THE FAMA-FRENCH FIVE-FACTOR MODEL: EVIDENCE FROM THE JSE

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Abstract

The desire to explain the returns of shares listed on stock exchanges has long driven research into asset pricing models. The formation of the Capital Asset Pricing Model (CAPM) served as the starting point for almost all models derived to describe share returns. Further research into returns discovered that there were various risk factors that impacted share returns; these could be captured by market value (size) and the book-to-market ratio (value). In 1993, Eugene Fama and Kenneth French used these to expand on the CAPM. The Fama-French three-factor model created a framework to model returns and allowed for other researchers to explore asset pricing further.

In 2015, Fama and French augmented their profitability and investment factors onto their three-factor model. The author of this study postulated that these additional factors may proxy for quality, a formally undefined characteristic of shares that is used in selecting investments. This study tested the effectiveness of the five-factor model and the additional factors in explaining returns on the Johannesburg Securities Exchange (JSE). The five-factor model was compared to the traditional size-value three-factor model as well as other three-factor models that incorporated the additional factors. Furthermore, the study looked at what premiums are present in the returns on the JSE and captured by the various models tested.

The results show that the size-value and size-profitability three-factor models best describe time-series returns when comparing models. The five-factor model best explains the cross-section of returns and overall, the results identify a strong inverse size effect and value and market premiums. Interestingly, the strength of the original size-value three-factor model is reinforced. The additional factors of profitability and investment contribute to explaining the returns on the JSE. Furthermore, profitability could be seen as a contributing factor in a “quality premium”. In addition to being a risk factor, quality could also be used as a filter through which the traditional premiums like size or value can be unlocked.

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1. Introduction

The development of asset pricing models has been of interest to researchers for many years. The desire to capture the underlying risk factors that drive returns has driven the formation of a variety of models that can, to some degree, capture those risk factors in share price returns. Underpinning asset pricing theory is the belief that markets are either efficient or inefficient (or some combination of both incorporating periods of efficiency and inefficiency). Market participants cite the efficient market hypothesis (EMH) to explain returns, use models that highlight "anomalies" in returns to contradict the EMH, or use a combination of both the EMH and models to better understand returns. One such model is the Capital Asset Pricing Model (CAPM), as theorised collectively (but independently) by Treynor (1961, 1962), Sharpe, (1964), Lintner, (1965), Mossin (1966) and Black (1972), and is based on the work of Harry Markowitz and the concept of mean-variance efficiency. Asset pricing models provide useful information in explaining returns by capturing the underlying risk factors that influence returns. Proxies for the risk factors are used as it is unclear as to what the underlying factors actually are. A model incorporating this idea was formulated by Eugene Fama and Kenneth French in 1993 and includes factors capturing the size and value effects that the CAPM was unable to explain. This was seen as a major improvement on the CAPM and asset pricing as a whole. Since then, both the CAPM and Fama-French three-factor model have been expanded and adjusted to include liquidity, momentum, cash flows, and a variety of other factors that could explain some of the share returns we see on many security exchanges globally. In 2015, Fama and French added the factors of profitability and investment to their three-factor model to improve the explanatory power of their model. These models can be used in asset pricing tests, the prediction of returns, and for calculating the cost of capital, to name but a few of the applications. Of interest to researchers in countries besides the United States is the applicability of the models on exchanges that are not as efficient or developed compared to the United States. Furthermore, the additional factors of profitability and investment can be used to evaluate whether a company could be seen as a 'quality' company. Quality is often referred to when looking for companies to invest in and can be a subjective consideration. The aim of this study is to evaluate the Fama-French five-factor model on the Johannesburg Securities Exchange (JSE) and determine whether the additional factors proxy for quality, a characteristic that is not formally defined but has ties to growth and profitability.

The applicability of the model is tested on the JSE in two ways. Firstly, the relative ability of the model to explain the returns on portfolios sorted on a variety of characteristics is evaluated.

The characteristics include combinations of size and value, profitability or investment. The models are subjected to several test measures in a relative comparison analysis whereby their performance is compared to one another. This will highlight whether the five-factor model offers an improved asset pricing performance compared to other models on the JSE. The models include the CAPM, the five-factor model, and three-factor models using combinations of size and value, profitability and investment. The measures include the pricing error and the proportion of returns left unexplained by the model as in Fama and French (2016). Three-factor models incorporating size and either value or profitability performed best in the relative comparison analysis. The five-factor model showed no improvement on those three-factor models in terms of pricing but performed better than the CAPM. The five-factor model outperformed in terms of adjusted R-squared across all the portfolio sorts tested. Robustness tests confirmed these results and showed that over long estimation periods, the investment factor improved on the ability of the CAPM and other three-factor models in explaining returns. The additional factors, overall, improved on the explanatory power of the models, while the traditional size-value model still performed well. The five-factor model did not offer an improvement when tested on JSE listed companies.

The second part of the analysis tested whether the various models could price the cross-section of returns, whether there are still significant premiums associated with size and value on the JSE, and whether the new factors are significantly priced. The five-factor model best captured the variety of cross-sectional premiums associated with the market, size, value, profitability and investment factors. A significant inverse size premium was found, and the models captured a significant value premium in returns on the JSE. Furthermore, the market premium was also significant, but beta showed an inverse relationship with returns. Premiums associated with profitability and investment were captured, but not to the same extent as the others. Furthermore, investment premiums were captured more significantly over longer time periods.

A final postulation in testing the five-factor model was that the new characteristics may proxy for quality. Quality is not formally defined and thus the new factors may shed light on how quality is priced into returns, given that asset base growth and strong profitability are two of the signs of a quality company. Evidence of a consistent profitability or investment premium may mean that quality is priced into returns. In general, robust profitability is significantly priced, which means that as an aspect of quality, it is sought out by the market. Both conservative and aggressive investment is significantly priced. Thus, determining whether it proxies for quality is difficult because it depends on the interpretation of the premiums.

Therefore, the author concludes that it will not proxy for quality unless more thorough analysis is done on what drives the investment premium.

This study proceeds as follows: section 2 reviews prior literature on the topic, including asset pricing and risk factors from a local and international perspective. Section 3 outlines the methodology employed in this study. Sections 4 and 5 present the results and related robustness tests. Section 6 summarises the findings and provides a conclusion to the study.

2. Literature Review

2.1 The Capital Asset Pricing Model and Efficient Markets Hypothesis

The Capital Asset Pricing Model (CAPM) was built on portfolio theory as developed by Harry Markowitz. Markowitz (1959) outlined that investors are rational, utility maximising, and will choose an efficient portfolio based on the mean and variance of the expected return. Investors will seek the lowest variance for a given return, and the highest return for a given variance (this is mean-variance efficiency). This relationship between risk and return drove the formation of the CAPM; however, the CAPM is attributable to several authors who each contributed independently to the formation of the CAPM as it is known today.

The efficient markets hypothesis (EMH) poses that a security's price "fully reflects" all available information. Thus, it should be impossible to outperform the market and expected prices should not be predictable. Prices will change instantly in reaction to new information. The CAPM and EMH can be seen to be linked in that for the CAPM to hold, the market must be efficient at allocating capital. The CAPM and EMH are discussed further below.

2.1.1 The origins

Treynor (1961, 1962) is credited by French (2003) as a significant contribution to the development of the CAPM due to the development of an expected return comprised of the return on capital at the risk-free rate and the return due to risk premia. The CAPM has often been quoted as the Sharpe, (1964); Lintner, (1965) (Sharpe-Lintner) CAPM due to their respective contributions in establishing the model. The Sharpe-Lintner CAPM establishes that the expected return is a function of the risk-free rate and the sensitivity to the excess return on the market portfolio. Mossin (1966) outlined a model of general equilibrium of exchange in a market for risky assets, including the introduction of compensation per unit of risk for an asset's return. A final, notable contribution was made by Black (1972) where restricting the borrowing capacity of investors produced a model of the CAPM that was more consistent with empirical results.

The CAPM is represented by the following formula:

$$E(R)_i = R_f + B_i[E(R_m) - R_f], \qquad (1)$$

where $E(R)_i$ is the expected return on the share, i; R_f is the risk-free rate; $E(R_m)$ is the expected return on the market portfolio of assets; and $[E(R_m) - R_f]$ is the market risk premium. B_i, the sensitivity of share i to the excess return on the market, can be decomposed as follows:

$$B_i = \frac{Cov(R_i, R_m)}{Var(R_m)}, \tag{2}$$

where beta is the covariance of share i with the return on the market divided by the variance of the return on the market portfolio.

As stated above, the relationship between expected return and risk is linear. Risk can be defined as having two components: a systematic component and an idiosyncratic component. Systematic risk is undiversifiable risk that is pervasive in the market. These risks are faced by all investors and include economic or political risks. Beta captures the systematic risk of a share. If the CAPM holds, all impacts of systematic risk should be captured by the sensitivity to the market premium. Idiosyncratic risk is the risk that is specific to a company or share. This type of risk is borne out of a company's size, operations, industry, or any other company specific factor that increases the uncertainty of investing in that share. Idiosyncratic risk is thus diversifiable and can therefore be reduced. The CAPM does not price idiosyncratic risk as it is assumed that by holding the market portfolio, all idiosyncratic risk is diversified away. Thus, all that is relevant is the sensitivity of that share to the excess return on the market portfolio. Therefore, one can only earn greater returns by taking on more risk (Sharpe, 1964) (in other words, by having an increased sensitivity to the excess market return).

In order for this to work, the market should allocate capital efficiently. In 1965, Eugene Fama looked at the theory of random walks. An efficient market was defined as: "a market where there are large numbers of rational, profit-maximisers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants" (Fama, 1965a, p.76). Significantly, it contributed that the price changes are almost instantaneous and that the effect of new information is reflected instantly in prices. The implication of this is that the successive price changes are independent and identically distributed, and thus prices follow a random walk; therefore, historical prices cannot be used to predict future prices. It was said that unless the investor can consistently outperform the market, the random walk hypothesis holds. Fama (1965b) presented substantially strong evidence of the random walk hypothesis holding true using the daily prices for the Dow-Jones Industrial Average from 1956/58 to 1962. Around the same time, Samuelson (1965) formed a general theory that showed that the next period's price difference was shown to be uncorrelated with the previous period's price differences. This analysis focused on the martingale property of changes as opposed to a random walk. He found that there was no way to use prior price history to profit in futures trading.

Fama (1970) further defined an efficient market as one in which, at any time, security prices "fully reflect" all available information. Weak form, semi-strong form, and strong form efficiency was also tested. Weak form efficiency is where historical price changes are already reflected in the price, and thus cannot be used to forecast future prices. Semi-strong form efficiency means that all publicly available information is reflected in the price and that prices change instantly to reflect newly available information; therefore, analysis of publicly available information (such as fundamentals) will not predict future stock prices. Strong form efficiency means that all information is reflected, including inside information; therefore, no information, both public and inside information, could be used to forecast prices in the future. Fama (1970) looked at both the random walk and submartingale model, where the expected value of the next period's price is greater than or equal to the current price given the information set. While not expected to be literally true, the theory held firm in tests. Weak form tests resulted in strong support for the EMH. Using stock splits, it was found that the price adjustment to the split was efficient and 'fully reflected' the publicly available information. Finally, strong form tests of efficiency mostly held true as it was found that while specialists and corporate insiders had monopolistic access to information and could have used this to profit, there was no trickle-down effect to the rest of the investment community. Thus, strong form efficiency will hold for the majority of other participants (Fama, 1970).

The underlying risk factors that influence the flow of information cause share prices to fluctuate. The fluctuations are determined by the content of the information and timing thereof. The flow of information is random and will cause prices to move as and when the information event occurs in the market. That information is, theoretically, instantaneously imputed into prices. It follows that a company which is less sensitive to the market and these information events will feel a smaller impact on its returns compared to companies that are more sensitive to the market. This idea has been assessed through testing the implications of the CAPM and EMH and is discussed next.

2.1.2 Testing the CAPM and EMH

The CAPM can be seen as the foundation of asset pricing; however, it is not a perfect model. Criticism is aimed at the formation and implications of the model (and simplifying assumptions) as well the empirical failings of the model when it is tested. Fama and French (2004) reviewed the theory and empirical evidence relating to the CAPM and noted that tests of the CAPM are based on three implications of the relation between beta and expected return inferred by the model. Firstly, expected returns are linearly related to beta and no other variables can explain returns. Secondly, the expected return on the market portfolio always exceeds the expected return on shares that are uncorrelated with the market return as beta is positive. Finally, those shares uncorrelated with the market (where beta is zero) will only return the risk-free rate (Fama & French, 2004).

Tests of the EMH often involve the use of a model to disprove some of the implications of the EMH. The opposite is also true: tests of asset pricing models require an assumption of market efficiency, inextricably linking asset pricing tests and the EMH. In tests of the EMH using CAPM or another model, the joint hypothesis problem is ever present: the results from a test of market efficiency show that either the market is inefficient, the asset pricing model is misspecified or incomplete, or both (Roll, 1977). Therefore, theoretically, it may be impossible to test for market efficiency or truly test an asset pricing model. Despite these problems, tests of the CAPM and EMH do provide useful information regarding stock returns and what drives them. The efficiency of the market implies that those who bear more risk should earn higher returns. This linear relationship informs the CAPM and the linear relationship between risk and return; if the market is efficient, it will reward those who bear more risk.

Roll (1977) outlined that while tests have been performed on the validity of the CAPM, it is not possible to wholly and accurately test it. He concluded that the only test that is possible is to see whether the market portfolio is mean-variance efficient. The concept of a market portfolio is difficult to define as the 'investable market' can be broader than just the financial market (Roll, 1977). Thus, investors use a proxy for the market that is usually an index of the market being tested. Therefore, one could argue that tests of the CAPM merely test the mean-variance efficiency of the proxy (Fama & French, 2004). This may lead to a result that says that the CAPM holds; however, the true market portfolio may not be mean-variance efficient. However, even when viewing the proxy as the market portfolio (and accepting that if the proxy is mean-variance efficient, the market is mean-variance efficient) tests have found that CAPM does not estimate the returns well. Following this, beta and expected return will only hold if the market portfolio is mean-variance efficient (Roll, 1977). Despite these shortcomings, using assumptions such as the proxy equates to the true market portfolio allows for a degree of testing to be done.

Furthermore, several early studies cast doubt over the CAPM's ability to explain returns. This includes evidence of the value effect in Basu (1977), the size effect in Banz (1981) and momentum in stock returns in Jegadeesh and Titman (1993). The Fama and French (2004) review of the CAPM highlighted several cases where the CAPM fell short. The review outlined that in both cross-sectional and time-series tests, there was a positive relation between beta and return, but the relation was too small (the slope is too "flat"). Using data from the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and Nasdaq Stock Market (NASDAQ), Fama and French (2004) formed ten value-weight portfolios based on estimated pre-ranking betas and calculated the monthly returns for the next 12 months from 1928 to 2003. These return results were plotted against the post-ranking beta, estimated by regressing those monthly returns on the return of the Centre for Research in Security Prices (CRSP) value-weighted portfolio of U.S. stocks. The results showed that the returns on the low beta portfolios were too high, and the returns on the high beta portfolios were too low; the relation between beta and return was too flat. Fama and French (2006) noted that from 1926 to 1963, the CAPM could capture the size and value premiums. However, from 1963 to 2004, beta could not capture the variation in expected returns due to size or book-to-market or a risk factor linked to them.

Using prior research that tested the CAPM on U.S. stock exchanges as a base, van Rensburg and Robertson (2003) noted that the beta was inversely related to returns. Using one-way size sorted quintile portfolios, small size companies earned a higher return on the JSE but had a lower beta. This was a direct contradiction of the notion that higher returns can only be earned by taking on more risk (i.e. a higher beta). Portfolios comprising of low price-to-earnings stocks also had lower betas but higher returns. Sorting on size and the price-to-earnings ratio resulted in support for at least two style-based factors present in the cross-section of returns on the JSE; this contradicted the idea that returns are only related to the exposure to the excess returns on the market portfolio.

Strugnell, Gilbert, and Kruger (2011) built on van Rensburg and Robertson (2003) with a larger sample (just over thirteen and a half years versus ten years respectively), different but slightly overlapped time periods (January 1994 to October 2007 versus July 1990 to June 2000) and altered methodology for beta estimation for comparison to the van Rensburg and Robertson (2003) methodology. When using the same methodology, they also found that the size and value effects exist on the JSE and that beta was inversely related to returns. However, when using the Dimension Aggregated Coefficients method that takes thin trading effects and ordinary least squares (OLS) estimation weakness into account by incorporating lags, the relation between beta and return lost its statistical significance. It was concluded that beta had no predictive power on the JSE, thus invalidating the CAPM as a returns model on the JSE.

Ward and Muller (2012) tested the appropriateness of the CAPM on the JSE using four different methodologies. They found that there was an inverse relationship between the estimated beta and returns on the JSE for their sample period of 1985-2011. This contradicted the CAPM where beta should be positive. Charteris (2014) analysed the influence of bull and bear markets on the measure of beta. When adjusted for market segmentation, it was seen that there was a positive relationship between risk and return in bull markets and a negative relationship between risk and return in bear markets. It was proposed that the beta of a share can vary between each market (therefore, each share has dual beta). While not significant over the whole sample, the industrials and financials sample showed that the estimated dual beta was significant in explaining the variation in returns for the sample (January 2000 to December 2009).

2.1.3 Adjustments to the CAPM

Adjustments to the CAPM have been proposed to improve its explanatory power. One such adjustment to the CAPM beta can be made using cash flow betas (Cohen, Polk, & Vuolteenaho, 2009). Using cash flow betas (measured by regressing the firm's log return on equity (ROE) on the market's log of ROE), the authors found that the CAPM explained the prices and long-term returns on price-to-book sorted portfolios. Furthermore, these betas aligned closely with the long-term value and growth returns in the study and were allowed to change with the implied changes in cash flow as time progressed. Page and Auret (2014) applied this methodology to the JSE over the period January 1995 to December 2011 and found that cash flow betas explained the value effect but not the size effect. Finally, Kolari, Liu, and Pynnonen (2016) derived a covariance adjustment term and specified what they termed as an *observed market beta* for each stock. As opposed to being related to the market portfolio, this beta was computed as a theoretical approximation of the betas between a stock and each of the other stocks in the market (pairwise betas). The traditional market beta multiplied by a simple covariance adjustment resulted in the *observed market beta*. Using U.S. stock returns from February 1959 to December 2014, this beta was tested in the CAPM. The observed market beta resulted in strong support for the CAPM even after various cross-sectional and robustness tests. The traditional market beta resulted in a rejection of the CAPM. Adapting Liu (2006) to the JSE, McClelland (2014) applied a liquidity augmented two-factor CAPM to explain the premiums associated with size, value, liquidity and beta portfolios. This was relevant to the

JSE given the concerns regarding the liquidity of the market. It was found that this model adequately captured the returns and better explained the value premium compared to the traditional CAPM and Fama-French three-factor model.

It can be said that while there are issues with the CAPM as it is known, adjusting the beta in the model yields some support for the CAPM. However, there are underlying risk factors associated with certain anomalies that the CAPM cannot explain; this prompted the formulation of other models that seem to better capture the returns of exchange traded shares. Furthermore, these anomalies seem to contradict the EMH to some degree as they provide an investing methodology that, during some time periods, provides returns over and above the market return. This is contrary to the idea that public information cannot be used to earn excess returns. These anomalies in returns (risk factors) could be as a result of market inefficiency or model misspecification. Nonetheless, they are relevant in explaining returns. Some have termed these effects as anomalies as they go against the CAPM or market efficiency depending on the test. However, it has also been said that these are proxies for common risk factors affecting all firms in the market (see Fama and French (1993)). Thus, these risk factors are undiversifiable, if they are indeed systematic. There have been many documented anomalies/risk factors that have led to challenges to the CAPM and EMH as well as the development of new models for explaining returns. These are discussed next.

2.1.4 Anomalies, risk factors, and challenges to the EMH

Benjamin Graham and David Dodd introduced a theory of investing in their 1934 book: *Security Analysis*. Value investing has been derived from Graham and Dodd (1934) and involves buying stocks that are priced at less than their intrinsic values as indicated by fundamental analysis (the discount between the price and the minimum intrinsic value as determined by an analyst is part of the 'margin of safety' principle of stock selection[1]). The 'value effect' is where stocks underpriced in terms of their fundamental values earn returns greater than the market over the long term. The value effect is one of the most well-known factors in asset pricing and is one of the explanatory factors in the Fama-French three- and five-factor models (Fama & French, 1993) (Fama & French, 2015).

Basu (1977) studied the price-earnings (P/E) ratio as an indicator of future investment performance using NYSE listed stocks from 1956 to 1971. The P/E ratio was chosen as it was hypothesised that due to exaggerated investor expectations, the P/E ratio could be an indicator

[1] *"Security Analysis", Graham and Dodd (1934), p. 372-373*

of future performance. It was thought that low P/E stocks perform better than suggested by the underlying risks; thus, the returns from a low P/E portfolio were compared to a portfolio of randomly selected stocks with the same overall risk. It was found that low P/E stocks earned greater risk-adjusted abnormal returns than high P/E stocks. Therefore, there seemed to be information in the P/E ratio that was not instantaneously incorporated into the stock price. This was early empirical evidence of the value effect where low P/E stocks (value stocks) should outperform high P/E stocks which are known as growth stocks. Growth stocks have lower expected returns as future growth is already priced in.

Roll and Ross (1980) empirically tested the arbitrage pricing theory where more than one risk factor is present in the return generating process. It was found that a stock's expected return was related to the covariance between its return and the factors extracted from a multivariate regression, with suitable loadings on those factors (French, Schwert, & Stambaugh, 1987). There were thus risk premia attached to these factors that influenced a stock's return. It was noted that this model could include the P/E ratio to explain returns as found by Basu (1977).

Banz (1981) examined the relationship between return and the total market value of a stock (the market capitalisation). Using NYSE stocks, it was found that small stocks had significantly higher risk-adjusted returns than large firms over the period 1936-1975; although, it was not stable through time. This result was termed the 'size effect'. Markedly, there was no theoretical foundation for the size effect and thus it was noted that the effect may be a risk factor in itself or it could be a proxy for a more appropriate factor that was correlated with size. Both P/E and book-to-market were observed to be proxies for the size effect. Banz (1981) also pointed out that the effect could have been due to model misspecification. The size effect is also one of the most tested and well-known risk factors in return analysis literature and is a main factor in the Fama-French three- and five-factor models (Fama & French, 1993) (Fama & French, 2015).

Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) found that U.S. stock returns were positively related to the book-to-market ratio. Rosenberg et al. (1985) employed a strategy of buying stocks with high book-to-market ratios and selling stocks with low book-to-market ratios. Although the period tested was short (mostly NYSE data from 1980 to 1984 was used), the returns on the strategy were statistically significant. It was noted by the authors that this result rejected the null hypothesis of an efficient market and that the strategy used an instrumental variable for pricing error.

Jegadeesh and Titman (1993) documented what is now known as the momentum effect, where stocks that have performed well in the past perform well in the future, and vice versa for poorly performing stocks. This is contrary to De Bondt and Thaler (1985) who hypothesized that investors overreact to unexpected news; this resulted in a mean reversion effect where initially prices would move substantially to the news, then revert back to a more appropriate level once the overreaction had subsided. There was thus a level of predictability in the results that confirmed weak-form inefficiencies in the market. Their test of buying stocks that performed poorly over the previous three to five years resulted in higher returns than buying stocks that had performed well over that period. This, however, could have been due to systematic risk. Jegadeesh and Titman (1993) used NYSE and AMEX stocks over the period 1965 to 1989. Stocks that performed well over the previous three to twelve months were bought and those that performed poorly were sold. The holding period of between three and twelve months and the strategy of buying winners and selling losers yielded abnormal excess returns. However, the effect mostly disappeared with holding periods longer than twelve months. Importantly, the effect was not due to any systematic risk of the strategies; however, the evidence is consistent with delayed price reactions to information specific to the firm.

Fama (1998) reviewed the evidence against the EMH, namely long-term return anomalies, and split them between over- and underreaction. It was found that overreaction and underreaction occurred with the same frequency, thus the net result was long-term efficiency, especially if anomalies are split randomly between under- and overreaction. Methodology also influences whether anomalies are found. When using altered or different models, many anomalies are minimal or eliminated. Thus, many anomalies are attributed to chance. Notably, it is observed that tests of the EMH do not test an alternative to the EMH, but rather simply state that the market is inefficient. It is suggested that an alternative hypothesis be proposed, yet this is not that case. The alternative hypothesis would have to explain biases in information reaction, explain returns better than the original EMH as well as account for the anomalies present. Thus, in the absence of this, Fama (1998) concluded that market efficiency held up well to empirical tests and was still relevant. Further, the bad-model problem still existed as testing the EMH must be done with a model, which itself is subject to misspecification and bias. Finally, while the two behavioural models tested explained the anomalies they were designed to explain, they did not explain any others, which lead to the conclusion that the results were more consistent with the EMH than those two models.

Robins, Sandler, and Durand (1999) observed that from 1986 to 1995, no combination of a value (in this case, price-to-book) and size factor resulted in abnormal risk-adjusted returns; this was in line with the CAPM. An implication from this result was that returns on the JSE adhered to the EMH, bar the January effect (this could have been due to the short data horizon). From 1986 to 1995, there were no abnormal excess returns to be earned using a combination of value and size portfolios. Van Rensburg (2001) decomposed style-based risk for JSE industrial shares from February 1983 to March 1999. It was concluded that earnings yield, momentum, market capitalisation, dividends, leverage, cash-flow-to-debt and turnover were all style factors in JSE returns. These persisted after risk-adjusting returns using an arbitrage pricing theory (APT) model. Overall, there was evidence of a value (E/P), momentum (past twelve months positive returns) and size (market capitalisation) aspect to the returns. When expanding the style factors across sectors, Robertson and van Rensburg (2003) observed that size and various measures of value such as price-to-earnings and cash-flow-to-price were significant in explaining returns across all sectors. Reinforcing the conclusion regarding value in van Rensburg (2001), the value effects were strongest in the industrials and financials sectors.

Malkiel (2003) also looked at the evidence against the EMH. It was noted that there was a difference between statistical and economic significance and that in the majority of cases, the gains to be made from the various strategies to outperform the market will be almost, if not completely, eliminated by transaction costs. A buy-and-hold strategy, for example, performed better than a momentum strategy even when there was significant statistical evidence of momentum. Furthermore, these strategies are not dependable in the long-term. This could also be due to practitioners trying to take advantage of newly found effects that result in these effects disappearing after discovery (Malkiel, 2003). Mean reversion was found to be evident only during certain time periods, and it could be said that reversals are consistent with market efficiency in that prices revert to their 'truer' values. In looking at the January effect, where excess returns are abnormally high in January, it was found that it was not consistent from period to period. Fundamental analysis was examined and it was concluded that these variables' predictive power could be reflecting general economic conditions and were also period dependent. The size and value effects were inconsistent in time and could have been a result of a bad-model problem. The shortcomings of the CAPM in not capturing these underlying factors could lead to these effects being prevalent.

Following this, once-off events that severely impacted prices were analysed as it was hypothesized that if markets were efficient, these events could not have happened. Rational investors could not have set the market prices during these events and that psychological factors must have played a part. Malkiel (2003) considered the crash of October 1987 and the Internet Bubble of the late 1990's. In looking at the October Crash, while noted that it was impossible to ignore behavioural effects, factors such as increasing bond yields, regulatory changes and potential dollar devaluation would have contributed to an adjustment to risk premiums and discount rates, which could partly explain the large drop in prices. In terms of the Internet bubble, in retrospect it was clear that the allocation of capital to technology companies was exaggerated, and that the forecast growth rates that impacted the valuations were incorrect. However, Malkiel (2003) noted that given the growth of the Internet at the time, no one was entirely sure about the possible growth in the technology. It was observed that even the most highly respected independent analysts were recommending technology companies. In addition, no predictable profitable arbitrage opportunities were present. While it was conceded that prices were incorrect for a time, the bubble was noted as "the exception rather than the rule, and acceptance of such occasional mistakes is the necessary price of a flexible market system that usually does a very effective job of allocating capital to its most productive users" (Malkiel, 2003, p.76).

Finally, in both Malkiel (2003) and Malkiel (2005), the performance of investment managers was tested to ascertain whether or not they can beat the market. It was concluded that these managed funds did not beat the market/benchmark fund consistently, and this was evidence that in general, market prices reflect all available information. Lewellen (2014) investigated the cross-sectional properties of forecasts of returns based on Fama-MacBeth regressions. It was found that the forecasts held significant power in predicting actual stock returns over the period tested, contrary to the ideas of the EMH.

Pástor and Stambaugh (2003) investigated whether market-wide liquidity was an explanatory variable for returns. The variable was based on the average of a stock's liquidity measures and incorporated signed volume where signed volume was viewed as order flow. Thus, lower liquidity was reflected in an order flow in one direction for the day to be followed by a price change in the opposite direction the following day. It was observed that returns were sensitive to liquidity and that higher sensitivity to liquidity resulted in greater returns.

Cubbin, Eidne, Firer, and Gilbert (2006) investigated mean reversion on the JSE as this is viewed as a key challenge to the EMH. The period analysed was 31 October 1983 to 31 December 2005. The data was corrected for survivorship bias and the winner/loser portfolios were sorted using the P/E ratio, which was interpreted as an advantage as P/E's are forward looking and thus determine the expected attractiveness of the share. The result confirmed mean reversion in that the loser portfolios greatly outperformed the winner portfolios, and further suggested that a value strategy would produce returns above the expected market return which was contrary to the EMH.

Fama and French (2008) revisited the cross-section of returns by evaluating prominent anomalies such as momentum and net stock issues. It was found that they are still pervasive whereas growth and profitability are less so. They also further reinforced the idea that the anomalies seen were due to underlying risk factors or mispricing.

Investment styles are often inconsistent and cannot be profitably exploited due to this inconsistency. Over the period 1 January 1997 to 31 December 2007, it was observed that firm specific attributes in terms of fundamental values relative to price, fundamental growth, size, return momentum and analyst forecasts resulted in consistently higher returns for JSE listed shares in the following period (Hodnett, Hsieh, & van Rensburg, 2012). In investigating the consistency of style anomalies, Kruger and Toerien (2014) compared the consistency of anomalies in JSE returns during a crisis with a non-crisis period. Most previously identified anomalies such as the size effect, dividend yield, cash-flow-to-price and the book-to-market ratio (eight anomalies were tested in total) were significant and consistent over the stable period (February 2002 to October 2007). However, during the crisis period from November 2007 to December 2009, it was found that only cash-flow-to-price was significant and stable, which highlighted the fact that anomalies do not hold up well in periods of crisis.

Van Heerden and van Rensburg (2015) investigated the cross-section of returns of JSE listed stocks from 1997 to 2011 and found strong and consistent value and momentum effects and an inconsistent size effect, confirming the conclusion of Kruger and Toerien (2014) that anomalies can be inconsistent. It also confirmed that cash-flow-to-price is a consistent value factor. Van Heerden and van Rensburg (2015) noted that either the JSE is inefficient or the models to measure returns are misspecified.

When testing the interaction between long-term reversal and value, Britten, Page, and Auret (2016) examined the overreaction hypothesis where recent events are more heavily weighted in investors' minds and they subsequently ignore historical fundamental information. This means that over the long term, prices should be reasonably predictable as they tend towards their actual fundamental value. It was found that there was significant overreaction in winner and loser portfolios and that value and overreaction were independent of each other. This confirmed the result in Cubbin et al. (2006).

As mentioned previously, size and value are two risk factors that are prevalent on local and international exchanges. The derivation of the Fama-French three-factor model, in which size and value factors are present, as well as the theory and application of the size and value effects, is presented next.

2.2 The Fama-French three-factor model

Fama and French (1992) informed the formulation of the Fama-French three-factor model (FF3F). The authors investigated which variables explained the cross-section of average stock returns of non-financial firms on the NYSE, AMEX and NASDAQ as listed in CRSP and COMPUSTAT database. The period tested was 1963 to 1990. It was found that size (in terms of market capitalisation) and book-to-market equity captured the cross-section of average returns associated with size, E/P, book-to-market equity and leverage; these had all been found to explain returns in prior literature, contradicting the traditional Sharpe-Lintner CAPM model. Importantly, it was noted that underlying stock risks are proxied by size and the book-to-market ratio; they are not seen as anomalies. This model represented an important expansion on the CAPM and a transition in research from single-factor models to multifactor models.

2.2.1 Forming the model

Fama and French (1993) assessed the common risk factors in stocks and bonds and found that mimicking portfolios for size and book-to-market equity captured the majority of the variation in returns. However, the model mispriced small growth firms' returns, overestimated the returns on small firms and underestimated large firms' returns. The mimicking portfolios were constructed as follows: the stocks in the sample (all NYSE, AMEX and NASDAQ stocks from 1963 to 1991) were ranked on size per the median NYSE size and split into two groups, small and big (S and B). The stocks were also broken into three book-to-market equity groups based on a 30/40/30 split low, medium and high (L, M and H). Six portfolios were formed from this (S/L, S/M, S/H, B/S, B/M, B/H) where, for example, S/L contained small companies with low book-to-market equity ratios. The size factor (SMB) is the average return of the three small-stock portfolios minus the average return of the three big-stock portfolios. The book-to-market factor (HML) is the average returns of the two high book-to-market portfolios minus the average returns for the low book-to-market portfolios. These mimic the risks associated with size and book-to-market. The market factor is the excess return on the market (return on the market less the risk-free rate). A size, book-to-market equity and market factor explained the returns in the three-factor model and can be shown as follows:

$$R = a + R_f + b\left[R_M - R_f\right] + s * SMB + h * HML + e, \tag{3}$$

where the return R can be explained by the risk-free rate R_f, the excess return on the market $R_M - R_f$, the size factor SMB, and book-to-market equity factor HML. The sensitivities to each factor are given by b, s, h respectively. The size and book-to-market factor capture risks associated with size and book-to-market while the market factor explains the additional return earned above the risk-free rate. It can be rewritten as follows to incorporate different factors for the size and value effects:

$$R = R_f + b\lambda^{Market} + s\lambda^{Size} + h\lambda^{Value}, \tag{4}$$

where b, s, h are the sensitivities to each factor, R_f is the risk-free rate, λ^{Market}, λ^{Size}, λ^{Value} represent the premiums on each type of risk.

This model has provided a new way to explain returns and has been applied in a variety of contexts. It also allows for further examination of the size and value effects as the factors in the model proxy for these effects or underlying risks associated with the effects (for example, HML can be replaced with book-to-market equity or the P/E ratio to measure value). The applications of the Fama-French three-factor model are analysed next along with further discussion on the size and value effects.

2.2.2 Applications of the Fama-French three-factor model

As the size and value effects could be interpreted proxies for underlying risk factors, Fama and French (1995) looked at whether size and book-to-market equity reflected the behaviour of earnings. It was found that book-to-market equity was linked to earnings in that a high book-to-market ratio was linked to lower earnings (a sign of distress). These companies were less profitable for four years before and five years after ranking date. Size was related to earnings (small stocks have low earnings); however, this effect was only seen post 1980, during and after the recession in the two years following 1980. This study showed evidence of how size and value factors linked to economic fundamentals like earnings.

A value premium has been observed in markets around the world (Fama & French, 1998). Value stocks had higher returns than growth stocks for the test period of 1975 to 1995 when using the book-to-market ratio, earnings-to-price ratio, cash-flow-to-price ratio and dividend-to-price ratio to proxy for the value effect. Twelve of the thirteen markets tested across the U.S., Europe, Australia and the Far East showed that value stocks had higher returns than growth stocks (Fama & French, 1998). It was also noted that a risk factor for relative distress captured the value premium in country and global returns. Linked to distress, Gharghori, Chan, and Faff (2007) examined whether the size and value factors in the FF3F were proxying for default risk. Using equities from the Australian Securities Exchange, it was found that default risk was not priced into equity returns, contrary to the suggestions of previous literature. Therefore, the risk that the value and size factors might be capturing is unknown. Furthermore, the value and size factors in the FF3F did not proxy for default risk. Gharghori et al. (2007) also concluded that the FF3F was far superior to the CAPM in explaining returns. The results could be due to differences in the underlying risks between the U.S. and Australian stock markets.

Carhart (1997) extended the FF3F by incorporating an additional momentum factor to the FF3F factors that enabled a better capturing of the returns of mutual fund performance from January 1962 to December 1993. Carhart's momentum factor is formed by taking the returns on a portfolio of winner shares and subtracting from it the returns on a portfolio of loser shares. Winner and loser shares are the shares that performed the best or worst, respectively, in the last twelve months (they are categorised by percentiles: winner shares will be in the top 30% of returns and loser shares will be in the bottom 30% of returns). By extension and following up on prior international research, Fama and French (2012) investigated size, value and momentum in international stock returns. It was observed that there was a value premium in all regions tested (North America, Europe, Japan, Asia Pacific) and momentum was evident in all regions expect Japan. Value premiums were larger for small stocks and momentum returns declined as company size declined. The four-factor model explained global size-value and size-momentum portfolio returns well, but could not be used for regional returns. Local four-factor models captured the returns on size-value portfolios but not size-momentum portfolios. While the Carhart four-factor model reasonably captured returns, its usefulness for international returns was limited.

Robertson and van Rensburg (2003) concluded that a size and price-to-earnings based model captured most of the returns across the JSE. This was consistent with the FF3F where size and value captured the cross-section of returns. In a comparative study, Auret and Sinclaire (2006) studied the role of book-to-market equity in explaining returns on the JSE. It was found that there was a significant positive relationship between book-to-market equity and returns. When included in the Robertson and van Rensburg (2003) size and P/E model, the book-to-market factor actually subsumed the effect of size and P/E in the regressions and had more explanatory power.

Further support for the size and value effects is found in Basiewicz and Auret (2009) which re-examined the cross-section of returns on the JSE. It was confirmed that there is a size and value premia that is present even after adjusting for transaction costs. The magnitudes of the effects were smaller than in previous studies due to transaction costs and illiquidity; the size premium was most affected by this. Confirming Fama and French (1993), it was observed that the book-to-market ratio best captured the underlying risk associated with value and subsequently the value premium.

Contrary to the above, Bartens and Hassan (2009) tested real-time trading strategies based on size, value and momentum on the JSE. They found that the strategy would not outperform a passive index of JSE stocks. It was concluded that the risk factors were not stable in explaining returns. It is noted that both studies cover virtually the same periods: Basiewicz and Auret (2009) used data from December 1989 to July 2005 while Bartens and Hassan (2009) covered July 1987 to June 2004. The results may therefore be due to the different methodologies where Basiewicz and Auret (2009) use Fama-MacBeth regressions and Bartens and Hassan (2009) employ a recursive out-of-sample method using simulated out-of-sample returns.

Basiewicz and Auret (2010) go on to directly test the FF3F on JSE listed shares in comparison to the CAPM and an APT model. The FF3F generally explained the returns on the JSE over the period June 1992 to July 2005. In time-series tests, the FF3F accounted for the value and size effects for grouped data; however, the size effect was less prominent on ungrouped data. It was noted that when testing the model on the JSE, small, value firms were mispriced whereas in Fama and French (1993) small, growth firms were mispriced. Furthermore, the model overestimated the large firms' returns and underestimated small firms' returns, the opposite effect to the U.S. data. Despite this, the magnitudes of the premia were very similar between the U.S. and South African markets.

Strugnell et al. (2011) confirmed that there was a size and value effect on the JSE, but noted that there was a reduction in the size premium over time in the period 1994 to 2007. The size effect was concentrated in the smallest stocks while the value effect was pervasive across all the shares tested. The authors noted that transaction costs may have eliminated the economic significance of their results. In stark contradiction to this study and others, Auret and Cline (2011) expanded on Robins et al. (1999) in looking at the value, size and January effects on the JSE. Despite a similar test period of January 1996 to December 2006 (which was the second period tested, the first being January 1988 to December 1995), there was no significant size, value or January effects in either period tested. It was noted that this could have been due different datasets being used.

The results in Hoffman (2012) support the notion that the Fama-French factors and momentum can explain returns on the JSE. It was concluded that size, book-to-market equity and momentum were all significant in capturing the underlying risks in share returns. This study used dividend-adjusted returns, accounted for survivorship bias and corporate actions, and used a market value and equal weighted portfolio methodology. Muller and Ward (2013) used a graphical time-series approach to investigate style-based effects on the JSE. The size effect was not significant, possibly due to excluding the smallest stocks from the sample to represent the interests of institutional investors. Momentum and some value related effects such as cash-flow-to-price and earnings yield resulted in abnormal excess returns and reinforced the idea that value and momentum capture underlying risks in JSE listed shares.

Inconsistent with the conclusion in Muller and Ward (2013) regarding the size effect, Hammar (2014) found that not only did small capitalisation value stocks generate the highest mean returns in the test period, the size effect prevailed no matter which valuation multiple was used. The value effect was evident when value stocks outperformed growth stocks when sorted based on P/E and cash-flow-to-price. It was concluded that the value and size effects were independent of each other. It was also noted that one of size, value or growth effects did not always outperform the other effects, showing how these effects may be period specific on the JSE.

Further evidence of period specific effects is observed in Hsieh (2015) where it was noted that value effects are weak for the whole sample, despite there being some evidence of a value effect when looking at earnings-to-price and sales-to-price portfolios. The study included the period of the financial crisis of 2007 to 2009 and thus this may have impacted the results. It highlighted that the value effect may be inconsistent on the JSE due to the liquidity issues and transaction costs. The size effect was significant and persistent no matter how the portfolio was formed, and supports the conclusion in Hammar (2014) that size is a prominent factor in returns. Barnard and Bunting (2015) specifically investigated whether the size and value effects were evident during the financial crisis and periods following. Contrasting Hammar (2014), there was no overall size effect during the period; this may have been due to the shorter period tested. Small, value stocks showed evidence of a significant size and value effect, showing that the two factors should be considered together. This may mean that the two effects are becoming less independent over time.

Boamah (2015) directly tested the robustness of the FF3F and Carhart four-factor model on the JSE. It was concluded that there were independent size and value (book-to-market) effects; however, the FF3F only partially captured the size and value effects on the JSE. The Carhart four-factor model also only partially explained returns; however, it performed better than the FF3F due to its capturing of a prevalent momentum factor. Vermeulen (2016) investigated nine prominent explanatory variables for returns on the JSE. It was found that a three-factor model consisting of the book-to-market ratio, dividend payout and leverage explained returns best. This provides further evidence for a value effect and is consistent with Fama and French (1993) in using book-to-market to capture risk factors in returns. In analysing a variety of effects that have been prevalent on the JSE, Flint, Seymour, and Chikurunhe (2016) concluded that momentum has been the most rewarded factor on the JSE. In contradiction to various other studies, the size effect was not prevalent in South Africa from December 2002 to August 2016. This provides evidence that the size effect on its own is becoming less prevalent on the JSE.

There have been noted issues with the FF3F despite its widespread success in asset pricing tests. Risk factors relating to profitability and investment, in addition to the traditional size and value factors, have been hypothesized to impact share returns and informed the formation of the Fama-French five-factor model. This further expansion of multi-factor asset pricing models to include profitability and investment is discussed next.

2.3 Additional risk factors

Li (2004) studied the link between investment, profitability and stock returns from the perspective of overinvesting when managers have greater discretion over their investments. It was found that investment is negatively related to future profitability and stock returns. This was mostly due to overinvestment where managers engage in empire building. Titman, Wei, and Xie (2004) also studied the relation between capital investments and stock returns. They observed a negative relation between abnormal capital investments and future returns. The firms that increased their expenditure the most have the worst returns over the next five years. This effect was independent of previously identified anomalies.

Fama and French (2008) investigated whether asset growth and profitability, among others, could explain anomalous returns. In evaluating microcaps, asset growth had a strong negative relation to returns and was weaker but still significant for small stocks. There was no relation to big stocks. Profitability was noted to contain unique information about returns across company size. For univariate sorts, asset growth and profitability did not show any abnormal returns. It was observed that the results suggest that when controlling for book-to-market equity, higher net cash flows (defined as earnings minus investment, per dollar of book value) implied higher expected returns (Fama & French, 2008). It can be said that the negative relationship between accruals and returns can be explained by the fact that higher accruals means lower net cash flow (high investment relative to earnings); this shows how the variables tested could be seen as proxies for cash flow. The disappearance of the size effect in U.S. stock returns can be attributable to unexpected profitability shocks to U.S. listed firms (Hou & Dijk, 2008). Small firms had negative shocks while large firms had positive shocks to profitability in the mid 1980's. After controlling for the profitability shocks, the size effect was still present. Cash flow shocks thus have a material impact on explaining returns.

Cooper, Gulen, and Schill (2008) found that asset growth at the firm level also had a negative relationship with returns, where low asset growth rates (measured as the growth in total assets) resulted in significant positive abnormal returns, confirming the link between investment and returns documented in prior literature.

Chen and Zhang (2010) proposed that a three-factor model of a market factor, investment factor and return on assets factor better captures return than the traditional FF3F for their more recent dataset. This was confirmed in their results where anomalies that were left unexplained by the FF3F like net stock issues or financial distress were now priced and it performed as well as the FF3F in explaining portfolios based on book-to-market equity.

In contradiction to the majority of the theory of the value premium, whereby firms are seen to be distressed, Novy-Marx (2013) analysed gross profitability as a proxy for the value effect. Measured as gross profit-to-assets, it was found to have the same explanatory power as the book-to-market ratio. It was observed that profitable firms earned significantly higher returns than unprofitable ones despite the higher valuation ratios such as market-to-book and market capitalisation. If one controls for profitability, value-based strategies show dramatically increased performance, especially in large, liquid stocks. These stocks are less distressed, have less operating leverage and longer cash flow durations (Novy-Marx, 2013).

Aharoni, Grundy, and Zeng (2013) provided backing to the theoretical idea that the Miller Modigliani (1961) valuation formula implies a set of relations between future stock returns, current book-to-market, firm-level expected profitability, and firm-level expected investment as observed in Fama and French (2006a). The tests performed in Fama and French (2006a) did not find a negative relation between expected investment and returns as per share measures of expected investment were used; the Miller Modigliani valuation formula is at the firm level not share level. When measuring the variables at the firm level and not the share level, Aharoni, Grundy, and Zeng (2013) found that the negative relation was confirmed and reinforced the theory proposed by (Fama & French, 2006a). This links to Fama and French (2015) where book-to-market equity can be seen as a proxy for expected return.

Additional factors such as profitability and investment, when considered together, could be proxies for quality. Earnings may be an indicator of how well the business is operating and its ability to fund investment and grow can signal strong underlying operations. The concept of quality and its link to asset pricing is discussed next.

2.4 Is quality a common risk factor?

Despite Fama and French (2015) citing the dividend discount model and previous literary evidence of profitability and investment explaining the cross-section of returns as the basis for the additional variables, the additional FF5F factors may be aiding in capturing the returns related to the '*quality*' of the company. The definition and measurement of quality is subjective yet there is an investment ideology termed 'quality investing'. The idea of quality has been around but is followed in a less systematic way than other factors. Quality could refer to the quality of management or a competitive advantage, or to more quantitative aspects like earnings growth, cash generation or earnings quality. The characteristics of what may define quality are analysed next.

2.4.1 The concept of quality

Quality can be ascribed to quantitative and qualitative considerations; however, most literature has focused on the quantitative side. Accounting numbers and ratios can be used to determine proxies for quality such as cash-flow-to-profit (quality of earnings). Good companies (these could also be perceived as quality companies) may not always be good investments as the perception of these companies may be different to their financial health. The use of accounting and other measures of quality are deliberated below along with some analysis of whether a good company is a good investment.

Ou and Penman (1989) performed financial statement analysis that creates one summary measure from the financial statements that predicted whether the one-year-ahead earnings will increase or decrease. An estimate of firm value could be derived from financial statement analysis given the information content in financial statements and that financial statement information is priced into securities (Ball & Brown, 1968). The authors highlighted that fundamental analysis yielded an intrinsic value against which a market price could be compared, and abnormal returns could be earned by identifying undervalued stocks (Ou & Penman, 1989). The analysis period was 1973-1983 using the firms in the COMPUSTAT database. Long positions were taken where the measure indicated that future earnings were higher and short positions taken where earnings would be negative (zero net investment). It was found that the measure predicted stock returns that are robust to risk factors such as size, leverage, market-to-book and E/P ratio. However, as one part of their study, Holthausen and Larcker (1992) implemented the same measure as Ou and Penman (1989) over 1978-1988 and found that the strategy did not earn excess returns. This was most likely due to differing time periods. Holthausen and Larcker (1992) then used a different approach by directly predicting the sign of one-year excess returns from accounting ratios. Again, a zero-investment strategy was taken where long positions were taken in firms with positive predicted excess returns and short positions in firms with negative predicted excess returns. The period analysed was 1978-1988 which contain NYSE, AMEX and over-the-counter (non-listed) firms. The strategy yielded significant positive excess returns showing the ability of financial statement information to help forecast future returns. Greig (1992) also looked at Ou and Penman (1989) and found that after controlling for cross-sectional differences in CAPM beta and firm size, their zero investment strategy no longer yielded abnormal excess returns. The summary measure was seen as a proxy for expected return differences rather than evidence of a systematic underreaction to the direction of future earnings indicated by the financial statements (Greig, 1992).

The sustainable earnings of a company have been used to analyse the quality of the operations. Booth, Broussard, and Loistl (1997) investigated the relationship between earnings and stock returns in Germany by analysing the links between various measures of earnings and stock returns. It was observed that the change in earnings due to adjusting for once-off and extraordinary items as per the Deutsche Vereinigung für Finanzanalyse und Analgeberatung (DVFA) (similar to the headline earnings adjustments) were significant in predicting share price returns. Furthermore, the change in the difference between published earnings and DVFA earnings was also significant. There is thus information content in the earnings that could signal the quality and sustainability of the operations.

Antunovich and Laster (1998) studied whether investors confuse the quality of the firm with its appeal as an investment. It was hypothesized that if investors do confuse the two, then well-managed companies will have their prices bid up too high; subsequently, these companies earn negative abnormal returns. Contrary to this, it was found that the most admired firms per the Fortune Magazine survey of ‘America’s Most Admired Companies’ earned positive abnormal returns for at least three years after date of publication. These results were persistent and significant and outlined the return behaviour of good companies. Anderson and Smith's (2006) findings support the conclusion in Antunovich and Laster (1998) by looking at the returns of companies identified in the Fortune Magazine survey from 1983 to 2004. A portfolio of the most admired stocks substantially and significantly outperformed the market.

In further examination of this idea, Shefrin (2001) analysed whether investors expect higher returns from safer or riskier stocks. Traditional finance theory suggests that riskier stocks should earn higher returns. Using a survey similar to Fortune Magazine’s, participants were asked to judge the riskiness of the stock on a scale from 1 to 10 and to specify the expected return for the stock over the next year. It was observed that participants expected lower returns from riskier stocks, a result that means that beta has a negative relationship with return. It was concluded that the behavioural heuristic representativeness was the cause of this result (Shefrin, 2001). Representativeness is the over-reliance on stereotypes where, in an investing context, it is believed that “good stocks are stocks of good companies”[2]. Shefrin (2001) tested this by comparing the answers to the questions in the Fortune survey with his own; the questions related to the goodness of the company and the goodness of the stock. The idea that good companies are good stocks was supported as the responses to the questions correlated with the survey at 90%. The underlying reason stemmed from companies having a sound financial position; these were seen as safe companies. Therefore, investors saw good companies as safe companies that were well managed. Higher returns were thus associated with these companies

[2] *Page 179, “Do Investors Expect Higher Returns From Safer Stocks Than From Riskier Stocks?”, Shefrin (2001)*

due to the influence of representativeness. Kaustia, Laukkanen, and Puttonen (2009) surveyed Finnish financial advisers regarding the risk premium required and the expected returns in separate analyses. It was found that risk factors (such as leverage) caused inconsistent responses in each analysis (i.e. it may cause a response of an increase in risk premium but lower expected returns). The results showed that safe stocks were associated with lower discount rates (higher valuations) but also higher expected returns. Supporting the above, it seems that companies perceived as good are associated with higher returns; however, this depends on the perspective of the question and how it is framed. The perception that safe stocks generate greater returns may stem from Fama and French (1993) where the default risk and the term spread capture the cross-section of returns when a market factor is excluded. The inherent risk in the company may be reflected by its loading on the default risk factor, which is formed based on a portfolio corporate bond returns that varies depending on the economic climate (which in turn impacts company profitability and returns). The return on those bonds individually reflects the riskiness of the company. This also ties into optimal levels of leverage when considering quality.

Penman and Zhang (2002) linked accounting conservatism to earnings quality and returns. Earnings were of a higher quality if conservatism in accounting is practised. Sustainable earnings are perceived as high quality as they better represent the future earnings of the company (there are no once-off items). An earnings quality score was formulated per company and it was observed that this predicted future stock returns after controlling for common risk factors.

Another indication of earnings quality is the level of accruals and the relationship with increases in earnings. Accruals are the difference between earnings and underlying cash flow. Earnings increases with high accruals are considered poor quality earnings due to the decreased cash generation. Chan, Chan, Jegadeesh, and Lakonishok (2006) tested this and concluded that accruals were negatively related to future stock returns. Thus, high quality earnings earned greater returns; high quality earnings that are backed by cash flows are indicative of a quality company. This was supported in Fama and French (2008) where abnormal returns were negatively related to accruals. The reversal of large accruals also explained stock returns after growing accruals (Allen, Larson, & Sloan, 2010).

Gallagher, Gardner, Schmidt, and Walter (2014) studied quality investing in the Australian market. The measure of quality used was the aggregate of the following fundamental accounting metrics: return on equity, return on assets, operating cash flow, asset turnover, shares outstanding and total equity. Investing in the highest quality stocks produced significant excess returns of 6.37% while investing in the poorest quality stocks produced -7.98%. Excess returns were significant across all size groups but strongest in the small stocks (micro, small and large were the three size groups). Kalesnik and Kose (2014) tested U.S. stocks and supported this finding by observing that high quality stocks earned positive excess returns but only when combined with a value tilt. This is further supported in Novy-Marx (2014) where quality, value stocks significantly outperformed value-only strategies. It was noted that gross profitability was a strong indicator of quality and persists in large stocks as well. The specification of quality does not seem to impact the quality premium. It was also noted that a quality strategy performs well in an economic downturn and is inversely related to value and thus quality and value can lead to beneficial diversification.

McLachlan (2014) outlines that when trying to find a quality company that has the potential to grow exponentially (generate 1000%+ share price appreciation), strong fundamentals and an attractive valuation are vital. It is proposed that profitability, cash generation, optimal debt levels, strong management, high barriers to entry, excellent competitive advantage and blue-sky growth potential are key factors to consider. This shows that quality may be a filter, instead of a risk factor, that allows the investor to take advantage of traditional anomalies like the size effect. Further to these characteristics, McLachlan (2015a) also outlines pricing power and the fact that only quality businesses will survive in the long term aid in the investment philosophy for the fund he manages. Management that is capable and properly incentivised is also an indicator of quality.

McLachlan (2015b) highlights how a strong balance sheet, high profitability, high growth and low volatility of profits are all signs of a quality company. A strong balance sheet has more cash and less debt than competitors and has real assets, as opposed to accounting assets like goodwill. High growth and profitability link as a high return on equity stock with a growing return on equity, growing revenue and expanding margins signal that the company is sound. Where management can control what impacts profit volatility, one should look for companies that have low volatility. Low volatility leads to consistent returns that allow for compounding to generate exceptional long-term performance.

When controlling for quality, the size effect became stronger, more stable through time (and not only present in January), consistent across markets, industries and size groups and not captured by an illiquidity premium (Asness, Frazzini, Israel, Moskowitz, & Pedersen, 2015). Large firms tend to be quality firms while smaller firms could be seen as 'junk'. The performance of the lowest quality stocks explains the inconsistencies and failures seen in the size effect over time because they are small with low returns and are illiquid and distressed. Controlling for quality was performed by taking the covariation of a stocks return with a quality minus junk (QMJ) factor or other measure such as profitability into account. High quality small stocks drove the size effect. The results could show two things: quality itself, regardless of size, could be a characteristic that drives returns. Alternatively, because small quality firms outperformed large quality firms, quality is a useful screening tool for traditional factors such as size as opposed to being a risk factor itself. This could possibly be extended to value shares, as mentioned above, where quality can be used to find value shares that should outperform.

Sokolowski (2015) outlined the Cambridge Trust's definition of quality as a firm that can sustainably grow cash flows in varying economic climates, while being good stewards of capital and earning good returns on that capital, and being able to create and maintain a strong competitive position. Indicators of these characteristics are return on equity, stable or increasing profit margins, balance sheet capacity and strong cash flow generation. There was also a strong emphasis on long-term investing, where the aim is to limit turnover and trading costs. Furthermore, it was noted that while quality on its own may not offer a significant premium, quality combined with value produced significant abnormal returns that are consistent. This quality, value tilt is prevalent in the AlphaWealth Prime Small & Mid Cap Fund investment selection process (McLachlan, 2015a).

Consistent with other descriptions of quality, MSCI (2015) used profitability, earnings quality, financial leverage, asset growth and corporate governance to construct the MSCI Quality Index. There is an emphasis on financial metrics as well as the governance of the company by management. The FTSE Russell (2015) report on quality investing further supports the conclusions in (SIMNAI, 2014) and MSCI (2015). Mimicking portfolios for a quality factor also produced positive abnormal returns (Norges Bank Investment Management, 2015).

Quality factors could add to the explanatory power of asset pricing models as observed in Campbell (2015) where accruals and cash flow return on equity (among others) were significant in explaining returns on the JSE between January 1994 and November 2014. These effects persisted at the country level where quality factors like profitability and debt ratios explained country level returns; the higher quality the stock was, the greater the returns

(Zaremba, 2016). Furthermore, size and value strategy returns were improved by incorporating quality, which supports Kalesnik and Kose (2014), Novy-Marx (2014) and Asness et al. (2015).

Similarly to Ou and Penman (1989), a composite index (one summary measure) consisting of nine indicators based on accounting ratios was statistically significantly correlated to equity returns on the JSE (Bunting & Barnard, 2016). It was observed that this result was strongest in portfolios that had no value or growth characteristics and were mid-sized in terms of market capitalisation. It was noted that the lack of correlations with value/growth and large/small cap stocks should be considered. This could be seen as a challenge to the notion that portfolios must sit within a growth or value strategy. It could also indicate that the market has effectively exploited the value/growth trading opportunities and thus the effects are no longer prevalent (Bunting & Barnard, 2016).

2.5 Does the choice of factor matter?

Kyosev, Hanauer, Huij, and Lansdorp (2016) examined the predictive power of quality by comparing the power of industry-defined quality factors with academically-defined quality factors. Industry quality factors included return on equity, profit margins, leverage and earnings volatility while academic definitions included accruals, net stock issues and gross profitability. It was noted that academic definitions all had significant explanatory power while industry factors showed no significant predictive power.

Hou, Xue, and Zhang (2015, 2017) (HXZ) defined their own size, value, investment and profitability factors. It was discovered that a six-factor model of the FF3F market and size factors, HXZ investment and profitability factors, Asness and Frazzini (2013) value factor and Carhart momentum factor best described the returns of stocks. Thus, the specification of the factor can influence the results or provide a better model. This could be market or time-period specific.

2.6 The Fama-French five-factor model

Fama and French (2015) outlined a five-factor asset pricing model (FF5F) that aims to better capture the cross-section of returns compared to their three-factor model (FF3F). It draws inspiration from the dividend discount model where book-to-market equity can be seen as a proxy for return because market capitalisation reacts to forecasts of investments and earnings (Fama & French, 2015). Furthermore, the idea to include a profitability and investment factor is linked to the following studies as discussed below.

2.6.1 Theoretical background to the Fama-French five-factor model

The dividend discount model states that the value of share is the present value of the expected dividends attributable to that share as follows:

$$m_t = \sum_{\tau=1}^{\infty} E(d_{t+\tau})/(1+r)^{\tau}, \tag{5}$$

where m_t is the share price at time t, $E(d_{t+\tau})$ is all the expected dividends on the share, and r is the internal rate of return on expected dividends or the cost of equity. Manipulating the equation above, the relationship between expected return and expected profitability, investment and book-to-market equity can be derived. Miller and Modigliani (1961) show that at time t, the total market value of a firm is:

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}, \tag{6}$$

where $Y_{t+\tau}$ is the total equity earnings for $t+\tau$ period and $dB_{t+\tau}$ is the change in total book equity. Dividing by the book equity at time t shows market-to-book as:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{7}$$

This equation highlights three characteristics of stock returns. Firstly, hold all constant except the current value of the stock and the expected return; the lower the value of the stock (or greater the book-to-market ratio), the greater the expected return (Fama & French, 2015). Secondly, hold the value of the stock constant as well as everything else except for expected earnings and return; higher expected earnings imply a higher expected return (Fama & French, 2015). Thirdly, holding book and market value of equity constant and expected earnings constant, higher expected growth in book equity (investment) implies a lower expected return (Fama & French, 2015). Thus, given the above relationships, book-to-market equity be a noisy proxy for expected return.

2.6.2 Deriving the Fama-French five-factor model

The FF5F takes into account factors omitted from FF3F that cause it to miss some of the variation in returns that are driven by profitability and investment. Fama and French (2015) thus add a profitability factor and investment factor to create the FF5F. The FF5F can be denoted as follows:

$$R = a + R_f + b[R_M - R_f] + s * SMB + h * HML + r * RMW + c * CMA + e, \quad (8)$$

where a, R_f, $b[R_M - R_f]$, $s * SMB$, $h * HML$ and e are defined as before, r and c are the loadings on the profitability and investment factors respectively, and RMW and CMA are the profitability and investment factors respectively.

RMW and CMA are also constructed using mimicking portfolios as in Fama and French (1993). RMW (robust minus weak) takes the average returns on a portfolio of stocks with robust profitability minus the average returns on a portfolio of stocks with weak profitability. CMA (conservative minus aggressive) is derived from the difference between the return on a portfolio of stocks with low investment and the return on a portfolio of stocks with high investment (conservative and aggressive). The profitability sort is based on the prior year accounting data taking revenue less cost of goods sold, interest expense and selling, general and administrative expenses (to get operating profitability), all divided by book equity at the end of that year. The investment sort is based on the growth in total assets in the previous year (*t-1*) divided by total assets at the end of the year prior to the previous year (*t-2*).

The models were tested in various combinations, such as three-factor models that combine the market factor and size factor with HML, RMW, or CMA; four-factor models combining the market and size factor with pairs of HML, RMW, and CMA; and finally the five-factor model including all the factors. Fama and French (2015) observed that the five-factor model showed an improvement on the FF3F, producing lower intercepts. The portion of returns left unexplained by the FF3F was also lower. It was found that when using a four-factor model excluding HML, the five-factor model was not any better in explaining returns. Thus, the HML factor was redundant for describing average returns in this dataset as the returns associated with HML were captured by the other factors. The main issue for the five-factor model was that small stocks with negative exposures to RMW and CMA were mispriced. The returns behaved like stocks that invest a lot despite low profitability; it was noted that the high investment alone could be the main reason for this problem (Fama & French, 2015).

2.6.3 Tests of the five-factor model

Barillas and Shanken (2015) tested various asset pricing models to compare the ability to explain returns. Using data from 1972 to 2013, the FF3F, Carhart and FF5F were tested along with other adaptations of the models such as including momentum. Only one version of a factor was used per model. The Fama and French (2015) factors, the Asness and Frazzini (2013) (AF) value factor (HML^M)[3] as well as the Hou, Xue, and Zhang (2015, 2017) (HXZ) factors (their own version of size, investment and profitability – ME, IA and ROE) were considered. It was found that a six-factor model of the FF3F market and size factors, HXZ investment and profitability factors, AF value factor and Carhart momentum factor best described the returns of stocks. Contrary to Fama and French (2015), the value factor was not redundant, which could be due to the different specification of the factor whereby book-to-market rankings incorporating the most recent monthly share price in the denominator were used as opposed to the mimicking portfolio. It was noted that the study performed a relative test of the models and that the validity of the models is still questionable.

Fama and French (2016) tested their five-factor model on return anomalies not targeted by the FF5F and that proved troublesome for the FF3F such as accruals, net stock issues, momentum, volatility and the issues with low/high beta. Except for momentum and accruals, the FF5F shrunk the anomalies left unexplained by FF3F (Fama & French, 2016). The high returns associated with a low beta, share repurchases and low volatility were absorbed by positive exposures to RMW and CMA and the low returns associated with high beta, large share issues and high volatility were explained by negative exposures to RMW and CMA. Once more, small stocks that invest aggressively despite low profitability caused issues for the FF5F. Contrary to Barillas and Shanken (2015), momentum did not add to the explanatory power of the model.

Chiah, Chai, Zhong, and Li (2016) also investigated the ability of the FF5F to explain returns, but used Australian equities over the period 1982 to 2013. The FF5F was compared to the FF3F, the Carhart four-factor model, a U.S. version of the Carhart model, five-factor model and four- and five-factor models augmented with orthogonalised Australian factors. It was concluded that the FF5F better captured returns compared to the FF3F, and could explain more asset pricing anomalies. However, it still failed to fully explain the time-series variation in portfolio returns. Contrary to Fama and French (2015), the value factor (book-to-market equity) was still an explanatory variable when included in a model with profitability and investment. Fama and French (2017) further applied the model to international stock returns and found that the FF5F fared better than the FF3F and still observed that the returns of small stocks that behave like low profitability, high investment firms were not explained.

[3] The AF value factor is constructed using book-to-market rankings based on the most recent monthly stock price in the denominator.

Walkshausl (2016) investigated whether a systematic misvaluation factor still explained average returns in comparison to the FF5F. The misvaluation factor was constructed as the average returns of a portfolio of stocks that were undervalued minus the average returns of a portfolio of overvalued stocks. This was based on the return difference between repurchasing and issuing firms. A two-factor CAPM augmented with the misvaluation factor explained returns as well as FF5F for the period 1972 to 2014 for U.S. stocks. It was noted that this augmented CAPM contained a rational, risk-based factor as well as a behavioural mispricing factor (the reaction to stock repurchases and issues), which may provide a new way to model returns.

In reviewing the FF5F, Huang (2016) noted that the way the factors were defined did not materially impact the results of the study. This contradicted Kyosev et al. (2016) and Hou, Xue, and Zhang (2015, 2017) in that the broad spectrum of definitions for quality or other factors impacted the explanatory power of a factor.

When augmenting the Pástor and Stambaugh (2003) into the FF5F, Racicot and Rentz (2017) tested the robustness of the FF5F and analysed whether a liquidity factor could help to explain returns better. Using a generalised method of moments to combat the issues surrounding model specification and measurement errors, it was found that only the market factor was consistently significant. The other factors display varying, inconsistent significance.

Charteris, Rwishema, and Chidede (2018) investigate whether an alternative three-factor model proposed by Chen, Novy-Marx, and Zhang (2011) and a five-factor model developed by Fama and French (2015) can explain momentum on the JSE. These models both include profitability and investment as pricing factors in the models. While these models showed an improvement in pricing error in explaining momentum compared to the CAPM, Fama-French three-factor model, and the Carhart four-factor model, they errors were still significant. Returns and profitability were significantly positively correlated, consistent with the results of prior studies.

To the author’s knowledge, looking at the applicability of the five-factor model on JSE listed share returns in the manner proposed in Fama and French (2015) and assessing the link between the additional factors and quality has not been done in a South African context. Thus, this study will be the first to perform this analysis.

2.7 Summary

This chapter summarised the literature on asset pricing and the development of asset pricing models from a single factor model into a multi-factor model that can incorporate a variety of risk factors. The next section explains how these models will be tested on the JSE.

3. Methodology

The research methodology follows that of Fama and MacBeth (1973) and Fama and French (1993) carried over as in Fama and French (2015, 2016). The data, models tested, factors and test portfolios are discussed next.

3.1 Data

Johannesburg Securities Exchange (JSE) listed companies were obtained for the study. The data was obtained for 1991 to 2017 from the iNET Expert (Iress) database, supplemented by Bloomberg and the Findata@Wits database as supplied by the School of Economic and Business Sciences Finance division where the data was not available or incomplete. The factors were constructed based on yearly data, with the monthly factor returns calculated using monthly data. To allow for the construction of test portfolios before the actual testing period, data was collected from 1991 to 2017, with the actual testing period being from 1994 to 2017. Portfolios and factors were formed in July of each year based on the information in the iNET database as at December year end to allow for an information lag to be taken into account in the final sample and portfolio formation. This was to mitigate look-ahead bias and ensure that financial statement information was available to all market participants. Companies that have delisted were included for the years that they were listed to mitigate survivorship bias. For simplicity, if a company delisted during a year, a nil return was assigned. Monthly value-weighted returns for the test portfolios were calculated using monthly share price data. Companies must have been listed for at least three years to be included in the sample. The investment factor used two years of data, thus an additional year is required to ensure that the company has reported at least one full year of final results.

Shares that had a price lower than 50c were eliminated as the large returns on these microcap shares due to small price movements could skew the results. To further mitigate the impact of outlier returns, a trimming procedure was employed, similar to van Rensburg and Robertson (2003) and Kruger and Toerien (2014). The 1^{st} and 99^{th} percentiles were calculated and those returns below or above these percentiles were assigned the value at the 1^{st} or 99^{th} percentile. The above papers as well as Page and Auret (2014) apply a liquidity filter to mitigate the impact of large returns that can occur in thinly-traded shares. The volume of shares traded for the month was divided by the shares outstanding at the end of the previous month. Shares that traded less than 0.01% of the shares in issue in the prior month were removed.

As can be seen in Figure 1, there was always at least 200 shares in the sample, with an average per year of 240 shares. This enabled test portfolios to be formed with a sufficient number of shares and could be split into different percentiles to analyse the spread of premiums across differently formed portfolios[4].

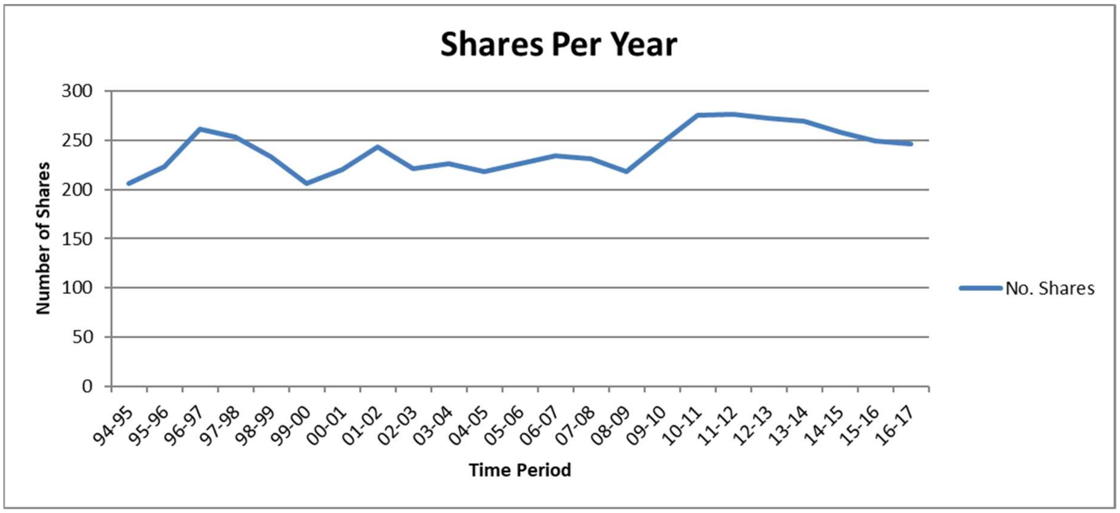

Figure 1. Shares per year

3.2 Models

The CAPM, three-factor and five-factor models were tested. This research looked specifically at Fama-French models and variations based on the Fama-French factors of size, value, profitability and investment.

3.2.1 The Capital Asset Pricing Model (CAPM)

The CAPM was included as it was one of the first formal models for asset returns. The CAPM is represented by the following regression equation derived from equation 1:

$$R = a + R_f + b\left[R_M - R_f\right] + e, \tag{9}$$

where R is the return on a test portfolio.

where b is the premium to the risk factor $[R_M - R_f]$.

$R - R_f$ is the excess return on a test portfolio.

[4] The minimum number of shares in a portfolio was 8. While there is some industry concentration on the JSE, the effect of concentration on the results is not the focus of this study and is an avenue of future research.

$[R_M - R_f]$ is the excess return on the value-weighted market portfolio.

The market factor in Fama and French (2015) is constructed by creating a value-weighted portfolio of all sample stocks for the year and calculating its excess return. This was attempted but the resulting returns did not even closely resemble that of the JSE market as a whole. Thus, the JSE All-Share Index (ALSI) was used as a proxy for the market return. The 3-Month Treasury Bill rate was used as a proxy for the risk-free rate. This was attained from the South African Reserve Bank website. Monthly excess returns for the ALSI above the risk-free rate were calculated to construct the market factor.

3.2.2 The Fama-French three-factor model

The Fama-French factors use mimicking portfolios to represent the underlying risk factors that influence share returns. The original Fama-French three-factor model was constructed as follows per equation 3:

$$R = a + R_f + b\left[R_M - R_f\right] + s * SMB + h * HML + e, \qquad (3)$$

where b, s, h are the premia to the risk factors $[R_M - R_f]$, SMB, HML respectively.

$R - R_f$ is the excess return on a test portfolio.

$[R_M - R_f]$ is the excess return on the value-weighted market portfolio.

SMB (size factor) is the value-weighted returns on a diversified portfolio of small stocks minus the value-weighted returns on a diversified portfolio of big stocks.

HML (value factor) is the value-weighted returns on a diversified portfolio of high book-to-market equity stocks minus the value-weighted returns on a diversified portfolio of low book-to-market equity stocks.

e is the error term and should the premia b, s, h capture all the variation in expected returns, the intercept a will be zero for all stocks and portfolios.

The factors were rebalanced annually.

Three other three-factor models were tested given the inclusion of additional factors in the Fama-French Five-factor model. The first two augmented the profitability (RMW) and investment (CMA) factors with the size and market factor (the first will be a size, profitability and market factor model; the second a size, investment and market factor model). The final model augmented the two new factors from the prior two models, profitability and investment, with the market factor. The formation of the factors in these models used the same methodology as described above for size and value where two size groups were formed with two profitability/investment groups. The profitability factor included the returns of robust profitability shares minus the returns of weak profitability shares. The investment factor included the returns of shares that invest conservatively minus the returns of shares that invest aggressively. The formal definitions are given in the next section.

3.2.3 The Fama-French five-factor model

The Fama-French Five-factor model using mimicking portfolios as the factors per Fama and French (2015) is described below:

$$R = a + R_f + b[R_M - R_f] + s * SMB + h * HML + r * RMW + c * CMA + e, \qquad (8)$$

where b, s, h, r, c are the premia to the risk factors $(R_M - R_f)$, SMB, HML, RMW, CMA respectively. $R - R_f$, $(R_M - R_f)$, SMB, and HML are defined as before.

RMW is the value-weighted returns on a diversified portfolio of stocks with robust profitability minus the value-weighted returns on a diversified portfolio of stocks with weak profitability. Profitability is defined as Operating Profit divided by the book value of equity.

CMA is the value-weighted returns on a diversified portfolio of stocks of firms that invest conservatively minus the value-weighted returns on a diversified portfolio of stocks of firms that invest aggressively. Investment is defined as the growth in total assets from period *t-2* to period *t-1*.

e is the error term and should the premia b, s, h, r, c capture all the variation in expected returns, the intercept a will be zero for all stocks and portfolios.

Factors were rebalanced annually.

3.3 Factor definitions

The factor definitions are explained below. Size was based on the JSE median and the second sort was based on the 33rd and 66th percentiles of the second characteristic[5].

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[5] An analysis as performed whereby the 30% and 70% breakpoints were used. The resulting portfolio returns did not change materially given that the sample size per year is over 200 shares.

3.3.1 Three-factor models

Size, Value, Market Model

SMB_v[6] and HML were formed as follows:

Six portfolios were formed using size and book-to-market equity; two size groups (small and big) and three book-to-market groups (high, medium and low) intersect to form the six portfolios (S/H, S/M, S/L, B/H, B/M, B/L).

SMB_v – the simple average of the returns of the small portfolios (S/H, S/M, S/L) and big portfolios (B/H, B/M, B/L) was calculated. The difference between the small and big portfolio average returns is the SMB factor.

HML – the simple average of the returns of the high book-to-market (S/H and B/H) and low book-to-market (S/L and B/L) portfolios was calculated. The difference between the high and low book-to-market portfolio average returns is the HML factor.

Size, Profitability, Market Model

SMB_p[7] and RMW were formed as follows:

Six portfolios were formed using size and profitability; two size groups (small and big) and three profitability groups (robust, average and weak) intersect to form the six portfolios (S/R, S/A, S/W, B/R, B/A, B/W).

SMB_p – the simple average of the returns of the small portfolios (S/R, S/A, S/W) and big portfolios (B/R, B/A, B/W) was calculated. The difference between the small and big portfolio average returns is the SMB factor.

RMW – the simple average of the returns of the robust profitability portfolios (S/R and B/R) and weak profitability (S/W and B/W) portfolios was calculated. The difference between the robust and weak portfolio average returns is the RMW factor.

Size, Investment, Market Model

[6] SMB_v is the size factor as formed in a size and value sort. It is the size factor associated with the value characteristic in forming the factor portfolio.

[7] SMB_p is the size factor as formed in a size and profitability sort. It is the size factor associated with the profitability characteristic in forming the factor portfolio.

SMB_i[8] and CMA were formed as follows:

Six portfolios were formed using size and investment; two size groups (small and big) and three investment groups (conservative, moderate and aggressive) intersect to form the six portfolios (S/C, S/M, S/A, B/C, B/M, B/A).

SMB_i – the simple average of the returns of the small portfolios (S/C, S/M, S/A) and big portfolios (B/C, B/M, B/A) was calculated. The difference between the small and big portfolio average returns is the SMB factor.

CMA – the simple average of the returns of the conservative investment portfolios (S/C and B/C) and aggressive investment (S/A and B/A) portfolios was calculated. The difference between the conservative and aggressive portfolio average returns is the CMA factor.

Profitability, Investment, Market Model

This augments the profitability and investment factors above with the market factor.

The size factors were not averaged across the models. The motive for this was that each model is formed and tested as if it is a standalone model based on the particular underlying risk factors the model aims to capture. Thus, the goal was to isolate the effect of company size on returns when using a size and value/profitability/investment based model, without influence from the other factor formations. It is not only looking at the effect of size in isolation but also how it interacts with different risk factors specifically. This gives the specific premium or coefficient that is specific to the model and unique to the formation of that model. Therefore, each model was formed as in Fama and French (1993) and tested as such.

3.3.2 The five-factor model

More formally, due to the addition of the profitability and investment factors, SMB, HML, RMW and CMA were formed as follows in the five-factor model:

Sixteen portfolios were formed using the intercepts from independent sorts into two size groups, two book-to-market groups, two profitability groups, and two investment groups (2x2x2x2 sort). Each factor group was determined using the JSE median as the breakpoint. This allows one to isolate the premiums associated with each factor individually, without

[8] SMB_i is the size factor as formed in a size and investment sort. It is the size factor associated with the investment characteristic in forming the factor portfolio.

influence from the others. The intercepts provide us with two sets of portfolios to form each factor as explained below:

Given the above, small shares (*S*) have market capitalisations below the JSE median market capitalisation and big shares (*B*) have ones that are larger than the median. *SMB* is the average return on the eight small stock portfolios from the sort minus the average return on the eight big stock portfolios.

High book-to-market shares (*H*) have book-to-market ratios greater than the JSE median book-to-market ratio while low book-to-market shares (*L*) have ratios lower than the median. *HML* is the average return on the eight high book-to-market portfolios from the sort minus the average return on the eight low book-to-market portfolios.

Robust profitability shares (*R*) have profitability (scaled by equity) that is higher than the JSE median profitability and weak profitability shares (*W*) have profitability lower than the median. *RMW* is the average return on the eight robust profitability portfolios from the sort minus the average return on the eight weak profitability portfolios. Profitability was measured as Operating Profit per the financial statements, divided by the book value of equity.

Conservative investment shares (*C*) invest amounts below the JSE median investment while aggressive investment shares (*A*) invest amounts greater than the median. *CMA* is the average return on the eight conservative investment portfolios from the sort minus the average return on the eight aggressive investment portfolios. Investment was measured as the percentage change in total assets from period *t-2* to period *t-1*.

The logic behind this sort was to be able to isolate the effects of each factor without influence of the others. By controlling for the other factors, one can better explain the reasons for the results found.

Again, the value-weighted simple average returns of the big, low, weak and aggressive portfolios were subtracted from the value-weighted simple average returns of the small, high, robust and conservative portfolios respectively.

3.4 The test portfolios

The aim of Fama and French (2015) was to test how well the five-factor model explained the returns of portfolios sorted on several different characteristics. Fama and French (2015) used

5x5 and 2x4x4 sorts in testing the five-factor model. Given the smaller investment universe on the JSE, smaller sorts were used. These are as follows:

1. 9 size/book-to-market portfolios (3x3 sort)
2. 9 size/profitability portfolios
3. 9 size/investment portfolios
4. 12 size/book-to-market portfolios (3x4 sort)
5. 12 size/profitability portfolios
6. 12 size/investment portfolios
7. 12 size/value/profitability portfolios (2x2x3 sort)
8. 12 size/value/investment portfolios
9. 12 size/investment/profitability portfolios
10. 12 size/profitability/investment portfolios

The sorts mimic Fama and French (2015) in that there are two-way and three-way sorts. 2x3x3 sorts were performed but resulted in some portfolios having no shares for a particular period; thus, this sort was abandoned. 3x3 and 3x4 two-way sorts were performed to see how the portfolio construction may impact the results. Furthermore, sorts nine and ten above were both performed because the final two ways in the 2x2x3 sorts are not the same. Because sorting only profitability or investment three ways may have led to incomplete results, both were tested.

The two-way sorts were performed using the same methodology as the factor construction for the three-factor models. Shares were sorted independently into three size groups and then three or four value, investment or profitability groups. The intersections of these form the nine or twelve portfolios in the sort. The 2x2x3 sorts were performed by sorting independently into two size groups, then into two groups of the next factor (value/profitability/investment), and finally into three groups of the last factor (profitability/investment). The sorts were based on the financial information available as at the end of the prior December. Value-weighted monthly excess returns were calculated for each portfolio from July to July and portfolios were rebalanced annually.

The question of whether to rebalance monthly or annually is a frequent consideration in research. Yearly rebalancing is more appropriate in this study as it is in line with the methodology of Fama and French (2015). Furthermore, yearly rebalancing is more closely related with practical holding periods for portfolios. Fama and French (1992) highlighted that the rebalancing frequency did not change their results. Basiewicz and Auret (2010) and Chen and Zhang (2010) both used annual rebalancing in tests of the three-factor model, with the former advising against monthly rebalancing as it may pick up on a short-term reversal effect. Finally, in terms of investment and profitability, Cooper et al. (2008) and Novy-Marx (2013) used annual rebalancing in their tests. Novy-Marx (2013) noted that the profitability effect that is being tested for is a long-term, persistent effect that shouldn't be confused with short-term earnings announcement effects. Given the above, annual rebalancing was used.

3.5 Factor model analyses

The study conducted two sets of tests. The first was a model comparison that tested which of the models presented best explained the returns of portfolios based on different sorts. The second employed the methodology first used by Fama and MacBeth (1973) to test for significant cross-sectional premiums associated with each of the factors tested.

This study followed the objectives of Fama and French (2015) in that the result of interest is the relative performance of the models compared to one another as opposed to the statistical rejection or confirmation of the model in tests with power. The identification of the best (but imperfect) model to describe the returns of the test portfolios will be performed. The performance of each model can be assessed to determine whether the additional factors improve the explanatory power of prior models. However, tests of each model will still be performed and subject to statistical tests of significance. This provides a complete picture of the applicability of new models or factors in explaining JSE share returns.

The tests are performed in two ways. Firstly, in the spirit of Fama and MacBeth (1973), the regressions are performed per yearly period and averages are attained over the test periods (23 regressions in total per portfolio per model). Secondly, as a robustness test, regressions are performed over the entire sample period (one regression per portfolio per model). The Fama-MacBeth results are presented in *Section 4. Results* with the full-period regressions presented under the robustness tests in *Section 5. Robustness Tests*.

3.5.1 Model comparison – relative performance

Six statistics were calculated to perform a comparison of the models to be tested. This included updated versions of the Fama and French (2015) statistics per Fama and French (2016). These were estimated from time-series regressions as follows:

1. The average absolute value of alpha $A|\alpha|$ (the intercept) for each model was determined. This showed the error in pricing returns left by the model. If a model

explained returns, the intercept should have been indistinguishable from zero. Thus, the lower the alpha, the more that was explained by the factors in the model.

2. The second statistic used was one of three statistics that measured the proportion of returns left unexplained by the model. These measured the dispersion of the intercepts (unexplained test portfolio average excess returns) relative to the dispersion of test portfolio average excess returns. It was calculated as the average of the absolute alphas $A|\alpha|$ as a proportion of the average absolute deviation $A|\gamma|$ of each test portfolio's average time-series return from the market portfolio excess returns $\frac{A|\alpha|}{A|\gamma|}$. The lower the statistic, the less that was left unexplained.

   The time-series average return was estimated per portfolio, and market portfolio excess return subtracted therefrom. The average absolute value of this deviation was used as the denominator. This differed from Fama and French (2015) where the deviation was measured from the cross-sectional average return of the test portfolios; the difference is because the market portfolio is the aggregate of portfolios chosen by investors and any model that includes a market factor will perfectly explain that market portfolio excess return. The choice of the average cross-sectional return holds no special place in asset pricing as it changes with sample, period, and portfolio formation. The market portfolio (or its proxy), however, is central to asset pricing theory and thus it is a better reference point from which to measure deviation (Fama and French, 2016).

3. The next statistic used was simply the ratio of the average squared intercepts and deviations above $\frac{A|\alpha|^2}{A|\gamma|^2}$ (point 2).

4. This statistic was another measure of returns left unexplained by the model due to sampling error. The higher the statistic, the more the dispersion in intercepts was due to sampling error rather than the actual dispersion of true values of the intercepts. It was calculated as the average squared standard error of alpha As^2 divided by the squared average absolute intercept $\frac{As^2}{A|\alpha|^2}$.

5. The GRS[9] statistic of Gibbons, Ross, and Shanken (1989) tests whether the intercepts of all test portfolios are jointly zero. The lower the statistic, the better the model. This accounts for models that have a different number of factors. It was calculated per the following formula:

$$\frac{T-N-K}{N} * \hat{\alpha}'\hat{\Sigma}\hat{\alpha} * [1 + E_T(f_t)'\hat{\Omega}^{-1}E_T(f_t)]^{-1}, \tag{10}$$

where there are T observations, N portfolios and K factors. $E_T(f_t)$ is the expected return on the factors, $\hat{\Omega}^{-1}$ is the inverse covariance matrix of factor returns, $\hat{\alpha}$ is the vector of alphas from the regressions, and $\hat{\Sigma}$ is the covariance matrix of estimated residuals. An apostrophe indicates the transposition of an item in the equation.

6. An average adjusted R-squared was also calculated. This took into account the fact that models with more factors have higher R-squared values by default. This enabled a comparison between models on an equivalent level to show which explained more of the returns.

These statistics allowed for a comparison of the models even with differing numbers of factors or test portfolios.

3.5.2 Fama-MacBeth regressions

The cross-sectional premiums associated with each risk factor are of interest in asset pricing models. Thus, in order to facilitate comparison, the two-stage Fama and MacBeth (1973) regressions were performed. This outlined whether any significant premiums were priced into the cross-section of returns. The two-stage regression is described below.

In stage one, the time-series coefficients were estimated per the following equation:

$$R_i - R_{f,i} = a + B_{1,i} * F_{1,i} + B_{2,i} * F_{2,i} + \cdots + B_{n,i} * F_{n,i} + e_i, \tag{11}$$

where $R_i - R_{f,i}$ is the monthly excess return on a test portfolio and B_n is the time-series coefficient on the factor F_n per portfolio for n factors for each period i for that portfolio. The coefficients were estimated for each portfolio over 36 month rolling periods. The coefficients became the independent variables in the second stage regression.

[9] The GRS statistic is widely recognised and used as a test of a model's ability to price returns.

The second stage regression was estimated as follows:

$$r_i - r_{f,i} = a + \lambda_{1,i}B_{1,i} + \lambda_{2,i}B_{2,i} + \cdots + \lambda_{n,i}B_{n,i} + e_i, \qquad (12)$$

where the average excess return for the period on each portfolio is regressed on the coefficients from stage one. The resulting lambdas (λ) are the cross-sectional premiums associated with each factor. The cross-sectional premiums (λ) were averaged and the standard deviations and the relevant t-statistics and p-values were extracted therefrom. This is consistent with Goyal (2012) as a cross-sectional regression is run in each period, allowing for the time-series averaging of the cross-sectional premiums and pricing error estimates across all periods. This allowed for an analysis of significant cross-sectional premiums picked up by each model that are associated with the underlying risk factors.

4. Results

The results are presented as follows: summary statistics and analysis are provided for portfolios and factors. The results of the model comparison will be discussed, followed by the Fama-MacBeth test results. Robustness tests are then presented for comparison.

4.1 Summary statistics

Summary statistics are presented for the factors and portfolios:

4.1.1 Factor statistics

Tables 1 and 2 show the summary statistics for each factor for July to July of each year shown as well as the average for the entire sample period of 23 years. The returns presented are the average monthly returns for the years shown and subsequently for the entire period. The market factor is included in both tables for completeness. The market provides a positive average monthly excess return of 0.116%.

Comparing the size factors in the three-factor models (*Table 1.*) shows a notable difference between the returns when the second factor is value as opposed to profitability or investment. The returns of big shares outweighed those of smaller shares by a much larger degree in the size-value model compared to the size and profitability or investment models. The return in the size-value model was -0.450% compared to -0.298% and -0.288% for profitability and investment respectively. The standard deviation is lower for the size-value model compared to the other two. Interestingly, given the factor construction, there was a premium associated with holding big shares compared to small shares, which is an unusual result given the prior evidence on the size effect. High book-to-market and robust profitability shares show a stronger premium compared to the low book-to-market and weak profitability shares, which is consistent with Novy-Marx (2013). These are 0.510% and 0.085% respectively. Companies that invest conservatively have a premium over those that invest aggressively by an average of 0.301% per month, which is consistent with the results of Cooper et al. (2008) and Fama and French (2015) where lower asset growth led to positive abnormal returns. The market and investment factors have the highest standard deviations. The largest absolute premiums are associated with the size-value model.

Table 1. Factor returns for the CAPM and three-factor models

Year	MKT	SMB_v	HML	SMB_p	RMW	SMB_i	CMA
1994-1995	-0.723%	1.110%	0.141%	1.100%	0.797%	1.256%	-0.876%
1995-1996	0.429%	-0.651%	-0.114%	-0.618%	0.088%	-0.634%	0.533%
1996-1997	-0.570%	-0.632%	1.268%	-0.973%	0.136%	-0.774%	0.228%
1997-1998	-1.549%	-0.203%	-1.046%	-0.246%	-0.620%	-0.581%	-2.455%
1998-1999	-0.316%	-2.436%	1.759%	-2.105%	-0.587%	-2.335%	2.907%
1999-2000	0.130%	-2.540%	2.470%	-2.357%	-1.241%	-2.521%	3.606%
2000-2001	0.044%	-0.761%	1.784%	-0.083%	-2.696%	0.101%	1.481%
2001-2002	-0.669%	0.030%	1.671%	1.576%	0.970%	0.830%	1.448%
2002-2003	-2.418%	1.460%	2.073%	2.645%	0.155%	2.766%	-0.079%
2003-2003	0.961%	0.944%	0.893%	1.560%	-0.219%	1.435%	0.109%
2004-2005	2.535%	0.552%	1.375%	0.767%	0.560%	0.800%	-0.026%
2005-2006	2.439%	-0.734%	0.290%	-0.869%	-0.342%	-0.853%	-0.706%
2006-2007	1.607%	0.764%	0.424%	0.818%	0.806%	0.580%	-1.097%
2007-2008	-0.989%	-2.041%	-0.500%	-2.162%	0.029%	-2.214%	0.349%
2008-2009	-2.496%	0.870%	0.858%	0.650%	-1.006%	0.906%	-0.343%
2009-2010	1.379%	-1.055%	1.441%	-1.121%	0.530%	-0.919%	0.923%
2010-2011	0.866%	-0.685%	-0.160%	-0.563%	1.019%	-0.451%	0.837%
2011-2012	0.187%	-0.757%	-0.805%	-0.780%	1.702%	-0.909%	0.248%
2012-2013	1.151%	-1.171%	-0.004%	-0.789%	1.545%	-0.715%	-0.810%
2013-2014	1.565%	-1.178%	1.228%	-0.944%	-1.322%	-0.750%	-0.195%
2014-2015	-0.316%	-0.073%	-2.745%	-0.745%	0.555%	-0.232%	-1.397%
2015-2016	-0.401%	-0.165%	-0.962%	-0.568%	0.534%	-0.687%	1.334%
2016-2017	-0.166%	-0.997%	0.402%	-1.047%	0.567%	-0.728%	0.910%
Average Return	**0.116%**	**-0.450%**	**0.510%**	**-0.298%**	**0.085%**	**-0.288%**	**0.301%**
Standard Deviation	**1.338%**	**1.081%**	**1.225%**	**1.266%**	**1.009%**	**1.255%**	**1.342%**
t-statistic	**0.417**	***-1.996**	***1.998**	**-1.129**	**0.405**	**-1.101**	**1.077**

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

The five-factor model (*Table 2.*) shows a similar (inverse) size effect whereby big shares outperformed small shares in the sample period. The premium is large at -0.502%. As in the three-factor model, the value factor shows a positive return in the five-factor model formation. This means that value shares outperformed growth shares in this sort. Notably, it is the largest in absolute terms at 0.523%. There is a much stronger profitability premium compared to the three-factor model at 0.308%; it too is thus consistent with prior literature. The investment premium confirms the results in other studies but is weaker in the five-factor model compared to the three-factor model. The standard deviations are all lower in the five-factor model except for the size factor in the size-value model, which is lower than in the five-factor model.

Table 2. Factor returns for the five-factor model

Year	*MKT*	*SMB*	*HML*	*RMW*	*CMA*
1994-1995	-0.723%	1.080%	0.363%	0.838%	0.038%
1995-1996	0.429%	-0.454%	0.096%	-0.269%	-0.160%
1996-1997	-0.570%	-0.721%	1.484%	0.823%	0.076%
1997-1998	-1.549%	-0.020%	-0.795%	-1.157%	-0.501%
1998-1999	-0.316%	-1.945%	0.822%	0.257%	0.981%
1999-2000	0.130%	-2.278%	1.574%	-0.332%	1.351%
2000-2001	0.044%	-0.560%	2.086%	-0.490%	1.227%
2001-2002	-0.669%	-0.461%	1.331%	0.379%	0.894%
2002-2003	-2.418%	1.473%	1.864%	0.794%	0.191%
2003-2003	0.961%	1.950%	-0.052%	-0.791%	-0.066%
2004-2005	2.535%	0.677%	0.980%	0.037%	-0.370%
2005-2006	2.439%	-0.522%	0.120%	-0.122%	-0.332%
2006-2007	1.607%	0.238%	0.905%	1.296%	-0.492%
2007-2008	-0.989%	-2.903%	0.646%	0.961%	-0.498%
2008-2009	-2.496%	0.232%	0.416%	-0.421%	-0.685%
2009-2010	1.379%	-1.099%	0.820%	0.537%	0.158%
2010-2011	0.866%	-0.162%	0.127%	0.895%	0.605%
2011-2012	0.187%	-0.665%	-0.343%	1.147%	-0.387%
2012-2013	1.151%	-1.684%	0.820%	2.088%	-0.164%
2013-2014	1.565%	-1.147%	0.459%	-0.380%	-0.084%
2014-2015	-0.316%	-1.136%	-1.672%	0.428%	-1.413%
2015-2016	-0.401%	-0.711%	-0.056%	0.376%	0.907%
2016-2017	-0.166%	-0.719%	0.040%	0.201%	1.088%
Average Return	**0.116%**	**-0.502%**	**0.523%**	**0.308%**	**0.103%**
Standard Deviation	**1.338%**	**1.148%**	**0.861%**	**0.753%**	**0.706%**
t-statistic	**0.417**	****-2.096**	*****2.915**	***1.964**	**0.699**

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Tables 3 and 4 show the correlations between the factors for each set of models. In terms of the three-factor models, unsurprisingly, the size factors are all are closely correlated. Notable correlations are with the investment factor, where the factors from the size-value model are quite negatively correlated with investment. The correlation between investment and value is similar to what is noted in Fama and French (2015) but is smaller in this sample. Fama and French (2015) note that the correlation could be explained by the fact that high book-to-market firms tend to be low investment firms. Size is also quite negatively correlated with investment. This means that small companies are more associated with aggressive investment, which is intuitive in that small companies grow from a smaller base and thus may expand quicker and to a greater degree than large, established companies.

Table 3. Factor correlation matrix - three-factor models

Factors	MKT	SMB_v	HML	SMB_p	RMW	SMB_i	CMA
MKT	1.000	-0.185	0.055	-0.193	0.145	-0.191	0.005
SMB_v	-0.185	1.000	-0.093	0.924	0.216	0.945	-0.567
HML	0.055	-0.093	1.000	0.161	-0.416	0.120	0.571
SMB_p	-0.193	0.924	0.161	1.000	0.153	0.979	-0.379
RMW	0.145	0.216	-0.416	0.153	1.000	0.130	-0.254
SMB_i	-0.191	0.945	0.120	0.979	0.130	1.000	-0.429
CMA	0.005	-0.567	0.571	-0.379	-0.254	-0.429	1.000

Table 4. Factor correlation matrix - five-factor model

Factors	MKT	SMB	HML	RMW	CMA
MKT	1.000	-0.056	-0.028	0.085	-0.045
SMB	-0.056	1.000	-0.030	-0.209	-0.171
HML	-0.028	-0.030	1.000	0.141	0.552
RMW	0.085	-0.209	0.141	1.000	-0.106
CMA	-0.045	-0.171	0.552	-0.106	1.000

The five-factor model mirrors the results from the three-factor model except for the market and profitability factors, which show the opposite correlations to value. The relationship makes sense as robust profitability shares are more likely to be large, established companies with less growth opportunity.

4.1.2 Test portfolio statistics

Portfolio summary statistics are as follows:

4.1.2.1 3x3 Sorts – 9 Portfolios

Table 5 shows the average monthly excess return for the entire period of 23 years on portfolios sorted independently by size (three groups) and three groups of either value, profitability or investment. Standard deviations are presented in italics.

Table 5. Average monthly excess returns and standard deviations for 3x3 portfolio sorts

		Size		
		Small	Mid	Big
	High	0.091%	0.101%	0.332%
		4.320%	*4.985%*	*6.782%*
Value	Medium	-0.218%	-0.267%	0.105%
		4.876%	*4.796%*	*5.723%*
	Low	-0.650%	-0.670%	0.093%
		5.510%	*5.278%*	*5.771%*
	Robust	-0.110%	-0.428%	-0.251%
		4.871%	*5.039%*	*5.874%*
Profitability	Average	-0.237%	-0.323%	0.172%
		4.925%	*5.202%*	*5.984%*
	Weak	-0.428%	-0.558%	-0.310%
		4.787%	*4.948%*	*5.957%*
	Conservative	-0.064%	-0.249%	-0.052%
		4.530%	*4.547%*	*5.576%*
Investment	Moderate	0.016%	-0.469%	-0.296%
		4.629%	*4.803%*	*5.835%*
	Aggressive	-0.916%	-0.528%	0.021%
		5.265%	*5.408%*	*5.980%*

There is some variability among the portfolio returns. High book-to-market portfolios showed greater returns than low book-to-market portfolios across all size groups, which is consistent with the value effect. Small- and mid-sized growth portfolios performed poorly. Across value categories, big share portfolios outperform small share portfolios, with big, high book-to-market shares showing an average return of 0.332% per month. This mirrors the effect seen in the factor summary statistics. The big portfolios also had larger standard deviations compared to the mid-size and small portfolios.

While most of the average monthly excess returns are negative, a similar pattern for company size and returns is seen in profitability sorts where big portfolios generally outperform small portfolios (they have smaller negative returns than small portfolios), with the exception being small portfolios with robust profitability and big portfolio shares with average profitability which has a positive monthly average excess return of 0.172%. In terms of profitability, portfolios with average profitability outperform in mid- and big-sized portfolios. An interesting observation is that the small portfolio with robust profitability outperforms its bigger counterpart. It is also noted (but partly expected) that average to robust profitability leads to greater returns. Perhaps companies in the average profitability portfolios are still growing. Given that high book-to-market portfolios and average or better profitability leads to better returns, one could suggest that quality (with profitability as the proxy) and value are linked (McLachlan, 2014). Once again, the portfolios of big stocks were more volatile than the smaller sized portfolios.

Big portfolio shares outperformed in both the conservative and aggressive sorts, with small companies with moderate investment outperforming the other two size groups. Small portfolio shares that invest aggressively performed the worst, with an average monthly excess return of -0.916%. It is also evident that as shares get larger and invest more aggressively, the standard deviations increase. The best performing investment category depends on company size. Small firms that invest moderately outperform firms that invest to a greater or lesser degree. Mid-sized firms that invest conservatively outperform firms that invest more heavily. Finally, big firms that invest aggressively outperform. This is quite inconsistent with what Cooper et al. (2008) and Aharoni, Grundy, and Zeng (2013) found on investment levels and returns. It is evident that unless a company is quite large, aggressive investment can be detrimental. This is intuitive as large companies should have the required cash generating ability to fund the investment whereas smaller firms may not. Similarly, small firms that invest moderately perform well because they are growing at a reasonable rate.

4.1.2.2 3x4 Sorts – 12 Portfolios

Table 6 shows the average monthly excess return for the entire 23-year test period on portfolios sorted independently by three size groups and four groups of either value, profitability or investment.

In the size and value sorts, the big share portfolios follow same pattern to the 3x3 sorts; big shares generally outperform small shares. Again, high book-to-market portfolios outperform low book-to-market portfolios. Big share portfolios are more volatile than small share portfolios if they are value shares. Small portfolios with lower book-to-market ratios are more volatile than their larger counterparts. It is notable that in the small size group, growth portfolios are more volatile than value portfolios. In the big size group, value portfolios have lower standard deviations than growth portfolios.

Table 6. Average monthly excess returns and standard deviations for 3x4 portfolio sorts

		Size		
		Small	Mid	Big
Value	High	0.107%	-0.086%	0.287%
		4.418%	*5.834%*	*7.565%*
	Medium High	-0.308%	-0.140%	-0.041%
		5.212%	*4.739%*	*6.200%*
	Medium Low	-0.819%	-0.461%	-0.145%
		5.791%	*5.292%*	*5.787%*
	Low	-0.691%	-0.891%	-0.100%
		5.928%	*5.572%*	*5.894%*
Profitability	Robust	-0.226%	-0.460%	-0.181%
		5.005%	*5.093%*	*6.021%*
	Average Robust	-0.115%	-0.337%	0.017%
		5.174%	*5.307%*	*6.293%*
	Average Weak	-0.095%	-0.198%	-0.039%
		5.166%	*5.134%*	*6.336%*
	Weak	-0.521%	-0.820%	-0.431%
		5.027%	*5.450%*	*6.063%*
Investment	Conservative	-0.128%	-0.264%	0.062%
		4.803%	*5.024%*	*5.555%*
	Moderate Conservative	0.243%	-0.374%	-0.174%
		5.194%	*4.829%*	*5.883%*
	Moderate Aggressive	-0.252%	-0.254%	-0.138%
		5.388%	*5.168%*	*5.942%*
	Aggressive	-1.089%	-0.697%	-0.194%
		5.531%	*5.537%*	*6.376%*

The size and profitability sorts show a familiar pattern in terms of size. Big portfolios outperform small portfolios. Mid-size portfolios underperform both small and big portfolios across all profitability groups. In terms of profitability, the two average profitability portfolios always show better returns than either the robust or weak portfolios. The interaction with size may cause this effect to occur. Once again, portfolios of big shares have greater standard deviations than the other size groups. Generally, the average profitability portfolios also show higher standard deviations compared to the other profitability groups. There is a pattern in the data of the better performing portfolios having higher standard deviations.

Big share portfolios generally show better performance over the other size groups when the second sort is investment. The only exception is the portfolio of small shares that invest moderately conservatively. This portfolio shows a 0.243% average monthly excess return. In general, more conservative investment portfolios have improved returns over the other investment groups. A similar general pattern is seen in the standard deviations: bigger company portfolios that invest aggressively tend to have higher standard deviations. Perhaps the market sees greater investment as riskier given the long-term nature of investment and the impact that politics and economic health has on investment in the country. The analysis in *Section 4.3 Fama-MacBeth Regressions* sheds light on this assessment of the data.

4.1.2.3 2x2x3 Sorts – 12 Portfolios

The next set of summary statistics shows the average monthly excess return for the entire 23-year test period on portfolios formed in three-way sorts. Table 7 shows sorts based on size and value, and then either profitability or investment, respectively. Table 8 shows sorts based firstly on size, then on either investment or profitability and then on profitability or investment, respectively.

In Table 7, big portfolios in the profitability sort show higher returns than small portfolios, supporting the evidence in Auret and Cline (2011) and McClelland (2014). Across both size groups, high book-to-market portfolios still outperform low book-to-market portfolios. In the small portfolios, there is a large negative return for growth portfolios that have weaker profitability. This may be evidence of a small cap company that may be in a growth phase but has little earnings to back up the investment case. In general, the average profitability portfolios show the best performance. An exception is the small size, low book-to-market portfolio with average profitability. It performs worst in the small size group. High book-to-market portfolios with robust or average profitability have higher standard deviations and returns than those with weak profitability. In terms of standard deviations, the opposite is true for low book-to-market portfolios: weaker profitability portfolios across both size groups have greater standard deviations. This is intuitive as growth stocks such as a small, newly listed company will not have strong earnings but future earnings are already priced in. This effect, however, is seen in both size groups.

In the investment sort, big portfolios mostly have higher returns than small portfolios. There is some pattern in terms of investment levels: in small portfolios, moderate and conservative investment outperforms whereas in big portfolios, the aggressive investment portfolios outperform. This is similar to other sorts and supports the idea that big companies can afford to invest more aggressively and show that they are trying to innovate and not stagnate. The big, high book-to-market, aggressive investment portfolio performs best at 0.136% average excess return per month. Once more, high book-to-market portfolios perform better than low book-to-market portfolios. The exception is the small, conservative group, where the low book-to-market portfolio outperforms. Small growth portfolios have greater standard deviations than small value portfolios, while large value portfolios have greater standard deviations than large growth portfolios. This characteristic could possibly be explained by the fact that small growth shares with future earnings already priced in could be seen as riskier, given that they would not have produced sustainable earnings yet. The large value shares may be indicative of "value traps", where undervalued shares seem to be underpriced yet there is a fundamental reason as to why the share is lowly rated. Moderate or aggressive investment levels tend to show the greatest standard deviations.

Table 7. Average monthly excess returns and standard deviations for 2x2x3 portfolio sorts using size and value as the fixed sorting characteristics

		Small			***Big***	
		Value			***Value***	
		High	Low		High	Low
Profitability	Robust	-0.445%	-0.475%	Robust	0.194%	-0.262%
		7.092%	*4.853%*		*7.183%*	*5.779%*
	Average	-0.145%	-1.362%	Average	0.257%	0.194%
		4.972%	*6.634%*		*6.430%*	*6.204%*
	Weak	-0.305%	-0.950%	Weak	-0.155%	-0.314%
		4.650%	*7.364%*		*6.205%*	*6.436%*
		High	Low		High	Low
Investment	Conservative	-0.359%	-0.187%	Conservative	0.116%	-0.057%
		4.848%	*5.223%*		*6.752%*	*5.760%*
	Moderate	0.084%	-0.759%	Moderate	-0.094%	-0.305%
		4.974%	*5.977%*		*6.761%*	*5.908%*
	Aggressive	-0.531%	-1.075%	Aggressive	0.136%	0.020%
		5.438%	*5.911%*		*6.707%*	*6.238%*

Table 8 shows the sorts on size, investment and profitability as well as size, profitability and investment. As explained in *Section 3.4 The Test Portfolios*, both sets of portfolios were formed as the final sort was not equal to the second sort (2x2x3). Thus, by only including one of the sorts, there may be information that is not discovered which could impact the results.

In the size, investment and profitability sort, big portfolios outperformed small portfolios except in the small, conservative, robust sort. Of all the portfolios, it performed the best. The interaction of small size, conservative investment and robust profitability, all of which should provide greater returns than their counterparts, seems to be a more profitable portfolio combination. This interaction between profitability and investment could be an indication of quality in small shares. Conservative investment outperforms aggressive investment except in the small size and weak profitability portfolio. This makes sense as small shares with weak profitability may not be able to sustain the investment or repay the financing required to fund the investment. In terms of profitability, there is no real pattern. The robust profitability shares in the small portfolio that invest conservatively outperform the weak profitability portfolios, but the weak profitability small portfolios outperform when they invest aggressively. In the big portfolios, average profitability outperforms robust and weak profitability portfolios regardless of investment levels. Overall, big portfolios have larger standard deviations in the average monthly excess returns. Conservative investment portfolios show lower standard deviations compared to aggressive investment portfolios. This pattern was seen in the other sorts as well. Profitability does not show any discernible patterns regarding standard deviations.

Table 8. Average monthly excess returns and standard deviations for 2x2x3 portfolio sorts using size as the fixed sorting characteristic

		Small			***Big***	
		Investment			***Investment***	
		Conservative	Aggressive		Conservative	Aggressive
Profitability	Robust	0.104%	-0.789%	Robust	-0.027%	-0.380%
		4.906%	*5.499%*		*5.779%*	*6.342%*
	Average	-0.237%	-0.757%	Average	0.068%	0.010%
		5.546%	*5.512%*		*6.122%*	*6.384%*
	Weak	-0.622%	-0.358%	Weak	-0.279%	-0.302%
		4.903%	*5.715%*		*6.104%*	*6.415%*
		Profitability			***Profitability***	
		Robust	Weak		Robust	Weak
Investment	Conservative	-0.020%	-0.523%	Conservative	-0.011%	-0.073%
		5.075%	*4.740%*		*6.086%*	*6.242%*
	Moderate	-0.339%	-0.002%	Moderate	-0.301%	-0.257%
		5.421%	*5.157%*		*5.727%*	*6.400%*
	Aggressive	-0.800%	-0.958%	Aggressive	-0.118%	0.051%
		5.725%	*5.759%*		*6.685%*	*5.798%*

In the size, profitability and investment sort, big portfolios mostly outperform. The small, weak and moderate portfolio outperforms its larger counterpart. Robust profitability portfolios outperform weak profitability portfolios about half the time. It is interesting to note that big shares with weak profitability and aggressive investment outperforms all other portfolios. This combination could be associated with growth shares. Because the sort was based on the JSE median, the companies in the portfolio may not all be Top 40 companies. Thus, a growth tilt in the slightly smaller shares in the "big" portfolio may be present. In robust profitability portfolios, conservative investment portfolios outperform aggressive investment portfolios. Perhaps these portfolios contain shares that are more established and are now extracting good profits from their investments. In weak profitability portfolios, moderate investment performs best for small portfolios and aggressive investment performs best in big portfolios. Perhaps the search for improved returns that weak profitability necessitates is rewarded by the market.

Several portfolio average monthly excess returns were negative. The test period covers a variety of bull and bear markets including the technology bubble and credit crisis. The JSE is also exposed to significant influence from the political situation in the country. The replacing of the Minister of Finance at the end of 2015 and in early 2017 led to large declines on the JSE. Credit downgrades more recently would also contribute to this performance. The exact cause of the average negative returns, however, is difficult to pinpoint. Because the portfolios are value-weighted, strong declines in the largest shares would influence the performance of the portfolio as a whole. This may be presented as a case for equal-weighting, but this has its own challenges, such as increasing the weighting of the returns of smaller shares that may influence the portfolio returns in an unrealistic way. Value-weighting follows the methodology of Fama and French (2015) and, as will be mentioned in *Section 6.2 Areas for Future Research*, alternative methodologies can be tested in future studies. Furthermore, despite the negative returns, it is the relationship with the risk factors that is most interesting; that relationship can still guide portfolio selection in a market downturn; for example, to limit the impact of the downturn.

4.2 Model comparison – relative performance

As mentioned above, the aim of this analysis is to compare the relative performance of the models as opposed to statistically testing for rejection or confirmation of the model in tests with power. The analysis helps determine which is the best (but imperfect) method of modelling returns on the JSE. Table 9 below presents the results.

Before analysing each sort, some general results can be presented as they are common to every sort. Firstly, a three-factor model augmenting profitability and investment with the market factor performs the worst of all the models in terms of pricing error and returns left unexplained. It always has the highest intercept; it falls between 1.057% and 1.298%. Subsequently, it also performs worst in tests of the dispersion of the intercepts. The relative dispersion of the intercepts is always greater than 1, thus the dispersion of intercepts is always greater than the dispersion of test portfolio average returns. It does perform better than the CAPM in terms of the dispersion of intercepts due to sampling error and in the adjusted R-squared measure, but falls short of all the other models tested. Failing to account for size has a large impact in a model's ability to explain returns. The CAPM performs second worst across the first three measures and has the lowest adjusted R-squared across all the sorts.

A notable improvement is seen in the adjusted R-squared in every sort when using the five-factor model. The five-factor model explains between 58% and 65% of returns across the various sorts. It is noted that the three-factor models of size and either value, profitability or investment also perform well in the adjusted R-squared measure (often only 1% or 2% lower than the five-factor model).

4.2.1 3x3 Portfolio sorts

In each of the 3x3 sorts, the model that performs best in terms of the first three measures (relating the portion of returns left unexplained by the model) is the three-factor model that matches the factors used in the sort. For example, the three-factor model with size and value performs best in explaining the 3x3 size and value sorted portfolios. This result is expected and is intuitive in that should a portfolio manager want to invest in a particular style, the best model to use would be one that matches that style.

In the 3x3 size and value sorted portfolios, the three-factor model containing size and value performs best. It produces an alpha of 0.779%, narrowly outperforming the size and profitability three-factor model. Three-factor models containing size and either value, profitability or investment perform better than the CAPM or five-factor model. The five-factor model does not improve the pricing error, but the additional factors offer better performance when augmented with size compared to the five-factor model. The same result is evident for the measure of returns left unexplained using the dispersion of the intercepts. The three-factor model of size and value performs best, leaving the least left unexplained. While high at 78.2%, it is the relative performance compared to other models that is of interest. The squared measure shows that the size-value model clearly outperforms the others. In analysing the dispersion of intercepts due to sampling error rather than true dispersion, the three-factor model of size and profitability performs best with a statistic of 1.949.

Table 9. Model comparison results *Profitability is shortened to profit to enable neat presentation*

Portfolio Sorts	$A\|\alpha\|$	$\frac{A\|\alpha\|}{A\|\gamma\|}$	$\frac{A\|\alpha\|^2}{A\|\gamma\|^2}$	$\frac{As^2}{A\|\alpha\|^2}$	*Adjusted R-squared*	*GRS*
3x3 size/value						
MKT	1.029%	1.034	1.068	1.022	43.023%	2.378
SMB HML MKT	0.779%	0.782	0.612	1.565	62.440%	1.377
SMB RMW MKT	0.782%	0.786	0.617	1.949	59.261%	1.551
SMB CMA MKT	0.842%	0.846	0.715	1.773	59.193%	1.426
RMW CMA MKT	1.088%	1.092	1.193	1.238	47.289%	3.703
SMB HML RMW CMA MKT	0.864%	0.868	0.753	1.817	62.591%	0.835
3x3 size/profitability						
MKT	1.015%	1.026	1.054	0.935	44.669%	5.123
SMB HML MKT	0.768%	0.776	0.603	1.601	61.308%	2.183
SMB RMW MKT	0.737%	0.745	0.554	1.739	63.498%	2.120
SMB CMA MKT	0.824%	0.833	0.694	1.574	61.162%	2.153
RMW CMA MKT	1.057%	1.068	1.141	1.083	50.901%	4.785
SMB HML RMW CMA MKT	0.801%	0.810	0.656	2.010	63.546%	2.251
3x3 size/investment						
MKT	1.044%	1.015	1.031	0.807	46.295%	6.311
SMB HML MKT	0.774%	0.752	0.566	1.440	62.619%	2.850
SMB RMW MKT	0.783%	0.762	0.580	1.483	62.942%	3.248
SMB CMA MKT	0.760%	0.740	0.547	1.541	64.525%	2.323
RMW CMA MKT	1.079%	1.049	1.101	0.935	53.042%	8.066
SMB HML RMW CMA MKT	0.800%	0.778	0.606	1.815	65.522%	1.344
3x4 size/value						
MKT	1.145%	1.025	1.051	1.006	39.985%	2.565
SMB HML MKT	0.896%	0.802	0.644	1.556	57.618%	1.479
SMB RMW MKT	0.932%	0.834	0.696	1.755	54.811%	1.758
SMB CMA MKT	0.961%	0.860	0.740	1.735	55.034%	1.635
RMW CMA MKT	1.187%	1.063	1.130	1.256	44.217%	3.617
SMB HML RMW CMA MKT	1.008%	0.903	0.815	1.698	58.493%	1.405
3x4 size/profitability						
MKT	1.067%	1.027	1.055	1.017	41.187%	6.609
SMB HML MKT	0.855%	0.823	0.677	1.638	56.150%	2.283
SMB RMW MKT	0.846%	0.814	0.663	1.699	58.041%	3.420
SMB CMA MKT	0.929%	0.894	0.799	1.555	56.251%	4.428
RMW CMA MKT	1.149%	1.106	1.223	1.115	47.149%	6.915
SMB HML RMW CMA MKT	0.911%	0.876	0.768	1.966	58.313%	2.867

3x4 size/investment						
MKT	1.088%	1.039	1.079	0.922	42.679%	3.411
SMB HML MKT	0.880%	0.840	0.705	1.500	56.451%	1.860
SMB RMW MKT	0.910%	0.869	0.755	1.456	57.164%	2.305
SMB CMA MKT	0.864%	0.825	0.681	1.663	57.922%	1.590
RMW CMA MKT	1.167%	1.114	1.242	1.025	48.262%	3.972
SMB HML RMW CMA MKT	0.884%	0.844	0.713	1.976	59.385%	1.486
2x2x3 size/value/profitability						
MKT	1.244%	1.006	1.013	1.005	40.952%	1.907
SMB HML MKT	0.993%	0.803	0.645	1.470	59.591%	0.680
SMB RMW MKT	0.962%	0.778	0.605	1.768	59.196%	0.721
SMB CMA MKT	1.085%	0.878	0.771	1.614	56.471%	0.890
RMW CMA MKT	1.298%	1.050	1.102	1.136	48.663%	1.758
SMB HML RMW CMA MKT	1.040%	0.841	0.708	1.699	64.585%	0.569
2x2x3 size/value/investment						
MKT	1.183%	1.023	1.047	0.977	42.487%	3.989
SMB HML MKT	0.981%	0.849	0.721	1.362	60.791%	1.970
SMB RMW MKT	1.042%	0.902	0.813	1.413	58.446%	3.080
SMB CMA MKT	1.016%	0.879	0.773	1.546	61.138%	1.998
RMW CMA MKT	1.220%	1.056	1.114	1.178	50.525%	5.741
SMB HML RMW CMA MKT	0.983%	0.851	0.724	1.722	65.501%	1.954
2x2x3 size/investment/profit*						
MKT	1.087%	0.994	0.989	1.046	42.895%	4.097
SMB HML MKT	0.873%	0.799	0.638	1.635	59.598%	2.117
SMB RMW MKT	0.893%	0.817	0.668	1.589	61.881%	2.298
SMB CMA MKT	0.917%	0.839	0.703	1.709	60.563%	1.730
RMW CMA MKT	1.093%	1.000	1.000	1.240	51.932%	7.694
SMB HML RMW CMA MKT	0.958%	0.876	0.768	1.847	63.613%	4.926
2x2x3 size/profit/investment						
MKT	1.120%	0.997	0.994	0.998	42.610%	1.965
SMB HML MKT	0.968%	0.862	0.742	1.351	58.751%	2.104
SMB RMW MKT	0.950%	0.845	0.715	1.446	60.043%	1.965
SMB CMA MKT	0.954%	0.849	0.721	1.505	60.912%	1.951
RMW CMA MKT	1.164%	1.036	1.074	1.098	51.804%	3.967
SMB HML RMW CMA MKT	1.015%	0.903	0.816	1.584	64.548%	3.284

The five-factor model outperforms the other three-factor models with a statistic of 1.817, with the size and investment model the next best performer at 1.773. The new factors show an improvement in looking at what is the most likely cause of the dispersion of intercepts. Thus, the new factors improve the modelling of the returns. As expected, the adjusted R-squared is highest for the size-value model compared to the other three-factor models, though it performs only slightly worse than the five-factor model.

In the size and profitability sort, the size-profitability three-factor model produces the lowest intercept. This intercept of 0.737% is lower than the size-value model intercept from the previous sort and is the lowest pricing error of all models across all the sorts. A three-factor model of size and value produces the second lowest intercept with the five-factor model coming in next best. In measuring the returns left unexplained, the size-profitability model leaves the least unexplained in the size and profitability sort. The three-factor model of size and profitability is improved upon by the five-factor model in terms of the dispersion of intercepts due to sampling error as opposed to true dispersion. The five-factor model has a statistic of 2.010, the highest across all models and sorts. The size-profitability model performs next best compared to the other models in terms of dispersion due to sampling error. As expected, the three-factor model of size and profitability has a high adjusted R-squared at 63.498%, but cannot improve on the five-factor model's performance in this regard.

In the size and investment sort, the size-investment model produces the lowest intercept. Size and value outperforms the size-profitability model, but both still improve on the performance of the five-factor model. In terms of returns left unexplained, the three-factor model of size and investment produces the best result across all models and sorts with a statistic of 74%. The other three-factor models of size and either profitability or investment perform very well. In the size and investment sort, the five-factor model performs best once more in terms of the sampling error measure. Of the three-factor models, it is unsurprising that the size-investment model trumps the other combinations as well as the CAPM. Finally, the size-investment model explains returns best out of the three-factor models and CAPM, only outperformed by the five-factor model. The three-factor models of size and one of the other factors all have adjusted R-squares above 62%.

The GRS statistics can be summarised as follows for the 3x3 sorts: the test is rejected that all the alphas are jointly zero, and thus all the models are rejected. However, the relative performance is of interest and not the statistical rejection of the model. The CAPM and profitability-investment three-factor model perform the worst, having the highest GRS statistics across all three sorts. Consistent with Fama and French (2015), the five-factor model shows an improved GRS statistic in two of the three sorts. The next best performing statistic is based on the second sort: for example, in the size and value sort, the size-value three-factor model has the lowest GRS statistic. This is intuitive as a model designed to explain certain factors should, in theory, best describe portfolios formed using those same factors.

Overall, each three-factor model performs best depending on the second characteristic used in the sort. It is observed that generally, the size-value and size-profitability models work well in explaining the returns across the various sorts. Their intercepts are usually smaller than the other models and leave less of the returns unexplained. There may be evidence for value and profitability complementing each other in explaining returns. The only notable improvement that the five-factor model brings is in terms of the adjusted R-squared and the dispersion of intercepts due to sampling error rather than true dispersion. The five-factor model is not an improvement but it still does reasonably well in explaining returns.

4.2.2 3x4 Portfolio sorts

Because these portfolios are also constructed on a two-way sort, the results are almost identical to the 3x3 sorts. In general, the intercepts are larger and more is left unexplained compared to the 3x3 sorts. Adjusted R-squareds are also lower across all 3x4 sorts. The five-factor model produces the highest adjusted R-squareds across all 3x4 sorts. The CAPM still underperforms in most measures across the 3x4 sorts. Overall, the models analysed perform worse on a bigger sort.

In the size and value portfolios, the size-value model produces the lowest intercept at 0.896%, and as noted above, the size-profitability model performs well in modelling the returns in this sort. Less is left unexplained in the size-value model compared to the other models. The size-profitability model produces the highest statistic when looking at what causes the dispersion of intercepts (sampling error or true dispersion). The size-value model has the highest adjusted R-squared of the three-factor models at 57.6%. The size-investment model performs better than the size-profitability model in this sort in terms of the adjusted R-squared.

In the size and profitability portfolios, the size-profitability model results in the smallest intercept at 0.846%. This is the lowest intercept across all the sorts in the 3x4 category and mirrors the result in the 3x3 sorts. Again, the size-value model performs well and the five-factor model produces the next lowest intercept after the size-value model. The size-profitability model leaves the least deviation unexplained, and the size-value model only performs slightly worse at 82.3%. Once more, in terms of dispersion of intercepts, a five-factor model results in the best statistic at 1.966, with the size-value and size-profitability models performing well at 1.699 and 1.638 respectively. The adjusted R-squared is highest for the size-profitability model when comparing the three-factor models and CAPM, and is only outperformed by the five-factor model.

In the size and investment 3x4 sorts, the size-investment model produces the lowest pricing error at 0.864%. As in the 3x3 sorts, the size-value three-factor model outperforms the size-profitability model when looking at the intercepts. It is noted that the five-factor model outperforms the size-profitability model in terms of pricing error. The size and investment model leaves the least of the deviation unexplained at 82.5%. In terms of dispersion due to sampling error, the five-factor model outperforms as it did in the 3x3 sorts. The adjusted R-squareds are all quite high, with the five-factor model once again performing best.

The GRS result is the same as the 3x3 sorts in terms of the models tested: the models are rejected and the alphas are not jointly zero. Further to this, the CAPM and profitability-investment model again perform the worst, having the highest GRS statistics. Again, the five-factor model performs best in two of the three sorts, with the size-value model performing best in the size and profit sort. In the other sorts, the next best model once more depends on the second sort, with the size-value and size-investment three-factor models performing best when the second sort is value or investment, respectively.

As in the 3x3 sorts, the size-value and size-profitability models generally perform well. The five-factor model performed as well as in the 3x3 sort. It seems that with a greater dispersion of portfolios, the models perform slightly worse. This could be because they must explain a greater range of difference in the larger sorts. As in the 3x3 sorts, profitability and value, when augmented with size, tend to produce the most consistent results. If one were to invest based on a certain factor, the best model to use would be a three-factor model that augments size and that chosen factor. While it seems intuitive, the result has been extended to the new factors that are being tested. Because profitability and value seem to work well as models, there may be a case for postulating that value may capture some of the returns explained by investment. The interaction of value and profitability may be evidence of an aspect of quality being priced into returns, and the new factors help in capturing it.

4.2.3 2x2x3 Portfolio sorts

Once more, the CAPM and profitability-investment model perform the worst. The five-factor model produces the highest adjusted R-squared values.

In the size, value and profitability sort, the size-profitability and size-value models perform best with intercepts of 0.962% and 0.993% respectively. This result is extended to the returns left unexplained as both leave the least unexplained by the model. The five-factor model performs better than the size-investment model, which may be further support for the use of profitability and value to capture investment. The size-profitability model performs best in terms of dispersion due to sampling error with a statistic of 1.768, with the five-factor model producing the next highest statistic. As before, the five-factor model produces the highest adjusted R-squared, with the size-profitability and size-value models producing the next best measures.

In the size, value and investment sort, the size-value model produces the lowest intercept, followed closely by the five-factor model. This is consistent with other results obtained above and shows how value captures investment. The five-factor model has the highest adjusted R-squared followed by the size-investment and size-value models at 61.1% and 60.8% respectively. Profitability is not useful in explaining investment-based portfolios in this analysis. Finally, the five-factor model works best in terms of the dispersion of intercepts due to sampling error.

In the size, investment and profitability sort, the size-value model describes returns best with an intercept of 0.873%. This is followed closely by the size-profitability model at 0.893%. Following from this, less is left unexplained by these two models compared to the others. The sampling error measure shows that the five-factor model performs best, followed by the size-investment model, and finally the size-value model. Interestingly, the size-profitability model has the highest adjusted R-squared of the three-factor models, with the five-factor model performing best.

In the size, profitability and investment sort, the size-profitability model performs best, with the lowest intercept and least left unexplained at 0.950% and 84.5% respectively. Once again, size-investment and the five-factor model has the best performance in terms of dispersion due to sampling error and in terms of adjusted R-squared. However, it is noted that the size-profitability three-factor model still produces an adjusted R-squared above 60%.

In terms of the GRS statistic, when the second sort is value, the five-factor model performs best with the lowest statistic. The size-value three-factor model performs second best in both sorts, with the size-investment three-factor model's performance being close to that of the size-value three-factor model. Again, the CAPM and profitability-investment three-factor model perform worst. When the second sort is investment or profitability, the size-investment three-factor model performs best. The five-factor model performs poorly in both these sorts. The size-value three-factor model performs well in the sorts. Surprisingly, the CAPM performs as well as the size-profitability three-factor model in the size, profitability and investment sort. Once more, the models are all rejected in testing whether the alphas are jointly zero.

The results of the 2x2x3 sorts further reinforce the developing theme that size, value and profitability could explain a broad range of differently formed portfolios.

4.2.4 Observations

The model comparison provides several important common results. The five-factor model best describes returns from a statistical perspective due to having the highest adjusted R-squared across all sorts. It also produced several measures of dispersion due to sampling error that were the highest for a particular sort. The five-factor model showed a reduced the GRS statistic, as in Fama and French (2015), for most sorts tested. Thus, from a statistical perspective, it performed reasonably well. However, the results of interest were the relative performance using various measures of returns left unexplained. Overall, while it was expected that a model using specific factors would perform best in explaining the returns of portfolios formed on the same factors, the size-value and size-profitability models produced reasonably consistent results throughout the sorts tested. In particular, those two models did best in explaining three-way sorts on combinations of size, value, profitability and investment. This lead to postulation of the idea that perhaps value and profitability capture the returns associated with investment. Novy-Marx (2013) showed that a profitability effect had the same power as value in explaining the cross-section of returns. The idea that profitability and value can work together is discussed in the paper and the conclusion reached that using profitability to hedge out a value strategy enabled the investor to capture both premiums with no additional risk. The potential redundancy of investment is contrary to the results of Fama and French (2015) which found that value was subsumed by the additional factors, but is similar to Chiah et al. (2016) which observed that value retained its explanatory power on the Australian equities market. This implication is tested in the next section, where a four-factor model of size, value, profitability and the market is evaluated on the same criteria as in *Section 4.2 Model Comparison – Relative Performance*.

4.2.5 A better model?

As discussed in the prior sections, the size-value and size-profitability models generally perform the best of all the models tested. Notably, these two models result in the smallest pricing errors in the 2x2x3 sorts and still perform admirably in the two-way sorts, even when explaining portfolios formed on other factors. Thus, it was postulated that a four-factor model that drops the investment factor and includes size, value and profitability may produce improved results compared to the five-factor model or even the three-factor models from which this hypothesis was derived. The results are presented in Table 10:

Table 10. Four-factor model results **Profitability is shortened to profit to enable neat presentation**

Portfolio Sorts	$A\|\alpha\|$	$\frac{A\|\alpha\|}{A\|\gamma\|}$	$\frac{A\|\alpha\|^2}{A\|\gamma\|^2}$	$\frac{As^2}{A\|\alpha\|^2}$	*Adjusted R-squared*	*GRS*
3x3 size/value	0.825%	0.829	0.687	1.794	61.473%	1.085
3x3 size/profit	0.775%	0.784	0.614	1.807	63.192%	2.777
3x3 size/investment	0.824%	0.801	0.642	1.513	63.293%	1.845
3x4 size/value	0.969%	0.867	0.752	1.663	56.851%	1.401
3x4 size/profit	0.903%	0.869	0.755	1.693	58.014%	4.034
3x4 size/investment	0.923%	0.881	0.776	1.593	57.700%	1.616
2x2x3 size/value/profit*	0.972%	0.786	0.618	1.733	63.095%	0.595
2x2x3 size/value/investment	1.038%	0.898	0.806	1.415	61.906%	2.669
2x2x3 size/investment/profit	0.939%	0.859	0.738	1.654	61.341%	4.058
2x2x3 size/profit/investment	1.009%	0.898	0.807	1.424	61.053%	2.247

The four-factor model outperforms the size-value model in the 2x2x3 size, value and profitability sort when looking at the intercept. It still does not outperform the size-profitability model which has an intercept of 0.962%. In the size, value and investment sort, it marginally outperforms the size-profitability model but it falls behind the size-value and five-factor model with an intercept of 1.038%. In comparing it to the five-factor model, the intercepts are lower for seven of the ten sorts. It is observed that two-way sorts based on investment and the size, value and investment sort result in the five-factor model having lower intercepts. This is expected to some degree as the investment factor is still present in the five-factor model and those sorts are based on investment. Overall, the four-factor model offers smaller pricing errors compared to the five-factor model but does not quite fully capture the investment risk factor to the same degree as the five-factor model. It does not do as well in modelling portfolios with the final sort on investment.

The four-factor model does not improve of the size-value and size-profitability models in terms of the returns left unexplained by the model. The only minor improvement is evident in the size, value and profitability sort where it outperforms the size-value model. As in the intercept analysis, the four-factor model improves on the returns left unexplained in seven of the ten sorts in comparison to the five-factor model, with sorts involving investment being the exceptions.

In terms of sampling error explaining the dispersion of intercepts, the five-factor model generally performed well. The four-factor model only improves on these results in the size, value and profitability sort. Thus, the five-factor model does a better job in that the dispersion of intercepts is probably due to sampling error rather than true dispersion as modelled with certain risk factors.

The five-factor model showed the highest adjusted R-squared regardless of sort. The four-factor model performs roughly the same as both the size-value and size-profitability in the two-way sorts, with the five-factor model still outperforming it in each case regardless of sort. In the three-way sorts, the four-factor model performs better in all but the 2x2x3 size, investment and profitability sort.

When analysing the GRS statistic in the two-way sorts, the four-factor model performs well, but not best, in sorts based on value and investment, despite there being no investment factor. It performs best in the 3x4 size and value sort. In the three-way sorts, the four-factor model only performs well in the size, value and profitability sort. The model is rejected across all portfolio sorts in terms of the test that the alphas are jointly zero. While showing minor improvements in one or two cases, the four-factor model does not, in an overall sense, improve on the performance of the five-factor model or the three-factor models using value or investment.

This analysis shows that even though there were some pricing error improvements over the five-factor model, the three-factor models of size-value and size-profitability still perform best in terms of intercepts and proportion of returns left unexplained. The five-factor model still shows the highest adjusted R-squared as more risk factors are captured. The strength of the size-value and size-profitability models is reinforced by this analysis.

4.2.6 Summary

The theory that value and profitability may jointly capture investment does not seem to hold in the analysis using a four-factor model that omits investment. From the analysis above, the size-value and size-profitability models work best in leaving as little pricing error as possible in the modelling of returns as well as leaving as little return unexplained as possible. The five-factor model does not improve on these two models, contrary to Fama-French (2015). However, this study tested a slightly different set of models, rather incorporating the new factors into three-factor models as opposed to four-factor models. This resulted in equivalent performance compared to the original three-factor model. One could use a size-value or size-profitability model to explain returns on a variety of portfolios sorted on different characteristics. In general, the dispersion of intercepts was mostly due to sampling error as opposed to true dispersion, meaning the models generally explained returns well. Furthermore, while the adjusted R-squareds were not as high as for the five-factor model, that can be as a result of the methodology and the fact that more factors will generally give a more complete explanation of returns.

4.3 Fama-MacBeth regressions

Fama and MacBeth (1973) pioneered a methodology to test for significant premiums in the cross-section of average returns. The same methodology was applied in assessing whether there are significantly priced premiums in the cross-section of returns on the JSE. The results are presented per model to determine how well each model picks up on the various factors being tested for and to see which premiums are present in the various portfolio sorts. The p-value of the t-stat is shown in italics under the t-stat. The t-stat is based on the normal distribution given the averaging procedure as noted in Fama and MacBeth (1973). Premiums are in bold if significant, with the significance level indicated by the asterisk.

4.3.1 The CAPM

Table 11 shows the results from the Fama-MacBeth analysis using the CAPM to explain the returns on portfolios sorted by different characteristics.

Table 11. CAPM results from the Fama-MacBeth analysis

CAPM *MKT*	*Alpha*	*t-stat*	*MKT Premium*	*t-stat*	*Adjusted R-squared*
3x3 size/value	-0.462%	-0.867	-0.071%	-0.144	13.844%
		0.193		*0.443*	
3x3 size/profitability	-0.576%	-1.032	0.080%	0.154	15.522%
		0.151		*0.439*	
3x3 size/investment	-0.534%	-1.075	0.045%	0.100	7.610%
		0.141		*0.460*	
3x4 size/value	-0.402%	-0.827	-0.184%	-0.391	9.143%
		0.204		*0.348*	
3x4 size/profitability	-0.399%	-0.837	-0.188%	-0.447	12.651%
		0.201		*0.328*	
3x4 size/investment	-0.515%	-1.063	-0.012%	-0.031	5.703%
		0.144		*0.488*	
2x2x3 size/value/profitability	-0.440%	-0.916	-0.216%	-0.466	10.339%
		0.180		*0.321*	
2x2x3 size/value/investment	-0.412%	-0.873	-0.163%	-0.405	8.830%
		0.191		*0.343*	
2x2x3 size/investment/profitability	-0.427%	-1.056	-0.119%	-0.314	14.633%
		0.145		*0.377*	
2x2x3 size/profitability/investment	-0.209%	-0.547	-0.393%	-1.019	10.887%
		0.292		*0.154*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

In line with much of prior research, the CAPM does not perform well in explaining the cross-section of returns. It is noted that the alphas are all negative. In this analysis, the CAPM overprices portfolio returns across all the sorts evaluated. The alpha is largest (in absolute terms) for the 3x3 size and profitability sort at -0.576% (*0.151*). The alphas are, for the most part, larger compared to the other models. There are no significant market premiums; the most significant market premium is in the 2x2x3 size, profitability and investment sorted portfolios. The premium is negative at -0.393% (*0.154*). The majority of premiums are negative showing that portfolios of stocks with larger betas earned lower returns. This is consistent with van Rensburg and Robertson (2003). The negative relation to the market factor is a function of the data: the sorts and related returns. In the Summary Statistics, it was noted that the majority of portfolio returns were negative, and that the average market excess return was positive. Thus, statistically, there would be an inverse relationship between the portfolio returns and the market factor. In this case, it is only significant in one set of portfolios. The adjusted R-squareds are all relatively low. Six of the ten sorts have adjusted R-squareds above 10%, which is a reasonable result for the CAPM.

4.3.2 The size-value three-factor model

Table 12 presents the results for the original Fama-French three-factor model as applied to the various portfolio sorts of JSE listed stocks. Contrary to the CAPM, the size-value model underprices most portfolios, with only the 3x4 size and profitability sort having a small negative intercept. The intercepts are generally not significant. There is a significant unexplained portion of returns in the 2x2x3 size, value and investment sort which is significant at the 1% level. The size-value three factor model also showed significant intercepts in the 3x4 size and value sort and the size, profitability and investment sort.

The Market Premium

Once more, there is a consistently negative market premium across the various sorts analysed. There is an inverse relationship between beta and returns. The premiums are stronger in the three-way sorts compared to the two-way sorts and seven of the ten sorts show significant premiums. The size and value 3x4 sort has a significant premium of -0.872% (*0.025*) and both the 3x3 size and value and 3x3 size and profitability sorts has significant premiums at the 10% level. The 2x2x3 size, value and investment sort has the largest premium of -1.389% significant at the 1% level. The final two sorts, based on size, investment and profitability and size, profitability and investment, show significant premiums of -0.565% (*0.088*) and -0.922% (*0.025*). The market is significantly negatively priced in a size-value model when looking at three-way sorts. Once more, this could be because of the negative average excess returns displayed by many of the portfolios. In these two sorts, the market factor is the most significant premium, meaning that the role of size and value is diminished when the sort is focused on profitability and investment, even though size is considered. These portfolios could be used as a hedge against the market given the inverse relationship.

The Size Premium

There is a significant size premium in six of the ten sorts. The size premium is not present in portfolios based on size and profitability. As mentioned above, there is also no size premium in the size, investment and profitability sort and an almost insignificant premium in the size, profitability and investment sort (-0.427% with a significance of *0.097*). The strongest size premium is seen in the 2x2x3 size, value and profitability sort with a premium of -0.529%

(*0.026*); this mirrors the strength of the market premium in sorts based on three characteristics as opposed to two. The other premiums are significant at the 10% level and the 5% level, with the 3x4 size and investment sort showing the largest absolute premium of -0.602% (*0.044*). Investment and value based sorts generally show significant size premiums while sorts including profitability only show a size premium in the three-way sorts. This may be due to the differing levels of investment between small and big companies as well as the previously-mentioned evidence of a growth effect (which was sort specific). The size premiums are all negative. This is evidence that big firm portfolios outperformed small firm portfolios. This is contradictory to the size effect first observed by Banz (1981) as well as Fama and French (1993, 2015), but is supported by Auret and Cline (2011) and McClelland (2014) who also tested the companies on the JSE.

The Value Premium

Three sorts showed a significant value premium. Of the three sorts, two of them include investment, the strongest being in the 3x4 size and investment sort at 1.094% (*0.020*). The majority of the premiums are positive, meaning that high-book-to-market portfolios outperformed low book-to-market portfolios. This is consistent with much of the prior research into the value effect on the JSE including Kruger and Toerien (2014) and van Heerden and van Rensburg (2015). In the cross-section of returns, there does not seem to be evidence of a growth effect. None of the two-way sorts on size and value or profitability produced a significant value premium. Overall, the value premium is less prevalent as captured by a size-value three-factor model and it is generally similar in significance to the size premium when it is captured.

The adjusted R-squareds are higher than in the CAPM, which is expected as more risk premiums are captured by the additional factors. The model describes the returns in the size and profitability sort best with an adjusted R-squared of 37.054%.

Table 12. Size-value three-factor model results from the Fama-MacBeth analysis

Size-value three-factor model *MKT SMB HML*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***SMB Premium***	***t-stat***	***HML Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	0.322%	0.514	-0.707%	-1.310	-0.349%	-1.645	0.341%	1.112	29.873%
		0.304		****0.095***		*****0.050***		*0.133*	
3x3 size/profitability	0.554%	0.737	-0.896%	-1.312	-0.180%	-0.563	-0.279%	-0.529	37.054%
		0.231		****0.095***		*0.287*		*0.298*	
3x3 size/investment	0.073%	0.148	-0.418%	-0.839	-0.345%	-1.063	0.638%	1.257	10.960%
		0.441		*0.201*		*0.144*		*0.104*	
3x4 size/value	0.493%	1.347	-0.872%	-1.966	-0.362%	-1.385	0.440%	1.216	27.300%
		****0.089***		*****0.025***		****0.083***		*0.112*	
3x4 size/profitability	-0.004%	-0.011	-0.325%	-0.907	-0.227%	-0.775	0.286%	0.539	28.762%
		0.496		*0.182*		*0.219*		*0.295*	
3x4 size/investment	0.136%	0.306	-0.531%	-1.117	-0.602%	-1.707	1.094%	2.061	24.271%
		0.380		*0.132*		*****0.044***		*****0.020***	
2x2x3 size/value/profitability	0.391%	0.842	-0.721%	-1.311	-0.529%	-1.942	0.408%	1.347	30.558%
		0.200		****0.095***		*****0.026***		****0.089***	
2x2x3 size/value/investment	1.048%	3.105	-1.389%	-2.738	-0.473%	-1.629	0.491%	1.442	30.509%
		******0.001***		******0.003***		****0.052***		****0.075***	
2x2x3 size/investment/profitability	0.186%	0.510	-0.565%	-1.355	-0.340%	-1.140	0.377%	1.050	31.754%
		0.305		****0.088***		*0.127*		*0.147*	
2x2x3 size/profitability/investment	0.555%	1.441	-0.922%	-1.966	-0.427%	-1.298	0.437%	0.916	27.854%
		****0.075***		*****0.025***		****0.097***		*0.180*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

4.3.3 The size-profitability three-factor model

Table 13 presents the results from using a size-profitability three-factor model. The intercepts are generally greater than when the portfolio sorts were modelled with the size-value three-factor model. There are significant unexplained returns in the six of the ten sorts, with all the 2x2x3 sorts showing strongly significant intercepts. This model is thus weaker than the size-value model in describing returns and has the most significant intercepts of all the models.

The Market Premium

The market premium is significant in each of the four 2x2x3 sorts and is significant at the 1% level in three of the four sorts. The 3x3 size and profitability sort no longer has a significant market premium. The premium is largest and strongest in the 2x2x3 size, value and investment and size, profitability and investment sorts at -1.639% (*0.001*) and -1.622% (*0.000*). These results are like those in the size-value model. There is a stronger market premium in the 3x4 size and value sort when using the size-profitability model compared to the size-value model. This may be because the market factor in the size-profitability model captures the majority of the cross-section of returns; this is evidenced by the lack of a significant size premium in that size and value sort.

The Size Premium

The size-profitability model fails to capture a significant size-premium to the same extent as the size-value model. The only significant premium is in the size, value and profitability sort at -0.404% (*0.081*), which is indicative of big size portfolios outperforming small size portfolios. The premiums are negative in all ten sorts. This is consistent with the summary statistics. The size-profitability model does not capture a size effect, meaning that perhaps there is an interaction between size and value that better enables the modelling of returns, or that the market and profitability factors subsume the size factor in this model.

The Profitability Premium

The model captured significant profitability premiums in the size and value 3x3 sort and the 2x2x3 size, value and profitability sort. Thus, both the value and profitability factors captured premiums in this 2x2x3 sort. No other sorts showed a significant profitability premium.

There is a strong negative profitability premium of -0.885% (*0.024*) in the size and value sort. Firms with weaker profitability thus outperformed strong profitability firms in this sort. A value premium was also found in the 2x2x3 size, value and profitability sort when using the size-value model. This could be because weak profitability firms might be in distress; this can be associated with the value effect because distressed firms could have depressed share prices, causing them to be undervalued. There is a pattern in the results (despite the majority being insignificant): value two-way sorts have negative profitability premiums while profitability based sorts have positive premiums. In the three-way sorts, if the final sort is profitability, the premium is positive while if the final sort is based on investment, the premium is negative. Based on profitability, robust portfolios outperform, which is consistent with Novy-Marx (2013); Novy-Marx (2013) also confirms the link between value and profitability.

The adjusted R-squareds are generally greater in the size-profitability model than in the size-value model. Both models struggle to explain the 3x3 size and investment cross-sectional returns. This is explained by the fact that investment is not specifically captured by a size, value or profitability factor. This model also best describes the 3x3 size and profitability sort with an adjusted R-squared of 42.283%.

Table 13. Size-profitability three-factor model results from the Fama-MacBeth analysis

Size-profitability three-factor model *MKT SMB RMW*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***SMB Premium***	***t-stat***	***RMW Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	1.132%	1.922	-1.502%	-2.814	-0.172%	-0.544	-0.885%	-1.981	37.899%
		*****0.027***		******0.002***		*0.293*		*****0.024***	
3x3 size/profitability	0.167%	0.301	-0.509%	-1.012	-0.185%	-0.699	0.217%	0.826	42.283%
		0.382		*0.156*		*0.242*		*0.204*	
3x3 size/investment	-0.175%	-0.383	-0.049%	-0.104	-0.225%	-0.789	0.104%	0.263	11.162%
		0.351		*0.459*		*0.215*		*0.396*	
3x4 size/value	0.965%	2.303	-1.257%	-2.264	-0.292%	-1.005	-0.221%	-0.574	28.718%
		*****0.011***		******0.012***		*0.158*		*0.283*	
3x4 size/profitability	-0.072%	-0.224	-0.245%	-0.678	-0.124%	-0.444	0.210%	0.870	33.679%
		0.411		*0.249*		*0.329*		*0.192*	
3x4 size/investment	0.152%	0.336	-0.410%	-0.836	-0.278%	-0.958	-0.016%	-0.049	16.655%
		0.368		*0.202*		*0.169*		*0.481*	
2x2x3 size/value/profitability	1.123%	2.772	-1.332%	-2.284	-0.404%	-1.396	0.373%	1.608	37.579%
		******0.003***		*****0.011***		****0.081***		****0.054***	
2x2x3 size/value/investment	1.471%	2.997	-1.639%	-3.095	-0.305%	-1.123	-0.403%	-1.038	22.528%
		******0.001***		******0.001***		*0.131*		*0.150*	
2x2x3 size/investment/profitability	1.089%	3.191	-1.414%	-3.101	-0.190%	-0.701	0.132%	0.635	37.870%
		******0.001***		******0.001***		*0.242*		*0.263*	
2x2x3 size/profitability/investment	1.314%	3.717	-1.622%	-3.390	-0.168%	-0.648	-0.173%	-0.643	34.856%
		******0.000***		******0.000***		*0.258*		*0.260*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

4.3.4 The size-investment three-factor model

The results from testing the size-investment three-factor model are shown in Table 14. The alphas are a bit closer to zero compared to the size-profitability model but are greater than the alphas in the size-value model. Two of the 2x2x3 sorts have significant alphas, with four sorts showing negative alphas and six showing positive alphas. The largest alpha is 1.128% in the size, value and investment sort. Similarly to the other three-factor models, the size-investment model also leaves returns unexplained in the three-way sorts.

The Market Premium

There is a similar pattern to the market premium when returns are explained using the size-investment model. Both two-way size and profitability sorts have significant market premiums at the 10% level. In total, there are 6 portfolio sorts that show a significant premium, and the sorts that do show a premium are mostly common across all three-factor models tested thus far. The size-investment model captures a premium on all three-way sorts, consistent with the other two three-factor models tested. The strongest premium is -1.422% (*0.001*) in the size, value and investment sort. There are also significant market premiums in the size, value and profitability and size, profitability and investment sorts at -0.911% (*0.035*) and -0.848% (*0.033*) respectively. The premiums are all negative, as observed in the prior models.

The Size Premium

There is no significant size premium in any of the sorts when using the size-investment model. The premiums are all negative, once more indicating that, in general, big company portfolios outperform small company portfolios. The most significant premium is in the size, value and profitability sort at -0.357% (*0.131*). The size premium found using other models is not really picked up by the market or the investment factor.

The Investment Premium

Only three sorts show a significant investment premium. The 3x4 size and investment sort, the size, investment and profitability sort, and the size, profitability and investment sort have significant premiums at the 10% level, the most significant being 0.439% (*0.052)* in the size, investment and profitability sort.

The positive premiums across all sorts indicate that conservative investment portfolios will outperform aggressive investment portfolios. This is consistent with Cooper et al. (2008)

whereby conservative investment leads to greater returns. The three-way sorts generally have greater premiums than the two-way sorts. Sorts based on size and profitability show smaller premiums than those based on investment. Perhaps there is a relationship between profitability and investment levels that is being priced into the returns. Weaker profitability may be associated with aggressive investment that is common to growth stocks. Further to this, perhaps companies with stronger profits are more established and thus investment levels are lower and used for maintenance rather than expansion of the asset base.

The adjusted R-squareds are roughly the same as in the size-value model. Once again, the model has trouble explaining the returns on the 3x3 size and investment sort. This is surprising as the model includes a factor to specifically capture the premiums associated with investment. Perhaps total asset growth itself is not the best factor to capture a premium associated with investment. The model best explains the returns on the 3x3 size and value sort with an adjusted R-squared of 37.134%.

Table 14. Size-investment three-factor model results from the Fama-MacBeth analysis

Size-investment three-factor model *MKT SMB CMA*	*Alpha*	*t-stat*	*MKT Premium*	*t-stat*	*SMB Premium*	*t-stat*	*CMA Premium*	*t-stat*	*Adjusted R-squared*
3x3 size/value	-0.019%	-0.046	-0.352%	-0.819	-0.157%	-0.704	0.335%	0.514	37.134%
		0.482		*0.206*		*0.241*		*0.304*	
3x3 size/profitability	0.320%	0.575	-0.787%	-1.544	-0.225%	-0.827	0.018%	0.043	30.473%
		0.283		***0.061**		*0.204*		*0.483*	
3x3 size/investment	-0.147%	-0.361	-0.182%	-0.373	-0.146%	-0.591	0.224%	0.741	18.131%
		0.359		*0.355*		*0.277*		*0.229*	
3x4 size/value	0.109%	0.319	-0.476%	-1.059	-0.157%	-0.673	0.118%	0.227	30.401%
		0.375		*0.145*		*0.250*		*0.410*	
3x4 size/profitability	-0.030%	-0.076	-0.460%	-1.335	-0.047%	-0.186	0.230%	0.624	24.384%
		0.470		***0.091**		*0.426*		*0.266*	
3x4 size/investment	-0.066%	-0.165	-0.228%	-0.484	-0.176%	-0.691	0.465%	1.454	23.543%
		0.434		*0.314*		*0.245*		***0.073**	
2x2x3 size/value/profitability	0.633%	1.711	-0.911%	-1.807	-0.357%	-1.124	0.489%	1.206	30.191%
		****0.044**		****0.035**		*0.131*		*0.114*	
2x2x3 size/value/investment	1.128%	3.884	-1.422%	-3.112	-0.273%	-1.023	0.229%	0.794	26.307%
		*****0.000**		*****0.001**		*0.153*		*0.214*	
2x2x3 size/investment/profitability	0.413%	1.092	-0.709%	-1.601	-0.239%	-0.883	0.439%	1.621	36.384%
		0.138		***0.055**		*0.189*		***0.052**	
2x2x3 size/profitability/investment	0.476%	1.234	-0.848%	-1.842	-0.257%	-0.918	0.317%	1.288	35.106%
		0.109		****0.033**		*0.179*		***0.099**	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

4.3.5 The profitability-investment three-factor model

Table 15 displays the results for the Fama-MacBeth analysis using a profitability-investment model. The motivation behind this model was to test whether the new factors could replicate the results of the other models, and in particular, the original size-value three-factor model. The alphas are similar to the other models tested. As in the model comparison analysis, it is the worst performer of all the three-factor models tested. Only one portfolio sort showed a significant intercept which was at the 10% level. In terms of the return left unexplained by the other factors, the profitability-investment three-factor model performs best. However, its explanatory power in terms of the various premiums is lacking.

The Market Premium

The were no significant market premiums in any of the sorts. The most significant premium is in the 3x3 size and value sort at -0.487% (*0.114*). There is no discernible pattern in the direction of the premium, with seven sorts showing a negative premium and three showing a positive premium. The model does not capture the market premium but it does support the evidence for an inverse relationship between the market factor and returns.

The Profitability Premium

The model captures a profitability premium in two of the ten sorts, with the 3x3 size and value sort showing a premium in the size-profit and profit-investment three-factor models. The profitability-investment model also captures a profitability premium in the 3x3 size and investment sort. The premiums are -0.588% (*0.080*) and -0.566% (*0.044*) and are the largest and most significant premiums respectively. In the two-way sorts, size and value and size and investment have negative profitability premiums, whereas the two size and profitability sorts have positive premiums. This provides evidence that profitability based portfolios will earn returns that are positively correlated to profitability. Both significant premiums are negative which suggests that the weak profitability portfolios outperform when sorted on value and investment. The premiums captured are observed to a slightly lesser degree compared to the size-profitability model.

The Investment Premium

An investment premium is captured to a greater extent than the size-investment model. The premiums are all positive, consistent with prior literature and results, indicating that conservative investment portfolios outperform aggressive investment portfolios. This is the same result as in the size-investment model. Three of the four three-way sorts exhibit significant premiums, with both sorts based on size, investment and profitability having significant premiums captured by both the size-investment and profitability-investment three-factors models. The most significant premium is in the 2x2x3 size, investment and profitability sort at 0.458% (*0.035*). The largest premium is 0.728% (*0.043*) in the 2x2x3 size, value and profitability sort.

The adjusted R-squareds are generally weaker than the other three-factor models, but follow the same pattern in that the returns in the investment based sorts are the least explained. This model best describes the 3x3 size and value sorted portfolios with an adjusted R-squared of 34.824%.

Of the three-factor models, the size-value model captures more significant premiums than the other models. It also best captures a size premium whereby portfolios consisting of big shares outperform small portfolios. The size-profitability model captures stronger market premiums than the size-value model but fewer across the sorts. One could conclude that there is a significant size and value premium in portfolios formed with JSE listed shares. There is also a significant market premium picked up in most of the three-factor models. The size-value and size-profitability models still perform best; in terms of capturing premiums, the size-value model captures more of the size premium and a significant value premium, but the size-profitability model finds a stronger market premium. The size-profitability model generally has higher adjusted R-squareds that the size-value model. Once again, these two outperform the other three-factor models.

Table 15. Profitability-investment three-factor model results from the Fama-MacBeth analysis

Profit-Investment three-factor model *MKT RMW CMA*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***RMW Premium***	***t-stat***	***CMA Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	0.023%	0.047	-0.487%	-1.208	-0.588%	-1.408	0.177%	0.274	34.824%
		0.481		*0.114*		***0.080**		*0.392*	
3x3 size/profitability	-0.122%	-0.238	-0.299%	-0.594	0.126%	0.470	0.317%	0.819	27.357%
		0.406		*0.276*		*0.319*		*0.206*	
3x3 size/investment	-0.175%	-0.389	-0.133%	-0.326	-0.566%	-1.698	0.191%	0.627	16.539%
		0.349		*0.372*		****0.045**		*0.265*	
3x4 size/value	-0.212%	-0.487	-0.292%	-0.766	-0.353%	-1.162	0.039%	0.075	24.793%
		0.313		*0.222*		*0.123*		*0.470*	
3x4 size/profitability	-0.232%	-0.511	-0.216%	-0.516	0.137%	0.511	0.361%	0.933	25.864%
		0.305		*0.303*		*0.305*		*0.175*	
3x4 size/investment	-0.312%	-0.845	0.013%	0.036	-0.356%	-0.997	0.556%	1.614	15.322%
		0.199		*0.486*		*0.159*		***0.053**	
2x2x3 size/value/profitability	-0.559%	-1.046	0.107%	0.200	-0.017%	-0.063	0.728%	1.720	21.104%
		0.148		*0.421*		*0.475*		****0.043**	
2x2x3 size/value/investment	-0.228%	-0.561	-0.217%	-0.578	-0.250%	-0.897	0.257%	0.956	17.777%
		0.287		*0.282*		*0.185*		*0.169*	
2x2x3 size/investment/profitability	-0.544%	-1.332	0.208%	0.530	0.130%	0.533	0.458%	1.812	31.136%
		***0.091**		*0.298*		*0.297*		****0.035**	
2x2x3 size/profitability/investment	-0.357%	-0.810	-0.059%	-0.139	-0.102%	-0.388	0.369%	1.448	31.331%
		0.209		*0.445*		*0.349*		***0.074**	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

4.3.6 The five-factor model

Table 16 presents the results from the Fama-MacBeth analysis using the five-factor model containing factors for the market, size, value, profitability and investment. Overall, the model produces intercepts that are generally in line with the other models. There are three significant intercepts as in the size-value three-factor model. The alphas are significant for both two-way size and value sorts and the size, value and profitability. The most significant intercept is the last sort mentioned above at 0.857% (*0.*039). The alphas are almost all positive, meaning the model generally underprices the returns.

The Market Premium

The market premium is significant in four of the ten sorts, and is strongly significant in two of the three significant two-way sorts. Consistent with prior results, there are significant market premiums in the size and value sorts. The three-way sort of size, value and profitability has a significant premium of -1.067% (*0.045*). The most significant premium is in the 3x4 size and value sort at -1.133% (*0.023*). Similar to the results from other models, the market premium is mostly negative, indicating an inverse relationship between beta and portfolio returns. There are positive market premiums for both two-way size and investment sorts indicating that portfolios based on investment show a positive correlation with the market. In general, the market factor is not as prevalent as with previous models and is not picked up in the three-way sorts to the same extent.

The Size Premium

The five-factor model picks up on more size premiums across the sorts than the other models, except for the size-value model. Four of the ten sorts have significant size premiums with the 3x3 size and value sort showing the strongest premium at -0.817% (*0.013*) and the size, value and profitability sort having a premium of -0.568% (*0.026*). The model captures a significant premium in the same sorts as the size-value model, except for two of the three-way sorts which do not show a premium when using the five-factor model. Nine of the ten premiums are negative, meaning that in general, big size portfolios outperform small size portfolios, as seen in the premiums picked up in other models. It is noted that the 3x3 two-way sort on profitability shows a positive premium, but this is insignificant.

Table 16. Five-factor model results from the Fama-MacBeth analysis

The five-factor model MKT SMB HML RMW CMA	Alpha	t-stat	MKT Premium	t-stat	SMB Premium	t-stat	HML Premium	t-stat	RMW Premium	t-stat	CMA Premium	t-stat	Adjusted R-squared
3x3 size/value	1.221%	1.422	-1.507%	-1.703	-0.817%	-2.215	-0.605%	-1.389	0.359%	0.826	-0.698%	-1.652	44.386%
		*0.078		**0.044		**0.013		*0.082		0.204		**0.049	
3x3 size/profitability	0.319%	0.467	-0.613%	-1.068	0.106%	0.231	0.864%	1.738	-0.215%	-0.569	0.508%	1.011	55.362%
		0.320		0.143		0.409		**0.041		0.285		0.156	
3x3 size/investment	-0.465%	-0.543	0.206%	0.210	-0.038%	-0.120	-0.280%	-0.727	-0.143%	-0.379	-0.040%	-0.089	9.873%
		0.293		0.417		0.452		0.234		0.352		0.464	
3x4 size/value	0.795%	1.493	-1.133%	-1.992	-0.501%	-1.716	0.571%	2.033	0.442%	1.066	-0.632%	-2.378	39.443%
		*0.068		**0.023		**0.043		**0.021		0.143		***0.009	
3x4 size/profitability	0.249%	0.546	-0.625%	-1.449	-0.117%	-0.384	0.189%	0.677	-0.043%	-0.170	0.064%	0.165	44.144%
		0.292		*0.074		0.351		0.249		0.433		0.435	
3x4 size/investment	-0.829%	-0.886	0.604%	0.620	-0.419%	-1.463	1.339%	2.014	0.887%	1.692	0.508%	1.174	29.590%
		0.188		0.268		*0.072		**0.022		**0.045		0.120	
2x2x3 size/value/profitability	0.857%	1.768	-1.067%	-1.696	-0.568%	-1.946	0.236%	1.180	0.252%	0.966	-0.147%	-0.603	48.611%
		**0.039		**0.045		**0.026		0.119		0.167		0.273	
2x2x3 size/value/investment	0.323%	0.720	-0.541%	-1.034	-0.319%	-0.941	0.338%	1.303	0.185%	0.606	0.043%	0.154	37.149%
		0.236		0.151		0.173		*0.096		0.272		0.439	
2x2x3 size/investment/profit	0.154%	0.403	-0.359%	-0.764	-0.082%	-0.221	-0.140%	-0.346	-0.007%	-0.024	0.025%	0.107	51.814%
		0.343		0.223		0.413		0.365		0.491		0.457	
2x2x3 size/profit/investment	0.236%	0.422	-0.540%	-0.950	-0.342%	-1.166	0.226%	0.746	0.007%	0.029	0.243%	0.950	38.968%
		0.337		0.171		0.122		0.228		0.488		0.171	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

The Value Premium

There is a much stronger value premium across more sorts when modelled using the five-factor model. Five of the ten sorts show significant value premiums with three of those significant at the 5% level. The size and value sorts have significant market, size and value premiums. The strongest premiums are observed in the 3x4 size and value sort and the 3x4 size and investment sort at 0.571% (*0.021*) and 1.339% (*0.022*) respectively. When modelled with the five-factor model, the positive premiums suggest that value shares outperform growth shares; this is evident in seven of the ten sorts tested using the five-factor model and is mostly consistent with the size-value three-factor results. The negative premiums are generally not significant. The results suggest that bigger firms that are established and potentially undervalued have significant premiums associated with its size and relative value compared to the market price.

The Profitability Premium

There is only one significant profitability premium found in the 3x4 size and investment portfolios. This is not as strong as the market, size or value premiums but is still significant at the 5% level. It is positive at 0.887% (*0.045*). Overall, the evidence suggests that there is a premium associated with robust profitability, but it is not particularly strong in the cross-section of returns on the JSE. This is consistent with the size and value premiums as big companies that are established will most likely have strong profitability; they may be undervalued compared to the median and thus there is a premium associated with investing in them.

The Investment Premium

The investment premium is generally positive, suggesting that conservative investment is rewarded. However, the strongest premiums are negative at -0.698% (*0.043*) and -0.632% (*0.009*). The size and value sorts show a value and investment premium. The reason why big companies may be undervalued is that substantial growth in assets may not be immediately priced by the market. The change in total assets is used, thus pinpointing the reason for the effect is difficult. The investment premium is stronger than the profitability premium.

The adjusted R-squareds are generally larger using the five-factor model. Once more, the model struggles to explain portfolios sorted on size and investment. Five sorts have adjusted R-squareds above 40% with three of the models in the high thirties. This is a strong result and is consistent with conclusions in prior research regarding the explanatory power of the model.

4.3.7 Observations

The five-factor model captures strong cross-sectional premiums associated with all the factors except for profitability. The premiums captured are more significant across the sorts compared to the size-value three-factor model. In general, it performs as well or better than the other models in capturing premiums, and the fact that the three-factor size-value model performs best of the three-factor models is supported by the strong premiums found in the five-factor model. There is perhaps some interaction between the additional factors in a five-factor model that separates out some of the returns captured by factors in a three-factor model. This may enable the model to better isolate the premiums associated with value. The five-factor model captures the market premium to a lesser extent as the other models. In terms of size, one could conclude that the size-value three-factor model best captures the size premium found in this study: big companies generally outperform small companies. The five-factor model captures this to a lesser extent but shows a significant value premium across half of the sorts. High book-to-market shares outperform low book-to-market shares. Large, established companies that are undervalued could be associated with significant premiums. This mirrors the results in the summary statistics where big high book-to-market portfolios generally showed greater returns. The profitability and investment premiums are not as prevalent but there is some significant pricing of these factors. It is also noted that the five-factor model does not capture premiums to the same extent when explaining the returns of three-way sorted portfolios.

When sorting on two characteristics, the five-factor model outperforms the size-value model because it includes additional factors and can thus capture more underlying risks. It also prices the value factor to a greater extent. The five-factor model captures fewer premiums across the three-way sorts compared to the size-value model (the model capturing more size and value premiums without considering the additional factors). Thus, the conclusion is two-fold: there is a significant market, size and value premium in the cross-section of returns. This is noticeably stronger when sorting on three characteristics as opposed to two. The second part of the conclusion is that the five-factor model does well in capturing significant premiums that are evident in the cross-section of returns. However, the strength of the size-value three-factor model is reinforced based on the results above.

4.3.8 Summary

Thus far, the additional factors have provided some improvements over the traditional factors of size and value. A size-profitability model works well in explaining returns while the five-factor model performs best in capturing the variety premiums associated with the underlying risk factors in the cross-section of returns. Even though the five-factor model did not improve on the size-value or size-profitability models in terms of relative performance, it performed reasonably well in the analysis and provided a statistical improvement in terms of returns explained. It also had less dispersion as a result of dispersion of true intercepts (i.e. it would be due to sampling error). Profitability, as a factor, offers an additional pricing element in return models.

5. Robustness Tests

The methodology applied thus far has followed the principles of Fama and MacBeth (1973) whereby regressions are run in many shorter periods and the coefficients averaged across those periods in order to determine the various statistics needed for analysis. The robustness tests thus focus on results obtained from one regression over the entire sample period of 23 years as opposed to the averaging of separate regression results. The first test looks at the results from the model comparison analysis and the second looks at the cross-sectional return analysis[10].

5.1 Model comparison – relative performance

One regression was run over the entire 23-year sample period per model per portfolio. The results are displayed in Table 17 in the appendix. Overall the results are stronger than in the Fama-MacBeth analysis. This is because the results are estimated over a much longer time-period. There are some general observations: the alphas are all substantially lower. There is less pricing error when the model is estimated over a longer period. Subsequently, the portion of returns left unexplained is lower. The measure of dispersion of the true intercepts is very low across all models and sorts. This may mean that the dispersion is due to true dispersion of intercepts rather than sampling error; however, one can still draw comparisons between the models as all the measures are low. The importance of this analysis is relative performance. A further function of the long time-period of the single regression is that adjusted R-squareds are higher; this is particularly noticeable in the CAPM. Each sort is discussed next.

5.1.1 3x3 Portfolio sorts

It must be noted that given the methodology behind the GRS statistic, the GRS statistic could not be calculated using one full period regression because one cannot determine the covariance matrix of residuals from one regression. Meaningful analysis can still be drawn from the other measures. In the 3x3 sorts, there is no real pattern to the results. In the size and value sort, the size-investment model performs best across all measures except for the adjusted R-squared. This differs to the initial results where the size-value model worked best. The alpha is 0.411% with the proportion of returns left unexplained at 0.688. The size-value model has the highest adjusted R-squared at 69.610%. Overall, the size-value and size-profitability models still outperform the other models.

[10] Note that the four-factor model tested in *Section 4.2.5 A Better Model?* is not analysed in the robustness tests because it did not outperform in the model comparison test.

In the size and profitability sort, the size-investment model also outperforms in all measures except adjusted R-squared, with the size-value and size-profitability models again outperforming the remaining models. This differs to the initial results whereby the size-profitability model outperformed. The alpha is 0.419% and the dispersion measure is 0.19. It leaves the least returns unexplained. The size-profitability model has the highest adjusted R-squared at 71.106%.

In the size and investment sort, the size-value model has the best performance across all measures except adjusted R-squared. This is contrary to initial results but is consistent with the overall conclusion regarding the performance of the size-value model. The size-investment model has the highest adjusted R-squared. The size-profitability model has the second lowest alpha at 0.427%.

Overall, the size-investment model performs best in the 3x3 sorts, but the power of the size-value and size-profitability models is still evident. The size-investment model only reduces pricing error by 0.003% at most when it is the best performing model. It is noted that the five-factor model does not have the highest adjusted R-squared in any of the 3x3 sorts. This is contrary to the results in Fama and French (2015).

5.1.2 3x4 Portfolio sorts

In the size and value sort, the size-investment model once again outperforms with an alpha of 0.446% and a dispersion due to sampling error estimate of 0.255. The size-value model has the highest adjusted R-squared at 64.196%. The size-value and size-profitability models still perform well. This is contrary to initial results but the performance of the size-value and size-profitability model is still noticeable.

The size and profitability sort restores some consistency to the results. Although different to initial results, the size-value model far outperforms, with the lowest alpha across all models and sorts at 0.396%. It also leaves the least returns unexplained and performs best in each of the other measures. The exception to this is the adjusted R-squared whereby the five-factor model outperforms with an adjusted R-squared of 64.344%.

In the size and investment sort, the size-value model once again outperforms across the various measures. The alpha is 0.460%, followed by the size-investment and size-profitability model, which are only 0.003% and 0.006% higher than the size-value model. The size-investment model has the highest adjusted R-squared. Again, this contradicts the initial results but supports the conclusion regarding the strength of the size-value model.

In the 3x4 sorts, the size-value model outperforms. The size-profitability and size-investment models also do well across the measures. The five-factor model does not improve performance and has the highest adjusted R-squared in one sort only, contrary to Fama and French (2015). The results support the observations in *Section 4.2* in that the size-value and size-profitability models are generally the best performing of the models tested. In the two-way sorts, it is noted that the size-investment model performs admirably, suggesting that the investment factor is more suited to long-term modelling of returns. The model prices the investing behaviour of companies over time, once the investment begins to mature.

5.1.3 2x2x3 Portfolio sorts

The alphas are generally higher compared to the two-way sorts. The size-profitability model outperforms in the size, value and profitability sort, leaving the least returns unexplained. This is consistent with initial results and reinforces the strength of the model in explaining returns. The alpha is 0.511%, with the size-investment and size-value model performing well. The size investment model has the best dispersion due to sampling error estimate at 0.233, and the five-factor model has the highest adjusted R-squared at 65.539%, which is consistent with Fama and French (2015).

In the size, value and investment sort, the size-profitability model once again outperforms across all measures except for the adjusted R-squared; the five-factor model performs best at 66.211%. This contradicts initial results but is once more in-line with the main conclusion of the initial results that the size-value and size-profitability models perform best. The size-investment model performs well once more with an alpha of 0.466% and a dispersion measure of 0.224.

The size, investment and profitability sort shows the strength of the investment factor once more, with the size-investment model producing an alpha of 0.498%. The size-value model shows the best dispersion due to sampling error result at 0.177, with the size-profitability sort having the highest adjusted R-squared at 68.101%.

Finally, in the size, profitability and investment sort, the tests are consistent with the initial results with the size-profitability model performing best with an alpha of 0.469%. It also leaves the least returns unexplained. The five-factor model has the highest adjusted R-squared at 66.774%. Once again, size-value and size-investment perform well.

In the 2x2x3 sorts, the results show consistency with the overall conclusion that the size-value and size profitability models perform the best. The strength of the investment factor is also apparent.

5.1.4 Observations

The results do not match the initial tests exactly due to the different time frame of the regressions. However, the overall conclusion remains the same: the size-value and size-profitability models perform best in the model comparison analysis. When outperformed by the size-investment model, both still outperform the five-factor model and CAPM, reinforcing their robustness. It is noted that the size-investment model performs best in four of the ten sorts and still performs well when the size-value or size-profitability model excels. It does, however, fail to perform in the two-way sorts on size and investment. One can conclude that the size-value and size-profitability models are the best models to use on JSE listed company returns. Over longer modelling periods, the investment factor works as well in explaining returns. Significantly, when looking at longer periods, investment as a factor is not redundant; in fact, it adds substantial power to a three-factor model. The five-factor model fails to improve on three-factor models, contrary to Fama and French (2015). Importantly, over longer periods, the additional factors of profitability and investment are useful in explaining returns. The strength of value-, profitability- and investment-based models may point to an underlying risk that each model is able to more accurately price. The link between these factors could point to quality being that risk-factor, and that the factors, when coupled with size, pick up on this element of returns.

5.2 Fama-MacBeth regressions

The same two-stage regression methodology was applied in the robustness tests. However, the stage one coefficients were estimated on one regression spanning the full 23-year sample period. The second stage was then estimated using the full period average returns per portfolio and the estimated coefficients from stage one. Thus, the coefficients are not averages but the actual time-series coefficients for the full period. The t-stat is based on the t-distribution as the coefficients are not averaged to attain the relevant statistics but drawn straight from the regression itself[11]. The results are presented in Tables 18 to 23 in the appendix.

5.2.1 The CAPM

The CAPM performs poorly over the period. Six of the ten sorts show significant pricing error. There are no significant premiums which is consistent with the initial results. The market premium is positive (but insignificant) compared to mostly negative premiums in the preliminary tests. This contradicts the initial results but is due to the substantially longer time-period tested. The adjusted R-squareds are low or negative, indicating that the CAPM is a poor model. While the results do not match the initial tests, the same conclusion is drawn: the CAPM performs poorly in capturing the cross-section of returns.

5.2.2 The size-value three-factor model

The size-value model picks up on fewer premiums compared to the initial results. There is similar pricing error, with a significant alpha in the 3x4 size and value sort. The market premium is not as prevalent but is significant in the 3x4 size and value sort at -2.378% (*0.047*). The market premium is generally large and negative, which coincides with initial results. Fewer sorts exhibit size premiums but the premiums are stronger. The 3x4 size and value sort has the strongest premium at -0.587% (*0.009*). The largest premium in absolute terms is -0.642% (*0.046*) in the 3x4 size and investment sort. The other premiums are significant at the 5% and 10% level (the second premium has a probability of 0.056 attached to it). The value premium is evident in fewer sorts but is significant in the 3x4 size and value sort once again at 0.472% (*0.020*). The value premium is generally positive, consistent with initial results and confirms that high book-to-market portfolios outperformed. The adjusted R-squareds display greater variation in the robustness tests; however, they are generally higher. Overall, while weaker, similar premiums and premium directions are seen in the size-value model compared to earlier results.

5.2.3 The size-profitability three-factor model

There are more premiums captured compared to the size-value model, but they are only present in three sorts: the 3x3 size and value sort, the 3x4 size and value sort, and the size, value and investment sort. The premium is generally weaker but there is still evidence of a market, size and profitability premium. The 3x3 size and value sort shows a significant negative profitability

[11] A negative adjusted R-squared means that the additional factors do not add any additional explanatory power to the model.

premium at -1.558% (*0.013*). The other sorts show a market, size and profitability premium. Mirroring earlier results, the market premium is negative but larger in magnitude. The largest significant premium is seen in the 3x4 size and value sort at -2.062% (*0.094*). In line with initial results, the size premium is negative, indicating that big company portfolios outperform smaller company portfolios. The premiums are only significant at the 10% level but this is similar to preliminary results. The profitability premium is almost as strong as initial results: two sorts are significant at 5% and one at 10%. There is an even split between positive and negative premiums, but the significant premiums are all negative, the largest being -1.558% (*0.013*). This coincides with earlier results and suggests that weak profitability shares earn greater returns. The adjusted R-squareds show mixed results in that there is a greater variation in the result but they are much larger in some portfolios, up to nearly 69% compared to the high of 42% in initial tests.

5.2.4 The size-investment three-factor model

The size-investment model capture premiums to a similar degree as initial tests. The premiums captured are, however, in different factors compared to the initial tests. The intercepts are not significant which is similar to most of the sorts in earlier results. There is only one significant market premium compared to earlier results. The premiums are also larger in absolute terms. The model picks up on some significant size premiums in the sorts, with the most significant in the 3x4 size and value sort at -0.547% (*0.008*) There is a strong investment premium, with two significant at the 5% level and one at the 1% level at 1.540% (*0.000*). The size and investment premiums are in line with earlier results in that big company portfolios outperform and conservative investment portfolios outperform. There is some substantial variability in adjusted R-squareds, ranging from -16% to 81%. This is similar to the other models in the robustness tests. Overall, while the model picks up on different premiums in the test, the ability to capture some significantly priced cross-sectional premiums is comparable to earlier results. The results are consistent with those in *Section 5.1 Model Comparison – Relative Performance* in that investment is quite significantly priced in the cross-section of returns when estimating over a longer period.

5.2.5 The profitability-investment three-factor model

The results are similar in the profitability-investment model robustness tests. The intercept is significant in more sorts than in other models. There are some premiums captured, but not to the same extent as the other models. Markedly, the 3x4 size and value sort has a significant investment premium at 1.400% (*0.007*). It does not perform as well as the other three-factor models. The directions of the premiums are almost identical to earlier results but are more in favour of prior research in that robust profitability portfolios and conservative investment portfolios outperform.

5.2.6 The five-factor model

There are some differences in the five-factor model that mirror the results in other robustness tests but contradict the power of the model seen in initial results. Notably, the adjusted R-squareds are mostly negative, meaning that over long time-periods, the additional factors do not provide any additional explanatory power. As is seen in the results, there are no significant profitability or investment premiums when using the five-factor model in a long-term analysis. However, the weaker results are evident across all models and thus there is still some validity in the use of a five-factor model in pricing returns. The intercepts are less significant in the robustness tests. The premiums captured are mostly significant at the 5% level. There is a significant market, size and value premium in the 3x4 size and value sort, as was seen in the analysis using the size-value model. The market and size premiums are generally negative, which coincides with prior results. The size premiums are significant at the 5% level. The value premiums are generally positive but to a lesser extent compared to earlier results. However, this is a function of the long time period used in the robustness tests. Given that the coefficients are estimated over a substantial period, the variability of the coefficients through time is not taken into account to the same degree compared to when they are averaged. The strongest value premium is 0.573% (*0.048*).

5.2.7 Observations

The results in the Fama-MacBeth analysis matched those in the model comparison analysis in that the size-investment model is a good model for returns over a substantial time-period. This is seen by the premiums captured and the performance in the comparative analysis. The size-value and size-profitability models still perform well in capturing the cross-section of returns, even when estimated over the full sample period. The five-factor model did not perform to the same degree as initial tests but still priced some significant premiums. The direction of the premiums matched those in the initial results. The ability of the five-factor model to capture returns over a very long period is lower compared to the other models.

5.3 Summary

While the five-factor model did not perform as well in the full period analysis, it still captured some significant premiums. Overall, it only captured two fewer than the best performing model in the long-term analysis. One can conclude that the change in methodology resulted in much weaker premiums, but the evidence for a size and value premium, as found in the initial results, remains firm. The size-value three-factor model best captures the size premium found in this study: big companies generally outperform small companies; this is the same result as the initial tests. The five-factor model captures the value premium better than the size-value model. Profitability and investment are significantly priced over the long-term. This matches the results in the model comparison analysis but also has important implications for the models tested.

The results of the robustness tests compared to the initial tests show that perhaps size and value are associated with shorter time horizons while investment and profitability are only priced in over time. This could be as a result of the mechanics of each factor. The size factor, based on market capitalisation, changes with price movements almost every day. These changes could lead to temporary periods where the company is either smaller or bigger than it should be. This is linked to value whereby the book-to-market ratio also changes with daily movements. Thus, the premiums on these factors could be linked to the "reversal" of some of the temporary fluctuations in market value. This would be picked up in a short-term analysis, where the coefficients are estimated over three years, but will be hidden in an analysis using the full sample period. Shorter estimation periods allow for the effects of varying coefficients over time to be considered. The fact that the models tested still pick up on a size and value premium reinforces the idea that there are significant premiums and that the size and value factors, when paired with the additional factors, price the respective premiums. Profitability and investment can only be reliably gauged at year end with audited results. Thus, the changes take much longer to be imputed into market prices. The trends and premiums priced into returns due to changes in profitability and investment would thus be picked up in a longer period of analysis as the changes are not as frequent. Furthermore, the two factors are linked; investment increases productive capacity and the success of the investment depends on the incremental profitability that the investment brings. A longer-term view of the effects thus enables the model to better price the premiums associated with profitability and investment.

Overall, the robustness tests show that, in line with earlier conclusions, the additional factors add to the explanatory power of models. The five-factor model is still a good model for pricing returns, but one can utilise a size-investment model to model returns over a longer period. Given that most holding periods are substantially shorter than 23 years, the five-factor model would still be the best model to use to price the cross-section of returns. This allows for the variation in premiums over time to be captured while allowing for those premiums to be captured for a wide variety of portfolio formation methods. Overall, the most accurate models in terms of returns explained would still be the size-value and size-profitability models. However, once again, over longer time periods, a size-investment model would work as well. The models in the robustness tests mainly captured premiums in the size and value based test portfolio sorts; this may mean that the models, in general, are not good at pricing risk factors over a substantial period of time. Given that holding periods are usually shorter, and the fact that anomalies are time-period dependent as concluded in Kruger and Toerien (2014), the models overall performance in the robustness tests does not change the conclusion that the additional factors improve the ability of asset pricing models to capture returns. In terms of relative performance, this improvement is seen in three-factor models as opposed to a five-factor model. In the cross-section of returns, a five-factor model is best for more practical holding periods.

6. Conclusions

6.1 Conclusion

The desire to explain returns on listed shares has long been an area of interest for researchers. Various models have been developed to capture the underlying risk factors that drive returns. The CAPM and Fama-French three- and five-factor models are three well-known models that have been theorised to capture those risk factors. The CAPM could be seen as the starting point for asset pricing models and is based on the work of Harry Markowitz and the concept of mean-variance efficiency. This model has been expanded to include factors that attempt to capture the underlying risks such as size or a firm's book-to-market position, and more recently the strength of its profitability and investment levels. These are perceived as an improvement on the CAPM's beta, the 'catch-all' factor that should describe returns. The ability of these models to explain returns on the JSE was tested.

The aim of this study was to evaluate the Fama-French five-factor model on the JSE and determine whether the additional factors proxy for quality, a characteristic that is not formally defined but has ties to growth and profitability. Furthermore, the study determines whether asset return effects such as the size or value effect are still present on the JSE. The five-factor model has not been tested in this way on the JSE to the author's knowledge, and thus provided an opportunity to explore new ways of capturing underlying risk factors. Two tests were employed: as in Fama and French (2015, 2016), a relative comparison between the models was conducted. This involved analysing the pricing error of the models as well as the proportion of returns left unexplained. The measures used in the analysis were such that the number of factors or portfolios tested in the sort did not bias the results. The second test followed the methodology of Fama and MacBeth (1973) where the ability of each model to explain the cross-section of returns was analysed. The two tests were employed due to their importance in asset pricing; the model should capture the premiums associated with the underlying risk factors in the cross-section of returns and ensure that it adequately explains the time-series returns with minimal pricing error. These cross-sectional premiums determine the presence of the size and value effect, for example.

In terms of relative performance, the additional factors improve on current models in explaining portfolio returns. However, the strength of the original size-value three-factor model is reinforced. The size-value and size-profitability models outperform the other models in terms of explaining returns, even over longer periods. Investment can be used in a three-factor model should one want to estimate returns over longer periods. However, given the holding periods used in practice, the size-value and size-profitability models would be preferred. The five-factor model showed no improvement on those three-factor models in terms of pricing, but performed better than the CAPM. The five-factor model outperformed in terms of adjusted R-squared across the portfolio sorts tested. Robustness tests confirmed these results and showed that over long estimation periods, the investment factor improved on the ability of the CAPM and other three-factor models in explaining returns. Thus, profitability and investment add new dimensions in pricing returns on the JSE.

The five-factor model outperforms in explaining the cross-section of returns on portfolios of JSE listed shares sorted by various characteristics. The five-factor model captures premiums associated with the market, size and value factors as does the size-value model. It also captures profitability and investment and can price a relatively significant premium across most of the sorts tested. The model does not perform as well over longer periods but once more, given that shorter holding periods are used in practice, one could conclude that it best captures cross-sectional premiums. Overall, there are significantly priced market, size and value premiums in the cross-section of returns. The negative market premium is captured by the five-factor model to a lesser degree compared to the size-value and size-profitability models. Returns are inversely related to the market factor and larger companies outperform smaller companies. These results are consistent with the time-series return analysis performed in the summary statistics and with van Rensburg and Robertson (2003), Strugnell et al. (2011) and Ward and Muller (2012). High book-to-market shares outperform on the JSE, consistent with much of prior literature. This is well captured by the five-factor model.

In terms of the additional factors, the premiums were not as consistent as the size or value premiums in terms of direction. Robust profitability outperforms weak profitability and this may be evidence of quality being priced into returns. The result is consistent with Novy-Marx (2013), which Fama and French (2015) cite as inspiration for the additional factors. Given that the five-factor model best captured the premiums, the overall result that big, undervalued firms with robust profitability outperform is consistent with the analysis performed on the portfolio returns. While the premium on investment varies between positive and negative, it is generally associated with firms that invest conservatively, mirroring prior literature. The negative premium is mainly evident in value sorts; perhaps undervalued firms are as such because their aggressive investment policy is not fully priced in when the financial statements are released. When the estimation period was extended in robustness tests, the profitability and investment premiums are more significant and are better captured, despite there being fewer significant premiums. This also explains why the market may not fully price in aggressive investment, leaving those firms undervalued. One can conclude that the additional factors improve asset pricing models on the JSE and provide some evidence that different factors could be priced over different lengths of time. The robustness tests mostly confirm this.

Do the additional factors proxy for quality? One could conclude that profitability could proxy for quality and perhaps explains part of a premium that may be associated with investing in quality companies. Robust profitability is generally significantly priced, thus providing evidence for this conclusion. Investment is both positively and negatively priced. Both could be considered signs of quality: conservative investment is significantly priced because management are wary of making the wrong investment decision. It also may mean that the growth in assets may be organic and not due to acquisitions, in which case, the steady growth of assets is a sign of quality operations. On the other hand, aggressive investment may be evidence that the firm has sufficient cash flows to maintain high levels of investment and thus can grow earnings quickly. This is also a sign of quality. The investment factor is the growth in total assets; thus, it is difficult to pinpoint the cause of the changing premium. The author thus concludes that it cannot proxy for quality in its current form. If the factor is derived from more specific constituents, a more informed conclusion could be made. Quality could also be seen as a filter, rather than a risk factor, that is used to unlock the abnormal returns associated with size or value.

6.2 Areas for future research

Areas for future research are considerations that have been taken into account when analysing the prior literature on the topic but is not vital to the conclusions of this study given the nature of the study. These are ideas that could be tested when using this study as a springboard for future research.

Factor definitions can be altered to test whether their formation is material in asset pricing tests. Furthermore, mimicking portfolios could be tested against using the actual factor itself (i.e. using the actual book-to-market ratio versus a mimicking portfolio for the value effect). The way factors are sorted or the frequency of rebalancing could also be adjusted and tested. The investment factor could be derived from more specific assets rather than total assets. This could result in a more detailed conclusion on what drives the premiums associated with the differing levels of investment.

The coefficients could be estimated over varying time-periods to see how the market prices the various risk factors in different time periods and economic conditions, something which was not in the scope of this study. This could confirm the observations made in the study that profitability and investment are priced over longer periods.

Current models could be augmented with the new factors and applied in a South African context. For example, four-factor models using combinations of momentum and size, value, profitability, investment and the market could be tested. Furthermore, other factors such as momentum, liquidity or cash-flow-based characteristics could be included in addition to the new factors to create a multi-factor model that incorporates a variety of risk factors. The interaction of profitability and other quality-based measures such as cash conversion could be tested to further examine whether quality is significantly priced.

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Appendix

A1. Robustness tests

The following tables present the results from robustness tests using one, full sample period regression to analyse relative model performance and the cross-sectional premiums associated with each factor.

A1.1 Model comparison – relative performance

Table 17. Model comparison robustness test results

3x3 size value	$A\|\alpha\|$	$\frac{A\|\alpha\|}{A\|\gamma\|}$	$\frac{A\|\alpha\|^2}{A\|\gamma\|^2}$	$\frac{As^2}{A\|\alpha\|^2}$	***Adjusted R-squared***
MKT	0.559%	0.935	0.875	0.162	51.703%
SMB HML MKT	0.455%	0.762	0.580	0.160	69.610%
SML RMW MKT	0.413%	0.691	0.477	0.218	66.584%
SML CMA MKT	0.411%	0.688	0.473	0.230	65.037%
RMW CMA MKT	0.541%	0.906	0.821	0.169	53.240%
SMB HML RMW CMA	0.468%	0.783	0.613	0.172	67.095%
3x3 size profit					
MKT	0.563%	0.938	0.879	0.141	54.178%
SMB HML MKT	0.424%	0.705	0.497	0.185	67.797%
SML RMW MKT	0.422%	0.703	0.494	0.162	71.106%
SML CMA MKT	0.419%	0.698	0.487	0.190	67.165%
RMW CMA MKT	0.543%	0.904	0.817	0.142	57.398%
SMB HML RMW CMA	0.440%	0.732	0.536	0.175	68.143%
3x3 size investment					
MKT	0.564%	0.935	0.875	0.132	55.143%
SMB HML MKT	0.420%	0.697	0.486	0.175	68.903%
SML RMW MKT	0.427%	0.708	0.501	0.160	69.966%
SML CMA MKT	0.431%	0.715	0.512	0.151	70.654%
RMW CMA MKT	0.552%	0.916	0.839	0.127	58.616%
SMB HML RMW CMA	0.436%	0.723	0.523	0.165	69.236%
3x4 size value					
MKT	0.586%	0.939	0.882	0.184	47.810%
SMB HML MKT	0.482%	0.773	0.598	0.191	64.196%
SML RMW MKT	0.449%	0.719	0.518	0.243	61.248%
SML CMA MKT	0.446%	0.715	0.512	0.255	59.903%
RMW CMA MKT	0.570%	0.914	0.835	0.190	49.458%
SMB HML RMW CMA	0.490%	0.786	0.617	0.202	62.859%

3x4 size profit					
MKT	0.584%	0.940	0.883	0.156	50.193%
SMB HML MKT	0.396%	0.607	0.368	0.275	59.002%
SML RMW MKT	0.417%	0.639	0.409	0.225	61.919%
SML CMA MKT	0.411%	0.631	0.398	0.255	58.392%
RMW CMA MKT	0.582%	0.893	0.797	0.165	45.279%
SMB HML RMW CMA	0.467%	0.750	0.563	0.195	63.511%
3x4 size investment					
MKT	0.568%	0.937	0.878	0.160	51.233%
SMB HML MKT	0.460%	0.759	0.577	0.192	63.239%
SML RMW MKT	0.466%	0.768	0.590	0.180	64.079%
SML CMA MKT	0.463%	0.764	0.584	0.180	64.344%
RMW CMA MKT	0.555%	0.916	0.839	0.159	53.898%
SMB HML RMW CMA	0.467%	0.770	0.593	0.191	63.493%
2x2x3 size value profitability					
MKT	0.665%	0.957	0.917	0.173	48.070%
SMB HML MKT	0.537%	0.773	0.597	0.204	63.260%
SML RMW MKT	0.511%	0.737	0.543	0.216	64.137%
SML CMA MKT	0.518%	0.747	0.558	0.233	59.870%
RMW CMA MKT	0.645%	0.929	0.863	0.172	52.283%
SMB HML RMW CMA	0.544%	0.783	0.614	0.188	65.539%
2x2x3 size value investment					
MKT	0.578%	0.946	0.895	0.195	48.370%
SMB HML MKT	0.489%	0.800	0.639	0.199	64.723%
SML RMW MKT	0.450%	0.736	0.541	0.251	61.891%
SML CMA MKT	0.466%	0.763	0.582	0.224	63.538%
RMW CMA MKT	0.577%	0.943	0.889	0.182	52.440%
SMB HML RMW CMA	0.504%	0.824	0.679	0.186	66.211%
2x2x3 size investment profit					
MKT	0.620%	0.952	0.907	0.150	51.430%
SMB HML MKT	0.504%	0.774	0.600	0.177	64.616%
SML RMW MKT	0.501%	0.769	0.591	0.159	68.101%
SML CMA MKT	0.498%	0.765	0.585	0.175	65.147%
RMW CMA MKT	0.603%	0.926	0.857	0.140	57.595%
SMB HML RMW CMA	0.501%	0.768	0.590	0.173	67.077%
2x2x3 size profit investment					
MKT	0.598%	0.948	0.898	0.164	50.301%

SMB HML MKT	0.476%	0.755	0.569	0.201	63.914%
SML RMW MKT	0.469%	0.743	0.552	0.194	65.562%
SML CMA MKT	0.479%	0.759	0.576	0.185	65.428%
RMW CMA MKT	0.590%	0.935	0.873	0.149	56.357%
SMB HML RMW CMA	0.498%	0.790	0.623	0.174	66.774%

A1.2 Fama-MacBeth regressions

Table 18. Fama-MacBeth analysis robustness test – CAPM

CAPM *MKT*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	-0.812%	-1.829	0.490%	0.832	-4.006%
		0.110		*0.433*	
3x3 size/profit	-0.652%	-2.667	0.262%	0.822	-4.232%
		*****0.032***		*0.438*	
3x3 size/investment	-0.585%	-1.484	0.172%	0.330	-12.538%
		0.181		*0.751*	
3x4 size/value	-0.854%	-2.063	0.507%	0.929	-1.266%
		****0.066***		*0.375*	
3x4 size/profit	-0.605%	-2.447	0.170%	0.527	-7.024%
		*****0.034***		*0.609*	
3x4 size/investment	-0.543%	-1.451	0.109%	0.221	-9.467%
		0.177		*0.830*	
2x2x3 size/value/profitability	-1.067%	-1.799	0.649%	0.893	-1.873%
		0.102		*0.393*	
2x2x3 size/value/investment	-0.835%	-2.018	0.477%	0.915	-1.499%
		****0.071***		*0.382*	
2x2x3 size/investment/profitability	-0.833%	-2.596	0.414%	1.047	0.870%
		*****0.027***		*0.320*	
2x2x3 size/profitability/investment	-0.709%	-1.971	0.287%	0.635	-5.738%
		****0.077***		*0.540*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Table 19. Fama-MacBeth analysis robustness test – Size-value three-factor model

<u>Size-value three-factor model</u> *MKT SMB HML*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***SMB Premium***	***t-stat***	***HML Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	1.880%	0.980	-2.282%	-1.186	-0.532%	-1.978	0.427%	1.694	38.837%
		0.372		*0.289*		*0.105*		*0.151*	
3x3 size/profit	-0.756%	-0.810	0.389%	0.422	0.127%	0.614	-0.872%	-1.382	-2.653%
		0.455		*0.691*		*0.566*		*0.225*	
3x3 size/investment	1.002%	0.594	-1.358%	-0.810	-0.518%	-1.666	0.526%	0.449	10.131%
		0.578		*0.455*		*0.157*		*0.672*	
3x4 size/value	1.992%	1.956	-2.378%	-2.343	-0.587%	-3.453	0.472%	2.887	65.273%
		****0.086***		*****0.047***		******0.009***		*****0.020***	
3x4 size/profit	-1.431%	-1.692	1.001%	1.184	0.200%	0.987	-0.612%	-1.079	-1.378%
		0.129		*0.271*		*0.353*		*0.312*	
3x4 size/investment	0.043%	0.034	-0.414%	-0.330	-0.642%	-2.360	1.432%	1.732	27.265%
		0.974		*0.750*		*****0.046***		*0.121*	
2x2x3 size/value/profitability	0.701%	0.509	-1.057%	-0.767	-0.563%	-2.236	0.603%	1.725	35.631%
		0.625		*0.465*		****0.056***		*0.123*	
2x2x3 size/value/investment	1.668%	1.344	-2.033%	-1.634	-0.439%	-2.645	0.293%	1.215	44.509%
		0.216		*0.141*		*****0.029***		*0.259*	
2x2x3 size/investment/profitability	1.328%	0.834	-1.694%	-1.073	-0.170%	-0.689	-0.491%	-0.664	3.798%
		0.428		*0.315*		*0.510*		*0.525*	
2x2x3 size/profitability/investment	1.453%	0.938	-1.867%	-1.191	-0.173%	-0.695	-0.260%	-0.382	5.089%
		0.376		*0.268*		*0.507*		*0.713*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Table 20. Fama-MacBeth analysis robustness test – Size-profitability three-factor model

Size-profitability three-factor model *MKT SMB RMW*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***SMB Premium***	***t-stat***	***RMW Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	1.404%	1.124	-1.651%	-1.354	-0.386%	-1.917	-1.558%	-3.783	68.705%
		0.312		*0.234*		*0.113*		*****0.013***	
3x3 size/profit	-0.796%	-0.677	0.398%	0.338	-0.101%	-0.521	0.192%	0.756	-29.121%
		0.529		*0.749*		*0.625*		*0.483*	
3x3 size/investment	1.102%	0.768	-1.428%	-0.981	-0.310%	-1.093	-0.869%	-0.727	17.216%
		0.477		*0.372*		*0.324*		*0.500*	
3x4 size/value	1.838%	1.643	-2.062%	-1.896	-0.485%	-2.242	-1.178%	-2.710	48.364%
		0.139		****0.094***		****0.055***		*****0.027***	
3x4 size/profit	-0.768%	-0.715	0.319%	0.298	-0.060%	-0.325	0.351%	1.337	-6.134%
		0.495		*0.774*		*0.754*		*0.218*	
3x4 size/investment	1.539%	1.206	-1.877%	-1.482	-0.360%	-1.301	-0.392%	-0.406	10.551%
		0.262		*0.177*		*0.230*		*0.695*	
2x2x3 size/value/profitability	1.905%	1.366	-2.144%	-1.559	-0.419%	-1.548	0.290%	0.921	25.660%
		0.209		*0.158*		*0.160*		*0.384*	
2x2x3 size/value/investment	1.568%	1.562	-1.860%	-1.862	-0.348%	-1.976	-0.820%	-1.862	46.138%
		0.157		****0.100***		****0.084***		****0.100***	
2x2x3 size/investment/profitability	1.104%	1.028	-1.480%	-1.381	-0.281%	-1.702	0.153%	0.740	15.529%
		0.334		*0.205*		*0.127*		*0.481*	
2x2x3 size/profitability/investment	1.350%	1.375	-1.750%	-1.747	-0.252%	-1.412	0.037%	0.134	18.118%
		0.206		*0.119*		*0.196*		*0.896*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Table 21. Fama-MacBeth analysis robustness test – Size-investment three-factor model

Size-investment three-factor model *MKT SMB CMA*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***SMB Premium***	***t-stat***	***CMA Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	1.283%	1.162	-1.501%	-1.390	-0.540%	-2.174	1.659%	3.900	66.452%
		0.298		*0.223*		***0.082**		*****0.011**	
3x3 size/profit	-0.991%	-1.083	0.571%	0.621	-0.075%	-0.316	-0.822%	-0.977	-15.813%
		0.328		*0.562*		*0.765*		*0.373*	
3x3 size/investment	1.732%	0.876	-2.124%	-1.053	-0.518%	-1.356	0.282%	0.867	9.344%
		0.421		*0.341*		*0.233*		*0.426*	
3x4 size/value	1.007%	1.702	-1.231%	-2.142	-0.547%	-3.544	1.540%	6.431	81.330%
		0.127		***0.065**		*****0.008**		******0.000**	
3x4 size/profit	-1.560%	-1.722	1.085%	1.193	0.129%	0.558	-0.260%	-0.408	-9.457%
		0.123		*0.267*		*0.592*		*0.694*	
3x4 size/investment	-1.142%	-0.656	0.801%	0.452	-0.087%	-0.271	0.762%	2.447	27.988%
		0.530		*0.663*		*0.793*		*****0.040**	
2x2x3 size/value/profitability	0.053%	0.038	-0.324%	-0.231	-0.467%	-1.541	1.264%	1.536	10.124%
		0.971		*0.823*		*0.162*		*0.163*	
2x2x3 size/value/investment	0.984%	0.690	-1.293%	-0.903	-0.426%	-1.875	0.397%	1.623	22.305%
		0.510		*0.393*		***0.098**		*0.143*	
2x2x3 size/investment/profitability	-1.277%	-0.963	0.904%	0.676	-0.194%	-0.991	0.446%	1.647	15.289%
		0.364		*0.518*		*0.351*		*0.138*	
2x2x3 size/profitability/investment	-0.499%	-0.260	0.113%	0.057	-0.214%	-1.017	0.348%	1.435	5.931%
		0.801		*0.956*		*0.339*		*0.189*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Table 22. Fama-MacBeth analysis robustness test – Profitability-investment three-factor model

Profit-investment three-factor model *MKT RMW CMA*	***Alpha***	***t-stat***	***MKT Premium***	***t-stat***	***RMW Premium***	***t-stat***	***CMA Premium***	***t-stat***	***Adjusted R-squared***
3x3 size/value	-0.259%	-0.832	-0.046%	-0.118	-1.188%	-1.712	0.811%	1.337	62.947%
		0.443		*0.910*		*0.148*		*0.239*	
3x3 size/profit	-0.896%	-2.501	0.477%	1.190	0.127%	0.513	-0.954%	-0.823	-15.990%
		***0.054**		*0.288*		*0.630*		*0.448*	
3x3 size/investment	-0.182%	-0.365	-0.147%	-0.260	-1.356%	-1.173	0.397%	1.288	6.799%
		0.730		*0.805*		*0.293*		*0.254*	
3x4 size/value	-0.545%	-1.962	0.219%	0.624	-0.065%	-0.131	1.400%	3.637	64.971%
		***0.085**		*0.550*		*0.899*		*****0.007**	
3x4 size/profit	-0.703%	-2.513	0.259%	0.763	0.370%	1.375	0.000%	0.000	-5.924%
		****0.036**		*0.467*		*0.206*		*1.000*	
3x4 size/investment	-0.651%	-1.492	0.306%	0.602	0.090%	0.097	0.730%	2.548	27.532%
		0.174		*0.564*		*0.925*		****0.034**	
2x2x3 size/value/profitability	-1.051%	-1.824	0.774%	1.110	0.036%	0.108	1.420%	1.717	12.893%
		0.106		*0.299*		*0.917*		*0.124*	
2x2x3 size/value/investment	-0.580%	-1.326	0.225%	0.425	-0.820%	-1.277	0.305%	1.210	18.795%
		0.222		*0.682*		*0.237*		*0.261*	
2x2x3 size/investment/profitability	-0.881%	-3.117	0.510%	1.472	0.123%	0.644	0.457%	1.893	25.556%
		****0.014**		*0.179*		*0.537*		***0.095**	
2x2x3 size/profitability/investment	-0.785%	-2.221	0.407%	0.925	0.006%	0.019	0.359%	1.566	6.591%
		***0.057**		*0.382*		*0.985*		*0.156*	

* Significant at 10% level
** Significant at the 5% level
*** Significant at the 1% level

Table 23. Fama-MacBeth analysis robustness test – Five-factor model

The five-factor model *MKT SMB HML RMW CMA*	*Alpha*	*t-stat*	*MKT Premium*	*t-stat*	*SMB Premium*	*t-stat*	*HML Premium*	*t-stat*	*RMW Premium*	*t-stat*	*CMA Premium*	*t-stat*	*Adjusted R-squared*
3x3 size/value	3.405%	1.488	-3.716%	-1.617	-0.707%	-1.635	0.342%	0.653	-0.459%	-0.526	-0.185%	-0.321	58.450%
		0.234		*0.204*		*0.201*		*0.561*		*0.635*		*0.769*	
3x3 size/profit	-0.276%	-0.146	-0.047%	-0.026	0.015%	0.024	-0.260%	-0.154	-0.053%	-0.072	-1.782%	-0.678	-72.124%
		0.893		*0.981*		*0.983*		*0.887*		*0.947*		*0.546*	
3x3 size/investment	0.284%	***0.071**	-0.667%	-0.168	-0.205%	-0.218	-0.081%	-0.038	-0.360%	-0.266	0.228%	0.188	-79.984%
		0.948		*0.877*		*0.841*		*0.972*		*0.808*		*0.863*	
3x4 size/value	2.151%	2.124	-2.527%	-2.518	-0.642%	-3.418	0.521%	2.040	-0.007%	-0.016	0.186%	0.471	69.580%
		****0.078***		*****0.045***		*****0.014***		****0.087***		*0.987*		*0.654*	
3x4 size/profit	-1.770%	-1.248	1.311%	0.924	0.153%	0.350	-0.235%	-0.233	0.074%	0.163	-0.012%	-0.014	-36.195%
		0.259		*0.391*		*0.738*		*0.824*		*0.876*		*0.989*	
3x4 size/investment	-0.561%	-0.253	0.243%	0.110	-0.576%	-1.048	1.258%	1.097	0.260%	0.288	0.334%	0.503	-0.950%
		0.809		*0.916*		*0.335*		*0.315*		*0.783*		*0.633*	
2x2x3 size/value/profitability	0.505%	0.426	-0.821%	-0.684	-0.608%	-2.537	0.573%	2.481	0.370%	1.511	-0.153%	-0.198	46.051%
		0.685		*0.519*		*****0.044***		*****0.048***		*0.181*		*0.850*	
2x2x3 size/value/investment	-0.080%	-0.032	-0.312%	-0.124	-0.492%	-1.300	0.468%	0.939	0.474%	0.429	0.280%	0.664	-1.972%
		0.976		*0.905*		*0.241*		*0.384*		*0.683*		*0.532*	
2x2x3 size/investment/profit	0.375%	0.192	-0.724%	-0.372	-0.745%	-1.455	1.168%	0.912	0.694%	1.119	0.040%	0.105	2.662%
		0.854		*0.723*		*0.196*		*0.397*		*0.306*		*0.920*	
2x2x3 size/profit/investment	-1.300%	-0.474	0.864%	0.311	0.056%	0.138	-0.753%	-0.876	-0.222%	-0.543	0.691%	1.140	-2.253%
		0.653		*0.766*		*0.894*		*0.415*		*0.606*		*0.298*	

* Significant at 10% level

** Significant at the 5% level

*** Significant at the 1% level