Wits School of Education

Investigating Grade 12 probability online learning resources for multiple representations

Master of Education Research Report

by

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Abstract

This research explores multiple representations of Grade 12 probability in online teaching and learning resources. The research is motivated by the observation that probability is poorly performed in Grade 12, with education authorities proposing multiple strategies, including the use of representational tools, as possible solutions to the problem of poor performance. Due to the closure of schools in 2020 as a result of the Covid-19 pandemic, the study relied entirely on online videos and electronic textbooks and study guides as sources of data. Data for the research are obtained from three open-source online YouTube video channels, two of which are South African video channels and one is an international channel. Other data are gathered from online learning materials, consisting of an electronic textbook (e-textbook) and two online study guides (e-study guides). Data from video channels were captured using the Mathematical Classroom Observation Protocol for Practice (MCOP^2) instrument and data from online learning materials were captured using the Mathematical Task Framework (MTF) instrument. Numerical data obtained through the use of the above mentioned instruments were entered into Microsoft Excel software to obtain descriptive statistics as well as to enable me to carry out hypothesis tests. The hypothesis tests to assess whether there is a difference in the means of the video channels based on the MCOP^2 item scores, and whether there is a difference in the means of cognitive demand levels based on MTF scores, were performed using the analysis of variance (ANOVA) test statistic.

Findings from the data suggest that there was no statistical significant mean difference in the MCOP^2 item scores for the three video channels. However, when the three video channels were analysed individually in terms of the MCOP^2 instrument that promote multiple representations of probability, the data revealed that only one episode from each of the two South African video channels promoted multiple representations of probability, while all episodes from the international video channel promoted multiple representations of probability. In terms of the online learning materials, only the e-textbook and one e-study guide promoted multiple representation of probability as the means of cognitive demand levels for these two e-learning sources were statistically different from the mean of the other e-study guides. The results from the video instructions suggest that currently few lessons from South African video learning channels promote multiple representations of counting and probability compared to the international learning channel. The result from online learning materials implies that two out of three e-learning text materials have reasonable quantity and

quality of tasks that promote multiple representations of counting and probability. These results confirm that there are gaps between what the department of education recommends in its yearly Grade 12 diagnostic reports and the instructional reality in the classroom. As such, this study recommends that in-service teachers undergo regular professional development training that are initiated and organised by the schools themselves but facilitated by universities that train secondary mathematics teachers. The training should be inclusive, irrespective of whether a teacher will deliver instructions through the online platforms or through the traditional classrooms. Moreover, pre-service teachers should also be conversant with multiple representations of probability concepts before they graduate from teacher training universities.

DEDICATION

To the loving memory of my late father, Patrick Msizeni Mlotshwa, and my late mother, Sibongile Evelyn Mlotshwa. Thank you for everything.

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Abbreviations and Acronyms

Full name	Abbreviations and Acronyms
Common Content Knowledge	CCK
Corona Virus of 2019	Covid-19
Curriculum and Assessment Policy Statement	CAPS
Department of Basic Education	DBE
e-Study Guide	SG
Episode	Ep.
Further Education and Training	FET
General Education and Training	GET
Intermediate Phase	IP
Knowledge of Content and Curriculum	KCC
Knowledge of Content and Students	KCS
Knowledge of Content and Teaching	KCT
Mathematics Classroom Observation Protocol for Practices	$MCOP^2$
Mathematical Knowledge for Teaching	MKT
Mathematical Task Framework	MTF
National Curriculum Statement	NCS
National Senior Certificate	NSC
Senior Phase	SP
Specialised Content Knowledge	SCK
Statistics South Africa	Stats SA
Subject Matter Knowledge	SMK
Video Channel	VC

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CHAPTER 1: INTRODUCTION

1.1 Introduction

The worldwide spread of the Corona Virus disease of 2019 (Covid-19) in the year 2020 has brought changes to the way societies live, work and study. Governments around the world, including the government of the Republic of South Africa, responded to this health crisis by implementing measures such as social distancing, the wearing of masks in public, and the shutdown of public and private institutions including sites and spaces for mass gathering. The institutions affected by Covid-19 responded by moving away from physical, human-to-human interaction platforms to virtual and online platforms. This shift highlighted the importance of technology as a tool especially for learning centres such as schools and universities. By technology, I mean devices such as mobile phones, tablets, desktop and laptop computers that are used to access online instructional resources and learning materials (Greefrath, Hertleif, & Siller, 2018, p.234).

This research report will explore selected learning resources available online for Grade 12 mathematics. The learning materials are selected because of their popular use by mathematics teachers and learners at Grade 12 level. The research report will focus on the mathematics learning area of probability. Probability is chosen in this study for two reasons. First, from the National Senior Certificate (NSC) diagnostic reports, probability has been highlighted as an area where learners have major challenges in understanding multiple representations of concepts, hence the poor performance (Department of Basic Education, 2019, 2020a). Second, I have not come across any recent studies in the main South African mathematics education journals such as Pythagoras and the African Journal of Research in Mathematics, Science and Technology Education (AJRMSTE) on multiple representations of probability concepts in secondary school mathematics. By multiple representations I refer to different ways of expressing probability procedures as follows: logical (i.e. inclusion/exclusion), graphical procedures (e.g. tree diagrams), tabular (i.e. tables and arrays), numerical (e.g. multiplication and factorial notation), and algebraic (Batanero, Godino, & Navarro-Pelayo, 1997). The above reasons as well as my own experience in teaching secondary mathematics motivated me to research the topic of probability. Although learners are not directly involved in this study due to the lockdown of schools, the outcome of this investigation will inform our understanding of online learning resources and will therefore contribute toward improved learner performance.

Following this brief introduction, I next present the topic of probability in the South African basic education curriculum. Thereafter, the goal of the research will be defined based on the identified research gap. Research questions for this study will be informed by the identified gap in research. This chapter will conclude by outlining the structure of the research report.

1.2 Probability in basic education curriculum

The Curriculum and Assessment Policy Statements (CAPS) is the policy statement for the teaching and learning of mathematics in South Africa. The policy document outlines the knowledge that learners are expected to attain in primary and secondary schools. CAPS is divided into four phases, namely, the Foundation Phase (FP) which covers Grades R to 3; the Intermediate Phase (IP) which covers Grades 4 to 6; the Senior Phase (SP)-also called the General Education and Training (GET) Phase-which covers Grades 7 to 9; and the Further Education and Training (FET) Phase which covers Grades 10 to 12.

Statistics and probability are two learning areas of mathematics that the curriculum addresses starting from Grade 4 and form part of the curriculum in each grade up to Grade 12 (DBE, 2011, 2012a). For learners electing mathematical literacy instead of mathematics in Grade 10, statistics and probability remain part of the mathematical literacy curriculum in each grade of the FET Phase. The focus of this research is on Grade 12 probability as a learning area of mathematics. Statistics and mathematical literacy do not form part of the research.

The coverage of probability in CAPS only starts at the Intermediate Phase (Grades 4 – 6). Probability is not a separate content area in the IP but is included in the data handling content area (DBE, 2012a). Data handling is the preferred word for statistics in the South African school curriculum. At this phase probability is expected to “enable the learner to develop skills and techniques for making informed predictions and describing randomness and uncertainty” (DBE, 2012a, p. 11). During the IP, learners are not expected to calculate probabilities of events, but they perform repeated trials of events (such as tossing a coin and rolling a die) up to 50 times so that they can count and make predictions on the number of times that the expected outcomes will occur in those trials. The recommended teaching and learning time for probability is 2 hours for each grade in this phase.

In the Senior Phase (Grades 7 – 9), probability is again a subsection of the data handling content area. Probability of events appears as both single and compound events, where learners determine probabilities of outcomes of equally likely events. Learners also compare theoretical probabilities with relative frequencies in repeated trials (DBE, 2012b). Events forming part of probability include those covered in the Intermediate Phase, with the addition of deck of cards, different coloured balls, and other hypothetical events. In Grade 9, learners are introduced to tree diagrams and contingency tables, which are used to determine probabilities of compound events. In the SP, the recommended learning and teaching time for probability increases to 4.5 hours.

In the FET Phase (Grades 10 to 12), probability is no longer a sub-topic of data handling. In addition to probability of events covered at the IP and SP, probability of events at the FET Phase include the categorisation of events as either dependent and independent; the determination of whether events are mutually exclusive or not; complementary events; the use of Venn diagrams, tree diagrams and contingency tables as representations to solve probability problems (DBE, 2011). In Grade 12 in particular, the sub-topics of probability include the fundamental counting principle, the factorial notation (permutations), combinations (with and without replacement); and the probability of events arising from the application of fundamental counting principle. Grade 12 end of year examinations assess both Grade 11 and Grade 12 probability topics. However, in this research report, the focus will solely be on Grade 12 curriculum. Two weeks is the recommended teaching and learning duration for probability in the FET Phase.

1.3 The South African context

The socioeconomic status of South African families is an important consideration in children accessing online instruction and learning resources. Parents will be expected to purchase technological devices such as mobile phones and computers that will be used to access online instructions and learning materials. According to Statistics South Africa (Stats SA), sources of income for all households in South Africa during 2018 indicated that about 45.2% of households received government social grants; about 64.8% received salaries; about 16.6% listed remittances as a source of income; and about 13.6% of households listed business as a source of income (Statistics South Africa, 2018). The numbers do not sum up to 100% as one household can have more than one sources of income. Despite some households having more than one sources of income, there remain a high percentage of households that rely on social grants and remittances to support their families. The socioeconomic status of households is very important as it determines the kind of assets that are prioritised by households including technological devices that may be used to access online instructions and learning resources.

Assets ownership by households (excluding immovable, motor vehicle and financial assets) indicate priorities set by families depending on their needs as well as the income levels to acquire such assets. According to the Stats SA, most families prioritised the ownership of electric stoves, followed by television sets and refrigerators (Statistics South Africa, 2018). However, only about 21.5% of households owned one or more computer devices (including desktops and laptops) as assets (Statistics South Africa, 2018). In terms of mobile phones ownership, about 89.5% of households in South Africa had mobile phones in 2018, indicating extensive accessibility to this technological device. What is not indicated though is the percentage ownership of mobile phones that have internet access features or applications.

Taken together, this indicates that very few families have technological devices such as computers and mobile phones which are tools that can be used to access online instructions and learning resources. The ownership of these technological resources does not guarantee access to the internet which is a prerequisite to access online instructions and learning resources. According to Stats SA, 64.7% of households in South Africa had one or more of its members having access to the internet either at home, work, place of study or internet cafés during 2018 (Statistics South Africa, 2018). However, only 10.4% of households had access to the internet at home through platforms such as Wi-Fi networks or ADSL fibre networks, showing the low levels of internet connectivity of South Africans at home (Statistics South Africa, 2018). With respect to connectivity to the internet using mobile phones, about 60.1% of households had connection to the internet on their mobile phones (Statistics South Africa, 2018). This is a respectable number showing the utility of mobile phones as means to access online instructions and learning resources. Lastly, internet connectivity through internet cafes and schools by members of households was at 10.1% in 2018 (Statistics South Africa, 2018).

Despite the relatively good number of members of households having the ability to connect to the internet, it is widely accepted that the costs of data used to connect to the internet either at home, through mobile devices, or at internet cafés remain expensive. Internet connection from public institutions such as libraries and schools come with restrictions in terms of time and the availability of space. There are also differences in internet connectivity according to urban or rural classification of households, with households based in metropolitan municipalities generally showing high connectivity to the internet (Statistics South Africa, 2018). Overall, the quoted figures from Stats SA point to the potential that learners have in accessing online instructions and learning materials using mobile phones as they are cheaper compared to computers. The only hindrance is the cost of data owing to the socioeconomic status of households.

1.4 Learning remotely during the Covid-19 pandemic

On 18 March 2020, South African schools were shut down for two and half months until the 8th of June 2020 when Grade 12 and Grade 7 learners were permitted to return to school. During the shutdown period, learning continued virtually for some schools, especially wealthy schools (Mohohlwane, Taylor, & Shepherd, 2020). In response to this inequality, DBE issued a guideline that made recommendations for learners to take advantage of online learning resources to continue with schooling from home after certain websites were zero rated in partnership with mobile phones companies (DBE, 2020b). Accessing online resources comes with costs of purchasing data to access websites that are not zero rated such as the YouTube platform which contains a wealth of instructional videos on all topics covered in Grade 12 mathematics curriculum. There is also the challenge of many learners that do not have mobile phones or computers, having to rely on other members of their households to access online learning materials. Other challenges such as hunger experienced by households during the lockdown period had direct influence on the effectiveness of learning as a result of massive job losses (Mohohlwane, Taylor, & Shepherd, 2020). So too did other factors such as the lack of adequate space in homes for meaningful learning.

What is clear is that a health crisis such as Covid-19 will result in the majority of learners from the no-fees paying schools staying at home. At home little learning will takes place as parents will be out of jobs, making it challenging to access online instructions and learning resources using mobile phones and/or computers.

Connectivity to the internet using internet cafés will be futile as either financial resource will be scarce or these facilities will be closed as a response to the health safety measures put in place in public spaces.

1.5 Problem statement

The performance of Grade 12 learners in mathematics remains poor in South Africa (DBE, 2020a). In 2014, around 53.5% of learners writing mathematics end of year examinations passed the subjects (i.e. they obtained a mark above 30% on average between papers 1 and 2) (DBE, 2015a). The performance in mathematics peaked in the year 2018 with a performance of 58.0%, before subsiding to a performance of 54.6% in the year 2019 (DBE, 2020a).

Probability content area is poorly performed in Grade 12 mathematics examinations. In the 2014 national mathematics examination, the average mark for the question on counting and probability was 29% (DBE, 2015a). The reasons cited for the poor performance were that "candidates did not know how many letters there are in the English alphabet" (DBE, 2015a, p. 120) and "candidates displayed poor understanding of the arrangement of a subgroup of objects within an entire group of objects" (DBE, 2015a, p. 120). In the 2015 examination, the average mark for the question on counting and probability was 28% (DBE, 2016). The reasons for the poor performance were that "candidates were unable to distinguish between the scenarios where repetition is allowed and where it is not allowed" and "candidates knew that the vowels and consonants were fixed and were able to identify that the remaining could be arranged…but wrote it incorrectly" (DBE, 2016, p. 161). Some of these reasons suggest a lack of conceptual knowledge of multiple representations and other related errors on the part of learners.

The average marks for questions on counting and probability were in general quite low (i.e. 2% (2016), 25% (2017) and 34% (2018)) (DBE, 2017, 2018, 2019). Reasons for poor performance included the lack of conceptual understanding relating to multiplication and arrangements, as well as avoidable errors such as the exclusion of zero as a numeric digit by learners. In the most recent mathematics examination (i.e. 2019), the average mark for the section on probability and counting was 26% (DBE, 2020). The reason for poor performance was attributed to learners' inability to distinguish between multiplication and the factorial notation.

In all the years considered, the performance of learners remained very low in counting and probability suggesting the lack of conceptual understanding and systematic errors by learners as evidenced by what examination markers highlighted in these reports. On the errors that learners make, the 2015 diagnostic report, recommends that “learners ought to know certain information that is regarded as general knowledge, e.g. that there are 26 letters in the English alphabet, that the English alphabet consists of 5 vowels and 21 consonants” (DBE, 2015a, p. 120).

The diagnostic reports also present suggestions on how teachers can intervene to improve the performance of learners. There is a suggestion that “fundamental counting principle needs to be taught as clearly and simply as possible, steer away from formulae and reasoning out scenarios, using diagrams to explain scenarios” (DBE, 2017, p. 163) which was repeated in the years 2016, 2017, 2018 and 2019, suggesting that the same problem has persisted over the years. Another suggestion is that teachers must “choose practical scenarios to demonstrate the concepts of ‘repetition is allowed’ and ‘repetition is not allowed’” (DBE, 2018, p. 142) which is also mentioned by reports for the years 2015, 2017, 2018 and 2019.

These clearly identified problems provide the motivation and the basis for the current study, in which multiple representations of probability concepts is explored as one possible intervention. Other instructional support from teachers mentioned in some of these reports includes the suggestion that “teachers should teach the counting principle by real-life demonstrations” (DBE, 2015a, p. 120), “terminology needs to be explained in greater depth” (DBE, 2016, p. 162), and “teachers should only introduce factorial notation once learners have a good understanding of the fundamental counting principle” (DBE, 2018, p. 163).

The other most important reason cited by the diagnostic reports is the inadequacy of teacher content knowledge to teach counting and probability. The 2015 diagnostic report acknowledged that counting principles learning area “is still new to a number of teachers” and that “teachers should work through as many questions from textbooks and past examination papers as they can” (DBE, 2016, p. 163). This was reiterated by the 2016 diagnostic report which mentioned that “teachers still need assistance to improve their own content knowledge in probability” (DBE, 2017, p. 163). Hence, content knowledge that teachers have plays an important role in the teaching and learning of a challenging content area of mathematics such as probability.

Several researchers have argued that teachers' lack of subject matter knowledge (SMK) leads to poor learner performance (Baumert et al., 2010). In South Africa, research on teacher SMK has primarily focused on primary schools and pre-service teachers, with emerging research (but still low) on secondary school teacher subject matter knowledge (Ibeawuchi, Ogbonnanya, & Mogari, 2015). Learning deficits have been identified in poor schools in South Africa where many learners are performing well below their grades-appropriate levels (Spaull & Kotze, 2015). These learning deficits are expanded when learners progress to secondary school. In fact, Spaull and Kotze show that Grade 12 learners from the poor schools are five grades-appropriate levels below learners from affluent schools. This means that Grade 12 learners from poor schools are at Grade 8 level, which partly explains the poor performance of Grade 12 mathematics learners.

Studies on teacher subject matter knowledge conducted in South Africa have shown pre-service teacher educators displaying poor SMK on functions (Ibeawuchi, Ogbonnanya, & Mogari, 2015; Ndlovu, 2019); and no improvement in pre-service teachers' performance across their studies (Fronseca, Maseko, & Roberts, 2018). None of the studies conducted explored pre-service or in-service teachers' SMK on probability. The poor performance in Grade 12 probability, as indicated earlier, is also attributed to learners' inability to make meaning of multiplication in counting principles which impair their ability to work with the factorial notation, a specialised form of multiplication.

There are few studies that have been performed to study the existence of multiple representations in probability lessons and tasks (see chapter 2 for details). Studies that have been carried out include the investigation of models for statistical literacy and statistical thinking (Ben-Zvi & Garfield, 2004). Lockwood (2013) developed a framework to explain combinatorial (i.e. permutations and combinations) thinking of students. Lockwood's framework has three parts, namely, formulae/expressions, counting processes and sets of outcomes. *Formulae/expressions* are the algorithms of mathematics that produce outcomes or answers to counting problems. *Counting processes* are the iterative or enumerative steps that are considered by learners to find solutions to counting problems. *Sets of outcomes* are the lists of possible solutions generated by the counting processes. Lockwood's framework can only explain how learners think in terms of counting. However, the model does not explain whether learning resources that are available online or in classrooms use multiple representations to represent counting and probability. A framework is thus needed to explain whether there is presence of multiple paths to solutions in observed lessons and whether tasks in textbooks and study guides available online promote high levels of cognitive demand, which points to those tasks that can be answered using multiple strategies.

1.6 Objective of the research

The effective learning of Grade 12 mathematics (including the learning area of probability) is a function of teacher mathematical knowledge of teaching and learning resources that promote conceptual understanding through the use of tasks promoting multiple representations (Ball, Thames, & Phelps, 2008; Stein, Smith, Henningsen, & Silver, 2000). Teachers with strong subject matter knowledge will enable learners using online resources to make meaning and construct probability knowledge. Learning materials accessible online should stay away from using formulae as primary tools for learning, but they should promote reasoning, the use of diagrams and other forms of representation. The aim of the research is thus to firstly investigate the presence of multiple representations in online Grade 12 probability video instructions. Secondly, the research seeks to investigate whether textbooks and study guides that are accessible online, and are aligned to the CAPS curriculum, promote the solving of Grade 12 probability problems through multiple representations.

1.7 Research questions

The research report seeks to provide answers to the following questions:

1. In what ways do the observed online video lessons promote multiple representations in Grade 12 probability tasks?
2. How does higher order thinking necessarily promote multiple representations of probability in Grade 12 online mathematics textbooks and study guides?

1.8 The structure of the research report

This research report is structured as follows: Chapter 1 introduces the aims of the research. The chapter starts by giving background of the South African context where some social challenges around access to online learning resources are outlined. Also outlined in chapter 1 is the mathematics curriculum in the South African schooling system. It is shown that learners start learning probability in Grade 4 and learn to calculate probability from Grade 7. The poor historical performance of Grade 12 in probability is also laid to bear. Gaps in literature are identified such as the absence of a study that explores multiple representations of probability in popular learning materials that can be accessed online. Lastly, research questions arising from the gap in literature are asked. Answers to these research questions are provided in chapter 5 of this report.

Chapter 2 reviews the academic literature on probability and provides for the theoretical frameworks and an analytical framework concerning the learning probability. The second chapter starts by providing explanations to concepts of counting (a subset of combinatorics) and probability which are widely used in this research report. Since the approach of this research report is to look at how students learn in remote environments (similar to Covid-19 prompted lockdowns), a constructivist theoretical framework is adopted. Also adopted in this chapter is the theoretical framework of mathematical knowledge for teaching explaining probability knowledge for teaching. Lastly, chapter 2 considers the Mathematical Task Framework developed by Stein, Smith, Henningsen and Silver (2000) as the analytical framework used to explore multiple representations of probability tasks found in textbooks and study guides.

Chapter 3 begins by comparing lesson observation instruments that are used to collect data. The Mathematics Classroom Observation Protocol for Practices (MCOP^2) is the preferred instrument for observing video lessons. The rationale for preferring the Mathematical Task Framework for classifying the cognitive demand levels of tasks found in e-textbook and e-guides is then outlined. The chapter also lists the sources of data that are used, consisting of three YouTube video channels, as well as a textbook and two study guides. The chapter concludes by explaining how ethics protocols were complied with.

In Chapter 4 data captured on the MCOP^2 instrument and the Mathematical Task Framework are used to make statistical calculations. Data generated by the MCOP^2 instrument is used to calculate statistical numeric measures such as the mean, mode, median, standard deviation and the quartiles. Thereafter, the analysis of variance (ANOVA) test is performed to determine whether the null hypothesis that there is no mean difference between the three online video sources can be accepted or rejected. The same process of calculating summary statistics and performing hypothesis tests using the ANOVA test is performed for the three sources constituting online learning materials.

Chapter 5 provides answers to the two research questions. The analysis of data performed in chapter 4 is used to answer the research questions. In addition, for the video instructions, screenshots of the video lessons are used in providing answers to research question 1. The rest of chapter 5 outlines challenges that were experienced during the research, provide reasons to the limitations of the research, and identify areas for future research.

1.8 Chapter summary

The first chapter of this research report outlined the background, the purpose, the research questions as well as the structure of the research report. The chapter started by giving context to the South African schooling system of grades at which probability appears in the mathematics curriculum. The historical poor performance of Grade 12 learners in the learning areas of counting and probability was shown to have not improved since 2014, with about the same feedback provided to teachers by the department of education which made various recommendations in its diagnostic reports. Gaps in the teaching and learning of probability were identified especially within the context of remote learning, where online video lessons and learning materials are used by learners. Two research questions were posed and the structure that this research report is adopting was then outlined. The next chapter reviews literature on how students learn probability, what teacher subject and pedagogical content knowledge are needed for probability instructions, as well as the framework describing tasks that promote multiple ways of representing probability.

CHAPTER 2: LITERATURE REVIEW, THEORETICAL AND ANALYTICAL FRAMEWORKS

2.1 Introduction

Chapter 2 starts by defining the concepts of counting and probability. The constructivist approach to learning is introduced with a particular focus on how learners engage with counting at different stages of mental development. The constructivist approach is relevant as learners are actively constructing their own learning, using online learning resources. The mathematical knowledge for teaching as a theoretical framework is then outlined as teachers in the video lessons are expected to have the subject matter knowledge and the pedagogical content knowledge to assist learners with probability. The chapter concludes with the mathematical task framework (MTF) as the analytical framework to be used by the research report for assigning levels of cognitive demands to tasks found in the online learning materials.

2.2 Counting and probability.

Before providing a review of literature on how students learn counting and probability as well as the discussion on the role of instruction, let me first explain both the concepts of counting and probability.

2.2.1 The concept of counting

Combinatorics is the oldest branch of discrete mathematics which was developed in the 16th century and it is primarily about counting (English, 2005). Batanero, Godino and Navarro-Pelayo (1997) use Jacob Bernoulli's metaphor to define combinatorics "as the art of enumerating all the possible ways in which a given number of objects may be mixed and combined so as to be sure of not missing any possible result" (p. 181). In fact, combinatorics is viewed as a calculus tool for probability owing to its wide application in other branches of mathematics such as number theory, topology, graph theory, combinatorial geometry, operational research and artificial intelligence (Batanero & Sanchez, 2013). Batanero, Godino and Navarro-Pelayo (1997) list different ways of expressing combinatorial procedures, what I refer to as multiple representations in this research report, as follows: logical (inclusion/exclusion); graphical procedures (tree diagrams); tabular (tables and arrays);

numerical (multiplication and factorial notation); and algebraic. This research report pays close attention to whether teachers use these different forms of representation for counting in their instructional practice, and whether online learning materials such as textbooks portray counting and probability using two or more of these representations.

2.2.2 The concept of probability

Probability involves a degree of belief of some events occurring relative to all possibilities of events (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). Bryant and Nunes (2012) allude that the concept of probability is best explained by viewing it in terms of the concepts of randomness and sample space. Randomness entails variation, unpredictability or uncertainty of events occurring. A sample space is the set of all possibilities of events occurring. To calculate sample space, a combinatorial schema is required (Bryant & Nunes, 2012). By combinatorial schema Bryant and Nunes refer the internalised ability of learners to develop or list all elements constituting events. Batanero, Chernoff, Engel, Lee and Sanchez (2016) further explain that "combinatorics is used in listing all the events in a sample space or in counting (without listing) all its elements" (p.15).

Quantifying probabilities involves the computation of events occurring expressed as a proportion of the sample space. Probabilities vary between values 0 and 1, where 0 represents an event that will not occur, while 1 represents an event that will certainly occur. Probabilities are expressed as decimals, fractions, percentages or as ratios.

Probability can also be classified in terms of its orientations. There are at least three philosophical orientations to probability, namely, the classical orientation, the frequentist orientation and the subjective orientation (Gomez-Torres, Batanero, Diaz, & Contreras, 2016). The classical view of probability is where probabilities are presumed to be fixed based on prior generalisation such as the probability of $\frac{1}{2}$ assigned to an outcome of finding a head when a coin is flipped (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). Posteriori probabilities are determined through trials or experiments and are aligned to the frequentist approach. In the frequentist approach, probabilities converge to classical probabilities after a large number of trials are performed.

As it will later be shown, probabilities based on both the classical and frequentist views may not always be equal because of the small number of repeated trials (Kazima & Adler, 2006). It would have made sense if learners were permitted by the South African mathematics curriculum, and through textbook tasks, to repeat trials in large numbers to show that probabilities based on many repeated trials converge to classical probabilities.

Lastly, the subjective view of probability is based on personal beliefs about outcomes of future events subject to certain conditions (Gomez-Torres, Batanero, Diaz, & Contreras, 2016). These orientations of probabilities are important when computing probabilities of events involving repetition and replacement.

2.3 Students learning of counting and probability

Resources that are available online must be viewed as learning resources by learners. Learners, in most cases, will have access to these resources outside of the classroom environment where they will have to construct their own meaning and make sense of these resources. Lockdown regulations enacted from the spread of Covid-19 have made individual learning from home more likely. The starting point to understand how learners make meaning of mathematical concepts is the constructivist learning theory.

2.3.1 Constructivist learning theory

Piaget (1964, 2003) posits that mental structures or schemas develop through four stages, namely (i) the sensory-motor, pre-verbal stage, (ii) the pre-operational representation stage, (iii) the concrete operational stage, and (iv) the formal, hypothetico-deductive, operational stage. Piaget further identified four factors that explained the movement between these stages of mental knowledge structures. These four factors are maturation, experience, social transmission and equilibration or self-regulation. Piaget showed that, out of these four factors, only equilibration or self-regulation was adequate to explain movements between the stages of mental development. The other three factors were inadequate in ensuring movements between the stages of mental development. The learning of counting and probability resembles the age dependent Piaget's stages of mental development (Batanero, Godino, & Navarro-Pelayo, 1997).

Learning, in the constructivist learning theory, is said to occur when a new reality, or new knowledge, is integrated into mental schemas. This process is called assimilation (Piaget, 1964, 2003). When new knowledge cannot be assimilated to the schemas, cognitive conflict or misconception ensues. It is through repeated reinforcement of this new knowledge or through assistance by a teacher (who guides the thinking process) or through assistance from learning resources that this new knowledge gets accommodated by the schemas (Piaget 1964, 2003). Once accommodated, this new knowledge becomes part of the schemas through a process of equilibration or self-regulation (Piaget, 1964, 2003).

This current research will view learners as constructing their own knowledge, using online resources at their disposal. These resources include online textbooks, study guides and videos accessible on popular platforms such as YouTube. The constructivist theory of learning posits that as learners use these resources available to them, new knowledge will be matched to existing schemas. If this new knowledge of probability matches existing knowledge in the schemas, then learning will occur. However, if this new knowledge is not assimilated by the schemas, reinforcement in the form of various online learning resources will need to occur until such time that knowledge is accommodated by the schemas (Piaget, 1964, 2003). These different types of resources serve to assist learners to accommodate the learning area of probability until the knowledge becomes part of their schemas through the process of self-regulation.

As noted in Chapter 1, probability is introduced in Grade 4 in the South African curriculum through the use of physical devices such as the dice, coins, spinners, and balls in bags (Department of Basic Education, 2012a), which is consistent with the concrete operational stage of cognitive development by Piaget, where 7 to 11-year olds use physical devices to attach meaning to concepts. These physical devices are basically used to explain the concept of chance and randomness to learners by assuming that the chances of particular outcomes occurring are equal (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). Learners then use these concrete objects to perform activities (either of flipping a coin or throwing a die) where they count the number of outcomes and make meaning to themselves that the outcomes of events are equal after many trials.

These physical devices could be combined to get compound events such as when learners throw two dice simultaneously. To count the possibilities of outcomes of compound events, representational tools such as tables and tree-diagrams are useful. These representational tools are also used to compute probabilities. Compound events are introduced in the senior phase in the South African mathematics curriculum when learners are above 11 years (DBE, 2012b), which corresponds to the formal, hypothetico-deductive, operational stage of development in Piaget's framework. At this stage of mental development, especially in Grade 12, the use of representational tools such as the tree diagrams can be retained by learners. However, learners are expected to explore other forms of representations, such as numeric or algebraic, to generalise and make abstraction of probability in order to apply these concepts in novel situations.

In Piaget's cognitive development framework, learners move from the concrete operational stage to the formal operational stage from the age of 11 years. However, researchers have recommended that it is best for learners to first reach the age of 15 years before the concept of permutation (i.e. arrangements of objects) is introduced to them (Batanero & Sanchez, 2013). The late introduction of permutation is supported by the premise that learners have to first assimilate the concepts of counting in compound events before developing the conception of arranging within the compound events. Permutations also involve choosing events when ordering of such events matters, adding to the complexity of combinations. In the South African mathematics curriculum, permutations are introduced in Grade 12 when learners are mostly between the ages of 16 and 18 years, which is a slight delay when compared to the recommended age of 15 years. Researchers have also suggested that some learners, and even adults, never reach the level of permutations, despite working at the formal operational level of mental development, without the intervention of teachers (Batanero & Sanchez, 2013).

While Batanero and Sanchez (2013) recommend that permutations be introduced when learners are 15 years old (or are in Grade 10), Kimani, Gibbs and Anderson (2013) have shown that Grade 6 learners (or 11-year-olds) in the United States have managed to solve problems on permutations using representational tools such as tree diagrams and unifix cubes (i.e. the interlinking cubes that are used as manipulative for counting). In South Africa, permutations are usually introduced in an abstract form using the factorial formula. This calls for a review of the curriculum and the early introduction of permutations in the Senior Phase where the use of graphical or concrete representations should be encouraged in line with the findings from the study by Kimani, Gibbs and Anderson.

English (1991) conducted research involving 50 children between the ages of 4 to 9 years to ascertain the age at which they could solve problems involving counting principles. This experiment was inspired by Piaget's (1964, 2003) works on stages of mental development. English showed that 17 out of 24 children between the ages of 4 to 6 years could not perform a simple problem using counting principles without the assistance of adults. However, for the age group of 7 to 9 years there were children who performed basic counting using concrete objects, without the assistance of adults. In the task learners were asked to dress teddy bears using pants and tops of various colours. Learners were asked to indicate the number of ways in which these teddy bears could be dressed when the number of colours of pants and tops were varied. The result was that a number of learners (4 out of 26) randomly fitted pants and tops of different colours on the bears, while a substantial number of learners (22 out of 26) developed a systematic approach such as fitting teddy bears with one colour of the pants and varying the colours of tops. This was repeated for another colour of pants while colours of tops were varied. The last group of learners developed combinatorial thinking in counting. This also shows that learners can work independently from the ages of 7 to 9 years, although the role of the teacher cannot be substituted for children aged 4 to 6 years. This important result shows that basic counting principles could be introduced in elementary grades in schools when children are in the concrete operational stage of cognitive development. This systematic reasoning would form the basis of generalising counting principle when children reach the formal operational stage of cognitive development.

In another study, English (1999) conducted a research based on thirty-two 10 year-olds who were given tasks with a goal of testing their ability to solve two and three-dimensional combinatorics problems. English showed that middle grade school going children could represent two-dimensional problems (i.e., combining two groups) based on counting principle using graphs (drawing, listing and tree diagrams) and in terms of its multiplication property. With regards to three-dimensional tasks (i.e., counting 3 groups), only few learners correctly represented the solution using graphs, while even a smaller number of children managed to use the multiplication property correctly. However, these learners could not explain how they reached solutions to combinatorial problems. Multiplication was represented as repeated addition instead of the generalised form. This result is consistent with the South African mathematics curriculum at the Intermediate Phase where learners can count and make predictions on the number of times outcomes occur in repeated trials (DBE, 2012a).

However, the curriculum does not encourage the use of graphic representation nor the numerical representation to count the outcomes of trials. This result from English (1999) study also shows that learners can represent counting principles and probability in multiple forms (using graphic and numerical procedures) when they reach 10 years of age, at Grade 5 level.

As mentioned in Chapter 1, it is the department of basic education that provides the recommendation that teaching and learning of counting and probability should involve multiple representations. This recommendation is in line with English (2005) who provided a list of suggestions to increase children's conceptions of combinatorics. According to English (2005), combinatorics should foster independent thinking, encourage flexibility in approaches and representations, focus on problem structures, encourage the sharing of solutions, provide problem-posing opportunities and provide novel probability problems. Other researchers such as Batanero, Godino and Navarro-Pelayo (1997) have suggested that "both the teaching and assessment of combinatorics should be based on solving different combinatorial problems in which students need systematic enumeration procedures, recurrence, classification, tables, and tree diagrams" (p. 185), which is more specific and closely aligned to the recommendations of multiple ways of representing counting and probability in South Africa.

2.4 Probability knowledge for teaching

2.4.1 Mathematical knowledge for teaching

Video instructions on counting and probability can be viewed on popular online platforms such as YouTube. Teachers in these video instructions should have knowledge of the mathematics topic that is presented. I make the assumption upfront that the online video instructions are delivered by qualified educators, motivated by the profile of the three video channels I have sampled for this research (see chapter 3 for more details).

In two seminal addresses, Shulman (1986) distinguished knowledge that a teacher has on a subject which is deeper than knowledge prescribed by the curriculum. He called this subject matter knowledge or content knowledge (CK). Shulman also defined knowledge that teachers have on a subject that include knowledge of students' misconceptions, errors, learning difficulties and tasks set for lessons (Krauss, et al., 2008). He called this knowledge pedagogic content knowledge (PCK). Researchers have shown that teachers who possess strong content knowledge of mathematics produce students who score high marks in standardised tests (Baumert, et al., 2010). In the South African context, the education department has used this result to prescribe standards to develop teachers who have strong content knowledge so that outcomes in the basic education sector are improved (Department of Basic Education & Department of Higher Education and Training, 2011).

The mathematical knowledge for teaching (MKT) is the amalgamation of subject matter (content knowledge) and pedagogical content knowledge. MKT assumes that learning is situated in the practice of teaching (Kazima & Adler, 2006). MKT is defined as the mathematical knowledge that teachers use in their instructional practice which allows students to flourish (Hill, Ball, & Schilling, 2008). Ball, Thames and Phelps (2008) further outline content knowledge subdomains, which consist of common content knowledge (CCK) and specialised content knowledge (SCK). CCK is defined as knowledge of content of mathematics that is common to everyone familiar with mathematics, while SCK is the knowledge of content unique to teachers. SCK is thus the knowledge of content that only teachers of mathematics would have.

Features of SCK include the teachers' ability to "make representations and giving explanations of curricular items" as well as "how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill, Ball, & Schilling, 2008, pp. 377-378). Flores Medrano, Escudero and Carrillo Yanez (2013) explain SCK as "selecting and designing class activities, and making representations and giving explanations of curricular items [and] one skill demanded by SCK is that of interpreting mathematical productions, both those generated by students and those to be found in materials" (p. 3057). It is expected that teachers in online video instructions should possess SCK, particularly those who are teaching probability at Grade 12 level where the teachers would be expected to present probability through multiple representations.

Subdomains of pedagogic content knowledge include knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) (Ball, Thames, & Phelps, 2008). While content knowledge unique to teachers of mathematics cannot be substituted, teachers also need KCS which entails the knowledge of how students learn mathematics, how students use semiotic tools and reasoning in different mathematics learning areas. For this research, KCS entails the knowledge of teachers on learners' thinking and reasoning with regards to counting and probability including selecting tasks that will address misconceptions that may arise, whose context will accommodate all learners. KCT on the other hand has to do with how teachers select, sequence and pace tasks in appropriate ways such that all learners are given sufficient opportunities to work through the tasks.

2.4.2 Probability teaching in South Africa

Brijlall (2014) studied the mathematical knowledge for teaching (MKT) in the learning area of probability for Grade 9 in-service teachers in South Africa. Brijlall asked in-service teachers to come up with a sequence of lessons when introducing probability. In the first three lessons, the majority of teachers responded by claiming that they start probability lessons with definitions followed by lesson two where expressions of probability as fractions, decimals or even ratios were outlined. It was during lesson three where teachers started giving learners probability tasks.

Results of Brijlall's study showed that teachers revealed their common content knowledge on probability only in lesson three where tasks needed application of procedures that anyone familiar with probability could do. According to Brijlall lessons one, two and three all required teachers' specialised content knowledge as only a teacher could provide definitions, explanations and meaning of what probability values denote, and issues related to tasks such as the use of language and the meaning of the word "then" in a probability tasks to signify that there are events that precede the current event (i.e. conditional probability). The use of such words in conditional probability tasks points to a teacher who knows the meaning of words used by learners, which is knowledge of the content and of students (KCS). The relevance of Brijlall's (2014) study is that it shows how teachers structure their lessons when introducing probability and which part of the MKT domains are activated during instructions.

Kazima and Adler's (2006) paper on mathematical knowledge for teaching (MKT) probability in South Africa starts from the premise that probability is a relatively new topic that made appearance in schools only in 1992, and formally became part of the curriculum when Curriculum 2005 was introduced in 1997. Teachers' CCK and SCK during the 1990s were low as they had no prior knowledge and experience in teaching probability. Teachers could only acquire CCK through university training and professional development in probability, and acquire SCK through instructional experience and professional development. What made it even more challenging were reforms to the curriculum when Curriculum 2005 was discontinued, with the National Curriculum Statement (NCS) enacted.

NCS made it optional for Grade 12 mathematics learners to write paper 3 where probability was assessed (Mutodi & Ngirande, 2014). In other words, teachers did not have mandatory duty to instruct learners on probability in Grade 12 save for few learners who elected to write paper 3. This meant pre-service teachers at university who did not write paper 3 in Grade 12 were at a disadvantage at universities. Those teachers who did not take the optional paper 3 in Grade 12 may have elected not to teach probability once trained as teachers due to insufficient training in probability at universities or due to them being uncomfortable to teach probability (Jones & Tarr, 2007). This instructional disjuncture was rectified with the enactment of the current CAPS document which discontinued the optional paper 3 for mathematics (DBE, 2011).

2.4.3 Content knowledge and pedagogical content knowledge for probability

While mathematical knowledge for teaching is essential, teachers also need probability knowledge for teaching (PKT) (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). Like MKT, PKT can be divided into probability content knowledge and pedagogical content knowledge. Probability content knowledge requires teachers to have specialised training in probability beyond the content area covered in high school (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). This requires teacher training universities to have programs that focus on both theoretical and procedural aspects of probability.

Pedagogical content knowledge for probability on the other hand differs with mathematical PCK in that the reversibility of probability experiments or trials from abstract to concrete does not always hold compared to the reversibility in areas of algebra or geometry (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). In other words, it is difficult to generalise and make an abstraction of probability of events arising from experiments such as rolling of dice unless these experiments are repeated many times. Moving from the theoretical probability to the concrete result, confirmed through experiments, may result in misconceptions to learners especially since the concept of law of large numbers is never covered in South African school curriculum. Another challenge that makes pedagogical knowledge of probability different to mathematical PCK is that teachers must present both concepts and applications of probability while at the same time make clear the meaning of probability concepts as well as controversies that are associated with these concepts (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). Lastly, probability knowledge for teaching entails teachers knowing how learners reason probabilistically and how attitudes towards probability affect learners' achievement (Batanero, Chernoff, et al., 2016).

Gomez-Torres, Batanero, Diaz and Contreras (2016) studied primary school prospective teachers' probability content knowledge. Gomez-Torres and colleagues created their own questionnaire to measure the CCK, SCK and horizontal content knowledge of prospective teachers. Results from Gomez-Torres et al.'s study show that these prospective teachers performed better in questions that measured their CCK, followed by questions measuring their horizontal knowledge for teaching, and lastly, the questions measuring their SCK. Performance on the questionnaire items, classified in terms of the three probability philosophical orientations (i.e. classical, frequentist and subjective), results show that prospective teachers performed well on the classical oriented questions. However, the performance was below par on items that were based on the frequentist and subjective views, showing teachers' lack of proportional reasoning and the inability to distinguish between theoretical probability and probability from trials.

Knowledge of content and students in probability include teachers knowing how factors such as gender and cultural background affect the way students learn. Mutodi and Ngirande (2014) carried a study in 5 different schools in one region in the Limpopo province, South Africa, in which learners between Grade 10 to 12 were given a common test. Mutodi and Ngirande classified learners according to grade levels, gender and home languages. The results from this study showed that female learners underperformed male learners, meaning that the difference in performance along gender lines was statistically significant. However, the authors did not investigate the cause(s) explaining the unequal performance along the gender line. The performance of learners did not show statistical difference along their grades level. Mutodi and Ngirande explained that despite the expectation that Grade 12 learners would outperform Grades 10 and 11, this did not materialise. Lack of teacher training was one of the factors listed by the authors explaining this surprising result. Further, the results of the study revealed statistical insignificance in performance differences along home languages, meaning that learners' home languages were not explainers of different performances. Although the results of this study are useful to teachers to show that female learners require instructional support, however without knowing the exact factors responsible for this difference, teachers may not use effective instructional support strategies to assist female learners.

Batanero, Godino and Navarro-Pelayo (1997) outline some of the instructional considerations teachers should take heed on counting and probability. Batanero and colleagues mention that the tree diagram is one of the forms of representation that is widely recommended for use by learners to solve probability problems. Despite this usefulness, teachers should be aware that learners find the use of tree diagrams challenging and is the cause of many errors that learners make (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016).

Batanero, Godino and Navarro-Pelayo also suggest teachers should be aware that learners usually use "non-systematic" representation that does not lead to generalisation and abstraction. This is useful for the South African context where learners' performance on counting and probability is poor. Further, according to Batanero, Godino and Navarro-Pelayo, learners usually make errors with ordering events, which is consistent with the observation of diagnostic reports of the department of education with regards to counting and probability (DBE, 2015a). Repetition is listed as an error that learners make in counting according to Batanero and colleagues, which is again congruent to the observation that is made in the DBE's diagnostic reports. Other errors that learners make, and are unique to counting and probability, are confusions on subset, misconceptions with regards to partitioning, and misunderstanding of objects to be counted (Batanero, Godino, & Navarro-Pelayo, 1997). All these are errors that learners make that teachers should be aware of as part of specialised content knowledge on probability.

2.4.4 Recommendations on teaching probability

English (2005) lists six instructional recommendations that teachers can employ to make combinatorial ideas accessible. Firstly, teachers need to promote independent thinking of learners, only intervening through scaffolding and revoicing of learners' original ideas. Secondly, teachers need to support learners to use flexible strategies and representations as many counting and probability problems are novel in nature. Thirdly, learners are to focus on structures of problems or on how different objects are related instead of focusing on objects themselves. Fourthly, teachers are to encourage learners to share solutions with their peers so that their work is put under scrutiny and to share different strategies used by other learners to find the correct solution. Fifthly, learners must be allowed to construct their own problems so that they can see the structure of combinatorial problems, depicting the known and unknown. Lastly, teachers can give novel tasks to enable learners to apply learned strategies to new situations. All these six instructional recommendations are useful in a classroom set-up where there is teacher-learner interaction. In online video instructions, teachers can only recite these recommendations to learners without any form of direct mediation. It appears that only the second, third and the sixth recommendations can be implemented by teachers in online video instructions.

2.5 Textbook and study guide tasks

Textbooks and study guides are used as learning resources. Textbooks in particular are used as primary learning resources in South Africa. Study guides on the other hand are additional resources complementing textbooks. Online textbooks are additions to physical (hardcopy) textbooks supplied to learners. For online learning materials to be widely used, they must at least match the standard of quality of physical textbooks and be recommended by education authorities.

There is usually a disjuncture in what the mathematics curriculum on counting and probability expects from teachers and learners, with how learning materials outline counting and probability, as well as a disjuncture to what education authorities expect teachers' instructions to look like. All these three (i.e., curriculum, teachers and learning materials that are accessible to learners) must pull in the same direction of representing probability in multiple ways.

2.5.1 Textbook and study guides as teaching and learning resources

One important teacher attribute mentioned in the section on mathematical knowledge for teaching is the knowledge of content and curriculum. Teachers need to ensure that what the curriculum prescribes and the tasks outlined in the learning materials are communicated clearly to learners (Jones & Tarr, 2007). In fact "school textbooks provide examples and teaching resources, some texts present too narrow a view of probabilistic concepts or only one approach to probability" (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016, p. 23). Any gap therein needs teachers' intervention. Also, teachers might misinterpret the intentions of authors of mathematics textbooks (Jones & Tarr, 2007).

Teachers may also not cover probability tasks in the learning materials due to time constraints (Jones & Tarr, 2007). This arises from the challenge that probability is the last topic in the South African curriculum, both in the General Education and Training (GET) as well as in the Further Education and Training (FET) phases. Jones and Tarr mention that only about 75% of textbook content gets covered by teachers, which places probability at risk of not being taught in the GET and FET phases. Although there are no studies suggesting that the reason for poor performance in probability by Grade 12 learners is because of insufficient curriculum coverage, it can be inferred that little probability coverage takes place in earlier grades which could be one of the reasons for poor probability performance in Grade 12.

2.5.2 Mathematical task framework

In their seminal book, Stein, Smith, Henningsen and Silver (2000) outline the mathematical task framework (MTF). The MTF seeks to explain how students use tasks to learn over three phases, (i) tasks of the curriculum found in learning materials such as textbooks (the primary unit of instruction), (ii) teachers who select tasks that are used in the classroom, and (iii) students who implement such tasks (Stein, Grover, & Henningsen, 1996; Stein, Smith, Henningsen, & Silver, 2000). Within the context of this research report, the online learning materials (e-textbook and e-study guides) constitute the first phase of the MTF. Since teachers are not within reach of learners in remote settings, the second phase of MTF is largely negligible. It can be argued that online video instructions could be used as a substitute for teachers. However, online videos have teachers who select tasks from sources that are different to those learners use. Also, online teachers in video lessons demonstrate strategies to find solutions to tasks. Teachers in online videos are in the same position as learners in the sense that they are both implementers of tasks, which is the third phase of MTF. However, in this research report only learners are assumed to be the implementers of tasks found in e-textbook and e-study guides.

This research report aligns itself with the definition of a probability task offered by Jones and Tarr (2007). Jones and Tarr define a task as "an activity, exercise, or set of exercises in a textbook [and study guides] that has been written with the intent of focusing a student's attention on a particular idea from probability" (p. 13). What is not a probability task is also well defined by Jones and Tarr where they explain that the "sections of probability lessons that contain narrative, such as definitions or written explanations of concepts and procedures, are not considered as probability tasks" (p. 13).

2.5.3 Cognitive demands of tasks

The mathematical task framework (MTF) classifies tasks found in learning resources in terms of either high or low cognitive demand levels. Tasks that are set at a high cognitive demand level use multiple strategies to get towards the solution (Stein, Grover, & Henningsen, 1996). Low cognitive demanding level tasks are preoccupied with reproducing known knowledge. The MTF classifies tasks into four cognitive demand levels, namely, memorisation, procedures without connections, procedures with connections and doing mathematics (Stein, Grover, & Henningsen, 1996). Tasks promoting memorisation and procedures without connections require less thinking. Tasks that can be solved through procedures with connections and doing mathematics are cognitive highly demanding. Tasks of high cognitive demand level will have more than one way of finding solutions (Stein, Grover, & Henningsen, 1996). Thus, probability tasks found in online learning materials must be of high cognitive demand level, which is congruent to the recommendations of DBE that advises the use of multiple strategies and representations in probability tasks.

While Stein, Smith, Henningsen and Silver (2000) started from tasks as a unit of instruction where only textbook exercises where considered, Mellor, Clark and Essien (2018) considered not only textbook exercises in their comparative study of German and South African textbooks, but also considered the content of explanations. Mellor, Clark and Essien also added a fifth category, called "reading for understanding" (p. 5), in addition to the four categories of the MTF of Stein et al. (2000). The objective of Mellor and her colleagues was to compare whether textbooks used by learners in Germany and South Africa focused on procedural and conceptual understanding, and whether multiple representations and real-world examples are used by these textbooks to explain the mathematical concept of linear functions. One result from this study was that the South African Grade 9 textbook had exercises where each question had subsections to guide learners towards answers. This style of posing questions reduced task's cognitive demand. Also, the South African textbook used less real-world exercises compared to the German textbook, resulting in learners' mathematical proficiency to be low on productive disposition (Kilpatrick, Swafford, & Findell, 2001).

These two results may appear to be in contradiction with each other. Mellor, Clark and Essien (2018) however argue that questions of similar cognitive demand level when repeated (using multiple representations or not) tend to lower the overall cognitive demand level of tasks as learners end up being proficient in applying procedural knowledge across different representations instead of choosing their own representation to answer questions. This is an important result for this research as learning materials that form part of the sample are from South Africa where there might be tendencies to repeat questions of similar cognitive demand level, leading to learners applying similar procedures on such questions, reducing the level of cognitive demand in the process. I acknowledge that Mellor et al. study was based on a Grade 9 textbook, instead of a Grade 12 textbook which is the focus of this research.

An advantage of the mathematical task framework in online learning materials is that tasks' cognitive demand levels are preserved. Teachers cannot intervene to lower high cognitive demanding tasks. This is consistent with the constructivist theoretical framework. Teachers appearing in online video lessons can be viewed as mediators to select tasks and help learners to move through the zone of proximal development (ZPD) (Vygotsky, 1978). However, tasks found in online videos are different to tasks found in learning materials, thus discarding the role of the teacher to intervene within the MTF structure.

2.6 Chapter summary

Chapter 2 introduced counting, which includes the fundamental counting principles, permutations and combinations. The concept of probability was then explained together with the way it is related to counting. The chapter then reviewed the literature centred on how learning takes place from a learner's perspective when she or he is alone (i.e. no immediate mediation from a teacher or a knowledgeable other). A constructivist theoretical framework is adopted to show that the learning of counting and probability is directly linked to Piaget's cognitive development stages. However, at the formal operational stage, the reversibility property that can be seen in mathematics content areas such as algebra and geometry does not always hold true for probability when the classical and the frequentist philosophies are reconciled. This is so because by the time learners start to calculate probabilities in the Senior

Phase of the curriculum, they would have only performed limited trials to count outcomes of events. The law of large numbers, if covered in school curriculum, could restore the reversibility property between the classical and frequentist approaches.

Teachers should have subject matter knowledge for the area of probability that will not only be restricted to fluency in solving problems, but also being able to use multiple representations to teach probability. Learning material accessible online must have probability tasks that lead to high cognitive demand levels as it has been argued that such tasks involve multiple ways of finding solutions. The focus now shifts to outlining methodologies that are used to collect data for multiple ways of representing probability in observed video lessons and in online learning materials.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter is concerned with the design of the research and methods used to collect data from a variety of sources. The chapter starts by outlining reasons for the use of the quantitative approach to collect and analyse data. Data collection instruments are then presented. Thereafter, data sources for video instructional lessons and online learning materials are outlined. The manner in which the research complies with ethical standards is then explained. The sequence that is followed in this chapter is aligned with Hitchcock and Hughes' logical sequence of research from their 1995 study, as cited by Cohen, Manion and Morrison (2007), where it is held that "ontological assumptions give rise to epistemological assumptions; these, in turn, give rise to methodological considerations; and these, in turn, give rise to issues of instrumentation and data collection" (p. 8).

3.2 Research design: a quantitative approach

In the first quarter of the year 2020, the world, including South Africa, was concerned with finding innovative ways of teaching and learning in remote environments due to the widespread of Covid-19. This meant collecting data from primary sources such as teachers and learners was out of reach. Rather, secondary data sources are preferred for data collection. Resources for teaching and learning such as online videos, e-textbooks and e-study guides are the sources of data. The challenge that confronts this research is how to organise and make the use of these resources effectively.

A lesson observation protocol and a task framework are used to collect and sort data such that it is ready for analysis. A quantitative approach to research is adopted. This approach entails the use of statistical techniques to present results in a robust way such that hypotheses that are formulated are either accepted or rejected. These hypothetical claims must be free of the researcher's subjective beliefs and bias so that results are presented in a credible manner (Scott & Morison, 2006). Corroboration of evidence from the different data sources ensures that there is triangulation of data. In this research, data is collected from three online video sources and from three online learning material sources.

Video lessons require transcription to explain words that teachers use for explanations. Also, pictorial segments are required to provide evidence to what the teachers say. In that respect, screenshots (also called static video clips) of what teachers write to represent explanations are made available in chapter 5 where research questions are answered. These screenshots are also reproduced to support finding from chapter 4 of this research report.

3.3 Research instruments

3.3.1 Pre-recorded video instructions

This research involved the collecting of data from pre-recorded video lessons that are available on the internet. The research makes use of video instructions from organisations that have good track records, whose reputations are regarded highly by the Department of Education (DBE, 2020b). This is done purely on the premise that there are thousands of videos on probability on the internet, and it would make sense to only focus on video lessons that comply with the Grade 12 CAPS curriculum and are recommended by the Department of Education (DBE, 2020b). The learning area of focus on these videos is Grade 12 counting and probability.

In pre-recorded video lessons there are no learners, making the teacher-learner interaction impossible. Also, in pre-recorded lessons teachers are not confronted with situations where learners may interrupt lessons to show a general lack of understanding. A benefit of pre-recorded video lesson is that observers act independently to measure the quality of the instruction and can replay the recording to pay attention to specific instances of a lesson (Boston, Bostics, Lesseig, & Sherman, 2015). Another advantage of a video lesson is the ability of observers to measure the quality of the instruction without the need to be present in class (Boston, et al., 2015).

3.3.2 Observation instruments

The choice of the observation instrument to use for measuring the quality of instructions for multiple representations of probability requires careful consideration. Boston, Bostics, Lesseig and Sherman (2015) reviewed three widely used observation instruments. First is the Reformed Teaching Observation Protocol (RTOP) which consists of 25 items on a Likert-scale questionnaire and is used for measuring adherence to reform standards of instructions in mathematics and science for K-12 education levels. RTOP is anchored on three pillars, namely, student-centred instructions, inquiry approach to lessons and adherence to reform standards of instructions that were enacted before the year 2000.

Second is the Instructional Quality Assessment Mathematics Toolkit (IQA) which is based on the cognitive demands of tasks set by teachers, how learners implement these tasks and the level of interactions between learners and teachers. IQA is a rubric instrument based on a Likert-scale and it is used to observe two or more K-12 lessons. Third is the Mathematical Quality of Instruction (MQI) which measures both the talk and actions of a teacher and learners relative to what the curriculum prescribes. The MQI measured dimensions include the link of class work to mathematics, the richness of mathematical work performed in class, how the teacher works with students and mathematics, errors made by students and adherence of students to standard practices. Also, MQI is suitable for observation of K-9 video lessons.

3.3.3 Limitations of the observation instruments

The weakness of the RTOP instrument, according to Boston et al.(2015), is that it is based on the reform standards of instruction that were enacted more than 20 years ago. There have been other reforms to standards of instruction enacted recently, rendering the RTOP instrument out of date. The IQA instrument is not suitable for pre-recorded video lessons as it assumes the teacher assigns tasks to learners who have to be physically present in class, then monitors these tasks for implementation, and thereafter gets involved in the ensuing discussions with learners involving the same tasks (Boston, et al., 2015). The weaknesses of the MQI instrument are that it is only suitable for K-9 (Grades R to 9) lessons and that observed lessons are divided into smaller segments to determine the overall richness of mathematical lessons (Hill, et al., 2008).

An alternative noticing instrument is desirable that is based on latest reforms to standards of instruction, suitable for video lessons, that separates the actions of the teacher from students and can be used for observing Grade 12 lessons. Ultimately, the research questions that this research report is asking lead to the rejection of these three observation instruments in line with the argument of Boston et al. (2015) where they assert that “[t]he choice of an observation tool should be driven by the alignment between the research questions in a given study and the aspects of instruction made salient by that particular tool” (p. 165).

3.3.4 Mathematics Classroom Observation Protocol for Practices (MCOP^2)

The Mathematics Classroom Observation Protocol for Practices (MCOP^2) is used as the instrument to measure the quality of teaching practices on observed video instructions with a particular focus on the presence of multiple representations of probability.

The MCOP^2 instrument was developed to capture teacher facilitation and students' engagements during lessons (Gleason, Livers, & Zelkowski, 2018). The instrument is a collection of instructional practice standards set by various organisations and professional bodies such as:

- Common Core State Standards in Mathematics: Standards for Mathematical Practice of 2010;
- Mathematical Association of America (MAA): CUPM Curriculum Guide of 2004;
- American Mathematical Association of Two-Year Colleges (AMATYC): "Crossroads" of 1995 and "Beyond Crossroads" of 2006; and
- National Council of Teachers of Mathematics (NCTM): Process Standards of 2000.

What also makes this instrument desirable is that important factors such as the context of the school, the teacher and learners' backgrounds (including their socio-economic status), as well as the assessment standards do not form part of the instrument. The implication is that in online video instructions there is no school context; the backgrounds of instructors and targeted learners are not known in advance, and assessments standards may not be known.

The MCOP^2 instrument does not measure the quality of teaching over a single episode or lesson, but it is designed to measure the quality of teaching in a series of 3 to 6 lessons (Gleason, Livers, & Zelkowski, 2018). The advantage of measuring the quality of instructions over a series of episodes or lessons is that comparison could be made to measure the quality of teaching between two or more teachers on the same content area such as probability.

The MCOP^2 instrument measures the classroom interaction between the teacher practices and learners' engagement. The instrument is grounded in the constructivist theory of learning. It consists of 16 items where nine items relate to teacher facilitation and the other nine items relate to students' engagement (see Table 1 below). There is overlap of two items relating to both teacher facilitation and students' engagement. The instrument measures the observed teacher practices and learners' engagement separately, and results are scored and analysed separately. The design of this research is on video lessons which do not involve learners, so in the current study the classroom interaction envisaged by $MCOP^2$ does not hold. However, the research will use the teacher facilitation part of the instrument. Hence, only nine questions relating to the role of the teacher facilitation are scored on the $MCOP^2$ instrument.

Table 1: $MCOP^2$ items corresponding to observations relating Students Engagement and Teacher Facilitation

Item	Student Engagement	Teacher Facilitation
1	X	
2	X	
3	X	
4	X	X
5	X	
6		X
7		X
8		X
9		X
10		X
11		X
12	X	
13	X	X
14	X	
15	X	
16		X

Note: The table is from J. Gleason, S.D. Livers, and J. Zelkowski (2015), Mathematics classroom observation protocol for practices: Descriptors manual. (Retrieved from http://jgleason.people.ua.edu/mcop2.html)

The $MCOP^2$ instrument is preferred over other instruments because it fits the description of multiple representations which is the core theme of my research. Examples of items from $MCOP^2$ relating to the teacher facilitation include the following items: "teachers critically assessed mathematical strategies", "the lesson involved fundamental concepts of the subject to promote relational/conceptual understanding", "the lesson promoted modelling with mathematics", "the lesson provided opportunities to examine mathematical structure (symbolic notation, patterns, generalizations, conjectures, etc.)", "the lesson included tasks that have multiple paths to a solution or multiple solutions" (Gleason, Livers, & Zelkowski, 2018). Another reason for preferring the MCOP^2 instrument over other instruments is that the instrument was designed for K-14 education levels, equivalent to Grade R to Grade 12 in the South African basic education schooling system and the first two years of university mathematics.

The MCOP^2 instrument uses a four level scoring approach for each item on the instrument. An example of one item on the teacher facilitation part is given in Table 2 below (the full instrument is given in APPENDIX I):

Table 2: Item 6 of the MCOP^2 instrument: The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding. TF= Teacher facilitation.

TF	**Description**
3	“The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.”
2	“The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.”
1	“The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.”
0	“The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the “why” behind the procedures.”

Source: Gleason, J., Livers, S.D., & Zelkowski, J. (2015). Mathematics classroom observation protocol for practices: Descriptors manual. Retrieved from http://jgleason.people.ua.edu/mcop2.html

The instrument was validated by 125 experts and re-validated again by 26 of those experts after suggested changes were implemented (Gleason, Livers, & Zelkowski, 2018) (Gleason, Livers, & Zelkowski, 2018). The reliability measure (Cronbach alpha) of whether the questions relating to teacher facilitation are asking questions in a consistent manner is 0.85 (Gleason, Livers, & Zelkowski, 2018). A Cronbach alpha measure that is greater than 0.70 is considered to be reliable.

3.3.5 The Mathematical Task Framework

The mathematical tasks framework (MTF) is used to classify tasks in the e-textbook and e-study guides according to levels of cognitive demand (Stein, Grover, & Henningsen, 1996). These levels are: (i) memorisation, (ii) procedures without connections, (iii) procedures with connections, and (iv) doing mathematics. Memorisation and procedures without connections are lower cognitive level tasks while procedures with connections and doing mathematics are higher cognitive level tasks. The detailed description of the four cognitive demand levels is provided in Figure 1 below.

"Levels of Demands

Lower-level demands (Memorization):

• Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.

• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.

• Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.

• Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (Procedures without Connections):

• Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.

• Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.

• Have no connection to the concepts or meaning that underlie the procedure being used.

• Are focused on producing correct answers instead of on developing mathematical understanding.

• Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (Procedures with Connections):

• Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.

• Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
• Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
• Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and develop understanding.

Higher-level demands (Doing Mathematics):
• Require complex and non-algorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
• Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
• Demand self-monitoring or self-regulation of one's own cognitive processes.
• Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
• Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions.
• Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required."

Figure 1: Cognitive Demand Levels in Mathematical Task Framework

Source: Stein, M. K., Smith, M. P., Henningsen, M., & Silver, E. (2000). Implementing standards-based mathematics instruction: A casebook for professional development. New York, NY: Teachers College Press.

Coding is applied on MTF in line with Boston et al. (2015) recommendations. However, Boston et al. includes a code for tasks that are not completed by learners. This research does not view tasks in terms of completion by learners, but it seeks to allocate a level of cognitive demand to tasks as they appear in learning materials. The coding applied to MTF is as follows.

- 1 = memorisation
- 2 = procedures without connections
- 3 = procedures with connections
- 4 = doing mathematics

3.4 Data collection and sampling

The Department of Education (DBE) has compiled a list of resources obtainable from various platforms such as YouTube videos, television, radio, podcasts, and many others (DBE, 2020b). This research only considers video instructions that are obtainable from the YouTube online platform. Instructions broadcasted on television, radio and in podcasts do not form part of the sample. The weakness in television broadcast arises from the inability of the researcher to record these instructions due to the lack of resources as well as stringent licence conditions from broadcasters on reproduction of their materials. Radio lessons and podcasts are not suitable as they will not show teacher's representations of probability. The anonymity of YouTube video producers is maintained to protect the content producers from research findings that may be unfavourable to their reputation. The researcher has however provided names of the video channels used in this research to his supervisor who has undertaken to maintain confidentiality of these names.

Four video sources (or channels) are recommended by DBE for Grade 12 mathematics consisting of three South African sources and one international source (DBE, 2020b). Only two South African channels are selected as samples for this study. The video channels, pseudo-named VC1 and VC2, are selected because they are well established in terms of their reputations as well as the large number of video lessons that are available in their channels. From the international list, only one video channel is available (DBE, 2020b). This channel is selected and is pseudo-named VC3.

Learning materials are restricted to e-textbooks and e-study guides as they are the primary resources that allow learners to access mathematics content. Out of the large number of learning resources recommended by DBE for Grade 12 mathematics, the only textbook that is accessible online is selected and named e-textbook (DBE, 2020b). There are only two e-study guides that are freely accessible online from the DBE list based on their track record and reputation (DBE, 2020b). These e-study guides are pseudo-named SG1 and SG 2 to protect the reputations of the publishers and any consequence that may result from this research. Both the e-textbook and the two e-study guides represent 100% samples of learning materials from the list provided by DBE in the online learning materials' category.

3.5 Data Sources

3.5.1 e-Videos channels

Two South African education organisations and one international education organisation are used to collect data from video instructions posted in their YouTube channels. These videos can be accessed on the YouTube platform so as long learners have data on their smart mobile phones, tablets or computers.

AlthoughVC1 does not explicitly state whether their mathematics video lessons comply with the CAPS curriculum, given that the organisation was established in 2015 and is sponsored by reputable organisations including some provincial Departments of Education of South Africa, I considered it reasonable that VC1 complies with mathematics CAPS.

The VC1 YouTube channel was created almost 5 years ago and its videos have been viewed 58 040 times as of 23 November 2020. Four video lessons are relevant for the purpose of this research report. Three of the four videos are based on counting, namely, (i) fundamental counting principles, (ii) grouping, repetition and selection, and (iii) counting application to probability. The other video is based on probability where the focus is on tree diagrams.

The organisation producing VC2 content on YouTube was formed 24 years ago with the initial purpose of making education materials aligned to the curriculum available to educators. The scope of this organisation was expanded to make learning materials available for students. There are specialised learning and teaching resources produced by the organisation that are available for sale. There are also online resources such as video instructions and study guides that can be downloaded by members of the public without a need to subscribe to the organisation's services.

VC2 YouTube channel has a 13-year track record. The content on this YouTube channel has been viewed 24 823 932 times (almost 25 million) as of 23 November 2020. When compared to VC1, VC2 is over 400 times bigger in terms of viewership largely due to its long period of existence. Ten video lessons (episodes) from VC2 match the scope covered by this research.

Video instructions from VC3 are chosen for this study over other international education organisations because it is sponsored by a reputable international organization (OECD). The

Organisation for Economic Co-operation and Development (OECD) advised departments of education from its member states (of which South Africa is a member) to encourage schools, teachers and learners to access video resources from VC3 during the period of worldwide Covid-19 lockdowns (Reimers, Schleicher, Saavedra, & Tuominen, 2020). South Africa responded by making online instructions from the organisation producing VC3 content to be zero rated on local mobile phone service providers as a response to Covid-19 restriction on school attendance (DBE, 2020b). VC3 is rated highly internationally in providing free online education resources, from Grades R up to university undergraduate level.

VC3 YouTube online channel has a 14-year track record. Its video lessons have been viewed 1 820 977 633 (1.8 billion) times as of 23 November 2020 reflecting the popularity of video instructions from the organisation. VC3 has twelve video instructions on counting and probability, suitable for Grade 12 level. It should be noted that selecting video instructions aligned to what is prescribed by the South African Grade 12 curriculum needed careful sorting as VC3 videos include counting and probability instructions suitable for university curriculum.

3.5.2 e-Textbook and e-study guides

Tasks form the basic unit of lessons in a remote online environment where teachers are not within reach of learners. Tasks on the content area of probability are sourced from one textbook and two study guides. All these electronic learning materials are published by South African organisations.

The textbook used in this research report has been given the pseudonym e-textbook to protect the brand of the publisher. This textbook is freely downloadable from the publisher's website and is highly recommended by the Department of Education (DBE, 2020b). This textbook is the only free textbook available online that is aligned to the South African CAPS documentation.

The first part of the chapter on probability in the e-textbook is revision of Grade 11 probability. The next part contains the topics on counting principles, the factorial nations, applications to counting principle and applications to probability. Counting principles are introduced through the use of sets and tree diagrams as means of representation.

As it shall be shown later, the e-textbook contains many tasks compared to the e-study guides combined. The high number of tasks in each activity makes a strong case for a sample to be used as a representation of the total population. However, no matter what kind of sampling techniques are considered, it will not cure the fact that tasks are not randomly formulated in each activity of the e-textbook. Authors usually start activities with tasks that are not difficult and the tasks' difficulty progresses throughout these activities. All 35 tasks contained in the exercises on four sections of e-textbook constitute the sample. These tasks will then be compared with the tasks on e-study guides for multiple representations.

The first e-study guide (SG1) that is used in this research report is aligned to the CAPS curriculum. SG1's chapter on probability provides a summary of the various such as independent events. In addition, summaries of the factorial notation and arrangement of letters are provided through a set of rules. There is no summary provided using other representations besides the use of formulae. The chapter has only 8 exercises of which all are selected for this research.

SG2 has been developed and is published DBE. Users can freely download the e-study guide. The first nine sections of the chapter on probability in SG2 contain revision of probability prescribed for Grade 11 but examinable in Grade 12. The next (and last) two sections contain tasks on counting principle with probability applications. The major limitation of this chapter is the fact that tasks are only given for the part on counting principles. There are no tasks on counting principles with probability applications.

Regarding multiple representations of counting and probability, SG2 uses the odometer strategy (Batanero & Sanchez, 2013) to introduce counting concepts before generalising with formulae. The odometer strategy is about the list of sets of all possible combinations. This is a clear departure from SG1 which emphasises rule based learning. Chapter 4 of this research explores whether tasks from SG2 provide opportunities for multiple representations when compared to SG1. With regards to task selection, SG2 has two activities on counting and probability. The first activity has three questions and the last activity has eight questions. Due to the low number of questions in these two activities, all the eleven questions form part of the sample.

3.6 Inter-observer and inter-rater reliability

It is desirable that scores assigned on video lessons and codes assigned on the learning materials are seen as reliable. One way to achieve this inter-observer and inter-rater reliability is to have more than one person assigning scores and codes on the instruments that are used in this research. If the scores and codes assigned do not differ materially between the assigners, then those instruments are said to be reliable (Cohen, Manion, & Morrison, 2007). The researcher initially observed video lessons and then assigned scores on the teacher facilitation part of the MCOP^2 instrument. A teacher, who is also a postgraduate student at University of the Witwatersrand, observed the video lessons followed by assigning scores on the MCOP^2 instrument. Scoring differed between the researcher and the teacher on items 6, 13 and 16 in VC1 observed lessons. There were also differences in scores assigned to lessons in VC2. There were no differences in scores assigned to VC3 lessons. The researcher and fellow teacher had a telephone meeting where they agreed on the final scores to be assigned on all items of the MCOP^2 instrument where they differed.

The same procedure was undertaken for assigning the levels of cognitive demand based on the mathematical tasks framework. The researcher initially assigned cognitive demand level codes on mathematical tasks from e-learning materials using the MTF instrument. The same teacher, who assigned scores on MCOP^2, assigned codes on the tasks from e-learning materials. With respect to cognitive demand level codes assigned to e-learning materials, the researcher and fellow teacher differed only in one task with respect to each one of the online learning materials used in this research. Disagreements on coding were resolved through a telephone meeting.

3.7 Ethics consideration

The MCOP^2 instrument that is used to observe the presence of multiple representations of probability is freely available online to use in educational research. MCOP^2 can be modified to suit the context of the research (Gleason, Livers, & Zelkowski, 2018). Researchers who developed the instrument prefer to be notified when the instrument is used as a form of courtesy. The researcher was granted access to use the instrument on 14 December 2020 (see APPENDIX II).

The mathematics task framework classifies tasks according to set criteria of cognitive demand levels (Stein, Grover, & Henningsen, 1996). The framework is available in the research papers of its developers. No consent was sought from its developers and there is no indication that the developers of the framework require prior notification before it is used for research. Wherever MTF is reproduced in this research, full source citation is made to acknowledge its developers.

The current research uses secondary data. All video learning channels from YouTube and electronic learning materials were given pseudonyms to protect the names of organisations producing the contents from finding emanating from this research that may be interpreted as detrimental to their reputations. For the e-video channels and e-learning materials, copyrights and permissions received from publishers are attached starting from APPENDIX XI to APPENDIX XV. Where the names of the organisations are stated, I have redacted these names to protect the reputation of these organisations. In most cases, the copyright agreements from these organisations state that content can be copied and used for educational purposes for non-commercial purpose. In cases where there were no copyrights agreements found on these organisations' websites, email consent was sought. Such email consent has also been attached in APPENDIX XI with emails addresses and names of individuals associated with these organisations redacted.

Application for ethics clearance has been approved by the University of the Witwatersrand. The ethics clearance protocol number is: 2020ECE016M (see APPENDIX III).

3.8 Chapter summary

This chapter presented the instruments used for data collections together with detailed sources of data. The approach adopted is quantitative, which ensures that the researcher's subjective biases are kept at bay throughout the research. Data from video instructions are collected using the MCOP^2 instrument and data from learning materials are collected using the mathematical task framework. Video data sources are selected from YouTube online channels. The e-textbook and two e-study guides comply with CAPS and can be downloaded from the internet. Anonymity has been ensured to protect the names of video sources and electronic materials at all times. The supervisor of the researcher will also maintain confidentiality of the names of data sources in line with the Ethics application for the study. Chapter 4 provides the analysis of data collected.

CHAPTER 4: ANALYSIS OF DATA AND RESULTS PRESENTATION

4.1 Introduction

This chapter is concerned with the analysis of data that is collected using two instruments, inter alia, mathematical classroom observation protocol for practice (MCOP^2) and mathematical task framework (MTF). The video lessons from three YouTube channels were firstly observed, followed by scoring using the MCOP^2 instrument. Thereafter, tasks from the three online learning materials were coded using the MTF instrument. Statistical analysis performed in this chapter uses data collected from the two instruments. In particular, hypothesis tests are formulated in this chapter to assess whether there is a mean difference in the scores from video lessons and learning materials respectively.

4.2 Video results analysis

Each video channel has different features and different lessons. VC1 and VC2 show the face of the instructor, giving the option of assigning a score directly to the instructor using the MCOP^2 instrument. VC3 does not show the face of the instructor, making it challenging to assign a score directly for the instructor. Moreover, the MCOP^2 instrument requires different classroom settings or different lessons, that range from 3 to 6, to find a reliable and valid construct (Gleason, Livers, & Zelkowski, 2018).

Recalling that VC1 has four video lessons on counting and probability, consisting of two different instructors, all four lessons will be combined to find one rating for VC1. The situation is different for VC2 as it has ten video lessons, making the direct use of MCOP^2 to all ten lessons inappropriate. Moreover, the ten lessons from VC2 are delivered by five different instructors. Two instructors deliver three lessons each, another instructor has two lessons and the last two instructors have one lesson each. To resolve the problem of which lessons to choose among the five instructors, only lessons from the instructors with three lessons each are chosen in order to comply with MCOP^2's 3 to 6 lessons requirement. One instructor with three lessons is referred to as Teacher 1 (to preserve anonymity) and the other instructor with three lessons is named Teacher 2, also to ensure anonymity.

VC3 has 12 video lessons on counting and probabilities. Only the voice of the instructor is audible in all these videos. The voice of the instructor is the same, meaning that the same instructor is giving 12 video lessons. To reduce the video lessons to the 3 to 6 lessons range permitted by MCOP^2 so that the construct remains reliable and valid, I chose six lessons whose topics are aligned to the CAPS curriculum. These six topics of the lessons are (i) counting using tree diagrams, (ii) factorial formula, (iii) factorial count on seat arrangements, (iv) the probability of getting exactly two heads, (v) zero factorial, and (vi) introduction to combinations. The excluded lessons are concerned with combinations and the application of combinations to probability, which are all advanced topics, beyond the scope of Grade 12.

4.2.1 Analysis of video channel 1

Four video lessons, hereafter referred to as episodes, are observed in video channel 1 (VC1). Each episode is assigned a score based on the MCOP^2 instrument (see APPENDIX IV). The summary of statistics for each episode of the teacher facilitation was generated using Microsoft Excel software. Table 3 shows the summary statistics for each episode of VC1 as well as the combined statistics for VC1. Small letters *n* and *t* in brackets represent the number of items of the MCOP^2 instrument and the duration of each episode in minutes and seconds respectively. It can be seen that the mean of MCOP^2 scores is lowest in episodes 2 and 3, and highest in episode 1. The mean for the combined episodes is 1.25, which is closer to a score of 1 on the scale of MCOP^2 instrument (which has codes that range between 0 and 3). The standard deviation is lowest in episode 2 and 3, highest in episodes 4. The standard deviation for the combined episodes is 0.99, showing little variability of MCOP^2 scores around the mean.

Table 3: Summary statistics of teacher facilitation for VC1 lessons based on MCOP^2, where n is the number of items and t represents the duration of each episode in minutes and seconds

Summary Statistics	Episode 1 (n=9, t=12:07)	Episode 2 (n=9, t=11:29)	Episode 3 (n=9, t=15:31)	Episode 4 (n=9, t=16:26)	Combined statistics
Mean	1,56	1,11	1,11	1,22	1,25
Standard deviation	0,88	0,93	0,93	0,97	0.88
Mode	2	2	2	2	2
Median	2	1	1	2	1,75
Minimum	0	0	0	0	0
Q1	1	0	0	0	0,25
Q3	2	2	2	2	2
Maximum	3	2	2	2	2

There are many items that are assigned a score of 2 in each episode of VC1 as shown by the mode. The combined mode for VC1 episodes is also 2. The median is 1 for episodes 2 and 3, while for episodes 1 and 4 the median is 2. The overall median for VC1 is 1.75. All episodes had at least one item that received a score of zero such as item 4 of MCOP^2 that requires "students [to] critically assess mathematical strategies" when prompted by the teacher (see APPENDIX I for the full MCOP^2 instrument). Due to the fact that pre-recorded video lessons are used in this research, as well as the fact that students are absent in these videos, all item 4 in VC1 received a score of zero (see APPENDIX IV). It is only in episode 1 were there is a score of 3 assigned to item 9 on the MCOP^2 instrument. A score of 3 on item 9 of MCOP^2 pertains to "the lesson [that] includes tasks that have multiple paths to a solution or multiple solutions". I now consider video channel 2.

4.2.2 Analysis of video channel 2

Video channel 2 (VC2) is divided into two parts based on the premise that two teachers had 3 lessons each on counting and probability. Details of the scores assigned to each teacher based on the MCOP^2 instrument with 9 items on teacher facilitation is shown in APPENDIX V. Starting with the teacher pseudo-named Teacher 1, it can be seen that the mean of MCOP^2 scores is lowest in episode 1, highest in episode 3 (see Table 4 below). The overall mean for Teacher 1 is 1.44. The variability of MCOP^2 scores around the mean is lowest in episodes 1, highest in episode 3. The standard deviation for the combined episodes is 0.73. Most of episode 1 scores are assigned a value of 1; while the score of 2 appears the most in episodes 2 and 3. For the combined episodes, the mode is 2.

Table 4: Summary statistics of teacher facilitation for VC2 (Teacher 1) lessons based on MCOP^2, where n is the number of items and t represents the duration of each episode in minutes and seconds

Summary Statistics	Episode 1 (n=9, t=7:30)	Episode 2 (n=9, t=7:41)	Episode 3 (n=9, t=5:16)	Combined statistics
Mean	1,22	1,44	1,67	1,44
Standard deviation	0,67	0,73	1,00	0,73
Mode	1	2	2	2
Median	1	2	2	2
Minimum	0	0	0	0
Q1	1	1	1	1
Q3	2	2	2	2
Maximum	2	2	3	2

The median of MCOP^2 scores is 1 for episode 1 of Teacher 1's lessons; it is 2 for episodes 2 and 3. The overall median is also 2 for the combined episodes. Just like in VC1, item 4 is responsible for the minimum value of zero due to there being no students in all Teacher 1's episodes. Only items 6 and 11 of the MCOP^2 instrument, in episode 3, are assigned a score of

3 for Teacher 1 (see APPENDIX V for details). This implies that in those two items, Teacher 1 focused on relational understanding of concepts and used the teacher talk to increase the cognitive demand of students.

Table 5 below shows the summary statistics for Teacher 2 lessons in VC2. The mean of MCOP^2 scores was lowest in episode 3, highest in episode 2. The combined mean for all Teacher 2's episodes is 1.33. Episode 3 has the lowest variability of MCOP^2 scores around the mean, while variability around the mean is highest in episode 2. The standard deviation for all episodes is 0.67.

Table 5: Summary statistics of teacher facilitation for VC2 (Teacher 2) lessons based on MCOP^2, where n is the number of items and t represents the duration of each episode in minutes and seconds

Summary Statistics	Episode 1 (n=9, t=9:42)	Episode 2 (n=9, t=8:49)	Episode 3 (n=9, t=6:24)	Combined statistics
Mean	1,33	1,44	1,22	1,33
Standard deviation	0,71	0,73	0,67	0,67
Mode	2	2	1	2
Median	1	2	1	1,33
Minimum	0	0	0	0
Q1	1	1	1	1
Q3	2	2	2	2
Maximum	2	2	2	2

The score of 2 appears the most in episodes 1 and 2, while for episode 3 the score of 1 appears the most. The mode for Teacher 2's three lessons is also 2. The median is 1 for episodes 1 and 3; it is 2 for episode 2. The combined median is 1.33. Item 4 is responsible for the scores of zero (i.e. the minimum score) as the absence of the teacher-learner interaction implies no learners were active participants in the video lessons. I now the shift focus to video channel 3 episodes.

4.2.3 Analysis of video channel 3

Unlike VC1 and VC2 which are produced by South African organisations, video channel 3 (VC3) is produced by an international organisation. VC3 has six episodes. Details of how each of the 9 items in all episodes is assigned a score on the MCOP^2 instrument are given in APPENDIX VII. The mean of MCOP^2 scores is lowest in episodes 4 and 5, highest in the other episodes (see Table 6 below). The combined mean is 1.70. Variability of MCOP^2 scores around the mean is also lowest in episodes 4 and 5, highest in other episodes. The combined standard deviation for VC3 is 1.34. The mode is 3 for episodes 1, 2, 3 and 6; it is zero for episodes 4 and 5. The combined mode is 3, indicating most items were assigned a score of 3 in the episodes. The median is 2 for all six episodes leading to a combined median of 2 as well.

Table 6: Summary statistics of teacher facilitation for VC3 lessons based on MCOP^2, where n is the number of items and t represents the duration of each episode in minutes and seconds

Summary Statistics	Ep. 1 (n=9, t=4:31)	Ep. 2 (n=9, t=7:34)	Ep. 3 (n=9, t=9:01)	Ep. 4 (n=9, t=10:00)	Ep. 5 (n=9, t=4:50)	Ep. 6 (n=9, t=6:17)	Combined Average
Mean	1,78	1,78	1,78	1,56	1,56	1,78	1,70
Standard deviation	1,39	1,39	1,39	1,33	1,33	1,39	1,34
Mode	3	3	3	0	0	3	3
Median	2	2	2	2	2	2	2
Minimum	0	0	0	0	0	0	0
Q1	0	0	0	0	0	0	0
Q3	3	3	3	3	3	3	3
Maximum	3	3	3	3	3	3	3

Items 4, 13 and 16 of VC3 episodes are assigned scores of zero in all six episodes due to the premise that in this video channel there is no learner participation. No learners share ideas in the video lessons. Also, no learners pose questions in VC3 videos (see APPENDIX VI for details of how these items were scored). These reasons explain the minimum score of zero. The maximum score of 3 that is shown in episodes 1 to 6 is due to a full score assigned to items 6, 9 and 11, as the teacher in these episodes focuses on the conceptual understanding, used tasks that have multiple paths to solutions, and employed teacher-talk that trigger the thinking of students watching the videos. I now turn my attention to ask the question whether there is a difference in the means of MCOP^2 scores for all video channels.

4.2.4 Analysis of the combined video channels

It can be seen that VC3 had a higher mean based on MCOP^2 scores compared to the means of VC1 and VC2 (for both Teacher 1 and Teacher 2), while the means for VC1 and VC2 seem to be close to each other. However, it is essential that hypothesis tests be formulated that will seek to confirm or disprove these claims.

When comparing two means from two different population groups, a student t-test is normally used. However, a student t-test is not suitable for use when comparing the mean difference between video lessons in the current study for two reasons. Firstly, a student t-test prefers a minimum sample size of 30 items to be used, which exceeds the 9 items in the teacher facilitation part in the $MCOP^2$ instrument. Secondly, a student t-test compares the mean difference of two groups. In this case we have three video channels whose means are to be compared, making the student t-test unsuitable. Rather, the analysis of variance test statistics (ANOVA) is preferred.

ANOVA measures whether there is a difference in the variance of the combined video channels and a difference in the variance of each video channel. Statistical significance in the means will be indicated by the p-value that is below a defined critical value. A p-value in statistics indicates the probability of getting an extreme value from actual data observed. A critical value is a selected point on a test statistics distribution (ANOVA for this research) used to compare the null and alternative hypotheses. A critical value of 5% is preferred in this research report due to the low number of items in the $MCOP^2$ instrument.

ANOVA only requires a minimum of three items before it is used. Further, ANOVA can be used to compare a minimum of two samples. The video lessons in this research meet the criteria of ANOVA as there are 9 items in the $MCOP^2$ instrument and 3 different video channels (i.e. the samples) whose means will be compared. In addition, the ANOVA assumption of independence of the video channels is met as all three video channels are produced by different organisations. Also, the tasks used by each video channel are different, making the tasks independent from each other. I also assume that scores of items from the $MCOP^2$ instrument are normally distributed to align with the ANOVA test suitability criteria.

The following hypothesis test is formulated to test the mean difference of $MCOP^2$ scores for VC1, VC2 Teacher 1, VC2 Teacher 2 and VC3:

H_0: $\mu_{VC1} = \mu_{VC2\ Teacher1} = \mu_{VC2\ Teacher2} = \mu_{VC3}$
H_1: not all μ_{VC1}, $\mu_{VC2\ Teacher\ 1}$, $\mu_{VC2\ Teacher\ 2}$ and μ_{VC3} are equal (i.e. some of the two means can be equal, but not all means are equal)

where:
H_0 is the null hypothesis;
H_1 is the alternate hypothesis;

μ_{VC1} is the population mean of VC1;

$\mu_{VC2\ Teacher1}$ is the population mean of VC2 Teacher 1;

$\mu_{VC2\ Teacher2}$ is the population mean of VC2 Teacher 2; and

μ_{VC3} is the population mean of VC3.

If the p-value of the F-test (for ANOVA) is greater than the 5% critical value, the null hypothesis is not rejected, suggesting that the difference between the three means is not statistically significant. However, if the p-value is found to be below the critical value, then the null hypothesis is rejected to the favour of the alternative hypothesis. That will suggest that not all the means of VC1, VC2 and VC3 are equal (i.e. some of the two means can be equal, but not all means are equal).

4.2.5 ANOVA test for the combined video channels

Table 7 shown below provides a summary of data for the ANOVA test for VC1, VC2 (for both Teacher 1 and Teacher 2) and VC3 combined.

Table 7: Summary of one-way ANOVA test comparing the difference in the means of VC1, VC2 and VC3, where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	VC1	VC2 (Teacher 1)	VC2 (Teacher 2)	VC3	5	Total
N	9	9	9	9		36
∑X	11.25S	13	12	15.33		51.58
Mean	1.25	1.4444	1.3333	1.7033		1.433
$\sum X^2$	20.1875	23.2178	19.5578	40.4289		103.392
Std.Dev.	0.875	0.745	0.6669	1.3378		0.9179
Result Details						
Source	***SS***	***Df***	***MS***			
Between-treatments	1.0497	3	0.3499	*F* = 0.3937		
Within-treatments	28.4396	32	0.8887			
Total	29.4893	35				
The *f*-ratio value is 0.3937. The *p*-value is .758364.						

It can be seen that the F-ratio value is 0.3937, while the p-value is 0.758364. These statistics were calculated using the online software found at www.socscistatistics.com. This result is not statistically significant since the p-value is > 0.05. This means that the null hypothesis cannot be rejected at 5% level. According to this result, there is no evidence to suggest that the means of VC1, VC2 Teacher 1, VC2 Teacher 2 and VC3 are all not equal. This result is affirmed despite the mean of VC3 clearly exceeding the means of VC1 and VC2 (for both Teacher and Teacher 2), and 3 items of VC3 receiving full $MCOP^2$ scores. The reason for this could be the three items (i.e. items 4, 13, and 16) that were assigned a score of zero in the $MCOP^2$ instrument due to the absence of teacher-student interaction in the video lessons.

Table 8 below shows the summary of data for the ANOVA test when items 4, 13 and 16 are removed from the sample of VC1, VC2 (both Teacher 1 and Teacher 2) and VC3 as there is minor teacher-student interaction in these videos.

Table 8: Summary of one-way ANOVA test comparing the difference in the means of VC1, VC2 and VC3, excluding 3 items from $MCOP^2$, where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	VC1	VC2 (Teacher 1)	VC2 (Teacher 2)	VC3	5	Total
N	6	6	6	6		24
$\sum X$	10.75	11	10	15.33		47.08
Mean	1.7917	1.8333	1.6667	2.555		1.962
$\sum X^2$	20.0625	21.2178	17.5578	40.4289		99.267
Std.Dev.	0.4005	0.4585	0.4222	0.5021		0.5482
Result Details						
Source	***SS***	***df***	***MS***			
Between-treatments	2.9066	3	0.9689	*F* = 4.83822		
Within-treatments	4.0051	20	0.2003			
Total	6.9117	23				
The *f*-ratio value is 4.83822. The *p*-value is .010851.						

There are now six items in each sample. The F-ratio value is 4.83822 which gives the p-value of 0.010851 (calculated using the online software found at www.socscistatistics.com). This result is statistically significant at p-value < 0.05. There is now a rejection of the null hypothesis, to the favour of the alternative hypothesis. This means that at least one of the means of the three video channels is different from the other means.

It is useful to carry out a Post-Hoc Tukey test to assess which of the four treatments are responsible for differences in the means. Table 9 below provides a summary of Post Hoc Tukey data calculated using the online software found at www.socscistatistics.com.

Table 9: Summary of Post Hoc Tukey HSD (beta) test comparing the pairwise mean differences between VC1, VC2 and VC3, excluding 3 items of MCOP[2], where $T_{1,2,3 \text{ and } 4}$ represents treatments 1, 2, 3 and 4 respectively, and statistical outputs are represented in columns 2, 3 and 4.

Pairwise Comparisons		$HSD_{.05} = 0.7231$ $HSD_{.01} = 0.9167$	$Q_{.05} = 3.9583$ $Q_{.01} = 5.0180$
T_1:T_2	$M_1 = 1.79$ $M_2 = 1.83$	0.04	Q = 0.23 (*p* = .99846)
T_1:T_3	$M_1 = 1.79$ $M_3 = 1.67$	0.13	Q = 0.68 (*p* = .96181)
T_1:T_4	$M_1 = 1.79$ $M_4 = 2.56$	0.76	Q = 4.18 (*p* = .03625)
T_2:T_3	$M_2 = 1.83$ $M_3 = 1.67$	0.17	Q = 0.91 (*p* = .91611)
T_2:T_4	$M_2 = 1.83$ $M_4 = 2.56$	0.72	Q = 3.95 (*p* = .05057)
T_3:T_4	$M_3 = 1.67$ $M_4 = 2.56$	0.89	Q = 4.86 (*p* = .01277)

It can be seen that T4 (standing for treatment 4 or VC3) when compared to treatments 1 and 3 (i.e. VC1 and VC2 Teacher 2) lead to a statistical significant p-value <0.05. Further, comparing treatments 2 (VC2 Teacher 1) and 4, one gets a sense that the mean difference is insignificant at 5% level. However, the p-value is 5.057% which is closer to the critical value of 5%. This result suggests that when the three items that account for the role of students are removed from the teacher facilitation in the MCOP[2] instrument, the mean of video channel 3 becomes statistically different to the means of other video channels. Thus, the mean of VC3 (the internationally channel) appears to be superior to the means of VC1 and VC2 (both local channels).

Removing the three items from the MCOP^2 instrument relating to students was done only to compare the effect of the teacher actions in the video channels. It should be noted that removing any item from the teacher facilitation part in the MCOP^2 instrument may distort the validity and reliability of the instrument. Even if we get significant statistical results by removing these three items, the results should be accepted with caution.

4.2.6 ANOVA test for South African video channels

It is desirable to compare the mean difference of the two South African video channels based on data generated by the MCOP^2 instrument. A hypothesis test is again formulated as follows:

H_0: $\mu_{VC1} = \mu_{VC2\,Teacher1} = \mu_{VC2\,Teacher2}$
H_1: not all μ_{VC1}, $\mu_{\text{VC2}\,Teacher1}$ and $\mu_{\text{VC2}\,Teacher2}$ are equal (i.e. some of the two means can be equal, but not all means are equal)

where
H_0 is the null hypothesis;
H_1 is the alternate hypothesis;
μ_{VC1} is the population mean of VC1;
$\mu_{VC2\,Teacher1}$ is the population mean of VC2 Teacher 1; and
$\mu_{VC2\,Teacher2}$ is the population mean of VC2 Teacher 2.

Table 10 below provides the summary of data for the ANOVA test (calculated using the online software found at www.socscistatistics.com). The F-ratio value is 0.14555 whereas the p-value is 0.865305. The result is *not* statistically significant at p-value < 0.05. This implies that there is a failure to reject the null hypothesis at 5% level. This suggests that the means of VC1 and VC2 (for both Teacher 1 and Teacher 2) are equal.

Table 10: Summary of one-way ANOVA test comparing the difference in the means of VC1, VC2 (Teacher 1) and VC2 (Teacher 2), where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	VC1	VC2 (Teacher 1)	VC2 (Teacher 2)	4	5	Total
N	9	9	9			27
∑X	11.25	13	12			36.25
Mean	1.25	1.4444	1.3333			1.343
$\sum X^2$	20.1875	23.2178	19.5578			62.9631
Std.Dev.	0.875	0.745	0.6669			0.7415
Result Details						
Source	***SS***	***df***	***MS***			
Between-treatments	0.1713	2	0.0856	*F* = 0.14555		
Within-treatments	14.1228	24	0.5885			
Total	14.2941	26				
The *f*-ratio value is 0.14555. The *p*-value is .865305.						

The attention now turns to the analysis of data for online learning materials.

4.3 Analysis of online learning materials

4.3.1 Analysis of e-Textbook

The e-textbook has 35 questions on counting principles, permutations and probability applications. Considering that each question (and its subcomponents) is treated as a task on its own, it implies that 35 tasks from the textbook are assigned cognitive demand levels using the mathematical task framework. The 35 tasks and corresponding cognitive demand level are shown in APPENDIX VIII of this research report. Table 11 below details the tally (or count) of each cognitive demand level as well as the corresponding frequency, expressed as a percentage.

Table 11: Frequency table representing the cognitive demand levels of the e-textbook tasks

Cognitive levels	Cognitive level	Tally of Tasks	Percentage
Memorisation	1	0	0%
Procedures without connections	2	10	29%
Procedures with connections	3	22	63%
Doing mathematics	4	3	9%
Total		35	100%

There is no task whose cognitive demand level is at memorisation level. This means, when it comes to counting and probability, there are no questions that require learners to reproduce definitions or facts that were previously learnt. There are 10 tasks out of 35, corresponding to 29%, with a cognitive demand level that is classified as procedures without connections. Tasks at this cognitive level involve using procedures without reference to underlying concepts. Since both memorisation and procedures without connections are at the lower cognitive levels, it can be concluded that 29% of the textbook tasks are at the lower cognitive level.

Procedures with connections constitute 22 of the 35 tasks, which is equivalent to 63%. These kinds of tasks involve a considerable level of thinking, connecting procedures to underlying concepts and use multiple representations to arrive at the required solutions. Moreover, procedures with connections constitute the majority of tasks in the textbook, and can be answered using multiple problem solving strategies. Lastly, 3 tasks out of the 35 tasks, corresponding to 9%, are at doing mathematics cognitive level.

Table 12 below provides a summary of statistics for cognitive demand levels of the e-textbook. These summary statistics were calculated using the Microsoft excel software.

Table 12: Summary statistics of the cognitive demand levels for e-textbook tasks

Summary Statistics	Value (n=35)
Mean	2,80
Standard deviation	0,58
Median	3
Minimum	2
Q1	2
Q3	3
Maximum	4

The mean score of cognitive demand level is 2.8, which is closer to procedures with connections than procedures without connections. The standard deviation is 0.58, suggesting little variation among the cognitive levels. I interpret a standard deviation of less than 1 as acceptable variation. The median is 3, meaning that half of tasks' cognitive demand level comprise a score of 3 or more. This is also confirmed by the frequency of 63% for procedures with connections (see Table 11 above).

As mentioned earlier, no task is assigned a memorisation cognitive level, thus the minimum cognitive level is 2. The minimum value is equal to the first-quartile as confirmed by the box-and-whisker plot in Figure 2 below. This is also confirmed by the fact that 29% of textbook tasks are assigned a score of 2. The median is equal to the third-quartile as seen also in Figure 2. This is also confirmed by the frequency count of 63% assigned to procedures with connections tasks. The maximum is 4, as there are tasks corresponding to doing mathematics cognitive level.

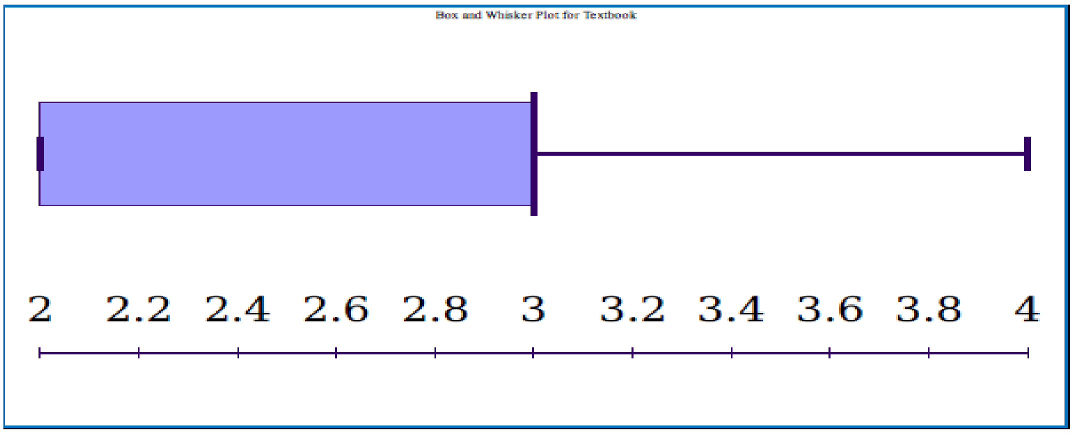

Figure 2: Box-and-whisker plot showing the cognitive demand levels of the e-textbook tasks

4.3.2 Analysis of e-study guide 1

Study guide 1 (SG 1) has 8 tasks in counting and probability applications. The 8 tasks as well as their corresponding cognitive demand levels are shown in detail in APPENDIX IX of this research report. Table 13 below gives the tally of each cognitive demand level and the corresponding frequency, expressed as a percentage.

Table 13: Frequency table representing the cognitive demand levels of SG1 tasks

Cognitive levels	Cognitive level	Tally of Tasks	Percentage
Memorisation	1	0	0%
Procedures without connections	2	3	37.5%
Procedures with connections	3	4	50%
Doing mathematics	4	1	12.5%
Total		8	100%

None of the tasks in SG1 are assigned the memorisation cognitive demand level. There are three (3) tasks out of 8, corresponding to 37.5%, that are assigned the procedures without connections cognitive demand level. Since both memorisation and procedures without connections tasks, taken together, constitute a lower cognitive level, it can be said that 37.5% of tasks were of lower cognitive level.

Four (4) of the 8 SG1 tasks are assigned a code of 3, which corresponds to the procedures with connections cognitive demand level, meaning that 50% of SG1 tasks require multiple representations to reach the required solutions. There is only 1 task out of 8 (12.5%) that corresponds to the doing mathematics cognitive level. In total, 62.5% of SG1 tasks are of high cognitive level, meaning these tasks involve numerous strategies that can be employed to reach solutions to the tasks. This also corresponds to recommendations from various DBE diagnostic reports which emphasise the use of different representations in order to solve counting and probability tasks at Grade 12 level.

Table 14 below gives the summary statistics of the SG1 tasks. The mean of cognitive demand levels is 2.75, which is closer to the code of 3 representing procedures with connections. The standard deviation of SG1 is higher than that of the e-textbook, but still below the value of 1. This is congruent with the concentration of 87.5% of the tasks which are assigned cognitive demand levels of either 2 or 3. The median of SG1 is 3, meaning that 50% or more of the tasks are assigned higher cognitive levels.

Table 14: Summary statistics of the cognitive demand levels for SG1 tasks

Summary Statistics	Value (n=8)
Mean	2,75
Standard Deviation	0,66
Median	3
Minimum	2
Q1	2
Q3	3
Maximum	4

The box-and-whisker plot in Figure 3 below confirms that the minimum value of SG1 tasks is equal to the first-quartile (Q1). The median is equal to the third-quartile (Q3) as there are 50% of tasks assigned a code of 3 starting from the 37.5% of tasks assigned a code of 2. The maximum value of 4 confirms the presence of doing mathematics tasks. In sum, the box-and-whisker plot confirms the variation of cognitive demand levels assigned to the SG1, where tasks are concentrated in both procedures without connections, and procedures with connections.

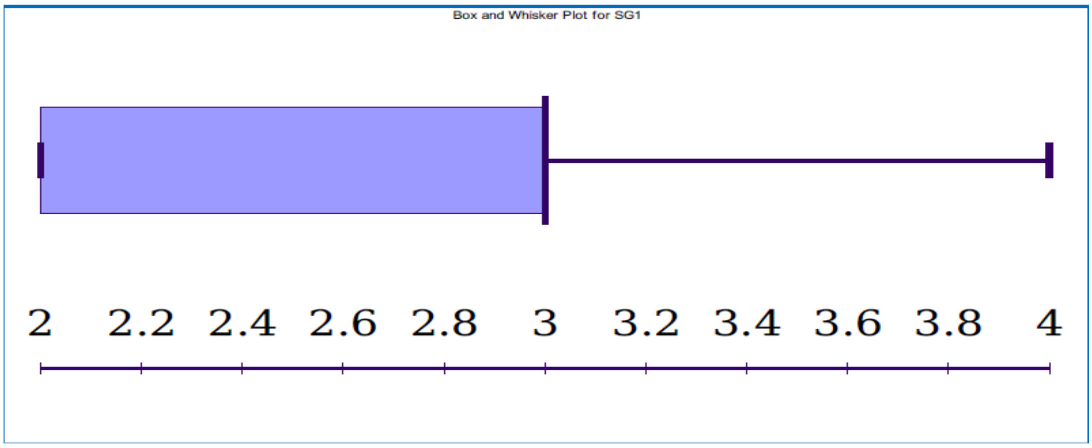

Figure 3: Box-and-whisker plot showing the cognitive demand levels of SG1 tasks

4.3.3 Analysis of e-study guide 2

Study Guide 2 (SG 2) has 11 questions on counting and probability applications. These 11 tasks and their corresponding cognitive demand levels are shown in detail in APPENDIX X of this research report. Table 15 below shows in detail the tally of each cognitive demand level as well as the corresponding frequency, expressed as a percentage.

Table 15: Frequency table representing the cognitive demand levels of SG2 tasks

Cognitive levels	Cognitive level	Tally of Tasks	Percentage
Memorisation	1	0	0%
Procedures without connections	2	7	64%
Procedures with connections	3	4	36%
Doing mathematics	4	0	0%
Total		11	100%

There is no task that requires the recall of concepts or facts (i.e., memorisation). Seven (7) tasks out of 11, corresponding to 64%, are assigned the procedures without connections cognitive demand level. Since both memorisation and procedures without connections tasks constitute lower cognitive level, this means that 64% of SG2 tasks are at a lower cognitive level. This is worrying as study guides are expected to best prepare learners for examinations where counting and probability questions require multiple representations of solutions.

Four (4) tasks out of 11, corresponding to 36%, represent the procedures with connections cognitive demand level. There is no task assigned the cognitive demand level corresponding to doing mathematics. In total, 36% of tasks in SG2 are at a higher cognitive level, compared with 72% of tasks in the e-textbook and 62.5% in SG1, respectively.

The mean of cognitive demand levels for SG2 tasks is 2.36 as shown in Table 16 below. The mean is closer to the cognitive level code of 2, which represents procedures without connections. The standard deviation is 0.48, representing little variation in cognitive level codes. The standard deviation value is also consistent with the concentration of cognitive level codes around the values 2 and 3. The median is 2, confirming that 64% of tasks are assigned a cognitive demand level of 2.

Table 16: Summary statistics of the cognitive demand levels for SG2 tasks

Summary Statistics	Value (n=11)
Mean	2,36
Standard Deviation	0,48
Median	2
Minimum	2
Q1	2
Q3	3
Maximum	3

Overall, the shape of the box-and-whisker plot (see Figure 4 below) confirms two observations. First, SG2 tasks are assigned to only two codes, namely codes 2 and 3. Secondly, more than 50% of tasks are assigned to the cognitive demand level of procedures without connections, which is represented by code 2.

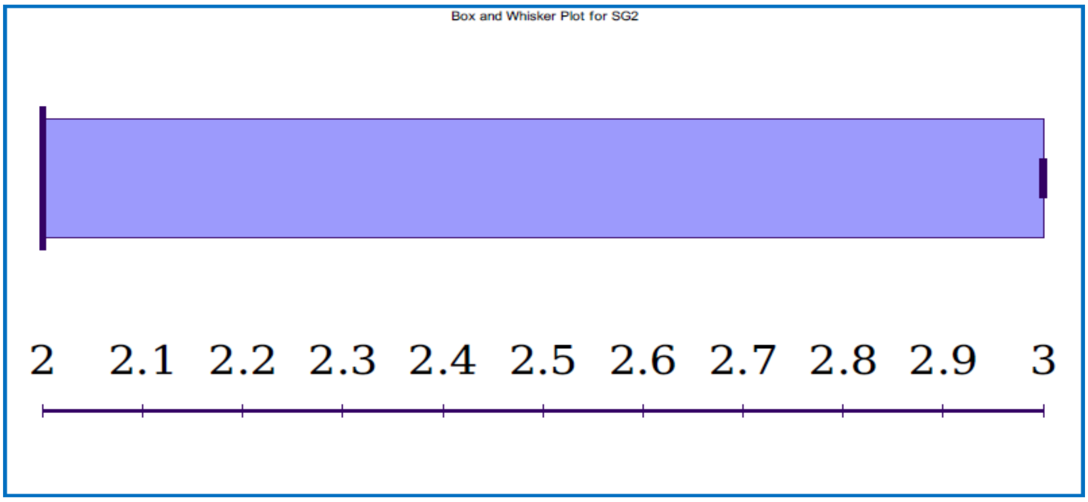

Figure 4: Box-and-whisker plot showing the cognitive demand levels of SG2 tasks

4.3.4 Hypothesis tests for the combined e-textbook and e-study guides

I now turn my attention to hypothesis testing to assess whether there is significance differences to the mean values of the cognitive demand levels in three online learning materials, namely, the e-textbook and the two e-study guides

It is important to compare the three online learning materials to see if the means of their cognitive demand levels differ statistically. This result is important as it will indicate whether learning materials available online promote multiple representations of counting and probability tasks at Grade 12 level.

To compare whether there is a mean difference between cognitive demand levels of the e-textbook, SG1 and SG2, the analysis of variance (ANOVA) statistical test is performed. The ANOVA test measures whether there is a difference in the variance of the combined textbook and the two study guides, and a difference in individual variance of the e-textbook, SG1 and SG2. Differences in the mean will be indicated by a p-value that is below the critical value. A p-value in statistics indicates the probability of getting an extreme value from observed data. A critical value of 5% is preferred in this report due to the low number of tasks in the two study guides.

The ANOVA test assumption of independence is met as the textbook and the two study guides are published and authored independently. Also, there are no tasks that overlap in these three samples, making the tasks independent from each other.

The hypothesis test is formulated to test the difference in the means of the three samples:

$H_0: \mu_T = \mu_{SG1} = \mu_{SG2}$

H_1: At least one of the means is not equal to the other two means

where

H_0 is the null hypothesis;

H_1 is the alternate hypothesis;

μ_T is the population mean of the e-textbook;

μ_{SG1} is the population mean of SG1; and

μ_{SG2} is the population mean of SG2.

If the p-value of the F-ratio (from ANOVA test) is greater than the 5% critical value, there will be failure to reject the null hypothesis. However, if the p-value is found to be below the critical value, then the null hypothesis will be rejected in favour of the alternative hypothesis. Below is Table 7 showing the summary of data from the ANOVA test obtained by using the online software found at www.socscistatistics.com.

Table 17: Summary of one-way ANOVA test comparing the difference in the means of the e-textbook, SG1 and SG2, where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	Textbook	SG 1	SG 2			Total
N	35	8	11			54
$\sum X$	98	22	26			146
Mean	2.8	2.75	2.3636			2.704
$\sum X^2$	286	64	64			414
Std.Dev.	0.5841	0.7071	0.5045			0.6028
Result Details						
Source	*SS*	*df*	*MS*			
Between-treatments	1.6138	2	0.8069	*F* = 2.33216		
Within-treatments	17.6455	51	0.346			
Total	19.2593	53				

The F-ratio value is 2.33216, while the *p*-value is 0.107357 (also found using the online software at www.socscistatistics.com). The result is *not* statistically significant at p-value <0.05. This means that there is failure to reject the null hypothesis due to the lack of evidence for differences in the means of the e-textbook, SG1 and SG2. Combined, there is no evidence that the means of cognitive demand levels for the e-learning materials are different.

It is also desirable to compare the means of cognitive demand levels between two e-learning materials. Although a t-test would have sufficed, however due to the low number of tasks for

SG1 and SG2 (minimum of 30 tasks would be required for a t-test), the ANOVA test is still preferred. The test between the e-textbook and e-study guide 1 is formulated as follows:

$H_0: \mu_T = \mu_{SG1}$

$H_1: \mu_T \neq \mu_{SG1}$

Table 18 below provides a summary of data of the ANOVA test (calculated using the online software found at www.socscistatistics.com).

Table 18: Summary of one-way ANOVA test comparing the difference in the means of the e-textbook and SG1, where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	Textbook	SG 1				Total
N	35	8				43
$\sum X$	98	22				120
Mean	2.8	2.75				2.791
$\sum X^2$	286	64				350
Std.Dev.	0.5841	0.7071				0.5999
Result Details						
Source	***SS***	***Df***	***MS***			
Between-treatments	0.0163	1	0.0163	*F* = 0.0442		
Within-treatments	15.1	41	0.3683			
Total	15.1163	42				

The F-ratio value is 0.0442, whereas the *p*-value is 0.834521 (found using the online software at www.socscistatistics.com). The result is *not* statistically significant at the p-value of 5%. Thus, there is failure to reject the null hypothesis, meaning that there is no evidence to suggest that the means of cognitive demand levels for the textbook and study guide 1 are different at 5% statistical level. This is consistent with evidence available showing that the means of cognitive demand level for the two e-learning materials are closer to the cognitive level representing procedures with connections.

Formulating another similar hypothesis test between the e-textbook and SG2, we get the ANOVA test results shown in Table 19 below (found using the online software at www.socscistatistics.com).

$H_0: \mu_T = \mu_{SG2}$

$H_1: \mu_T \neq \mu_{SG2}$

Table 19: Summary of one-way ANOVA test comparing the difference in the means of the e-textbook and SG2, where the left most column represent numerical statistics measures.

Summary of Data						
	Treatments					
	Textbook	SG 2				Total
N	35	11				46
∑X	98	26				124
Mean	2.8	2.3636				2.696
$\sum X^2$	286	64				350
Std.Dev.	0.5841	0.5045				0.5914
Result Details						
Source	***SS***	***Df***	***MS***			
Between-treatments	1.5937	1	1.5937	*F* = 4.95719		
Within-treatments	14.1455	44	0.3215			
Total	15.7391	45				

The F-ratio value is 4.95719. The p-value is given by 0.03115 (also found using the online software at www.socscistatistics.com), meaning that the result is statically significant at p-value < 0.05. What this implies is that there is rejection of the null hypothesis in favour of the alternative hypothesis. The means of the textbook and study guide 2 are not equal. This should not be surprising as the mean of the textbook is closer to the cognitive level code of 3 compared to the mean of the e-study guide 2 which is closer to the cognitive level code of 2.

Now a comparison is made between the means of e-study guide 1 and e-study guide 2. The hypothesis test is test is formulated as follows:

$H_0: \mu_{SG1} = \mu_{SG2}$

$H_1: \mu_{SG1} \neq \mu_{SG2}$

Results are shown in Table 20 below (calculated using the online software found at www.socscistatistics.com).

Table 20: Summary of one-way ANOVA test comparing the difference in the means of SG1 and SG2, where the left most column represent numerical statistics measures

Summary of Data						
	Treatments					
	SG 1	SG 2				Total
N	8	11				19
∑X	22	26				48
Mean	2.75	2.3636				2.526
$\sum X^2$	64	64				128
Std.Dev.	0.7071	0.5045				0.6118
Result Details						
Source	***SS***	***df***	***MS***			
Between-treatments	0.6914	1	0.6914	*F* = 1.9442		
Within-treatments	6.0455	17	0.3556			
Total	6.7368	18				

The F-ratio value is 1.9442, whereas the *p*-value is 0.18117 (calculated using the online software found at www.socscistatistics.com). The result is *not* statistically significant at p-value < 0.05, meaning that there is failure to reject the null hypothesis. According to this result, the means of the cognitive demand levels for SG1 and SG2 are not different from each other. Despite the mean of SG1 being closer to the cognitive level code of 3, and while the mean of SG2 is closer to the cognitive level code of 2, the small number of tasks for each study guide are a contribution to the lack differences in the means. (Note: Large samples tend to induce type one error, whereas small samples tend to contribute to a type 2 error, i.e. we are failing to reject the null hypothesis when it should be rejected (Scott & Morison, 2006)).

4.4. Chapter summary

The video channels were assigned scores using the MCOP^2 instruments. Summary statistics were then generated using the Microsoft Excel software. To compare the means of the three video channels, an ANOVA test was performed using the statistical software found at www.socscistatistics.com. The ANOVA test shows that there are no mean differences in the lessons from the three video channels on counting and probability.

However, these video channels only show the teachers, with little to non-existent voices of students. Three items relating to students on the teacher facilitation part on the MCOP^2 instrument were removed. Subsequent results show that at least one of the means in the three video instructional channels is different to the means of other video channels. The international channel was solely responsible for the difference in the means, with no mean difference between the South African channels using the Post-Hoc Tukey test. Since the mean of the international channel exceeds all other means, this suggests that, when accounting for students' involvement in the lessons and other instructional qualities, the international channel promotes multiple representations of counting and probability compared to the local channels. However, removing items from the MCOP^2 instrument may distort the results, affecting the validity and reliability of the instrument. Thus, results when three items are removed from the MCOP^2 instrument should be interpreted with caution.

This chapter has also shown that when viewed together, the three learning materials have cognitive levels that are not different from each other. Even when pairing two of the three learning materials together, similarities of means of the cognitive demand levels is confirmed save for the comparison between e-textbook and e-study guide 2. The unequal means between the e-textbook and e-study guide 2 could be explained by the big difference in their means as well as the big gap in the number of tasks between the two e-learning materials. Attention now turns to chapter 5 where the research questions are answered, and where conclusions and recommendations are made.

CHAPTER 5: CONCLUSION

5.1 Introduction

This chapter answers the research questions outlined in chapter 1. These research questions are the following:

1. In what ways do observed online video lessons promote multiple representations in Grade 12 probability tasks?
2. How does higher order thinking necessarily promote multiple representations of probability in Grade 12 online mathematics textbooks and study guides?

When answering research question 1, the report will take into account data analysis performed in chapter 4 and the screenshots of the lessons from the video channels as evidence to support positions taken. In answering research question 2, the report will also make reference to the findings from chapter 4. The rest of this chapter will focus on the limitations arising from the research, the journey that this researcher experienced while doing this research, as well as recommendations and areas for future research.

5.2. Answers to research question 1

The first research question is, "[i]n what ways do observed online video lessons promote multiple representations in Grade 12 probability tasks?" I showed in chapter 4 that the means of MCOP^2 scores from video channels 1, 2 and 3 are the same using the ANOVA test statistics. I also showed in chapter 4 that the mean of MCOP^2 scores from video channel 3 is different and exceeds the means of MCOP^2 scores from video channels 1 and 2 if three items relating to students' involvement in lessons are removed from the observation instrument. I now focus on the items of the MCOP^2 instrument that directly implicate multiple representations to answer research question 1. As a result, I will show that all lessons from VC3 promote multiple representations of counting and probability compared to one lesson each from VC 1 and VC 2 that promote multiple representations.

5.2.1 Video channel 1

Items 6, 8, 9 and 11 of the MCOP^2 instrument relate to "[t]he lesson involved fundamental concepts of the subject to promote relational/conceptual understanding", "lesson provided opportunities to examine mathematical structure", "lesson included tasks that have multiple paths to a solution or multiple solutions", and "teacher's talk encouraged student thinking" respectively (see APPENDIX I). A score of 3 on these four items means that counting and probability are represented in multiple ways, involving higher order thinking. A score of 2 implies that certain attributes of higher order thinking, or multiple problem solving strategies, are missed by the teacher.

For item 6, instructions were assigned a score of 2 in all four episodes of VC1. A score of 2 on item 6 of $MCOP^2$ means that "the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students", which is short of the maximum score of 3 where the "teacher/lesson uses these concepts to build relational/conceptual understanding of the students".

For item 8, all four episodes of VC1 are assigned a score of 1 based on the $MCOP^2$ instrument. A score of 1 denotes that "students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves", which is very short of what this research is investigating. For item 11, all four episodes of VC1 are assigned a score of 2 on $MCOP^2$, which means that "the teacher's talk focused on mid-levels of mathematical thinking". It is desirable for the teacher's talk to be focused at higher levels of thinking so that learners are given opportunities, enabling learners to have confidence to use original thinking to complete counting and probabilities tasks.

For item 9 of the $MCOP^2$ instrument, only episode 1 is assigned a score of 3. A score of 3 represents "a lesson which includes several tasks throughout… multiple paths to a solution and which increases the cognitive level of the task". In this episode of VC1, the teacher uses both the tree diagram and algebraic methods to determine probabilities of events. Viewed in terms of Batanero, Godino and Navarro-Pelayo's (1997) framework of classifying multiple representations, tree diagrams are classified as graphical procedures whilst algebraic methods are algebraic procedures. The use of at least two forms of representations in a task is therefore consistent with how multiple representation is defined in this research report (Batanero, Godino, & Navarro-Pelayo, 1997).

The fact that a score of 3 is assigned only to one episode out of a total of four episodes, suggests that VC1 missed the opportunity to follow the recommendations from the

Department of Education (DBE) diagnostic reports for teachers to use multiple strategies to arrive at solutions to probability problems.

5.2.2 Video channel 2

In the previous sub-section of this research report (i.e. 5.2.1), I argued that only items 6, 8, 9 and 11 of the MCOP^2 instrument provide opportunities for the teacher to increase the cognitive thinking of students and use multiple strategies to solve counting and probability tasks. The VC2 lessons have two instructors: Teacher 1 and Teacher 2. It is only episode 3 of Teacher 1's lessons that has two items with the score of 3 assigned to them (i.e. items 6 and 11). A score of 3 on item 6 of MCOP^2 means that "the teacher/lesson uses these concepts to build relational/conceptual understanding of the students"; and a score of 3 on item 11 of MCOP^2 means that the question "requires original, creative thinking". None of the Teacher 2's episodes are assigned a score of 3.

Figure 5 below shows a screenshot (also called a static video clip) taken from episode 3 of Teacher 1's instructions. In the screenshot it can be seen that Teacher 1 uses visual representation to show the four blacks cars (represented by the letter B) that are parked next to each other in a parking bay that has 9 spaces. Looking at the algebraic solution, it is clear that the 6 represents the number of positions that any of the 4 B's could be partitioned into when they are placed next to each other. The 4 factorial that is multiplied with the 6 represents the number of ways that the 4 B's could be arranged when attached together. And the 5 factorial that is multiplied to $6 \times 4!$ represents the number of arrangements that can be made in the remaining 5 empty slots of the parking bay. The visual representation is aligned with the graphical procedures in Batanero et al.'s (1997) framework of classifying multiple representations and the multiplication of factorials is the numerical procedure.

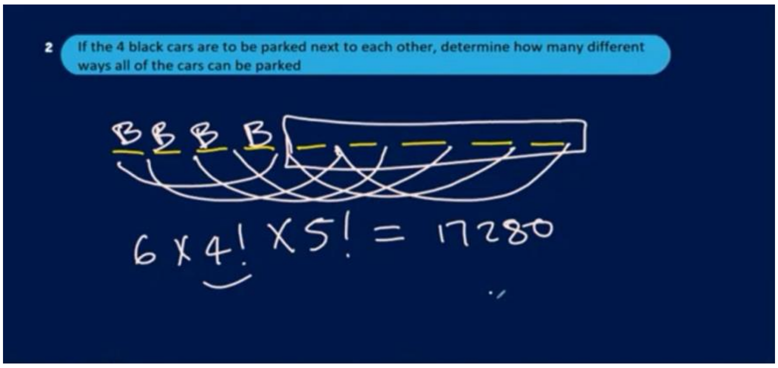

Figure 5: VC2 Episode 3 of Teacher 1 on arrangements. (Used with permission from the content producer who prefers to remain anonymous – see APPENDIX XI)

Item 8 of the MCOP^2 instrument for both Teacher 1 and Teacher 2's lessons are assigned a score of 1. The score of 1 for item 8 indicates that "students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves", suggesting that video lessons that are continuous, without breaks, provide little time for students to discover for themselves relationships and make generalisations. No lesson from Teacher 1 and Teacher 2 is assigned a score of 3 for item 9. It is only episode 3 of Teacher 1 that comes close to using multiple strategies to find a solution to a counting and probability task as shown in Figure 5 above.

Lastly, only episode 3 of Teacher 1's lessons is assigned a score of 3 for item 11 on the MCOP^2 instrument. None of Teacher 2's episodes are assigned a score of 3 for item 11. Score of 3 for item 11 means that the teacher "examines/ interprets the pattern, order or relationship of the mathematics" which promotes higher order thinking of students. Also, according to Batanero, Godino and Navarro-Pelayo (1997), the use of at least two forms of representations is considered multiple representations. Now I consider video lessons from video channel 3.

5.2.3 Video channel 3

For VC3, all six episodes are assigned the score of 3 on the MCOP^2 instrument for items 6, 9 and 11 (see APPENDIX VII). For item 8 of MCOP^2, only episodes 1, 2, 3 and 6 are assigned the maximum score of 3 as the instructor in these episodes clearly asked learners to pause the video so that they could work on their own to discover mathematical relationships, followed by making generalisations to get the solutions. I now show what make these six episodes of VC3 exemplar and why all these episodes promote multiple representations of counting and probability. Batanero, Godino and Navarro-Pelayo (1997) also argued that two or more ways of representing a task is considered as multiple representations. I will attach screenshots on three episodes out of the total of six to support the argument that these lessons promote multiple representations of counting and probability.

Episode 1 of VC3 is concerned with using tree diagrams (i.e. the graphical procedure in Batanero et al.'s (1997) framework of classifying representation) to find the number of ways of arranging objects. This episode used two strategies to arrive at the same answer, implicating item 9 of MCOP^2. This implies that this episode promotes multiple representations of counting. Also, episode 1 uses the tree diagram to promote relational understanding of students (implicating item 6 of MCOP^2). Episode 1 requires creative thinking to show the number of ways of arranging objects (corresponding to item 11 of MCOP^2).

Figure 6 below is a screenshot from VC3 episode 2 and shows how the permutations (or factorial) formula is derived. There are about three distinct strategies or forms of representations used to find the number of ways of partitioning three letters from five letters (implicating item 9 of MCOP^2). These forms of representations are using relational arguments (which is item 6 of MCOP^2 and referred to as logical procedure in Batanero at al.'s framework), multiplication formula (called numerical procedure in Batanero et al.'s framework) and algebraic representation. The last representation shows three ways of writing the permutation formula ${}_5P_2$ (implicating item 9 of MCOP^2). Also, it is apparent that creativity and original thinking is needed to derive the factorial formula (item 11 of MCOP^2).

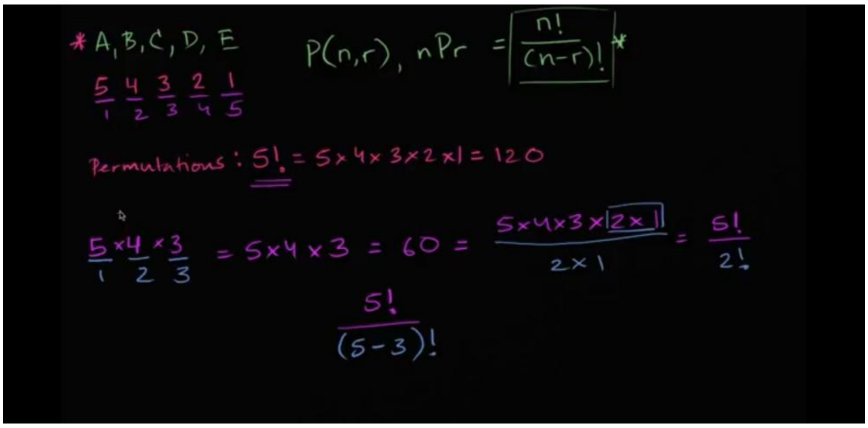

Figure 6: Episode 2 of VC3 showing how to derive the permutation formula. (Used with permission from the Khan Academy – see APPENDIX XII)

Figure 7 below represents episode 3 of VC3. This figure shows the number of ways of partitioning letters A, B and C in 3 slots. Multiple representations are used to partition these letters consisting of the counting strategy (multiplication or numerical procedure in Batanero et al.'s framework), the odometer strategy (logical procedure in Batanero et al.'s framework) and the tree diagram - all implicating item 9 of MCOP^2). In particular, the use of visual diagrams promotes relational understanding, linking the multiplication procedure to the tree diagram (item 6 of MCOP^2). Once the teacher has shown how to arrange the 3 letters, she generalises by arranging 5 and 6 letters using the factorial formula (item 11 of MCOP^2).

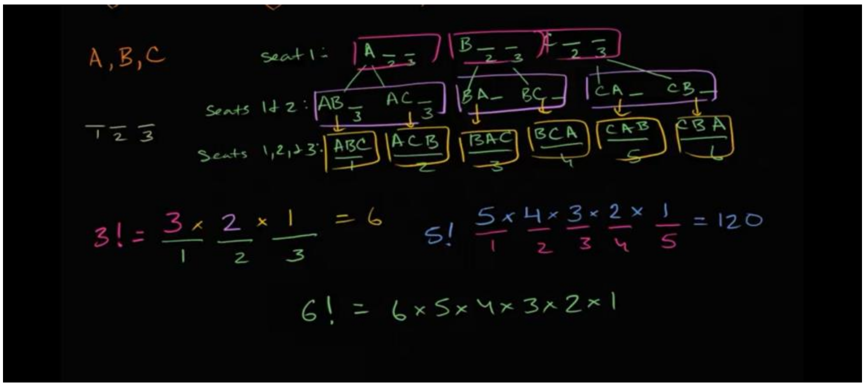

Figure 7: Episode 3 of VC3 showing factorial count of seating arrangements (Used with permission from the Khan Academy – see APPENDIX XII)

Episode 4 of VC3 is about finding the probability of getting exactly two heads when a coin is flipped four times. The instructor in the episode used the listing strategy (graphical procedure in Batanero et al.'s framework), counting principles and algebraic methods to find the probability (implicating item 9 of MCOP^2). Finding the probability using different strategies promotes relational understanding (item 6 of MCOP^2) as students get to make a link between the odometer strategies to counting principles. In this lesson the teacher shows creative thinking (item 11 of MCOP^2) to finding the probability of two heads by employing multiple strategies.

Episode 5 of VC3 is concerned with showing that the value of zero factorial is one. As it can be seen in Figure 8 below, the value of zero factorial is found from first principles by generalising the factorial formula and thereafter using the permutations formula to show that dividing by zero factorial cannot be undefined (implicating item 9 of MCOP ^{2}). Relating the factorial formula to the permutations formula and thereafter showing that dividing by zero factorial is permissible, this promotes conceptual understanding, which is item 6 of MCOP^2. Again, this task requires creative and original thinking (logical procedure in Batanero et al.'s framework) as it is not obvious that zero factorial yields a value of one (item 11 of MCOP^2).

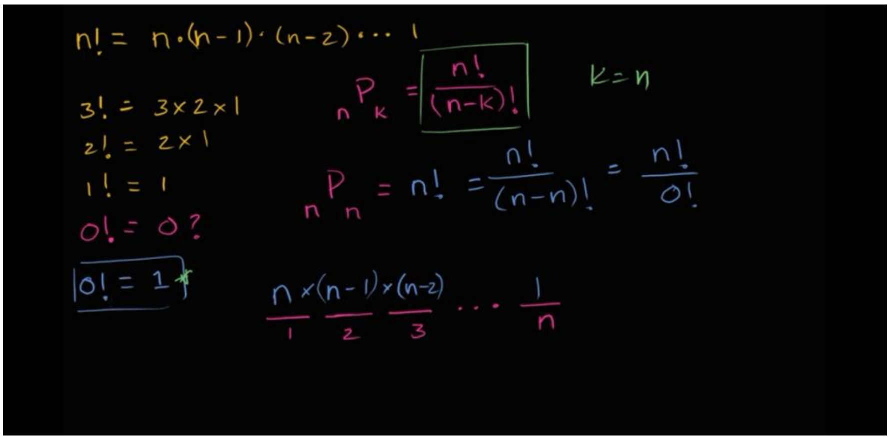

Figure 8: Episode 5 of VC3 showing how to get zero factorial (Used with permission from the Khan Academy – see APPENDIX XII)

The last episode from VC3 is on the introduction to combinations. The lesson links the permutation concept to the concept of combinations (implicating item 6 of MCOP[2]). The lesson starts by partitioning three letters from a total of six letters to find the permutation. Then it is shown that these three letters can be arranged six times, but this arrangement is equivalent to just one combination as the order of arrangement does not concern combinations. So, in essence, combinations are found by finding permutations and diving by the number of arrangements of the chosen letters to account for the ordering of these letters, which does not matter in combinations. Episode 6 of VC3 uses creative and original thinking (item 11 of MCOP[2]) and there are multiple representations used in this lesson (item 9).

5.2.4 Conclusion to research question 1

Overall, all counting and probability lessons from VC3 promote multiple representations of counting and probability. Only one lesson, each from VC1 and VC2, promotes multiple representations of counting and probability. Despite numerous DBE diagnostic reports between the years 2015 and 2020 advising teachers to use various strategies and forms of representations in their instructions on counting and probability, South African based video teaching channels show that teachers still have not implemented this recommendation. I now answer research question 2.

5.3 Answers to Research Question 2

The second research question asks "[h]ow does higher order thinking necessarily promote multiple representations of probability in Grade 12 online mathematics textbooks and study guides?” To answer this question, I will refer to the result for the e-textbook, study guide 1(SG1) and study guide 2 (SG2) from section 4.3 in chapter 4. In particular, I will focus on high level cognitive tasks as these tasks promote multiple representations to find solutions.

Before considering whether each of the three e-learning materials promote multiple representations of probability in Grade12, it is useful to recall the codes assigned to each cognitive demand level. Memorisation is assigned a code of 1, procedures without connections is assigned a code of 2, procedures with connections is assigned a code of 3, and doing mathematics is assigned a code of 4.

5.3.1 e-Textbook

Procedures with connections and doing mathematics constitute high cognitive demand levels (Stein, Grover, & Henningsen, 1996). From chapter four, 72% of counting and probability tasks from the e-textbook were of high cognitive level (see Table 11). Of these tasks, 63% were procedures with connections. In procedures with connections, procedures are connected to the underlying concepts, and tasks are represented in multiple ways, using multiple strategies to finding solutions. Exactly 9% of textbook tasks were doing mathematics. Doing mathematics tasks require non-routine solutions and self-monitoring on the part of learners.

The mean of the cognitive demand levels for the e-textbook tasks is 2.8, as shown in Table 12, which is a value closer to a cognitive demand level of 3 (procedures with connections). The median of the cognitive demand levels for tasks is 3, confirming that 50% or more of tasks were procedures with connections. From this it can be concluded that 72% of tasks from the e-textbook promote multiple representations of Grade 12 probability.

5.3.2 e-Study guide 1

Table 13 shows that 63% of SG1 tasks are of high cognitive level. Of these tasks, 50% are procedures with connections and 13% are doing mathematics. The mean of cognitive demand levels for SG1 tasks is 2.75 (from Table 14), while the median is 3, implying that 50% or more of tasks are procedures with connections. To answer research question 2, the majority of tasks (63% to be exact) found in e-study guide 1 promote multiple representations of probability.

5.3.3 e-Study guide 2

For SG2, only 38% of probability tasks were of high cognitive level (see Table 15 in chapter four). There are no doing mathematics tasks in SG2; meaning 38% of the tasks are solely procedures with connections. The mean of cognitive demand levels for SG2 tasks is 2.36 (from Table 16), which is closer to the level of 2 (procedures without connections). The median of cognitive demand level is 2, meaning that 50% or more of the tasks are procedures without connections. To answer the second research question, SG2 has few tasks that promote multiple representation of probability.

5.3.4 Conclusion to research question 2

In summary, the e-textbook and e-study guide 1 have a significant number of tasks that promote multiple representation of probability. E-Study guide 2 has few tasks that promote multiple representation of probability. It might be useful for authors of e-study guide 2 to update probability tasks in order to comply with the recommendations of the Department of Education that Grade 12 probability tasks be solved using multiple strategies and solutions be represented in multiple ways.

5.4 Generalisation and limitations of the study

Only online video instructions that are recognised by the Department of Education have been used in this research (DBE, 2020b). Online video platforms such as YouTube have thousands of videos that are of educational value. Most of these videos are not accredited by DBE. Some of these videos could better than the videos forming part of the sample for this research, others could be worse. Results from this research should only be read as pertaining to those video channels that are accredited by DBE. The results should not be generalised to video lessons broadcasted on television channels, or to audio lessons. These results do not apply to classrooms lessons conducted by teachers in person.

The same applies to online learning materials. The findings should apply only to online learning materials that are recommended by the Department of Education (DBE, 2020b). Results from the research cannot be generalised to other equivalent learning materials available on the internet. The results cannot be generalised to textbooks used in classrooms taking the form of hard copies nor to study guides that are usually available for a fee.

Covid-19 resulted in this research being limited to online video instructions. School visits to observe teachers in person are always desirable in normal circumstances. Both the teacher facilitation part of the $MCOP^2$ instrument and the student participation part of $MCOP^2$ would have been completed to get a complete sense of whether probability lessons in schools promote multiple representations, but this was not possible during the lockdown situation.

In ensuring that learning materials comply with CAPS curriculum, only a local e-textbook and e-study guides are used in this research. This means that, without international e-textbooks or e-study guides, the replication of the results will have to be designed carefully and executed based on the prevailing circumstances. Nevertheless, the mathematical tasks framework used in this study is also applicable to tasks from international learning materials.

5.5 Implication and recommendations

The results from the video instructions imply that currently few lessons from South African learning channels promote multiple representations of counting and probability compared to the international video instructions. This indicates that there is a gap between what DBE recommends in its yearly Grade 12 diagnostic reports and the instructional reality. If this gap persists, then poor performance in the content area of probability will remain, affecting the overall mathematics outcomes. It is thus recommended that producers of local video instructions use results from this research report to improve the teaching of counting and probability by including multiple strategies and different forms of representation to increase relational understanding of learners.

Two out of three online learning materials have a substantial number of tasks that promote multiple representations of counting and probability. This implies that when learners use tasks from these learning materials to work on their own, without the intervention of teachers, then learners are presented with the opportunity to use multiple strategies and representations to solve counting and probability problems. However, in a classroom setting, there is the danger of teachers reducing the cognitive level of tasks by scaffolding or revoicing questions to make it obvious to the learners the strategies that must be applied to arrive at solutions to probability problems. It is recommended that teachers allow learners to struggle through the tasks, similar to the Covid-19 situations, when learners are working alone at home.

It is also recommended that in-service teachers attend professional development workshops that are designed to promote multiple representations of counting and probability. These workshops should be organised by DBE in partnership with universities. Moreover, lectures on counting and probability for pre-service teachers should be designed in such a way that the use of different strategies and representations to formulate solutions is always encouraged. This will ensure that the next cadre of mathematics educators are sufficiently trained to use reformed and informed instructional practices on counting and probability.

5.6. Reflection on research journey

The year 2020 brought a lot of disruptions in how this research was conducted. I initially intended to observe the teaching and learning of counting and probability in schools to get a better sense on why Grade 12 results in that learning area have persistently been poor. However, lockdown measures were enacted in March 2020 because of Covid-19. These lockdown measures remained in place for three months in schools, making it challenging to collect primary data. I resorted to collecting secondary data. The results from secondary data, especially on video instructions, confirm observations by DBE that the teaching of probability is largely procedural, focused on the use of rules to formulate solutions (Department of Basic Education, 2019, 2020a). The research could be extended by including classroom observations of probability instructional practices under normal learning conditions.

5.7 Areas for future research

This research was solely focused on Grade 12 counting and probability. However, as it has been argued in this research, counting can be introduced in the curriculum in the Senior Phase when learners are at 11 years of age (Grade 7), using representations such as the tree diagrams and counting strategy such as the odometer (listing objects). Research is needed on how counting could be introduced earlier in the curriculum linked to other forms of representations that learners encounter in earlier grades such that by the time learners reach Grade 12, they would have had many years of learning counting and probability using different representations.

As mentioned earlier, similar research is needed to explore whether Grade 12 teachers use multiple representations for counting and probability lessons in classrooms, and if so, to what extent do learners benefit from such instructional strategies in terms of in-school assessment results. This research would require classroom observations in different schools. Another research that could be conducted, linked to classroom observations, is the observation of pre-service teaching lectures on counting and probability. Focus should be on whether pre-service teachers are taught using procedures and rules, which they then replicate in their classrooms once they start teaching.

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APPENDIX I: Mathematics Classroom Observation Protocol for Practices (MCOP^2)

1) Students engaged in exploration/investigation/problem solving.

SE		Description
3		Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.
2		Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.
1		Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate.
0		Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

Comments

2) Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts.

SE		Description
3		The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.
2		The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were **not** explicitly discussed by the teacher or students.
1		The students manipulated or generated one representation of a concept.
0		There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

Comments

3) Students were engaged in mathematical activities.

SE		Description
3		Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
2		Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1		Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students' mathematical engagement.
0		Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.

Comments

4) Students critically assessed mathematical strategies.

SE	TF	Description
3	3	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
2	2	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
1	1	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.
0	0	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.

Comments

5) Students persevered in problem solving.

SE		Description
3		Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
2		Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1		Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a road block to score above a 0.
0		Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem solving situation no students persevered. That is to say, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy they stopped working.

Comments

6) The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.

	TF	Description
	3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
	2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
	1	The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.
	0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the "why" behind the procedures.

Comments

7) The lesson promoted modeling with mathematics.

	TF	Description
	3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).
	2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); <u>or</u> modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
	1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.
	0	The lesson does not include any modeling with mathematics.

Comments

8) The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)

TF	Description	Comments
3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.	
2	Students are given some time to examine mathematical structure, but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.	
1	Students are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.	
0	Students are given no opportunities to explore or understand the mathematical structure of a situation.	

9) The lesson included tasks that have multiple paths to a solution or multiple solutions.

TF	Description	Comments
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.	
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; <u>or</u> more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
1	Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; <u>or</u> a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.	
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.	

10) The lesson promoted precision of mathematical language.

TF	Description	Comments
3	The teacher "attends to precision" in regards to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.	
2	The teachers "attends to precision" in all communication during the lesson, but the students are not always required to also do so.	
1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.	
0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.	

11) The teacher's talk encouraged student thinking.

TF	Description	Comments
3	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. ***Analysis*** *:* examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.	
2	The teacher's talk focused on **mid-levels** of mathematical thinking. **Interpretation**: discovers relationships among facts, generalizations, definitions, values and skills. **Application**: requires identification and selection and use of appropriate generalizations and skills	
1	Teacher talk consists of **"lower order"** knowledge based questions and responses focusing on recall of facts. **Memory**: recalls or memorizes information. **Translation**: changes information into a different symbolic form or situation.	
0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.	

12) There were a high proportion of students talking related to mathematics.

SE	Description	Comments
3	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	
2	More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.	
1	Less than half of the students were talking related to the mathematics of the lesson.	
0	No students talked related to the mathematics of the lesson.	

13) There was a climate of respect for what others had to say.

SE	TF	Description
3	3	**Many** students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
2	2	The environment is such that **some** students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.
1	1	Only a **few** share as called on by the teacher. The climate supports those who understand or who behave appropriately. **Or** Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.
0	0	No students shared ideas.

Comments

14) In general, the teacher provided wait-time.

SE		Description
3		The teacher **frequently** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
2		The teacher **sometimes** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
1		The teacher **rarely** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
0		The teacher **never** provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.

Comments

15) Students were involved in the communication of their ideas to others (peer-to-peer).

SE		Description
3		**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
2		**Some** class time **(less than half, but more than just a few minutes)** was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics.
1		The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.
0		No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.

Comments

16) The teacher uses student questions/comments to enhance conceptual mathematical understanding.

	TF	Description
	3	The teacher **frequently** uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.
	2	The teacher **sometimes** uses student questions/ comments to enhance conceptual understanding.
	1	The teacher **rarely** uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.
	0	The teacher **never** uses student questions/ comments to enhance conceptual mathematical understanding.

Comments

Additional Notes: Preservice or Inservice. Live or Video. #Students, Grade Level, topic/subject, date, other demographics, school, etc.

Source: Gleason, J., Livers, S.D., & Zelkowski, J. (2015). Mathematics classroom observation protocol for practices: Descriptors manual. Retrieved from http://jgleason.people.ua.edu/mcop2.html

APPENDIX II: Permission to use the MCOP^2 instrument.

Gmail **Simphiwe Mlotshwa <simphiwefortune.mlotshwa@gmail.com>**

MCOP2 instrument

3 messages

Simphiwe Mlotshwa <simphiwefortune.mlotshwa@gmail.com> Mon, Dec 14, 2020 at 3:17 PM

To: jgleason@ua.edu

Dear Dr Gleason

I am a Master of Education student with the University of Witwatersrand, South Africa. I intend using the Mathematics Classroom Observation Protocol for Practice (MCOP2) as part of my research report. As part of my research, video lessons would be assigned a score that is based on MCOP2 codes.

I would like to receive permission from you to use the MCOP2 instrument for my research. I look forward to favorable response and permission.

Kind regards

Simphiwe Mlotshwa
Student: University of the Witwatersrand

Virus-free. www.avast.com

Gleason, Jim <jgleason@ua.edu> Mon, Dec 14, 2020 at 4:49 PM
To: Simphiwe Mlotshwa <simphiwefortune.mlotshwa@gmail.com>

You have our permission as we have made it an open source instrument.

From: Simphiwe Mlotshwa <simphiwefortune.mlotshwa@gmail.com>
Sent: Monday, December 14, 2020 5:17:22 AM
To: Gleason, Jim <jgleason@ua.edu>
Subject: [EXTERNAL] MCOP2 instrument

[Quoted text hidden]

Simphiwe Mlotshwa <simphiwefortune.mlotshwa@gmail.com> Mon, Dec 14, 2020 at 5:01 PM
To: "Gleason, Jim" <jgleason@ua.edu>

Thank you
[Quoted text hidden]

APPENDIX III: University of the Witwatersrand Ethics Clearance.

WITS SCHOOL OF EDUCATION

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: 2020ECE016M**
PROJECT TITLE	Investigating Multiple Representation of Grade 12 Counting Principles and Probability in a Virtual Learning Environment
INVESTIGATOR	Fortune Mlotshwa
SCHOOL/DEPARTMENT OF INVESTIGATOR	WITS SCHOOL OF EDUCATION
DATE CONSIDERED	17 June 2020
DECISION OF THE COMMITTEE	Approved unconditionally
EXPIRY DATE	Date of submission of the project report

ISSUE DATE OF CERTIFICATE 29 June 2020 **CHAIRPERSON** ____________ (Dr Paul Goldschagg)

cc: Supervisor: Dr George Ekol

DECLARATION OF INVESTIGATOR

To be completed in duplicate and **ONE COPY** emailed to the Ethics Office: Matsie.Mabeta@wits.ac.za .

I fully understand the conditions under which I am authorized to carry out the abovementioned research and I guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to resubmit the protocol to the Committee.

_______________________ _______________________

Signature **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

APPENDIX IV: Data obtained after observing video channel 1 and the scores are assigned using the $MCOP^2$ instrument.

Item	Description	Ep 1 Code	Ep 2 code	Ep 3 code	Ep 4 code	Combined Average
4	Students critically assessed mathematical strategies	0	0	0	0	0
6	The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.	2	2	2	2	2
7	The lesson promoted modeling with mathematics.	2	2	2	2	2
8	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	1	1	1	1	1
9	The lesson included tasks that have multiple paths to a solution or multiple solutions.	3	1	1	2	1,75
10	The lesson promoted precision of mathematical language.	2	2	2	2	2
11	The teacher's talk encouraged student thinking.	2	2	2	2	2
13	There was a climate of respect for what others had to say	1	0	0	0	0,25
16	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	1	0	0	0	0,25

APPENDIX V: Data obtained after observing video channel 2 (teacher 1) and the scores are assigned using the MCOP^2 instrument.

Item	Description	Ep 1 Code	Ep 2 code	Ep 3 code	Combined Average
4	Students critically assessed mathematical strategies	0	0	0	0
6	The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.	1	2	3	2,00
7	The lesson promoted modeling with mathematics.	2	2	2	2
8	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	1	1	1	1
9	The lesson included tasks that have multiple paths to a solution or multiple solutions.	1	2	2	1,67
10	The lesson promoted precision of mathematical language.	2	2	2	2
11	The teacher's talk encouraged student thinking.	2	2	3	2,33
13	There was a climate of respect for what others had to say	1	1	1	1
16	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	1	1	1	1,00

APPENDIX VI: Data obtained after observing video channel 2 (teacher 2) and the scores are assigned using the MCOP^2 instrument.

4	Students critically assessed mathematical strategies	0	0	0	0
6	The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.	2	2	1	1,67
7	The lesson promoted modeling with mathematics.	2	2	2	2
8	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	1	1	1	1
9	The lesson included tasks that have multiple paths to a solution or multiple solutions.	1	2	1	1,33
10	The lesson promoted precision of mathematical language.	2	2	2	2
11	The teacher's talk encouraged student thinking.	2	2	2	2
13	There was a climate of respect for what others had to say	1	1	1	1
16	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	1	1	1	1,00

APPENDIX VII: Data obtained after observing video channel 3 and the scores are assigned using the MCOP² instrument.

Item	Description	Ep 1 Code	Ep 2 code	Ep 3 code	Ep 4 code	Ep 5 code	Ep 6 code	Combined Average
4	Students critically assessed mathematical strategies	0	0	0	0	0	0	0
6	The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.	3	3	3	3	3	3	3
7	The lesson promoted modeling with mathematics.	2	2	2	2	2	2	2
8	The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.)	3	3	3	1	1	3	2,33
9	The lesson included tasks that have multiple paths to a solution or multiple solutions.	3	3	3	3	3	3	3
10	The lesson promoted precision of mathematical language.	2	2	2	2	2	2	2
11	The teacher's talk encouraged student thinking.	3	3	3	3	3	3	3
13	There was a climate of respect for what others had to say	0	0	0	0	0	0	0
16	The teacher uses student questions/comments to enhance conceptual mathematical understanding.	0	0	0	0	0	0	0

APPENDIX VIII: Cognitive demand level data for the e-textbook tasks obtained using the Mathematical Task Framework instrument

Questions/Tasks	Level of Cognitive Demand	Score of cognitive demand	Features and explanation of tasks
1. Tarryn has five different skirts, four different tops and three pairs of shoes. Assuming that all the colours complement each other, how many different outfits can she put together?	Procedure without connections	2	Procedures applied with limited thinking
2. In a multiple-choice question paper of 20 questions the answers can be A, B, C or D. How many different ways are there of answering the question paper?	Procedure without connections	2	Procedure applied by recalling repetition rule
3. A debit card requires a five digit personal identification number (PIN) consisting of digits from 0 to 9. The digits may be repeated. How many possible PINs are there?	Procedure without connections	2	Procedure applied by recalling repetition rule
4. The province of Gauteng ran out of unique number plates in 2010. Prior to 2010, the number plates were formulated using the style LLLDDDGP, where L is any letter of the alphabet excluding vowels and Q, and D is a digit between 0 and 9. The new style the Gauteng government introduced is LLDDLLGP. How many more possible number plates are there using the new style when compared to the old style?	Procedures with connection	3	Procedure (or any type of representation) applicable, accompanied by cognitive effort on varied concepts
5. A gift basket consists of one CD, one book, one box of sweets, one packet of nuts and one bottle of fruit juice. The person who makes the gift basket can choose from five different CDs, eight different books,	Procedure without connections	2	Procedures applied with limited

three different boxes of sweets, four kinds of nuts and six flavours of fruit juice. How many different gift baskets can be produced?			cognitive effort ($5 \times 8 \times 3 \times 4 \times 6$)
6. The code for a safe is of the form XXXXYYY where X is any number from 0 to 9 and Y represents the letters of the alphabet. How many codes are possible for each of the following cases: a) The digits and letters of the alphabet can be repeated. b) The digits and letters of the alphabet can be repeated, but the code may not contain a zero or any of the vowels in the alphabet. c) The digits and letters of the alphabet can be repeated, but the digits may only be prime numbers and the letters X, Y and Z are excluded from the code.	Procedures with connections	3	Connection between the knowledge of rules of repetitions as well as knowledge of properties of numbers and alphabets needed to complete the task
7. A restaurant offers four choices of starter, eight choices for the main meal and six choices for dessert. A customer can choose to eat just one course, two different courses or all three courses. Assuming that all courses are available, how many different meal options does the restaurant offer?	Procedures with connections	3	Procedures connected and applied with due regard to restrictions, requires cognitive effort
8.	Procedures without connections	2	Effortless application of algorithms

Work out the following without using a calculator: a) $3!$ b) $6!$ c) $2!3!$ d) $8!$ e) $\frac{6!}{3!}$ f) $6! + 4! - 3!$ g) $\frac{6! - 2!}{2!}$ h) $\frac{2! + 3!}{5!}$ i) $\frac{2! + 3! - 5!}{3! - 2!}$ j) $(3!)^3$ k) $\frac{3! \times 4!}{2!}$			requiring limited cognitive effort
9. Calculate the following using a calculator: a) $\frac{12!}{2!}$ b) $\frac{10!}{20!}$ c) $\frac{10! + 12!}{5! + 6!}$ d) $5!(2! + 3!)$ e) $(4!)^2(3!)^2$	Procedures without connection	2	Use of calculator on algorithms
10. Show that the following is true: a) $\frac{n!}{(n-2)!} = n^2 - n$ b) $\frac{(n-1)!}{n!} = \frac{1}{n}$ c) $\frac{(n-2)!}{(n-1)!} = \frac{1}{n-1}$ for $n > 1$	Procedures with connections	3	Procedures applied with considerati on of properties of algebraic division rules
11. How many different possible outcomes are there for a swimming event with six competitors?	Procedures without connections	2	Procedure applicable with knowledge permutatio ns
12. How many different possible outcomes are there	Procedures	3	Use of

for the gold (1st), silver (2nd) and bronze (3rd) medals in a swimming event with six competitors?	with connections		visual representati ons needed
13. Susan wants to visit her friends in Pretoria, Johannesburg, Phalaborwa, East London and Port Elizabeth. In how many different ways can the visits be arranged?	Procedures without connections	2	Procedure of permutatio ns applicable
14. A head boy, a deputy head boy, a head girl and a deputy head girl must be chosen out of a student council consisting of 18 girls and 18 boys. In how many ways can they be chosen?	Procedures with connections	3	Visual representati ons provide clarity; cognitive effort needed
15. Twenty different people enter a golf competition. Only the first six of them can win prizes. In how many different ways can the prizes be won?	Procedures with connections	3	More than one type of representati on can be used; careful use of procedure given the restrictions
16. Three letters of the word 'EMPTY' are arranged in a row. How many different arrangements are possible?	Procedures with connections	3	Combinati on of alphabets with careful considerati on of

			restrictions
17. Pool balls are numbered from 1 to 15. You have only one set of pool balls. In how many different ways can you arrange: a) All 15 balls. Write your answer in scientific notation, rounding off to two decimal places. b) Four of the 15 balls.	Procedures with connections	3	Careful procedures application with due regard to restrictions
18. The captains of all the sports teams in a school have to stand next to each other for a photograph. The school sports programme offers rugby, cricket, hockey, soccer, netball and tennis. a) In how many different orders can they stand in the photograph? b) In how many different orders can they stand in the photograph if the rugby captain stands on the extreme left and the cricket captain stands on the extreme right? c) In how many different orders can they stand if the rugby captain, netball captain and cricket captain must stand next to each other?	Procedures with connections	3	Procedures applicable with due regard to restrictions
19. How many three-digit numbers can be made from the digits 1 to 6 if: a) Repetition is not allowed? b) Repetition is allowed?	Procedures without connections	2	Procedures applicable by recalling repetition rules
20. There are two different red books and three different blue books on a shelf. a) In how many different ways can these books be arranged? b) If you want the red books to be together, in how many different ways can the books be arranged? c) If you want all the red books to be together and all the blue books to be together, in how many different	Procedures with connections	3	Procedures applicable with lot of cognitive effort caused by restrictions on

ways can the books be arranged?			arrangement
21. There are two different Mathematics books, three different Natural Sciences books, two different Life Sciences books and four different Accounting books on a shelf. In how many different ways can they be arranged if: a) The order does not matter? b) All the books of the same subject stand together? c) The two Mathematics books stand first? d) The Accounting books stand next to each other?	Doing mathematics	4	Complex task where all factors are changed or are restricted
22. You have the word 'EXCELLENT'. a) If the repeated letters are regarded as different letters, how many letter arrangements are possible? b) If the repeated letters are regarded as identical, how many letter arrangements are possible? c) If the first and last letters are identical, how many letter arrangements are there? d) How many letter arrangements can be made if the arrangement starts with an L? e) How many letter arrangements are possible if the word ends in a T?	Procedures with connections	3	Procedures applicable requiring cognitive effort with due regards to restrictions
23. You have the word 'ASSESSMENT'. a) If the repeated letters are regarded as different letters, how many letter arrangements are possible? b) If the repeated letters are regarded as identical, how many letter arrangements are possible? c) If the first and last letters are identical, how many letter arrangements are there? d) How many letter arrangements can be made if the arrangement starts with a vowel? e) How many letter arrangements are possible if all the S's are at the beginning of the word?	Procedures with connections	3	Different representations can be used, required cognitive effort with due regard to restrictions

24. On a piano the white keys represent the following notes: C, D, E, F, G, A, B. How many tunes, seven notes in length, can be composed with these notes if: a) A note can be played only once? b) The notes can be repeated? c) The notes can be repeated and the tune begins and ends with a D? d) The tune consists of 3 D's, 2 B's and 2 A's.	Procedures with connections	3	Procedures followed, not mindlessly, with lot of cognitive effort
25. There are three black beads and four white beads in a row. In how many ways can the beads be arranged if: a) same-coloured beads are treated as different beads? b) same-coloured beads are treated as identical beads?	Procedures without connections	2	Procedures applied by recalling rules of identities or non-identities
26. There are eight balls on a table. Some are white and some are red. The white balls are all identical and the red balls are all identical. The balls are removed one at a time. In how many different orders can the balls be removed if: a) Seven of the balls are red? b) Three of the balls are red? c) There are four of each colour?	Procedures with connections	3	Cognitive effort needs, procedures not applied aimlessly
27. How many four-digit numbers can be formed with the digits 3, 4, 6 and 7 if: a) There can be repetition? b) Each digit can only be used once? c) If the number is odd and repetition is allowed?	Procedures with connections	3	Cognitive effort needed, procedures not followed aimlessly
28. A music group plans a concert tour in South Africa. They will perform in Cape Town, Port Elizabeth, Pretoria, Johannesburg, Bloemfontein,	Procedures with connections	3	Cognitive needed, probability

<table>
<tr><td>Durban and East London.
a) In how many different orders can they plan their tour if there are no restrictions?
b) In how many different orders can they plan their tour if their tour begins in Cape Town and ends in Durban?
c) If the tour cities are chosen at random, what is the probability that their performances in Cape Town, Port Elizabeth, Durban and East London happen consecutively? Give your answer correct to 3 decimal places.</td><td></td><td></td><td>connected to arrangements of cities with restrictions</td></tr>
<tr><td>29.
A certain restaurant has the following course options available for a three-course set menu:<table><tr><th>STARTERS</th><th>MAINS</th><th>DESSERTS</th></tr><tr><td>Calamari salad</td><td>Fried chicken</td><td>Ice cream and chocolate sauce</td></tr><tr><td>Oysters</td><td>Crumbed lamb chops</td><td>Strawberries and cream</td></tr><tr><td>Fish in garlic sauce</td><td>Mutton Bobotie</td><td>Malva pudding with custard</td></tr><tr><td></td><td>Chicken schnitzel</td><td>Pears in brandy sauce</td></tr><tr><td></td><td>Vegetable lasagne</td><td></td></tr><tr><td></td><td>Chicken nuggets</td><td></td></tr></table>a) How many different set menus are possible?
b) What is the probability that a set menu includes a chicken course?</td><td>Procedures with connections</td><td>3</td><td>Mental effort, procedures applied not aimlessly</td></tr>
<tr><td>30. Eight different pairs of jeans and 5 different shirts hang on a rail.
a) In how many different ways can the clothes be arranged on the rail?
b) In how many ways can the clothing be arranged if all the jeans hang together and all the shirts hang together?
c) What is the probability, correct to three decimal places, of the clothing being arranged on the rail with a shirt at one end and a pair of jeans at the other?</td><td>Doing mathematics</td><td>4</td><td>Task requires non-algorithmic thinking, many constraints requiring exploration of relationships</td></tr>
<tr><td>31. A photographer places eight chairs in a row in his studio in order to take a photograph of the debating</td><td>Procedures with</td><td>3</td><td>Procedures applied not</td></tr>
</table>

team. The team consists of three boys and five girls. a) In how many ways can the debating team be seated? b) What is the probability that a particular boy and a particular girl sit next to each other?	connections		aimlessly, cognitive effort essential
32. If the letters of the word 'COMMITTEE' are randomly arranged, what is the probability that the letter arrangements start and end with the same letter?	Procedures with connections	3	Cognitive effort required, mindful of procedures followed, multiple paths to solutions
33. Four different Mathematics books, three different Economics books and two different Geography books are arranged on a shelf. What is the probability that all the books of the same subject are arranged next to each other?	Procedures with connections	3	Cognitive effort needed, multiple paths to solutions, mindful of procedures followed
34. A number plate is made up of three letters of the alphabet (excluding F and S) followed by three digits from 0 to 9. The numbers and letters can be repeated. Calculate the probability that a randomly chosen number plate: a) Starts with the letter D and ends with the digit 3. b) Has precisely one D. c) Contains at least one 5.	Procedures with connections	3	Critical thinking needed, multiple paths to solution, mindful of procedures followed
35. In the 13-digit identification (ID) numbers of South African citizens:	Doing mathematic	4	Requires non-

The first six numbers are the birth date of the person in YYMMDD format. The next four digits indicate gender, with 5000 and above being male and 0001 to 4999 being female. The next number is the country ID; 0 is South Africa and 1 is not. The second last number used to be a racial identifier but it is now 8 for everybody. The last number is a control digit, which verifies the rest of the number. Assume that the control digit is a randomly generated digit from 0 to 9 and ignore the fact that leap years have an extra day. a) Calculate the total number of possible ID numbers. b) Calculate the probability that a randomly generated ID number is of a South African male born during the 1980s. Write your answer correct to two decimal places.	s		algorithmic thinking, no clear pathway to solution,

Source: *Jenkins, A., Van Zyl, M., & Kannemeyer, L. (2011). Everything Maths: Grade 12 Mathematics, version 1 CAPS. Cape Town: Siyavula Education.*

Coding

- 1 = memorisation
- 2 = procedures without connections
- 3 = procedures with connections
- 4 = doing mathematics

APPENDIX IX: Cognitive demand level data for SG1 tasks obtained using the Mathematical Task Framework instrument

Questions/Tasks	**Level of Cognitive Demand**	**Score of cognitive demand**	**Features and explanation of tasks**
1. How many different 074- cell phone numbers are possible if the digits may not repeat?	procedures without connections	2	7!=5040
2. How many different 082- cell phone numbers are possible if the digits may only be integers?	procedures without connections	2	10^7 $= 10000000$
3. What is the probability that you will draw a queen of diamonds from a pack cards?	procedures without connections	2	P(Queen of diamonds) $=\frac{1}{52}$
4. How many different arrangements can be made with the letters of the word TSITSIKAMMA, if: a) Repeating letters are regarded as different letters. b) Repeating letters are regarded as identical.	procedures with connections	3	Cognitive effort required when answering part b. $\frac{11!}{2!\,2!\,2!\,2!\,2!}$
5. Four different English books, three different German books and two different Afrikaans books are randomly arranged on a shelf. Calculate the number of arrangements if: a) The English books have to be kept together. b) All books of the same language have to be kept together. c) The order of the books does not matter.	procedures with connections	3	Solution can be represented in multiple ways. Lot of cognitive thinking involved.
6. In how many different ways can a chairman and a vice-chairman be chosen from a committee of 12 people?	procedures with connections	3	Tricky, procedure applied with cognitive

			effort. $\frac{12!}{10!}$
7. The letters of the word MATHEMATICS have to be rearranged. Calculate the probability that the "word" formed will not start and end with the same letter.	doing mathematics	4	Task require active exploration of alternative solutions
8. In how many different ways can the letters of the word MATHEMATICS rearranged so that: a) The H and the E stay together. b) The E keeps its position.	procedures with connections	3	Procedure applied with cognition of the underlying maths

Source: *Malan, M. (n.d.). Via Afrika Mathematics Grade 12 Study Guide. Via Afrika Publishers.*

Coding

- 1 = memorisation
- 2 = procedures without connections
- 3 = procedures with connections
- 4 = doing mathematics

APPENDIX X: Cognitive demand level data for SG2 tasks obtained using the Mathematical Task Framework instrument

Questions/Tasks	**Level of Cognitive Demand**	**Score of cognitive demand**	**Features and explanation of tasks**
1. Determine the number of permutations that can be formed from all the letters of the word **ABRACADABRA**.	procedures without connections	2	Procedures applicable by
2. Determine the number of permutations that can be formed from all the letters of the word **ABRACADABRA**. This time, the first and last letters must be **A**.	procedures with connections	3	Procedures applicable with considerati on to underlying properties of permutatio ns
3. Determine the number of permutations that can be formed from all the letters of the word **ABRACADABRA**. This time, all the As have to be next to each other.	procedures with connections	3	Careful considerati ons of properties of permutatio ns while procedures are applied
4. At Angelo's pizza place you can choose from 6 different types of pasta and 28 different sauces. How many different meals of 1 type of pasta and 1 type of sauce can you have?	procedures without connections	2	Procedure applied with limited cognitive effort
5. In how many different ways can we arrange 7 books on a shelf?	procedures without connections	2	Procedure applied with

			limited cognitive effort (7!)
6. In how many different ways can 9 girls sit on one side of a table?	procedures without connections	2	Algorithm applied with limited cognitive effort (9!)
7. In how many ways can a three-letter word be made from the letters c; d; e; f without repeating any letters?	procedures with connections	3	Cognitive effort needed, with help of multiple representati on
8. How many possible choices can be made in a multiple choice quiz if there are 4 questions each with 3 answers?	procedures without connections	2	Procedure applied by recalling repetition principle
9. How many different words can be made using the letters from **LIMPOPO**?	procedures without connections	2	Procedure applied by dividing with repeated letters
10. How many 3-digit numbers can be made with the digits 1 – 5 if: a) Repetitions are allowed. b) Repetitions are not allowed.	procedures without connections	2	Procedures applied by recalling repetition rules
11. A code is made using the format XYY, where the	Procedures	3	Multiple

X is any letter in the alphabet and Y represents any digit from 0 to 9. a) How many possible codes can be formed if the letters and digits are repeated? b) How many possible codes can be formed if the letters and digits are not repeated?	with connections		representations needed with properties of alphabets and digits

Source: *Department of Basic Education. (2015b). Grade 12 Mind the Gap study guide for Mathematics. Pretoria: Department of Basic Education.*

Coding

- 1 = memorisation
- 2 = procedures without connections
- 3 = procedures with connections
- 4 = doing mathematics

APPENDIX XI: Copyright permission for video channel 2

Re: {Info} Permission to use a screenshot of a video lesson from [redacted] for research

10 messages

[redacted] 15 January 2021 at 08:49

To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

Cc: [redacted]

Hi Simphiwe and many thanks for your email. Is it likely that you will publish this video clip at all?

Let me know and I will then send the request to our exco.

Regards

[redacted]

On Thu, 14 Jan 2021 at 18:14, Fortune Mlotshwa <0313218x@students.wits.ac.za> wrote:

Dear [redacted]

My name is Simphiwe Fortune Mlotshwa. I am Master of Education student at the University of the Witwatersrand. I am currently doing a research project that seek to investigate whether video lessons on counting and probability in Grade 12 make use of multiple strategies to find solutions to tasks. My research is titled "Investigating Grade 12 probability online learning resources for multiple representations". My research supervisor at Wits is Dr. George Ekol. He can be contacted in his email, george.ekol@wits.ac.za.

I would like your consent to use a screenshot from one of the video lessons in your YouTube channel for research purposes. The lesson is dated 7 July 2020 and it is.5 minutes and 15 seconds long. The part that I am interested in appears in 4 minutes 35 seconds in the video. The screenshot is attached below.

The name of your organization will be kept confidential during the research process and in the final report. Teachers appearing in the videos will not be shown in the screenshot.

I would greatly appreciate your consent to my request. Let me know if you need clarity on my request, I will gladly respond.

Kind Regards
Simphiwe Mlotshwa
Cell number 072 614 7872

Virus-free. www.avast.com

--

[redacted]

[redacted]

Fortune Mlotshwa <0313218x@students.wits.ac.za> 15 January 2021 at 09:46
To: [redacted]>

Dear [redacted]

No, I will only publish the screenshot. The video will not be published. The name of your organisation will be kept confidential. Only myself and my research supervisor will know the name of your organisation.

Kind Regards
Simphiwe

[redacted] 15 January 2021 at 10:11
To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

Hi again

The screenshot you mention is actually a video clip is it not? A screenshot is static so I am not completely clear. Who will this be exposed to during the research? Where will you publish?

[redacted]

Fortune Mlotshwa <0313218x@students.wits.ac.za> 15 January 2021 at 10:31
To: [redacted]

Dear [redacted]

Yes, the screenshot is a video clip and is static. I am interested in that screenshot as part of my research that seeks to investigate whether counting and probability in Grade 12 instructions is presented in ways that promote multiple representation. By multiple representation I mean showing solutions to a question using two or more of the following methods: algebraic, numeric, graphical, pictorial, and others. The full video will not be saved nor will it be reproduced in the research.

The research will be submitted to the University of the Witwatersrand for examination as part of my Master of Education degree. As part of ethics protocol of the University, I will maintain confidentiality of your organisation and teachers appearing in the video. Myself and my supervisor will know about the name of your organisation. Both of us will maintain confidentiality of your organisation and any person appearing in the video. Even examiners of the research will not know the name of your organisation and any person who appeared in the video.

I hope this provides clarity to your questions.

Best regards
Simphiwe

[redacted] 15 January 2021 at 10:41
To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

Okay then please go ahead.

Fortune Mlotshwa <0313218x@students.wits.ac.za> 15 January 2021 at 10:45
To: [redacted]

Thank you for the permission.

Best regards
Simphiwe

[redacted] 15 January 2021 at 10:49
To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

You are welcome.

[redacted] 15 January 2021 at 11:14
To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

Hi Simphiwe

My exco have asked that you share the paper with us as well.

Regards
[redacted]

Fortune Mlotshwa <0313218x@students.wits.ac.za> 15 January 2021 at 12:05
To: [redacted]

Dear [redacted]

I will share the research report once it has been examined by the University.

Regards
Simphiwe

[redacted] 15 January 2021 at 12:31
To: Fortune Mlotshwa <0313218x@students.wits.ac.za>

Noted with thanks.
[Quoted text hidden]

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Curriculum and Assessment Policy Statement (CAPS) Grade 12
[redacted] study guide for Mathematics
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