

The effect of suction and blowing at the base on the horizontal spreading under gravity of a two-dimensional thin fluid film and an axisymmetric liquid drop is investigated. The velocity  $v_n$  which describes the suction/injection of fluid at the base is not specified initially. The height of the thin film satisfies a nonlinear diffusion equation with  $v_n$  as a source term. The Lie group method for the solution of partial differential equations is used to reduce the partial differential equations to ordinary differential equations and to construct group invariant solutions. For a group invariant solution to exist,  $v_n$  must satisfy a first order linear partial differential equation. The two-dimensional spreading of a thin fluid film is first investigated. Two models for  $v_n$  which give analytical solutions are analysed. In the first model  $v_n$  is proportional to the height of the thin film at that point. The constant of proportionality is  $\beta$  ( $-\infty < \beta < \infty$ ). The half-width always increases to infinity as time increases even for suction at the base. The range of  $\beta$  for the thin fluid film approximation to be valid is determined. For all values of suction and a small range of blowing the maximum height of the film tends to zero as time  $t \rightarrow \infty$ . There is a value of  $\beta$  corresponding to blowing for which the maximum height remains constant with the blowing balancing the effect of gravity. For stronger blowing the maximum height tends to infinity algebraically, there is a value of  $\beta$  for which the maximum height tends to infinity exponentially and for stronger blowing, still in the range for which the thin film approximation is valid, the maximum height tends to infinity in a finite time. For blowing the location of a stagnation point on the centre line is determined by solving a cubic equation approximately by a singular perturbation method and then exactly using a trigonometric solution. A dividing streamline passes through the stagnation point which separates the flow into two regions, an upper region consisting of fluid descending due to gravity and a lower region consisting of fluid rising due to blowing. For sufficiently strong blowing the lower region fills the whole of the film. In the second model  $v_n$  is proportional to the spatial gradient of the height with constant of proportionality  $\beta^*$  ( $-\infty < \beta^* < \infty$ ). The maximum height always decreases to zero as time increases even for blowing. The range of  $\beta^*$  for the thin fluid film approximation to be valid is determined. The half-width tends to infinity algebraically for all blowing and a small range of weak suction. There is a value of  $\beta^*$  corresponding to suction for which the half-width remains constant with the suction balancing the spreading due to gravity. For stronger suction the half-width tends to zero as  $t \rightarrow \infty$ . For even stronger suction there is a value of  $\beta^*$  for which the half-width tends to zero exponentially and a range of  $\beta^*$  for which it tends to zero in a finite time but these values lie outside the range for which the thin fluid film approximation is valid. For blowing there is a stagnation point on the centre line at the base. Two dividing streamlines pass through the stagnation point which separate fluid descending due to gravity from fluid rising due to blowing. An approximate analytical solution is derived for the two dividing streamlines. A similar analysis is performed for the axisymmetric spreading of a liquid drop and the results are compared with the two-dimensional spreading of a thin fluid film. Since the two models for  $v_n$  are still quite general it can be expected that general results found will apply to other models. These include the existence of a dividing streamline separating descending and rising fluid for blowing, the existence of

a strength of blowing which balances the effect of gravity so the maximum height remains constant and the existence of a strength of suction which balances spreading due to gravity so that the half-width/radius remains constant.