An Investigation into *the* ARITHMETICAL COMPETENCE OF STUDENTS *entering the professional courses of training for teaching*

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1. *Introduction*

Those who are responsible for the training of teachers in the field of arithmetic constantly complain that students entering the training colleges are lacking in basic arithmetical knowledge. The arithmetical incompetence of their students precludes lecturers from carrying out an effective training programme, and topics such as how children think and reason in arithmetic, new experiments in the field of the methodology of arithmetic, the use and interpretation of diagnostic and standardized tests, discussions on contemporary literature on the subject of arithmetic, etc., simply cannot be handled. The little time available in an overcrowded curriculum for teacher-training must be utilized mainly to instruct students how to do for themselves the sums which they will in turn be called upon to teach their future pupils in the classrooms.

These are not idle assertions.

At the beginning of this year (1962), the author initiated a pilot investigation into the arithmetical competence of students entering the professional courses of training at the Johannesburg College of Education.

During the period of orientation at the commencement of the academic year, all first year professional students (men and women) were given a test involving the four basic processes in addition, subtraction, multiplication and division of integers, and also simple computations involving common fractions and decimals.

The test given is of about Standard V level, i.e. what would be expected of pupils at the end of their primary school course.

2. *The Actual Test Given*

The actual test given is reproduced here.(1)

No time limit was set, but students had to indicate on the answer sheet how long (in minutes) they had taken to complete the test.

The test was not a speed test, but rather a diagnostic test, designed to assess accurately the actual competence and understanding of the testees of the essential basic processes in arithmetic and their methods of attacking simple computations of a mechanical nature.

THE TEST

(*All working must be done on these sheets. No working on separate paper is allowed.*)

Section 1: *Addition*

1.	2.	3.	4.
3	64	16,837	1,000
0	57	84,273	100
2	93		10
8	15		101
1	41		9,789
7	26		
5	77		
9			
6			
4			

5. $8+4+1+9+6+5=$
6. $24+89+708+50=$
7. $5{,}500+3{,}344+5{,}588+7{,}711=$
8. $66+66+66+66+66=$

(1) The detailed instructions that accompanied the test are not included.

Section 2: Subtraction

1. $\begin{array}{r} 79{,}865 \\ -43{,}201 \\ \hline \end{array}$ 2. $\begin{array}{r} 45{,}321 \\ -6{,}897 \\ \hline \end{array}$ 3. $\begin{array}{r} 100{,}002 \\ -76{,}893 \\ \hline \end{array}$

4. $\begin{array}{r} 777{,}777 \\ -599{,}999 \\ \hline \end{array}$ 5. $\begin{array}{r} 101{,}010{,}010 \\ -90{,}000{,}101 \\ \hline \end{array}$ 6. $100-69=$

7. $4{,}326-2{,}654=$ 8. $70{,}006-29{,}148=$

Section 3: Multiplication

1. $\begin{array}{r} 537 \\ \times 9 \\ \hline \end{array}$ 2. $\begin{array}{r} 478 \\ \times 12 \\ \hline \end{array}$ 3. $\begin{array}{r} 8{,}050 \\ \times 11 \\ \hline \end{array}$ 4. $\begin{array}{r} 629 \\ \times 16 \\ \hline \end{array}$ 5. $\begin{array}{r} 84{,}196 \\ \times 23 \\ \hline \end{array}$

6. $\begin{array}{r} 625 \\ \times 84 \\ \hline \end{array}$ 7. $\begin{array}{r} 70 \\ \times 707 \\ \hline \end{array}$ 8. $\begin{array}{r} 6 \\ \times 46{,}823 \\ \hline \end{array}$ 9. $\begin{array}{r} 23 \\ \times 456 \\ \hline \end{array}$ 10. $\begin{array}{r} 5{,}205 \\ \times 804 \\ \hline \end{array}$

11. 204×139 12. $1{,}212\times 12=$ 13. $456\times 99=$

Section 4: Division

1. $10{,}068\div 12$ 2. $813{,}627\div 9$

3. $2{,}700\div 54$ 4. $33{,}888{,}921\div 11$

Section 5: Fractions

1. $3\frac{3}{5}-1\frac{3}{4}-\frac{7}{10}$ $=$ 2. $\frac{1}{4\frac{1}{2}}+\frac{5}{2\frac{1}{4}}-\frac{1}{22\frac{1}{2}}$ $=$

3. $3\frac{7}{8}\div 5\frac{1}{6}\times 1\frac{1}{9}$ $=$ 4. $3\frac{7}{8}\div 5\frac{1}{6}$ of $1\frac{1}{9}$ $=$

5. After a baker had sold $\frac{3}{8}$ of his bread, he still had 875 loaves over. How many loaves had he at first?

Section 6: Decimals

1. $0{\cdot}2\times 0{\cdot}2\times 0{\cdot}2$ 2. $0{\cdot}2\times 0{\cdot}3\times 0{\cdot}4$

3. $300\times 0{\cdot}003\times 0{\cdot}03$ 4. $6{\cdot}4\div 4$

5. $0{\cdot}00012\div 0{\cdot}04$ 6. $2\div 4{,}000$

7. $\frac{6\times 0{\cdot}5\times 0{\cdot}04}{0{\cdot}025}$ 8. $\frac{1}{0{\cdot}2}\times\frac{1}{0{\cdot}4}\times\frac{1}{0{\cdot}5}$ (Answer as a common fraction)

9. $14{\cdot}6\times 0{\cdot}29$ 10. $13{\cdot}8\div 4$

3. *Some Details About the Testees*

The following details concerning the testees are of importance.

No. of students who were tested	231
No. of students in possession of the T.S.S.C.	124
No. of students in possession of the N.S.C.	23
No. of students in possession of the U.E.C.	75
No. of students in possession of other school leaving certificates	9

The testees were drawn from 99 schools. The schools included State and private institutions, commercial and technical colleges, and were representative of all the provinces of the Republic of South Africa. In addition a few testees came from schools in Southern Rhodesia and Portuguese East Africa.

Of the total number of testees, 11 only had taken arithmetic at school to Std. VI level, 13 to Std. VII level, 167 to Std. VIII level, 4 to Std. IX level and 36 to Std. X level.

It will thus be seen that 207 out of the 231 testees had studied arithmetic to Junior Certificate level or beyond.

A further significant point is that 135 out of 231 testees had taken mathematics up to matriculation level.

The testees represented, therefore, a good sample and with their background in arithmetic, one would have expected to find very few unable to cope with the test, but as the subsequent analysis of the results show, this was not the case.

4. *An Analysis of the Test Results*

4.1: ADDITION

Though the testees on the whole succeeded well in adding columns of integers (both horizontal and vertical), their methods of working are worthy of analysis.

(a) Of the 231 testees, 127 or 55%([2]) indicated that they added seriatim, i.e. as the numbers appear on the paper. The remainder did the additions by seeking out tens or other combinations.

The inference that can be drawn from this is that teachers do stress on the whole the learning of the addition bonds.

(b) Of the 231 testees no fewer than 58% stated that they found it necessary at times during the test to count on their fingers or make marks on paper.

This may be attributed to the fact that many testees were out of practice as far as rapid addition was concerned, and were, therefore, compelled to resort to more infantile methods.

The test does show how deeply ingrained these early methods of learning become.

(c) Of the 231 testees, there were at least 87 or 38% of the total number, who made use of carrying figures. This is of considerable interest, if it is borne in mind that most teachers and educational authorities frown on the use of crutches! In fact,

([2]) All percentages are given in round figures merely for the convenience of the reader.

the Education Department of the Cape of Good Hope has gone so far as to forbid the use of crutches from Standard II upwards except in the case of the weakest pupils and those in special classes.(3)

Some testees, faced as it were with a situation which in a sense was of a critical nature to them, resorted to a method looked upon with disfavour by their former teachers, but which nevertheless from their point of view brought the desired results.

There is, the author believes, an important lesson to be learned from the results in this test. The ability to dispense with crutches must depend upon the pupil himself. Some children (and indeed many adults) may never be able to operate successfully without them. The author is of the opinion that no hard and fast rules ought to be laid down.

4.2: SUBTRACTION

The most significant fact illustrated by the results of this test is that no fewer than 159 out of the 231 testees, or 69% of the total made use of the *decomposition method* of subtraction and employed the word *borrowing*. (One testee spelt it "borough-ing"!)

The word *borrowing* is a misnomer, for borrowing implies giving back, which is not the case in subtraction. One gains the impression, especially after having had discussions with students, that the *borrowing* of a number in subtraction is learned without any explanation of the logic on which the performance is based.

The following are some typical errors in equations made by candidates in a recent matriculation examination:

(*a*) $7t^2-7t^2=15, \therefore t^2=15$

(*b*) $x+2y = 7, \therefore x=\frac{7}{-2y}$ (4)

(*c*) $4a = 8, \therefore a=\frac{8}{-4}$

It is certainly worthwhile investigating whether the incorrect use of the negative sign in algebra at high school level, might not very well have its roots in the rather loose and incorrect meaning attached to the word *borrowing* in the primary primary school.

So ingrained is the word *borrowing* in the minds of many students, that lecturers give up in frustration after having tried in vain to eradicate this misnomer in a three year course of training!

4.3: MULTIPLICATION

In this test, as in all others concerned with the fundamental processes, the author did not concern himself with the correctness of the answers, but essentially with the methods of attack, i.e. how the testee approached the solution to each multiplication sum.

From an analysis of the results it will become clear to the reader that each sum is not treated on its merits. In many instances the testee merely does as he was told by his teacher and through years of practice has established definite habits of reaction in given situations. Arithmetic then becomes the mechanical manipulation of number without any raionalization.

The following ought to substantiate what the author has just stated.

Ex. 7:

```
    70
 ×707
 -----
   490
   000
 490
 -----
 49,490
```

No fewer than 99 out of the 231 testees (i.e. 43%) did the sum as shown alongside. A few left out the second partial product consisting of noughts.

This sum could have, in fact, been done mentally had the testees realized that 70 should have been used as the multiplier instead of the multiplicand.

Ex. 8:

```
        6
  ×46,823
  -------
  240,000
   36,000
    4,800
      120
       18
  -------
  280,938
```

Fortunately only 10 out of the 231 testees (i.e. 4%) did the sum as shown alongside.

The fact that the sum was done as indicated shows lack of insight, and far too much dependence on rule of thumb methods.

Ex. 9:

```
     23
   ×456
  ------
    138
  1,150
  9,200
  ------
 10,488
```

Once again 99 out of the 231 testees (i.e. 43%) did the sum as shown alongside.

The principle applied successfully in example 8 was not extended to example 9.

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(3) See *The Education Gazette*, Vol. LIX, No. 7—17.3.1960, p. 510.

(4) Knobel, J. C.: *Die Betekenis en Hantering van Foute en Leermoelikhede in Algebra.* (HAUM Boekhandel, Pretoria, 1962).

Two of the examples given involve a nought in the multiplier or multiplicand. The solutions given by some testees are as follows:—

Ex. 10:

```
  5,205
 ×804
 20,820
 00,000
420,000
440,820
```

Of the 231 testees, 25 or 11% did the sum as indicated; many of these (and others) had the answer incorrect, because of the omission or addition of noughts.

Ex. 11:

```
   204
  ×139
20,400
 6,120
 1,836
28,356
```

Of the 231 testees as many as 151, or 65%, did the sum as shown alongside.

The similarity between examples 10 and 11 was not realized.

The author was interested to know how the testees would react to multiplication sums involving 11, 12 and 16 as multipliers. In all schools much attention is given to the eleven and twelve times tables, and in many schools the sixteen times table is also taught.

When multiplying by 11 and 12, pupils are not expected to multiply by 10 and 1 or 10 and 2 separately, but as one operation. Instructions to this effect have been incorporated in the syllabuses issued to teachers in the Transvaal.

Here are the efforts of some of the testees:

```
Ex. 2:        Ex. 3:        Ex. 4:
  478          8,050          629
  ×12            ×11          ×16
  956          8,050        6,290
4,780         80,500        3,774
5,736         88,550       10,064
```

Example set out as above by 21% of the testees.

Example set out as above by 17% of the testees.

Example set out as above by 82% (189 out of 231) of testees.

Horizontal multiplication is hardly taught in our schools. Very few testees, indeed, did this sum as follows: 1,212×12=14,544.

Much time is devoted in our schools to the teaching of short methods of multiplication, especially with multipliers such as 99, 101, 201, etc. The author was therefore particularly interested in how the testees would react to the following sum: Ex. 13: 456×99.

No fewer than 175 out of the 231 testees, i.e. 76%, did the sum by the long method.

Of the remainder, who attempted to work the sum by the short method, 13 failed completely. Here are some of the efforts:

(i) 456×99=4,560—99=4,461

(ii) 456×99=36,040+36,040=72,080

(iii) 456×99=4,140+4,104=8,244

(iv) 456×99=45,600—99=45,601

4.4: FRACTIONS

The testees' responses to the questions on fractions were, to say the least, appalling.

The following is a summary of the results:

	Example	*No. of testees out of 231 who did the sum incorrectly*	*Percentage of testees who did the sum incorrectly*
1.	$3\frac{3}{5}-1\frac{3}{4}-\frac{7}{10}$	111	48%
2.	$\frac{1}{4\frac{1}{2}}+\frac{5}{2\frac{1}{4}}-\frac{1}{22\frac{1}{2}}$	192	83·1%
3.	$3\frac{7}{8}\div5\frac{1}{6}\times1\frac{1}{9}$	109	47·2%
4.	$3\frac{7}{8}\div5\frac{1}{6}$ of $1\frac{1}{9}$	180	77·9%
5.	After a baker had sold $\frac{3}{8}$ of his bread, he still had 875 loaves over. How many had he at first?	170	73·6%

Here is a list of typical errors:—

Ex. 1:

$$3\frac{3}{5}-1\frac{3}{4}-\frac{7}{10}$$
$$=\frac{\overset{9}{\cancel{18}}}{5}-\frac{7}{4}-\frac{7}{\underset{5}{\cancel{10}}}$$
$$=\frac{201}{100}$$

Ex. 2:

$$\frac{1}{4\frac{1}{2}}+\frac{5}{2\frac{1}{4}}-\frac{1}{22\frac{1}{2}}$$
$$=\frac{6}{6\frac{3}{4}}-\frac{1}{22\frac{1}{2}}$$
$$=\frac{5}{-15\frac{3}{4}}$$

Ex. 2:

$$\frac{1}{4\frac{1}{2}}+\frac{5}{2\frac{1}{4}}-\frac{1}{22\frac{1}{2}}$$
$$=\frac{5}{(4\frac{1}{2}+2\frac{1}{4})-22\frac{1}{2}}$$
$$=\text{etc.}$$

Ex. 2: $\frac{1}{4\frac12}+\frac{5}{2\frac14}-\frac{1}{22\frac12}$
$=4\frac12+(5\times 2\frac14)-22\frac12$

Ex. 2: $\frac{1}{4\frac12}+\frac{5}{2\frac14}-\frac{1}{22\frac12}$
$=\frac{1}{0}\times\frac{2}{9}+\frac{5}{0}\times\frac{4}{9}-\frac{1}{0}\times\frac{2}{22}$

Ex. 5: $875=\frac58$
$\therefore\ \frac18=5\div 875$
$=\ldots$ loaves

Ex. 5: $\frac58=875$ loaves
$\frac{8}{8}=x$
$=\frac{875\times 8\times 8}{5\times 8}$

Ex. 5: $\frac38=875$
$\frac58=\frac{\frac58}{\frac38}\times 875$ loaves
Originally he had
$1{,}458\frac13$ loaves

Ex. 5: $\frac58=875$
$\frac18=177$
$\frac{8}{8}=1{,}416$ loaves

Ex. 5: $109\frac38\times 8$
$\frac{955}{8}\times 3$
$\frac{2{,}865}{8}$
$358\frac18+875$
$=1{,}233\frac18$ loaves

Ex. 5: $875=\frac58$
$\therefore\ \frac18=875\div 5$
$=171$
$\frac{8}{8}=1{,}368$

Ex. 5: $x-\frac38x=875$
$x=875+\frac38x=8x=7{,}000+3x$
$\therefore\ 5x=7{,}000,\ \therefore\ x=1{,}400$

From example 5 it is clear that the average professional student entering the teaching profession does not know how to set out a simple problem, and that the equality sign (=) has no meaning to him.

4.5: DECIMALS

In all fields of scientific work, and in the newer branches of computation such as cost accounting and statistics, a knowledge of decimals is indispensable.

Much attention will have to be given to a study of decimals at school level and at Colleges of Education.

The results of the test in decimals indicate that professional students on entering the teachers' training course have a very inadequate knowledge of the computational aspects of decimals.

The following is a summary of the results.

	Example	*No. of testees out of 231 who did the sum incorrectly*	*Percentage of testees who did the sum incorrectly*
1.	$0\cdot2\times0\cdot2\times0\cdot2=$	140	60·6%
2.	$0\cdot2\times0\cdot3\times0\cdot4=$	127	55%
3.	$300\times0\cdot003\times0\cdot03=$	173	74·9%
4.	$6\cdot4\div4=$	39	16·9%
5.	$0\cdot00012\div0\cdot04=$	144	62·3%
6.	$2\div4{,}000=$	164	71%
7.	$\frac{6\times0\cdot5\times0\cdot04}{0\cdot025}$	186	80·5%
8.	$\frac{1}{0\cdot2}\times\frac{1}{0\cdot4}\times\frac{1}{0\cdot5}$ (Answer as a common fraction)	205	88·7%
9.	$14\cdot6\times0\cdot29$	142	61·5%
10.	$13\cdot8\div4$	76	32·9%

Here is a list of typical errors. Only the incorrect answers are given alongside each sum.

	Example	*List of typical incorrect answers*
1.	$0\cdot2\times0\cdot2\times0\cdot2$	(i) 0·8; (ii) 0·12; (iii) 0·6; (iv) ·006
2.	$0\cdot2\times0\cdot3\times0\cdot04$	(i) ·24; (ii) 2·4; (iii) ·00024
3.	$300\times0\cdot003\times0\cdot03$	(i) 9; (ii) ·00009; (iii) ·00027; (iv) ·02700; (v) 300·009; (vi) 300·0009; (vii) 300·033; (viii) 3·033; (ix) 900·009; (x) 2700000
4.	$6\cdot4\div4$	No significant errors
5.	$0\cdot00012\div0\cdot04$	(i) 0·03; (ii) ·00000048; (iii) ·00003; (iv) 000·003; (v) ·0000003
6.	$2\div4{,}000$	(i) 2,000; (ii) 8,000; (iii) ·002; (iv) 2; (v) $\frac{1}{2000}$; (vi) ·5
7.	$\frac{6\times0\cdot5\times0\cdot04}{0\cdot025}$	(i) $4\frac45$; (ii) 4·08; (iii) $0\cdot04\frac45$
8.	$\frac{1}{0\cdot2}\times\frac{1}{0\cdot4}\times\frac{1}{0\cdot5}$	(i) $\frac14$; (ii) $\frac52$; (iii) $\frac{1}{\cdot040}$; (iv) $\frac{25}{0}$; (v) $\frac{1}{25}$; (vi) $\frac{1}{\cdot4}$

9. 14·6×0·29	(i) 42·340; (ii) 4,234; (iii) ·42340
10. 13·8÷4	(i) 3·405; (ii) ·345; (iii) 340·5; (iv) 34·25; (v) ·03405; (vi) 34½

Why do the testees perform so badly in this test on decimals, particularly if one considers the fact that decimals are taught in all classes from Std. IV upwards?

The author is of the opinion that in the teaching at school level too much emphasis is placed on the learning and applying of rules without ensuring that the basic principles on which the rules depend are fully grasped and mastered.

Some authorities in fact recommend that the manipulation of decimals should be taught by rule of thumb methods. The children should simply be told that the decimals be ignored until the multiplication is completed. Thereafter the number of places in the multiplicand and the multiplier are counted and the decimal point is inserted.[5]

A rule of thumb for the division of decimals is also given.

If a pupil is not made to understand the principle, the rule is really meaningless and easily forgotten. This then appears to have been the case with the testees. Many did not have occasion to use decimals for a year or longer and, confronted with examples, they attempted to rely on memory instead of tackling the examples *de novo* from basic principles.

5. *Where Does the Remedy Lie?*

(a) The primary school syllabuses in arithmetic must be recast to stress an understanding of basic principles, and the present emphasis on soul-destroying and monotonous drill of processes involving long, complicated and abstruse manipulation should be discarded. Oral work should be the key to all primary school arithmetic, and should, in the opinion of the author, occupy one half of the time of each lesson period.

([5]) Potter, F. E.: *The Teaching of Arithmetic* (Pitman & Sons, London, 1932) p. 246.
See also: *The Education Gazette*, Province of the Cape of Good Hope, Vol. LIX, 17.3.1960, p. 511.

The new syllabus for arithmetic for primary schools in the Transvaal reflects a new trend and shows a movement in the right direction. It is gratifying to read, for example, that in the division of decimals by decimals, teachers are instructed to "stress the fact that the divisor must be converted to a whole number and explain why this is done."[6]

(b) The present curriculum for the training of primary school teachers needs to be amended. While not denying the importance of the humanities in any training course for teachers, ours in this scientific and technological age is far too heavily weighed down by the so-called skills, i.e. Art and Crafts, Music, Physical Education, Writing and Blackboard Work, etc. The general atmosphere is still far too much orientated towards the traditional classical approach.

All students following a three-year general professional course for the primary school with or without specialization in a particular subject, should be given a much more elaborate and challenging syllabus in arithmetic than is the case at present. To provide them with instruction that is merely an extension and an elaboration of the primary school syllabus will no longer do. Instruction in arithmetic alone is not enough. Our students require a well co-ordinated, adequately planned course in basic modern mathematics that will give them insight into the rigours of mathematical thinking, and will make them aware of the meaningfulness of absolute truths, precision of statement and deductive logic. It is this mental equipment which our future teachers need.

This, then, is a challenge we in the teacher training colleges of South Africa must face.

The Johannesburg College of Education is attempting to meet this challenge by introducing, with effect from 1963, *a new course in modern mathematics for all first year students. More time is to be devoted to the subject, and the emphasis will be a distinctly different one.*

([6]) Transvaal Education Department: *Syllabus for Arithmetic Grade I—Std. V* (1960, p. 41 op. cit.)