A SuperNEC Implementation of Model Based Parameter Estimation by Interpolating the Method of Moments Impedance Matrix

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***Abstract*— *SuperNEC* is a method of moments (MoM) electromagnetic field solver based on the Numerical Electromagnetics Code (NEC). Much of the simulation time can be attributed to the filling of the impedance matrix, which is performed at each frequency point of interest. Impedance matrix interpolation methods have been implemented in *SuperNEC*to reduce the computational time required to fill the impedance matrix $[Z]$. Elements in $[Z]$ vary predictably over frequency and can be approximated by a second order polynomial. A second improved method is implemented where the dominant frequency variation term is removed prior to calculating the fitting function. A method of determining the optimum sample range relative to simulation range and maximum interaction distance has been developed. Given the correct choice of sample range the mean error in the MoM solution is less than 10$\%$ over the frequency range and the input impedance can be reproduced with good agreement over a wide bandwith. Improvement in the simulation efficiency of 1.7 times can be expected if sufficient frequency points are of interest to account for the computational time required to sample the matrix and determine fitting function coefficients. This method has been applied to a dipole antenna, an LPDA and a horn antenna. To increase the simulation bandwidth and retain an acceptable level of accuracy, the bandwidth is split into multiple sub-bands.**

Index Terms*— MoM efficiency, Impedance matrix interpolation, *SuperNEC

I. Introduction

THE use of the Method of Moments (MoM) as a frequency domain field solver is a well used technique to accuracy predict the electromagnetic (EM) characteristics of large structures [1]. When designing antennas, observable parameters such as input impedance, VSWR and radiation patterns are required over a wide frequency range [2]. Generation of wide bandwidth EM information using MoM is a computationally expensive task as the MoM procedure is performed at each frequency point of interest. Much of the computational time used is in the filling and solving of the $N \times N$ impedance matrix ($[Z]$), where N is the number of unknowns in the problem.

Miller in [3] introduced Model-Based Parameter Estimation (MBPE) as a form of "smart" curve fitting. Where a fitting model (FM) is applied to a observable parameters itself [4] or in the formulation field solver [5] thus reducing the computational time in generating the wide bandwith information with minimal loss in solution accuracy. MBPE was applied in [6] to increase the computational efficiency of determining EM transfer function with the use of a rational function which is solved using the Padé procedure [7].

MBPE was applied to the efficient generation of radiation patterns by Werner in [8] spatially and over the frequency. This method uses the Padé rational function to approximate the radiation pattern. The rational function coefficients are determined by sampling far electric field values at various spacial and frequency points. This method was implemented by the author in *SuperNEC* in [9] with poor results. Correct rational function order is critical in obtaining an accurate fitting model, this requires *apriori* knowledge of the number of poles in the transfer function.

Werner further improved this method in [10] to fixed order rational function over smaller sub-bands. The method can however be used to reduce the amount of storage required in storing wideband three dimensional radiation patterns as the FM can be stored instead of all the numerical values.

Each of the impedance matrix elements is determined by evaluating a integral over the segment length [11]. While simple numerical integration techniques have been developed [12] these integrals are evaluated for each source-observation point in the structure at every observation frequency. Computational efficiency can be improved if the evaluation of the integral is replaced with a simple FM model. Impedance matrix interpolation as a form of MBPE was implemented by Newman [13] where the impedance matrix elements where approximated by simpler quadratic fitting functions thus reducing the time required to fill the impedance matrix. Newman applied the method to dipole antenna cases producing accurate approximations. The method was futher applied to mobile communication antennas by Rahmat-Samii [14], a method of interpolating the admittance matrix ([Y]) with rational functions was also implemented in this paper thus removing the need to invert the impedance matrix. A high order rational function was needed to accurately approximate the unpredictable admittance matrix elements and with improved matrix invertions methods such as LU decomposition [15] make the efficiency improvement in this method negligible. The method has been applied to planar microstrip antennas in [16], [17] with accurate results and good improvements in computational efficiency.

SuperNEC is an object-oriented (OO) C++ [18] implementation of the MoM Numerical Electromagnetics Code (NEC2) originally developed by the Lawrence Livermore Laboratory

[11]. The OO nature of the code makes it possible to easily add functionality to the code, additions include a Uniform Geometric Theory of Diffraction (UTD) MoM hybrid [19], dielectric coated wires and a MoM parallel implementation [20].

Impedance matrix interpolation methods have been implemented in the *SuperNEC* code, this is achieved by approximating the elements in the impedance matrix with a quadratic function. Two methods have been implemented: a standard method where all the elements are approximated with a quadratic function directly, and improved method where the frequency variant component is removed prior to approximation and a windowed sub band method. A detailed explanation of the theory and implementation of these methods can be found in [21]. Section II outlines the MoM procedure and impedance matrix interpolation method, Section III is on how the method was implemented in *SuperNEC* and determining correct sample range. Results applied to antenna cases and computational efficiency of the methods are discussed in Section IV and Section V respectively.

II. Interpolation Method

The Electric Field Integral Equation (EFIE) is solved in MoM by splitting the structure into N short thin pieces or segments such that the thin wire approximation [11] can be applied. The currents on each of the segments are solved by a linear system in *Eq. (1)* where $[Z]$ is the generalised impedance matrix, $[I]$ is the unknown structure current vector and $[V]$ is the known excitation.

$$[Z][I] = [V] \tag{1}$$

The impedance matrix is can be referred to the as matrix of mutual impedance between source segments m and observation segment n, notation used to refer to a single entry in the matrix is $Z_{m,n}$. Each element in $[Z]$ is determined by:

$$Z_{m,n} = \int_L f(s) \left[\frac{\delta^2}{\delta s \delta s} g(r_m, r_n) + k^2 g(r_m, r_n) \right] ds' \tag{2}$$

Where $g(r_m, r_n) = \dfrac{e^{-jk|\overrightarrow{r_m} - \overrightarrow{r_n}|}}{|\overrightarrow{r_m} - \overrightarrow{r_n}|}$
is the free space Green's function and $f(s)$ is a set of sine, cosine and constant basis functions used the MoM procedure [11].

Much of the computational time in MoM is in the filling of the $N \times N$ impedance matrix $[Z]$ which is performed at every frequency point of interest. If the matrix filling function in *Eq. (2)* can be replaced with a simple FM the computational time used to fill $[Z]$ can be greatly reduced. Approximating the MoM solution $[I]$ is difficult due to the unpredictability of the frequency variation, a high order rational function is needed to for an accurate FM.

The elements in the impedance matrix vary predictably over frequency, hence it is possible to approximate the elements in the matrix with a low order polynomial function or Taylor series. Consider a 1 Metre long dipole antenna segmented in 30 equal pieces, the input impedance of the antenna over a wide frequency range, 100MHz to 800MHz, is shown in Fig. 1. By

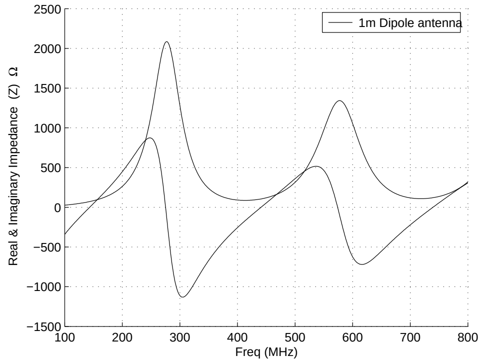


Fig. 1. Input impedance of 1m dipole antenna over wide frequency band

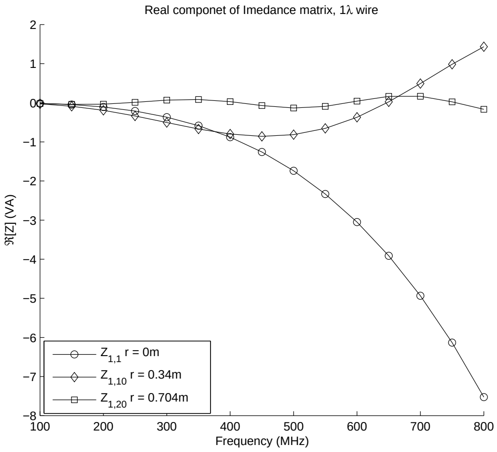
contrast consider three element of the impedance matrix in Fig. 2: $Z_{1,1}$ represents the self impedance of the first segment, $Z_{1,10}$ is the mutual impedance between the first and tenth segment (0.34m appart) and $Z_{1,20}$ is the mutual impedance between the first and twentieth segment (0.7m appart). While the frequency variation increases with the interaction distance it is still significantly less than is the case of the input impedance. The input impedance could be easily approximated with the use of a rational function given the correct order for this case, however for an antenna with a greater level of variation in the observable the FM order would not hold. Variation in impedance matrix elements is constant for any given antenna characteristics, this factor makes it possible to apply MBPE to accurately approximate the observables for a generalised structure. Interpolation the elements in the impedance matrix directly over frequency is known as the standard interpolation method.

From *Eq. (2)* it is clear that frequency variation term $e^{-j2\pi\lambda\overrightarrow{R}}$, where $\overrightarrow{R}$ is the interaction distance between the source and observations segments, begins to dominate as the interaction distance increases. For interaction distances greater than 0.5λ removing this terms results in the impedance matrix elements becoming more predictable over frequency. Thus a new matrix is $[Z]'$ generated by:

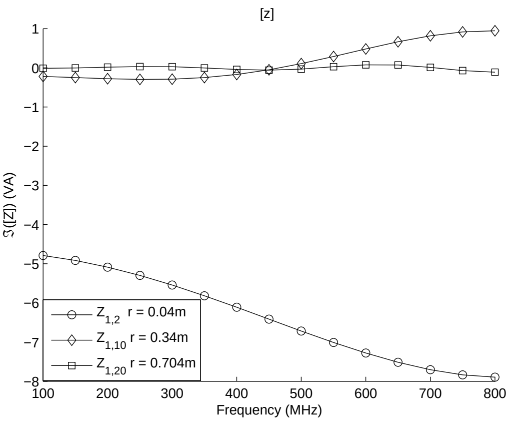
$$[Z]' = \frac{[Z]}{e^{-j2\pi\lambda\overrightarrow{R}}} \tag{3}$$

Fig. 3 shows the same impedance matrix elements as Fig. 2 with the variant component removed. It is clear that the frequency variation is significantly less and a simple approximation model could be used to accurately approximate the elements over a wide frequency bandwidth. Fitting a second order polynomial function to the $[Z]'$ matrix is referred to as the improved interpolation method. The improvement in simulation efficiency is not as great as the standard method explained above as computational effort is required to remove the variant componets after sampling and reintroducing it when filling the matrix by evaluating the quadratic functions.

Coefficients of the fitting function are determined by sampling the matrix at specified frequency points, this is achieved by filling the matrix in the standard fashion for the standard interpolation method. Samples for the improved method have the variant component removed before the coefficients are

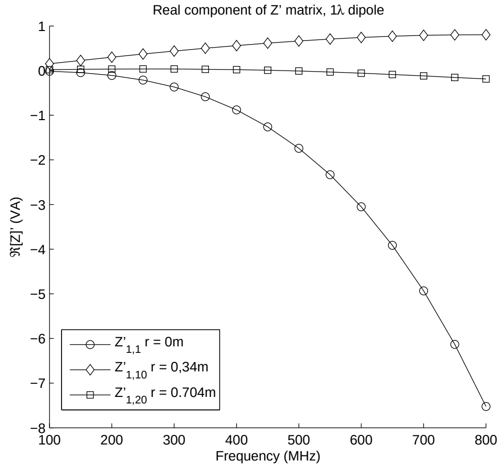


(a) Real Components

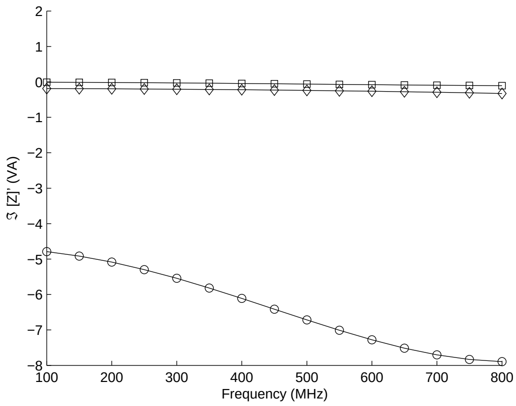


(b) Imaginary Componets

Fig. 2. Three $[Z]$ elements over frequency



(a) Real Components



(b) Imaginary Componets

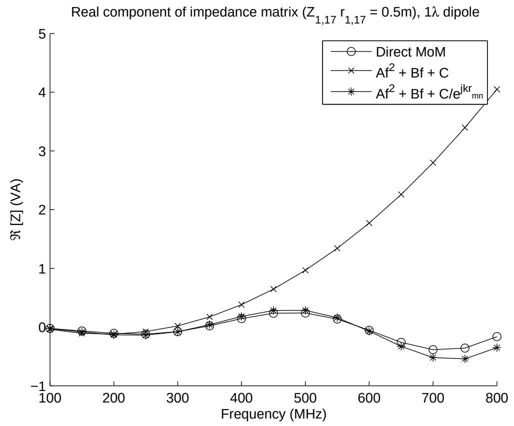
Fig. 3. Three $[Z]'$ elements over frequency

solved. As the elements in the impedance matrix are complex two fitting functions are required to approximate each of the elements. Newman used a quadratic function to approximate the real components and a logarithmic expression to account for the logarithmic singularity with frequency of element self impedance [13]. However it was found that a quadratic function could approximate the imaginary component was well without the need for introducing logarithmic functions. The method of least squares was used to determine the coefficients of the polynomial fitting functions as the method is flexible as to polynomial order and number of samples used to determine the coefficients [23]. A better approximation was expected when using a higher order polynomial approximation or when sampling a higher number of sample points. In testing with various polynomial orders and sample frequencies it was found that the sample range (Δf) of the sample points had the greatest effect on the accuracy in the solution [21]. The error level converges when the optimum sample range is used, the method of determing this range is covered in Section III.

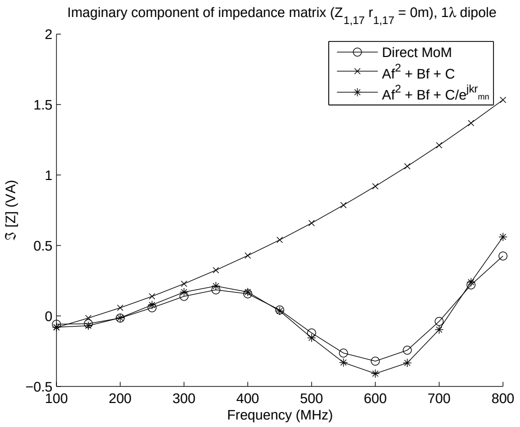
Fig. 4 shows a second order polynomial function approximating $Z_{1,17}$ for a wire structure, the separation distance of the two elements is 0.5m. The standard interpolation method holds over a small frequency range but becomes highly inaccurate toward the upper end of the range. By contrast the improved interpolation method shows and accurate approximation of the element over the entire frequency range. While these figures show that the standard method is highly inaccurate the method should not be discounted for electrically small structures and small frequency bandwidths due to the superior efficiency increase compared with the improved method.

III. Implementation for wire structures

Wire structures in *SuperNEC* are modelled by interconnected segments and plates as mesh segments grids [24]. It is of utmost importance that the structure is segmented correctly. Restrictions in the segments length, radius and interconnection are due to the assumptions in reducing the EFIE from a vector



(a) Real Components



(b) Imaginary Components

Fig. 4. $Z_{1,17}$ with polynomial approximation over wide frequency range

to a scalar equation, the restrictions themselves can be found in [25]. An incorrectly segmented structure will result in errors in $[Z]$ and ultimately the MoM solution. When approximating $[Z]$ elements the error is compounded as they no longer follow a non-pole relationship over frequency. Neither the standard nor improved methods of approximating the elements can be used. When using impedance matrix interpolation the structure should be segmented at the highest frequency of interest. A higher segmentation frequency will result in a shorter segments and hence more unknowns in the solution. A trade off between accuracy in the MBPE solution and simulation efficiency results as segmentation at a lower frequency may produce less accurate results but a greater simulation speedup will be achieved. Where possible frequency scaling should be used to reduce the problem size and the minimum possible upper frequency should be used for segmentation. Another method to counteract the segmentation paradox is to use multiple sub-bands, where the simulation frequency range is split into smaller bands with impedance matrix interpolation performed on each of the smaller bands. Each sub-band upper frequency should be used for the segmentation of the structure thus reducing the problem size for the all but the upper most frequency band. Discontinuities will result between band junctions, a trade off which may be acceptable given a sufficient speedup in simulation time. Discontinuities can be minimised by using overlapping bands with a window function to smooth the junction, as was implemented by the Author with impedance matrix elements in [21].

A. *Determing sample range*

Interaction distance between source and observations segments has the greatest effect on the variation of the elements in the impedance matrix, while the elements in the reduce variation matrix are as dependent on interaction distance the effect is still apparent.

MoM solution accuracy depends on choice of correct sample and simulation range. The method used by Newman in [13] was to apply the Nyquist sampling criteria such that wavenumber step (Δk) size is less than π. Maximum sample range can thus determined by:

$$\Delta f_s = \frac{f}{2R_\lambda^{\max}} \tag{4}$$

This specifies the upper step size for the standard interpolation method and as a result a smaller step size should be used when applying standard interpolation and a greater one for the improved method. This method does not however specify the bandwidth over which the interpolation function can be accurately extrapolated. *Eq. (4)* also only relates the sample range to the maximum interaction distance of the structure and not the simulation range. If a narrow frequency band is required it would be advantageous to sample over a narrower band than specified by Eq. 4. Yeo and Mittra in [16] related the sample range to the upper frequency value by specifying f in *Eq. (4)* to be the upper most frequency value of interest ($f_{\max}$). Given an upper frequency value too high this method would produce inaccurate results as the fitting functions would be poorly conditioned. For structures with a maximum interaction distance of less than 1λ a too wide sample range would be achieved, again resulting in poorly conditioned fitting functions. Segmentation must be done at the highest frequency of interest whether it be a sample or simulation frequency, thus there is no point in sampling at higher frequencies than are of interest.

The sample range from *Eq. (4)*is reduced by the addition sample range coefficient (k_s):

$$\Delta f_s = k_s \times \frac{f_{\text{centre}}}{2R_\lambda^{\max}} \tag{5}$$

(k_s) is determined by the total simulation range (f_M) in order to minimise the error for specified simulation range and maximum interaction distance. The simulation range is specified as an input parameter and related to the maximum interaction distance distance by a simulation range coefficient (k_f):

$$f_M = k_f \times \frac{f_{\text{centre}}}{R_\lambda^{\max}} \tag{6}$$

In general it was found that if the required simulation range coefficient (k_f) was greater than one the simulation band is too large and should be split into sub-bands. As mentioned before if the maximum interaction distance is less than 1λ unity should be used for the maximum interaction distance. The curves in Fig. 5 are used to obtain (k_f) from the simulation range coefficient calculated in *Eq. (6).* These curves where produced by simulation an arbitrary wire structure over the simulation ranges varying the sample range, the point where minmum error was achieved was noted. This process was repeated for increasing interaction distance case, the number of unknowns in the structure where kept constant as increasing the problem size would change the level of error achieved. Fig. 5 is for the improved interpolation case, the curve for the standard interpolation allthough similar can be found in [26].Mean square structure current error over the entire simulation range was used to calculate the error in the simulation, the method will be discussed in greater depth in Section IV.

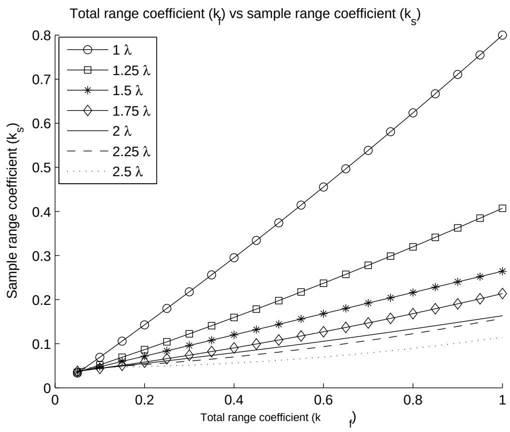


Fig. 5. Graph relating the simulation range coefficient (k_f) to the sample range coefficient (k_f)

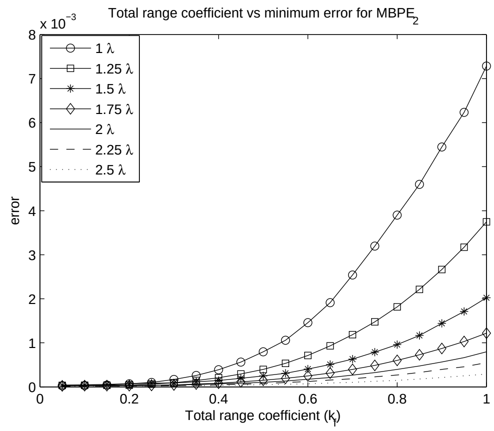


Fig. 6. Graph relating the simulation range coefficient (k_f) to mean root square error over the simulation range

Fig. 6 shows the mean root square error over simulation range coefficient (k_f) for increasing maximum interaction distance. The error in the solution steadily increases with increasing simulation range and given a too large simulation range the error in the solution becomes unacceptable. Note that the above curves must only be used as a guide as to the error in the solution. Other factors such as the number of segments in the structure, position of all the segments and segment connection will effect the final accuracy of the solution.

B. Required storage

The amount of additional memory used in the MBPE scheme is worth noting. Since three impedance matrices must be filled and stored in order to calculate fitting function coefficients the amount of additional memory increases rapidly with problem size. Another source of additional memory required is in the storing of the fitting function coefficients themselves, three coefficients are required for the fitting function. Impedance matrix elements and fitting function coefficients are complex values each expressed by two *float* value, the amount of additional memory in bytes is:

$$\mathrm{MEM}_{\max} = (8 \times 7.5)N^2 \tag{7}$$

This is for the improved interpolation case, where three samples are taken for second order polynomial fitting function. As the problem size exceeds 800 unknowns 1GB of additional memory is required.

IV. Results

The usefullness of any optimisation method is determined by how accurately it can be used and the improvement in efficiency. When using approximation functions as is the case with impedance matrix interpolation a certain level of accuracy will be lost, in general this error will increase with extrapolation past the centre frequency. One of impedance matrix interpolation's best attributes is it's the ability to apply the technique to any structure. The method has been applied to various wire and grid structures, these are a Dipole antenna, Log periodic dipole array and a horn antenna. Each of these introduce different elements in the simulation, being size,make up of structure and addition of transmission lines. In order to specify error it is important to have an error norm, which must be applied to a element of the simulation being either in the simulation domain or the solution domain. Looking at the error of the impedance matrix approximation functions is not useful as error in the impedance matrix will be increased when the matrix is inverted and solution found. On the other hand looking at an observable parameter such as input impedance will also not give a realistic error norm as input impedance is only calculated on a single segment (as is the case when a single excitation is used). Error in segments far away from the excitation would not have a great effect the input impedance however would have a significant effect on the radiation pattern. Error in radiation pattern is a good method as it incorporates all the effect of all the segments, radiation pattern specified in dB gives a intuitive value for error. Calculation of wideband three dimensional radiation patterns however is not only computationally expensive a large amount of storage is needed is needed to post process the results. The error norm is thus calculated with the MoM solution, the current vector $[I]$. Relative root mean square error is used as it gives the most intuitive results and offsets in phase do not cause an unrealistic amount of solution error. The error as a function of frequency is defined by:

$$e(f) = \frac{1}{N}\sqrt{\sum_{k=1}^{N}\frac{|I_k - I_k^m|^2}{|I_k|^2}} \tag{8}$$

A norm as the mean error over the frequency range is then:

$$\|e\| = \frac{1}{n_f}\sum_{f=1}^{n_f} e(f) \tag{9}$$

A confidence interval is defined as the frequency range extending on either side of the centre frequency where the error is less than 10%. It is often found that the confidence interval is greater on the upper frequency range than the lower however is expressed by a single value. The confidence interval gives a user an idea of the simulation range that the technique can be used and still reproduce accurate results.

A. Dipole Antenna

The impedance matrix interpolation technique was applied to a 0.5m dipole antenna with a centre frequency of 500MHz. The dipole antenna is used as it's simplicity in structure and in observables [2] makes it good for benchmarking the technique. In general MBPE techniques would not be applied to cases this small and simple as simulation from first principles can be done quickly with modern computers.

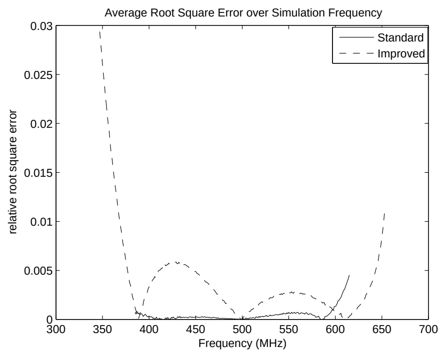


Fig. 7. Root square structure current error over simulation range for dipole antenna

Fig. 7 shows the mean root square error over frequency for the dipole antenna, both method show a low level of error over the entire range being 0.04% and 0.4% for the standard and improved methods respectively. In cases such as this where the maximum interaction distance is small the standard method outperforms the improved one. While the error is small for both case the simulation time improvement is better of the standard case being 36% apposed to 34% with improved method. However the gain in seconds is so slight that the loss in accuracy is not worth the speedup in simulation time.

B. Log Periodic Dipole Array

Broadband antennas such as the LPDA are ideally suited to MBPE method due to the wide frequency range required and size of the structure [27]. As there are many variable parameters in the structure numerous computer simulations are required to optimise the design. Even a small increase in simulation efficiency will result in a large time reduction when computing observables for a large number of cases. The LPDA brings transmission lines into the structure which effect the performance if the impedance matrix fitting models.

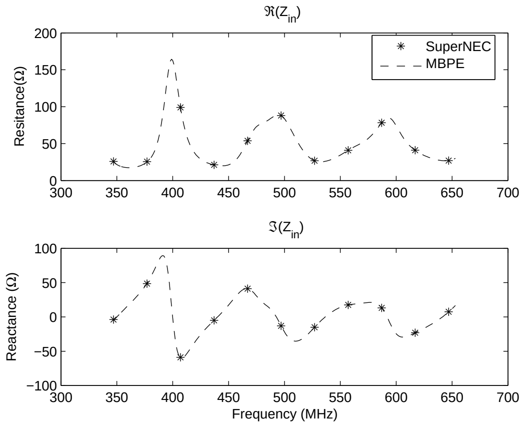


Fig. 8. Input impedance of LPDA antenna via direct *SuperNEC* computation and Improved interpolation method

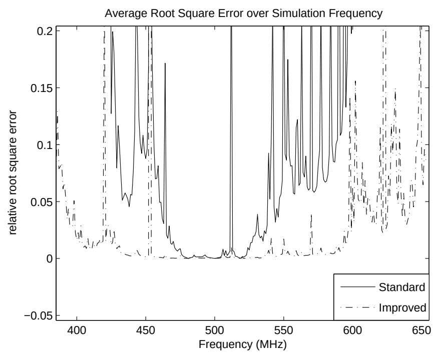


Fig. 9. Root square structure current error over simulation range for LPDA antenna

Fig. 8 shows the input impedance the LPDA antenna and the impedance generated with the improved interpolation method. The observable curve is well conditioned over the entire range. Consider the root square error in Fig. 9, the standard method shows a high level of inaccuracy while the improved method has acceptable error over the simulation range. Sparse points of increased error are attributed to the transmission line in the solution, further work is needed to generate a fitting model to account for the elements. However the a mean error of 2.8 % can be expected in the confidence interval of 147 MHz with a simulation time speedup of 41%. The radiation pattern is also well reproduced when using the improved method, the radiation pattern can be found in [26].

C. Horn Antenna

Like the LPDA horn antennas are used in broad-band applications, they consist of a waveguide element connected to a conical or pyramid radiation element [28]. Metal plates are modelled as grid mesh of wire segments, a large number of segments are required modelling such plates. Simulation time is further increased in NEC as the number of junctions at the segment ends is greater than wires. The basis functions are evaluated over all the adjacent connected segments [11], however when filling the matrix from the fitting model only a single calculation is needed for each element independent of connection. To increase the simulation bandwidth this total simulation bandwidth was split into two sub-bands the first extending from 207-390 MHz with a center frequency of 400 MHz and a second extending from 395-500 MHz with a center frequency of 450MHz. To reduce the problem size the first band was segmented at 300MHz and the second at 500MHz resulting in a problem size of 1289 and 3535 unknowns respectively. Only the improved interpolation method was used for this example as the maximum interaction distance was 2.2 metres.

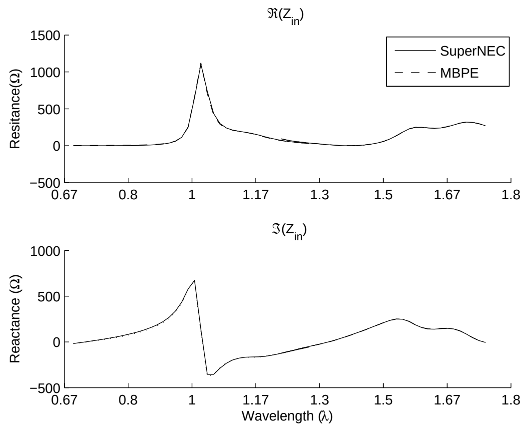


Fig. 10. Root square structure current error over simulation range for horn antenna

Fig. 10 show the input impedance of the antenna over both the frequency bands, the impedance matrix interpolation method reproduces the curve well over the entire frequency band. There is a discontinuity between bands however it is the same for the *SuperNEC* and MBPE cases, this is due to the different segmentation frequencies. Mean root square error for the first band is 18.9% and the second 19.8%. This level of error is higher than in previous examples due to the large number of segments in the structure and large interaction distances. The error however in the input impedance is relatively small in comparison due to the reduced effect the elements with large interaction distances have on the input impedance.

V. Improvement in computational efficiency

The computational time required to in the MoM is proportional to $N^2 f$ [29] where N is the probelm size and f the number of frequency values of interest. Whether the impedance matrix elements are calculated from direct computation or by evaluating a fitting function the simulation time will be proportional to this factor. Additional time which must be made up when using impedance matrix interpolation is in the sampling of the impedance matrices and solving fitting function coefficients. A break even point can be found, for small problems (less than 500 unknowns) 17 and 38 frequency points must be of interest for the standard and improved method respectively in order to obtain an increase in efficiency [30]. This is a guide a matrix filling time differs for different structure geometries. Solving of the matrix is performed with LU decomposition: an N^3 process [31], this term is expected to dominate simulation time are the problem size increases sufficiently. However problems sizes exceeding 5000 unknowns have been tested with the dominant time component remaining the matrix filling time.

Impedance matrix interpolation was applied to various wire and grid antenna structures, the simulation time performance is shown in Table I

TABLE I
Simulation time for antenna cases

			$[Z]$ fill time		Time Gain	
Antenna	N	f	Direct	MBPE	Direct/MBPE	(s)
Dipole	15	307	0.016	0.01	1.71	1.23
Yagi	50	290	0.047	0.031	2.16	6.85
LPDA	221	240	0.568	0.454	1.68	65.18
Horn	1289	46	24.43	13.5	1.71	359.9
Horn	3535	37	145	78	1.7	1372

VI. Discussion

Impedance matrix interpolation can be used to increase the computational efficiency of the method of moments. While simulation time improvements are made with small structures (less than 50 segments) the improvement in efficiency only becomes relevant with dealing with large structures over wide bandwidths. Broadband antennas such as the LDPA and horn antenna are well suited to the use of impedance matrix interpolation, however to obtain a wide enough bandwidth numerous sub-bands may be required resulting in discontinuities. This may be acceptable as MBPE is used in infancy of the design process where computational efficiency is traded for accuracy. In general it is possible to extrapolate to higher frequencies than is predicted by the method in Section III however not lower, this is especially true when input impedance is of concern. As input impedance is calculated on a single segment the effect of segments with large interaction distances have little effect on the solution. However when calculating radiation patterns the extrapolation range should not be exceeded as all the segment currents are used in the far field calculation. Simulation efficiency improvement of approximately 1.7 can generally be expected. When simulating electrically small structures the time again, in seconds, is not significant however when the problem size is large and many frequency points are of interest the use of impedance matrix interpolation results in a large reduction in simulation time.

A possible extention to the method is to treat each fitting function separately. The polynomial coefficients would be calculated from samples for the specific element before sampling the next element. This would reduce the additional storage required, as impedance matrix sample need not be stored. Another benefit of this method is that the sample range for all the fitting function samples need not be the same. The sample range could then be specified per element, depending on interaction distance and connection, as to achieve the most accurate fitting function.

VII. Conclusion

Two impedance matrix interpolation methods have been effectively implemented in *SuperNEC* as a form of MBPE. The standard method approximates all the elements in the matrix with quadratic functions directly and an improved method where the dominant frequency variation terms is removed prior to approximation. The impedance matrix is then filled from the simpler fitting functions this reducing the simulation time as the fitting function is evaluated with less computational expense than direct computation. Choice of correct sample points in important in obtaining an accurate fitting model, the sample range can be determined by the maximum interaction distance of the structure, the centre frequency and simulation range. Curves where developed to obtain a relationship between simulation range and sample range. Given correct sample and simulation ranges a mean root square error of less than 10% can be expected of the entire range and significantly less error when the input impedance if the only parameter of interest. To extend the simulation bandwidth the band should be split up into sub bands, thus reducing the problem size for each of the sub-bands by segmenting at a highest frequency of the band. The use of impedance matrix interpolation methods have been found to be 1.7 times faster than using direct *SuperNEC* computation providing enough frequency points are of interest to make up for the time used in sampling the matrix and solving fitting function coefficients.

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