Comparison of accounting-based financial distress prediction models of companies listed on the JSE

A research report submitted by

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Contents

Abstract

The study compared the forecasting accuracy of binary state corporate failure prediction models (multiple discriminant analysis and logit), and multistate models (multinomial and mixed logit models), to assess which models were more reliable in predicting financial distress and corporate failure from a South African context. The study used a sample of 108 firms listed on the Johannesburg Stock Exchange (JSE) for the period between 2010 to 2019. The sample was sub-divided into a testing and a validation sample and the accuracy of the models was tested 5 years prior to failure, 3 years prior to failure and 1 year prior to failure. The empirical results indicate that the binary models performed relatively well up to 3 years prior to failure; but their performance dropped considerably beyond that. The multistate models produced better results overall and their performance did not materially drop as the lead time from failure increased. This study provides evidence that multistate corporate failure prediction models can be used to predict corporate failure.

Keywords: Financial distress prediction; multiple discriminant analysis; logit, multinomial logit; mixed logit; bankruptcy

Chapter 1 – Introduction

1.1 Introduction

Investors and lending institutions globally have become increasing more cautious since the fall of companies such as Arthur Andersen, Enron and WorldCom (Jones & Hensher, 2008; Muller, Steyn-Bruwer, & Hamman, 2009). The degree of cautiousness increased considerably after the global financial crisis with the fall of large financial institutions, including Lehman Brothers, Washington Mutual and Bear Stearns (Muller et al., 2009). It was the collapse of these firms that resulted in many organisations globally to concentrate on corporate governance and ethics with the aim to reduce the risk of financial distress (Muller et al., 2009). While corporate governance plays an essential role in minimising financial distress, it is reasonable to also have an early warning system that is capable of timely predicting such situations with high confidence (Caggiano, Calice, & Leonida, 2014; Mihalovic, 2016).

In March 2020, the World Health Organisation (WHO) officially declared the coronavirus (COVID-19) outbreak to be a global pandemic. The short-term impact will include a significant slow down of economic activity as many countries adopt strict quarantine policies; however, the long-term impact of the pandemic is likely to be increasingly significant as a consequence of mass unemployment and business failure (Zhang, Hu, & Ji, 2020). Therefore, corporate failure prediction models are likely to become more important as firms navigate the pandemic.

Similar to when a rock is thrown into a lake with the shockwaves reaching far beyond the point of initial impact; so to when a company becomes financially distressed or insolvent there are significant adverse impact for its investors, creditors, customers, employees and the wider economy and society (Jackson & Wood, 2013).

1.2 Statement of the problem

It is important for stakeholders to predict the probability of corporate failure, accurately and reliably, so that they can react before the event occurs. There are several techniques that have been developed to predict financial distress. Beaver (1966) used univariate analysis for selected ratios and detected that some of them had very good predictive power. Altman (1968) argued that the univariate approach employed a single ratio which could lead to misinterpretation and potentially confusion. Altman (1968) developed a multiple discriminate analysis (MDA) approach, called the z-score, with 5 ratios which also had high predictive accuracy. Ohlson (1980) raised questions about the MDA model, particularly regarding the restrictive statistical requirements imposed by the model and developed a binary logit model to predict financial distress. Johnsen and Melicher (1994) did not subscribe to the binary notion of failure / non-failure dichotomy; instead, argued that financial distress is a continuum ranging from being financially weak to bankrupt with the possibility of various degrees of financial weakness and developed a multistate multinomial logit model to predict corporate failure. Jones and Hensher (2004) on the other hand have used mixed logit models to predict corporate failure as they argue that these models offer substantial improvements compared to binary logit and multinomial logit models.

Whilst corporate failure prediction models have evolved from univariate financial ratios to multivariate models, and from discriminate models to logit models that greatly relax restrictive statistical assumptions (Johnsen & Melicher, 1994); there is lack of consensus on which technique is most accurate. Bellovary, Giacomino, and Akers (2007) found that new techniques are not necessarily more promising than earlier techniques. There is limited research that compares the earlier techniques to newer techniques in this field from a South African context.

1.3 Purpose and objective

The main purpose of this study was to assess which accounting-based financial distress prediction models are more accurate in predicting financial distress from a South African context. To achieve this, the study compared the performance of rudimentary modelling techniques, that is, MDA and binary logit, to more advanced modelling techniques, that is, multinomial and mixed logit models. The models were tested on a sample of selected firms listed on the Johannesburg Stock Exchange (JSE) for the period between 2010 to 2019. By comparing the various accounting-based corporate failure prediction models, it might be possible to identify firms that could be experiencing a decline and it may be possible for the firm to be restructured, thus avoiding failure.

1.4 Significance of the study

The results of this study are expected to contribute to existing knowledge of financial distress models, that will further enhance investors and lenders decision-making (Altman, 1968).

Financial distress prediction models are especially important for the banking sector. According to April 2020 data released by the South Africa Reserve Bank (SARB), there are 19 registered banks in South Africa with an aggregate of R4.4 trillion gross loans advanced (SARB, 2020). To put it in another way, the total loans advanced by the banking sector in South Africa are equivalent to 86% of the country’s Gross Domestic Product (GDP). Banks are therefore interested in minimising the level of non-performing loans and early warning systems play a critical role for banks to minimise credit risk. Similarly, asset managers and investors are interested in reliable tools that can assist them in their decision-making process and select appropriate companies to invest in. While financial distressed companies might be detrimental to investor returns, some asset managers and investors may be less risk averse and attracted to such companies that could provide opportunities for high returns (Altman, Iwanicz-Drozdowska, Laitinen, & Suvas, 2017).

The interest in financial distressed prediction models is not confined to lenders and investors; there are many other interested stakeholders, for example employees of companies, or governments in areas where the society is dependent on a few companies and credit rating agencies (Altman et al., 2017; Beaver, 1966).

1.5 Research question

The research question that this study aims to answer is as follows: Which financial distress models are more accurate in predicting financial distress for JSE listed companies?

1.6 Assumptions and delimitations of the study

The independent variables (that is, the financial ratios) were extracted from the company's financial statements; and it is assumed that these financial statements are accurate and represents the company's true financial position several years before financial distress.

The sample excluded companies that operate in the financial and property sector. The reason is because of their high volatility ratios as a result of heavily reliance on the economy (Bunyaminu & Issah, 2012). Furthermore, financial services sector is highly regulated and typically have higher levels of gearing and leverage ratios which could skew the dataset. This approach is consistent with previous studies (Abdullah, Halim, Ahmad, & Rus, 2008; Bunyaminu & Issah, 2012; Muller et al., 2009; Rowlings, 2016). Muller et al. (2009) also excluded mining companies from their sample, but provided no motivation for this decision. For the current study, the decision was made to retain mining companies in the sample since this industry is subject to various degrees of financial distress as shown in Table 2.

1.7 Definition of terms

This study adopted a multistate approach to financial distress (Johnsen and Melicher (1994); Jones and Hensher (2004)):

> State 0: failed firms. The legal definition of *failure* is adopted which is consistent with previous studies (Abdullah et al., 2008; Altman, 1968; Bunyaminu & Issah, 2012; Mihalovic, 2016). The South African Companies Act, 2008 no. 71 (Companies Act) defines a company financially distressed if it cannot meet its debt obligations when they are due and payable, or it will become insolvent. Therefore, a firm is considered

to have failed if it entered into Business Rescue or Liquidated under Chapter 6 of the Companies Act.

State 1: non-failed firms

State 2: financially weak. A firm is financially weak when it experience a decline (Pretorius, 2009), that is, when its financial performance deteriorates (i.e., net losses) over consecutive periods. For this study, consecutive periods means 3 years or more since prolonged losses is a sign of financial distress (DeAngelo & DeAngelo, 1990).

Jones and Hensher (2004) defined the different states of financial distress as follows: State 0 as “non-failed”; State 1 as “insolvent” and State 2 as “bankrupt”. Although the arrangement of the definition in this study differs slightly from Jones and Hensher (2004); it is applied consistently.

Bankruptcy, corporate failure and financial distress are used interchangeably to describe when a company has failed.

“t” represents time. “t=0, t=1… n” represents the financial years from financial distress. For example, t=5 means 5 years prior to going into business rescue.

1.8 Structure of the study

This study is organised in several chapters. Chapter 2 contains the literature reviewed with a focus on examining the definition of corporate failure, the different categories of corporate failure prediction models, the modelling techniques deployed in this study and finally a survey empirical studies focusing on comparing the accuracy of MDA and logit. Chapter 3 discusses the methodology adopted in this study. It also discusses the partitioning of the sample (estimation and validation), the plan for analysing the data and the specification of the models. Chapter 4 presents the results of the study and Chapter 5 contains the conclusion.

Chapter 2 – Literature Review

2.1 Introduction

This chapter provides a review of literature related to corporate failure prediction models. The literature review focuses on (1) the definition of failure; (2) different categories of corporate failure prediction models; (3) discussion on the statistical models used in this study; and (4) empirical studies comparing the accuracy rate of MDA and binary logit.

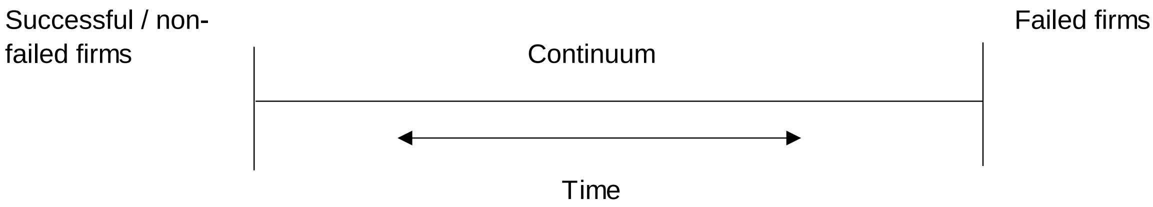
2.2 Definition of corporate failure

The primary objective of financial distress prediction models is to accurately and reliably classify ‘failing’ firms from ‘non-failing’ firms and such classification, to a large extent, is contingent on the definition of ‘failure’. In their systematic literature review of corporate failure prediction models ranging between 1966 to 2012 in peer review journals, Appiah, Chizema, and Arthur (2015) found that one of the main issues in this research area is that studies arbitrarily define corporate failure. This problem was also confirmed in earlier studies (Ohlson, 1980; Pretorius, 2009). Adopting an inappropriate definition of failure could have considerable impact on data sample collection and misclassification errors. Misclassification errors can be categorised as Type I and Type II (Altman, 1968). Type I errors incorrectly classify failed firms as being healthy and Type II errors incorrectly classify non-failed firms as being financially distressed. Appropriately defining failure contributes to better comparability of results (Pretorius, 2009).

Corporate failure occurs when a firm encounters certain types of financial difficulties (Sun, Li, Huang, & He, 2014). Traditionally corporate failure has been defined as the inability of a firm to pay its financial obligations as they become due (Beaver, 1966). Beaver’s definition of corporate failure is based on the theoretical framework of the cash flow model. According to Beaver (1966), a financially distressed firm exhibits similar qualities as a reservoir whose water is being drained.

Pretorius (2009) performed an extensive review of the literature that examined the definition of corporate failure. Between the dichotomous state of “non-failed” and “failed” firms is a state of “decline” (Pretorius, 2009). A firm is deemed to be in a state of “decline” when its financial performance “deteriorates over consecutive periods” and experiences financial distress in the ordinary course of business (Pretorius, 2009). A company is considered to have “failed” when it is unable to raise new debt or attract additional equity to reverse the decline, and decline is a natural prerequisite to failure (Pretorius, 2009). Therefore, corporate failure can be argued as a continuum that ranges from non-failed, financially weak (firms experiencing "decline") and failed firms (Figure 1).

Figure 1: Financial distress continuum



Source: author

Appiah et al. (2015) found that there is consensus in the literature on the legal definition of corporate failure. According to the South African Companies Act no. 71 of 2008 a firm is financially distressed if (1) it is unable to pay all its debts as they become due and payable within six months; or (2) the firm will become insolvent within six months. There are benefits of adopting the legal definition of corporate failure since it allows an objective criterion for dating the failing firms and splitting the sample into 'failed' and 'non-failed' (Appiah et al., 2015). The following section provides an overview of the most common accounting-based methods of predicting corporate failure.

2.3 Categories of financial distress prediction models

Financial distress prediction models have evolved over time; from univariate ratio analysis, to multiple discriminant analysis, to logit and probit analysis, to recursive partitioning algorithm, and lastly to neural networks (Bellovary et al., 2007). Aziz and Dar (2006) performed a critical analysis of various types of financial distress prediction models and narrowed the techniques down to three broad categories: (1) statistical models; (2) artificially intelligent expert system (AIES) models; and theoretical models.

Table 1: Three categories of financial distress prediction models

Statistical models	Artificially Intelligent Expert System (AIES) models	Theoretical models
• Univariate analysis	• Recursively partitioned decision trees (Inductive learning model)	• Balance Sheet Decomposition Measure (BSDM) / Entropy theory
• Multiple Discriminant Analysis (MDA)	• Case-Based Reasoning (CBR) model	• Gambler's Ruin theory
• Linear Probability Model (LPM)	• Neural Networks (NN)	• Cash management theory
• Logit model	• Genetic Algorithms (GA)	• Credit risk theories
• Probit model	• Rough sets models	
• Cumulative Sums (CUSUM) procedure		
• Partial adjustment process		

Source: Aziz and Dar (2006)

Aziz and Dar (2006) found that statistical models are the most popular techniques used in predicting corporate failure; and within this category, the most frequently used technique was MDA followed by binary logit. The authors reported that all the modelling techniques are comparative with respect to their predictive power; however, AIES models reflect slightly better accuracy rates. Aziz and Dar (2006) cautioned that on closer examination of the individual AIES models; It was noted that statistical models (especially MDA and logit) provide consistently better predictive accuracies. This suggests that MDA and logit models, although marginally less accurate when compared to AIES, may provide the most reliable techniques for predicting corporate failure.

Considering their popularity in the literature; the following section will examine statistical models, with a focus on MDA and logit, used to predict financial distress.

2.4 Statistical models

The use of statistical models, incorporating financial statement data (also referred to as accounting-based models), to predict financial distress began in the 1930's and 1940's when Fitzpatrick (1932) and Merwin (1942) studied corporate failure (Altman, 1968; Ohlson, 1980). The revolutionary studies in the field took place in the late 1960's when Beaver (1966) and Altman (1968) developed the first univariant and multivariate discriminate analysis models to predict corporate failure. Although many decades have since passed, the use of financial statement data to predict corporate failure is still relevant and in use (Almamy, Aston, & Ngwa, 2016; Tian, Yu, & Guo, 2015).

Whilst financial statement data continue to be in use, it has been subject to considerable criticism. The main criticism is summarised by Agarwal and Taffler (2008): (1) financial statements represents the historic financial performance of a firm and might not be appropriate in predicting the future, (2) historical cost accounting might not reflect the true fair value of assets, (3) data reported in the financial statements might be manipulated by management, and (4) the going concern assumption is applied when financial statements are prepared. In spite of these limitations Agarwal and Taffler (2008) concluded that there is little difference in terms of predictive accuracy between accounting-based and market-based corporate failure prediction models. Whilst there is a lack of real economic theory underpinning financial distress prediction models (Appiah et al., 2015); there are overriding conclusions from earlier studies using accounting-based models that demonstrate that financial ratios provide significant indications of the likelihood of corporate failure (Altman, 1968; Johnsen & Melicher, 1994; Ohlson, 1980; Zmijewski, 1984). More recently Altman et al. (2017), Hensher and Jones (2007) and Jones and Hensher (2004) have also shown strong correlation between financial ratios and corporate failure.

2.4.1 Multiple discriminant analysis

The literature review on MDA dates back to the sixties when Altman (1968) developed the z-score model which remains very popular in the literature today. The objective of the study was to find out which combinations of financial ratios predict corporate failure best. The sample comprised of 66 publicly held manufacturing companies in the United States of America (USA) between 1946 and 1965. Small (< USD 1 million total assets) and very large (> USD 25 million total assets) companies were eliminated from the sample because corporate failure was rare at the time for very large companies and there as limited data for small firms. Altman (1968) split the sample into two groups (1) Group 1 comprising 33 firms as “bankrupt”; and (2) Group 2 comprising 33 firms as “non-bankrupt”. The legal definition of corporate failure was adopted for Group 2; and constituted of a “paired sample”, that is, they were matched, as close as possible, to Group 1 by industry and asset size (Altman, 1968). The data was extracted from the company’s financial statements. The discriminant function contained five independent variables (see 3.3.1.1); and the selection of the variables was based on popularity in previous literature and relevancy to the study (Altman, 1968).

To interpret the results of the z-score, Altman (1968) used a classification scale. On one hand, companies with a z-score more than 2.99 were considered non-bankrupt. On the other hand, firms with a z-score less than 1.81 were considered as bankrupt. A z-score between 1.81 and 2.99 were said to be in a “grey area” where classification between bankrupt and non-bankrupt was susceptible to error classification. To overcome this issue, Altman (1968) proposed a z-score cut-off point of 2.675. The z-score model was highly accurate and classified 95 percent of the total initial sample correctly. Type I error was 6 percent and Type II error was 3 percent.

Whilst MDA models have proven to be quite accurate for assessing the financial health of firms, they have been subject to criticism. The main criticisms are based on (1) statistical drawbacks making them difficult to apply (Eisenbeis, 1977; Ohlson, 1980; Sheppard, 1994), and (2) their predictive accuracy (Moyer, 1977). Despite the criticisms, MDA method and Altman (1968) original z-score model have proven to be quite accurate over the past few decades and remain an established tool for assessing the financial health of firms (Almamy et al., 2016).

2.4.2 Logit model

Ohlson (1980) raised questions about the MDA model, particularly regarding the restrictive statistical requirements imposed by the model. As a direct response Ohlson (1980) developed the logit model (also referred to as binary logit). The binary logit model estimated the probability of a company failing (Ohlson, 1980). The sample consisted of 105 bankrupt firms and 2 058 non-bankrupt firms between 1970 and 1976. Ohlson (1980) developed three logit models that used nine independent variables, six of which were financial ratios (see 3.3.1.2). Model 1 and 2 estimated the probability of failure within one and two years respectively; and Model 3 estimated the probability of failure within one or two years (Ohlson, 1980).

The overall accuracy rate for the estimation sample was 96% and for the hold-out sample 85%. Ohlson (1980) found that the cut-off point that minimised the classification error (that is, the sum of Type I and Type II) rate was 0.038. In other words, firms with a probability below 0.038 are predicted not to fail and firms with a probability over 0.038 are predicted to fail. At this point, 17.4% bankrupt and 12.4% non-bankrupt firms were misclassified at t-1. The study showed that factors such as the “size” and “financial structure of a company” as well as the “current liquidity” play an important role in predicting corporate failure (Ohlson, 1980).

2.4.3 Multinomial logit model

Johnsen and Melicher (1994) did not subscribe to the binary notion of failure / nonfailure dichotomy; instead, argued that financial distress is a continuum ranging from being financially weak to bankrupt with the possibility of various degrees of financial weakness. As a result, Johnsen and Melicher (1994) proposed a multinomial logit model to predict corporate failure with memberships ranging between (1) bankrupt; (2) nonbankrupt; and (3) financially weak. The sample consisted of 112 bankrupt firms, 293 nonbankrupt firms and 255 financially weak firms from 1970 to 1983. Johnsen and Melicher (1994) used 13 independent variables of which 7 were selected from Altman, Haldeman, and Narayanan (1977) and 6 were selected from

Beaver (1966). There were a total of six possible types of misclassification errors and were decomposed as follows:

> Component 1: the misclassification of bankrupt firms as nonbankrupt plus the misclassification of nonbankrupt firms as bankrupt.
>
> Component 2: the misclassification of bankrupt firms as weak plus the misclassification of weak firms as bankrupt.
>
> Component 3: the misclassification of nonbankrupt firms as weak plus the misclassification of weak firms as nonbankrupt.

The multinomial logit model yielded very accurate results for component 1 (97.2% for the Altman et al. (1977) financial ratios and 96.0% for the Beaver (1966) financial ratios). This was a 73.2% and 57.9% improvement from the binary logit model using the Altman et al. (1977) and Beaver (1966) financial ratios respectively. The accuracy rate decreased for component 2 (84.7% and 86.1% for Altman et al. (1977) and Beaver (1966) financial ratios) and only showed a 6.7% and 13.6% improvement over binary logit model. The analysis was only confined to component 1 and 2, however, Johnsen and Melicher (1994) noted that for component 3, the misclassification were not significantly different. The introduction of a third state (i.e., financially weak firms) improved the overall accuracy of the binary logit model (Johnsen & Melicher, 1994).

2.4.4 Mixed logit model

Jones and Hensher (2004) recognised that there has been limited innovation in the field and the literature on failure prediction models has not kept up with the major advances in discrete choice modelling. Jones and Hensher (2004) developed a mixed logit model and argued that it offers substantial improvements compared to binary logit and multinomial logit models as their model allows for the measurement of observed and unobserved heterogeneity. The mixed logit model adopted a multistate notation as follows: failed firms (state 0); non-failed firms (state 1) and insolvent firms (state 2). The estimation sample consisted of 2 838 non-failed firms, 78 insolvent firms and 116 failed firms. The mixed logit model was substantially better at predicting state 1 and 2 firms with an overall hit rate of 80.7% and 95.5%. In contrast, a multinomial logit model only obtained a hit rate of 24.8% and 5.1% for state 1 and 2 respectively.

One of the advantages of the mixed logit model is that they allow for the complete relaxation of the Identically Distributed Errors (IID) and Independence from Irrelevant Alternatives (IIA) conditions by allowing all unobserved variances and covariances to be different, up to identification (Jones & Hensher, 2008). This means, the mixed logit model, is highly flexible in representing sources of firm-specific observed and unobserved heterogeneity through the incorporation of random parameters; compared to binary logit and multinomial logit that only allow for fixed parameter estimates (Jones & Hensher, 2008). Although modelling financial distress in a multi-state setting can present major conceptual and econometric challenges (Jones & Hensher, 2004); the practical risk assessment by lending institutions and other stakeholders usually cannot be reduced to a simple pay-off space of just “failed” and “non-failed” dichotomy of corporate failure.

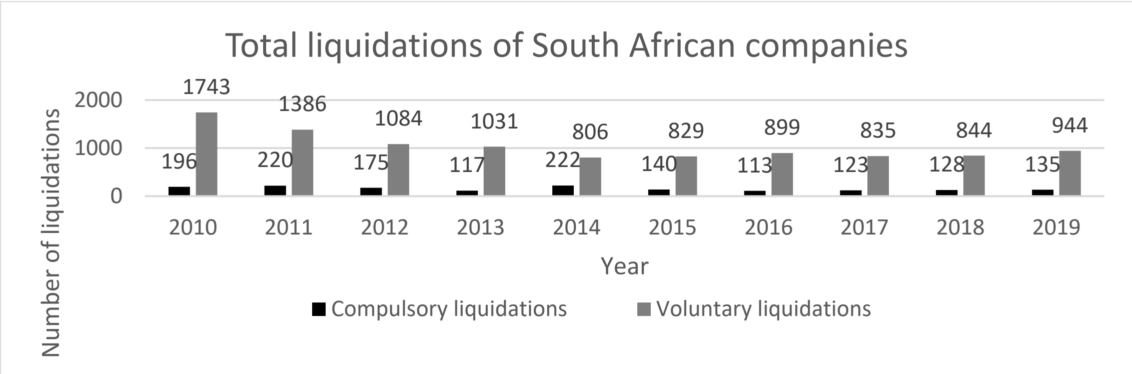
Since most studies have focused on modelling failure as a simplistic binary classification of failure versus non-failure, that is, the dependent variables can only take on one of two possible states (Jones & Hensher, 2008), the following section examines studies which compare the accuracy of MDA and logit.

2.5 Empirical studies

MDA and binary logit model comparisons forms the backbone of the empirical analysis; and such comparisons in recent studies show that logit models, on average, have higher accuracy in predicting corporate failure. Abdullah et al. (2008) compared the performance accuracy between three different models including MDA and logit model and found that the predictive accuracy of logit model (80.8%) is higher than MDA (73.1%) from a Malaysian context. Lin (2009) also found similar results from a Taiwanese context but the performance difference was not considered material (84.3% and 86.4% for MDA and logit model respectively). Interestingly, from a United-Kingdom (UK) perspective, Bunyaminu and Issah (2012) found that the predictive accuracy for one year prior to failure was identical (i.e., 71.4%) for both MDA and the logit model, however, the logit model outperformed MDA for years 2 and 3 prior to failure. The MDA performed poorly from a Slovak context with a hit ratio of 61.8% compared to 73.73% for logit model (Mihalovic, 2016).

To highlight the extent of corporate failures in South Africa, the following graph shows the number of company liquidations from 2010 to 2019 (StatsSA, 2020).

Figure 2: South African company liquidations from 2010 to 2019



Muller et al. (2009) compared the predictive accuracy of various accounting-based models, including MDA and logit from a South African context. They did not select a sample, instead, they selected all companies listed on the JSE that “failed” between 1997 and 2002. The results were inconsistent with similar studies noted above, with predictive accuracy of only 39.6% and 1.9% for MDA and logit model respectively for one year prior to failure. An important observation in Muller et al. (2009) study is that the researchers arbitrarily used “delisting” as one of the criteria to define corporate failure. This is a significant problem because a company might have legitimate reasons for delisting which might not necessarily mean the company has failed. The inappropriate definition of corporate failure would have adversely impacted the data collection and skewed the results in Muller et al. (2009).

Cassim (2016) used a Bayesian model to predict bankruptcy. The sample size of this study was 132 JSE listed companies between 2000 and 2013 which were split into 66 pairs of bankrupt and non-bankrupt. The results were also compared with MDA and logit models. Although the Bayesian model performed better when compared to the MDA and logit model; the overall results of all 3 models were not very encouraging as it was found that the predictive accuracy for bankrupt firms did not exceed 70%. The accuracy rate for the Bayesian model did not exceed 67%; and for the MDA and logit, this did not exceed 41% and 32% respectively.

Rowlings (2016) examined the performance of MDA from a South African context. The sample comprised of 44 failed firms listed on the JSE between 2007 and 2015 and this was matched with 44 non-failed firms, for the same period, which resulted in a final sample of 88 firms. The study excluded Financial Institutions from the sample noting that these companies generally have higher leverage than a typical company and were also subject to different regulations and capital requirements. The findings from this study was very encouraging. The overall accuracy rate of the MDA was 82.95%. The author noted that the MDA model’s performance did not materially change (80.68%) when refined to only include three variable model comprising Revenue, Debt / Market capitalisation and Debt / EBITDA as these variables were found to be the most statistically significant. This suggests that reducing the number of independent variables in the MDA model does not materially impact the performance of the model. However, it is interesting to note that the accuracy rate had a notable improvement with the three variable MDA model, increasing from 79.55% to 93.18%.

Quantitative corporate failure prediction models have evolved as new statistical modelling techniques have improved. Multinomial logistic regression is considered less restrictive than binary logit models since it does not assume normality, linearity or homoscedasticity (Starkweather & Moske, 2011). Although empirical seminal research has demonstrated that multinomial logit and mixed logit models have higher predictive power than binary logit models (e.g., Johnsen and Melicher (1994); Jones and Hensher (2004)); there has been limited research in the field that compares the performance of multinomial logit and mixed logit model to earlier corporate failure prediction techniques.

2.6 Conclusion

Existing empirical studies show that there is lack of consensus on what constitutes failure, with definitions varying significantly and arbitrarily across studies (Appiah et al., 2015; Ohlson, 1980; Pretorius, 2009). The adoption of the legal definition allows of an objective and consistent criterion for sample and data collection (Appiah et al., 2015) which could reduce misclassification errors. Pretorius (2009) noted that a firm first experience a decline in financial performance before it is considered to have failed. An important observation noted in the study done by Muller et al. (2009) was that the researchers inappropriately defined corporate failure as all companies that delisted from the JSE. Furthermore, there has been limited research that compares the predictive power of earlier corporate failure prediction techniques (MDA and logit) to later approaches (multinomial logit and mixed logit). This study addresses this gap by (1) appropriately defining corporate failure phenomenon; (2) improving the sample selection methods so that misclassification errors are minimised; and (3) testing the predictive accuracy of the models using more recent data (i.e., post IFRS and post global financial crisis era).

Chapter 3 – Methodology

3.1 Introduction

As outlined earlier, only few studies compared the accuracy rate of several corporate failure prediction models from a South African context. This study uses quantitative research methods to answer the following research question: Which financial distress models are more accurate in predicting financial distress in a South African context?

3.2 Population and sampling

The population for this study is defined as all failed, non-failed and financially weak companies listed on the JSE between January 2010 and December 2020. A comprehensive list of companies that were delisted or suspended from the JSE was obtained from various sources: EquityRT, the JSE and INET BFA (formerly known as McGregor BFA). Firms that were suspended or delisted for reason that were not associated with financial distress (e.g., voluntary winding up, mergers and schemes of arrangements) were removed from the analysis. A total of 42 firms that met the criteria of failed (State 0) were identified. Firms operating in the property and financial sector were removed for reasons described above, resulting in a final list of 36 failed firms. A sample of 36 non-failed (State 1) firms was matched (using the Standard Industrial Classification code and asset size) with that of the failed companies. The one-to-one match minimise size bias. This approach is consistent with previous studies (Abdullah et al., 2008; Altman, 1968; Beaver, 1966; Bunyaminu & Issah, 2012; Lin, 2009; Mihalovic, 2016). EquityRT was used to identify firms that were financially weak (State 2) that reported net losses for 3 or more consecutive reporting periods. A total of 40 firms were identified that met State 2 criteria, however, this was reduced to 36 when Special Purpose Vehicle (SPV) companies were removed. SPVs were removed because they are created for specific purposes (e.g., raising of capital and bankruptcy protection) and typically have no employees, make no substantive decisions, have no physical location and cannot go bankrupt (Gorton & Souleles, 2007). The final sample comprised of 108 firms: 36 failed, 36 non-failed and 36 financially weak. Table 2 summarise the sample of firms by industry.

Table 2: Sample of firms by industry

Industry	State 0	State 1	State 2
Construction	9	9	4
Energy	0	0	1
Information Technology	4	4	1
Leisure	1	1	1
Manufacturing	7	7	6
Mining	7	7	14
Other	1	1	6
Pharmaceutical	1	1	0
Retail	1	1	0
Telecommunication	4	4	1
Transportation	1	1	2
Total firms	**36**	**36**	**36**

The study was conducted by relying on secondary sourced data provided by EquityRT and INET BFA in a standardised format. Altman (1968) and Beaver (1966) noted that financially distressed firms have different characteristics for up to five years prior to failure. Attempts were made to collect up to five annual reporting periods of data on all firms in State 0, 1 and 2. However, there were instances where financial data was not available for all five reporting periods. There were seven firms that did not contain all five reporting periods of financial data, six of which were from State 0 and one from State 1. One firm contained only one reporting period of financial data and the remaining six contained between two and four. Therefore, instead of referring to the sample as "firms", this study refers to the sample as "firm years" which is consistent with the earlier studies of Jones and Hensher (2004) and Hensher and Jones (2007). This produced a final usable sample of 526 firm years which was split as follows:

- State 0 (failed): 167
- State 1 (non-failed): 179
- State 2 (financially weak): 180

3.2.1 Estimation and validation sample

The holdout methodology was applied to develop two sample sets for the purposes of model estimation and validation. The holdout method partitions the sample into two mutually exclusive subsets called estimation (also called training data) and validation (also called testing data) (Kohavi, 1995). The estimation sample is based on data collected between 2010 and 2015; whereas, the validation sample is based on 2016 to 2020. This approach is consistent with earlier studies of Jones and Hensher (2004) and Abdullah et al. (2008). The purpose of partitioning the sample into two subsets is to assess how the results of the statistical analysis will generalise to an independent dataset (Kohavi, 1995).

The full sample of 526 firm years, comprising State 0, 1 and 2, was used for the multistate models (multinominal and mixed logit). The sample was partitioned into estimation and validation sample of 412 and 114 firm years respectively. For the binary models (MDA and logit), the sample for State 2 (financially weak) was dropped since the dependent variable for these models is dichotomous (Altman, 1968; Hosmer Jr, Lemeshow, & Sturdivant, 2013; Kleinbaum, Dietz, Gail, Klein, & Klein, 2002). This resulted in a final sample size of 342 firm years for these models. The sample was then partitioned into an estimation and validation sample of 272 and 74 firm years respectively.

3.3 Analysis plan

The accuracy rate of the MDA, logit, multinomial logit and mixed logit models was compared. The independent variables were obtained from INET BFA. Appiah et al. (2015) noted there is no uniformity in variable selection in the field of failure prediction models. However, there is an overriding body of literature that select variables based on (1) popularity in literature, (2) simplicity and (3) performance based on previous studies (Altman, 1968; Beaver, 1966; Hensher & Jones, 2007; Johnsen & Melicher, 1994; Jones & Hensher, 2004; Ohlson, 1980; Zmijewski, 1984). Bellovary et al. (2007) performed an extensive review of bankruptcy prediction studies from 1930 to 2007 and found 42 frequently used financial ratios. He concluded that a higher number of variables does not guarantee a higher predictive ability. This study used the independent variables which are popular in the literature that have yielded very accurate results.

3.3.1 Models and equations

The following models and equations were used in this study:

3.3.1.1 Multiple discriminant analysis

$$\mathrm{Z} = 0.012(\mathrm{a1}) + 0.014(\mathrm{a2}) + 0.033(\mathrm{a3}) + 0.006(\mathrm{a4}) + 0.999(\mathrm{a5}) \quad (1)$$

Where: a1=working capital/total assets; a2=retained earnings/total assets; a3=earnings before interest and taxes (EBIT)/total assets; a4=market value of equity/book value of total liabilities; a5=sales/total assets (Altman, 1968). *Z* may be regarded as discriminant score and can take any value range between -∞ and +∞ (Frank, Massy, & Morrison, 1965).

When using MDA it is necessary to check that the normality of the data assumption is met. The violation of the normality assumption is one of the main problems that arise when using financial ratios to predict corporate failure (Mihalovic, 2016). Diagnostic tests to test the normality of data assumption can be performed by examining the skewness and kurtosis values as follows (Ho, 2013):

$Z_{skewness} = \frac{skewness}{\sqrt{s.e.\ skewness}}$ and $Z_{kurtosis} = \frac{kurtosis}{\sqrt{s.e.\ kurtosis}}$

Multiple discriminant analysis is highly sensitive to the inclusion of outliers (Mihalovic, 2016; Tabachnick, Fidell, & Ullman, 2013). IBM SPSS Statistics Version 26 (SPSS) software was used to test the dataset for outliers. Tabachnick et al. (2013) proposed using the Box plots (a feature available in SPSS software) to identify outliers since it is simpler and literally boxes in observations that are around the median; cases that fall far away from the box are extreme and visible. Nyitrai and Virág (2019) performed a review of the literature on how bankruptcy prediction research deals with outliers and found that the researchers either (1) excluded the outliers from the dataset; (2) used winsorization in an ad hoc way; or (3) the authors did not report how they handled outliers. Nyitrai and Virág (2019) also found that the predictive power of MDA and logit can be considerably enhanced by using winsorization. Therefore in the present research outliers in the dataset were winsorized using Tukey (1962) approach, that is, the outlier was transformed by replacing its original value by the nearest value of an observation that was not seriously suspect.

SPSS was used to identify missing values in the dataset. Hot deck imputation was employed to transform the dataset for missing values. Hot deck imputation involves replacing a missing value with the value of a similar "donor" in the dataset that closely matches the characteristics (e.g., similar values of other independent variables) of the "donee" (Myers, 2011).

Unequal covariance matrices might negatively affect the results of significant testing when the sample size is small and unequal (Ho, 2013). The Box's M test was used to determine if covariance matrices were equal (Tabachnick et al., 2013). The following equation was employed to test the hypothesis that the population covariance matrices were all equal (Mihalovic, 2016):

$H_0: \sum_1 = \sum_2 = \cdots \sum_m,$

Where m is the number of independent populations. Now, assume that $S_1 \dots S_m$ presents sample covariance matrices from the m populations, df is degrees of freedom and every S_j is of n_j independent observations comprising of $ks\,1$ column vector. We now define the covariance matrix of sample:

$S = \frac{1}{n-m}\sum_j^m (n_j - 1)S_j,$

$M = (n-m)ln \mid S \mid - \sum_{j=1}^m (n_j - 1)ln \mid S_j \mid,$

$c = \frac{2k^2 + 3k - 1}{6(k+1)(m-1)}\left(\sum_j^m \frac{1}{n_j - 1} - \frac{1}{n-m}\right),$

The test statistic of Box's M test is then:

$M(1-c) \sim X^2(df)$

A one-way Analysis of Variance (ANOVA) was used to test the individual discriminating ability of the independent variables.

3.3.1.2 Logit model

$$O = -1.32 - 0.407(\text{b1}) + 6.03(\text{b2}) - 1.43(\text{b3}) + 0.0757(\text{b4}) - 2.57(\text{b5}) - 1.83(\text{b6}) + 0.285(\text{b7}) - 1.72(\text{b8}) - 0.521(\text{b9}) \qquad (2)$$

Where: b1=Size (LOG (Total Assets / GDP Index)); b2=Debt Ratio (Total Liabilities / Total Assets); b3=Working Capital / Total Assets; b4=Current Liabilities / Current Assets; b5= dummy variable (1 if Total Liabilities > Total Assets; 0 otherwise); b6=Net Income / Total Assets; b7=Funds Provided by Operations / Total Liabilities; b8= dummy variable (1 if Net Income was Negative for The Last Two Years; 0 otherwise); b9 = Delta Net Income Divided by the Sum of the Absolute Net Income (Ohlson, 1980). The logit model is obtained from the logistic function which is a linear sum of the constant plus the independent variables (b1, b2 …n) multiplied by their respective coefficients (Kleinbaum et al., 2002) and can take any value rage between -∞ and +∞ (Hosmer Jr et al., 2013). The logit model is derived as follows (Kleinbaum et al., 2002; Ohlson, 1980):

$$P = \frac{1}{1 + e^{-(\alpha + \sum \beta_i X_i)}}$$

Where:

P is the probability function, $P \leq 0 \leq 1$

β and α are constant terms representing unknown parameters

X is the independent variables

The cut-off point for classifying a firm as failed, non-failed or financially weak was set to 0.5 (Abdullah et al., 2008; Bunyaminu & Issah, 2012; Johnsen & Melicher, 1994; Lin, 2009). There is no consensus on determining the optimal cut-off point for logit models since this depends on the decision context and payoff functions (Bunyaminu & Issah, 2012).

3.3.1.3 Multinomial logit model

Unlike binary logit models whereby the dependent variable is dichotomous (0 and 1 for failed and non-failed firms respectively); multinomial logit models provide J probabilities of entering into J possible states of financial distress. In general, for the J states the probabilities are specified as (Johnsen & Melicher, 1994):

$$P_{tj} = \frac{e^{(a_j + \beta_j x_j)}}{\sum e^{(a_j + \beta_j x_j)}} \quad (3)$$

Where:

P_{tj} = the ith firm is classified as the jth state

t = 1, 2, …, T firms

j = 0 (failed), 1 (non-failed) or 2 (financially weak)

x = independent variables, that is: b1=log(total assets)/gross national product; b2=total liabilities/total assets; b3=working capital/total assets; b4=current liabilities/current assets; b6=net income/total assets; b7=cash flow from operations/total liabilities; b9=change in net income/absolute change in net income; c6=equity/(equity+debt)

$a_1 = \beta_1 = 0$ and state 1 is the benchmark for comparison (Johnsen & Melicher, 1994).

This study estimated the probability of a particular firm in the jth state. Borrowing from Johnsen and Melicher (1994), let $Y_{tj} = 1$, if the tth firm is classified as the jth state. Assuming no particular ordering of the choices and $P_{tj} = 1$, the multinomial logit model becomes:

$$\log(P_{t2} / P_{t1}) = \alpha + \beta_\alpha X_t$$

$$\log(P_{t3} / P_{t1}) = \alpha_3 + \beta_\alpha X_t$$

$$P_{t1} = 1 - P_{t2} - P_{t3}$$

3.3.1.4 Mixed logit model

Mixed logit model (Hensher & Jones, 2007; Jones & Hensher, 2004) is a general statistical model for examining discrete choices. Mixed logit eliminates three limitations of binary logit by allowing for random taste variation, unrestricted substitution patterns and correlation in unobserved factors over time (Chaudhuri, 2013). In mixed logit models the utility associated with each observation i can be expressed as follows (Jones & Hensher, 2004):

$$U_{iq} = \beta' x_{iq} + (\eta_{iq} + \varepsilon_{iq}) \quad (4)$$

$$L_i(\eta) = \frac{e^{(\beta' x_i + \eta_i)}}{\sum_i e^{(\beta' x_i + \eta_i)}}$$

(10)

$$P_i = \int L_i(\eta)\ f(\eta \mid \Omega)d\eta \quad (11)$$

Where:

η_{iq} = a random term with zero mean whose distribution over firms and alternative outcomes depends in general on underlying parameters and observed data relating to alternative outcome i and firm q.

ε_{iq} = a random term with zero mean that is an Independent and Identically Distributed (IID) alternative outcome and does not depend on underlying parameters or data.

Ω = fixed parameters of the distribution.

x = independent variables, that is: b1=log(total assets)/gross national product; b2=total liabilities/total assets; b3=working capital/total assets; b4=current liabilities/current assets; b6=net income/total assets; b7=cash flow from operations/total liabilities; b9=change in net income/absolute change in net income; c6=equity/(equity+debt)

itq = financial state (i), time (t) and firm (q)

β = Unobserved variable and treated as stochastic influences (Jones & Hensher, 2004).

The mixed logit probability is a weighted average of the logit formula evaluated at different values of β, with the weights given by the density f (β). The weighted average of several functions is called a mixed function, and the density that provides the weights is called the mixing distribution. Mixed logit is a mixture of the logit function evaluated at different β's with f (β) as the mixing distribution (Train, 2009).

3.4 Comparison of Models

Once the predictions of each model were calculated the accuracy of the models was assessed by comparing their predictions to what actually happened. This entailed determining whether the models correctly classified firms as failed or non-failed, or whether failed firms were classified as non-failed (Type I Error), and non-failed firms were classified as failed (Type II Error).

3.5 Reliability and validity

This study used SPSS to perform all the tests for the MDA and logit models. EViews 11 software was employed to perform all the statistical analysis for the multinomial and mixed logit models. SPSS was also used to perform all diagnostic test and data cleaning procedures.

Internal validity was enhanced by the matching sample of failed firms with non-failed firms described above. Furthermore, the selection of independent variables was based on previous studies which have demonstrated high predictive ability (Altman, 1968; Beaver, 1966; Hensher & Jones, 2007; Johnsen & Melicher, 1994; Jones & Hensher, 2004; Ohlson, 1980; Zmijewski, 1984).

The presence of multicollinearity was assessed using the Variance Inflation Factor (VIF). The chi-square of the likelihood ratio test was performed to check model fit for the multinomial logit and mixed logit models. Whilst the chi-square test is prone to inflation as sample size increases (Starkweather & Moske, 2011), there are merits to using this statistical test since it is robust with respect to the distribution of the data, and it is capable of handling data from both two groups and multiple groups (McHugh, 2013).

Chapter 4 – Results

The results of the econometric data analysis are presented in this chapter which is based on the research methods described in the previous chapter. This chapter is organised as follows: (1) analysis of the two state models (i.e., MDA and logit); (2) analysis of the multistate models (i.e., multinomial and mixed logit); and (3) comparison of the predictive performance of each model. Each section begins with the descriptive statistics, followed by individual analysis of the models.

4.1 Two state models

4.1.1 Descriptive statistics

The descriptive statistics of the independent variables for the two state models are presented in Table 3 for State 0 (failed) and State 1 (non-failed) firm years. It should be noted that the descriptive statistics presented here only represents the estimation sample which is consistent with the seminal study of Altman (1968). The aim of the descriptive statistics presented below is to examine the different independent variables and observed mean differences between State 0 (failed) and State 1 (non-failed) firms.

By examining the independent variables of failed (State 0) and non-failed (State 1) firms, the mean values are, on average, lower for failed firms than is for non-failed firms. In general, the mean values for non-failed firms are positive which is consistent with the seminal studies of Altman (1968) and Ohlson (1980). In addition, the independent variables suggest that, in aggregate, non-failed firms are in a better financial position in terms of liquidity and profitability.

For State 0 firms (failed), there are four independent variables with a negative mean value. The first one is a2 (retained earnings / total assets) which suggest that failed firms experienced losses for one or more financial periods that consumed a portion of its equity base. Variable a3 (EBIT / total assets) indicates that failed firms earnings are, on average, negative. Similarly, b6 (net income / total assets) ratio is negative for failed firms. Lastly, it is observed that mean value of b9 (change in net income / absolute change in net income) is negative for failed firms, suggesting that, in general, failed firms experience a decline in profitability in the preceding years prior to failure. Variable b1 (log(total assets / gross national product)) is, on average, negative for both failed and non-failed firms and this is mainly due to the logarithmic transformation.

Table 3: Descriptive statistics of two state models (estimation sample)

Independent variables	State 0 - failed firm years						
	Mean	N (firm years)	Std. Deviation	Minimun	Maximum	Kurtosis	Skewness
MDA							
a1-working capital/total assets	0.08386	132	0.22493	-0.35000	0.70000	0.38485	-0.01743
a2-retained earnings/total assets	-0.13455	132	0.44339	-0.75000	0.71000	-1.21487	-0.13523
a3-EBIT/total assets	-0.01492	132	0.17990	-0.31000	0.40000	-0.57795	-0.10731
a4-Equity/total liabilities	1.29697	132	1.21583	-0.80000	4.17000	0.59619	1.16260
a5-sales/total assets	1.26408	132	0.61042	0.05000	2.86000	-0.42751	0.08805
Logit							
b1-log(total assets)/gross national product	-2.36916	132	1.56680	-9.27222	0.89371	2.36892	-0.89188
b2-total liabilities/total assets	0.53189	132	0.25745	0.01000	1.12000	-0.27478	0.15950
b3-working capital/total assets	0.08386	132	0.22493	-0.35000	0.70000	0.38485	-0.01743
b4-current liabilities/current assets	0.80250	132	0.42047	0.07000	1.48000	-0.84896	0.22075
b5-1 if total liabilities > total assets; 0 otherwise	0.04545	132	0.20909	0.00000	1.00000	17.75830	4.41468
b6-net income/total assets	-0.02629	132	0.14427	-0.22000	0.31000	-0.57401	0.21567
b7-cash flow from operations/total liabilities	0.09977	132	0.32405	-0.45000	0.92000	0.69333	0.54070
b8-1 if net income < 0 for last two years; 0 otherwise	0.34091	132	0.47582	0.00000	1.00000	-1.56289	0.67899
b9-change in net income/absolute change in net income	-0.11630	132	0.62739	-1.00000	1.00000	-1.09476	-0.00430
Independent variables	State 1 - non-failed firm years						
	Mean	N (firm years)	Std. Deviation	Minimun	Maximum	Kurtosis	Skewness
MDA							
a1-working capital/total assets	0.24984	140	0.21253	-0.23000	0.70000	-0.47469	0.31421
a2-retained earnings/total assets	0.24793	140	0.33388	-0.75000	0.93000	2.30642	-1.09202
a3-EBIT/total assets	0.13557	140	0.12015	-0.28000	0.40000	1.58706	-0.35114
a4-Equity/total liabilities	1.69050	140	1.26178	-0.60000	4.17000	-0.52892	0.69593
a5-sales/total assets	1.32047	140	0.63143	0.10000	2.86000	-0.04411	0.48836
Logit							
b1-log(total assets)/gross national product	-1.31699	140	1.93079	-5.05835	3.51029	-0.48035	0.15567
b2-total liabilities/total assets	0.44793	140	0.21769	0.05000	1.12000	-0.03001	0.43670
b3-working capital/total assets	0.25084	140	0.21323	-0.23000	0.70000	-0.50640	0.30432
b4-current liabilities/current assets	0.59279	140	0.31280	0.02000	1.48000	-0.03910	0.51790
b5-1 if total liabilities > total assets; 0 otherwise	0.01429	140	0.11909	0.00000	1.00000	67.44171	8.27517
b6-net income/total assets	0.09679	140	0.09347	-0.22000	0.31000	2.03571	-0.45111
b7-cash flow from operations/total liabilities	0.39907	140	0.34201	-0.45000	0.92000	-0.34405	-0.16192
b8-1 if net income < 0 for last two years; 0 otherwise	0.02143	140	0.14533	0.00000	1.00000	43.26077	6.68153
b9-change in net income/absolute change in net income	0.06569	140	0.43393	-1.00000	1.00000	1.09669	-0.16194

4.1.2 Analysis of MDA

MDA is a discriminant function used to determine group membership of samples from a group of predictors by finding linear combinations of the variables which maximize the differences between the populations being studied, with the objective of establishing a model to sort objects into their appropriate populations (Brown, 2012; Ho, 2013). The primary advantage of MDA in dealing with classification problems is the potential of analysing the entire variable profile of the object simultaneously rather than sequentially examining its individual characteristics (Altman, 1968). The dependent variables are mutually exclusive and binary (i.e., State 0 (failed) and State 1 (non-failed)); and the independent variables are the five financial ratios as per the seminal study of Altman (1968).

This section is organised as follows. Firstly, data screening procedures are discussed together with diagnostic test performed to ensure the assumptions of MDA are met. This is followed by discussing the results generated from the original MDA model developed by Altman (1968) using the estimation sample only with a cut-off value of 2.67. A recalibrated MDA model using an optimal cut-off value derived from the group centroids was then developed. Finally, the results generated from the recalibrated MDA model (for both estimation and validation sample) are discussed.

4.1.2.1 Data screening

Inference based on a fitted statistical model often may be criticised because the features of the data are not in congruence with the model assumptions (Frees, 2004). It is therefore necessary to perform procedures that examine this congruence. Data screening and diagnostic tests were performed to check that the statistical assumptions relating to MDA were met; including checking the dataset for outliers, testing of variables for normality, multicollinearity and homogeneity of variance-covariance (Ho, 2013; Tabachnick et al., 2013).

The procedure started by screening the dataset. Firstly, the sample size of State 0 and 1 firm years is unequal given that n is 132 and 140 respectively. However, no special problems are posed by unequal sample sizes since MDA is typically a one-way analysis of variance and naturally occurring groups rarely occur or are sampled with equal number of cases in groups (Tabachnick et al., 2013). Secondly, there were 15 missing values from the estimation sample, which is not considered material since it represents 1.1% of the sample (5 independent variables x 272 firm years = 1 360). The missing values were transformed by employing the hot deck imputation technique as discussed in the previous chapter. Finally, there were 98 outliers identified and again, this was not considered material since it accounted for 7.2% of the total sample. The outliers were winsorized using the technique described in the previous chapter.

Diagnostic tests were performed using SPSS to assess if there were any violations to the statistical assumptions. Shapiro Wilk test was first performed which showed a significant departure from normality for all five independent variables since the p-values < 0.05. However, the Shapiro Wilk test is calculated when the sample size is small ($n < 50$). Diagnostic tests to test the normality of data assumption can be performed by examining the skewness and kurtosis values (Ho, 2013). The test is dependent on the sample size and in a large sample ($n > 200$), the normality of data assumption is not a problem since a variable with a statistically significant skewness, on average, does not deviate enough from normality to make a significant difference in the analysis (Tabachnick et al., 2013). The calculated Z values for all variables were < 3.29 suggesting that the assumption of normality at the 0.05 critical probability (alpha) level was not rejected (Kim, 2013).

The violation of the linearity assumption tends to lead to reduced power of the MDA function rather than increase Type I error. Linearity was diagnosed by inspecting bivariate scatterplots (Ho, 2013; Tabachnick et al., 2013). Nothing substantive was noted in this regard. A multicollinearity diagnostics test was also performed. For all independent variables, the tolerance value was > 0.10 and the Variance Inflation Factors (VIF) < 10 suggesting that multicollinearity is not a problem (Ho, 2013).

A Box's M test was performed which tests the hypothesis that the covariance matrices of the dependent variables are significantly different across levels of the independent variables (Ho, 2013). The Box's M value of 64.177 (F = 4.194) with a significant p-value (p = 0.000) means that the assumption of equality of covariance matrices has been violated. However, the large sample size (272 firm years) makes this violation not too important for the further model interpretation and analysis (Memić, 2015).

4.1.2.2 Results per Altman (1968)'s original model

To test the individual discriminating ability of the variables, a one-way Analysis of Variance (ANOVA) was conducted as reported in Table 4. ANOVA tests the statistical significance of mean differences among different groups (Tabachnick & Fidell, 2007; Tabachnick et al., 2013). Variables a1 through a4 are all significant at the 0.001 level, indicating extremely significant differences in these variables between groups. Variable a5 does not show a significant difference between groups which is consistent with the seminal study of Altman (1968). Although variable a5 is not statistically significant; it is particularly important because of its unique relationship to the other variables in the model and Altman (1968) noted that it ranked second in its contribution to the overall discriminating ability of the model. On average, all of the financial ratios indicate higher mean values for State 1 (non-failed) firms. The discriminant coefficients of the z-score equation display positive signs which is expected, and therefore, the greater the firms bankruptcy potential, the lower its discriminant score (Altman, 1968).

Table 4: MDA: One-way ANOVA

Variable Means and Test of Significance				
Variable	**State 0 Group mean**	**State 1 Group mean**	**F Ratio**	**Sig**
	N = 132	N = 140		
a1	0.0839	0.2498	39.156	0.000
a2	-0.1345	0.2479	65.056	0.000
a3	-0.0149	0.1356	66.515	0.000
a4	1.2970	1.6905	6.846	0.009
a5	1.2641	1.3205	0.560	0.455

Firm years with a cut-off value ≤ 2.67 are classified as failed whereas firm years with a cut-off value > 2.67 are classified as non-failed (Altman, 1968). The classification matrix for the MDA model is presented in Table 5. The overall performance of the model was poor since it only correctly classified 50.0% of the sample. On one hand failed firms (State 0) were 98.5% accurately classified; however, on the other hand, non-failed firms (State 1) were only 4.3% accurately classified, which dropped the overall performance of the model. The results suggest that the model can classify failed firms with a very high degree of accuracy; however, caution should be exercised because the false positive rate was extremely high. Contrary to prior studies (Altman, 1968; Gavurova, Packova, Misankova, & Smrcka, 2017; Memić, 2015) whereby the performance of the MDA model deteriorates as the lead time from bankruptcy increases, the performance of the model was inconsistent. For instance, at t-1, the model correctly classified 48.2% of the overall sample; increased to 51.8% at t-3 and slightly decreased to 49.1% at t-5.

Table 5: MDA classification matrix

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	130	98.5%	1.5%	132
Type II	6	4.3%	95.7%	140
Total	136	50.0%	50.0%	272

The overriding conclusion from the initial test is that Altman (1968)’s original model is not a good fit and recalibration with an optimal cut-off value is warranted. The following section examine whether the Altman (1968) model could be improved upon.

4.1.2.3 Recalibrated MDA model

The results derived from Altman (1968)’s original model were not encouraging, particularly given extremely high false positive rate. In this section attempts have been made to recalibrate the MDA model and examined if it could be improved.

4.1.2.3.1 Model development

As discussed in the previous chapter, the total sample of 346 firm years was partitioned into estimation and validation sample of 272 and 74 respectively. The estimation sample was used for purposes of developing the MDA model. Once the MDA model was calibrated, it was then tested on the validation sample to assess how well it can generalise to an independent dataset (Kohavi, 1995). The model was developed using the same five independent variables (financial ratios) per Altman (1968)'s model.

The Wilks' Lambda and univariate F test was employed to examine the difference between the mean values of the independent variables for failed and non-failed firms. Except for variable a5, significant group differences were found for all variables ($p \leq 0.05$). Therefore, variables a1 to a4 are important for discriminating between failed and non-failed firms. The Wilks' Lambda shows that the discriminant function is statistically significant by the chi-square test, χ^2 (df = 5) = 92.203, p < .001, indicating that the model was a good fit and was able to distinguish between failed and non-failed firms. The standardised canonical discriminant function coefficients (Table 6) show that variables a2 and a3 contribute the most to the discriminant function in discriminating between the two groups of firms. Variable a5 was the lowest contributor, suggesting that it is not a strong predictor variable. Based on the canonical discriminant function coefficients, the equation derived for the recalibrated MDA is as follows:

$$\mathrm{Z} = -0.695 + 1.801(\mathrm{a1}) + 1.224(\mathrm{a2}) + 3.621(\mathrm{a3}) - 0.113(\mathrm{a4}) + 0.199(\mathrm{a5})$$

Table 6: Canonical Discriminant Function Coefficients

Variable	Function
	1
a1	1.801
a2	1.224
a3	3.621
a4	-0.113
a5	0.199
(Constant)	-0.695

Group centroids are standardised "joint means" based on the linear combination of the independent variables for each group and are used to interpret group differences (Ho, 2013). The group centroid for failed firm years is -0.685, whereas the group centroid for non-failed firm years is 0.639. The optimal cut-off value can be calculated as follows (Mihalovic, 2016):

$$CSopt = \frac{N_A Z_B + N_B Z_A}{N_A + N_B}$$

Where:

$CSopt$ = optimal cut-off value

$N_A N_B$ = size of group A and B

$Z_A Z_B$ = centroids for group A and B

The optimal cut-off value derived from the above equation is -0.037, and this cut-off point was used to test the predictive capacity of the model. Companies with Z values lower than -0.037

were classified as failed and companies with Z values greater than -0.037 were classified as non-failed.

4.1.2.3.2 Results from MDA estimation sample

Inspection of the classification matrix, Table 7, shows that 73.9% of the estimation sample were accurately classified. In other words, the discriminant function correctly predicted 73.9% of the group memberships. Examination of the individual group results illustrate that there was 61.4% correct classification (Type I error 38.6%) for failed firm years and 85.7% (Type II error 14.3%) correct classification for non-failed firm years. Therefore, the two groups of firms appear exert different characteristics with State 1 firms more likely to be correctly classified by the discriminant function than State 0 firms. The model's accuracy rate decreased as the lead time from bankruptcy increased which is consistent with prior studies (Altman, 1968; Gavurova et al., 2017; Memić, 2015). At t-1 85.19% of the overall sample was correctly classified and decreased to 69.64% and 64.15% at t-3 and t-5 respectively. It can therefore be concluded that the recalibrated MDA model performed significantly better than Altman (1968)'s original model.

Table 7: MDA classification matrix (estimation sample)

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	81	61.4%	38.6%	132
Type II	120	85.7%	14.3%	140
Total	201	73.9%	26.1%	272

To test if the discriminant function is statistically better than chance (i.e., 50% assignment) a Press's Q test was performed (Ho, 2013):

$$Press's\ Q = \frac{[N - (nK)]^2}{N(K-1)}$$

Where:

N = total sample size

n = number of observations correctly classified

K = number of groups

Therefore,

$$Press's\ Q = \frac{[272 - (201 * 2)]^2}{272(2-1)} = 64.1$$

The results from Press's Q equation is compared with the chi-square critical value of 6.63 with 1 degree of freedom ($p < 0.01$). It can be concluded that the classification results derived from the discriminant function performs better, at a statistically significant level ($p < 0.01$), than the classification accuracy expected by chance because the Press's Q = 64.1 which is greater than 6.63.

4.1.2.3.3 Results from MDA validation sample

The recalibrated MDA model was tested on the validation sample of 74 firm years to assess how well it can generalise to an independent dataset (Kohavi, 1995). The results from the validation sample shown in Table 8 illustrates that the model performed slightly better than the estimation sample. Overall, the model correctly classified 55 of 74 firm years, representing 74.3% of the validation sample. The misclassification percentage was lower for State 1 firms represented by 17.9% Type II error compared to 34.3% for State 0 firms. This mean that the discriminant function was more likely to correctly classify non-failed firms than failed firms. Assessing the results from different lead times (Appendix 7); the model performed best at t=3 with an overall accuracy rate of 86.67%, however, as the lead time increased to t=5, the accuracy rate dropped to 61.54% which is consistent with the seminal study of Altman (1968). It is also encouraging to note that the classification results from the discriminant function is greater than the classification accuracy expected by chance since Press's Q = 17.51 ($p < 0.01$).

Table 8: MDA classification matrix (validation sample)

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	23	65.7%	34.3%	35
Type II	32	82.1%	17.9%	39
Total	55	74.3%	25.7%	74

The results derived from using Altman (1968)'s original discriminant function were not encouraging since the model only correctly classified 50.0% of the sample with very high Type II errors. Recalibrating the discriminant function yielded improved results overall.

4.1.3 Analysis of logit

Logistic regression (logit) is a statistical technique that can be used to describe the relationship between several independent variables to a dichotomous dependent variable (Kleinbaum et al., 2002). The advantage of logit is that it is less restrictive from a statistical point of view when compared to MDA because the assumptions with respect to prior probabilities of group memberships and the distribution of independent variables are relaxed (Ohlson, 1980). The independent variables are the nine financial ratios as per the seminal study of Ohlson (1980) and the dependent variables are the different states of financial distress.

The section on the logit is organised similar to the previous section on MDA. It starts by discussing the data screening procedures and diagnostics tests that were performed to ensure the assumptions of the logit are met. This is then followed by discussing the results derived from the original logit model developed by Ohlson (1980) using the estimation sample only with a cut-off value of 0.38. A recalibrated logit model using a cut-off value of 0.5 which is popular in the literature is developed. Lastly, the results generated from the recalibrated logit model (both estimation and validation sample) are presented.

4.1.3.1 Data screening

The dataset was screened to identify outliers and any missing values. There were 19 missing values from the estimation sample, which is not considered material since it represents 0.8% of the sample (9 independent variables x 272 firm years = 2 448). The missing values were transformed by employing the hot deck imputation technique as discussed in the previous chapter. There were 107 outliers identified and again, this was not considered material since it accounted for 4.4% of the sample. The outliers were winsorized using the technique described in the previous chapter.

Although logit is less restrictive than MDA, that is, the assumptions with respect to the distribution of independent variables are relaxed; linearity among the independent variables and multivariate normality may enhance its predictive power (Ho, 2013; Tabachnick et al., 2013). Except for variables b5 and b8, the Z values derived from skewness and kurtosis test did not exceed ±2.58 which suggest that normality is not a problem. Variables b5 and b8 were excluded from the skewness and kurtosis test since these are discrete dummy variables, that is, coded 0 and 1; and the assumption of multivariate normality is concerned with continuous variables (Malkovich & Afifi, 1973). The predictor variables, b1 to b9, were also tested a *priori* to verify that there was no violation of the assumptions of the linearity of the logit by examination of scatterplots between the pairs of independent variables (Ho, 2013; Pallant & Manual, 2010). Furthermore, multicollinearity diagnostic tests were performed and the VIF values of all independent variables were less than 10 suggesting that multicollinearity is not a problem (Ho, 2013).

4.1.3.2 Results per Ohlson (1980)'s original model

The seminal logit model developed by Ohlson (1980) was examined to assess if it was accurate in terms of classifying failed and non-failed firms using the estimation sample of 272 firms. Microsoft Excel software was used to perform the analysis. The results, incorporating 1 to 5 years prior to the event measured, are presented in Table 9. The cut-off value of 0.38 was used (Ohlson, 1980). Firms years with $P_i < 0.38$ are predicted to be non-failed or financially healthy and firm years with $P_i \geq 0.38$ are predicted to be failed or financially distressed.

The performance of the model was extremely poor since it correctly classified only 6.8% of failed firms year; however, it performed better in terms of classifying non-failed firms with 67.1% accuracy in this regard. Overall, the model's performance was not appealing, only 37.9% of the firm years were correctly classified. Contrary to prior studies (Bunyaminu & Issah, 2012; Gavurova et al., 2017; Memić, 2015; Ohlson, 1980) whereby the overall classification error rate deteriorates as the lead time prior to the event increases; the model's performance was inconsistent with the overall accuracy at t-1of 37% and changing to 38% and 42% at t-3 and t-5 respectively.

Table 9: Classification matrix - Ohlson (1980)'s original model

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	9	6.8%	93.2%	132
Type II	94	67.1%	32.9%	140
Total	103	37.9%	62.1%	272

The model was therefore recalibrated to fit the data set and the results are presented in the following section.

4.1.3.3 Result from recalibrated logit model

Since the seminal model of Ohlson (1980) performed poorly and especially given that it was incapable of classifying non-failed firms reliably; the model parameters and the cut-off point were re-estimated to assess if the overall classification rates could be improved.

4.1.3.3.1 Model development

After screening the dataset and performing the diagnostic tests discussed earlier, SPSS was used to develop the logit model. The estimation sample was used, containing 272 firm years which is divided into 132 and 140 for State 0 (failed) and State 1 (non-failed) firm years respectively.

A test of the full logit model with all nine (i.e., b1 to b9) predictor variables against a constant-only model was statistically significant, $\chi 2$ (9, N = 272) = 119.901, $p < 0.001$, indicating that the model had a good fit and was able to distinguish between failed and non-failed firms. The model as a whole, explained 35.6% (Cox and Snell R square) and 47.5% (Nagelkerke R square) of the variance in the different groups. According to Table 10, only four of the independent variables (b1, b3, b7 and b8) were statistically significant, $p < 0.05$. The strongest predictor of reporting financial distress was variable b3, recording an odds ratio (Exp(B)) of 113.304. Given that the relationship between b3 and the dependent variable (0 = failed and 1 = non-failed) is positive (β = 4.73), this shows that a firm with higher liquidity ratio (working capital / total assets) increases the odds of that firm not failing. Effectively, each one-unit increase in b3 increases the odds of not failing by a factor of 113. The Wald chi-square statistic tests the unique contribution of each predictor variable and the statistical significance of each coefficient (β) in the model (Ho, 2013). On the basis of the Wald test, the following equation can be derived for the recalibrated logit model:

$$O = -1.391 + 0.455(b1) + 0.68(b2) + 4.73(b3) + 1.041(b4) + 0.968(b5) + 2.57(b6) + 1.708(b7) - 1.523(b8) + 0.236(b9)$$

Table 10: Logit predicting likelihood of financial distress

Independent variables (reference)	Independent variables (long name)	B	Sig.	Exp(B)
b1	log(total assets)/gross national product	0.455	0.000	1.576
b2	total liabilities/total assets	0.68	0.522	1.975
b3	working capital/total assets	4.73	0.004	113.304
b4	current liabilities/current assets	1.041	0.324	2.831
b5	1 if total liabilities > total assets; 0 otherwise	0.968	0.441	2.633
b6	net income/total assets	2.57	0.226	13.071
b7	cash flow from operations/total liabilities	1.708	0.003	5.518
b8	1 if net income < 0 for last two years; 0 otherwise	-1.523	0.042	0.218
b9	change in net income/absolute change in net	0.236	0.490	1.266
Constant		-1.391	0.157	0.249

To determine the financial condition predicted by the logit model, the probability of distress cut-off $P_i = 0.5$ was used (Abdullah et al., 2008; Bunyaminu & Issah, 2012; Johnsen & Melicher, 1994; Lin, 2009). There is no consensus on determining the optimal cut-off point for logit models since it depends on the decision context and payoff functions (Bunyaminu & Issah, 2012).

4.1.3.3.2 Results from estimation sample

The classification matrix Table 11 shows that 73.5% of the failed firms were correctly classified, while 85.0% of the non-failed firms were correctly classified. Overall, the classification results show that the model predicted correctly 216 of the estimation sample of 272 firm years, for an overall success rate of 79.4%. Furthermore, the accuracy rate of the model diminished as the lead time from the event increased which is consistent with the prior studies noted above. At t-1, the overall accuracy of the model was 83% and decreased to 79% and 74% for t-3 and t-5 respectively. This is a significant improvement from Ohlson (1980)'s original model.

Table 11: Logit classification matrix (estimation sample)

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	97	73.5%	26.5%	132
Type II	119	85.0%	15.0%	140
Total	216	79.4%	20.6%	272

4.1.3.3.3 Results from validation sample

The results derived from the estimation sample of 74 firm years shows that the model improved marginally with an overall accuracy rate of 82.4%. The model was highly accurate in predicting State 1 firms (non-failed) with 37 of 39 firm years correctly classified, representing 94.9%. However, the model's predictive power diminished in terms of predicting Sate 0 firms (failed) and was only able to predict 24 of 35 (or 68.6%) firm years in this group. It is not surprising to note that the model's predictive power decreased as the lead time to financial distress increased (Appendix 8). At t=1 and t=3, the model correctly classified 93% of the validation sample, but decreased significantly to 62% at t=5. This is consistent with the seminal study of Ohlson (1980).

Table 12: Logit classification matrix (validation sample)

Classification	Number Correct	Percent Correct	Percent Error	N
Type I	24	68.6%	31.4%	35
Type II	37	94.9%	5.1%	39
Total	61	82.4%	17.6%	74

4.2 Multistate models

State 2 (financially weak) firms are introduced into the analysis of the multistate models (i.e., multinominal and mixed logit). Recalling from earlier, a State 2 firms is defined as *financially weak* when it experiences a decline (Pretorius, 2009), that is, when its financial performance deteriorates (i.e., net losses) over consecutive periods. For this study, consecutive periods means 3 years or more since prolonged losses is a sign of financial distress (DeAngelo & DeAngelo, 1990). Therefore, the dependent variable of the multistate models comprise of three mutually exclusive discrete groups: State 0 (failed), State 1 (non-failed) and State 2 (financially weak) firms.

As discussed in the previous chapter, the full sample of 526 firm years is used in the analysis of the multistate models. The sample was partitioned into (1) an estimation sample of 412 firm years which is based on firm financial distress data collected between 2010 and 2015 and (2) a validation sample of 114 firm years that is based on data collected between 2016 and 2020.

The estimation sample was used for developing the models and the estimation sample was used to assess how well the models generalise to an independent dataset (Kohavi, 1995).

This section is organised as follows. Firstly, procedures performed to screen the dataset are discussed together with any diagnostic tests carried out to ensure the assumptions of the logit are met. This is followed by a discussion of the descriptive statistics. Finally, the analysis of both the multinominal and mixed logit model is presented; which includes, model development and the results thereof from the estimation and validation sample respectively.

4.2.1 Data screening

Data screening procedures performed on the multistate models followed a consistent approach adopted for the binary models (logit and MDA), that is, the dataset was screened to identify outliers and any missing values. There were 19 missing values from the estimation sample, which is not considered material since it represents 0.6% of the sample (8 independent variables x 412 firm years = 3 296). The missing values were transformed by employing the hot deck imputation technique as discussed in the previous chapter. There were 184 outliers identified and again, this was not considered material since it accounted for 5.6% of the sample. The outliers were winsorized using the technique described in the previous chapter.

Similar to binary logit; and although it is not a requirement, the assumption of multivariate normality and linearity among the independent variables were tested since these assumptions may enhance the predictive power of the respective multistate models (Ho, 2013; Tabachnick et al., 2013). The Z values of all the independent variables derived from the skewness and kurtosis test did not exceed ±2.58 and therefore can be concluded that the distribution for these variables does not depart significantly from normality. The predictor variables were also tested a *priori* to verify that there was no violation of the assumptions of the linearity of the logit by examination of scatterplots between the pairs of independent variables which showed no serious nonlinearity (Ho, 2013; Pallant & Manual, 2010). The multicollinearity diagnostic tests performed (Table 13) showed that the VIF values of all independent variables were < 10 therefore suggesting that multicollinearity does not appear to be a problem (Ho, 2013); however, the correlation matrix between the independent variables showed that the Pearson Correlation between b3 (working capital / total assets) and b4 (current liabilities / current assets) and between b2 (total liabilities / total assets) and c6 (equity / (equity + debt)) was above 0.7. Variables b4 and c6 are deemed important. Variable b4 is a liquidity ratio and it is found to be popular the literature of financial distress (Appiah et al., 2015; Bellovary et al., 2007). Variable c6 represents capital structure, and generally, when a firm possesses high levels of debt, it is generally associated with greater risk of failure; and was found to be a good predictor of financial distress (Bunyaminu & Issah, 2012). Although variable b2, which is a measure of a firm's leverage, is present in several studies, it is not as popular as variable b4 or c6 (Bellovary et al., 2007). Variable b3 measures a firm's liquidity; however, it is not as popular in the literature as b4 and furthermore it was not statistically significant in the seminal study of Ohlson (1980). Therefore a decision was made whereby variables b4 and c6 were retained and variables b2 and b3 were dropped.

Table 13: Multicollinearity diagnostic tests

Coefficients	
Variables	Collinearity Statistics
	VIF
b1	1.147
b2	3.340
b3	3.537
b4	4.289
b6	2.389
b7	1.629
b9	1.301
c6	2.659

4.2.2 Descriptive statistics

The descriptive statistics of the independent variables for the multistate models are presented in Table 14 for State 0 (failed); State 1 (non-failed) and State 2 (financially weak) firm years. It should be noted that the descriptive statistics presented here is only for the estimation sample which is consistent with Altman (1968). Similar to the binary models; the mean values are, on average, lower for State 0 and State 2 firms than State 1 firms. By the introduction of State 2 firms, it can be seen that there is a transition of the mean values from non-failed to financially weak and then failed. The non-failed group are in a better financial position which is represented by all positive mean values. There are two financial ratios that are negative for State 0 and State 2 firms, notably: b6 (net income / total assets) and b9 (change in net income / absolute change in net income) and are likely negative for similar reasons discussed under 4.1.1.

Table 14: Descriptive statistics of multistate models (estimation sample)

Independent variables	State 0 - failed firms				
	N (firm years)	Mean	Minimum	Maximum	Standard Deviation
b1-log(total assets)/gross national product	132	13.00759	7.12393	18.10740	1.93259
b2-total liabilities/total assets	132	0.53190	0.01000	1.12000	0.25745
b3-working capital/total assets	132	0.08360	-0.35000	0.67000	0.22432
b4-current liabilities/current assets	132	0.82200	0.07000	1.61000	0.45347
b6-net income/total assets	132	-0.02570	-0.21000	0.27000	0.13855
b7-cash flow from operations/total liabilities	132	0.09570	-0.46000	0.85000	0.31776
b9-change in net income/absolute change in net income	132	-0.11630	-1.00000	1.00000	0.62739
c6-equity/(equity+ debt)	132	0.69540	0.04000	1.44000	0.28020
Independent variables	**State 1 - non failed firms**				
	N	**Mean**	**Minimum**	**Maximum**	**Standard Deviation**
b1-log(total assets)/gross national product	140	14.05429	9.30790	19.30988	2.30895
b2-total liabilities/total assets	140	0.44790	0.05000	1.12000	0.21769
b3-working capital/total assets	140	0.25041	-0.23000	0.67000	0.21235
b4-current liabilities/current assets	140	0.59460	0.02000	1.61000	0.31844
b6-net income/total assets	140	0.09530	-0.21000	0.27000	0.08993
b7-cash flow from operations/total liabilities	140	0.38890	-0.46000	0.85000	0.32784
b9-change in net income/absolute change in net income	140	0.06569	-1.00000	1.00000	0.43393
c6-equity/(equity+ debt)	140	0.77630	0.04000	1.44000	0.23169
Independent variables	**State 2 - financially weak firms**				
	N	**Mean**	**Minimum**	**Maximum**	**Standard Deviation**
b1-log(total assets)/gross national product	140	14.44358	8.09905	20.57720	2.92294
b2-total liabilities/total assets	140	0.42480	0.00000	1.12000	0.26559
b3-working capital/total assets	140	0.13490	-0.22000	0.67000	0.20331
b4-current liabilities/current assets	140	0.70670	0.00000	1.61000	0.46766
b6-net income/total assets	140	-0.00166	-0.21000	0.27000	0.10043
b7-cash flow from operations/total liabilities	140	0.07990	-0.46000	0.85000	0.29420
b9-change in net income/absolute change in net income	140	-0.02702	-1.00000	1.00000	0.64283
c6-equity/(equity+ debt)	140	0.73370	0.04000	1.00000	0.28267

4.2.3 Analysis of the multinomial logit model

Multinomial logit models are used to predict categorical placement in or the probability of category membership on a dependent variable based on multiple independent variables (Starkweather & Moske, 2011). The independent variables are continuous and the dependent variables are categorical (i.e., State 0, 1 and 2) and are mutually exclusive. Unlike binary logit models; multinomial logit models allow for more than two categories of the dependent variable outcome which is appropriate for this study. Similar to binary logit models; multinomial logit models uses maximum likelihood estimation to evaluate the probability of categorical membership (Starkweather & Moske, 2011).

4.2.3.1 Model development

EViews was employed to develop the multinominal logit model. For purposes of developing the multinominal logit, the estimation sample was used comprising 412 firm years that is split between the three groups of financial distress: 132 in State 0 (failed); 140 each for State 1 (non-failed) and State 2 (financially weak) firm years respectively.

The chi-square statistic was used to test the overall goodness-of-fit of the multinomial model. The chi-square test is statistical test which measures the association between categorical variables (McHugh, 2013), and is most often used with nominal data, where observations are grouped into several discrete, mutually exclusive groups, and where one counts the frequency of occurrence in each group (Ho, 2013). The Pearson chi-square value was not statistically significant, $\chi 2$ (df = 802) = 815.28, $p > 0.05$. This means that the model fits the data well.

To assess the model fit, the -2 log likelihood statistic was used (Table 15). The -2 log likelihood statistic tests the significance of the difference between any two models, provided that one model is nested inside the other (Ho, 2013). For the purposes of developing the multinomial logit model, the base model (containing only the intercept) is compared to the full model (containing the six independent variables, see Table 14 above). Comparison of the -2 log likelihood ratios for the models with and without the independent variables showed statistically significant improvement with the addition of predictor variables, $\chi 2$ (12, N=412) = 767.51, $p < 0.05$. This mean that the independent variables explain the dependent variable significantly and better than the intercept only model. Variables b1 (log(total assets)/gross national product), b6 (net income/total assets) and b7 (cash flow from operations/total liabilities) were statistically significant, therefore explaining the dependent variable significantly, $p < 0.05$. One would expect that b9 (delta of net income) would be statistically significant since State 0 and State 2 firm’s mean values are negative, that is, on average loss-making, which is associated with some degree of financial distress. The liquidity ratio, b4 (current liabilities/current assets), was also not a strong predictor (individually), which is consistent with earlier study of Ohlson (1980). An interesting observation is that the capitalisation ratio, c6 (equity/(equity+ debt)), is not a strong predictor which is consistent with an earlier study of Johnsen and Melicher (1994).

Table 15: Likelihood Ratio Tests

Effect	Model Fitting Criteria	Likelihood Ratio Tests	
	-2 Log Likelihood of Reduced Model	Chi-Square	df
Intercept	778.504	10.999	2
b1	791.964	24.459	2
b6	788.409	20.904	2
b7	784.399	16.894	2
b9	770.265	2.76	2
b4	772.84	5.335	2
c6	768.713	1.208	2

4.2.3.2 Results from estimation sample

The overall classification was not very accurate. Table 16 shows 47.0% of State 0 (failed) firms years were correctly classified; and 61.4% and 49.3% for State 1 (non-failed) and State 2 (financially weak) firms years respectively. Overall, the accuracy rate for the multinomial logit is 52.7%.

Table 16: Multinomial logit classification matrix (estimation sample)

Classification: estimation sample				
Observed	**Predicted**			
	State 0	State 1	State 2	Percent Correct
State 0	62	27	43	47.0%
State 1	23	86	31	61.4%
State 2	40	31	69	49.3%
Overall Percentage	30.3%	35.0%	34.7%	52.7%

4.2.3.3 Results from validation sample

The results derived from the validation sample showed an improvement in the model's overall classification to 72.8%. The model performed well, particularly for State 0 (failed) firms with an accuracy rate of 80.0%; but decreased to 76.9% and 62.5% for State 1 and 2 firms respectively.

Table 17: Multinomial logit classification matrix (validation sample)

Classification: validation sample				
Observed	**Predicted**			
	State 0	State 1	State 2	Percent Correct
State 0	28	1	6	80.0%
State 1	0	30	9	76.9%
State 2	7	8	25	62.5%
Overall Percentage	30.7%	34.2%	35.1%	72.8%

The model performed well, even when the lead time of finance distress increased (Appendix 9). At t=1 and t=3, the overall classification rate was 87.0%; however, deteriorated marginally to 85.7% at t=5.

4.2.4 Analysis of the mixed logit model

Jones and Hensher (2004) argued that the literature on financial distress prediction modelling techniques have been slow to develop; and unlike other fields of the social sciences, the commonly used techniques are primitive and dated. The authors therefore proposed the mixed logit model for predicting corporate failure. Mixed logit is a highly flexible model that can approximate any random utility model (McFadden & Train, 2000). It overcomes three important limitations of the binary logit model by allowing for random taste variation, unrestricted substitution patterns across choices, and correlation in unobserved factors over time (Train, 2009). Similar to the multinomial logit model; the independent variables are continuous and the dependent variables are categorical (i.e., State 0, 1 and 2) and are mutually exclusive.

4.2.4.1 Model development

EViews was employed to develop the mixed logit model. An estimation sample was used comprising 412 firm years. The sample was split as follows: 132 firm years in State 0 (failed); 140 firm years for State 1 (non-failed) and 140 firm years for State 2 (financially weak).

The mixed logit model was developed using panel data. A panel, or longitudinal, data is one that follows a given sample of individuals over time, and therefore provides multiple observations on each individual in the sample (Hsiao, 2014; Landau & Everitt, 2003). Therefore, the panel data presented here are the individual firms financial ratios observed over the five reporting periods. The advantages of using panel data includes (1) the ability to study dynamic relationships; and (2) to model the differences, or heterogeneity, among subjects (Frees, 2004).

Firstly, a model which included both fixed and random effects was estimated (Model 1, see Appendix 1, 2 and 3) (Frees, 2004). Having done this it was found that two of the variables displayed random effects (variables c6 and b9) while the rest of the variables displayed fixed effects. This led to the estimation of a second model (Model 2, see Appendix 4, 5 and 6) in which variables c6 and b9 were treated as random effects and the remaining variables as fixed effects. The -2 Log Likelihood of Model 2 (3178.135) was lower than the -2 Log Likelihood of Model 1 (3226.182), indicating that Model 2 fitted the data better. This difference was also found to be statistically significant: $\chi 2$ (df = 3, n= 412) = 3226.182 – 3178.135 = 48,047, $p < 0.01$.

Results from estimation sample

The mixed logit model containing all variables in fixed and random effects predicted 50.80% of State 0, 62.90% State 1 and 49.30% State 2 for the estimation sample correctly, achieving an overall accuracy of 54.4%.

Table 18: Classification: Model 1 estimation sample

Classification: Model 1 (fixed and random effects)				
Observed		Predicted		
		1	2	0
1	Count	88	29	23
	% within Observed	62.90%	20.70%	16.40%
2	Count	33	69	38
	% within Observed	23.60%	49.30%	27.10%
0	Count	24	41	67
	% within Observed	18.20%	31.10%	50.80%

Model 2, in which variables b9 and c6 were treated as random effects, predicted 47.0% of State 0, 62.1% State 1 and 50.0% State 2 for the estimation sample correctly, achieving an overall accuracy of 53.2%.

Table 19: Classification: Model 2 estimation sample

Classification: Model 2 (b9 and c6 as random effects)				
Observed		Predicted		
		1	2	0
1	Count	87	29	24
	% within Observed	62.10%	20.70%	17.10%
2	Count	29	70	41
	% within Observed	20.70%	50.00%	29.30%
0	Count	27	43	62
	% within Observed	20.50%	32.60%	47.00%

4.2.4.2 Results from validation sample

The two models were also estimated using the validation sample data. The results show that the correct classification rates for the two models improved. Model 1 predicted 80% of State

0, 82.10% State 1 and 65% State 2 for the validation sample correctly, achieving an overall accuracy of 75.4%.

Table 20: Classification: Model 1 validation sample

Classification: Model 1 (fixed and random effects)				
Observed		**Predicted**		
		1	2	0
1	Count	32	7	0
	% within Observed	82.10%	17.90%	0.00%
2	Count	8	26	6
	% within Observed	20.00%	65.00%	15.00%
0	Count	1	6	28
	% within Observed	2.90%	17.10%	80.00%

Model 2 predicted 77.1% of State 0, 69.2% of State 1 and 55.0% of State 2 for the validation sample, therefore achieving an overall accuracy of 66.7%.

Table 21: Classification: Model 2 validation sample

Classification: Model 2 (b9 and c6 as random effects)				
Observed		**Predicted**		
		1	2	0
1	Count	27	11	1
	% within Observed	69.20%	28.20%	2.60%
2	Count	13	22	5
	% within Observed	32.50%	55.00%	12.50%
0	Count	1	7	27
	% within Observed	2.90%	20.00%	77.10%

4.3 Comparison of accounting-based financial distress prediction models

One of the objectives of corporate failure prediction models is to act as an early warning system that is capable of timely predicting financial distress with high confidence (Caggiano et al., 2014; Mihalovic, 2016). That is, models must be able to reliably and accurately differentiate 'failing' firms from 'non-failing' firms. This can provide valuable information to different stakeholders; including management and shareholders who can make corrective decisions to restore the financial health of the company. This study therefore examined different statistical financial distress prediction models to answer the research question: Which financial distress models are more accurate in predicting financial distress in a South African context?

4.3.1 Pooled data

Table 22 presents a summary of the accuracy rate of each model for the validation sample when pooled data was used. When examining the overall results of the validation sample, the binary logit model produced the highest accuracy with 82.4% of the sample correctly classified. This was better than its fellow two-state model (MDA) which produced an overall accuracy rate of 74.3%. It is interesting to observe that with the introduction of a State 2 firms (financially weak), the mixed and multinomial logit models did not produce significantly better results on pooled data than the binary models. A possible reason for this could be that these firms (State 2, financially weak) do not represent borderline cases of financial distress, that is, they have relatively similar characteristics as State 1 (non-failed) firms. However, a closer examination of the results show that the mixed and multinomial logit models were better at correctly classifying State 0 firms with an accuracy rate of 80% for the multinomial model, 80% for the mixed logit model 1 and 77.1% for the mixed logit model 2. This was significantly better than the results obtained by the binary logit and MDA models, which achieved accuracy rates of 68.6% and 65.7% respectively.

The mixed and multinomial logit models produced respectable results for State 1 (non-failed) firms with 76.9% accuracy rate for the multinomial logit, 82.1% for mixed logit model 1 and 69.2% for the mixed logit model 2. The multistate models produced less accurate results for State 2 firms; with accuracy rates of 62.5%, 65% and 55% respectively. Overall the mixed logit model 1 achieved the highest accuracy rate of all the estimated multistate models, with a total accuracy level of 75.4%. This model also had the best accuracy levels for State 1 and State 2 firms when compared to the other multistate models.

Table 22: Summary of classification (pooled data)

Classification Validation sample	**State 0**	**State 1**	**State 2**	**Overall**
MDA	n=35	n=39	-	n=74
Correct %	65.70%	82.10%	-	74.30%
Logit	n=35	n=39	-	n=74
Correct %	68.60%	94.90%	-	82.40%
Multinomial logit	n=35	n=39	n=40	n=114
Correct %	80.00%	76.90%	62.50%	72.80%
Mixed logit - Model 1	n=35	n=39	n=40	n=114
Correct %	80.00%	82.10%	65.00%	75.40%
Mixed logit - Model 2	n=35	n=39	n=40	n=114
Correct %	77.10%	69.20%	55.00%	66.70%

4.3.2 Lead time from financial distress

Table 23 contains the results of the accuracy rates of all the models at t=1, t=3 and t=5. As can be seen for the two-state models (logit and MDA) their predictive power, on average, drops as the lead time from financial distress increases, which is consistent with the seminal studies of Altman (1968) and Ohlson (1980). At t=1 and t=3, the binary logit model produced an accuracy rate of 85.7% and 100% for State 0 and 1 firms respectively. This was notably better than what was produced by its fellow two-state model (MDA) with 71.43% at t=1 and 85.71% at t=3 for State 0, and 75% at t=1 and 87.5% at t=3 State 1. On one hand, the binary logit model produced relatively high accuracy rate (85.71%) at t=5 for State 1 firms; however, on the other hand, it produced relatively low accuracy rate (33.33%) at t=5 for State 0 which suggests its predictive power is less reliable in this regard. Similar results are observed for the MDA model. It can therefore be concluded that the two-state models produce reliable results for State 0 (failed) firms up to t=3, but beyond that the results becomes less reliable.

Unlike the two-state models, the predictive power of the multistate models, on average, does not drop significantly as the lead time from financial distress increases. The multinomial logit model correctly classified 100% of State 0 (failed) firms at t=1 and t=3, and maintained a relatively high accuracy rate (83.3%) at t=5. However, for State 1 and 2 firms, its accuracy rate fluctuated but still remained relatively high, at no point dropping below 75%. The mixed logit (Model 1) correctly classified 85.7% of State 0 firms at t=1 and t=3 and marginally dropped to 83.3% at t=5 which suggest that the model is capable of reliably predicting financial distress, even when the lead time increases. Although its accuracy rates remained moderately high (> 71.4%) for State 1 firms; its predictive power for State 2 firms, remained relatively low (< 62.5%) for t=3 and beyond.

Mixed logit (Model 2) produced some interesting results. On average, as the lead time from financial distress increased, its classification rate increased. For State 0 firms, it did not correctly classify any of the firms; however, at t=3 and t=5, this increased to 85.7% and 83.3%. Similar patten were observed for State 2 firms. For State 1 firms, it produced 100% accuracy rate at t=1 which was maintained at t=5.

Table 23: Summary of classification (t=1, t=3, t=5)

Model	State 0	State 1	State 2
t=1			
MDA	71.4%	75.0%	-
Logit	85.7%	100.0%	-
Multinomial logit	100.0%	87.5%	75.0%
Mixed logit - Model 1	85.7%	75.0%	87.5%
Mixed logit - Model 2	0.0%	100.0%	0.0%
t=3			
MDA	85.7%	87.5%	-
Logit	85.7%	100.0%	-
Multinomial logit	100.0%	75.0%	87.5%
Mixed logit - Model 1	85.7%	75.0%	62.5%
Mixed logit - Model 2	85.7%	75.0%	75.0%
t=5			
MDA	33.3%	85.7%	-
Logit	33.3%	85.7%	-
Multinomial logit	83.3%	100.0%	75.0%
Mixed logit - Model 1	83.3%	85.7%	50.0%
Mixed logit - Model 2	83.3%	100.0%	87.5%

Although there is no one best model that performed well in every respect; it is interesting to note that the multinomial logit model produced very high accuracy rates for State 0 (failed) firms, even when the lead time from financial distress was increased, which is of particular interest in this study. The mixed logit (Model 1) also produced very encouraging results for up to 3 years prior to financial distress; and Model 2 performed well at t=5. This is important finding, since longer lead time will provide management and shareholders with sufficient time to take corrective actions to restore the financial health of the company and potentially evade financial distress.

Chapter 5 – Conclusion

The principal objective of this study was to assess which financial distress prediction models are more accurate in predicting financial distress for JSE listed companies. This entailed comparing rudimentary modelling techniques, that is, MDA and binary logit, to more advanced modelling techniques, that is, multinomial and mixed logit models. The binary logit model had the highest overall accuracy rate; however, caution must be exercised since this model performed well in correctly classifying State 1 (non-failed) firms but performed poorly with regards to State 0 (failed) firms which was of particular interest in this study. The multistate models performed best with regards to correctly classifying State 0 and 1 firms; however, they performed poorly with regards to State 2 (financially weak) firms. With the exception of the mixed logit Model 2, the predictive power, on average, of the models dropped as the lead time from financial distress increased, which is consistent with earlier studies. However the Mixed logit Model 2 did not perform that well at t=1. By taking a broad approach and estimating both multinomial and mixed logit models it may be possible to provide the best information for managers, especially as they try to predict corporate failure several years before it happens.

Although the selection criteria of the predictor variables was based on popularity in the literature and therefore each model had its own set of predictor variables; there were overlapping variables occurring in each model. Liquidity (variables a1, b3 and b4); size (variable b1) and profitability (variable a3 and b6) were found to be statistically significant and important predictors of financial distress. These findings are consistent with the literature and previous studies. Abdullah et al. (2008); Altman (1968); Bellovary et al. (2007); Bunyaminu and Issah (2012); Mihalovic (2016); Ohlson (1980) found that liquidity ratios (Current Ratio and Working Capital / Total Assets); size (Total Assets) and profitability (Net Income to Total Assets) are important predictor of financially distressed companies. It is interesting to note that leverage (variable b2 and b5) and solvency (variable c6) were not found to be statistically significant. These variables are borrowed from the seminal studies of Ohlson (1980) and Altman et al. (1977) which found them to be statistically significant. However, the finding from this study is consistent with Johnsen and Melicher (1994) which found that solvency was not an important predictor of financial distress and leverage was only significant for bankrupt versus nonbankrupt firms but fails to discriminate between bankrupt and financially weak firms. It is therefore plausible to generate early warning signals for South African JSE listed companies which are accurate and reliable when using liquidity, firm size and profitability ratios.

Even though financial distress prediction models are not underpinned by any economic theory; the results derived from this study are encouraging and can assist various stakeholders, including, managers, company auditors, employees and lenders to identify firms that are likely to fail and provide early warnings and take corrective actions to lower the probability of those firms failing.

There are several possibilities for future research arising from this study. Firstly, the population of this study was confined to companies listed on the JSE, however, Figure 2 illustrates that there are significantly more companies which are not listed that face financial distress and end up being liquidated. However, there could be challenges in terms of collecting the data since these firms are not listed. Secondly, it may be possible to integrate qualitative variables, such as corporate governance indicators and audit opinions as independent variables in the various models. Finally, the continuum of financial distress could be extended to include other categories of financial distress such as distressed restructures and mergers, defaults on loan repayments and financial covenant breaches.

Appendixes

Appendix 1: Mixed logit Model 1 (Fixed effects)

Source	F	df1	df2	Sig.
Corrected Model	6.626	12	398	0
b1	11.139	2	398	0
b4	2.643	2	398	0.072
b6	9.54	2	398	0
b7	6.119	2	398	0.002
b9	0.661	2	398	0.517
c6	0.392	2	398	0.676

Appendix 2: Mixed logit Model 1 (Fixed Coefficients)

State	Model Term	Coefficient	Std. Error	t	Sig.	95% Confidence Interval		Exp (Coefficient)	95% Confidence Interval for Exp(Coefficient)	
						Lower	Upper		Lower	Upper
1	Intercept	-1.052	1.0644	-0.988	0.324	-3.145	1.04	0.349	0.043	2.83
	b1	0.108	0.0611	1.768	0.078	-0.012	0.228	1.114	0.988	1.256
	b4	-0.866	0.4216	-2.054	0.041	-1.695	-0.037	0.421	0.184	0.964
	b6	6.826	1.7864	3.821	0	3.314	10.338	921.577	27.497	30887.328
	b7	1.639	0.668	2.453	0.015	0.326	2.952	5.149	1.385	19.146
	b9	-0.109	0.3104	-0.352	0.725	-0.72	0.501	0.896	0.487	1.65
	c6	-0.521	0.6471	-0.805	0.421	-1.793	0.751	0.594	0.166	2.119
2	Intercept	-3.076	0.9866	-3.118	0.002	-5.016	-1.136	0.046	0.007	0.321
	b1	0.272	0.0583	4.662	0	0.157	0.386	1.312	1.17	1.471
	b4	-0.647	0.3629	-1.784	0.075	-1.361	0.066	0.523	0.256	1.068
	b6	-0.721	1.5691	-0.459	0.646	-3.805	2.364	0.487	0.022	10.637
	b7	-0.735	0.5081	-1.447	0.149	-1.734	0.264	0.48	0.177	1.302
	b9	0.328	0.3415	0.961	0.337	-0.343	1	1.389	0.71	2.717
	c6	-0.016	0.5603	-0.029	0.977	-1.118	1.085	0.984	0.327	2.96

Appendix 3: Mixed logit Model 1 (Random effects)

State	Random Effect Covariance	Estimate	Std. Error	Z	Sig.	95% Confidence Interval	
						Lower	Upper
1	b1	1.498E-7[a]	.	.	.	.	.
	b4	2.485E-9[a]	.	.	.	.	.
	b6	2.795E-11[a]	.	.	.	.	.
	b7	0.792	0.938	0.844	0.399	0.078	8.072
	b9	0.114	0.338	0.336	0.737	0	38.802
	c6	9.503E-7[a]	.	.	.	.	.
2	b1	4.530E-11[a]	.	.	.	.	.
	b4	.000[a]	.	.	.	.	.
	b6	1.018E-11[a]	.	.	.	.	.
	b7	2.178E-12[a]	.	.	.	.	.
	b9	0.296	0.362	0.817	0.414	0.027	3.26
	c6	.000[a]	.	.	.	.	.

a. This parameter is redundant.

Appendix 4: Mixed logit Model 2 (Fixed Effects)

Fixed Effects[a]

Source	F	df1	df2	Sig.
Corrected Model	12.618	8	402	0
b1	11.044	2	402	0
b6	10.053	2	402	0
b7	8.561	2	402	0
b4	2.762	2	402	0.064

Probability distribution: Multinomial

Link function: Generalized logit

a. Target: State

Appendix 5: Mixed logit Model 2 (Fixed Coefficients)

Fixed Coefficients[a]

State	Model Term	Coefficient	Std. Error	t	Sig.	95% Confidence Interval		Exp(Coefficient)	95% Confidence Interval for Exp(Coefficient)	
						Lower	Upper		Lower	Upper
1	Intercept	-1.383	0.8478	-1.631	0.104	-3.05	0.284	0.251	0.047	1.328
	b1	0.099	0.0602	1.639	0.102	-0.02	0.217	1.104	0.981	1.242
	b6	6.509	1.6004	4.067	0	3.363	9.655	671.324	28.88	15605.154
	b7	1.405	0.5137	2.736	0.006	0.396	2.415	4.077	1.485	11.193
	b4	-0.688	0.3603	-1.909	0.057	-1.396	0.021	0.503	0.248	1.021
2	Intercept	-3.032	0.8057	-3.763	0	-4.615	-1.448	0.048	0.01	0.235
	b1	0.266	0.0576	4.621	0	0.153	0.38	1.305	1.165	1.462
	b6	-0.371	1.4767	-0.251	0.802	-3.274	2.532	0.69	0.038	12.579
	b7	-0.739	0.5105	-1.447	0.149	-1.743	0.265	0.478	0.175	1.303
	b4	-0.627	0.3083	-2.034	0.043	-1.233	-0.021	0.534	0.291	0.979

Probability distribution: Multinomial

Link function: Generalized logit

a. Target: State

Appendix 6: Mixed logit Model 2 (Random Effects)

Random Effect

State	Random Effect Covariance	Estimate	Std. Error	Z	Sig.	95% Confidence Interval	
						Lower	Upper
1	Var(b9)	0.041	0.264	0.154	0.878	1.21E-07	13659.296
	Var(c6)	0.02	0.107	0.19	0.849	6.64E-07	617.414
2	Var(b9)	0.305	0.338	0.901	0.368	0.035	2.685
	Var(c6)	.000[a]	.	.	.	.	.

Covariance Structure: Variance components

Subject Specification: Yearsbeforeeventdate

a. This parameter is redundant.

Appendix 7: Summary of validation sample results (MDA)

MDA	State 0	State 1	Overall
Pooled data (reporting periods 1 to 5)			
n	35	39	74
Correct %	65.7%	82.1%	74.3%
t=1			
n	7	8	15
Correct %	71.4%	75.0%	73.3%
t=3			
n	7	8	15
Correct %	85.7%	87.5%	86.7%
t=5			
n	6	7	13
Correct %	33.3%	85.7%	61.5%

Appendix 8: Summary of validation sample results (Logit)

Logit	State 0	State 1	Overall
Pooled data (reporting periods 1 to 5)			
n	35	39	74
Correct %	68.6%	94.9%	82.4%
t=1			
n	7	8	15
Correct %	85.7%	100.0%	93.3%
t=3			
n	7	8	15
Correct %	85.7%	100.0%	93.3%
t=5			
n	6	7	13
Correct %	33.3%	85.7%	61.5%

Appendix 9: Summary of validation sample results (Multinomial)

Multinomial	State 0	State 1	State 2	Overall
Pooled data (reporting periods 1 to 5)				
n	35	39	40	114
Correct %	80.0%	76.9%	62.5%	72.8%
t=1				
n	7	8	8	23
Correct %	100.0%	87.5%	75.0%	87.0%
t=3				
n	7	8	8	23
Correct %	100.0%	75.0%	87.5%	87.0%
t=5				
n	6	7	8	21
Correct %	83.3%	100.0%	75.0%	85.7%

Appendix 10: Summary of validation sample (Mixed logit: Model 1)

Mixed logit: Model 1	State 0	State 1	State 2	Overall
Pooled data (reporting periods 1 to 5)				
n	35	39	40	114
Correct %	80.0%	82.1%	65.0%	75.4%
t=1				
n	7	8	8	23
Correct %	85.7%	75.0%	87.5%	82.6%
t=3				
n	7	8	8	23
Correct %	85.7%	75.0%	62.5%	73.9%
t=5				
n	6	7	8	21
Correct %	83.3%	85.7%	50.0%	71.4%

Appendix 11: Summary of validation sample (Mixed logit: Model 2)

Mixed logit: Model 2	State 0	State 1	State 2	Overall
Pooled data (reporting periods 1 to 5)				
n	35	39	40	114
Correct %	77.1%	69.2%	55.0%	66.7%
t=1				
n	7	8	8	23
Correct %	0.0%	100.0%	0.0%	34.8%
t=3				
n	7	8	8	23
Correct %	85.7%	75.0%	75.0%	78.3%
t=5				
n	6	7	8	21
Correct %	83.3%	100.0%	87.5%	90.5%

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