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Confinement and diffusion time-scales of CR hadrons in AGN-inflated bubbles

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ABSTRACT

While rich clusters are powerful sources of X-rays, γ-ray emission from these large cosmic structures has not been detected yet. X-ray radiative energy losses in the central regions of relaxed galaxy clusters are so strong that one needs to consider special sources of energy, likely active galactic nucleus (AGN) feedback, to suppress catastrophic cooling of the gas. We consider a model of AGN feedback that postulates that the AGN supplies the energy to the gas by inflating bubbles of relativistic plasma, whose energy content is dominated by cosmic-ray (CR) hadrons. If most of these hadrons can quickly escape the bubbles, then collisions of CRs with thermal protons in the intracluster medium (ICM) should lead to strong γ-ray emission, unless fast diffusion of CRs removes them from the cluster. Therefore, the lack of detections with modern γ-ray telescopes sets limits on the confinement time of CR hadrons in bubbles and CR diffusive propagation in the ICM.

Key words: radiation mechanisms: non-thermal – gamma rays: galaxies: clusters.

1 INTRODUCTION

X-ray radiation observed from clusters of galaxies is emitted by hot, low-density and metal-enriched plasmas filling the space between galaxies (for a review, see Sarazin 1986). We focus attention on relaxed galaxy clusters that are not disturbed by ongoing mergers with other groups (or clusters) of galaxies. The measured X-ray surface brightness and derived plasma density profiles of such clusters are sharply centrally peaked and decrease with radius, while the derived plasma temperature profiles have minima at the centres (for a review, see e.g. Peterson & Fabian 2006, and references therein). These central low-temperature dense regions are coined as cool-cores (Molendi & Pizzolato 2001, formerly known as ‘cooling flows’) and galaxy clusters having cool cores are called cool-core clusters. If no external energy sources are present in cool cores, the radiative losses in clusters’ centres would lead to gas cooling well below X-ray temperatures in a fraction of the Hubble time. The absence of the ‘low temperature’ emission lines in the spectra of cool-core clusters (e.g. Tamura et al. 2001) and the lack of intense star formation suggest that external energy sources for plasma heating do exist in the cool cores. A likely explanation is that the energy produced by the central active galactic nuclei (AGNs) balances radiative losses of the X-ray emitting plasma (e.g. Churazov et al. 2000; McNamara et al. 2000). The activity of the central AGNs manifests itself via the presence of X-ray ‘cavities’ (or bubbles) inflated by relativistic outflows from the AGNs that are present in majority of cool-core clusters (for a review, see Fabian 2012; Bykov et al. 2015). These bubbles are typically bright in the radio band (e.g. Boehringer et al. 1993), implying that relativistic electrons (and, perhaps, positrons) are present inside them. The bubbles rise buoyantly in cluster atmospheres and pure mechanical interaction can provide efficient energy transfer from the bubble enthalpy to the gas (e.g. Churazov et al. 2000, 2001, 2002). These buoyancy arguments suggest that the energy supplied by the AGNs matches approximately the gas cooling losses. This conclusion has been confirmed by the analysis of many systems that differ in X-ray luminosity by several orders of magnitude (e.g. Bîrzan et al. 2004; Hlavacek-Larrondo et al. 2012).

So far there is no firm conclusion on the content of the observed bubbles. We only know that there are relativistic electrons that are responsible for the synchrotron radiation of the bubbles. Very hot, but still non-relativistic, matter could also make significant contribution to the bubbles’ pressure and the most direct way to probe this component observationally is to use the Sunyaev–Zeldovich effect (Pfrommer, Enßlin & Sarazin 2005; Prokhorov, Antonuccio-Delogu & Silk 2010). It is also possible that relativistic protons dominate the energy content of the bubbles (Dunn & Fabian 2004), similar to the situation in the Galaxy, where cosmic-ray (CR) protons dominate electrons by a factor of the order of 100. The signature of the presence of CR hadrons in AGN-inflated bubbles might be established through γ-ray emission produced due to a decay of neutral pions created in the inelastic interactions of CR hadrons with thermal protons. Inside the bubbles, this mechanism is unlikely to be

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important, since the density of protons is small, and the relativistic hadrons have first to escape from bubbles into the intracluster medium (ICM; Mathews 2009).

Diffuse γ-ray emission from galaxy clusters has not been detected yet even with the modern γ-ray pair conversion telescope, *Fermi*-LAT (e.g. Huber et al. 2013; Ackermann et al. 2014; Prokhorov & Churazov 2014) or the Cherenkov telescopes, including HESS (e.g. Aharonian et al. 2009), MAGIC (e.g. Aleksić et al. 2012b; Ahnen et al. 2016) and VERITAS (Arlen et al. 2012). However, the X-ray brightest cool-core clusters, Perseus and Virgo, in which AGN-inflated bubbles are very prominent, do contain bright central γ-ray emitting sources. These γ-ray sources detected with *Fermi*-LAT and Cherenkov telescopes are identified with the cores of radiogalaxies, NGC 1275 (Abdo et al. 2009a; Aleksić et al. 2012a) and M 87 (Abdo et al. 2009b; Acciari et al. 2008; Aleksić et al. 2012c), respectively. The Perseus cluster is expected to be one of the most promising cool-core clusters to constrain the pressure of CR hadrons due to its high X-ray flux and high central concentration of the expected γ-ray flux (Aleksić et al. 2012b; Ahnen et al. 2016). The observations of the Perseus cluster with MAGIC result in the upper limit on the average CR-to-thermal pressure ratio of $<$2 per cent. The tight bounds on the ratio of CR proton-to-thermal pressures of $\lesssim$1.5 per cent have also been obtained with *Fermi*-LAT through the stacking of $\simeq$50 galaxy clusters of smaller angular sizes (e.g. Huber et al. 2013; Ackermann et al. 2014; Prokhorov & Churazov 2014).

In this paper, we show that the flux upper limits provided by the modern γ-ray telescopes can strongly constrain the models of both the CR hadron confinement in AGN-inflated bubbles and diffusive propagation of CR hadrons in the ICM. We will come to the conclusion that if CR hadrons are injected by AGN jets in bubbles, these CR hadrons have to be confined in buoyantly rising bubbles much longer than the sound crossing time of cool cores.

The structure of the paper is the following. In Section 2, we provide the order-of-magnitude estimates and rough scaling of the γ-ray flux. In Section 3.1, we present a more elaborate but still very straightforward model for γ-ray flux computation. Section 3.2 summarizes our findings.

2 ORDER-OF-MAGNITUDE γ-RAY FLUX ESTIMATES

An overall concept of our model is the following; We assume that the energy provided by the AGN feedback matches the gas cooling losses L_X. We further assume that relativistic protons dominate in the feedback energy balance. Thus, the total production rate of CR protons by the AGNs is known and equal to the observed X-ray luminosity coming from the region within the cooling radius. This injection of relativistic protons by itself is not associated with the γ-ray flux, since the protons are initially confined within the bubbles. The bubbles rise buoyantly through the cluster atmosphere with the velocity $u_{adv} \sim \eta c_s$, where c_s is the sound speed in the ICM and η is in the range 0.2–0.5. During the rise, the CR protons experience adiabatic losses, thus reducing the total energy associated with them. A fraction of protons can escape from the bubble as it rises and enter the ICM. The rate of escape is characterized by the confinement time t_{conf}, so that the fraction of protons escaping during a short interval Δt is $\Delta t/t_{conf}$. Once the relativistic proton is in the ICM, it diffuses in space with a diffusion coefficient D. The proton in the ICM can collide with the thermal protons and generate γ-ray flux.

Apart from diffusive propagation of CR protons in the ICM, the CRs can stream relative to the bulk plasma along magnetic field lines (see e.g. Wentzel 1974). The streaming of the CRs with respect of the ICM excites magnetohydrodynamic waves, on which the CRs scatter. Generation of waves and scattering on them limits CR streaming speeds to the speed of the waves, which is the Alfvén speed, but if the waves are strongly damped, highly super-Alfvénic streaming is possible. If the streaming can efficiently transport the CRs down to their pressure gradient from the central regions of cool-core clusters, it can be an alternative mechanism to quench diffuse γ-ray emission, while dissipation of the waves can provide additional heating of the ICM plasma in cool cores. We do not include CR streaming in this study. The effect of CR streaming in the context of galaxy clusters was studied by e.g. Pfrommer (2013), Wiener, Oh & Guo (2013) and Ruszkowski, Yang & Reynolds (2017).

Thus, in the frame of the model considered here, the most important (unknown) parameters affecting the γ-ray flux are the confinement time t_{conf} and the diffusion coefficient D. Below we begin our analysis by considering the most extreme values of these parameters that should lead to the maximal γ-ray flux.

2.1 Maximal γ-ray flux

The γ-ray flux F_γ (in units of phot s^{-1} cm^{-2}) coming from a volume V with the density of thermal protons n_{pl} can be estimated as

$$F_\gamma \simeq \frac{q_\gamma \epsilon_{CRp} V n_{pl}}{4\pi d^2}, \quad (1)$$

where q_γ is the γ-ray emissivity normalized to the unit CR hadron energy density, which is given by Drury, Aharonian & Voelk (1994), and ϵ_{CRp} is the energy density of CR hadrons. It is obvious that the maximal γ-ray flux can be expected if all hadrons are released immediately into the ICM (without being trapped inside the bubble, i.e. $t_{conf} \to 0$) and they do not diffuse outside the central region (i.e. $D \to 0$). Of course this is not a realistic setup, but it serves as an absolute upper limit on the γ-ray flux. The rate of energy losses by relativistic protons weakly depends on proton energy $E \gtrsim 100$ GeV. We assume

$$\frac{\mathrm{d}\ln E}{\mathrm{d}t} \approx 3.85 \times 10^{-16} \left(\frac{n_{pl}}{\mathrm{cm}^{-3}}\right) \ \mathrm{s}^{-1}, \quad (2)$$

see e.g. Krakau & Schlickeiser (2015). The total energy released by the AGNs over the lifetime of the clusters, which we assume to be comparable to the Hubble time t_H, is $\epsilon_{CRp} V \sim L_X t_H$. If the plasma density $n_{pl} \geq 6 \times 10^{-3}$ cm^{-3} then equation (2) implies that the lifetime of relativistic protons $t_{loss} = \left(\frac{\mathrm{d}\ln E}{\mathrm{d}t}\right)^{-1} \sim \frac{8 \times 10^7}{n_{pl}}$ yr is shorter than t_H. This condition is satisfied in the centres of all cool-core clusters. In this case, $\epsilon_{CRp} V \sim L_X t_{loss}$ and the plasma density n_{pl} term in equation (1) cancels out:

$$F_{\gamma,max} \sim \frac{q_\gamma L_X}{4\pi d^2 \left(\frac{1}{n_{pl}} \frac{\mathrm{d}\ln E}{\mathrm{d}t}\right)} \sim \frac{L_X}{4\pi d^2 \ 78 \ \mathrm{erg}}, \quad (3)$$

where we use $q_\gamma = 4.9 \times 10^{-18}$ s^{-1} erg^{-1} cm^3 (H-atom)$^{-1}$ (for $\Gamma = -2.2$ from Drury et al. 1994) for the production of γ-rays above 1 TeV. For the Perseus cluster, using the X-ray luminosity within the cooling radius L_X and distance to the cluster d from Bîrzan et al. (2012), the estimated γ-ray flux above 1 TeV equals to 1×10^{-11} cm^{-2} s^{-1} and exceeds the MAGIC upper limit (for the *point-like* model), 4×10^{-14} cm^{-2} s^{-1} above 1 TeV (Ahnen et al. 2016), by about 250 times. Therefore, in the absence of streaming losses, the observational constraints permit us to rule out the case corresponding to the short confinement time of CR hadrons and the slow CR diffusion in the ICM.

2.2 Pure diffusion case

The same arguments can be easily extended to the case of a finite diffusion coefficient, while keeping the confinement time small $t_{\rm conf} \to 0$. The γ-ray flux will be smaller than predicted by equation (3), if diffusion can transport a large fraction of CR protons from the central region into lower density outer plasmas, where $t_{\rm loss} \gtrsim t_{\rm H}$. In this case, $\epsilon_{\rm CRp} V \sim L_{\rm X} t_{\rm H}$, but now the effective value of $n_{\rm pl}$ in equation (1) depends on the size of the region occupied by diffused CR protons. Consider a stationary solution of the diffusion equation with a steady point source of CRs at the centre. In the central region, the constant diffusive flux through any radius r implies that $\epsilon_{\rm CRp}(r) \propto r^{-1}$. Assuming that $n_{\rm pl}(r)$ declines with radius slower than r^{-2}, one can conclude that the integral $\int \epsilon_{\rm CRp}(r) n_{\rm pl}(r) r^2 {\rm d}r$ is dominated by the largest radii $r_{\rm max}$. One can choose $r_{\rm max} \sim \sqrt{D t_{\rm H}}$. Thus,

$$F_{\gamma,{\rm dif}} \sim \frac{q_\gamma L_{\rm X} t_{\rm H} n_{\rm pl}(r_{\rm max})}{4\pi d^2}. \quad (4)$$

If $r_{\rm max} \sim 1$ Mpc, the MAGIC flux upper limit for the *point-like* model of γ-ray emission from the Perseus cluster is inapplicable since it was obtained within the reference radius of 0°.15 (or 200 kpc at z=0.0179). The MAGIC flux upper limit obtained within the virial radius for the *extended* model is a factor of 15 larger than that for the *point-like* model and, given that predicted γ-ray flux slowly increases with radius, the constraints obtained on a basis of the flux from the cool-core region are tighter.

It takes about $t_{\rm D} \sim R_{\rm core}^2/D$ for CR protons to spread out by diffusion from the cluster cool core. During this time interval, the energy released in CR hadrons in the central region is $L_{\rm X} t_{\rm D}$. The γ-ray flux produced owing to CR protons not having a sufficient time to diffuse beyond the cool-core region is

$$F_\gamma \sim \frac{q_\gamma L_{\rm X} t_{\rm D} n_{\rm pl}(R_{\rm core})}{4\pi d^2}. \quad (5)$$

From equation (5), one can find that to satisfy the MAGIC flux upper limit for the *point-like* model, the diffusion coefficient has to be $D > 5 \times 10^{31}$ cm^2 s^{-1}.

2.3 Pure advection case

For the finite $t_{\rm conf}$, the calculations are slightly more complicated, since one has to take into account adiabatic losses during the buoyant rise of the bubbles. This is done in Section 3 below. Here, we consider a special (and rather artificial) case of zero diffusion ($D = 0$) and very long $t_{\rm conf}$, such that only a small fraction of protons escape the bubble over its entire lifetime. In this approximation, we can assume that the energy content of the rising bubble is changing only due to adiabatic expansion. The momentum of a relativistic proton p inside the bubbles changes as $p = p_0 (P(r)/P_0)^{(\gamma-1)/\gamma}$, where p_0 is the initial particle momentum, P_0 and $P(r)$ are the initial and current pressure (at radius r) of the ICM and $\gamma = 4/3$. Accordingly, the number of relativistic particles, N, producing γ-rays above given energy $E_\gamma = p_0 c$ is

$$N(r) \propto N_0 \left(\frac{P_0}{P(r)}\right)^{\frac{\gamma-1}{\gamma}(1+\Gamma)}, \quad (6)$$

where N_0 is the initial number of such particles with $p > p_0$ and the CR hadron power-law spectrum, $\propto p^\Gamma$. With this correction, the expression for the γ-ray flux can be written as

$$F_{\gamma,{\rm adv}} \sim \frac{q_\gamma L_{\rm X}}{4\pi d^2} \int \frac{N(r)}{N_0} n_{\rm pl}(r) \frac{t_{\rm min}}{u_{\rm adv} t_{\rm conf}} {\rm d}r, \quad (7)$$

where $t_{\rm min}$ is the minimal time among $t_{\rm H}$ and $t_{\rm loss} \sim \frac{8 \times 10^7}{n_{\rm pl}(r)}$ yr. The integral in expression (7) is dominated by the range of radii where the radial density profile is shallower than r^{-1}, i.e. the inner ~100 kpc of the Perseus cluster, especially taking into account the adiabatic losses that further reduce the contribution of the outer regions. Using $r \sim 100$ kpc and $n_{\rm pl} \sim 7 \times 10^{-3}$ cm^{-3} for estimates, one gets an order-of-magnitude constraint on $\frac{r}{u_{\rm adv} t_{\rm conf}} \lesssim 5 \times 10^{-3}$, or, equivalently, $t_{\rm conf} \gtrsim 4 \times 10^{10}$ yr.

2.4 Most constraining galaxy clusters

In this Section, we use simple estimates to pre-select the most promising clusters in terms of the possible constraints on the confinement time and the diffusion coefficient.

Most of the gas properties of cool-core clusters are known from X-ray observations. Since γ-ray diffuse emission from cool-core clusters has not been detected yet, both the amount and distribution of CR hadrons are unknown. We apply a scaling law to select cool-core clusters expected to be bright in γ-rays. To deduce the γ-ray scaling law, we use four basic assumptions:

(i) The production rate of CR hadrons is proportional to the X-ray luminosity of the cooling radius region, $\dot{N}_{\rm CR} \propto L_{\rm X}$ (energy constraint);

(ii) The X-ray luminosity within the cooling radius is $L_{\rm X} \propto \int_0^{R_{\rm cool}} n_{\rm pl}^2 {\rm d}^3 r$, where $R_{\rm cool}$ and $n_{\rm pl}$ are the cooling radius and the plasma number density in the central region, respectively;

(iii) The γ-ray flux is proportional to the product of CR hadron and thermal plasma number densities, $F_\gamma \propto \dot{N}_{\rm CR} n_{\rm pl}/d^2$; and

(iv) The lifetime of CR protons due to hadronic losses is inversely proportional to the plasma density.

Using these assumptions, we expect the approximate γ-ray scaling relation,

$$F_\gamma \propto \frac{L_{\rm X}}{d^2}. \quad (8)$$

Putting the values of parameters for the sample of cool-core clusters taken from Bîrzan et al. (2012), we find that the highest γ-ray flux is expected from the central region of the Perseus cluster. The second highest γ-ray flux is expected from the central region of the Ophiuchus cluster (which is about 4 times lower than that from the Perseus cluster) and the third highest γ-ray flux is expected from M87 (which is about 5 times lower than the expected γ-ray flux from the Perseus cluster). Other clusters expected to be bright in γ-rays are Abell 2029, Abell 2199, Abell 478, Centaurus, 2A 0335+096 and Abell 1795 (those fluxes are about 10 times lower than that expected from the Perseus cluster).

Observational upper limits on the γ-ray fluxes are different for nearby galaxy clusters because these limits depend on the presence of γ-ray emitting central radio galaxies in cool cores, on how sensitive various γ-ray telescopes are, and on how strong the Galactic foreground emission is in the direction of galaxy clusters. To investigate which cool-core cluster can provide us with the tightest constraints on the CR hadron confinement time and on the CR diffusion coefficient, one needs to calculate the flux upper limits scaled to the flux above the same energy (above 1 TeV in this paper). These scaled fluxes have to be computed assuming the production of γ rays via a neutral pion decay. To scale the flux from 1 GeV to 1 TeV, we used equation (19) from Pfrommer & Enßlin (2004) derived in the framework of Dermer's model (Dermer 1986) and

Table 1. The scaled flux upper limits (in cm^{-2} s^{-1}) above 1 TeV for the selected cool-core clusters (and the estimator values).

Cluster name	Scaled flux limit, $\Gamma = -2.2$	Scaled flux limit, $\Gamma = -2.5$
Perseus	3.8×10^{-14} (1.00)	3.8×10^{-14} (0.34)
Abell 1795	1.2×10^{-14} (0.24)	1.0×10^{-15} (1.00)
Abell 478	2.0×10^{-14} (0.18)	1.8×10^{-15} (0.68)
Ophiuchus	6.2×10^{-14} (0.15)	5.4×10^{-15} (0.57)
Abell 2199	3.9×10^{-14} (0.11)	3.4×10^{-15} (0.42)
Centaurus	3.2×10^{-14} (0.11)	2.8×10^{-15} (0.42)
Abell 2029	4.7×10^{-14} (0.08)	4.1×10^{-15} (0.32)
2A 0335+096	4.1×10^{-14} (0.07)	3.5×10^{-15} (0.31)
Virgo (M87)	4.6×10^{-13} (0.01)	–

assumed that the values of the CR hadron power-law spectral index, Γ, are -2.2 and -2.5. The upper limit on the γ-ray flux (Table 1) from the Perseus cluster above 1 TeV was derived for the *point-like* model by Ahnen et al. (2016). The flux upper limits above 1 GeV for cool-core clusters (including Abell 478, Abell 2199 and Abell 1795) derived from the *Fermi*-LAT data are taken from Ackermann et al. (2014), while the flux upper limits above 1 GeV for the Ophiuchus cluster, 2A 0335+096, Centaurus and Abell 2029 are taken from Selig et al. (2015). The γ-ray flux from the Virgo cluster is that of M87 at a low flux state (Aleksić et al. 2012c). The scaled flux upper limits above 1 TeV for these cool-core clusters are also shown in Table 1.

To select the most constraining targets for studying the CR hadron confinement in the bubbles, we introduce an estimator defined as the ratio of the expected flux[1] to the upper limit of the scaled observational γ-ray flux. The values of the estimator for the selected cool-core clusters are shown in parentheses in Table 1. We normalized the estimator to its highest value for each of the values of the CR hadron spectral index, Γ. We found that the Perseus cluster corresponds to the highest value of the estimator, if $\Gamma = -2.2$. For the softer power-law index, $\Gamma = -2.5$, the situation is different and the cool-core clusters with the scaled fluxes obtained from the *Fermi*-LAT data give estimator values similar to (or even higher than) that derived for the Perseus cluster by using the MAGIC observations.

3 MODELLING OF γ-RAY EMISSION PRODUCED BY ESCAPED CR HADRONS

In this Section, we introduce the toy model that combines constraints on the CR hadron escape from AGN-inflated bubbles and CR diffusive propagation in the ICM. In the framework of this model, we will compute the γ-ray flux expectation and will set limits on the confinement and diffusion time-scales of CR hadrons.

Both the confinement and the diffusion control transport of CR hadrons in cool-core clusters. Confined CR hadrons can be transported inside the bubbles from the cluster centre to outer cluster regions. The longer the confinement time, the fewer CR hadrons escape from the bubbles into the ICM when passing the densest cooling region. After CR hadrons escape their parent bubbles, they then diffuse and fill the ICM. In this case, CR hadron propagation is specified by the diffusion coefficient. After time, τ, the CR hadrons diffuse a distance $l \sim (D\tau)^{1/2}$. If the diffusion coefficient is sufficiently large, e.g. $D \sim 10^{31}$ cm^2 s^{-1}, CR hadrons injected into the ICM within the cooling region cannot be retained in the cooling region for 10^{10} yr. Therefore, both the long confinement time and the fast diffusion cause a decrease in the containment of CR hadrons within the cooling radius. Apart from confinement and diffusion, hadronic losses also affect the containment of CR hadrons within the cooling radius, because CR hadrons cannot remain in the cooling radius longer than their lifetime due to hadronic losses. Beyond the cooling radius, the lifetime of CR protons due to hadronic losses is much longer because of the lower plasma density in the outer regions. Although CR protons can exist in the outer regions for the Hubble time, γ-ray emission from the outer regions is comparably small because of the same reason.

[1] Computed using the scaling relation from equation (8).

3.1 Toy model

To calculate the γ-ray fluxes from the cool-core galaxy clusters expected due to the interactions of CR hadrons that escaped from rising bubbles with the ICM, we introduce a toy model having the following properties:

(i) Mechanical work done by jets to inflate bubbles and that done by rising bubbles during their adiabatic expansion in the ICM compensates X-ray radiative energy losses within the cooling radius.

(ii) Isolated galaxy clusters are assumed to be spherically symmetric, i.e. plasma temperature and density profiles only depend on a radius. These profiles are determined from X-ray observations and to simplify the case we ignore the evolution of these profiles in time.

(iii) We also assume that AGN-inflated bubbles are filled with CR hadrons and that CR pressure inside the bubbles equal ambient plasma pressure. Thus, the CR hadron energy density in the bubbles is determined solely by the radial plasma pressure profile.

(iv) AGN-inflated bubbles rise with a constant sub-sonic speed, u_{adv}, along the radius-vector from the centre of a cool-core cluster. This determines the spatial profile of the CR hadron source function that drops as the square of the radius from the centre of a cool-core cluster.

(v) The CR hadron escape from the bubbles (in other words, the injection of CR hadrons in the ICM) is described by the source term in the CR diffusion equation. For the sake of simplicity, we assume that the time of CR hadron confinement, t_{conf}, does not depend on energy.

(vi) The number density and low energy bound of CR hadrons in each bubble change with time and, therefore, depend on the radial position of the bubble. To calculate how these parameters change with time, we solved the system of equations

$$\frac{N_{CR}\epsilon}{3V} = P_{pl}, \quad (9)$$

$$\frac{\partial N_{CR}}{\partial t} = \frac{-N_{CR}}{t_{conf}}, \quad (10)$$

$$\epsilon = \epsilon_0 \left(\frac{V}{V_0}\right)^{1-\gamma}, \quad (11)$$

where N_{CR}, ϵ, V and P_{pl} are the number of CRs, the energy per particle in a bubble and the volume of a bubble, and the plasma pressure as a function of radius from the centre of the cluster, respectively.

(vii) We assume that the CR hadron spectrum has a power-law shape. The change of a low energy bound of CR hadrons with time leads to the injection of CR hadrons into the ICM at different low energy bounds. This would affect the shape of the CR hadron spectrum at low energies. We do not include this effect and consider the model under the condition that the initial minimal kinetic energy of CR hadrons, $E_{\mathrm{kin},0}$, is lower than a typical energy of CR hadrons, $E_{\mathrm{p}} \gtrsim 10E_{\gamma,\mathrm{min}}$, producing γ-rays above the minimal observed energy $E_{\gamma,\mathrm{min}}$ (e.g. if $E_{\gamma,\mathrm{min}} = 1$ GeV then $E_{\mathrm{kin},0} \lesssim 10$ GeV). Note that the minimal kinetic energy of CR hadrons inside the bubbles decreases with time due to the bubble adiabatic expansion. Therefore, if the latter condition is satisfied at the initial moment, it will also be satisfied as AGN bubbles evolve in time. To compute the energy budget, the minimal kinetic energy of a few GeV was assumed, thus the γ-ray emissivity values used in this paper are conservative.

(viii) Under the assumption that escaped CR hadrons are well mixed with the ICM, CR hadrons undergo proton–proton hadronic interactions with an ICM plasma. Owing to these interactions, CR hadrons produce neutral pions, each decaying into two γ-rays. The expected γ-ray flux is expressed as $F_\gamma = \sigma_{\mathrm{pp}} c f(E_\gamma) \int n_{\mathrm{cr}}(r) n_{\mathrm{pl}}(r) r^2 \mathrm{d}r/d^2$, where σ_{pp} is the inelastic p–p cross-section and $f(E_\gamma)$ is a function of γ-ray energy, which is obtained from a simple fitting form given by Pfrommer & Enßlin (2004, section 3.2.2). Note that the CR hadron number density, $n_{\mathrm{cr}}(r)$, is derived by solving the diffusion equation and the plasma number density profile, $n_{\mathrm{pl}}(r)$, is taken from X-ray observations.

(ix) The transport of CR hadrons through the ICM is diffusive. The CR diffusion coefficient, D, is also assumed not to depend on energy. This corresponds to the case of passive advective transport in a turbulent flow with the diffusion coefficient of $D = v_{\mathrm{turb}}\lambda_{\mathrm{turb}}/3 \sim 10^{29}$ cm^{2}s^{-1}, where $v_{\mathrm{turb}} \sim 100$ km s^{-1} and $\lambda_{\mathrm{turb}} \sim 10$ kpc are the turbulent velocity and coherence length, respectively (see e.g. Pfrommer & Enßlin 2004). In addition to a transport in cluster turbulent flows, wave-CR interactions result in CR propagation with the energy dependent CR diffusion coefficient. In the latter case, the CRs are scattered by the magnetic fluctuations of the microscopic scales. The random component of magnetic field with a Kolmogorov spectrum of inhomogeneities leads to the diffusion coefficient of $D \propto E^{1/3}$ (see e.g. Völk, Aharonian & Breitschwerdt 1996, in the context of galaxy clusters). If the microscopic CR transport is dominant, then the constraints obtained in the framework of the toy model can be re-estimated.[2] The lifetime of CR protons due to hadronic losses is inversely proportional to the plasma density and only weakly depends on energy in the high energy regime (Krakau & Schlickeiser 2015). To simplify the model, we assume that the CR hadron lifetime due to hadronic losses does not depend on energy (in this case, the energy term cancels off in the diffusion equation). We solve the diffusion equation numerically using the Crank–Nicolson method.

These basic properties of the toy model allow us to calculate the γ-ray fluxes expected from cool-core galaxy clusters, if CR hadron escape from the bubbles into the ICM takes place. This toy model has two free parameters: the confinement time of CR hadrons in the bubble and the CR diffusion coefficient in the ICM.

[2] Note that the higher the CR diffusion coefficient is, the more extended the distribution of CR protons becomes. In the case of the diffusion coefficient increasing with energy, the lowest γ-ray flux is expected for the highest value of the diffusion coefficient (due to a plasma density profile decreasing with radius). If the integral γ-ray flux at energies higher than E_{min} is determined by CR protons with energies between $10E_{\mathrm{min}}$ and 10^4E_{min} (see e.g. Kelner, Aharonian & Bugayov 2006, for γ-ray spectra produced at p–p collisions), then the constraints obtained on the diffusion coefficient, D, in the framework of the toy model can conservatively be converted into the constraints on the CR diffusion coefficient at energy $10E_{\mathrm{min}}$ by dividing them by a factor of $\sim(10^4/10)^{1/3} \sim 10$ in the case of a Kolmogorov spectrum of magnetic field fluctuations.

3.2 Constraints on CR hadron confinement and diffusion

In this Section, we present the results of numerical computations obtained in the framework of the toy model. We are pursuing the goal of computing the expected γ-ray fluxes from cool-core clusters as functions of the two parameters, the CR confinement time, t_{conf}, and the CR diffusion coefficient, D. We select the three cool-core clusters, Perseus, Abell 1795 and Abell 478, expected to be the most constraining (see Table 1) for the investigation.

Observational γ-ray flux upper limits are used to constrain the two free parameters of the model. The gas parameters of the model, including the radial profiles of plasma density and pressure in the ICM, are taken from Churazov et al. (2003) and Zhuravleva et al. (2013) for the Perseus cluster, and from Vikhlinin et al. (2006) for the Abell 1795 and Abell 478 clusters. For the Perseus cluster, the γ-ray flux is estimated within the cylindrical region of a reference radius, $0.^{\circ}15$, to make comparisons with the flux upper limit obtained for the *point-like* model with MAGIC (see Section 2.2). The other two cool-core clusters under investigation, Abell 1795 and Abell 478, located at about 270 and 385 Mpc from Earth, respectively, are point-like sources for *Fermi*-LAT. For Abell 1795 and Abell 478, the γ-ray fluxes are estimated within the region of virial radii to make comparisons with the flux upper limits obtained with *Fermi*-LAT.

We found that the MAGIC observations of the Perseus cluster provide the tightest constraints, if the spectral index is hard, $\Gamma = -2.2$. Fig. 1 shows the constraints obtained for the Perseus cluster for the spectral index values of $\Gamma = -2.2$ and -2.5. Both these constraints are based on the observations above 1 TeV. The constraints are much tighter for $\Gamma = -2.2$ than those for $\Gamma = -2.5$. It is because of a much higher production rate (normalized to the CR hadron energy density) of γ-rays above 1 TeV for the former hadron spectral index (see table 1 from Drury et al. 1994). This shows that AGN-inflated bubbles keep CR hadrons for a time, $t_{\mathrm{conf}} > 10^{10}$ yr, if $D < 3 \times 10^{30}$ cm^2 s^{-1} and $\Gamma = -2.2$.

We checked the compatibility of the obtained constraints with those estimated in Section 2 for the Perseus cluster. The computed maximal flux, 1.5×10^{-11} ph cm^{-2} s^{-1}, is close to that estimated above. The computed constraints obtained for the pure diffusion and pure advection cases are also similar to those estimated above. Therefore, the order of magnitude γ-ray flux estimates are compatible with the γ-ray fluxes obtained in the framework of the toy model.

The results of the computations in the case of $\Gamma = -2.5$ for Abell 1795 and Abell 478 are shown in Fig. 2. We found that the constraints obtained using the *Fermi*-LAT flux upper limits for these cool-core clusters (bottom white lines in the panels of Fig. 2) are similar to or even tighter than those obtained for the Perseus cluster (if $\Gamma = -2.5$). It agrees with the result obtained by means of the estimators in Section 2 (see Table 1). The constraints obtained for Abell 1795 and Abell 478 for the spectral index of $\Gamma = -2.2$ are similar to those obtained for these clusters if $\Gamma = -2.5$. To strengthen the constraints, we applied the γ-ray flux upper limit obtained through the stacking of 50 cool-core clusters (e.g. Huber et al. 2013; Ackermann et al. 2014; Prokhorov & Churazov 2014). The upper white curves in the panels of Fig. 2 show this γ-ray flux upper limit. We found that the bubbles keep CR hadrons for a time, $t_{\mathrm{conf}} > 5 \times 10^9$ yr, if $D < 1 \times 10^{30}$ cm^{2}s^{-1} and $\Gamma = -2.5$. Note that the γ-ray flux upper limit for Abell 1795 is close to that obtained through the stacking (the scaled flux upper limit of Abell 1795 in Table 1 is the tightest).

The sound crossing times of the cooling regions in the Perseus cluster, Abell 1795 and Abell 478 are about 100 Myr. Therefore,

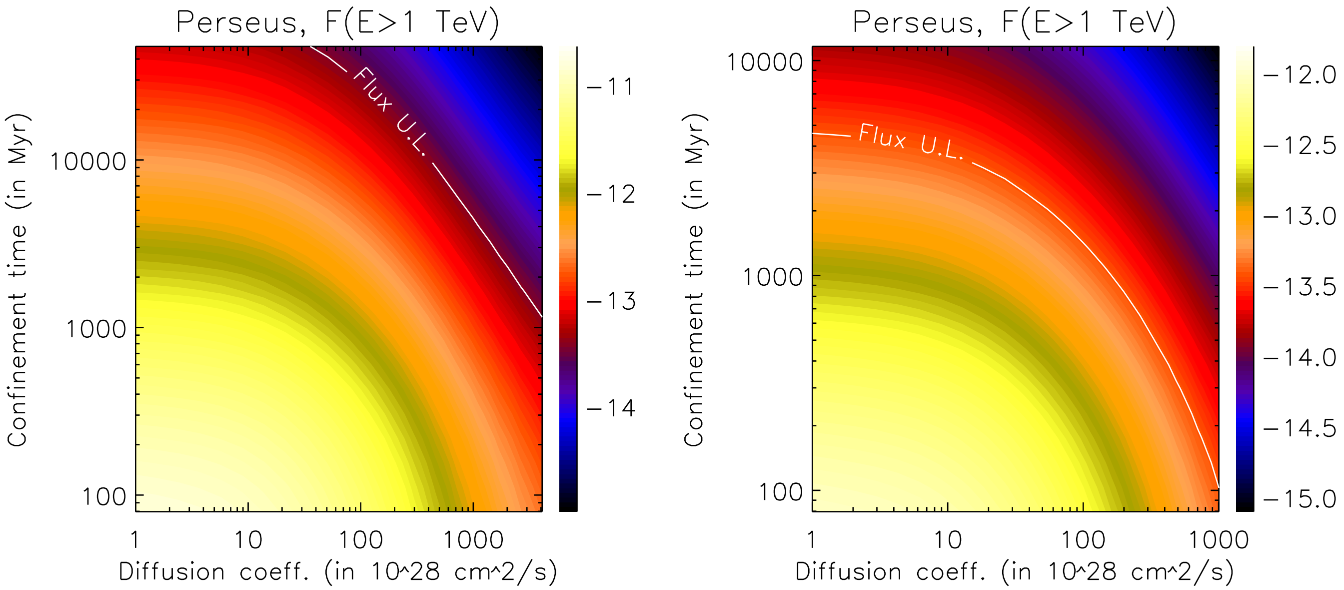

Figure 1. Logarithm of the γ-ray flux in ph cm^{-2} s^{-1} and constraints on the parameter space ($t_{\rm conf}$, D) obtained for the Perseus cluster for the spectral index values of $\Gamma = -2.2$ (left-hand panel) and $\Gamma = -2.5$ (right-hand panel)

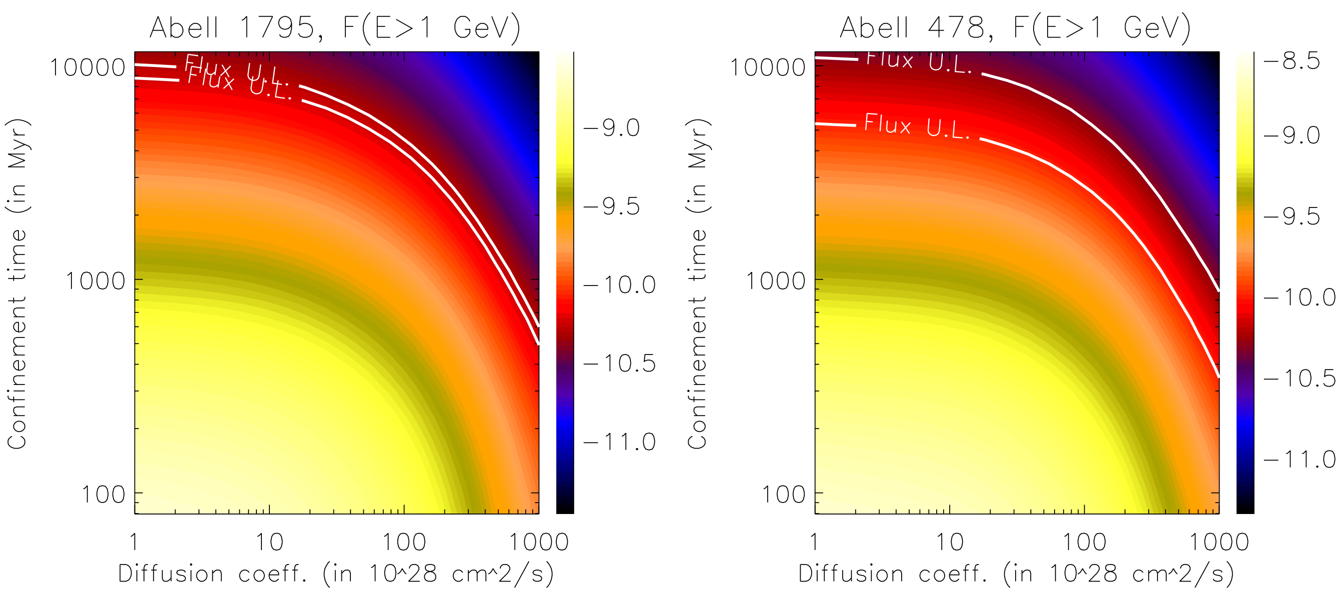

Figure 2. Logarithm of the γ-ray flux in ph cm^{-2} s^{-1} and constraints on the parameter space ($t_{\rm conf}$, D) obtained for Abell 1795 (left-hand panel) and Abell 478 (right-hand panel) for the spectral index value of $\Gamma = -2.5$.

CR hadrons are confined in the bubble over times much longer than the sound crossing time under the condition that the diffusion coefficient, D, is less than 10^{31} cm^2 s^{-1} (see Figs 1 and 2).

We cross-checked and confirmed these constraints using an alternative method of computations based on the solution of the non-homogeneous diffusion equation by means of the Green function. The Green function was obtained by applying Duhamel's principle to the textbook solution of the homogeneous diffusion equation in spherical coordinates. The advantage of the main method, the Crank–Nicolson finite difference method, used in this paper is that it allows the inclusion of hadronic losses in a simple way, while hadronic losses were not included using the alternative method.

4 CONCLUSIONS

AGNs are the likely heat source of the cool cores in relaxed clusters of galaxies. Observational evidence of the AGN interaction with the gas at the centres of cool-core clusters is the presence of bubbles in the hot surrounding medium. Mechanical power of the bubbles is sufficient to balance radiative X-ray cooling in the cool cores. If the bubbles consist of CR hadrons, γ-rays produced in collisions between CR hadrons, escaped from the bubbles, and the ICM protons can be measured by modern γ-ray telescopes. High-energy observations of the cool-core clusters provide important insights into the physical processes of CR hadron confinement in the inflated bubbles and into CR transport models.

This paper reports results of modelling of γ-ray emission expected owing to the escape of CR hadrons from the bubbles in cool-core clusters. The order of magnitude γ-ray flux estimates were performed and showed that the maximal γ-ray flux obtained for the Perseus cluster under the conditions (all hadrons are released immediately into the ICM and do not diffuse outside the central region) strongly exceeds the observed flux upper limit. The pure diffusion and pure advection cases were investigated and the order of magnitude limits on the diffusion coefficient and on the CR confinement time-scale were derived. The γ-ray scaling relation was derived and used to select galaxy clusters whose existing γ-ray observations are expected to result in the tightest constraints on the parameter space of CR confinement and transport models. The toy model was introduced to carry out more quantitative calculations (see Section 3.1). This model was applied to the selected cool-core clusters and numerical computations using the Crank–Nicolson method were performed. Comparing the computed γ-ray fluxes with the observed upper flux limits, the two free parameters of the model, CR confinement time-scale and diffusion coefficient, were constrained. CR hadrons are confined in the bubbles for a time of, $t_{\rm conf} > 10^{10}$ yr, if $D < 3 \times 10^{30}$ cm^2 s^{-1} and $\Gamma = -2.2$, and for a time of $t_{\rm conf} > 5 \times 10^9$ yr, if $D < 1 \times 10^{30}$ cm^2 s^{-1} and $\Gamma = -2.5$. These time-scales are much longer than the sound crossing times of the cooling regions in the investigated clusters. These constraints are tight, but could possibly be relaxed if other physical processes, such as streaming, complicate the simple model used in this paper.

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