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Abstract

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Master of Science in Engineering

Friction Compensation in the Swing-up Control of Viscously Damped Underactuated Robotics

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In this research, we observed a torque-related limitation in the swing-up control of underactuated mechanical systems which had been integrated with viscous damping in the unactuated joint. The objective of this research project was thus to develop a practical work-around solution to this limitation.

The n^{th} order underactuated robotic system is represented in this research as a collection of compounded pendulums with $n-1$ actuators placed at each joint with the exception of the first joint. This system is referred to as the $\mathbf{PA}_{n-1}$ robot (**P**assive first joint, followed by $n-1$ **A**ctive joints), with the Acrobot (PA_1 robot) and the PAA robot (or PA_2 robot) being among the most well-known examples. A number of friction models exist in literature, which include, and are not exclusive to, the Coulomb and the Stribeck effect models, but the viscous damping model was selected for this research since it is more extensively covered in existing literature. The effectiveness of swing-up control using Lyapunov's direct method when applied on the undamped PA_{n-1} robot has been vigorously demonstrated in existing literature, but there is no literature that discusses the swing-up control of viscously damped systems. We show, however, that the application of satisfactory swing-up control using Lyapunov's direct method is constrained to underactuated systems that are either undamped or actively damped (viscous damping integrated into the actuated joints only). The violation of this constraint results in the derivation of a torque expression that cannot be solved for (*invertibility problem*, for systems described by $n > 2$) or a torque expression which contains a conditional singularity (*singularity problem*, for systems with $n = 2$). This constraint is formally summarised as the *matched damping condition*, and highlights a clear limitation in the Lyapunov-related swing-up control of underactuated mechanical systems. This condition has significant implications on the practical realisation of the swing-up control of underactuated mechanical systems, which justifies the investigation into the possibility of a work-around. We thus show that the limitation highlighted by the *matched damping condition* can be overcome through the implementation of the partial feedback linearisation (PFL) technique. Two key contributions are generated from this research as a result, which include the *gain selection criterion* (for Traditional Collocated PFL), and the *convergence algorithm* (for noncollocated PFL).

The gain selection criterion is an analytical solution that is composed of a set of inequalities that map out a geometric *region of appropriate gains* in the swing-up gain space. Selecting a gain combination within this region will ensure that the fully-pendent equilibrium point (FPEP) is unstable, which is a necessary condition for swing-up control when the system is initialised near the FPEP. The convergence algorithm is an experimental solution that, once executed, will provide information about the distal pendulum's angular initial condition that is required to swing-up a robot with a particular angular initial condition for the proximal pendulum, along with the minimum gain that is required to execute the swing-up control in this particular configuration. Significant future contributions on this topic may result from the inclusion of more complex friction models. Additionally, the degree of actuation of the system may be reduced through the implementation of energy storing components, such as torsional springs, at the joint.

In summary, we present two contributions in the form of the *gain selection criterion* and the *convergence algorithm* which accommodate the circumnavigation of the limitation formalised as the *matched damping condition*. This condition pertains to the Lyapunov-related swing-up control of underactuated mechanical systems that have been integrated with viscous damping in the unactuated joint.