Modelling the dependence structures among international stock markets using copulas: Evidence from South Africa and advanced economies

A Research Report submitted in partial fulfillment of the Degree
of Master of Commerce (Economic Science)
in the School of Economics and Finance,
University of the Witwatersrand

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Word Count: 11843

Abstract

Growth in financial linkages has led to similar movements between financial markets. This increase in financial market interdependence or comovement raises questions about the transmission of risk through conventional channels (balance sheet, trade dependence and portfolio flows), particularly the methods policymakers and institutional investors use to measure the strength and intensity of these transmission channels. This study will add to the literature by empirically assessing the dependence structure between South Africa and Advanced Economies (AE) stock prices, to determine if diversification is still possible. These are represented by the Johannesburg Securities Exchange (JSE TOP 40 index for South Africa, the Standard and Poor's 500 (S&P 500) for the United States, Financial Times Stock Exchange 100 (FTSE 100) for the UK, and the Deutscher Aktienindex for Germany. This study will model the dependence structure using both static and time-varying copula methods. The time-varying copula model specification is adapted from the Generalized Autoregressive Score (GAS) model of Creal et al (2013). This study finds little evidence to support the benefit of diversification between South Africa's stock market and advanced economies. The results are consistent with the stylised facts in the literature: asset returns are asymmetric and leptokurtic and the dependence between markets intensifies during crisis periods. The findings of this study are consistent with prior literature on African markets (Mensah and Alagidede, 2017, Bello et al, 2022).

Key Words: Association, Dependence, Copula, Conditional Tail-Dependence, Diversification, Contagion

JEL Code: F3, G11, G15

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1. **Introduction**

The nature of the association, or dependence structure, between international stock markets explains the relationship between these markets and the effects they have on one another. Studies of international stock market behaviour suggest, that during periods of high volatility the extent of association between these markets is higher (Longin and Solnik, 2001, Chesnay and Jondeau, 2001, Ang and Chen, 2002, Ang and Bekaert, 2002, Karolyi *et al*, 2003). The dependence structure plays a critical role in asset allocation, risk diversification, as well as the formulation of economic policy regarding financial stability. The association between stock markets is subject to change through time due to evolving market conditions (Longin and Solnik, 1995). The literature on the behaviour of joint stock market distributions identifies several distinct characteristics of this association; including asymmetric distributions or skewness, and excess kurtosis implying that there is greater activity in the tails of a distribution (Patton, 2004). Asymmetric dependence is best described as a differing degree of association between the upper and lower tails of the distribution of returns.

Contagion is propagated through well-established channels; including trade channel dependency, bank balance sheets, and portfolio investment.[1] However, there is no agreed upon econometric technique that appropriately measures contagion or interdependence.[2]This study uses extreme-value theory to measure the dependence structure between financial markets, capturing the asymmetric behaviour of financial time series. Modern portfolio theory is propped up by the assumption of normality in asset returns bolstering the linear correlation coefficient as a measure of dependence or association between two assets. However, this measurement as shown in Embrechts *et al* (2002) would be misleading in the presence of non-normality as it is only able to explain the linear properties in a relationship without consideration for the entire variation of the dependence structure.

The marked increase in dependence structures among advanced economies has seen investors turn to emerging markets (EMs) for diversification benefits during periods of economic downturn

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[1] See King and Wadhwani (1990), Barberis et al (2005) for a detailed summary of the channels through which risk and uncertainty are transmitted.
[2] Measures include but are not limited to, probability analysis, cross market correlations, vector autoregressive models and extreme value theory.

(Christofferson et al, 2012). Raising the question, whether EMs particularly those in Africa have seen an increase in interdependence as their markets become more sophisticated and financially liberated?[3] This study investigates the dependence structure of the Johannesburg Stock Exchange Top 40 (JSE TOP 40) and selected advanced economy (AE) share price returns over the period 2000-22, a sample that includes periods of both market turmoil: The Great Financial Crisis (GFC), European debt crisis, Covid 19 and the Russia-Ukraine war, and tranquility. The AEs are represented by the Standard and Poor's 500 (S&P 500) index for the USA, the Financial Times Stock Exchange 100 (FTSE 100) index for the United Kingdom, and the Deutscher Aktienindex (DAX) index for Germany.

This research study will address the following question. What is the nature of the dependence structure between the JSE TOP 40 index and AE share price returns? The key sub-questions for this inquiry are: How does the dependence structure vary through time and for different markets? How do these findings compare with the preceding literature? Are asymmetries present in the dependence structure and how does this influence the behaviour at the tails of the distribution? Do we find strong evidence of contagion from AEs to the JSE TOP 40 during periods of financial crises? What are the driving forces behind this finding?

This study finds evidence of a strong, stable dependence structure between the JSE TOP 40 and AEs equity markets. There are marked increases in interdependence during periods of financial crisis supporting the existence of contagion between these markets. Furthermore, regarding diversification benefits – there is little to no evidence supporting the potential for risk minimisation for a foreign investor in one of the AEs in this study.

The research report is structured as follows. Section 2 provides a theoretical review of copulas and dependence structures, their application in empirical work and how they relate to the study of financial contagion. Section 3 sets out the research methodology, and Section 4 provides a preliminary analysis of the data. Section 5 presents the empirical results, and Section 6 provides further analysis regarding conditional tail dependence. Section 7 concludes.

[3] The evidence on African markets for both intraregional and global interdependence have shown a significant increase in comovements and reduction in the potential for diversification (Gil-Alana et al, 2018, Abakah et al 2021, Bello et al, 2022)

2. Literature Review

Building upon the theoretical discussions of financial contagion and dependence, this section considers the measurement of dependence and the extent of associations between financial equity markets. Longin and Solnik (1995) define the dependence structure as a description of the risk relationship between assets and estimate the risk. The measurement and management of risk are fundamental in portfolio analysis. From the seminal paper on portfolio theory by Markowitz (1952), the relevance and implication of risk exposure to wealth gains have been of interest to investment practitioners and policymakers. Grubel (1968) countered the home bias of international finance by shedding light on the welfare gains that could be had through international portfolio diversification. The existing literature on portfolio theory and asset allocation has focused on diversifying risk on a global scale to reduce the investor's exposure (Black and Litterman, 1992). A sophisticated understanding of risk and how to measure it is relevant to practitioners and policymakers alike to mitigate the potentially devastating effects of financial contagion and spillovers.

Modern portfolio theory generally assumes that asset returns follow a Gaussian distribution (Ane and Kharoubi, 2003). The correlation coefficient measures the strength and association of a linear relationship between two variables. The popularity of the simple linear correlation measure in the literature is partly due to its ease of computational ease and suitability to the assumption of normality. However, its use as a measure of dependence for ‘‘extreme’’ observations is problematic (e.g. Embrechts *et al*, 2002). In the instances where normality does not hold, and the distribution is far from elliptical, linear correlation fails to explain the entire variation of the dependence structure resulting in it being a misleading estimate. A further drawback of linear correlation is that it is not invariant under positive nonlinear transformations, such as logarithmic transformations (Embrechts *et al*, 2002). A convenient way around these problems is to adopt a conditional modelling approach such as the Dynamic Conditional Correlation (DCC) GARCH model of Engle (2000). A great proportion of the literature has turned to GARCH models as a tool to measure the magnitude and direction of volatility transmissions (Chesnay and Jondeau, 2001). However, the drawback of these models is their reliance on the assumption of multivariate Gaussian returns that do not explain the asymmetry and nonlinearities present in financial data (Patton, 2006).

2.1. Financial Contagion

Economic policymakers have long been interested in the communication, transmission, and transfer of shocks during economic crises. Following the seminal paper of King and Wadhwani (1990), a large body of literature exists to explain, decipher, and interpret both theoretically and empirically the phenomenon of financial contagion. The time-varying nature of dependence structures and the strength of financial linkages may interest policymakers from a perspective of spillovers and the transmission of shocks from financial markets into the real economy. While there is a large literature on contagion, there is no consensus on what the terms mean. Definitions range from an increase in cross-market linkages ex-post a shock in one country (Forbes and Rigobon, 2002) to irrational co-movements that are residual after fundamental and rational investor behaviour co-movements have been properly accounted for (Karolyi, 2003). Rigobon (2019) discusses the tenuous relationship between interdependence, spillover, and contagion, indicating little to no distinction can be made and if there are any what does it matter? Diebold and Yilmaz (2016) avoided the formalities of definition and attempted to measure the magnitude and significance of contagion rather than engaging in theoretical rhetoric.

Contagion is transmitted through various channels and linkages both in the financial and broader economic markets. The primary channels in the literature that propagate contagion are existing trade linkages, portfolio investors minimising downside risk by shorting assets or offloading debt of certain countries exacerbating capital flow problems, international bank linkages and financial channels have strengthened interdependence and resulted in complicated intra and interregional networks (Forbes, 2012). Barberis *et al* (2005) postulate theories of co-movement, rational investor behaviour in which fundamental asset values are maintained and an alternative where rigidities, frictions and irrational behaviour disassociated with fundamentals. In assessing investor behaviour, Shiller (2005) and his theory of bubble formation concluded that psychological biases, herding behaviour and positive sentiment in the market generate comovements in asset prices. However, with this information it therefore must be possible through sound policy formation to insulate against contagion and by appropriately influencing expectations (Makrelov *et al*, 2019). The strength of financial interdependence and linkages has steadily risen due to globalisation and the diversification benefits associated with new emerging markets. With this growing complexity in mind, we therefore postulate, what policy is a good one? How can a country avoid these spillovers, and is such a thing possible? What can be found in the literature is that policy safeguards and principles can be implemented to dampen the impact of these transmissions through the financial sector towards the real economy. Policies which ensure financial market liquidity, debt sustainability, and bank balance sheet stability are closely related to regulating the risk-taking behaviour of banks (Forbes, 2012). Controls on capital flows may intensify the magnitude of distortions in the economy leading to illicit financial flows as investors shirk and avoid these controls.

Studies on African markets present evidence of weak dependence with advanced economies (Alagidede, 2009, Adams and Opoku, 2015, Mensah and Alagidede, 2017). Bello *et al* (2022) find evidence that African equity markets have become more integrated and dependent beyond intraregional linkages. Gil-Alana *et al* (2018) used a cointegration approach to assess the strength of financial interlinkages between many African and AEs. The results suggest there is a great degree of persistence, implying permanent effects from external shocks. However, the evidence suggests low cointegration between African markets and AEs, leading to the possibility of potential gains through diversification. The findings in Gil Alana *et al* (2018) is not consistent with the recent literature on African market interlinkages, particularly Mensah and Alagidede (2017) and Bello *et al* (2022). South Africa's financial market is highly developed and mature implying deeper, well-established links with foreign financial markets. The South African financial market is reliant on these interconnections for capital inflows, which are necessary for liquidity, leverage and financing for public and private institutions. This greater level of financial development coupled with long standing relationships with AEs like the USA, UK and Germany are likely to prop up debt and equity flows into South Africa's financial market. Makrelov *et al* (2019) highlight the impact on the South African economy of a capital flow reversal. The authors find evidence of a reduction in financial market liquidity, exacerbating risk perspectives, which generates a squeeze on credit. The credit squeeze widens the spread between lending and deposits leading to a fall in equity prices. Foreign investor sentiment regarding risk, liquidity and stability are drivers of comovements in South Africa's financial market.

This studies a priori expectation is that the dependence between the JSE TOP 40 and FTSE 100 will be the strongest based on historical linkages. Such as the FTSE/JSE All Share Index which was a joint launch fixed income index encompassing the Bond index and Inflation-Linked index.

This partnership increased exposure and access to South African debt. Further financial linkages include joint company listings on the London Stock Exchange (LSE) and the JSE. Therefore, we expect similarities in investors (foreign and domestic) decisions regarding jointly listed companies equities resulting in stronger comovements between these two financial markets. A similar but less robust result is expected for the association between the DAX and the JSE TOP 40. This study does not expect a similar result for the S&P 500 and JSE TOP 40, rather the primary source of comovements between these two markets will be the policy position of the US Federal Reserve. Movements in the US federal funds rate affects real variables such as inflation, potential GDP and trade which have implications on the likelihood of contagion (Barberis *et al*, 2005, Forbes, 2012). The position of the US dollar as a vehicle currency for all international trade transactions implies that the stability and strength of the US dollar is crucial. This generates persistence in US monetary policy dominance, policy actions impacting the stability and value of the US dollar have implications for trade globally.

The work of Helene Rey (2013) provided further evidence on the cyclicality of financial capital flows and its comovement with global assessments of risk and uncertainty. The author found evidence of US monetary policy dominance as movements in the federal funds rate had a direct effect on global risk appetite, which had implications on the direction and magnitude of international financial capital flows. Heightened financial linkages and capital mobility act as a constraint on monetary policy sovereignty, reducing monetary independence and inhibit a countries ability to insulate itself from transmission channels that directly affect financial asset prices, liquidity, and leverage. Global financial cycle theory ties in with the theory of financial contagion, spillovers, and interdependence as well as the literature on bubble formation in asset prices. Miranda-Agrippino and Rey (2020) used a dynamic factor model to capture the effects of US monetary policy on the rest of the world. The authors found that contractionary policy led to tightening of credit channels domestically resulting in falling asset prices, furthermore, the shock induced greater uncertainty about risk globally leading to a contraction in price of risky asset prices, reducing liquidity and deleveraging financial institutions globally. These findings are consistent with previous literature that the US federal funds rate is an important determinant of global risk perceptions, which directly impact the mobility and flow of capital assets globally.[4]

[4] See Barberis *et al* (2005), and Forbes (2012).

The measurement of contagion in the literature is dependent upon the boundaries of the definition used. The most common methods, tools and procedures include probability analysis (Forbes and Warnock, 2012, Constancio, 2012), correlation analysis (Forbes and Rigobon, 2002), VAR models and variance decompositions (Diebold and Yilmaz, 2016), GARCH models (Chesnay and Jondeau, 2001, Dungey and Renault, 2018) and extreme value analysis (Rodriguez, 2007). Each of these approaches has been capable of providing evidence that supports the existence of contagion during crisis periods. However, there are also a set of drawbacks such as weak endogeneity and strong assumptions regarding the distribution of asset returns that do not correspond to the behaviour of financial data. The study of correlation breakdowns in equity returns exhibit contagion in the spirit of Forbes and Rigobon (2002). Overall dependence and cross market linkages have increased however these studies suffer in that these measures are too linear and do not properly explain all the variation in the comovements of asset prices. Rodriguez (2007) showed using an extreme value approach that a breakdown in the structure of the tail dependence via a switching copula model is a facet of the contagion story.

2.2. Copulas and Dependence

The standard definition of a copula is that it is a multivariate distribution with uniformly distributed marginals over the [0,1] domain. This definition is quite dry and does not provide intuition regarding copulas. We can view copulas as links between distinct univariate distributions, which form new multivariate distributions. This is why they are often termed dependence functions (Joe, 1997).[5] Sklar's theorem is the most crucial result for using copulas as a tool to model dependence structures (Sklar, 1959). This states that if the joint distribution H is an n-dimensional distribution function with marginal distributions $F_1, \dots, F_n$, then there exists a unique function C such that,

$$H(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n))$$

If the marginal distributions F_i are all continuous, the copula is a unique n-dimensional distribution with marginals F_i. What Sklar's theorem implies is that for a continuous multivariate distribution

[5] For a deeper understanding of the mathematical background of copula functions, Joe (1997) and Nelsen (2006) are fundamental resources.

the marginals are separable from the multivariate distribution. This separability implies that the copula or dependence structure is independent of any scaling, transformation or manipulation of the univariate marginals, a property that is relevant when measuring nonlinearities and asymmetries in dependence structures.

The copula provides greater flexibility to model developments since the marginal distributions and the copula need not be the same type (Fan and Patton, 2014). This property is related to the inverse of Sklar's theorem, which infers that two univariate distributions not of the same type can be linked together to form a valid copula (Patton, 2006). The marginal $F(X) \sim U(0,1)$ undergoes a probability integral transform over the $[0,1]$ domain, which keeps the marginals consistent during the copula estimation while separating the uniform variables from the marginals. This separability makes the copula invariant to the marginals' linear and nonlinear positive transformations. If U is the probability integral transform of $F(X)$, define Vas the probability integral transform of $G(Y)$. If the joint distribution of $(X, Y) \sim H = C(F, G)$ then it implies that $(U, V) \sim C$.

When it comes to the measurement of dependence, copulas are a suitable tool and possess unique properties that remove bias and inaccuracies in the measurement statistic. The most important of these properties is separability, which implies that any scaling, transformation of the univariate marginals has no bearing on the copula. An alternative measure, perhaps most relevant in portfolio optimisation, is Pearson's correlation coefficient which this study will refer to as linear correlation. The popularity of linear correlation as a measure of dependence is largely due to its ease of computation and its natural application to symmetrical distributions with Gaussian distribution properties. However, as shown in Embrechts *et al* (2002), the linear correlation coefficient is often misleading for the following reasons: it can only measure the linear component of the relationship between variables, and it is not invariant to nonlinear transformations of the marginal distributions. The presence of nonlinearities, asymmetries and excess kurtosis in the data renders the linear correlation coefficient as an unsuitable measure which will produce misleading results.

There exists a large family of copulas that can be broken down into two branches, namely elliptical and archimedean copulas. Common elliptical copulas are the Gaussian and Student's' t. However, in the empirical analysis of financial asset data, elliptical copulas do not have the shape and necessary kurtosis to accurately explain the behaviour of the data. Economic time series data is characterised by asymmetric returns, skewed distributions, and heavy tails (excess kurtosis).

Exceptions include Hansen's skewed t (Hansen, 1994) and the generalised skewed t of McDonald and Newey (1988), which have been applied successfully to financial data to fit the skewness and excess kurtosis characteristics.

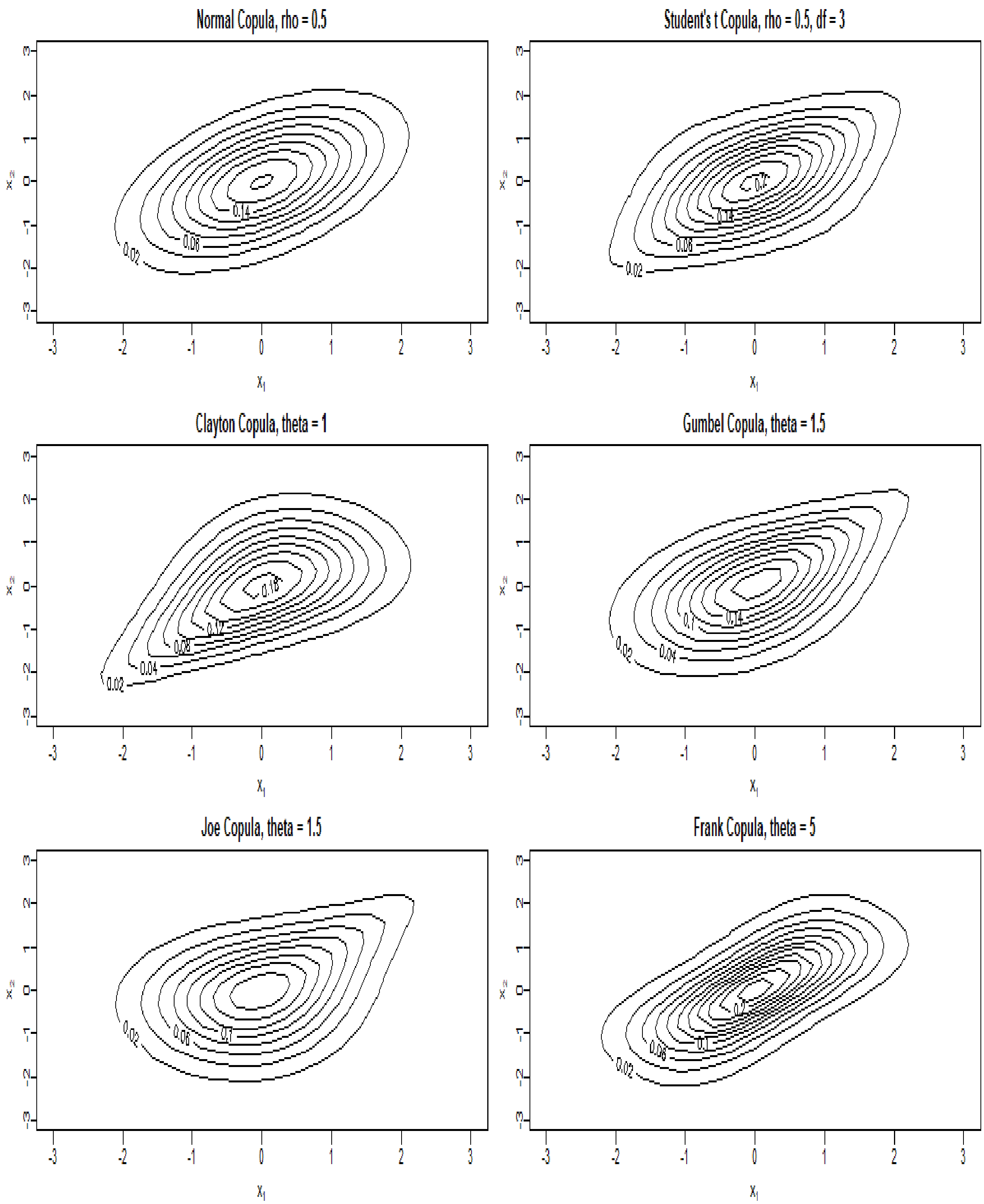
Archimedean copulas offer greater variety in the shape and kurtosis of their distributions, making them more suitable candidates for empirical work on financial time series data. These copulas are generated by functions and not multivariate distributions like elliptical copulas. If φ is a continuous and decreasing function from $[0,1]$ to $[0,\infty]$, $\varphi(1) = 0$ and φ^{-1} is the inverse function, then the copula (C) with marginals u and v takes the following functional form,

$$C(u,v) = \varphi^{-1}\big(\varphi(u) + \varphi(v)\big)$$

The Gumbel and Clayton copulas are examples of copulas with asymmetric shapes and greater dependence in the tails. The Gumbel copula exhibits a greater dependence in the upper tail of the distribution, whereas the Clayton copula has a greater dependence towards the lower tail. However, both possess the property of exchangeability which allows for the distribution to be rotated 180 degrees. This property of exchangeability not only gives the researcher more flexibility it enables the comparison between both the Gumbel and Clayton copula directly.

In Figure 1 we present various elliptical and archimedean distributions, each calibrated with the same linear correlation coefficient of 0.5. The importance of tail dependence, its measurement and magnitude are clear. We can also see that assumptions of normality and measures that only consider the linear component of a joint distribution are not suitable when describing joint multivariate distributions (Patton, 2004). This is evident in each subplot as the contours are limited in how far they stretch into the extreme values. Suggesting that the assumption of Gaussian marginals is inappropriate when assessing joint distributions of financial asset data. The middle panels show that the Clayton and Gumbel copulas have contours that are asymmetric and place a greater emphasis on the negative and positive quadrants making them suitable for measuring the clustering of extreme events such as a market crash.

Figure 1: Contour Plots of distributions with Gaussian marginals and a linear correlation coefficient of 0.5



2.3. Empirical Review of Copulas

The application of copula models to financial time series data is far from novel. Even prior to Patton (2006)'s seminal paper on the application of copulas to exchange rate dynamics, many applications in the literature applied copula models to measure the dynamics of risk in insurance and portfolio management. These earlier adoptees include Li (2000), Clemen and Reilly (1999), and Cherubini and Luciano (2000). However, a common trend in these earlier works was the persistent assumption of normality, A well-known example of this is Li (2000), who used survival copulas to measure the dependence between two financial assets to determine the likelihood of both assets defaulting simultaneously. The Gaussian copula was used to construct the dependence relation between the time-to-default events. The mainstream adoption of the Gaussian copula to price credit products such as Collateralised Debt Obligations (CDO) and Credit Default Swaps (CDS) can be attributed to Li (2000). The drawback of Gaussian models is that they do not account for any tail dependence and instead provide a conservative assessment of clustering around extreme events. The implication is devastating losses for portfolios comprised of assets with strong associations during market downturns due to the underestimation of the risk exposure. Other applications of copula models include portfolio risk measurement and volatility (Messaoud and Aloui, 2015, Jondeau and Rockinger, 2001, Jondeau and Rockinger, 2006, McNeil, 2021, Patton, 2004), forecasting conditional volatility and tail dependence (Sokolinskiy and van Dijk, 2011, Patton, 2012), structural breaks and regime changes (Rodriguez, 2007) as well as VaR and CoVaR (Mensah and Alagidede, 2017).

Investigations of dependence structures between financial assets have found that they are time-varying and tighten during extreme events, implying a stronger dependence between financial assets during market crashes (Ang and Chen, 2002). Clustering of observations during extreme events is referred to as tail dependence. Time-varying tail dependence implies the dependence structures between financial assets fluctuate due to systematic risk and macroeconomic conditions. The *a priori* expectation is that during the Great Financial Crisis (GFC) and the Covid 19 pandemic, the dependence structures will tighten, resulting in significant comovement and a greater association at the distribution's tails. A potential explanation for this behaviour is provided by Brunnermeier (2008), who argues that investors tend to ignore fundamentals and pursue a strategy motivated by market feedback. The broader market is characterised by herding behaviour in that they follow signals or positive feedback from institutional investors or high-volume traders. Herding behaviour is prevalent during extreme events such as recessions or market booms, all investors tend to pursue similar strategies simultaneously.

Breymann *et al* (2003) were among the early researchers to apply copula models to high-frequency financial data. Their findings were that the data exhibited heavier tails than the normal distribution with dependence structures that were nonlinear and asymmetric. These findings tie into the literature from that period (Ang and Chen, 2002). For example, Jondeau and Rockinger (2006) used a Copula-GARCH approach not too dissimilar to Patton (2006). However, Jondeau and Rockinger (2006) found that the dependence structure was greatly affected when the returns between the equity markets were moving in the same direction. These findings substantiate the earlier work on asset market correlation tightening during turbulent periods and the tendency of markets to crash together (Chesnay and Jondeau, 2001).

For papers with more of an economic focus, one can refer to Patton (2006). Here the focus is on whether asymmetries in exchange rate dependence exist and whether they result from central bank behaviour. The study used the US Dollar, Japanese Yen, and Deutsche Mark (DM) for the empirical analysis and focused on two alternative policy strategies, competitiveness of exports and price stability. The findings suggested maintaining the competitiveness of Japanese exports to the US with rival exports from Germany was a contributing factor to asymmetric in the exchange rate dynamics. The Japanese central bank would depreciate the Yen against the US dollar whenever the DM depreciated against the US dollar. Alternatively, a policy focused on price stability meant the Japanese central bank would appreciate the Yen against the US dollar whenever the DM appreciated against the US dollar. The central bank of Japan and its preference between export competitiveness and price stability produced asymmetries in the dependence structure.

There is also research that analyses the dependence between emerging African nations and developed ones from an African perspective, using copulas. Mensah and Alagidede (2017) assessed the dependence of many African countries with the US and the UK. The authors determined using a Conditional Value at Risk measure that no significant spillovers were present from advanced markets into African markets. The only exception being South Africa, the asymmetric nature of returns suggested differing behaviour during market booms and busts. Their results are consistent with later work on African financial market interdependence (See Bello *et* *al*, 2022). Abakah *et al* (2021) find evidence of asymmetric dependence in bond markets using a pair copula framework for developed economies. The results suggest a degree of heterogeneity in the magnitude of the dependence structure across pairs, with the eurozone being the most interconnected with all the other bond market pairs. A further study on the risk-return paradigm in international equity markets looked at many advanced economies including South Africa (Abakah *et al*, 2022). This study used a Markov-switching copula to account for geopolitical risks and uncertainty in the financial equity markets. The evidence suggests that risk and return dependence structure is positive for many equity markets including South Africa, implying that risk and return move in the same direction. Additionally, only four out of the fifteen markets assessed possessed negative dependence structures in the risk-return paradigm. Abakah *et al* (2022) further assert that macroeconomic factors and global risk perspectives negatively affect the risk-return paradigm, leading to a redirection of diversification benefits.

3. Research Methodology

3.1. Modelling the conditional marginal distributions

This study will adopt an approach similar to that of Patton (2012), who allows each series to have a potentially time-varying conditional mean and variance. For each index, the conditional mean will be modelled using an autoregressive moving average (ARMA(p,q)) model. The conditional variance will be modelled using a variation of the generalised autoregressive conditional heteroskedasticity (GARCH(p,q)) model (Bollerslev, 1986). We transform each of the indices into continuously compounded returns (Y_t), taking the log-differences of their value (P_t) over the interval $[t-1, t]$.

$$Y_t = \log\left(\frac{p_t}{p_{t-1}}\right) \tag{1}$$

Equations 1 and 2 represent the specifications for the conditional mean and variance, respectively. In the mean equation the dependent variable is simply the index returns (Y_t).

The order of both the mean and variance equations is determined using an information criterion. The Bayesian-Schwartz information criterion or BIC is preferred here as it performs well in larger samples and does not run the risk of overspecification, producing a parsimonious model.[6]

$$Y_t = c + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \varepsilon_t \text{, Where } \varepsilon_t = \sigma_t Z_{\nu,t}, \ \tilde{Z}_{\nu,t} \ i.i.d\ SkewT(0,1) \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} + \sum_{i=1}^{p} \theta_i \sigma_{t-i}^2 \tag{3}$$

This step of modelling the marginal distributions serves as a filter for our econometric procedure. The purpose is to extract the standardised residuals, obtained by dividing the residuals at each point in time t by the conditional standard deviation obtained in our variance equation. This

[6] See Chakrabarti and Ghosh (2011) for a detailed explanation on model selection.

technique is useful when encountering data with heterogenous volatility as we can easily identify potential outliers and extreme observations.

3.2. Paramaterising the Standardised Residuals

Patton (2012) applies a variety of methods to testing time-varying copula models. These include fully parametric, semi-parametric and non-parametric estimation methods. This study will adopt a semi-parametric approach for the following reasons. The data exhibit a leftward skewed shape with excess kurtosis implying we need to determine a good distributional fit that can accommodate these characteristics during the marginal distribution modelling. Following Hansen (1994), the higher moments of the distribution should contain features of the conditioning information set. Extending this analysis to the standardised residuals obtained in the marginal modelling process, the data can be parameterised to preserve the leptokurtic and asymmetric distribution. The skewed t distribution of Hansen (1994) is a suitable choice as it accommodates for the leftward skewing data by placing a greater emphasis on the lower tail of the distribution but does not neglect any upper tail clusters.

3.3. Testing for Asymmetric Dependence

One of the major questions postulated in this study is: to what extent does behaviour differ during extreme events? Do we find evidence of a stronger association - tightening dependence structure - during periods of economic recession versus a period of economic tranquility or boom? Our *a priori* expectation is that during the GFC and Covid 19 the dependence structures between the JSE TOP 40 and the AEs' indices would have been tighter and asymmetric (leaning towards a stronger negative tail dependence in comparison to the upper tail). To investigate this, we test the null hypothesis that $\tau^L = \tau^U$ versus the alternative that they are significantly different. The test will be based upon the chi-squared distribution, however this initial analysis may not provide us with the entire story as it covers the entire sample period without consideration for structural breaks. Therefore, it is useful to assess whether the upper and lower tails are equivalent or unique as a preliminary statistic regarding the joint distributions. However, it should not be our only means of answering whether asymmetries are present in the joint distributions. The data used for this test will be the standardised residuals from the marginal modelling process prior to any parametric or probability integral transform.

3.4. Modelling the Conditional Copula

The dependence or association between the JSE TOP 40 and the AEs' indices will be measured using copulas. The conditional copula model is an extension to Sklar (1959) and takes on the following functional form,

$$H_t(x|\mathcal{F}_{t-1}) = C_t(F_{1,t}(x_1|\mathcal{F}_{t-1}), \dots, F_{n,t}(x_n|\mathcal{F}_{t-1})|\mathcal{F}_{t-1}).$$

This specification requires consistency in the way in which the marginals and copula are conditioned. Problems will arise if the conditioning is inconsistent such as nonsense regressions and invalid or unsuitable results. A further issue concerning a study such as this one that is common to empirical studies on financial time series data is the relevance of all the information. In many instances the datasets may not be functions of one another, implying that information in the conditioning set may be relevant to one but not all series jointly. A way around this potential problem is to define subsets within the broader information set to uphold the copula models validity but also to ensure that the relevant information to the individual variables is preserved (Patton, 2009). If $\mathcal{F}_{i,t-1} \in \mathcal{F}_{t-1}$, we can model the marginals using the subsets and then use the broader conditioning information set for the copula. This study is particularly interested in the behaviour of the dependence structure at the tails of a joint distribution. These clusters of observations at the extremes of a distribution inform us as to the behaviour of equity markets during market turmoil and periods of tranquility. Tail dependence between two variables can be explained mathematically in the following way,

$$\lim_{\vartheta \to 0} \Pr(U \leq \vartheta | V \leq \vartheta) = \lim_{\vartheta \to 0} \frac{C(\vartheta, \vartheta)}{\vartheta} = \tau^L$$

$$\lim_{\vartheta \to 1} \Pr(U \geq \vartheta | V \geq \vartheta) = \lim_{\vartheta \to 1} \frac{(1 - 2\vartheta + C(\vartheta, \vartheta))}{(1 - \vartheta)} = \tau^U .$$

Our *a priori* expectation concerning the constant conditional copulas is that the archimedean family - in particular the Gumbel, Clayton and Rotated Gumbel - will perform better in explaining the extreme clustering at the tails due to their shape. These copulas are more suitable for capturing properties of asymmetric dependence and excess kurtosis. The elliptical copulas applied in this study will be the Normal and Student's t, with the expectation that the latter will be a better fit than the former due to its fatter tails. However, we do not expect the Student's t copula to be a better fit to the data than any of the archimedean copulas mentioned above. The copulas will be estimated using Maximum Likelihood, applied in stages (multi-stage maximum likelihood estimation), which renders the best results when dealing with time-varying conditional copulas (Patton, 2012).

3.5. Generalized Autoregressive Score Models

The issue of time-varying dependence is driven by the need to understand the dynamic evolution of the relationship between two equity markets. The first paper to model time-varying dependence using copulas was Patton (2006), which developed an autoregressive evolution equation that governs the variation in the parameters of the copula. The method has some computational difficulties, particularly regarding implementing a forcing variable in the evolution equation. More recently, the methodology used to estimate time-varying copulas has developed further, following important papers by Creal *et al* (2011) and Creal *et al* (2013) that develop a model for updating parameters over time using the scaled score of the log likelihood. These Generalized Autoregressive Score (GAS) models have a wide range of applications pertaining to nonlinear datasets. By incorporating the scaled log likelihood, which essentially acts as a line of steepest ascent these models provide a much clearer and computationally viable fit to time-varying copula models. The GAS models are specified below in equations 3 through 6.

$$\kappa_t = f(\lambda_t) \tag{4}$$

$$\kappa_{t+1} = \eta + \varphi\kappa_t + \mu I_t^{-\frac{1}{2}} s_t \tag{5}$$

$$s_t \equiv \frac{d}{d\rho} \log c(u_t, v_t; \lambda_t) \tag{6}$$

$$I_t \equiv E_{t-1}[s_t s_t'] \tag{7}$$

Equations 5 and 6 are the scaled score of the copula log-likelihood and the information set. Equation 4 is the primary equation, and models the evolution of the copula parameter as a function of its past value, a constant a forcing variable made up of the scaled score and the information set.

This model provides insight into the time-varying conditional tail dependence that is of interest to this study. Determining whether the JSE TOP40 and AEs' equity markets dependence is significantly different during periods of economic downturn, if there is a tightening of the relationship and whether we expect any spillovers or transmissions emanating from one of these markets into South Africa.

4. Data and preliminary analysis:

4.1 Overview of the data

The study uses daily closing prices over the period from 3 January 2000 to 28 February 2022, providing a sample that includes periods of both market turmoil and tranquility. The data were sourced from *Bloomberg*. On days in which no trading took place, the previous day's closing price was carried over. To serve as proxies for the equity markets of South Africa, USA, UK, and Germany the following benchmark indices were chosen; JSE TOP 40, S&P 500, FTSE 100 and the DAX. Each index is value weighted based on the market capitalisation of the constituent companies. The JSE TOP 40 consists of the 40 largest companies actively trading on the JSE, furthermore this index accounts for close to 100 percent of the market capitalisation and liquidity. The S&P 500 is the value weighted index of the 500 largest companies trading on both the New York stock exchange and the Nasdaq. The FTSE 100 consists of the 100 largest companies on the London stock exchange, and the DAX is made up of the 40 largest companies trading on the Frankfurt stock exchange. The reason for these choices is that these indexes account for most of the trade that takes place on these markets from a volume standpoint.

Figure 2: Plot of each series through time.

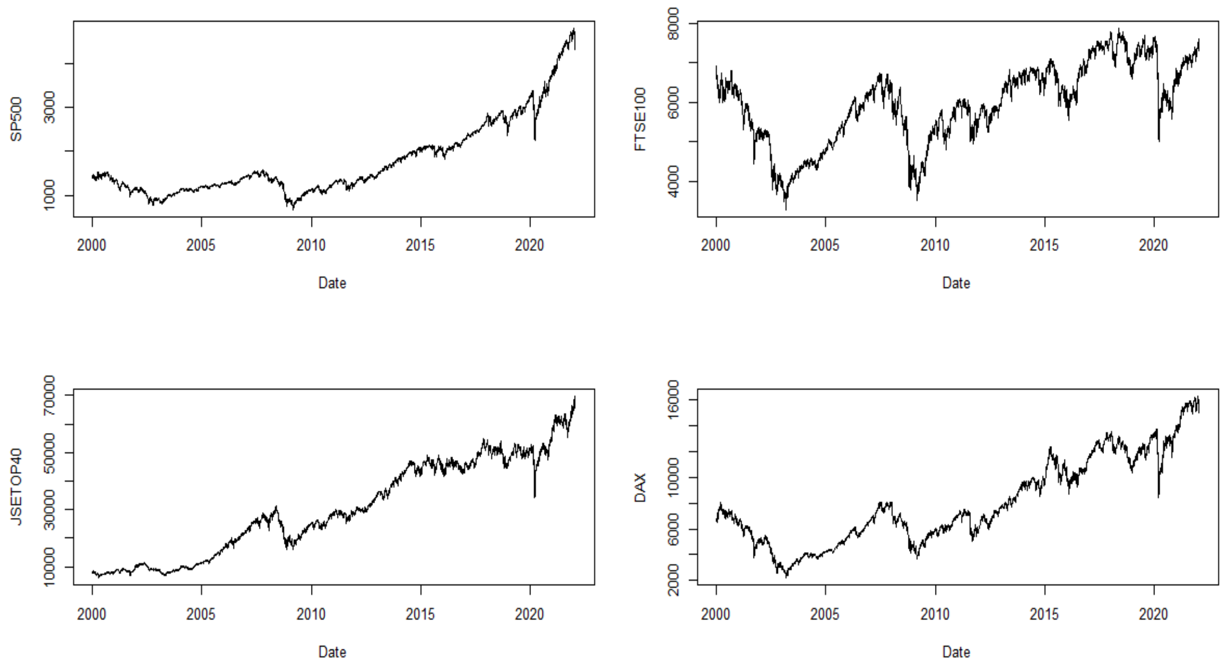


Figure 2 provides plots of each index over the sample period. From this, we can see that each index has trended upwards over time with expected declines during major financial crises (these include the GFC in 2008, European Debt Crisis in 2013 and the Covid-19 pandemic starting in 2020).

Table 1 shows the descriptive statistics for the returns of each index. We can see that the average returns over the sample are quite low, between 0.01 and 0.02%. The market with the greatest variance in its returns is the DAX, although it is only marginally greater than the JSE TOP 40. In terms of the shape and symmetry of the return distributions it is evident each index is negatively skewed, the S&P 500 having the largest skewness parameter at -0.4029 and the DAX possessing the lowest at -0.1806. The excess kurtosis measure (kurtosis – 3) indicates that the return distribution for each market appears to have 'fatter' tails than normal (leptokurtic tails), with the S&P 500 having the largest kurtosis and the JSE TOP 40 the least excess kurtosis. These findings of asymmetry and leptokurtosis are in line with the 'stylised facts' provided by the existing literature. The excess kurtosis implies that there may be significant interaction between equity markets during extreme events, as there is a significant clustering of observations in the tails of the distributions.

Table 1: Descriptive Statistics

	S&P 500	***FTSE 100***	***JSE TOP 40***	***DAX***
Mean	0.0002	0.00001	0.0004	0.0001
Median	0.0003	0.000005	0.0001	0.0005
Minimum	-0.1277	-0.1151	-0.1045	-0.1305
Maximum	0.1096	0.0938	0.0791	0.1080
Std. Deviation	0.0122	0.0116	0.0129	0.0145
Variance	0.0001	0.0001	0.0002	0.0002
Skewness	-0.4029	-0.3247	-0.2530	-0.1806
Kurtosis	14.4449	11.5123	7.7602	9.0836

A preliminary analysis of the association between each market is provided by the matrix of linear correlation coefficients in Table 2. What is of importance in this correlation matrix is the association between the JSE TOP 40 and all the other markets. In line with our expectations the linear correlation between the FTSE 100 and JSE TOP 40 is the strongest followed by the DAX and the S&P 500.

Table 2: Linear Correlation Matrix

	S&P 500	***FTSE 100***	***JSE TOP 40***	***DAX***
S&P 500	1			
FTSE 100	0.5167	1		
JSE TOP 40	0.3685	0.5927	1	
DAX	0.5891	0.7831	0.5504	1

The lower correlation coefficient from the perspective of an American investor might be indicative of potential diversification benefits. However, bearing in mind the earlier warning that this measure of dependence is linear in nature and tells us a broad story over the entire sample and not during specific periods of turmoil, it would be potentially misleading to make inferences based solely on this.

4.2. Unit Root Tests

Figure 2 suggested that each series trends upwards over the sample period. From these plots it is difficult to determine whether these trends are deterministic or stochastic. Our expectation is that the process of transforming the data into returns using a log-difference is sufficient to ensure our data are covariance stationary. However, in performing our due diligence a test on the stability properties in levels and on the log transformation of the data was carried out using unit root tests. In Table 3 we report the results for the Dickey-Fuller GLS test of Elliot, Rothenberg, and Stock (1996). All the series were estimated inclusive of a trend term in the Dickey-Fuller equation, apart from the FTSE 100 which only retained the constant term, based upon the evidence in Figure 2. For the given sample size and maximum lag specification of four the critical values in the distribution were -3.48, -2.89 and -2.57, representing the values for the 1, 5 and 10 percent critical levels. The lag length for each series was chosen using the BIC.

Table 3: DF-GLS Tests on Levels and Returns of the series

	S&P 500	***FTSE 100***	***JSE TOP 40***	***DAX***
Levels	0.0782	-1.5824	-2.6527	-1.3456
Log-Difference	-8.1685**	-4.0260**	-33.7849**	-14.9088**

Significance at the 5% level is given by a superscript *, and significance at the 1 and 5% levels is given by a superscript **.

Our expectations about the stationarity properties of the data appear to be correct. All the series are integrated of order one; we cannot reject the unit root null in levels but after the log-difference transformation to generate returns we reject the unit root null at the 1% significance level.

5. Results:

5.1. Mean and Variance Model Estimates

For the purposes of modelling the conditional mean an ARIMA model specification was chosen. The selection process used a maximum lag of five autoregressive and moving average components respectively. The key metric for model comparison was the BIC. Therefore, the model which minimised the BIC was selected as the optimal marginal model for the mean. The results of the algorithm search are as follows; the S&P 500 was best represented by an MA(1) process, the FTSE 100 which appeared in Figure 2 to be the most volatile and susceptible to swings was chosen to be an ARMA(2,3), whereas the JSE TOP 40 and DAX simply included a constant in the mean (i.e. an ARMA(0,0)). The coefficients for these models are presented below.

Table 4: Mean model coefficients

	Constant	α_{t-1}	α_{t-2}	β_{t-1}	β_{t-2}	β_{t-3}
S&P 500	0.00019			-0.1140		
	(0.00015)			(0.0066)		
FTSE 100	0.00004	-0.6643	-0.4479	0.6329	0.3983	-0.1022
	(0.00031)	(0.05584)	(0.05011)	(0.05659)	(0.05069)	(0.0075)
JSE TOP 40	0.00037					
	(0.0002)					
DAX	0.00014					
	(0.0002)					

The values in parenthesis are standard errors, * represents significance at the 5% level.

From the table it is evident that all the parameters are significant at the 5 percent level and possess low standard errors.

The variance modelling process is a bit more complicated given the wide variety of ARCH and GARCH models. The models chosen for comparison were the ARCH(1), GARCH(1,1), GJR GARCH(1,1) and EGARCH(1,1) models. A constant volatility model was included as a benchmark for the other models since we expect the conditional volatility to be heteroskedastic and time-varying. The values for the BIC's of each of the models are presented in Table 5. The model which minimises the information criteria is the EGARCH(1,1), narrowly outperforming the GJR-GARCH(1,1). Surprisingly the constant volatility performed better than the ARCH(1) for the S&P 500 and FTSE 100, implying perhaps a lower degree of time-varying volatility.

Table 5: BIC Values for different Volatility Models

	S&P 500	***FTSE 100***	***JSE TOP 40***	***DAX***
Constant Volatility	-5.9982	-6.0907	-5.8628	-5.6240
ARCH(1)	-5.4453	-5.4641	-5.9221	-5.6974
GARCH(1,1)	-6.4914	-6.4792	-6.1233	-5.9867
GJR-GARCH(1,1)	-6.5301	-6.5162	-6.1460	-6.0274
EGARCH(1,1)	-6.5301	-6.5201	-6.1456	-6.0313

Values in the table are for the Bayesian-Schwartz Criterion.

With our volatility model chosen we can now proceed with estimation, the results of which are captured in Table 6 (see Appendix A for details regarding the GJR-GARCH model of Glosten, Jagannathan and Runkle (1993), along with the EGARCH model of Nelson (1991)).

Table 6: EGARCH parameters

	S&P 500	***FTSE 100***	***JSE TOP 40***	***DAX***
$\boldsymbol{\omega}$	-0.2381	-0.1578	-0.1723	-0.1872
$\boldsymbol{\varphi}$	-0.1406	-0.1173	-0.0924	-0.1227
$\boldsymbol{\gamma}$	0.1462	0.1001	0.1352	0.1144
$\boldsymbol{\theta}$	0.9739	0.9826	0.9804	0.9784

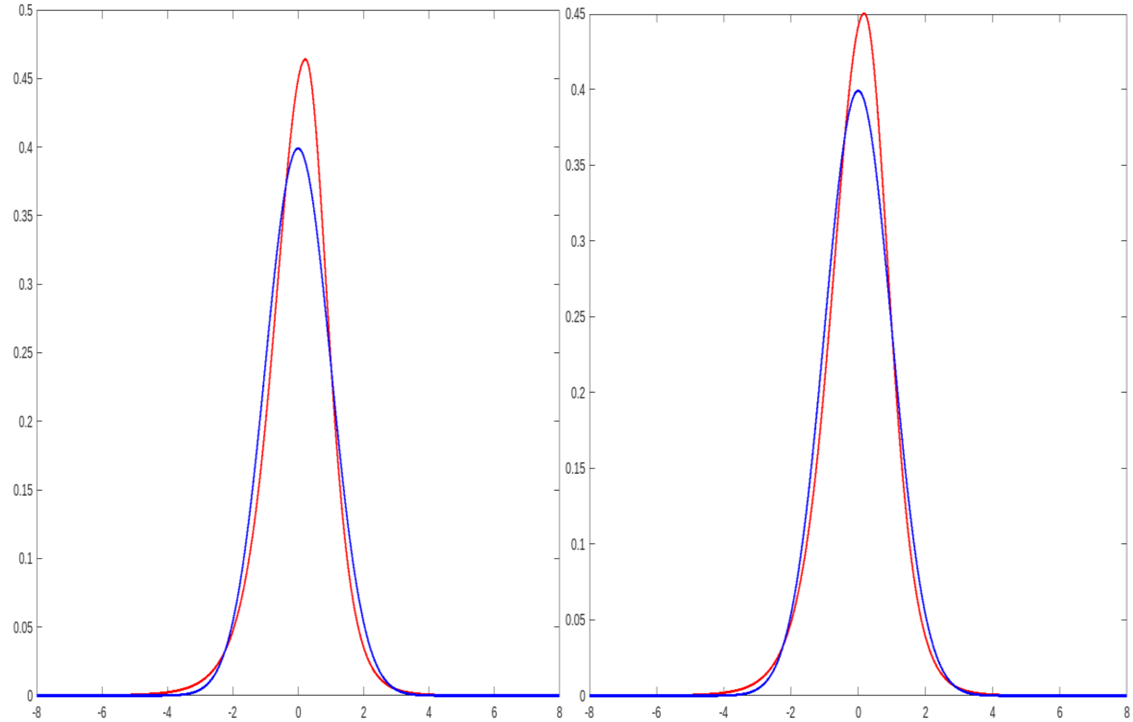
The parameter φ represents the leverage effect, essentially the impact of losses on the levels of future volatility. The S&P 500 and JSE TOP 40 have greater leverage effects than the FTSE 100 and DAX as we expect losses today to increase volatility in the S&P 500 and JSE TOP 40 by 0.23% and 1.94%. Note that the persistence in the volatility of the daily returns measured by $\gamma + \theta$ is found to be high, in line with Nelson (1991).

The final step of our marginal modelling is to estimate the standardised residuals. This is done by normalising the residuals in our conditional mean models by the conditional variance. Further diagnostic tests of asymmetric dependence mentioned in the research methodology are conducted on the standardised residuals. We also apply our parametric transformation using the skewed t of Hansen (1994) to account for the leftward skewing characteristics as well as the asymmetric dependence leaning towards the lower tail.

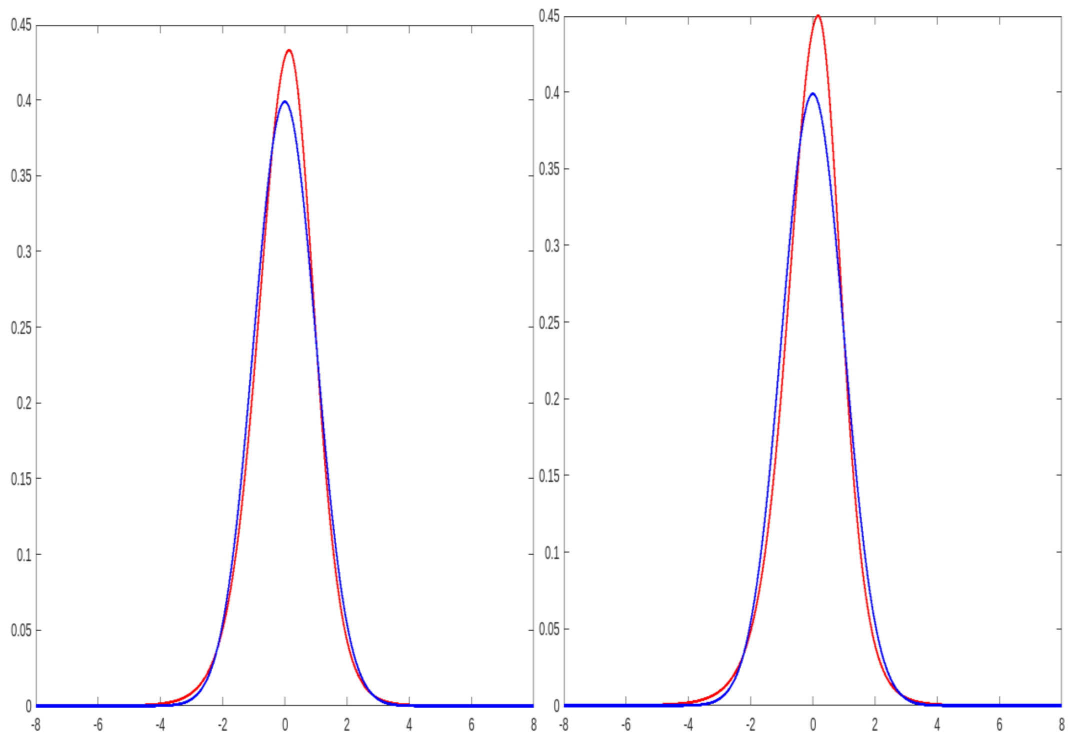
The shape and skewness of the paramaterisation can be observed in Figure 3. These figures graph the Skewed t standardised residuals (red curve) for all four indices against a superimposed Gaussian distribution (blue curve). The figures suggest that the standardised residuals have the properties of excess kurtosis and asymmetry, verified by the curve having a leptokurtic shape with a greater dependence on the lower tail of the distribution. Furthermore, we observe that the S&P 500 has the largest lower tail inferring greater clusters of observations during periods of negative returns, typically characterised by economic recessions.

Figure 3: Skewed t standardised residuals (red) vs Gaussian distribution (blue)

(a) S&P 500 (Left) and (b) FTSE 100 (Right)



(c) JSE TOP 40 (Left) and (d) DAX (Right)



In Table 7, we report tests of the null hypothesis of equality between the upper and lower tails of the joint distributions (at the 0.025, 0.05, 0.1 and 0.975, 0.95, 0,9 levels): $\tau^L = \tau^U$. The alternative is that they are different over the full sample. We find that the null could not be rejected for any of the joint distributions. However, since we are comparing across the entire sample without accounting for structural breaks or potential regime switches, these results are not confirmation that asymmetric dependence is not present and further testing using the copula models needs to be conducted.

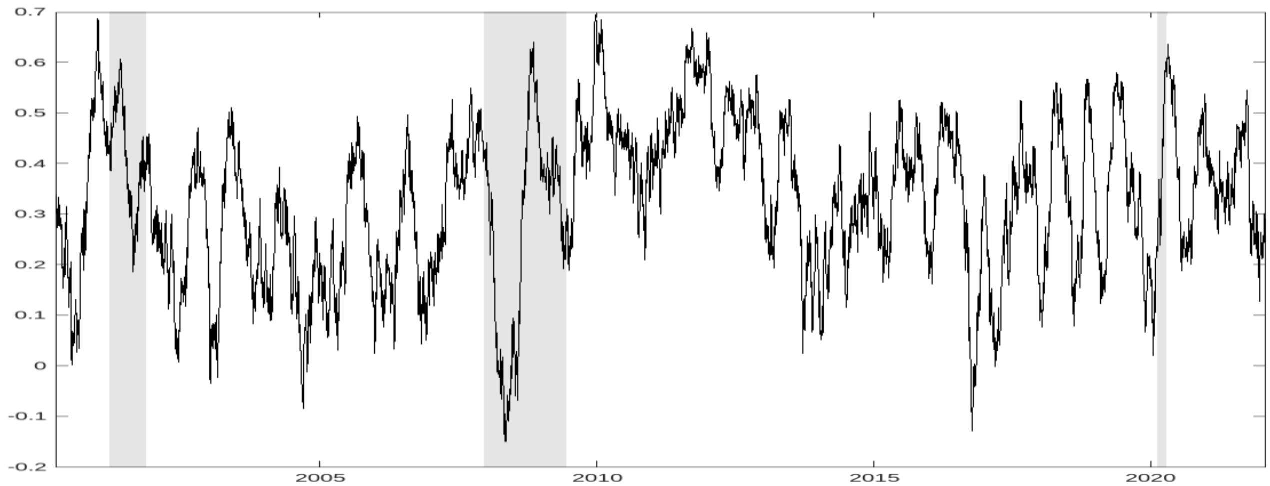
Table 7: Chi Squared Test for Asymmetric Dependence

	S&P 500 & JSE TOP 40	***FTSE 100 & JSE TOP 40***	***DAX & JSE TOP 40***
Test Statistic	0.6194	4.4589	1.0862
(P-value)	(0.8920)	(0.2160)	(0.7804)

A 60-day rolling window rank correlation on the standardised residuals was also estimated, in order to assess the stability of the associations over time. The choice of a longer rolling window was based upon the high frequency of the data and the large sample available. The results are presented in Figure 4. Additionally, the plots also contain information about periods of US economic recession consistent with the National Bureau of Economic Research's (NBER) dating cycle. These are the grey shaded areas in the background of the graphs. The behaviour of the association between the JSE TOP 40 and AEs' equity markets is consistent with the literature during the great financial crisis of 2008.[7] At the onset of the recession there is a weaking of the associations and in the case of the S&P 500 and JSE TOP 40 the relationship becomes slightly negative. This suggests that at the early stages of a recession these two markets are moving in opposing directions and there is potential for risk diversification. However, as the recession progresses and develops the associations appear to tighten and gain momentum suggesting greater comovements and dependence during periods of crisis. These findings are corroborated with the literature and are consistent across all three relationships. The recessions in early 2001 and 2020 do not have sharp declines at the initial onset of the economic downturn, rather there is a tightening and greater dependence between the markets as shown by the greater association.

[7] See Longin and Solnik (2001); Ang and Chen (2002) and Ang and Bekaert (2002).

<u>Figure 4A: Rolling window correlation for the JSE TOP 40 and the S&P 500</u>



<u>Figure 4B: Rolling window correlation for the JSE TOP 40 and the FTSE 100</u>

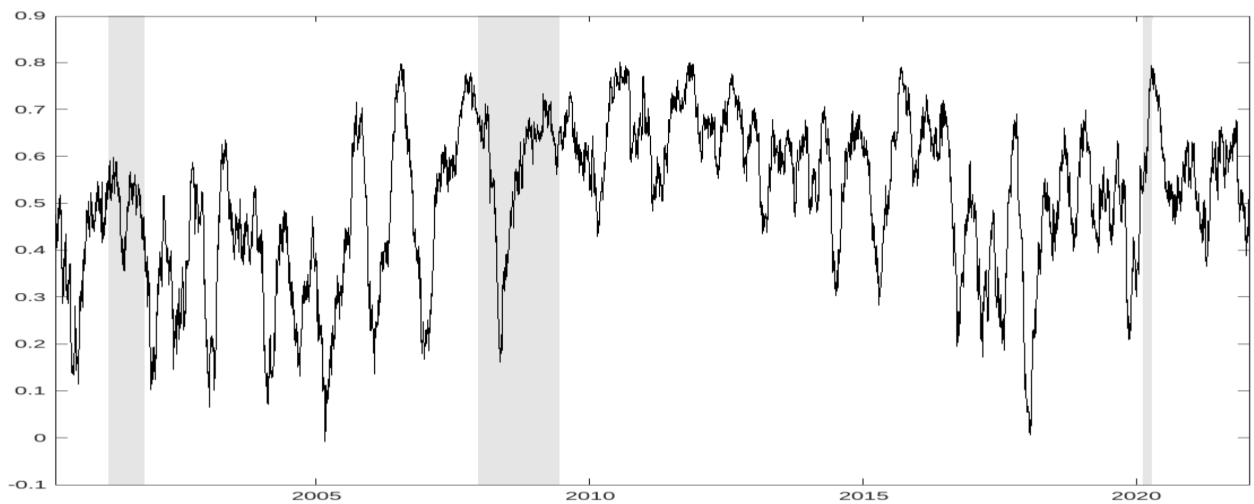
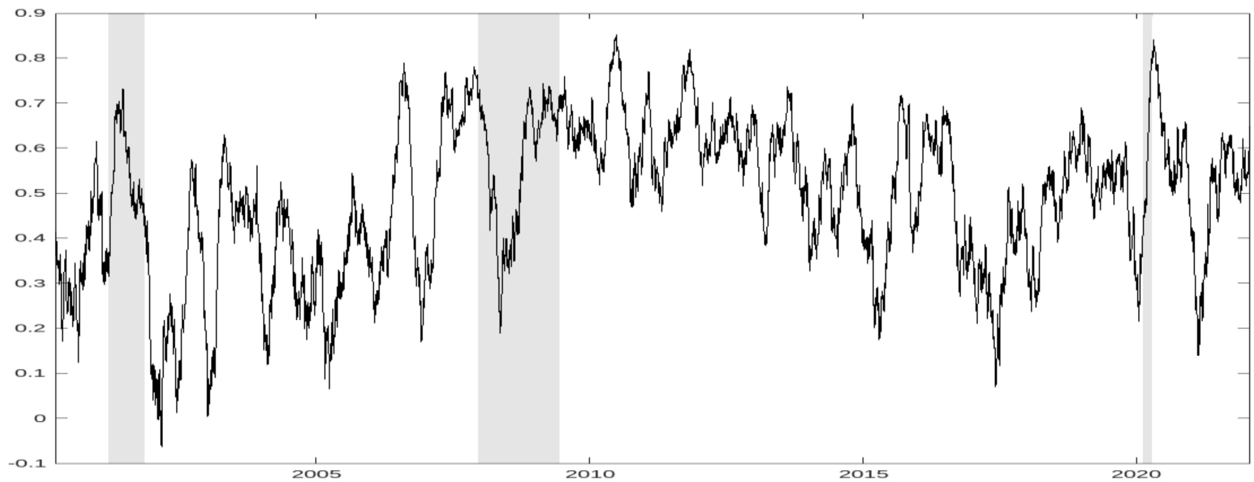


Figure 4C: Rolling window correlation for the JSE TOP 40 and the DAX



5.2. Constant Copula Estimates

The parametric standardised residuals from our marginal modelling procedure need to undergo one final process before they can be applied to our copula models. This involves a probability integral transform over the [0,1] domain. The reason we do this is to test whether our data can be modelled as being derivative of some specific population level distribution. In the case of this study the distribution is Hansen's skewed t.

The copulas considered in this study for static analysis are the Normal, Student's t, Gumbel, Clayton, and Rotated Gumbel. The reason for including the Rotated Gumbel is that we expect it to provide a better fit of the lower tail compared to the Clayton model due to its shape, with much deeper contours that can pick up clustering at the extreme tails of the distribution (This can be verified by Figure 1 in section 2.2). The results of these copulas are presented in Table 8. Each copula model is specified as a joint distribution, conditioned using relevant marginal information as specified in section 3.4.

Table 8: Copula Parameters.

	JSE TOP 40 & S&P 500	***JSE TOP 40 & FTSE 100***	***JSE TOP 40 & DAX***
Normal	0.3552*	0.5417*	0.5168*
Student's' t	0.3538* (0.0672)	0.5434* (0.0766)	0.5191* (0.0990)
Gumbel	1.2596*	1.4969*	1.4672*
Clayton	0.4380*	0.8279*	0.7682*
Rotated Gumbel	1.2626*	1.5241*	1.4853*

The values in parenthesis are the degrees of freedom. The superscript * indicates significance at the 5% level.

For all the copula parameters in Table 8, the greater the value the stronger the level of dependence between them.[8] From section 2.2 we know that the copula parameter is a measure of the association, the strength of this association is gauged by how great the value is. Therefore a larger parameter is indicative of a greater dependence in the joint distribution. With regard to all the models it is quite evident that there appears to be a greater dependence between the JSE TOP 40 and both the European markets. This may be for a few reasons, historical ties, stronger trade agreements resulting in larger capital inflows and outflows between these specific countries, similarities in market composition and possibly that major companies in the UK and Germany might have subsidiaries on the JSE TOP 40 resulting in similar investment patterns. These results dampen any possibilities for diversification between these three markets due to the strong linkages present. A similar finding of co-movement is reported for the S&P 500 and the JSE TOP 40, although the relationship is notably weaker across all the models.

A further assessment of the tail dependence was carried out on each copula. This is so that we can identify which copula model best identifies the clustering of observations in the extremities of the joint distribution. From these results we can formulate further how to proceed with our time-varying GAS copula analysis.

Table 9 presents the results for the conditional tail dependence of the constant copula models estimated in Table 8. These estimates are based on the equations for tail dependence in section 3.4, the copula specific measure of tail dependence can be found in Table 12 of the appendix.

[8] Table 12 in the appendix has information on the parameter space for each copula model used in this study. This should provide the reader with a deeper appreciation for the size and strength of the associations between the markets.

The estimates in the table measure the probability of extreme values in the tails of the distributions. A greater value implies a stronger association during extreme events (lower tail measures market crashes and upper tail market booms).

<u>Table 9: Upper and Lower Tail Dependence for Constant Copulas</u>

	JSE TOP 40/S&P 500	***JSE TOP 40/FTSE 100***	***JSE TOP 40/DAX***
Student's t (τ^U & τ^L)	0.5992	0.6674	0.6534
Clayton (τ^L)	0.2055	0.4329	0.4056
Gumbel (τ^U)	0.2662	0.4111	0.3961
Rotated Gumbel (τ^L)	0.2685	0.4242	0.4053

The results in Table 9 suggest that there is a more intense association between the JSE TOP 40 and both the FTSE 100 as well as the DAX. Further inspection of copula specific models we find that the Student's t is similar across all three joint distributions, only marginally greater for both European markets in comparison to the S&P 500. The differential between the joint distribution of both European markets and the US index becomes far larger when we apply asymmetric copula models. In addition, for the joint distribution between the JSE TOP 40 and the S&P 500 there is significant increase in the lower tail dependence when using the Rotated Gumbel compared to the Clayton copula. However, these results are not conclusive for this study to infer that one model should outperform the other. The Clayton, Gumbel and Rotated Gumbel are asymmetric models which implies that their estimates can distinguish between contrasting market events (crisis vs. boom). Whereas the Student's t cannot distinguish between these two events, its symmetrical design treats both extremes in the tails homogenously. For instances where the data is not characterised by nonlinearities, we should expect a symmetric model (Student's t) to provide a policymaker or investor with the necessary information to make informed decisions on risk. However, when that is not the case a model that can distinguish between asymmetric behaviour might be more suitable for policymakers and investors.

5.3. Time-Varying Copula Estimates

With our previous findings in mind, we now turn to the time-varying estimation procedure which will use the GAS model of Creal *et al* (2013) introduced in Section 3.5. Equation 5 is the workhorse for all estimation in this section, each copula model is fitted with the time-varying specification as detailed in equation 5. The results of these models are presented in Table 10.

We find that most of the models are highly significant as the parameters tend to have very low standard errors in conjunction with low p-values. The time-varying Gumbel copula is consistent for each joint distribution with a large proportion of the movement in the copula parameter attributable to the autoregressive component (μ). The scaled score parameter also has a significant impact ranging from 4.5% for the JSE TOP 40 and S&P 500 to 6.7% for the JSE TOP 40 and DAX with the highest being the JSE TOP 40 and FTSE 100 at 7.3%. These results appear to contrast in some cases with the results of the time-varying Rotated Gumbel copula. For the JSE TOP 40, S&P 500 and DAX joint distributions there are many similarities in that the parameters are all highly significant, although the values are slightly different. For the JSE TOP 40 and the FTSE 100 the results show a different story. This model is highly insignificant and does not contain a single significant parameter. This could imply two things, either the Rotated Gumbel is a poor fit for modelling the lower tail of this joint distribution or there is a weak lower tail relationship in this joint distribution. The latter does not seem plausible given the results for our constant copula estimation in Table 9. We therefore must consider that the models fit might be inappropriate in this instance at measuring the lower tail of this joint distribution. The models all suggest that much of the movement in the copula parameter over time is because of the autoregressive component. These findings are consistent with those in the literature that applied similar techniques.[9] There is a great deal of persistence or hysteresis in the joint distributions which is in line with the long memory that financial assets tend to exhibit. The impact of the scaled score parameter appears to be dampened in comparison to the Gumbel and Rotated models. The results in these tables confirm the suitability of this model in measuring the time-varying nature of the dependence in these joint distributions.

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[9] See Patton (2012), Mensah and Alagidede (2017) for papers with a similar empirical methodology.

Table 10: Time-varying Copula Parameters

	JSE TOP 40/S&P 500	***JSE TOP 40/FTSE 100***	***JSE TOP 40/DAX***
	Gumbel Time-Varying Copula		
η	-0.0169*	-0.0134*	-0.0117*
	(0.0076)	(0.0008)	(0.0003)
φ	0.0458*	0.0736*	0.0674*
	(0.0194)	(0.0052)	(0.0079)
μ	0.9876*	0.9816*	0.9850*
	(0.0057)	(0.0001)	(0.0010)
	Rotated Gumbel Time-Varying Copula		
η	-0.0135*	-0.0067	-0.0103*
	(0.0293)	(0.2081)	(0.0160)
φ	0.0335	0.0550	0.0726*
	(0.4038)	(1.16)	(0.0005)
μ	0.99*	0.99	0.9864*
	(0.021)	(0.3506)	(0.0006)
	Student's' t Time-Varying Copula		
η	0.0049*	0.0155*	0.0168*
	(0.0004)	(0.0009)	(0.0012)
φ	0.0179*	0.0516*	0.0561*
	(0.0037)	(0.0072)	(0.0076)
μ	0.9936*	0.9875*	0.9856*
	(0.0000)	(0.0000)	(0.0000)

Values in parenthesis are standard errors, A superscript * indicates significance at the 5% level.

5.4. Goodness of fit Tests

Before proceeding to an investigation of the time-varying conditional tail dependence of the joint distributions, we consider the goodness of fit of the models estimated so far. To test for goodness of fit in copula models the consensus in the literature gravitates towards two high-powered tests: the Kolmogorov-Smirnov (KS) test and the Cramer von Mises (CvM) test. Both are similar in construction in that they compare the distance between the empirical copula and the conditional distribution. The null hypothesis of these tests is that the sample is a part of a pre-specified population distribution. Similar to the work of Genest and Remillard (2009) this study applies a simulation-based method of estimation. We use this method for the following reasons, given our parametric approach the asymptotic distributions of the KS and CvM tests are unsuitable in the presence of estimated parameters. However, the conditional distribution we need to compare our empirical copula against has been well defined in our marginal modelling process, allowing us to use simulation-based methods to compute the test statistics. More information on these methods can be accessed in Remillard (2010). The results for both the constant and time-varying copulas are presented in Table 11. The values in the table are the p-values and not the KS and CvM test statistics. In each case the p-values were very low leading to a rejection of the null, which implies that the sample is drawn from the conditional distribution. In the case of this study that reference distribution is Hansen's skewed t. This suggests that there is a lower chance that the joint distributions in both the constant and time-varying case, were drawn from the same reference distribution, being the skewed t of Hansen. This finding provides evidence that perhaps the skewed t is not entirely representative of the data as a parametric conditional distribution.

Table 11: Goodness-of-Fit Statistics

Kolmogorov-Smirnov Test		Cramer von Mises Test	
JSE TOP 40 and S&P 500 Joint Distribution			
Normal	0.000	Normal	0.000
Student's' t	0.000	Student's' t	0.000
Clayton	0.000	Clayton	0.000
Gumbel	0.000	Gumbel	0.000
Rotated Gumbel	0.000	Rotated Gumbel	0.000
T-V Gumbel	0.000	T-V Gumbel	0.000
T-V Rotated Gumbel	0.000	T-V Rotated Gumbel	0.000
T-V Student's' t	0.000	T-V Student's' t	0.000
JSE TOP 40 and FTSE 100 Joint Distribution			
Normal	0.000	Normal	0.000
Student's' t	0.000	Student's' t	0.000
Clayton	0.000	Clayton	0.000
Gumbel	0.000	Gumbel	0.000
Rotated Gumbel	0.000	Rotated Gumbel	0.000
T-V Gumbel	0.000	T-V Gumbel	0.000
T-V Rotated Gumbel	0.000	T-V Rotated Gumbel	0.000
T-V Student's' t	0.000	T-V Student's' t	0.000
JSE TOP 40 and DAX Joint Distributions			
Normal	0.000	Normal	0.000
Student's' t	0.000	Student's' t	0.000
Clayton	0.000	Clayton	0.000
Gumbel	0.000	Gumbel	0.000
Rotated Gumbel	0.000	Rotated Gumbel	0.000
T-V Gumbel	0.000	T-V Gumbel	0.000
T-V Rotated Gumbel	0.000	T-V Rotated Gumbel	0.000
T-V Student's' t	0.000	T-V Student's' t	0.000

5.5. Conditional Tail Dependence

This section looks at the time-varying conditional tail dependence of the joint distributions. The results are consistent with those of Rodriguez (2007), and Mensah and Alagidede (2017) in that we find strong evidence of tightening and comovement during periods of economic crisis. The starting point for our analysis is Figure 5, which graphically illustrates the evolution of the conditional upper and lower tails.

The figures are in line with our *a priori* expectations regarding the performance of the Student's t time-varying copula. The elliptical shape infers that the upper and lower tails are treated identically, furthermore the Student's t copula does not possess the far-reaching contours that can identify clustering during extreme events and thus provides a much weaker degree of comovement in the tails. For these reasons our analysis will primarily focus on the Gumbel and Rotated Gumbel. Both can estimate and explain far more of the variation and dependence in the upper and lower tails.

In all three panels there does not appear to be any identifiable pattern or trend. For all three joint distributions the degree of dependence is quite robust throughout the sample with fluctuations occurring during cyclical swings in global economic performance and attitudes towards risk. The dependence is relatively consistent and there are no signs of a dampening or even further increasing in financial dependence, there is a substantial degree of persistence across the sample period. In all three panels we identify that asymmetries are present as the upper and lower tails are significantly different during cyclical swings. As in Figure 4 the shaded regions in the background of the model correspond to periods of US recession according to the NBER's dating cycle. There are also periods where the lower and upper tails are indistinguishable, this symmetry implies the market was in a relatively tranquil state. The figures are also consistent with our expectations, the dependence between the JSE TOP 40 and the FTSE 100 (Panel B) appears to be the strongest followed closely by the DAX (Panel C) and lastly the S&P 500 (Panel A). The reasons include historical linkages such as trade, partnerships, and joint listings, generating a stronger dependence. At this point a key distinction needs to be made. We are unable to infer in which direction the risk is being transferred, what has thus far been established is the strength and stability of the dependence structure throughout the sample period. There is sufficient evidence supporting the presence of contagion, particularly during the periods of financial crises as there is a marked tightening (increase) of the association in the joint distributions.

The primary focus areas will be the GFC in 2008 and the Covid 19 pandemic in early 2020. Prior to 2008 the upper tail was stronger for two out of the three joint distributions, the exception being the DAX (panel c). This implies that the returns during this period greater on average. At the onset of the financial crisis there is a dramatic swing downwards, implying a significant change in the dependence structure for all three joint distributions. During this period the associations between the JSE TOP 40 and AEs was at its weakest throughout the entire sample period. Furthermore, the conditional lower tail was strong which is to be expected, returns on all four financial markets was far below average. As the crisis prolonged the dependence structure began to tighten which is in line with the stylised facts. There is strong evidence of comovement between the JSE TOP 40 and the AEs, further suggesting that contagion was present.

By comparison, the impact of Covid 19 on the conditional tail dependence is more in line with the literature. At the initial stages of the crisis there was upward pressure on both the lower and upper tails. This tightening across all three joint distributions is potential evidence of contagion as there was an instantaneous shift and greater comovement during this crisis period. Similarly at the early stages of the crisis the lower tail was greater, implying asset returns that were falling. However, there was a much faster transition in financial markets as the upper tail far exceeded the lower tail towards the end of the crisis, indicating asset returns were more positive. In contrast to the GFC the Covid 19 financial crisis was not only short lived but propagated through existing channels in a different manner. The GFC can best be described as a credit squeeze, tightening US monetary policy lowered the supply of credit domestically and globally which directly affected market risk sentiments resulting in investors behaving more conservatively and limiting capital flows into financial markets. The lower capital flows lead to a fall in asset prices, reduced market liquidity and source of financing putting downward pressure on economic output. The Covid 19 crisis was driven by global macroeconomic factors. The implementation of a lockdown directly affected capacity utilization, lower production levels resulted in growing unemployment which decreased disposable income, dampening demand for goods and services both domestic and foreign (lower levels of trade, reduced demand for oil), putting downward pressure on GDP.[10]

Figure 35A: Conditional Tail Dependence for the JSE TO P40 and the S&P 500

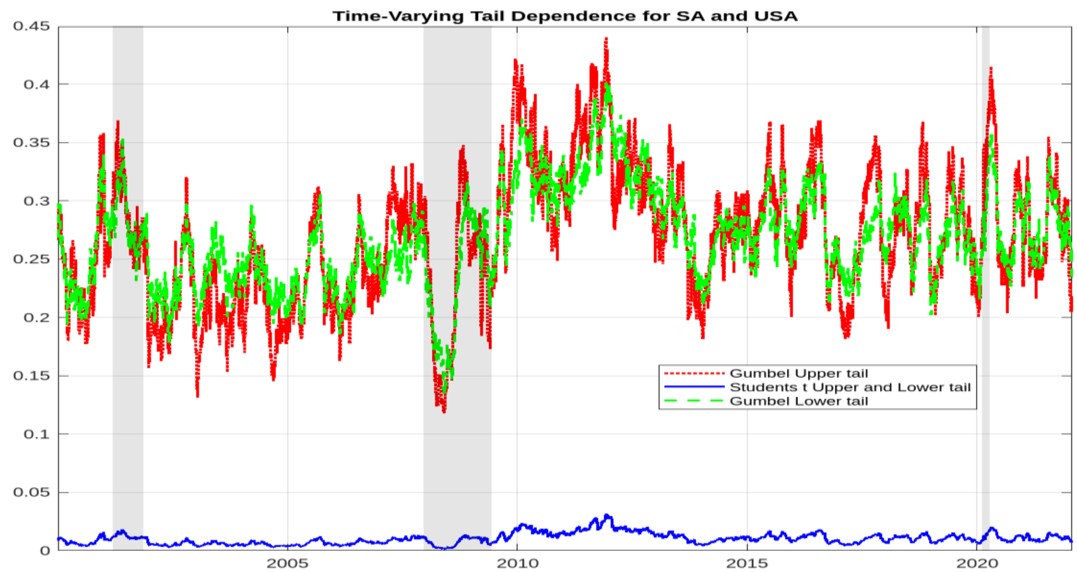
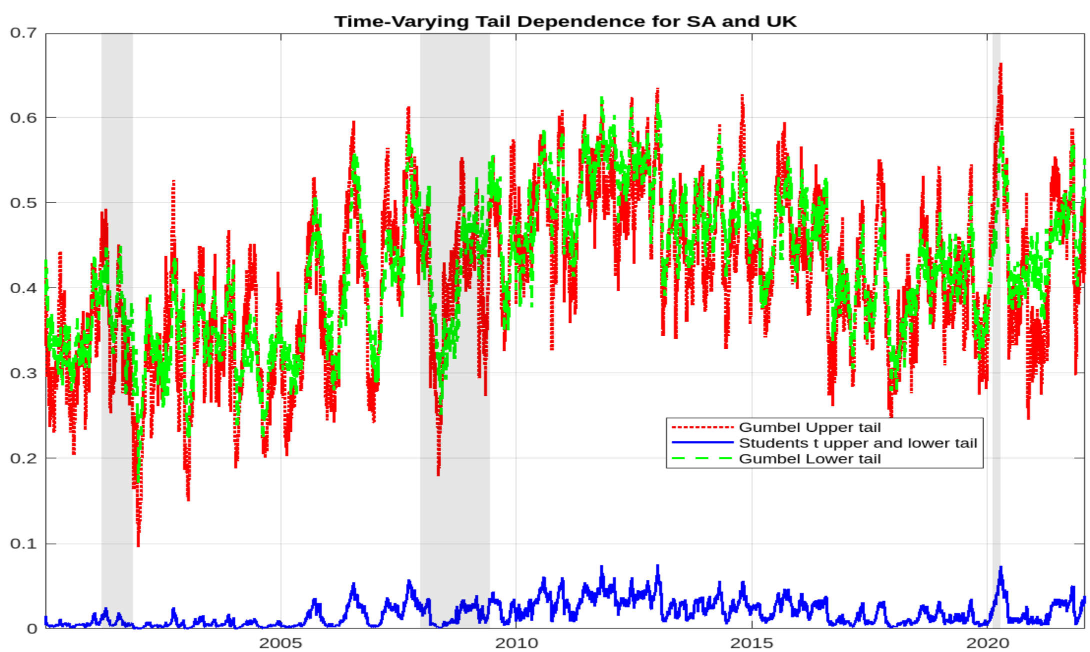
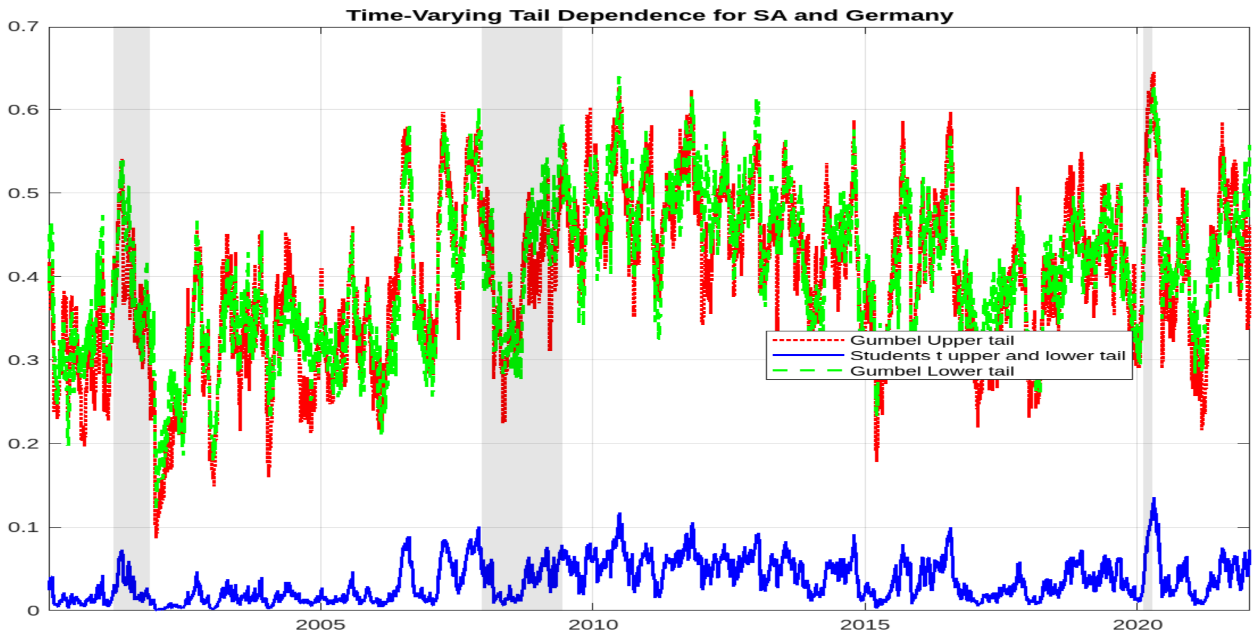


Figure 5B: Conditional Tail Dependence for the JSE TOP 40 and the FTSE 100



[10] See Abakah *et al* (2022) for the effects of geopolitical risk and uncertainty on the risk and return paradigm.

Figure 5C: Conditional Tail Dependence for the JSE TOP 40 and the DAX



6. Model Implications

This section provides further detail on investor and policy perspectives given the results in section 5.5. We have established that the dependence structure is different across the two crises mentioned, what this section seeks to do is discuss the implications for the potential diversification benefits for investors and policymakers.

Prior literature has stated that African financial markets in their fledgling state are a viable investment due to the diversification benefits (Alagidede, 2009; Adams and Opoku, 2015; Gil-Alana, 2018). This can be interpreted in the following way: these markets have fewer financial and trade linkages resulting in a lower degree of association or dependence implying fewer comovements with advanced financial markets, creating the possibility for diversification and potential growth. Unlike other African countries South Africa exhibits a much stronger association and is a far less isolated financial market. This is evident by the greater degree of comovements throughout the sample but is also found in the literature (Mensah and Alagidede, 2017, Bello *et al* 2022). These comovements imply that there is greater potential for transfers, spillovers and shocks transmitting through well-defined linkages into the South African financial market and ultimately influence the real economy.

For policymakers, global financial cycle theory suggests that monetary policy independence is under question during periods of financial crisis. This is partly due to the impact of US monetary policy dominance, any sharp hike in the federal funds rate is likely to generate conservative global risk perspectives leading to fewer capital flows into emerging markets that are deemed as risky investments. The implications are lower levels of investment and financing into the South African financial market, leading to falling asset prices. In this situation it can be difficult for South African monetary policy to initiate easing measures if the response from the US federal reserve is to tighten. The effects of interest rate pass through from the centre country can dominate, undermining the sovereignty of monetary policy (Rey, 2013). Especially when focusing on the GFC, a period in which US monetary policy underwent significant tightening. This restrictive Fed policy resulted in a reduction in credit globally, putting significant downward pressure on capital flows as a source of liquidity and leverage, this period was one of high-risk aversion leading to investors avoiding emerging markets like South Africa. A fall in the value of South African assets (e.g. Rand denominated sovereign bonds) leads many foreign investors to sell their assets, placing downward pressure on the Rand, further worsening global investors risk prospects leading to them taking up an even greater short position on South African assets.[11]

From an investors perspective the results appear to be consistent with the literature.[12] The financial linkages between South Africa's financial market and those of the AEs is much stronger. These strong linkages imply a greater dependence which lends itself to much greater comovements. This implies that in most circumstances unlike other African countries South Africa's financial market is not a suitable investment to diversify portfolio risk.[13] This does appear to be the case when assessing Figure 5, the dependence across all three joint distributions is persistent and robust. The only exception is the initial period of the GFC. In this period the

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[11] See Shin (2022). Article on the original sin redux.
[12] See Mensah and Alagidede (2017) and Bello *et al* (2022).
[13] This finding is consistent with Mensah and Alagidede (2017) & Bello *et al* (2022). However, it is not in agreement with results in Gil-Alana (2018) who find that African markets have weak structural linkages both regionally and internationally.

dependence structure appears to have undergone a significant change as both the lower and upper tails experience a dramatic downward swing across all three panels. The precise reason for this sharp swing could be attributed to capital flight, asymmetric monetary policy responses. What is of interest is of interest is that there was a period in which gains through diversification were present. The diversification benefits do not exist across all three joint distributions, the JSE TOP 40 and DAX while lower than in the period preceding it still possess a robust dependence structure. The JSE TOP 40 and the S&P 500 (Panel A) have the lowest association during this period, implying that this would be the most likely AE for a South African investor to diversify risk. The upper tail drops as low as 0.12 while the lower tail is slightly higher at 0.15 which signals that the association while positive is relatively weak. At this initial point possibility of comovement between these financial markets is low. Diversifying through the FTSE 100 presents more difficulty, while the associations do weaken the presence of joint listings, partnerships as mentioned in Section 2.1 make this an unlikely prospect. Foreign invests however, are likely driven away from the prospect of investing in the South African financial market during a period of financial crises due to the risks associated with it. These include lower market liquidity, declining asset prices and an increase in conservatism regarding emerging markets as an investment.

7. Conclusion:

This study sought to provide clarity on the time-varying nature of the dependence structure between the JSE TOP 40 and AEs equity prices. The paper found evidence supporting the findings in previous literature on the subject. This includes the significant linkages and dependence between South Africa and AEs given the strong nature of comovement in the dependence structure. There is also evidence of asymmetric behaviour in the tails of the distribution during periods of economic crisis, as well as a surging degree of association or tightening during crisis periods. These findings also support the occurrence of contagion and spillovers during periods of economic crisis, as the magnitude of the comovements in the lower tail is far greater than the upper tail at the beginning of a financial crisis. As far as diversification, there is little to no evidence in support of South Africa as a haven for a foreign investor looking to diversify their risk. This is mainly due to the existing linkages between these markets that have led to frequent comovements and greater dependence. The only brief period wherein the dependence structure is dampened is at the onset of a market crash driven by a credit squeeze and declining asset prices (GFC of 2008). However, this sort of knife-edge equilibrium is difficult to predict as it would require a detailed log of the holdings of specific foreign assets by South African market participants in order to determine the potential for contagion. This strategy would not only be unsuitable but expensive from a monitoring perspective for a South African investor. The findings in this study should provide further evidence to the limitations of South Africa as a diversification hub for American and European investors. These results imply that there is little to no risk immunity given the strong degrees of association. Furthermore, for a policymaker the evidence supports the need to regulate and implement suitable safeguards against the transmission of risks in the financial sector. However, given the strong links perhaps a more suitable approach would be to limit the exposure these shocks have from the real economy by regulating the risky behaviour of banks and financial intermediaries who may offload their debt in the form of financing on other institutions and individuals in the real economy.

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9. Appendix

The GJR GARCH model of Glosten, Jagannathan and Runkle (1993) has the following functional form:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i + \gamma_i \Pi_{t-i}) \varepsilon_{t-i} + \sum_{i=1}^{p} \theta_i \sigma_{t-i}^2$$

In order to ensure the conditional volatility is non-negative similar to the standard GARCH model it has the following parameter restrictions.

$\omega > 0, \alpha \geq 0, \theta \geq 0, \frac{(\alpha+\theta)}{2} \geq 0$ and for weakly stationary processes $\alpha + \theta < 1$. The last restriction ensures model stability and convergence.

The addition of the indicator function gives the model leverage effects. The indicator function takes on a binary value, $\Pi_{t-i} = 1$ if event A occurs and is 0 if it does not occur. The model offers a more detailed interpretation of the stylised facts. By incorporating leverage effects, it can explain why losses or negative returns have a greater impact on future volatility than positive returns. This model better captures asymmetry in financial data.

The EGARCH model of Nelson (1991) is similarly able to capture leverage effects and better model the asymmetry present in financial data. It differs from the GJR-GARCH specification as the conditional volatility is in logarithmic form. The logarithmic transformation on the left-hand side ensures that the conditional volatility is nonnegative. This implies that parameter restrictions are not required.

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} [\alpha_i (|\varepsilon_{t-i}| - E[|\varepsilon_{t-i}|]) + \gamma_i \varepsilon_{t-i}] + \sum_{i=1}^{p} \theta_i \ln(\sigma_{t-i}^2)$$

Table 12: Copula Model Characteristics

	Parameter(s)	Parameter Space	Independence	Pos & Neg Dependence	Lower tail	Upper tail
Normal	ρ	$(-1,1)$	0	Yes	0	0
Student's t	(ρ, ν)	$(-1,1)\ x\ (2, \infty)$	$(0, \infty)$	Yes	$2\ x\ F_{Stud}(-\sqrt{(\nu+1)\frac{\rho\ -\ 1}{\rho+1}, \nu+1}$	
Clayton	γ	$(0, \infty)$	0	No	$2^{\frac{-1}{\gamma}}$	0
Gumbel	γ	$(1, \infty)$	1	No	0	$2\ -\ 2^{\frac{1}{\gamma}}$
Rotated Gumbel	γ	$(1, \infty)$	1	No	$2\ -\ 2^{\frac{1}{\gamma}}$	0