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Exploring Learner Errors and Misconceptions in Algebraic Expressions with Grade 9 Learners Through the use of Algebra Tiles

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South African learners' performance in mathematics, both locally and internationally, has raised significant concerns, particularly in the realm of algebra. To address this issue, a mixed-methods study was conducted to explore the effectiveness of using algebra tiles to teach algebraic expressions. The study aimed to investigate algebraic expression errors and misconceptions among Grade 9 learners and evaluate the impact of an intervention involving algebra tiles on learners' post-test results. Data were collected through tests administered to a class of 22 Grade 9 learners. The findings of the study confirmed the error types identified in the literature and demonstrated a notable improvement in performance on the post-test following the intervention using algebra tiles. The results indicated that the intervention successfully rectified pre-existing errors and misconceptions, resulting in an 18% enhancement in overall performance among participants. The study aligned with a Vygotskian sociocultural perspective, emphasizing the pivotal role of manipulatives in facilitating learning within the zone of proximal development. The use of manipulatives aids learners in constructing conceptual understanding by reinforcing abstract ideas. Therefore, the study contributes to existing research highlighting the utility of manipulatives in mathematics classrooms, underscoring their effectiveness in enhancing learning outcomes.

Keywords: *Algebraic expressions; algebra tiles; manipulatives; intervention*

Introduction

Background of the Study

This article reports the findings of a Master's study (Stemele, 2023) that explored the efficacy of employing an intervention using algebra tiles to improve performance on the post-test in the area of simplifying and factorising algebraic expressions. Algebra is a foundational skill that prepares learners for other areas of mathematics. Success in mathematics is highly contingent upon understanding algebraic concepts (McIntyre, 2005). Proficiency in algebra is a prerequisite for entry into advanced mathematics and is correlated with positive life outcomes, such as college graduation (Adelman, 2006). Learners in Grade 9 should be able to simplify, multiply, divide and factorise algebraic expressions among other computations (Department of Basic Education, 2011). Grade 9 is an important year in high school. The results of Grade 9 mathematics determine whether a learner continues with mathematics or moves on to mathematical literacy in Grade 10. In South Africa, Grades 7–9 constitute the senior phase, while Grades 10–12 are classified as Further Education and Training Phase. Despite the expectation that learners in the senior phase should be able to comprehend algebraic language and be able to expand and simplify algebraic expressions, it was reported that learner mathematics achievements in the Annual National Assessments were still at an unacceptable level (Department of Basic Education, 2014). A 2019 Trends in International

Mathematics and Science Study reported that 59% of Grade 9 learners did not have basic mathematics skills. Following reports of this poor performance, South Africa's Grade 9 learners are rated last worldwide in terms of academic achievement in mathematics and science (Mullis et al., 2020).

To improve these results, research in mathematics education recommends that errors and misconceptions in algebraic expressions be understood and addressed (Lim, 2010; Luneta & Makonye, 2010; Egodawatte, 2011; Brodie, 2014; Pournara et al., 2016; Booth et al., 2017; Enu & Ngcobo-Ndlovu, 2020; Marpa, 2019; Cholily et al., 2020). Suggestions include that teachers comprehend the origins of the errors and misconceptions by probing learners' understanding of them. Teachers can encourage learners to explain correct and incorrect worked examples to help them focus on developing a solid conceptual foundation as well as the necessary procedural skills (Booth et al., 2017). Moreover, the literature suggests that manipulatives like algebra tiles be employed as a teaching device to assist learners in understanding algebraic expressions (Ergene & Haser, 2021; Pranada et al., 2019; Salifu, 2022; Sharp, 1995). However, there is limited research employing manipulatives such as algebra tiles to address errors and misconceptions in algebraic expressions within the South African context. Therefore, this study explored the effectiveness of the employment of algebra tiles as a manipulative to address the errors and misconceptions that Grade 9 learners in this study made in a pre-test on algebraic expressions.

An algebraic expression is a combination of numeric and symbolic notation with a variety of variables and signs (Sfard, 1995). For example, $5x - 2y + 10$ is recognised as an algebraic expression. Algebraic expressions present numbers by means of letters, without indicating the values that those letters represent (McNeil et al., 2010). Providing learners in Grades 7–9 with manipulatives to help address the errors and misconceptions they hold is a necessary skill to assist in converting common language into algebraic expressions that will later aid learners in forming algebraic equations to solve for unknowns. The research question was: what are the changes in Grade 9 post-test results following the intervention using algebra tiles to address the errors and misconceptions that learners hold?

The study used the term 'errors and misconceptions'. Errors are systematic and are grounded at a deeper conceptual level. Learners commit errors when their responses are inaccurate, as explained by Brodie (2014). Brodie notes that when learners make multiple errors and gain confidence from them, misconceptions can develop. These misconceptions, in turn, lead to further errors. Understanding the errors made in tests is crucial because they unveil misconceptions. The manner in which errors are addressed holds significance, as it can either enhance learners' conceptual understanding or deepen misconceptions (Brodie, 2014).

Pournara (2020) and Pournara et al. (2016) conducted extensive studies on the challenges faced by Grade 9 learners in mathematics, irrespective of school resources. They identified common issues such as difficulties with conjoining, negatives and brackets, and a tendency to evaluate expressions prematurely rather than maintaining them in open form. Furthermore, Pournara et al. (2016) highlighted persistent errors in multiplication, exponents, letter evaluation and mastering exponent basics among learners across various schools. These errors, if not addressed effectively, can hinder progression to higher levels and negatively impact the final Grade 12 paper 1 results. In a related study, Pournara et al. (2022) examined errors in solving linear equations with two unknowns on both sides using data from the Wits Mathematics Connect Secondary project. They found that learners in Grades 7–9 encounter significant challenges in mastering such equations, including algebraic simplification and problems involving negatives and subtraction. The researchers suggested improved teaching approaches to address these challenges, highlighting the ongoing struggle for learners to grasp fundamental algebraic concepts, which could impede their mathematical progression. In our study, we concentrate on the use of algebra tiles as a manipulative in intervention lessons. We explore how these tools can assist learners in rectifying errors and addressing misconceptions when solving algebraic expressions.

Theoretical Background and Literature Review

This study draws on Vygotsky's sociocultural theory since concepts of zone of proximal development and scaffolding support ways that errors and misconceptions can be addressed. Vygotsky described the zone of proximal development (ZPD) as 'the area between actual and potential development, as judged by problem-solving with adult guidance or more skilled peers' (1978: 86). The ZPD represents the gap between the independent and assisted skills of a learner. Algebra tiles can be seen as the manipulative to close this gap. Learners can build their mathematical comprehension using algebra tiles, which are tactile objects that they can manipulate. Algebra tiles are two-dimensional shapes used to represent variables and constants (Salifu, 2022). The four colours that make up algebra tiles are yellow, blue, green and red. Yellow, blue and green tiles represent positives, while only red tiles represent negatives (Salifu, 2022). The colours and shapes of the different algebra tiles are represented in Figure 1.

Algebra tiles facilitate mental visualisation by incorporating concrete objects (Chaurasia, 2019; Pranada et al., 2019; Ergene & Haser, 2021; Gabina, 2019; Hall, 1999; Salifu, 2022; Thornton, 1995; Wingett, 2019). They facilitate the transition between manipulating algebraic expressions and manipulating concrete examples by allowing the creation of transitional situations (Sharp, 1995). The tiles provide alternative representation systems that are internalised by learners in a way that memorised facts and rote manipulations do not. Learners can add, subtract, multiply, divide, simplify and factorise algebraic expressions with the aid of algebra tiles, which prove to be a highly effective tool for assisting learners in manipulating algebraic expressions (Salifu, 2022).

The first image from the left in Figure 2 depicts the addition of two integers, adding 2 to -7. As shown in the image, the positive 2 is represented by two yellow tiles, while the negative 7 is represented by red tiles. A pair of yellow tiles is paired with a pair of red tiles (the cancellation procedure), and there are only five red tiles remaining, so the answer is -5. Hence $2 - 7 = -5$.

The second image from the left depicts addition and subtraction expressions in algebra. The positive 5 illustrated by yellow squares is added to the negative 5 illustrated by red squares. The negative $2x$ red rectangle tiles are like the negative $4x$ red rectangle tiles. The positive 5 and negative 5 will pair using the procedure for cancellation. With negative $2x$ and negative $4x$, the tiles do not pair because they are all negative like terms, so they are added together to obtain $-6x$.

The third and fourth images illustrate how to multiply a binomial by a binomial using tiles, as well as how to factorise a trinomial, which is the opposite of multiplying a binomial, using the tiles. To obtain the solution, one must multiply each tile in the first column by each tile in the top row.

Ergene and Haser (2021) illustrated the positive impact of algebra tiles on performance in their study conducted in Turkey. In their research, they analysed the efficacy of the tiles for sixth-grade learners. In an experiment involving classes taught by the same mathematics teacher, one class was taught algebra without the use of algebra tiles, whereas the other class utilised algebra tiles throughout their seven-hour algebra lesson. Learners in the experimental group performed better in the following areas: writing algebraic expressions for the given questions; determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions; performing

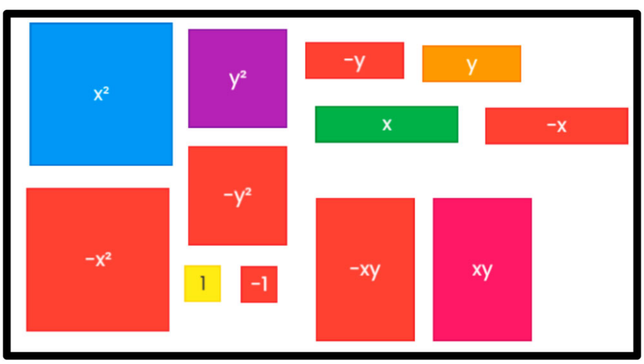

Figure 1. A representation of algebra tiles

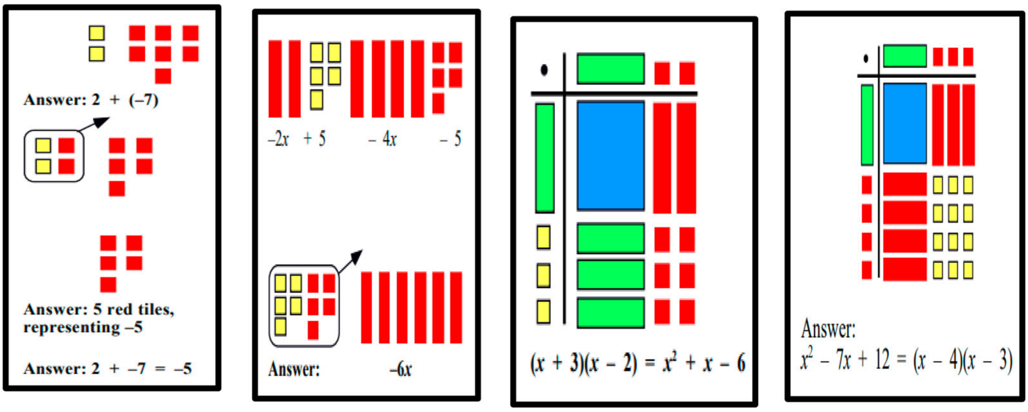

Figure 2. The manipulation of algebra tiles (Hall, 1999)

operations with the given models of algebraic expressions; performing addition and subtraction with the given models of algebraic expressions; and writing given algebraic expressions as multiplicative expressions (Ergene & Haser, 2021).

Chaurasia (2019) worked with secondary school learners in India and demonstrated that algebra tiles can engage uninterested learners in the classroom by accommodating various learning styles. The study revealed that by manipulating algebra tiles, learners were able to interact and engage with one another in groups. The research comprised nine exercises performed by the participants. These exercises included integer addition and subtraction, addition and subtraction of like terms, simplification of algebraic expressions, linear equations, simplification of polynomials and factoring. The use of concrete materials improved learners' confidence and understanding, and this was true for all learning style groups. Learners were able to adapt knowledge obtained from concrete experiences to abstract situations (Chaurasia, 2019).

Wingett (2019) evaluated algebra tiles for Grade 9 learners in the United States who failed Algebra One for at least one semester. Two learner groups were given a pre- and post-test of a specific ability (binomial multiplication). The second group, but not the first, was instructed using algebra tiles. The second group used algebra tiles for manipulating concrete, abstract and representational models. The learners were expected to have drawn the tiles and written the algebraic terms indicated by the tiles in response to the exercise during the intervention lessons. Wingett (2019) showed that the algebra tile manipulators enhanced test scores. The experimental group learners grew more at ease with the manipulatives once they recognised that the tiles represented a concrete depiction of an abstract ability.

Gabina (2019) observed that when learners utilised algebra tiles during algebraic expression lessons taught with manipulatives, their level of participation and comprehension of operations on algebraic expressions increased. Salifu (2022) also asserted that the use of algebra tiles is extremely beneficial in solving linear equations with one variable. The experimental group (algebra tiles and balance model) outperformed their counterparts in the control group, who did not use any manipulatives, according to a post-test achievement comparison conducted by Salifu (2022).

Salifu's (2022) study corroborates Gabina's (2019) findings that learners who utilize algebra tiles while solving algebraic expression problems demonstrate higher levels of performance compared with those who do not employ this strategy. Both researchers conclude that the use of algebra tiles promotes active learner engagement. This literature underscores how algebra tiles enhance learners' comprehension and cognition in solving algebraic expression tasks. The aim of this research is to foster learner engagement in identifying and addressing errors and misconceptions in algebraic expressions through the utilisation of algebra tiles. Furthermore, the use of algebra tiles is expected to instill in learners an intrinsic motivation to independently tackle exercises involving algebraic expressions.

Research Material and Methods

The study utilised the pre-experimental one-shot case study (Komala, 2018) in which an experimental treatment (e.g. on a single group) is performed, followed by observation of the experimental participants (e.g. learners). The use of these research methodologies allowed a better understanding of whether the intervention of the algebra tiles was successful in addressing learner errors and misconceptions.

Participant Selection

This research involved 22 Grade 9 learners in a public school in Pretoria, South Africa, with the principal's permission granted for after-school hours. The first author conducted a 10 day intervention. Parental consent for the study, involving minors, was obtained through signed letters. Identities have been concealed, and participants are referred to as 'learners 1–22' to maintain confidentiality.

The Intervention Using Algebra Tiles

The Grade 9 participants were given a pre-test to investigate and analyse the errors they made and misconceptions they held. Following the pre-test, an intervention programme with 10 lessons was implemented with the same group of learners. They then completed a post-test to determine whether the misconceptions and errors identified in the pre-test had been addressed in the intervention lessons.

In the initial research stage, we analysed and identified common errors and misconceptions from the pre-tests, using Ncube's (2016) findings in Table 1 as a reference. The first author conducted the analysis and shared the results with learners to enhance their understanding of the errors during the intervention. The objective was to raise awareness among learners in the study about prevalent errors and misconceptions when solving algebraic expressions. Prior to intervention sessions, these misconceptions and errors were reviewed to shape the content of the lessons.

Table 1. Categorisation and description of errors (adapted from Ncube, 2016)

Categories	Description for errors	Examples of common errors
Conjoin error	Lack of understanding of like terms and unlike terms.	(1) $3c + 4d = 7cd$ (2) $8a + 6 = 14a$
Misapplication of rules	Adding instead of multiplying, incorrect application of previously learned procedures.	(1) $4m \times m = 5\text{m}$ (2) $\frac{x}{y} + \frac{w}{z} = xz + yw$
Misinterpretation of symbolic notation	Partial misunderstanding of factorisation.	(1) $\frac{ma + mb}{m + md} = \frac{a + b}{d}$
Invalid distribution of brackets	Incomplete expansion of brackets and overgeneralised the distributive law.	(1) $2(3a + 4) = 6a + 4$ (2) $(2m - n) + n = 2mn - n^2$
Sign errors	Problems working with integers and misunderstanding operation signs.	(1) $(8x^2 + 3x + 4) - (5x^2 - 7x + 2)$ $= 3x^2 - 4x + 6$
Substituting letters with numeric values	Substituting letters with random numbers.	(1) if $b + d = 6$ *then* $b + d + e = ?$ Learner solution $b + d + e = 9$ Learner assuming b, d, and e are identical

The first author further categorised the error types into slips, sign errors, misconceptions, substitution errors and multiplication errors based on the identification derived from the data. Slips are incorrect responses that originate from carelessness, whereas errors are incorrect responses that reflect a lack of conceptual understanding (Moru et al., 2014). Luneta and Makonye (2010) explain that slips are unintended mistakes. This means that the learner made a small error in the solution steps, which affected their final answer. If this error is rectified, the learner will be able to complete the exercise without further problems.

Sign errors are possible outcomes of performing operations such as subtracting numbers or adding integers in the wrong order (Seng, 2010). This error is caused by improper sign usage while simplifying algebraic expressions.

Mixing the rules is identified as a misconception because the learners are applying their prior knowledge to the new information in an overly generic manner (Brodie, 2014). The misconceptions are because of the overgeneralisation of existing knowledge, and they are pervasive; therefore, they will continue to arise because of inadequate conceptual knowledge (Egodawatte, 2011; Moyo, 2020).

Learners make substitution errors when they substitute correctly but are unable to determine the numerical value of the expression, showing a lack of fundamental knowledge on the correct use of signs while doing addition and subtraction (Ncube, 2016). In the substitution error, letters are substituted for numbers in an algebraic expression to find the expression's value.

Multiplication errors are caused by learners' misunderstanding of the BODMAS rules. Learners often tend to add and subtract before performing multiplication (Ncube, 2016). Learners demonstrate difficulty with multiplication of algebraic expressions involving many terms, especially when stated in a different format with brackets.

The analysis of error types from the pre-test resulted in the design of an intervention, following which 10 lessons were enacted over a period of two weeks. Each lesson lasted an hour. Each day, the intervention took a different format determined by the algebraic expressions that had to be solved on that day. As outlined in Table 2, the design and implementation of the intervention progressed through preparation of activities that embed the learning topics in the stages of development. The intervention lessons included exercises from the Grade 9 Mindbourne textbook, the current curriculum resource for the learners.

Using the required algebra tiles, each lesson was devoted to addressing errors and misconceptions. In addition, several opportunities for practising and demonstrating with the tiles were provided. Learners in this study were also encouraged to use algebra tiles for the exercises.

Rigour

Our research primarily focused on utilising algebra tiles to address errors and misconceptions in algebraic challenges. The pre- and post-tests were meticulously crafted to ensure fairness, age-appropriateness, non-discrimination, and adherence to quality standards. Aligned with the South African Grade 9 curriculum, both tests contained identical content, although not identical in structure. The intervention's effectiveness was gauged by measuring learner performance improvement. To maintain reliability, the post-test underwent changes in structure and question count. Despite occasional challenges in a classroom setting, all learners completed both tests, ensuring consistency. The study aligns with the intervention model suggested by Bryant et al. (2020), showcasing promise for future investigations. Although sessions were limited to afternoons, the validity and reliability of the tests affirm that the study meets the criteria for validity. The results depict how Grade 9 learners employed algebra tiles to address challenges, showcasing diverse methodologies before and after the intervention.

Data Analysis Results

Pre- and Post-test Performance of Learners

All learners in this study were required to take a pre- and post-test, each worth 40 points and based on the content of algebraic expressions. The contents of both tests were similar, but not identical. After

Table 2. Intervention lessons

Lesson	Focus of the lesson	Summary of each lesson
1	Examining pre-test errors	To make learners in this study aware of their errors and use them as a teaching point, the pre-test assessed learners' work and revealed the errors.
2	Algebra tiles add and subtract like terms	Learners combine tiles to produce 0s by pairing like terms. Exercise 8.1 question (a) focuses on adding and subtracting like terms.
3–4	Multiplying binomial products and squares	Learners often multiply binomials by squaring the first and last term, e.g. $(x-2)^2 \neq x^2+4$ This activity aimed to clarify for learners that all terms must be multiplied. Learners solved exercise 8.2 (a) and (c) by multiplying binomials with algebra tiles.
5–6	Polynomial division by integers and monomials	Modeling the divisor vertically. Learners utilised dividend tiles to form a rectangle whose height equalled the divisor's length and whose quotient was the horizontal width. In addition, learners conducted exercise 8.1 question (d).
7	Factorisation: highest common factor	Learners built a rectangle with the tiles: rectangle length is the solution; width is the highest common factor. Exercise 8.5, part (a) was solved using the factorisation method.
8	Factorisation: common bracket	This lesson showed the learners that distributing a number outside the bracket is like finding the highest common factor, but in expanded form. Exercise 8.5 (b) required factoring, hence the common bracket must be removed. In certain instances, learners must alter the symbol to fit the bracket. Learners did not mind the obligation to change signs.
9	Factorisation: difference of two squares	Learners were told to first assemble the squares and then add the 'missing' pieces to complete the rectangle to find the difference of perfect squares. As part of their work, learners completed exercise 8.8 (a) and (b).
10	Factorisation-trinomial	The solutions for trinomial factorisation were illustrated with algebra tiles. Learners solved exercise 8.9 (a) and (b). Learners who use algebra tiles effectively appear to understand the relationship between multiplication, division and factorisation of algebraic expressions.

the intervention lessons had taken place, the pre-test results were compared with the post-test results. Table 3 summarises the performance of learners on their pre- and post-tests. The pre-test results indicated that learners initially performed poorly, with an average score of 48%; however, the post-test results indicate remarkable progress, with an 18 percentage-point gain, resulting in an average score of 66%. The gains indicate that the algebra tile-based 10-lesson intervention strategy was successful. Considering this, it is vital to analyse the magnitude of the disparities in greater depth, as well as the various methods by which learners acquired the capacity to solve algebraic expressions. Table 4 displays the performance of all 22 learners on both the pre-test and post-test. The results of each learner's performance on both tests are displayed in Figure 3.

Table 3. The average overall performance on the pre- and post-tests

22 Learners	Pre-test (%)	Post-test (%)	Gains (%)
Mean (%)	48	66	18

Table 4. A summary of the results from both the pre- and post-tests that demonstrate learner achievement

	Algebraic expressions				
	Pre-test		Post-test		Increase/decrease
Learner	40	%	40	%	%
1	8	20	19	47.5	27.5
2	14	35	27	67.5	32.5
3	15	37.5	27	67.5	30
4	27	67.5	31	77.5	10
5	19	47.5	22	55	7.5
6	6	15	30	75	60
7	24	60	23	57.5	–2.5
8	24	60	30	75	15
9	18	45	20	50	5
10	18	45	20	50	5
11	21	52.5	30	75	22.5
12	18	45	29	72.5	27.5
13	16	40	25	62.5	22.5
14	11	27.5	18	45	17.5
15	26	65	32	80	15
16	25	62.5	16	40	–22.5
17	27	67.5	27	67.5	0
18	12	30	24	60	30
19	15	37.5	25	62.5	25
20	26	65	35	87.5	22.5
21	28	70	31	77.5	7.5
22	27	67.5	38	95	27.5
Average	19.31818	48.29545	26.31818	65.79545	17.5

In the final column of Table 4 the percentage of growth for each learner is displayed. The table revealed that there were only two negative gains and one learner who neither gained nor lost. At least 11 learners' post-test improvements exceeded 20%. The average increase of 18% for all learners in this study implies that the learners benefited from the intervention classes.

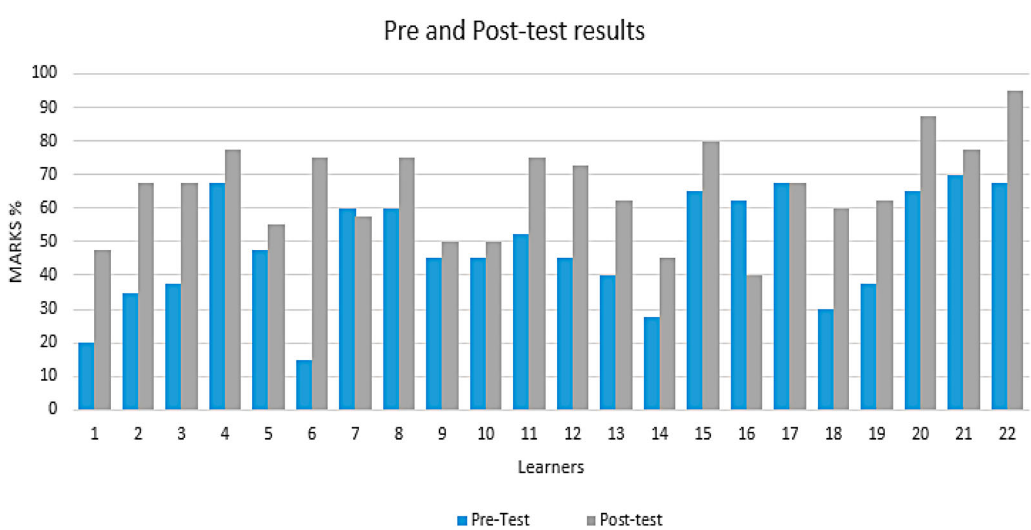

Figure 3. Learners' performance on each test

Table 5. The dependent sample test results

Pre-test	Post-test	Difference (post-test – pre-test)	Standard deviation (Diff-M)	Sq-Dev
20	47.5	28	10.55	111.21
35	67.5	33	15.55	241.66
37.5	67.5	30	12.55	157.39
67.5	77.5	10	–7.45	55.57
47.5	55	7	–10.45	109.3
15	75	60	42.55	1810.12
60	57.5	-2	–19.45	378.48
60	75	15	–2.45	6.02
45	50	5	–12.45	155.12
45	50	5	–12.45	155.12
52.5	75	22	4.55	20.66
45	72.5	28	10.55	111.21
40	62.5	23	5.55	30.75
27.5	45	17	–0.45	0.21
65	80	15	–2.45	6.02
62.5	40	-23	–40.45	1636.57
67.5	67.5	0	–17.45	304.66
30	60	30	12.55	157.39
37.5	62.5	25	7.55	56.93
65	87.5	23	5.55	30.75
70	77.5	6	–11.45	131.21
	95	27	9.55	91.12
		Mean: 17.45		S: 5757.45

Difference scores calculations

Mean: 17.45
$\mu = 0$
$S^2 = SS/d.f. = 5757.45/(22 - 1) = 274.16$
$S_M^2 = S^2/N = 274.16/22 = 12.46$
$S_M = \sqrt{S_M^2} = \sqrt{12.46} = 3.53$

t-Value calculation

$t = (M - \mu)/S_M = (17.45 - 0)/3.53 = 4.94$

Note: Column 4 is (difference-mean)
t-statistic -hypothesis test statistic;
Sq-Dev-Squared deviations from the mean;
s, variance;
μ - Population mean;
M-sample mean;
S^2 sample standard deviation

The dependent sample test was employed to compare the results of the same learners before and after the intervention. The results in Table 5 were obtained using a significance level of 0.005 and a two-tailed hypothesis.

The value of t is 4.944408. The value of p is 0.00007. The result is significant at $p < 0.05$. The errors and misconceptions committed throughout the tests had an impact on the results attained by the learners. Algebra tiles were then employed to address the errors and misconceptions highlighted on the pre-test in each of the following lessons.

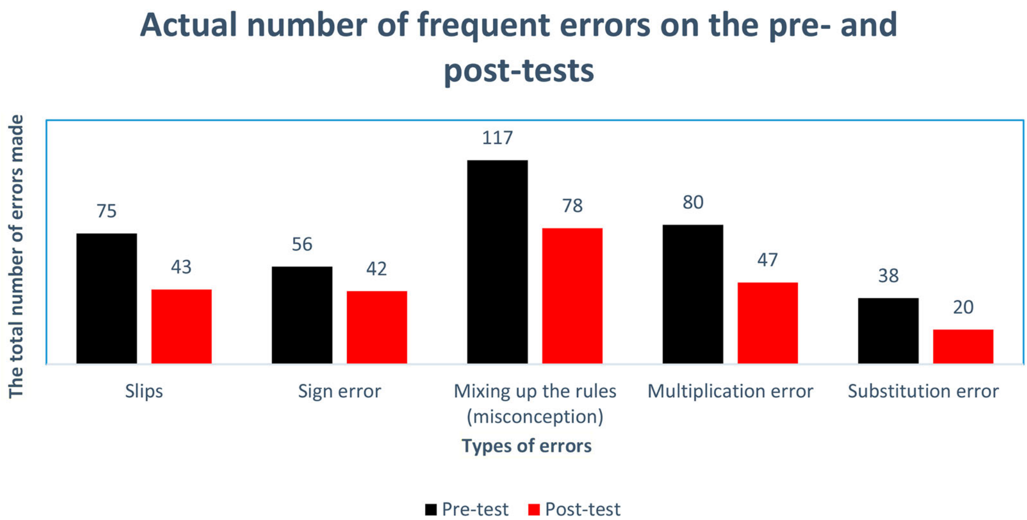

Figure 4. Learners' common errors on the tests

Description of Identified Frequent Errors

The test results were rigorously evaluated to identify common errors and misconceptions of learners in this study when simplifying algebraic expressions. The frequency of each type of error was also recorded. There was a total of 366 errors committed on the pre-test; however, there were only 230 errors committed on the post-test. To further organise the data, the first author divided common errors into five main categories. These errors consist of slips, sign errors, misconceptions, substitution errors and multiplication errors. The bar graph in Figure 4 illustrates the identified common errors on the pre- and post-tests. As shown in Table 6, the percentage decrease in errors on the post-test compared with the pre-test was also recorded.

The pattern of the results indicates a slight decline in sign errors and misconceptions, while slips, multiplication errors and substitution errors decreased by at least 40% on average. The intervention lessons using algebra tiles were effective in addressing errors and misconceptions, according to the findings. Since the gains were modest, the results indicated that algebra tiles should be utilised more often in algebraic expressions exercises. A discussion of the findings pertaining to each of the five categories of errors discovered is provided in the following section.

Slips

Slips, in this context, denote minor errors that learners have the capacity to correct. In the pre-test, all learners collectively made 75 slip errors, a number that decreased to 43 slip errors in the post-test. Figure 5 demonstrates some slip errors on the tests.

Table 6. The average percentage decrease between the pre- and post-tests for common errors

	Actual number of errors		
Errors	Pre-test	Post-test	Percentage decrease
Slips	75	43	43%
Sign error	56	42	25%
Misconception	117	78	33%
Multiplication error	80	47	41%
Substitution error	38	20	47%
Total	366	230	37%

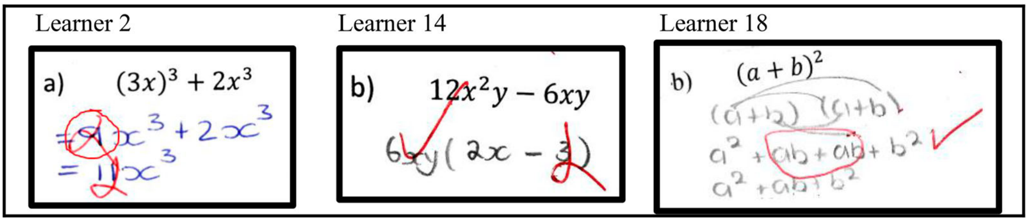

Figure 5. Some common slips on the tests

The number of slip errors decreased by 43%, as displayed in Table 6. Most of these slips were due to carelessness on the part of the learner. The most frequent slip made by learners in this study was insufficient simplification of solutions. In addition, the slip included omitting variables from the submitted answers. Some learners factorise the expression $x^2 - 16$ incorrectly as $(x-4)(x-4)$, when it should be factorised as $(x-4)(x+4)$. Another common slip with the exponents was writing $(3x)^3 = 3x^3$ when the correct form is $27x^3$. A further common slip was arranging an algebraic expression with increasing rather than decreasing powers of y, as requested by the question. The slips as suggested by Luneta and Makonye (2010) can be reduced easily if they are called out.

Sign Errors

Figure 6 demonstrates in detail, using the tiles, how to add and subtract unlike terms. Learners in this study addressed the sign error by using algebra tiles during the intervention lessons. The image on the left depicts the expressions as tiles, the image on the right demonstrates how the zero-pairs cancel, and the figure below indicates the remaining solution.

$$-2x^2 + 2x - 4 + (3x^2 - x + 1) = x^2 + x - 3$$

According to the results, there was not a significant difference between the pre-test and post-test for sign errors. There were only 25% fewer errors on the post-test than on the pre-test. The primary reason for this error, according to the findings, is that learners in this study struggle with integer addition and subtraction. There is a direct relationship between the concepts of integers and the

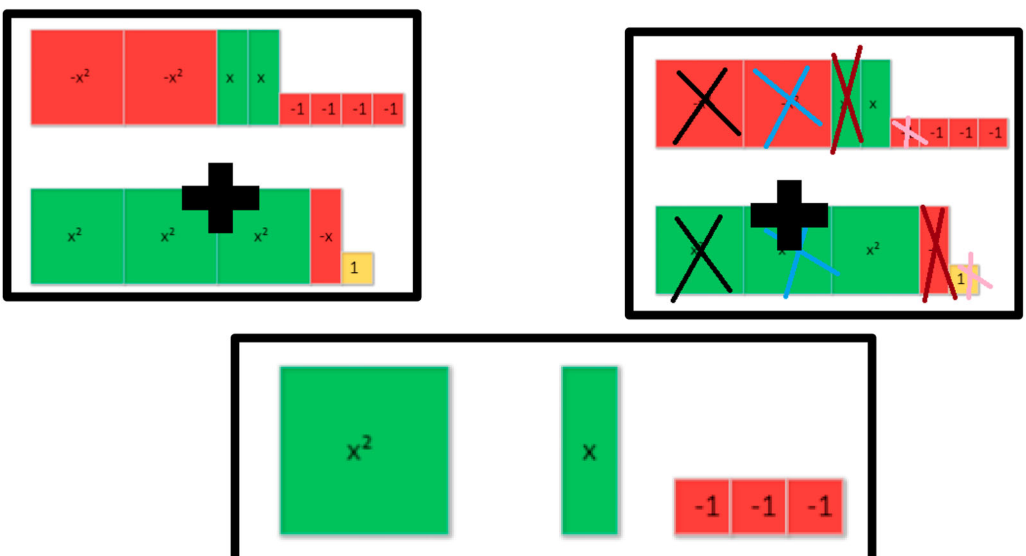

Figure 6. A representation of addition and subtraction of like terms using the algebra tiles

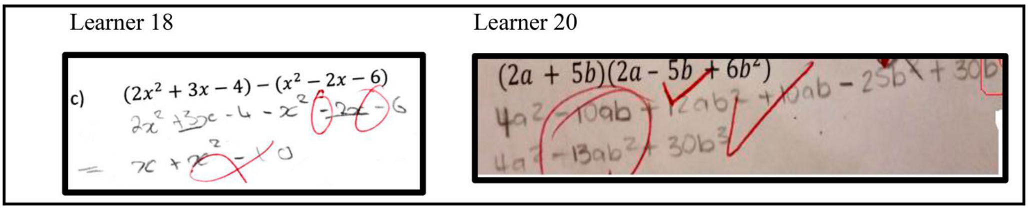

Figure 7. Some sign errors that were recorded

addition and subtraction of algebraic terms. The findings indicate that since the learners in this study struggle with integers, they also tend to struggle with addition and subtraction of like terms. Lim (2010) found comparable results. The primary focus of Lim's (2010) investigation was an error analysis of learners in Grade 7 as they attempted to simplify algebraic expressions. According to Lim (2010), the addition and subtraction of like terms make up the fundamentals of algebra, and if learners are unable to master these concepts, it will impact their performance in other areas of the mathematics curriculum.

It was a challenge for learners in this study to collect like terms in $2x - 4y + 3xy + 2y - x$. The main challenge was that the signs of the terms were different. In another expression learner 20 in figure 7 was able to distribute the expression $(2a + 5b)(2a - 5b + 6b^2)$ but could not simplify their answers by collecting like terms or added the like terms incorrectly. The expression $(2p - 3)(p + 4)$ was correctly expanded to $2p^2 + 8p - 3p - 12$, but because $8p$ and $-3p$ contain different signs, a variety of answers were provided, including $11p$, $-5p$ and $5p^2$. Some learners added all terms containing variable p and obtained the answer $7p$. Additionally, some learners added all three terms in the expression, obtaining $-5p$.

Various types of algebraic expressions must be simplified by addition and subtraction of like terms. Since this knowledge is applicable to the addition and subtraction of like terms, it is crucial that learners have a solid foundation in the addition and subtraction of integers. The results indicate that learners have difficulty identifying like and unlike terms. In some instances, learners added algebraic expressions to constants. The findings also indicate that the learners in this study demonstrated difficulty in performing addition and subtraction operations on algebraic expressions. The outcomes of the study are like those of Faramarzpoor and Fadaii (2020). They conducted a study to determine the causes of learners' errors in simplifying algebraic expressions and discovered that improper integer addition and subtraction calculations are the root cause of numerous algebraic errors and misconceptions.

Muchoko et al. (2019) researched factoring and simplifying algebraic expressions by middle school learners. The learners' work was evaluated. Analyses revealed that the learners were unable to combine like terms and rearrange expressions in a manner that made simplification easier for them. Learners work from left to right without considering BODMAS. The findings from Muchoko et al. (2019) show that Grade 9 learners struggle with addition and subtraction of like terms and suggest that the concept was not fully understood in Grade 8. They will probably continue to struggle with the same concept in Grade 10. Taban and Cadorna (2018) explain how learners in Grades 8–10 struggle with addition and subtraction involving like terms. They examined the structure of learners' algebraic solutions. The authors found that the errors and misconceptions were caused by learners' inability to add and subtract like terms across three grade levels.

Misconceptions

In this study, learners often offer the following solution in response to the given expression:- $(x - 3)^2 = x^2 + 9$. This misconception may result from the fact that $(4x)^2 = 16x^2$. Learners use their understanding of the laws of exponents to factorise, but this understanding is applied incorrectly. To correct this misconception, it was first explained to the learners that $(x - 3)^2$ means that you have two of the brackets. Learners arrange $x - 3$ on the far left-hand side. On top, learners will have the other $x - 3$ as depicted in Figure 8.

The aim now is to fill in the inside in such a way that the square is formed. Learners in this study accomplish this by multiplying the x on the left by the x on the right to obtain x^2. This answer is

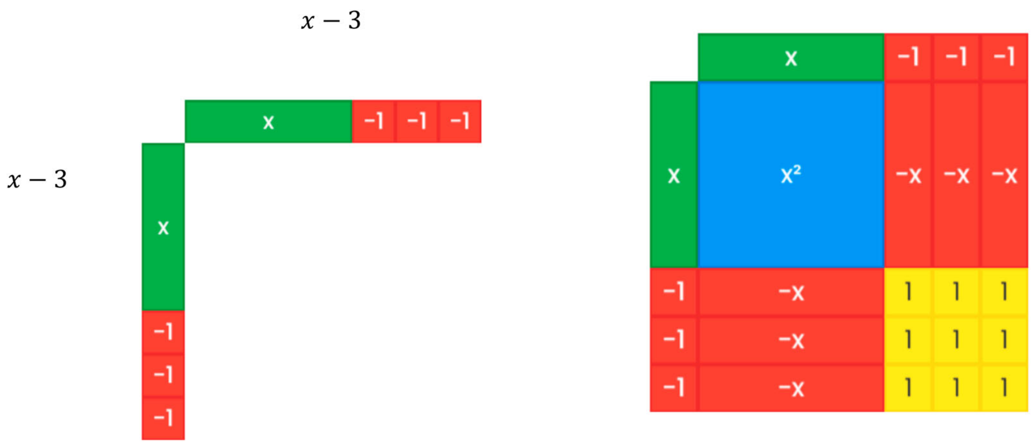

Figure 8. Squaring a binomial using algebra tiles: $(x - 3)^2 = x^2 - 3x - 3x + 9$

then placed inside the empty shell; the next step is to multiply the *x* on the left with the remaining terms at the top. The same procedure was applied for the three negatives ones on the left.

Figure 9 represents a selection of the misconceptions that were encountered on the pre-test. On the post-test, misconceptions decreased by only 33% compared with the pre-test. The findings of the study indicate that Grade 9 learners in the study frequently had misconceptions. The learners applied irrelevant rules or overgeneralised a concept and consequently developed misconceptions (Brodie, 2014). The learners were simply memorising the rules without a solid understanding of them, resulting in misconceptions. Luneta and Makonye (2010) explained that most algebra errors

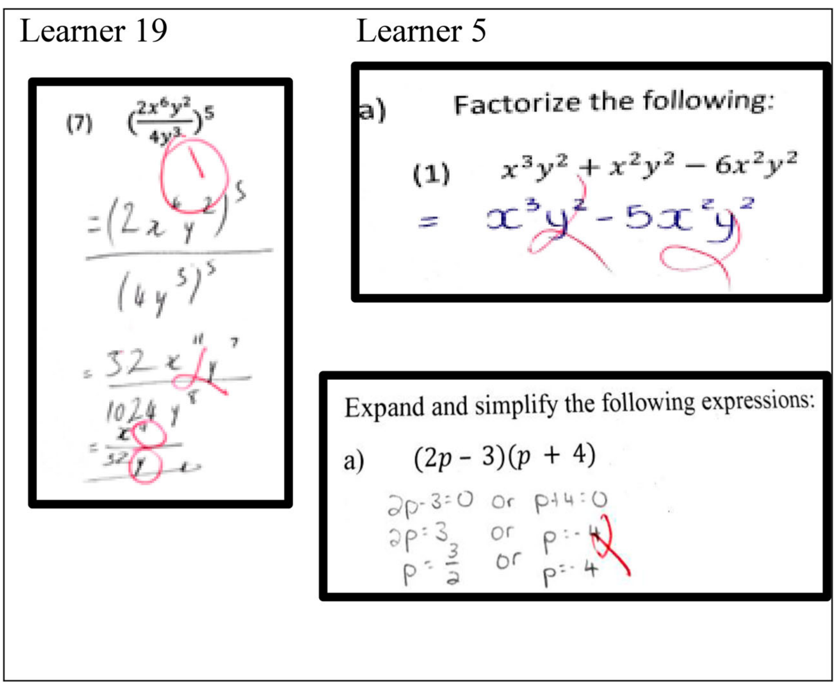

Figure 9. Typical misconceptions that were encountered on the pre-tests

and misconceptions can be traced back to a lack of proficiency in elementary algebra, which is a topic that should have been mastered in earlier grades.

The findings also indicate that learners in this study created their own rules to solve questions, which is consistent with the findings from Ncube (2016). Learners identified $-2x$ and $-\frac{5}{x}$ as like terms and added them for the pre-test question $-2x+3-\frac{5}{x}$. The results show that learners changed $-\frac{5}{x}$ to $-5x$; learners were unable to take the LCD (lowest common denominator) and simplify appropriately, so they altered the question and the original form. The learners did not understand the concept of the LCD and instead treated fractions as normal numbers. Irawati et al. (2018) found that most learners multiplied $\frac{x}{2}+\frac{x}{3}=\frac{x^2}{6}$ instead of finding an LCD and adding. Ncube (2016) found a similar pattern of simple cross-multiplication when learners had to add algebraic fractions: $\frac{x}{y}+\frac{w}{z}=xz+yw$. The researchers' two examples show that learners misapply rules they understand (Brodie, 2014).

The findings also revealed that learners in this study frequently misapplied the rules of exponents. For example, Learner 19 (see Figure 9), for the post-test question $\left(\frac{2x^6y^2}{4y^3}\right)^5$, added the powers rather than multiplied them. Gardee (2015) identified the misapplication of rules error as well. According to Gardee (2015) most learners in Grade 9 concluded that $a.a=2a$ by using addition rather than multiplication. Similarly, the findings show that learners multiplied 2 by 15 in the expression $2a+a+15=30a$ because they assumed the numbers in the question to be like terms. Based on the findings, learners continue to struggle with identifying like terms and overgeneralise the rules for integer addition and subtraction to apply to the addition and subtraction of like terms.

The findings also indicate that, despite being taught a new method for solving algebraic questions using tiles, some learners in this study continued to hold the misconception discovered in the pre-test.

Multiplication Errors

Figure 10 shows some of the multiplication errors made on the tests. In a comparison of pre-test and post-test scores, the multiplication error ranked third among those that decreased. On post-tests, learners in this study made 41% fewer multiplication errors. In accordance with what Ncube (2016) referred to as invalid multiplication, the findings indicate that when multiplying, learners omitted negative signs or failed to fully distribute. In instances such as $(2a+5b)(2a-5b+6b^2)$ learners would collect the like terms $2a$ and $2a$ along with $5b$ and $-5b$ as like terms. They neglected the brackets, with nine learners in the study omitting the $6b^2$ term and directly multiplying the two binomials instead. Alongside this, another common expansion error was multiplying the variables' powers instead of adding them. Ncube (2016) and Egodawatte (2011) report that while learners can apply the distributive property rule to simple statements without terms, they encounter difficulties when dealing with complex expressions comprising multiple terms. A learner can answer $3(n+7)=3n+21$, but not $2p-4p(2p^2-1)$.

The findings of this study also indicate that the learners in this study were proficient at multiplying less complex terms but struggled with complex ones. For the question $(t+3)^2-2(t+1)(t-10)$, learners would multiply and simplify, but they would not correctly distribute the 2. In some cases, learners would distribute the 2 to the bracket prior to simplifying the binomial. Multiplying variable powers when expanding instead of adding them was another common multiplication error.

Substitution Errors

What will the value of $3x-2$ be if $x=3$? To solve the problem each green x tile must be replaced with three yellow positive one-unit tiles. The two negative one-unit red tiles will combine with the

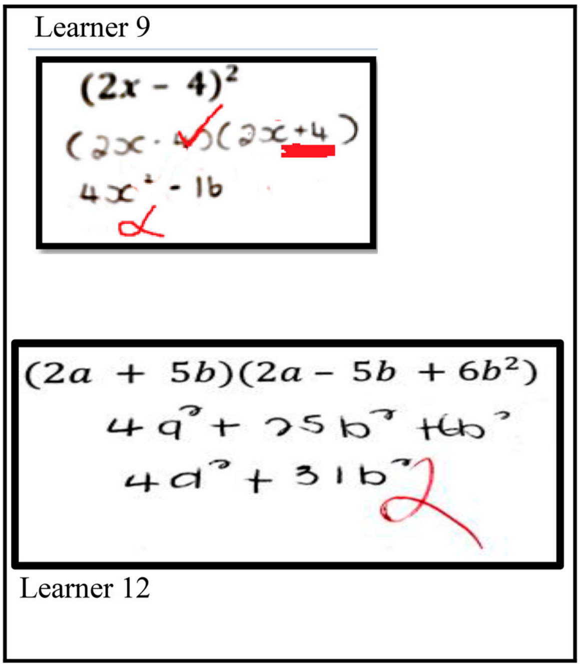

Figure 10. Some of the most common multiplication errors discovered on the tests

two positive one-unit yellow tiles to produce zeros, which will cancel out, resulting in a solution of seven (Figure 11).

Figure 12 illustrates some examples of these substitution errors. There was a 47% decrease in substitution errors on the post-test. The findings show that the learners in this study could substitute numbers in algebraic expressions; the problem was performing the correct calculations and obtaining the correct value. Again, the findings indicate that understanding the addition and subtraction of integers is the key challenge. The use of algebra tiles in the intervention made it significantly simpler for learners in this study to cancel zero pairs.

The substitution question on the pre-test to determine the value of $x^2 - 3y$ if $x = -1$ and $y = -2$ was presumably straightforward; however, learners in this study struggled greatly owing to the substitution of negative numbers. The post-test question on substitution for calculating the value of

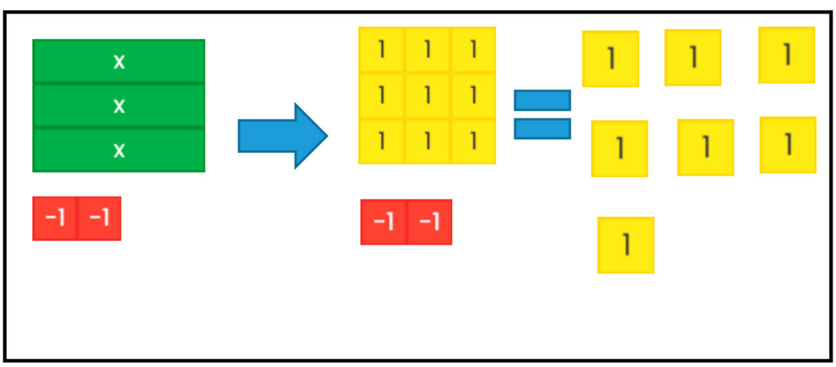

Figure 11. Substitution using algebra tiles: $3(3) - 2 = 7$

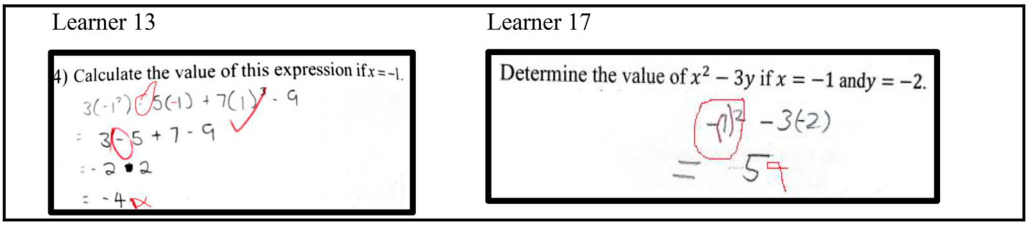

Figure 12. Example of substitution errors observed on the pre-tests

$8y \times 7y^3 - \frac{2y^2}{3} - 4$ if $y = -2$ was less poorly answered, but a few learners omitted the negative sign again, while others correctly substituted but were unable to obtain the correct expression value. Comparing the pre-test and post-test, the substitution error was the most improved.

Limitations

The study utilised a relatively small sample size, involving only one class with a maximum of 22 learners, limiting the generalizability of the findings. The research spanned two weeks on weekdays, with one-hour classes each afternoon. However, this timeframe proved insufficient for in-depth engagement with learners. Owing to time constraints, only one error was addressed each day, with the following day dedicated to a different error type. Consequently, some errors and misconceptions may not have been thoroughly addressed owing to these time limitations. A longer time frame could permit learners to be exposed to algebra tiles more frequently, thereby boosting their confidence when solving algebraic expressions. Even though definitive claims cannot be drawn from this one-time, relatively small-scale study, it does provide a useful lens and sufficient evidence to indicate the important role of algebra tiles in addressing errors and misconceptions in algebraic expressions.

Conclusions and Recommendations

The study explored the efficacy of using algebra tiles to address the errors and misconceptions that Grade 9 learners hold on the topic of algebraic expressions. The statistics of this study indicate that the 22 Grade 9 learners' work contained a high number of errors prior to their exposure to the intervention lessons. Learners in this study struggled with slips, sign errors, multiplication errors, misconceptions and substitution errors. According to the analysis of the tests, the most frequent error made by learners was misconceptions. When comparing the pre-test with the post-test, it was discovered that the learners' overall performance had improved by 18% after receiving the intervention sessions. The results of the post-test indicate that the intervention lessons addressed the errors and misconceptions effectively. The algebra tiles as manipulatives employed in the intervention classes led to significantly higher post-test scores for the learners in this study. The findings of this study were consistent with the literature (e.g. Chaurasia, 2019; Ergene & Haser, 2021; Furner & Worrell, 2017; Hall, 1999; Pranada et al., 2019; Salifu, 2022; Thornton, 1995; Pournara, 2020; Pournara et al., 2016, 2022). Pournara's research (2020, 2016) reveals persistent challenges in Grade 9 math, including errors in conjoining, negatives, brackets, premature evaluation and algebraic operations. Pournara et al. (2022) advocate improved teaching to overcome these hurdles. The results of integer addition and subtraction challenges are identical to Pournara's (2020) findings. Pournara's (2020) study revealed that learners frequently made errors in calculation when attempting to solve algebraic expressions, specifically by attributing values to unknown letters.

Our study demonstrates the efficacy of algebra tiles in aiding learners' comprehension of abstract algebraic expression concepts. Using algebra tiles, learners were able to demonstrate their own solutions and justify supplied solutions. Vygotsky's (1978) social constructivism perspective with a specific focus on the ZPD demonstrates how algebra tiles closed the gap between the independent and assisted skills of a learner. Largely, the findings present other ways to assist learners in addressing the errors that they make and misconceptions that they hold in areas including expression simplification, factorisation, distribution and integer rules. In relation to the findings, the study advocates the enactment of manipulatives like algebra tiles for teaching the topic of algebraic expressions. The study recommends further studies be conducted with larger sample groups across different grades to investigate whether the findings can be replicated.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

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