INVESTIGATING PARTICLE SIZE SEGREGATION IN A BATCH JIG

by

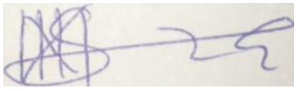
Silwamba Marthias

A research report submitted to the faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Science in Engineering

Johannesburg, 16th May, 2016

Declaration

I declare that this research report is entirely my own work. It is being submitted for the Degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. This work has never been submitted for any degree or examination to this or any other university.



Marthias Silwamba

16th day of May 2016

Abstract

Particle size and size range are among the characteristics that affect the segregation of particles in a jig hence they affect the separation efficiency. The effects of these variables on segregation of particles are not fully understood. This work aimed at contributing to knowledge in this area. To better understand how particle size and size range influence segregation, tests were conducted in which the effects of the density and shape of the particles on segregation were minimized by using as the feed material spherical glass beads of uniform shape and density.

Batch experiments of two components systems of various particle sizes were conducted under the same set of jigging conditions: the jigging frequency and jigging time were respectively maintained at 60 cycles per minute and 999 seconds (16.65 minutes). The effect of these operating conditions on segregation was not investigated. At the end of each test run, the jig bed was split into horizontal slices and the composition of each slice was determined.

The experimental results showed that below a particle size ratio of 1.50:1, the driving force for the segregation of particles, i.e. the particle size difference, was small hence a low degree of segregation was obtained. The degree of segregation increased above this ratio. However, above the size ratio of 2.00:1, interstitial trickling occurred. With the smaller particles tested (8, 6 and 4mm) poor segregation was observed when the size ratios were of 1.50:1 or less along with what is believed to have been remixing due to convective currents within the jig chamber. It was found that the particle size range had a more pronounced effect on size segregation than the particle size. From the results, it can be said that above a size ratio of about 1.50:1, size segregation is very pronounced. This suggests that density separations of real ores, where both the density and size of particles vary, would be impaired if the particle size range of the material fed to the jig exceeds this ratio. However, this needs further confirmation by testing multiple component systems.

Dedication

This work is dedicated to:-

The Almighty God

My Dad, H. E. Silwamba and Mam, J. Nambukwa

All my brothers and sisters

My friend, Salome

Acknowledgements

First and foremost, I wish to give praise and thanks to the almighty God for making it possible for me to finish my Master of Science in Engineering. Indeed, my God you are the Master of Everything.

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To Dr. M. Bwalya I say thank you for supporting my application. You were also an inspiration for one year I worked with you.

To my family and friends in Zambia, thank you for being there for me. Talking to you in times of stress and hearing you say "you will make it with time" was so uplifting.

Lastly but not the least, I would like to thank the University of Maastricht and the University of Zambia through the STRECC programme for financial support. University of the Witwatersrand is also acknowledged for financial support via Postgraduate Merit Award (PMA).

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CHAPTER 1

Introduction

1.1 Background

Valuable minerals are concentrated by separating them from the associated gangue minerals before subjecting them to subsequent metal extraction processes. Similarly, coal is beneficiated before use by separating it from the minerals which contribute to the ash left after the coal is burned. There are quite a number of technologies available for the concentration process. Jigging is one of the oldest concentration technologies that has been used. Although jigging technology now faces competition with newer technologies such as flotation and dense media separators, it has many advantages. Among these advantages are that jigs have relatively low operating costs per tonne of throughput, low power requirement per tonne of concentrate produced, and are environmentally friendly owing to the fact that do not use chemicals (Burt, 1999).

One would expect technology which has been in use for many centuries with such aforementioned advantages to be fully understood and the particle segregation process to be theoretically well described. However, that is not the case with jigs. The challenges faced by many researchers who have attempted to study the particle segregation process in jigs, have been the strong interaction among variables that affect segregation of particles (Mishra and Adhikari, 1999). Karantzavelos and Frangiscos (1984) classified the independent variables that affect stratification of particles into two major categories namely manipulated and disturbance variables. Disturbance variables are the physical characteristics of the feed material including feed particle size, particle size range, particle shape, and the density of the particles. Manipulated variables are those associated with the operating conditions of the process.

There have been many attempts by researchers to study the stratification process by considering the previously mentioned variables. A review by Mehrotra and Mishra (1997) provides some researchers' approaches and assumptions made to study the stratification process of particles in a jig. In an effort to concentrate on density segregation and not to complicate the study, many investigators considered only the effect of variation of density in their models and assumed mono-sized particles and uniform shape (King, 1987; Mayer, 1964; Tavares and King, 1995; Vetter et al., 1987). This is contrary to what actually happens in real concentration process where the jig feed contains particles of variable sizes. Very few researchers such as Rao (2007) and Tavares (1999) have attempted to do some simulation studies on the effect of variation of both particle size and density on stratification. However, even with these attempts the influence of particle size on segregation still remains not fully understood.

1.2 Problem statement

Environmental laws have recently become stringent and the cost of chemicals and energy continue to skyrocket. This has led to many practitioners in mineral processing, particularly in iron ore and coal plants to continue using already existing jigs or install new ones if the plant has none. Jigs are, however, challenging to control and optimise. This is due to inadequate understanding of particle segregation process which is affected by a number of variables (Karantzavelos and Frangiscos,1984). Among the variables that are not yet fully understood are the effects of variable particle size and size range on the segregation of particles. In industrial practice, the effects of these variables are suppressed by screening the feed into different size classes and jigging them separately (Myburgh and Nortje, 2014). However, even with this effort particle sizes are never uniform and the choice of the particle size range remains critical. Therefore, understanding the effect of particles size and size range on segregation of particles is imperative for better operation, control, and optimisation of jigs. It is for this reason that this study was devoted to contribute to the knowledge of the effect of particle size and size range on segregation of particles in a jig.

To better understand how particle size and size range influence segregation, the variation of density and shape of particles was constrained by using artificial feed material of uniform shape (spherical glass beads) and the same density. We were fully aware that concentration of wanted material (valuable material) by jig concentration technology is hinged on the density differences between particles of wanted material (valuable mineral particles) and particles of unwanted material (gangue mineral particles). However, this was ignored in order to better investigate the effect of particle size. As such, this is to our knowledge the first study of its kind to be found in the literature.

1.3 Research objectives

As stated in Section 1.2, the overall expected end result of this study is the contribution to the knowledge about the effect of particle size and size range on particle segregation in a jig.

Therefore, this study addresses the following research questions:-

- How does particle size range affect segregation of particles in a batch jig under a particular set of jigging conditions?
- How does particle size influence segregation of particles in a batch jig under those conditions?

1.4 Summary of the project report layout

This research report is divided into five chapters. The first chapter is an introduction to the study which entails the background, problem statement, and the research questions.

In the second chapter, an overview of the jigging concentration process and industrial application of jigs is presented. Factors that affect stratification, theories of motion of particles in a jig and models that predict stratification process are also reviewed in this chapter.

A description of the experimental equipment, experimental set up and procedure used to achieve the set objective and answer the research questions are presented in the third chapter.

In the fourth chapter the results are presented and discussed. Finally, in the fifth chapter, a conclusion is drawn based on the literature review and the results obtained. Also recommendations and further work are given in that chapter.

CHAPTER 2

Literature Review

2.1 Introduction

An overview of the stratification processes in a jig is presented in the first section of this chapter. The second section highlights the industrial application of jigs. Factors that affect stratification of particles and their impact on concentration performance of jigs are presented in the third section of this chapter. Theories that describe motion of particles in a jig bed are presented in section four. In the fifth section, models that have been developed to predict segregation of particles in a jig are reviewed. The last section looks at models that describe the segregation of particles where effects of particle size are taken into consideration.

2.2 Jigging and particle stratification process

The feed to jig concentrators is generally composed of reasonably well liberated valuable particles and gangue particles of different specific gravities, sizes and shapes. The feed is continuously added (for continuous jigging processes) or intermittently added (for batch jigging processes) to the jig chamber. Particles in a jig chamber are supported by the jig support screen or ragging material and form a bed of particles. The size of the jig chamber and the thickness of the bed are many times larger and thicker, respectively, than the largest particle in the bed (Burt, 1984).

Fluid, water in traditional jigging or air in pneumatic jigging, is repeatedly pulsated vertically through the bed of particles. The upward stroke (pulsion stroke) makes the bed expand while the downward stroke (suction stroke) makes the bed to compact. Each stroke is further divided into two stages and together they make four stages namely inlet, expansion, exhaust and compression (Mehrotra and Mishra, 1997). The inlet stage is the initial stage of the upstroke; the bed of particles is lifted up as a whole. Near the end of the upstroke (expansion), heavy-large particles decelerate while the light-small particles continue moving up with fluid. Exhaust and compression stages are the initial and final stages of the suction stroke, respectively. Near the end of the down stroke, particles maintain their positions till the bed is compacted. After compaction of the bed, heavy-small particles may continue moving towards the bottom of the bed by trickling through the interstices between coarse particles. Interstitial trickling happens provided that heavy-fine particles are smaller than the voids and the period between the end of the suction strokes and start of the pulsion strokes is long (Burt, 1984).

In summary, pulsating fluid through a bed of particles of variable density and size causes coarse and high density particles to migrate towards the bottom of the bed while fine and low density particles move towards the top of the bed. Particles with intermediate properties lie somewhere between the top and bottom of the bed. The bed is then cut at an appropriate height to separate a concentrate product (or valuable particles) from the gangue. In the case of coal beneficiation, the top part of the bed is composed of the lighter, lower ash yield components of the coal, that is, the higher quality coal fraction. It is cut and separated from the bottom part which is composed of the denser, high mineral content components of the coal, that is, the lower quality coal fraction. The opposite occurs for most mineral beneficiation processes where the bottom part is composed of valuable particles and the top part composed of the gangue particles. The description of particle stratification and concentration by jigging is well covered in a number of books (Burt, 1984; King, 2001; Wills, 2006; Yan and Gupta, 2006).

2.3 Industrial application of jigs

Jigs are used in some instances to pre-concentrate mineral or coal that is fed to the next stage in a concentration process while in other instances they are used as the sole concentration method to produce final product. The industrial applications of jigs are as follows:-

(i) Mineral and metal concentration

Heavy valuable minerals that are liberated from gangue minerals at coarse particle sizes have been concentrated through the utilisation of jigs for many centuries. Examples of such minerals as well as metals include iron minerals, tin, diamonds, gold and manganese. A good example of a local South African mine where jigs are still used for concentration is Sishen mine where approximately 20% of the total iron mineral produced per annum is concentrated through jigging (Myburgh and Nortje, 2014).

(ii) Coal beneficiation by jigs

Jigs have been in use in coal washing for many centuries. In the United States of America, for example, about 37% of coal beneficiation plants clean their coal using jigs (Mehrotra and Mishra, 1997). In South Africa, Leeuwpan coal mine is currently the only coal mine that is using jigs alongside Dense Media Separator (DMS) to beneficiate coal (Lundt, 2010). Jigs are suitable for coal seams where the density difference between the lower ash components coal (high quality coal) and high ash coal component (low quality coal) is significant and the amount of intermediate density material is minimal (Botha, 2009). Near density material in coal beneficiation consists of particles having densities in a range of 0.1 kg/L above or below the cut density of the separation.

(iii) Some new application of jigs

Due to the advantages discussed in chapter one, jigs continue to find other applications apart from mineral and coal beneficiation. Jigs have also found application in concentration of non-magnetic scrap metal and cleaning of contaminated soil (Witteveen, 1995). Another promising application of jigs is the separation of inert construction and demolition materials from concrete. This has been and is still under investigation (see for example Cazacliu et al. 2014).

2.4 Factors that affect particle stratification processes in a jig

Identifying and understanding the variables as well as parameters that affect a given process is important not only for operation but also for design, control, and optimisation. It is for this reason that a number of variables that affect jigging process have been identified and studied. These variables are broadly classified into two major categories namely independent and dependent variables. Dependent variables are the performance variables whereas independent variables are the variables whose changes by either manipulation or by unwanted external factors affect the dependent variables. Independent variables are further broken down into two classes, namely manipulated and disturbance variables. These variables are diagrammatically presented as shown in Figure 1(Karantzavelos and Frangiscos, 1984).

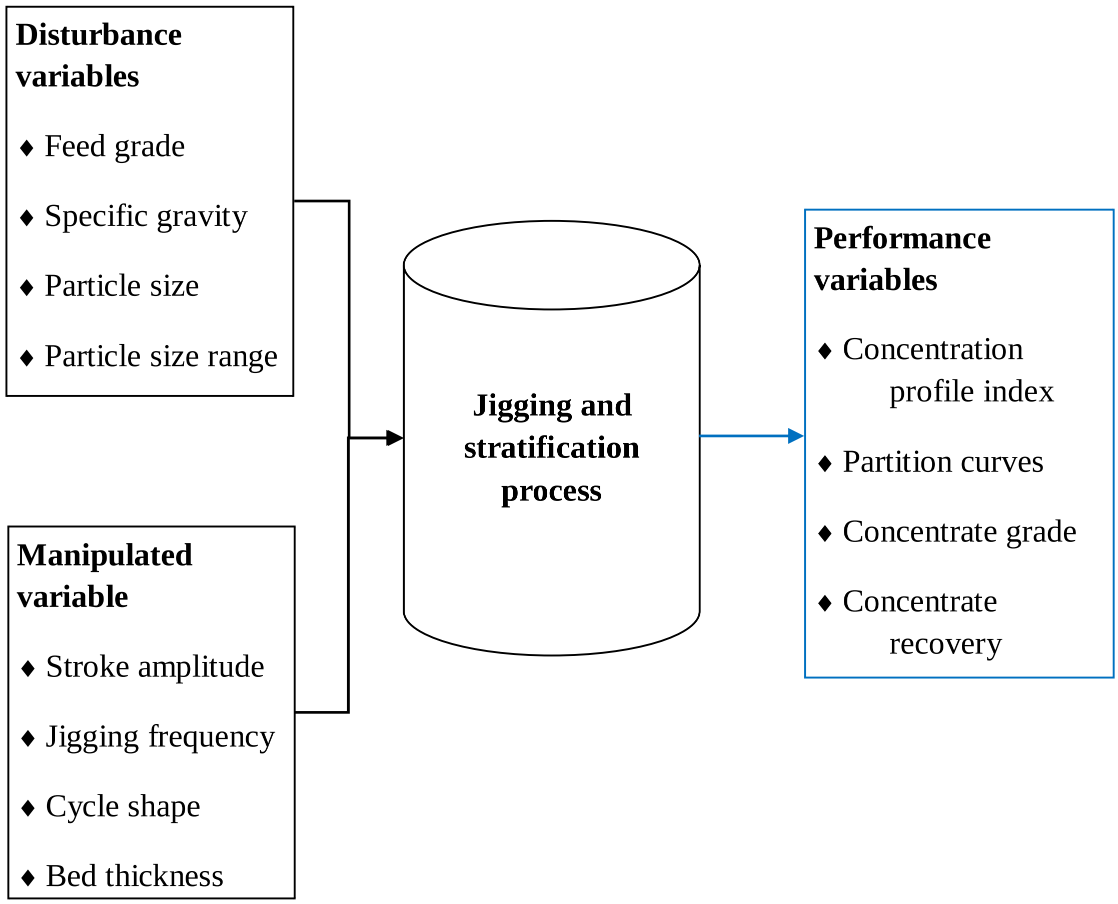


Figure 1: Categories of variables that affect stratification and performance variables (Karantzavelos and Frangiscos, 1984)

2.4.1 Feed grade

Under ideal conditions the feed grade to a jig should remain more or less constant with time and the same cut height for separating concentrate from gangue should be maintained. However, in industrial practice the grade of the feed varies over time. The variation of the feed grade does not only affect bed cut height but also affects the concentrate grade and yield (Karantzavelos and Frangiscos, 1984).

2.4.2 Specific gravity

As stated previously, jigs concentrate liberated valuable particles from gangue particles by taking advantage of the difference between the densities of valuable particles and gangue particles, the separation efficiency increasing as increase of the differences between densities of valuable particles and gangue particles. In other words, the increase of the difference between the densities of valuable particles and gangue particles increases the driving force for the density segregation. The principle is that jiggability, that is, the ability to achieve a good separation in jig, increases with increase of absolute value of the concentration criterion (CC) given as (Wills, 2006):-

$$CC = \frac{\rho_{sgh} - \rho_{sgf}}{\rho_{sgl} - \rho_{sgf}} \quad \dots (1)$$

Here, ρ_{sgh}, ρ_{sgl}, and ρ_{sgf} are respectively, the specific gravity of the heavy mineral, light mineral, and fluid. The larger the absolute value of the concentration criterion the higher the efficiency of separation. One point to note is that the concentration criterion is a feed property that is constant throughout the separation process for a particular feed composition.

2.4.3 Particle size and particle size range

Segregation of particles and concentration efficiency are affected by the size and size range of particles in a jig. The efficiency increases with increase of the particles size in the feed. This is the reason why jigs are particularly suitable for minerals that can be liberated at coarser sizes (Chatterjee, 1998).

The particle size ratio (size range), that is, the ratio of top particle size to bottom particle size in the feed, introduces another driving force for segregation of particles in a jig. Because jigs exploit density differences to achieve the desired separation of particles, segregation by size leads to misplacement of particles. The degree of misplacement depends on the magnitude of both the concentration criterion and the size ratio. In the simulation study by Taveres (1999), it was shown that the percentage of misplaced particles was large for systems where both the density difference and size ratio were small. Minimal particle misplacement was obtained for systems of large density difference and small size ratio. Xia et al. (2007) showed that the proportion of misplaced particles in a bed composed of mono-sized particles is relatively lower than when it is composed of multiple sized particles. In industry, the misplacement of particles in the jig bed due to size range effects is partially suppressed by screening the feed into different size fractions with each size fraction being jigged separately. For example, Shishen mine classifies iron ore feed going to the jig concentrator into three different size fractions, that is, coarse (-25mm +8mm), medium (-8mm +3mm), and fine (-3mm +1mm) (Myburgh and Nortje, 2014; Voigt and Twala, 2012). The critical parameter for deciding on this split is the ratio of the top particle size to the bottom particle size, that is, the size ratio of the largest to smallest particles for each size class. The smaller the size ratio the higher the concentrate grade and recovery obtained. In other words, a small size ratio reduces the misplacement of particles during jigging.

2.4.4 Particle shape

Differences in the shapes of particles is another driving force for the segregation of particles in a jig. In industrial applications of jigs, it is often observed that fine, flat and elongated particles move towards the top of the jig bed (Burt, 1984). Brozek and Surowiak (2007) have studied and quantified the effect of the shape of particles on settling velocity and separation efficiency in a jig. They compared settling velocity and separation efficiency of irregular particles to spherical particles and found that the degree of irregularity of particles has negative effects on settling velocity and separation efficiency.

2.4.5 Jigging frequency and stroke amplitude

The mobility of particles, especially during pulsion, affects stratification. It is directly related to the degree of bed expansion which is also directly proportional to the velocity of fluid through the bed. Studies have shown that the higher the jigging frequency and amplitude the higher the velocity of fluid through the bed and the higher the rate of stratification (Ahmed, 2011; Karantzavelos and Frangiscos, 1984; Mukherjee et al., 2005; Srinivasa et al., 1999). However, better stratification is obtained at neither too high nor too low frequency and amplitude values but at intermediate values. The optimal values of frequency and amplitude depend on the feed particle size.

2.4.6 Jig bed height

Stratification of particles is also affected by the bed height (bed thickness) of the jig. The rate and degree of stratification is inversely proportional to the bed height. This is due to the fact that particles in a thin bed have relatively great chances and short times to move either to the top or bottom of the bed depending on their attributes. However, it is always advisable not to make the bed thickness so thin that it leads to particles remixing instead of stratifying (Ahmed, 2011; Karantzavelos and Frangiscos, 1984).

2.4.7 Hutch water

In the case where water is used as the fluid in a jig, the hutch water addition rate and water level affect particle stratification and the recovery of valuable particles. The relationship is direct, i.e. both stratification and recovery of valuable particles improve with the level of water in the bed and flow rate of hutch water. The reason for this is the fact that at high hutch water addition rate and water level, bed suction intensity is reduced. However, the addition rate should not be so high that it causes remixing of particles during the suction stage of the jigging cycle (Ahmed, 2011; Karantzavelos and Frangiscos, 1984).

2.5 The motion of particles in a jig bed

The variables and parameters highlighted in the previous section (Section 2.4) affect the motion of particles in a jig bed. The study of the movement of particles in a jig bed is a microscopic study in the sense that the forces that affect the movement of a single particle are considered first before relating to the mass movement of particles as a whole. This section reviews theories that have been developed to describe this motion.

2.5.1 Classical theory

Classical theory describes the mechanisms that affect movement and stratification of particles in a jig. According to this theory, particles stratify by three mechanisms which may each occur during jigging. These are (Burt, 1984; Gaudin, 1939):-

- Differential acceleration
- Hindered settling
- Interstitial trickling

Differential acceleration

Differential acceleration is based on the settling laws of particles in a fluid. It is stated that particles accelerate at the beginning of the pulsion and suction of the jigging cycle. The initial acceleration of each particle for both stages of the cycle depends only on its specific gravity and the specific gravity of the fluid. This is due to the fact that resistance to the movement of particle which depends on viscosity of fluid and the size of particles is low enough at low velocities to be ignored.

In the case where the stroke amplitude is high and the jigging frequency is low, particles may reach their terminal velocities, i.e. they reach a state of zero acceleration. Under this condition, their movements depend on their sizes, densities, shapes and the density of the fluid. On the other hand, when the stroke amplitude is small and jigging frequency is high enough, particles may not reach their terminal velocities; differential acceleration occurs (Equation 2) (Gaudin, 1939). According to Equation 2, denser particles can be separated from less dense particles regardless of their shapes and sizes under these conditions.

$$\frac{dv}{dt} = \left(1 - \frac{\rho_{sgf}}{\rho_{sgs}}\right) g \qquad \ldots (2)$$

Here v, t, g, ρ_{sgs} and ρ_{sgf} are respectively the settling velocity of a particle, time, acceleration due to gravity, the density the particle and the density of the fluid.

Hindered settling

Hindered settling is also based on the laws of settling but assumes the particles have reached an equilibrium settling rate. If particles settle in a fluid containing few solids, i.e. free settling, their settling rate can be predicted using particle-fluid dynamics – i.e. Stokes Law. However, if the concentration of solids is high, as it is in a jig bed, settling is 'hindered' by particle-particle interactions and these have to be taken into account as well as particle-fluid interactions. However, this is a highly non-trivial challenge, and, pragmatically, has often been handled by simply using the free settling equations but assuming the density of the fluid to be the pulp density of the fluid-solid system. This appears to give the right kind of trends namely that the settling rates of particles under hindered settling conditions are lower than under free settling conditions, and that hindered settling enhances the effect of differences in particle density and reduces the effect of particle size on settling rates. the effect of the density differences on segregation is enhanced while the effect of particle size is reduced (Wills, 2006). Figure 2 suggests how hindered settling enhances the effect of the density differences on settling.

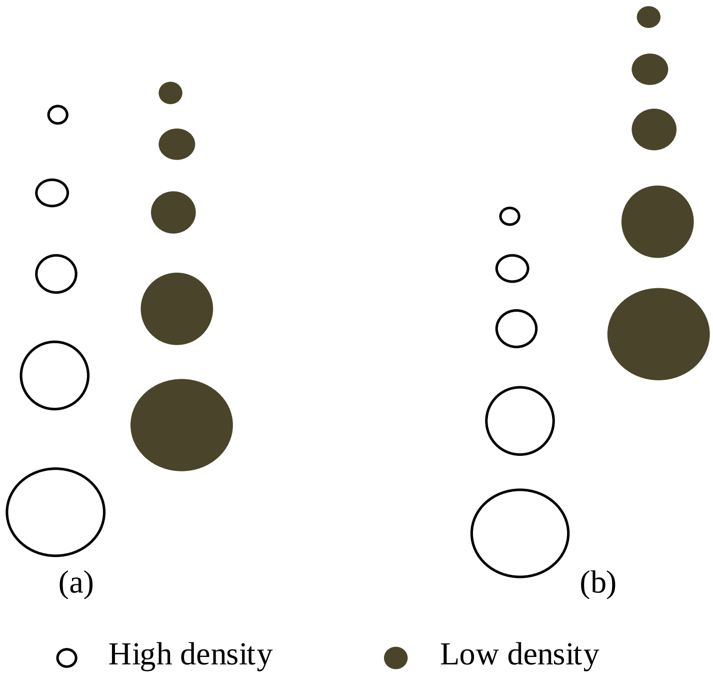


Figure 2: Classification (a) Free settling (b) Hindered settling (Wills, 2006)

Figure 2 suggests that although hindered settling enhances separation by density differences, coarse light particles cannot be separated from fine dense particles by hindered settling.

Interstitial trickling

At the end of the suction phase of the jigging cycle, coarse particles interlock and leave interstitial spaces filled with moving fluid. Particles can trickle through these interstices if they are small enough to move toward the bottom of the bed under the influence of gravity, momentum, and fluid velocity. Denser particles are likely to penetrate further than less dense particles because they have more momentum. In this way, fine more dense particles and even fine less dense particles may be recovered along with the coarse heavy particles. Interstitial trickling depends on particle density, and size as well as on the fluid velocity and the duration between the suction and pulsion stages of the jigging cycle. The extent of interstitial trickling increases with the size range of particles and with the length of the hold time between the suction and the pulsion stages of the jigging cycle (Gaudin, 1939). For spherical particles, it is determined geometrically that fine particles percolate interstitially at and above the size ratio of about 2.4:1 (Burt, 1984).

2.5.2 Studies aimed at describing the movement of particles in a jig bed

Many researchers have studied the motion of particles in a jig bed from different perspectives and by using different and sophisticated techniques (see for example, de Jong et al., 1996; Kuang et al., 2009, 2008; Mukherjee and Mishra, 2006; Roux and Naudé, 2014; Xie and Kuang, 2004).

Xie and Kuang (2004) used the High Speed Dynamic Analysis System (HSDAS) to study the movement of particles and water in a laboratory jig. They observed that during pulsion strokes, the positions of particles were interchanged in the jig bed depending on their physical characteristics. That is, if the light particle is below the heavy particle, the positions of these two would be swapped during the pulsion stroke. During the suction stroke it was observed that particles maintained their positions till the bed compacts. After compaction of the bed, fine particles were observed to be moving horizontally and vertically through the voids between the coarse particles. The conclusion was that particle stratification happens during the pulsion stroke but not during the suction stroke. This was, however, contradictive to the findings by Beck and Holtham (1993) who concluded from their study that stratification occurs during the suction stroke as well.

Movement of heavy particle relative to the light particles in a jig bed was analysed by de Jong et al. (1996). The displacement of the heavy particle in a jig bed was traced and analysed. From the study they showed that particles displacement increased with jigging frequency till an optimum frequency was reached. Above that optimum frequency the displacement of particles started decreasing.

Further, Kuang et al. (2009) mimicked a damped vibrator to develop a differential equation of the motion of particles in a jig bed. According to the model, it was found that the displacement (jumping height as stated by the authors) of particles during pulsion and suction increased with decrease of the mass of a particle. Since the particle mass is a product of volume and density it means that the displacement of particles decreases with an increase of the size and density of the particle. However, the relationship between displacement of particles and mass of particles is not linear. The model equations that describe particle displacement (jump heights) at three stages of jigging cycle are given as:-

$$x(t) = \begin{cases} x_0 + \dfrac{F_1 T^2}{2m_d\sqrt{\left[\frac{g}{l}T^2 - 2\pi^2\right]^2 + \left(\frac{\epsilon\pi T}{m_d\theta}\right)^2}}\cos\left(\frac{2\pi t}{T} - \varphi\right) & 0 < t \le T_1, Pulsion \\ x_0 + \dfrac{F_1 T^2}{2m_d\sqrt{\left[\frac{g}{l}T^2 - 2\pi^2\right]^2 + \left(\frac{\mu d\varepsilon\pi T}{m_d\theta}\right)^2}}\cos\left(\frac{2\pi t}{T} - \varphi\right) & 0 < t \le T_2, Suction \\ x_{T_2} & t > T_2\text{, } Time\ between\ pulsion\ and\ suction \end{cases} \quad \ldots(3)$$

Here $x(t)$ is the particles displacement at time t, x_0 is the distance from the fixed jig screen when particles are at equilibrium position, θ is the mobility of the jig bed, ϵ is the mobility of the bed during the pulsion stage, d is the jig bed height, ε is the volume coefficient jig chamber, φ is the initial air valve position, m_d is the mass of the particle, T is the jigging period, $F_1$and F_2 are the external force on particles during pulsion and suction respectively. Equation (3) is complex with multiple parameters to evaluate. However, it suggests that if particles of different sizes and densities are exposed to the same external force their movements are determined by those two physical characteristics.

In a similar study done by Kuang et al. (2008), it was postulated that particles of equivalent physical properties in a jig tend to move to a particular height in the bed which depends on the composition of the jig bed. The classical settling velocity of particles was used to estimate this 'inherent height'. Equation (4) was derived from the settling velocity as an estimate for bed layer number for a particular particle of a given density and size.

$$\gamma_{ij} = \sqrt{d_{ij}\left(\rho_{ij} - \rho_o\right)} \quad i \text{ is } 1,2,\ldots,\text{n size class, } j \text{ is } 1,2,\ldots,\text{n density class} \quad \ldots (4)$$

Here γ_{ij} is an estimate of the layer number according to the properties of the particles making up the jig bed, d_{ij} is the diameter of the particle, ρ_{ij} is the density of the particle, ρ_o is the density of fluid.

In a recent study by Roux and Naudé (2014), positron emission tracking was used to track the movement of a single particle of higher and lower density through the other particles in jig a bed. The movement of the particles from different starting positions were studied and the results demonstrated the existence of convection flow patterns in the bed during jigging. It was observed that the light particle which started at the bottom centre of the jig bed moved to the side of the bed on its way to the top while the light particle which started at the bottom side tended to maintain its position at the side of the bed. In contrast, a heavy particle which started at the top side of the jig bed moved to the centre of the bed and then moved down while the heavy particle which started at the top centre of the bed maintained its position on its way to the bottom of the bed. This kind of convective flow was also observed by Williams et al. (1998) (as cited by Roux and Naudé, 2014). Figure 3 below shows the convection flow of particles in a jig bed that they reported.

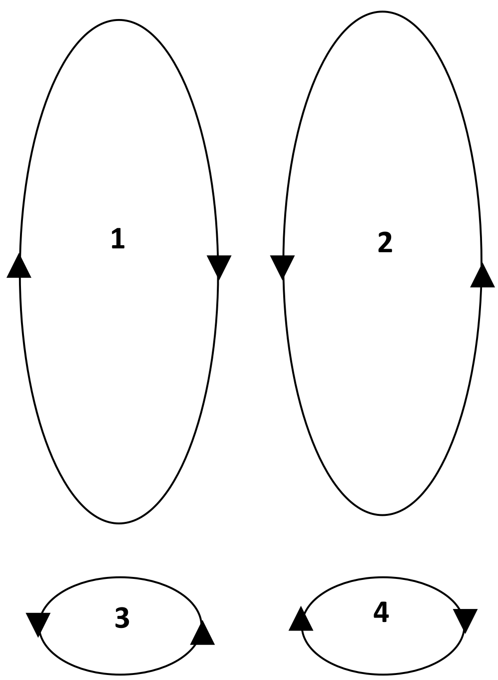


Figure 3: Particle flow pattern due to convection effect (Williams et al., 1998 as cited by Roux and Naudé, 2014)

2.6 Models for predicting stratification of particles in a jig

An ideal model capable of predicting stratification of particles in a jig should incorporate all the factors discussed in Section 2.4 while remaining as simple as possible. Such a model would be of great value for operation, designing, controlling, and optimisation of the jigging process. However, finding such a model has been a challenge for many decades. This section reviews some of the models that have been developed to predict and describe segregation processes of particles in a jig. The models considered are classified into empirical models, potential energy based models, dispersion of suspended particles, and energy dissipation models.

2.6.1 Empirical models

Some researchers have avoided the complexity of trying to establish a theoretical basis for describing the effect of the interacting variables in their models. They have based their models only on experimental data (Karantzavelos and Frangiscos, 1984; Rong and Lyman, 1992), i.e. the relationship of the performance variables to operating and disturbance variables is drawn from the results obtained experimentally. For instance, the model by Karantzavelos and Frangiscos (1984), Equation 5, is based on the two parameters Weibull distribution function to model the yield of valuable particles to the concentrate zone with time.

$$Y(t) = 1 - \exp(\frac{-\beta t}{\theta}) \qquad \ldots (5)$$

Here $Y(t)$, t, β and θ are respectively the yield at a given time, time, the 'delay' of the segregation process and the natural tendency of particles to segregate into different layers according to their attributes. The two parameters, β and θ, are obtained experimentally and depend on the physical attributes of the particles (density and size), bed thickness, hutch water, stroke amplitude, jigging frequency and feed grade. Although empirical models are useful in operation, they contribute less to the fundamental theory of the jig and their applications are limited.

2.6.2 Potential energy based model

Mayer (1964) postulated that particles in a jig bed stratify in order for the bed to attain a state of lowest possible potential energy. According to this theory, a perfectly mixed bed of particles (i.e. an unstratified bed) has a relatively high potential energy compared to a stratified bed hence it is unstable. Conversely, a stratified bed has a relatively low potential energy and hence is stable. According to Mayer (1964), stratification of particles is not caused by pulsion and suction. Instead, he proposed that pulsion and suction function only to free the particles so that they can move and reduce the bed potential energy by stratification. The potential energy theory has some demerits (Mehrotra and Mishra, 1997; Vetter et al., 1987). These include

- ♦ It only accounts for two states, unstratified and stratified bed states. The rate at which high and low density particles move towards the bottom and top of the bed, respectively, is left out.
- ♦ It does not account for the poorly defined (un-sharp) boundaries obtained in practice between two layers of particles of certain physical attributes. In other words, Mayer's theory assumes perfect separation where there is no remixing of particles at the boundaries of particles with different physical characteristics.

2.6.3 Models based on the dispersion of particles

Mayer's potential energy theory assumed perfect separation as illustrated in Figure 4 (a). However, what is actually obtained in practice is more like what is illustrated in Figure 4 (b).

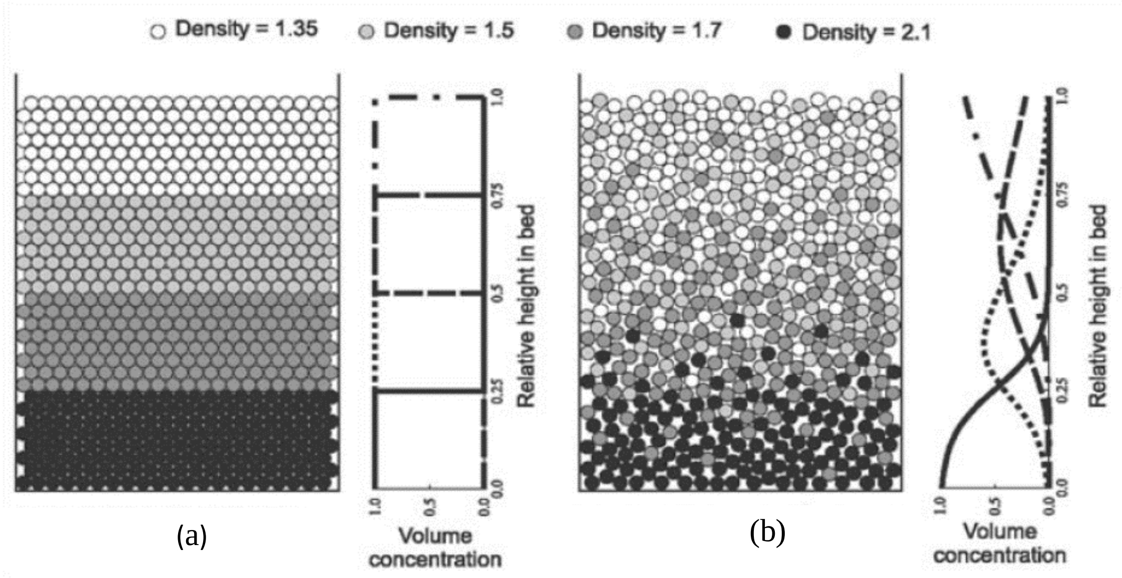


Figure 4: Ideal stratification by Mayer's potential energy theory and the actual stratification obtained in practice (King, 2001)

A number of researchers (King, 2001, 1987; Tavares and King, 1995; Vetter et al., 1987) have attempted to address the shortcomings of the potential energy theory highlighted above. The extension of Mayer theory to incorporate the remixing of particles of different physical characteristics at the boundaries was done by King (1987), and Tavares and King (1995). The model obtained is referred to as the King model in some literature (Mutibura, 2015; Woollacott et al., 2014). According to this model, particles in a bed face two opposing fluxes - stratification and diffusion. The stratification flux is due to the tendency of the bed of particles to attain stability by the reduction of potential energy according to Mayer's theory. The diffusive flux is caused by a dispersive process which can be described by Fickian diffusion. Equilibrium is reached when these two fluxes balance. The principle equation is given as:-

$$\frac{dC_i(h)}{dh} = -\alpha C_i(h)[\rho_i - \bar{\rho}(h)] \qquad \text{for } i \text{ 1,2,3... } n \text{ density classes} \qquad \dots (5)$$

Here, h is the normalized bed height, $C_i(h)$ is the volumetric concentration of particles of density ρ_i in a particular layer of thickness dh, $\bar{\rho}(h)$ is the mean density of the particles in that layer, α is the specific stratification parameter dependent on the sizes of the particles and is given as;

$$\alpha = \frac{\vartheta g V_p H_{bd}}{D} \quad \ldots (6)$$

where ϑ is the specific penetration velocity of the particles, g is the acceleration due to gravity, V_p is the volume of the particle, H_{bd} is the bed height and D is the diffusion coefficient.

Experimental validation of the King model has been done by several workers (King,2001, 1987; Mutibura, 2015; Tavares and King, 1995; Woollacott et al., 2014). For example, King (1987) used experimental data obtained by Vetter (1987) to fit the model. Mono-sized cubic particles made of polyvinyl chloride (PVC) of variable densities were used as feed to validate his model. The model agreed well with the experimental data obtained for this binary system. King further compared the specific stratification constant between coal of narrow density and PVC cubes. The specific stratification constant obtained in a binary system of coal was lower than the one obtained in binary systems of PVC cubes due, according to King, to the influence of shape. Recent validation by Woollacott et al. (2014) showed excellent fits of the King model using artificial particles of uniform shape and size for up to seven components systems. Mutibura (2015) fitted the model for complex systems of coal of variable sizes, densities and shape. Although the fits were not as good as in previous studies, it was shown that the model is fairly good at describing stratification of particles as long as the size difference between the top size and bottom size is not more than 2.4 (i.e. the ratio of top size particles to bottom size not more than 2.4:1). Mutibura's findings suggests that even if the effects of size and size range on stratification are ignored, the King model provides a reasonable basis for modelling stratification in real systems. However, it still stands that the King model does not explicitly account for the size or the size distribution of particles.

Vetter and co-researchers (Vetter et al., 1987) also studied and modelled the segregation of particles in a jig by taking into account the natural dispersion of particles that results into undefined boundaries between layers. According to their dispersion model, the particle movement in the bed was probabilistically modelled where the influence of fluid was divided into two main parts, the dynamic friction and noise. The noise was due to particle-particle interactions as particle move through the bed. The final equation obtained is given as:-

$$\frac{\partial C_i}{\partial t} = \frac{\partial[k_\mu(\rho_i - \bar{\rho})C_i]}{\partial y} + k_D \frac{\partial^2 C_i}{\partial y^2} \qquad \text{for } i \text{ 1,2,3,..., } n \text{ density classes} \qquad \ldots (7)$$

Here C is the volumetric concentration, ρ is the density, $\bar{\rho}$ is the mean density for a particular space coordinate, y, k_μ is the drift coefficient that measures the penetration velocity of particles through the bed, and k_D is the diffusion coefficient that accounts for dispersive mixing of particles in the bed. The limitations of the above model are that uniform particle size and shape are assumed. In addition, ideal conditions are assumed due to the fact that the terms for particle drag and gravitational segregation are linear.

2.6.4 Energy dissipated model of particles stratification

An energy dissipation model of particles segregation in a jig bed has been developed by paying attention to the energy balances in a jig bed that is being stratified. Rong and Lyman (1993a, 1993b) have worked extensively on developing the relationship between the total energy dissipated and stratification. They found that the total energy dissipated in a given cycle which is defined as the difference between energy supplied and energy recovered as the bed collapses has a direct relationship with the particle stratification in the jig bed. The total energy dissipated is a function of operating parameters (jigging frequency, stroke amplitude, and jigging time), physical attributes of the feed material (density, size, and shape of particles), and the jig structure. Therefore, similar stratification is obtained even when feed characteristics have changed provided that the total dissipated energy is maintained by manipulating the operating parameters. The parameters that have a strong influence on the dissipated energy are the jigging frequency and the operating air pressure (Rong and Lyman, 1993a, 1993b). However, the energy dissipation model is a complex empirical involving a large number of parameters whose values must be determined experimentally, and so are difficult to apply in practice.

2.7 Integration of particle size effects in modelling the stratification processes in a jig

The study of particle size and particle size range's influence on stratification of particles in jigs is not just an academic exercise but a step in the direction of developing greater understanding of jigging and by this means to foster better practical control, design and optimisation of the process. There have been a few attempts to model the influence of particle size on particle stratification: one by Tavares (1999) and the other by Rao (2007).

2.7.1 Integration of the effect of particle size in Mayer's potential energy model

Tavares (1999) extended Mayer's gravitational potential energy theory of particles stratification by incorporating the effect of particle size. The principle equation is given as

$$E = g h_b V_{cell} \emptyset \left[\frac{M}{N}\sum_i^N \left(\hat{\rho}_i + \rho_f \frac{1-\emptyset_i}{\emptyset_i}\right)\left(\frac{1}{2\emptyset_i} + \sum_{j=1}^{i-1}\frac{1}{\emptyset_i}\right)\right] \quad \ldots (8)$$

where, E is the potential energy of the bed, g is the acceleration due to gravity, $\emptyset$ is the packing density of the whole bed, $\emptyset_i$ is the packing density of the ith bed layer, h_b is the bed height, N is number the vertical slices, M is the number of cells in each slice with equal volume, V_{cell} is the volume of particles (particles of uniform density and size) in each cell, $\hat{\rho}_i$ is the mean density of the particles in a cell and ρ_f is the density of the fluid. From Equation (8), two variables, $\emptyset_i$ and $\hat{\rho}_i$ account for variation of particle size and density respectively.

Obtaining a solution numerically for Equation (8) was difficult hence a Monte Carlo simulation technique was used. The simulation results showed that coarse-heavy particles move toward the bottom of the bed while the fine-light particles move towards the top of the bed. The misplacement of particles was found to be highly affected by the distribution of density and size of particles (Tavares, 1999).

2.7.2 Incorporation of particle size in the King stratification model

The King stratification model is strictly applied to feed composed of particles of variable density but of uniform size and shape. In order to make the King model as practical as possible, Rao (2007) extended it by accounting empirically for the effect of particle size. The feed particles were discretised into size and density classes. The stratification parameter, α, which entirely depends on the particle size was proposed to be a power function of the particle size given as

$$\alpha_j = A(d_j)^b \qquad \text{for} \quad j=1,2,3,...,m \text{ size classes} \qquad \ldots (9)$$

Here, A and b are the parameters of the feed distribution and stratification, and d_j is the particle diameter. The principle equation that takes into consideration the effects of both the size and density of the particles is given as

$$\frac{dC_{ij}(h)}{dh} = -A(d_j)^b C_{ij}(h)[\rho_i - \bar{\rho}(h)] \quad \text{for } i = 1,2,\ldots,n \text{ density classes} \qquad \ldots (10)$$

where the definitions of symbols have the same meaning as given in Section 2.6.3. Equation (10) was used to simulate the partition surfaces at different jig bed height. It was found that the separation of coarse multi-sized particles of distributed density was better than fine multi-sized particles (Rao, 2007). However, as far as the author is aware this extension of the King model has not been validated experimentally.

2.7.3 Model based on settling rate of particles

Professor Woollacott has embarked on modelling the settling rates of particles in a jig (Woollacott, in preparation). For uniform density material, the Woollacott model reduces to a model similar to the King model. Equation 11 below shows the model developed for describing segregation of particles of uniform density but variable sizes.

$$\frac{dC_{si}(h)}{dh} = -\alpha_s C_{si}(h)[l_i^b - \bar{l}^b(h)] \qquad \dots (11)$$

Here, $C_{si}(h)$ is the concentration of particles of size i in the thin layer at a height h in the jig bed, α_s is the stratification parameter which is independent of particle size but dependent on bed expansion, l is the size of the particle, b is a parameter determined experimentally, and $\bar{l}$ is the mean of the particle sizes in the layer but raised to the power b.

It is important to note that the above model has not been validated with experimental data. In addition, the model has not been published yet but will be used as a means to fit some of the experimental data generated in this study.

2.8 Summary

Factors that affect particle stratification processes in a jig were reviewed in this chapter. These factors are classified into two major categories namely manipulated and disturbance variables. The interactions between these factors is generally significant. It is for this reason that many researchers have faced difficulties in quantitatively describing particle stratification processes in a simple and systematic manner while considering as many relevant factors as possible.

The literature review showed that the effect of particle size and particle size distribution on stratification and separation efficiency has received little attention and that most models of stratification assume that the feed to the jig consists of mono-sized particles. The models that attempt to account for the effect of particle size on stratification are essentially empirical in nature and have not been adequately validated. Therefore, this study is intended to establish more precisely than was previously done how particle size affects stratification in a jig.

CHAPTER 3

Experimental Equipment, Design, and Procedure

3.1 Introduction

This chapter provides a description of the equipment, feed material, jigging conditions and the experimental procedure used to gather experimental data for answering the research questions and meeting the research objectives.

3.2 Description of the laboratory batch jig

Figure 5 shows a photograph of the batch jig in which all the experimental tests were carried out. The jig was located in the School of Chemical and Metallurgical Engineering laboratory at University of the Witwatersrand. It was similar to Mintek's mineral density separator (MDS) which had been used by investigators such as Naudé et al. (2013), Mutibura (2015) and Woollacott et al. (2014) in their investigations into jigging. The jig chamber consisted of a bed support screen on which rings that could be built up for each test to make a cylindrical jig chamber. Rings with of heights 20 mm and 15 mm were available. The rings were clamped tightly to prevent leakage of water. The internal diameter of the rings (hence of the jig chamber) was 200 mm which was more than fourteen times larger than the coarsest particles used in the study. The system of rings allowed the jig bed to be split into small horizontal segments as described shortly.

The bed support screen apertures were many times smaller than the finest particle size used. Pulsion and suction of the jig bed were caused by means of a bellows and pneumatic system situated below the support screen. The jigging frequency and stroke amplitude were set at the desired values by means of the Programmable Logic Controller (PLC).

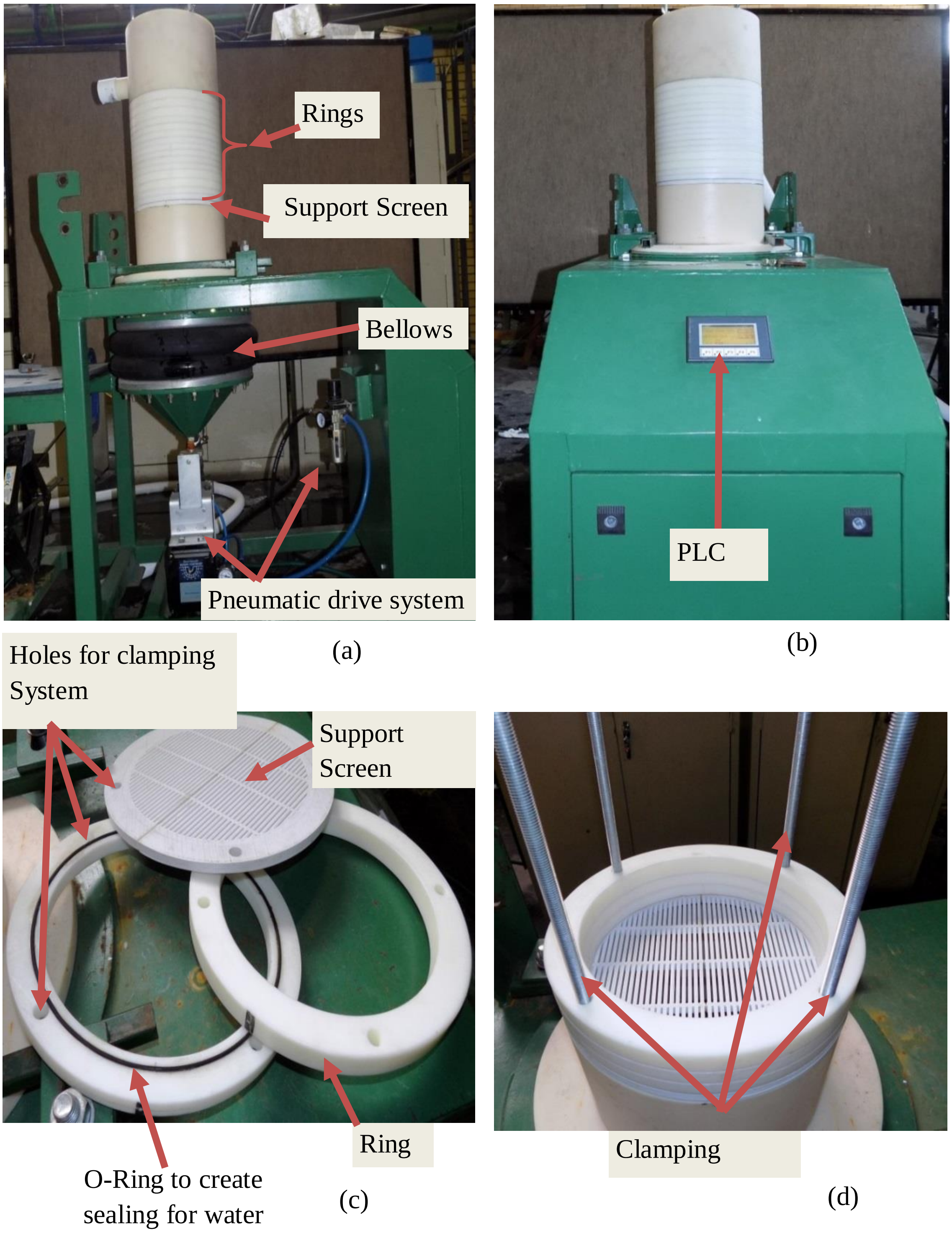


Figure 5: Photograph of batch jig used for all experiments (a) Front view, (b) Rear view, (c) Rings and bed support screen, (d) Clamping system

3.3 Feed material and feed preparation

The study was limited to investigating the effects of particle size and size range on stratification. To this effect, particles of uniform shape (spherical glass beads) and density (2520 kg/m^3) but of different sizes were used as feed. Figure 6 shows a sample of the glass beads that were available for the tests; grouped according to their diameters: 14 mm, 12 mm, 10 mm, 8 mm, 6 mm, and 4 mm.

Even with this limited range of particle sizes, an infinite number of combinations is possible. Accordingly, the scheme used to select combinations appropriate for the study was as follows.

a) Only binary combinations of the beads were selected.
b) Each binary combination would consist of a 50-50% mix (on a volumetric basis) of the relevant particle sizes.
c) A test number was assigned to each test in order of decreasing size ratio, that is, the diameter of top size (D_{tp}) to diameter of bottom size (D_{bm}). Table 1 shows the fifteen particles size combinations selected using this scheme.

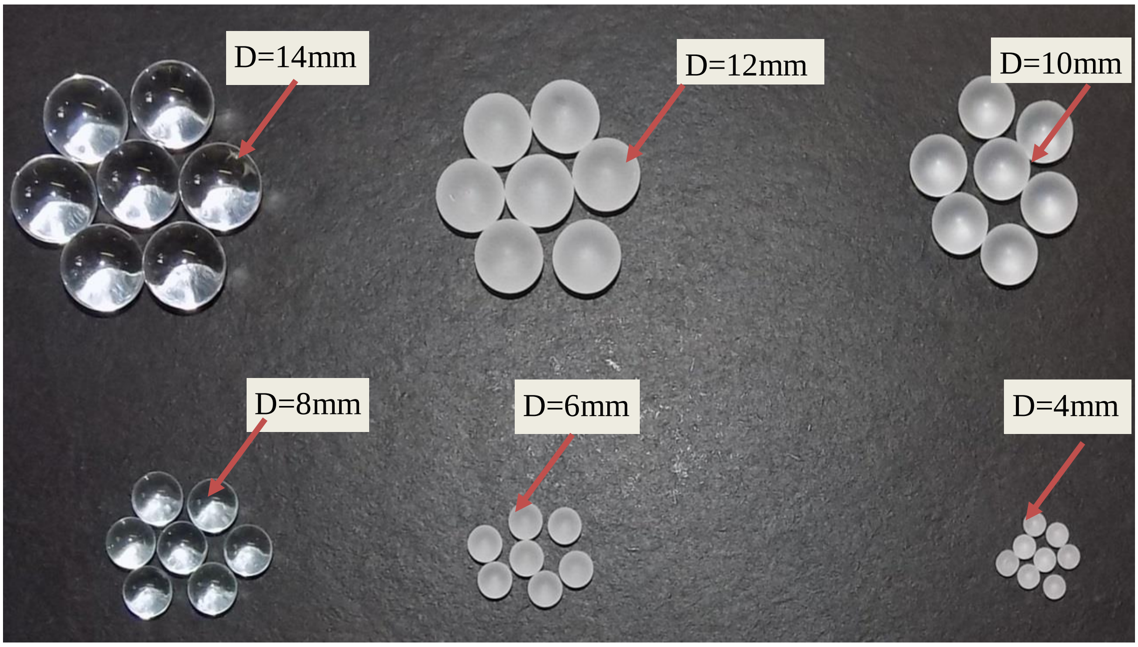


Figure 6: Photograph of a sample of the glass beads used as feed

Table 1: Particles size combination for each test run

	Particles size combination per each test run		
Test run #	**Top size, D_{tp} (mm)**	**Bottom size, D_{bm} (mm)**	**Ratio (D_{tp}:D_{bm})**
TR1	14	4	3.50 : 1
TR2	12	4	3.00 : 1
TR3	10	4	2.50 : 1
TR4	14	6	2.33 : 1
TR5	12	6	2.00 : 1
TR6	8	4	2.00 : 1
TR7	14	8	1.78 : 1
TR8	10	6	1.67 : 1
TR9	12	8	1.50 : 1
TR10	6	4	1.50 : 1
TR11	14	10	1.40 : 1
TR12	8	6	1.33 : 1
TR13	10	8	1.25 : 1
TR14	12	10	1.20 : 1
TR15	14	12	1.17: 1

3.4 Experimental procedure

Each test involved setting up the jig chamber as explained in section 3.2. A mass of 6000g of the feed material selected for the test was then weighed out, mixed thoroughly, and was then poured into the jig chamber which was three quarters filled with water. In this way, essentially the same bed height – approximately 130 mm - was maintained in each test.

The feed was jigged for 999 seconds (16.65 minutes) – the maximum allowed by the PLC controller. Preliminary tests showed that this was sufficient for the system to reach an equilibrium state. (To check this, several tests were repeated with the smaller particles poured in first to form a layer at the bottom of the bed after which the larger particles were added. These repeats achieved the same results as when the particles were homogeneously mixed before being added to the jig chamber.)

The same jigging cycle was used in all tests. It was trapezoid in shape as shown in Figure 7. As can be seen from the figure, the frequency was 60 cycles per minute (1 Hz).

After each test, the rings were unclamped and the bed height was measured. Thereafter, the bed was split into horizontal segments by progressively removing one ring at a time and scraping off all the glass beads in that particular ring as indicated in Figure 8. The samples collected in this way were dried, screened, and the mass of each size component in the sample was determined. The plot of these compositions as a function of bed height gave the concentration profile of each component in the bed achieved in that test.

All tests were done in duplicate in order to see how similar the concentration profiles from replicate tests were. However, a repeat test was intentionally conducted with a different sequence of ring heights so that the points defining the concentration profile were slightly different from the original. This did not alter the jigging conditions, only the points at which the bed was split. The benefit of this approach is that not only could the desired comparison of the replicated profiles be made, but also, the concentration profile could now be better defined when the results from a test and its replicate were combined.

Two tests were not replicated. This was done in error, but is not considered critical because the reproducibility of the experimental procedure was well established by the tests that were replicated.

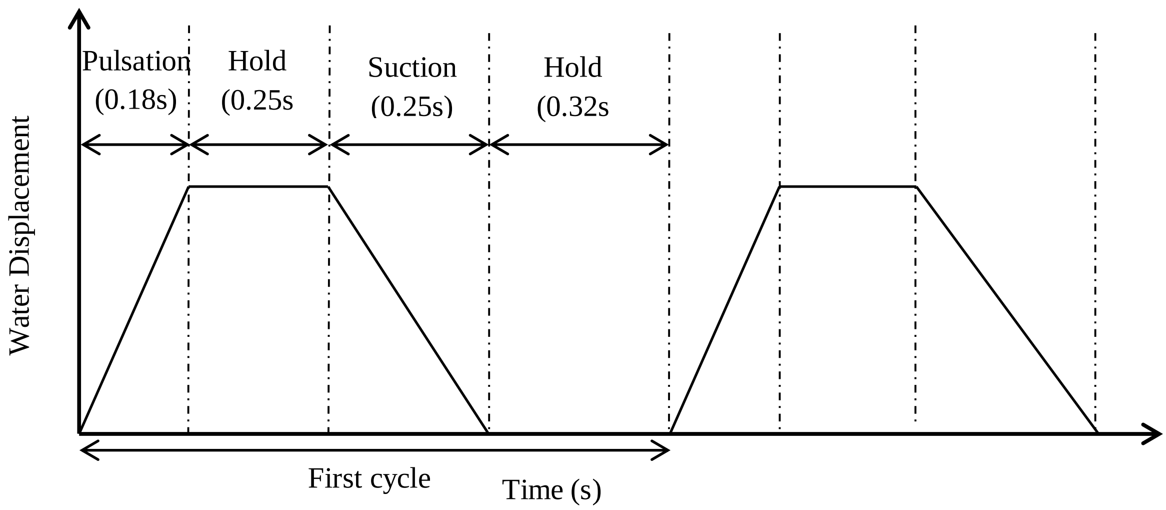
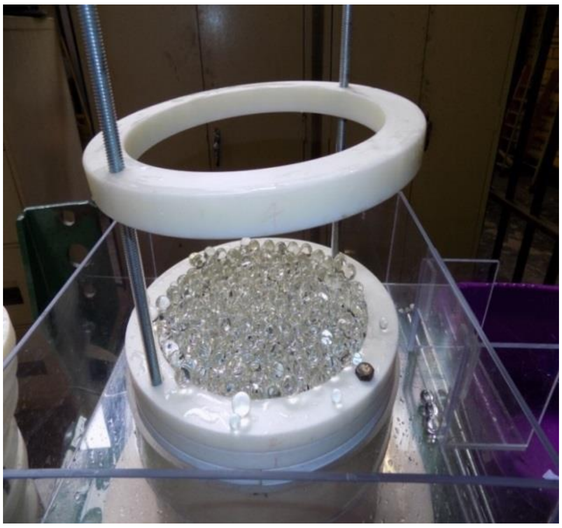
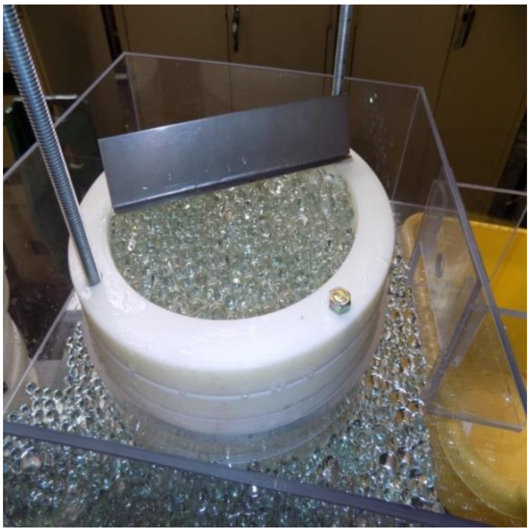


Figure 7: Jigging cycle used



(a)



(b)

Figure 8: Jig bed sampling. (a) Removing the ring. (b) Scraping off the glass beads

3.5 Applying the Woollacott settling rate model to binary systems

The procedure used to solve the Woollacott model (Equation 11) was the same as the procedure used to solve the King model (King, 2001, 1987; Tavares and King, 1995). After simplifying, the two equations used to fit the experimental data are :-

$$C_{tp}(h_i) = \frac{\frac{C_{tp}(0)}{C_{bm}(0)}\exp\left[-\alpha(l^b-\bar{l}^b)\right]}{1+\frac{C_{tp}(0)}{C_{bm}(0)}\exp\left[-\alpha(l^b-\bar{l}^b)\right]} \quad \ldots (12)$$

$$\frac{C_{tp}(0)}{C_{bm}(0)} = \frac{1-\exp\left[-\alpha(l^b-\bar{l}^b)C_{tp}^f\right]}{\exp\left[-\alpha\left(l^b-\bar{l}^b\right)C_{bm}^f\right]-1}\exp\left[-\alpha(l^b-\bar{l}^b)\right] \quad \ldots (13)$$

where $C_{tp}(h_i)$ is the volumetric concentration of top size particles in jig bed of layer, i, C_{tp}^f is the volumetric concentration of feed of top size, and C_{bm}^f is the volumetric concentration of feed of bottom size. Other symbols have the same meaning as defined in equation 11.

The model was simplified by assuming b =1. The solver function in Microsoft Excel was used to estimate the value of the stratification parameter, α, that gives a minimum value of the Sum of Square the Difference (SSD) between the experimental concentration and predicted volumetric concentration.

3.6 Experimental limitations

Although experimental tests were successfully carried out, there were some limitations that should be noted. These include:-

- Because the jig chamber was built by clamping rings, the boundaries between the rings created a slightly rough surface for the inside chamber. Therefore, the motions of particles near the wall – and hence the shape of the concentration profiles obtained – may have been affected slightly.
- Scouring of the bed of particles was observed when scrapping off particles coarser than 6mm. This was due to the fact that larger particles straddled the split plane – i.e. many of these particles were located with part lying in the segment being removed from the jig bed and part of it in the segment below it. Therefore, these particles were either pushed up or down by the scrapper during sampling which sometimes left a scour in the bed.
- Trickling of the smaller particles down into the jig bed was sometimes observed during the scrapping off of particles, especially in binary systems where the size range was large.

CHAPTER 4

Results

4.1 Introduction

In this chapter, experimental results are presented and discussed. The chapter is divided into three sections. Preliminary general information regarding the presentation of results is given in the first section. It includes four types of concentration profiles of particles that were obtained for different binary systems, measures of the degree of segregation, and data on the repeatability of the experimental results. In the second section, results are presented and discussed in an attempt to address the first research question, "how does particle size range affect segregation of particles in a batch jig under a particular set of jigging conditions?" The second research question, "how does particle size influence segregation of particles in a batch jig under those conditions?" is addressed in the third section of this chapter. In the last section, the Woollacott settling rate model is fitted to some of the experimental data and the fits are discussed.

4.2 Concentration profiles of binary systems and repeatability of experimental results

4.2.1 Types of concentration profiles

The method which was chosen to analyse and evaluate the degree of segregation of particles in a batch jig was to plot the concentration profiles obtained from the experimental data. Figure 9 shows four types of concentration profiles that were obtained. In each plot, the profile associated with a perfectly mixed bed and ideal segregation is represented by dotted vertical and horizontal lines at 50% concentration and 50% of the bed height, respectively. For differentiating sake, the four types of concentration profiles were arbitrary called type I, II, III and IV. These are shown in figure 9 and discussion thereafter.

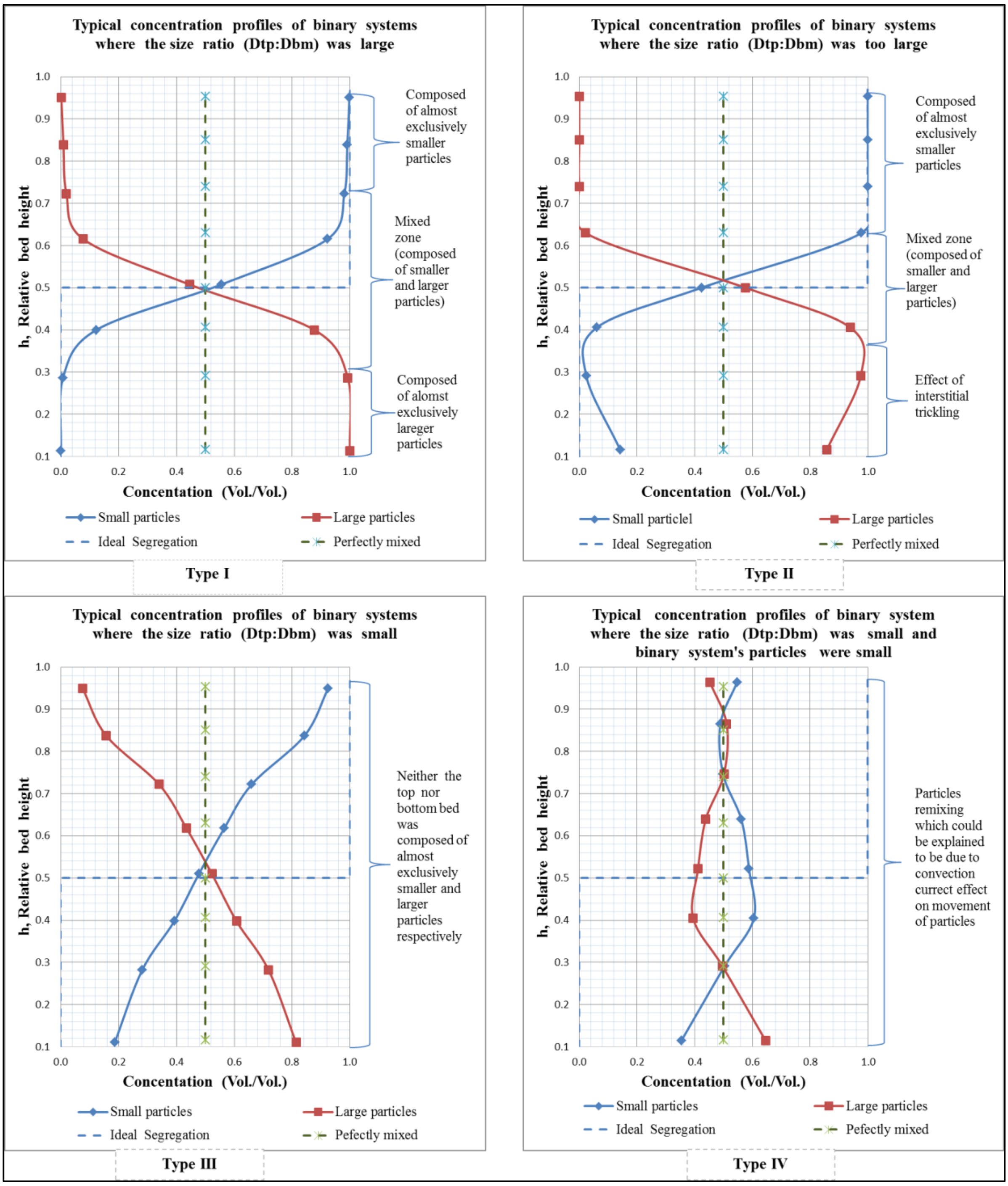


Figure 9: Four types of concentration profiles

(i) Type I: Normal

Type I of Figure 9 is what is intuitively expected if a bed segregates by size. The top part of the jig bed is composed almost exclusively of the smaller particles followed by a 'mixed zone' composed of larger and smaller particles in the middle of the bed. The bottom part is composed almost exclusively of the larger particles as shown in the figure. This type of concentration profile is typical of systems where the particle size ratio (D_{tp}:D_{bm}) was large but not so large that interstitial trickling becomes significant as seen in the next type.

(ii) Type II: Normal with trickling

Interstitial trickling occurs when larger particles interlock leaving interstices through which smaller particles can 'trickle'. This trickling can occur during jigging but can also occur when particles are scrapped from the bed during bed splitting process – in which case the trickling constitutes the experimental error. The result is that unexpected high concentrations of the smaller particles are found in the lower part of the bed as seen in Figure 9 type II. This type of the concentration profile occurs when the size ratio is so large that significant interstitial trickling occurs.

There is no quantitative description of interstitial trickling only the qualitative one that states that interstitial tricking is the function of particle size, particle size range and operating conditions (Burt, 1984; Wills, 2006).

(iii) Type III: Poor segregation

As can be seen in Figure 9, in a type III profile neither the top nor bottom part of the jig bed is composed of predominantly smaller or larger particles; significant proportions of both occur throughout the bed. Segregation does occur – in that the concentration of coarse particles is greater near the bottom of the bed and the concentration of smaller particles is greater near the top of the jig bed. This type of segregation occurred for systems where the size range was small

(iv) Type IV: Poor segregation with remixing

The Type IV concentration profile is unexpected. The segregation is poor in that the concentration profiles are much closer than the other types of profiles to what is expected for a perfectly mixed bed – i.e. a vertical profile where the concentration of both large and small particles is close to 50% everywhere. However, some segregation does occur in that the concentration of the larger particles is greater in the lower part of the bed and the concentration of the smaller particles is greater in the middle portion of the bed. However, it appears that some remixing is occurring in that the upper part of the bed has unexpected high concentrations of the larger particles and that part of the bed appears to be very close to being perfectly mixed. As discussed in Section 4.3, this type of profile occurred with systems that (1) had a small particle size ratio and (2) consisted only of relatively small particles, i.e. 8mm and smaller.

4.2.2 Measuring the degree of segregation

To be able to quantify the degree of segregation achieved in the various binary systems tested, it was necessary to develop appropriate measures of some kind. To do this, attention was focused on how much a concentration profile deviated from both ideal segregation and a perfectly mixed bed. Figure 10 shows the two methods developed which are explained thereafter.

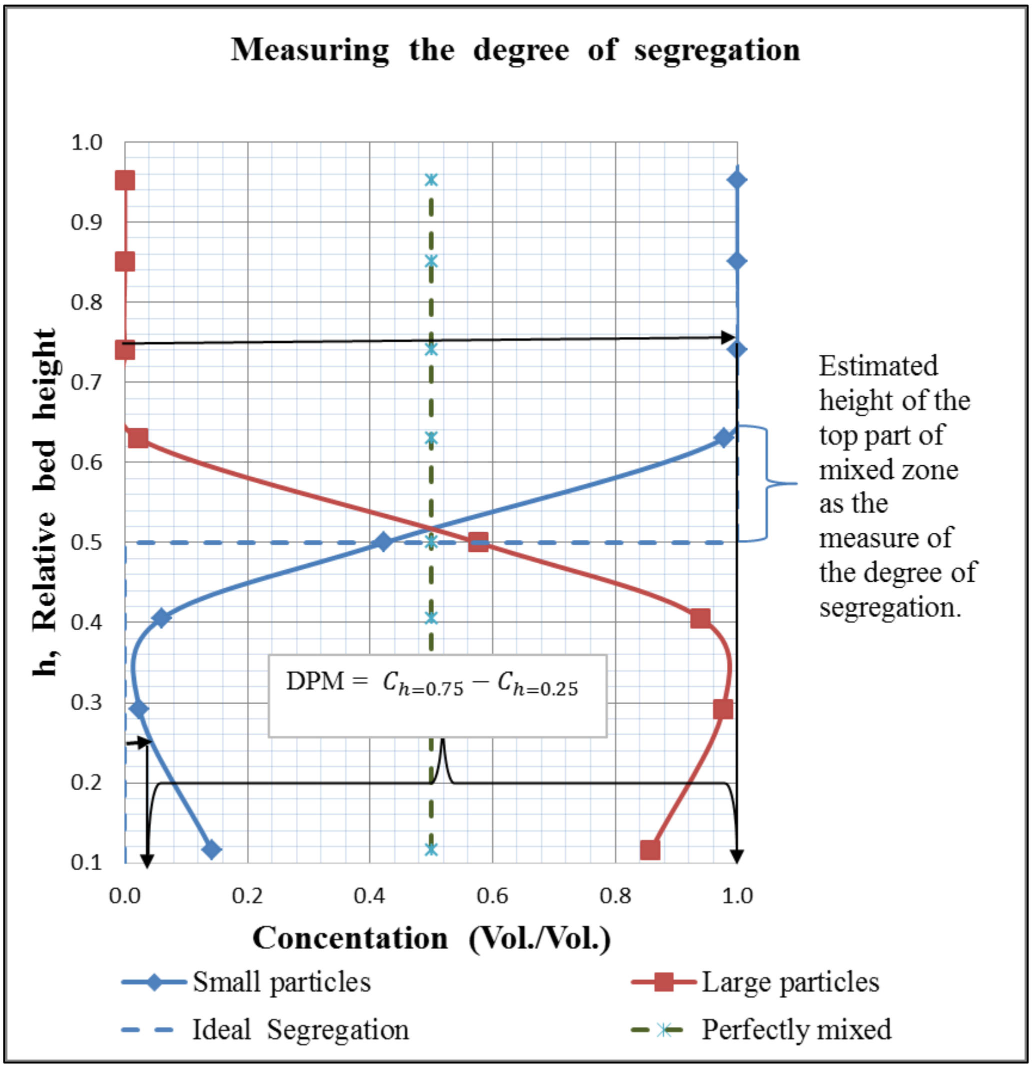


Figure 10: Two methods used to measure the degree of segregation

(i) The mixed zone bed height as a measure of the degree of segregation

The height of the mixed zone in the bed (Figures 9 and 10) was used as a measure of the degree of segregation. In other words, the sharper the degree of segregation, the smaller the mixed zone should be. Perfect segregation would be associated with no mixed zone. Therefore, an estimation of the mixed zone bed height could be used as an indication of the deviation of the concentration profile of a given system from an ideal segregation profile. The higher the mixed zone bed height obtained the poorer the degree of segregation for that particular binary system. As shown Figure 10, only the top part (above 50% of the bed height) was used to estimate the mixed zone bed height. The focus on only the top part was due to the fact that interstitial trickling was observed in some systems and this affected the concentration profiles in the bottom part of the bed obscuring where the bottom of the mixed zone was located.

(ii) Deviation from the perfectly mixed bed (DPM) as a measure of the degree of segregation

A second method – the 'deviation from a perfectly mixed' bed (DPM) – was developed to give an indication of the degree of segregation. In a perfectly mixed bed consisting of a 50-50% volumetric mix of larger and smaller particles, the concentration of all particles should be the same everywhere in the bed and the concentration profile would be a vertical line indicating a concentration of 50% at any level in the bed. As shown in Figure 10, this method involved finding the difference between the normalized concentration of either the larger or smaller particles between 75% and 25% of the bed height. A low degree of segregation would be associated with a DPM that was close to zero, i.e. a small DPM meant concentrations of both smaller and larger particles were close to that in a perfectly mixed bed. On the other hand, a high degree of segregation would be associated with a DPM that was close to one.

4.2.3 Reproducibility of experimental results

It can be observed in any of the plots in Figures 9 and 10 that the concentration profiles of smaller particles are mirror images of the concentration profiles of the larger particles. This is because the systems involve only two components. Therefore, from this point onwards only the concentration profiles of the smaller particles are plotted.

To establish how reproducible the test procedure was, replicate tests were done as indicated in Section 3.4. Figure 11 gives examples of these results. The full set of results is presented in Appendix B. From the figure, it can be seen that the concentration profiles obtained from replicate tests were very reproducible when segregation was very marked (Type I and II profiles) but less reproducible when segregation was less well developed (Type III and IV profiles). However, even in the latter case, the general trends of the concentration profiles were reproducible.

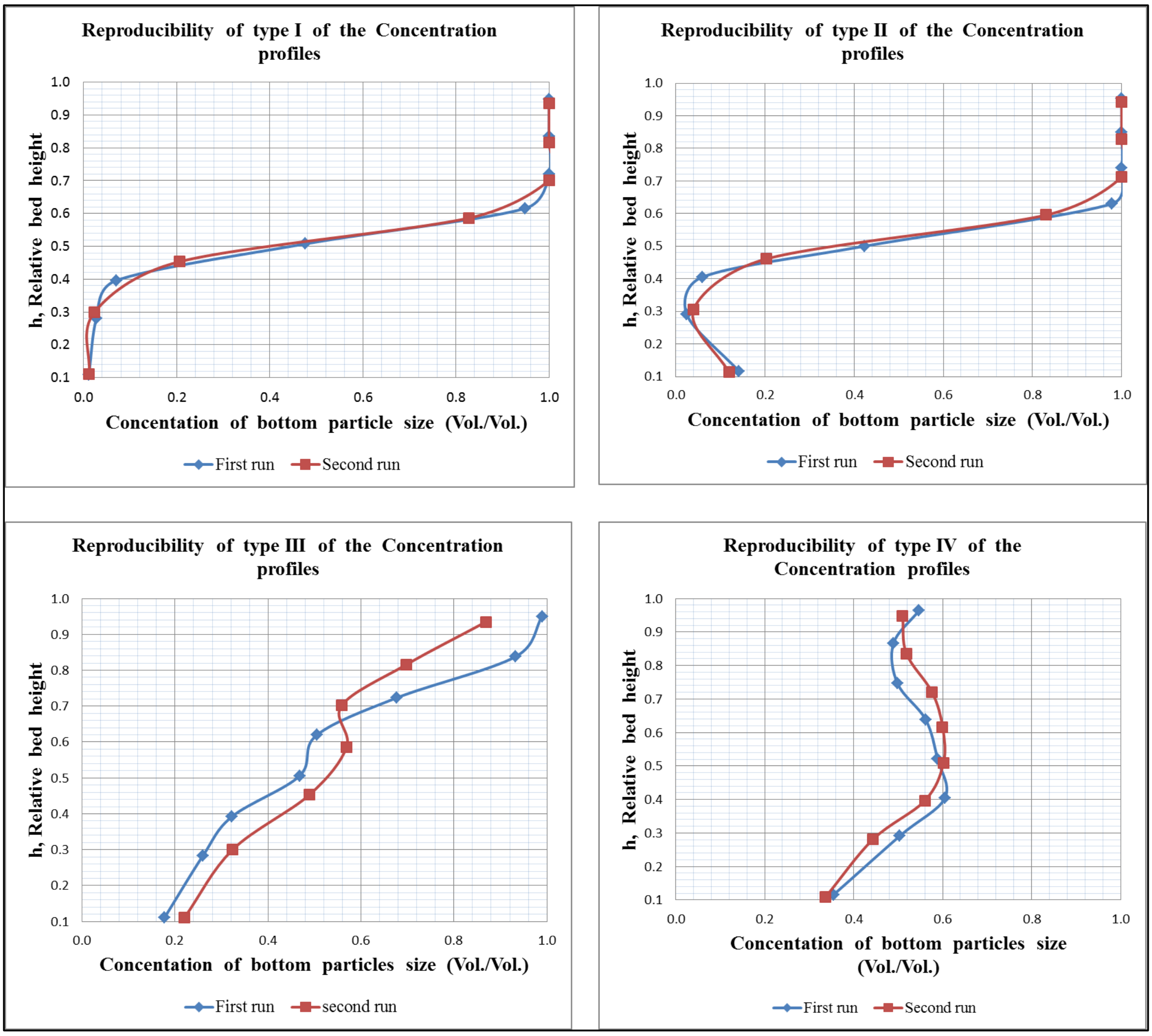


Figure 11: The repeatability of the experimental results

It appears that when the driving force for segregation is relatively low (leading to less well developed segregation), small differences from one test to another (such as in the alignment, sequencing and verticality of the rings) have a more significant impact on the segregation dynamics in the bed and the resulting concentration profile compared to when the driving force is much stronger (leading to higher degrees of segregation).

The research questions focus on the impact of size and size range on size segregation and so require a comparison of the concentration profiles obtained from systems with different feed compositions. Accordingly, it was concluded that the reproducibility of the test results was satisfactory: it was excellent where segregation was strong, and, where segregation was somewhat weaker, the reproducibility was adequate to indicate the general shape and trends of the concentration profiles.

4.3 The effect of the particles size range on segregation

In order to address the first research question, "How does particle size range affect stratification of particles in a batch jig?" the test results were presented in the following way. All the tests results were considered and the effect of the size range for each top particle size was analysed by varying the size ratio, i.e. plotting the concentration profiles of the available bottom particle size on the same graph. For example, in the case where 12 mm was the top size, concentration profiles of particle size 10 mm, 8 mm, 6 mm and 4 mm were plotted on the same graph. Table 2 shows the complete set of systems that were considered and the variation of size range (size ratio) for each top particle size. For each case, the mixed zone bed height and DPM were used to quantify the degree of segregation. The concentration profiles and the measures of the degree of segregation are presented in Figures 12 to 15 and are discussion thereafter.

Table 2: Variation of size range for combinations with the same top size

Figures	Top size, D_{tp} (mm)	Bottom size, D_{bm} (mm)	Ratio (D_{tp}:D_{bm})	Test run number
Figure 12	14	12	1.17 : 1	TR15
		10	1.40 : 1	TR11
		8	1.75 : 1	TR7
		6	2.33 : 1	TR4
		4	3.50 : 1	TR1
Figure 13	12	10	1.20 : 1	TR14
		8	1.50 : 1	TR9
		6	2.00 : 1	TR5
		4	3.00 : 1	TR2
Figure 14	10	8	1.25 : 1	TR13
		6	1.67 : 1	TR8
		4	2.50 : 1	TR3
Figure 15	8	6	1.33 : 1	TR12
		4	2.00 : 1	TR6

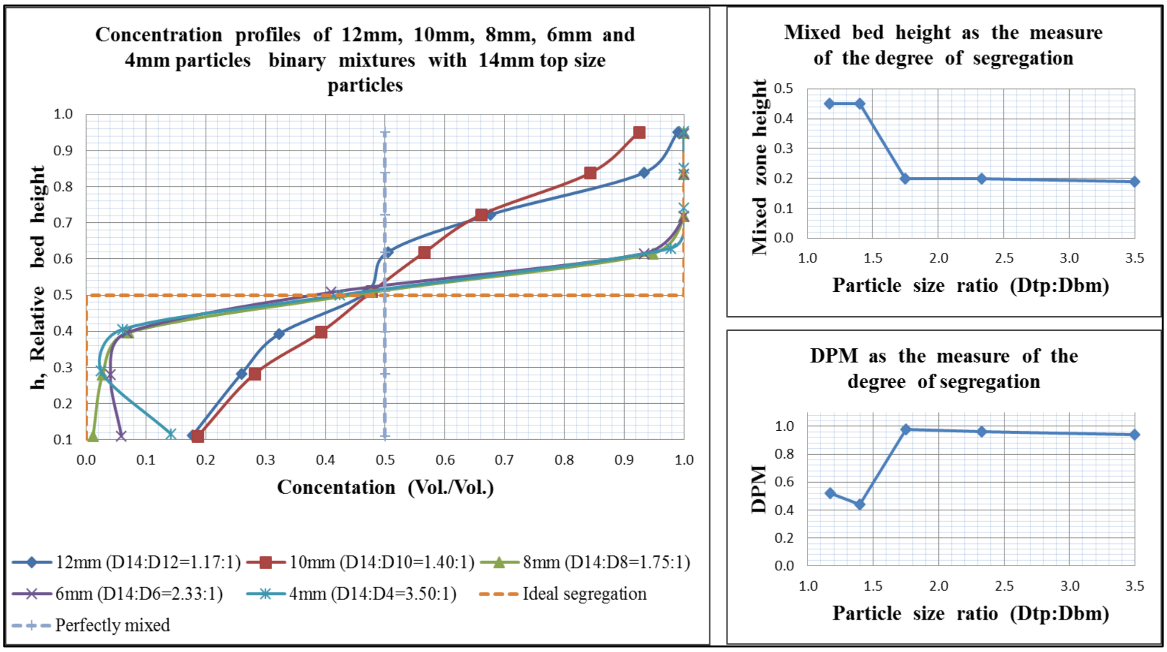


Figure 12: Concentration profiles of the small particles in the binary systems with 14mm top size particles

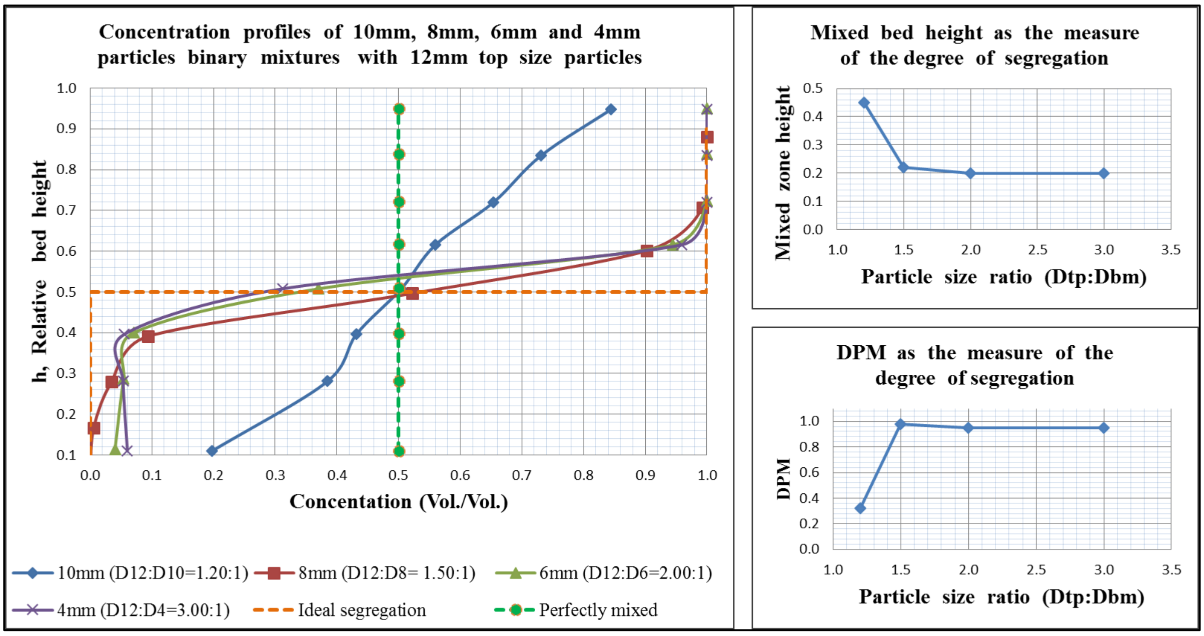


Figure 13: Concentration profiles of the smaller particles in the binary systems with 12mm top size particles

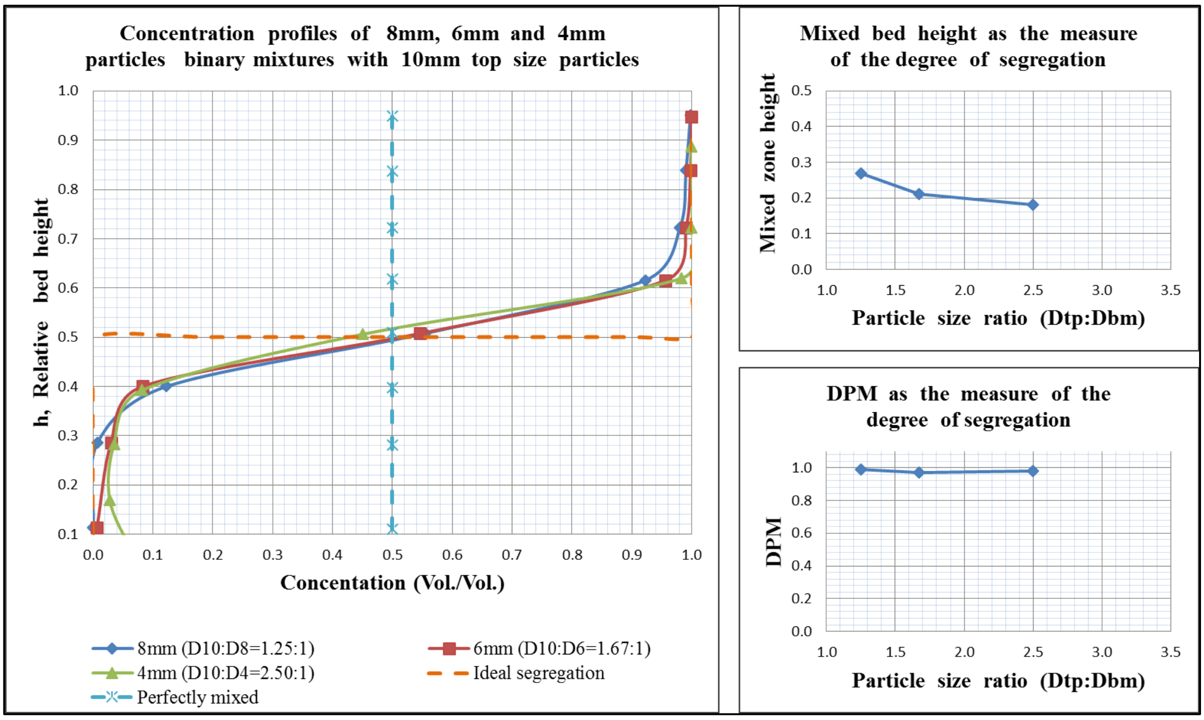


Figure 14: Concentration profiles of the smaller particles in the binary systems with 10mm top size particles

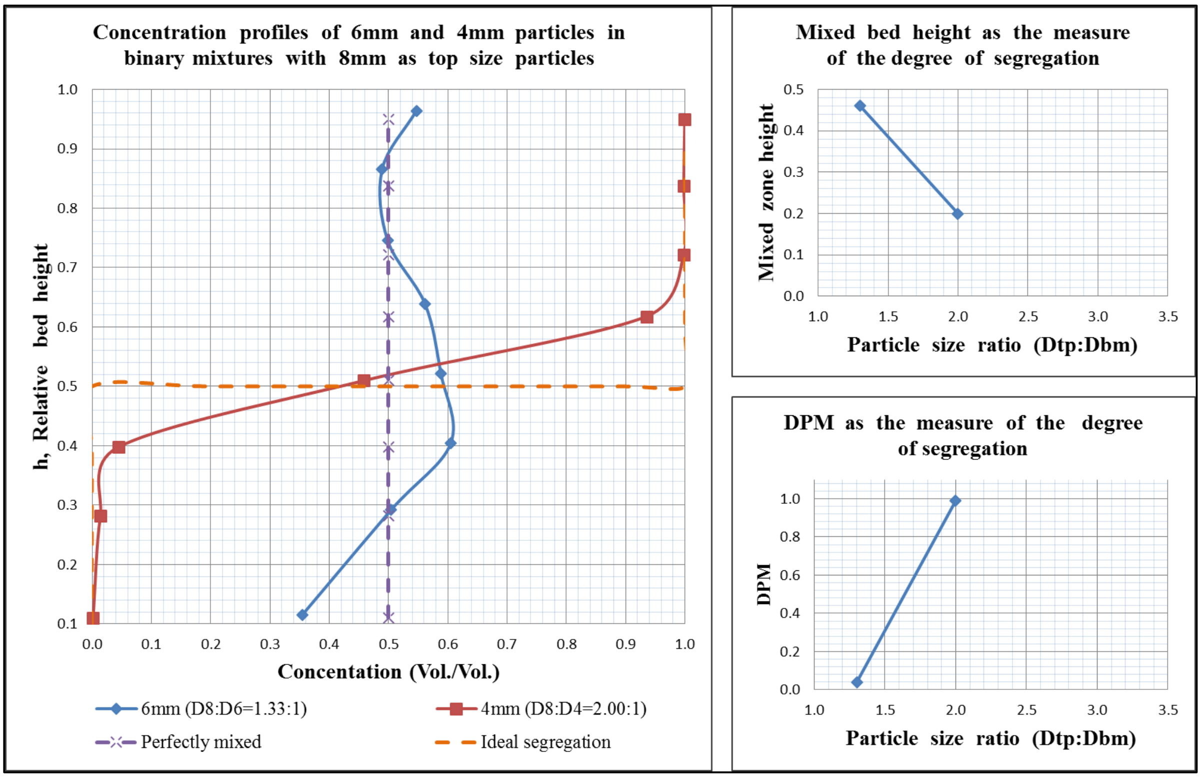


Figure 15: Concentration profiles of the smaller particles in the binary systems with 8mm top size particles

Figure 12 depicts the change of concentration profile from type III to type I to type II as the particle size ratio increases. It can be seen that poor segregation (type III) was obtained for the binary systems of size ratio 1.40:1 and below. Normal segregation (type I) was obtained for the systems with a size ratio from 1.40:1 up to 1.75:1. Above this size ratio (1.75:1), interstitial trickling occurred (type II). The methods used to quantify the degree of segregation shows that almost a constant degree of segregation (DPM = 0.94 ±0.02) was obtained for binary systems of size ratio above 1.75:1.

The segregation trends of Figure 13 are similar to that of Figure 12. Poor segregation (type III) was obtained for the binary system of size ratio 1.20:1. Normal segregation (type I) was obtained for system of size ratio 1.50:1. Interstitial trickling (Type II) occurred for binary systems of size ratio 2.00:1 and above. Almost a constant degree of segregation (DPM = 0.94±0.02) was achieved for binary systems of size ratio above 1.50:1

Figure 14 depicts a fairly good degree of segregation for all the three binary systems. Although the particle size ratio of binary system composed of 10mm and 8mm (size ratio 1.25:1) particles was smaller than 1.50:0, normal segregation (type I) was achieved. Actually, from Figure 14 it can be seen that the concentration profiles of all the three are almost lying on top of one other. If attention is paid to the measure of degree of segregation it can be seen a DPM of around 0.95±0.02 for all the three profiles. The amount of smaller particles trickled down for system of particle size ratio of 2.50:1 was relatively minimal.

In Figure 15, the degree of segregation was compared for two systems. Although only two data points are available for comparison, it can be seen the difference of the degree of segregation between these two binary systems was huge. A peculiar concentration profile (type IV) was obtained for the system with a particle size ratio of 1.33:1. The DPM for this system (≈ 0.05) is very small indicating that the concentration profile was relatively close to a perfectly mixed bed.

General observations can be made from Figures 12 to 15. The degree of segregation was poor for binary systems with particle size ratios below about 1.50:1. Poor segregation with remixing was obtained for the systems consisted of relatively small particle sizes (for binary systems of 8 mm particles and smaller) and size ratio below about 1.50:1. Normal segregation was obtained for the systems of size ratio about 1.50:1 to 2.00:1. Above the size ratio 2.00:1, interstitial trickling occurred.

The driving force for the segregation of particles for systems composed of particles of uniform density and shape is the difference in particle size. In other words, the difference between the top particles size and bottom particles size is the only driving force for segregation of particles in a system composed of particles of uniform density and shape. Therefore, poor segregation (type III) was expected in the systems of smaller size ratio. It can be said that at and above the size ratio of 1.5:1, the driving force was sufficiently high to cause very significant segregation of particles by size.

The poor segregation with remixing (type IV) obtained for the system composed of 8 mm and 6 mm was peculiar in the sense that the small size ratio (small driving force for the segregation) would suggest the profile should be more like a type III concentration profile. It is difficult to state with certainty what could have led to the remixing that was observed. However, the fact that this type of profile was not found in systems involving large particles (particles of diameter 10mm and larger) and small size ratios, would suggest that some kind of momentum effect was at play. It appeared that, in the fine binary systems of small size ratio, particles were more prone to being affected by momentum effects than in coarser binary systems of small size ratio. The explanation offered here is based on reports in the literature about the existence of a convective current within a jig bed during jigging (Knight et al., 1993; Roux and Naudé, 2014; Williams et al., 1998). Convective current movement of the particles is said to be set up when particles from the bottom of the jig bed move up the wall of jig chamber to the top of the bed and once they reach the top they move to the centre of the bed.

Interstitial trickling occurred as the size ratio (size range) of larger particle to smaller particle became too larger. The results show that interstitial trickling occurred at around and above the size ratio of 2.00:1. The size ratio at which trickling of smaller particles occurred was lower than the theoretical particle size ratio limit, 2.4:1, below which percolation of spherical fine particles is said to be minimal (Burt, 1984; Formisani et al., 2001). This could be attributed to the method used to split the bed as percolation of smaller particles was observed during scrapping off of particles. This was one of the experimental limitations and errors highlighted in section 3.5. Another reason could be that since no hutch water was added, the intensity of suction was significant.

A high degree of segregation by size is expected when the driving force for segregation is large, i.e. when the particle size ratio is large. Therefore, in the case where feed particles vary in both sizes and densities, the large size ratio could lead to particles misplacement due to the fact that two driving forces, i.e. density and size difference, taking place and influence the segregation of particles. Hence the particles do not only segregate by density only but by size also, which results in poorer recovery of either the heavy density materials or light materials. This would happen especially for materials where the concentration criterion is not very large.

This is in agreement with studies found in literature where concertation efficiency was found to decrease with increase in size range (Mutibura, 2015; Rao, 2007; Tavares, 1999; Myburgh and Nortje, 2014).

4.4 The Effect of particle size on segregation

To answer the second research question, "How does particle size influence segregation of particles in a batch jig?" the concentration profiles of binary systems of equal or very similar size ratio but of different sizes were compared. Table 3 shows the binary systems of approximately equal or very similar size ratios. The concentration profiles are plotted in Figure 16 and discussed thereafter.

Table 3: Binary systems with equal or similar size ratio but different sizes

Figures 16	Top size, D_{tp} (mm)	Bottom size, D_{bm} (mm)	Exact size ratio (D_{tp}:D_{bm})	Approx. ratio (D_{tp}:D_{bm})
(a)	14	12	1.17:1	≈1.2:1
	12	10	1.20:1	
(b)	14	10	1.40:1	≈1.3:1
	10	8	1.25:1	
	8	6	1.33:1	
(c)	12	8	1.50:1	≈1.5:1
	6	4	1.50:1	
(d)	14	8	1.75:1	≈1.7:1
	10	6	1.67:1	
(e)	12	6	2.00:1	≈2.0:1
	8	4	2.00:1	
	14	6	2.33:1	
(f)	14	4	3.50:1	≈3.0:1
	12	4	3.00:1	
	10	4	2.50:1	

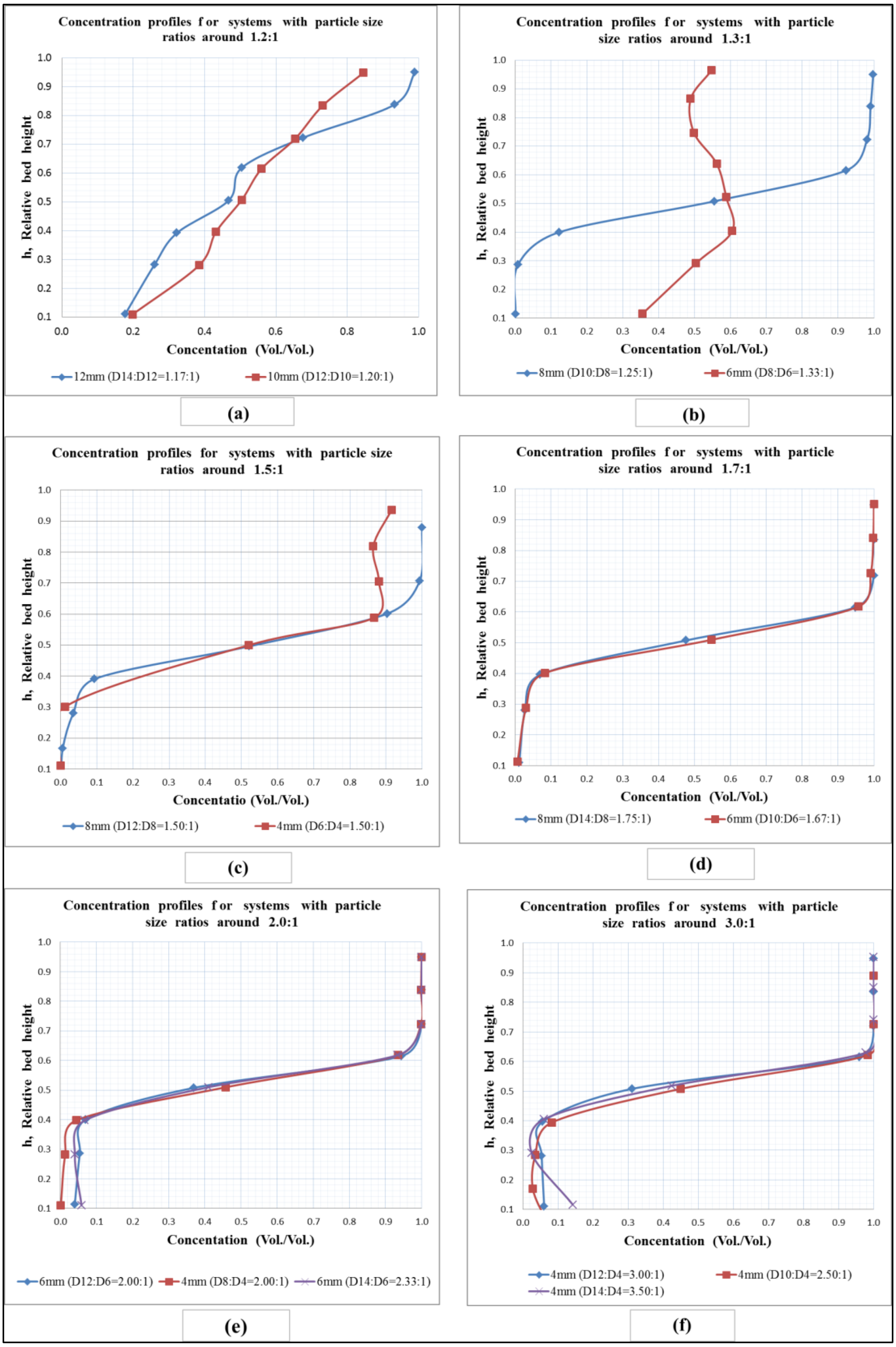


Figure 16: Concentration profiles for binary systems with similar size ratio, (a) 1.2:1, (b) 1.3:1, (c) 1.5:1, (d) 1.7:1, (e) 2.0:1 and (f) 3.0:1

Figure 16 (a) shows that the segregation of particles by size in coarse binary systems of narrow size range (1.2:1) was poor. If the concentration profiles of Figure 16 (a) are compared, then it can be seen that particles segregated by size to some degree in the coarser binary system (14mm and 12mm) and did so a little more extensively than in the finer binary system (12mm and 10mm). However, this effect is not certain because (1) the experimental errors associated with the two concentration profiles resulted in a relatively poor definition of the two concentration profiles associated with these profiles and (2) the actual size ratios of these two systems were not exactly the same; the former binary system (1.17:1) having a lower size ratio than the latter (1.20:1).

Figure 16 (b) and (c) shows the effect of 'remixing' of particles in the top half of the bed in the finer binary system. When the size ratio was 1.3:1 (Figure 16 (b)), remixing occurred in the system composed of particles sizes of 8mm and 6mm but not in system composed of particles sizes of 10mm and 8mm. When the size ratio was 1.5:1 (Figure 16 (c)), remixing occurred in system composed of particle sizes of 6mm and 4mm, but not in a systems composed of particles sizes of 12mm and 8mm.

When the size ratios were 1.7:1 or greater there was an insignificant difference of the degree of segregation irrespective of the size of the particles (Figures 16 (c) to (f)). The concentration profiles of these systems were almost lying on top of one another – i.e. they were experimentally indistinguishable. In other words, the segregation of particles by size between the size ratios lie 1.5:1 to 2.5:1 appears to be independent of the particles size in the binary systems. This is illustrated in Figure 17.

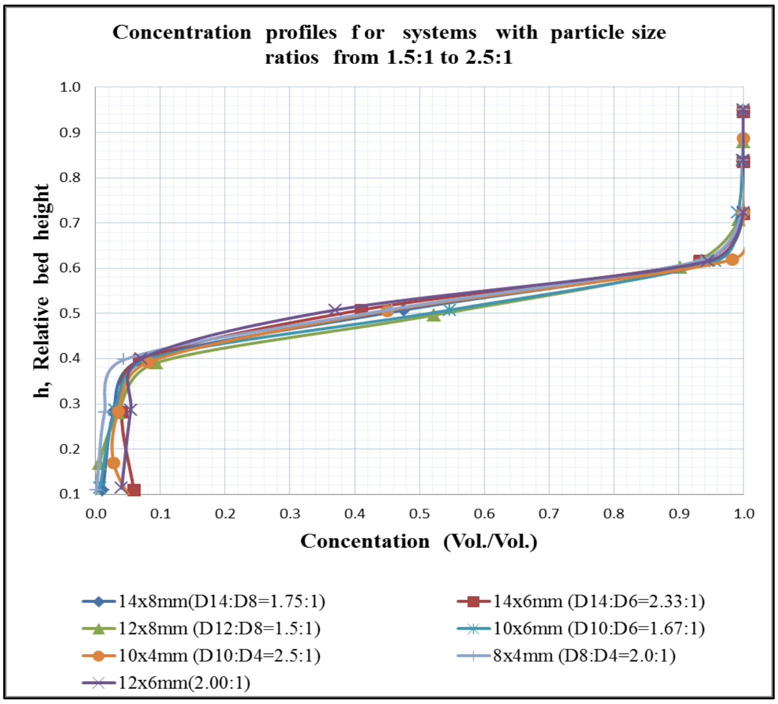


Figure 17: Concentration profiles of different binary systems with size ratios from 1.50:1 to 2.50:1

Therefore, it could be generally stated that with the coarser binary systems of narrow size range, particles segregated by size but segregation was poor. Similarly, poor segregation of particles was obtained with finer binary systems of narrow size range. With size ratios of 1.5: 1 and above, the degrees of segregation of particles were essentially the same for the coarser and finer particle systems. If the effects of the particle size and size range on segregation were to be compared for the sizes tested, it can be said that size range had a more pronounced influence on segregation than the size of the particles (Figure 17).

In industry where jigs are used to separate particles of variable densities and sizes, the above statement would mean that particle size range has more pronounced effects on particle misplacement than the particle size. However, it is important to note that the misplacement of particles does not only depend on particles size and size range but on density difference. Therefore, the general observation may not be valid in the case where the difference between the density of valuable particles and unwanted particles is large. For example, Mutibura (2015) compared the influence of size and size range on the performance of the jig on beneficiation of South African coal. His finding was that size had a greater influence on segregation of coal than the size range. However, the systems he studied were multi-component not binary systems. This suggests that the effects noted in this study may be masked when the particle system becomes more complex, i.e. multiple components systems and when density effects seem to be more dominant.

4.5 Fitting of the Woollacott settling rate model

The Woollacott settling rate model is only fitted for type I and III of the concentration profiles because the model does not take into consideration interstitial trickling or remixing of particles. The solver function in Microsoft Excel was used to find the value of the stratification parameter, α, that gives minimum Sum of Square Difference (SSD) between the experimental data and the model data. Table 4 shows the value of SSD and α for the binary systems considered. Figure 18 shows the model prediction and experimental concentration obtained in each layer. The fits are discussed thereafter.

Table 4: Comparison of fits of the binary systems considered

Concentration profile type	Top size, D_{tp} (mm)	Bottom size, D_{bm} (mm)	Size ratio (D_{tp}:D_{bm})	Alpha, α[1]	SSD	Fit comment
Type III	14	12	1.17:1	2.46	0.0452	Fair fit
Type III	14	10	1.40:1	1.0	0.00479	Good fit
Type I	14	8	1.75:1	3.864	0.0063	Good fit
Type III	12	10	1.20:1	1.568	0.0064	Good fit
Type I	12	8	1.50:1	5.259	0.0017	Excellent fit
Type I	10	8	1.25:1	10.259	0.0004	Excellent fit
Type I	10	6	1.67:1	5.984	0.0020	Excellent fit
Type I	8	4	2.00:1	6.311	0.0078	Good fit

[1] The bed height was almost constant in all the tests. Hence the value of alpha was not normalized.

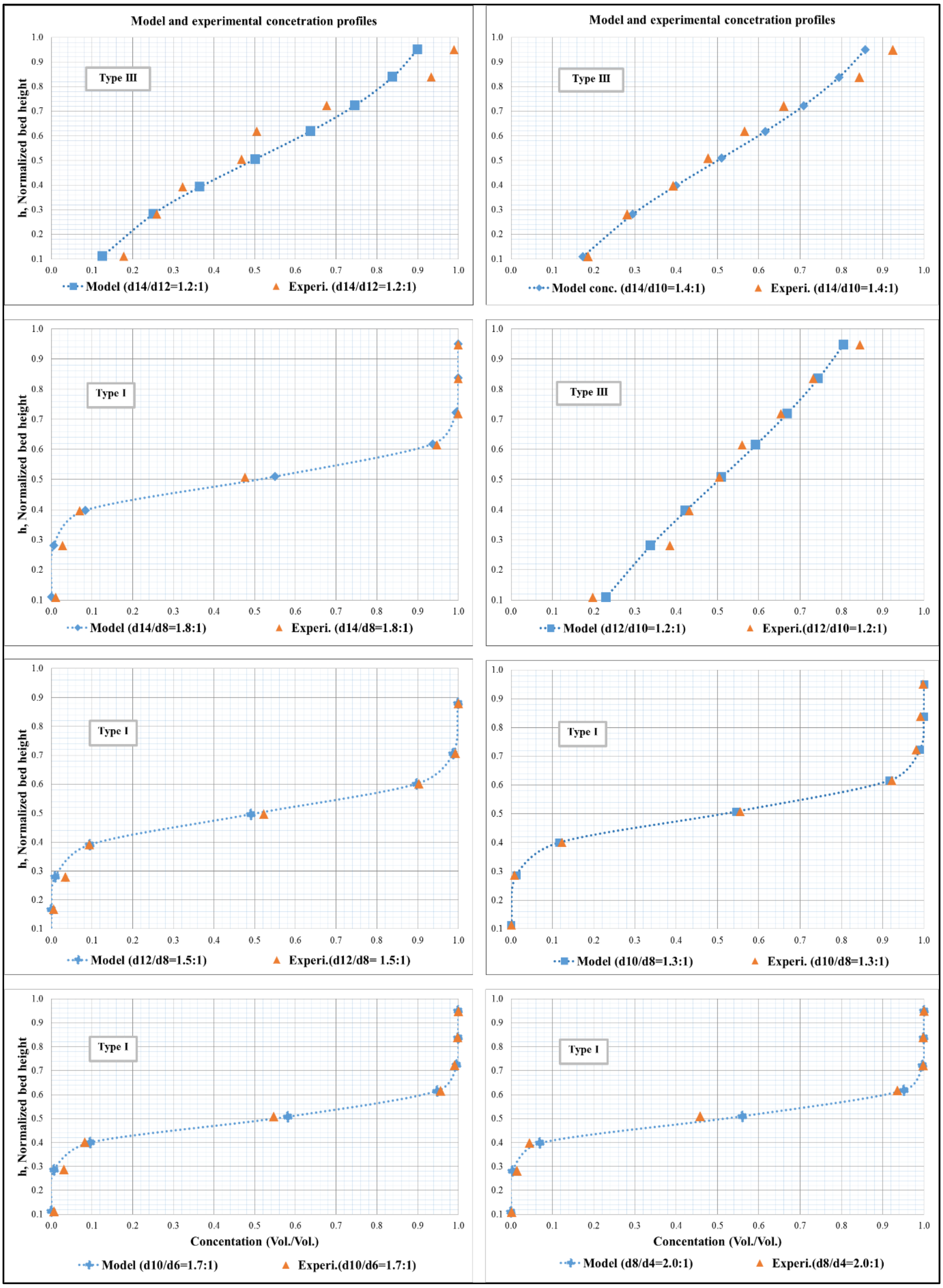


Figure 18: Fits for systems shown in Table 4: Concentration of smaller particles in each layer from bottom to top

It can be seen from Table 4 as well as Figure 18 that the Woollacott settling rate model fitted the experimental data very well for type I profiles and reasonably well for type II profiles. However, it can be seen from Table 4 that no relationship appears to exist between the stratification parameter, α and particle size and size range.

CHAPTER 5

Discussion and Conclusion

5.1 Discussion

The problem which this study addresses is the limited amount of knowledge about the nature of size segregation in jig beds. A body of general knowledge in this regard has been developed and has informed jigging technology for many years. As mentioned in chapters 1 and 2, this includes the existence of several different, size dependent, segregation mechanisms that may operate in jig beds; the rule of thumb that in order to minimise the detrimental effects of size segregation on density separations, the ratio of top to bottom size in the feed to a jig should be between 2:1 and 3:1; and the rule of thumb that interstitial trickling becomes more pronounced as the ratio of top to bottom size in the feed increases above about 2.4:1. With regard to a deeper theoretical understanding of size segregation in jig beds, however, the literature is thin and attempts to model size segregation have been less than satisfactory. Even from an experimental point of view, it does not appear that any study has been reported in the literature that shows the nature of the concentration gradients that result from size segregation either when size variation is the only variable or when both the size and density of the particles in the feed vary. From this perspective, the concentration gradients presented in this study constitute a significant contribution to knowledge.

The study presented in this research report was an experimental study to gain a deeper understanding of the nature of size segregation in jig beds. To this end, an experimental study was conducted that addressed the following two research questions:-

- How does particle size range affect segregation of particles in a batch jig under a particular set of jigging conditions?
- How does particle size influence segregation of particles in a batch jig under those conditions?

Tests were conducted using glass beads with diameters in the range from 14 to 4mm. In order to be able to make meaningful comparisons of the concentration profiles obtained from these tests, they were all conducted under the same jigging conditions – a 200mm diameter batch jig, a bed height of 130mm, a jigging frequency of 60 cycles per second, and a trapezoid pulsion pattern. Density as a variable was eliminated as a factor in the tests in that the beads were all made from soda lime glass with the same density, 2.52 kg/L. Shape too was eliminated as a factor in that all the beads where spherical. In addition, only binary systems of the beads were tested and the concentration of smaller to larger sized beads in the feed was set at 50-50% on a volumetric basis.

The following conclusions were drawn when the concentration profiles from these tests were analysed and compared.

1) *Types of concentration profiles*: Four distinctly different types of concentration profile were identified when the test results were considered as a whole. Normal size segregation (type I), segregation with trickling (type II), poor segregation (type III), and segregation with remixing (type IV). The regimes in which these different profiles occurred is discussed shortly. In all of these profiles, the expected general sequence of smaller particle migrating upwards and larger particles migrating downwards was evident.

2) *Size segregation mechanisms*: By considering the nature of the four types of concentration profiles, it was possible to say something about the stratification mechanisms behind each one. The most straight forward was one of the mechanisms associated with the type II profile, segregation with trickling, in that relatively high concentrations of smaller beads occurred in the bottom of the bed compared to normal segregation (type I); and that the concentration of the smaller beads in the lower part of the bed increased as the size ratio (top/bottom size) increased. The only segregation mechanism that could explain these two phenomena is that of interstitial trickling.

Remixing was postulated as occurring in the type IV profile, because in this case higher than expected concentrations of larger beads occurred in the top part of the bed, and, in some instances, exceeded the concentration of smaller beads. In other words, the expected sequence of smaller beads migrating upwards and larger beads downwards was to some extent reversed. The only mechanism that could explain this was that some remixing within the bed had occurred. To support this conclusion, literature was cited which reported the existence of convective currents in small batch jigs. These would impact smaller particles more than larger particles which is what was observed – the 'remixing' occurred only with smaller particles.

The mechanism postulated as giving rise to type I segregation – i.e. normal segregation with zones of small particles at the top of the bed and larger particles at the bottom with a mixed zone in between – was the equilibrium between stratification and dispersive forces in a jig bed. This mechanism, which I will term to be equilibrium mechanism, was first identified by King (1987), and has been very successful in explaining density segregation when all particles have the same size, and in predicting the concentration profiles in that context. Type I concentration profiles have the same general shape. The Woollacott size segregation model is based on the same principle as the King model except that the driving force for segregation is the difference in the settling rates of particles. It was found that this model fitted type I profiles very well indicating that the mechanism behind the model – the equilibrium between stratification and dispersive forces – could explain the shape of the concentration profile. In addition, it provided reasonable fits to type III profiles (poor segregation) which were associated with systems in which the size ratio in the feed (top/bottom size) was small – i.e. relatively close to 1. In this context the driving force for stratification would be expected to be small and the concentration profile not as markedly developed as in type I profiles. The Woollacott model therefore suggests a continuity between type 1 and type III profiles.

Taken all together, it appeared that three mechanisms were operative in the tests. The primary mechanism, the stratification/dispersion equilibrium, appeared to dominate. Firstly, it fitted both type I and type III profiles. Secondly, it appeared to underlie the other two mechanisms in that the general shape of the profile was similar to either type I or type III but was distorted because of trickling, in type II profiles, and by 'remixing' in Type IV profiles. To some extent these conclusions are speculative, and would need to be tested by a more thorough experimental programme. However, they do provide a coherent basis for understanding the nature of the size segregation observed in the tests.

3) *The impact of size range on segregation*:

By comparing the concentration profiles of systems which had the same top size but different bottom sizes in the jig feed, it was possible to establish a rough indication about where the nature of the profiles changed from one type to another, and, by implication, from one set of mechanisms to another, as the particle size range (top/bottom size) changed. The conclusions reached are as follows.

a) When the size ratio (top/bottom size) was small, the driving force for stratification is small and poor segregation occurs (type III or type IV). In this composition regime the equilibrium mechanism segregation is operative.
b) As the size ratio increases above about 1.5:1 the segregation becomes more marked and normal segregation (type I) occurs. In this composition regime the equilibrium mechanism segregation is operative, but now with a stronger stratification driving force.
c) As the ratio increases above about 2.0:1, interstitial trickling comes more into play distorting the normal stratification pattern (type I) so that smaller particles have higher than expected concentrations in the lower parts of the bed (type II profiles).

Although the trends described appear to be generally valid, they appear to also be affected by the actual size of the particles in the systems. This is discussed next.

4) *The impact of particle size on segregation*:

   By comparing the concentration profiles of systems which had the same or similar size ratios of large to small particles in the jig feed but different top sizes, it was possible to get an idea about the extent to which the actual size of the particles influenced the nature of segregation and influenced the trends described in (3). The conclusions reached are as follows.

   a) The onset of 'remixing': Type IV profiles (poor stratification with remixing) appears to occur when both the size range of the feed and the actual size of the particles is small. The limited data available made it difficult to estimate at which particle size this effect would begin to manifest. It appeared to be somewhere between 8 and 6 mm for the systems tested.

   b) The transitions between Type III, Type I and Type II profiles: As indicated in point (3), the general trend as the feed size ratio (top/bottom size) increases is from poor segregation when the size ratio is small, normal segregation at larger size ratios, to segregation with trickling at higher size ratios. However, the transitions from one type of profile to another appear to be size dependent to some degree, in particular the transition from poor to normal segregation (type II to type I). The discrete nature of the experimental data does not allow a very precise determination of how the transition shifts as the feed size decreases. What can be said from Figures 12 to 16, however, is that with 14 mm and 12mm top sizes the transition appears to be in the region from 1.2:1 to 1:5:1, whereas with a 10mm top size it occurs at lower feed size ratios – i.e. below 1.3:1. The transition to segregation with trickling (from type I to type II profiles) appears to occur at about the same size ratio, i.e. above 2:1, for feed top sizes greater than 8mm. The data is insufficient to indicate where this transition might be for top sizes of 8mm or less.

5) *Additional conclusions*:

Two additional observations can be made from the experimental data. The first relates to the onset of interstitial trickling. As has been stated this occurs when the feed size ratios increase above about 2:1, with no observable trickling at lower size ratios. However, this applies to binary systems. For ternary or more complex systems, it might well be the case that it may occur at higher feed size ratios because the average size of the interstices between particles would be expected to decrease as the systems become more complex and involve particles with sizes intermediate between the largest and smallest particles.

A more significant observation is that the type I profile (i.e. normal, well established segregation) appears to be independent of size and size range. This can be seen in Figure 17 where profiles for systems of different feed top sizes and different feed size ratios essentially are experimentally indistinguishable.

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5.2 Recommendation and further work

Quite clearly, the experimental conditions used in the tests presented in this report are artificial, and served only to provide a window on the dynamics that affect size segregation in jig beds. It is well known that in order to achieve optimum jigging performance, the jigging conditions need to change as the feed size decreases. This was beyond the scope of the project and so was not investigated. In addition, the feed size distributions in real jigging applications are invariably not binary so this work could be expanded by investigating how the picture of size segregation provided by this study might change for ternary or more multi-component systems.

Much the same can be said with regard to the shape of the feed particles and the influence of particle shape on size segregation in jig beds.

A further point to make is that the smallest particle size tested in this study was 4mm. The susceptibility of particles of this size and smaller to momentum effects such as convective remixing and, more generally, to a deterioration in the degree of segregation and jigging performance is well known. The study gave a glimpse of unusual effects at small feed sizes but these occurred at the edge of the compositional range tested. Because of the growing interest in fine jigging, it would be helpful to extend this study into that composition region.

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APPENDICES

Appendix A: Experimental Data

Volume of particles in each layer of the bed was calculated using the formula below

$$Voulme = \frac{mass}{density} \qquad \ldots \text{A1}$$

From the tables below, Ht is the height of each layer, Hc is the height from the bottom to the centre of each layer, and h is the normalized height of Hc.

Table 5: Test data for binary system of particle size 14mm and 12mm

Test Run Data TR15				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		14mm	12mm	mm	mm	mm	mm/mm	14mm	12mm	14mm +12mm	14mm	12mm
1	29.00	946.70	204.40	29.00	0.00	14.50	0.11	375.67	81.11	456.79	0.82	0.18
2	44.50	620.50	217.60	15.50	29.00	36.75	0.28	246.23	86.35	332.58	0.74	0.26
3	57.50	463.10	220.30	13.00	44.50	51.00	0.39	183.77	87.42	271.19	0.68	0.32
4	74.00	336.50	295.80	16.50	57.50	65.75	0.51	133.53	117.38	250.91	0.53	0.47
5	87.00	340.30	346.70	13.00	74.00	80.50	0.62	135.04	137.58	272.62	0.50	0.50
6	101.00	211.90	443.00	14.00	87.00	94.00	0.72	84.09	175.79	259.88	0.32	0.68
7	117.00	47.40	660.70	16.00	101.00	109.00	0.84	18.81	262.18	280.99	0.07	0.93
8	130.00	6.80	643.10	13.00	117.00	123.50	0.95	2.70	255.20	257.90	0.01	0.99

Table 6: Repeat test data for binary system of particle size 14mm and 12mm

Test Run Data TR15				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		14mm	12mm	mm	mm	mm	mm/mm	14mm	12mm	14mm +12mm	14mm	12mm
1	29.00	911.60	257.80	29.00	0.00	14.50	0.11	361.75	102.30	464.05	0.78	0.22
2	49.00	647.20	309.50	20.00	29.00	39.00	0.30	256.83	122.82	379.64	0.68	0.32
3	69.00	517.80	496.90	20.00	49.00	59.00	0.45	205.48	197.18	402.66	0.51	0.49
4	83.50	283.20	374.00	14.50	69.00	76.25	0.59	112.38	148.41	260.79	0.43	0.57
5	99.00	322.00	407.30	15.50	83.50	91.25	0.70	127.78	161.63	289.40	0.44	0.56
6	113.00	202.30	465.20	14.00	99.00	106.00	0.82	80.28	184.60	264.88	0.30	0.70
7	130.00	109.10	720.30	17.00	113.00	121.50	0.93	43.29	285.83	329.13	0.13	0.87

Table 7: Test data for binary system of particle size 14mm and 10mm

Test Run Data TR11				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		14mm	10mm	mm	mm	mm	mm/mm	14mm	10 mm	14mm + 10mm	14mm	10mm
1.00	28.5	991.60	227.20	28.50	0.00	14.25	0.11	393.49	90.16	483.65	0.81	0.19
2.00	44.5	579.40	226.90	16.00	28.50	36.50	0.28	229.92	90.04	319.96	0.72	0.28
3.00	58.5	400.10	258.40	14.00	44.50	51.50	0.40	158.77	102.54	261.31	0.61	0.39
4.00	73.5	336.50	307.30	15.00	58.50	66.00	0.51	133.53	121.94	255.48	0.52	0.48
5.00	86.5	294.20	382.20	13.00	73.50	80.00	0.62	116.75	151.67	268.41	0.43	0.57
6.00	100.5	230.10	447.10	14.00	86.50	93.50	0.72	91.31	177.42	268.73	0.34	0.66
7.00	116.5	120.40	647.50	16.00	100.50	108.50	0.84	47.78	256.94	304.72	0.16	0.84
8.00	129.5	41.00	502.60	13.00	116.50	123.00	0.95	16.27	199.44	215.71	0.08	0.92

Table 8: Test data for binary system of particle size 14mm and 8mm

Test Run Data TR7				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		14mm	8mm	mm	mm	mm	mm/mm	14mm	8mm	14mm + 8mm	14mm	8mm
1.00	28.5	1173.10	13.30	28.50	0.00	14.25	0.11	465.52	5.28	470.79	0.99	0.01
2.00	44.5	847.70	24.00	16.00	28.50	36.50	0.28	336.39	9.52	345.91	0.97	0.03
3.00	58.5	574.20	43.20	14.00	44.50	51.50	0.40	227.86	17.14	245.00	0.93	0.07
4.00	73.5	364.70	331.70	15.00	58.50	66.00	0.51	144.72	131.63	276.35	0.52	0.48
5.00	86.5	33.70	613.80	13.00	73.50	80.00	0.62	13.37	243.57	256.94	0.05	0.95
6.00	100.5	0.00	653.10	14.00	86.50	93.50	0.72	0.00	259.17	259.17	0.00	1.00
7.00	116.5	0.00	742.00	16.00	100.50	108.50	0.83	0.00	294.44	294.44	0.00	1.00
8.00	130.0	0.00	588.90	13.50	116.50	123.25	0.95	0.00	233.69	233.69	0.00	1.00

Table 9: Repeat test data for binary system of particle size 14mm and 8mm

Test Run Data TR7				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		14mm	8mm	mm	mm	mm	mm/mm	14mm	8mm	14mm + 8mm	14mm	8mm
1.00	29.0	1106.60	13.20	29.00	0.00	14.50	0.11	439.13	5.24	444.37	0.99	0.01
2.00	49.0	1011.00	24.30	20.00	29.00	39.00	0.30	401.19	9.64	410.83	0.98	0.02
3.00	69.0	760.10	198.30	20.00	49.00	59.00	0.45	301.63	78.69	380.32	0.79	0.21
4.00	83.5	115.70	558.20	14.50	69.00	76.25	0.59	45.91	221.51	267.42	0.17	0.83
5.00	99.0	0.00	715.60	15.50	83.50	91.25	0.70	0.00	283.97	283.97	0.00	1.00
6.00	113.0	0.00	667.60	14.00	99.00	106.00	0.82	0.00	264.92	264.92	0.00	1.00
7.00	130.0	0.00	831.70	17.00	113.00	121.50	0.93	0.00	330.04	330.04	0.00	1.00

Table 10: Test data for binary system of particle size 14mm and 6mm

Test Run Data TR4				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		14mm	6 mm	mm	mm	mm	mm/mm	14mm	6 mm	14mm + 6mm	14mm	6mm
1.00	28.5	1110.40	69.60	28.50	0.00	14.25	0.11	440.63	27.62	468.25	0.94	0.06
2.00	44.5	792.50	33.50	16.00	28.50	36.50	0.28	314.48	13.29	327.78	0.96	0.04
3.00	58.5	646.80	47.30	14.00	44.50	51.50	0.40	256.67	18.77	275.44	0.93	0.07
4.00	73.5	399.40	277.20	15.00	58.50	66.00	0.51	158.49	110.00	268.49	0.59	0.41
5.00	86.5	44.10	613.50	13.00	73.50	80.00	0.62	17.50	243.45	260.95	0.07	0.93
6.00	100.5	0.00	645.10	14.00	86.50	93.50	0.72	0.00	255.99	255.99	0.00	1.00
7.00	116.5	0.00	733.10	16.00	100.50	108.50	0.83	0.00	290.91	290.91	0.00	1.00
8.00	129.5	0.00	593.10	13.00	116.50	123.00	0.946	0.00	235.36	235.36	0.00	1.00

Table 11: Repeat test data for binary system of particle size 14mm and 6mm

Test Run Data TR4				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		14mm	6 mm	mm	mm	mm	mm/mm	14mm	6 mm	14mm + 6mm	14mm	6mm
1.00	29.0	1099.00	38.80	29.00	0.00	14.50	0.11	436.11	15.40	451.51	0.97	0.03
2.00	49.0	1056.00	52.20	20.00	29.00	39.00	0.30	419.05	20.71	439.76	0.95	0.05
3.00	69.0	756.50	197.80	20.00	49.00	59.00	0.46	300.20	78.49	378.69	0.79	0.21
4.00	83.5	81.70	607.30	14.50	69.00	76.25	0.60	32.42	240.99	273.41	0.12	0.88
5.00	99.0	0.00	749.00	15.50	83.50	91.25	0.71	0.00	297.22	297.22	0.00	1.00
6.00	113.0	0.00	652.70	14.00	99.00	106.00	0.83	0.00	259.01	259.01	0.00	1.00
7.00	128.0	0.00	714.50	15.00	113.00	120.50	0.94	0.00	283.53	283.53	0.00	1.00

Table 12: Test data for binary system of particle size 14mm and 4mm

Test Run Data TR1				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		14mm	4 mm	mm	mm	mm	mm / mm	14mm	4 mm	14mm + 4mm	14mm	4mm
1.00	29.5	1174.30	194.00	29.50	0.00	14.75	0.12	465.99	76.98	542.98	0.86	0.14
2.00	44.5	672.80	16.80	15.00	29.50	37.00	0.29	266.98	6.67	273.65	0.98	0.02
3.00	58.5	763.10	49.10	14.00	44.50	51.50	0.41	302.82	19.48	322.30	0.94	0.06
4.00	73.0	369.20	271.60	14.50	58.50	65.75	0.50	146.51	107.78	254.29	0.58	0.42
5.00	87.0	13.80	605.90	14.00	73.00	80.00	0.63	5.48	240.44	245.91	0.02	0.98
6.00	101.0	0.00	646.20	14.00	87.00	94.00	0.74	0.00	256.43	256.43	0.00	1.00
7.00	115.0	0.00	719.70	14.00	101.00	108.00	0.85	0.00	285.60	285.60	0.00	1.00
8.00	127.0	0.00	503.20	12.00	115.00	121.00	0.95	0.00	199.68	199.68	0.00	1.00

Table 13: Repeat test data for binary system of particle size 14mm and 4mm

Test Run Data TR1				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		14mm	4 mm	mm	mm	mm	mm / mm	14mm	4 mm	14mm + 4mm	14mm	4mm
1.00	29.0	1097.20	148.80	29.00	0.00	14.50	0.11	435.40	59.05	494.44	0.88	0.12
2.00	49.0	1029.70	42.80	20.00	29.00	39.00	0.30	408.61	16.98	425.60	0.96	0.04
3.00	69.0	750.60	191.50	20.00	49.00	59.00	0.46	297.86	75.99	373.85	0.80	0.20
4.00	83.5	116.00	570.10	14.50	69.00	76.25	0.60	46.03	226.23	272.26	0.17	0.83
5.00	99.0	0.00	721.80	15.50	83.50	91.25	0.71	0.00	286.43	286.43	0.00	1.00
6.00	113.0	0.00	665.20	14.00	99.00	106.00	0.83	0.00	263.97	263.97	0.00	1.00
7.00	128.0	0.00	666.00	15.00	113.00	120.50	0.94	0.00	264.29	264.29	0.00	1.00

Table 14: Test data for binary system of particle size 12mm and 10mm

Test Run Data TR14				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		12 mm	10mm	mm	mm	mm	mm / mm	12 mm	10mm	12mm +10mm	12mm	10mm
1	28.50	1090.00	268.70	28.50	0.00	14.25	0.11	432.54	106.63	539.17	0.80	0.20
2	44.50	427.50	267.50	16.00	28.50	36.50	0.28	169.64	106.15	275.79	0.62	0.38
3	58.50	386.80	293.80	14.00	44.50	51.50	0.40	153.49	116.59	270.08	0.57	0.43
4	73.50	324.20	329.90	15.00	58.50	66.00	0.51	128.65	130.91	259.56	0.50	0.50
5	86.50	270.60	344.10	13.00	73.50	80.00	0.62	107.38	136.55	243.93	0.44	0.56
6	100.50	250.00	471.50	14.00	86.50	93.50	0.72	99.21	187.10	286.31	0.35	0.65
7	116.50	189.70	516.20	16.00	100.50	108.50	0.83	75.28	204.84	280.12	0.27	0.73
8	130.00	93.20	507.40	13.50	116.50	123.25	0.95	36.98	201.35	238.33	0.16	0.84

Table 15: Repeat test data for binary system of particle size 12mm and 10mm

Test Run Data TR14				Calculation for h								
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		12 mm	10mm	mm	mm	mm	mm / mm	12 mm	10mm	12mm +10mm	12mm	10mm
1	29.00	1001.40	289.70	29.00	0.00	14.50	0.11	397.38	114.96	512.34	0.78	0.22
2	49.00	611.60	299.50	20.00	29.00	39.00	0.30	242.70	118.85	361.55	0.67	0.33
3	69.00	524.80	412.40	20.00	49.00	59.00	0.46	208.25	163.65	371.90	0.56	0.44
4	83.50	328.80	329.50	14.50	69.00	76.25	0.59	130.48	130.75	261.23	0.50	0.50
5	99.00	276.50	472.60	15.50	83.50	91.25	0.71	109.72	187.54	297.26	0.37	0.63
6	113.00	169.70	452.30	14.00	99.00	106.00	0.82	67.34	179.48	246.83	0.27	0.73
7	129.00	118.10	741.60	16.00	113.00	121.00	0.94	46.87	294.29	341.15	0.14	0.86

Table 16: Test data for binary system of particle size 12mm and 8mm

Test Run Data TR9				Calculation for h									
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		12 mm	8 mm	mm	mm	mm	mm / mm	12 mm	8 mm	12mm + 8 mm	12mm	8mm
1.00	15.0	502.80	0.00	15.00	0.00	7.50	0.06	199.52	0.00	199.52	1.00	0.00
2.00	29.5	893.70	5.20	14.50	15.00	22.25	0.17	354.64	2.06	356.71	0.99	0.01
3.00	45.0	673.00	24.40	15.50	29.50	37.25	0.28	267.06	9.68	276.75	0.97	0.03
4.00	59.0	597.70	61.90	14.00	45.00	52.00	0.39	237.18	24.56	261.75	0.91	0.09
5.00	73.0	294.80	322.00	14.00	59.00	66.00	0.50	116.98	127.78	244.76	0.48	0.52
6.00	87.0	64.60	600.40	14.00	73.00	80.00	0.60	25.63	238.25	263.89	0.10	0.90
7.00	101.0	4.50	608.80	14.00	87.00	94.00	0.71	1.79	241.59	243.37	0.01	0.99
8.00	133.0	0.00	1388.10	32.00	101.00	117.00	0.88	0.00	550.83	550.83	0.00	1.00

Table 17: Repeat test data for binary system of particle size 12mm and 8mm

Test Run Data TR9				Calculation for h									
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		12 mm	8 mm	mm	mm	mm	mm / mm	12 mm	8 mm	12mm + 8 mm	12mm	8mm
1.00	29.0	1317.50	3.00	29.00	0.00	14.50	0.11	522.82	1.19	524.01	1.00	0.00
2.00	49.0	889.60	19.80	20.00	29.00	39.00	0.30	353.02	7.86	360.87	0.98	0.02
3.00	69.0	694.40	259.30	20.00	49.00	59.00	0.45	275.56	102.90	378.45	0.73	0.27
4.00	83.5	123.10	493.50	14.50	69.00	76.25	0.59	48.85	195.83	244.68	0.20	0.80
5.00	99.0	6.60	700.80	15.50	83.50	91.25	0.70	2.62	278.10	280.71	0.01	0.99
6.00	113.0	0.00	640.30	14.00	99.00	106.00	0.82	0.00	254.09	254.09	0.00	1.00
7.00	130.0	0.00	891.90	17.00	113.00	121.50	0.93	0.00	353.93	353.93	0.00	1.00

Table 18: Test data for binary system of particle size 12mm and 6mm

Test Run Data TR5				Calculation for h									
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height	Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
		12 mm	6 mm	mm	mm	mm	mm / mm	12 mm	6 mm	12mm + 6mm	12mm	6mm
1.00	29.5	1287.20	54.00	29.50	0.00	14.75	0.11	510.79	21.43	532.22	0.96	0.04
2.00	45.0	652.30	37.50	15.50	29.50	37.25	0.29	258.85	14.88	273.73	0.95	0.05
3.00	59.0	670.20	51.10	14.00	45.00	52.00	0.40	265.95	20.28	286.23	0.93	0.07
4.00	73.0	385.90	226.90	14.00	59.00	66.00	0.51	153.13	90.04	243.17	0.63	0.37
5.00	87.0	35.60	606.60	14.00	73.00	80.00	0.62	14.13	240.71	254.84	0.06	0.94
6.00	101.0	0.00	647.30	14.00	87.00	94.00	0.72	0.00	256.87	256.87	0.00	1.00
7.00	117.0	0.00	735.70	16.00	101.00	109.00	0.84	0.00	291.94	291.94	0.00	1.00
8.00	130.0	0.00	653.40	13.00	117.00	123.50	0.95	0.00	259.29	259.29	0.00	1.00

Table 19: Test data for binary system of particle size 12mm and 4mm

Test Run Data TR2				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		12 mm	4 mm	mm	mm	mm	mm / mm	12 mm	4 mm	12mm + 4mm	12mm	4mm
1.00	28.5	1216.80	78.10	28.50	0.00	14.25	0.11	482.86	30.99	513.85	0.94	0.06
2.00	44.5	714.10	40.30	16.00	28.50	36.50	0.28	283.37	15.99	299.37	0.95	0.05
3.00	58.5	632.00	37.10	14.00	44.50	51.50	0.40	250.79	14.72	265.52	0.94	0.06
4.00	73.5	441.50	200.00	15.00	58.50	66.00	0.51	175.20	79.37	254.56	0.69	0.31
5.00	86.5	26.80	622.90	13.00	73.50	80.00	0.62	10.63	247.18	257.82	0.04	0.96
6.00	100.5	0.00	629.40	14.00	86.50	93.50	0.72	0.00	249.76	249.76	0.00	1.00
7.00	116.5	0.00	720.90	16.00	100.50	108.50	0.83	0.00	286.07	286.07	0.00	1.00
8.00	130.0	0.00	678.80	13.50	116.50	123.25	0.95	0.00	269.37	269.37	0.00	1.00

Table 20: Test data for binary system of particle size 10mm and 8mm

Test Run Data TR13				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to	Hc (Slice centre to	h, Normalized bed height					
		10mm	8 mm	mm	mm	mm	mm / mm	10mm	8 mm	10mm +8 mm	10mm	8mm
1	29.50	1266.90	0.70	29.50	0.00	14.75	0.11	502.74	0.28	503.02	1.00	0.00
2	45.00	790.10	6.30	15.50	29.50	37.25	0.29	313.53	2.50	316.03	0.99	0.01
3	59.00	579.40	81.20	14.00	45.00	52.00	0.40	229.92	32.22	262.14	0.88	0.12
4	73.00	291.90	363.90	14.00	59.00	66.00	0.51	115.83	144.40	260.24	0.45	0.55
5	87.00	50.80	604.70	14.00	73.00	80.00	0.62	20.16	239.96	260.12	0.08	0.92
6	101.00	12.00	609.70	14.00	87.00	94.00	0.72	4.76	241.94	246.71	0.02	0.98
7	117.00	6.50	697.70	16.00	101.00	109.00	0.84	2.58	276.87	279.44	0.01	0.99
8	130.00	1.30	646.70	13.00	117.00	123.50	0.95	0.52	256.63	257.14	0.00	1.00

Table 21: Test data for binary system of particle size 10mm and 6mm

Test Run Data TR8				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		10 mm	6 mm	mm	mm	mm	mm/mm	10 mm	6 mm	10mm + 6 mm	10mm	6mm
1.00	29.5	1260.90	8.40	29.50	0.00	14.75	0.11	500.36	3.33	503.69	0.99	0.01
2.00	45.0	793.30	24.90	15.50	29.50	37.25	0.29	314.80	9.88	324.68	0.97	0.03
3.00	59.0	612.50	55.10	14.00	45.00	52.00	0.40	243.06	21.87	264.92	0.92	0.08
4.00	73.0	296.00	357.20	14.00	59.00	66.00	0.51	117.46	141.75	259.21	0.45	0.55
5.00	87.0	28.50	624.70	14.00	73.00	80.00	0.62	11.31	247.90	259.21	0.04	0.96
6.00	101.0	6.50	706.00	14.00	87.00	94.00	0.72	2.58	280.16	282.74	0.01	0.99
7.00	117.0	1.30	706.00	16.00	101.00	109.00	0.84	0.52	280.16	280.67	0.00	1.00
8.00	129.5	0.00	589.80	12.50	117.00	123.25	0.95	0.00	234.05	234.05	0.00	1.00

Table 22: Repeat test data for binary system of particle size 10mm and 6mm

Test Run Data TR8				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		10 mm	6 mm	mm	mm	mm	mm / mm	10 mm	6 mm	10mm + 6 mm	10mm	6mm
1.00	29.0	1231.60	3.60	29.00	0.00	14.50	0.11	488.73	1.43	490.16	1.00	0.00
2.00	49.0	977.80	16.40	20.00	29.00	39.00	0.30	388.02	6.51	394.52	0.98	0.02
3.00	69.0	716.40	273.80	20.00	49.00	59.00	0.46	284.29	108.65	392.94	0.72	0.28
4.00	83.5	60.00	581.70	14.50	69.00	76.25	0.59	23.81	230.83	254.64	0.09	0.91
5.00	99.0	6.50	708.50	15.50	83.50	91.25	0.70	2.58	281.15	283.73	0.01	0.99
6.00	113.0	5.20	659.00	14.00	99.00	106.00	0.82	2.06	261.51	263.57	0.01	0.99
7.00	129.5	0.00	770.90	16.50	113.00	121.25	0.94	0.00	305.91	305.91	0.00	1.00

Table 23: Test data for binary system of particle size 10mm and 4mm

Test Run Data TR3				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		10 mm	4 mm	mm	mm	mm	mm/mm	10 mm	4 mm	10mm + 4mm	10mm	4mm
1.00	15.0	538.50	39.00	15.00	0.00	7.50	0.06	213.69	15.48	229.17	0.93	0.07
2.00	29.0	711.60	20.50	14.00	15.00	22.00	0.17	282.38	8.13	290.52	0.97	0.03
3.00	44.5	753.20	27.40	15.50	29.00	36.75	0.28	298.89	10.87	309.76	0.96	0.04
4.00	57.5	622.90	55.60	13.00	44.50	51.00	0.39	247.18	22.06	269.25	0.92	0.08
5.00	74.0	361.90	296.10	16.50	57.50	65.75	0.51	143.61	117.50	261.11	0.55	0.45
6.00	87.0	10.70	603.50	13.00	74.00	80.50	0.62	4.25	239.48	243.73	0.02	0.98
7.00	101.0	0.00	659.00	14.00	87.00	94.00	0.72	0.00	261.51	261.51	0.00	1.00
8.00	129.5	0.00	1306.30	28.50	101.00	115.25	0.89	0.00	518.37	518.37	0.00	1.00

Table 24: Repeat test data for binary system of particle size 10mm and 4mm

Test Run Data TR3				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		10 mm	4 mm	mm	mm	mm	mm/mm	10 mm	4 mm	10mm + 4mm	10mm	4mm
1.00	29.0	1233.10	22.00	29.00	0.00	14.50	0.11	489.33	8.73	498.06	0.98	0.02
2.00	49.0	956.10	25.80	20.00	29.00	39.00	0.30	379.40	10.24	389.64	0.97	0.03
3.00	69.0	773.10	229.00	20.00	49.00	59.00	0.46	306.79	90.87	397.66	0.77	0.23
4.00	83.5	36.40	616.40	14.50	69.00	76.25	0.60	14.44	244.60	259.05	0.06	0.94
5.00	99.0	0.00	722.50	15.50	83.50	91.25	0.71	0.00	286.71	286.71	0.00	1.00
6.00	113.0	0.00	654.30	14.00	99.00	106.00	0.83	0.00	259.64	259.64	0.00	1.00
7.00	128.0	0.00	736.90	15.00	113.00	120.50	0.94	0.00	292.42	292.42	0.00	1.00

Table 25: Test data for binary system of particle size 8mm and 6mm

Test Run Data TR12				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to	Hc (Slice centre to	h, Normalized bed height					
		8 mm	6 mm	mm	mm	mm	mm/mm	8 mm	6 mm	8 mm + 6 mm	8mm	6mm
1	29.00	843.70	463.20	29.00	0.00	14.50	0.12	334.80	183.81	518.61	0.65	0.35
2	44.50	370.00	376.20	15.50	29.00	36.75	0.29	146.83	149.29	296.11	0.50	0.50
3	57.50	263.40	403.80	13.00	44.50	51.00	0.40	104.52	160.24	264.76	0.39	0.61
4	74.00	279.00	398.70	16.50	57.50	65.75	0.52	110.71	158.21	268.93	0.41	0.59
5	87.00	291.10	373.10	13.00	74.00	80.50	0.64	115.52	148.06	263.57	0.44	0.56
6	101.00	328.80	326.80	14.00	87.00	94.00	0.75	130.48	129.68	260.16	0.50	0.50
7	117.00	386.30	369.90	16.00	101.00	109.00	0.87	153.29	146.79	300.08	0.51	0.49
8	126.00	249.10	301.00	9.00	117.00	121.50	0.96	98.85	119.44	218.29	0.45	0.55

Table 26: Repeat test data for binary system of particle size 8mm and 6mm

Test Run Data TR12				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		8 mm	6 mm	mm	mm	mm	mm/mm	8 mm	6 mm	8 mm + 6 mm	8mm	6mm
1	28.50	849.20	432.20	28.50	0.00	14.25	0.11	336.98	171.51	508.49	0.66	0.34
2	44.50	397.20	316.70	16.00	28.50	36.50	0.28	157.62	125.67	283.29	0.56	0.44
3	58.50	295.50	376.40	14.00	44.50	51.50	0.40	117.26	149.37	266.63	0.44	0.56
4	73.50	262.30	396.60	15.00	58.50	66.00	0.51	104.09	157.38	261.47	0.40	0.60
5	86.50	261.60	392.20	13.00	73.50	80.00	0.62	103.81	155.63	259.44	0.40	0.60
6	100.50	290.50	395.00	14.00	86.50	93.50	0.72	115.28	156.75	272.02	0.42	0.58
7	116.50	357.60	384.90	16.00	100.50	108.50	0.84	141.90	152.74	294.64	0.48	0.52
8	129.80	306.30	318.90	13.30	116.50	123.15	0.95	121.55	126.55	248.10	0.49	0.51

Table 27: Test data for binary system of particle size 8mm and 4mm

Test Run Data TR6				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		8 mm	4 mm	mm	mm	mm	mm/mm	8 mm	4 mm	8 mm + 4 mm	8mm	4mm
1.00	28.5	1250.00	2.00	28.50	0.00	14.25	0.11	496.03	0.79	496.83	1.00	0.00
2.00	44.5	717.60	10.20	16.00	28.50	36.50	0.28	284.76	4.05	288.81	0.99	0.01
3.00	58.5	627.10	29.00	14.00	44.50	51.50	0.40	248.85	11.51	260.36	0.96	0.04
4.00	73.5	372.10	314.70	15.00	58.50	66.00	0.51	147.66	124.88	272.54	0.54	0.46
5.00	86.5	42.20	608.10	13.00	73.50	80.00	0.62	16.75	241.31	258.06	0.06	0.94
6.00	100.5	1.40	645.50	14.00	86.50	93.50	0.72	0.56	256.15	256.71	0.00	1.00
7.00	116.5	1.30	724.60	16.00	100.50	108.50	0.84	0.52	287.54	288.06	0.00	1.00
8.00	129.5	0.00	672.40	13.00	116.50	123.00	0.95	0.00	266.83	266.83	0.00	1.00

Table 28: Repeat test data for binary system of particle size 8mm and 4mm

Test Run Data TR6				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		8 mm	4 mm	mm	mm	mm	mm/mm	8 mm	4 mm	8 mm + 4 mm	8mm	4mm
1.00	29.0	1252.30	2.00	29.00	0.00	14.50	0.11	496.94	0.79	497.74	1.00	0.00
2.00	49.0	909.80	10.20	20.00	29.00	39.00	0.30	361.03	4.05	365.08	0.99	0.01
3.00	69.0	740.30	202.00	20.00	49.00	59.00	0.45	293.77	80.16	373.93	0.79	0.21
4.00	83.5	92.30	558.80	14.50	69.00	76.25	0.59	36.63	221.75	258.37	0.14	0.86
5.00	99.0	7.90	703.10	15.50	83.50	91.25	0.70	3.13	279.01	282.14	0.01	0.99
6.00	113.0	3.20	653.90	14.00	99.00	106.00	0.82	1.27	259.48	260.75	0.00	1.00
7.00	130.0	2.20	875.40	17.00	113.00	121.50	0.93	0.87	347.38	348.25	0.00	1.00

Table 29: Test data for binary system of particle size 6mm and 4mm

Test Run Data TR10				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		6mm	4 mm	mm	mm	mm	mm/mm	6 mm	4 mm	6 mm + 4 mm	6 mm	4 mm
1	28.50	1323.10	1.00	28.50	0.00	14.25	0.11	525.04	0.40	525.44	1.00	0.00
2	44.50	712.80	20.70	16.00	28.50	36.50	0.28	282.86	8.21	291.07	0.97	0.03
3	58.30	435.60	239.20	13.80	44.50	51.40	0.40	172.86	94.92	267.78	0.65	0.35
4	73.50	129.60	514.70	15.20	58.30	65.90	0.51	51.43	204.25	255.67	0.20	0.80
5	86.50	96.00	557.00	13.00	73.50	80.00	0.62	38.10	221.03	259.13	0.15	0.85
6	100.50	109.00	523.80	14.00	86.50	93.50	0.72	43.25	207.86	251.11	0.17	0.83
7	116.50	129.80	602.20	16.00	100.50	108.50	0.83	51.51	238.97	290.48	0.18	0.82
8	130.00	78.10	547.10	13.50	116.50	123.25	0.95	30.99	217.10	248.10	0.12	0.88

Table 30: Repeat test data for binary system of particle size 6mm and 4mm

Test Run Data TR10				Calculation for h				Volume (cm3)		Total Volume	Volumetric Concentration (Vol./Vol)	
Bed Layer Number	Bed height (mm)	Mass of particles (g)		Ht (slice thickness)	Hb (slice thickness to bottom)	Hc (Slice centre to bottom)	h, Normalized bed height					
		6 mm	4 mm	mm	mm	mm	mm/mm	6 mm	4 mm	6 mm + 4 mm	6 mm	4 mm
1	29.00	1312.70	0.00	29.00	0.00	14.50	0.11	520.91	0.00	520.91	1.00	0.00
2	49.00	925.00	11.40	20.00	29.00	39.00	0.30	367.06	4.52	371.59	0.99	0.01
3	69.00	445.70	484.10	20.00	49.00	59.00	0.46	176.87	192.10	368.97	0.48	0.52
4	83.50	85.50	558.00	14.50	69.00	76.25	0.59	33.93	221.43	255.36	0.13	0.87
5	99.00	88.80	652.50	15.50	83.50	91.25	0.70	35.24	258.93	294.17	0.12	0.88
6	113.00	87.30	557.80	14.00	99.00	106.00	0.82	34.64	221.35	255.99	0.14	0.86
7	129.50	68.70	742.10	16.50	113.00	121.25	0.94	27.26	294.48	321.75	0.08	0.92

Appendix B: Other repeatability experimental results

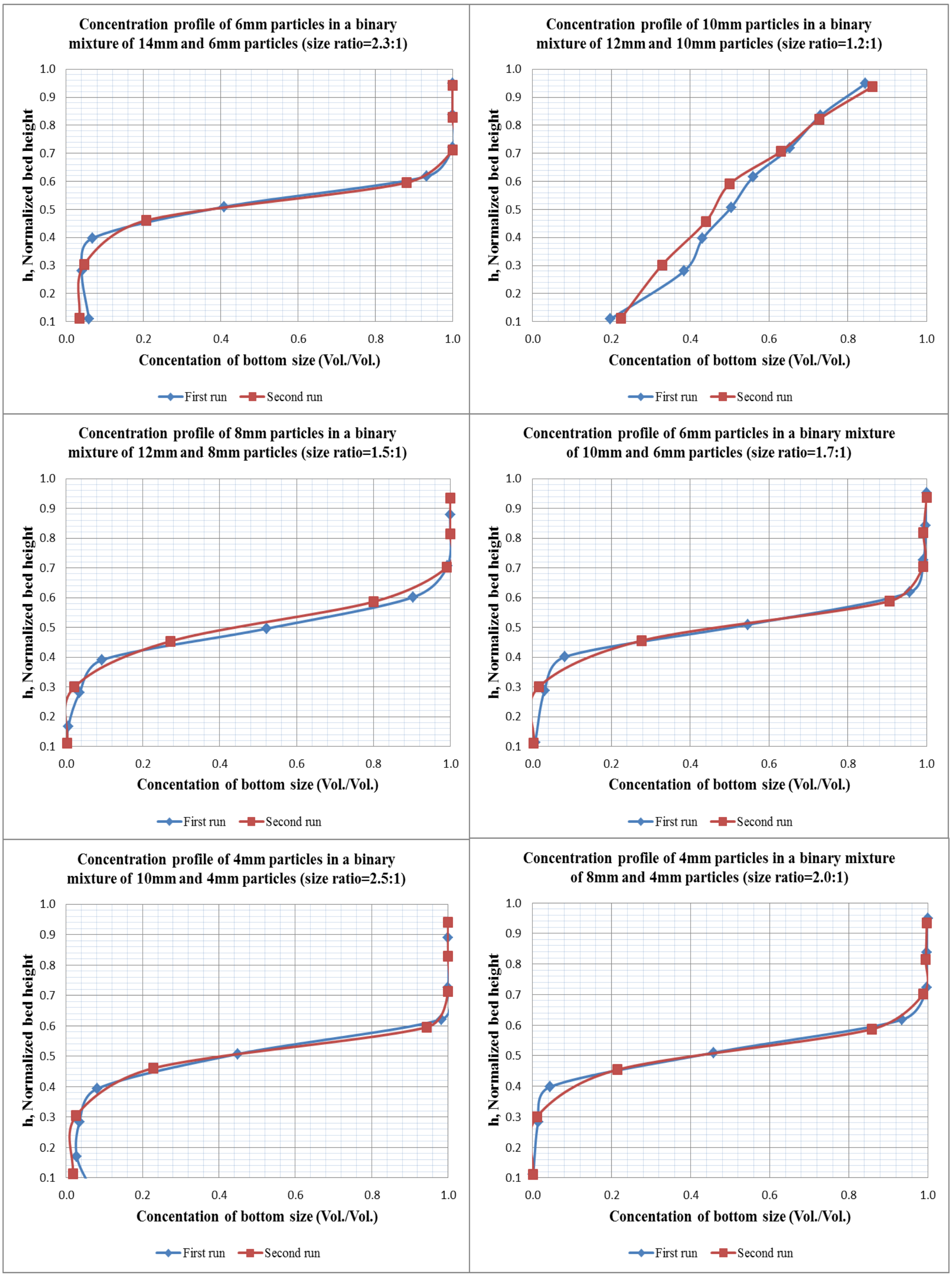

Figure 19: Repeatability of the experimental results