Professional noticing of learner's mathematical thinking by the teacher in teaching and learning of directed numbers in grade 8.

by

Peter Kushanda

Student No: 0714381Y

PROTOCOL NUMBER: 2020ECE002

A thesis submitted in fulfillment of the degree of

Masters of Education

University of the Witwatersrand

School of Education, Faculty of Humanities

JOHANNESBURG

SOUTH AFRICA

January 2021

Supervisor:

Professor Judah Makonye

PLAGIARISM DECLARATION

1. I know that plagiarism means taking and using the ideas, writings, works or inventions of another as if they were one`s own. I know that plagiarism not only includes verbatim copying, but also the extensive use of another person's ideas without proper acknowledgement (which includes the proper use of quotation marks). I know that plagiarism covers this sort of use of material found in textual sources and from the Internet.

2. I acknowledge and understand that plagiarism is wrong.

3. I understand that my research must be accurately referenced. I have followed the rules and conventions concerning referencing, citation and the use of quotations as set out in the Departmental Guide.

4. This assignment is my own work, or my group's own unique group assignment. I acknowledge that copying someone else's assignment, or part of it, is wrong, and that submitting identical work to others constitutes a form of plagiarism.

5. I have not allowed, nor will I in the future allow, anyone to copy my work with the intention of passing it off as their own work.

Student signature: __________ Date: December 2020

ACKNOWLEDGEMENTS

First and foremost, I wish to express my sincere great thanks to my supervisor Professor Judah Makonye for his encouragement, patience and professional guidance. Your continued support and constructive criticism provided me with the means of improving my work to my level best. You have been a truly great mentor who went an extra mile in assisting your students to achieve great results in their reports, I remember you making us as your students to bring meals collectively on one of the Saturdays so as to spend the whole day pushing our reports at the same time supervising us. I thank you for your part in my Masters journey and I would like to say I am really fortunate and privileged to have had you, as, my supervisor.

I want to say, thank you to my family for the encouragement, kind words and standing alongside me during all my days of studies. While I have always been away from you, your continued support and motivation too inspired me to complete my Master's degree. You encouraged me and kept a smile on my face. To friends and colleagues, I want to say thank you to all for the support you rendered to me, you were professional in every aspect of encouragement and corrections. You constantly checked in on me to make sure I was doing great.

Last but not least, thanks to the four teachers and students that took part in this research project. You welcomed me to your classrooms and helped me to learn a lot about mathematics teaching and learning.

ABSTRACT

This study explored the significance of teacher professional noticing of learners' mathematical thinking in teaching and learning of the most confusing operations on directed numbers in Grade 8. The focus of this study was on the use of Wallach et al.'s (2005) forms of learner hearing together with Jacob et al.'s (2010) three interrelated skills on teacher professional noticing. The three aspects of teacher professional noticing are attending learners' techniques, interpreting learning understanding, and determining how to respond on the basis of learners' understanding. According to Wallach et al. (2005), five forms of learner hearing are non-hearing, biased-hearing, compatible-hearing, over-hearing, and under-hearing.

Does the application of teacher professional noticing have an impact on teaching and learning performance? To answer this question the study specifically addressed three critical questions namely, 1. What skills of noticing does the teacher embark on when using teacher professional noticing theory to teach and identify errors/misconceptions experienced in the classroom? 2. How does teaching based on professional noticing of errors/misconceptions assist teachers in teaching and learning new knowledge? 3. What form of learner hearing is the best associate when practicing professional noticing?

The data was collected from two experimental classes and two control classes in two different schools located in the inner-city of Johannesburg. The data was collected in the form of Pre-test marked scripts, Post-tests marked scripts, and transcribed audio-recordings. The data was then analysed on the basis of the concepts emerging from the conceptual framework.

The findings revealed that both Jacob et al.'s (2010) interrelated skills on professional noticing and Wallach et al.'s (2005) form of learner hearing namely compatible-hearing had the potential to improve and address the problem of teaching and learning operation on directed numbers. As a result, the three interrelated skills of professional noticing make it easier for teachers to recognize the learners' Zone of Proximal Development (ZPD). Professional noticing is like a teacher seeking to find out about a learner's ZPD. And if the teacher listens to the learner, hears the learner, and understands what the learner does, it allows the teacher to find the learners' ZPD and that s/he can help the learners. The value of using powerful knowledge in conjunction with teacher professional noticing among learners

in the experimental classes can, therefore, be realized in this situation through a pedagogy that is sensitive to their life-worlds.

Table of Contents

List of tables

List of figures

LIST OF ABBREVIATIONS

ALS : Attending to learner's strategies

BH : Biased-hearing

CAPS : Curriculum and Assessment Policy Statement

CH : Compatible-hearing

DBE : Department of Basic Education

DRLU or DoHrtBoLU: Deciding on how to respond basing on learner's understanding.

FET : Further Education and Training

GET : General Education and Training

ILU : Interpreting learner's understandings

KCS : Knowledge of Content and Students

Lnr : Learner

Lnrs : Learners

MKT : Mathematical Knowledge for Teaching

NH : Non-hearing

NSC : National Senior Certificate

OH : Over-hearing

PP : Progressive pedagogy

SSS / LSS: Shongamiti Secondary School / Lundi Secondary School

UH : Under-hearing

ZPD : Zone of Proximal Development

CHAPTER 1: INTRODUCTION TO THE STUDY AND RESEARCH QUESTIONS

1.1 Introduction and background to the study

In this chapter, the researcher discusses the problem statement; the significance of teacher professional noticing of the learners' mathematical thinking in teaching operations on directed numbers (integers) in Grade 8; and also discusses the aims of this research project. Generally, the researcher's concern in the study was on the effects of practicing teacher professional noticing of learners' mathematical thinking as errors and misconceptions are not meant to go unnoticed and unaddressed so that learners can repair their errors and misconceptions.

According to Makonye and Fakude (2016), ordering, adding, subtracting, multiplying, and dividing directed numbers is initially taught at senior phase and General Education and Training bands, but learners often fail to perform calculations involving all four operations with directed numbers in Further Education and Training bands. Learning mathematics is like building a house by laying bricks one on top of the other, so without the first level of support, one may not attempt the second level with success. In other words, if a learner does not have a good conceptual understanding of operations on directed numbers at the senior phase (grade 7-9), then, using alternate routes will make it difficult for such learners to make the transition from one stage of generalization to another (Hativa & Cohen, 1995). Alternative routes on generalization can lead to a continuous display of errors and misconceptions by learners.

Continuous display of errors and misconceptions by learners while conducting calculations involving operations on directed numbers, inspired the researcher to do a study on how learner hearing and teacher's professional noticing of learners' mathematical thinking in teaching directed numbers in Grade 8 work (Wallach & Even, 2005; Jacobs, Lamba, & Philipp, 2010).

Thus errors and misconceptions showed by the Grade 12 learners usually go unnoticed by most senior phase teachers. Mason (2001) explains that teachers sometimes do not understand or know that in a learning situation they need to pay attention to certain features, which means that the cognitive development of learners is incorrect without being corrected. Works carried out by some researchers on errors and misconceptions in dealing with directed numbers operations indicate that it is important to recognize that errors are in the work of misconceptions, therefore errors and misconceptions are considered to be part of the learning resources for learners (Nesher, 1987). According to Forsnot (2005) and Nesher (1987), learning new knowledge as a natural field has paths that twist the truth and progressively cross each other, meaning that the learners are not supposed to initially develop ideas in an organized sequence. Instead, learners go in many directions by making some experiments and inaccuracies as they explore and struggle to understand concepts and ideas. Schunk (1991) states that "learning is an enduring change in behaviour, or in the capacity to behave in a given fashion, which results from practice or other forms of experience" (p. 2).

However, the foundation laid by the constructivism theory is important to teachers as it informs them that learners come into the classroom with existing knowledge and will match external reality with their current cognitive structures (assimilation). In other situations, learners may display errors and misconceptions, then the accommodation process that relates to changing internal structures to ensure consistency with external reality will take place (Piaget, 1964). The researcher believes that errors and misconceptions showed by learners in Grade 7-9 need to be noticed, acknowledged, addressed, and used as a teaching tool to help learners accommodate and assimilate mathematical knowledge.

According to Piaget (1964), cognitive development relies on four key factors: the physical world experience (errors and misunderstandings), biological development, social situation, and balance (assimilation and accommodation). It is therefore important to explore how teacher professional noticing of learner's mathematical thinking involving teaching and learning of directed number operations in Grade 8 can make learners learn new knowledge. Brown and Drouhard (1989) define cognitive learning as an approach that aims to enculturate learners into credible activities through activity and social interaction, hence learners need to be encouraged to go into the field and practically do things. Thus, it will pave a way for learners to display errors and misconceptions that will be noticed by teachers and learners and later be used to improve teaching and learning by both teachers and learners (Nesher, 1987).

Vygotsky (1978) with the same ideology of constructivism provides another perspective on learning and development indicating that learning is a well-organized exercise that moves learners from the level of actual growth to that of potential growth under the guidance of the teacher or a knowledgeable person, and this he called it the Zone of Proximal Development (ZPD). When learners display errors and misconceptions, they occur at the level of actual development, and this must be noticed by teachers in the classroom and be addressed so that learners can move into the level of potential development through the practice of professional teacher noticing of mathematical thinking of learners (Jacobs, Lamba, and Philipp, 2010).

According to constructivism, moving from actual development to potential development confirms that the learner does not come to class empty-headed instead the ZPD is made possible with the help of a more knowledgeable person. Through interaction with the social environment, ZPD will be experienced by learners.

According to Brown et al. (1989), the theory of situated learning is the core basis of constructivism and involves a relationship between a potential learner and a situation. According to psychologists, philosophers, and sociologists, learning is caused by situations such as a behavioural experimenter or an educator with a certain pedagogical point or an actual physical situation (Piaget, 1964). A classroom is, therefore, a place where the theory of situated learning can be implemented in the noticing of the learner's mathematical thinking. According to a situated perspective, "mathematics is a set of the practice of inquiry and sense-making that include communication, questioning, understanding, explaining, and reasoning" and that "learning mathematics is marked by increasing participation in an expanding range of such practices" (Greeno & MMAP, 1997, p. 104).

However, the study was implicitly inspired by the poor performance of Grade 12 learners in mathematics NSC examinations. Question 1 of paper 1 in Grade 12 mathematics covers algebraic expressions and equations dealing with operations of directed numbers, but learners struggle to add, subtract, multiply, and divide directed numbers. This research also seeks to identify challenges faced by senior phase teachers in teaching operations on directed numbers, resulting in learners failing to perform well in matric examinations, yet the addition, subtraction, multiplication, and division of directed numbers are taught at the primary level. This study was interested in exploring teacher professional noticing of learners' mathematical thinking in teaching and learning of the topic that made the use of directed numbers in Grade 8. Thus study foregrounds the teacher's professional noticing of learners' mathematical thinking in teaching and learning operations with directed numbers. The study focused on the teaching of the operation of integers in South African secondary schools. Hopefully, the problems identified will not be unique to mathematics learners in Johannesburg in South Africa but will be something common to all mathematics learners in South Africa as a country and the world at large.

1.2 Problem statement

Lack of proper foundational teaching of basic four operations in mathematics leads to poor performance on integers. According to Balbuena & Buayan (2015) state that there are "four of the most confusing operations in mathematics under integers are: (1) adding a positive integer and a negative integer: $11 + (-14)$, (2) adding two negative numbers: $(-12) + (-7)$, (3) subtracting a negative integer from a positive number: $5 - (-8)$, and (4) subtracting a negative number from a negative number :$-8 - (-20)$ " (Balbuena & Buayan, 2015, p.15). This is supported by Makonye and Fakude (2016) when they report that 83% of learners commit errors in the addition and subtraction of integers due to misconception, 67% due to strategic errors, 28.6% due to logical errors, and 16.7% due to procedural errors.

Chang (1985) states that, "many of us learned to obtain the correct answers to problems with integers by applying rules. Few of us had a good grasp of the meanings behind these rules. Yet we may be offering the golden rules, with little meaning, to our learners" (Chang, 1985, p. 14). Therefore, as teachers, we have a problem of transmitting this so-called golden rule to our learners because we usually claim that two negatives mean positive in a problem, then teachers teach learners to solve problems such as $5 - -9 = 5 + 9 = 14$ ignoring duplicate signs of the negative sign (Chang, 1985). The challenge of school pedagogy is to monitor the interaction between the object of learning and its referent in the learner's life setting and to expose the learner to abstract environments made up of objects whose significance does not originate from a connection with the real world as it is encountered (Charlot, 2009). Teachers also teach learners to respond to problems like $8 - +4 = 8 - 4 = 4$ without allowing them to grasp the meaning behind these "golden" laws which say that combining minus and positive signs implies negative as a correct way forward to solve the expression (Chang, 1985). In general, these basic rules must be introduced once learners have understood the concepts involved through conceptualising given concepts. According to Makonye and Fakude (2016), learners encounter problems in dealing with secondary-level directed number operations because they did not assimilate directed number concepts into the scheme of directed numbers they built up from primary school.

Mason (2001) explains that teachers sometimes do not notice or realise that in a learning situation they need to pay attention to certain features, which means that cognitive development among learners goes wrong without being noticed and corrected. Sherin and van Es (2005) urges teachers to make connections in noticing between certain classroom interactions and the broader teaching and learning of concepts (principles) they present. Sherin and van Es (2005) suggest that the capacity to notice is very important in the context of recent reforms in mathematics education, requiring teachers to make educational decisions based on learning experience-in-progress and to notice practical problems that learners encounter in their specific context of learning to reason about a particular situation.

The cause of failure to notice may be the lack of powerful knowledge among teachers on how to teach this main content area or the failure of teachers to attend to learners professionally during the learning process. Ball, Thames, and Phelps (2008) note that in teaching, mathematical knowledge for teaching (MKT) is the knowledge needed by teachers to carry out teaching work as it includes; showing the learners how to solve problems, answering the questions of the learner, and monitoring the work of the learner. MKT is no longer a challenge in South African schools as the Department of Education has set the entrance qualification for teaching to be at a degree level unlike in the old days when unqualified teachers were hired to teach mathematics since mathematics educators were considered to be scarce skills. Currently, a challenge in teaching mathematics is that of practicing teacher professional noticing where Jacobs et al. (2010) suggest that, “teachers must be able to attend to children’s strategies, interpret their understanding, and use these understandings in deciding how to respond” (p. 192).

Currently, the reason why teachers do not find learning difficulties among learners is that they rely on traditional methods of teaching like rotary learning and “telling” without conceptualizing the subject area instead of teaching a reformulation of telling that involves initiating, probing, and eliciting as for facilitation purpose (Lobato, Clarke, & Ellis, 2005). It is important that proper noticing take place during teaching and learning as it is considered a learner-centred approach while teachers are also considered facilitators at the same time. Constructivist theories argue that, when teachers impart learners with the knowledge or give them resources, learners are stimulated to do and create understanding without showing or asking them what to do. Teachers should, therefore, withdraw from conventional methods of teaching and become facilitators. According to Lobato et al. (2005), teachers are encouraged to become facilitators so that they may listen more closely to their learners, present problematic issues, and allow learners to engage in independent mathematical reasoning and problem-solving activities.

South African curriculum calls for learners to participate in mathematics lessons and share their mathematics ideas and be learners-centric. In short, to encourage participation in the classroom by learners, teachers must facilitate the construction of ideas and making meaning of mathematical problems by learners. Learner participation is considered as important because:

> *…….it (i) shows that learners are attending to the lesson; (ii) allows learners to express and clarify their own ideas; (iii) enables learners to share ideas with each other; and (iv) provides teachers with information about what learners know and don't know, and how learners are thinking and trying to make sense of the ideas (Brodie, 2007, p. 3).*

Teachers, however, have little guidance on how to deliver and manage learner-centered lessons in South Africa, because very little training is provided as to what such "facilitation" would look like during their pre-service training.

In other words, in the new era of reformulation of telling Lobato et al. (2005), encourage teaching and learning to be learner-centred. Lobato et al. (2005) see teaching and learning as learners-centred because they reformulated telling in three ways when they say:

> *We reformulate telling in three ways: (a) in terms of the function (which involves attention to the teacher's intention, the nature of teaching action, and the student's interpretations of the action) rather than the form of teachers' communicative acts ;(b) in terms of the conceptual rather than procedural content of new information; and (c) in terms of its relationship to other actions rather than as an isolated action (Lobato et al., p. 100).*

Therefore, reformulation of telling changes the role of the teacher from being a "teller" to being a "facilitator" that will see the definition of "noticing" playing its role in this kind of teaching. Lobato et al. (2005) suggest that mathematics education is reforming from traditional teaching that practiced rote learning that relies too heavily on telling to "not telling". Traditional teaching was the first method used to transmit knowledge to the learners who were not considered as thinking individuals that can construct knowledge on their own and this was informed by behaviourist theories.

Education in mathematics has been structured in such a way that the traditional teaching method makes it difficult for teachers to notice errors and misconceptions encountered by their learners during mathematical operations of directed numbers. It has been under pressure to the point that it faces ongoing reforms on worldwide teaching and learning. This rote learning, which encourages the use of golden rules when adding, subtracting, multiplying, dividing directed numbers, does not succeed in educating our learners. There are two methods of teaching used by teachers to teach in primary and secondary schools namely the traditional "chalk and talk'' method, considered to be teacher-centred, and the learner-centred approach where teachers act as facilitators (Lobato et al., 2005). Worldwide curriculum advancements promote learner participation in mathematics classrooms, where teachers will serve as facilitators to "facilitate" and notice learners creating sense out of given directed mathematical operations. Therefore, there is a need to explore professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8 learners.

1.3 The significance of professional noticing of learner's mathematical thinking

The researcher believes that mathematics can be best taught in higher grades if a sound base or strong foundation has been laid on calculations that involve directed numbers at all levels of the learners' educational pursuit right from primary to senior phase. As learners leave the senior phase with a good understanding of dealing with directed numbers in all four operations, it will be much easier and more efficient to build on that and apply previous experience in the later phase of problem-solving (in Grade 10-12). Practicing teacher professional noticing of learners' mathematical thinking at the senior phase will help the teacher to notice learners' errors/misconceptions and use them as a tool to teach in the classroom. Learners encounter problems in dealing with integer number operations at the secondary level because they did not assimilate operation concepts into their scheme on the directed numbers they built up from primary school (Makonye & Fakude, 2016). Since there is poor performance in mathematics, the purpose of this study is to explore and explain what and how teacher professional noticing of learners' mathematical thinking must be conducted to identify and use errors/misconceptions identified among learners to improve the teaching of directed numbers in Grade 8.

Noticing is important in developing mathematical knowledge for teaching (MKT) particularly with the caption for knowledge of content and students (KCS). Ball et al (2008) note KCS is the knowledge that incorporates learning about learners and mathematical knowledge at the same time, for example, they say that teachers will predict what learners are likely to think and what they will find challenging through experimenting with or practicing mathematics teaching. Ball et al. (2008) on the KCS domain urges that "when choosing an example, teachers need to predict what learners will find interesting and motivating. When assigning a task, teachers need to anticipate what learners are likely to do with it and whether they will find it easy or hard" (p. 401). Therefore, among the things that are needed to be "noticed" by teachers are learner errors and misconceptions in this KCS domain. Thus, "teachers must also be able to hear and interpret students' emerging and incomplete thinking …." (Ball, 2008, p. 401). Therefore, awareness of teacher professional noticing must be made known among teachers.

1.4 The purpose and aim of this research project.

The aim of this study was to explore teacher professional noticing of learners' mathematical thinking in teaching directed numbers. Misconceptions and learners' errors should be "noticed" by teachers or any other knowledgeable individuals during the teaching and learning operation with directed numbers in Grade 8. Teachers should address errors and misconceptions on operations with directed numbers before learners move to the next Grade or phase. Teachers need to know the subject matter they are teaching so that they can quickly identify or note the shortcomings of their learners when they provide wrong answers by misunderstanding or by showing errors. Generally speaking, teachers must be in a position to do their job of teaching for conceptualization. According to Jacobs et al. (2010), skilled teachers are not well aware of the mathematical thinking of the learner.

Therefore, the study is framed in the following overall self-asked question derived from the main title of the research study: Does the application of teacher professional noticing have an impact on teaching and learning performance? To answer this question, the following critical research questions must be addressed.

1.4.1 Critical Research Questions

1.4.1.1 What skills of noticing does the teacher embark on when using teacher professional noticing theory to teach and identify errors/misconceptions experienced in the classroom?

1.4.1.2 How does teaching based on professional noticing of errors/misconceptions assist teachers in teaching and learning new knowledge?

1.4.1.3 What form of learner hearing is the best associate when practicing professional noticing?

The objective of this research was to explore ways of teacher professional noticing of learners' mathematical thinking being practiced by Grade 8 teachers on directed number operations and observe its impact on how it should be used in teaching and learning in Grade 8.

1.5 Conclusion

The above chapter addressed the context of the study citing the issue that the study is addressing, which is the lack of basic operations as the main cause of poor performance on integers in upper grades. Noticing has been identified as one approach that is meant to assist in the development of learners in integers. The study aims to explore and explain what and how teacher professional noticing of learners' mathematical thinking must be conducted to identify and use errors /misconceptions identified among learners to improve the teaching of directed numbers in Grade 8. The study research questions discussed above were set to guide the research study, especially on data collection. Therefore, in Chapters 2, the researcher will provide a relevant literature review for the study.

CHAPTER 2: LITERATURE REVIEW

2.1 Literature Review Introduction.

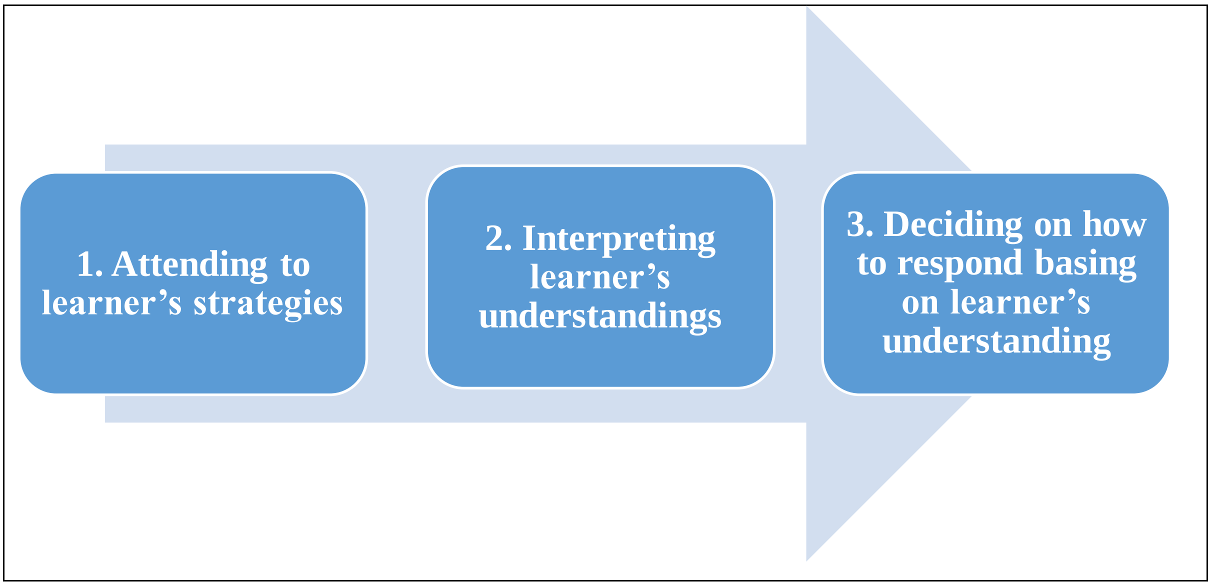
This research project aims to explore teacher professional noticing of learner's mathematical thinking in the teaching of directed numbers in the class of Grade 8. Therefore, in this chapter, the study will initially discuss literature on how some researchers view noticing, professional noticing, misconceptions, errors, and powerful knowledge in detail. The study will further discuss all the types of hearing brought up by Wallach et al. (2005) and Davis (1997). Noticing is always linked to hearing and listening, it also includes action in response to what one notices. Noticing, hearing, and listening can be formative and transformative, in other words, they can lead to teacher changing in terms of how they teach. The research will then focus on literature related to learners' algebraic thinking, and included are learner's errors and misconceptions usually encountered by learners when dealing with operations on directed numbers under the four operations namely addition, subtraction, multiplication, and division. Lastly in this chapter, the researcher will also discuss some important researched models for teaching addition, subtraction, and multiplication of directed as means of using powerful knowledge.

2.2 What is professional Noticing?

Noticing is a verb that means being conscious of or treating someone or something worthy of recognition. In everyday language, this term is used to refer to general observations one makes either consciously or unconsciously (Qi, 2001). We simply plug messages on the notice board to bring things to the attention of people, so that they will notice them. Qi (2001) loosely defines "noticing" as awareness of a stimulus which in a particular way evokes one's attention. Sherin and Van (2005) define noticing as something that involves identifying what is relevant in the teaching environment because the classroom is a complex environment with multiple simultaneous interactions.

Figure 1 below illustrates Jacobs et al.'s (2010) professional noticing of children's mathematical thinking "as a set of interrelated skills including (a) attending to learners' strategies, (b) interpreting learners' understandings, and (c) making a decision on how to respond based on learner's understandings" (p. 169). Jacobs et al. (2010) treat learner strategies as the degree to which teachers deal with a particular aspect of learning situations, perceive learner understandings as expressed in their strategies, and then determine how to respond to learner strategies based on learner understanding.

Figure 1: Jacob et al. 2010's three interrelated skills on professional noticing.



Jacobs et al. (2010) also notice classroom events through four observation categories namely classroom situation, mathematics content, learners' written work, and communication.

Jacobs et al. (2010) noted that researchers interpret noticing in various ways, based on how individuals make sense of the complex situation that prevails. Teachers have to pay attention to errors and misconceptions in teaching and learning mathematics through teacher professional noticing practice. Thomas, Jong, Fisher, and Schack (2017) cite hypothesising that successful professional noticing occurs at the intersection of established mathematical knowledge for teaching and a high level of sensitivity with regard to learners' mathematical activities. According to Thomas et al. (2017), professional noticing is a skill that teachers can use to identify and act on learners' (salient) mathematical action, and it also consists of three components, namely the process of attendance, interpretation, and decision-making. Teachers need to see noticing as a collection of practices designed to sensitize one to notice future opportunities to act freshly rather than out of habit automatically (Thomas et al., 2017).

Teachers need to be conscious so that they can identify any misconceptions made by learners rather than unintentionally discover some of those errors/misconceptions among learners. Gattegno (1987) used the word consciousness to mean the reaction that makes action, and therefore knowledge has to be conscious. Mason (2011) suggests "in fact, that there is a great deal that we do not notice, either because we are not attuned or sensitised or because our attention is directed and occupied elsewhere" (p. 35). Mason (2002) explains that teachers sometimes do not notice or realize that in a learning situation they need to pay attention to some of the features, with the matter of fact that cognitive development between many learners went wrong without it being corrected.

Mason (2008) seems to be in agreement with other scholars by suggesting that consciousness, attention, awareness, and noticing need to be interwoven strands that teachers can work together to help learners learn from their mistakes, errors, and misconceptions they display. Teachers need to be conscious so that attention is paid since it is the central concern in tracking learning progress among learners, so attention is the drive of intense awareness (Mason, 2008). The term conscious is usually used later in conjunction with some degree of awareness if the teacher is incapable of recognizing specific core awareness they will not be able to select correct tasks or actions to fix misconceptions, mistakes, and errors experienced by learners (Mason, 2008). Teachers, therefore, need to have the ability to notice and interpret the errors of learners in the act of teaching that allows them to provide the best possible individual support in their learning.

Teacher's professional noticing is crucially important to learner's cognitive development. Goleman (1985) sees noticing as the most important thing for teachers to practice and he states that,

> *The range of what we think and do is limited by what we fail to notice. And because we fail to notice that we fail to notice, there is little we can do to change until we notice how failing to notice shapes our thoughts and deeds (p. 24).*

Bennett (1976) suggests that, unless we notice, we cannot be in a position to choose or act for ourselves or our learners. Mason (2002), states that noticing what learners are doing and say is more complex and requires the noticing discipline known as teacher professional noticing that requires a complete set of practices to increase sensitivity to notice different aspects of learner experience. Teachers who are capable of noticing learner's errors and misunderstandings are most likely to perform well both in teaching and learning because they will not let misconceptions go unrepaired.

2.3 What are the characteristics of hearing learners?

On learner hearing, the study will define what is meant by hearing and listening using Davis (1997) and Wallach et al. (2005) papers.

Hearing is the sense that individuals interpret sound, and listening is the act of listening carefully or giving an ear to hear something, and this describes "noticing". Hence, hearing works by listening hand in hand. Evaluative, interpretive, and transformative (Hermeneutic) listening are three types of listening discussed by Davis. According to Davis (1997), the notion of listening suggests that listening is a very important path to follow for them to understand learners and also helps teachers to better understand teaching.

Evaluative listening, especially, pays much attention to something and it is more than listening to the speaker. If teachers and learners listen in an evaluative way, they will conclude what others say in terms of the wrong and the right, these are the contributions of learners that are considered either right or wrong (Coles, 2002; Davis, 1997). This is an effort to understand while attempting to make sense of what the speaker said when answering the question or argument.

Interpretive listening is described by a conscious awareness of the "fallibility of the sense being made..." by learners and teachers (Davis, 1996, p. 53). In this case, a learner or teacher may give a brief response that could lead one of them to ask for further elaboration on the brief answer given.

Transformative (Hermeneutic) listening "demands the willingness to interrogate the taken for granted and the prejudices that frame our perceptions and action" (Davis, 1997, p. 369-370). Coles (2002) went on to explain that transformative listening in a classroom would include the willingness to change ideas in a constructive dialogue to consider other points of view and to hold them as real.

Wallach et al. (2005) studied teaching practice related to hearing learners where teachers perceive the talk and behaviour of learners and examined what it can mean for a teacher to hear and explain their talk and actions. In recent years the function of the learner evaluation has been explored and updated. Wallach et al. (2005) assert that teachers are required to evaluate the comprehension of learners by watching learners solve mathematics tasks in a class by attending to their conceptual discussions throughout the lesson, by taking care of the essence of their in-class participation activities, and also by being responsive to their feelings. Wallach et al. (2005), urges teachers to experience four types of interpretation after they observe learners solving a math problem: description, explanation, assessment, and justification.

They went on to further explain the teacher's actions by stating the characteristics of each of the four interpretation as follows:

> *Describing –the teacher describes learners' talk, thoughts, feelings, and actions by direct (or almost direct) "quotation" or portrayal, (b) Explaining – the teacher explains the learners' talk or actions. This includes ideas about the learners' thoughts, reasoning, knowledge, and assumptions, (c) Assessing – the teacher assesses the students' talk and action, (d) Justifying –the teacher justifies her assessment of, or the meaning she attributes to, learners' talk and actions (Wallach et al., p. 401).*

Examining the forms of perception identified and discussed above according to Wallach et al. (2005) there are five different characteristics of teacher hearing learners: over-hearing, compatible-hearing, under-hearing, non-hearing, and biased-hearing. The table below offers a brief explanation of the various teacher hearing learner characteristics.

Table 1: Five forms of Hearing Learners (Wallach et al., 2005)

Forms of Hearing	**Statistics**	**Description**
• Over-Hearing	MORE	Add what the teacher expects
• Compatible-Hearing	ON PAR	Hear what is said
• Under-Hearing	LESS	Ignore a part
• Non- Hearing	LESS	Ignore the whole
• Biased-Hearing	LESS	Hear without proof

Wallach et al. (2005), describe overhearing as "hearing" stuff that the learners have not said that we would consider being an accurate reflection of what the learners have said, compatible hearing is when the teacher hears and accepts all the responses/answers, under-hearing is when the teacher "ignores" some of the things done or said by the learners, and non-hearing is when the teacher senses the wrong answer being raised amongst the learners and chooses the right answers without also reflecting on the wrong answers. Biased-hearing recognizes the work of the learners as taking risks and accepting answers without much consideration seems unsubstantiated (Wallach & Even, 2005).

2.4 What are errors and misconceptions?

Unlike the behaviourist perspective where errors and misconceptions are not significant because they do not consider the current concepts of learners to be applicable to learning, the constructivism theory finds misconceptions and errors to be crucially important to learning and teaching (Nesher, 1987). Errors and misconceptions are important to learning and teaching because they are part of the conceptual structure of the learners' thinking that will interact with new concepts (Nesher, 1987).

Generally speaking, an error means a simple slip of care or ability to concentrate which is made at least occasionally by almost everyone. According to Young and O' shea (1981), an error means the deviation from the correct solution of a problem in mathematics. An error is considered to be a mistake shown in the process of resolving a mathematical procedurally, algorithmically, or by other methods. It could be found in incorrectly answered problems that have faults in the response generating process (Young & O'shea, 1981). According to Riccomoni (2005), an error is a mistake, slip, blunder or inaccuracy, and unsystematic errors are unanticipated, recurring, incorrect responses that learners can easily correct on their own. Errors can be seen in the artifacts of learners (Riccomoni, 2005).

Riccomoni (2005) describes misconceptions as an idea that is incorrect because it was founded on a lack of understanding of a situation or conceptual awareness. He also states misconceptions arise when a learner does not construct protocols from conceptual understanding and also appears to come from prior knowledge as a learner tries to create mathematical significances. Misconceptions are sometimes concealed in the right answers, for example in multiple-choice questions, a learner can have everything right by guessing, and so most teachers will be unaware of the conceptual misconceptions that the learners hold (Riccomoni, 2005).

2.5 Previous studies on errors and misconceptions encountered by learners in solving algebraic problems involving directed numbers

Primary school teachers often tell their learners that they always need to subtract smaller numbers from larger numbers (Bofferding, 2014). This results in instances such as $4 - 12\ or\ 4 - -12\ or - 12 + 100$ becoming a challenge to learners, particularly when they are dealing with negative numbers (Bofferding, 2014). According to Makonye and Fakude (2016), learners face problems in dealing with the secondary level integral operation of directed numbers because they did not assimilate integer concepts of the directed numbers into their schemas they built up from primary school. As a result, learners tend to stick to their primary school number schemas, for example, they can solve the following as $8 - (-6) = 2$, and when learners are asked to organize the set of four ascending directed numbers, some learners write as follows:$-3, -4, -9, 2$ as their response, (Makonye & Fakude, 2016). In other words, learners with the above-mentioned primary school experience assume that negative nine is greater than negative three, and therefore they consider the inequality $-9 > -3$ as mathematical correct.

Comprehension skills of directed number operations in grades 7 and 8 are important for learners to do well in high school and tertiary education. The traditional teaching approach makes it difficult for learners to do well because they have been taught to obey rules and procedures without conceptualization. Hayes and Stacey (1998) discovered that some learners have fundamental misconceptions when basic exercises on addition, subtraction, and multiplication on directed numbers are expected to be solved, for example: $(a) - 3 + 5 = 8\ (b) - 3 - 5 = 8\ (c) - 3 \times 5 = 15\ (d) - 3 \times (-5) = -15$. According to Hayes and Stacy (1998), explains the first three problems (a-c) show that the learners did not fully recognize negative signs as fundamental tools whenever it comes to addressing operations of directed numbers. For (d) part, the learner could have figured it out like this: $(-1)(3) \times (-1)(5) = (-1)(3 \times 5) = -15$, the learner seems to have been factored out -1 (Hayes and Stacey, 1998). Hayes and Stacey (1998) postulate that negative numbers are difficult to teach and understand because they are not incorporated in early schooling phases.

Seng (2010) cites the 1999 study of the Third International Mathematics and Science which showed that less than 46% of Malaysian grade 7 learners did well in the three algebraic problems. Similarly in 1994, 1995, and 1996 surveys showed that Grade 8 learners did not do well on simplifying algebraic expression problems (Ministry of Education Malaysia, 2000). In Malaysia, these learners encounter some learning difficulties in the field of negative integers, exponents, and extension of brackets (Seng, 2010). According to Seng (2010), learners made two types of disconnection from negative sign errors in a complicated algebraic expression such as (i) learners worked the following as $-6a + 3a = -9a$ were learners performed the operation without taking into account the negative sign attached to their integer, (ii) disconnection from negative sign error was noted when learners simplified a complicated algebraic expression as follows $5ab - 6 + 4ba + 7 = 9ab - 13$. In the above case, integers have been added to the solution and brought their negative signs back. Seng (2010) says that there are two types of subtraction of integer errors and that they occur more frequently in the resolution of integer items and simple algebraic expression which are (i) a learner worked out the expression as follows: $3a - 6a = 3a$, taking 6a minus 3a to get 3a as evidence that learners used the concept of subtraction in arithmetic bigger value minus smaller value, and (ii) learners misapplied the golden rule "negative multiplied by another negative is positive" when they were solving expression involving two successive negative signs. In this case, an addition was carried out while using the multiplication golden rule for integers applied. Seng (2010) went to identify two types of the addition of integer errors: (i) learners simplified the following expression as $-6a + 3a = 3a$ where the positive sign loses its value as an operation sign, and (ii) learners simplified $5ab - 6 + 4ab + 7$ as $9ab - 1$ where the learners used procedural approach rather than the approach of a number line.

Seng (2010) found two types of multiplication of variable errors, (i) learners used the golden rules of multiplication or addition in simplifying $a \times a\ as\ 2a \times a$ through multiplying and adding the coefficients of a and then copyng the variable without change, and (ii) learners simplified $2 \times a + a + 15$ as $30 + a + a$ through multipying the coefficient of a with the constant of the expression $(2 \times 15 = 30)$.

Although the scholars have researched the topic of integers and how errors are commited by the learners across the world, this study also pays attention to the errors and misconceptions made by both learners and teachers in the Gauteng context in South Africa.

2.6 Powerful knowledge

Powerful knowledge in civilized societies is advanced, academic, or specialized knowledge that offers a more precise explanation (Zipin, Fataar, & Brennan, 2015; Young & Muller, 2013). Young (2007) states that context-independent knowledge is usually part of the normal experience gained and learned in schools and tertiary institutions, and it is called powerful knowledge because it enables learners to comprehend why lifeworld tends to work when it is communicated to them. The "specialised knowledge is knowledge about natural kinds - that is, knowledge about nature" (Young & Muller, 2013, p. 238). Bernstein (1975) stipulates that it is procedures, methods, and decisions that the learners are required to follow in order to learn new information, and indeed the process is called instrumental order. Instrumental order is not powerful knowledge instead, it is part of teaching methods (pedagogy) that is made ready for use by learners to get the correct answer and it is derived from powerful knowledge which they might probably not know at that stage. For instance, in mathematics learners can be asked to solve a quadratic equation using the quadratic formula: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and then do the substitution in order to get the correct answer without conceptualising. Generally, the quadratic formula is an instrumental order which is derived from powerful knowledge. Powerful knowledge has been used to come up with that formula in order to generalise the original knowledge.

2.6.1 Everyday knowledge and its differences from powerful knowledge

Everyday knowledge or common-sense knowledge (horizontal discourse or non-academic knowledge) is the knowledge that in the context of daily life may have been generated outside the school environment and is usually insufficient for knowledge of education (Morais & Neves, 2018).

Everyday knowledge is context-dependent knowledge acquired during the process of solving specific everyday problems (Young, 2007). According to Young (2007), context-dependent knowledge tells the learner how to do concrete things and there is no explanation or generalization. Everyday knowledge consisting of repetitive actions such as knowing how to make a fire or how to prepare coffee can be learned openly and free.

Everyday knowledge is also referred to as common sense in that if a child, who is unable to use or count positive numbers, places his or her three sweets on the table and you steal one of them he or she might weep for the missing one because it is common sense that tells her that the quantity is no longer the same (Young, 2007). Therefore, positive integers can be used to count the number of items in real-life situations. When incorporating positive numbers to grade one pupil, teachers use counters to teach learners to count numbers from zero to infinity. The value of developing powerful knowledge among learners can, therefore, be realized in this situation through a pedagogy that is sensitive to their life-worlds.

However, everyday knowledge is usually insufficient for knowledge of education. Conceptual understanding of positive numbers is not enough to deal with all forms of directed numbers, as directed numbers consist of positive and negative numbers. So what is the sense or meaning of negative two (-2); negative five (-5) or negative hundred (-100) when it comes to the conceptual understanding among learners? These negative numbers do exist in mathematics but not in the real life-worlds of the learner yet there are supposed to be learned by all the learners. Hence, we cannot always develop powerful knowledge of negative numbers among learners through a pedagogy that is responsive to their everyday knowledge.

Powerful knowledge was born because of the existence of everyday knowledge. The movement from the particular to the general, or from context-dependent meanings to context-dependent ones, is essentially a move from everyday knowledge to more vertical, codified, and abstract knowledge, such movement is normally achieved in schools and universities (Hoadley, 2007). The movement of knowledge from everyday knowledge to powerful knowledge shows that there is a great distinction between the two types of knowledge.

Hoadley (2007) states that "the initial distinction between these different types of knowledge was made by Durkheim (1915), who distinguishes between sacred and profane knowledge" (p. 682) The profane stands for the reaction of learners to their daily lives, and scared is knowledge, like that of negative numbers, that is not normally experienced physically from the everyday world, meaning it is not directly related to any real-world problem (Young, 2008a; Zipin et al., 2015). Throughout South Africa, the relationship between these two forms of knowledge in the curriculum and schooling is widely discussed (Muller & Taylor, 2000; Hoadley, 2007). The value of distinguishing between everyday knowledge and powerful knowledge is its focus on the uneven distribution of knowledge among learners which is usually based on socio-economic status (Hoadley, 2007). For example, urban learners have different tastes of everyday knowledge from those in rural farming areas. This can certainly affect the rate at which learners acquire powerful public school knowledge. In the progressive platform, the daily experience is often demonstrated to inspire and encourage the access of learners to school information (Hoadley, 2007). Therefore, many learners in different cultural contexts have access to different fields of knowledge (Hoadley, 2007).

The connection between everyday knowledge and academic concepts is mutually beneficial as they mediate powerful knowledge in order to maintain meaningful meaning (Zipin et al., 2015). Generally speaking, knowledge develops every day, in the field of learners, thus gaining meaning and purpose (Zipin et al., 2015). We need to turn everyday knowledge into a verticalising system by gaining disciplinary (powerful) knowledge. The educator must be responsible for transferring knowledge from horizontal to vertical discourse, scaffolding where the context of the field of life of learners can be used meaningfully as a gateway into disciplinary knowledge (Fataar, 2012).

Understanding the vertical and disciplinary knowledge by educators empowers them to better teach and impact knowledge to the learners more consciously and strategically.

2.6.2 Transmission of powerful knowledge

Young and Muller (2013) are against the fact that knowledge is only gained during formal teaching and learning as they argue that as part of the daily lives of learners powerful knowledge is not gained or generated informally since this knowledge requires specialized institutions such as schools and universities to be passed on to the next generations.

However, based on Bernstein's concepts of classification and framing, teaching can mean more or less separation of powerful knowledge and everyday knowledge (Zipin et al., 2015; Bernstein, 1975).

Morais et al. (2018) are in agreement with Young and Muller in treating academic knowledge (vertical discourse) as powerful knowledge and non-academic knowledge (horizontal discourse) as everyday knowledge. Horizontal discourse can contribute positively to the acquisition of vertical discourse to some extent. Fataar et al. (2015) argue that knowledge of everyday life can best be used as steps to scaffold into vertically specialized codes of knowledge, leaving behind life-based horizontal codes. In fact, powerful knowledge is vertical relative to the discourses of everyday life. If understanding is vertical relative to everyday knowledge, it means that a learner can procure powerful knowledge efficiently from low to high (Zipin et al., 2015). Young and Muller (2010) states that "vertical in horizontal knowledge structures [thus] occurs not through integration but through the introduction of ... apparently new problematic[s]…." (p. 128). The day-to-day knowledge is marked as horizontal and the marking of vertical knowledge is needed for deeper theoretical knowledge. Therefore, to avoid the use of everyday practical experience on a curriculum does not mean that there is no use of pedagogy for life-world knowledge (Zipin et al., 2015). Spontaneous ideas arising in the experiences of learners with everyday life-worlds are undoubtedly rich inexperience, but they do not provide any definitions and can be learned through a hierarchical process of disciplinary knowledge acquisition (Fataar et al., 2015).

Now it is the right time for the researcher to discuss researched models that will be used for teaching learners in the experimental classes of this project. Hence, the following section below is a program that will discuss the Professional Staff Development Seminar on developing Topic Specific Mathematics Teacher Knowledge (TSMTK) on teaching operations with directed numbers to Grade 8 learners.

2.7 Some Models for teaching addition, subtraction, and multiplication of directed numbers

Developing Topic-Specific Mathematics Teacher Knowledge (TSMTK) Programme was based on:

1. Professional mathematics teacher noticing on learners' mathematical thinking;
- listening and Hearing learners; probing and eliciting their thinking; try to make sense of anything a learner says or writes
- Being aware of learner errors and misconceptions- these are important milestones for learning
- Being authentic listeners to learners' voices
2. Teaching for conceptual understanding rather than remembering rules and formulas; which is mechanical learning- procedural mathematics.

This calls for;

- Use of models and representations that help teachers to unpack mathematics so that it is understandable to learners who are meeting it for the first time.

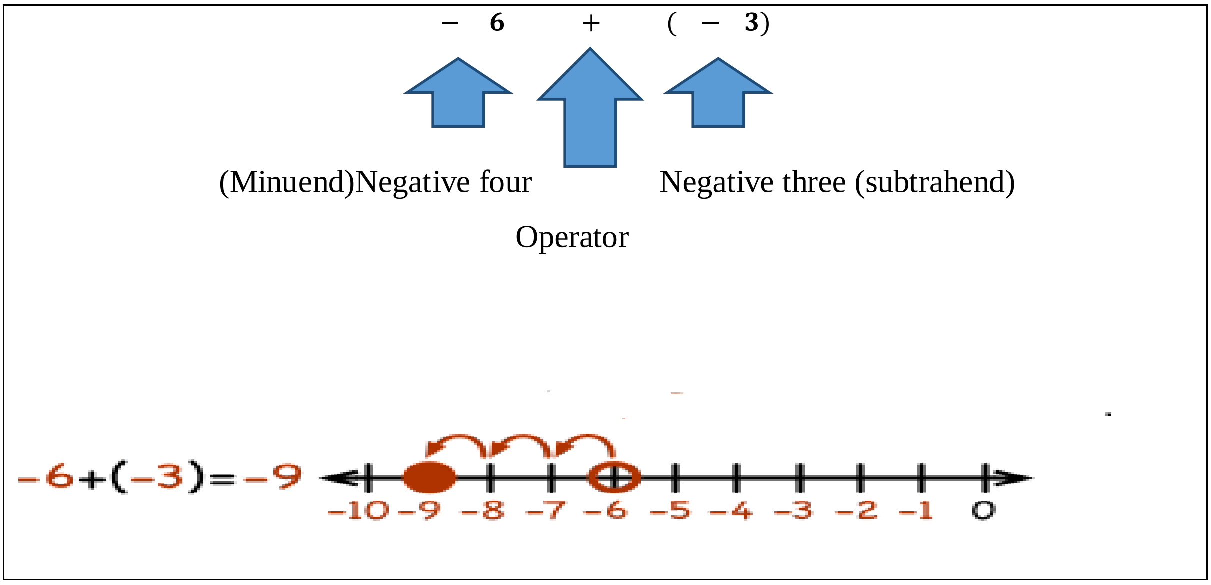
Ball et al. (2008) state that "knowledge of content and teaching (KCT), combines knowing about teaching and knowing about mathematics" (p.401), and "many of the mathematical task of teaching requires a mathematical knowledge of the design of instruction" (p. 401). The following models: models 1, 2, 3, and 4 were used as mathematical knowledge in teaching operations with directed numbers in Grade 8.

2.7.1 Model 1: Numbers line jumps

Example 1: Solve $-6 + (-3)$

Solution: Using the technique of number-line-walking

Figure 2: addition of two directed negative numbers



Step 1: The learner must stand up and start walking from the negative six when solving the expression.

Step 2: The positive operator instructs a learner to face right while standing at negative six.

Step 3: The negative sign on the subtrahend means to reverse and face right when at negative six, then shift 3 steps to the left, resulting in a stop at negative nine as the answer.

The researcher partially agrees with the model on the approach. The challenge is on the specification of facing right or left. The direction is not clear because it is not clarified whether the child is standing on 6 facing north or south direction. The applicability of the model is therefore limited in its lack of clarity.

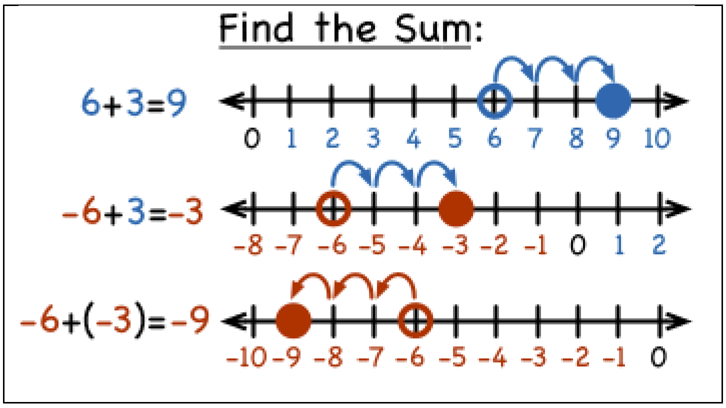
Example 2: Solve: $-4-(+3)$

Solution:

In general, following the number-line-walking technique, the negative operator on the expression instructs a learner to face left on a number line and the positive sign on the subtrahend or addend instructs a learner not to reserve but move forward and move three steps stopping at negative seven (-7) as the answer. That is $\mathbf{-4-(+3)=-7.}$

The figure below shows us how to find the sum of directed numbers by applying the above-mentioned three steps.

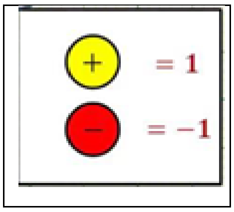
Figure 3: Find the sum of two directed numbers



Ball et al. (2008) suggest that "knowledge of content and students (KCS), is the knowledge that combines knowing about students and knowing about mathematics" (p. 401). Teachers need to predict what learners are likely to think and find it confusing when trying to define the meaning of an operator and the signs on the addend and subtrahend on given expressions (Ball, 2008).

2.7.2 Model 2: Use of positive and negative colour counters

Figure 4: The Zero principles



Zero principles mean that a **y**ellow counter plus a negative counter is equal to zero or $\mathbf{-1+1=0}$

2.7.2.1 Subtracting integers (directed numbers) using colour counters:

Solve: $3-(-4)=$

The expression above $3-(-4)$ means that negative four must be subtracted from positive three.

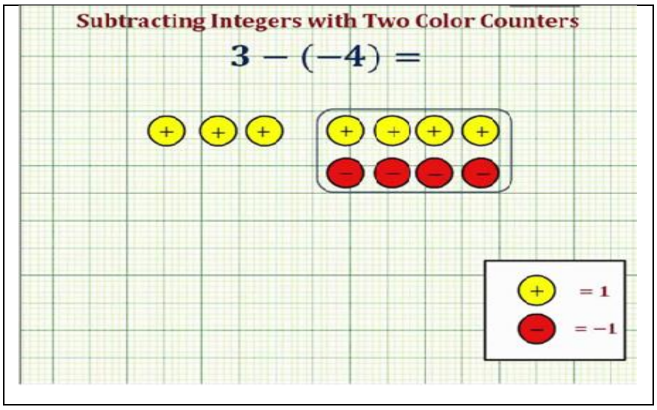


Figure 5: Subtraction of two directed numbers

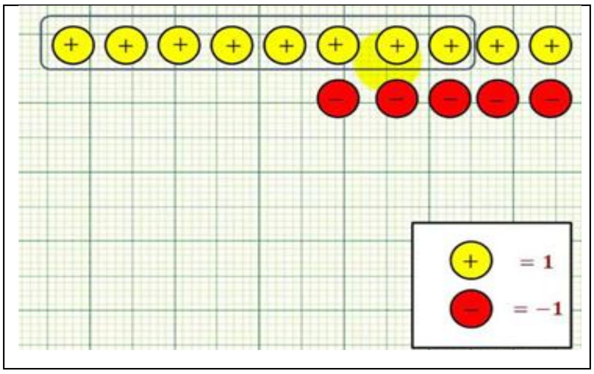
When finding the difference between positive three and negative four using counters, there must be three positive counters and four zeros in order to have four negative counters as shown in figure 5.Therefore, by removing four negative counters from the grouped counters, seven positive counter will remain as the solution of the above expression: $3 - (-4) = +7$.

2.7.2.2 Adding integers using colour counters:

Solve: $\mathbf{10} + (-\mathbf{5})$

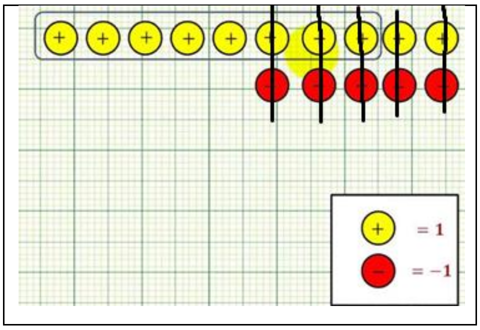
To solve $\mathbf{10} + (-\mathbf{5})$, gather ten positive counters and five negative counters together as shown in figure 6 below.

Figure 6: Adding two directed numbers using colour counters



To find the solution, re-align positive and negative discs to show zero principles (pairs) and count the remaining counters as depicted in figure 7 below. There will be five zeros and five positive counters will remain and become the answer to the expression: $\mathbf{10} + (-\mathbf{5}) = +\mathbf{5}$

Figure 7: Realigning to show zeros

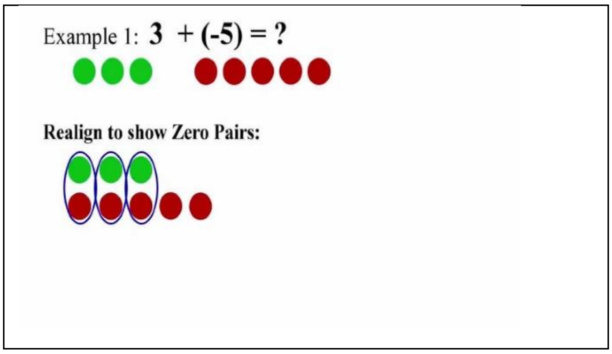


$$\mathbf{10} + (-\mathbf{5}) = +\mathbf{5}$$

To familiarise with the use of discs or chips the researcher provides one more example of adding directed numbers as shown in figures 8 and 9 below

Example 1. Solve: $3 + (-5) =$

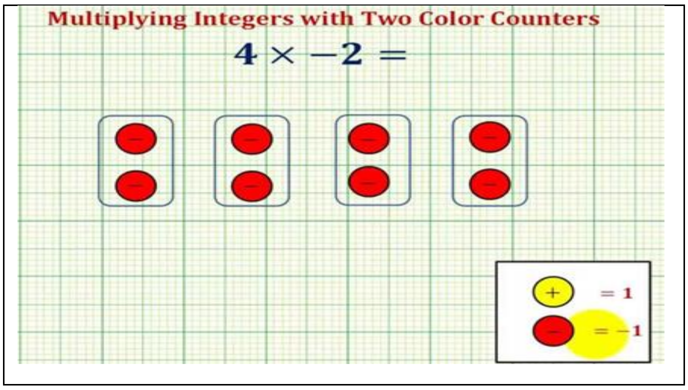
Figure 8: Adding two integers using colour counters



2.7.3 Model 3: Teaching multiplication of Directed numbers using positive and negative coloured counters

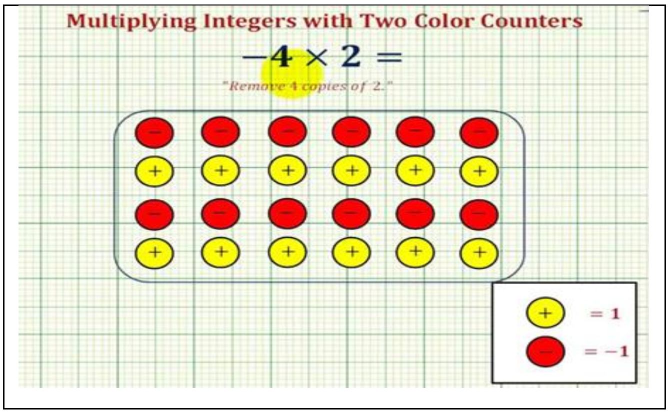
Multiplying directed numbers with 2 colour counters: $4 \times (-2)$ means add 4 copies of -2 as shown below the table: $4 \times (-2) = \mathbf{-8}$

Figure 9: Multiplying two integers using colour counters



The expression -4×2 instructs the learner to gather a set of zeros sufficient to remove four copies of positive 2s.

Figure 10: Multiplying two integers using colour counters

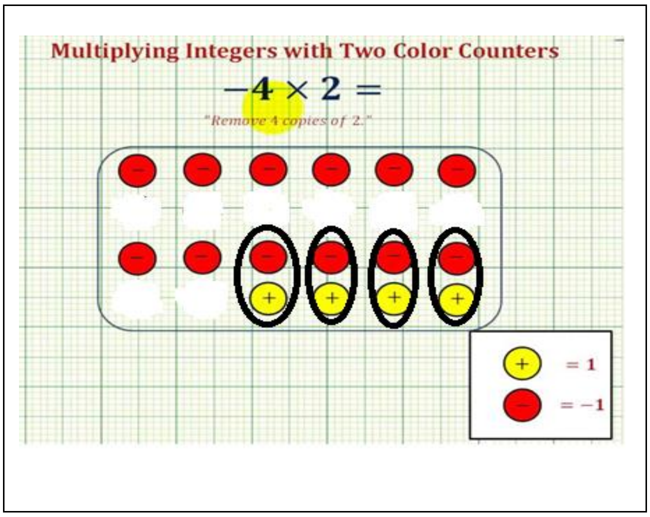


When four copies of positive 2s are removed the above figure number 10 will look like this:

8 negative discs plus four zeros are left resulting in having negative 8 as the answer:

$$-4 \times 2 = -8$$

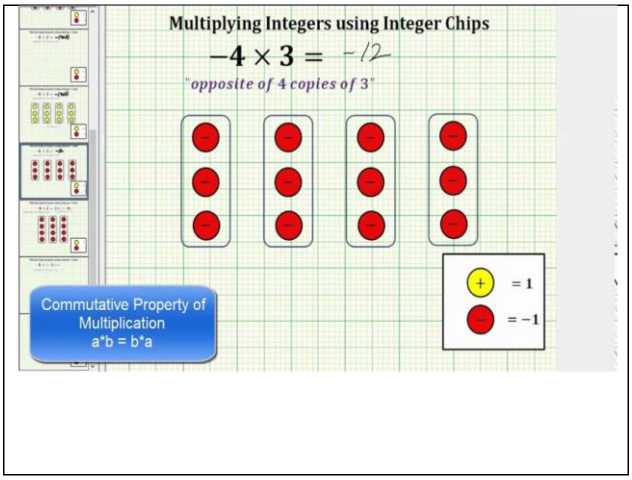
Figure 11: Multiplying two integers using colour counters



Commutative property of multiplication means that:

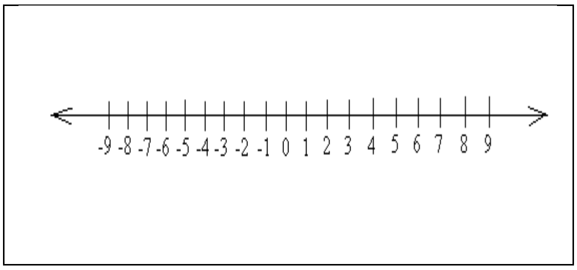
-4×3 $(remove\ four\ copies\ of\ positive\ three) = 3 \times -4$ (add three copies of negative) as shown in figure 12 below:

Figure 12: Multiplying two integers using colour counters



2.7.4 Model 4: Switching on the number line

Figure 13: Number Line



Multiplying by a negative uses the idea that negative numbers are a reflection of positive numbers on 0 in the number line. Every positive number has a negative counterpart. In general, multiplying a number by a negative one (-1) just switches it to the other side of zero (0) as shown by figure 14 below.

Figure 14: Reflection counterparts of positive and negative numbers

Positive $(+)$ → Negative $(-)$

For instance: $6 \times -1 = -6$

$8 \times -1 = -8$

$12 \times -1 = \ ?$

Therefore, what is the value of? $(-1) \times (-1) = ?\ and\ (-1) \times (-1) \times (-1) = ?$

$$4 \times (-1) = -4$$

$$4 \times (-1) \times (-1) = 4$$

$$4 \times (-1) \times (-1) \times (-1) = -4$$

$$4 \times (-1) \times (-1) \times (-1) \times (-1) = 4$$

$$4 \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -4$$

Task activity: Now, replace ■ with $-\ or +$

$$4 \times 3 = ■12$$

$$-4 \times 3 = ■12$$

$$4 \times -3 = ■12$$

$$-4 \times -3 = ■12$$

2.8 Conclusion

In this chapter, the researcher discussed, with the help of articles, the meaning of noticing in general; professional noticing; everyday knowledge; powerful knowledge; errors; and misconceptions. The aim of this chapter was also to look at professional noticing as the main character that will contribute to the creation of the theoretical framework for this study. Five forms of learner hearing found in the Wallach et al. (2005) article were discussed in detail and the researcher is interested in using these five forms of learner hearing theories in conjunction with powerful knowledge theory as supplements towards the formation of the theoretical framework of the study. In chapter 3 the researcher will then develop a theoretical framework based on the definition of powerful knowledge, professional noticing, and five forms of learner hearing (Zipin et al., 2015; Young & Muller, 2013; Jacob et al., 2010; Wallach et al., 2005).

CHAPTER 3 THEORETICAL FRAMEWORK

3.1 Introduction

This chapter tackles the theoretical framework of the study. Lester (2005) states that "The notion of a research framework is central to every field of inquiry" (p. 458). The theoretical framework guides and conceptualizes the research study. A theoretical framework enables the researcher to make sense of the collected data. According to Lester (2005), there are three types of frameworks that a researcher can use to explore or investigate a particular problem or experience namely; conceptual, theoretical, and practical. A theoretical framework "guides research activities by its reliance on a formal theory; that is, a theory that has been developed by using an established, coherent explanation of certain sorts of phenomena and relationships" (Lester, 2005, p. 458). In the case of this study, the main observed phenomenon to be described is teacher professional noticing in teaching and learning operations of directed numbers in a classroom context. This study is informed by relevant literature discussed in chapter 2, which leads to the use of Jacobs et al. (2010) professional noticing theory as a major central theory of this study, and also the use of Wallach et al. (2005) forms of hearing and powerful knowledge (Zipin et al., 2015; Young et al., 2013) compliment the central theory of this study.

Teachers perform all forms of hearing in our schools and tertiary institutions, of which some are supposed to be positive towards teaching and learning among learners and others are not. Wallach et al. (2005) forms of hearing play a key role in choosing one of the relevant forms of learner hearing that can be used by teachers to practice professional noticing of learners' mathematical thinking in teaching and learning mathematics.

Therefore, a study on professional noticing, forms of hearing, and powerful knowledge from scholars will guide the researcher in designing a meaningful theoretical framework.

The literature on different types of noticing of learners' mathematical thinking by the teacher in teaching and learning operations on directed numbers was reviewed in the previous chapter. According to Jacobs et al. (2010), a literature review on professional noticing is an ability that teachers can use to recognize and act on learners' mathematical thinking, and it is made up of three interrelated components namely the method of attending, interpreting, and making decisions.

Therefore the review of literature on teacher professional noticing, learner hearing by the teacher, and the use of powerful knowledge has become part of the theoretical framework for describing teachers' discourses in teaching and learning operations with directed numbers in Grade 8. This literature review on powerful knowledge guided the researcher in designing relevant tasks for experimental classes. Literature review on teacher professional noticing and learner hearing also guided the experimental class teacher in practicing a proper form of hearing to learner's responses and activities.

3.2 Professional noticing and learner hearing

3.2.1 Professional noticing

This study is informed by professional noticing: (i) attending to learners' strategies, (ii) interpreting learners' understandings, and (iii) deciding on how to respond based on learner's understandings (Jacobs et al., 2010). The three interrelated skills of attending, interpreting, and remediating (deciding) the mathematical thinking of learners while teaching and learning directed numbers in Grade 8 are three key phases that enable teachers to practice professional noticing.

In the first phase of Jacob et al.'s (2010) study of attending to learners' strategies, the teacher analyses learners' responses that arise in the classroom during the lesson. In this skill, the focus on learners' strategies is on mathematical context as this generates a window into the understanding of the learners' thinking. In this case, the teacher will ask herself a question that needs her to describe in detail what she thinks each learner did in response to the problem. For instance, learners can be asked to work out a problem such as $-2 + (-7) = \blacksquare$ in a way that makes sense to them. The mathematical information in the learners' response would, therefore, concentrate on how the learners computed the answers rather than the positions they took when carrying out the task.

The second phase is interpreting the learners' understanding or responses. In this step, the emphasis is on interpreting the understandings or responses of learners by teachers as expressed during their strategies. The question to be asked by teachers: Please clearly explain the conceptual understanding of the learners' mathematical thinking.

The third phase is on deciding how to respond based on learners' understanding or responses. The emphasis in this process is on the logic used by teachers when determining how to answer. This is, to what degree do teachers use what they have learned from the particular situation about the comprehension of the learners and whether their rationale is compatible with the mathematical development of the learners' study.

3.2.2 Hearing Learners: What is it that we hear when we listen?

To master the practice of professional noticing teachers should listen and exclusively hear learners' ideas. Hearing learners fall within the complexity of understanding what they say, show, and do therefore by understanding Wallach et al.'s (2005) five forms of hearing, namely over-hearing, compatible-hearing, under-hearing, non-hearing, and biased-hearing, teachers can select the best form of listening and hearing learners' ideas.

Wallach et al. (2005), defines five forms of hearing:

- Over-hearing is to 'hear' things that were not said by the learners which we would expect to be an accurate representation of what the learners said.
- Compatible-hearing is when the teacher hears and considers all the responses/answers raised by learners.
- Under-hearing is when the teacher 'ignores' some of the things done or said by the learners.
- Non-hearing is when the teacher hears the wrong answer being raised among the learners and picks on the correct answers without also commenting on the wrong answers
- Biased-hearing is understanding learners' work as taking a risk and accepting answers without much thought seems unsubstantiated.

3.3 Powerful knowledge

A review of literature on errors, misconceptions, and difficulties learners experience on operations with directed numbers made the researcher develop topic-specific mathematics knowledge (powerful knowledge) on teaching operations with directed numbers in Grade 8 using number lines and counters.

Shulman (1986) argued that understanding a teaching subject involves more than knowing the facts and concepts. Ball et al. (2008) argue that teachers also need to understand the principles and systems of organization and the rules for deciding what is appropriate to do and say in the classroom. Not only does the teacher need to understand that something is so; the teacher needs to understand more *why* it is so, on what grounds his warrant can be sought, and under what conditions our trust in his argument can be diminished or refused (Ball, 2008). Therefore, with powerful knowledge, researchers focus on the work of teaching to frame the conceptualization of teachers' and learners' mathematical knowledge and skills.

Ball et al. (2008) explored the essence of technical subject matter knowledge in mathematics through the study of real mathematics teaching and the identification of teaching mathematical knowledge based on the analysis of mathematical problems in teaching. It is not enough to learn certain teaching procedures, you need to know exactly what you are going to say (Shulman, 1986). The literature review affirms that powerful knowledge in civilized societies is advanced academic or specialized knowledge that offers a more detailed interpretation of procedural rules (Zipin et al., 2015; Young et al., 2013). Therefore, Ball et al. (2008) indicate that teachers need to learn mathematics in ways that are useful in making the learner's mathematical sense of work and in choosing powerful ways to portray the subject so that learners can grasp it.

Therefore, teaching for learner comprehension involves applying mathematics to everyday life interactions of the learners for sense-making purposes; and that it encompasses the conceptual understanding of the learners. Hence, a teacher with a strong mathematical background is important as learners need to learn and explain every step taken while solving mathematical problems through defining the mathematical topic's requirements in context. A teacher also needs to evaluate the comprehension of the learners by the physical expression, and stop using shortcuts in mathematics teaching since the essential thing is to illustrate and explain concepts. Teachers without powerful knowledge or academic knowledge do not seem to understand the importance of demonstrating and explaining actions taken to solving mathematics in schools. The reasons why the importance of demonstrating and clarifying is not understood are because teachers do not practice their ability.

Since the researcher has gathered enough of the necessary theories, it is now time to move on to the establishment of the theoretical framework of this study.

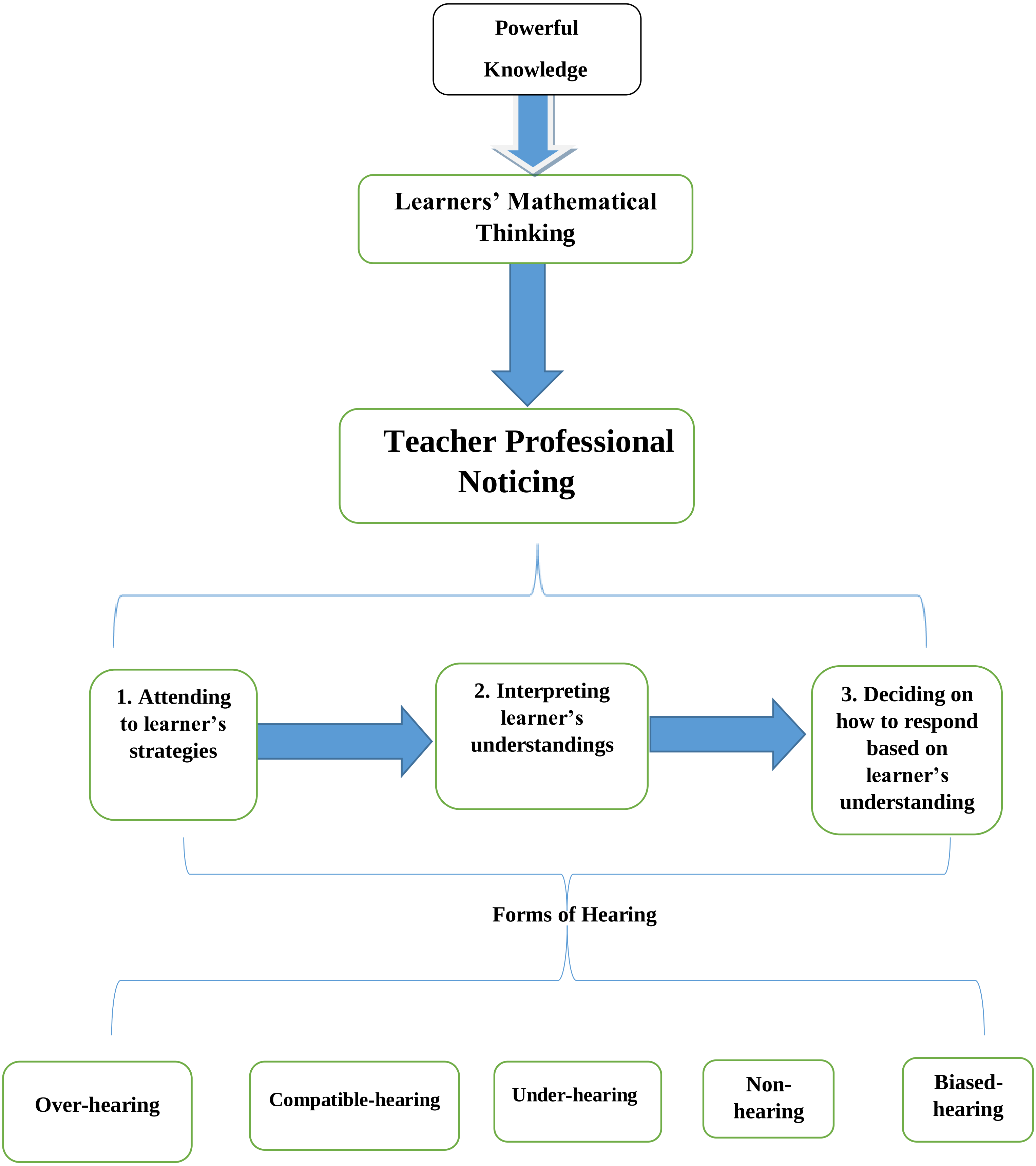
3.4 Proposed theoretical framework to explore teacher professional noticing in the teaching of directed numbers.

The theoretical framework is mainly extracted from the insights obtained from the definition of powerful knowledge, professional noticing, and five forms of learner hearing (Zipin et al., 2015; Young & Muller, 2013; Jacob et al., 2010; Wallach et al., 2005).

Figure 15 below is treated as a dynamic interrelationship between powerful knowledge (Zipin et al., 2015; Young & Muller, 2013), Jacobs et al.'s (2010) theory, and Wallach et al.'s (2005) theory. As illustrated in figure 15 below, the use of powerful knowledge is mandatory for teaching and learning for conceptual understanding among learners in schools. Also, the researcher noted that Jacobs et al.'s (2010) professional noticing interrelated skills are not sufficient to stand on their own because there are several forms of learner hearing which are assumed to have a negative and positive effect when they are exercised by teachers and these have been described by Wallach et al. (2005) as a mode of non-hearing, compatible-hearing, over-hearing, biased-hearing, and under-hearing.

Figure 15: A dynamic interrelationship between powerful knowledge (Zipin et al., 2015; Young & Muller, 2013), Jacobs et al.'s (2010) theory and Wallach et al.'s (2005) theory.

3.4.1 Theoretical framework



Teacher professional noticing on learners' mathematical thinking must be based on the use of powerful knowledge in teaching and learning mathematics so that learners can have a better conceptual understanding of operation with directed numbers or any other concepts.

When teaching learners in the classroom, a teacher can overhear or under hear learners' mathematical thinking, or can simply be biased in hearing learners' responses, and this can harm the use of the three interrelated skills of teacher professional noticing. Once the teacher experience these forms of hearing, automatically the teacher will not be able to attend and interpret learners' strategies and understandings. Therefore, s/he will not be able to decide on how to respond based on learners' understanding.

When the teacher hears and considers all the responses/answers (compatible-hearing), he will be in a position to attend and interpret learners' strategies and understandings. Therefore, s/he will be to decide on how to respond based on learners' understanding.

In general, the three interrelated skills of professional reporting make it easy for teachers to identify the ZPD learners. So, if the teacher listens to what the learners say, he or she will be in a position to hear what the learners say. As a result, he/she will know what the learners are doing, and it will help the teacher to locate their ZPD so that he/she can help the learners. Attending, interpreting, and remediating (deciding) the mathematical thinking of learners will help the teacher to teach with conceptual understanding.

3.5 Conclusion

Having surveyed the field (literature review) and developed a theoretical framework that basically shows a dynamic interrelationship between powerful knowledge, teacher professional noticing and five forms of learner hearing, the researcher will now move on to the next chapter to discuss the methodology and design of the research study. In the next chapter, the researcher will describe in detail the methods and the processes that were involved in the gathering of data for the research project.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

The previous chapter explored a dynamic interrelationship between Jacobs et al. (2010), Wallach et al. (2005) theories, and academic knowledge, or powerful knowledge bodies shaping each other in teaching and learning of integers. The main aim of this study was to explore teacher professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. Specifically, the study was focusing on the dynamic interrelationship between teacher professional noticing, learner hearing, and powerful knowledge. Thus, this chapter discusses the research methodology and procedures that were followed to achieve the objectives of this research including the research ethics, data collection, data analysis, data trustworthiness, and limitations of the study. Clive (2004) defines methodology as, "the theory of getting knowledge, to the consideration of the best ways, methods or procedures, by which data will provide an evidence basis for the construction of knowledge about whatever it is, that is being researched, or, is obtained" (p .16).

4.2 Methodology

The research employs case study and experiment approaches to tap from the benefits of both methods as well as strengthen the validity of the data.

They were two types of classes that were used to carry out the study, namely the experimental class, and the control class. The experimental class teachers were trained to practice teacher professional during the class interventions for the study and they were also trained to teach for conceptual understanding through using skills that they acquired at the Professional Staff Development Seminar on developing Topic Specific Mathematics Teacher Knowledge (TSMTK) on teaching operations with directed numbers to Grade 8 learners. The research project studied two qualified secondary maths teachers who received training to practice teacher professional noticing of learner's mathematical thinking in teaching and learning operations on directed numbers in Grade 8. The control class had two qualified secondary school maths teachers who did not train for teacher professional noticing of learners' mathematical thinking. Therefore, the research paradigm of this study was an interpretive one, which makes use of qualitative methods to answer the research questions.

Qualitative research can help researchers gain access to research participants' thoughts and feelings, which can allow them to develop an understanding of the meaning people attach to their experiences (Sutto & Zubin, 2015). Thorn (2000) states that qualitative study depends on inductive reasoning processes to structure and interpret the meanings that can be obtained from the data, whereas deductive reasoning begins with the ideas and uses the data on making ideas and conclusions (Thorne, 2000). In a qualitative study, the researcher goes into the field or community (site) to collect data enabling the researcher to observe teaching and learning in a natural setting through video or audio recordings. The researcher will then analyse the collected data to obtain findings and make a conclusion.

This project aimed to capture the impact of practicing teacher professional noticing of learner's mathematical thinking in teaching and learning operations on directed numbers. Thus, this enabling teachers who have been taught in the professional noticing of learners' mathematical thinking to describe and understand learners' actions and subsequently the consequences of their actions. The interpretative approach to this research is acceptable because the researcher understands that the study results are generated by interpreting the data collected (Bertram & Christiansen, 2014), and thus lends itself to interpretations informed by the theoretical framework that was discussed in Chapter Three. The interpretive approach assisted the researcher in describing the teacher's professional noticing of learner's mathematical thinking in teaching and learning operations on integers.

According to Halliday and Hasan (1985), learning takes place in a social cycle. The social background which facilitates learning occurs in the school system among learners and teachers and even between the learners themselves. The nature of the research problem is exploratory and seeks to understand how the participants in the study respond to the professional noticing of their learners' mathematical thinking during the teaching of the topic using certain researched teaching methods. The purpose of this study was to improve the teaching and learning of subtraction, addition, and multiplication of integers through an intervention where teachers teach the topics using certain researched teaching techniques. Therefore, two case studies of experimental and the control classes were used to address the problem statement of this study that was mentioned in chapter 1.

4.3 Context of the Study and Participants

The research was conducted in two secondary schools that are located in the inner city of Johannesburg. The pseudonymous names of the two schools used for the study data collection were Lundi Secondary School (LSS) and Shongamiti Secondary School (SSS). Lundi is a public secondary school, and Shongamiti is a private secondary school, which also receives funding from the government. The two above-mentioned schools use English as a medium of instruction. Lundi Secondary School was relatively well-equipped with mathematics facilities and well-sponsored because it is a government school and has classrooms that were up to standard and well ventilated. The infrastructure of Shongamiti Secondary School was not up to standard, as the buildings were too small and clustered together, resulting in overcrowding.

The researcher had two types of classes namely the control and the experimental class in each school. A total of four classes of Grade 8 learners were selected to participate in the research project from the above-mentioned schools and each class had a maximum of 45 learners. The four classes were made up of two experimental classes and two control classes and among the four Grade 8 mathematics class teachers, they were three qualified male teachers and one qualified female teacher.

This research project is considered an experimental work. In experimental work, Opie (2004) suggests that learners' actions can be controlled or conditioned by external circumstances such that they are the products of the environment in which they find themselves. Personality, feelings, imagination are meaningless in this case (Opie, 2004). In this study, stages of experimental work will enable researchers to select the independent variables (control classes) and dependent variables (experimental classes).

Unlike 'true' experimental, 'quasi' experimental does not use randomisation to seek to ensure a greater probability of learners' equivalence being tested (Opie, 2004). In other words, the best that can be done is a balance in which groups being studied are the most similar (Opie, 2004). Within this report, the groups referred to here are the experimental classes and control classes contained in these schools. Therefore, the non-equivalent control group called 'quasi' experimental was used to set up the experimental and control classes.

4.3.1 Sampling of research participants

4.3.1.1 Teachers

The researcher applied the random probability sampling method to select the subject teachers who made the sample size of two teachers. The sampling method was fair as all the educators had an equal chance to be selected.

Out of the four teachers in both schools, only two Grade 8 mathematics teachers namely Mr. Ngele and Mrs. Zitha participated in this study. In other words, one Grade 8 teacher from each school was selected to conduct the research study. Mr. Ngele was a teacher at Lundi Secondary School (LSS) and Mrs. Zitha was a teacher at Shongamiti Secondary School (SSS). Mr. Chauke (the researcher) and Mr. Manana were University Researchers who participated in the research by teaching Grade 8 learners that belonged to experimental classes during data collection in both schools.

Mr. Ngele and Mrs. Zitha were maths teachers who participated in the research project by teaching Grade 8 learners during the data collection of the two control classes at Lundi Secondary School and Shongamiti Secondary School. Mr. Ngele and Mrs. Zitha were control-class maths teachers at Lundi and Shongamiti Secondary schools respectively.

Mr. Chauke was a master's student and Mr. Manana was a lecturer at a university located in Gauteng, and they both received professional development training on the development of topic-specific mathematics teacher knowledge (TSMTK) in Grade 8 teaching operations. Mr. Chauke and Mr. Manana also received professional development training on awareness of teacher professional noticing, learner hearing, and methods of conceptual teaching before they were observed and audio recorded while teaching Grade 8 experimental mathematics classes in both schools.

Mr. Ngele and Mrs. Zitha belong to controlled classes that were observed and audio recorded and they did not receive professional staff development training on developing (TSMTK) on teaching operations on directed numbers in Grade 8. Mr. Ngele and Mrs. Zitha did not attend technical staff engagement workshops on teacher professional noticing and learning hearings.

Teachers were chosen on the basis that they were eligible to teach Grade 8 mathematics in both schools. The table below contains relevant information about the teachers (participants' biographical data)

Table 2: Information about participants (teachers)

Participants	Age	Number of years teaching mathematics in Grade 8	Qualification	Total number of years teaching
Mr. Chauke (the researcher)	42	4	1. Diploma (mathematics) 2. A.C.E (FET Mathematics) 3. BSc Honours (mathematics) 4. M.Ed. masters(mathematics pending)	16
Mr. Manana	50	10	PhD (mathematics)	28
Mr. Ngele	28	4	B.Ed. degree (mathematics)	4
Mrs. Zitha	31	6	B.Ed. degree (mathematics)	6

4.3.1.2 Learners

Lundi School has six classes of Grade 8 learners, and only two of them were randomly selected to participate in the research project. Of the two classes, one was the experimental class and the other one was the control class. SSS had three classes and only two classes were used for the study, one for experimental and the other for control class. The classes were randomly selected to avoid grading learners according to their performance.

There was no grading of learners according to their performance hence they were all treated equally. Most of the learners at Lundi Secondary School had mathematics textbooks, whereas most of the learners at Shongamiti Secondary School did not have them at all. The use of calculators was strictly prohibited in both the experimental and control classes.

A total of 161 Grade 8 learners consisting of both boys and girls participated in this research project in both schools. Table 3 contains the number of boys against girls and the total number of learners per Grade 8 class.

Table 3: Information about learners at Lundi Secondary School and Shongamiti Secondary School.

Class	Number of boys	Number of girls	Total number of learners
Grade 8A at LSS	18	23	41
Grade 8B at LSS	17	20	37
Grade 8A at SSS	20	24	44
Grade 8B at SSS	23	19	42

4.4 Research instruments

Instruments of research are strategies to find facts. They are the data collection tools. In this study, they included document analysis (CAPS), pre-test, post-test, observation (participant observer and non-participant observer), audio recording, and transcribing. The researcher ultimately ensured that the chosen instrument is valid and reliable, as a research project's validity and reliability depend largely on the appropriateness of the instruments **(**Abowitz & Toole, 2010).

Observation, audio recording, and learners' written work in form of pre-test and post-test provided the researcher with valuable data that enabled the provision of answers to the research questions for this research study. The method of data collection used is called triangulation meaning, it is "the use of two or more methods of data collection in the study of some aspect of human behaviour" (Cohen, 2000, p. 112).

4.4.1 Document analysis: Caps document

The Curriculum and Assessment Policy Statement (CAPS) document is an official document from the Department of Education that is used to assess learners in South African schools. In the Grade 8 CAPS document, item 1.3 highlights an overview of what should be taught about the addition, subtraction, and multiplication of integers. The Caps document presents some teaching guidelines and clarification notes on the topic of integers.

See appendix C: Grade 8 CAPS document for more information on operation with integers.

4.4.2 Pre and Post-test

According to Cohen, Manion, and Morrison (2000), a test is a powerful method of data collection which is an impressive way for gathering data of a numerical rather than verbal kind. Learners did write a pre-test and post-test in both schools to see what form of misconceptions and errors were displayed by learners in the classroom. The Grade 8 caps document was used by the researcher to set or develop the two tests because it highlighted what should be taught at the Grade 8 level concerning operation with integers. The pre-test had a total of 20 questions and the total number of questions in the post-test was reduced to 18 questions because the researcher noticed that four questions found in the pre-test were testing similar concepts. In general, pre-test and post-test had the same questions but the numbers were altered. See appendix D for the samples of the pre-test and post-test. The pre-test was also meant to check their level of performance before the research intervention and the post-test was meant to check on their level of performance after the research intervention.

4.4.3 Observation and audio recording lessons in progress

Observation is one of the effective methods of obtaining comprehensive data in qualitative research, particularly when a combination of both oral and visual data is crucial to research and involves the study of images, videotapes, tape recordings, art objects, computer software (Abowitz & Toole, 2010). The technique should be inoffensive to allow participants to freely share their experience with researchers, as a result, researchers will be able to obtain first-hand information about the study area (Abowitz & Toole, 2010).

There are two types of observing in the classroom namely participant observer and non-participant observer. As a participant-observer, the researcher lives as a member of the study subject when witnessing and taking records of the characteristics of the subject being researched so that the phenomenon being examined can be observed directly (Abowitz & Toole, 2010). Despite being seen as a member of the study subject, he marks his real identity as a researcher, and that bears the advantage of receiving first-hand experience with informants (Abowitz & Toole, 2010). Therefore, Mr. Chauke and Mr. Manana were participant observers in the experimental classes meant for this study.

A non-participant observer is not a member of the study subject. The researcher is observing his / her study subject with their knowledge of his / her role as a researcher, but without taking an active part in the situation being studied (Abowitz & Toole, 2010). Therefore, Mr. Chauke and Mr. Manana were non-participant observers in the control classes of this study. However, there are some disadvantages to using this means of collecting information. Sometimes this approach is questioned because the very fact that they are being monitored can cause people to behave submissively and thus invalidate the collected data (Abowitz & Toole, 2010). According to Clive (2004) in observational research, teachers and learners can change the way they behave when being observed as a result biased information may be collected. Therefore, in this study, the researcher observed all lessons so that learners and the teachers got used to the researcher’s presence and did not change their normal way of behaving during the research process.

The researcher collected data by practicing all the two types of observation mentioned above. Clive (2004) sees observation as a very important tool to collect data since the researcher can directly gather information about the physical environment and human behaviour. The researcher acted as a non-participant observer for controlled classes and as a participant-observer for experimental classes held in both schools. Clive (2004) suggests that observation helps researchers to gain knowledge and information that informs our everyday actions.

Observation and the presence of audio recorded lesson-in-progress helped the researcher to identify five different forms of teacher-learner hearing practiced by the teachers while he was exploring the set of interrelated skills of professional noticing of learners' mathematical thinking as defined by Jacobs et al. (2010).

4.4.4 Transcribing and Checking

The class lessons done during data collection were audio-recorded and used for data analysis. Transcription is an exhausting process, even to the most experienced transcribers, but to facilitate analysis, it must be done by converting the spoken word to the written word (Sutto & Zubin, 2015). The transcription of at least one observed lesson was done for the two control classes taught by Mr. Ngele and Mrs. Zitha and was also done for experimental classes that were taught by Mr. Chauke and Mr. Manana. Lesson observations transcripts were then developed into scenarios for data analysis as shown in Appendix F. Once the researcher had transcribed and created scenarios, the coding of major theories and important aspects were done for data analysis purposes. Coding refers to defining problems, similarities, and differences that were identified by the narratives of the participants and interpreted by the researcher concerning the main title and its critical research questions of the study mentioned in chapter 1 (Sutto & Zubin, 2015).

4.5 Limitations experienced in data collection

During data collection, the researcher experienced some challenges in setting up the project. All the learners were willing to take part in the research as participants except for one teacher who by the last minute refused to teach the learners although she had initially agreed to take part in the research process as an experimental class teacher. Another challenge was, that, the researcher was unable to capture some of the work that was displayed by learners on the chalkboard since he was using an audio recording gadget.

The class lessons held at SSS were severely affected in the sense that they were not running every day as per the project schedule because the community was suffering xenophobic attacks on foreign nationals. Such xenophobic assaults led to learners knocking off early on school days, and for several other days until schools faced forced closures. Despite the xenophobic-inspired disturbances experienced on some of the days at SSS, the researcher managed to do some interventions to make sure that all the required lessons were carried out.

4.6 Reliability and validity

The reliability of the interpretation and representation of the narratives of the participants is one of the questions that arise in qualitative research. According to Sutto and Zubin (2015), as there is in quantitative research, there are no statistical measures that can be used to verify reliability and validity. Nevertheless, Lincoln and Guba's research (1985) shows that there are other ways to build confidence in the ‘reality’ of the results and they call this faith, trustworthiness (Lincoln, 1985).

Since the design of this study is qualitative, the reliability and validity will be consistent with the use of the main source of data collected through audio recordings, lesson observations, and learners’ pre and post-tests transcripts.

4.6.1 Validity in data collection

The validity type is centered on the authenticity of the collected and used data. Wellington (2000) states that “validity refers to the degree to which a method, test or a research tool measures what it is supposed to measure” (p. 201). The data that was used to write this report was collected from Grade 8 learners who attended formal schools that fall under the Department of Education regulations in South Africa. According to the research project schedule, Grade 8 learners were given a pre-test, class lesson interventions, and a post-test. The pre-test and the post-test were written by learners on strict supervision similar to that of sit-down examination. During the class lesson interventions, an audio-recorder was used by the researcher to collect data. In this way, having pre-test and post-test transcripts from the Grade 8 learners, observational audio recordings allowed for a more trustworthy data collection.

The audio recordings and transcripts automatically created a trustworthy data collection, but to maintain validity in the learner transcripts and observed audio-recorded, lessons required descriptive validity. Descriptive validity is a term of research that refers to the exactness and integrity of the gathered information (Bamber, Ramsay, & Tubbs, 1997). Descriptive validity ensures that the researcher maintains the credibility of what they have heard and seen, and does not distort, exaggerate, or create bias data (Bamber et al., 1997). Therefore, to maintain descriptive validity the researcher had to ensure that all the transcripts were completely marked by the researcher or qualified personnel and then moderated. Audio observed class lessons must be transcribed as accurately as possible using the exact words without modifying the structures of the participants.

Once the marking and transcriptions had been completed accurately, the data was analysed and interpreted. Interpretive validity relates to the occurring events and the meanings derived from those events. Maxwell (1992) stresses "qualitative researchers are not concerned solely, or even primarily, with providing a valid description of the physical objects, events, and behaviors in the settings they study; they are also concerned with what these objects, events, and behaviors mean to the people engaged in and with them" (p. 288). Hence, interpretive validity is built from the viewpoint of the research participants and must accurately characterize the behaviour of the individuals as described to them. As such, the researcher participated in the transcriptions to understand the meanings of the conversations and discourses that have taken place. The next segment discusses attempts to preserve interpretive validity in the data review.

4.6.2 Validity in data Analysis:

If, as mentioned earlier, doing qualitative research is about putting yourself in another person's shoes and seeing the world from another person's perspective, the first and most critical element of data analysis and interpretation is about being true to the participants (Sutto & Zubin, 2015). The researcher wanted to hear their voices and ideas so that they can be understood and recorded to read and learn for others and based on the analytical point of view of the researchers to interpret data or evaluate the situation (Sutto & Zubin, 2015).

Interpretations rely heavily on the transcribed lesson observation, audio recordings, and learners' marked transcripts prepared by the researcher in response to the actions of the participants. To maintain validity in my interpretations, the researcher had to ensure that the meanings, established from the transcripts were based on the theoretical frameworks established in this project. The literature review, the theoretical frameworks raised in this report aided my decisions in the data analysis and findings.

4.6.3 Reliability

Wellington (2000) defines reliability as, "The extent to which a test, a method or a tool gives consistent results across a range of settings if used by a range of researchers" (p. 200). Reliability is a word that works hand in hand with the word validity meaning "the degree to which a method, a test or a research tool measures what it is supposed to measure" (Wellington, 2000, p. 21). Cohen (2000) explains that "if a piece of research is invalid then it is worthless" (p. 105). Therefore, Jacobs et al (2010) and Wallach et al.'s (2005) findings have been used in this study to construct a theoretical framework that was used to collect data. The audio lesson recordings and marked pre-test/post-test transcripts enabled the researcher to select appropriate scenarios for the data analysis.

4.7 Research Ethics considerations

Sieber (1992) states that "Ethics have to do with the application of moral principles to prevent harming or wronging others, to promote the good, to be respectful and to be fair" (p. 14). The researcher applied for ethics clearance that was considered and authorized by the Ethics Committee of the Witwatersrand School of Education (WSoE). Upon recommending that the study was ethical, an ethical clearance certificate was granted by WSoE for the study to be conducted as was proposed. The researcher was aware that the participants had the right to decide to participate or not to, discontinue their participation in the study if they feel like not doing it. Therefore, participants were informed that if they decide not to participate or discontinue their participation during the progress of the study, they were free to withdraw.

Confidentiality and anonymity were assured through the use of numbers and pseudonyms. Participants were told that their privacy was covered and that the organizations will remain anonymous in documenting the results for any publication or presentation. Also, participants were advised that raw data that may contain their names will be stored in a locked cabinet and will be destroyed if no longer necessary.

Participants' performance did not affect school processes and production. Learners were informed that the pre and post-test marks were not going to be used for school reports or publications. Teachers were also informed that the study was not meant to undermine their performance instead the study was meant to explore the newly discovered professional noticing theory of learners' mathematical thinking by the teachers in the teaching of directed numbers to Grade 8 learners.

The schools' written permissions allowing the researcher to access the study sites and perform the analysis were obtained from the relevant authorities. Letters seeking permission from the school administrators, the school Governing Boards (SGB), and all participants (see Appendix B) to perform this research were sent and obtained.

4.8 Conclusion.

In this chapter, the researcher discussed the research paradigm, conditions for the study, and data collection methods. The processes involved in conducting the study were described and explained. Justifications for the selected methods and limitations involved with these methods were addressed. All ethical considerations were openly discussed and taken into account. Lastly, issues concerning the validity and reliability of data collection and data analysis were brought to the attention of the reader, and a strategy to reduce these risks was discussed. The next chapter presents the analysis and conclusions of the study's exploration of professional noticing of learner's mathematical thinking in the teaching of directed numbers on Grade 8 learners basing the theoretical framework on Jacobs et al.'s (2010) work and Wallach et al.'s (2005) work.

CHAPTER 5: DATA ANALYSIS

5.1 Introduction

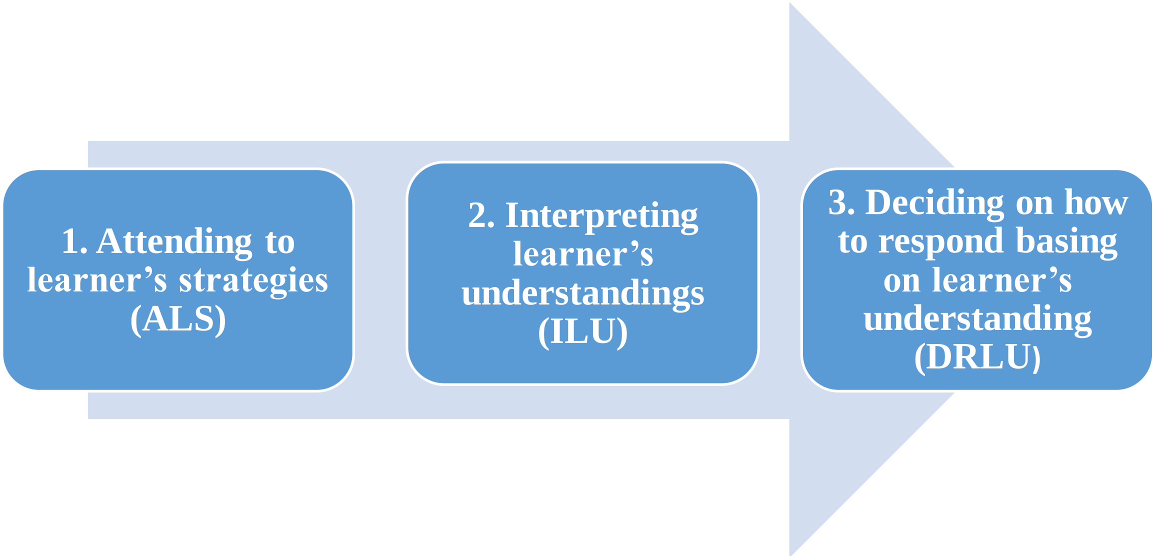
This chapter aims to concentrate on the pre-test, classroom observations transcripts, and post-test data analysis. It presents a discussion of the results from the data collected by exploring teacher professional noticing of learners' mathematical thinking during teaching and learning operations on directed numbers to Grade 8 mathematics learners. Audio-recorded lesson observations were transcribed and then divided into scenarios. See appendix E for the samples of the scenarios. The scenarios extracted from the transcribed lesson observations are essentially designed to provoke discussions on teacher professional noticing and non-professional noticing of learners' mathematical thinking in teaching and learning operations on directed numbers. These scenarios will also help the researcher to find answers to the three critical research questions of this study and these answers will be later discussed in chapter 6.

The theoretical framework used to analyse the collected data is mainly informed by Jacob et al.'s (2010) work on professional noticing and Wallach et al.'s (2005) five categories of what discourse learner hearing entails. According to Jacobs et al. (2010), professional noticing regard attending learner's strategies (ALS) as the degree to which teachers approach a particular aspect of learning circumstances, interpret learner's understandings (ILU) as reflected in their strategies, and then decide on how to respond to learner strategies based on learner's understanding (DRLU) as discussed in chapters 2 and 3. Therefore, in this chapter, the researcher is going to analyse in detail the data collected to inform all the studies. In addressing answers to the three critical research questions mentioned in chapter 1, the researcher uses the data collected from two different types of classes namely the experimental class and the control class. The following items in this chapter will explore teacher professional noticing through initially analysing learners' pre-test scripts followed by analysing scenarios extracted from transcribed lesson observations recorded during class interventions and lastly through analysing learners' post-test scripts.

5.2 Analysis of learners' pre-test scripts before class interventions.

In this section, the researcher will analyze learners' responses to the pre-test questions on directed numbers in preparation for the class interventions. Before class interventions, learners' pre-test marked transcripts were analyzed so that the researcher will know the level of learners' mathematical thinking when it comes to operations on directed numbers at Grade 8 level. Pseudonyms are used for confidentiality. The following Jacob et al.'s (2010) coded interrelated skills on professional noticing will be used to analyse learners' responses to the pre-test as follows:

Figure 16: Coding Jacob et al. 2010's three interrelated skills on professional noticing.

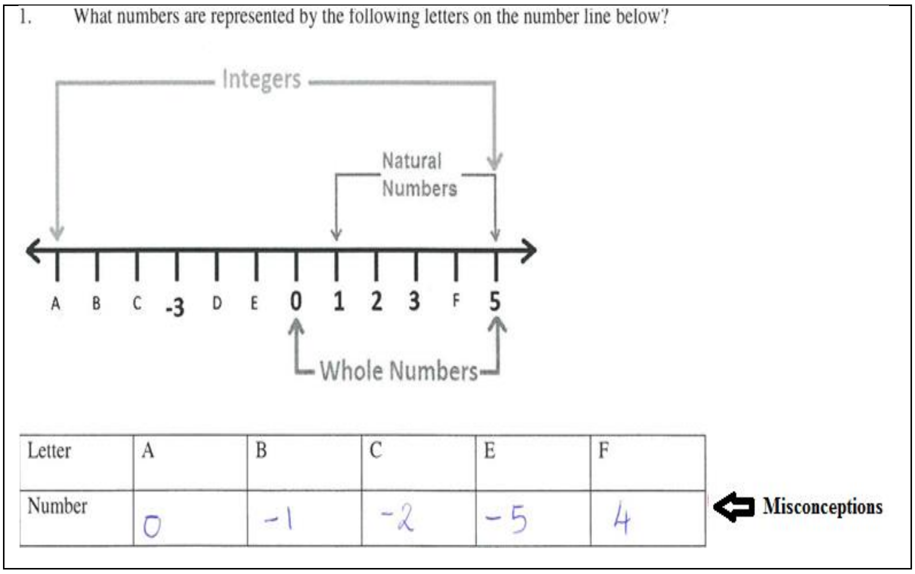


The following is an analysis of four learners' mathematical thinking responses to the pre-test questions namely Dimpo, Themba, Phumzile, Peter and, a group of 6 learners. The researcher selected these four learners' marked scripts from the four participating classes on the basis that it was possible to apply *all* three interrelated skills of professional noticing when analysing their responses.

5.2.1 Learner 1: Dimpo's responses in ordering directed numbers from the experimental class were incorrect.

Figure 17 below illustrates Dimpo's responses in ordering numbers that were extracted from her pre-test script showing some misconceptions and errors she committed.

Figure 17: Dimpo's responses on ordering integers


1. What numbers are represented by the following letters on the number line below?

Letter	A	B	C	E	F
Number	0	-1	-2	-5	4

5.2.1.1 Teacher professional noticing of Dimpo's mathematical thinking in teaching and learning ordering of directed numbers in Grade 8 mathematics.

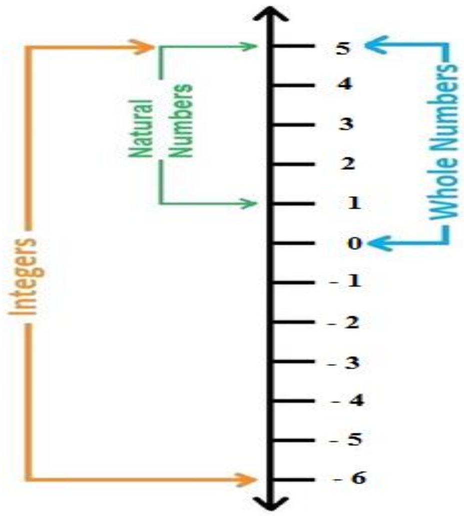
The three interrelated skills for teacher professional noticing on Dimpo's responses was done by Mr. Chauke (the researcher) as follows:

1. Attending to Dimpo's strategies (ALS), the above figure indicates that Dimpo as a learner from an experimental class was unable to order or arrange negative numbers according to their sizes or in ascending order.

2. Interpreting the learner understandings (ILU), the researcher discovered that misconceptions Dimpo experienced showed that she believed that counting always starts at zero for all number counting sessions. The correct answers for letters A, B, C, E and, F in the table are $-6, -5, -4, -1$ respectively. Comparing Dimpo's responses against correct answers shows that linking negative numbers with daily experience has been challenging for this Grade 8 learner.

3. Deciding on how to respond based on Dimpo's understanding (DRLU), both teachers (the researcher and Mr. Manana) decided to use a vertical calibrated number line as shown below in teaching and learning of ordering directed numbers.

Figure 18: Vertical Number line.



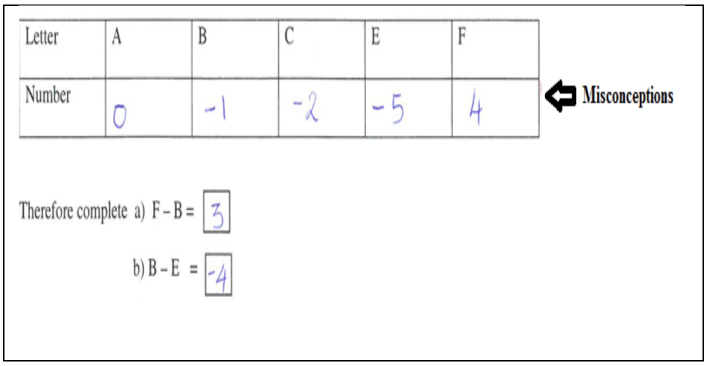
Seeing that most of the learners in the experimental classes had the misconception of ordering negative numbers, Mr. Manana had to use vertical number lines in teaching and learning of ordering integers according to their sizes as shown by the above figure. By using these vertical number lines, Mr. Manana was trying to show how numbers descend from the highest to the lowest or ascend from the lowest to the highest so that eventually, the learner will be able to order numbers in ascending or descending order. By going up the vertical number line shows that the numbers are getting bigger and bigger. By taking the opposite direction the learner can discover and understand that numbers get smaller and smaller as you go down the vertical number line. Since Dimpo had a problem ordering numbers, she also had a challenge in subtracting two integers.

5.2.2 Dimpo's responses on operations with two directed numbers in figure 19 were also recorded as incorrect answers.

Since Dimpo had a misconception of ordering integers, she also failed to subtract two integers as shown by the figure below on questions (a) and (b). Therefore, Dimpo's responses on operations with two directed numbers were also recorded as incorrect. In responding to questions (a) and (b) Dimpo substituted F by 4, B by -1, and E by -5 according to her incorrect answers she obtained on ordering integers on the number line.

Figure 19 below shows Dimpo's answers that she obtained for questions (a) and (b).

Figure 19: Dimpo's responses on the subtraction of two directed numbers.



Letter	A	B	C	E	F
Number	0	-1	-2	-5	4

Therefore complete a) F – B = 3

b) B – E = -4

5.2.2.1Professional noticing of Dimpo's mathematical thinking by the researcher in teaching and learning subtraction of two integers in Grade 8 mathematics.

The researcher used the three interrelated skills for teacher professional noticing to analysis Dimpo's responses by:

1. Attending to Dimpo's strategies (ALS), the above figure indicates that Dimpo's mathematical thinking on solving questions (a) and (b) was through substituting the letter F by 4, B by -1 , and E by -5 : (a) $F - B = 4 - -1 = 3$ (b)$B - E = -1 — 5 = -4$.

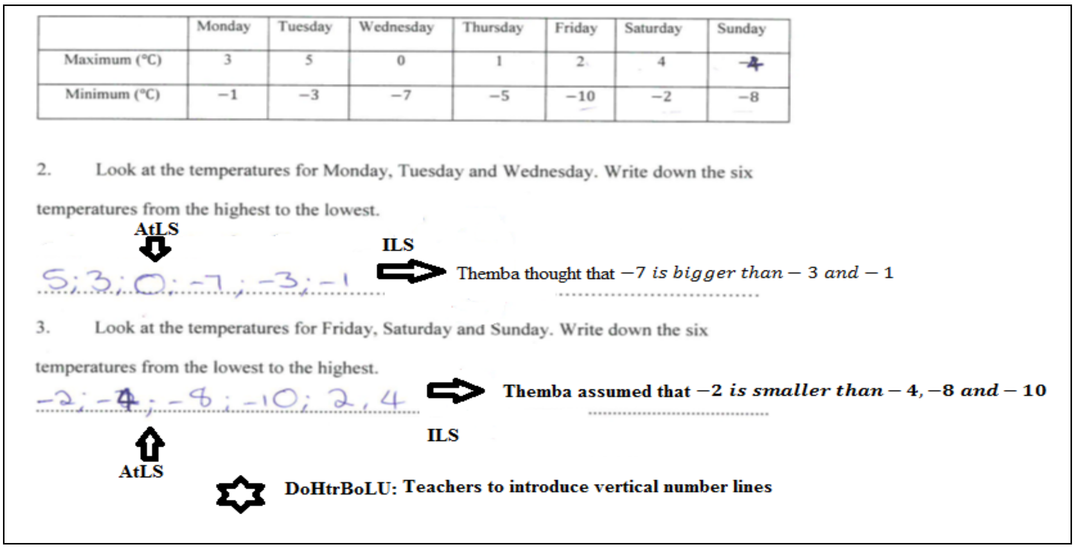
2. Interpreting the learner understandings (ILU), the researcher noticed that misconceptions Dimpo experienced showed that she worked question (a) as $4 - -1 = 4 - 1 = 3$ and for (b) as$-1 - -5 = -1 + 5 = -4 \ or -1 - 5 = -4$.

3. Deciding on how to respond based on Dimpo's understanding (DRLU), experimental class teachers (the researcher and Mr. Manana) through interventions decided to re-teach the subtraction and addition of two integers by using certain researched teaching techniques that were discussed in Chapter 3 yet control class maths teachers resorted to using golden rules that of$-- = +$.

5.2.3 Learner 2: Themba's responses in ordering directed numbers from an experimental class

Themba's had misconceptions about the meaning of temperatures that are responsive to his life-worlds. In figure 20 below, Themba was not able to arrange given temperature ranges from highest to lowest for question 2 and, for question 3 he also failed to arrange the same given temperature ranges from lowest to highest.

Figure 20: Themba's responses in ordering temperatures and the three interrelated skills for teacher professional noticing.



	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (°C)	3	5	0	1	2	4	−4
Minimum (°C)	−1	−3	−7	−5	−10	−2	−8

2. Look at the temperatures for Monday, Tuesday and Wednesday. Write down the six temperatures from the highest to the lowest.

3. Look at the temperatures for Friday, Saturday and Sunday. Write down the six temperatures from the lowest to the highest.

5.2.3.1 Professional noticing of Themba's mathematical thinking by the researcher in teaching and learning ordering of integers in Grade 8 mathematics:

The researcher used the three interrelated skills for teacher professional noticing to analyse Themba's responses by:

1. Attending to learner's strategies (ALS), the researcher noticed that Themba's strategy was to write temperatures for Monday, Tuesday, and Wednesday from the highest to the lowest. His answer to question 2 in figure 20 was $5; 3; 0; -7; -3; -1$. Themba was able to correctly identify all the temperatures for the required days. The first 3 positive temperatures were correctly arranged from highest to lowest. Negative temperatures of $-7; -3$ and -1 were wrongly answered because there were written from the lowest to the highest.

2. Interpreting the learner's understandings (ILU), the researcher noticed that Themba thought that numerically-7 $is\ bigger\ than\ -3\ and\ -1$. And also that Themba knew that negative numbers are on the other side of zero on the number line and his misconception was to believe that the size of the number is determined by counting upward regardless of whether it has a negative or positive sign against it.

3. Deciding on how to respond based on learner's understanding (DRLU), the researcher noticed that Themba's misconceptions needed the teacher to re-teach the topic using a certain researched teaching approach like that of teaching integers patterns using vertical drawn number lines.

5.2.3.2 Discussions on the nature of directed Numbers according to Dimpo and Themba's views.

Taking a close look at figure 20 above, questions 2 and 3 found in the pre-test script shows that Temba had the same misconception faced by Dimpo because they did not have a conceptual understanding of the actual meaning of negative numbers. Charlot (2009) informs us that a "school is a place where the world is treated as an object and not as an environment, place of experience. At times, this object of thought has a referent outside school, in the environment of the pupil's life" (p. 91). In this case, the object of thought is powerful knowledge or academic knowledge that may be important or irrelevant to everyday knowledge, depending on the learner's daily experience. The phrase "at times" in the above quote takes the position of partially agreeing with the notion that learners must develop powerful knowledge through a pedagogy that is responsive to their life-worlds (Charlot, B., 2009). For instance, directed numbers are positive and negative numbers such as $-5, -4, -3, -2, -1, 0 , 1, 2, 3, 4, 5, 6, and$ 7 but we cannot use negative numbers to represent the total number of oranges, instead one can use positive numbers to count the total number of oranges. Positives numbers like 0, 1, 2, 3, 4, and, 5 and so on can be used to develop powerful knowledge that is responsive to learners' life-worlds because they can be physically demonstrated. In this case, we cannot rule out the importance of negative numbers since they are not sensitive to the everyday life of the learners. Hence, the point of view Charlot (2009) brought up in his article is that very often in the sense of the learner's life, the focus of school thinking is not always connected to his/her life-worlds as it belongs to a scientific world created by science and school. Therefore, integers must be taught for conceptual understanding even though there are not related to learners' life-worlds through using objects such as counters or thermometers readings.

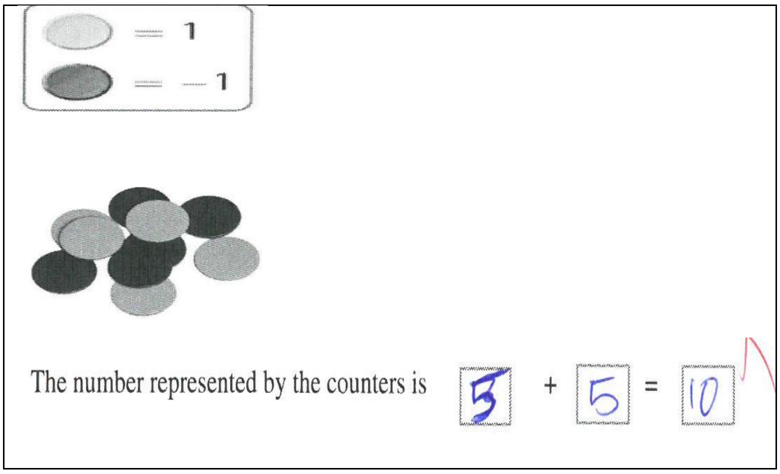
Figure 21, shows an attempt to address the issue of negative numbers in a way that is responsive to their life-worlds using discs or chips. Most of the learners failed to get the correct answers to pre-test questions because they never considered negative as non-existing numbers.

5.2.4 Themba's responses in dealing with negative integer numbers from an experimental class

Themba's misconceptions on negative numbers were noted as follows:

The pre-test question answered by Themba required him to use the following counter-models to find the numbers represented by the discs in questions 9 and 10 considering only the colour of the top of the counter.

Figure 21: Themba's responses in dealing with negative numbers.



5.2.4.1 Professional noticing of Themba's mathematical thinking by the researcher in teaching and learning the nature and the meaning of negative integer numbers in Grade 8 mathematics:

The researcher used the three interrelated skills for teacher professional noticing to analyse Themba's responses by:

1. Attending to learner's strategies (ALS), the researcher noticed that Themba had a misconception of ordering negative numbers and had challenges when it came to the addition and subtraction of integers. Attending to Temba's strategies on the pre-test question that of using discs, he physically counted all the ten discs without taking note of the meaning of the negative signs. He added the five black discs and five white discs and got a total of ten discs.

2. Interpreting the learner's understandings (ILU), the researcher noticed that Themba thought that negative numbers were not used for counting objects. This shows that negative numbers were considered as non-existing by most of the learners.

3. Deciding on how to respond based on learner's understanding (DRLU), the researcher noticed that Themba's misconception needs the teacher to re-teach the topic using certain researched teaching techniques like that of using discs to represent positive and negative integers. The zero principles can be introduced by using discs as $-+= 0 \; or + -=$

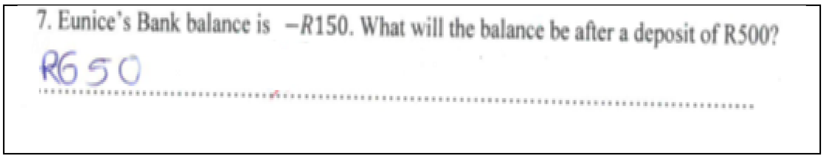
0 meaning that it is the same as $-1+1=0$ so that learners can have a conceptual understanding of negative numbers.

Question 7 in figure 22 below shows that negative numbers can be used to develop powerful knowledge that is not responsive to learners' life-worlds. If the concept of negative numbers is properly introduced among learners without rote learning they will be in a position to solve financial problems that deal with negative numbers: If Eunice's Bank balance is $-R150$. What will the balance be after a deposit of $R500$. The answer to this question is $R250$.

5.2.5 Learner 3: Phumzile's responses in dealing with negative integer numbers from a control class.

Just like Themba, Phumzile had a misconception of negative numbers upon this question on Eunice's bank account. Phumzile's answer to question 7 on the pre-test paper was as follows:

Figure 22: Phumzile's response.

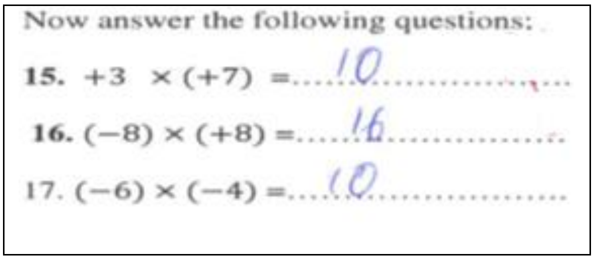

7. Eunice's Bank balance is $-R150$. What will the balance be after a deposit of R500?
R650

To address Phumzile's misconception, the same teacher professional noticing of Themba's mathematical thinking in teaching and learning the nature and the meaning of negative integer numbers in Grade 8 mathematics mentioned above must also be applied.

5.2.6 Learner 4: Peter's responses on operations with directed numbers from a control class.

Peter was supposed to do the multiplication of the two integers on questions 15, 16, and 17 in the figure below. The correct answers for question 15 is $+3\times(+7)=+21$, 16. $(-8)\times(+8)=-64$, and 17. $(-6)\times(-4)=+24$. Peter's responses were incorrect as shown in the figure below.

Figure 23: Operation on multiplication with two directed numbers.



5.2.6.1 Professional noticing of Peter’s mathematical thinking by the researcher in teaching and learning operations of integers in Grade 8 mathematics:

The researcher used the three interrelated skills for teacher professional noticing to analyse Peter’s responses by:

1. Attending to learner’s strategies (ALS), the researcher noticed that the learner was adding the two integers without considering negative signs instead of multiplying.

2. Interpreting the learner’s understandings (ILU), the researcher noticed that according to Peter’s strategies, the multiplication operator tends to work as a positive operator. Interpreting the cause of these wrong answers, it can be a misconception, error, or slip. An error is considered to be a mistake shown in the process of resolving a mathematical procedurally, algorithmically, or by other methods (Riccomoni, 2005), and hence, the learner might have mistaken the multiplication operator for the addition operator.

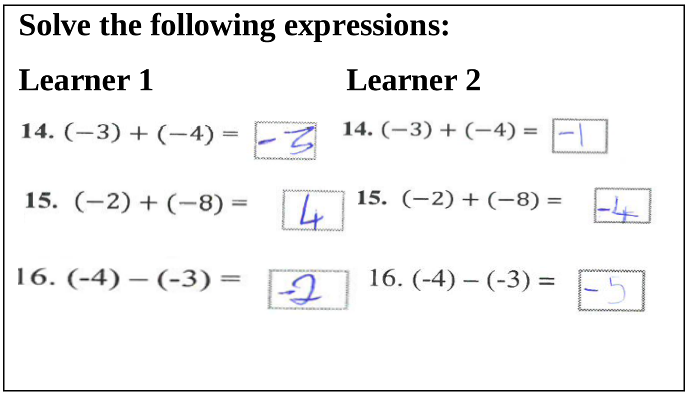
3. Deciding on how to respond based on the learner’s understanding (DRLU), the researcher needed to provoke Peter, and ask him how he got all the answers because besides being a misconception it might be a slip or an error.

In the pre-test results, most of the learners had challenges dealing with the addition and subtraction of integers as shown in the following discussion.

5.2.7 Different learners’ responses to the same questions.

Learners’ misconceptions/or errors on the addition of directed numbers. Figure 24 below shows how two different learners answered questions 14, 15, and 16 of the pre-test.

Figure 24: Different learners' responses to the same questions.



5.2.7.1 Professional noticing of learners' mathematical thinking by the researcher in teaching and learning operations of integers in Grade 8 mathematics:

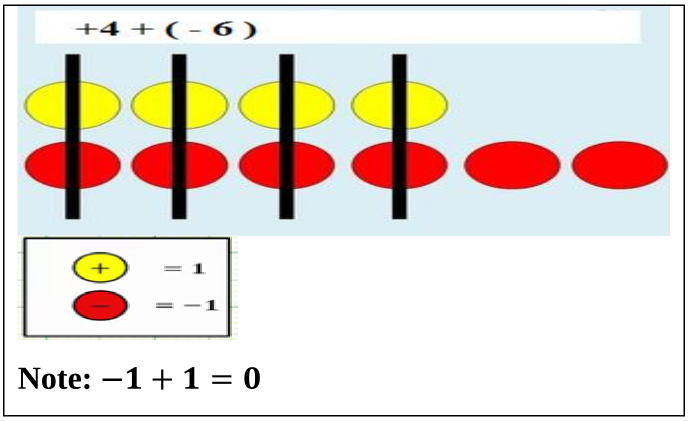
The researcher used the three interrelated skills for teacher professional noticing to analysis the learners' responses by:

1. Attending to learner's strategies (ALS), the researcher noticed that by considering Jacob et al.'s (2010) interrelated skills of professional noticing in such scenarios it will not be easy to attend learner strategies (ALS) as the degree to which teachers approach a particular aspect of learning circumstances because the answers provided by learner 1 and leaner 2 did not show any links with the expressions.

2. Interpreting the learner's understandings (ILU), the researcher noticed that, it was difficult to interpret working strategies shown by learner 1 and learner 2 in figure 24, and then, as a result, it becomes difficult for the teacher to decide on how to respond to both learners' challenges, using strategies based on their understanding (DRLU) simply because their responses were not related to the above subtraction and addition expressions. Learners' misconception needs the teacher to re-teach the topic using certain researched teaching approach of using chips or discs instead of using golden rules, for instance, example 2: $+4 + (-6)$can be solved in a conceptual way such as shown below.

Example 2: $+4 + (-6)$

Figure 25: Adding integers with two colour counters



Therefore, $+4 + (-6) = -2$

5.3 Analysis of transcribed lesson observations

In this section, pseudonyms are used and the researcher to analyse the transcribed lesson observations for experimental and control classes held in both schools using the aspects of teacher professional noticing in teaching and learning.

To provide the experimental-class teacher's review of exploring professional noticing, Mr. Chauke (the researcher) transcribed the entire lesson on teaching addition and subtraction of directed numbers as shown in Appendix E. After transcribing, scenarios were selected that had something to do with teacher professional noticing and the importance of learner talk in classrooms as shown in the tables that will later follow. Learner talk is considered important because it demonstrates that the learners are attending the lesson since it allows the learners to communicate and explain their ideas, and it also allows the learners to share ideas with everyone; and this supply the teachers with knowledge as to what the learners know and does not understand, as well as how the learners think and try to make sense of ideas (Brodie, 2007).

Many teachers assumed that learners engage in the lesson as they ask questions and provide responses, yet some research shows that many question-and-answer exchanges are not effective in improving mathematical thinking for learners (Brodie, 2007). Therefore, the use of teacher professional noticing must do justice for question-and-answer exchanges to help develop learners' mathematical thinking in the classroom. Scenarios of teacher professional noticing extracted from experimental class lesson observation transcripts will be used to explore the three interrelated skills of professional noticing. The scenarios have been divide into four columns: turns, time-intervals, transcript and codes of professional noticing (ALS, ILU, and DRLU), and codes of learner hearing (CH, BH, NH, OH, and NH). All codes of teacher professional noticing and leaner hearing fall under the last column of the scenario.

Two scenes among these scenarios helped the researcher to explore teacher professional noticing of learner's mathematical thinking in teaching and learning of directed numbers by Grade 8 learners. Among the chosen scenarios, the interrelated skills on professional noticing scenarios are coded then selected from the scenes, and then they are used to analyse the observed data. The table below offers brief explanations and the coding of various forms of teacher hearing learners' strategies that are going to be used in analysing the transcribed data from lesson observation. The coding of the five forms of Hearing Learners (Wallach et al., 2005) is shown below.

Table 4: Codes and summaries of Wallach et al. (2005) forms of learner hearing.

Forms of Hearing	**Forms of Hearing (Codes)**	**Statistics**	**Description**
• Over-Hearing	OH	MORE	Add what the teacher expects
• Compatible-Hearing	CH	ON PAR	Hear what is said
• Under-Hearing	UH	LESS	Ignore a part
• Non- Hearing	NH	LESS	Ignore the whole
• Biased-Hearing	BH	LESS	Hear without proof

Below are scenarios, numbers 6 and 8, extracted from a transcribed experimental class lesson observation for Mr. Chauke and Mr. Manana.

5.3.1. Exploring teacher professional noticing from observing Mr. Chauke and Mr. Manana lesson scenarios for Grade 8 experimental class at Lundi Secondary School.

5.3.1.1 Experimental class scenario number 6 extracted from lesson observation in Appendix E on introducing the Zero principles using discs $(+-=\mathbf{0}\ or-+=\mathbf{0})$

Scenario number 6 reads as follows:

Table 5: Introducing the Zero principles using discs (+-=0 or-+=0)

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
100	11:07	Mr. Manana	Minus 3 [Mr. Manana agrees with the learners who said 3 negatives $(---)$ means minus 3 (represent-3 as a number)]. Each one of these is plus one $(+)$, plus one $(+)$ and plus one $(+)$ (teacher referring to these signs:$+++$), and this one is minus 1 $(-)$, this is minus one $(-)$, minus one $(-)$ (teacher referring to these signs: $---$) right, and we are going to use and consider all these discs together, is that right?	CH
101	11:28	Lnrs	Yes. (Chorus response)	
102	11:28	Mr. Manana	Right, if we are considering all those signs discs. Or what I want you to do now is to take one plus disc and one negative disc, one minus one, and put them on the table in front of you. (learner group the discs)	
103	12:08	Mr. Manana	Okay, now what I want you to do is to put what, 3 pluses $(+,+,+)$ and 3 minuses $(---)$ in front of you. Can you put them together? 3 pluses and 3 minuses (Mr. Manana let the learners group the discs)	
104	12:36	Lnrs	(Grouping the discs)	
105	12:46	Mr. Manana	So, we have them. Yes, I can see there are very neat. Ok, so you know if you have one plus and one minus like this $+-$. What number is	

			represented by this? Can you guess? Yes (Teacher pointing at the learner)	
106	13:01	Lnr	Zero	
107	13:02	Mr. Manana	Yes, it is zero because it is the same as plus one minus one and this you gives you what?	ALS ILU
108	13:11	Lnrs	Zero (Chorus response)	
109	13:12	Mr. Manana	So, they cancel each other, a plus and a minus give us zero. So now if you have three positive discs $+, +, +$ and three negative discs $- - -$, what number therefore do we have there? What number do we have there? Yes (Teacher pointing to Sam)	DRLU
110	13:30	Sam	Zero	
111	13:31	Mr. Manana	You have zero, yes, because, actual we have a what? A plus 3 right, plus what? A minus 3 and therefore we have zero so this and this one (Mr. Manana pointing at pairs of $+ -$) give us zero, this one and this one $+ -$ give us zero, this one and this one $+ -$ give us zero. So, it is zero plus zero plus zero is zero, they neutralise each other. If you think of science example, a proton and a what?	ALS ILU DRLU
112	13:59	Lnrs	Neutrons	
113	14:00	Mr. Manana	One proton and one neutron. Then that chemical is neutral, isn't it?	
114	14:10	Lnrs	Yes. (Chorus response)	
115	14:11	Mr. Manana	Very good, so you can think of it like that. These discs are very important for you to master. We don't want you to remember the rules without understanding them. We want you to understand directed numbers meaningfully because this is a very key concept and a key topic in mathematics. If you don't understand	

			directed numbers then you are gone, your mathematics does not make sense, ok, so now can you please take positive 2 discs right and also have one negative. 2 plus disc and one negative what number is represented by that. Yes. (teacher pointing at the learner)	
116	15:03	Lnr	1	
117	15:05	Mr. Manana	Yes, it is one definitely because this one and that one $(+-)$ they do conceal, okay?	ALS ILU DRLU
118	15:12	Lnrs	Yes (Chorus response)	

5.3.1.1.1 Professional noticing of learners' mathematical thinking by the researcher and Mr. Manana in teaching and learning the zero principles:

Scene 1: Zero principles means that by adding one positive and one negative disc the answer will be zero$(+-= 0\ or - += 0)$. Inline 103, Mr. Manana asked learners to work in groups in pairing 3 positive discs $(+++)$ and 3 negative discs$(---)$. ALS, Mr. Manana checks on their prior knowledge by asking learners what number is represented by$+-$. One of the learners, Sam, responded by saying that the answer is equal to zero. ILS, Mr. Manana agrees that the answer for $+- or -+$ discs is equal to zero because it is the same as plus one $(+1)$ minus one (-1) since $+- or -+$ concealing each other. DRLU, inline 109 Mr. Manana made learners find the number represented by adding three positive discs $(+++)$ and three negative discs$(---)$. Using the zero principles all learners in line 108 were able to raise the answer of zero. DRLU, Mr. Manana went to clarify the point that $(+++)$ plus $(---)$ will led to the creation of three zeros: $-+$ plus $-+$ plus $-+$ equals to three zeros because $-+$ neutralises each other leading to having zero as the final answer$(0 + 0 + 0 = 0)$.

5.3.1.2 Scenario number 8 extracted from lesson observation in Appendix E on subtracting directed numbers using discs.

Scenario number 8 reads as follows:

Table 6: Subtracting directed numbers using discs

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
135	17:46	Mr. Manana	Negative 2 right (Manana re-voices), ok, I want you to work out a problem, which is $-3-(-5)$ is that right?	
136	18:01	Lnrs	Yes (Chorus response)	
137	18:03	Mr. Manana	How are you going to do that? Listen, right, listen, remember subtracting is taking away, isn't it? So, what we have is minus 3 at the beginning, then we want to take away minus 5 from this. Right, quite challenging, isn't it?	
138	18:25	Lnrs	Yes. (Chorus response)	
139	18:26	Mr. Manana	Right, don't tell me the answer listen. What we do in this case, is to say, aaaah, we have got a 5 here, right, so let's introduce 5 zeros, is that right?	
140	18:39	Lnrs	Yes. (Chorus response)	
141	18:40	Mr. Manana	So, we say a plus and its negative, a plus and its negative, a plus and its negative, a plus and its negative, a plus and its negative$(\pm \pm \pm \pm \pm)$, isn't it?	
142	19:02	Lnrs	Yes. (Chorus response)	
143	19:03	Mr. Manana	Right, very well, this is zero, zero, zero, zero, zero, we have not change anything, have we? $(- - - \pm \pm \pm \pm\pm)$	
144	19:12	Lnrs	Zero. (Chorus response)	
145	19:13	Mr. Manana	Right, but now let's work this, because we are saying we have minus 5 and minis 3, and we want to take away 5 negatives is that ok?	NH
146	19:21	Lnrs	Yes. (Chorus response)	
147	19:23	Mr.	Right, we had minus 3 here, these is minus 3	

		Manana	(teacher pointing at 3 negative signs), here we know that these are just 5 zeros (teachers pointing to($-,-,-,+-,+-,+-,+-,+-$) so its zero it does not change anything, it is still negative 3. So, we now say minus 3 minus 5 so now I think it's time to take out the minuses isn't it?	
148	19:42	Lnrs	Yes (Chorus response)	
149	19:44	Mr. Manana	Oh, let us count them, one minus taken away, two minuses, three minuses taken away, four minuses away, and five minuses taken away. Eeh, so what are we remaining with now? Yes. (teacher pointing at John)	
150	20:04	John	Minus 3.	
151	20:o5	Mr. Manana	Right, minus 3, where is it now, where is our minus 3. Yes, can you come and show us our minus 3. Yes. (teacher pointing at John)	CH; ALS
152	20:20	John	(John goes up to the board to show the 3 negatives)	
153	20:29	Mr. Manana	Aaa, What about these ones (Mr. Manana pointing to the signs on the board) What's your name?	ILU DRLU
154	20:41	John	John	
155	20:43	Mr. Manana	John, this and this is it not zero? (Mr. Manana pointing at $+-$)	DRLU
156	20:44	Lnrs	Its zero.	
157	20:45	Mr. Manana	This is zero, this is zero isn't it? What do we really remain with?	DRLU
158	20:51	Lnrs	2	
159	20:53	Mr. Manana	Can you see?	DRLU
160	20:54	Lnrs	Yes. (Chorus response)	
161	20:55	Mr. Manana	Therefore $-3-(-5)$ Is what? Plus?	

162	20:59	John	2	

5.3.1.2.1 Professional noticing of learners' mathematical thinking by the researcher and Mr. Manana in teaching and learning subtraction of directed numbers.

In analysing the above scenario on misconceptions and errors encountered by learners, the researcher considered the first two critical research questions such as, (1). What skills of noticing does the teacher embark on when using teacher professional noticing theory to teach and identify errors/misconceptions experienced in the classroom? And (2). What form of learner hearing is the best associate when practicing professional noticing?

In the above-extracted scene (lines 147 to 162), the Grade 8 class and the teacher were solving $-3-(-5)$ using discs that are linked to their daily knowledge and experience. Progressive pedagogy (PP) has the idea that learners' academic knowledge must be linked to their daily knowledge and experiences (Sikoyo & Jacklin 2009). Sikoyo and Jacklin (2009), who researched the notion of progressive pedagogy and found that schools with teachers who promoted every knowledge as a source of learning mathematics performed well in comparison with schools that did not have such teachers. Therefore, expression $-3-(-5)$ instructed learners to take away 5 negatives $(-----)$ from negative $3(---)$. Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education that was first introduced and developed by the Freudenthal Institute in the Netherlands. According to Freudenthal (1991)'s two important points of view are that mathematics must be connected to reality and the education system must take mathematics as a human activity. During the process of mathematising learners are allowed to make errors through experiencing misconceptions Nesher (1987) and they are given the opportunity to search for methods/patterns, even asks questions, construct ideas and ways to mathematically model their daily activities (strategies). Hence, in line 147, since it was impossible to take away 5 negatives from 3 negatives, the teacher decided to add five zeros to negative 3 to represent negative 3 as set of $(-,-,-,+-,+-,+-,+-,+-)$. Therefore, having $(---+-+-+-+-+-)$ representing negative 3, it thus became easy for the learner to solve the problem because it was possible to practically take away 5 negatives. In line 149, the teacher and the learners took away five negatives from the above set and remained with the set of $(+++-+-+-)$ discs. DRLU, the teacher asked learners to find out the answer using the remaining discs $(+++-+-+-)$. John raised the answer of minus $3(-3)$. Inline 151 the teacher experienced CH by revoicing the answer obtained by John(-3). ALS, the teacher seeing that the learner had a misconception on deriving the correct answer he asked John to explain how he came up with minus 3 from the remaining set discs: $(+++-+-+-)$. Fosnot (2005) explains that learners do not construct ideas in an ordered sequence instead they go in different directions as they explore, struggle to conceptualize (misconceptions/errors) and make sense of RME. ILU, Mr. Manana noticed that John ignored the 5 positives signs as if they mean nothing and only counts 3 negatives as the answer. According to Nesher (1987), the role of a learner is to become an "expert" in making errors since it is his/her contribution to the process of learning. This set of misconceptions can be not easy to detect in case the tasks given to learners are in a form of multiple-choice, whereby learners can get correct answers through guessing. As a result, the teacher can move on to teach new concepts assuming that learners have grasped the required information. But if learners' misconceptions and errors are physically seen on their classwork and homework it will be easy for the teacher to revise with them what they do not understand. Nesher (1987) claims that "the road to a state of expertise is paved with errors and misconceptions. Each error has the potential to become a significant milestone in learning. Let these errors be welcomed" (p. 39). DRLU, Mr. Manana then asked John the meaning of $+-$ on the remaining discs $(+++-+-+-)$. The learner through noticing that $+-$ means zero, he changes his answer to positive two and it was considered to be the correct answer by the teacher. In this case, Jacob et al.'s (2005) work on professional noticing does add value to the teaching and learning of mathematics.

5.4.1 Exploring teacher professional noticing from observing Teacher Ngele lesson scenarios for Grade 8 control class at Lundi Secondary School.

The scenarios below are extracted from the transcript of lesson observation of a controlled class taught by Teacher Ngele (Mr. Ngele) at Lundi Secondary School. Mr. Ngele did not attend classes on noticing but it seems that when teaching and dealing with misconceptions/errors made by Grade 8 learners he was embarking on teacher professional noticing. Here in this control class, it was assumed that the teacher was not aware of the existence of professional noticing theory but he applied the theory of teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8 and also the teacher was not aware of the importance of learning and teaching of mathematics through the use of powerful knowledge. The researcher noticed that the teacher who attended the workshop on professional noticing might have shared the acquired knowledge with his colleagues.

5.4.1.1 Scenario number 20 extracted from lesson observation in Appendix F on adding directed numbers using a number line

Scenario number 20 reads as follows:

Table 7: Lesson Introduction (Adding directed numbers using a number line)

Turns	Time-intervals	Transcript		Codes of professional noticing and learner hearing
1	00:00	Mr. Ngele	A big question is to see if you really understood what we did yesterday. Right, now let's try this question. Can you answer this: $(-11) + (-5)$? Negative eleven plus negative five. How do we answer that? Try it and then if you are confident enough you can come here in front then show us how you got the answer. (A Learner was given time to work out the given problem on the chalkboard.)	
2	02:11	Ashton	$(-11) + (-5) = 6$ (learner writes incorrect answer on the board)	
3	03:20	Mr. Ngele	Alright, now do you agree with that?	
4	03:22	Learners	No. (chorus response)	
5	03:30	Randy	$(-11) + (-5) = -6$ (another learner suggest a different solution on the chalkboard)	
6	06:15	Mr. Ngele	Do you want to explain to us how you got the answer? Anyone to explain this? Anyone who can explain this? (Teacher pointing at another learner). Yes, you want to explain it?	ALS
7	06:32	Siphokazi	Yes, sir	
8	06:35	Mr. Ngele	Try.	

9	06:50	Siphokazi	Sir, I think he forgot the negative at 5. Positive times negative is equal to negative $(+\times-=-)$then the answer is negative 16(-16). That is $(-11)+(-5)=-11-5=-16)$	ALS. ILU.
10	07:05	Mr. Ngele	Do we agree with this answer?	DRLU
11	07:07	Lnrs	Yes, sir. (chorus response)	
12	07:10	Mr. Ngele	Ok, and any other explanations?	
13	07:15	Londie	My explanation is that negative 11 plus negative 5 is negative 16. She said that a positive multiplied by a negative which is equals to negative. Then she puts a negative to be $-11-5=-16$. (Mr. Ngele accept the method used of solving the problem).	ALS. ILU.

5.4.1.2 Professional noticing of learners' mathematical thinking by the researcher in teaching and learning addition of two directed numbers.

Looking at the above scenario in line 5, a learner (Randy) displayed a misconception by saying $(-11)+(-5)=-6$. ALS, Mr. Ngele asked learner(s) to explain how he got the answer of negative six. ILU, one of the learners (Siphokazi) suggested that Randy obtained a negative six because he forgot to take note and make use of the negative sign against 5 on the expression inline 9 on the extract. While Mr. Ngele was ILU, Siphokazi suggested that Randy was supposed to use a golden rule of multiplying a positive sign and a negative sign to obtain negative signs as the answer $(+\times-=-)$. Hence, Siphokazi re-expressed $(-11)+(-5)\ as-11-15$ and obtained the answer of -16. DRLU, Mr. Ngele went on to further ask learners to suggest other alternatives to solving the same problem. Thus eliciting their mathematical thinking, meaning that when learners are struggling to solve a problem instead of showing them how to do it, the teacher must probe learners into a new leading mathematical idea. Instead, "teachers may relegate telling actions to their students by asking them for the idea or by waiting until a more knowledgeable learner articulate the idea" (Lobato et al., 2005, p.106). In line 13, Mr. Ngele later discovered that the other learner, Londie, shares the same procedural method with Siphokazi. Mr. Ngele accepted the use of the golden rule in the class by learners without proof and according to Wallach et al. (2010), this kind of hearing is called biased-hearing (BH).

Since the researcher noticed that the control teacher (Mr. Ngele) used the interrelated skills of professional noticing, it is thus imperative to discuss how teaching based on professional noticing of errors/misconceptions on adding two directed numbers using a number line assist teachers in teaching and learning new knowledge.

5.4.1.3 Discussion: Randy's misconception

Randy got the answer wrongly as, $(-11) + (-5) = -6$. Mr. Ngele had to ALS by asking him to explain how he got the answer of negative six. ILU, Siphokazi suggested that Randy obtained a negative six because he forgot to take note and make use of a negative sign against 5 on the expression. Siphokazi suggested that Randy was supposed to use a golden rule of multiplying a positive sign and a negative sign to obtain a negative sign as the answer $(+ \times - = -)$. Hence, Siphokazi re-expressed $(-11) + (-5)\ as - 11 - 15$ and obtained the answer of -16 . Chapin et al. (2009) state that, "… the teacher often refrains from providing the correct answer. He or she does not reject incorrect reasoning, instead attempts to get students to explore the steps in their reasoning, with the aim that they will gain practice in discovering where their thinking falls short" (p 17). Nesher (1987) suggests that learners' contribution to the process of learning is to become experts in making mistakes since they learn from mistakes. Chapin et al. (2009) explain that learners learn by processing information, getting ideas from classmates, reasoning, and connecting what they already know with new thinking. Small-group discussions and partner talk also give the same privilege towards promoting thinking among learners. Therefore, this process will be important to the victim since it will be directed to correct Randy's misconception and the victim will be getting some facts through listening to their peers. Peer teaching has been found to have force as learners are free to interact with others of the same age, child-friendly schools encourage child-sensitive learning hence group activities are one of the most relevant pedagogy in child-sensitive schools. DRLU, Mr. Ngele went on to further ask learners to suggest other alternatives to solving the same problem. Inline 13, Mr. Ngele later discovered that the other learner shares the same procedural method with Siphokazi. If Mr. Ngele had happened to ignore Randy's misconception and move on to look for another answer from a different learner, the above misconception was not going to be addressed. Therefore, embarking on professional noticing adds value to the teaching and learning of new knowledge among learners. Mr. Ngele accepted the use of the golden rule in the class by learners without proof, hence rote learning is in control although powerful knowledge or academic knowledge is supposed to be preferred in such situations.

5.4.2 Control class scenario number 23 extracted from lesson observation in Appendix F on distinguishing three different types of subtraction expressions:

$$(a) + 8 — 5, \qquad (b)\ \ 8 - 5, \qquad (c)\ \ -3 - 9$$

Scenario number 23 reads as follows:

Table 8: Distinguishing subtraction expressions

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
27	15:20	Mr. Ngele	Yesterday, we were supposed to do addition of two integers only but we also ended up introducing subtraction integers. So today we want to focus on subtraction of two integers only. (teacher writes problems on subtraction on the board)	
28	16:29	Mr. Ngele	Alright, so this will take few minutes. So let's say we have this example: a) $+8 - -5$ *, and we have another one* $b)$ $8 - 5$ and we have another one c) $-3 - 9$.Now let's look at the three problems. Can we identify the difference between among the three? Like, what is the different? Compare the first one and the second one (teacher ask a learner to compare)	ALS
29	17:27	Akeem	Sir , the first one is 8 subtract negative 5 ($8 - (-5)$) and the second one is positive 8 subtract 5	ILU
30	17:43	Mr. Ngele	Ok, do read out by spotting the difference I am not saying that you should read out what is written here. Can you try and identify the difference among the three?	DRLU

31	18:00	Jayne	Number (a) if you subtract a whole number by a negative number, you are subtracting two positive numbers.	ILU
32	18:15	Mr. Ngele	Which one? Where are we subtracting number (a) ,on (a)	CH
33	18:17	Jayne	On (b) as well	
34	18:18	Mr. Ngele	(b), we are subtracting positive numbers, ah?	CH
35	18.26	Jayne	At (a) we are subtracting a positive number with a negative number (teacher also agreeing with learner)	
36	18:30	Mr. Ngele	So, what you saying is that on (b) we are subtracting two positive numbers and then on (a) we are subtracting positive and a negative number. What about (c) now, do we agree with what he is saying here?	ALS ILU DRLU
37	18:48	Lnrs	Yes. (chorus response)	
38	18:50	Mr. Ngele	What about C, what's happening here on C (teacher pointing at the learner)	
39	18:57	Dimpo	Positive number is subtracted from a negative number (Teacher re-voices the statement)	
40	19:01	Mr. Ngele	A positive number is being subtracted from a negative number, alright let's forget about this and look at 9 minus 2. I am interested in your knowledge but at the same time your language. Now how do you comment? We see that it is 9 minus 2. Which is being subtracted from what?	ALS ILU DRLU
41	19:30	Lnrs	2 is being subtracted from 9 (chorus response and the teacher agrees with learner and re-voices what they have just said)	
42	19:35	Mr. Ngele	2 is being subtracted from 9. Right, let's go back here 2 is being subtracted from 9. So what about here (pointing at A). What is the difference again?	ILU

43	19:46	Jayne	Ah, on A, positive 8 is being subtracted from negative 5 and ……	
44	20:35	Mr. Ngele	Can you please help him out?	CH ILU
45	20:37	Nathi	Negative 5 is being subtracted from positive 8 and b) 5 is subtracted from 8 and c) 9 is being subtracted from negative 3	ILU
46	20:53	Mr. Ngele	So, now let's look at the first one right, the first one, we have a positive number and a negative number. So, we are subtracting a negative number from a positive number. The second one, we have a positive number and a negative?	ALS ILU DRLU
47	21:16	Lnrs	No (chorus response), positive	CH
48	21:17	Mr. Ngele	And a positive number right, so we are subtracting a positive number from a positive number, right. Now, look at this one (pointing at b), we have a negative number and a positive number (learners' seconds). So, we are subtracting a positive number from a negative number.	ALS ILU DRLU

5.4.2.1 Professional noticing of learners' mathematical thinking by the researcher in distinguishing given subtraction expressions.

In the above Scenario number 23, we see the theory of variation being implemented in designing mathematical expressions and from line 28, Mr. Ngele and learners are interpreting or distinguishing different kinds of subtraction expression: $a. +8 - -5$ $\quad b.\ 8 - 5$ $\quad c. -3 - 9$. Variation theory is a studying theory and understanding which demonstrates how much a learner could come to see, comprehend, or encounter a particular phenomenon in some way, as well as why learners could come to realize an idea differently while learning in the very same class (Orgill, 2012; Bussey et al., 2012). ALS, Mr. Ngele inline 28 asks learners to identify the difference between the three expressions. Learners were supposed to compare the first two expressions against each other. In responding to the teacher's question, Akeem said that the first one was 8 subtract negative 5, while the second one was positive 8 subtract 5. DRLU, Mr. Ngele probed learners to read out spotted differences rather than mere reading from left to right. Jayne comes up with a different response stating that $+8 - -5$ means that you are subtracting a whole number by a negative and $8 - 5$ means that you are subtracting a positive number from a positive number. Mr. Ngele inline 36 tries to ILU by saying that, Jayne described number $a. +8 - -5$ as subtracting a positive number and a negative number and $b. 8 - 5$ as subtracting two positive numbers. Jayne showed a conceptual understanding of the meaning presented by the two expressions. The theory of variation enables learners to better understand and contrast different scenarios so that they can make sense of the phenomenon (Bussey et al., 2012). According to Bussey et al. (2012) "phenomenography describes the limited number of qualitatively different ways individuals can experience the same phenomenon" (p. 10). DRLU, Mr. Ngele had to allow the learners to interpret number $c. -3 - 9$ based on their understanding of the meaning of numbers $a\ and\ c$. The theory of variation promotes empirical learning and according to Karpov et al. (1995), empirical learning is about, "comparing a number of different objects, picking out their common observable characteristics, and formulating on his basis a general concept about this class of objects" (p. 62).

5.4.3 Control class scenario number 24 extracted from lesson observation in Appendix E on subtraction of two directed numbers using a number line.

Scenario number 24 reads as follows:

Table 9: Subtraction of two directed numbers using a number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
49	21:50	Mr. Ngele	(Teacher introduces brackets on the last numbers of each previous question): a) $+8-(-5)$, $b)$ $8-(5)$ and c)$-3-(9)$. We now have brackets in there. So, let's look at the first one, the one which inside the brackets is negative. With this one (Pointing at number b) is positive, and this one this number is also positive (with learner re-voicing).	ALS ILU DRLU

50	22:13	Mr. Ngele	Now, 8 here is positive (pointing at number a), so in the first question we subtracting a negative number from a positive number, right. And on the second question we are subtracting a positive number from a positive number. On the third question, we are subtracting a positive number from a negative number (learners re-voicing what the teacher is saying), negative number. I know that from grade 7, grade 6 you been doing subtraction, now could you please with an aid of a number line, (school bell goes), can you please subtract negative 5 from 8, 5 from 8 as well 9 from negative 3 right. Right, let's do the first one. Can I have someone to demonstrate? (A learner comes upfront to demonstrate on the chalkboard). Leaner's response : $+8-(-5)=-3$ (Zachariah's answer)	ALS ILU DRLU
51	23:43	Mr. Ngele	(Learners clapping hands). Right ok, ok is the answer negative three.	
52	23:52	Lnrs	No. (chorus response)	
53	24:00	Mr. Ngele	Ok, you come and help.	
54	24:05	Philip	Positive 3. (Others learners say no)	
55	24:06	Mr. Ngele	Positive 3. (Other learner say no while others say yes). Positive 3	CH
56	20:12	Lnrs	No. (chorus response)	
57	20:13	Mr. Ngele	Ok, can you come and show us using a number and show us how you are getting positive 3?	ALS
58	24:30	Philip	(Learner demonstrating)	
59	25:32	Mr. Ngele	Alright, ok , can you please explain to us how you get the answer	ALS
60	25:40	Philip	(using a number line) I move 8 times to the right then move 5 times to the left: $+8-(-5)=+3$ (Teacher re-voicing what the learner	

			is saying)	
61	26:01	Mr. Ngele	So, you move 8 times to the left and then 5 times to the left. Ok, do you agree with that (teacher asking the whole class and pointing at one learner to help others)?	ALS ILU
62	27:05	Lnrs	No. (Chorus response disagreeing with the answer of positive 3) (selected learner was not able to explain but got the incorrect answer) : $+8-(-5)=-13$	
63	27:20	Mr. Ngele	Apparently, you can't explain. Do you agree with negative 13? (Learners made a chorus of response : No) You want to come and correct him (another learner, Kevin, wrote 13 instead of -13) : $+8-(-5)=+8+5=13$	ALS ILU DRLU
64	27:58	Mr. Ngele	13, positive 13.	
65	27:59	Lnrs	Yes. (chorus response)	
66	28:00	Mr. Ngele	How did you get the answer?	
67	28:11	Kevin	I said sir, negative plus negative, negative plus negative is equal to positive. Then I said 8 plus 5 equals positive 13. (clapping hands of learner)	
68	28:42	Mr. Ngele	Since negative multiplied by negative.	
69	28:43	Lnrs	Is positive.	
70	28:45	Mr. Ngele	Right, here is my suggestion before you can calculate a problem deal with a bracket first. So, you have 8 minus negative 5, which is equivalent to $8\ - \times - = +5$=$8+5$. Then you can get the answer of positive 13. Ok, who can do the second one ($8-(5)$), it should be easy.	ALS ILU DRLU BH
71	30:23	Mr. Ngele	What is your answer?	

72	30:24	Veli	3.	
73	30:25	Mr. Ngele	3. (Teacher re-voicing).Do we agree with answer?	CH
74	30:27	Lnrs	Yes. (Chorus response).	
75	31:40	Mr. Ngele	The last one: $-3-9=-12$ .Any questions	
76	31:43	Lnrs	No. (chorus response as the learners were preparing to leave the class)	
			End of the lesson	

5.4.3.1 Professional noticing of learners' mathematical thinking by the researcher in distinguishing given subtraction expressions.

In examining the above situation, the researcher considered addressing the first two important research questions such as 1. What skills of noticing does the teacher embark on when using teacher professional noticing theory to teach and identify errors/misconceptions experienced in the classroom? And 2. What form of learner hearing is the best associate when practicing professional noticing?

The following paragraph is meant to be the response to the two above mentioned critical research questions

According to line 50 on the above scenario, a learner (Zachariah) had a misconception of subtracting a negative 5 from a positive 8. Zachariah's response was $+8-(-5)=-3$ and the answer was incorrect. The class rejected the answer and a learner (Philip) shouted the answer as positive 3. Hearing what learners say (CH), Mr. Ngele reaffirmed the answer raised in the class. ALS, Mr. Ngele asks Philip to explain to the class how he came up with a positive 3 after solving the above expression. ILU, Mr. Ngele listened to Philip's strategy of solving the expression. Inline 61, Philip's strategy was to start at zero on a number line, then move 8 units to the right and thereafter move 5 units to the left. Philip's movement description leads him to obtain a positive 3 as the answer and this happened to be a misconception. The incorrect answer of negative 13 was also arrived at until Kevin guesses a correct answer of positive 13. DRLU, Mr. Ngele asked Kevin to show how he got the answer of positive 13. Inline 67, Kevin's work was as follows: $+8-(-5)=+8+5=13$ since he knew that negative multiplied by negative is equal to positive. Mr. Ngele accepted Kevin's strategy and used it to clarify the misconception experienced by other learners in the classroom. There was BH on the teacher's side because he accepted the use of the golden rule idea of $- \times - = +$ without conceptual understanding or proof.

5.4.4 Exploring teacher professional noticing from observing Mrs. Zitha lesson scenarios for Grade 8 control class at Shongamiti Secondary School.

The scenarios below are extracted from the transcript of lesson observation of a control class taught by Mrs. Zitha at Shongamiti Secondary School. Mrs. Zitha did not attend classes on teacher professional noticing.

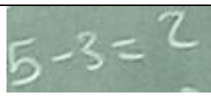
5.4.4.1 Control class scenarios analysis for Shongamiti Secondary School

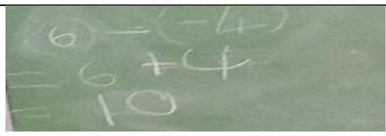
Scenario number 29 extracted from lesson observation in Appendix E on addition and subtraction of directed numbers.

Scenario number 29 reads as follows:

Table 10: Addition and subtraction of directed numbers.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
67	15:45	Mrs. Zitha	Ok, let me give four questions that I want you to answer and then we discuss them. Ok, you have three minutes to answer $5 + (-3)$ (teacher goes on to write the other three questions on the chalk board silently)	
68	16:38	Mrs. Zitha	Ok, so you have four minutes, one minute on each to answer those 4 questions. To answer those question you have 4 minutes one minute per question so its 1. $5 + (-3)$, 2. $10 — (-15)$, 3. $-11 - (-9)\ then$ 4. $6 - (-4)$	
69	17:30	Tasha	"Ma'am" (The learner ask the teacher about question 3)	
70	17:34	Mrs. Zitha	Yes its negative eleven minus negative nine.	
71	17:40	Lnrs	(learners start to answer questions while charting and discussing)	

72	18:30	Mrs. Zitha	There you now have four minutes gone so far, let’s stop writing.	
73	19:30	Mrs. Zitha	So its $1.5+(-3), 2.10—(-15), 3.-11-(-9)\ then\ 4.6-(-4)$	
74	20:35	Mrs. Zitha	Ok, hurry up.	
75	22:33	Mrs. Zitha	Ok, let’s answer the following questions before we can go to the new session. What is five plus minus 3?	
76	22:40	James	2.	
77	22:44	Lihle	−2.	
78	22:46	Mrs. Zitha	 So basically , That is five minus three gives a what?	Not ALS. Not ILU. Not DRLU
79	22:53	Itai	2.	
80	23:00	Mrs. Zitha	(The teacher writes: ). Ok in number 2 what is the answer?	Not ALS. Not ILU. Not DRLU. BH;NH
81	23:05	Mercy	Negative five.	
82	23:05	Lucky	Positive 5.	
83	23:06	Samson	Negative 5.	
84	23:09	Sibo	Positive 5.	
85	23:10	Mrs. Zitha	So, basically, ten plus 15 will give you a positive 25. Ok these signs (teacher pointing to 2 negatives signs) here it will emerge to be a positive so basically negative eleven plus nine gives us a negative 2	Not ALS. Not ILU. Not DRLU NH
86	23:45	Mrs. Zitha	Ok six minus negative four.	

87	23:47	Jackie	 10	
88	23:53	Mrs. Zitha	So, I want you to complete the classwork at the back of your exercise books and complete the class work	Not ALS. Not ILU. Not DRLU
89	24:01	Lnrs	(learners copy the classwork into their exercise books)	
			End of the lesson.	

5.4.4.2 Professional noticing of learners' mathematical thinking by the researcher in addition and subtraction of two directed number expressions.

Mrs. Zitha is an 8-Grade mathematics teacher who served in a controlled class during the data collection at Shongamiti Secondary School. In line 73, Mrs. Zitha presented learners with classwork consisting of four additive and subtraction problems on directed numbers: 1. $5+(-3)$, 2. $10—(-15)$, 3. $-11-(-9)$ 4 . $6-(-4)$

Five minutes were given to the learners to go over the classwork. After a few minutes of writing the discussion on the exercise begins in line75, whereby Mrs. Zitha asks the learner to provide responses to the classwork. The teacher went on to ask learners to give the solution to five-plus negative three. James gave the answer of 2 and learner 2 gave the answer of negative 2. This shows that learners were experiencing some misconceptions in terms of adding these two directed numbers of positive five and negative three. Mrs. Zitha did not ALS (misconceptions) because she did not comment on any of the answers given by Lihle instead in line 78 she went on to do some calculations without ILU. Mrs. Zitha did not DRLU because she simplified the expression $5+(-3)\ to\ 5-3$ without explanation. Therefore, it seems that Mrs. Zitha assumed that learners already know how to use the golden rules: $+\times -=-$ in simplifying $5+(-3)$ to $5-3$ and from this point learners were able to easily say the answer is 2 and the teacher accepted the answer without proof (BH). Learners also had misconceptions on solving question 2 in the above exercise because two incorrect answers of negative five and positive five were suggested by Mercy, Lucky, Samson, and Sibo during class discussion. Mrs. Zitha did not ALS that led them in getting the wrong answers, as a result, the teacher won't be able to ILU in the classroom if he/she does not ALS. During the process of teaching, Mrs. Zitha was not able to DRLU because she did not ALS and also she did not ILU. Mrs. Zitha seeing that learners are not getting the correct answer, eventually told learners that $10-(-15)$ is$10+15\ is\ equal\ to + 25$.

5.5 Analysis of Jacob et al.'s (2010) interrelated skills of professional noticing recorded over two combined experimental class lessons in Lundi Secondary School.

Figure 26: Interrelated skills of professional noticing

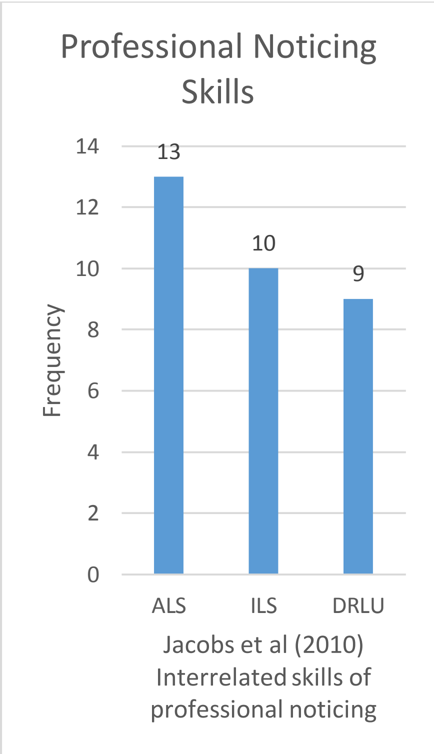
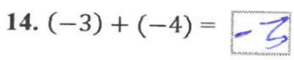


Figure 26 shows that there are 13 ALS instances, 10 ILS instances, and 9 DRLU instances recorded over two experimental class lessons. Therefore, this shows that Jacob et al.'s (2010) three interrelated skills of professional noticing cannot be equally scored by the teacher when practicing professional noticing during the lesson discussion. For instance, there are some cases whereby a teacher can attend to learner's strategies (ALS) but fail to interpret the learner's strategies (ILS) because of some serious misconceptions that cannot be traced among the struggling learners. For instance, it should be noted that not all attending to learner's strategies (ALS) are always meant to pave the way to the interpretation of learner's strategies (ILS), for example, after ALS such as 

we can see that it is

difficult to interpret the learner's strategy (ILS) of what led him/her to obtain the answer of negative three. In this case, the only answer to deciding on how to respond is based on the learner's understanding (DRLU), is to re-teach the concept of adding two negative integers. Therefore, this is why we see that the total numbers against the three interrelated skills are decreasing as we move from ALS to DRLU as shown above in figure 26.

At times if we consider or ALS on number 15 below:

15. $(-2) + (-8) =$ -4 , we can see that by ILS the teacher can realise that the learner might have obtained the answer of negative four simply by dividing negative eight by negative two. To decide on how to respond based on the learner's understanding (DRLU) in this case is to explain the meaning of the positive operator on the expression to the learner. A simple way to explain this is to inform the learner that the positive operator on the expression informs us to add two negatives discs and eight negatives discs together so that he/she can obtain the correct answer of negative ten.

5.6 Analysis of Wallach et al. (2005) learner hearing.

Figure 27 shows the forms of learner hearing recorded in selected experimental scenarios analysed above:

Figure 27: Forms of learner hearing recorded in selected experimental class scenarios.

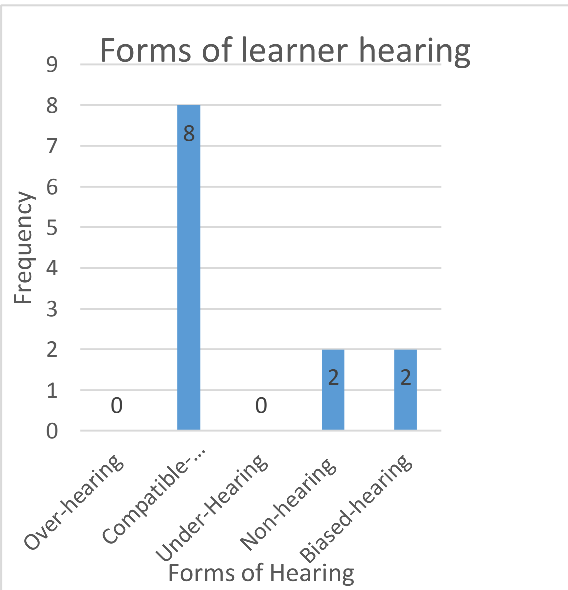


Figure 27 shows that 0 over-hearing instances, 8 compatible-hearing instances, 0 under-hearing instances, 2 non-hearing instances, and 2 biased-hearing instances were recorded from selected experimental scenarios. Therefore, in all the above-discussed forms of hearing, compatible-hearing is one of the most prevailing forms of hearing being used to implement teacher professional noticing in the experimental classes. Teachers must always practice compatible-hearing to attend to learner's ways of thinking, interpret the understanding of the learner, and determine how to answer based on the understanding of the learner. The teacher cannot ALS, ILU, and DRLU if he/she overhears the learner, under-hear the learner or ignore some of the things he/she do or say.

5.7 Analysis of post-test after class interventions.

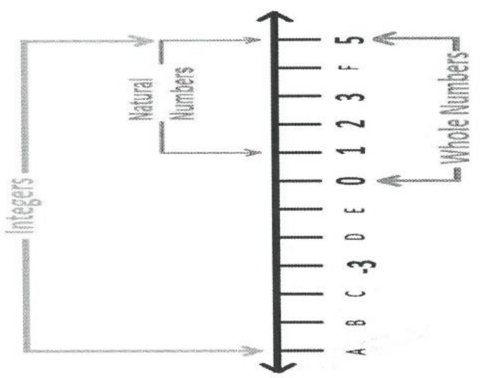
In this section, the researcher will analyse learners' responses to the post-test questions on directed numbers. Pseudonyms are used are also going to be used. Post-test was given to learners after class interventions to check on their learning progress.

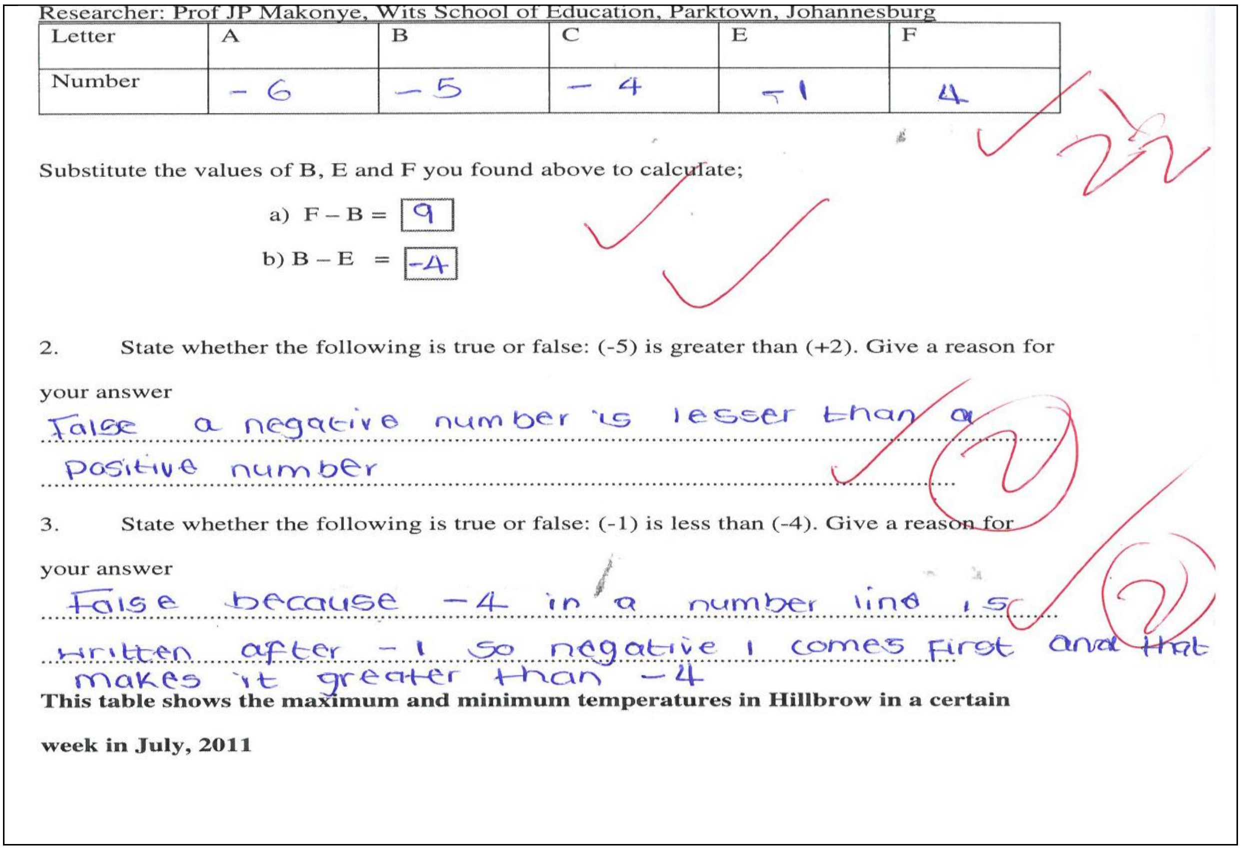
5.7.1 Class interventions had a positive impact on Dimpo's misconceptions about ordering directed numbers from the experimental class.

When experimental class teachers decided to teach using vertical number lines in teaching and learning how to order integers, Dimpo was able to get correct answers as shown in the figure below.

Figure 28: Corrected misconceptions and errors obtained from the post-test scripts

1. What numbers are represented by the following letters on the number line below?





Researcher: Prof JP Makonye, Wits School of Education, Parktown, Johannesburg

Letter	A	B	C	E	F
Number	-6	-5	-4	-1	4

Substitute the values of B, E and F you found above to calculate;

a) F – B = 9

b) B – E = -4

2. State whether the following is true or false: (-5) is greater than (+2). Give a reason for your answer

False a negative number is lesser than a positive number

3. State whether the following is true or false: (-1) is less than (-4). Give a reason for your answer

False because -4 in a number line is written after -1 so negative 1 comes first and that makes it greater than -4

This table shows the maximum and minimum temperatures in Hillbrow in a certain week in July, 2011

5.7.1.1 Professional noticing of Dimpo's mathematical thinking in ordering numbers in ascending order or descending order by the researcher.

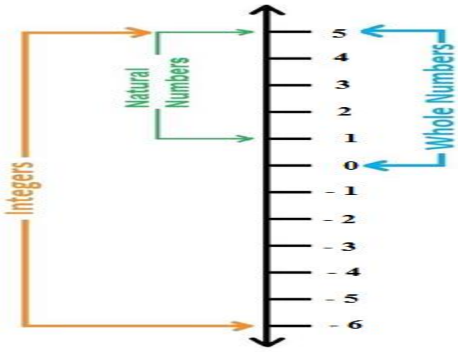
The researcher used the three interrelated skills for teacher professional noticing to analyse Dimpo's responses by:

1. Attending to Learner's strategies (ALS), the researcher noticed that Dimpo no longer had the misconceptions that existed on ordering numbers because she managed to replace all the letters with correct numbers.

2. Interpreting learner's understandings (ILU), the researcher noticed that learners did quite well in the post-test after the class interventions.

3. Deciding on how to respond based on learner's understanding (DRLU), the researcher and Mr. Manana decided to use a vertically drawn number line as shown in the figure below to teach directed numbers. The use of vertical drawn numbers lines makes it easy for the learner to learn how to order directed numbers in ascending or descending order correctly. Figure 29 below is an illustration of a vertical number line. Most of the learners in the experimental classes were able to represent unknown letters with correct integers on any form of a number line after the interventions.

Figure 29: Sample of a vertical number line



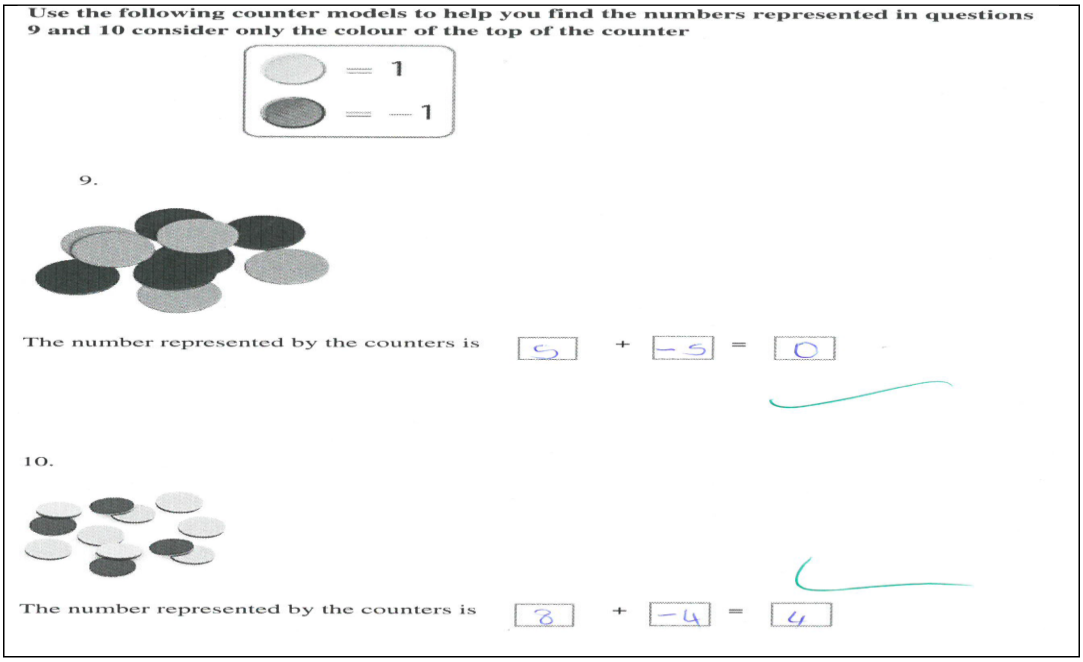
1. ALS and ILS on question 2 in the figure above, by looking at the learner's reasoning when responding to question 2 on figure 24, Dimpo said that a negative number is lesser than a positive number from the sense that by using vertical drawn number lines the learner tends to realize that the numbers become smaller and smaller as you go down along the vertical number lines.

2. ALS and ILU on question 3 in the figure above, more evidently, on question 3 Dimpo said that negative four is smaller than negative one because " -4 in a number line is written after -1, so negative 1 comes first and makes it greater than -4''.

5.7.2 Themba's responses on ordering directed numbers from experimental class intervention where teachers teach the topics using certain researched teaching approaches.

Corrected Themba's misconceptions/or error on the addition of directed numbers

Figure 30: Corrected misconceptions and errors obtained from the post-test scripts



5.7.2.1 Professional noticing of Themba's mathematical thinking after class interventions by the researcher in teaching and learning the nature and the meaning of negative integer numbers in Grade 8 mathematics:

The researcher used the three interrelated skills for teacher professional noticing to analyse Themba's responses by:

1. Attending to Learner's strategies (ALS), the researcher noticed that Themba's responses of $5 + -5 = 0 \; and \; 8 + -4 = 4$ were correctly answered.

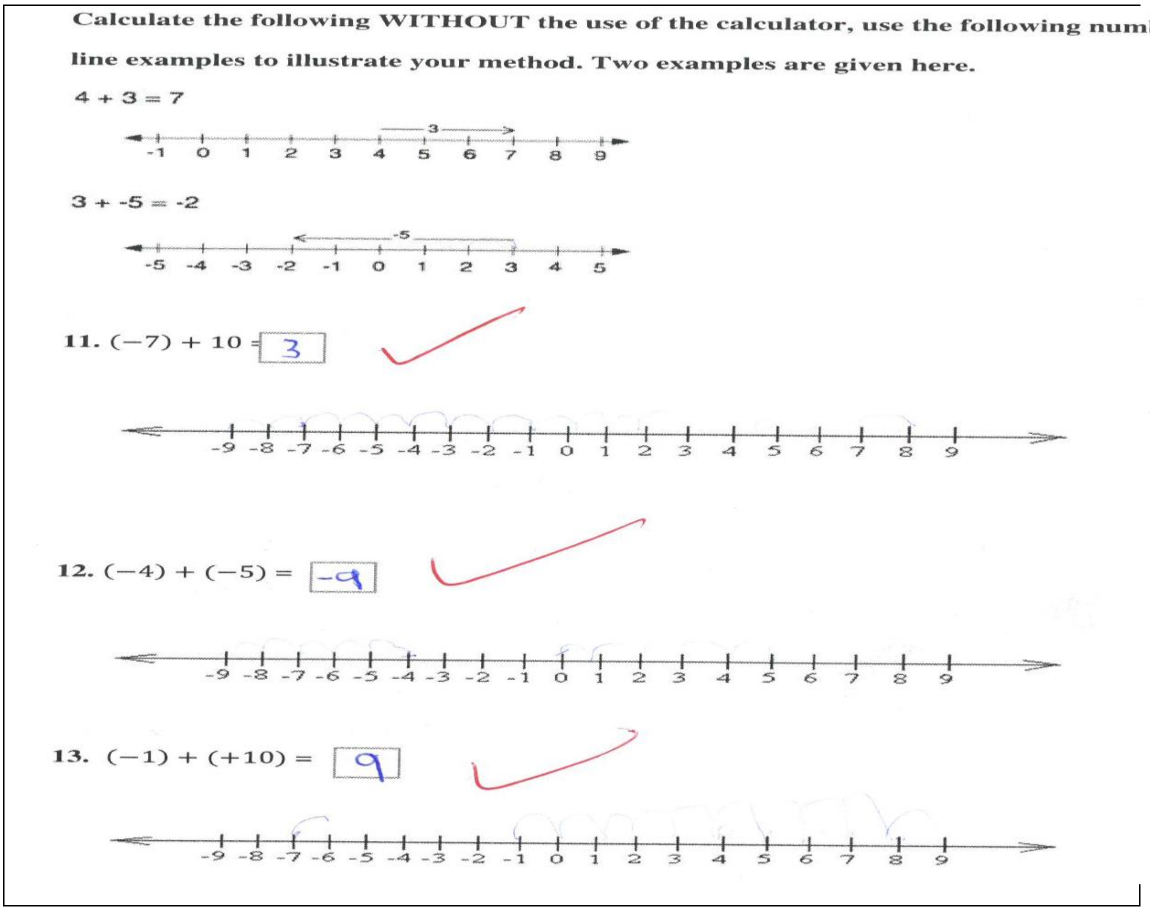
2. Interpreting learner's understandings (ILU), the researcher noticed that, in figure 30, questions 9 and 10 recommend learners to use discs in solving addition expressions, and this method of using discs in solving expressions on operation with integers was done in the experimental classes. The black disc represents a negative one $(-1)$and the white disc represents a positive one$(+1)$. For instance, two black discs represent positive two numerically. Therefore, if someone aligns one white disc and one black disc they will give a numerical value of zero and this is considered as a zero principle. In questions 9 and 10, Themba was able to align the discs according to the zero principles so that they can calculate the answer for each expression by counting the remaining discs. In control classes, teachers and learners did not conceptualise the meaning of the two expressions because they used golden rules such as $+ \times -= -$ to solve the problems. By applying golden rules $5 + (-5)$ was simplified and solved as $5 - 5 = 0$ same applies to $8 + (-4) = 8 - 4 = 4$. Some

learners in control classes were able to get correct answers without conceptualising the ideas behind these operations.

5.7.3 Adding two directed numbers using certain researched teaching approaches.

Adding two integers using number lines.

Figure 31: Misconceptions and errors obtained from the Pre-test scripts



5.7.3.1 Professional noticing of Randy's mathematical thinking by the researcher in teaching and learning the addition of directed numbers.

The researcher used the three interrelated skills for professional noticing for the analysis of Randy's responses by:

1. Attending to learner's strategies (ALS), the researcher noticed that Randy's responses of $(-7) + 10 = 3, (-4) + (-5) = -9$ & $(-1) + (+10) = 9$ were correctly answered. Since Randy did not show the methods he used in solving the above expression, it will be difficult to ILS. Therefore, to ILS the researcher opted to discuss all the methods used in experimental and control classes.

3. Deciding on how to respond based on learner's understanding (DRLU), the researcher noticed that besides using golden rules in solving question 12, in, $-4 + (-5)$ as $(-4) + (-5) = -4 - 5 = -9$, the experimental class teachers were teaching learners to answer these expressions with an aid of number lines to solve questions 11, 12, and 13. For example in solving question 12 [$(-4) + (-5)$] for conceptual understanding in figure 31 the positive sign between negative four and negative five was treated as the operator, meaning that when the operator is positive you must be prepared to take the right direction on a number line, and if it is negative be prepared to move to the left.

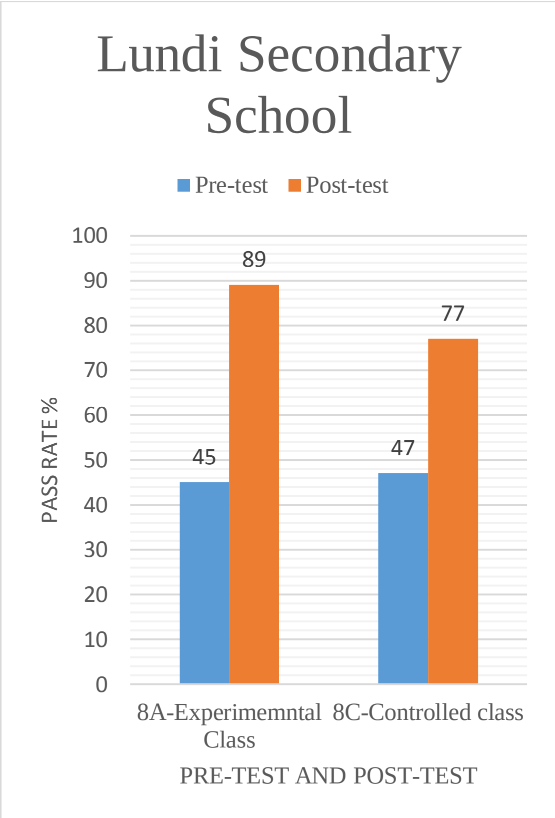
Therefore, when solving $(-4) + (-5)$ using a number line the learner must stand at negative four on the number line and because of the positive operator be prepared to go in the right direction along the number line. The negative sign on negative five will inform the learner to reverse direction and go 5 units to the left. By moving 5 units to the left the learner will stop at negative nine as the answer of $(-4) + (-5)$.

5.8 Findings based on pre and post-test analysis

This section relates to observations made by the researcher from the performance results of learners after they had a pre-test and post-test as scheduled by the research process. It gives a clear knowledge of the performance of learners before and after the experimental and controlled research classes which were held in the two schools.

5.8.1 Number of learners who passed the pre-test and post-test as percentages for Lundi Secondary School

Figure 32: Pass rate for Pre and Post-test results of Lundi Secondary School

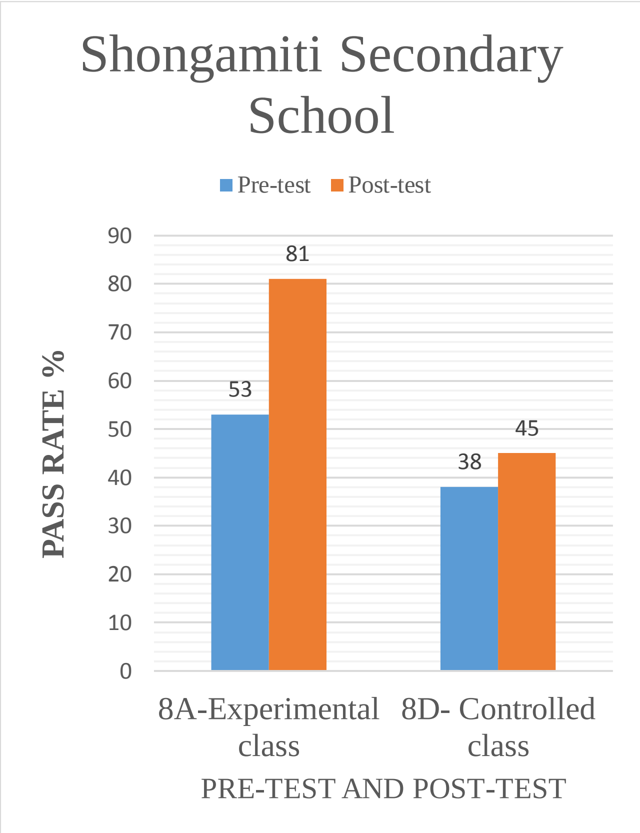


At Lundi Secondary School, Grade 8 learners in both classes did not perform well as shown by the pre-test results. Figure 32 shows that the overall pass rate for experimental and controlled pre-tests results was below 50%. This overall performance was similar to findings made by Makonye and Fakude (2016) when they also reported that 83% of learners commit errors and subtract integers due to misconception, 67% due to strategic errors, 28.6% due to logical errors, and 16.7% due to procedural errors. Therefore, if 83% of learners often commit errors in subtraction and addition of directed numbers due to misconception then it becomes difficult for learners to have a good pass rate because only 17% do not commit errors due to misconception. Hence the pass rate in an experimental class for a pre-test is 45% and the controlled class is 47% as shown in figure 32. The pre-test results had a bad pass rate depiction the very reason that encouraged the researcher to conduct the research study on how to improve the teaching and learning of operation with integers.

The post-test pass rate was awe-inspiring in both classes. The experimental class pass rate in figure 32 is 89% and the control class pass rate is 77%. The use of professional noticing and powerful knowledge by the teacher did add value to the teaching and learning of operations with directed numbers in experimental classes, hence it increased the level of understanding and participation by learners during lesson delivery.

The number of learners who passed the pre-test and post-test as percentages at Shongamiti Secondary School is shown below in figure 33.

Figure 33: Pass rate for Pre and Post-test results of Shongamiti Secondary School



Looking at figure 33 above it shows that the experimental class at Shongamiti Secondary School performed better than all the other three classes designed for the research project studies because learners' pre-test pass rate was pegged at 53% initially, and went up to score a pass rate of 83% according to the post-test results. The control class did not perform well in tests because the learners got a 38% pass-rate for pre-test and 45% pass-rate for post-test. There was an improvement in both classes at Shongamiti Secondary School because the pass rate for the post-test result was higher than the pass rate results of the pre-tests. Though there was an improvement in Grade 8D, 55% of learners failed the post-test, and only 19% failed the post-test in Grade 8A at Shongamiti. This shows that the activities that were taking place in the experimental classes in both schools added value to the teaching and learning of operations with directed numbers during the class interventions.

5.8.2 General Discussions on data analysis

The data analysed indicates that there is no proper communication in the classroom. The teacher does not engage the learners to fully understand their logic in answering questions, rather the teacher presents the problem and allows the learners to resolve then which after he points out errors if any. To alleviate the communication challenge observed in the classroom, the researcher recommends teachers identify a good communication approach that will be associated with teacher professional noticing so as to add value in teaching and learning mathematics.

Teachers and learners are supposed to communicate in the classroom for teaching and learning to take place. The big question is 'Which type of communication approach can be the best approach for teaching and learning?' Generally, there is one type of communication approach that can appropriately associate with teacher professional noticing so that it adds value in teaching and learning classroom mathematics.

5.8.2.1 Types of communication approaches that can prevail in South African schools and the world at large.

There are two types of communication approaches namely interactive and non-interactive (Scott, Mortimer, and Aguilar, 2006). The talk can be interactive in the sense that it is allowing for the participation of more than one learner, or non-interactive in the sense that it excludes the participation of other learners (Scott et al., 2006).

Closed and open-chain patterns are two types of interaction approaches to communication. Normally, open-chain patterns do not have a final evaluation of learners' responses by the teacher, for instance, there are considered as I-R-P-R-P-R patterns. Where 'I' stand for "Initiation" (from the teacher), R stands for "Response" (from the learner), P stand for Prompt and E stands for "Evaluation" (from the teacher). Therefore, the closed chain, I-R-P-R-P-R-E, is supposed to be practiced by teachers who use professional noticing as a tool for assessing learners' mathematical thinking because the evaluation (E) of learners' ideas is very important towards learners cognitive development. When the teacher evaluates the learner's mathematical thinking, he will be able to decide on how to respond based on the learner's understanding (Lobato et al., 2005).

Generally, there are four classes of communication namely the interactive/dialogic, non-interactive/dialogic, interactive/authoritative, or non-interactive/authoritative (Scott et al., 2006). These four social classes of communication are normally used by school teachers in teaching and learning mathematics. Normally, teachers and learners share different views of ideas and pose legitimate questions as they discuss mathematics through a form of communication known as interactive/dialogic (Scott et al., 2006). Non-interactive/dialogic happens when the teacher completes and summarizes the lesson by analysing and summarizing what was learned during the lesson in this case the learners will not be participating (Scott et al., 2006). Interactive/authoritative happens when teachers ask learners questions searching for answers to consolidate and develop the point of view of the topic of the mathematics lesson (Scott et al., 2006). Non-interactive/ authoritative occurs mainly at the start of the key lesson when new concepts are implemented (Scott et al., 2006). The authoritative class debate is when most of the talking is from the teacher and learners participate by answering the teacher's questions and providing assessment responses (Scott et al., 2006).

Interactive/dialogic communication can be in a form of constructive debate in a learning environment in all our institutions. The interactive/dialogic debate occurs when the majority of the speakers are given a chance to talk, meaning that both the learner and the teacher are allowed to build and evaluate ideas relating to their existing evidence (Scott et al., 2006) and this kind of communication approach can bring favourable conditions for the practice of the three interrelated skills on teacher professional noticing. Learners' existing/everyday views must be considered useful by teachers in every dialogical discourse.

5.8.2.2 Four kinds of teaching techniques.

The researcher noticed that upon the introduction of teacher professional noticing and powerful knowledge there are 4 kinds of classroom teaching techniques that can be practiced by teachers in South African schools. Drawing on the lesson observations held in Grade 8 mathematics classes for this study, the researcher discovered that there are four different kinds of teaching practices associated with teacher professional noticing and powerful/academic knowledge. These four kinds of teaching inform us that in every learning and teaching situation there is either:

1. Use/presence of professional noticing and use/presence of powerful knowledge $(PN^{+}; PK^{+})$ .In this case, the teacher applies professional noticing theory and also makes use of powerful knowledge in teaching and learning mathematics in the classroom. This kind of teaching was used by Mr. Chauke and Mr. Manana who were teaching learners in experimental classes in both schools or,

2. Use/presence of professional noticing and absence in the use of powerful knowledge $(PN^{+}; PK^{-})$. Teachers embarked on the use of professional noticing in teaching and learning of mathematics and made use of golden rules in teaching operations without conceptual understanding. Mr. Ngele who was a teacher under the control class at Lundi Secondary School showed some elements of use and application of interrelated skills of professional noticing but he did not use powerful knowledge in teaching addition, subtraction, and multiplication of integers/directed numbers or,

3. Absence of professional noticing and use/presence of powerful knowledge $(PN^{-}; PK^{+})$. In this case, there was no use of teacher professional noticing in teaching and learning mathematics but the teacher made use of the powerful knowledge in teaching for conceptual understanding, or

4. The absence of professional noticing and the absence of powerful knowledge $(PN^{-}; PK^{-})$. In this case, teachers are not in a position to practice the three stages of professional noticing in teaching and learning and also no powerful knowledge /academic knowledge is used in teaching addition/subtraction of directed numbers. The use of golden rules normally leads to rote learning. During data collection, the researcher noticed that Mrs. Zitha under the controlled class at Shongamiti Secondary School, to some certain extent used aspects of professional noticing and academic knowledge in teaching and learning operations on directed numbers.

5.8.2.3 A general discussion on pre and post-test results

These findings suggest that the practice of teacher professional noticing on learners' mathematical thinking should be accompanied by the use of powerful knowledge for it to be effective in teaching and learning mathematics. This suggestion has been proven accurate by a review of the data collected during the research process. The suggestion was certified by the fact that all learners who attempted the post-test showed great remarkable progress in the experimental classes. Great improvement was noted when the results of the experimental class pass rate at Lundi Secondary School and Shongamiti Secondary School that were pegged at 45%, and 53% respectively for the pre-test increased to 89% and 81% respectively for the post-test at both schools.

Less improvement was noted when the results of the control class pass rate for the pre-test at Shongamiti Secondary School that was pegged at 38% increased to 45% for its post-test. The control class teacher (Mrs. Zitha) in Shongamiti Secondary School never made use of professional noticing theory and powerful knowledge during class interventions. Moreover, she did not have good learner hearing skills.

The control class pass rate at Lundi Secondary School increased from 47% to 77%. The researcher suggests that the reason for having improved the pass rate was because the topic was being re-taught in this class for the second time and most of the learners in that class were very intelligent and active. Though Mr. Ngele had good listening skills, he did not use powerful knowledge but he used professional noticing techniques in teaching and learning operations on directed numbers. If the teacher made use of powerful knowledge in teaching and learning operations on directed numbers, learners were going to score a minimum pass rate of 90% for their post-test test. The problem faced by Mr. Ngele was that he did not use powerful knowledge to teach operations on directed numbers for conceptual understanding.

Therefore, the researcher suggests that professional noticing and powerful knowledge must be considered to be important by teachers during their teaching and learning of mathematics lessons so as to leverage learners' grasp of concepts. Professional noticing of learners helps to build bridges and therefore help struggling learners to cross over the math phobia challenge that affects many learners due to misconceived directed number operations. Professional noticing also helps to improve the relationship between the teacher and learners, including parents who will be inspired to pay school fees on time because their children have academically shown progress and change.

Thus, professional noticing of learners done during the research process contributed positively to the improvement of the learners as the learners in experimental classes also did quite well in their post-test results.

In suggesting ways of how teachers should help their learners develop the required mathematical thinking, it is imperative to always use powerful knowledge when teaching addition and subtraction of integers so that learners will do well in mathematics through improved concepts conceptualising of what they are being taught or what they are being asked to do in the classroom.

Lastly, when referring to types of learner hearing, most teachers must exercise compatible-hearing to effectively apply the three interrelated skills of professional noticing. The concepts of overhearing, under-hearing, non-listening, and biased listening cannot provide the teacher with an abstract understanding of the individual learner's thinking so that the teacher can be able to ALS, ILU, and DRLU.

5.9 Conclusion

This chapter presented the results forthcoming from the analysis of professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations on directed numbers. The analysis of teacher professional noticing has been done using learner pre-test scripts, transcribed lesson observations, and post-test respectively for experimental and control classes for both schools. Jacobs et al.'s (2010) three interrelated skills (ALS, ILU, and DRLU) and Wallach et al.'s (2005) forms of learner hearing played a major role in analysing the collected data. The next chapter, chapter six, presents the answers to the three critical research questions basing on the analysis and findings discussed in this chapter.

CHAPTER 6: SUMMARY OF FINDINGS AND CONCLUSION

6.1 Introduction

This research aimed to explore teacher professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. The inspiration for doing this work stemmed from the problem discussed in chapter 1. The problems discussed such as adding a positive integer and a negative integer, adding two negative numbers, subtracting a negative integer from a positive number, and subtracting a negative number from a negative number are four of the most confusing operations in mathematics under directed numbers. To resolve these most challenging behaviours among learners, the research examined the significance of learner hearing together with teacher professional noticing of the mathematical thought of learners on these operations. Two experimental classes and two control classes were formed to address the problem statement by exploring teacher professional noticing theory and learner hearing theory. In the experimental classes, the study made use of professional noticing theory and learner hearing theory, counters, and number lines for conceptual understanding to encourage understanding and support the teaching and learning operations on directed numbers in Grade 8. On the other hand, the control classes were created to prove the existence of the problem statement. Teachers under the control classes did not receive workshops on professional noticing and learner hearing theories and they were not obliged to use these theories.

This chapter summarizes and reflects the results presented in chapter 5. The summary from the instruments used to collect data will be discussed as well as the significance of the study. The chapter continues to consider the limitations of the research and their relevance, while at the same time recommending directions for further analysis and suggestions on conclusion.

6.2 Summary of the findings.

There are four issues that the researcher seeks to highlight in this discussion section. The three issues hinge on the research questions and lastly on the meaning of the findings concerning the literature.

6.2.1 Research questions

The study was about exploring teacher professional noticing of learners' mathematical thinking in teaching and learning operations on directed numbers in Grade 8. To achieve this, the three critical research questions were addressed by relating the responses with the analysed collected data. At the beginning of this report, the researcher asked the following question:

6.2.1.1 What skills of noticing does the teacher embark on when using teacher professional noticing theory to teach and identify errors/misconceptions experienced in the classroom?

According to Jacob et al. (2010), there are three interrelated skills of noticing that make a professional noticing a theory and these skills are ALS, ILU, and DRLU. Referring to scenario 8 that was analysed in chapter 5, the Grade 8 experimental class and the teacher were solving $-3-(-5)$ using discs with positive and negative signs on them. Negative 3 was written as a set of $(---+-+-\ +-\ +-+-)$ through adding the 5 zeroes $(+-\ +-\ +-\ +-+-)$ on negative 3 so that it will be possible to subtract 5 negative signs. The teacher and the learners took away 5 negative signs from the above set and remained with a set of $(+++-+-+-)$ discs. ALS, Mr. Manana asked learners to find out the answer using the remaining discs$(+++-+-+-)$. One of the learners gave the answer as minus $3(-3)$. When teacher saw that, learners had misconceptions in coming up with the correct answer, he asked John to explain how he came up with minus 3 from the remaining set discs: $(+++-+-+-\ )$. ILU, the teacher noticed that the learner ignored the 5 positives signs as if they meant nothing and only counted 3 negatives as the answer. DRLU, Mr. Manana then asked John the meaning of $+-$ on the remaining discs $(+++-+-+-\ )$. In this case, the learner was made to realise his mistake and to recall based on the previous lesson experiences that $+-$ means zero according to zero principles, he then changed his answer to positive two and it was the correct answer. Therefore, the researcher noted that ALS, ILU, and DRLU added value to the teaching and learning mathematics because most of the learners were able to have a better understanding of difficult concepts based on the results of the post-test.

Teachers are encouraged to embark on the use of Jacob et al. 2010's interrelated skills since they assist teachers in teaching and learning for better comprehension among the learners. Therefore, the three above-mentioned interrelated skills on teacher professional noticing are what the teacher should embark on in teaching and learning in order to identify and deal with misconceptions and errors made in the classroom. Professional noticing can also be used as a tool for teaching and learning new knowledge.

6.2.1.2 How does teaching based on professional noticing of errors/misconceptions assist teachers in teaching and learning new knowledge?

Referring to how Mr. Manana and the learners came up with the solution of $-3-(-5)$ as discussed above, the researcher noticed that the use of three interrelated skills of professional noticing played a significant role in teaching and learning for better understanding. Teaching based on professional noticing of errors/misconceptions assists teachers in teaching and learning mathematics in three ways. Firstly, attending learner strategies as the degree to which teachers approach a particular aspect of learning circumstances, the teacher will be in a position to get an opportunity to study learners' mathematical thinking and reasoning during the lesson delivery. When ALS, the teacher will be giving learners authority over teaching and learning of new or challenging concepts. If the authority is granted to learners this means they are given control over powerful knowledge rather than imposing knowledge on them. Therefore, learners must have authority over what they are learning so that there can be producers and authors of knowledge rather than be mere consumers of it.

Secondly, interpreting learner understandings as reflected in their strategies, will assist the teacher to work on errors and misconceptions faced and experienced by the learners. ILS is meant to hold learners accountable to themselves and others when navigating their misconceptions.

Thirdly, in a learner-centered approach to teaching, teachers can decide how to respond to learner strategies based on their learner understanding. DRLU is meant to provide relevant resources that will assist learners to conceptualize a new or difficult concept basing on learners' understandings. Also, in the case of content DRLU or problematisation, learners have the right to ask questions, make recommendations, and challenge, rather than waiting for answers and repetitive procedures and information from teachers. If learners are responding to what they are learning, they are then empowered not to acknowledge new or difficult concepts automatically, but to respond to it.

Generally, the notion of dialogic discourse between the teacher and learners seems to promote conceptual understanding among learners because they are allowed to air their views and ideas among themselves from all angles. If learners are allowed to air their ideas and views, it becomes possible for the subject teacher to ALS, ILS, and DRLU. The use of professional noticing by the teacher allows learners to take control of their learning and increase their participation in the class (Sikoyo et al., 2009; Koosimile, 2004).

Scott et al. (2006) suggest that there are four concepts in place, which are meant to promote successful disciplinary engagement: "problematising content, giving students authority, holding students accountable to others and disciplinary norms, and providing relevant resources" (p. 607). ALS, ILU, and DRLU project teaching and learning in schools as being based around learners (learner-centred). The explanation for progressive pedagogy (PP) is that learner-centred activities are more successful because they are responsive to the needs of learners, experience, and current expertise, contributing to the creation of skills that improve competitiveness in a global economic order (Sikoyo & Jacklin, 2009). This PP approach and professional noticing approach encourages or empowers learners to build self-confidence and promote social relevance (Sikoyo et al., 2009; Brodie, 2002). In South Africa, curriculum policies encourage reference in school mathematics to daily knowledge. There are so many advantages of using PP and the professional noticing approach in schools.

6.2.1.3 What form of learner hearing is the best associate when practicing professional noticing?

As from line 50 in scenario 24, a learner (Zachariah) had a misconception of subtracting a negative 5 from a positive 8. Zachariah's response was $+8-(-5)=-3$ and the answer was incorrect. The class rejected the answer and a learner (Philip) shouted the answer of positive 3. Hearing what learners say (CH), Mr. Ngele re-voices the answers being raised by Philip and his classmates. When the teacher listens and acknowledges (CH) all the explanations/answers provided by the learners then he or she *will be able* to ALS, ILS, and DRLU. To avoid under-hearing some of the things that the learners do or say, Mr. Ngele ALS by asking Philip to explain in front of the class how he came up with positive 3 as the answer after solving the above expression. ILU, Mr. Ngele listens to Philip's strategy of solving the expression. Inline 61, Philip's strategy was to start at zero on a number line, then move 8 units to the right and thereafter move 5 units to the left. Philip's movement description leads him to get the answer as positive 3 and this happened to be a misconception. The incorrect answer of negative 13 was also raised until Kevin guesses the correct answer of positive 13. The under-hearing of the learner's mathematical thinking was not practiced by Mr. Ngele because he sensed the wrong answers being posed among the learners and did not prefer the right answers without also focusing on the wrong answers. DRLU, Mr. Ngele asked Kevin to show how he got the answer positive 13. Inline 67, Kevin's work was as follows: $+8-(-5)=$

+8 + 5 = 13 since he knew that negative multiplied by negative is equal to positive. Mr. Ngele accepted Kevin’s strategy and used it to clarify the misconception experienced by other learners in the classroom. There was BH on the teacher’s side because he accepted the use of the golden rule idea of $- \times -= +$ without conceptual understanding or proof.

So, in schools, for professional noticing to add value to the teaching and learning of new mathematics knowledge in the classroom, the teacher must not practice over-hearing, under-hearing, biased-hearing, or non-hearing. Compatible hearing is the best practice of learner hearing because the teacher will be in a position to hear exactly what each learner will be saying when responding to the questions while implementing teacher professional noticing.

The key summary findings of the study are:

1. Jacob’s interrelated skills add value to the teaching and learning of integers
2. Attention to learner errors and misconceptions by teachers is imperative to the learners’ cognitive development
3. Compatible hearing is the best hearing technic to be applied in teaching integers

6.3 Limitations of the study

The system used in this study did not take into account the total use of powerful knowledge or academic knowledge in teaching and learning operation on directed numbers in both two types of classes. According to Jacob et al. (2010), the concept of professional noticing did not stress the use of powerful knowledge or academic expertise when teaching and learning take place in the classroom. It is concerned only with attending to the strategies of the learners, analysing the understandings of the learners, and making a decision on how to react is based on the understandings of the learners (Jacobs et al., 2010). The researcher made use of both powerful knowledge and teacher professional noticing principles in teaching experimental class learners in both schools. Most of the teaching proceedings in control classes made use of procedural methods instead of using powerful knowledge. Qualified teachers are trained to make use of powerful knowledge from universities but they tend to resort to drilling learners using golden rules. Therefore, if all teachers managed to use powerful knowledge in teaching all the learners, it was going to be easy to only evaluate the impact of teacher professional noticing theory of teaching used in experimental classes against other teaching theories used by teachers who taught learners in control classes.

The study investigates the application of professional noticing in learning mathematics in the South African context. Due to the limited resources (like finance, manpower, and time) the study was narrowed down to two schools in the Gauteng province. The sample used is meant to represent the learners in grade 8 across the country. This limitation of the study means a similar study carried out in a different province could bear differing results due to the different geographic and resourcefulness differences. The ideal sample could have been the one that entails representations (Of teachers and learners) from all South African provinces to give a better view. The limitations confined the study to only Gauteng province.

6.4 Recommendation from the findings

i. According to the researcher's view after analysing the collected data, teacher professional noticing cannot be practiced effectively without using powerful knowledge in teaching and learning mathematics. Teachers must have strong academic knowledge in their major subjects before they can apply Jacob et al.'s (2010) interrelated skills. Therefore, teachers who are using professional noticing as a tool for teaching and learning, must not use golden rules if they want to teach learners for conceptual understanding. Mr. Ngele from Lundi Secondary School also used professional noticing in teaching mathematics but did not continuously use powerful knowledge for conceptual understanding, as a result, the pass rate was lower than that of learners who were taught by Mr. Chauke and Mr. Manana in Lundi Secondary School.

   Teacher professional noticing cannot work in isolation with powerful/academic knowledge. Mr. Ngele in the control class of Lundi Secondary School was practicing teacher professional noticing but he lacked topic-specific mathematics teacher knowledge (TSMTK) on teaching operations on directed numbers to Grade 8 learners. In other words, Mr. Ngele was a practicing teacher professional noticing in teaching and learning operation on directed numbers through using golden rules. Although Mr. Ngele was practicing teacher professional, learners did not conceptualize what was being taught about the operation on directed numbers because they were using golden rules to add, substrate, multiply two integers. Learners taught by Mr. Ngele in the control class of Lundi Secondary School showed signs of improvement because the topic was being taught for the second time during the same academic year but they lacked a conceptual understanding of operation with integers as they used golden rules to get correct answers. The use of teacher professional noticing and powerful knowledge by Mr. Chauke and Mr. Manana in experimental classes for both schools which made use of certain researched teaching approaches brought a significant improvement among the learners in both schools.

ii. Teacher professional noticing arguments must be interactive/dialogical as it requires both the teacher and the learner to be actively be involved in the teaching and learning of new knowledge. Therefore, out of the four social classes of communication mentioned earlier, teachers can apply Jacob et al. (2010) professional noticing theory in teaching and learning mathematics by only practicing interactive/dialogic communication with learners during the lesson.

Summary of recommendations.

Key finding	Recommendation
1. Jacob's interrelated skills	Professional noticing to be used as a tool for teaching and learning, not the golden rules to teach learners for conceptual understanding. The tools to be coupled with TSMTK (Strong powerful knowledge in Maths)
2. Errors and misconceptions	Conscious attention (Noticing) to be paid during lessons to identify errors and misconceptions. Not to be ignored.
3. Compatible hearing	Practicing interactive/dialogic communication with learners during the lesson.

Action plan

Teacher workshops:

What	**topic**	**Who**	**When (duration) and cost**	**Return On Investment**
Teacher workshop	Professional Noticing	Department of Education with Wits	2 days R80K per 50 educators	Empowered and motivated educators, 20% improvement of the learners' pass rate.

Teacher workshop, by the Department of education on professional noticing over 2days it would cost r80000 to workshop 50 educators within Gauteng. NB: the monetary figures given above are estimates based on the 2020 financials on key resource costs to conduct a workshop within the Gauteng province. The pass rate increase is based on the weighting of the questions that relate to the integers in the grade 8-12 syllabus.

6.5 Conclusion

By practicing professional noticing of learners' mathematical thinking at the senior phase and all other phases of learning will help the teacher to notice learner's errors/misconceptions and use them as a tool to teach in the classroom. Teachers must have good hearing skills. The researcher had learned much about the impact of practicing teacher professional noticing it has on the learners' conceptual understanding. Compactible hearing is meant to hear everything that has been said by the learner so that the teacher will be able to apply all the three interrelated skills of teacher professional noticing.

References

Abowitz, D. A., & Toole, T. M. (2010). Mixed method research: Fundamental issues of design, validity, and reliability in construction research. *Journal of construction engineering and management*, *136*(1), 108-116.

Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for research in mathematics education*, 37(4), 270-296.

Adler, J., & Patahuddin, S. M. (2012). Recontexualising items that measure mathematical knowledge for teaching into scenario based interviews: an investigation. *Journal of Education*, *56*(1), 17-43.

Adler, J., & Pillay, V. (2007). An investigation into mathematics for teaching: Insights from a case. *African Journal of Research in Mathematics, Science and Technology Education*, *11*(2), 85-101.

Arcavi, A. (1995). Teaching and Learning Algebra: Past, Present, and Future. *Journal of mathematical behavior*, *14*(1), 145-162.

Balbuena, S. E., & Buayan, M. C. (2015). Mnemonics and gaming: Scaffolding learning of integers. *Asia Pacific Journal of Education, Arts and Sciences*, *2*(1), 14-18.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59(5), 389-407.*

Bamber, E. M., Ramsay, R. J., & Tubbs, R. M. (1997). An examination of the descriptive validity of the belief-adjustment model and alternative attitudes to evidence in auditing. *Accounting, Organizations and Society*, *22*(3-4), 249-268.

Bell, A. (1995). Purpose in school algebra. *The Journal of Mathematical Behavior*, *14*(1), 41-73.

Bell, J. (2014). *Doing Your Research Project: A guide for first-time researchers*. McGraw-Hill Education (UK).

Bernstein, B. (2000). *Pedagogy, symbolic control, and identity: Theory, research, critique* (Vol. 5). New York: Rowman & Littlefield Publishers, INC.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers’ knowledge. *Journal of Mathematics Teacher Education*, *15*(2), 131-157.

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders' mental models. *Journal for Research in Mathematics Education*, *45*(2), 194-245.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the learning of Mathematics*, *7*(3), 2-8.

Brodie, K. (2004). Re-thinking teachers' mathematical knowledge: A focus on thinking practices. *Perspective in Education, 22(1),* 65-80.

Brodie, K. (2007). Dialogue in mathematics classrooms: Beyond question-and-answer methods. *Pythagoras*, *2007*(66), 3-13.

Brodie, K., Lelliott, A., & Davis, H. (2002). Forms and substance in learner-centred teaching: Teachers’ take-up from an in-service programme in South Africa. *Teaching and teacher Education*, *18*(5), 541-559.

Brown, L., & Drouhard, J. P. (2004). Responses to ‘The Core of Algebra’. In: Stacey K., Chick H., Kendal M. (Ed.), *The Future of the Teaching and Learning of Algebra the 12thICMI Study* (pp. 35-44). New York: Springer, Dordrecht.

Bussey, T. J., Orgill, M., & Crippen, K. J. (2013). Variation theory: A theory of learning and a useful theoretical framework for chemical education research. *Chemistry Education Research and Practice*, *14*(1), 9-22.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational researcher*, *18*(1), 32-42.

Chang, L. (1985). Multiple methods of teaching the addition and subtraction of integers. *The Arithmetic Teacher*, *33*(4), 14-19.

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Sausalito, California, USA: Math Solutions.

Charalambous, C. Y., Hill, H. C., & Ball, D. L. (2011). Prospective teachers’ learning to provide instructional explanations: how does it look and what might it take? *Journal of Mathematics Teacher Education*, *14*(6), 441-463.

Coles, A. (2002). Teaching strategies related to listening and hearing in two secondary classrooms. *Research in mathematics education*, *4*(1), 21-34.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for research in mathematics education*, 28(3), 355-376.

Department of Basic Education (2011). *Curriculum and Assessment Policy statement, Grades 7-9, Mathematics*. South Africa, Pretoria: Government Printing works.

Fosnot, C. T. (2005). Constructivism revisited: Implications and reflections. *The Constructivist*, *16*(1), 1-17.

Hativa, N., & Cohen, D. (1995). Self-learning of negative number concepts by lower division elementary students through solving computer-provided numerical problems. *Educational Studies in Mathematics*, *28*(4), 401-431.

Hoadley, U. (2007). The reproduction of social class inequalities through mathematics pedagogies in South African primary schools. *Journal of Curriculum Studies*, *39*(6), 679-706.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27(1), 59-78.*

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic - specific knowledge of students. *Journal for research in mathematics education, 39(4),* 372 - 400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers’ mathematical knowledge for teaching on student achievement. *American educational research journal*, *42*(2), 371-406.

Jacobs, V. R., Lamb, L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education*, 41(2), 169-202.

Kazima, M., Pillay, V., & Adler, J. (2008). Mathematics for teaching: Observations from two case studies. *South African Journal of Education*, *28*(2), 283-299.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics* (Vol. 2101). National research council (Ed.). Washington, DC: National Academy Press.

Koosimile, A. T. (2004). Out-of-school experiences in science classes: problems, issues and challenges in Botswana. *International Journal of Science Education*, *26*(4), 483-496.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for research in mathematics education*, 36(2), 101-136.

Makonye, J. P., & Fakude, J. (2016). A study of errors and misconceptions in the learning of addition and subtraction of directed numbers in Grade 8. *SAGE Open*, *6*(4), 1-10

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: RoytledgeFalmer.

Maxwell, J. (1992). Understanding and validity in qualitative research. *Harvard educational review*, *62*(3), 279-301.

Morais, A. M., & Neves, I. P. (2018). The quest for high-level knowledge in schools: revisiting the concepts of classification and framing. *British Journal of Sociology of Education*, *39*(3), 261-282.

Morgan, C., Tsatsaroni, A., & Lerman, S. (2002). Mathematics teachers' positions and practices in discourses of assessment. *British Journal of Sociology of Education*, 23(3), 445 - 461.

Muller, J. (1998). The well-tempered learner: Self-regulation, pedagogical models and teacher education policy. *Comparative education*, *34*(2), 177-193.

Muller, J. (2012). *Reclaiming knowledge: Social theory, curriculum and education policy*. London: RoutledgeFalmer.

Nesher, P. (1987). Towards an instructional theory: The role of student's misconceptions. *For the learning of mathematics*, *7*(3), 33-40.

Opie, C., & Sikes, P. J. (2004). *Doing educational research*. London: Sage.

Orgill, M., Bussey, T. J., & Crippen, K. J. (2013). Variation theory: A theory of learning and a useful theoretical framework for chemical education research. *Chemistry Education Research and Practice*, *14*(1), 9-22.

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the learning of mathematics*, *29*(3), 22-25.

Peressini, D., Borko, H., Romagnano, L., Knuth, E., & Willis, C. (2004). A conceptual framework for learning to teach secondary mathematics: A situative perspective. *Educational Studies in Mathematics*, *56*(1), 67-96.

Qi, D. S., & Lapkin, S. (2001). Exploring the role of noticing in a three-stage second language writing task. *Journal of second language writing*, *10*(4), 277-303.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, *28*(3), 233-242.

Rowland, T., & Turner, F. (2008). How shall we talk about 'subject knowledge'for mathematics teaching? *Proceedings of the British Society for Research into Learning Mathematics*, *28*(2), 91-96.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of mathematics teacher education*, *8*(3), 255-281.

Ryan, J., & Williams, J. (2007). *Children'S mathematics 4-15: learning from errors and misconceptions: learning from errors and misconceptions*. McGraw-Hill Education (UK): Open University Press.

Schunk, D. H. (1991). Self-efficacy and academic motivation. *Educational psychologist*, *26*(3-4), 207-231.

Scott, P. H., Mortimer, E. F., & Aguiar, O. G. (2006). The tension between authoritative and dialogic discourse: A fundamental characteristic of meaning making interactions in high school science lessons. *Science education*, *90*(4), 605-631.

Segall, A. (2004). Revisiting pedagogical content knowledge: the pedagogy of content/the content of pedagogy. *Teaching and teacher education*, *20*(5), 489-504.

Seng, L. K. (2010). An Error Analysis of Form 2 (Grade 7) students in Simplify Algebraic Expressions: A Descriptive Study. *Edycation in Psychology* , 8(1), 139-162.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational researcher*, *27*(2), 4-13.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The journal of the learning sciences*, *16*(4), 565-613.

Sherin, M. G. (2005). Using Video to Support Teachers' Ability to Notice Classroom Interactions. *Jl. of Technology and Teacher Education 13(3)*, 475-491.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review*, *57*(1), 1-23.

Sikoyo, L. N., & Jacklin, H. (2009). Exploring the boundary between school science and everyday knowledge in primary school pedagogic practices. *British Journal of sociology of Education*, *30*(6), 713-726.

Singh, P. (2002). Pedagogising knowledge: Bernstein's theory of the pedagogic device. *British journal of sociology of education*, *23*(4), 571-582.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *77*(1), 20-26.

Smith III, J. P., Disessa, A. A., & Roschelle, J. (1994). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The journal of the learning sciences*, *3*(2), 115-163.

Sutton, J., & Austin, Z. (2015). Qualitative research: Data collection, analysis, and management. *The Canadian journal of hospital pharmacy*, *68*(3), 226.

Talasi, T. (2007). *Mathematical knowledge for teaching: a focus on the mathematical work of a Grade 8 teacher when introducing algebra* (Doctoral dissertation).

Thorne, S. (2000). Data analysis in qualitative research. *Evidence-based nursing*, *3*(3), 68-70.

Tirosh, D., Even, R., & Robinson, N. (1998). Simplifying algebraic expressions: Teacher awareness and teaching approaches. *Educational studies in mathematics*, *35*(1), 51-64.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education*, 41(2), 169-202.

Vygotsky, L. S. (1978). *Mind in Society : The development of higher psychological processes.* Cambridge : MA: Harvard Univesity Press.Chapters 4 and 6.

Wallach, T., & Even, R. (2005). Hearing students: The complexity of understanding what they are saying, showing, and doing. *Journal of Mathematics Teacher Education*, *8*(5), 393-417.

Wertsch, J. V. (1984). The zone of proximal development: Some conceptual issues. *New Directions for Child and Adolescent Development*, *1984*(23), 7-18.

Young, M., & Muller, J. (2013). On powers of powerful knowledge. *Review of Education, 1(3)*, 229-250.

Zazkis, R., & Leikin, R. (2010). Advanced mathematical knowledge in teaching practice: Perceptions of secondary mathematics teachers. *Mathematical Thinking and Learning*, *12*(4), 263-281.

Zipin, L., Fataar, A., & Brennan, M. (2015). Can social realism do social justice? Debating the warrants for curriculum knowledge selection. *Education as Change*, *19*(2), 9-36.

APPENDIX A: LETTERS OF PERMISSION

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**18 February 2020**
Validity of Research Approval:	**04 February 2020 – 30 September 2020** **2019/403**
Name of Researcher:	**Kushanda P**
Address of Researcher:	**Stand Number : 4765**
	Lekazi, Kanyamazane
	Mpumalanga, 1248
Telephone Number:	***072 808 3960***
Email address:	***0714381y@students.wits.ac.za***
Research Topic:	**Professional noticing of learner's mathematical thinking by the teacher in teaching and learning of directed numbers in Grade 8**
Type of qualification	**Master's in Education**
Number and type of schools:	**Two Secondary Schools**
District/s/HO	**Johannesburg East**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

18|02|2018

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 18/02/2020

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX A: LETTERS OF PERMISSION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG | WSoE Wits School of Education

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NONMEDICAL)

CLEARANCE CERTIFICATE **PROTOCOL NUMBER: 2020ECE002**

PROJECT TITLE	Professional noticing of learner's mathematical thinking by the teacher in teaching and learning of directed numbers in grade 8.
INVESTIGATOR	Peter Kushanda
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	09 March 2020
DECISION OF THE COMMITTEE	Approved unconditionally
RISK LEVEL	LOW RISK
EXPIRY DATE	Date of submission of the project Research Report

ISSUE DATE OF CERTIFICATE 22 MAY 2020 CHAIRPERSON: Dr. P Goldschagg

cc: Supervisor: Prof. J Makonye

27 St Andrews Road, Parktown, Johannesburg, 2193 | Pvt Bag 3, WITS, 2050 | T +27 11 717 3007 | www.wits.ac.za/education

IYUNIVESITHI YASEWITWATERSRAND | YUNIVESITHI YA WITWATERSRAND

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** returned to the Chairperson of the School/Department ethics committee.

I fully understand the conditions under which I am are authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

______________________________ **30 / 05 / 2020**

Signature **Date**

APPENDIX B: INFORMATION AND CONSENT LETTERS.

INFORMATION SHEET PRINCIPALS 2020ECE002

University of the Witwatersrand, Wits School of Education, 27 St Andrews Rd, Parktown, Johannesburg,

<u>The Principal</u>

St Endas/Barnato Park High School

Hillbrow

Date: ------------------------------------

Dear Sir/Madam

<u>REF: Request for permission to research on mathematics teaching and learning at your school</u>

My name is Peter Kushanda, a master's student at the University of the Witwatersrand. As part of my research, I am exploring professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics. The research aims to improve the teaching and learning of subtraction, addition, and multiplication of integers through an intervention where teachers teach the topics using certain researched teaching approaches.

I kindly ask for your permission to carry out this research that will ultimately benefit teachers and learners in teaching and learning mathematics. If permission is granted the research will be carried out at St Endas and Barnato Park high schools. Two grade8 teachers per school and their learners will participate in the research project.

The research will take up to 8 weeks and I will be working with two Grade 8 maths teachers and their learners. A pre-test will be given to learners to check their level of understanding then re-teaching the operations. There will be two types of classes' namely experimental and control. Experimental classes consist of teachers practicing professional noticing of learners' mathematical thinking during their lessons, and the control class will be an ordinary class. In the event of collecting data, they will be some lesson observations and audio-recordings. In the end, a post-test will be given to Grade 8 learners to assess their progress. Participation is voluntary and no incentives or penalties will be given for participation. Participation will not affect classroom activities and assessments. All the research will be done confidentially. Learners will be fully respected, they can freely refuse to answer any questions they are uncomfortable with. They can also withdraw from the study at any time.

Learners will be observed in research lessons and will write mathematical exercises and tests on the focused topics: Directed Numbers and Introduction to Algebra. I hope that with time, all learners will academically do well on these topics. Learners will benefit from the innovative methods used in teaching these topics.

Teachers and learners will not receive any direct benefits from participating in the research, besides knowledge; and there are no disadvantages and penalties for not participating in the research. Teachers and learners may withdraw at any time or not answer any questions if they do not want to. In the event of collecting data, they will be some lesson observations and audio-recordings. Reporting will be anonymous. I will not ask for any names or any identification. I will use false names to represent teachers' and learners' participation in my resultant research publications. If teachers and learners experience any distress or discomfort at any stage of this process, I will stop the observation or resume another time. If teachers and learners need some support or counselling services following the lessons observation, these are free of charge from the Wits University Counselling and Careers Development Unit. Please contact me or my supervisor on the details below so that I will help you arrange that. No incentives/penalties will be given to the participants.

This research project will have low risks to the learners.The results of the research will be shared at educational conferences and the report will be anonymous. Your school's name will not be mentioned in any publications or conferences. All research data will be securely stored in locked up cupboards or in computers using passwords and it will be destroyed within 3-5 years of completion of this project. You can contact me or my supervisor on the details below if you have any questions about the research. If you wish, a more detailed summary of the completed research that can be shared with your school upon request. Any concerns or complaints regarding ethical procedures of this study, are to be referred to the University Research Ethics Committee (Non-Medical) telephone +27(0) 11 717 1408.

Your completion of the consent form allows me to conduct this study at your school.

Yours sincerely,

Peter Kushanda (Student No: 0714381Y)

Wits School of Education, 27 St Andrews Rd, Parktown

Researcher: Peter Kushanda

Phone: 072 8083 960

Email:0714381y@students.wits.ca.za

Research supervisor: Professor J. Makonye.

Wits School of Education, 27 St Andrews Rd, Parktown

Phone : +27 (0) 11 717 3086

Email : Judah.Makonye@wits.ac.za

Principal Consent Form 2020ECE002

Title of research: Professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

I ----------------------------------- ------------am the principal of --school. I have read and understood the content of the letter seeking permission to do research on professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

I permit/ do not permit………………………………………………………….. to do the above named research at this school

Signature ______________________________________

Signed at ______________________________on this day of ___________ 2020

Cell Number____

University of the Witwatersrand, Wits School of Education, 27 St Andrews Rd, Parktown, Johannesburg,

The Chair SGB

St Endas/Barnato Park High School

Hillbrow

Date: -------------------------------------

Dear Sir/Madam

REF: Letter seeking for permission to conduct research on mathematics teaching and learning at your school

My name is Peter Kushanda, a master's student at the University of the Witwatersrand. As part of my research, I am exploring professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics. The research aims to improve the teaching and learning of subtraction, addition, and multiplication of integers through an intervention where teachers teach the topics using certain researched teaching approaches.

I kindly ask for your permission to carry out this research that will ultimately benefit teachers and learners in teaching and learning mathematics. If permission is granted
the research will be carried out at St Endas and Barnato Park high schools. Two grade8 teachers per school and their learners will participate in the research project.

The research will take up to 8 weeks and I will be working with two Grade 8 maths teachers and their learners. A pre-test will be given to learners to check their level of understanding then re-teaching the operations. There will be two types of classes' namely experimental and control. Experimental classes consist of teachers practicing professional noticing of learners' mathematical thinking during their lessons, and the control class will be an ordinary class. In the end, a post-test will be given to Grade 8 learners to assess their progress. Participation will not affect classroom activities and assessments. This research project will have low risks to the learners.The results of the research will be shared at educational conferences and the report will be anonymous. Your school's name will not be mentioned in any publications or conferences. You can contact me or my supervisor on the details below if you have any questions about the research. If you wish, a more detailed summary of the completed research that can be shared with your school upon request.

Teachers and learners will not receive any direct benefits from participating in the research, besides knowledge; and there are no disadvantages and penalties for not participating in the research. Teachers and learners may withdraw at any time or not answer any questions if they do not want to. In the event of collecting data, they will be some lesson observations and audio-recordings. Reporting will be anonymous. I will not ask for any names or any identification. I will use false names to represent teachers' and learners' participation in my resultant research publications. If teachers and learners experience any distress or discomfort at any stage of this process, I will stop the observation or resume another time. If teachers and learners need some support or counselling services following the lessons observation, these are free of charge from the Wits University Counselling and Careers Development Unit. Please contact me or my supervisor on the details below so that I will help you arrange that. No incentives/penalties will be given to the participants.

If the SGB chair has any questions during or after this research, they are encouraged to feel free to contact me or my research supervisor on the details listed below. If he/she wishes to receive summaries of these reports, I will gladly send them. All research data will be securely stored in locked up cupboards or in computers using passwords and it will be destroyed within 3-5 years of completion of this project.

Any concerns or complaints regarding ethical procedures of this study, are to be referred to the University Research Ethics Committee (Non-Medical) telephone +27(0) 11 717 1408.

Yours sincerely,

Peter Kushanda (Student number: 0714381Y).

Researcher: Peter Kushanda

Phone: 072 8083 960

Email:0714381y@students.wits.ca.za

Research supervisor: Professor J. Makonye.

Wits School of Education, 27 St Andrews Rd, Parktown

Phone : +27 (0) 11 717 3086 Email : Judah.Makonye@wits.ac.za

SGB Consent Form 2020ECE002

Title of research: Professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

I ----------------------------------- ------------am the SGB chairperson of --- school. I have read and understood the content of the letter seeking permission to do research on professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

I permit/ do not permit………………………………………………………….. to do the above named research at this school

Signature ______________________________________

Signed at _____________________________on this day of ___________ 2020

Cell Number____

2020ECE002

Information LETTER TO THE PARENTS DATE……….……..

Re: Seeking consent to do a research study with your child as a participant.

Dear Parent of ……………. ……………………….. (St Endas /Barnato Park high schools)

My name is Peter Kushanda, a master's student at the University of the Witwatersrand. As part of my research, I am exploring professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics. The research aims to improve the teaching and learning of subtraction, addition, and multiplication of integers through an intervention where teachers teach the topics using certain researched teaching techniques.

The research will take up to 8 weeks and I will be working with two Grade 8 maths teachers and their learners. A pre-test will be given to learners to check their level of understanding then re-teaching the operations. There will be two types of classes' namely experimental and control. Experimental classes consist of teachers practicing professional noticing of learners' mathematical thinking during their lessons, and the control class will be an ordinary class. In the end, a post-test will be given to Grade 8 learners to assess their progress. Participation will not affect classroom activities and assessments. This research project will have low risks to the learners.The results of the research will be shared at educational conferences and the report will be anonymous. Your child school's name will not be mentioned in any publications or conferences. No incentives/penalties will be given to your child.

This research project will have low risks for your child. He/she will be allowed to withdraw at any time during this project without any penalties. Your child's name and identity will remain confidential and anonymous in all academic writings of this study. If your child experience any distress or discomfort at any stage of this process, I will stop the observation or resume another time. If he/she need some support or counselling services following the lessons observation, these are free of charge from the Wits University Counselling and Careers Development Unit. Please contact me or my supervisor on the details below so that I will help you arrange that His/her privacy will be maintained in all published and written data resulting from the study. You can contact me or my supervisor if you have any questions about the research. If you wish, a more detailed summary of the completed research that can be shared with you upon request.

All research data will be securely stored in locked up cupboards or in computers using passwords and it will be destroyed within 3-5 years of completion of this project.

Yours sincerely,

Peter Kushanda (Student No: 0714381Y)

Researcher: Peter Kushanda

Phone: 072 8083 960

Email:0714381y@students.wits.ca.za

Research supervisor: Professor J. Makonye.

Wits School of Education, 27 St Andrews Rd, Parktown

Phone : +27 (0) 11 717 3086

Email : Judah.Makonye@wits.ac.za

Parent/Guardian Consent Form 2020ECE002

Name of researcher: Peter Kushanda (Student No: 0714381Y).

Title of research project: Professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

I……………………………………..consent that my child …………………………………to participate in this research project.

- I report that the above research has been explained to me and I understand it **YES** **NO**
- I have been promised that; participation is voluntary and I can withdraw my child from the study anytime we are not comfortable with it YES NO
- I understand the study lasts for at most 8 weeks YES NO
- I can communicate with Peter Kushanda or his supervisor anytime for clarity **YES** **NO.**

I agree that my child will participate in the lessons on Directed Numbers and Algebra

YES NO (please circle)

I agree that my child will write exercises and pre and post-test on Directed Numbers and Algebra

YES NO

I agree that the lesson observation may be audio recorded YES NO

I agree that the information provided may be used YES NO

anonymously by other researchers following this study

…………………………………… (Signature)

…………………………………… (Name of parent/guardian)

…………………………………… (Date)

……………………………………… (Cell Number)

2020ECE002

LETTER TO THE TEACHERS DATE…………..

Re: Seeking permission to do a research study with you at your school.

Dear Sir/Madam

My name is Peter Kushanda, a master's student at the University of the Witwatersrand. I am a researcher who seeks to improve the teaching and learning techniques of mathematics in class.

Would you mind being part of this research project that is meant to be of benefit to the teacher, learners, and community as a whole?

This research will take up to 8 weeks and I will be working with two Grade 8 maths teachers and their learners. A pre-test will be given to learners to check their level of understanding then re-teaching the operations. In the event of collecting data, they will be some lesson observations and audio-recordings. In the research, learners will be taught and thereafter write maths exercises and tests on the focused topics (Directed Numbers and Introduction to Algebra). Learners are likely to benefit from the innovative methods used in teaching these topics. In the end, a post-test will be given to Grade 8 learners to assess their progress. Participation will not affect classroom activities and assessments.

Your name and identity will be kept confidential at all times and in all academic writings of this study. All research data will be securely stored in locked up cupboards or in computers using passwords and it will be destroyed within 3-5 years of completion of this project. The results of the research will be shared at educational conferences and the report will be anonymous. Your school's name will not be mentioned in any publications or conferences.

Your participation is voluntary and you can withdraw at any time during this project without any penalties. There are no foreseeable risks in participating. No incentives will be given to the participants. If teachers and learners experience any distress or discomfort at any stage of this process, I will stop the observation or resume another time. If teachers and learners need some support or counselling services following the lessons observation, these are free of charge from the Wits University Counselling and Careers Development Unit. Please contact me or my supervisor on the details below so that I will help you arrange that. Any concerns or complaints regarding ethical procedures of this study, are to be referred to the University Research Ethics Committee (Non-Medical) telephone +27(0) 11 717 1408.

Please let me know if you require any further information.

Your help will be greatly appreciated.

Yours sincerely,

Peter Kushanda (Student No: 0714381Y)

Researcher: Peter Kushanda

Phone: 072 8083 960

Email:0714381y@students.wits.ca.za

Research supervisor: Professor J. Makonye.

Wits School of Education, 27 St Andrews Rd, Parktown

Phone : +27 (0) 11 717 3086 Email : Judah.Makonye@wits.ac.za

Teacher’s Consent Form 2020ECE002

Title of research project: Professional noticing of learner’s mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project.

I, ________________________ give my consent for the following:

I agree to administer pre and post-tests YES/NO

I give the researcher permission to observe me in my class YES/NO

I agree to be audiotaped during the observation lesson YES/NO

Name: …………………………………………..

Signature: …………………………………………

Date: ………………………………………………

LETTER TO THE LEARNER DATE…………..… 2020ECE002

Re: Seeking permission to do a research study with you at your school.

Dear Learner

My name is Peter Kushanda, a master's student in the School of Education at the University of the Witwatersrand. I am researching on teacher professional noticing of learner's mathematical thinking in teaching and learning addition, subtraction and multiplication of integers in Grade 8. Integers are a set of whole numbers that are positive or negative.

Would you mind being part of the project to discover how best we can teach addition, subtraction, and multiplication of integers?

My research involves finding reasons why learners have difficulties in using positive and negative whole numbers when adding, subtracting, or multiplying them. A Pre-test and a post-test will be given to you to check on your level of understanding. Lessons on addition, subtraction and multiplication of positive and negatives numbers will take place in your classroom. The lessons will take from 40 to 60 minutes. In collecting data, they will be some lesson observations and audio-recordings.

Remember writing these tests is by choice and not for marks or school reports. Also, if you decide halfway through that you prefer to stop, your choice will be respected as this will not affect you negatively in any way. No gifts or penalties will be given to the participants.

When announcing the research all information about you will be kept confidential in all my writings on this study. All research data will be securely stored in locked up cupboards or in computers using passwords and it will be destroyed within 3-5 years of completion of this project. If you experience any distress or discomfort at any stage of this process, I will stop the observation or resume another time. If you also need some support or counselling services following the lessons observation, these are free of charge from the Wits University Counselling and Careers Development Unit. Please contact me or my supervisor on the details below so that I will help you arrange that

An information sheet and consent form have been given to your guardians, but after all the decision to join us be yours.

I look forward to working with you and feel free to contact me if you have any questions.

Thank you.

Yours sincerely, Peter Kushanda (Student number: 0714381Y).

Researcher: Peter Kushanda

Phone: 072 8083 960

Email:0714381y@students.wits.ca.za

Research supervisor: Professor J. Makonye.

Wits School of Education, 27 St Andrews Rd, Parktown

Phone : +27 (0) 11 717 3086

Email : Judah.Makonye@wits.ac.za

Learner ASSENT Form 2020ECE002

Title of project: Exploring professional noticing of learner's mathematical thinking by the teacher in teaching and learning operations with directed numbers in Grade 8 mathematics.

Name of researcher: Peter Kushanda.

I ……………………………………….. assent to participate in this research project. The research has been explained to me and I understand what my participation will involve.

I agree to participate in the research project YES NO (please circle)

I agree that the lesson observation may be audio recorded YES NO

I agree to write the Pre and Post Tests YES NO

I agree that the information I provide may be used YES NO

anonymously by other researchers following this study

…………………………………… (Signature)

…………………………………… (Name of participant)

…………………………………… (Date)

APPENDIX C: CAPS DOCUMENTS

TOPIC: Integers

CONTENT AREA	TOPICS	CONCEPTS AND SKILLS	SOME CLARIFICATION NOTES OR TEACHING GUIDELINES	DURATION (in hours)
	1.3 **Integers**	**Counting, ordering and comparing integers** • Revise: - counting forwards and backwards in integers for any interval - recognising, ordering and comparing integers **Calculations with integers** • Revise addition and subtraction with integers • Multiply and divide with integers • Perform calculations involving all four operations with integers • Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers	**What is different to Grade 7?** • Multiply and divide with integers • All four operations with integers • All four operations with squares, cubes, square and cube roots of integers In Grade 8 learners consolidate number knowledge and calculation techniques for integers, developed in Grade 7. **Counting, ordering and comparing integers** • Learners should continue practising counting, ordering and comparing integers. Counting should not only be thought of as verbal counting. Learners can count using: - structured, semi-structured or empty number lines - chain diagrams for counting • Learners should be given a range of exercises • Arrange given numbers from the smallest to the biggest: or biggest to smallest • Fill in missing numbers in - a sequence - on a number grid - on a number line - fill in <, = or > e.g. – 425 • – 450; **Calculations using integers** • Start calculations with integers using small number ranges. • Develop an understanding that subtracting an integer is the same as adding its additive inverse. **Example:** 7 – 4 = 7 + (– 4) = 3 OR –7 – 4 = –7 + (–4) = –11 So too, 7 – (– 4) = 7 + (+4) = 11 OR –7 – (– 4) = –7 + (+ 4) = – 3. Here the use of brackets around the integers are useful. • A useful strategy is to use repeated addition and number patterns to show learners the reasonableness of rules for the resultant sign for multiplication with integers.	Total time for Integers: 9 hours

APPENDIX C: CAPS DOCUMENTS

TOPIC: Integers

CAPS

CONTENT AREA	TOPICS	CONCEPTS AND SKILLS	SOME CLARIFICATION NOTES OR TEACHING GUIDELINES	DURATION (in hours)
	1.3 **Integers**		**Example:** a) Repeated addition of (–3): (–3) + (–3) + (–3) = –9 = 3 x (–3) b) Repeated addition of (–2): (–2) + (–2) + (–2) + (–2) = –8 = 4 x (–2) c) Counting down in intervals of 3 from 9: 3 x 3 = 9 3 x 2 = 6 3 x 1 = 3 3 x 0 = 0 3 x –1 = –3 3 x –2 = ? 3 x –3 = ? Hence the rule: a positive integer x a negative integer = a negative integer d) Using the rule that a positive integer x a negative integer = a negative integer, established from examples above, the following pattern can be used: –1 x 3 = –3 –1 x 2 = –2 –1 x 1 = –1 –1 x 0 = 0 –1 x –1 = 1 –1 x –2 = ? –1 x –3 = ? Hence the rule: a negative integer x a negative integer = a positive integer • Use the inverse operation for multiplication and division to develop a rule for the resultant sign for division with integers. **Example:** a) If 4 x (–2) = –8, then –8 ÷ 4 = –2 and –8 ÷ (–2) = 4 b) If (–1) x (–3) = 3, then 3 ÷ (–1) = –3 and 3 ÷ (–3) = –1 Hence the rules: division of a positive and negative integer equals a negative integer and division of two negative integers equal a positive integer.	

79

APPENDIX C: CAPS DOCUMENTS

TOPIC: Integers

CONTENT AREA	TOPICS	CONCEPTS AND SKILLS	SOME CLARIFICATION NOTES OR TEACHING GUIDELINES	DURATION (in hours)
	1.3 Integers	**Properties of integers** • Recognize and use commutative, associative and distributive properties of addition and multiplication for integers • Recognize and use additive and multiplicative inverses for integers **Solving problems** • Solve problems in contexts involving multiple operations with integers	• Finding the squares, cubes, square roots and cube roots of integers are also opportunities to check that learners know the rules for resultant signs when multiplying integers. • Therefore, make sure that learners understand why you cannot find the square root of a negative integer, and that the square of a negative integer is always positive. **Example:** a) $(-5)^2 = (-5) \times (-5) = 25$ b) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$ c) $\sqrt[3]{-27} = -3$ because $-3 \times -3 \times -3 = -27$ **Properties of integers** • Learners should investigate the properties for operations with whole numbers on the set of integers. • These properties should serve as motivation for the operations they can perform with integers. • Learners should see that the commutative property for addition and multiplication holds for integers, e.g. $8 + (-3) = (-3) + 8 = 5$; $8 \times (-3) = (-3) \times 8 = -24$ • Learners should see that they can still use subtraction to check addition or vice versa, e.g. if $8 + (-3) = 5$, then $5 - 8 = -3$ and $5 - (-3) = 8$ • Learners should see that the associative property for addition holds for integers, e.g. $[(-6) + 4] + (-1) = (-6) + [4 + (-1)] = -3$ • Learners should see that the inverse operation for multiplication and division holds for integers, e.g. if $5 \times (-6) = -30$, then $-30 \div 5 = -6$ and $-30 \div (-6) = 5$ • Learners should develop the rules, through patterning, for resultant signs when multiplying and dividing integers: $(+5) \times (+5) = (+25)$; $(-5) \times (-5) = (+25)$; $(-5) \times (+5) = (-25)$; $(+25) \div (+5) = (+5)$; $(-25) \div (-5) = (+5)$; $(-25) \div (+5) = (-5)$;	

APPENDIX D: PRE-TEST AND POST-TEST

Pre-test 2020ECE002

<u>Researcher: Peter kushanda, Wits School of Education, Park town, Johannesburg</u>

GRADE 8 MATHEMATICS PRE-TEST QUESTIONS ON INTEGERS.

School Name……………………………………………

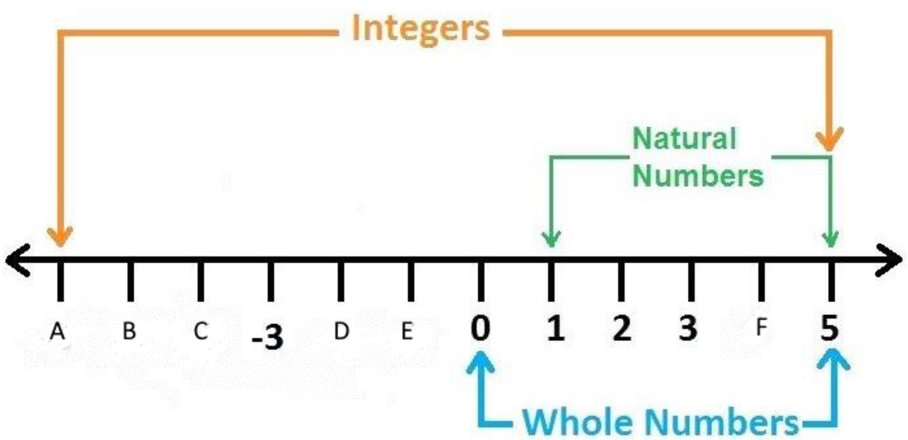
Name of learner……………..……………………..Grade……………….

Date………………………………….. Time 1 hour

ANSWER ALL QUESTIONS ON THE SPACES PROVIDED. YOU ARE NOT ALLOWED TO USE THE CALCULATOR.

<u>Answer all the questions and show all working</u>

1. What numbers are represented by the following letters on the number line below?



Letter	A	B	C	E	F
Number					

Therefore complete a) F – B =

b) B – E =

This table shows the maximum and minimum temperatures in Hillbrow in a certain week in July, 2011

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (℃)	3	5	0	1	2	4	-2
Minimum (℃)	-1	-3	-7	-5	-10	-2	-8

2. Look at the temperatures for Monday, Tuesday and Wednesday. Write down the six temperatures from the highest to the lowest.

………

………………………………………………………………………………

3. Look at the temperatures for Friday, Saturday and Sunday. Write down the six temperatures from the lowest to the highest.

……

……………………………………………………………………………………

4. Rewrite the following words as number symbols: **positive four; negative twenty; negative fifty-three; positive one**

………

……………………………………………………………………..

5. Write the following numbers in words: (-5); (+7); 0; (+10); (-1))

………

………………………………………………………………………………

6. State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer

………………………………………………………………………………………………

7. Eunice's Bank balance is $-R150$. What will the balance be after a deposit of R500?

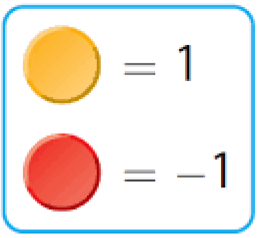
……………………………………………………………………………………………

8. Tandy's DSTV due balance is R400 in her account and she pays R450. What is her balance in the account after the payment?

………

………………………………………………………………………………

Use the following counter models to help you find the numbers represented in questions 11 and 12 consider only the colour of the top of the counter



11.

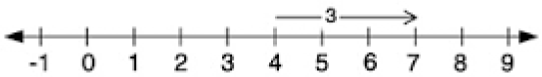
The number represented by the counters is =

12.

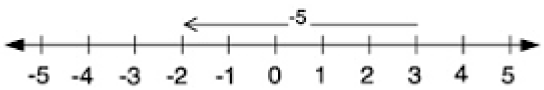
The number represented by the counters is =

Calculate the following WITHOUT the use of the calculator, use the following number line examples to illustrate your method. Two examples are given here.

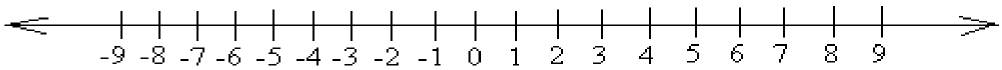
4 + 3 = 7



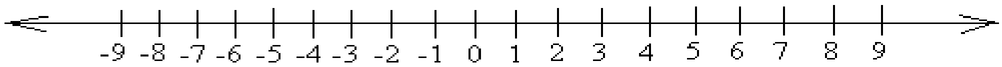
3 + -5 = -2



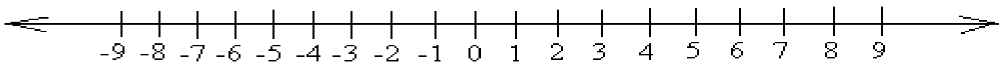
13. $(-8) + 5 =$



14. $(-3) + (-4) =$ ☐

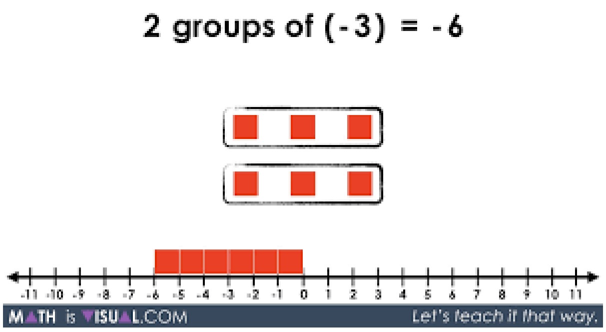


15. $(-2) + (-8) =$ ☐



16. (-4) – (-3) = ☐

Look at this number line example to calculate 2 x (-3)



Now answer the following questions;

17. -2 x $(+5)$ =……………………..

18. $(-5) \times (+5)$ =……………………..

19. $(-6) \times (-4)$ =……………………..

20. Make up a word problem or draw a number line to show how to work out (– 10) – (– 3)

………

………

……………………………………………………….

THANK YOU.

Researcher: Peter kushanda, Wits School of Education, Park town, Johannesburg

Pre-test memorandum 2020ECE002

GRADE 8: MATHEMATICS ANSWERS ON INTEGERS PRE-TEST.

School Name……………………………………………

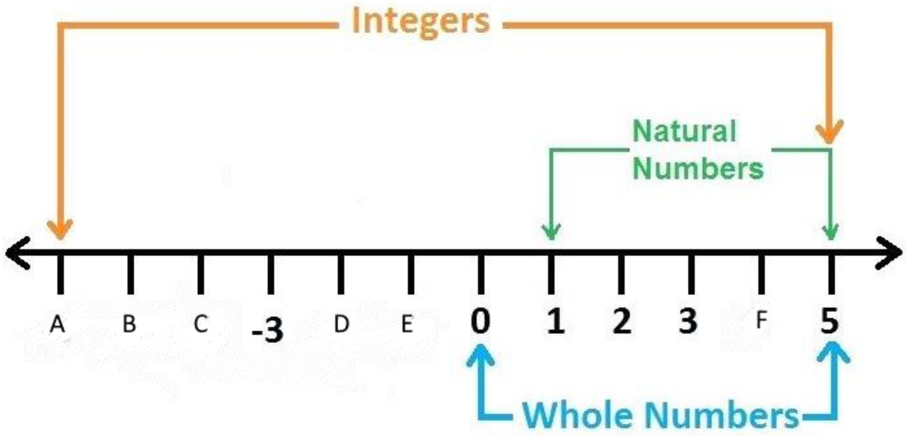
Name of learner……………..……………………..Grade……………….

Date………………………………….. Time 1 hour

ANSWER ALL QUESTIONS ON THE SPACES PROVIDED. YOU ARE NOT ALLOWED TO USE THE CALCULATOR.

Answer all the questions and show all working

1. What numbers are represented by the following letters on the number line below?



Letter	A	B	C	E	F
Number	-6	-5	-4	-1	4

Therefore complete a) F – B = 9

b) B – E $= -4$

This table shows the maximum and minimum temperatures in Hillbrow in a certain week in July, 2011

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (℃)	3	5	0	1	2	4	-4
Minimum (℃)	-1	-3	-7	-5	-10	-2	-8

2. Look at the temperatures for Monday, Tuesday and Wednesday. Write down the six temperatures from the highest to the lowest.

$$\mathbf{5\,;3\,;0\,,-1\,;\ -3\,;\ -7}$$

3. Look at the temperatures for Friday, Saturday and Sunday. Write down the six temperatures from the lowest to the highest.

$$\mathbf{-10\,;\ -8\,;\ -4\,;\ -2\,;2\,;4}$$

4. Rewrite the following words as number symbols: **positive four; negative twenty; negative fifty-three; positive one**

$$\mathbf{+4\,;\ -20\,;\ -53\,;\ +1}$$

5. Write the following numbers in words: (-5); (+7); 0; (+10); (-1))

= negative five; positive seven; zero; positive ten; negative one

6. State whether the following is true or false: (+2) is greater than (-5). Give a reason for your answer :

True because all positive numbers are greater than negative numbers since negative numbers are below zero and positive numbers are above zero.

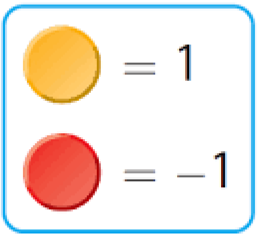
7. Eunice's Bank balance is $-R150$. What will the balance be after a deposit of R500?

R 350.

8. Tandy's DSTV due balance is R400 in her account and she pays R450. What is her balance in the account after the payment?

$$\mathbf{-\ R\ 50}$$

Use the following counter models to help you find the numbers represented in questions 11 and 12 consider only the colour of the top of the counter



11.

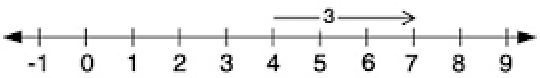
The number represented by the counters is $\mathbf{-5 + +5 \text{ or } -5 + 5 \text{ or } +5 + -5 \text{ or } 5 - 5 = 0}$

12.

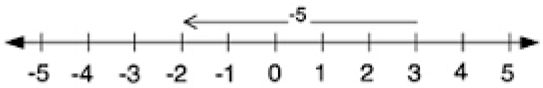
The number represented by the counters is $\mathbf{8 + -4}$ **or** $\mathbf{-4 + +8}$ **or** $\mathbf{-4 + 8 = 4}$

Calculate the following WITHOUT the use of the calculator, use the following number line examples to illustrate your method. Two examples are given here.

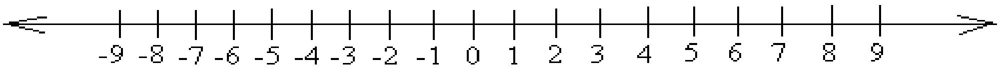
4 + 3 = 7



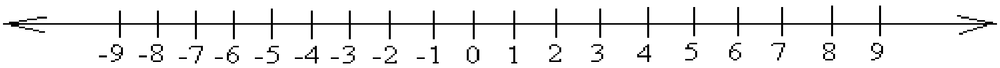
3 + -5 = -2



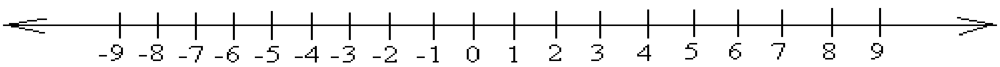
13. $\mathbf{(-8) + 5 = -3}$



14. $\mathbf{(-3) + (-4) = -7}$

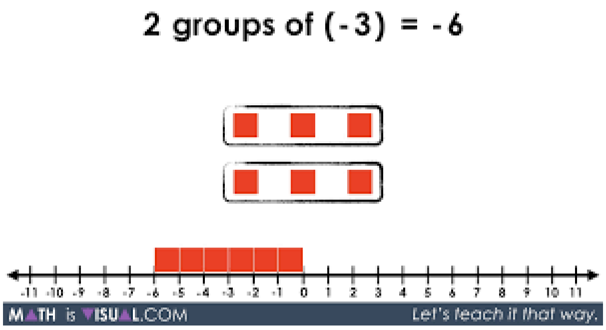


15. $\mathbf{(-2) + (-8) = -10}$



16. $\mathbf{(-4) - (-3) = -1}$

Look at this number line example to calculate 2 x (-3)



Now answer the following questions;

17. $\mathbf{-2\ x\ (+5) = -10}$

18. $(\mathbf{-5}) \times (\mathbf{+5}) = \mathbf{-25}$

19. $(\mathbf{-6}) \times (\mathbf{-4}) = \mathbf{24}$

20. Make up a word problem or draw a number line to show how to work out $(-10) - (-3)$

$$(10) - (-3) = -7$$

APPENDIX E: POST TEST 2020ECE002

Researcher: Peter kushanda, Wits School of Education, Park town, Johannesburg

GRADE 8 MATHEMATICS POST-TEST QUESTIONS ON INTEGERS.

School Name……………………………………………

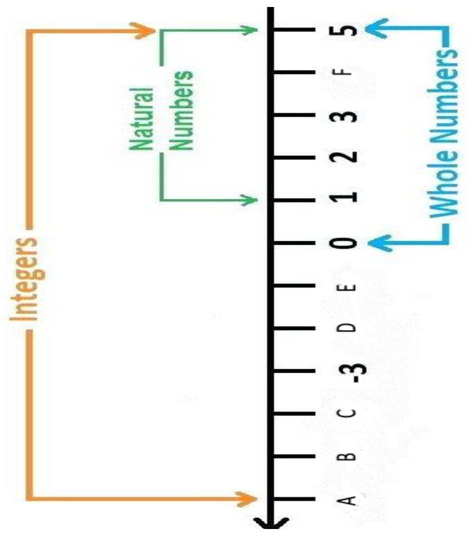
Name of learner…………….……………………..Grade……………….

Date………………………………….. Time 1 hour

ANSWER ALL QUESTIONS ON THE SPACES PROVIDED. YOU ARE NOT ALLOWED TO USE THE CALCULATOR.

Answer all the questions and show all working

1. What numbers are represented by the following letters on the number line below?



Letter	A	B	C	E	F
Number					

Therefore complete a) F – B = ☐

b) B – E = ☐

2. State whether the following is true or false: (-5) is greater than (+2). Give a reason for your answer

………

……………………………………………………………………………………..

3. State whether the following is true or false: (- 1) is less than (-4). Give a reason for your answer

………

…………………………………………………………………………………….

This table shows the maximum and minimum temperatures in Hillbrow in a certain week in July, 2011

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (°C)	4	8	0	1	3	2	-4
Minimum (°C)	-3	-2	-9	-4	-12	-3	-6

4. Look at the temperatures for Monday, Tuesday and Wednesday. Write down the six temperatures from the highest to the lowest.

………

…………………………………………………………………………………

5. Rewrite the following words as number symbols: **positive four; negative twenty; negative fifty-three; positive one**

………

…………………………………………………………..…………………..

6. Write the following numbers in words: (-8); (+11)

………

…………………………………………………………………………………

7. Patric's Bank balance is $-R300$. What will the balance be after a deposit of R700?

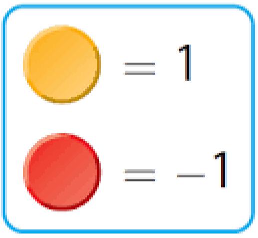
…………………………………………………………………………………………

8. If Phumzile's DSTV due balance is R200 in her account and she pays R 450. What is her balance in the account after the payment?

………

………………………………………………………………………………

Use the following counter models to help you find the numbers represented in questions 9 and 10 consider only the colour of the top of the counter



9.

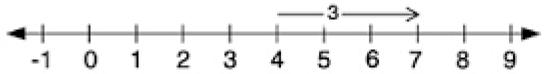
The number represented by the counters is =

10.

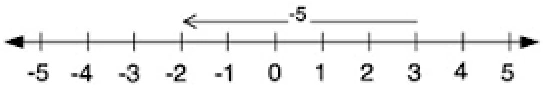
The number represented by the counters is =

Calculate the following WITHOUT the use of the calculator, use the following number line examples to illustrate your method. Two examples are given here.

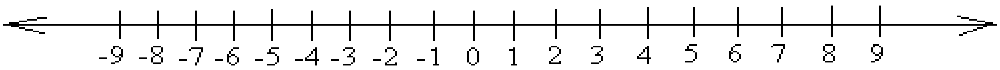
4 + 3 = 7



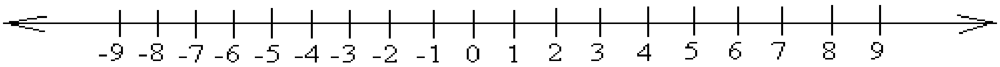
3 + -5 = -2



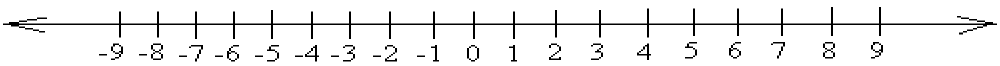
11. $(-7) + 10 =$ ☐



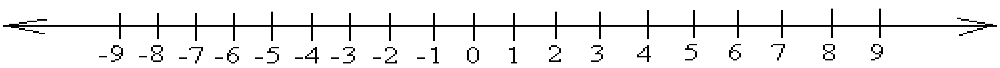
12. $(-4) + (-5) =$ ☐



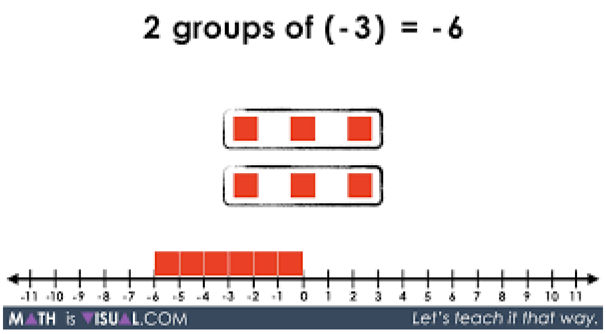
13. $(-1) + (+10) =$ ☐



14. $(-6) - (-8) =$ ☐



Look at this number line example to calculate 2 x (-3)



Now answer the following questions;

15. $+3 \times (+7)$ =…………………….

16. $(-8) \times (+8)$ =…………………….

17. $(-6) \times (-4)$ =…………………….

18. Make up a word problem or draw a number line to show how to work out $(-13) - (-8)$

………

………

…………………………………………………….

THANK YOU.

<u>Post-test memorandum</u> 2020ECE002

<u>Researcher: Peter kushanda, Wits School of Education, Park town, Johannesburg</u>

GRADE 8 MATHEMATICS POST-TEST QUESTIONS ON INTEGERS.

School Name……………………………………………

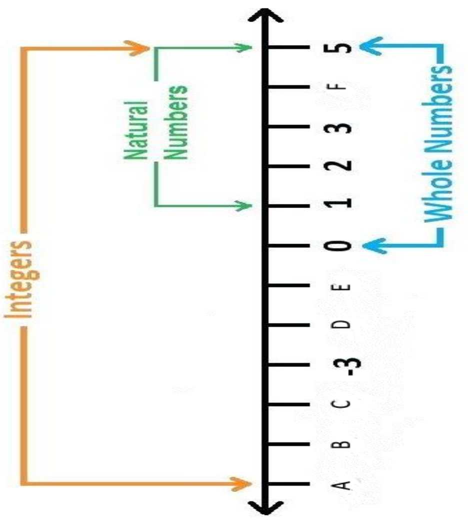
Name of learner……………..……………………..Grade……………….

Date………………………………….. Time 1 hour

ANSWER ALL QUESTIONS ON THE SPACES PROVIDED. YOU ARE NOT ALLOWED TO USE THE CALCULATOR.

<u>Answer all the questions and show all working</u>

1. What numbers are represented by the following letters on the number line below?



Letter	A	B	C	E	F
Number	-6	-5	-4	-1	4

Therefore complete a) F – B = 9

b) B – E = -4 --

2. State whether the following is true or false: (-5) is greater than (+2). Give a reason for your answer

False

1. State whether the following is true or false: (- 1) is less than (-4). Give a reason for your answer

False

This table shows the maximum and minimum temperatures in Hillbrow in a certain week in July, 2011

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (℃)	4	8	0	1	3	2	-4
Minimum (℃)	-3	-2	-9	-4	-12	-3	-6

2. Look at the temperatures for Monday, Tuesday and Wednesday. Write down the six temperatures from the highest to the lowest.

$$\mathbf{8; 4; 0;\ -3;\ -2;\ -9}$$

3. Rewrite the following words as number symbols: **positive four; negative twenty**

$$\mathbf{+4;\ -20}$$

4. Write the following numbers in words: (-8); (+11)

negative eight; positive eleven

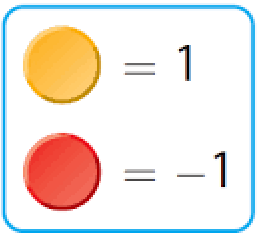
7. Patrick's Bank balance is $-R300$. What will the balance be after a deposit of R700?

R400

8. If Phumzile's DSTV due balance is R200 in her account and she pays R 450. What is her balance in the account after the payment?

$$\mathbf{-R250}$$

Use the following counter models to help you find the numbers represented in questions 9 and 10 consider only the colour of the top of the counter



9.

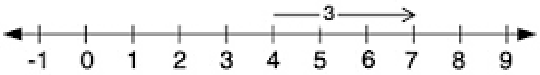
The number represented by the counters is -5 = 5 0

10.

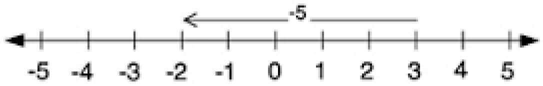
The number represented by the counters is $\mathbf{-4 + (8) = 4}$

Calculate the following WITHOUT the use of the calculator; use the following number line examples to illustrate your method. Two examples are given here.

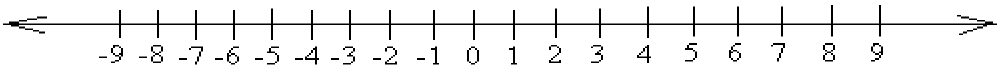
4 + 3 = 7



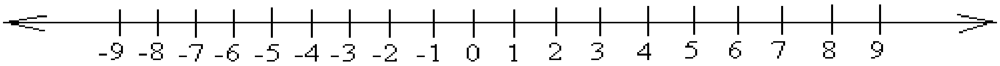
3 + -5 = -2



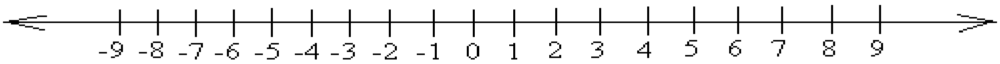
11. $(-7) + 10 = \mathbf{+3}$



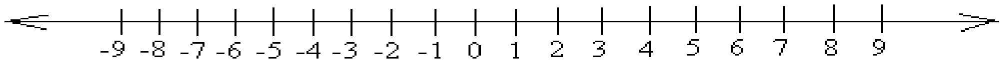
12. $(-4) + (-5) = \mathbf{-9}$



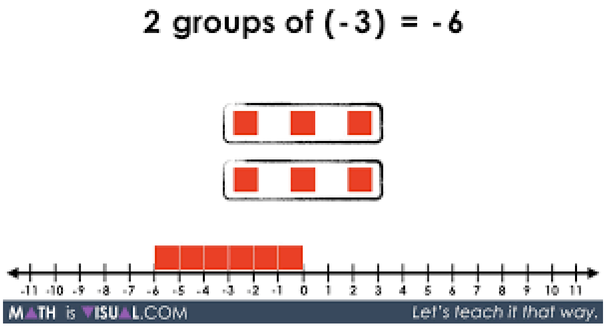
13. $(-1) + (+10) = \mathbf{+9}$



14. $(-6) - (-8) = +\mathbf{2}$



Look at this number line example to calculate 2 x (-3)



Now answer the following questions;

15. $+3 \times (+7) = \mathbf{21}$

16. $(-8) \times (+8) = \mathbf{64}$

17. $(-6) \times (-4) = \mathbf{24}$

18. Make up a word problem or draw a number line to show how to work out $(-13) - (-8)$

$$\mathbf{-5}$$

THANK YOU.

APPENDIX E: LESSON OBSERVATIONS TRANSCRIPTS

Transcripts: Experimental class using counters – (Mr. Mr. Chauke & Mr. Mr. Manana) at Lundi Secondary School

Below is a transcribed experimental class observation lesson delivered by a secondary qualified mathematics teacher recorded during class interventions. This transcribed lesson was a result of the research study that focused on teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. This focus included attending to the strategies of the learners, analyzing the understanding of the learners, and making decisions about how to respond based on the learner's understanding (Jacobs et. al., 2010). Here the teacher was well aware of the importance of knowing mathematics by using powerful knowledge.

Scenario number 1: Introduction of the lesson topic on counting.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
1	00.13	Mr. Chauke	Right let's begin the lesson. We are going to talk about counting and comparing directed numbers neh. Counting and comparing directed numbers right, at times there are known as integers and this is our main topic today. Right before we talk about this topic everyone knows how to count isn't it?	
2	00:40	Lnrs	Yes (Chorus response)	
3	00:56	Mr. Chauke	I know when you started grade one you used to count from zero to ten	
4	01:00	Ls	Yes (Chorus response)	
5	01:02	Mr. Chauke	Then from zero to twenty and I don't know to what extend did you go on counting	
6	01:11	Lnrs	10 000 , million , Billion (Ls shouting)	
7	01:13	Mr. Chauke	You mean if I ask you to count from one, can get to a billion?	
8	01:16	Lnrs	No (Chorus response)	
9	01:24	Mr. Chauke	So today we are going to talk about how to count these numbers. But before we talk about that one, do you still remember we used to have numbers from 0,1,2,3 up to where ever it can end neh?	

10	01:42	Lnrs	Yes (Chorus response)	
11	01:43	Mr. Chauke	But aah which are other numbers do we have other than 1,2,3.Yes (Mr. Chauke nominating a L)	
12	01:49	Lnr	minus	
13	01:50	Mr. Chauke	Right we got minus .So those minuses there come under zero.	
14	01:55	Lnrs	Zero	
15	01:56	Mr. Chauke	Right, from minus one we have got minus?	
16	01:59	Lnrs	2 ,	
17	01:59	Mr. Chauke	Minus?	
18	02:00	Lnrs	3	
19	02:01	Mr. Chauke	Right is goes on like that, so those numbers are above zero we call them what?	
20	02:10	Lnrs	negative	
21	02:11	Mr. Chauke	Eeh which are above zero are called?	
22	02:12	Lnrs	Positive	
23	02:14	Mr. Chauke	Which are above zero there are called positive isn't it?	
24	02:15	Lnrs	Yes (Chorus response)	
25	02:16	Mr. Chauke	And then aah what about which are below zero?	
26	02:21	Lnrs	Negative (Chorus response)	
27	02:22	Mr. Chauke	Right, so I can say let's do it this way if there are positive there are above zero and are below zero are negative right	
28	02:30	Lnrs	Yes (Chorus response)	
29	02:31	Mr. Chauke	Right now these numbers that have got signs we are saying there are positive and negative which means there what? There are directed numbers. What we	

			mean by directed numbers is that there show direction. Right each and every number has a sign in front of it. Normally we normally write these numbers like 20, 30, 40 or whatever, even 60, all these numbers have signs. Numbers that does not have signs there called positive because they have positive sign in front isn't it.	
30	03:08	Lnrs	Yes (Chorus response)	

Scenario number 2: Introduction to number line

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
31	03:10	Mr. Chauke	Right, so now let's do this way I am going to introduce a number line. We all know what is number line?	
32	03:16	Lnrs	Yes (Chorus response)	
33	03:17	Mr. Chauke	Right do we know a number line which is like (teacher drawing a vertical number line) this whereby we start at zero going to what?	
34	03:27	Lnrs	1,2,3,4	
35	03:30	Mr. Chauke	Then it goes on, there are plus numbers?	
36	03:37	Lnrs	$+1, +2, +3$	
37	03:42	Mr. Chauke	Right, all these numbers going up are positive then going down we have got what?	
38	03:44	Lnrs	Minus 1,minus 2,minus 3,minus 4	
39	03:50	Mr. Chauke	Minus 4 goes on like that right. So now aah if we are counting numbers like this we know that if we are at zero we have got nothing isn't it?	
40	03:59	Lnrs	Yes (Chorus response)	
41	04:00	Mr. Chauke	Zero cannot have hold sign because it doesn't show us any direction right?	
42	04:08	Lnrs	Yes (Chorus response)	
43	04:10	Mr. Chauke	So the numbers on top of zero are positive and these ones below zero are negative (Teacher pointing numbers that are below zero)	

44	04:13	Lnrs	Negative (Chorus response)	

Scenario number 3: Comparing numbers

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
45	04:14	Mr. Chauke	So now we want to talk about how do we compare numbers right? Let's say we have got 4, plus 4 and then we have minus 1. Which one is great? Yes (Mr. Chauke nominating a L)	
46	04:32	Lnr	Plus 4	
47	04:43	Mr. Chauke	Plus 4 it's greater because it is above?	
48	04:38	Lnrs	Zero	
49	04:39	Mr. Chauke	Isn't it? Right let's say we have got minus 4 and plus 1 which one is great? Which one is greater than the other one?	
50	04:52	Lnrs	Plus one (Learners shout with confidence)	
51	04:54	Mr. Chauke	Plus one neh. That's correct. So this how we compare using number line isn't it, and then now aah. (Teacher 2 comes up front to add on what Mr. Chauke was saying)	
52	05:21	Mr. Manana	Let me add a little bit. Right what you need to do when you are comparing numbers right aah these numbers are higher (pointing at numbers above zero) like 4 is higher all those numbers right, and then here we have minus 4 lower. We are talking of height isn't it we are talking of height here isn't it? So it means that a number which is higher is bigger than a number which is	

			lower, is that right?	
53	05:55	Lnrs	Yes (Chorus response)	
54	05:56	Mr. Manana	For example we can have a number like say one here. Can you see (1) one?	
55	06:01	Lnrs	Yes (Chorus response)	
56	06:02	Mr. Manana	One is higher, one is higher than say minus 3 cause plus 1 is here right, it's here and we have minus three here it is higher therefore we say one is what bigger than what?	
57	06:29	Lnrs	Minus 3	
58	06:30	Mr. Manana	That's minus 3 because it is higher, if you are having for example this one being a ground zero for a building isn't it, someone on the first floor is higher than someone on the ground floor like minus 3 isn't it? Right and then we can even go on to say ok what about if we compare minus 1 and minus 4 which one is larger? Yes	
59	07:03	Lnr	Minus 1	
60	07:04	Mr. Manana	Yes minus 1 is what? Is higher than minus 4.Therefore, minus 1 is what is larger than what? minus 4	
61	07:20	Lnrs	Minus 4 (Say simultaneously with Mr. Manana)	
62	07:21	Mr. Manana	Because if it is higher if your friend is on the basement minus 4 downstairs and you're on the floor basement minus 1 then you are than higher than what? That friend of yours so that is why we say minus 1 is higher than even zero, zero is higher is someone on ground floor is higher than someone at minus 4 is that right?	
63	07:50	Lnrs	Yes (Chorus response)	
64	07:51	Mr. Manana	Thank you (Mr. Manana went to sit down for Mr. Chauke to take over.)	

Scenario number 4: Movement

Turns	Time intervals	Transcript	Codes of professional noticing and learner hearing

65	07:55	Mr. Chauke	Thank you very much sir. Right aah hope everyone have understood that neh? Right let's look at the movement right? Let's say I am at 4, I want to move backwards by 2 units where will I be?	
66	08:11	Lnrs	2	
67	08:12	Mr. Chauke	I am at 4 neh, then I want to move backwards by 2 units. We count one, then two then we will be at what?	**ALS** **ILU** **DRLU**
68	08:21	Lnrs	2 (Chorus response)	
69	08:22	Mr. Chauke	2 which means moving backwards, if you moving backwards you are what?	
70	08:29	Lnrs	Munising, Subtracting (Chorus response)	
71	08:29	Mr. Chauke	Subtracting that's correct. Then if you are moving the other direction	**CH**
72	08:35	Lnrs	Adding (Chorus response)	
73	08:36	Mr. Chauke	So I said if you are at 4 and the you move backwards by 2 units what do you get	
74	08:43	Lnrs	2 (Chorus response)	
75	08:44	Mr. Chauke	You get plus 2 isn't it	
76	08:46	Lnrs	Yes (Chorus response)	
77	08:47	Mr. Chauke	Right let's say I have 5, let's say plus 5 minus 3 the answer going to be	
78	09:03	Lnrs	2	
79	09:05	Mr. Chauke	Going to be	**CH**
80	09:06	Lnrs	Plus 2	
81	09:07	Mr. Chauke	Positive 2.Let's say I am at negative 2 and then I move aah let's say 4 unites going up. It's going to be plus 2	
82	09:26	Lnr	Plus 2 (learner agreeing with the teacher)	
83	09:29	Mr. Chauke	Let's check it, from minus 2 we going where? If we are adding we are going up isn't it?	**ALS** **ILU** **DRLU**
84	09:33	Lnrs	Yes (Chorus response)	
85	09:34	Mr. Chauke	From minus 2 we go how many steps	**DRLU**
86	09:35	Lnrs	4	

87	09:36	Mr. Chauke	(Counting) 1, 2, 3, 4.The we go to what? To Positive	**DRLU**
88	09:46	Lnrs	2	

Scenario number 5: Introduction using discs or chips ($+ \ or -$)when counting numbers

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
89	09:46	Mr. Chauke	2 (revoices).Okay, right now I want to introduce some chips here. Let's say the first one we have minus 2 isn't it?	
90	09:53	Lnrs	Yes (Chorus response)	
91	09:54	Mr. Chauke	This is a minus 2. How many minuses do we have here?	
92	10:06	Lnrs	(Ls were quiet because they couldn't answer the question)	
93	10:08	Mr. Chauke	Ok minus 2 can be written as $- -$ isn't it? Minus 2. What about plus 4 can be written as what?	
94	10:09	Lnrs	Four Pluses $(+ + ++)$(Chorus response)	
95	10:11	Mr. Chauke	Four Pluses $(+ + ++)$ (teacher writes 4 pluses on the board) Right I am going to, aah lets use this chips to present these numbers (Mr. Chauke starts to distribute counters)	**ALS** **ILU** **DRLU**
96	10:29	Mr. Manana	Right what he is saying is that you know we can have a chip like this (Mr. Manana drawing three chips with positive signs) $+ , + , +$ so the numbers represented here is what?	
97	10:51	Lnrs	Plus 3	
98	10:52	Mr. Manana	Plus 3 $(+3)$ (Mr. Manana revoices) is that right? And then we have another disc right (Mr. Manana picking up three negative discs, right so this one will also be what?	**ALS** **ILU** **DRLU**
99	11:07	Lnrs	Minus 3 (-3)	

Scenario number 6: Introducing the Zero principles using discs $(+-= \mathbf{0} \ \boldsymbol{or} - += \mathbf{0})$

Turns	Time	Transcript	Codes of

	intervals			professional noticing and learner hearing
100	11:07	Mr. Manana	Minus 3 [Mr Manana agrees with the learners who said 3 negatives $(---)$ means minus 3 (represent-3 as a number)]. Each one of these is plus one $(+)$, plus one $(+)$ and plus one $(+)$ (teacher referring to these signs:$+++$), and this one is minus 1 $(-)$, this is minus one $(-)$, minus one $(-)$ (teacher referring to these signs: $---$) right, and we are going to use and consider all these discs together, is that right?	CH
101	11:28	Lnrs	Yes. (Chorus response)	
102	11:28	Mr. Manana	Right, if we are considering all those signs discs. Or what I want you to do now is to take one plus disc and one negative disc, one minus one and put them on the table in front of you. (learner group the discs)	
103	12:08	Mr. Manana	Okay, now what I want you to do is to put what, 3 pluses $(+,+,+)$ and 3 minuses $(---)$ in front of you. Can you put them together? 3 pluses and 3 minuses (Mr. Manana let the learners group the discs)	
104	12:36	Lnrs	(Grouping the discs)	
105	12:46	Mr. Manana	So, we have them. Yes, I can see there are very neat. Ok, so you know if you have one plus and one minus like this $+-$. What number is represented by this? Can you guess? Yes (Teacher pointing at the learner)	
106	13:01	Lnr	Zero	
107	13:02	Mr. Manana	Yes, it is zero because it is the same as plus one minus one and this you gives you what?	ALS ILU
108	13:11	Lnrs	Zero (Chorus response)	

109	13:12	Mr. Manana	So, they cancel each other, a plus and a minus gives us zero. So now if you have three positive discs +,+,+ and three negative discs − − −, what number therefore do we have there? What number do we have there? Yes (Teacher pointing to Sam)	DRLU
110	13:30	Sam	Zero	
111	13:31	Mr. Manana	You have zero, yes, because, actual we have a what? A plus 3 right, plus what? A minus 3 and therefore we have zero so this and this one (Mr. Manana pointing at pairs of + −) give us zero, this one and this one + − give us zero, this one and this one + − give us zero. So, it is zero plus zero plus zero is zero, they neutralise each other. If you think of science example, a proton and a what?	ALS ILU DRLU
112	13:59	Lnrs	Neutrons	
113	14:00	Mr. Manana	One proton and one neutron. Then that chemical is neutral, isn't it?	
114	14:10	Lnrs	Yes (Chorus response)	
115	14:11	Mr. Manana	Very good, so you can think of it like that. These discs very important for you to master. We don't want you to remember the rules without understanding them. We want you to understand directed numbers meaningfully because this is a very key concept and a key topic in mathematics. If you don't understand directed numbers then you are gone, your mathematics does not make sense, ok, so now can you please take positive 2 discs right and also have one negative. 2 plus disc and one negative what number is represented by that. Yes. (teacher pointing at the learner)	
116	15:03	Lnr	1	
117	15:05	Mr.	Yes, it is one definitely because this one and	ALS

		Manana	that one (+−) they do conceal, okay?	ILU DRLU
118	15:12	Lnrs	Yes (Chorus response)	

Scenario number 7: Adding directed numbers using discs

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
119	15:13	Mr. Manana	So now I want you to work out this problem $+3+(-5)$ is that right? Did you see how you work it?	
120	15:58	Lnrs	Yes (Chorus response)	
121	15:59	Mr. Manana	Right you take 3 discs a +,+,+ for (+3) is that right?	**ALS**
122	16:04	Lnrs	Yes (Chorus response)	
123	16:05	Mr. Manana	And you put what? Minus 5 (− − − − −) is that right? So this represent what 3 pluses and 5 negatives is that so?	**ALS**
124	16:20	Lnrs	Yes (Chorus response)	
125		Mr. Manana	Right can we do that, represent that on you. I can see that you can do it, do that it's very important for you my dear. Very important for you. Right 3 positives and five aaaa	
126	16:43	Lnrs	negatives	
127	16:44	Mr. Manana	Negative, yes. Mr. Chauke somebody have enough (Discs). Right have you done that?	
128	16:51	Ls	Yes (Chorus response)	
129	16:55	Mr. Manana	Right 3 positives and 5 negatives. I can see you doing nicely. Everyone is participating that's what we want. We learn by participating not by hearing are we together, hearing yes it's important but also being involved. Right so here we are, what do we get from that? Yes	**ILU**
130	17:22	Lnrs	Negative 2	
131	17:27	Mr. Manana	Negative 2 (revoicing) ooh yes she got negative 2. Can you explain how you got negative 2	

132	17:33	Lnrs	You conceal positive and negative, positive and negative, positive and negative	
133	17:38	Mr. Manana	Yes, she says this is zero$+\,-$, this is zero$+\,-$, this is zero$+\,-$ isn't it and what is left is negative what?	**ALS** **ILU** **DRLU**
134	17:45	Lnrs	Negative 2 (Chorus response)	

Scenario number 8: Subtracting directed numbers using discs

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
135	17:46	Mr. Manana	Negative 2 right (Manana re-voices), ok, I want you to work out a problem, which is $-3-(-5)$ is that right?	
136	18:01	Lnrs	Yes (Chorus response)	
137	18:03	Mr. Manana	How are you going to do that? Listen, right, listen, remember subtracting is taking away isn't it? So, what we have is minus 3 at the beginning, then we want to take away minus 5 from this. Right, quite challenging isn't it?	
138	18:25	Lnrs	Yes. (Chorus response)	
139	18:26	Mr. Manana	Right, don't tell me the answer listen. What we do in this case, is to say, aaaah, we have got a 5 here, right, so let's introduce 5 zeros, is that right?	
140	18:39	Lnrs	Yes. (Chorus response)	
141	18:40	Mr. Manana	So, we say a plus and its negative, a plus and its negative, a plus and its negative, a plus and its negative, a plus and its negative($\pm\pm\pm\pm\pm$), isn't it?	
142	19:02	Lnrs	Yes. (Chorus response)	
143	19:03	Mr. Manana	Right, very well, this is zero, zero, zero, zero, zero, we have not change anything, have we? ($---\pm\pm\pm\pm\pm$)	
144	19:12	Lnrs	Zero. (Chorus response)	
145	19:13	Mr. Manana	Right, but now let's work this, because we are saying we have minus 5 and minis 3, and we want to take away 5 negatives is that ok?	**NH**

146	19:21	Lnrs	Yes. (Chorus response)	
147	19:23	Mr. Manana	Right, we had minus 3 here, these is minus 3 (teacher pointing at 3 negative signs), here we know that these are just 5 zeros (teachers pointing to$(-,-,-,+-,+-,+-,+-,+-)$ so its zero it does not change anything, it is still negative 3. So, we now say minus 3 minus 5 so now I think it's time to take out the minuses isn't it?	
148	19:42	Lnrs	Yes (Chorus response)	
149	19:44	Mr. Manana	Oh, let us count them, one minus taken away, two minuses, three minuses taken away, four minuses away, and five minuses taken away. Eeh, so what are we remaining with now? Yes. (teacher pointing at John)	
150	20:04	Lnr	Minus 3.	
151	20:05	Mr. Manana	Right, minus 3, where is it now, where is our minus 3. Yes, can you come and show us our minus 3. Yes. (teacher pointing at John)	**CH; ALS**
152	20:20	Lnr	(John goes up to the board to show the 3 negatives)	
153	20:29	Mr. Manana	Aaa, What about these ones (Mr. Manana pointing to the signs on the board) What's your name?	**ILU** **DRLU**
154	20:41	Lnr	John	
155	20:43	Mr. Manana	John, this and this is it not zero? (Mr. Manana pointing at $+-$)	**DRLU**
156	20:44	Lnrs	Its zero.	
157	20:45	Mr. Manana	This is zero, this is zero isn't it? What do we really remain with?	**DRLU**
158	20:51	Lnrs	2	
159	20:53	Mr. Manana	Can you see?	**DRLU**
160	20:54	Lnrs	Yes. (Chorus response)	
161	20:55	Mr. Manana	Therefore $-3-(-5)$ Is what? Plus?	
162	20:59	L	2	
163	21:05	Mr. Manana	Can you see? Right I want you to do the same thing. Now I want to say$-2-(-4)$. Use what? Use your discs. Start with what? You start with	

			minus 2. Right	
164	21:23	Lnr	What?	
165	21:24	Mr. Manana	Minus 2.Is that right? Start with minus 2 because it is what we start with is that right? Yes good, started with minus 2. Right so when you have started with minus 2, what do you do? You introduce four zeros. Isn't it?	**CH**
166	21:42	Lnrs	Yes (Chorus response)	
167	21:44	Mr. Manana	$-+, -+, -+, -+$ Can you do that? Start with minus 2.Start with minus 2 I want to see minus 2 first. Start with minus 2 is that right and introduce 4 zeros. Don't do anything, we want minus 2 and 4 zeros(Mr. Manana and Mr. Chauke go around the class helping learner to build the required set using discs)	
168	22:22	Lnrs	(Ls build the sets in groups with the assistance from the Mr. Chauke and Mr. Manana)	
169	25:22	Mr. Manana	Right ok boys and girls some of you they know the answer isn't it?	
170	25:27	Lnrs	Yeeeeeees (Chorus response in a jovial way)	
171	25:28	Mr. Manana	We want to understand the method, right? I have seen what you have done, you have your minus 2 very good, minus two discs. So we want to take away minus 4 from there.so our method we go indirectly so we say let's put 4zeros there which is $-+, -+, -+, -+$ (Ls say it out also)	**ALS** **ILU** **DRLU**
172	26:00	Lnrs	$-+, -+, -+, -+$ (Ls repeat after the teacher)	
173	26:2o	Mr. Manana	Right, we know that this is zero ,zero, zero, zero that helps us to work out our problem	
174	26:28	Lnrs	Yes (Chorus response)	
175	26:32	Mr. Manana	Now we can take away now, this number is still what? Is minus 2 because other are zeros (Mr. Manana pointing to set of$(--, -+, -+, -+, -+)$ So now we can take away the minus 4 now. So let's take away the minus 4. Who can come and conceal them for me? Yes, conceal the minus 4	**DRLU**
176	27:00	Lnr	(L come up front to the chalk board and conceal the all the 4 negatives) remaining with $+, +, -+, -+$	**ILU**

177	27:05	Mr. Manana	Thank you L thank you. So we have now taken out the minus 4 so now we are now looking at what remains is that right. Can you work with your set up take away the minus 4 from your counters? Throw them away or on the side .So what number do you have? Yes	**ALS** **ILU** **DRLU**
178	27:33	Lnrs	Positive 2	
179	27:35	Mr. Manana	Right will then have positive 2 isn't it?	
180	27:36	Lnrs	Yes (Chorus response)	
181	27:40	Mr. Manana	Why? Because this one is zero and this is zero (Mr. Manana pointing at the last zeros on the set: $+, +, \pm, \pm$ you are only left with what the positive ++ therefore $-2-(-4)$ is equal to what?	**ILU**
182	27:52	Lnrs	2	
183	27:53	Mr. Manana	Right can you see?	
184	27:55	Lnrs	Yes (Chorus response)	
185	27:57	Mr. Manana	Are there any questions? I don't want answers unless I ask you to give me. I want you to learn the methods. Yes do you have any questions?	
186	29:04	Lnrs	(Ls show not to have questions)	
187	29:04	Mr. Manana	When you are given question like this, when you are given questions like this, the best thing for is to draw isn't it? You can draw the discs on paper on the paper before you work out. That is what we want. If just give an answer I don't appreciate it. I want to know how you got the answer. I want you to write you names and work out question number quickly, try to feel numbers on number line right for number one. So what is A , what is B what is C.(on conclusion Ls are given classwork)	

<u>Transcript: Experimental class using number line – Teacher (Mr. Chauke & Mr. Manana) at Lundi Secondary School</u>

Below is a transcribed experimental class observation lesson delivered by a secondary qualified mathematics teacher recorded during class interventions. This transcribed lesson was a result of the research study that focused on teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. This focus included attending to the strategies of the learners, analyzing the understanding of the learners, and making decisions about how to respond based on the learner's understanding (Jacobs et. al., 2010). Here the teacher was well aware of the importance of learning mathematics through making use of powerful knowledge.

Scenario number 9: Introduction of the lesson topic on counting.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
1	00:05	Mr. Chauke	Afternoon guys	
2	00:09	Lnrs	Afternoon sir	
3	00:10	Mr. Chauke	Today we are going to continue from where we left last week. Last week we talked about ordering numbers, talked about adding integers and talk about subtracting integers isn't it?	
4	00:27	Lnrs	Yes (Chorus response)	
5	00:29	Mr. Chauke	When we were adding integers what did we use last time?	
6	00:35	Lnrs	Chips (Counters)	
7	00:36	Mr. Chauke	We used chips isn't it?	
8	00:35	Lnrs	Yes (Chorus response)	
9	00:38	Mr. Chauke	We used chips to find answers for addition and subtraction right?	
10	00:43	Lnrs	Yes (Chorus response)	

Scenario number 10: Addition of directed numbers using number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
11	00:44	Mr. Chauke	Today we going to do something different. Right so, I am going to introduce another way	

			on how to add directed numbers. Since we all know that there are many ways of killing a cat. Right, let's say I have this diagram. Let's play a game. Using arithmagons methods diagram right. So I have got -8, I have got 12, I have 5 as our directed numbers isn't it?	
12	01:18	Lnrs	Yes (Chorus response)	
13	01:19	Mr. Chauke	And then now I want us to add those numbers using that diagram to build operation expressions. In this case, we are talking about addition only. Which expression can we have from this diagram, let's say we are only using two numbers, any expression of addition that we can have? Yes	
14	01:41	Lnr	8 and 12	
15	01:45	Mr. Chauke	8 and 12. Ok, so we can say $-8 + 12$ isn't it?	OH
16	01:53	Lnrs	Yes (Chorus response)	
17	01:54	Mr. Chauke	So it's minus 8 plus twelve. Right, so we have minus sign which goes together with 8 as a number and this one we call it a what? (Mr. Chauke pointing at the positive sign on the expression)	
18	02:03	Lnrs	Operation (Chorus response)	
19	02:07	Mr. Chauke	Understand, it tells us that we must add 12 to?	
20	02:11	Lnrs	Minus 8	

Scenario number 11: Drawing and calibrating a number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
21	02:12	Mr. Chauke	So we going to use a number line jump method. Most of us are familiar with number line I know. Today, by using our number line we are going to put it this way (Mr. Chauke drawing a horizontal number line)	
22	02:33	Mr.	We are going to draw it in a horizontal way. Still	

		Chauke	remember that last time we used it in a vertical manner isn't it?	
23	02:38	Lnrs	Yes (Chorus response)	
24	02:39	Mr. Chauke	That vertical way helps you to order the numbers. To know which one is higher than the other one, that is if you put it in a vertical way. But today we are going to use it in a horizontal way but it didn't change its meaning, it is still a what? It's still a number line. So at the centre what we going to have?	
25	02:56	Lnrs	Zero	
26		Mr. Chauke	This side? (Mr. Chauke pointing at the right hand side of zero)	
27	03:00	Lnrs	1, 2,3 ,4,5	
28	03:07	Mr. Chauke	So this side (Mr. Chauke pointing at the left hand side of zero)	
29	03:06	Lnrs	$-1, -2, -3, -4, -5$	

Scenario number 12: Solving: $-8 + 12$ using number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
30	03:16	Mr. Chauke	Right let's try to accommodate these numbers (Mr. Chauke pointing at -8 and 12 on the expression) we have minus 5	
31	03:20	Lnrs	$-6. -7, -8$ (Ls counting continuing from -5)	
32	03:25	Mr. Chauke	Then we are going to have 12 on the side isn't it? (Mr. Chauke pointing at the right hand side of zero) So we are going to have what? 6,7,8,9,10,11 and	
33	03:40	Lnrs	12	

34	03:41	Mr. Chauke	Right, we want to add those two integers right we have minus 8, which means we have to start at what?	
35	03:47	Lnrs	Minus 8 (Chorus response)	
36	03:49	Mr. Chauke	Start at minus 8 and then we add how many units?	
37	03:49	Lnrs	12	
38	03:50	Mr. Chauke	12 so if it positive (Mr. Chauke pointing at + on the expression.) This is an operation, it tells you what to do isn't it?	
39	03:55	Lnrs	Yes (Chorus response)	
40	03:55	Mr. Chauke	It tells you to add, if you still remember if you are adding do we go to the left or right? (Pointing on a number line)	
41	04:02	Lnrs	We go to the right (Chorus response)	
42	04:03	Mr. Chauke	We go to the right (Revoicing) Right, which means from -8 we are going to the right how many steps?	
43	04:11	Lnrs	12 (Chorus response)	
44	04:12	Mr. Chauke	12 lets count together (Using a number line)	
45	04:13	Lnrs & Mr. Chauke	1,2,3,4,5,6,7,8,9,10,11,12 (Ls and Mr. Manana counting together)	
46	04:27	Mr. Chauke	So the answer is what?	
47	04:29	Lnrs	4	
48	04:30	Mr. Chauke	Negative	
49		Lnrs	4 Positive	
50	04:33	Mr. Chauke	Sorry, Positive 4.Right, what is another expression can we have from the diagram? Let us have someone else eeh. Yes	

Scenario number 13: Solving: $-8 + 5$ using number line.

Turns	Time intervals	Transcript	Codes of professional noticing and learner hearing

51	04:51	Lnr	$-8+5$	
52	04:52	Mr. Chauke	Aah $-8+5$ it's still the same can we have something different, its fine ok$-8+5$. Who can come and show us how to work it out .Yes	
53	05:15	Lnr	We start at minus 8, 1,2,3,4,5 (L counts 5 units to the right from negative 8)	
54	05:28	Mr. Chauke	So you say we start from minus 8	**ALS**
55	05:30	Lnr	Yes	
56	05:33	Mr. Chauke	And then why are you saying we add 5	**ILU**
57	05:34	Lnr	Aaa because the operator is positive	
58	05:38	Mr. Chauke	Right	
59	05:39	Lnr	That tells us to add	
60	05:40	Mr. Chauke	To add, that's good. Did you get what he said?	
61	05:41	Lnrs	Yes (Chorus response)	
62	05:42	Mr. Chauke	We are adding 5 because the operator tells us what to do. Then the answer is what?	**DRLU**
63	05:52	Lnrs	3, Negative 3	
64	05:53	Mr. Chauke	Negative 3. Ok can we have another one. another, Yes	
65	06:00	Lnr	$12-5$	
66	06:02	Mr. Chauke	$12-5$, Twelve minus 5?	
67	06:06	Lns	No (Chorus response)	

Scenario number 14: Solving: $12+5$ using number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
68	06:10	Mr. Chauke & Lnr	$12+5$ Aah this one is simple.	
69	06:17	Mr. Chauke	$12+5$ Yes can you explain how do we get the answer	

70	06:20	Lnr	To get the answer at 12 we must move 5 places forward (Mr. Chauke ask the L to demonstrate)	**ALS**
71	06:57	Mr. Chauke	Tell the class	
72	07:03	Lnr	From 12 you count 5 units to the right	
73	07:10	Lnrs	17 wow (Ls clapping hands)	
74	07:31	Mr. Chauke	Did you hear what he said?	
75	07:33	Lnrs	yes (Chorus response)	
76	07:34	Mr. Chauke	He said from 12 we add 5 isn't it? Starting at 12, let's say we have 12 here: 13,14,15,16,17 . Then positive operator tells us to add 5.So from 12 we going to add what?	**ILU** **DRLU**
77	08:10	Lnrs	5 (Chorus response)	
78	08:12	Mr. Chauke	So it's (Counting from 12) 1,2,3,4,5 then we have a what?	
79	08:19	Lnrs	17	
80	08:20	Mr. Chauke	Positive 17.Alright now, aaah I want you to do the following. Someone will come up front and show us how to solve the problem, but for now you do it with your friend. (Mr. Chauke writes problems on the chalkboard) do this one with your friend there. I will give 2 minutes	
81	08:75	Lnrs	(Ls start to work in pairs)	
82	12:23	Mr. Chauke	Right, before I ask one of you to come and demonstrate this, I wanted you to understand this before I teach you, Right?	
83	12:26	Lnrs	Yes (Chorus response)	

Scenario number 15: Addition two directed negative numbers using number line: $-3+(-4)$.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
84	12:28	Mr. Chauke	I know some of you had a problem of having two signs there (Mr. Chauke pointing at the middle two signs on the expression: $-3+(-4)$). Right, number 2 says $-3+(-4)$, so if you are	

			trying to get an answer for this one, this is how you do it, this is minus 3, and it's a number with a sign isn't it? And then this one is a what?(Mr. Chauke pointing at plus sign on the expression: $-3+(-4)$	
85	12:53	Lnrs	Operator	
86	12:55	Mr. Chauke	It's an operator neh and this is a sign for this negative 4(Mr. Chauke pointing at negative sign against 4). So if you come across this one, what you do, you say minus 3, let's go to minus 3 .This is our minus 3(Mr. Chauke pointing at minus 3 on the number line). Right, this is our minus 3, we start here isn't it? Then the operator is asking us to do what?	
87	13:16	Lnrs	Add (Chorus response).	
88	13:17	Mr. Chauke	Add, so if you are adding which direction do you take?	ILU
89	13:20	Lnrs	Right (Chorus response)	
90	13:21	Mr. Chauke	To the right neh so if you are taking the right direction, you found that there is negative here (Mr. Chauke pointing at the negative sign which is against 4) you see the negative?	
91	13:28	Lnrs	Yes (Chorus response)	
92	13:29	Mr. Chauke	Right this negative means that you must reverse four steps going to the left. Do you get it? For those people who are writing you are going to miss what I am saying right. I am saying that you go to minus 3 here (Mr. Chauke pointing at minus 3 on the number line) right and then the operator is telling you what to do here, it says add what?	
93	13:50	Lnrs	4	
94	13:51	Mr. Chauke	Negative 4, which means you are going which direction	OH
95	13:53	Lnrs	Left. Right, right	
96	13:55	Mr. Chauke	Right correct, you are going to the right, but this sign (Mr. Chauke pointing at negative sign against 4) will tell you to reverse. So let's go to minus 3 on the number line, go to the right (positive operator) but we have a negative what?	CH **ILU** **DRLU**

97	14:08	Lnrs	4	
98	14:09	Mr. Chauke	4, which means we will reverse going this side (Mr. Chauke pointing to the left) by how many steps?	
99	14:10	Lnrs	4	
100	14:13	Mr. Chauke	Which means the answer is going to be what?	
101	14:15	Lnr	Minus 7 (L shouts)	
101	14:15	Mr. Chauke	1,2,3,4 Which means the answer will be what?	
102	14:20	Lnrs	Minus 7	
103	14:21	Mr. Chauke	Minus 7 right. I think you now know and understand on how to solve such kind of expressions. Can you do this one and that one (Mr. Chauke pointing to expressions on the chalkboard) number 3	
104	15:12	Mr. Chauke	I think you are done neh?	
105	15:13	Lnrs	Yes (Chorus response)	
106	15:14	Mr. Chauke	Who can come and do number 3.Yes, come and do number 3 fast	
107		Lnrs	(L work out quietly on the chalkboard)	
108	15:52	Mr. Chauke	Aaa I would like you to explain the sign ,how do you come up with that direction	
109	15:53	Lnr	Ok ,it it's a negative you go to the left hand side so let me go to the left hand side aaa how many times	ALS
110	16:18	Mr. Chauke	That's correct, you get it?	
111		Lnrs	Yes (Chorus response)	
112		Mr. Chauke	The first one is not difficult, what is the answer?	
113	16:26	Lnrs	Negative 4	

Scenario number 16: Deriving expression of operation with directed using algorithms

Turns	Time intervals	Transcript	Codes of professional noticing and learner

				hearing
114	16:28	Mr. Chauke	Negative 4 that's correction let's look at the other handout you have there, let's talk about subtraction now using the worksheet, let's have the worksheet. Right, our worksheet looks like this we have what on top	
115	16:50	Lnrs	$5 , 4 , -3$	
116	16:52	Mr. Chauke	And then we see that there are arrows going both direction	
117	17:00	Lnrs	Yes (Chorus response and chanting)	
118	17:05	Mr. Chauke	Anyone who wants to say something? It means what?	
119	17:06	L	The first expression on this side means $-5 - (3)$	
120	17:13	Mr. Chauke	The first one says $-5 - (3)$ right	
121	17:19	Lnr	$-5 - (-3)$	
122	17:22	Mr. Chauke	You are say $-5 - (-3)$ that's the first one, we can have something like this.	
123	17:30	Mr. Chauke	Right, we are on subtraction now isn't it? The other one can be?	
124	17:35	Lnr	$-5 - 4$	
125	17:37		Minus 5, no let's try to use this direction (Mr. Chauke encouraging learning to create expression using opposite direction). We have been using this and this isn't it?	
126	17:45	Lnrs	Yes (Chorus response)	
127	17:46	Mr. Chauke	So it can be what?	
128		Lnrs	$-5 + (-3)$	
129	17:53	Mr. Chauke	You saying $-5 + (-3)$ now let's look at operator, what is your operator?	
130	17:56	Lnrs	Plus	
131	17:59	Mr. Chauke	But we said now we are doing subtraction isn't it?	
132	18:00	Lnrs	Yes (Chorus response)	
133	18:01	Mr. Chauke	Yah, what can be the best expression we can draw from the diagram?	
134	18:03	Lnrs	$-3 - (-5)$	

Scenario number 17: Subtracting two directed negative numbers using number line: $-5-(-3)$.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
135	18:07	Mr. Chauke	Right $-3-(-5)$, let's do this way, let's find the answer for the first one then the answer for the second one. The first one is -5 the operator tells to what? $(-5-(-3))$	
136	18:30	Lnrs	Subtract	
137	18:33	Mr. Chauke	Subtract what?	
138	18:35	Lnrs	Negative 3	
139	18:36	Mr. Chauke	Right if we are subtracting there, we started at negative 5	
140	18:40	Lnrs	Yes (Chorus response)	
141	18:41	Mr. Chauke	This is our negative five, lets us the bottom part for writing, negative 5 and then the operator telling us to?	
142	18:45	Lnrs	Subtract (Chorus response)	
143	18:49	Mr. Chauke	Which direction are we going to take?	
144	18:50	Lnrs	Left, left (Chorus response)	
145	18:51	Mr. Chauke	Left, then there is a minus against 3.	
146	18:53	Lnrs	Reverse direction	
147	18:56	Mr. Chauke	Reverse direction. Isn't it?	
148	18:55	Lnrs	Yes (Chorus response)	
149	18:58	Mr. Chauke	So what are we going to have there, if we reverse direction, which direction are we going to take?	
150	19:02	Lnrs	Right(Chorus response)	
151	19;04	Mr. Chauke	Right, so we have to move how many steps	
152	19:07	Lnrs	3	
153	19:08	Mr. Chauke	(Counting) $1,2, and\ then$ 3 right is it correct?	

154	19:15	Lnrs	Yes (Chorus response)	
155	19:20	Mr. Chauke	$-5 - (-3)$ what are we going to get	
156	19:24	Lnrs	Minus 2	

Scenario number 18: Subtracting two directed negative numbers using number line: $-3-(-5)$.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
157	19:27	Mr. Chauke	Right who can do this one, who can explain this one? Yes come and show us (The class is on number 2: $-3-(-5)$)	
158	19:43	Ln	We start from minus 3 (Mr. Chauke says right in agreeing with the learner) the operator tells us to subtract but there is the other minus so we reverse direction (counting) 1,2,3,4,5 the answer is 2	**ALS** **ILU**
159	20:02	Mr. Chauke	Positive , yah that's good, Positive 2 right so do you see what it means on this diagram (Mr. Chauke explaining the work sheet) there 2 arrows that shows you that on each side you can two expressions right?	**DRLU** CH
160	20:15	Lnrs	Yes (Chorus response)	
161	20:16	Mr. Chauke	Like the first one has two expression, another one 2 and then third one two. So can you do the other two sides	
162	20:30	Lnrs	Yes (Chorus response)	

Scenario number 19: Subtracting two directed negative numbers using number line: $\mathbf{-5-4}$ and $\mathbf{4-(-5)}$

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
163	20:31	Mr. Manana	If you have any question or what just call us will come and help you. There two answers for each side. (Mr. Chauke and Mr. Manana were assisting learners on how to have addition and subtraction expression using the worksheets and also how to solve)	
164	31:58	Mr. Manana	Can you round up Mr. Chauke	
165	32:12	Mr. Chauke	Let's finalise this thing right	
166	32:18	Lnrs	Yes (Chorus response)	
167	32:20	Mr. Chauke	Aah can we have any volunteers who can come and do those two sets neh.	
168	32:25	Lnrs	Yes (Chorus response)	
169	32:26	Mr. Chauke	The two sides because the third one we have done it ,the first one .Yes, come and explain .Before you explain can you write the expression first	
170	33:40	Lnrs	(Ls workout on the body quietly)	
171	37:01	Mr. Chauke	Right let's look here this is minus 5 minus 4 ($-5-4$). So now from minus five and the operator what sign do we have here? (Mr. Chauke pointing just before 4)	
172	37:16	Lnrs	4	
173	37:17	Mr. Chauke	I mean in front of 4	
174	37:19	Lnrs	Positive 4	
175	37:20	Mr. Chauke	There is a positive 4, right let's say if I can write it as:$-5-(+4)$ it's still the same isn't it?	CH
176	37:27	Lnrs	Yes (Chorus response)	
177	37:28	Mr. Chauke	Because this number has a positive sign even though we can't see it physically (Mr. Chauke pointing at 4 on the expression) but this is our operator since we are subtracting two directed	

			numbers Right.	
178	37:36	Lnrs	Yes (Chorus response)	
179	73:37	Mr. Chauke	So, now from minus 5, we start from minus five. There is minus 5 then our operator tells what?	
180	37:43	Lnrs	Subtract, subtract	
181	37:44	Mr. Chauke	Go which direction	
182	37:46	Lnr	Left	
183	37:48	Mr. Chauke	Go left and there is a positive there. Do we still remember when we had a negative we should do what?	
184	37:54	Lnrs	We reverse direction	
185	37:56	Mr. Chauke	We reverse direction right?	
186	37:57	Lnrs	Yes (Chorus response)	
187	37:58	Mr. Chauke	So if there is a positive we don't reverse direction you still continue right?	
189	38:00	Lnrs	Yes (Chorus response)	
190	38:01	Mr. Chauke	So we go how many steps	
200	38:02	Lnrs	4	
201	38:04	Mr. Chauke	(Counting) 1,2,3,4 then our answer is what?	
202	38:08	Lnrs	Minus 9	
203	38:09	Mr. Chauke	Negative 9. What about this one: $4-(-5)$. This is one is simple, this one is complete simple right?	
204	38:14	Lnrs	Yes (Chorus response)	
205	38:14	Mr. Chauke	Its positive 4 even if there is no sign in front of it, its positive 4 minus for the operator minus	
206	38:23	Lnrs	5	
207	38:24	Mr. Chauke	5 right, from there we start at 4 then	
209	38:26	Lnrs	Left	
210	38:27	Mr. Chauke	Left	
211	38:28	Lrns	Reverse direction	
212	38:30	Mr. Chauke	Then we start at positive 4, this is our 4 then what, negative operator tells us to go left	

			direction on the number line. Then negative sign on 5 is meant for? $(4-(-5))$.	
213	38:43	Lnrs	Reverse direction	
214	38:46	Mr. Chauke	So what are you going to get after moving 5 steps to the right: 1,2,3,4,5 steps then stops at 9 right?	
215	38:53	Lnrs	Yes (chorus respond)	
216	38:58	Mr. Chauke	Right what about this one, let's look at this one is the same. Is it correct this one? Its fine?	
217	39:05	Lnrs	Yes (chorus respond)	
218	39:07	Mr. Chauke	Anyone with a problem or who didn't understand what we have done?	
219	39:10	Lnrs	No (chorus respond)	
220	39:16	Mr. Chauke	Its fine (Mr. Chauke give them home work from the work sheets)	
			The End	

Transcript: Control class – Teacher Mr. Ngele at Lundi Secondary School

Transcribed control class observation lesson below has been delivered by a secondary qualified mathematics teacher and it was recorded during class interventions. This transcribed lesson was a result of the research study that focused on teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. This focus included attending to the strategies of the learners, analyzing the understanding of the learners, and making decisions about how to respond based on the learner's understanding (Jacobs et. al., 2010). Here in this control class, it was assumed that the teacher was not aware of the existence of professional noticing theory but he applied the theory of teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8 and also the teacher was not aware of the importance of learning and teaching of mathematics through the use of powerful knowledge. The researcher noticed that the teacher who attended the workshop on professional noticing might have shared the acquired knowledge with his colleagues.

Lesson introduction that deals the existing knowledge and previous experience of learners on addition of directed numbers

Scenario number 20: Lesson Introduction (Adding directed numbers using a number line)

Turns	Time-intervals	Transcript		Codes of professional noticing and learner hearing
1	00:00	Mr. Ngele	A big question is to see if you really understood what we did yesterday. Right, now let's try this question. Can you answer this: $(-11) + (-5)$? Negative eleven plus negative five. How do we answer that? Try it and then if you are confident enough you can come here in front then show us how you got the answer. (A Learner was given time to work out the given problem on the chalkboard.)	
2	02:11	Ashton	$(-11) + (-5) = 6$ (learner writes incorrect answer on the board)	
3	03:20	Mr. Ngele	Alright, now do you agree with that?	
4	03:22	Learners	No. (chorus response)	
5	03:30	Randy	$(-11) + (-5) = -6$ (another learner suggest a different solution on the chalkboard)	
6	06:15	Mr. Ngele	Do you want to explain to us how you got the answer? Anyone to explain this? Anyone who can explain this? (Teacher pointing at another learner). Yes, you want to explain it?	ALS
7	06:32	Siphokazi	Yes, sir	
8	06:35	Mr. Ngele	Try.	
9	06:50	Siphokazi	Sir, I think he forgot the negative at 5. Positive times negative is equal to negative $(+ \times - = -)$then the answer is negative 16(-16). That is $(-11) + (-5) = -11 - 5 = -16)$	ALS. ILU.
10	07:05	Mr. Ngele	Do we agree with this answer?	DRLU
11	07:07	Lnrs	Yes, sir. (chorus response)	

12	07:10	Mr. Ngele	Ok, and any other explanations?	
13	07:15	Londie	My explanation is that negative 11 plus negative 5 is negative 16. She said that a positive multiplied by a negative which is equals to negative. Then she puts a negative to be $-11 - 5 = -16$. (Mr. Ngele accept the method used of solving the problem).	ALS. ILU.

Scenario number 21: Adding directed numbers using a number line

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
14	07:37	Mr. Ngele	Then solved right! Aah do we need a number line to show how we got the answer here.	**DRLU.** **BH: golden rules.**
15	07:39	Ls	Yes (chorus response)	
16	07:42	Mr. Ngele	Aah, Who wants to come and draw a number line? Yes come forward I need different people. Can erase please (teacher pointing at the chalkboard.)	**ALS**.
17	08:05	Lnr	(learner drew a number line ranging from $-20\ to + 5$while others were watching)	**ILU.**
18	10:07	Mr. Ngele	Ok, let me show you a simplest way to avoid very a long, long number line. So let me show you the simplest way of doing this problem. (Teacher draws a number line).Right now this, you know the rule BODMAS, right?	**DRLU.**
19	10:39	Lnr	Yes (chorus response)	
20	10:41	Mr. Ngele	The rule BODMAS, before you can do anything, before you can add, before you can multiply, before you can divide you need to solve the bracket first right (learners agrees with teacher). This is what is meant by B on BODMAS, so you need to solve the Brackets first before you can add or multiply or divide or subtract, right? So we need to solve this bracket first, right? Is the first bracket necessary does it have to be there?	**DRLU.** **ALS.**
21	11:08	Lnr	No (chorus response from learners)	

22	11:20	Mr. Ngele	So we can write this as negative 11, so what about the second one: $-11 + (-15)$. What do we do, we say positive multiplied by negative which we get $-11 - 15$. So negative 15 to solve this you go to your number line. Since you are starting at negative 11 to make things easy and simple for you to avoid drawing a very, very long number line, you can start your number line at negative 11 right. So we start at negative. Then we go 15 units to the left , so it will be 1 , 2 , 3,4 ,5 ,6,7,8,9,10,11,12,13, 14, 15 then you land on negative 26.Now meaning that negative 11 minus negative 15 equals negative 26. So to avoid drawing a very, very long number line that has negative 11 and positive numbers you can look at your first number on your question. So here we have negative 11 then you can start your number line at negative 11 and then you count 15 units to the left as it states. The number that you will lend on will be your answer for this question, Right?	**ILU: Use of golden rules** $(+ \times -= -)$ **DRLU**
23	13:30	Lnr	Yes (chorus response)	

Scenario number 22: Mr. Chauke **using a conceptual way of adding negative 11 and negative 15:** $\mathbf{-11 + (-15)}$

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
24	13:30	Mr. Chauke	Hi class how are you? I just want to add from what our teacher has been saying. There are many ways of calculating these problems like the one the teacher has shown us. But the other one those pupil who don't understand, you see the number that says minus 11 plus minus 15 its matter of plus being an operator that tells you what to do with negative 11. So we have got negative 11, the operator (plus) is telling us to add another negative 15. So we have 11 negatives, you draw your 11 negative signs $(- - - - - - - - - - -)$ and add another 15 negatives	

			signs $(- - - - - - - - - - - - - - - -)$. So you draw your 11 negatives and then you are going to add another 15 negatives. So if you have those 11 negatives and 15 negatives what are you going to have altogether?	
25	14:45	Lnr	Negative 26 (chorus response)	
26	14:47	Mr. Chauke	So that is how the first girl got the answer ,she went to negative 11 then added another 15 negatives and then they added to negative 26. So (plus) is an operator it tells you that you must add 11 negatives plus 15 negatives then you get 26 negatives. Hence this one counts as a short way of getting the answer.	

Scenario number 23: Distinguishing subtraction expressions.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
27	15:20	Mr. Ngele	Yesterday, we were supposed to do addition of two integers only but we also ended up introducing subtraction integers. So today we want to focus on subtraction of two integers only. (teacher writes problems on subtraction on the board)	
28	16:29	Mr. Ngele	Alright, so this will take few minutes. So let's say we have this example: a) $+8 - -5$, *and we have another one* $b)$ $8 - 5$ and we have another one c) $-3 - 9$.Now let's look at the three problems. Can we identify the difference between among the three? Like, what is the different? Compare the first one and the second one (teacher ask a learner to compare)	ALS
29	17:27	Akeem	Sir , the first one is 8 subtract negative 5 ($8 - (-5)$) and the second one is positive 8 subtract 5	ILU

30	17:43	Mr. Ngele	Ok, do read out by spotting the difference I am not saying that you should read out what is written here. Can you try and identify the difference among the three?	DRLU
31	18:00	Jayne	Number (a) if you subtract a whole number by a negative number, you are subtracting two positive numbers.	ILU
32	18:15	Mr. Ngele	Which one? Where are we subtracting number (a) ,on (a)	CH
33	18:17	Jayne	On (b) as well	
34	18:18	Mr. Ngele	(b), we are subtracting positive numbers, ah?	CH
35	18.26	Jayne	At (a) we are subtracting a positive number with a negative number (teacher also agreeing with learner)	
36	18:30	Mr. Ngele	So, what you saying is that on (b) we are subtracting two positive numbers and then on (a) we are subtracting positive and a negative number. What about (c) now, do we agree with what he is saying here?	ALS ILU DRLU
37	18:48	Lnrs	Yes. (chorus response)	
38	18:50	Mr. Ngele	What about C, what's happening here on C (teacher pointing at the learner)	
39	18:57	Dimpo	Positive number is subtracted from a negative number (Teacher re-voices the statement)	
40	19:01	Mr. Ngele	Positive number is being subtracted from a negative number, alright let's forget about this and look at 9 minus 2. I am interested in your knowledge but at the same time your language. Now how do you comment? We see that it is 9 minus 2. Which is being subtracted from what?	ALS ILU DRLU
41	19:30	Lnrs	2 is being subtracted from 9 (chorus response and the teacher agrees with learner and re-voices what they have just said)	

42	19:35	Mr. Ngele	2 is being subtracted from 9. Right, let's go back here 2 is being subtracted from 9. So what about here (pointing at A). What is the difference again?	ILU
43	19:46	Jayne	Ah, on A, positive 8 is being subtracted from negative 5 and ……	
44	20:35	Mr. Ngele	Can you please help him out?	CH ILU
45	20:37	Nathi	Negative 5 is being subtracted from positive 8 and b) 5 is subtracted from 8 and c) 9 is being subtracted from negative 3	ILU
46	20:53	Mr. Ngele	So, now let's look at the first one right, the first one, we have a positive number and a negative number. So, we are subtracting a negative number from a positive number. The second one, we have a positive number and a negative?	ALS ILU DRLU
47	21:16	Lnrs	No (chorus response), positive	CH
48	21:17	Mr. Ngele	And a positive number right, so we are subtracting a positive number from a positive number, right. Now, look at this one (pointing at b), we have a negative number and a positive number (learners' seconds). So, we are subtracting a positive number from a negative number.	ALS ILU DRLU

Scenario number 24: Subtraction of directed numbers using a number line.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
49	21:50	Mr. Ngele	(Teacher introduces brackets on the last numbers of each previous question): a) $+8-(-5)$, $b)$ $8-(5)$ and c)$-3-(9)$. We now have brackets in there. So, let's look at the first one, the one which inside the brackets is negative.	ALS ILU DRLU

			With this one (Pointing at number b) is positive, and this one this number is also positive (with learner re-voicing).	
50	22:13	Mr. Ngele	Now, 8 here is positive (pointing at number a), so in the first question we subtracting a negative number from a positive number, right. And on the second question we are subtracting a positive number from a positive number. On the third question, we are subtracting a positive number from a negative number (learners re-voicing what the teacher is saying), negative number. I know that from grade 7, grade 6 you been doing subtraction, now could you please with an aid of a number line, (school bell goes), can you please subtract negative 5 from 8, 5 from 8 as well 9 from negative 3 right. Right, let's do the first one. Can I have someone to demonstrate? (A learner comes upfront to demonstrate on the chalkboard). Leaner's response : $+8-(-5)=-3$ (Zachariah's answer)	ALS ILU DRLU
51	23:43	Mr. Ngele	(Learners clapping hands). Right ok, ok is the answer negative three.	
52	23:52	Lnrs	No. (chorus response)	
53	24:00	Mr. Ngele	Ok, you come and help.	
54	24:05	Philip	Positive 3. (Others learners say no)	
55	24:06	Mr. Ngele	Positive 3. (other learner say no while others say yes). Positive 3	CH
56	20:12	Lnrs	No. (chorus response)	
57	20:13	Mr. Ngele	Ok, can you come and show us using a number and show us how you are getting positive 3?	ALS
58	24:30	Philip	(Learner demonstrating)	
59	25:32	Mr. Ngele	Alright, ok , can you please explain to us how you get the answer	ALS

60	25:40	Philip	(using a number line) I move 8 times to the right then move 5 times to the left: $+8-(-5)=+3$ (Teacher re-voicing what the learner is saying)	
61	26:01	Mr. Ngele	So, you move 8 times to the left and then 5 times to the left. Ok, do you agree with that (teacher asking the whole class and pointing at one learner to help others)?	ALS ILU
62	27:05	Lnrs	No. (Chorus response disagreeing with the answer of positive 3) (selected learner was not able to explain but got the incorrect answer) : $+8-(-5)=-13$	
63	27:20	Mr. Ngele	Apparently, you can't explain. Do you agree with negative 13? (Learners made a chorus of response : No) You want to come and correct him (another learner, Kevin, wrote 13 instead of -13) : $+8-(-5)=+8+5=13$	ALS ILU DRLU
64	27:58	Mr. Ngele	13, positive 13.	
65	27:59	Lnrs	Yes. (chorus response)	
66	28:00	Mr. Ngele	How did you get the answer?	
67	28:11	Kevin	I said sir, negative plus negative, negative plus negative is equal to positive. Then I said 8 plus 5 equals positive 13. (clapping hands of learner)	
68	28:42	Mr. Ngele	Since negative multiplied by negative.	
69	28:43	Lnrs	Is positive.	
70	28:45	Mr. Ngele	Right, here is my suggestion before you can calculate a problem deal with a bracket first. So, you have 8 minus negative 5, which is equivalent to $8\ -\times-=+5$=$8+5$. Then you can get the answer of positive 13. Ok, who can do the second	ALS ILU DRLU BH

			one ($8 - (5)$), it should be easy.	
71	30:23	Mr. Ngele	What is your answer?	
72	30:24	Veli	3.	
73	30:25	Mr. Ngele	3. (teacher re-voicing).Do we agree with answer?	CH
74	30:27	Lnrs	Yes. (Chorus response).	
75	31:40	Mr. Ngele	The last one: $-3 - 9 = -12$ .Any questions	
76	31:43	Lnrs	No. (chorus response as the learners were preparing to leave the class)	
			End of the lesson	

Transcript: Control class – Teacher Mrs. Zitha in Shongamiti Secondary School

Below is a transcribed control class observation lesson delivered by a secondary qualified mathematics teacher recorded during class interventions. This transcribed lesson was a result of the research study that focused on teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8. This focus included attending to the strategies of the learners, analyzing the understanding of the learners, and making decisions about how to respond based on the learner's understanding (Jacobs et. al., 2010). Here the teacher is not aware or he/she is aware of teacher's professional noticing of learners' mathematical thinking in the teaching of directed numbers in Grade 8 and also the teacher is not aware or he/she aware of the importance of learning and teaching of mathematics through the use of powerful knowledge.

Scenario number 25: Introduction of the lesson on integers.

Turns	Time intervals	Transcript: Participants		Codes of professional noticing and learner hearing
1	00:00	Mrs. Zitha	So we are going to go back to integers. Hurry up sir (teacher ask incoming learners to sit down)	
2	00:35	Lnrs	Integers (learners shouts)	

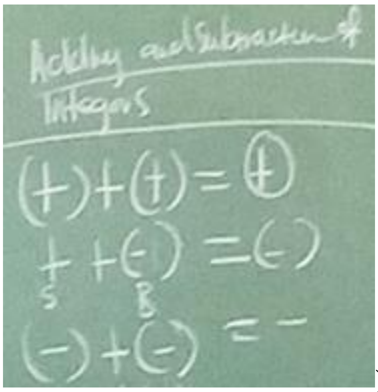
3	01:35	Mrs. Zitha	(teacher asks leaners to get seated and open their maths exercise book for them to take down notes) At the back of maths exercise books write the date and the word integers.	
4	02:00	Mrs. Zitha	So who still remember what integers are? Who still remember what integers are? (Teacher asks the whole class). Learner A hurry up, who still remember what integers are? Let's recall what are integers?	
5	02:30	Lnr	Ma'am, are numbers less than or greater than zero (Teacher repeating after the learner)	
6	02:40	Mrs. Zitha	Number less than or greater than zero. Yes learner B (teacher pointing at another learner to try and define integers)	ALS ILU CH
7	02:45	Lnr	A set of positive numbers and negative numbers (teacher re-voice the given definition)	
8	03:00	Mrs. Zitha	A set of positive and numbers. Ok, can someone give me an example? Can someone give an example? Yes learner C	ALS ILU DRLU CH
9	03:12	Lnr	10 and −10	
10	03:18	Mrs. Zitha	Yes 10 and−10. So it's positive and a negative .Someone else give me another number. Yes, Learner	ALS ILU DRLU
11	03:28	Lnr	12 and −13 (teacher re-voices)	
12	03:30	Mrs. Zitha	12 and−13. Someone else	
13	03:40	Lnr	Positive 100 and negative 100 (teacher re-voices)	
14	03:45	Mrs. Zitha	Positive 100 and negative 100. So these our set of positive numbers and negative numbers including zero ok. So if we are given temperature: 80 degrees. Let's say 21degrees, zero degrees and negative 1degrees. So one is the coldest day or which one is the coldest temperature?	
15	04:35	Lnr	Negative 1	
16	04:37	Mrs. Zitha	Negative 1 is the coldest and then which one is the highest?	

17	04:55	Lnr	21℃	

Scenario number 26: Comparing directed numbers.

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
18	05:00	Mrs. Zitha	21℃ is the hottest one .Ok if you are given different sets of temperatures let's say you have zero , 15 , −5℃, 71℃, −21℃, 15℃, 29℃, 50℃. Ok, I want you to arrange these figures, temperatures in ascending order so that is your (a), (b) descending order. Use the same set of numbers to arrange according to question (a) and (b). What is the meaning of the word ascending?	
19	06:10	Lnr	Going up	
20	06:11	Mrs. Zitha	From the smallest to the	ALS ILU
21	06:13	Lnr	Biggest	
22	06:15	Mrs. Zitha	What is the meaning of the word descending? Biggest to the	ILU
23	06:17	Lnr	Smallest	
24	06:19	Mrs. Zitha	So, I want you to arrange those set of numbers firstly start with ascending them and secondly starts with descending (teacher give learners a chance to write) So I will give you 2 minutes 30 seconds to write. You only have 2 minutes exactly. Ok, you are left with a minute. Ok 30 seconds	DRLU
25	10:20	Mrs. Zitha	Ok, let's starts which one will be first number	
26		Lnr	−71℃, −21℃, −5℃, 0℃, 7℃ ,15℃, 29℃, 50℃ (chorus). Mrs. Zitha re-voice the answers and writes them down	
27		Mrs. Zitha	−71℃, −21℃, −5℃, 0℃, 7℃ ,15℃, 29℃, 50℃ That was our ascending order. So descending order	
28		Lnr	50℃, 29℃, 15℃, 7℃, 0℃, −5℃, −21℃, −71℃ finish (chorus responses)	

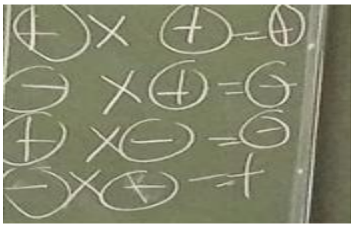
Scenario number 27: Adding and subtraction of directed numbers (integers)

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
29	11:18	Mrs. Zitha	(Mrs. Zitha writes on the chalkboard: ) So now we are going to look at adding and subtracting of integers, adding and subtracting of integers (teacher writes on the board). Ok if you add positive integers your answer will be	
30	11:52	Lnr	Positive (chorus)	
31	12:00	Mrs. Zitha	Your answer will be positive. But then, if you are adding smaller positive number plus adding a negative number	
32		Lnr	Negative	
33			If you are adding a smaller positive number and you negative number is bigger and you are adding a positive and a negative what will be your answer	
34	12:20	Lnr	Positive	
35	12:26	Lnr	Negative	
36	12:28	Mrs. Zitha	Why negative?	UH
37		Mrs. Zitha	Yes these two signs will make a negative and our answer always the sign of the biggest number. If you have negative plus negative what do you think our answer will	Not ALS. Not ILU. Not DRLU.

			be	
38	12:57	Lnr	Positive (chorus response)	
39		Lnr	Negative (chorus response from other group of learners)	
40			Positive	
41			Negative	
42	13:00	Mrs. Zitha	Ok, because we are arguing about it. Ok let's say we are given negative 2 plus negative 3: $-2+(-3)$. I want you to give the answer	NH
43	13:18	Lnr	1	
44	13:20	Mrs. Zitha	$-2+(-3)$ I want you to calculate and give me the answer there. Let's give others a chance	NH
45	13:30	Lnr	-5	
46	13:32	Mrs. Zitha	Your answer will be :  (chorus response from learners) Your answer is negative but you are adding negatives but your answer is negative 5	BH
47	13:40	Lnr	Oooh, negative 5 (chorus response)	
48	13:47	Mrs. Zitha	Ok, so if you have minus and negative what do you think your answer will be	
49	13:50	Lnr	Positive	
50		Mrs. Zitha	(teacher not commenting on the learners response continues to ask questions) What do you think your answer will be	NH
51	14:02	Lnrs	Negative	
52	14:01	Mrs. Zitha	Ok, let's say we have $-2-(-3)$ you answer will be	NH
53	14:12	Lnr	Positive 1, negative 1, positive 5, negative 1, minus 1 (learners gives different answers showing confusion). It's one. positive 1	
54	14:35	Mrs. Zitha	No we have to work it out and tell me the true reflection. It's what , so basically $-2+3$ (teacher pointing at consecutive negative on the expression) because these two the match together to create a positive ,so its $-2+3$	Not ALS. Not ILU. Not DRLU
55	14:47	Lnr	-5	
56	14:48	Mrs. Zitha	No $-2+3$ we give you	Not ALS. Not ILU. Not DRLU

57	14:52	Lnr	Positive 1	
58	14:53	Mrs. Zitha	Yes it will give a positive. So you answer is a positive 1. Ok can I rub this one off	Not ALS. Not ILU. Not DRLU

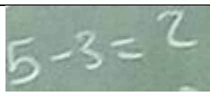
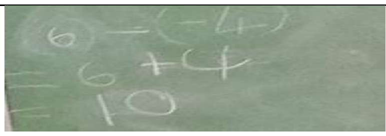
Scenario number 28: Teaching golden rules

Turns	Time intervals	Transcript		Codes of professional noticing and learner hearing
59	15:16	Mrs. Zitha	(The teacher writes the four golden rules that she taught earlier and ask she questions)  So what if you have a positive and a negative	
60	15:20	Lnr	Negatively	
61	15:22	Mrs. Zitha	What if you have positive minus a negative .Sorry a positive minus a positive. A positive minus a positive what will be your answer?	NH
62	15:32	Lnr	Positive (chorus response)	
63	15:34	Mrs. Zitha	Your answer will be what	
64	15:35	Lnr	Positive (chorus response)	
65	15:40	Mrs. Zitha	Ok let's say $5 - 3$ that gives us a what?	Not ALS. Not ILU. DRLU
66	15:43	Lnr	Positive 2 (chorus response)	

Scenario number 29: Addition and subtraction of directed numbers.

Turns	Time intervals	Transcript	Codes of professional noticing and learner hearing

67	15:45	Mrs. Zitha	Ok, let me give four questions that I want you to answer and then we discuss them. Ok, you have three minutes to answer $5 + (-3)$ (teacher goes on to write the other three questions on the chalk board silently)	
68	16:38	Mrs. Zitha	Ok, so you have four minutes, one minute on each to answer those 4 questions. To answer those question you have 4 minutes one minute per question so its $1. \quad 5 + (-3)$, $2. \quad 10 — (-15)$, $3. \quad -11 - (-9)$ $then$ $4. \quad 6 - (-4)$	
69	17:30	Tasha	"Ma'am" (The learner ask the teacher about question 3)	
70	17:34	Mrs. Zitha	Yes its negative eleven minus negative nine.	
71	17:40	Lnrs	(learners start to answer questions while charting and discussing)	
72	18:30	Mrs. Zitha	There you now have four minutes gone so far, let's stop writing.	
73	19:30	Mrs. Zitha	So its $1. 5 + (-3)$, $2. 10 — (-15)$, $3. -11 - (-9)$ $then$ $4. 6 - (-4)$	
74	20:35	Mrs. Zitha	Ok, hurry up.	
75	22:33	Mrs. Zitha	Ok, let's answer the following questions before we can go to the new session. What is five plus minus 3?	
76	22:40	James	2.	
77	22:44	Lihle	-2.	
78	22:46	Mrs. Zitha	 So basically , That is five minus three gives a what?	Not ALS. Not ILU. Not DRLU
79	22:53	Itai	2.	

80	23:00	Mrs. Zitha	(The teacher writes: ). Ok in number 2 what is the answer?	Not ALS. Not ILU. Not DRLU. BH;NH
81	23:05	Mercy	Negative five.	
82	23:05	Lucky	Positive 5.	
83	23:06	Samson	Negative 5.	
84	23:09	Sibo	Positive 5.	
85	23:10	Mrs. Zitha	So, basically, ten plus 15 will give you a positive 25. Ok these signs (teacher pointing to 2 negatives signs) here it will emerge to be a positive so basically negative eleven plus nine gives us a negative 2	Not ALS. Not ILU. Not DRLU NH
86	23:45	Mrs. Zitha	Ok six minus negative four.	
87	23:47	Jackie	 10	
88	23:53	Mrs. Zitha	So, I want you to complete the classwork at the back of your exercise books and complete the class work	Not ALS. Not ILU. Not DRLU
89	24:01	Lnrs	(learners copy the classwork into their exercise books)	
			End of the lesson.	