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Inventory Dynamics, Asset Prices and Business Cycles

by

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Declaration

I declare that this thesis, which is titled *Inventory Dynamics, Asset Prices and Business Cycles*, is solely my own work. I confirm that, not a single portion of the thesis has been used for any other professional, academic or educational purposes.

Signature:

Name: Sedjro Aaron Alovokpinhou
Date: 28/06/2021

Dedication

To my family and friends.

Acknowledgements

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Abstract

The thesis has three main papers.

The first paper presents a model of inventory dynamics to explain the sign puzzle–the positive relationship that is often found between inventories and interest rates. Firstly, the study updates the empirical evidence on the puzzle in the United States, Canada and South Africa by means of a log linear inventory model and an inventory growth model. The empirical results reveal that the positive effect of interest rates on inventories still exists in the United States and Canada, especially in the data sample period that precedes the global financial crisis of 2007-2009. However, the study finds no evidence-in support of the sign puzzle in South Africa. Secondly, the theoretical analysis of the structural model that we develop in this study reveals that the sign puzzle is mainly driven by the elasticity of benefits for inventory holdings and the sensitivity of stockouts cost to changes in the cost of capital.

The second paper shows that the New Keynesian dynamic stochastic general equilibrium (NK DSGE) model which incorporates inventory dynamics can generate a hump-shaped response of macro variables to shocks, even without habit-formation. Furthermore, we find that only a moderate level of price indexation, far below those normally estimated at the macro level, is needed to generate a gradual response of inflation to a monetary policy shock. Moreover, the model replicates stylized facts about inventory dynamics, such as the procyclicality of inventories and the countercyclical nature of inventory-sales ratio. Finally, the results are robust to variations in the probability of price stickiness and the inventory adjustment cost parameter.

The third paper shows that adding inventory dynamics rather than habit formation to a small open New Keynesian dynamic stochastic general equilibrium (NK DSGE) model, amplifies the immediate response of asset prices to a monetary policy shock. Specifically, and without the financial accelerator effect, a 100 basis-point increase in the nominal interest rate leads the stock price to decline by 17.06% in the model with inventories and the stock price declines by 4.38% in the model without inventories. The results suggest that inventories magnify the financial accelerator effect and habit formation reduces the amplifying role of inventories. The model also replicates a procyclical response in inventories and a countercyclical response in the inventory-sales ratio and the equity premium.

In summary, the first paper provides a solution to the sign puzzle and the second paper shows that inventories play the role of habit formation in a New-Keynesian DSGE model. The third paper reveals that the response of asset prices in macro-finance models without inventories could be underestimated.

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1 Introduction

1.1 Background of the study

We are in the midst of a pandemic and this global health crisis, also known as Covid-19 epitomizes almost every aspect of the role inventories play in the real economy. The body of the literature that studied inventories up to the end of 1990 notes that, at the firm level, inventories can be used to smooth production, avoid stockouts, speculate on price changes and to avoid delays in delivery (Blinder and Maccini, 1991). This implies that inventories could become extremely important during economic and financial crises, as well as during any other crisis that disrupts the supply and demand factors of an economy. For instance, as noted by Maccini *et al.* (2015), inventories accounted for almost one third of the decline in the US output during the global financial crisis of 2007-2009. The ongoing crisis (Covid-19) has affected both demand and supply sides factors. Therefore, inventories may be of great importance to the economies with highly developed supply chain management systems.

At the macro level, inventories serve as a transmission channel for monetary policy. This is because firms that hold inventories incur the opportunity cost of doing so. Therefore, an increase in the real or nominal interest rates is expected to lead to a decline in inventories and therefore output. This channel is known as the supply side effect of monetary policy and in sources such as Barth and Ramey (2001), Ravena and Wash (2006) and Henzel *et al.* (2009) is termed the cost channel. The demand side effect of the monetary policy shock works through the relationship between sales and inventories. Thus, if a contractionary monetary policy shock leads to a decline in sales, inventories would be the first to absorb the negative demand shock. If the shock persists, the decline in inventories would depend on expectations about future demand and, the motives of inventory holdings. However, the increase in the opportunity cost of holding inventories would make it less important to accumulate stocks, while the decline in both sales and inventories would lead to a significant decline in output.

It is important to note that empirical studies on the relationship between inventories and interest rates report mixed results. Some report a negative effect of interest rates on inventories (Irvine, 1981; Akhtar, 1983), while others report a positive effect (Maccini et al, 2004; Jones and Tuzel, 2013). The positive effect has been termed the sign puzzle and needs close attention. Furthermore, even though inventories represent a smoothing device in macro models (Blinder and Fischer, 1982; Boldrin *et al.*, 2001; Wen,2005), the modern macroeconomic models such as the New-Keynesian general equilibrium models of Christiano *et al.* (2005) and Smets and Wouters (2007) rely heavily on habit formation to introduce a delayed response in consumption and thus output. In addition, the role of inventories in macro-finance models can be traced back to the financial accelerator theory presented by Bernanke *et al.* (1996, 1999). Nonetheless, the impact of inventories on the response of asset prices to a monetary policy shock is not considered in the literature.

1.2 Motivation for the study

This study is motivated by (a) the scarcity of both theoretical and empirical inventory research in emerging economies, and (b) the gap that exists in the literature where the theoretical relationship between inventories and interest rates is not supported by the data, and (c) the gap that exists in the New Keynesian literature, where models that incorporate inventory behaviour are rare[1], and (d) the gap that exists in the New-Keynesian literature, where general equilibrium models that incorporate the asset pricing role of inventories are rare.

1.3 Objectives of the study

a) To formulate a model of inventory dynamics to explain the possibility of the sign puzzle
b) To test whether inventories can play the role of habit formation in a New-Keynesian model

[1] For example, though the real business cycle (RBC) model of Kydland and Prescott (1982) incorporates the behaviour of inventories, the importance of inventories in the dynamic stochastic general equilibrium models of the New-Keynesian type is not much studied.

c) To extend the New-Keynesian model to study how inventories influence the response of asset prices to a monetary policy shock.

1.4 Research questions

a) What are the structural parameters that drive the sign puzzle?
b) Can inventories play the role of habit formation in a new Keynesian model?
c) How do asset prices respond to a monetary policy shock in a New-Keynesian model with inventory dynamics?

1.5 Research gaps

The study addresses the following research gaps. Firstly, we extend the empirical evidence on the relationship between inventories and interest rates and then formulate a model of inventory dynamics to explain the possibility of the sign puzzle. Secondly, we use inventories to explain the hump-shaped responses in aggregate variables. Thirdly, we employ the financial accelerator mechanism to link inventories and asset prices and sudsequently study how the presence of inventories limits or amplifies the impact of monetary policy shock on asset prices.

1.6 Contribution of the study

This study provides both theoretical and empirical contributions to the existing literature. First of all, we extend the empirical evidence on the sign puzzle using a log linear model and an inventory growth model. We then explicitly model the dynamics of inventory fluctuations in order to expose the structural parameters that drive the puzzle. Secondly, we employ a dynamic stochastic general equilibrium (DSGE) model to test whether inventories play the role of habit formation. Finally, using the financial accelerator mechanism, we significantly add to the literature by testing whether the presence of inventories amplifies the response of asset prices to a monetary policy shock.

2 Chapter. Inventory Dynamics, Interest Rates and the Sign Puzzle

2.1 Introduction

This study develops a model of inventory dynamics to explain the sign puzzle– the positive relationship that is found between inventories and the cost of capital as measured by the real interest rate. Inventories are known to play an important role in explaining economic fluctuations (Metzler, 1941; Blanchard, 1983). For instance, Wen (2005) notes that the $252 billion fall in the Gross Domestic Product (GDP) of the United States in 1982 was accompanied by $219 billion fall in inventory stock. Furthermore, Maccini *et al.* (2015) show that during the 2007–2009 recession, inventories accounted for one third of the decline in the US output. In addition, the current global health crisis (Covid-19) has led the inventory-sales ratio of the US economy to jump from its level of 1.39 in December 2019 to 1.67 in April 2020, and then to drop to 1.51 by the end of May 2020. These rises and falls show the shock-absorbing nature of inventories.

Inventories are also known to serve as transmission channels for the real effect of monetary policy (Frankel, 2006; Barth and Ramey, 2001). Inventories are expected to decline following a rise in the real short term interest rate (Maccini et al., 2004). This negative relationship arises from the fact that the real short term interest is the opportunity cost that firms incur by accumulating inventories. On the relationship between inventories and interest rates, early studies such as those by Gertler and Gilchrist (1994) and Bernanke *et al.* (1996) find that the inventory stock of small manufacturing firms responds negatively to positive changes in the monetary policy rates. Maccini *et al.* (2004) show that the negative relationship between finished goods inventories and interest rates exists only in the long term, but Bivin (2010) finds that the long term effect for the period 1997-2007 for finished goods, and the results for the raw materials and work in progress inventories are less supportive of the negative long term relationship.

The gap we intend to fill in this study is to update and extend the empirical evidence on the puzzle and also to provide a theoretical explanation for the sign puzzle. Theoretical explanations of the sign puzzle are rare in the literature. Nonetheless, Benati and Lubik (2014) note that the total effect of monetary policy on inventories depends on whether its effect on sales and therefore inventories dominates its direct effect on inventories. Bivin (2011) also notes that when the benefit of inventory holdings (in terms of markups) is significant, firms will still accumulate inventories. This view is in line with the relationship noted between stockouts cost and markups in Bils (2016). A more recent study by Jones and Tuzel (2013) shows that risk premiums rather than short term interest rates should be the appropriate costs of capital for inventory holdings. What is rare in the literature, however, is evidence of the puzzle in countries other than the US and also a structural model to explain the sign puzzle.

This research study contributes both theoretically and empirically to the existing literature. Firstly, we provide empirical evidence on the sign puzzle in economies such as the United States, Canada and South Africa. This is accomplished by estimating a log linear inventory model and an inventory growth model for these countries where we employ short term real and nominal interest rates as the cost of capital for inventory holdings. For example, previous studies such as those by Maccini *et al.* (2004) and Jones and Tuzel (2013) focus on the US data. Therefore, extending the empirical evidence to include countries like Canada and South Africa would shed more light on the status of the puzzle in small open economies. Secondly, we expose the structural parameters that govern the sign puzzle by calibrating the coefficient of the cost of capital of the structural inventory model in the countries that present with the sign puzzle.

The remainder of this paper is structured as follows: Section 2 uses data on inventories to provide evidence on the micro behaviour of inventories and the relationship between inventories and interest rates. Section 3 updates the empirical evidence on the puzzle, and Section 4 presents the structural model, as well as the estimates of the structural model. Section 5 exposes the structural parameters that govern the sign puzzle. Section 6 concludes the paper and provides some policy implications.

2.2 The economics of inventories and the role of interest rates

2.2.1 The importance and role of inventories

The role of inventories in the real economy can be looked at from both micro and macro perspectives. According to Blinder and Maccini (1991), the microeconomic role of inventories can be epitomized by the motives for holding inventories at the firm level. In this regard, these scholars note the incentives for the firm to ensure a smooth production process, to reduce costs related to stockouts, to use inventories to take advantages of price fluctuations and to reduce delays in delivery. The role of inventories at the macro level can be summarized in terms of their role in driving the up and down fluctuations in the aggregate variables. Inventories represent a source of persistence in output (Blinder and Fischer, 1981). They also allow firms to make fewer price changes to restor equilibrium to the goods market. In other words, inventories play an important role in real price stickiness (Blinder, 1982).

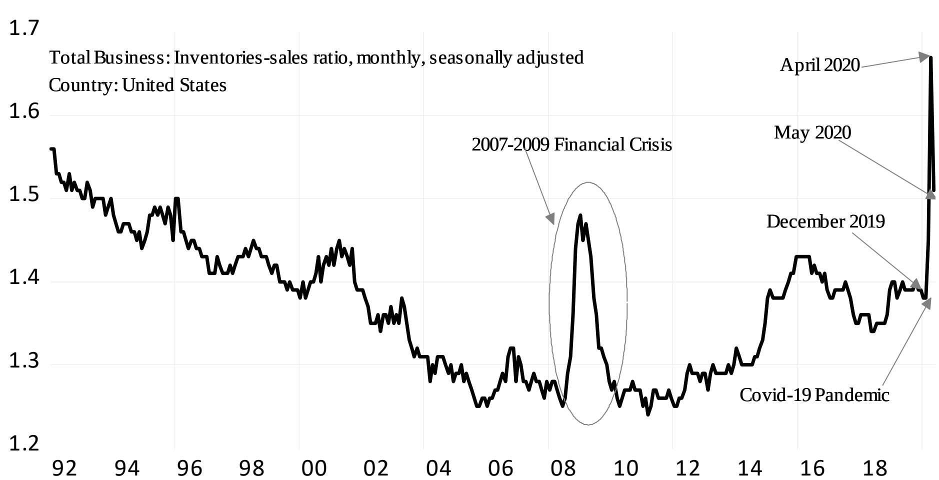


Figure 1: The role of inventories in crises

In crises, inventories act as shock absorbers and at the same time allow firms to mitigate the impact of the crisis on revenues. Figure 1 shows the monthly trend of inventory-sales ratio for the United States total business inventory from 1992. During the global financial crisis of 2007-2009, this ratio rose significantly from nearly 1.28 by the end of 2007 to approximately 1.47 in March, 2009, before declining to a level of 1.28 in December, 2009. The rise in inventories between December, 2007 and March, 2009 demonstrates the shock-absorbing nature of inventories and is a sign of a worsening economic depression. The drop between March, 2009 and December, 2009 could be a sign of a slow recovery where firms first eliminate the surplus inventories, or a sign of worsening depression, involving a decline in both output and inventories.

Like the financial crisis, the same jump in the monthly inventory-sales ratio during the 2007-2009 crisis is also noted in the case of the current global health crisis also known as the Covid-19 pandemic. The total business inventory-sales ratio was, on average, 1.39 in December, 2019. It rose to a level of 1.67 by the end of April, 2020 after dropping to 1.51 by the end of May, 2020. We are still in the midst of the health crisis and the fluctuations in inventory-sales ratio are tied to the fluctuations in demand or sales. What is important to note here is that, the rise in the inventory-sales ratio noted for both crises does not imply that inventories are rising, but instead demand (sales) is falling faster than inventories.

The literature on inventory behaviour notes that two basic theories are employed by researchers to investigate the motives of inventory holdings at the firm level (Wen, 2005). These are the production smoothing model and the stockout avoidance model. According to Blinder (1986) and Fitzgerald (1997), the assumptions that underlie the production smoothing model are: (i) demand is required to vary over-time, (ii) the production cost function is required to be convex and (iii) the goods produced are required to be inventoriable or storable. Thus, under these assumptions, the production smoothing model predicts that inventory investment is countercyclical, and inventory-sales ratio is procyclical, and that the variance of sales is greater than the variance of output. Moreover, if demand has a random component then inventories might act as a buffer stock.

The failure of the production smoothing model to reproduce the procyclical behaviour of inventory data and the higher variance of output over the variance of sales has led to many modifications, especially those relating to the introduction of the stockout avoidance model and the introduction of observable and unobservable cost shocks. Empirical evidence on how the production smoothing model has failed to match the data is noted in at least Blanchard (1983) and Blinder (1986). The stockout avoidance model reconciles the behaviour of the data and predicts a procyclical behaviour in inventories and a countercyclical movement in the inventory-sales ratio. Wen (2005) notes that the stockout avoidance model is better able to explain the key inventory stylized facts than the production smoothing model.

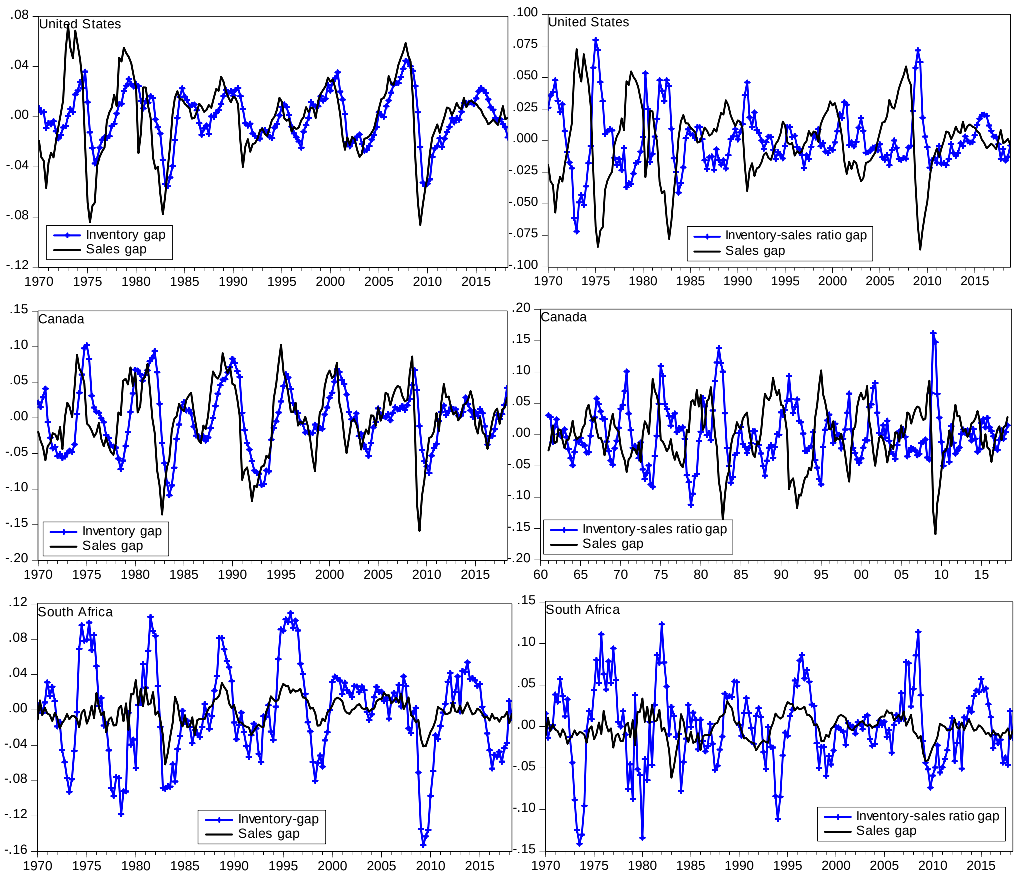


Figure 2: Cyclical fluctuations in inventories, inventory-sales ratio and sales

A chronological review of the evolution of the modifications to the production smoothing model are summarized in Blinder and Maccini (1991) and Fitzgerald (1997). These studies provide more detail, as well as the chronological transformation of inventory literature. However, it is important to note that the stockout avoidance model, although noted in Blanchard (1983), is mainly attributed to the work of Khan (1987, 1992). In order to show how the predictions of the production smoothing model have failed the data, Figure 2 shows how inventories as deviation from the HP-trend fluctuate with the cyclical variations in sales. We also document how inventory-sales ratio gap fluctuate with sales. We provide evidence on these relationships in three countries namely the United States, Canada and South Africa.

Although, the lack of data on inventory stocks limits our choice of countries, the chosen countries summarize all types of economies. We follow Sarte et al. (2015) and use the composition of inventory data that includes raw materials, work in progress and finished goods. The data for the United States and South Africa consist of industrial and trade inventories while the data for Canada pertain mainly to the manufacturing inventory. We collect the South African data from the South African Reserve Bank (SARB) database and the data for the United States and Canada from the Reserve Bank of St Louis' database (FRED). The data covers the quarterly period 1970Q1–2018Q2 for all the countries.

The results indicate that inventories are procyclical in the United States, Canada and South Africa and inventory-sales ratios are countercyclical in the United States and Canada. The behaviour of inventories and inventory-sales ratio in the United States and Canada are consistent with the findings in Kryvtsov and Midrigan (2013). However, the inventory-sales ratio for South Africa displays a very weak type of countercyclical behaviour. These findings are in line with the predictions of the stockout avoidance model.

2.2.2 Interest rates and inventories

In the same era in which micro and macro behaviour of inventories were investigated, very few studies focusing on the relationship between inventories and interest rates were found. However, most inventory studies do mention the role of interest rates in passing or include interest rates as a measure of the cost of capital for holding a desired level of inventories. Among the few studies that pay a particular attention to interest rates, Irvine (1981) and Akhtar (1983) find that interest rates have a significant and negative effect on inventory investment. On the other hand, Maccini and Rossana (1981) do not find any significant effect of real interest rates on inventories.

Recent studies by Maccini *et al.* (2004) and Jones and Tuzel (2013) report insignificant effects of interest rates on inventories in the short term. Maccini *et al.* (2004) use the regime switching approach and show that the relationship between inventories and interest rates is negative and significant in the long run. Jones and Tuzel (2013) argue that risk premiums constructed from bond and equity portfolios constitute the appropriate costs of capital for inventory holdings. Their findings are in line with the control variables suggested by Frankel (2006) and the findings in Gortz *et al.* (2019, 2020) that debt and equity costs move countercyclically with inventory demand. However, Jones and Tuzel do mention that interest rates variables would be poor proxies of the cost of capital only if investment in inventories were to be considered risky. Further, Maccini *et al.* (2004) note that there is no consensus on which between the real and the nominal interest rate to use in inventory equations.

Table 1: Correlation with the growth rate of inventory

	United States			Canada			South Africa		
	s_1	s_2	s_3	s_1	s_2	s_3	s_1	s_2	s_3
ρ_{ny}	0.52	0.46	0.60	0.42	0.43	0.42	0.66	0.62	0.81
ρ_{ny^s}	0.58	0.50	0.71	0.50	0.51	0.46	0.57	0.53	0.74
ρ_{nr}	0.17	0.11	−0.08	0.07	0.02	0.06	−0.16	−0.25	−0.04
ρ_{nR}	0.09	0.08	−0.29	0.02	−0.003	−0.15	0.02	0.01	0.06

ρ_{ny} (inventory and output), ρ_{ny^s} (inventory and sales)
ρ_{nr} (inventory and nominal rates), ρ_{nR} (inventory and real rates)

To show the relationship between inventories and interest rates, we compute the growth rate of inventory stock in each country and also compute the correlation coefficeints between the growth rate of inventory and the variables such as output growth, sales growth and the real and nominal short term interest rates. The correlation metrics are presented for three different samples. The first is the full sample (s_1) which covers the period 1970QQ1-2018Q2, (s_2) ends before the global financial crisis, that is 1970Q1-2006Q4 and (s_3) covers the period 2007Q1-2018Q2. The correlation metrics are presented in Table 1.

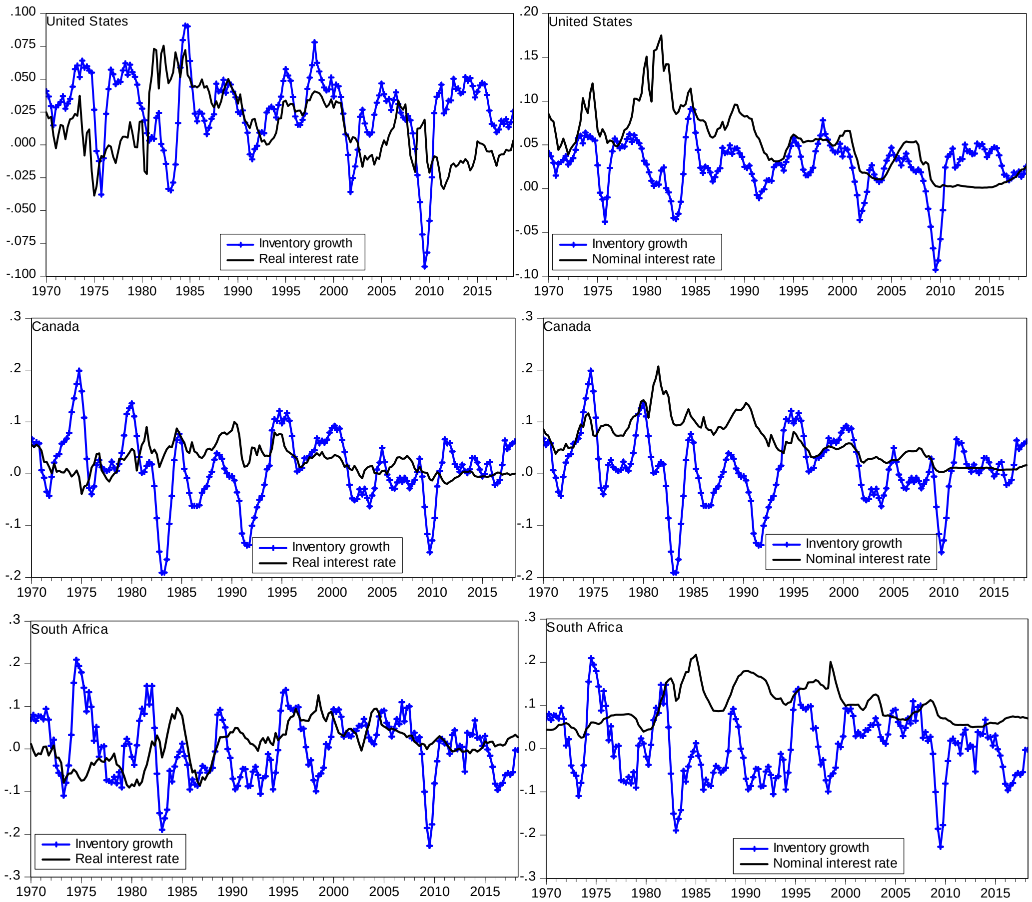


Figure 3: The relationship between inventory growth and interest rates

We also plot the growth rate of the inventory stock and the interest rate variables, which are presented in Figure 2. The results in Table 1 and Figure

2 show that in the United States and Canada, the nominal interest rate has a positive and weak correlation with inventory growth for the first and second samples, but a negative for the third sample. The real interest rate displays a positive but also a weak correlation with inventory growth across all samples for the United States while the relationship is weakly negative for the first and second samples for Canada, and weakly positive for the third sample. It is also important to note that in the United States and Canada, both output growth and sales growth are positively correlated with inventory growth. This suggests that the stockout avoidance motive dominates the data for the United States and Canada.

When it comes to South Africa, the nominal interest rate shows a negative correlation with inventory growth across all samples, while the real interest rate displays a weakly positive correlation with inventory growth. We note, however, that the relationships between inventory growth and both output and sales growth respectively is positive across all samples. This implies that the stockout avoidance motive also dominates the inventory data in South Africa. Although, we find mixed evidence of positive and negative coerrelation between interest rates and inventories, the main conclusion from the analysis above is that the relationship between interest rates and inventory growth is weak in most of the countries.

2.3 Inventory dynamics and the sign puzzle

2.3.1 Empirical evidence from the United States, Canada and South Africa

First of all, we extend the empirical evidence on the sign puzzle to Canada and South Africa. This allows us to compare the respective relationships between inventories and interest rates in these countries to that of the US. The empirical evidence on the puzzle in the literature focuses on the US data and is rarely applied to other developed and emerging economies. We follow Maccini et al. (2004), Frankel (2006), and Jones and Tuzel (2013) and present reduced form estimates. We employ a log linear inventory model and an inventory growth model. The econometric specification is presented as follows:

$$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 \left(y_t^s, y_t\right) - \beta_4 \left(R_t, r_t\right) + \varepsilon_t \qquad (1)$$

where n_t can be in log form or as the growth rate of inventory stock. The variables y_t^s, y_t are sales and output respectively and can also be in log form or as the growth rate, depending on whether it is a log linear or a growth model. The variables R_t, r_t are the real and nominal interest rates respectively, and ε_t is the disturbance term. Given the samples already defined in the previous section, the use of output or sales and nominal or real interest rates implies four estimations for each sample and for each type of model. That is each cost of capital enters the model one at a time. Furthermore, output and sales and output growth and sale growth also enter the model one at a time.

The inclusion of the first and second lags of the inventory variable allow us to correct for serial correlations. The specification also shows the expected signs of the coefficients. The coefficient on sales/output β_3 can be positive or negative. It is negative when the production smoothing motive dominates, and positive when the stockout avoidance motive dominates (Maccini *et al.*, 2004). β_4 is positive when there no sign puzzle and negative when the puzzle is present. We follow Jones and Tuzel (2013) and report reduced form coefficients and HAC standard errors calculated with the Newey and West (1987) approach.

Assuming that the coefficient of a variable is g^j, the results are reported such that g^{ys} is the coefficient when sales are used and g^y when the output series are used. The standard errors are reported in parentheses below the estimated coefficients. The adjusted coefficients of determination (Adj-R^2) are also reported, where this metric for the models when output/output growth series are used are presented in square brackets. The results for the first, second and third samples are reported in Tables 2,3 and 4 respectively. The discussion of the results is presented country by country.

United States

The results pertaining to the first sample (S_1), which is also the full sample, show that the coefficients of sales/output, sales growth/output growth are all positive and significant regardless of the cost of capital used in each model. The coefficients of the real and the nominal interest rates in the log linear model are negative and significant regardless of whether output or sales is used. This implies that if substituted in eq.(1), changes in the real or nominal short term interest rate will have a positive effect on inventories. This confirms the sign puzzle.

The coefficient of the nominal interest rate for the growth model is also negative and significant. However, though negative, it is not significant when the real interest rate is used as the cost of capital in the growth model. The results pertaining to the sub-samples (S_2) are similar to the one derived from the full sample (S_1). We note in the third sample (S_3) that the coefficient of the nominal interest rate in the log linear model is positive and significant. As such if substituted in eq.(1), changes in the nominal interest rate will have a negative effect on inventories. Nonetheless, this coefficient is not significant in the inventory growth model.

Canada

The results pertaining to the full sample (S_1) reveal that the coefficients of sales/output and sales growth/output growth are positive and significant for all of the specifications. The coefficient of the real interest rate displays a negative sign in the log linear model but not significant. The coefficient of the nominal interest also displays a negative sign but is significant when the output series is replaced by the sales series. This finding is consistent with the sign puzzle phenomenon. In the growth model, the coefficients of both the real and the nominal interest rates are positive but are not significant. The results pertaining to the sub-samples (S_2) are also consistent with the results of the full sample. Like the results of the third sample for the United States, we also note that in the third sample (S_3) for Canada, the nominal interest rate has a significant and a negative effect on inventories in the log linear model.

South Africa

The results pertaining to the full sample reveal that the models that describe the South African data best are, the log linear and the growth models with the nominal interest rate as the cost of capital. The coefficients of output/sales and sales/output growths are significant and are acompanied by the correct signs. The coefficients of the nominal interest rate variable are positive and also significant. This implies that when the coefficients are substituted in eq.(1), an increase in the nominal interest rate will lead to a decline in inventories. In the growth model, the coefficients of the real interest rate variable are positive but not significant.

Table 2: Estimates of the first sample

$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t(\mathrm{S}_1$, log linear model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.38* (0.10)	1.25* (0.08)	0.45* (0.09)	0.35* (0.08)	0.07* (0.01)	0.09* (0.01)	−0.06* (0.02)	−0.07* (0.02)	0.99 [0.99]
Canada	1.64* (0.06)	1.59* (0.06)	0.66* (0.06)	0.62* (0.06)	0.016** (0.006)	0.03* (0.01)	−0.01 (0.04)	−0.01 (0.04)	0.98 [0.98]
South Africa	1.23* (0.10)	1.23* (0.10)	0.23** (0.10)	0.23** (0.10)	0.00 (0.01)	0.00 (0.01)	0.01 (0.05)	0.02 (0.05)	0.96 [0.96]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t(\mathrm{S}_1$, log linear model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.39* (0.09)	1.17* (0.08)	0.40* (0.08)	0.28* (0.07)	0.09* (0.01)	0.11* (0.01)	−0.06* (0.01)	−0.07* (0.02)	0.99 [0.99]
Canada	1.62* (0.06)	1.54* (0.06)	0.65* (0.06)	0.58* (0.06)	0.02* (0.00)	0.04* (0.01)	−0.05 (0.04)	−0.10* (0.04)	0.98 [0.99]
South Africa	1.22* (0.10)	1.22* (0.10)	0.22** (0.10)	0.22** (0.10)	0.00 (0.01)	0.00 (0.01)	0.09** (0.04)	0.10** (0.04)	0.96 [0.96]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t(\mathrm{S}_1$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	0.87* (0.07)	0.52* (0.09)	0.16* (0.06)	−0.19** (0.08)	0.24* (0.02)	0.24* (0.02)	−0.001 (0.02)	−0.001 (0.02)	0.91 [0.92]
Canada	1.22* (0.05)	0.88* (0.07)	0.43* (0.04)	0.09 (0.06)	0.19* (0.02)	0.19* (0.02)	0.04 (0.03)	0.05 (0.03)	0.94 [0.94]
South Africa	1.11* (0.10)	1.03* (0.10)	0.34* (0.10)	0.26* (0.09)	0.29* (0.08)	0.28* (0.08)	0.02 (0.05)	0.02 (0.05)	0.77 [0.77]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t(\mathrm{S}_1$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	0.87* (0.07)	0.53* (0.09)	0.19* (0.06)	−0.15*** (0.08)	0.24* (0.02)	0.24* (0.02)	−0.03** (0.01)	−0.03** (0.01)	0.92 [0.92]
Canada	1.21* (0.05)	0.88* (0.07)	0.43* (0.04)	0.09 (0.06)	0.18* (0.02)	0.19* (0.01)	0.02 (0.02)	0.02 (0.02)	0.94 [0.94]
South Africa	1.09* (0.09)	0.96* (0.09)	0.38* (0.10)	0.24** (0.09)	0.53* (0.11)	0.52* (0.11)	0.09* (0.02)	0.09* (0.02)	0.79 [0.79]

*,**,*** are significance at 1%, 5% and 10% respectively; HAC standard errors in parentheses

Table 3: Estimates of the second sample

$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t(\mathrm{S}_2$, log linear model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.25* (0.09)	1.15* (0.08)	0.33* (0.08)	0.24* (0.08)	0.08* (0.01)	0.09* (0.01)	−0.07* (0.02)	−0.07* (0.02)	0.99 [0.99]
Canada	1.67* (0.06)	1.62* (0.06)	0.68* (0.06)	0.65* (0.06)	0.02* (0.00)	0.03* (0.00)	0.01 (0.04)	0.02 (0.05)	0.99 [0.99]
South Africa	1.24* (0.11)	1.23* (0.11)	0.25** (0.12)	0.25** (0.11)	0.01 (0.01)	0.02 (0.01)	0.04 (0.06)	0.06 (0.06)	0.96 [0.96]

$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t(\mathrm{S}_2$, log linear model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.16* (0.08)	1.04* (0.08)	0.27* (0.08)	0.16* (0.07)	0.10* (0.02)	0.12* (0.01)	−0.09* (0.02)	−0.09* (0.02)	0.99 [0.99]
Canada	1.66* (0.06)	1.59* (0.06)	0.68* (0.06)	0.62* (0.06)	0.02** (0.007)	0.04* (0.01)	−0.02 (0.04)	−0.07 (0.04)	0.99 [0.99]
South Africa	1.18* (0.11)	1.17* (0.11)	0.21*** (0.11)	0.20*** (0.11)	0.02* (0.001)	0.02* (0.008)	0.19* (0.05)	0.19* (0.05)	0.96 [0.96]

$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t(\mathrm{S}_2$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	0.89* (0.08)	0.57* (0.10)	0.16** (0.08)	−0.15 (0.09)	0.24* (0.02)	0.22* (0.02)	−0.02 (0.02)	−0.02 (0.02)	0.89 [0.89]
Canada	1.28* (0.05)	0.94* (0.07)	0.48* (0.04)	0.14** (0.05)	0.18* (0.02)	0.19* (0.02)	0.04 (0.03)	0.05 (0.03)	0.94 [0.94]
South Africa	1.12* (0.09)	1.04* (0.09)	0.35* (0.11)	0.27** (0.10)	0.31* (0.09)	0.30* (0.09)	0.01 (0.05)	0.01 (0.05)	0.77 [0.78]

$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t(\mathrm{S}_2$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	0.87* (0.08)	0.55* (0.09)	0.19* (0.07)	−0.12 (0.09)	0.22* (0.02)	0.22* (0.02)	−0.04* (0.01)	−0.04* (0.01)	0.89 [0.92]
Canada	1.27* (0.05)	0.93* (0.07)	0.47* (0.04)	0.13** (0.06)	0.18* (0.02)	0.18* (0.02)	0.02 (0.02)	0.02 (0.02)	0.94 [0.94]
South Africa	1.12* (0.09)	0.98* (0.09)	0.38* (0.11)	0.25** (0.09)	0.49* (0.11)	0.48* (0.11)	0.08* (0.02)	0.08* (0.02)	0.79 [0.79]

*,**,*** are significance at 1%, 5% and 10% respectively; HAC standard errors in parentheses

Table 4: Estimates of the third sample

Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t$(S$_3$, log linear model)									
United States	1.49* (0.13)	1.32* (0.10)	0.59* (0.12)	0.44* (0.10)	0.10* (0.03)	0.12* (0.02)	0.09 (0.06)	0.06 (0.05)	0.99 [0.99]
Canada	1.17* (0.09)	1.06* (0.09)	0.36* (0.08)	0.23* (0.08)	0.19* (0.02)	0.17* (0.02)	0.22 (0.14)	0.19 (0.13)	0.94 [0.96]
South Africa	1.13* (0.17)	1.13* (0.17)	0.16 (0.20)	0.17 (0.20)	0.02 (0.04)	0.03 (0.04)	0.01 (0.23)	0.00 (0.23)	0.86 [0.86]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t$ (S$_3$, log linear model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.44* (0.11)	1.26* (0.09)	0.57* (0.11)	0.40* (0.08)	0.13* (0.03)	0.14* (0.02)	0.14* (0.04)	0.14* (0.03)	0.99 [0.99]
Canada	1.17* (0.10)	1.06* (0.09)	0.38* (0.09)	0.24* (0.08)	0.21* (0.02)	0.19* (0.02)	0.32** (0.12)	0.30* (0.10)	0.94 [0.96]
South Africa	1.14* (0.14)	1.14* (0.14)	0.14 (0.15)	0.15 (0.15)	−0.00 (0.04)	0.01 (0.04)	0.33 (0.33)	0.26 (0.33)	0.86 [0.86]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 R_t + \varepsilon_t$(S$_3$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.00* (0.10)	0.91* (0.08)	0.22** (0.09)	0.09 (0.07)	0.23* (0.04)	0.21* (0.03)	0.02 (0.05)	−0.01 (0.05)	0.95 [0.97]
Canada	1.16* (0.08)	1.07* (0.07)	0.36* (0.08)	0.21* (0.08)	0.18* (0.02)	0.17* (0.02)	−0.11 (0.17)	−0.15 (0.13)	0.89 [0.93]
South Africa	1.06* (0.26)	0.89* (0.21)	0.28 (0.25)	0.23 (0.20)	−0.02 (0.21)	0.78** (0.31)	0.08 (0.32)	0.86*** (0.46)	0.70 [0.74]
$n_t = \beta_1 n_{t-1} - \beta_2 n_{t-2} + \beta_3 (y_t^s, y_t) - \beta_4 r_t + \varepsilon_t$(S$_2$, growth model)									
Parameters	β_1^{ys}	β_1^{y}	β_2^{ys}	β_2^{y}	β_3^{ys}	β_3^{y}	β_4^{ys}	β_4^{y}	R^2
United States	1.00* (0.11)	0.91* (0.08)	0.22** (0.10)	0.08 (0.08)	0.24* (0.04)	0.20* (0.03)	0.01 (0.04)	0.01 (0.03)	0.95 [0.97]
Canada	1.15* (0.09)	1.06* (0.08)	0.35* (0.09)	0.21* (0.08)	0.17* (0.02)	0.17* (0.02)	−0.12 (0.08)	−0.11 (0.08)	0.90 [0.93]
South Africa	0.82* (0.17)	0.57* (0.14)	0.19 (0.18)	0.01 (0.12)	0.73*** (0.40)	1.59* (0.31)	0.29*** (0.15)	0.51* (0.09)	0.72 [0.82]

*,**,*** are significance at 1%, 5% and 10% respectively; HAC standard errors in parentheses

The full sample results of the log linear model reveal that the coefficients of the nominal interest rate variable are also positive and significant. This communicates the same information as the growth model with the nominal interest rate as the cost of capital. The difference in the results is that sales and output variables are not quantitatively important in the log linear model. Most importantly, although the coefficients of the real interest rate variables are not significant for South Africa, most of these coefficients are positive and if substituted in eq.(1) would imply the negative effect of interest rates on inventories. Based on these results, we might say that the puzzle is not present in South Africa and that the nominal interest rate is apparently better able to explain inventory behaviour than the real interest rate.

2.3.2 Summary of the main findings

In summary, the sign puzzle is applicable in the US and Canada, with the only exception being in the third sample, where the nominal interest rate is used in the log linear model. The existence of the puzzle in these two countries is in line with the findings in Maccini et al. (2004) and Jones and Tuzel (2013). Thus, if one were to follow the theoretical justification provided in Benati and Lubik (2014), we would conclude that firms derive more benefits from accumulating inventories and avoiding stockouts. This conclusion emanates from the fact the stockout avoidance motive dominates the production smoothing in the United States and Canada. Thus, the demand side effect of a contractionary monetary policy will lead to a decline in inventories, but the supply side effect would be positive if the financial cost of holding inventories is less than the benefits firms expect from avoiding stockouts.

Alhough all specifications indicate that interest rates have a negative effect on inventories in South Africa, the most appropriate models of inventory dynamics in South Africa are the log linear and growth models with the nominal interest rate as the cost of capital. Thus, changes in the nominal interest rate would have a negative and significant effect on inventories should these models be adopted for South Africa. To shed more light on the puzzle, the following section presents a model of inventory dynamics in order to expose the structural parameters that govern the sign puzzle or the coefficient of the cost of capital.

2.4 A model of inventory dynamics

The components of the cost structure that we employ in this study are well explained in Irvine (1981) and were used in Blanchard (1983), Maccini and Pagan (2013) and Maccini et al.(2015). We augment their inventory holding costs with the financing costs of holding inventories. Furthermore, we factor in the view that stockout firms can still accumulate inventories in the face of rising financing costs as long as the benefit emanating from doing so is higher than the cost associated with it. This modification yields an ambiguous sign on the cost of capital variable and shows the structural parameters that lead to the sign puzzle.

Following Pindyck (1994), we employ convex unit cost of storage as a function of a lagged inventory-sales ratio and express it as a proportion of the current inventory stock. This implies that it would be beneficial for firms to hold a large stock of inventories, as in doing so, the storage costs would be reduced. Storage costs are often expressed as the rate of depreciation of inventories in the DSGE models without an explicit functional form of storage. Studies in this category include the work of Jung and Yun (2005) and Kryvtsov and Midrigan (2010, 2013).

Like the presentation in eq.(1), we assume that Q_t can take the value of sales Y_t^s or output Y_t. The aggregate inventory holdings cost H_t is therefore determined as follows:

$$H_t = A_t \left(\frac{N_t}{Q_t}\right)^{-\gamma} Q_t e^{\delta R_t} + R_t N_t + \left(\frac{N_{t-1}}{Q_t}\right)^{-\gamma_s} N_t + \frac{\chi}{1+\vartheta} \left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} N_t \quad (2)$$

where A_t captures the optimism of stock managers, which also allow firms to control the probability of stockout. N_t is the level of inventory stock, R_t is a measure of the cost of capital and it is the real interest rate, which can also be replaced by the nominal interest rate. The parameter $\gamma > 0$ is the sensitivity of the probability of stockout to the inventory-sales ratio, also known as the elasticity of benefits of inventory holdings, as presented in Maccini and Pagan (2013). $\gamma_s > 0$ is the sensitivity of storage cost to the lagged inventory-sales ratio and $\delta > 0$ is the sensitivity of stockout costs to changes in the cost of capital. The first term in eq.(2) represents the stockout cost. The second term stands for the opportunity cost of holding inventories and the third term is the storage cost. The fourth component is the inventory adjustment cost, which is similar in form to the one in Belo and Lin (2012). ϑ determines the degree of the adjustment cost function and χ is the adjustment cost parameter.

Assuming that stock managers choose the level of inventory in order to minimize the total cost of holding inventories, the first order condition is derived as follows:

$$\gamma A_t N_t^{-(1+\gamma)} Q_t^{(1+\gamma)} e^{\delta R_t} = R_t + \left(\frac{N_{t-1}}{Q_t}\right)^{-\gamma_s} + \chi \left(\frac{2+\vartheta}{1+\vartheta}\right) \left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} \tag{3}$$

The level of inventories at time t that is consistent with eq.(3) is the desired inventory at time t. Following Flaschel et al. (2001), the following supply side identities are assumed at firm level.

$$N_t = N_t^d + Y_{t-1} - Y_{t-1}^s \tag{4}$$

$$Y_t = Y_t^s + N_t - N_{t-1} \tag{5}$$

The identity in eq.(4) constitutes inventory investment demand by the inventory management unit, where N_t^d represents the desired inventories and $\left(Y_{t-1} - Y_{t-1}^s\right)$ determines how the inventory management unit measures inventory investment. The identity in eq.(5) is the normal output identity and is equal to the sales plus inventory investment. We combine the first order condition in eq.(3) with the identities in eqs.(4) and (5) to obtain the linearized inventory dynamics as follows:

$$\widehat{n}_t = \left(\frac{1}{(1+g)} + \theta_n\right) \widehat{n}_{t-1} - \frac{1}{(1+g)^2} \widehat{n}_{t-2} + \theta_q \widehat{q}_t - \theta_R \widehat{R}_t + \widehat{\varepsilon}_t \tag{6}$$

where $\widehat{q}_t$ represents the log deviation of output or sale from its steady state level and $\widehat{R}_t$ is the deviation of the measure of the cost of capital from its steady state level. The parameters of eq.(6) are defined as follows:

$$\theta_n = \frac{\omega_i + \zeta}{\varphi + \zeta}, \theta_q = \frac{\varphi - \omega_i}{\varphi + \zeta}, \theta_R = \frac{(\phi - \gamma\delta p_0^s)}{\phi(\varphi + \zeta)}, \varphi = \gamma(1+\gamma)\frac{p_0^s}{\phi}, p_0^s = A_0\frac{e^{(\delta R_0)}}{\phi^\gamma}$$
$$\omega_i = \frac{\gamma_s}{(1+g)(\phi)^{\gamma_s}}, \widehat{\varepsilon}_t = \frac{1}{\varphi + \zeta}\widehat{a}_t, \zeta = \chi(2+\vartheta)(1+g)^{(1+\vartheta)}$$

In Eq.(6), ϕ is the steady state inventory-sales/output ratio depending on the series that is used. The inventory model reveals that the signs of θ_q and θ_R are ambiguous. $\theta_q > 0$ implies that the stockout avoidance motive dominates and $\theta_q < 0$ implies that the production smoothing motive dominates. Furthermore, $\theta_R > 0$ (or $\phi > \gamma\delta p_0^s$) implies that changes in the cost of capital have a negative effect on the inventories. $\theta_R < 0$ (or $\phi < \gamma\delta p_0^s$)implies a positive effect of costs of capital on the inventories which is termed the sign puzzle. The main contribution here is that the sign of the coefficient of the cost of capital variable is ambiguous. Eq.(6) can further be reduced to the following:

$$\widehat{n}_t = \lambda_1\widehat{n}_{t-1} - \lambda_2\widehat{n}_{t-2} + \theta_q\widehat{q}_t - \theta_R\widehat{R}_t + \widehat{\varepsilon}_t \tag{7}$$

where the parameters of eq.(7) are described as follows:

$$\lambda_1 = \left(\frac{1}{(1+g)} + \theta_n\right), \lambda_2 = \frac{1}{(1+g)^2}$$

Eq.(7) is similar to eq.(1), the only difference being that variables are expressed as deviations from the steady state. λ_1 and λ_2 are comparable to β_1 and β_2 in eq.(1) respectively and θ_q and θ_R are comparable to β_3 and β_4 respectively. We take this one step further by incorporating how inflation influences inventory fluctuations, as presented in eq.(7). In doing so, we assume that the cost of capital in eq.(7) is the nominal interest rate r_t and therefore specify a simple monetary policy rule where the central bank smooths the interest rate and react to both output and inflation. The equations are presented as follows:

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1-\rho_r)\left(\rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t\right) + \widehat{\xi}_t \tag{8}$$

$$\widehat{n}_t = \lambda_1 \widehat{n}_{t-1} - \lambda_2 \widehat{n}_{t-2} + \psi_y \widehat{y}_t - \psi_r \widehat{r}_{t-1} - \psi_\pi \widehat{\pi}_t + \widehat{\eta}_t \tag{9}$$

where the parameters in eq.(9) are defined as follows:

$$\psi_r = \theta_r \rho_r, \psi_\pi = \theta_r (1-\rho_r)(\rho_\pi), \psi_y = \left[\theta_y - \theta_r (1-\rho_r)(\rho_y)\right], \widehat{\eta}_t = \widehat{\varepsilon}_t - \theta_r \widehat{\xi}_t$$

The policy rule in eq.(8) is substituted in eq.(7) to obtain the inventory gap equation with the lagged nominal interest rate and inflation gap in eq.(9). The signs of the coefficients of the lagged nominal interest rate gap ψ_r and the inflation gap ψ_π are still ambiguous owing to the presence of θ_r. Note that θ_r is the coefficient of the cost of capital when the nominal interest rate is used.

2.4.1 Reduced form estimates of the structural models

Before providing a theoretical illustration of the puzzle, we first present the reduced form estimates of eqs.(7) and (9) and compare the results to the estimates that were obtained using eq.(1). The results are presented in Table.5. The top and middle panels of the table show the results for eq.(7) and the bottom panel the results for eq.(9). Firstly, the results pertaining to eq.(7) reveal that the first and the second lags of inventory gap are significant for the United States, Canada and South Africa regardless of the cost of capital used. Secondly, regardless of the cost of capital, the coefficients on the output and sales gap are significant for the United States and Canada.

We note that both the real and the nominal interest rates display negative coefficients for the United States and Canada. This implies that when substituted in eq.(7), the changes in interest rate gaps will have a positive effect on the inventory gap. When the sales gap series is used, the coefficient of the real interest rate gap is significant for the United States but weakly significant for Canada. The coefficients of output and sales gaps are significant when the nominal interest rate gap is used as the cost of capital.

Table 5: Reduced form estimates of the structural model

$$\widehat{n}_t = \lambda_1 \widehat{n}_{t-1} - \lambda_2 \widehat{n}_{t-2} + \theta_q \widehat{q}_t - \theta_R \widehat{R}_t + \widehat{\varepsilon}_t \text{ eq.(7)}$$

	λ_1^{ys}	λ_1^{y}	λ_2^{ys}	λ_2^{y}	θ_q^{ys}	θ_q^{y}	θ_R^{ys}	θ_R^{y}	R^2
United States	1.04*	0.92*	0.24*	0.09***	0.17*	0.17*	−0.03*	0.00	0.92
	(0.06)	(0.09)	(0.06)	(0.05)	(0.02)	(0.01)	(0.04)	(0.04)	[0.95]
Canada	1.17*	1.02*	0.35*	0.15*	0.18*	0.19*	−0.09***	−0.01	0.93
	(0.06)	(0.04)	(0.05)	(0.03)	(0.02)	(0.01)	(0.05)	(0.04)	[0.96]
South Africa	1.07*	0.81*	0.22**	0.07	−0.01	0.87*	0.07	0.04	0.78
	(0.10)	(0.09)	(0.10)	(0.09)	(0.18)	(0.13)	(0.05)	(0.05)	[0.83]

$$\widehat{n}_t = \lambda_1 \widehat{n}_{t-1} - \lambda_2 \widehat{n}_{t-2} + \theta_q \widehat{q}_t - \theta_r \widehat{r}_t + \widehat{\varepsilon}_t \text{ eq.(7)}$$

	λ_1^{ys}	λ_1^{y}	λ_2^{ys}	λ_2^{y}	θ_q^{ys}	θ_q^{y}	θ_r^{ys}	θ_r^{y}	$R^{2,}$
United States	1.00*	0.91*	0.24*	0.11**	0.16*	0.17*	−0.11*	−0.06**	0.93
	(0.07)	(0.05)	(0.06)	(0.05)	(0.01)	(0.01)	(0.02)	(0.02)	[0.95]
Canada	1.13*	1.01*	0.34*	0.15*	0.16*	0.18*	−0.23*	−0.08***	0.93
	(0.05)	(0.04)	(0.04)	(0.04)	(0.02)	(0.01)	(0.03)	(0.04)	[0.96]
South Africa	1.09*	0.80*	0.21**	0.07	−0.13	0.85*	0.26*	0.05	0.79
	(0.10)	(0.09)	(0.10)	(0.09)	(0.19)	(0.13)	(0.08)	(0.07)	[0.83]

$$\widehat{n}_t = \lambda_1 \widehat{n}_{t-1} - \lambda_2 \widehat{n}_{t-2} + \psi_y \widehat{y}_t - \psi_r \widehat{r}_{t-1} - \psi_\pi \widehat{\pi}_t + \widehat{\eta}_t \text{ eq.(9)}$$

	λ_1	λ_2	ψ_y	ψ_r	ψ_π	$R^{2,}$
United States	0.85*	0.08	0.18*	−0.11*	−0.01	0.96
	(0.05)	(0.05)	(0.01)	(0.02)	(0.03)	
Canada	0.97*	0.14*	0.18*	−0.21*	−0.00	0.96
	(0.04)	(0.03)	(0.01)	(0.04)	(0.04)	
South Africa	0.81*	0.06	0.79*	0.12	−0.02	0.83
	(0.09)	(0.09)	(0.12)	(0.08)	(0.07)	

*,**,*** are significance at 1%, 5% and 10% respectively; HAC standard errors in parentheses

In the case of South Africa, both the coefficients of the first and second lags of the inventory gap are significant regardless of the cost of capital used. Compared to the sales gap, the output gap significantly explains the fluctuations in the inventories. We note that the coefficient of the real interest rate is positive and not significant, but positive and significant when the nominal interest gap is used. This implies that when substitutions are made in eq.(7), the changes in the interest rate gap will have a negative effect on the inventory gap in South Africa. The results for all three of the countries are consistent with the ones provided in Section 2 using the log inventory model and the inventory growth model. Compared to the results of eq.(7), the results pertaining to eq.(9) reveal that inflation does not play a significant role in explaining inventory fluctuations.

In summary, the structural models presented in eq.(7) and eq.(9) respectively generate results that are similar to the findings in Section 2. Most importantly, we find that the sign puzzle is present in the United States and Canada but absent in South Africa.

2.5 Numerical calibration of the sign puzzle

We start off from the argument postulated by Benati and Lubik (2014). These scholars note that the total effect of changes in the monetary policy rate on inventories depends on whether its effect on inventories through sales (or demand channel) dominates its direct effect on inventories (or inventory channel). The components of the total effect can be shown by considering the first derivative of eq.(7) with respect to changes in the cost of capital. This is presented in eq.(10) below.

$$\frac{\partial n}{\partial r} = \underbrace{\overset{-,+}{\theta_s}\left(\overset{-}{\frac{\partial y^s}{\partial r}}\right)}_{\text{demand channel}} - \underbrace{\overset{-,+}{\theta_R}}_{\text{inventory channel}} \tag{10}$$

What is important to note about eq.(10) is that inventories may still increase even if $\theta_R > 0$, that is $-\theta_R < 0$. Thus, the focus of our paper is on the structural parameters that determine the sign of θ_R. In order to understand what makes the coefficient of the cost of capital turn positive or negative, we provide a numerical calibration of the structural parameters that determine this coefficient.

Table 6: Calibration of the sign puzzle

Parameters	United States	Canada
ϕ	1.4140	1.5774
	Case 1 (A_0= 0.055, $\gamma = 0.676, \delta = 0.5$)	
p_0^s	0.0438,	0.0413
$\gamma\delta p_0^s$	0.0148	0.0139
φ	0.0351	0.0297
$\phi - \gamma\delta p_0^s$	1.3992	1.5635
χ	1.6700	0.9500
g	0.0100	0.0100
ϑ	1.0000	1.0000
ζ	5.1107	2.9073
θ_R	**0.1923**	**0.3375**
Parameters	United States	Canada
ϕ	1.4140	1.5774
	Case 2 (A_0= 1.30, $\gamma = 1.676, \delta = 1.5$)	
p_0^s	0.7445	0.6571
$\gamma\delta p_0^s$	1.8716	1.6519
$\phi - \gamma\delta p_0^s$	−0.4576	−0.0745
φ	2.3614	1.8683
χ	1.6700	0.9500
g	0.0100	0.0100
ϑ	1.0000	1.0000
ζ	5.1107	2.9073
θ_R	**−0.0433**	**−0.0099**
	$\theta_R = \frac{\left(\phi - \gamma\delta p_0^s\right)}{\phi(\varphi+\zeta)}$, $\varphi = \gamma\left(1+\gamma\right)\frac{p_0^s}{\phi}$	

Given the empirical evidence that the puzzle persists in the United States and Canada, we now set to illustrate what drives the puzzle in these two countries. We follow Maccini et al.(2015) and Dasgupta et al. (2019) to calibrate the different levels of the steady state elasticity of the benefits of holding inventories. We provide two case examples to show how the coefficient of the cost of capital could become positive or negative. The most important parameters are the ones that define the probability of stockout. In our first case, we set $\gamma = 0.676$, which is the calibrated value provided in Maccini *et al.* (2015). In our second case, we set $\gamma = 1.676$, which is closer to the value $\gamma = 1.5$ used in Dasgupta *et al.* (2019). The values of A_0 are set at 0.055 and 1.30 respectively for the first and second cases. The value of $A_0 = 0.055$ was first used to obtain a frequency value for stockout that is closer to the one provided by Bils (2016) based on the US micro data.

The sensitivity of stockouts cost to changes in the nominal interest rate is set at 0.5 and 1.5 respectively for the first and second cases. We use the historical average of the nominal interest rate to define the steady state interest rate. The calibrated parameters above endogenously determine the probability of stockout and therefore drive the sign puzzle. However, given the steady state interest rate and the steady state level of optimism, the coefficient of the cost of capital shows that the sensitivity of stockouts cost to changes in the cost of capital and the elasticity of the benefits for inventory holdings are the two main parameters driving the sign puzzle. Thus, the willingness to accumulate inventories in the face of a rising cost of capital does increase with the term $\gamma\delta p_0^s$.

The numerical calibration provided in Table.6 shows that the values of θ_R in the first case ($A_0 = 0.055, \gamma = 0.676, \delta = 0.5$) are 0.1923 and 0.3375 for the US and Canada respectively. In the second case ($A_0 = 1.30, \gamma = 1.676, \delta = 1.5$), the values are -0.0433 and -0.0099 for the US and Canada respectively. These results confirm that the puzzle is driven by the elasticity of the benefits of firms for inventory holdings and the sensitivity of stockout cost to changes in the cost of capital.

2.6 Conclusion and policy implications

2.6.1 Conslusion

In this paper, we first update the empirical evidence on the sign puzzle and using a log linear inventory model and an inventory growth model provide evidence for the US, Canada and South Africa. We find that the puzzle still exists in the United States and Canada, especially in the sample researched before the global financial crisis of 2007-2009. The sample that starts with the period of the crisis shows that the nominal interest rate has a negative and significant effect on inventories for both the United States and Canada when the log linear model is used. We find that the puzzle is not present in South Africa, and that the nominal interest rate has a significant and negative effect on inventory growth in South Africa.

Secondly, we present a structural model of inventory dynamics to explain the puzzle, with this model being further extended to include the role of inflation. The empirical evidence based on the structural models confirms the results documented using the log linear model and the growth model. Finally, we numerically calibrate the structural parameters that determine the sign of the coefficient of the cost of capital in the structural model. We note that the elasticity of benefits to firms for holding inventories and the sensitivity of stockouts cost to changes in the cost of capital determine the sign of the coefficient of the cost of capital, and therefore drive the sign puzzle.

2.6.2 Policy implications

The real effect of monetary policy is known to be transmitted through several channels, which include the inventory channel. The inventory channel is a supply-side channel and has attracted attention in the literature owing to the role inventories play in driving the real business cycles. Although the sign puzzle has cast doubts on the effectiveness of the inventory channel, studies such as those by Barth and Ramey (2001) and Copeland et al. (2019) have demonstrated that the inventory channel exists.

This study has extended the empirical evidence on the puzzle in countries such as the United States, Canada, and South Africa, and finds that the puzzle still exists in the United State and Canada, but not in South Africa. The absence of the puzzle in South Africa offers an additional channel for the transmission of the real effect of monetary policy. Therefore, monetary policy authorities should consider evaluating how the inventory channel contributes to the transmission of the real effect of monetary policy in South Africa.

3 Chapter. Inventory Dynamics and Endogenous Persistence in a New-Keynesian Model

3.1 Introduction

This paper uses a standard New Keynesian (NK) model to investigate the role of inventories in generating a hump-shaped response of macroeconomic variables to shocks. NK models rely on a number of frictions in order to replicate the stylized facts of the data. Fuhrer (2000) finds that habit formation is necessary to generate a hump-shaped response of aggregate spending to shocks. Consequently, habit formation has become a standard feature of the NK consumption function. In influential models, such as those by Smets and Wouters (2003) and Christiano *et al.* (2005), the estimate of habit formation fluctuates between 0.60 and 0.71. An additional friction that is used in these models on the aggregate demand side is the investment adjustment cost.

On the supply-side, NK models apply price stickiness in order to generate the short-run effects of monetary policy on output and to induce aggregate persistence. For instance, the specification in Christiano *et al.* (2005) yields a range of 0.28 to 0.89 for the price stickiness parameter, while the range estimated in Rabanal (2007) is 0.59 to 0.86. Justiniano and Preston (2010a) estimate it to be 0.79 for Australia and 0.68 for Canada and New Zealand. Various indexation schemes are also specified in order to generate persistence in inflation. The estimate of price indexation in Smets and Wouters (2003, 2007) is 0.905 and 0.24 respectively. These frictions have been criticised, by among others Fuhrer (2004), for their ad-hoc nature and the limited amount of microeconomic evidence pertaining to them.

Some of the criticisms of the NK models (e.g. Chari *et al.*, 2000), highlight the fragility of nominal rigidities in these models. For example, in order for the model to generate persistence, price stickiness has to be unrealistically high (in some instances, prices have to be sticky for more than a year). In Christiano *et al.* (2005), price stickiness needs to be very high when wage stickiness is absent. Chari et al. (2009) also criticize the ad hoc nature of price indexation and other frictions in these models. As already noted by Fuhrer (2004), microeconomic studies find mixed results about the significance of habit formation. Dynan (2000) finds the habit formation estimate to be -0.038 and Iwamoto (2013) reports a value of -0.37. Nevertheless, Kapteyn and Teppa (2003) estimate habit formation to be 0.77 and Ravina (2019) estimates it to be 0.503 and 0.29 for internal and external habit parameters respectively. A review by Havranek *et al.* (2017) notes that the gulf between macro and micro estimates of this parameter is 0.50.

In this paper, we show that incorporating inventory dynamics in the standard NK model goes a long way to reducing the role of many of these frictions. This is contrary to Kryvtsov and Midrigan (2013), who find that nominal price rigidities are needed to replicate key stylized facts such as the procyclical behaviour of inventories and the countercyclical nature of inventory/sales ratio in the data. Most NK models do not take into account the role of inventories as a smoothing device, a point already acknowledged in Boldrin *et al.*(2001). Inventories play important roles at both micro and macro levels (Blinder and Maccini, 1991). For instance, Blinder and Fischer (1981) and Wen (2005) describe inventories as a natural source of persistence in output. Furthermore, Blinder (1981) posits that real price stickiness prevails in industries with inventorable goods. What this means is that the presence of inventories in industry allows firms to maintain stable prices in the face of shocks.

In modelling the dynamics of inventories, we minimize the inventory holding cost function proposed by Blanchard (1983) and used in Maccini et al. (2015). This cost function is augmented with the inventory adjustment cost, as motivated by the work of Jung and Yun (2005). The desired inventory gap, which is obtained from the cost minimization problem, is combined with an inventory accumulation identity of the firm's inventory management unit to obtain the dynamics of the actual inventory. The demand side of the model is an extension of the studies such as those by Smets and Wouters (2007), Gali and Monacelli (2005) and Lubik and Schorfheide (2007). However, the closest studies in a spirit similar to ours are the works of Boileau and Letendre (2011), Kryvtsov and Midrigan (2010; 2013) and Kim (2015).

The remainder of this paper is structured as follows: Section 2 presents the model and its linearized version, while Section 3 presents the parameters of the model economy. Section 4 presents the results and Section 5 concludes the paper and provides some policy implications.

3.2 Model

3.2.1 Households

We assume that households hold their wealth in terms of short-term domestic and foreign bonds. The nominal returns on the bonds are r_t and r_t^f for domestic and foreign respectively. In line with Gali and Monacelli (2005) and Lubik and Schorfheide (2007), we assume that households consume domestic and foreign goods and supply labour hours. They maximize utility with the following technology:

$$U_t(C_t) + U_t(Z_t) - U_t(L_t) = \frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} + \frac{Z_t^{1-\theta_z}}{1-\theta_z} - \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \quad (1)$$

where U_t stands for utility and the parameters σ_c,θ_z and σ_l are the respective intertemporal elasticity of substitution for domestic consumption C_t, imported goods Z_t and the elasticity of labour L_t, and h is habit formation in consumption. Households are paid wage incomes for supplying labour hours and their networth is written as follows:

$$\frac{B_t}{P_t} + \frac{E_t^e B_t^f}{P_t} = \frac{(1 + r_{t-1}) B_{t-1}}{P_t} + \frac{\left(1 + r_{t-1}^f\right) E_t^e}{P_t} B_{t-1}^f + \frac{W_t}{P_t} L_t - C_t - \frac{E_t^e P_{zt}^f}{P_t} Z_t \quad (2)$$

In eq.(2), B_t and B_t^f are the risk-free domestic and foreign bonds respectively. P_t is the domestic price index, P_{zt}^f is the foreign price index, which is the same as the import price index, W_t is the nominal wage and E_t^e is the nominal exchange rate. The term $\frac{E_t^e P_{zt}^f}{P_t}$ is the real exchange rate, which is further defined in this paper as S_t. The equilibrium conditions from the household's utility maximization problem are derived as follows:

$$(C_t - hC_{t-1})^{-\sigma_c} = \lambda_t \tag{3}$$

$$\frac{L_t^{\sigma_l}}{\lambda_t} = \frac{W_t}{P_t} \tag{4}$$

$$\lambda_t = \beta E_t \left(\frac{1 + r_t}{1 + \pi_{t+1}} \right) \lambda_{t+1} \tag{5}$$

$$\beta E_t \left(\frac{1 + r_t}{1 + \pi_{t+1}} \right) S_t = \beta E_t \left(\frac{1 + r_t^f}{1 + \pi_{t+1}^f} \right) S_{t+1} \tag{6}$$

$$Z_t^{-\theta_z} = S_t \lambda_t \tag{7}$$

The variable λ_t is the Lagrangian multiplier and β stands for factor discount. In the presence of inventories, we define aggregate demand to be equal to aggregate sales ($Y_t^D = Y_t^s$) but aggregate demand differs from output ($Y_t^D \neq Y_t$). Aggregate demand (sales) is therefore determined as follows:

$$Y_t^s = C_t + X_t - Z_t + G_t \tag{8}$$

where X_t is exports and G_t government spending, which we transform into a spending shock in the linearized version of the model.

3.2.2 Production unit

Following Batini et al. (2005) and Malikane and Mokoka (2014), we assume that firms produce goods using labour, and apply an exogenously determined technology and fixed factors of production in fixed proportion to output. That is $M_{jt} = Y_t^{\delta_j}$ where M_{jt} represents the quantity of non-labour inputs, which we assume are nonlinear in output. The production technology is therefore determined as follows:

$$Y_t = \xi_t^{'} L_t^{\sigma} \left[\Pi_{j=1}^{n} Y_t^{\delta_j \theta_j} \right] \tag{9}$$

$$Y_t = \xi_t L_t^{\alpha_l} \tag{10}$$

where $0 < \sigma < 1$, Y_t is the real output, and L_t is labour input. The reduced form of eq.(9) is presented in eq.(10) where $\xi_t = \xi_t'^{\frac{1}{1-\alpha_0}}$ captures the state of technology, $\alpha_l = \frac{\sigma}{1-\alpha_0}$, $\alpha_0 = \sum_{j=1}^{n} \theta_j \delta_j$. The parameter δ_j is the coefficient of input requirement and θ_j is the elasticity of output with respect to each input. The total cost of production and the real marginal cost are presented in eqs.(11) and (12). The production cost is composed of labour cost and costs related to other inputs in the production process such as domestic and foreign inputs.

$$TC_{t,Y} = \frac{W_t}{P_t}\left(\xi_t^{-\frac{1}{\alpha_l}} Y_t^{\frac{1}{\alpha_l}}\right) + \frac{E_t^e P_{zt}^f}{P_t} Y_t^{\delta_z} + \sum_{j=1}^{n-1} \frac{P_{jt}}{P_t} Y_t^{\delta_j} \tag{11}$$

$$\psi_t = \frac{1}{\alpha_l} w_t \left(\xi_t^{-\frac{1}{\alpha_l}} Y_t^{\frac{1-\alpha_l}{\alpha_l}}\right) + \delta_z S_t Y_t^{\delta_z - 1} + \sum_{j=1}^{n-1} \delta_j p_{jt} Y_t^{\delta_j - 1} \tag{12}$$

The variable P_{jt} is the price of domestic input, P_{zt}^f is the price of imported inputs and $w_t = \frac{W_t}{P_t}$ is the real wage.

3.2.3 Inventory management unit

Following Blanchard (1983) and Maccini *et al.*(2015), we formulate the aggregate inventory holding cost as the combination of stockout cost, the financing cost of holding inventories, the storage cost and the inventory adjustment cost. We augment their inventory holdings cost function with the opportunity cost of holding inventories, which is the real expected interest rate. Gertler and Gilchrist (1994) note that a tight monetary policy shock weakens the value of the assets of firms. This reduces the incentive for firms to accumulate inventories, which implies that the probability of stockout will increase when the cost of capital goes up. In line with the notion of the marginal value of storage in Pindyck (1994), we employ a convex unit cost of storage as a function of the lagged inventory-sales ratio and express it as a proportion of the current inventory stock. We also introduce inventory adjustment cost which is motivated by the works of Jung and Yun (2005) and Kryvtsov and Midrigan (2010).

The aggregate inventory holdings cost H_t is therefore determined as follows:

$$H_t = A_t \left(\frac{N_t}{Y_t^s}\right)^{-\gamma} Y_t^s e^{\delta R_t} + R_t N_t + \left(\frac{N_{t-1}}{Y_t^s}\right)^{-\gamma_s} N_t + \frac{\chi}{1+\vartheta}\left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} N_t \tag{13}$$

where A_t captures the optimism of stock managers, N_t is the level of inventories, Y_t^s is sales, and R_t is the real expected interest rate, which is determined by the Fischer equation. The parameter $\gamma > 0$ is the sensitivity of the probability of stockout to the inventory-sales ratio also known as the benefit of stockout in Maccini and Pagan (2013). $\gamma_s > 0$ is the sensitivity of storage cost to the lagged inventory-sales ratio and $\delta > 0$ is the sensitivity of the probability of stockout to changes in the cost of capital. The first term in eq.(13) represents stockouts cost. The second term stands for the opportunity cost of holding inventories and the third term is storage cost. The fourth component is the inventory adjustment cost, which is similar in form to the one in Jones and Tuzel (2013). ϑ is the degree of convexity of the adjustment cost function and χ is the adjustment cost parameter.

Assuming that stock managers choose the level of inventory in order to minimize the total cost of holding inventories, the first order condition is derived as follows:

$$\gamma A_t N_t^{-(1+\gamma)} Y_t^{s(1+\gamma)} e^{\delta R_t} = R_t + \left(\frac{N_{t-1}}{Y_t^s}\right)^{-\gamma_s} + \chi\left(\frac{2+\vartheta}{1+\vartheta}\right)\left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} \tag{14}$$

The level of inventories at time t that is consistent with eq.(14) is the desired inventory at time t. Following Flaschel et al. (2001), the following supply side identities are assumed at firm level.

$$N_t = N_t^d + Y_{t-1} - Y_{t-1}^s \tag{15}$$
$$Y_t = Y_t^s + N_t - N_{t-1} \tag{16}$$

The identity in eq.(15) states that the gap between output and sales in the previous period constitutes the inventory investment demand by the inventory management unit. Thus, the actual inventory is equal to the desired inventory plus inventory investment. The identity in eq.(16) is the normal output identity. These identities show that within a firm, the production unit and the inventory management unit can be separated.

3.2.4 Linearized model economy

We present the linearized version of the model in terms of linearized quantity variables, followed by cost and financial variables. The expression in eq.(17) describes sales gap and the consumption gap in eq.(18) is obtained by linearizing the first order condition (FOC) in eq.(5). The import gap in Eq.(19) and the export gap in eq.(20) are obtained from the FOC in eq.(7). The output expression in eq.(21) is obtained by linearizing the identity in eq.(16), where g is the steady state growth rate.

$$\widehat{y}_t^s = \gamma_c \widehat{c}_t + \gamma_x \widehat{x}_t - \gamma_z \widehat{z}_t + \gamma_G \widehat{\varepsilon}_t^G \tag{17}$$

$$\widehat{c}_t = \frac{1}{1+h} E_t \widehat{c}_{t+1} + \frac{h}{1+h} \widehat{c}_{t-1} - \frac{(1-h)}{\sigma_c (1+h)} \left(\widehat{r}_t - E_t \widehat{\pi}_{t+1} \right) \tag{18}$$

$$\widehat{x}_t = \frac{\sigma_c}{\theta_z (1-h)} \widehat{c}_t^f - \frac{\sigma_c h}{\theta_z (1-h)} \widehat{c}_{t-1}^f + \frac{1}{\theta_z} \widehat{s}_t \tag{19}$$

$$\widehat{z}_t = \frac{\sigma_c}{\theta_z (1-h)} \widehat{c}_t - \frac{\sigma_c h}{\theta_z (1-h)} \widehat{c}_{t-1} - \frac{1}{\theta_z} \widehat{s}_t \tag{20}$$

$$\widehat{y}_t = (1 - \lambda g) \widehat{y}_t^s + \lambda \widehat{n}_t - \frac{\lambda}{1+g} \widehat{n}_{t-1} \tag{21}$$

$$\widehat{n}_t = \frac{1}{(1+g)} \widehat{n}_{t-1} - \frac{1}{(1+g)^2} \widehat{n}_{t-2} + \widehat{n}_t^d \tag{22}$$

$$\widehat{n}_t^d = \theta_n \widehat{n}_{t-1} + \theta_s \widehat{y}_t^s - \theta_R \widehat{R}_t + \widehat{\varepsilon}_t \tag{23}$$

$$\phi_t = \widehat{n}_t - \widehat{y}_t^s \tag{24}$$

$$\widehat{\psi}_t = \vartheta_y \widehat{y}_t + \vartheta_w \widehat{w}_t + \vartheta_s \widehat{s}_t + \sum_{j=1}^{n-1} \vartheta_{p_j} \widehat{p}_{jt} - \vartheta_\xi \widehat{\xi}_t \tag{25}$$

$$\widehat{w}_t = \frac{\sigma_l}{\alpha_l} \widehat{y}_t + \frac{\sigma_c}{(1-h)} \widehat{c}_t - \frac{\sigma_c h}{(1-h)} \widehat{c}_{t-1} - \frac{\sigma_l}{\alpha_l} \widehat{\xi}_t \tag{26}$$

$$\widehat{\pi}_t = \lambda_f \widehat{\pi}_{t+1} + \lambda_b \widehat{\pi}_{t-1} + \kappa \widehat{\psi}_t \tag{27}$$

$$\widehat{s}_t = E_t \widehat{s}_{t+1} + \left(\widehat{r}_t^f - \widehat{r}_t \right) + \left(E_t \widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1}^f \right) \tag{28}$$

$$\widehat{r}_t = \phi_r \widehat{r}_{t-1} + (1 - \phi_r) \left(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{y}_t + \phi_s \widehat{s}_t \right) + \widehat{\varsigma}_t \tag{29}$$

The parameters are defined as follows:

$$\theta_n = \frac{\omega_i + \zeta}{\varphi + \zeta}, \theta_s = \frac{\varphi - \omega_i}{\varphi + \zeta}, \theta_R = \frac{(\phi - \gamma \delta p_0^s)}{\phi (\varphi + \zeta)}, \varphi = \gamma (1 + \gamma) \frac{p_0^s}{\phi}, p_0^s = A_0 \frac{e^{(\delta R_0)}}{\phi^\gamma}$$

$$\omega_i = \frac{\gamma_s}{(1+g)(\phi)^{\gamma_s}}, \widehat{\varepsilon}_t = \frac{1}{\varphi + \zeta} \widehat{a}_t, \zeta = \chi (2 + \vartheta)(1+g)^{(1+\vartheta)}$$

$$\vartheta_y = \left(\left(\frac{1 - \alpha_l}{\alpha_l^2} \right) \frac{w_o L_o}{Y_0 \psi_o} + \frac{\sum_{j=1}^{n-1} (\delta_j - 1) \delta_j \omega_j}{\psi_o} + (\delta_z - 1) \vartheta_s \right)$$

$$\vartheta_w = \frac{w_o L_o}{\alpha_l Y_0 \psi_o},\ \vartheta_{p_j} = \frac{\delta_j \omega_j}{\psi_o},\ \vartheta_s = \frac{\delta_z \omega_s}{\psi_0},\ \vartheta_\xi = \frac{w_o L_o}{\alpha_l^2 Y_0 \psi_o}$$

$$\alpha_l = \frac{\sigma}{1-\alpha_0},\ \alpha_0 = \sum_{j=1}^{n} \theta_j \delta_j,\ M_{j0} = Y_0^{\delta_j},\ \frac{p_{j0} M_{j0}}{Y_0} = \omega_j,\ \omega_s = \frac{S_0 Y_0^{\delta_z}}{Y_0}$$

$$\lambda_f = \frac{\beta\theta}{\{\theta + \omega\left[1 - \theta\left(1-\beta\right)\right]\}},\ \kappa = \frac{\left(1-\alpha\right)\left(1-\theta\right)\left(1-\beta\theta\right)\left(1-\omega\right)}{\left[1+\alpha\left(\mu-1\right)\right]\{\theta + \omega\left[1-\theta\left(1-\beta\right)\right]\}}$$

$$\lambda_b = \frac{\omega}{\{\theta + \omega\left[1 - \theta\left(1-\beta\right)\right]\}}$$

The actual inventory gap in eq.(22) is derived from the combination of eqs.(15) and (16). The desired inventory gap in eq.(23) is obtained by linearizing the FOC in eq.(14). The expression in eq.(24) describes the dynamics of the inventory-sales ratio. The parameters γ_c, γ_x, γ_z and γ_G are the respective steady state consumption-sales ratio, export-sales ratio, import-sales ratio and spending-sales ratio.

The real marginal cost gap in eq.(25) is obtained by linearizing the real marginal cost function in eq.(12). The real wage gap in eq.(26) is derived from the combination of eq.(4) and eq.(10). Lubik and Teo (2012) estimate a NKPC with and without inventories. They conclude that the NKPC with inventories performs just as well as the standard NKPC. This conclusion allows us to simply adopt the standard NKPC as in Gali and Gertler (1999). The presence of the real exchange rate in the real marginal cost makes our NKPC a small open-economy version.

The real interest rate variable in the desired inventory equation is defined by the Fischer relation, that is $\widehat{R}_t = \widehat{r}_t - E_t \widehat{\pi}_{t+1}$. The parameter θ captures the degree of price stickiness, ω is the degree of price indexation, β is the discount factor, μ is goods elasticity of substitution and α is the capital share in output. The UIP condition in eq.(28) is derived from eq.(6). We follow Lubik and Schorfheide (2007) to specify the monetary policy rule where the parameters $\phi_{r,}$ ϕ_π, ϕ_Y, ϕ_s are the smoothing parameter of the central bank interest rate, the central bank reaction to inflation gap, the output gap and the real exchange rate gap respectively. The model is driven by five major first order autoregressive processes, namely the government spending shock

$\widehat{\varepsilon}_t^G$, the optimism shock $\widehat{\varepsilon}_t$, the technology shock $\widehat{\xi}_t$ and the monetary policy shock $\widehat{\varsigma}_t$. All foreign variables are converted into a first order autoregressive process. In all of these shocks, only the monetary policy and the technology shocks are central to our analysis.

3.3 Parameterization of the model economy

The share of consumption in demand is set at 0.58, which is derived from the Canadadian consumption data, the share of government spending is set at 0.20. Export/import share in sales is set at 0.22, which is the range used in Justiniano and Preston (2010a). Owing to the small open-economy nature of our model, we set the annual interest rate at 6%, that is 1.5% per quarter. This corresponds to a discount factor of 0.985. We set the price indexation parameter at $\theta = 0.09$, the same as the value estimated in Kryvtsov and Midrigan (2010), by introducing inventories in Smets and Wouters (2007) model. We chose this value to gain insights on whether inflation inherits some level of persistence from the inventories. We follow Christiano *et al.* (2005) and set the price stickiness parameter at 0.60. This matches a price reset duration of two and a half quarters. The goods elasticity of substitution is set at 11, which corresponds to a price markup rate of 10%. In line with Chang *et al.* (2009), we set the share of capital in production at 0.33.

We follow Justiniano and Preston (2010b) to define the domestic and foreign elasticity of substitution and follow Sugo and Ueda (2008) to set the elasticity of labour hours to 2. The values $\delta_j = \delta_z = 2$, $\omega_j = \omega_s = 0.5$ and $\theta_j = 0.40$ are set to mimic the estimates in Batini *et al.* (2005) for United Kingdom and Malikane and Mokoka (2014) for both developed and emerging economies. In line with the Taylor rule principle, we follow Smets and Wouters (2007) and set the central bank reaction to inflation and output at 1.50 and 0.125 respectively. The reaction to the exchange rate variable is set at 0.25, which is not far from the value 0.30 used in Justiniano and Preston (2010b). The interest rate smoothing parameter is set at 0.85 and the persistence of the shocks at 0.85, with these values having been taken from Smets and Wouters (2003).

Table 7: Calibrated parameters

Symbol	Description	Values
	Non inventory related parameters	
γ_c	Steady state consumption/sales ratio	0.58
γ_G	Steady state spending/sales ratio	0.20
γ_x/γ_z	Steady state imports,exports/sales ratio	0.22
β	Discount factor	0.985
θ	Degree of price stickiness	0.60
ω	Degree of price indexation	0.09
μ	Goods elasticity of substitution	11.00
α	Capital share in output	0.33
σ_c/θ_z	Elasticity of substitution for domestic and imported goods	1.00
σ_l	Elasticity of labour	2.00
δ_j/δ_z	Fixed input requirement coefficient	2.00
θ_j	Output's elasticity of fixed input	0.40
ϕ_r	Interest rate smoothing	0.85
ϕ_π	Central bank's reaction to inflation	1.50
ϕ_y	Central bank's reaction to output	0.125
ϕ_s	Central bank's reaction to exchange rate	0.25
	Inventory related parameters	
ϕ	Steady state inventory/sales ratio	1.40
λ	Steady state inventory/output ratio	1.38
χ	Inventory adjustment cost parameter	1.67
γ	Elasticity of the probability of stockout to changes in ϕ	0.676
γ_s	Sensitivity of storage	1.39
δ	Sensitivity of the probability of stockout to cost of capital	0.50

In line with Kryvtsov and Midrigan (2010), we set the inventory sales ratio ϕ=1.40 and the inventory adjustment cost parameter $\chi = 1.67$. The inventory-sales ratio is approximately the same as the manufacturing inventory-sales ratio for Canada over the period (1992Q1-2020Q1). We follow Maccini et al.(2015) and set $\gamma = 0.676$. In line with the estimates in Pindyck (1994), we set the sensitivity of storage cost at $\gamma_s = 1.39$. The annual steady state growth rate is set at 5%. The value of the sensitivity of the probability of stockout to changes in the cost of capital is set at $\delta = 0.50$ and the steady state optimism at $A_0 = 0.06$. These values are set to ensure that the steady state probability of stockout matches the 5% used in Kryvtsov and Midrigan (2013).

3.4 Results for calibration

3.4.1 Results of the model without inventories

The results are presented for three different degrees of habit parameter, that is $h = 0.00$ (the solid black line), $h = 0.35$ (the dotted yellow line) and $h = 0.70$ (the purple dashed line).[2] Furthermore, the value of $h = 0.35$ is set to compare our results to micro level evidence cases on habit formation. Figure 4 shows the impulse responses with respect to a monetary policy contraction and a positive technology shock.

The results show that to obtain a hump-shaped response of output and consumption to a monetary policy contraction, a minimum value of $h = 0.70$ is required. This demonstrates why the value of this parameter is high in monetary based DSGE models. For example, the values of habit formation reported in Rabanal (2007) are above 0.73. A value of 0.98 is reported in Bouakez *et al.* (2005), and one of 0.71 is reported in Smets and Wouters (2007). Compared to output, inflation failed to display a hump-shaped response to the monetary policy contraction. This implies that the value of price indexation used here is too low to drive a gradual response of inflation in the model without inventories.

[2] We chose the maximum level to be $h = 0.70$ because the average estimate of habit parameter in DSGE models fluctuates around 0.70.

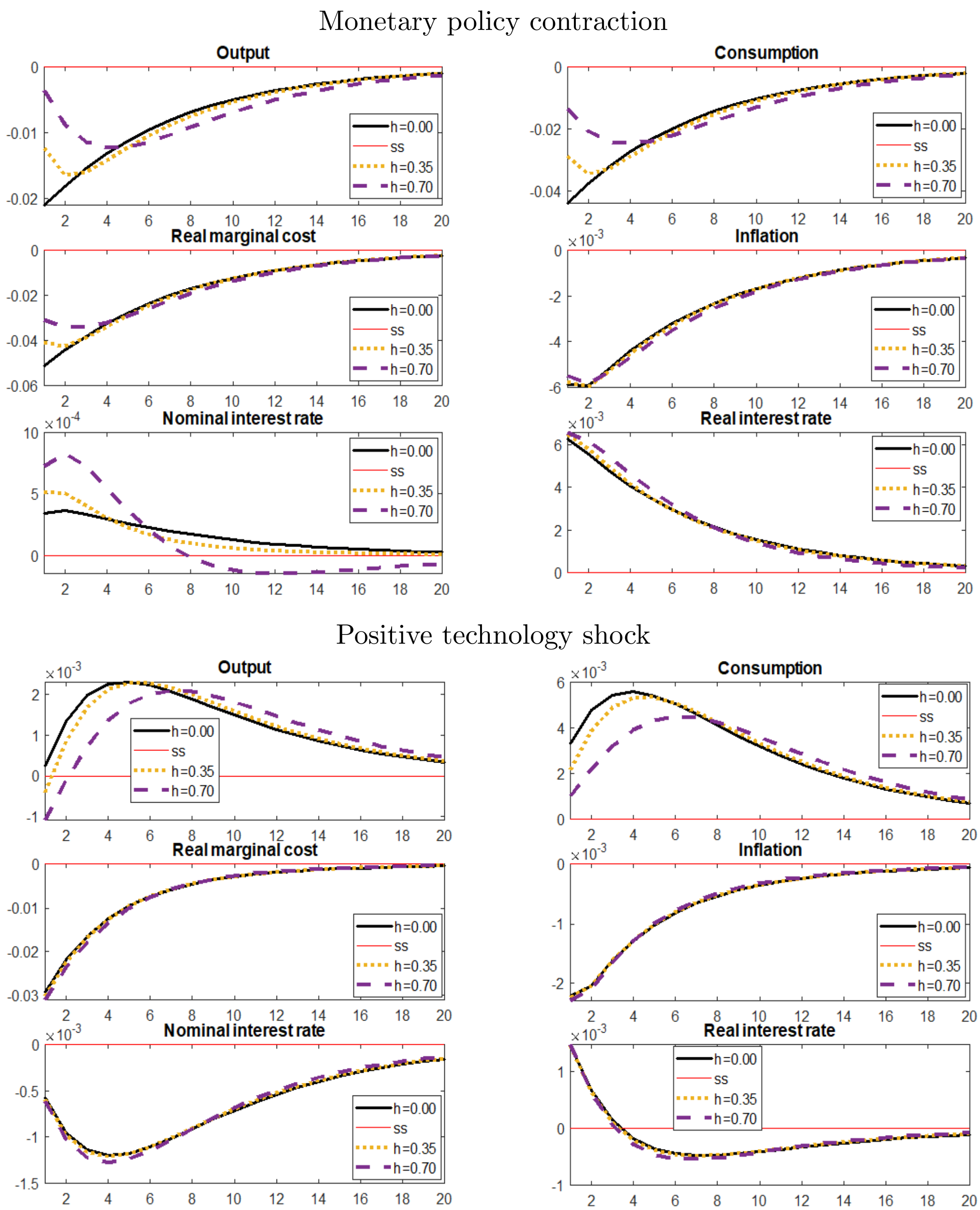


Figure 4: Impulse responses of the model without inventories

It is also observed that the real marginal cost displays a hump-shaped response as the habit parameter increases above zero. The interest rate variables increase after the shock and nominal interest also shows a hump-shaped response as habit formation increases above zero.

Table 8 reports the cyclical properties of the model. These are the standard deviation of each variable, the correlation of the variables with output and the first order autocorrelation of each of the variables. The results presented with the impulse responses are also reflected in these statistics. The role of habit formation is to smooth the dynamics of consumption and therefore output. This is shown by the statistics of the first order autocorrelation in Table 8. It can be seen that the autocorrelations of output and consumption increase from 0.85 at $h = 0.00$ to 0.96 at $h = 0.70$. Furthermore, the volatilities of these two variables decrease from 4.09% to 3.36% and from 8.66% to 6.86% respectively. This indicates that adding habit formation does indeed increase the persistence of output and consumption.

Table 8: Cyclical properties of the model without inventories

	Habit levels			Habit levels			Habit levels		
h	0.00	0.35	0.70	0.00	0.35	0.70	0.00	0.35	0.70
	Standard deviation (SD)			Correlation with output			Autocorrelation		
y	4.09	3.82	3.36	1	1	1	0.85	0.93	0.96
c	8.66	8.07	6.86	0.98	0.98	0.97	0.85	0.92	0.96
ψ	10.90	10.59	9.92	0.85	0.84	0.79	0.83	0.87	0.88
π	1.35	1.37	1.38	0.90	0.90	0.82	0.88	0.88	0.89
r	0.36	0.37	0.40	−0.37	−0.39	−0.32	0.96	0.96	0.95
R	1.26	1.29	1.36	−0.98	−0.96	−0.83	0.85	0.85	0.86

We reported the first order autocorrelation

The results with respect to a positive technology shock reveal that output, consumption and both the nominal and the real interest rates respond gradually to the shock even at $h = 0.00$. Output and consumption increase after the shock and reach their respective peaks at nearly four quarters when

$h = 0.00$, five quarters when $h = 0.35$ and approximately 7 quarters when $h = 0.70$. The real marginal cost and inflation on the other hand drop below their steady states and then start increasing. In terms of the response of inflation to the positive technology shock, Dupor *et al.*(2009) also document that inflation falls as a response to a positive technology shock. Altig *et al.* (2011) introduce firm-specific capital and also find that inflation decreases after a positive technology shock.

3.4.2 Results of the model with inventories

We maintain the parameterization of the model without inventories and allow inventory related parameters to drive the difference between the two versions of the model. The results of the model with inventories are also presented for $h = 0.00$ (the solid black line), $h = 0.35$ (the dotted yellow line) and $h = 0.70$ (the purple dashed line). Table 9 reports the cyclical properties and Figure 5 reports the impulse responses.

Table 9: Cyclical properties of the model with inventories

	Habit levels			Habit levels			Habit levels		
h	0.00	0.35	0.70	0.00	0.35	0.70	0.00	0.35	0.70
	Standard deviation (SD)			Correlation with sales			Autocorrelation		
y	9.72	6.51	5.32	0.99	0.98	0.96	0.97	0.95	0.97
ys	7.18	3.60	2.51	1	1	1	0.97	0.93	0.94
ϕ	6.20	2.54	1.56	−0.99	−0.99	−0.96	0.96	0.90	0.90
c	47.03	13.69	4.83	0.92	0.93	0.94	0.89	0.93	0.95
n	1.01	1.12	1.09	0.97	0.95	0.92	0.98	0.97	0.97
ψ	13.19	10.62	11.11	0.99	0.84	0.86	0.69	0.88	0.89
π	1.91	1.60	1.54	0.95	0.89	0.89	0.95	0.91	0.90
r	3.38	0.71	0.45	−0.98	−0.79	0.25	0.96	0.94	0.95
R	5.14	2.01	1.33	−0.99	−0.91	−0.82	0.96	0.90	0.83

We reported the first order autocorrelation

The results show that at $h = 0.00$, most of the variables, with the exception of consumption, display a humped-shaped response to the monetary policy shock. Output, sales and inventory gaps decline after the shock while output and sales gaps reach their trough nearly a year after the monetary policy contraction and inventory after a year and a quarter. The reason why no hump-shaped response is observed in consumption at $h = 0.00$ is obvious. The hump-shaped responses in output, sales and the inventory-sales ratio at this habit level can be attributed to the presence of inventories. Owing to their shock absorbing nature in the short run, inventories allow these variables to be persistent. The smoothing role that inventories offer is also shown by the first order autocorrelations presented in Table 9. The table shows that the first order autocorrelation of output at $h = 0.00$ is 0.97, which is much higher than the autocorrelation of output at $h = 0.35$ and approximately the same as the autocorrelation at $h = 0.70$ in the model without inventories.

The response of inventories and the inventory-sales ratio in the model are consistent with theoretical expectations. As such, inventories are procyclical and the inventory-sales ratio is countercyclical. Table 9 also shows that inventories are positively correlated with sales and the inventory-sales ratio is negatively correlated with sales. Furthermore, the magnitude of response of output and sales is higher at $h = 0.00$ compared to their responses at $h = 0.35$ and $h = 0.70$. Thus, adding habit formation smoothes the dynamics of consumption and reduces its volatility as well as the volatilities of sales and output. Table 3 also reveals that even in the absence of inventories, consumption is the most volatile component of the quantity variables in the model. The main lesson from the results above is that inventories play the same role assigned to habit formation in the NK models.

The results also show that the real marginal cost, inflation and the interest rate variables respond gradually. Inflation falls after the shock and reaches its trough at about a year. This implies that inflation inherits some level of persistence from the presence of inventories. Therefore, only a moderate level of price indexation, far below those levels estimated in the macro models without inventories, is required to generate a gradual response of inflation to a monetary policy shock. The results are in line with the findings in Boileau and Letendre (2011) which show that adding inventories to a shopping cost model adds to the persistence of output and inflation.

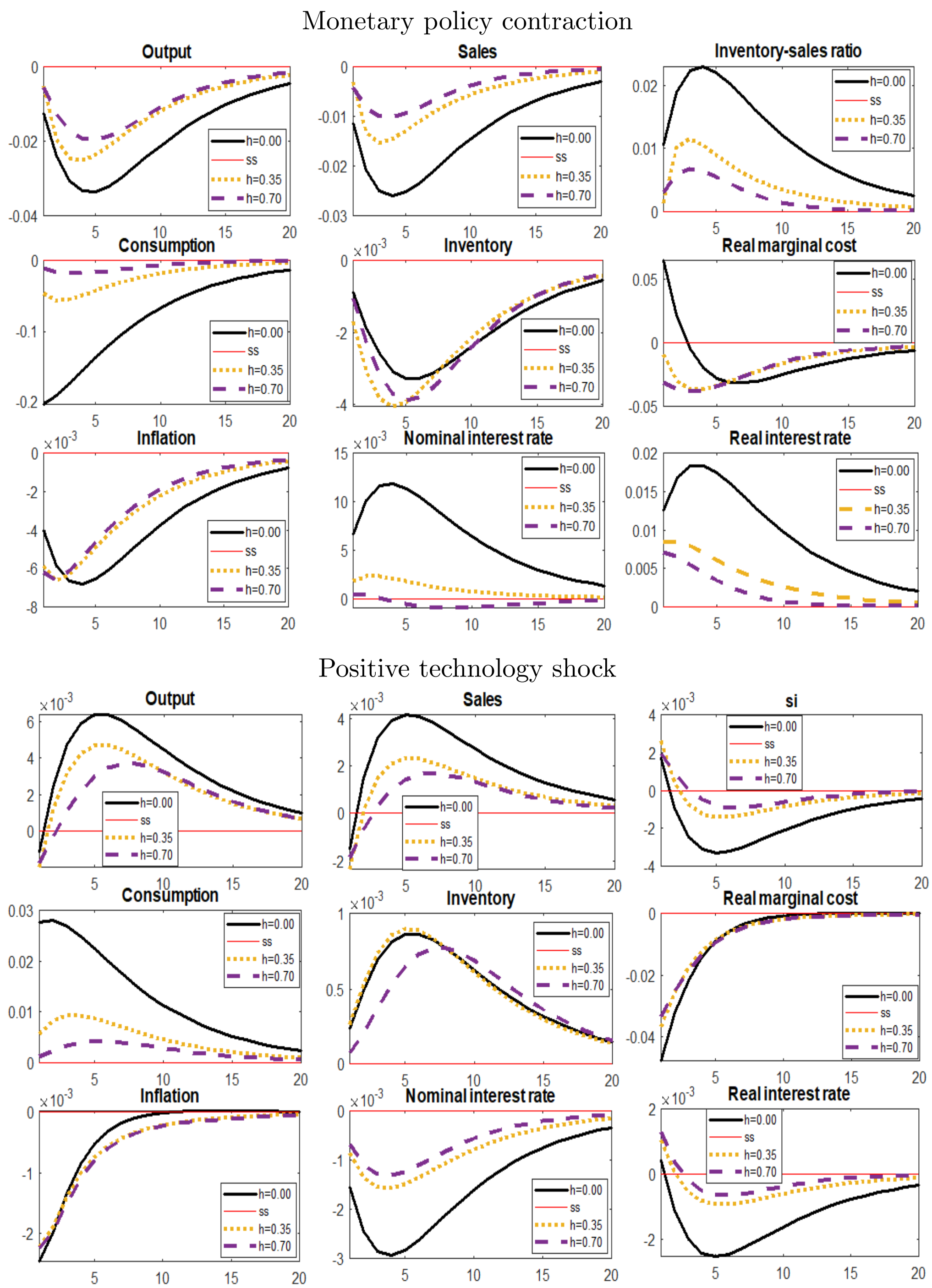


Figure 5: Impulse responses of the model with inventories

Kim (2015) also demonstrates that introducing inventories and margins in the labour market into a general equilibrium model, results in a hump-shaped response in output. His model with inventory alone is not able to generate a hump-shaped response in output, however. Furthermore, he documents counterfactual responses in inventories. What these studies neglect to consider is what would happen under different degrees of habit formation. These models are not based on the NKPC, where price indexation and nominal price stickiness contribute to the persistence of inflation and output respectively.

Our results in terms of the response of output and inventory to the contractionary monetary policy shock are in line with the findings in Copeland and Maccini (2019) and Kryvtsov and Midrigan (2013). Gertler and Gilchrist (1994) also find that small firms deplete inventories after a monetary policy contraction, whereas large firms increase their inventory stock because they have access to more open sources of funding.

On the other hand, Jung and Yun (2005) and Kryvtsov and Midrigan (2010) find that inventories decrease after an expansionary monetary policy shock. It is important to note that these two studies follow inventory in the demand function approach, which is proposed by Bils and Kahn (2000), and may be called models of finished goods inventories. However, our results approve the decline in habit formation from $h = 0.71$ to $h = 0.59$ and the drop in price indexation from $\omega = 0.24$ to $\omega = 0.09$ in the version of Smets and Wouters (2007) with inventories, which is assessed in Kryvtsov and Midrigan (2010). As such, the level of habit formation and price indexation in NK models with inventories should reflect more cases of micro evidence.

We also report the impulse responses to a positive technology shock. At $h = 0.00$, our results show an increase and a delayed response of output, sales and inventory to a positive technology shock, while inflation shows disinflationary behaviour in the short run. The results are also in line with Chang *et al.* (2009). This source argues that when inventories take on the cost smoothing role in an environment that has been subjected to a positive technology shock, this allows the associated firms to expand output in order to accumulate inventories, which are used in turn to smooth production over time. Iacoviello *et al.* (2011) estimate a DSGE model by introducing input inventories in the production function and finished goods inventories in the utility function. They also find that both input and output inventories increase after a positive goods sector specific technology shock.

3.4.3 Model variations

Despite the role of price indexation and price stickiness in the NK literature, some fragilities are noted with these models. One of the most important critiques identified by Cogley and Sbordone (2008) and Chari *et al.* (2009) is the ad hoc nature of price indexation in the NK models. In addition, Chari *et al.* (2000) also point to the fragile role of nominal rigidities in generating persistence in the NK models. Jung and Yun (2005) and Kryvtsov and Midrigan (2010) note that inventory adjustment cost help to reproduce the cyclical behaviour of inventories. As a result, we test the robustness of our results to the variations in these parameters. We have already shown above that inflation inherits some level of persistence from the presence of inventories. That allows us to focus only on the variations in price stickiness and the inventory adjustment cost parameters.

We reset the price stickiness and the inventory adjustments cost parameters, but only one at a time. First of all, we follow Rabanal (2007) and reset the price stickiness parameter to $\theta = 0.0001$, which leads the duration of price reset to approximately a quarter. This is closer to the evidence presented in Bils and Klenow (2004) and Cogley and Sbordone (2008). This version of the model is called the *"flexible price model"*. Secondly, we reset the inventory adjustment cost parameter to $\chi = 0.00$ and call it the *"model without adjustment cost"*. The goal in this case is that, we want to test whether by specifying a convex storage cost as we have done, there is enough room to reproduce the results in the model with inventories as presented in the previous section. The results are also presented for three different levels of habit formation and in response to both monetary and technology shocks.

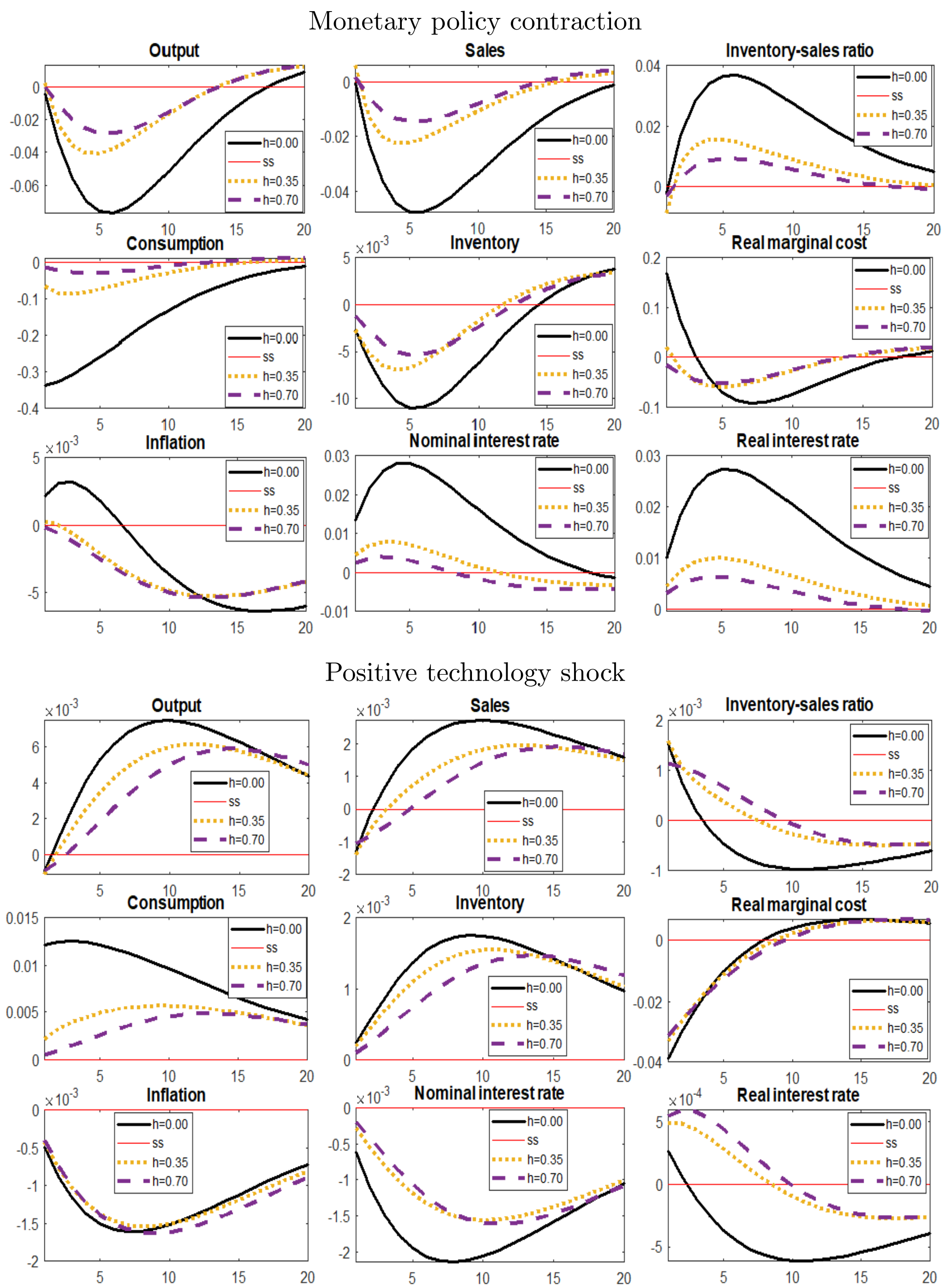


Figure 6: Impulse responses of the flexible price model with inventories

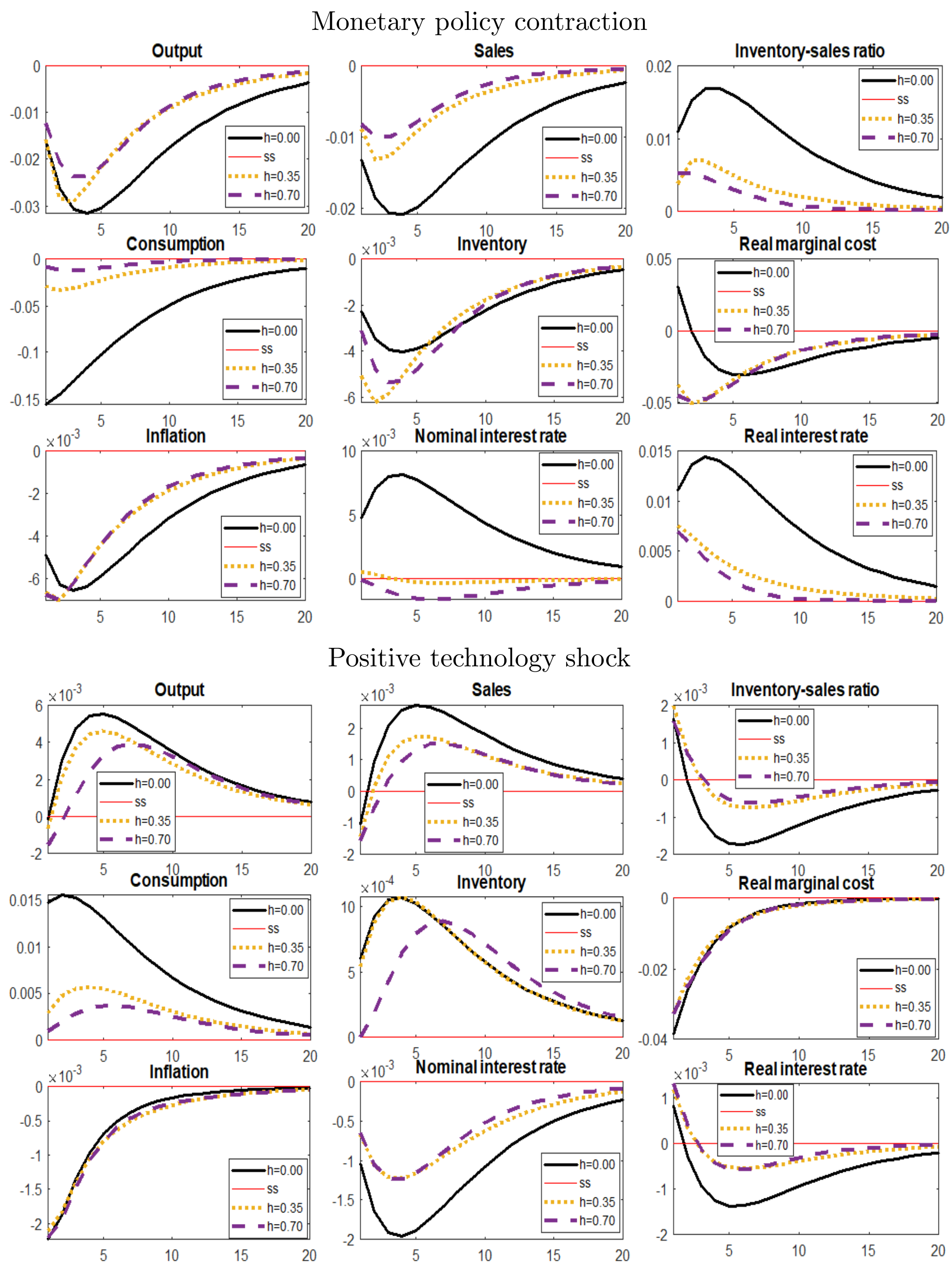


Figure 7: Impulse responses of the model without adjustment cost

Figure 6 shows the results for the flexible price model. The results reveal that output, sales, inventories and the inventory-sales ratio still display a hump-shaped response to the monetary policy shock. Furthermore, the response of inventories is procyclical with sales, while the response of the inventory-sales ratio is still countercyclical. This signifies that the presence of inventories in the NK models is enough to generate a hump-shaped response in output. In addition, the observed price puzzle displayed by inflation and in response to the monetary policy shock is well explained in Rabanal (2007, pp 918-919). Rabanal (2007) shows that low levels of price stickiness and price indexation increase the positive response of inflation to monetary policy contraction in the short run. The responses of the interest rate variables are still consistent with theoretical expectations.

With respect to the positive technology shock, the responses of the variables in the flexible price model are still consistent but are prolonged. Furthermore, inflation shows a persistent disinflationary response to the positive technology shock, which is consistent with the literature. The results of the model without adjustment cost are presented in Figure 7. There is not much difference in the response of the variables to shocks when compared to the response of the variables in the baseline model with inventory adjustment cost. Therefore, a convex storage cost, which we modelled by following Pyndick (1994), is enough to create gradual responses in the aggregate variables. In summary, the ability of the model to spawn a hump-shaped response in the variables at $h = 0.00$ and a gradual response in inflation at a low level of price indexation suggests that DSGE models can be misspecified if inventory adjustments are excluded.

3.5 Conclusion and policy implications

3.5.1 Conclusion

In this study, we investigate whether the addition of inventories to a standard New Keynesian (NK) DSGE model would help spawn endogenous persistence in the aggregate variables. We study whether the dynamic response of macro variables is sensitive to variations in the habit formation, price indexation, nominal price stickiness and the inventory adjustment cost parameter.

These exercises, undertaken with respect to a monetary policy shock and a technology shock, reveal that the hump-shaped response of output to a contractionary monetary policy is mainly driven by habit formation in the model without inventories.

In the version of the model that includes inventory adjustments, the results reveal that output, sales and inventory display a hump-shaped response to both shocks at zero degrees of habit formation. Moreover, only a moderate level of price indexation, far below those estimated in the macro models without inventories, is required to generate a gradual response of inflation to a monetary policy shock. We also note that it would not be necessary to introduce inventory adjustment cost into the total cost of holding inventories. Furthermore, inventories are procyclical, while the inventory-sales ratio is countercyclical. These results are robust and resilient in terms of the variations in the probability of price stickiness and the inventory adjustment cost parameters

3.5.2 Policy implications

This study has shown that inventories are a natural source of persistence in output, while inflation inherits some level of persistence in the presence of inventories. We document that inventories play the role of habit formation when they are incorporated in the new Keynesian models. Thus, using inventories rather than habit formation in NK models not only allow central bankers to successfully reproduce the behaviour of the data but also to avoid the use of free parameters when modelling the behaviour of the real economy. The implications of the findings for the existing literature is that DSGE models without inventories can result in the wrong parameterization of calibrated models and incorrect estimates in estimated models. Should these decision be based on models without inventory dynamics, this, aspect would consequently, lead to costly or erroneous policy decisions.

4 Inventories and Asset Prices Dynamics in a New-Keynesian Model

4.1 Introduction

The reaction of asset prices to monetary policy shocks has been studied extensively using event studies, structural VAR as well as DSGE models. For instance, Bernanke and Kuttner (2005) use an event study to report that a 0.25% cut in the monetary policy rate increases the stock price by approximately 1%. Bjørnland and Leitemo (2009) use a structural VAR model and find that the S&P500 declines by 7% to 9% following a 1% increase in the Fed's rate. Alessi and Kerssenfischer (2019) note that the reaction of asset prices is stronger in a dynamic factor model (DFM) than in a standard VAR model. Challe and Giannitsarou (2014) employ a New-Keynesian (NK) model and document that stock prices react by approximately 3.06% to 6.57% to a 100 basis-point increase in the interest rate. However, none of these studies considers the role that inventories might play in macro models with asset prices and the financial accelerator.

Inventories are known for their role in business cycle fluctuations (Maccini and Pagan, 2013). They are also known to play an intertemporal smoothing role similar to habit formation (Boldrin et al., 2001). Despite these acknowledgements, the use of habit formation to explain the asset pricing implications of macro models, dominates the literature. Nonetheless, recent studies such as Belo and Lin (2012) and Jones and Tuzel (2013) have also documented that inventory based portfolios exhibit a substantial spread in future excess returns. They also show that risk premia are the appropriate costs of capital for inventory holdings. These findings, in combination with the relationship between inventories and the firm's networth as documented in Carpenter et al. (1994), motivate for the use of the financial accelerator (FA) mechanism to link inventories and asset prices. Theoretical and empirical evidences on the FA mechanism can be found in Bernanke *et al.* (1999) and Christensen and Dib (2008).

The gap we intend to fill in this study is to use a NK model with inventories and the FA mechanism to explain the reaction of asset prices to monetary policy shocks. We show that when inventories rather than habit formation, are used as a smoothing device, this not only produces hump-shaped responses of aggregate variables but also it amplifies the reaction of asset prices to monetary policy shocks. Studies such as Li and Palomino (2014) and De-Paoli *et al.* (2010) focus on the role of real and nominal rigidities in explaining asset price responses to both monetary policy and productivity shocks. Gali and Gambetti (2015) study the reaction of the components of asset prices, namely fundamental and bubbles, while Laopodis (2013) examines the interconnection between the stock market and the US monetary policy regimes. Models with the FA mechanism and asset prices such as those devised by Aoki et al. (2004) focus on the housing market. Scholars have not paid much attention to the role of inventories in these studies, and that issue, constitutes the focus of this paper.

We contribute to the existing literature by linking inventories and asset prices and calibrating four versions of our open economy model. This allows us to quantify and compare asset price multipliers and to detect how inventory adjustment and the FA mechanism amplify shocks to the economy. Secondly, the calibrated versions of our model are subject to habit formation. This allow us to detect which, inventory or habit formation explains asset prices fluctuations more effectively. Although our study is related to asset price models such as Castelnuovo and Nistico (2010), Nistico (2012), and FA models such as those of De Graeve (2008) and Hansen (2018), the papers more closely aligned with ours are the work of Li *et al.* (2010) and Challe and Giannitsarou (2014). The work of Challe and Giannitsarou (2014) is a NK DSGE model. However, it is not a small open economy model. In addition, the small open economy nature of our model is motivated by the work of Li et al. (2010) but they employ a structural VAR model. The main difference between our study and these two is that we incorporate inventory dynamics and the FA mechanism.

The remainder of the paper is structured as follows: In Section 2, we present both the NK model and its linearized version. Section 3 explains the parameters and the calibrated values, Section 4 presents the results of calibration and Section 5 concludes the paper and provides some policy implications.

4.2 Model

4.2.1 Households

Like the model of Adolfson *et al.* (2008, 2014) and Christiano *et al.* (2011), we assume that the household consumption is based on domestically produced and imported goods. Households also supply labour hours in a competitive labour market and earn a competitive labour income. They maximize their utility using the following technology:

$$U_t(C_t) + U_t(Z_t) - U_t(L_t) = \frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} + \frac{Z_t^{1-\theta_z}}{1-\theta_z} - \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \quad (1)$$

In Eq.(1), C_t stands for domestic consumption, Z_t is imported consumption and L_t represents labour hours, and σ_c,θ_z, and σ_l are their respective elasticity parameters. Following Castelnuovo and Nistico (2010) and Challe and Giannitsarou (2014), we assume that from period to period households transfer wealth using domestic and foreign government bonds and equity shares. For simplicity's sake, we assume a domestic (home) bias and postulate that households hold only domestic equity shares E_t^k, and do not invest in foreign equity shares. Furthermore, households interact with firms through the stock market and require a premium over the riskless bond rate in order to hold the risky equity shares of firms. This risk premium is comparable to an equity premium. Thus, following Bernanke *et al.*(1999), Gilchrist and Leahy (2002) and Aoki *et al.* (2004), we use the FA mechanism to model the dynamics of the gross value of the external finance premium or equity premium as follows:

$$(1 + \rho_t) = \left(Q_t \frac{E_t^k}{B_t} \varepsilon_t^n\right)^{-\gamma_n} \quad (2)$$

where $Q_t = \frac{P_{et}}{P_t}$ is the real equity price, P_{et} is the nominal equity price, P_t is the domestic price index, B_t is the domestic short term debt instrument or risk free domestic bond, $\frac{E_t^k}{B_t}$ is networth, γ_n is the elasticity of the external finance premium and the disturbance term ε_t^n stands for the networth's shock. Following the FA literature, especially the work of Christensen and Dib (2008), we assume that networth is procyclical to output, which allows us to rewrite eq.(2) as follows:

$$(1+\rho_t) = \left[Q_t f\left(Y_t\right)\varepsilon_t^n\right]^{-\gamma_n} \tag{3}$$

where Y_t is output. We now formulate the household's budget constraint as follows:

$$\begin{aligned} Q_t E_t^k + \frac{B_t}{P_t} + \frac{E_t^e}{P_t}B_t^f = Q_t E_{t-1}^k + \frac{\left(1+r_{t-1}\right)\left(1+\rho_{t-1}\right)B_{t-1}}{P_t} \\ + \frac{\left(1+r_{t-1}^f\right)E_t^e}{P_t}B_{t-1}^f + \frac{W_t}{P_t}L_t - C_t - \frac{E_t^e P_{zt}^f}{P_t}Z_t \end{aligned} \tag{4}$$

where the variable B_t^f stands for short term foreign bond, r_t represents the nominal domestic interest rate and r_t^f is the nominal foreign interest rate. E_t^e stands for the nominal exchange rate and the term $\frac{E_t^e P_{zt}^f}{P_t}$ is the real exchange rate which we further denote by S_t. $\frac{W_t}{P_t}$ is the real wage and W_t is the competitive nominal wage that households earn by supplying labour hours. The introduction of ρ_t captures the variations in the stock price.

It is important to point out that the mechanism in eq.(3) applies to both the household's portfolio as well as the firm's balance sheet. From the household perspective, shocks that increase output and stock prices will increase the household's networth and reduce the premium required to hold equity shares. From the firm's perspective, shocks that increase the firm's output and equity price, increase the firm's networth and put a downward pressure on the equity premium required to raise equity finance using the stock market. The first order conditions (FOC) from the combination of eqs.(1) and (4) are presented as follows:

$$(C_t - hC_{t-1})^{-\sigma_c} = \lambda_t \tag{5}$$

$$\lambda_t = \beta E_t \frac{(1+r_t)(1+\rho_t)}{(1+\pi_{t+1})} \lambda_{t+1} \tag{6}$$

$$\frac{L_t^{\sigma_l}}{\lambda_t} = \frac{W_t}{P_t} \tag{7}$$

$$\beta E_t \frac{(1+r_t)(1+\rho_t)}{(1+\pi_{t+1})} S_t = E_t \frac{\left(1+r_t^f\right)}{\left(1+\pi_{t+1}^f\right)} S_{t+1} \tag{8}$$

$$\lambda_t Q_t = \beta E_t (\lambda_{t+1} Q_{t+1}) \tag{9}$$

$$Z^{-\theta_z} = S_t \lambda_t \tag{10}$$

The FOC in eq.(5) is with respect to consumption while eq.(6) presents the FOC with respect to the domestic bond. Eq.(7) displays the FOC with respect to labour hours and eq.(8) represents the FOC with respect to the foreign bond. Eqs(9) and (10) are the FOCs with respect to equity shares and imported goods respectively. The variable λ_t is the Lagrangian multiplier and β is the subjective factor discount. In the presence of inventories, demand is equal to sales $\left(Y_t^D = Y_t^s\right)$ but demand differs from output $\left(Y_t^D \neq Y_t\right)$. We therefore specify the aggregate demand (sales) as follows:

$$Y_t^s = C_t + X_t - Z_t + G_t \tag{11}$$

where Y_t^s and X_t are sales and exports respectively and G_t is government spending, which is further used as a demand shock in the linearized version of the model economy.

4.2.2 Production unit

The production unit of our model economy is based on Batini *et al.* (2005) and Malikane (2014). Building on these studies allows us to derive a real marginal cost equation that applies to small open economies. We assume that firms produce goods using labour, technology and other fixed domestic and foreign factors of production in fixed proportion to output. These other inputs are nonlinear in output and the production technology is determined as follows:

$$Y_t = \xi_t^{'} L_t^{\sigma} \left[\Pi_{j=1}^{n} Y_t^{\delta_j \theta_j} \right] \tag{12}$$

$$Y_t = \xi_t L_t^{\alpha_l} \tag{13}$$

where $0 < \sigma < 1$, Y_t is the real output, L_t is labour input and ξ_t captures the state of technology. The parameter δ_j governs the coefficient of input requirement and θ_j is the output's elasticity of other inputs in the production process. The reduced form of eq.(12) is presented in eq.(13) where $\alpha_l = \frac{\sigma}{1-\alpha_0}$, $\alpha_0 = \sum_{j=1}^{n} \theta_j \delta_j$, $\xi_t = \xi_t^{'\frac{1}{1-\alpha_0}}$, $L_t = \xi_t^{-\frac{1}{\alpha_l}} Y_t^{\frac{1}{\alpha_l}}$. The production cost is composed of the cost of labour and costs related to other domestic and foreign inputs. The total cost of production and the real marginal cost are therefore determined as follows:

$$TC_t = \frac{W_t}{P_t} \left(\xi_t^{-\frac{1}{\alpha_l}} Y_t^{\frac{1}{\alpha_l}} \right) + \frac{E_t^e P_{zt}^f}{P_t} Y_t^{\delta_z} + \sum_{j=1}^{n-1} \frac{P_{jt}}{P_t} Y_t^{\delta_j} \tag{14}$$

$$\psi_t = \frac{1}{\alpha_l} w_t \left(\xi_t^{-\frac{1}{\alpha_l}} Y_t^{\frac{1-\alpha_l}{\alpha_l}} \right) + \delta_z S_t Y_t^{\delta_z - 1} + \sum_{j=1}^{n-1} \delta_j p_{jt} Y_t^{\delta_j - 1} \tag{15}$$

where TC_t is the total cost of production and ψ_t is the real marginal cost. P_{jt} is the price of other domestic inputs and P_{zt}^f is the price of imported inputs, $w_t = \frac{W_t}{P_t}$ is the real wage, and $S_t = \frac{E_t^e P_{zt}^f}{P_t}$ is the real exchange rate.

4.2.3 Inventory management unit

In line with Maccini and Pagan (2013) and Maccini et al.(2015), we formulate the aggregate inventory holding cost as the combination of the stockout cost, the financing cost of holding inventories, the storage cost and the inventory adjustment cost. Furthermore, we model the probability of stockout to capture the view that if firms place much weight on stockouts, they would still accumulate stocks of goods to maintain their desired level of inventory even if the cost of capital for inventory holdings were to increase. In other words, if the benefit for the inventory holdings is high, firms would still accumulate inventories in the face of the increasing cost of capital. This modification captures the sign puzzle as explained in Maccini et al. (2004, 2015). However, Jones and Tuzel (2013) argue that it is not really a puzzle and that risk premiums should constitute the appropriate cost of capital for inventory holdings. This is what motivates the use of equity premium as the cost of capital for inventory holdings in this study.

We define the unit cost of storage following Pindyck (1994), which is a convex function of the lagged inventory-sales ratio and expressed as a proportion of the current inventory stock. The aggregate inventory holding cost H_t is therefore determined as follows:

$$H_t = A_t \left(\frac{N_t}{Y_t^s}\right)^{-\gamma} Y_t^s e^{\delta \rho_t} + \rho_t N_t + \left(\frac{N_{t-1}}{Y_t^s}\right)^{-\gamma_s} N_t + \frac{\chi}{1+\vartheta}\left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} N_t \quad (16)$$

where A_t captures the optimism of stock managers, N_t is the level of inventories, Y_t^s represents sales,ρ_t denotes the cost of capital or the risk premium, and $\gamma > 0$ is the sensitivity of the probability of stockout to changes in the inventory-sales ratio.[3] $\delta > 0$ is the sensitivity of the probability of stockout to changes in the cost of capital and $\gamma_s > 0$ is the sensitivity of storage cost to the lagged inventory-sales ratio. The first term in eq.(16) represents stockout cost. The second term stands for the opportunity cost of holding inventories and the third term is the storage cost. The fourth component is the inventory adjustment cost. ϑ is the degree of convexity of the adjustment cost

[3] Maccini and Pagan (2013) note that it captures the firm's elasticity of benefits for inventory holdings.

function and χ is the adjustment cost parameter. The first order condition using eq.(16) is derived as follows:

$$\gamma A_t N_t^{-(1+\gamma)} Y_t^{s(1+\gamma)} e^{\delta \rho_t} = \rho_t + \left(\frac{N_{t-1}}{Y_t^s}\right)^{-\gamma_s} + \chi \left(\frac{2+\vartheta}{1+\vartheta}\right) \left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} \qquad (17)$$

The level of inventories at time t that is consistent with eq.(17) is the desired inventory at time t. Following Flaschel *et al.* (2001), we assume that the supply side has two inventory accumulation identities which are specified as follows:

$$N_t = N_t^d + Y_{t-1} - Y_{t-1}^s \qquad (18)$$

$$Y_t = Y_t^s + N_t - N_{t-1} \qquad (19)$$

The identity in eq.(18) is related to the inventory management unit and states that the level of the actual inventory is equal to the desired inventory plus inventory investment. Eq.(18) is formulated using adaptive expectations. The identity in eq.(19) is the normal output identity.

4.2.4 Linearized model economy

The demand side of the model is comprised of aggregate sales, consumption, exports, imports, the equity premium and the stock price. The supply side is composed of output, inventories, inventory-sales ratio, the real marginal cost, the real wage and inflation. The aggregate sales gap in eq.(20) is obtained from eq.(11), while the consumption gap in eq.(21) is obtained from the FOCs in eqs.(5) and (6). Exports and imports in eqs.(22) and (23) respectively are obtained from the FOC in eq.(10) where exports are simply foreign imports as in McCallum and Nelson (2000). The equity premium in eq.(24) is obtained by linearizing eq.(3) and the stock price gap in eq.(25) is obtained from the FOC in eq.(9). The output gap in eq.(26) is derived from the identity in eq.(18). The dynamics of inventory gap in eq.(27) is obtained from eq.(19)

and the FOC in eq.(17), where the desired inventory gap is obtained from eq.(17) and substituted in the linearized form of eq.(19). Eq.(28) signifies the inventory sales ratio gap.

$$\widehat{y}_t^s = \gamma_c \widehat{c}_t + \gamma_x \widehat{x}_t - \gamma_z \widehat{z}_t + \gamma_G \widehat{\varepsilon}_t^G \tag{20}$$

$$\widehat{c}_t = -\frac{(1-h)}{\sigma_c (1+h)} \left(\widehat{r}_t - E_t \widehat{\pi}_{t+1} + \widehat{\rho}_t\right) + \frac{1}{1+h} E_t \widehat{c}_{t+1} + \frac{h}{1+h} \widehat{c}_{t-1} \tag{21}$$

$$\widehat{x}_t = \frac{\sigma_c}{\theta_z (1-h)} \widehat{c}_t^f - \frac{\sigma_c h}{\theta_z (1-h)} \widehat{c}_{t-1}^f + \frac{1}{\theta_z} \widehat{s}_t \tag{22}$$

$$\widehat{z}_t = \frac{\sigma_c}{\theta_z (1-h)} \widehat{c}_t - \frac{\sigma_c h}{\theta_z (1-h)} \widehat{c}_{t-1} - \frac{1}{\theta_z} \widehat{s}_t \tag{23}$$

$$\widehat{\rho}_t = -\gamma_n \left(\widehat{q}_t + \theta_y \widehat{y}_t + \widehat{\varepsilon}_t^n\right) \tag{24}$$

$$\widehat{q}_t = -\left(\widehat{r}_t - E_t \widehat{\pi}_{t+1}\right) + E_t \widehat{q}_{t+1} - \rho_t \tag{25}$$

$$\widehat{y}_t = (1 - \lambda g) \widehat{y}_t^s + \lambda \widehat{n}_t - \left(\frac{\lambda}{1+g}\right) \widehat{n}_{t-1} \tag{26}$$

$$\widehat{n}_t = \left(\theta_n + \frac{1}{1+g}\right) \widehat{n}_{t-1} - \frac{1}{(1+g)^2} \widehat{n}_{t-2} + \theta_s \widehat{y}_t^s - \theta_\rho \widehat{\rho}_t + \widehat{\varepsilon}_t \tag{27}$$

$$\phi_t = \widehat{n}_t - \widehat{y}_t^s \tag{28}$$

$$\widehat{\psi}_t = \vartheta_y \widehat{y}_t + \vartheta_w \widehat{w}_t + \vartheta_s \widehat{s}_t + \sum_{j=1}^{n-1} \vartheta_{p_j} \widehat{p}_{jt} - \vartheta_\xi \widehat{\xi}_t \tag{29}$$

$$\widehat{w}_t = \frac{\sigma_l}{\alpha_l} \widehat{y}_t + \frac{\sigma_c}{(1-h)} \widehat{c}_t - \frac{\sigma_c h}{(1-h)} \widehat{c}_{t-1} - \frac{\sigma_l}{\alpha_l} \widehat{\xi}_t \tag{30}$$

$$\widehat{\pi}_t = \lambda_f \widehat{\pi}_{t+1} + \lambda_b \widehat{\pi}_{t-1} + \kappa \widehat{\psi}_t \tag{31}$$

$$\widehat{s}_t = E_t \widehat{s}_{t+1} + \left(\widehat{r}_t^f - \widehat{r}_t\right) + \left(E_t \widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1}^f\right) - \rho_t \tag{32}$$

$$\widehat{r}_t = \phi_r \widehat{r}_{t-1} + (1 - \phi_r) \left(\phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t + \phi_s \widehat{s}_t\right) + \epsilon_t \tag{33}$$

The linearized real marginal cost presented in eq.(29) is from eq.(15). The real wage gap in eq.(30) is derived from the FOC in eq.(7) and eq.(13). We follow Gali and Gertler (1999) to specify the hybrid New Keynesian Phillips curve (NKPC) in eq.(31), this because the NKPC with inventories presented in Lubik and Teo (2012) performs just as well as the standard NKPC. The uncovered interest parity (UIP) condition in eq.(32) is obtained by linearizing the FOC in eq.(8), which is the same as the one in Gali and Monacelli (2005). To close the model, we follow Justiniano and Preston (2010a) and specify the interest rate rule in eq.(33), where the central bank reacts to inflation, output and the exchange rate. Complex parameters in the model are summarized as follows:

$$
\begin{aligned}
\theta_n &= \frac{\omega_i+\zeta}{\varphi+\zeta}, \theta_s = \frac{\varphi-\omega_i}{\varphi+\zeta}, \theta_\rho = \frac{(\phi-\gamma\delta p_0^s)}{\phi(\varphi+\zeta)}, \varphi = \gamma(1+\gamma)\frac{p_0^s}{\phi}, \\
p_0^s &= A_0\frac{e^{(\delta\rho_0)}}{\phi^\gamma}, \ \omega_i = \frac{\gamma_s}{(1+g)(\phi)^{\gamma_s}}, \widehat{\varepsilon}_t = \frac{1}{\varphi+\zeta}\widehat{a}_t, \zeta = \chi(2+\vartheta)(1+g)^{(1+\vartheta)} \\
\vartheta_y &= \left(\left(\frac{1-\alpha_l}{\alpha_l^2}\right)\frac{w_oL_o}{Y_0\psi_o} + \frac{\sum_{j=1}^n(\delta_j-1)\delta_j\omega_j}{\psi_o} + \frac{(\delta_z-1)\delta_z\omega_s}{\psi_0}\right) \\
\vartheta_w &= \frac{w_oL_o}{\alpha_lY_0\psi_o}, \ \vartheta_{p_j} = \frac{\delta_j\omega_j}{\psi_o}, \ \vartheta_s = \frac{\delta_z\omega_s}{\psi_0}, \ \vartheta_\xi = \frac{w_oL_o}{\alpha_l^2Y_0\psi_o}, \ \theta_y = f_YY_0 \\
\omega_j &= \frac{p_{j0}Y_0^{\delta_j}}{Y_0}, \ \omega_s = \frac{S_0Y_0^{\delta_z}}{Y_0}, \ \lambda_f = \frac{\beta\theta}{\{\theta+\omega[1-\theta(1-\beta)]\}} \\
\kappa &= \frac{(1-\alpha)(1-\theta)(1-\beta\theta)(1-\omega)}{[1+\alpha(\mu-1)]\{\theta+\omega[1-\theta(1-\beta)]\}}, \ \lambda_b = \frac{\omega}{\{\theta+\omega[1-\theta(1-\beta)]\}}
\end{aligned}
$$

The parameters of the NKPC are defined as: θ is the degree of price stickiness, ω is the degree of price indexation, β is the discount factor and μ is goods elasticity of substitution. The interest rate rule parameters $\phi_{r,}$ ϕ_π, ϕ_Y, ϕ_s include the central bank interest rate smoothing parameter, the central bank reaction to the inflation gap, the output gap and the exchange rate respectively. The model is driven by six autoregressive processes. These are the government spending shock $\widehat{\varepsilon}_t^G$, the optimism shock $\widehat{\varepsilon}_t$, the networth shock $\widehat{\varepsilon}_t^n$, the technology shock $\widehat{\xi}_t$ and the monetary policy shock $\widehat{\varsigma}_t$. All foreign variables are converted into a first order autoregressive process. However, the main focus of the paper is the monetary policy shock.

4.3 Parameterization of the model economy

In line with Smets and Wouters (2007), we set the government spending/sales ratio to 0.18, which is also closer to the value 0.20 used in Bernanke et al. (1999). Following Justiniano and Preston (2010a), we calibrate the import/export share in sales to 0.27, which is closer to the prior values of 0.28 these authors used for Canada as a small open economy. These values imply a consumption/sales ratio of 0.55, which is also consistent with the consumption-sales ratio of Canada.

The steady state annual interest rate is set at 6%, which corresponds to a discount factor of 0.985. The price stickiness parameter is taken from Challe and Giannitsarou (2014) and the price indexation is set closer to the estimated value in Lubik and Teo (2012). The elasticity of substitution for domestic and imported goods are given a value of unity, which is consistent with the literature. The share of capital in output is set at the same value in Aoki *et al.* (2004). The elasticity of labour is set at 2, which is below the value of 2.5 used in the FA model of Agenor *et al.* (2014). We follow Christiano *et al.* (2005) and set the goods elasticity of substitution to 21, which corresponds to a markup rate of 5%. We employ standard interest rate rule parameters but take the central bank reaction to the exchange rate from Justiniano and Preston (2010b). Parameters such as the coefficient of input requirement, the elasticity of other inputs to output are set following Malikane and Mokoka (2014).

The parameters for the inventory-sales ratio and the inventory adjustment are taken from Kryvtsov and Midrigan (2010, 2013). The value 1.40 adopted in this study is closer to the average value 1.38 of the manufacturing inventory-sales ratio for Canada for the period 1992Q1-2020Q1. This period also matches the period of inflation targeting in Canada and justifies the use of an interest rate rule in this study. The value of the inventory-sales ratio is set to also reflect the recent trends in the manufacturing inventory-sales ratio for Canada. We choose to match the frequency of stockout of 5% used in Kryvtsov and Midrigan (2013) which is approximately the average value reported in Bils (2016). This requires us to set the steady state optimism parameter at $A_0 = 0.055$, which yields a frequency of stockout of 4.93%.

Table 10: Calibrated parameters

Symbol	Description	Values
	Non inventory and non FA related parameters	
γ_c	Steady state consumption/sales ratio	0.55
γ_G	Steady state spending/sales ratio	0.18
γ_x/γ_z	Steady state imports,exports/sales ratio	0.27
β	Discount factor	0.985
θ	Degree of price stickiness	0.60
ω	Degree of price indexation	0.40
μ	Goods elasticity of substitution	21.0
α	Capital share in output	0.33
σ_c/θ_z	Intertemporal elasticity for domestic and imported goods	1.00
σ_l	Elasticity of labour	2.00
δ_j/δ_z	Fixed input requirement coefficient	2.00
θ_j	Output's elasticity of fixed input	0.38
ϕ_r	Interest rate smoothing	0.85
ϕ_π	Central bank's reaction to inflation	1.50
ϕ_y	Central bank's reaction to output	0.125
ϕ_s	Central bank's reaction to exchange rate	0.25
	Inventory and FA related parameters	
ϕ	Steady state inventory/sales ratio	1.40
λ	Steady state inventory/output ratio	1.38
χ	Inventory adjustment cost parameter	1.67
γ	Sensitivity of stockout to inventory/sales ratio	0.35
γ_s	Sensitivity of storage cost	1.39
δ	Sensitivity of stockout to changes in the cost of capital	0.50
γ_n	Elasticity of the external equity premium	0.05
θ_y	Output's elasticity of the external equity premium	0.58
ρ_0	Steady state equity premium/annum	0.06

We set the elasticity of the probability of stockout to the inventory-sales ratio in the range provided in the literature. For example, Maccini and Pagan (2013) estimate this parameter to be 0.026 for finished goods inventories and 0.015 for input inventories. Furthermore, Maccini *et al.* (2015) provide a value of 0.676 for this parameter while Dasgupta et al. (2019) calibrate it at 1.5. Therefore, the value 0.35 adopted here is within the range reported in recent literature. The value of the inventory adjustment cost parameter that we employ is the estimated value in Kryvtsov and Midrigan (2010). The sensitivity of the probability of stockout to changes in the cost of capital is set at 0.50 and the steady state growth rate at 5% per annum. It is important to note that inventory related parameters are set so that the coefficient on the cost of capital $\theta_\rho = \frac{\left(\phi - \gamma\delta p_0^s\right)}{\phi(\varphi+\zeta)}$ is positive. The combination of inventory related parameters yields a value of $\theta_\rho = 0.1928$, which is consistent with the estimates in Jones and Tuzel (2013).

We use the elasticity of external finance premium provided in Bernanke et al. (1999) and Gilchrist and Leahy (2002). The value of this parameter often fluctuates around 0.05 in the literature. For instance, Christensen and Dib (2008) estimate it to be 0.042 and De Graeve (2008) reports it at 0.10, while Aoki et al. (2004) calibrate it to be 0.10. We set the steady state equity premium at 6%, which is consistent with the recent statistics of the equity premium provided in Horvarth (2020). He reports an average equity premium of 4.9% for developed economies and 9.7% for emerging economies. This value is consistent with the average value often reported in the literature pertaining to the equity premium puzzle (see for example Boldrin *et al.*, 2001).

We follow Aoki et al. (2004) and set the standard deviation of the monetary policy shock at 0.0025, which correspond with an annualized 1% or a 100 basis-point increase in the nominal interest rate. The use of this value is common in the literature that studies the interdependence between monetary policy and asset prices. We follow Challe and Giannitsarou (2014) and set the standard deviation of technology shock at 0.025, while the standard deviation of all the other shocks in the model is set at 0.0025. Finally, we follow Challe and Giannitsarou (2014) and report the results for $h = 0$ and for $h = 0.80$. Setting $h = 0$ allows us to firstly focus on the role of inventories. Increasing it to $h = 0.80$ helps to determine the role of habit formation in a NK model with inventories and asset prices.

4.4 Results for calibration

First of all, the system is reduced to the nine aggregate variables we deem important to the objectives of the study. These are output gap, sales gap, inventory-sales ratio gap, inventory gap, the stock price gap, equity premium, nominal interest rate, real expected interest rate and inflation. The real expected interest rate, that is the difference between the nominal interest rate and expected inflation, is measure with the Fischer equation. The reported variables are simply the ones reported in Challe and Giannitsarou (2014) to which we add the dynamics of inventory and the inventory-sales ratio. The versions of the model for which we report results are (a) with financial accelerator and inventories (henceforth, WFAN), (b) without financial accelerator (henceforth, WTFA), (c) without inventories (henceforth, WTN) and (d) without financial accelerator and inventories (henceforth, WTFAN).

The most important statistics pertaining to our objectives are the impulse responses and the asset prices multipliers. The impulse responses for the baseline model are presented in Figures 8. The black solid line displays the results for WFAN, the dotted yellow line shows the results for WTFA, the dashed purple line presents the results for WTN and the green dotted-dashed line presents the results for WTFAN. For $h = 0.00$, the results show that the magnitude of response of output gap in the model with the FA mechanism is greater than that reflected in the model without FA. This holds true, regardless of the presence of inventories. We observe similar responses in sales gap, inventory-sales ratio gap, equity price gap, equity premium and inflation.

The responses of the interest rate variables are persistent and consistent with the theory and do not display any liquidity puzzle. However, we note that the decline in inventories following the shock is less with the FA mechanism and greater without the FA mechanism, which implies that inventories are less volatile in the version WFAN than in WTFA.

The stock price and equity premium multipliers are reported in Table 11. At $h = 0.00$, the stock price declines by 33.36% in the model with both FA and inventories (WFAN) compared to 17.06% in the model without FA (WTFA). The baseline model without FA corresponds to the model with inventories. In the presence of habit formation, the model without inventories (WTN) generates a multiplier of 5.27%, while the model without both inventory and the FA mechanism (WTFAN) generates a stock price multiplier of 4.38%.

In the baseline model with both the FA mechanism and inventories (WFAN), equity premium responds by 1.70% but only 0.34% in the model without inventories (WTN). Furthermore, it is noted that the reactions of the inventory and stock price gap are procyclical with sales, while that of the inventory-sales ratio and equity premium is countercyclical.

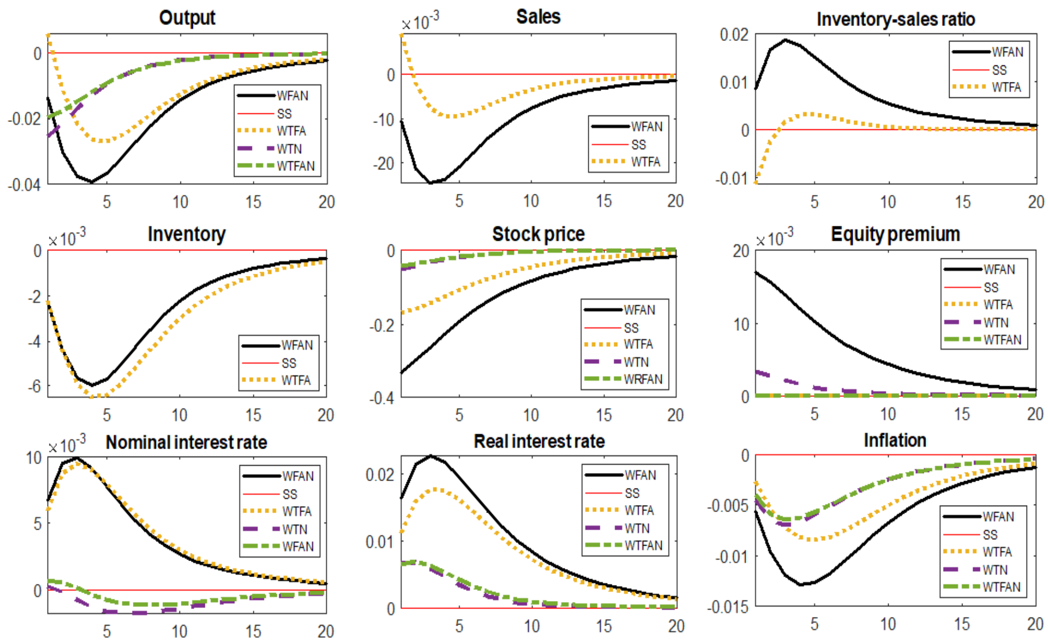
Table 11: Asset price multipliers of the baseline model

	$WFAN$	$WTFA$	WTN	$WTFAN$
$M_q\,(h=0)$	-33.36	-17.06	-5.27	-4.38
$M_\rho\,(h=0)$	1.70	.	0.34	.
$M_q\,(h=0.80)$	-5.44	-4.97	-5.16	-4.42
$M_\rho\,(h=0.80)$	0.26	.	0.26	.

M_q = stock price multiplier (in percentage)
M_ρ = equity premium multiplier(in percentage)

When we increase habit formation to above zero, that is $h = 0.80$, we note that the magnitude of response of the variables in the model with both FA and inventories (WFAN) is not much different from those values generated in the model without inventories (WTFA). Thus, adding habit formation nullifies the amplifying role of inventories and reduces the magnitude of response of the aggregate variables. We note that in the presence of inventories, the smoothing role of habit formation in output through consumption and sales dominates in the model. This brings the stock price multipliers of the versions calibrated with habit formation to 5.44% in WFAN, 4.97% in WTFA, 5.16% in WTN and 4.42% in WTFAN. The equity premium multiplier drops to 0.26% in both WFAN and WTN.

Monetary policy contraction $(h = 0.00)$



Monetary policy contraction $(h = 0.80)$

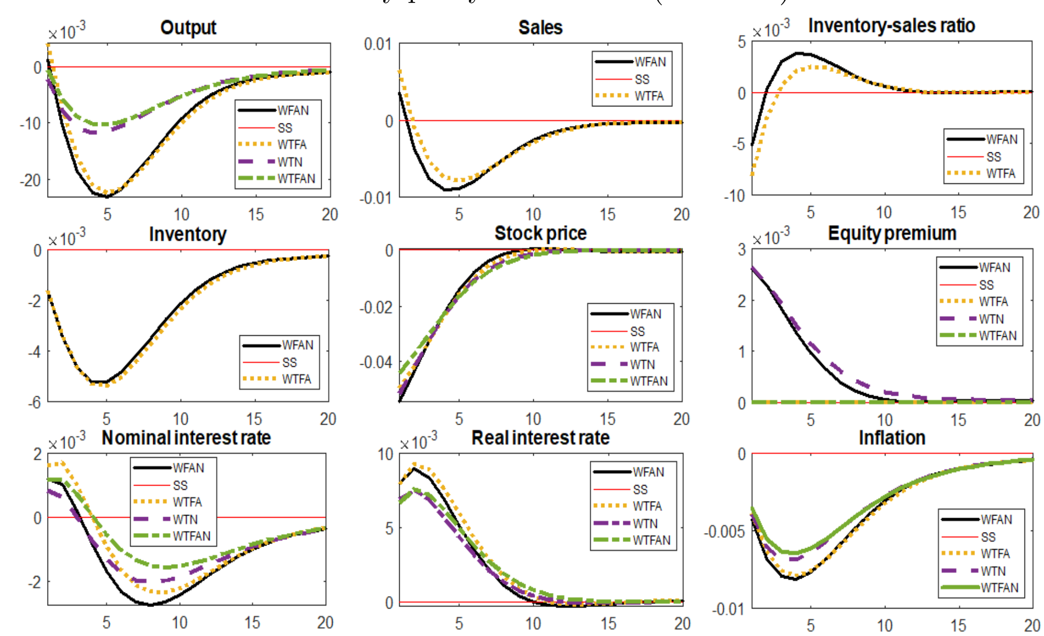


Figure 8: Responses of aggregate variables in the baseline model

The main innovations at this stage can be summarized as follows: (i) By adding inventory dynamics to our standard small open NK DSGE model without habit formation, the stock price response, on impact, is amplified by 12.68% (calculated as 17.06% minus 4.38%) and only 0.47% (calculated as 5.44% minus 4.97%) when habit persistence is added in the model. On impact, the response of the external finance premium, which in our case is the equity premium, is magnified by 1.36%. (ii) Inventories also magnify the amplifying role of the FA mechanism. Thus, in the presence of both the FA mechanism and inventories, and without habit formation, the response of stock price increases from 17.06% to 33.36%, a difference of 16.30%. Inventories magnify the effect of the FA because the contribution of the FA mechanism in the absence of inventories is only 0.89% (computed as 5.27% minus 4.38%) and 0.74% (computed as 5.16% minus 4.42%) in the presence of habit formation.

The results of our model without inventories are consistent with the results presented in the DSGE literature, the VAR literature, as well as in event studies. For example, Rigobon and Sack (2004) employ an event study and report that in absolute values, the S&P500 reacts by 6.8%, WILD5000 by 6.5%, and NASDAK and DJIA by 9.42% and 4.85% respectively to an annualized 100 basis-point increase in the monetary policy rate. These results are not far from the range reported by Bernanke and Kuttner (2005), who combine event study and VAR methodology. The range documented in Bjørnland and Leitemo (2009) for quarterly data using a VAR analysis is 3.80%-7.20% in absolute values. Li et al. (2010) also use a structural VAR model and find that the response of the stock price in Canada to a surprising 100 basis-point increase in the Canadian repo is 0.01% , while the same percentage increase in the Fed rate yields a response of 2.2% in the US stock price.

Using a NK DSGE setup, Challe and Giannitsarou (2014) find that the stock price reacts by approximately 3.06% to an annualized 1% increase in the policy rate. This finding is closer to the ones in our model without inventories. Kontonikas *et al.* (2013) also document conventional reactions of the stock price to a surprise cut in the monetary policy rate. They find significant reaction prior to the global financial crisis of 2007-2009 but no reaction during the crisis itself. Paul (2019) finds that before the financial crisis, the reaction of both stock and house prices were not sizeable, while Alessi and Kerssenfischer (2019) find that the response of stock and house prices are stronger in a dynamic factor model than in a VAR setup.

The cyclical behaviour of inventories and the inventory-sales ratio is also consistent with the findings in Kryvtsov and Midrigan (2013). The cyclical behaviour of the equity premium is consistent with the literature that uses the financial accelerator mechanism of Bernanke et al. (1999) and Gilchrist and Leahy (2002). Ravn (2014), for example, documents a countercyclical response of the external finance premium. While Gomes *et al.* (2003) document a procyclical behaviour of the finance premium, Agenor et al. (2014) document a countercyclical behaviour of the external finance premium and note that the studies with procyclical finance premium are the ones in which the entrepreneur has no capital stock.

4.4.1 Model variations

We now subject the results to variations in key NK parameters, namely the price indexation and the price stickiness, as well as the inventory adjustment cost parameter. We first set the price indexation $\omega = 0.00$ and price stickiness $\theta = 0.011$ such that the duration of the price reset is approximately a quarter. This variation is called the *flexible price model*. Secondly, we reset these parameters to their benchmark levels and reset $\chi = 0.00$. This variation is called the *model without adjustment cost*. We present the results of the two sets of variations for both $h = 0.00$ and $h = 0.80$. Figure 9 presents the results for the flexible price model. The results show that the dynamics response of the aggregate variables is consistent with the relevant theories. However, the response of asset prices in the flexible price model are slightly more amplified while those of the model without adjustment cost are less amplified as opposed to the results in the baseline model.

Multipliers of stock price and equity premium are reported in Table 12. The reaction of stock price at $h = 0.00$ in the flexible price model with both FA and inventories is 5.56% higher than the response generated in the baseline model. The flexible price model without FA, generates a response in the stock price which is 0.94% less compared to the baseline model without FA (WTFA). This implies that the sticky price model with inventories and the flexible price model with inventories produce responses of approximately the same magnitude. The response in the flexible price model without inventories is nearly 2.52% greater compared to the baseline model without inventories. The stock price multiplier in the flexible price model without FA and without inventories is 5.53%, which is 1.15% greater than the 4.38% generated with the sticky price model. The flexible price model also amplifies the response of the equity premium.

Table 12: Asset price multipliers of the variated models

The flexible price model				
	$WFAN$	$WTFA$	WTN	$WTFAN$
$M_q\,(h=0.00)$	-38.92	-16.12	-7.79	-5.53
$M_\rho\,(h=0.00)$	1.97	.	0.47	.
$M_q\,(h=0.80)$	-5.31	-4.48	-7.66	-5.65
$M_\rho\,(h=0.80)$	0.25	.	0.37	.
Model without inventory adjustment cost				
	$WFAN$	$WTFA$		
$M_q\,(h=0.00)$	-23.15	-12.24		
$M_\rho\,(h=0.00)$	1.20	.		
$M_q\,(h=0.80)$	-4.16	-3.87		
$M_\rho\,(h=0.80)$	0.21	.		

M_q = stock price multiplier (in percentage)
M_ρ = equity premium multiplier(in percentage)

When we increase habit formation in the flexible price model, the results do not differ much from the responses in the sticky price model with habit formation. The only version with a significantly amplified response is the flexible price model without inventories. Figure 10 shows the impulse responses of the *model without adjustment cost*. The stock price multipliers in the lower panel of Table 12 reveal that the reaction of stock price at $h = 0.00$ in WFAN is 23.15% in absolute value compared to 33.36% in the model with inventory adjustment costs. The response is 12.24% in WTFA compared to 17.06% in the baseline model with adjustment costs. The responses of asset prices with habit formation are similar to the ones in the baseline model.

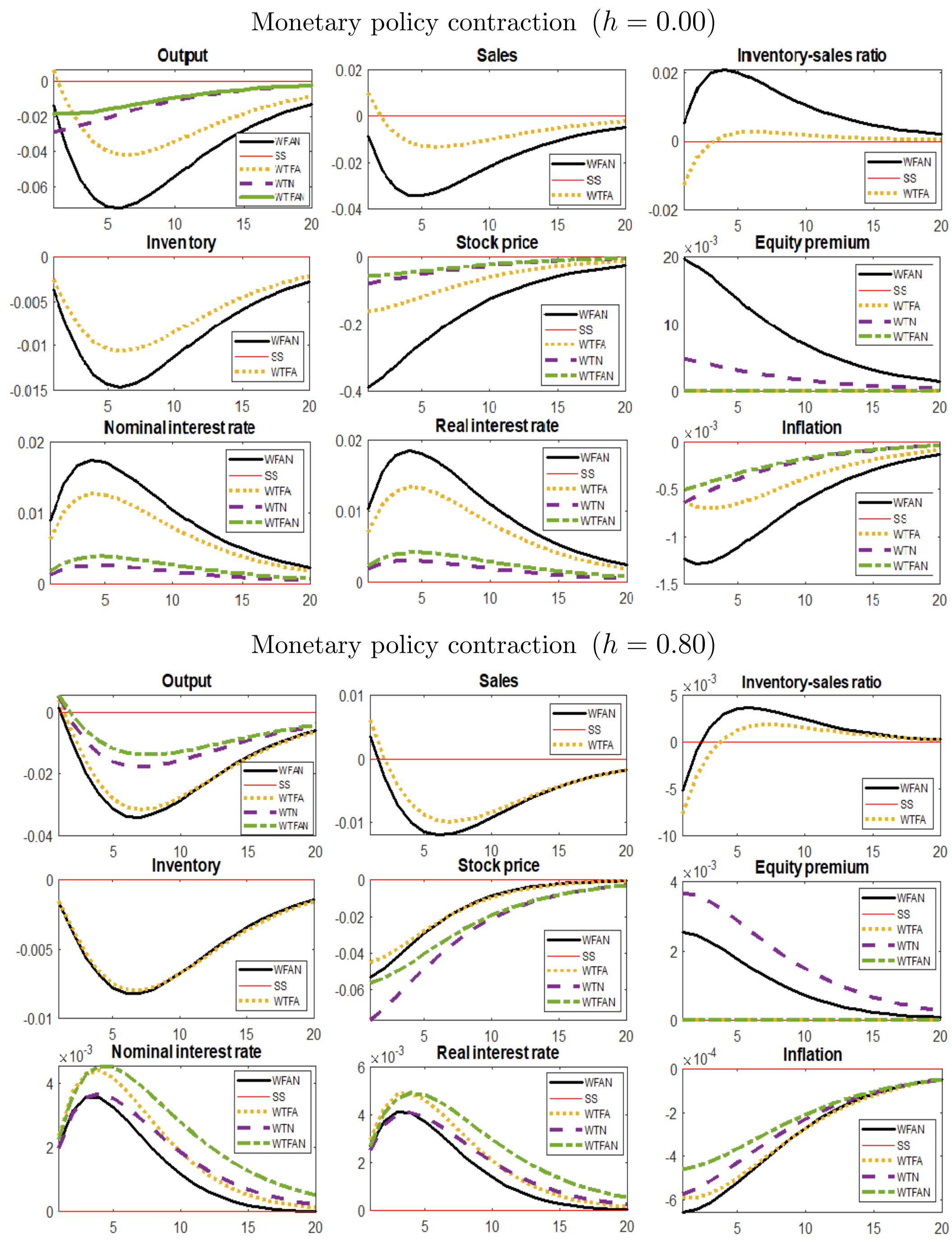


Figure 9: Responses of aggregate variables in the flexible price model

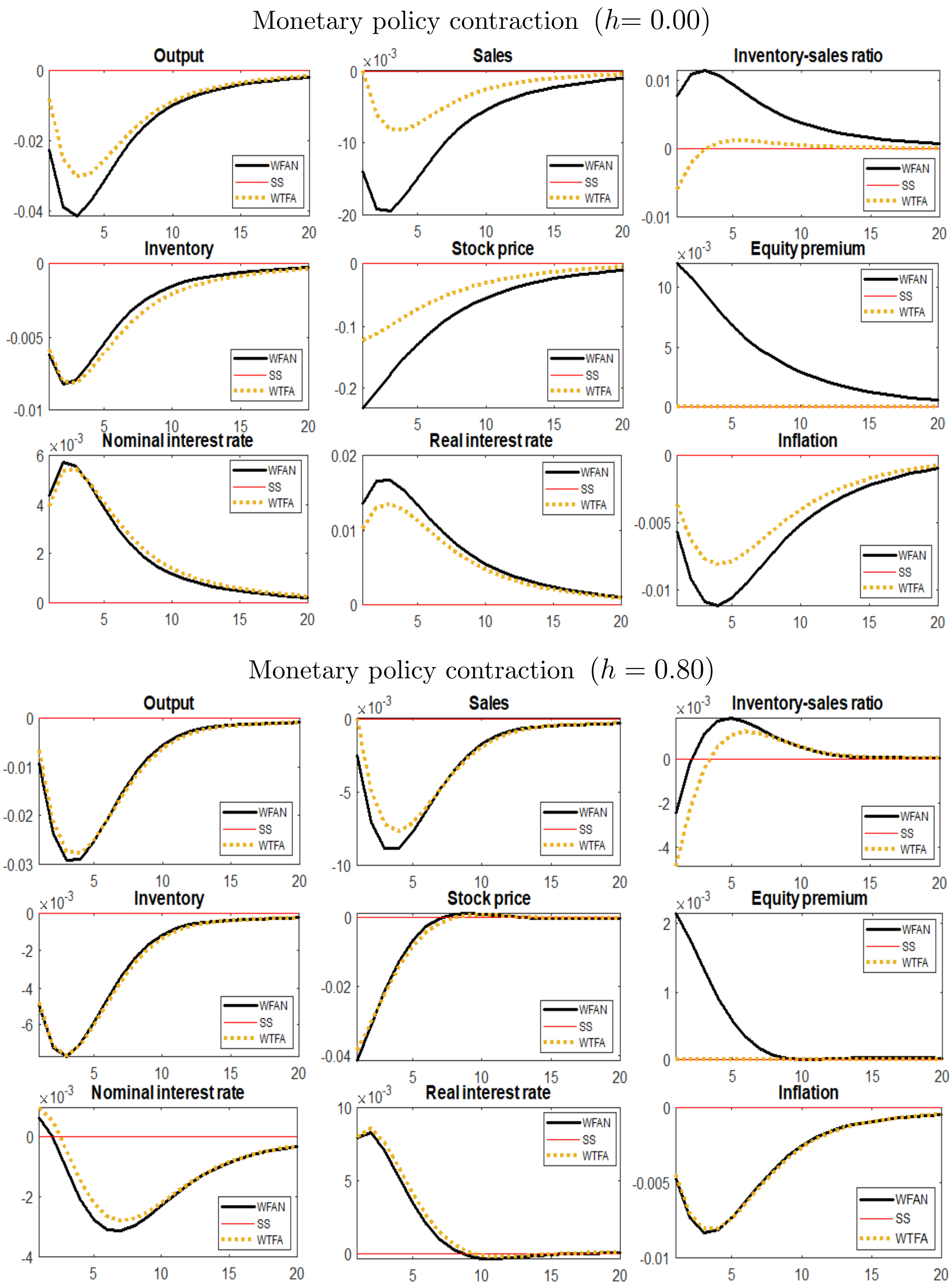


Figure 10: Responses of variables in the model without adjustment cost

The response of the equity premium drops from 1.70% in the baseline model to 1.20% in the model without adjustment cost. The remark here is that inventory adjustment costs play an important role in the response of asset prices to a monetary policy shock.

In summary, we find that inventories amplify the response of asset prices to a monetary policy shock and that the results generated from the model without inventories are consistent and comparable to the ones documented using event studies, VAR models and those documented in Challe and Giannitsarou (2014) using a NK DSGE model. Furthermore, in comparison to Li *et al.* (2010), a source which studies a small open economy like Canada *versus* a large and closed economy like the United States, our results, which are based on a NK DSGE model, reveal a much higher reaction of the stock price to a monetary policy shock.

4.5 Conclusion and policy implications

4.5.1 Conclusion

This study uses a New-Keynesian DSGE model to investigate the manner in which inventories contribute to the reaction of asset prices to a monetary policy shock and whether the models account for the cyclical behaviour of key macro-finance variables. The financial accelerator mechanism is used to link the demand side and the supply side through the effect of the external finance premium or equity premium on inventories. We subject the dynamics of the aggregate variables to an annualized 100 basis-point increase in the nominal interest rate.

The results show that the response of variables such as output and sales are consistent with theoretical expectations. Specifically, we find that the response of stock price and equity premium are amplified in the presence of inventories and the FA mechanism. Moreover, we note that the amplifying role of the FA mechanism is magnified by inventories. Furthermore, we find that adding an additional smoothing device, such as habit formation, nullifies the amplifying role of inventories and thus reduces the magnitude of response of the aggregate variables.

We reproduce the results using a flexible price model and a sticky price model without inventory adjustment cost. The main findings are that, in the absent of the the financial accelerator mechanism, the sticky price and the flexible price models reveal a similar contribution of inventories in respect of the reaction of stock price to a monetary policy shock. Furthermore, we note that inventory adjustment cost play a crucial role in the response of stock price to a monetary policy shock. With the exception of these innovations, the results in the models without inventories are consistent with the existing literature. Our macro model therefore displays important asset pricing implications, and inventories should be considered in future macro models with asset prices.

It is important to point out that we do not distinguish between small and large firms and do not incorporate the dynamics of capital stock and therefore investment as often noted in the financial accelerator literature. However, these issues have constituted the cores of previous FA models with investments. Moreover, the main objective of this study is to investigate the asset pricing implications of a macro model with inventories. We therefore recommend further extension in these directions, as well as an acknowledgement of the ability of our model to shed light on whether the central bank should react to asset prices.

4.5.2 Policy implications

Bernanke and Kuttner (2005) note that asset prices constitute a transmission channel for monetary policy shocks. They argue that the most immediate effects of monetary policy shocks are often reflected in asset prices. On the other hand, other scholars have pointed to the need for monetary policy authorities to control the volatility of asset prices, as it significantly contributes to the volatility of the economy. Therefore, understanding the interconnection between asset prices and monetary policy shocks is important to comprehend the mechanism behind the transmission of monetary policy.

This study shows that inventories amplify the reaction of equity prices to monetary policy shocks. The decline in equity premium following a contractionary monetary policy shock is amplified by the negative effect of monetary policy contraction on sales and inventories. The decline in inventories, in turns, amplifies the response of equity prices. Given these findings, it becomes imperative for monetary policy authorities to consider models that incorporate inventory behaviour when quantifying the real effect of monetary policy on the economy.

5 Concluding remarks and directions

5.1 Introduction

This thesis answers three major research questions. Firstly, it provides both empirical evidence and theoretical justification for the sign puzzle. Secondly, the thesis investigates whether inventories can be used as a smoothing device in the New-Keynesian model. Thirdly, the thesis investigates how asset prices respond to a monetary policy shock in the presence of inventories. This chapter summarizes the major conclusions and provides directions for future research.

5.2 Inventory dynamics, interest rates and the sign puzzle

Firstly, this paper employs a log linear inventory model and an inventory growth model to provide empirical evidence on the sign puzzle. While data on inventory stock is still the main barrier to the lack of empirical evidence on the sign puzzle in both developed and emerging economies, we provide evidence in respect of the United States, Canada and South Africa. Secondly, we develop a model of inventory dynamics to explain the puzzle. In line with Bivin (2011), the main assumption made in the formulation of the structural model is that firms, especially the stockout avoidance firms, will still accumulate inventories in the face of the increasing cost of capital.

The results based on the log linear model and the inventory growth model show that the puzzle still exists in the United States and Canada, but not in South Africa. The reduced form estimates of the structural model confirm the results based on the log linear inventory model and the inventory growth model. The structural model shows that the coefficient on the cost of capital variable is ambiguous. We calibrate this coefficient to show the conditions under which the sign puzzle can materialize. The study concludes that the elasticity of benefits for inventory holdings and the sensitivity of stockouts cost to change in the cost of capital determine the sign of the coefficient of the cost of capital and therefore drive the sign puzzle.

Inventories are known to serve as monetary policy transmission channels (Frankel, 2006). The inventory channel was acknowledged in Barth and Ramey (2001) to function as a supply side channel. Thus, if the inventory channel does in fact exists, its size would depend on the elasticity of benefits of stockouts cost and the sensitivity of stockouts cost to changes in the cost of capital. Our findings, therefore, complement the view in Benati and Lubik (2014) that the effect of monetary policy on inventories depends on whether its effect on inventories through sales, dominates its direct effect on inventories. This direct effect of interest rates on inventories is known as the inventory channel and its size depends on the two parameters that drive the sign puzzle.

5.3 Inventory dynamics and endogenous persistence in a New-Keynesian model

This paper investigates whether inventories could play a similar role to habit formation in the New-Keynesian (NK) model. In doing so, we introduce the dynamic of inventories into a small open dynamic stochastic general equilibrium (DSGE) model. The fact that NK models rely on habit persistence to spawn a gradual response in aggregate variables such as consumption and output has been the motivation for this paper. However, the role of inventories as a smoothing device or in generating persistence in output has long been acknowledged, as in Blinder and Fischer (1982), Boldrin *et al.* (2001) and Wen (2005), and empirically supported by Boileau and Letendre (2011). We calibrate a small open economy model and study its sensitivity to changes in the price indexation, price stickiness and the inventory adjustment cost parameter.

We find that the NK model with inventories but without habit formation generates a hump-shaped response of aggregate variables to both monetary policy and technology shocks, while the version of the model without inventories and without habit formation fails to spawn a hump-shaped response in consumption and output. We also find that inflation inherits some level of persistence from the presence of inventories and, therefore, only a moderate level of habit formation is required to generate a gradual response of inflation to a monetary policy shock. Furthermore, our flexible price model with inventories generates similar results when compared to the baseline model. We note that the addition of inventory adjustment costs to the total inventory holdings cost is not necessary. Finally, we note that our DSGE model replicates the cyclical behaviour of inventories and the inventory-sales ratio.

These results suggest that DSGE models without inventories can suffer from wrong specifications and may therefore lead to unreliable policy decisions if employed by policy makers in policy decision making.

5.4 Inventories and asset prices dynamics in a New-Keynesian model

This paper investigates the response of asset prices to a monetary policy shock in the presence of inventories. We calibrate a small open economy DSGE model with inventories and asset prices. The relationship between inventories and asset prices is embedded using the financial accelerator mechanism. We find that a 100 basis-point increase in the nominal interest rate causes the equity price gap to respond by 17.06% as opposed to 4.38% in the absence of inventories. We also note that inventories amplify the response of asset prices to a monetary policy shock, while habit formation nullifies the role of inventories. The implications of the findings in this paper for the existing literature are that (i) macro-finance models without inventories can underestimate the response of asset prices to shocks and that (ii) macro-finance models without inventories may therefore mislead policy makers in making macro policy decisions.

5.5 Directions for future research

The research questions posed in this study focus on the role of inventories while ignoring the role of long term investments. Thus, inventories are considered as short term assets and inventory investment merely constitutes only a portion of the firm's total investment. Future studies should consider advancing the ideas developed in this study to also include long term assets.

6 Appendix

This appendix presents the derivation of the model of inventory dynamics in section 2.4.

The total cost of inventory holdings is presented as follows:

$$H_t = A_t \left(\frac{N_t}{Q_t}\right)^{-\gamma} Q_t e^{\delta R_t} + R_t N_t + \left(\frac{N_{t-1}}{Q_t}\right)^{-\gamma_s} N_t + \frac{\chi}{1+\vartheta}\left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} N_t \quad (1)$$

The first order condition from eq.(1) is presented as follows:

$$\gamma A_t N_t^{-(1+\gamma)} Q_t^{(1+\gamma)} e^{\delta R_t} = R_t + \left(\frac{N_{t-1}}{Q_t}\right)^{-\gamma_s} + \chi\left(\frac{2+\vartheta}{1+\vartheta}\right)\left(\frac{N_t}{N_{t-1}}\right)^{1+\vartheta} \quad (2)$$

We assume that the level of inventories at time t that is consistent with eq.(2) is the desired inventory at time t. The linearized version of eq.(2) gives the desired inventory gap as follows:

$$\widehat{n}_t^d = \theta_s \widehat{q}_t - \theta_R \widehat{R}_t + \theta_n \widehat{n}_{t-1} + \widehat{\varepsilon}_t \quad (3)$$

$$\theta_n = \frac{\omega_i + \zeta}{\varphi + \zeta}, \theta_q = \frac{\varphi - \omega_i}{\varphi + \zeta}, \theta_R = \frac{(\phi - \gamma\delta p_0^s)}{\phi(\varphi+\zeta)}, \varphi = \gamma(1+\gamma)\frac{p_0^s}{\phi}, p_0^s = A_0\frac{e^{(\delta R_0)}}{\phi^\gamma}$$
$$\omega_i = \frac{\gamma_s}{(1+g)(\phi)^{\gamma_s}}, \widehat{\varepsilon}_t = \frac{1}{\varphi+\zeta}\widehat{a}_t, \zeta = \chi(2+\vartheta)(1+g)^{(1+\vartheta)}$$

The inventory accumulation identities of the inventory management and the production units are presented as follows:

$$N_t = N_t^d + Y_{t-1} - Y_{t-1}^s \quad (4)$$

$$Y_t = Y_t^s + N_t - N_{t-1} \quad (5)$$

By substituting eq.(5) into eq.(4) and linearizing around the steady state, the dynamics of the actual inventory gap is obtained as follows:

$$\widehat{n}_t = \frac{1}{(1+g)}\widehat{n}_{t-1} - \frac{1}{(1+g)^2}\widehat{n}_{t-2} + \widehat{n}_t^d \quad (6)$$

By substituting the desired inventory gap in eq.(3) into eq.(6), the dynamics of the actual inventory gap is presented as follows:

$$\widehat{n}_t = \left(\frac{1}{(1+g)} + \theta_n\right)\widehat{n}_{t-1} - \frac{1}{(1+g)^2}\widehat{n}_{t-2} + \theta_q\widehat{q}_t - \theta_R\widehat{R}_t + \widehat{\varepsilon}_t \quad (7)$$

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