From fraction to ratio: Exploring the features of first-year students' percent discourse

Jaqueline Marques Luksmidas

Johannesburg

June 2019

A research report submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in partial fulfilment of the requirements for the degree of Master of Education by combination of coursework and research

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ABSTRACT

Percent is a familiar, yet complex topic that is found to be difficult for both adults and children. The question of why percent has been persistently difficult has spurred much research, the most notable of which was conducted in the early 1990's. Those studies have adopted a cognitive perspective. This study adds a *commognitive* perspective to the discussion by proposing a model for the development of percent discourse (PD-Model). The model rests on Sfard's premise that learning mathematics is synonymous with modifying and extending one's discourse.

I begin by employing a cognitive framework of percent for the design of written tests to identify the areas of percent that first-year university students experience difficulty with. The quantitative analysis of the written tests shows that less than half the students obtained a score of 50% or more. Later, in search of the features of students' discourse that hinder their access to percent discourse, I examine the discourse of two pairs of students in interview sessions. I illustrate the application of the PD-Model as an interpretive analytical tool that offers an explanation for the insufficiency in their objectification of percent as a comparative ratio.

This study confirms the results of Parker's (1994) study, that is: percent is difficult for students to work with. The key findings of the discursive analysis show that students' discourse of percent is narrow and deeply rooted in a percent-as-fraction notion. The students' discourse is predominantly additive in nature and does not show signs of recognising the underlying multiplicative structures of percent tasks. As such, a fully-fledged objectification of percent as a comparative ratio is not evident in the students' discourse of percent.

Keywords: Commognition, mathematical discourse, percent.

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Jaqueline Marques Luksmidas

28th day of June in the year 2019

DEDICATION

To my daughters, Maia and Meera.

ACKNOWLEDGEMENTS

To the students that agreed to participate in this study: thank you. None of this would have been possible without your willingness to give up your time for my benefit.

To my colleagues that took over my projects when it mattered most to me. Thank you too for agreeing to pilot my tests. Your input, feedback and open discussion are deeply appreciated.

To my supervisor, Dr Craig Pournara: thank you for your consistent support and level-headed approach. I am privileged to have had the opportunity to work with you and am especially grateful for your patience in allowing me to pursue an approach that differed from yours.

To my friends and family: thank you for your support, for lifts for my children and for sticking around despite the deafening silence from me. To Joy, for planting the seed so many years ago. To Arielle, for all the times that you saved the day by being there for my children. To Anne-Marie, for your curiosity, support and many cups of tea. To Kath and Clare, for your sense of humour, support and lessons in comma usage.

To my daughters, Maia and Meera: thank you for your patience with me. For all the matches, galas and gymnastics competitions that I could not attend. This would not have been possible without your ability to be so independent.

To my husband, Dharmendra: thank you for all the times that you cooked, lifted the girls, helped with homework and had to go it alone. From our early beginnings in a Chaos Theory lecture more than two decades ago, you have always believed in me and encouraged me to aim higher. Thank you.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1 | INTRODUCING THE STUDY

1.1 Introduction

In 2018, this study was born out of a personal interest in financial mathematics. Once I started reviewing the literature it became apparent that the computation of annuities, future values, present values and such were dependent on a solid foundation in the mechanics of percent. It occurred to me that in my experience as a financial model-builder I had encountered many situations in which my peers had grappled with interpreting and making sense of percent. Through conversations over coffee with my husband (who shares my passion for mathematics) I realised that a study in financial mathematics would only lead me back to percent. What started out as a research study on financial mathematics then soon evolved into a study about people making sense of the object percent.

1.2 Why percent?

We are inundated daily with data and statistics that are expressed as a percent. Newspaper headlines quote matric pass rates, levels of unemployment and increases in crime rates. We are tempted with discounts, special offers and vehicle financing options with or without balloon payments. Yet, despite the prevalence of percent in our daily lives, it is still a challenging mathematical notion to grasp (Parker, 1997).

As consumers of financial data, marketing campaigns and statistics we need to have access to a comprehensive discourse of percent to make informed decisions. Percent is familiar to most adults and for many, it might be assumed that they know how to ‘work it’. However, as will become apparent from the findings of this study, there are first-year university students who have a severely restricted view of percent. The motivation for this study consequently stems from the pivotal role that percent plays in supporting financial literacy.

1.3 The focus and purpose of this study

This study had its humble beginnings as a quantitative study into student knowledge of percent. Armed with a framework for percent developed by Melanie Parker in 1994, I designed a test instrument and set to work conducting tests on first-year university students. The analysis of the written tests, however, painted a deficit picture. Unsatisfied with reporting that first-year students did not know how to work with percent, I felt compelled to meet the people behind the test scripts to delve deeper into their thinking.

Parker's (1994) framework had seen me through the design and analysis of test items. It had also helped me to pinpoint exactly what areas of percent students had found most difficult. However, the framework was limited to student cognition rendering it an *acquisitionist* approach (Sfard, 1998). If I was to uncover the roots of the observed difficulty with percent I needed a change in course.

I turned to Anna Sfard's communicational framework in which she advocates a *participationist* approach. An approach that considers changes in how *people* are participating in the learning environment (Sfard, 1998). Sfard claims that discourse is a special form of communication that does not have to be verbal. Sfard's framework considers thinking as "an individualised form of the activity of communicating that is, as communication with oneself" (Sfard, 2007, p. 569). Sfard's communicational framework showed promise as a means for accessing student discourse of percent.

1.4 Research questions

This study employed both quantitative and qualitative research methods in analysing the degree of objectification of students' discourse of percent. The specific questions that framed the analysis were:

- What areas of percent do students experience difficulty with?
- What features of student discourse contribute to this difficulty?

To address these questions, I have investigated student knowledge of percent from two theoretical perspectives. On the one hand, I adopted Parker's (1994) cognitive framework of percent in the design and analysis of written tests. Parker's framework provided a tool for investigating what knowledge of percent had been acquired and retained by first-year students, that is an *aquisitionist* perspective (Sfard, 1998). The quantitative analysis of written tests located the areas of difficulty that students experienced with percent.

On the other hand, I employed Sfard's (2007) communicational framework to construct a proposed model for the development of percent discourse (*PD-Model*). The proposed *PD-Model* provided a map of the hierarchical levels of percent discourse (Caspi & Sfard, 2012; Kim, Ferrini-Mundy, & Sfard, 2012). This model equipped me with a tool for the classification of the students' discursive activity from a *participationist* perspective. In the qualitative analysis of the features of the discourse of students, the *PD-Model* furnished an explanation for the difficulties experienced in percent tasks.

The findings of this study will show that the degree of objectification of student discourse was limited to a persistent additive construct of percent and a deeply rooted *percent-as-fraction* perspective. I will argue that these features of student discourse hindered their access to the fully-fledged multiplicative structure of percent that elicits the comparative nature of *percent-as-ratio*.

1.5 Contextual background

The research participants were drawn from first-year Bachelor of Education (B.Ed.) students with specialisation in secondary mathematics at the University of the Witwatersrand in South Africa. Students would have fulfilled the course requirements of a minimum of 60% for mathematics in the National Senior Certificate (NSC) assessments and should have received instruction in percent as part of the South African school curriculum. South African education, however, is not without its challenges and it is therefore necessary to clarify the context from which this study emerges.

1.5.1 The language of teaching and learning in South Africa

South Africa is a multilingual society, recognising 11 official languages, with less than 10% of the population speaking English at home (Robertson & Graven, 2018). According to Setati (2008, p. 114), teachers and learners view English as the medium for accessing "social goods". With English positioned as the language of social and economic power, it is the dominant choice as the language of learning and teaching in South African schools (Robertson & Graven, 2018; Setati, 2008). This places many learners in the position of learning mathematics in their second or third language. The most recent Trends in International Mathematics and Science Study (TIMSS) (2015) reports the notable advantage enjoyed by learners across the cohort that speak the language of instruction at home (Robertson & Graven, 2018). Although language is not the focus of this study, this provides some context for the research participants most of whom were not first-language English speakers.

1.5.2 Location of percent within the school curriculum

South African learners are introduced to percent in Grade 6 as part of the topics on common fractions and decimal fractions where the focus lies in expressing equivalent forms of these fractions as a percent. Under the topic of data handling, learners are introduced to pie charts with data expressed in percentages. The revision of equivalent forms of common fractions, decimal fractions and percent is continued in Grades 7 to 9 where it is extended to compute percent change.

In Grade 7 and 8 learners are introduced to the financial applications of percent in relation to profits, simple interest and loans. The financial applications of percent are extended to include compound interest in Grade 9.

From Grade 10 to 12 the curriculum no longer references *percent* as a particular topic to be taught but refers to interest, growth rates or inflation in relation to the financial applications of growth and decay. Learners are also introduced to the statistical application of percent as percentiles.

1.5.3 School leavers' performance on NBTs

In addition to meeting minimum requirements on the NSC assessments, students entering higher education institutions in South Africa may be required to write the National Benchmark Tests (NBTs) for certain programmes. All NBT candidates write the Academic and Quantitative Literacy (AQL) component of the NBTs while students intending to study mathematical programmes are further required to write the Mathematics (MAT) component. The quantitative literacy (QL) component of the NBT is designed to assess students' ability to interpret and reason with quantitative data within realistic contexts (Frith & Prince, 2018).

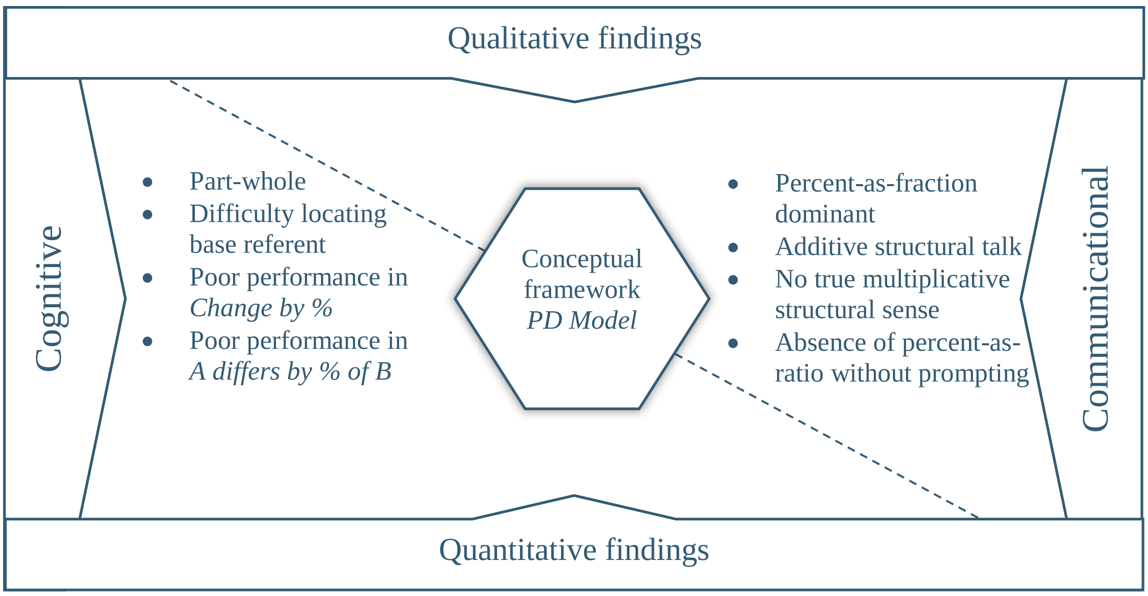
In their analysis of 2014 school-leavers across South Africa who wrote the NBTs and met the requirements for entry into higher education in 2015, Prince and Frith (2017) reported that nearly 70% of the students fell within the lower two proficiency bands of the NBT QL test. Their analysis of patterns of performance found that students had difficulty with the quantitative language used to describe percent and struggled with proportional reasoning. Bohlmann, Prince and Deacon (2017) analysed common errors identified in the MAT component of the 2012 and 2013 cohort of NBT candidates. Their analysis showed that many candidates did not associate percent with its referent quantities. Bohlman et al (2017) conclude that, although percent is taught in the earlier grades, even high-performing school leavers appear to have forgotten the applications of percent.

The research participants in this study fall within the same high-performance band as the samples analysed in Prince and Frith's (2017) and Bohlman et al's (2017) studies. This suggests that although they should have received instruction in percent at school, even as high-performing school leavers, they are likely to experience difficulty with making sense of percent.

1.6 Overview of the research report

The diagram in Figure 1.1 relates the cognitive and the communicational aspects of this study. In the chapters that follow, I will unpack the relationship between the two approaches as I address the research questions.

Figure 1.1: Overview of research report



Chapter 1: An introduction to the study and the research questions.

Chapter 2: A review of cognitive studies in percent. In this chapter, I introduce the reader to Parker's (1994) framework of percent in which she defines the nine contexts of percent.

Chapter 3: An introduction to Sfard's communicational framework.

Chapter 4: The formulation of a model for the development of percent discourse (*PD-Model*). This framework is used in Chapter 7 for the analysis of students' discursive activity.

Chapter 5: Research design and methodology.

Chapter 6: A quantitative analysis of the findings of the written test. In this chapter, I address the research question: "What areas of percent do students experience difficulty with?".

Chapter 7: A discursive analysis of the activities of two pairs of students as they work their way through four real-world percent tasks. In this chapter, I address the research question: "What features of student discourse contribute to the difficulty that they experience with percent?"

Chapter 8: Findings, recommendations, and reflections.

CHAPTER 2 | LITERATURE REVIEW

2.1 Introduction

Much of the research on the learning of percent predates Sfard's (2008) communicational framework. The studies on percent that inform my research are largely cognitive in nature and focus on instruction in percent, misconceptions, the evolution of solution strategies and the language of percent. A common finding of these studies is that percent has been a source of confusion for learners of all ages for many years (Parker, 1994; Parker & Leinhardt, 1995).

In this chapter, I review the cognitive studies that inform the design of my research instruments. In Chapter 3 I consider research studies that employ Sfard's communicational approach. I place emphasis on the tools that have been developed to uncover the discursive actions that impede students' participation in mathematical discourse. In chapter 4 I draw on both the cognitive and communicational perspectives to construct a conceptual framework for the analysis of the degree of objectification in the students' discourse of percent.

2.2 The evolution of percent over time

Two types of mathematical quantities can be distinguished: extensive and intensive quantities. *Counts, measures* or *values* are classified as extensive quantities while *relational quantities* are classified as intensive quantities (Schwarts, 1988 in Parker & Leinhardt, 1995). For example: extensive quantities include measures such as 500 ml or 25 km or counts such as 20 beans in a bag. Intensive quantities are numbers such as speed measured in kilometres per hour or rates expressed as Rands per hour. Parker (1994) describes percent as a comparative index between two referent quantities expressed as $percent = \frac{percentage}{base}$. Percent is therefore classified as an intensive quantity.

The terms percent and percentage are often used interchangeably but according to Schwarts' classification, they fall into different categories of mathematical quantities. Percent is a special case of an intensive quantity that has no label or unit of measurement. Since percent relates two referents that have the same unit of measurement (such as km/km) the units cancel each other out through division. Percentage, however, is classified as an extensive quantity.

Evidence of early commercial uses of percent can be traced back to Babylon as early as 2100 BC. This "proto-percent" notion emerged as an additive construct in the form of a unit fraction for deriving interest or tax due on a transaction (Parker & Leinhardt, 1995, p. 429). For example: if $\frac{1}{5}$ was due in interest then the commodity was split into 5 parts and an amount equivalent to one part was deemed payable as interest. This was extended later to cater for larger amounts where, for example, 14 units were owed for every 300 borrowed[1] (Delaporte, 1925, in Parker & Leinhardt, 1995).

Evidence of percent as a measure per hundred can be traced back to India in 300 BC and China in 200 BC although the application of a base of 100 did not appear to have been privileged as yet (Mikami, 1913). The privileging of percent as a base of 100 was found in 15th-century European texts with explicit examples such as: "10 lb. von 100, 10 fl. mit 100 fl., libre .30 per 100" (Parker & Leinhardt, 1995, p. 432). It is not until 1650 that the percent symbol made its appearance in Italy, but it continued to be used as a rate per hundred well into the 19th century.

It is thought that the shift in the discourse of percent from a rate per hundred to a comparative ratio was closely linked to the growth and development of statistics in the late 19th century. With the increased quantity of data being reported, percent offered a standardised method for expressing comparative relationships between data without the need for quantifying the underlying referents (Parker & Leinhardt, 1995). It is believed that the visual representation in the form of the pie chart was born with this new application of percent (Parker & Leinhardt, 1995). Percent, however, was still limited to a part-whole comparison. Documentation of percent values greater than 100 has been traced back to 1845 where percent was first used as a non-monetary, non-part-whole comparative tool (Cruttenden, 1845, in Parker & Leinhardt, 1995).

This account of the evolution of percent shows an increase in the flexibility of the application of percent with the compression of the comparative relationships into the mathematical symbol of "%".

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[1] Parker and Leinhardt (1995) note that it is unclear whether the 14 units were taken *from* the 300 or *on* the 300.

2.3 The view of percent today

Studies reveal that percent is dominated by a part-whole construct that is rooted in a percent-as-fraction notion (Parker, 1994; Parker & Leinhardt, 1995). Parker (1994), however, argued the case for percent-as-ratio, while specifically avoiding the use of conversions and fractions

2.3.1 Percent: fraction or decimal?

Over time, percent has been reduced to a collection of conversion rules between decimals and fractions, placing emphasis on the acquisition of how to 'work it' quickly without emphasizing the underlying multiplicative structure of percent. Early encounters with percent at school are often focussed on expressing a percent such as 25% as a common fraction $\frac{25}{100}$, $\frac{1}{4}$ or decimal 0.25. As such, learners are inclined to interpret percent as division (most often by 100) and not as a comparative index between any two referent quantities (Cincinatus & Sheffet, 2016; Parker & Leinhardt, 1995). Parker and Leinhardt (1995) advocate that this is the leading cause of difficulty in percent. Based on the findings of the analysis of student discourse, I will present evidence in support of Parker and Leinhardt's claim.

2.3.2 Percent as a part-whole

Most often, early encounters with percent associate the visual representation of a pie chart with percent. Where the circular pie represents the whole that is broken into fractional parts (Lembke & Reys, 1994; Parker, 1994; Parker & Leinhardt, 1995). This representation of percent places an upper limit of 100 on the values of percent, limiting it to a part-whole structure.

Studies have found that school learners and adults alike are inclined to experience difficulty with percent values greater than 100% (Cincinatus & Sheffet, 2016; Ginsburg, 1995; Parker, 1994). This is echoed in the words of a student interviewed in this study: "we usually only deal with percentages up to 100 percent". The misconception of an upper limit of 100% is a direct consequence of the part-whole notion of percent.

2.3.3 Percent as a comparative ratio

Parker and Leinhardt eloquently describe percent as “a language of privileged proportion which simplifies and condenses descriptions of multiplicative comparisons" (Parker & Leinhardt, 1995, p. 427). Parker (1994) explains that the language of percent is additive featuring phrases such as “increase by”, “more than” or “less than”. These phrases suggest a symmetry of increase and decrease as with addition and subtraction (Parker, 1997). However, in the case of percent the amount that is “added” or “subtracted” is relative to the referent. For example, if 20% is added to 100 we obtain a value of 120. If we now wish to deduct 20% from 120 we arrive at an answer of 96 and not 100. The 20% deduction is relative to the new referent of 120 and not the original value of 100. Although the language suggests an additive structure, the mathematics is based on a multiplicative construct. Students in this study experienced difficulty in realising the multiplicative structures of percent tasks. I will argue that this finding was closely linked to their persistent use of additive language in their discourse.

In order to make sense of percent as a multiplicative structure, it is necessary to unpack what is meant by the condensed descriptions of comparison (Perez, 2014). Parker’s (1994) framework of percent achieves this by presenting the comparative contexts of percent as a visual model. In contrast with studies that focussed on conversions and procedures, Parker (1994) conducted a study with pre-service teachers in which she instructed an experimental group on the concept and language of percent with the aid of her visual model. (Parker, 1994; Parker & Leinhardt, 1995). A description of Parker’s framework follows.

2.4 Parker’s framework

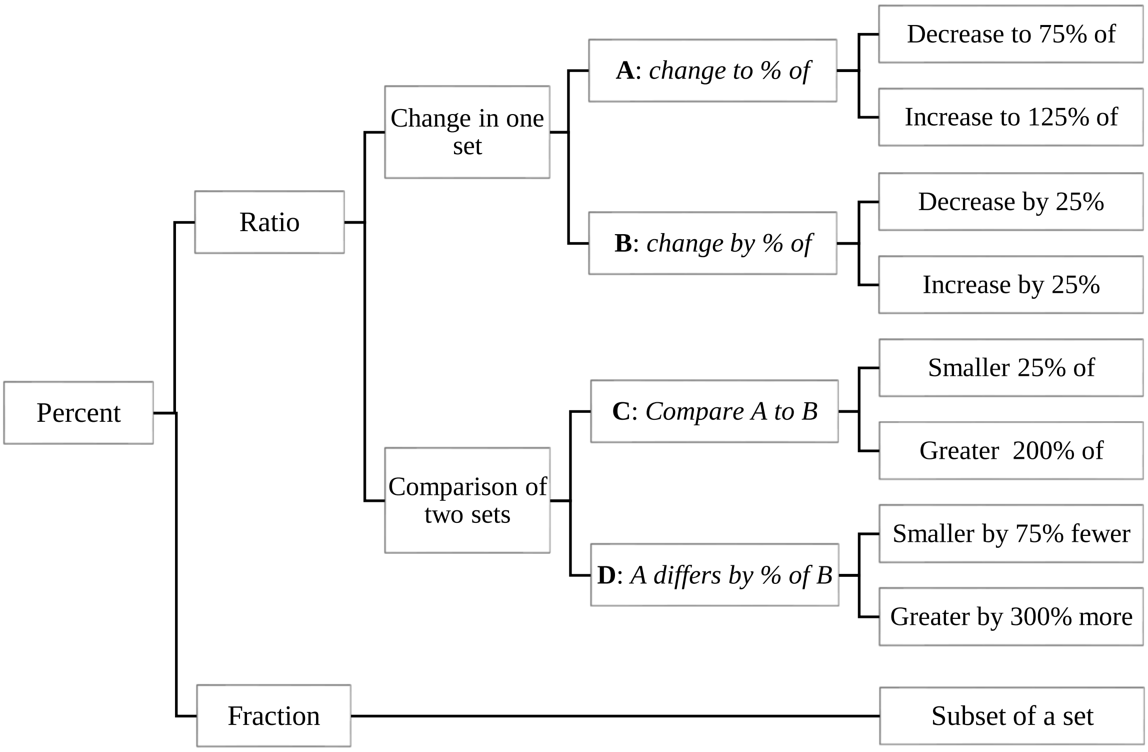
2.4.1 Overview

Parker’s (1994) framework was derived from the nine comparative contexts of percent and represented the additive nature of percent tasks within a multiplicative framework. The visual model, as illustrated in Figure 2.1, was used to analyse the structure and context of the task before proceeding with computations. Furthermore, Parker emphasized percent as a ratio while avoiding the use of conversions and fractions.

Initial testing of the pre-service teachers showed that less than half of the participants scored more than 50% on percent tasks. The study found that the experimental group outperformed the control group in a post-instruction test confirming that a focus on the language and structure of percent tasks opens learner access to knowledge of percent.

According to Parker's (1994) framework, percent is categorised into two main contexts: *fraction* and *ratio*. Parker claims that the difference in the two comparative contexts is a pivotal feature of percent. Where *fraction* measures a subset of a set or a whole, *ratio* measures the relationship between two sets.

Figure 2.1: The nine comparative contexts of percent according to Parker (1994)



2.4.2 Percent-as-fraction

The *fraction* context of percent represents the part-whole model of percent that is traditionally encountered in textbooks. The keyword *of* often appears in tasks of this type. These tasks are concerned with a closed set of data that is not changing, most often with at least one subset: a net salary that needs to be shared amongst expenses, a bag with red and blue balls, a classroom of boys and girls[2] or the score on a test. The percent values in a part-whole task have an upper limit of 100% and cannot take on a value greater than this without some change occurring to the underlying set. In this context, percent is described as taking on a value between 0% and 100% and is often represented as a pie chart.

2.4.3 Percent-as-ratio

Under the *ratio* context percent is used to measure the change in one set over time or the difference between two disjoint sets. In this sense, percent describes either the *change* in a single set or the *comparative* relationship between two different sets. Percent values greater than 100% are possible in these contexts.

2.4.3.1 Percent-as-ratio: change

Change tasks measure the extent to which the size of an individual set has changed over time. The population growth of the southern right whale between the years 1990 and 2017 is an example of this. In contrast to the *fraction* context of percent, it is possible to have values greater than 100% under the *ratio* context of percent.

There are two ways in which to measure the change that has occurred within the individual set. Change is measured as either a *change to a percent* of the original set or a *change by a percent* of the original set.

A *change to a percent of* task (ratio context A in Figure 2.1) measures how much a set has changed in size by describing the relationship between the original set (base referent) and the new set. For example: an item priced R160 has been reduced to a new price of R120. The ratio of the new price to the old price is expressed as $120/160$. In percent terms, the new price is 75% of the old price. Alternatively, the price of R80 may be increased to a new price of R100. The ratio of the new price to the old price in this scenario is $100/80$ and is referred to as 150% of the old price. Both tasks are based on a single product or set and how it has changed over time. In this context, we are measuring the relationship between the old set and the new set as illustrated in Figure 2.2.

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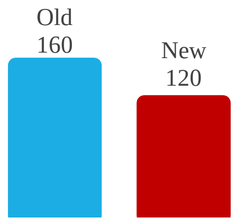
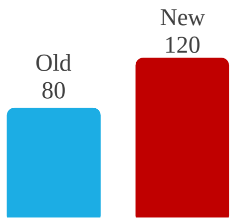
[2] If there are no subsets, then there is just one whole and no parts. This is 100%.

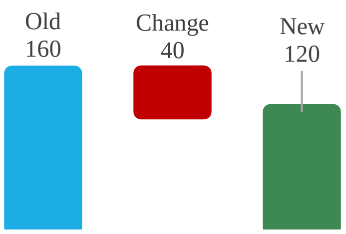
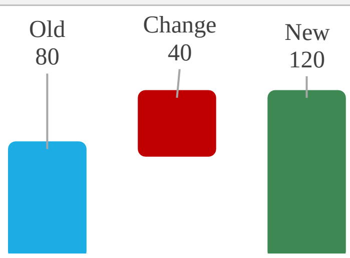
Figure 2.2: Ratio context A

***Ratio context A*: Change to a percent of**	
Decrease	Increase
	
$ratio = 120/160$	$ratio = 120/80$
New price is 75% of the old price	New price is 150% of the old price

A *change by a percent of* task (ratio context B in Figure 2.1) measures the amount of change in a set and describes the relationship between the change and the original set. In the previous example, the price decreased by R40 from R160 to R120. The ratio of the change in price to the old price is expressed as $40/160$. In percent terms, the price changed by 25%. In the second scenario, the price increased by R40 from R80 to R120.

The ratio of the change in price to the old price is expressed as $40/80$. In percent terms, the price changed by 50%. In this context we are measuring the relationship between the old set and the amount the set has changed by as illustrated in Figure 2.3.

Figure 2.3: Ratio context B

***Ratio context B:* Change by a percent of**	
Decrease	Increase
	
$ratio = 40/160$	$ratio = 40/80$
Old price decreased by 25%	Old price increased by 50%

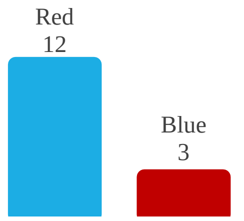
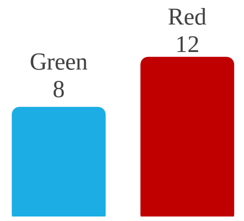
2.4.3.2 Percent-as-ratio: comparison

Comparison tasks measure the relationship between two disjoint sets at a point in time. There are two sub-contexts under this category: *compare set A to set B* and compare the difference between set A and set B (*A differs by % of B*). A *compare set A to set* B task (ratio context C in Figure 2.1) measures the size of one set against the size of another set and describes the proportional relationship between the two sets.

For example: suppose there are 12 red balls, 8 green balls and 3 blue balls in a jar. The ratio of the number of red balls to green balls is expressed as $12/8$. In percent terms, the red balls are 150% of the number of green balls.

Alternatively, the ratio of the number of blue balls to red balls is expressed as $3/12$. In percent terms, the number of blue balls is 25% of the number of red balls in the jar. In this context, we are measuring the relationship between the size of two distinct sets as illustrated in Figure 2.4.

Figure 2.4: Ratio context C

***Ratio context C:* Compare the size of set A to the size set B**	
Smaller	Larger
	
$ratio = \frac{blue}{red} = \frac{3}{12}$	$ratio = \frac{red}{green} = \frac{12}{8}$
Blue balls are 25% of red balls	Red balls are 150% of green balls

Although Parker has chosen to describe ratio context A and ratio context C as two distinct contexts of percent, the mathematical treatment of the two contexts are identical[3] in that both can be expressed as $B = A \times p\%$. Specifically, in the case of ratio context A if we consider a change in a set to result in the generation of a new set

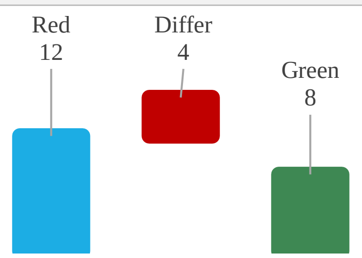
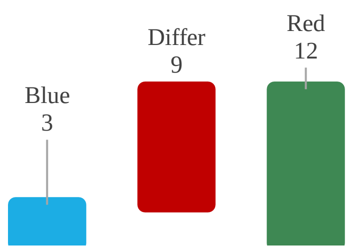
[3] Refer to Table 4.1, level *PD2.*

then ratio context A is comparing the sizes of two separate and distinct sets in the same way as ratio context C.

An *A differs by % of B* task (ratio context D in Figure 2.1) compares the difference in size between two sets by describing the relationship between this difference and either one of the reference sets. Suppose we have 12 red balls, 8 green balls and 3 blue balls in a jar. There are 9 more red balls than blue balls. The ratio of the difference between red balls and blue balls to the number of blue balls is expressed as $9/3$. In percent terms we say that there are 300% more red balls than blue balls in the set.

Alternatively, there are 4 fewer green balls than red balls in the jar. The ratio of the difference between green balls and red balls to the number of red balls is expressed as $4/12$. In percent terms there are 33% fewer green balls than red balls in the set. In this context we are measuring the relationship between the difference between two distinct sets and one of the sets as illustrated in Figure 2.5.

Figure 2.5: Ratio context D

Ratio context D: **A differs by % of B**	
Fewer than	More than
	
$ratio = \frac{difference}{red} = \frac{4}{12}$	$ratio = \frac{difference}{blue} = \frac{9}{3}$
There are 33% fewer green balls than blue balls	There 300% more red balls than blue balls

Although Parker has chosen to describe ratio context B and ratio context D as two distinct contexts of percent, the mathematical treatment of the two contexts is identical[4] in that both can be expressed as $B = A(1 \pm p\%)$. Specifically, in the case of ratio context B if we consider a change in a set to result in the generation of a new set, then

[4] Refer to Table 4.1, level *PD3*.

ratio context B is comparing the difference in the size of two distinct sets in the same way as ratio context D.

2.5 The case view of percent tasks

In their review of studies on percent, Parker and Leinhardt (1995) reported that traditional percent exercises required solving for one of three unknowns: *percent*, *percentage* or *base*. Tasks were classified into cases based on the unknown that had to be solved for. The three common case types were:

- Tasks of type *Case 1* required the learner to determine the percentage where $percentage = percent \times base$. For example: what is $20\%\ of\ 50$?
- Tasks of type *Case 2* required the learner to determine the percent where $percent = \frac{percentage}{base}$. For example: what percent is $10\ of\ 50$?
- Tasks of type *Case 3* required the learner to rearrange the percent equation to determine the base. In this case $base = \frac{percentage}{percent}$. For example: $10\ is\ 20\%\ of$ what number?

After research had confirmed that percent was an area of difficulty for learners, much was done to change the way in which it was taught (Parker & Leinhardt, 1995). The case method was one of the instruction methods employed to remedy the situation. This method provided learners with rules for the identification and classification of percent tasks according to case types. Each case type had a set of rules or procedures for computing the task.

Case 1 tasks require multiplication of two values. Since multiplication is commutative, the order of operations is insignificant. For this reason, students were rarely found to struggle to compute tasks of this type.

Case 2 and *Case* 3 tasks require division to solve. Since division is not commutative learners often performed erroneous computations as they were not able to identify the appropriate divisor for the ratio. As a consequence of this, *Case 2* and *Case 3* type tasks were typically found to be more difficult.

Montgomery, however, claimed that by reducing the computational complexity of percent tasks, learners were better able to apply the appropriate procedure regardless of the case type. He cautioned against making claims that "*case 3* type tasks are the most difficult" (Montgomery, 1958 as cited in Parker & Leinhardt, 1995, p. 427).

In this study, I have taken the case type into consideration for both the design and analysis of the research instruments. The case classification of percent tasks proved useful in designing an instrument that varied the unknown value in the task. I elaborate on this further in Chapter 5 and again in the findings in Chapter 6.

2.6 Strategies for solving percent tasks

Lembke and Reys (1994) analysed the shift in learners' approaches to percent tasks between grades 5, 7, 9 and 11. They tracked the transition from informal to formal procedural methods.

Lembke and Reys (1994) found that students in grade 5 made use of a wider variety of approaches. Once formal instruction on percent had been given in grade 7 and grade 9 they found that the variety of strategies used by learners narrowed to reflect formal school-taught routines and procedures. However, by grade 11 the formal approaches appeared to have been forgotten or inappropriately applied. Often, learners abandoned informal approaches for faulty learned procedures.

Lembke and Reys identified eight types of strategies that were used by the learners in the study. Table 2.1 provides definitions and examples of strategies encountered in the study (Lembke & Reys, 1994, p. 243).

Table 2.1: Strategy types according to Lembke and Reys (1994)

Strategy	*Description*	*Example*
Benchmark	Use of common percent values such as 10%, 25% or 50% to produce a reference point. Then divide or multiply to obtain final value.	**What is 30% of 50?** "If 10% = 5 then: 30% = 5 × 3 = 15"
Fraction	Percent values are rewritten as a fraction to solve the task.	**What is 25% of 48?** "$\frac{1}{4} \times 48 = 12$"

Strategy	*Description*	*Example*
Equation	Set the unknown value to a letter or write an equation to represent the task. Solve algebraically.	**20 is 40% of what number?** "$40\% \times x = 20$"
Ratio	Percent value is rewritten as a proportion to solve the task.	**What is 22% of 500?** "22% of 100 is 22 so 22% of 500 is 5×22."
Compute and check	Multiplies and divides numbers and selects the most reasonable answer.	**What is 22% of 500?** "$500 \div 0.22 = 2\ 272.73$ while $500 \times 0.22 = 110$. 110 is the more reasonable answer"
Draw a picture	Draws a picture as the only method for solving the task	**What is 22% of 500?** Learner places numbers totalling 500 in a pie chart.
Trial and error	Guesses a value. Tests the value and then revises the guess.	**If you have 72% on a test, how many marks did you get out 25?** "60% is 15, 80% is 20, so 72% is 18"
Unclassified	Guesses or provides an answer without explanation	**What is 22% of 500?** "Somewhere between 100 and 120"

For the *benchmark* strategy the learners made use of a common percent value such as 10% or 50% to produce a reference point. They then either divided or multiplied to obtain a final value. For example: to determine 30% of 50 the learners computed 10% of 50 to be 5 and then multiplied by 3 to obtain 15.

Where learners rewrote percent values as a fraction to solve the task their strategies were classified as *fraction*. For example: to compute 25% of 48 the learners expressed 25% as $\frac{1}{4}$ to obtain $\frac{1}{4} \times 48 = 12$. Learners that made use of an *equation* strategy assigned a letter to the unknown value and then solved the equation algebraically. For example: to compute "20 is 40% of what number?" the learners wrote the equation as $40\% \times x = 20$.

When learners rewrote the percent value as a proportion to solve the task this was classified as a *ratio* strategy. For example: to compute 22% of 500 the learners determined that 22% of 100 is 22, so 22% of 500 is 5×22. The *compute and check* strategy was assigned to workings where students had tested both multiplication and division of the quantities and then selected the more reasonable answer.

Where students presented a graphical solution to the task the strategy was classified as *draw a picture.* Workings were classified as *trial and error* if their workings showed evidence of guessing and revision of answers. Where answers were provided without any workings, the strategy was deemed to be *unclassified.*

Lembke and Reys (1994) concluded that formal instruction in percent narrowed the strategies employed by learners in working with percent. They found that from grade 11, learners abandoned many of the procedures that they had been taught in favour of a wider variety of approaches to solve percent tasks.

This finding was relevant to my study as the participants were first-year university students. The participants in this study were expected to approach percent tasks with a broad range of computational strategies that were not necessarily the product of formal instruction. In Chapter 5 I discuss how the strategies of Lembke and Reys were adapted to this study for the data analysis process. The student responses were analysed according to the adapted strategies and reported on in Chapter 6.

2.7 Conclusion

Parker's framework is well suited to the design and analysis of test items as it provides a mechanism for identifying the nature of percent tasks that students experience difficulty with. In Chapter 5 I discuss how the cognitive studies from this chapter have informed the design of the research instrument and the data analysis process.

In Chapter 3 I present Sfard's commognitive theory and consider several studies that have employed her approach. In Chapter 4 I illustrate how Parker's (1994) cognitive framework and Sfard's (2008) commognitive perspective are drawn on to provide a comprehensive approach to analysing student knowledge of percent.

CHAPTER 3 | THEORETICAL FRAMEWORK

3.1 Introduction

As seen in Chapter 2, much of the research on percent is predominantly cognitive in nature. While Parker's (1994) framework is well-suited to the analysis of test items and the categorisation of areas of difficulty with percent tasks, the framework is limited to student cognition rendering it an 'acquisitionist' approach (Sfard, 1998). For this purpose, Sfard's commognitive framework has been adopted to analyse the discourse of percent in students in an interview context.

I begin this chapter with a discussion of my view on knowledge and learning. I then introduce Sfard's (2008) theory of commognition: a theory that considers learning mathematics as synonymous with modifying and extending one's discourse.

3.2 On knowledge and learning: a sociocultural perspective

In this study I take on a largely sociocultural perspective (Vygotsky, 1978), drawing on commognition as a lens for the research (Sfard, 2008). My perspective of learning is that it is not wholly individual, nor is it solely based on social interaction. Learning is an active process of creating meaning.

Vygotsky proposed that the individual cannot be separated from his social environment. He developed sociocultural theory which acknowledges features of cognitive and affective processes (Karpov, 2003). Under this theory the mind metaphor is that of "persons in conversation" with knowledge being socially constructed (Ernest, 1996, p. 342).

Central to Vygotsky's account of social formation is mediation through the use of psychological tools. These tools are invented by society and acquired by children by virtue of their interaction with adults and peers (Karpov, 2003). Psychological tools include language, signs and symbols capable of influencing the mind and behaviour of the individual and of others, bringing language to the fore as an indispensable factor of learning (Daniels, 2009; Weegar & Pacis, 2012).

Sociocultural theory is non-positivist in its philosophy – where the truth is dependent on the viewer (Aliyu, Bello, Kasim, & Martin, 2014). The theory is based on the fallibilist view that "all mathematical knowledge is based on cultural, social and political forces which are inherently flawed, evolving and biased" (Chesky, 2006, p. 20). The epistemology is that of true knowledge, if it exists, cannot be accessed. Meaning is a derivative of the process of interacting with the world and with others.

Ontology describes where we believe knowledge can be found and what this knowledge is made up of (Chesky, 2006). Ontologically, sociocultural theory shares the view that reality is grounded in a world constructed by beliefs, purposes and social makeup of the classroom. However, from the sociocultural perspective the reality is shaped primarily by society and the individual's relationship to society (Schuh & Barab, 2008).

3.3 A commognitive approach to learning

Sfard, initially an advocate of the cognitive acquisitionist approach, gravitated towards a more sociocultural context in adopting the *participationist* approach (Sfard, 1998, 2007). Vygotsky's sociocultural theory provides an inclusive analysis of learning processes as it views learning as a social undertaking and failure in learning as "a product of collective doing" (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005). Where the acquisitionist approach is deeply personal in nature, the *participationist* approach considers changes in what and how *people* are participating in the learning environment.

Sfard's (2007) theory of *commognition* is based on the premise that learning mathematics is synonymous with modifying and extending one's discourse. Sfard (2007) introduced the word *commognition* as the fusion of the concepts *cognition* and *communication*. Sfard (2007) claims that under the framework of *commognition*, discourse is a special form of communication that does not have to be verbal. She posits that thinking is "an individualised form of the activity of communicating that is, as communication with oneself" (Sfard, 2007, p. 569).

Discourse, according to Sfard (2007), can be categorised as either everyday (*colloquial*) or *literate*. Everyday discourse is closely related to the spontaneous concepts referred to by Vygotsky (1978). Everyday discourse develops as if by itself and without explicit instruction from an expert (Ben-Yehuda et al., 2005).

3.4 Percent as a mathematical object

Sfard (2012) explains that the formalisation of mathematics was a direct result of mathematicians' quest for disambiguation in their communication. This was achieved through the compression of the discourse resulting in the construct of *mathematical objects:* a construct that serves the purpose of being able to "say more with less".

Parker and Leinhardt (1995) describe the language of percent as concise. Percent is a means of describing the relationship between two referents without having to quote the actual referents. For example, consider the statement: The matric pass rate is 78%. In this example the two underlying referents are unknown. Through the *mathematical object* of percent, it is inferred that the two referents in this ratio are the total number of students that wrote matric and the number of students that passed matric.

3.5 Objectification

Sfard (2008) refers to objectification as a discursive process that results in the production of phenomena (objects) that are more permanent than any human activity that brings them into being. Sfard (2008) speaks of two discursive moves that result in objectification: *reification* and *alienation*. Objectification begins with the process of *reification* in which interlocutors substitute talk about actions on objects with talk about objects (Sfard, 2008). The linguistic markers for objectification include the use of nouns instead of verbs: "20 percent of 50 is 10" instead of "I divide 20 by 100 times by 50 to get 10" (Ben-Yehuda et al., 2005; Newton, 2012; Sfard, 2008). The final stage of objectification is reached when interlocutors present new phenomena in an 'authorless' manner (*alienation)*. By this Sfard means that the human subject is eliminated from the discourse as in the phrase "20 percent of 50 is 10".

In this study, the discursive activity of students is analysed for evidence of objectification. This is possible by analysing the features of students' mathematical discourse.

3.6 Mathematical discourse: the four tenets

Sfard (2007) defines mathematical discourse as a formal discourse that deals with mathematical objects of quantity or shape. According to Sfard, mathematical discourse is distinguishable through four characteristics: *words* and their application, *visual mediators*, *routines* and *endorsed narratives* (Sfard, 2007, 2008; Tabach & Nachlieli, 2015).

3.6.1 Words and visual mediators

Any academic discipline has a distinct vocabulary where *words* and their uses determine what can be said about the world (Sfard, 2008; Tabach & Nachlieli, 2015). In mathematics, the *words* predominantly represent quantities or shapes (Sfard, 2008). Some *words* may have been encountered before in everyday language but within mathematical discourse have specific meanings that must be adhered to (Sfard, 2007). For example: in everyday language the words "99 percent" mean "almost all"; the words "100 percent" mean "great" or "could not be better"; while "110 percent" describes an extra-ordinary performance (Parker & Leinhardt, 1995, p. 474). In the mathematical discourse of percent, the words "99 percent", "100 percent" and "110 percent" refer to exact quantities.

In this study the discursive patterns of students were analysed for evidence of *objectified word* use. *Objectified word* use is indicated by utterances in which mathematical objects become the subject or the object of a sentence. For example: "20 percent of 50 is 10" where the verb "is" links the subject "20 percent of 50" to the predicate "10". In contrast, procedural utterances such as "I divide 20 by 100 and multiply it by 50" are considered unobjectified. In this case the utterance has a human subject "I" and makes use of verbs such as "divide" and "multiply".

Visual mediating tools are the visible objects acted upon through communication. They communicate the relationships and operations with mathematical objects (Sfard, 2007). Everyday discourse is mediated through independent images of concrete objects while the *visual mediators* of mathematical discourse have been created for the explicit purpose of communication about quantity and shape (Ben-Yehuda et al., 2005; Tabach & Nachlieli, 2015). The *visual mediators* of mathematical discourse can be likened to the symbolic artefacts in Vygotsky's (1978) sociocultural theory. Sfard differentiates between; *symbolic* mediators such as formulae and algebraic expressions, *concrete* mediators such as counter beans and *iconic* mediators such as graphs and diagrams (Sfard, 2007, 2008; Tabach & Nachlieli, 2015).

The level of objectification of an interlocutor's discourse is evidenced through the richness, depth and flexibility in which *visual mediators* are applied and combined (Ben-Yehuda et al., 2005; Sfard, 2008). In this study the discursive patterns of students were analysed for evidence of varied and flexible use of *mediators*.

3.6.2 Narratives

According to Sfard (2007), *keywords* and *visual mediators* are used to produce *narratives*. A *narrative* is any spoken or written text "framed as a description of objects, or of relations between objects or activities with or by objects" (Sfard, 2007, p. 572). *Narratives* are deemed to be either true or false through consensual endorsement. Theories, definitions, proofs and theorems fall into this category and are endorsed by the mathematical community.

3.6.3 Routines

Routines are a set of well-defined rules that describe repetitive, patterned ways of performing tasks characteristic of a particular discourse (Sfard, 2012). The patterns can be discerned from the use of mathematical *words* and *mediators,* in categorising tasks as same or different and in the process of substantiating *narratives* about quantities or shapes (Sfard, 2007). Routines are the performance aspect of mathematical communication (Sfard, 2008).

3.6.3.1 The *how* and *when* of routines

The pattern-defining rules of routines are divided into the *how* and the *when* of routines. The *how* rules indicate the pattern of discursive moves that may be taken in response to a prompt. Within the school context prompts include requests, questions, tasks or problems that are familiar to the learners such as: "show that the following expressions are equivalent*"*. Prompts frame the discourse, setting the scene for what the appropriate response should be (Ben-Yehuda et al., 2005).

The set of rules that determine the appropriate discursive action in response to prompts are the *when* rules (Sfard, 2008). The *when* rules of a routine are governed by the context in which the prompt occurs. Sfard (2008) refers to this aspect of *when* rules as the *applicability* conditions of the routine. The *applicability* conditions of a routine are an individual construct and may change with a change in context (Sfard, 2008). For example, students may face some of the contextual tasks in this study in real life but could potentially respond differently when not in a 'school' setting. For this reason, although I have tested students on real-world contextual tasks, we cannot conclude that they would have the same discursive response in their everyday lives.

In addition to the *applicability* conditions, the *when* rules define the conditions for *closure* of a routine. The *closure* conditions identify the circumstances under which the routine is considered complete (Sfard, 2008).

3.6.3.2 Explorations, deeds and rituals

Together, the rules of *how* and *when* determine what type of routine will be called upon. Sfard (2008) distinguishes between three types of routines; *explorations*, *deeds* and *rituals*.

Explorations are routines for which the end result (*closure* condition) is the proof or substantiation of an endorsed narrative (Sfard, 2008). It is through explorations that narratives are changed, and we get to know the world. Realisation routines such as: numerical calculations 52 + 3 = 55; solving equations; defining objects or proving are examples of such explorations.

Explorations can further be categorised into *constructions, substantiation* and *recall*. *Constructions* are discursive processes that result in new endorsable narratives. The discursive action of *substantiation* is how mathematicians decide whether to endorse a narrative. Routines that summon previously endorsed narratives are referred to as *memorisation* or *recall.* Sfard claims that the developmental precursors to *explorations* are *deeds* and *rituals* (Sfard, 2008).

Where *explorations* serve to change narratives, *deeds* are discursive actions that change objects. *Deeds* operate on objects (object level) where *explorations* operate at the discursive level (meta level). *Deeds* can at times appear to have the characteristics of *explorations* but under closer inspection it becomes evident that the discursive action is object-focussed and not discourse-focused.

Rituals enable access to the discourse as thoughtful imitation of 'more-knowledgeable' others. The closing conditions of *rituals* are social in nature where the discursive action serves to create a bond with others. Sfard (2008) claims that *rituals* are often the first steps of learning as they are thoughtful imitation that could culminate in new endorsed narratives.

In this study, the discursive actions of students were analysed for applicability and evidence of flexibility and corrigibility in routine use.

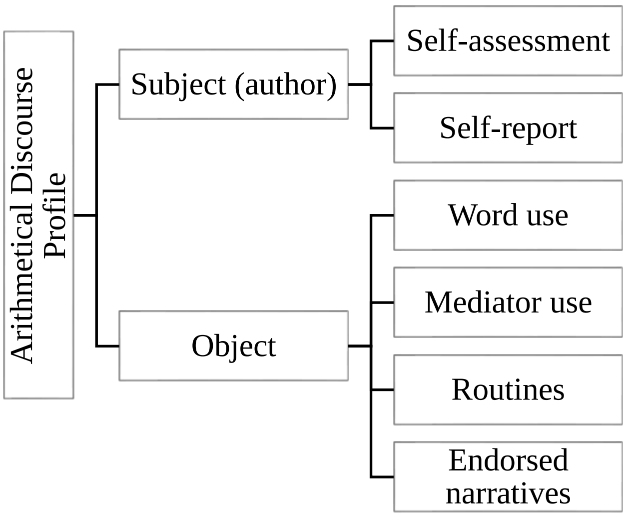
3.7 Discourse profiling

Ben Yehuda et al (2005) adopt a communicational approach in their study of the arithmetic discourse of two 18-year old girls that have experienced learning difficulties. In their study, discourse is a broad term that includes any act of communication: spoken or not, with oneself or with others, face-to-face or through an asynchronous exchange (Ben-Yehuda et al., 2005).

They describe discourse profiling as the discovery and depiction of discursive activities through observation. The observed rules of discourse are referred to as *meta-discursive* since they are propositions about the discourse (Ben-Yehuda et al., 2005).

Ben Yehuda et al (2005) construct an *arithmetical discourse profile (ADP)* as an analytical tool for eliciting the underlying differences in the discursive activities of the two girls. They consider two dimensions in the ADP, that is, the *subject* and the *object* dimensions. The structure of the ADP is represented in Figure 3.1.

Figure 3.1: Arithmetical Discourse Profile (ADP) (Ben-Yehuda et al., 2005)



Ben Yehuda and her colleagues describe numbers as objectified when "we begin experiencing the implied mathematical entities as 'happening to us' rather than caused by us” (Ben-Yehuda et al., 2005, p. 195). The analysis of the object-dimension of the ADP is structured around the question of the degree of objectification of the interlocutors’ discourse. With this in mind, the discursive activities of the participants are mapped according to Sfard’s four characteristics of mathematical discourse: *word* use, *mediator* use, *routines* and the endorsement of *narratives*. A discourse profile is intended to classify learner discourses and not the individual learner themselves.

Since the percent tasks in this study are computational in nature, there are many aspects of Ben Yehuda et al’s (2005) ADP that can be applied. I have chosen to consider only the object dimension in this study since the focus of this study is student knowledge of the object of percent. In Chapter 5 I illustrate how discourse profiling is used in the discursive analysis of the interviews.

3.8 Summary

In this chapter I introduced Sfard’s (2008) theory of commognition: a theory rich in analytical tools that serve to constrain the subjectivity that can so easily influence the researcher’s perspective in discursive analysis. Sfard’s commognitive framework poses a viable union between the cognitive and communicational perspectives of learning as it provides an integrated system of tools that bridge the gap between these two traditionally separate perspectives. (Sfard, 2007, 2012).

In Chapter 4 I draw on Parker’s (1994) cognitive framework and Sfard’s (2008) commognitive framework to construct a proposed model for the analysis of percent discourse.

CHAPTER 4 | CONCEPTUAL FRAMEWORK

4.1 Introduction

In this chapter I construct a framework for the development of the discourse of percent. I draw on the work of Caspi and Sfard (2012) and the historical evolution of percent in section 2.2 to define a model for the hierarchical development of the discourse of percent.

In Chapter 7 the percent discourse profiles of students are analysed for evidence of the degree of objectification in their discourse of percent. The discursive analysis is based on the model in Table 4.1 of this chapter and the discourse profiling criteria laid out in Table 5.4 of Chapter 5.

4.2 The development of percent discourse

Sfard describes the development of mathematical discourse as a hierarchical, layered structure where each level of the hierarchy annexes the preceding meta-discourse thus extending it to include increasingly more complex and reified versions of the discourse (Caspi & Sfard, 2012; Kim et al., 2012; Sfard, 2008).

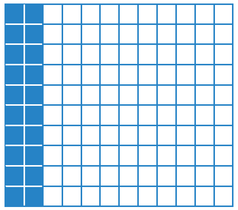
Caspi and Sfard (2012) illustrated, through their study of the development of informal algebraic talk in 7th and 5th graders, that the most likely hypothetical trajectory towards formal algebra is closely related to the possible path historically taken by mathematicians.

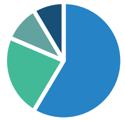
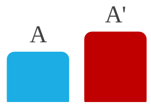
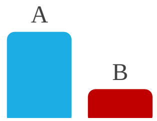
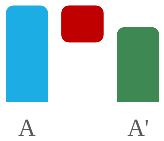
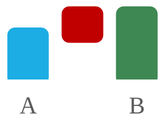
To define a model of the hierarchical development of the discourse of percent, it is useful to unpack the historical development of the discourse into its composite layers (Caspi & Sfard, 2012; Kim et al., 2012; Sfard & Linchevski, 1994). In the section that follows, I draw on the historical evolution of percent in section 2.2 to define the developmental succession of percent discourse.

4.3 Model of the development of percent discourse (PD-Model)

The history of the mathematical discourse of percent is a gradual evolution from talk about fractions to talk about abstract comparative ratios. This transition can be represented as a succession of four layers as illustrated in Table 4.1.

Table 4.1: Model of development of percent discourse (PD)

Level	*Main theme*	*Nature*	*Keywords*	*Use – examples*	*Visual Mediators*	*Routines*	*Procedure*	*Narratives*
PD0	Fraction	Part-whole Extensive quantity Concrete	Number	Unit fraction as $\frac{1}{b}$	Numerals 1/5 	Split quantity into equal number of parts. One part is due as interest.	Division into *b* equal parts	"One part per *b*" "a quarter". "sum of the parts is a *b*"
		Part-whole Extensive quantity Concrete	Number	Proper fraction as $\frac{a}{b}$ where $b \geq a$	Numerals 13/65 	For every 65 parts, 13 parts are due as interest	Division into *b* equal parts	"*a* parts per *b*" "three quarters" "sum of the parts is *b*"
		Part-whole Extensive quantity Concrete	Number	Fraction base 100 as $\frac{a}{100}$ where $b = 100$	Numerals 20/100 	For every 100 parts, 20 parts are due as interest	Division into 100 equal parts	"*a* parts per 100" "20 divided by 100" "Sum of the parts is 100"

Level	*Main theme*	*Nature*	*Keywords*	*Use – examples*	*Visual Mediators*	*Routines*	*Procedure*	*Narratives*
PD1	Percent as part-whole	Part-whole Comparative Abstract Intensive quantity	Percent Subset of set	Percent as base 100 $p \in [0,100]$	20% Pie chart 	Multiply the whole by the percent	Multiplicative $b = B \times p\%$	p% of 'that' "sum of the parts is 100"
PD2	Percent as ratio	Ratio Comparative Abstract Intensive quantity	Compare New set *A'* versus old set *A*. Increase Decrease	Comparison of a set that has changed from *A* to *A'*. $p \in (-\infty, +\infty)$	*A* increased to *A'* 	Multiply the old set by the percent	Multiplicative $A' = A \times p\%$	"*A'* is 120% of *A*" "*A'* is 60% of *A'*"
			Compare Set *A* versus set *B* Compare *A: B* Compare *B: A*	Comparison of disjoint sets *A* and *B* $p \in (-\infty, +\infty)$	*B* is smaller than *A* 	Multiply the base referent by the percent	Multiplicative $B = A \times p\%$ $A = B \times p\%$	"*A* is 120% of *B*"
PD3	Percent as ratio: Change or Differ by %	Ratio Comparative Abstract Intensive quantity	Compare Change verses old set *A* Increase by Decrease by	Compare change in set *A* to set *A'* $p \in (-\infty, +\infty)$	20% decrease 	Multiply the old set by the percent increase/decrease	Multiplicative $A' = A(1 \pm p\%)$	*p*% decrease of old set "*A* has decreased by 20%"
			Compare Difference versus set A or set B Differs by	Comparisons of disjoint sets *A* and *B* $p \in (-\infty, +\infty)$	20% more than 	Multiply the base referent by the percent difference	Multiplicative $B = A(1 \pm p\%)$ $A = B(1 \pm p\%)$	*p*% more than the other set "*B* is 20% more than *A*"

4.3.1 Percent discourse level 0 (*PD0*)

The *unreified* discourse of percent as a fraction prevailed from its early commercial roots in 300 BC to the introduction of the percent symbol in 1650. Even still, percent was used as a rate per hundred well into the 19th century. For this reason, the hierarchical model for the development of percent discourse begins at level 0 (*PD0*) with the *proto-percent* notion of fraction. This level of percent discourse features percent as an extensive quantity that is part-whole in nature. The discourse-generating tasks involve measuring out a share of a total amount which is typically associated with the operation of division. There are three types of fraction that fall into level *PD0*: unit fraction; proper fraction and fraction with base 100.

The first type of fraction is a unit fraction. A typical routine would be to split a quantity into b equal parts where "one part of b" is represented as $\frac{1}{b}$. An example of a narrative at this level is "25% is $\frac{1}{4}$". The second type of fraction is the proper fraction in the form of "a parts per b" represented as $\frac{a}{b}$. In this case $b \geq a$ and the sum of the parts is equal to b. The final type of fraction is the fraction with a base of 100. In this case the fraction takes the form of "a parts per 100" represented as $\frac{a}{100}$. In this case $b = 100$ and the sum of the parts are equal to 100.

4.3.2 Percent discourse level 1 (*PD1*)

As level 1 (*PD1*) is traversed the succession towards objectification begins. At this point, the discourse of 'per hundred' changes and is compressed to the symbol "%". The nature of the discourse is still part-whole and bounded by an upper limit of 100, yet the discourse becomes that of an abstract intensive quantity that represents the relationship between a set B and its subsets. The relationship between the set B and a subset b can be expressed as $b = B \times p\%$. Level 1 is equivalent to Parker's *fraction* (FR) context of percent.

At level 1, it is possible to compare two sets of data using percent. For example: Johnny's Juice claims to have 20% of the recommended daily allowance of vitamin C whereas Katy's Juice has only 15%; I would choose to buy Johnny's juice if I wanted to increase my intake of vitamin C. I can make this decision without having to know the quantity of vitamin C in Johnny's juice. It is sufficient to know that it has comparatively more vitamin C than Katy's Juice.

4.3.3 Percent discourse level 2 (*PD2*)

It is at level 2 (*PD2*) that the part-whole nature of the discourse is shed allowing for percent values that are greater than 100. The sum of the parts is no longer limited to 100. Two of Parker's ratio contexts fall into level 2: *change to a percent of* task (ratio context A in Figure 2.1) and *compare set A to set* B task (ratio context C in Figure 2.1)[5].

At this level the discourse evolves to that of comparative ratio. It is possible to express the transformation of a set *A* as a function of the new set *A'* where $A' = A \times p\%$. For example, an item priced R160 has been reduced to a new price of R120. In percent terms the new price is 75% of the old price $120 = 160 \times 75\%$.

Comparisons between two disjoint sets A and B can be expressed as $\mathrm{B} = A \times p\%$ or $\mathrm{A} = B \times p\%$. For example, suppose there are 12 red balls, 8 green balls and 3 blue balls in a jar. In percent terms the red balls are 150% of the number of green balls $12 = 8 \times 150\%$. The discourse of comparison between disjoint sets is similar to the discourse of changing sets with the added complexity of having to determine which set is the base referent for the comparative ratio. In the case of a set that has undergone a change, the base referent is more easily identified as the "old" set *A* when measuring a change in *A.* With disjoint sets a decision needs to be made regarding the base referent, that is: are we measuring the ratio of the change from *A* to *B* or from *B* to *A?*

4.3.4 Percent discourse level 3 (*PD3*)

At level 3 (*PD3*) the discourse is fully objectified as the ratio between a change or a difference is measured against a base referent. Two of Parker's ratio contexts fall into level 2: *change by a % of* task (ratio context B in Figure 2.1) and *A differs by % of* B task (ratio context D in Figure 2.1)[6].

Objectification is evident in the realisation that the comparative relationships of relative increase or decrease are not symmetric. Consider the example of an item that is priced

[5] Note that ratio contexts A and C were identified as mathematically equivalent in Parker's (1994) model. Refer to section 2.4.3.2.

[6] Note that ratio contexts B and D were identified as mathematically equivalent in Parker's model. Refer to section 2.4.3.2.

R120 including 15% Value Added Tax (VAT). What is the price of the item excluding VAT?

It is well known that to determine the VAT inclusive price of an item, you multiply the cost of the item by 15% and add the result to the cost of the item. When given a task of this type, most students are likely to erroneously multiply the 15% VAT by R120 and then subtract the result from R120.

Since addition and subtraction are opposites it is assumed that the same symmetrical relationship applies to tasks of percent increase ("addition") or decrease ("subtraction")[7]. In fact, percent increase and decrease tasks are relative change rates and require a different approach based on the multiplicative structure of the relationships (Parker, 1997). In the example above, the appropriate procedure is to divide 120 by (1+15%). Careful attention needs to be paid to identifying the base referents in comparisons of this type.

Once the relationship can be expressed as $\mathrm{B} = A(1 \pm p\%)$ or $\mathrm{A} = B(1 \pm p\%)$ the realisation of the multiplicative structure of percent is evidence of objectification.

4.4 Summary

The proposed model provides a snapshot of the hierarchical layers of the development in the discourse of percent. According to Caspi and Sfard (2012), each level is intended to show evidence of increased complexity and generalisability in the mathematical discourse with the order of the discursive layers resembling the historical development of percent. Transitions from one level to the next are seen as developmental milestones in the discourse of percent.

I do not claim that the model proposed provides a fully-fledged representation of the discourse of percent. The model serves as an outline and remains subject to corroboration and modification. Furthermore, the model is but a part of the bigger picture that links to the discourses of arithmetic, fraction, ratio and rational numbers. The model could be extended to higher levels where percent is annexed by the discourses of financial mathematics and statistics.

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[7] I refer to this as an *additive* perspective of percent.

CHAPTER 5 | RESEARCH DESIGN AND METHODOLOGY

5.1 Introduction

This chapter outlines the research design and methodology of this study. I elaborate on the data collection procedures and discuss how test items were selected to be representative of the nine contexts of percent within Parker's (1994) framework of percent. Parker's framework provides structure to the data collection by identifying the areas of difficulty with percent. The study then turns the focus onto the discursive actions of participants as they navigate their way through a selection of percent tasks. Sfard's (2008) theory of commognition provides the analytical tools to illuminate the links between student discursive processes and student performance.

5.2 Overview of research design

This study is based on both quantitative and interpretive qualitative research methods. In the first phase of the study, I used a test instrument to numerically quantify student performance. The test instrument was designed according to Parker's (1994) framework of percent which facilitates the identification and categorisation of areas of difficulty that students experience. The results of the test were used for the selection of interview participants and interview tasks.

Qualitative data were collected by means of semi-structured interviews based on task items that emerged as most difficult in the written test. This process was followed by a discursive analysis of student participation framed by Sfard's (2008) sociocultural theory of commognition.

5.3 Research participants

The participants were drawn from first-year Bachelor of Education (B.Ed.) students with specialisation in secondary mathematics at the University of the Witwatersrand for the 2018 academic year. The sample was selected as a matter of convenience for the timing of the study. Although the students were prospective teachers, this study does make any claims about their pedagogic practice or mathematical content knowledge apart from that reported in this study pertaining to percent.

I did not specifically gather biographical data from the research participants as this was not the focus of this study. Based on the demographics of the student sample however, it can be inferred that between 25 to 29 of the 32 students who wrote the test did not have English as their first language. Of the four interview participants, only one student may have had English as a first language. In an effort to support students that may have struggled with the language demands of contextual items in the interviews, I read the items out loud and discussed the item with them before any computation took place. When students struggled to express themselves in English, they were encouraged to illustrate their thinking with objects on the desk or to write down their thinking on paper.

The students are considered representative of mathematically literate young adults on the basis that they would have fulfilled the course requirements of a minimum of 60% on their matric mathematics results. This suggests that students would have received instruction in percent as part of the South African school curriculum[8] and, as young adults, should have had some exposure to percent in their everyday lives.

5.4 Data collection: Instrument design

The study is based on a written test and an interview. The written test contained fifteen items thoughtfully designed to reflect a combination of contexts, task cases and benchmark percent values embedded within Parker's (1994) nine comparative contexts of percent. Following a quantitative analysis of the written test, four test items were selected for the interviews. A description of the written and interview instruments follows.

5.4.1 The written test

There are three main dimensions that have been considered in the design of the research instruments. The first dimension of task type is informed by Parker's (1994) framework in which she places emphasis on the language and representation of percent. Her framework is well suited to the design and analysis of test items as it provides a mechanism for identifying the nature of percent tasks that students experience difficulty with.

[8] See section 1.5.2 for a description of percent in the South African school curriculum.

The second dimension categorised test items according to context so that an even distribution of contextual and non-contextual tasks could be represented. The arithmetic of non-contextual items is explicit in the task and requires no decision to be made about what operation is required to complete the task. Non-contextual items include typical exercises such as "Find 5% of 60". Contextual items present percent in a verbal situation such as a word problem. In the case of contextual items, students need to recognise the relative relationships of the information provided and decide on the appropriate referent to be used in the computation. The categorisation of test items by context provides the necessary link to Parker's (1994) framework in which contextual items are classified into nine contexts of percent (see Section 2.4). In this study, contextual items have been designed to reflect real-world situations such as: measuring the percent increase in the national minimum wage rate or determining the price of an item including VAT.

The third dimension of case type provided an alternate perspective on the nature of percent tasks. Within each of the contexts, tasks were selected to reflect a range of *case 1*, *case 2* and *case 3* tasks. *Case* 1 tasks required the participant to determine the percentage given the base and percent where $percentage = percent \times base$. For example: what is 10% of 60? Tasks of type *Case 2* required the participant to determine the percent where $percent = \frac{percentage}{base}$. For example: what percent is $10\ of\ 50$? Tasks of type *Case 3* required the participant to rearrange the percent equation to determine the base. In this case $base = \frac{percentage}{percent}$. For example: $10\ is\ 20\%\ of$ what number?

The selection and categorisation of test items was a recursive process. Test items were either modified or excluded to meet the requirements of an even distribution of the categories. The selection of items included tasks with benchmark (BM) percent values such as 10%, 20%, 25% or 50% and considered whether the percent values in the task were greater than or less than 100%. The intersection of the three dimensions provided a detailed description of the types of percent tasks that had been selected for the research instrument and how students approached such tasks. The categorisation of test items by the three dimensions is illustrated in Table 5.1.

Table 5.1: Written test instrument

Written test instrument				< 100%			> 100%		
	#	**Test Question**	**BM[9]**	**Case**			**Case**		
			y/n	**1**	**2**	**3**	**1**	**2**	**3**
Non-contextual	1	200% of 70 = ___	No				✓		
	2	42 = __% of 70	No		✓				
	3	40% of ___ = 20	No			✓			
	4	75 = ___% of 60	No					✓	
	5	78% of 65 = ___	No	✓					
	6	120% of ___ = 96	No						✓
Contextual	7	Sumaya spends 25% of her monthly salary for rent. If she spends R2 700 a month for rent, what is Sumaya's monthly salary?	Yes						FR
	8	A packet of rice is selling for 175% of its 1995 price. If a packet of rice sold for R16 in 1995 what is the selling price today?	No				$R_{A}i$		
	9	Government have offered workers a new hourly wage rate of R20/hr. If the minimum wage in 2017 was R16/hr what is the percentage increase?	No		$R_{B}i$				
	10	A store is offering a 20% discount on scarves for Women's Day. On a Saturday, all ladies automatically get an additional 5% discount. What is the total discount that that they will get if they buy a scarf on a Saturday?	Yes	$R_{B}d$					
	11	Your sister just got a raise and now earns R12 000 a month. Now your salary is only 85% of your sister's salary. How much do you make?	Yes		$R_{C}s$				
	12	When you buy a 440ml can of Coca Cola you get ___ % more volume than when you purchase a 330ml bottle of Coca Cola?	No			$R_{D}g$			
	13	If Jacob earns 25% more than Ramona, then Ramona earns ___% less than Jacob.	Yes		$R_{D}s$				
	14	VAT increased from 14% to 15% on 1 April 2008. If an item costs R100 excluding VAT. You will pay __% more for the item in May 2018.	No	$R_{B}i$					
	15	The store is offering a 25% discount on a jacket that costs R560 excluding VAT. A customer asks the shop assistant to add VAT first and then apply the discount so that she gets a bigger discount. The shop assistant explains that VAT is automatically added to the FINAL price, so the discount has to be applied before VAT. How much money does the customer lose?	Yes	$R_{B}i$ $R_{B}d$					
Totals			***5***	***4***	***4***	***2***	***2***	***1***	***2***

9 Benchmark percent values such as 20%, 25% or 50%

Symbol	Description		
BM	Benchmark percent values such as 20%, 25% or 50%		
Case 1	Find the percentage given the base and percent: $percentage = percent \times base$.		
Case 2	Find the percent given the base and percentage: $percent = percentage/base$.		
Case 3	Find the base given percent and percentage: $base = percentage/percent$.		
FR	Fraction or part-whole relationship		
$\boldsymbol{R_{\mathrm{A}}i}$	Increase to __% of	$\boldsymbol{R_C s}$	Smaller is __% of
$\boldsymbol{R_{\mathrm{A}}d}$	Decrease to __% of	$\boldsymbol{R_C g}$	Greater is __% of
$\boldsymbol{R_{\mathrm{B}}i}$	Increase by __% of	$\boldsymbol{R_D s}$	Smaller by __%
$\boldsymbol{R_{\mathrm{B}}d}$	Decrease by __% of	$\boldsymbol{R_D g}$	Greater by __%

5.4.2 Interview instrument

Sfard (2007) considers thinking as an act of communication with oneself, subsuming it under the umbrella of mathematical discourse. Paper-based tests alone cannot provide an extensive reflection of the participants' thinking. For this reason, interviews were crucial to access the participants' mathematical discourse of percent. The interview instrument was designed with an interpretive qualitative study in mind. Data was collected through detailed transcripts of interviews, written workings and observation.

The interview instrument was based on areas of difficulty that became evident from the analysis of written tests. The intention of the interview sessions was to foreground the students' thinking and approaches to percent tasks. Since interview items were selected from written test items, students who participated in the interview sessions had already been exposed to the interview questions in the written test.

In addition to the four questions that were discussed in the interview session, I asked the participants questions relating to their notion of percent such as: "How would you explain what 20% is to someone that has never worked with percent before?" and "How would you explain what 20% discount means?" A brief discussion of each interview item follows.

5.4.2.1 The Increase task

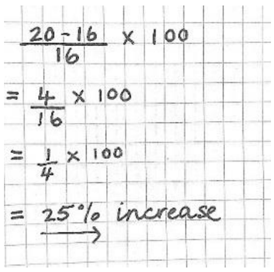
> *Government have offered workers a new minimum hourly wage rate of R20/hr. If the rate last year was R16/hr what is the percentage increase?*

The *increase task* falls into the *Change by a % of* class according to Parker's (1994) framework. A *change by a % of* task (ratio context B in Figure 2.1) measures the amount of change in a set and describes the relationship between the change and the original set.

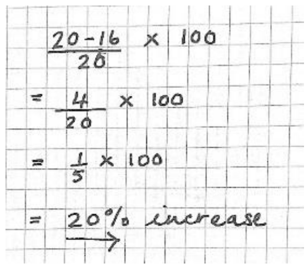
The interview item presented the students with two possible solutions, A and B (see Figure 5.1). The students were asked to review the two solutions and to select the appropriate response for the task.

Figure 5.1: The increase task

A



B



Students needed to determine that the change in rate had to be expressed as a percent of the base referent quantity. The task required the students to identify this referent quantity as R16/hr and not R20/hr. What became evident from the written test was that students regularly used the incorrect base referent. This item was included in the interview session to elicit a conversation about which referent was appropriate and why.

5.4.2.2 The More Than task

If Jacob earns 25% more than Ramona, then Ramona earns ___% less than Jacob.

The *more than task* is categorised as an *A differs by % of B* item according to Parker's (1994) framework. This item type (ratio context D in Figure 2.1) compares the difference in size between two sets by describing the relationship between this difference and either one of the reference sets.

This item presented students with a relationship between two unknown quantities Jacob's salary (J) and Ramona's salary (R). The ratio of J: R was given as 25% *more* and students were asked to consider the ratio of R: J. In the absence of any quantities for these variables, this relationship could either have been expressed algebraically or an arbitrary value assigned to either one of the referents. Furthermore, students had to correctly identify that the referent quantity had shifted from Jacob (J) to Ramona (R).

5.4.2.3 The Double Discount task

A store is offering a 20% discount on jelly beans. This weekend only, all shoppers will get an extra 5% discount. What is the total discount that they will get?

The *double discount task* is categorised as a *Change by %* (ratio context B in Figure 2.1) item according to Parker's (1994) framework. This task is similar to the *increase task* as they are both classified under the same ratio context according to Parker's (1994) framework. This item differs from the *increase task* in three aspects: it is a decrease task; two decreases are applied, and no referent quantities have been provided.

In the absence of a referent quantity (jelly bean price) for the task, students could either solve the task algebraically or assign an arbitrary value to the unknown quantity. The discursive moves of students within the context of this task provided evidence of the structure of percent that students had constructed. An additive structure of percent would result in a total discount of $20\% + 5\% = 25\%$ whereas the appropriate multiplicative structure is that the total discount can be expressed as $(1 - 20\%)(1 - 5\%) = (1 - 24\%)$. Students needed to realise that the referents for the two discounts were different and it is therefore not correct to simply deduct 25% off the original price (Parker, 1997).

5.4.2.4 The Jacket task

A jacket costs R560 excluding VAT and is now discounted by 25%. A customer asks the teller to add VAT first and then apply the discount so that she gets a bigger discount. The teller explains that the system automatically adds VAT after the discount has been applied. Would the customer pay less for the jacket if VAT was added before the discount?

The *jacket* task is categorised as a *Change by %* (ratio context B in Figure 2.1) task according to Parker's (1994) framework. In this case, however, the referent quantity (the jacket price) has been provided. Students could approach this task as a computational task. However, the intention was for the students to reflect on the fact that the result is the same regardless of the order in which VAT and the discount are applied. With the realisation that percent is a multiplicative structure, the commutative property of multiplication proves the equivalence of $(1-25\%)(1+15\%)$ and $(1 + 15\%)(1-25\%)$.

5.5 Data collection: Process

5.5.1 Ethical considerations

Ethical clearance was obtained from the University of the Witwatersrand Ethics Committee (protocol number 2018ECE016M) in June 2018. The research was conducted on the university's Education Campus. In July 2018 a research information session was held for all first-year B.Ed. students on the secondary mathematics programme. Consent forms were distributed to students at the information session and students were advised that their participation was entirely voluntary. A sample of the consent form can be found in Appendix B1.

Those willing to participate in the research returned the forms either at the information session or over the course of the next week. Of the forty-seven (47) students who agreed to participate in the research study, thirty-two (32) students attended the written test sessions.

My ethical obligation, first and foremost, was to not produce a deficit description of the students. This was a key driver in my decision to conduct interviews once I had reviewed the written test responses.

In all my interactions with the students, I set out to ensure that students were not inconvenienced by my research. For this reason, written tests and interviews were conducted at times that fell within the hours of the daily academic program. This was necessary since many students relied on public transport to move between campus and their places of residence.

To minimise discomfort, written tests and interviews were conducted in a familiar environment on campus. Refreshments were provided for students to snack on during the test sessions.

To guarantee anonymity in the written test, I randomly allocated numbers to students' scripts. All references to the written test scripts in this report are based on student numbers and not on student names. Any electronic communication with the group of students was conducted under blind copy.

Since interviews were conducted in pairs, it is not possible to guarantee anonymity. However, I have allocated pseudonyms to the interview participants that reflect the gender and the ethnicity of each participant.

5.5.2 Conducting written tests

Students were contacted by email after the information session and invited to participate in one of two written test sessions in the first week of August 2018. The two test sessions were held one day apart in tutorial rooms on the campus. Students that attended the first written session were requested to not discuss the test with students that were writing the next day. All pen and paper workings were collected to manage the risk of information sharing between the two sessions. There is no guarantee that information sharing did not occur but there was no evidence to suggest that students in the second written session performed any better than students in the first session.

Students were given an hour to complete the pen and paper test without the use of calculators. Calculators were excluded from the written test to encourage participants to communicate their thinking by writing down as much detail as possible. Most students were able to complete the written test within the designated time. Approximately 15% of students were given an additional ten minutes to complete the test.

5.5.3 Written test analysis process

The objective of the test was primarily to identify types of percent tasks that participants experienced difficulty with according to Parker's (1994) nine contextual types of percent tasks.

Students were randomly assigned numbers from one to thirty-two. The test scripts were numbered and arranged according to this numbering sequence. Test items were analysed one at a time in this order. This was to ensure that each item was analysed according to the same principles.

5.5.3.1 The scoring rubric

The written tests were analysed three times with the aid of the scoring rubric in Table 5.2. The rubric was an adapted version of the scoring rubric used by Parker (1994) in her study.

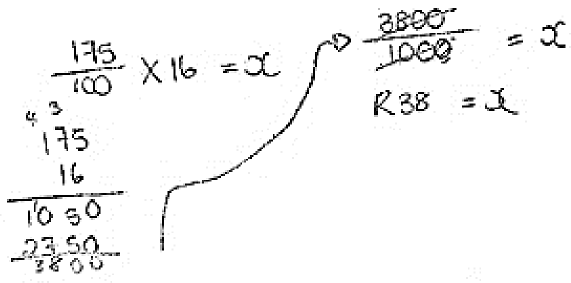
Each question had a maximum potential mark of three where the answer was *fully correct with or without supporting workings*. A score of two was allocated to a response that was either *correct with reasonable workings, or incorrect due to an earlier computational error*. A score of one was allocated if the *answer or the workings were reasonable*.

Table 5.2: Scoring rubric for written test analysis

Score	*Description*
0	No answer provided. No workings are shown and answer incorrect and unreasonable. Answer incorrect and unreasonable. Working shown but incorrect strategy applied.
1	Answer incorrect, but reasonable. Workings reasonable, no final answer provided.
2	Answer correct; with reasonable workings. Answer incorrect due to computational error, but workings correct.
3	Answer correct with or without workings shown.

The response to written test item eight in Figure 5.2 demonstrates an unreasonable answer.

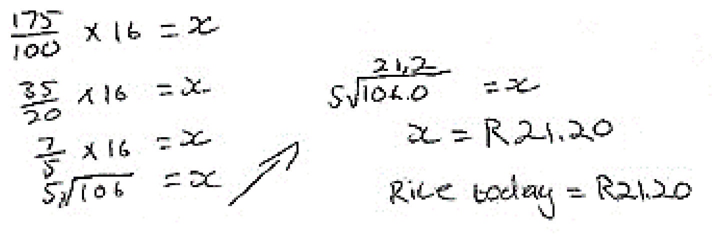
Figure 5.2: Sample of unreasonable answer



In this case, the answer of R38 is more than double (200% of) the original price of R16. In this situation, the student had provided the appropriate workings but made a computational error that resulted in an unreasonable result. The item was allocated a score of 1.

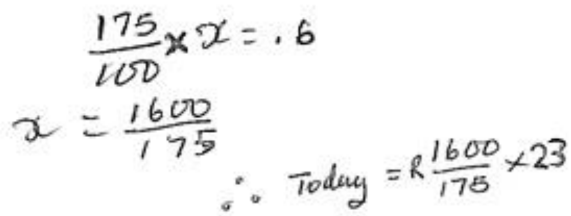
Figure 5.3 illustrates an example of a computational error that results in a reasonable answer. Since the value of R21.20 lies within the 25% margin of error and the workings provided were appropriate to the item, a score of 2 was allocated.

Figure 5.3: Sample of a reasonable answer



A score of zero was allocated to a response that was unreasonable or showed no workings to support the answer. Figure 5.4 is an example of a response that was allocated a score of zero.

Figure 5.4: Sample of a zero-score response



5.5.3.2 Analysis of students' strategies

During the scoring process, each test script was analysed and categorised according to the strategy that students used in solving the task. The workings were categorised according to the strategies identified in the study by Lembke and Reys (1994). Some slight modifications were made to the strategies used by Lembke and Reys. The strategies of *compute and check, draw a picture* and *trial and error* were excluded as there was no evidence of these strategies in the written scripts.

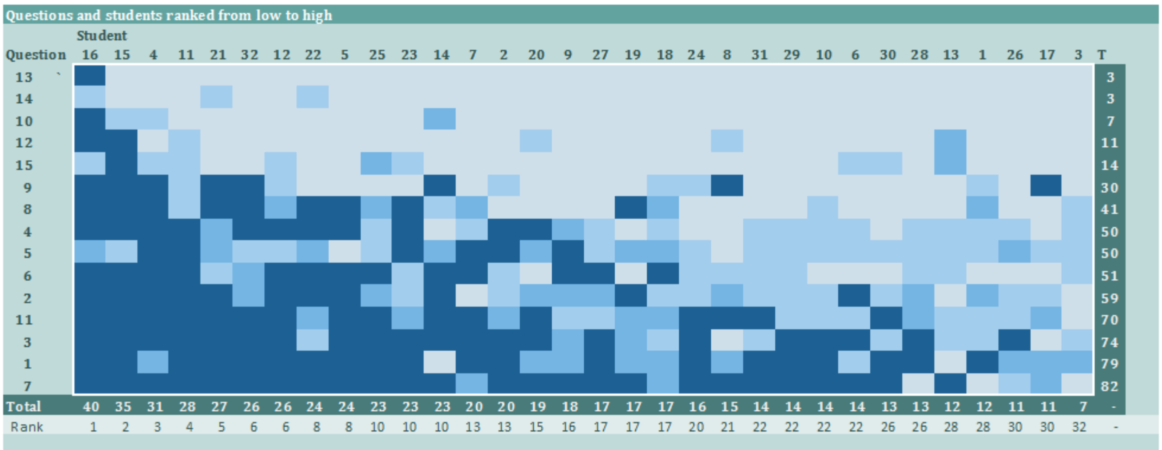
Table 5.3: Strategies for solving percent tasks adapted from Lembke and Reys (1994)

Strategy	*Description*	*Example*
Benchmark	Use of common percent values such as 10%, 25% or 50% to produce a reference point. Then divide or multiply to obtain the final value.	**What is 30% of 50?** "If 10% = 5 then: $30\% = 5 \times 3 = 15$"
Fraction	Percent values are rewritten as a fraction to solve the task.	**What is 25% of 48?** "$\frac{1}{4} \times 48 = 12$"
Equation	Set the unknown value to a letter or write an equation to represent the task. Solve algebraically.	**20 is 40% of what number?** "$40\% \times x = 20$"
Ratio	Percent value is rewritten as a proportion to solve the task.	**What is 22% of 500?** "22% of 100 is 22 so 22% of 500 is 5×22."
Guess / estimate	Reasoned estimate	**What is 22% of 500?** "Somewhere between 100 and 120"

5.5.4 Interview participant selection

Final written test scores were arranged from the lowest scoring question to the highest scoring question. Students were then ranked according to their scores with the highest scoring student ranked as 1st and the lowest scoring student ranked as 32nd. A scoring grid was generated to reflect the test scores by item on the vertical axis and the test scores by student on the horizontal axis. The scoring grid is illustrated in Figure 5.5.

Figure 5.5: Scoring grid for the written test



The coloured squares in the grid indicate the score obtained for each student by test question according to the scoring rubric in Table 5.2. The darker the square, the higher the score. The arrangement of the grid provides the scoring patterns for the test and facilitates the selection of candidates for the interview sessions.

The two lowest and two highest ranking students were eliminated from the selection process as they were outliers. The scoring patterns were then analysed for irregular patterns. Candidates who displayed differing scoring patterns to their direct neighbours on the scoring grid were added to the interview selection pool. For example, students 8, 18, 19 and 14 had scored three for more challenging questions where adjacent candidates had scored lower. Student 26 scored only one three in the test where adjacent candidates had scored lower. Students 13, 30 and 6 scored higher than the adjacent candidates for two of the most difficult tasks. Students 2 and 11 displayed unusual patterns in their scoring.

Once the candidates were highlighted for selection, their written work was reviewed for the level of detail they provided in their workings. Only candidates who provided workings on their written test were kept in the selection pool. Of the eleven candidates initially earmarked for selection, ten were invited to the interview sessions. All candidates initially agreed to be interviewed. One candidate had to withdraw from the study due to personal commitments; another candidate withdrew as he no longer wished to participate. Five of the remaining eight candidates participated in the interview sessions.

5.5.5 Conducting interviews

The analysis revealed that students found the test difficult. With only 12 students passing the test I felt unsatisfied with reporting that students did not know how to 'work' percent. I felt compelled to meet the people behind the papers to dig deeper into their thinking and to gain insight into what might be hindering their access to the discourse of percent.

At first, I intended to hold one-on-one interviews but then I chose to group students instead. I hoped that by pairing the students, they would be more likely to interact with each other.

Three interview sessions were held at the end of October 2018. Three to four participants were invited to each session. The first session had one participant and this session was treated as the pilot for the study. The other two sessions each had two participants.

The pilot and the first interview were held one week apart. The gap between these sessions provided the opportunity to listen to, transcribe and analyse the audio recordings. The interview instrument was refined based on the transcription of the pilot interview. The wording of the *jacket* task was simplified to make the task less overwhelming and a semi-structured script was included as part of the interview instrument. A sample of the interview script is included in Appendix A2.

The next two interviews were held four days apart. Each session was set up for 45 minutes to an hour. Participants were asked to complete four pen and paper tasks with the aid of a calculator. A calculator was allowed in the interview sessions since the focus of these sessions was not numerical calculation, but student discourse. To this end, participants were encouraged to ask questions and discuss the tasks with each other.

The basic type of data for a discursive analysis is human talk. In the case of commognitive research, this includes any written communication and requires a verbatim account of what was said (Sfard, 2008). The interviews were audio-recorded and transcribed on the same day to ensure that all communication was recorded as accurately as possible. Careful attention was given to ensuring that words such as "a quarter" was not rewritten as $\frac{1}{4}$. Utterances that included the word "percent" were recorded as "percent" and not as the symbol "%".

The transcriptions included notes on my observations of the students as they worked through the tasks. Students were assigned gender- and culturally-appropriate pseudonyms in the transcripts to protect their identities. All written work was collected for analysis.

5.5.6 Interview analysis process

Full transcripts from the two interviews were produced. However, for the analysis of the transcripts, I omitted any talk that was not clearly audible. At times, participants spoke at the same time and it was not always possible to identify the speakers. Repetitive words such as "ja", "ok" or "oh" were also omitted from the transcripts in the analysis.

To encourage unhindered discussion, I often allowed the talk to steer off task. For the purpose of the analysis, I regarded only talk that was on-task.

The transcripts of on-task talk were analysed and coded according to the characteristics of the mathematical discourse, that is *visual mediators*, *words*, *routines* and *narratives* (Sfard, 2008). Ben Yehuda and her colleagues (2005) describe discourse profiling as the discovery and depiction of discursive activities through observation. In this study, the discursive actions of each student were recorded and summarised by task according to the criteria laid out in Table 5.4. The summary provided a snapshot of the developmental level of each student's percent discourse according to the model of the development of percent discourse (*PD-Model*) in Table 4.1.

5.5.6.1 Word use

The transcripts were scanned for evidence of objectified *word use* in the discursive patterns of the students. Utterances were classified as either *structural* or *procedural* on this basis. Utterances were classified as *personal* if they showed evidence of human agency (Ben-Yehuda et al., 2005)

The discursive patterns of students also provided an indication of the mathematical structure that students associated with percent. Utterances such as "100 minus 20 percent" regarding a 20% discount were considered *additive* while "100 multiplied by a 20% discount" was considered *multiplicative*.

The coding of the *word use* according to these characteristics was taken into consideration in classifying the students' developmental level of percent discourse.

5.5.6.2 Mediational modes

The transcripts were analysed to assess the quality and level of objectification of the discourse based on patterns in mediator use. According to Sfard (2008), the richness, depth and flexible use of mediators are indicators of objectification.

The discursive moves of students were scanned for evidence of the three types of mediators; *symbolic*, *iconic*[10] and *concrete*. Since the interview items were largely computational tasks, *symbolic* mediators featured prominently in the student discourse. For this reason, the *symbolic* mediators were closely scrutinised and classified

[10] No occurrences of iconic mediational use were observed in this study.

according to the following categories; syntactic verbal, syntactic written and objectified. The syntactic symbolic mediational mode is concerned with the replacement of symbols with other symbols (either verbally or written). For example: “20% of 50 is 10”.

Table 5.4: Summary of student discourse profile features

Discourse feature			Description and example
Word use How are the keywords of the discourse of percent used by students? What is the user able to say about the percent world?	Objectified	Structural	Mathematical object is the object or subject of the sentence “20 percent of 50 is”
		Impersonal	No human subject evidenced in the utterance “20 percent of 50 is”
	Unobjectified	Procedural	Use of verbs to describe the process “Multiply by 20 and divide by 100”
		Personal	Evidence of a performing subject “I subtract 20% from 100”
Mediational mode Assessing the richness, depth and cross-situational stability of mediators to gauge the quality of discourse	Symbolic	Syntactic: Oral/written	Replacement of symbols with other symbols (either verbally or written). “20 percent of 50 is 10”
		Objectified	Different words are used to describe the same phenomena “140 percent is 40 percent more than your initial value”.
	Concrete	Concrete	Manipulation of an object (either physical or imagined)
Routines Are the routines applicable to the task at hand?	Type	Ritual	The closing condition is to create a social bond
		Exploration	The closing condition is to create a new narrative
		Deed	The closing condition is to change an object
	Flexibility	Routines and mediator modes	Is there evidence of flexible and varied use of routines and mediator modes?
	Corrigibility	Propensity for erring	Do they show a high propensity for erring?
		Retracing	Can they retrace their steps and self-correct?
		Successful correcting	Are they able to assess and correct their own performance?
Endorsed narratives	Type	Derivation	The discursive process that produces a new narrative
		Substantiation	Discursive action that helps decide whether to endorse a new narrative or not
		Memorisation Recall	Routines to recall previously encountered endorsed narratives
Overall percent discourse	Nature		Fraction or ratio?
	Structure		Multiplicative or additive
	PD Level	PD0 – PD3	See Table 4.1

Mediator use was considered objectified if different words were used to describe the same phenomena. For example: “140 percent is 40 percent more than your initial value”.

5.5.6.3 Routines and narratives

Routines were categorised as either *explorations, rituals* or *deeds* (Sfard, 2008). The routines were then considered for their *flexibility* and *corrigibility*[11] If students used multiple routines or were inclined to work with different mediational modes their routines were considered *flexible*. The way students corrected their work (*corrigibility*) was analysed for evidence of retracing, mode-switching and successful correcting.

The endorsed narratives of the discourse were categorised as either *derivation, substantiation* or *recall* (Sfard, 2008). Discursive processes that resulted in new endorsable narratives were regarded as *derivation* or *construction*. Discursive actions that helped the students decide whether to endorse a narrative were regarded as *substantiation*. Where previously encountered narratives were summoned, the discursive actions were classified as *memorisation* or *recall.*

5.6 Validity and trustworthiness

5.6.1 Validity

For the quantitative analysis of the written test, I consulted both colleagues and my supervisor for feedback on test items. Specific care was taken to ensure that the wording of the items was not ambiguous or misleading. The written test items were included in my research proposal and the feedback from the examiner was considered in the final version of the test. A group of colleagues at work piloted the written test and provided feedback on the time required to complete the items. The pilot sample for the written test consisted of young graduates with strong mathematical and statistical academic backgrounds and as such were not representative of the sample for this study.

For the qualitative analysis of the study, I made use of Maxwell’s (2013) “rich” data approach where the data was thick and diverse enough to provide a clear picture of what

[11] In the study conducted by Ben Yehuda and her colleagues (2005) they considered the *applicability (when)* of routines as part of their discourse profiling. I have chosen not to do so on the basis that Sfard (2008) claims that tests and interviews, although useful in examining the *how* of mathematical routines, are insufficient in assessing the *when* of routines.

was taking place. This was achieved through the production of verbatim transcripts of the interviews. According to Sfard (2008), researcher bias can be addressed by ensuring that participant utterances are reported verbatim.

In addition, the transcripts were paralleled against the observation notes made during the interviews and the written workings of the students in both the interviews and in the written tests. A process that Maxwell (2013) refers to as triangulation. In support of any conclusions relating to the prevalence of observed discursive actions, I have referred to quasi-statistics derived from the transcript coding exercise.

Although I am inherently an insider to the discourse of percent, I have strived to view the discursive activities of the students as an outsider by focussing only on what was present in the discourse.

5.6.2 Trustworthiness and reliability

A research study is considered trustworthy if there is consistency in the study's findings if the research is repeated with the same participants.

For the quantitative analysis, the written tests were analysed and coded three times. Through this iterative process, the coding of written scripts was revisited and refined where necessary to obtain a standardised scoring rubric.

Since qualitative analysis is open to interpretation by readers and the narratives of researchers alike, standardisation is pivotal in limiting the subjective nature of this unit of analysis. To this end, all data has been viewed as integral to this study. To afford the reader the opportunity to decide whether they agree with my interpretations, I have included lengthy, detailed transcripts of a portion of each interview. The appended transcripts are intended to provide very comprehensive accounts of what the students could or could not actually do.

5.7 Conclusion

Although this study has both quantitative and qualitative research elements, it predominantly employs an interpretative qualitative paradigm. Framed within Parker's (1994) cognitive comparative contexts of percent, quantitative research methods are called upon to identify student areas of difficulty with percent in the written test. The quantitative findings of the written test inform the qualitative research design. Interview transcripts, against the backdrop of student discursive activity, are then analysed from Sfard's (2008) commognitive perspective.

CHAPTER 6 | FINDINGS: THE WRITTEN TEST

6.1 Introduction

In this chapter, I report on the findings of the written test. I first identify the areas of difficulty in percent by comparing the performance from three perspectives: contextual versus non-contextual items, case type and Parker's classification of task type. Later, I present an analysis of how students approached the most difficult test items.

The purpose of the written test was threefold. Firstly, it served as a yardstick for what the students were capable of. Secondly, it informed the design of the interview instrument as the findings of the written test identified students' areas of difficulty with percent. The percent tasks that were identified as most challenging based on the findings of the written test were chosen for the interview instrument.

Finally, it provided a means for identifying students for the interview sessions. Students with unusual response patterns were approached to participate in interviews affording me the opportunity to access the thinking behind the responses that students had provided in the written tests. Since the written test was designed according to the same principles as Parker's (1994) study, it is comparable to the findings of her study.

The analysis presented in this chapter will show that students were limited to a part-whole notion of percent. Students did not appear to notice the multiplicative structure of the percent items and experienced difficulty in identifying the appropriate referents in contextual tasks. The analysis confirms that percent items that were categorised as c*hange by % of* and *A differs by % of B* were the most challenging to solve.

6.2 Identifying areas of difficulty with percent

6.2.1 Overview

The written test contained 15 questions designed to reflect a combination of contexts, task cases, benchmark percent values and Parker's (1994) nine comparative contexts of percent (see Table 5.1).

A total of 32 students wrote the test. Student scores ranged between 15.6% and 88.9%. Only 12 of the students obtained a score of 50% or more. The average score for the test was 43.3% with a median of 38.9%. These results provide a deficit view of the students' knowledge of percent. However, they align with the findings of Parker's (1994) study in which she found that less than half her pre-service teachers obtained a score of 50% or more in her pre-test.

In the sections that follow I compare the test scores of non-contextual items to the scores on contextual items. I then report on the findings of the analysis of test items by case type. Finally, a comparison of test scores by task type highlights the specific areas of students' difficulty.

6.2.2 Comparison of test scores by contextual and non-contextual test items

The written test provided a selection of contextual and non-contextual items. Contextual items presented percent in verbal situations such as word problems. Non-contextual items included typical exercises such as "Find 5% of 60". Table 6.1 provides a comparison between contextual and non-contextual tasks by performance segment.

Table 6.1: A comparison of contextual and non-contextual test items sores

Measure	*Non-contextual*	*Contextual*	*Total score*
Mean score	63.0%	30.1%	43.3%
No. of items (N)	6	9	15

When the results are analysed according to the context of the task we see that 25 of the students obtain a score of 50% or more in non-contextual type tasks. Non-contextual tasks had an average score of 63.0%, while contextual tasks had an average score of 30.1%. Students performed more than twice as well in non-contextual tasks as they did in contextual tasks. This is in line with the findings of earlier studies on percent (Lembke & Reys, 1994; Parker, 1994).

With non-contextual items, no decisions need to be made about what operation is required since the arithmetic is explicit in the task. For example: 200% of 70 can be interpreted as multiplication by virtue of the keyword 'of'.

Contextual items require the students to make decisions regarding the relative relationships of the information in the task. In the case of percent tasks, decisions need to be made regarding the appropriate referent to be used in the computation. These decisions will be influenced by the language of the task. Based on the demographics of the student sample, it can be inferred that most of the students that wrote the test did not have English as their first language. Although language is not the focus of the analysis, the poor performance on contextual items may have been linked to the language demands of contextual items. This is further complicated by the fact that most of the contextual items in the written test were financial in nature. Only three of the 32 students obtained a score of 50% or more for contextual tasks.

6.2.3 Comparison of test scores by case type

The test instrument provided a range of *case 1, case 2* and *case 3* problems. *Case* 1 problems required the participant to determine the percentage given the base and percent where $percentage = percent \times base$. Problems of type *Case 2* required the participant to determine the percent where $percent = \frac{percentage}{base}$. Problems of type *Case 3* required the participant to rearrange the percent equation to determine the base. In this case $base = \frac{percentage}{percent}$. The results of the analysis are illustrated in Table 6.2.

Table 6.2: Comparison of test scores by case type

Measure	*Case 1*	*Case 2*	*Case 3*	*Total score*
Non-contextual	67.2%	56.8%	65.1%	63.0%
Contextual	43.1%	11.3%	85.4%	30.1%
Mean score	52.7%	24.3%	71.9%	43.3%
No. of items (N)	3	9	3	15

According to the test scores, students performed considerably better overall in *case 3* type problems than in *case 2* and *case 1* type problems. However, when the analysis is split between contextual and non-contextual items a different pattern emerges. Performance by case type for non-contextual items is fairly consistent with only minor dips in the scores for *case 2* and *case 3* type problems. The contextual performance by case type, however, is erratic with large variances between the scores.

I posit that there are two factors that influence these findings. First, the non-contextual items were evenly weighted by case type with exactly two items per case type. This means that the results for non-contextual items are comparable to each other. The contextual items were not evenly weighted since there was only one item each for *case 1* and *case 3* but there were seven *case 2* type problems. This means that the contextual items are not comparable to each other since there was not an equal weighting of case type items.

Second, the *case 2* type contextual items featured the more difficult task types according to Parker's classification. The *case 3* type contextual item was the highest scoring test item overall. The analysis of the test scores according to Parker's (1994) categorisation of percent tasks in the next sections provides more insight into these findings. Since the test instrument was not evenly weighted in terms of case types and complexity no claims can be made regarding the influence of case types on student performance.

6.2.4 Comparison of test scores by task category: Parker's Classification

Parker's (1994) framework (see Figure 2.1) classifies contextual, real-world percent tasks into two main contexts: *fraction* and *ratio*. Where *fraction* measures a subset of a set or a whole, *ratio* measures the relationship between two referent sets. In the case of *ratio-change* items, the relationship between a set A and a modified version of this set A' is measured. In the case of *ratio-comparison* items, the relationship between two disjoint sets A and B is measured. The test instrument was designed to include percent tasks from each of Parker's categories to facilitate the identification of areas of difficulty in percent. Table 6.3 illustrates the relationship between the average test score by the type of percent task as defined by Parker.

Table 6.3: A comparison of test scores by task category

Parker classification	*Total scores*	*No. of items (N)*
Fraction	**85.4%**	**1**
Ratio	**23.3%**	**8**
Change		
Change to % of	*42.7%*	*1*
Change by %	*14.1%*	*4*
Comparison		
Compare A to B	*72.9%*	*1*
A differs by % of B	*7.3%*	*2*
Contextual tasks	***30.2%***	***9***

The mean score for the *fraction* item was the highest. This suggests that students identify with percent as a *fraction* more so than as a *ratio* between two referent quantities. The association of percent with the part-whole representation is strongly held by children and adults alike and is supported in this study by the average of 85.4% in the *fraction* category (Lembke & Reys, 1994; Parker, 1994; Parker & Leinhardt, 1995).

Under the *ratio* context of percent, students performed considerably well on the *Compare A to B* item scoring an overall average of 72.9%. The second highest performance under the *ratio* context was on the *Change to a % of* item with an average of 42.7%. Both categories of percent can be computed by applying the multiplicative relationship $B = A \times p\%$.

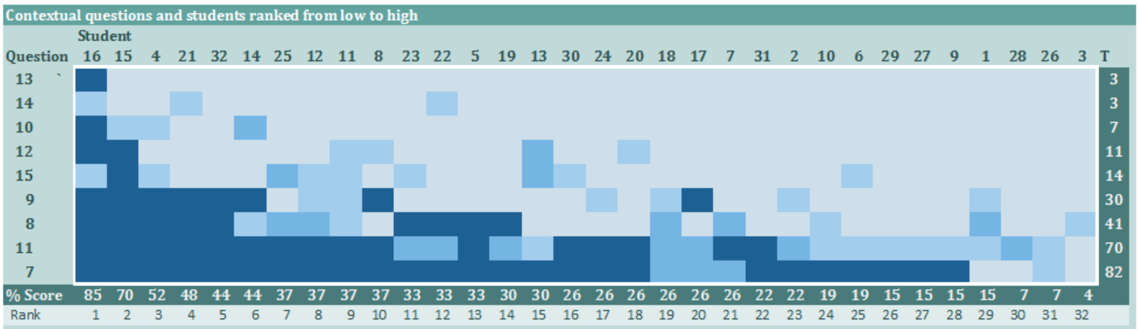
Students had considerable trouble with *Change by %* and *A differs by % of B* tasks scoring averages of only 13.8% and 7.3% respectively. The application of the incorrect base referent proved to be the leading cause of the poor performance in these tasks.

Change by % tasks are based on the relationship between a set *A* and the transformed set *A'* where $A' = A(1 \pm p\%)$. Students needed to be able to identify that the base referent is *A*, the initial set, and not *A'*. Tasks of type *A differs by % of B* are based on the relationship between two disjoint sets *A* and *B* where the relationship can be expressed as $\mathrm{B} = A(1 \pm p\%)$ or $\mathrm{A} = B(1 \pm p\%)$. Careful attention needed to be paid in identifying the base referents in comparisons of this type.

6.3 How students approached contextual items

For the analysis of test scores, students were randomly numbered from one to thirty-two. The number allocated to each student had no bearing on the student's score. Test scores for contextual items were arranged from lowest to highest by question and by student number to obtain the grid in Figure 6.1. Only three students achieved a score greater than 50% for the contextual questions in the written test.

Figure 6.1: Contextual tasks scoring grid



6.3.1 Percent as '*fraction*'

In this section, I discuss the results of test items that are "*fraction-like*". Although the tasks are grouped together they are not strictly classified as *fraction* under Parker's (1994) framework. However, the tasks bear a close resemblance to each other as they could all be computed by applying the multiplicative relationship $B = A \times p\%$.

6.3.1.1 Percent as part-whole

Item 7 was the only task that was classified as *fraction* according to Parker's (1994) framework.

Item 7*: Sumaya spends 25% of her monthly salary for rent. If she spends R2 700 a month for rent, what is Sumaya's monthly salary?*

This item was the highest scoring item across both contextual and non-contextual tasks with 25 students scoring three. The association of percent with the part-whole representation is strongly held by children and adults alike and is supported in this study by the average of 85.4% for this item (Lembke & Reys, 1994; Parker, 1994; Parker & Leinhardt, 1995).

Students were required to determine the base to solve this task, classifying this as a *case 3* item. This item featured a benchmark percent of 25% which could have made it arithmetically easier to solve. Although research has shown that tasks of type *case 3* are more difficult to solve, Montgomery claimed that this might not necessarily be the case for arithmetically easier items (Parker, 1994; Montgomery, 1925 as cited in Parker & Leinhardt, 1995).

Since 25% is typically associated with $\frac{1}{4}$, the task could have been computed by multiplying 2 700 by 4. Nineteen students used this approach, another two doubled 2 700 twice and one student determined 2700×3 and then added 2 700 to the result.

6.3.1.2 Percent as compare A to B

Under the *ratio* context of percent, students performed considerably well on the *compare A to B* item scoring an average of 72.9%.

> ***Item 11:*** *Your sister just got a raise and now earns R12 000 a month. Now your salary is only 85% of your sister's salary. How much do you make?*

Students were required to determine the percentage to solve this item, classifying this as a *case 1* task. Twenty-one students approached this item with a fraction[12] strategy. In these cases, the percent was written as $\frac{85}{100}$ and multiplied by 12 000 and simplified.

From a mathematical perspective, this *ratio* context item is not any different to the *fraction* context since the part-whole framework is still maintained with a percent value that is less than 100%. This could be an attributing factor to the high average achieved for this item.

6.3.1.3 Percent as *change to a % of*

The second highest performance under the *ratio* context was on the *Change to a % of* item with a mean of 42.7%.

> ***Item 8:*** *A packet of rice is selling for 175% of its 1995 price. If a packet of rice sold for R16 in 1995 what is the selling price today?*

Students were required to determine the percentage to solve this item, classifying this as a *case 1* task. Thirteen students approached this item with a fraction[12] strategy by writing the percent as $\frac{175}{100}$. Ten students made use of an equation strategy by assigning a symbol to the unknown amount and solving algebraically for the percentage. Fewer students were inclined to use the fraction strategy to solve this task than in the other two items under this section. I infer that this was due to the percent value being greater than

[12] The fraction strategy according to Lembke and Reys should not be confused with the fraction context in Parker's model.

100 – a notion that conflicts with the part-whole construct of percent (Parker & Leinhardt, 1995).

6.3.2 Percent as the variance between two referents

According to the contextual scoring grid in Figure 6.1 the lowest scoring questions were of the *A differs by % of B* and *Change by %* type scoring averages of only 7.3% and 13.8% respectively. Both *ratio* contexts compare the variance (increase/decrease or more than/less than) between two sets of data and are of the form $\mathrm{B} = A(1 \pm p\%)$.

A summary of the six lowest scoring items is provided in Table 6.4. Of the six lowest scoring questions, items 13, 10, 15 and 9 were selected for the further study in the interview sessions. The sections that follow investigate how students approached these items in the written test.

Table 6.4: Lowest scoring contextual questions

Rank	*Item*	*Parker category*	*Tag*[13]	*Case*	*Average*
1	13	Differ by %	R_Ds	2	3.0%
2	14	Change by %	$R_B i$	2	3.0%
3	10	Change by %	$R_B d$	2	7.0%
4	12	Differ by %	$R_D g$	2	11.0%
5	15	Change by %	$R_B(i, d)$	1	13.0%
6	9	Change by %	$R_B i$	2	30.0%

6.3.2.1 The increase item (item 9)

The *increase* item falls into the *Change by %* category according to Parker's (1994) framework. The average score for this item was 30.0%.

> ***Item 9:*** *Government have offered workers a new minimum hourly wage rate of R20/hr. If the rate last year was R16/hr what is the percentage increase?*

Twenty-three students correctly established that the task was one of change by calculating the difference between the two rates. Seventeen students identified the

[13] Refer to Table 5.1 for a detail on the item types.

correct referent quantity of R16/hr with thirteen students basing their calculations on the R20/hr as a base.

Seventeen of the students approached this item with a fraction strategy expressing the task as either a $\frac{1}{4}$ or $\frac{1}{5}$ increase and then re-writing the value as 25% or 20% respectively. Five of the students that approached the item with a fraction strategy obtained the correct final answer.

Nine students assigned the unknown amount a symbol (mostly x) expressing the task as an equation. None of the students that approached the item in this manner obtained a correct final answer.

6.3.2.2 The more than item (item 13)

The *more than* item is categorised as an *A differs by % of B* item according to Parker's (1994) framework. The mean score for this item was 3.0%.

Item 13: *If Jacob earns 25% more than Ramona, then Ramona earns ___% less than Jacob.*

This item presented students with a relationship between two unknown quantities Jacob's salary (J) and Ramona's salary (R). The ratio of J: R was given as 25% *more* and students were asked to consider the ratio of R: J. In the absence of any quantities for these variables, this relationship could either have been expressed algebraically or an arbitrary value assigned to either one of the referents. For the most part, students did not show any workings and responded with either 25% or 75% as answers.

Eighteen students answered 25% to this item indicating a perception that 'percent more than' is the symmetrical equivalent of 'percent less than'. Students needed to realise that percent more than and percent less than are relative rates of comparison and cannot be considered symmetrical (as is the case in the additive world) (Parker, 1997).

The response of 75% is evidence of a part-whole construction of percent, where if one part was 25% more than the remaining part must be 100% less 25%. Nine students answered 75% to this item. One student obtained the correct final answer of 20% for this item.

6.3.2.3 The double discount item (item 10)

The *double discount* item is categorised as a *Change by %* (decrease) task according to Parker's framework. The average score for this item was 7.0%.

> ***Item 10****: A store is offering a 20% discount on jelly beans. This weekend only, all shoppers will get an extra 5% discount. What is the total discount that they will get?*

Twenty-seven (27) students responded with an answer of 25% in the written test. This response is indicative of an additive construction of percent where 20% + 5% = 25%. Three students responded with answers that were indicative of a multiplicative structure of percent, with only one student providing the correct final answer of 24%. Students needed to realise that the referents for the two discounts were different and it was therefore not correct to simply deduct 25% off the original price (Parker, 1997).

6.3.2.4 The jacket item (item 15)

The *jacket* item is categorised as a *Change by %* task according to Parker's (1994) framework. The average score for this item was 13.0%.

> ***Item 15:*** *A jacket costs R560 excluding VAT and is now discounted by 25%. A customer asks the teller to add VAT first and then apply the discount so that she gets a bigger discount. The teller explains that the system automatically adds VAT after the discount has been applied. Would the customer pay less for the jacket if VAT was added before the discount?*

Students performed marginally better on this item than they did on the *double discount* and *more than* tasks. Given that all three items were of the same multiplicative construct where $B = A(1 \pm p\%)$, I infer that the leading cause for the improved performance in this item is the fact that a quantity (the jacket price) was given for the base referent. For the other two items, no quantity was provided for either of the referents. As such, students could solve the *jacket* item computationally.

However, most computations were completed as a two-step process where the discount and VAT amounts were first computed and then added or subtracted from the price of the jacket. No student was able to express the equivalence in terms of the commutative property of the multiplicative structure, that is: $560(1-25\%)(1+15\%)$ is equivalent to $560(1+15\%)(1-25\%)$.

6.4 Conclusion

The written tests served three purposes: a yardstick for what the students were capable of; informed the design of the interview instrument and it provided a means for selecting students for the interview sessions.

Students in this study were undergraduates at first-year university level and were on the secondary mathematics programme of the B.Ed. degree. To enter this course, students were required to obtain a minimum of 60% on their matric mathematics results. This suggests that students would have received instruction in percent and, as young adults, should have had some exposure to percent in their everyday lives. However, the written test was based on a topic that the students had most likely not recently reviewed. As such, some students had to recall procedures while others relied on benchmarks or a fraction approach to solving percent tasks. The exclusion of calculators also placed them at a disadvantage since they would have had access to calculators during their school instruction on percent.

With an overall average of 43.3% for the written test and only 12 of the students obtaining a score of 50% or more, the results of the written test paint a deficit picture of student knowledge of percent. Upon analysing the results by context, it was evident that the poor performance was predominantly driven by student scores in the contextual percent tasks. This suggests that students were well versed in approaching non-contextual percent tasks. However, contextual items required students to make decisions regarding the relative relationships of the information in the task and to identify the appropriate operations. These decisions were influenced by the language of the task. Although language was not the focus of the analysis, the poor performance on contextual items may have been linked to the language demands of these items. This is further complicated by the fact that most of the contextual items in the written test were financial in nature.

At closer inspection, it became apparent that students were limited to a part-whole fraction construct of percent and exhibited difficulty in identifying the appropriate base referent in percent tasks. Students did not appear to realise the multiplicative structure of percent and this was particularly evident with the poor performance in c*hange by % of* (ratio context B in Figure 2.1) and *A differs by % of B* (ratio context D in Figure 2.1).

These findings warranted further investigation into the possible contributing factors to the poor performance. For this reason, interview sessions were necessary to gain access to the student discourse of percent with specific reference to *change by % of* and *A differs by % of B* type tasks.

CHAPTER 7 | FINDINGS: THE INTERVIEWS

7.1 Introduction

Although students would have been exposed to percent throughout their schooling, what became evident from the written tests is that students had a narrow construction of percent that was limited to a part-whole definition of percent. Students experienced difficulty in identifying the appropriate base referent in c*hange by % of* (ratio context B in Figure 2.1) and *A differs by % of B* (ratio context D in Figure 2.1) type items. Furthermore, the multiplicative structure of these items did not appear to be present in student workings. Interviews were necessary to gain access to the student discourse of percent with specific reference to *change by % of* and *A differs by % of B* type tasks. Interviews were conducted with two pairs of students (four students in total).

In this chapter, I present the findings of the qualitative analysis of the students' discursive actions in the interviews. I draw on Sfard's commognitive theory from Chapter 3 and the model of percent discourse constructed in Table 4.1.

The two interviews are analysed in succession under sections 7.3 and 7.4, with the analysis of each interview presented by task. I conclude the analysis of each interview with a summary of the discourse profile of each student, presented according to the criteria laid out in Table 5.4.

7.2 Overview of the interview analysis

7.2.1 Discourse analysis approach

The analysis that follows is aimed at providing an indication of the degree of objectification in the students' discourse of percent as it tracks the evolution of their activities. The analysis is organised in terms of the four characteristics of mathematical discourse, that is; *visual mediators*, *words*, *routines* and *narratives* (Sfard, 2008). The analysis is structured around the discourse profile features from Table 5.4 and the development levels of the discourse of percent from Table 4.1.

The analysis of students' discourse revealed that development had taken place for some students through the interaction in the interview sessions. The evidence lies in the way in which the discourse changed for some students – evidence that will be presented in the sections that follow. This chapter therefore is not only an account of *what was said* but of how *what was said* and *what was done* changed through social interaction. It should be noted however, that I did not seek to study the change in students' discourse. For this reason, the observed changes in student discourse have been noted but no further analysis of the relationship between the interactions and the change in discourse has been undertaken.

7.2.2 Structure and layout of the analysis

For the analysis, I have taken transcripts from interviews of two pairs of students and coded the discourse as they worked through the items. A full description of the transcription coding process can be found in section 5.5.6. Although both interviews were fully transcribed and coded, the analysis presented here covers only a selection of the data. I have omitted any talk that was not clearly audible. At times, participants spoke at the same time and it was not always possible to identify the speakers. Repetitive words such as "ja", "ok" or "oh" were also omitted from the transcripts in this chapter. To encourage unhindered discussion, I often allowed the talk to steer off task. Topics included what exams they were writing that week, to their work experience and even the inflation rate in Argentina. For the purpose of this analysis, I have regarded these discussions as off-task.

The transcripts have been divided into separate episodes, with each episode referring to a segment of the conversation that deals with a change in focus related to the same task. For example: in the increase task of the first interview, the students first discussed which referent was the appropriate base referent. This was followed by a discussion of how their choice of base referent would change if they were asked to determine the decrease in rate instead. The discussions are classified as two separate episodes as the focus had shifted. Each episode is given a descriptive name that ties back to the focal point of the episode.

The analysis of each interview is first presented chronologically by episode and is then summarised by student according to the four characteristics of mathematical discourse, that is; *visual mediators*, *words*, *routines* and *narratives* and the *PD-level*.

7.3 Interview 1: The case of Nisha and Johan:

7.3.1 Performance on the written test

Nisha ranked 4th with a score of 62.2% in the written test; 100% on non-contextual questions, and 37.0% on contextual questions. Johan ranked 10th with a score of 48.9% in the written test; 72.2% on non-contextual questions, and 33.3% on contextual questions.

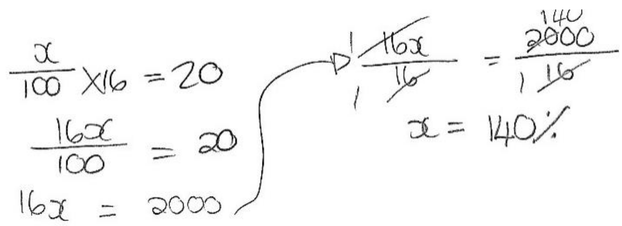
7.3.2 The increase task

7.3.2.1 Written test responses

Neither student scored more than one in the written test for this task. Johan identified that the task was related to the difference between the two rates as he calculated this to be R4, but then he expressed the percent increase against the incorrect base referent of 20.

Nisha expressed the task algebraically assigning the variable x to the unknown percent (Figure 7.1). However, the expression was written as a *change to %* task and resulted in a value for x that was greater than 100%.

Figure 7.1: Nisha's written work - increase task



7.3.2.2 Episode 1.1: Locating the base referent

Students were given two images with possible solutions for this task and were asked to choose the most appropriate solution (refer to section 5.4.2.1). All communication in Episode 1.1 was verbal and neither student wrote anything down on their interview scripts.

Table 7.1: Episode 1.1

Line		What was said	What was done
5	J	I would say B	
6	I	B? and for you?	
7	N	I'd go with A	

Line		What was said	What was done
8	I	You would go with A?	
9	J		Mumbles softly.
10	I	OK. Think. Look at it again.	Addressing both students.
11	N	Well B does seem logical because it is the new hourly rate right. The 20 rand. But then also 16 is the old, right? So, obviously you want to find out how much it has increased by. So, I would take A because it would be 20 minus the 16.	
12	J	Ja. I would also take A.	
13	N	Which is 4 and then the 4 divided by the 16 which is multiplied by the 100. If that makes sense?	
16	J	In science I also do it this way. So, I would have done it this way. So.	
27	J	So. That's why I would have done it like A. Because in science we were taught to use the original. So, I don't know really why they would do B.	

Johan, having visually scanned the images, initially chose option B which was the same routine that he had used in the written test (line 5). Nisha's utterances in line 7 to line 11 appeared to have swayed his choice and he reverted to option A. He recalled the *rates of change* formula from science and substantiated his choice through this connection stating that "we were taught in science to use the original [base referent]" (line 16, 27). The key aspect of his substantiation is that he appeared to have endorsed the narrative based on what he had been taught. His substantiation does not come from within the mathematical discourse of percent - not because it came from science but because it came from another person's actions. His substantiation is classified as extra-discursive.

Nisha's utterances were predominantly procedural as indicated by the words "subtract" and "divide by". Her actions were symbolically mediated as she performed syntactic numerical calculations aloud (lines 11, 13). Her substantiation for her choice in routine appears to have been "logic", classifying it as extra-discursive.

7.3.2.3 Episode 1.2: Determining the applicability conditions

I asked the students to explain how they would calculate a percent decrease from a rate of R20/hr to R16/hr. The purpose of this question was, firstly, to determine if the students were aware of the applicability conditions of the routine that they had selected in Episode 1.1. Secondly, the question was intended to provoke some thought on what the appropriate base referent would be.

Table 7.2: Episode 1.2

Line		What was said
30	N	Well, decrease means obviously subtraction, right? Firstly. So, I would…. So obviously if you take 20 and you minus it from 16 you are going to get a negative answer so I would work that way and work out how you would … you would obviously get a negative percentage but then you can always just say it decreased by this and you can write it as a positive percentage
31	I	OK
32	J	Ja. That makes sense to me also. Ja.
33	I	Are you being swayed?
34	J	No. I can't think of another way though.

Nisha's response was largely procedural and personal with statements such as: "decrease means obviously subtraction"; "you take 20 and you minus it from 16" (line 30). I did not prompt her to specify what base referent would be appropriate, so no further inference can be made.

Johan agreed with Nisha's response, endorsing her narrative, yet providing no substantiation other than "I can't think of another way". Johan's endorsement of Nisha's narrative without clear substantiation could indicate that the routine was ritual. Lack of an alternative is not considered a literate substantiation of the narrative. In retrospect, I should have prompted the students to write down their responses as this might have led to further discussion on the appropriate base referent for a decrease scenario.

Johan and Nisha's discourse in the increase task did not provide enough evidence of the percent discourse level of the students. For this reason, no *PD-level* has been assigned for this task.

7.3.3 The more than task

This task[14] presented students with a relationship between two unknown quantities Jacob's salary (J) and Ramona's salary (R). The ratio of J: R was given as 25% *more* and students were asked to consider the ratio of R: J.

7.3.3.1 Written test responses

For the most part, students in the written test did not show any workings and responded with either 25% or 75% as answers to this task. The response of 25% signifies that 'percent more than' is perceived to be the symmetrical equivalent of 'percent less

[14] Please refer to section 5.4.2.2 for a detailed description of the *more than* task.

than'[15] and suggests an additive construction of percent. The response of 75% is evidence of a part-whole construction of percent. In the written test, both Nisha and Johan answered 25% without showing any workings.

7.3.3.2 Episode 1.3: It's not 25 percent

In the interview, Nisha substantiated her written test response by explaining that a symmetrical relationship "make[s] logical sense" (line 48). Johan suggested approaching the task with the use of a "normal number like 10" (line 49). This utterance suggests that he viewed percent as something other than a "normal number". This could signify a percent discourse level of at least *PD1*.

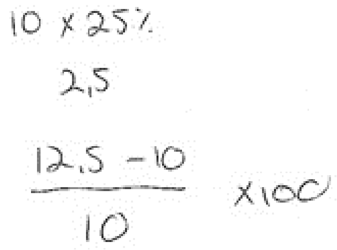
Table 7.3: Episode 1.3

Line		What was said	What was done
48	N	Because it would make logical sense if he is earning 25 per cent more than she should be earning 25 per cent less than he is.	
49	J	I remember I had quite a … um … how I worked it out quite a long working out. So. But I used just a normal number like 10 rand.	
60	J	So, what I just did I just took 10 rand and times it with 25 percent to get then 12.5 and then that is the amount Ramona would have. So, then you would take the 12.5 minus with the 10 and then divide it by the 10 and then get the percentage that Ramona earns less.	
61	I	Can you show me how?	
62	J	Ja… I said 10 times 25 per cent	Writing $10 \times 25\%$
64	J	And then I got to 2.5 so I added that with the 10 rand and I got 12.5 so then you subtract the 12.5.	Writing 2.5
68	J	Right. Then I would divide that with the original amount which is 10 ….	Writing $(12.5 - 10)/10 \times 100$
70	J	And then times it with 100 … and that would be … minus 10 times 100. Oh no.	Working on the calculator
71	N	It's still …	Spoken softly
72	J	So, it will still be 25. So …. I don't know. Is that correct?	
73	I	It's … It's not 25	
74	N	I had a feeling	

Johan described his routine making predominant use of procedural words such as: "times it with 25 percent"; "12.5 minus with the 10"; "divide it by the 10" (line 60). When prompted to illustrate his method, he switched to writing down the steps: illustrating flexibility in his use of mediational modes. He continued to describe his process verbally as he wrote, concluding with the expression $\frac{12.5-10}{10} \times 100$. His written workings are illustrated in Figure 7.2.

[15] Please refer to section 6.3.2.2 for more detail on the results of the written test.

Figure 7.2: Johan's written work for the more than task



7.3.3.3 Episode 1.4: Why would you use a ratio?

Sfard (2007) defines commognitive conflict as a situation in which interlocutors act according to different discursive rules, often using the same words in different ways. I infer that Johan's commognitive conflict in line 76 of Table 7.4, stems from his word use in Episode 1.3. In line 68 of Episode 1.3 Johan refers to the "original amount" as the base referent. The initial ratio presented by the task used *Ramona*[16] (10) as a base referent. Johan's process did not account for the need to switch the base referent from *Ramona* (10) to *Jacob* (12.5) and so he continued to work with the "original amount" (*Ramona*) as the base referent.

Table 7.4: Episode 1.4

Line		What was said	What was done
76	J	So, why would it not work if I gave if I have like a set amount of money. Like I have a 10 rand and then I would times it with the 25 percent and add it to the 10 rand	
79	I	Well let's do this, right. Let's say what if …. I am just going to choose 100 because I find it easier to work with 100 …but let's say Jacob earns 100 …	
80	N	OK	
81	I	OK. Um …. No actually, if we make him 125. So, Jacob earns 125 that means then that Ramona earns 100. Because 25 per cent more than a 100 is 125.	
82	J	Yes	
83	I	So now work out Ramona's salary as a ratio of Jacob's	
84	J	As a ratio?	
85	I	Hmm	
86	J	So, then you would say 125 divided by 100.	$125/100$ on the calculator
87	N	Mm	Agreeing
88	J	And then times that with 100 ….	$125/100 \times 100$ on the calculator
89	I	And what do you get?	
91	J	125 divided by 100 and times that with 100	$125/100 \times 100 = 25$ on the calculator
92	I	Mm	

[16] I have expressed Ramona in italics as the variable for Ramona's salary.

Line		What was said	What was done
94	N	It will still be 25.	
95	I	Ja. But that is still Jacob… that is saying Jacob's salary to Ramona.	
96	J	Oh, so it is 100 divided by 125	$100/125$ on the calculator
97	I	What do you get then?	
98	J	You get 80	J using the calculator N writing "125:100 80% more"
99	I	80. OK so hers is 80 per…	
100	J	80 percent of his.	
101	I	Which means it is what percentage less?	
102	J	20 per cent	
103	N	OK. That's clever	
104	J	So why would you use the ratio's then?	
105	I	Because that is … that's what … we are not actually doing an increase here are we?	
106	J	Oh, OK .	
107	N	And remember ratios also work like division. Like when you think of proportionality theorem … when they are over …. when they divide each other? It also works with ratios.	
109	N	So, ratios mean divide also.	

I prompted the students to relook at the relationship between the two referents as a ratio (line 83). The students were unconvinced by this suggestion but proceeded to express the two salaries as a ratio. Johan proceeded in this manner working verbally, but his selection of a base referent in line 91 was again the "original number". At this point, both methods that Johan had used resulted in the same erroneous answer. I felt that if I did not interject again, that the students would not realise the non-symmetrical nature of the task. After my correction in line 95 Johan recalculated the ratio as $\frac{100}{125}$ but did not show signs of determining the final answer independently. Some further scaffolding from me resulted in a final answer of 20% (lines 99 to 101).

The repeated interjection from me suggests that the routine was most likely a ritual for Johan. His discursive actions were syntactically mediated with symbols replacing words such as "original number". In retrospect, I felt that I should have left the students to explore the relationship independently for longer. For this reason, I avoided providing scaffolding for this task in the second interview.

Although Nisha had been a quiet participant throughout much of this episode, her utterance in line 107 aptly related the task to the Proportionality Theorem. Although this mathematically literate connection to geometry suggests a multiplicative construct of percent, Nisha continued to use additive language for the remainder of the interview. This suggests that she had not truly realised the connection between geometry and the multiplicative structure of percent.

7.3.4 The double discount task

This task[17] differs from the *increase* task in three aspects: it is a decrease task; two decreases are applied; and no referent quantities have been provided.

In the absence of a referent quantity (jelly bean price) for the task, students could either solve the task algebraically or assign an arbitrary value to the unknown quantity. The discursive moves of students within the context of this task provided evidence of the structure of percent that students had constructed.

7.3.4.1 Written test responses

In the written test[18], most students responded to this task with an answer of 25%. This response is indicative of an additive construct of percent where 20% + 5% = 25%. The appropriate multiplicative structure is expressed by $(1-20\%)(1-5\%)$. Both Johan and Nisha responded with an answer of 25% in the written test with no workings to support their answers.

7.3.4.2 Episode 1.5: Ratio becomes ritual

Johan selected the quantity of 10 for the task. His utterances were procedural with some sense of a multiplicative structure evidenced in "then it won't be 25 percent" (line 135) as he came to the realisation that the 5% discount should be applied to the 8 and not the 10. These utterances show evidence of symbolic objectification as the symbolic string $10 - 10 \times 20\%$ was replaced with the equivalent symbol 8. In line 141 Johan refers to the ratio routine from the previous task. This reference indicates that the final steps in his routine were potentially ritualised in nature, that is: thoughtful imitation of a new narrative that has just been introduced in the interaction (Sfard, 2008).

Nisha's participation was limited to nods in agreement with Johan's narrative. Her utterances in line 140, 145 and 148 appear to indicate that she was just going along with Johan's approach, either repeating Johan's responses or providing the answers to a

[17] Please refer to section 5.4.2.3 for more detail on the *double discount* task.
[18] Please refer to section 6.3.2.3 for more detail on the written test results.

computational step in Johan's process. This would suggest that her discursive process was ritual (Sfard, 2008). It is unclear what step her utterance in line 138 of "20 divided by 5 is 4" relates to but the phrase is procedural. She had written the computation $10 - \frac{20\%}{100}$ which, besides the inappropriate notation, is evidence of an additive construct to the task.

Table 7.5: Episode 1.5

Line		What was said	What was done
135	J	So. Let's say if the jelly beans initially cost 10 rand. And then you subtract, you take the 20 percent off. Um. Take it off from 10 rand then it would be 2 rand. So, then you have 8 rand left. So, then you take the … 5 …	Written $10 \times 20\%$
136	J	… oh ja … then it won't be 25 percent. So, then you take that 8 per cent out … not 8 per cent … 5 per cent from 8 rand and that would be …. 5 per cent	Working on calculator Written $8 \times 5\%$
137	J	Let's hope its …	Mumbles
138	N	20 divided by 5 is 4.	Written $10 - (20\%) / 100$
139	J	Ja. So that would be 0.4. So, then that would be seven rand sixty.	
140	N	Seven rand sixty.	
141	J	Then that you take a ratio with the 10.	
143	J	Ok. So then that and 10. You have seven rand sixty. Taking ratios.	
144	J	So that would be 7.6 divided by 10 times 100. So that is whoa! I don't know if that is correct.	Working on calculator
145	N	Seventy six percent? Is that possible?	
146	I	You are paying 76 percent of the price. So, then what is the discount?	
147	J	Oh, then you just. Ok. Then the discount ..	
148	N	Is 24	
149	J	Ja it would be 24 percent	
150	N	Wow. OK. OK.	Surprised. But unconvinced
152	J	It makes sense now because I did the previous problem differently.	

7.3.4.3 Episode 1.6: Nisha's generalisation

I asked the students if they thought it was possible to use algebra to prove the result. Johan gave a verbal account of how he would attempt to generalise the relationship by setting the price of the jelly beans equal to x. Although he did not write the expression down, it could be inferred from his utterance in line 176 that his expression for the price after the first discount could be written as $x - x \times 20\%$. This shows signs of the early stages of a multiplicative construct, where a fully-fledged multiplicative structure would be represented by $x(1 - 20\%)$. The routine was explorative in nature as he was attempting to construct a new narrative. Johan did not pursue extending the expression to determine the total discount, so no further inference can be made.

Table 7.6: Episode 1.6

Line		What was said	What was done
174	J	Algebra? Yes, you can just make the x ten ag the 10 x.	
176	J	And then times that with … your … 20 per cent … so x would be your initial value …	
178	J	And then you want to get the total discount …	
179	N	You could also make the total discount x. To solve for x	
180	I	Yes. You could.	
181	J	Ja. That would be easier though	
183	J	I don't know how to … like … write that in an equation.	
185	N	So, I would start by saying let total discount equal x. OK. So, we are looking for x. So, then the question is how to write out the equation more than anything else.	
187	N	To find it. So … we'll obviously you would say that x is the total discount. So, then x will equal to the 10 rand minus the 20 per cent right? And then …	Written $x = 10 - 20\%$
188	J	But we don't have the 10 rand .	
189	N	No. I'm just using a theoretical value. Right?	
191	N	So obviously that would be 8 rand and then you will have that answer. But then it will be a bit tricky because that won't be the final answer of x. If that makes sense because you still have to subtract the 5 per cent from it. And then eventually you will end up dividing by the 10 to get the answer.	
192	J	I don't think that is ... For me … this … if you just look at the question without using the 10 that I used.	
194	J	You can't use algebra for me.	

Nisha suggested an alternative approach of setting the total discount equal to x and expressed this in writing as $x = 10 - 20\%$. In addition to the inappropriate notation, Nisha's symbolic expression is additive in nature. Nisha had also inadvertently assigned a specific value to x. Her discourse was procedural in nature and she was stymied when she needed to account for the second discount in the expression. Johan eventually claimed that "you can't use algebra".

7.3.5 The jacket task

The *jacket* task[19] is categorised as a *Change by %*. Unlike the *more than* and *double discount* tasks, the referent quantity (the jacket price) has been provided. Students could approach this task as a computational task. However, the intention was for the students to reflect on the fact that the result is the same regardless of the order in which VAT and the discount are applied.

[19] Please refer to section 5.4.2.4 for a detailed description of the *jacket* task.

7.3.5.1 Written test responses

Student performance on this task in the written test was slightly higher than in the *more than* and *double discount* tasks. The improved performance is attributed to the presence of a referent quantity (the jacket price)[20]. Student responses were computational and did not show evidence of any realisations of the multiplicative structure of the task.

Neither Nisha nor and Johan produced the correct final answer in the written test. Johan correctly computed the two discounts in Rand terms and then claimed that the customer lost the difference of R21 between the two discount values. Nisha's written test response was a computation of the discount-first scenario only and did not consider this in relation to the VAT-first scenario.

7.3.5.2 Episode 1.7: Computing the task

In the interview both students worked independently in solving the task (see Figure 7.3 and Figure 7.4). The written work shows evidence of both additive and multiplicative structures. It is interesting to note Johan's computation for the VAT as a multiple of 114%. He had not expressed any other increases as a percentage greater than 100%. Although this approach is multiplicative, I infer that the inclusion of VAT is a special and familiar case that had been summoned from a previously endorsed narrative for adding VAT. For this reason, I infer that this may not necessarily be evidence of a fully-fledged realisation of the multiplicative construct of percent. This view is supported by his utterance in line 274 of Episode 1.8.

Figure 7.3: Episode 1.7 - Neo's jacket task

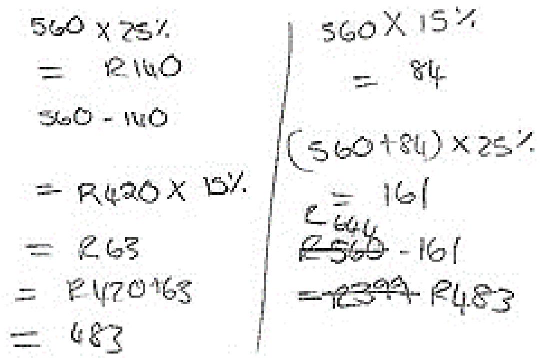
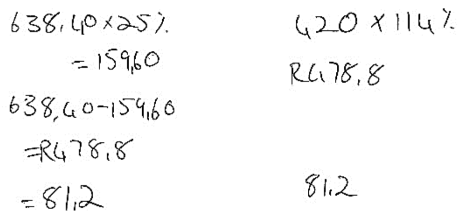


Figure 7.4: Episode 1.7 - Johan's jacket task



[20] Please refer to section 6.3.2.4 for more detail on the written test results.

7.3.5.3 Episode 1.8: Explaining the equivalence

The students noticed that both scenarios were equal in the *jacket* task but were not sure that this was the correct outcome. I asked them to explain why they thought their answers were the same for the two scenarios. The discussion that followed is illustrated in Table 7.7.

Table 7.7: Episode 1.8

Line		What was said
274	J	Ja. I understand that. Ja. Um … so, if you first add the 14 percent. Then Ok with my 14 percent I first got the 638.40. So, and then subtract the 25 percent which was the 159.60. And then I got my 478 and I subtracted it and got my 81.2. So, for me when I think … first adding the 14 percent and adding it afterwards doesn't change the fact that it's still only 14 percent that is added initially.
277	N	I think it has something to do with that cumulative properties I think it is called. I don't know what it is called. Say when you add A + B or B + A where if you say 1 over 2 times 1 over 4 it is not the same as 1 over 2 times 4 over 1.
285	J	I think it has to do with the subtraction.
286	N	The addition and subtraction one.
287	J	Ja
288	I	Because you're … adding and then subtracting? But you are adding and subtracting different amounts?
289	J	But you … when you subtract the different amount from this one you already discounted it 25 percent.
291	J	And when you subtracted it here you already added the 14 percent. So, it cancels out the whole time.

Johan's discourse was procedural and personal and showed signs of an additive construct of the task with word use such as "subtract the 25 percent", "I subtracted it and got my 81.2". Nisha referred to the commutative property (although she used the incorrect terminology) as an explanation for the equivalence (line 277). It is not clear how Nisha's reference to the commutative property is linked to her utterance: "if you say 1 over 2 times 1 over 4 it is not the same as 1 over 2 times 4 over 1", but her reference to "A + B or B + A" may suggest that she was referring to the commutative property of addition. It can be inferred that neither student realised the multiplicative structure of the task. For this reason, their discourse is classified as, at most, *PD2* level.

7.3.6 Summary of Nisha and Johan's discourse of percent

Word use and mediational mode: the word use for both students was predominantly procedural and personal with mediation limited to syntactic oral and syntactic written modes. No objectified word use was observed. Despite Nisha's literate reference to the commutative property in the *jacket* task, she was referring to the commutative property of addition and not multiplication.

Routines: in the *more than* task, both students reverted to ritual routines as they grappled with the comparative relationship between the two unknown referents. Much scaffolding was required to assist them through the task. I first prompted the students to relook at the relationship between the two referents as a ratio but they both appeared to be unconvinced. Johan attempted to express the relationship as a ratio but was unsuccessful. I felt that if I did not interject again, that the students would not realise the non-symmetrical nature of the task. I suggested a correction to Johan's work, but he still did not show signs of determining the final answer independently.

Reflecting on this, I felt that I should not have interjected as much as I had and should rather have allowed the discussion to emerge between them. However, the scaffolding appeared to assist Johan in the next task as he experimented with the new routine to solve the *double discount* task. Nisha's routine for the *double discount* task continued to be predominantly ritual. Her attempt at generalising the problem (explorative activity) for this task was unsuccessful.

Johan's routines showed some flexibility as illustrated by his attempts to adopt a ratio approach to the interview tasks. The routines were predominantly ritual, triggered by prompts from me. Sfard (2008) claims that rituals are often the first steps of learning as they are thoughtful imitation that could culminate in new endorsed narratives. This could suggest that Johan was taking his first steps towards the development of new narratives.

The routines for both the students in the *jacket* task were explorative numerical computations. Since the referents were provided in this task, both students worked independently to determine the solution. However, neither of the students were able to provide an explanation for the equivalence of the two scenarios. Although capable of the numerical computations, the students did not identify the multiplicative structure of the task.

Narratives: in his substantiation of the routine in the *increase* task, Johan based his decision on what he had been taught in science. In the *more than* task, I prompted the students to consider the task as a ratio. Johan proceeded to solve the *more than* task by applying a ratio approach and then repeated this approach in his solution to the *double discount* task. Johan's substantiation for the endorsed narrative of percent-as-ratio is therefore inferred to be extra-discursive. Johan's discourse profile is summarised in Table 7.8.

Table 7.8: Johan's discourse of percent

Johan's discourse of percent			*Tasks*			
			Increase	**More than**	**Double discount**	**Jacket**
Word Use			Unobjectified	Procedural and personal	Predominantly procedural and personal	Predominantly procedural and personal
Mediational mode	Symbolic	Syntactic	Oral	Oral and written	Oral and written	Oral and written
		Objectified	Not observed	Not observed	Not observed	Not observed
Routines	Type	Ritual or Explorative	Ritual	Explorative and Ritual	Predominantly explorative	Explorative
	Flexibility	Routines	Spontaneously switches routine	Routine switching occurs after prompting	Spontaneously switches to 'ratio routine'.	Not observed
		Mode switching	Not observed	Oral - written	Oral – written Oral – objectified	Written – oral
	Corrigibility	Propensity for erring	Medium	Medium	Low	Low
		Retracing	Observed	With prompting	Some	Not observed
		Successful correcting	Observed	With prompting	With prompting	Not observed
Endorsed narratives	Derivation		Not observed	Attempted use of a ratio approach	Attempted generalisation. But then gives up.	Not observed
	Substantiation		Extra-discursive (Science teacher)	Extra-discursive (Interviewer)	Extra-discursive (Interviewer)	Known procedure
	Memorisation / Recall		"It is how we were taught in science"	Not observed	Recalls ratio approach from the previous task	Not observed
Development of percent discourse	Nature		Too little talk	Fraction	Early ratio	Early ratio
	Structure		Too little talk	Additive	Some sense of multiplicative	Additive
	Level		Too little talk	PD1	PD2	PD1/PD2

Despite Nisha's occasional literate references to mathematical theorems and properties, her percent discourse was largely substantiated through the extra-discursive notion of what she referred to as 'logic'. Her attempts to derive narratives in the *double discount* and *jacket* tasks were unsuccessful. Nisha's discourse profile has been mapped in Table 7.9.

Table 7.9: Nisha's discourse of percent

Nisha's discourse of percent			*Tasks*			
			Increase	**More than**	**Double discount**	**Jacket**
Word Use			Procedural and personal	Minimal word use	Predominantly procedural	Some evidence of structural
Mediational mode	Symbolic	Syntactic	Oral	Written	Written	Written and Oral
		Objectified	Not observed	Not observed	Unsuccessful attempt at generalising	Unobjectified
Routines	Type	Ritual or Explorative	Explorative Numerical computation	Ritual Adopts Johan's approach (social)	Ritual Attempted explorative	Explorative Numerical computations
	Flexibility	Routines	Not observed	Written attempt at ratio approach	Attempted	Not observed
		Mode switching	Not observed	Not observed	Attempts Written - object	Oral – written Attempted objectified
	Corrigibility	Propensity for erring	Not observed	Medium	High	Low
		Retracing	Not observed	No evidence	No evidence	Evidenced
		Successful correcting	Not observed	Not observed	No evidence	Observed
Endorsed narratives	Derivation		Not observed	Not observed	Unsuccessful attempt to construct a new narrative	Not observed
	Substantiation		"Logic" Extra-discursive	Logic Extra-discursive	None	Known procedure
	Memorisation / Recall		Not observed	Endorses the ratio approach by relating it to the proportionality theorem	Not observed	Refers to commutative property of addition (though it should be multiplication)
Development of percent discourse	Nature		Fraction	Unclear	Unclear	Some notion of proportions
	Structure		Additive	Unclear	Additive	Additive
	Level		Too little talk	Unclear	PD1	PD1/PD2

PD-level: Johan's written workings in the *jacket* task suggest a multiplicative approach to adding VAT. However, when asked to explain the equivalence of the two scenarios in the *jacket* task his discourse is distinctly additive. As such, I conclude that his routine for "adding VAT" was a special case of computing a percent increase for which the procedure may have been taught.

Johan had the same score for the contextual tasks in the written test as Nisha (37%) yet through his flexible approach to routines, his discourse of percent began to show signs of *PD2*-level discourse. Since his percent-as-ratio routines remained ritual in nature, I have classified his discourse as *PD1/PD2.*

Nisha displayed proficiency in the computation of percent tasks when the referents were provided. This was particularly evident in her 100% score for the non-contextual tasks in the written test. However, she only scored 37% for contextual tasks in the written test. This gap between contextual and non-contextual tasks is evident in her difficulty in approaching the *more than* and *double discount* tasks. In the absence of referent quantities, she did not realise the relative relationship between the referents exhibiting, instead, an additive construct of percent. As such, Nisha's developmental level of percent discourse has been classified as *PD1*.

7.4 Interview 2: The case of Mandla and Bheki

7.4.1 Performance on the written test

Bheki ranked 17th with a score of 37.8% in the written test; 55.6% on non-contextual questions, and 25.9% on contextual questions. Mandla ranked 28th with a score of 26.7% in the written test; 22.2% on non-contextual questions, and 29.6% on contextual questions.

7.4.2 The increase task

This task presented the students with two possible solutions, A and B (see Figure 5.1), for an increase in wage from R16/hr to R20/hr. The students were asked to review the two solutions and to select the appropriate response for the task.

7.4.2.1 Written test responses

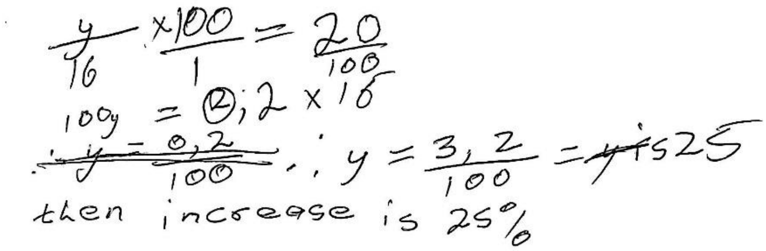
Neither Mandla nor Bheki scored more than one for the *increase* task in the written test. Mandla identified that the task was related to the difference between the two rates as he calculated this to be R4, but he did not express this as a percent.

In the written test, Bheki responded with the correct final answer, but the written workings were disjoint from the final answer. During the interview session, it became evident that the student could mentally compute basic tasks but had difficulty in communicating his thinking as a written response. I reviewed his written test again after the interview and found that he had, more than once, obtained the final correct answer despite erroneous written workings.

Figure 7.5 illustrates that he had written the correct final answer of 25% for the *increase* item, yet his workings show that he obtained this result from the expression $y = \frac{3.2}{100}$.

It is unlikely that he derived an answer of 25% from this expression. This suggests that he may have mentally computed the answer first and then attempted to write down a mathematically literate solution to support his answer.

Figure 7.5: Bheki's written test response to the increase task



7.4.2.2 Episode 2.1: Locating the base referent

The episode begins with a prompt from me for a comparison task. Recall from section 5.4.2.1 that students were given two images with possible solutions for this task and were asked to choose the most appropriate solution. Bheki first confirmed with me that it was "the increase" that had to be computed. He then rephrased this as "the amount that it [the rate] goes up by", in this way offering different *words* to describe the same phenomenon (line 7). In both cases, the phrases were about the object, percent, or more specifically the percent increase. This discursive move, according to Sfard (2008), is a possible signifier of objective word use. It should be noted, however, that the substantiation for his choice in routine was based on an empirical argument classifying it as colloquial.

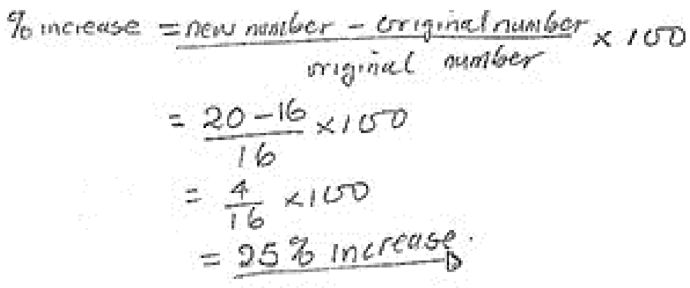
Table 7.10: Episode 2.1

Line		**What was said**	**What was seen/done**
7	B	So, in this case we are looking for the new increase. How much did it go up by?	
8	I	Yes	
9	B	Hmm	
10	M		Writing
11	B	OK. Eish.	
12	M		M is writing

Line		What was said	What was seen/done
13	I	What are your thoughts?	
14	M	I'm think the first one is correct.	
15	I	OK. And for you?	Addressing B. M continues writing
21	B	This one makes sense.	Pointing at solution A
22	I	Makes sense to you?	
23	B	Hmm	
24	I	How have you done it there? Oh, you did the same …	Referring to the written work of M as shown in Figure 7.6
25	M	Hmm … ja	
26	B	It makes sense.	
27	I	Ja	
28	B	The reason being that… Can I elaborate?	
29	I	Yes. Please. Please do.	
30	B	Hmm … before this 20 earned per hour there was actually the one that was already there. Which is 16.	
31	I	Uh ha	
32	B	And then from it … to understand how much it is we have to subtract the new offer… Ja … ja … we have to subtract the old one from the new offer. And then from there the difference will be divided by what was already there.	
33	B	And then from there … hmm … what we are going to get is a quarter multiplied by 100 that's what we are looking for.	Taps on the paper

Mandla wrote a solution to the task as illustrated in Figure 7.6. Mandla's written work was an imitation of the routine that he has been presented within the task. He incorporated the descriptors "new number" and "original number" to represent the symbols 20 and 16 respectively. His process was mediated through written *symbolic* forms and was *syntactic* in nature. I interpret this as testing the applicability conditions for the routine by following a "compute and check" process. Thereafter, he endorsed the routine in option A as the applicable routine (line 14).

Figure 7.6: Mandla: The Increase Task



Bheki agreed with Mandla's endorsement of the routine (line 21). Bheki substantiated the endorsed routine in lines 30 to 33 offering a predominantly procedural sense of *word use*. Bheki described the old rate (base referent) as "the one that was already there" (noun) (line 30). His talk in line 32 featured procedural words such as "subtract" and “divided”. The utterance “quarter multiplied by 100” suggests that the symbol $\frac{4}{16}$ was replaced with $\frac{1}{4}$ or a quarter (line 33) further eluding to a process of symbolic objectification. The reference to “quarter”, however, suggests that Bheki’s discourse of percent may be limited to fraction[21] as a unit fraction (*PD0*).

While speaking, Bheki gestured to the two rates in the task and in the routine by underlining them but he did not write anything more. His communication was strictly oral with colloquial phrases such as “the one that was already there” and “what was already there” (lines 30 and 32). Despite his inclination towards colloquial discourse, he had a tendency to refer to the objects of the discourse as entities in themselves. For example the phrase “the one that was already there” was a reference to a noun. He also used the word “difference” in line 32 as the objectification of the operation of subtraction.

7.4.2.3 Episode 2.2: Determining the applicability conditions

In this episode, Mandla questioned the applicability conditions of the increase routine of Episode 2.1 (see Table 7.11). I selected this episode to illustrate Mandla’s process for substantiating his choice in Episode 2.1.

Table 7.11: Episode 2.2

Line		**What was said**	**What was done**
1	M	[Quietly mumbling to himself]	
2	M	But I am also thinking … with this one. I was … OK. The only difference is that we divided by the new offer instead of by the old offer.	
3	I	Hmm	
4	M	But it is also part of an increase. OK. So, I was looking at something in the morning before I came here.	
5	I	Hmm	
6	M	This one. Then when they divide by the new offer I am not so sure I was confusing it with the negative increase. Whether it is You can say … If you don’t want to say 16 minus 20. You don’t want to say old offer minus new offer divided by the new offer. You can still do it this way, but it will give you more like a decrease.	Pointing at image B on paper
7	I	A decrease. Going backwards. Yes. OK.	
8	B	But it’s this one here.	Pointing at image A on paper

At this point, I had not endorsed either of the routines. Mandla revisited options A and B, scanning them for differences in lines 38 to 42. I infer that the purpose of this discursive move was to verify the applicability conditions (*when*) of the percent increase

[21] There is further evidence of the fraction perspective of percent in the next task.

routine. His utterances were largely personal and procedural such as “we divided by” and “you don’t want to say 16 minus 20”. From this, it can be inferred that his substantiation for using option B in a percent decrease task appears to be a preference for working with positive numbers.

7.4.3 The more than task

This task[22] presented students with a relationship between two unknown quantities Jacob’s salary (J) and Ramona’s salary (R). The ratio of J: R was given as 25% *more* and students were asked to consider the ratio of R: J.

In the written test, both Bheki and Mandla answered 25% without showing any workings.

7.4.3.1 Episode 2.3: The specific versus the general approach

In the absence of underlying quantities for the relationship between Jacob and Ramona’s salaries, students could either have expressed the task algebraically (generalised) or assigned specific values to either of the referents[15]. Bheki began by allocating the larger referent (Jacob) a value of 400 (line 104). Instead of deriving the smaller referent quantity he assigned a specific value of 100 to it having mentally determined 25% of 400 to be 100 (line 106). This choice of quantities suggests that Bheki’s discourse of percent was at the *PD0* level of *fraction.*

Table 7.12: Episode 2.3

Line		What was said	What was done
102	B	I think what we can do is that we can use an example like … say that … using numbers. Playing around with numbers.	
103	M	Hmm	
104	B	You can say, let’s make it up where Jacob’s salary say it is 400, right?	
105	M	Hmmm	
106	B	And then we let Ramona’s be 100. It’s going to be much easier when you use that. The reason being that … remember when you do that …. When you are going to calculate its gonna ...	
111	M	Ja. OK. That is fine. That’s fine.	
112	B	Unless you want to use 4000 and 1000.	
113	M	I’m thinking of x, y x, y. But it is still fine 4000 and 1000.	
114	B	So, can we use … uh … functions as well?	Laughs
123	M	[Writing: Refer to Figure 7.7]	Written: Let Jacob’s earn be $x + 25\%$ Ramona = $x - 25\%$

[22] Please refer to section 5.4.2.2 for a detailed description of the *more than* task.

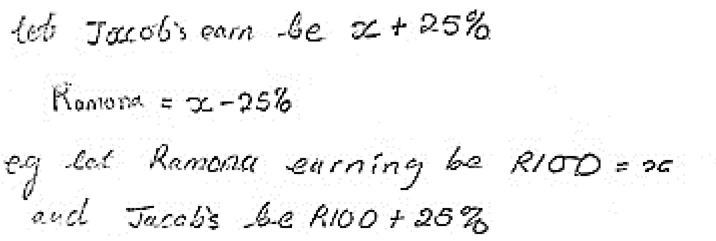
Sfard (2008) defines *deeds* as routines that result in a change in an object, whereas an exploration results in a new narrative. Bheki's discursive actions did not show evidence of the construction of a new narrative. A narrative for this task would consider the relative relationship between the two referents. Instead, he appeared to simply know what combination of numbers would give him the relationship that he required in the same way that he might know what combination of coins would give him 80 cents. His focus appeared to be exclusively on the combination of numbers (as concrete objects) as opposed to the mathematical object of "percent more than". This is further supported by his utterance "So, can we use … uh … functions as well?" in line 114. He laughed at the notion of using "functions" and continued to work with the numbers that he had chosen.

Given the context of the discussion, I have assumed that he used the word "functions" to refer to algebraic expressions. His hesitation in the utterance could suggest that he was searching for the appropriate terminology for an algebraic expression. I infer that Bheki's routines were deeds, despite the appearance of being explorative numerical calculations. Bheki had chosen the two numbers 100 and 400, knowing that the combination of numbers could be manipulated to obtain the desired number (object) of 25%.

Mandla disagreed with the approach endorsed by Bheki preferring to work with variables (line 113). He continued to work without speaking from this point on, writing, erasing and rewriting down his thoughts. He later revealed his written work[23] where he had assigned Jacob's salary the expression $x + 25\%$ (line 177 of Episode 2.4 and Figure 7.7). This discursive move suggests an erroneous additive procedure, however, there is evidence later in line 177[b] of a correction to this procedure. Mandla's routines were exploratory, showing signs of attempted construction of new narratives through the use of algebraic expressions.

[23] Ramona's earnings are expressed as $x - 25\%$ but this does not appear to hinder the student's discourse.

Figure 7.7: Episode 2.3 Mandla's written work



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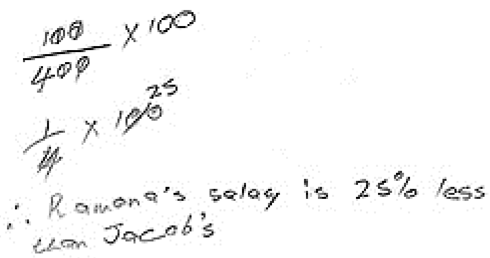
7.4.3.2 Episode 2.4: Finding an expression for "more than"

Having noticed the fractional approach that Bheki had taken (see Figure 7.8), I prompted him to explain his choice of 100 and 400 for the two variables (line 131).

Figure 7.8: Episode 2.4 - Bheki's writing



While explaining his decision, Bheki referred to "the guy behind him … below that". Despite the use of adjectives in this utterance, I infer that Bheki's discourse was mediated through concrete "mind's eye" images of people and have categorised this as concrete imagined (line 136) (Sfard, 2008).

Table 7.13: Episode 2.4

Line		**What was said**	**What was done**
131	I	I see that you have chosen 400 for Jacob's salary.	
132	B	Yes	
133	I	Why is that your choice for Jacob's salary?	
134	B	I … uh … The reason being … that … uh … as I said … it will be much easier to be … not … Let me put it in this manner. The reason why Jacob's 400 I thought that if maybe Jacob's salary can be whereby everybody starts here … for example.	
135	I	Hmm	
136	B	Let me put it in this way. Let's say that Jacob is the big boss within this company and he earns that. And then with Jacob as the guy behind him. Uh … below that.	
137	I	Hmm	
138	B	with … uh … 25 percent whereby … in this case it can be 100 rand	
[…]			
175	M	Me. I said let Ramona's earning be 100.	
176	I	Uh ha	

Line			What was said	What was done
177	M	a.	And because Jacob we are saying he is earning 25 percent more … then I said it is 100 plus 25 percent.	Written: Jacob: 100 + 25%
		b.	Somehow, I am stuck. But I am getting to understand that if he is earning more than her then it's the 100 plus 25 percent of this one (pointing at 100).	Points at 100 ("this one")

Having initially chosen to generalise the task, Mandla reverted to assigning a specific quantity to the one variable. Ramona's salary, represented as x, was assigned a value of R100. Jacob's salary was expressed as $R100 + 25\%$ in his written work and in utterance 177[a], continuing with the additive construct. In line 177[b] his utterances shifted from additive to "pseudo-multiplicative"[24] as he came to the realisation that it was not 25% that was added. Instead, it was "25% of this one" where "this one" referred to Ramona's salary. His word use was structural in nature showing signs of objectification through the replacement of symbols with other expressions or symbols. With the shift from an additive process to a "pseudo-multiplicative"[24] process, Mandla's percent discourse showed signs of traversing from level *PD1* to *PD2*.

7.4.4 The double discount task

This task[25] differs from the *increase* task in three aspects: it is a decrease task; two decreases are applied, and no referent quantities have been provided.

In the absence of a referent quantity (jelly bean price) for the task, students could either solve the task algebraically or assign an arbitrary value to the unknown quantity. The discursive moves of students within the context of this task provided evidence of the structure of percent that students had constructed.

Both Bheki and Mandla responded with an answer of 20% + 5% = 25%. in the written test.

7.4.4.1 Episode 2.5: Clarifying the task

Mandla began by clarifying the terms for the discounts. He first interpreted that the extra discount of 5% applied to products other than jelly beans, meaning a 20% discount on jelly beans and 5% discount on anything else. To illustrate his interpretation of the

[24] Student responses are regarded as "pseudo-multiplicative" where the relationship $B = A(1 \pm p\%)$ is expressed as $B = A \pm Ap\%$. I refer to Mandla's expression of $100 + 100 \times 25\%$ as "pseudo-multiplicative" as he does not express this as the fully-fledged multiplicative expression $100(1 + 25\%)$.

[25] Please refer to section 5.4.2.3 for more detail on the *double discount* task.

task, he manipulated juice boxes and packets of crisps that were on the desk. In his demonstration juice boxes represented the jelly beans that were discounted by 20%, while crisps represented any other item besides jelly beans that were discounted by 5%. In this way, he was mediating his discourse through concrete objects (line 252).

Having clarified the mechanism for the two discounts he switched to a syntactic oral mode by assigning a value of 100 to the price of the jelly beans (line 269). He continued to talk through his process of calculating the net discount with the aid of his calculator (lines 269 to 287). He first deducted 20% from 100 to get 80. In line 280 he exclaims "Oh … then they take 5 percent off the 80" showing potential of realising the structure of the problem. Despite this, his utterance of "you minus 4 then it becomes 76" in line 284 is still predominantly procedural. During this time, he had not written anything down. This is evidence of a shift in his mediation from previous tasks where he first wrote and then verbalised his thinking.

Table 7.14: Episode 2.5

Line		**What was said**	**What was done**
251	M	Because … OK … cos they are saying only for jelly beans is 20 percent. But if ever I think the second sentence is saying … if ever you are buying anything else for all of them I think the total of all the things that you will buy. The discounts for that one is 5 percent. But however for jelly beans if ever they add there jelly beans in it than it is 20.	
252	M	Let's say you are buying all of these other things now.	Pointing at a juice boxes and chips on the desk.
253	I	Ja?	
254	M	The total discount for all of them is going to be 5 percent.	
255	I	Oh I see. OK. No, that's not actually … not actually what the question is saying. It's saying … OK … it's actually saying you are only buying jelly beans.	
256	I	OK? So you are only buying this. So you are only buying jelly beans.	Illustrates with juice boxes only
269	M	OK. I am trying to think of the …. Let's say the price is 100 rand.	
270	I	OK. 100 is a good one.	
271	M	They take 25? OK … they take 20?	
272	I	20. Yes.	
273	M	They take 20 percent off.	
274	B	It's gonna be … it's not going to be 80 bucks?	
278	M	Then it's going to be 80.	
279	I	Mm-hm.	
280	M	And then again they …. Oh … then they take the 5 percent off the 80?	All worked out on calculator. No writing yet.
281	I	Mm-hm.	
282	B	5 percent of 80 you just minus 2. Is it 2? No … 4. You minus 4.	
283	I	Hmmm	

Line		What was said	What was done
284	M	…..you minus 4 then it becomes 76	
285	I	OK. So, you paid 76 rand now for this that was 100 rand. So what was your discount actually?	
286	B	Hm?	
287	M	OK. 24.	
288	I	24	
297	M	Cos also … beside having the confusion that I was having … I was just gonna add 20 and 5 and use 25 percent.	

Bheki's participation was limited to calling out the answers to Mandla's computational steps (lines 247, 276, 282). Very little can be inferred from the fragments of Bheki's discourse, but there is evidence of an additive approach in line 282 with the phrase "5 percent of 80 you just minus 2". The word use in this utterance is predominantly procedural and personalised. Bheki's discursive processes continue to appear to be deeds with an explicit focus on the new numbers that he computes at each step.

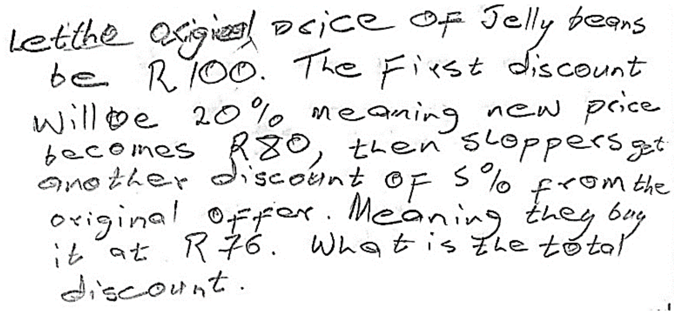
7.4.4.2 Episode 2.6: Constructing new narratives

Having solved the task verbally, Mandla proceeded to write out his thinking on paper, this time replacing the quantity of 100 with 200 (line 348). He concluded that "whatever the price [base referent] is, it does not affect what the discount [percent] is", showing early signs of a ratio view of percent (line 366). Mandla's consideration of a range of values from 2000 to 1 for which this relationship would be true is indicative of symbolic objectification (lines 368 to 379). His discourse of percent began to take shape as a comparative ratio.

Bheki showed signs of commognitive conflict with Mandla's discourse in his reference to using 100 as the "starting point" (line 369). His utterances in lines 371 of "or maybe 200?" and line 377 "even if you use 1?" provide further evidence of commognitive conflict as he appeared to be unconvinced by Mandla's generalisation.

Bheki continued to write out a detailed problem statement as illustrated in Figure 7.9.

Figure 7.9: Episode 2.6 Bheki's problem statement


Let the original price of Jelly beans be R100. The First discount will be 20% meaning new price becomes R80, then shoppers get another discount of 5% from the original offer. Meaning they buy it at R76. What is the total discount.

Although Bheki's discourse had shifted from oral to written, the statement was written as "it would be spoken". He attempted to express the relationship mathematically by using the algorithm from the *increase* task. He first wrote $\frac{(100-80)-76}{100} \times 100$. Only after prompting from me, did he erase the expression and replace it with $\frac{(100-76)}{100} \times 100$. From this, I infer that for Bheki the discourse of percent was not a "discourse for oneself" (Sfard, 2007, p. 607).

Table 7.15: Episode 2.6

Line		**What was said**	**What was done**
348	M	I tried a different number here.	Student uses 200 instead of 100 to calculate.
349	I	I see that. And what did you find?	
350	M	It's 24 still.	Continues writing
353	I	I like that you tried a different number	
354	M	Ja. I actually tried to increase it to 200 to see if …	
355	I	To see if it still works?	
356	B	[Laughs]	Laughing
357	M	Sometimes 100 becomes a problem statement.	
358	I	Yes	
359	I	So it is showing you that it actually doesn't matter what the price is, you get the same discount?	
360	M	It doesn't matter. Ja.	Mumbled.
361	B		Continues writing.
362	I	Why do you think that is?	
363	M	That the answer is the same?	
364	I	Yes. Why? How come the answer is the same?	
365	B		Still writing
366	M	Cos I don't think it. Whichever price it is it does not affect what the discount is.	
367	I	Hm.	
368	M	Maybe when they talk of 2000, it will still … have the same discount?	
369	B	So, the starting point will have to be 100?	Taps pencil hard on desk.
370	M	It …	Mumbles uncertainly
371	B	In this case … or maybe 200?	
372	I	Yes. So …	
373	M	It doesn't matter what number you use.	

Line		What was said	What was done
374	I	Yes.	
375	B	OK …	Seems uncertain.
376	M	Even if you use 1. It will work.	
377	B	Even if you use 1?	Inaudible
378	I	Yes.	
379	M	Even if you use 1.	

The switch to a written mode and the use of the algorithm from the *increase* task may have been an attempt at ritual imitation for social acceptance within the interview group (Sfard, 2008). However, Bheki did not show evidence of objectification of the symbolic mediators. Instead, he relied heavily on his ability to mentally compute steps in the task. His propensity for erring was quite high with little evidence of reflection or retracing of steps independently.

7.4.5 The jacket task

Unlike the *more than* and *double discount* tasks, the referent quantity (the jacket price) had been provided in this task[26]. Students could approach this task as a computational task. However, the intention was for the students to reflect on the fact that the result is the same regardless of the order in which VAT and the discount are applied.

In the written test, Bheki's response consisted of two expressions written as $560 \rightarrow x$ and $25 \rightarrow \_\_$. He did not provide a final answer and was allocated a score of zero. Mandla determined the discount amount before VAT to be R140. His final response was "Thus the customer loses 15% of 140. ∴ After adding VAT in the discount the customer did not lose money." The two contradictory responses were not supported by workings, so he was allocated a score of two.

7.4.5.1 Episode 2.7: Unpacking the equivalence

This episode begins with a verbal description from Mandla on how to approach the *jacket* task. Bheki recalled an example of how he had previously explained "percentage". The utterance was colloquial and supported by the concrete mediation of the juice boxes that were on the table in the interview room. Bheki's explanation of percent was fractional in nature as evident in the phrase "How many did he get out of that 4" (line 495). He then attempted to approach the task through abstraction by

[26] Please refer to section 5.4.2.4 for more detail on the *jacket* task.

assigning the symbol x to the percentage (line 501). In previous episodes, Bheki had resorted to working with specific quantities when abstractions would have been more appropriate (See Episodes 2.3 and 2.6). This utterance is evidence of an attempt at a new routine, however, the applicability conditions for generalisation do not match the task at hand because a referent quantity of 560 had already been provided for the jacket price.

Table 7.16: Episode 2.7

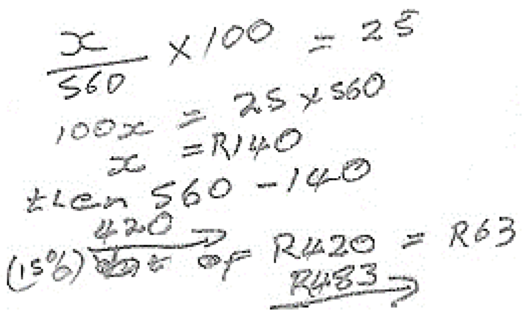
Line		What was said	What was done
489	M	OK. So, it's like we are going to calculate the total price of the jacket including the VAT.	
490	I	Hmm	
491	M	And then we discount … we include the discount.	
495	B	As I was telling somebody that working with percentages you must also understand it if they give you a certain number and a percentage of this. This number is the total (the 4 juices). And then .. we wanna know if this person got 25 percent. How many did he get out of that 4?	Illustrates using 4 juice boxes on the desk
501	B	Maybe you can say is we are working with functions. We can say that the value of that 25 percent be x and then you have x over 560 times 100 which is comes to 25. Or … am I … ?	
502	I	Um	
503	M	I'm thinking plainly we can just calculate. Find the VAT of this one. VAT is 15, right? 15 percent?	
504	I	Hmm	
505	M	So you can just find 15 percent of this one (pointing at 560). Then we add it onto this one (pointing at 560)	
506	B	Uh	
507	M	Then we apply the discount. Oh!	
508	B		Mumbles
509	M	We can also try to … before the discount. Sorry … before we add the VAT we find the discount of this one and then we add VAT. Just to compare.	
510	I	Hmm	
511	M	But then we will answer this one. To see is it less or what?	
512	B	So we do what? We gonna …	
513	M	Let's find VAT and then add it … or the discount. And then we have the discount before the VAT.	
528	B	Hmm … what is half of 56?	
529	M	Hmm …. I am just going to do the calculation.	

Mandla repeated the process that he intended to follow. His utterances from line 489 to 507 were initially predominantly structural with the words "include", "15 percent of this one". After some time, it becomes evident that Bheki was not performing the mathematical task according to the same rules as Mandla. Mandla's discourse began to take on a more procedural nature in lines 509 to 513. I infer that he was adapting his discourse to a level at which Bheki could participate. Sfard refers to this phenomenon as the "mutual adjustment of their discursive ways" (Sfard, 2008, p. 145). In the case where one discourse is privileged over another, interlocutors will begin to adjust their discourse to align with the more privileged discourse. The opposite appears to be true in the case of Mandla and Bheki since Mandla has taken a discursive step backwards to resolve the conflicting discourses.

Bheki spent approximately ten minutes attempting to formulate the task with pen and paper. He showed no workings for his VAT calculations. However, in Figure 7.10 he has expressed the discount of 25% as $\frac{x}{560} \times 100$ which illustrates a part-whole view of percent.

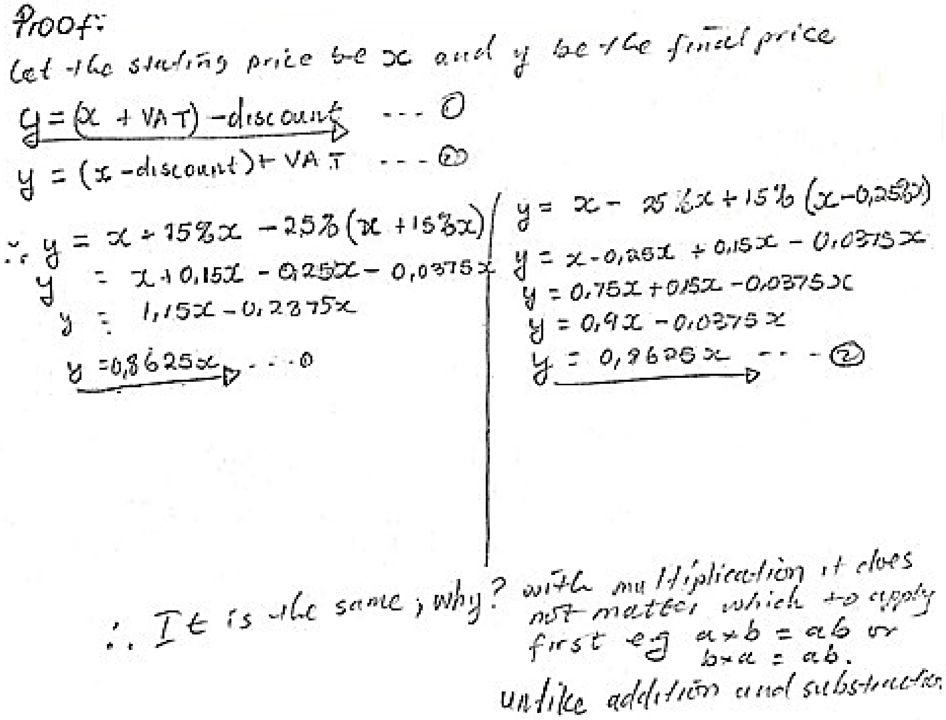
His work regularly featured errors and he did not show evidence of retracing his steps to correct his work. I assisted him on a few occasions to work his way through the task.

Figure 7.10: Bheki's discount



Mandla attempted to prove that the relationship was independent of the price of the jacket (Figure 7.11). Although the initial expression of $y - (x + VAT) - discount$ appears to be additive in nature, he switches to a "pseudo-multiplicative" [24] approach with the expression $y = x + 15\%x - 25\%(x + 15\%x)$. The fully-fledged multiplicative expression should have been $y = x(1 + 15\%)(1 - 25\%)$.

Figure 7.11: Episode 2.7 Mandla's proof



At the end of his proof in Figure 7.11. Mandla had written a note qualifying that the equivalence was due to the commutative property of multiplication since $a \times b = ab$ and $b \times a = ab$. Although he did not formulate a truly multiplicative expression, his workings show early signs of traversing the PD2 to PD3 level of percent discourse.

7.4.6 Summary of Mandla and Bheki's discourse of percent

The interview with Bheki and Mandla was rich with talk and the transcript provided many opportunities for me to analyse the discourse. Bheki and Mandla had similar scores in the written test: 27.8% and 26.7% respectively. But this is where their similarities end. Bheki's score of 55.6% for non-contextual tasks was more than double his 25.9% score for contextual tasks. Whereas Mandla's scores were not much different. Mandla's consistently poor performance on tasks in the written test suggests that he had most likely forgotten school-taught procedures for percent tasks. I posit that this counted in his favour in the interview session as he did not first have to undo any buggy learnt-procedures before considering percent as a comparative ratio. This may explain his unhindered, early adoption and reliance on the routine established in the *increase* task. Mandla's discourse profile has been summarised in Table 7.17.

Table 7.17: Mandla's discourse profile

Mandla's discourse of percent			*Tasks*			
			Increase	More than	Double discount	Jacket
Word Use			Procedural and personal	Procedural and personal	Procedural and personal Some objective word use	Both structural and procedural
Mediational mode	Symbolic	Syntactic	Predominantly written, oral occasionally	Predominantly written, oral occasionally	Oral first and then written	Oral first and then written
		Objectified	Renames objects in the task.	Some evidence of replacing expressions with symbols	Provides a range of alternate values for which the ratio will hold	Attempts to prove the relationship by generalising the task.
	Concrete				Manipulates juice boxes	
Routines	Type	Ritual or Explorative	Ritual: imitation	Ritual: imitation Explorative	Explorative	Explorative
	Flexibility	Routines	Spontaneously adopts a new routine	Spontaneously switches between routines	Change in routine	Routine switching observed
		Mode switching	Written - Oral	Written – Oral Written - Objectified	Concrete - Oral Oral – Written	Oral – Objective Objective – Written Written – Objective
	Corrigibility	Propensity for erring	Works cautiously	Works cautiously	Low	Low
		Retracing	Regularly observed	Regularly observed	Regularly observed	Regularly observed
		Successful correcting	Spontaneous	Spontaneous	Spontaneous	Spontaneous
Endorsed narratives	Derivation		Reconstructs the given routine using his own descriptors	Attempts to generalise the relationship	Not observed	Attempts to generalise the relationship
	Substantiation		Compute and check	The Method	Literate	Literate
	Memorisation / Recall		Observed	Observed	Not observed	Not observed
Development of percent discourse	Nature		Not clear		Early signs of comparative ratio	Early signs of comparative ratio
	Structure		Not clear	Additive with some pseudo-multiplicative[24]	Additive with some pseudo-multiplicative	Additive with some pseudo-multiplicative
	Level		Not clear	PD1 – PD2	PD2	PD2/PD3

Word use and mediational mode: Mandla, at first, worked cautiously and limited his verbal discourse preferring to think through the tasks before speaking. Mandla's discourse profile in Table 7.17 illustrates a change in his discourse when viewed from left to right. There is a distinct switch in his mediational mode from syntactic written to syntactic oral in the second half of the interview.

Bheki's word use remained procedural and personal throughout the interview. He displayed heavy reliance on mental computations and resisted writing down his solutions on paper. His view of percent as fraction was unyielding and evident in both his syntactic oral discourse and in his written work. Bheki's discourse profile has been summarised in Table 7.18.

Table 7.18: Bheki's discourse profile

Bheki's discourse of percent			*Tasks*			
			Increase	**More than**	**Double discount**	**Jacket**
Word Use			Predominantly procedural and personal	Procedural	Procedural and personal	Procedural and personal
Mediational mode	Symbolic	Syntactic	Oral only	Written and oral	Oral. Written attempt unsuccessful	Oral. Written attempt with prompting
		Objectified	One instance of referring to 'a quarter' to replace 25%.	Not observed	Not observed	Not observed
	Concrete			Imagined		Manipulates juice boxes
	Gestures		Points at or underlines the text			
Routines	Type	Ritual or Explorative	Explorative	Deeds Ritual: imitation	Deeds Ritual: imitation	Ritual
	Flexibility	Routines	Not observed	Attempted switching is unsuccessful	Attempted switching is unsuccessful	Attempted new routine
		Mode switching	Syntactic oral – Objectified Syntactic - Gestures	Syntactic oral – written	Attempted switching is unsuccessful	Attempted switching between objective and symbolic unsuccessful
	Corrigibility	Propensity for erring	Not observed	High	High	Very high
		Retracing	Not observed	Required prompting	Required prompting	Required prompting
		Successful correcting	Not observed	Required prompting	Required prompting	Required prompting
Endorsed narratives	Derivation		Not observed	Not observed	Not observed	Not observed
	Substantiation		Colloquial – empirical argument	Colloquial – empirical argument	None	Extra-discursive
	Memorisation / Recall		Not observed	Not observed	Not observed	Not observed
Development of percent discourse	Nature		Unit fraction	Fraction	Fraction	Part-whole Fraction
	Structure		Additive	Additive	Additive	Additive
	Level		PD0	PD0	PD0	PD1

Routines: Bheki's earlier routines, despite the appearance of being explorative, were classified as deeds. Instead of establishing a relative relationship between the referents in the tasks, he appeared to simply know what combination of numbers would give him the relationship that he required in the same way that he might know what combination of coins would give him 80 cents. He later showed signs of shifting into ritual routines, but his attempts at imitation showed a high propensity for error and were seldom successful. At times when the task could have been solved through abstraction, he chose to use specific values. In the *jacket* task, when specific quantities were given he attempted to use abstraction.

Mandla's routines in the first half of the interview were predominantly rituals as he mimicked the routine from the *increase* task. With the distinct switch in mediational modes, Mandla's routines became explorative in nature. He experimented with generalising the relationships that emerged from the tasks. Although his proof was not a fully-fledged generalisation of the relationship, it indicated that he had come to realise that the equivalence of the two scenarios in the *jacket* task was due to the commutative property of multiplication. According to Sfard (2008), this change in discourse signifies that development occurred as a consequence of the interaction in the interview.

PD-levels: Bheki maintained an additive approach to percent tasks despite the lengthy discussions of the multiplicative nature of percent. I posit that his deeply rooted perspective of percent-as-fraction prevented him from accessing percent discourse levels beyond *PD1*. Mandla showed evidence of a change in discourse from *PD1* in the early stages of the interview to potentially traversing the gap between *PD2* and *PD3*.

7.5 Conclusion

In this chapter I analysed the discourse of two pairs of students as they worked their way through four real-world percent tasks. The analytical tools developed by Sfard and her colleagues, paint a picture of the characteristics of the students' discourse and illustrate the aspects that potentially hinder the students from accessing a fully-fledged discourse of percent, that is: percent as a comparative ratio.

For Nisha, Johan and Bheki, it is evident that their discourse was narrow and deeply rooted in a percent-as-fraction perspective. The students' discourse was predominantly additive in nature and did not show signs of recognising the underlying multiplicative structures of percent tasks. This restricted view of percent impeded their development of percent discourse beyond the *PD1* level.

Mandla's performance in the written test was consistently poor across all contexts. I infer that he may have forgotten school taught procedures for percent tasks. There was evidence to suggest that Mandla's discourse changed through the course of the interview from *PD1* to *PD2/PD3*. Despite this, Mandla's discourse did not show evidence of a fully-fledged objectification of percent as a comparative ratio.

CHAPTER 8 | CONCLUSION

8.1 Introduction

We know from earlier research that percent has for many years been a source of confusion for learners of all ages (Parker, 1994; Parker & Leinhardt, 1995). Students in this study were undergraduates at first-year university level and were on the secondary mathematics programme of the B.Ed. degree. Students would have received instruction in percent at school and, as young adults, would have had some exposure to percent in their everyday lives.

The research questions that guided the analysis in this study were:

- What areas of percent do students experience difficulty with?
- What features of student discourse contribute to this difficulty?

I have taken a two-pronged approach to address these research questions. Firstly, Parker's (1994) cognitive framework of percent was adopted for the design and analysis of written tests. The second approach employed a combination of Parker's (1994) cognitive framework of percent and applied Sfard's (2007) communicational framework to construct a model for the development of percent discourse (*PD-Model*) (Caspi & Sfard, 2012; Kim et al., 2012).

8.2 Conceptual framework

Parker's (1994) framework[27] identifies nine comparative contexts of percent categorised into two main contexts: *percent-as-fraction* and *percent-as-ratio*. Percent-as-fraction represents the part-whole model of percent. Percent-as-ratio describes either a *change* in a set or provides a *comparison* of two disjoint sets. Change is measured as either a *change to a % of* the original set or a *change by a % of* the original set. Each change sub-context considers both increase and decrease changes. There are two further sub-contexts under the comparison contexts: *compare A to B* and *A differs by % of B.* Each comparison sub-context includes comparisons of greater than and smaller than.

[27] Please refer to Figure 2.1.

Parker's framework equipped me with a tool for designing a balanced test instrument. Balanced in terms of the three dimensions of the test: the contexts of percent tasks, case types of percent and task types according to Parker's classification. The analysis of written tests identified the areas of difficulty that students experienced with percent.

The *PD-model* served as a tool for the analysis of the features of the discourse of students. The first two levels of the hierarchical model feature Parker's *percent-as-fraction* context. The *PD-model* begins at level 0 (*PD0*) with the *proto-percent* notion of fraction. This level of percent discourse features percent as an extensive quantity that is part-whole in nature. As level 1 (*PD1*) is traversed the nature of the discourse is still part-whole and bounded by an upper limit of 100, however the discourse becomes that of an abstract intensive quantity that represents the relationship between a set B and its subsets.

It is at level 2 (*PD2*) that the part-whole nature of the discourse is shed allowing for percent values that are greater than 100. At this level the discourse evolves to that of comparative ratio where it is possible to express the transformation of a set A as a function of the new set *A'* where $A' = A \times p\%$.

At level 3 (*PD3*) the discourse is fully objectified as the ratio between a change or a difference is measured against a base referent. The realisation of the multiplicative structure of percent is evidence of objectification. At this level, the relationship can be expressed as $\mathrm{B} = A(1 \pm p\%)$ or $\mathrm{A} = B(1 \pm p\%)$.

The *PD-Model* provides an explanation for the difficulties experienced in percent tasks based on the students' degree of objectification of percent discourse.

8.3 Findings

8.3.1 The written test

The students' test responses presented a restricted view of percent that was limited to a persistent notion of percent-as-fraction. The results of the written test showed that students were mostly well-versed in procedures for non-contextual percent tasks but were not able to apply this knowledge to real-world contextual tasks. Notably, students experienced difficulty in identifying the appropriate base referent for an item.

Performance was particularly poor in c*hange by % of* (ratio context B in Figure 2.1) and *A differs by % of B* (ratio context D in Figure 2.1) items. Students did not appear to notice the multiplicative structure of these tasks.

8.3.2 The interviews

The discursive moves of two pairs of students were analysed with respect to their *word use, mediational modes, routines* and *endorsed narratives*. Through the detailed analysis of the features of student percent discourse, it was possible to identify the *PD-level* of their discursive activity and explain the insufficiency in their objectification of percent as a comparative ratio.

Nisha and Johan, despite their proficiency in written non-contextual tasks, showed great difficulty in working with contextual tasks. Although they showed early signs in the interview of realising percent as a comparative ratio, their discourse persistently exhibited percent as an additive construct. Nisha's routines were predominantly substantiated extra-discursively suggesting that she had not made percent a "discourse for herself" (Sfard, 2007, p. 607). Johan displayed some flexibility in his routines although his substantiation was evoked through prompting by me and not from the discourse itself. The students' discourse did not show signs of recognising the underlying multiplicative structures of percent tasks. This restricted view of percent impeded their development of percent discourse beyond the *PD2* level.

Bheki and Mandla both had written test scores below the average for the group. Bheki showed more proficiency in non-contextual tasks than real-world tasks while Mandla's performance was equally poor regardless of context.

In the interview, Bheki displayed a deeply-rooted part-whole construct of percent. His discourse was severely restricted to percent-as-fraction. Despite numerous prompts from me, his discourse showed no indication of the realisation of a multiplicative structure of percent. I maintain that his discourse did not traverse beyond the *PD1* level due to the persistence of a part-whole construct.

Mandla's consistently poor performance in the written test could suggest that he had forgotten school-taught procedures for percent tasks. It is possible that this may have counted in his favour in the interview session as he did not have to first undo any learnt procedures before considering percent as a comparative ratio.

There was evidence to suggest that Mandla's discourse evolved during the interview. Although his discourse was distinctly additive to begin with, he started to independently experiment with the relationships between referents. By the end of the interview, he had attempted a proof to illustrate the commutative nature of the *jacket* task. Although his attempted proof did not display a fully-fledged multiplicative structure, his discourse showed evidence of the emergence of a "pseudo-multiplicative" construct. For this study, student responses were regarded as "pseudo-multiplicative" where the relationship $B = A(1 \pm p\%)$ was expressed as $B = A \ \pm Ap\%$.

The findings of this study present Bheki and Mandla as distinctly different with respect to percent discourse. Bheki, weighed down by his unrelenting percent-as-fraction construct, was unable to realise the comparative relationships between the referents of percent. Mandla, on the other hand, appeared to be free of the percent-as-fraction construct affording him unhindered access to percent as a comparative ratio. This did not benefit him in the written test, but his discourse in the interview showed evidence of development that could be directly attributed to the interactions between the interviewees and the interviewer.

8.4 What is implied by the findings?

All students interviewed in this study had existing knowledge of percent. From the discursive analysis, it is evident that this existing knowledge was based on restrictive school-taught procedures that were firmly grounded on a percent-as-fraction notion. This study was not about instruction, so I cannot make claims about what instructive practices might better support the development of percent discourse. However, I can make claims about the features of percent discourse that support the development of percent as a comparative ratio.

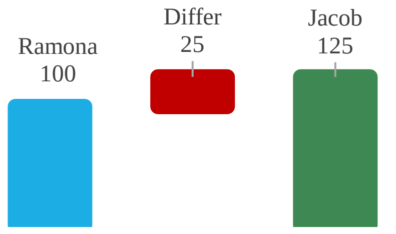
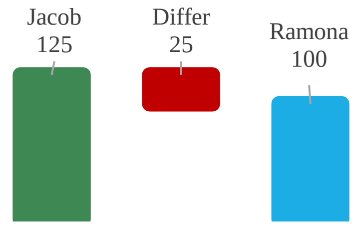
Instruction in percent has traditionally focussed on procedures and conversions between percent, decimal and fraction (Parker & Leinhardt, 1995). While this might be useful in establishing connections between different symbolic forms of percent, it does not bring the relative relationships between referents into focus. Appropriate routines that foreground the multiplicative structure of the object of percent need to be constructed and substantiated from within the discourse itself. This is in keeping with the practice of literate mathematical discourse in which the substantiation of mathematical theorems, definitions and axioms is based on the *intradiscursive* manipulation of existing narratives (Sfard, 2008).

The prevalence of additive word-use by the students further supports the claim that instruction should be framed according to the multiplicative structure of percent with careful attention paid to word use. This is particularly important given that the language of percent has additive phrases such as “increase by” or “more than” whereas the underlying relationships are multiplicative. This study does not analyse the impact of a change in word use, but reinterpreting phrases such as “more than” to signify “proportionally more than” or “increase by” to signify “growth” could possibly contribute to the development of multiplicative constructs of percent.

While percent-as-fraction is an important developmental pre-cursor to percent-as-ratio, the findings of this study suggest that instruction in percent as a comparative ratio should be foregrounded over percent-as-fraction. Furthermore, percent-as-ratio should be contrasted with percent-as-fraction to draw out the distinction between the two. Parker’s (1994) study illustrated the positive effect that visual representations had on students’ construction of percent as a comparative ratio. A shift in the use of visual mediators from the restrictive pie-chart to representations that elicit the relative relationships between referents could potentially serve this purpose well. To illustrate this, consider the *more than* task as an example.

This *more than* task presented students with a relationship between two unknown quantities, Jacob’s salary (J) and Ramona’s salary (R). The ratio of J: R was given as 25% *more* and students were asked to consider the ratio of R: J. This relationship can be demonstrated with Parker’s visual model in Figure 8.1. The left-hand side of the visual model illustrates the relative relationships between the two referents in a “more than” comparison and the right-hand side illustrates the relationships in a “less than” comparison.

Figure 8.1: Visual representation of the more than task

More than	**Less than**
	
$ratio = \frac{difference}{Ramona} = \frac{25}{100}$	$ratio = \frac{difference}{Jacob} = \frac{25}{125}$
Jacob earns 25% more than Ramona	Ramona earns 20% less than Jacob
$J = R(1 + 25\%)$	$R = J(1 - 20\%)$

The diagram brings into focus the base referent that changes between the two comparisons. For the “more than” comparison, the smaller amount is used as the base referent. For the “less than” comparison, the larger amount serves as the base referent. The diagram illustrates that although the difference between the two referents remains unchanged at 25, its relationship to the base referent in each scenario shifts.

8.5 Critical reflection

As is the case with all research, the findings of this study do not paint a complete picture of what was taking place. Firstly, I have deliberately chosen to focus explicitly on the object dimension of the discourse profiles thus ignoring the subject dimension of the participants’ implied identities. I have not analysed the interactions between the participants nor have I considered turn-taking in the episodes. This was by design and was a necessary omission given the time-frames of the study.

My choice of sample was limited to the availability of students on campus at the time of the study. The possibilities for expansion of this research abound. For example: a comparison between samples of first-year students from different faculties at the university could relate the study to students’ mathematical backgrounds or a comparison of samples of students at different levels of the B.Ed. the programme could measure the development of percent discourse over the course of the programme.

The poor performance on the written test suggested that the items may have been more difficult than I had anticipated. However, I maintain that the level of complexity in the written test could possibly have encouraged a greater variety of approaches to the tasks. Upon reflection, however, I would have eliminated item 14 from the written test[28]. The wording was ambiguous, and the item was challenging without the use of a calculator.

The tests and interviews were conducted on the university's campus. Although a number of tasks were modelled on real-world situations, Sfard (2008) warns that students may not necessarily respond as they would in their everyday lives when the context of the test is *institutionalised* by virtue of the location of the test and the presence of a researcher. For this reason, I cannot draw comparisons between the findings of the study and the students' approach to similar problems encountered in everyday life.

Transcribing and analysing the interview data presented me with many opportunities to reflect on my own discursive habits. There were instances in the first interview where I could have encouraged more talk or probed a little further for answers. At the same time, I feel that I provided too much scaffolding in other areas of this interview. For example: I repeatedly interjected in the discussion of the *more than* task following my suggestion to consider the relationship between the referents of the task as a ratio[29]. Reflecting on this, I felt that I should rather have allowed the discussion to emerge between the students. At least one additional interview with another pair of students could have been beneficial to the study to allow for my role as interviewer to mature.

I made many attempts at refining Parker's (1994) framework, however, in the end, I found that her model was superior to any modification that I had attempted. Parker's framework was unambiguous in its categorisation of percent tasks providing a mathematically sound foundation for the construction of the model for the development of percent discourse (*PD-Model).* I do not, however, claim that the proposed *PD-Model* provides a comprehensive representation of the discourse of percent. The model serves as an outline and remains subject to corroboration and modification. Furthermore, the

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[28] Refer to table Table 5.1 for detail on item 14.
[29] Refer to Episode 1.4 for further detail.

model is but a part of the bigger picture that links percent to the discourses of arithmetic, fraction, ratio and rational numbers.

This study could have been a purely qualitative study that focussed only on the discursive analysis. However, the quantitative analysis provided insight and depth to the study that would have taken much longer to achieve with a purely discursive analysis. The quantitative analysis ensured that the interviews were strictly focussed on areas of difficulty. Since the quantitative analysis was complete before the interviews began, it also provided some context for what to concentrate on in the interview. In contrast, had I pursued a purely quantitative analytical approach, the findings would have painted a deficit picture of student knowledge of percent.

The two-pronged approach to the analysis of data was time-consuming and challenging. However, in electing to parallel the quantitative analysis with the qualitative analysis, I have gained access to a more complete picture of the underlying features of percent discourse that influenced the student's objectification of percent discourse.

The motivation for this study was grounded in the pivotal role that percent has to play in financial literacy. It was not my intention to instruct students in the real-world application of percent. However, at the end of the interview sessions, several students remarked that they had a newfound awareness of the mechanics of percent and would be more mindful in their approach to everyday applications of percent. Thus, in addressing the question of the degree of objectification in students' percent discourse, the study also presented the students with an opportunity to become more knowledgeable consumers of financial data.

REFERENCES

Aliyu, A. A., Bello, M. U., Kasim, R., & Martin, D. (2014). Positivist and non-positivist paradigm in social science research: Conflicting paradigms or perfect partners? *Journal of Management and Sustainability*, *4*(3), 79–95.

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Sfard, A. (2005). Doing wrong with words, what bars students' access to arithmetical discourses. *Journal for Research in Mathematics Education*, *36*(3), 176–247. https://doi.org/10.2307/30034835

Bohlmann, C. A., Prince, R. N., & Deacon, A. (2017). Mathematical errors made by high performing candidates writing the National Benchmark Tests. *Pythagoras*, *38*(1), 1–10. https://doi.org/10.4102/pythagoras.v38i1.292

Caspi, S., & Sfard, A. (2012). Spontaneous meta-arithmetic as a first step toward school algebra. *International Journal of Educational Research*, *51–52*, 45–65. https://doi.org/10.1016/j.ijer.2011.12.006

Chesky, N. (2006). Beyond epistemology and axiology: Locating an emerging philosophy of mathematics education. *Analytical Teaching and Philosophical Praxis*, *34*(1), 16–25.

Cincinatus, R. B., & Sheffet, M. (2016). "With percentages the 100 is always in the denominator": From the field to pre-service teachers. *International Journal of Research in Education and Science*, *2*(1), 143–155. Retrieved from http://ezproxy2.utwente.nl/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=EJ1105176&site=ehost-live

Daniels, H. (2009). An introduction to Vygotskian theory. In H. Daniels (Ed.), *Vygotsky and research* (pp. 1–28). London: Routledge.

Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In L. P. Steffe, P. Nesher, P. Cobb, B. Sriraman, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 335–350). Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Frith, V., & Prince, R. (2018). The National Benchmark Quantitative Literacy Test for Applicants to South African Higher Education. *Numeracy*, *11*(2).

https://doi.org/10.5038/1936-4660.11.2.3

Ginsburg, L. (1995). What does “100% juice” mean? Exploring adult learners’ informal knowledge of percent., (November 2016).

Karpov, Y. V. (2003). Vygotsky’s doctrine of scientific concepts. In A. Kozulin, B. Gindis, V. Ageyev, & S. Miller (Eds.), *Vygotsky’s Educational Theory in Cultural Context* (pp. 65–82). New York: Routledge.

Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students’ discourses on infinity. *International Journal of Educational Research, 51–52*, 86–108. https://doi.org/10.1016/j.ijer.2012.01.004

Lembke, L. O., & Reys, B. J. (1994). The Development of, and interaction between, intuitive and school-taught ideas about percent. *Journal for Research in Mathematics Education*, *25*(3), 237–259. https://doi.org/10.2307/749337

Maxwell, J. (2013). *Qualitative research design: An interactive approach*. https://doi.org/https://doi.org/10.4324/9780203431917

Mikami, Y. (1913). *The development of mathematics in China and Japan*. New York: Chelsea Publishing Company.

Newton, J. A. (2012). Investigating the mathematical equivalence of written and enacted middle school standards-based curricula: Focus on rational numbers. *International Journal of Educational Research*, *51–52*, 66–85. https://doi.org/10.1016/j.ijer.2012.01.001

Parker, M. (1994). *Instruction in percent: Moving prospective teachers under procedures and beyond conversions*. *ProQuest Dissertations and Theses*. https://doi.org/10.16953/deusbed.74839

Parker, M. (1997). The ups and downs of percent (and some interesting connections). *School Science and Mathematics*, *97,* 406–412.

Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, *65*(4), 421–481.

https://doi.org/10.3102/00346543065004421

Perez, S. (2014). Thinking aloud together : A teacher's semiotic mediation of a whole-class conversation about percents. *Educational Studies in Mathematics*, *73*(1), 21–53. https://doi.org/10.1007/sl0649-009-9203-3

Prince, R., & Frith, V. (2017). The quantitative literacy of South African school-leavers who qualify for higher education. *Pythagoras*, *38*(1), 1–14. https://doi.org/10.4102/pythagoras.v38i1.355

Robertson, S. A., & Graven, M. (2018). Exploratory mathematics talk in a second language: a sociolinguistic perspective. *Educational Studies in Mathematics*, 215–232. https://doi.org/10.1007/s10649-018-9840-5

Schuh, K. L., & Barab, S. A. (2008). 7 Philosophical perspectives. In J. M. Spector, M. . Merrill, J. van Merriënboer, & M. . Driscoll (Eds.), *Handbook of Research on Educational Communications and Technology* (Third, pp. 67–82). Routledge. https://doi.org/10.2307/2184941

Setati, M. (2008). Access to mathematics versus access to the language of power: the struggle in multilingual mathematics classrooms. *South African Journal of Education*, *28*(1), 103–116.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Research*, *27*(2), 4–13. Retrieved from http://www.jstor.org/stable/1176193

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, *16*(4), 565–613. https://doi.org/10.1080/10508400701525253

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing* (Kindle). New York: Cambridge University Press.

Sfard, A. (2012). Introduction: Developing mathematical discourse - Some insights from communicational research. *International Journal of Educational Research*, *51–52*, 1–9. https://doi.org/10.1016/j.ijer.2011.12.013

Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics*, *26*(2–3), 191–228. https://doi.org/10.1007/BF01273663

Tabach, M., & Nachlieli, T. (2015). Classroom engagement towards using definitions for developing mathematical objects: the case of function. *Educational Studies in Mathematics*, *90*(2), 163–187. https://doi.org/10.1007/s10649-015-9624-0

Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Weegar, M., & Pacis, D. (2012). A comparison of two theories of learning - behaviorism and constructivism as applied to face-to-face and online learning. *E-Leader Manila*, 1–20.

APPENDICES

Appendix A1: Written Instrument

Percent (1 hour)

Name:	
Student number:	
Email address:	
Telephone number:	

Instructions

Thank you for volunteering to participate in this research project.

- Please solve the problems on the pages overleaf.
- Please feel free to solve the problems in any way that you wish but remember to show all your work.
- I am interested in how you approach solving the problems.
- Even if you cannot come to a final answer, please indicate any attempts that you may have made.
- No calculators are allowed for this paper
- Two additional blank pages have been included for rough work.

Good luck.

1	200% of 70 = ___
2	42 = ___% of 70
3	40% of ___ = 20
4	75 = ___% of 60

5	78% of 65 = ___

6	120% of ___ = 96

7	Sumaya spends 25% of her monthly salary for rent. If she spends R2 700 a month for rent, what is Sumaya's monthly salary?

8	A packet of rice is selling for 175% of its 1995 price. If a packet of rice sold for R16 in 1995 what is the selling price today?
9	Government have offered workers a new hourly wage rate of R20/hr. If the minimum wage in 2017 was R16/hr what is the percentage increase?
10	A store is offering a 20% discount on scarves for Women's Day. On a Saturday, all ladies automatically get an additional 5% discount. What is the total discount that they will get if they buy a scarf on a Saturday?

11	Your sister just got a raise and now earns R12 000 a month. Now your salary is only 85% of your sister's salary. How much do you earn?
12	When you buy a 440ml can of Coca Cola you get ___ % more volume than when you purchase a 330ml bottle of Coca Cola?
13	If Jacob earns 25% more than Ramona, then Ramona earns ___% less than Jacob.

14	VAT increased from 14% to 15% on 1 April 2018. If an item costs R100 excluding VAT. You will pay ___% more for the item in May 2018.
15	A store is offering a 25% discount on a jacket that costs R560 excluding VAT. A customer asks the shop assistant to add VAT first and then apply the discount so that she gets a bigger discount. The shop assistant explains that VAT is automatically added to the FINAL price, so the discount has to be applied before VAT. How much money does the customer lose?

Appendix A2: Sample interview schedule

Conversations about percent (1 hour)

Name:	
Student number:	
Email address:	
Telephone number:	
Date:	

Instructions

Thank you for volunteering to participate in this research project.

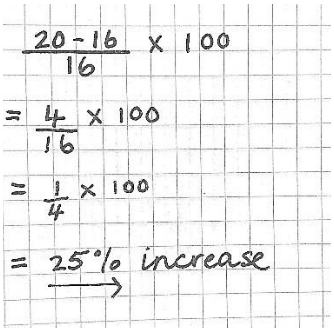
- You may use calculators for this session.
- You may ask questions to clarify the problems.
- Please feel free to share your ideas with the group.

Problem 1

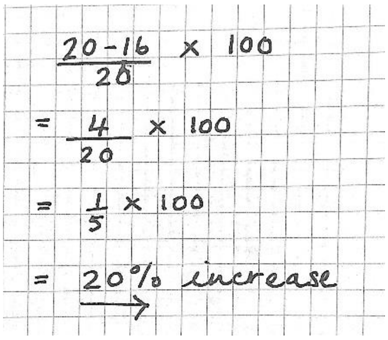
- The text below contains examples of two methods that were used to solve a percent problem.
- Please read through the solutions and consider which solution, if any, you agree with.

1	Government have offered workers a new minimum hourly wage rate of R20/hr. If the rate last year was R16/hr what is the percentage increase?

A



B



1. What are your thoughts?

2. Explain to me why this is the correct answer?

3. This question was in the test that we wrote in August. Most people answered B. Why do you think that is the case?

4. If you were asked to measure the decrease in rate from R20 to R16 how would you calculated that?

Problem 2

2.	If Jacob earns 25% more than Ramona, then Ramona earns ___% less than Jacob.

1. What is your gut feel?

2. In the test, many people answered 25%. Why do you think that is?

3. How would you illustrate to them that it is 20%? Can you use maths to explain this?

Problem 3

3	A store is offering a 20% discount on jelly beans. This weekend only, all shoppers will get an extra 5% discount. What is the total discount that they will get?

1. What is your gut feel? Why?

2. How would you explain this to someone? Example or algebra or both?

3. When consumers see a special like this advertised in the stores what do you think they assume their discount will be?

4. How would you explain what 20 percent is to someone that had never worked with percent before? You can use pictures if you like.

5. How would you explain what a 20% discount means?

Problem 4:

4	A jacket costs R560 excluding VAT and is now discounted by 25%. A customer asks the teller to add VAT first and then apply the discount so that she gets a bigger discount. The teller explains that the system automatically adds VAT after the discount has been applied. Would the customer pay less for the jacket if VAT was added before the discount?

1. What would you guess the answer might be?
2. Work it out.
3. Why do you think the answers are the same? Can you use maths to help you prove this?
4. Is this true for all discount amounts?
5. Is this true for all prices?

Appendix B1: Student Consent Forms

Please fill in the reply slip below if you agree to participate in my study called: "Percent – Exploring the approaches of pre-service teachers".

My name is: ____

Please tick the relevant boxes for each of the questions below:	Yes	No
Permission for a test		
I agree to write a test for this study	☐	☐
Permission to be interviewed		
I agree to be interviewed for this study if invited to participate	☐	☐
I know that I can stop the interview at any time and don't have to answer all the questions asked	☐	☐
Permission to be audiotaped		
I agree to be audiotaped during the interview	☐	☐
I know that the audiotapes will be used for this project only	☐	☐

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- my written work in the interview will be used for research purposes
- the test does not form part of the normal assessment for my studies
- all the data collected during this study will be kept confidential and stored in locked cabinets and/or password protected folders.
- all data will be destroyed within 3-5 years after completion of the project.

____ ______________________

Signature Date

____ ______________________

Email address Contact number

Appendix B2: Participant information sheet

30 July 2018

Dear Student,

My name is Jaqui Luksmidas. As a Masters student at the School of Education at the University of the Witwatersrand I am studying how people work with percent. As part of my research study I will be exploring your approaches to working with percent.

My investigation involves the completion of a set of fifteen problems using percent. The problems are a mix of typical school test items and real-world problems. My intention is not to measure the number of correct answers. Instead, I am interested in how participants go about solving the problem. This will provide me with the most accurate picture of the participants' approach to the problem. For this reason, the set of questions will need to be completed without the use of a calculator.

I would like to invite you to participate in this study by completing the set of problems in the form of a test. This should take approximately 60 minutes to complete. I will invite some students to participate in interviews based on a selection of responses. The interviews are voluntary and will be conducted on campus on an individual basis at your convenience. Interviews will be audio-taped and calculators will be provided for this part of the study. Interviews will be limited to 45 minutes.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. The test will not form part of the normal assessment in your studies. All research data will be stored safely and destroyed between 3-5 years after I have completed my project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.

Should you wish to participate in the study, please complete the student consent form. Interviews will be audio-taped, but should you prefer not to have your interview audio-taped please indicate this on the consent form.

Please feel free to contact me if you have any questions.

Thank you for your help.

Yours sincerely,

Jaqueline Luksmidas
9302535R@students.wits.co.za
0824931472
Unit 2a, Regent Place, The Zone of Rosebank

This research project has been given ethics clearance under the protocol number 2018ECE016M.

Appendix B3: Letter to the Head of the Wits School of Education

30 July 2018

Dear Professor Maringe,

My name is Jaqui Luksmidas. I am a Masters student at the School of Education at the University of the Witwatersrand. I am conducting research on student knowledge of percent.

The objective of the study is to determine to what extent formal school-taught methods for solving percent problems are retained and used after school. The study should illuminate the methods employed by students in their approach to percent problems and identify the types of percent problems that they experience difficulty with. Specifically, the research study intends to address the following questions:
What strategies do students employ to solve percent problems?
Are the strategies employed formal or informal?
What types of percent problems do students experience difficulty with?

Participants will be required to complete a test with a range of percent problems. A sample of students will be selected based on their test responses for interviews. Interviews will be audio recorded and transcribed. The purpose of the interviews is to gain insight into the approaches used by the participants as a means to understanding the participants' concept of percent.

I am approaching the Wits School of Education as the B.Ed. students on the Mathematics programme are representative of individuals that will be entering the field to teach learners the concept of percent. Furthermore, percent is an essential component in teachers' administration roles as they make use of percent in computing learner performance. Finally, the students are representative of adults that experience contextual applications of percent in the real world.

I am inviting the Wits School of Education to participate in this research study.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they may withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

This research project has been given ethics clearance under the protocol number 2018ECE016M.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,
Jaqueline Luksmidas
9302535R@students.wits.co.za 082 493 1472
Unit 2a, Regent Place, The Zone of Rosebank

Appendix B4: Letter to the Faculty Registrar

30 July 2018

Dear Ms Julie Poyser,

My name is Jaqui Luksmidas. I am a Masters student at the School of Education at the University of the Witwatersrand. I am conducting research on student knowledge of percent.

The objective of the study is to determine to what extent formal school-taught methods for solving percent problems are retained and used after school. The study should illuminate the methods employed by students in their approach to percent problems and identify the types of percent problems that they experience difficulty with. Specifically, the research study intends to address the following questions:
What strategies do students employ to solve percent problems?
Are the strategies employed formal or informal?
What types of percent problems do students experience difficulty with?

Participants will be required to complete a test with a range of percent problems. A sample of students will be selected based on their test responses for interviews. Interviews will be audio recorded and transcribed. The purpose of the interviews is to gain insight into the approaches used by the participants as a means to understanding the participants' concept of percent.

I am approaching the Wits School of Education as the B.Ed. students on the Mathematics programme are representative of individuals that will be entering the field to teach learners the concept of percent. Furthermore, percent is an essential component in teachers' administration roles as they make use of percent in computing learner performance. Finally, the students are representative of adults that experience contextual applications of percent in the real world.

I am inviting the Wits School of Education to participate in this research study.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they may withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

This research project has been given ethics clearance under the protocol number 2018ECE016M.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,
Jaqueline Luksmidas
9302535R@students.wits.co.za 082 493 1472
Unit 2a, Regent Place, The Zone of Rosebank

Appendix B5: Ethics clearance certificate

Wits School of Education

WITS UNIVERSITY

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

11 June 2018

Student Number: 9302535R

Protocol Number: 2018ECE016M

Dear Jaqueline Luksmidas

Application for Ethics Clearance: Master of Education

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Unpacking pre-service teachers' knowledge of percent

The committee recently met and I am pleased to inform you that **clearance was granted**. However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

The following comments were made:

- A letter to the Registrar needs to be added to the application and of course sent to the Registrar.
- It will be useful to indicate in the information sheets that the tests to be conducted will not be part of the normal assessment of the students.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mabeta

Wits School of Education

011 717-3416

Cc Supervisor: Dr Craig Pournara

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Appendix C1: Sample transcript Interview 1

The double discount task

Line		*What was said*	*What was seen/done*
115	I	A store is offering a 20% discount on ……	Reads question
116	I	What’s your gut feel.	
117	J	Well it would be 25.	Laughs.
118	N		Mumbles softly
119	J	Did we have that one?	Referring to the written test
120	I	Ja we did.	
121	N	Yup it was the last one.	
122	I	It wasn’t the …. It was close to the last one … let me tell you which one it was … um ..	
123	N	Oh number 10.	
124	J	Number 10.	
125	J	Oh, yes. I also just said 25.	
126	N	I also said 25 per cent	
127	I	Hmm	
128	J	OK.	
129	N	Ja. I said 25 per cent. I added the 20 and the 5	
130	I	Yes. Because that looks ….	
131	N	That looks right	
132	I	Do you want to try some maths? Use your maths …	
133	J	Yes.	
134	I	… to see if you can work out what it actually is?	
135	J	So. Let’s say if the jelly beans initially cost 10 rand. And then you subtract, you take the 20 per cent off. Um. Take it off from 10 rand then it would be 2 rand. So, then you have 8 rand left. So, then you take the … 5 …	
136	J	… oh ja … then it won’t be 25 per cent. So, then you take that 8 per cent out … not 8 per cent … 5 per cent from 8 rand and that would be …. 5 per cent	Working on calculator
137	J	Let’s hope its …	Mumbles
138	N	20 divided by 5 is 4.	
139	J	Ja. So that would be 0.4. So, then that would be seven rand sixty.	
140	N	Seven rand sixty.	
141	J	Then that you take a ratio with the 10.	
142	I	Hmm	
143	J	Ok. So then that and 10. You have seven rand sixty. Taking ratios.	
144	J	So that would be 7.6 divided by 10 times 100. So that is whoa! I don’t know if that is correct.	Working on calculator
145	N	Seventy six percent? Is that possible?	
146	I	You are paying 76 percent of the price. So, then what is the discount?	
147	J	Oh, then you just. Ok. Then the discount …	
148	N	Is 24	
149	J	Ja it would be 24 per cent	
150	N	Wow. OK. OK.	Surprised. But unconvinced
151	I	Does it make sense?	
152	J	It makes sense now because I did the previous problem differently.	
153	I	Yes. Yes.	

Line		*What was said*	*What was seen/done*
154	N	I think also it is because maybe when we are looking at it from a mathematical point of view. When we were answering, we were answering it from a logical point of view.	
155	I	Yes	
156	J	Ja	
157	N	From what our brain is telling us is that way.	
158	I	Yes. Yes. Um …	
159	J	So …	
160	I	When you … when consumers see a special like this what do you think they automatically think their discount is?	
161	J N	Also 25 per cent	
162	J	But I … I don't think when I did this sum … I did not think that first the 20 per cent was taken off and then the 5 per cent because when you take the 20 per cent off then you have a smaller …	Referring to the written test
163	N	Yes	
164	I	Yes	
165	J	… amount. So …	
166	I	That's true though. Because even as a shopper you are not always certain that that is the way they do it.	
167	NJ	Ja	
168	I	Cos … you've got to read the … you've actually got to read the fine print quite carefully.	
169	NJ	Ja	
170	I	Hey?	
171	NJ	Hmm	
172	I	And this is actually why we are sitting here having a conversation. That's why we are having a conv … you know … because you could interpret it in different ways just by reading it on your own. Um.	
173	I	How … OK … wait … before I come back to this. Do you think there is any way that you could use algebra to show …	
174	J	Algebra? Yes, you can just make the x ten ag the 10 x.	
175	I	OK.	
176	J	And then times that with … your … 20 per cent … so x would be your initial value …	
177	I	Hmm. Yes.	
178	J	And then you want to get the total discount …	
179	N	You could also make the total discount x. To solve for x	
180	I	Yes. You could.	
181	J	Ja. That would be easier though	
182	I	Ja.	
183	J	I don't know how to … like … write that in an equation.	
184	I	Hmm …. You can think about it for a little while. You don't absolutely have to, if it does not come to you.	
185	N	So, I would start by saying let total discount equal x. OK. So, we are looking for x. So, then the question is how to write out the equation more than anything else.	
186	I	Hmm	
187	N	To find it. So … we'll obviously you would say that x is the total discount. So, then x will equal to the 10 rand minus the 20 percent right? And then …	
188	J	But we don't have the 10 Rand .	
189	N	No. I'm just using a theoretical value. Right?	
190	I	Hmm	

Line		*What was said*	*What was seen/done*
191	N	So obviously that would be 8 rand and then you will have that answer. But then it will be a bit tricky because that won’t be the final answer of x. If that makes sense because you still have to subtract the 5 per cent from it. And then eventually you will end up dividing by the 10 to get the answer.	
192	J	I don’t think that is ... For me … this … if you just look at the question without using the 10 that I used.	
193	I	Yes	
194	J	You can’t use algebra for me.	
195	I	For you. OK. Is it always going to be 24 per cent regardless of what that starting value is here? This 10 rand. Whether it is 10 rand or 20 rand.	
196	J	It should be. Ja. It should be	
197	N	Ja	
198	I	OK. Now a slightly different question. If you came across someone who had never dealt with percent before, how would you explain to them what 20 percent means?	
199	J	I would say 20 percent is … ah … a small piece of the initial value. So, a small part of the initial value. So, your whole initial value is 100 and then that would be … the 20 would be 20 of the 100.	
200	I	Hmm	
201	J	And when you separate that then you would have 20 and 80. And Ja. That’s what how I would explain it.	
202	I	Hmm	
203	N	I think I would use an example without like using one. So …	
204	I	Hmm. You can draw pictures if you want to.	
205	N	Ja. So, maybe if they is living in a rural area for example and they are used to going to the spaza shop or something. So, you see the packet of pap that you are going to buy? They are charging you 10 rand for it. But they say that there is a discount of 20 per cent on it. So then what means is that you are going to pay less for something.	
206	I	Hmm	
207	N	Or you will end up paying more. For example, whether they are increasing or decreasing. And then you will be saving money or something like that. Like use an example … like consider where they are from kind of thing.	
208	I	Yes yes yes. OK. OK. You kind of answered my next question in one step. Cos you answered what 20 per cent is and my next question was going to be how do you explain what a discount is. But you sort of did in your explanation as well cos you said you have got a 20 per cent and an 80 per cent.	
209	I	OK. What about percentages that are … can a percent be greater than 100?	
210	N	Ja. That’s a good question. Because I remember the first one that we did I got 200 per cent. Right.	Pointing at her answer to question 1 of the written test.
211	I	Hmm	
212	N	No, I got 140 percent. It said 200 per cent of 70 and we usually only deal with percentages up to 100 percent. So, then that would have meant that it would be double for example.	
213	I	Hmm	
214	N	So, I was confused on that one.	
215	J	Well. If you … as soon as its 140 percent, it is 40 percent more than your initial value. That’s how I always saw it. It was never for me a difficult…	

Line		*What was said*	*What was seen/done*
216	I	Yes. OK.	
217	J	… thing to grasp. I don’t know if I am wrong in seeing it like that?	
218	I	No no no. Not at all.	
219	J	So, I also got 140. So, because you have your initial value of 100 which is 70 and then you add 70 so then it has to be 140.	
220	I	Yes. OK. And if it was … if the question was asking for 150 per cent?	
221	J	Then I would say 70 divided by 2 and add it with my 70.	
222	N	Ja. So, he used subtraction and I think addition, right? And then I used multiplication because I know Of means multiply.	
223	I	Ja. OK. OK that’s great. OK. So now, the last question.	

Appendix C2: Sample transcript Interview 2

The double discount task

Line		What was said	What was seen / done
240	I	OK. Next one. OK. A store is offering a 20 percent discount on jelly beans. This weekend only, all shoppers will get an extra 5 percent discount. What is the total discount that they will get?	
241	I	So, when consumers see a special like this advertised in the stores what do you think they immediately think their discount will be?	
242	M	They think it is going to be 25.	
243	I	Yes. Do you think that the actual discount is more than or less than 25?	
244	M	It’s less than.	
245	I	Less than?	
246	M	Ja. I’m thinking it’s less than.	
247	I	You’re thinking it’s less than.	
248	I	Is that your gut feel?	Addressing the other student
249	B	(Sighs) I think I must my own use experience …. Because	
250	I	Yes!	
251	M	Because … OK … cos they are saying only for jelly beans is 20 percent. But if ever I think the second sentence is saying … if ever you are buying anything else for all of them I think the total of all the things that you will buy. The discounts for that one is 5 percent. But however for jelly beans if ever they add there jelly beans in it than it is 20.	
252	M	Let’s say you are buying all of these other things now.	Pointing at a juice boxes and chips on the desk.
253	I	Ja?	
254	M	The total discount for all of them is going to be 5 percent.	
255	I	Oh I see. OK. No, that’s not actually … not actually what the question is saying. It’s saying … OK … it’s actually saying you are only buying jelly beans.	
256	I	OK? So you are only buying this. So you are only buying jelly beans.	Illustrates with juice boxes only
269	M	OK. I am trying to think of the …. Let’s say the price is 100 rand.	
270	I	OK. 100 is a good one.	
271	M	They take 25? OK … they take 20?	
272	I	20. Yes.	
273	M	They take 20 percent off.	
274	B	It’s gonna be … it’s not going to be 80 bucks?	
278	M	Then it’s going to be 80.	
279	I	Mm-hm.	
280	M	And then again they …. Oh … then they take the 5 percent off the 80?	All worked out on calculator. No writing yet.
281	I	Mm-hm.	
282	B	5 percent of 80 you just minus 2. Is it 2? No … 4. You minus 4.	
283	I	Hmmm	

Line		What was said	What was seen / done
284	M	…..you minus 4 then it becomes 76	
285	I	OK. So, you paid 76 rand now for this that was 100 rand. So what was your discount actually?	
286	B	Hm?	
287	M	OK. 24.	
288	I	24	
297	M	Cos also … beside having the confusion that I was having … I was just gonna add 20 and 5 and use 25 percent.	Confusion of one different items priced at different rates
298	I	Ja	
299	B	Oh … OH! Yeah ….	Exclaims .
300	I	Whereas if it was 25 percent …. If it was a straight 25 percent then what would you pay?	
301	B	75 bucks.	Laughs.
302	I	75. Ja. Ja.	
303	M	OK	
304	B	So, you pay what? 76?	
305	I	Ja	
306	B	Ok …	
307	M	Ja you take …	
308	B	So here we should calculate how much discount does that?	Interrupting
309	M	Ja. You take the 25 percent	
310	I	Yes. Well … we've just … we've … well … you don't have to …	M continues to write independently.
311	B	Ja.	
312	I	You don't have to write it down. Unless you really want to. If you want to … to see the numbers work. Absolutely, you can. You don't have to though.	
313	M	… and then you take …	Speaking softly as he is writing
324	B	So, we can use the one of 100 rand.	
325	I	Yes. 100 works.	
326	B	It's much easier. Then …	Laughing
327	I	It's easier.	
340	M	OK. Now. Well, at least I am understanding something here.	Laughs
341	I	You will look a little more closely at discounts when you are in the stores, hey?	
342	B	So … Uh … here … when you say this we should … be very careful. Can't we like that … the original price of …	
343	I	Yes the original price.	
344	B	Cos the minute I put it like this way …	
345	I	Yes. Is that the price before or after?	
346	B	Someone will be confused.	
347		[Silence]	Both students writing.
348	M	I tried a different number here.	Student uses 200 instead of 100 to calculate.
349	I	I see that. And what did you find?	
350	M	It's 24 still.	Continues writing
353	I	I like that you tried a different number	
354	M	Ja. I actually tried to increase it to 200 to see if …	
355	I	To see if it still works?	
356	B	[Laughs]	Laughing
357	M	Sometimes 100 becomes a problem statement.	

Line		What was said	What was seen / done
358	I	Yes	
359	I	So it is showing you that it actually doesn't matter what the price is, you get the same discount?	
360	M	It doesn't matter. Ja.	Mumbled.
361	B		Continues writing.
362	I	Why do you think that is?	
363	M	That the answer is the same?	
364	I	Yes. Why? How come the answer is the same?	
365	B		Still writing
366	M	Cos I don't think it. Whichever price it is it does not affect what the discount is.	
367	I	Hm.	
368	M	Maybe when they talk of 2000, it will still … have the same discount?	
369	B	So, the starting point will have to be 100?	Taps pencil hard on desk.
370	M	It …	Mumbles uncertainly
371	B	In this case … or maybe 200?	
372	I	Yes. So …	
373	M	It doesn't matter what number you use.	
374	I	Yes.	
375	B	OK …	Seems uncertain.
376	M	Even if you use 1. It will work.	
377	B	Even if you use 1?	Inaudible
378	I	Yes.	
379	M	Even if you use 1.	
380	I	It will be very small numbers you will be working with. Yes.	
381	M	It will … It will still work	
382	B	So … divide by …	Writing
383	M	Hmm. Understanding the question first is always a problem.	
384	I	Hmm.	
385	M	So, when you look at this thing you think of … I am getting 25 percent, yet you are not getting 25.	
406	I	How is that going there?	Addressing B. Has been writing intermittently all along.
407	B	I'm still trying to … hmm …	
408	B	What did I do?	
409	I	Um … Yes. So, you've got the first 20 percent there, right?	Looking over written calculations.
410	B	Hmm	
411	I	Oh. You are trying to work out what the full discount is?	
412	B	Hmm.	
413	I	I see. I see. I see. So to work that out you actually need to say 100 minus that full discount.	
414	B	Oh.	
415	I	Ja.	
416	B	Not …	
417	M	Not. Not …	Not quite audible.
418	I	Ja. Cos this 80 you have gone and you have discounted, hey?	Student had written (100 – 80) - 76
419	B	Hmm. Oh!	
420	I	Yes. Yes.	

Line		What was said	What was seen / done
421	B	Eish.	Erases the 80.
422	I	I hadn’t actually noticed the 76 there before.	
423	B	So … 24.	
424	I	Hmm.	
425	B	That still gives me that.	
426	I	Hmm. You’ve got the same answer and you have used different starting points.	
427	B	Hmm.	
428	B	You get punished for not writing all the steps	Continues writing divide by 100 and multiply by 100. Then cancels the two.
429	I	Who is going to punish you here? Hey?	Laughs
447	I	I wanna ask you. So … um … if you had to explain to someone … just before we start the next question. If you had to explain to someone who had never heard of percent before. Never worked with it. To try and explain to someone like that what percent is … so … I mean … you … could think of someone who lives in a rural area and isn’t seeing all the adverts that we are exposed to. Or a 10 year-old child …who doesn’t … hears the word but doesn’t know what it means?	
448	B	Hmm.	
449			
450	I	How would you?	
451	M	OK.	
452	B	What is a percentage?	
453	I	What is a percent?	
454	M	Well …It comes to like buying things. I will always tell them. Like now that I know. Percentage is unlike numbers where when you add a number it just becomes plain clear that I am having 10 time … 10 times things more than you have. Therefor how many less do you have …	
455	I	Hmm mm.	
456	M	With the numbers it’s just clear, but with percentage now we have just proved it that that is not always the case.	
457	I	Hmm.	
458	M	And … uh … also, percentage is … it’s a way in which the retailers ... somehow, they play around … and it’s like … that’s how they gain. In a hidden way that we think they are not gaining … or we think we are gaining or they are losing out whenever there is a discount or something.	
459	I	Yes	
460	M	But in either way they are gaining without us seeing it.	
461	I	Yes.	
462	B	I remember that …	In audibly.
463	M	Ja of course it is not like it is a bad thing. It’s … but in the end in the eye of the public they are being …	
464	B	I remember …. I remember that as well for you as a student teacher to understand how many learners uh have passed your subject. In this case maths. You have to use this whereby you will be dividing a number of learners that passed from the num … total number of children that you have. Let’s say in this case …	
465	M	Hmm.	
466	B	… you find that. Let’s use this. You find that that all of this is your student … and then this one you pass … they passed … which is 4. You have to … add the total number …	
467	I	Hmm.	
468	B	… and then calculate … divide that and then from there …	

Line		What was said	What was seen / done
469	M	… by the total of children …	
470	B	So, in most cases for me. I think percentage-wise they don't just work in terms of retailers. They are there in our everyday meaning. For example: one can ask … themself that … how did the government come to the unemployment rate of 27.7 percent?	Using the 4 juice boxes to illustrate
471	I	Hmm. Hmm.	
472	B	It comes to this …they looked at how many people were retrenched in this case. Let's not say fired. Retrenched. They look at that number … I don't know … are they using the total population or they using the total number of adults that we do have in this country?	
473	I	To be honest, I don't know that answer. But it's important. Cos what you are saying is that the total you are looking at … if that number changes, your percent changes doesn't it?	
474	B	Hmm. Yeah it does.	
	M	Hmm.	Inaudible
475	I	OK. Because 10 out a 100 is very different to 10 out of a 120. Isn't it?	
476	M	Hmmm	
477	I	Um … and those are questions that … that are important to look at. So, when you are reading um newspaper articles and they say unemployment has … has dropped by 5 percent you need to be asking what was the base?	
478	M	What was the actual …	
479	I	What … what were they putting in the total in the first instance … in the first year and what are they putting in the total in the second year. And maybe you find in the first year they included people from the age of 16 and in the second year they included people only form the age of 18. That's gonna give you a different number.	