UNIVERSITY OF THE WITWATERSRAND

Channel Assembling and Resource Allocation in Multichannel Spectrum Sharing Wireless Networks

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2017

CHANNEL ASSEMBLING AND RESOURCE ALLOCATION IN MULTICHANNEL SPECTRUM SHARING WIRELESS NETWORKS

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Submitted in fulfilment of the academic requirements for the degree of Doctor of Philosophy (Ph.D.) in Engineering, in the School of Electrical and Information Engineering, Faculty of Engineering and the Built Environment, at the University of the Witwatersrand, Johannesburg, SOUTH AFRICA.

JUNE 2017

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DECLARATION

I, <u>CHABALALA STEPHEN CHABALALA</u> declare that

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DEDICATION

To my family and friends

PREFACE

THIS thesis presents my Ph.D. research study on dynamic resource allocation and channel assembling techniques in spectrum sharing wireless networks. The work has been done under the joint supervision of Professor Fambirai Takawira and Professor Rex van Olst, in the School of Electrical and Information Engineering, at the University of the Witwatersrand (Wits University), Johannesburg, South Africa. This work was supported in part, by the Center for Telecommunications Access and Services (CeTAS) at Wits University.

In a nutshell, this work is focused on the development of dynamic and adaptive resource allocation and management techniques for spectrum sharing and the emerging wireless networks. Accordingly, this research has resulted in the new concepts and techniques on how to perform effective spectrum characterization with respect to time varying nature of wireless channels and the activity patterns of licensed users; how to perform power control and interference management techniques for efficient use of the limited radio spectrum resource; as well as establishing mathematical models for performance analysis over fading wireless channels.

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ACKNOWLEDGEMENTS

THE work presented in this thesis would not have been accomplished if it were not because of the generous support and the assistance I received from many people, who throughout the challenging years of my Ph.D. study, have been kind. To all, I am very grateful.

First of all, I would like to express my sincere gratitude and appreciation to my principal supervisor, Professor Fambirai Takawira, who has always been the anchor, and offered me the incredible support throughout the entire ordeal. Without the requisite assistance I received from him, the work presented herein would not have seen the light of the day. I would also like to thank my co-supervisor, Professor Rex van Olst for his insightful guidance and support, which I received even beyond his retirement during my research study.

I would like to extend my gratitude to the entire academic staff, the administrative staff and the technical staff members in the School of Electrical and Information Engineering (EIE) for all their support and assistance. Many thanks to my fellows in the Center for Telecommunications Access and Services (CeTAS) research group for the technical discussions and great collaboration.

I would also like to thank my proposal defence committee, for the initial approval of my Ph.D. research study. In particular, thanks to Dr. Olutayo Oyerinde for organising my proposal defence presentation, and Prof. Ekow Otoo for consolidating feedback and comments from the review panel. To Prof. Ivan Hofsajer, the 'MVP' (*i.e.* minimum viable project) approach has been helpful, thank you for your advice. To Jacques Naude, thank you for the insightful discussions on probability theory. Many thanks to Prof. Estelle Trengove for her recommendation, encouragement and assistance in teaching whilst conducting my research study. Great appreciation also goes to the anonymous examiners for their meticulous review and insightful comments.

Special thanks to my wonderful wife 'Matabane Chabalala, and amazing son, Tabane Chabalala, for their outstanding commitment to make this Ph.D. study a success. My heartfelt gratitude goes to my father who could not see this thesis completed, Ntate Matšerane Chabalala, and my caring mother, 'M`e 'Mankalimeng Chabalala, for her ongoing love and endless support. To my siblings, Thato, 'Mantho and Refiloe, I thank you all for your great support. *Kea leboha*.

ABSTRACT

THE continuous evolution of wireless communications technologies has increasingly imposed a burden on the use of radio spectrum. Due to the proliferation of new wireless networks applications and services, the radio spectrum is getting saturated and becoming a limited resource. To a large extent, spectrum scarcity may be a result of deficient spectrum allocation and management policies, rather than of the physical shortage of radio frequencies. The conventional static spectrum allocation has been found to be ineffective, leading to overcrowding and inefficient use. Cognitive radio (CR) has therefore emerged as an enabling technology that facilitates dynamic spectrum access (DSA), with a great potential to address the issue of spectrum scarcity and inefficient use. However, provisioning of reliable and robust communication with seamless operation in cognitive radio networks (CRNs) is a challenging task. The underlying challenges include development of non-intrusive dynamic resource allocation (DRA) and optimization techniques.

The main focus of this thesis is development of adaptive channel assembling (ChA) and DRA schemes, with the aim to maximize performance of secondary user (SU) nodes in CRNs, without degrading performance of primary user (PU) nodes in a primary network (PN). The key objectives are therefore four-fold. Firstly, to optimize ChA and DRA schemes in overlay CRNs. Secondly, to develop analytical models for quantifying performance of ChA schemes over fading channels in overlay CRNs. Thirdly, to extend the overlay ChA schemes into hybrid overlay and underlay architectures, subject to power control and interference mitigation; and finally, to extend the adaptive ChA and DRA schemes for multiuser multichannel access CRNs.

Performance analysis and evaluation of the developed ChA and DRA is presented, mainly through extensive simulations and analytical models. Further, the cross validation has been performed between simulations and analytical results to confirm the accuracy and preciseness of the novel analytical models developed in this thesis. In general, the presented results demonstrate improved performance of SU nodes in terms of capacity, collision probability, outage probability and forced termination probability when employing the adaptive ChA and DRA in CRNs.

Index-Terms—Channel assembling (ChA); cognitive radio networks (CRNs); Dynamic spectrum access (DSA); multiuser multichannel access; radio resource management (RRM).

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LIST OF ACRONYMS

AWGN:	Additive White Gaussian Noise
BnB:	Branch-and-Bound
BnBKOP:	Branch-and-Bound-K Optimal Power
ChA:	Channel Assembling
CDF:	Cumulative Distribution Function
CR:	Cognitive Radio
CRN:	Cognitive Radio Network
CSA:	Concurrent Spectrum Access
CSI:	Channel Status Information
CTMC:	Continuous Time Markov Chain
DRA:	Dynamic Resource Allocation
DSA:	Dynamic Spectrum Access
FCC:	Federal Communications Commission
FixedKEP:	Fixed-K Equal Power
FixedKOP:	Fixed-K Optimal Power
HunKOP:	Hungarian-K Optimal Power
IEEE:	Institute of Electrical and Electronics Engineering
IET:	Institute of Engineering and Technology
i.i.d.:	Independent and Identically Distributed
i.n.i.d:	Independent Non-Identically Distributed
KKT:	Karush-Kuhn-Tucker
LB:	Lower Bound
MAC:	Medium Access Control

MATLAB:	Matrix Laboratory
OFDM:	Orthogonal Frequency Division Multiplexing
OSA:	Opportunistic Spectrum Access
PDF:	Probability Distribution Function
PGF:	Probability Generating Function
PN:	Primary Network
POMDP:	Partially Observable Markov Decision Process
PU:	Primary User
QoS:	Quality of Service
RndKOP:	Rounded-K Optimal Power
RRM:	Radio Resource Management
SF:	Sequential-Fixing
SNIR:	Signal-noise and Interference Ratio
SNR:	Signal-to-Noise Ratio
SN:	Secondary Network
SU:	Secondary User
UB:	Upper Bound
URC:	Ultra-Reliable Communication

LIST OF SYMBOLS

$\mathcal{B}$	Bandwidth
$\boldsymbol{\mathcal{S}}$	Set of total PU channels
$\boldsymbol{\mathcal{N}}$	Set of occupied PU channels
N	Number of busy PU channels
$\boldsymbol{\mathcal{M}}$	Set of free PU channels
M	Number of free PU channels
$\boldsymbol{h}$	Channel power gain vector
$\mathcal{K}$	Number of assembled channels
$\boldsymbol{x}$	Channel selection vector
$\boldsymbol{p}$	Power vector
$\boldsymbol{\mathcal{J}}$	Set of SU nodes
$\mathcal{J}$	Number of SU nodes in $\boldsymbol{\mathcal{J}}$
$r_i(x_i, p_i)$	Capacity over channel i
$\mathcal{N}_0$	Gaussian background noise
γ_i	SNR on channel i
$\mathbb{E}\{\cdot\}$	Expectation operator
$\mathcal{U}(\cdot)$	Unit-step function
$\mathcal{R}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p})$	Sum capacity of assembled channels
Δ_f	Number of function evaluations
Δ_k	Number of iterations
k	Iteration index
$\boldsymbol{\mathcal{A}}$	Set of assembled channels
Δt	Time interval
$\mathcal{P}_{\mathcal{A}}(\cdot)$	Probability of no collision

T_{Tx}	Transmission time
L_{Data}	Data length
λ_{pu}	PU arrival rate
$\mathcal{P}_{\phi}$	Collision probability threshold
β	Collision probability characterization
$\mathcal{R}_{\phi}$	Minimum capacity threshold
$P_{\max}$	Maximum available transmit power
$\max\{a, b\}$	Maximum of a and b
$\mathbb{M}_{s}\{\cdot\}$	Mellin transform
$\Gamma(s, \alpha)$	Complementary incomplete gamma function
$\mathcal{H}_{c,d}^{a,b}[\cdot]$	Fox-$\mathcal{H}$ function
$\mathcal{W}(\cdot)$	Lambert-$\mathcal{W}$ function
$G_{c,d}^{a,b}[\cdot]$	Meijer-G function
$\mathbb{Pr}\{\cdot\}$	Probability operator
$T_{\lambda,\mathrm{pu}}$	PU arrival time
$\mathcal{G}_{n}(\cdot)$	Probability generating function
P_{pu}	PU transmit power
σ_{pu}^{2}	PU noise variance
σ_{su}^{2}	SU noise variance
γ_{pu}	SNR at the PU receiver
γ_{th}	SNR threshold
$p_{\max,n}$	SU maximum allowable transmit power
ε_{th}	Outage probability threshold
δ_{f}	Newton function step-size

CHAPTER ONE

1 INTRODUCTION

1.1 General Background

THE rapid advances in both hardware and software technologies fostered the proliferation of wireless networks devices, as well as fueling the exponential growth of new applications and services [1]-[2]. The next-generation wireless networks and the emerging technologies are envisioned to provide reliable and robust communication with seamless operation. Further, the emergence of bandwidth-hungry network applications and services with varying quality-of-service (QoS) requirements has further resulted in explosive demands for ubiquitous high-speed wireless networks, which underline the need for new communication techniques that are capable of transporting large amounts of data with varying QoS requirements [1], [3]-[4].

In recent research efforts, accessibility and reliability of communication services have been established as crucial aspects for future wireless networks [5]-[6]. However, the proliferation of wireless networks devices poses significant challenges that lead to scarcity of radio spectrum, together with inefficient utilization due to the traditional static spectrum management policies. To a large extent, spectrum scarcity is due to deficient spectrum allocation and management policies, rather than of the physical shortage of radio frequencies [2], [5] [7].

Efficient radio resource management (RRM) techniques are mandatory for provisioning of robust and ultra-reliable communication (URC). In essence, RRM techniques are required to maintain high system utilization while satisfying users QoS requirements [8]-[11]. URC is one of the challenging concepts for which the emerging wireless networks are envisaged to address. In principle, URC concept refers to the provisioning of communication services at the certain QoS level with a high degree of dependability, whereby dependability is attributed to high service availability and reliability [7]-[8], [11]. Consequently, the main focus of this research is to study and quantify performance of, as well as developing new concepts and techniques for adaptive channel assembling (ChA) and resource allocation schemes in spectrum sharing wireless networks.

1.1.1 Cognitive Radio Networks

Cognitive radio (CR) constitutes an inventive technology that enables intelligent operation in wireless networks, whereby a CR node can adapt its operating parameters based on environmental conditions and network constraints as shown in Fig. 1-1 [11]-[12]. The two main components that are significant for successful operation of cognitive radio networks (CRNs) are spectrum sensing, which detects spectrum holes in a primary network (PN); and spectrum allocation, which deals with how to allocate spectrum holes to secondary user (SU) nodes [1], [7]. Spectrum holes are the unoccupied bands of frequencies that are assigned to primary user (PU) nodes.

Through CR concept, dynamic spectrum access (DSA) allows seamless coexistence of PU nodes and SU nodes sharing radio spectrum. The coexistence offers a great potential to improve spectral efficiency and network performance [9]. Nonetheless, provisioning of reliable communication for both PU and SU nodes is a challenging task, as PU nodes have preemptive priority over SU nodes for spectrum access; wherefore the key issue is to ensure that SU nodes do not degrade performance of PU nodes. In general, DSA can be categorized into: *overlay*, *underlay* and *hybrid* modes [1], [14]. In overlay, SU nodes select free PU channels and instantly refrain from further transmissions on detecting PU arrival on any of the occupied channels. In underlay, SU nodes transmit through channels that are occupied by PU nodes, but at lower power levels for which the resulting co-channel interference is below tolerable limits as seen by PU nodes [14]-[15]. Hybrid mode merges the merits of overlay and underlay modes by adopting both for SU transmissions.

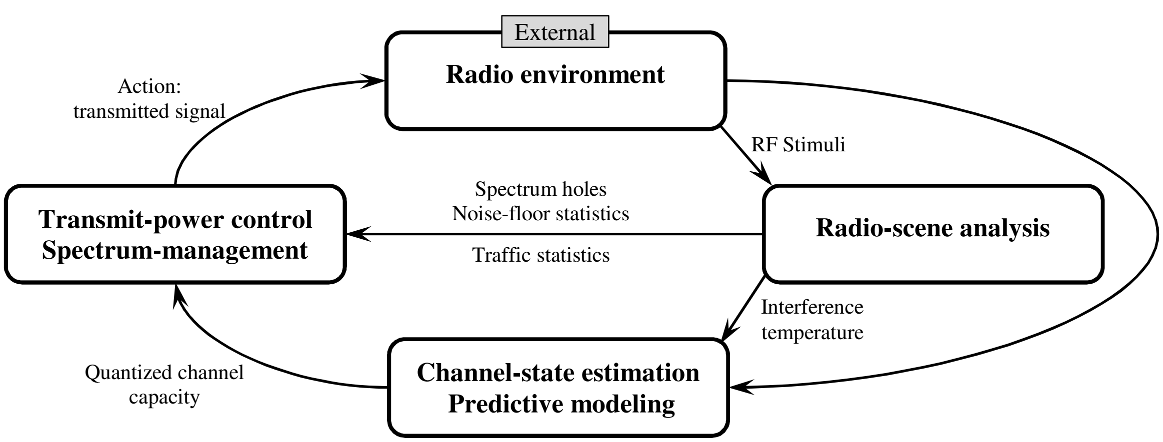

Figure 1-1: Basic cognitive radio functions and operational cycle [13].

1.1.2 Radio Resource Management

Radio resource management (RRM) techniques mainly deal with maximizing radio resource utilization and efficiency in wireless networks, while guaranteeing the varying QoS requirements for different users. In the case of CRNs, the issue of spectrum agility complicates RRM, making it even more challenging. Information from both the PN and CRN is required for efficient spectrum allocation in the CRN, as shown in Fig. 1-2. Thus, efficient spectrum management and resource allocation are some of the cornerstones on which performance of SU nodes can be enhanced, while protecting PU nodes at the same time. The commonly employed methods for resource allocation and optimization techniques are based on (*i*) *heuristic methods*, (*ii*) *game theory*, (*iii*) *graph theory*, (*iv*) *convex optimization* and (*v*) *stochastic modelling* to mention a few [1], [7].

1.1.2.1 Heuristic Methods

Heuristic based resource allocation techniques in wireless networks provide relatively easy solutions that can be computed within acceptable time and complexity [7]. Although heuristic methods are mostly useful when dealing with optimization problems that are very hard to solve, they usually provide suboptimal solutions without any guarantee for convergence and optimality. Furthermore, heuristic methods have generally been found to be more suitable for obtaining quick solutions when employed CRNs, whereby resource allocation problems are otherwise mainly characterized by challenging optimization problems with high complexity [16].

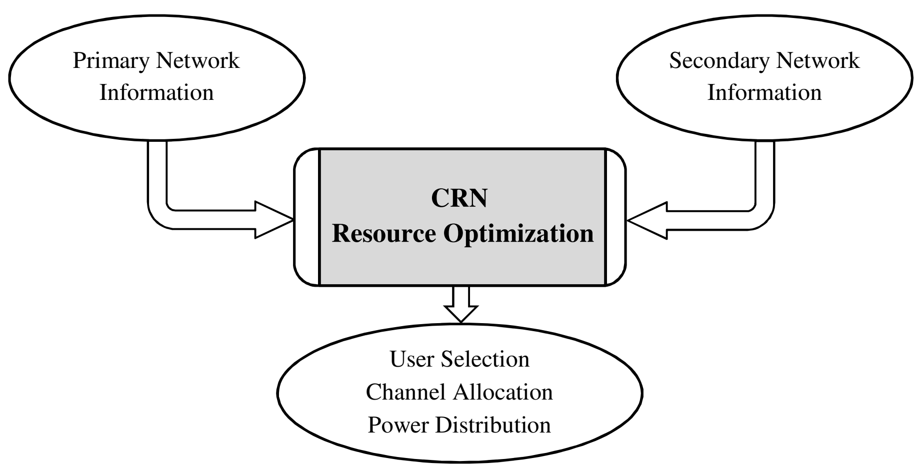

Figure 1-2: Resource allocation and optimization process in CRNs.

1.1.2.2 Game Theory

In general, game theory provides a mathematical framework to model interaction of multiple entities (*i.e.* players) in a competition (*i.e.* game), whose decisions affect one another [3], [17]. The players aim to maximize their utility on each decision taken. Game theory can be categorized into cooperative and non-cooperative models. In cooperative models, players collaborate to exchange information and make decisions that improve the overall network utility, in which case the solution point is commonly known as Nash bargaining [18]. In the case of non-cooperative models, players make selfish decisions to maximize their individual utility irrespective of how their decisions affect other players, and the common solution point is referred to as Nash equilibrium [19].

1.1.2.3 Graph Theory

Graph theory is one of the techniques that have been employed to solve scheduling and resource optimization problems in wireless networks, whereby a graph $G(V,E)$ represents an optimization model with vertices V representing network entities, and the edges E for their interactions. In particular, there are various types of graph modelling for resource allocation problems, such as bipartite graph, vertex coloring and conflict graph, as discussed in detail in [20]. Moreover, the types of graphs can further be classified as directed (*i.e.* whereby the edges have directions) or undirected, and weighted (*i.e.* whereby each edge is assigned a nonnegative weight) or unweighted [21]. The graph based models have mainly been employed in infrastructure based networks, with a central controller for gathering network information and performing resource allocation.

1.1.2.4 Machine Learning Algorithms

Machine learning concepts such as neural networks, fuzzy logic and genetic algorithms are not uncommon for RRM and spectrum allocation in CRNs. Neural networks have been found to be more suitable for highly dynamic environments with frequent radio spectrum changes, to which SU nodes are required to promptly respond [22]. Fuzzy logic is based on human understandable fuzzy sets and inference rules to obtain resource allocation solutions. Fuzzy logic based techniques are also relatively simple as they are not based on complicated mathematical models [23]-[24]. These are also appropriate for real-time CRN applications with stringent constraints on system response time. Genetic algorithms are based on evolutionary biological processes to determine optimal solutions for complex problems, but with fast convergence and ease of implementation [25].

1.1.2.5 Convex Optimization

Resource allocation problems are formulated as optimization models with objective functions for which an optimal solution can be obtained among all the feasible solutions. In the case of CRNs, resource allocation problems are usually formulated as constrained optimization problems that take resource constraints into consideration [1]-[3], [7], [9], [11]. The optimization problems can further be classified into *linear*, *non-linear*, *convex*, *non-convex integer* or *mixed-integer non-linear programming* (MINLP) problems [26]. Different optimization tools can be employed to determine optimal solutions based on the structure and nature of the optimization problems. For example, Lagrangian framework with dual decomposition can be employed to solve convex optimization problems, while linear programming techniques can be employed to solve linear optimization problems where the objective function and the associated constraints are all linear [27].

1.1.3 Channel Assembling in Wireless Networks

Channel assembling (ChA) constitutes one of the emerging spectrum access techniques with a great potential to support the envisaged heterogeneity in future wireless networks. This is a technique through which a user performs concurrent transmissions through multiple channels to maximize capacity and improve spectral efficiency [27]-[29]. In general, ChA techniques can be classified into *static* and *dynamic* schemes. A predetermined fixed number of channels is employed in the case of static schemes, whereas the number of assembled channels varies in the case of dynamic ChA schemes; hence, the number of assembled channels can be adjusted based on QoS requirements for different applications and network constraints. In this thesis, ChA refers to both spectrum bonding and aggregation, wherefore the arrangement of assembled channels with respect to one another in frequency domain is insignificant. In a strict sense however, a set of contiguous non-overlapping channels is required for channel bonding, as illustrated in Fig. 1-3; while channel aggregation does not necessarily require the assembled channels to be contiguous [29]-[30].

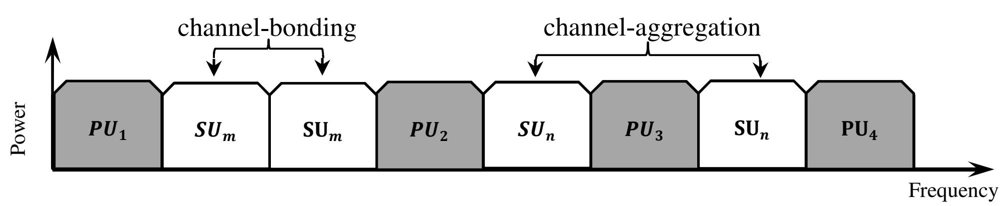

Figure 1-3: Illustration of channel-bonding and channel-aggregation.

Notwithstanding the disregard of the arrangement of assembled channels in this thesis, it has been established in the previous studies that channel aggregation is relatively subject to higher complexity and overhead than channel bonding, as a result of the need for channel management and scheduling policies across aggregated channels [29], [31]-[32]. In the case of CRNs, PU nodes have exclusive rights to access radio spectrum. Thus, increasing the number of assembled channels for SU nodes increases the probability that a PU node may appear on any of the selected channels, which therefore increases collision probability between PU services and SU services [27]-[28], [31], [33]. In general, maximizing capacity for SU services through ChA schemes is a challenging task that conflicts with the issue of spectrum agility with respect to the PU nodes.

1.2 Problem Statement and Research Objectives

1.2.1 Problem Statement

The ubiquity of wireless devices and the proliferation of network applications with stringent QoS requirements are in contrast to the issue of the limited radio spectrum, in which case the conventional fixed spectrum management and allocation techniques have been found to be a bottleneck for high spectral efficiency and utilization by the Federal Communications Commission (FCC) [3]. The concept of CR is continuously evolving, but fulfilling QoS requirements for SU nodes in CRNs remains is a critical concern, that is also extremely challenging due to spectrum agility whereby preemptive priority is granted to licensed users. The applicability of CRNs in addressing the increasing demands for high speed wireless networks therefore requires meticulous and insightful design considerations for development of efficient RRM techniques and DRA schemes. Efficient RRM techniques are imperative for reliable and robust communication.

The central issue in this thesis is therefore development of efficient resource allocation and ChA schemes for CRNs. In particular, this focuses on how to perform adaptive resource allocation and ChA taking into account PU activity patterns, fading wireless channels, interference constraints to protect PU transmissions, QoS requirements for SU services, network dynamics and resource constraints. This compels development of appropriate spectrum characterization techniques for selection of channels that offer optimal performance for CRNs, as well as reducing susceptibility to performance degradation due to PU activity patterns and network dynamics.

1.2.2 Primary Objectives

The primary aims and objectives of the research presented in this thesis are:

(a) To study and identify the challenges associated with efficient RRM in CRNs, and investigate the intrinsic factors that affect performance of ChA schemes and RA techniques.

(b) To develop adaptive ChA and RA schemes that determine the optimal number of channels and power distribution for overlay CRNs, subject to power constraint, QoS requirement on capacity, PU activities and collision probability threshold over fading channels.

(c) To develop statistical characterization of assembled channels in fading channels, and derive compact closed-form analytical models to quantify and gauge performance of ChA and RA schemes on average capacity, outage probability and forced termination probability.

(d) To develop criteria on how to perform adaptive ChA and RA for hybrid overlay and underlay CRNs architectures, with respect to PU spectrum occupancy and interference constraints imposed on SU transmissions to protect ongoing PU services.

(e) To develop adaptive and dynamic ChA and RA schemes for multiuser multichannel access in CRNs, with the aim to maximize total network capacity, subject to time varying wireless channels, individual users QoS requirements, PU activities and resource constraints.

In summary, this research is mainly focused on the development of new techniques for adaptive ChA and RA schemes in CRNs, with the aim to improve performance and provide reassurance for satisfying QoS requirements for SU services, without degrading performance of PU nodes.

1.3 Original Contributions and Thesis Organization

Unlike the presented work in this thesis, the main research studies on RA techniques based on the Lagrangian framework do not incorporate PU activity patterns as a constraint in formulating optimization problems. In the case of ChA schemes, the key studies in literature are mainly based on continuous time Markov chain (CTMC) modeling with a predetermined and fixed number of channels to assemble. Moreover, the issue of transmit power optimization has been generally overlooked in CTMC based studies. In this regard, the main contributions of this work are derived from the aforementioned primary aims and objectives, accordingly outlined as follows:

(a) A criterion for determining the optimal channel selection, together with the associated power distribution for ChA schemes in overlay CRNs has been developed. This takes into account, the time varying and fading nature of wireless channels, PU nodes activity patterns, QoS requirements on capacity for SU transmissions, and the constraint on SU nodes total transmission power to make judicious ChA decisions.

(b) The statistical characterization of assembled channels in terms of probability distribution function (PDF) and cumulative distribution function (CDF) has been derived. This characterization provides the foundation on which closed-form mathematical models are derived towards theoretical performance analysis and evaluation of ChA schemes.

(c) Closed-form analytical models have been developed to quantify and evaluate performance of ChA schemes in terms of the average capacity, outage probability and forced termination probability for SU transmissions. Further, the correctness and preciseness of the analytical models have been confirmed through cross-validation with extensive simulations.

(d) Adaptive hybrid overlay and underlay ChA scheme that aims to maximize SU capacity in spectrum sharing wireless networks has been developed. The developed scheme determines the optimal number of channels to assemble in both overlay mode and underlay for SU transmissions. Multilevel maximum allowable SU transmit power in underlay mode has been derived to ensure that SU transmit power is constrained under the noise floor of PU nodes.

(e) As an extension to the optimal ChA technique mentioned in (a) above, optimal ChA scheme for multiuser multichannel access in overlay CRNs has been developed. The optimal scheme is based on convex optimization and Lagrangian relaxation framework. Alternatively, a suboptimal ChA scheme has been developed based on the modified Hungarian algorithm.

Although the developed ChA and RA schemes may be complex in terms of practical implementation issues, the work in this thesis presents in-depth analysis and evaluation which provide an exposition for significant theoretical insights into the performance gains that can be obtained by employing optimization techniques in CRNs and future wireless networks in general.

1.4 Author Publications

The following are peer-reviewed journals and conference publications:

- **C.S. Chabalala** and F. Takawira, "Hybrid channel assembling and power allocation for multichannel spectrum sharing wireless networks," accepted for the *IEEE Wireless Communications and Networking Conference* (*WCNC'2017*), San Francisco, CA, March 2017.
- **C.S. Chabalala**, R. van Olst and F. Takawira, "Optimal channel selection and power allocation for assembling in cognitive radio networks," in *Proceedings of the IEEE Global Communications Conference (GLOBECOM'2015)*, San Diego, CA, pp. 1-6, December 2015.
- **C.S. Chabalala** and F. Takawira, "Performance analysis of channels selection and power allocation for channel assembling in multichannel cognitive radio networks," in review for publication in *IEEE Transactions on Vehicular Technology*, submitted November 2016.
- **C.S. Chabalala** and F. Takawira, "Hybrid overlay and underlay spectrum aggregation with optimal channel selection and adaptive power allocation in cognitive radio networks," submitted for publication in the *IET Communications Journal*, submitted February 2016.
- **C.S. Chabalala** and F. Takawira, "Adaptive spectrum aggregation for opportunistic resource allocation in multichannel wireless networks," to be submitted for the *Proceedings of the IEEE AFRICON'2017*, Cape-Town, 18-20 September, 2017.

1.5 Thesis Organization

The following provides the roadmap for the remainder of this thesis:

Chapter 2: Presents the overview of the investigation into the related works in literature, mainly focusing on the DSA techniques for CRNs, RRM and optimization techniques in wireless communications, and ChA schemes with more emphasis on CRNs.

Chapter 3: Presents the adaptive ChA and RA scheme which determines the optimal number of channels to assemble, as well as the associated optimal power profile for SU transmissions in overlay CRNs. This chapter also provides the details of the employed Lagrangian relaxation framework that is based on convex optimization theory, together with the branch-and-bound (BnB) technique with sequential fixing (SF) for optimal solution to the original MINLP problem. Moreover, simulation based performance evaluation is presented for various ChA schemes.

Chapter 4: This chapter presents performance modelling of ChA schemes over fading channels. The closed-from expressions for the PDF and CDF are derived for statistical characterization of assembled channels. The characterization forms the basis on which the compact closed-form mathematical models for average capacity, outage probability and forced-termination probability are derived. Then the preciseness and correctness of the derived analytical models are established by performing cross-validation between simulation results and analytical results, wherefore the cross-validation is employed to confirm the accuracy of the developed analytical models.

Chapter 5: This chapter presents adaptive ChA schemes for hybrid overlay and underlay SU transmissions. This allows SU nodes to opportunistically assemble free PU channels in overlay mode, and adaptively select the occupied PU channels with controlled transmit power levels in underlay mode. A multilevel maximum allowable SU transmit power for channels assembled in underlay mode is derived based on various assumptions about the knowledge of channel status information (CSI) availability at the SU transmitter, which ensures that SU transmit power in underlay mode is constrained under the noise floor of PU nodes. Further, the Lambert-$\mathcal{W}$ function is employed to derive closed-form expressions for channel selection. Then finally, the simulation based performance evaluation for hybrid ChA schemes is presented.

Chapter 6: In this chapter, the adaptive ChA schemes for multiuser multichannel access CRNs are presented. In particular, two ChA schemes are discussed. First, an optimal scheme that is based on convex optimization to determine the overall optimal solution for multiple SU nodes is presented, followed by a suboptimal ChA scheme that employs the modified Hungarian algorithm. Then, the simulation based performance analysis is presented to evaluate performance of the ChA schemes for multiuser multichannel access CRNs, subject to PU activity patterns.

Chapter 7: This chapter provides the main points on which the work presented in this thesis is summarised, together with the concluding remarks. This also highlights the possible directions on how the accomplished work can be extended for future research on adaptive ChA schemes and efficient RA techniques for CRNs and the emerging wireless networks.

CHAPTER TWO

2 LITERATURE REVIEW

2.1 Introduction

THIS chapter presents a comprehensive review of the related works on recent advances on ChA and RA techniques in literature. The previous research trends and accomplishments are therefore summarised with more emphasis on CRNs, wherein the susceptibility to performance degradation due to PU activity patterns and network dynamics compel development of robust ChA and RA techniques to improve performance of SU nodes. Since the emergence of CR technology, development of appropriate communication techniques for efficient operation of CRNs has always been a challenge due to radio spectrum dynamics as dictated by the activities of PU nodes. Thus, the presented review provides the core pillars and foundations which serve as a preamble to the research ideas and contributions that are discussed in subsequent chapters.

The rest of the chapter is organized as follows: Section 2.2 presents the related works on DSA techniques in CRNs, where in particular, the overlay mode, underlay mode and hybrid access mode are discussed. Section 2.3 delves into research accomplish on ChA schemes, underlining the open issues and challenges in the context of CRNs. Then Section 2.4 presents the related works on RA techniques in wireless communications and spectrum sharing networks, as well as highlighting the optimization frameworks that are usually employed; followed by Section 2.5 which summarizes the main points to conclude the work presented in this chapter.

2.2 Dynamic Spectrum Access in Cognitive Radio Networks

One of the major challenges in CRNs is the provisioning of efficient and robust communication without degrading performance of PU nodes. Robust communication refers to the ability of a network to operate continuously under changing environments and resource constraints without failure [1], [3], [7]. To address the issue of robust communication amid the looming overcrowding of radio spectrum, the subsections in sequel provide DSA techniques and research efforts in CRNs.

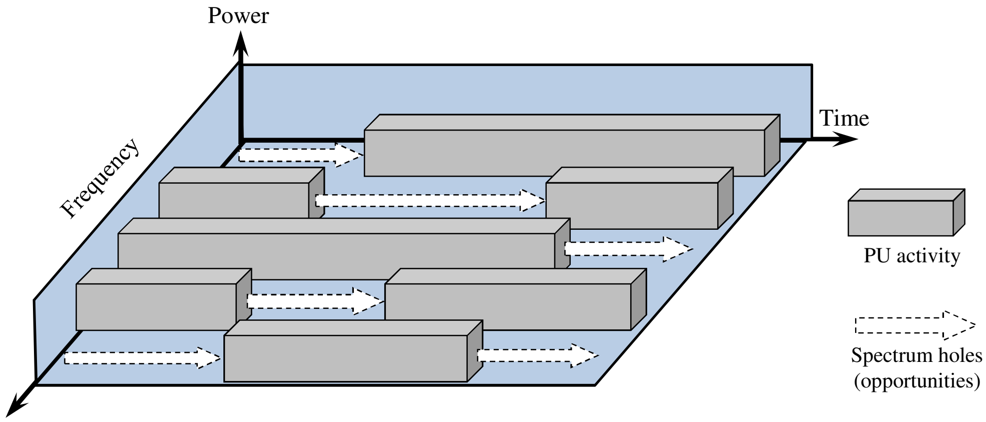

Figure 2-1: The concept of spectrum holes in CRNs.

2.2.1 Overlay Spectrum Access

In overlay spectrum access mode, SU nodes opportunistically select the free PU channels, and instantly vacate the occupied channels on detection of new arrivals for PU services. This is usually referred to as opportunistic spectrum access (OSA) [13]. The vacation of SU nodes as a result of PU arrivals results in forced terminations, which therefore degrades performance of SU nodes. The overlay spectrum access forms the basis of operation for the original inception of CR technology. SU nodes are therefore strictly required to detect spectrum holes (white spaces) defined in space, time and frequency as illustrated in Fig. 2-1. This also requires accurate online spectrum sensing techniques to promptly detect arbitrary arrivals of new PU services. However, the previous works in literature have shown that accurate spectrum sensing is difficult to implement [1], [34].

The main objective in OSA is collision avoidance between SU and PU services, which is analogous to interference mitigation whereby PU nodes have exclusive rights to spectrum access. One of the constraints on which SU nodes can make spectrum access decisions is the collision probability [27]-[28], [31]. This is the probability that a PU service arrives on a channel that is occupied by SU nodes. To optimize performance in OSA CRNs, joint spectrum sensing and access problem has been formulated as a partially observable Markov decision process (POMDP) in [35]; whereby PU activities have been modeled as a two-state Markov chain process (*i.e.* idle or busy). The key focus was to maximize throughput while maintaining collision probability below a threshold. However, the state space of the POMDP model increases with the number of channels; hence, increase in complexity, based on which a suboptimal scheme was developed to reduce complexity [35].

In [34], the authors proposed OSA scheme whereby reactive PU nodes take the history of SU activity patterns into consideration to determine the probability that a particular channel can be accessed. The PU channel occupancy is modeled as a 4-state discrete Markov chain process. The main focus of the study was to maximize SU throughput by formulating the optimization problem as a constrained finite-horizon POMDP. Then the numerical results were presented to illustrate that the proposed scheme guaranteed satisfactory QoS level on throughput for PU nodes, while making a tradeoff to increase spectrum access opportunities for SU transmissions in a CRN. Similar works that are based on POMDP have also been reported in [36]-[38], where in general, POMDP based problems were found to be computationally prohibitive to solve.

In addition to sensing outcomes about channel occupancy status, the works in [27], [31], [39]-[43] proposed robust spectrum access policies that incorporate the time varying quality of wireless channels. The key objective in these works is to maximize effective throughput of SU nodes subject to channel variations and collision constraints. In general, it was established that capturing the quality of wireless channels for spectrum access improves performance under tights collision constraints. However, in the case where PU activity patterns are high, spectrum is hardly available for SU nodes while operating in overlay mode, to the extent that SU nodes would completely have no access to spectrum when PU nodes are always busy [27], [31]. Development of appropriate DSA access techniques that can cope with high PU activities is therefore required.

2.2.2 Underlay Spectrum Access

In contrast to the overlay spectrum access, SU nodes in underlay mode employ interference threshold to mitigate performance degradation in high PU activity patterns [7], [44]-[45]. This facilitates concurrent spectrum access for both PU and SU transmissions; where SU nodes transmit through the channels that are occupied by PU nodes, subject to transmit power and interference control as illustrated on Fig. 2-2 on the next page. Thus, minimizing interference for SU transmissions is the most common criterion for efficient communication. In particular, interference constraint is imposed on SU transmissions to protect PU services. Nonetheless, co-channel interference is usually bidirectional, therefore affects both PU and SU transmissions alike. How to maintain interference threshold for SU transmissions while guaranteeing QoS requirements for both PU services and SU services is therefore a technically challenging task [2], [7].

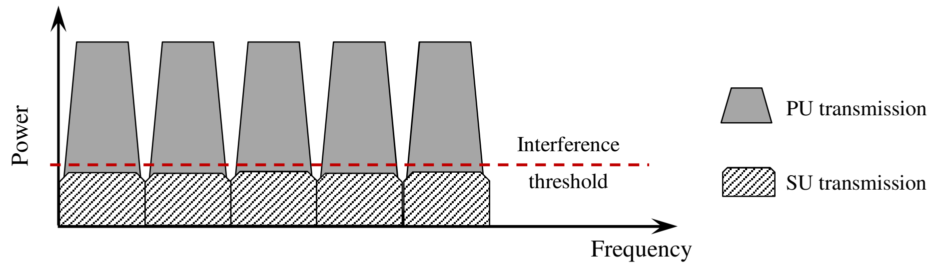

Figure 2-2: Example of underlay spectrum access in CRNs.

Various studies on underlay CRNs have incorporated PU outage probability as a basis for interference management and power control. In most studies, interference constraints and transmit power constraints are usually employed to protect PU nodes from SU transmissions. Interference constraint refers to the maximum amount of interference that a PU node can tolerate without QoS degradation [1], [3], [15]. In [46]-[50], peak interference constraint has been employed to analyse performance and improve capacity of SU nodes in underlay CRNs over fading channels, where in particular, adaptive power allocation for SU transmissions has been mainly based on the SNR at the PU receiver. The results in these studies revealed significant capacity gain for CRNs over fading channels, where full knowledge of CSI is available at the SU transmitter.

Other studies in [51]-[55] investigated the impact of imperfect knowledge of CSI at the SU transmitter, whereby a closed-form expression for SU capacity was derived based on peak power constraint. Moreover, transmission power allocation techniques have been proposed in [56] to minimize outage probability in Rayleigh fading wireless channels. From the previous studies, it has been established that using outage constraint at the PU node receiver offers better performance and protection for PU services than using interference power constraint [56].

As noted in [56], full CSI at the SU transmitter is required to protect PU transmissions using interference power constraint; yet full CSI may be difficult to obtain as full cooperation between PU nodes and SU nodes is required. Outage probability constraint may be relatively easier to implement as it rather based on statistical information [57]. The impact of PU transmissions on SU nodes was also investigated under Rayleigh fading channels in [58]-[59], where it was revealed that PU transmissions can also result in severe fading against SU transmissions.

2.2.3 Hybrid Overlay and Underlay Spectrum Access

Hybrid spectrum access mode mainly aims to merge the merits of the overlay and underlay modes by adopting both for SU transmissions as they have their respective advantages [3], [15], [60]-[61]. Hence, hybrid schemes jointly exploit the overlay and the underlay spectrum access techniques as illustrated in Fig. 2-3. In previous studies, it has been shown that hybrid schemes outperform either overlay-only or underlay-only schemes in terms of achievable system capacity and bit-error-rate (BER) [14]-[15], [62]-[65]. In [66]-[67], hybrid overlay and underlay schemes were studied for CRNs over additive white Gaussian noise (AWGN) channels, where it was generally established that hybrid schemes achieve significant performance improvement in CRNs.

Spectrum sensing forms the critical component for transmit mode selection and switching in hybrid schemes [8], [61], [68]. The study in [63] proposed a Markov chain model that facilitates the switching between overlay and underlay spectrum access modes. The proposed model detects PU activity patterns based on a double-threshold energy detection technique to mitigate collision and interference between PU and SU services. In [14], a hybrid overlay and underlay spectrum access was investigated, with the aim to maximize data-rate subject to power constraint; whereby an auction-based power allocation technique was employed for competing SU nodes.

Moreover, a location-aware spectrum access scheme was proposed in [8], whereby SU nodes which are close to a PU node access spectrum in overlay mode, while concurrent spectrum access in underlay mode is allowed for SU nodes that are located far from a PU node. It was generally established that incorporating location information in spectrum access techniques improves spectrum efficiency. However, it was also found that the location-aware approach mainly depends

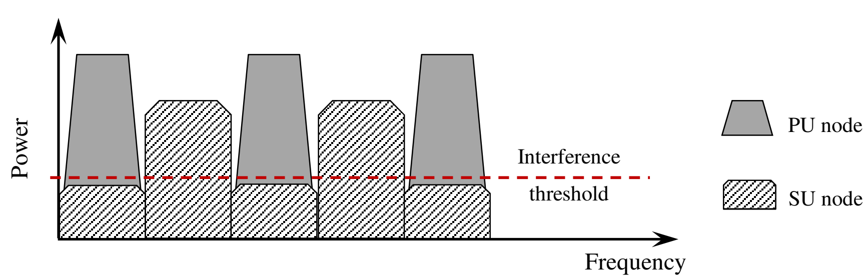

Figure 2-3: Example of hybrid spectrum access in CRNs.

on network topology, especially in terms of the distance between SU transmitter in a CRN, and a PU receiver in a PN [3], [7]-[8]. The study in [70] proposed a hybrid scheme that aims to improve throughput and spectrum efficiency, as well as reducing congestion in CRNs. SU services with minimum flow are allowed to bypass sensing phase to access spectrum under hybrid mode subject to signal-to-noise and interference ratio (SNIR) constraints; while unoccupied channels are left for SU services with higher flows. The authors presented simulation results, which revealed improved performance and spectrum utilization. Another study in [71] presented a power control scheme based on game theory, where SU nodes compete for spectrum access. A repeated game model was used to ensure that SNIR at the PU receiver is kept below a predefined threshold [70]-[72].

2.3 Channel Assembling in Wireless Communications

This section presents the related works on ChA schemes in literature. In recent works, channel ChA technique has been proposed and implemented in practical networks, as a mechanism to maximize capacity in wireless communications [27]-[28], [33], [73]-[75]. This is a technique through which a user performs concurrent transmissions through multiple channels. Arguably, ChA is the only effective solution for network applications with large amounts of data to send, but through narrow frequency bands [76]. In literature, static and dynamic ChA techniques have been proposed [28], [31], [73]-[78]. For static ChA schemes, the number of assembled channels is prefixed, while the number of channels may vary for dynamic ChA schemes. In an effort to establish the performance benefits and limitations associated with ChA schemes, experimental and theoretical studies have been reported in [27]-[33], [64], [73]-[85], where it was revealed that naïve ChA decisions can greatly degrade performance in wireless networks, as the benefits of ChA are greatly influenced by various network factors such as noise levels and interference from neighboring links.

The key research studies on ChA in CRNs have mainly concentrated on CTMC modeling to provide significant insights into performance of ChA schemes [27]. Most of the reported studies are thus, predominantly focused on medium access control (MAC) based performance analyses. In [80], the authors proposed a spectrum sensing scheme which employs channel bonding and maintains a list of backup channels for redundancy, with the aim to meet various QoS requirements for heterogeneous SU services while reducing spectrum access latency. A similar study has also been reported in [81] for construction of a network backbone aiming to improve network reliability and throughput. In accord with the study [80], it has been demonstrated in [81] that channel bonding with backup-list maintenance provides significant performance improvement. In [27]-[28], [73]-[75], dynamic ChA where SU nodes adjust the number of channels based on heterogeneous traffic and channel availability during transmissions has been presented. In general, increasing the number of assembled channels for SU nodes increases capacity, hence reduces transmission time and latency. Reducing transmission time reduces probability of collisions between SU nodes and PU nodes. However, increasing the number of channels decreases SNR per channel, and also increases susceptibility to collision with PU service arrivals. Thus, determining the optimal number of channels subject to collision constraints in CRNs is also a challenging task [27].

A recent study on the performance of ChA in underlay CRNs has been presented in [86]. Similar to majority of the existing studies in literature, the work in [86] is mainly based on CTMC modeling in a multiuser environment. Further, comprehensive studies based on CTMC analytical framework to investigate performance of channel bonding under various network conditions have been presented in [31], [76], [79], [82]. In these studies, factors that affect performance, and conditions under which channel bonding schemes improve performance in wireless networks were established, whereby it has been generally concluded that significant performance benefits of ChA schemes can be achieved by adaptively adjusting channel-width with respect to QoS requirements and network conditions among other parameters. However, the CTMC based analyses in literature do not necessarily account for the issues pertaining to network dynamics and constraints such as the varying nature of wireless channels and adaptive power control. Moreover, the study in [83] proposed the channel aggregation diversity (CAD) scheme with joint channel selection and power allocation, which aims to improve spectrum efficiency and energy efficiency bounded by total power constraint. Through simulations studies, the CAD scheme has been found to offer improved performance. Unlike the work presented in this thesis, a node using CAD scheme selects a set of channels with predetermined power levels through the exchange of control packets.

Further, ChA scheme based on queuing and channel fragmentation was proposed in [77], and another with priority based queuing in [78]. Thus, queuing and fragmentation were found to reduce blocking probability of SU nodes significantly. However, the results are also limited to MAC schemes without appropriate spectrum characterization for efficient ChA. Optimizing channel selection and power distribution for ChA schemes in CRNs requires spectrum assessment and characterization to determine the quality of available channels. In general, stronger signal strength is usually required for transmission through assembled channels to achieve the same reliability of a single channel [30]-[31], which therefore mandates judicious design considerations for development of efficient ChA schemes. For instance, doubling the number of channels decreases signal strength by 3dB [32]. This is because the same amount of transmit power is distributed across all the channels; wherefore the power allocated per channel is reduced. Moreover, allocation of several channels to a single SU node through ChA schemes reduces the probability of channel availability for arrival of new SU services, hence may have detrimental effects on the performance of SU nodes in a CRN as a result of increased blocking probability [27], [30, [31]-[32], [79].

In general, the major efforts in prior works mainly focused on a preset number of assembled channels, with the same power levels across the selected channels, independent of the number of channels assembled. Hence, the total power increases linearly with the number of channels. However, results obtained in such manner are misleading if applied to SU nodes with a total power constraint as illustrated in Fig 2-4. Also, impact of the fading nature of wireless channels have been generally overlooked, yet ChA schemes are inherently prone to fading as it results in performance degradation. Thus, how to optimize channel selection and power distribution for ChA schemes is a

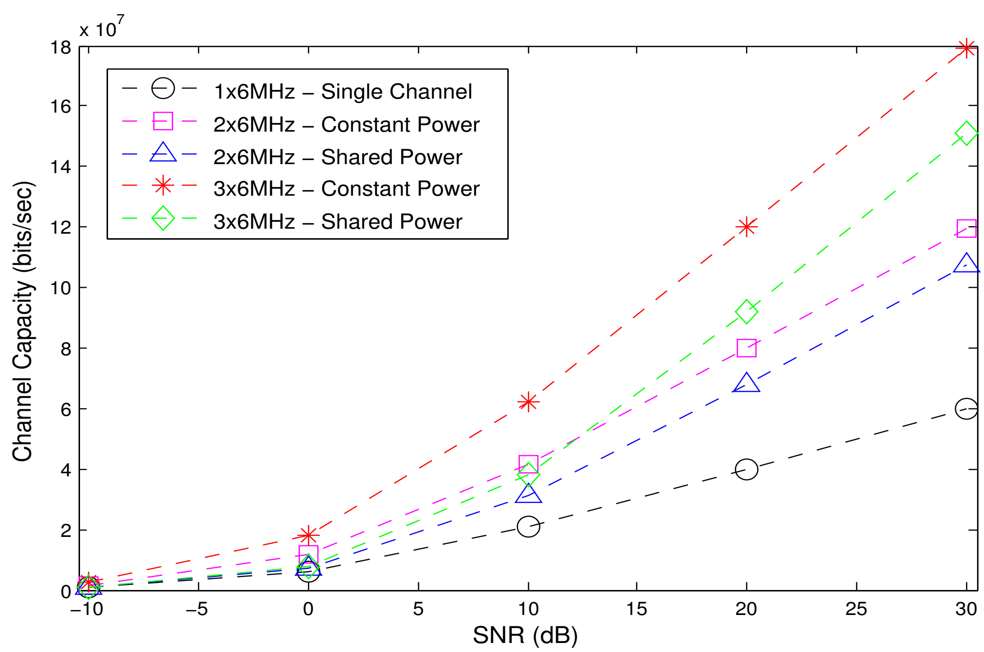

Figure 2-4: Illustration of the impact of power allocation on ChA techniques.

crucial issue that needs to be investigated. Accordingly, the optimization compels development of appropriate spectrum assessment and characterization criteria for available channels. To augment the highlighted limitations, this thesis extends the studies in literature, with the main objective to develop criteria on how to perform adaptive ChA with the optimal number of channels and power profile. This entails selection of the best channels to assemble, together with the optimal power distribution to maximize SU capacity, taking into account: channel variations, total power constraint, minimum capacity threshold, and collision probability threshold to protect PU services. This aims to reassure QoS requirements satisfaction while minimizing outage probability under fading channels, reduce collision probability between PU and SU services, hence reduce forced termination probability for SU services. The departure of SU nodes as a result of PU service arrivals results in forced terminations, which generally degrades performance of SU nodes.

2.4 Dynamic Resource Allocation in Wireless Networks

This section highlights the related works on optimization techniques for dynamic resource allocation (DRA) in wireless networks, which also includes CRNs. Different techniques for development of optimal and suboptimal solutions have been studied in previous works. For instance, greedy and heuristic resource allocation schemes have been proposed, whereby the greedy schemes have been found to be effective in homogenous environments where all the users require the same amount of spectrum [87]; in which case, the amount of spectrum allocated to users is predetermined and fixed, whereas adapting the allocated spectrum based on QoS requirements has been found to be more effective and efficient; especially in spectrum sharing networks where the spectrum allocation schemes also incorporate PU traffic patterns [2], [4], [8]-[11], [87].

Moreover, various studies have looked into development of mathematical models to establish performance insights of resource allocation techniques based on: game theory for problem formulation, CTMC modelling and convex optimization [1], [2]-[3]. Many of the studies based on CTMC modelling in spectrum sharing networks mainly overlooked the effects of the varying nature of wireless channels [27], [64]. As a result, these studies do not account for the issues related to adaptive RRM and network dynamics such as PU activity patterns in the case of spectrum sharing wireless networks, where adaptive resource allocation schemes based on PU activity patterns are otherwise crucial to improve performance of both PUs and SUs. Nonetheless, the dependence of the performance of RRM schemes on wireless channel links and transmit power optimization highlights the necessity for development of appropriate schemes that are resilient to performance degradation [3], [7], [88]-[89]. One of the popular approaches on DRA techniques in wireless communications is convex optimization technique, whereby convex reformulation is performed by relaxing the binary integer constraints such as $x_{n,m} \in \{0,1\}$, which usually denotes channel selection status for user n on channel m. Convex reformulation is mainly performed by the following relaxation $\{0 \leq x_{n,m} \leq 1\}$, which is normally interpreted as sharing factor [26]. In general, constrained optimization problems are usually expressed as follows [1], [87], [90]:

$$\begin{aligned} &\underset{\boldsymbol{x}\in\mathbb{X}}{\text{minimize}}\ f(\boldsymbol{x}) \\ &\text{subject to: } \begin{cases} g_i(\boldsymbol{x}) \leq 0, & \text{for } i = 1,\cdots,m \\ h_j(\boldsymbol{x}) = 0, & \text{for } j = 1,\cdots,n, \end{cases} \end{aligned} \tag{2.1}$$

where $f(\boldsymbol{x})$ is the objective function for the optimization problem, $g_i(\boldsymbol{x})$ and $h_j(\boldsymbol{x})$ are respectively inequality and equality constraints, and $\boldsymbol{x}$ is the optimization parameter vector with the feasible range $\mathbb{X}$. In most studies, convex optimization framework is employed with Lagrangian decomposition, whereby the Lagrangian reformulation for (2.1) can be expressed as

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{\lambda},\boldsymbol{\mu}) = f(\boldsymbol{x}) - \sum_{i=1}^{m} \lambda_i g_i(\boldsymbol{x}) - \sum_{j}^{n} \mu_j h_j(\boldsymbol{x}) \tag{2.2}$$

where $\forall\lambda_i \in \boldsymbol{\lambda}$ and $\forall\mu_j \in \boldsymbol{\mu}$ are Lagrangian multipliers [26], [41], [91]-[94]. This is usually solved by determining the Karush-Kuhn-Tucker (KKT) conditions which are necessary and sufficient for optimality; based on which closed-form expressions can be developed, which are however, difficult to obtain in most cases [26]-[27]. It has been shown in previous works such as in [95]-[97] that most near-optimal heuristic approaches that solve (2.1) would have a complexity order of $O(nm^2)$. The Lagrangian relaxation with dual optimization framework on the other hand, achieves 99.9999% of the optimal solution with complexity order of $O(nm)$; hence the optimality gaps are usually less than 10^{-4} with Lagrangian duality theorem [97]. In orthogonal frequency division multiplexing (OFDM) systems, convex optimization with dual decomposition has been mostly employed for joint subcarrier allocation and power distribution in multiuser systems [95]-[99].

One of the popular issues on DRA techniques in wireless networks is fairness, which indicates how equally resources are allocated in multiuser systems [8], [10], [100]. For instance, fairness may refer to equal allocation of channels to multiple users, or allocation of equal portions of transmit power based on the total power budget, or rate proportionality which aims to achieve the same rate for each user in a network [101]-[103]. In terms of rate proportionality, *max-min* techniques have been employed in various studies in literature, whereby the main objective is to maximize the minimum data rate of users [103]. The rate proportionality based methods have also been employed for efficient and fair distribution of resources in heterogeneous environments [100], [104].

In the case of CRNs, DRA and optimization techniques are more challenging than in conventional wireless networks as a result of the imperative necessity to protect PU nodes against SU transmissions. Most of the works on resource allocation in CRNs in general, overlap with the studies discussed in Section 2.2. Some of the previous works studied rate and power allocation problems in multichannel access CRNs, where the optimization problems considered rate and power allocation schemes to satisfy QoS requirements for SU services, and power control to minimize interference imposed on PU services from SU transmissions [2], [7]-[9], [41], [55]-[56]. Performance of SU nodes is absolutely dictated by PU activity patterns in CRNs, which are however, mostly overlooked in the existing works on resource allocation techniques.

Notwithstanding, QoS-aware resource allocation in CRNs is a challenging task due to network dynamics and time varying wireless channels, as well as the obligation to provide efficient communication for SU services without degrading performance of PU nodes. Further, it is worth reiterating once again that, the main studies on resource allocation techniques in spectrum sharing networks are mainly focused on user selection to maximize sum capacity, and power control to protect PU nodes by minimizing interference in underlay SU transmissions [45]-[50], [52]-[55], [59], [105]. Thus, the resource allocation techniques do not address issues related to performance of ChA schemes in spectrum sharing networks. Moreover, the research work on user selection and power allocation in wireless networks has generally been focused on the facets of resource allocation and scheduling in multiuser systems [8], [10], [53], [88]-[89], [95], [97]-[104], [106]. These studies do not account for issues related to ChA and network dynamics such as PU activity patterns in the case of CRNs. In general, adjusting the number of assembled channels based on PU activity patterns is crucial to reduce probability of forced termination for SU services.

2.5 Chapter Summary

In this chapter, background research and information on dynamic spectrum access in CRNs, ChA in wireless communications and DRA in wireless networks have been presented; highlighting some significant research accomplishments with more emphasis on spectrum sharing wireless networks. The presented investigation provides the core pillars and foundations which serve as a preamble to the research ideas and contributions that are discussed in the subsequent chapters. Given the underlying challenges inherent to spectrum sharing wireless networks, the emerging applications underline the need for new network technologies and communication protocols which are capable of transporting large amounts of data from heterogeneous sources. The central issue in this Ph.D. research is therefore development of robust and reliable resource allocation and ChA schemes in CRNs, taking into consideration, networks dynamic and resource constraints.

The major studies on dynamic RRM schemes are mainly focused on the facets of resource allocation and scheduling in multiuser systems, which mainly deal with the distribution of radio resources for different users in wireless networks. The main works are therefore focused on user selection with power control and interference management. In the case of spectrum sharing wireless networks, it has been shown that network performance can be greatly improved by performing adaptive resource allocation techniques with respect to network dynamics and constraints. Indisputably, the issue of interference and collision between PU and SU services have been found to be a major caveat, and a fundamental concern for network performance improvement for both PNs and CRNs. Consequently, adaptive resource allocation and ChA techniques that take into account transmit power constraints and management, the varying nature of wireless channels, PU activity patterns, and QoS requirements are presented in subsequent chapters.

CHAPTER 3

3 OVERLAY MULTICHANNEL SPECTRUM ACCESS

3.1 Introduction

THIS chapter presents the optimal channel selection and power distribution for ChA in overlay spectrum access for SU transmissions in CRNs. The main objective is to maximize SU channel capacity over fading wireless channels, subject to total transmit power constraint, minimum QoS requirement on capacity, and collision probability threshold to protect PU transmissions. The nature of the optimization problem turns out to be in the form of nonconvex mixed integer nonlinear programming (MINLP), which is generally known to be nondeterministic in polynomial time. A Lagrangian framework is therefore employed to reformulate the optimization problem, based on which dual decomposition with subgradient and Newton-Raphson methods are used to determine a relaxed optimal solution. The relaxed optimal solution serves as the upper-bound for the optimal solution to the original MINLP problem. Thereafter, accelerated branch-and-bound (BnB) with sequential fixing (SF) is applied to determine an optimal solution to the original MINLP problem. The BnB technique reduces complexity of the otherwise nondeterministic combinatorial problem. In general, the simulation results presented in this chapter demonstrate that adaptive ChA with channel characterization improves network performance in terms of channel capacity, outage probability and collision probability between PU and SU services. Part of the work presented in this chapter has been presented and published in the *Proceedings of the IEEE Global Communications Conference* (*GLOBECOM'2015*), San Diego, CA, 6-10 December 2015; and submitted in part to the *IEEE Transactions on Vehicular Technology*, 15 November 2016.

The remainder of this chapter is organized as follows: Section 3.2 once again, highlights the related works for overlay ChA schemes in literature; followed by the system model and the outline of the basic assumptions, then the collision probability constraint and the optimization problem formulation in Section 3.3. Section 3.4 presents the optimal ChA scheme based on the Lagrangian optimization framework and the BnB technique. Then Section 3.5 presents the simulation based performance evaluation and discussion for various ChA schemes. Finally, Section 3.6 highlights the main points on which the work presented in this chapter is concluded.

3.2 Related Works

The key research studies on ChA in CRNs have mainly concentrated on CTMC modeling to provide significant insights into performance of ChA schemes [28], [33], [73]-[75], [77]-[78]. These studies focused on a preset number of assembled channels, without optimal power distribution whatsoever. Besides, the same power levels across selected channels have been assumed, independent of the number of channels assembled [27]. As it has already been pointed out in Section 2.3, the total power increases linearly with the number of assembled channels; whereas power per channel actually decreases as the number of assembled channels increases for a given total power constraint. Notably, the results obtained by assuming the same power levels irrespective of the number of assembled channels are misleading when applied to SU nodes with a total transmit power constraint as a realistic assumption under resource constrained CRNs.

The study presented in [83] for the CAD technique investigated the joint power allocation and channel selection to improve spectrum efficiency and energy efficiency in CRNs, subject to total transmit power constraint. The optimization problem was reformulated into multiple-choice knapsack problem and solved using dynamic programming. Unlike the work presented in this chapter, a CR node using CAD technique selects a set of channels with predetermined power levels through the exchange of control packets. Except the study presented in [27], PU arrivals are generally not incorporated in resource allocation schemes to mitigate the issue of collision between PU and SU services. In [33], the authors formulated an optimization problem to minimize the rate of collisions, in which case it was generally concluded that selection of a single channel offers an optimal solution for minimal collision. Nevertheless, the work presented in this chapter shows that improved performance can still be obtained by assembling multiple channels to maximize capacity for SU nodes subject to collision probability constraint, instead of a single channel.

The fundamental issue addressed in this chapter is therefore how to develop a criterion on how to optimally select the best channels together with the optimal power profile for overlay SU transmissions, with the aim to meet SU QoS requirements, while at the same time protecting PU services against collisions due to SU transmissions. This requires appropriate spectrum characterization techniques for assessment of available channels; which in this work, involves the incorporation of PU activity patterns and the time varying nature of wireless channels.

3.3 System Model and Problem Formulation

3.3.1 System Model and Basic Assumptions

A CRN environment is considered as illustrated in Fig. 3-1, where SU nodes have access to multiple PU channels to maximize capacity. The fading channel conditions are assumed to be quasi-static, hence change from frame to frame, but remain constant for a given frame interval. The network model comprises a primary network (PN) spectrum with a set $\boldsymbol{\mathcal{S}}$ for all PU channels, whereby channel status for a set of free PU channels $\boldsymbol{\mathcal{M}} = \{1, \cdots, M\} \subseteq \boldsymbol{\mathcal{S}}$ is represented by the channel gain vector $\boldsymbol{h}_M = \{h_1, \cdots, h_M\}$, and $\bar{h}_i \triangleq \mathbb{E}\{h_i\}$ denotes the average channel power gain for the i-th channel, and M signifies the cardinality of $\boldsymbol{\mathcal{M}}$. The subset $\boldsymbol{\mathcal{N}} = \boldsymbol{\mathcal{S}}/\boldsymbol{\mathcal{M}}$ represents the channels that are occupied by PU nodes. The work in this chapter considers arrival of a single SU node at any time t to assemble multiple PU channels for SU transmissions. The arrival of multiple SU nodes at the same time is otherwise considered to be a medium access control (MAC) protocol issue. Also, channel state vector has a feasible binary integer vector $\boldsymbol{x} \triangleq \{x_i \mid x_i \in \{0,1\}, \forall i \in \boldsymbol{\mathcal{M}}\}$, which represents the channel selection status, whereby

$$x_i = \begin{cases} 1 & \text{if channel } i \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

Furthermore, channel selection vector has a corresponding feasible set of non-negative power distribution vector for the assembled channels $\boldsymbol{p} \triangleq \{p_i \mid p_i \geq 0, \ \forall i \in \boldsymbol{\mathcal{M}}\}$. In wireless communication systems, the well-known Shannon-Hartley channel capacity theorem characterizes

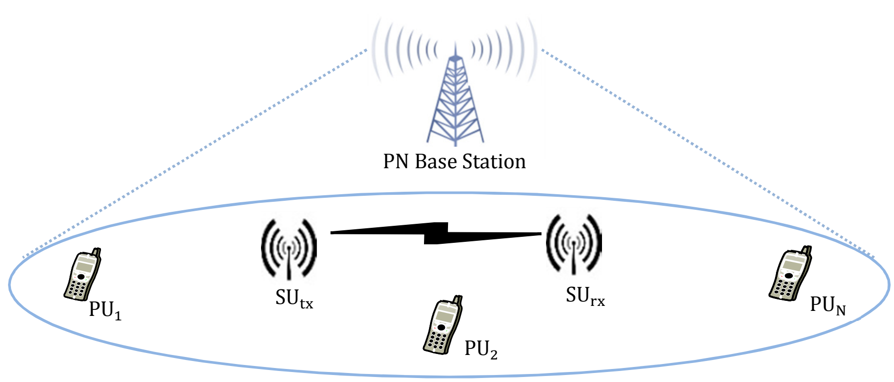

Figure 3-1: Network model for coexistence of a PN and CRN.

the maximum rate at which information can be reliably transmitted with arbitrarily small probability of error across a noisy channel [107]. The theorem plays a crucial role in quantifying the fundamental performance limits in design and development of digital communication systems. Thus, transmit power and bandwidth are some of the key resources in identifying the information capacity limits for digital communication systems. In the case of the development of optimal ChA schemes, the dependence of channel capacity on both channel selection criteria and power distribution techniques is generally not uncomplicated for maximizing network performance, more especially in CRNs. The instantaneous channel capacity limit $r_i(x_i, p_i)$ for any SU node over a single PU channel i based on the Shannon theorem can be expressed as follows [27], [82], [108]:

$$r_i(x_i, p_i) \triangleq x_i \mathcal{B} \log_2\left(1 + \frac{p_i h_i}{x_i}\right), \qquad \forall i \in \boldsymbol{\mathcal{M}} \tag{3.2}$$

whereby $h_i \triangleq |g_i|^2/\mathcal{N}_0\,\mathcal{B}$ represents the instantaneous channel-to-noise ratio power gain, $\mathcal{N}_0$ represents the Gaussian background noise power spectral density, $\mathcal{B}$ represents bandwidth, and g_i denotes channel fading coefficient. Furthermore, the channel coefficients are assumed to be independent non-identically distributed (*i.n.i.d*) random variables that follow Rayleigh fading model, wherefore the PDF of the SNR $\gamma_i = p_i h_i$ is a function of the square envelope of the channel gain, which follows an exponential distribution characterized by [109]:

$$f_\gamma(u) = \frac{1}{\bar{\gamma}_i} \exp\left(-\frac{u}{\bar{\gamma}_i}\right) \mathcal{U}(u), \tag{3.3}$$

where $\bar{\gamma}_i \triangleq \mathbb{E}\{p_i h_i\}$ and $\mathcal{U}(u)$ denotes a unit-step function. Statistical characterization of assembled channels facilitates performance evaluation of various ChA schemes with respect to timescale variations of wireless channels due to fading characteristics. In particular, the assembled channels are characterized by a joint multivariate distribution (*i.e.* convolution of individual PDFs for individual channels) from the statistics of the independent PU channels. Without loss of generality, it can be deduced from (3.2) that $\{x_i = 0\} \Rightarrow \{r_i(x_i, p_i) = 0\}$. The *L'Hopitals* rule can also be applied to show that the following holds: $\lim_{x_i \to 0} r_i(x_i, p_i) = 0$. This is also consistent with the fact that transmit power is not allocated for channels that are not selected for assembling; therefore nonzero capacity cannot be obtained from such channels. For convenience purpose in the derivatives of $r_i(x_i, p_i)$ in subsequent sections, the following is defined: $\mathcal{w} = \mathcal{B}/\ln(2)$, and further assumed that $\mathcal{w} = 1$; based on which (3.2) can further be simplified into:

$$r_i(x_i, p_i) \triangleq x_i \ln\left(1 + \frac{p_i h_i}{x_i}\right), \qquad \forall i \in \boldsymbol{\mathcal{M}}. \tag{3.4}$$

Following the works in [82] and [108], it can be easily deduced from (3.4) that $r_i(x_i, p_i)$ is jointly concave and unimodal with respect to $\{x_i, p_i\}$. In the case of ChA schemes, the capacity for each SU node depends jointly on both the number of selected channels for assembling, and the transmit power allocated in each of the channels. The total instantaneous channel capacity $\mathcal{R}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p})$ obtained from the assembled channels can therefore be simply defined as the sum of instantaneous capacity from the individual channels, given by the following:

$$\mathcal{R}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i=1}^{\mathcal{K}} r_i(x_i, p_i), \qquad \mathcal{K} \leq M. \tag{3.5}$$

where $\mathcal{K}$ is the ChA order out of M available PU channels. Considering the channel power gains from $\boldsymbol{h}_M$, PU activity patterns (*i.e.* arrival rates) and SU QoS requirements for minimum capacity, SU nodes can adaptively adjust the number of assembled channels, as well as optimizing the transmit power levels in those channels. However, increasing the number of channels (*i.e.* channel assembling order) increases the likelihood of PU service appearance on any of the assembled channels, thereby increasing collision probability between PU and SU services; which may degrade performance for both PU nodes and SU nodes as a result. In general, high ChA order and high PU arrival rates increase susceptibility to collisions and forced terminations for SU services. When a PU service arrives on any of the assembled channels, the SU node may also terminate the ongoing transmissions from all the other channels. Alternatively, the SU node with adaptive and dynamic ChA scheme may continue transmission over the remaining PU channels if the constraints are not violated as a result of handing-over one of the PU channels [27], [77]-[78].

Further, in time varying wireless channels, ergodic capacity is one of the metrics used to quantify and evaluate performance of wireless communication systems [46], [72]. In literature, ergodic channel capacity is mainly defined as the maximum achievable data rate averaged over all fading channel realizations [87], [95], [97]. Then based on the Jensen's inequality, the following holds for ergodic capacity $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x},\boldsymbol{p})$ as a function of $\mathcal{K}$ assembled PU channels for SU services [110]:

$$\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x},\boldsymbol{p}) = \mathbb{E}\left\{\sum_{i=1}^{\mathcal{K}} r_i(x_i,p_i)\right\} \;\; \leq \;\; \sum_{i=1}^{\mathcal{K}} \mathbb{E}\{r_i(x_i,p_i)\}, \tag{3.6}$$

where $\mathcal{K}$ denotes ChA order. Therefore the maximum achievable average capacity derived from the expected channel realizations for fading wireless channels constitutes the upper bound in comparison to the average of instantaneous capacity as illustrated in (3.6).

3.3.2 Collision Probability Constraint

Suppose PU service arrivals follow a Poisson process with an average arrival rate λ_{pu} on any channel $i \in \boldsymbol{\mathcal{M}}$. Each channel can only be assigned to a single PU node at the time, but SU nodes can assemble more channels to perform concurrent transmissions in order to maximize the sum capacity; hence the multichannel spectrum access scheme for SU nodes. The probability $\mathcal{P}_i$ that a SU node finishes transmission on any of the PU channels i within a time interval Δt_i, without collision with a PU service can be defined as follows [27], [33]:

$$\mathcal{P}_i(\Delta t) = \exp\left(-\lambda_{\text{pu}}\Delta t_i\right), \qquad \forall i \in \boldsymbol{\mathcal{N}}. \tag{3.7}$$

Then following the study presented in [33], the PU service arrivals are assumed to be independent over individual channels. It can therefore be easily inferred from (3.7) above that the probability $\mathcal{P}_{\boldsymbol{\mathcal{A}}}$ that there is no collision between any ongoing SU service and a PU service arrival (*i.e.* SU node finishes transmission successfully without forced termination) over a set of assembled channels $\boldsymbol{\mathcal{A}} \triangleq \{1,\cdots,\mathcal{K}\}$ from the PN spectrum can be expressed as follows:

$$\mathcal{P}_{\boldsymbol{\mathcal{A}}}(\Delta t) = \exp\left(-\sum\nolimits_{\forall i\in\boldsymbol{\mathcal{A}}} \lambda_{\text{pu}}\Delta t_i\right). \tag{3.8}$$

It therefore follows from (3.8) that the probability that there is no collision between new PU service arrivals and ongoing SU services decreases with the increase in the number of assembled channels for SU transmissions within the transmission period Δt; based on which the following remark can be stated in line with the collision probability [27], [33]:

Remark 3-1: *Increasing the channel assembling order $\mathcal{K}$ for SU transmissions increases the likelihood of a PU service appearance on any of the assembled channels, therefore increases the probability of collision between PU service arrivals and the ongoing SU transmissions; hence increases forced termination probability for SU services*.

On the other hand, a dilemma surfaces based on the proposition that increasing the number of assembled channels for SU services increases the resulting sum capacity, and the increase for the sum capacity reduces the SU transmission period and latency. It follows therefore that, the reduction on SU transmission time should reduce collision probability $\{\mathcal{P}_{\text{col}} = 1 - \mathcal{P}_{\boldsymbol{\mathcal{A}}}\}$.

The SU transmission time can simply be defined as $T_{\text{tx}} = L_{\text{Data}}/\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x},\boldsymbol{p})$, where L_{Data} denotes the amount of SU data to transmit, and $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x},\boldsymbol{p})$ is the average capacity as a function of channel selection vector $\boldsymbol{x}$, and the associated power distribution vector $\boldsymbol{p}$. Then reducing the SU transmission time may also decrease the probability of forced terminations for SU services, *i.e.* reducing transmission time reduces susceptibility of SU transmissions to forced terminations due to arrival of new PU services. Therefore the probability that any SU node finishes transmission without collision with a PU service on any of the assembled channels is also a function of the sum capacity resulting from the assembled channels, given by the following expression:

$$\mathcal{P}_{\boldsymbol{\mathcal{A}}}(T_{\text{tx}}) = \exp\left(-\frac{\left(\sum_{i=1}^{\mathcal{K}} x_i\right)\lambda_{\text{pu}} L_{\text{Data}}}{\sum_{i=1}^{\mathcal{K}} x_i\, \mathbb{E}\left\{\ln\left(1 + \frac{p_i h_i}{x_i}\right)\right\}}\right). \tag{3.9}$$

It can therefore be easily established from (3.9) that channels with high power gain contribute more on reducing collision probability by provisioning of high channel capacity. Let $\mathcal{P}_{\phi}$ define the minimum probability threshold for SU service transmissions without collision with PU service arrivals, such that the following constraint is defined for SU service admission: $\left\{\mathcal{P}_{\phi} \leq \mathcal{P}_{\boldsymbol{\mathcal{A}}}\right\}$. The constraint on collision probability ensures that SU services are admitted without degrading performance of PU nodes as a result of collisions, as well as protecting the SU nodes from forced

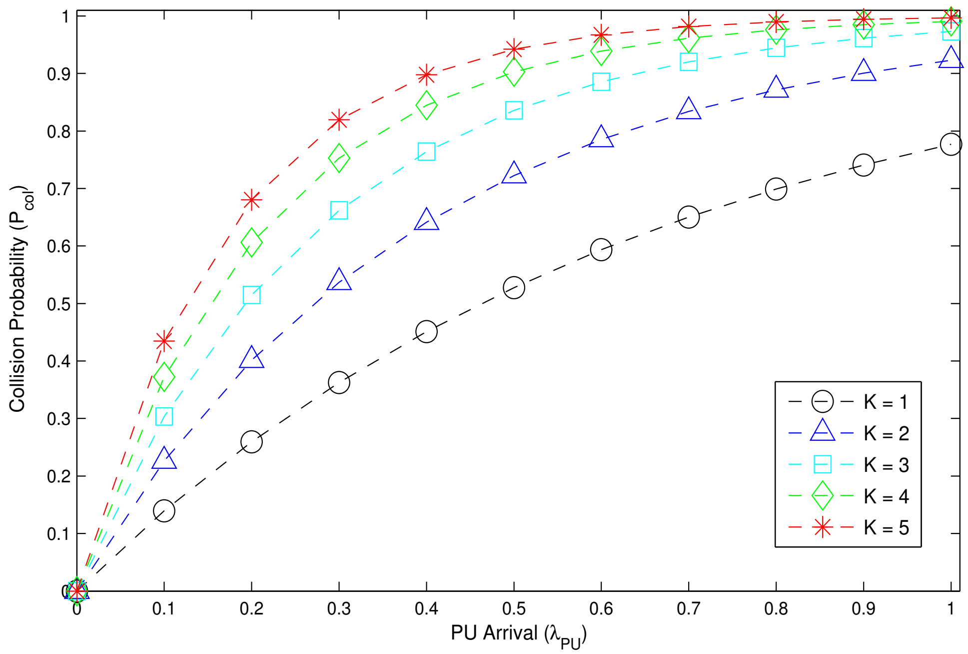

Figure 3-2: Collision probability between PU and SU services for increasing PU arrival rates.

terminations. Obviously, SU nodes must finish their transmissions before arrival of a PU service to avoid collisions. Then the following probability constraint can be derived from (3.9):

$$\mathcal{P}_{\phi} \leq \exp\left(-\frac{\left(\sum_{i=1}^{\mathcal{K}} x_i\right)\lambda_{\mathrm{pu}} L_{\mathrm{Data}}}{\sum_{i=1}^{\mathcal{K}} x_i \mathbb{E}\left\{\ln\left(1+\frac{p_i h_i}{x_i}\right)\right\}}\right). \tag{3.10}$$

Furthermore, the collision probability increases with the increase in PU services arrival rate as illustrated in Fig. 3-2 above. The antilog is then applied in (3.10), and the expression is further reformulated to derive the following, based on the sum channel capacity:

$$\beta \sum_{i=1}^{\mathcal{K}} x_i \leq \sum_{i=1}^{\mathcal{K}} x_i \mathbb{E}\left\{\ln\left(1+\frac{p_i h_i}{x_i}\right)\right\}, \tag{3.11}$$

where $\beta = \left(\lambda_{\mathrm{pu}} L_{\mathrm{Data}} / -\ln(\mathcal{P}_{\phi})\right)$. The next subsection formulates the optimization problem to maximize channel capacity for SU nodes through optimal ChA scheme in overlay spectrum access.

3.3.3 Overlay Channel Assembling Problem Formulation

The fundamental issue addressed in this work is stated as follows: given a set of free PU channels $M \triangleq |\boldsymbol{\mathcal{M}} \subseteq \boldsymbol{\mathcal{S}}|$ available for a single SU on service arrival, PU service arrival rates and the QoS requirements for SU services, determine how many and which of the available PU channels to assemble, together with how much power is distributed across the selected channels. This entails determining an optimal solution for channel selection vector $\boldsymbol{x}$, as well as the associated optimal power distribution vector $\boldsymbol{p}$ such that $\mathcal{R}_{\phi} \leq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p})$, for a minimum SU capacity threshold ($\mathcal{R}_{\phi}$). For any concave function $f(u)$ of a random variable u, the following holds: $\mathbb{E}\{f(u)\} \leq f(\mathbb{E}\{u\})$. It follows from (3.6) that: $\mathbb{E}\{\ln(1+p_i h_i)\} \leq \ln(1+p_i \mathbb{E}\{h_i\}) \Rightarrow \mathbb{E}\{\ln(1+p_i h_i)\} \leq \ln(1+p_i \bar{h}_i)$. Then the optimization problem to maximize average sum capacity can be formulated as:

$$\underset{\{\boldsymbol{x},\boldsymbol{p}\} \in \mathcal{M}}{\text{maximize}} \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right) \tag{3.12}$$

$$\text{subject to:} \quad \sum_{i=1}^{M} x_i p_i \leq P_{\max}, \quad p_i \geq 0, \qquad \forall i \in \boldsymbol{\mathcal{M}} \tag{C3.1}$$

$$\mathcal{R}_{\phi} \leq \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right), \tag{C3.2}$$

$$\beta \sum_{i=1}^{M} x_i \leq \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right), \tag{C3.3}$$

$$\left(\mathcal{K} \triangleq \sum_{i=1}^{M} x_i\right) \leq M, \qquad x_i \in \{0,1\}, \qquad \forall i \in \boldsymbol{\mathcal{M}} \tag{C3.4}$$

where $P_{\max}$ represents the total transmission power that can be distributed across all the assembled channels. The constraint in (C3.1) expresses the condition that only nonnegative powers can be assigned, and the sum of all the assigned powers must not exceed the total available power $P_{\max}$. The constraint in (C3.2) expresses the QoS requirement for channel capacity, which states that the resulting capacity from assembled channels must exceed the minimum threshold $\mathcal{R}_{\phi}$. Then (C3.3)

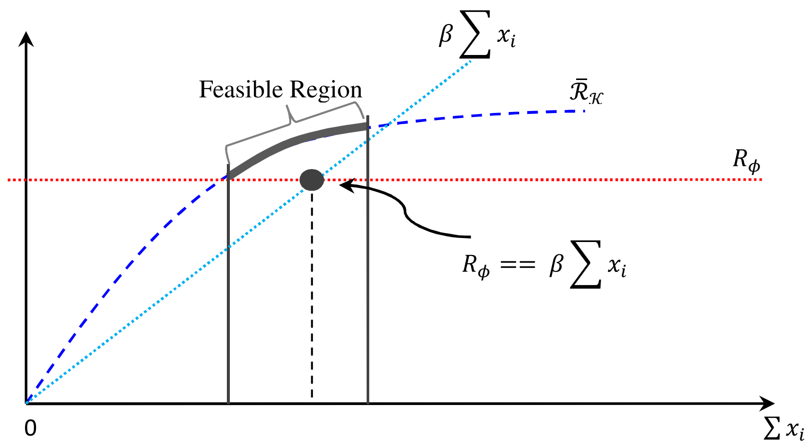

Figure 3-3: Illustration of feasibility-region for optimal solution.

expresses the condition on collision probability between SU and PU services, which is derived from the premise that: the probability that SU node services finish transmission without appearance of a PU service on any of the assembled channels must be above the minimum threshold $\mathcal{P}_\phi$. The constraint in (C3.4) denotes the channel selection status, which states that the number of selected channels cannot exceed the number of available channels from the PU spectrum. Accordingly, the optimization problem in (3.12) is a nonconvex MINLP problem, for which the optimal solution is generally known to be nondeterministic polynomial-time (NP) hard [108]. As shown in Fig. 3-3, any feasible region for the optimization problem defined in (3.12) must lie between the solution bounded by the minimum capacity constraint (C3.2) and the collision constraint (C3.3).

3.4 Overlay Channel Selection and Power Allocation

3.4.1 Lagrangian Convex Optimization Framework

This section reformulates (3.12) using the Lagrangian technique, and examines the optimality conditions based on the KKT conditions together with the complementary slackness [82], [91]-[92], [108]. To convert the MINLP problem into a convex optimization problem, the binary channel selection variables x_i's are relaxed to take real values in the interval $[0,1]$, which also renders the problem tractable [82], [108]. Hence, the relaxed x_i values illustrate a soft-decision (*i.e.* binary uncertainty) for channel selection. A solution to the relaxed problem provides an upper bound for the solution to the original MINLP problem, based on which the optimal solution is obtained by the BnB technique. The Lagrangian $\mathcal{L}(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\lambda})$ from (3.12) is given by

$$\begin{aligned}\mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}) = \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right) - \lambda_1 \left(\sum_{i=1}^{M} x_i p_i - P_{\max}\right) \\ - \lambda_2 \left(\beta \sum_{i=1}^{M} x_i \leq \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right)\right) - \lambda_3 \left(\mathcal{R}_{\phi} - \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right)\right)\end{aligned} \tag{3.13}$$

where $\boldsymbol{\lambda} = \{\lambda_m \geq 0 \mid m = 1, \cdots, 3\}$, in which case $\{\forall \lambda_m \in \boldsymbol{\lambda}\}$ represent the nonnegative Lagrangian multipliers for the constraints in (3.12). The dual function can be formulated as a pointwise infimum of the Lagrangian function in (3.13), which can be expressed as [91]-[92]:

$$\mathcal{D}(\boldsymbol{\lambda}) = \inf_{\{\boldsymbol{x}, \boldsymbol{p}\} \in \boldsymbol{M}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}). \tag{3.14}$$

In general, the dual problem is relatively easy to solve, and the solution to the primal problem (*i.e.* the optimization variables) can be analytically obtained from the solution of the dual problem [82], [91]-[92]. The dual problem defines the minimum value of the Lagrangian over the nonnegative dual variables, such that the optimal solution satisfies the following: $\mathcal{L}(\boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*) \geq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$. Thus, the dual problem can therefore be formulated as follows:

$$\begin{aligned}&\underset{\lambda_1, \lambda_2, \lambda_3}{\text{minimize}} \ \mathcal{D}(\boldsymbol{\lambda}) \\ &\text{subject to: } \{\lambda_i \geq 0 \mid i = 1, \cdots, 3\}.\end{aligned} \tag{3.15}$$

To establish the KKT conditions with complementary slackness, the Lagrangian function defined in (3.13) is partially derived with respect to $\{p_i, x_i\}$, and with respect to all the Lagrangian multipliers; then the derivatives are set to zero. Based on the complementary slackness conditions, if a constraint is not active at any feasible point (*i.e.* if the constraint is neither tight nor violated), the corresponding Lagrangian multiplier must be set to zero [91], [108]. Moreover, the orthogonality conditions suggest that either the constraint function is zero when it is active, or the associated

Lagrangian multiplier is zero when the inequality condition is active. However, the power allocation constraint defined in (C3.1) must always be tight and binding in order to maintain the optimal power distribution that maximizes SU sum capacity across all the selected channels. Otherwise, any feasible solution for the power distribution vector would be suboptimal [27].

Remark 3-2: *On reaching the optimal point, all the constraints that are active have nonzero Lagrangian multipliers to ensure that the search methods are directed to, and remain within the feasible subspace that contains the optimal solution.*

Using duality theory in convex optimization problems, closed-form expressions are sometimes obtained from the KKT conditions. However, as a result of the tight coupling between channel selection and power distribution vectors in determining the optimal solution for resource allocation problems, it is usually difficult to obtain the closed-form analytical solutions directly from the KKT conditions [91]-[92]. Consequently, iterative numerical search methods are not uncommon in solving optimization problems based on the KKT conditions.

Proposition 3-1: *The solution* $\{\boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*\}$ *provides an optimal solution from the Lagrangian problem if the following* $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}) \leq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$ *holds for any feasible primal solution* $\{\boldsymbol{x}, \boldsymbol{p}\}$*, whereby the following is also true:* $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*) \leq \mathcal{L}(\boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*)$.

Proof: The Lagrangian relaxation framework $\mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda})$ forms the upper bound on the optimal value of the objective function. Suppose $\{\boldsymbol{x}^*, \boldsymbol{p}^*\}$ is the optimal solution from (3.13) for a corresponding optimal dual solution vector $\boldsymbol{\lambda}^*$. By definition, the value of the objective function for any feasible solution is given by the following [91]-[92], [108]:

$$\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}) \leq \mathcal{L}(\boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*) \; = \; \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*) - (\boldsymbol{\lambda}^*)^{\mathrm{T}} \times \boldsymbol{g}(\boldsymbol{x}^*, \boldsymbol{p}^*). \tag{3.16}$$

From the KKT complementary slackness property, it follows from (3.16) that the following holds: $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}) \leq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$, whereby $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$ is the solution obtained by minimizing the upper bound of the Lagrangian function defined in (3.13). From which it suffices then, to infer that any other feasible solution for the average sum capacity must lie below, or equal to the value given by $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$, which concludes the proof. ∎

Then the partial derivatives of the Lagrangian function $\mathcal{L}(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{\lambda})$ from (3.13) are taken with respect to the primal variables $\{p_i, x_i\}$ to derive the KKT conditions, which are necessary and sufficient for optimality towards solving the optimization problem, and expressed as follows for $\{\forall i \in \boldsymbol{\mathcal{M}}\}$:

$$\left(\frac{x_i \bar{h}_i}{x_i + p_i h_i}\right)(1 + \lambda_2 + \lambda_3) - \lambda_1 x_i = 0, \tag{3.17a}$$

$$\left(\ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right) - \left(\frac{p_i \bar{h}_i}{x_i + p_i \bar{h}_i}\right)\right)(1 + \lambda_2 + \lambda_3) - \lambda_1 p_i - \lambda_2 \beta = 0, \tag{3.17b}$$

$$\lambda_1 \left(\sum_{i=1}^{M} P_i - P_{\max}\right) = 0, \tag{3.17c}$$

$$\lambda_2 \left(R_\phi - \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right)\right) = 0, \tag{3.17e}$$

$$\lambda_3 \left(\beta \sum_{i=1}^{M} x_i - \sum_{i=1}^{M} x_i \ln\left(1 + \frac{p_i \bar{h}_i}{x_i}\right)\right) = 0, \tag{3.17f}$$

where (3.17a) is derived from $\partial \mathcal{L}(\cdot)/\partial p_i$, (3.17b) from $\partial \mathcal{L}(\cdot)/\partial x_i$ and (3.17c)-(3.17f) from the constraints in (C3.1)-(C3.3) respectively. Then based on the KKT conditions, for a given feasible channel selection vector $\boldsymbol{x}$ and the dual variables vector $\boldsymbol{\lambda}$, the closed-form expression for the optimal power distribution vector $\boldsymbol{p}^*$ that maximizes the objective function for the average sum channel capacity for assembled channels is derived by solving (3.17a) for p_i, together with the restriction from (C3.1) that only nonnegative transmit power can be assigned. The closed-form solution is therefore given by the following expression:

$$p_i^*(x_i, \boldsymbol{\lambda}) = \max\left\{0, \ \left[\left(\frac{1 + \lambda_2 + \lambda_3}{\lambda_1}\right) - \frac{x_i}{\bar{h}_i}\right]\right\}, \quad \forall i \in \boldsymbol{\mathcal{M}}. \tag{3.18}$$

Accordingly, the expression for optimal transmit power presented in (3.18) follows a well-known waterfilling (WF) technique [82], [91]-[92], [108], [111], based on which it can be deduced that more power is allocated to the channels that have a relatively high channel power gain, hence good SNR. Moreover, iterative search methods such as Newton-Raphson technique can be employed to determine the relaxed channel selection vector variable as follows [108]:

$$x_i^{(k+1)} = x_i^{(k)} - \delta_f \frac{f\left(x_i^{(k)}\right)}{f'\left(x_i^{(k)}\right)}, \tag{3.19}$$

where k denotes an iteration index, δ_f is the Newton function step-size, $f\left(x_i^{(k)}\right)$ is formulated as a partial derivative of the Lagrangian from (3.13) with respect to the channel selection variables x_i's $f\left(x_i^{(k)}\right) = \partial\, \mathcal{L}(\cdot)/\partial x_i$, which is simplified from (3.17b) as follows:

$$f\left(x_i^k\right) = \ln\left(1 + \frac{p_i\bar{h}_i}{x_i}\right) - \left(\frac{p_i\bar{h}_i}{x_i + p_i\bar{h}_i}\right) - \left(\frac{\lambda_1 p_i + \lambda_2\beta}{1 + \lambda_2 + \lambda_3}\right), \tag{3.19a}$$

and $f'\left(x_i^{(k)}\right)$ is the derivative of $f\left(x_i^{(k)}\right)$given by

$$f'\left(x_i^{(k)}\right) = \frac{p_i\bar{h}_i}{x_i + p_i\bar{h}_i}\left\{\frac{1}{x_i + p_i\bar{h}_i} - \frac{1}{x_i}\right\}. \tag{3.19b}$$

Alternative to the Newton method, other iterative search methods such as the interior-point search algorithm can be applied to determine the optimal solution for channel selection [112]. Moreover, the optimal dual solution that minimizes (3.15) is obtained by using a subgradient based search method. Although fairly slow in convergence, subgradient methods generally provide low complexity solutions [91]-[92]. The search direction for optimal dual variables can be obtained directly from the dual function, based on whether the constraints have been violated or not. For instance, in the case of power allocation constraint, λ_1 is increased if $\left(P_{\max} < \sum_{\forall i \in \mathcal{M}} x_i p_i\right)$, or decreased otherwise. Thus, the Lagrangian dual variables are iteratively updated as [91]:

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} + \delta_\lambda\left(g_i(\boldsymbol{x}, \boldsymbol{p})\right), \qquad i = 1, \cdots, m \tag{3.20}$$

where δ_λ denotes a step-size for m contraints, and $g_i(\boldsymbol{x}, \boldsymbol{p})$ represents a constraint function. The choice of step-size affects complexity of the optimization problem [91]. At the initial stage, the primal variables $\{\boldsymbol{x}, \boldsymbol{p}\}$ and the dual variables $\boldsymbol{\lambda}$ are arbitrarily initialized to feasible values, and then iteratively updated until convergence towards a feasible optimal solution $\{\ \boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*\ \}$. In principle, if the feasible set for the optimization problem is empty, there is no channel selection vector and power allocation that exist such that all the constraints are not violated. Based on the relaxed optimal solution from the Lagrangian framework, the following are the properties that also hold for establishing the solution to the original MINLP problem:

Property 1: *The objective value of the relaxed problem for (3.12) provides the upper bound to the objective value of the original MINLP problem.*

Property 2: *For any feasible solution to the original MINLP problem, there is a corresponding feasible solution for the relaxed problem, but the opposite does not necessarily hold.*

Property 3: *In the case of infeasibility for the optimization problem, an infeasible relaxed problem indicates infeasibility for the original MINLP problem.*

Then the branch-and-bound (BnB) technique is presented in the next subsection to develop the solution to the original MINLP problem from the relaxed optimal solution.

3.4.2 Branch and Bound with Sequential Fixing

The main objective in this subsection is the recovery of the primal optimal solution from the relaxed optimal solution. An algorithm that preserves the discrete nature of the channel selection vector through a systematic enumeration on the relaxed x_i variables is therefore developed. In particular, the accelerated BnB algorithm with SF is employed, for which the details are presented in Algorithm 3-1 on the next page. The algorithm is executed by decomposing (*i.e.* partitioning) a problem set $\boldsymbol{\mathcal{X}}$ into smaller sets of subproblems $\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{X}}_0 \cup \boldsymbol{\mathcal{X}}_1 \cup \cdots \cup \boldsymbol{\mathcal{X}}_M$, which therefore facilitates a divide and conquer technique that reduces the complexity of the otherwise nondeterministic combinatorial problem [79], [82], [108], [113]-[114]. Thus, channel selection vector comprises partially fixed variables ($\bar{\boldsymbol{x}}$) and the free variables ($\hat{\boldsymbol{x}}$). This is imposed by fixing one of the fractional (*i.e.* non binary-integer) variables from the relaxed solution such that $x_i \leftarrow 0$ and $x_i \leftarrow 1$ as an additional constraint. The BnB technique facilitates partial enumeration to solve the original MINLP problem without explicit enumeration of the entire feasible discrete solution space $\{0,1\}^M$, which is otherwise in a worse case, difficult to solve in polynomial time, more especially as the number of channels increases [79], [108]. At the root node, the upper-bound (UB) for the optimal solution is directly obtained from the relaxed solution.

ALGORITHM 3-1: THE BRANCH-AND-BOUND WITH SEQUENTIAL FIXING

01: **Input**: $\mathcal{R}_\phi$, β, $P_{\max}$, $\bar{\boldsymbol{h}}_M$
02: **Output**: $\{\boldsymbol{x}^*, \boldsymbol{p}^*\}$ and $\bar{\mathcal{R}}_{\mathcal{K}}^*$
03: **initialize:** $\bar{\boldsymbol{x}} \leftarrow \emptyset$, $\hat{\boldsymbol{x}} \leftarrow \mathbf{1}_M$, $\boldsymbol{x} \leftarrow \{\bar{\boldsymbol{x}}, \hat{\boldsymbol{x}}\}$, $\boldsymbol{p}$, $\boldsymbol{\lambda}$ and ϵ
04: compute $\bar{\mathcal{R}}_{\mathcal{K}}(\hat{\boldsymbol{x}}, \boldsymbol{p})$ for the relaxed optimization problem
05: **if** $\left(\bar{\mathcal{R}}_{\mathcal{K}}(\hat{\boldsymbol{x}}, \boldsymbol{p}) < \mathcal{R}_\phi\right)$ **then**
06: exit: render the optimization problem infeasible.
07: **else if** $\left(\left(\bar{\mathcal{R}}_{\mathcal{K}}(\hat{\boldsymbol{x}}, \boldsymbol{p}) < \beta \sum_{i\in\mathcal{M}} x_i\right) \&\& \left(\bar{\mathcal{R}}_{\mathcal{K}}(\hat{\boldsymbol{x}}, \boldsymbol{p}) < \mathcal{R}_\phi\right)\right)$ **then**
08: **repeat**
09: fix minimum nonzero $x_i = 0$;
10: update $\bar{\boldsymbol{x}} \leftarrow \{\bar{\boldsymbol{x}} \cup x_i\}$, $\hat{\boldsymbol{x}} \leftarrow \{\hat{\boldsymbol{x}}/\bar{\boldsymbol{x}}\}$;
11: recalculate $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p})$;
12: **until** $\left(\max\left\{\mathcal{R}_\phi, \beta \sum_{i\in\mathcal{M}} x_i\right\} \le \bar{\mathcal{R}}_{\mathcal{K}}(\hat{\boldsymbol{x}}, \boldsymbol{p})\right)$
13: **end if**
14: **if** $(x_i \in \{0,1\}, \forall i \in \boldsymbol{x})$ **then**
15: set: $\{\boldsymbol{x}, \boldsymbol{p}\} \leftarrow \{\boldsymbol{x}^*, \boldsymbol{p}^*\}$ and go to *line* 37.
16: **else**
17: **repeat**
18: fix maximum $x_i \in \hat{\boldsymbol{x}} \leftarrow 1$;
19: update $\bar{\boldsymbol{x}} \leftarrow \{\bar{\boldsymbol{x}} \cup x_i\}$, $\hat{\boldsymbol{x}} \leftarrow \{\hat{\boldsymbol{x}}/\bar{\boldsymbol{x}}\}$;
20: **until** $(\sum_{i\in\bar{\boldsymbol{x}}} p_i \le P_{max})$
21: recalculate $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p})$;
22: **if** $(\exists\, x_i \in \hat{\boldsymbol{x}}$: $0 < x_i < 1$, $i = 1, \cdots, |\hat{\boldsymbol{x}}|)$ **then**
23: $\boldsymbol{x}' \leftarrow round(\boldsymbol{x} \leftarrow \bar{\boldsymbol{x}} \cup \hat{\boldsymbol{x}})$
24: set: $UB_\phi \leftarrow \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}), LB_\phi \leftarrow \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}', \boldsymbol{p})$
25: **repeat**
26: determine branching variable: $x_b \leftarrow \max_{\forall i\in\hat{\boldsymbol{x}}} x_i$
27: **for** the two subproblems $x_b \leftarrow \{0,1\}$ **do**
28: update $(\boldsymbol{x} \leftarrow \bar{\boldsymbol{x}} \cup \hat{\boldsymbol{x}} \mid \bar{\boldsymbol{x}} = \{\bar{\boldsymbol{x}} \cup x_b\}, \hat{\boldsymbol{x}} = \{\hat{\boldsymbol{x}}/\bar{\boldsymbol{x}}\})$
29: $\boldsymbol{x}' \leftarrow round(\boldsymbol{x} \leftarrow \bar{\boldsymbol{x}} \cup \hat{\boldsymbol{x}})$
30: calculate: $UB \leftarrow \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}, \boldsymbol{p}), LB \leftarrow \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}', \boldsymbol{p})$
31: **end for**
32: $UB_\phi \leftarrow \min\left\{UB_\phi, \max\left\{UB|_{x_b=0}, UB|_{x_b=1}\right\}\right\}$
33: $LB_\phi \leftarrow \max\left\{LB_\phi, \max\left\{LB|_{x_b=0}, LB|_{x_b=1}\right\}\right\}$
34: **until** $(x_i \in \{0,1\}\ \forall i,\ \ i.e. \hat{\boldsymbol{x}} \leftarrow \emptyset)$
35: **end if**
36: **end if**
37: set: $\{\boldsymbol{x}, \boldsymbol{p}\} \leftarrow \{\boldsymbol{x}^*, \boldsymbol{p}^*\}$, $\bar{\mathcal{R}}_{\mathcal{K}}^* \leftarrow \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$
38: **return** $\{\boldsymbol{x}^*, \boldsymbol{p}^*\}$

The lower-bounds (LBs) are computed by discretizing the relaxed optimal solution through a simple rounding $\boldsymbol{x}' \leftarrow round(\boldsymbol{x})$, which rounds fractional values of the channel selection vector $\boldsymbol{x}$ to the nearest binary integer vector $\boldsymbol{x}'$.

> **Proposition 3-2**: *For a sequence of the nonincreasing upper bounds (UBs) and the nondecreasing lower bounds (LBs) based on the Lagrangian function $\mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda})$, if there exists a feasible solution $\{\boldsymbol{x}, \boldsymbol{p}\}^k$ such that $(\forall x_i \in \boldsymbol{x}) = \{0,1\}$ at k-th node, $(UB_k - LB_k) \leq \epsilon$ for a small approximation error $\epsilon \geq 0$ and $UB_k = \max\{UBs\}$, then $\{\boldsymbol{x}, \boldsymbol{p}\}^k$ is an optimal solution with $\bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*) = UB_k$ for the original MINLP optimization problem.*

The Proposition 3-2 has been derived from the fact that sequences of subproblems for nonincreasing UBs based on the BnB technique are generated, such that the following holds:

$$UB_1 \geq UB_2 \geq \cdots \geq UB_k \geq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*), \tag{3.21}$$

as well as the corresponding sequences of subproblems for nondecreasing LBs that jointly converge towards an optimal solution such that the following holds:

$$LB_1 \leq LB_2 \leq \cdots \leq LB_k \leq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*). \tag{3.22}$$

From the root subproblem, the largest $x_i \in \boldsymbol{x}$ values are selected and set to $x_i \leftarrow 1$, upper bounded by the total transmission power constraint $P_{\max}$ based on (C3.1), *i.e.* select the best channels and sum the corresponding power levels subject to $\{\sum_{\forall i \in \mathcal{M}} p_i \leq P_{\max} |\, \forall x_i = 1\}$. Then the remainder of the channel selection variables are iteratively solved by employing the SF technique until $\boldsymbol{x}$ comprises binary integer values in its entirety (*i.e.* $x_i \in \{0,1\}$). When performing the proposed SF, the largest fractional values of the remaining x_i variables where $\{0 < x_i < 1\}$ are selected as a criterion for branching. This selection criterion is based on the fact that a fractional value that is close to 1 (*i.e.* $x_i \approx 1$) signifies that the channel is likely to be selected for optimal solution, while the value close to zero (*i.e.* $x_i \approx 0$) signifies that the channel is likely to be excluded for optimal solution. In general, the efficiency of BnB algorithms depends mainly on how to obtain the good initial global bounds, as well as the efficient bounding techniques [79], [113]-[114].

3.4.3 Computational Complexity Analysis

The brute-force approach for obtaining the optimal channel selection and power allocation is governed by exponential complexity in the order of the number channels M, given by $\mathcal{O}(M2^M)$ as shown in Table 3-1 [95], [97], [115]. The Lagrangian dual decomposition technique on the other hand, decouples the optimization problem; hence, reduces the otherwise exponential complexity to linear complexity in the number of channels M. Assuming arrival of a single SU service, the search dimension to solve the dual problem requires $\mathcal{O}(\Delta_k \Delta_f)$ computations, which involve Δ_k number of iterations with Δ_f function evaluations each time to determine the dual solution $\boldsymbol{\lambda}^* \geq 0$ that minimizes $\mathcal{D}(\boldsymbol{\lambda})$ in (3.15). Then given $\boldsymbol{\lambda}^*$, the optimal solution for the primal variables $\{\boldsymbol{x}^*, \boldsymbol{p}^*\}$ requires $\mathcal{O}(M)$ computations, which has linear complexity [96], [116]-[118]. The relaxed solution is projected back to the solution for the MINLP problem by applying BnB technique. In general, the complexity of BnB is unpredictable, and depends on the number of visited nodes and pruning operations. In a worse case, conventional BnB algorithm may generally degenerate into exhaustive enumeration search [96]. For the proposed BnB technique in this chapter (*i.e.* BnBKOP), one of the sub-problems is pruned at each node, wherefore $2M(M-1)$ relaxed sub-problems are solved with $\mathcal{O}(M^2)$ computational complexity [119]-[120]. Further, the suboptimal solution (*i.e.* RndKOP) obtained by rounding the relaxed solution requires $\mathcal{O}(M)$ computations. For the fixed-$\mathcal{K}$ ChA schemes with WF power (*i.e.* FixedKOP) and equal power (*i.e.* FixedKEP) allocation, the complexity is given by $\mathcal{O}(\mathcal{K})$ and $\mathcal{O}(1)$ respectively, independent of the number of channels M.

TABLE 3-1: COMPUTATIONAL COMPLEXITY FOR SINGLE-SU CHA SCHEMES

Algorithm	**Order of operations**	
	Initialization	Computational Complexity
Exhaustive Search	—	$\mathcal{O}(M2^M)$
Relaxed	$\mathcal{O}(\Delta_k \Delta_f)$	$\mathcal{O}(M)$
BnBKOP	$\mathcal{O}(\Delta_k \Delta_f M)$	$\mathcal{O}(M^2)$
RndKOP	$\mathcal{O}(\Delta_k \Delta_f M)$	$\mathcal{O}(M)$
FixedKOP	—	$\mathcal{O}(\mathcal{K})$
FixedKEP	—	$\mathcal{O}(1)$

3.5 Simulation Results and Discussion

3.5.1 Simulation Model

In this work, an ad-hoc CRN is considered, where SU nodes can access multiple channels for overlay transmissions. It is assumed that the SU nodes can perfectly sense free PN channels, where a total of 16 channels are assumed to be available for SU transmissions. To accommodate different values of the *i.n.i.d.* average channel gains, random $\bar{h}_i$ values are assumed across the channels for frequency selective fading, from which the instantaneous channel realizations are generated based on the exponential distribution as defined in Section 3.3.1 of this thesis. Performance evaluation is based on Monte-Carlo simulations, and the simulation results are averaged over 10^4 channel realizations. Arrival of a single SU node is assumed at the time; with simulation parameters shown in Table 3-2. For the fixed-$\mathcal{K}$ ChA order with equal power allocation (*i.e.* FixedKEP), power per channel is simply obtained by dividing the total transmit power by the number of channels, while in the case of fixed-K ChA order with optimal power allocation (*i.e.* FixedKOP), power per channel is given by the WF based power allocation in (3.17). Two variants of the adaptive ChA schemes are derived from the relaxed solution (*i.e.* Relaxed). First, the optimal solution (*i.e.* BnBKOP) for the original MINLP problem is obtained as described in Section 3.4. Second, the suboptimal solution (*i.e.* RndKOP) is obtained by a simple rounding heuristic, which rounds channel selection vector to the nearest binary integer vector. However, such rounding may lead to infeasible solutions that are also far from the actual solutions, therefore resulting in constraints violation, [26], [79].

TABLE 3-2: SIMULATION PARAMETERS FOR OVERLAY SPECTRUM ACCESS

Parameters	Value
Number of frames	10^4
Total Number of PU channels	16
Total SU transmit power	1W
Collision probability threshold	$0.1 - 0.2$
SU QoS requirement on capacity	6.5Kbps
Bandwidth ($\mathcal{B}$)	1 MHz

3.5.2 Results and Discussion

In this work, performance evaluation is mainly based on channel capacity, outage probability and collision probability as a function of PU arrival rates. Outage occurs when the obtained average channel capacity $\bar{\mathcal{R}}_{\mathcal{K}}^{*}$ is less than the minimum requirement R_{ϕ}. Collision probability denotes the likelihood that a new PU service arrives on any of the channels assembled by SU node, before the SU node finishes transmission. Fig. 3-4 and Fig. 3-5 present the average capacity against the increasing rate of PU arrivals, with the collision probability threshold kept at 0.1 and 0.2 respectively. At low PU arrivals, optimal ChA scheme based on BnB achieves significant improvement on capacity as a result of high ChA order. Then the capacity drops with the increasing PU arrivals in order to avoid collisions. At high PU arrivals, spectrum is hardly available for SU transmissions, such as above $\{\lambda_{\text{pu}} = 0.8\}$ in Fig. 3-4, and above $\{\lambda_{\text{pu}} = 1.6\}$ in Fig. 3-5. The results also illustrate that relaxing the collision constraint provides a wider range for PU arrival rates, within which SU nodes are allowed to access PN spectrum.

Outage probability increases as capacity drops with the increasing PU arrivals, even worse when collision constraint is tight as shown in Fig. 3-6 and Fig. 3-7. At low PU arrivals, the optimal ChA scheme based on the BnB technique has the lowest outage probability. This is also consistent with the results for average capacity. Outage probability increases at high PU arrival rates because of the decrease in capacity due to low spectrum access opportunities for SU nodes. Moreover, Fig. 3-8 and Fig. 3-9 present results for collision probability. As a result of taking PU activities into consideration, the optimal ChA scheme based on BnB technique avoids collisions by reducing ChA order $\mathcal{K}$; while the collision probability for the ChA schemes with the fixed $\mathcal{K}$ (*i.e.* FixedKEP and FixedKOP) increases with the increase in PU arrivals. Therefore at high rate of PU arrivals, the ChA schemes with fixed $\mathcal{K}$ may be subject to forced terminations and increased channel witching frequency due to high collisions, which therefore degrade performance of SU nodes.

In general, the results presented in this chapter reveal that taking PU activity patterns into consideration for ChA schemes is imperative to improve performance in overlay CRNs. Otherwise, the results reveal that high PU arrival rates degrade performance of ChA schemes. Accordingly, adaptive ChA schemes with quick reaction to PU arrival rates are relatively unsusceptible to performance degradation. Henceforth, spectrum characterization based on the quality of wireless channels and PU activities is significant to improve performance of ChA in CRNs.

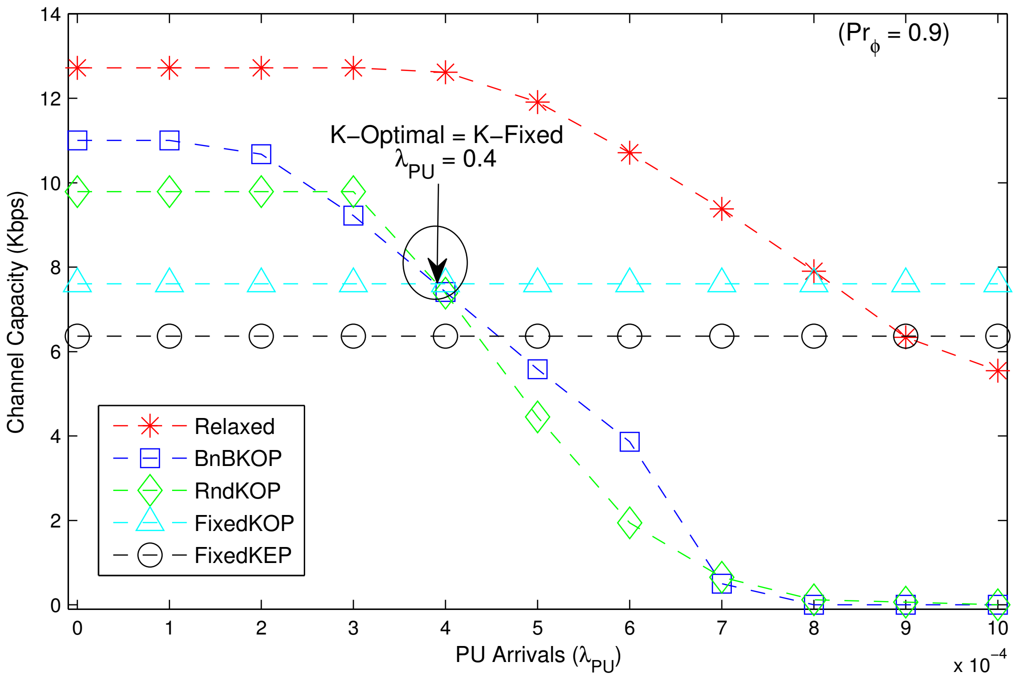

Figure 3-4: SU average capacity for increasing PU arrival rates at $\mathcal{P}_{\phi} = 0.9$.

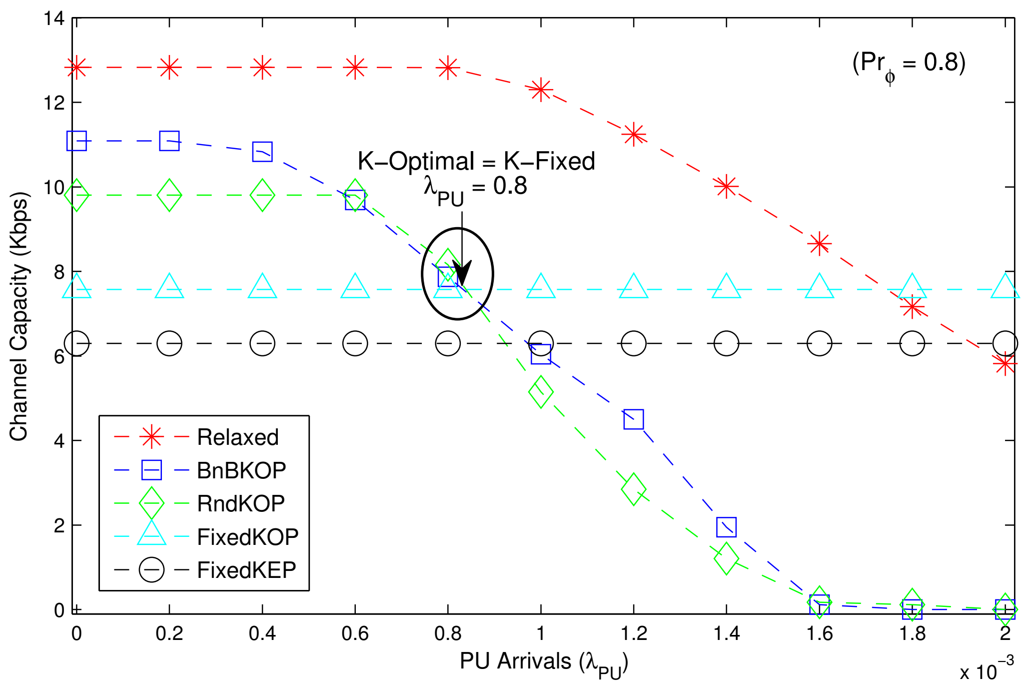

Figure 3-5: SU average capacity for increasing PU arrival rates at $\mathcal{P}_{\phi} = 0.8$.

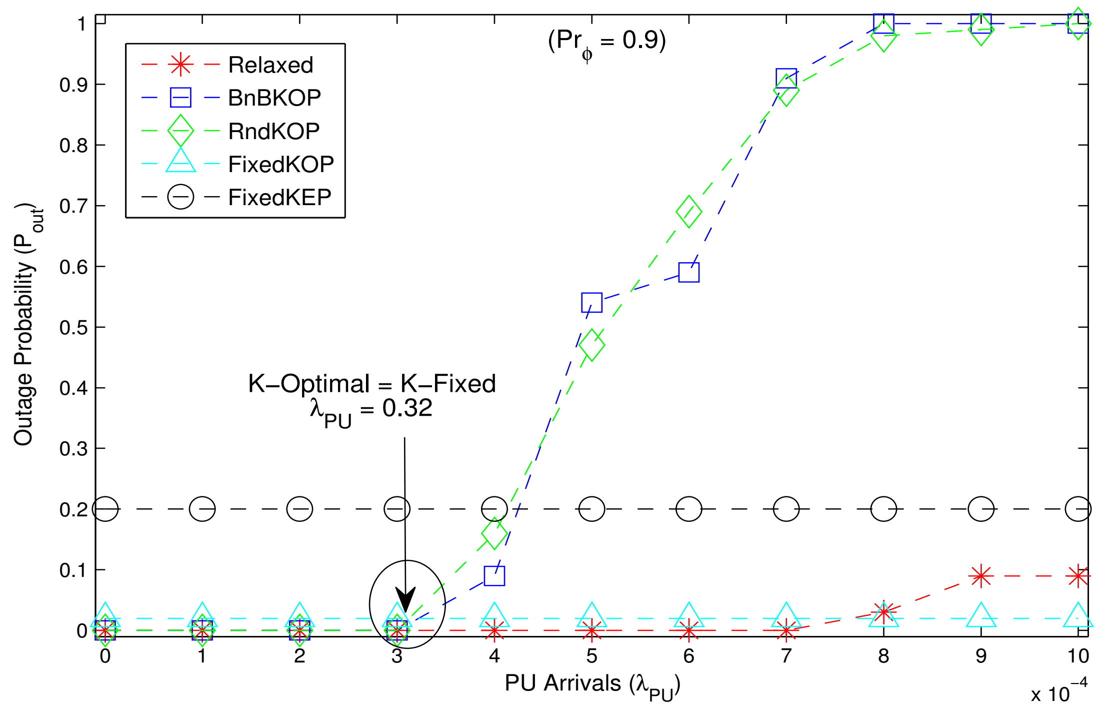

Figure 3-6: SU outage probability for increasing PU arrival rates at $\mathcal{P}_\phi = 0.9$.

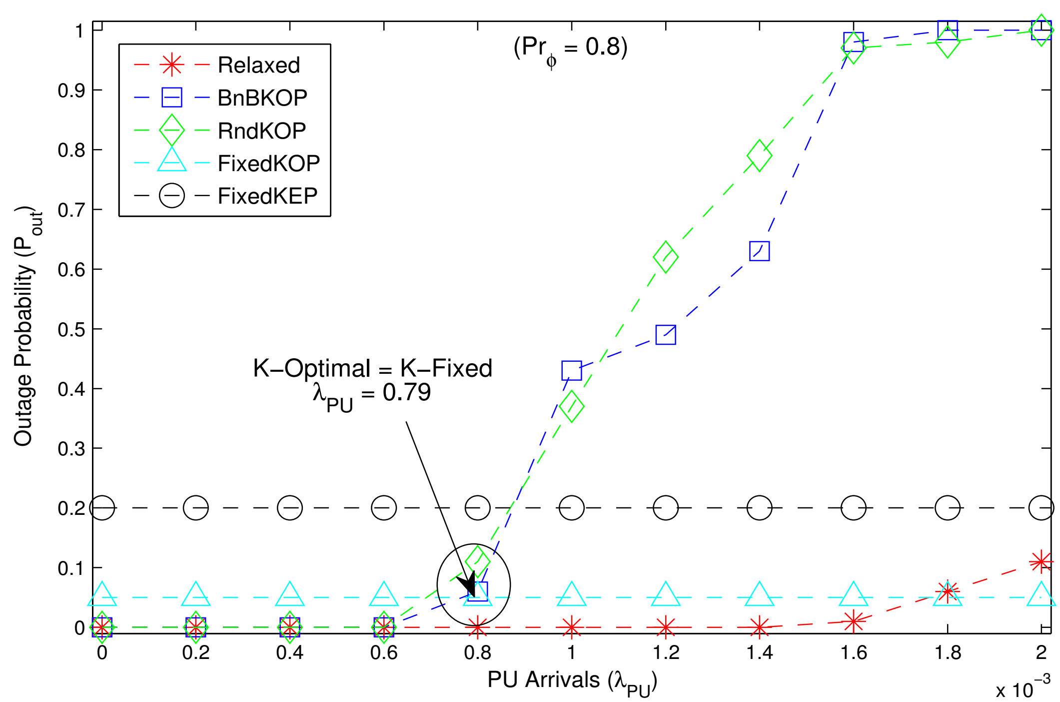

Figure 3-7: SU outage probability for increasing PU arrival rates at $\mathcal{P}_\phi = 0.8$.

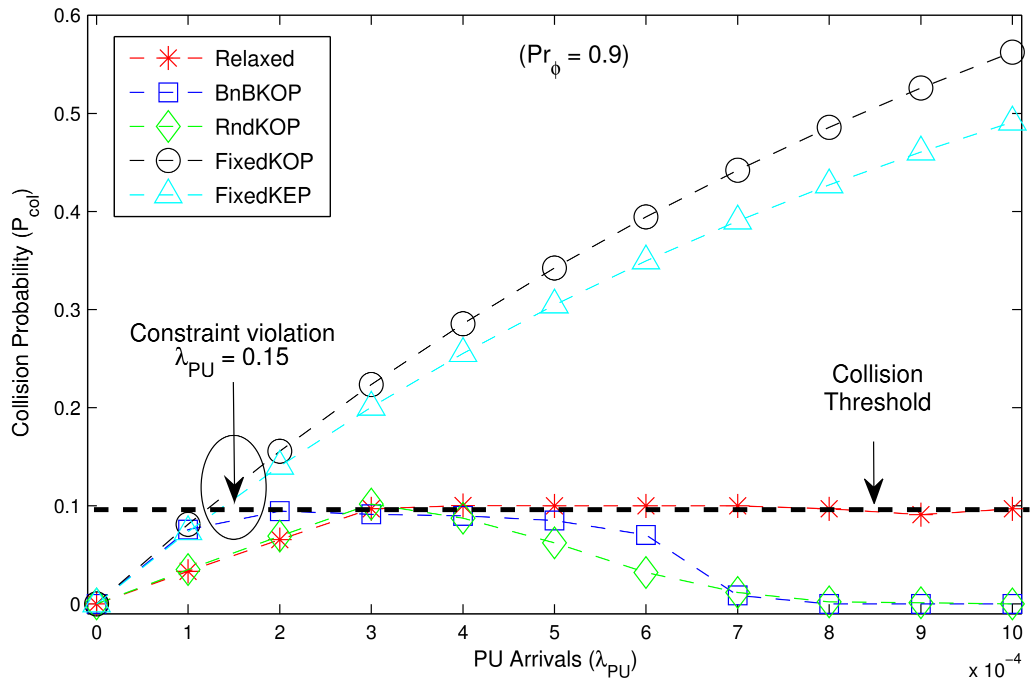

Figure 3-8: SU collision probability for increasing PU arrival rates at $\mathcal{P}_\phi = 0.9$.

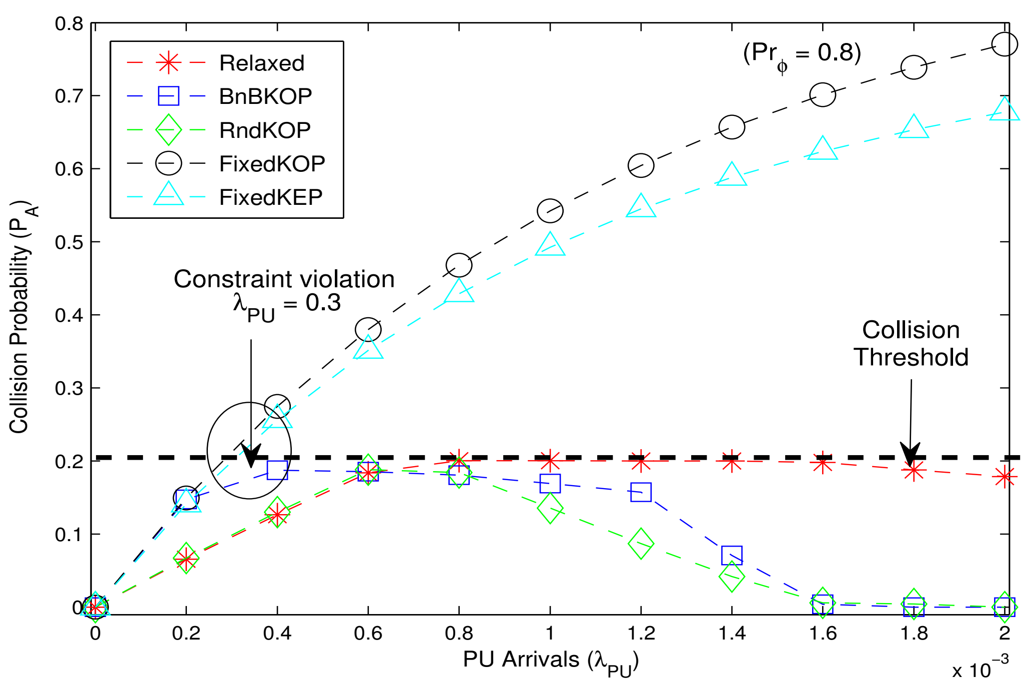

Figure 3-9: SU collision probability for increasing PU arrival rates at $\mathcal{P}_\phi = 0.8$.

3.6 Chapter Conclusion

In this chapter, a criterion to determine the optimal channel selection and power distribution for ChA has been developed, as opposed to the fixed number of channels without power optimization from the existing works in literature. Lagrangian optimization framework has been employed to derive the relaxed optimal solution, based on which accelerated BnB with SF was used to determine the solution for the original MINLP problem. In general, the presented simulation results illustrate that performance of ChA schemes is largely affected by PU activity patterns, wherefore significant performance degradation for SU services is encountered at high PU arrival rates. It can therefore be deduced from the presented simulation based performance evaluation that performance benefits of ChA schemes in overlay spectrum access are generally realized at low PU activity patterns. Moreover, adaptive ChA schemes with quick reaction to PU activities are relatively unsusceptible to performance degradation due to collisions. Accordingly, spectrum characterization based on time scale variations of wireless channels and the PU activity patterns is not insignificant to improve performance of ChA schemes, but comprises one of the crucial spectrum profiling techniques for development of dynamic and adaptive ChA schemes in CRNs.

CHAPTER FOUR

4 PERFORMANCE MODELLING AND EVALUATION

4.1 Introduction

THIS chapter extends the work presented in Chapter 3 by developing analytical models to quantify performance of ChA schemes over fading wireless channels, in terms of: (*i*) *average channel capacity*, (*ii*) *outage probability*, and (*iii*) *forced termination probability* for SU services. First, the statistical characterization of assembled channels is developed for the probability density function (PDF) and the cumulative distribution function (CDF). The characterization provides the foundation on which the compact closed-form analytical models are derived. In particular, Mellin integral transform theorems are employed to develop the closed-form analytical models, and expressed in terms of the generalized upper incomplete Fox-$\mathcal{H}$ function. Further, the preciseness of the analytical models is confirmed by cross validation through extensive simulations. In essence, the presented performance models uncover the underlying trade-offs towards development of efficient ChA schemes for spectrum sharing wireless networks. Part of the work presented in this chapter has been submitted in part, to the *IEEE Transactions on Vehicular Technology*.

The rest of the chapter is organised as follows: Section 4.2 provides the related works in literature, followed by Section 4.3 which presents the statistical characterization of assembled channels. Section 4.4 presents the derivations for the analytical performance models and evaluation. Then Section 4.5 presents both the simulation results and the analytical results for cross-validation and verification; and finally, the concluding remarks are presented in Section 4.6.

4.2 Related Works

The existing works on performance analysis and evaluation of ChA schemes in literature are mainly based on CTMC models [27], [32], [72]-[74], [76]-[77]. In [27], closed-form expressions for capacity of dynamic ChA schemes in quasistationary regimes were developed based on CTMC modeling. Another study that investigated the capacity upper bound of ChA schemes was presented in [73], where the authors developed closed-form expressions for the maximum capacity that can be obtained through ChA schemes in quasistationary PU activity patterns. In [77], CTMC based analysis has been employed to model the interaction of PU and SU nodes, based on which analytical models for throughput, blocking probability and forced termination probability for SU transmissions were developed. Further, CTMC models that included closed-form expression for spectrum utilization were developed in [78] for ChA schemes with priority-based queues.

The reported studies have mainly focused on MAC issues for development of analytical models in multiuser systems. None of these works have considered analytical models to quantify performance of ChA schemes over fading channels. In general, the fading and time varying nature of wireless channels dictate performance of ChA schemes, which has been largely overlooked. In the context of performance modeling for multicarrier systems, the studies in [121] and [109] developed the CDF and PDF for the product of the shifted Gamma variates and product of the shifted exponential variates respectively, based on which closed-form expressions for outage capacity for multicarrier systems was developed. The established models from the studies reported in [109] and [121] provide insights for development of analytical models to quantify performance of ChA schemes.

4.3 Channel Characterization of Assembled Channels

This section presents the statistical characterization of assembled channels as a foundation on which analytical performance models for ChA schemes in overlay spectrum access are derived. Thus, characterization of instantaneous sum channel capacity $\mathcal{R}_{\mathcal{K}} = \sum_{\forall i \in \boldsymbol{\mathcal{A}}} \mathcal{B} \log_2(1+\gamma_i)$ is required to quantify performance of ChA schemes, where $\gamma_i = p_i h_i$ denotes SNR per channel. In this case, capacity is a random variable due to the time varying nature of wireless channels, as well as the random number of assembled channels in the case of adaptive ChA schemes. Obtaining the joint statistical distribution of the sum capacity is therefore required.

From the laws of logarithms, it is known that: $\log_2(u) + \log_2(v) = \log_2(uv)$. Then the following can be established for sum capacity: $\mathcal{R}_{\mathcal{K}} = \sum_{i=1}^{\mathcal{K}} \mathcal{B} \log_2(1+\gamma_i) = \mathcal{B} \log_2 \prod_{i=1}^{\mathcal{K}}(1+\gamma_i)$. Suppose a random variable is defined by $z_i = (1+\gamma_i)$, where z_i is characterized by the shifted exponential distribution based on the distribution of γ_i. The product of $\mathcal{K}$ *i.n.i.d.* random variables can then be defined as: $Y_{\mathcal{K}} \triangleq \prod_{i=1}^{\mathcal{K}} z_i = \prod_{i=1}^{\mathcal{K}}(1+\gamma_i)$. It follows that: $\mathcal{R}_{\mathcal{K}} = \mathcal{B} \log_2(Y_{\mathcal{K}}) \Rightarrow Y_{\mathcal{K}} = 2^{(\mathcal{R}_{\mathcal{K}}/\mathcal{B})}$, whereby the random variable $Y_{\mathcal{K}}$ is distributed according to the PDF of the product of $\mathcal{K}$ *i.n.i.d.* random variables. Then given that γ_i follows exponential distribution, the change-of-variable technique can be employed to derive the PDF of $z_i = (1 + \gamma_i)$ from (3.3) in Chapter 3 as follows:

$$f_z(z_i) = \frac{1}{\overline{\gamma_i}} \exp\left(-\frac{z_i - 1}{\overline{\gamma_i}}\right) \mathcal{U}(z_i - 1). \tag{4.1}$$

The joint statistical distribution of $Y_{\mathcal{K}}$ is characterized by the multiplicative convolution of the PDF's for the z_i's. Thus, the closed-form expression for PDF of $Y_{\mathcal{K}}$ is derived and presented in the form of a generalized upper incomplete Fox-$\mathcal{H}$ function in sequel [109], [121]-[124]. From (4.1), the Mellin-transform of the PDF of $\{z_i \mid i = 1,2, \cdots, \mathcal{K}\}$ is given by the following [109]:

$$\mathbb{M}_s\{f_z(z_i)\} = \int_0^\infty z_i^{s-1} \left(\frac{1}{\overline{\gamma_i}} \exp\left(-\frac{z_i - 1}{\overline{\gamma_i}}\right) U(z_i - 1)\right) dz_i \; = \; \exp\left(\frac{1}{\overline{\gamma_i}}\right) \overline{\gamma_i}^{\,s-1}\, \Gamma\left(s, \frac{1}{\overline{\gamma_i}}\right), \tag{4.2}$$

whereby s is a complex number, and $\Gamma(s, \alpha) \triangleq \int_\alpha^\infty {z_i}^{s-1} \exp(-z_i) dz_i$ defines the complementary incomplete gamma function [109]. Then based on (4.2), the Mellin-transform of the PDF of assembled PU channels $Y_{\mathcal{K}}$ can be derived by applying the property of the multiplicative convolution of individual PDF's for each of the independent random variables z_i's, wherefore the applicable Mellin-transform theorem in this case can be stated as follows [112]:

Theorem 4-1: *The Mellin transform of a product of independent random variables is equal to the product of the individual Mellin transforms of the independent random variables, and the PDF of the product is given by inverse of the Mellin transform* (Proof in Appendix-A).

Then given that $Y_{\mathcal{K}} \triangleq \prod_{i=1}^{\mathcal{K}} z_i$ for the product of shifted random variables, the Mellin-transform of the PDF of $Y_{\mathcal{K}}$ is obtained by applying Theorem 4-1 as follows:

$$\mathbb{M}_s\{f_{Y_{\mathcal{K}}}(y)\} = \prod_{i=1}^{\mathcal{K}} \mathbb{M}_s\{f_z(z_i)\}. \tag{4.3}$$

Hence, the Mellin transform provides a versatile technique to derive statistical distributions that involve product of random variables. Then using (4.2) in (4.3) results in:

$$\mathbb{M}_s\{f_{Y_\mathcal{K}}(y)\} = \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_\iota}}\right) \prod_{i=1}^{\mathcal{K}} \left(\overline{\gamma_\iota}^{s-1}\, \Gamma\left(s, \frac{1}{\overline{\gamma_\iota}}\right)\right). \tag{4.4}$$

From (4.4), the PDF of $Y_\mathcal{K}$ can be obtained by taking the inverse of the Mellin-transform as follows:

$$f_{Y_\mathcal{K}}(y) = \frac{1}{2\pi i} \oint_c \; y^{-s} \mathbb{M}_s\{f_{Y_\mathcal{K}}(y)\} ds, \tag{4.5}$$

where $i \triangleq \sqrt{-1}$ and c denotes the contour of integration on a complex plane. From (4.5), the closed-form expression for the PDF $f_{Y_\mathcal{K}}(y)$ is derived as follows:

$$f_{Y_\mathcal{K}}(y) = \frac{1}{2\pi i} \oint_c \; y^{-s} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_\iota}}\right) \prod_{i=1}^{\mathcal{K}} \left(\overline{\gamma_\iota}^{s-1}\, \Gamma\left(s, \frac{1}{\overline{\gamma_\iota}}\right)\right) ds, \tag{4.6a}$$

$$= \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_\iota}}\right) \left\{ \frac{1}{2\pi i} \oint_c \; y^{-s} \left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_\iota} \right)^{s-1} \prod_{i=1}^{\mathcal{K}} \Gamma\left(s, \frac{1}{\overline{\gamma_\iota}}\right) ds \right\}, \tag{4.6b}$$

$$= \frac{1}{\left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_\iota}\right)} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_\iota}}\right) \left\{ \frac{1}{2\pi i} \oint_c \; \left(\frac{y}{\prod_{i=1}^{\mathcal{K}} \overline{\gamma_\iota}} \right)^{-s} \prod_{i=1}^{\mathcal{K}} \Gamma\left(s, \frac{1}{\overline{\gamma_\iota}}\right) ds \right\}. \tag{4.6c}$$

The PDF is then expressed in terms of the upper incomplete Fox-$\mathcal{H}$ function, given by the following expression (the definition of the Fox-$\mathcal{H}$ presented in Appendix A) [122]-[124]:

$$f_{Y_\mathcal{K}}(y) = \frac{1}{\left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_\iota}\right)} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_\iota}}\right) \mathcal{H}_{0,\mathcal{K}}^{\mathcal{K},0}\left[\frac{y}{\prod_{i=1}^{\mathcal{K}} \overline{\gamma_\iota}} \middle| \begin{matrix} ---\\ \left(0\,,1\,,\frac{1}{\overline{\gamma_1}}\right),\left(0\,,1\,,\frac{1}{\overline{\gamma_2}}\right),\cdots,\left(0\,,1\,,\frac{1}{\overline{\gamma_\mathcal{K}}}\right) \end{matrix} \right]. \tag{4.7}$$

Furthermore the CDF $F_{Y_\mathcal{K}}(y_{th})$, which in this case is the probability that a random variable y falls below y_{th}, can be obtained from the distribution of $Y_{\mathcal{K}}$ from (4.7) by using the integral properties of the Fox-$\mathcal{H}$ function for $F_{Y_{\mathcal{K}}}(y_{th}) \triangleq \int_0^{y_{th}} f_{Y_{\mathcal{K}}}(y)dy$, given by the following:

$$F_{Y_{\mathcal{K}}}(y_{th}) = \int_0^{y_{th}} \frac{1}{\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right)} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\bar{\gamma}_i}\right) \mathcal{H}_{0,\mathcal{K}}^{\mathcal{K},0}\left[\frac{y}{\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i}\middle| \begin{matrix} - - - \\ \left(0,1,\frac{1}{\bar{\gamma}_1}\right),\cdots,\left(0,1,\frac{1}{\bar{\gamma}_{\mathcal{K}}}\right)\end{matrix}\right] dy, \tag{4.8a}$$

$$= \frac{1}{\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right)} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\bar{\gamma}_i}\right) \int_0^{y_{th}} \mathcal{H}_{0,\mathcal{K}}^{\mathcal{K},0}\left[\frac{y}{\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i}\middle| \begin{matrix} - - - \\ \left(0,1,\frac{1}{\bar{\gamma}_1}\right),\cdots,\left(0,1,\frac{1}{\bar{\gamma}_{\mathcal{K}}}\right)\end{matrix}\right] dy, \tag{4.8b}$$

$$= \frac{1}{\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right)} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\bar{\gamma}_i}\right)\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right) \mathcal{H}_{1,\mathcal{K}+1}^{\mathcal{K},1}\left[\frac{y_{th}}{\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right)}\middle| \begin{matrix} (1,1,0) \\ \left(1,1,\frac{1}{\bar{\gamma}_1}\right),\cdots,\left(1,1,\frac{1}{\bar{\gamma}_{\mathcal{K}}}\right),(0,1,0)\end{matrix}\right]. \tag{4.8c}$$

The CDF of $Y_{\mathcal{K}}$ is also expressed in terms of the Fox-$\mathcal{H}$ function as follows:

$$F_{Y_{\mathcal{K}}}(y_{th}) = \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\bar{\gamma}_i}\right) \mathcal{H}_{1,\mathcal{K}+1}^{\mathcal{K},1}\left[\frac{y_{th}}{\left(\prod_{i=1}^{\mathcal{K}} \bar{\gamma}_i\right)}\middle| \begin{matrix} (1,1,0) \\ \left(1,1,\frac{1}{\bar{\gamma}_1}\right),\cdots,\left(1,1,\frac{1}{\bar{\gamma}_{\mathcal{K}}}\right),(0,1,0)\end{matrix}\right]. \tag{4.9}$$

Also, the Fox-$\mathcal{H}$ function representation provides an efficient framework for simplifying otherwise a tedious process dealing with non-uncomplicated expressions to develop the closed-form analytical models for evaluating the performance of ChA schemes in fading wireless channels. The analytical models for the aforementioned performance metrics are derived in the next section.

4.4 Analytical Performance Metrics

4.4.1 Average Channel Capacity

Due to fading in wireless channels, channel power gains vary from one frame to another during transmissions. If one or more of the selected channels experiences extremely deep fading, the resulting sum capacity from all the channels is affected, in which case the capacity may also fall below the minimum capacity threshold that defines the QoS requirement. The mean channel capacity $\bar{\mathcal{R}}_{\mathcal{K}}$ for $\mathcal{K}$ assembled channels is given by the following [125]:

$$\bar{\mathcal{R}}_{\mathcal{K}} = \frac{\mathcal{B}}{\ln(2)} \sum_{i=1}^{\mathcal{K}} \int_0^{\infty} \ln(1+\gamma_i)\, f_{\gamma}(\gamma_i) d\gamma_i \quad \langle b/s \rangle, \tag{4.10a}$$

$$= \frac{\mathcal{B}}{\ln(2)} \sum_{i=1}^{\mathcal{K}} \int_0^{\infty} \ln(1+\gamma_i) \left(\frac{1}{\bar{\gamma}_i} \exp\left(-\frac{\gamma_i}{\bar{\gamma}_i}\right)\right) d\gamma_i \quad \langle b/s \rangle. \tag{4.10b}$$

Then the logarithm function can be expressed in the form of a Meijer-G function $G[\cdot]$ as follows:

$$\ln(1+\gamma_i) = G_{2,2}^{1,2}\left(\gamma_i \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right.\right). \tag{4.11}$$

Also the Meijer-G function representation for the exponential function is given by the following:

$$\exp\left(-\frac{\gamma_i}{\bar{\gamma}_i}\right) = G_{0,1}^{1,0}\left[\frac{\gamma_i}{\bar{\gamma}_i} \left| \begin{matrix} - \\ 0 \end{matrix} \right.\right]. \tag{4.12}$$

Then for simplification in solving the integral in (4.10), the expression can be transformed as the integral of the product of two Meijer-G functions as follows [122], [124]:

$$\bar{\mathcal{R}}_{\mathcal{K}} = \frac{\mathcal{B}}{\ln(2)} \sum_{i=1}^{\mathcal{K}} \int_0^{\infty} \frac{1}{\bar{\gamma}_i} \left(G_{2,2}^{1,2}\left[\gamma_i \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right.\right] G_{0,1}^{1,0}\left[\frac{\gamma_i}{\bar{\gamma}_i} \left| \begin{matrix} - \\ 0 \end{matrix} \right.\right]\right) d\gamma_i\,. \tag{4.13}$$

The integral property of the product of two Meijer-G functions is applied to establish the following:

$$\bar{\mathcal{R}}_{\mathcal{K}} = \frac{\mathcal{B}}{\ln(2)} \sum_{i=1}^{\mathcal{K}} \left(\frac{1}{\bar{\gamma}_i} G_{2,3}^{3,1}\left[\frac{1}{\bar{\gamma}_i} \left| \begin{matrix} -1,0 \\ 0,-1,-1 \end{matrix} \right.\right]\right). \tag{4.14}$$

The average sum capacity for ChA schemes is mainly governed by the number of assembled channels, together with the average SNR per channel as illustrated in (4.14).

4.4.2 Outage Probability

Outage probability is one of the fundamental standard metrics to measure performance over fading channels. In this thesis, outage probability defines the probability that the achievable sum capacity for assembled channels falls below a predefined threshold; which in particular, is the event that $\{\mathcal{R}_{\mathcal{K}} < \mathcal{R}_{\phi}\}$. The probability is given by: $\mathcal{P}_{\text{out}} = \mathbb{P}\text{r}\{\mathcal{B}\log_2(Y_{\mathcal{K}}) < \mathcal{R}_{\phi}\}$, such that

$$\mathcal{P}_{\text{out}} = \mathbb{P}\text{r}\left\{Y_{\mathcal{K}} < 2^{\frac{\mathcal{R}_{\phi}}{\mathcal{B}}}\middle|\,\mathcal{K}\right\}. \tag{4.15}$$

Then from the derived CDF of $Y_{\mathcal{K}}$ in (4.9), the closed-form expression for the outage probability is expressed in terms of capacity threshold $\mathcal{R}_{\phi}$ as $\mathcal{P}_{\text{out}} = F_{Y_{\mathcal{K}}}\left(2^{(\mathcal{R}_{\phi}/\mathcal{B})}\right)$, given by the following:

$$\mathcal{P}_{\text{out}} = \exp\left(\sum_{i=1}^{\mathcal{K}}\frac{1}{\overline{\gamma}_i}\right)\mathcal{H}_{1,\mathcal{K}+1}^{\mathcal{K},1}\left[\frac{2^{\left(\frac{\mathcal{R}_{\phi}}{\mathcal{B}}\right)}}{\left(\prod_{i=1}^{\mathcal{K}}\overline{\gamma}_i\right)}\middle|\begin{matrix}(1,1,0)\\ \left(1\,,1\,,\frac{1}{\overline{\gamma}_1}\right),\cdots,\left(1\,,1\,,\frac{1}{\overline{\gamma}_{\mathcal{K}}}\right),(0\,,1\,,0)\end{matrix}\right]. \tag{4.16}$$

Thus, it can also be deduced from (4.16) that the outage probability is also governed by the number of assembled channels, as well as the corresponding SNR levels.

4.4.3 Forced Termination Probability

Forced termination probability ($\mathcal{P}_{\text{term}}$) is based on the event that $\{T_{\lambda,\text{pu}} < T_{\text{Tx,su}}\}$, which defines the probability that SU transmission time exceeds PU arrival time interval, *i.e.* forced termination occurs when a PU arrives before the ongoing SU transmission finishes. Let the sum capacity over $\mathcal{K}$ channels and n frames be: $\mathcal{R}_{n,\mathcal{K}} = \sum_{i=1}^{n}\sum_{j=1}^{\mathcal{K}}\mathcal{B}\log_2(1+\gamma_{ij})$, which can be expressed as

$$\mathcal{R}_{n,\mathcal{K}} = \mathcal{B}\log_2\left(\prod_{i=1}^{n}\prod_{j=1}^{\mathcal{K}}(1+\gamma_{ij})\right), \tag{4.17}$$

where $n = \{1,2,3,\dots\}$ denotes jointly independent, discrete random variables with a geometric distribution for the number of frames. Then given the transmission slot interval ΔT_{Tx}, let the actual amount of data transmitted by SU node just before a PU node arrives be given by the following: $L_{n,\mathcal{K}} = \mathcal{R}_{n,\mathcal{K}} \times \Delta T_{\mathrm{Tx}}$, based on which, given the total amount of data to be transmitted by the SU node L_{Data}, the forced termination probability for SU transmission due to PU arrival can be defined as $\mathcal{P}_{\mathrm{term}} = \mathbb{Pr}\{L_{n,\mathcal{K}} < L_{\mathrm{Data}}\}$, which can also be expressed as:

$$\mathcal{P}_{\mathrm{term}} = \mathbb{Pr}\left\{T_{\lambda,\mathrm{pu}} \times \mathcal{B}\log_2\left(\prod_{i=1}^{n}\prod_{j=1}^{\mathcal{K}}(1+\gamma_{ij})\right) < L_{\mathrm{Data}}\right\}, \tag{4.18}$$

and further reformulated as:

$$\mathcal{P}_{\mathrm{term}} = \mathbb{Pr}\left\{\prod_{i=1}^{n}\prod_{j=1}^{\mathcal{K}}(1+\gamma_{ij}) < 2^{\left(\frac{L_{\mathrm{Data}}}{\mathcal{B}\,T_{\lambda,\mathrm{pu}}}\right)}\right\}. \tag{4.19}$$

Then let $\mathcal{X}_n = \prod_{i=1}^{n} Y_{i,\mathcal{K}}$ be a compound product of a random number of *i.i.d.* random variables. Thus, the forced termination probability in (4.19) depends on the distribution of the compound variate $\mathcal{X}_n$. Therefore the Mellin-transform of the compound random product of a random number of random variables $\mathcal{X}_n$ is given by the following expression [122]:

$$\mathbb{M}_{\mathrm{s}}\{f_{\mathcal{X}_n}(x)\} = \mathcal{G}_n\left(\mathbb{M}_{\mathrm{s}}\{f_{Y_{\mathcal{K}}}(y)\}\right), \tag{4.20}$$

where $\mathcal{G}_n(\cdot)$ denotes the probability generating function (PGF) for the independent probabilities $\mathcal{P}_1, \mathcal{P}_2, \cdots,$ for the corresponding number of terms $n = \{1,2,\cdots\}$ in $\mathcal{X}_n$, and $\mathbb{M}_{\mathrm{s}}\{f_{Y_{\mathcal{K}}}(y)\}$ denotes the common Mellin transform as it is based on the statistical characterization of assembled channels $\{Y_{i,\mathcal{K}} \mid \forall i \in n\}$. Then the following can be established [122]:

$$\mathbb{M}_{\mathrm{s}}\{f_{\mathcal{X}_n}(x)\} = \mathcal{P}_1\mathbb{M}_{\mathrm{s}}\{f_{Y_{\mathcal{K}}}(y)\} + \mathcal{P}_2\left(\mathbb{M}_s\{f_{Y_{\mathcal{K}}}(y)\}\right)^2 + \mathcal{P}_3\left(\mathbb{M}_s\{f_{Y_{\mathcal{K}}}(y)\}\right)^3 + \cdots \tag{4.21}$$

Without loss of generality, (4.21) provides the result towards deriving the distribution of product of random variables, where the number of random variables is also random [122]. Then the general theorem for the Mellin-transform of the compound product can be stated as follows [122]:

Theorem 6-2: *Suppose* $\mathcal{X}_n = \prod_{i=1}^{n} Y_{i,\mathcal{K}}$ *is a compound product of* $i.i.d.$ *nonnegative variates with the Mellin-transforms* $\mathbb{M}_s\{f_{Y_\mathcal{K}}(y_i)\}$, *and probabilities* $\{\mathcal{P}_1, \mathcal{P}_2, \cdots\}$ *corresponding to the number of terms* $n = \{1,2,\cdots\}$ *in* $\mathcal{X}_n$; *Mellin-transform of the compound product is given by*

$$\mathbb{M}_s\{f_{\mathcal{X}_n}(x)\} \triangleq \sum_{n=1}^{\infty} \mathcal{P}_n \left(\mathbb{M}_s\{f_{Y_\mathcal{K}}(y_i)\}\right)^n. \tag{4.22}$$

Proof: The theorem stems directly from (4.20) and (4.21). ■

Then the PDF of the compound variable $\mathcal{X}_n$ can be recovered by applying Theorem 4-1, based on which the inverse Mellin transform of $\mathbb{M}_s\{f_{\mathcal{X}_n}(x)\}$ in (4.22) is given by

$$f_{\mathcal{X}_n}(x) = \frac{1}{2\pi i}\oint_c x^{-s}\left(\sum_{n=1}^{\infty} \mathcal{P}_n \left(\mathbb{M}_s\{f_{Y_\mathcal{K}}(y_i)\}\right)^n\right) ds, \tag{4.23a}$$

$$= \sum_{n=1}^{\infty} p_n \left\{\frac{1}{2\pi i}\oint_c x^{-s}\left(\mathbb{M}_s\{f_{Y_\mathcal{K}}(y_n)\}\right)^n ds\right\}. \tag{4.23b}$$

Using the common Mellin transform $\mathbb{M}_s\{f_{Y_\mathcal{K}}(y_n)\}$ from (4.3), the following is established:

$$f_{\mathcal{X}_n}(x) = \sum_{n=1}^{\infty} p_n \frac{1}{2\pi i}\oint_c x^{-s}\left\{\exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_i}}\right)\prod_{i=1}^{\mathcal{K}}\left(\overline{\gamma_i}^{s-1}\,\Gamma\left(s, \frac{1}{\overline{\gamma_i}}\right)\right)\right\}^n ds, \tag{4.24a}$$

$$= \sum_{n=1}^{\infty} p_n \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_i}}\right)^n \left\{\frac{1}{2\pi i}\oint_c x^{-s}\left\{\prod_{i=1}^{\mathcal{K}}\left(\overline{\gamma_i}^{s-1}\,\Gamma\left(s, \frac{1}{\overline{\gamma_i}}\right)\right)\right\}^n ds\right\}. \tag{4.24b}$$

Obviously, the integral transform in (4.24) is complex. Then for simplification, the joint PDF for the random phenomenon $\mathcal{X}_n$ is therefore expressed in the form of the Fox-$\mathcal{H}$ function as follows:

$$f_{\mathcal{X}_n}(x) = \sum_{n=1}^{\infty} \left(\frac{\mathcal{P}_n \left\{ \left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_i} \right)^{-1} \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_i}} \right) \right\}^n}{\times\ \mathcal{H}_{0,n\mathcal{K}}^{n\mathcal{K},0} \left[\frac{x}{\left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_i} \right)^n} \middle| \begin{matrix} --- \\ \left\{ \left(0,1,\frac{1}{\overline{\gamma_1}}\right), \cdots \right\}, \cdots, \left\{ \left(0,1,\frac{1}{\overline{\gamma_{\mathcal{K}}}}\right), \cdots \right\} \end{matrix} \right]} \right). \quad (4.25)$$

The forced termination probability for SU transmissions is then obtained by evaluating the following integral: $\mathcal{P}_{\text{term}} = F_{\mathcal{X}_n}(\phi) \triangleq \int_0^{\phi} f_{\mathcal{X}_n}(x)dx$. Given that $\mathcal{P}_n = p_{\text{pu}}(1 - p_{\text{pu}})^{n-1}$ for geometrically distributed random number of transmission slots, where p_{pu} is a function of PU arrival rates, the forced termination probability is therefore given by

$$\mathcal{P}_{\text{term}} = F_{\mathcal{X}_n}\left(2^{\left(\frac{L_{\text{Data}}}{BT_{\text{Tx}}} \right)} \right), \quad (4.26)$$

for which the novel result for the close-form expression is given by

$$\mathcal{P}_{\text{term}} = \sum_{n=1}^{\infty} \left(\frac{p_{\text{pu}}\left(1 - p_{\text{pu}}\right)^{n-1} \left\{ \exp\left(\sum_{i=1}^{\mathcal{K}} \frac{1}{\overline{\gamma_i}} \right) \right\}^n}{\times\ \mathcal{H}_{1,n\mathcal{K}+1}^{n\mathcal{K},1} \left[\frac{2^{\left(\frac{L_{\text{Data}}}{BT_{\text{Tx}}} \right)}}{\left(\prod_{i=1}^{\mathcal{K}} \overline{\gamma_i} \right)^n} \middle| \begin{matrix} (1,1,0) \\ \left\{ \left(0,1,\frac{1}{\overline{\gamma_1}}\right), \cdots \right\}, \cdots, \left\{ \left(0,1,\frac{1}{\overline{\gamma_{\mathcal{K}}}}\right), \cdots \right\}, (0,1,0) \end{matrix} \right]} \right). \quad (4.27)$$

Thus, performance evaluation based on the derived analytical models for assessment of various ChA schemes is presented and discussed in the next section. Unlike the reported studies in literature, the novel analytical models presented in this chapter capture the impact of fading wireless channels on the performance of ChA schemes in CRNs with overlay spectrum access.

4.5 Performance Analysis and Evaluation

4.5.1 Simulation Model

Performance evaluation is mainly based on the (i) average SU capacity, (ii) outage probability and (iii) forced termination probability; for which the analytical models have been derived in the previous section. The optimal ChA scheme with the optimal channel selection and power allocation (BnBKOP) is evaluated with three other opportunistic spectrum access techniques based on the existing works in literature. These include a single channel access technique without ChA (NoChA), which only selects the best channel with total power allocation. In the case of predefined ChA order, two main aspects are evaluated. First, the fixed number of assembled channels with equal power distribution (*i.e.* $p_i = P_{\max}/\mathcal{K}$) across the channels (FixedKEP). Second, the fixed number of channels with optimal power allocation based on the WF technique (FixedKOP) as derived in Chapter 3. The fixed ChA order with optimal power distribution has not necessarily been investigated in literature, but also part of the work proposed in this thesis, to put more emphasis on the significance of transmit power optimization for ChA schemes. To validate the accuracy of the developed analytical models, the simulation results in this chapter are averaged over 10^4 frames. The simulation model has been implemented in MATLAB, while the incomplete Fox-$\mathcal{H}$ function has been implemented in MATHEMATICA to obtain the analytical results [122].

4.5.2 Results and Discussion

The results for the aforementioned performance metrics are presented in Fig. 4-1 to Fig. 4-3 as a function of PU arrival rates. As Fig. 4-1 illustrates, the proposed BnBKOP ChA scheme achieves the highest average capacity at low PU arrivals. As the PU arrival rates increase, the BnBKOP scheme reduces the number of assembled channels in order to satisfy the collision constraint between PU and SU services, defined by (C3.3) in Chapter 3. Reducing the number of assembled channels reduces the resulting sum capacity, hence the low capacity for the BnBKOP scheme at high PU arrivals. Also, Fig. 4-1 shows a better performance for the FixedKOP scheme in comparison to FixedKEP scheme, which generally illustrates the importance of optimizing power allocation for ChA schemes. Fig. 4-2 presents results for outage probability, which in accordance with Fig. 4-1, outage probability for the proposed BnBKOP scheme increases with the increasing PU arrival rates. This is simply because of the reduction in SU capacity at high PU arrivals. Most importantly, Fig. 4-3 illustrates that as a result of incorporating PU arrival rates to perform ChA, the BnBKOP has the lowest forced termination probability, which vanishes at high PU arrivals as no channels may be selected for assembling. Although the FixedKEP and FixedKOP ChA schemes may appear to have relatively higher capacity (*i.e.* Fig. 4-1) and lower outage (*i.e.* Fig. 4-2) at high PU arrival rates, such performance may not be realized due to frequent collisions, which also leads to high rate of forced terminations at high PU arrivals as Fig. 4-3 illustrates.

Furthermore, Fig. 4-4 to Fig. 4-6 presents results for varying SU total transmit power. Obviously, capacity increases with the increase in SU transmit power $P_{\max}$. Based on the constraint defined in (C3.3), which has been derived from (3.8) in Chapter 3, the BnBKOP scheme can assemble more channels to maximize SU capacity with increasing $P_{\max}$, without violating the collision constraint in (C3.3). Hence, the best performance for the BnBKOP scheme, more as $P_{\max}$. Moreover, Fig. 4-4 illustrates the significance of transmit power optimization for ChA schemes, as FixedKOP scheme outperforms the FixedKEP scheme. As a result of the increase on capacity for increasing SU total transmit power, outage probability decreases as illustrated by Fig. 4-5, whereby the BnBKOP scheme has the lowest outage probability in comparison to other ChA schemes.

Fig. 4-6 presents the results for forced termination probability as a function of increasing $P_{\max}$, which reveals that the proposed BnBKOP scheme has the lowest forced termination probability due to the increase in average capacity, followed by the FixedKOP scheme. Increasing the average capacity reduces SU transmission time. Intuitively, the reduction in SU transmission time reduces the collision probability between PU service arrivals and the ongoing SU services; hence the low forced termination probability by the BnBKOP scheme as Fig. 4-6 illustrates.

Fig. 4-7 presents results for varying the SU threshold for QoS requirement on capacity. Inasmuch as a CRN may comprise heterogeneous SU nodes with varying QoS requirements, the results in Fig. 4-7 illustrate that the BnBKOP scheme proposed in this work has a better performance with the lowest outage probability for varying QoS requirements on the average capacity, again followed by the FixedKOP scheme. In general, the analytical results for the SU average capacity, outage probability and forced termination probability have been obtained from the analytical in (4.14), (4.16) and (4.27) respectively. The presented results from Fig. 4-1 to Fig. 4-7 also provide the cross validation between both the simulation results and analytical results.

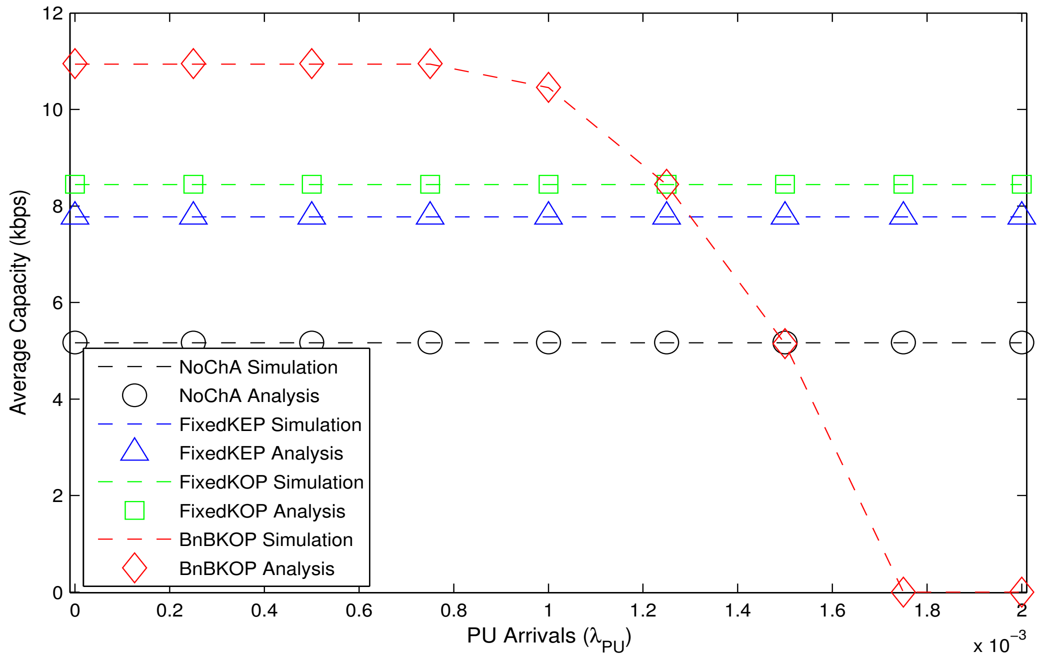

Figure 4-1: SU average capacity for increasing PU arrival rates.

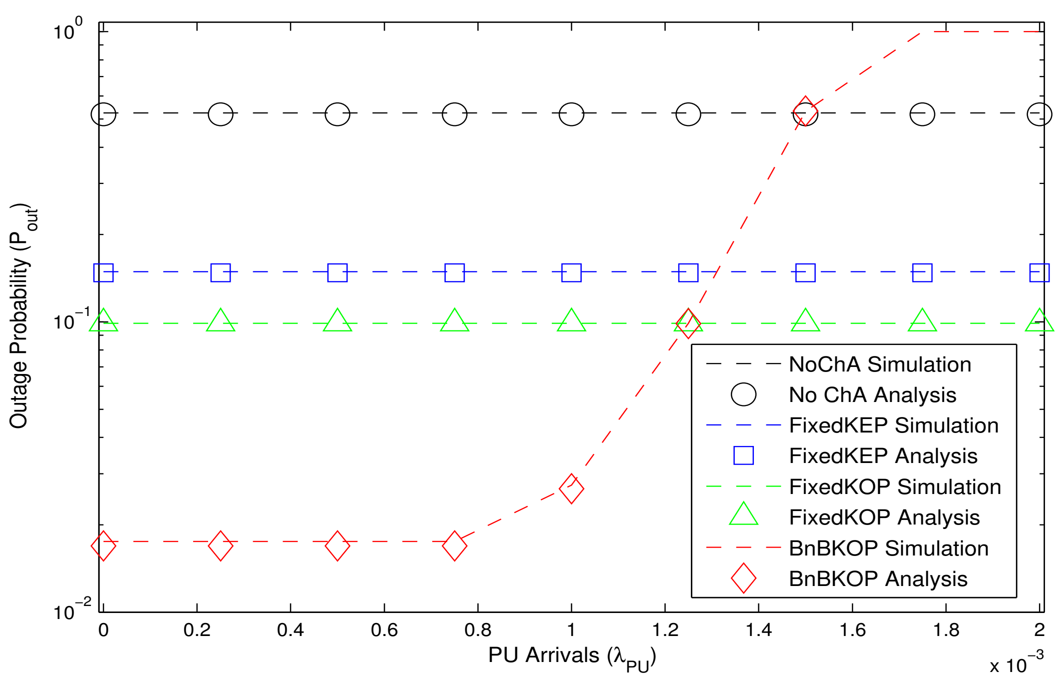

Figure 4-2: SU outage probability for increasing PU arrivals.

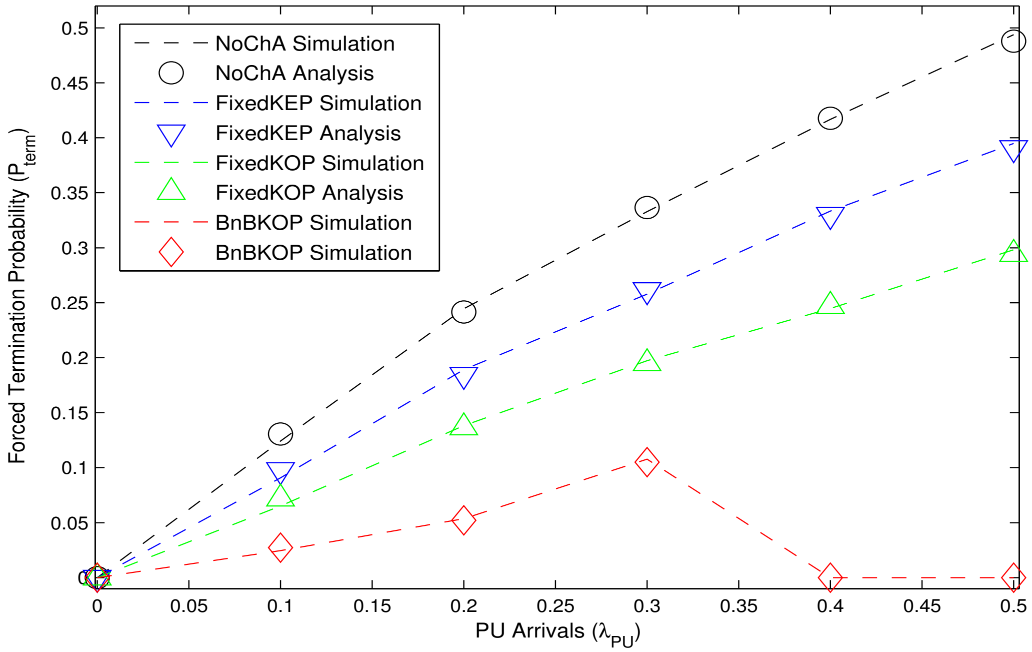

Figure 4-3: Forced-termination probability for increasing PU arrivals.

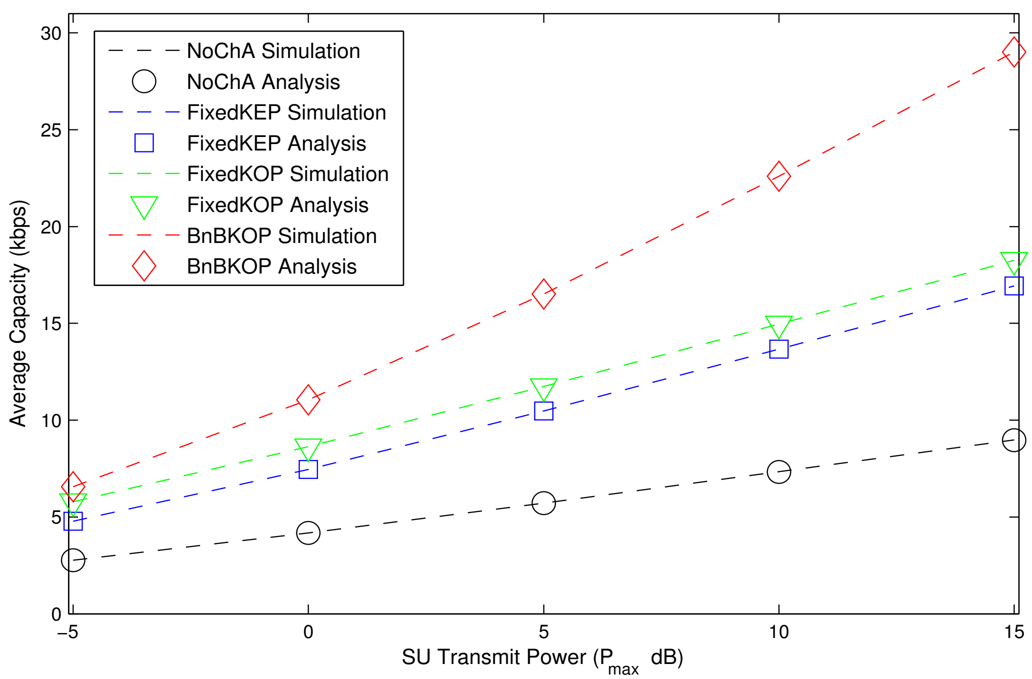

Figure 4-4: Average capacity against transmit power.

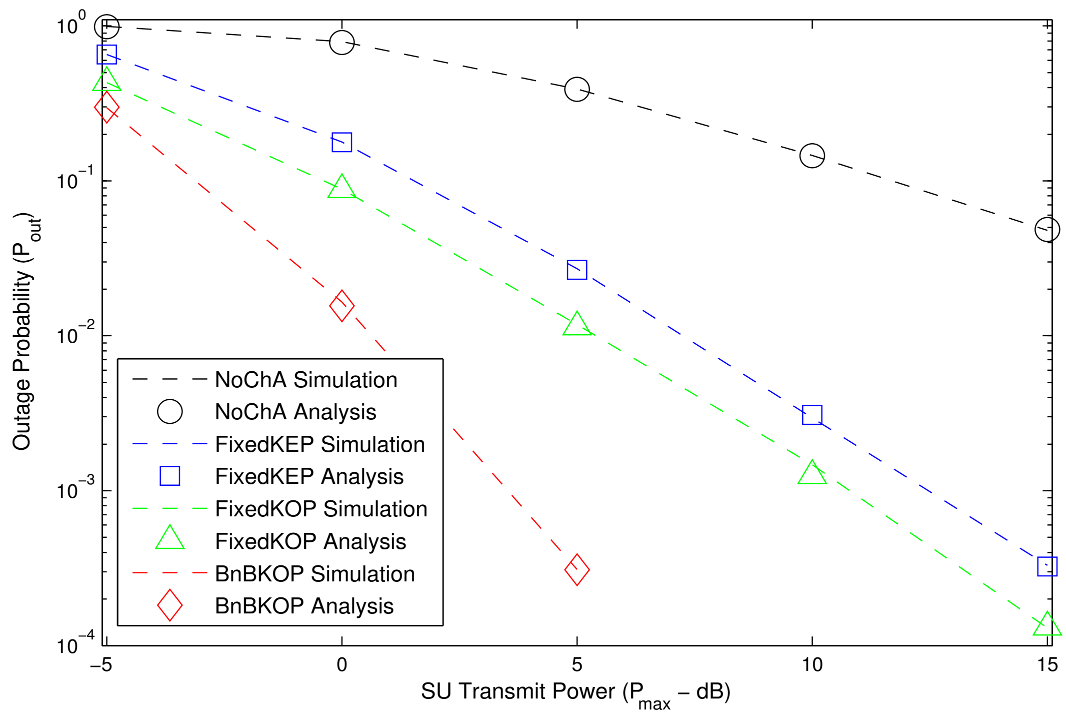

Figure 4-5: SU outage probability against transmit power.

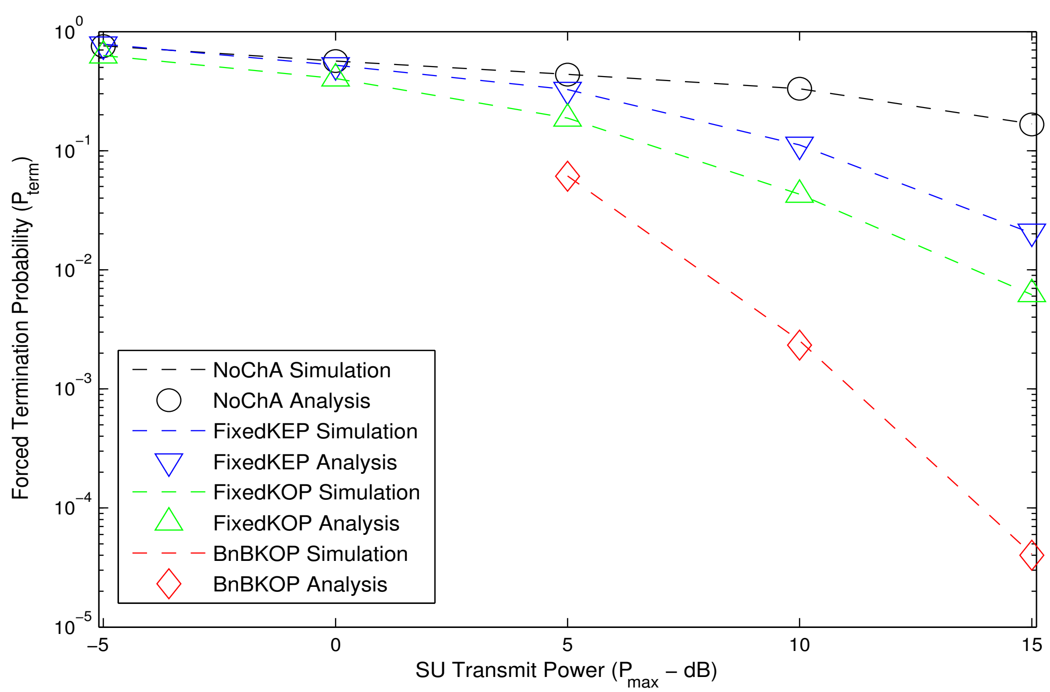

Figure 4-6: Forced-termination against transmit power.

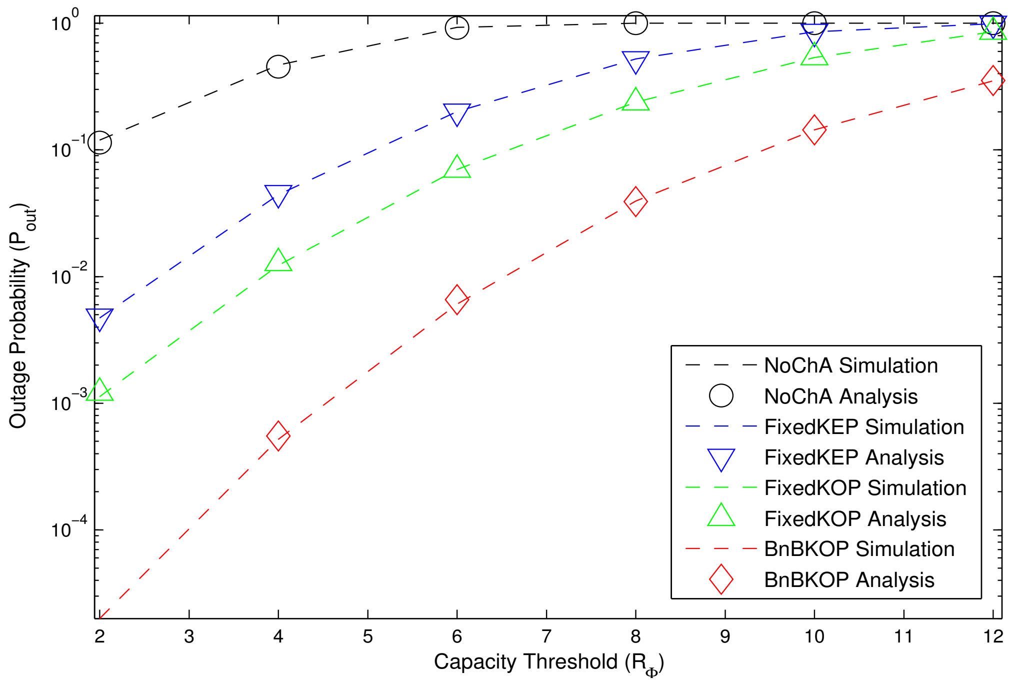

Figure 4-7: Outage probability against capacity threshold.

4.6 Chapter Conclusion

In this chapter, statistical characterization of assembled channels has been developed in terms of PDF and CDF. The characterization is based on the Mellin integral transform, and expressed in terms of the generalised upper incomplete Fox-$\mathcal{H}$ function. The then analytical models were derived and expressed in the form of Meijer-G and Fox-$\mathcal{H}$ functions to quantify performance of ChA schemes over fading wireless channels. Thus, the closed-form expressions for SU average capacity, outage probability and forced termination probability have been developed.

Furthermore, the cross validation to determine the accuracy and preciseness of the developed analytical models has been performed through extensive simulations and analytical results. More importantly, the analytical and simulation results were found to match, validating the correctness of the derived analytical models. In general, analytical and simulation results illustrate that performance of ChA schemes is largely affected by PU activity patterns, wherefore significant performance degradation for SU services is encountered at high PU arrival rates.

CHAPTER FIVE

5 HYBRID MULTICHANNEL SPECTRUM ACCESS

5.1 Introduction

IN this chapter, adaptive ChA schemes for hybrid overlay and underlay transmissions are developed. The hybrid schemes determine the optimal channel selection and power allocation for both overlay and underlay spectrum techniques. This allows SU nodes to opportunistically assemble free PU channels in overlay mode, and adaptively select the occupied PU channels with controlled transmit power levels in underlay mode. The primary aim is to maximize SU capacity subject to the total power constraint, QoS requirement on sum capacity, collision probability constraint and the maximum allowable transmit power over each channel in underlay transmission mode. Hence, the technical challenges include protecting PU services from interference caused by SU transmissions in underlay mode, while maintaining the QoS requirements for SU services under the stringent interference constraints imposed for PU protection.

Convex optimization based on the Lagrangian framework is employed to reformulate the MINLP problem; based on which Newton-Raphson technique with dual decomposition are employed to determine the optimal solution. Further, the Lambert-$\mathcal{W}$ function is employed to derive closed-form expressions for optimal channel selection as an alternative to the Newton technique. In general, the presented simulation results illustrate significant performance improvement in terms of channel capacity, outage probability and collision probability when employing the proposed hybrid ChA schemes. The work presented in this chapter has been presented and published in the *Proceedings of the IEEE Wireless Communications and Networking Conference* (*WCNC'2017*), San Francisco, CA, March 2017, and submitted in part for publication in the *IET Communications Journal*.

The remainder of this chapter is organized as follows: Section 5.2 highlights the related works in literature, followed by Section 5.3 which presents the system model and the problem formulation for hybrid multichannel spectrum access. Then Section 5.4 presents the optimization framework based on Lagrangian technique. Section 5.5 presents simulation based performance evaluation for various ChA techniques, and finally the concluding remarks in Section 5.6.

5.2 Related Works

The work presented in this chapter extends the overlay ChA schemes developed in Chapter 3. The key studies on ChA schemes in literature have already been highlighted [28], [31]-[32], [73]-[75], [77]-[78], [84], [86]. It can therefore be restated that the dependence of ChA schemes on the quality of fading wireless channels and power constraints has generally been overlooked, as well as the incorporation of PU activity patterns to perform spectrum characterization for ChA schemes. Inasmuch as hybrid spectrum access techniques have been studied in literature, ChA schemes for hybrid transmissions have not received much attention. Rather, the major studies on ChA schemes only select channels that are not occupied by PU nodes in overlay mode [27]-[28], [31], [76]. However, the channels which are occupied by PU nodes may relatively have good SNR for SU nodes, in which case their exclusion may result in suboptimal performance.

A recent study on the performance of ChA in underlay CRNs has been presented in [86]. Similar to majority of the existing studies in literature, the work in [86] is mainly based on CTMC modeling in a multiuser environment. Nonetheless, CTMC based analyses do not necessarily account for the challenges inherent in network dynamics and constraints. Generally, a technical challenge is to protect PU transmissions from interference caused by SU nodes, while at the same time maintaining the QoS requirements for SU services under stringent interference constraints imposed for PU protection. In this work, a multilevel maximum allowable SU transmit power for channels assembled in underlay mode is derived based on various assumptions about the knowledge of CSI at the SU transmitter. This ensures that SU transmit power in underlay mode is constrained under the noise floor of PU nodes. This is different from the approaches in literature where interference constraint is imposed, rather than power threshold. Hence, the maximum allowable transmit power is obtained from analytical expressions derived based on PU outage probability constraint.

5.3 System Model and Problem Formulation

5.3.1 System Model

In this section, the coexistence of a licensed PN with a CRN is assumed, where SU nodes in a CRN can access PN spectrum in both overlay and underlay modes with reference to Fig. 5-1, where the dashed lines denote interference links between PU nodes and SU nodes. Suppose $\boldsymbol{S}$ represents a set

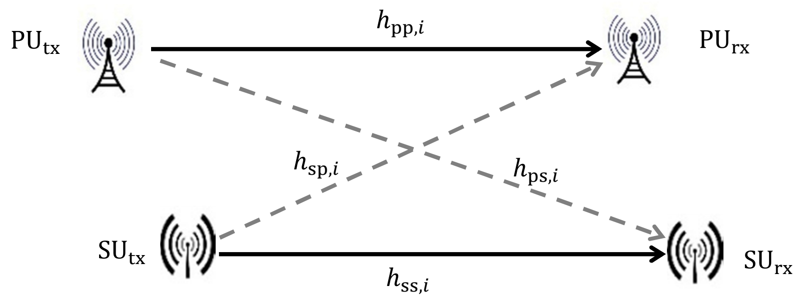

Figure 5-1: Network model for hybrid overlay and underlay SU transmissions.

of channels from the PN spectrum, whose cardinality $S = \{M + N\}$ for the total number of channels in the PN, where $\boldsymbol{\mathcal{M}}$ is a subset of free PU channels with cardinality M, and $\boldsymbol{\mathcal{N}}$ represents a subset of busy channels occupied by PU nodes, with cardinality N.

It is assumed that SU nodes have a priori knowledge of CSI to accordingly adapt transmit power levels, while a fixed transmit power P_{pu} is employed by PU nodes. It is further assumed that all the links in both the PN and CRN are statistically *i.n.i.d.* variates with channel fading coefficients g_i on any channel i characterized by Rayleigh distribution. Also, the noise for both PN and CRN is assumed to be AWGN with zero mean and variance σ_{pu}^2 for PN and σ_{su}^2 for CRN respectively. The channel gains $h_i = |g_i|^2$ are therefore assumed to be exponentially distributed, with a mean given by $\overline{h_i} = \mathbb{E}\{h_i\}$, where $\mathbb{E}\{\cdot\}$ denotes the statistical expectation operator [90], [105]. Furthermore, the instantaneous channel gains are assumed to be constant within a frame interval, but change from one frame to another due to wireless channel variations during transmissions. Channel selection for the optimal ChA scheme in overlay mode is denoted by $\boldsymbol{x}_M \triangleq \{x_m \mid x_m \in \{0,1\}, \forall m \in \boldsymbol{\mathcal{M}}\}$, and selected channels in underlay mode by $\boldsymbol{x}_N \triangleq \{x_n \mid x_n \in \{0,1\}, \forall n \in \boldsymbol{\mathcal{N}}\}$, whereby

$$\{x_m, x_n\} = \begin{cases} 1 & m \to \text{overlay}, \;\; n \to \text{underlay} \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

Moreover, the corresponding feasible set of non-negative power distribution vector for channels assembled in overlay mode is denoted by $\boldsymbol{p}_{\mathrm{M}} \triangleq \{p_m \mid p_m \geq 0, \forall m \in \boldsymbol{\mathcal{M}}\}$, and for the underlay mode by $\boldsymbol{p}_N \triangleq \{p_n \mid p_n \geq 0, \forall n \in \boldsymbol{\mathcal{N}}\}$. The power allocation for each channel n selected over underlay mode is upper bounded by $p_{\max,n}$ for SU transmissions. Then based on Shannon Hartley theorem, the expected channel capacity expression over any PU channel $\{m \in \boldsymbol{\mathcal{M}}\}$ selected for transmission in overlay mode can be expressed as follows [27], [108]:

$$\bar{r}_m(x_m, p_m) = \mathbb{E}\left\{x_m \mathcal{B} \log_2\left(1 + \frac{p_m h_{\mathrm{ss},m}}{x_m \sigma_{\mathrm{su}}^2}\right)\right\}, \qquad \forall m \in \boldsymbol{\mathcal{M}} \tag{5.2}$$

where $h_{\mathrm{ss},m}$ is the channel m gain between SU transmitter and the SU receiver nodes as illustrated in Fig. 5-1. On the other hand, the expected channel capacity on any channel $\{n \in N\}$ selected in underlay transmission mode from the PN spectrum can be expressed as

$$\bar{r}_n(x_n, p_n) = \mathbb{E}\left\{x_n \mathcal{B} \log_2\left(1 + \frac{p_n h_{\mathrm{ss},n}}{x_n\left(P_{\mathrm{pu}} h_{\mathrm{ps},n} + \sigma_{\mathrm{su}}^2\right)}\right)\right\}, \qquad \forall n \in \boldsymbol{\mathcal{N}} \tag{5.3}$$

where $h_{\mathrm{ps},n}$ is channel n gain between PU transmitter and SU receiver, and $h_{\mathrm{ss},n}$ is the gain between SU nodes. The x_m and x_n variables in (5.2) and (5.3) are included for the purpose of convex optimization. It follows from (5.1) that $\{x_i = 0\} \Rightarrow \{\bar{r}_i(x_i, p_i) = 0\}$, which is consistent with the fact that power is zero for channels that are not selected, for which capacity is also zero. The average sum capacity $\bar{\mathcal{R}}_{\mathcal{K}}$ from $\mathcal{K}$ channels in both underlay and overlay modes is given by

$$\bar{\mathcal{R}}_{\mathcal{K}} = \left(\sum_{m=1}^{M} \bar{r}_m(x_m, p_m) + \sum_{n=1}^{N} \bar{r}_n(x_n, p_n)\right). \tag{5.4}$$

Thus, the sum capacity in (5.4) is generally governed by how many channels are selected in underlay mode and overlay mode, as well as the transmission power policy across the selected channels. Channel selection in overlay mode is even more significant for performance improvement of SU nodes under high PU arrivals, where spectrum is hardly available for SU nodes. Further, overlay channel selection for SU services in high PU arrivals is susceptible to collisions, as the ongoing SU services must terminate on arrival of new PU services on any of the selected channel.

5.3.2 Hybrid Channel Assembling Problem Formulation

The optimization problem for ChA scheme can be stated as follows: given the PN spectrum, the knowledge of CSI at the SU transmitter, PU outage constraint and channel occupancy status, SU

QoS requirements on capacity and collision probability constraint, determine the optimal channel selection and power distribution for ChA over hybrid underlay and overlay transmissions to maximize the SU average sum capacity. The optimization problem can be formulated as

$$\underset{\{\boldsymbol{x},\boldsymbol{p}\}}{\text{maximize}}\left\{\sum_{m=1}^{M}\bar{r}_m(x_m,p_m)+\sum_{n=1}^{N}\bar{r}_n(x_n,p_n)\right\}, \tag{5.5}$$

$$\text{subject to: }\left(\sum_{m=1}^{M}x_m p_m+\sum_{n=1}^{N}x_n p_n\right)\leq P_{\max}, \tag{C5.1}$$

$$\mathcal{R}_\phi\leq\left(\sum_{m=1}^{M}\bar{r}_m(x_m,p_m)+\sum_{n=1}^{N}\bar{r}_n(x_n,p_n)\right) \tag{C5.2}$$

$$\beta\sum_{m=1}^{M}x_m\leq\left(\sum_{m=1}^{M}\bar{r}_m(x_m,p_m)+\sum_{n=1}^{N}\bar{r}_n(x_n,p_n)\right) \tag{C5.3}$$

$$x_n p_n\leq\ p_{\max,n},\qquad\forall n\in\boldsymbol{\mathcal{N}} \tag{C5.4}$$

$$\sum_{m=1}^{M}x_m\leq M,\qquad\sum_{n=1}^{N}x_n\leq N,\qquad\forall\{x_m,x_n\}\in\{0,1\}, \tag{C5.5}$$

$$0\leq p_m,\forall m\in\boldsymbol{\mathcal{M}},\qquad 0\leq p_n,\forall n\in\boldsymbol{\mathcal{N}} \tag{C5.6}$$

where $\mathcal{R}_\phi$ is the minimum SU capacity for QoS requirement, $P_{\max}$ denotes the total available power at the SU transmitter, $p_{\max,n}$ is the adaptive transmit power threshold on any channel n selected in underlay mode, and β has been derived in [27] as $\beta=\left(\lambda_{\text{pu}}L_{\text{Data}}/-\ln\left(\mathcal{P}_\phi\right)\right)$, whereby λ_{pu} is the PU arrival rate, L_{Data} is the SU data-length, and $\mathcal{P}_\phi$ represents the minimum collision threshold between PU and SU services. The constraint in (C5.1) expresses the condition that the sum of allocated transmit powers across all the selected channels cannot exceed the total available power. (C5.2) denotes a constraint that the resulting sum capacity must satisfy the minimum QoS requirement. (C5.3) has been derived in [27] based on the condition that the probability that SU services finish transmissions before arrival of a PU node on any of the channels assembled in overlay mode must be above the minimum threshold. (C5.4) constrains the SU transmit power in underlay mode below multilevel threshold to protect PU nodes. (C5.5) expresses the constraint that the sum of assembled channels cannot exceed the number of available channels, and (C5.6) denotes the condition that only nonnegative powers can be assigned across all the selected channels.

5.4 Hybrid Channel Selection and Power Allocation

5.4.1 Lagrangian Convex Optimization Framework

The MINLP problem in (5.5) is reformulated into a convex problem by relaxing the channel selection variables $\{x_m, x_n\}$ to take real values in the interval $[0,1]$. The relaxed values illustrate a soft-decision (*i.e.* binary uncertainty) for channel selection. Thus, the solution to the relaxed problem provides an upper-bound for the solution to the original MINLP problem, based on which an optimal solution is obtained using the BnB technique as presented in Chapter 3. The Lagrangian $\mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu})$ with nonnegative dual variables can be formulated as follows:

$$\begin{aligned}\mathcal{L}(\cdot) = \boldsymbol{\lambda}_{(2+3)}\left\{\sum_{m=1}^{M} \bar{r}_m(x_m, p_m) + \sum_{n=1}^{N} \bar{r}_n(x_n, p_n)\right\} - \lambda_1\left\{\left(\sum_{m=1}^{M} x_m p_m + \sum_{n=1}^{N} x_n p_n\right) - P_{\max}\right\} \\ - \lambda_2 R_\phi - \lambda_3 \beta \sum_{m=1}^{M} x_m - \sum_{n=1}^{N} \mu_n\left(x_n p_n - p_{\max,n}\right),\end{aligned} \tag{5.6}$$

where $\boldsymbol{\lambda} = \{\lambda_i \geq 0, i = 1, \cdots, 3\}$, $\boldsymbol{\lambda}_{(2+3)} = (1 + \lambda_2 + \lambda_3)$ and $\boldsymbol{\mu} = \{\mu_n \geq 0, \forall n \in \boldsymbol{\mathcal{N}}\}$, in which case $\{\boldsymbol{\lambda}, \boldsymbol{\mu}\}$ represent non-negative Lagrangian multipliers vectors. A dual function is formulated as a pointwise infimum of the Lagrangian from (5.6) as [91]:

$$\mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\{\boldsymbol{x}, \boldsymbol{p}\} \in \boldsymbol{\mathcal{S}}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}). \tag{5.7}$$

The dual problem is relatively easy to solve than the primal problem, and the solution to the primal problem can be analytically obtained from the solution of the dual problem [27], [91]-[91], [108]. Thus, the dual problem defines the minimum value of the Lagrangian function over the nonnegative dual variables, such that the optimal solution that maximizes sum capacity satisfies the following expression: $\mathcal{L}(\boldsymbol{x}^*, \boldsymbol{p}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) \geq \bar{\mathcal{R}}_{\mathcal{K}}(\boldsymbol{x}^*, \boldsymbol{p}^*)$, where $(\cdot)^*$ denotes the optimal variables for the optimal solution. The dual problem can therefore be formulated as follows:

$$\begin{gathered} \underset{\boldsymbol{\lambda},\boldsymbol{\mu}}{\text{minimize}} \ \mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ \text{subject to: } \{\forall \lambda_i \in \boldsymbol{\lambda}\} \geq 0, \{\forall \boldsymbol{\mu}_n \in \boldsymbol{\mu}\} \geq 0 \, . \end{gathered} \tag{5.8}$$

The dual decomposition decouples the Lagrangian function into a series of parallel subproblems that are relatively easy to solve. Following the works in [27] and [108], it can be shown that the objective function is jointly concave in both x_i and p_i, for i signifying any channel selected in either overlay mode or underlay mode. Thus, the dual function is unimodal, and the duality gap between the primal solution and dual solution is zero. It can therefore be deduced that a unique optimal solution can be obtained from the KKT conditions [91]-[92], [94], [111].

To establish the KKT conditions, the Lagrangian function is partially derived with respect to $\{p_i, x_i\}$, as well as all the Lagrangian multipliers; then the derivatives are set to zero. As stated in Chapter 3, the orthogonality conditions suggest that either the constraint function is zero when active, or the associated Lagrangian multiplier is zero when the inequality condition is active [27], [91], [108]. However, the power constraint defined in (C5.1) must always be tight and binding in order to maintain optimal power distribution to maximize capacity, hence the following remark:

> **Remark 4-1**: *Any feasible optimal power distribution must satisfy the total transmit power constraint with equality for hybrid overlay and underlay spectrum access. Otherwise, any other feasible power distribution would be suboptimal, unless the entire PN spectrum is occupied by PU nodes, and the assembled channels are selected in underlay mode.*

As mentioned in Chapter 3 also, the coupling between channel selection and power distribution leads to some difficulty in obtaining the closed-form analytical solutions directly from the KKT conditions [27], [108]. Accordingly, iterative search methods are usually employed to determine the optimal solutions for resource allocation problems in wireless networks, which mainly involve user selection, scheduling and power allocation in multiuser systems, as well as subcarrier allocation techniques in the case of OFDM systems [20], [42], [95]-[99], [104], [112]-[114].

5.4.2 Overlay Channel Selection and Power Allocation

The Lagrangian dual function is decomposed into M individual subproblems for overlay channel selection and power allocation. To establish the KKT conditions for optimal solution, each of the subproblems is partially derived with respect to $\{x_m, p_m\}$, and the derivatives are set to zero to solve for the primal variables $\{x_m, p_m\}$. In the case of p_m, the following is established:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial p_m} = \frac{\mathcal{B}}{\ln(2)}\left(\frac{(1+\lambda_2+\lambda_3)x_m\bar{h}_{\mathrm{ss},m}}{p_m\bar{h}_{\mathrm{ss},m}+x_m\sigma_{\mathrm{su}}^2}\right) - \lambda_1 x_m = 0, \qquad \forall m \in \boldsymbol{\mathcal{M}}. \tag{5.9}$$

Then solving for p_m, the closed-form expression for the optimal power allocation that maximizes SU capacity for channels that are selected in overlay transmission mode is given by

$$p_m = \max\left\{0, \left[\frac{\mathcal{B}}{\ln(2)}\left(\frac{1+\lambda_2+\lambda_3}{\lambda_1}\right) - \frac{x_m\sigma_{\mathrm{su}}^2}{\bar{h}_{\mathrm{ss},m}}\right]\right\}, \qquad \forall m \in \boldsymbol{\mathcal{M}} \tag{5.10}$$

which is in the form of the classical WF power allocation technique in fading wireless channels. The power allocation policy in overlay mode is largely dictated by the channel quality for the direct link between SU transmitter and SU receiver $\bar{h}_{\mathrm{ss},m}$. It can therefore be easily deduced from (5.10) that more power is allocated to channels that have a good power gain. Moreover, the following can be established for channel selection in overlay mode:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial x_m} = \frac{\mathcal{B}}{\ln(2)}\left\{\ln\left(1+\frac{p_m\bar{h}_{\mathrm{ss},m}}{x_m\sigma_{\mathrm{su}}^2}\right) - \left(\frac{p_m\bar{h}_{\mathrm{ss},m}}{p_m\bar{h}_{\mathrm{ss},m}+x_m\sigma_{\mathrm{su}}^2}\right)\right\} - \left(\frac{\lambda_1 p_m+\lambda_3\beta}{1+\lambda_2+\lambda_3}\right), \qquad \forall m \in \boldsymbol{\mathcal{M}}. \tag{5.11}$$

Let $f(x_m) = \partial\mathcal{L}(\cdot)/\partial x_m$, from which the second derivative is given by

$$f'(x_m) = \frac{\mathcal{B}}{\ln(2)}\left(\frac{p_m\bar{h}_{\mathrm{ss},m}}{p_m\bar{h}_{\mathrm{ss},m}+x_m\sigma_{\mathrm{su}}^2}\right)\left\{\frac{\sigma_{\mathrm{su}}^2}{\left(p_m\bar{h}_{\mathrm{ss},m}+x_m\sigma_{\mathrm{su}}^2\right)} - \frac{1}{x_m}\right\}, \qquad \forall m \in \boldsymbol{\mathcal{M}}. \tag{5.12}$$

Then to establish the optimal solution for channel selection in underlay spectrum access mode, the iterative Newton method can be applied as follows [27], [108]:

$$x_m^{(k+1)} = x_m^k - \delta_f \frac{f\left(x_m^{(k)}\right)}{f'\left(x_m^{(k)}\right)}, \qquad \forall m \in \boldsymbol{\mathcal{M}} \tag{5.13}$$

where k represents an iteration index, and δ_f denotes the Newton function step-size. Alternatively, the closed-form expression for channel selection can be established by equating (5.11) to zero and solving for the channel selection $\boldsymbol{x}_M$, from which the following expression can be established:

$$\ln\left(\frac{x_m\sigma_{\mathrm{su}}^2}{x_m\sigma_{\mathrm{su}}^2 + p_m\bar{h}_{\mathrm{ss},m}}\right) - \left(\frac{x_m\sigma_{\mathrm{su}}^2}{x_m\sigma_{\mathrm{su}}^2 + p_m\bar{h}_{\mathrm{ss},m}}\right) = -\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{\lambda_1 p_m + \lambda_3\beta}{1 + \lambda_2 + \lambda_3}\right)\right], \forall m \in \boldsymbol{\mathcal{M}} \tag{5.14}$$

Then taking the anti-log and rearranging (5.14), the following holds:

$$-\frac{x_m\sigma_{\mathrm{su}}^2}{x_m\sigma_{\mathrm{su}}^2 + p_m\bar{h}_{\mathrm{ss},m}} \exp\left(-\frac{x_m\sigma_{\mathrm{su}}^2}{x_m\sigma_{\mathrm{su}}^2 + p_m\bar{h}_{\mathrm{ss},m}}\right) = -\exp\left(-\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{\lambda_1 p_m + \lambda_3\beta}{1 + \lambda_2 + \lambda_3}\right)\right]\right), \tag{5.15}$$

from which applying the Lambert-$\mathcal{W}$ function results into the following:

$$-\frac{x_m\sigma_{\mathrm{su}}^2}{x_m\sigma_{\mathrm{su}}^2 + p_m\bar{h}_{\mathrm{ss},m}} = \mathcal{W}\left(-\exp\left(-\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{\lambda_1 p_m + \lambda_3\beta}{1 + \lambda_2 + \lambda_3}\right)\right]\right)\right). \tag{5.16}$$

The closed-form expression for channel selection in underlay mode is therefore established by rearranging (5.16) to solve for the x_m as follows:

$$x_m = -\frac{p_m\bar{h}_{\mathrm{ss},m} \times \mathcal{W}\left(-\exp\left(-\left[1 + \left(\frac{\ln(2)}{\mathcal{B}}\right)\left(\frac{\lambda_1 p_m + \lambda_3\beta}{1+\lambda_2+\lambda_3}\right)\right]\right)\right)}{\sigma_{\mathrm{su}}^2\left(1 + \mathcal{W}\left(-\exp\left(-\left[1 + \left(\frac{\ln(2)}{\mathcal{B}}\right)\left(\frac{\lambda_1 p_m + \lambda_3\beta}{1+\lambda_2+\lambda_3}\right)\right]\right)\right)\right)}, \qquad \forall m \in \boldsymbol{\mathcal{M}} \tag{5.17}$$

where $\mathcal{W}(\cdot)$ denotes the Lambert-$\mathcal{W}$ function, which is generally defined as a multivalued inverse of the function of the form $f(x) = xe^x$ [126]-[127]. The solution for the optimal dual variables that

ALGORITHM 5-1: THE LAGRANGIAN BASED OPTIMIZATION MODEL

Input: $\boldsymbol{\mathcal{S}}, P_{\max}, \mathcal{R}_{\phi}, P_{\phi}$
Output: $\{\boldsymbol{x}^{*}, \boldsymbol{p}^{*}, \bar{\mathcal{R}}_{\mathcal{K}}^{*}\}$
initialize: $\{\{\boldsymbol{\lambda}, \boldsymbol{\mu}\}, \{\boldsymbol{x}, \boldsymbol{p}\}\}$
formulate Lagrangian: relax channel vector: $\{0 \leq x_i \leq 1\}\ \forall x_i \in \boldsymbol{x}$.
repeat
    solve the dual problem for dual variables $\{\boldsymbol{\lambda}, \boldsymbol{\mu}\}$.
    obtain power vector based on water-filling $\{\boldsymbol{p}\}$.
    employ Newton or Lambert-$\mathcal{W}$ function channel selection $\{\boldsymbol{x}\}$.
until: the optimal solution (assuming feasibility)
employ BnB to obtain solution of MINLP problem $\{\boldsymbol{x}^{*}, \boldsymbol{p}^{*}\}$.
return

minimize (5.8) are obtained by applying the subgradient search method, which is updated with respect to constraints violation as follows [91]:

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} + \delta_{\lambda}\left(g_i(\boldsymbol{x}, \boldsymbol{p})\right), \qquad i = 1, \cdots, z \tag{5.18}$$

where $g_i(\boldsymbol{x}, \boldsymbol{p})$ denotes a constraint function for each of the z constraints, and δ_{λ} denotes a gradient step-size. At first, the primal variables $\{\boldsymbol{x}, \boldsymbol{p}\}$ and dual variables $\{\boldsymbol{\lambda}, \boldsymbol{\mu}\}$ are initialized to arbitrary feasible values, and then iteratively updated until convergence towards an optimal solution $\{\boldsymbol{x}^{*}, \boldsymbol{p}^{*}, \boldsymbol{\lambda}^{*}, \boldsymbol{\mu}^{*}\}$ for both the primal and dual domains, as shown in Algorithm 5-1.

5.4.3 Underlay Channel Selection and Power Allocation

In the case of underlay mode, the closed-form expression for power distribution is also obtained from the partial derivative of the Lagrangian with respect to p_n. Let $I_{\mathrm{pu},n} = \left(P_{\mathrm{pu}} h_{\mathrm{ps},n} + \sigma_{\mathrm{su}}^2\right)$, then the following can be established from Lagrangian function in (5.6)

$$\frac{\partial \mathcal{L}(\cdot)}{\partial p_n} = \frac{\mathcal{B}}{\ln(2)} \left\{ \frac{(1 + \lambda_2 + \lambda_3) x_n \bar{h}_{\mathrm{ss},n}}{p_n \bar{h}_{\mathrm{ss},n} + x_n I_{\mathrm{pu},n}} \right\} - \lambda_1 x_n - \mu_n x_n = 0, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.19}$$

Then the closed-form expression for optimal power allocation in underlay mode can be directly obtained by solving for p_n from (5.19), which is also in a form of the WF technique given by

$$p_n = \max\left\{0, \left[\frac{\mathcal{B}}{\ln(2)}\left(\frac{1+\lambda_2+\lambda_3}{\lambda_1+\mu_n}\right) - \frac{x_n I_{\text{pu},n}}{\bar{h}_{\text{ss},n}}\right]\right\}, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.20}$$

In addition to the Lagrangian multipliers, the power allocation policy in (5.20) depends on the CSI for the direct link between SU transmitter and SU receiver $h_{\text{ss},n}$, interference link between the SU receiver and the PU transmitter $h_{\text{ps},n}$, together with the PU transmission power P_{pu}.

Moreover, the Lagrangian function is derived with respect to x_n to determine the solution for optimal selection of channels in underlay transmission mode as follows:

$$\begin{aligned}\frac{\partial\mathcal{L}(\cdot)}{\partial x_n} = \frac{\mathcal{B}}{\ln(2)}&\left\{\ln\left(1+\frac{p_n\bar{h}_{\text{ss},n}}{+x_n I_{\text{pu},n}}\right) - \left(\frac{p_n\bar{h}_{\text{ss},n}}{p_n h_{\text{ss},n}+x_n I_{\text{pu},n}}\right)\right\} \\ &- p_n\left(\frac{\lambda_1+\mu_n}{1+\lambda_2+\lambda_3}\right), \qquad \forall n \in \boldsymbol{\mathcal{N}}\end{aligned} \tag{5.21}$$

where $I_{\text{pu},n} = \left[P_{\text{pu}}h_{\text{ps},n} + \sigma_{\text{pu}}^2\right]$ in the interference term. Let $f(x_n) = \partial\mathcal{L}(\cdot)/\partial x_n$, from which the second derivative of the Lagrangian function is given by the following:

$$f'(x_m) = \frac{\mathcal{B}}{\ln(2)}\left(\frac{p_n\bar{h}_{\text{ss},n}}{p_n\bar{h}_{\text{ss},n}+x_n I_{\text{PU},n}}\right)\left\{\frac{I_{\text{pu},n}}{p_n\bar{h}_{\text{ss},n}+x_n I_{\text{pu},n}} - \frac{1}{x_n}\right\}, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.22}$$

Similar to (5.13), the Newton iterative search method can be applied as follows:

$$x_n^{(k+1)} = x_n^{(k)} - \delta_f \frac{f\left(x_n^{(k)}\right)}{f'\left(x_n^{(k)}\right)}, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.23}$$

Moreover, similar to the approach presented in Section 5.4.2, the closed-form expression for optimal channel selection in underlay mode can alternatively be expressed in the form of the

Lambert-$\mathcal{W}$ function by equating the expression in (5.21) to zero to solve for the x_n's; based on which the following rearrangement can be easily established:

$$\ln\left(\frac{x_n I_{\mathrm{pu},n}}{p_n \bar{h}_{\mathrm{ss},n} + x_n I_{\mathrm{pu},n}}\right) - \frac{x_n I_{\mathrm{pu},n}}{p_n \bar{h}_{\mathrm{ss},n} + x_n I_{\mathrm{pu},n}} = -\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{(\lambda_1 + \mu_n)p_n}{1 + \lambda_2 + \lambda_3}\right)\right], \quad \forall n \in \boldsymbol{\mathcal{N}} \quad (5.24)$$

and further expressed in a form of a Lambert-$\mathcal{W}$ as follows:

$$-\frac{x_n I_{\mathrm{pu},n}}{p_n \bar{h}_{\mathrm{ss},n} + x_n I_{\mathrm{pu},n}} = \mathcal{W}\left(-\exp\left(-\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{(\lambda_1 + \mu_n)p_n}{1 + \lambda_2 + \lambda_3}\right)\right]\right)\right), \qquad \forall n \in \boldsymbol{\mathcal{N}}. \qquad (5.25)$$

Then from (5.25), the closed-form solution for optimal channel selection in underlay mode is given by the following expression, as an alternative to the Newton method:

$$x_n = -\frac{p_n \bar{h}_{\mathrm{ss},n} \times \mathcal{W}\left(-\exp\left(-\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{(\lambda_1+\mu_n)p_n}{1+\lambda_2+\lambda_3}\right)\right]\right)\right)}{I_{\mathrm{pu},n}\left(1 + \mathcal{W}\left(-\exp\left(-\left[1 + \frac{\ln(2)}{\mathcal{B}}\left(\frac{(\lambda_1+\mu_n)p_n}{1+\lambda_2+\lambda_3}\right)\right]\right)\right)\right)}, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \qquad (5.26)$$

Accordingly, the likelihood of any channel n to be selected in underlay mode increases with high channel power gain for the link between SU transmitter and SU receiver; while on the other hand, the likelihood decreases with high power gain for the interference link between PU transmitter and SU receiver. Moreover, the transmit power in underlay mode is bounded by the maximum power threshold $p_{\max,n}$, based on which the optimal transmit power is given by

$$p_n^* = \min\{p_n, p_{\max,n}\}, \qquad \forall n \in \boldsymbol{\mathcal{N}} \qquad (5.27)$$

The maximum power threshold $p_{\max,n}$ is derived from the outage constraint that is imposed to protect PU services. If the PU outage constraint cannot be satisfied by SU transmission, the corresponding channel cannot be allocated to protect PU nodes. In both underlay and overlay modes, the classical WF power distribution technique allocates more power to channel that have high power gains and less interference as it can be deduced from (5.10) and (5.20).

5.4.4 SU Transmit Power Threshold

For more emphasis in this chapter, the optimal ChA schemes aim to maximize SU capacity while keeping PU outage probability at minimum. For clarity of exposition therefore, four scenarios about the knowledge of CSI are investigated as discussed in sequel. This also follows from various scenarios investigated in various studies in literature [46], [49], [51]-[53], [56]-[57], [128]-[131]. In particular, this adapts the study presented in [128] for similar scenarios.

5.4.4.1 Full CSI at SU Transmitter

In the first scenario, full CSI is assumed at SU transmitter, which may be unrealistic for practical purposes as it would otherwise require cooperation with a high degree of sophistication between a PN and a CRN. Thus, obtaining the full knowledge of CSI may not always be possible. However, the results obtained based on the assumption of full CSI serve as a benchmark for performance evaluation of ChA schemes [68]. For the perfect CSI at the SU transmitter side, signal-to-noise and interference ratio (SNIR) threshold is imposed to protect any PU node, such that

$$\gamma_{\mathrm{pu}} = \frac{P_{\mathrm{pu}} h_{\mathrm{pp},n}}{p_{\mathrm{max},n} h_{\mathrm{sp},n} + \sigma_{\mathrm{pu}}^2}, \qquad \gamma_{\mathrm{pu}} \geq \gamma_{\mathrm{th}} \tag{5.28}$$

where γ_{pu} is the instantaneous SNIR at the PU receiver, and γ_{th} denotes the SNIR threshold. SU transmitter adapts its power with respect to instantaneous channel gains to protect PU services. The maximum allowable power is directly obtained by solving for $\{\gamma_{\mathrm{pu}} = \gamma_{\mathrm{th}}\}$ as follows:

$$p_{\mathrm{max},n} = \max\left\{0, \left[\frac{P_{\mathrm{pu}} h_{\mathrm{pp},n} - \gamma_{\mathrm{th}} \sigma_{\mathrm{pu}}^2}{\gamma_{\mathrm{th}} h_{\mathrm{sp},n}}\right]\right\}. \tag{5.29}$$

Then the optimal SU transmit power over channel n is obtained by applying (5.21) to (5.19). The adaptive transmit power for underlay SU transmissions is mainly governed by the channel gains $h_{\mathrm{pp},n}$ and $h_{\mathrm{sp},n}$ from both the PN and the CRN [61], [68], [105], [128]. In essence, SU transmitter adapts $p_{\mathrm{max},n}$ to a maximum value such that PU node SNIR does not fall below the predefined threshold. It is obvious from (5.29) that $p_{\mathrm{max},n}$ is inversely proportional to $h_{\mathrm{sp},n}$.

5.4.4.2 Partial CSI: Perfect PU Link

In the second scenario, the CSI for the direct link between PU transmitter and PU receiver $h_{\text{pp},n}$ is known, while only the average value is available for the interference link between the SU transmitter and the PU receiver $\bar{h}_{\text{sp},n}$. Due to the time varying nature of wireless channels, SU nodes cannot guarantee to satisfy the constraint in (5.28) during transmissions. Thus, the probability that (5.28) is satisfied is capped with outage probability threshold ε_{th}. The outage probability constraint at the PU receiver can therefore be formulated in conjunction with (5.28) as follows:

$$P_{\text{pu,out}} = \mathbb{Pr}\left\{\frac{P_{\text{pu}}\, h_{\text{pp},n}}{p_{\max,n}\, h_{\text{sp}} + \sigma_{\text{pu}}^2} \le \gamma_{\text{th}} \middle| h_{\text{pp},n}, \bar{h}_{\text{sp},n}\right\} \le \ \varepsilon_{\text{th}}, \tag{5.30}$$

where $\mathbb{Pr}\{\cdot\}$ denotes probability. It can therefore be stated that the probability that the constraint in (5.28) is violated must be less than ε_{th}. The expression in (5.30) can also be written as [128]

$$P_{\text{pu,out}} = \mathbb{Pr}\left\{h_{\text{sp}} \le \frac{P_{\text{pu}}\, h_{\text{pp},n} - \gamma_{\text{th}}\sigma_{\text{pu}}^2}{p_{\max,n}\gamma_{\text{th}}} \middle| h_{\text{pp},n}, \bar{h}_{\text{sp},n}\right\} \le \ \varepsilon_{\text{th}}. \tag{5.31}$$

Given that the channel power gains in both the PN and the CRN are exponentially distributed with the CDF given by: $F_{h_{\text{sp}}}(u) = 1 - \exp\left(u/\bar{h}_{\text{sp},n}\right)$, then it follows that the outage probability at the PU receiver due to SU transmissions is given by the following expression:

$$P_{\text{pu,out}} = 1 - \exp\left(-\frac{P_{\text{pu}} h_{\text{pp},n} - \gamma_{\text{th}}\sigma_{\text{pu}}^2}{\gamma_{\text{th}}\, p_{\max,n}\, \bar{h}_{\text{sp},n}}\right). \tag{5.32}$$

It can be easily inferred from (5.32) that the PU outage probability increases with deep fading for the interference link $h_{\text{sp},n}$ between the PU receiver and the SU transmitter. The maximum allowable power can be derived by substituting (5.32) into (5.31) as follows:

$$p_{\max,n} = -\frac{P_{\text{pu}} h_{\text{pp},n} - \gamma_{\text{th}}\sigma_{\text{pu}}^2}{\ln(1-\varepsilon_{\text{th}})\, \gamma_{\text{th}}\, \bar{h}_{\text{sp},n}}, \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.33}$$

Thus, the SU transmit power threshold is reduced in the case of deep fading for the SU interference link between SU transmitter and PU receiver, or increased otherwise as illustrated by (5.33).

5.4.4.3 Partial CSI: Perfect SU Interference Link

In the third scenario, the full knowledge of CSI for the SU interference link $h_{\mathrm{sp},n}$ is assumed, together with the average value of the direct link $\bar{h}_{\mathrm{pp},n}$ between the PU transmitter and the PU receiver in the PN. The PU outage probability constraint to protect PU services in this case can therefore be expressed as follows [49], [128], [130]:

$$P_{\mathrm{pu,out}} = \mathbb{Pr}\left\{\frac{P_{\mathrm{pu}}\, h_{\mathrm{pp},n}}{p_{\mathrm{max},n}\, h_{\mathrm{sp}} + \sigma_{\mathrm{pu}}^2} \le \gamma_{\mathrm{th}} \middle| \bar{h}_{\mathrm{pp},n}, h_{\mathrm{sp},n}\right\} \le \varepsilon_{\mathrm{th}}, \tag{5.34}$$

which can further be expressed as

$$P_{\mathrm{pu,out}} = \mathbb{Pr}\left\{h_{\mathrm{pp},n} \le \frac{\gamma_{\mathrm{th}}\left(p_{\mathrm{max},n}\, h_{\mathrm{sp}} + \sigma_{\mathrm{pu}}^2\right)}{P_{\mathrm{pu}}} \middle| \bar{h}_{\mathrm{pp},n}, h_{\mathrm{sp},n}\right\} \le \varepsilon_{\mathrm{th}}. \tag{5.35}$$

Then following from the previous scenario about the CDF of exponentially distributed random variable $h_{\mathrm{pp},n}$, the closed-form expression for the PU outage probability is then given by

$$P_{\mathrm{pu,out}} = 1 - \exp\left(-\frac{\gamma_{\mathrm{th}}\left(p_{\mathrm{max},n} h_{\mathrm{sp},n} + \sigma_{\mathrm{pu}}^2\right)}{P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}}\right). \tag{5.36}$$

Obviously, PU outage probability decreases with high channel power gains for the direct link between the PU transmitter and the PU receiver $h_{\mathrm{pp},n}$ as illustrated by (5.36), and increases with deep fading otherwise. The maximum allowable SU transmit power is inversely proportional to the interference link $h_{\mathrm{sp},n}$, as given by the following expression:

$$p_{\mathrm{max},n} = -\frac{1}{h_{\mathrm{sp},n}}\left(\frac{\ln(1-\varepsilon_{\mathrm{th}})\, P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}}{\gamma_{\mathrm{th}}} + \sigma_{\mathrm{pu}}^2\right), \qquad \forall n \in \boldsymbol{\mathcal{N}}. \tag{5.37}$$

It can be easily deduced from (5.37) that good channel power gains for $h_{\mathrm{pp},n}$ in the PN provide more degrees of freedom for SU transmit power threshold; hence beneficial for SU nodes.

5.4.4.4 Statistical CSI at SU transmitter

In the fourth scenario, only the statistical knowledge of CSI about the PN and CRN is assumed; hence $\bar{h}_{\mathrm{pp},n}$ and $\bar{h}_{\mathrm{sp},n}$ are assumed to be available at the SU transmitter. The outage probability constraint to protect PU services can therefore be expressed as follows [128]:

$$P_{\mathrm{pu,out}} = \mathbb{Pr}\left\{\frac{P_{\mathrm{pu}}\, h_{\mathrm{pp},n}}{p_{\mathrm{max},n}\, h_{\mathrm{sp}} + \sigma_{\mathrm{pu}}^2} \leq \gamma_{\mathrm{th}} \middle| \bar{h}_{\mathrm{pp},n}, \bar{h}_{\mathrm{sp},n}\right\} \leq \varepsilon_{\mathrm{th}}. \tag{5.38}$$

Suppose $f_h(u) = 1/\bar{h}_{\mathrm{sp},n} \times \exp\left(-u/\bar{h}_{\mathrm{sp},n}\right)$; then the following can be established from (5.38):

$$P_{\mathrm{pu,out}} = \int_0^\infty \mathbb{Pr}\left\{\frac{P_{\mathrm{pu}}\, h_{\mathrm{pp},n}}{u \times p_{\mathrm{max},n} + \sigma_{\mathrm{pu}}^2} \leq \gamma_{\mathrm{th}}\right\} f_h(u) du, \tag{5.39a}$$

$$= \int_0^\infty \mathbb{Pr}\left\{P_{\mathrm{pu}}\, h_{\mathrm{pp},n} \leq \gamma_{\mathrm{th}}\left(u\, p_{\mathrm{max},n} + \sigma_{\mathrm{pu}}^2\right)\right\} \left(\frac{1}{\bar{h}_{\mathrm{sp},n}} \exp\left(-\frac{u}{\bar{h}_{\mathrm{sp},n}}\right)\right) du, \tag{5.39b}$$

$$= \int_0^\infty \left\{1 - \exp\left(-\frac{\gamma_{\mathrm{th}}\left(u\, p_{\mathrm{max},n} + \sigma_{\mathrm{pu}}^2\right)}{P_{\mathrm{pu}}\, \bar{h}_{\mathrm{pp},n}}\right)\right\} \left(\frac{1}{\bar{h}_{\mathrm{sp},n}} \exp\left(-\frac{u}{\bar{h}_{\mathrm{sp},n}}\right)\right) du. \tag{5.39c}$$

Then the closed-form expression for outage probability is given by

$$P_{\mathrm{pu,out}} = 1 - \frac{P_{\mathrm{pu}}\, \bar{h}_{\mathrm{pp},n}}{\gamma_{\mathrm{th}}\, p_{\mathrm{max},n}\, \bar{h}_{\mathrm{sp},n} + P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}} \exp\left(-\frac{\gamma_{\mathrm{th}} \sigma_{\mathrm{pu}}^2}{P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}}\right). \tag{5.40}$$

From (5.40), the following is applied $\left\{P_{\mathrm{pu,out}} = \varepsilon_{\mathrm{th}}\right\}$ in (5.38) towards solving for $p_{\mathrm{max},n}$. Then the closed-form expression for the upper-bound on the average SU transmit power is given by

$$p_{\mathrm{max},n} = \frac{P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}}{\gamma_{\mathrm{th}} \bar{h}_{\mathrm{sp},n}} \left[\frac{1}{(1 - \varepsilon_{\mathrm{th}})} \exp\left(-\frac{\gamma_{\mathrm{th}} \sigma_{\mathrm{pu}}^2}{P_{\mathrm{pu}} \bar{h}_{\mathrm{pp},n}}\right) - 1\right]. \tag{5.41}$$

Thus, deep fading for the interference link is beneficial to improve SU performance as more power can be allocated for SU transmission. Without loss of generality, the Lagrangian optimization framework provides a relaxed solution that serves as the upper-bound to the solution of the original problem. The BnB technique presented in [27] and Chapter 3 of this thesis is then employed to recover the optimal solution of the original MINLP problem. Thus, the BnB facilitates partial enumeration to solve the MINLP problem without explicit evaluation of the entire feasible discrete solution space, which would otherwise in a worse case, be difficult to solve in polynomial time, especially as the number of channels increases [27], [79], [113]-[114].

5.5. Simulation Results and Discussion

5.5.1 Simulation Model

In this chapter, performance evaluation is mainly based on SU average capacity, outage probability and collision probability. Similar to the work presented in Chapter 3, outage probability refers to the probability that the SU average capacity falls below a minimum threshold $\mathcal{R}_{\phi}$; while collision refers to the probability that the collision constraint is violated, with reference to the constraint defined in (C5.3). Except in the case where the performance metrics are investigated against varying SU transmit power, SU transmit power and other parameters are shown in Table 5-1. Further, seven variants of ChA schemes are investigated as presented in Table 5-2, with respect to various assumptions about the knowledge of CSI at the SU transmitter.

5.5.2 Results and Discussion

This subsection presents the main discussions on performance analysis and evaluation of the hybrid ChA schemes. In Fig. 5-2, the results for average capacity against PU arrival rates are presented. At low PU arrival rates, ChA schemes that incorporate PU arrivals to make ChA decisions have better capacity, *i.e.* ChA-C to ChA-G. As it was also shown from Chapter 3, this is based on the flexibility to assemble more channels without violating the collision constraint, which in this case is defined in (C5.3). As PU arrivals increase, collision probability increases; as a result, the proposed optimal ChA schemes reduce the number of assembled channels. The hybrid overlay and underlay schemes (*i.e.* ChA-D to ChA-G) outperform the overlay optimal scheme (*i.e.* ChA-C) by achieving relatively higher capacity. This is because the hybrid ChA schemes select more channels in underlay mode to increase the sum capacity. Moreover, the hybrid ChA scheme with full CSI (*i.e.* ChA-G) results in more performance gain by achieving the highest capacity for SU transmissions.

TABLE 5-1: HYBRID OVERLAY AND UNDERLAY CHA SCHEMES

Parameters	Value
Number of frames	10^4
Total Number of PU channels	16
PU Transmit power	2W
Total SU transmit power	1W
Collision probability threshold	$0.1 - 0.2$
SU QoS requirement on capacity	5.5Kbps
Bandwidth ($\mathcal{B}$)	1 MHz

TABLE 5-2: HYBRID OVERLAY AND UNDERLAY CHA SCHEMES

Scheme	Channel Access	No. of Channels	Power Profile
ChA-A (FixedKEP)	Overlay full CSI	Pre-fixed	Equal-power
ChA-B (FixedKOP)	Overlay full CSI	Pre-fixed	Waterfilling
ChA-C (BnBKOP)	Overlay full CSI	Optimal	Waterfilling
ChA-D (HybStatistics)	Hybrid with partial CSI	Optimal	Waterfilling
ChA-E (HybFullSU)	Hybrid with full SU link CSI	Optimal	Waterfilling
ChA-F (HybFullPU)	Hybrid with full PU link CSI	Optimal	Waterfilling
ChA-G (HybFullCSI)	Hybrid with full CSI	Optimal	Waterfilling

In accordance with Fig. 5-2, Fig. 5-3 illustrates that outage probability increases with the increase in PU arrival rates. Hence, as a result of low capacity for optimal ChA schemes at high PU arrivals, outage probability increases. Fig. 5-4 illustrates that the optimal ChA schemes do not violate the collision constraint as PU arrivals increase. In essence, the optimal ChA schemes adapt the number of assembled channels accordingly, in order to meet the collision constraint; whereas prefixing the number of channels increases collision probability at high PU arrivals, as illustrated in Fig. 5-4 for the ChA-A and ChA-B. It can therefore be deduced that, the fixed ChA schemes which are based on the prefixed number of channels (*i.e.* fixed-$\mathcal{K}$) are highly susceptible to performance degradation, mainly as a result of forced terminations at high PU activity patterns.

In Fig. 5-5 to Fig. 5-7, results for the aforementioned performance metrics are presented against the increasing SU total transmit power. Fig. 5-5 presents results on capacity, which shows that the adaptive schemes (*i.e.* ChA-C to ChA-G) have zero capacity. This is because at low SU transmit power, a feasible solution that satisfies all the constraints may not be obtained, especially for the QoS requirement on capacity. The sum capacity increases significantly as transmit power increases, with ChA-F and ChA-G achieving the highest capacity. However, performance of the adaptive ChA schemes becomes the same as transmit power increases, *i.e.* above 10dB in Fig. 5-5.

Fig. 5-6 presents results on outage probability against increasing SU transmit power. Consistent with Fig. 5-5, the results in Fig. 5-6 illustrate that outage probability deceases as the SU transmit power increases, with the ChA-F and ChA-G schemes achieving the lowest outage probability. Then Fig. 5-7 presents results for collision probability, which illustrates that although the ChA schemes with fixed number of channels (*i.e.* the ChA-A and ChA-B) may access the PU spectrum, performance is worse in terms of collision probability. At high SU transmit power, adaptive schemes assemble more channels to maximize capacity subject to the collision constraint as shown in Fig. 5-7 where the collision probability threshold was capped at 0.2 for SU transmissions.

Finally, Fig. 5-8 presents results for SU outage probability as a function of QoS requirement on capacity $\mathcal{R}_\phi$. Obviously, outage probability is zero when $\mathcal{R}_\phi = 0$, and increases as the $\mathcal{R}_\phi$ increases. As Fig. 5-8 shows, outage probability increases fastest for the ChA-A with fixed number of channels but without power optimization, followed by the ChA-B with fixed number of channels and power optimization. The results illustrate the significance of power optimization even in the case where the number of assembled channels is fixed. Among the adaptive ChA schemes, outage probability increases faster for the overlay scheme (*i.e.* ChA-C) than hybrid schemes (*i.e.* ChA-D to ChA-G), with ChA-F and and ChA-G showing the lowest outage probability.

In general, the results illustrate the importance of hybrid spectrum access techniques for improving performance of SU nodes, as well as the impact of the knowledge of CSI available at the SU transmitter. As the results illustrate, the perfect knowledge of CSI at the SU transmitter is significantly beneficial for improving performance of SU nodes. However, such may require full cooperation for sharing of information between a PN and a CRN, which would without any doubt, be technically challenging to implement in practical networks [64], [128], [130].

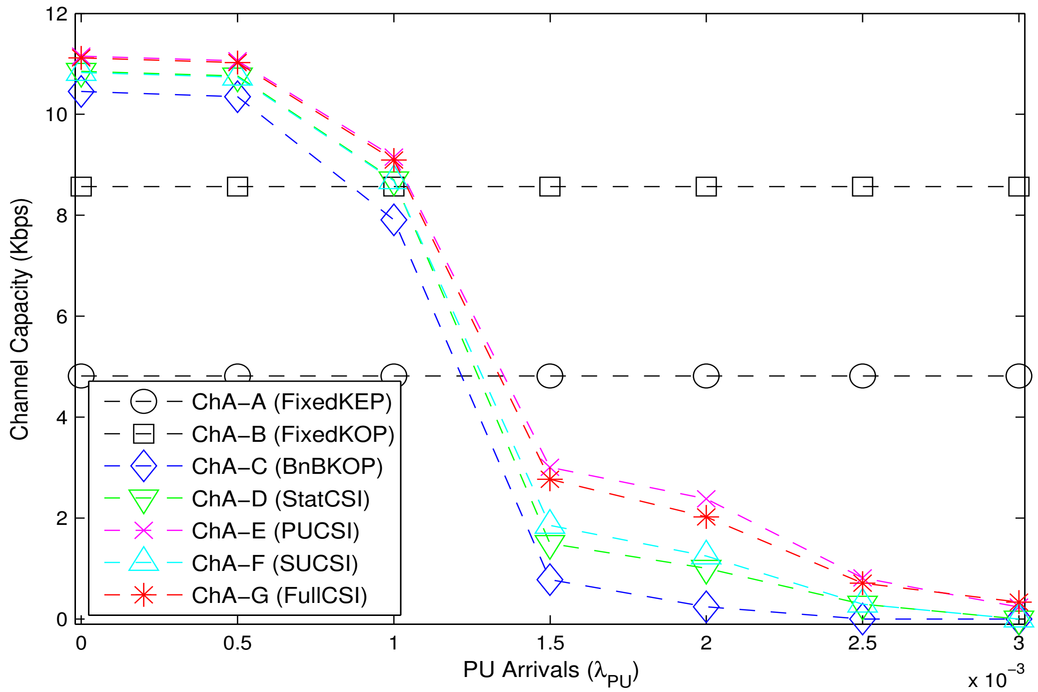

Figure 5-2: Channel capacity for increasing PU arrival rates.

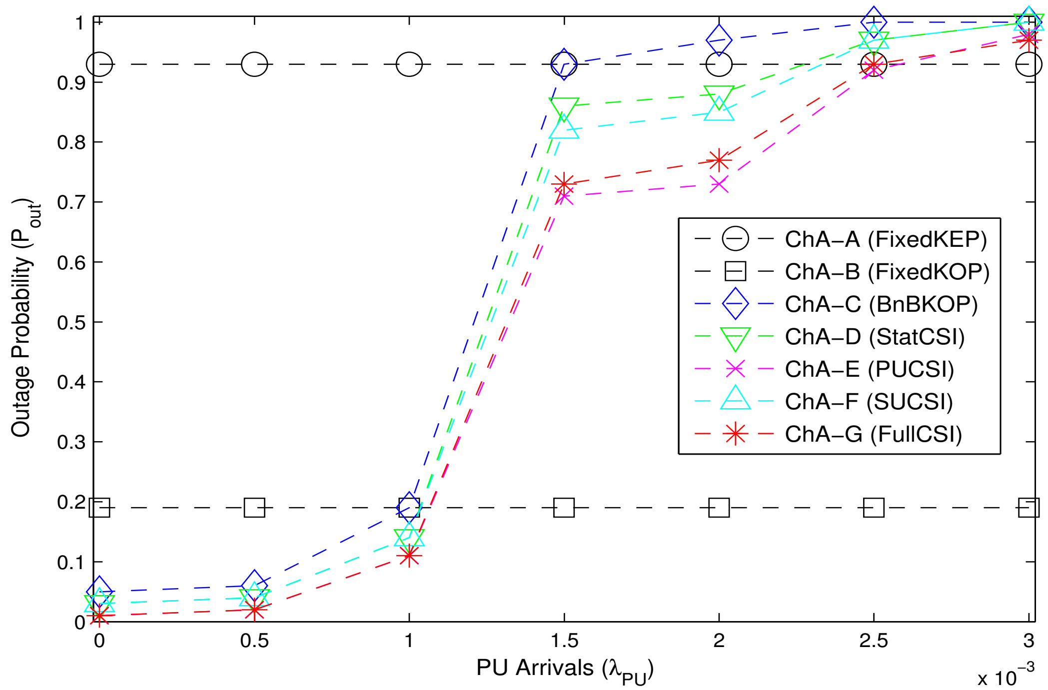

Figure 5-3: SU Outage probability for increasing PU arrival rates.

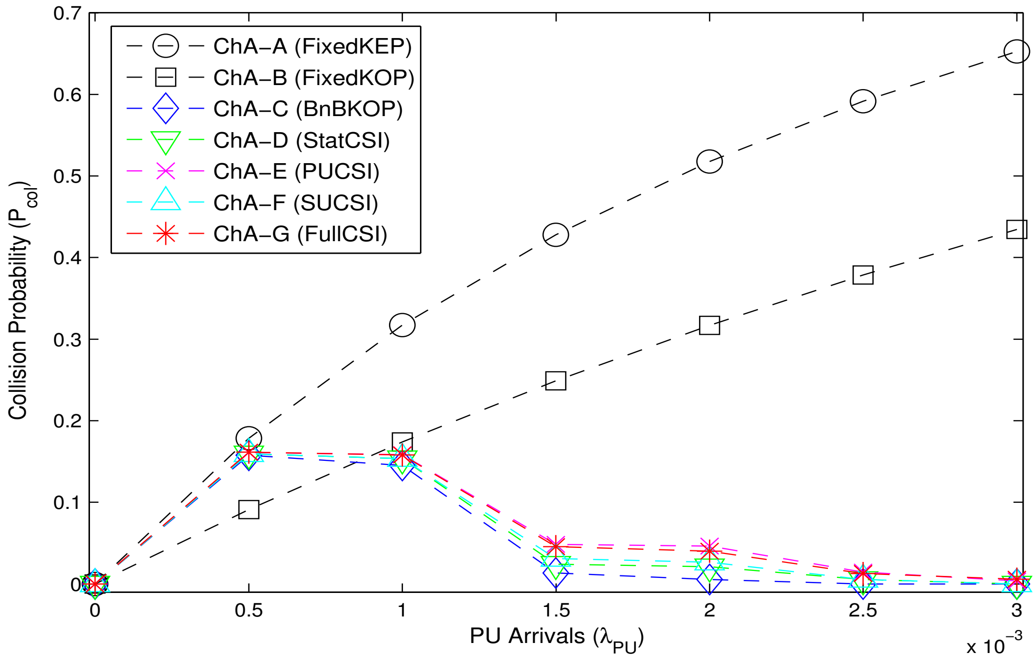

Figure 5-4: Collision probability for increasing PU arrival rates.

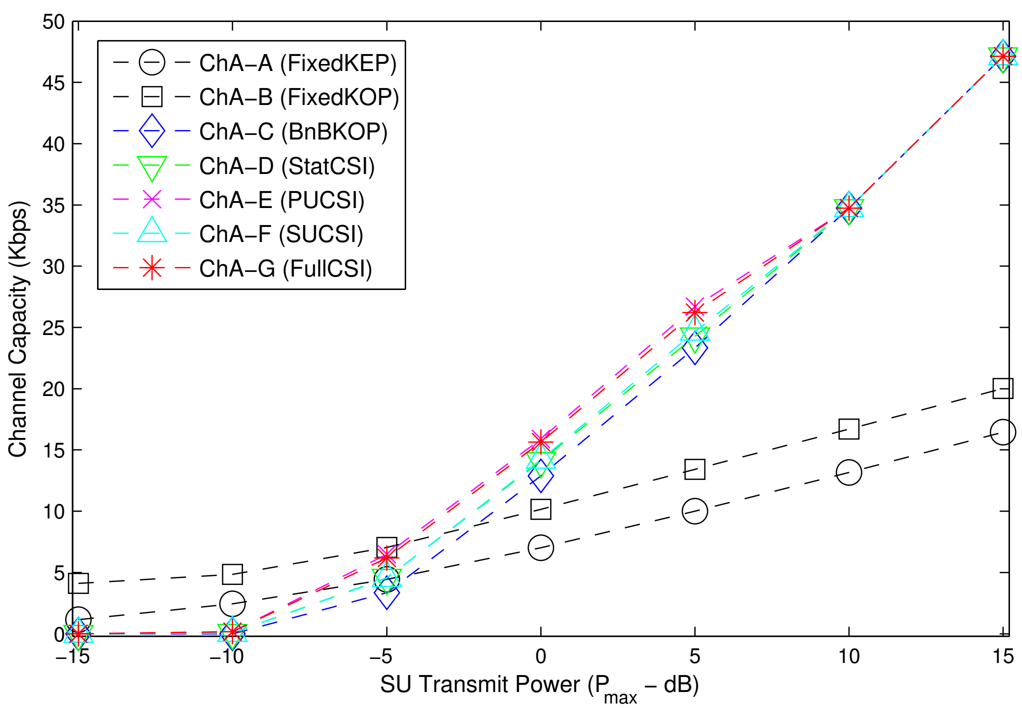

Figure 5-5: SU channel capacity versus SU total transmit power.

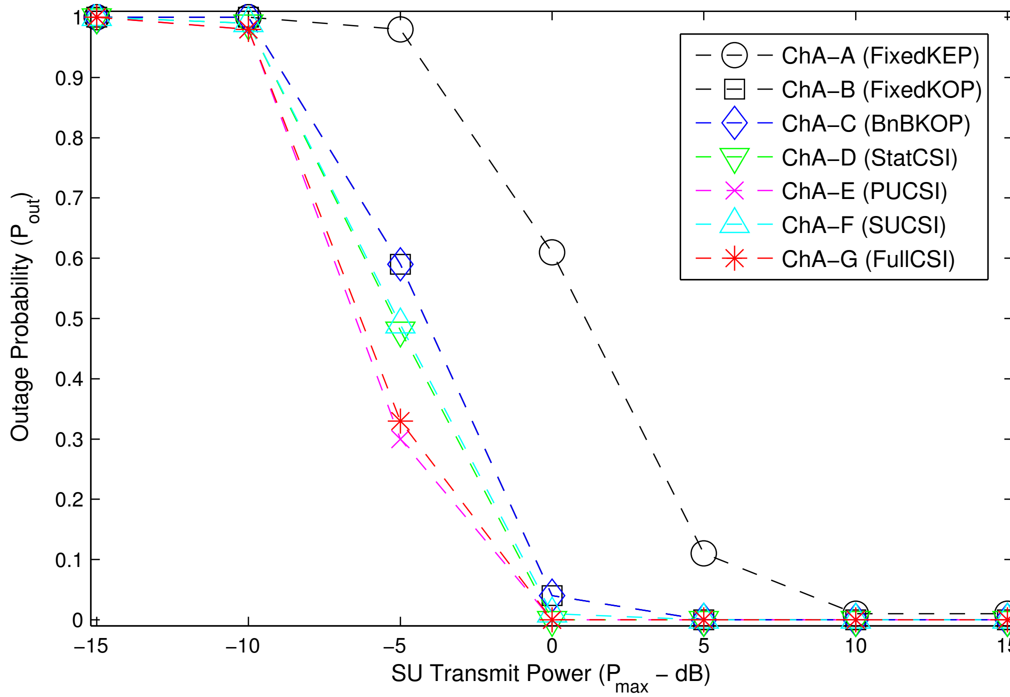

Figure 5-6: Channel capacity for increasing PU arrival rates.

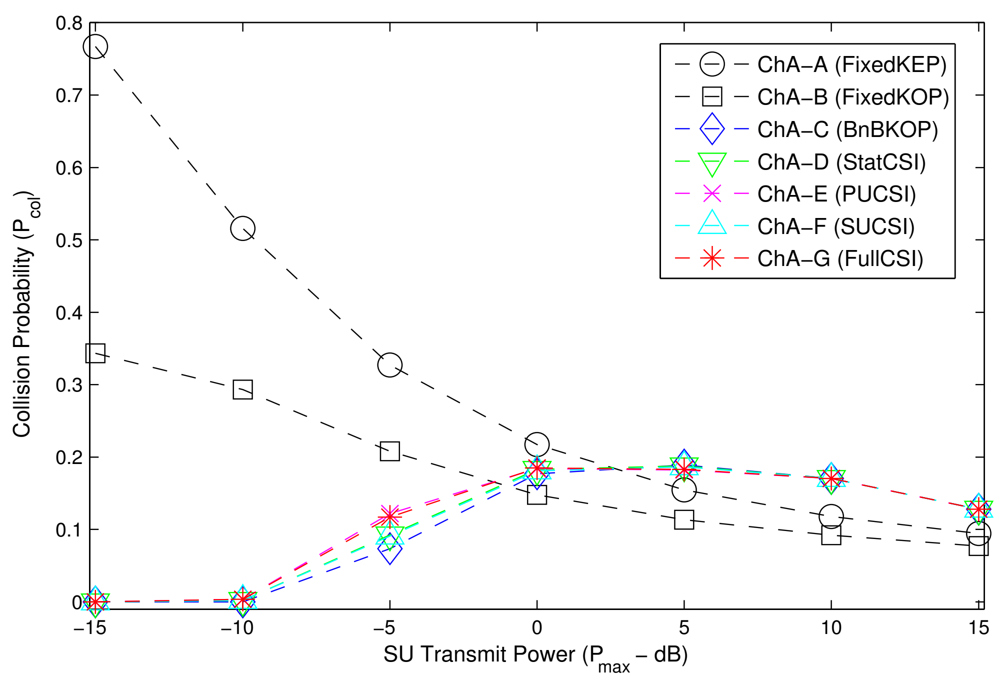

Figure 5-7: Collision probability versus SU total transmit power.

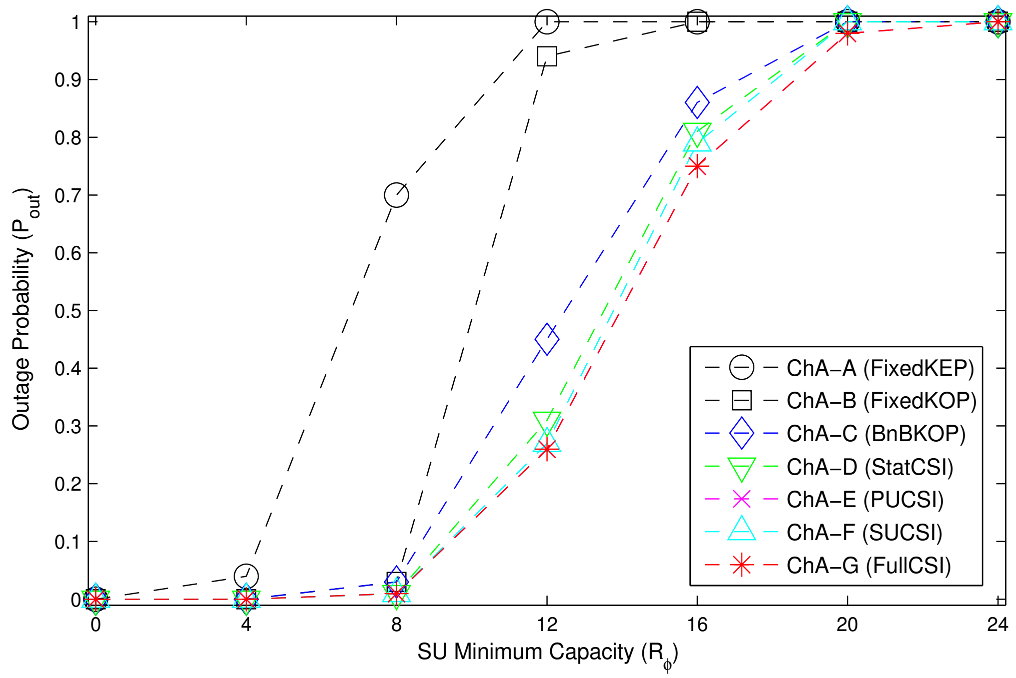

Figure 5-8: Outage probability versus SU QoS requirement on capacity.

5.6. Chapter Conclusion

This chapter presented the hybrid ChA schemes that select channels in both overlay mode and underlay mode, with the aim to maximize capacity in spectrum sharing networks. PU activity patterns have been incorporated to determine the optimal solutions for hybrid spectrum access. Multilevel adaptive transmit power over the channels selected in overlay mode has been developed based on PU outage. Further, as an alternative to the Newton technique and other iterative search methods, the closed-form expressions for channel selection have been developed based on the Lambert-$\mathcal{W}$ function. In general, the presented results illustrate that hybrid ChA schemes offer significant performance improvement in CRNs. The results also reveal that the type of the knowledge of CSI available at the SU transmitter affects performance of ChA schemes, in which case the perfect knowledge of CSI offers best performance. Inasmuch as spectrum may be hardly available for SU nodes at high PU arrivals, hybrid overlay and underlay spectrum access technique comprises one of the effective mechanisms to circumvent performance degradation in CRNs.

CHAPTER SIX

6 MULTIUSER MULTICHANNEL SPECTRUM ACCESS

6.1 Introduction

THIS chapter presents the optimal ChA techniques for multiuser multichannel spectrum access in CRNs. This facilitates efficient allocation of PN radio spectrum to multiple SU nodes with adaptive channel selection and optimal power distribution. The key objective is to maximize capacity subject to network and resource constraints in a multiuser CRN, where multiple SU nodes contest for PU spectrum access. ChA technique allows each of the competing SU nodes to use multiple channels in the PN spectrum. Notwithstanding, efficient ChA schemes in multiuser CRNs are essential, as it is possible that increasing the number of channels assigned to a single SU node may reduce the overall network capacity; while on the other hand, the number of SU nodes contending for spectrum access affects the number of channels that each SU node may assemble.

Thus, the appropriate number of channels assigned to a single SU node should be determined for efficient spectrum access in multiuser environments. In this regard, two adaptive ChA schemes are proposed: first, the optimal ChA scheme based on Lagrangian framework; second, the suboptimal ChA scheme based on the Hungarian algorithm. The Hungarian algorithm has mainly been applied in scenarios where users are allocated the same amount of resources. In general, the presented simulation results illustrate significant performance improvement for the proposed ChA schemes in terms of capacity, outage probability and collision probability. Part of the work presented in this chapter has been submitted for publication in the proceedings of the *IEEE AFRICON'2017*.

The rest of this chapter is organised as follows: Section 6.2 highlights the related works on dynamic resource allocation (DRA) and ChA schemes for multiuser multichannel access in CRNs, followed by Section 6.3 which presents the system model together with the problem formulation. Then Section 6.4 presents the proposed solutions for the optimal and suboptimal ChA schemes for multiuser multichannel access in CRNs. Section 6.5 presents the simulation based performance analysis and discussion, and finally the concluding remarks in Section 6.6.

6.2 Related Works

This section provides an overview of the existing works on resource allocation and ChA schemes in multiuser CRNs. In [27], a study on optimal ChA scheme was presented, which also forms part of the work presented in Chapter 3 of this thesis. The ChA scheme presented in [27] selects the optimal number of channels and power distribution to maximize SU capacity in overlay CRNs. However, the work in [27] is restricted to arrival of a single SU service to optimize ChA schemes in CRNs. Further, the key studies on MAC protocols with channel bonding and aggregation in CRNs have inherently investigated issues relating to multiuser multichannel access [28], [31]-[32], [84], [86]. Nonetheless, the CTMC modeling employed in these studies assume homogenous channels, without spectrum profiling and power optimization [26], [76]. These works also assume a predetermined number of channels assembled by each SU node in multiuser CRNs.

Some of the existing works on resource allocation include adaptive techniques in OFDM systems, which are mainly concentrated on user selection and subcarrier allocation to maximize data rates, subject to transmit power constraint [42], [95]-[98], [112]-[114]. Moreover, the main studies on DRA in CRNs mainly focused on user selection and scheduling to maximize sum capacity, as well as power allocation techniques and control to mitigate interference for protection of PU [3], [49], [51]-[53], [57], [130]. The works do not account for issues related to performance of ChA in CRNs. In addition to the challenges inherent to fading, performance of ChA schemes is mainly dictated by network dynamics such as PU activity patterns. Accordingly, this chapter proposes efficient ChA schemes with adaptive number of channels and optimal power distribution in multiuser CRNs.

6.3 System Model and Problem Formulation

6.3.1 System Model

In this chapter, a centralized CRN architecture is considered, where a central controller coordinates resource allocation for a set of J SU nodes $\boldsymbol{J}$, that coexist with PU nodes as shown in Fig. 6-1. The adopted network model has also been considered in [78], where individual SU nodes send their CSI to the central controller for coordination. Unlike the work presented in this chapter, the study in [78] is mainly focused on CTMC modelling for analysis of ChA schemes. A set of M available PU

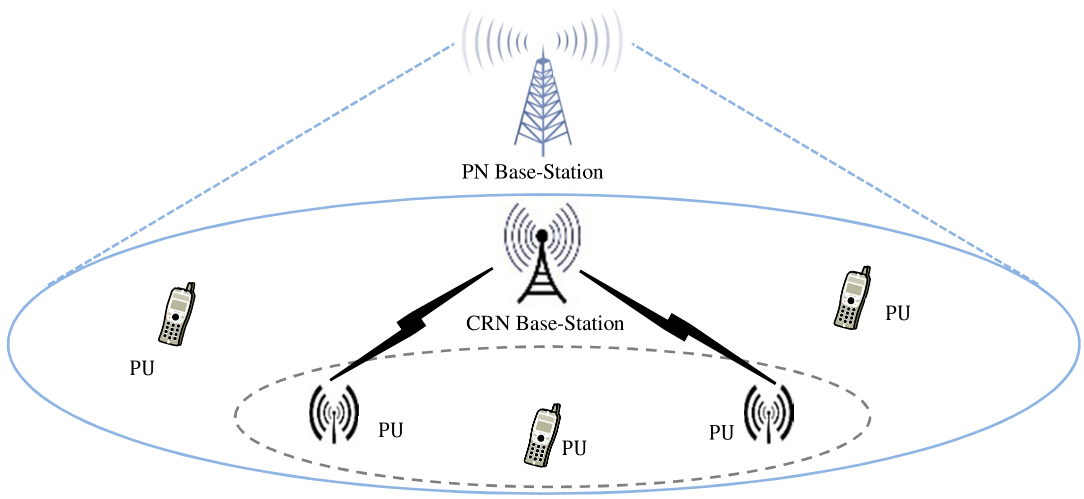

Figure 6-1: Network model for infrastructure based multiuser multichannel CRN.

channels is denoted by $\boldsymbol{\mathcal{M}}$. Furthermore, the channel state information is denoted by a $\{\mathcal{J} \times M\}$ matrix $\boldsymbol{H}$, for which a corresponding feasible binary integer matrix $\boldsymbol{\mathcal{X}} \triangleq \{x_{j,m} | x_{j,m} \in \{0,1\}\}$ represents channel selection for each node j, on any channel m for ChA schemes, where

$$x_{j,m} = \begin{cases} 1 & \text{if channel } m \text{ is assigned to node } j \\ 0 & \text{otherwise.} \end{cases} \tag{6.1}$$

Also, the channel selection matrix has a corresponding nonnegative matrix for power allocation matrix $\boldsymbol{\mathcal{P}} \triangleq \{p_{j,m} \mid p_{j,m} \geq 0\}$. Based on the Shannon theorem, expected channel capacity ($\bar{r}_{j,m}$) for SU_j on any channel m is given by the following expression [107]:

$$\bar{r}_{j,m} = \mathbb{E}\{\mathcal{B} \log_2(1 + p_{j,m} h_{j,m})\}, \tag{6.2}$$

where $h_{j,m} = |g_{j,m}|^2 / N_0 \mathcal{B}$, N_0 denotes the Gaussian background noise power spectral density, $\mathcal{B}$ represents the bandwidth, and $g_{j,m}$ denotes the channel fading coefficient which is characterized by Rayleigh distribution, based on which $h_{j,m}$ is therefore assumed to be exponentially distributed [27], [109]. In the case of ChA schemes, the sum capacity for each SU node is a function of the number of assembled channels. The sum capacity for SU_j obtained over $\mathcal{K}$ channels is given by

$$\bar{\mathcal{R}}_{j,\mathcal{K}} = \sum_{m=1}^{\mathcal{K}} \mathbb{E}\{\mathcal{B}\log_2(1 + p_{j,m}h_{j,m})\}, \qquad \forall j \in \boldsymbol{\mathcal{J}} \tag{6.3}$$

where $\mathcal{K} \leq M$. Obviously, the sum capacity depends on which channels are selected for assembling by each SU, as well as the power distribution across the selected channels. The optimization problem to maximize the sum capacity is formulated in sequel.

6.3.2 Multiuser Problem Formulation

Given a set of J SU nodes in a CRN, contending for M PU channels, the optimization problem to maximize the average sum capacity can be formulated as follows [27], [108]:

$$\underset{\{x_{j,m},p_{j,m}\}}{\text{maximize}} \sum_{j=1}^{\mathcal{J}} \sum_{m=1}^{M} x_{j,m}\mathcal{B}\log_2\left(1 + \frac{p_{j,m}\bar{h}_{n,m}}{x_{j,m}}\right) \tag{6.4}$$

$$\text{Subject to: } \sum_{m=1}^{M} x_{j,m}p_{j,m} \leq P_{j,\max}, \quad x_{j,m}p_{j,m} \geq 0, \tag{C6.1}$$

$$\mathcal{R}_{j,\phi} \leq \sum_{m=1}^{M} x_{j,m}\mathcal{B}\log_2\left(1 + \frac{p_{j,m}\bar{h}_{j,m}}{x_{j,m}}\right), \tag{C6.2}$$

$$\beta \sum_{m=1}^{M} x_{j,m} \leq \sum_{m=1}^{M} x_{j,m}\mathcal{B}\log_2\left(1 + \frac{p_{j,m}\bar{h}_{j,m}}{x_{j,m}}\right), \tag{C6.3}$$

$$\sum_{j=1}^{\mathcal{J}} x_{j,m} \leq 1, \qquad x_{j,m} \in \{0,1\}, \tag{C6.4}$$

where $P_{j,\max}$ is the total power distributed across all the channels assembled by SU_j, $\mathcal{R}_{j,\phi}$ is the minimum QoS requirement on capacity. The Constraint in (C6.1) ensures that the total power does not exceed the available power for each SU node, and only nonnegative power can be allocated.

(C6.2) ensures that the sum capacity for each SU node exceeds the minimum threshold. (C6.3) ensures that the collision probability for assembled channels is maintained below a minimum threshold, where $\beta = \{\lambda_{\text{pu}} L_{\text{Data}} / -\ln(\mathcal{P}_{\phi})\}$ is a function of collision probability threshold $\mathcal{P}_{\phi}$, the PU arrival rate λ_{pu}, and SU packet size L_{Data}. (C6.3) has been derived from the condition that: $\{\mathcal{P}_{\phi} \leq \mathcal{P}_{\mathcal{A}}\}$, where $\mathcal{P}_{\mathcal{A}}$ is the probability that any SU finishes transmission without collision on any of the assembled channels within the period ΔT_{Tx}, given by [27], [33]:

$$\mathcal{P}_{\mathcal{A}}(\Delta T_{\text{Tx}}) = \exp\left(-\frac{\lambda_{\text{pu}} L_{\text{Data}} \sum_{m=1}^{M} x_{j,m}}{\sum_{m=1}^{M} x_{j,m} \mathcal{B} \log_2\left(1 + \frac{p_{j,m}\bar{h}_{j,m}}{x_{j,m}}\right)}\right). \tag{6.5}$$

Then (C6.4) ensures that a channel can only be occupied by a single SU node, which enforces exclusive channel selection constraint. As a result of the binary integer variables for channel selection, the constrained optimization problem in (6.4) is also in a form of MINLP problem; which as it has already been mentioned, is generally known to be NP-hard [25]-[26], [78], [107]. Thus, the complexity increases exponentially with the increasing number of SU nodes and the number of available channels. For example, solving the problem in (6.4) with the integer constraints would generate $(M^{\mathcal{J}})$ possible channel selection solutions using exhaustive search [112]-[113].

6.4 Multiuser Channel Allocation and Power Distribution

6.4.1 Lagrangian Convex Optimization

In this section, the Lagrangian technique is employed to solve the constrained optimization problem. To convert the MINLP problem into a convex optimization framework, the channel selection indicator variable is relaxed to take a continuous real value such that $\{0 \leq x_{j,m} \leq 1\}$. The relaxed $x_{j,m}$ can be considered as a soft decision for uncertainty into whether a channel is assigned a user or not. The objective function is jointly concave with respect to the primal variables, $\{x_{j,m}, p_{j,m}\}$, for which the Hessian matrix is negative semi-definite, and the inequality constraints are convex [91]. Thus, the relaxed optimization problem is unimodal with a unique optimal solution that has zero duality gap, and the solution can be obtained in polynomial time [27], [91]-[92], [108].

Further, all the feasible solutions to the original MINLP problem are within the solution space of the relaxed problem. The Lagrangian $\mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta})$ from (6.4) can be formulated as

$$\mathcal{L}(\cdot) = \sum_{j=1}^{\mathcal{J}} (1 + \nu_j + \mu_j) \left(\sum_{m=1}^{M} x_{j,m} \mathcal{B} \log_2 \left(1 + \frac{p_{j,m} \bar{h}_{j,m}}{x_{j,m}} \right) \right) - \sum_{j=1}^{\mathcal{J}} \lambda_j \left(\sum_{m=1}^{M} x_{j,m} p_{j,m} - P_{j,\max} \right)$$
$$- \sum_{j=1}^{\mathcal{J}} \nu_j \mathcal{R}_{j,\phi} - \sum_{j=1}^{\mathcal{J}} \mu_j \left(\beta \sum_{m=1}^{M} x_{j,m} \right) - \sum_{m=1}^{M} \eta_m \left(\sum_{j=1}^{\mathcal{J}} x_{j,m} - 1 \right) \tag{6.6}$$

where $\{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}\}$ are Lagrangian nonnegative multiplier vectors associated with each of the constraints. Then the dual function is defined by the following [91]:

$$\mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}) = \inf_{\{\mathcal{X}, \mathcal{P}\}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}), \tag{6.7}$$

from which the dual problem can be formulated as

$$\begin{gathered} \min_{\{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}\}} \mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}), \\ \text{Subject to: } \{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\eta}\} \geq 0. \end{gathered} \tag{6.8}$$

The dual function can be decomposed into $\{\mathcal{J} \times M\}$ manageable subproblems that can be solved efficiently [68], [92]. To facilitate the decomposition, the Lagrangian can then be reformulated as

$$\mathcal{L}(\cdot) = \sum_{j=1}^{\mathcal{J}} \left(\sum_{m=1}^{M} \ell_{j,m}(\cdot) + \lambda_j P_{j,\max} - \mu_j \mathcal{R}_{j,\phi} - \eta_m \right), \tag{6.9}$$

where $\mathcal{L}_{j,m}(\cdot)$ is defined as:

$$\ell_{j,m}(\cdot) = \psi_j x_{j,m} \mathcal{B} \log_2 \left(1 + \frac{p_{j,m} \bar{h}_{j,m}}{x_{j,m}} \right) - \lambda_j x_{j,m} p_{j,m} - \mu_j \beta x_{j,m} - \eta_m \sum_{j=1}^{\mathcal{J}} x_{j,m}, \tag{6.10}$$

where $\psi_j = (1 + \nu_j + \mu_j)$. Then to establish the necessary and sufficient KKT conditions for optimal solutions for each of the subproblems, the partial derivatives are obtained in terms of the primal variables $\{x_{j,m}, p_{j,m}\}$ as follows:

$$\frac{\partial \ell_{j,m}(\cdot)}{\partial p_{j,m}} = \frac{\mathcal{B}}{\ln(2)} \psi_j \left(\frac{x_{j,m} \bar{h}_{j,m}}{x_{j,m} + p_{j,m} \bar{h}_{j,m}} \right) - \lambda_j x_{j,m}. \tag{6.11}$$

The optimal power allocation is obtained by equating (6.11) to zero and solving for $p_{j,m}$, and the solution is in a form of the WF technique [108], [111]:

$$p^*_{j,m} = \max\left\{0, \left[\frac{\mathcal{B}}{\ln(2)} \left(\frac{\psi_j}{\lambda_j}\right) - \frac{x_{j,m}}{\bar{h}_{j,m}}\right]\right\}, \qquad \forall j. \tag{6.12}$$

Hence, the power allocation scheme constitutes a multilevel WF technique for different SU nodes in CRNs. Also, the following can be established:

$$\begin{aligned} \frac{\partial \ell_{j,m}(\cdot)}{\partial x_{j,m}} = \frac{\mathcal{B}}{\ln(2)} \psi_j \left(\ln\left(1 + \frac{p_{j,m} \bar{h}_{j,m}}{x_{j,m}}\right) - \left(\frac{p_{j,m} \bar{h}_{j,m}}{x_{j,m} + p_{j,m} \bar{h}_{j,m}}\right) \right) \\ - \lambda_j p_{j,m} - \mu_j \beta - \eta_m \times \mathcal{J}. \end{aligned} \tag{6.13}$$

Furthermore, the Newton technique can be employed to determine the solution for the relaxed channel selection matrix based on (6.13), where the iterative update is given by [27], [108]

$$x^{(k+1)}_{j,m} = x^{(k)}_{j,m} - \delta_f \frac{f\left(x^{(k)}_{j,m}\right)}{f'\left(x^{(k)}_{j,m}\right)}, \tag{6.14}$$

where k represents an iteration index, δ_f denotes step-size, and $f'\left(x^{(k)}_{j,m}\right)$ is the derivative of $f\left(x^{(k)}_{j,m}\right) = \partial \ell_{j,m}(\cdot)/\partial x_{j,m}$. The $x^*_{j,m}$ values for each user in a channel have distinct values. Based on the constraint in (C6.4), the user with the largest $x^*_{j,m}$ may be selected for optimal assignment. However, simply selecting the user with the highest $x^*_{j,m}$ from the relaxed solution may violate the collision constraint. Then the BnB technique is employed to obtain the integer solution for the channel selection subproblem. The dual variables are updated by employing the subgradient technique based on the constraints as follows [27], [92], [108]:

$$\lambda_j^{(k+1)} = \left(\lambda_j^{(k)} - \delta_\lambda \left(P_{j,\max} - \sum_{m=1}^{M} x_{j,m} p_{j,m}\right)\right)^+, \qquad \forall j \tag{6.15}$$

$$\nu_j^{(k+1)} = \left(\nu_j^{(k)} - \delta_\nu \left(\sum_{m=1}^{M} r_{j,m} - \mathcal{R}_{j,\phi}\right)\right)^+, \qquad \forall j \tag{6.16}$$

$$\mu_j^{(k+1)} = \left(\mu_j^{(k)} - \delta_\mu \left(\sum_{m=1}^{M} r_{j,m} - \beta \sum_{m=1}^{M} x_{j,m}\right)\right)^+, \qquad \forall j \tag{6.17}$$

$$\eta_m^{(k+1)} = \left(\eta_m^{(k)} - \delta_\eta \left(1 - \sum_{j=1}^{\mathcal{J}} x_{j,m}\right)\right)^+, \qquad \forall m \tag{6.18}$$

where $(x)^+ = \max(0, x)$ and $\{\delta_\lambda, \delta_\nu, \delta_\mu, \delta_\eta\}$ denote the iteration step-sizes for the corresponding dual variables. In essence, the dual variables are updated towards the feasible optimal solution based on whether the constraints have been violated or not. As outlined in Chapter 3, the BnB technique is applied to recover the solution of the original MINLP problem from the relaxed optimal solution. From the root subproblem, the largest $x_{j,m} \in \boldsymbol{\mathcal{X}}$ value is selected and set to $x_{j,m} \leftarrow 1$, while the rest of the other variables in the channel selection vector for each SU are set to 0. The branching and fixing of non-integer values is done until integer solution is obtained.

6.4.2 The Modified Hungarian Channel Assignment

This section presents the proposed suboptimal ChA scheme based on the modified Hungarian algorithm [68], [133]-[137], which has a relatively lower complexity in comparison to the optimal solution that is based on the Lagrangian optimization framework. The modified Hungarian technique facilitates a heuristic algorithm that performs disjoint channel selection and power allocation for ChA in multiuser CRNs. In essence, this comprises two independent subproblems:

ALGORITHM 6-1: THE MODIFIED HUNGARIAN BASED CHA SCHEME

Input: $\boldsymbol{\mathcal{H}}$, $\mathcal{R}_{\jmath,\phi}$, β, $P_{\jmath,\max}$,
Output: $\{\boldsymbol{\mathcal{X}}^{\wedge}, \boldsymbol{\mathcal{P}}^{\wedge}\}$ and $\mathcal{R}_{\mathcal{J}}^{\wedge}(\boldsymbol{\mathcal{X}}^{\wedge}, \boldsymbol{\mathcal{P}}^{\wedge})$
repeat
    solve for $\boldsymbol{\mathcal{X}}^{\wedge}$ using Hungarian algorithm.
    determine $\boldsymbol{\mathcal{P}}^{\wedge}$ based on the WF technique from (5.19).
    for ($\forall \jmath \in \boldsymbol{\mathcal{J}}$ not blocked)
        calculate $R_{\jmath,\phi}(\boldsymbol{x}, \boldsymbol{p})$
        if($\bar{\mathcal{R}}_{\jmath,\mathcal{K}} < \mathcal{R}_{\jmath,\phi}$)
            block $\mathrm{SU}_{\jmath}$
        end if
        if($\bar{\mathcal{R}}_{\jmath,\mathcal{K}} < \beta\mathcal{K}$)
            drop channel m with min gain $\{\bar{h}_{\jmath,m} \in \boldsymbol{H}\}$.
        end if
    end for
until convergence for the suboptimal solution $\{\boldsymbol{\mathcal{X}}^{\wedge}, \boldsymbol{\mathcal{P}}^{\wedge}\}$
return $\{\boldsymbol{\mathcal{X}}^{\wedge}, \boldsymbol{\mathcal{P}}^{\wedge}\}$

(*i*) channel selection and (*ii*) optimal power allocation. The channel power gain matrix is used to allocate channels for multiple users, as shown in Algorithm 6-1. After obtaining the channel selection matrix, the corresponding optimal power allocation is determined by solving the following subproblem for each SU node, with respect to the solution for channel selection:

$$\underset{\{p_{\jmath,m}\}}{\text{maximize}} \sum_{m=1}^{\mathcal{K}} x_{\jmath,m}\mathcal{B}\log_2\left(1 + p_{\jmath,m}\bar{h}_{\jmath,m}\right) \tag{6.19}$$

$$\text{subject to:} \sum_{m=1}^{\mathcal{K}} x_{\jmath,m}p_{\jmath,m} \leq P_{\jmath,\max}, \qquad \{\forall m: \ x_{\jmath,m} = 1\} \tag{C6.1}$$

$$p_{\jmath,m} \geq 0, \qquad \{\forall m: \ x_{\jmath,m} = 1\} \tag{C6.2}$$

In this case, the number of channels selected by each user has been resolved. The objective function is to maximize capacity by optimizing power allocation over the selected channels. Consequently, the Lagrangian function to facilitate power optimization is formulated as follows:

$$\mathcal{L}(\boldsymbol{p}, \lambda_{j}) = \sum_{m=1}^{\mathcal{K}} \ell(p_{j,m}, \lambda_{j}) - \lambda_{j} P_{j,\max}\,, \tag{6.20}$$

where $\ell(p_{j,m}, \lambda_{j})$ facilitates Lagrangian decomposition, given by

$$\ell(p_{j,m}, \lambda_{j}) = x_{j,m}\mathcal{B}\log_2(1 + p_{j,m}\bar{h}_{j,m}) - \lambda_{j} x_{j,m} p_{j,m}, \qquad \{\forall m :\ x_{j,m} = 1\}, \tag{6.21}$$

from which the following is established:

$$\frac{\partial \ell(\cdot)}{\partial p_{j,m}} = \frac{\mathcal{B}}{\ln(2)}\left(\frac{x_{j,m}\bar{h}_{j,m}}{1 + p_{j,m}\bar{h}_{j,m}}\right) \ - \ \lambda_{j}\, x_{j,m}, \qquad \{\forall m :\ x_{j,m} = 1\}. \tag{6.22}$$

Then equating (6.22) to zero and solving for $p_{j,m}$, the power allocation is based on the multilevel WF technique corresponding to each user, given by the following:

$$p_{j,m}^{\wedge} = \max\left\{0, \left[\left(\frac{\mathcal{B}}{\ln(2)}\right)\left(\frac{1}{\lambda_{j}}\right) - \frac{1}{\bar{h}_{j,m}}\right]\right\}, \qquad \forall j \in \boldsymbol{\mathcal{J}} \tag{6.23}$$

where $(\cdot)^{\wedge}$ signifies the optimal power allocation for suboptimal channel selection. Thus, the modified Hungarian algorithm performs heuristic channel selection based on the minimum rate and collision constraints, as well as optimal power distribution to over the selected channels.

6.4.3 Computational Complexity Analysis

In the case of multiuser multichannel ChA, performing the exhaustive search (*i.e.* MU-EX) to find the optimal solution for ChA requires $\mathcal{O}(M\mathcal{J}^{M})$ operations as shown in Table 6-1 [95], [97], [115]. The computational complexity for the Lagrangian dual optimization requires the initial root-finding search with the complexity order of $\mathcal{O}(\Delta_t\Delta_f\mathcal{J})$ for solving the dual problem for $\mathcal{J}$ SU nodes, where

TABLE 6-1: COMPUTATIONAL COMPLEXITY FOR MULTI-SU CHA SCHEMES

Algorithm	**Order of operations**	
	Initialization	Computational Complexity
MU-EX	—	$\mathcal{O}(M\mathcal{J}^M)$
MU-BnBKOP	$\mathcal{O}(\Delta_t \Delta_f \mathcal{J} M)$	$\mathcal{O}(\mathcal{J}M^2)$
MU-HunKOP	$\mathcal{O}(M^2 \log M)$	$\mathcal{O}(\mathcal{J}\mathcal{K}_j)$
MU-FixedKOP	—	$\mathcal{O}(\mathcal{J}\mathcal{K})$
MU-FixedKEP	—	$\mathcal{O}(\mathcal{J})$

$\Delta_{\mathcal{K}}$ denotes the number of iterations, and Δ_f is the number of function evaluations per iteration. In general, the rate of convergence depends highly on the choice of initial conditions, step-size and the order of iterations [95], [116]. Then complexity to obtain the relaxed solution for the primal problem is linear in the number of SU nodes and the number of available PU channels, in the order of $\mathcal{O}(\mathcal{J}M)$ computations. Following from Section 3.4.3 in Chapter 3, the worst case scenario for the BnB technique (*i.e.* MU-BnBKOP) involves $2M(M-1)$ relaxed subproblems for $\mathcal{J}$ SU nodes, which results in $\mathcal{O}(\mathcal{J}M^2)$ computational complexity [119]-[120].

Suppose $\{\mathcal{J} \ll M\}$ for multiuser ChA. The modified Hungarian ChA scheme (*i.e.* MU-HunKOP) has initial complexity given by $\mathcal{O}(M^3)$ for channel assignment, which can be reduced to $\mathcal{O}(M^2 \log M)$ [134], [137]. Then WF power allocation is employed based on the Hungarian based channel assignment, with the complexity $\mathcal{O}(\mathcal{J}\mathcal{K}_j)$ in the order of the number SU nodes, together with the number of channels $\mathcal{K}_j$ assigned to each node $j \in \boldsymbol{\mathcal{J}}$. Then for the fixed-$\mathcal{K}$ ChA schemes, the computational complexity is in the order of the number of SU nodes as shown in Table 6-2.

6.5 Simulation Results and Discussion

6.5.1 Simulation Model

In this section, simulation results for average sum capacity, average outage probability and average collision probability are presented against: (*i*) PU arrival rates, and (*ii*) SU total transmit power. The

TABLE 6-2: MULTIUSER ACCESS CHA SCHEMES

Scheme	No. of Channels	Power Profile
MU-FixedKEP	Fixed	equal-power
MU-FixedKOP	Fixed	Waterfilling
MU-HunKOP	Hungarian	Waterfilling
MU-BnBKOP	Optimal	Waterfilling

investigated multiuser ChA schemes are shown in Table 6-2, which are described as follows: (*i*) the multiuser ChA scheme with prefixed number of channels and equal power allocation for the assembled channels (MU-FixedKEP); (*ii*) multiuser ChA scheme with prefixed number of channels and optimal power allocation based on WF technique (MU-FixedKOP); (*iii*) the suboptimal multiuser ChA scheme based on Hungarian algorithm for channel assignment with WF power allocation (MU-HunKOP) and (*iv*) the proposed optimal SA scheme based on convex optimization framework (MU-BnBKOP). Table 6-2 shows the simulation parameters. It is assumed that a central controller coordinates the allocation of resource resources for the presented ChA schemes.

6.5.2 Results and Discussion

This section provides discussion for the presented results. In Fig. 6-2 to Fig. 6-4, results for the various performance metrics are presented for increasing PU arrivals rates. The ChA schemes

TABLE 6-3: SIMULATION PARAMETERS FOR MULTIUSER CHA SCHEMES

Parameters	Value
Number of frames	10^4
Total Number of PU channels	16
Number of SU nodes	4
Collision probability threshold	$0.1-0.2$
SU QoS requirement on capacity	5.5Kbps
Bandwidth ($\mathcal{B}$)	1 MHz

with the fixed number of channels are not affected by PU arrival rates. For the adaptive ChA schemes, the number of channels is a function of PU arrival rates; in which case the number of selected channels is reduced at high PU arrivals. Reducing the number of channels at high PU arrivals reduces the average capacity for SU nodes. As shown in Figure 6-2, both the MU-BnBKOP and MU-HunKOP schemes have the highest capacity at low PU arrivals, with the MU-OKOP scheme outperforming the HunKOP scheme; while the capacity for the MU-FixedKEP scheme and MU-FixedKOP schemes seem to be unaffected by the increasing PU arrival rates.

Fig. 6-3 presents the results for the average outage probability for the SU nodes, which like in the previous chapters, is the probability that the sum capacity $\mathcal{R}_{\jmath,\mathcal{K}}$ resulting from the assembled channels falls below the minimum capacity $\mathcal{R}_{\jmath,\phi}$ requirement. In agreement with Fig. 6-2, Fig. 6-3 shows that the MU-BnBKOP and MU-HunKOP schemes have the lowest outage probability at low PU arrivals, which increases with the increase in PU arrival rates. In the case of MU-FixedKEP and MU-FixedKOP schemes, the outage probability remains constant as a function of PU arrival rates, which is also consistent with Fig. 6-2. Moreover, Fig. 6-4 presents results for the average collision probability between SU services and new PU arrivals. This is a function of the number of assembled channels, as well as the PU arrival rates as established in Chapter 3. Essentially, Fig. 6-4 reveals that the MU-FixedKEP and MU-FixedKOP schemes may appear to have a higher average capacity and lower outage probability at high PU arrivals; however, they are subject to high collisions against PU services, hence forced terminations to protect the PU services.

Fig. 6-5 to Fig. 6-7 provide results for varying total transmit power per SU node. The results illustrate that at low power levels, the MU-BnBKOP and MU-HunKOP schemes have zero capacity as shown in Fig. 6-5, *i.e.* below -5dB; while the MU-FixedKEP and MU-FixedKOP schemes with the fixed number of channels have non-zero average capacity. Above -5dB, the MU-BnBKOP scheme has the highest average capacity, followed by the MU-HunKOP scheme. This is because at high transmit power levels, the solution space for feasible channel selection increases for the adaptive ChA schemes. Further, in Fig. 6-6, the MU-FixedKEP has the highest outage probability, followed by the MU-FixedKOP. In the case of the adaptive ChA schemes, outage probability decreases faster as power increases due the increase in ChA order. Thus, the MU-BnBKOP and MU-HunKOP schemes have the lowest outage probability at high power levels. Fig. 6-7 illustrates that in general, increasing SU transmit power reduces collision probability.

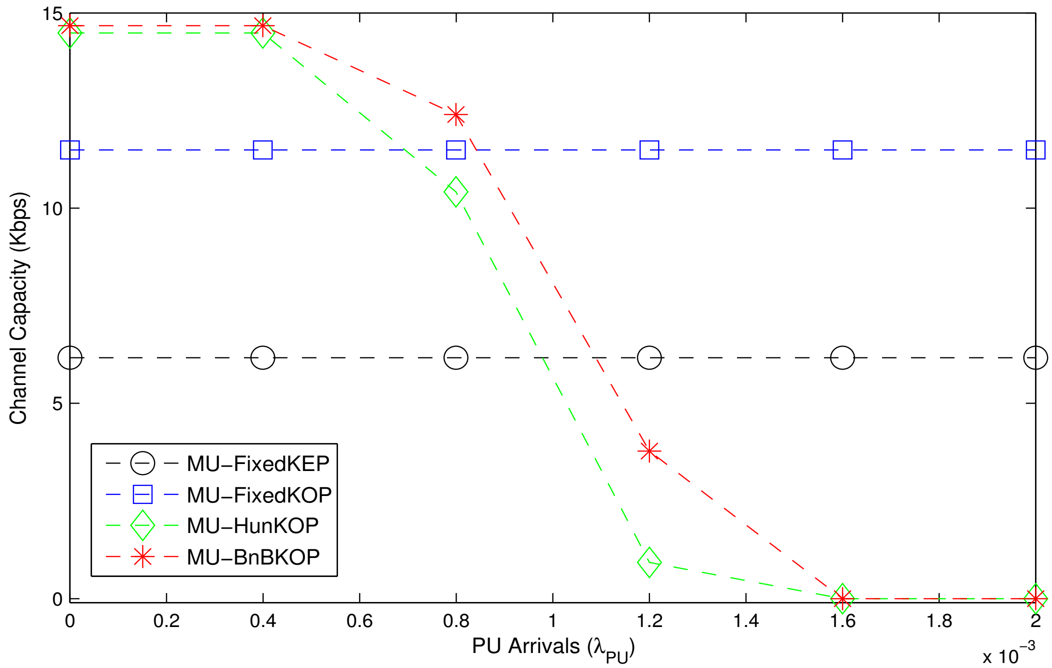

Figure 6-2: Average SU capacity vs PU arrivals.

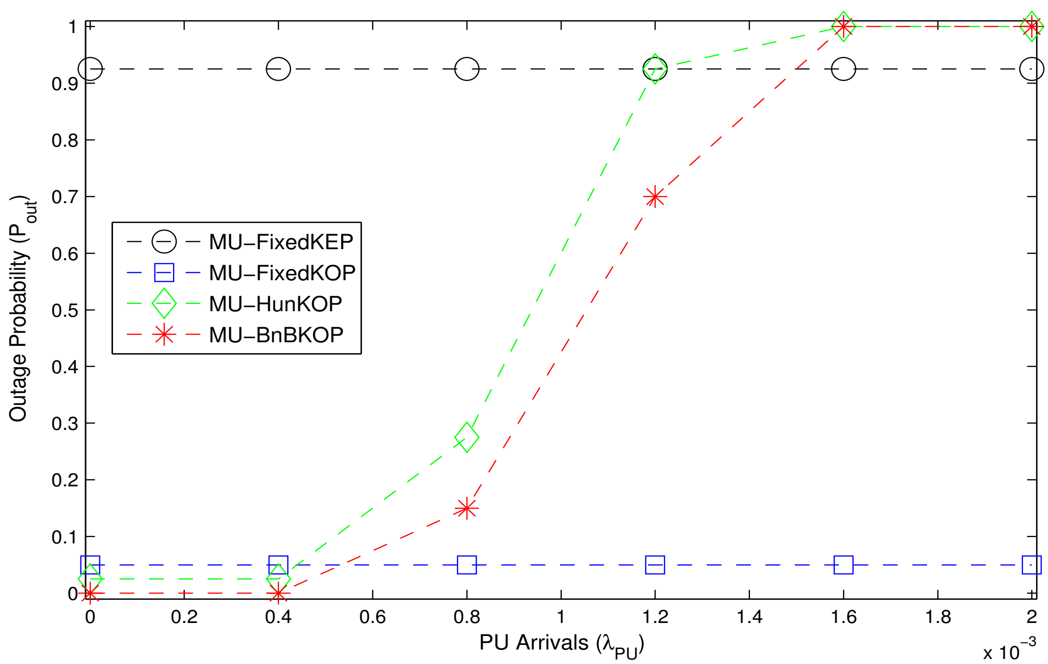

Figure 6-3: SU outage probability vs PU arrivals.

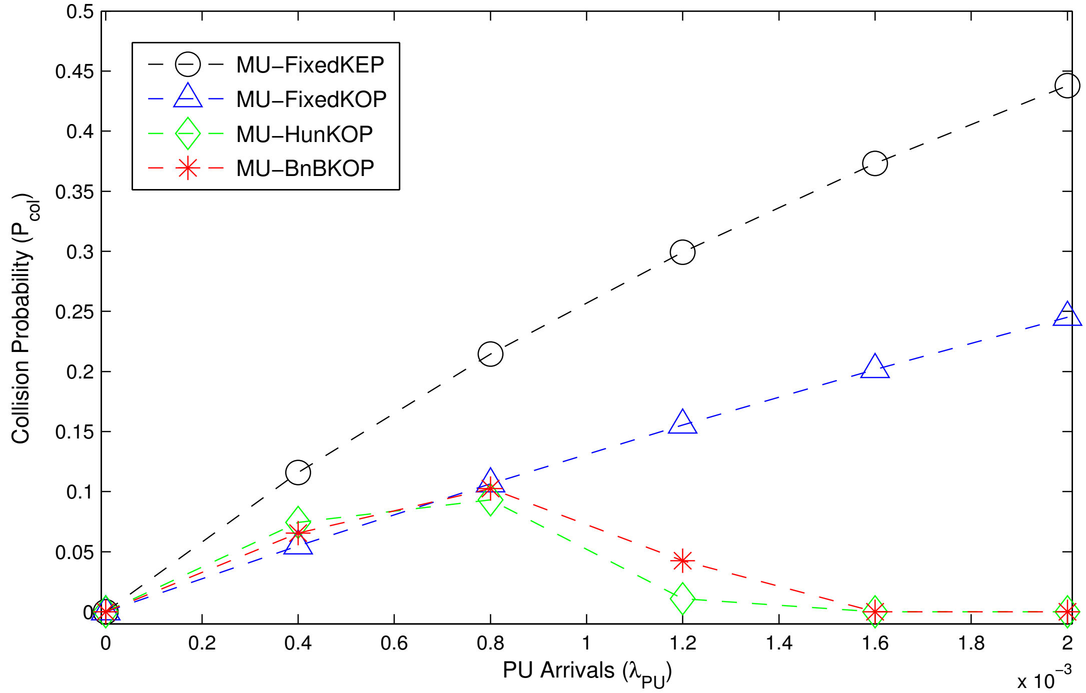

Figure 6-4: SU collision probability vs PU arrivals.

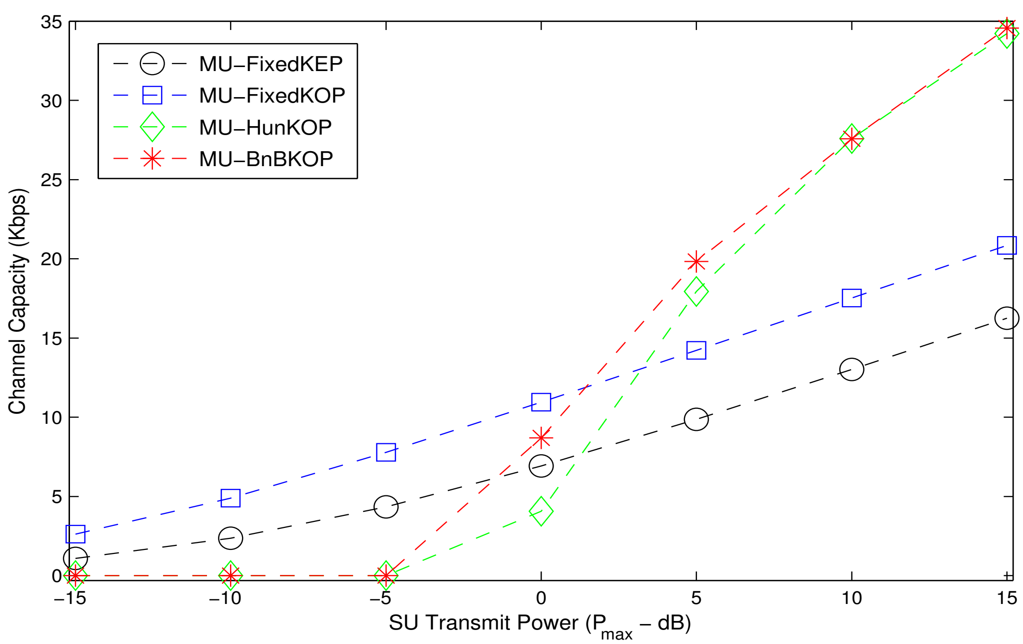

Figure 6-5: SU average capacity vs total transmit power.

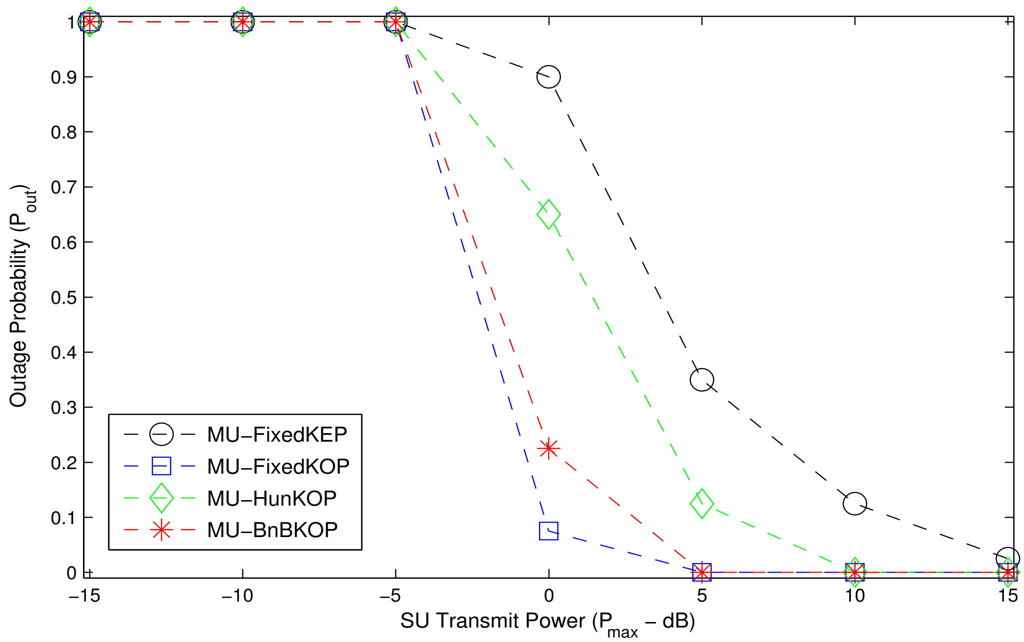

Figure 6-6: SU outage probability vs total transmit power.

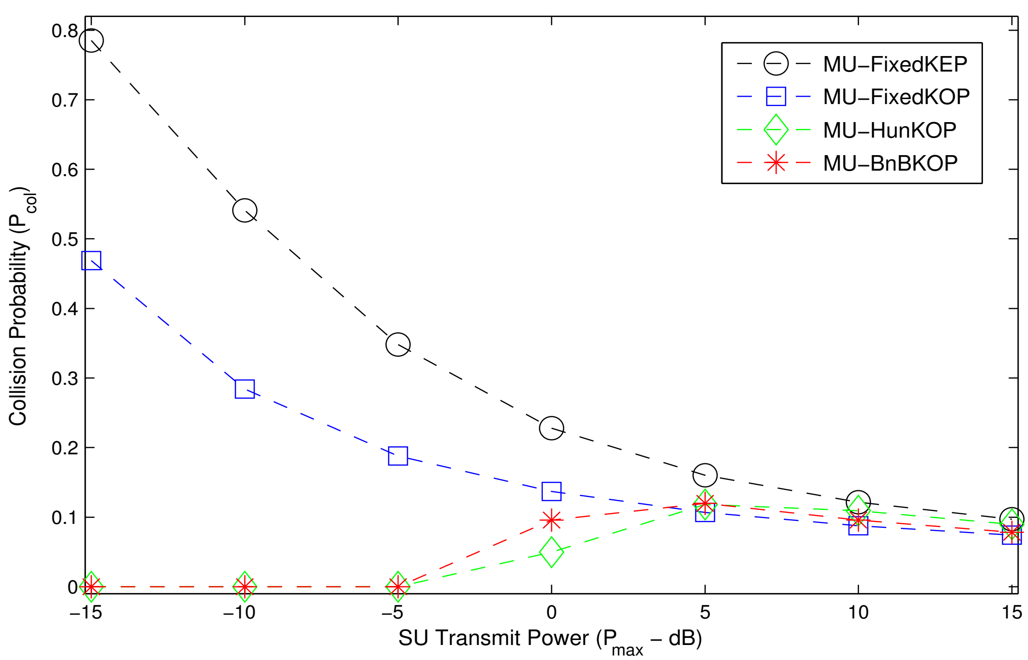

Figure 6-7: SU collision probability vs total transmit power.

6.6 Chapter Conclusion

In this chapter, ChA schemes for opportunistic resource allocation in multiuser multichannel access networks have been presented, with the main focus to improve multiple SU nodes performance subject to resource constraints and network dynamics. In particular, the proposed ChA schemes take into account PU arrival rates, power constraint, QoS requirement on capacity, and the varying nature of wireless channel links to perform optimal selection of channels and power allocation for multiple SU nodes in a CRN. In general, the presented simulation results illustrate significant performance improvement for the proposed optimal and suboptimal ChA schemes. Further, the results reveal that ChA schemes are highly susceptible to performance degradation due high PU activity patterns. Thus, spectrum characterization in terms of PU activities and power optimization are significant considerations towards realizing performance benefits of dynamic ChA schemes in multiuser multichannel spectrum sharing wireless networks.

CHAPTER SEVEN

7 CONCLUSIONS AND FUTURE WORKS

FINALLY, this chapter summarises the main points on which the work presented in this thesis is concluded, together with the possible investigations and directions for future research. The main introduction and background for the work presented in this thesis has been provided in Chapter 1, followed by Chapter 2 which presented the related studies in literature. Then Chapter 3 to Chapter 6 presented main contributions derived from the conducted research for development of adaptive ChA schemes with DRA techniques in multichannel access CRNs.

7.1 Concluding Remarks

DSA and adaptation techniques are envisaged to facilitate efficient use of radio spectrum for CRNs and future wireless networks. In CRNs, provisioning of robust and reliable communication is a challenging task as a result of spectrum agility with exclusive preemptive priority given to PU nodes in a PN, wherefore the performance of SU nodes in a CRN is largely dictated by PU activity patterns. Besides, PU activity patterns amplify the inherent challenges associated with stringent resource constraints and network dynamics in wireless communication systems. Accordingly, the work presented in this thesis developed the new techniques for adaptive ChA schemes and DRA in CRNs; with the key focus to improve performance, and provide reassurance for fulfillment of QoS requirements for SU services, without degrading performance of PU nodes in PNs.

In summary, the work presented in this thesis demonstrated the benefits of adaptive ChA schemes with optimal channel selection and power distribution, through which performance improvement can be achieved in CRNs. In general, the presented performance evaluation uncovers the underlying trade-offs towards development of efficient ChA schemes in CRNs. Although the proposed optimal ChA schemes in this research study may be complex in terms of implementation issues, the in-depth analyses presented in this thesis provide an exposition for significant theoretical insights into the performance gains that can be obtained by optimizing ChA schemes and DRA techniques in constrained multichannel CRNs, as well as the future wireless networks.

In Chapter 2, an overview of the related works in literature has been provided, highlighting the recent works and accomplishments on DSA techniques in CRNs and ChA schemes in wireless networks. The investigation provided the basis for identifying the limitations of the existing works in literature, based on which the main contributions of the work presented in this thesis were derived. In general, it was established that, amid the recent research accomplishments in previous studies, the underlying challenges in CRNs and the emerging wireless networks underline the need for efficient and effective communication techniques. Albeit the recently reported research efforts and contributions, achieving efficient communication in CRNs still remains a challenging task.

In Chapter 3, the optimal channel selection and power allocation for adaptive ChA scheme in overlay CRNs was presented, with the key objective to maximize SU capacity over fading wireless channels. The optimization problem incorporated SU transmit power constraint, minimum QoS requirement on capacity, as well as collision probability threshold to determine optimal solutions for ChA schemes. Convex optimization based on the Lagrangian relaxation framework was employed to determine the relaxed optimal solution, based on which BnB technique was applied to determine the solution to the original MINLP problem. Moreover, simulation based performance analysis and evaluation was presented for various ChA schemes. In this chapter, it has therefore been illustrated that significant performance improvement can be obtained through adaptive ChA schemes in overlay CRNs. Henceforth, spectrum characterization that incorporates PU activity patterns is not only a considerable issue, but absolutely necessary for ChA schemes in CRNs.

In Chapter 4, the analytical models to quantify performance of ChA schemes over fading wireless channels were developed. The compact closed-form expressions for average channel capacity, outage probability, and forced termination probability for SU services have been presented. The PDF and CDF have been developed for statistical characterization of assembled channels over fading wireless channels; based on which the analytical models were derived, and expressed in terms of the Meijer-G function and the generalized upper incomplete Fox-$\mathcal{H}$ function. Then the preciseness and correctness of the derived analytical models was established by performing cross validation through simulation results and analytical results, wherefore the cross validation confirmed the accuracy of the developed analytical models. In principle, with the correctness of the analytical models verified, the work presented in Chapter 4 provides the mathematical tools to gauge performance of ChA schemes over fading wireless channels.

In Chapter 5, the adaptive ChA schemes for hybrid overlay and underlay CRNs have been presented, which allow SU nodes to opportunistically assemble unoccupied PU channels in overlay mode, as well as the occupied PU channels subject to transmit power control in underlay mode. For channels assembled in underlay mode, multilevel maximum allowable SU transmit power control technique has been derived from PU outage constraint, also based on various assumptions about the knowledge of CSI available at the SU transmitter. The underlay power control techniques mainly ensure that SU transmit power is constrained under the noise floor of PU nodes. Through simulation based performance evaluation, the adaptive hybrid ChA schemes were shown to achieve significant performance improvement in comparison to overlay ChA schemes. In general, this work illustrated that hybrid spectrum access with appropriate power control and interference management is one of the effective techniques to improve performance of ChA in CRNs.

In Chapter 6, the adaptive ChA techniques for multiuser multichannel access CRNs have been presented. In the case of multiuser CRNs, the developed ChA schemes facilitate efficient allocation of PN radio spectrum to multiple SU nodes. In particular, two variants of adaptive ChA schemes for multiuser CRNs were presented. The optimal ChA scheme based on the Lagrangian framework, and the suboptimal ChA scheme based on the modified Hungarian technique. Further, simulation based performance analysis has been presented; which in general, demonstrated improved performance by the proposed ChA schemes in terms of capacity, outage probability and collision probability. This work illustrated that even in the case of multiuser multichannel access scenarios, ChA schemes with the prefixed number of channels may be inefficient in maximizing capacity in CRNs; despite the fact that the key objective for ChA techniques is to maximize capacity. Accordingly, adaptive ChA schemes with optimal power allocation in multiuser wireless networks provide an effective mechanism to improve performance in spectrum sharing networks.

7.2 Future Directions

This section highlights the open issues to extend the work presented herein for future research. In this thesis, adaptive ChA and DRA schemes have been developed based on convex optimization for optimal channel selection and power allocation. The issue of complexity mitigation techniques constitute possible extension for future research. The mitigation techniques would facilitate the necessary trade-offs to balance the issue of complexity and optimality of ChA schemes.

Spectrum negotiation and synchronization is one of the significant issues which has been generally overlooked by the existing works in literature, nor addressed in this thesis. Typically, channels are assembled in an ad-hoc manner between SU transmitter and SU receiver nodes. Thus, how to perform spectrum negotiation and synchronization, together with the associated performance implications are some of the compelling issues that are yet to be investigated for ChA in spectrum sharing wireless networks. Most of the reported works in literature are generally based on the assumption that spectrum synchronization and negotiation is perfect.

Furthermore, the adaptive ChA schemes in this thesis determine feasible solutions for spectrum access only prior to transmissions. Even in the case of hybrid schemes, spectrum access is resolved only before transmissions take place. However, rapid changes in spectrum occupancy status with respect to PU nodes may occur during SU transmissions. Most studies are mainly focused on the changes as a result of new PU service arrival; yet, exploiting spectrum access on departure of PU nodes may create more opportunities to improve performance of SU nodes. Accordingly, this work can be extended to investigate dynamic spectrum access techniques that can perform online spectrum switching with respect to the dynamics and change in spectrum availability. Such requires investigation of joint spectrum sensing and ChA in CRNs.

APPENDIX A

A.1 Mellin Integral Transform

A.1.1 Mellin Transform Definition

Let $f(x)$ be a single valued real function defined by a positive real variable $0 \leq x \leq \infty$, the Mellin transform of a function $f(x)$ is the mapping given by the following integral transform:

$$\mathbb{M}_s\{ f(x)\} = \int_0^\infty x^{s-1} f(x)dx \quad (A.1)$$

where s is a complex number. Then $f(x)$ can be obtained by evaluating the corresponding inversion integral. The inverse Mellin transform (*i.e.* Mellin inversion integral) is given by

$$f(x) = \frac{1}{2\pi i}\oint_C x^{-s}\, \mathbb{M}_s\{ f(x)\}ds \quad (A.2)$$

where C is a contour in a complex s-plane given by $C = \{c + i\infty \text{ to } c - i\infty\}$, and $i = \sqrt{-1}$. Thus, the inverse Mellin transform uniquely determines $f(x)$ as shown in (A.2). By definition also, the gamma function $\Gamma(s)$ is given by the following Mellin transform integral:

$$\Gamma(s) = \int_0^\infty x^{s-1}\exp(-x)\, dx \quad (A.3)$$

and the upper incomplete gamma function is defined by

$$\Gamma(s, \phi) = \int_\phi^\infty x^{s-1}\exp(-x)\, dx \quad (A.4)$$

Thus, the $\Gamma(\cdot)$ function can be described as the Mellin transform of the negative exponential function as shown in (A.3), and is commonly used to represent the generalized hypergeometric functions such the Meijer- G and the Fox-$\mathcal{H}$ functions as defined in subsequent sections.

A.1.2 Mellin Transform Convolution

Proof of Theorem 4-1: *For a product of independent nonnegative random variables defined by* $Y_N = \prod_{n=1}^{N} x_n$, *with Mellin transforms* $\mathbb{M}_s\{f_X(x_1)\}, \mathbb{M}_s\{f_X(x_2)\}, \cdots, \mathbb{M}_s\{f_X(x_N)\}$, *by definition, the Mellin transform* $\mathbb{M}_s\{f_Y(y)\}$ *of* Y_N *is given by*

$$\mathbb{M}_s\{f_Y(y)\} = \mathbb{E}\{y^{s-1}\} \tag{A.5a}$$

$$= \mathbb{E}\{(x_1 x_2 \cdots x_N)^{s-1}\} \tag{A.5b}$$

$$= \mathbb{E}\{{x_1}^{s-1} {x_2}^{s-1} \cdots {x_N}^{s-1}\} \tag{A.5c}$$

$$= \mathbb{E}\{x_1^{s-1}\}\, \mathbb{E}\{x_2^{s-1}\} \cdots \mathbb{E}\{x_N^{s-1}\} \tag{A.5d}$$

$$= \mathbb{M}_s\{f_X(x_1)\}\mathbb{M}_s\{f_X(x_2)\} \cdots \mathbb{M}_s\{f_X(x_K)\} \tag{A.5e}$$

$$= \prod_{i=1}^{K} \mathbb{M}_s\{f_X(x_i)\}. \tag{A.5f}$$

Thus in general, the Mellin transform of a product of nonnegative random variables is given by the product of the Mellin transforms of the random variables, which concludes the proof. ■

A.2 The Meijer-G Function

A.2.1 Meijer-G Function Definition

The Meijer-G function is a generalized hypergeometric function, defined by contour integral representation with the following standard notation [122]-[134]:

$$G_{p,q}^{m,n}\left(z \middle| \begin{matrix} a_1, \cdots, a_p \\ b_1, \cdots, b_q \end{matrix}\right) \triangleq \frac{1}{2\pi i} \int_C \frac{\prod_{j=1}^{m} \Gamma(b_j + s) \prod_{j=1}^{n} \Gamma(1 - a_j - s)}{\prod_{j=n+1}^{p} \Gamma(a_j + s) \prod_{j=m+1}^{q} \Gamma(1 - b_j - s)} z^{-s} ds \tag{A.6}$$

where z, $\{a_i\}_{i=1}^{p}$ and $\{b_i\}_{i=1}^{q}$ are complex numbers, $i = \sqrt{-1}$ and $C \triangleq \{c - i\infty, c + i\infty\}$, and $\Gamma(\cdot)$ denotes the gamma function. Hence, Meijer-G function is defined by the inverse Mellin transform.

A.2.2 Properties of the Meijer-G Function

The following are properties of the Meijer-G function that can be employed to solve for the associated integral functions [122]:

***Property* 1:** Meijer-G function integral formula

$$\int_0^x x^{\alpha-1}\, G_{p,q}^{m,n}\left(cx \,\middle|\, \begin{matrix} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{matrix}\right) dx = x^{\alpha} G_{p+1,q+1}^{m,n+1}\left(cx \,\middle|\, \begin{matrix} 1-\alpha, a_1,\cdots,a_p \\ b_1,\cdots,b_q,-\alpha \end{matrix}\right) \tag{A.7}$$

***Property* 2:** Product of two Meijer-G functions formula

$$\int_0^x G_{u,v}^{s,t}\left(\sigma x \,\middle|\, \begin{matrix} c_1,\cdots,c_u \\ d_1,\cdots,d_v \end{matrix}\right) \times G_{p,q}^{m,n}\left(\omega x \,\middle|\, \begin{matrix} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{matrix}\right) dx$$
$$= \sigma^{-1} G_{p+v,q+u}^{m+t,n+s}\left(\frac{\omega}{\sigma} \,\middle|\, \begin{matrix} (a_n),-(d_v),a_{n+1},\cdots,a_p \\ (b_m),-(c_u),b_{m+1},\cdots,b_q \end{matrix}\right) \tag{A.8}$$

Among many other properties for the Meijer-G function, these properties have been applied to derive closed-form expressions for average channel capacity in Chapter 4 of this thesis.

A.3 Upper Incomplete Fox-$\mathcal{H}$ Function

A.3.1 Incomplete Fox-$\mathcal{H}$ Function Definition

As defined in [121], the generalized upper incomplete Fox-$\mathcal{H}$ function is given by

$$\mathcal{H}_{p,q}^{m,n}[x] \triangleq \mathcal{H}_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix} (a_1,\alpha_1,A_1,\varphi_1)_{1,p} \\ (b_1,\beta_1,B_1,\phi_1)_{1,q} \end{matrix}\right] = \mathcal{H}_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix} (a_1,\alpha_1,A_1,\varphi_1),\cdots,(a_p,\alpha_p,A_p,\varphi_p) \\ (b_1,\beta_1,B_1,\phi_1),\cdots,(b_q,\beta_q,B_q,\phi_q) \end{matrix}\right] \tag{A.9}$$

where $\{m,n,p,q\}$ are integers such that $\{0 \le m \le q\}$ and $\{0 \le n \le p\}$, with $\{(a_i,b_i) \in \mathbb{C}\}$ and

$\{(\alpha_i, A_i, \varphi_i, \beta_j, B_j, \phi_j) \in \mathbb{R}^+\}$, also $\{1 \leq i \leq p\}$ and $\{1 \leq j \leq q\}$. This is based on the Mellin transform inversion defined as follows [122]:

$$\mathcal{H}_{p,q}^{m,n}[x] = \frac{1}{2\pi i} \oint_c M_{p,q}^{m,n}[s]\, x^{-s} ds \tag{A.10}$$

where $i = \sqrt{-1}$, c is a complex plain contour, and $M_{p,q}^{m,n}[s]$ is given by

$$M_{p,q}^{m,n}[s] = \frac{\prod_{j=1}^{m} \Gamma(b_j + \beta_j s, B_j) \prod_{j=1}^{n} \Gamma(1 - a_i - a_i s, A_i)}{\prod_{j=n+1}^{p} \Gamma(a_i + a_i s, A_i) \prod_{j=m+1}^{q} \Gamma(1 - b_j - \beta_j s, B_j)}, \tag{A.11}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function [109].

A.3.2 Distribution of Product of Fox-$\mathcal{H}$ Variates

For independent Fox-$\mathcal{H}$ function random variables $X_1, X_2, \cdots, X_n$ defined by the respective PDF's $f_1(x_1), f_2(x_2), \cdots, f_n(x_n)$, where the PDF's are given by

$$f_i(x_i) = k_i \mathcal{H}_{p_i,q_i}^{m_i,n_i}\left[c_i x_i \left| \begin{matrix} (a_{i1}, \alpha_{i1}), \cdots, (a_{ip_i}, \alpha_{ip_i}) \\ (b_{i1}, \beta_{i1}), \cdots, (b_{iq_i}, \beta_{iq_i}) \end{matrix} \right. \right], \qquad \forall x_i \geq 0 \tag{A.10}$$

for $i = 1,2, \cdots, n$, the PDF $f_y(y)$ of the product of the random variables $Y_n = \prod_{i=1}^{n} X_i$ can be simply established by applying the following [122]:

$$f_y(y) = \left(\prod_{i=1}^{n} k_i \right) \mathcal{H}_{\sum_{i=1}^{n} p_i, \sum_{i=1}^{n} q_i}^{\sum_{i=1}^{n} m_i, \sum_{i=1}^{n} n_i} \left[\prod_{i=1}^{n} c_i y \left| \begin{matrix} (a_{11}, \alpha_{11}), \cdots, (a_{np_i}, \alpha_{np_i}) \\ (b_{11}, \beta_{11}), \cdots, (b_{nq_i}, \beta_{nq_i}) \end{matrix} \right. \right], \qquad y \geq 0 \tag{A.11 }$$

This property can be applied to establish complex distributions of the product of independent random variables whose PDF's can be defined by Fox-$\mathcal{H}$ functions.

APPENDIX B

B.1 Fox-$\mathcal{H}$ function Implementation

The following implements the MATHEMATICA code for the generalized upper incomplete Fox-$\mathcal{H}$ function that was used to produce the analytical results in Chapter 4 [106], [121].

```
****************************************************************************************
Clear[FoxH,FL,FR,G,MT];
FoxH[A_, B_, z_, R_, W_]:= Module [
        {AL, AR, BR, BL, V, CF, s}, CF = True;
        AL = A[[1]]; AR = A[[2]];
        BL = B[[1]]; BR = B[[2]];
        FL[s_,ai_,aii_]:= ai + aii s ;
        FR[s_,ai_,aii_]:= 1 - ai - aii s ;
        Z[μ_]:=If[μ==0.0,$MachineEpsilon,μ];
        G[s_,α_] := Gamma[s,α];
        MT[s_]:= Module[{F}, F = 1.0;
                Do[F *= G[FL[s,BL[[n]][[1]],BL[[n]][[2]]],BL[[n]][[3]]],{n,Length[BL]}];
                Do[F *= G[FR[s,AL[[n]][[1]],AL[[n]][[2]]],AL[[n]][[3]]],{n,Length[AL]}];
                Do[F /= G[FR[s,BR[[n]][[1]],BR[[n]][[2]]],BR[[n]][[3]]],{n,Length[BR]}];
                Do[F /= G[FL[s,AR[[n]][[1]],AR[[n]][[2]]],AR[[n]][[3]]],{n,Length[AR]}];
                Return[F];
        ];
        V = 1/(2π I) NIntegrate[MT[s]z^-s, {s, R-I W, R+I W}];
        Return[V];
];
****************************************************************************************
```

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