THE USE OF GEOGEBRA IN ENHANCING GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY

Innocent Moyo and Judah P Makonye
University of the Witwatersrand
innoetho@gmail.com
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Abstract

This study explored the impact of the use of GeoGebra on Grade 10 learners' understanding of probability. The theory of semiotic mediation (TSM) was used to explain how the interaction between a class of Grade 10 learners and the GeoGebra artefact in a probability learning activity could lead to mathematical knowledge creation. A mixed methods design was adopted to collect data from 36 students through a pre-test, individual semi-structured interviews and a post-test. Findings indicate that using GeoGebra positively affected learners' understanding of probability. It particularly reduced item difficulty and error frequency while improving the incidence of correct methods in probability problem-solving and performance scores. Furthermore, using GeoGebra created an active learning environment where learners' errors and misconceptions were addressed. This study highlights a pedagogical framework for harnessing the semiotic affordances of GeoGebra to leverage the potential of errors and to support learners' understanding of probability.

Keywords: Probability; ICT; Semiotic Mediation; GeoGebra; Mathematics teaching; Mathematics learning.

Introduction

In high school, teachers and learners often perceive probability as a challenging and demanding topic (Department of Basic Education (DBE), 2019, 2020, 2021; Batanero, Chernoff, Engel, Lee, & Sánchez, 2016). A detailed question-by-question analysis of the National Senior Certificate (NSC) Mathematics Paper 1 examination reveals a contrasting performance in probability questions as compared to other topics. While average scores for topics like equations, inequalities, number patterns, finance and calculus exceeded 50%, the average score for probability questions was below 39% in 2014 and below 28% in 2015 (DBE, 2014, 2015). This pattern is not unique to South Africa. Globally, students' understanding and performance in statistics and probability are consistently low (Chiesi & Primi, 2010; Kazemi, Shahmohammadi & Sharei, 2013).

Probability is a challengimg topic to learn and to teach, not only due to its abstract concepts, but also due to the learners' intuitive and informal ideas that are brought to the classroom (Batanero & Diaz, 2012). Other reasons that are cited include inadequate knowledge base and misuse of representations (Mutara & Makonye, 2016), the irreversibility of probability experiments and the representativeness heuristic bias (Batanero et al., 2016).

Learners' performance in probability assessments is impacted by the errors they make and the misconceptions they have. These errors and misconceptions require careful analysis and targeted intervention. Misconceptions are beliefs, conceptions or knowledge that are inconsistent with accepted disciplinary interpretations. According to Keeley (2012), misconceptions need not be considered bad. However, teachers should use them to plan their teaching and learning activities. Literature reveals that errors indicate learners' reasoning patterns (Herholdt & Sapire, 2014). They may occur in probability when learners hold certain probability misconceptions such as equiprobability, outcome approach, compound approach (Jun & Pereira-Mendoza, 2002), representativeness heuristics bias, and outcome orientation (Khazanov & Prado, 2010).

Despite these challenges, probability is vital for everyday use, commercial enquiry and scientific enquiry, justifying its inclusion in the school mathematics curriculum. For this reason, the use of technology to enhance deeper understanding of mathematical concepts is receiving increasing attention worldwide. According to the National Council of Teachers of Mathematics (NCTM, 2000), classroom practices should embrace technological tools to promote student-centredness, critical thinking, collaboration, communication and problem-solving. The use of technology, particularly GeoGebra, can provide interactive and dynamic learning experiences for students to explore mathematical concepts in an engaging way. This requires the teacher's knowledge of both mathematics and the methodology with which to teach it.

Extensive research on teaching mathematics shows that technology-enhanced instruction positively impacts educational outcomes (Istenic Starčič, Cotic, Solomonides, & Volk, 2016). GeoGebra exemplifies an artefact that can be integrated into teaching to enhance students' understanding of mathematical concepts. The potential of GeoGebra lies in its computer algebra systems (CAS) and dynamic geometry software (DGS) functionalities, which provide a rich learning experience for students to discover mathematical relationships (Hohenwarter & Jones, 2007). According to Prodromou (2014), GeoGebra can generate huge quantities of data in a short time, allowing students to analyse the data for conceptual understanding. Studies also show that GeoGebra can be integrated into the curriculum to address specific learning outcomes. Prodromou (2014) examined how GeoGebra can be used to engage students in investigations, while Aizikovitsh-Udi and Radakovic (2012) explored how the use of GeoGebra can support high school students' understanding of probability. They found that the learning environment created by GeoGebra, through its interactive visualisations, enabled students to apply critical thinking and develop a deep conceptual understanding of probability. In another study of mathematics teachers' views on using GeoGebra, Kuzu (2021) found that GeoGebra can be useful for developing mathematical problem-solving skills.

Statement of the Problem

The persistent difficulty in understanding probability concepts and the resulting low performance scores in probability examinations highlight the need for effective instructional strategies. While many studies have explored the impact of Information Communication Technologies (ICTs) on learners' conceptual understanding, there is limited research specifically examining the effects of GeoGebra on specific learning outcomes, such as item difficulty and the use of methods in probability problem-solving. Additionally, in the South African context, the use of technology in classrooms is still limited due to various barriers, including inadequate ICT resources (Graham, Stols, & Kapp, 2020), a lack of necessary ICT skills, and insufficient technological pedagogical teacher training (Mishra & Koehler, 2006; Koehler & Mishra, 2009; Padayachee, 2017). This study investigates how the use of GeoGebra, a dynamic mathematics software, can enhance Grade 10 learners' understanding of probability. It aims to explore the impact of GeoGebra on Grade 10 learners' understanding of probability, specifically focusing on identifying and addressing common errors and misconceptions. The study proposes a pedagogical approach that leverages the affordances of GeoGebra to enhance students' problem-solving skills and conceptual understanding in probability.

The Aim and Objectives of the Study

The overarching aim was to explore how using GeoGebra as an alternative ICT intervention tool could impact Grade 10 learners' understanding of probability. The study had three objectives:

- To identify errors and misconceptions that Grade 10 learners make when modelling and solving probability problems.
- To explore how using GeoGebra can support the learning of probability concepts.
- To explore how GeoGebra can be used to address Grade 10 learners' errors and misconceptions in probability problem-solving.

The Theoretical Framework

This study adopted the theory of semiotic mediation (TSM) (Vygotsky, 1978) to highlight the interaction between the learner and the GeoGebra artefact in the activity of learning probability. According to Vygotsky (1978), knowledge is constructed when learners interact with other people using mediating cultural tools (e.g., computers) and psychological tools (e.g., signs, language, and symbols) to evoke specific mathematical knowledge (Mariotti & Maffia, 2018). The potential of an artefact is exploited through the orchestrating role of the teacher and realised when learners complete mathematical tasks using the artefact.

This study considered the key TSM constructs that support learning when organising learning activities and assigning learner and teacher roles. First, Vygotsky argued that learning occurs through social sources of individual thinking (Kalina & Powell, 2009). This implies that individuals learn through interaction in a sociocultural setting using available tools. Second, learning occurs at the zone of proximal development (ZPD), implying that a child will learn mathematical concepts with the necessary help in the classroom (Kalina & Powell, 2009). Third, language is used as a cultural tool to construct meaning. As learners use language to accomplish a task, how they think about the activity is shaped, and signs begin to emerge, which develop into acceptable mathematical meanings.

Materials and Methods

Research Design

This study adopted a mixed methods approach within an interpretive paradigm. Quantitative and qualitative data strands were mixed for purposes of triangulation and complementarity (Creswell & Plano Clark, 2011). This helped offset each strand's weaknesses and enhanced the research design. The research methods were grounded within the interpretive paradigm to “yield insight and understandings of behaviour and explain actions from the participant’s perspective” (Scotland, 2012, p.12).

Research Setting and Sample

The research site was a co-educational high school in Gauteng (South Africa), with enrolled learners from Grades 8 to 12. Participants were conveniently recruited from Grade 10 learners in the Further Education and Training (FET) phase. The grade was convenient since the probability concepts that were studied are introduced in Grade 10. Two mixed-ability classes were assigned to the control group (n = 22) and the treatment group (n = 14). The groups were well-matched in terms of ability, as evidenced by their average scores in their Grade 9 results. This minimised pre-existing differences, which could jeopardise the reliability of the results.

Data Collection

Qualitative data were collected through semi-structured individual interviews, while a pre-test and post-test collected quantitative data. In the first phase, both classes were taught probability for three weeks in February/March before writing a pre-test in July. The pre-test was quantitatively analysed using error analysis which focused on item difficulty, using correct methodologies and the most commonly occurring errors (Herholdt & Sapire, 2014). In the second phase, interviews were conducted with learners from whom permission was obtained to better understand their pre-test responses. The third phase commenced with four intervention lessons taught three weeks before the post-test in November.

GeoGebra intervention was given to the treatment group, while the control group was taught without using ICT. The researcher planned all the lessons and discussed them with a colleague who taught the control group to ensure uniformity and adherence to the teaching standards set by the school. The colleague was a qualified teacher with more than ten years of teaching experience and was also the head of mathematics at the school. Both groups also wrote a post-test which was quantitatively analysed using error analysis. In the final phase, five participants from the treatment group were interviewed to solicit their perspectives and experiences of learning probability using GeoGebra. All interviews were audio-recorded and transcribed verbatim by the researcher.

GeoGebra Activities

Probability models were constructed through GeoGebra simulations of tossing coins and rolling dice. The simulations developed by Sturr (2014a, 2014b) and Lindenmuth (2016) were accessed by learners through their respective URLs. The activities were designed to assist learners 1) in comparing the relative frequency with theoretical probability, 2) in drawing Venn diagrams and 3) to conceptualise the rule $P(A\,or\,B) = P(A) + P(B) - P(A\,and\,B)$. GeoGebra's interactive CAS and DGS functionalities provided feedback in the form of numerical data and animated visualisations. These enabled learners to find patterns in the data to answer posed questions.

Data Analysis

Quantitative data were analysed using error analysis. The pre-test's error analysis results were compared with those of the post-test to determine the impact of GeoGebra intervention. Paired Samples t Test statistics were calculated to explain the impact of GeoGebra intervention on the prevalence of learner errors, levels of difficulty of question items and use of methods and strategies in probability problem-solving (Herholdt and Sapire, 2014). To address the different group sizes between the control and treatment groups, the researchers used proportion/rates to report the results. For example, item difficulty was obtained by calculating the proportion of learners who had the item correct using the following formula: $difficutly\,index\,(d) = \frac{number\,of\,correct\,answers}{number\,of\,students}$. An item was considered difficult if d ≤ 0.5, moderate if 0.51 ≤ d ≤ 0.84 and easy if d ≥ 0.85. Similarly, the average rate of error occurrence and prevalence of the use of correct methods between the pre-test and post-test for each group were compared to determine the impact of GeoGebra use. The Difference-in-Differences (DD) estimate was also used to measure the impact of the GeoGebra intervention. Goodman-Bacon (2021, p. 1) describes DD as "the difference between the change in outcomes before and after a treatment in a treatment versus a control group."

Interview data were qualitatively analysed using coding to identify underlying themes in the dataset (Leech & Onwuegbuzie, 2007). According to Saldaña (2009), coding is an interpretive act which requires the researcher's ability to summarise what the participant says using descriptive words. As suggested by Leech and Onwuegbuzie (2007), transcripts of interview data were read and divided into meaningful parts, which were labelled using the same code. The themes that emerged from these codes were identified and used to answer the research questions.

Ethical Considerations

The British Educational Research Association (2011) offers guidelines regarding the voluntary informed consent of participants, the guarantee of anonymity, openness and disclosure, the right to withdraw and many other issues. These guidelines were followed to ensure that the research met all the standards of ethics. Permission was sought from the research site to conduct the study. The purpose of the research was discussed with the school authorities and the participants, assuring them that participation in the study would not compromise the quality of their learning. This helped to ensure that participants made informed decisions. Permission from the participants' parents to participate in the research was also sought. Participation was voluntary, and participants were assured that any data collected from them would be kept anonymous and confidential. Participants were also assured that they could withdraw from the research at any stage, and the researchers guaranteed their safety and comfort during data collection.

Results

The error analysis framework proposed by Herholdt and Sapire (2014) was adopted to analyse the pre-test and post-test. The results were used to determine the impact of GeoGebra intervention on item difficulty, the use of correct methods, the incidence of errors in learners' work and performance scores.

Item Difficulty Indices

Table 1 shows that item difficulty indices for all questions increased for the treatment group, suggesting that participants found the post-test questions easier than the pre-test questions. An independent sample *t*-Test showed that the reduction in the difficulty levels of questions for the treatment group was statistically significant ($t(3) = 2.644$, $p = .039$, $d = 1.322$). The mean item difficulty index for the post-test was .42 while the mean item difficulty index for the pre-test was .16. The mean item difficulty index for the control group decreased from .43 in the pre-test to .33 in the post-test, indicating that the group found the post-test more difficult than the pre-test.

Table 1

Pre-test and Post-test difficulty indices (d) for the treatment and control groups

Question	Control Group (n = 22)		Treatment group (n = 14)	
	Pre-test difficulty indices	Post-test difficulty indices	Pre-test difficulty indices	Post-test difficulty indices
1	.00	.27	.07	.50
2	.18	.025	.21	.25
3	.73	.68	.21	.64
4	.81	.36	.14	.29
Mean	.43	.33	.16	.42

Note: Questions were easy if $d \geq .85$, moderate if $.51 \leq d \leq .84$ and difficult if $d \leq .50$

It was unclear what the average item difficulty would look like for the control group if they had also received GeoGebra intervention. However, the Difference-in-Differences (DD) estimate of .36 between the two groups showed that GeoGebra intervention provided a positive treatment effect in item difficulty for the experimental group.

Use of Strategies/Methods

Some test questions required the use of a strategy or method. A Venn diagram strategy was considered correct if the data of the question, namely elements, number of elements or probabilities, were shown in their correct regions of the Venn diagram. An analysis of students' use of methods revealed four categories of results. Table 2 shows the category of results.

Table 2

Category of results showing the use of Venn diagram strategies

Category	Results
A	A correct Venn diagram was drawn and correctly interpreted, leading to a correct answer.
B	A correct Venn diagram was drawn and incorrectly interpreted, leading to a wrong answer.
C	An incorrect Venn diagram was drawn and correctly interpreted to get some answer.
D	An incorrect Venn diagram was drawn and incorrectly interpreted to get some answer.

Table 3 shows the percentage of cases that were found under each category.

Table 3

Number of cases found in each category of students' use of methods in the pre-test and post-test

Test	Group	Category A		Category B		Category C		Category D	
		No. of Cases	%	No. of Cases	%	No. of Cases	%	No. of Cases	%
Pretest	Treatment	5	36	9	64	3	21	11	79
PostTest	Treatment	10	71	4	29	0	0	14	100
PreTest	Control	8	36	7	32	4	18	25	114
PostTest	Control	9	41	10	45	1	5	5	23

Treatment group (n = 14). Control group (n = 22).
(Pretest = 2 items; Posttest = 2 items).
Number of cases = total number of times all students used a correct method.

Results indicate that the use of GeoGebra effectively improved the proportion of the use of correct methods leading to the correct answer (Category A). The frequency of Category A methods for the treatment group improved from 36% in the pre-test to 71% in the post-test, while the frequency of Category B methods dropped from 64% in the pre-test to 29% in the post-test. On the other hand, the frequency of Category A for the control group increased from 36% in the pre-test to 41% in the post-test, while Category B methods increased from 32% in the pre-test to 45% in the post-test.

Incidence of Errors in Learners' Work

Learners made factual, procedural and conceptual errors in the written tests. Despite the intervention, learner errors persisted in the post-test for both groups. Table 4 shows that errors for the treatment group decreased from 35.2% in the pre-test to 33.6%. The incidence of errors for the control group increased from 64.8% in the pre-test to 66.4% in the post-test. The results indicate that GeoGebra use had no negative impact on the incidence of errors in students' written work.

Table 4
Total number of errors made by both groups in the pre-test and post-test

	Number of errors in the pre-test		Number of errors in the post-test		Total number of errors	
	n	%	n	%	n	%
Treatment Group (n =14)	58	35.2	44	33.6	102	34.5
Control Group (n = 22)	107	64.8	87	66.4	194	65.5
Total	165	100	131	100	296	100

Performance Success Rate

The mean scores in the pre-test and post-test were calculated to determine learners' success rates. The average success rate for the treatment group in the pre-test and the post-test were 49% and 52%, respectively, while the respective rates for the control group were 45% and 44%. Table 5 provides a summary of the success rate of learners in the pre-test and post-test. The improvement in scores for the treatment group was not statistically significant (Paired Samples *t* Test, $t(13) = .372$, $p = .716$, $d = .099$). However, the results showed using GeoGebra had no negative impact on scores.

Table 5
Success rate of learners in the pre-test and post-test

	Success rate in the pre-test	Success rate in the post-test	Difference
	%	%	%
Treatment Group (n =14)	49	52	+3
Control Group (n = 22)	45	44	-1
Difference	+4	+8	+4

Students' Challenges when Solving Probability Problems

Analysis of pre-test responses identified the following challenges: 1) students had difficulty in drawing or interpreting Venn diagrams to solve problems, 2) students confused probability concepts, 3) students failed to apply probability rules correctly in context (e.g., some students used $P(A \cup B) = P(A) + P(B)$ when A and B were not mutually exclusive), and 4) some students found probability content difficult. GeoGebra intervention was aimed at remediating these challenges. An analysis of interview data led to the discovery of themes which summarised the potential of GeoGebra in addressing some of these challenges. The results are reported below.

Learning Barriers Due to Language/Communicative Registers

Some students had difficulty understanding certain probability terms (e.g., bias). This forced them to skip some questions or to give incorrect solutions. Students admitted in the interviews that their learning difficulties were caused by the difficult nature of the language used in probability problems. As a result, the misunderstanding of words led to reduced levels of motivation to learn probability. GeoGebra remediation helped boost learners' confidence. Some students' interview responses revealed a positive gain in understanding due to using GeoGebra. One student stated: "Yes, relative frequency. I didn't understand it initially, but you made it clear for me to understand it."

GeoGebra's Potential for the Development of Mathematical Competencies

Interview responses showed that learners perceived GeoGebra usage to be effective in providing opportunities for developing necessary competencies for overcoming challenges in probability. First, students pointed out that using GeoGebra gave them more time to analyse raw data, thus providing them with opportunities to develop their mathematical thinking and reasoning. One learner stated: "It [GeoGebra] definitely gave us a lot of time to work with the numbers. It gave us time to work with unrealistically high numbers. It's a tool that makes us do things that would otherwise be very impossible." In order to build probability models, raw data is needed. However, much time would be needed to get raw data using physical artefacts such as coins and dice. Since GeoGebra simulations efficiently provided the raw data, learners spent less time on this aspect. The available time was then used to make sense of the data, thus providing opportunities for learners to develop their mathematical thinking and reasoning. Second, interview responses showed that students perceived GeoGebra as a useful digital platform for conceptual understanding and probability knowledge acquisition. One student stated: "For me, the lessons opened me up, … it [GeoGebra] lit a light bulb." This indicates that GeoGebra's use enhanced learners' cognitive processes and led to a better understanding of probability and its related concepts. Third, the artefact allowed learners to exchange ideas based on individually held opinions.

For example, when a die was tossed, the number of heads and tails obtained were displayed both numerically and visually as animated bar graphs. Initially, after a small number (n) of tosses (e.g., n = 20, n = 80), there was no clear pattern to conclude how the outcomes (head or tail) would look. Some learners argued that heads would outnumber tails. However, they changed their mind after observing the outcomes after many tosses (e.g., n = 6739). Thus, some preconceived ideas that were held were either refuted or validated through the help of GeoGebra.

GeoGebra's Potential to Enhance Learners' Problem-Solving Strategies

Interview data attested to the potentiality of GeoGebra to develop problem-solving strategies. Some questions in the pre-test and post-test were answered using Venn diagrams or probability rules. Raw data were obtained using GeoGebra simulations and represented in different modes such as Venn diagrams or outcome tables. This is consistent with Lesh's translation model (Suh & Moyer, 2007), which argues that deeper mathematical understanding is developed when students are able to translate information into different forms. For example, when two dice were rolled, Events A and B were defined as A = {sum of numbers on the dice is 7} and B = {at least one of the dice shows a 2}. Learners used a GeoGebra simulation of rolling two dice to generate several possible sample spaces for the experiment. The outcomes were then represented in outcomes tables or Venn diagrams. The potential for GeoGebra to probe students' understanding of probability methods was evident in the questions students asked. One student asked, "When we draw the Venn diagram, do we have to put all the other outcomes that don't fit in A and/or B outside the circles?"

GeoGebra's Potential to Support Active Learning

Students were observed working through planned GeoGebra activities in class. The technology stimulated their active engagement and interest. One learner described their experience as fun, and another linked the experience to that of playing a game. GeoGebra lessons also initiated valuable conversations among students. Thus, using GeoGebra supported learner interaction, leading to information sharing and collaboration.

Discussion

The study explored the impact of the use of GeoGebra on Grade 10 learners' understanding of probability. The findings indicate that the use of GeoGebra had a positive effect on probability problem-solving. The impact was evident in item difficulty, performance scores, use of strategies/methods and occurrence of errors in students' work. The results also show that opportunities for developing probability thinking and reasoning were provided through active learning.

The above findings are consistent with results from studies by Aizikovitsh-Udi and Radakovic (2012), Kuzu (2021), Shadaan and Leong (2013), Saha, Ayub and Tarmizi (2010), and Zulnaidi and Zakaria (2012). They found that the use of GeoGebra was effective in achieving specific learning outcomes in mathematics. In their investigation of students' understanding of circles using GeoGebra, Shadaan and Leong (2013) found that GeoGebra effectively enhanced performance. In another study, Saha et al. (2010) found that the experimental group's use of GeoGebra in learning coordinate geometry resulted in improved mathematical achievement. Zulnaidi and Zakaria (2012) determined the effect of GeoGebra on conceptual and procedural knowledge of functions. Their study found that participants who received GeoGebra treatment gained more knowledge than those who did not. This study's findings provide a pedagogical framework which exploits the potential of GeoGebra to support probability learning. In the era when technological solutions are needed, the results go a long way in finding answers around the development of mathematical skills that learners need in order to participate in a global community (DBE, 2011).

The probability topic is among many others examined in the Further Education and Training (FET) phase. Studies confirm that students perform differently in these topics depending on the extent to which they experience difficulty with each of them (e.g., Jacobs, Mhakure, Fray, Holtman & Julie, 2014). This study found that the use of GeoGebra had a positive effect on item difficulty and performance. This is significant considering that, according to the FET subject assessment guidelines (DBE, 2011), examination questions are posed to test cognition at various levels. Reduced difficulty levels suggest moderate and difficult questions were accessible to more students due to GeoGebra use. This is in line with the suggestion of Jacobs et al. (2014) that mathematics teaching should focus on complex questions in preparing students for examinations. The GeoGebra CAS and DGS functionality, which simultaneously yields numerical and visual feedback, offers an opportunity for learners to develop conceptual understanding, a requisite to coping with higher-order questions.

This study also found that students made factual, procedural and conceptual errors in their written work, even after the GeoGebra intervention. However, the prevalence of these errors decreased in the post-test for the treatment group. Brodie (2014) argues that errors are conceptual structures constructed when students make sense of the world around them. For this reason, errors are not easy to eradicate since they become "truths" in the mind of the learner. For this reason, errors in probability should be expected in students' work, regardless of teaching or even re-teaching. Misconceptions about some aspects of probability caused errors in this study, such as the confusion between $A \cup B$ and $A \cap B$. The researchers argue that errors should be viewed as opportunities for learning in which students can be guided through planned didactical activities to restructure their misconceptions and fit them in their current cognitive domain.

The use of an ICT artefact such as GeoGebra can be a valuable intervention to identify improved results.

The interactive functionality of the GeoGebra tool created an active learning environment which enabled students to apply critical thinking. Thus, GeoGebra allowed students to develop their conceptual understanding of probability. This was evidenced by a significant improvement in the treatment group's use of correct strategies and methods. The above findings are consistent with findings from previous studies. Aizikovitsh-Udi and Radakovic (2012) and Kuzu (2021) found that using GeoGebra helped students develop problem-solving skills and improve their conceptual understanding. Some questions in the pre-test and post-test modelled real-life situations, which could be represented in various forms such as Venn diagrams and probability rules. According to Suh and Moyer (2007), representing information in different modes can enhance conceptual understanding. Learners used numerical and visual data produced through GeoGebra to create possibility spaces and Venn diagrams.

Necessarily, effective teaching of mathematics should provide learners with opportunities to acquire problem-solving skills and conceptual understanding. A pedagogical framework (Figure 1) for ICT mediation in resolving learner misconceptions in probability is recommended based on the findings of this study. It comprises four components which interact to support probability learning using GeoGebra, namely diagnostic teaching and feedback, learner and teacher decisions, probability content and knowledge of GeoGebra affordances.

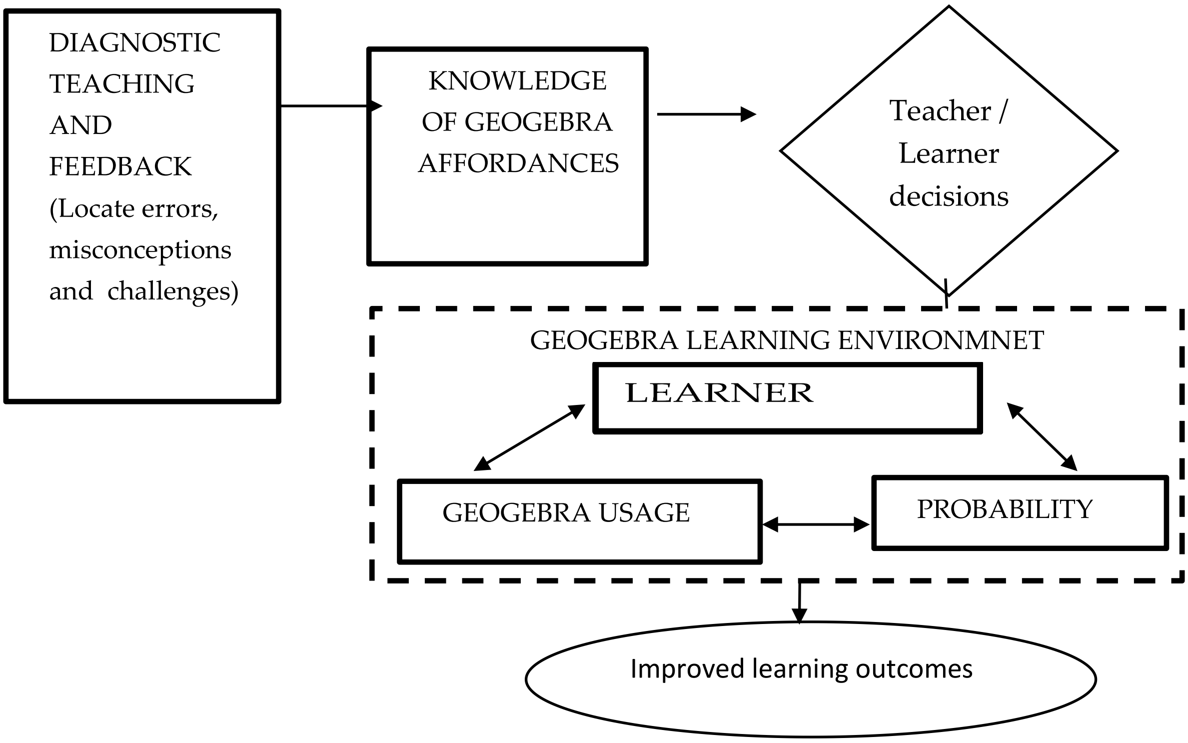

Figure 1: A pedagogical framework for teaching probability with GeoGebra

Diagnostic Teaching and Feedback: Diagnostic teaching and testing are essential to locate existing knowledge gaps. For Bray (2013), some teachers may not communicate learner errors openly for fear of embarrassing the learner. However, an open analysis of students' errors can promote conceptual understanding. This is supported by Makonye and Khanyile (2015), who recommend that teachers probe learners on errors in their work to overcome them.

Learner and teacher decisions: Piaget (1952) argues that mental disequilibrium occurs when individuals encounter new information. It causes them to make some adjustments in their thinking to resolve the conflict. The researchers opine that students who are aware of their knowledge gaps should feel the urge to take appropriate action to fix the problem. Thus, both the teacher and the student have important decisions to make in order to address existing challenges. Awareness of learning gaps and the knowledge of GeoGebra affordances to address those gaps can potentially motivate both the teacher and the learner to intentionally use the tool.

Probability content/knowledge: The researchers argue that probability content should be learnt through discovery. This requires that the selected learning activities should not be too easy. Otherwise, learners will get bored or find it difficult to avoid anxiety and unnecessary perceived difficulty.

GeoGebra learning environment: GeoGebra integration in probability teaching should be an intentional decision by the teacher and the learner. This decision should be driven by the teacher's pedagogical content knowledge and understanding of how GeoGebra supports knowledge creation. The constructivist theory of semiotic mediation (TSM) (Vygotsky, 1978), which underpins this study, provides useful guidelines for organising GeoGebra-supported intervention.

Implications and recommendations

This study provided a foundation for mathematics teachers to rethink their classroom practice. The CAPS curriculum is clear about exposing learners to "mathematical practices that give them many opportunities to develop their mathematical reasoning and creative skills" (DBE, 2011, p.10). These skills are essential for a successful transition into the technology-driven global economy. Based on the findings, the following recommendations are made:

1. Curriculum planners should consider revising the number of topics taught to create sufficient time for technology integration. Currently, teachers' attempts to deliver technology-based instruction are frustrated by the demands of the curriculum. Teachers are expected to complete a lengthy syllabus in a short space of time. This leaves them with little choice but to adopt poor teaching strategies.

2. Educators should rethink teaching and assessment to emphasise mathematical understanding over and above performance scores. This study argues that integrated technology in a student-centred learning environment is needed to teach well. Teachers will be encouraged to integrate technology into their teaching by placing more emphasis on mathematical understanding.

Conclusion

This study aimed to explore the impact of the use of GeoGebra on Grade 10 learners' understanding of probability. Understanding how probability can be organised to support technology integration is critical for effective mathematics teaching. GeoGebra integration in probability teaching potentially creates a student-centred environment which supports active learning. When students use GeoGebra to model probability situations, they engage in discussions and information-sharing, which results in a deeper conceptual understanding of the object of learning. The researchers argue that a pedagogical framework for teaching probability using GeoGebra should clarify the interacting roles of the learner, the teacher, the object of learning (probability content), and the mediating artefact (GeoGebra). On its own, GeoGebra, like any other ICT tool, cannot bring about any change. Therefore, a deliberate decision to use it to teach and learn is a critical starting point for harnessing its potential.

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