

Figure 6.10 Results of a DIGS simulation near a stope abutment. Quarter symmetry, friction angle across bedding planes is 30 degrees

These results are qualitatively similar to the results from the FLAC simulations which have been described in Section 6.2.2 and can therefore be used with more confidence for the simulation of fracture growth. Figure 6.11 shows the results of such a simulation in which tensile fracturing has been reproduced. Fracture initiation points (seeds) have been placed at the bedding planes, as the potential for tensile stresses is the highest there. Although this is not clear from the contours in these Figures, the locations of shear slip along the bedding planes are associated with infinitely high stress gradients across the discontinuities with a tensile component on the one and a compressive component on the other side of these discontinuities. The contouring program cannot cope with these high stress gradients and does therefore not correctly represent the situation.

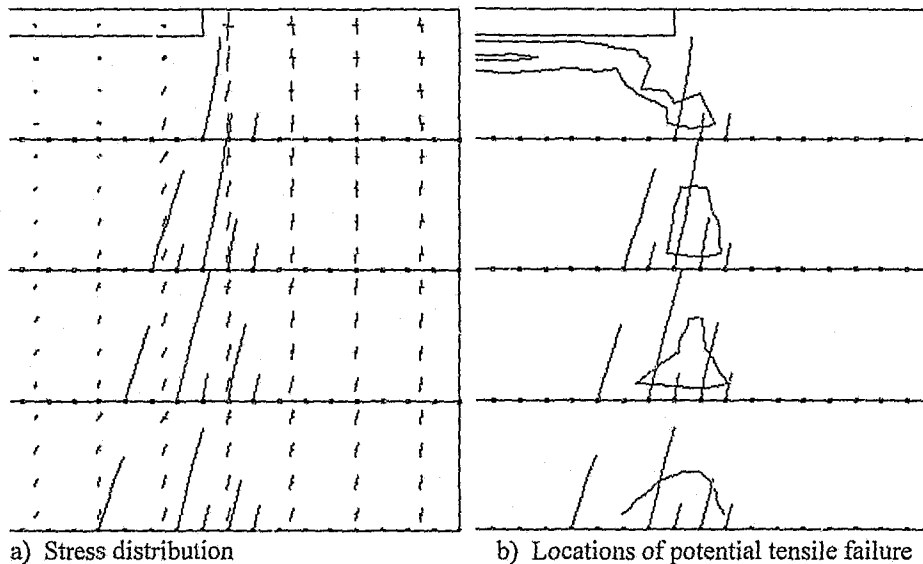


Figure 6.11 (tensile) Fracture formation due to the stress field induced by a stope in a layered medium as in the previous example

Figure 6.11 shows a typical result of fracturing induced around a highly stressed stope abutment in a rock mass in which bedding planes are present. Similar results have been reported by Napier and Hildyard (1992) who demonstrated that bedding planes can encourage the nucleation of extension fractures near the abutments of a stope. They also showed that the effective stoping width can be increased by these fractures, if such fractures are intersected by the excavation upon further mining and in such a way assist in the creation of detachable blocks. Figure 6.12 shows an alternative possibility for the increase of the effective stoping width. In this case the tensile fractures, which have been generated from bedding planes at various levels, are artificially joined by allowing the narrow regions which separates them to crush. The idea is to reproduce fracture coalescence in a very simple fashion, assuming that such coalescence can occur under these conditions.

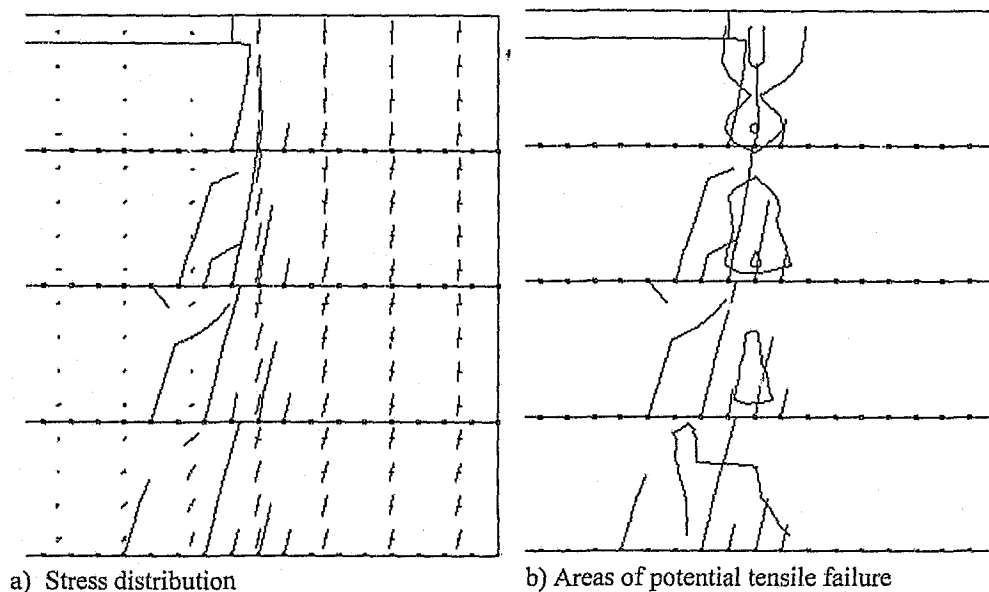


Figure 6.12 Effect of induced fracture coalescence. The narrow zone between those fractures, which completely split individual layers, is allowed to “crush”

Figure 6.12 shows how the presence of fractures can lead to the effective widening of an excavation as the hanging- and footwall regions become de-stressed. De-stressing has been achieved by artificially joining those fractures which intersect individual layers completely, if the distance between such fractures is relatively small (Figure 6.12). This may be of relevance to failure and stability around mining excavations. Another practical application of this mechanism is a scenario in which symmetry conditions are not applicable and the effective widening of the excavation may take place in either the hangingwall or the footwall both not in both. In such a situation it may be found that the excavation is not in the centre of the “effective” opening, but may be located at either the top or the bottom, depending on the geological situation. This mechanism may therefore be of assistance in explaining the change of fracture patterns in various geological areas.

Bedding planes are obviously not the only geological structures which can affect stress distributions and subsequent fracturing. Faults and joints are also likely candidates and should be considered when any failure or fracture analysis is undertaken. Such structures are most likely of more relevance to the failure of a rock mass than the strength of the rock (UCS) itself, as they present weaknesses which will be preferentially employed for the absorption of inelastic deformations and associated stress relieve. Figure 6.13 shows the effect of an inclined joint set on the generation of fractures around an opening as another example of the influence of pre-existing (geological) structures.

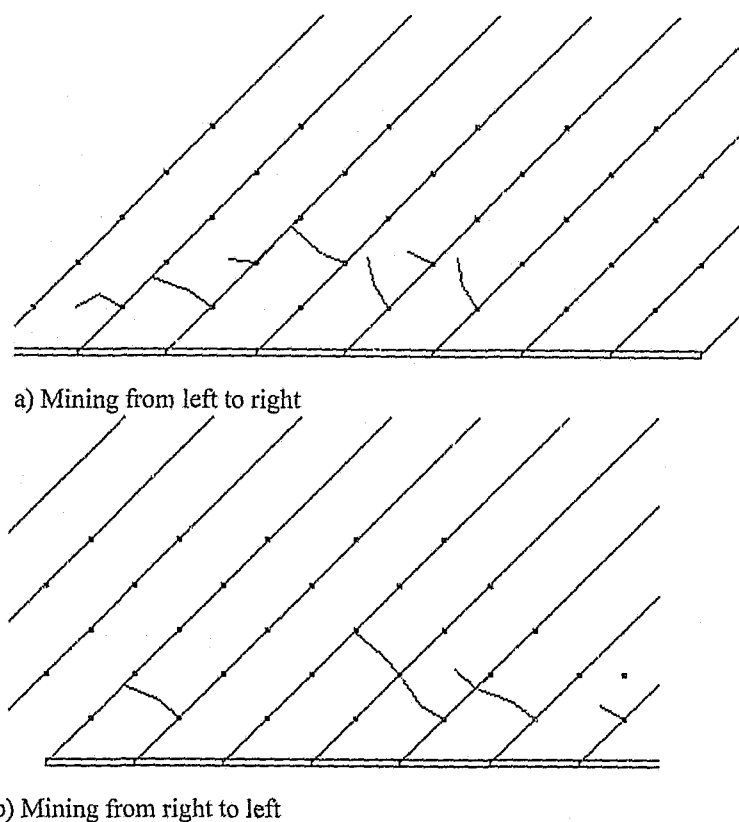


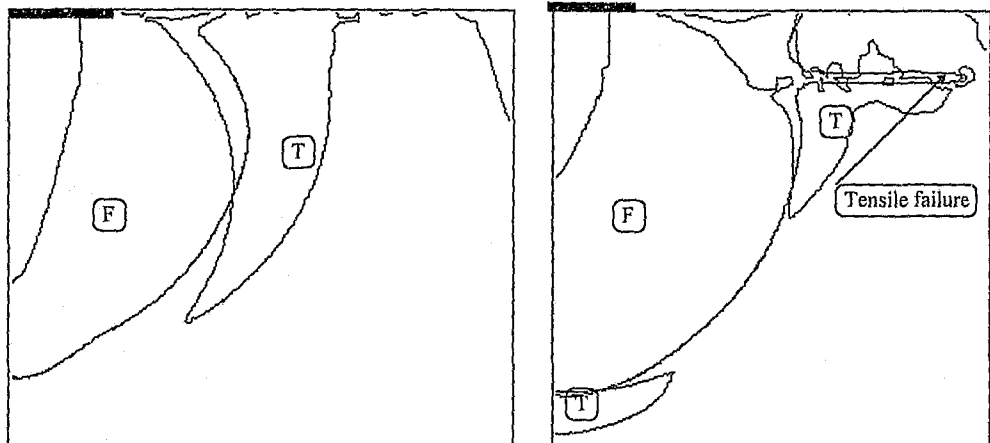
Figure 6.13 Effect of a joint set on mining induced (extension) fracturing

It can be observed from Figure 6.13 that the fracture orientations and locations are mainly determined by the position of the joints. Even the direction of mining (excavating) does not have a substantial influence on the mechanism. This strongly suggests that the presence of such structures can have a major effect on the mining induced fracturing similar to the effect of bedding planes. As these effects are purposely excluded in most laboratory tests, it cannot be expected that the results of such laboratory tests are representative of underground situations. In order to understand the occurrence of fracturing and failure in a realistic mining environment, geological structures need to be identified and appreciated as primary sources of failure and fracture generation. The orientation of the field stresses could also have an important influence on fracture formation as has been reported by Seliers (1995).

6.3 PUNCHING

The subject of punching is quite elaborate and will not be addressed in detail in this thesis. However, the concepts of plastic and brittle failure, as discussed previously, may contribute to the understanding of the mechanism involved. Figure 6.14 shows the result of a FLAC model in which the left hand boundary is used as an axis of symmetry. A perfectly plastic material has been assumed and a non-associative flow rule has been employed. The boundary conditions which have been used for this model are as follows:

- Vertical boundaries are subjected to a horizontal stress of 10MPa
- The bottom surface is not allowed to deform in the vertical direction
- The top surface is free except for the area underneath the punch
- The punch area is subjected to uniform vertical displacements



a) Contours of plastic failure (F) and tensile minor principal stresses (T) at an initial stage of loading

b) Contours of plastic failure (F) and tensile minor principal stresses at a later stage

Figure 6.14 FLAC simulation of punching in a perfectly plastic material. Half symmetry; angle of internal friction = 35° ; dilation angle = 35° ; cohesion = 10MPa

From Figure 6.14 it can be observed that intensifying plastic deformations lead to increasing tensile stresses which ultimately appear directly underneath the punch, below the area (F) which is plastically deformed. Tensile failure is already initiated at another location as shown in Figure 6.14b. Localisation of tensile failure is not properly simulated in FLAC due to mesh sensitivity, as has been demonstrated in section 5.3.3.1. The tensile failure in Figure 6.14b may for this reason not represent a realistic tensile fracture.

In order to allow for the representation of tensile fracturing, the results of this FLAC simulation have been used to construct a model in DIGS in such a way that the plastic deformations are captured to a certain extent. The results of that model are shown in Figure 6.15 where it can be seen how similar boundary conditions as in the FLAC model lead to a

fracture pattern which consists mainly of near vertical fractures underneath the intersecting discontinuities which form a wedge. This wedge is supposed to represent the plastic region in the FLAC simulation (Figure 6.14).

Discontinuities have been placed in an incremental, stepwise fashion, along the surface of the future wedge, initiating from the boundary. Tensile fracturing was allowed to take place after each increment of this wedge formation, until both sides of the wedge intersected and the wedge was completed. The fracture pattern in Figure 6.15 resulted from that process. The load on the surface, representing the punch, has been maintained constant during the incremental emplacement of the discontinuities forming the wedge.

Simulation of punching, using the tessellation scheme, has been reported by Napier and Dede (1997) and their results also indicated that the formation of the wedge structure was best represented by plastic (non softening) behaviour, while the formation of the vertical fractures could be viewed as tensile, brittle failure. They also reported that incremental fracture simulation, as has been used in all the numerical models in this thesis, leads to different results as a simulation in which fractures are allowed to grow simultaneously (so called parallel growth). In the first case of incremental fracture growth, the formation of an individual fracture influences the stress field, and in such a way affects the formation of the next one, etc. In the second case of parallel fracture growth such an interaction does not take place and the resulting fracture pattern can be different in both cases. It can be argued that incremental growth is more suitable for stable fracture growth processes, while parallel growth allows for a better representation of violent, dynamic fracture formation.

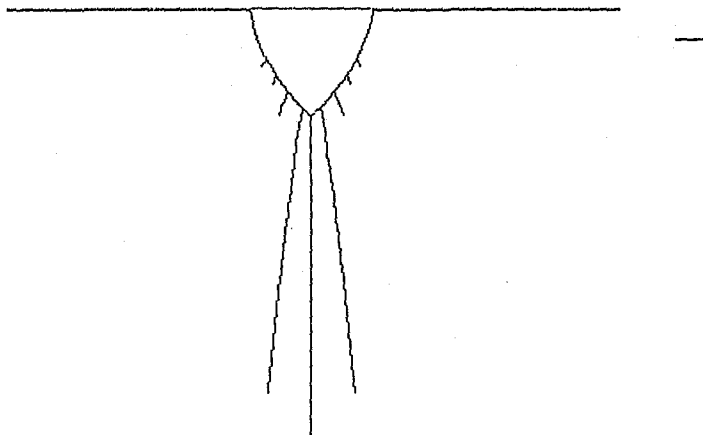


Figure 6.15 DIGS simulation of punch failure in an elastic material

While the boundary conditions which have been selected for both the FLAC and the DIGS model were identical, a noticeable difference between the results of these models could be observed. The tensile region, indicated in Figure 6.14a could not be reproduced with the DIGS simulations. This appears to point to a difference between the behaviour of a plastic zone as represented in FLAC and a discrete discontinuity as represented in DIGS. While it is not immediately obvious to what mechanism(s) this difference can be attributed, it is of importance to realise that the representation of a plastic zone by a discontinuity may not be entirely appropriate. The results as shown in Figure 6.15 could therefore be affected if a more realistic representation of the plastic zone could be formulated. The results do nevertheless indicate how the presence of such a plastic zone could lead to the formation of tensile, brittle fractures.

6.4 EFFECT OF LOADING HISTORY UNDER INELASTIC CONDITIONS.

The loading path, or history, has no influence on results under elastic conditions, as the so-called principle of superposition will be applicable in such a case. Irrespective of how a particular loading condition and geometry is reached, the response of the medium to that loading condition will always be the same. This principle is, however, not valid under inelastic conditions where the potential for irreversible deformations and stresses exists. During deep level mining operations, where the strength of a rockmass is exceeded and inelastic deformations are induced near the highly stressed stope faces, the rockmass around the excavations typically experiences loading and unloading conditions when the mining excavation is advanced. Inelastic deformations which are generated during loading may not be totally reversible upon unloading and the effects of such 'locked-in' deformations on the overall deformations and stresses are analysed in this section.

Salomon (1968) analysed the stress distributions around a slit in an elastic medium. The horizontal stress which is induced over and above the primitive horizontal stress along the centre line of the slit is given by:

$$\sigma_{xx}(0, y) = \left\{ -1.0 + \frac{y(2l^2 + y^2)}{(l^2 + y^2)^{1.5}} \right\} P_y \quad 6.6$$

where l = half span of the panel,
 y = vertical distance from the panel
 and P_y = primitive vertical stress

Equation 6.6 predicts induced horizontal stresses along the skin of a panel, which are directly proportional to the depth of the excavation and which are tensile. The total horizontal stress will be affected by the primitive horizontal stress as well, but remains tensile as long as the primitive horizontal stress does not exceed the primitive vertical stress. If the (K)-ratio of primitive horizontal to vertical stress is assumed to be 0.5, resulting tensile tangential stresses which are equal in magnitude to the primitive (compressive) stress are predicted from this elasticity theory. Since these stresses can exceed the tensile strength of the rock, it would be expected that large extension fractures would open normal to stope hanging- and footwalls. Although such behaviour may take

place at relatively shallow mining depths, it is not observed in deep level mining, where it is evident that horizontal stresses are compressive.

The explanation for the presence of horizontal compressive stresses is at present based on a number of conjectured hypotheses. These have largely been based on detailed observation of the fracture patterns that form in the vicinity of an advancing stope face. (Adams et al., 1981; Brummer and Rorke, 1984; Legge, 1984) It is basically inferred that slip on steeply dipping, mining induced fractures must be accompanied by dilation. It is postulated that this, in turn, is transmitted to the layers of rock formed by reef parallel parting planes. In order to test this hypothesis and to investigate potential alternative explanations, the numerical simulations which are presented in this section have been analysed.

In "The effect of loading history on stress generation due to inelastic deformations around deep-level tabular stopes" (Kuijpers and Napier, 1991), a potential mechanism for the locking in of inelastic deformations under mining conditions is discussed. The same mechanism is active in the simulations which are shown in this section and can basically be described as 'ratchetting'. Inelastic shear deformations which are induced during loading are not completely reversed when the affected area is undermined, or unloaded. As a consequence these deformations remain locked into the material. Additional inelastic shear deformations are generated at new locations due to the mining process. These deformation also remain partly locked into the material upon further mining and a cumulative process of locked in deformations is activated. The effect of the locked in deformations is twofold; firstly, they affect the global deformations and displacements such as the closure of the excavation and secondly they induce additional stresses.

In the example which is presented here the inelastic shear deformations are assumed to occur along vertical discontinuities. While this is not a requirement for the mechanism to be operative, this example has been selected as it offers a relatively simple geometry, allowing a clearer demonstration of the principles involved. The shear deformations do not necessarily have to be absorbed by discrete discontinuities, as this mechanism is also active with plastic deformations in a homogeneous medium. This has been confirmed with FLAC and the finite element code ABAQUS (Kuijpers and Napier, 1991).

Figure 6.16 shows the elastic stress distribution around a longwall stope in a homogeneous material. The assumed values for the in-situ, virgin stresses are: 60MPa vertical and 30MPa horizontal which results in a K-ratio of 0.5. Compressive stresses are indicated in black and tensile stresses are indicated in white.

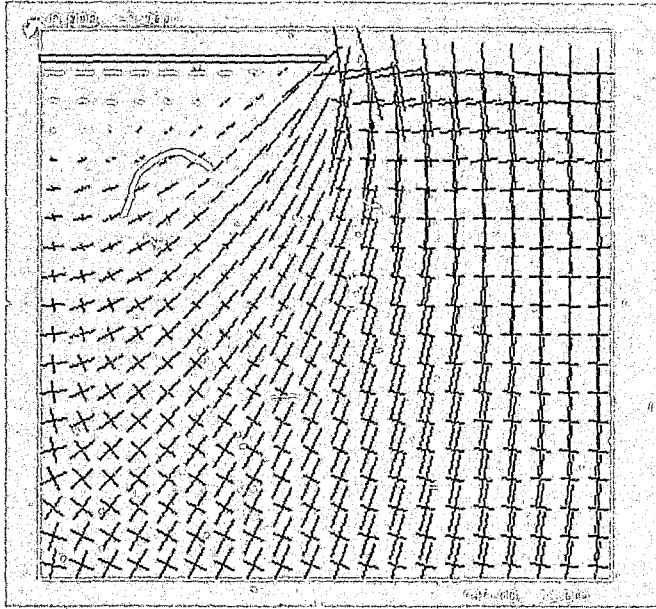


Figure 6.16 Elastic stress distribution around a slope (quarter symmetry; K-ratio = 0.5)

Figure 6.17 shows the resulting stress distribution after a vertical discontinuity, which is placed at the face of the longwall stope, has been activated. The vertical discontinuities have been given a frictional resistance only ($\tan 30^\circ$) and relatively large shear deformations are absorbed by this discontinuity. The discontinuity which is located further towards the centre of the stope did not experience any shear deformation.

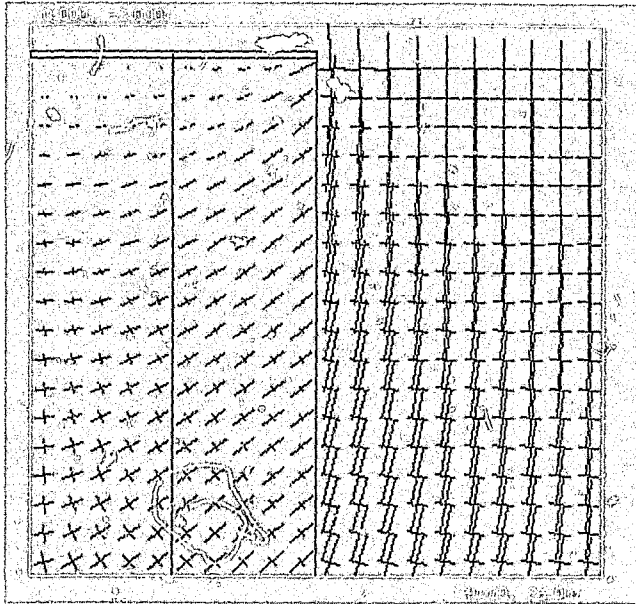


Figure 6.17 Stress distribution after activation of the first vertical discontinuity at the abutment (Quarter symmetry)

Figure 6.18 shows the resulting stress distribution in one quarter of a double symmetric geometry which represent a stope and various vertical discontinuities. The vertical discontinuities are placed at the intersection of the face at each mining step, as this allows the maximum amount of shear deformation. The situation depicted in Figure 6.18 is the situation after 7 mining steps and it can be seen how this stress distribution has been affected by the mining steps (or loading history)

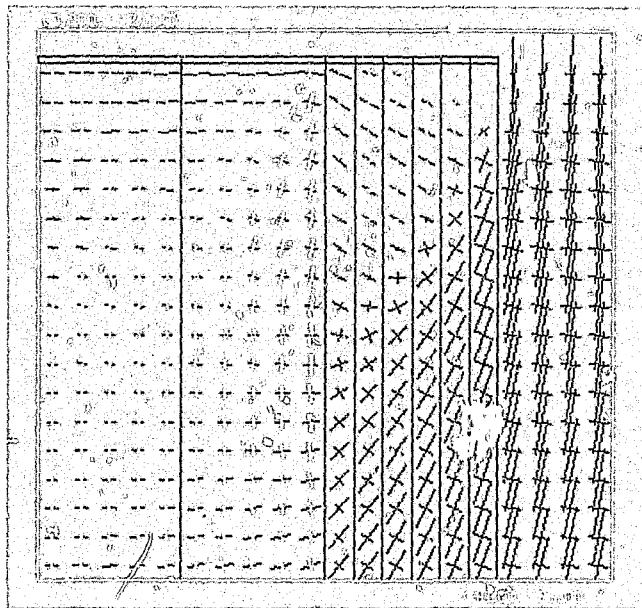


Figure 6.18 Stress distribution after 7 mining steps, no tensile fracturing simulated

The most striking difference with Figure 6.17, in which only a single mining step has been simulated, is the presence of relatively large horizontal compressive stresses in the immediate skin around the stope and the presence of tensile stresses of varying magnitude, which are more or less vertically orientated. As has been argued before, the stress distribution is influenced by the presence of residual strains which become locked in after each mining step. Shear deformations (slip) along the vertical discontinuities do not completely reverse upon further mining and this results in a 'ratchetting' process of accumulating locked in deformations and associated stresses.

In the following simulation, the tensile stresses are allowed to induce tensile fracturing. Fractures initiate from locations of maximum tensile stress, which are found along the vertical discontinuities. The resulting fractures are approximately oriented in a horizontal direction and they do not intersect the excavation. Their effect on the stress distribution, other than the relieve of tensile stresses, appears to be limited but stability conditions may obviously be affected.

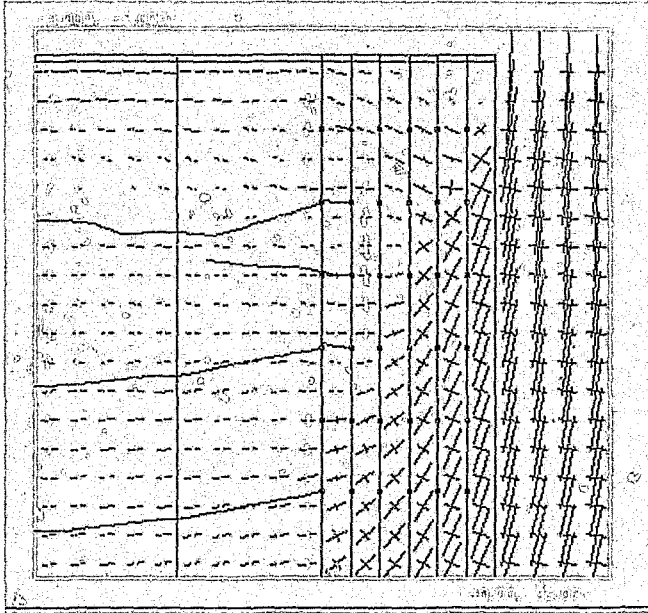


Figure 6.19 Stress distribution after 7 mining steps, with generation of tensile fracturing

Figure 6.20 shows the effect of accumulated residual deformations on the development of hanging- and footwall stresses at the centre of the stope. It is clear that every mining step contributes and causes these stresses to increase continuously. The tensile fractures do apparently influence the horizontal stress distribution to a certain extent, although this is not immediately obvious from Figure 6.19. The explanation for this behaviour may be found in the fact that the horizontal fractures promote the formation of beams which, as has been shown in section 6.2, can have a large influence on stresses and deformations. The relation between the number of mining steps and the tangential stress in the skin of the excavation is markedly different for the two cases which have been simulated. The case in which tensile fracturing is represented shows a more rapid development of compressive horizontal stresses than the case in which no additional fracturing is simulated.

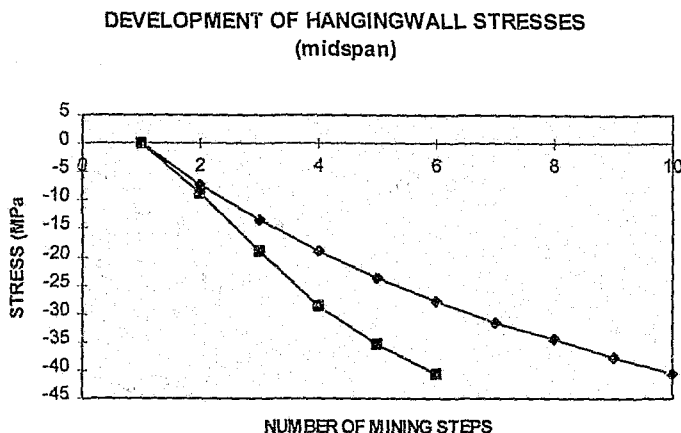


Figure 6.20 Development of horizontal tangential stresses in the skin of the stope due to the 'ratchetting' mechanism of locked-in deformations; top graph is the situation without additional fracturing (Figure 6.18) and the bottom one represent the case in which tensile fracturing is simulated (Figure 6.19)

Figure 6.21 shows the vertical displacements along the skin of the excavation for different scenario's. The elastic deformations, which occur after a stope with a half span of 20m has been mined in a homogeneous rockmass, shows the typical smooth distribution. The introduction of a single vertical discontinuity causes additional deformations which are mainly concentrated at the abutment, along the discontinuity. While the vertical deformations are larger than in the elastic case, the curvature of the deformation profile is reduced. This reduction accounts for a decrease in the induced tensile tangential stresses along the skin of the excavation.

The execution of additional mining steps results in a reversed curvature of the skin of the stope in Figure 6.21. This reversal accounts for the induction of compressive tangential stresses in this skin. The locked in deformations can also readily be observed in Figure 6.21 as the staggered deformation profile is caused by these locked in shear deformations along the vertical discontinuities. The formation of (horizontal) tensile fractures results in some additional vertical deformation, but does not have a substantial effect on the profile of the deformation curve.

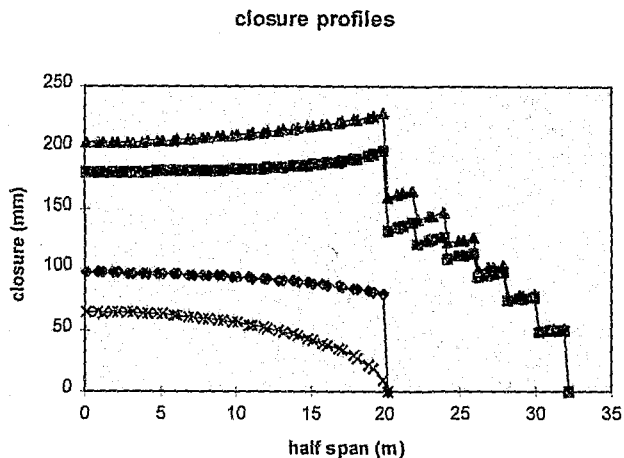


Figure 6.21 Vertical deformations at the skin of a stope for different scenarios:
 -elastic, after 1 mining step (half span of 20m)
 -vertical discontinuity introduced, after 1 mining step
 -vertical discontinuities introduced, after 6 additional mining steps
 -same plus simulation of additional tensile fractures

What these simulations have demonstrated is that inelastic deformations may not be released upon unloading under certain conditions. While the phenomenon of irreversible and locked in deformations is nothing extraordinary and is in fact typical for plasticity, its effects are not always appreciated. One of these effects, which has been shown here, is the cumulative influence of consecutive mining steps on the global stress distribution and the associated deformations. Unlike elastic deformations, which are completely recoverable upon unloading and which follow the same stress-strain relation during loading as during unloading, inelastic deformations are sensitive to the exact history of loading and unloading as deformations are typically not recoverable. This implies that the loading history needs to be considered when inelastic deformations occur. For mining applications such a requirement means that numerical simulations which represent inelastic behaviour have to take into account the detailed mining sequence.

6.5 SUMMARY

It is clear that the numerical representation of shear fractures still requires further study. Numerical problems associated with the simulation of shear failure in a continuum formulation are directly related to mesh dependency in cases of material softening. While such a mesh sensitivity does not occur when discrete fractures are represented explicitly by shear fractures as displacement discontinuities, it is not obvious if the empirical shear fracture criteria are associated in any way with real shear fracture behaviour. The critical issue is the lack of sufficiently detailed information on the behaviour of these shear fractures. Unless such information becomes available, meaningful progress in this field cannot be expected. The (empirical) results which have been presented in this chapter nevertheless show the potential for localised shear failure simulation and possible models have been investigated.

The simulations of bedding planes proved to involve numerical problems in the Boundary Element models (DIGS). Although it is not obvious what the cause of these problems is, it appeared that the results for slender beams simulations are not correct and the results of similar simulations, in which beam like structures occur, are for that reason suspect. The presented results did however show how the presence of bedding planes can influence the deformations and stress distributions around a longwall stope. The effect of the redistributed stresses on subsequent fracture formation has also been investigated and this resulted in a substantial change in fracture geometry. The presence of joints has also been investigated and their effect on subsequent fracture generation has been analysed as well. It is obvious from these numerical simulations that the presence of geological discontinuities can have a major influence on fracture formation and needs to be incorporated in any realistic analysis.

Numerical models in which punching of a flat indenter into a solid half space was simulated showed that the mechanism of punching could be reproduced in a fairly realistic way. The best results were obtained by representing the wedge structure underneath the punch as a ductile failure zone. Subsequent tensile, brittle fracturing was also reproduced and the combination of ductile (shear) failure and brittle, tensile failure led to results which appear to capture the main features of the punching mechanism. The ductile shear failure was most conveniently simulated with *FLAC* in a continuum analysis, while the explicit tensile fractures could only be represented by *DIGS* in a realistic way.

The stress path, or loading history, may have a substantial influence when inelastic deformations occur. This influence has been investigated by numerical models in which inelastic deformations are concentrated along vertical discontinuities which intersect a horizontal slot. Incremental enlargement of the horizontal slot initiates (inelastic) deformations along some of these discontinuities, while deformations which were induced previously, on other discontinuities, may be partly, but not completely, reversed. The effect of incomplete reversal of inelastic deformations is that inelastic deformations remain locked into the system. The model demonstrates that these locked in deformations accumulate with each mining step, or loading cycle, and that their influence on the stress distribution is also cumulative. This effect has major implications for the simulation of inelastic deformations in general, as it indicates the need to consider realistic loading paths. Unlike linear elastic behaviour in which the principle of superposition is always valid; results cannot be superimposed when inelastic deformations occur. This is especially relevant in the context of the numerical simulation of mining excavations, which cannot be represented by a single "mining step" if the potential for locked-in deformations exists.

7.0 DISCUSSION AND CONCLUSIONS

Different numerical techniques and constitutive models have been analysed with respect to their applicability to the simulation of inelastic deformation processes in rock. One of the major problems in the evaluation of these techniques and models is the fact that the inelastic processes themselves are not well understood. The fundamental processes which control the inelastic behaviour of rock are undoubtedly related to the micro structure of a particular rock type. Any attempt to represent this complicated micro structure will require that certain simplifying assumptions have to be made. In all of the models used in this thesis, elasto-dynamic effects are *not* represented, as failure processes have been assumed to take place in a quasi static way. Such an assumption may in some cases be too simplistic, especially if violent post failure behaviour is involved, but it avoids the complexity of dynamic stress calculations.

The simplest constitutive model is based on linear elasticity and can successfully be applied to many situations. Such applications are however strictly limited to those cases in which the yield strength of the material is not exceeded, or where very small regions in the material are damaged. A rockmass around deep level excavations is easily subjected to stresses which may induce deformations on pre-existing discontinuities, fracturing in intact rock and even local crushing of the rockmass. These inelastic processes influence the behaviour of the material around an excavation and need therefore to be incorporated in any analysis which aims to assess this influence.

While linear elasticity requires only two parameters in the form of an Elasticity modulus and a Poisson's ratio to describe the material behaviour of a continuum, inelastic material behaviour by definition cannot be represented by such a simple model, especially if the assumption of a continuum is not valid. This is in fact a crucial assumption and it becomes important to know how the discontinuous nature of a particular material controls its behaviour at a particular scale of interest.

In an attempt to represent the discontinuous nature of rock at a micro scale, a tessellation model has been used. The model is based on the assumption of plane strain and the rockmass is simulated as an assembly of randomly oriented cracks. Inelastic deformations are assumed to take place along crack boundaries, due to excess tensile and/or shear stresses along such boundaries (intra-granular fracturing). Although not explicitly demonstrated here, the model also allows for trans-granular fracturing so that individual grains can split as well. This tessellation model does not represent the three dimensional nature of the granular structure at the moment (a 3-D version is being developed) and the matrix material of the rockmass between cracks is assumed to be elastic. Crushing of individual grains is not catered for as inelastic behaviour is only assumed to take place along (predefined) discontinuities.

Whether these simplifying assumptions have a substantial effect on the results is difficult to judge, especially if one considers that the main purpose of these models was to simulate large scale fracturing. The only criterion by which the results have been evaluated is therefore the resemblance of the simulated fracture patterns with observed fracture patterns in selected laboratory experiments. Although many of the numerical results could be interpreted as realistic, it is still unclear whether the simulation of explicit shear fractures

has been achieved in a realistic fashion. Results appear to be extremely sensitive with respect to the choice of parameters such as the friction angle and the effects of cohesion softening. This, in combination with the fact that the results are also very sensitive with respect to the geometry of the tessellation, leads to a complex situation in which a wide variety of choices is available. The results therefore show a wide variation of fracture patterns as they are sensitive to the choice of input parameters. From a modelling point of view, this is not an effective scenario as this approach requires the same parameters as the continuum approach, while the segmentation of the continuum into discrete elements introduces additional variables.

The parameters associated with a continuum model are effectively "smeared" properties which describe the global behaviour of the material. As the detailed micro mechanical processes which underlie the global behaviour are not represented explicitly, these parameters should not be related directly to any physical processes. Properties such as cohesion, internal friction angle and dilation are empirical and are not meaningful on a scale at which fundamental processes are assumed to take place. This is not always appreciated and the Mohr Coulomb criterion, which employs cohesion and friction, is often treated as if it were based on fundamental physics. The fact that a cohesive strength and a frictional resistance are assumed to operate simultaneously in order to achieve an effective shear strength cannot be justified on mechanical grounds. Surely friction is only mobilised once bonds have broken and cohesion has been dissipated. Cohesion itself is not a fundamental parameter, but must be associated with stretching (tension) at smaller scales. It is important that these issues are considered if such smaller scales are to be represented.

In the case of the tessellation models it is not clear whether fundamental processes are captured by the representation of grain boundaries as planar surfaces. For this reason the conventional, empirical properties are still used in the tessellation models. A possible alternative to avoid the use of these empirical properties is the introduction of flaws in the tessellation mesh. Such flaws act as stress concentrators and would allow tensile failure to initiate from locations which would otherwise not be prone to tensile fracturing. Simulations which have been done with additional flaws demonstrated the potential for coalescence and fracture propagation. The fact that the associated results are sensitive to the distribution of these flaws is encouraging and appears to be realistic. The failure (fracture) criterion which is used in these simulations is the tensile strength of a primitive crack in the random grid. This is a fundamental property and its magnitude would be the only variable with respect to failure. The mesh geometry and the distribution of flaws are the other parameters controlling the global failure behaviour in that case. It can be argued that such a model is based on a solid physical basis as it does not require additional empirical failure rules. None of these simulations has however incorporated the effect of deformation on changes in geometry so that second order effects have not been included.

A continuum representation with the finite element and finite difference method requires a mesh in order to compute displacements and stresses. Failure localisation can be reproduced by both methods, but is often influenced by the geometry and density of the chosen mesh. Although this is an artefact of the numerical method, it again demonstrates the effect of a basic structure on post failure behaviour. In order to avoid the influence of numerical factors, irregularities can be imposed upon a mesh. Such irregularities could consist of deviations in strength or stiffness etc. and these irregularities can determine the

post failure behaviour to a large extent, especially if some form of material softening is taking place. (Small) variations in the distribution of such irregularities may lead to substantial differences in results as the system is effectively "chaotic". In principle it is not possible to predict the detailed evolution of such a chaotic system and the sensitivity to perturbations in initial conditions of the computer models may well be representative of real behaviour.

Although it is possible to construct a distorted mesh in either Finite Element or Finite Difference applications, this has not been done in the simulations which are presented in this thesis. The tessellation models based on the Boundary Element method have however been shown to be sensitive to the mesh geometry. The difference between a random mesh and a regular mesh appeared to be most pronounced when Delaunay triangles were used and brittle behaviour was simulated. No attempt has been made to investigate the sensitivity towards changes in the random mesh. The size distribution of the individual triangles or the mesh density could for instance have an influence, but this has not been analysed specifically. The Voronoi mesh was far less prone to failure localisation and the results for the models based on the Voronoi mesh did not show much sensitivity toward changes in mesh geometry or failure parameters.

While the mesh geometry has been identified as a very important factor controlling the localisation of failure, only a limited choice of mesh variations have been analysed. No attempt has been made to represent the micro structure of a particular rock in any detail; the representations used here were merely conceptual models. It is impossible to assess at the moment if these conceptual models capture real behaviour in a satisfactory way as no comparison with more realistic models or real materials is available. For most of the simulations a particular random mesh of Delaunay triangles has been used, as this allowed for failure localisation and brittle behaviour.

The chaotic nature of the softening models can lead to large variations in the resulting failure behaviour, but this does not imply that results are completely random and that any result is in principle valid or invalid. In general, a range of possible results may be found and the distribution of such results may be best described by statistical methods. A similar statistical distribution should be imposed on various input parameters as well in order to capture natural variations in real materials. The results of such computer simulations can then be compared and calibrated against the results of laboratory experiments. The laboratory experiments should also be designed according to statistical methods. This implies that variations in laboratory results are not ignored, but are treated as a fundamental part of such results. The practical effect of these requirements is of course that large numbers of laboratory and numerical experiments have to be conducted in order to obtain meaningful results.

The simulation of large scale fractures as explicit discontinuities in an elastic continuum is well established in the case of tensile fractures. Numerous examples can also be found throughout this thesis and many of these simulations produce realistic results. The representation of so-called shear fractures requires however a different failure criterion. In this thesis two alternative criteria have been presented, both empirical and both involving friction and cohesion. The results obtained with these criteria were difficult to judge. The Energy criterion proved to produce the most stable results because tensile stress conditions

were adequately catered for. Especially under conditions of brittle failure the occurrence of tensile stresses at the fracture tip is not uncommon and has to be accounted for. Simulations with the Energy criterion led to results which could be judged to be satisfactory, in the case of the punch through experiments, but highly unlikely in the case of stope simulations. The formation of shear fractures is still not well understood as the processes which control their nucleation and growth are associated with very complex micro fracturing and fracture coalescence processes at various scales.

Even the representation of large scale discontinuities such as joints, faults and bedding planes is not based on exact realistic behaviour. These structures have been simplified in the numerical models to planar features with uniform properties. While these simulations demonstrated the potential influence of such discontinuities on the stress distribution and associated (tensile) fracture formation around a longwall stope, it is unclear how this influence would vary if a more realistic (3D) representation of such features were to be used. The presented results show clearly that the presence of large scale weaknesses can have a dominating effect on fracture formation. It is therefore important that these structures should be identified and included in any numerical model which attempts to simulate rockmass behaviour.

Localisation of failure is not of practical relevance as long as localised fractures do not manifest themselves at a scale large enough to have any influence. Manifestation occurs only when the resulting fracture geometry has an important effect at a particular scale of practical relevance. If that is the case, it is necessary to identify the mechanism which controls the localisation of failure in order to simulate these processes in a realistic way. The scale at which this mechanism operates thus becomes the scale of interest and this scale may vary from the atomic level to a scale which is measured in tens and hundreds of metres. In the case of granular material for instance, it makes sense to represent the granular structure if localisation of failure is initiated at that particular scale and not at a larger scale of (pre-existing) weaknesses.

In the case that localisation of failure is *not* relevant to a particular problem, a much simpler approach can be taken and a continuum formulation based on empirical failure rules would be perfectly suitable. Localisation of failure should then purposefully be avoided in the numerical simulations, as that would introduce an unwanted complexity. It is important that a clear distinction is made beforehand between problems which are, and problems which are not sensitive to failure localisation, in order to select an appropriate numerical model.

Although no definite conclusions can be drawn as to the absolute validity of the models used in this study, many of the results indicated potential mechanisms of failure. The effect of plastic deformations on subsequent tensile, brittle failure for instance may point to a similar behaviour in reality. Although plasticity is represented in a very simplified manner in the various numerical models, either as a continuum property or as the behaviour along discontinuities, these results may capture the essence of real material behaviour. Plasticity has to be treated in an empirical fashion as the fundamental processes controlling this behaviour operate on a scale which is too small to be of practical assessment. The interaction between brittle and plastic processes appears to be of practical relevance as many of the results suggest.

An extremely relevant and practical issue has been addressed in Chapter 6, namely the influence of the loading history, or excavation sequence. Inelastic deformations can become locked into the system during loading cycles and such locked in, or residual, deformations can have an effect on the stress distribution. In the relevant examples it has been demonstrated that consecutive mining cycles lead to an accumulation of residual deformations and associated stresses, when inelastic deformations are represented within a non linear model. This mechanism introduces an additional complexity to the simulation of inelastic behaviour around mining excavations. Any excavation sequence has to be represented explicitly in order to include the residual stresses and strains which are generated during the excavation process. The creation of a longwall stope for instance would require the simulation of each mining increment if the residual deformations and stresses have to be represented. It is shown that the associated effects can be very pronounced in the immediate vicinity of the excavation and thus in the area which is of most practical interest.

The main conclusions can be summarised as follows:

- A serious review of current modelling paradigms in relation to chaotic behaviour is required
- Brittle failure is shown to be very sensitive to perturbations in model parameters and in the outcomes of physical experiments. It is essential that the statistical nature of such perturbations and the associated variations in resulting behaviour should be properly identified and accounted for
- The interaction between plastic and brittle behaviour has been identified as a potential mechanism for brittle failure localisation; further analysis is required
- The representation of shear fractures and shear failure in general requires a better understanding of the processes which are involved in brittle shear fracture formation.
- The geometry of the micro structure controls to a large extent the localisation of failure in brittle material; it is therefore important that this geometry is realistically represented in any numerical model in order to represent localisation of failure.
- It is important to decide if localisation of failure is relevant to a particular problem scale before complex numerical analyses are carried out. In case localisation of failure is not an issue, the simulation of localisation should be avoided
- Most of the numerical modelling results are based on empirical failure rules, such as the Mohr-Coulomb criterion; it is therefore doubtful whether fundamental failure processes are captured by such models
- The presence of geological structures are likely to have a dominating effect on subsequent fracture formation; it is important that such structures are identified and represented
- The representation of a realistic loading path can be essential when inelastic deformations are simulated as such deformations can become locked into the system during consecutive loading cycles and have a major influence on the associated stress distribution

8 RECOMMENDATIONS

It has been demonstrated that the numerical simulation of brittle failure processes can lead to a wide variation of results, due to the chaotic nature of brittle systems subjected to compressive stresses. As such a wide variation is typical for these kind of simulations, it needs to be accounted for by appropriate techniques. In order to obtain a representative distribution of results, it is required to compute the response to variations in input parameters, which is supposed to reflect the natural distribution of flaws and properties in real materials.

A typical problem would therefore require a large number of numerical simulations. In order to ensure the validity of the distribution in input parameters, a large number of laboratory experiments needs to be conducted as well. These laboratory experiments are only required to calibrate the intrinsic material behaviour and do not necessarily have to resemble the problem being analysed. This type of statistical analysis is not common, as most problems are perceived to have unique solutions. However, hard rock is a material which does not have unique properties. The distribution of flaws, grains and matrix material can cause wide variations in properties for different samples in a particular rock.

Conditions which are conducive to chaotic systems may occur for additional reasons as well in deep level mining applications. Not only will the rock itself have variable properties, but the distribution of geological structures, the geological history and the excavation processes may introduce additional variations. The prediction of unique solutions may therefore be unrealistic under those circumstances.

The acceptance of the non existence of unique solutions and the insight that unstable behaviour can result in chaotic behaviour, is a prerequisite for further progress in this field. Models need to be able to represent the natural distribution of properties of real materials and it may well be that the models which have been used in this thesis are not capable of doing that. This is however impossible to judge at this stage as an appropriate range of laboratory tests was not available and also because the models have not yet been used in three dimensional simulations. It is suggested therefore that three dimensional simulations of relevant laboratory experiments will be done with existing numerical models, in a statistically meaningful way.

Shortcomings in existing models will thus manifest themselves and alternatives can be designed accordingly if this proves to be necessary. The ideal model should ultimately allow the representations of relevant parameters in a statistical sense and will in that way be able to reproduce a variation of results which will match observations from similar experiments on real materials. Practically meaningful results cannot be expected unless such models are available and calibrated.

The distribution of relevant input parameters can take place at different scales. From a practical point of view it would make sense to focus on those parameters which present a weakness or a flaw at the largest scale. In mining applications such weaknesses or flaws frequently occur at a relatively large scale of metres up to kilometres in the form of geological discontinuities. Such discontinuities are often the "weakest link" in the rock mass and are therefore susceptible to failure initiation. It is therefore important that such

discontinuities are detected and included in any type of failure analysis as a first priority. The distribution of weaknesses at smaller scales can be included as well obviously, but this is only required if the large scale model does not provide sufficient detail. The most efficient representation of relevant flaws should be in accordance with the ideal model as defined in the previous paragraph.

Both the simulation of failure on a granular scale and the continuum simulations have identified the need for an appropriate combination of plastic and brittle failure. A potential solution, which appears to resemble rock like materials, could for instance be the representation of matrix material as (ideally) plastic with embedded, hard grains. Plastic deformations are in that case restricted by the limits for grain transformation and rotation. This restriction on plastic deformations must result in the localisation of failure according to the geometry and distribution of grains. Brittle failure may ultimately be associated with the occurrence of excessive plastic strains and the (tensile) splitting of grains. This model could be suggested for materials which demonstrate some form of plastic behaviour.

The representation of fundamental brittle behaviour can only be achieved by a tensile failure criterion. In order to allow for pure brittle failure in a global compressive stress field, it is necessary to introduce local flaws which can induce micro fracturing. Subsequent fracture coalescence may lead to macro fracture localisation and brittle failure as has been demonstrated in some of the models in this thesis. It would appear that such a model is the only alternative to avoid the use of empirical failure rules. The distribution of flaws could obviously be treated in a statistical way. This model is suggested for materials which do not demonstrate any plastic behaviour. Second order effects, which involve the adjustment of the geometry in relation to induced deformations (large strain), have not been taken into account in the brittle models presented in this thesis. It is important that these second order effects are analysed in future as they may influence the stress distribution and associated fracture formation.

In case the localisation of failure is not critical to a particular problem, the use of empirical failure rules is in fact recommended. In other words if, at the scale of interest, explicit fracturing is not relevant, or at least not part of the problem to be addressed, the simulation of fracturing and failure localisation should be avoided. Instead numerical models should be selected which do not cause localisation of failure. Although localisation of failure has been the main subject in this thesis it is recognised that this may be irrelevant in many practical applications. In those cases a much simpler approach can be adopted by selecting empirical models which do not necessarily incorporate the physical structure at a smaller scale than the scale of interest.

It is also suggested that failure of extremely brittle materials may be associated with the violent release of (elastic) energy which would induce dynamic stress waves. Such behaviour would especially be expected from class II materials which demonstrate self sustaining fracture processes. In order to simulate such fracture processes it may be necessary to account for the dynamic stress waves as well.

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