Van Rensburg's (1996, 2000) studies *suggest* that the APT framework can be applied in the South African context. Although, priced factors and those that feature in the return generating process of the South African stock market differ from those in other markets, the APT framework is nevertheless successfully applied in an investigation of the pricing and return generating process of South African stock returns. However, there is a limitation to Van Rensburg's (1996, 2000) application of the APT framework in investigating the return generating process; the framework is applied to a *single* return series, namely returns on the JSE All-Share Index. This motivates for an application of the APT framework to an *extended* number of South African return series – an endeavour that is undertaken in this study.

3.2.3. Beyond stock returns

Elton *et al*. (1995) state that although bond markets play an important role in the US economy, there is relatively little interest in bond pricing models and the APT framework has yet to be applied to bond pricing. The authors address this gap by developing a set of models within the APT framework which explain *both* the time series behaviour of returns and expected returns on bonds. It is further argued that tests of the APT model are also a test of the return generating process – an important proposition suggesting that the seminal studies of Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988) indirectly confirm the validity of the underlying multifactor return generating process. Two models are suggested to explain returns on three bond samples over the January 1980 to December 1992 period. The four-factor model incorporates excess returns on the S&P 500 Index, returns on a modified Lehman Brothers bond index, unexpected changes in Real GNP and inflation. The six-factor model incorporates the default spread and an option factor in addition to the factors in the four-factor model.[44] In motivating for these specifications, Elton *et al*. (1995: 1237) acknowledge the role of the APT as a conceptual framework by stating that their hypothesis that these models describe returns on bonds is "in the tradition of Chen *et al*. (1986), that returns are generated by a mixture of tradable portfolios and fundamental (macroeconomic) economic factors." Furthermore, the authors also acknowledge that the APT can be decomposed into time series and cross-sectional explanations of returns – an important acknowledgment of both aspects of the APT framework.

[44] The difference between returns on the Lehman Brothers Government National Mortgage Association Index and returns on a government bond series with the same duration.

Using the NLSUR methodology, Elton *et al*. (1995) find that each factor explains a proportion of expected returns. For example, the four-factor model explains a total of 87.05 percent of the variation in expected returns. In this model, the S&P 500 accounts for 3 percent of explained variation, the aggregate bond index accounts for 73 percent of explained variation, unexpected changes in the GNP and inflation account for 7 percent and 17 percent of explained variation respectively. The six-factor model explains 82.47 percent of the variation in expected returns. Comparatively, a single-factor model incorporating only an aggregate bond index explains a total of 40.32 percent of the variation in expected returns. Results indicate that both the four and six-factor models, on average, explain over 90 percent of the time series variation in bond returns, with most of the time series explanatory power attributable to the aggregate bond index. While the aggregate bond index has the greatest explanatory power in a time series context, in the cross-section, the additional factors play an important role. This suggests that systematic risk plays different roles in the return generating process and in the APT model (Elton *et al*., 1995). Furthermore, it also suggests that, as with stock returns, there are extra-market sources of risk in bond returns. These important inferences provide further motivation for directly investigating the role of systematic risk in the return generating process as opposed to considering it only within the context of the APT model as in Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988). Elton *et al*. (1995) also apply the APT framework in an exploratory context to examine the sensitivity of returns on corporate, mortgage and government bond funds to the factors describing the return generating process. According to the six-factor model, mortgage funds are more sensitive to the option index relative to the other funds whereas government bonds are least sensitive to this factor. The corporate bond fund is the only fund which is positively affected by growth in the GNP and inflation has the least adverse impact upon the government fund. The positive impact of increases in GNP on corporate bonds is attributed to decreases in risk, which more than compensate for rising interest rates which follow increases in the GNP. The negative impact of unexpected inflation on the three categories of funds is attributed to increases in interest rates which decrease returns.

Elton *et al*.'s (1995) extension of the APT framework to the description of the return generating process and cross-sectional behaviour of bond returns demonstrates the flexibility and applicability of the APT framework. The impact of pre-specified factors on returns is interpreted in the context of economic theory. It is shown that factors have a differing impact upon the time series behaviour of returns and expected returns.

3.3. Applying the APT as a conceptual framework

Beenstock and Chan (1988), McElroy and Burmeister (1988), Van Rensburg (1996, 2000), and Elton *et al*. (1995) investigate *both* aspects of the APT. These studies acknowledge that assets can be priced *and* the return generating process can be investigated within the APT framework. It is however evident that studies employing the APT framework primarily focus on asset pricing. Nevertheless, the APT framework is credited with providing a conceptual framework for multifactor models of the return generating process. Elton *et al*. (2003) credit Chen *et al*. (1986) with laying the foundation for models attributing the variation in returns to a broad set of factors. It is suggested by the authors that the APT framework has been applied in academic and commercial contexts to describe the return generating process. Elton *et al*. (2003) further acknowledge that estimating the linear factor model underlying the APT model in a time series context yields a model of the return generating process. The influence of the APT on Chen *et al*.'s (1986) work is evident and motivation for multifactor models of the return generating process stems from Chen *et al*.'s (1986) acknowledgement that the return generating process can be described by a set of pre-specified macroeconomic factors. Shanken and Weinstein (2006) suggest that the APT is often cited as a motivation for models employing a set of macroeconomic factors to explain returns. Caporale and Perry (2006: 3) state that the APT framework has been proven to be a "very flexible framework to analyze asset returns since it allows an asset be exposed to several risk factors." The authors further state that the variability - as opposed to pricing - of returns has been analyzed within the context of the APT framework. Connor (1995) directly credits Chan *et al*. (1985) and Chen *et al*. (1986) with laying the basis for models of returns which employ macroeconomic factors to explain return behaviour.

The discussion that follows is based upon studies that demonstrate how the APT framework is applied *directly* to investigate the return generating process without taking into consideration the pricing aspect of the APT framework. The influence of the APT framework in these studies is apparent; aside of each study referring to the APT framework to motivate for a multifactor model, these studies refer to the APT and studies based thereupon to identify the category of factors used to explain the return generating process.

3.3.1. Modelling the return generating process within the APT framework

Burmeister and Wall (1986) argue that the linkage between returns and macroeconomic activity has long been mired in controversy. The lack of understanding of the linkages between return behaviour and macroeconomic factors is considered to be problematic as return behaviour is central to measuring discount rates, understanding business cycles and answering questions relating to efficient resource allocation. It is within this context that the authors seek to examine the time series behaviour of US stock returns over the December 1971 to November 1981 period. With reference to Ross' (1976) work on the APT and the linear factor model, the dividend discount model and the work of Chen *et al.* (1986), Burmeister and Wall (1986) consider innovations in the default spread, the term structure, unexpected inflation and real final sales as factors in a multifactor model of the return generating process. Although mention is made of the cross-sectional APT model, Burmeister and Wall (1986) focus upon describing the return generating process of returns on the S&P500 Index, an equally weighted portfolio of stocks and a mutual fund.[45] The return generating process specification proposed by Burmeister and Wall (1986) is as follows:[46]

$$r_t = b_{j0} + b_{j1}UPR(t) + b_{j2}UTS(t) + UI(t) + UGS(t) + UM(t) + \varepsilon_t \tag{3.3}$$

where r_t is the return on a given series in Burmeister and Wall's (1986) sample, b_{j0} is the intercept term and the residuals are denoted by ε_t. $UPR(t)$ is the default spread, $UTS(t)$ is the term structure, $UI(t)$ is unanticipated inflation and $UGS(t)$ is the unexpected change in real final sales. $UM(t)$ is the residual market factor – a notable contribution of Burmeister and Wall (1986) who are the first to employ this factor within the APT framework. Each return series is initially regressed on the first four factors and then on all factors including the residual market factor in equation (3.3). Results of the time series LS regressions are (faithfully) reproduced in Table 3.2.

[45] T. Rowe Price New Horizons Fund.
[46] The notation of Burmeister and Wall (1986) is retained for demonstrative purposes.

Table 3.2: Results of time series regressions

r_t	Const.	$UPR(t)$	$UTS(t)$	$UI(t)$	$UGS(t)$	$UM(t)$	$\bar{R}^2$
1) S&P 500 total returns index	0.0094 (2.64)	1.54 (4.57)	0.50 (4.39)	-3.03 (-2.64)	1.30 (4.38)	-	0.29
2) Total Return on equally weighted portfolio of 20 randomly selected stocks.	0.0110 (2.43)	2.19 (5.02)	0.58 (4.00)	-4.20 (-2.82)	1.60 (4.15)	-	0.29
3) Total Return on T. Rowe Price New Horizons Fund	0.0120 (2.08)	1.74 (3.30)	0.49 (2.75)	-5.78 (-3.22)	1.89 (4.07)	-	0.22
4) Same as row (2)	0.0110 (4.76)	2.19 (9.84)	0.58 (7.75)	-4.20 (-5.53)	1.60 (8.13)	1.12 (18.10)	0.82
5) Same as row (3)	0.0120 (3.90)	1.74 (6.20)	0.49 (5.16)	-5.78 (-6.05)	1.89 (7.64)	1.32 (17.10)	0.78

Notes:
1. *t*-statistics are reported in parentheses as in Burmeister and Wall (1986)

Source: Burmeister & Wall (1986)

The results above indicate that $UPR(t)$, $UTS(t)$, $UI(t)$ and $UGS(t)$ jointly explain 29 percent of variation in returns on the S&P 500 Index. Moreover, the model explains 29 percent and 22 percent of the variation in returns on the portfolio and the mutual fund in rows 2) and 3) respectively. The estimated factor sensitivities in the multifactor model reveal the impact of innovations in factors on each return series. Burmeister and Wall (1986) then proceed to model returns on the two latter series with the residual market factor incorporated into the model (rows 4) & 5)). The residuals of the regression of S&P 500 returns on the four factors in row 1) constitute the residual market factor – the unexpected change in the market index *not* explained by the first four factors in the model. The residual market factor has come to play a central role in the modelling of the return generating process within the APT framework by fulfilling the role of a *catch-all* proxy for omitted factors measuring other market risk (Berry *et al*., 1988; Van Rensburg, 1996).

The amount of variation explained for these two series increases substantially to 82 percent and 78 percent respectively suggesting that although the market index can explain a significant amount of variation in returns, there are *at least* four different types of distinct risk aside from market risk in returns (Burmeister & Wall, 1986). This is evident from rows 4) and 5) in Table 3.2 where innovations in each factor, aside from the residual market factor, continue to have a statistically significant impact upon returns. This points towards the presence of significant extra-market sources of systematic risk in the return generating process. Burmeister and Wall (1986) extend this approach to individual stocks and demonstrate that the return generating process of individual stocks can also be modelled within the framework. Returns on eight individual stocks are regressed onto the four risk factors and the residual market factor. Results reveal that most of the estimated factor sensitivities (36 out of 40), with a few exceptions, are statistically significant. The authors note that the size of factor sensitivities differs across stocks suggesting that different assets respond differently to innovations in macroeconomic factors. The widespread statistical significance of factors suggests that a multifactor model can be used to not only describe the return generating process of aggregates but also to describe the return generating process of individual stocks. Burmeister and Wall (1986: 16) conclude that the APT "offers a most promising line of research for the better understanding stock market behaviour and its linkages with macroeconomic variables." An analysis of stock market behaviour conducted within the context of the APT framework, together with the introduction of the residual market factor, represents a direct application of the APT framework and an extension of the theory to an investigation of the return generating process.

Berry *et al*. (1988) state that the APT framework permits a variety of risks to affect returns and seek to examine the importance of five risk factors that have been shown to influence returns in US markets. It is argued that by being able to measure the differences in exposures to risk across economic groups and industries, it is possible to construct strategies to manage risk and achieve superior returns. Although, there is no single set of risk factors that explain returns, and a number of equivalent sets of factors may exist, what is required is that returns are plausibly related to these factors. The authors go onto argue that although a given set of factors may not be fully exhaustive in its explanation of returns, the residual market factor will proxy for any missing factors. Berry *et al*. (1988) frame their investigation of the return generating process within the APT framework. The risk factors chosen to explain returns are almost identical to those proposed by Chan *et al*. (1985) and Chen *et al*. (1986) in their respective asset pricing studies. These factors are the term structure, the default spread, changes in unexpected deflation and the growth rate in real sales. The residual market factor is employed to measure the portion of S&P 500 returns not explained by the other four factors. As in Burmeister and Wall (1986), Berry *et al*. (1988) use innovations[47] in these macroeconomic factors to describe the return generating process. Having defined these factors, returns on portfolios representing seven economic groups and eighty-two industrial

[47] Berry *et al*. (1988) report that statistical tests reveal that these risk factors cannot be predicted from past values implying that they qualify as innovations.

sectors of the US stock market are regressed onto the unexpected risk factor series using time series LS regressions over the January 1972 to December 1982 period.

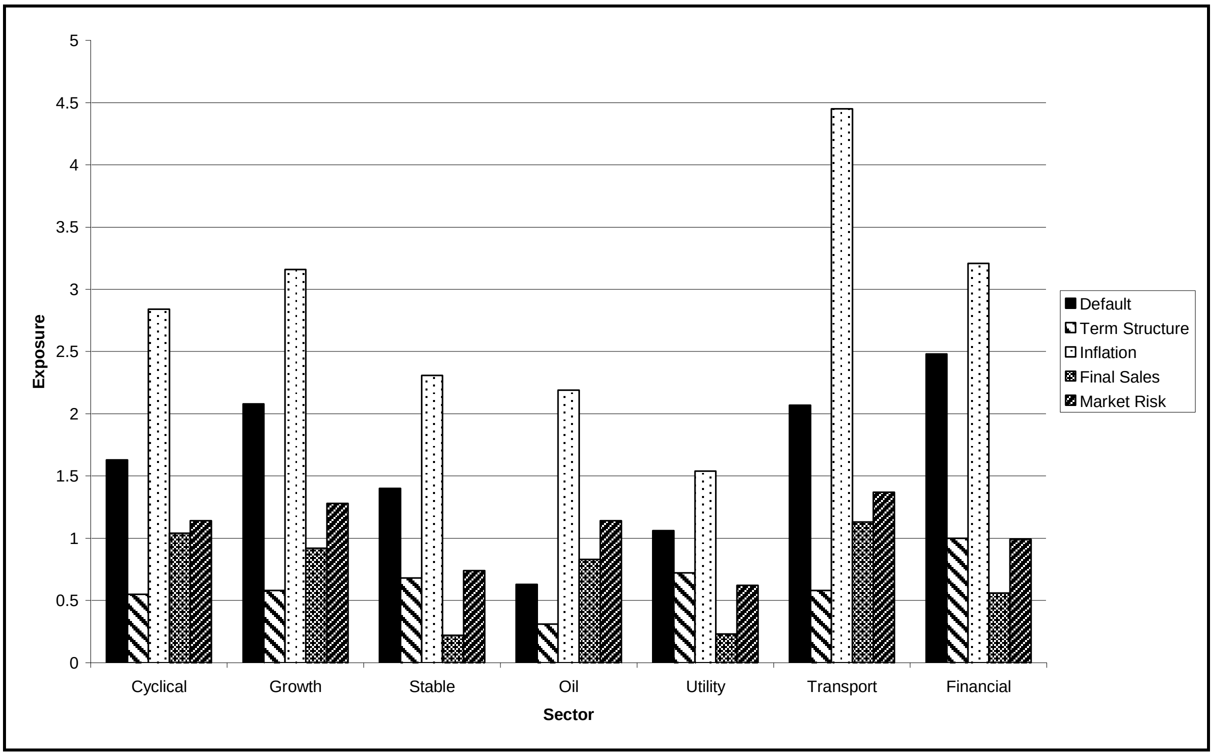
Table 3.3: Quantities of different types of risk for seven US economic sectors

Sector Name	Type-1 Risk (default)	Type-2 Risk (term structure)	Type-3 Risk (inflation or deflation)	Type-4 Risk (unexpected change in growth rate of profits)	Type-5 Risk (residual market risk)	R^2 (adjusted R-squared)	DW (Durbin Watson statistic)
Cyclical	-1.63	0.55	2.84	-1.04	1.14	0.77	1.67
	(-6.93)	(6.97)	(3.55)	(-3.64)	(18.47)		
Growth	-2.08	0.58	3.16	-0.92	1.28	0.84	1.94
	(-9.80)	(8.21)	(4.38)	(-3.57)	(23.05)		
Stable	-1.40	0.68	2.31	-0.22	0.74	0.73	1.81
	(-7.09)	(10.25)	(3.43)	(-0.93)	(14.20)		
Oil	-0.63	0.31	2.19	-0.83	1.14	0.50	1.79
	(-1.62)	(2.42)	(1.65)	(-1.75)	(11.12)		
Utility	-1.06	0.72	1.54	0.23	0.62	0.67	1.84
	(-4.93)	(10.02)	(2.11)	(0.87)	(11.03)		
Transport.	-2.07	0.58	4.45	-1.13	1.37	0.66	2.01
	(-5.65)	(4.75)	(3.57)	(-2.53)	(14.24)		
Financial	-2.48	1.00	3.20	-0.56	0.99	0.72	1.85
	(-8.44)	(10.21)	(3.21)	(-1.57)	(12.86)		

Notes:
1. *t*-statistics are reported in parentheses as in Berry *et al*. (1988).

Source: Berry, Burmeister & McElroy (1988)

The results in Table 3.3 indicate that the five-factor model identifies sources of risk in US returns and that the five risk factors explain a substantial amount of variation in returns. The model explains between 50 percent and 84 percent of the variation in the returns on economic groups with thirty-one out of the thirty-five estimated factor sensitivities being statistically significant. This suggests that a multifactor model of the return generating process can be used to explain returns on economic sectors of the US economy and to relate returns to multiple sources of systematic risk. Berry *et al*. (1988) interpret the estimated sensitivities as different risk types. Risk profiles denoting the levels of risk exposure to each factor are reproduced in Figure 3.1. This permits a comparison of exposures faced by each economic group to given risk categories.



Source: Berry, Burmeister & McElroy (1988)

Figure 3.1: Risk profile of economic sectors

Berry *et al*. (1988) ascribe economic interpretation to the observed results. For example, the low exposure of utilities to inflation may arise from the ability of utilities to pass on inflation induced costs in the form of higher product prices. On the other hand, this ability is not observed for the transportation sector which is highly sensitive to inflation risk. The authors extend the proposed multifactor model to returns at the industry level. Results indicate that the model explains between 15 percent and 72 percent of the variation in returns on industrial portfolios. A closer analysis of the estimated factor loadings reveals that *overall*, all factors have a statistically significant impact on industrial sector returns. Moreover, this suggests that the return generating process of US industrial sector returns can be described by a multifactor return generating process specification constructed within the APT framework. Similarly to Burmeister and Wall (1986), Berry *et al*.'s (1988) approach represents a generalization and extension of a return generating process specification constructed within the APT framework to an extended number of series. That a single specification derived within the APT framework can be used to describe returns on a number of series is suggested by Antoniou *et al*. (1998) in asset pricing literature.

Aside from relying upon the APT framework to explore the return generating process, other applications dependent upon the APT framework are suggested. Berry *et al*. (1988) state that if exposures to risk are known, a policy of risk sterilization can be followed. Alternatively, a strategy may be employed whereby unexpected components of the risk factors are forecast and according to these forecasts, a selection of stocks is made so as to maximize returns or outperform a benchmark.[48] Finally, the authors suggest that the influence of more complicated combinations of factors can also be considered within this framework. This suggests a degree of flexibility and emphasizes the APT framework's role as a conceptual basis - as opposed to a prescript - for investigating the return generating process. However, a limitation of Berry *et al*.'s (1988) study is that although it is acknowledged that factors in excess of the residual market factor can add new information, the amount of new information added is not investigated.

Chan, Hendershott and Sanders (1990) analyze returns on equally-weighted indices of real estate investment trusts (REITs) traded on exchanges in the US over the January 1973 to December 1987 period. The APT framework is the conceptual basis of the multifactor model employed by the authors and the factors employed in the model are those of Chen *et al*. (1986). As Chan *et al*. (1990) do not incorporate a market factor or a residual market factor into the return generating process specification, the explanatory power of the Chen *et al*. (1986) risk factors is investigated in isolation from the market factor. The multifactor model is estimated by regressing equally-weighted REIT returns onto industrial production growth, changes in expected inflation, unexpected inflation, the default spread and the term structure. Notably, this return generating process specification is identical to the one proposed by Chen *et al*. (1986) in their seminal study. However, unlike Chen *et al*. (1986) who only investigate the pricing of these factors, Chan *et al*. (1990) employ the model to investigate the structure of the return generating process. For comparative purposes, returns are also regressed onto the returns on equally and value-weighted NYSE indices.

Results indicate that over the entire sample, the equally-weighted NYSE index explains over 60 percent of the variation in REIT returns whereas the value-weighted index explains 37.3 percent. These results provide a benchmark which permits a comparison of the ability of the Chen *et al*. (1986) factors to explain the return generating process against a simpler alternative in the form of a single-factor model. Chan *et al*. (1990) find that over the entire

[48] For example, a forecast of negative unexpected inflation is made for the next period and returns are positively related to unexpected inflation. By knowing the risk exposures of given assets, a selection of assets or portfolios with the lowest exposure or even a negative exposure to unexpected inflation is made thus reducing losses or even increasing returns (Berry *et al*., 1988).

sample period, the five factors explain 16.6 percent of the variation in REIT returns. Although, the multifactor model explains a lower amount of variation relative to the single-factor model, it is evident that factors aside from the market index contribute significantly to the explanation of returns. Furthermore, what can be inferred from these results is that although a single-factor model is superior in terms of explanatory power, relying solely upon a market index to explain returns is a gross abstraction of the return generating process. Chan *et al*. (1990) also use the five-factor model to explain returns on the equally-weighted NYSE index. Results indicate that the model explains 34.8 percent of the variation in returns on the equally-weighted NYSE index. This discrepancy in explanatory power is attributed to greater unique risk inherent in REITs. Unexpected inflation, negative changes in the term structure of interest rates and the default spread are identified as having a negative impact upon REITs and stocks in general as represented by the NYSE index. The factors that are statistically significant in the return generating process of REIT returns for the *entire* sample period are changes in industrial production, changes in the default spread and changes in the term structure. Returns on the equally-weighted NYSE index are significantly related to unexpected changes in inflation, changes in the default spread and changes in the term structure over the entire sample period (see Chan *et al*., 1990: Table 5). Similarly to Berry *et al*. (1988), the risk profiles of the REIT and equally-weighted NYSE index return series are analyzed by comparing and contrasting estimated exposures of returns to the five factors. Returns on REITs are found to be 40 percent less sensitive to innovations in the five factors relative to aggregate returns. Chan *et al*. (1990) suggest that this implies that REITs are less risky than stocks in general.

Aside from demonstrating further applications of the APT framework and applying the framework to model the return generating process operational in a specific market sector, Chan *et al*.'s (1990) findings lead to important inferences. One such inference is that the risk categories suggested by the Chen *et al*. (1986) and the factors representative thereof *can* explain a substantial proportion of time series variation in returns as opposed to only variation in expected returns. Furthermore, these findings also confirm the validity of the five-factor model of the return generating process proposed by Chen *et al*. (1986). In doing so, these findings re-affirm the specific role of Chen *et al*. (1986) and the APT in general in laying a conceptual basis for models of the return generating process which employ macroeconomic factors representative of systematic risk.

3.3.2. Beyond developed markets

Burmeister and Wall (1986), Berry *et al*. (1988) and Chan *et al*. (1990) successfully model the return generating process of securities compromising the US market. However, it is by no means certain that the APT framework can successfully be applied to developing markets. Burmeister (2003) acknowledges that while linear factor models of the return generating process can successfully be constructed for a number of developed markets (e.g., the UK, Germany and Japan), there are possible difficulties for developing markets.

Kandir (2008) investigates the role of macroeconomic factors in explaining returns on Turkish stock portfolios over the July 1997 to June 2005 period. The APT framework is employed to model the return generating process and Kandir (2008) first traces the development of asset pricing models and their contribution to providing a conceptual framework. In doing so, the influence of asset pricing models and notably the APT on contemporary literature dealing with time series models of stock returns is demonstrated. Chen *et al*. (1986) are credited for motivating the extensive study of linkages between returns and macroeconomic factors representative of systematic risk within a multifactor framework. The influence of the APT framework is further evident in the identification of risk factors; Kandir (2008) refers to Chen *et al*. (1986) and Clare and Thomas (1994) amongst others[49] to identify macroeconomic factors for inclusion in the return generating process specification of Turkish stock returns. A multifactor model incorporating changes in industrial production, consumer prices, money supply, the exchange rate, short-term interest rates and returns on the MSCI World Index is proposed to explain Turkish stock returns. Aside from Kandir's (2008) direct references to the APT framework, it is clearly evident that this model reflects the influence of the framework in structure and factor composition.

Returns to be explained are the returns on size, book-to-market, earnings-to-price and leverage ratio sorted portfolios. The model is estimated by regressing returns on the abovementioned factors using the LS methodology. Kandir (2008) reports that the factors that are statistically significant in the return generating process are the exchange rate, the short-term interest rate and returns on the MSCI World Index. Together, these factors explain approximately 30 percent of variation in returns. It is noteworthy that the identity of significant factors does *not* change according to portfolio formation criteria. This suggests

[49] One of these studies is that of Chen (1991) who draws upon APT framework and Chen *et al*. (1986) in his study of the predictability of stock returns.

that the limitation of the APT model noted by Clare and Thomas (1994) whereby the identity of priced factors differs according to portfolio formation criteria, does not affect inferences relating to the return generating process.[50] Kandir (2008) refers to Chen *et al*. (1986) to establish the consistency of the results. The finding that industrial production and the inflation rate are statistically insignificant is inconsistent with the findings of Chen *et al*. (1986) who find that these factors explained *expected* returns. However, these findings suggest that these factors do not explain the time series behaviour of returns or are irrelevant to the Turkish market. Further interpretations of significant relationships are provided by Kandir (2008); exchange rates are significant because of increasing trade and tourist activities, interest rates impact Turkish stock returns because of their role as a proxy for alternative investment opportunities and the MSCI Word Index plays an important role as a result of Turkey's increasing integration within world markets (see Clare & Priestley, 1998). In contrast, it is argued that oil prices do not have a significant impact upon returns as there may be more important factors of production for Turkish companies and industrial production does not affect returns because of the under-development of the Turkish stock market. The insignificant relationship between stocks and inflation suggests that Turkish stocks are not a hedge against inflation. The money supply does not appear to impact real activity.

Kandir (2008) applies the APT framework within the context of a developing market. In modelling the return generating process, the APT framework is extensively treated as a conceptual basis for the structure and composition of the return generating process specification. While a number of macroeconomic factors are identified in the return generating process, the impact of factors such as industrial production, oil prices and inflation – factors which play an important role in developed markets – is statistically insignificant. This provides support for Burmeister (2003) postulation that there are difficulties in constructing linear factor models of the return generating process in developing markets. The foremost difficulty is the identity of the risk factors themselves; factors that describe the return generating process are likely to differ across markets and need to be identified in these markets.

While Kandir (2008) applies the APT framework to a single developing market, Bilson *et al*. (2001) investigate the return generating process of twenty emerging stock markets over the

[50] Not only are the same factors statistically significant for returns on portfolios formed according to differing criteria, explanatory power and estimated factor sensitivities are consistent.

January 1985 to December 1997 period.[51] As such, Bilson *et al*.'s (2001) study is far more extensive in scope. It is argued that two variants of multifactor models exist. The first assumes perfect integration with returns being driven by global risk factors whereas the second, primarily informed by the work of Chen *et al*. (1986), assumes complete segmentation whereby domestic factors drive returns. Bilson *et al*. (2001) list goods prices, oil prices, the money supply, real activity, exchange rates, interest rates and trade factors as some of the risk factors that drive returns. Although the authors recognize that the selection of factors is subject to criticism on the grounds that selection is subjective and arbitrary, guidance is provided by prior research and the role of APT literature in factor identification is evident in the abovementioned list. This list includes factors considered by Chen *et al*. (1986), Hamao (1988), Beenstock and Chan (1988), Clare and Thomas (1994) and Priestley (1996). A further problem is the identity of the international risk factor which Bilson *et al*. (2001) argue can be represented by returns on a value-weighted world index. The use of an international risk factor again finds support in APT literature (see Clare & Priestley, 1998). Bilson *et al*. (2001) however extend the theory by suggesting that regional influences may also play a role if countries are regionally integrated.

An initial a five-factor model motivated by APT literature incorporating four domestic factors and returns on a value-weighted world market index, the MSCI World Index, is chosen by Bilson *et al*. (2001) to model returns. Changes in the money supply, the prices of goods (inflation), real activity and exchanges rates represent domestic factors. Bilson *et al*. (2001) acknowledge that while the APT framework employs unexpected components, factors in the study are employed in their raw form.[52] The base specification is further extended to include a political risk measure, a trade sector factor, interest rates, a regional factor, the price-to-earnings ratio and the (aggregate) dividend yield. Although, the last two factors are not fully consistent with the approach of Chen *et al*. (1986) who hypothesize returns to be a function of macroeconomic factors and non-equity factors, their inclusion is justified by the ability of factors constructed from the same market to better explain returns relative to macroeconomic factors (see Chen, 1991; Van Rensburg, 2000).

[51] Argentina, Brazil, Chile, Colombia, Greece, India, Indonesia, Jordan, Korea, Malaysia, Mexico, Nigeria, Pakistan, Philippines, Portugal, Taiwan, Thailand, Turkey, Venezuela and Zimbabwe.

[52] Bilson *et al*. (2001) use this term to describe factor series that consist of expected *and* unexpected components. In other words, "pure" innovations are not used. The use of expected and unexpected components of factors is common place in recent studies of the return generating process (see Sadorsky, 2001; Sadorsky & Henriques, 2001). This represents a relaxation of the assumptions of the APT framework.

For the restricted five-factor model, results indicate that returns on ten markets are positively and significantly influenced by returns on the MSCI World Index whereas the exchange rate is statistically significant for eleven markets with a predominantly negative impact. The money supply is statistically significant for five markets, with the relationship between returns and changes in the money supply being predominantly positive. Inflation and real activity significantly impact returns on two individual markets, namely Mexico and Portugal. Reported *F*-statistics indicate that the factors in the model are jointly significant for ten out of the twenty markets and the average $\bar{R}^2$ is 0.136 ranging between -0.00 and 0.38 for Columbia and Indonesia respectively. A number of important inferences arise from these results. Firstly, the same risk factors *do not* explain returns across markets suggesting that there may be risk factors that are relevant to specific markets. In other words, the structure of the return generating process *differs* across markets. Secondly, as the factors are jointly significant in ten out of the twenty markets, a multifactor model may *not* be applicable to all markets. Finally, the spread of explanatory power also suggests that not all emerging stock markets can be adequately described by a multifactor return generating process derived within the APT framework. These results suggest that the identification of the return generating process is not a straightforward task. Such a proposition is consistent with the findings of Poon and Taylor (1991) who suggest that factors that explain returns are likely to vary across markets. Furthermore, these findings highlight a potential limitation of the multifactor APT framework; it may impossible to fully describe the return generating process relying *only* upon macroeconomic factors (Van Rensburg, 1996; Bilson *et al*., 2001).

The unrestricted model is estimated next. Results indicate that the MSCI World Index loses much of its significance; returns for three markets are significantly related to returns on the index. However, returns are significantly related to returns on regional indices in twelve out of twenty instances suggesting that regional risk factors are more important than international factors. The exchange rate is statistically significant for fourteen markets – a finding re-affirming this factor’s importance. Money supply and real activity are significant for two markets, and the inflation, country risk and the trade factors are significant for three markets. Interest rates have a statistically significant impact upon returns in four markets. Notably, the price-to-earnings ratio and dividend yield are statistically significant for returns in sixteen and ten markets respectively suggesting that factors constructed out of the same market are *better* at explaining returns relative to macroeconomic factors used in place of unidentified APT risk factors. This suggests that the APT framework should be extended to consider equity related aggregates. On average, the unrestricted model explains 60 percent of the variation in returns. Bilson *et al*. (2001) conclude by stating that within a multifactor setting, a larger set of factors improves the fit of the model.

Like Kandir's (2008) approach, Bilson *et al*.'s (2001) approach strongly reflects the influence of the APT framework. However, the sporadic significance of certain factors across markets and inconsistent explanatory power emphasizes the argument that the same risk factors are not applicable across markets and that not *all* markets can be described by an APT framework derived model. These findings – extensive in scope - suggest that there are possible limitations and difficulties in applying the APT framework to developing markets. This is especially pertinent given that the South African stock market can be classified as a developing market. Moreover and in light of the limitations of Van Rensburg's (1996, 2000) analysis, it remains to be seen whether the APT framework can be applied and generalized to an *extended* number of South African return series. If the APT framework can be applied in an extensive investigation of the behaviour of South African stock returns, only then can it be said that the framework is applicable within the South African context.

3.3.3. Beyond the return generating process

The APT framework can be applied to a range of financial phenomena – any phenomena that require multifactor specifications. While there are numerous studies that draw upon the APT framework to study financial phenomena, two noteworthy examples are the studies of Chen (1991) and Caporale and Perry (2006). These studies directly cite the APT framework as their motivation unlike other studies that reflect the influence of the framework but are short on directly acknowledging its important contribution (see Sadorsky, 2001; Sadorsky & Henriques, 2001; Elyasiani & Mansur, 1998).

Chen (1991) argues that because the equilibrium models of Chan *et al*. (1985) and Chen *et al*. (1986) suggest that asset returns are related to macroeconomic factors, it is important to determine whether these factors are related to the state of the macroeconomy in a manner that is consistent with their forecasts of returns. Chen's (1991) hypothesis is that returns are a function of macroeconomic factors, which proxy for the future growth in economic activity.

To test hypothesis that the predictive power of the default spread, term structure, short-term interest rates, industrial production and the dividend yield arises from the ability of these factors to predict changes in the US macroeconomic environment, successive and future GNP growth rates are regressed onto these factors over the 1954 to 1986 period. Individually, the dividend yield and default spread predict the future GNP growth rate for up to one quarter, the term structure and short-term interest rate predict future GNP growth rates for up to four quarters and the industrial production growth rate predicts GNP growth rates for up to four quarters. To directly investigate the hypothesis that returns reflect changes in future output, Chen (1991) uses fitted values of GNP growth rates[53] as explanatory factors to explain excess returns on the value-weighted NYSE index. Results indicate that both past and future expected and unexpected GNP growth rates explain returns. Multifactor models of GNP growth rates – recent and future – are also estimated by regressing GNP growth rates on the default spread, short-term interest rate, dividend yield, industrial production and the term structure. Results indicate that GNP growth rates are significantly related to the dividend yield, the term structure, industrial production growth and the default spread implying that these factors *are* related to direct measures of macroeconomic activity. Finally, excess returns are regressed onto the dividend yield, default spread, term structure, short-term interest rates and industrial production growth within a multifactor model. Chen (1991) reports that this specification predicts 47.6 percent of the variation in returns with the short-term interest rate, industrial production and the dividend yield having statistically significant predictive power. These three significant factors are then augmented with an *unexpected* GNP growth factor[54] in a four-factor model. The explanatory power of this model increases to 61 percent and the GNP growth rate is statistically significant in the model.

Not only is Chen's (1991) study directly motivated by Chen *et al.*'s (1986) work, the influence of the APT framework is evident; the combination of the dividend yield, short-term interest rates, industrial production growth and an unexpected GNP growth factor yield a multifactor model incorporating factors representative of systematic risk factors. The predictive power of these factors is attributed to their relationship with changes in the US macroeconomic environment. Although Chen (1991) does not seek to investigate the return

[53] Estimated by regressing GNP growth rates on the five factors. See Chen (1991: 547) for a more detailed account of the methodology and results of the five factor model of GNP growth rates.
[54] The unexpected future GNP growth rate factor is the residual from a regression of the average of GNP growth rates over future quarters on the term structure, interest rate, dividend yield, industrial production and the default spread factors (see Chen, 1991: 547).

generating process or asset pricing directly, the APT framework forms the conceptual basis of the study.

Caporale and Perry (2006) nest their investigation of the impact of monetary policy shocks on stock returns within the APT framework. It is argued that the framework is appealing as it relates the variation in returns to risk factor exposures, considers multiple risk factors and constrains the interpretations of model coefficients. To study monetary policy shocks, excess returns on large US firms over the January 1971 to December 1996 period are regressed onto the term structure, monetary policy shocks as measured by changes in the Bernanke and Mihov quantitative index and changes in treasury bond yields which proxy for interest rates. Reflecting the influence of the APT framework, Caporale and Perry (2006) refer to the proposed specification as the APT model without formally making a distinction between the linear factor model and the cross-sectional APT model. Factors in the model are referred to as "common (systematic) risk factors" in line with APT terminology (Caporale & Perry, 2006: 4). Results indicate that returns are significantly and positively related to the term structure and monetary policy shocks suggesting that these are positive risk factors. Innovations in the bond yield have a negative and statistically significant impact upon returns. The positive impact of monetary policy shocks is attributed to expansionary monetary policy being instituted during periods when the economy enters a recession. Based upon the observed impact of expansionary monetary policy and the impact of the term structure on returns, Caporale and Perry (2006) conclude that monetary policy as conducted by the Federal Reserve Bank carries information about the future state of the US economy.

Whereas Caporale and Perry's (2006) model represents a linear factor model of the return generating process consistent with the APT framework, the authors apply the APT framework to conduct policy analysis and to analyze the transmission mechanism of monetary policy in the US market. Similarly to Chen (1991), Caporale and Perry's (2006) study demonstrates an application of the APT framework for purposes other than a direct investigation of the return generating process and asset pricing.

3.3.4. Proprietary applications of the APT framework

Testimony to the APT framework's role as a conceptual framework and its applicability is borne by its application in proprietary macroeconomic factor models. Two examples of proprietary application are the better known Salomon Brothers' model and the more recent Northfield APT Model.[55]

The Salomon Brothers' model is a model in the spirit of multifactor APT models similar to those of Chen *et al*. (1986). The purpose of the model is to explain the return generating process of US stocks by employing seven macroeconomic factors. This set of factors is almost identical to those used by Chen *et al*. (1986) and Hamao (1988) to explain expected returns; namely, the year on year change in total industrial production, the default spread, long and short-term interest rates, the inflation rate and changes in a 15-country trade-weighted currency basket (Fabozzi, 1998; Elton *et al*., 2003). The last factor is the uncorrelated residual market factor.[56] Elton *et al*., (2003) report that on average, the model explains 41 percent of the variation in returns on individual stocks. It is worth noting that the authors refer to the model as a model of the return generating process indicating the important role of the APT framework in deriving the return generating process specification (as opposed to deriving the cross-sectional APT model). The importance and influence of the framework is evident; Elton *et al*., (2003) discuss the Salmon Brothers' model within the context of Chen *et al*.'s (1986) model. The influence of Chen *et al*.'s (1986) macroeconomic APT model is reflected in its multifactor functional form and factor composition. Estimated factor coefficients (factor loadings, sensitivities in APT terminology) are interpreted as sensitivities of returns to specific factors and the more a stock's sensitivity to a factor deviates from zero, the more responsive are returns to innovations in that factor. A positive sensitivity to a factor implies that a stock is likely to outperform the market whereas a negative sensitivity to a factor implies that a stock will underperform the market if the innovation in the factor is positive and all other influences are held constant (Fabozzi, 1998).

The Northfield APT Model is based upon the premise that a stock's exposure to pervasive macroeconomic factors captures risk where exposures (coefficients) are estimated using regression techniques. It is argued that because the single-factor model underlying the CAPM is misspecified owing to the failure of the market portfolio to capture all risks borne by investors, the APT provides a framework for extending the single-factor model and relating returns to several macroeconomic factors. The choice of factors in the Northfield APT model

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[55] Information on this model is obtained from a document titled *US Macroeconomic Equity Risk Model* published by Northfield Information Services, Inc. (www.northfield.com). The model is also referred to as the Northfield Macroeconomic Equity Model.

[56] Uncorrelated with the other six factors.

is based upon factors that have previously been shown to be priced in APT literature. The first four factors are those of Chen *et al*. (1986); namely, changes in unexpected inflation, industrial production, the default spread and the change in the term structure. Additional factors in the form of changes in housing starts, exchange rates and oil prices are also included. The Northfield APT Model introduces a somewhat unique approach to estimating factor exposures within a multifactor time series framework; returns are first regressed onto the Chen *et al*. (1986) factors and then in the second step, the residuals of this regression are regressed onto the remaining three factors. This approach preserves the essence of Chen *et al*.'s (1986) model and mitigates possible multicollinearity. Reflecting the influence of the APT, factors are referred to as APT factors. As the Northfield APT Model relates a given stock's past performance to changes in the economy and in doing so explains the return generating process, investors can use the model to identify stocks that will perform well or poorly in different economic environments. The model is considered to be well-suited for portfolio management where measures of exposure to innovations in macroeconomic factors are required.

The Salomon Brothers' and Northfield APT models constitute yet another example of how the APT can be applied as a framework to model the return generating process and indicate the widespread application of the framework.

3.4. Conclusion

While the focus of the studies dealing with the macroeconomic APT model (see Chan *et al*., 1985; Chen *et al*., 1986; Hamao, 1988; section 3.1 & 3.1.1) is asset pricing and not describing the return generating process of stock returns, it is acknowledged that the return generating process can be described in terms of macroeconomic factors. Limited insight into the time series relationships between returns on market aggregates and macroeconomic factors is provided. This constitutes partial support for the proposition that returns can be described by innovations in macroeconomic factors. Studies based upon the macroeconomic APT consider the role of different sets of macroeconomic factors (section 3.1.2), portfolio formation criteria (section 3.1.3), the role of innovations within the framework (section 3.1.4), the generalizability of the approach (section 3.1.5) and a departure from relying solely upon domestic risk factors (section 3.1.6).

Beenstock and Chan (1988), McElroy and Burmeister (1988), and others consider both aspects of the APT framework and do not solely focus upon the asset pricing implications of the APT (section 3.2). These studies first relate returns to macroeconomic risk factors and consider the structure of the return generating process within the APT framework. Equilibrium relationships are also established indicating that the APT model and underlying linear factor model of the return generating process are linked. Elton *et al*. (1995) investigate the return generating process and equilibrium relationships for bond returns within the APT framework (section 3.2.3). This constitutes a departure from a focus upon equity. These studies serve as direct acknowledgement of the framework's role in providing a conceptual basis for investigating the return generating process.

Burmeister and Wall (1986), Berry *et al*. (1988) and Chan *et al*. (1990) apply the APT framework directly to model the return generating process; it is explicitly acknowledged in these studies that the APT framework is the motivation for a multifactor approach to investigating and modelling the return generating process (section 3.3 & 3.3.1). Macroeconomic factors found to explain expected returns in APT literature are shown to describe the time series variation in returns. Kandir (2008) and Bilson *et al*. (2001) investigate the return generating process operational in developing markets within the APT framework (section 3.3.2). The flexibility of the APT framework is demonstrated by Chen (1991) and Caporale and Perry (2006), who rely upon the framework in its entirety or parts thereof to investigate financial phenomena which require multifactor specifications (section 3.3.3). Further testament to the flexibility and applicability of the framework is borne by its application in proprietary macroeconomic factor models (section 3.3.4).

There is one aspect of the APT framework that has not been addressed in depth in this chapter; a theory upon which the identification and selection of factors can be based. However, APT literature alludes to a theoretical model – the dividend discount model - for the selection and identification of APT risk factors within the APT framework (see section 3.1.1: 44) . Chapter 4 considers risk factor selection within the APT framework and discusses a number of core factors used to describe the behaviour of stock returns.

4. RISK FACTOR SELECTION WITHIN THE APT FRAMEWORK

4.1. Identifying and selecting risk factors

The class of models introduced by Chen *et al*. (1986) belongs to the category of models commonly known as macroeconomic factor models (section 3.1 & 3.1.1) and the linear factor model underlying the APT model is the conceptual basis of these multifactor models (section 3.3; Connor, 1995). In these models, systematic risk factors are represented by macroeconomic factors (see section 3.1 & 3.1.1). As evident from Chapter 3, factors that are widely employed to explain returns in APT literature include the inflation rate, industrial production growth rates, interest rates, the term structure of interest rates, the default spread and a market factor. However, there is no set of universal factors that explain returns and it is noted that the factors that explain returns differ across markets (Seneque, 1987; Berry *et al*., 1988; Poon & Taylor, 1991). Although, Bilson *et al*. (2001) state that the selection of explanatory factors is somewhat an arbitrary and subjective process, the APT literature, by adapting existing theory, provides guidance to the selection and incorporation of legitimate APT risk factors.

Chapter 4 aims to elaborate upon a theory that aids the selection of risk factors and to introduce a number of core factors. The dividend discount model and criteria which must be satisfied before a factor can be considered as a legitimate APT risk factor are outlined next in section 4.2. Having outlined how the dividend discount model can aid the identification and selection of risk factors for use within the APT framework, this chapter proceeds by providing a glossary of systematic risk factors. Factors considered include market indices (section 4.3.1), inflation (section 4.3.2), real activity (section 4.3.3), the term structure and default spread (section 4.3.4), oil prices and exchange rates (section 4.3.5) and factors representative of monetary policy (section 4.3.6). Other somewhat less used factors are also considered (section 4.3.7) and a summary is provided in the conclusion (section 4.4).

4.2. A theoretical framework for the identification and selection of risk factors

APT literature relies upon a formula central to numerous theoretical models, namely the dividend discount model, to establish linkages between macroeconomic factors and returns. This basic valuation formula assumes that stock prices are a function of expected cash flows, represented by the dividends of a firm, discounted by an appropriate discount rate (Chen *et al*., 1986; Burmeister & Wall, 1986; Poon & Taylor, 1991; Clare & Thomas, 1994):

$$P(t) = \frac{E(c)}{k} \tag{4.1}$$

where $P(t)$ is the price of a given stock at time *t*, $E(c)$ is the present value of expected cash flows and *k* is the discount rate. Combining the assumptions underlying equation (4.1) and the diversification argument implies that systematic factors that influence expected cash flows and/or the discount rate influence stock market returns (Chen *et al*., 1986; Clare & Thomas, 1994). Furthermore, as current beliefs are already incorporated into prices, it is only unexpected changes or innovations in systematic factors that affect prices (Priestley, 1996; Elton *et al*., 2003).

The discount rate is assumed to be an average of rates over time, sensitive to the magnitude of rates and the term structure of spreads attributable to different maturities. Changes in the interest rate, default spread and a number of other factors will impact the discount rate, which in turn will influence the time value of expected cash flows and therefore returns. Furthermore, expected cash flows are assumed to be affected by both *nominal* and *real* forces. Therefore, these forces may be changes in expected inflation, which will impact nominal cash flows and nominal interest rates or alternatively, changes in *real* activity which will influence expected cash flows within the framework (Chen *et al*., 1986; Clare & Thomas, 1994). In essence, *any* factor that influences expected cash flows or the discount rate will have an impact upon stock returns. Chen *et al*. (1986) state that even factors that do not influence *current* cash flows, but are indicative of changing investment opportunities are also relevant. By this reasoning, any factor that describes the economic state qualifies as a systematic risk factor. The approach of Chen *et al*. (1986) is consistent with that expounded by Chen (1983) who suggests that one approach to solving the problem of which factors determine returns is to outline a theory that guides which factors should enter the pricing function. Chen *et al*. (1986) do not introduce a new theory, but reinterpret an existent one so as to serve that purpose in the context of the APT framework. Berry *et al*. (1988) set out an additional three criteria that factors must meet to qualify as legitimate APT risk factors:

1) Each factor must be unpredictable at the beginning of each period.
2) Each factor must have a pervasive influence upon stock returns.
3) Relevant factors must influence expected returns - they must be priced.

The first criterion requires that factors are unpredictable from either prior information or publicly available information, implying that at the start of every period, the expected value of a given factor is zero and the series of observations for a given factor is uncorrelated (Berry *et al*., 1988; Priestley, 1996). Hence, a factor such as inflation is not a legitimate APT risk factor because it is partially predictable. However, this does not preclude unexpected components of inflation from being considered as a candidate risk factor. Unexpected components will be serially uncorrelated and have a mean value of zero by construction; they will qualify as innovations (Priestley, 1996). The second criterion implies that firm specific factors are not valid candidate risk factors as firm specific risks can be diversified away. By this reasoning, firm specific factors will not have a pervasive influence on the returns on a *large* number of assets (Berry *et al*., 1988). In light of this, only systematic risk factors must be considered in the set of candidate risk factors. The final criterion can only be investigated through econometric endeavour. Berry *et al*. (1988: 30) state that in practice there is no "correct" set of factors, although, an extended number of factors may give rise to equivalent results because of substitutability. The choice of factors should be made on empirical grounds; factors should be able to explain returns and pass statistical tests to qualify as valid APT factors. Furthermore, stock returns should show credible sensitivities to realizations of these factors and these factors should be priced. Stated differently, the former argument suggests that risk factors should explain the return generating process of stock returns. Though Berry *et al*. (1988) set out criteria which candidate APT risk factors must meet, no generalized theory to guide factor selection prior to empirical testing is outlined. Fortunately, this question is already addressed by Chen *et al*. (1986).

4.3. A glossary of systematic risk factors

In keeping with the approach undertaken in this study of adapting the APT framework to explain the return generating process, candidate APT risk factors are discussed. As the list of possible risk factors is extensive, not all factors can be discussed at length. The discussion therefore centres on a number of factors that have been widely shown to explain expected and realized returns. Moreover, each of these factors has the potential to explain the return generating process or is indicative of a category of factors that feature in the return generating process. As the glossary of systematic risk factors that follows is not exhaustive, it should be treated as indicative of factors that can describe the return generating process rather than a definitive guide.

4.3.1. Market indices

One of the first systematic risk factors proposed is the market index. Sharpe (1963) suggests that a market index should be considered as a candidate factor for the index factor in his single-factor model (the “single-index model”). Teall (1999) states that a simple observation reveals that stock returns are affected by systematic factors and especially movements in the market index. Kwon and Yang (2008) state that traders in the market refer to market indices when undertaking investment decisions, suggesting that stock prices reflect the movements of a market index or a number of market indices. The authors show that there is a bidirectional flow of information between daily returns on individual stocks, the S&P 500 and the DJIA - an instance of transfer entropy. Whereas it is accepted that returns reflect market movements, what is of interest is the nature of informational content of market indices.

Chen *et al*. (1986) and Hamao (1988) suggest that market indices capture unexpected shocks to macroeconomic factors. As stock prices respond quickly to public information, returns on market indices are related to innovations in macroeconomic factors (Chen *et al*., 1986). Elton *et al*. (1995) state that the market index is an indicator of expectations of general economic conditions. Fabozzi (1998) suggests that the market index can be used to measure market timing risk and exposure to this factor provides information as to how stocks respond to changes in market conditions. Burmeister (2003) suggests that the market index proxies for events such as natural disasters, political developments, and bear and bull markets. The role of the residual market factor (which is derived from the market index) is similar to that of the market index; this factor acts as catch-all proxy for unspecified and omitted risk factors (Van Rensburg, 1996; Berry *et al*., 1988).

The preceding discussion suggests that the market index and residual market factor reflect innovations in systematic risk factors and measure risk inherent within the general economic and political environment. The role of international and/or foreign equity indices can be seen in a similar context; international and foreign indices capture innovations in the international economic environment and political developments. Clare and Priestley (1998) rely upon the MSCI World Index to separate international influences on the Malaysian stock market from domestic ones, and Van Rensburg (1996, 2000) shows that returns on the DJIA influence South African stock market returns. In terms of equation (4.1), movements in the domestic market and international indices reflect innovations in risk factors that influence expected cash flows and/or the discount rate domestically.

4.3.2. Inflation

Clare and Thomas (1994) suggest that changes in the expected rate of inflation affect expected cash flows and discount rates. The latter part of this hypothesis is especially pertinent in the South African context, given that the South African Reserve Bank (SARB) follows a policy of inflation targeting which relies upon setting appropriate levels of short-term interest rates (van der Merwe, 2004). By this policy, higher inflation expectations imply higher short-term interest rates. Higher short-term interest rates in turn translate into lower expected cash flows associated with a higher discount rate. With regard to the former part of this hypothesis, Chen (1991) suggests that lower inflation expectations are indicative of greater future economic growth. Fabozzi (2008) suggests that increases in inflation erode real incomes and negatively impact consumer demand for goods and services. However, it is recognized that the impact of inflation is likely to vary across stocks. Fama (1981) argues that a negative relationship between stock returns and inflation arises as a result of the so called proxy-effect; the negative relationship between stock returns and inflation that is attributable to the negative relationship between real activity and inflation. Fama's (1981) propositions are supported by J. Vanderhoff and M. Vanderhoff (1986), but not by Wei and Wong (1992) who suggest that the negative return-inflation relationship is not fully explained by the proxy hypothesis.

4.3.3. Real activity

The role of real activity in explaining stock returns is complicated by the multitude of measures of real activity. The growth rate in industrial production and the growth in GNP find support in their own right and can be considered as two popular and competing measures of real activity (Fama, 1990; Kandir, 2008). Lee (1992) identifies the industrial production growth rate as a measure of economic activity. Chen (1991) considers lagged production growth as an indicator of the *current* health of the economy. Fama (1990) suggests that the relationship between stock returns and industrial production growth rates reflects information relating to cash flows. These arguments taken together with the assumption that expected cash flows vary with real activity imply that stock prices will be positively related to changes in industrial production (Elton *et al*., 2003). The impact of GNP growth rates on returns is expected to be similar to that of industrial production as it is also a measure of real activity. Canova and De Nicolo (1995) treat GNP growth rates as proxies for shocks to expected cash flows, suggesting that GNP growth rates are reflected in stock prices through their relationship with expected cash flows. Cheung and Ng (1998) suggest that real GNP measures real activity and is a proxy for cash flow shocks to stock markets. It can be hypothesized that any factor (not necessarily industrial production or GNP) that directly measures or represents real activity will impact returns through the abovementioned mechanism.

4.3.4. Term structure and default spread

Fama (1990) suggests that the term structure has a business-cycle pattern; it is low around business peaks and high around troughs. It is suggested that the relationship between changes in the term structure and returns arises from the response of returns to business conditions whereby business conditions are measured by the term structure. Caporale and Perry (2006) also consider the term structure to be a proxy for cyclical variation in economic activity. The authors state that the spread between long-term and short-term interest rates decreases near peaks of economic activity, and increases near downturns. Burmeister and Wall (1986) and Van Rensburg (1996) suggest that the term structure is representative of a discount rate effect.

Fama (1990) suggests that the default spread also tracks the response of returns to business conditions. The spread between yields on corporate bonds and government bonds increases during adverse economic conditions and decreases during favourable economic conditions. Elton *et al*., (2003) state that the default spread captures the cyclical behaviour of the economy. Chen *et al*. (1986) and Burmeister and Wall (1986) suggest that the default spread captures a leverage effect. It is further suggested that the default spread is a measure of aggregate risk for the economy. Chan *et al*. (1985) suggest that changes in the default spread reflect changing business conditions. The authors find that a business cycle indicator (growth in the NBF series) is an (imperfect) substitute for the default spread. This finding supports the hypothesis that the default spread is a measure of the business cycle. As such, the role of the default spread is less ambiguous than that of the term structure. In terms of equation (4.1), it can be hypothesized that expected cash flows and discount rates vary with the business cycle. This cyclical variation is reflected by the term structure and default spread.

4.3.5. Oil prices and exchange rates

The impact of oil prices and exchange rates on market returns is acknowledged in the literature. Chen *et al*. (1986: 390) state "that oil prices must be included in any list of the systematic factors that influence stock market returns and pricing." Hamao (1988) suggests that oil prices *and* exchanges rates will influence market returns. Every modern economy is highly dependent upon oil and almost all countries take part in international trade, suggesting that these factors are almost universally applicable. Oil prices and exchange rates affect returns by impacting expected cash flows through varying input costs and revenues.

Kaul and Seyhun (1990) state that supply side shocks are related to stock price variability, which is reflected in market returns. The authors attribute the negative impact of oil price shocks to a negative response in output growth suggesting that oil prices affect stock returns through their impact upon real activity. Jones and Kaul (1996) hypothesize that the impact of oil price shocks on stock prices stems from the importance of oil to the world economy. Oil price shocks are assumed to reflect the impact of news on current and expected cash flows. However, the effects of oil price shocks are likely to differ from country to country depending upon the production and consumption of oil (Jones & Kaul, 1996). Nandha and Faff (2008) postulate that oil price shocks affect economic conditions through increases in the costs of production, transfers of wealth between oil producers and consumers, inflationary pressures and consumer confidence. Higher oil prices negatively impact real output, which implies lower corporate profits in industries where oil is an input in the production process. Oil price shocks also impact stock prices through indirect channels such as interest rates and consumer confidence. It is further argued by Nandha and Faff (2008) that although generally higher oil prices are bad news for economic growth, the size and direction of the impact of oil price shocks is dependent upon whether industries produce or use oil as an input or whether higher oil prices can be passed onto consumers (the pass-through effect). Poon and Taylor (1991) suggest that variability in oil prices influences stock prices through its impact upon industry costs, induced macroeconomic policy responses and output.

Griffin and Stultz (2001) report that the impact of exchange rates differs across industries; exporting industries are adversely affected by (domestic) currency appreciation while importing industries benefit from currency appreciation. Exchange rate fluctuations are hypothesized to affect firm value by impacting the demand for products, which in turn affects expected cash flows and therefore, stock prices. Jorion (1990) shows that the impact of exchange rate fluctuations on stock prices is dependent upon the level of operations in foreign markets. The larger the scale of foreign sales and operations, the more sensitive a firm (analogously industry) is to fluctuations in the exchange rate. Firms with high levels of foreign operations will be positively affected by a depreciation of the (domestic) currency as expected cash flows from foreign operations increase. Alternatively, firms with low levels of foreign operations will be negatively affected by a depreciation of the domestic currency as a result of higher input costs. Poon and Taylor (1991) suggest that changes in the exchange rate influence stock prices through their impact upon foreign earnings and export performance.

4.3.6. Monetary policy

Two widely considered factors in models of stock returns are changes in the money supply and interest rates. In economic theory, these factors relate to monetary policy (Parkin, Powell & Matthews, 2008).

Cutler, Poterba and Summers (1989) suggest that changes in the money supply, short and long-term interest rates act as proxies for economic news. Geske and Roll (1983) propose that stock returns are negatively related to interest rates which are a proxy for expected inflation. Muradoglu, Taskin and Bigan (2000) argue that increases in interest rates are incorporated into stock price evaluations. An increase in the interest rate is assumed to reduce the present value of expected cash flows resulting in a decrease in stock prices. Moreover, the impact of interest rates can be directly observed through equation (4.1); changes in the interest rate lead to changes in the discount rate and hence impact the value of expected cash flows (Poon & Taylor, 1991; Thorbecke, 1997). Thorbecke (1997) suggests monetary policy tightening, in the form of increasing interest rates, decreases a firm’s net worth and limits its ability to borrow and therefore, invest. Since investors are forward looking, lower investment expenditure implies lower expected cash flows, and as a result, a decline in stock prices.

Rozeff (1974) considers monetary shocks (measured by the narrow money supply in the US) in the context of the efficient market hypothesis and suggests that these are of an informational nature. It is postulated that markets extract information about future stock returns from data relating to current monetary shocks, and this information is reflected in current returns (Rozeff, 1974). Mookerjee and Yu (1997) also postulate that changes in the money supply possess policy information content. Higher money growth is assumed to imply higher inflation resulting in a negative relationship between returns and money supply growth. Cheung and Ng (1998) suggest that money supply fluctuations influence stock returns through their impact upon inflation uncertainty. Bilson *et al*. (2001) and Kandir (2008) argue that changes in the money supply impact real activity which in turn influences stock prices. Economic theory postulates that increases in the quantity of money result in a decrease in interest rates, which in turn drives interest rate sensitive expenditure. This results in an increase in aggregate demand which is matched by an increase in real activity. Increasing aggregate demand, associated with increasing real activity, implies higher expected cash flows (Clare & Thomas, 1994; Parkin *et al*., 2008). Günsel and Çukur (2007) suggest that growth in the money supply may have a varying impact upon stock prices, depending upon the industry. A negative relationship may arise when money supply growth leads to uncertainty about inflationary pressures. Alternatively, a positive relationship may result from falling discount rates associated with an increase in the money supply (Günsel & Çukur, 2007).

4.3.7. Other risk factors

The abovementioned systematic risk factors are widely considered in the literature and should be seen as core factors – factors which provide the basis for the search and selection of candidate risk factors. This set of factors is not exhaustive; any factor that plausibly impacts expected cash flows or the discount rate, or both will influence returns (Chen *et al*., 1986; Berry *et al*,. 1988).

Cheung and Ng (1998) consider consumption as a measure of aggregative real activity suggesting that the role of this factor is analogous to that of other measures of real activity. The high level of correlation between industrial production, GNP and consumption growth[57] rates in Chen (1991) suggests that this is indeed the case. Burmeister and Wall (1986) hypothesize that changes in final sales directly influence expected cash flows. This factor is considered to be a substitute for industrial production. Chan and Faff (1998) suggest that the impact of changes in the gold price on returns arises as a result of gold being a hedge against market-wide uncertainties. It is suggested that gold is a proxy for other factors within a multifactor setting. Clare and Thomas (1994) suggest that gold prices influence returns through their impact upon the outlook for interest rates. Kaneko and Lee (1995) state that the influence of the terms of trade upon stock prices is attributable to the importance of

[57] Chen (1991) reports that the level of correlation between industrial production and consumption growth is 0.75 and the level of correlation between GNP and consumption growth is 0.8.

international trade in a specific economy. An improvement in the terms of trade suggests increasing inflows and a deterioration in the terms of trade points towards increasing outflows. Increasing inflows imply higher cash flows and increasing outflows imply lower cash flows for domestic firms. Clare and Thomas (1994) suggest that the trade balance is representative of levels of economic activity. Other factors that are hypothesized to fulfil this role are unemployment, credit extension and the stock market turnover. Sadorsky and Henriques (2001) suggest a role for general commodity prices as a risk factor in the return generating process. Van Rensburg (1996) considers the growth rate in building plans passed. Van Rensburg (2000) lists the level of gold and foreign exchange reserves and the money market shortage amongst potential risk factors. Beenstock and Chan (1988) consider a broad set of candidate risk factors. Amongst these are fuel and material cost measures of manufacturing, measures of retail prices and wages, industrial stoppages, measures of retail volumes and relative export prices.

4.4. Conclusion

In the course of developing the macroeconomic APT model, Chen *et al*. (1986) elaborate upon a theory that aids the selection of systematic risk factors which are central to the return generating process under the APT framework (see section 3.1.1: 44). Berry *et al*. (1988) further elaborate upon criteria that candidate risk factors must meet to qualify as APT risk factors (section 4.2: 86). Factors identified in the literature that are consistent with the theory aside from the market index and the residual market factor (section 4.3.1) are inflation (section 4.3.2), measures of real activity (section 4.3.3), the term structure and default spread (section 4.3.4), exchange rates, oil prices (section 4.3.5), interest rates and the money supply (section 4.3.6). Although, these factors are considered to be core factors owing to their widespread usage and relevance to particular economies, they can be seen as a basis for a more extensive set of macroeconomic factors measuring systematic risk within the APT framework.

The APT framework provides a comprehensive framework for investigating the return generating process and asset pricing. It suggests a multifactor return generating process characterized by innovations in systematic risk factors identified through theory. Moreover, as the APT framework can be used to investigate the time series behaviour of stock returns, consideration must be given to the properties and behaviour of stock returns. Such consideration is given in Chapter 5.

5. PROPERTIES AND BEHAVIOUR OF STOCK RETURNS

5.1. Introduction

Models of the return generating process constructed within the APT framework aim to explain return behaviour by relying upon factors representative of systematic risk (see section 3.2 & 3.3; Chapter 4). Although, Burmeister and Wall (1986), Berry *et al*. (1988), Chan *et al*. (1990) and others demonstrate the adequacy of the APT framework as a conceptual framework for modelling and investigating the return generating process, these studies give little consideration to the properties and behaviour of stock returns (see section 3.3.1). This lack of consideration points towards a conspicuous gap in the literature; studies seeking to explain the return generating process fail to consider the characteristics of the very returns that they seek to explain. A failure to consider the characteristics and behaviour of returns not only results in the application of an inappropriate econometric methodology in estimating models of the return generating process, but also potentially results in misleading inferences and general statistical inadequacy (Gujarati, 2003). It is therefore necessary to examine the properties and behaviour of return distributions when investigating the return generating process.

Although, returns are assumed to conform to a set of *a priori* assumptions relating to their distribution and volatility - assumptions which are crucial for model specification and estimation - these assumptions may not hold in practice (Xiao & Aydemir, 2007). The theory of random walks[58] formalizes theory relating to the behaviour of stock prices by assuming that successive price changes (returns) are *normally, independently and identically distributed (n.i.i.d)* (Mandelbrot, 1963; Fama, 1965). Fama (1995) attributes the random walk to competition between numerous participants in the market. It is suggested that disagreements between market participants about the true intrinsic value of a stock result in discrepancies between the intrinsic value and the actual value. Competing actions of market participants – which are attributable to these discrepancies – result in the stock price wandering randomly about its intrinsic value.

[58] A pure random walk process is a random walk process without drift and is therefore non-stationary. It is considered to be a difference stationary process implying that a stationary time series can be obtained by differencing a non-stationary time series (see Gujarati, 2003).

Returns can further be described as Gaussian or normally distributed with a mean of 0 and variance proportional to the differencing interval, Δt, implying that the distribution of returns is described by the mean, μ, and an innovation from the mean, ε_t (Mandelbrot, 1963). However, the literature has cast doubt upon the validity of assumptions relating to return behaviour and empirical evidence suggests that the first two moments of the return distribution are not "well-behaved" as implicitly assumed in empirical studies dealing with the structure of the return generating process.

Chapter 5 investigates the assumptions underlying stock returns and the behaviour of stock returns. The assumptions of normality (section 5.2.1), independence (section 5.2.2) and stationarity (section 5.2.3) are stated and their validity is discussed. The properties of volatility are considered next with the aim of describing the behaviour of stock returns. The properties considered are volatility clustering (section 5.3.1), volatility persistence (section 5.3.2), the leverage effect (section 5.3.3) and mean reversion (section 5.3.4). The main points relating to the assumptions underlying stock returns and the behaviour of stock returns are summarized in the conclusion (section 5.4). In light of this chapter's findings, it is noted that an empirical framework that takes the properties and behaviour of stock returns into account must be considered.

5.2. Assumptions underlying stock returns

5.2.1. The normal distribution

The development of the theory of random walks from which the assumption of normally distributed returns stems began with Bachalier (1914). The normal distribution is described by a symmetric "bell-shaped" curve and under the conditions of the central limit theorem; daily, weekly and monthly returns are expected to follow a normal distribution (Fama, 1965). Fama (1965) states that prior to Mandelbrot's (1963) work, the assumption of normality was not widely questioned and according to Officer (1972: 807), the normal distribution was seen as "a good working hypothesis." Mandelbrot (1963) is credited with re-examining the distributional assumptions of stock returns. Initially, the focus was upon the thickness of the tails of the distribution whereas the presence of *both* excess kurtosis and skewness is considered later (Choi & Nam, 2008). Mandelbrot (1963) contends that the normal distribution fails to account for the excess kurtosis and the long tails exhibited by return distributions. As an example, the distribution of the changes in wool prices is cited with

Mandelbrot (1963: 395) noting that “there are typically so many ‘outliers’ that ogives fitted to the mean square of price changes are much lower and flatter than the distribution of the data themselves.” Mandelbrot (1963: 395) goes on to state that “the tails of the distributions of price changes are in fact so extraordinarily long that the sample second moments typically vary in an erratic fashion.” In a subsequent paper, Mandelbrot (1967: 396) reiterates his position regarding the high levels of kurtosis observed in financial time series and notes that “Bachalier’s assumption, that the marginal distribution of $L(t,T)$ (returns) is Gaussian with vanishing expectation, might be convenient, but virtually every student of the distribution of prices has commented on their leptokurtic (i.e., very long-tailed) character.”

Fama (1965) undertakes an extensive study of the properties of returns for the 1956 to 1958 period using data on thirty stocks compromising the DJIA. The frequency distribution of price changes for individual stocks within given standard deviations of the mean is compared with what is expected under a normal distribution. On average, a greater proportion of observations are found to be centred around the mean and a greater number of observations are observed in the tails of the empirical distribution relative to that implied by the normal distribution. Fama (1965) reports that the actual level of excess frequency beyond five standard deviations is almost 2000 times greater than that implied by the normal distribution. Empirical distributions are more peaked around the centre and have longer tails in every instance relative to the normal distribution. This points towards the presence of leptokurtosis. Fama (1965) concludes that the normal distribution is not an accurate representation of the return distribution. Praetz (1972) employs formal goodness-of-fit tests to establish whether returns on indices on the Sydney Stock Exchange (SSE) for the 1958 to 1966 period follow a normal distribution. It is argued that while theory[59] suggests that the distribution of share price changes should be normal, a large body of evidence shows that this is not the case; the typical return distribution is characterized by a peaked centre and fat-tails (Praetz, 1972). After fitting the normal distribution to returns on the indices in the sample, the normal distribution is rejected in almost every instance.[60] Praetz (1972) finds that a better description of returns is provided by the fat-tailed (leptokurtic) *t*-distribution.

[59] Praetz (1972) refers to the work of Osborne (1959). Osborne (1959), like Bachalier (1914), suggests that the distribution of share price changes (returns) is normally distributed (see Praetz, 1972; Fama, 1965).

[60] Praetz (1972) rejects the hypothesis that returns are normally distributed in 12 out of 13 instances at a 1 percent level of significance.

Officer (1972), similarly to Praetz (1972), seeks to describe the distributional properties of stock returns and considers formal tests of the distributional properties of stocks compromising the CRSP database over the January 1926 to June 1968 period. Results indicate that return distributions are non-normal,[61] and as in other studies, distributions exhibit fat-tails pointing towards the presence of leptokurtosis. Westerfield (1977) provides an indication of the level of excess kurtosis relative to that of the normal distribution. Sample kurtosis for stocks listed on the NYSE over the January 1968 to September 1969 period is almost always *greater* than 3 where 3 is the value under a normal distribution and *on average,* the return distributions exhibit kurtosis of almost 5. Westerfield's (1977) findings are based upon a sizeable sample of 315 stocks and therefore, imply that the phenomenon of leptokurtosis is widespread. Brown and Warner (1985), in their paper on how the properties of the return distribution affect event study methodologies, also quantify the level of kurtosis inherent in securities in the CRSP database over the July 1962 to December 1979 period. The authors find that kurtosis is more than double that of the normal distribution. Furthermore, departures from normality differ according to the frequency of data used. Daily returns for individual stocks exhibit substantially greater departures from normality relative to monthly returns.

The abovementioned studies are a small fraction of numerous studies recognizing that return distributions are characterized by leptokurtosis and thus depart from normality. Widespread recognition of the presence of excess kurtosis is acknowledged by Xiao and Aydemir (2007) who state that the level of kurtosis for many studies is above 3, and by Engle and Patton (2007) who state that it is well-established that return distributions have fat-tails and that typical estimates of kurtosis range between 4 to 50.

Peiró (1999) suggests that the assumption of symmetry implies that upside and downside risks are considered equally by investors. It is further argued that while high levels of kurtosis are a well-recognized feature of return distributions, less consideration is given to the symmetry of the distribution as it is considered to be less significant. This is problematic given that leptokurtosis is usually accompanied by asymmetry. Arditti (1967), Simkowitz and Beedles (1980) and Kon (1984) are amongst those that recognize and acknowledge the presence of asymmetry in return distributions. Arditti (1967) relates the concept of skewness

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[61] In Officer's (1972) study, the characteristic exponent ($\hat{\alpha}$) is 1.51 (less than 2) implying that leptokurtosis is present in the return distributions of the stocks considered.

to risk. It is hypothesized that a risk averse investor will be unwilling to undertake an investment that will potentially yield a larger loss relative to a limited gain. This asymmetry in outcome is assumed to be captured by skewness. Skewness, whether negative or positive, suggests that a given outcome is more likely overall and skewed distributions reflect the likelihood of this outcome. Arditti (1967) does not consider skewness as a characteristic of the return distribution directly. Instead, the author aims to establish how returns are related to several risk factors with skewness being amongst these factors. Using cross-sectional regressions, returns on firms compromising the S&P Composite Index over the 1946 to 1963 period are found to be significantly and negatively related to skewness, implying that investors prefer positive skewness. Arditti (1967) concludes that skewness is a reasonable measure of risk. Although, these findings shed little light upon asymmetry as a deviation from normality, this analysis nevertheless serves as an important early acknowledgement that return distributions may be asymmetric.

Simkowitz and Beedles (1980) argue that the normal distribution is not an adequate description of stock returns as skewness is so pervasive that the assumption of a symmetric and therefore normal distribution must be questioned. It is suggested that asymmetry may be interpreted as an unusually large number of positive or negative outcomes. Using returns on constituents of the DJIA over the January 1951 to December 1976 period, the authors find that the majority of return distributions are positively skewed; although, a substantial number of returns series are characterized by negatively skewed distributions.[62] Furthermore, it is established that the frequency of statistically significant skewness is not the result of chance and positive skewness is pervasive as evident from significance tests. Simkowitz and Beedles (1980) extend their analysis to a large (400) sample of US stocks so as to avoid biases arising from correlation and lower return bounds. Prior results are confirmed; positive skewness is more prevalent than negative skewness and the majority of the return series exhibit statistically significant skewness.[63] As with the constituents of the DJIA, returns compromising the enlarged sample tend to be characterized by positive skewness. Simkowitz and Beedles' (1980) findings provide strong evidence that return distributions are not

[62] Out of 30 return series, 18 are characterized by positive skewness. The rest are negatively skewed. For positively skewed return series, skewness is statistically significant for 8 series. For negatively skewed return series, skewness is statistically significant for 4 series. The mean (estimated) level of skewness is 0.077.

[63] Statistically significant positive skewness is observed for 167 return series and statistically significant negative skewness is observed for 35 return series at the 5 percent level of significance.

symmetric as postulated by the normal distribution; skewness is observed with returns exhibiting both positive and negative skewness.

Kon (1984) shows that returns on the S&P 500 Index and CRSP value and equally-weighted indices are significantly skewed *and* leptokurtic with the equally-weighted index being the only negatively skewed index. The presence of statistically significant skewness is also observed in a larger sample of individual US stocks with almost all return series showing statistically significant positive skewness.[64] Kon (1984) ascribes the observed skewness to shifts in the time series mean whereas leptokurtosis is attributed to a time-varying variance. It is suggested that a discrete mixture of normal distributions should be used to explain the observed levels of kurtosis and significant skewness in returns on individual stocks and indices. A rejection of symmetry, as with the presence of leptokurtosis, translates into a rejection of the hypothesis of normality. A finding that return distributions are characterized by both leptokurtosis *and* skewness poses a further challenge to the assumption of normality.

5.2.2. Independence

According to Cont (2001), it is a well-known fact that there is no significant linear correlation in returns. Therefore, the independence assumption assumes that the serial correlation function of returns as denoted by equation (5.1) decays rapidly to zero (Cont, 2001):

$$C(\tau) = corr(r(t), r(t+\Delta t) = 0 \qquad (5.1)$$

where $C(\tau)$ is the serial correlation coefficient of order τ, $r(t)$ is the return on a given series at time t and Δt is the time scale. The absence of (linear) serial correlation is often cited as evidence in favour of the efficient market hypothesis (Cont, 2001). Campbell *et al*. (1997) state that if equation (5.1) holds, returns may be considered as serially uncorrelated and therefore, mutually independent. Returns are assumed to show little or no linear serial correlation and if serial correlation is present, it is short-lived. Independence can further be defined from two perspectives. The first relates to *statistical* independence in returns. The second relates to whether investors can use knowledge of past returns to increase expected profits (Fama, 1965; Mandelbrot, 1967). The independence assumption in this study is of interest primarily from a statistical viewpoint. As much of the discussion relating to the

[64] Positive skewness is statistically significant for 26 out of 30 stocks.

validity of the independence assumption centres on testing for serial correlation in returns, the discussion that follows focuses upon this aspect (Giannoplolous, 2000).

Kendall and Hill (1953) conduct an early analysis of the properties of returns and find that the pattern of events in price series is less systematic than generally accepted. Changes from one period to another behave almost like a "wandering series" implying that subsequent returns follow a random walk and are therefore, independent (Kendall & Hill, 1953: 11). Kendall and Hill (1953) first report findings relating to the Chicago Wheat Series.[65] The series follows a random walk with changes from one period to the other appearing to be independent and thus making serial correlation unlikely. This is confirmed by a finding of small and mostly negligible serial correlation in the series.[66] An analysis of the serial correlation in series constituting what the authors define as British Industrial Share Prices[67] yields similar results; for the most part, changes in prices are independent and where dependence is observed, it is too low to exploit for predictive purposes. These findings suggest that returns are independent from a statistical viewpoint and from a practical perspective.

Fama (1965) states that even though it is difficult to find a series that fully conforms to the assumption of independence, statistical independence holds even if some level of dependence is present but insufficient to account for certain properties of the return distribution. It is proposed that the most basic explanation for the assumption of independence arises from the arrival of new information, which does not follow any consistent pattern.[68] To test for dependence in returns, Fama (1965) relies upon the serial correlation model and the runs test. An analysis of the serial correlation structure for the entire sample indicates that overall, the level of serial correlation is low. Only about a third of the series compromising the DJIA show statistically significant serial correlation at the first and second orders, with the proportion of return series showing statistically significant serial correlation decreasing steadily at higher orders.[69] Fama (1965) notes that even in instances where correlation is statistically significant, the level of dependence implied by a statistically significant serial

[65] Basic cash wheat in US cents per bushel in Chicago.
[66] This finding is confirmed by low serial correlation up to the 10th order over the entire sample period from 1883 to 1934 and omitting the period from 1915 to 1920.
[67] Each index constituting the sample is an aggregate. For example, one of the series is "Insurance Companies."
[68] According to Fama (1965) these are rather extreme assumptions. Estimates of intrinsic values may be dependent upon the estimates of others and the arrival of information need not be independent; often good news is followed by more good news.
[69] 11 stocks exhibit statistically significant first order serial correlation and 9 exhibit statistically significant second order serial correlation.

correlation coefficient is so low that it is unimportant from both a statistical and practical perspective. These inferences are applicable when larger differencing intervals are considered; the average size of correlation coefficients decreases with the size of the differencing interval. Fama (1965) states that these findings, based upon the serial correlation model, indicate that dependence is of an extremely low magnitude suggesting that the independence assumption is a valid working assumption. Runs tests support the results of the serial correlation analysis; overall percentage differences between actual and expected runs are small, there is no pattern in the signs of the differences, the lengths of the runs are similar and the number of long runs does not exceed the expected number under the assumption of independence. Fama (1965) concludes that there is little evidence of dependence in returns.

Akgiray (1989) investigates whether returns can be represented by a linear white noise process with independent increments. Based upon Fisher's kappa and Bartlett's test,[70] the assumption of independence is rejected for the CRSP value-weighted index over the entire sample period between January 1963 and December 1986. These findings are supported by periodograms and Ljung-Box *Q*-statistics (see Ljung & Box, 1978). Moreover, the serial correlation function of the return series indicates high first order serial correlation. According to Akgiray (1989), this permits a conclusive rejection of the hypothesis that return series are white noise suggesting that returns do not approximate independent observations. However, the serial correlation function also reveals that dependence is short-lived. It is hypothesized that the presence of a common market factor, thin trading, a day of the week effect and adjustments to the arrival of new information may be responsible for the presence of statistical dependence in returns on the CRSP value-weighted index.

Campbell *et al*. (1997) argue that the independence assumption is often violated over the long-run with returns exhibiting long-run dependence. To test whether there is long-run dependence coupled with predictability in returns, the serial correlation structure of returns on the CRSP value and equally-weighted indices is investigated over the period from July 1962 to December 1994. Both indices are found to exhibit substantial first order serial correlation in daily returns. Substantial first order serial correlation is also found in weekly and monthly returns on the CRSP equally-weighted index. Returns on the equally-weighted

[70] According to Akgiray (1989), both procedures are primarily designed to test whether a series is white noise and in large samples, these procedures are tests of independence. Six year sub-periods are also considered. The independence assumption is rejected for two sub-periods when Fisher's test is used and for all four sub-periods by Bartlett's test.

index exhibit a higher level of serial correlation at all differencing intervals and the serial correlation in returns on this index decays at a slower rate relative to the serial correlation in returns on the value-weighted index. For both indices, serial correlation declines rapidly after the first order, again suggesting that dependence is short-lived regardless of the differencing interval.[71] Although, dependence is observed in both series and regardless of the differencing interval, evidence of dependence is weaker for the CRSP value-weighted index and weaker still at larger differencing intervals for both series. Campbell *et al*.'s (1997) findings are in line with Akgiray's (1989) findings of short-lived statistical dependence but contrast with those of Kendall and Hill (1953) and Fama (1965).

Lo and MacKinlay (1988) investigate the serial correlation structure of weekly returns on equally and value-weighted CRSP NYSE-AMEX indices, size based portfolios and returns on individual stocks over the October 1962 to December 1985 period. Weekly data as opposed to daily data is used to minimize biases associated with non-trading, the bid-ask spread and asynchronous prices. Based upon the variance ratio test (q=2), Lo and MacKinlay (1988) find evidence of statistically significant positive first order serial correlation in returns on both CRSP indices and this is seen as evidence in favour of rejecting the random walk hypothesis. Results for size based portfolios are similar; statistically significant positive first order serial correlation is present in returns on three portfolios consisting of firms of similar market value.[72] These findings again imply a rejection of the random walk hypothesis and thus, the rejection of the independence assumption. Unlike the positive serial correlation in returns on indices and size based portfolios, serial correlation in returns on individual stocks is negative and not statistically significant. Lo and MacKinlay (1988) attribute this to company specific noise which complicates the detection of predictable components. These findings suggest that the independence assumption does not hold for returns on aggregates. However, there is ambiguity relating to serial correlation in returns on individual stocks.

Unlike the assumption of normality, which is widely rejected, the independence assumption continues to be debated. As indicative of Kendall and Hill (1953) and Fama (1965), a body of literature finds support for the independence assumption and limited evidence of dependence in returns. As is indicative of Akgiray (1989) and Campbell *et al*. (1997), any statistical

[71] For example, first order serial correlation for daily returns on the CRSP value-weighted index is 17.6 percent. Second order serial correlation is -0.7 percent. For the CRSP equally-weighted index, first order serial correlation is 35.0 percent and second order serial correlation is 9.3 percent.
[72] The three quintiles are the smallest, central and largest market value quintiles.

dependence in returns is (very) short-lived, although its presence nevertheless challenges the independence assumption. Matters are further complicated by Lo and MacKinlay (1988) who find that while returns on aggregates show statistically significant first order serial correlation, returns on individual stocks *appear* to be independent. Given these findings, it is impossible to conclusively pronounce upon the validity of the assumption of independence. Perhaps the best approach is to investigate the independence assumption on a "case-by-case" basis.

5.2.3. Stationarity

Returns are assumed to be identically distributed implying that certain statistical properties of time series data remain invariant over time. This is known as the stationarity hypothesis (Cont, 2001). According to Mandelbrot (1967), stationarity implies that sample moments do not vary substantially from sample to sample. Gibbons and Hess (1981) argue that the assumption of identically distributed returns requires that the mean and variance are constant over time. Giannopoulos (2000) states that changes in these two sample moments are often cited as the reason for excess kurtosis in return distributions. Cont (2001) argues that it is not clear whether this assumption holds as evident from seasonal effects such as the January, weekend and the day of the week effect. Giannopoulos (2000) states that while evidence regarding the non-stationarity of the mean is inconclusive, the non-stationarity of variance is widely recognized.

Two studies indicative of the debate relating to the stationarity of the mean are those of Gibbons and Hess (1981) and Peiró (1994). Gibbons and Hess (1981) state that it is generally assumed that the distribution of stock returns is identical for all days of the week. However, there is increasing evidence that the distribution varies across the days of the week. An often cited example is that of the so called Monday effect whereby Monday returns exhibit a higher mean and variance. The authors investigate the day of the week effect using return data on the S&P 500 Index, value and equally-weighted portfolios constructed from the CRSP database, and individual stocks compromising the DJIA between July 1962 and December 1978. The hypothesis of equality is rejected[73] for returns on the S&P 500 Index and both

[73] The conventional test for the equality of means is conducted using a dummy regression specification, $R_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \alpha_4 D_{4t} + \varepsilon_t$ (Gibbons & Hess, 1981; Kiymaz & Berument, 2003). The coefficients of this specification represent mean returns for each day of the week. By showing that coefficients α_1 through to α_4 are equal, it can be shown that returns are from identical distributions (Mookerjee & Yu, 1999).

portfolios suggesting that the return distribution is not identical across time. Returns for Mondays are lowest although a degree of variation in the mean is also observed for other days of the week. The equality hypothesis is also rejected for all individual stocks compromising the DJIA. Gibbons and Hess' (1981) findings relating to returns on the aggregates considered and individual stocks suggest that the assumption of identically distributed returns does not hold. The authors conclude that daily seasonality is evident in stock returns and this is manifested by persistently negative *mean* returns on Mondays.

Peiró (1994) states that one of the most interesting seasonal effects observed is daily seasonality, which manifests itself in a differing distribution across days of the week. The author seeks to establish whether seasonality and day of the week effects are present in six major stock market indices; namely, the DJIA, Nikkei, Financial Times Ordinary Share 30 (FT 30), Commerzbank, Compagnie des Agents de Change (CAC) General and the General index for the period from December 1987 to December 1992.[74] As in Gibbons and Hess (1981) the standard dummy regression approach is employed to test for seasonality. The null hypothesis of equality is not rejected for the DJIA, Nikkei, Commerzbank and the General indices suggesting that the distribution of returns does not differ in the mean across days of the week for these indices. However, seasonality is observed in returns on the FT 30 and the CAC General indices. For the FT 30 Index, seasonal behaviour is attributed to a strong Monday effect. Peiró (1994) concludes that these findings question the validity of widespread seasonal patterns observed in prior literature. In contrast to Gibbons and Hess' (1981) findings, these results mostly support the assumption of identically distributed returns.

While Peiró (1994) does not find widespread evidence of seasonality in the mean, the same does not hold for variance. Tests of the equality of variance across days of the week indicate that the null hypothesis of equal variance is rejected for all indices with the exception of the DJIA. This suggests that variance exhibits widespread seasonal effects and is of time-varying nature. Moreover, unlike the ambiguous debate regarding the stationarity of the mean, it is widely accepted that the variance of stock returns is of a time-varying nature (Giannopoulos, 2000). Evidence suggesting the variance is not stationary is found in the literature as early as Bachalier (1914), Mandelbrot (1967) and Praetz (1972). Bachalier (1914) notes that the evidence diverges from his original theoretical formulation in that sample variance differs

[74] Indices on exchanges situated in New York, Tokyo, London, Frankfurt, Paris and Madrid.

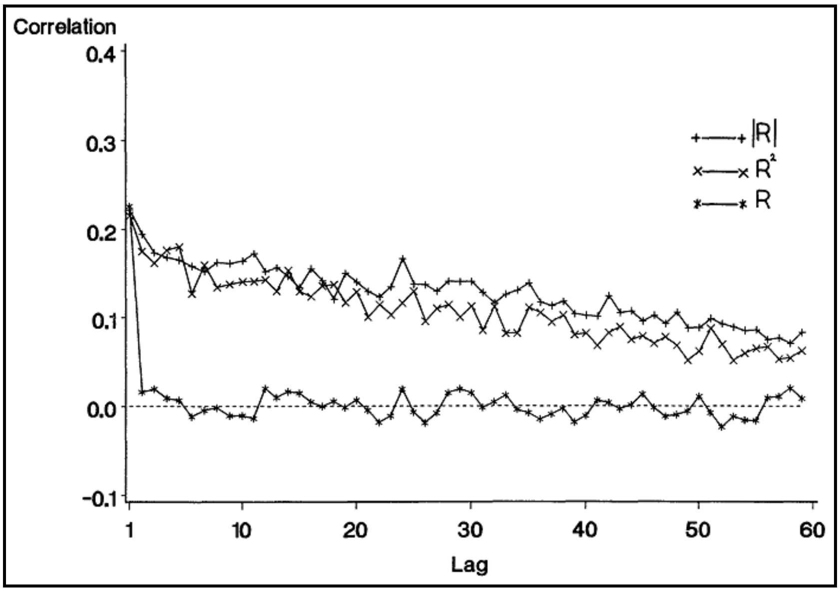
over time. Mandelbrot (1967) presents a plot of the variance of cotton price changes which indicates that variance differs over time (see Mandelbrot, 1967: Figure 1). It is suggested that seasonal effects, changes in the macroeconomic environment and economic policy are behind changing variance. Praetz (1972) states that it is widely assumed that the variance of returns is constant over time. This assumption however is contradicted by observed extended periods of market activity which are followed by extended periods of inactivity. These transitions in the magnitude of variance are attributed to information clustering around certain dates. Praetz (1972) further notes that evidence suggests that variance varies from year-to-year as market activity varies.

It is however Taylor (2008)[75] who popularized the notion of time-varying variance in his extensive study of the properties of returns. Taylor (2008) reports that absolute and squared transformations[76] of US stock return series - both proxies for volatility - exhibit high levels of first order serial correlation and continue to be positively correlated over extended periods of time.[77] It is suggested that this serial correlation structure is attributable to changes in the variance of returns implying that variance is of a time-varying nature for the January 1966 to December 1976 period. Akgiray (1989) arrives at a similar conclusion. While first order serial correlation in returns on the value-weighted CRSP index is high, it becomes statistically insignificant at longer lags. However, this is not so for absolute and squared returns, which are highly correlated for extended periods of time, as evident from Figure 5.1:

[75] This paper references the second edition of Taylor's seminal work, *Modelling Financial Time Series,* owing to the unavailability of the original text. The first edition was published in 1986.

[76] See Poon (2000), McMillan and Ruiz (2009). The presence of non-linear dependence in returns is interpreted as correlation in volatility and does not in itself imply that returns are serially correlated (Cont, 2001).

[77] For example, for the Kodak return series, first order serial correlation for absolute and squared returns is 0.146 and 0.178 respectively. For Alcoa, first order serial correlation is 0.194 and 0.144 respectively. Whereas Taylor (2008) finds that less than 10 percent of correlation coefficients for the linear series are outside the -0.05 and 0.05 interval for 1 to 30 lags, for squared returns, the percentage of series for which correlation coefficients exceed 0.05 is 58 percent.



Source: Akgiray (1989)

Figure 5.1: Returns and non-linear transformations of returns

Whereas the serial correlation function of the return series falls below zero after the first lag, the serial correlation functions of absolute and squared returns decay slowly and are still above zero after 60 lags. Akgiray (1989) states that this implies that large price changes are followed by large price changes and small price changes are followed by small price changes of either sign – an example of volatility clustering. Furthermore, the non-linear dependence observed in absolute and squared return series is attributed to changing variance and is cited by Akgiray (1989) as an explanation for leptokurtosis in return distributions. It is suggested that these changes in variance are related to the rate of information arrival, levels of trading activity and corporate financial and operating leverage decisions. Notably, Akgiray (1989) suggests that any model of returns must be compatible with this characteristic (changing variance) and take into account non-linear dependence in returns. Taylor's (2008) and Akgiray's (1989) findings confirm the propositions of Mandelbrot (1967) and Praetz (1972) that variance is non-stationary.

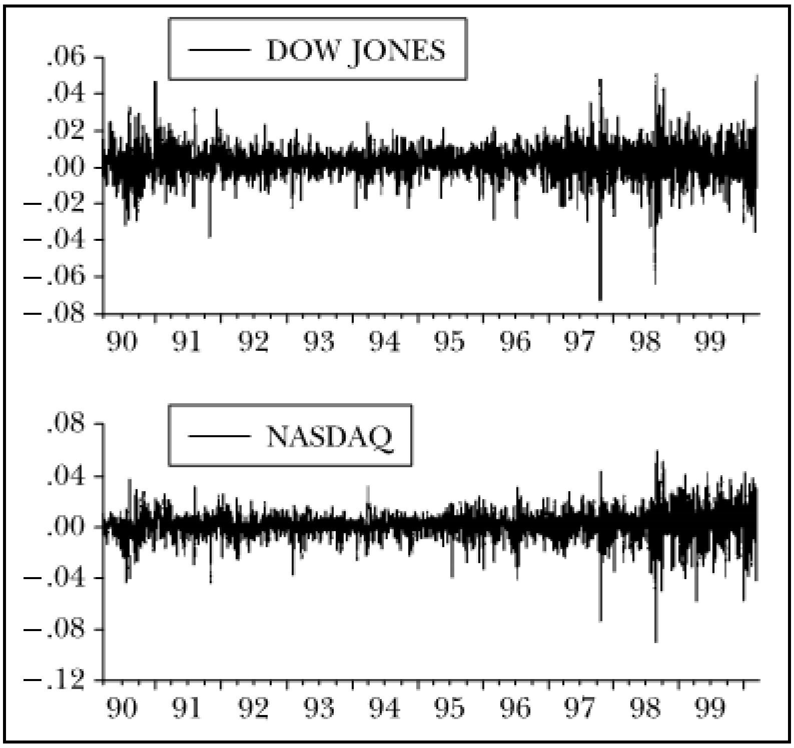
The literature suggests that although there is debate surrounding the stationarity of the mean, it is almost a certainty that the variance differs over time. Tang (1997) goes even further. Using return data on industrial sectors compromising the Hong Kong stock market and returns on the Hong Kong Index (HKI) over the January 1984 and March 1992 period, Tang (1997) finds that seasonality extends into the higher moments of the return distribution implying that the higher moments are non-stationary. Whether this is related to non-stationarity in the variance warrants further investigation. However, given the evidence relating to the non-stationarity of the variance, the validity of the assumption of identically distributed returns is questionable; it can be argued that return distributions are stationary in the mean but not in the variance.

5.3. Behaviour of stock returns

Poon (2005:1) defines volatility as a “spread of all likely outcomes of an uncertain variable,” suggesting that volatility is a measure of the *spread* of a distribution but not its shape. For the normal distribution, the mean and standard deviation are deemed sufficient to reproduce the empirical distribution and although, volatility is not the only descriptor of the distribution, it plays an important role in a number of financial applications such as investing, portfolio construction, option pricing, risk management and hedging (Poon, 2005). Therefore, an investigation of volatility yields further insight into the behaviour of returns and the return distribution. Similarly to stock returns, volatility is characterized by a number of stylized facts, namely volatility clustering (section 5.3.1), persistence (section 5.3.2), leverage effects (section 5.3.3) and mean reversion (section 5.3.4).

5.3.1. Volatility clustering

The phenomena of volatility clustering, although not referred to by that name at the time, is acknowledged early on by Mandelbrot (1963: 418) who notes that “large changes (in price) tend to be followed by large changes – of either sign – (and) small changes tend to be followed by small changes.” Volatility clustering implies that volatility exhibits alternating periods of tranquillity and heightened amplitude suggesting that fluctuations in returns are lumped together (Poon, 2005; Chan & Cryer, 2008). The presence of volatility clustering is further evidence in favour of the proposition that variance is of a time-varying nature (Jacobsen & Dannenburg, 2003). Engle (2001) states that time-varying variance is easily observed and an examination of a time series plot of returns is all that is required to establish whether volatility clustering is present.



Source: Engle (2001)

Figure 5.2: Time series plot of DJIA and NASDAQ returns

Plots of the DJIA and NASDAQ[78] returns in Figure 5.2 illustrate that the amplitude of returns varies over time; the amplitude of returns is greater around initial observations (1990 – 1991), declines towards the middle (1992 – 1996) and increases greatly towards the end of the sample period (1997 – 2000). This is an observable example of volatility clustering, also referred to as the "ARCH effect," often cited as an explanation for leptokurtosis (Akgiray, 1989; Engle, 2001). Engle (2001) interprets the variance as the risk level of returns and volatility clustering implies that some time periods are riskier than others. Additionally, these riskier times are not random and are serially correlated.

Jacobsen and Dannenburg (2003) state that evidence of volatility clustering arises in the form of highly serially correlated squared returns with confirmation provided by corresponding Ljung-Box *Q*-statistics which indicate statistically significant correlation at extended lag lengths. The authors go on to show that volatility clustering is present in the return series of six national indices representative of the stock markets of France, Germany, Italy, the Netherlands, the UK and the US. Results suggest that all markets show volatility clustering at daily and weekly frequencies at all lags over the January 1973 to May 1993 period and this is

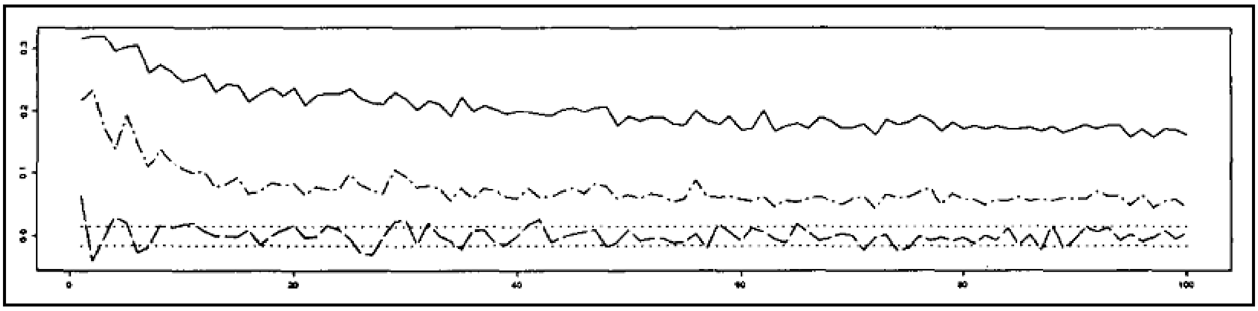
[78] The NASDAQ as it is commonly referred to today, stands for the National Association of Securities Dealers Automated Quotation.

supported by statistically significant Ljung-Box *Q*-statistics, which indicate that serial correlation differs from zero at various orders. These findings continue to hold for bi-weekly observations but not for monthly observations with the exception of the US, for which serial correlation is statistically significant at the monthly frequency. Jacobsen and Dannenburg (2003) attribute the latter observation (of a perceived absence of serial correlation) to either the series being too short or the absence of volatility clustering in monthly return series. However, serial correlation functions of squared monthly returns are similar to those of squared daily returns, supporting the explanation of return series being too short. These findings suggest that volatility clustering is a prevalent feature of financial time series, regardless of the frequency of the data, and that variance is of a time-varying nature (Jacobsen & Dannenburg, 2003; Akgiray, 1989).

5.3.2. Volatility persistence

The concepts of volatility clustering and volatility persistence are closely related to the extent that some authors do not make an explicit distinction between these two phenomena (see Engle & Patton, 2007). Volatility clustering implies volatility persistence; if *extended* periods are characterized by greater variability in returns and other periods by lower variability, then this suggests that variability must be persistent to create identifiable periods of greater and lower volatility. Perhaps, a more fitting term for persistence is “long memory.” Whereas volatility clustering implies that extended periods of volatility arise from the clustering of news or the clustering of information arrivals, the persistence or long memory property of volatility implies that a single shock will have an impact upon future volatility in periods to come (Engle, 2004; Engle, Focardi & Fabozzi, 2008; McMillan & Ruiz, 2009). McMillan and Ruiz (2009) state that the standard approach to examining the long memory property in time series is to examine the sample serial correlation function for non-linear transformations of returns. Whereas non-linear serial correlation of any length is a symptom of volatility clustering, in the context of long memory, what is of interest is *how* long it takes for a shock to die out. If it takes the sample serial correlation function an extended period of time to decline to zero, the process exhibits long memory. In other words, levels of heightened volatility *persist* and shocks do not die out immediately.

Ding, Granger and Engle (1993) investigate the long memory properties of the S&P 500 by considering the serial correlation structure of absolute returns and squared returns over the January 1928 to August 1991 period. The authors note that although first order serial correlation in the (linear) return series is positive and statistically significant, implying some degree of memory, it is of small magnitude and short-lived. However, this is not the case for the absolute and squared return series; the serial correlation functions of these transformations are positive over longer lags, as is evident from Figure 5.3.



Source: Ding, Granger & Engle (1993)

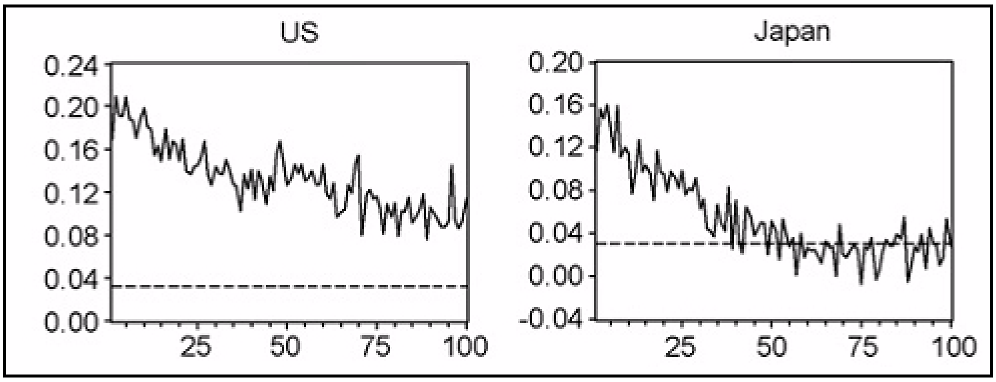
Figure 5.3: Serial correlation structure of S&P 500 returns

The level of serial correlation is highest for absolute returns (top in Figure 5.3), $|r|$, followed by squared returns (middle), r^2, and then for the untransformed return series (bottom). Both the absolute and squared return series remain above the 95 percent confidence interval and are positive for the (reported) 100 orders. If fact, Ding *et al*. (1993) report that absolute returns are positively correlated for over 2500 orders. Substantial and heightened serial correlation in absolute returns is also observed by the authors for the NYSE and the German Deutscher Aktien IndeX *(*DAX) suggesting that volatility is highly persistent in these markets and that shocks take an *exceptionally* long period of time to subside.[79]

McMillan and Ruiz (2009) investigate the long memory property of volatility for 10 national indices[80] over the January 1990 to December 2005 period by examining the serial correlation functions for absolute returns and the *half-lives* of shocks. Similarly to Ding *et al*. (1993), serial correlation functions for all series with the exception of Japan decay slowly and are statistically significant for 100 orders. Figure 5.4 reproduces the serial correlation functions for the US and Japan:

[79] Ding *et al*. (1993) state that the long memory property is mainly attributable to pre-World War 2 events such as the Great Depression in 1929.

[80] National indices for exchanges situated in Canada, France, Germany, Hong Kong, Italy, Japan, Singapore, Spain, the UK and the US.



Source: McMillan & Ruiz (2009)

Figure 5.4: Serial correlation functions for US and Japanese returns

In McMillan and Ruiz's (2009) sample, shocks persist for the longest period of time (relative to other markets) in the US market, as evident from the serial correlation function which remains positive and statistically significant for over 100 orders. The *half-life* for shocks in this market is almost 150 days. In contrast, volatility is not as persistent in the Japanese market with the serial correlation function becoming statistically insignificant at approximately 50 orders and shocks exhibiting a *half-life* of approximately 36 days. On the basis of the serial correlation functions, McMillan and Ruiz (2009) state that volatility is characterized by long memory and exhibits a hyperbolic decay; decaying relatively quickly at low orders but levelling out at higher orders. This again implies that shocks do not die out immediately. The authors however caution that the observed persistence may be attributable to time-variation and structural breaks in the unconditional mean variance and not only the result of shocks. Regardless of the source, it is evident from Ding *et al*.'s (1993) and McMillan and Ruiz' (2009) studies that volatility exhibits persistence.

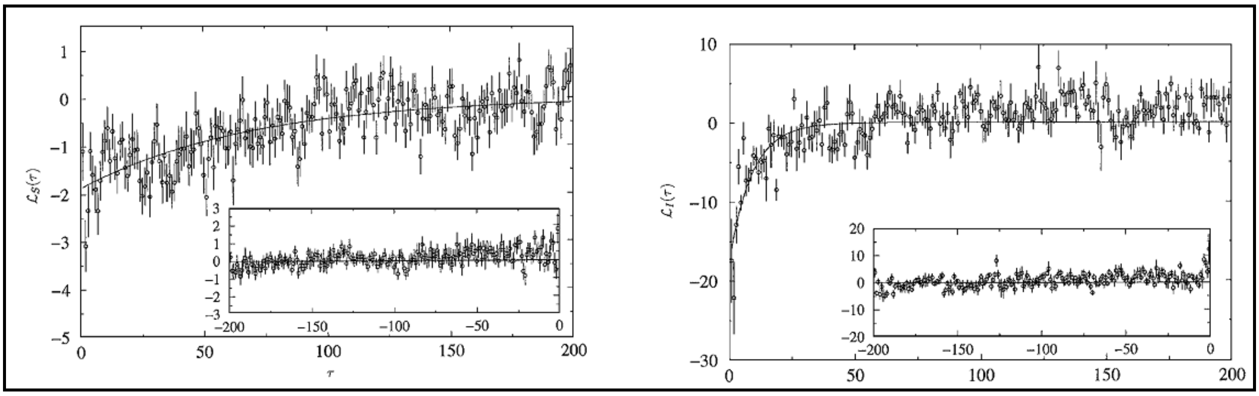
5.3.3. Leverage effect

The term, leverage effect, is attributed to Black (1976). In the classical sense, the term refers to the observation that as stock prices fall, the debt-to-equity ratio increases leading to increased volatility. In the modern context, the leverage effect refers to negative correlation between returns and volatility (Engle & Patton, 2007). The leverage effect implies that the relationship between returns and volatility is asymmetric; negative shocks have a greater impact upon volatility than do positive shocks (Kirchgässner & Wolters, 2007). The presence of the leverage effect is evident from a negative correlation between returns and volatility suggesting that negative returns are accompanied by increases in volatility (Cont, 2001). Schrager (2001) attributes the leverage effect to investor preferences for positive returns and states that investors become unsettled following negative returns which leads to increases in volatility.

Haugen, Talmor and Torous (1991) present evidence of a negative relationship between volatility and returns, and investigate the reaction of returns to changes in volatility using return data for the DJIA over the January 1897 to July 1988 period. It is postulated that increased volatility is associated with future declines in stock prices, implying a negative relationship between volatility and returns. Results indicate that over the entire sample period, negative average returns and negative average excess returns are observed over a four-week period following an increase in volatility. Notably, a statistically significant decrease in returns and excess returns is observed around periods of increased volatility. In contrast, average returns and average excess returns are higher following decreases in volatility. In light of these findings, Haugen *et al.* (1991) state that market response is greater to increases in volatility relative to decreases in volatility. It is further evident that there is an asymmetric response[81] to changes in volatility which the authors attribute to non-linear risk aversion. This argument supports Schrager's (2001) contention that investors favour positive returns over negative returns.

Bouchaud, Matacz and Potters (2001) investigate and measure the leverage effect and suggest that the negative return-volatility correlation explains negatively skewed distributions. The causality of the leverage effect is also questioned; does increased volatility lead to a decline in stock prices as suggested by Haugen *et al.* (1991) or does volatility increase following a decline in stock prices as suggested by Schrager (2001). It is hypothesized that a decline in the stock price of an individual firm increases the possibility of financial distress, which in turn increases the volatility of returns on given stock. Conversely, increased volatility makes a given stock less attractive leading to a decrease in the price (Bouchaud *et al.*, 2001). Using return data on a sample of individual US companies compromising the S&P 500 and seven major indices,[82] Bouchaud *et al.* (2001) find evidence of a leverage effect in both individual stocks and the indices. This is evident from the respective return-volatility correlation functions reported in Bouchaud *et al.* (2001):

[81] Haugen *et al.* (1990) base this statement upon a finding that the mean adjustment in stock prices is -2.62 percent following an increase in volatility and 1.70 percent following a decrease in volatility. It is evident, that in absolute terms, the response is greater following an increase in volatility.
[82] The seven indices considered by Bouchaud *et al.* (2001) are the S&P 500, NASDAQ, CAC 40, FTSE, DAX, the Nikkei and the Hang Seng. The dataset spans the January 1990 to May 2000 period for individual stocks and the January 1990 to October 2000 period for indices.



Source: Bouchaud, Matacz & Potters (2001)

Figure 5.5: Return-volatility correlation functions

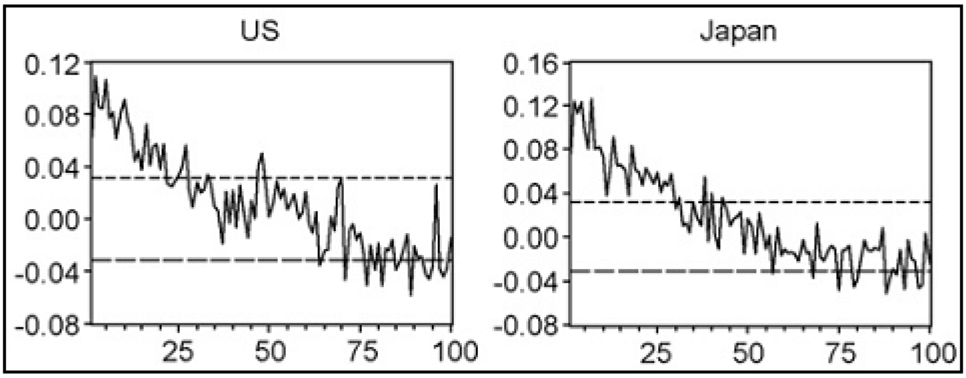
Figure 5.5 illustrates the (averaged) return-volatility correlation functions for individual stock returns ($\mathrm{L}s(\tau)$) and returns on the indices ($\mathrm{L}i(\tau)$) respectively. Both functions start from negative values and decay towards zero implying a negative relationship between returns and subsequent volatility - evidence of the leverage effect. Bouchaud *et al*. (2001) attribute the presence of a leverage effect to a feedback mechanism whereby anticipated increases in volatility trigger sell orders which drive down stock prices. As in Haugen *et al*. (1991), risk aversion is credited for the negative relationship between returns and volatility. Haugen *et al*.'s (1991) and Bouchaud *et al*.'s (2001) studies constitute empirical evidence of the leverage effect.

5.3.4. Mean reversion

Engle and Patton (2007) state that volatility clustering implies that volatility comes and goes. A period of high volatility will be followed by a period of lower volatility and a period of low volatility will be followed by a period of higher volatility, implying that volatility *reverts* to some long-run level with time. A further interpretation of mean reversion provided by the authors is that current information has no effect on long-run volatility, suggesting that shocks die out with time. The authors go on to state that mean reversion in volatility is widely interpreted as relating to the eventual convergence of volatility upon a *normal* level of volatility. Figlewski (1997) distinguishes between mean reversion in volatility and mean reversion in stock prices. Mean reversion in volatility implies that levels of extremely high or low volatility tend to give way to a reversion towards a more moderate long-term level. In contrast, the value of an underlying stock tends to a long-run mean level over time. Moreover, whereas mean reversion is an accepted characteristic of volatility, debate centres upon *what* the normal level of volatility is and whether it is constant over time (Engle & Patton, 2007).

Evidence of mean reversion is indirect in that studies that concentrate on volatility clustering and persistence are also suggestive of mean reversion. Although not noted directly, evidence of mean reversion is nevertheless abundant as evident in Figure 5.2. Reversion is most pronounced for the DJIA; high volatility observed between 1990 and 1991 gives way to a *protracted* period of low volatility lasting until approximately 1996. This lower level of volatility during this period is likely to be closer to the mean than the shorter periods of heightened volatility. While further transient periods of heightened volatility are observed during this period, in each instance, a degree of reversion to a protracted lower level is observed for both return series. This interpretation is consistent with that of Figlewski (1997). Further evidence of mean reversion stems from studies of volatility persistence where the underlying assumption in these studies is that although volatility is persistent, it reverts to some constant mean level (McMillan & Ruiz, 2009). Evidence of mean reversion in volatility may therefore be seen in the decay of serial correlation functions of non-linear transformations of returns; if shocks eventually die out, then this implies that periods of high volatility following a shock transition into periods of lower volatility and therefore, revert to a long-run or normal mean level. McMillan and Ruiz' (2009) second set of results, adjusted for time variation in the unconditional variance,[83] suggests that shocks do not persist infinitely and die out, implying that periods of higher volatility give way to more tranquil periods. This is evident from the serial correlation function of absolute returns for Japan in Figure 5.4, and in Figure 5.6 below where time variation in the unconditional variance is controlled for:

[83] To adjust for time variation in unconditional variance, a moving average of absolute returns is estimated for each series. A slowly changing unconditional mean may result in apparent long memory when using a methodology that assumes a constant unconditional mean. The procedure does not detract from the behaviour of the data; the *unadjusted* absolute return series and the *adjusted* absolute return series show relatively high correlations of around 0.95 (see McMillan & Ruiz, 2009).



Source: McMillan & Ruiz (2009)

Figure 5.6: Serial correlation functions for US and Japanese returns using adjusted data

Figure 5.6 indicates that while shocks persist in the US and Japanese markets, they do so for shorter periods of time (than in Figure 5.4) and become statistically insignificant after approximately 25 orders for both countries. Therefore, while shocks *persist* - implying higher volatility for a limited period of time - these shocks *do* die out eventually giving way to lower volatility. This suggests that volatility reverts to some mean or normal level and is consistent with Engle and Patton's (2007) interpretation whereby shocks die out and, as a result, current information has no impact upon long-run volatility. Engle and Patton (2007) provide further evidence of mean reversion relying upon a GARCH(1,1) model (discussed extensively in Chapter 6). Results indicate that while volatility has a long memory, it exhibits mean reversion. This is evident from the finding that the ARCH and GARCH parameters of the model are significantly less than one, implying that although volatility is persistent, volatility does return to its mean.[84]

From the above discussion, it is evident that volatility exhibits mean reversion and this can be demonstrated using a variety of approaches. The question that nevertheless remains is whether mean reversion is full or partial and if it is partial, then this implies that the long-run variance may not be constant and that variance does not fully revert to the mean.

[84] Thus far, the present discussion has attempted to use model-free approaches to demonstrate mean reversion in volatility. While it is argued that this is a valid approach, an acknowledgement must be made that the use of stochastic volatility models greatly simplifies the study of volatility and its properties. One such class of models (ARCH/GARCH models) is extensively discussed in the following chapter (Chapter 6) and applied in the empirical analysis (Chapter 8).

5.4. Conclusion

Stock returns are assumed to be “well-behaved.” However, this is not the case in practice. Deviations from the assumptions are observed in the form of non-normality characterized by excess kurtosis, fat-tails and skewness (section 5.2.1). Returns appear to be uncorrelated suggesting that they are independent and where dependence is observed, it is short lived (section 5.2.2). Moreover, volatility is characterized by volatility clustering (section 5.3.1), persistence and long memory (section 5.3.2), asymmetry in the form of the leverage effect (section 5.3.3) and reversion (section 5.3.4).

While it remains for it to be established that South African stock returns violate the assumptions discussed above, the characteristics of stock returns and volatility observed in literature call for an econometric framework that accounts for these characteristics. This framework must ensure that estimated models of the return generating process are statistically adequate. Such a framework, which is discussed in Chapter 6, is found in the ARCH/GARCH model framework with its numerous extensions.

6. THE ARCH AND GARCH FRAMEWORK: AN ECONOMETRIC MODELLING FRAMEWORK

6.1. Introduction

The aim of this study is to investigate the return generating process of South African stock returns within the APT framework. An investigation of this nature requires an appropriate econometric framework for model estimation. The preceding discussion in Chapter 5 shows that returns are not well-behaved. Returns are characterized by non-normality in the form of excess kurtosis, fat-tails and skewness (section 5.2.1). Volatility is characterized by volatility clustering (section 5.3.1), long memory, persistence (section 5.3.2), the leverage effect (section 5.3.3) and mean reversion (section 5.3.4). Furthermore, volatility clustering implies that volatility is heteroscedastic. These properties of returns and volatility require an econometric methodology that accounts for these characteristics.

The LS methodology is widely employed in estimating models of the return generating process within the APT framework (see section 3.3.1; Burmeister & Wall, 1986; Berry *et al*., 1988). This approach is based upon the assumption that the expected value of all squared residuals terms is the same at any point in time – residuals terms are assumed to be homoscedastic (Engle, 2001; Gujarati, 2003). While the assumption of homoscedasticity is convenient, it does not hold in reality; the variance of the residual terms tends to differ across time and is therefore heteroscedastic (Engle *et al*., 2008). If this is the case, and the LS methodology is applied, regression coefficients (factor loadings, exposures, sensitivities or betas in APT terminology) will be unbiased and consistent. However, regression coefficients will be inefficient suggesting that estimators will not have minimum variance. Standard errors will be overstated and as a result, confidence intervals will be unnecessarily large, resulting in misleading inferences. Because the variance of an estimate is exaggerated, coefficients that are statistically significant may appear to be statistically insignificant if confidence intervals fail to account for heteroscedasticity (Gujarati, 2003; Barreto & Howland, 2006).[85] Importantly and in the context of this study, the structure of return generating process will be misidentified. This is especially pertinent for return data where non-normalities and heteroscedasticity may carry over into the residuals suggesting that the

[85] The width of the confidence interval is proportional to the standard error of a coefficient. Larger standard errors will result in wider confidence intervals translating into greater uncertainty of the true value of a parameter (Gujarati, 2003).

characteristics of the return distribution have a direct bearing upon modelling and inference making (Roll, 1992; Wong & Bian, 2000). In this context, Gujarati (2003: 399) states that "if we persist in using the usual testing procedures despite heteroscedasticity, whatever conclusions we draw or inferences we make may be very misleading." Furthermore, if the residuals of a model are not normally distributed, then *t*-tests and *F*-tests based upon estimated errors used in inference making will be misleading (Ford, 2003). This emphasizes the need to consider an alternative to the LS methodology when modelling the return generating process.

The ARCH/GARCH framework is designed to capture the observed characteristics of returns and discards the restrictive assumptions of normality, independence and constant variance (Zakoian, 1994; Palm, 1996; Elyasiani & Mansur, 1998; Engle, 2004). Dowd (2005: 131) emphasizes the appropriateness of this framework by stating that the ARCH/GARCH framework can readily accommodate leptokurtosis and volatility clustering, and is "tailor-made" for volatility clustering, which is responsible for fat-tails (section 5.2.1 & 5.3.1). Furthermore, the ARCH/GARCH framework treats heteroscedasticity as variance to be modelled and thereby corrects a deficiency of the LS framework and estimates the variance of each residual term (Engle, 2001). By modelling residuals, it is possible to obtain a more accurate description of the return generating process; Engle (1982: 1004) states that the ARCH model "comes closer to truly random residuals after standardizing for their conditional distributions." Not only do ARCH/GARCH models appear to be well-suited to modelling returns, they contribute to a superior description of the return generating process and mitigate erroneous inferences.

This chapter proceeds by providing a background to the ARCH/GARCH framework (section 6.2). A number of generalizations of ARCH/GARCH models and extensions are then discussed (section 6.3.1 – 6.3.6) and it is the ARCH (section 6.3.1), GARCH (section 6.3.2), IGARCH (section 6.3.3) and EGARCH (section 6.3.5) generalizations that are applied to estimate models of the return generating process (see section 7.4.2 & 7.4.3; Chapter 8). The literature is reviewed to provide an overview of the applications of ARCH/GARCH models in finance (section 6.4) and limitations of the framework are noted (section 6.5). The discussion is summarized in the conclusion (section 6.6).

6.2. ARCH and GARCH framework

According to Xiao and Aydemir (2007), the starting point to modelling variance is to treat innovations in the mean as a series of independent and identically distributed random variables, z_t, with a mean of zero and unit variance scaled by the standard deviation $\sqrt{h_t}$:

$$\varepsilon_t = \sqrt{h_t} z_t, \quad z_t \sim i.i.d(0,1) \tag{6.1}$$

In this context, ARCH and GARCH models describe variance in terms of past observations and the stochastic residual term, ε_t, is interpreted as an innovation in the mean (Xiao & Aydemir, 2007). The simplest return generating process specification can be denoted by (Engle *et al*., 2008):

$$r_{it} = \mu_t + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t} z_t \tag{6.2}$$

where r_{it} is the return on stock *i* at time *t*, μ_t is the mean and ε_t is the innovation in the mean. Similarly to $E(R_i)$ in equation (2.1), the mean in equation (6.2), can be interpreted as the expected value of r_{it}, with returns being a function of a conditional mean value and an innovation in the form of the residual term (Engle *et al*., 2008). Elyasiani and Mansur (1998) suggest a generalized specification for the conditional mean equation describing the return generating process in terms of a multifactor specification consistent with the APT framework:

$$r_{it} = \mu_t + \sum \beta x_t + \varepsilon_t \tag{6.3}$$

where $\sum \beta x_t$ is a vector of pre-specified or exogenous factors, which under the APT framework will constitute innovations in systematic risk factors. Thus, according to equation (6.3), returns are a function of the conditional mean, a residual term *and* a vector of systematic risk factors. In the second equation, equation (6.4), the *conditional variance* is modelled as a by-product of the conditional mean equation. The conditional variance equation can be denoted in generalized form as:

$$h_t = \psi(\Omega_t, x_t) \tag{6.4}$$

where h_t is the conditional variance as per ARCH/GARCH literature convention, Ω_t is the information set at time *t* and x_t is a set of endogenous lagged factors and exogenous factors. Equation (6.4) implies that the conditional variance is dependent upon prior information, endogenous and exogenous factors. The exact specification of equation (6.4) differs according to the type of ARCH/GARCH model employed to describe the conditional variance. Moreover, by describing conditional variance, the ARCH/GARCH framework not only permits a description of sample variance, it also allows the testing of economic models which seek to identify the causes of volatility by incorporating factors into the conditional variance equation (Engle, 2004).

As returns are described by the mean, a vector of predetermined endogenous or exogenous factors, and a residual term which can be decomposed into a random variable scaled by the conditional variance, an accurate description of returns requires an accurate model of the conditional variance. The multitude of ARCH/GARCH models which capture various characteristics of returns and volatility make modelling returns with greater accuracy and statistical adequacy possible. In the context of the APT framework, equations (6.3) and (6.4) are estimated within the ARCH/GARCH framework with equation (6.3) representing the return generating process and equation (6.4) representing the underlying conditional variance.

6.3. ARCH and GARCH models and extensions

Engle (1982) introduced the first ARCH model and this was followed by Bollerslev's (1986) generalization, the GARCH model. A number of notable extensions soon followed; Engle and Bollerslev's (1986) Integrated GARCH (IGARCH) model, Engle *et al*.'s (1987) ARCH-in-Mean (ARCH-M) model, its generalization, the GARCH-in-Mean (GARCH-M) model and Nelson's (1991) Exponential GARCH (EGARCH) model. Since then, a great number of generalizations have been proposed constituting what Engle (2004: 329) refers to as "an alphabet soup of ARCH models."[86] These models capture the non-linearity, asymmetry and the long memory properties of volatility and the non-normality of returns. This chapter outlines some of the better known generalizations utilized in financial literature and it is the

[86] Engle (2004) cites AARCH, APARCH, FI-GARCH, FIEGARCH, STARCH, SWARCH, GED-ARCH, GJR-GARCH, TARCH, MARCH, NARCH, SNPARCH, SPARCH, SQGARCH, CESGARCH, CARCH and ACARCH as *some* examples.

ARCH, GARCH, IGARCH and EGARCH generalizations that are applied in this study in the modelling of the return generating process in Chapter 8.

6.3.1. ARCH

The first model of the ARCH/GARCH framework is the ARCH model of Engle (1982) who sought to introduce a model in which variance is dependent upon prior information. According to Engle (1982), the conventional approach to heteroscedasticity at the time was to introduce an exogenous factor that would predict the variance. Engle's (1982) proposed solution, the ARCH regression model, assumes that the conditional mean of the dependent factor is a linear combination of lagged endogenous and exogenous factors, and that the evolution of the conditional variance is described by the ARCH(*p*) model:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{6.5}$$

where $\omega > 0$ and $\alpha_i \geq 0$ so that h_t is strictly positive variance. According to the ARCH model, the conditional variance is a function of *past* squared residual terms, ε_{t-i}^2, with p denoting the order of the squared residual terms in the model (Poon, 2005). Optimal weights of ω and α_i are estimated using the maximum likelihood (ML)[87] methodology making it possible for the ARCH model to describe time-varying volatility at any point in time (Engle, 2004). Using weighted averages of past squared residuals gives more recent observations greater weighting relative to more distant observations. The ARCH model is considered to be a *short* memory model with high values of α_i indicating that volatility is "spiky" and reacts quickly to market movements. A positive intercept ω permits mean reverting volatility (Dowd, 2005). Poon (2005) states that p is usually of a high order as a result of the persistence of volatility in financial markets (see section 5.3.2). Tsay (2002) and Xiao and Aydemir (2007) suggest that the ARCH model and specifications based upon the ARCH model possess the ability to capture volatility clustering in return series (see section 5.3.1). The ARCH methodology, therefore, permits for a conventional regression model with time-varying variance and residuals that follow an ARCH process. The ARCH model can be

[87] See Li, Ling and McAleer (2002) for a more detailed outline of estimation techniques.

further generalized by incorporating contemporaneous and lagged endogenous and exogenous factors into the conditional variance equation (Engle, 1982).

Engle (1982) applies the ARCH and LS models to describe inflation in the UK over the 1958 to 1977 period. Outliers of the respective models are examined and although, the number of outliers is found to be reasonable under the LS model, the timing of the occurrence of outliers is far from random. However, outliers under the ARCH model come closer to truly random residuals suggesting a more realistic description of the data. Tsay (2002) elaborates upon this finding by suggesting that the ARCH model is more likely to produce the outliers observed in returns than those suggested by an *n.i.i.d.* sequence, implying that the ARCH model can capture the fat-tails of return distributions.[88] Engle (1982) concludes that the ARCH model represents an improvement over the performance of the LS model and describes variance more realistically.

Notwithstanding these favourable findings, a number of criticisms have been levelled at the ARCH model. The ARCH model assumes that positive and negative shocks have the same influence upon volatility. However, volatility responds differently to negative and positive shocks, as evident from the leverage effect (section 5.3.3). Also, the ARCH model by itself does not provide insight into the sources of variation in returns; all that it does is provide a methodology to describe conditional variance. Furthermore, the model does not describe volatility parsimoniously and an extended number of parameters is required to describe the volatility process accurately (Tsay, 2002).

6.3.2. GARCH

Bollerslev (1986) argues that in applications of ARCH models, a long lag structure is often required in the conditional variance specification and a fixed lag structure is imposed to avoid problems associated with negative variance parameters. These limitations motivate for an ARCH-type model that permits a more flexible structure and a longer memory. Both ARCH and GARCH models treat conditional variance as a function of past shocks and thus, permit shock persistence and the evolution of volatility over time. As in the ARCH model, the GARCH model incorporates a weighted average of past squared residual terms (Elyasiani &

[88] This is an example of how the ARCH/GARCH framework can accommodate the characteristics of returns and volatility discussed in Chapter 5. In this example, the ARCH model is seen accommodating leptokurtosis (discussed in section 5.2.1).

Mansur, 1998; Engle, 2001). However, to achieve a more flexible structure and to allow for a longer memory (section 5.3.2), the GARCH model incorporates a set of lagged conditional variance terms and thus permits an adaptive learning mechanism, which implies that the best predictor of variance is a weighted average of the long-run variance (Bollerslev, 1986; Engle, 2001). Through the inclusion of lags of the conditional variance in addition to past squared residual terms, the GARCH model becomes a long memory model. This contrasts with the (relatively) short memory underlying the ARCH model (Elyasiani & Mansur, 1998). As with the ARCH model, optimal weights are assigned to the squared residual and lagged conditional variance terms. The GARCH(p,q) specification is denoted by:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{6.6}$$

where $\alpha_i > 0$ and $\beta_j > 0$, and q denotes the order of conditional variance terms, h_t. A high value of β_j indicates that volatility is persistent and takes a long time to change (Dowd, 2005). The sum of the coefficients of the conditional variance specification, $\alpha_i + \beta_j$, is less than unity if unconditional variance is finite, implying mean reversion (section 5.3.4: 116; Engle, 2001; McMillan & Ruiz, 2009). The GARCH model is considered to be more parsimonious relative to the ARCH model; parameter values of $p \leq 2$ and $q \leq 2$ are deemed to be sufficient for most financial applications (Bollerslev, Chou & Kroner, 1992; Xiao & Aydemir, 2007). Xiao and Aydemir (2007) state that the most crucial characteristic of the GARCH model is linearity, which assumes an Autoregressive Moving Average (ARMA) specification for the squared innovation process. This property permits a comprehensive study of the squared residual terms and simplifies statistical inference.

To assess the performance of the GARCH model in relation to the LS methodology and an ARCH specification, Bollerslev (1986) applies the proposed GARCH methodology to model the growth rate in the US GNP deflator for the 1948 to 1983 period. A more parsimonious GARCH(1,1) model is found to provide a better fit relative to an ARCH(8) model. The observed high order of the ARCH model is attributed to long memory in the conditional variance - a problem addressed by the more parsimonious GARCH(1,1) model. Bollerslev (1986) also finds that the GARCH(1,1) model is better at capturing long memory relative to the ARCH(8) model.[89] Finally and importantly, LS intervals are found to be too wide, potentially leading to erroneous inferences. This suggests that the application of the GARCH model is more appropriate.

While Bollerslev's (1986) GARCH model appears to be an improvement over the LS and ARCH model, there are shared limitations. As in the ARCH model, conditional variance responds equally to positive and negative shocks (Tsay, 2002). This arises from the non-negativity constraints placed upon coefficients of the GARCH model and implies that regardless of direction, shocks have a positive impact upon volatility. As a result, the model does not account for the cyclical behaviour and non-linear characteristics of volatility (Xiao & Aydemir, 2007). Finally, according to Tsay (2002), the tails of GARCH models remain too short when high frequency financial time series data is used.

6.3.3. IGARCH

Bollerslev *et al*. (1992) state that persistence is commonly observed in estimates of conditional variance specifications when using high-frequency data. Moreover, parameters of the GARCH model tend to sum to unity, $\sum a_i + \sum \beta_j = 1$, implying that variance is not finite and therefore, that the unconditional variance does not exist. In the presence of an approximate unit root, the conditional variance of the residuals does not converge towards its unconditional variance suggesting that conditional variance grows linearly with the forecast horizon (Bollerslev & Engle, 1993; Kirchgässner & Wolters, 2007). To capture infinite persistence in volatility, Engle and Bollerslev (1986) propose the IGARCH(*p*,*q*) model:

$$h_t = \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{6.7}$$

Tsay (2002) describes the IGARCH model as a unit-root GARCH model and attributes continued persistence to occasional shifts in volatility. The IGARCH model captures infinite variance by describing an $I(1)$ process where current information remains important for all forecast horizons of the conditional variance (Xiao & Aydemir, 2007; Kang, Kang & Yoon, 2009). Under the IGARCH model, the parameter of integration, *d*, is assumed to be 1

[89] The ARCH model restricts the mean lag of the conditional variance to 3.5. Bollerslev (1986) states that the mean lag for the GARCH(1,1) model is estimated to be 5.848 suggesting that the GARCH(1,1) model is better at capturing persistence.

implying that the series has infinite persistence and is non-stationary. If d is equal to zero, the effects of shocks die out rapidly suggesting a short memory. On the other hand, if $0 < d < 0.5$, then the series exhibits a long memory but is stationary (McMillan & Ruiz, 2009). As the IGARCH is an infinite memory model that assumes that shocks never die out, it does not allow for long memory in volatility (Kang *et al.*, 2009). Together, the ARCH, GARCH and IGARCH models capture the different types of memory which characterize the volatility underlying stock returns.

6.3.4. ARCH-M and GARCH-M

The next notable extension to the ARCH/GARCH framework is the ARCH-in-Mean (ARCH-M) model introduced by Engle *et al*. (1987). The ARCH-M model is well-suited to the modelling of the risk-return relationship that is central to econometric research on time-varying expected returns and posited by financial theories (Bollerslev *et al*., 1992; Xiao & Aydemir; 2007). Backus and Gregory (1993) state that the theoretical justification for the use of ARCH-M models to model the relationship between returns and time-varying conditional variance stems from the CAPM. Elyasiani and Mansur (1998) state that the ARCH-M specification, by establishing a link between returns and a measure of risk in the form of the conditional variance, brings empirical models closer to asset pricing theories such as those underlying the CAPM and the APT. Central to the model is the premise that as the level of risk varies over time, the compensation for bearing risk must also vary. By this line of reasoning, the ARCH-M model permits the conditional mean to be affected by the conditional variance (Engle *et al*., 1987). By doing so, the ARCH-M extension permits the consideration of a time-varying premium (Elyasiani & Mansur, 1998). The conditional mean equation incorporating conditional variance is denoted as:

$$r_{it} = \mu + \sum \beta x_t + \lambda h_t + \varepsilon_t \tag{6.8}$$

where h_t is the conditional variance term in the mean equation and λ, the *coefficient of relative risk aversion*, quantifies the impact of the (time-varying) conditional variance on returns (Xiao & Aydemir, 2007). Engle *et al*. (1987) suggest that the conditional standard deviation and logarithm of the conditional variance may be used in place of the conditional variance. The conditional variance equation remains the same as that of the ARCH model. This yields an ARCH-M model although in practice, the conditional variance equation can be based upon any ARCH or GARCH type model. By this line of reasoning, the GARCH-M is a natural extension to the ARCH-M model (see Elyasiani & Mansur, 1998: 541).

Engle *et al.* (1987) provide empirical support for the ARCH-M model using holding yields on US treasury bills[90] from 1960 to 1984. Yields are regressed onto a constant with ARCH residual terms and the results indicate a strong ARCH effect in holding yields (see section 5.3.1: 109). Because of this, Engle *et al.* (1987) state that a misspecification of the model arises in the form of a time- varying risk premium which is incorporated into the residual terms instead of appearing in the conditional mean equation. The model is therefore re-estimated with the standard deviation ($\sqrt{h_t}$) incorporated into the conditional mean equation. The coefficient of relative risk aversion (λ) is found to be positive and statistically significant suggesting a positive trade-off between risk and returns. The authors also find that incorporating the logarithm of the standard deviation ($\log\sqrt{h_t}$) into the conditional mean equation in place of the standard deviation improves the fit of the model. The statistically significant coefficients of relative risk aversion in these variants of the ARCH-M model suggest that the ARCH-M specification provides a better fit and description of returns relative to a conventional ARCH model. Engle *et al.* (1987) recognize that the trade-off between risk and returns is dependent upon risk preferences and therefore, the coefficient of relative risk aversion can take on a positive, negative or zero value. Although, ARCH-M type specifications are not always supported by theory, the application of these models to financial data is nevertheless widespread (Backus & Gregory, 1993; Xiao & Aydemir, 2007).

6.3.5. EGARCH

Nelson (1991) suggests that the functional form and non-negativity coefficient constraints placed upon parameters of ARCH and GARCH models translate into a number of limitations. ARCH and GARCH models assume that only the magnitude and not the sign of returns determines conditional variance; implying that linear models are unable to capture the negative correlation between returns and changes in volatility. Non-negativity constraints imply that increasing values of ε_t^2 lead to increases in conditional variance suggesting that conditional variance does not show oscillatory behaviour. Furthermore, non-negativity constraints also complicate the estimation of ARCH and GARCH models (Nelson, 1991;

[90] Salomon Brothers data on 3 and 6 month treasury bills from the Analytical Records of Yields.

Xiao & Aydemir, 2007). Finally, Nelson (1991) suggests that it is difficult to evaluate whether shocks to conditional variance are persistent under the GARCH model. To address these limitations, Nelson (1991) proposes the EGARCH(*p,q*) model which permits conditional variance to respond asymmetrically to positive and negative residuals:

$$\ln(h_t) = \omega + \sum_{i=1}^{p} \alpha_i g(z_{t-i}) + \sum_{j=1}^{q} \beta_j \ln h_{t-j}\,, \quad z_t = \varepsilon_t / \sqrt{h_t} \tag{6.9}$$

$$g(z_t) = \theta z_t + \gamma[\,|\,z_t\,| - E\,|\,z_t\,|\,]$$

where $\ln(h_t)$ is a function of time and lagged ε_t s, and the *logarithm* of conditional variance permits the relaxation of the non-negativity constraints placed upon model coefficients. To capture the asymmetric relationship between returns and volatility, $g(z_t)$ is assumed to be a function of the magnitude *and* sign of z_t. This permits the relationship between the residuals and conditional variance to be negative (Nelson, 1991; Tsay, 2002). Moreover, given appropriate conditioning of the parameters, the EGARCH model captures the asymmetric relationship between negative shocks and variance (Poon, 2005).[91] When $\gamma = 0$ and $\theta < 0$, innovations in conditional variance are positive (negative) when returns are negative (positive) (Nelson, 1991). This addresses the criticism pertaining to the ARCH and GARCH models that only the magnitude of return innovations and not the sign is considered. Furthermore, as there are no inequality constraints, β_j can be either positive or negative and thus permit cycling and oscillatory behaviour.

Nelson (1991) applies the EGARCH model to investigate the risk-return relationship, the asymmetric relationship between positive and negative returns and conditional variance, the persistence of shocks to volatility, the presence of leptokurtosis in returns and the impact of non-trading days upon the conditional variance. The return data employed is for the CRSP value-weighted market index over the July 1962 to December 1987 period.[92] The conditional mean equation incorporates the conditional variance and an endogenous factor in the form of

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[91] The two components of $g(z_t)$, θz_t and $\gamma[\,|\,z_t\,| - E\,|\,z_t\,|\,]$, are assumed to have a mean of zero. If z_t is symmetrically distributed, then these two components are orthogonal. $g(z_t)$ is linear in z_t over the range $0 < z_t < \infty$ with a slope of $\theta + \gamma$ and with a slope of $\theta - \gamma$ over the range $-\infty < z_t \le 0$. According to Nelson (1991), this allows $g(z_t)$ to respond asymmetrically to stock price movements.

[92] Nelson (1991) performs a parallel analysis using a capital gain series, which ignores dividends and riskless interest rates. Results are almost identical.

lagged excess returns.[93] Results indicate that the coefficient of relative risk aversion is negative and statistically insignificant. More importantly however, the relationship between returns and volatility is found to be asymmetric; θ is negative and statistically significant implying that volatility increases (decreases) when returns are negative (positive).[94] This provides empirical support for the EGARCH model suggesting that the model captures the leverage effect. Although, volatility shocks appear to be persistent, Nelson (1991) warns that this result must be interpreted with caution owing to the limited length of the dataset. Nevertheless, these findings suggest that the EGARCH model captures the persistence of shocks *in addition* to the asymmetric relationship between returns and volatility. Nelson (1991) also finds that the conditional distribution of the residual terms has thicker tails than that implied by the normal distribution suggesting that the EGARCH model is able to capture the characteristics of the return distribution rather well. Finally, non-trading days are found to contribute less than a fifth to daily volatility. Nelson's (1991) application of the EGARCH model to return data demonstrates the usefulness and applicability of the model; the model captures the asymmetric relationship between returns and volatility, volatility persistence and the properties of the return distribution.

6.3.6. Extensions

The ARCH, GARCH, their in-mean extensions, the IGARCH and EGARCH models are considered to be the better known and widely used ARCH/GARCH specifications. However, a (rather large) number of other models also feature prominently in the literature.[95]

Engle and Lee (1999) propose the Component GARCH (CGARCH) model which captures both long-run and short-run volatility. The model permits a slow mean reverting component of conditional variance and a more volatile short-run component. By making a distinction between the short-run and long-run components of volatility, the CGARCH model provides a better description of volatility dynamics relative to the GARCH model (Guo & Neely, 2008). The Fractionally Integrated GARCH (FIGARCH) model of Baillie, Bollerslev and Mikkelsen (1996) captures long-run dynamic relationships in conditional variance and is seen as an extension of the IGARCH model. Under the FIGARCH model shocks eventually die out; although, the impact of shocks decays at a hyperbolic and not exponential rate (Mills &

[93] This is to correct for serial correlation induced by discontinuous trading in stocks which constitute the index.
[94] Nelson (1991) reports that $\theta = -0.118$ and is highly significant for CRSP excess returns.
[95] For an extensive glossary of ARCH and GARCH models see Bollerslev (2008).

Markellos, 2008). The model introduces the fractional integration parameter, *d*, which measures the persistence of shocks and permits an intermediate range of persistence in the model (McMillan & Ruiz, 2009; Kang *et al*., 2009).

The GJR-GARCH model proposed by Glosten, Jagannathan and Runkle (1993) assumes different GARCH specifications for negative and positive shocks. This allows the model to captures the leverage effect (Kirchgässner & Wolters, 2007; Bollerslev, 2008). Zakoian (1994) introduces the Threshold GARCH (TGARCH) model, another asymmetric extension to the GARCH framework. Bollerslev (2008) states that the TGARCH model is closely related to the GJR-GARCH model. In this model, the conditional *standard deviation* is a function of the (untransformed) positive and negative components of the residual term and the conditional standard deviation. By modelling the conditional standard deviation, non-negativity constraints are not necessary in the definitions of model parameters (Zakoian, 1994). Unlike the EGARCH model, the TGARCH model permits additive modelling, different orders to yield opposite contributions and a linear equation. Ding *et al*. (1993) propose a more flexible and general specification that encompasses seven other models. The Asymmetric Power ARCH (A-PARCH) model incorporates absolute residuals, which capture the long memory property of stock returns and imposes a Box-Cox power transformation of the standard deviation and asymmetric absolute residuals allowing a linearization of non-linear models. The model is generalizable into the ARCH, GARCH, GJR-GARCH, TGARCH, non-linear ARCH (NARCH) and log-ARCH models.

Whereas the abovementioned models aim to capture different characteristics of returns and volatility, a number of models make specific assumptions relating to the distribution of the residual terms. Examples of these are Bollerslev's (1987) GARCH-t model which assumes residuals follow a Student's *t*-distribution instead of a normal distribution and Nelson's (1991) GED-GARCH which assumes that the residuals follow a generalized error distribution (GED) (Bollerslev, 2008). Finally, there are the regime switching models which aim to model the effects of extreme political and economic events upon financial time series (Xiao & Aydemir, 2007). An example of such a model is the regime Switching ARCH (SWARCH) model of Hamilton and Susmel (1994). The development of this model is motivated by the poor forecasting performance resulting from structural changes in the ARCH process. The model treats changes in the regime as changes in the parameters and the scale of the ARCH process (Hamilton & Susmel, 1994).

6.4. Applications of ARCH and GARCH models

6.4.1. Forecasting volatility

Xiao and Aydemir (2007) state that volatility forecasts are important for financial markets and have received considerable attention in recent decades. ARCH and GARCH models, by assigning optimal weights to parameters, make it possible to obtain forecasts that are closest to the volatility or variance for the next period (Engle, 2004). Akgiray (1989) makes use of the ARCH/GARCH framework to forecast volatility. It is argued that the predictive capabilities of the ARCH/GARCH framework are evidence of its usefulness and applicability to stock returns. To evaluate predictive capabilities, volatility forecasts are compared against those of the Historical Average (HIS) model and an Exponentially Weighted Moving Average (EWMA) forecast. Akgiray (1989) reports that the ARCH and GARCH specifications employed yield superior forecasts of monthly volatility relative to the benchmark HIS and EWMA models and capture volatility spikes not captured by the benchmarks. A visual comparison of actual volatility and volatility predicted by the ARCH and GARCH specifications, indicates that these models realistically model the time series behaviour of volatility.[96] By Akgiray's (1989) own criteria, the predictive capabilities of the ARCH and GARCH models are evidence of the applicability of the models and point towards the ARCH/GARCH framework's usefulness and applicability in forecasting volatility.

6.4.2. Investigating the risk-return relationship

The importance of the ARCH/GARCH framework is partially attributable to the direct association of variance and risk, and the risk-return relationship. The ARCH-M and GARCH-M class of models greatly simplifies the study of the price of risk and risk-return relationships (Bollerslev *et al*., 1992; Engle, 2004). French, Schwert and Stambaugh (1987) investigate whether daily returns on the S&P Composite Index over the January 1928 to December 1984 period are positively related to risk as measured by the volatility of returns on the index. A GARCH-M model is used to estimate the relationship between returns and volatility, and French *et al*. (1987) find a statistically significant and positive relationship between returns on US common stocks and predicted volatility. Chou (1988: 280) states that the ARCH/GARCH framework is considered to be an "effective tool" in modelling the behaviour of economic time series and especially financial market data. A GARCH-M model is applied

[96] This finding is confirmed by the mean error (ME), root mean square error (RMSE), mean absolute error (MAE) and the mean absolute percentage error (MAPE) statistical loss functions. These results are generalizable to the sub-periods.

to study the relationship between volatility and market movements with changing risk measured by volatility. It is acknowledged that the in-mean extensions of the ARCH/GARCH framework present a link between the conditional mean and the volatility. Using return data for the NYSE value-weighted index over the April 1988 to March 1991 period, Chou (1988) finds a positive risk-return relationship as indicated by a positive and statistically significant coefficient of relative risk aversion. The author states that this confirms the existence of a changing risk premium in the US. By linking the conditional variance and the conditional mean and estimating model parameters simultaneously, ARCH-M and GARCH-M models provide a simplified approach to investigating the risk-return relationship.

6.4.3. Modelling the return generating process

Roll (1992) applies the ARCH/GARCH framework in a study of the time series behaviour of twenty-four national market indices[97] in response to global industry influences and exchange rate movements over the April 1988 to March 1991 period. The model employed to study market behaviour is initially estimated using the LS methodology and then as a GARCH specification. The use of a GARCH specification is motivated by the presence of the ARCH effect in the majority of indices in the sample and the presence of leptokurtosis in almost all indices. Roll (1992) argues that the presence of heteroscedasticity and leptokurtosis can lead to incorrect inferences as a result of biased and inconsistent standard errors in LS regression. The ARCH/GARCH framework is deemed to address these problems. Results indicate that the explanatory power of the GARCH(1,1) model is comparable to that of the LS model and more importantly, global industry influences and exchange rate movements continue to explain a significant amount of the variation in returns. Even though the GARCH(1,1) model does not show an improvement over the LS methodology in terms of explanatory power, Roll's (1992) application of the model suggests that the ARCH/GARCH framework may be applied as a more robust model estimation methodology. It also suggests that the ARCH/GARCH framework can be applied as a robustness and adequacy check on model specifications and results.

In a similar vein, Sadorsky and Henriques (2001) apply the ARCH/GARCH framework in their study of the return generating process underlying the Canadian paper and forest

[97] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Malaysia, Mexico, Netherlands, New Zealand, Norway, Singapore, South Africa, Spain, Sweden, Switzerland, the UK and US.

products industry over the January 1971 to January 1999 period. Using the LS methodology, monthly excess returns on the Canadian paper and forest products industry index are regressed onto excess market returns, an exchange rate factor, the growth rate in commodity prices and the term structure. Although, the model provides a relatively good fit by explaining more than half of the variation in excess returns, diagnostic tests revealed that ARCH effects are present in the residuals suggesting that the LS coefficient estimates are inefficient. Motivated by the turbulence of the sample period and the presence of ARCH effects in LS residuals, Sadorsky and Henriques (2001) re-estimate the model as a GARCH(1,1) model. Results suggest that market returns and commodity prices have a positive and statistically significant impact upon returns whereas changes in the exchange rate have a negative and statistically significant impact upon returns. Whereas results are consistent across the LS and GARCH(1,1) methodologies, diagnostic tests indicate that ARCH effects are no longer present in the residuals of the GARCH(1,1) model. This implies that the potential for misleading inferences is mitigated by the ARCH/GARCH framework and that the framework is appropriate for the modelling of returns on the Canadian paper and forest products industry index (Sadorsky & Henriques, 2001).

Similarly to Roll (1992), Sadorsky and Henriques (2001) demonstrate the applicability of the ARCH/GARCH framework as an alternative to the LS methodology in modelling the return generating process. As the ARCH/GARCH framework is able to model ARCH effects, the contention that the ARCH/GARCH methodology is a more appropriate estimation methodology for models of returns finds further support. Together, Roll's (1992) and Sadorsky and Henriques' (2001) studies suggest the ARCH/GARCH framework is an attractive and robust alternative econometric framework.

6.4.4. Modelling the return generating process and conditional variance

The ARCH/GARCH framework permits variation in *both* the mean and variance to be investigated simultaneously. Elyasiani and Mansur (1998) employ a GARCH-M model to investigate the impact of interest rates and interest rate volatility upon the returns and the conditional variance of three size based portfolios[98] of commercial banks listed on the NYSE between January 1970 and December 1992. The GARCH-M model is favoured as it permits more flexibility by discarding the restrictive assumptions of linearity, independence and

[98] Designated as the Money Center, Large and Regional Bank portfolios.

constant conditional variance, and also allows for the modelling of the interdependence of time-varying risk and returns. Reflecting the influence of the APT framework, Elyasiani and Mansur (1998: 544) refer to the proposed time series specification as a “two-factor APT model” with the interest rate changes and conditional variance as risk factors. In addition, lagged excess returns on the portfolios are incorporated into the mean equation. In the conditional variance equation, dummy factors are used to capture the impact of shifts in volatility arising from changes in the monetary policy regime over the sample period. The conditional interest rate volatility is also incorporated as an exogenous factor into the conditional variance equation. Results indicate that there is a *negative* and statistically significant trade-off between volatility and returns for all portfolios[99] and that the impact of monetary policy shifts upon volatility is statistically significant for two portfolios. Fluctuations in the interest rate have a statistically significant and negative impact upon the returns on two portfolios and interest rate volatility has a negative and statistically significant impact upon the volatility of two portfolios. Elyasiani and Mansur’s (1998) approach illustrates the application of the ARCH/GARCH framework in a simultaneous study of the return generating process underlying returns *and* the conditional variance process. Not only is the impact of quantitative factors upon volatility considered within the ARCH/GARCH framework, but so is the impact of qualitative factors in the form of monetary policy shifts.

Aga and Kocaman (2006) investigate the impact of changes in the inflation rate and industrial production upon the returns and the volatility of an index of Turkish stocks[100] using an EGARCH(1,1) model. The application of the EGARCH model is motivated by the GARCH model’s inability to account for the asymmetric relationship between returns and volatility. Lagged changes in industrial production and inflation are incorporated as explanatory factors in *both* the mean and the conditional variance equations. Results indicate that the relationship between returns and changes in inflation and industrial production is negative but not statistically significant. The same holds for conditional variance; coefficients on both factors are negative and statistically insignificant. This suggests that these two factors do not influence the returns *and* volatility of Turkish stocks. The EGARCH(1,1) conditional variance specification, however, indicates that the relationship between returns and volatility is asymmetric. Similarly to Elyasiani and Mansur (1998), Aga and Kocaman (2006) employ

[99] The negative sign on the coefficient of relative risk aversion is attributed to investors’ risk preferences (Elyasiani & Mansur, 1998).

[100] The sample consists of 20 stocks with the highest trading volume and full price sequences over each year for the January 1986 to November 2005 period.

the ARCH/GARCH framework to investigate the return generating process and conditional variance simultaneously.

6.4.5. Further applications

The ARCH/GARCH framework is flexible enough to permit the study of a range of financial phenomena associated with the behaviour of stock returns and volatility. McMillan and Ruiz (2009) employ the ARCH/GARCH framework to study the persistence and long memory properties of volatility in ten national stock market indices over the January 1990 to December 2005 period.[101] The persistence and the long memory property can be *easily* quantified within the framework; the sum of the ARCH and GARCH coefficients, $\alpha+\beta$, indicates whether shocks are persistent. The authors find that in four of the markets considered (Canada, Italy, Singapore and the US), the hypothesis that the parameters of a GARCH(1,1) model are jointly equal to one cannot be rejected suggesting that shocks to volatility never die out and therefore exhibit infinite persistence. For the remaining six markets, the hypothesis of ARCH and GARCH parameters jointly equalling one is rejected; although, their sum is over 0.98 indicating that volatility is characterized by long memory. McMillan and Ruiz (2009) suggest that the IGARCH model can further simplify this analysis and that the GARCH specification employed confirms the widely recognized long memory property of volatility. The authors' approach demonstrates how the ARCH/GARCH framework simplifies the analysis of the behaviour of volatility.

Kiymaz and Berument (2003) employ the ARCH/GARCH framework to investigate the day of the week effect in returns *and* volatility using return data for five national markets over the January 1988 to June 2002 period.[102] It is hypothesized that patterns in volatility reflect the arrival of both public and private information and that these arrivals are tied to increased volatility. The choice of the econometric framework is motivated by the inability of the LS methodology to capture heteroscedasticity and it is acknowledged that the ARCH/GARCH framework permits a simultaneous study of the day of the week effect in returns and volatility. Results suggest that there is a statistically significant day of the week effect in returns for three of the five markets (Japan, Canada and the United Kingdom), with returns being significantly lower on Monday relative to returns for other days. The day of the week

[101] Canada, France, Germany, Hong Kong, Italy, Japan, Singapore, Spain, the UK and US.
[102] The TSE-Composite (Canada), DAX (Germany), the Nikkei-225 (Japan), the NYSE-Composite (NYSE) and the FT-100 (UK).

effect in return volatility is far more pronounced; volatility differs across the days of the week for each of the five markets considered and this is confirmed by log-likelihood ratio tests.[103] Kiymaz and Berument (2003) conclude that the day of the week effect is present in both returns (although, not for all markets in the sample) and volatility (see section 5.2.3 & 5.3.1). Based upon a finding that volatility is highest on Fridays for two markets (Canada and the US), the public information release hypothesis is not refuted.

Bollerslev *et al*. (1992) state that in assessing abnormal returns in event studies, it is necessary to correctly estimate standard errors so to avoid misleading inferences. This is especially important in the presence of time-varying variance and the ARCH/GARCH framework provides a systematic approach to dealing with time-varying variance. Corhay and Rad (1996) show that accounting for ARCH effects in the single-factor (market) model affects parameter coefficient estimates, which results in different conclusions regarding the economic significance of disinvestitures.[104] The data sample used in the study consists of Dutch firms that have undertaken disinvestitures over the January 1989 to December 1993 period. The CBS General Index is used as a proxy for the market index. Abnormal returns are obtained from the single-factor model estimated using the LS methodology assuming a constant variance and then from an ARCH-corrected model based upon a GARCH(1,1) specification.[105] Corhay and Rad (1996) find that under the LS model, there are statistically significant cumulative abnormal returns around the event (disinvestiture) period. This however is not the case under the ARCH effect corrected single-factor model – cumulative abnormal returns are statistically insignificant. The observed contradiction is attributed to the inefficiency of the $\hat{\alpha}$ and $\hat{\beta}$ estimates arising from a failure to consider ARCH effects, which are present in the LS residuals. The authors go onto to state that even if the conditional variance is of no interest to the researcher, maximum likelihood should be used to obtain more efficient estimates of regression parameters. Corhay and Rad (1996) conclude that failing to adjust for ARCH effects may result in the wrong conclusion being reached as a result of LS inefficiency. The robustness and statistical adequacy of the ARCH/GARCH framework is demonstrated in this study.

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[103] Note that as with prior findings, the non-stationarity in variance is far more pronounced than the non-stationarity in the mean.

[104] Disinvestiture (alternatively divesture) refers to the disposal of assets by a firm.

[105] $A_{it} = R_{it} - \hat{\alpha} - \hat{\beta}R_{mt}$. Abnormal returns, A_{it}, are obtained by subtracting returns predicted by the model from actual returns, R_{it}. R_{mt} is the return on the market index whereas $\hat{\alpha}$ and $\hat{\beta}$ are the parameters of the single-factor model.

6.5. Limitations of ARCH and GARCH models

Although, the ARCH and GARCH framework appears to be a flexible, robust and widely applicable econometric framework for the study of stock return behaviour *and* volatility, it is not without limitations.[106]

Within the ARCH/GARCH framework, conditional variance is modelled as a "by-product" of the return generating process, which implies that the ARCH/GARCH framework is somewhat restrictive and less responsive to changes in volatility dynamics relative to other methodologies, such as the HIS model[107] class. Furthermore, this also implies that well-specified models of the return generating process are required to model or forecast volatility accurately (see Poon & Granger, 2005). Poon and Granger (2005) survey literature on the (forecast) accuracy of ARCH and GARCH models in relation to simpler volatility models in the form of HIS models, Stochastic Volatility (SV) models and the Implied Standard Deviation (ISD) model. A (narrow) majority of studies comparing HIS, and ARCH and GARCH models indicate that HIS models outperform ARCH and GARCH models. However, the ISD model decisively outperforms ARCH and GARCH models. These findings suggest that there are other frameworks that are more appropriate than the ARCH/GARCH framework. This is especially pertinent if the nature and properties of variance affect the estimation of return generating process specifications.

Nwogugu (2006) suggests that ARCH and GARCH models are inaccurate when an inappropriate assumption relating to the conditional distribution is made. Baillie and DeGennaro (1990) suggest that a failure to capture the leptokurtosis in stock returns may result in misleading inferences relating to the risk-return relationship posited by asset pricing theory (see section 3.2, 3.3 & 5.2.1). The authors state the ARCH/GARCH framework (by default) assumes a conditional normal distribution. However, this may be inappropriate as typically return data exhibits high levels of leptokurtosis. Using return data on the value-weighted CRSP index over the January 1970 to December 1987 period, GARCH-M specifications are estimated, first assuming a conditional normal distribution and then a

[106] For a comprehensive critique of ARCH/GARCH models and other volatility models, see Nwogugu (2006).

[107] Historical Volatility models do not require return information and therefore, are less restrictive and more responsive to changing volatility dynamics. This class of models includes the Random Walk, Moving Average, Exponentially Weighted Moving Average and AR models (Poon & Granger, 2005).

Student's *t* distribution. Results indicate that for two models[108] where conditional normality is assumed, the conditional variance and standard deviation parameters are statistically significant. This result does not hold under the assumption of a conditional Student's *t* distribution as implied by statistically insignificant coefficients of relative risk aversion in the mean equation. As excess sample kurtosis is above that suggested by conditional normality, Baillie and DeGennaro (1990) state that the conditional normality assumption is inappropriate. Furthermore, Nwogugu (2006) state that although a specific distribution or mixtures of distributions may be assumed (as deemed appropriate), these will provide only a rough approximation. This, together with Baillie and DeGennaro's (1990) findings, suggests that the ARCH/GARCH framework is susceptible to producing spurious results stemming from its dependence upon a *correctly* specified distribution.

The estimation and fitting of ARCH and GARCH models is further complicated by the need to impose the number of ARCH and GARCH terms, which is often done in an *ad hoc* manner (Xiao & Aydemir, 2007). For example, Bollerslev (1986) in fitting an ARCH model to the growth rate of the GNP deflator settles for an ARCH (8) specification while acknowledging this is a rather *ad hoc* structure motivated by the long memory property of the conditional variance. Akgiray (1989) is faced with the same problem when fitting an ARCH model to returns on the CRSP value-weighted index. This limitation extends to the GARCH specification with various combinations of *p* and *q* being tested by Akgiray (1989) until there are no improvements in likelihood-ratio tests measuring the goodness-of-fit.[109] This suggests that specifying the order of parameters in ARCH and GARCH models is not straightforward and the same can be said about the choice of the appropriate conditional distribution (Tsay, 2002). Such limitations make model estimation within the ARCH/GARCH framework more complicated relative to estimation by the LS methodology.

Stocks prices are assumed to reflect the expected profitability of a firm, which changes whenever there is an arrival of new information and the market is uncertain about the value of a given stock. The process of price discovery leads to upward and downward revisions of the price and this results in volatility clustering as market participants are uncertain about the true

[108] Four models are first estimated with assumed conditional normality. A conditional Student's *t* distribution is assumed for the next four models. The GARCH-M specifications incorporate different transformations of the conditional variance in the mean equation.
[109] Akgiray (1989) fits models with p = 1, 2, 3, 4 and 5 and q = 1, 2 and 3 to determine the best fitting model. This yields 15 possible GARCH specifications. The best fitting specification is found to be the GARCH (1,1) model.

value of a given security (Engle, 2004; Engle *et al*., 2008). Engle *et al*. (2008) suggest that ARCH and GARCH models describe the evolution of this uncertainty by measuring the intensity of the news process. According to Nwogugu (2006), this suggests that the major causes of volatility are arrivals of new information and trading intensity. However, this assumption ignores a number of important factors such as information processing capabilities, different values attached to information by participants, cognition, perception, the cost and availability of capital, and traders' willingness to accept losses. If this is the case, then ARCH and GARCH models only describe volatility partially as the above factors are not taken into consideration. Furthermore, this implies that the conditional variance equation *needs* to incorporate exogenous factors to provide an accurate description of volatility. In light of this, the ARCH/GARCH framework is a mere simplification of the problem. This simplification is further compounded by the assumption that the coefficients on ARCH and GARCH parameters are sufficient for modelling return volatility. These coefficients are derived from regressions and therefore, highly sensitive to the selection period and dataset under consideration (Nwogugu, 2006). These arguments suggest that the ARCH/GARCH framework is based upon a number of assumptions that are either incorrect or oversimplify the nature of volatility.

Finally, a number of limitations relating to conventional ARCH and GARCH models must be noted. The standard ARCH model may at times require a great number of parameters to describe the volatility process of stock returns (Tsay, 2002). For example, Tsay (2002) reports that an ARCH(9) models is required to describe the volatility of returns on the S&P 500 Index. If the ARCH model does indeed require such a long lag structure, then it may be considered as unparsimonious and computationally burdensome. Furthermore, the ARCH model is restrictive; the ARCH parameter in the ARCH model must lie within the interval $[0,\frac{1}{3}]$ for the series to possess a finite fourth moment. This constraint becomes problematic for ARCH models of high order. Both the ARCH and GARCH models share a common drawback in that under both models, the conditional variance responds equally to positive and negative shocks (Tsay, 2002). The failure of these models to capture asymmetry is the motivation behind a number of asymmetric models, notably those of Nelson (1991) and Zakoian (1994). Moreover, a large number of other models addresses these and other limitations of the conventional ARCH and GARCH models; Bollerslev (2008) in his glossary of ARCH and GARCH models lists over 100 models. Although, this multitude of models demonstrates the flexibility of the ARCH/GARCH framework, it can also be seen as a limitation. Selecting the optimal model that best fits the data out of all the available alternatives introduces a degree of uncertainty, is likely to be time consuming (given the large number of variants) and also computationally burdensome. This is likely to force the practitioner to settle for an abstraction in the form of an ARCH or GARCH model or an immediate extension. Therefore, while the multitude of ARCH and GARCH models can accommodate the various characteristics of returns and volatility discussed in Chapter 5, the application of these models is complicated and potentially restricted in practice.

6.6. Conclusion

Engle (2004) attributes the popularity of ARCH/GARCH models to their wide range of applications in finance. ARCH/GARCH models are designed to capture unpredictability, excess kurtosis, fat-tails and volatility clustering observed in stock returns (section 5.2.1, 5.2.2 & 5.3.1). A wide number of generalizations capture different aspects of returns and volatility, such as asymmetry (section 5.3.3 & 6.3.5) and long memory (section 5.3.2 & 6.3.2), making the ARCH/GARCH framework an attractive alternative to the LS econometric framework. The suitability and flexibility of the framework is evident from the wide range of applications in the literature. These include forecasting volatility (section 6.4.1), modelling the risk-return relationship (section 6.4.2), investigating the conditional variance and determinants thereof (section 6.4.4), investigating seasonalities in returns and volatility, and application in event studies (section 6.4.5). More importantly, this study seeks to investigate the return generating process of South African stock returns within the APT framework (see Chapter 2 & 3; section 6.4.3 & 6.4.4). The ARCH/GARCH framework provides a robust econometric framework within which the return generating process of South African stock returns can be investigated.

As with any econometric framework, the ARCH/GARCH framework is subject to limitations; ARCH and GARCH models are restricted by their dependence upon a mean specification, may underperform in relation to simple volatility models and may be computationally burdensome (section 6.5). Nevertheless, it can be argued that the ARCH/GARCH framework finds redemption in its flexibility, robustness and applicability to a multitude of financial problems. It is hoped that these favourable aspects of the framework will contribute meaningfully to the estimation of models of the return generating process of South African stock returns in light of the characteristics of returns and volatility noted in

Chapter 5. It is further hypothesized that the application of the ARCH/GARCH framework constitutes a statistically more robust approach to investigating the return generating process relative to studies that rely upon the LS methodology.

However, it remains to be established whether South African stock returns exhibit behaviour that requires the application of the ARCH/GARCH framework for model estimation. This is addressed in Chapter 7 where an analysis of the statistical properties of South African stock returns is conducted and the approach to modelling the return generating process is outlined.

7. PRELIMINARY ANALYSIS AND METHODOLOGY

7.1. Data

The data sample consists of monthly returns on the FTSE/JSE All-Share Index (henceforth JSE All-Share Index) and the FTSE/JSE Africa economic group and industrial sector indices over the July 1995 to March 2011 period. The data is sourced from the INET Bridge Database and as in Van Rensburg (1996), month end return data is used. As in Berry *et al*. (1988), total returns - returns adjusted for dividend payments - are used. According to the Ground Rules for the Management of the FTSE/JSE Africa Index Series document, the indices are designed to represent the performance of South African companies by measuring the performance of major economic groupings and industrial sectors of the South African stock market. The economic group and industrial sector indices considered in the study are listed in Table 7.1.

Table 7.1: Economic groups and industrial sector FTSE/JSE All-Africa Series Indices

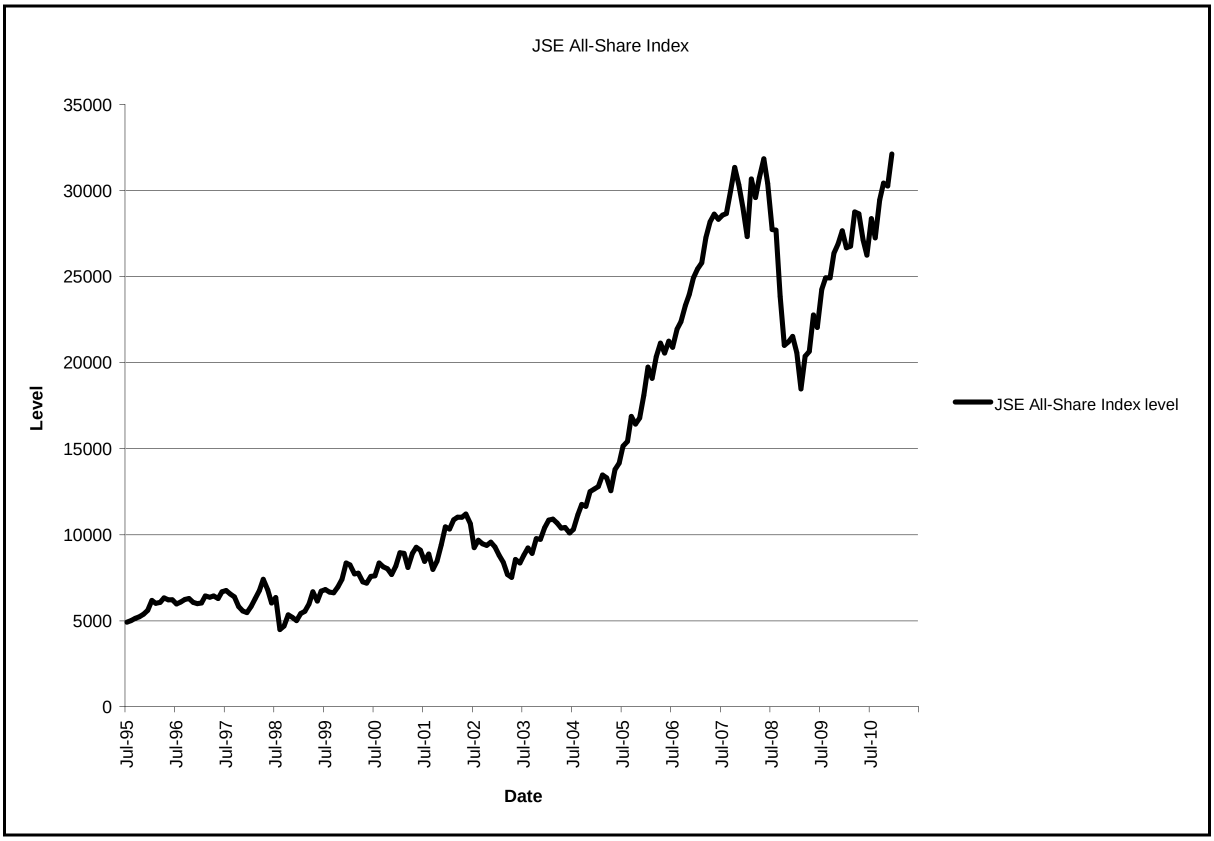
Economic Group Index	Industrial Sector Index
1.Oil & Gas	1.1. Oil & Gas Producers
2.Basic Materials	2.1. Chemicals
	2.2. Forestry & Paper
	2.3. Industrial Metals
	2.4. Mining
3.Industrials	3.1. Construction & Materials
	3.2. General Industrials
	3.3. Electronic & Electrical Equipment
	3.4. Industrial Engineering
	3.5. Industrial Transport
	3.6. Support Services
4.Consumer Goods	4.1. Automobiles & Parts
	4.2. Beverages
	4.3. Food Producers
5.Health Care	5.1. Health Care Equipment &Services
	5.2. Pharmaceuticals & Biotechnology
6.Consumer Services	6.1. Food & Drug Retailers
	6.2. General Retailers
	6.3. Media
	6.4. Travel & Leisure
7. Telecommunication	7.1. Fixed Line Telecommunications
8. Financials	8.1. Banks
	8.2. Non-life Insurance
	8.3. Life Insurance
	8.4. General Financial
	8.5. Equity Investment Instruments
9. Technology	9.1. Software & Computer Services

Notes:
1. Economic group and industrial sector classification based upon the FTSE/JSE Global Classification system.

Source: Compiled by author

The FTSE/JSE Industry Indices (economic group indices) incorporate all constituents of the JSE All-Share Index that belong to an economic group whereas the FTSE/JSE Sector Indices (industrial sector indices) consist of all constituents of the JSE All-Share Index that belong to a specific industrial sector. The JSE All-Share Index is representative of 99 percent of the full market capital value of ordinary securities listed on the main board of the JSE and therefore, the economic group and industrial sector indices are representative of economic groupings and industrial sectors constituting the overall South African stock market.

As the sample spans the period from July 1995 to March 2011, this study traces the growth of the economic groups and industrial sectors constituting the South African stock market over this period. A plot of monthly JSE All-Share Index levels over this period is shown in Figure 7.1:



Source: Compiled by author

Figure 7.1: JSE All-Share Index levels: July 1995 - March 2011

The sample period coincides with a number of significant events. These are the Asian Financial Crisis (1997-1998), the bursting of the dot-com bubble in 2000, an unprecedented terrorist attack on the twin towers of the World Trade Centre in New York in 2001, the sub-prime mortgage crisis of 2008 and its aftermath, growing trade liberalization and the emergence and consolidation of the economic clout of the “Asian Tigers” in the form of India and China.

Formally, the returns used are continuously compounded total returns - the natural logarithm of monthly total returns over the sample period (Tsay, 2002):

$$r_{it} = \ln S_{it} - \ln S_{it-1} \qquad (7.1)$$

where r_{it} is the total return on index *i* at time *t*, and S_{it} is the level of index *i* at time *t*. Excess total returns (henceforth referred to as returns), R_{it},[110] are obtained by subtracting the risk-free rate, as measured by the yield on the R157 government bond, from the logarithm of total returns in equation (7.1).[111]

Having defined returns, Chapter 7 proceeds by investigating the statistical properties of South African stock returns and outlining the approach used in applying the APT framework to investigate the return generating process. To determine whether the ARCH/GARCH framework is appropriate for the modelling of South African stock returns, the statistical properties of South African stock returns are first considered (section 7.2). The set of candidate risk factors is then presented in section 7.3.1. Having done so, the approach to deriving innovations in risk factors as required by the APT framework (see section 3.1.4: 52) and selecting risk factors is outlined (section 7.3.2 & 7.3.3). The modelling methodology is discussed (section 7.4) prior to its application in Chapter 8. A summary is provided in the conclusion (section 7.5).

7.2. Preliminary analysis of statistical properties

Preliminary analysis is conducted on each return series to investigate the properties of the first two moments of the distribution *and* to determine the appropriateness of the proposed ARCH/GARCH econometric framework (see Chapter 5 & 6). The mean, kurtosis, skewness and standard deviation are reported for each series in Table 7.2. To formally test whether each series conforms to the normality assumption (section 5.2.1), the Jarque-Bera (JB) test which assumes that a normal distribution is characterized by a skewness (S) coefficient of zero and a kurtosis (K) coefficient of three is applied to test the joint hypothesis that S = 0

[110] Preliminary analysis of the statistical properties of the return series and volatility suggests that using excess returns and returns does not alter results.
[111] Nel (2011) states that although a number of proxies can be used for the risk-free rate, the most widely used proxy in academia is the R157 government bond. This finding is based upon a survey of twelve South African universities. Whereas the appropriateness of the risk-free rate proxy can be debated, it is beyond the scope of this study.

and K = 3 (Cryer & Chan, 2008). As outliers are likely to bias normality tests towards a rejection of the normality assumption, box plots are used to identify outliers and far (extreme) outliers are excluded (Hodge & Austin, 2004; Poon, 2005; Agung, 2009). Near outliers are not excluded as these may be the result of volatility clustering and not unusual events (Galpin, 2009). A rejection of the null hypothesis implies that a return series is not normally distributed (Gujarati, 2003).

The results in Table 7.2 indicate widespread departures from normality in the form of skewness and excess kurtosis. Although, returns on the JSE All-Share Index appear to be normally distributed (surprisingly),[112] almost all economic group and industrial sector return series show a level of kurtosis that is in excess of that expected under a normal distribution. This suggests that return distributions are characterized by peakedness and fat-tails. For the economic group indices, the average level of kurtosis is 3.627. Only for the consumer services economic group is the level of kurtosis under 3. The same holds for the industrial sector indices. The average level of kurtosis is 3.765, although there are two exceptions; namely, the support services and food producers industrial sectors for which kurtosis coefficients are under 3. While excess kurtosis is not of a large magnitude, it is evident that returns on series constituting the sample are characterized by leptokurtosis.

[112] When outliers are not excluded, returns on the JSE All-Share Index are highly skewed and leptokurtic.

Table 7.2: Distributional properties

	Obs.	Mean	Std Dev	Skewness	Kurtosis	JB Test Statistic
JSE All-Share Index	188	-0.003	0.024	-0.259	2.881	2.210
Economic Group Index						
1. Oil & Gas	189	-0.003	0.035	-0.115	3.374	1.513
2. Basic Materials	188	-0.005	0.033	-0.604	3.894	17.706***
3. Industrials	188	-0.003	0.026	-0.427	3.421	7.090**
4. Consumer Goods	188	-0.002	0.029	-0.420	4.113	15.234***
5. Health Care	188	-0.003	0.027	-0.314	3.320	3.881
6. Consumer Services	188	-0.003	0.028	-0.422	2.962	5.604**
7. Telecommunication	188	-0.001	0.040	0.021	4.085	9.241***
8. Financials	188	-0.003	0.025	0.097	3.323	1.113
9. Technology	187	-0.004	0.046	-0.614	4.147	21.981***
Industrial Sector Index						
1.1: Oil & Gas Producers	189	-0.002	0.042	-0.167	3.773	5.591*
2.1: Chemicals	189	-0.004	0.029	-0.416	3.722	9.564***
2.2. Forestry & Paper	187	-0.008	0.046	-0.043	3.455	1.668
2.3: Industrial Metals	188	-0.001	0.049	-0.133	4.338	14.581***
2.4: Mining	189	-0.003	0.036	-0.216	3.608	4.377
3.1: Const & Materials	189	-0.006	0.037	-0.697	4.065	24.247***
3.2: General Industrials	188	-0.002	0.027	-0.378	3.438	5.981*
3.3: E & E Equipment	188	-0.004	0.031	-0.634	4.300	25.843***
3.4: Industrial Engineering	186	-0.003	0.030	-0.441	3.817	11.197***
3.5: Industrial Transport	188	-0.006	0.031	-0.193	3.403	2.446
3.6: Support Services	188	-0.005	0.027	-0.412	2.949	5.335*
4.1: Automobiles & Parts	188	-0.010	0.042	-0.273	3.688	6.053**
4.2: Beverages	188	-0.004	0.030	-0.290	3.666	6.104**
4.3: Food Producers	187	-0.003	0.023	0.036	2.991	0.041
5.1: Health Care E & S.	188	-0.001	0.034	-0.027	3.664	3.479
5.2: Pharma & Biotech.	189	-0.002	0.037	-0.026	4.561	19.223***
6.1: Food & Drug Retailers	189	-0.001	0.034	-0.488	4.779	32.409***
6.2: General Retailers	188	-0.003	0.034	-0.373	3.194	4.654*
6.3: Media	187	-0.001	0.043	-0.741	4.406	32.509***
6.4. Travel & Leisure	188	-0.006	0.030	-0.517	4.356	22.798***
7.1: Fixed Line Telecom.	188	-0.004	0.046	-0.132	4.194	11.720***
8.1: Banks	188	-0.002	0.031	0.262	3.207	2.492
8.2: Non-life Insurance	188	-0.002	0.028	-0.202	3.580	3.908
8.3: Life Insurance	188	-0.005	0.029	-0.171	3.332	1.777
8.4: General Financial	188	-0.002	0.032	-0.132	3.493	2.451
8.5: Equity Investment Inst.	188	-0.004	0.025	0.118	3.646	3.703
9.1: Soft & Comp Services	187	-0.003	0.049	-0.585	4.039	19.078***

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. Obs. refers to the number of observations. Numbers below 189 indicate that outliers have been omitted.

Source: Compiled by author

Xiao and Aydemir (2007), and Engle and Patton (2007) state that it is common to find that levels of kurtosis in financial return series are above 3. These findings attest to that. The presence of widespread leptokurtosis suggests that the variance is non-stationary (Akgiray, 1989). Return distributions are also asymmetric; the average level of skewness for the economic group and industrial sector return series is -0.311 and -0.260 respectively, suggesting that negatively skewed distributions are more prevalent than positively skewed distributions. Nevertheless, isolated instances of positive skewness are observed for returns on the telecommunication and financials economic group indices and for the food producers, banks and equity investment instruments industrial sector indices.

In contrast to the findings of Simkowitz and Beedles (1980:10) who state that “securities display a habitual tendency to positive skewness,” returns on South African economic group and industrial sector indices display a habitual tendency towards negative skewness. Based upon the JB test, the null hypothesis of normally distributed returns is rejected for six out of the nine economic group indices and for seventeen out of the twenty-seven industrial sector indices. Departures from normality in the form of excess kurtosis, skewness or both are also observed for series where the null hypothesis is not rejected.[113]

To investigate the assumption of (statistical) independence (section 5.2.2), the approach of Fama (1965) and Campbell *et al*. (1997) is adopted in the form of the serial correlation model:

$$\rho_{\tau} = \frac{\mathrm{cov}(R_{it}, R_{it-\tau})}{\mathrm{var}(R_{it})} \tag{7.2}$$

where ρ_{τ} is the serial correlation coefficient, τ is the lag order and R_{it} is the return on series *i* at time *t*. Serial correlation coefficients not only provide insight into whether the assumption of independence holds, but also reveal the magnitude of dependence (Fama, 1965). The assumption of independence is further investigated using Ljung-Box *Q*-statistics (henceforth *Q*-statistics). Unlike serial correlation coefficients which indicate the level of serial correlation at individual lags, the *Q*-statistic indicates whether serial correlation coefficients up to a certain order are *jointly* equal to zero (Gujarati, 2003). The *Q*-statistic is denoted by:

$$LB = n(n+2)\sum_{k=1}^{m}\left(\frac{p_{\tau}}{n-k}\right) \tag{7.3}$$

[113] It must however be emphasized that departures from normality are far more pronounced and are understated due to the exclusion of outliers in Table 7.2. For a brief discussion of how outliers affect measures of kurtosis and skewness, see Poon (2005).

where n is the sample size and m is the lag length. Campbell *et al*. (1997) apply the Q-statistic[114] with five and ten serial correlation orders to test whether daily, weekly and monthly CRSP stock return indices exhibit statistically significant serial correlation and Gujarati (2003) suggests that the Q-statistic tests whether a time series is white noise. White noise in the context of returns implies independence as in Akgiray (1989). On the basis of a statistically significant Q-statistic, Akgiray (1989) rejects the null hypothesis of strict white noise for returns on the CRSP value-weighted index and concludes that this return series violates the assumption of independence.

Using the Q-statistic to test whether serial coefficients are jointly equal to zero complements the serial correlation model; while individual correlation coefficients may be statistically significant, jointly they may be equal to zero suggesting a negligible level of dependence (see Fama, 1965). An analysis of the serial correlation structure of stock returns also provides preliminary insight into the validity of the assumption of identically distributed returns; if a time series is white noise, then the series is mostly likely stationary (Gujarati, 2003). However, the more formal Augmented Dickey-Fuller (ADF) unit root test is employed to test the stationarity of each series. The ADF test is based upon the following specification:

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-\tau} + \varepsilon_t \tag{7.4}$$

where ΔY_t is the dependent factor, t is the time trend and τ represents the lag order. Lagged differenced factors, $\sum_{i=1}^{m} \alpha_i \Delta Y_{t-\tau}$, are incorporated into the regression model to ensure that the residuals, ε_t, are serially uncorrelated. The null hypothesis of $\delta = 0$ implies that the series has a unit root and is therefore, non-stationary. A rejection of the null hypothesis implies that the series is stationary (Gujarati, 2003). While returns are most likely to be stationary in the mean, results of the ADF test for returns are reported for comprehensiveness (see Henriques, 2001; Sadorsky & Henriques, 2001). Individual serial correlation coefficients for the first five orders are reported for each series together with the Q-statistics for the first five and ten orders.

[114] Campbell *et al*. (1997) apply the Box-Pierce Q-statistic (see Box & Pierce, 1970). The Ljung-Box Q-statistic is applied in this study as it is considered to be statistically more powerful (Gujarati, 2003).

Table 7.3: Serial correlation structure and ADF test statistics

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	$Q(5)$	$Q(10)$	ADF Test
JSE All-Share Index	0.007	0.020	0.116	-0.049	-0.109	5.522	9.538	-13.541***
Economic Group Index								
1. Oil & Gas	-0.056	0.035	0.185**	-0.037	-0.085	9.154	14.707	-14.424***
2. Basic Materials	0.053	0.031	0.113	0.006	-0.038	3.500	12.543	-12.933***
3. Industrials	0.089	0.033	-0.008	-0.122	-0.011	4.700	9.742	-12.464***
4. Consumer Goods	-0.078	0.077	0.159**	-0.117	-0.067	10.778*	21.595**	-14.744**
5. Health Care	0.101	-0.012	0.028	-0.041	0.075	3.550	10.916	-12.323***
6. Consumer Services	0.211**	0.071	-0.031	-0.098	0.000	11.591**	17.681*	-11.008***
7. Telecommunication	0.096	-0.043	0.167**	-0.042	-0.020	7.948	12.510	-12.340***
8. Financials	0.014	-0.084	0.019	0.008	-0.058	2.125	8.5282	-13.453***
9. Technology	0.152**	0.026	0.078	0.008	0.033	5.971	14.724	-11.706***
Industrial Sector Index								
1.1: Oil & Gas Producers	-0.098	0.044	0.174**	-0.047	-0.005	8.521	12.887	-15.077***
2.1: Chemicals	0.012	-0.041	0.053	0.042	-0.034	1.455	3.158	-13.483***
2.2. Forestry & Paper	0.003	-0.142**	0.143**	0.044	-0.047	8.700	16.332*	-13.597***
2.3: Industrial Metals	0.027	0.109	0.072	0.113	0.055	6.554	10.321	-13.274***
2.4: Mining	-0.006	0.060	0.126	0.005	-0.103	5.880	10.672	-13.714***
3.1: Const & Materials	0.199**	0.074	0.025	0.015	0.087	10.303*	14.617	-11.134***
3.2: General Industrials	0.023	-0.008	0.010	-0.154**	-0.019	4.819	12.755	-13.331***
3.3: E & E Equipment	0.178**	0.044	0.020	-0.138**	-0.103	12.336**	24.804**	-11.397***
3.4: Industrial Engineering	0.358**	0.199**	0.139**	0.200**	0.223**	53.562***	61.264**	-9.3858***
3.5: Industrial Transport	-0.022	0.088	-0.057	0.000	0.050	2.699	6.759	-13.946***
3.6: Support Services	0.129	0.025	-0.088	-0.075	-0.034	6.134	15.420	-11.956***
4.1: Automobiles & Parts	0.181**	-0.026	-0.044	-0.080	0.197**	15.643***	29.570***	-11.293***
4.2: Beverages	0.018	-0.063	0.087	-0.153**	0.033	7.088	17.519*	-13.396***
4.3: Food Producers	0.170**	-0.015	0.080	-0.016	-0.059	7.540	11.099	-11.480***
5.1: Health Care E & S	0.153**	0.008	0.072	0.079	0.106	8.964	20.796**	-11.689***
5.2: Pharma & Biotech	-0.044	0.017	0.047	0.014	0.013	0.942	2.350	-14.245***
6.1: Food & Drug Retailers	-0.075	-0.026	-0.029	-0.031	-0.096	3.371	10.742	-14.725***
6.2: General Retailers	0.261**	0.068	-0.019	-0.078	0.024	15.386***	18.816**	-10.426***
6.3: Media	0.154**	-0.041	-0.003	-0.069	0.073	6.823	11.164	-11.652***
6.4. Travel & Leisure	0.166**	0.002	0.016	0.006	0.059	5.979	7.734	-12.453***
7.1: Fixed Line Telecom	0.095	-0.027	0.108	-0.044	0.011	4.565	7.134	-12.439***
8.1: Banks	-0.036	-0.083	0.003	0.014	-0.100	3.584	9.479	-14.132***
8.2: Non-life Insurance	0.070	-0.076	-0.052	-0.046	-0.003	3.012	5.247	-12.720***
8.3: Life Insurance	0.050	-0.051	-0.011	0.007	0.003	1.018	3.170	-12.981***
8.4: General Financial	0.057	-0.071	0.038	0.030	0.062	2.806	10.218	-12.924***
8.5: Equity Investment Inst	-0.021	0.033	0.051	-0.070	-0.008	1.753	12.351	-13.926***
9.1: Soft & Comp Services	0.164**	0.043	0.076	0.011	0.034	6.896	17.071*	-11.559***

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. The ADF test applied here assumes a strict random walk process. Lag selection is based upon the SIC. The results of the ADF test are validated by the Phillips-Perron (PP) test as in Sadorsky and Henriques (2001). The PP supports the conclusions of the ADF test for all return series. See Table A1.1 in Appendix 1.
3. Outliers *not* excluded.

Source: Compiled by author

Results in Table 7.3 indicate that returns on the JSE All-Share Index are uncorrelated and the Q-statistics for the first five and ten orders are statistically insignificant suggesting that the assumption of independence holds for the JSE All-Share Index return series.[115] For the economic group and industrial sector indices, two and ten serial correlation coefficients are

[115] This that does not mean that serial correlation is not present in non-linear transformations of returns.

statistically significant at the first order respectively. Although, isolated instances of statistically significant serial correlation are observed at higher orders, most are of a small magnitude. The largest higher order statistically significant serial correlation coefficients for the economic group and industrial sector indices are observed for returns on the oil and gas economic group index at 0.185 (ρ_3) and for the industrial engineering industrial sector index at 0.233 (ρ_5). The magnitude of correlation at these higher orders is comparable to that found in Poon and Taylor (1991) who consider correlation of a similar magnitude as unimportant. The null hypothesis that the first five and ten serial correlation coefficients are jointly equal to zero is rejected in only two instances for the economic group return series. The null hypothesis is rejected in five and eight instances respectively for the industrial sector return series. In instances where *Q*-statistics are statistically significant, an analysis of serial correlation coefficients suggests that the respective *Q*-statistics may be biased upwards by large individual serial correlation coefficients.[116] The only notable exception is the industrial engineering industrial sector where the first five and the first ten serial correlation coefficients are statistically significant individually and jointly. A potential explanation for the high levels of serial correlation observed in this series and in isolated instances in other series is the presence of a common factor (Akgiray, 1989; Poon & Taylor, 1991). These results suggest that while *overall* the assumption of independence holds, violations of this assumption occur in isolated instances. Finally, results of the ADF test suggest that the presence of a unit root may be rejected for returns on the JSE All-Share Index, the economic group and industrial sector indices. This implies that the return series constituting the sample are stationary *as expected.*

If serial correlation coefficients are high and statistically significant for an extended number of orders, then the series is non-stationary. The source of non-stationarity can originate from non-stationarity in the mean or the variance or both (section 5.2.3 & 5.3.1; Gujarati, 2003). In the context of stock returns, a lack of serial correlation in *linear* returns implies that returns are stationary in the mean. This appears to be the case given the results in Table 7.3 and this is further supported by the rejection of the null hypothesis of a unit root for all series. However, a lack of serial correlation in linear returns provides little insight into the

[116] For example, although the null hypothesis of joint statistical significance of the first ten serial correlation coefficients is rejected for the health care equipment and services industrial sector, the largest statistically *significant* serial correlation coefficients are observed at the first and ninth orders (unreported) respectively. Although, eight out of the ten serial correlation coefficients are individually statistically insignificant, the null hypothesis is rejected on the basis of two statistically significant serial correlation coefficients.

stationarity of the variance. An investigation of the stationarity of the variance translates into a simultaneous investigation of ARCH effects in returns (see section 5.3.1). A model free approach is to test whether squared returns - a proxy for volatility - are serially correlated using the *Q*-statistic. Statistically significant *Q*-statistics indicate that the ARCH effect is present in the return series (Poon, 2005; Cryer & Chan, 2008). Cryer and Chan (2008) apply the *Q*-statistic to squared returns for College Retirement Equity Funds (CREF) and find that test statistics for an extended number of orders are statistically significant suggesting that the ARCH effect is present in the data. Engle (2001) also makes use of this approach for returns on a composite portfolio consisting of NASDAQ, DJIA stocks and bonds. The null hypothesis of "no ARCH effects" is rejected on the basis of statistically significant serial correlation and the *Q*-statistic. As in Engle (2001), a *Q*-statistic for the first fifteen serial correlation coefficients of squared returns is reported for each series in the sample. The presence of ARCH effects implies that the ARCH/GARCH framework is appropriate for modelling and analyzing the return generating process of South African stock returns (see section 6.4.3; Elyasiani & Mansur, 1998). Another test applied to determine whether the ARCH/GARCH framework is appropriate is the ARCH Lagrange Multiplier (LM) test. In this test, squared residuals from a LS regression are regressed on a constant and lagged squared residual terms (Engle, 1982). The null hypothesis assumes that:

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = ... = \alpha_p = 0 \tag{7.5}$$

whereas the alternative hypothesis postulates that:

$$H_0 : \alpha_1 \neq \alpha_2 \neq \alpha_3 \neq ... \neq \alpha_p \neq 0 \tag{7.6}$$

where the α s are coefficients on the lagged squared residual terms and *p* denotes the order of the lag. The null hypothesis in equation (7.5) assumes that the coefficients on the squared residual terms are jointly equal to zero, implying that there are no ARCH effects in the residuals. If this is not the case, as in equation (7.6), residual terms differ across time suggesting time-varying variance. The approach undertaken in this study is to determine whether the residual terms are conditionally heteroscedastic. If residuals are conditionally heteroscedastic, then it can be argued that the residuals reflect volatility clustering and time-varying variance in returns. The conventional approach is to use an AR(1) model of returns to generate residual terms for testing (see Akgiray, 1989):

$$R_{it} = \alpha + b_1 R_{it-1} + \varepsilon_{it} \tag{7.7}$$

where R_{it} is the return on index *i* and time *t* and R_{it-1} is the autoregressive term. Tests are conducted to determine whether ARCH(1), (5) and (10) effects are present in the residuals. Sadorsky and Henriques (2001) rely upon this approach to determine whether the LS or ARCH/GARCH methodology should be applied when estimating a multifactor model of returns. A rejection of the null hypothesis implies that ARCH effects are present in the residual terms - ARCH errors are significant.

Another feature of volatility aside from the presence of time-variation and ARCH effects is the asymmetric relationship between returns and volatility - the leverage effect - which has been cited as an explanation for negatively skewed return distributions (Black, 1976; Bouchaud *et al*., 2001). While the causality of the effect is questioned, the presence of leverage effects may be established by considering the correlation between squared returns representative of volatility and prior returns (section 5.3.3: 113; Cont, 2001):

$$L(\tau) = corr(R_{it}^2, R_{it-\tau}) \tag{7.8}$$

where squared returns are denoted by R_{it}^2, returns are denoted by $R_{it-\tau}$ and τ is the lag order. If the correlation function $L(\tau)$ starts from a negative value and decays to zero, negative (positive) returns result in increases (decreases) in volatility.

The results in Table 7.4 show that while returns on the JSE All-Share Index are free from non-linear dependence, suggestive of time-varying variance and ARCH effects, returns on most economic group and industrial sector indices are characterized by some form of non-linear dependence in returns or ARCH effects in the residuals of the AR(1) model. Statistically significant non-linear dependence in returns *or* statistically significant ARCH effects in the residuals are detected in seven of the nine economic groups and in sixteen of the twenty-seven industrial sectors.

Table 7.4: ARCH effects and the leverage effect

	$Q^2(15)$	ARCH(1)	ARCH(5)	ARCH(10)	Lev Effect
JSE All-Share Index	8.646	0.039	0.900	0.612	-0.029
Economic Group Index					
1. Oil & Gas	33.625***	3.680*	0.766	1.853	0.064
2. Basic Materials	54.229***	3.069*	2.500**	3.973***	-0.007
3. Industrials	25.584**	0.012	3.689***	2.267**	-0.060
4. Consumer Goods	42.717***	0.067	4.858***	3.767***	-0.074
5. Health Care	30.292***	0.275	7.427***	4.137***	-0.111
6. Consumer Services	7.433	0.137	1.736	0.900	-0.095
7. Telecommunication	53.008***	15.229***	6.182***	3.076***	-0.122*
8. Financials	5.598	0.047	0.886	0.502	0.000
9. Technology	32.174***	0.337	1.927*	1.040	-0.138*
Industrial Sector Index					
1.1: Oil & Gas Producers	53.459***	5.432**	2.187*	2.758***	0.108
2.1: Chemicals	65.286***	5.613**	3.668***	2.792***	-0.129*
2.2. Forestry & Paper	22.192	0.107	1.308	1.681*	0.045
2.3: Industrial Metals	5.726	0.588	0.465	0.387	-0.073
2.4: Mining	31.031***	3.576*	0.853	2.139**	-0.046
3.1: Const & Materials	28.033**	0.057	0.472	1.613	-0.143**
3.2: General Industrials	20.143	0.001	2.866**	1.735*	-0.060
3.3: E & E Equipment	54.264***	0.042	11.377***	6.359***	-0.068
3.4: Industrial Engineering	30.419**	0.556	1.537	1.209	-0.278***
3.5: Industrial Transport	10.422	0.694	1.241	0.718	-0.053
3.6: Support Services	9.151	0.078	0.769	0.507	-0.066
4.1: Automobiles & Parts	18.504	0.175	0.774	0.744	-0.198***
4.2: Beverages	18.771	0.078	3.464***	1.879*	-0.096
4.3: Food Producers	60.667***	1.173	11.620***	6.160***	-0.182***
5.1: Health Care E & S	29.473***	1.398	1.258	2.023**	-0.218***
5.2: Pharma & Biotech	14.364	0.450	0.989	0.969	0.006
6.1: Food & Drug Retailers	72.928***	0.772	8.545***	4.946***	-0.102
6.2: General Retailers	17.015	0.361	2.293**	1.406	-0.147**
6.3: Media	20.377	1.430	1.297	1.574	-0.143**
6.4. Travel & Leisure	18.834	0.339	2.290**	1.853*	-0.102
7.1: Fixed Line Telecom	43.502***	15.357***	4.469***	2.492***	-0.066
8.1: Banks	9.389	0.228	1.642	0.842	-0.017
8.2: Non-life Insurance	10.627	0.054	1.476	0.858	-0.054
8.3: Life Insurance	10.031	0.092	1.416	0.824	-0.054
8.4: General Financial	3.634	0.615	0.260	0.174	0.038
8.5: Equity Investment Inst	6.4817	0.123	0.107	0.568	0.003
9.1: Soft & Comp Services	32.511***	0.300	1.701	1.001	-0.145**

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. Leverage effect established by testing whether the correlation between squared returns and lagged returns is statistically significant. The lag order is 1 $(\tau = 1)$.
3. The results of the ARCH LM test for ARCH effects at the 1st order are validated by the White test. While the results of the ARCH LM and White tests are for the most part consistent, the results of the White test suggest that the ARCH LM test may (slightly) understate the presence of ARCH effects. See Table A1.1 in Appendix 1.

Source: Compiled by author

Higher order ARCH(5) and (10) effects are more prevalent than ARCH(1) effects; three economic groups and four industrial sectors exhibit ARCH(1) effects in comparison to six economic groups and ten industrial sectors which exhibit ARCH(5) effects. The frequency of statistically significant ARCH(10) effects is approximately equal to that of ARCH(5) effects in both the economic group and industrial sector return series. Finally, there is evidence of a weak leverage effect as evident from the high number of (mostly statistically insignificant) negative correlation coefficients reported in the last column of Table 7.4. Observed negative correlation between squared returns and lagged returns is statistically significant for only two of the nine economic groups and nine of the twenty-seven industrial sectors. Positive correlation between squared returns and lagged returns is limited to two economic groups and five industrial sectors.

The preceding discussion suggests that the ARCH/GARCH framework is appropriate for modelling returns on the JSE All-Share Index, the economic group and industrial sector indices. Table 7.2 indicates widespread departures from normality in the form of leptokurtosis and skewness, Table 7.3 suggests that overall, the independence assumption holds and Table 7.4 reveals instances of non-linear dependence in returns and ARCH effects in LS residuals. These observed characteristics of returns and the residuals suggest that the ARCH/GARCH econometric framework is appropriate for the modelling of South African stock returns (Elyasiani & Mansur, 1998; Sadorsky & Henriques, 2001).

7.3. Risk factor selection and analysis

7.3.1. Candidate risk factors

The candidate systematic risk factors considered in constructing the multifactor model of the return generating process find support in the literature (see section 4.3). It can be argued that each of these factors directly impacts or is representative of systematic forces that impact expected cash flows and/or the discount rate (see equation (4.1); section 4.2; Chen *et al*., 1986). Table 7.5 lists by no means an exhaustive list of *categorized* risk factors. Although, an attempt is made to consider as many candidate risk factors as possible in the South African context, certain widely used factors are omitted. Factors such as the growth rate in GDP are reported quarterly and other factors, such as unemployment figures, have series that either do not coincide with the period under consideration or only coincide with part of the sample period. This renders these factors inapplicable in this study.[117]

[117] Less popular factors (such as Electric Current Generated, a proxy for real activity) are also considered, but not included in the list once it has been established that these factors have no impact upon returns *and* are not widely considered in the literature.

Table 7.5: Candidate risk factors

Factor	Symbol	Form	Reference
1. Market Indices			Section 4.3.1, Van Rensburg (1996, 2000), Barr (1989), Clare & Priestley (1998), Kandir (2008), Bilson *et al*. (2001), Berry *et al*. (1988)
1.1. JSE All-Share Index (Total returns)	M_t	FDL	
1.2. Dow Jones Industrial Average	DJ_t	FDL	
1.3. FTSE World Index	FTW_t	FDL	
1.4. FTSE 100 Index	$FTSE_t$	FDL	
1.5. MSCI World Index	$MSCI_t$	FDL	
1.6. MSCI World Index (Local Currency)	$MSCIR_t$	FDL	
1.7. Nikkei 225	NK_t	FDL	
2. Inflation			Section 4.3.2, Geske & Roll (1983), Clare & Thomas (1994), Chen (1991), Van Rensburg (1996), Chen (1991), Fabozzi (2008), Fama (1981), Wei & Wong (1992)
2.1. Consumer Price Index	CPI_t	PC	
2.2. Inflation Expectations	$RBAS_t$	L	
2.3. Producer Price Index	PPI_t	PC	
3. Real Activity			Section 4.3.3, Chen *et al*. (1986), Van Rensburg (1996), Fama (1990), Elton *et al*. (2003), Cheung & Ng (1998), Berry *et al*. (1988), Canova & De Nicolo (1995), Lee (1992)
3.1. Industrial Production	MP_t	FDL	
3.2. Building Plans Passed	BP_t	FDL	
3.3. Retail Sales	SLS_t	FDL	
4. Money Supply			Section 4.3.6, Cutler *et al*. (1989), Cheung & Ng (1998), Rozeff (1974), Bilson *et al*. (2001), Mookerjee & Yu (1997), Kandir (2008), Günsel & Çukur (2007)
4.1. M1A (Narrow) Money Supply	$M1A_t$	FDL	
4.2. M3 (Broad) Money Supply	$M3_t$	FDL	
5. Interest Rates			Section 4.3.4 & 4.3.6, Clare & Thomas (1994), Fama (1990), Elton & Gruber (1988), Van Rensburg (1996), Cutler *et al*. (1989), Geske & Roll (1983), Murdagolu *et al*. (2000), Thorbecke (1997)
5.1. Three Month Treasury Bill Rate	$TBT3_t$	L	
5.2. 10 Year Government Bond Yield	$SAGB10_t$	L	
5.3. 30 Year Government Bond Yield	$SAGB30_t$	L	
5.4. Changes in the Term Structure	DTS_t	L	
6. Commodities			Section 4.3.5 & 4.3.7, Chen *et al*. (1986), Hamao (1988), Van Rensburg (2000), Kaul & Seyhun (1990), Jones & Kaul (1996), Nandha & Faff (2008), Poon & Taylor (1991), Sadorsky & Henriques (2001)
6.1. Rand Brent Crude Price	OIL_t	FDL	
6.2. Rand Gold Price	$GOLR_t$	FDL	
6.3. All Commodity Index	COM_t	FDL	
6.4. Metal Index	MET_t	FDL	
6.5. Non-Fuel Commodity Index	$NFCI_t$	FDL	
7. Exchange Rates			Section 4.3.5, Van Rensburg (2000), Hamao (1988), Griffin & Stultz (2001), Jorion (1990), Poon & Taylor (1991)
7.1. Rand-Dollar Exchange Rate	$ZARUS_t$	FDL	
7.2. Rand/Currency Basket Exchange Rate	$ZARBA_t$	FDL	
8. International Trade			Section 4.3.7, Hamao (1988), Kaneko & Lee (1995), Clare & Thomas (1994), Beenstock & Chan (1988)
8.1. Terms of Trade	TT_t	FDL	
8.2. Composite Lead. Index of Trad. Partners	LTT_t	FDL	
8.3. Composite Coinc. Index of Trad. Partners	CTT_t	FDL	
9. Business Cycle Indicators			Section 4.3.4, Chan *et al*. (1985), Caporale & Perry (2006), Fama (1990), Elton *et al*. (2003)
9.1. Composite Leading Index	LI_t	FDL	
9.2. Composite Coincident Index	CI_t	FDL	

Notes:
1. L = Level, FD= First Difference, FDL= First Logarithmic Differences, PC= Percentage Changes
2. The Industrial Production series is seasonally adjusted.
3. Equally-weighted currency basket consisting of the Euro (backcast), British Pound, US Dollar, Australian Dollar, Japanese Yen, Chinese Yuan and Indian Rupee.

Source: Compiled by author

7.3.2. Derivation of innovations

As it is innovations in risk factors that are assumed to drive returns within the APT framework, it is necessary to postulate a methodology for the derivation of innovations (section 3.1.4; Berry *et al*., 1988; Clare & Thomas, 1994). The rate of change methodology used by Chen *et al*. (1986) is widely employed, although Priestley (1996) has shown that this methodology fails to generate uncorrelated series. An alternative approach utilized by Clare and Thomas (1994) and suggested by Priestley (1996) is the autoregressive time series methodology, which assumes that agents form expectations based upon prior information.[118] This methodology is employed in this study. Least Squares AR models are estimated for each factor by regressing changes in factor *k*, ΔF_{kt}, onto twelve autoregressive terms. The generalized form of the autoregressive specification is as follows:

$$\Delta F_{kt} = \alpha_0 + b_k \Delta F_{kt-\tau} + \varepsilon_{kt} \tag{7.9}$$

where ΔF_{kt} is the change in risk factor *k* at time *t* and τ is the lag order. After estimating equation (7.9), insignificant lags are omitted to arrive at a more parsimonious version of the model and the residuals, ε_{kt}, are treated as innovations in risk factor *k*. AR models for each factor are estimated over the January 1994 (not July 1995) to March 2011 period so as to ensure that the use of lagged terms in the modelling of innovations does not consume degrees of freedom over the sample period (see Van Rensburg, 2000). The Breusch-Godfrey LM test statistics and *Q*-statistics for 12$^{\text{th}}$ order serial correlation in the residuals of each AR model are reported in Table 7.6 alongside other summary statistics to demonstrate that the residuals approximate uncorrelated series of innovations (Clare & Thomas, 1994; Van Rensburg, 2000). The prefix *U* indicates that a given residual series is in terms of innovations.

[118] Priestley (1996) suggests that the autoregressive time series methodology is a compromise between the rate of change methodology and Kalman filter techniques in terms of simplicity, accuracy and robustness (see section 3.1.4: 53).

Table 7.6: Series of innovations

Factor	*LM test*	*Q(12)*	Lags	Mean	Std Dev	ADF Test
UM_t	-	-	-	-0.004	0.027	-13.541***
UDJ_t	0.806	12.682	0	0.002	0.020	-13.758***
$UFTW_t$	0.430	6.157	1, 3	8.81E-19	0.021	-12.785***
$UFTSE_t$	0.435	3.802	4	6.84E-20	0.018	-13.616***
$UMSCI_t$	0.815	10.656	1	-2.36E-19	0.018	-14.135***
$UMSCIR_t$	1.529	10.385	7, 10	-2.13E-18	0.021	-12.764***
UNK_t	0.451	5.652	6	-7.59E-19	0.026	-12.820***
$UCPI_t$	1.576	5.439	1-2,6-7,11-12	2.85E-19	0.004	-13.369***
$URBAS_t$	0.909	11.353	1,3	7.58E-18	0.005	-12.646***
$UPPI_t$	0.858	9.708	1	1.48E-18	0.007	-14.696***
UMP_t	1.296	11.632	1,3,7	-1.73E-20	0.008	-14.761***
UBP_t	0.477	5.043	1-2,4	1.70E-18	0.050	-14.422***
$USLS_t$	0.789	8.489	1,12	-2.46E-19	0.008	-14.643***
$UM1A_t$	0.735	6.154	1,11-12	-1.97E-18	0.013	-14.541***
$UM3_t$	0.889	10.376	12	-5.07E-19	0.006	-15.059***
$UTBT3_t$	0.454	4.999	1-2	-5.93E-17	0.005	-14.263***
$USAGB10_t$	1.138	8.590	1, 6-7	3.48E-18	0.006	-14.058***
$USAGB30_t$	1.173	9.883	1, 6-7	-3.16E-17	0.006	-14.494***
$UDTS_t$	0.694	6.265	1,4	-3.08E-18	0.006	-13.829***
$UOIL_t$	0.629	5.682	7, 11	-2.19E-18	0.045	-14.717***
$UGOLR_t$	0.635	8.855	0	0.004185	0.023	-15.118***
$UCOM_t$	1.101	9.248	2,8,11	-7.79E-19	0.018	-12.588***
$UMET_t$	0.834	8.891	1,8	3.31E-19	0.027	-14.214***
$UNFCI_t$	1.182	8.202	1, 4, 8 11	4.25E-19	0.011	-13.596***
$UZARUS_t$	0.973	11.814	6	3.45E-19	0.019	-13.168***
$UZARBA_t$	1.090	12.083	6,8	-2.63E-19	0.017	-14.306***
$UDTT_t$	1.750*	5.550	1-11	1.82E-17	0.046	-13.977***
$ULTT_t$	0.763	6.407	1, 3, 6	2.20E-19	0.002	-13.867***
$UCTT_t$	0.851	8.376	1, 6	1.81E-19	0.001	-15.401***
ULI_t	0.973	8.750	1-2,6 ,11-12	-4.45E-20	0.004	-14.721***
UCI_t	0.976	5.988	1,3,6-7, 11–12	-2.91E-19	0.003	-14.689***

Notes:

1. ADF test conducted with constant only (Sadorsky & Henriques, 2001).
2. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
3. Innovations extracted over 1994M01 – 2011M03.
4. No residual series is generated for returns on the JSE All-Share Index. The series is uncorrelated and therefore approximates innovations. UM_t is also referred to as R_{UM_t}. This notation is used to distinguish between the role of the market aggregate as risk factor and the role of market returns as a dependent factor in subsequent analysis.

Source: Compiled by author

Table 7.6 indicates that all the residual series are uncorrelated with the exception of changes in the terms of trade, $UDTT_t$. For $UDTT_t$, the Breusch-Godfrey LM test statistic indicates that there is statistically significant 12th order serial correlation in the residuals of the AR(12) model used to generate unexpected components. However, an inspection of the correlogram does not reveal statistically significant serial correlation coefficients up to the 12th order suggesting that the residual series *does* represent innovations.[119] Additionally, if the residual series generated by the AR models are truly innovations as required by the APT framework

[119] Results available upon request.

and as suggested by the apparent lack of serial correlation, then the expected (mean) value of the hypothesized innovations must be zero, $E(\varepsilon_{kt})=0$. This is the second assumption that must be satisfied by the candidate risk factor series to qualify as legitimate APT risk factors (Berry *et al*., 1988; Priestley, 1996). The results in Table 7.6 indicate that this is indeed the case; no mean value is greater than two standard deviations and the extremely low mean values are the likely result of the methodology used to construct the series of innovations (see Priestley, 1996). ADF test statistics indicate that like the return series to be modelled, each series of innovations is stationary at first differences.

7.3.3. Risk factor selection

Aside from being in agreement with the theory embodied by equation (4.1), the APT framework requires that risk factors have a pervasive influence upon returns (section 3.1.1, 3.2.2 & 4.2; Berry *et al*., 1988). This can be demonstrated by showing that there is a level of correlation between major indices and the candidate risk factors. To identify risk factors that impact returns on the South African stock market, the approach of Van Rensburg (2000) of establishing correlation between the risk factors and returns on market aggregates is adopted. This approach is almost identical to the approach employed by Chan *et al*. (1985), Chen *et al*. (1986) and Hamao (1988).

Candidate risk factors found to be correlated with returns on the market index are retained and used in further testing. If a risk factor is correlated with the market index, then this factor should have a pervasive influence on other indices and also individual series. This follows from the fact that the market index compromises various indices, which represent economic groupings and industries which in turn consist of individual stocks. The market index, the JSE All-Share Index, is therefore representative of the South African stock market. Following Poon and Taylor (1991), Van Rensburg (2000) and Clare and Thomas (1994), risk factors are entered contemporaneously *and* with lags into the correlation matrix. Factors such as interest rates or exchange rates are known instantaneously and therefore, can be considered contemporaneously. On the other hand, measurements of factors such as inflation or industrial production are reported with a lag. For example, January's inflation rate is announced in February, hence stock prices react to January's inflation in February. Therefore, incorporating lags ensures that prices respond to announcements of macroeconomic factors (see Clare & Thomas, 1994). However, as Poon and Taylor (1991) and Van Rensburg (1996) suggest, risk factors for which data is not instantaneously available may *also* enter into the model contemporaneously. It is however the former approach (coinciding with announcements, lags), rather than the latter approach, that is more in-line with the APT framework. The level of correlation between each risk factor and returns on the JSE All-Share Index is reported in Table 7.7.

Table 7.7: Correlation of JSE All-Share Index returns with candidate risk factors

Factor	UM_t		
UM_t	1.0000	$UTBT3_t$	-0.291***
UDJ_t	0.619***	$USAGB10_t$	-0.406***
$UFTW_t$	0.652***	$USAGB30_t$	-0.410***
$UFTSE_t$	0.608***	$UDTS_t$	-0.088
$UMSCI_t$	0.638***	$UOIL_t$	0.189***
$UMSCIR_t$	0.467***	$UGOLR_t$	0.018
UNK_t	0.549***	$UCOM_t$	0.386***
$UCPI_t$	-0.038	$UMET_t$	0.429***
$UCPI_{t-1}$	-0.159**	$UNFCI_t$	0.168**
$UCPI_{t-2}$	0.049	$UZARUS_t$	-0.180***
$URBAS_t$	-0.284***	$UZARBA_t$	-0.081
$UPPI_t$	-0.027	UTT_t	0.124*
$UPPI_{t-1}$	0.031	UTT_{t-1}	0.013
$UPPI_{t-2}$	-0.061	$ULTT_t$	0.393***
UMP_t	0.217***	$ULTT_{t-1}$	0.056
UMP_{t-1}	0.005	$UCTT_t$	-0.005
UMP_{t-2}	-0.005	$UCTT_{t-1}$	0.148**
UBP_t	0.136*	ULI_t	0.222***
UBP_{t-1}	0.226***	ULI_{t-1}	0.119
UBP_{t-2}	-0.036	UCI_t	0.271***
$USLS_t$	-0.079	UCI_{t-1}	0.092
$USLS_{t-1}$	0.022		
$USLS_{t-2}$	0.158**		
$UM1A_t$	0.095		
$UM1A_{t-1}$	0.144**		
$UM1A_{t-2}$	-0.029		
$UM3_t$	0.040		
$UM3_{t-1}$	0.164**		
$UM3_{t-2}$	-0.225***		

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance.
** Indicates statistical significance at the 5 percent level of significance.
* Indicates statistical significance at the 10 percent level of significance.
2. Correlation coefficients indicate the level of correlation over the period 1995M07-2011M03.

Source: Compiled by author

The correlation coefficients in Table 7.7 indicate that returns on the JSE All-Share Index are, as expected, positively and significantly correlated with returns on the international and foreign indices; namely, the DJIA, UDJ_t, the FTSE World Index, $UFTW_t$, the FTSE 100 Index, $UFTSE_t$, the MSCI World Index, $UMSCI_t$, and the MSCI World Index in local currency (Rands), $UMSCIR_t$, and the Nikkei 225, UNK_t. These indices can be interpreted as catch-all proxies for international risk (Van Rensburg, 1996; Clare & Priestley, 1998; Kwon & Yang, 2008).

The fist lag of the *unexpected changes* representative of innovations in the inflation rate, $UCPI_{t-1}$, as well as unexpected changes in inflation expectations as measured by the bankers acceptance rate, $URBAS_t$, are both negatively and significantly correlated with market returns. Unexpected changes in industrial production, UMP_t, are positively and significantly correlated with returns on the JSE All-Share Index. Another measure of real activity, the unexpected changes in the number of building plans passed, UBP_t, is positively and significantly correlated with returns contemporaneously and at the first lag. The statistically significant correlation at the first lag potentially reflects the publication lag or the delayed availability of information. Returns are positively and significantly correlated with the unexpected growth rate in retail sales, $USLS_t$, at the second lag. This statistically significant relationship also potentially reflects a publication lag or the delayed availability of information. UMP_t, UBP_t and $USLS_t$ are all proxies for real activity suggesting that stock prices respond positively to unexpected changes in real activity.

Whereas unexpected changes in the narrow and broad money supply (monetary aggregates), $UM1A_t$ and $UM3_t$, are positively and significantly correlated with returns at the first lag, the correlation between returns and the second lag of $UM3_t$ is statistically significant and negative. This suggests that while positive changes in the money supply may signal falling discount rates or increased real activity, increases in the broad money supply may also result in uncertainty about inflationary pressures in the future (Günsel & Çukur, 2007; Parkin *et al*., 2008). The correlation between short-term interest rates and long-term interest rates as measured by yields on $UTBT3_t$, $USAGB10_t$, $USAGB30_t$ and returns on the JSE All-Share Index is highly negative and statistically significant implying a strong discount rate effect. Notably, although Chen *et al*. (1986) find that the term structure of interest rates is correlated with aggregate returns in their study, unexpected changes in the term structure of interest rates, $UDTS_t$, as measured by the difference between long-term and short-term interest rates are not significantly correlated with South African stock market returns.

There is a statistically significant and positive correlation between returns and growth in commodity prices; namely, the growth in the prices of oil, $UOIL_t$, metals, $UMET_t$, non-fuel commodities, $UNFCI_t$, and commodity prices in general, $UCOM_t$. Within the commodities risk category, the level of correlation is strongest between returns and $UMET_t$. Returns and unexpected changes in the Rand-Dollar exchange rate, $UZARUS_t$, are negatively and significantly correlated whereas there is a positive and statistically significant contemporaneous relationship between returns and unexpected changes in the terms of trade, UTT_t. Returns on the JSE All-Share Index are positively correlated with local and foreign business cycle indicators, both leading and coincident, as denoted by statistically significant correlation between returns and ULI_t, UCI_t, $ULTT_t$ and $UCTT_{t-1}$ respectively. This suggests that South African stock returns respond to variations in the domestic business cycle and variations in the business cycles of South Africa's trading partners.

Factors found to be significantly correlated with returns on the JSE All-Share Index in Table 7.7 are retained and risk factors that are not significantly correlated with returns on the JSE All-Share Index are omitted from further analysis.

The (unreported) correlation matrix[120] for the retained risk factors indicates that in most instances correlation coefficients are below 0.5 and therefore, the level of correlation is not large enough to result in a multicollinearity problem (Poon & Taylor, 1991). In most instances where statistically significant, the level of correlation remains well below 0.5 as in the instance of $USAGB30_t$ and $UCPI_{t-1}$ where the correlation between these two factors is 0.163. However, high levels of correlation are observed between $URBAS_t$, $UTBT3_t$, $USAGB10_t$ and $USAGB30_t$ with correlation coefficients nearing 0.5 and even over 0.8 for $UTBT3_t$ and $URBAS_t$. As expected, $URBAS_t$ is highly correlated with the interest rate factors as this measure of inflation expectations is itself based upon short-term interest rates (see Van Rensburg, 1996). A number of other factors are also notably correlated with each

[120] The correlation matrix is not reported in-text owing to its size. It is however available from the author upon request.

other. Measures of the money supply at the first lag, $UM1A_{t-1}$ and $UM3_{t-1}$, are significantly correlated. $UZARUS_t$ is highly correlated with the interest rate factors, $USAGB10_t$ and $USAGB30_t$. However, the level of correlation between these factors is well below 0.5. Commodity risk factors, $UCOM_t$, $UMET_t$ and $UNFCI_t$, exhibit levels of correlation of over 0.5 amongst themselves. The leading cyclical indicator for South Africa's trading partners, $ULTT_t$, is highly correlated (correlation coefficients of around 0.5) with the international and foreign indices, suggesting that these indices also reflect changes in the economic climate prevailing within South Africa's trading partners. Statistically significant correlation is observed between the domestic business cycle indicators and commodity prices. The leading domestic business cycle indicator , ULI_t, and coincident business cycle indicator, UCI_t, are both positively correlated with $UOIL_t$, $UCOM_t$ and $UMET_t$ suggesting that changes in commodity prices are also indicative of future or current states of the business cycle. Correlation between the cyclical indicators and the commodity price risk factors is well below 0.3 and usually around 0.2 and therefore, unlikely to result in multicollinearity. Nevertheless, it is borne in mind when model building that high levels of correlation between risk factors will result in some multicollinearity which may weaken the influence of individual risk factors within a multifactor model (Chen *et al*., 1986; Blanchard, 1987; Van Rensburg, 2000).

7.4. Modelling methodology

As the modelling methodology is particularly involved and outlining it in minute detail will unnecessarily inflate the discussion, intricate details such as diagnostic tests are relegated to the body of the next chapter (Chapter 8), the footnotes and appended to tables therein. Therefore, the main steps which form the basis of the empirical analysis are outlined below.

7.4.1. Explanatory power of risk factors

In the first step, returns on the JSE All-Share Index are regressed onto each risk factor found to be significantly correlated with returns in Table 7.7. This is done so as to establish the explanatory power of each risk factor within a univariate (single-factor) context and to show that each factor explains returns on the JSE All-Share Index. Notably, this approach determines whether each risk factor has *meaningful* explanatory power and serves to avoid the pitfall of estimating a complex specification that does not convey more information relative to a simpler one (see Chen, 1991; Reinganum, 1981). An example of such a specification is that of Sadorsky and Henriques (2001) who estimate a four-factor model with only a 0.02 increase in the $\bar{R}^2$ relative to a single-factor model employing excess returns on the TSE (Toronto Stock Exchange) Index as the only explanatory factor. This approach also permits for the role of each factor in the return generating process to be examined in isolation from other risk factors. The specification of the single-factor model relating returns to individual risk factors is as follows:

$$R_{UM_t} = \alpha + b_k F_{kt} + \varepsilon_{UM_t} \tag{7.10}$$

where R_{UM_t} (see note in Table 7.6) is the (excess total) return on the JSE All-Share Index at time *t* and b_k is the sensitivity of R_{UM_t} to *innovations* in risk factor F_{kt} at time *t*. Equation (7.10) is estimated using LS so as to permit a direct comparison of explanatory power not attributable to different ARCH/GARCH specifications. The $\bar{R}^2$ is reported for each regression as a measure of the explanatory power of each risk factor.

7.4.2. The market model

Having determined the explanatory power of each risk factor with respect to market returns, a multifactor model of South African stock market returns is specified by selecting factors that are representative of the risk categories considered (see Table 7.5). This model can be thought of as a *generalized* description of the return generating process of South African stock returns that is used to identify risk factors that drive South African stock returns (see section 3.1.5; Antoniou *et al*., 1998).

The generalized multifactor specification incorporating an international or foreign equity index and domestic risk factors can be denoted as follows:

$$R_{UM_t} = \alpha + b_G F_{Gt} + \sum_{k=1}^{K} b_k F_{kt} + \varepsilon_{UM_t} \tag{7.11}$$

where R_{UM_t} is the return on the JSE All-Share Index at time *t*, b_G is the sensitivity of R_{UM_t} to returns on an international or foreign equity index, F_{Gt}, at time *t*, and b_k is the sensitivity of

R_{UM_t} to *innovations* in risk factor F_{kt} at time *t*. The use of returns on a foreign or international index to explain returns on the domestic market is motivated by the studies of Van Rensburg (1996), Clare and Priestley (1998), Bilson *et al*. (2001) and Kandir (2008) and indicates the level of integration of the domestic stock market with foreign stock markets.

Equation (7.11) reflects the essence of the APT framework; returns are described by a linear factor model representative of a multifactor return generating process, featuring innovations in risk factors. The APT framework informs the structure of the model, the category of factors used to describe returns and the manner in which these factors enter the model.

Equation (7.11) is initially estimated using LS and then within the ARCH/GARCH framework (see Chapter 6). Although the preliminary analysis supports the appropriateness of the ARCH/GARCH framework, both methodologies are applied so as to further establish and confirm the appropriateness of this framework (section 6.4.3; Sadorsky & Henriques, 2001). As before, the $\bar{R}^2$ is considered as an indicator of explanatory power. Engle's (1982) ARCH (section 6.3.1), Bollerslev's (1986) GARCH (section 6.3.2), Engle and Bollerslev's (1986) IGARCH (section 6.3.3) and Nelson's (1991) EGARCH (section 6.3.5) models are considered for the conditional variance specification.[121] This selection of models is made with the hope that this set of ARCH/GARCH models is sufficient to model leptokurtosis, independence, volatility clustering, the long memory and persistence properties of variance, and the leverage effect (section 5.2.1, 5.2.2, 5.3.1, 5.3.2 & 5.3.3; Ding *et al*., 1993; Palm, 1996; Engle, 2001; Engle, 2004; Engle & Patton, 2007; Zakoian, 1994; McMillan & Ruiz, 2009). To select the best fitting ARCH/GARCH model, the number of ARCH and GARCH terms and the appropriate conditional error distribution, the Akaike Information Criterion (AIC)[122] is employed (see Lütkepohl, 2004; Cryer & Chan, 2008; Cornish, 2007). The three conditional error distributions considered in estimating models within the ARCH/GARCH framework are the normal, the Student's *t* and the generalized error distribution. The application of the ARCH/GARCH framework addresses the failure of studies that describe the return generating process within the APT framework to employ an econometric

[121] Denoted by equations (6.5), (6.6), (6.7) and (6.9) respectively.
[122] The AIC is chosen as it is one of the most widely known, studied and applied approaches to model specification (see Cryer & Chan, 2008). Other alternative approaches include the Bayesian Information Criterion (BIC) and the Hannan-Quinn Information Criterion (HQIC) (see Jiang, 2007). For a detailed discussion of the impact of these approaches upon model selection see Cryer and Chan (2008) and Kirchgässner and Wolters (2007).

framework that takes into account the characteristics of returns and volatility (see section 3.3.1; Burmeister & Wall, 1986; Berry *et al*., 1988).

7.4.3. Economic group and industrial sector models

The model of returns on the economic group and industry sector indices is based upon the assumption that the risk factors that feature in the return generating process underlying the JSE All-Share Index have a pervasive influence on other return series. This argument is based upon logic; risk factors that explain returns on a market aggregate consisting of economic groups and industrial sectors *should* be relevant to returns on the economic groups and industrial sectors that are part of this market aggregate (see Van Rensburg, 1996). To model returns on the economic group and industrial sector indices, the model is augmented with a residual market factor where the residual market factor is the residual term in equation (7.11). The residual market factor, denoted by $UM\varepsilon_t$[123] represents returns on the JSE All-Share Index which are uncorrelated with returns on the international or foreign index, F_{Gt}, and the vector of domestic risk factors denoted by $\sum_{k=1}^{K} b_k F_{kt}$ (section 3.3.1: 68; Burmeister & Wall, 1986; Elton *et al*., 2003). It is unlikely that the set of factors in the model – especially the domestic risk factors - will fully explain returns. If this is the case, the residuals of equation (7.11) will be correlated with omitted risk factors and the residual market factor will capture the impact of identified and unidentified omitted risk factors (Berry *et al*., 1988). The generalized model describing the return generating process of the economic group and industrial sector indices is therefore given by:

$$R_{it} = \alpha + b_{UM\varepsilon} UM\varepsilon_t + b_G F_{Gt} + \sum_{k=1}^{K} b_k F_{kt} + \varepsilon_{it} \qquad (7.12)$$

where R_{it} is the return on economic group or industrial sector *i* at time *t*, b_G is the sensitivity of R_{it} to returns on a international or foreign equity index, F_{Gt}, at time *t*, and b_k is the sensitivity of R_{it} to innovations in risk factor F_{kt} at time *t*. The residual market factor is denoted by $UM\varepsilon_t$, $b_{UM\varepsilon}$ is the sensitivity of returns to $UM\varepsilon_t$ and the residuals are denoted by ε_{it}. As before, each model is estimated within the ARCH/GARCH framework and the

[123] ε_{UM_t} in equation (7.11).

appropriate ARCH/GARCH model, the number of ARCH and GARCH terms and the conditional error distribution are selected by employing the AIC.

7.5. Conclusion

This chapter begins by defining the data that will be used in the study. The dataset compromises monthly returns on the JSE All-Share Index and the economic groups and industrial sectors that form part of the JSE All-Share Index (section 7.1). A preliminary analysis of statistical properties is then conducted on each return series in the sample. (section 7.2). While returns on the JSE All-Share Index are found to be normally distributed, almost all economic group and industrial sector return series exhibit levels of excess kurtosis and skewness. While the assumption of independence is violated in isolated instances, overall, the assumption of independence holds (see Table 7.2). The results of the ADF test indicate that as expected, all return series considered are stationary (see Table 7.3). While returns on the JSE All-Share Index are found to be free from non-linear dependence, most return series for the economic group and industrial sector indices exhibit some form of non-linear dependence or ARCH effects in the residuals (Table 7.4). This suggests that the ARCH/GARCH framework is appropriate for the modelling of South African stock returns (section 7.2; Elyasiani & Mansur, 1998). There is weak evidence of the leverage effect (section 7.2: 154).

To derive innovations, the autoregressive time series methodology is applied (see section 7.3.2; equation (7.9); Priestley, 1996) and the results in Table 7.6 indicate that this methodology successfully derives innovations in the form of the residuals of an AR model. To select risk factors, the correlation between returns on the JSE All-Share Index and innovations in the candidate risk factors listed in Table 7.5 is examined. Results indicate that factors representative of international risk, inflation, real activity, the money supply, interest rates, commodity prices, the exchange rate and local and foreign business cycles are significantly correlated with returns on the JSE All-Share Index (see section 7.3.3; Table 7.7).

The modelling methodology which is applied in the analysis in Chapter 8 is discussed next (section 7.4). The first part of the analysis regresses returns on the JSE All-Share Index onto each of the individual factors found to be statistically correlated with JSE All-Share Index returns in Table 7.7 (equation (7.10); section 7.4.1). The market model, which incorporates domestic risk factors and a factor representative of international risk, is outlined next (equation (7.11); section 7.4.2). The residuals of this model are treated as the residual market factor (see Burmeister & Wall, 1986). Consideration is given to the AIC in selecting the ARCH/GARCH model specification, the number of ARCH/GARCH parameters and the underlying conditional distribution. The economic group and industrial sector models are elaborated upon in section 7.4.3 (also see equation (7.12)). While the specification of these models is identical to that of the market model in that these models incorporate a set of domestic risk factors and an international risk factor, these models also incorporate the residual market factor derived from the market model in equation (7.11). The results of the analysis are reported in Chapter 8.

8. RESULTS AND DISCUSSION

8.1. Conducting the analysis

Chapter 8 brings together the concepts developed throughout the study; the multifactor APT framework discussed in Chapter 2 and Chapter 3 is applied to model and investigate the return generating process of South African stock returns using the category (systematic risk) of risk factors outlined in Chapter 4. In light of the properties and behaviour stock returns outlined in Chapter 5 and the specific findings relating to South African stock returns (where South African stock returns are found to exhibit deviations from normality in the form of excess kurtosis, skewness, non-linearity and ARCH effects; see section 7.2), the ARCH/GARCH econometric framework discussed in Chapter 6 is applied in modelling the return generating process of South African stock returns.

Prior to reporting the results of the multifactor models of the return generating process of South African stock returns, a univariate analysis is undertaken in section 8.2 (also see section 7.4.1; equation 7.10). Having identified factors with substantive explanatory power (section 8.2; Table 8.1), the return generating process underlying the South Africa stock market in investigated in section 8.3. The model specification (equation (8.1)) is first estimated within the LS framework for the purposes of preliminary analysis (section 8.3.1). ARCH/GARCH modelling is undertaken next with extensive consideration being given to the structure of the return generating process and the underlying conditional variance (section 8.3.2 & 8.3.3). As the proposed model specification in equation (8.1) does not include *all* the factors found to be significantly correlated with returns on the JSE All-Share Index in Table 7.7 and all factors with substantive explanatory power (section 8.2; Table 8.1), an investigation of additional risk factors in South African stock returns is undertaken in section 8.3.4.

A similar approach is followed in investigating the return generating process (equation 8.2) underlying economic group (section 8.4) and industrial sector returns (section 8.5). Results of the respective models are discussed in detail in section 8.4.1 and section 8.5.1. Conditional variance is considered in section 8.4.3 and section 8.5.2 and possible specification problems are addressed in section 8.4.4 and section 8.5.3. An investigation of gains in explanatory power from combining the various risk factor categories is undertaken in section 8.4.5 and section 8.5.4 respectively. The omission of risk factors is discussed in section 8.4.6 and section 8.5.5 and additional risk factors in the respective return series are investigated in section 8.4.7 and section 8.5.6. A synthesis for each level of analysis is provided in section 8.3.5, section 8.4.8 and section 8.5.7 and the conclusion follows in section 8.6.

8.2. Univariate analysis

Table 8.1 reports the results of the single-factor model in equation (7.10) whereby returns on the JSE All-Share Index are regressed onto the retained risk factors using LS regressions.

Table 8.1: Univariate regressions

Panel A: International & Foreign Market Indices						
	UDJ_t	$UFTW_t$	$UFTSE_t$	$UMSCI_t$	$UMSCIR_t$	UNK_t
b_k	0.826***	0.861***	0.902***	0.896***	0.578***	0.570***
$\bar{R}^2$	0.380	0.422	0.370	0.404	0.214	0.300
AR(1)	0.141	0.433	1.098	0.269	0.045	0.505
ARCH(1)	0.004	0.102	0.532	0.278	0.586	0.149
Panel B: Inflation						
	$UCPI_{t-1}$	$URBAS_t$				
b_k	-0.999**	-1.393***				
$\bar{R}^2$	0.020	0.076				
AR(1)	0.019	0.064				
ARCH(1)	0.002	0.062				
Panel C: Real Activity						
	UMP_t	UBP_t	UBP_{t-1}	$USLS_{t-2}$		
b_k	0.708***	0.072*	0.119***	0.506**		
$\bar{R}^2$	0.042	0.013	0.046	0.020		
AR(1)	0.004	0.064	0.058	0.013		
ARCH(1)	0.049	0.058	0.022	0.030		
Panel D: Money Supply						
	$UM1A_{t-1}$	$UM3_{t-1}$	$UM3_{t-2}$			
b_k	0.279*	0.776**	-1.065***			
$\bar{R}^2$	0.015	0.022	0.046			
AR(1)	0.001	0.235	0.775			
ARCH(1)	0.085	0.045	0.045			
Panel E: Interest Rates						
	$UTBT3_t$	$USAGB10_t$	$USAGB30_t$			
b_k	-1.532***	-1.953***	-1.954***			
$\bar{R}^2$	0.080	0.160	0.164			
AR(1)	0.012	0.166	0.324			
ARCH(1)	0.391	0.220	0.018			
Panel F: Commodities						
	$UOIL_t$	$UCOM_t$	$UMET_t$	$UNFCI_t$		
b_k	0.110***	0.576***	0.430***	0.413**		
$\bar{R}^2$	0.030	0.144	0.180	0.023		
AR(1)	0.004	0.009	0.228	0.585		
ARCH(1)	0.115	0.077	0.021	0.051		

Table 8.1: Univariate regressions (continued)

Panel G: Exchange Rates			
	$UZARUS_t$		
b_k	-0.241**		
$\bar{R}^2$	0.027		
AR(1)	0.049		
ARCH(1)	0.068		
Panel I: International Trade			
	UTT_t	$ULTT_t$	$UCTT_{t-1}$
b_k	0.072*	5.573***	3.852**
$\bar{R}^2$	0.010	0.150	0.017
AR(1)	0.015	0.877	0.008
ARCH(1)	0.011	0.002	0.077
Panel J: Business Cycle Indicators			
	ULI_t	UCI_t	
b_k	1.461***	2.425***	
$\bar{R}^2$	0.044	0.068	
AR(1)	1.228	0.036	
ARCH(1)	0.021	0.122	

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. AR(1) are Breusch-Godfrey LM test statistics for residual serial correlation at the 1st order.
3. ARCH(1) are LM test statistics for residual ARCH effects at the 1st order.

Source: Compiled by author

The results in Table 8.1 indicate that returns on international and foreign market indices explain between 21.4 percent and 42.2 percent of the variation in returns on the JSE All-Share Index. This suggests that international risk plays an important role in the South African stock market and that the South African stock market is highly integrated with world markets (see section 3.1.6, 3.3.2 & 4.3.1; Clare & Priestley, 1998; Bilson *et al*., 2001). Innovations in the inflation factors have a negative and statistically significant impact upon returns. Unexpected changes in inflation, $UCPI_{t-1}$, explain approximately 2 percent of the variation in returns. Unexpected changes in inflation expectations, $URBAS_t$, explain almost 8 percent of the variation in returns. It is plausible that the impact of inflation expectations is through the discount rate; Geske and Roll (1983) argue that interest rates are a proxy for inflation expectations suggesting that higher expected inflation results in increased interest rates and therefore, depresses stock prices. By this reasoning, inflation expectations are reflected in the banker's acceptance rate, which is a proxy for inflation expectations (see section 4.3.2; Van Rensburg, 1996). Returns are significantly and positively related to all four measures of real activity, UMP_t, UBP_t, UBP_{t-1} and $USLS_{t-2}$. The contemporaneous relationship between UMP_t, UBP_t, and returns on the JSE All-Share Index suggests that the incorporation of information relating to real activity may be instantaneous. However, it is UBP_{t-1} that carries the most explanatory power. Regardless of which measure of real activity is considered, it is evident that real activity is an important risk factor for the South African stock market (see section 4.3.3). Measures of real activity explain between 1.3 percent and 4.6 percent of the variation in returns.

Returns are positively and significantly related to both measures of the money supply, $UM1A_{t-1}$ and $UM3_{t-1}$, at the first lag. A positive relationship suggests that $UM1A_{t-1}$ and $UM3_{t-1}$ reflect falling interest rates and/or rising real activity. However, returns are negatively and significantly related to the second lag of the broad money supply, $UM3_{t-2}$, suggesting that increases in the money supply are also associated with uncertainty relating to inflationary pressures (Günsel & Çukur, 2007; Parkin *et al*., 2008). Changes in the money supply explain between 1.5 percent and 4.6 percent of the variation in stock returns. The negative and statistically significant relationship between returns and the interest rate factors, $UTBT3_t$, $USAGB10_t$ and $USAGB30_t$, suggests that there is a strong discount rate effect in returns (section 4.3.6; Thorbecke, 1997; Muradoglu *et al*., 2000). The long-term interest rate, $USAGB30_t$ has the largest impact upon returns, explaining over 16 percent of the variation in returns. This suggests that South African stock returns are more sensitive to changes in long-term interest rates relative to changes in short-term interest rates as measured by the yield on three month treasury bills.

Returns are positively and significantly related to the various measures of commodity prices, $UOIL_t$, $UCOM_t$, $UMET_t$, and $UNFCI_t$. Innovations in general commodity prices, $UCOM_t$, and world metal prices, $UMET_t$, explain a sizeable amount of variation in returns on the JSE All-Share Index. The explanatory power of these two factors is potentially attributable to the importance of resources for South Africa's economy (see section 4.3.5 & 4.3.7; Roberts & Zalk, 2004). That $UMET_t$ plays an important role in explaining South African stock returns is demonstrated by Barr (1990). However, as both $UCOM_t$ and $UMET_t$ are significantly correlated with international and foreign market indices,[124] it is plausible that these factors reflect elements of international risk. The relationship between returns and unexpected changes in oil prices, $UOIL_t$, is positive – an unexpected finding given that literature (mostly)

[124] Correlation between $UCOM_t$ and the international and foreign indices ranges between 0.175 and 0.355. Correlation between $UMET_t$ and the international and foreign indices ranges between 0.204 and 0.413.

proposes that increases in oil prices negatively impact expected cash flows through increased costs of production and have an adverse impact upon real activity (section 4.3.5; Jones & Kaul, 1996; Nandha & Faff, 2008). However, it may be that certain economic groups and industrial sectors that dominate the JSE All-Share Index respond positively to increases in oil prices and this results in a positive relationship between $UOIL_t$ and returns on the market aggregate. Alternatively, there is a pass through effect (Jones & Kaul, 1996; Van Rensburg, 1996; Nandha & Faff, 2008). A further hypothesis that warrants investigation is that oil prices are positively related to *global* real activity; rising global real activity implies higher demand for oil leading to an increase in oil prices. Therefore, rising oil prices are a proxy for rising global real activity which results in higher returns on the JSE All-Share Index. Fluctuations in the Rand-Dollar exchange rate, $UZARUS_t$, explain 2.7 percent of the variation in returns. The negative and statistically significant relationship between returns and $UZARUS_t$ suggests that the South African stock market is adversely affected by a depreciation of the Rand. Fluctuations in the exchange rate affect stock prices through their impact upon the demand for a given firm's or industry's product and thus, expected cash flows. Alternatively, fluctuations in the exchange rate may affect input costs – a depreciation of the domestic currency will increase input costs and reduce expected cash flows (section 4.3.5; Jorion, 1990).

Innovations in factors related to trade have a positive and statistically significant impact upon returns. Their explanatory power however is weak; unexpected changes in the terms of trade, UTT_t, and unexpected changes in the coincident business cycle indicator for South Africa's trading partners, $UCTT_{t-1}$, explain only 1 percent and 1.7 percent of the variation in returns on the JSE All-Share Index respectively. In contrast, unexpected changes in the leading business cycle indicator for South Africa's trading partners, $ULTT_t$, explain 15 percent of the variation in returns on the JSE All-Share Index. The impact of these factors can be explained through their role as indicators of demand for South African products. An improvement in the terms of trade suggests that the demand for exports relative to imports has increased whereas improvements in the economic climate of South Africa's trading partners imply an increase in the demand for South African goods (Parkin *et al*., 2008). However, $ULTT_t$ is highly correlated with returns on international and foreign indices[125] suggesting that this factor

[125] Correlation ranges between 0.468 and 0.547.

reflects international risk as well as the economic conditions experienced by South Africa's trading partners. The final category of factors consists of South African business cycle indicators. Returns are significantly and positively related to unexpected changes in the leading cyclical indicator, ULI_t, and the coincident cyclical indicator, UCI_t. These factors explain 4.4 percent and 6.8 percent of the variation in returns respectively. The role of business cycle indicators is similar to the role of the default spread as an indicator of business conditions (see section 4.3.4; Chan *et al.*, 1985; Fama, 1990). Although, Van Rensburg (2000) omits the default spread in his study due to perceived poor availability of data, this study treats ULI_t and the UCI_t as a substitutes for the default spread. A positive relationship between returns and the business cycle indicators is expected; improving business conditions translate into higher expected cash flows.

8.3. South African stock market

8.3.1. Preliminary estimation

Having identified a set of macroeconomic factors assumed to be representative of systematic risk, a number of specifications were considered so as to investigate potential combinations of risk factors that feature in the return generating process of South African stock returns. Equation (7.11) was first estimated without the international and foreign indices so as to establish the explanatory power of the risk factors in the absence of a single factor that may dilute or obscure the explanatory power of a set of domestic risk factors. All models were first estimated using LS and then within the ARCH/GARCH estimation framework (see Chapter 6). The number of possible combinations is extensive.

After testing a number of combinations, it was decided that the model should permit returns to be explained by as many different categories of risk as possible while avoiding problems associated with multicollinearity. There are limitations associated with this approach, which must be recognized. The *representative* set of risk factors may not be the set that maximizes $\overline{R}^2$. However, the purpose of this approach is to identify risk factors in the return generating process and not necessarily to achieve a high $\overline{R}^2$ (see Van Rensburg, 1996). Furthermore, this approach identifies risk factors at the expense of parsimony and may introduce factors that are relevant to specific economic groups and industrial sectors (Kryzanowski & To, 1983). Although this approach suffers from some limitations – partially addressed at a later stage – it provides insight into the structure of the return generating process underlying South

African securities as an *aggregate.* After careful consideration, the specification chosen to represent the multifactor return generating process is given by:

$$R_{UM_t} = \alpha + b_{UFTW}UFTW_t + b_{UCPI}UCPI_{t-1} + b_{URBAS}URBAS_t + b_{UBP}UBP_{t-1} + b_{UM3}UM3_{t-1} + b_{UOIL}UOIL_t$$
$$+ b_{UZARUS}UZARUS_t + b_{UCI}UCI_t + \varepsilon_{UM_t} \tag{8.1}$$

where R_{UM_t} is the return on the JSE All-Share Index at time *t*. Although equation (8.1) manages to incorporate factors representative of almost every risk factor category, two important risk categories are excluded. These are the interest rate and international trade risk categories. $UTBT3_t$, $USAGB10_t$ and $USAGB30_t$ are excluded due to their high correlation with $URBAS_t$.[126] Although $URBAS_t$ is assumed to be a proxy for unexpected changes in inflation expectations, this factor is also indicative of a discount rate effect and therefore captures some of the impact of the omitted interest rate factors (see Geske & Roll, 1983). Furthermore, $USAGB10_t$ and $USAGB30_t$ - long-term interest rate factors with relatively high levels of explanatory power - are excluded so as not to obscure the role of other factors. This omission is addressed at a later stage. Factors representative of international trade are also excluded. Preliminary analysis shows that UTT_t is statistically insignificant in a multifactor context and even in the univariate context, its explanatory power is almost negligible. $ULTT_t$ is excluded due to its high correlation with the international and foreign indices suggesting that these indices account for the impact of $ULTT_t$. This is plausible in that the international and foreign indices are likely to reflect the economic conditions experienced by South Africa's trading partners. Finally, preliminary analysis indicates that $UCTT_{t-1}$ is statistically insignificant in a multifactor context.

To represent commodity price risk, $UOIL_t$ is chosen over $UMET_t$ and $UCOM_t$. Although these two factors explain a sizable proportion of the variation in returns on the JSE All-Share Index, they are significantly correlated with the international and foreign indices suggesting that some of their impact can be accounted for by these indices. As the level of correlation between $UOIL_t$ and these indices is substantially lower, it is an ideal representative of the

[126] $URBAS_t$ exhibits a correlation of 0.822, 0.479 and 0.469 with $UTBT3_t$, $USAGB10_t$ and $USAGB30_t$ respectively suggesting that it is more representative of a short-term interest rate effect.

commodities risk category. Out of the six international and foreign indices, the FTSE World Index, $UFTW_t$, is chosen to represent global economic conditions and hence to proxy for international risk. This specific index is chosen over the traditionally used MSCI World Index (section 3.1.6, 3.3.2 & 4.3.1; Clare & Priestley, 1998; Bilson *et al*., 2001; Kandir, 2008) as it has the highest $\bar{R}^2$ and because it represents between 90 percent and 95 percent of investable constituent market capitalisation[127] thus encompassing a sizeable portion of the world's stock markets. Returns on this index are however highly correlated with $UZARUS_t$.[128] Given the size of South Africa's economy relative to the world economy and the economies of its trading partners, it is plausible that domestic policy has little impact upon fluctuations in the exchange rate which are driven by international developments reflected in $UFTW_t$. This may explain the high level of correlation between these two factors. Preliminary analysis reveals that combining $UZARUS_t$ and $UFTW_t$ in a two-factor model does not render either of these factors statistically insignificant, although the coefficient on $UZARUS_t$ becomes positive when $UFTW_t$ is incorporated into the model. Equation (8.1) denotes the *unrestricted* model (see section 3.3.2: 76 for a discussion of restricted and unrestricted models) of the return generating process of South African stock returns with $UFTW_t$ fulfilling the dual role of an international risk factor and a proxy for the numerous other international risk factors that influence returns on the South African stock market (Clare & Priestley, 1998)

Equation (8.1) is first estimated using LS and preliminary analysis is conducted upon various aspects of the model. $UFTW_t$ is excluded in a *restricted* version of the model in Panel A of Table 8.2 and then included in the unrestricted model in Panel B so as to establish the explanatory power of the seven domestic risk factors without the presence of $UFTW_t$. Panel C reports the results of a single-factor model incorporating only $UFTW_t$.

[127] Information reproduced from the FTSE website (www.ftse.com).
[128] The level of correlation is -0.435.

Table 8.2: Least Squares model of JSE All-Share Index returns

	Panel A	Panel B	Panel C
Intercept	-0.004**	-0.004***	-0.004***
$UFTW_t$	-	0.885***	0.861***
$UCPI_{t-1}$	-0.921**	-0.570*	-
$URBAS_t$	-1.142***	-1.318***	-
UBP_{t-1}	0.079**	0.053**	-
$UM3_{t-1}$	0.606*	0.619***	-
$UOIL_t$	0.090**	0.075**	-
$UZARUS_t$	-0.182*	0.225***	-
UCI_t	1.840***	1.132**	-
$\bar{R}^2$	0.224	0.583	0.422
AIC	-4.624	-5.238	-4.949
F-Statistic	8.771***	33.804***	138.384***
AR(1)	0.006	0.000	0.433
AR(5)	2.220**	0.766	1.022
ARCH(1)	0.293	1.748	0.102
ARCH(5)	0.290	3.036**	2.783**

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. AR(1) and AR(5) are Breusch-Godfrey LM test statistics for residual serial correlation at the 1st and 5th orders.
4. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

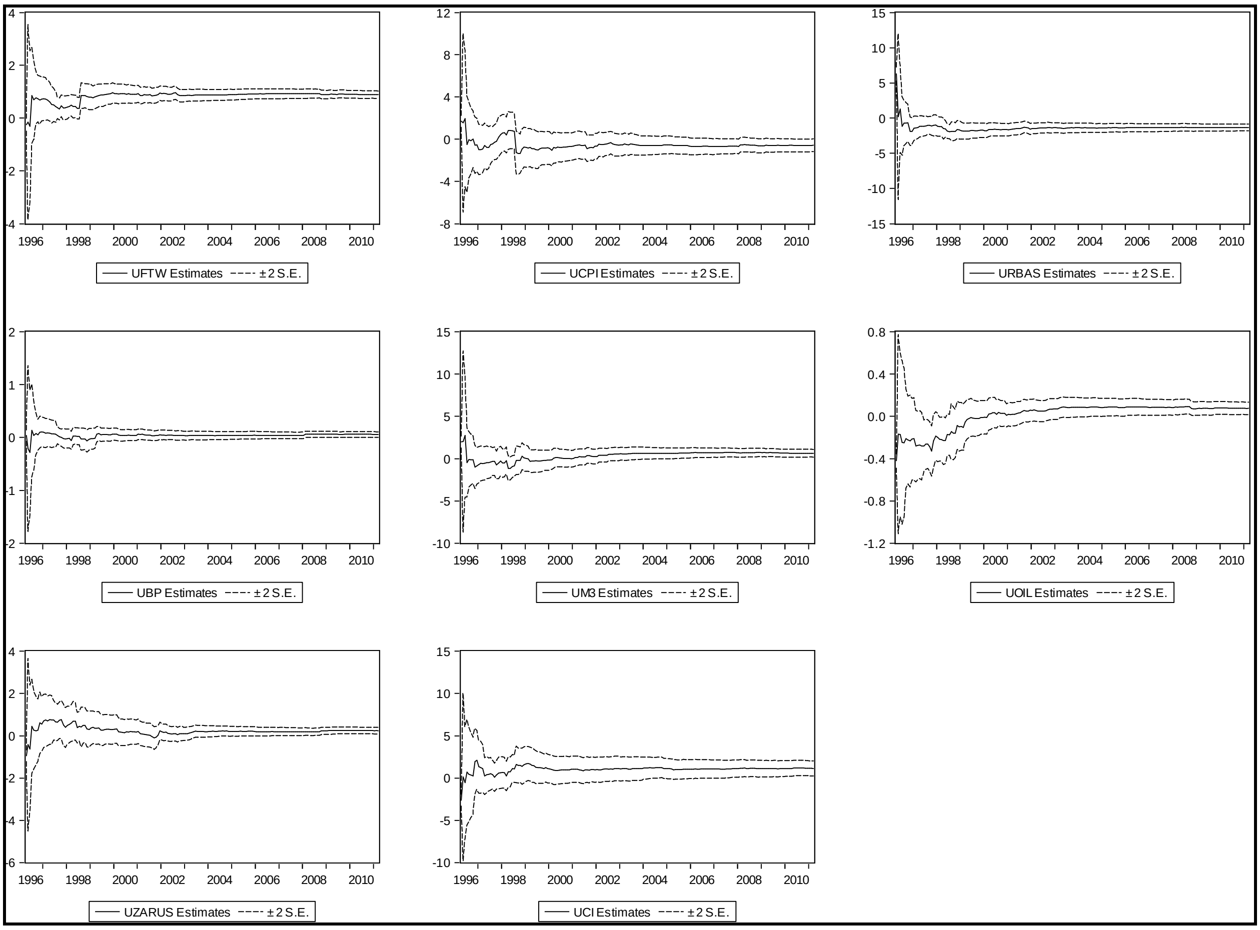
Source: Compiled by author

The results in Panel A of Table 8.2 indicate that the seven domestic risk factors by themselves explain 22.4 percent of the variation in returns on the JSE All-Share Index. All factors are individually statistically significant with negative coefficients (factors loadings, betas, exposures or sensitivities in APT terminology) on $UCPI_{t-1}$, $URBAS_t$ and $UZARUS_t$ and positive coefficients on UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and UCI_t. To test for joint statistical significance, Wald's test of coefficient restrictions is applied and all coefficients are constrained to zero. Based upon the resultant *F*-statistic, the null hypothesis is rejected confirming the significance of including all factors in the model (Sadorsky & Henriques, 2001; Brooks, 2008). Although no ARCH(1) and ARCH(5) effects are present, the Breusch-Godfrey LM test indicates statistically significant residual serial correlation at the fifth order suggesting that the estimated LS coefficients are inefficient (Gujarati, 2003).

Next, the model is estimated with $UFTW_t$ included (Panel B). All coefficients remain statistically significant although for certain factors, coefficients decrease in size suggesting that $UFTW_t$ captures information contained in the domestic risk factors. The coefficients on $UCPI_{t-1}$, UBP_{t-1}, $UOIL_t$ and UCI_t all decrease in absolute size relative to the coefficients of the restricted model in Panel A. The direction of the relationships remains unchanged with the exception of $UZARUS_t$, which now has a positive (and still statistically significant) impact on returns. This suggests a potential multicollinearity problem arising from the high levels of correlation between $UFTW_t$ and $UZARUS_t$. For this reason, the impact of $UZARUS_t$ on South African stock returns should be interpreted with caution. It is however certain that $UZARUS_t$ plays a significant role in the return generating process. As before, all factors are jointly statistically significant. This provides further support for a multifactor model of the return generating process of South African stock market returns. The $\bar{R}^2$ increases to 0.583 suggesting that $UFTW_t$ significantly contributes to explaining South African stock returns. This improvement in fit is confirmed by a lower AIC statistic suggesting that the unrestricted model is more adequate (see section 7.4.2 & 7.4.3). However, there is evidence of higher order ARCH effects in the residuals (see section 7.2: 151; equations (7.5) and (7.6)) implying that LS coefficient estimates are inefficient (Gujarati, 2003).

Finally, returns on the JSE All-Share Index are regressed onto $UFTW_t$ (Panel C). As before, the coefficient on $UFTW_t$ is positive and statistically significant. By itself, $UFTW_t$ explains 42.2 percent of the variation in returns on the JSE All-Share Index. This is in comparison to the 58.3 percent explained by the unrestricted model suggesting that $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t explain *at least* 16.1 percent of the variation in returns in the presence of $UFTW_t$. Whereas the AIC statistic in Panel C is lower than that of the restricted model in Panel A, it is higher than that of the unrestricted model in Panel B suggesting that the unrestricted model combining $UFTW_t$ and the domestic risk factors is the most appropriate model of the return generating process underlying South African stock returns. Regression diagnostics again suggest that there are higher order ARCH effects in the residuals implying that the LS methodology is not appropriate for modelling South African stock returns.

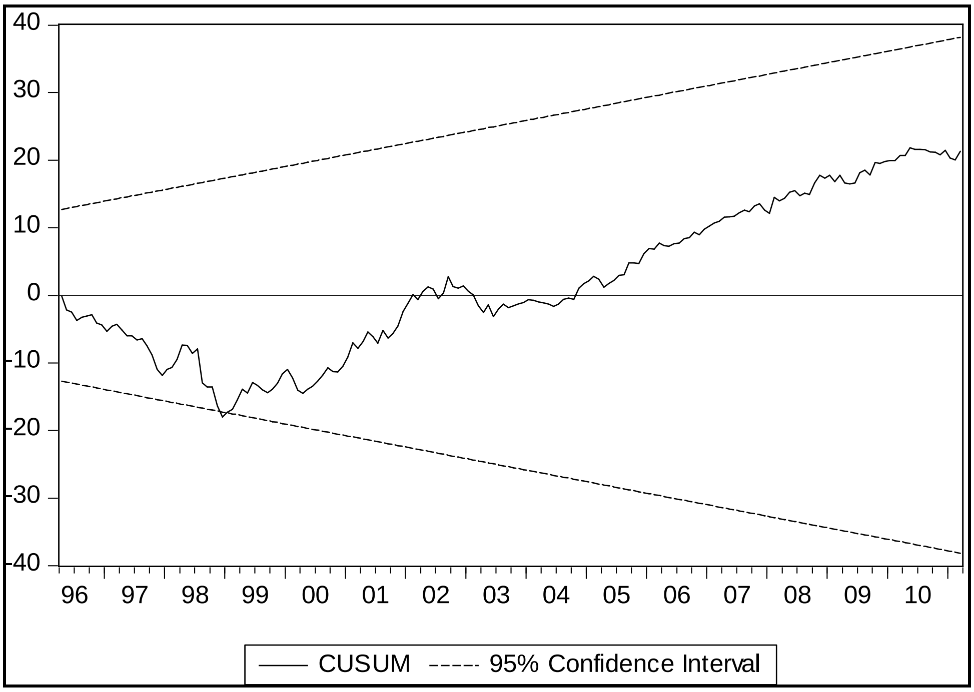
Following Sadorsky and Henriques (2001), parameter stability is first tested by recursive estimation of the unrestricted model. The recursively estimated coefficients and associated standard error bands are reported in Figure 8.1. Results suggest that the estimated coefficients are stable over time. Although, the coefficient estimates at the beginning of the recursive procedure appear to be unstable, this is expected as these estimates are obtained using few observations. What is important is that coefficient estimates are similar for most of the sample period suggesting that there are no structural stability problems (Sadorsky & Henriques, 2001; Brooks, 2008).



Source: Compiled by author

Figure 8.1: Recursive coefficient estimates

To further test the stability of the model, the more formal CUSUM test is applied (Brooks, 2008). Figure 8.2 indicates that the CUSUM test statistic moves beyond the 95 percent confidence interval suggesting that the null hypothesis of stability can be rejected. However, as the CUSUM test statistic remains within the 95 percent confidence interval for most of the sample period, parameter instability does not appear to be a severe problem.



Source: Compiled by author

Figure 8.2: CUSUM test graph

A potential source of parameter instability may arise from the presence of outliers which can have an undue influence upon coefficient estimates. To investigate this, the unrestricted model is re-estimated with a dummy factor taking on a value of 1 for August 1998 and a value of 0 for all other observations (see Brooks, 2008). Brooks (2008) argues that the removal of outliers is appropriate for extreme events. The negative return of 16.9 percent observed during August 1998 coincides with the Russian Default of 1998 and it can be argued that this constitutes an extreme event. While there have been a number of crises over the sample period, none have had such a significant impact upon returns on the JSE All-Share Index.[129]

The results in Table 8.3 suggest that the model is generally robust to outliers as all factors in the specification with the exception of $UCPI_{t-1}$ remain statistically significant. The signs of the coefficients remain unchanged and there are no large changes in the magnitude of any of the coefficients with the exception of $UCPI_{t-1}$ and UCI_t, which decrease in absolute size.

[129] For a detailed outline of the Russian Default crisis see Chiodo and Owyang (2002). For a discussion on the appropriateness of removing outliers see Brooks (2008). Arguments have been made in favour of retaining outliers and in favour of excluding outliers from models. After August 1998, the next lowest (negative) return is 7 percent (July 2002). It is evident that returns for August 1998 are a once-off occurrence - even other crises, such as September 2001 and the market crash of 2008, do not result in negative returns of this magnitude.

The coefficient on the dummy factor, $DUM98M08$, is highly significant suggesting that this outlier *does* have an impact upon model results.

Table 8.3: Least Squares model of JSE All-Share Index returns incorporating dummy factor

	Panel A
Intercept	-0.004***
$UFTW_t$	0.810***
$UCPI_{t-1}$	-0.176
$URBAS_t$	-1.138***
UBP_{t-1}	0.063***
$UM3_{t-1}$	0.541**
$UOIL_t$	0.084***
$UZARUS_t$	0.200***
UCI_t	0.793*
$DUM98M08$	-0.090***
$\bar{R}^2$	0.633
AIC	-5.362
F-Statistic	37.023
AR(1)	0.229
AR(5)	0.371
ARCH(1)	7.974***
ARCH(5)	2.243***

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. AR(1) and AR(5) are Breusch-Godfrey LM test statistics for residual serial correlation at the 1st and 5th orders.
4. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

Source: Compiled by author

The $\bar{R}^2$ increases marginally and the AIC statistic decreases suggesting that the exclusion of the outlier improves the fit of the model. However, regression diagnostics indicate the presence of ARCH(1) and ARCH(5) effects suggesting that the LS methodology fails to account for heteroscedasticity even after controlling for outliers (see section 6.4.3, 6.4.4 & 7.2; Gujarati, 2003; Brooks, 2008). It must however be emphasized that although the coefficient on the dummy factor is statistically significant, this finding does not automatically translate into a confirmation that outliers are the source of parameter instability. Thus, to formally test for parameter instability after controlling for outliers, the CUSUM test is applied again. Results (unreported) indicate that the CUSUM test statistic lies within the 95 percent confidence interval throughout the entire sample period suggesting that the null hypothesis of stability is not rejected. This finding suggests that the instability of parameters is not inherent to the model, but possibly attributable to a single outlier which is not fully explained by the risk factors incorporated into the unrestricted model. Even when outliers are not controlled for, the model still performs well – the CUSUM test statistic falls within the 95 percent confidence interval for almost the entire duration of the sample period as evident in Figure 8.2.

8.3.2. ARCH/GARCH modelling

The preceding preliminary analysis suggests that the proposed multifactor model of the return generating process provides a reasonable description of the process driving South African stock market returns. The model explains almost 60 percent of the variation in returns with the domestic risk factors explaining at least 16.1 percent of the variation in returns in addition to $UFTW_t$. All factors are statistically significant in the unrestricted model, the model is generally robust to outliers and parameter instability does not appear to be inherent to the model. However, regression diagnostics indicate that ARCH effects are present in both versions of the unrestricted model (Table 8.2, Panel B & Table 8.3, Panel A). This suggests that the ARCH/GARCH framework is more appropriate for modelling returns on the JSE All-Share Index (see section 6.4.3 & 6.4.4). Furthermore, the framework also provides insight into the conditional variance of South African stock returns. Table 8.4 reports the results of the unrestricted and restricted versions of the model. Selection of the ARCH/GARCH model (ARCH (section 6.3.1), GARCH (section 6.3.2), IGARCH (section 6.3.3) or EGARCH (section 6.3.5)), the number of ARCH and GARCH terms and the conditional error distribution is based upon the unrestricted model (including outliers) using AIC. The ARCH/GARCH specification, the number of ARCH and GARCH parameters and the conditional error distribution applied in the estimation of the unrestricted model, are extended to the restricted versions of the model.

The results in Panel A of Table 8.4 are those of the unrestricted model whereas the results in Panel B are those of the unrestricted model after controlling for outliers. The model in Panel C excludes $UFTW_t$ and the model in Panel D is a single-factor model incorporating only $UFTW_t$. The results in Panel A of Table 8.4 are comparable to those of Panel B in Table 8.2. All factors are individually and jointly statistically significant. As before, returns on the JSE All-Share Index are positively related to $UFTW_t$, $UBP_{t-1}, UM3_{t-1}$, $UOIL_t$ $UZARUS_t$ and UCI_t and negatively related to $UCPI_{t-1}$ and $URBAS_t$. Whereas most coefficients remain similar in magnitude, the coefficient on $URBAS_t$ decreases from an absolute value of 1.318 in Panel B of Table 8.2 to 0.800 in Panel A of Table 8.4. Together, these factors explain 56.2

percent of the variation in returns and the AIC statistic of -5.322 is lower than the AIC statistic in Panel B of Table 8.2 suggesting that the ARCH/GARCH framework yields a better description of returns on the JSE All-Share Index relative to the LS model. Notably, regression diagnostics indicate that the GARCH(1,1) model with normally distributed errors provides an adequate description of the return generating process. Residuals and squared residuals are white noise and the ARCH LM test suggests that ARCH effects are no longer present (see Sadorsky & Henriques, 2001; Gujarati, 2003). This strengthens the argument that the ARCH/GARCH framework is a more appropriate econometric framework for the modelling of returns.

Table 8.4: ARCH/GARCH model of JSE All-Share Index returns

	Panel A	Panel B	Panel C	Panel D
Intercept	-0.002*	-0.002**	-0.003**	-0.002
$UFTW_t$	0.888***	0.835***	-	0.802***
$UCPI_{t-1}$	-0.617**	-0.402	-0.961**	-
$URBAS_t$	-0.800***	-0.951***	-1.049***	-
UBP_{t-1}	0.071***	0.072***	0.090***	-
$UM3_{t-1}$	0.697***	0.641***	0.785***	-
$UOIL_t$	0.057**	0.069***	0.084**	-
$UZARUS_t$	0.242***	0.207***	-0.268***	-
UCI_t	1.069***	0.928***	1.858***	-
$DUM98M08$	-	-0.088	-	-
$\bar{R}^2$	0.562	0.626	0.217	0.416
AIC	-5.322	-5.393	-4.653	-5.039
F-Statistic	46.463***	32.676***	17.619***	155.915***
$Q(1)$	1.077	1.104	0.014	0.136
$Q(5)$	4.422	3.247	8.080	1.856
$Q^2(1)$	0.444	0.064	0.023	0.907
$Q^2(5)$	0.583	2.660	1.643	3.278
ARCH(1)	0.438	0.063	0.022	0.885
ARCH(5)	0.125	0.481	0.292	0.890
ARCH/GARCH	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
Distribution	Normal	Normal	Normal	Normal

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. F-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. $Q(1)$ and $Q(5)$ are Ljung-Box test statistics for residual serial correlation at the 1st and 5th orders.
4. $Q^2(1)$ and $Q^2(5)$ are Ljung-Box test statistics for squared residual serial correlation at the 1st and 5th orders.
5. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

Source: Compiled by author

The results in Panel B indicate that there are no substantial changes in the magnitudes and signs of the estimated coefficients after controlling for outliers. However, the coefficient on

$UCPI_{t-1}$ is now marginally statistically insignificant (*p*-value of 0.131). Notably, the decrease in the absolute size of the coefficient on $UCPI_{t-1}$ in Table 8.4 is less pronounced than the decrease under the LS methodology after controlling for outliers. The question that arises is whether the unrestricted model in Panel A or the unrestricted model which excludes outliers in Panel B should be chosen as the multifactor model of the return generating process of South African stock returns. The $\bar{R}^2$ indicates that after controlling for outliers in Panel B, the model explains 62.6 percent of the variation in returns – an improvement over the unrestricted model in Panel A. Furthermore, the AIC statistic is now -5.393 suggesting that removing outliers improves the fit of the model. Regression diagnostics indicate that the unrestricted model after controlling for outliers is also appropriate. However, although this version of the unrestricted model explains a greater amount of the variation in returns and the AIC statistic is lower, the coefficient on the dummy factor is statistically insignificant suggesting that the ARCH/GARCH framework, unlike the LS framework, is robust to outliers. Furthermore, because the coefficient on the dummy factor is statistically insignificant, it can be argued that it is unnecessary to control for outliers under the ARCH/GARCH framework. Following from this argument, it can be further postulated that the strongest argument for retaining outliers is that the removal of outliers will artificially improve the characteristics of the model (Brooks, 2008). In light of these arguments, the unrestricted model in Panel A is accepted as the most appropriate multifactor model of the return generating process.

The results of the restricted models are reported in Panel C and Panel D. The purpose of these models in the *present* context is to act as benchmarks and these models can be considered as "naive models" representative of simpler models. This is motivated by Reinganum's (1981) argument that there is no justification for accepting a more complex model that does not convey more information relative to a simpler model . In a similar vein, Elton *et al*. (1995) suggest that a more complex model can only be considered for further use if it outperforms a simpler model. One approach to determining whether a multifactor model is more appropriate relative to a single-factor model or a simpler multifactor model, is to determine whether the unrestricted model encompasses the two models (Brooks, 2008). In the context of this study, this suggests that an unrestricted model should be accepted if and only if it provides more insight into the return generating process of South African stock returns by explaining a greater amount of variation relative to simpler specifications and results in a more adequate fit (see section 2.2: 15 & 2.2.4).

The results in Panel C - those of the restricted model incorporating the domestic risk factors - indicate that these factors explain 21.7 percent of the variation in returns on the JSE All-Share Index. This suggests that there *is* value in incorporating these factors into a multifactor model. The results in Panel D indicate that $UFTW_t$ explains 41.6 percent of the variation in returns. The AIC statistic of -5.039 is lower than the AIC statistic of -4.653 of the restricted model in Panel C suggesting that the restricted single-factor model provides a better fit relative to the seven-factor model. However, the unrestricted model in Panel A performs better relative to both restricted models in Panel C and Panel D. The $\bar{R}^2$ suggests that the unrestricted model explains a higher percentage of variation in returns than either of the restricted models in Panel C and Panel D. The AIC statistic is lower than that of the two restricted models suggesting that this model is more suitable relative to these simpler specifications. Furthermore, if the unrestricted model is considered to be a direct extension of the single-factor model in Panel D, then the seven domestic risk factors explain at least an additional 14.6 percent of the variation in returns on the JSE All-Share Index under the ARCH/GARCH framework. This represents an increase of 35.096 percent in $\bar{R}^2$.[130] These findings suggest that the unrestricted model in Panel A encompasses the restricted models in Panel C and Panel D in terms of explanatory power and adequacy of fit (addressed further in section 8.4.5 & 8.5.4). Therefore, the unrestricted multifactor model *is* a more appropriate model of the return generating process (Reinganum, 1981; Brooks, 2008).

[130] Estimated as a percentage change between the $\bar{R}^2$ of the unrestricted model and that of the single-factor model.

8.3.3. Conditional variance

Table 8.5 reports the results of the ARCH/GARCH models of conditional variance underlying the unrestricted and restricted versions of the model in Table 8.4.

Table 8.5: ARCH/GARCH model of JSE All-Share Index conditional variance

	Panel A	Panel B	Panel C	Panel D
ARCH/GARCH	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)	GARCH(1,1)
Distribution	Normal	Normal	Normal	Normal
ω	1.22E-05	1.38E-05	3.08E-05	5.09E-05
α_1	0.188**	0.145	0.143	0.199***
β_1	0.780***	0.802***	0.812***	0.682***
F-Statistic	201.193***	105.504***	143.534***	75.544***

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).

Source: Compiled by author

The results in Panel A of Table 8.5 indicate that both ARCH and GARCH terms are individually and jointly statistically significant, suggesting that the conditional variance of returns on the JSE All-Share Index is of a time-varying nature. This confirms that the GARCH(1,1) specification is appropriate for the modelling of returns on the South African stock market. In Panel B, only the coefficient on the GARCH term is statistically significant, although the *F*-statistic suggests that the ARCH and GARCH terms are jointly statistically significant. This also holds true for the conditional variance model underlying the restricted version of the model in Panel C of Table 8.4. In Panel D, both ARCH and GARCH terms are individually and jointly statistically significant. These results suggest that the conditional variance is of a time-varying nature and therefore, the ARCH/GARCH framework is appropriate. Following these findings, all further analysis is based upon the *unrestricted* model estimated within the ARCH/GARCH econometric framework.

8.3.4. Additional risk factors in South African stock returns

The multifactor model of returns on the JSE All-Share Index suggests that returns are driven by eight risk factors with each factor representative of one of several risk categories. While the unrestricted model estimated within the ARCH/GARCH framework provides an adequate description of the return generating process of returns on the JSE All-Share Index, it is possible that there are other factors that explain returns and therefore, should also be considered as valid APT factors. It is assumed under the APT framework that the covariance between the residuals, ε_{it}, and factor realizations, F_{kt} , is zero (see equation (2.3); section

2.1; Burmeister *et al*., 1994). This assumption will hold for factors incorporated into the model of the return generating process. The covariance between factors *not* included in the model and the residuals should *ideally* be zero if all systematic risk has been captured by risk factors in the model. *If this is the case, then this indicates that the set of factors used to explain returns proxies for and captures information in the remaining factors not incorporated into the model.* However, if this assumption does not hold, then factors which show significant covariance with the residuals should be considered as APT risk factors that can also be used to explain the return generating process. It is possible that in this study, this assumption does not hold, as the model is constructed with the intention of identifying risk categories and not maximizing the amount of variation explained (see Van Rensburg, 1996).

To test whether the eight factors in the model exhaust all explanatory power and to address the exclusion of factors with high levels of explanatory power, the *correlation* between the remaining factors and the residuals of the model is examined. Instead of considering the *covariance* between the remaining factors and the residuals, correlation is considered, as correlation indicates the direction and strength of the linear relationship (Galpin, 2009).[131] Correlation coefficients indicating the level of correlation between the remaining risk factors and residuals obtained from the unrestricted model are reported in Panel A of Table 8.6. Correlation coefficients indicating the level of correlation between returns on the JSE All-Share Index and the risk factors are reported in Panel B for comparative purposes.

The results in Panel A of Table 8.6 indicate that only seven candidate risk factors remain out of a total of twenty risk factors not included in the model. Factors that are significantly correlated with the residuals are UNK_t, UMP_t, $UTBT3_t$, $USAGB10_t$, $USAGB30_t$, $UCOM_t$ and $UMET_t$. However, in each instance, the level of correlation decreases suggesting that the risk factors incorporated into the model explain most of the variation in returns. For example, whereas the level of correlation between UNK_t and returns on the JSE All-Share Index is 0.549, the level of correlation between UNK_t and the residuals is only 0.120. This suggests that although UNK_t contains *some* relevant information, most of the information is captured by the risk factors incorporated into the model. As the correlation coefficient of

[131] Correlation coefficients are bounded by values between -1 and 1.

UNK_t and the residuals is marginally statistically significant (*p*-value of 0.0998) and its explanatory power for the residuals is almost negligible ($\bar{R}^2 = 0.0105$), it can be ignored.

Table 8.6: Correlation of JSE All-Share Index residuals with omitted risk factors

	Panel A	Panel B
	ε_{UM_t}	UM_t
UDJ_t	0.045	0.619***
$UFTSE_t$	0.080	0.608***
$UMSCI_t$	0.010	0.638***
$UMSCIR_t$	-0.007	0.467***
UNK_t	0.120*	0.549***
UMP_t	0.126*	0.217***
UBP_t	0.020	0.136*
$USLS_{t-2}$	-0.012	0.158**
$UM1A_{t-1}$	0.005	0.144**
$UM3_{t-2}$	-0.118	-0.225***
$UTBT3_t$	-0.158**	-0.291***
$USAGB10_t$	-0.291***	-0.406***
$USAGB30_t$	-0.304***	-0.410***
$UCOM_t$	0.167**	0.386***
$UMET_t$	0.129*	0.429***
$UNFCI_t$	0.016	0.168***
UTT_t	0.047	0.124*
$ULTT_t$	-0.019	0.393***
$UCTT_{t-1}$	0.054	0.148**
ULI_t	0.047	0.222***

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.

Source: Compiled by author

Correlation between returns and UMP_t declines from 0.217 to 0.126, but remains statistically significant suggesting that UBP_{t-1} may not fully capture risk associated with unexpected changes in real activity. Another factor of interest, which is significantly correlated with the residuals, is $UMET_t$. However, as with UMP_t, the correlation coefficient declines from 0.429 to 0.129 suggesting that most of the impact of $UMET_t$ is captured by the eight risk factors in the model. As the correlation between $UFTW_t$ and $UMET_t$ is 0.413, it is plausible that $UFTW_t$ captures a significant proportion of the impact of $UMET_t$ on returns. As with UNK_t, the explanatory power of UMP_t and $UMET_t$, after controlling for the eight risk factors is almost negligible; single-factor regressions of the residuals on these factors indicate that individually, these factors explain just over 1 percent of the variation in the residuals.

Changes in general commodity prices, $UCOM_t$, retain some explanatory power; although, the correlation coefficient decreases from 0.386 to 0.167. A single-factor regression of the residuals onto $UCOM_t$ shows that this factor explains 2.274 percent of the variation in the residuals. This is comparable to the amount of variation explained by $UM3_{t-1}$ and $UCPI_{t-1}$ in Table 8.1. The high level of correlation between $UCOM_t$ and $UMET_t$ (0.579) suggests that $UCOM_t$ can be incorporated into the model to capture general commodity price risk *and* the impact of $UMET_t$. Furthermore, as $UCOM_t$ has greater *residual* explanatory power relative to $UMET_t$, it is a more appropriate candidate risk factor for explaining time series variation if a parsimonious description of the return generating process is sought. Correlation between the residuals and $UTBT3_t$ decreases by approximately half (in absolute terms) after controlling for the eight risk factors, but is still statistically significant. $USAGB10_t$ and $USAGB30_t$ - the two measures of long-term interest rates - stand out. There is only a relatively small decrease in absolute correlation between these factors, returns and the residuals; correlation decreases from 0.406 to 0.291 for $USAGB10_t$ and from 0.410 to 0.304 for $USAGB30_t$ after controlling for the eight risk factors. Single-factor regressions of the residuals onto these factors indicate that the explanatory power of these factors declines; $USAGB10_t$ explains 7.984 percent of the variation in the residuals whereas $USAGB30_t$ explains 8.725 percent. This represents a decline in the explanatory power of these factors of approximately 8 percent and 7.6 percent respectively (see Table 8.1) indicating that a significant portion of the impact of long-term interest rates is captured by the eight factors incorporated into the model. Nevertheless, the presence of residual explanatory power suggests that at the very least, the long-term interest rate factors should be considered as important risk factors in the South African financial environment.

Remaining factors – factors with residual explanatory power - can be incorporated into the return generating process within a two-stage framework (section 2.2.3: 25; Yli-Olli & Virtanen, 1992). In the first stage, returns are regressed on the eight factors used in the model of the return generating process. In this way, the essence of the model is preserved. In the second stage, residuals are regressed onto any remaining factors. These remaining factors will be either a long-term interest rate factor or, $UCOM_t$ or both. A two-stage time series approach will avoid a multicollinearity problem and the associated dilution of the impact of factors that is likely to arise with the inclusion of these factors in the original specification (see Blanchard, 1987). Alternatively, $USAGB10_t$ or $USAGB30_t$ and $UCOM_t$ can be substituted for a number of factors in the eight-factor model. This approach however will deviate from the approach undertaken, which seeks to ascribe variation within the return generating process to several risk categories. Finally, a third solution is to treat the residuals in *generalizations* of the model as a residual market factor that captures the impact of the omitted factors in Panel A of Table 8.6 and that of other unspecified omitted risk factors (Burmeister & Wall, 1986; Van Rensburg, 1996; Berry *et al*., 1988).

8.3.5. South African stock market: A synthesis

After testing a number of combinations of factors and on the basis of the univariate analysis conducted in section 8.2, it was decided that a model representative of the return generating process of South African stock returns should include as many risk factor categories as possible (section 8.3.1: 173). This approach results in a eight-factor model for the South African stock market (equation 8.1). An identical model is applied for the economic groups and industrial sectors with the residuals of the market model treated as the residual market factor, $UM\varepsilon_t$ (additional factor in equation 8.2; discussed in section 8.4 & 8.5). The factors chosen to represent the return generating process of the South African stock market are international risk ($UFTW_t$), inflation ($UCPI_{t-1}$), inflation expectations ($URBAS_t$), real activity (UBP_{t-1}), money supply ($UM3_{t-1}$), oil prices ($UOIL_t$), the exchange rate ($UZARUS_t$), and the business cycle (UCI_t).

Preliminary analysis relying upon the LS framework confirms the appropriateness of the ARCH/GARCH framework (section 8.3.1; Table 8.2 & 8.3); ARCH effects are present in the unrestricted model. Nevertheless, the model of the return generating process explains returns on the JSE All-Share Index rather well; the R^2 suggests that almost 60 percent of the variation in returns is explained (see section 8.3.1: 177; Table 8.2, Panel B). ARCH/GARCH modelling is undertaken next in section 8.3.2 and on the basis of the AIC, a GARCH(1,1) model is chosen (see Table 8.4). The unrestricted version of the model explains over 56 percent of the variation in returns with $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$, and UCI_t found to be statistically significant individually and jointly (see section 8.3.2 for a detailed discussion; Table 8.4, Panel A for main results). A lower AIC (relative to the AIC for the LS model) for the unrestricted GARCH(1,1) model confirms that the ARCH/GARCH framework is more appropriate for the modelling of South African stock returns. Notably, both the R^2 and the AIC suggest that the unrestricted model outperforms restricted versions of the model in terms of explanatory power and fit (see Table 8.4). This confirms the appropriates of the multifactor model specification used to describe the return generating process (see section 8.3.2: 183; section 2.2.4). The results of the ARCH/GARCH models of conditional variance (both restricted and unrestricted; section 8.3.3; Table 8.5) further confirm the appropriateness of using the ARCH/GARCH framework for modelling South African stock market returns. The results in Panel A of Table 8.6 suggest that there *are* other factors that feature in the return generating process (see section 8.3.4: 186). The failure of the multifactor approach utilized to capture the impact of these factors suggests a limitation of the approach. Factors with residual explanatory power are the long-term interest rate factors, $USAGB10_t$ and $USAGB30_t$. It is suggested that further consideration be given to these factors (section 8.3.4: 189). General commodity prices, $UCOM_t$, also carry some residual explanatory power.

8.4. Economic groups

8.4.1. Economic group model

The preceding discussion indicates that the return generating process of South African stock returns can be described by a multifactor model incorporating innovations in eight risk factors, representative of international risk, inflation, real activity, money supply, commodity prices, exchange rates and the domestic business cycle. This multifactor model – estimated within the ARCH/GRACH framework - explains more than half of the variation in returns on the JSE All-Share Index and encompasses simpler specifications in terms of explanatory power and adequacy of fit. It is assumed that because these factors explain returns on an aggregate, they have a pervasive impact upon the South African stock market implying that these factors will describe returns on economic groups and industrial sectors (section 3.2.2: 62: Van Rensburg, 1996, 2000).

The economic group model is based upon the unrestricted general market model in equation (8.1) and incorporates the same factors used to model returns on the JSE All-Share Index *and* a residual market factor denoted by $UM\varepsilon_t$[132] obtained from the unrestricted GARCH(1,1) model. $UM\varepsilon_t$ is assumed to reflect the impact of factors that are significantly correlated with the residuals in Panel A of Table 8.6 and to act as a *catch-all* proxy for omitted risk factors (Van Rensburg, 1996; Berry *et al*., 1988). The specification is as follows:

$$R_{it} = \alpha + b_{UM\varepsilon}UM\varepsilon_t + b_{UFTW}UFTW_t + b_{UCPI}UCPI_{t-1} + b_{URBAS}URBAS_t + b_{UBP}UBP_{t-1} + b_{UM3}UM3_{t-1}$$
$$+ b_{UOIL}UOIL_t + b_{UZARUS}UZARUS_t + b_{UCI}UCI_t + \varepsilon_{it} \qquad (8.2)$$

where R_{it} is the return on economic group index *i* at time *t*, $UM\varepsilon_t$ is the residual market factor and $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t are innovations in the respective risk factors incorporated into the model. Residuals for economic group *i* are denoted by ε_{it} and the model is estimated within the ARCH/GARCH framework.

The results in Table 8.7 indicate that of the 81 estimated factor coefficients, 59 are statistically significant at the 10 percent, 5 percent or 1 percent levels of significance. An average $\bar{R}^2$ of 0.570 suggests that on average, the model explains 57 percent of the variation in returns on the economic group indices. With regard to the maximum and minimum amount of variation explained by the model, the model explains 72.7 percent of variation in returns on the oil and gas economic group index and 39.5 percent of variation in returns on the technology economic group index. *F*-statistics for Wald's test of coefficient restrictions suggest that the null hypothesis of all coefficients jointly equalling zero can be rejected for each economic group. As with the results of the unrestricted model of returns on the JSE All-Share Index in Panel A of Table 8.4, the rejection of the null hypothesis for all economic group indices points towards the significance of having multiple factors in the model. This provides further support for a *multifactor* model of the return generating process of the South African stock market (see Sadorsky & Henriques, 2001).

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[132] Where $UM\varepsilon_t$ in equation (8.2) is ε_{UM_t} in equation (8.1) and equation (8.1) is estimated within the ARCH/GARCH framework.

Table 8.7: ARCH/GARCH model of economic group returns

	Oil&Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecom.	Financials	Technology
Intercept	-0.001	-0.004***	-0.003***	-0.001	-0.002**	-0.003***	-0.000	-0.002*	0.002
$UM\varepsilon_t$	1.221***	1.153***	0.995***	0.732***	0.822***	0.845***	0.760***	0.675***	0.920***
$UFTW_t$	0.893***	0.934***	0.678***	0.791***	0.680***	0.623***	0.785***	0.834***	1.461***
$UCPI_{t-1}$	-0.183	-0.844***	-0.791***	-0.398	-0.947***	-1.139***	-0.717*	-0.446*	-0.399
$URBAS_t$	-0.676**	-1.107***	-0.982***	-0.590**	-0.812***	-0.830***	-1.044***	-1.086***	-0.591*
UBP_{t-1}	0.072***	0.098***	0.052***	0.021	0.034*	0.013	0.140***	0.056***	0.024
$UM3_{t-1}$	1.204***	0.767***	0.246	0.653***	0.109	0.377*	-0.334	0.394**	0.357
$UOIL_t$	0.166***	0.155***	-0.019	-0.001	0.023	-0.023	-0.084*	-0.056**	0.084*
$UZARUS_t$	0.490***	0.229***	-0.106**	0.313***	0.091	-0.224***	-0.214**	0.073	0.699***
UCI_t	2.168***	1.282***	0.385	1.226**	0.006	0.264	-0.301	-0.084	0.021
$\bar{R}^2$	0.727	0.574	0.700	0.489	0.543	0.609	0.434	0.659	0.395
AIC	-5.228	-4.851	-5.515	-4.911	-5.204	-5.161	-4.292	-5.433	-3.727
F-Statistic	93.181***	80.366***	81.897***	21.283***	151.514***	53.623***	41.710***	56.127***	36.419***
$Q(1)$	3.537*	0.024	0.029	4.302**	0.1780	1.994	0.088	1.292	2.851*
$Q(5)$	7.961	4.567	3.612	10.151*	4.704	5.876	12.534**	9.720*	10.162*
$Q^2(1)$	0.410	0.000	0.0442	0.3151	0.020	0.036	0.841	0.267	0.041
$Q^2(5)$	2.557	3.073	1.082	1.1540	2.470	1.508	2.766	1.294	2.731
ARCH(1)	0.398	0.000	0.043	0.306	0.020	0.035	0.826	0.259	0.039
ARCH(5)	0.659	0.701	0.189	0.222	0.573	0.288	0.534	0.251	0.458
ARCH/GARCH	GARCH(2,2)	IGARCH(2,2)	GARCH(1,2)	GARCH(1,2)	EGARCH(2,2)	EGARCH(2,1)	GARCH(2,1)	EGARCH(1,2)	EGARCH(1,2)
Distribution	GED	GED	Student's *t*	Student's *t*	Normal	Normal	GED	Student's *t*	Normal

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. $Q(1)$ and $Q(5)$ are Ljung-Box test statistics for residual serial correlation at the 1st and 5th orders.
4. $Q^2(1)$ and $Q^2(5)$ are Ljung-Box test statistics for squared residual serial correlation at the 1st and 5th orders.
5. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

Source: Compiled by author

The results in Table 8.7 indicate that almost all of the factors with the exception of UCI_t have a pervasive impact on returns on the economic group indices. The most important factors are $UM\varepsilon_t$ and $UFTW_t$. The relationship between each return series and $UM\varepsilon_t$ is positive and statistically significant, suggesting that returns on the economic group indices move positively with returns on the JSE All-Share Index. The positive and statistically significant relationship between returns for each economic group and $UFTW_t$ suggests that returns on economic groups respond positively to innovations in the global financial environment. Furthermore, this finding suggests that the South African stock market is highly integrated with international and foreign markets (see Clare & Priestley, 1998; Bilson *et al*., 2001). $UCPI_{t-1}$ has a negative and statistically significant impact upon six out of nine economic groups suggesting that, on a balance of probabilities, it is a pervasive factor in the South African stock market. Economic groups unaffected by innovations in $UCPI_{t-1}$ are the oil and gas, consumer goods and technology economic groups. A possible reason for the lack of a significant relationship between $UCPI_{t-1}$ and returns on these three economic groups is that inflation costs can be passed on in the form of higher prices for these economic groups and therefore, expected cash flows are not affected by inflation (Berry *et al*., 1988; Nandha & Faff, 2008). Berry *et al*. (1988) also find that inflation risk does not impact returns on the oil economic group in the US. This suggests that a pass-through effect explanation may be applicable to the South African stock market, even if this explanation is limited to returns on the oil and gas economic group. Returns on all economic groups are negatively and significantly related to $URBAS_t$, suggesting that unexpected changes in inflation expectations are more important relative to unexpected changes in inflation in explaining South African stock returns.

Real activity, UBP_{t-1}, has a positive and statistically significant impact on the returns on six out of nine economic groups. The economic groups for which returns are not significantly related to UBP_{t-1} are the consumer goods, consumer services and technology economic groups. It may be that these specific economic groupings are simply not influenced by changes in real activity or that there are *other* proxies for real activity that are better at measuring the risk associated with real activity for these groups. Innovations in the broad money supply, $UM3_{t-1}$, have a statistically significant and positive impact upon the returns on five out of nine economic groups. The coefficient on $UM3_{t-1}$ is positive but statistically insignificant for returns on the industrials, health care and technology economic groups, and negative but statistically insignificant for returns on the telecommunications economic group. The overwhelmingly positive relationship between returns and $UM3_{t-1}$ may arise for a number of reasons. Mookerjee and Yu (1997) suggest that monetary aggregates carry policy information content. Bilson *et al*. (2001) and Kandir (2008) argue that increases in the money supply positively impact real activity, which suggests greater expected cash flows. Günsel and Çukur (2007) suggest that an increase in the money supply implies falling interest rates, which in turn translate into higher expected future cash flows. The generally positive relationship implies that whatever information is contained in $UM3_{t-1}$ has a positive impact upon returns and this factor captures *certain* aspects of real activity *or* the policy stance of the SARB.

The impact of $UOIL_t$ is interesting. The results of the unrestricted model of returns on the JSE All-Share Index in Panel A of Table 8.4 suggest that changes in oil prices have a positive impact upon returns. This is somewhat contrary to theory, which generally stipulates that oil price shocks have a *negative* impact upon returns (Kaul & Seyhun, 1990; Nandha & Faff, 2008). Results in Table 8.7 indicate that returns for five out of nine economic groups are *negatively* related to $UOIL_t$. Having made this observation, it is surprising that returns on the JSE All-Share Index are *positively* related to $UOIL_t$. A clue to the positive impact of $UOIL_t$ on returns on the JSE All-Share Index is offered by the positive (and statistically significant) coefficient on $UOIL_t$ for the oil and gas and basic materials economic groups. The coefficient on $UOIL_t$ for these two economic groups is 0.166 and 0.155 respectively. The most *negative* coefficient on $UOIL_t$ is -0.084 for returns on the telecommunications economic group. This suggests that the high levels of positive sensitivity of the oil and gas and basic material economic groups to $UOIL_t$ bias the results of the model of JSE All-Share Index returns in favour of a positive relationship between $UOIL_t$ and returns. In their entirety, results indicate a negative and statistically significant relationship between returns on the telecommunications and financial economic group indices and $UOIL_t$ and a positive and statistically significant relationship between returns on the oil and gas, basic materials and technology economic group indices. That the direction of the relationship between returns and a specific factor may differ across economic groups is recognized in the literature; events that negatively impact returns on one economic group can positively impact returns on another economic group (Beenstock & Chan, 1988; Jones & Kaul, 1997). As $UOIL_t$ has a statistically significant impact on the returns of five out of nine economic groups, it may be considered as a pervasive factor. $UZARUS_t$ has a statistically significant impact on the returns of seven out of nine economic groups. Returns on the oil and gas, basic materials, consumer goods, and technology economic groups are positively and significantly related to $UZARUS_t$ whereas returns on the industrials, consumer services and telecommunications economic groups are negatively and significantly related to $UZARUS_t$. As with $UOIL_t$, the inconsistency in the direction of the relationship is not unexpected. The results in Table 8.7 are certainly consistent with the hypothesis that the nature and composition of an economic group determines the direction of the relationship between exchange rate fluctuations and returns (Jorion, 1990; Griffin & Stultz, 2001).[133]

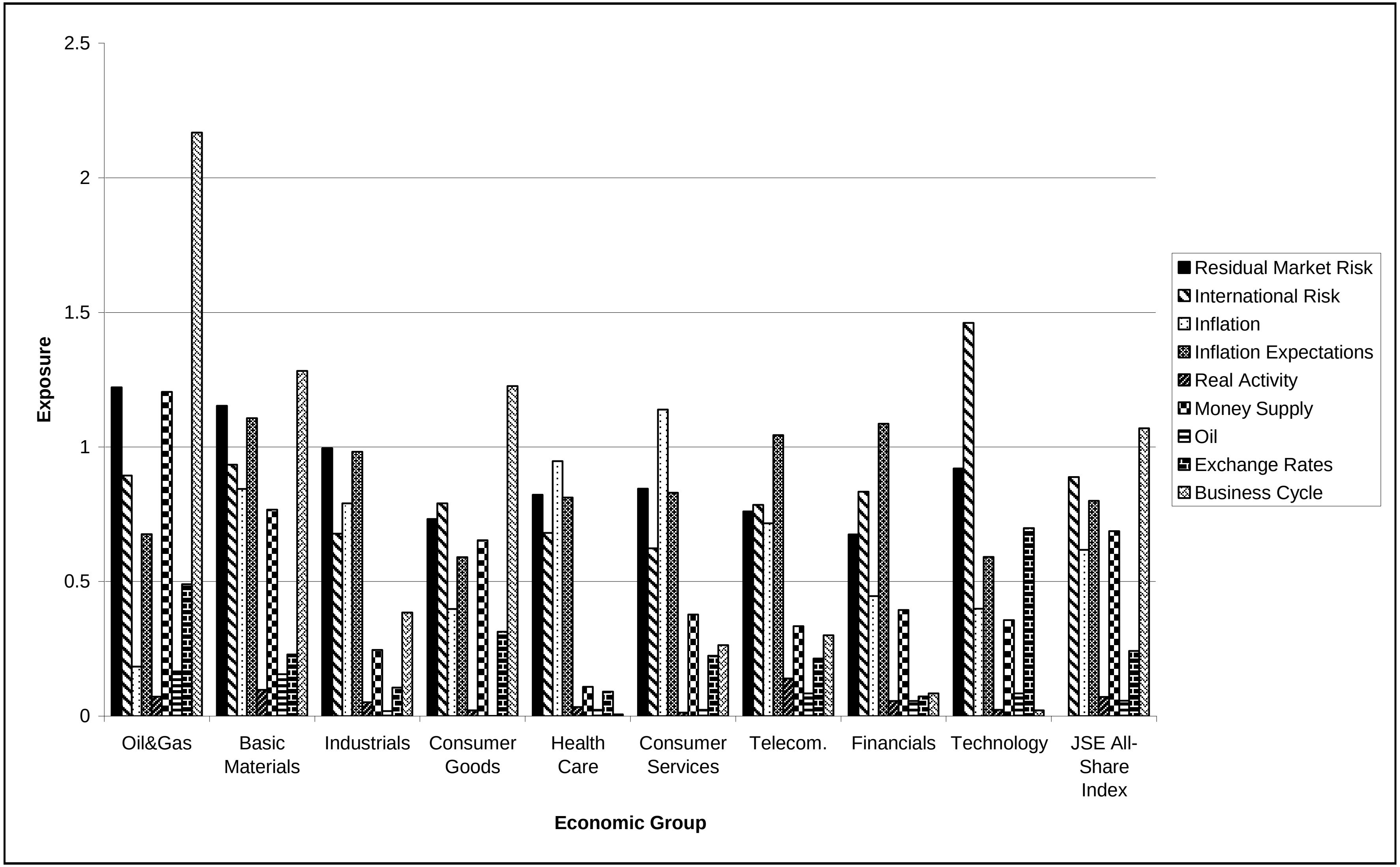
UCI_t does not have a pervasive impact upon returns; only three economic groups are significantly affected by UCI_t. Returns on the oil and gas, basic materials and the consumer goods economic group indices are positively and significantly related to UCI_t. The relationship is positive but statistically insignificant for four out of the remaining six groups and negative but statistically insignificant for two out of the remaining six groups. The results in Table 8.7 contrast with the results of the model of returns on the JSE All-Share Index in Panel A of Table 8.4, which suggest that UCI_t has a systematic impact. As with $UOIL_t$, it is possible that the high levels of sensitivity of returns on certain economic groups which compromise the JSE All-Share Index bias the results of the unrestricted model in Panel A of Table 8.4 in favour of a positive and statistically significant relationship between returns and UCI_t at the aggregate level. The

[133] A more definitive pronouncement upon the nature of the relationship requires a detailed investigation of the constituents of the economic groups and the nature of their operations, an endeavour that is beyond the scope of this study. Examples of studies concerned with a more detailed overview of specific industries as opposed to a generalized overview of the return generating process of an extended number of economic groups and industrial sectors are those of Sadorsky (2001) and Sadorsky and Henriques (2001).

coefficients on UCI_t for the oil and gas, basic materials and consumer goods economic groups are 2.168, 1.282 and 1.226 respectively. The economic group with the next highest exposure to UCI_t is the industrials economic group with a coefficient of 0.385. This suggests that returns on the former three economic group indices show a disproportionate sensitivity to UCI_t, which biases the results of the unrestricted model in Panel A of Table 8.4 in favour of a statistically significant relationship. Furthermore, this implies that the impact of UCI_t is specific to a subset of South African stocks (see Kryzanowski & To, 1983). It must be recalled that UCI_t, the coincident composite index – an indicator of the business cycle – is used in place of the default spread to capture the impact of business cycles upon returns (Chan *et al*., 1985). As the impact of UCI_t is limited to three economic groups, it may be hypothesized that the role of the default spread will also be limited in the South African context. This contrasts with its relative importance in US markets (see Figure 3.1; Burmeister & Wall, 1986; Berry *et al*., 1988).

8.4.2. Risk exposure profile

Following Berry *et al*. (1988) , the risk exposure profile of the economic groups and the JSE All-Share Index is reported in Figure 8.3 (also see Figure 3.1). Estimated exposures – factor coefficients – are reported as absolute values. Returns on the oil and gas economic group are especially sensitive to $UM\varepsilon_t$ while returns on other economic groups are less sensitive to $UM\varepsilon_t$. $UFTW_t$ has a disproportionate impact upon returns on the technology economic group whereas returns on all other economic groups are affected to a lesser extent by this factor. Whereas returns on the consumer services economic group are highly responsive to $UCPI_{t-1}$, returns on the oil and gas economic group are least responsive to innovations in this factor. While returns on the financials economic group are not very sensitive to $UCPI_{t-1}$, they are highly sensitive to $URBAS_t$. Returns on the financials, basic materials and telecommunication economic groups exhibit similar levels of sensitivity to $URBAS_t$. Returns on the telecommunication economic group are highly sensitive to UBP_{t-1} relative to other groups followed by returns on the basic materials and oil and gas economic groups which show a similar level of sensitivity to UBP_{t-1}. Returns on the consumer goods and consumer services economic groups are least sensitive to UBP_{t-1}. Returns on the oil and gas economic group are highly sensitive to $UM3_{t-1}$ whereas returns on the health care economic group are least sensitive to $UM3_{t-1}$. The level of exposure to $UOIL_t$ is similar for returns on the oil and gas, and the basic materials economic groups suggesting that these two groups share similar characteristics. As can be expected, returns on the oil and gas economic group are the most sensitive of all groups to $UOIL_t$.



Source: Compiled by author

Figure 8.3: Risk exposure profile for economic groups

An interesting observation in Figure 8.3 is that returns on the technology economic group exhibit the highest level of sensitivity out of the nine economic groups to $UZARUS_t$ as well as the highest level of sensitivity to $UFTW_t$. Given this observation and that $UZARUS_t$ potentially captures elements of international risk, it may be hypothesized that returns on this economic group are mainly driven by international risk factors as opposed to domestic risk factors. Returns on the oil and gas economic group are most sensitive to UCI_t out of the nine economic groups. This suggests that oil prices – which have a strong impact on returns for this group - are also related to fluctuations in the business cycle. This argument is supported by a statistically significant correlation coefficient (unreported in-text) of 0.233 for UCI_t and $UOIL_t$. In contrast, returns on the health care economic group are almost insensitive to UCI_t. It can be argued that risk exposures for these economic groups generally correspond to the intuition behind the distribution of different risk types.

The risk exposure profile of returns on the JSE All-Share Index reported in Figure 8.3 can be used to draw comparisons between risk inherent in returns on the economic groups and the South African stock market as a whole. For example, returns on the oil and gas economic group index are more sensitive to UCI_t than returns on South African stocks in general. Whereas the South African stock market is less responsive to $UOIL_t$ relative to returns on the oil and gas economic group, returns on the this group are relatively less sensitive to $UCPI_{t-1}$. One can also gauge the level of *overall* riskiness of certain economic groups relative to the South African stock market by comparing estimated exposures to risk factors in the return generating process. Returns on the consumer goods economic group exhibit exposures to the risk factors that are *generally* lower for most risk types relative to the exposures of the South African stock market. This suggests that the consumer goods economic group is less risky relative to the South African stock market (see Chan *et al.*, 1990). The converse is true for groups that exhibit exposures to risk factors that are *generally* greater than those of returns on the JSE All-Share Index such as the oil and gas, and basic materials economic groups. Risk exposure profiles reveal a number of interesting patterns that warrant further investigation and should be of great interest for further research.

8.4.3. Conditional variance

The type of ARCH/GARCH specification selected to describe the conditional variance of returns on the economic group indices provides insight into the nature of the conditional variance of the respective series. The AIC statistic indicates that the EGARCH specification is appropriate for the conditional variance of four economic groups; namely the health care, consumer services, financials and technology economic groups (see section 5.3.3 & 6.3.5). As the EGARCH model captures the asymmetric relationship between returns and conditional variance, this suggests that there is an asymmetric relationship between returns on these economic groups and conditional variance. The asymmetric relationship can be investigated further by examining the coefficient of asymmetry, γ_1, for each estimated EGARCH conditional variance specification (Nelson, 1991; Aga & Kocaman, 2006). Results in Table 8.8 indicate that the coefficient of asymmetry is negative and statistically significant for the conditional variance models of the health care, consumer services and technology economic groups suggesting that these series exhibit a leverage effect. Statistically significant *F*-statistics for Wald's test of coefficient restrictions for the four EGARCH models reveal that conditional variance is not constant for each economic group and therefore, ARCH/GARCH modelling is appropriate. However, whereas both ARCH and GARCH parameters are statistically significant at various orders in the EGARCH model for the health care, consumer services and technology economic groups, only the first order GARCH parameter is statistically significant for the financials economic group.

Table 8.8: ARCH/GARCH models of economic group conditional variance

	Oil&Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecom.	Financials	Technology
ARCH/GARCH	GARCH(2,2)	IGARCH(2,2)	GARCH(1,2)	GARCH(1,2)	EGARCH(2,2)	EGARCH(2,1)	GARCH(2,1)	EGARCH(1,2)	EGARCH(1,2)
Distribution	GED	GED	Student's *t*	Student's *t*	Normal	Normal	GED	Student's *t*	Normal
ω	1.48E-05	-	1.28E-5***	4.51E-6***	-2.717***	-0.374***	2.44E-05	-0.651	-2.560*
α_1	-0.077***	-0.079***	0.017***	0.002*	0.523**	0.308***	0.379**	0.121	0.346***
α_2	0.164***	0.129***	-	-	0.201	-0.526***	-0.283	-	-
β_1	1.377***	1.422***	1.917***	1.992***	-0.110*	0.931***	0.884***	1.506***	-0.120
β_2	0.505***	-0.472	-0.989***	-1.003***	0.848***	-	-	-0.574	0.790***
γ_1	-	-	-	-	-0.219**	-0.132***	-	-0.076	-0.202***
F-Statistic	163.693***	33.925***	76053.95***	1067145***	1019.140***	1.66E+11***	780.526***	254.176***	481.120***

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).

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An IGARCH(2,2) model is employed to describe the conditional variance for the basic materials group. The IGARCH model is based upon the assumption that finite variance does not exist and can be used to capture infinite persistence in conditional variance (section 6.3.3; Kang *et al.*, 2009; McMillan & Ruiz, 2009). Although persistence is most often observed with high-frequency data, there is no reason to believe that the conditional variance of lower frequency data will not exhibit infinite persistence. As the AIC statistic indicates the best fitting model out of a number of alternatives, a finding that the conditional variance of the basic materials economic group is described by an IGARCH model suggests that the conditional variance of this series is characterized by infinite persistence. The first and second order ARCH parameters are statistically significant together with the first order GARCH parameter and the *F*-statistic indicates that the ARCH and GARCH parameters are jointly statistically significant. This implies that the conditional variance of returns on the basic materials economic group is of a time-varying nature and therefore, the fitted IGARCH model is appropriate.

The GARCH model is chosen to describe the conditional variance of returns on the oil and gas, industrials, consumer goods and telecommunications economic groups. The GARCH model specification permits a more flexible lag structure with a longer memory relative to the ARCH model by incorporating lagged conditional variance terms (section 6.3.2; Bollerslev, 1986; Elyasiani & Mansur, 1996). The results indicate that the ARCH and GARCH parameters of the GARCH models fitted to the oil and gas, industrials, consumer goods and telecommunications economic groups are statistically significant for each series. For the oil and gas economic group, both the first and second order ARCH and GARCH parameters are statistically significant whereas for the telecommunications economic group, the first ARCH and the sole GARCH parameter is statistically significant. The single ARCH parameter and both GARCH parameters for the industrials and consumer goods economic groups are statistically significant. The *F*-statistic is statistically significant for all GARCH models implying that the conditional variance of returns on the oil and gas, industrials, consumer goods and telecommunications economic groups is of a time-varying nature and therefore, ARCH/GARCH modelling is appropriate. As the GARCH specification is selected to describe the conditional variance of these four economic groups, it may be inferred that the conditional variance of these economic groups exhibits long memory. It is also worth noting that the GARCH models for these economic groups require more than one lagged conditional variance (h_t) term suggesting that the conditional variance of these economic groups exhibits *exceptionally* long memory (see Bollerslev, 1986).

8.4.4. Possible specification problems

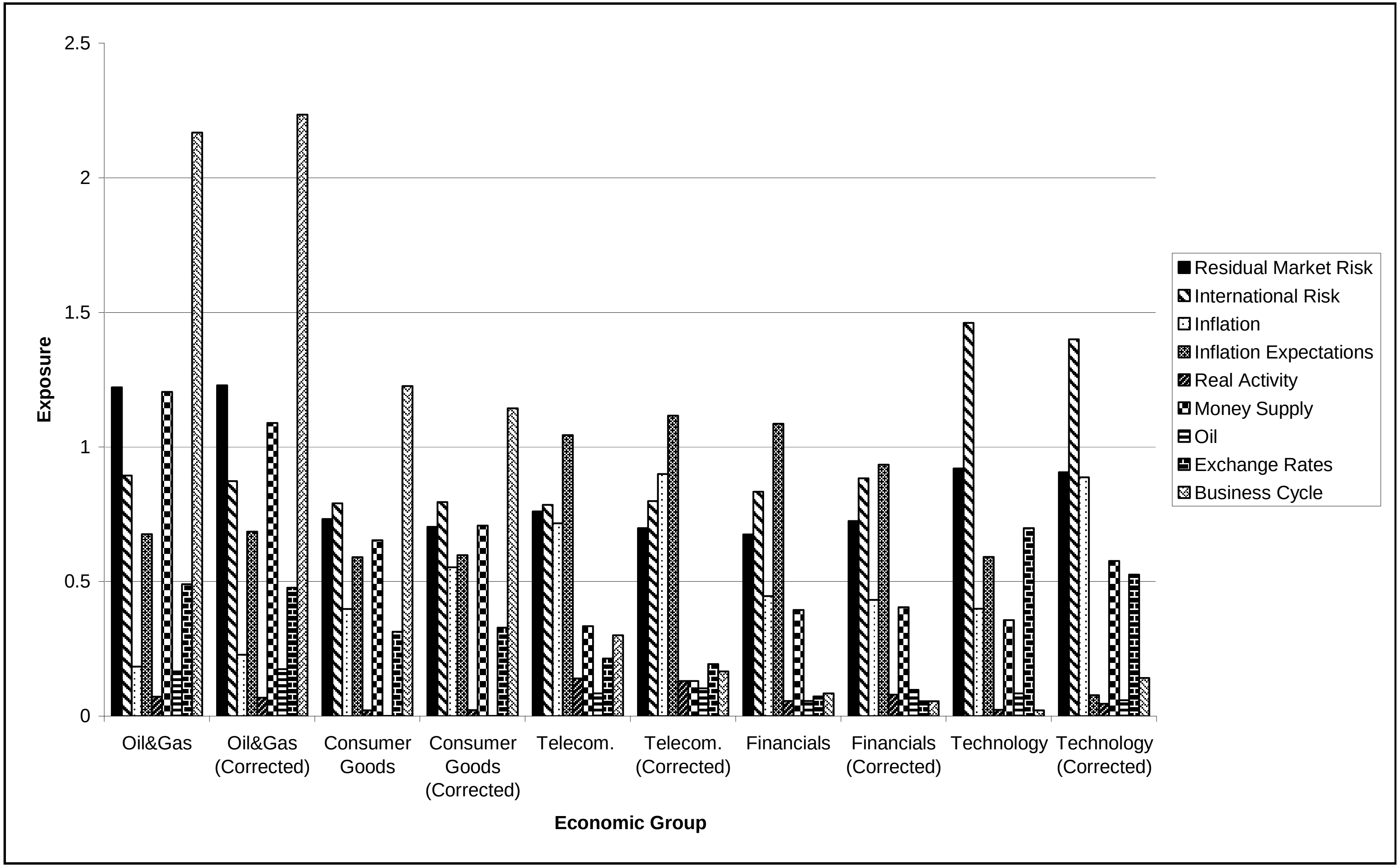
Regression diagnostics, however, reveal that although ARCH effects are not present in the residuals, the model may not be adequate. *Q*-statistics indicate that the residuals for five out of the nine economic groups are not white noise. The null hypothesis of residuals jointly equalling zero at the first and/or fifth orders is rejected for the oil and gas, consumer goods, telecommunications, financials and technology economic groups (see Table 8.7). A number of reasons are cited in econometric literature for residual serial correlation. One reason is that the model excludes a factor that should have been included in the return generating process specification. Alternatively, the functional form is incorrect because it is assumed that returns are generated by a *linear* return generating process as postulated by the APT framework whereas the return generating process is in fact *non-linear* (Gujarati, 2003). It must be noted that as a *single* model specification is generalized to a number of return series, it is possible that not all return series are described by a single specification. There is merit in these arguments. If the misspecification is the result of an omitted factor, then this suggests that the residual market factor is not a sufficient proxy for omitted factors (section 3.2.1, 3.2.2 & 3.3.1; Van Rensburg, 1996; Berry *et al*., 1988). The assumption that returns are generated by a linear factor model is directly noted in APT literature (section 2.1; Ross, 1980; Burmeister *et al*., 1994). If the misspecification is the result of the true return generating process not being linear, then this suggests that the APT framework is not suited to modelling South African stock returns. A finding that South African stock returns are generated by a non-linear return generating process would be an important finding in its own right. Alternatively, a single model specification cannot be generalized to an extended sample of South African return series. This perhaps is indicative of the difficulties noted by Burmeister (2003) relating to building models of the return generating process for different markets. Whereas a single model specification may be adequate to describe an extended number of series in the US as in Berry *et al*. (1988), this may not be the case for the South African stock market.

It is tempting, but premature and inappropriate, to accept that the model is misspecified and that the results in Table 8.7 and Table 8.8 are misleading prior to investigating the robustness of the results in the presence of residual serial correlation. To correct for residual serial correlation and test the robustness of the results, two approaches are employed. The first approach to correct for serial correlation is to re-estimate the models where residual serial correlation is noted. In re-estimating these models, the ARCH/GARCH model of the conditional variance is re-specified and so is the number of ARCH and GARCH parameters and the underlying conditional error distribution (where necessary) (see section 7.4.2: 164). The second approach is based upon that of Kiymaz and Berument (2003) and involves the addition of lagged returns denoted by $R_{it-\tau}$ to correct for residual serial correlation, where *i* represents an economic group return series and τ represents the order of the lag. The lag order, τ, is not fixed and may differ from series to series as required. The number of autoregressive terms introduced is kept to a minimum so as not to significantly deviate from the essence of the multifactor model. Whereas the first approach maintains the essence of the APT framework, the second approach is inconsistent with the APT framework. This is because the introduction of autoregressive terms represents the introduction of series specific factors into the model. Therefore if the first approach of re-specifying the ARCH/GARCH conditional variance model fails to correct for residual serial correlation, only then is the second approach employed.

The first approach eliminates residual serial correlation in the oil and gas, financials and technology economic groups. The second approach is applied to the consumer goods and the telecommunications economic groups (see Table A1.2 in Appendix 1). *F*-statistics are statistically significant for all five economic groups indicating that the multifactor model continues to explain returns on these five economic groups. Regression diagnostics do not reveal evidence of residual serial correlation suggesting that these approaches successfully correct for residual serial correlation. With the exception of the results for the technology economic group, results appear to be robust to the presence of residual serial correlation. Although regression diagnostics indicate that the corrected model for the technology economic group is adequate, closer inspection suggests that the results are not robust. Whereas the results in Table 8.7 indicate that returns on this economic group are significantly related to $UM\varepsilon_t$, $UFTW_t$, $URBAS_t$,

$UOIL_t$ and $UZARUS_t$, the results of the corrected model indicate that returns are significantly related to $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $UM3_{t-1}$ and $UZARUS_t$. $URBAS_t$ and $UOIL_t$ are no longer statistically significant whereas $UCPI_{t-1}$ and $UM3_{t-1}$ are statistically significant after correcting for residual serial correlation. The coefficient attaching to $UCPI_{t-1}$ more than doubles in absolute magnitude in the corrected model and the coefficient on $URBAS_t$ decreases in absolute terms. The coefficient on UCI_t is now positive. Whereas the results for this economic group are somewhat ambiguous, the relationship between returns and $UM\varepsilon_t$, $UFTW_t$ and $UZARUS_t$ is consistent in magnitude, direction and statistical significance.

The results of the remaining corrected models are closely comparable to those in Table 8.7; relationships are consistent and of a similar magnitude. For example, both models indicate that returns on the oil and gas economic group are positively and significantly related to $UM\varepsilon_t$, $UFTW_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t; and negatively and significantly related to $URBAS_t$. The average $\bar{R}^2$ for the nine economic groups including the corrected models is 0.575 whereas the average $\bar{R}^2$ for the models in Table 8.7 is 0.570. The average $\bar{R}^2$ is therefore comparable. As before, the oil and gas economic group has the highest (and unchanged) $\bar{R}^2$ of 0.727 whereas the technology economic group has the lowest $\bar{R}^2$ of 0.410. The consistency of the results is further evident from Figure 8.4, which reports the estimated risk exposures (factor coefficients) for the models in Table 8.7 and those of the corrected models. It is evident from Figure 8.4 that the magnitude of estimated risk exposures - with the (partial) exception of the technology economic group - does not change substantially for the corrected models. Even for the technology economic group return series, estimated risk exposures are roughly comparable for all factors with the exception of $UCPI_{t-1}$, $URBAS_t$, and UCI_t.



Source: Compiled by author

Figure 8.4: Risk exposure profile for economic groups after correcting for residual serial correlation

As models of the consumer goods and telecommunications economic groups are augmented with autoregressive terms and then re-estimated, the ARCH/GARCH specifications of the conditional variance equation remain the same. As in Table 8.8, the ARCH and GARCH parameters of the conditional variance equation for these two groups remain statistically significant individually and jointly (see Table A1.2 in Appendix 1). Residual serial correlation for the oil and gas, financials and technology economic groups is removed by employing the IGARCH specification for each series and assuming different conditional error distributions for the latter two economic groups. IGARCH(2,1), IGARCH(1,2) and IGARCH(1,2) specifications are employed to model the conditional variance of returns on these three economic groups respectively. That the use of an IGARCH specification is able to remove residual serial correlation suggests that the respective GARCH and EGARCH specifications initially employed to model the conditional variance of these three economic groups fail to capture certain characteristics of returns and variance. This finding also suggests that residual serial correlation is not the result of a model misspecification but rather due to the inadequacy of the ARCH/GARCH models fitted. Furthermore, in the corrected model of the financials and technology economic groups, conditional errors are assumed to follow the generalized error distribution in contrast to the Student's *t* and normal distributions underlying the respective original models. Brooks (2008) states that the generalized error distribution encompasses a broad family of distributions that can be used to model many types of series. This argument, together with a lack of residual serial correlation in the corrected models, suggests that the generalized error distribution is more appropriate for these two economic groups.

While the ARCH and GARCH parameters in the conditional variance equation are not directly comparable under the corrected models as the ARCH/GARCH specifications differ to those in Table 8.8 for the oil and gas, financials and technology economic groups, the *overall* result is the same for two of the three economic groups. *F*-statistics for the oil and gas and technology economic groups are statistically significant, suggesting that the conditional variance of returns on these two economic groups is of a time-varying nature (see Table A1.2 in Appendix 1). Therefore, the application of the ARCH/GARCH framework is appropriate for these economic groups. However, a different conclusion is reached for the financials economic group. Whereas the *F*-statistic in Table 8.8 is statistically significant, the *F*-statistic of the corrected model is statistically insignificant suggesting that the conditional variance of this series is not of a time-varying nature. Nevertheless, the corrected models reveal that the main conclusions drawn from the results in Table 8.8 remain the same. Four out of the five economic groups exhibit evidence of time-varying variance in the conditional variance specification, suggesting that the presence of residual serial correlation does not detract from the appropriateness of the ARCH/GARCH framework.

8.4.5. Gains in explanatory power

As results are generally robust to residual serial correlation and residual serial correlation is not an inherent problem, further analysis is based upon the original (uncorrected) return generating process specification. This approach is methodologically consistent in that the AIC is used to select the most appropriate ARCH/GARCH specification for the conditional variance and retains the essence of the APT framework in relying only upon systematic risk factors to explain returns. To deduce the explanatory power of the domestic risk factors in isolation from the explanatory power of the residual market factor and the international risk factor, restricted models are estimated and the resultant $\bar{R}^2$s are compared to the $\bar{R}^2$ of the unrestricted model for each economic group. In estimating the restricted versions of the model, the ARCH/GARCH specification selected upon the basis of the unrestricted model is applied to each economic group.

The second column in Table 8.9 reports the $\bar{R}^2$ of a single-factor model incorporating $UM\varepsilon_t$. The third column reports the $\bar{R}^2$ of a single-factor model incorporating only $UFTW_t$. These two models indicate the respective explanatory power of the residual market factor and international risk. The fourth column reports the $\bar{R}^2$ of a restricted model incorporating the domestic risk factors. This indicates whether the set of domestic risk factors chosen to describe the return generating process of South African stock returns carries explanatory power that is independent of $UM\varepsilon_t$ and $UFTW_t$ (see Burmeister & Wall, 1986). The fifth column reports the $\bar{R}^2$ of the model combining the domestic risk factors *and* $UM\varepsilon_t$ (Excluding $UFTW_t$). The explanatory power of the unrestricted model (Unrestricted) is reported in the sixth column and the seventh column reports the explanatory power of a single-factor model incorporating only the market index, UM_t. It is important to reiterate the distinction between $UM\varepsilon_t$ and UM_t. The residual market factor, $UM\varepsilon_t$ - a proxy for omitted risk factors - is the return on the JSE All-Share Index that is uncorrelated with $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t (Berry *et al*., 1988; Elton *et al*., 2003). The market factor, UM_t, on the other hand reflects the impact of these and other factors not incorporated into equation (8.1). Therefore, when UM_t is used by itself to explain returns, it will have greater explanatory power relative to $UM\varepsilon_t$. What is of interest is whether the unrestricted model incorporating $UM\varepsilon_t$ and $UFTW_t$ and the domestic risk factors has greater explanatory power than a single-factor model incorporating only UM_t. If the single-factor model has greater explanatory power relative to the unrestricted multifactor model, then a single-factor model provides a more parsimonious and superior description of the return generating process of South African stock returns (see section 2.2.4).

Table 8.9: Gains in explanatory power for economic groups

	$UM\varepsilon_t$	$UFTW_t$	Domestic Risk	Exc. $UFTW_t$	Unrestricted	UM_t
Oil&Gas	0.323	0.171	0.187	0.559	0.727	0.671
Basic Materials	0.248	0.183	0.244	0.466	0.574	0.577
Industrials	0.336	0.309	0.214	0.525	0.700	0.627
Consumer Goods	0.126	0.266	0.057	0.180	0.489	0.485
Health Care	0.282	0.219	0.100	0.376	0.543	0.503
Consumer Services	0.248	0.293	0.205	0.434	0.609	0.522
Telecom.	0.148	0.256	0.146	0.282	0.434	0.342
Financials	0.228	0.364	0.133	0.316	0.659	0.585
Technology	0.091	0.265	-0.011	0.113	0.395	0.384
Average $\bar{R}^2$	0.225	0.269	0.142	0.361	0.570	0.522

Source: Compiled by author

Table 8.9 indicates that on average, $UM\varepsilon_t$ explains 22.5 percent of the variation in economic group returns. Notably, this is less than $UFTW_t$, which explains an average of 26.9 percent of the variation in returns. The relatively important role of $UFTW_t$ in explaining returns again suggests that the South African stock market is highly integrated with global markets and that international events have a strong influence upon the South African stock market. For certain economic groups, the explanatory power of $UFTW_t$ is greater than that of $UM\varepsilon_t$. For example, whereas 12.6 percent of the variation in returns on the consumer goods economic group is explained by $UM\varepsilon_t$, $UFTW_t$ explains 26.6 percent. Other economic groups for which $UFTW_t$ has greater explanatory power relative to $UM\varepsilon_t$ are the consumer services, telecommunications, financials and technology economic groups. Economic groups for which $UM\varepsilon_t$ has greater explanatory power relative to $UFTW_t$ are the oil and gas, basic materials, industrials and health care economic groups.

The domestic risk factors explain on average 14.2 percent of the variation in returns on the economic groups. The average $\bar{R}^2$ is likely to be understated owing to the low explanatory power of these factors for returns on the consumer goods economic group and a *lack* of explanatory power for the technology economic group. In the latter case, this finding supports the hypothesis that there are other factors that are relevant to the technology economic group aside from the domestic risk factors. This is attested to by the high explanatory power of $UFTW_t$ in the single-factor model for this group, which suggests that returns on the technology economic group are predominantly influenced by international risk factors rather than domestic risk factors. Nevertheless, the explanatory power of the domestic risk factors for the economic groups should not be underestimated. The explanatory power of the domestic risk factors is directly comparable to the explanatory power of $UM\varepsilon_t$ in the single-factor model for the basic materials and telecommunications economic groups. It is *slightly* lower than the explanatory power of $UM\varepsilon_t$ for the consumer services economic group and lower still but nevertheless substantial for the oil and gas and industrials economic groups. Domestic risk factors have explanatory power for returns on the consumer goods, health care and financials economic groups that is approximately half or less than half of the explanatory power of $UM\varepsilon_t$ by itself in the single-factor model. Combined, $UM\varepsilon_t$ and the domestic risk factors (Exc. $UFTW_t$) on average explain 36.1 percent of the variation in returns on the economic group indices. The explanatory power of these factors ranges between 11.3 percent for the technology economic group and 55.9 percent for the oil and gas economic group. For certain economic groups, the explanatory power of this combination of factors is lower than the explanatory power of $UFTW_t$ in the single-factor model. For example, whereas $UM\varepsilon_t$ and the domestic risk factors explain 18 percent of the variation in returns on the consumer goods economic group, $UFTW_t$ by itself explains 26.6 percent of the variation in returns on this economic group. This also applies to the financials and technology economic groups, supporting the argument that international risk plays a central role in explaining South African stock returns (see section 3.2.2, 3.3.2 & 4.3.1).

On average, the unrestricted model explains 57 percent of the variation in returns on the economic group indices. This is greater than the amount explained by $UM\varepsilon_t$, $UFTW_t$ and the domestic risk factors alone, suggesting that there is a substantial gain in explanatory power when $UM\varepsilon_t$, $UFTW_t$ and the domestic risk factors are combined in the unrestricted multifactor model. It is evident that the unrestricted model performs well in explaining South African stock returns; on average the unrestricted model explains more than half of the variation in returns on the economic groups compromising the South African stock market. The average $\overline{R}^2$ in the final column indicates that returns on the JSE All-Share Index account for 52.2 percent of the variation in the returns on the economic group indices. This average $\overline{R}^2$ is lower than that of the unrestricted model suggesting that on average, the unrestricted multifactor model conveys more information relative to a single-factor alternative. The explanatory power of the single-factor model and the unrestricted model is comparable for the basic materials, consumer goods and the technology economic groups. The unrestricted model has greater explanatory power for the oil and gas, industrials, health care, consumer services, telecommunications and financials economic groups. Two important conclusions can be reached from these observations. Firstly, the unrestricted model proposed in this study conveys information over and above the information conveyed by a single-factor model relying only upon UM_t. This suggests that UM_t does *not* capture all sources of risk and that there are other risk factors that contribute to explaining the return generating process operational in the South African stock market. Finally and perhaps most importantly, the evidence presented in Table 8.9 again suggests that a multifactor model constructed within the APT framework provides an appropriate description of the return generating process of South African stock returns.

8.4.6. Omitted risk factors

As the structure of the return generating process is imposed upon the data with the aim of representing as many risk categories as possible, it is plausible that factors have been omitted. This is the case with the model of the returns on the JSE All-Share Index; the residuals of the unrestricted model of the South African stock market are significantly correlated with UNK_t, UMP_t, $UTBT3_t$, $USAGB10_t$, $USAGB30_t$, $UCOM_t$ and $UMET_t$ (see Table 8.6) suggesting that the impact of these factors is not captured by the factors used to explain returns on the JSE All-Share Index (see section 2.1: 8).

Underlying the linear factor model suggested by the APT framework is the assumption that the covariance between ε_{it} and ε_{jt} is zero. It follows from this assumption that after controlling for systematic risk factors in the return generating process, only asset specific risk remains. If this is indeed the case and systematic risk has been fully accounted for, then the covariance between ε_{it} and ε_{jt} is zero. If this assumption is violated and residuals move together, then this implies that there are other systematic factors aside from those directly incorporated into the model of the return generating process that explain the time series behaviour of returns (Roll & Ross, 1980). Although, $UM\varepsilon_t$ should capture the impact of omitted risk factors, Van Rensburg (2000) states that the assumption of independence between ε_{it} and ε_{jt} is likely to be violated for models employing pre-specified macroeconomic factors. To test whether this assumption is violated, pairwise correlation coefficients are estimated for the residuals of the economic groups (see Elton & Gruber, 1988).

Table 8.10: Residual correlation matrix

	Oil&Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecom.	Financials	Technology
Oil&Gas	-								
Basic Materials	0.242***	-							
Industrials	-0.320***	-0.086	-						
Consumer Goods	-0.131*	-0.137*	0.158**	-					
Health Care	-0.186*	-0.169**	0.283***	-0.001	-				
Consumer Services	-0.549***	-0.287***	0.488***	0.040	0.310***	-			
Telecom.	-0.325***	-0.132*	0.127*	-0.026	-0.021	0.342***	-		
Financials	-0.607***	-0.434***	0.269***	-0.009	0.291***	0.446***	0.231***	-	
Technology	-0.298***	-0.302***	0.088	0.075	-0.012	0.278***	0.307***	0.208***	-

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.

Source: Compiled by author

The results in Table 8.10 suggest that the assumption of independence between ε_{it} and ε_{jt} is widely violated. Estimated correlation coefficients are statistically significant for most paired series. While some correlation can be expected, the *high* levels of correlation between certain residual series suggest that there are other factors aside from those included in equation (8.2) that account for systematic variation in the return generating process (section 2.1; Roll & Ross, 1980). Instances where pairwise residual correlation is over 0.4 in absolute terms are of particular concern.[134] Pairwise residual correlation of over 0.4 is observed between the residual series of the oil and gas and consumer services, the oil and gas and financials, the basic materials and financials, the industrials and consumer services, and the consumer services and financials economic groups. Given the inclusive approach undertaken in selecting factors, it is possible that there are superior specifications that yield residuals that are either uncorrelated or show lower levels of pairwise correlation (Elton & Gruber, 1988).

As noted by Van Rensburg (2000), the violation of the assumption of independence of the residuals is widely observed in multifactor models of the return generating process. This points towards a general limitation of the APT framework suggesting that a fixed set of risk factors is unable to fully account for systematic variation in the return generating process. A further potential explanation for the observed levels of pairwise residual correlation follows from Kryzanowski and To's (1983) and Beenstock and Chan's (1986) suggestion that there are omitted factors that are specific to economic groups and therefore, not systematic in nature. If this is indeed the case, then high levels of pairwise residual correlation will be limited to a few economic groups suggesting that there is a factor that is *common* to these *specific* groups. However, the *widespread* statistically significant pairwise residual correlation presented in Table 8.10 provides evidence to the contrary.

It is tempting to accept Van Rensburg's (2000) suggestion that the violation of the assumption of independent residuals is a limitation inherent to specifications of the return generating process that employ pre-specified factors. Accepting this argument; however, fails to acknowledge that the violation of this assumption leads to a number of important conclusions. The first and

[134] In *this* study, pairwise correlation of between 0.4 and 0.5 is considered to be noteworthy; correlation of 0.5 is considered to be problematic. Admittedly, these values are somewhat arbitrary

obvious conclusion – already stated – is that there are other factors in the return generating process aside from those in equation (8.2). The second conclusion is that the residual market factor, $UM\varepsilon_t$, is not an adequate proxy for omitted risk factors. This is especially pertinent given that the residual market factor plays an important role in models of the return generating process and pricing within the APT framework (section 3.2.2, 3.3.1 & 4.3.1; Burmeister & Wall, 1986; McElroy & Burmeister, 1988; Berry *et al*., 1988; Van Rensburg, 1996). This second conclusion by itself is important and suggests that a *single* residual market factor fails to account for the impact of omitted risk factors. This implies that two or more residual market factors may be required. Such an approach is undertaken by Van Rensburg (2000) who constructs two residual market factors from returns on the JSE Industrial and All-Gold Indices. Not only does the fit of Van Rensburg's (2000) model of the returns on the JSE All-Share Index improve significantly, a number of risk factors that were previously statistically insignificant are now significant. It is suggested that a failure to account for omitted factors results in an estimation bias which leads to erroneous inferences that certain factors are unimportant in explaining the return generating process.

The question of the adequacy of the residual market factor *and* that of omitted risk factors can be investigated within the APT framework (see equation (2.3)). It must be noted that nine risk factors out of an extended set of candidate risk factors are incorporated into the model. Moreover, a number of omitted risk factors are found to be significantly correlated with the residuals of the general market model (equation (8.1)) in Table 8.6 suggesting that $UM\varepsilon_t$ captures *some* of the impact of these factors. If $UM\varepsilon_t$ is an adequate proxy for omitted risk factors, then the residuals of each economic group should be uncorrelated with factors omitted from the model implying that $UM\varepsilon_t$ is a proxy for *some* omitted factors. If the residuals are correlated with omitted risk factors, then this suggests that $UM\varepsilon_t$ is not an adequate proxy and some of these omitted factors may be responsible for the pairwise residual correlation observed in Table 8.10. However, even if the residuals are uncorrelated with these omitted factors, then in light of the results in Table 8.10, there are additional factors in the return generating process not considered in the extended set of risk factors.

8.4.7. Additional risk factors in economic group returns

The results in Table 8.11 are somewhat ambiguous given that no omitted risk factors, aside from $USAGB10_t$ and $USAGB30_t$, exhibit a consistent pattern of statistically significant correlation with the residuals. $USAGB10_t$ and $USAGB30_t$ are significantly correlated with the residuals of the oil and gas, basic materials, industrials, consumer services and financials economic groups. This suggests that the impact of these two interest rate factors is not fully accounted for by $UM\varepsilon_t$. Correlation between $USAGB10_t$, $USAGB30_t$ and the residuals is significant, but not of a exceptionally high magnitude. However, the financials economic group is an exception; the level of correlation between these two factors and the residuals is above 0.4 in absolute terms. The next highest correlation is between $USAGB10_t$, $USAGB30_t$ and the residuals of the basic materials economic group at 0.217 and 0.225 respectively. Given these levels of correlation, it is unlikely that the omission of $USAGB10_t$ and $USAGB30_t$ fully accounts for the high levels of pairwise residual correlation reported in Table 8.10. Nevertheless, the noteworthy levels of correlation between $USAGB10_t$, $USAGB30_t$ and the residuals again suggest that these factors are important in the return generating process of South African stock returns (see section 2.1: 8).

Another factor that shows noteworthy correlation with the residuals is $UCOM_t$. Whereas the pattern of correlation for this factor is not systematic, the residuals of the basic materials, industrials, health care and financials economic groups are significantly correlated with $UCOM_t$ implying that the impact of $UCOM_t$ is not fully captured by $UM\varepsilon_t$. The level of correlation between $UCOM_t$ and the residuals of the abovementioned economic groups is however of a lower magnitude than the correlation observed between $USAGB10_t$, $USAGB30_t$ and the residuals. Correlation between this factor and the residuals ranges between -0.158 for the financials economic group and 0.093 for the oil and gas economic group. Therefore, the omission of $UCOM_t$ is not of great concern even if its impact is not fully accounted for by $UM\varepsilon_t$.

Table 8.11: Correlation of economic group residuals with omitted risk factors

	Oil&Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecom.	Financials	Technology
UDJ_t	-0.027	-0.105	0.047	0.093	-0.018	0.066	0.049	0.152**	0.067
$UFTSE_t$	-0.021	-0.179**	0.045	0.112	-0.016	0.123*	0.061	0.063	0.111
$UMSCI_t$	-0.097	-0.184**	0.073	0.144**	-0.005	0.129*	0.111	0.138*	0.115
$UMSCIR_t$	-0.109	-0.165***	-0.014	0.212***	-0.065	0.108	0.083	0.027	0.036
UNK_t	-0.064	-0.154**	0.085	0.093	0.028	0.116	0.040	0.120*	0.095
UMP_t	0.013	-0.084	0.085	-0.041	0.006	0.045	0.076	0.068	0.013
UBP_t	-0.022	-0.019	-0.002	-0.089	0.022	0.005	0.176**	-0.74	0.049
$USLS_{t-2}$	-0.069	-0.123*	-0.112	-0.041	0.039	0.075	-0.056	0.084	-0.061
$UM1A_{t-1}$	-0.025	0.048	-0.017	-0.075	-0.004	0.018	-0.027	0.073	0.038
$UM3_{t-2}$	-0.067	-0.017	0.041	-0.055	0.056	0.067	0.104	-0.047	-0.055
$UTBT3_t$	0.000	0.008	0.016	-0.070	0.073	0.011	0.087	-0.058	0.041
$USAGB10_t$	0.167**	0.217***	-0.149**	0.094	-0.105	-0.187***	-0.069	-0.449***	-0.105
$USAGB30_t$	0.163**	0.225***	-0.163**	0.105	-0.097	-0.191***	-0.066	-0.447***	-0.098
$UCOM_t$	0.093	0.136*	-0.133*	-0.101	-0.149**	-0.093	-0.060	-0.158**	-0.080
$UMET_t$	0.065	0.104	-0.099	-0.100	-0.091	-0.030	-0.028	-0.060	0.016
$UNFCI_t$	-0.009	0.111	-0.983	-0.111	-0.026	-0.114	-0.067	-0.052	0.008
UTT_t	-0.109	-0.057	0.005	-0.063	-0.110	0.103	0.104	0.098	0.091
$ULTT_t$	0.036	-0.058	-0.141*	0.096	-0.026	0.017	-0.027	-0.127*	-0.009
$UCTT_{t-1}$	-0.036	-0.051	0.061	0.035	-0.184**	-0.002	0.012	0.009	0.019
ULI_t	-0.069	-0.002	-0.019	-0.023	-0.067	-0.013	0.078	0.001	0.055

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance.
* Indicates statistical significance at the 10 percent level of significance.

Source: Compiled by author

The residuals of the basic materials, consumer goods, consumer services and financials economic groups are significantly correlated with one or more international and foreign indices. For example, residuals of the consumer services economic group are significantly correlated with $UFTSE_t$ and $UMSCI_t$ whereas the residuals of the financials economic group are significantly correlated with UDJ_t and $UMSCI_t$. The purpose of including international and foreign indices in the set of candidate risk factors is to account for international risk factors and in the preceding analysis, $UFTW_t$ is assumed to be the most suitable proxy for international risk (see Clare & Priestley, 1998). However, a finding that the residuals are significantly correlated with other international and foreign indices indicates that $UFTW_t$ may not fully capture international risk.

The results in Table 8.11 and the preceding discussion suggest that $UM\varepsilon_t$ does not fully capture the impact of omitted risk factors upon the returns on economic groups. This is suggested by pervasive statistically significant correlation between the residuals and the interest rate factors and notable correlation between the residuals and $UCOM_t$. Therefore, $UM\varepsilon_t$ is not an adequate proxy for omitted factors in the return generating process of economic groups. Furthermore, although the residuals are significantly correlated with the interest rate factors, $UCOM_t$ and other factors on a more sporadic basis, the level of correlation is not high enough to account for the high and pervasive levels of pairwise residual correlation outlined in Table 8.10. This points towards the presence of factors in the return generating process which are not considered in the initial set of candidate risk factors. This also implies that the complexity of the return generating process of South African stock returns cannot be fully described by the proposed specification and the model should be augmented with a second residual market factor to capture omitted and unidentified systematic risk factors (see Van Rensburg, 2000).

8.4.8: Economic groups: A synthesis

On average, the model specification of the return generating process denoted in equation (8.2) explains 57 percent of the variation in returns on the economic group indices (section 8.4.1 ; Table 8.7). For each economic group, the significance of having multiple factors in the return generating process is confirmed (see section 8.4.1: 191). The return generating process is described by $UM\varepsilon_t$,

$UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$. In contrast to the results in Table 8.4, UCI_t has a limited impact upon economic group returns (see Table 8.7). The ARCH/GARCH models chosen for the economic group return series consists of a variety of specifications, namely GARCH (section 6.3.2), IGARCH (section 6.3.3) and EGARCH (section 6.3.5) models, suggesting that the conditional variance of these return series displays a number of differing characteristics (discussed extensively in section 8.4.3; section 5.3).

Risk exposure profiles reported in Figure 8.3 indicate that exposure to risk factors in the return generating process differs across economic groups (also see Figure 3.1 in section 3.3.1). Furthermore, some economic groups such as the consumer goods economic group, are less risky relative to the South African stock market, whereas other economic groups, such as the oil and gas economic group, are more risky relative to the South African stock market (also see section 3.3.1; Chan *et al*., 1990). The results of the ARCH/GARCH models of the conditional variance of economic group returns reported in Table 8.8 again point towards the appropriateness of the ARCH/GARCH framework (section 8.4.3). Possible specification problems are addressed in section 8.4.4 and the results for economic groups where specification problems are noted and addressed are reported in Table A1.2 (see Appendix 1); most of the conclusions reached upon the basis of the results in Table 8.7 and Table 8.8 hold. Notably, it is found that the IGARCH model is able to correct the residual serial correlation for the oil and gas, financials and technology economic groups (see Table 8.7 & Table A1.2). Table 8.9 (section 8.4.5) indicates that combining domestic risk factors with the international risk factor ($UFTW_t$) results in an increase of the R^2. Notably, the unrestricted model (Unrestricted) outperforms the single-factor model relying only upon UM_t to explain returns. It is also found that international risk plays an important role in explaining South African stock returns.

Table 8.10 suggests that the assumption of independence between the model residuals is violated (see section 2.1; equation (2.2)). Statistically significant pairwise residual correlation is present, and a number of reasons for this finding are suggested, namely; the existence of superior model specifications (which include omitted risk factors), a general limitation of the APT framework, the existence of sector specific factors and the inadequacy of the residual market factor (see section

8.4.6: 214). The results in Table 8.11 indicate that $USAGB10_t$ and $USAGB30_t$ show a consistent pattern of statistically significant correlation with the residuals of the economic groups. This suggests that these factors feature in the return generating process and that $UM\varepsilon_t$ does not fully account for omitted risk factors. This latter finding suggests that a second residual market factor may be required (section 8.4.7: 218)

8.5. Industrial sectors

8.5.1. Industrial sector model

The final stage of the analysis generalizes the model of the return generating process in equation (8.2) to returns on industrial sectors compromising the South African stock market. As before, each model is estimated within the ARCH/GARCH framework with the ARCH/GARCH specification, the number of ARCH and GARCH parameters and the conditional error distribution selected upon the basis of the AIC. As a total of twenty-seven industrial sectors are analyzed, the generalization of the multifactor model to these series represents an extension of a model grounded in the APT framework to series that are widely representative of the dynamics of the South African stock market. Owing to the substantial amount of statistical output, the results in Table 8.12 and Table 8.13 are in summarized form.

Results indicate that out of the 243 estimated factor coefficients, 147 are statistically significant at the 10 percent, 5 percent or 1 percent levels of significance. *F*-statistics are statistically significant for each industrial sector suggesting that the multifactor model of the return generating process has explanatory power for South African industrial sector returns. This result also indicates the significance of having this set of risk factors in the model specification and supports a multifactor return generating process specification (see section 2.2.1 & 6.4.3; Chen, 1983; Sadorsky & Henriques, 2001). The average $\bar{R}^2$ is 0.422, suggesting that on average the model explains 42.2 percent of the variation in industrial sector returns. The explanatory power of the model is variable; $\bar{R}^2$ ranges between 0.170 for the pharmaceuticals and biotechnology industrial sector to 0.752 for the mining industrial sector. In their entirety, these results suggest that the model can explain industrial sector returns.

Table 8.12: ARCH/GARCH model of industrial sector returns

	Intercept	$UM\varepsilon_t$	$UFTW_t$	$UCPI_{t-1}$	$URBAS_t$	UBP_{t-1}	$UM3_{t-1}$	$UOIL_t$	$UZARUS_t$	UCI_t	$\bar{R}^2$	F-statistic
Oil & Gas Producers	0.005***	0.877***	0.949***	-0.613	-0.663	-0.008	0.833***	0.327***	0.415***	1.459**	0.511	31.786***
Chemicals	-0.002*	0.537***	0.478***	-0.190	-1.211***	-0.045	0.497**	0.081***	-0.121	0.005	0.391	15.814***
Forestry & Paper	-0.007***	0.833***	1.005***	0.334	-1.658***	0.024	0.756***	0.055**	0.478***	-0.130	0.311	75.549***
Industrial Metals	-0.002	0.902***	0.901***	-0.508	-0.339	0.180***	1.064***	0.103**	-0.178	0.843	0.241	34.759***
Mining	-0.001	1.322***	1.005***	-0.619**	-0.714**	0.103***	1.014***	0.153***	0.500***	2.286***	0.752	90.841***
Const & Materials	-0.004*	0.665***	0.389***	-0.238	-0.813**	0.025	0.780*	-0.007	-0.321**	0.775	0.278	17.670***
General Industrials	-0.002	1.020***	0.606***	-0.736***	-0.804***	0.051**	0.289	-0.004	-0.190***	0.651	0.633	97.270***
E & E Equipment	-0.003*	0.721***	0.721***	-1.655***	-0.848***	0.023	0.275	0.013	-0.085	-0.333	0.453	27.064***
Industrial Engineering	-0.001	0.777***	0.581***	-0.263	-0.869***	0.021	-0.504	-0.025	0.043	0.431	0.279	12.967***
Industrial Transportation	-0.006***	0.827***	0.660***	-1.452***	-0.627**	0.082**	0.052	0.007	-0.107	0.429	0.468	51.104***
Support Services	-0.004***	0.797***	0.706***	-0.780***	-0.741**	0.010	0.322	-0.094***	0.013	0.256	0.557	42.925***
Automobiles & Parts	-0.007***	0.811***	0.465***	-1.380***	-1.551***	0.078*	-0.492	-0.006	0.080	0.098	0.201	19.209***
Beverages	-0.001***	1.054***	0.663***	-1.187***	-0.703**	0.072**	-0.089	-0.036	0.203**	-0.029	0.537	29.020***
Food Producers	-0.002	0.764***	0.463***	-1.026***	-0.648***	0.041*	0.067	-0.063**	-0.017	-0.219	0.530	27.146***
Health Care E & S	0.001	0.775***	0.487***	-1.717***	-1.064***	0.060	0.588***	-0.061*	0.117	0.656	0.292	27.549***
Pharma & Biotech	-0.000	0.461***	0.447***	-2.072***	-0.237	-0.014	0.098	0.081	-0.441***	-1.567*	0.170	6.998***
Food & Drug Retailers	0.004***	0.614***	0.302***	-0.598**	-0.544***	0.062**	-0.040	-0.114***	-0.154*	1.610***	0.252	21.115***
General Retailers	-0.005***	0.776***	0.702***	-1.072***	-1.118***	0.014	0.581	-0.041	-0.267**	0.169	0.473	32.038***
Media	0.003*	1.214***	0.904***	-1.049***	0.484	0.070**	0.197	-0.032	-0.182*	0.399	0.375	115.770***
Travel & Leisure	-0.004***	0.692***	0.459***	-1.302***	-0.194	0.007	0.542**	-0.055	-0.067	1.104***	0.344	15.414***
Fixed Line Telecom	-0.001	0.872***	0.889***	-1.216***	-1.640***	0.140***	-0.571*	-0.113**	0.052	-0.871*	0.314	40.174***
Banks	-0.002	0.784***	0.818***	-0.798**	-0.849**	0.068**	0.557**	-0.073**	-0.083	-0.127	0.540	27.062***
Non-Life Insurance	-0.003**	0.794***	0.625***	-1.214***	-1.096***	0.064**	-0.346	0.050	-0.042	-0.286	0.437	33.049***
Life Insurance	-0.005***	0.858***	0.967***	-0.214	-0.967***	0.064**	0.085	-0.026	0.038	-0.493	0.644	47.023***
General Financial	-0.001	0.875***	1.019***	-0.974***	-0.622	0.016	0.388	-0.117***	0.170*	0.227	0.527	1083.980***
Equity Investment Inst	-0.004***	0.690***	0.695***	-0.213	-1.139***	0.023	0.266*	-0.041	0.307***	0.262	0.490	157.686***
Soft & Comp Services	-0.000	1.003***	1.495***	-0.940	-0.354	0.026	0.378	0.056	0.674***	-0.037	0.399	19.119***

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. F-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).

Source: Compiled by author

To permit a concise analysis and interpretation of the results, the significance of the mean value of the 27 coefficient estimates for each factor is tested using a *t*-test (Beenstock & Chan, 1988).

Table 8.13: Summarized ARCH/GARCH industrial sector model results

	Mean	%Sig	Positive	Negative
Intercept	-0.002***	59.259%	4 (3)	23 (13)
$UM\varepsilon_t$	0.827***	100%	27 (27)	0 (0)
$UFTW_t$	0.719***	100%	27 (27)	0 (0)
$UCPI_{t-1}$	-0.877***	66.666%	1 (0)	26 (18)
$URBAS_t$	-0.797***	74.074%	1 (0)	26 (20)
UBP_{t-1}	0.050***	48.148%	24 (13)	3 (0)
$UM3_{t-1}$	0.281***	40.741%	21 (10)	6 (1)
$UOIL_t$	0.001	44.444%	10 (5)	17 (7)
$UZARUS_t$	0.031	48.148%	13 (7)	14 (6)
UCI_t	0.277*	22.222%	17 (4)	10 (2)

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *t*-test applied to mean of coefficients to establish whether factor coefficients are significantly different from zero (Beenstock & Chan, 1988).
3. Values in brackets ( ) are the number of statistically significant instances at the 10 percent level of significance.

Source: Compiled by author

The results in Table 8.13 indicate that the *means* of factor coefficients on $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$ and UCI_t are statistically significant suggesting that these risk factors are important for returns on industrial sectors. The signs of the means are generally consistent with *a priori* expectations; $UM\varepsilon_t$, $UFTW_t$, UBP_{t-1}, $UM3_{t-1}$ and UCI_t have an *overall* positive impact upon returns and the inflation factors, $UCPI_{t-1}$ and $URBAS_t$ have an *overall* negative impact upon returns. The coefficient means of $UOIL_t$ and $UZARUS_t$ are not statistically significant. Caution must however be exercised with regard to the means of coefficients on $UOIL_t$ and $UZARUS_t$; it may be that positive and negative coefficient estimates offset each other bringing the mean value close to zero and thus, rendering these factors *seemingly* insignificant. Therefore, it is more appropriate to establish the importance of risk factors in explaining returns by establishing the number of instances in which each factor is statistically significant. The third column (%Sig) indicates that $UM\varepsilon_t$ and $UFTW_t$ have a statistically significant impact upon the returns on all (100 percent) industrial sectors in the sample. The fourth and fifth columns indicate the number of positive and negative coefficient estimates and the values in brackets indicate the number of statistically significant estimates of each sign.

The estimated coefficients on both $UM\varepsilon_t$ and $UFTW_t$ are positive in 27 instances with 27 positive statistically significant estimates. $UCPI_{t-1}$ and $URBAS_t$ have a statistically significant impact upon more than half of the industrial sectors, with coefficients on these factors statistically significant for 66.667 percent (18/27) and 74.074 percent (20/27) of industrial sectors respectively. The impact of these factors upon returns is almost exclusively negative as evident from the fifth column. As $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$ and $URBAS_t$ are statistically significant for more than half of the industrial sectors, these factors may be considered as the most important and pervasive factors in the return generating process of South African industrial sector returns. UBP_{t-1} is statistically significant in just under a half of the industrial sector return series and has a predominantly positive impact upon returns. Out of the 24 positive factor coefficient estimates, 13 are statistically significant for UBP_{t-1}. $UM3_{t-1}$ is statistically significant for 40.471 percent (11/27) of industrial sectors with the relationship between $UM3_{t-1}$ and returns being predominantly positive. $UOIL_t$ and $UZARUS_t$ are statistically significant for 44.444 percent (12/27) and 48.148 percent (13/27) of industrial sectors respectively. In contrast to the lack of overall significance of these two factors in the second column, it is evident from the third column that these factors *are* important for returns on industrial sectors. The direction of the relationship between $UOIL_t$, $UZARUS_t$ and returns is ambiguous. Estimated coefficients on $UOIL_t$ are positive in 10 instances and negative in 17 instances whereas coefficients on $UZARUS_t$ are positive in 13 instances and negative in 14 instances. This suggests that good news for some industrial sectors is bad news for other industrial sectors (Jones & Kaul, 1996; Griffin & Stultz, 2001; Jorion, 2001; Nandha & Faff, 2008).

Although, the impact of UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$ on industrial sector returns is not as pervasive as that of $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$ and $URBAS_t$, it is evident that these risk factors play an important role in explaining South African industrial sector returns. Finally, UCI_t has a statistically significant impact upon only 22.222 percent (6/27) of industrial sectors. This suggests that this factor is not as important in explaining South African stock returns as the other eight factors. It is worth noting that while the results in the second column suggest that UCI_t is significant overall, the results in third column suggest that the impact of this factor is limited. These results are consistent with those for the economic groups; $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$ are important factors in the return generating process of South African stock returns and the explanatory power of UCI_t is limited to subsets of the sample. These results provide further support for a multifactor model of the return generating process of South African stock returns and suggest that specific risk categories feature in the return generating process.

8.5.2. Conditional variance

The results in Table 8.14 are for the ARCH/GARCH models of the conditional variance. Based upon the AIC, the conditional variance of eleven series is best described by an EGARCH model, the conditional variance of ten series is described by a GARCH model and the conditional variance of six series is described by an IGARCH model.

The prevalence of the EGARCH specification suggests that the leverage effect is widespread in South African industrial sector returns (see section 5.3.3 & 6.3.5). Out of the eleven series to which an EGARCH model is fitted, seven exhibit a negative and statistically significant coefficient of asymmetry. These are the electronic and electrical equipment, industrial engineering, industrial transportation, automobiles and parts, food producers, general retailers and media industrial sectors. The coefficient of asymmetry is positive and statistically significant for the food and drug retailers industrial sector. The ARCH and GARCH parameters of the EGARCH specifications exhibit statistical significance across industrial sectors. The conditional variance of the chemicals, mining, construction and materials, support services, beverages, pharmaceuticals and biotechnology, fixed line telecommunications, banks, non-life insurance, and software and computer services industrial sectors is described by the GARCH model. The adequacy of the GARCH model suggests that the conditional variance of these industrial sectors is characterized by a long memory (section 5.3.2 & 6.3.2; Bollerslev, 1986; Elyasiani & Mansur, 1998; Engle, 2001; Kang *et al*., 2009). As with the EGARCH model, the ARCH and GARCH parameters of the GARCH model are statistically significant across industrial sectors.

Table 8.14: ARCH/GARCH models of industrial sector conditional variance

	ω	α_1	α_2	β_1	β_2	γ_1	F-Statistic	ARCH/GARCH	Distribution
Oil & Gas Producers	-	-0.050***	-	1.050***	-	-	2146020.***	IGARCH(1,1)	GED
Chemicals	4.77E-06***	-	-	1.997***	-1.007***	-	3094549.***	GARCH(0,2)	Normal
Forestry & Paper	-1.335***	-1.606***	1.901***	0.836***	-	-0.058	196.636***	EGARCH(2,1)	GED
Industrial Metals	-	0.425***	-0.382***	0.957***	-	-	6.627***	IGARCH(2,1)	GED
Mining	6.76E-06**	0.032*	-	1.757***	-0.811***	-	2702.422***	GARCH(1,2)	Student's t
Const & Materials	3.62E-05***	-0.006	0.034	1.868***	-0.933***	-	28384.22***	GARCH(2,2)	Normal
General Industrials	-	0.095***	-	-0.092***	0.996***	-	3213.346***	IGARCH(1,2)	Normal
E & E Equipment	-0.659*	0.045	-	0.917***	-	-0.166***	263.930***	EGARCH(1,1)	Normal
Industrial Engineering	-3.323***	0.700***	-	-0.102*	0.718***	-0.096**	18887.80***	EGARCH(1,2)	Normal
Industrial Transportation	-0.521	-0.233**	-	0.906***	-	-0.176***	442.244***	EGARCH(1,1)	Normal
Support Services	4.88E-05***	-0.126***	0.229***	1.535***	-0.768***	-	155.353***	GARCH(2,2)	Normal
Automobiles & Parts	-0.589***	-0.384***	-	0.186	0.682***	-0.294***	507.961***	EGARCH(1,2)	Student's t
Beverages	6.87E-06	0.184	-0.225	1.437***	-0.411	-	1445.697***	GARCH(2,2)	Normal
Food Producers	-0.735***	-0.460***	0.203	0.419	0.465*	-0.246***	557.953***	EGARCH(2,2)	Normal
Health Care E & S	-	0.226***	-	0.774***	-	-	14.663***	IGARCH(1,1)	Normal
Pharma & Biotech.	5.44E-06***	-	-	1.989***	-0.994***	-	5705264.***	GARCH(0,2)	Normal
Food & Drug Retailers	-3.734***	-0.218	0.757***	1.282***	-0.727***	0.084*	1140.955***	EGARCH(2,2)	Normal
General Retailers	-0.284***	-0.105***	-	1.475***	-0.526***	-0.125***	1.87E+11***	EGARCH(1,2)	Normal
Media	-0.911*	0.699***	-	0.088*	0.865***	-0.062*	248.002***	EGARCH(1,2)	Normal
Travel & Leisure	-5.806	0.485***	-	-0.290	0.572*	-0.064	4488.598***	EGARCH(1,2)	GED
Fixed Line Telecom.	-5.64E-05	0.122	-	0.844***	-	-	214.520***	GARCH(1,1)	GED
Banks	7.55E-05	-0.030	0.169*	0.719***	-	-	19.821***	GARCH(2,1)	Student's t
Non-Life Insurance	6.12E-07***	-	-	2.004***	-1.005***	-	12943561***	GARCH(0,2)	Student's t
Life Insurance	-	-0.047***	-	-0.623***	-	-	1139.891	IGARCH(1,1)	Normal
General Financial	-	-0.020***	-	1.643***	-0.623***	-	8.43E+10***	IGARCH(1,2)	Normal
Equity Investment Inst.	-2.311***	-0.759***	0.967***	1.431***	-0.697***	0.057	537.845***	EGARCH(2,2)	GED
Soft & Comp Services	2.84E-05***	0.012	-	1.877***	-0.908***	-	10256.35***	GARCH(1,2)	Student's t

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. F-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).

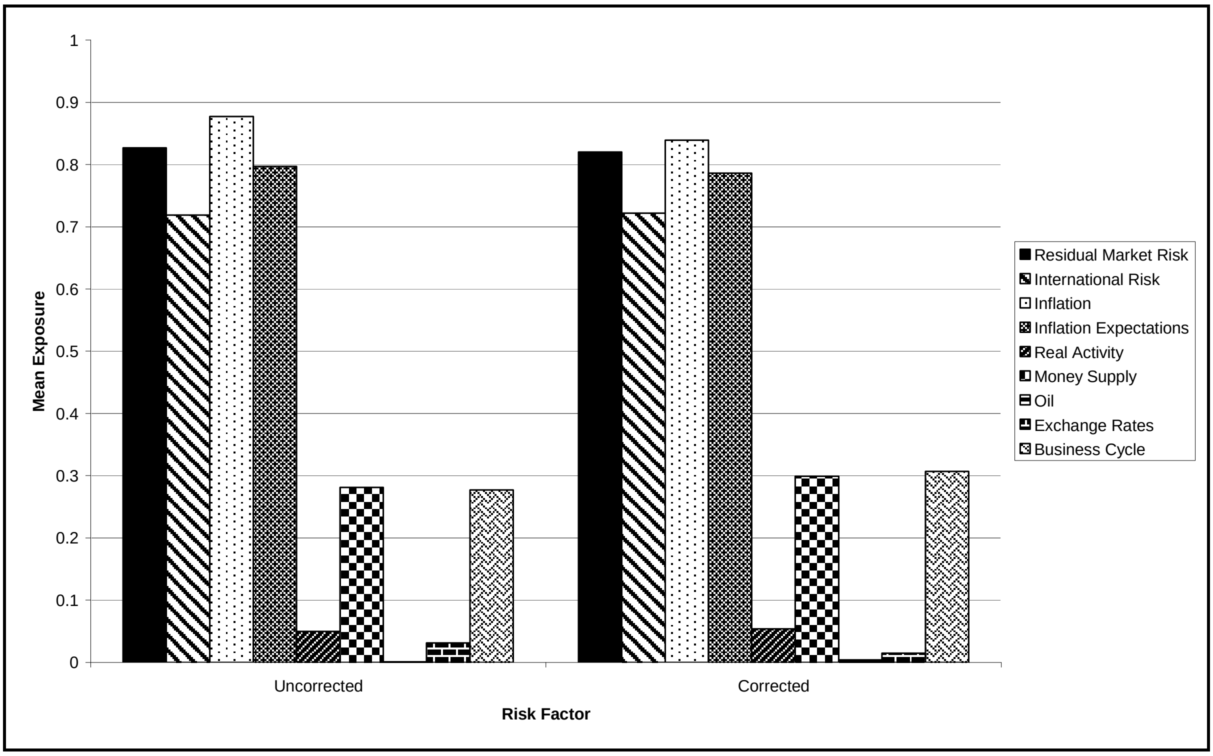
Source: Compiled by author

The IGARCH model describes the conditional variance of the oil and gas produces, industrial metals, general industrials, health care equipment and services, life insurance and general financial industrial sectors. The choice of this model for these industrial sectors suggests that the conditional variance of these sectors is characterized by the infinite persistence of shocks (Cryer & Chan, 2008; Kang *et al*., 2009; McMillan & Ruiz, 2009). The ARCH and GARCH parameters of the IGARCH specifications are statistically significant across industrial sectors. *F*-statistics for Wald's test of coefficient restrictions are statistically significant for each industrial sector return series in the sample suggesting that variance is of a time-varying nature. The conclusion is therefore, that the ARCH/GARCH framework is appropriate for the modelling of South African industrial sector returns.

8.5.3. Possible specification problems

While the model in equation (8.2) appears to provide an adequate description of the return generating process of South African industrial sector returns, regression diagnostics indicate the presence of serial correlation or ARCH effects in the residuals of the chemicals, forestry and paper, industrial engineering, support services, food producers, general retailers, non-life insurance, and software and computer services industrial sectors. Whereas residual serial correlation and ARCH effects are not a pervasive problem as their presence is observed in under a third of the industrial sectors, models are nevertheless re-estimated with different ARCH/GARCH specifications or augmented with autoregressive terms to correct for residual serial correlation and ARCH effects (Kiymaz & Berument, 2003). ARCH/GARCH specifications are re-estimated for the forestry and paper, support services, non-life insurance and the software and computer services industrial sectors and the model of returns on the industrial engineering, food producers and general retailers industrial sectors is augmented with autoregressive terms. Re-estimating the model of returns on the chemicals industrial sector with a different ARCH/GARCH specification and/or augmenting the model with autoregressive terms does not eliminate fifth order residual serial correlation. This suggests that for this industrial sector, the model is misspecified or that the true return generating process underlying returns on this industrial sector is non-linear (Gujarati, 2003).

F-statistics for the corrected models indicate that all factors are jointly statistically significant for each industrial sector (see Table A1.3 in Appendix 1). The average $\bar{R}^2$ increases marginally from 0.422 to 0.431. Summarized results incorporating the re-estimated models indicate that the same factors are statistically significant overall as in Table 8.13 and there is no difference in the general direction of impact (see Table A1.4 in Appendix 1). With the exception of the mean of coefficients on $UOIL_t$ which increases in size, and the mean of coefficients on $UZARUS_t$ which decreases in size; there is no substantial change in the size of the coefficient means of the remaining factors. This is evident from Figure 8.5 which indicates that the absolute coefficient means are relatively unchanged after correcting for residual serial correlation and ARCH effects.



Source: Compiled by author

Figure 8.5: Mean risk exposure for industrial sectors including corrected models

The number of industrial sectors for which $UM\varepsilon_t$ and $UFTW_t$ are statistically significant remains unchanged and therefore, the important role of these factors is re-affirmed (see Table A1.4 in Appendix 1). The number of industrial sectors for which $UCPI_{t-1}$ is statistically significant decreases from 66.666 percent (18/27) to 62.963 percent (17/27), increases for UBP_{t-1} from 48.148 percent (13/27) to 55.556 percent (15/27), and decreases for $UOIL_t$ from 44.444 percent (12/27) to 40.741 percent (11/27). The number of industrial sector return series for which the remaining factors are statistically significant remains unchanged. As before, $UCPI_{t-1}$ and $URBAS_t$ play a pervasive and important role in explaining stock returns. However, these factors are now joined by UBP_{t-1} which increases in importance.

In terms of importance, $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$ and UBP_{t-1} are followed by $UZARUS_t$ which continues to significantly impact returns on 48.148 percent (13/27) of industrial sectors. The number of positive coefficient estimates for $UZARUS_t$ decreases from 13 to 12 and the number of negative coefficient estimates increases from 14 to 15. The number of positive and negative coefficient estimates for $UM3_{t-1}$ is unchanged as is its statistically significant impact on 40.741 percent (11/27) of industrial sectors. For $UOIL_t$, the direction of the estimated relationships remains the same. Both $UM3_{t-1}$ and $UOIL_t$ continue to play an important role in explaining industrial sector returns. As before, the impact of these two factors is important but less pervasive than that of $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$ and now UBP_{t-1}. UCI_t continues to play a relatively minor role in explaining returns – statistically significant relationships are limited to a fifth of industrial sectors. The number of positive coefficient estimates for this factor increases from 17 to 18 and the number of negative coefficient estimates decreases from 10 to 9. These results suggest that the presence of residual serial correlation and/or ARCH effects does not substantially detract from the main findings in Table 8.13. The model described in equation (8.2) explains a substantial amount of variation in returns on South African industrial sectors; returns are described by a multifactor model motivated by the APT framework (see section 2.1 & 2.2).

The ARCH/GARCH specifications for the industrial engineering, food producers and general retailers industrial sectors remain unchanged (see Table A1.3 in Appendix 1). While all the ARCH and GARCH parameters of the conditional variance specification of the industrial engineering sector are statistically significant in Table 8.14, the corrected model indicates that none of the ARCH and GARCH parameters are individually statistically significant. Furthermore, according to the *F*-statistic, ARCH and GARCH parameters are also not jointly significant suggesting that conditional variance is not of a time-varying nature and therefore, ARCH/GARCH modelling is unnecessary for this industrial sector. This is not the case for the respective EGARCH(2,2) and EGARCH(1,2) models fitted to the food producers and general retailers industrial sector series. ARCH and GARCH parameters of the EGARCH(2,2) model remain jointly significant. The first order ARCH parameter becomes statistically insignificant whereas the second order ARCH parameter is now statistically significant. There is also a discrepancy in the magnitude and signs of the coefficients on the ARCH and GARCH parameters after correcting for residual serial correlation. Although, the direction of the estimated relationships is consistent for the EGARCH(1,2) model, relatively small discrepancies in the size of the coefficients on the ARCH and GARCH parameters are evident. The first ARCH parameter becomes statistically insignificant though the *F*-statistic indicates that the ARCH and GARCH parameters are jointly statistically significant. The conclusion relating to the appropriateness of modelling returns within the ARCH/GARCH framework for the food producers and general retailers industrial sectors therefore remains unchanged; conditional variance is of a time-varying nature and ARCH/GARCH modelling is appropriate for these return series.

Results of the EGARCH(2,2) model fitted to the forestry and paper industrial sector contrast with those of the EGARCH(2,1) model in Table 8.14 (see Table A1.3 in Appendix 1). None of the ARCH and GARCH parameters is individually statistically significant and the *F*-statistic indicates that the ARCH and GARCH parameters are jointly insignificant. The model of the support services industrial sector is re-estimated as an EGARCH(2,2) specification. Both ARCH and GARCH parameters are statistically insignificant although the coefficient of asymmetry is statistically significant and positive. The *F*-statistic is statistically insignificant. The case for ARCH/GARCH modelling of the forestry and paper, and the support services industrial sector return series is weaker after correcting for ARCH effects (forestry and paper) and residual serial correlation (support services). For the non-life insurance industrial sector, a GARCH(1,2) model is estimated. Results differ from those in Table 8.14; none of the ARCH and GARCH parameters are individually statistically significant. *Surprisingly*, the null hypothesis of ARCH and GARCH coefficients jointly equalling zero is rejected upon the basis of a statistically significant *F*-statistic. This is perhaps attributable to the marginally insignificant coefficient on the second GARCH parameter; omitting this coefficient from Wald's test of linear restrictions renders the *F*-statistic insignificant. An IGARCH(1,2), for which conditional errors are assumed to follow the generalized error distribution, is fitted to the software and computer services industrial sector. Although, not directly comparable to the results of the GARCH(1,2) model in Table 8.14, the results of the IGARCH(1,2) model lead to the same conclusion for this sector; conditional variance is of a time-varying nature and the IGARCH(1,2) model is appropriate. Although, the ARCH parameter is not statistically significant, both GARCH parameters are statistically significant.

The finding that conditional variance may not be of a time-varying nature for the forestry and paper, industrial engineering, and support services industrial sectors marginally detracts from the case for ARCH/GARCH modelling. ARCH/GARCH modelling remains appropriate for the food producers, general retailers, and software and computer services industrial sector return series. The results for the non-life insurance industrial sector are ambiguous. Nevertheless, these findings, taken *together* with the findings for industrial sectors where ARCH effects or residual serial correlation are not observed, provide widespread support for ARCH/GARCH modelling. It must however be acknowledged that the presence of residual serial correlation and/or ARCH effects impacts the results of ARCH and GARCH models of the conditional variance.

8.5.4. Gains in explanatory power

Gains in explanatory power from employing a multifactor specification are established by utilizing the same approach as for economic groups in Table 8.9. To assess the explanatory power of the model of the return generating process for industrial sectors, the explanatory power ($\bar{R}^2$) of the restricted versions of the model is compared to the explanatory power of the unrestricted model.

The results in Table 8.15 indicate that the unrestricted model (Unrestricted) on average explains 42.2 percent of the variation in industrial sector returns - an amount that is approximately 4 percent higher than that explained by UM_t alone. This suggests that the unrestricted model is superior in terms of explanatory power and conveys more information relative to a single-factor alternative relying only upon the market index to explain return behaviour. The $\bar{R}^2$ of the unrestricted model ranges between 0.752 for the mining industrial sector and 0.170 for the pharmaceuticals and biotechnology industrial sector. The unrestricted model outperforms the single-factor alternative for most industrial sectors with the exception of the forestry and paper, industrial metals, industrial engineering, automobiles and parts, health care and equipment services, and equity investment instruments industrial sectors where the single-factor model is characterized by a *marginally* higher $\bar{R}^2$. Although, the single-factor model is superior in terms of $\bar{R}^2$ for these industrial sectors relative to the unrestricted model, it does not identify the numerous risk factors embodied by UM_t. This limitation must be borne in mind when interpreting these results.

Table 8.15: Gains in explanatory power for industrial sectors

	$UM\varepsilon_t$	$UFTW_t$	Domestic Risk	Exc. $UFTW_t$	Unrestricted	UM_t
Oil & Gas Producers	0.211	0.061	0.264	0.463	0.511	0.441
Chemicals	0.161	0.158	0.182	0.318	0.391	0.357
Forestry & Paper	0.125	0.156	0.056	0.176	0.311	0.360
Industrial Metals	0.076	0.126	0.076	0.153	0.241	0.268
Mining	0.321	0.205	0.250	0.550	0.752	0.707
Const. & Materials	0.082	0.102	0.198	0.264	0.278	0.216
General Industrials	0.359	0.225	0.163	0.505	0.633	0.568
E&E Equipment	0.203	0.205	0.115	0.315	0.453	0.405
Industrial Engineering	0.121	0.166	0.085	0.193	0.279	0.306
Industrial Transportation	0.183	0.245	0.137	0.300	0.468	0.431
Support Services	0.238	0.278	0.135	0.363	0.557	0.487
Automobiles & Parts	0.066	0.127	0.070	0.104	0.201	0.235
Beverages	0.320	0.181	0.027	0.367	0.537	0.503
Food Producers	0.270	0.165	0.138	0.392	0.530	0.440
Health Care E & S	0.162	0.116	0.037	0.198	0.292	0.309
Pharma & Biotech	0.034	0.097	0.104	0.128	0.170	0.116
Food & Drug Retailers	0.091	0.133	0.137	0.228	0.252	0.189
General Retailers	0.165	0.219	0.241	0.377	0.473	0.365
Media	0.177	0.207	0.023	0.227	0.375	0.371
Travel & Leisure	0.178	0.109	0.096	0.287	0.344	0.329
Fixed Line Telecom	0.114	0.187	0.071	0.179	0.314	0.269
Banks	0.172	0.287	0.184	0.331	0.540	0.456
Non-Life Insurance	0.207	0.158	0.146	0.337	0.437	0.386
Life Insurance	0.194	0.414	0.151	0.336	0.644	0.565
General Financial	0.211	0.295	0.084	0.291	0.527	0.471
Equity Investment Inst	0.207	0.180	0.089	0.320	0.490	0.511
Soft & Comp Services	0.071	0.249	-0.014	0.084	0.399	0.364
Average $\bar{R}^2$	0.175	0.187	0.120	0.288	0.422	0.386

Source: Compiled by author

Domestic risk factors and $UM\varepsilon_t$ (Exc. $UFTW_t$) on average explain 28.8 percent of the variation in returns suggesting that international risk plays an important role in explaining the return generating process of industrial sector returns; incorporating $UFTW_t$ to arrive at the unrestricted model increases the average $\bar{R}^2$ from 0.288 to 0.422. Without $UFTW_t$, the explanatory power of the model declines substantially; the $\bar{R}^2$ ranges between 0.550 for the mining sector and 0.084 for the software and computer services industrial sector. The software and computer services industrial sector is most affected by the exclusion of $UFTW_t$; the $\bar{R}^2$ declines from 0.399 to 0.084. Domestic risk factors by themselves on average explain 12 percent of the variation in industrial sector returns with the $\bar{R}^2$ of the restricted model incorporating these factors ranging between -0.014 for the software and computer services industrial sector and 0.264 for the oil and gas producers industrial sector. The $\bar{R}^2$ for the restricted model incorporating domestic risk factors is greater than 0 but less than 0.1 for eleven industrial sectors, greater than 0.1 but less than 0.2 for twelve industrial sectors and greater than 0.2 for three industrial sectors. The average $\bar{R}^2$ of 0.12 suggests that domestic risk contributes meaningfully to describing industrial sector return behaviour.

The explanatory power of the domestic risk factors is best appreciated when compared to the explanatory power of $UM\varepsilon_t$ and $UFTW_t$; the average $\bar{R}^2$s of the single-factor models incorporating these factors are 0.175 and 0.187 respectively. Although, the average explanatory power of the domestic risk factors is lower than the average explanatory power of these two factors, it is nevertheless noteworthy. For certain industrial sectors, domestic risk factors are more important than $UFTW_t$ or $UM\varepsilon_t$. The explanatory power of the domestic risk factors is greater than the explanatory power of $UFTW_t$ for the oil and gas produces, chemicals, mining, construction and materials, pharmaceuticals and biotechnology, food and drug retailers and general retailers industrial sectors. Domestic risk factors have greater explanatory power relative to $UM\varepsilon_t$ for the oil and gas producers, chemicals, construction and materials, automobiles and parts, pharmaceuticals and biotechnology, food and drug retailers, general retailers and the banks industrial sectors. Nevertheless, the high explanatory power of $UFTW_t$ again points towards high levels of integration with world markets (section 3.1.6 & 3.3.2: 76; Clare & Priestley, 1998; Bilson *et al.*, 2001; Kandir, 2008).

The results in Table 8.15 suggest that factors aside from the market index explain South African stock returns. This is evident from the explanatory power of the domestic risk factors. Combining domestic risk factors with $UM\varepsilon_t$ and $UFTW_t$ in the unrestricted model yields a multifactor model of the return generating process that provides greater insight into the behaviour of South African industrial sector returns relative to a single-factor model relying solely upon the market index to describe return behaviour.

8.5.5. Omitted risk factors

To establish whether risk factors important for industrial sector returns have been omitted from the model, pairwise correlation coefficients are estimated for the residuals of the industrial sector models. This procedure produces 351 (unique) estimates. Rather than report all these, the means of correlation coefficients[135] and the distribution of the estimated correlation coefficients are reported in Table 8.16.

Table 8.16: Mean correlation and distribution of correlation coefficients for industrial sector residuals

Panel A		Panel B		
	Mean Corr.	Interval	Obs.	Cumulative
Oil & Gas Producers	-0.074	$-0.7 < \rho_{ij} \leq -0.6$	1 (0.285%)	0.285%
Chemicals	0.138	$-0.6 < \rho_{ij} \leq -0.5$	5 (1.425%)	1.709%
Forestry & Paper	0.016	$-0.5 < \rho_{ij} \leq -0.4$	3 (0.855%)	2.564%
Industrial Metals	-0.011	$-0.4 < \rho_{ij} \leq -0.3$	5 (1.425%)	3.989%
Mining	-0.304	$-0.3 < \rho_{ij} \leq -0.2$	8 (2.279%)	6.268%
Const & Materials	0.143	$-0.2 < \rho_{ij} \leq -0.1$	25 (7.123%)	13.390%
General Industrials	0.139	$-0.1 < \rho_{ij} \leq 0$	38 (10.826%)	24.217%
E & E Equipment	0.185	$0 < \rho_{ij} \leq 0.1$	62 (17.664%)	41.880%
Industrial Engineering	0.115	$0.1 < \rho_{ij} \leq 0.2$	91 (25.926%)	67.806%
Industrial Transportation	0.179	$0.2 < \rho_{ij} \leq 0.3$	68 (19.373%)	87.179%
Support Services	0.221	$0.3 < \rho_{ij} \leq 0.4$	35 (9.972%)	97.151%
Automobiles & Parts	0.111	$0.4 < \rho_{ij} \leq 0.5$	8 (2.279%)	99.430%
Beverages	0.063	$0.5 < \rho_{ij} \leq 0.6$	1 (0.285%)	99.715%
Food Producers	0.151	$0.6 < \rho_{ij} \leq 0.7$	1 (0.285%)	100%
Health Care E & S	0.156	Maximum:	0.617	
Pharma & Biotech	0.104	Minimum:	-0.662	
Food & Drug Retailers	0.148	Mean:	0.104	
General Retailers	0.202	Total	351 (100%)	
Media	0.150			
Travel & Leisure	0.184			
Fixed Line Telecom.	0.068			
Banks	0.125			
Non-Life Insurance	0.142			
Life Insurance	0.129			
General Financial	0.138			
Equity Investment Inst.	0.140			
Soft & Comp Services	0.059			

Source: Compiled by author

The results in Panel A of Table 8.16 suggest that, with a few exceptions, the level of mean correlation between the residuals is low. Sectors that show relatively high levels of pairwise residual correlation are the mining industrial sector with a mean correlation of -0.304, the support services industrial sector with a mean correlation of 0.221 and the general retailers industrial sector with a mean correlation of 0.202. Although, these three series exhibit relatively high levels of correlation, mean correlation for these sectors is below 0.4 in

[135] Mean correlation is defined as the sum of pairwise correlation coefficients (excluding the diagonal) for each industrial sector residual series divided by 26.

absolute terms suggesting that the omission of factors is not a severe problem for the industrial sector return series.

An inspection of the correlation matrix reveals that high levels of correlation are mostly confined to specific sectors.[136] These isolated instances of high levels of correlation point towards the presence of *sector specific* factors in the return generating process. This hypothesis is supported by the results in Panel B of Table 8.16. The highest level of positive residual correlation is observed between the residuals of the general retailers and support services industrial sectors at 0.617. The lowest (most negative) level of correlation is observed between the residuals of the banking and mining industrial sectors at -0.662. However, these observations represent extremes; 28.490 percent of pairwise correlation coefficients lie within the -0.1 and 0.1 interval (encompassing the -0.1 to 0, 0 to 0.1 intervals) and are therefore statistically insignificant.[137] Furthermore, 61.539 percent of correlation coefficients lie within the -0.2 and 0.2 interval suggesting that residuals are not highly correlated across sectors. Correlation coefficients falling within the -0.2 to 0.2 interval can be considered to be of a low magnitude and therefore of no particular concern. Over 21 percent of correlation coefficients lie within the -0.3 to -0.2 and 0.2 to 0.3 intervals suggesting that there are instances in which there is some notable pairwise residual correlation. Over 11 percent of estimated correlation coefficients lie within the -0.4 to -0.3 and 0.3 to 0.4 intervals. Correlation of this magnitude gives rise to concern. A closer inspection of the correlation matrix reveals that these relatively high levels of pairwise residual correlation are confined to the electronic and electrical equipment, support services, travel and leisure, and the general financial industrial sectors. The pairwise residual correlation for these industrial sectors accounts for almost two-thirds of correlation coefficients in the 0.3 to 0.4 interval. As these relatively high levels of correlation are mostly confined to specific sectors, this implies that there are sector specific factors in the return generating process. As these (implied) omitted and unidentified factors are sector specific, they do not qualify as legitimate APT risk factors (section 2.2.1: 16 & 4.2; Kryzanowski & To, 1983). The number of pairwise residual

[136] For the mining industrial sector, pairwise correlation coefficients of over 0.5 in absolute terms are observed between this sector and the general retailers, banks, life insurance, general financials and the equity investment instruments industrial sectors. For the support services industrial sector, pairwise correlation coefficients of over 0.5 in absolute terms are observed between this sector and the mining and the general retailers industrial sectors. The residuals of the general retailers industrial sector are highly correlated with those of the support services industrial sector.

[137] Coefficients of approximately 0.12 and above in absolute terms are statistically significant at the 10 percent level of significance.

correlation coefficients that are greater than 0.4 and lower than -0.4 is 5.414 percent. An inspection of the correlation matrix again suggests that these high levels of pairwise residual correlation are confined to individual industrial sectors implying the omission of sector specific factors.[138] While pairwise residual correlation of this magnitude is substantial and should not be ignored, it is not widespread. Out of the 351 estimated correlation coefficients, 233 are statistically significant at the 10 percent level of significance.

The results in Panel A and Panel B of Table 8.16 suggest that there may be sector specific factors that are not considered in the return generating process. This is supported by the finding that high levels of pairwise residual correlation are usually confined to specific industrial sectors. However, as two-thirds of pairwise correlation coefficients are statistically significant, it can also be argued that there is a degree of ambiguity as to whether the omission of sector specific factors or systematic risk factors is responsible for pairwise residual correlation.

8.5.6. Additional risk factors in industrial sector returns

An examination of the correlation between the industrial sector residual series and omitted risk factors is undertaken to further investigate the possibility of omitted risk factors, the adequacy of $UM\varepsilon_t$ as a catch-all proxy and to resolve the ambiguity of the preceding discussion. Results are aggregated owing to the amount of statistical output.

The second column of Table 8.17 reports the mean correlation between the residuals and omitted risk factors. For example, the mean correlation of industrial sector residuals with UDJ_t is 0.044. For *most* factors, the mean level of correlation with the industrial sector residuals is below 0.05 in absolute terms. Nevertheless, mean correlation between UNK_t, $USAGB10_t$, $USAGB30_t$ and the industrial sector residuals is 0.057, -0.090 and -0.094 respectively.

[138] For example, out of the 9 coefficients that are less or equal to -0.4 ($\rho_{ij} \leq -0.4$), 8 are confined to the mining industrial sector.

Table 8.17: Correlation of industrial sector residuals with omitted risk factors

	Mean Corr.	%Sig	Positive	Negative
UDJ_t	0.044	7.407%	21 (2)	6 (-)
$UFTSE_t$	0.035	11.111%	19 (3)	8 (-)
$UMSCI_t$	0.045	18.519%	22 (3)	5 (2)
$UMSCIR_t$	0.021	7.407%	20 (1)	7 (1)
UNK_t	0.057	11.111	23 (3)	4 (-)
UMP_t	0.027	3.703%	22 (1)	5 (-)
UBP_t	-0.002	7.407%	9 (2)	18 (-)
$USLS_{t-2}$	0.008	18.519%	16 (2)	11 (3)
$UM1A_{t-1}$	0.035	11.111%	20 (3)	7 (-)
$UM3_{t-2}$	0.011	18.519%	14 (3)	13 (2)
$UTBT3_t$	-0.004	3.703%	10 (-)	17 (1)
$USAGB10_t$	-0.090	48.148%	5 (2)	22 (11)
$USAGB30_t$	-0.094	48.148%	6 (2)	21 (11)
$UCOM_t$	-0.037	37.037%	8 (4)	19 (6)
$UMET_t$	-0.011	18.519%	11 (2)	16 (3)
$UNFCI_t$	-0.009	22.222%	12 (3)	15 (3)
UTT_t	0.017	7.407%	16 (1)	11 (1)
$ULTT_t$	-0.016	14.814%	11 (1)	16 (3)
$UCTT_{t-1}$	-0.000	7.407%	14 (1)	13 (1)
ULI_t	0.014	3.703%	17 (1)	10 (-)

Notes:
1. Values in brackets ( ) are the number of statistically significant instances at the 10% level of significance.

Source: Compiled by author

The two interest rate factors show the highest level of mean correlation with the residuals suggesting that interest rate factors should be considered as risk factors in the return generating process of South African stock returns. The results reported in the third column (%Sig) confirm that these two factors are important; 48.148 percent (13/27) of industrial sector residual series are significantly correlated with $USAGB10_t$ and $USAGB30_t$. This suggests that both factors have explanatory power after controlling for variation attributable to the nine risk factors incorporated into the model. Furthermore, this also suggests that the impact of these factors is not fully accounted for by $UM\varepsilon_t$. As expected, most residual series are negatively correlated with $USAGB10_t$ and $USAGB30_t$. The relatively high number of instances of statistically significant correlation between $USAGB10_t$, $USAGB30_t$ and the industrial sector residual series is analogous to the results in Table 8.11; the results in Table 8.11 indicate that these two factors are significantly correlated with the economic group residual series in five out of nine instances. Taken together, both sets of results point towards the importance of long-term interest rates in the return generating process of South African stock returns and indicate that $UM\varepsilon_t$ fails to indirectly account for the impact of these factors.

Another factor that is also notably correlated with both industrial sector and economic group residuals is $UCOM_t$. Statistically significant correlation between $UCOM_t$ and the residual series is observed for 37.037 percent (10/27) of industrial sectors suggesting that the impact of this factor is not fully accounted for by any of the extra-market risk factors in equation (8.2) and the residual market factor, $UM\varepsilon_t$. $UNFCI_t$ is significantly correlated with 22.222 percent (6/27) of the industrial sector residual series. Other factors, $UMSCI_t$, $USLS_{t-2}$, $UM3_{t-2}$ and $UMET_t$ are significantly correlated with 18.519 percent (5/27) of the industrial sector residual series. The limited instances of statistically significant correlation between the residual series and $UMSCI_t$ suggests that $UFTW_t$ by itself may not fully capture international risk. The correlation between the remaining factors and the industrial sector residual series is limited suggesting that these factors are not important for industrial sector returns *or* that their influence has been accounted for by the nine risk factors in equation (8.2).

Together, the results in Table 8.16 and Table 8.17 suggest the pairwise residual correlation suggested by the results in Table 8.16 *may* arise from omitted long-term interest rate factors. As with the economic group series, one solution to this omission is to substitute $USAGB10_t$ or $USAGB30_t$ for any of the factors in equation (8.2). As the impact of UCI_t on industrial sector returns is limited to just over a fifth of industrial sectors, this factor is a candidate for substitution for either of these factors. Alternatively, a two-step time series approach can be employed to incorporate these factors into the return generating process while retaining the original specification (Yli-Olli & Virtanen, 1992). $UCOM_t$ is also a candidate risk factor for inclusion in the model. These recommendations are similar to those made with regard to $USAGB10_t$, $USAGB30_t$ and $UCOM_t$ for the economic groups. The approach of Van Rensburg (2000) of augmenting the model in equation (8.2) with a second residual market factor to capture the impact of omitted risk factors is another approach to accounting for omitted risk factors. This approach acknowledges that $UM\varepsilon_t$ by itself is not an adequate proxy for omitted risk factors – a hypothesis supported by results in Table 8.17.

Finally, given the results in Table 8.17, it may be argued that *some* of the pairwise correlation between the residual series discussed in the context of the results in Table 8.16 is the result of the presence of sector specific risk factors. The presence of sector specific risk factors is suggested by pairwise residual correlation of a high magnitude that is *limited* to individual industrial sectors. However, the results in Table 8.17 suggest that the nine risk factors do not fully account for systematic variation in the return generating process. In light of this, it *is* plausible that the inclusion of systematic risk factors excluded from the original model specification will account for pairwise residual correlation. Therefore, these preceding findings are ambiguous; pairwise residual correlation may arise as a result of omitted systematic risk factors *or* as a result of omitted sector specific factors *or* both. It does however appear that $UM\varepsilon_t$ is an inadequate proxy for omitted risk factors.

8.5.7. Industrial sectors: A synthesis

The final part of the analysis involves investigating the return generating process of industrial sectors compromising the South African stock market (section 8.5). As for the economic groups, the model specification is denoted by equation (8.2).

Results indicate that $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$ and $URBAS_t$ followed by UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$ feature prominently in the return generating process underlying industrial sector returns; a finding similar to that made with respect to economic group returns (also see section 8.4.1 & 8.4.8; Table 8.12 & 8.13 for main results). The average R^2 suggests that the model explains over 42 percent of the variation in industrial sector returns. However, the direction of the relationship between industrial sector returns and risk factors differs across industrial sectors (see section 8.5.1: 223; Beenstock & Chan, 1988). The conditional variance of industrial sector returns is described by GARCH, IGARCH and EGARCH models with the EGARCH model featuring most prominently (section 8.5.2; Table 8.14). As with economic group returns, the appropriateness of the ARCH/GARCH framework is evident; widespread significance of ARCH and GARCH parameters is observed across sectors. Specification problems are addressed in section 8.5.3. In their entirety, the findings continue to be consistent; the above-mentioned factors remain relevant as does the application of the ARCH/GARCH framework after addressing specification problems. This can be seen from the results in Table A1.3 and (notably) Table A1.4 in Appendix 1. Furthermore, Figure 8.5 indicates (at a glance) that the presence of possible specification problems, as suggested by the presence of residual serial correlation or ARCH effects in industrial sector returns, does not detract from the results.

An comparison of the explanatory power of restricted versions of the model indicates that the unrestricted model (section 8.5.4; Table 8.15) outperforms a single-factor model relying solely upon UM_t to explain returns. While international risk ($UFTW_t$) plays an important role in explaining the time series behaviour of returns, combining the residual market factor, international risk and the set of domestic risk factors into the unrestricted model, yields a model with superior explanatory power (see section 8.5.4: 231). In investigating omitted risk factors in section 8.5.5, it is found that the mean level of pairwise residual correlation is generally low (see Table 8.16). Where high levels of mean pairwise residual correlation are observed in isolated instances, an inspection of the correlation matrix suggests that sector specific factors are present (see section 8.5.5: 234). Nevertheless, as two-thirds of pairwise correlation coefficients are statistically significant, there is a degree of ambiguity as to whether the omission of sector specific or systematic risk factors is responsible for pairwise residual correlation (section 8.5.5: 235). The presence of additional risk factors is investigated in section 8.5.6 and the results in Table 8.17 indicate that almost half of the residual series are significantly correlated with $USAGB10_t$ and $USAGB30_t$. This suggests that these factors are not accounted for by $UM\varepsilon_t$ and that they continue to have explanatory power. Nevertheless, the results are somewhat ambiguous; pairwise residual correlation may arise as a result of omitted systematic risk factors or omitted sector specific factors or both. What does appear to be certain is that $UM\varepsilon_t$ is not an adequate proxy for omitted risk factors (section 8.5.6: 238).

8.6. Conclusion

The APT framework is successfully applied in this chapter to investigate the return generating process underlying South African stock returns. It is shown on three levels – market (section 8.3), economic groups (section 8.4) and industrial sectors (section 8.5) - that a multifactor model informed by the APT framework, as presented in Chapter 2 and Chapter 3 can be used to investigate the return generating process. Moreover, the application of a multifactor framework yields improved results; a greater amount of variation is explained by a multifactor model relative to a single-factor model and a multifactor model is a better fit (see section 2.2.4, 8.3.1, 8.4.1 & 8.5.1; Table 8.4, 8.7, 8.9, 8.12 & 8.15). Furthermore, returns on the South African stock market are described by factors representative of systematic risk, as postulated by the APT framework (see section 2.2, 3.1 & 3.1.1; Chapter 4).

The ARCH/GARCH framework presented in Chapter 6 is appropriate for the modelling of South African stock returns. This is especially true in light of the discussion relating to the general properties and behaviour of stock returns in Chapter 5 and specifically, the statistical properties of South African stock returns presented in the preliminary analysis in Chapter 7 (see section 7.2).

The final chapter, Chapter 9, concludes the study; a general summary of the study is provided, findings are re-iterated and areas for research are identified.

9. CONCLUSION AND FURTHER RESEARCH

9.1. The APT framework and return generating process of South African stock returns

The APT is a response to acknowledgments that there are other factors aside from a market factor that feature in the return generating process and explain expected returns (King, 1966). It is assumed that returns are generated by a linear *k*-factor model and returns in equilibrium reflect multiple risk premia (section 2.1; Ross, 1976; Roll & Ross, 1980; Berry *et al*., 1988). In this study, the APT is not considered exclusively as an asset pricing model. Instead, the APT fulfils the role of a conceptual framework – the APT framework – within which the return generating process of South African stock returns is investigated.

This study investigates the return generating process of South African stock returns within the multifactor APT framework at three levels; namely, the market (section 8.3), economic group (section 8.4) and industrial sector level (section 8.5). As the APT framework assumes that *innovations* in systematic risk factors drive stock returns, innovations are generated for an extended number of macroeconomic factors assumed to proxy for systematic risk (Priestley, 1996) A total of twenty-eight factors, which are significantly correlated with returns on the JSE All-Share Index, are considered as candidate risk factors for explaining the return generating process of South African stock returns. These risk factors are hypothesized to represent nine risk categories; namely international, inflation, real activity, monetary policy, interest rate, commodity price, exchange rate, international trade and cyclical risk (see section 4.3; Table 7.5).

The model of the return generating process of South African stock market returns incorporates eight risk factors representative of seven risk categories. As the JSE All-Share Index is representative of aggregate market behaviour, returns on this series are modelled first and it is assumed that factors that impact returns on this series have a pervasive influence throughout the South African stock market. Results indicate that unexpected changes in international risk ($UFTW_t$), inflation ($UCPI_{t-1}$), inflation expectations ($URBAS_t$), real activity (UBP_{t-1}), money supply ($UM3_{t-1}$), oil prices ($UOIL_t$), the exchange rate ($UZARUS_t$), and the business cycle (UCI_t) significantly impact and explain returns on the JSE All-Share Index (Table 8.2 & 8.4). The model explains more than half of the realized variation in South African stock market returns and the residuals of this model are treated as the residual market factor, $UM\varepsilon_t$.

The model is then generalized to economic groups compromising the JSE All-Share Index. Factors that feature prominently in the return generating process of returns on economic groups are $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$. UCI_t has a limited impact upon economic group returns (see Table 8.7). The explanatory power of these factors is further investigated by estimating restricted versions of the model and results indicate that jointly the domestic risk factors, $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t, explain a substantial amount of the variation in returns on economic groups (Table 8.9). $UFTW_t$ plays a *very* important role in explaining returns suggesting that the South African stock market is highly integrated with world markets and heavily influenced by international developments. The model is generalized to industrial sectors returns. As with returns on the economic groups, multiple risk factors feature in the return generating process. Of particular importance are $UM\varepsilon_t$, $UFTW_t$, $UCPI_{t-1}$ and $URBAS_t$ followed by UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$ and $UZARUS_t$. The role of UCI_t is relatively minor (see Table 8.12 & 8.13). An analysis of the explanatory power of the domestic risk factors for industrial sector returns reveals that $UCPI_{t-1}$, $URBAS_t$, UBP_{t-1}, $UM3_{t-1}$, $UOIL_t$, $UZARUS_t$ and UCI_t explain a significant amount of variation in realized returns. $UFTW_t$ continues to play an important role, this highlighting the importance of international events for the South African stock market. Combining the residual market factor, $UM\varepsilon_t$, $UFTW_t$ and the domestic risk factors yields a multifactor model of the return generating process that explains a significant amount of the variation in aggregate returns, returns on economic groups and industrial sectors. The model explains a greater amount of variation at all levels of analysis relative to restricted specifications and a single-factor alternative incorporating UM_t.

The hypothesis that South African stock returns are characterized by a multifactor return generating process cannot be rejected (see section 1.2: 3). A residual market factor, $UFTW_t$ and domestic risk factors representative of six other risk categories have a pervasive influence on the time series behaviour of South African stock returns and explain a significant amount of variation in returns. The APT framework *can* therefore be applied to investigate the return generating process of South African stock returns. This finding is important given that Bilson *et al*. (2001) and Burmeister (2003) acknowledge that multifactor models grounded in the APT framework are not always easily constructed and applied to developing markets. The hypothesis that a multifactor model of the return generating process provides a better description of the time series behaviour of stock returns relative to a single-factor alternative cannot be rejected (see section 1.2: 3). On average, the unrestricted model explains a greater amount of variation in returns on the economic groups and industrial sectors relative to restricted single-factor models employing only $UM\varepsilon_t$, $UFTW_t$ and UM_t This also applies to the unrestricted model of JSE All-Share Index returns when compared against a single-factor alternative incorporating $UFTW_t$ (Table 8.2, 8.4, 8.9 & 8.15). The hypothesis that international risk plays an important role in explaining South African stock returns cannot be rejected (see section 1.2: 3). $UFTW_t$ explains a substantial amount of variation at all levels. In fact, the *average* explanatory power of $UFTW_t$ is greater than that of the domestic risk factors and $UM\varepsilon_t$ for the economic group and industrial sector return series. This suggests that the role and influence of international risk on the South African stock market cannot be overstated.

9.2. Properties and behaviour of stock returns and the ARCH/GARCH framework

Returns are assumed to be normally, independently and identically distributed (section 5.2). However, observed behaviour does not conform to these assumptions. Furthermore, the literature recognizes that volatility exhibits clustering, persistence, leverage effects and mean reversion (section 5.3). These observed properties of returns and volatility require an econometric framework that can account for these characteristics and yield robust model estimates. Such an econometric framework is the ARCH/GARCH framework which discards the assumptions of normality, independence and constant variance (Chapter 6). The multiple extensions of this framework capture the various properties of variance such as persistence, long memory and asymmetry.

Preliminary evidence shows that the return distributions of South African economic groups and industrial sectors depart from normality (section 7.2; Table 7.2). Distributions are characterized by excess kurtosis and negative skewness. It is likely that the extent of departures from normality is understated as outliers are excluded in the preliminary analysis. The conclusion is nevertheless the same; South African stock returns are not adequately described by the normal distribution. Although, the assumption of independence is not widely violated and returns are stationary in the mean, there is substantial evidence of time-varying variance in returns and ARCH effects in the residuals of an AR(1) model estimated using the LS methodology. There is limited evidence of the leverage effect (Table 7.4). Departures from normality, evidence of time-varying variance and the presence of ARCH effects in the residuals of an AR(1) model strengthen the case for model estimation within the ARCH/GARCH framework. Four ARCH/GARCH-type models are considered, namely the ARCH, GARCH, IGARCH and EGARCH models.

The model of returns on the JSE All-Share Index is first estimated using the LS methodology. However, regression diagnostics reveal that higher order ARCH effects are present in the residuals (Table 8.2). ARCH/GARCH modelling resolves this and the results also suggest that the ARCH/GARCH econometric framework is more robust to outliers relative to the LS framework (Table 8.3 & 8.4). ARCH/GARCH modelling is applied to the economic group and industrial sector return series (Table 8.7 & 8.13). Results indicate that the conditional variance is of a time-varying nature and therefore, ARCH/GARCH modelling is appropriate for most economic groups and industrial sectors (Table 8.8 & 8.14). Where present, residual serial correlation and ARCH effects are eliminated by re-specifying the ARCH/GARCH model or by augmenting the return generating process specification with autoregressive terms. Notably, the IGARCH model corrects for residual serial correlation in a number of economic groups. This suggests that certain ARCH/GARCH specifications may be more appropriate than others in the modelling of South African stock returns. The evidence supports the application of the ARCH/GARCH framework as an econometric framework for estimating models of the return generating process of South African stock returns.

9.3. Further research

The approach undertaken in this study and the findings of the analysis motivate for further research. Some of the possible avenues of further research are a direct outcome of findings made in this study whereas other avenues are related.

The risk exposure profile of economic groups in Figure 8.3 permits a comparison of sensitivities to risk factors *across* economic groups. However, a comparison of *within*-series sensitivity to innovations in risk factors is also of interest. Fabozzi (2008) suggests that this can be investigated by using *standardized* regression coefficients. This study sought to investigate the return generating process by identifying risk factor categories in the return generating process and factors with an excessive amount of explanatory power were excluded. However, the results in Table 8.1, Table 8.6, Table 8.11, and Table 8.17 suggest that there are other factors in the return generating process of South African stock returns aside from those incorporated into equation (8.1) and equation (8.2). Factors that show pervasive and statistically significant correlation with the residuals of the JSE All-Share Index, the economic groups and industrial sectors are $USAGB10_t$, $USAGB30_t$ and $UCOM_t$. Further research should consider the role of these factors *and* alternative sets of risk factors in the return generating process.

The results in Table 8.10 and Table 8.16 indicate that economic group and industrial sector residuals are correlated suggesting that there are systematic risk factors that feature in the return generating process but have not been incorporated into the model. Furthermore, given the relatively weak level of correlation between the economic group, industrial sector residuals and the omitted risk factors in the risk factor set, it is unlikely that the omitted risk factors will completely account for pairwise residual correlation. This is especially true for the economic groups where pairwise residual correlation is strong. Thus, the presence of pairwise residual correlation calls for the exploration of possible *unidentified* systematic risk factors. For the industrial sectors, there is ambiguity as to whether pairwise residual correlation arises as a result of sector specific factors. This is an area for further research. Moreover, these findings imply that the residual market factor fails to fully account for omitted risk factors (e.g. see section 8.4.6: 215). The solution suggested by Van Rensburg (2000) is to use a second residual market factor. Further research should consider this approach in modelling the return generating process and therefore, a closely associated direction of research is the identity of a second residual market factor. The residual market factor need not be derived from an aggregate market index such as the JSE All-Share Index. For example, Van Rensburg (2000) derives two residual market factors from the (old) JSE Industrial and All-Gold Indices. Further research should investigate the identity of alternative residual market factors that will result in the lowest levels of pairwise residual correlation and thereby, fulfil the role of c*omprehensive* proxies for omitted risk factors.

While the residual market factor can *partially* account for omitted risk factors, both identified and unidentified, it is impossible to definitively determine whether omitted risk factors will improve the overall performance of the model (e.g. see section 8.4.7: 218). Addressing this limitation requires that alternative sets of *identified* but omitted risk factors are considered in the model of the return generating process. This approach will alter the structure of the model of the return generating process presented here. Furthermore, this limitation also requires that *unidentified* risk factors are identified and incorporated into the model. This further emphasizes the need for an ongoing effort to identify systematic risk factors that drive South African stock returns. While the residual market factor partially accounts for omitted and unidentified risk factors, it does not shed light on how these factors will impact model performance directly.

The conventional approach to asset pricing and modelling the return generating process in studies based upon the APT framework relies upon *macroeconomic* factors (see section 3.1 & 3.1.1; Chapter 4). However, Chen (1991) and Bilson *et al*. (2001) suggest that factors constructed from the same market – factors such as *aggregate* dividend yields, earnings-price ratios, aggregate earnings and the stock market turnover – are better at explaining stock market returns. The impact of these factors is likely to also be systematic in nature. Further research should be undertaken into the explanatory power of such factors for South African stock market returns. The explanatory power of these factors can be assessed against that of factors derived from outside of the equity market.

The assumption that innovations in systematic risk factors drive returns is central to the APT framework (section 3.1.4 & 4.2; Priestley, 1996). Although this study employs innovations, it is worth investigating and comparing the explanatory power of innovations and factor series that consist of both unexpected and expected components. This is especially relevant given that a substantial number of studies do not make use of innovations and that it has been postulated that the pre-whitening process may remove information relevant to stock returns (see Poon & Taylor, 1991; Sadorsky & Henriques, 2001; Sadorsky, 2001). Finally, this study is concerned with the APT framework; a framework within which the return generating process can be investigated and the cross-section of expected returns explained. An obvious extension of this study is to establish which factors are priced in the cross-section of expected returns.

9.4. Concluding remarks

This study provides insight into the return generating process of the South African stock market and demonstrates how the APT can be applied as a conceptual framework in investigating the return generating process. The behaviour of South African stock returns is taken into account and an appropriate econometric framework is employed to model the return generating process. It is suggested that aspects of the proposed return generating process specification are investigated further and it is hoped that the findings of this study provide a basis for further research into the linkages between South African stock returns and systematic risk factors.

APPENDIX 1: STATISTICAL APPENDIX

Table A1.1: Phillips-Perron and White test statistics

	PP Test	White Test
JSE All-Share Index	-13.556***	0.173
Economic Group Index		
1. Oil & Gas	-14.416***	2.413*
2. Basic Materials	-12.994***	3.069*
3. Industrials	-12.437***	0.448
4. Consumer Goods	-14.705***	0.967
5. Health Care	-12.320***	1.367
6. Consumer Services	-10.933***	0.871
7. Telecommunication	-12.360***	10.385***
8. Financials	-13.454***	0.034
9. Technology	-11.733***	1.314
Industrial Sector Index		
1.1: Oil & Gas Producers	-15.007***	4.301**
2.1: Chemicals	-13.481***	3.712**
2.2. Forestry & Paper	-13.597***	0.262
2.3: Industrial Metals	-13.484***	0.596
2.4: Mining	-13.760***	1.905
3.1: Const & Materials	-11.127***	0.703
3.2: General Industrials	-13.327***	0.436
3.3: E & E Equipment	-11.304***	0.144
3.4: Industrial Engineering	-9.807***	5.352***
3.5: Industrial Transport	-13.946***	1.048
3.6: Support Services	-11.846***	0.663
4.1: Automobiles & Parts	-11.508***	1.802
4.2: Beverages	-13.392***	1.161
4.3: Food Producers	-11.488***	1.662
5.1: Health Care E & S	-11.735***	3.520**
5.2: Pharma & Biotech	-14.237***	0.202
6.1: Food & Drug Retailers	-14.841***	1.125
6.2: General Retailers	-10.338***	1.654
6.3: Media	-11.587***	1.365
6.4. Travel & Leisure	-11.017***	0.549
7.1: Fixed Line Telecom	-12.439***	9.285***
8.1: Banks	-14.151***	0.246
8.2: Non-life Insurance	-12.698***	0.248
8.3: Life Insurance	-12.971***	0.497
8.4: General Financial	-12.911***	0.669
8.5: Equity Investment Inst	-13.925***	0.077
9.1: Soft & Comp Services	-11.597***	1.457

Notes:
1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. Outliers *not* excluded.

Source: Compiled by author

Table A1.2: ARCH/GARCH economic group models corrected for residual serial correlation

	Oil&Gas	Consumer Goods	Telecom.	Financials	Technology
Intercept	-0.002*	-0.001	-0.001	-0.002***	-0.000
$UM\varepsilon_t$	1.229***	0.702***	0.699***	0.725***	0.906***
$UFTW_t$	0.873***	0.795***	0.799***	0.883***	1.400***
$UCPI_{t-1}$	-0.228	-0.553	-0.899**	-0.431***	-0.887*
$URBAS_t$	-0.685***	-0.598**	-1.117***	-0.934***	-0.078
UBP_{t-1}	0.068***	0.022	0.130***	0.080***	0.046
$UM3_{t-1}$	1.089***	0.708***	-0.131	0.404***	0.577*
$UOIL_t$	0.174***	-0.001	-0.103**	-0.098***	0.059
$UZARUS_t$	0.477***	0.329***	-0.194*	0.055	0.526***
UCI_t	2.235***	1.144**	-0.166	-0.055	-0.142
$R_{it-\tau}$	-	- 0.082	0.124**	-	-
$\bar{R}^2$	0.727	0.493	0.457	0.660	0.410
AIC	-5.204	-4.904	-4.271	-5.411	-3.685
F-Statistic	148.011***	21.388***	39.662***	149.703***	26.976***
$Q(1)$	2.339	0.697	0.103	0.000	0.743
$Q(5)$	7.171	6.795	4.653	8.191	5.123
$Q^2(1)$	0.037	0.599	0.433	0.226	0.089
$Q^2(5)$	4.141	1.579	2.758	3.166	3.807
ARCH(1)	0.031	0.583	0.425	0.220	0.086
ARCH(5)	0.938	0.262	0.531	0.640	0.634
ARCH/GARCH	IGARCH(2,1)	GARCH(1,2)	GARCH(2,1)	IGARCH(1,2)	IGARCH(1,2)
Distribution	GED	Student's *t*	GED	GED	GED
ω	-	4.79E-6***	2.41E-05	-	-
α_1	-0.045***	0.002*	0.336*	0.002	0.029*
α_2	0.181***	-	-0.253	-	-
β_1	0.864***	1.991***	0.893***	-	1.736***
β_2	-	-1.003***	-	1.153	-0.765***
γ_1	-	-	-	-0.155	-
F-Statistic	81.543***	720689.1***	511.003***	0.286	859.706***

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. $Q(1)$ and $Q(5)$ are Ljung-Box test statistics for residual serial correlation at the 1st and 5th orders.
4. $Q^2(1)$ and $Q^2(5)$ are Ljung-Box test statistics for squared residual serial correlation at the 1st and 5th orders.
5. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

Source: Compiled by author

Table A1.3: ARCH/GARCH industrial sector models corrected for residual serial correlation

	Forestry & Paper	Industrial Engineering	Support Services	Food Producers	General Retailers	Non-life Insurance	Soft & Comp Services
Intercept	-0.007***	-0.002	-0.005***	-0.002*	-0.004**	-0.003**	0.000
$UM\varepsilon_t$	1.072***	0.591***	0.834***	0.792***	0.634***	0.790***	0.845***
$UFTW_t$	1.004***	0.677***	0.750***	0.500***	0.758***	0.579***	1.414***
$UCPI_{t-1}$	0.531	-0.123	-0.519	-1.148***	-0.814**	-1.184***	-0.660
$URBAS_t$	-1.380***	-0.803**	-0.714**	-0.939***	-1.193***	-1.067***	-0.083
UBP_{t-1}	0.101**	0.040	0.042*	0.058***	-0.029	0.077**	0.027
$UM3_{t-1}$	0.479	-0.235	0.378	0.230	0.592*	-0.262	0.560
$UOIL_t$	0.112**	-0.009	-0.106***	-0.046	-0.034	0.047	0.073
$UZARUS_t$	0.272*	-0.014	0.022	-0.055	-0.220**	-0.066	0.513***
UCI_t	0.796	0.433	0.215	-0.200	0.357	-0.319	-0.274
$R_{it-\tau}$	-	0.285***	-	0.092**	0.198***	-	-
$R_{it-\tau}$	-	0.112*	-	-	-	-	-
$\bar{R}^2$	0.345	0.430	0.561	0.539	0.513	0.438	0.392
AIC	-3.529	-4.376	-4.951	-5.213	-4.605	-4.646	-3.555
F-Statistic	22.369***	9.428***	35.181***	43.252***	27.088***	26.996***	22.717***
$Q(1)$	0.140	2.188	2.419	0.124	0.143	2.528	0.871
$Q(5)$	5.631	7.885	7.058	6.450	2.103	6.550	5.267
$Q^2(1)$	0.078	0.002	0.149	0.000	0.000	0.674	0.164
$Q^2(5)$	0.534	4.843	6.448	1.584	3.948	1.259	4.004
ARCH(1)	0.075	0.002	0.146	0.000	0.000	0.657	0.159
ARCH(5)	0.112	0.942	1.059	0.296	0.767	0.256	0.642
ARCH/GARCH	EGARCH(2,2)	EGARCH(1,2)	EGARCH(2,2)	EGARCH(2,2)	EGARCH(1,2)	GARCH(1,2)	IGARCH(1,2)
Distribution	GED	Normal	Normal	Normal	Normal	Student's *t*	GED

Table A1.3: ARCH/GARCH industrial sector models corrected for residual serial correlation (continued)

ω	-6.392	-5.068	-12.589***	-14.785***	-0.260*	3.91E-05	-
α_1	-0.378	-0.026	-0.051	-0.156	-0.049	0.110	0.022
α_2	0.322	-	-0.077	0.356*	-	-	-
β_1	0.201	0.346	-0.178	-0.215	1.431***	0.111	1.780***
β_2	-0.194	-0.034	-0.415	-0.558**	-0.471*	0.697	-0.802***
γ_1	-0.035	-0.178	0.237**	0.199*	-0.141**	-	-
F-statistic	0.808	0.558	1.815	11.314***	1787.903***	37.988***	1000.142***

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *F*-statistics are reported for Wald's test of linear restrictions testing the null hypothesis of coefficients jointly equalling zero (Nelson, 1991; McMillan & Ruiz, 2009).
3. $Q(1)$ and $Q(5)$ are Ljung-Box test statistics for residual serial correlation at the 1st and 5th orders.
4. $Q^2(1)$ and $Q^2(5)$ are Ljung-Box test statistics for squared residual serial correlation at the 1st and 5th orders.
5. ARCH(1) and ARCH(5) are LM test statistics for residual ARCH effects at the 1st and 5th orders.

Source: Compiled by author

Table A1.4: Summarized ARCH/GARCH industrial sector model results incorporating corrected models

	Mean	%Sig	Positive	Negative
Intercept	-0.002***	59.259%	5 (3)	22 (13)
$UM\varepsilon_t$	0.820***	100%	27 (27)	0 (0)
$UFTW_t$	0.722***	100%	27 (27)	0 (0)
$UCPI_{t-1}$	-0.839***	62.963%	1 (0)	26 (17)
$URBAS_t$	-0.786***	74.074%	1 (0)	26 (20)
UBP_{t-1}	0.054***	55.556%	24 (15)	3 (0)
$UM3_{t-1}$	0.299***	40.741%	21 (10)	6 (1)
$UOIL_t$	0.004	40.741%	10 (5)	17 (6)
$UZARUS_t$	0.015	48.148%	12 (7)	15 (6)
UCI_t	0.307*	22.222%	18 (4)	9 (2)

Notes:

1. *** Indicates statistical significance at the 1 percent level of significance. ** Indicates statistical significance at the 5 percent level of significance. * Indicates statistical significance at the 10 percent level of significance.
2. *t*-test applied to mean of coefficients to establish whether factor coefficients are significantly different from zero (Beenstock & Chan, 1988).
3. Values in brackets ( ) are the number of statistically significant instances at the 10 percent level of significance.

Source: Compiled by author

APPENDIX 2: DATA SOURCES

Table A2.1: Data Sources

Factor	Symbol	Source
1. Market Indices		
1.1. JSE All-Share Index (Total returns)	M_t	INET Bridge Database
1.2. Dow Jones Industrial Average	DJ_t	INET Bridge Database
1.3. FTSE World Index	FTW_t	INET Bridge Database
1.4. FTSE 100 Index	$FTSE_t$	INET Bridge Database
1.5. MSCI World Index	$MSCI_t$	INET Bridge Database
1.6. MSCI World Index (Local Currency)	$MSCIR_t$	INET Bridge Database
1.7. Nikkei 225	NK_t	INET Bridge Database
2. Inflation		
2.1. Consumer Price Index	CPI_t	INET Bridge Database
2.2. Inflation Expectations	$RBAS_t$	INET Bridge Database
2.3. Producer Price Index	PPI_t	South African Reserve Bank
3. Real Activity		
3.1. Industrial Production	MP_t	South African Reserve Bank
3.2. Building Plans Passed	BP_t	South African Reserve Bank
3.3. Retail Sales	SLS_t	South African Reserve Bank
4. Money Supply		
4.1. M1A (Narrow) Money Supply	$M1A_t$	INET Bridge Database
4.2. M3 (Broad) Money Supply	$M3_t$	INET Bridge Database
5. Interest Rates		
5.1. Three Month Treasury Bill Rate	$TBT3_t$	INET Bridge Database
5.2. 10 Year Government Bond Yield	$SAGB10_t$	INET Bridge Database
5.3. 30 Year Government Bond Yield	$SAGB30_t$	INET Bridge Database
5.4. Changes in the Term Structure	DTS_t	INET Bridge Database[Construct]
6. Commodities		
6.1. Rand Brent Crude Price	OIL_t	INET Bridge Database
6.2. Rand Gold Price	$GOLR_t$	INET Bridge Database
6.3. All Commodity Index	COM_t	INET Bridge Database
6.4. Metal Index	MET_t	INET Bridge Database
6.5. Non-Fuel Commodity Index	$NFCI_t$	INET Bridge Database
7. Exchange Rates		
7.1. Rand-Dollar Exchange Rate	$ZARUS_t$	INET Bridge Database
7.2. Rand/Currency Basket Exchange Rate	$ZARBA_t$	INET Bridge Database[Construct]
8. International Trade		
8.1. Terms of Trade	TT_t	International Monetary Fund
8.2. Composite Lead. Index of Trad. Partners	LTT_t	South African Reserve Bank
8.3. Composite Coinc. Index of Trad. Partners	CTT_t	South African Reserve Bank
9. Business Cycle Indicators		
9.1. Composite Leading Index	LI_t	South African Reserve Bank
9.2. Composite Coincident Index	CI_t	South African Reserve Bank

Notes:
1. [Construct] indicates that factor is constructed from data obtained from source.
2. For certain data series, INET Bridge is a *secondary* provider.

Source: Compiled by author

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