Journal of Curriculum Studies Research

https://curriculumstudies.org

E-ISSN: 2690-2788

Volume: 6 Issue: 2 2024

pp. 259-278

Understanding the Importance of Numerical Context in the Transition from School to University Mathematical Writing

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Article Info
Received: January 14, 2024
Accepted: October 21, 2024
Published: December 28, 2024

doi 10.46303/jcsr.2024.21

How to cite
Matabane, M. E., & Inyangala, E. B. (2024). Understanding the Importance of Numerical Context in the transition from School to University Mathematical Writing. *Journal of Curriculum Studies Research*, 6(2), 259-278.
https://doi.org/10.46303/jcsr.2024.21

ABSTRACT

The transition from secondary school to university mathematics poses challenges for students, impacting their readiness and performance. The study explores the discrepancies in students' understanding and application of numerical context in mathematical discourse during this transition. The 2019 drop in mathematics enrollment and performance in South Africa prompted the investigation, revealing that even high-achieving students face difficulties in first-year university mathematics. The research delves into the significance of numerical context in students' mathematical writing, aiming to bridge the gap between school and university mathematics. Using a social theory of learning and Seo's model for analyzing mathematical texts, the study analyzes students' written expressions during the transition. Results highlight the inconsistency and conflation of numerical context in students' responses to mathematical tasks. The findings suggest that students struggle to adapt to the specific numerical contexts defined in university mathematics, leading to errors and misconceptions in their problem-solving processes. The paper emphasizes the importance of understanding and addressing these challenges in mathematical writing, as success and high achievement in mathematics depend on students' ability to effectively communicate their understanding. The study calls for increased attention to numerical context in mathematical communication, urging educators to focus not only on content but also on the construction of mathematical genres to enhance students' transition from school to university mathematics.

KEYWORDS

Community of practise discourse; mathematical writing; numerical context; transition.

INTRODUCTION AND BACKGROUND

The lack of readiness of pupils transitioning from secondary school to university mathematics has drawn attention of many scholars (Faulkner et al., 2014, Matabane et al., 2022). Despite having a unique place in the curriculum during high school, learning mathematics at university involves several changes that affect the knowledge and skills of arriving students. Just a few examples of these changes are the teaching and learning methods, the type of mathematics taught, conceptual comprehension, the procedural knowledge necessary to get through the content, and the amount of advanced mathematical thinking required (Faulkner et al., 2014).
In developing nations, first-year university students frequently find themselves in a learning environment that differs greatly from that of their secondary school (Thomas & Klymchuk, 2012). Mathematical communication, institutional, social, and content transitions, as well as those affecting the mathematical content, can all be considered as contributing to the shift from high school to university (Age & Machaba,2024; Alcock & Simpson, 2002; Machaba et al., 2024).

In 2011, South Africa's Department of Basic Education announced an 83% pass rate for the country's National Senior Certificate (NCS) results**.** The NCS results are commonly known as Grade 12 matric results**.** The passing of NCS determines school graduates' admission and placement into tertiary studies. The 83% pass was the highest recorded post-apartheid national pass rate and was well received and celebrated. Despite the celebrations over pass rate, there was a considerable drop in both mathematics enrolment and mathematics performance in 2019. Firstly, the number of students who wrote mathematics dropped from 270 516 in 2018 to 222 034 in 2019. Secondly, the number of students who passed mathematics dropped from 58% in 2018 to 54% in 2019. To pass mathematics the student needs to obtain at least 30%. This means that 46% of students who wrote the NCS exam for mathematics in 2019 obtained below 30%. From the 54% that passed mathematics, only 2% obtained a distinction (a score of at least 80%) (Makola et al.,2021).

While the drop in mathematics is of concern for the country, it is more concerning for universities that only 2% of NCS candidates obtained a distinction. A longitudinal study from the University of Cape Town suggests that only students who obtained a distinction in the NCS can cope with first-year university mathematics (Mathematics1) (Makola et al., 2021). For students to be admitted to Mathematics 1 at UCT they need to have obtained a minimum of 70% in the matric exams. The pass mark for Mathematics 1 is at least 50%. Based on several years of data, the students who are admitted to Mathematics 1 with a mark between 70% and 80%, fail Mathematics 1 with an average mark of 43%, and those who achieved between 80% and 89% in matric fail Mathematics 1 with an average of 47% (Makola et al., 2021). It is only those who achieved at least 90% in matric that pass Mathematics 1 with average mark of 64% (Makola et al., 2021). It is cause for concern that from the 2% that obtained a distinction in grade 12, only a few of them will be able to cope with and pass Mathematics 1. Students who fail Mathematics 1 will inevitably take longer to complete their degree and are at risk of being excluded from university. Student who is endorsed as a distinction candidate during the school leaving exam is not guaranteed to pass Mathematics 1, suggests there is a significant gap between what it means to study mathematics in schools and at university. Therefore, this study sought to understand first-year students' take care of numerical context in their transition from school to a university mathematical writing discourse.

Problem Statement

Even if the difference manifests in various ways, there is a sizable divide between secondary school and university mathematics in the genre of writing. The rules of discourse change, but nobody states it (Sfard, 2007). Students' participation in several communities of practice in school and at the university, with opposing engagement rules, suggests that they have diverse experiences with their identities (Sfard, 2006; Solomon, 2007). When students join university, some of those who thought they were "excellent" at mathematics at school start to feel uncomfortable around the subject. The discomfort may negatively marginalize them and can turn them against further studies in mathematics. The new community of practice that first-year students are asked to join may have contradictory characteristics.

This transition can be seen as "a question of identity in which persons see themselves developing due to the distinct social and academic demands that the new institution poses" (Hernandez-Martinez et al., 2011, p. 119). University as a whole and university mathematics are perceived by people as new worlds where they must adapt to new communication and participation norms, which may make a first-year student from school feel out of place (Gueudet, 2008). Students' initiation into a new practice of writing mathematics can be frustrating. Upon arrival at university, students are expected to shift their school mathematical writing discourse into that of university mathematics with little to no support during the transition (Gueudet, 2008).

Significance of the Study

The relationship between what students know or understand and what they write is critical. Success and high achievement in mathematics depend on the students' capacity to have their writing reflect their understanding (Cooper, 2012). Thus, the students' writing in mathematics can act as a resource from which teachers can make interconnections regarding students' mathematical knowledge and thinking. Teachers need to indicate why and how writing must be central in their instruction, regardless of the specialization. Therefore, it should be the teachers' duty to not only teach the content but also how the genre within the content is constructed and the realization of how writing should be patterned to communicate mathematics, taking care of the numerical context. It is important that academic teachers draw their attention to the importance of understanding the fact that, on the one hand, when constructing a written piece, the purpose of the text informs grammatical choices (Seo, 2015). When students write in mathematics, they can not only see where the mathematics they did comes from, but they also see where it is heading (Wilcox & Monroe, 2011). The idea of understanding where the mathematics is heading is crucial since mathematical knowledge is cumulative. Therefore, it is critical that the importance of mathematical writing be studied.

LITERATURE REVIEW

The problem of transition from secondary school to university mathematics learning has been recognised for some time (Faulkner et al., 2014; Matabane & Machaba, 2024; Sfard, 2020; Thoma & Nardi, 2018). The mathematical gap between secondary school and university is described in terms of students' thinking (Faulkner et al., 2014), approach and content (Thoma & Nardi, 2018), acceptance criteria for justification (Sfard, 2020; Tabach & Nacheili, 2011). The transition between secondary school and university is a significant barrier to effective mathematics instruction (De Guzmán et al., 1998). Despite mathematics having a prominent place in the curriculum in high school, it appears that new university students' knowledge and skills may not reflect this reality (Thomas & Klymchuk, 2012). One possible reason for this discrepancy is that several changes occur in the transition to university mathematics learning, including how mathematics is written and communicated (Seo, 2015; Sfard, 2016; Thomas & Klymchuk, 2012; Zetriuslita et al., 2024).

The capacity to communicate is necessary for learning mathematics, and this communication ability must be cultivated (Tinungki, 2015). When one practices mathematics, the goal is to express mathematical concepts and reasoning in an understandable manner. The mathematics community has recognised the importance of writing in learning and communicating mathematics knowledge (Gammill, 2006). As students communicate their ideas, they learn to refine, clarify, and consolidate their thinking (Shanahan & Shanahan, 2008; Vale & Barbosa, 2017). The clarification and refinement of thinking enables a student to have the ability to learn mathematics and to apply concepts and ideas in testing situations (National Council of Teachers of Mathematics [NCTM], 2008).

Writing and Learning Mathematics

It is believed that writing and studying mathematics are isomorphic, and writing can help people understand mathematical thought through interpersonal communication. Researchers contend that writing in mathematics enhances communication between students and teachers, which in turn helps students explain their omissions and misconceptions more clearly, and improves their critical thinking, comprehension, and problem-solving skills (Bhagwonparsadh & Pule, 2024; McMillan, 2017; Sibanda, 2023; Weinhuber et al., 2019). Many publications and journals mention the advantages of combining mathematics and writing (McMillan, 2017). Teachers who assign writing assignments in mathematics can examine their students' mathematical reasoning, identify their misconceptions, and assess their own teaching methods (Morgan, 2001). Writing in mathematics stresses the role of organising thought, comprehension and revising thinking (Morgan, 2001). Writing mathematics is complex since mathematical texts are more conceptually dense than other genres of writing (Adu-Gyamfi et al., 2010) and are replete with symbolic and linguistic conventions which make navigating the text challenging.

Writing and Critical Thinking in Mathematics

Writing enables students to develop critical thinking and teach pupils that knowledge is not a body of perfect responses, but rather is dynamic, dialogic, contextual, complicated, frequently ambiguous, and situational (Teo, 2019). The latter perspective on knowledge enables pupils to recognise the complex true nature of life's challenges within a wide range of potential solutions. Writing allows students to develop, clarify, broaden, and deepen their thoughts. When students struggle with their writing, they struggle with thought itself (Matabane & Machaba, 2023; Mulnix, 2012).

Writing is considered the most challenging of thinking experiences and how people think may largely depend on the kind of thinking experiences they have had. To develop critical thinking skills, it is important that writing be taught not as a product of learning but as a tool for discovery and learning. Research suggests that students' critical thinking skills can be fostered through communication. Writing is considered a real application of thinking as it involves collecting, analyzing, synthesizing, and evaluating information (Duron et al., 2006). Students can communicate their critical thinking through writing, and excellent writing is a manifestation of critical thinking (Karanja, 2021). Developing critical thinkers is necessary and should be a central goal of educational institutions, because improving students' critical thinking skills can be considered a universal goal of education (Gelder, 2005). Critical thinking abilities are essential for functioning as engaged and active citizens, and their development is crucial to excellent education (Karanja, 2021). Teachers would empower students to become better writers by assisting them in developing their thinking skills, and vice versa (Nejmaoui, 2019).

Writing and Mathematical Problem Solving

One of the most complicated human activities is writing. It is essentially a form of problem solving because writers need to produce an organised set of ideas by selecting and organising a manageable number of concepts and relations from a wide collection of information and tailor their knowledge to the reader's requirements (Jensen, 2005). Problem solving is facilitated by writing, not the other way around (Pulagee, 2001). When students are engaged in mathematical problem solving, writing, unlike speaking, allows the whole class to be actively engaged as it can be done simultaneously by everyone. While students could express themselves through speaking, not every student would risk making mistakes in front of their peers. Writing in mathematics is like individualising instruction in a group setting (Ruthven, 2018). The use of writing can help students to become problem solvers because when students get stuck during problem solving, writing out their thoughts often helps them to resolve the problem by themselves.

The steps involved in solving a mathematical problem, such as defining the unknowns, creating a plan, concluding, and then verifying the results, are also included in the writing process (Ruthven, 2018). When first-year students are asked to represent a mathematical idea using the three modes of communication (words, images, and symbols), the activity demand that the student retrospect to see if the ideas presented by images correspond with the ideas presented by symbols and words (and vice versa). Once the students can represent the problem in different ways, a greater understanding is evident, and the understanding is one step to solving the problem.

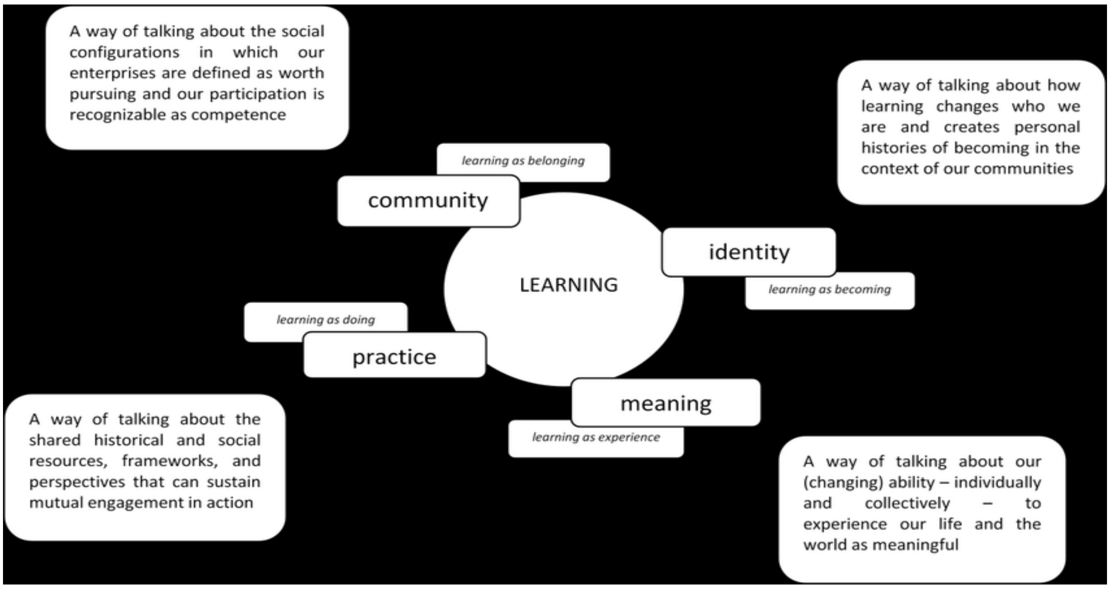
During the writing process, writers with experience take their time not just to compose but also to plan and edit their work. As a result, they engage in a process of "Knowledge Transforming," which includes linear text production but is structured more around goal-setting and problem-solving (Bereiter & Scardamalia, 1987). During the transition period, as students learn university ways of writing mathematics, students need to pay attention to how words are translated into other languages, how mathematical symbols are translated into words, and how English representations are translated into mathematical symbols and equations.

THEORETICAL FRAMEWORK

At the heart of Lave and Wenger's (1991) theory of situated learning is the notion that learning is fundamentally social and integrally related to an individual's evolving identity in a community of practice. To comprehend that students` involvement impacts they are (identity), what they do (practice), how they perceive what they do (meaning), and how they belong (community). The elements of identity, practice, meaning, and community stand for a different aspect of learning. Identity represents the evolving self-development gained through learning, practice stands for doing and participating in learning, meaning stands for understanding and learning from our experiences of living in the world, and community stands for the coalescing nature of people coming together with a common interest in learning (Lave & Wenger, 1991). Figure 1 summaries the four components.

Figure 1.

Components of Social Theory Learning



In summary, learning is considered as increasing participation in a CoP (Lave & Wenger, 1991). To fully participate in a community of practice, one needs to be able to communicate in the discourse of that community (Sfard, 2008). In this study, first-year university students needed to increase participation within the university mathematics community. For this participation to happen, students needed to be able to communicate their mathematics using endorsed ways to communicate mathematics and produce a written text that would be considered by the university mathematics community as mathematical. Such a text needs one to weave between symbols, images, and nominalization to produce a sound mathematical argument and takes care of define numerical contexts (Seo, 2009).

Analytical Framework

The main tools used to analyse the students' writings were based on Seo's (2009) model for analysing mathematical texts. Three components comprise mathematical writing: symbols, nominalizations, and images (Seo, 2015; 2019). Symbols are marks on a surface, and the context of the mark determines its meaning (Rotman, 2000). Nominalizations are terms with a precise mathematical meaning and, depending on the mathematical context, these words may have different meanings (Seo, 2015; 2019). Lastly, there are images. All mathematical writing that is neither symbols nor nominalizations are images. Their status as mathematical texts are a key component of the contexts for the texts under consideration here. While some linguistic characteristics may be considered typical or even unique to mathematical discourse, a variety of linguistic traits may also be anticipated, given the variety of situations and goals of mathematical writing. In the mathematical writing context of these university students' writing, this diversity is important as it may be related to teachers' judgement about their mathematical activity.

METHODOLOGY

This qualitative case study delves into the analysis of students' written expressions during their transition from high school to the first year of university mathematics. The primary focus of the study is to gain insights into how students incorporate numerical context in their problem-solving processes. Employing a purposive sampling method, 48 participants were chosen from the cohort of 160 first-year mathematics students. The chosen student had enrolled for the first at a university (no prior university experience) and were first generation students (first in their families to go to university).

Demographics

The 48 students, who were diverse in terms of their ages, genders, and study Programme, all had the desire to discuss and explore their mathematical writing experiences. To prevent the phenomena from being reduced to a stereotypical perspective of mathematics writing experience, a deliberately heterogeneous group was sought. This group consisted of 31 females and 14 males (3 did not indicate their gender), ranging from ages 17 years old to 38 years old. Of this group, 36 participants were aged between 17 years old to 20 years old, and 12 were between 21 years old to 38 years old. Of the participants, 30 were specialising in the senior and further education and training (FET) phase, while 18 were in the intermediate phase (IP)

Table 1.
Demographic Profile of Students

Variables	Frequency	Percentage
Gender		
Male	31	64.575
Female	14	29.175
Not indicated	03	6.25
Total	**48**	**100.0**
Age		
17-20	36	75
21-38	12	25
Phase		
Intermediate	18	37.5
Senior and FET	30	62.5

Data Collection Instruments

The main data collection instrument was document analysis (students written responses to tasks), followed by in-depth semi-structured interviews. The documents analyzed were students written responses to two tasks given within the first two week of teaching (transition period). The tasks covered the sections on functions and numbers (See appendix A). These sections were chosen because of their transitional nature when students move from school to university. In analysis of students' scripts, I first performed the within case analysis which focused on data from the students' scripts. I then performed the cross-case analysis to examine similarities on the commognitive conflicts evident on student scripts. Students' scripts were examined to understand how students considered numerical context in the problem-solving process.

Ten participants were selected for interviews based on the richness of their written responses to all two tasks, and their availability to participate in the interviews. Before the interview process, I compiled a list of themes and suggested questions to be covered in the interview. The first part of the interview questions was informed by the literature on the transition from secondary school to university mathematical discourse and the role of writing in the learning of mathematics. The second part of the interview was based on my reflections on students' written responses on different tasks. Using the interview technique, I was able to modify the order and format of my questions and add extra follow-up inquiries to learn more about what the participants were saying (Rowley, 2012). Each interviewee was given between 45 to 60 minutes to complete the process. Each interview was conducted by the researcher in a relaxed manner. The interviews developed based on the participants' growing needs, personalities, and methods of responding because each participant represented a distinct perspective, background, and variety of experiences. To retain the researcher's discipline and keep the emphasis on the research objectives, the questions and interactions with the participants were then tailored to everyone.

Triangulation was used to ensure credibility of the research. Triangulation is a method used by qualitative researchers to establish validity of their studies (Patton, 1999). Data triangulation and theory triangulation are the two sources of triangulation for qualitative research (Denzin, 2012; Patton ,1999). Data triangulation involves using different sources of data, in this case students' assignment scripts and interviews. Theory triangulation involves the use of multiple professionals' perceptions to interpret a single set of data. Two senior members in the department and a critical friend listened to the recordings and followed my transcripts to offer their perspectives on the data analysis.

RESULTS

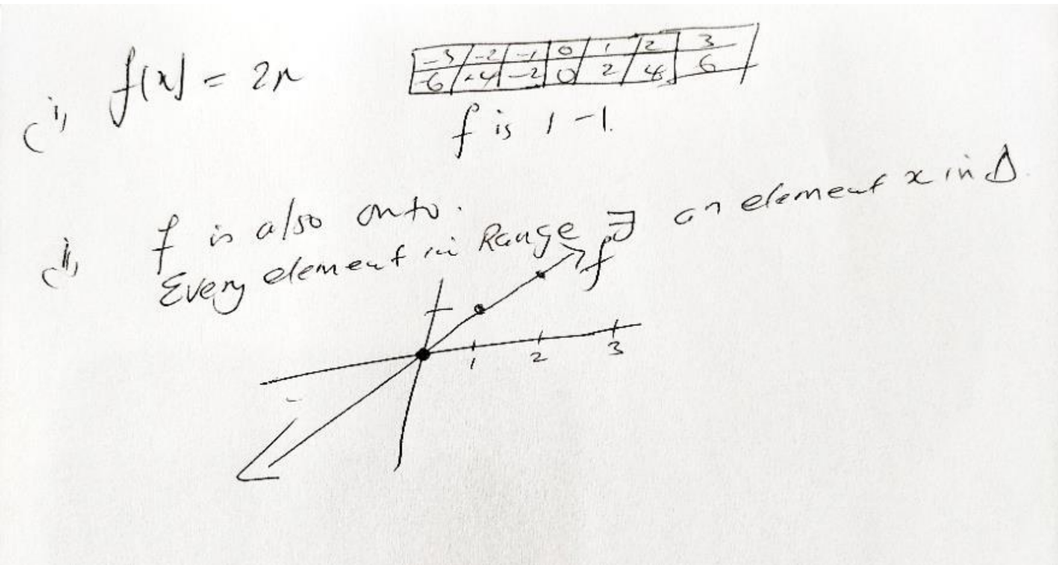
The findings of the study indicate that students encountered challenges when operating within the specified numerical setting and effectively communicating the numerical context they applied during the problem-solving process. This struggle was consistently observed across questions related to functions as well as those centered around numerical operations.

Inconsistency of Numerical Context: Case of Functions

Inconsistency on numerical context was visible on Fox's script in responding to Task 1(b). In this case, the conflation of the symbols is regarding the independent variables (see Figure 1).

Figure 1.

Fox's Response to Task 1(b)



The function f is defined as the numerical context of integers and the independent variable m is used. However, in trying to show that the function f is one-to-one (injective), Fox changed the independent variable to x. Usually, x is used in the numerical context of real numbers. The independent variable m was purposefully used to signal to the students that we were operating from a different numerical context, cautious to the fact that in schools, numerical context of real numbers is assumed almost all the time. Again, in justifying that indeed the function is one-to-one, Fox drew a straight-line graph, which is not aligned to the numerical context of integers. Integers cannot be represented by a straight-line graph. Fox also wrote "for every element in the range," while she should be talking about every element in the codomain. Thus, to Fox, the words "codomain" and "range" could be used interchangeably. The two are different and the range is always a subset of the codomain. As Fox stated,

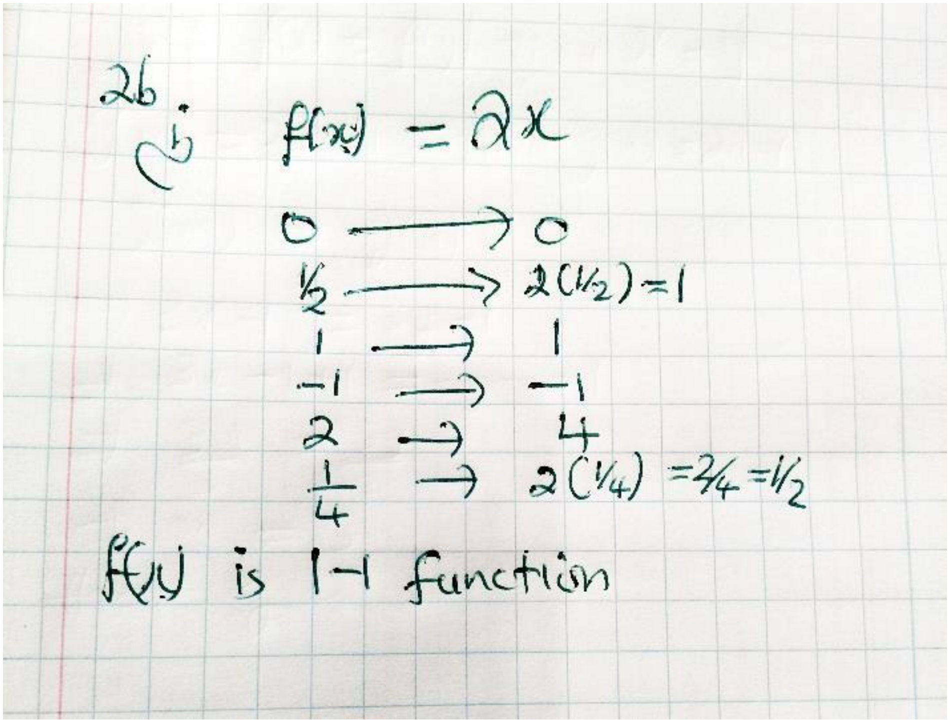
> *I never thought there was a difference between range and codomain; at the school the two are not differentiated and we hardly used the word codomain, we worked with range.*

The conflation between the symbols and numerical context suggest that the student does not see that due to the numerical context of the independent variable, the function g is one-to-one but not onto, because for $y = 1$ is in the codomain, then $x = {}^{1}/_{2}$ which does not belong to integers. An integer 1 in the codomain is a spectator element, which makes f not an onto. If the numerical context of the of 2(bi) is changed to a real number, the function f will be both one-to-one and onto, a bijection. Careful attention to the numerical context is very important as students transition from school to university mathematics. Their mathematical writing should carefully consider the numerical context.

The conflation of numerical context is also seen in students' responses to 1(b) (See Figure 2).

Figure 2.

Dimpho's Response to Task 1(b)



The students were asked to examine whether the two given functions are one-to-one, onto, both one-to-one and onto or neither one-to-one nor onto. In response to the questions, the students worked with elements from the domain and codomain and their different numerical contexts.

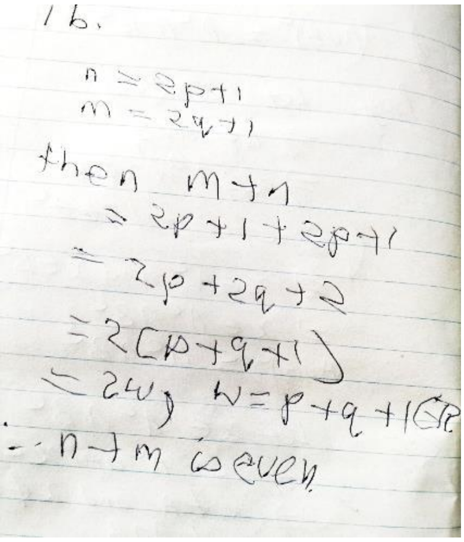
In responding to 2(bi), Dimpho did not carefully examine the context of the variables used. Instead of operating on the context of integers as the question demand, this student operated on the domain of real numbers, evident in picking $\frac{1}{4}$ in set A. The set of domains is given as integers, and $\frac{1}{4}$ is not an integer. In the range, there is an element $1/_2$ which is not an integer. The domain of real numbers is mostly assumed when solving mathematics problems at the school level. The students needed to transition to a new world of university mathematics where the numerical context gets to be defined, and one needs to carefully check how it is defined before attempting to solve the problem. As the father of problem solving and distinguished professor of mathematics at Stanford University, George Polya said, "The first step to solving a problem is understanding it" (Polya, 2020, p. 28). Therefore, students need not rush to solve the problem but take time to first understand it, including examining its numerical context. The university mathematical discourse defines the numerical context, and students do not have to trivialize it as real numbers. Dimpho's solution evidenced commognitive conflict.

Inconsistency of Numerical Context: Case of Numbers of Systems

In Task 2 (Appendix A), the students were asked to engage in the different discourse: the discourse of integers, naturals and real numbers. It also meant engaging with specific integers, namely even and odd numbers. The analysis of the students' scripts shows errors and misconceptions due to rote understanding of the number system and inconsistency in using the restricted number system. The students struggled to specify and retain their narratives within a specific numerical context.

Figure 3.

Thuto's Response to 2(b)



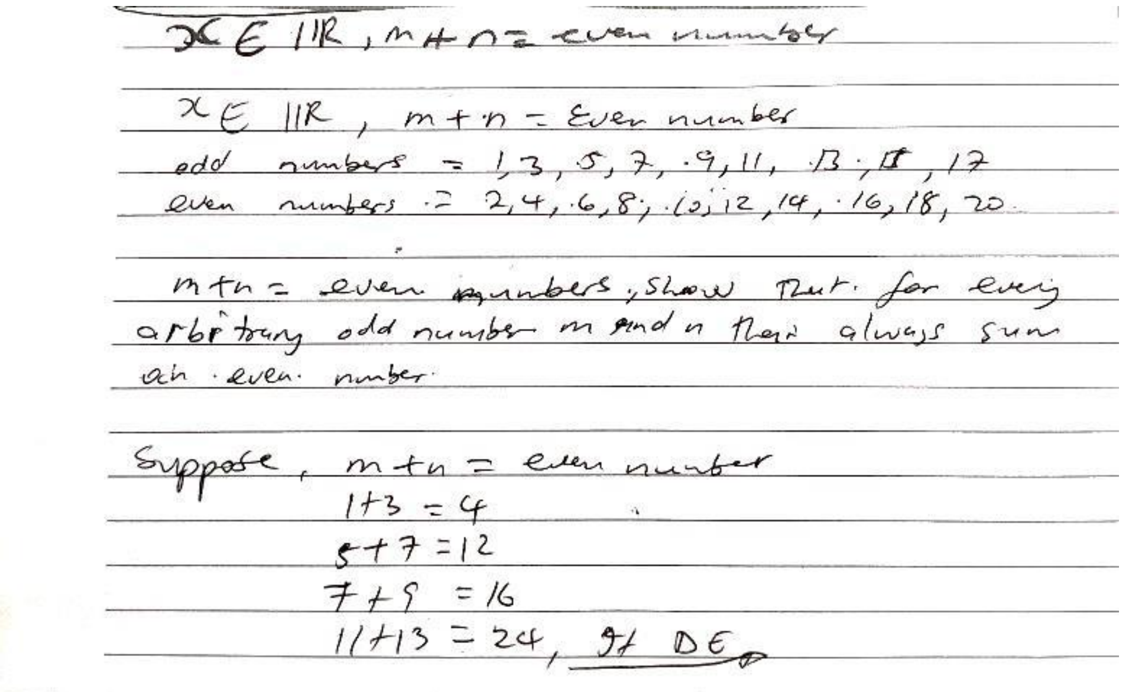
In Task 2(b), the students were asked to show that the sum of two odd integers is always an even integer. Therefore, the numerical context is that of odd integers. Thuto has an understanding that the integers m and n are odd if $n = 2p + 1$ and m=$2q + 1$ (see Figure 3).

However, Thuto, like many others, did not comment on the numerical context of p and q in the definition of an odd number. Not mentioning that p is an integer on $n = 2p + 1$, is problematic. For example, if the restriction of p is integer is not mentioned, one may check what happens if $p = 1/_2$. Then $n = 2(1/_2) + 1 = 2$, and two is not an odd number. By not commenting on the context of variable p and q, the student's script presented evidence of a problematic definition of an odd number.

Again, despite being told that that m and n belong to the set of integers, Mogale started his solution by writing "$x \in \Re$; $m + n$ is even" (see Figure 4).

Figure 4.

Mogale's Response to 1(b)



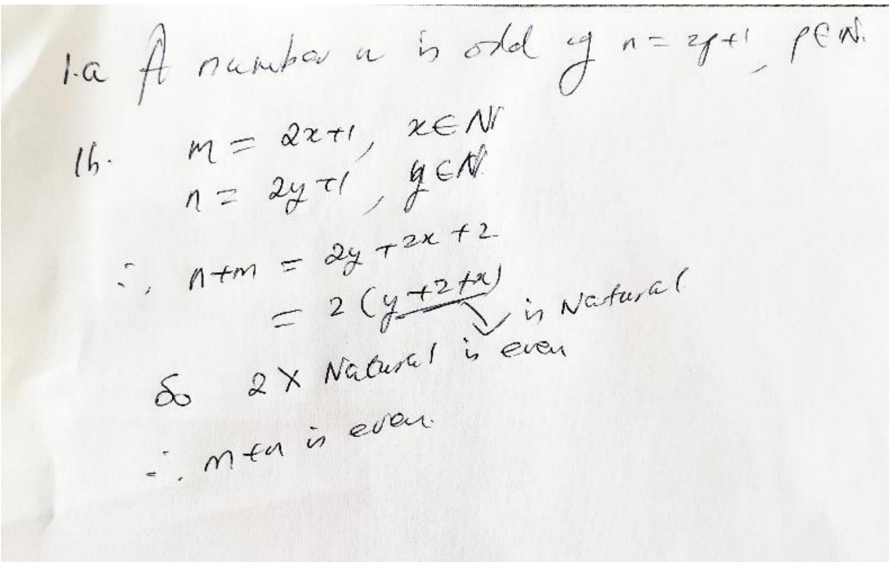
One wonders why Mogale started by explaining that x is a real number whereas the x is nowhere used on his argument. According to Mogale:

> *I used x to mean any randomly picked element, when we do not know something in mathematics, we call it x. So, m, n are also unknowns, so choosing x include them.*

He then continued and listed a series of numbers that were odd and illustrated their sum is even. His explanations showed that he is not cognizant that numbers cannot be used to prove general results.

In the definition of odd number, Jojo tried to provide the numerical context of the variables x and y (see Figure 5).

Figure 5.
Jojo's Response to Task 2(a)



However, she defined the variable x and y as natural numbers instead of integers. Restricting the variable x and y to natural numbers shows minimal understanding on the concept of odd number. Such a narrative does not take into consideration that odd numbers can also be negative. When asked what if p is negative, Jojo replied:

The question said we are using odd numbers, and we must prove the answer is even. We all know that odd numbers and even numbers are always positive, so we must add positive to get positives.

This student consciously used natural numbers. This response was not an error or lack of understanding, but evidence of misunderstanding and lack of conceptual understanding. To the student, the concepts of odd numbers and that of even numbers are understood to be always positive. Jojo was confident about this wrong information as seen by her writing that implicates the teacher as part of this wrong information or tries to be persuaded to agree. Jojo said:

We know that odd numbers and even numbers are always positive.

In many South African school textbooks, the example of odd numbers is almost always given on one side of the number line, but odd numbers include both negative and positive integers for as long as they can generated by $n = 2p + 1,$ where p is an integer. Using p as a natural number illustrates conflation between the students' numerical context.

DISCUSSIONS

The study reveals significant discrepancies in students' understanding and application of numerical context during the transition from secondary school to university mathematics. High-achieving students also face challenges in adapting to the specific numerical contexts defined in university mathematics (Almousa et al., 2022; Collie & Martin, 2017; Juter & Sriraman, 2011). Students struggle with the transition from school mathematical writing to university mathematical discourse (Sfard, 2020; Thoma & Nardi, 2018; Tinungki, 2015). The conflation of symbols, images, and nominalizations in mathematical writing indicates a lack of attention to numerical context (Matabane et al., 2022; Seo, 2015). Success and high achievement in mathematics depend on students' ability to communicate their understanding effectively (Tinungki, 2015).

For too long schoolteachers have taught mathematics through mastery of mechanical manipulation of mathematical symbols and not much on structure and clarity of written arguments including taking care of numerical context (Engelbrecht, 2010). Studies have shown that secondary school students face difficulties in writing mathematics when they transition to university (Chandrasegaran, 2013; Engelbrecht, 2010). The findings of this study show similar results on how students wrote their mathematics on arrival at university, ignorant of the numerical context. Upon arrival at university, the writings of the students evidenced difficulty in identifying the numerical context on which the problem was defined and consistently working within the defined numerical context (Morgan et al., 2014). For example, in Figure 1, the student, Fox, used a straight-line graph to demonstrate that the given function is onto. The use of a straight line suggests that Fox was operating in the context of real numbers, while the problem was clearly contextualized within the domain of integers. Similar confusion of numerical context can be seen from Dimpho`s response (see Figure 2). Dimpho picked an element half from the domain of integers and half is not an integer. University mathematics discourse involves different numerical contexts, and some results may be true under one numerical context and false in a different numerical context. The lack of skills to examine numerical context before attempting to solve the problem can be attributed to the fact that at school different numerical contexts are only studied at primary level and when students transition to secondary school, they start to exclusively use one numerical context, that of real numbers (Chandrasegaran, 2013). However, during the first year at university other numerical contexts are revised to include the numerical context of naturals, integers, and whole numbers (Reid & Knipping, 2010).

This study revealed that it is not mathematics content that is the greatest challenge for first-year university students. Instead, there are mathematical writing challenges on different levels that constrain and affect the students and they express disempowerment and an inability to communicate the mathematics they are learning. The mathematics content thus becomes secondary, due to language and writing constraints. Students’ understanding of mathematics can be promoted through better proficiency in mathematical writing and keeping in mind the numerical context in problem solving. Along with explicit instructions, educators need to teach mathematical vocabulary in context as the latter retains new concepts. Both mathematician Brian Rotman (2000) and linguist Roy Harris (1995) agree that the meaning of a text is dependent on how it is used, keeping in mind the contextual setting. Teaching students how to write is a way of teaching them how to organise their thoughts and ideas. It is not surprising that some of the world’s greatest mathematicians were also creative writers. For example, Lewis Carroll who wrote *Alice in Wonderland* and *Through the Looking-Glass* (Carroll, 2015).

Recommendations

The study recommend alignment between the mathematical curriculum in secondary schools and the expectations in first-year university mathematics, including teaching of different numerical contexts and not assume numerical context of real numbers all the time, as most school level textbooks do. This is because some results could be true in one numerical context and the same results do not hold once the numerical context is changed (Matabane et al., 2022). In addition, there is a need to implement bridge programs or orientation courses for incoming university students to familiarize them with the expectations of mathematical writing at the university level. Such a course will encourage students to articulate their mathematical reasoning clearly, considering the specific numerical contexts defined in the problems. More research should be conducted to develop instructional strategies to teach mathematical writing for first-year students and beyond. According to NCTM (2008), instructional programmes from prekindergarten through to grade 12 should enable all students to organize and consolidate their mathematical communication and use the language of mathematics to express their ideas precisely. It is important, therefore, that university teachers take instructional time to teach major forms of writing within the content area of mathematics including explicit mention of numerical contexts. Again, this study could be replicated in another context to give rich descriptions of first-year university transitional experiences of mathematical writing and their main contextual characteristics. I also aim to follow the students in their second year to understand further what their first-year experience and lessons learned mean for their mathematics learning in second year and to final year of undergraduate study.

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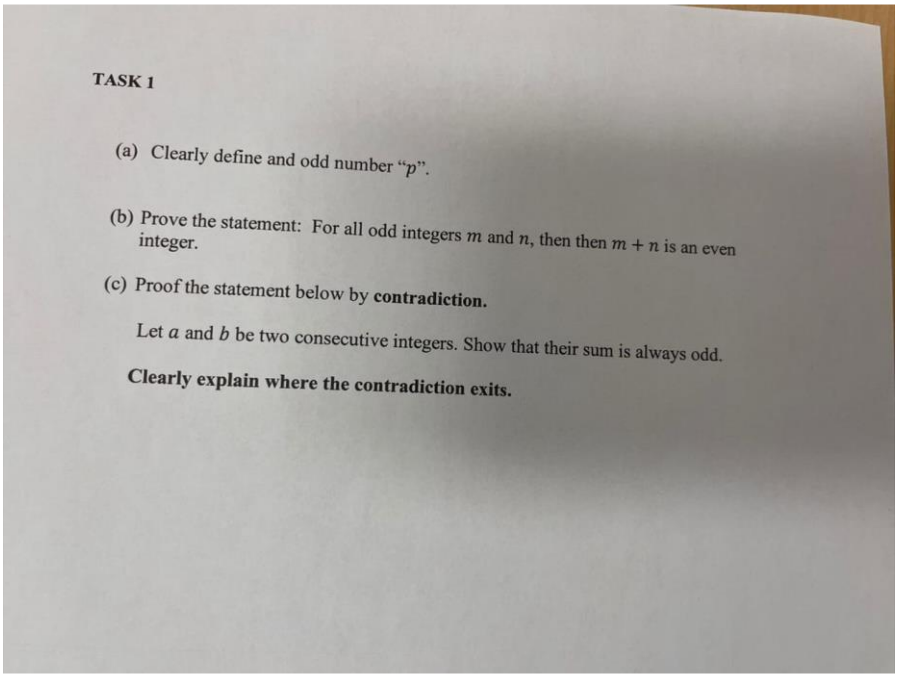
APPENDIX

Appendices A

Task 1



TASK 1

(a) Clearly define and odd number "p".

(b) Prove the statement: For all odd integers m and n, then then $m + n$ is an even integer.

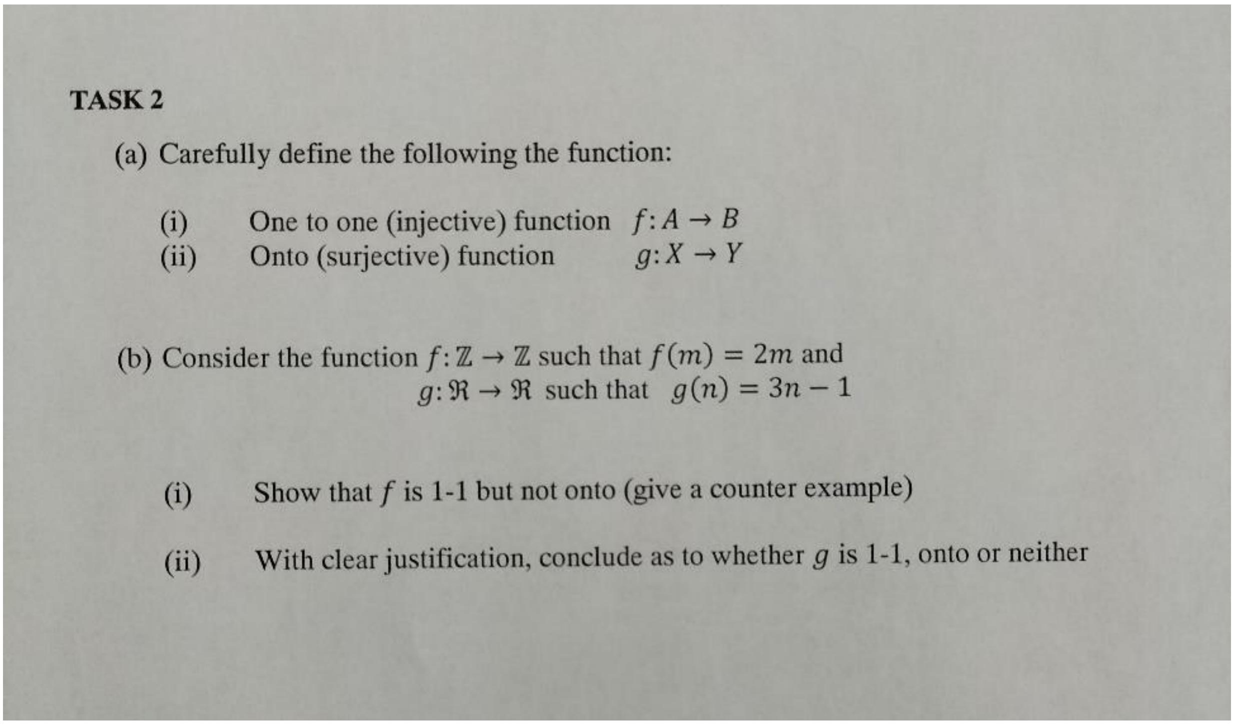
(c) Proof the statement below by **contradiction.**

Let a and b be two consecutive integers. Show that their sum is always odd.

Clearly explain where the contradiction exits.

Appendix B

Task 2



TASK 2

(a) Carefully define the following the function:

(i) One to one (injective) function $f: A \to B$

(ii) Onto (surjective) function $g: X \to Y$

(b) Consider the function $f: \mathbb{Z} \to \mathbb{Z}$ such that $f(m) = 2m$ and $g: \Re \to \Re$ such that $g(n) = 3n - 1$

(i) Show that f is 1-1 but not onto (give a counter example)

(ii) With clear justification, conclude as to whether g is 1-1, onto or neither