8.2 APPENDIX B - Dry-stack reinforced beam design

This appendix is referred to a step by step theoretical design of dry-stack reinforced beams. To evaluate if the theoretical results match the test ones, the load and the material resistance will not be increased or reduced by safety factors. All safety factors recommended by the code will be taken as unit value.

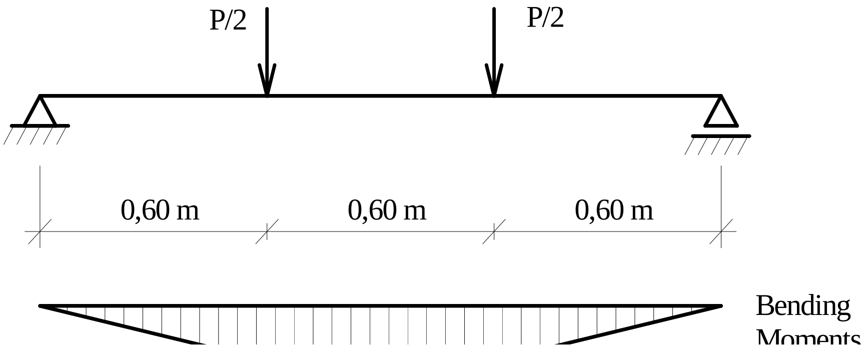

Figure - 8.1B: Bending moment and shear force diagrams

8.2.1 Design of cross section 1 beam

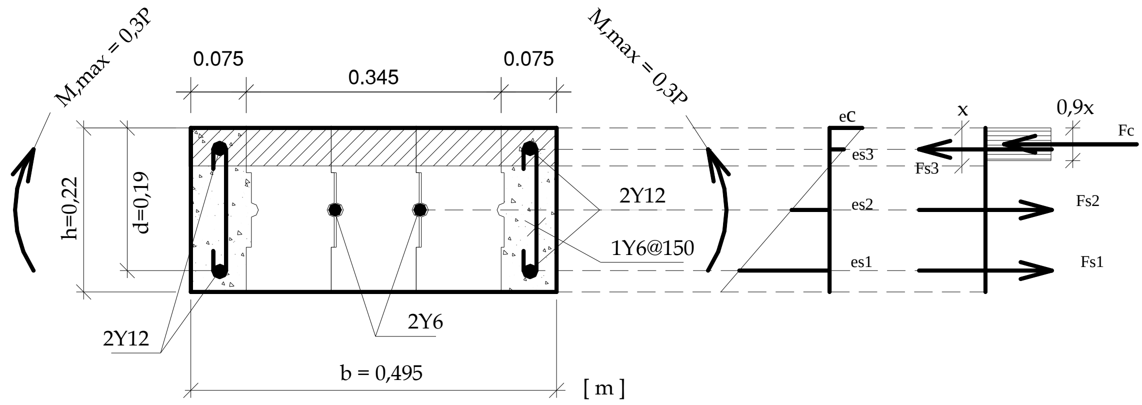

Figure - 8.2B: Cross section 1, stain distribution and rectangular stress block

8.2.1.1 Flexural design

From the strain distribution diagram, it can be shown that:

$$\begin{cases} \varepsilon_{s1} = 0{,}0035\dfrac{d-x}{x} = 0{,}0035\dfrac{0{,}19-x}{x} \\ \varepsilon_{s2} = 0{,}0035\dfrac{h/2-a}{x} = 0{,}0035\dfrac{0{,}11-x}{x} \end{cases}$$

Data

Compressive strength of dry-stack masonry ;

Yield strain of dry-stack masonry ;

Compressive strength of infill concrete ;

Yield strain of infill concrete $\varepsilon_u = 0{,}0035$;

Yield stress of reinforcement bars $f_y = 450{,}00 \ MPa$

Modulus of elasticity of reinforcement bars $E_s = 200{,}00 \ GPa$

Yield strain of reinforcement bars $\varepsilon_y = 0{,}002$

Assumptions:

A_{s1} is full stressed: $\varepsilon_{s1} > 0{,}002 \Rightarrow F_{s1} = A_{s1} f_y = 2{,}26*10^{-4}*450^3 \approx 101{,}70 \ kN$

A_{s2} is full stressed: $\varepsilon_{s2} > 0{,}002 \Rightarrow F_{s2} = A_{s2} f_y = 0{,}57*10^{-4}*450^3 \approx 25{,}65 \ kN$

A_{s3} is not full stressed:

$$\varepsilon_{s3} < 0{,}002 \Rightarrow F_{s3} = A_{s3}\sigma_{s3} = A_{s3}E_s\varepsilon_{s3} = 2{,}26*10^{-4}*200*10^6 \frac{x-0{,}03}{x}*0{,}0035 \approx 158{,}20\frac{x-0{,}03}{x} \ kN$$

$$F_c = 0{,}9xb\sigma_{cu} = 0{,}9x*0{,}495*9*10^3 \approx 3717{,}90x \ kN$$

From the force section equilibrium, it can be shown that:

$$\sum F_x = 0 \Rightarrow F_{s1} + F_{s2} - F_{s3} - F_c = 0 \Rightarrow 101{,}7 + 25{,}65 - 158{,}2\frac{x-0{,}03}{x} - 3717{,}9x = 0 \Rightarrow x \approx 0{,}0318 \ m$$

Then,

$$\begin{cases} F_{s1} = 101{,}7 \;\; kN \\ F_{s2} = 25{,}65 \;\; kN \\ F_{s3} = 158{,}20 \dfrac{0{,}0318 - 0{,}03}{0{,}0318} \approx 8{,}95 \;\; kN \\ F_c = 3717{,}90x \approx 118{,}23 \;\; kN \end{cases}$$

From the bending moment section equilibrium, it can be shown that:

$$\sum M_{F_{s3}} = 0 \Rightarrow 0{,}3P = F_{s1} * (d-a) + F_{s2}\left(\frac{h}{2} - a\right) + F_c\left(a - \frac{0{,}9x}{2}\right)$$

$$0{,}3P = 101{,}70(0{,}19 - 0{,}03) + 25{,}65(0{,}11 - 0{,}03) + 118{,}23(0{,}03 - 0{,}45 * 0{,}0318) \Rightarrow P \approx 67{,}26 \;\; kN$$

8.2.1.2 Shear design

$$v^{resis\tan e} = v_c + v_{sv} = \frac{0{,}75}{\gamma_m}\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{bd}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}} + \frac{0{,}87 f_{yv}}{b}\frac{A_{sv}}{s_v}$$

Taking only one portion of the concrete rectangular section (220 mm x 75mm) and once again a unit value for material safety factor, it can be shown that;

$$v^{resis\tan e} = \frac{0{,}75}{1}\left(\frac{9}{25}\right)^{\frac{1}{3}}\left(\frac{100*1{,}13}{75*190}\right)^{\frac{1}{3}}\left(\frac{400}{190}\right)^{\frac{1}{4}} + \frac{0{,}87*450}{75}\frac{*28}{150} \approx 0{,}59 + 0{,}97 \approx 1{,}56 \;\; MPa$$

Considering two identical portions of concrete we get;

The acting shear stress is given by

Equalling the acting and resisting stresses we get the allowable load;

8.2.1.3 Cause of beam failure

From flexural analysis it was found that $\begin{cases} P^{theo.} \approx 67{,}26 \;\; kN \\ P^{\exp.} \approx 69{,}67 \;\; kN \end{cases}$

From Shear analysis it was found that $\begin{cases} P^{theo.} \approx 88{,}92 \;\; kN \\ P^{\exp.} \approx 69{,}67 \;\; kN \end{cases}$

Even considering that the shear is applied to and resisted by the infill concrete portion (which is a non realistic and worst case scenario) it is clear that the beam has adequate shear resistance, therefore it failed by flexural stresses.

Assuming the testing value as the most accurate, the relative error is:

$$\delta = 100 \frac{69{,}67 - 67{,}26}{69{,}67} \approx 3{,}5 \;\; \%$$

8.2.2 Design of cross section 2 beam

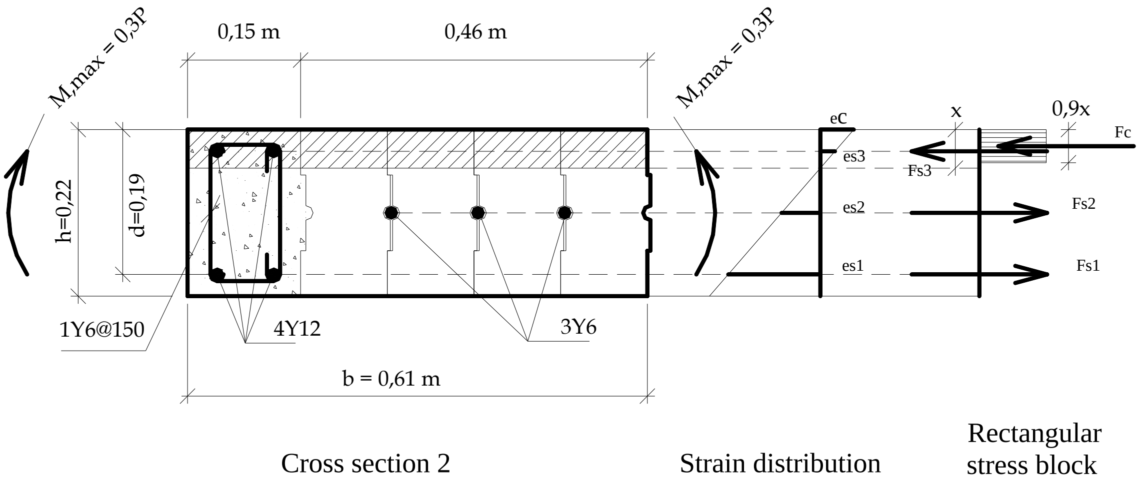

Figure - 8.3B: Cross section 2, stain distribution and rectangular stress block

8.2.2.1 Flexural design

From the strain distribution diagram, it can be shown that:

$$\begin{cases} \varepsilon_{s1} = 0{,}0035\dfrac{d-x}{x} = 0{,}0035\dfrac{0{,}19-x}{x} \\ \varepsilon_{s2} = 0{,}0035\dfrac{h/2-a}{x} = 0{,}0035\dfrac{0{,}11-x}{x} \end{cases}$$

Assumptions:

is full stressed:

is full stressed:

is not full stressed:

$$\varepsilon_{s3} < 0{,}002 \Rightarrow F_{s3} = A_{s3}\sigma_{s3} = A_{s3}E_s\varepsilon_{s3} 2{,}26*10^{-4}*200*10^6 \frac{x-0{,}03}{x}*0{,}0035 \approx 158{,}20\frac{x-0{,}03}{x} \; kN$$

$$F_c = 0{,}9xb\sigma_{cu} = 0{,}9x*0{,}61*9*10^3 \approx 4941{,}00x \; kN$$

From the force section equilibrium, it can be shown that:

$$\sum F_x = 0 \Rightarrow F_{s1} + F_{s2} - F_{s3} - F_c = 0 \Rightarrow 101{,}7 + 38{,}25 - 158{,}2\frac{x-0{,}03}{x} - 4941x = 0 \Rightarrow x \approx 0{,}0292 \; m$$

Since $x \approx 0{,}0292 < a = 0{,}03$, then A_{s3} is in tension rather than in compression as assumed early on. The sign of F_{s3} and ε_{s3} will become negative. All other assumptions remain valid. Then,

$$\begin{cases} \varepsilon_{s1} = 0{,}0035\frac{d-x}{x} = 0{,}0035\frac{0{,}19-x}{x} \approx 0{,}0193 > 0{,}002 \Rightarrow A_{s1} \; is \; full \; stressed \\ \varepsilon_{s2} = 0{,}0035\frac{h/2-a}{x} = 0{,}0035\frac{0{,}11-x}{x} \approx 0{,}0097 > 0{,}002 \Rightarrow A_{s2} \; is \; full \; stressed \\ \varepsilon_{s3} = 0{,}0035\frac{x-a}{x} = 0{,}0035\frac{x-0{,}03}{x} \approx -0{,}0010 < 0{,}002 \Rightarrow A_{s3} \; is \; not \; full \; stressed \end{cases}$$

$$\begin{cases} F_{s1} = 101{,}7 \; kN \\ F_{s2} = 38{,}25 \; kN \\ F_{s3} = 158{,}20\frac{0{,}0292-0{,}03}{0{,}0292} \approx -4{,}33 \; kN \\ F_c = 4941x \approx 144{,}28 \; kN \end{cases}$$

From the bending moment section equilibrium, it can be shown that:

8.2.2.2 Shear design

$$v^{resis\tan e} = v_c + v_{sv} = \frac{0,75}{\gamma_m}\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}\left(\frac{100A_s}{bd}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}} + \frac{0,87f_{yv}}{b}\frac{A_{sv}}{s_v}$$

Considering only the resistance of a rectangular concrete portion of the section (220 mm x 150mm), it can be shown that;

$$v^{resis\tan e} = \frac{0,75}{1}\left(\frac{9}{25}\right)^{\frac{1}{3}}\left(\frac{100*226}{150*190}\right)^{\frac{1}{3}}\left(\frac{400}{190}\right)^{\frac{1}{4}} + \frac{0,87*450}{150}\frac{*57}{150} \approx 0,59 + 0,99 \approx 1,58\ \ MPa$$

The acting shear stress is given by $v^{acting} = \frac{V_{\max}}{bd} = \frac{P/2}{bd}$

Equalling the acting and resisting stresses we get the allowable load;

$$\frac{P/2}{bd} = 1,58 \Rightarrow P = 1,58^3 * 2 * 0,22 * 0,19 \Rightarrow P \approx 132,09\ \ kN$$

8.2.2.3 Cause of beam failure

From flexural analysis it was found that $\begin{cases} P^{theo.} \approx 72,55\ \ kN \\ P^{\exp.} \approx 64,67\ \ kN \end{cases}$

From Shear analysis it was found that

It is clear that the beam has large reserve of shear resistance, therefore it failed by flexural stresses.

Assuming the testing value as the most accurate, the relative error is:

$$\delta = 100 \frac{72{,}56 - 64{,}67}{64{,}67} \approx 12{,}20 \ \%$$

8.2.3 Design of cross section 3 beam

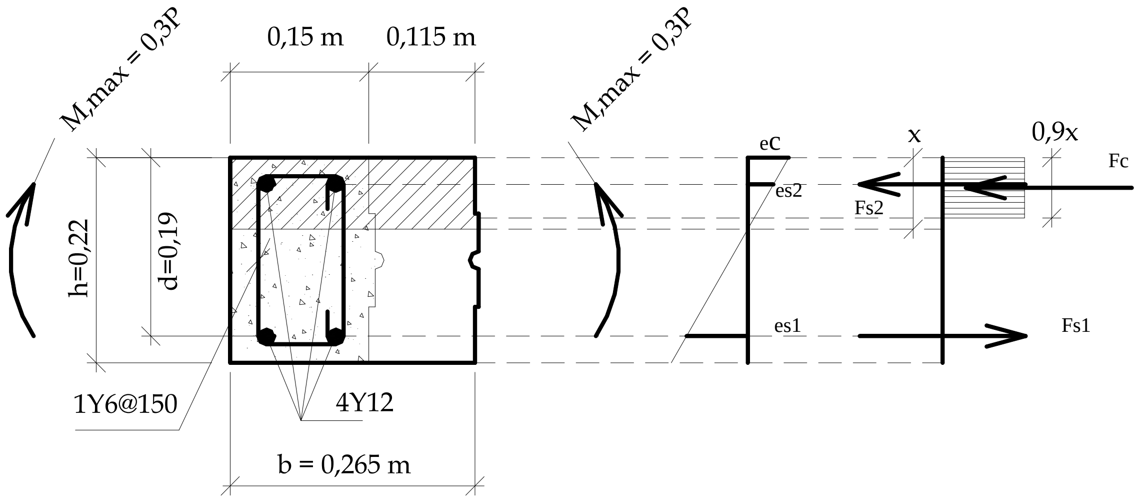

Figure - 8.4B: Cross section 3, stain distribution and rectangular stress block

8.2.3.1 Flexural design

From the strain distribution diagram, it can be shown that:

Assumptions:

is full stressed:

A_{s2} is not full stressed:

$$\varepsilon_{s2} < 0{,}002 \Rightarrow F_{s2} = A_{s2}\sigma_{s2} = A_{s2}E_s\varepsilon_{s2} 2{,}26*10^{-4}*200*10^6 \frac{x-0{,}03}{x}*0{,}0035 \approx 158{,}20\frac{x-0{,}03}{x} \;\; kN$$

$$F_c = 0{,}9xb\sigma_{cu} = 0{,}9x*0{,}265*9*10^3 \approx 214{,}65x \;\; kN$$

From the force section equilibrium, it can be shown that:

$$\sum F_x = 0 \Rightarrow F_{s1} - F_{s2} - F_c = 0 \Rightarrow 101{,}7 - 158{,}2\frac{x-0{,}03}{x} - 214{,}65x = 0 \Rightarrow x \approx 0{,}0670 \;\; m$$

Then,

$$\begin{cases} \varepsilon_{s1} = 0{,}0035\dfrac{d-x}{x} = 0{,}0035\dfrac{0{,}19-x}{x} \approx 0{,}0064 > 0{,}002 \Rightarrow A_{s1} \;\; is \;\; full \;\; stressed \\ \varepsilon_{s2} = 0{,}0035\dfrac{x-a}{x} = 0{,}0035\dfrac{x-0{,}03}{x} \approx 0{,}0019 < 0{,}002 \Rightarrow A_{s2} \;\; is \;\; not \;\; full \;\; stressed \end{cases}$$

$$\begin{cases} F_{s1} = 101{,}7 \;\; kN \\ F_{s3} = 158{,}20\dfrac{0{,}067-0{,}03}{0{,}067} \approx 87{,}36 \;\; kN \\ F_c = 214{,}65x \approx 13{,}95 \;\; kN \end{cases}$$

From the bending moment section equilibrium, it can be shown that:

$$\sum M_{F_{s2}} = 0 \Rightarrow 0{,}3P = F_{s1}*(d-a) - F_c\left(\frac{0{,}9x}{2} - a\right)$$

8.2.3.2 Shear design

Considering only the resistance of a rectangular concrete portion of the section (220 mm x 150mm), it can be shown that;

$$v^{resis\tan e} = \frac{0,75}{1}\left(\frac{9}{25}\right)^{\frac{1}{3}}\left(\frac{100*226}{150*190}\right)^{\frac{1}{3}}\left(\frac{400}{190}\right)^{\frac{1}{4}} + \frac{0,87*450}{150}\frac{*57}{150} \approx 0,59+0,99 \approx 1,58 \;\; MPa$$

The acting shear stress is given by $v^{acting} = \frac{V_{\max}}{bd} = \frac{P/2}{bd}$

Equalling the acting and resisting stresses we get the allowable load;

$$\frac{P/2}{bd} = 1,58 \Rightarrow P = 1,58^3 * 2 * 0,22 * 0,19 \Rightarrow P \approx 132,09 \;\; kN$$

8.2.3.3 Cause of beam failure

From flexural analysis it was found that $\begin{cases} P^{theo.} \approx 54,23 \;\; kN \\ P^{\exp.} \approx 50,00 \;\; kN \end{cases}$

From Shear analysis it was found that $\begin{cases} P^{theo.} \approx 132,09 \;\; kN \\ P^{\exp.} \approx 50,00 \;\; kN \end{cases}$

It is clear that the beam has large reserve of shear resistance, therefore it failed by flexural stresses.

Assuming the testing value as the most accurate, the relative error is:

8.2.4 Design of cross section 4 beam

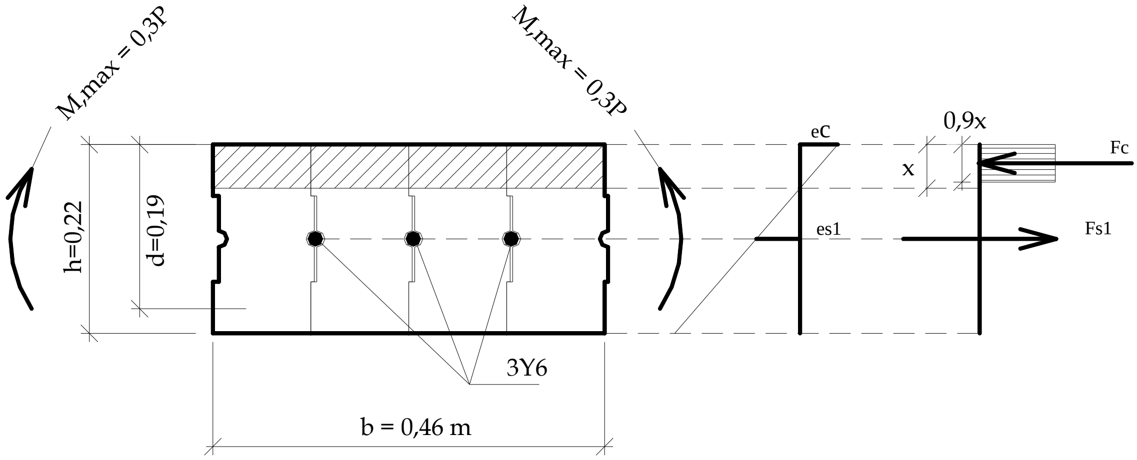

Figure - 8.5B: Cross section 4, stain distribution and rectangular stress block

8.2.4.1 Flexural design

From the strain distribution diagram, it can be shown that:

$$\varepsilon_{s1} = 0{,}0035\frac{h/2-a}{x} = 0{,}0035\frac{0{,}11-x}{x}$$

Assumption:

is full stressed:

From the force section equilibrium, it can be shown that:

Then,

$$\varepsilon_{s1} = 0,0035\frac{h/2-a}{x} = 0,0035\frac{0,11-x}{x} \approx 0,034 > 0,002 \Rightarrow A_{s2} \ \ is \ \ full \ \ stressed$$

$$\begin{cases} F_{s2} = 38,25 \ \ kN \\ F_c = 3726x \approx 38,38 \ \ kN \end{cases}$$

From the bending moment section equilibrium, it can be shown that:

$$\sum M_{F_{sc}} = 0 \Rightarrow 0,3P = F_{s1}\left(\frac{h}{2} - \frac{0,9x}{2}\right)$$

$$0,3P = 38,38(0,11 - 0,45 * 0,0103/2) \Rightarrow P \approx 13,78 \ \ kN$$

The experimental and theoretical values are:

$$\begin{cases} P^{theo.} \approx 13,78 \ \ kN \\ P^{\exp.} \approx 8,27 \ \ kN \end{cases}$$

Assuming the testing value as the most accurate, the relative error is:

$$\delta = 100\frac{13,78 - 8,27}{8,27} \approx 66,63 \ \%$$