STRUCTURAL OPTIMISATION USING THE PRINCIPLE OF VIRTUAL WORK

Richard Shaun Walls

A dissertation submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, in fulfilment of the requirements for the degree Masters of Science in Engineering.

Johannesburg, 2010.

DECLARATION

I declare that this dissertation is my own unaided work. It is being submitted to the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

………………………………………………………………………………………

Richard Shaun Walls

……… day of ………………… year …………….

ABSTRACT

This dissertation presents a new method for the automated optimisation of structures. The method has been developed to: (1) select sections to satisfy strength and deflection requirements using minimum material, and (2) efficiently group members.

The member selection method is based on the principle of virtual work, and is called the Virtual Work Optimisation (VWO) method. It addresses multiple deflection and load case constraints simultaneously. The method determines which sections provide the highest deflection and strength resistance per unit mass. When compared to several other methods in the literature, and designs from industry, the VWO method produced savings of up to 15.1%.

A parametric investigation of ungrouped, multi-storey frames is conducted using the VWO method to determine optimal mass and stiffness distributions. Unusual mass patterns have been found. Diagonal paths of increased stiffness are formed in the frames, which suggests truss behaviour.

A grouping algorithm is presented which determines how efficiently to create a specified number of groups in a structure. The VWO method has been incorporated into the automated algorithm to optimise the grouped structures. Members are grouped according to their mass per unit length. In the algorithm an exhaustive search of all feasible grouping permutations is carried out, and the lightest structure selected. Results produced are up to 5.9% lighter than those obtained using *ad hoc* grouping configurations found in the literature and based on experience.

ACKNOWLEDGEMENTS

I would like to thank:

- My supervisor, Prof. Alex Elvin, for the support and assistance received while completing this degree.
- My parents for the guidance and support throughout the course of doing all my studies.
- The SA Institute of Steel Construction (SAISC), especially Mr. Spencer Erling and Dr. Hennie de Clercq, for guidance received while conducting this research and developing ideas.
- Daniel Cromberge, for the assistance in 2008 during the initial stages of this research.
- BSM Baker, my bursary company, for allowing me to spend this year doing my masters degree.
- My Lord and Saviour Jesus Christ, for giving me the ability and opportunity to do this MSc.

CONTENTS **Page**

LIST OF FIGURES **Page**

LIST OF TABLES **Page**

LIST OF EQUATIONS **Page**

LIST OF SYMBOLS

A	Cross-sectional area
E	Young's modulus
Efficiency	The efficiency of a section change
f	Axial force in a member due a unit load
F	Axial force in a member due to an applied system of loads
$\underline{F}$	Virtual point force
G	Shear modulus
i	Number of initial sections in a structure
I	Second moment of area
J	Polar second moment of area
L	Length
m	Moment in a member due a unit load
m	Mass per unit length of a member
M	Moment in a member due to an applied system of loads
M	Total mass of a grouped structure
n	Number of groups to be created by the grouping algorithm
N	Number of grouping permutations
NC	Number of section changes that must be tested
q	Shear force in a member due a unit load
Q	Shear force in a member due to an applied system of loads
t	Torsion a member due a unit load
T	Torsion in a member due to an applied system of loads
V	Volume
X	Radius of search space to be investigated by the grouping algorithm
δ	Deflection contribution of a member
Δ	Deflection of a critical point
Δ^{Decrease}	Deflection decrease due to a section change
Δ^{Target}	Target deflection of a critical point
ΔM	Total mass change due to a section change

PREFACE – Note on the publication of journal articles

The following chapters from this dissertation have been submitted as papers to journals:

Chapter 2

Title: "Optimising Structures Using the Principle of Virtual Work"

Status: Published in the South Africa Institute of Civil Engineers (SAICE) Journal, October 2009 edition. (Vol. 51, No. 2, Pg 11-19, Paper 707).

Authors: Elvin, A.A., Walls, R.S. and Cromberge D.M.

Chapter 3

Title: "Optimizing Structures Subject to Multiple Deflection Constraints and Load Cases using the Principle of Virtual Work"

Status: Under review: Journal of Structural Engineering, ASCE.

Authors: Walls, R.S. and Elvin A.A.

Title: Automated Structural Design and Optimisation

Status: Accepted to be published in The Structural Engineer – Journal of the Institute of Structural Engineers (IStructE, UK).

Authors: Walls, R.S. and Elvin A.A.

Chapter 4

Title: "Mass and stiffness distributions in optimized ungrouped frames"

Status: Under review: International Journal of Steel Structures

Authors: Walls, R.S. and Elvin A.A.

Chapter 5

Title: "An Algorithm for Grouping Members in a Structure"

Status: Under review: Engineering Structures.

Authors: Walls, R.S. and Elvin A.A.

The following papers have been provisionally accepted to the Fourth International Structural Engineering, Mechanics and Computation Conference in 2010:

Title: The Virtual Work Optimisation Method Applied to Structures

Title: Grouping Members in a Structure

The following non-refereed paper is based on the research presented in this dissertation:

Title: Automating Structural Design – Getting Computers to Design
Status: Published in the Southern African Institute of Steel Construction Journal. Vol. 33, No. 3, May 2009.
Authors: Walls, R.S. and Elvin A.A.

CHAPTER 1: INTRODUCTION

This dissertation presents a new method for the automated design and optimisation of structures. The method is based on the principle of virtual work. Structural masses are minimised by efficiently selecting sections for members, and grouping members together.

1.1 Introduction to automated structural design

The design process in structural engineering is time-consuming, iterative and significantly affects the total cost of the project. Even though great advances have been made in automating the design process an effective and a general structural optimisation method is not available yet.

The design of a structure is primarily governed by strength and flexibility requirements, and a design must satisfy budget constraints. Automating the selection of sections to satisfy strength requirements is a straightforward task, and numerous software packages have this capability. However, satisfying flexibility, or deflection, constraints is much more complex, and is often not done at all or done poorly. For every change in a statically indeterminate structure a redistribution of force occurs which cannot be accurately predicted without reanalysing the structure. This makes it impossible to determine member sizes in a single step, and iterative methods are required. Structures with more than a few members have prohibitively large search spaces, so exhaustive searches cannot be carried out. However, even if members are correctly sized solutions obtained may not be optimal because of the member groupings defined by the user. If light and heavy members are grouped together then the lighter members will be assigned a larger than necessary section. This makes structures uneconomical. Thus, efficient methods for grouping members are also required.

1.2 The need for better optimisation methods

As construction materials increase in cost it is becoming more important that designers minimise material wastage in their designs. This necessitates the use of optimisation methods. Also, engineers often have limited time in which to design. Hence, only one structure is usually designed, rather than exploring a variety of structural configurations, to find which is best.

Structural engineers are generally unwilling to use optimisation methods which are computationally expensive, difficult to implement, can only be applied to certain structures or cannot be easily understood. The aforementioned problems must be overcome before optimisation methods can become viable.

Grouping is essential for reducing fabrication and erection costs. It must be considered in design. However, efficiently grouping a structure is a complex task and no automated methods are currently available.

1.3 An overview of optimisation literature

Numerous guidelines have been published relating to the optimisation of structures. Books by Wood (1960), Dowling et al. (1988) and the SA Institute of Steel Construction (2001) demonstrate how structures can be optimised by engineers through the use of good designs and careful member selections. Such methods are effective but must be manually implemented, and are generally more applicable when defining structural geometries and groups. Engineers require experience to implement these intuitive techniques. In larger, more complex structures such methods cannot easily be applied. Optimisation methods which can be automated are not addressed in this literature.

Books by Gallagher and Zienkiewicz (1973) or Haftka and Gürdal (1992) describe various computer optimisation methods that can be applied to structural design. However, all the methods presented are computationally expensive, which significantly limits their application. Some of the methods presented would take years, or even centuries, to optimise large structures.

Review papers on structural optimisation have been published by Arora and Huang (1994), Thanedar and Vanderplaats (1995) and Maalawi and Badr (2009), amongst others. These authors acknowledge that most methods are suitable only for specific types of structures, and a generic structural optimisation method is not yet available.

In technical and review papers methods are often compared against each other to determine which methods are superior. However, results presented are dependent on factors such as the design parameters chosen, algorithm used, the nature of the structures optimised, the number of sections considered, and the computational power available.

Recently researchers have developed optimisation methods based on genetic algorithms (Erbatur, 2009), harmony search algorithms (Saka, 2009), particle swarm optimisation (Li et al., 2009), ant colony optimisation (Camp et al., 2005), or tabu search (Kargahi et al., 2006). There is little agreement regarding which method is the most efficient and how these methods can be used in practice. It is possible that the aforementioned methods could be applied to the problem of optimising member groupings. However, this has not been reported in the literature.

The principle of virtual work has been used to determine which members should be selected to limit deflections by Park and Park (1997). However, the method developed only takes structural deflection requirements into account, and does not consider strength constraints. Optimality Criterion (OC) methods also use the principle of virtual work. However, OC methods select sections from a continuous spectrum, and a relationship between sectional parameters in databases, such as area and second moment of area, must be assumed (Chan, 1992; Pezeshk, 1998). Methods published by Makris and Provatidis (2002) and Makris et al. (2006) use strain energy criteria to optimise structures. Member selection is performed based on determining either cross-sectional areas for trusses or second moment of areas for frames, not both simultaneously. Most structures cannot be considered by such methods because bending, torsion and axial forces cannot be separated. Patnaik et al. (1997) proposed a methodology of satisfying stress constraints and then reducing deflections, but once again considered only trusses.

Few methods for automating the grouping of members can be found in the literature. Researchers have developed methods which group members according to the magnitude of internal forces (Krishnamoorthy et al., 2002; Toğan and Doloğlu, 2006, 2008), slenderness ratios (Toğan and Doloğlu, 2008), member lengths (Biedermann and Grierson, 1995), or sectional areas (Shea et al., 1997; Isaacs et al., 2008).

However, these methods suffer from one or more of the following limitations: only axial or bending forces are considered, one grouping configuration is tested, only one load case is considered and users must define empirical parameters which affect groupings.

1.4 Definitions of terms used

Is this dissertation an *optimal structure* is defined as one which satisfies all strength and deflection criteria using minimum material. A *group* is all the members in a structure which are constrained to have the same section. An *ungrouped structure* is one in which every member can have a different section. A *target deflection* is the maximum allowable amount a critical point is allowed to deflect, and is usually specified by codes. A *critical point* is the node in a structure that is being investigated at which deflections have to be limited, and is usually a point of maximum deflection.

1.5 Limitations of the research

This dissertation does not address the problem of optimising the geometric topology of a structure. The geometry significantly affects the efficiency of a structure. Refer to papers by authors such as Bendsøe et al. (1994), Kwak (1994), Fourie and Groenwold (2002) or Lee and Geem (2004) for more information.

Only steel structures have been considered. However, the methods developed would be suitable for other materials as well. Structures in which more than one material is used simultaneously have not been investigated, and this is a topic for further research.

In this dissertation it is assumed that a structure of minimum mass will be the most economical. This is an oversimplification and not always true. The problem of minimising total structural costs, including fabrication and erection, has not been considered, and is topic for further research.

1.6 Dissertation organisation

This dissertation develops and implements a method for automating the selection of structural sections, and grouping members. Chapter 2 presents the initial development of the method to address structures subject to a single deflection criterion and load case. The theory underlying the method is discussed. The methodology is modified and expanded in Chapter 3 such that structures with multiple deflection constraints and load cases can be addressed. Results obtained in Chapters 2 and 3 are compared to those in the literature and to practising engineers' designs. A parametric study of ungrouped, multi-storey frames is conducted in Chapter 4. It is observed that ungrouped, optimised frames tend towards specific, but unexpected, distributions of mass. In Chapter 5 an automated member grouping algorithm is presented. The algorithm utilises the member selection techniques developed in Chapter 3 to obtain optimised, ungrouped structures which can be grouped. Chapter 6 presents overall conclusions regarding the research, and discusses topics requiring future research. Results produced by the methods developed are compared to those found in the literature or from available civil engineering design practice to verify the solutions calculated and to show the effectiveness of methods.

1.7 References

Arora, J S and Huang, M W. (1994) Review Papers - Methods for optimisation of nonlinear problems with discrete variables: a review. *Struct. Optim.*, 8, 69-85.

Bendsøe, M P, Ben-Tal, A, and Zowe, J. (1994) Review Paper – Optimisation methods for geometry and topology design. *Struct. Optimisation*, 7, 141-159.

Biedermann, J.D., and Grierson, D.E. (1995) "A Generic Model for Building Design". *Engineering with Computers*, 11, 173-184.

Camp, C V, Bichon, B J and Stovall, S P. (2005) Design of Steel Frames Using Ant Colony Optimisation. *J. Struct. Div,* ASCE, 131(3), 369-379

Chan, C M. (1992) An optimality criteria algorithm for tall steel building design using commercial standard sections. *Structural Optimisation,* 5, 26-29.

Dowling, P J, Knowles, P, and Owens, G W. (1988) *Structural Steel Design.* The Steel Construction Institute, UK.

Erbatur, F, Hasançebi, O, Tütüncü, I and Kiliç, H. (2009) Optimal design of planar frames with genetic algorithms. *Computers and Structures*, 75, 209-224.

Fourie, P C, and Groewold, A A. (2002) The particle swarm optimisation algorithm in size and shape optimisation. *Struct, Multidisc. Optim.*, 23, 259–267.

Gallagher, R H. (1973) Fully Stressed Design in *Optimum Structural Design: Theory and Applications*. Editors Gallagher, R.H., and Zienkiewicz, O.C., Bristol, John Wiley & Sons.

Gallagher, R H, and Zienkiewicz, O C. (1973) *Optimum Structural Design: Theory and Applications*. Bristol, John Wiley & Sons.

Groenwold, A A, Stander, N and Snyman, J A. (1996) A pseudo-discrete rounding method for structural optimisation. *Structural Optimisation*, 11, 218-227.

Haftka, R T, and Gürdal, Z. (1992) *Elements of Structural Optimisation*. 3rd revised and expanded ed. London, Kluwer Academic Publishers.

Isaacs, A, Ray, T, and Smith, W (2008) An efficient hybrid algorithm for optimisation of discrete structures. *SEAL 2008*, LNCS 5361, 625–634.

Kargahi, M, Anderson, J C, and Dessouky, M M (2006) Structural Weight Optimisation of Frames Using Tabu Search. I: Optimisation Procedure. *Journal of Structural Engineering*, ASCE, 132(12), 1858-1868.

Krishnamoorthy C.S., Venkatesh P.P., Sudarshan R. (2002) "Object-oriented framework for genetic algorithms with application to space truss optimisation". *J Comput. Civil Eng*, ASCE, 16, 66–75.

Kwak, B M. (1994) A review on shape optimal design and sensitivity analysis. *Structural Eng. / Earthquake Eng*. Vol. 10, No. 4, 159s-174s.

Lee, K S, and Geem, Z W. (2004) A new structural optimisation method based on the harmony search algorithm. *Computers and Structures*, 82, 781–798.

Li, L J, Huang, Z B and Liu, F. (2009) A heuristic particle swarm optimisation method for truss structures with discrete variables. *Computers and Structures*, 87(7-8), 435-443.

Maalawi, Y M and Badr, M A. (2009) Design Optimisation of Mechanical Elements and Structures: A Review with Applications. *Journal of Applied Sciences Research*, 5(2), 221-231.

Makris, P A, and Provatidis, C G. (2002) Weight minimisation of displacement-constrained truss structures using a strain energy criterion. *Comput. Methods Appl. Mech. Engrg*., 191, 2159–2177.

Makris, P A, Provatidis, C G, and Rellakis, D A. (2006) Discrete variable optimisation of frames using a strain energy criterion. *Struct Multidisc Optim*, 31, 410–417.

Park, H S, and Park, C L. (1997) Drift control of high-rise buildings with unit load method. *The Structural Design of Tall Buildings,* 6, 23-25.

Patnaik, S, Gendy, A, Berke, L, and Hopkins, D. (1997) Modified Fully Utilized Design (MFUD) Method for Stress and Displacement Constraints. *NASA Technical Memorandum 4743,* August.

Pezeshk, S. (1998) Design of framed structures: An integrated non-linear analysis and optimal minimum weight design. *International Journal for Numerical Methods in Engineering*, 41, 459-471.

Saka, M.P. (2009) Optimum design of steel sway frames to BS5950 using harmony search algorithm. *Journal of Constructional Steel Research*, 65, 36-43.

Shea, K., Cagan, J., and Fenves, S.J. (1997) "A Shape Annealing Approach to Optimal Truss Design With Dynamic Grouping of Members." *Journal of Mechanical Design*, ASME, September, 119, 388-394.

Southern African Institute of Steel Construction (SAISC). (2001) *Introduction to the economics of steel structures*. Compiled and published by SAISC, Johannesburg.

Thanedar, P.B. and Vanderplaats, G.N. (1995) Survey of discrete structural optimisation for structural design. *J. Struct. Eng*., *ASCE.* 120(2), 301-306.

Toğan, V, and Doloğlu, A. (2006) "Optimisation of 3D trusses with adaptive approach in genetic algorithms". *Engineering Structures*, 28, 1019-1027.

Toğan, V, and Doloğlu, A. (2008) "An improved genetic algorithm with initial population strategy and self-adaptive member groupings." *Computers and Structures.*86, 1204-1218.

Wood, R H. (1960) *An economical design of rigid steel frames for multi-storey buildings*. National building studies, Department of Scientific and Industrial Research. Research Paper No. 10.

CHAPTER 2: OPTIMISING STRUCTURES WITH SINGLE DISPLACEMENT CRITERIA

2.1 Introduction

In general, the design of structures requires that each member and the structure as a whole meet two sets of requirements, namely strength and flexibility (or deflections) criteria. If the structure is designed to building codes then the strength requirement should be automatically met. On the other hand, it is not always clear how and where to stiffen the structure to meet the deflection criterion. In most cases, reducing deflection is based on the intuition and experience of the engineer. Often manual iterative trial and error type of approaches are used to reach the target deflection specified by the code.

This chapter presents a method for determining the stiffness of the identified member(s) within a structure in order to meet a single target deflection in an optimal way. Structures with single deflection criteria and load cases are addressed. This methodology is expanded and enhanced in Chapter 3 to address structures with multiple deflection criteria and load cases.

The problem addressed in this chapter can be stated as follows: to ***minimise*** the total mass of the structure while meeting strength ***and*** deflection requirements. The geometry of the structure, *i.e.* the position of the nodes and how they are connected, as well as the loading, are given; it is required to find each member's section in an overall optimal way. In this dissertation an optimal structure is defined as the lightest possible structure which satisfies all load resistance and deflection criteria. Since a minimum is sought, the method in general will require iteration, and to be tractable will have to be automated (with no human expertise required).

The optimisation of a structure with a given geometry has been extensively researched. A few examples of optimisation methods are: the genetic algorithm (Erbatur et al., 2009), tabu search (Kargahi et al., 2006), discrete effective optimisation (Gutkowski et al., 2006) and ant colony optimisation (Camp et al., 2005). None of these methods select the structural member's sections based on structural mechanics; rather, a search procedure is used. They require many (hundreds, thousands and in some cases tens of thousands) iterations to produce a solution. There is no guarantee that the solution is a global minimum. On the other hand, performing a straight forward exhaustive search of all possible combinations of member sections, to obtain the minimum mass, even of a simple structure, would take too much time (measured in centuries) with current modern computers. Thus it is well recognised that structural optimisation is a difficult problem.

To complicate matters, if the optimised structure has too many sections, it becomes difficult to construct, and prone to errors. For this reason, and to simplify the design process, in engineering practice, members are grouped together and assigned the same section. As the number of member groups decrease, so the overall structure’s mass increases. There should be a balance between the complexity of the design and the economy due to mass savings. Grouping members imposes constraints on the optimisation problem. Optimising the grouping of members is discussed in Chapter 5.

The principle of virtual work forms the basis of the optimisation algorithm. The developed method is called the Virtual Work Optimisation (VWO) method. This chapter is organised as follows. First the principle of virtual work is presented together with the assumptions made. The VWO method, in particular, how the strength requirements and deflection criteria are met, is described. The optimisation curve produced by the iterations of the VWO method, together with notes on increment size and member grouping constraints, are discussed. Next three case studies are considered: (a) The standard ten member benchmark truss; (b) a truss frame; and (c) a 24 storey frame. In all cases the VWO method is compared to published optimisation solutions. The chapter is concluded by identifying areas requiring future research, many of which are addressed in subsequent chapters.

2.2 The principle of virtual work

For any solid, the well-known principle of virtual work can be written as:

$$\delta \underline{F} \bullet \underline{\Delta} = \int_V \underline{\underline{\sigma}} \bullet \delta \underline{\underline{\varepsilon}} dV \qquad (2.1)$$

where δ stands for "variation in", and refers to the ***virtual*** load-displacement system. $\underline{F}$ is the virtual point force, $\underline{\Delta}$ is the actual displacement where the virtual force is applied, $\underline{\underline{\sigma}}$ is the stress in the real solid, and $\underline{\underline{\varepsilon}}$ is the virtual strain. Integration is performed over the entire volume, V, of the solid.

In structural mechanics, where the solid in Equation 2.1 is comprised of structural members, and for a ***unit*** virtual load, Equation 2.1 becomes:

$$\Delta = \sum \frac{Ff}{EA} L + \sum \int_L \frac{Mm}{EI} dx + \sum \int_L \frac{Qq}{GA} dx + \sum \int_L \frac{Tt}{GJ} dx \qquad (2.2)$$

The structure's deflection is at the point of application, and in the direction of the virtual unit load.

The small letters, f, q, m, and t refer to the virtual system's internal axial, shear forces, bending, and torsional moments, respectively. The capital letters, F, Q, M, and T refer to the real system's internal axial, shear forces, bending and torsional moments. Integration is performed over the length, L, of each member. Summation occurs over all members in the structure. The material and geometric section properties can vary along the length of the members, and are: the Young's modulus, E, the Shear modulus, G, the cross sectional area, A, the 2nd moment of area, I, and the polar 2nd moment of area, J.

Equation 2.2 can be viewed as a summation:

$$\Delta = \sum_{i=1}^{No.Members} \delta_i \qquad (2.3)$$

where δ_i is the deflection contribution of member i to the overall structural deflection Δ. The magnitude of the contribution is related to the amount of strain energy in the member.

If only two dimensional plane frames or trusses are considered, and shear deformation is neglected, then:

$$\delta_i = \frac{Ff}{EA} L + \int_L \frac{Mm}{EI} dx \tag{2.4}$$

or,

$$\delta_i = \delta_i^{Axial} + \delta_i^{Moment} \tag{2.5}$$

Please note that shear deformation is neglected because it is usually small compared to other terms, especially in steel structures.

In this chapter, only Equations 2.3, 2.4 and 2.5 are utilised, with the associated assumptions and limitations.

2.3 The Virtual Work Optimisation Method

The Virtual Work Optimisation (VWO) method finds the ***minimum mass*** structure for a given structural member configuration, by selecting member sections that satisfy strength and global deflection requirements. In structural design, the global deflection is an input parameter, often specified as a fraction of the structure's span or height. Not only is the magnitude of the global deflection required, but also the direction. The virtual unit point load is then placed at the point where the deflection is to be met in the direction of interest.

Whenever the internal forces or the global deflection is required, the standard stiffness matrix method is used. Most modern structural programs use this matrix method. It must be noted that the VWO method can use *any* method that computes the internal forces and deflections within the structure.

The VWO method is an iterative method. Although the iteration can start off assuming *any* section for each member, a more logical approach is to design each member to meet strength requirements.

2.3.1 Satisfying strength requirements

In the first iteration the members are chosen such that they satisfy strength requirements. The strength requirements are specified in building codes; the South

African steel code, SANS 10162 (2005) is used in this chapter. The internal forces within each member are checked against the code requirements.

The initial member selection for strength requires its own iteration for statically indeterminate structures. This is due to the fact that as member sections are changed, the internal forces within them change. The lightest section satisfying strength requirements is chosen for each member. If members are grouped into a set, then the section chosen for the set will be the lightest section satisfying strength requirement of every member in that set. For a general structure, perfect convergence of the strength iteration might not be achievable (*i.e.* achieving the lightest structure in which each member satisfies the strength criterion). Rather, several member sections can oscillate between possible solutions as the iteration continues. This occurs due to the force redistribution as the member sections change. After a predefined number of oscillations, and if a stable solution has not been achieved, the iteration is stopped and the optimisation process started.

It must be pointed out that the ultimate loads are used in the strength calculations; serviceability loads are used to check the deflection criteria. In some cases, the deflection criterion is met as soon as the strength requirement is satisfied. This is unusual for steel structures with long spans.

2.3.2 Meeting Deflection Criteria and Optimising the Structure

The first step in the optimisation iteration process (*i.e.* minimizing the structure's overall mass) is to determine the contribution of each member to the total deflection of the chosen point. The member's deflection contribution is calculated using Equation 2.4 and the total deflection by Equation 2.3. The internal forces due to the real and virtual load systems are calculated using any standard method or commercial software.

It is now assumed that the geometric sectional properties (2nd moment of area, I, and the cross section area, A) have a ***linear*** relationship with the member's deflection contribution. Thus considering member *i*, with current properties and deflection, and utilizing new sectional properties called *(new)*, the predicted deflection contribution is:

$$\delta_i^{new} = \frac{A_i}{A_i^{new}}\delta_i^{Axial} + \frac{I_i}{I_i^{new}}\delta_i^{Moment} \qquad (2.6)$$

For statically determinate structures this assumption is exact. For indeterminate structures the accuracy of the prediction depends on the ratios $\frac{I_i}{I_i^{new}}$ and $\frac{A_i}{A_i^{new}}$ and how far they are from unity. See Section 2.3.4 "A Note on Increment Size" for a brief discussion.

Two main questions arise:

1. Which member has to be changed?

2. By how much must the member be changed?

To answer these questions, Equation 2.7 is used to determine the ***efficiency*** of changing the sectional properties of member *i* to any other section. Efficiency of the change is defined as the change in deflection contribution of the member, versus the increase in the member's mass, *i.e.*

$$Efficiency = \frac{\delta_i - \delta_i^{new}}{\left(m_i^{new} - m_i\right)L_i} \qquad (2.7)$$

where *m* is the mass per unit length of sections. Equations 2.7 gives a rational basis to choose which member within a structure has to be changed and by how much. The efficiency of each cross section available from a data base (*e.g.* the Southern African Steel Construction Handbook (2005), or the "Red Book"), for each member in the structure, can be computed. (Restrictions such as selecting member changes only from one type of sections, *e.g.* selecting new sections only from angle irons, can be enforced). The most efficient section change, or the highest value in Equation 2.7, is now made. This completes the current iteration in the optimisation process.

The efficiency equation presented is suitable for a structure with a single deflection criterion. To address multiple deflection criteria the method would need to deal with one criterion at a time, or have the efficiency equation modified. This is explored in Chapter 3.

Any section database can be considered by the VWO method. Further, the database can be augmented with custom sections. As the data base increases so too does the computational cost. In the VWO method, since only Equations 2.6 and 2.7 have to be evaluated for new section sizes, the computational cost is linearly proportional to the size of the data base. Contrast this to most other optimisation methods, in which the computational cost increases exponentially (Gutkowski et al., 2006).

The iteration is continued until the deflection criterion, or target, is about to be met. In the last iteration, the section with the lowest mass increase which reaches the target deflection, and not necessarily the most efficient section, is chosen. This prevents deflection being reduced below the target.

It must be pointed out that the deflection contribution of a member (Equation 2.4) to the overall deflection can be ***negative***. This occurs when the internal forces due to the real and virtual loading system have opposite effects. In such a case, the member is designed to satisfy the strength requirement only.

Within each iteration the strength of each member is checked since section changes cause internal force redistribution. If required, the member size is adjusted to meet the strength requirement. At the end of the iteration, each member satisfies strength requirement and the overall structure is closer to meeting the deflection criterion.

2.3.3 The Optimisation Curve

The optimisation curve is updated at the end of each iteration by plotting the overall deflection of the node of interest versus the structure’s mass. An idealised optimisation curve is shown in Figure 2.1.

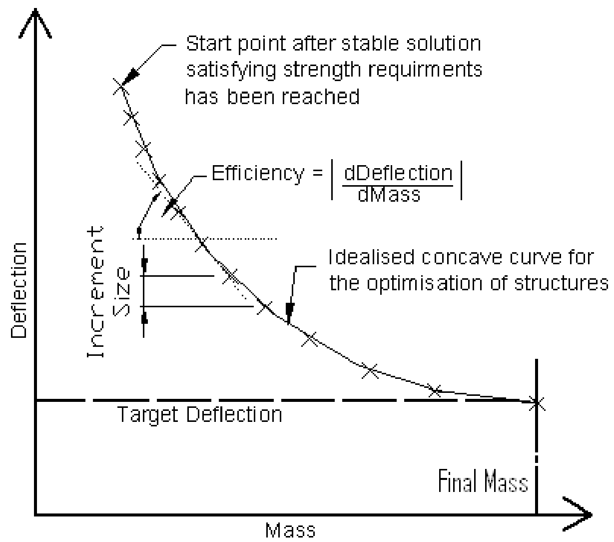


Figure 2.1: *Idealised optimisation curve*.

As the optimisation curve shows, as the structure is stiffened, it becomes increasingly difficult to reduce the deflection, *i.e.* a greater mass increase is required per unit deflection decrease, or the efficiency decreases.

In reality with discrete and finite number of sections available, the idealised curve in Figure 2.1 would not be smooth. The discrete nature of the section distribution, and the requirement for the member to meet strength criteria, leads to over-design of the members to some degree. If strength criteria were not enforced (or were not critical) the optimisation curve would be smoother.

The initial members' section choices, or the starting point of the optimisation, have little influence on the final structure reached. Members that are initially over-designed for strength are reduced in subsequent iterations both by the strength function, and by the efficiency iteration.

2.3.4 A Note on Increment Size

In each iteration the deflection of the critical point is approximately reduced by a fixed amount, the deflection increment, which users define. Sections are changed until this deflection decrease is reached. For large increments more changes are needed. However, as the deflection change increases so the assumption of Equation 2.6 for indeterminate structures becomes less valid, and this could lead to non-smooth and oscillatory optimisation curves. It has been found that increments of 1mm (per iteration) produce consistent optimisation curves. Please note that the deflection contribution reduction can only be a target since the section properties correspond to a finite data base and are discrete in nature. Throughout this chapter, the target deflection increment is set to 1mm; for comparison purposes, larger target increments of 10 and 20mm are also investigated.

2.3.5 A Note on Member Groups

One factor greatly affecting the optimised mass is how many different sections can occur in a structure. In practice, the economy of the structure (*i.e.* having as many sections as required) is weighed up against constructability and simplicity of the design. The members with the same sectional properties in a structure are grouped into sets. Structures with fewer groups will generally be heavier and many members will be larger than needed. The forced grouping of members imposes constraints on the optimisation process. This topic is discussed in depth in Chapter 5 and an automated grouping algorithm using the VWO method is proposed.

The VWO method can be applied directly when the optimisation is constrained by enforcing members to belong to groups. When groups are present, it is required that:

(a) the efficiency search (Equation 2.7) is performed for the whole group, and

(b) the biggest section calculated from the strength requirement of the group is adopted for the entire group.

In the above, the members belonging to groups or sets are specified at the start of the optimisation.

2.4 Case Studies

To demonstrate the VWO method, the optimisations of three different case studies are considered: (a) A benchmark ten member truss; (b) A truss frame that has been designed by a professional engineering company; and (c) A tall structure. Wherever possible the results are compared to published or obtained solutions. The case studies are solved assuming (a) no member grouping, (b) the same grouping as in the compared to solution, and (c) efficient grouping of members.

2.4.1 Ten Member Benchmark Truss

The ten member truss in Figure 2.2 is a standard benchmark structure used to test optimisation methods. This structure has been previously optimised by authors such as Gutowski et al. (2006) and Haug and Arora (1979). In Figure 2.2 the numbers indicate the node and element numbers. All the members have the following material properties: the stress is limited to 172.4MPa, the Young's Modulus is E = 68.95GPa, and the density is ρ = 2767.9 kg/m^3. In this standard problem the load is set to P = 445kN. Each member in the truss can support only axial load.

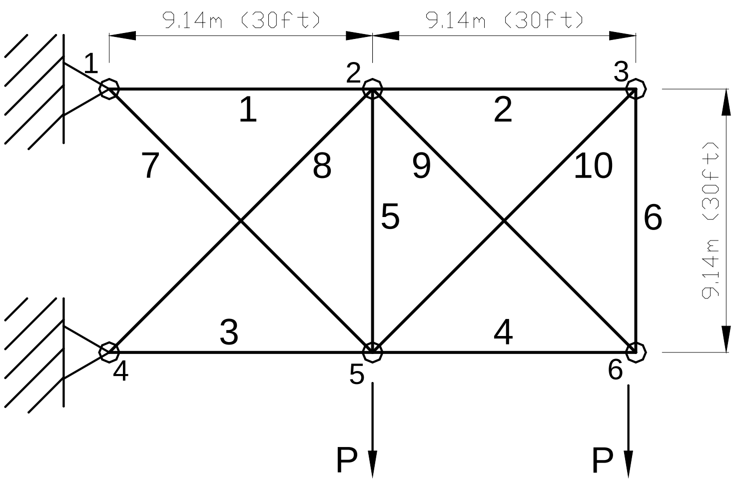


Figure 2.2: *The ten member truss used as a benchmark for optimisation methods.*

The vertical deflection of node 6 is limited to the target value of 50.8mm (after Haug and Arora, 1979).

A data base containing 61 sections was created with areas ranging from 64.55mm^2 (0.1 in^2) to 19419mm^2 (30in^2) in increments of 322.6 mm^2 (0.5 in^2) after Gutkowski et al. (2006).

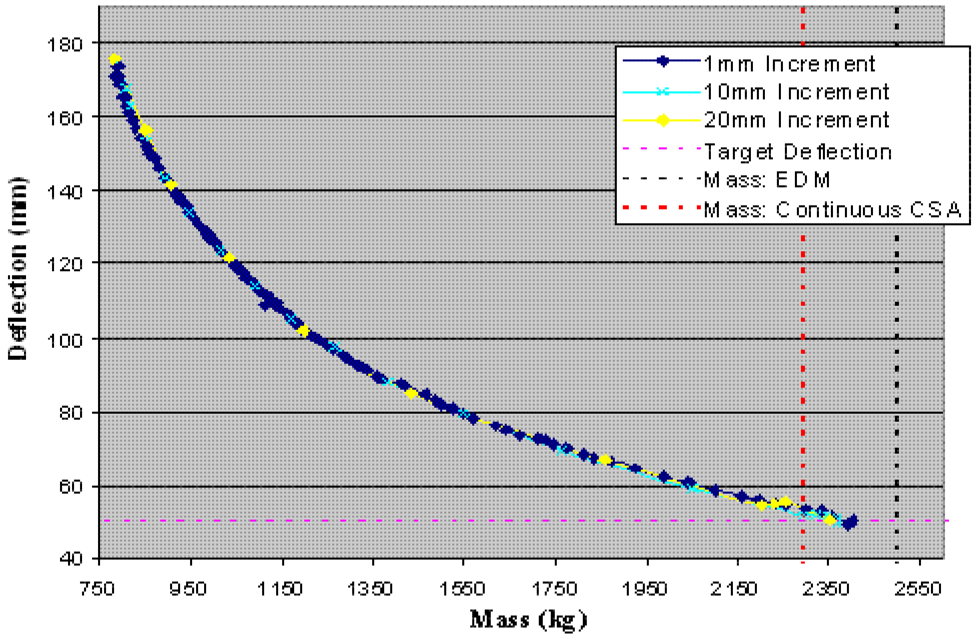


Figure 2.3: *The VWO method optimisation curve for the benchmark ten member truss with 1, 10 and 20mm deflection increments. The results of the EDM of Gutkowski et al. (2006) and the CSA method of Haug and Arora (1979) are shown as vertical dashed lines.*

This benchmark problem was analysed using the VWO method and the computed optimisation curves with target deflection increment of 1, 10 and 20mm are shown in Figure 2.3. The VWO method is compared to the “effective discrete method” (EDM) of Gutkowski et al. (2006), and the “continuous cross sectional area” (CSA) of Haug and Arora (1979). It must be pointed out that the latter reference assumes an infinite number of possible cross sections, while the VWO method and the EDM can select from a more realistic finite data base of sections as above. The results from these methods are summarised in Table 2.1.

Table 2.1 and Figure 2.3 show that VWO method produces a solution that is 4.6% lighter than EDM of Gutkowski et al. (2006). The number of iterations required to reach the solution is also significantly less. The VWO method solution is 4% heavier than the CSA due to the fact that Haug and Arora (1979) are not restricted to select from a finite discrete data base of cross sections.

Table 2.1: *The VWO method compared to the results of the EDM of Gutkowski et al. (2006) and the CSA method of Haug and Arora (1979).*

Method	Final Mass (kg)	Mass greater than VWO (kg)	% Greater than VWO	Number of Iterations
VWO	2394	-	-	93 (1mm increment) 18 (10mm increment) 10 (20mm increment)
EDM	2503	109	4.6	344 +Pre-processing
CSA	2296	-98	-4.0	Unreported

As can be seen in Figure 2.3 the different increments of target deflection produce optimum solutions within 1% of each other. As mentioned above, for statically determinate structures, the solution is independent of deflection increment size. The benchmark structure that is indeterminate initially tends to statically determinate as the optimisation process continues with members 2, 6, and 10 being reduced in size until they contribute negligibly to the overall strength and deflections of the structure. This is demonstrated in Figure 2.4(a) which shows the deflection contribution of each member to the vertical deflection of node 6. The line thickness represents the contribution of the member to the overall deflection of node 6. Figure 2.4(b) gives the cross sectional area of each member in mm^2. Here the line thickness is proportional to the cross sectional areas of the members. In Figure 2.4, as in the rest of the chapter, the colour scheme is as follows: members in red had their section sizes altered to satisfy the deflection criterion; green members have a negative contribution to the overall deflection and their size is determined by strength requirements; members shown in blue are sized based on strength criteria only.

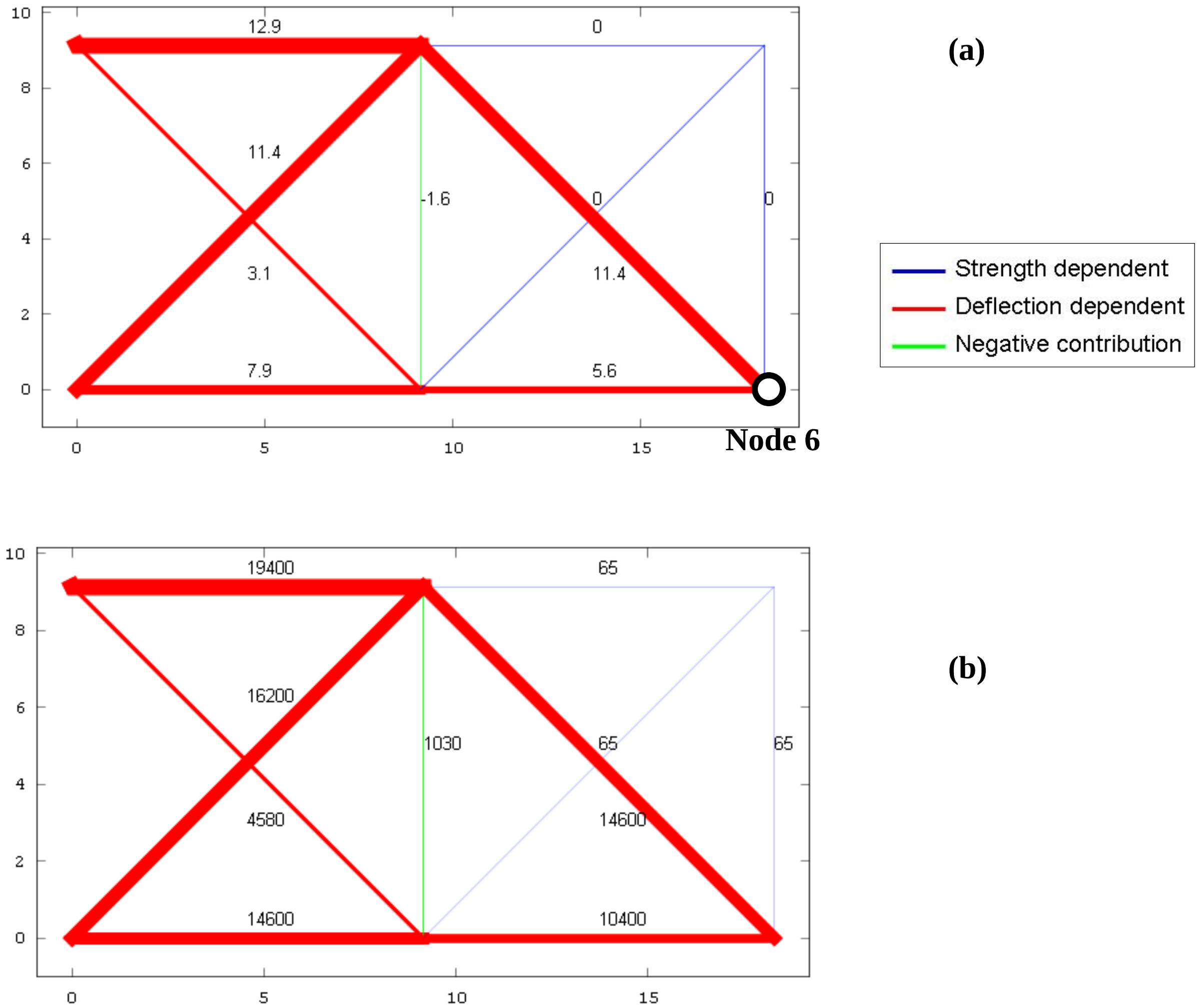


Figure 2.4: *The VWO method solution of the benchmark ten member truss showing: (a) The deflection contribution of each member (in mm) to the overall vertical deflection of node 6; (b) The cross sectional areas (rounded off and in mm^2) of each member. Line thickness represents magnitude of variable. Red members are sized based on deflection consideration; Green members have a negative contribution to the overall deflection and are sized based on strength; Blue members are controlled by strength criteria.*

2.4.2 Truss Frame

The truss frame shown in Figure 2.5 was designed by a firm of professional engineers to comply with the SANS 10162 (2005) code. All members were made of 350W steel, and the loading is W = 6.81kN. Please note that the structure is not perfectly symmetrical.

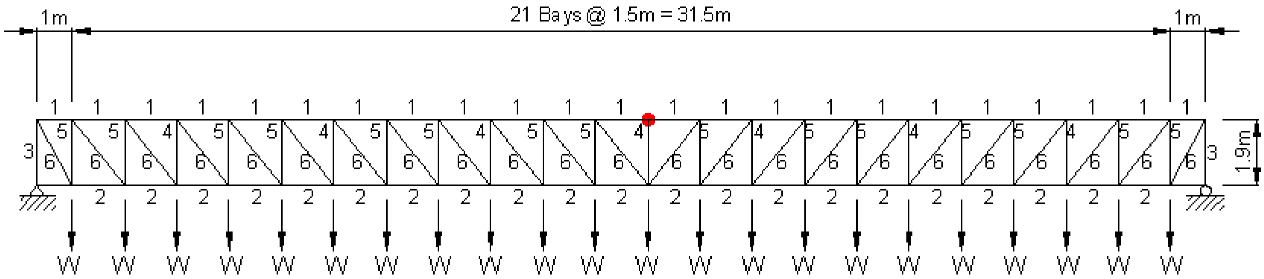


Figure 2.5: *Truss frame case study for the VWO method. The vertical deflection of the node identified by the circle is limited to 94.4mm**.***

In the design, the effective length factor for internal members was taken as 0.85, as specified by the original designers. The engineers specified the top and bottom chords as well as every third vertical member to be channel sections. The remaining members are angle irons. The group to which each member belongs is specified by a number in Figure 2.5. The maximum deflection occurs approximately at mid span at the node identified by the circle. By code requirements this deflection was limited to L/350.

The VWO method was used with the groups in Figure2.5 and with the same section type restriction as in the original design. In addition, the optimisation was performed assuming no member grouping *i.e.* each member can have its own section. The members were modelled as beam elements, *i.e.* bending and axial deformation is allowed. Figure 2.6 plots the optimisation curves as well as the design solution. The numerical results are presented in Table 2.2.

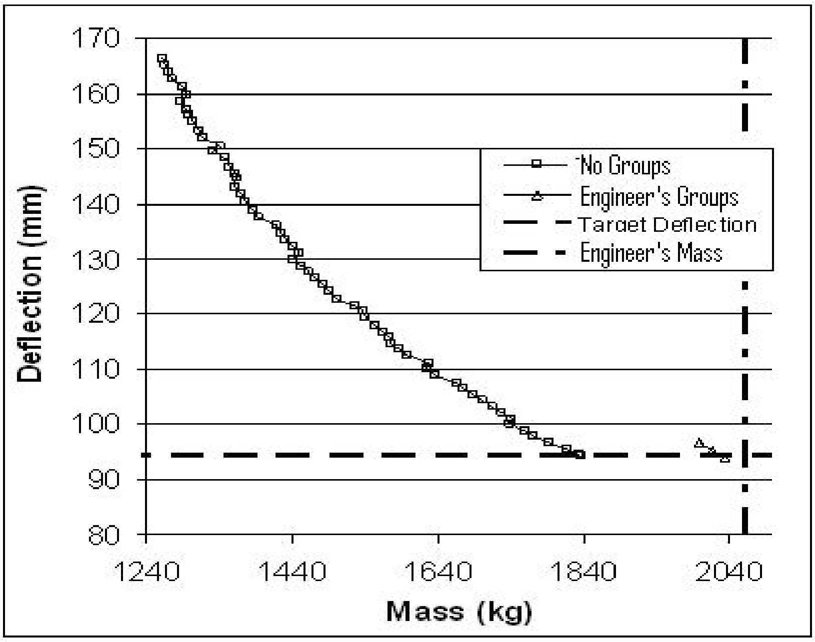


Figure 2.6: *Optimisation curves for the second case study of the truss frame. The members were grouped into 6 sets (triangles), and were ungrouped (squares). For comparison, the structure's mass as designed by the engineers is included.*

Table 2.2: *Comparison of the solutions for the truss frame case study.*

Solution Method	Final Mass (kg)	Mass saving (kg)	% Mass Saving	Number of Iterations
Engineer's design	2063.6	-	-	-
VWO method – Engineer's grouping	2033.5	30.1	1.5	2
VWO method – No grouping	1836.5	227.1	11.0	67

Table 2.2 shows that if the same groupings and section type constraints as the engineer's design are used, the VWO method produces a solution that is 1.5% lighter. If the members are not grouped, then the VWO method's solution is 11.0% lighter. It is interesting to note that the VWO method that can be automated, produces solutions that are slightly better than those of professional engineers.

Deflection increments of 1mm were used to produce the optimised solutions. Larger increments of 10mm and 20mm yield answers within 0.5% of the 1mm increment solution. This is due to the fact that although the structure is analysed as a frame, the geometry and loading configuration ensures that it is in effect a statically determinate truss.

Figure 2.7 plots the contribution of each member, in the optimised structure, with the professional engineer's member groupings shown in Figure 2.5, to the vertical deflection at the critical node. The section sizes are determined by strength requirements (identified in blue) for all members except the diagonals. Hence significant optimisation is not possible.

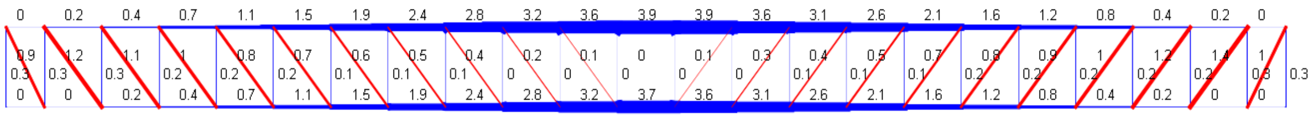


Figure 2.7: *The VWO method solution of the truss frame showing the deflection contribution of each member (in mm) to the overall vertical deflection of the critical node. The members are grouped as shown in Figure 2.5 consistent with the professional engineer's design. Line thickness represents magnitude of deflection. Red members are sized based on deflection consideration; Green members have a negative contribution to the overall deflection and are sized based on strength; Blue members are controlled by strength criteria.*

The contribution of each member to the vertical deflection of the critical node when the members are not grouped together is shown in Figure 2.8. Most sections are now determined by deflection criteria (identified in red), allowing for better optimisation.

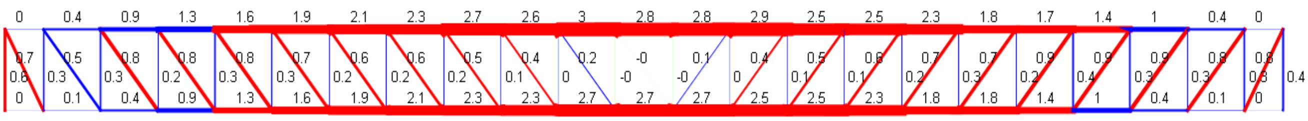


Figure 2.8: *The VWO method solution of the truss frame showing the deflection contribution of each member (in mm) to the overall vertical deflection of the critical node. The members are not grouped. Line thickness represents magnitude of deflection. Red members are sized based on deflection consideration; Green members have a negative contribution to the overall deflection and are sized based on strength; Blue members are controlled by strength criteria.*

Comparing the solutions with and without (Figure 2.7 to 2.8) member grouping suggests a more efficient grouping scheme. For example, adding just two more groups to those in Figure 2.5 leads to an optimised structure that is 10.3% lighter. This saving is close to the 14.0% when there are no groups at all! The two groups that are introduced are: the inner and outer four bays of the top chord, and the inner and outer six bays of the bottom chord. These observations form a theoretical basis for the grouping algorithm in Chapter 5.

2.4.3 Multi-Storey Building

The indeterminate multi storey frame designed by Davison and Adams (1974) and shown in Figure 2.9 is used as the third case study. The serviceability loads and the design parameters are presented in Figure 2.9; f_y is the yield stress, E is the Young's modulus, K_x and K_y are the effective length factors. The target horizontal deflection is limited to h/300 of the height of the building. The numbers next to the members represent the groups used by Davidson and Adams (1974). No vertical deflection criteria have been considered by the original designers.

The results of the VWO method is compared to the work of (a) Saka and Kameshki (1998) who used the "hybrid genetic algorithm" (HGA), and (b) Camp et al. (2005) who used the "ant colony optimisation" (ACO) method. The former reference utilised the United Kingdom standard BS5950 while the latter employed the United States load and resistance factor design (LRFD) AISC (2001). The present VWO method uses the South African SANS 10162 (2005) code. Each member in the multi storey frame is modelled as a beam that can deform axially and in bending.

The VWO method results and the comparison to the references are shown in Table 2.3. The base case is the VWO method using the member groups of the original design shown in Figure 2.9. The optimisation curves with and without groupings are shown in Figure 2.10.

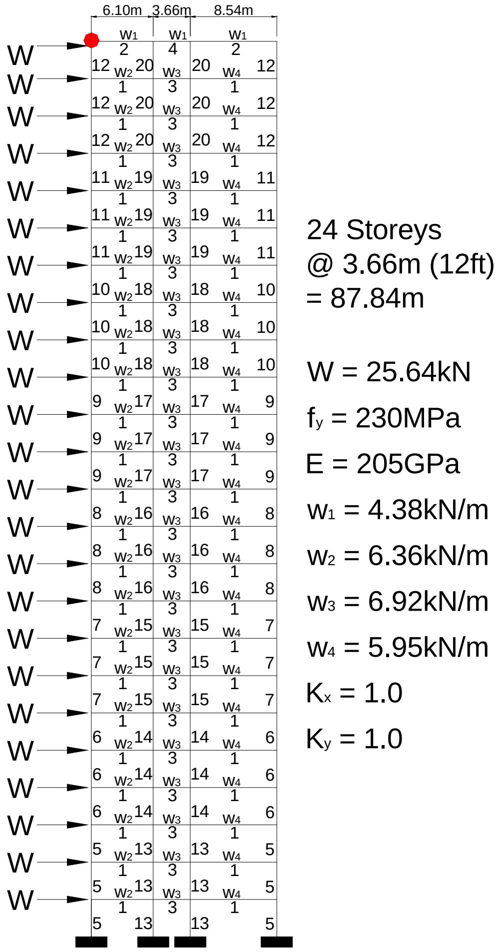


Figure 2.9: *Multi storey frame building to be optimised by the VWO method. Design loads and parameters are as shown. Target deflection of the point circled is 1/300 of the height of the building.*

Table 2.3: *Comparison of the VWO method to the published results for the multi-storey frame building.*

Solution Method	Final Mass (kg)	Mass greater than VWO method – Grouped (kg)	% Greater than VWO method	Number of Iterations
HGA	114101	14961	15.1	30 000
ACO	100002	862	0.9	12 500
VWO – Grouped	99140	-	-	23
VWO – No grouping	79775	-19365	-19.5	167

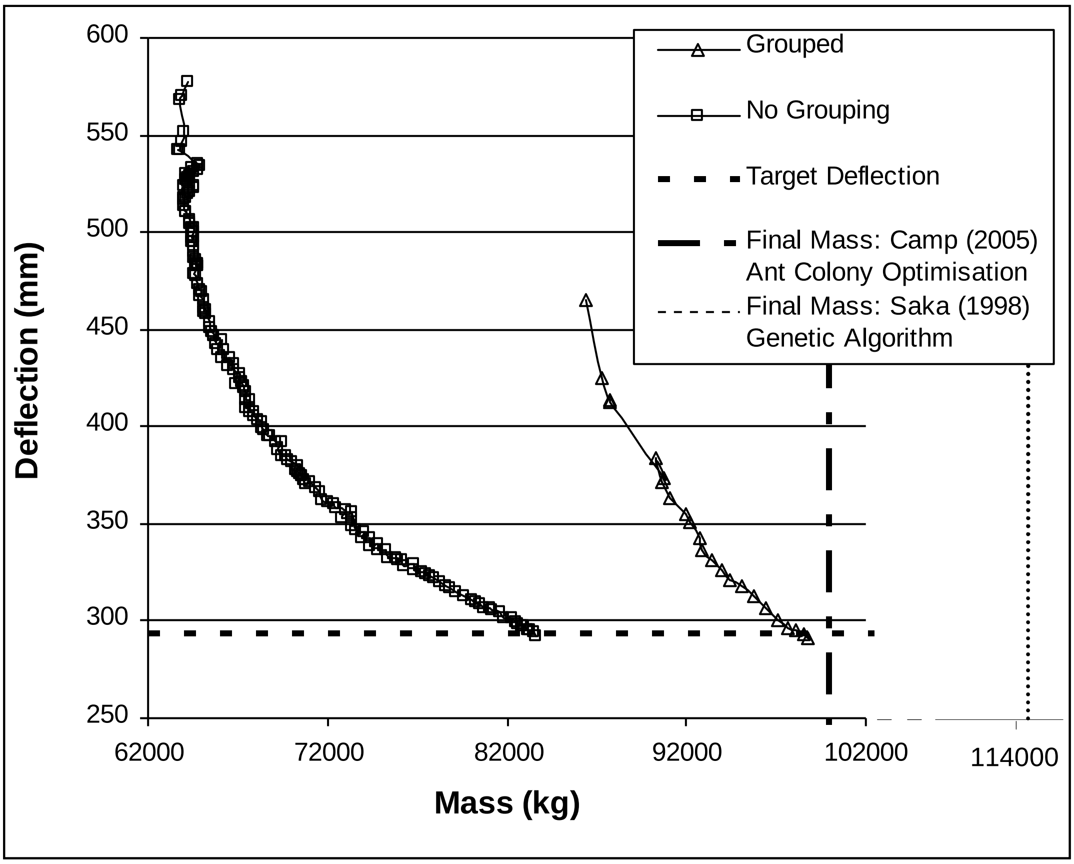


Figure 2.10: *Optimisation curves for the 24 storey frame structure with and without member groupings.*

Table 2.3 and Figure 2.10 show that the VWO method with member grouping produces a solution that is 0.9% lighter than Camp et al. (2005) and 15.1% lighter than Saka and Kameshki (1998). Since all the design parameters in the various methods were not published, and different design codes were adhered to, it can be argued that the VWO method produces similar results to the ACO and better results to the HGA. However, the number of iterations required by the VWO method is three orders of magnitude less than the references. Hence the VWO method is significantly less computationally expensive. Further, if the members are not grouped (*i.e.* each member can have a unique section) in the VWO method, a further 19.5% mass saving is realised.

Figure 2.11 shows the contribution of each member to the overall horizontal deflection of the top of the top storey at different stages in the optimisation process. Iteration 0 starts off with each member satisfying the strength criteria (members in blue). As the iterations progress more and more members are governed by deflection considerations (depicted in red). When the solution has been reached (Iteration 23), the member sections are tailored and the contributions to the overall deflection increase as the supports are approached.

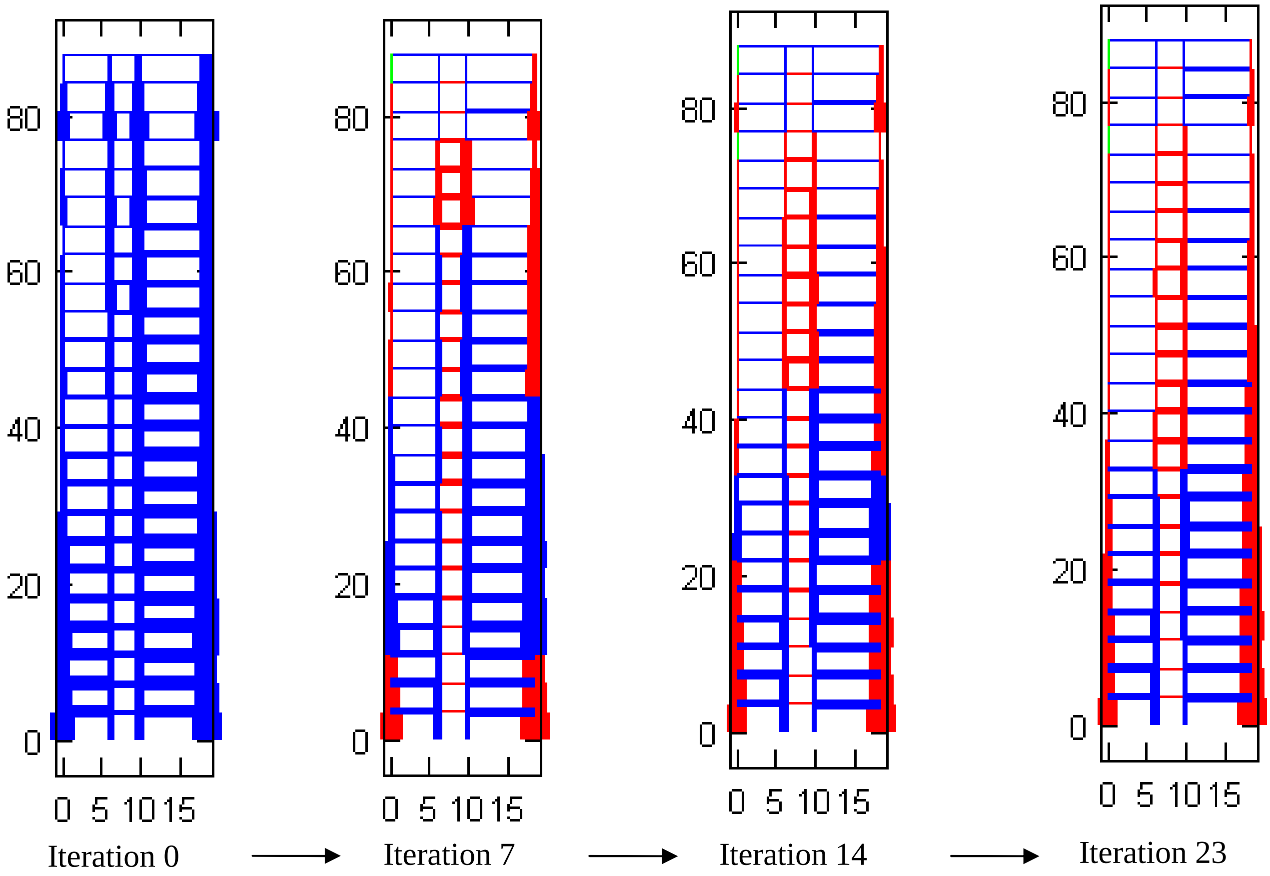


Figure 2.11: *The VWO method solution of the 24 storey frame showing the deflection contribution of each member to the overall horizontal deflection of the top of the top storey. The members are grouped. Line thickness represents magnitude of deflection. Red members are sized based on deflection consideration; Green members have a negative contribution to the overall deflection and are sized based on strength; Blue members are controlled by strength criteria.*

2.5 Effect of Initial Member Sections

The VWO method applied to the 24 storey frame, with member groupings, assumed three different initial distributions of members' sections: (a) every member having the lightest section in the data base; (b) every member having the heaviest section in the data base; and (c) a random mixture of sections from the data base. The first point of the optimisation curve shown in Figure 2.12 is plotted only after all the strength requirements have been satisfied. As can be seen in Figure 2.12, the path of the optimisation curve depends on the starting point, but the solutions converge to within 0.4% of each other.

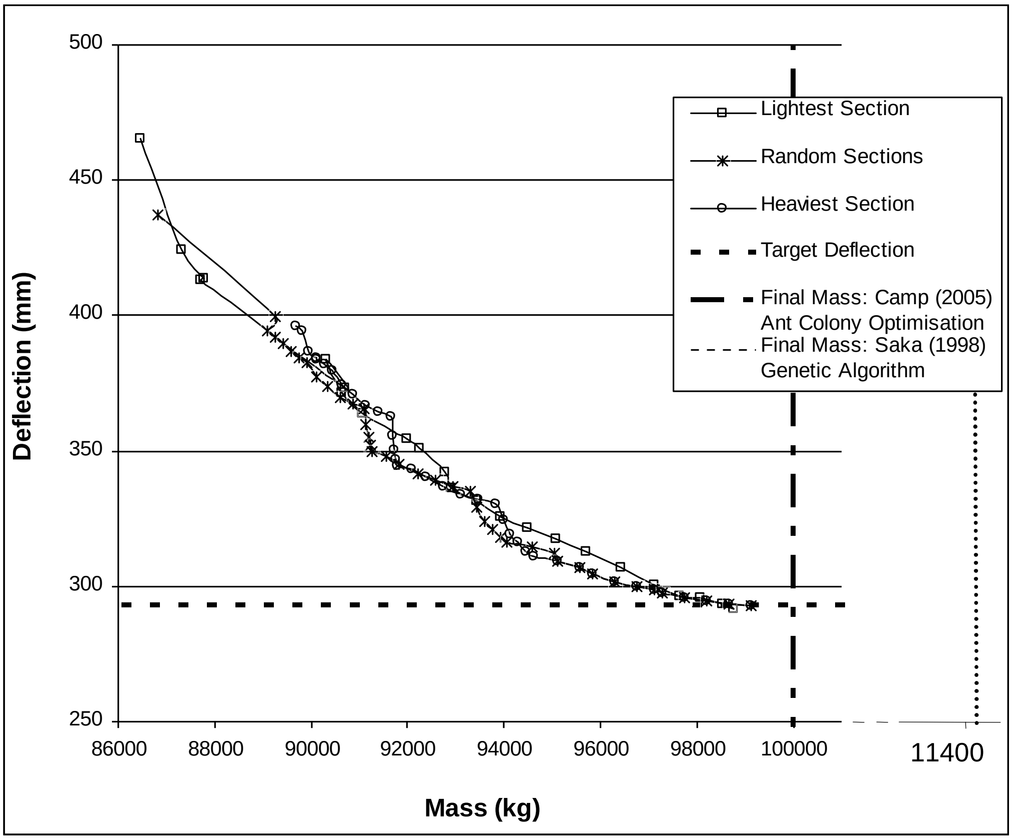


Figure 2.12: *Optimisation curves for assumed different distribution of members' sections: (a) all members have the lightest section; (b) all the members have the heaviest section; (c) random distribution of sections.*

2.6 Effect of Deflection Increment Size

Figure 2.13 plots the optimisation curve for the 24 storey frame assuming three different deflection target increments: 1, 10 and 20mm. The members are grouped as shown in Figure 2.9. Since the structure is statically indeterminate, the target deflection increment does affect the optimisation curve. If the increment is small enough, the final results are close to each other. For the three increment sizes considered, the optimisation curves follow a similar broad path and the results are within 0.9% of each other. The structure's indeterminancy produces non-smooth curves with force redistribution occurring after each iteration.

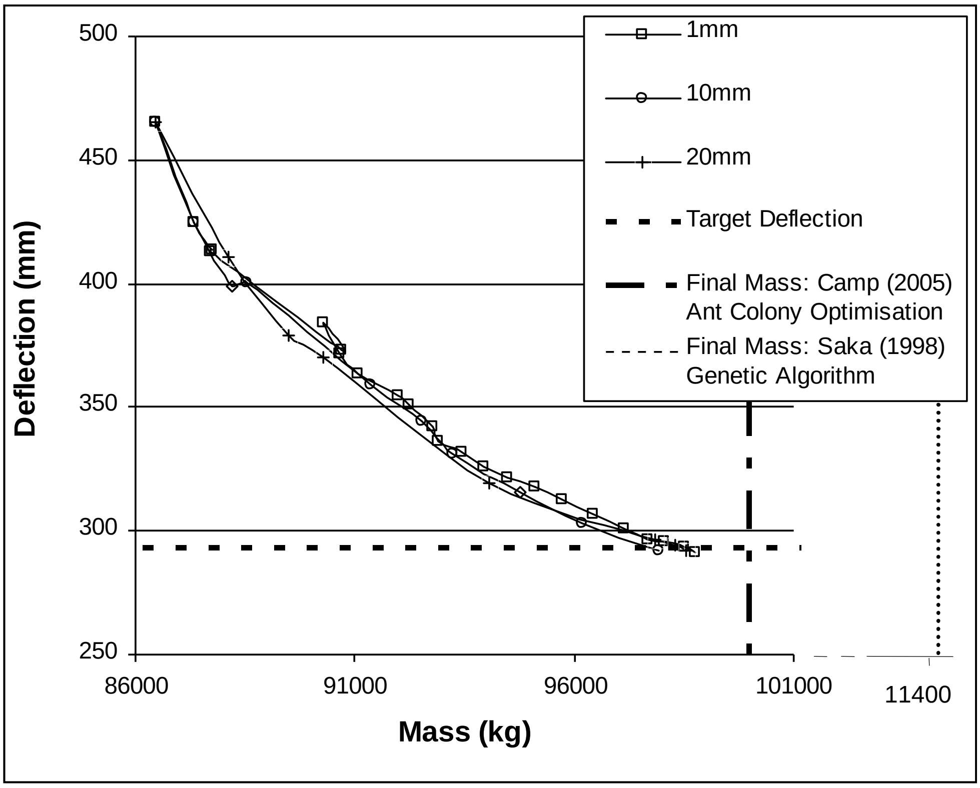


Figure 2.13: *Optimisation curves for different deflection increments: (a) 1mm; (b) 10mm; (c) 20mm.*

2.7 Conclusion

In this chapter the well known principle of virtual work was used as the framework to optimise a structure with a given geometry and loading. The developed Virtual Work Optimisation (VWO) method, was used to find the lightest structure that meets a prescribed deflection. While the design of members of a structure for strength can easily be automated to meet building code specification, to enforce deflection criteria requires the experience of an engineer. The VWO method can be used to automate not only the strength but also the deflection requirements.

The method was used on three case studies: (a) the benchmark optimisation ten member truss; (b) a truss-frame designed by professional engineers; and (c) a 24 storey frame. In all cases the VWO method produced solutions that were at least as efficient as published results. In some cases the solutions were significantly more economical. The computational effort (and hence time) of the method was less than the methods reported in the literature, requiring orders of magnitude fewer iterations to converge.

The optimisation can be constrained by grouping members into sets, and requiring that all members in a given set have the same sectional properties. In practice members are grouped together in order to simplify the design and the construction process. Allowing for member groups was incorporated in the VWO method. As expected, the constraint of grouping members together produced structures that were heavier than when each member could have its own unique section.

Further research on the VWO method will focus on the following areas:

(a) Addressing multiple deflection criteria and load cases. This if the focus of Chapter 3.

(b) How to select members belonging to a group. In the past this task relied on the experience of the engineer. The problem here is to choose groups most efficiently. This is addressed in Chapter 5.

(c) The uniqueness of the solution and the optimisation curve.

2.8 References

AISI (American Iron and Steel Institute) 2001. *North American specifications for the design of cold-formed steel structural members.* Washington D.C.

BS5950. (1995) Structural Use of Steelwork in Building, British Standards Institution.

Camp, C V, Bichon, B J and Stovall, S P. (2005) Design of Steel Frames Using Ant Colony Optimisation. *J. Struct. Div*, ASCE, 131(3), 369-379

Davison, J H, and Adams, P F. (1974) Stability of braced and unbraced frames. *J. Struct. Div.* ASCE, 100(2), 319-334.

Erbatur, F, Hasançebi, O, Tütüncü, I and Kiliç, H. (2009) Optimal design of planar frames with genetic algorithms. *Computers and Structures*, 75, 209-224.

Gutkowski, W, Bauer J, and Zawidzka, J. (2000) An effective method for discrete structural optimisation. *Engineering Computations*. Vol. 17. No. 4, pp. 417-426. MCB University Press.

Haug, E J and Arora, J S. (1979) *Applied Optimal Design*, Wiley, New York, NY.

Kargahi, M, Anderson, J C, and Dessouky, M M (2006) Structural Weight Optimisation of Frames Using Tabu Search. I: Optimisation Procedure. *Journal of Structural Engineering*, ASCE, 132(12), 1858-1868.

Mahachi, J 2004. *Design of Structural Steelwork to SANS 10162.* CSIR Building and Construction Technology. Pretoria

SAISC 2005. *Southern African Steel Construction Handbook – Fifth Edition, "The Red Book"*. Southern African Institute of Steel Construction.

Saka, M.P. (2009) Optimum design of steel sway frames to BS5950 using harmony search algorithm. *Journal of Constructional Steel Research*, 65, 36-43.

SANS 10162-1(2005), The Structural use of steel. Part 1: Limit-states design of hot-rolled steelwork. South African National Standard.

CHAPTER 3: OPTIMISING STRUCTURES SUBJECT TO MULTIPLE DEFLECTION CONSTRAINTS AND LOAD CASES

3.1 Introduction

In this chapter the Virtual Work Optimisation (VWO) method presented in Chapter 2 is expanded to address structures with multiple deflection constraints and load cases. The principle of virtual work guides the optimisation process, in a similar manner to that presented in Chapter 2. Discrete structural sections are selected to satisfy both strength and deflection criteria. An optimal structure is defined as one which satisfies all constraints using the minimum amount of material.

Many optimisation methods are capable of handling multiple deflection criteria, such as genetic algorithms (e.g. Erbatur et al, 2009), optimality criterion methods (e.g. Pezeshk, 1998) and tabu search (Kargahi et al., 2006). However, the difficulties encountered in optimisation include high computational costs where thousands or tens of thousands of iterations are needed. Methods using empirical optimisation constants require calibration specific to each structure. A relationship is often assumed between the sectional properties of members (e.g. Chan, 1992), which may not exist in standard section databases. The number of iterations required to optimise a structure can increase exponentially as the number of sections in a database increases. Methods can be geometry or material specific. Despite the fact that only discrete structural sections are available some methods choose sections from a continuous spectrum.

This chapter is organized as follows. First, the VWO methodology is presented using a simple portal frame as a case study. The theory underlying the method is discussed. The advantages and limitations of the method are shown. Three further case studies are presented to demonstrate the effectiveness of the method: (a) a 60 storey building, (b) an industrial warehouse with gantry cranes, and (c) a stepped cantilever. Results are compared to those found in the literature or produced by design engineers.

3.2 The Virtual Work Optimisation (VWO) Method

In Chapter 2 the VWO method for optimising structures with single deflection constraints was presented. Deformations were reduced by a fixed and prescribed amount each iteration. Variable numbers of section changes were made per iteration. The number of times a structure was reanalysed to satisfy initial strength constraints was user-defined. Frame analyses were done both before reducing deflections and before selecting sections to satisfy strength requirements.

In this chapter multiple deflection criteria and load cases are addressed. A fixed number of section changes are made per iteration. The number of times the analysis is performed to satisfy initial strength requirements is variable and dependent on the structure. Frame analyses are done only before reducing deflections, which substantially reduces computational requirements of the method in Chapter 2.

The new optimisation process can be summarized as follows: first, members are chosen to satisfy strength requirements. Second, members most critical for reducing deflections are identified and changed in an iterative manner until all deflection and strength criteria are satisfied. Although the method is explained for 2D structures, its application to 3D structures is identical.

To explain how the method works a portal frame with only four members will be optimised (Figure 3.1). This structure is subject to deflection constraints and strength requirements. The maximum deflection of the roof apex is limited to span/400 (25mm) when dead load is applied. The maximum horizontal sway of the columns is limited to height/200 (20mm) under wind load. Members are chosen to satisfy the South African structural steel code SANS 10162 (2005) using grade 350W steel. However, any design code and grade of steel can be used. I and H sections from standard AISC databases will be used for the rafters and columns respectively. The rafters and columns are grouped into two separate groups. All the members in each group will be adjusted rather than individual members.

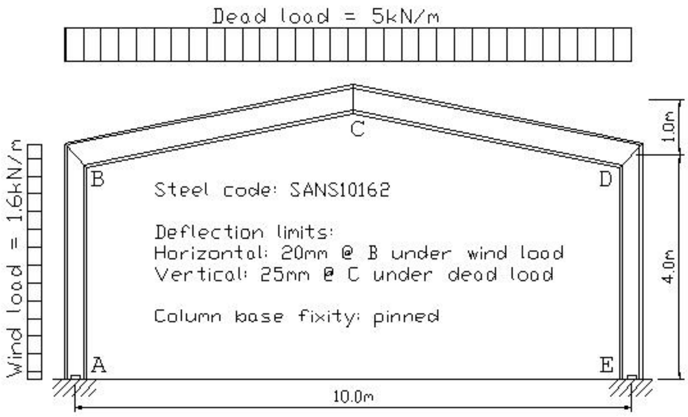


Figure 3.1: *Portal frame case study*

3.2.1 Step 0 – Setting optimisation parameters

The following information is required as input to the optimisation process: the structure's topology, loading, deflection requirements, design code to be used, and the effective length of members. Users must define points, referred to as critical points, at which deflections have to be limited. The automated optimisation process can now start.

The initial section selection can either be arbitrary, set by the user, or the median section from a database can automatically be chosen. The latter is implemented for all case studies in this chapter.

3.2.2 Step 1 – Satisfying strength requirements

Members are selected to satisfy strength requirements using the lightest sections possible. All load cases are considered. Members are resized after each iteration, accounting for the redistribution of force that occurs as the structure is changed. It is more accurate and computationally less expensive to have Step 1 repeated a variable number of times, rather than a predefined number as assumed in Chapter 2. Here Step 1 is repeated until the structure's mass has converged. In larger structures with high degrees of static indeterminancy between 3 and 10 iterations are generally needed to satisfy all strength requirements.

Portal frame – choosing initial sections

For the portal frame only one strength iteration was needed. The sections selected for the structure are W6x15 for the columns and W8x18 for the rafters.

3.2.3 Step 2 – Reducing deflections

Deflection constraints are now checked and if violated the deflection reduction process starts. The principle of virtual work is applied to determine which members should be changed.

The Principle of Virtual Work

This section briefly discusses the principle of virtual work. For a detailed explanation refer to Chapter 2, Section 2.2.

When loading is applied, a structure will deflect and internal forces will be setup. The amount that member *i* allows a point to deflect is defined as that member's deflection contribution, δ_i. The magnitude of the contribution is governed by the member's flexibility and internal forces. The total deflection at the critical point, Δ, is calculated as the summation of all member deflection contributions:

$$\Delta = \sum_{i=1}^{No.Members} \delta_i \tag{3.1}$$

For two dimensional structures the deflection contribution of each member is:

$$\delta_i = \frac{Ff}{EA} L + \int_L \frac{Mm}{EI} dx = \delta_i^{Axial} + \delta_i^{Moment} \tag{3.2}$$

The deflection contribution consists of axial and moment components. Shear is neglected because it is assumed to be small. For the portal frame example Figure 3.2 shows the deflection contribution of each member for the two load cases. In this and subsequent examples the thickness of the line is proportional to the deflection contribution of the member. The numerical value of the deflection contributions are shown. Note that each member's strength requirements have been satisfied.

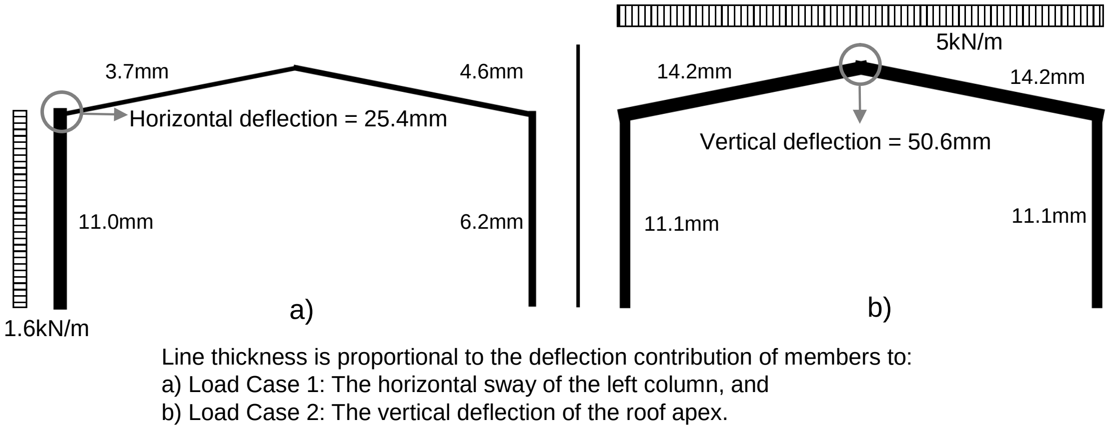


Figure 3.2: *Deflection contributions to the horizontal and vertical deflections in the portal frame*

It may seem obvious to stiffen members with the highest deflection contributions to reduce deformations of the critical points. However, such members may already be large and might require a substantial mass increase to stiffen them further. The ***efficiency*** of making any change is investigated next.

Predicting the effects of section changes

The deflection contribution of a member is inversely proportional to its area and 2nd moment of area (calculated using Equation 3.2). If a member's section is replaced, the new deflection contribution of that member would be changed in proportion to the ratio of sectional properties, i.e.:

$$\delta_i^{new} = \frac{A_i}{A_i^{new}}\delta_i^{Axial} + \frac{I_i}{I_i^{new}}\delta_i^{Moment} \tag{3.3}$$

Equation 3.3 is identical to Equation 2.6 in Chapter 2. Members in a structure are generally grouped together and an entire group's section properties are changed. The total deflection decrease, after a new section is selected for the group, is:

$$\Delta^{Decrease} = \sum_{i=1}^{No.Members.Changed} (\delta_i - \delta_i^{new}) \tag{3.4}$$

The deflection decrease given by Equation 3.4 is exact for a statically determinate structures. For an indeterminate structure the predicted deflection may be inaccurate.

The degree of inaccuracy is determined by the magnitude of the section change made, the size of the structure and the degree of static indeterminancy. Nonetheless, this prediction provides an excellent guide regarding which members should have their sections changed, and does not have to be precise.

The mass increase, ΔM, that occurs when a group of members is changed is given by:

$$\Delta M = (m^{new} - m)L_{group} \tag{3.5}$$

Where m denotes the mass per unit length of a section and L_{group} is the total length of the group of members.

An efficient section change is one that causes a large deflection decrease at all critical points per unit mass increase. Thus, it is necessary to determine the affect of any change relative to all critical points and quantify this to determine the best overall change.

Let the deflection decrease at a critical point j caused by a section change be $\Delta_j^{Decrease}$. Each critical point may have a different target deflection, Δ_j^{Target}, for any load case and will require different stiffening of the structure. Since all deflection constraints are equally important, a summed efficiency of change is proposed. Each critical point's deflection reduction is factored relative to its target deflection. The efficiency of a section change for N critical points is defined as:

$$Efficiency = \frac{\sum_{j=1}^{N} \left(\Delta_j^{Decrease} \middle/ \Delta_j^{Target} \right)}{\Delta M} \tag{3.6}$$

The efficiency of a section change can be viewed as the fraction of deflection reduction that will occur per unit mass increase.

3.2.4 Step 3 – Adjusting member sections

Every group of members is replaced by all eligible sections in a database. An eligible section has a larger cross-sectional area and/or second moment of area than the current section. The overall process is fast because the structure is not analyzed for each change. Instead, Equations 3.3 to 3.6 are used to predict the affect of adjusting section properties by calculating efficiencies. There is a linear relationship between the number of changes to be investigated, *NC*, and the number of eligible sections, S_i, for each group *i*. *NC* is determined by:

$$NC = \sum_{i=1}^{No.of\ .groups} S_i \tag{3.7}$$

Increasing the section database size or the number of groups does not result in an exponential increase in computational costs, as it does for many other methods. Large section databases and numbers of groups can be used.

After all eligible changes have been tested the one with the highest efficiency is selected. Once a group has been adjusted to reduce deflections it is considered a deflection dependent member. Such members are overdesigned in terms of strength. Deflection dependent members will not have their section sizes decreased during Step 1 in subsequent iterations. Equation 3.7 ensures that all critical point deflections are reduced simultaneously, and it has been observed that they reach their target deflections at approximately the same time. This prevents parts of the structure being over-stiffened.

Reducing deflections in the portal frame

In the first deflection iteration for the portal frame example the affect of changing columns and rafters to any H and I section is investigated. The most efficient change found is to replace the W8x18 rafters with a W14x22 section. This causes deflections to be reduced by approximately 2.1% (efficiency in percent) for each kilogram of material added. Table 3.1 shows the calculations used to determine the efficiency of this change. The predicted horizontal and vertical deflection reductions are 11.9mm and 18.6mm. The structure's mass increases by 64.3kg. Although axial strain energy has been taken into account, it is small and is not shown in Table 3.1.

Table 3.1: *Calculations for changing the section of the portal frame's rafters at iteration 1*

	Initial Member: W8x18	New Member: W14x22	Ratio of sectional properties		
I_x (x 10^6mm^4)	26.2	84.6	0.31		
A (x 10^3mm^2)	3.44	4.25	0.809		
	Initial contribution	New contribution (Predicted)	Approximate Change	Δ^{Target} (mm)	$\frac{\Delta^{Decrease}}{\Delta^{Target}}$
Moment deflection contribution to horizontal sway at B (mm)	17.2mm	5.3mm	-11.9mm	20mm	0.6
Moment deflection contribution to the vertical deflection at C (mm)	28.4mm	8.8mm	-18.6mm	25mm	0.74
Mass of columns (kg)	275.3kg	339.6kg	64.3kg	-	-
		Efficiency of change			**0.021**

3.2.5 Step 4 – Satisfying all deformation and strength constraints

Steps 1 to 3 are repeated until all user-defined criteria are satisfied. The number of iterations required to produce a final structure is dependent on the size of the structure, the number of groups and the amount that critical node deflections have to be reduced after strength criteria have been satisfied. A primarily strength dependent structure will require few iterations.

It has been found that making only one group section change per iteration usually produces the most optimal structures. In this way the effect of any section change on the rest of the structure is determined before more adjustments are made, preventing members from being over-stiffened. However, to speed up the process, multiple changes can be made per iteration. This may be necessary for large structures with numerous member groups. For larger structures increasing the number of changes made per iteration has little to no effect on the solution. However, this is case specific and convergence must be checked.

Results – Portal frame

The portal frame requires one strength and two deflection iterations to produce the solution. After the first deflection change, the section of the columns decreased from W6x15 to W4x13; this is due to redistribution of force. In the last deflection iteration this change is reversed to provide the most efficient deflection reduction. The mass of the structure increased 12.4% over the initial strength design (Step 1). The final horizontal and vertical deflections in the structure are 19.6mm and 24.9mm. This satisfies the target deflection constraints of 20mm and 25mm. Note that target deflections are seldom met exactly. Table 3.2 summarizes the optimisation results.

Table 3.2: *Summary of the optimisation of the portal frame*

<table>
<tr><th></th><th colspan="2">Strength satisfied configuration</th><th colspan="2">Final configuration</th></tr>
<tr><td>Column section</td><td colspan="2">W6x15</td><td colspan="2">W6x15</td></tr>
<tr><td>Rafter section</td><td colspan="2">W8x18</td><td colspan="2">W14x22</td></tr>
<tr><td>Total mass (kg)</td><td colspan="2">456.8</td><td colspan="2">521.2</td></tr>
<tr><td></td><td>Initial deflection (mm)</td><td colspan="2">Final deflection (mm)</td><td>Target deflection (mm)</td></tr>
<tr><td>Horizontal sway of B</td><td>25.4</td><td colspan="2">19.6</td><td>20</td></tr>
<tr><td>Vertical deflection of C</td><td>50.6</td><td colspan="2">24.9</td><td>25</td></tr>
<tr><td>Total iterations required</td><td colspan="4">1 strength + 2 deflection = 3</td></tr>
</table>

3.3 Other measures of efficiency

Besides Equation 3.7 other methods and equations for determining efficiency were investigated:

(1) Addressing one critical point at a time and superimposing solutions. The largest section for each group is chosen from the solutions obtained. It was found that this implementation in the VWO method leads to structures being over-stiffened.

(2) Making one section change per iteration considering one critical point at a time. The efficiency of a section change is calculated as:

$$Efficiency = \frac{\Delta_j^{Decrease}}{\Delta M} \tag{3.8}$$

Critical deflection points are considered in a given order, which influences the solutions obtained. The method fails to find section changes which are efficient relative to all critical deflection points.

(3) Determining efficiency by summing the deflection reductions:

$$Efficiency = \frac{\sum_{j=1}^{N} \left(\Delta_j^{Decrease} \right)}{\Delta M} \tag{3.9}$$

In this situation the optimisation process becomes biased towards critical points with large numeric deflection decreases, irrespective of the magnitude of target deflections.

3.4 Advantages of the VWO method

The main advantages of the VWO method are listed below. The method satisfies both strength and deflection requirements, which many methods do not. Any discrete section databases can be utilised by the method and no relationship is assumed between sectional properties. An increase in the size of a section database results in a small increase in overall computational costs. Any number of deflection points and load cases can be considered. No empirical optimisation constants must be set, except for the number of section changes made per iteration. The method does not require calibration. The initial choice of members does not have a great effect on the solutions. The method is applicable to all structures, irrespective of geometry or the material from which they are made.

The VWO method requires fewer iterations than many other optimisation methods. A one-bay ten-storey structure with 9 member groups was optimised by Camp et al. (2005) using Ant Colony Optimisation (ACO), by Pezeshk et al. (2000) using genetic optimisation and by the VWO method. The ACO required 8,300 frame analyses, the genetic optimisation 3000 and the present VWO method only 32 frame analyses. The methodology presented in Chapter 2 requires almost double the number of frame analyses.

3.5 Limitations to the VWO method

The VWO method has several limitations. As in all structural optimisation problems, there is no certainty that the global minimum has been found. To determine if a global minimum has been obtained an exhaustive search of all solutions needs to be carried out. Groenwold et al. (1996) notes that this is essential for convex problems. However, for average to large structures an exhaustive search produces a search area far too large for modern computers to analyze.

It is possible that situations arise where members alternate between being strength and deflection dependent as forces redistribute. Checks have to be included to prevent infinite loops from occurring in such instances. This can be done by artificially increasing the number of changes made in an iteration. Otherwise, if a group is alternating between being strength and deflection dependent it can be 'ignored' for a few iterations, and only adjusted once a certain number of member changes have been made.

It has been observed that in large multi-storey structures with no member grouping irregular distributions of mass can be produced, as will be seen in Chapter 4. When individual sections are stiffened it can alter load paths and cause regions of higher and lower internal forces. The grouping of members prevents individual sections from becoming over-stiffened and significantly changes load paths. This problem was not encountered in the case studies presented below

3.6 Case studies

Three case studies are presented below. The structures optimised are a 60-storey 7-bay frame, a warehouse designed by professional, structural engineers, and a stepped cantilever.

3.6.1 60 Storey Building

The 60-storey, 7-bay plane frame shown in Figure 3.3 was optimised by Chan (1992) using an efficient optimality criteria (OC) technique. Chan (1992) selected members assuming continuous section sizes and then converted these to discrete sections using: (a) a simple round-up method and (b) a pseudo-discrete method.

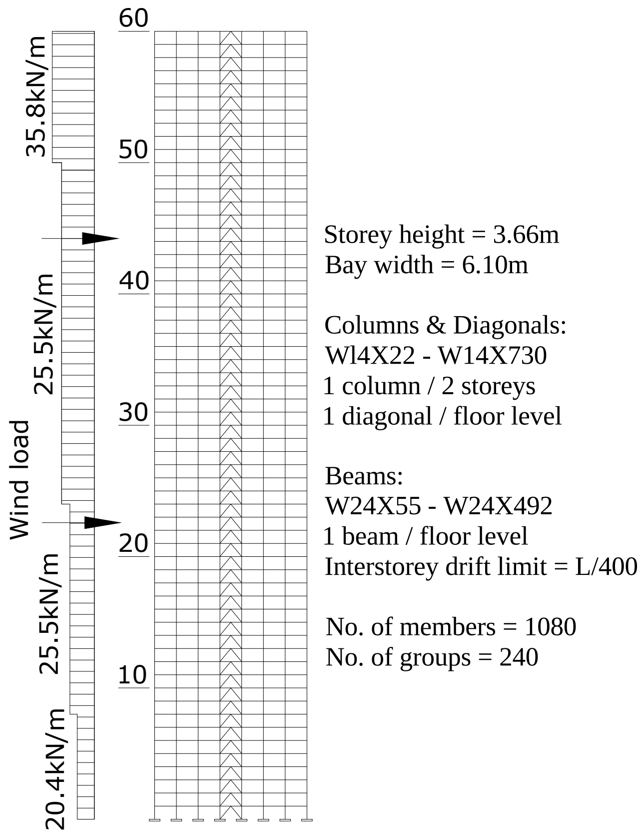


Figure 3.3: *60-storey, 7-bay frame example*

The VWO method will use the constraints set by Chan (1992). No vertical gravity loads are considered. The wind load is applied as point loads at each floor level. Interstorey drift is limited to floor height/400. Beams and diagonals are grouped together to have one section per floor. Columns are grouped together across two adjacent stories with exterior and symmetrical columns having the same section. Chan (1992) did not consider strength. The VWO method will satisfy strength requirements according to the SANS 10162 (2005) steel code using grade 300W steel. Columns and bracing are to be chosen from W14 sections ranging from

W14x22 to W14x730. Beams have to be chosen from W24 sections ranging from W24x55 to W24x492.

The VWO process requires 10 strength and 196 deflection iterations to optimise the structure. The number of section changes per iteration was set to 15. The solution converges to the same value when 5, 10 or 15 changes are made per iteration. An approximately 1% heavier solution is found if 20 to 25 changes are made per iteration.

Chan (1992) assumed a relationship between area and second moment of area for each section, which does not exist in many section databases. The OC method was tailored specifically for multi-storey buildings. Only deflection constraints have been satisfied in the OC solutions. The effect of satisfying strength criteria on the mass of the OC solutions is unknown.

Table 3.3 summarizes the results obtained by the VWO and the OC methods. The pseudo-roundup OC solution is 1.53% lighter than the VWO solution, while the simple roundup solution is 1.25% heavier. The heavier solution obtained by the VWO method might be due to the fact that not only the deflection requirements, but also the strength constraints, are met.

Table 3.3: *Comparison of results for the 60-storey building*

Method	Mass (tons)	% Greater than VWO	Constraints considered
OC Simple - Roundup	2316.5	1.25	Deflection
OC Pseudo - Roundup	2252.8	-1.53	Deflection
VWO	2287.8	-	Strength & Deflection

The optimisation graph of the building is shown in Figure 3.4. Interstorey drifts at stories 5 to 60, in intervals of 5 stories, are shown. The total mass of the structure increases as the optimisation process progresses. In the first 10 iterations strength constraints are satisfied. The stories reach their target drifts at approximately the same time. This suggests that the structure has not been over-stiffened. The efficiency of changes progressively decreases as the optimisation process progresses, i.e. it becomes more expensive to stiffen the structure per unit deflection decrease.

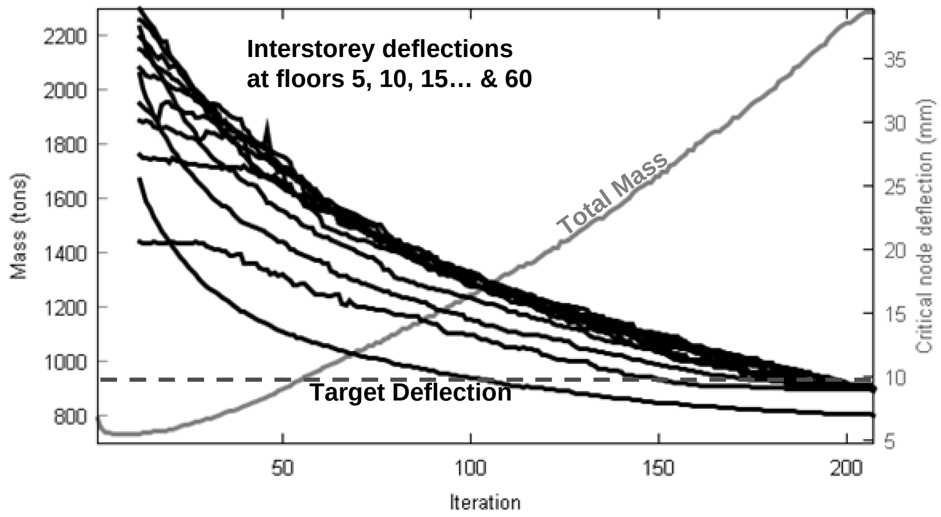
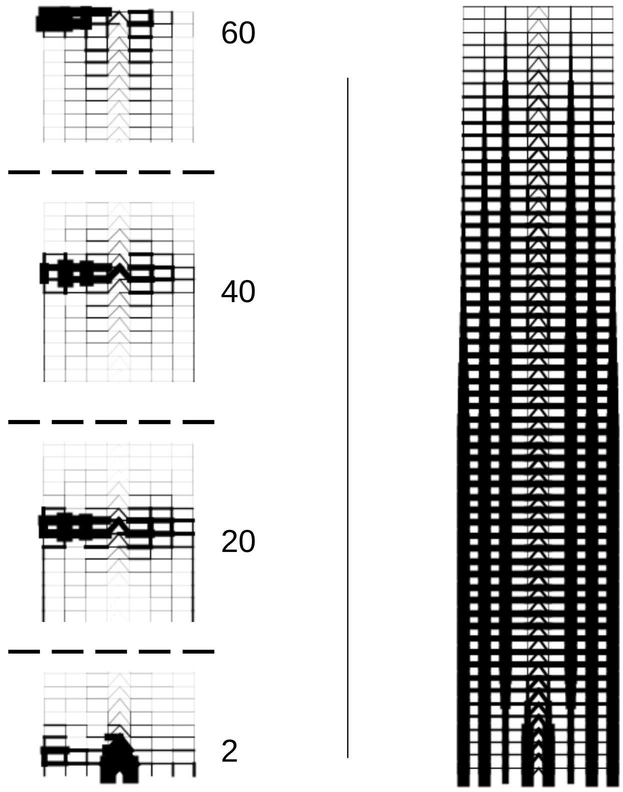


Figure 3.4: *Optimisation graph for the 60-storey framework*

Figure 3.5a shows the deflection contributions of members to the relative drift at floors 60, 40, 20 and 2. Line thickness is proportional to the deflection contribution of members. It can be seen that the stiffness of members more than 2 stories away from the level under consideration do not have a large effect. Figure 3.5b shows the distribution of mass in the final structure. The thickness of the line is proportional to the member's mass per length. As expected outer columns have greater stiffness and larger sections are used at the lower levels. Many columns have the maximum possible section size found in the database used. If a larger section database, or compound sections, could be selected the structure's weight could be reduced further.



a) Line thickness is proportional to a member's deflection contribution.

b) Line thickness is proportional to a member's mass per length

Figure 3.5:* (a)** *Deflection contributions of members to interstorey drift at floors 60, 40, 20 and 2.* ***(b) *Mass of members in the optimised structure*

3.6.2 Industrial warehouse with gantry cranes

In this case study the warehouse shown in Figure 3.6 is solved by the automated VWO method. Results are compared to the design produced by a firm of professional structural engineers. The structure's various load cases are shown schematically in Figure 3.6 and 3.8. The structure has seven load combinations dealing with dead, live, crane and wind loads. There are 14 deflection criteria and the designers specified 16 member groups. Sections must satisfy SANS 10162 (2005) strength requirements using grade 300W steel. Buckling of the latticed columns is taken into account.

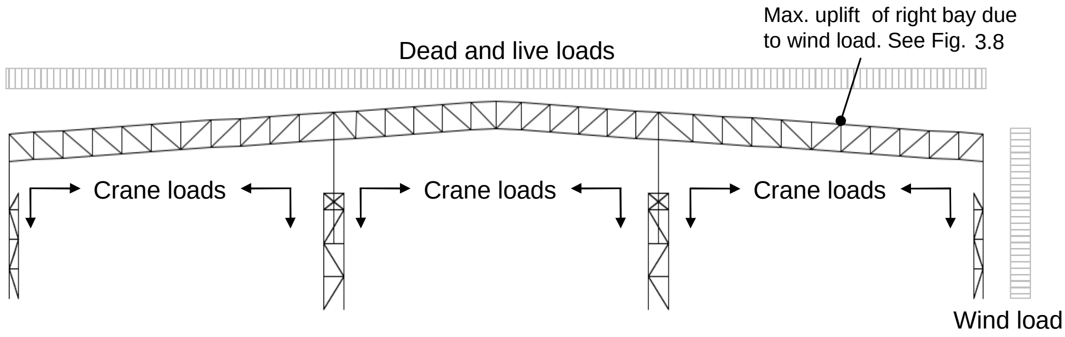


Figure 3.6: *Warehouse with gantry cranes designed by professional engineers*

Table 3.4 summarizes the VWO method and engineers' results. The structure was optimised in 12 iterations. The total optimisation process took 20 seconds on a 2 GHz Intel Centrino computer. The final solution is 4.5% lighter than the solution obtained by the design engineers.

Table 3.4: *Summary of the warehouse optimisation*

Comparison of results		**Optimisation speed**	
Engineers' mass (kg)	4153	Strength iterations	4
VWO mass (kg)	3936	Deflection iterations	8
% Saving using VWO	**4.5%**	Total iterations	12
		Total time needed to optimise structure	**20 sec.**

Figure 3.7 shows the optimised structure with 16 member groups. The thickness of the line is proportional to the member's mass per unit length. Deflection dependent members are depicted in black, strength dependent members are in grey. Figure 3.7 shows how mass should be distributed most efficiently to resist structural deformations. Note that the vertical members between the truss and the laced columns are deflection dependent members which play an important role in resisting crane and roof loads. The symmetry of the solution is due to the grouping of members.

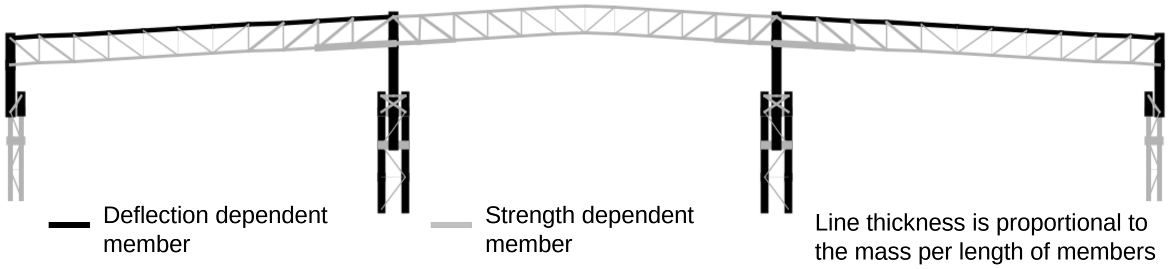


Figure 3.7: *Final mass distribution in the warehouse. Deflection and strength dependent members are shown.*

As a typical example, the deflection contributions of members to the uplift of the roof in the right-hand bay are shown in Figure 3.8. The uplift wind pressure is the dominant force in this load case. Plots such as Figure 3.8 are useful to visualize how deformations are resisted and determining how geometric topology and member groupings can be improved. In this instance, a more efficient roof system might be produced by introducing one additional member group at the mid-spans of the trusses, rather than stiffening entire chords.

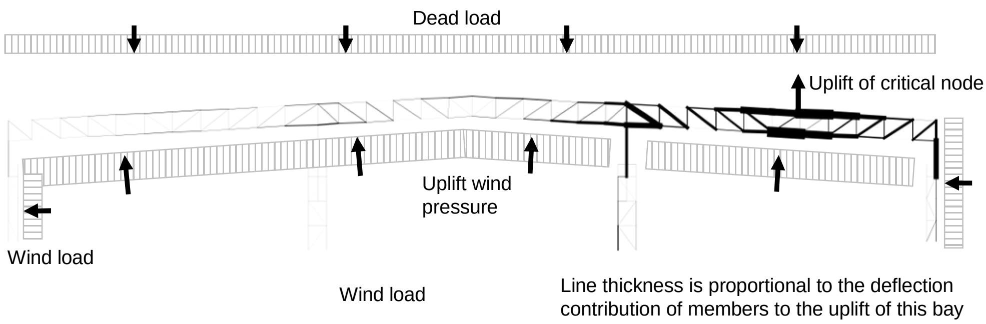


Figure 3.8: *Deflection contribution of members to the uplift of the left bay roof*

3.6.3 Stepped Cantilever

The stepped cantilever in Figure 3.9 is a statically determinant problem which has been optimised by Thanedar and Vanderplaats (1995) using: branch and bound methods, approximations based on branch and bound solutions and ad-hoc methods. The aim is to minimize the volume of the structure. The tip deflection is limited to

2.7cm. The sections for each member can only be rectangular, and the maximum ratio of height, H, to breadth, B, for each section is 20. The height and breadth of each section are the variables to be determined. The section dimensions must be integer centimeter values. The maximum allowable stress is limited to 140 MPa.

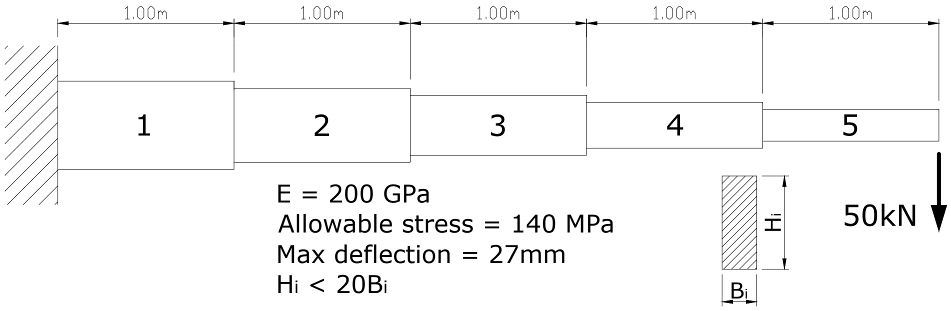


Figure 3.9: *Stepped Cantilever geometry and specifications*

The VWO method required 1 strength and 5 deflection iterations to optimise this structure. One frame analysis occurs within each iteration. The final volume is 68,100 cm^3 and the tip deflects 26.78mm. Members 2, 3 and 4 are deflection dependent while members 1 and 5 are strength dependent. Table 3.5 summaries these results.

Of the six solutions published by Thanedar and Vanderplaats (1995) it should be noted that four of them violate design constraints, as shown in Table 3.5. Of the two solutions that did not violate constraints, the conservative approximate discrete optimum found the same solution as the VWO method, but required 19.5 times more iterations. The continuous round-up solution needed only 1 iteration, but the solution was 14.4% heavier than the VWO solution. Of the solutions that violate design requirements the precise discrete optimum and linear approximate methods require 31.8 and 19.5 times more iterations. The continuous solution obtained a volume 7.3% lower than the VWO method in a single iteration. This lower volume and computational cost highlights the difficulties introduced by placing the additional, but necessary, constraint of selecting sections from a discrete database. Rounding off or up a continuous solution to produce discrete sections will not necessarily produce the lightest solution, and design constraints may be violated.

Table 3.5: *Optimisation results for the Stepped Cantilever*

Design Variable	Optimisation Method						
		Thanedar and Vanderplaats (1995) methods:					
	VWO (cm)	Continuous solution (cm)	Round-off continuous (cm)	**Round-up continuous (cm)**	Precise Discrete Optim. (cm)	Linear Approx.(cm)	**Conservative Approx. Discrete Optimum. (cm)**
B_1	**3**	3.06	3	**4**	3	3	**3**
B_2	**3**	2.81	3	**3**	3	3	**3**
B_3	**3**	2.52	3	**3**	3	3	**3**
B_4	**3**	2.2	2	**3**	3	3	**3**
B_5	**2**	1.75	2	**2**	2	2	**2**
H_1	**60**	61.16	61	**62**	60	60	**60**
H_2	**57**	56.24	56	**57**	57	59	**57**
H_3	**49**	50.47	50	**51**	49	46	**48**
H_4	**39**	44.09	44	**45**	38	37	**40**
H_5	**33**	35.03	35	**36**	33	33	**33**
Constraints satisfied	**Yes**	No: non-integer values	No: H/B ratio at 1 and 4.	**Yes**	No: Deflection	No: stress. 146MPa at 4, 142MPa at 3. Deflection	**Yes**
No. of Iterations	**6**	1	1	**1**	191	207	**117**
Tip Deflection (mm)	26.78	27	26.32	**21.47**	27.10 Violated	27.92 Violated	26.84
Volume (cm^3)	**68,100**	63,110	65,900	**77,900**	67,800	67,200	**68,100**
% Greater than VWO Vol.	**-**	-7.3	-3.2	**14.4**	-0.4	-1.3	**0**

3.7 Conclusion

This Virtual Work Optimisation (VWO) method has been expanded in this chapter for optimising structures subject to multiple deflection criteria and load cases. The method minimizes overall structural mass while satisfying multiple strength and deflection constraints simultaneously.

The case studies considered demonstrate that the VWO method can optimise structures in fewer iterations than other published methods. Methods which only consider deflection criteria can produce lighter structures in fewer iterations, but solutions obtained might not satisfy strength requirements. The VWO method produced solutions up to 14.4% lighter as compared to other techniques in the literature. The method does not require calibration and is applicable to all structures.

Future research should concentrate on determining the effect of multiple section changes per iteration. Minimising structures' cost, as opposed to mass, should be investigated. The section database entries and their distribution have an effect on the solution obtained. This effect must be characterized. An automated member grouping method based on VWO will be studied in Chapter 5. Finally, research should be conducted to determine the optimisation path followed and how this can be improved.

3.8 References

Camp, C V, Bichon, B J and Stovall, S P. (2005) Design of Steel Frames Using Ant Colony Optimisation. *J. Struct. Div*, ASCE, 131(3), 369-379

Chan, C M. (1992) An optimality criteria algorithm for tall steel building design using commercial standard sections. *Structural Optimisation*, 5, 26-29.

Erbatur, F, Hasançebi, O, Tütüncü, I and Kiliç, H. (2009) Optimal design of planar frames with genetic algorithms. *Computers and Structures*, 75, 209-224.

Groenwold, A A, Stander, N and Snyman, J A. (1996) A pseudo-discrete rounding method for structural optimisation. *Structural Optimisation*, 11, 218-227.

Kargahi, M, Anderson, J C, and Dessouky, M M (2006) Structural Weight Optimisation of Frames Using Tabu Search. I: Optimisation Procedure. *Journal of Structural Engineering*, ASCE, 132(12), 1858-1868.

Pezeshk, S. (1998) Design of frames structures: an integrated non-linear analysis and optimal minimum weight design. *International Journal for Numerical Methods in Engineering*, 41, 459-471.

Pezeshk, S., Camp C., and Chen, D. (2000). Design of nonlinear framed structures using genetic optimisation. *J. Struct. Eng.*, 126(3), 382-388.

SANS 10162-1(2005), The Structural use of steel. Part 1: Limit-states design of hot-rolled steelwork. South African National Standard.

Thanedar, P.B. and Vanderplaats, G.N. (1995) Survey of discrete structural optimisation for structural design. *J. Struct. Eng*., *ASCE.* 120(2), 301-306.

CHAPTER 4: MASS AND STIFFNESS DISTRIBUTIONS IN OPTIMISED UNRGOUPED FRAMES

4.1 Introduction

In this chapter the Virtual Work Optimisation (VWO) method is used to investigate the spatial distribution of mass in multi-storey, ungrouped frames.

An ungrouped structure is one in which every member can have a different section. Ungrouped structures are impractical to design and expensive to construct (Provatidis and Venetsanos, 2006). Thus, they are not used in practice. However, an optimized ungrouped structure is limited by few constraints. By investigating mass and stiffness distributions in ungrouped structures the topologies and grouping configurations of grouped structures can be improved. An optimal structure is defined as one which satisfies all strength and deflection criteria using minimal material.

Engineers group members in structures to simplify designs, and to reduce fabrication and erection costs. This is done based on experience, intuition and construction requirements. Thus, most grouping can be considered *ad hoc*. Inefficient member groupings make a structure more expensive. Grouped solutions are further from global mass minima.

Ungrouped structures have infrequently been optimized. For most optimization methods it is essential that designers group members to reduce search spaces and the number of design variables (Erbatur et al., 2000). There is a dearth of literature on optimized mass and stiffness distributions in ungrouped frames.

This chapter is organized as follows: first, the optimization method used is presented. A 60-storey structure from the literature is investigated to compare grouped to ungrouped results. Then, a parametric study is conducted with frames ranging from 5 to 30 stories. For each case study the optimization method selects from three section databases to determine the effect of using different discrete sections. The results obtained are first presented and then discussed.

4.2 The optimisation method

Although any building code can be used, strength requirements are satisfied according to the South African steel code, SANS 10162 (2005), with grade 350W steel. Axial and bending forces are considered. The Virtual Work Optimisation (VWO) method from Chapter 3 is used to optimise the structures.

4.3 A comparison between grouped and ungrouped structures

The 60-storey 7-bay structure shown in Figure 4.1 was originally optimised by Chan (1992), and by the VWO method in Chapter 3. The following conditions, as before, are adhered to. The structure is subject only to wind loads. Beams and diagonals are grouped together to have one section per floor. Columns are grouped together across two adjacent stories with symmetric columns having the same section. The optimisation is constrained to select from the following AISC sections (ASTM A6-81b, 2009). Columns and bracing are to be chosen from W14 sections ranging from W14x22 to W14x730. Beams have to be chosen from W24 sections ranging from W24x55 to W24x492. Interstorey drift is limited to floor height/400 (9.15mm).

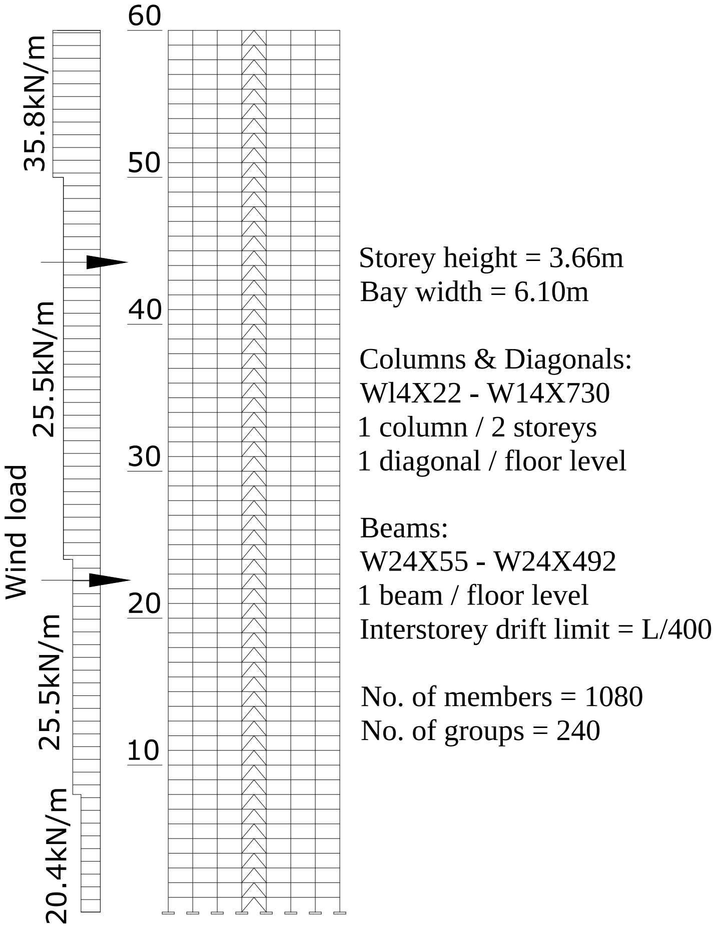
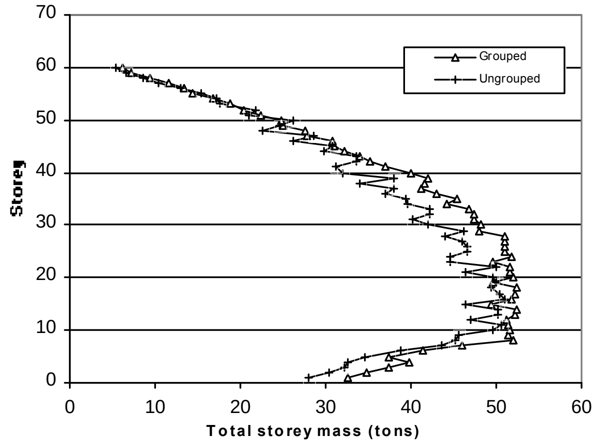
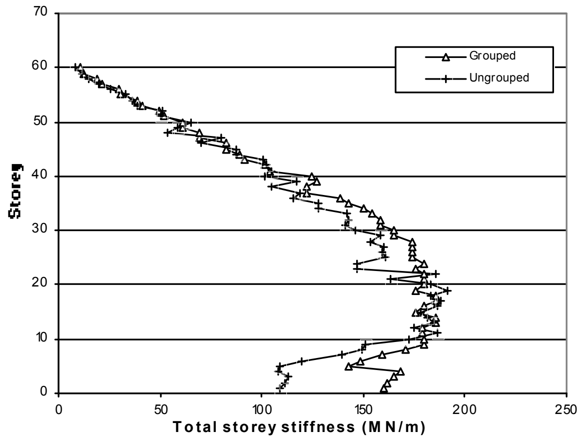


Figure 4.1: *60-storey 7-bay structure case study of Chan (1992)*

The stiffness and mass of each storey is plotted for the grouped and ungrouped optimised structure in Figures 4.2 and 4.3. The stiffness of each floor is defined as the sum of EI/L^3 for all the beams and columns, where E is the Young's modulus of the material, I is the second moment of area of each section and L is the length of each member.

Figures 4.2 and 4.3 show that there is an approximate linear increase in stiffness and mass from floor 60 to 25. The stiffness and mass then remain constant for the following 16 floors, before decreasing for the lowest levels. It is interesting to note that the summed mass and stiffness of each floor for the optimised grouped and ungrouped structure follow the same trend. As expected, the ungrouped solution oscillates more along the height of the building, and consistently has lower mass per floor. The optimum spatial distribution of mass within the structure is investigated next.





Figures 4.2: *Total mass of each floor for the grouped and ungrouped 60 storey structure.*

Figures 4.3: *Total stiffness of each floor for the grouped and ungrouped 60 storey structure.*

When the structure with the grouping of Chan (1992) is optimised by the VWO method it produces the mass distribution shown in Figure 4.4. The thickness of the line is proportional to the mass per unit length of the member. Members having the same thickness and shade have the same section. Figure 4.4 shows that the mass is distributed in a regular pattern.

The ungrouped structure of Chan (1992) is now re-optimised. Please note that the lateral wind load can be applied from either side, and since the structure is symmetric, symmetric members will be pre-grouped. This configuration is still referred to as "ungrouped" in this chapter. When the ungrouped 60 storey structure is optimised using the VWO method it produces the mass distribution shown in Figure 4.5.

The mass of the optimised, grouped structure is 2288 tons, while the ungrouped structure is 2100 tons. By removing grouping constraints an 8.2% saving is achieved. It must be emphasized that Chan (1992) assumed the grouped configuration. This grouping is thus *ad hoc*. The distribution of mass in the ungrouped structure (Figure 4.5), and the magnitude of the reduction in mass, suggests that the grouping selected might not be the most efficient. This is investigated in Chapter 5.

Figure 4.5 shows that in the central bays of the ungrouped optimised structure surrounding the shear wall, a distinct pattern has emerged: there are alternating stiff and slender areas, distributed in a checkered pattern. This checkered pattern is more prevalent at the core of the structure where moment effects are lower. These alternating regions are not reflected in the total storey mass plot (Figure 4.2).*

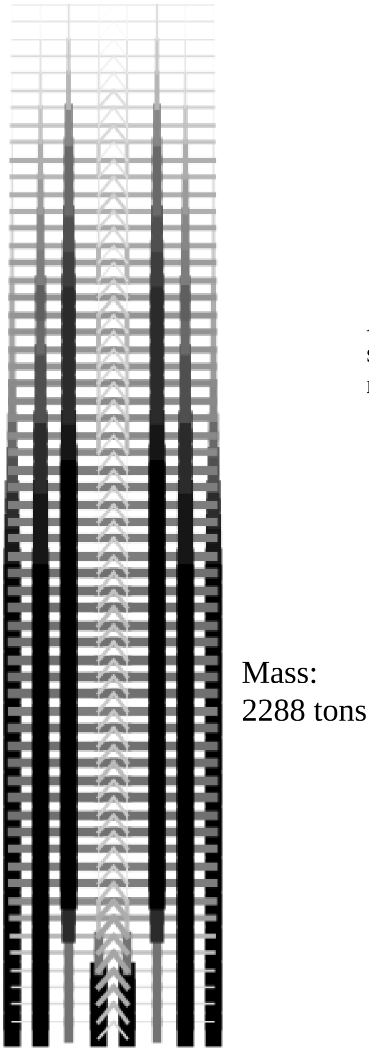


Figure 4.4: *Mass distribution in the grouped (Chan, 1992) 60-storey structure*

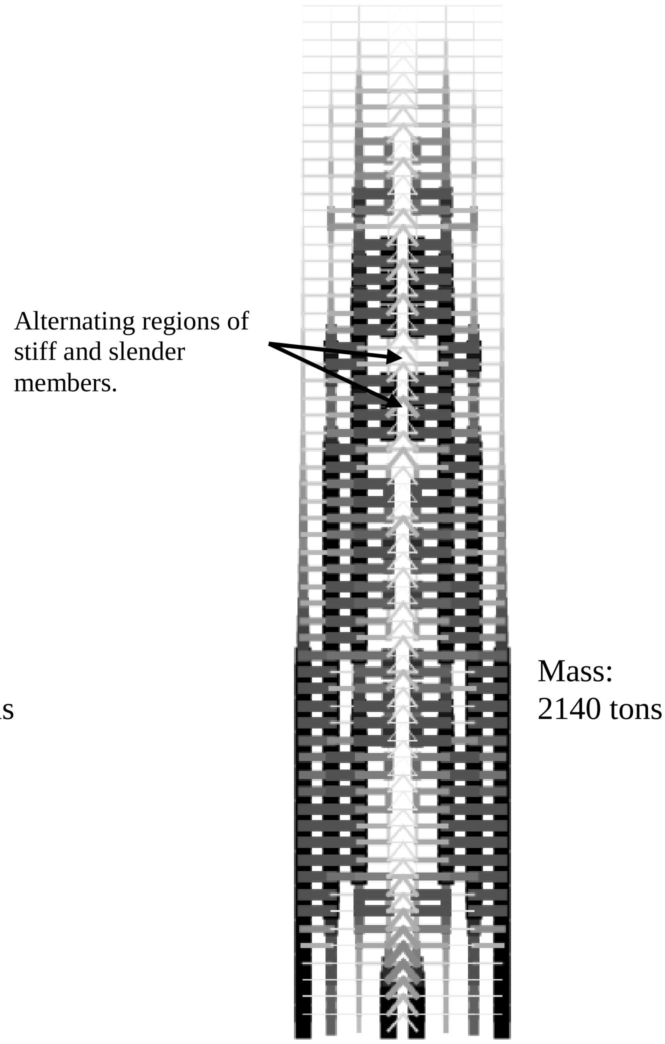


Figure 4.5: *Mass distribution in the ungrouped 60-storey structure.*

4.4 Parametric investigation of optimised ungrouped structures

The question now arises: is the distinct pattern of stiff and flexible regions particular to the ungrouped and optimised 60 storey building? To answer this question a parametric study is carried out on frames of various sizes. All structures to be investigated are optimised, ungrouped and have the general layout shown in

* The fluctuations of floor mass in Figure 4.2 occur from floor to floor, and hence have a different "period" to the checkered pattern in Figure 4.5.

Figure 4.6. The number of bays (X) and number of storeys (Y) is varied. A wind load of W=10kN is applied at each level. In order to identify how the structures respond to lateral forces no gravity loads are considered. Inter-storey drift is limited to L/400, or 7.5mm. H and I-sections are used for the columns and beams respectively. Columns are fixed at the foundations, allowing no rotation.

The nature and size of the section databases used when optimising structures is investigated. When optimising structures it is possible that the number, uneven distribution, and difference in size of sections can influence results. For this reason three section databases are used in each case study: Universal Beams (UB) and Columns (UC) (BS4: Part 1, 1993), AISC (ASTM A6-81b, 2009), and a theoretical, synthetic database. The databases contain 83 Universal sections, 187 AISC sections and 502 theoretical sections. To approximate a continuous section spectrum, the number of sections in the theoretical database is large, with small increments between sections. This database has been designed to have high bending resistance (i.e. second moment of area) per unit mass.

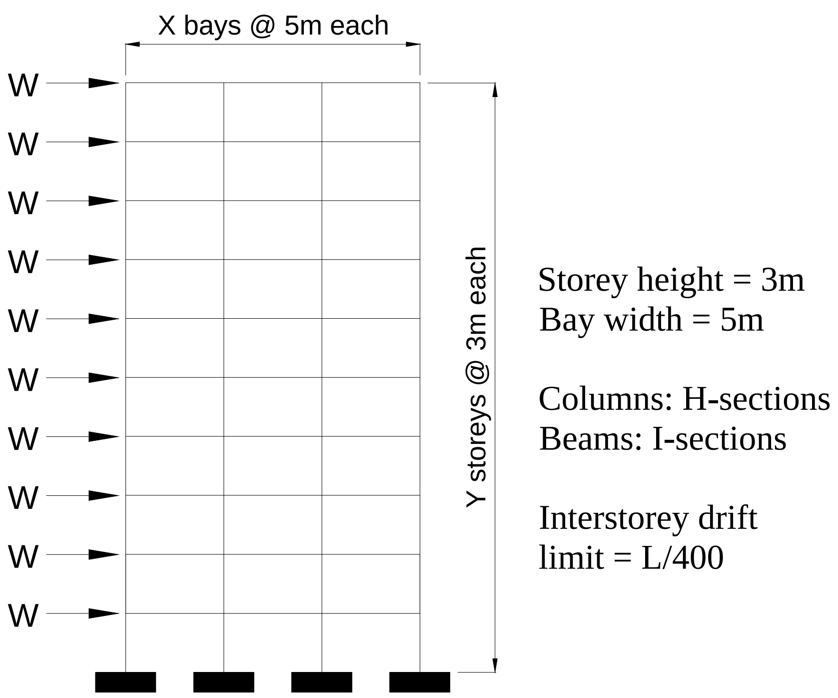


Figure 4.6: *Layout of structures to be optimised*

The parametric study consists of five frames with different numbers of bays and storeys as shown in Table 4.1. In all cases, except the one bay frame, symmetric members have been pre-grouped*. No other grouping was done. The structures' height to breadth ratio ranges from 3 to 4.5. Taller frames are analyzed to ensure that lateral resistance is the primary design consideration.

Table 4.1 summarises the computed masses of the ungrouped optimised structures.

Table 4.1: *Summary of case studies investigated and the optimisation results*

				Final mass (kg)		
Case Study	No. of bays (X)	No. of storeys (Y)	Symmetrically constrained	Universal Sections	AISC Sections	Theoretical Sections
1	1	5	No	2324	2413	2151
2	2	10	Yes	8563	8666	7855
3	3	20	Yes	29023	29905	27029
4	4	30	Yes	61372	63868	54113
5	6	30	Yes	67595	68553	59563

4.5 Distributions of stiffness and mass

Figures 4.7 to 4.10 plot the mass and stiffness of each storey of the case studies. Only the synthetic database solutions are presented in these graphs. The continuous spectrum in the synthetic database results in the greatest number of unique sections chosen for the members, and produces the lightest structures. These solutions are closer to the global minima. Please note that the Universal and AISC section databases produce similar stiffness and mass versus storey level distributions.

* A symmetric constraint of the members in the one bay frame would automatically produce a regular mass distribution.

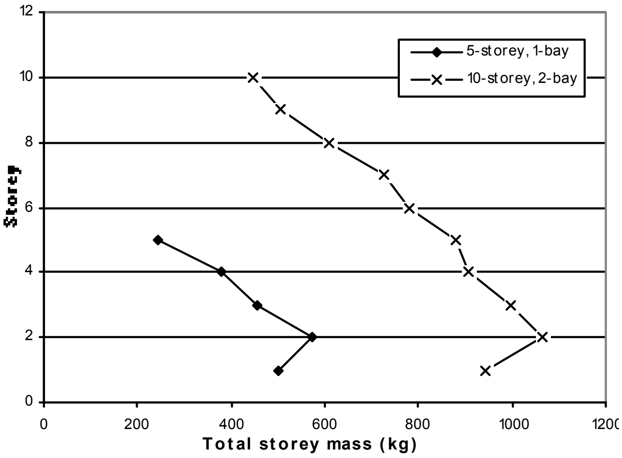


Figure 4.7: *Plot of the mass of each storey for the 5-storey 1-bay, and 10-storey 2-bay frames*

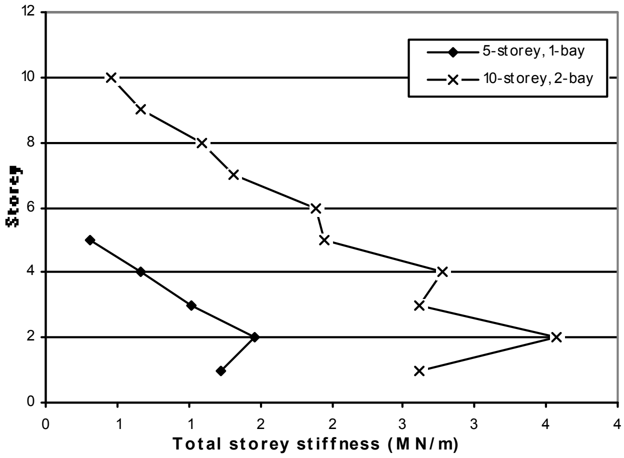


Figure 4.8: *Plot of the stiffness of each storey for the 5-storey 1-bay, and 10-storey 2-bay frames*

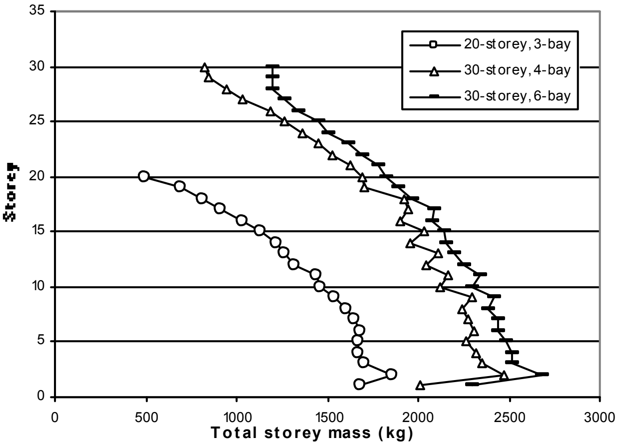


Figure 4.9: *Plot of the mass of each storey for the 20-storey 2-bay, 30-storey 4-bay and 30-storey 6-bay frames*

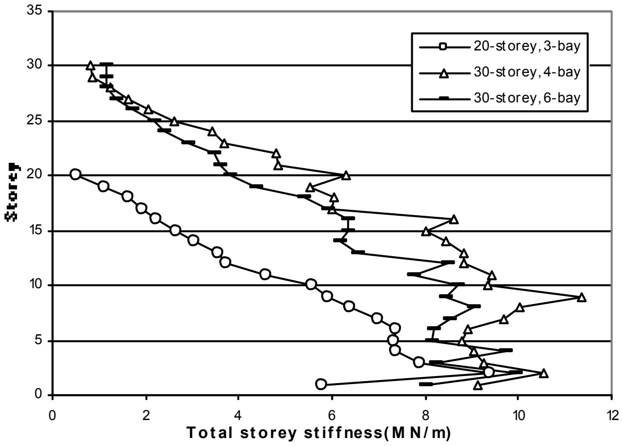


Figure 4.10: *Plot of the stiffness of each storey for the 20-storey 2-bay, 30-storey 4-bay and 30-storey 6-bay frames*

Although the case studies do not have a shear wall, the trends are similar and consistent with the 60-storey structure (Figures 4.2 and 4.3). For all the frames considered, there is a linear increase in mass and stiffness as the floor height decreases. All the case studies show a sharp decrease towards the base. For the 20 and 30 storey frames there is a zone above the support where the mass and stiffness is either increasing at a slower rate, or is constant (see Figures 4.9 and 4.10).

Figures 4.11 to 4.15 plot the spatial distribution of mass for the ungrouped, optimised case studies. The thicker lines, which represent members with higher masses per unit length, are chosen to resist either higher forces and/or to limit inter-storey drift. As can be seen from these figures, the mass distribution throughout the structure is not uniform, and once again a distinct pattern has emerged.

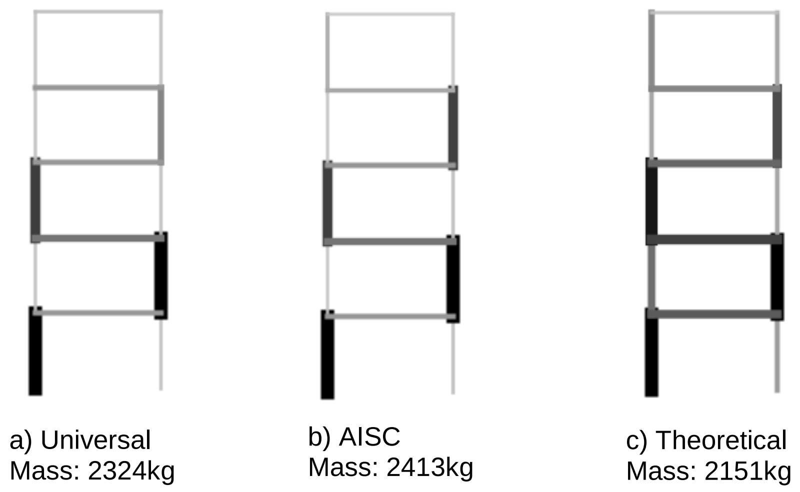


Figure 4.11: *Mass distribution for case study 1 - 5-storey 1-bay frame*

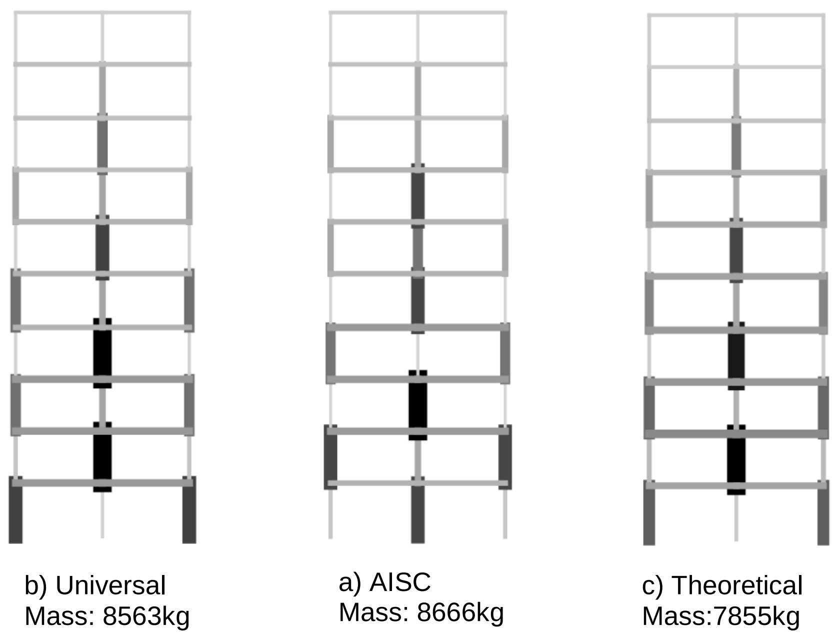


Figure 4.12: *Mass distribution for case study 2 - 10-storey 2-bay frame*

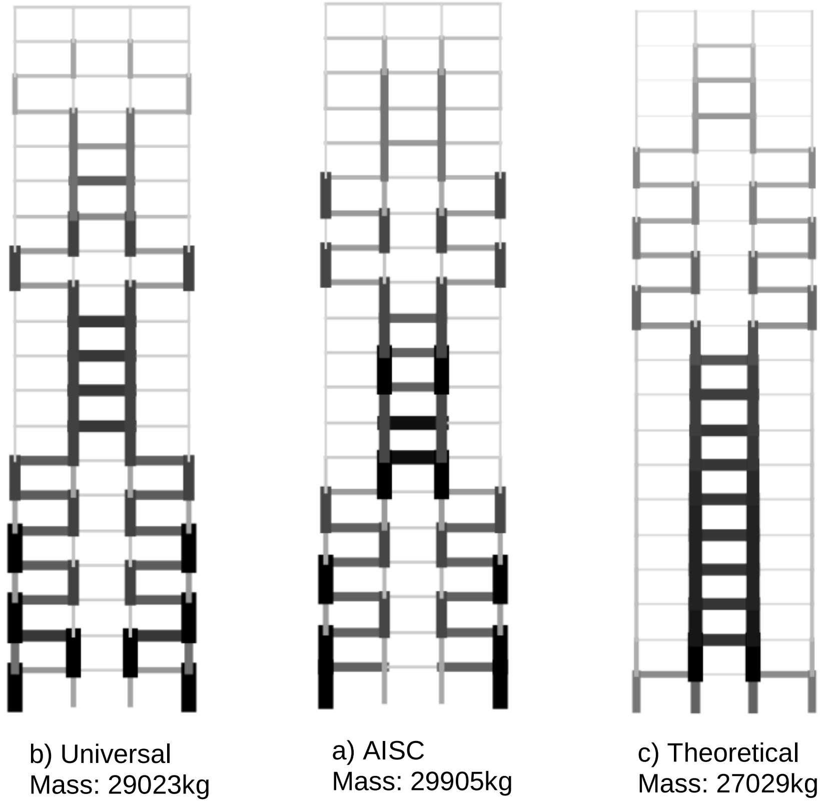


Figure 4.13: *Mass distribution for case study 3 - 20-storey 3-bay frame*

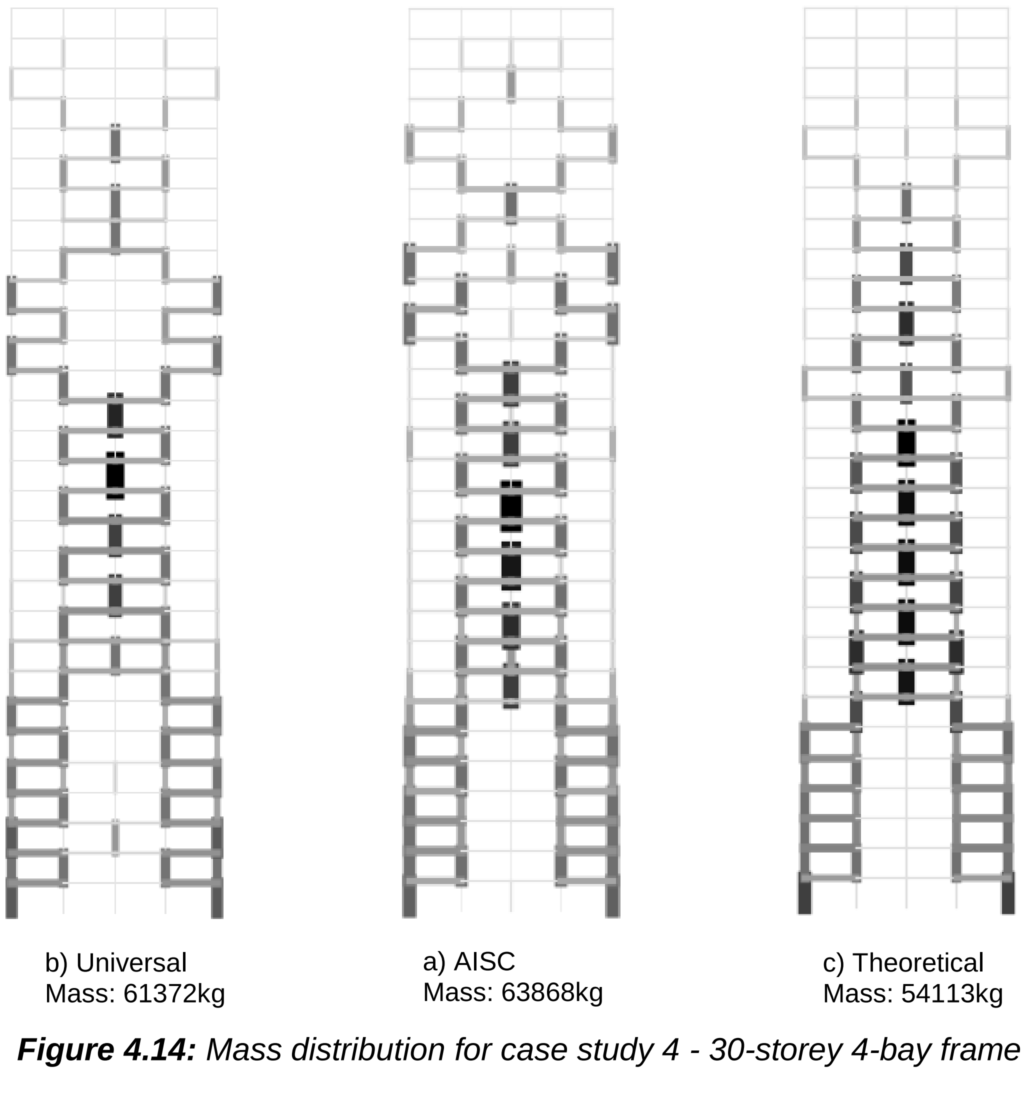


Figure 4.14: *Mass distribution for case study 4 - 30-storey 4-bay frame*

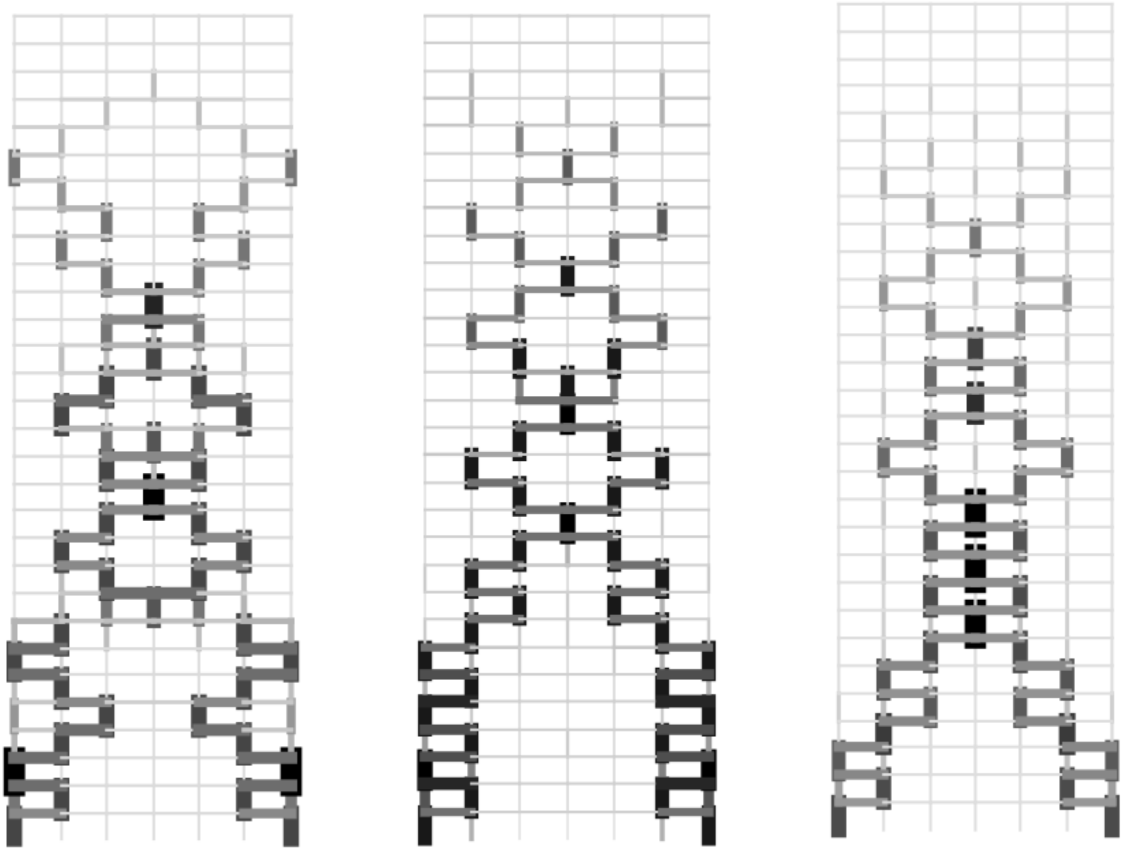


Figure 4.15: *Mass distribution for case study 5 - 30-storey 6-bay frame*

4.6 Discussion

The frame case studies can be approximated as a vertical cantilever with an uniformly distributed lateral load. Such a cantilever has a linearly increasing shear force diagram, and a parabolic bending moment diagram. Thus, it is expected that a multi-storey frame has an increasing stiffness and mass with decreasing height. The trends observed in Figures 4.7 to 4.10 show that the top floors approximate the shear force distribution more accurately. These upper floors display shear beam behaviour. In the 20, 30 and 60 storey structures there are broad transverse regions of stiffness at the higher levels. This suggests the belt\bandage bracing system, or “virtual outrigger” described by Kareem et al. (1999).

The lowest floors of each optimised frame has the outer beam and column regions stiffened. In the 60-storey optimised structure the shear core has been strengthened in the bottom 6 storeys. This shows that the structure behaves as a bending beam in the support region.

The upper and bottom most storeys are connected by a transition zone which behaves both as a shear and a bending beam. In this transition zone the 20 and 30 storey frames (Figures 4.9 and 4.10) have a decreasing rate of change in total floor mass and stiffness with height when compared to the upper floors. For the 5 and 10 storey frames the transition zone cannot be seen in Figures 4.7 and 4.8. In these cases either the entire structure falls into the shear and bending beam transition zone, or this zone is absent. In the 60-storey structure the transition zone overlaps with the region of constant mass and stiffness (Figures 4.2 and 4.3). However, this constant region may be due to the limited section database used: the VWO algorithm has selected the largest sections available in the database for most of the members in these floors.

The bottom floor in each frame case study has a significantly lower stiffness than the floors above it. This is due to the columns being fixed to the base, which increases effective stiffness. If rotation at the base is allowed, i.e. the supports are pinned, and

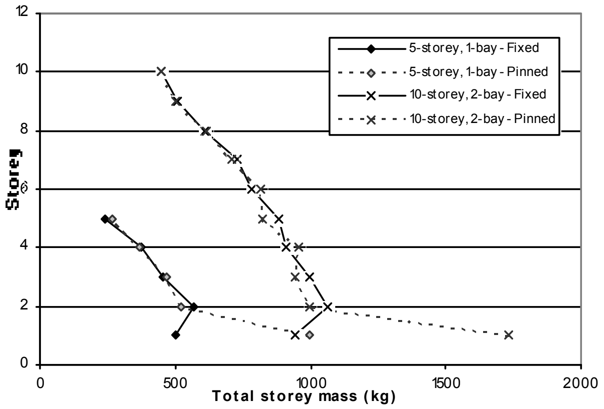


Figure 4.16: *Comparison of storey masses for the 5-storey, 1-bay and 10-storey, 2-bay frames with fixed and pinned bases*

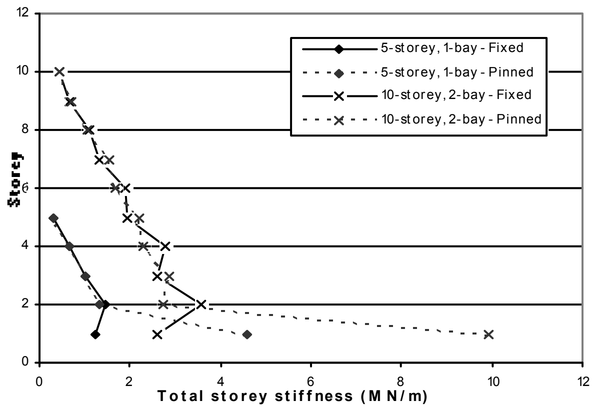


Figure 4.17: *Comparison of storey stiffnesses for the 5-storey, 1-bay and 10-storey, 2-bay frames with fixed and pinned bases*

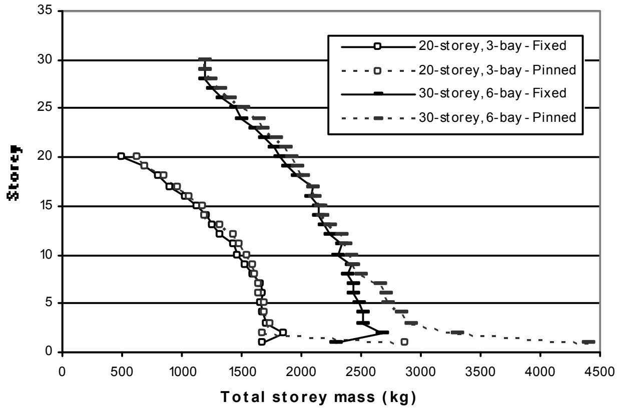


Figure 4.18: *Comparison of storey masses for the 20-storey, 3-bay and 10-storey, 4-bay frames with fixed and pinned bases*

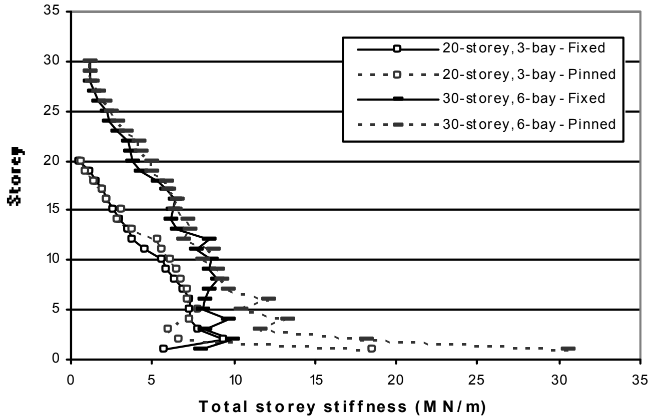


Figure 4.19: *Comparison of storey stiffnesses for the 20-storey, 3-bay and 10-storey, 4-bay frames with fixed and pinned bases*

the frames reoptimised, then the distribution of the floor mass and stiffness changes as shown in Figures 4.16 to 4.19. These figures show that the upper floors are not affected by the fixity of the foundations – the fixed and pinned optimum solutions are almost identical. However, the floors in the foundation region for the pinned structures require much more mass and stiffness to meet strength and flexibility requirements. The transition from beam to shear behaviour region, above the foundation zone, is also influenced by the nature of the foundations.

Figures 4.9 and 4.10 show that the masses and stiffnesses of the floors oscillate from storey to storey in the central floors of the ungrouped taller frames. This was also observed in the 60 storey structure (Figures 4.2 and 4.3). During the iterative optimisation procedure a member that is stiffened attracts load from adjacent floors, reducing the stiffness requirements of the proximal members. This creates the alternating stiff and slender regions of members. These regions can be local (individual members) or global (entire regions across multiple floors).

Figures 4.11 to 4.15 show that distinct force paths have formed in the optimised frames. Diagonal load paths are created by the stiff member regions in the structures. This distributes forces across the breadth of frames, increasing lateral resistance. The mass distributions suggest truss behaviour. This can be seen in Figure 4.20 where the diagonal lines have been fitted to the regions of high mass and stiffness*. The optimised distribution of mass suggests a megabrace structural configuration (Cross *et al*., 2007).

* Except for the 60-storey structure the theoretical database has been used for the structures in Figure 4.20.

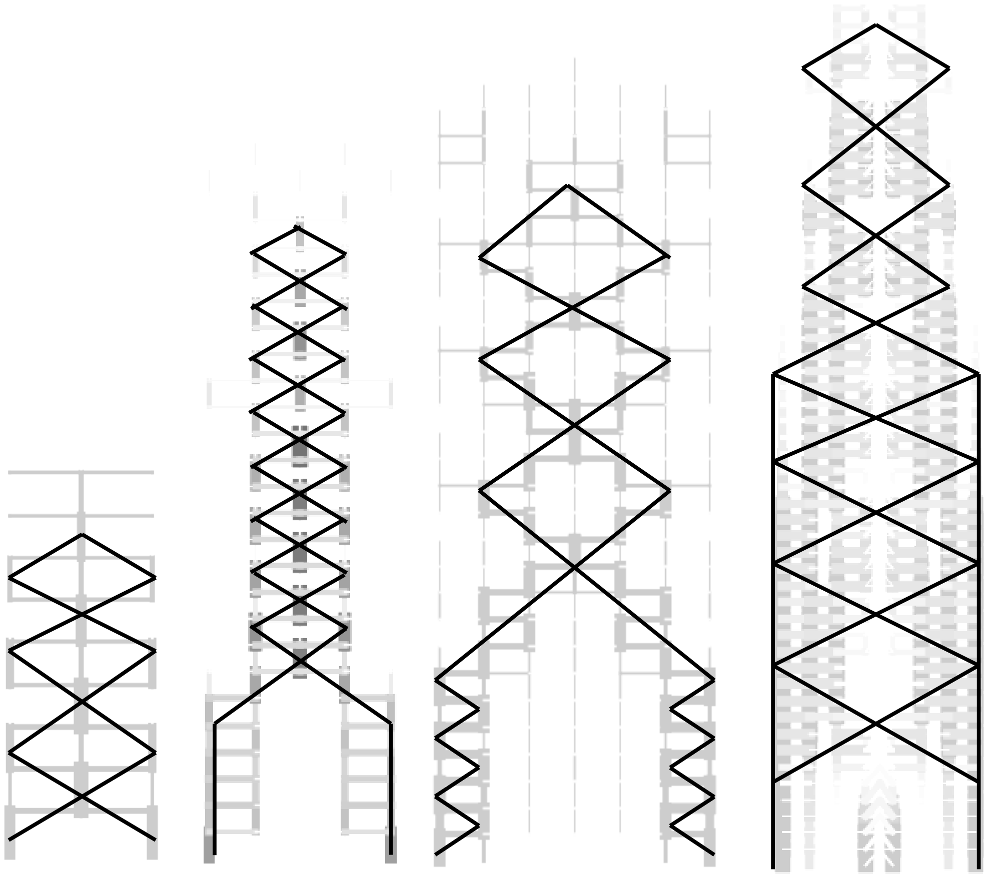


Figure 4.20: *Truss behaviour of the case studies. Black lines follow the regions of increased mass and stiffness.*

In the optimised structures the final masses obtained vary depending on which section database was used. The structures optimised with the synthetic database are on average 10.3% and 13.3% lighter than solutions produced with the Universal and AISC sections. This is to be expected because the theoretical section database is large and has been designed to resist bending forces. Even though total masses differ, the distributions of mass shown in Figures 4.11 to 4.15 are similar for the different section databases used. This shows that the nature of the section database does not significantly affect material and stiffness distribution patterns.

The unusual distributions of material may be an artefact of the VWO method. The method stiffens members in an iterative manner. Members which have their section sizes increased attract more load, creating areas of higher stress, and requiring section sizes to be increased even further. However, the fact that numerous structures have shown similar distributions, indicates that such configurations are efficient to resist lateral loads, and should not be strongly dependent on the optimisation method used.

4.7 Conclusion

In this chapter ungrouped, multi-storey frames have been optimised to investigate the spatial distribution of mass and stiffness. It has been found that measured from the top, the total storey mass and stiffness increase approximately linearly with decreasing height of the structure. This is followed in some cases by a region of lower rate of increase or constant floor mass and stiffness. The storeys at the foundation level show a sharp decrease in floor mass and stiffness. This is due to rotational fixity offered by the foundations. The section database used does effect the optimised total structural mass, but does not significantly effect mass distribution.

Unexpected spatial distributions of mass and stiffness have been computed in ungrouped, optimised frames. The mass configures in approximately diagonal patterns across the frames. This suggests the load paths can be resisted effectively by a truss or megabrace system. Members which are stiffened decrease the strength and flexibility requirements of members in their proximity. While the floor to floor total mass oscillates, larger spatial, checkered pattern of stiff and flexible regions are also produced.

It is possible that the Virtual Work Optimisation (VWO) method used influences the solutions obtained. However, the consistency of the results implies that the optimisation method has yielded correct (or acceptably correct) results. Thus the mass and stiffness distributions within the frames are efficient to resist lateral loads.

Future research should focus on comparing the results from other optimisation methods to the VWO solutions. Methods which can address a large number of design variables need to be used on such structures. This might be the reason why there is a dearth of literature investigating the spatial distribution of mass in ungrouped frames. The effect of grouping, or partially grouping, members should be characterized. Gravity loads must be included. How shear walls influence and alter the optimised pure frame behaviour should be investigated.

4.8 References

ASTM A6-81 (2009) Specifications from *ASTM A6 – Standard Specification for General Requirements for Rolled Structural Steel Bars, Plates, Shapes and Steel Piling*. American Society for Testing and Materials.

Barbosa, H J C, Lemonge, A C C and Borges, C C H. (2008) A genetic algorithm encoding for cardinality constraints and automatics variable linking in structural optimisation. *Engineering Structures*, 30, 3708-3723.

Barthelemy, J F M, and Haftka, R T. (1993) Approximation concepts for optimal structural design – a review. *Structural Optimisation*, 5, 129-144.

BS4:Part 1 (1993). *Structural steel sections. Specification for hot-rolled sections*. British Standard.

Chan, C M (1992). An optimality criteria algorithm for tall steel building design using commercial standard sections. *Structural Optimisation*, 5, 26-29.

Choi, B H, and Kim, Y M (2008) Decision support system in the design of steel moment-resisting frames using a realistic cost model. *The Structural Design of Tall and Special Buildings*. Published online at Wiley Interscience. www.interscience.wiley.com. Accessed 28 November 2009.

Erbatur, F, Oguzhan, H, Tutuncu, I, and Kilic, H. (2000). Optimal design of planar and space structures with genetic algorithms. *Computers & Structures*, 75, 209-24

Kareem, A, Kijewski, T, amd Tamrua, Y. (1999) Mitigation of Motions of Tall Buildings with Specific Examples of Recent Applications. *Wind and Structures*, 2(3), 201-251.

Cross, P, Vesey, D, and Chan C M. (2007) High Rise Buildings. *Modelling complex engineering structures.* Melchers, R.E. and Hough, R. (eds.). ASCE Press, Virginia.

Provatidis, C.G., and Venetsanos, D.T. (2006) Cost minimization of 2D continuum structures under stress constraints by increasing commonality in their skeletal equivalents. *Forsch Ingenieurwes*, 70, 159-169.

SANS 10162-1. (2005) *The Structural use of steel. Part 1: Limit-states design of hot-rolled steelwork*. South African National Standard.

Smith, B.S., and Coull, A. (1991). *Tall building structures – Analysis and design*. John Wiley and Sons. Canada.

CHAPTER 5: AN ALGORITHM FOR GROUPING MEMBERS IN A STRUCTURE

5.1 Introduction

This chapter presents an automated method for grouping discrete structural members. In Chapters 2, 3 and 4 mass distributions in optimised, ungrouped structures suggest ways in which structures can be efficiently grouped. The grouping algorithm presented is based on these observations.

A group is defined as all members in a structure which have the same section. Grouping is related to the principle of commonality (Provatidis and Venetsanos, 2006). The fewer section types a structure has, and the more similar the members are, the lower the construction costs become. The process of grouping elements is also known as variable linking (Barthelemy and Haftka, 1993). In a structure each time variables are linked the optimisation problem changes, producing different solutions. It is unclear how a structure's behaviour will change once it has been grouped, making it difficult to develop generalized grouping methods.

For construction purposes engineers group members together based on past experience, personal preferences and fabrication requirements. This is *ad hoc* grouping. In complex structures it may not be apparent how sections should be linked to reduce material costs. Inexperienced designers can create poor groupings.

Only a few grouping algorithms can be found in the literature. Krishnamoorthy et al. (2002) and Toğan and Doloğlu (2006, 2008) have developed methods which group members in trusses according to the magnitude of axial forces in members. A second method suggested by Toğan and Doloğlu (2008) is to group tension members together according to internal axial forces, and to group compression members according to slenderness ratios. Biedermann and Grierson (1995) group beams based on member lengths; beams with spans within 20% of each other are assigned a common section. Shea et al. (1997) group truss members according to similar sectional areas. Barbosa and Lemonge (2005) and Barbosa et al. (2008) have developed methods for variable linking using an adaptive penalty scheme. In general the methods in the literature suffer from either being only suitable for specific types of structures, such as trusses, or not taking both deflection and strength requirements into account. Most methods cannot consider multiple load cases. These weaknesses of the grouping techniques are addressed in this chapter.

The algorithm developed in this chapter determines how a user-specified number of groups can be created to minimise the mass of the structure. The number of groups is an independent variable and should be chosen to satisfy fabrication and construction requirements. Structures with fixed geometric topologies and loading conditions subject to multiple load cases are considered. The Virtual Work Optimisation (VWO) method, presented in Chapter 3, has been adopted in the grouping algorithm, but any optimisation method can be used.

This chapter is arranged as follows: first, the theories and limitations regarding various grouping techniques are discussed. The new method for grouping members is then presented. A simple frame is grouped to illustrate the algorithm. Four case studies are shown to demonstrate the effectiveness of the method. A simple, stepped cantilever is considered first. A 15-storey 5-bay frame, and a truss, are considered to compare *ad hoc* grouping to the results produced by the algorithm. Finally, a warehouse, as designed by professional engineers, is investigated and the results compared.

5.2 Limitations of grouping methods found in the literature

One aim of a good design should be to satisfy strength and deflection constraints whilst being as economical as possible. To standardize designs and reduce fabrication and erection costs members have to be grouped together. It is necessary to determine which parameters should be used as a basis for specifying groups. Either the geometric properties of members, or stresses induced by loads, have been considered. Specific properties which have been used include: axial forces in members (Krishnamoorthy et al., 2002), Toğan and Doloğlu (2006, 2008), sectional areas (Shea et al., 1997), or member lengths (Biedermann and Grierson, 1995). Other parameters which could be considered, but have not been explored in the literature, include second moment of areas, locations of members within the structure, stresses, or member energies per unit volume.

A major weakness of grouping members according to internal stresses, forces or energies is that in general only a single load case can be considered at a time. For grouping according to internal forces, members must have only one dominant type of force: either axial, bending, or torsion. It is difficult to combine multiple forces for grouping members. Further, a strength dependent member may have its section governed by a combination of internal forces, while a deflection dependent member's size is not only governed by the load it carries. Compression members and laterally unsupported beams require extra factors to take buckling into account.

When members are grouped together based on their length then geometric properties, forces in members, stress requirements and deflection criteria might not be accounted for. A member's length does not adequately represent its geometric properties.

If members are grouped together based on second moments of area then implcitly only bending forces are considered. The same limitations as using cross-sectional areas are encountered, as discussed above. There is a large variation in second moment of area in section databases making it difficult to group sections based on this parameter alone.

5.3 Grouping members according to mass per unit length

It is proposed that members should be grouped according to their mass per unit length, i.e. their cross-sectional area*. For a structure in which all design constraints have been satisfied it is assumed that members with similar mass per unit length have comparable section properties. Grouping members, which have been selected to satisfy all design criteria, according to section properties solves the problems associated with multiple load cases and strength requirements. It is important to note that when optimising structures for weight, the mass per unit length of members serves as part of the objective function.

* Please note that the grouping algorithm presented here is very different to that of Shea et al. (1997) who also used cross-sectional areas as the basis of grouping. This reference considered only trusses with members grouped according to pre-specified ranges. The proposed algorithm is more general and does not have these limitations.

5.4 Single and multi step grouping

It is possible to group members in either a single or multiple steps. For a multi-step process the number of sections used in the structure is reduced by one in each iteration, until the user-defined number of groups has been produced. Groups that are created are linked either with other members or groups. The problem encountered is that in one iteration it may be optimal to group certain members together, but in a later iteration such a group may need to be split to create a different, but more effective, configuration. It was found that a single step procedure is less computationally expensive, more effective and easier to implement. For these reasons the algorithm presented is based on a single step grouping method. The results from single versus multiple step methods are discussed in case study 2.

5.5 The Single Step Grouping Algorithm

The aim of the presented algorithm is to determine a grouping configuration which will result in the lightest structure. An overview of the grouping process is: first, an ungrouped structure is optimised to produce an initial solution. Second, all possible grouping configurations are investigated. The lightest, predicted configuration is chosen for the structure. The structure is then optimised again to satisfy all design criteria and produce the solution.

5.5.1 Step 0 – Setting grouping parameters

The following information is required for the grouping algorithm: the structure's geometric topology, loading, load combinations, deflection requirements, design code and the properties of the materials to be used. The user must define how many different groups, n, need to be created. The method will group the members in the structures such that the maximum number of groups is limited to n.

5.5.2 Step 1 – Obtaining the initial, ungrouped solution

If a structure in which every member can have a different section is optimised, the lightest solution is produced. The aim of the grouping method is to create a configuration that weighs as close to the ungrouped solution as possible.

The Virtual Work Optimisation (VWO) method (Chapter 3) is used to obtain the initial, ungrouped solution. The VWO method is based on the principle of virtual work, and selects members to satisfy both strength and deflection criteria to produce the lightest structure. Sections are chosen from standard databases by determining which sections provide the highest deformation and strength resistance per unit mass. The VWO method is chosen because it requires fewer iterations than other methods, and is influenced linearly by the number of optimisation variables. It must be emphasised that any optimisation method can be incorporated into the grouping algorithm.

Once sections have been selected for the ungrouped structure they are ordered from largest to smallest according to their mass per unit length. Members with identical sections are grouped together. This configuration is still referred to as the ungrouped structure, or the initial grouping configuration. The total number of different sections, *i*, selected for the ungrouped solution is less than or equal to the number of members in the structure.

5.5.3 Step 2 – Investigating grouping configurations

An exhaustive search is performed on the ungrouped structure by computing all possible member groupings. For each permutation the mass of the structure is calculated. The assumption in this step is that the section of a heavier member will satisfy the strength and deflection constraints of a lighter member. Thus, in any permutation a member cannot have its section size reduced from the one initially selected in Step 1. Also, the largest section of all the members in a group will be selected for each member in that group.

A structure with i initial sections will be reduced to the user-defined number of groups, n, where $1 \le n \le i$. Thus, the number of sections must be reduced by $(i - n)$. The total number of grouping permutations, N, is defined by the binomial coefficient:

$$N = \binom{i-1}{n-1} = \frac{(i-1)!}{(n-1)!(i-n)!} \tag{5.1}$$

The permutation process is illustrated in Table 5.1. The members with the sections listed on the left are placed into groups numbered on the right of the table (the unshaded region). In the table $m_1 > m_2 > \ldots > m_n > \ldots > m_i$, where m denotes the mass per unit length of a section. Members are distributed progressively into each group until all permutations have been investigated.

In the first permutation, $k=1$, all members retain their initial, ungrouped section size, except for the last $i–n$ sections which are incorporated into the n^{th} group. For the second permutation, $k=2$, section number n is incorporated into group n-1 rather than group n. This process continues until the size of group n reaches 1, at permutation $k=i-n+1$. Then the size of group n-2 increases by 1, and groups n and $n-1$ move one lower than they were in permutation $k=1$. This process of regrouping progresses until permutation $k = N$, where the additional $(i – n)$ sections are incorporated into the 1[st] group. As a numerical example consider how 7 sections can be placed into 3 groups, creating 15 grouping permutations (i=7, n=3, and N=15), as shown in Table 5.2.

Table 5.1: *Possible grouping configurations for creating n groups from i sections*

<table>
<tr><th rowspan="2">Section no.</th><th rowspan="2">Mass (kg/m)</th><th rowspan="2">Initial group no.</th><th colspan="8">Permutation number (k) and the distribution of sections into groups</th></tr>
<tr><th>Grouping k = 1</th><th>Grouping k = 2</th><th>---</th><th>Grouping k = i-n+1</th><th>Grouping k=i-n+2</th><th>Grouping k=i-n+3</th><th>---</th><th>Grouping k=N</th></tr>
<tr><td>1</td><td>m_1</td><td>1</td><td>Grp. 1</td><td>1</td><td rowspan="10">---</td><td>1</td><td>1</td><td>1</td><td rowspan="10">---</td><td rowspan="6">1</td></tr>
<tr><td>2</td><td>m_2</td><td>2</td><td>Grp. 2</td><td>2</td><td>2</td><td>2</td><td>2</td></tr>
<tr><td>3</td><td>m_3</td><td>3</td><td>Grp. 3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td></tr>
<tr><td>n-1</td><td>m_{n-1}</td><td>n-1</td><td>Grp.(n-1)</td><td rowspan="2">n-1</td><td rowspan="5">n-1</td><td>n-2</td><td>n-2</td></tr>
<tr><td>n</td><td>m_n</td><td>n</td><td rowspan="5">Grp. n (Extra i-n members initially placed in this group)</td><td>n-1</td><td rowspan="2">n-1</td></tr>
<tr><td>n+1</td><td>m_{n+1}</td><td>n+1</td><td rowspan="4">n</td><td rowspan="4">n</td><td>⋮</td></tr>
<tr><td>⋮</td><td>⋮</td><td>⋮</td><td rowspan="3">n</td><td>n-2</td></tr>
<tr><td>i-1</td><td>m_{i-1}</td><td>i-1</td><td>n-1</td></tr>
<tr><td>i</td><td>m_i</td><td>i-1</td><td>n</td><td>n</td></tr>
</table>

Table 5.2: *The possible permutations for creating 3 groups from 7 members.*

<table>
<tr><th rowspan="2">Section No.</th><th colspan="15">Permutation number (k) and the distribution of sections into groups</th></tr>
<tr><th>k=1</th><th>k=2</th><th>k=3</th><th>k=4</th><th>k=5</th><th>k=6</th><th>k=7</th><th>k=8</th><th>k=9</th><th>k=10</th><th>k=11</th><th>k=12</th><th>k=13</th><th>k=14</th><th>k=15</th></tr>
<tr><td>1</td><td>Grp.1</td><td>1</td><td>1</td><td>1</td><td>1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="3">1</td><td rowspan="3">1</td><td rowspan="3">1</td><td rowspan="4">1</td><td rowspan="4">1</td><td rowspan="5">1</td></tr>
<tr><td>2</td><td>Grp.2</td><td rowspan="2">2</td><td rowspan="3">2</td><td rowspan="4">2</td><td rowspan="5">2</td></tr>
<tr><td>3</td><td rowspan="5">Grp.3</td><td>2</td><td rowspan="2">2</td><td rowspan="3">2</td><td rowspan="4">2</td></tr>
<tr><td>4</td><td rowspan="4">3</td><td rowspan="4">3</td><td>2</td><td rowspan="2">2</td><td rowspan="3">2</td></tr>
<tr><td>5</td><td rowspan="3">3</td><td rowspan="3">3</td><td rowspan="3">3</td><td>2</td><td rowspan="2">2</td></tr>
<tr><td>6</td><td rowspan="2">3</td><td rowspan="2">3</td><td rowspan="2">3</td><td rowspan="2">3</td><td>2</td></tr>
<tr><td>7</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
</table>

The total predicted mass of the structure, M_k, for each permutation, k, is given by:

$$M_k = \sum_{j=1}^{J} L_j m_{j,new} \tag{5.2}$$

where J is the number of members in the structure and j is the member index. L_j is the length of each member. The new mass per unit length is $m_{j,new}$. The new mass of each member is taken as the mass of the largest section in its group.

5.5.4 Step 3 – Selecting a new grouping configuration

The member grouping selected out of all the permutations is the one that produces the minimum mass, $M_{\min}$,, where:

$$M_{\min} = \min\{M_1, M_2 \ldots M_N\} \tag{5.3}$$

$M_{\min}$ is the *estimated* mass of the grouped structure.

5.5.5 Step 4 – Ensuring design constraints are satisfied

The lightest grouped structure obtained in Step 3 may violate strength and/or design deflection criteria. This might occur because of the redistribution of forces resulting from changing members in indeterminate structures. Alternatively, a grouped structure may be over-designed because of the increase in section size of many members. The latter situation occurs more often. Thus, it is necessary to optimise the grouped structure once again. Although any method can be used, the VWO method is employed to give the final grouped solution.

The difference in the estimated and final masses will depend on numerous factors. In statically determinate structures, with strength dependent members, the estimation will be accurate. In statically indeterminate structures, which are predominantly deflection dependent, the estimated mass is usually inaccurate, and probably an over-estimate. However, the estimation provides an effective method for specifying groups, and not the final members sizes.

5.6 Using multiple section types – a further constraint

In most structures a further constraint can be imposed by selecting the type of section to be used for each member (I-section, angle, channel). These sections must be grouped separately, and are treated as subgroups. The user must specify the number of groups to be created for each type of section: $n_1, n_2,\ldots,n_\alpha$ where α is the number of different types of sections in the structure. The number of sections of each type in the initial ungrouped structure is $i_1, i_2,\ldots,i_\alpha$. The total number of sections in the ungrouped structure, i, is the summation of $i_1, i_2,\ldots,i_\alpha$. The number of permutations to be investigated for each section type is calculated using Equation 5.1, with the values of i_l and n_l of each type, where l is the section type index of each subgroup. The total number of permutations to be investigated is:

$$N = \sum_{l=1}^{\alpha} N_l \qquad (5.4)$$

The predicted mass of the structure is the summation of the minimum mass permutation of each section type:

$$M_{\min} = \sum_{l=1}^{\alpha} M_{\min,l} \qquad (5.5)$$

By considering different section types as subgroups the number of permutations to be investigated is limited. Please note that members having different section types, but the same mass per unit length, cannot be grouped together because of the possible large variation in geometric properties.

5.7 Illustrative Example

To illustrate the grouping method, the two-storey, 6 member frame (i = 6), shown in Figure 5.1, will have 3 groups (n = 3) created. The loading is as shown and is not symmetrical. The structure must satisfy the South African steel code requirements, SANS 10162 (2005), using grade 350W steel and AISC sections (ASTM A6-81b, 2009). Inter-storey drift is limited to L/300 (10mm). The VWO method calculates the ungrouped structure to have the mass per unit length shown in Table 5.3, and depicted in Figure 5.2. In Figure 5.2 the thickness of the line is proportional to the mass per unit length of the member. Figure 5.2 provides a graphical representation of the mass distribution which is used to assign member groups. The total mass of the ungrouped structure is 592.2kg.

Table 5.3: *Mass and lengths of members for the ungrouped, optimised structure shown in Figure 5.1.*

Section Number	Length (m)	Ungrouped Member	Ungrouped mass (kg/m)
1	3	W16x26	40.7
2	5	W14x22	33.3
3	5	W8x18	27
4	3	W8x13	19.8
5	3	W6x12	18.2
6	3	W10x12	18.2
Mass (kg)			592.2

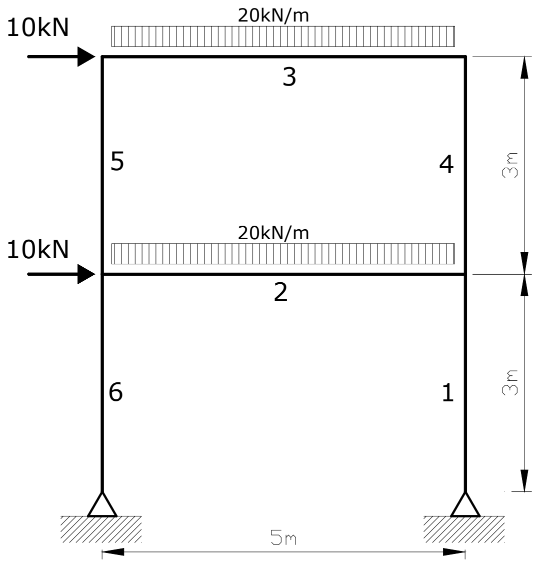


Figure 5.1: *Two-storey frame to be grouped. Numbers at mid-spans indicate section numbers.*

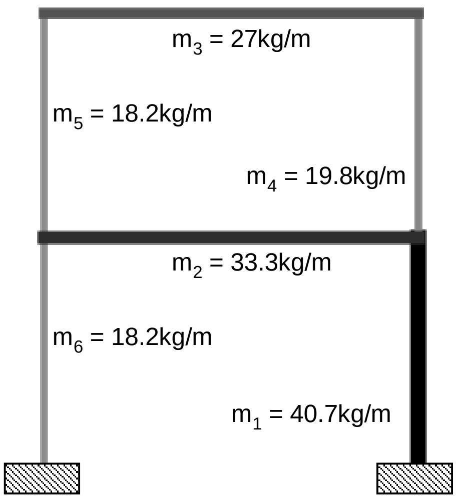


Figure 5.2: *Mass distribution in the two-storey ungrouped frame*

There are 10 possible grouping permutations for the structure (from Equation 5.1). Table 5.4 shows how the members are placed into different groups for each permutation. For each configuration the three extra sections (i – n=3) are included progressively in different groups.

Table 5.4: *Possible grouping configurations for the 2 storey frame and their mass estimates*

<table>
<tr><th rowspan="2">Member Number</th><th rowspan="2">Initial Section Number</th><th colspan="10">Grouping Configurations
New group numbers shown in un-shaded region</th></tr>
<tr><th>k=1</th><th>k=2</th><th>k=3</th><th>k=4</th><th>k=5</th><th>k=6</th><th>k=7</th><th>k=8</th><th>k=9</th><th>k=10</th></tr>
<tr><td>1</td><td>1</td><td>Grp.1</td><td>1</td><td>1</td><td>1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="2">1</td><td rowspan="3">1</td><td rowspan="3">1</td><td rowspan="4">1</td></tr>
<tr><td>2</td><td>2</td><td>Grp.2</td><td rowspan="2">2</td><td rowspan="3">2</td><td rowspan="4">2</td></tr>
<tr><td>3</td><td>3</td><td rowspan="4">Grp.3</td><td>2</td><td rowspan="2">2</td><td rowspan="3">2</td></tr>
<tr><td>4</td><td>4</td><td rowspan="3">3</td><td rowspan="3">3</td><td>2</td><td rowspan="2">2</td></tr>
<tr><td>5</td><td>5</td><td rowspan="2">3</td><td rowspan="2">3</td><td rowspan="2">3</td><td>2</td></tr>
<tr><td>6</td><td>6</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td colspan="2">Estimated Mass (kg)</td><td>667</td><td>633</td><td>664</td><td>710</td><td>639</td><td>651</td><td>677</td><td>698</td><td>703</td><td>760</td></tr>
</table>

The lightest grouped structure obtained is estimated to be 633kg (bold entries in Table 5.4). Members 2 and 3, as well as 4, 5 and 6 are grouped together. When the grouped structure is optimised using the VWO method the new mass obtained is 618.9kg. This is 4.5% heavier than the ungrouped structure, but contains 50% fewer sections. The predicted mass is 2.3% greater than the optimised mass.

5.8 Optimisation considerations

In symmetric structures with symmetric loading two options are possible to obtain the optimised member selection. Either (a) all load cases must be applied and considered separately, or (b) symmetric members can be constrained to have the same sections, and separate symmetric load cases need not be considered. It has been found that linking symmetric members produces more consistent results with lower computational costs. Symmetric members in case studies 2 to 4 have been constrained to be the same.

The number of sections in a database will influence the initial solution's number of sections, i, that have to be grouped together. The larger the database, the closer i will be to the number of members in the structure. If databases are small, numerous members may have the same section after the initial optimisation process, and will be pre-grouped together (see Step 1, Section 5.5.2). To prevent this, it is recommended that a large database is used in the initial selection process to minimize any initial grouping. For constructability only the available section database can then be used in Step 4.

Members with the same section type, but different requirements, can be isolated and grouped separately. This creates additional subgroups, which are addressed in the same manner as using different section types. An example of such a requirement is specifying that the chords of a truss must not be grouped with bracing or diagonal members (see case study 3, Section 5.12.3).

Linking existing groups is possible, and performed in an identical manner to linking individual members. In Step 1 the number of sections, *i*, is set to the number of existing groups.

5.9 Reducing computational costs

Large search spaces are rare because of the size and nature of existing section databases. It is unusual to find more than 20 different sections of each type in an optimised, ungrouped structure; this produces less than 100,000 permutations for each section type. In the case studies it was not necessary to reduce the search spaces. However, if large or continuous, synthetic section databases had been used to obtain the ungrouped solutions, it would have been essential to decrease computer time. If search spaces do become too large, two ways to reduce computational costs are proposed: (a) creating subgroups, and (b) investigating permutations only within a viable 'radius'.

Creating subgroups introduces extra constraints but reduces computational cost. For example if 80 sections have to be placed into 10 groups there would be 2.06×10^{11} permutations (Equation 5.1). However, creating 2 subgroups of 40 members and placing them into 5 groups each would only result in 1.64×10^{5} permutations.

Reducing computational cost by performing a radius search is based on the following observation: the lightest and heaviest members in a group are separated by only a few section sizes found in the initial solution. Permutations can thus be performed only a user defined radius, *X*, away from any one entry.

The two limits of the radius *X*, are: (a) $i - n$, and (b) the larger of 1 and i/n*. If *X* is equal to $i - n$ (or larger) then all permutations are performed (Equation 5.1). If *X* is less than $i - n$ then the number of permutations to be performed reduces. Figure 5.3 plots the number of permutations versus the number of initial sections for 10 groups with various radii *X*. Figure 5.3 shows that the number of permutations decreases rapidly as the radius, *X*, decreases. However, if *X* is set too low it is possible that the optimal solution may be missed. When grouping a large structure it may be necessary to test for convergence of solutions by investigating several values of *X*. More research is required to understand how to choose *X*.

Consider the example of 80 sections placed into 10 groups. If sections are not allowed to increase by more than 10 section sizes, the search space reduces from 2.06×10^{11} to 1.29×10^{6}

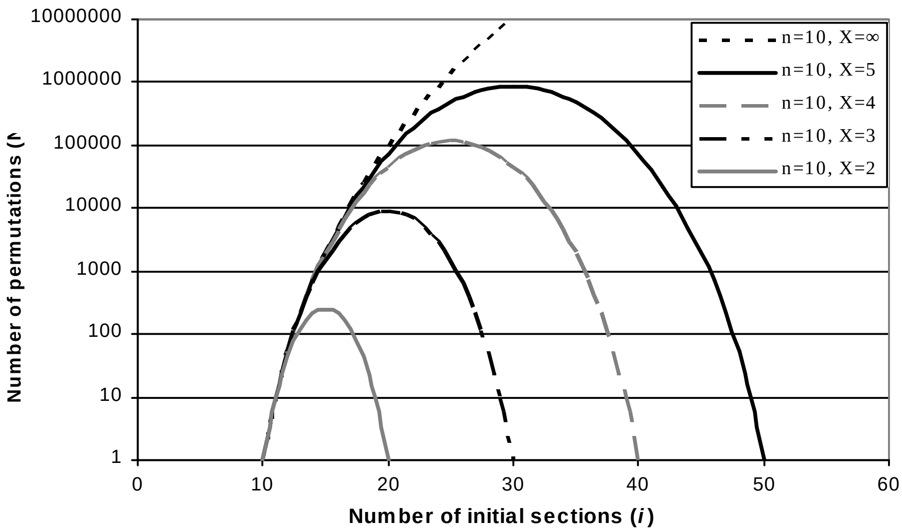


Figure 5.3: *Comparison of the number of initial sections to the number of configurations to be investigated for fixed values of i and n.*

5.10 Advantages of the algorithm

The grouping algorithm proposed is straight forward to implement and can be used for any structure. The method can group individual members or existing groups. Multiple internal forces arising from different load cases are considered. Strength and deflection criteria are satisfied by the optimisation method.

* The uninteresting case of $X = 0$ produces no permutations and the structure remains ungrouped.

The method is computationally inexpensive, even though large numbers of configurations are investigated. Structures are not analysed for each permutation, rather the algorithm predicts the structure's masses. Almost the entire computational cost is spent optimising the structure in Steps 1 and 4. However, if necessary, the grouping computational cost of Step 2 can be decreased as explained above.

5.11 Limitations of the method

The assumption that all section properties can be represented by the mass per unit length is an oversimplification. Large, non-linear variations in sectional properties relative to cross-sectional areas may cause members to be grouped incorrectly. These points, and how they interact, require further research.

5.12 Case Studies

Various aspects of the automated grouping algorithm are demonstrated by the four case studies considered. First, the stepped cantilever illustrates how masses increase when decreasing the number of groups. The 15-storey 5-bay frame and truss demonstrate how the grouping algorithm can produce lighter solutions than *ad hoc* grouping. Finally, the results of the grouping algorithm are compared to a warehouse designed by professional engineers.

In all the case studies the structures are steel with a density of 7,850kg/m^3.

5.12.1 Stepped cantilever

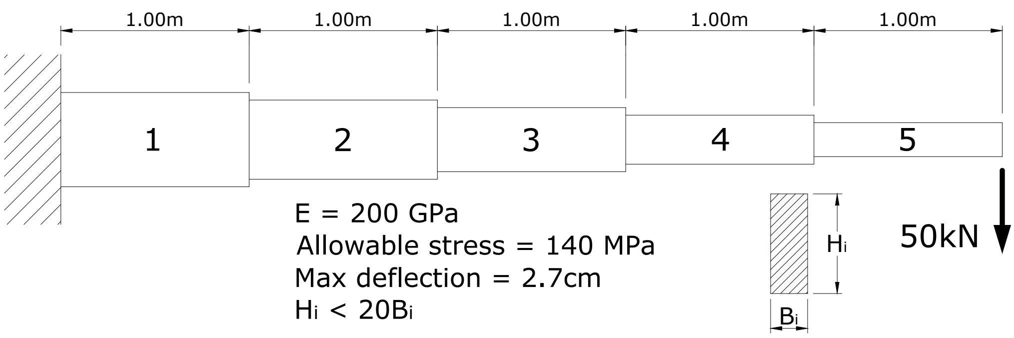


Figure 5.4: *Stepped Cantilever Beam (Thanedar and Vanderplaats, 1995)*

The cantilever shown in Figure 5.4 was optimised by Thanedar and Vanderplaats (1995), and in Chapter 3 with the following constraints. The tip of the cantilever is restricted to deflect a maximum of 2.7cm. The section of each member is rectangular and the maximum height, H, to breadth, B, ratio is limited to 20. Section dimensions must be integer centimeter values. The maximum allowable stress is 140 MPa.

First, the ungrouped structure was optimised with the VWO method to produce a solution of 531.3kg. The developed algorithm was applied to the stepped cantilever; 4 to 1 groups were specified. The results are summarized in Table 5.5. As expected, as the number of sections decrease so the structure's mass increases.

Table 5.5: *Final masses for various grouping configurations of the cantilever*

No. of groups	Final Mass (kg)	% Mass Increase from 5 groups	Members Grouped
5	534.6	-	-
4	534.6	0.0	1-2
3	555.8	3.9	1-2, 3-4
2	602.9	12.8	1-2-3, 4-5
1	706.5	32.2	1-2-3-4-5

The member lengths specified in Figure 5.4 by Thanedar and Vanderplaats (1995) introduce extra constraints. Lighter solutions can be found if the cantilever has more steps. To demonstrate this, the cantilever is discretised into 100 equal lengths, and then linked to form from 5 to 2 new groups. The results are summarized in Table 5.6. Comparing the solutions for the two levels of discretisation, shows that the finer discretisation produces lighter cantilevers for all levels of grouping. Figure 5.5 shows a comparison of the final masses of the grouped structures obtained from the initial 5 and 100 section configurations.

Table 5.6: *Final masses and section lengths for the cantilever. The structure was split into 100 members and regrouped.*

No. of groups	Final Mass (kg)	% Mass saving compared to the same no. of groups in Table 5.5	Lengths grouped
100	511.8	-	Each member 0.05m long
5	521.0	2.5	0-2m, 2-2.95m, 2.95-3.55m, 3.55-4.45m, 4.45-5m
4	533.9	0.1	0-2m, 2-2.95m, 2.95-3.55m, 3.55-5m
3	549.3	1.2	0-2.4m, 2.4-3.55m, 3.55-5m
2	592.7	1.7	0-3.55m, 3.55-5m

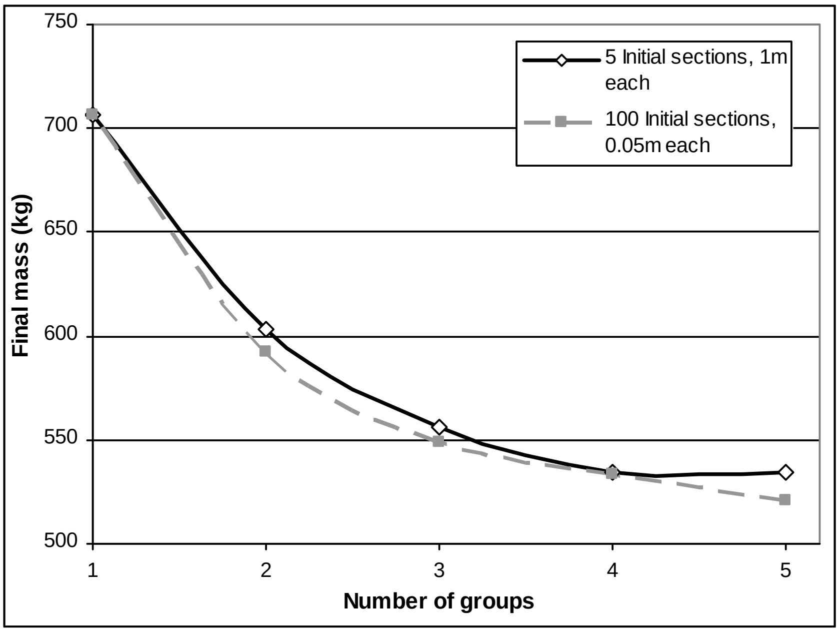


Figure 5.5: *Comparison of grouped masses for the cantilever with 5 and 100 initial sections*

5.12.2 15 Storey 5 bay frame

The 15 storey 5 bay frame, shown in Figure 5.6, has been included to compare the *ad hoc* grouping method found in the literature to configurations computed by the algorithm. The structure is subject to both strength and deflection constraints. Members must satisfy the South African steel code, SANS 10162 (2005), using grade 350W steel. Interstorey drift is limited to 9mm. Standard AISC I, H and angle sections are chosen for the beams, columns and braces respectively (ASTM A6-81b, 2009).

The following members in the frame are grouped together (a) all the beams in three consecutive stories, and (b) symmetric columns over 3 stories. This grouping was used by Camp et al. (1995), while a similar grouping was performed by Chan (1992). This *ad hoc* method produces a structure with an initial, *assumed* grouping of 5 I-sections, 15 H-sections and 5 angles.

Optimising this *ad hoc* grouping using the VWO method produces a structure with a mass of 32,954kg. If grouping constraints are removed, except for symmetry, a structure of 30,371kg is obtained (Step 1, Section 5.5.2). This structure is then grouped to have the same number of I-sections (15), H-sections (5) and angles (5) as the *ad hoc* grouping. The grouping algorithm solution is 31,104kg. When the multi-step algorithm is used the structure's optimised mass is 31,745kg. The multi-step result is 2.1% heavier than the single step result. Results are summarized in Table 5.7.

Figure 5.7 shows the final, optimised section selection for the structure with *ad hoc* grouping. Beams with the same thickness and shade of grey have been grouped together. The thickness of the line is proportional to the mass per unit length of the member. Figure 5.8 shows the grouping calculated by the algorithm.

The grouping algorithm produces a 5.9% lighter solution than the structure with the *ad hoc* grouping. Comparing Figures 5.7 and 5.8 shows that the *ad hoc* and algorithm's groupings and mass distributions are different. The distribution of mass is sufficiently uniform to allow the algorithm grouped structure to be fabricated.

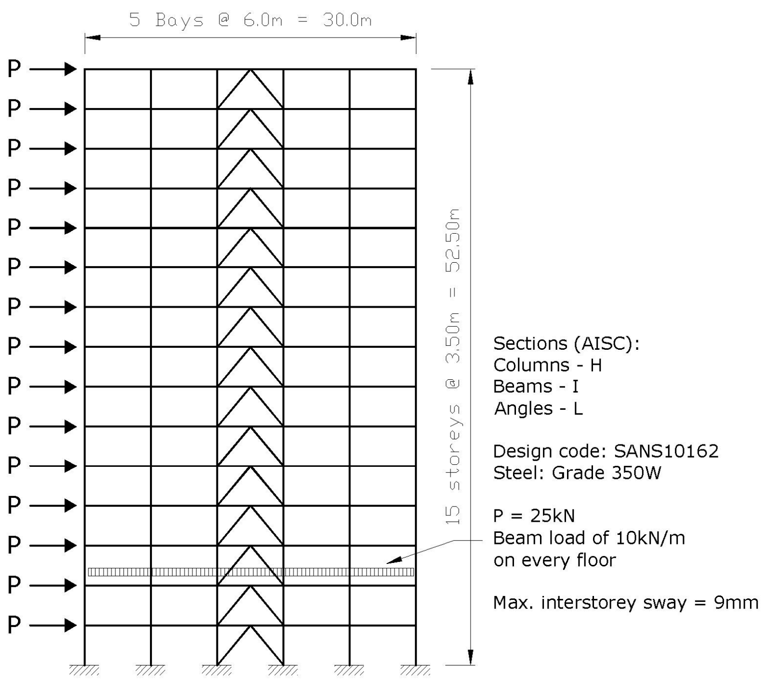
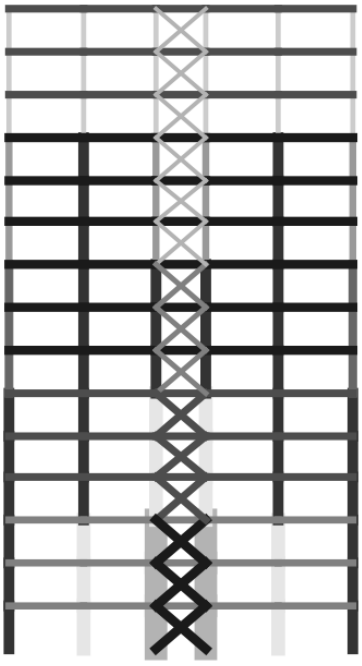


Figure 5.6: *20 storey 5 bay frame case study*

Table 5.7: *Results for the 15 storey frame*

Configuration	Final Mass (kg)	% Greater than ungrouped	Max no. of allowable groups: Beams	Columns	Braces
Ungrouped - symmetrical members the same	30371	-	∞	∞	∞
Single step grouping	31104	2.4	5	15	5
Multiple step grouping	31745	4.5	5	5	15
Ad hoc grouping across 3 floors	32954	8.5	5	15	5



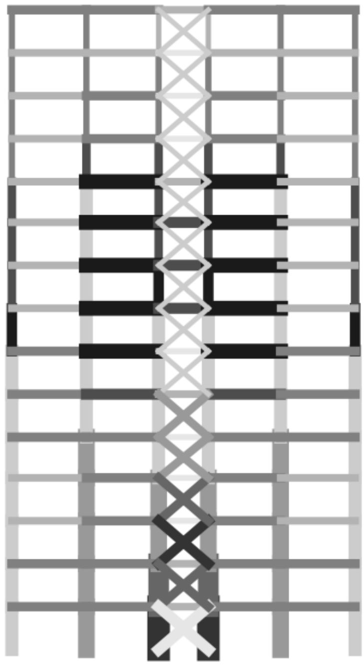


Figure 5.7: *Optimised 15 storey structure with groups across 3 floors (ad hoc grouping)*

Figure 5.8: *Optimised 15 storey frame with groups computed by the developed algorithm*

5.12.3 Truss

The truss shown in Figure 5.9 has to be designed to satisfy serviceability and ultimate limit state criteria. Groups have been defined (a) in two *ad hoc* ways, and (b) using the automated grouping algorithm. The maximum serviceability deflection is span/400 at the mid-span. Angles (BS4:Part 1, 1993) must be used for all members. Strength requirements must satisfy SANS 10162 (2005) using grade 350W steel.

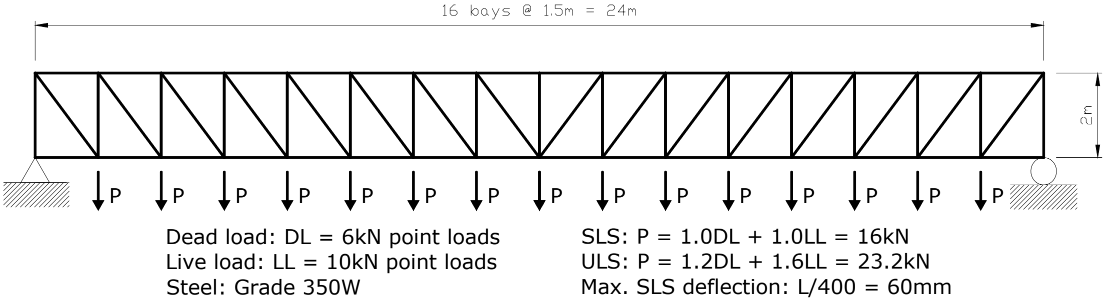


Figure 5.9: *Truss – geometry and loading*

The number of groups in the structure is limited to 4: 2 groups for the chords, and 2 groups for the vertical and diagonal members. Two *ad hoc* groupings are defined. *Ad hoc* grouping 1 consists of: setting the same member for the top and bottom chords in the middle 8 bays, a separate section for the outer 4 bays, the 4 verticals at each support are linked together, and the remaining members are grouped. *Ad hoc* grouping 2 consists of: the top chord, bottom chord, vertical members, and diagonal members each have a separate group. The optimised mass distributions of these grouping configurations are shown in Figures 5.10 and 5.11. The structures' final masses are 809.5kg and 788.2kg for *ad hoc* grouping 1 and 2 respectively.

The 4 group requirement specified above forms the input to the grouping algorithm. The ungrouped truss is optimised to create a structure of 660.2kg (Step 1, Section 5.5.2). The grouping calculated by the algorithm is shown in Figure 5.12. The optimised mass is 765.2kg (Step 4, Section 5.5.5). Table 5.8 summarizes the results obtained for the various grouping configurations.

The grouping algorithm produces a structure 5.8% lighter than *ad hoc* grouping 1, and 3% lighter than *ad hoc* grouping 2. Please note that the algorithm has stiffened the mid-span to limit deflections. Further, the algorithm has grouped the largest vertical sections in the end bays to resist the higher compressive forces found there.

Figure 5.10: *Ad hoc # 1 – mass distribution. Optimised mass: 809.5kg.*

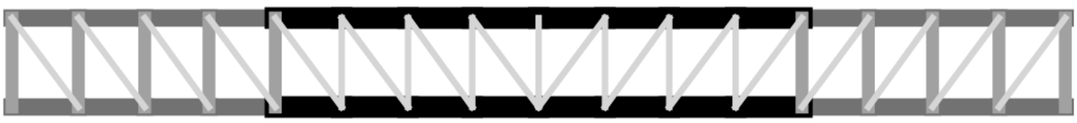


Figure 5.11: *Ad hoc # 2 – mass distribution. Optimised mass: 788.2 kg.*

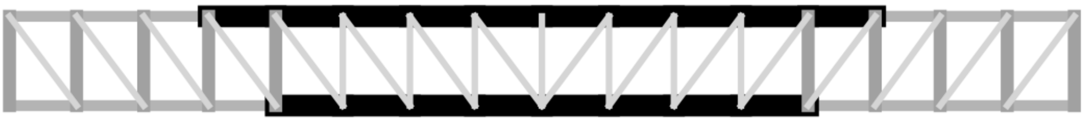


Figure 5.12: *Algorithm grouping. Optimised mass: 765.2kg.*

Table 5.8: *Results for the optimised the truss*

Configuration	Mass (kg)	% Greater than ungrouped
Ungrouped	660.2	-
Ad hoc grouping # 1	809.5	22.6
Ad hoc grouping # 2	788.2	19.4
Algorithm grouping	765.2	15.9

5.12.4 Warehouse

The warehouse shown in Figure 5.13 was designed by a South African company of professional engineers. The simplified loading is shown. Seven load combinations accounting for dead, live, crane and wind loads are considered. Fourteen deflection criteria are imposed. The structure is to consist of I, H, channel and angle section types (from BS4:Part 1 (1993) database). Sections are required to satisfy SANS 10162 (2005) strength requirements using grade 300W steel. Lateral buckling of latticed columns is taken into account.

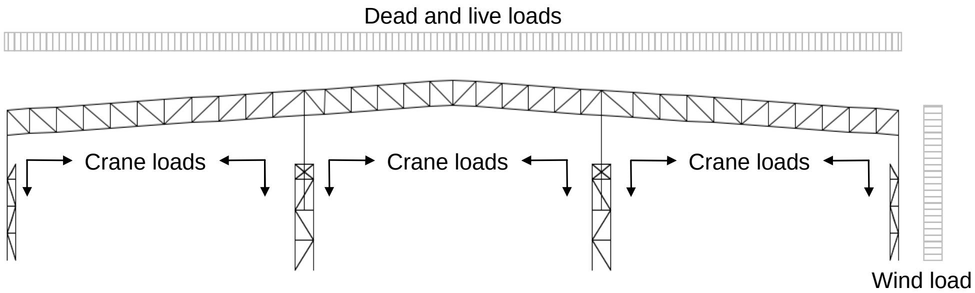


Figure 5.13: *Warehouse with dead, live, crane and wind loads*

Initially the engineers designed this structure to have 24 groups. Using their group configuration the VWO method produced a 3709.2kg structure. To decrease the number of sections further, the 24 groups were then placed into 17 groups by the engineers. The new optimised structure has a mass of 3777.5kg. These groupings were defined by the engineers based on experience, and are thus *ad hoc*.

The algorithm was applied to the structure with the 24 pre-selected groups in order to reduce the number to 17 groups. The mass calculated is 3759.5kg, or 0.5% lighter than the engineers' solution. When the ungrouped structure is optimised it has a mass of 3088.2kg (Step 1, Section 5.5.2). If 17 groups are now produced, the algorithm calculates a 3605.1kg structure, which is 4.6% lighter than the engineers' final design. This shows that the algorithm's solution is dependent on the starting configuration, i.e. starting with an ungrouped versus a pre-grouped structure. The results are summarized in Table 5.9.

Table 5.9: *Results for the warehouse*

Configuration	Optimised Mass (kg)	% Saving	Max. no. of sections
Engineers – Final 17 groups	3777.5	-	17
Engineers – Initial 24 groups	3709.2	1.8	24
Ungrouped	3088.2	18.2	∞
Algorithm – 17 new groups from the engineers' 24 sections	3759.5	0.5	17
Algorithm – 17 new groups from the ungrouped configuration	3605.1	4.6	17

Figure 5.14 shows the mass distribution in the warehouse with the 17 groups defined by the engineers. Figure 5.15 shows the warehouse with 17 groups computed by the algorithm, starting from the ungrouped configuration. It is interesting to note that the algorithm has optimised the lattice columns by stiffening their lower portions. It has also grouped the chords of the roof trusses at mid-span.

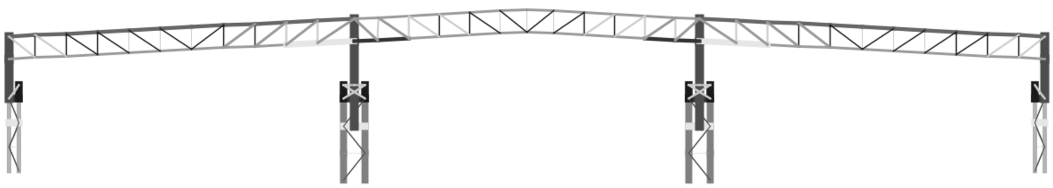


Figure 5.14: *Warehouse with final grouping specified by the professional structural engineers*

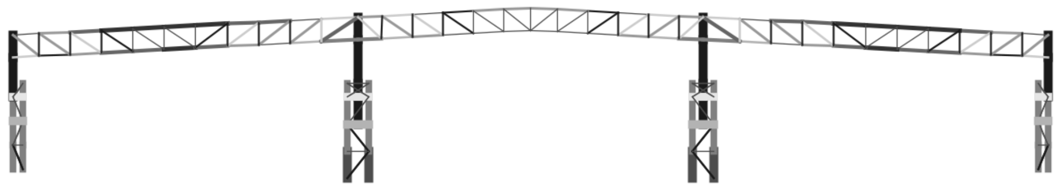


Figure 5.15: *Warehouse with final grouping computed by the algorithm*

5.13 Conclusion

Structural grouping is a complex task where solutions change with every perturbation in the system. This chapter presented an automated algorithm for optimising the grouping of discrete structural members. The algorithm groups members based on their mass per unit length. An exhaustive search of grouping permutations is carried out and the grouping which produces the lightest structure is selected. Any two-dimensional structure in which members carry axial and/or bending forces can be analyzed. Multiple load cases can be considered. The algorithm's solution is a grouped structure optimised for weight, which satisfies multiple strength and deflection requirements.

The algorithm is computationally inexpensive. Although the number of permutations can be large, for each trial grouping only the structure's mass is estimated, the structure's behaviour is not solved. If the search space is required to be reduced, as might happen for extremely large structures with members selected from a large database, two methods were proposed: (a) creating subgroups, and (b) only investigating permutations within a radius.

Four cases studies were investigated to compare the algorithm to *ad hoc* grouping configurations. In all cases lighter structures were computed by the algorithm. As expected, the algorithm solution is affected by the starting amount of pre-grouping.

The following topics require further research. The uniqueness of the solution obtained must be investigated. A multi-step algorithm should be developed further, and compared to the single step method presented. The effects of using different section databases for the initial and final optimisation steps should be characterized.

The algorithm's framework is suitable for three-dimensional structures, but this needs to be implemented and the performance investigated.

5.14 References

ASTM A6-81b. (2009). Specifications from *ASTM A6 – Standard Specification for General Requirements for Rolled Structural Steel Bars, Plates, Shapes and Steel Piling*. American Society for Testing and Materials.

Barbosa, H.J.C. and Lemonge, A.C.C. (2005) A Genetic Algorithm Encoding for a Class of Cardinality Constraints. *GECCO*, June, 25-29.

Barbosa, H.J.C., Lemonge, A.C.C., and Borges, C.C.H. (2008) A genetic algorithm encoding for cardinality constraints and automatic variable linking in structural optimisation. *Engineering Stuctures*, 30, 3708-3723.

Barthelemy, J.F.M, and Haftka, R.T. (1993) Approximation concepts for optimal structural design – a review. *Structural Optimisation*, 5, 129-144.

Biedermann, J.D., and Grierson, D.E. (1995) A Generic Model for Building Design. *Engineering with Computers*, 11, 173-184.

BS4:Part 1 (1993). *Structural steel sections. Specification for hot-rolled sections*. British Standard.

Camp, C V, Bichon, B J and Stovall, S P. (2005) Design of Steel Frames Using Ant Colony Optimisation. *J. Struct. Div*, ASCE, 131(3), 369-379

Chan, C M. (1992) An optimality criteria algorithm for tall steel building design using commercial standard sections. *Structural Optimisation*, 5, 26-29.

Provatidis, C.G., and Venetsanos, D.T. (2006) Cost minimization of 2D continuum structures under stress constraints by increasing commonality in their skeletal equivalents. *Forsch Ingenieurwes*, 70, 159-169.

SANS 10162-1(2005), The Structural use of steel. Part 1: Limit-states design of hot-rolled steelwork. South African National Standard.

Shea, K., Cagan, J., and Fenves, S.J. (1997) A Shape Annealing Approach to Optimal Truss Design With Dynamic Grouping of Members. *Journal of Mechanical Design*, ASME, September, 119, 388-394.

Thanedar, P.B. and Vanderplaats, G.N. (1995) Survey of discrete structural optimisation for structural design. *J. Struct. Eng., ASCE.* 120(2), 301-306.

Toğan, V, and Doloğlu, A. (2006) "Optimisation of 3D trusses with adaptive approach in genetic algorithms". *Engineering Structures*, 28, 1019-1027.

Toğan, V, and Doloğlu, A. (2008) "An improved genetic algorithm with initial population strategy and self-adaptive member groupings." *Computers and Structures.*86, 1204-1218.

CHAPTER 6: CONCLUSIONS

This dissertation has presented the Virtual Work Optimisation (VWO) algorithm for the optimisation of structures, which can be automated. The method selects sections for structures with fixed geometries. Strength and deflection criteria are satisfied through an iterative process. A parametric investigation of ungrouped, multi-storey frames was conducted using the VWO method to research optimal mass distributions. The grouping algorithm developed links members in ungrouped structures by determining efficient grouping configurations. Figure 6.1 summarises the layout and interaction of the chapters in this dissertation.

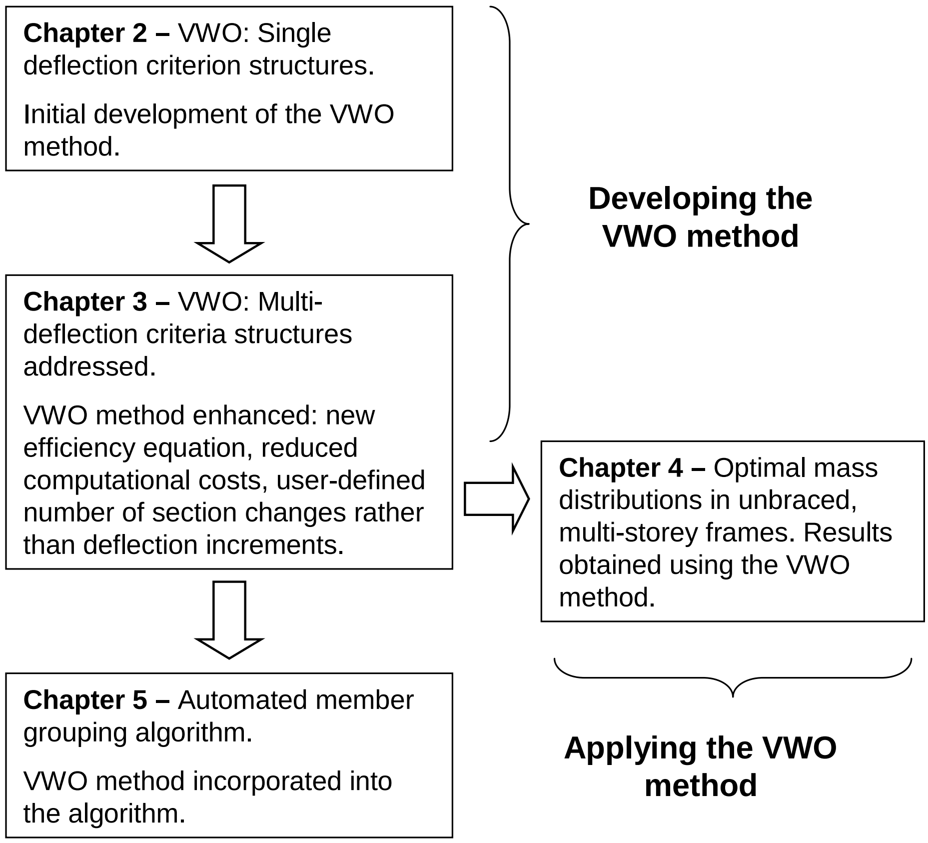


Figure 6.1: *Flow diagram of the development and application of the VWO method in this dissertation*

6.1 Initial development of the Virtual Work Optimisation Method

The primary development and implementation of the VWO method was presented in Chapter 2. The method selects sections which provide the highest deflection and strength resistance per unit mass. In this chapter the method only addressed structures with a single deflection criterion and one load case. Deflections were reduced by a user-defined increment, and a variable number of section changes were made in each iteration. Any database of discrete sections, design code or material can be considered by the method. Previous structural optimisation shortcomings such as only considering trusses are overcome. Savings of up to 15.1% were realised using the method, in comparison to methods in the literature.

6.2 The VWO method for multi-deflection criteria structures

In Chapter 3 the VWO method was expanded to address structures with multiple deflection criteria and load cases. A new efficiency equation was proposed and implemented. A user-defined number of section changes was made in each iteration, rather than reducing deflections by a specified amount. Computational costs were reduced by performing structural analyses only when calculating deflections, and not when checking strength criteria as well (as done in Chapter 2). Any number of strength and deflection criteria can be considered. Structures up to 14.4% lighter than those presented in the literature were computed by the method.

6.3 Applications of the VWO method – Mass distributions in ungrouped frames

The parametric investigation into the stiffness and mass distribution in ungrouped, multi-storey frames (Chapter 4) has demonstrated how mass should be configured to resist lateral loads efficiently. It has been found that measured from the top, the total storey mass and stiffness increase approximately linearly with decreasing height of the structure. In some cases this is followed by a region of lower increase or constant mass and stiffness. Distinct patterns were consistently seen in all structures tested. The mass distributes in diagonal paths across the breadth of structures, which seems to imitate truss behaviour.

6.4 Optimisation of member groupings

The grouping algorithm presented in Chapter 5 improves designs by optimally linking members. The algorithm first optimises ungrouped structures using the VWO method. Groups are then created by selecting the most optimal permutation obtained using an exhaustive search. Structures grouped by the algorithm were found to be up to 5.9% lighter than structures grouped using standard configurations presented in the literature. These configurations are based on experience and logic and considered *ad hoc*. Please note that only a small number of grouping algorithms have been found in the literature.

The grouping algorithm provides a way for efficiently grouping members in structures, independent of the engineer's experience. The method can be applied to any grouped or ungrouped structure and is generally not computationally expensive. However, if computational costs do become a problem they can be reduced by two methods. Subgroups can be created or permutations only within a viable 'radius' can be considered.

6.5 Limitations of the research

The optimisation method presented is limited in that it can only address structures with fixed geometries and loading. Topology optimisation has not been considered.

As with all other optimisation methods it is unknown whether results produced are local or global minima. This cannot easily be ascertained, because exhaustive searches are not feasible.

Only two-dimensional structures have been considered. However, the algorithms presented can be extended to three-dimensional structures.

Composite structures are not investigated in this dissertation. Only steel structures were optimised, even though the theory underlying the algorithms are applicable to all materials. The VWO method cannot address structures in which multiple materials are used simultaneously.

6.6 Future research

The VWO method and grouping algorithm can be developed further and improved in the following ways.

The VWO method has user-defined parameters, such as the number of changes made per iteration, which need to be investigated. These parameters affect solutions and computational costs, and so should be correctly defined. The efficiency equation should be tested to determine if there is any way in which it can be improved.

The developed algorithms have focussed on minimising structural masses. However, it is an over-simplification to assume a structure of minimum mass will be the most economic. Not all members have the same cost per kilogram. Factors such as fabrication and construction impact the final costs. Future research should focus on minimising overall costs.

The VWO method must be upgraded to address three-dimensional structures. Torsion and biaxial bending will need to be considered in the efficiency equation. No foreseeable changes will need to be made to the grouping algorithm.

In very large structures the grouping algorithm and VWO method may require substantial computational time. It should be investigated how the VWO method and grouping algorithm can be streamlined to converge to solutions in fewer iterations.

It appears that unusual distributions of mass are found in optimised, multi-storey frames with no grouping of members. These results need to be verified using other optimisation methods. Bracing or shear wall systems could be developed based on the distributions observed.

Solutions obtained are dependent on the section database used. If sections are uneconomical to satisfy design criteria then solutions are not optimal. It should be researched if and how databases can be altered to efficiently satisfy design criteria.

Only steel has been considered in the case studies presented. Other materials such as concrete and wood should be included in the method. Structures in which multiple materials are present should also be addressed. This may necessitate a change in the efficiency equation proposed in the VWO method.

Linear elastic analyses have been performed exclusively in this research. Non-linear behavior, both geometric and material, has to be considered in the future.