CONSTRUCTIVISM AND THE CREATIVE TEACHING AND LEARNING OF ALGEBRA IN SOUTH AFRICAN SCHOOLS

by

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RESEARCH REPORT

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Abstract

This research report compares the effect that the absolutist and constructivist pedagogies have on the creative teaching and learning of algebra. The concept of variable forms an integral part of this research because it is one of the basic building blocks of algebra.

This study is based on the belief that the student-centred approach of the constructivists, which encourages teachers to be creative and to adopt a variety of roles and approaches, will not only enable pupils to develop a greater understanding of the concept of variable but will ultimately contribute to the formation of a student who is filled with self-belief and confidence and who possesses a wide range of problem solving skills.

A controlled experiment was employed to investigate the effects of the different pedagogies on standard six pupils at two urban schools. The Ndfer Chelsea Diagnostic Test for Algebra was administered prior to and at the conclusion of the experiment.

The analysis of covariance of the data indicated that those students who had experienced the constructivist approach to teaching and learning algebra improved statisically significantly in relation to those pupils who had experienced the tradional approach to teaching and learning algebra.($p<0.0001$) No evidence was produced that showed that either gender outperformed the other. A frequency analysis of the data also indicated that the constructivist approach to teaching and learning algebra could lead to a reduction in the number of pupils that were experiencing certain misconceptions.

Declaration

I declare that this reseach report is my own, unaided work. It is being submitted in partial fulfilment for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University, nor has it been prepared under the aegis or with the assistance of any other body or organisation or person outside the University of the Witwatersrand, Johannesburg.

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INTRODUCTION AND OVERVIEW

1.1 INTRODUCTION

The purpose of this report is to show the importance of implementing an alternative strategy, based on a fallibilist epistemology, to the teaching and learning of algebra in our schools. The researcher believes that this process is inextricably linked with that of curriculum development, and evaluation and assessment. Furthermore, the ultimate success thereof depends to a large extent on the individual beliefs of the teacher and the prevailing social conditions.

Initially this report will discuss the context in which some of the aforementioned concepts will be utilised. The overview of curriculum which follows, will show that many of the ills that currently beset our educational system, stem from behaviourism. Hence the necessity of considering the merits of a philosophy of mathematics which emphasises a change in the approach to teaching and learning algebra. The new strategy that is adopted should allow the individual the opportunity to create situations that will lead to greater understanding and enjoyment of the individual components of mathematics, such as algebra.

1.2 FORMULATION OF CONCEPTS

1.2.1 Absolutism

Some of the roots of the philosophy of mathematics originated in classical Greece. In those days mathematics was geometry. Mathematics started with self evident truths and the platonists proceeded by careful reasoning to discover hidden truths (Davis and Hersh, 1981). Over the centuries the absolutists have believed that mathematical knowledge was eternal and objective. This perception was based on the euclidean myth, which implied that the books of Euclid contained statements about the universe that were clearly and indubitably true (Davis et al, 1981; Ernest, 1991; Lerman, 1989; Olivier, 1989a). The discovery of non-euclidean geometry by Riemann, Gauss and Lobachevsky and the development of analysis in the nineteenth century forced the absolutists to turn to arithmetic as the basis for the foundations of certainty in mathematics (Davis et al, 1981; Laridon, 1981). Logicism and formalism became two of the major absolutist schools concerned with the process of re-establishing the concept of certainty (Davis et al, 1981; Ernest, 1991). (See 2.1.1 for more detail.)

1.2.2 Constructivism

In the eyes of the constructivists, mathematical knowledge is fallible and corrigible, and as such is subject to revision and correction (Ernest, 1991). The naive, radical and social constructivists are three of the more prominent schools typifying this philosophy. (See 2.1.2 for more detail.)

1.2.3 Creative teaching

The ideas of T P Jones and J Foster on creativity, that are quoted in Laridon (1981), were the basis for the researcher's formulation of the attributes of a creative teacher. It is his belief that the creative teacher needs to be flexible, innovative, sensitive and open to the ideas of others. He or she, must have the innate ability to generate alternate strategies of teaching that will promote insight and understanding, and ultimately lead the student to experience a sense of self-satisfaction and joy. The latter traits are two prime requisites for self-motivation and empowerment.

1.2.4 Learning

Central to virtually all cognitive theories is the assumption that cognitive development takes place, not by the simple reception of information from the environment, but through the modification and building up of the individual's knowledge structures (Putnam, Lambert and Peterson, 1990). Thus, for a constructivist, all the mental material of a construction is the result of human activity. It is based on previous experiences and serves an important purpose in ordering and allowing for prediction of future events (Confrey, 1987).

If cognizance is taken of the above, then it seems likely that the constructivist teacher, in planning his approach to the teaching and learning of algebra, will incorporate a problem-solving and mathematical investigation approach in his lessons because it reflects both the conceptual growth view of mathematics and the nature of the learning process (Lerman, 1983).

In this report, the researcher will show that the philosophy of mathematics not only affects the teaching style but also impacts on the effectiveness of the comprehension of the individual. Greater attention is given to these aspects in 2.1; 2.2; 2.3 and 2.4

1.2.5 Algebra

Algebra is one of the main pillars of mathematics. The secondary school's algebra curriculum has traditionally focused on developing manipulative skills in the spheres of simplifying, factorising, solving equations and differentiation (De Villiers, 1994). It is generally recognised that the search for pattern, in conjunction with systemisation, abstraction and symbolism constitute what is known as algebra.

1.3 CURRICULUM OVERVIEW

1.3.1 Curriculum, evaluation and assessment

The curriculum is central to the educational process and broadly defined covers :
1) the selection of content and how it is arranged in syllabuses as well as what skills and processes are included,
2) the teaching and learning strategies,
3) the processes of assessment and evaluation which are used,
4) the counselling and guidance arrangements, and
5) the recording of pupil achievement (Roby, 1990; NEPI, 1991).

Macintosh and Hale (1976) regard assessment as a form of communication in which information is gathered, either by observation or by administering different measuring instruments. The results of which, depending on the purpose of the assessment, can be utilised for diagnosis, evaluation, guidance, grading, prediction and selection. Glencross and Fridjhon (1990) appear to concur with this definition when they express the belief that assessment is a means to an end. The end being the extent to which all pupils achieve a degree of mathematical (algebraic) competence, that is commensurate with their abilities and needs, and the formation of appropriate attitudes to the subject. The aims and objectives of a sound mathematics curriculum should be the vehicle whereby this end is realised. Furthermore, the pupil's progress should be measured and recorded in terms of these aims and objectives.

Bloom, Hastings and Madaus (1971) envisage evaluation as a method of acquiring and processing a variety of evidence, which ultimately leads to changes being implemented that improve the teaching-learning process. Evaluation also assists in clarifying the significant goals and objectives of education and helps to determine the extent of the students' development in these desired ways. Furthermore, evaluation is used as a tool in educational practice for ascertaining which procedures are more likely to achieve a set of educational ends.

> "It is inevitable that in an examination bound system, such as ours, the type of terminal examination largely determines the style of teaching. That is, the skills and abilities taught by the teachers will, more often than not, be restricted to those which were covered in the examination .."
> (Bishop, 1977)

These sentiments are echoed in NEPI (1991). Furthermore, the eurocentric nature of our system (Lambert, 1995; Laridon, 1981) accentuates their applicability to South Africa. A pupil's expectations about the nature of the mathematics examination that he will confront at the end of a course, significantly influences his learning strategy. Thus in order to overcome the present restrictive role of the final examination and its cultural bias (Laridon, 1990), we need to review the aims and objectives of the curriculum, and the types of tests and methods of constructing them, so that an improvement of learning can take place (Fraser & Gillam, 1972).

The above interpretations show that the teaching and learning of mathematics in general, and algebra in particular, are inextricably linked to evaluation, assessment and curriculum development.

1.3.2 Review of curriculum development in South Africa

> "... school mathematics is shaped and fashioned by social and historical conditions that have little to do with the meaning of mathematics as a discipline of knowledge. " (Carpenter, 1988:221)

The researcher believes that the sentiments, expressed by Carpenter, allude to vital areas that cognitive scientists need to explore in their attempts to resolve the primary question of epistemology:" How do we know what we know?" (Malone, 1988: 326) This is substantiated by the fact that the epistemological task of a philosophy of mathematics is not merely to determine how any mathematical knowledge is possible, but also to explain how the existing mathematical knowledge originated (Ernest, 1991). Thus, in the South African context a brief review of curriculum development and the implications thereof will be portrayed.

The arguments presented below with respect to mathematics are equally applicable to algebra as it is a significant and integral component thereof.

The absolutists have traditionally perceived mathematics as

> " a fixed and unchanging body of culturally-neutral, value-free, facts and procedures, that has afforded to it the luxury of maintaining its unwavering course through the centuries, blinded to the changing realities in society and indifferent to the revolution in the classroom." (Volmink, 1990: 13)

This perception of mathematics has dominated western society for thousands of years (Ernest, 1991). In South Africa, the early settler years were characterised by the authoritarian stance of the church towards education (Behr, 1978). Furthermore, South Africa's colonial inheritance has ensured that the roots of the mathematics curriculum in the country, have been strongly influenced by developments in Europe and the United Kingdom (Laridon, 1981; Maarschalk, 1975). Thus absolutism has, inevitably, become the dominant epistemological perspective of mathematics in South Africa, and, as a result, our country has not escaped the limiting effects thereof.

After the Declaration of the Union Of South Africa in 1910, the Universities and Provincial Administrations together with the Joint Matriculation Board (JMB) tended to dominate developments, with a top-down approach becoming prevalent (Laridon, 1981; 1990). This perception is endorsed by Mooney who believes that the educational scenario in South Africa is "characterised by imposition from above." (Mooney, 1985:46)

During the sixties the structural and deductive aspects of mathematics were emphasised (Jarvis, 1989). This procedure merely reflected the elegant way in which mathematics could be logically reconstructed without considering the process of learning mathematics (Ernest, 1985). Furthermore, an inspection of most school mathematics texts clearly showed that our pupils, like those in the western world, have been subjected primarily to a formalist or logicist approach (Pateman, 1989; Volmink, 1993). These far reaching developments have lead to traditional school mathematics becoming increasingly academic in nature and have heightened the importance of university mathematics (Jarvis, 1989; Laridon, 1993). The nett result has been that the vast majority of the population have become disillusioned, because they have been denied access to the domain itself and because of the lack of relevance (Laridon, 1993). This in turn has caused the perpetuation of instrumental (rote) teaching and learning with the products of mathematics being prioritised (Ernest, 1985; Jarvis, 1989; Laridon, 1992). The importance of pupils gaining access to university, and the emphasis placed by society on results (Breen, 1990), has led to the process of training pupils to succeed in examinations becoming paramount. Since assessment is a component of curriculum, the relationship that exists should be one of servant to master (Riding, 1990). However, in South Africa the reverse seems to apply, because

> "subject curricula are expressed only in terms of objectives and content, with no links made between objectives and assessment. The situation is exacerbated in the senior certificate, where the examination rather than the objectives may define the approach to learning."
> (NEPI, 1991: 25)

Thus it would appear that the exalted status accorded to the final matriculation examination has inevitably been to the detriment of curriculum development.

Certainly our examination-bound system and the failure of our curricula to specify assessment objectives has meant that the examiner, through the final examination, has played a crucial role in determining the emphasis that occurs in the classroom (Bishop, 1977; Butterfield, 1990). The unfortunate result has been that large numbers of teachers have become text-book orientated and rely extensively on chalk-and-talk together [illegible]ling examples from past examination papers in [illegible]sure acceptable results (Breen, 1986). These [illegible]nts may be ascribed to the fact that mathema[illegible], [illegible]nturies, has been dominated by the absolutists, with their emphasis on the child as a passive learner, whose qualities are shaped by the environment (Ernest, 1991). In particular, the narrow, utilitarian, formalist view of mathematics, that has been accepted by many absolutists, has played a most influential role in the mathematics curriculum (Volmink, 1993).

The advent of the differentiated 'core syllabus' of 1972 and the subsequent revisions in the eighties saw the tradition of a few academics assembling the syllabus, which emphasised content, being perpetuated. The modern topics were contained in watertight compartments with hardly any opportunity for spiral development to take place (Laridon, 1990). This process, in the researcher's opinion, has severely restricted the pupils' attempts to grasp the intricate concepts of algebra with any semblance of insight and understanding. The fact that only a small percentage of pupils actually proceed to university (Laridon, 1993), not only appears to vindicate his statement, but also implies that this linear model, which is based on an academic top-down approach (Jarvis, 1989), is not the most appropriate one for our country (Jarvis, 1989). This type of model was severely censured by the Cockcroft Report (1982) and certainly does not cater for the needs of the majority in South Africa (Laridon, 1993). This point is also illustrated by the fact that the needs of the higher level professions, such as engineering, accounting, medicine, computer programming and actuarial science seem to have been well catered for (Taylor, 1990). Indeed, it is certainly in the interests of the country for that sm , select band of talented, and highly motivated students to continue to acquire most of the traditional skills of algebra (Fey, 1992). However, the preponderance of formal algebra and euclidean geometry in the curriculum, hardly appears appropriate, when the needs of those scholars, who are leaving school in order to become apprentices, is taken into account (Taylor, 1990). The ultimate outcome has been that the realistic aspirations and career plans of the majority have been subordinated by the academic minority (Butterfield, 1990).

The proposed new senior secondary school curriculum, developed by the Department of Education and Culture, House of Assembly (DEC; HOA) (1990), in which modular components have been included, has not, as originally intended, been implemented in 1993. Typically this curriculum has been formulated with the interests of the white population, who dominated politically, being paramount (Julie, 1992). There is no doubt that the curriculum and the stereotyped instruction that has prevailed must be transformed in order to meet the goals of the emergent society (Volmink, 1993). Certainly the process of introducing the latest curriculum at a later juncture in other education departments, which has been the vogue in the past (Julie, 1992) must be discontinued. Julie (1992), after reviewing Jansen's analysis of curriculum history in South Africa, came to the conclusion that since the time of colonization, we have never had a mathematics curriculum that has been in the interests of all South Africans. The researcher hopes that the recently created Department of National Education, which is supposed to serve the interests of all pupils, will ractify that situation in the near future.

The researcher believes that the historical review, provided above, strongly indicates that behaviourism, prior to 1992, has played a significant role in the teaching and learning of mathematics in general, and algebra in particular, in South Africa. His beliefs are reinforced by a perusal of subject policy documents, study guides and syllabuses of the Transvaal Education Department, prior to 1990, which confirm that:

1) no distinction between aims and objectives occurs;
2) no mention is made of principles in the syllabuses;
3) a systems model of curriculum development, which is inherently behaviouristic and which is goals driven, has been utilised;
4) a liberal sprinkling of the authoritative word 'must' occurs;
5) the top-down approach, which uses the nesting principle in order to meet the requirements of differentiation, is prevalent (Laridon, 1990, 1993);
6) an emphasis is placed on content by the syllabuses and in teaching.

If our goal is mathematics for all by all (Volmink, 1990), and if we remember that the absolutists' views of mathematics provided the epistemological foundations of institutional racism (Ernest, 1991), then a pressing need to try to rectify the present imbalances via an alternative philosophy of mathematics becomes essential. Additional support for this departure is received from a number of mathematicians who state that

> " it is our contention that mathematics, not in itself, but in the way it is constituted, taught and applied at present, contributes specifically to cultural, class and gender discrimination and to the propagation of the authoritarian technocracy which influences every aspect of life in South Africa." (Taylor, Adler, Mazibuko & Magadla, 1987:1)

This is given further impetus by the fact that the information and technological explosion is a strong indicator of the necessity to bring about a fundamental change in the approach to the teaching of mathematics (Laridon, 1981; Roby, 1990). Furthermore the evidence provided by the Potsdam Mathematics Programme suggests that if we are able to create

> " an environment that is genuinely open to and supportive of all students and in which the style of teaching is true to the nature of enquiry " (Rogers, 1990 : 45),

then a number of the inequalities mentioned above, will cease to exist.

1.4 THE CONTEXT OF THE PROBLEM

The educational profession needs to initiate practices that will empower all pupils mathematically. Measures need to be implemented that help the primary school student to adapt to the rigours and challenges of algebra in the secondary school with renewed enthusiasm, interest and insight. It is the researcher's contention that this can be brought about by the adoption of a fallibilist epistemology, which lies at the heart of constructivism, with the associated changes in teaching and learning methods. This is supported by Taylor, Adler, Mazibuko & Magadla (1987), who stress the need for changing our approach to the teaching of mathematics in South Africa. Furthermore, the recent curriculum (DEC; HOA, 1990) that has been implemented in Standards 5 -7 seems to encourage a more constructivist approach to the teaching of algebra.

Mathematics is an excellent vehicle for solving problems. Algebra is a vital tool in this regard. The key areas of algebra have been identified as: real numbers, variables and functions, distributive property, equivalent fractions and expressions and sentences (NCTM, 1991). Certainly there are many applications of algebra that can only be implemented in the real life situation with any degree of success once the elementary concept of a variable has been mastered.

Paul Ernest, during his recent visit to South Africa, remarked on the fact that several youngsters in Umtata were experiencing great difficulty in coming to terms with the concept of a variable. A possible explanation for this fact is that a great deal of time is spent on acquiring proficiency in terminology and notational skills in standard six, but, unfortunately the pupil is not really presented with the opportunity of making the concept of a variable his or her own (Joubert, 1990). The researcher believes that this conceptual difficulty can be greatly reduced if we utilise modern technology, especially in the form of the relatively inexpensive calculator (Adler, 1989; Coburn, 1992; De

Vries, 1991; Laridon, 1993; Olivier, 1988; Stoker, 1991; Wheatley and Shumway, 1992), and adopt the informal, constructivist approach to algebra, as advocated by Osborne (1989) in his research that was conducted in the United States. Von Glasersfeld (1989b) lends his weight to this departure when he observes that the behaviourist approach to teaching and learning incorporates a training programme that pays scant attention to the generation of understanding.

The purpose of Osborne's numerical approach to algebra was to help pupils to gain a better understanding of the concept of variable (Osborne, 1989). This approach, which forms a major component of the researcher's approach to teaching and learning algebra, appears to be heartily endorsed by Olivier who says

> " pupils need to construct meaning for letters as numerical variables in order to cope with algebra." (Olivier, 1989a : 16-17)

In some primary schools in South Africa, a socio-constructivist approach to the learning of algebra appears to be gaining favour. Pupils are encouraged to develop an awareness of the importance of number properties in simplifying computational tasks. Human (1990), feels that this process will enable the individual to create a much better base for the conceptual understanding of algebra. Moreover, he claims that the socio-constructivist approach to whole number numeracy allows the concepts of variable and the relationships between variables to be introduced, non-symbolically, at a very early age. Thus this process, which seems to parallel Osborne's approach, could positively influence a pupil's perception of the functional relationships between numerical variables, a concept which lies at the heart of high school algebra and much of technologically-relevant higher mathematics (Human, 1990).

In the South African context, Olivier, Murray and Human (1990), have shown through their extensive research, that a constructivist approach to the teaching of arithmetic in the primary school, has positive implications for the future. Malan's research (1989), based on constructivistic principles, on the concept of number, adds further credence to this approach. Olivier (1989, 1989a) has highlighted the fact that a better understanding of the concept of variable may reduce the number of misconceptions that occur amongst pupils in the field of algebra. The vital role that language plays in this phenomenon was accentuated by Fujii (1988). Laridon (1992, 1993) has also emphasised the fact that constructivists' believe that communication plays a vital role in the early stages of learning algebra in a meaningful way.

The continuous changing face of mathematics, due to its fallible nature and the on-going development of language, necessitates a corresponding change in the approach to the teaching and learning of mathematics. The implementation of an ideology based on constructivism means that inquiry and open-ended investigations, human mathematical problem posing and solving must form an integral part of the school mathematics curriculum (Ernest, 1991; Laridon, 1992). The new syllabus for standards 8-10 (DEC; HOA, 1990) actually emphasises the approaches mentioned above. It also stresses the importance of activity-based learning and discussions between teacher and pupils as well as discussions between the pupils themselves (Laridon, 1990).

If these student-centred enquiry methods are utilised then the roles of intuition, imagination and creativity in the derivation of algebraic solutions will be more clearly understood (Burton, 1986).

1.5 THE PROBLEM

The present investigation explored the following possibilities:

1.5.1 Methodology

The level of understanding of the concept "variable" resulting from an absolutist as opposed to a constructivist pedagogy. The differential effect of the methodologies on boys and girls was explored as a secondary possibility in this respect.

1.5.2 Misconceptions

The effect that the absolutist and constructivist approaches to the teaching and learning of algebra have on the resolution of misconceptions.

1.6 PROCEDURE

The above issues were investigated by comparing the scores that the pupils, from two differe , schools in the same neighbourhood, attained in a number of tests over a period of three months.

1.6.1 Teaching methodology

In this context the teachers at the control school followed a traditional approach to the teaching and learning of algebra. They provided the pupils with an interpretation of the meaning of the different concepts, before allowing the pupils the opportunity of cementing the ideas, that had been generated, by means of drill and practice.

The experimental school adopted a constructivist approach. In this context, the teacher was not a dispenser of knowledge, but assumed a variety of roles in an endeavour to help the pupils to construct their own knowledge. The teachers were expected to utilize their ingenuity and creative talents in order to provide each pupil with the best opportunity of developing a sound understanding of the basic concepts of algebra. A variety of methods, including guided discovery, problem solving, problem posing and investigations, which focused on an exploratory and experimental mode of instruction were encouraged. The researcher felt that the different approaches would create more opportunities for the student to find a link between his informal knowledge and the more formal requirements of algebra. The pupils were allowed to use scientific calculators in order to create generalisations as a result of their active involvement with numerical problems. Class discussion and negotiation, which caused the pupils to reflect on their previously acquired knowledge and encouraged critical thinking, was also encouraged.

1.6.2 Programme of research

In order to investigate the issues listed above, the research was divided into the following sections:

1. In Chapter 2: a comparison of the philosophies of absolutism and constructivism; the respective learning theories and the corresponding teaching styles that were induced.
2. In Chapter 3: the research methodology.
3. In Chapter 4: the results of the controlled experiment.
4. In Chapter 5: the conclusions and recommendations.

LITERATURE REVIEW

2.1 THE PHILOSOPHY OF MATHEMATICS

> "The philosophy of mathematics is that branch of philosophy whose task is to reflect on, and account for the nature of mathematics." (Ernest, 1991:3)

2.1.1 Absolutism

The absolutists believe that knowledge consists of certain and unchallengeable truths (Olivier, 1989a). Mathematical knowledge (including algebraic knowledge) is established by means of logic applied to theorems or propositions (Ernest, 1991). Platonism has probably been **the** absolutist philosophy that has played the most influential role in the teaching and learning mathematics (Julie, 1991). Its effect has been to create the impression that mathematics is

> " made up of rules, formulae and proofs to be memorized; skills to be practised; methods to be followed precisely and where all questions have answers that are known to authority." (Julie, 1991:111)

Hilbert's formalism is one of the absolutist philosophies of mathematics, which has gained prominence in recent years. It adopts the view that mathematics is a meaningless (as relating to the physical universe) game played with marks on paper, following **specific rules** (Davis et al, 1981; Ernest, 1985). Thus there appears to be a striking analogy between the absolutist pedagogies and rote teaching and learning in mathematics (Ernest, 1985). (See 2.2.1). Kurt Godel's First Theorem of Incompleteness, clearly of the traditionalists' view of mathematics, when it showed that Peano's axioms, or any larger recursive axiomatic set, could not be used to validate all the truths of arithmetic (Davis et al, 1981; Ernest, 1985).

2.1.1.1 Mathematical Objects

A platonist believes that mathematical objects are real. The certainty of mathematics implies that their existence is an objective fact and quite independent of our knowledge of them. The platonists' believe that the function of a mathematician, is merely to discover. He cannot invent anything as it already exists (Davis et al, 1981).

A formalist, on the other hand, believes that there are no mathematical objects. Mathematics merely consists of axioms, definitions and theorems, which are linked by formulas that are derived by specific rules (Davis et al, 1981).

2.1.2 Constructivism

In order to fully comprehend how the implementation of the philosophy of constructivism will affect the teaching and learning of mathematics, and more specifically algebra, it is necessary to briefly review the individual constituents of constructivism. We need to distinguish between naive, radical and social constructivists and the impact that their philosophies will have on teaching and learning.

" (1) Knowledge is not passively received either through the senses or by way of communication; knowledge is actively built up by the cognizing subject.

(2) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability; cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality."
(Von Glasersfeld, 1989a : 5)

Radical constructivists ascribe to both hypotheses, whereas naive constructivists are adherents of the first hypothesis only, which is the broad view generally accepted by mathematical educators and most cognitively oriented psychologists (Lerman, 1989).

The naive constructivists tended to concentrate their efforts on those constructions that would lead to the formation of mathematical concepts that were perceived to be absolutely true (Lerman, 1989).

The second principle clearly implies that all mathematical knowledge is constructed and that no knowledge is certain (Ernest, 1991). This view is supported by Lerman who says that the acceptance of the second principle implies that if

> "the knowledge we have of the world is not forced on us (empiricism), nor do we have this knowledge innately (platonism), then what we know becomes conjecture, theory and hypothesis." (Lerman, 1989 : 216)

This lack of certainty of knowledge clearly shows that mathematics is fallible, which signifies a major difference between the naive and radical constructivists' philosophies.

Social constructivism is a blend of the philosophical perspectives of conventionalism, quasi-empiricism and radical constructivism (Ernest, 1991).

> "Social constructivism views mathematics as a social construction. It draws on conventionalism, in accepting that human language, rules and agreement play a key role in establishing and justifying the truths of mathematics. It takes from quasi-empiricism its fallibilist epistemology, including the view that mathematical knowledge and concepts develop and change. It also adopts Lakatos' philosophical thesis that mathematical knowledge grows through conjectures and refutations, utilizing a logic of mathematical discovery. Social constructivism is a descriptive as opposed to a prescriptive philosophy of mathematics, aiming to account for the nature of mathematics understood broadly, as in the adequacy criteria." (Ernest, 1991 :42).

2.1.2.1 Mathematical Objects

In Pateman's eyes, the naive constructivists are close to the position of the conceptualists who believe that

> "mathematical propositions are high level generalisations of the empirical world, but that mathematical things are things of the mind, developed from things of the real world." (Pateman, 1989 :16).

These mathematical objects are constructed by the individual reflecting on his sensual perceptions of reality or by relying on his intuitions of the world (Pateman, 1989).

The radical constructivists' belief that reflective abstraction, which allows mental operations to become objects of thought in their own right, accommodates the social constructivist thesis of mathematical objects as reifications. The social constructivist perceives the objects of mathematics as social constructs or cultural artifacts (Ernest, 1991).

2.1.2.2 Language

The role of language is yet another distinguishing feature of the philosophies. For the naive constructivists, language, which serves as a vehicle to transfer thought to others, is subsidiary to thought, whereas radical constructivists perceive language to be an imperfect tool of communication which gains meaning through shared social interpretation (Lerman, 1989). The social constructivists, on the other hand, believe that language is the corner-stone of mathematical knowledge (Ernest, 1991). They view mathematics as a social construction with the corresponding emphasis on language, social interaction and history.

2.2 Learning Theories

2.2.1 Behaviourist Learning Theory

The behaviourist model represents a narrow view of the learning process (Buckle, 1990), which is based on the perception that the learner is not an independent thinker, but a response system (Laridon, 1981). Behaviourists' believe that the pupil passively receives knowledge, which is transferred intact from one person to another (Laridon, 1981; Olivier, 1989a). This is possible because a person's senses enable their mind to produce a photocopy of any reality (Olivier, 1989a). Hence, we may conclude that behaviourists' believe that students absorb everything that they are taught. Thus it would appear that the behaviourists assume that a pupil's current knowledge is not relevant to the learning process (Olivier, 1989a), and that instrumental learning (understanding), as discussed below is the result of their learning theory.

Instrumental (rote) understanding of an item, whether concept, fact or skill, is usually manifested by a successful application of the knowledge, without relating the item to any previously acquired body of knowledge (Skemp, 1976). Thus to have instrumental understanding of an item is to have knowledge of the rules governing its correct use, without knowing what its meaning is in the relational sense (Ernest, 1985). Skemp (1971) showed that pupils, who relied on rote learning as opposed to meaningful (schematic) learning, experienced a lower level of performance on memorization tasks. Since the absolutist philosophies rely extensively on memory and rote learning (see 2.1.1), it is apparent that students who were taught by absolutists must have been severely disadvantaged.

Meaningful learning consists of establishing patterns of ideas and information in the mind of the learner. New ideas or information are assimilated by adding new examples to an existing idea or by becoming aware that an idea can be used in circumstances in which it has not previously been encountered (Ausubel, Novak and Haneslan, 1978; Olivier, 1989a).

Occasionally a more substantial change may occur and the learner's cognitive schemata are transformed to accommodate the new idea. This process facilitates the development of understanding and provides the student with an enriched basis for further learning. The basis is enhanced in two respects:

1) the new pattern can be used in the comprehension of a wider range of information and ideas;

2) the selection and recall of the appropriate ideas for use in solving problems is made easier (Ausubel et al, 1978).

Meaningful learning ensures that the pattern of ideas that have been generated, become the property of the individual student, reflecting the way in which the ideas and the particular examples, which helped to make those ideas meaningful, have been learned. The patterns and ideas generated by each student, who has understood the material in a meaningful fashion tends to be unique (Ausubel et al, 1978). By contrast, verbatim or rote learning lacks pattern and organization. At best, the information is gleaned from the textbook or teachers' notes and the resultant lack of appreciation of its nature and power does not facilitate understanding and retention. However, it must be remembered that instrumental understanding can lead to effective performance at appropriate tasks. Skemp (1976) states that instrumental understanding is attained more rapidly and efficiently than relational understanding, although the latter leads to longer retention and greater transferability.

The assertion that the past experiences of a pupil play a vital role in the individual's structural development of conceptual knowledge (Confrey, 1987), highlights a serious flaw in the learning theory of the behaviourists. The importance of this issue is given additional credence by Lerman who says

> "...in algebra and in the other topics investigated, the research has found that children frequently tackle mathematics problems with methods that have little or nothing to do with what has been taught. This may be because mathematics teaching is often seen as an initiation into rules and procedures which though very powerful (and therefore attractive to teachers), are often seen by children as meaningless."
> (Lerman, 1983: 64)

The following questions appear quite frequently in the literature:

1 Which is the larger, 2n or n + 2? (Becker, 1988; Brown M, Hart K & Kuchemann D, 1984; Joubert, 1990)

2 When is L + M + P = L + P + N true? (Brown et al, 1984; Olivier, 1989)

Pupil responses to these questions, in the respective research pieces, clearly indicated that a large proportion of the pupils did not realise that the value of the variables played an integral role in the solution. Thus it is quite apparent that, in spite of all the attention devoted in secondary schools to algebra and symbolic notation, students' command of the formal language of algebra is often instrumental rather than relational (Linchevski and Safard, 1991).

This position is strengthened by the fact that one of the major reasons for the difficulty that students have in learning algebra is their apparent inability to forge a link between their formal knowledge and their rich source of informal knowledge, which has been developed by working with quantities in everyday situations (Putnam et al, 1990). Regrettably, due to our historical background (See 1.3.2), many children appear to view school mathematics as a collection of arbitrary rules and procedures performed on meaningless algebraic symbols, in spite of the fact that they may have developed rather sophisticated concepts and strategies for solving quantitative problems encountered out of school (Putnam et al, 1990). This is clearly portrayed by Carraher's example of the street vendor selling coconuts. His mental arithmetic was excellent, however in the classroom his attempt to write down a formal algebraic account of the transaction was virtually incomprehensible (Carraher, Carraher & Schliemann, 1985). Thus, it would appear that conflicts between the informal use of language and formal algebraic usage could be seen as a source of confusion and misinterpretation for students (Confrey, 1987).

The confusion of literal symbols, which are used to represent variables, with letters of the alphabet that pervades English speaking nations, does not occur in Japan, because literal symbols in algebra differ from the Japanese written language. The Japanese seem to rely on instrumental understanding for their proficiency in solving equations and inequalities. Thus a large number of their students experience great difficulty, mainly as a result of their preconceptions about the meaning or existence of the symbol 'x' (Fujii, 1988).

2.2.2 Constructivist Learning Theory

Von Glasersfeld's first principle in 2.1.2 implies that a child is an active participant in the construction of his own knowledge. The extent to which his perception increases depends on the interaction between his experience and his current knowledge structures (Ernest, 1991), and the quality of ideas that the learner brings to the experience (Olivier, 1989a).

The epistemology of the naive constructivists focuses on what mathematics ought to be, emphasising those methods, statements and proofs in the construction of mathematics that will lead to certainty (Lerman, 1989). However, since the learner is not given knowledge but actually actively constructs that knowledge (Jaworski, 1991; Lerman, 1989; Pateman, 1989; Von Glasersfeld, 1989a), we may conclude that the naive constructivists' are moving to fallibilism.

The epistemology of the radical constructivists focuses on the activity of construction as the process by which individuals learn and by which knowledge is created (Lerman 1989; Von Glasersfeld, 1989a). The meaning that each pupil associates with a particular algebraic concept is unique as this subjective knowledge is essentially private (Ernest, 1991). Von Glasersfeld (1989b) and Stoker (1991), however, do acknowledge the importance of communication, by expressing the belief that a student's level of understanding is greatly advanced as a result of his or her reflections on discussions that have taken place with people who have different or opposing points of view. Unfortunately, the emphasis placed on the necessity of the availability of a rich source of resources (Volmink, 1993), could severely restrict radical constructivists in the pursuit of realising their aims in countries which are poor and are inhabited by people who have different cultures. This is exacerbated in South Africa by the fact that class size is considerably larger than those from which the constructivist model originally emanated (Stoker, 1991; Volmink, 1993).

The basis of the learning theory of both the naive and radical constructivists, may be found in Piaget's genetic epistemology (Lerman, 1989). The learning theory that emerges from Piaget's work can be summarised by saying that learning occurs when a change in the existing schemata takes place, that leads to the establishment of a new equilibrium. This process of accommodation is likely to ensue when a schema, instead of producing the expected result, leads to perturbation or conflict. This process occurs on two levels. On the sensory-motor level, action schemata are instrumental (utilitarian) in assisting organisms to realise goals during their interaction with the world of their experience. On the level of reflective abstraction, however, operative schemata are instrumental (epistemic) in helping organisms achieve a viable, coherent, conceptual network that reflects the paths of acting as well as thinking (Von Glasersfeld, 1989b).

Both the broad (naive) and narrow (radical) senses of constructivism offer a psychological parallel to social constructivism. The processes of accommodation and assimilation parallel the social constructivists' accounts of subjective and objective knowledge growth, for knowledge according to this account is hypothetico-deductive (Ernest, 1991).

The social constructivists' exposition of the nature and genesis of subjective knowledge is largely based on radical constructivism (Ernest, 1991). However, a unique feature of social constructivism is that it links subjective and objective knowledge, which are mutually dependent, in a creative cycle as illustrated in figure 2.1 below.

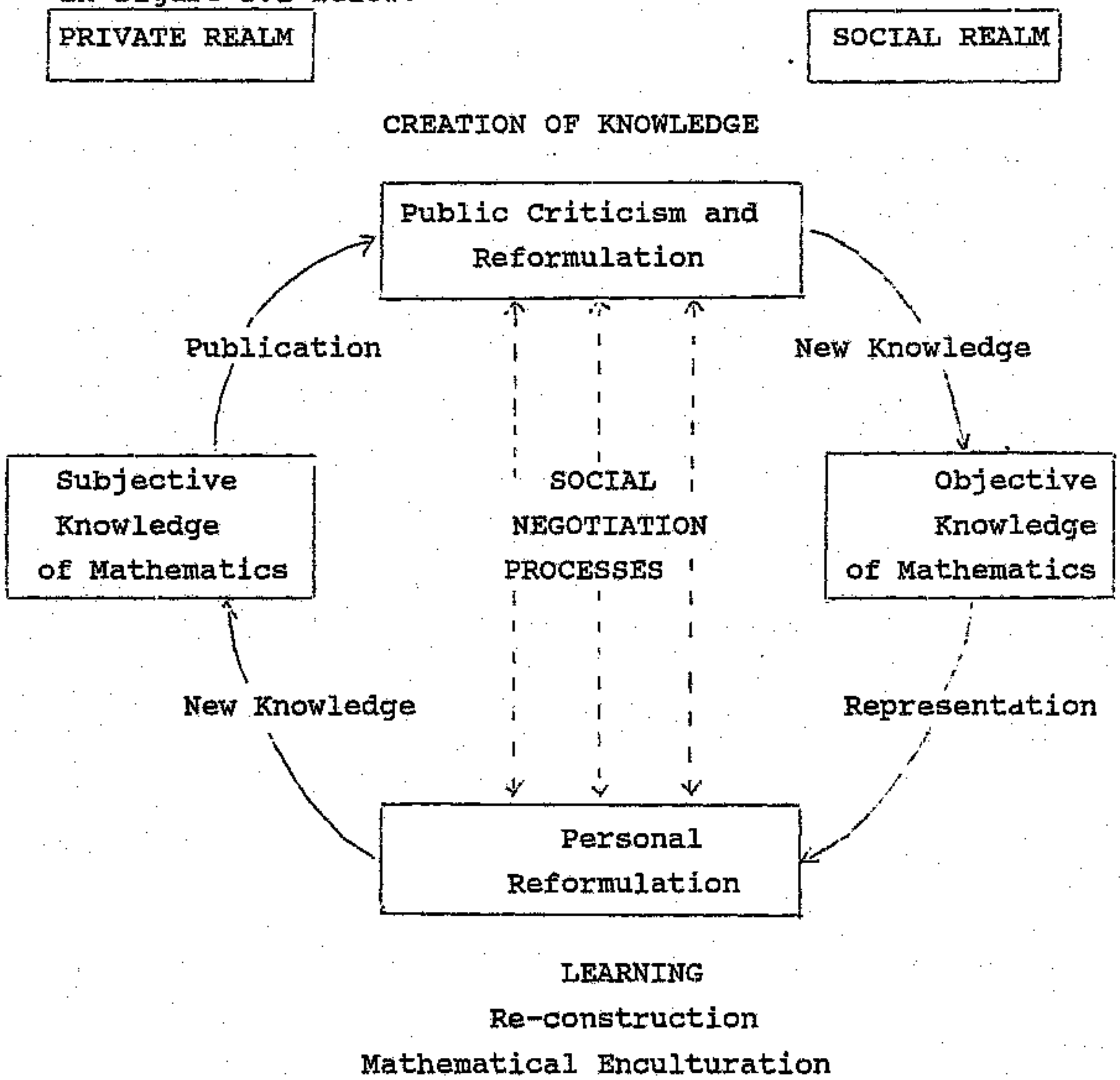


Figure 2.1

The relationship between Objective and Subjective Knowledge of Mathematics (Ernest, 1991 : 85)

This diagram clearly indicates that

> "children are not lonely voyagers in their construction of mathematical knowlege and their ideas are elaborated and authenticated through negotiation of individual meaning."
> (Steffe et al, 1988:102)

2.3 The Role of Language

The absolutists believe that there is no need for any discussion to take place as they already have the answer (Lerman, 1989). This claim should be treated sceptically since it is questionable whether different people are able to assign the same meaning to the printed or spoken word (Lerman, 1989). Furthermore, people are creative and as a result their minds need to react to information that is received. This is accentuated by the fact that " facility with language is vital for the development of the thinking processes." (Brodie, 1989)

During the sixth international congress on mathematical education, the Austrian, Gunther Malle emphasized that the construction of meaning associated with mathematical (and algebraic) concepts and methods should be the central goal of teaching and learning mathematics (and algebra) at all levels (Hirst & Hirst, 1988).

From a social constructivist point of view the subjective knowledge that each individual generates from his or her constructions is idiosyncratic (Ernest, 1991). Thus initially an algebraic concept may have several different interpretations, since meaning is created by the direct experiences of students (Volmink, 1993). A concept is identified by its use (Lerman, 1989). It ultimately becomes objective knowledge and gains its meaning and acceptance through shared social interpretation. (See figure 2.1). This in turn means that language, which itself is socially negotiated, plays a vital role in the notion of a concept (Lerman, 1989; Stoker, 1991). Objective algebraic knowledge is reconstructed as subjective algebraic knowledge by the individual, "through interaction with the teacher or other persons and by interpreting texts and other inanimate sources." (Ernest, 1991 : 81)

How are alg braic ideas communicated in the classroom? Interaction must be the key to this as objective knowledge is social and public, and resides in the shared rules and meanings of different members of society (Ernest, 1991). The very nature of mathematics and specifically algebra, means that the language that is used during the course of the lesson has important implications for the pupils' learning (Bliss and Sakomidis, 1988). Ernest (1991) lends his weight to the argument when he claims that mathematical knowledge begins with the acquisition of linguistic knowledge. There is no doubt that the meanings associated with the abstract concepts of algebra are inordinately difficult to communicate (Austin and Howson, 1979). This is compounded by the fact that teachers of algebra, must of necessity utilise a language that is full of symbolism. If teachers use symbolism extensively during the course of their lessons, then it could lead to an improvement in the pupils' competence to manipulate algebraic symbols. However, on the other hand, it could increase their difficulty of understanding the underlying meanings (Bliss et al, 1988).

Another important issue in the learning of algebra is the possibility that the structure of the learner's mother tongue could have a considerable influence on cognitive processes such as classification and recognition of equivalences and relationships (Presmeg, 1988). This is particularly relevant in the South African context because of the variety of languages that do exist. The problem has been exacerbated by the fact that numerous students are being forced to learn through the medium of their second or third language (Brodie, 1989; Laridon, 1992). This has been brought about by a combination of factors which include the following:

1) the acute shortage of materials in the venacular languages,

2) the limited availability of suitably qualified teachers who speak the languages proficiently, and

3) government policy (Brodie, 1989).

The limited role assigned to language in the naive constructivists' philosophy has far reaching implications in the sphere of learning. Cobb (in Ellerton, 1991), emphasises the fact that quality learning is more likely to take place through creative discussion which is generated through the expression of different points of view. Thus from Cobb's stance it is quite apparent that naive constructivism is not ideally suited to creating a quality learning environment, because the necessary verbal exchange of ideas, with the ensuing discussion of conflicting points of view between teacher-student and student-student is unlikely. The wider scope of radical constructivism or the even broader aspect of social constructivism, seem to offer better alternatives. Hence, the researcher concludes that the limited role of language and the lack of emphasis on social interaction will severely inhibit the naive constructivist in his endeavour to promote learning through understanding. These factors will, inevitably, increase the likelihood of misconceptions arising from this source. However, the naive constructivist's ultimate goal of providing the student with the opportunity of understanding the numerous concepts of algebra through active participation, although restricted in the manner specified above, is hopefully attainable, and must surely be preferable to lecturing or other methods that promote rote learning. This indeed is the focus of this research project.

Hanlie Murray in the foreword to a paper by Osborne (1989), on approaching algebra numerically, emphasises the relevance of his informal approach to the concept of variable, and stresses the fact that the new HOA (1990) syllabus calls for an informal, numerically based approach to algebra in standard 5, which should serve as an introduction to the more formal work in standard six. Osborne (1989) believes that teachers should encourage pupils, with the aid of scientific calculators, to generate numerical tables that illustrate the relationship that exists between the different variables in the problem that has been posed. The following example illustrates part of the procedure.

If the length of some rectangles is 3 metres more than their width then:

1) Complete the following table:

Width(m)	Length (m)	Perimeter (m)	Area (m^2)
5	5 + 3 = 8	2x5 + 2x8 = 26	5x8 = 15
8			
15			
20			
32,6			
W			

2) Find the width of a rectangle whose perimeter is 256 metres;

3) Find the width of a rectangle whose area is 396 square metres (adapted from Osborne, 1989).

Osborne also feels that it is important for pupils to verbalise their computational processes. The subsequent recognition of pattern could then help the pupil to use the variable to construct a mathematical phrase that describes a generalization of the relation (Osborne, 1989). De Jager (1990) emphasises the fact that the utilisation of pattern in the students' initial exploration of a problem enables them to gain greater insight into the problem. The study of patterns and their underlying structures provides both the teacher and the student with ample opportunity for originality and creative activity (Laridon, 1981).

After the numerical investigation, the pupil is encouraged to explore the initial problem in a graphical context before moving on to the more formal approach of solving equations. The technique of revisitation is used to elaborate on and extend the initial algebraic concepts that the pupil has formed (Osborne, 1989). The adoption of a methodology which enables pupils to approach a topic in algebra from a different stance, causing the pupil to reflect on his previous assumptions, could lead to retrospective relational understanding and provide them with the necessary insight for accommodation to take place (Padoa, 1987). This is supported by Ausubel, Novak and Haneslan (1978), who suggest that if new information and ideas are to be understood, rather than merely memorized, then they must be related to the learners existing knowledge and ideas. However, it is important to remember that for Ausubel it is the teacher who needs to create the meaningful links with the students' existing knowledge, whereas the constructivists believe that this process must be accomplished by the learner with the facilitation of the teacher. The importance of teachers utilising "the incredibly rich store of knowledge" (Fennema, Carpenter & Peterson, 1991: 30) with respect to algebra, that children bring to the classroom situation cannot be over stressed as this forms an integral part of the child's learning process. The uniqueness and significance of children's existing knowledge, beliefs, goals, and motivations in relation to their subsequent learning and problem-solving attempts, clearly demonstrates the need for negotiation skills (Ernest, 1991; Siemon, 1992). Thus it is apparent that verbalisation and discussion should be a significant part of any lesson if understanding is to be achieved (Osborne, 1989; Rogers, 1990). Support for this departure is received from Noddings. He (in Stoker, 1991) maintains that it is important for children to be challenged by their peers, because encounters of that nature could lead them to examine their own beliefs and strategies more closely, resulting in those that are untenable being discarded. This social negotiation process lies at the heart of social constructivism and is clearly portrayed in figure 2.1 above. This figure shows that the formation of subjective knowledge is a recursive one, which depends on the knowledge that we have gained as a result of our previous experiences (Ernest, 1991). Thus it is important for teachers to implement a spiral approach to the teaching of algebra. Vygotsky also supports this viewpoint because he feels that pupils not only internalise their knowledge, but are actually forced to reflect thereon when their peers challenge their assumptions (Stoker, 1991).

Since the construction of knowledge is a social process, communication skills are vital (Laridon, 1993). The language of communication used to convey instructions must be simple enough to be understood by all pupils with minimal teacher input. If this is not the case, then the value of active student learning is reduced and obstacles are placed in the way of pupils demonstrating what they can really do (Roby, 1990).

Small group or individual discussion, based on an outline agenda, which was developed in conjunction with the pupils could be most beneficial. The existence of such an agenda enables the students to prepare for the discussion, which gives them the opportunity to think about relevant experiences in addition to identifying the points that they are finding difficult. In this way the time spent in discussion can be used most effectively and efficiently (Entwistle, 1985).

Stephen Brown (1986) propagates the idea that pupils should be given situations to investigate. The broad nature of the problem will allow students the opportunity of posing questions about their respective fields of interest, thus stimulating discussion and widening the sphere of exploration into areas where everyone not only enjoys the work, but is also actively motivated to pursue personal concerns further. Teachers should encourage their students not only to share their ideas, but also to provide reasons for their conjectures. This process means that the pupil's ideas are being continually tested by the criticism of their peers, and replaced in the event that they are shown to be false, thereby leading to the resolution of a possible misconception. These conjectures form part of the pupil's subjective knowledge of the external world (Ernest, 1991). In particular, the process of reflection or looking back should be an active part of the learners' arsenal as this leads to unlimited opportunities for creative thought in algebra, particularly in the sphere of problem solving (Taback, 1988). Furthermore, the need to diagnose the strengths and weaknesses of a pupil's learning strategies requires a marked shift from teacher-directed courses to pupil-negotiated programmes. This is particularly the case with modular schemes (Roby, 1990). In order for the pupils to benefit to the fullest extent, the counselling skills of the teacher should be honed to perfection.

2.4 Teaching styles

> "The logical connection between the philosophy of mathematics and teaching styles, strongly influences students' attitudes to mathematics."
> (Lerman, 1983:59)

2.4.1 Behaviourism

The task of the behaviourist teacher is to provide the student with the necessary stimuli and reinforcements that will condition the student in such a way that he will respond in a manner that is deemed to be suitable by the teacher (Von Glasersfeld 1989b). The behaviourist model invariably leads to the adoption of a prescriptive approach to teaching and examining, which focuses on outcomes rather than the processes that are likely to occur (Buckle, 1990; Stoker, 1991). This severely restricts the didactical options available to the teacher and clearly inhibits a creative approach. Unfortunately, understanding is seldom achieved by this methodology (Von Glasersfeld, 1989b). Computer programmes, aimed at the South African primary child, are being developed that will optimize conditions for drill and practice routines (Stoker, 1991). This could, alas, give an added impetus to the behaviouristic approach advocated by Thorndike (1922).

The usual conventional (traditional) mathematics lesson may be typified as whole class teaching with the teacher as focal point (Brinkworth, 1988). The instruction is text-based and optimum use is made of drill and practice routines with increasing use of "board-centred" teacher talk (Brinkworth, 1988). A confident, assertive student often thrives in the competitive atmosphere that is created by the traditional style of teaching algebra. On the other hand, the more reticent and less confident are disadvantaged (Barnes and Coupland, 1990). The importance of the final algebraic examination has resulted in excessive drill work on certain types of examination questions. This approach generally does not induce a positive attitude towards the subject and invariably leads to hardly any improvement in the pupil's understanding of the material under consideration (Van Rooy, 1965).

Hence the behaviourists, instead of concentrating on teaching which

> "aims at the students' conceptual fit with the consensual domain of the particular field, a fit which, from the teacher's perspective, constitutes understanding;" (Von Glasersfeld, 1989a:9)

have placed their emphasis on training, which

> "aims at the students' behavioral fit which, from the teacher's perspective, constitutes acceptable performance." (Von Glasersfeld, 1989a: 9)

Algebra courses, should, instead of concentrating on the techniques of symbol manipulation, place far more emphasis on expressing and interpreting relations in symbolic and graphic form (Fey, 1992).

Laridon (1981) and De Villiers (1994) extend this idea by strongly advocating the use of mathematical modelling, as illustrated in figure 2.2 below, in order to solve real life problems.

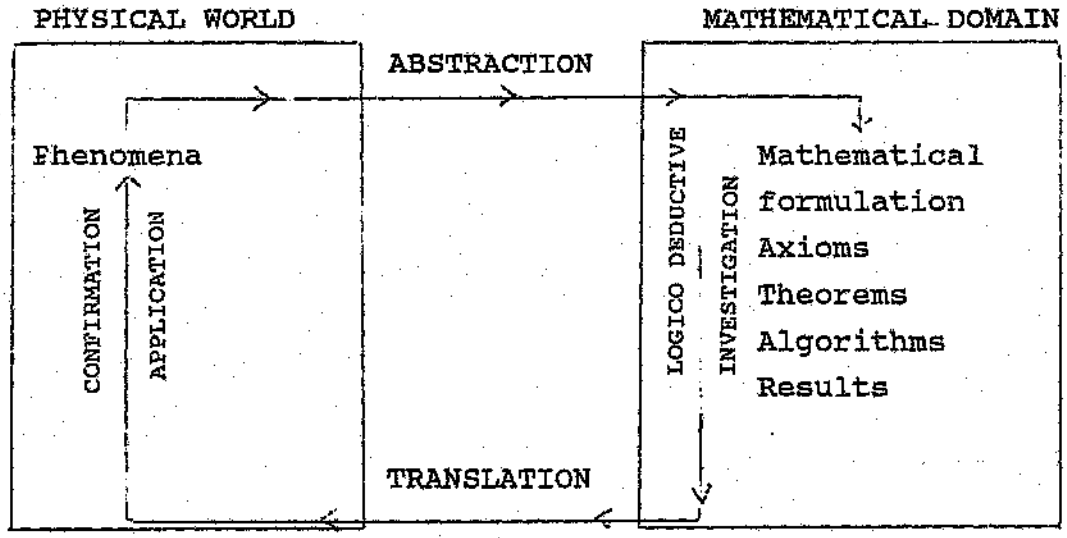


Figure 2.2
The mathematising cycle (Laridon, 1981 : 70)

Figure 2.2 shows that, when a physical phenomenon (problem) is analyzed through a series of abstractions and assumptions a corresponding mathematical model may be obtained (Laridon 1981). The process of mathematising (modelling) invariably means that data has to be collected, tabulated and graphed (De Villiers, 1994). This process often entails the use of equations or formulae in order to enable the student to evaluate and interpret the data by means of an investigation utilising deductive logic. The final solution needs to be critically assessed in terms of the real world situation (De Villiers, 1994; Laridon, 1981).

Mathematical modelling is truly creative as it often gives rise to genuinely new mathematics (Laridon, 1981). Hence, investigations involving mathematical modelling provide one of the avenues to satisfy Joubert's call to create more opportunities for pupils to develop their own understanding of the concept of a variable (Joubert, 1990).

The advent of the calculator, with algebraic logic, could promote the process of learning with greater insight (Laridon, 1993), and diminish the importance of mechanical operations. Coburn (1992) lends his voice to this departure when he states that the calculator can most certainly be utilised to:

1) assist in the development of concepts,
2) help reinforce skills,
3) promote higher level thinking and
4) enhance problem solving instruction.

The arguments presented above seem to indicate that if the emphasis in teaching is shifted towards generating understanding rather than computational skills then pupils will start to develop critical thinking, an essential component in a person's ability to resolve problems.

Support for this departure comes from Polya who says

> " a teacher of mathematics has a great opportunity. If he fulfils his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development and misuses his opportunity. But if he challenges the curiosity of his students by setting problems proportionate to their knowledge and helps them to solve their problems with stimulating questions, he may give them a taste for and some means of independent thinking."(Polya, 1973:v)

However, it must be remembered that to date no one best method for the teaching of mathematics in all circumstances exists. In fact Ellerton (1991) claims that researchers have shown that children taught by traditional methods, in which the teacher uses whole-class teaching, have often outperformed children from more individualised settings on standard tests emphasising computation. This phenomenon has also occurred in some tests involving mathematical problem solving. However one should also remember that these scores do not reflect the depth of understanding attained by these individuals. In addition researchers such as Ellerton (1991) have shown that a quality learning environment is characterised by classrooms where interactive communication between teacher-student and student-student is prevalent. These attributes, which are essential components of a social constructivists' approach, could have been present in the classes surveyed.

2.4.2 Constructivism

Bloom's taxonomy (1956) is a strong indicator that learning should not just be concerned with the mere acquisition of knowledge. Greater emphasis should be placed on the development of understanding, the application of knowledge, analysis, synthesis and evaluation. The emancipatory role of a problem posing pedagogy, which is based on fallibilism, will lead to the development of higher level cognitive skills in the individual, with enhanced beliefs of self-worth, ability and more in-depth understanding (Ernest, 1991). Constructivists' believe that mistakes are often the keys that unlock the doors to enhanced understanding. Hence, if a fallibilist epistemology is adopted then the pupil will not become demotivated because his attempt was incorrect (Von Glasersfeld, 1989a). This factor could be largely instrumental in encouraging pupils to adopt a positive attitude towards algebra.

The naive constructivist teacher acts as the mediator between the pupil and the algebraic concepts that have to be mastered. The teacher is responsible for interpreting the curriculum, for delivering the syllabus and for ensuring that pupils achieve their best possible results in examinations (Jaworski, 1989). Hence it may be construed that teachers would endeavour to provide the most suitable environment; containing, where possible, all the appropriate materials for their students to construct the necessary aspects of algebra. In this regard, it is important for the teacher to be aware of the particular learning style favoured by each pupil, since some students prefer to store information in a visual form whereas others think predominantly in words (Riding, 1991). Each student would be expected to generate mathematical knowledge from real-world problems, phrased in the natural language of the students. Abstract algebraic symbols should be introduced only after the relevant concepts have been formed (Pateman, 1989), thereby enabling the learner to internalize this knowledge (Putnam et al, 1990).

> "A radical constructivist approach emphasises the importance of experience but also emphasises the need for the thinker to develop reflective abstraction. It is not enough to act; one must also ratiocinate those actions, reflect on their consequences, and make subsequent modifications in both behaviour and thought." (Pateman, 1989 : 28)

The radical constructivist teacher acts as a guide and actively encourages the pupil's constructive effort, rather than curtailing their autonomy by presenting ready-made solutions (Von Glasersfeld, 1989a). Since knowledge is an individual construction and since language cannot be used to transmit knowledge to pupils, constructivists are often confronted with the task of explaining how it is possible for different people to have the same beliefs about the basic concepts of mathematics (Putnam et al, 1990). This obstacle has largely been overcome by the increasing significance that constructivist teachers of science and mathematics have been placing on co-operative group learning. This practice enables students to discuss different approaches to a given problem, with little or no input from the teacher (Von Glasersfeld, 1989b) and heralds the vital role that social interaction plays in the individual's progress.

The second hypothesis in 2.1.2 implies that all knowledge is constructed and none of it tells us anything certain of the world (Ernest, 1985). Hence, the implication is that a pupil could produce a sensible solution to a problem which makes no sense to the teacher. The teacher's role, in this instance, is to reflect on the approach of the pupil, in order to build a hypothetical model of the pupil's conceptual network. This highlights the need for teachers of algebra to cultivate the vital skill of listening (Steffe et al, 1988). A problem that may be encountered with this methodology is the fact that teachers are generally too busy during the day to observe and reflect on what is happening in the classroom (Taylor, 1986). However, this process of reflection should provide the creative teacher with the opportunity to ask specific questions, design activities or supply material that would introduce constraints that should challenge any inconsistencies that may have occurred (Jaworski, 1989). This approach should enable the pupil to make the necessary accommodation. The radical constructivist teacher is thus clearly a facilitator of learning, who promotes understanding and encourages problem solving through an investigatory approach based on group work. The facilitator teacher will know that he is not in a position to do the learning for his students. He must ensure that the wealth of experience that each child brings to the classroom is used to the full. Sufficient time must be devoted to the topic so that the student has the opportunity of making the knowledge his own (Breen, 1990), by seeing, saying and writing. With an extremely challenging problem, even high school students should be encouraged to talk aloud since words spoken out loud cause us to focus our attention more completely on the task at hand (Manning, 1984).

Ernest (1991) believes that the social constructivist teacher should, depending on circumstances, adopt a variety of roles, ranging from that of neutral chairman to being a devil's advocate. The extensive use of language and social interaction that this philosophy calls for, means that the teacher not only assumes the duties of the radical constructivist, but is also actively involved in teacher-pupil discussions as well as playing a pivotal role in ensuring that student interaction is prevalent. This implies that there is a clear and definite role for the teacher. The eacher will inevitably adopt the mantle of manager of the learning environment.

A child should be given an opportunity to utilise and to express his own unique creative potential (Maarschalk, 1981). Self-discovery, a teaching methodology that lies at the heart of constructivism, appears to be an appropriate vehicle for realising this objective. In this context, the teacher should create classroom situations that will not only stimulate the imagination of the children, but also encourage them to seek and to find solutions to problems that they encounter or to problems that they themselves have formulated. Pupils should be encouraged to use the guess and check technique to find solutions to these problems. This process is made feasible by the calculator and could serve to strengthen the replacement notion of a variable (Osborne. 1989). These components of self-discovery in the field of algebra may be regarded as the building blocks of the creative events in the didactical situation (Maarschalk, 1981).

Marianne de Vries (1991) places a great deal of emphasis on the importance that multiples and verbalisation can play in the teaching of the concept of a variable. The following approaches, generated by the researcher, from her article, can be effectively investigated and resolved using this approach.
1) Derive a generalisation for the sum of two multiples of seven.
2) Derive a generalisation for the sum of two multiples of a particular composite number.
3) Explain why $4x^2+3x^2 \neq 7x^4$.
4) Use numerical examples to formulate a generalisation for the addition of like terms. Hence, or otherwise determine whether $4a + 3b = 7ab$. Explain.
5) Derive a formula for the product of powers of the same base.

Wheatley and Shumway (1992) believe that the calculator could be an inspirational aid that transforms algebra from a set of dreary procedures to an exciting study of patterns and relations. The numerical approach to algebra by Osborne (1989) as mentioned in 2.3 places great emphasis on the role that pattern plays in the initial stages of the development of the concept of a variable. De Vries (1991) actually appears to incorporate his ideas in her programme. This pedagogic approach could serve as a good source for creative ideas for the students (De Jager, 1990). Geometry, which is a source of ideas (Hilton, 1990), seems to be the ideal vehicle whereby these patterns may be visualised. The Teaching Committee of the Mathematics Association (1974) believe that the study of patterns not only offers unbounded scope for originality and creative activity but can also be used to effectively link algebra and geometry. The following example is typical of the approach being advocated and is in keeping with Brown's recommendations in 2.3.

Investigate the sequence formed by determining the number of straight lines that can join 2, 3, 4, 5, 6 or 7 vertices. If the teacher feels that the students need some assistance then the question could be rephrased as follows:

Consider the sequence 1 3 6 10 15 21 ...

1) What is the next term?
2) What is the tenth term?
3) What is the hundredth term?
4) Why are these numbers called triangular numbers? Illustrate geometrically.
5) What else do the triangular numbers represent?
6) What is the nth triangular number? (Investigate 1 2 3 ... and 1 3/2 3 5/2 6 ...)
7) What happens when two triangular numbers are added?
8) What happens when two consecutive triangular numbers are added? (McGill, 1982: 1)

The uniqueness of the individual's learning strategies highlights the importance of utilising different approaches to teaching the concept of variable. The following methodologies could be used in conjunction with the approaches meantioned above.

1) Flow diagrams, which provide another way of conceptualising the variable (Giles, 1972; Joubert, 1990; McGill, 1982).

2) The Mathematics Workcard Booklets devised for the Fife Mathematics Project by Geoff Giles in 1972 can be used to consolidate the concept of variable. They also provide an admirable introduction to the solution of equations and to the idea of a mapping. The booklets also create an opportunity for the pupil to express his or her thoughts algebraically. An example which is adapted from Giles (1972) is provided below.

Programme in words: Double and take away from 40		
Programme in algebraic symbols:		?
IN		OUT
5	------------->	a
x	------------->	22
7	------------->	t
y	------------->	4

3) The use of tables, diagrams and/or centicube blocks could also provide the student with a way of visualising and eventually formulating the distributive law (De Vries, 1991; Joubert, 1990; McGill, 1982).

4) The use of puzzles and games (McGill, 19).

The emphasis that has been placed by the absolutists' on the learning of rules of algebra is directly opposed to the constructivists methodology and will undoubtedly create tensions for the teacher in his role as facilitator as he struggles to determine how much information he should impart (Cobb, 1989). This argument is supported by Lampert (1985) who says that

> " ....pushing his students to achieve and trying to create a comfortable learning environment, between covering the curriculum and attending to individual understanding" (Lampert, 1985: 183)

was a dilemma that he experienced on a daily basis in the classroom. This is substantiated by Ellerton (1991) who maintains that teachers are subjected to a host of tensions on a daily basis due to the need to cover algebraic content, the desire to use school time responsibly and the challenge to establish a successful learning environment. The ability of the algebra teacher to continually make the sensible choice is the factor that ultimately determines the success of the lesson and whether the aims of mathematics education, as perceived by the constructivists, have been realised at a micro level. Experience and a willingness to interact with colleagues should increase the probability of success in this field.

2.5 FACTORS AFFECTING THE CONSTRUCTIVISTS' APPROACH TO THE TEACHING AND LEARNING OF ALGEBRA

2.5.1 Curriculum, evaluation and assessment

One of the major problems confronting teachers of algebra today is the fact that large numbers of children do not enjoy the subject and are unable to apply their knowledge (Laridon, 1993). The De Lange Report (1981) showed that the overloading of syllabus and the inappropriateness of the content being taught had been identified as two of the factors that had contributed to the decline in the popularity of mathematics.

> "It is the teachers who play the key role of mediating between their students knowledge, experience and learning practices and the demands of the examiners."(Buckle, 1990: 74)

The conflict between teaching and examining is at the centre of most teaching practices and inevitably results in some of the quality that a process-based curriculum is striving for, being sacrificed on the sacrificial alter of the public examination, which is a legitimate social objective (Stenhouse, 1981). This is compounded by the fact that research has shown that, pupils who have been taught to engage in well-defined behaviours, perform higher on standardized achievement tests than pupils of teachers who have not (Fennema et al, 1991).

Teachers are often ranked on the achievements of their students in the final examinations, thus they resort to methods which rely almost exclusively on instrumental learning. Countless hours are devoted to drill and repetitive work (formalism), which leads to boredom, inefficiency and very little understanding, which in turn leads to fewer students, especially girls, continuing with the subject. The emphasis then shifts from teaching to training.

The set written examination favours the more extrovert and stable student, since they tend to perform in circumstances that require activity for short periods of time. By contrast the more introverted and nervous pupil does not enjoy the pressured situation of the examination. He or she prefers to reflect on the proceedings and tends to be more diligent and willing to concentrate for an extended period of time. Thus these students will probably cope far better with assignments such as investigations and mathematical modelling that rely on algebraic techniques for their resolution and which require an extended period of time for completion. Hence

> "it is important to allow a variety of types of assessment in order to permit pupils to demonstrate what th-- can accomplish in a variety of situati.. and also to reflect the range of activities that are encountered in everyday life situations." (Riding,1990:230)

2.5.2 Teacher Beliefs

One of the biggest obstacles to the successful implementation of a constructivist's approach to the teaching and learning of algebra in schools is the individual teacher's beliefs, because it is obvious that one's feelings about the nature and origins of mathematics in general and algebra in particular, will largely determine the teaching pedagogy that is adopted (Jaworski, 1989; Lerman, 1983; Pateman, 1989). This is substantiated by Fennema, who says that

> " teacher's beliefs, knowledge, judgements and thoughts, have a profound effect on the decisions they make, which in turn determines to a large extent what students learn in the classroom." (Fennema et al, 1991: 28)

Another proponent of this view is Ernest (1991), who describes how investigational work in algebra is often subverted by the teacher's preconceived view that there is a unique solution. This appears to confirm the fact that these teachers have approached the teaching and learning of algebra from the absolutists' point of view. Certainly if the constructivist approach to the teaching of the algebra was adopted, as elucidated in 2.4.2, then this subversion would not have taken place as teachers of this discipline are fully aware of the fact that each pupil will initially construct his own unique solution to the problem.

The teacher also has a preferred style of learning. This invariably means that the teacher will be predisposed to present and explain the concepts of algebra in a way which accords with his beliefs. His delivery, the methodology invoked, the choice of textbooks and other supporting materials such as visual aids, slides and videos will all be affected. A teacher with a strong preference for a verbal style may make little or no use of such aids. Thus, even the most dedicated teacher may unknowingly place a number of his students at a serious disadvantage (Buckle, 1990).

Historically we are aware of the fact that revisions of content and methodology have appeared on teachers' desks with disquieting regularity. In the past few decades teachers have been called upon to implement programmes that have ranged from new mathematics, back to basics, problem solving and the need to use technology (Pateman, 1989). These significant changes in emphasis are often met with a new chapter in the text-book and little else. The majority of teachers, because of their beliefs, have made no allowances for the changes and have continued to teach in their usual way (Ernest, 1991; Laridon, 1992).

The preceding paragraphs clearly depict the fact that the teaching and learning of algebra, has, and still is being influenced by the philosophy of the absolutists. The consistently changing face of mathematics, due to it's fallible nature and the continuous development of language, necessitates a corresponding change in the teaching approach of educators. Thus it is imperative for in-service courses to be held that will familiarise all teachers with the positive aspects of constructivism as mentioned in 2.1.1; 2.2.2; 2.3 and 2.4

2.5.3 Social Constraints

> "In considering the connections between constructivism and the teaching and learning of mathematics, the pragmatism forced by the classroom environment might be seen as throwing up constraints which have to be overcome before learning may be achieved. Thus there are both mathematical constraints and social constraints challenging viability in learning mathematics." (Jaworski, 1989: 294)

Until recently, education in South Africa was administered by a number of Educational Bodies, each funded differently, the main criterion being race. The standard of teaching and the facilities available, varied from exceptional in the private schools to abysmal or non-existent in the DET. The situation was further aggravated by the lack of suitably qualified teachers and the unrealistic teacher-pupil ratio in a number of schools (Adler, 1992a; Laridon, 1992). The strong sense of social injustice amongst black pupils prompted large scale attendance at political gatherings, resulting in non-attendance at school and the virtual breakdown of a culture of learning (Adler, 1992a). These factors must, inevitably, adversely affect the teaching of algebra in those schools.

Teachers often shift their pedagogical intentions and practices away from their espoused theories because of the social constraints imposed on them (Ernest, 1991). Lerman (1983) illustrates how effective this context is by pointing out the fact that teachers in the same schools often adopt similar classroom practices in spite of having distinct beliefs about algebra and the way in which it should be taught. Since teachers of today are the (successful) products of yesterday's system and are imbued with its characteristics (Howson, Keitel and Kilpatrick, 1981), there is a strong possibility that their beliefs about the teaching and learning of algebra could have been fashioned by our restrictive assessment system and the learning theory of the behaviourists. However, Laridon (1993) points out that there is an increasing tendency among educators internationally to generate meaning through social interaction, investigations and the forging of links with the informal methods of the child. The principles stated in the proposed modular curriculum, also stress these aspects (Department of National Education, 1991). Thus the possibility exists that a constructivist approach to teaching and learning algebra could become the norm in the near future.

The social context of the classroom and its power relations also need to be considered (Ernest, 1991). In order for the constructivist mode of teaching algebra to succeed, the teacher, as a result of his on going interaction with each pupil, should be aware of the different individual insights, particularly with regard to their thinking processes, which may be rather diverse because of their different cultural and social backgrounds. Jaworski points out that

> "a teacher may have a clear, coherent mathematical story which she wishes to convey to her pupils, and she may provide them with a good explanation as she sees it. However, it is only good for pupils if it fits their experience and needs."
> (Jaworski, 1989: 291)

These factors strongly indicate that the social constructivists' emphasis on verbalisation, which helps students to clarify their ideas (Brodie, 1991), and negotiation could play a very important role in helping the pupils to develop a sound understanding of the many intricate concepts of algebra. This is particularly relevant in South Africa today as a number of black youths are assimilated in what was originally the unique preserve of white education. Some of these youngsters are experiencing severe adaptation problems because of the differing standards of the curricula (See 1.3.2) and the limitations of their comprehension of the spoken language (Brodie, 1991). This has been exacerbated by the fact that the teachers and the majority of their peers do not have the necessary historical and cultural insights that are essential for algebraic concepts to be described in a meaningful way. Furthermore, Ohuche (In Steffe et al, 1988) questions the advisability of using the open-ended investigation approach in societies which frown on children asking questions. Social interaction, however, should slowly overcome these drawbacks. All good teachers need to know what mathematical sense their pupils are making of the lesson. Feedback is essential in this regard and certainly one of the most effective methods of so doing is to encourage interaction amongst learners, which is one of the basic tenets of constructivism.

2.5.4 Teacher shortage

In South Africa we are currently experiencing a definite shortage of scientifically-literate people in our society. The tremendous shortage of teachers in these specialised fields, particularly in black education (Science and Technology Commission, 1991; Adler, 1992a; Alfreds 1995), means that the epistemology embodied in mathematical texts often determines the didactical approach to teaching and learning algebra. The extent to which teachers slavishly adhere to the order and the manner in which the work is presented also plays a vital role in the nature of the implemented curriculum (Ernest, 1991). The teachers' apparent lack of self-confidence, due in all probability to a poor foundation in algebra, is a definite obstacle to the implementation of a creative teaching style based on the philosophy of constructivism (Stoker, 1991).

2.6 Misconceptions

From a constructivist point of view, students' misconceptions are never arbitrary or altogether unreasonable. Misconceptions are seen as emerging from some interaction between experience and other existing concepts the student has. Misconceptions are crucially important to teaching and learning of algebra for at least two reasons:

"* misconceptions form part of the student's conceptual structure that will influence further learning, mostly in a negative way, because misconceptions generate mistakes.

* misconceptions are highly persistent and resistant to change through instruction. They are maintained by their ability to distort or reject incompatible information and by the support from other concepts in the student's conceptual structure."
(Olivier, 1989: 25)

The behaviourists, on the other hand, attach hardly any importance to pupils' misconceptions because they believe that the pupils' current concepts are not relevant to the learning process (Olvier, 1989a).

Researchers have become increasingly aware that many students' misconceptions have evolved from their narrowness of representing phenomena (Confrey, 1990). Thus this researcher hopes that the implementation of a constructivist approach to the teaching and learning of algebra as stated in 2.3 and 2.4.2, will reduce this phenomenon. This approach incorporates the recommendations of the Cockcroft Report (1982) by including the key elements of exposition, discussion, practical work, practice, problem solving and investigations.

Probably the greatest constraint facing the teacher is one of time. The nature of the task is such that individual attention is very limited. As a result teachers are not able to resolve all the inconsistencies that each student develops, because they are either not aware of them, or they do not have the opportunity to develop sufficient conflicting strategies to enable the student to overcome them

(Jaworski, 1989). This is particularly relevant from the naive constructivists' point of view as their students are forming intuitions all the t e. Some of these unknown intuitions could lead to preconceptions which prevent pupils from accepting the fact that it is possible for the product of two numbers to be less than one of the factors (Pateman, 1990). Discussion, communication, reflection and negotiation of meaning are essential tools in the constructivist's armament when it comes to resolving pupil's misconceptions (Olivier, 1989a).

Misconceptions that result from over generalisation, as a result of erroneous preconception, appear to be well documented in the literature. For example:

1) $\cos(a + b) = \cos a + \cos b$

$\log(e + f) = \log e + \log f$

$a(bc) = (ab)(ac)$

$\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$

results from an overgeneralisation of the distributive law (Olivier, 1989a).

2) $9t - 5t = 5t - 9t$

results from an overgeneralisation of the commutative law for addition (Olivier, 1989a).

3) $(x + y)^2 = x^2 + y^2$

comes about because squaring does not preserve addition (Griffiths, 1978). This is simply another way of stating that it is another example of the overgeneralisation of the distributive law (Olivier, 1989a).

4) $3.\frac{4}{9}p^2 = \frac{12}{9}.p^6$

results from an incorrect generalisation of the distributive law to exponents (Becker, 1988)

5) $8a.7a = 15a^2$
results from the definition of multiplication as repeated addition (Steenkamp, 1982).

In each of the above examples the pupil's existing knowledge has been a stumbling block in accommodating an extension of the algebraic concept. It is interesting to note that nearly fifty percent of all mathematical errors are caused by the interference of the pupil's existing framework of knowledge (Steenkamp, 1982). The behaviourists' approach to the teaching and learning of mathematics probably plays a significant role in the formation in this type of error. The emphasis on rote learning and the preoccupation of teachers with drill and practice on discrete parts of the curriculum have lead to belief systems being formed that are resilient to change (Olivier, 1989a). In order to overcome this situation it is necessary to change the beliefs and attitudes of teachers to teaching algebra. Certainly the constructivist approach with their emphasis on understanding and critical thinking together with a more coherent curriculum could go a long way to alleviating this type of misconception.

> "For the constructivist, a misconception is identified when a relatively stable and functional set of beliefs held by an individual comes into conflict with an alternative position held by the community of scholars, experts, and teachers as a whole." (Confrey, 1987 : 96)

The above quotation creates the impression that misconceptions will normally only be perceived by someone with greater experience, either during a dialogue with the student concerned or when the written work of the student is critically examined. This implies that teachers will play a crucial role in identifying misconceptions. The concepts that are adversely influencing the accommodation of the new idea must be identified before a strategy can be evolved that will lead to the resolution of the misconception (Olivier, 1989a).

The radical constructivists have emphasized the personal nature of the meanings that individuals construct through interaction with others and the environment. The manner in which these meanings are transferred from one individual to another is a potential source for the formation of misconceptions.

Social interaction as illustrated in figure 2.1 should ensure the gradual elimination of the problem. In addition those preconceptions that form the belief system of the individual could be resilient to the accommodation of new concepts, since the existing knowledge of the individual often serves as a filter and a catalyst to the acquisition of new ideas (Confrey, 1987). It should be further emphasised that children's conceptions and skills are affected by both the instructional environment as well as their cultural backgrounds. Hence, if the constructivist teacher is to assist the child in overcoming any learning difficulties in the most efficient manner it is vital for him to be aware of as many facets of the child's background as possible.

It should also be remembered that the linguistic skills and algebraic knowledge of the educator could play a significant role in either promoting or limiting the formation of misconceptions. Thus teacher training and the selection of the best candidates for the profession play indirect but important roles in the field of misconceptions.

An important aspect that the majority of educators seem to forget is the likelihood that students' inconsistencies could diminish with time. This is certainly possible, because relational understanding could result from the effective use of retrospective reflection (Padoa, 1987). The art of looking back has remained the exclusive domain of our most skilful problem solvers, surely an added incentive for all aspirant mathematicians (Taback, 1988). Thus pupils should be encouraged to reflect on previous algebraic concepts after a section of new work has been completed. This could be particularly effective in countries where the spiral approach has been adopted for the curriculum development of algebra. In this manner additional insights are often gained resulting in conceptual adjustments which will enable accommodation of new ideas to take place.

When a student reaches a point in carrying out the procedure at which he or she does not know what to do next, a repair or patch is made, often resulting in a computational error. These repairs can be thought of as on-the-spot, invented procedures based on the student's existing algebraic knowledge that are necessary in order to obtain the prerequisite answer for the teacher. Thus, it is clear that children do not simply absorb algebraic knowledge as it is presented, but impose their existing frameworks of knowledge to incorporate and invent new knowledge. Hence, the method of teaching and testing algebra has far reaching consequences in this area.

The fact that similar error patterns are found across different symbol systems, equations, tables, word sentences and pictures clearly demonstrates the pervasiveness of the 'misconception' (Confrey, 1987).

The elusive character of the misconception creates several problems when teachers attempt to prevent or remediate them. The instructional strategy, designed by Nussbaum and Novick, for eliciting student conceptions that they hope will lead to cognitive restructuring appears to be most promising in this regard. They suggest that the teacher should :

a) create a situation that requires students to invoke their conceptual frameworks;

b) ask them to describe their framework both verbally and pictorially;

c) assist them in stating their ideas clearly, avoiding evaluation;

d) support the most highly generalized solution;

e) have students debate the pros and cons; (Confrey, 1990). The social negotiation process (figure 2.1) that ensues during the debate should help students to reflect on their perception of the different algebraic concepts that they hold and on the mental processes that they have used to formulate them. Negative feedback often provides an avenue for developing a fit between the pupil's subjective knowledge of algebra and the socially accepted objective knowledge of algebra (Ernest, 1991). Thus the cognitive conflict that results from students expressing different and contradictory points of view could lead to the successful remediation of the misconception (Olivier, 1989; 1989a).

The implementation of this basic teaching framework, which is at the heart of constructivism, must of necessity be accompanied by comprehensive in-service teacher education which leads to a positive change in the beliefs of the individual teacher.

2.7 CONCLUSION

The literature review strongly suggests that the behaviourists are responsible for the narrow restrictive outlook of our existing system, which ensures that "pupils become anxious, develop failure-induced apathy and negative attitudes." (Glencross & Fridjhon, 1990 : 309) The fact that there has been a growth in the number of teachers who feel that " power in mathematics resides with those who have knowledge of process " (Rogers, 1990 : 43), implies that the old methods of rote learning and rule-following should become less important and methods based on the formation of concepts and the development of the stages of logical thought should receive greater eminence (Walkerdine, 1989).

The variety of teaching methods that have been advocated in the review appear to give the teacher more options in ensuring that the wealth of experience that the child brings to the classroom is utilised in order to promote greater understanding of the concept variable.

The argument that has been presented in the literature review clearly indicates that language and social negotiation are two essential components in generating a higher level of understanding of the different concepts of algebra. The review has shown that these twc aspects are not prominent features of the traditional approach of the absolutists to teaching and learning algebra. Hence there is a strong possibilty that students from our multi-cultural, multi-lingual country are more likely to increase their conceptual understanding of variable if they are exposed to the constructivists' pedagogy.

The literature review indicates that the adoption of a constructivist approach to teaching and learning of algebra could play a significant role in reducing the number of misconceptions that confront pupils on a regular basis, because the additional emphasis that is placed on understanding will create better opportunities for the pupil to make the necessary conceptual adjustment. Thus although constructivism will not eliminate all misconceptions, the genesis of knowledge based on understanding and certainly the emphasis placed on critical thinking can only enhance the quality of the educational process.

The emphasis placed by society on the results of the final matriculation examination and the fact that a teacher's expertise is often seen in the light of her students' marks, means that problem-solving and investigative approaches, which are creative activities and are hampered by time constraints, have not received the attention they deserved.

The knowledge explosion that has taken place has greatly reduced the significance of recall, which was the main focus of past examinations, and highlighted the importance of a person's ability to reason and to solve problems (Roby, 1990). The advent of calculators and computers has intensified the need for educationists to promote methodologies that will encourage the critical thinking and reflective practices that prevail in the constructivist classroom.

The evidence of this report strongly indicates that the emancipatory role of a problem posing pedagogy will lead to the development of higher level cognitive skills in the individual with enhanced beliefs of self worth and ability and more in-depth understanding (Adler, 1992; Ernest, 1991; Laridon, 1981; De Villiers, 1994; Von Glasersfeld, 1989).

RESEARCH METHODOLOGY

3.1 AIM OF RESEARCH

Sections 1.1 and 1.4 drew the reader's attention to the fact that the aim of this report was to highlight the need for an alternate approach to the teaching and learning of algebra in South African schools. In particular a constructivist approach, which was referred to in sections 1.4, 1.6, 2.2, 2.3 and 2.4, was recommended.

3.1.1 Research problem

The research problem that was mentioned in 1.4 and stated in 1.5 lead to an investigation that tried to determine whether a standard six pupil's level of understanding of the concept of variable, was affected by the implementation of teaching and learning methodologies that were based on the different philosophies of absolutism and constructivism.

3.1.2 Research questions

The research questions that evolved from this problem were categorised into principal and secondary questions.

3.1.2.1 The principal research question

3.1.2.1.1 Methodology

How did the implementation of an absolutist, as opposed to a constructivist, pedagogy affect the pupils' levels of understanding of the concept "variable"?

It was also decided to investigate the following sub-questions:

1) How did these teaching methodologies affect the academic performance of the genders?

2) Was the attitude of the pupils related to the treatment or the achievement of the genders in the experiment?

3.1.2.2 The secondary research question

3.1.2.2.1 Misconceptions

What effect did the teaching methodologies of the absolutists and the constructivists have on the resolution of misconceptions?

3.2 RESEARCH DESIGN AND METHODOLOGY

3.2.1 Description of research method

This research employed a quantitative and an experimental approach. The experiment was conducted at two English medium schools on the northern periphery of Johannesburg. The control school, which was chosen at random, followed an absolutist (traditional) type of methodology (See 2.3.1). The other school adopted a constructivist approach (See 2.3.2).

The researcher's decision to compare the results of the total standard 6 group of one school with the results obtained by the total standard 6 group of another school meant that it was not possible to equate the groups by means of random selection or random assignment or matching. The fact that a non-equivalent control group design is extensively used in educational research (Campbell and Stanley; 1967) is a clear indicator that this form of equivalence is seldom possible in the school situation. Thus, in order to minimise this deficiency and to counter the problem of intact classes, the researcher attempted to meet Kerlinger's (1981) other criterion by using samples that were as alike as possible.(See 3.2.4 and 3.2.5) This in turn would increase the effectiveness of controlling those factors that influence the internal validity of the experiment (Campbell et al, 1967).

3.2.2 The model

The approach was based on an extended non-equivalent, pretest-posttest, experimental model. The mathematical representation thereof is as follows: $G(Y_1;Y_2) = f(X_1:Z/Y_3)$ A paradigm of the design of the model and an explanation of the variables used, follows in 3.2.3

3.2.3 The design

The design that was chosen can be represented diagramatically as follows:

Nonequivalent Control Group Analysis of Covariance Paradigm

	METHODS	
	X_1	X_2
MALE	Y_3 / $Y_1;Y_2$	Y_3 / $Y_1;Y_2$
FEMALE	Y_3 / $Y_1;Y_2$	Y_3 / $Y_1;Y_2$

Y_3 = PRETEST LEVEL OF UNDERSTANDING
Y_1 = POSTTEST LEVEL OF UNDERSTANDING
Y_2 = POSTTEST OF ATTITUDE

X_1 - absolutist methodology (placebo intervention) at school 1
X_2 - constructivist methodology at school 2

Figure 3.1 The experimental design

Initially all the students in standard six at both schools wrote the knowledge of variable pretest Y_3 (Chelsea Diagnostic Mathematics Test for Algebra (CDMTA)) at the beginning of the first term in 1994. The different teaching and learning strategies, as mentioned in 3.2.1, were then utilised in order to help the pupils to generate a better understanding of the concept of a variable. This section of the curriculum was completed by the end of the first term.

The test-retest method has been criticised in the past because it is possible that students' responses to the second test could be influenced by their memory of
1) their responses to the first test;
2) the discussions that took place between the students and/or their teacher on items that had provoked their interest (Ebel, 1972).

In order to minimise these effects it was decided to let the students write the knowledge of variable posttest Y_1 at the commencement of the second term in 1994. In addition some of the numerals and some of the variables in the original CDMTA were altered in an attempt to provide a similar but equivalent test. Remmer's attitudinal test Y_2 was also written at the beginning of the second term. The results were noted and tabulated. The different variables that were monitored are reflected below:

Independent variable : X: Teaching method
Z: Gender
Dependent variable : Y_1: Achievement of pupil
Y_2: Attitude of pupil
Covariate : Y_3: Pre-test of pupil's achievement

3.2.4 Schools, students and classes

Both schools are situated on the northern periphery of Johannesburg. The pupils who attend these schools come predominantly from white, urban, middle class families. Girls tend to out-number boys as there is a tendency for some parents to send their sons to any one of a number of boys' schools which exist in the area. Approximately 10% of the standard 6 pupils, at both schools, come from previously disadvantaged groups. The two schools that were chosen to participate in this experiment are served by a number of feeder schools. Some of these schools send pupils to both schools.

This experiment was targeted at standard 6 pupils in the first term of the academic year. The optional subjects that pupils had to choose, were the same at both schools. Furthermore, both schools used the same computer programme for allocating pupils to classes. The actual placement of pupils into class groups was largely determined by the subject choice of the pupils. The control school which was chosen at random had six classes that varied in size as follows; 20 (20), 24 (22), 24 (24), 27 (25), 28 (28) and 31 (30). The experimental school had 8 classes that varied in size as follows; 22 (16), 24 (18), 24 (23), 24 (24), 25 (24), 26 (24), 27 (25), 29 (28). The figures in brackets reflect the number of pupils who wrote the pretest as well as the posttest. An age analysis of the pupils, as reflected below, also shows a remarkable similarity between the two groups.

	Age	Boys		Girls		Total	
		No	%	No	%	No	%
Control	14+	12	19	9	11	21	14
	13+	37	58	50	59	87	58
	12+	15	23	26	30	41	28
	Total	64		85		149	
Experimental	14+	10	15	11	9	21	12
	13+	39	60	68	58	107	59
	12+	16	25	38	32	54	29
	Total	65		117		182	

Table 3.1 : Breakdown of the samples by age and gender

The researcher believes that the data from the pre-test adds credence to his claim that a marked similarity existed between the two groups. This data, which is summarised in the table below, presents a comparison of the means and standard deviations of the pre-test scores obtained by the control and experimental groups. The table also reflects the minimum and maximum scores obtained by individuals in the respective groups.

Variable	Group	Mean	Std Dev	Minimum	Maximum
PRETEST (BOYS)	C	20.94	9.20	5	57
	E	22.55	9.34	1	58
PRETEST (GIRLS)	C	20.31	8.51	3	51
	E	22.81	9.19	6	53

Table 3.2 : Comparison of gender scores on the pretest

Furthermore no significant difference in attitude was found to exist between the control and the experimental groups. (See 4.1)

3.2.5 Teachers

The pupils at the control school were taught by the Head of Department and two other teachers of several years standing. The pupils at the experimental school were taught by four teachers. Three of these teachers had several years teaching experience. The other was a first year teacher.
In both cases time-table restraints determined the allocation of teachers to specific classes.

3.3 THE INSTRUMENTS

3.3.1 The Algebra Test

3.3.1.1 Background

The CDMTA was designed as a diagnostic instrument that would be used to ascertain a child's level of understanding and to identify misconceptions. (See Appendix A). The tests were developed by the mathematics team of the Social Science Research Council Programme 'Concepts in Secondary Mathematics and Science', which was based at the Centre for Science Education, Chelsea College, University of London, during the period 1974-79. The aim of this project was to identify a developmental hierarchy of understanding, connecting the algebraic concepts commonly taught in the secondary school, in order to provide a structure which would help teachers and curriculum designers to plan appropriate teaching materials, and to match them to individual children (Brown et al, 1984).

The tests were used with a large sample of secondary and middle school children (ten thousand in all) in the United Kingdom, mainly in the summers of 1976 and 1977. Prior to the paper and pencil format being used, a series of interviews were carried out on each topic. For each topic, about thirty children from a number of schools were interviewed on various items, and those items judged suitable, and which appeared to provide information on understanding, were incorporated in a class test format. The items were continually modified and reviewed during these trials (Brown et al, 1984).

The fact that these tests were piloted in Great Britain raises the possibility that a cultural bias may exist. However, the composition of society in Great Britain at that time closely asembles the composition of the samples used in this experiment. On the other hand, those pupils, who had previously attended schools that had fallen under the Department of Education and Training, could have been seriously disadvantaged, because they would not have concentrated on the same sections of work as their counterparts from model C schools, as they were following different curricula. This is accentuated by the fact that all curricula have essentially been developed by whites for whites (Laridon, 1990).(See 1.3.2). This fact, together with the academic nature of the curriculum and the eurocentric slant given to algebra by the teachers and textbooks, has tended to alienate pupils of other cultures (Maqsud and Khalique, 1990). The situation is further aggravated by the fact that a number of students are being educated through their second or third language. There can be little doubt that the historical background of education in South Africa has led to the development of a cultural bias in education.(See 1.3.2). However the researcher contends that any test developed by a person from a particular culture must be culturally biased. Furthermore, the possibility exists that this effect may have been controlled by the fact that the number of second language speakers in the two samples was approximately the same. In addition a comparison of their results did not reveal a particular trend as their performance levels varied.

3.3.1.2 Uses of the CDMTA

The CDMTA are diagnostic instruments designed to be used either before or after a substantial amount of the topic is presented. Used before the teaching, the results should provide a guide to the present level of understanding of each child, enabling the teacher to match the child's work to what the child knows. This application could form an integral part in a constructivist's approach to the teaching and learning of algebra. After the topic has been taught, the tests can be used in a similar way to assess the degree of understanding displayed by individual children (Brown et al, 1984).

Some of the individual items were monitored in an attempt to ascertain whether the possibility existed that one of the methodologies adopted could lead to a reduction in the number of pupils that experienced misconceptions.

3.3.1.3 The objective of the CDMTA.

The objective of the CDMTA is to assess children's levels of understanding across a broad range of typical secondary school algebra tasks. These include substitution, simplifying expressions, and constructing, interpreting and solving equations. The assessment focuses on the different ways in which children use and interpret variables in generalised arithmetic. In devising the test an attempt has been made to minimise the need for remembered techniques and conventions (Brown et al, 1984). For the purposes of this experiment the levels of attainment as well as the actual achievement of the pupil on both tests were recorded. It was felt that the latter set of results would facilitate what was already a very complex statistical procedure.

The test as stated in 3.3.1.1 was compiled by experts, and modified over a period of time. The researcher felt that the test could serve a dual purpose because:

1) the items that were included in the test had been carefully selected and the pupils' understanding of different aspects of the concept of variable, in the different subsections of algebra, as mentioned above, was fairly and comprehensively tested;

2) the distribution of the scores, as reflected in table 3.2, shows that the individual questions effectively discriminated between the pupils of different abilities;

3) the questions were graded and required a specific competence in the understanding of the concept of variable;

4) two experts in the field verified the fact that the test measures the extent of a pupil's perception of the concept of a variable;

5) several experts in the field obtained perfect scores.

6) the correlation coefficient (Pre-test correlated with

post-test scores), which denotes the reliablity of the test is r = 0.78. The fact that the pupils were taught aspects of the the work between the two tests could have inflated the value of the coefficient. However other work was also taught during this period of time, which could have served as a source of confusion. Furthermore the school holidays could have adversely affected the pupils' results.

A short series of practice items preceded the main test. Its purpose was to remind children of certain conventions - for example, that 4a means 4 x a, and to show that letters can be used to rep[illegible]umbers. The practice items should be completed [illegible] test itself is attempted, and the answers should b[illegible]sed with the whole class (Brown et al, 1984).

3.3.1.4 Marking of scripts.

The level of understanding of the concept of variable in algebra was determined according to the following criteria:

CRITERION	LEVEL
Less than 4 correct: 5a,6a,7b,8,9a,13a	0
4 or more correct: 5a,6a,7b,8,9a,13a	1
5 or more correct: 7c,9b,9c,11a,11b,13d,15a	2
5 or more correct: 4c,5c,9a,13b,13h,14,15b,16	3
6 or more correct: 3,4e,7d,13e,17a,18b,20,21,22	4

Table 3.3 : Criteria needed to attain different levels of understanding

Initially three teachers marked the scripts. The researcher moderated all the scripts in an attempt to control inter-rater reliablity. No bias was found. This was to be expected, because of the objective nature of the answers.

The results of tests in the class test format were scored by the researcher according to a fixed memorandum. (See Appendix D). Two marks were scored for correct solutions to those items that were included in the rows of table 3.3 that could have led to a level of understanding of 3 or 4. One mark was scored for a correct solution to all other items. The only exception to this rule was item 23. Two marks were awarded for a correct solution to this item as the researcher felt that considerable insight was required in order to determine the final answer. The maximum mark that could be obtained was 72. The data from this source are referred to as scores as opposed to levels.

3.3.1.5 Age of student to be tested.

The test was developed for use with children of all abilities within the second, third and fourth year of secondary school i.e. ages 12+ to 15+. It can also be used with older children (Brown et al, 1984).

The age analysis of the pupils presented in 3.2.4 shows that this criterion was met.

3.3.1.6 Time required to do the test.

The series of practice items was be worked through for 2 or 3 minutes, and the answers discussed for another 5 minutes. The test itself took 30 - 35 minutes.

3.3.2 The attitude scale.

The pupils were required to complete the questionnaire prepared by H H Remmers which he developed in 1954.(See Appendix C). There was no time limit. This instrument has two noteworthy features:

1) it is wide enough to measure attitudes in a number of disciplines,
2) each item has been given a scale value that reflects its degree of positiveness (Tuckman, 1978).

Remmer originally asked a large group of students to rate the items in the attitude test on a scale from 0 to 10. The levels of favourableness were then obtained by averaging the values of the responses (Tuckman, 1978). For the sake of convenience, and in order to obtain a score out of 100, Remmers' original scale values were multiplied by 10 by the researcher. These values are reflected in Appendix C. The attitude score was obtained by simply averaging the scale values of the items that were endorsed by the pupil.

The attitude test was primarily used as a device to help to establish the similarity of the two groups. The comparatively short time-span of the experiment meant that there was little likelihood that a change in pupils' attitude would occur as a result of the intervention. However, it was important, from a statisical point of view, to ascertain whether their preknowledge could influence the outcome of the experiment. This proved to be the case. (See tables 4.4, 4.5 & 4.6). The nett result was that analysis of covariance had to be implemented and not simply an analysis of variance.

There was a possibility that second language speakers could have experienced interpretive difficulties with one or two items of this questionnaire.

3.4 Continuity

Regular discussions with all the teachers of the experimental school and the contact person of the control school were held. These sessions were utilized in order to:
1) assist the teachers with any difficulties that they were experiencing with the programme.
2) ascertain whether the envisaged methodology as discussed in 1.4, 1.6, 2.2, 2.3 and 2.4 was being followed. In the case of the experimental schools the researcher was able to confirm that this was indeed the case as he was present in

the classroom on several occasions. In the case of the control school his discussion with the teachers at the conclusion of the experiment served to reaffirm his initial impression that the teachers concerned were indeed following a tradional approach to the teaching and learning of algebra.

3) provide the necessary guidance and support.

STATISTICAL ANALYSIS

4.1 The results

Initially the level of understanding that each pupil achieved in the pretest and posttest was recorded. The respective change in levels of understanding were then noted and tabulated as reflected below.

CHANGE IN LEVELS	-1		0		+1		+2	
GROUP	Ex	C	Ex	C	Ex	C	Ex	C
NUMBER	2	12	52	67	115	56	13	14
%	1.1	8.1	28.6	45.0	63.2	37.9	7.1	9.4

Table 4.1 : Breakdown of the change in the levels of understanding of the experimental and control groups

The table clearly depicts the following:

1) the percentage of pupils that regressed or remained static was considerably lower for the experimental group. This appears to confirm the fact that drill work (see 2.4.1) does not necessarily lead to an improvement in the students' understanding of the material. Furthermore, the fact that so few pupils excelled seems to dispel the myth that pupils have the capacity to absorb everything that they are told. (See 2.4.1)

2) the constructivist pedagogy enabled a greater percentage of pupils to raise their level of understanding of the concept of variable. These initial results seem to substantiate the researcher's belief that the extensive interaction that the constructivists methodology calls for, has positive implications for the teaching and learning of algebra.(See 2.7)

3) the greater number of pupils, who advanced two levels of understanding, came from the group that had experienced the traditional style of teaching. This appears to confirm the fact, as stated in 2.4.1, that confident students often thrive in the competitive atmosphere that is frequently

created in the traditional classroom.

In order to ensure that these results could be analysed according to acceptable statistical procedures, it was necessary to record the results of the pupils in a normal class test format by way of the scoring procedure described in 3.3.1.4. The data from the experimental and control groups with respect to the pretest, posttest, change in achievement and attitude test was initially subjected to an analysis of variance. Tables 4.2 and 4.3 summarise the results of the performance scores that were attained by the different gender groups.

Variable	No. Boys		Mean of Group	
	CON	EXP	CONTROL	EXPERIMENTAL
PRETEST	67	76	20.9402985	22.5526316
POSTTEST	64	65	26.0468750	32.0153846
CHANGE	64	65	5.03125	8.7692308
ATTITUDE	62	48	71.9032258	74.4791667

Table 4.2 : Comparison of the mean scores achieved by the boys in the experimental and control groups.

Variable	No. Girls		Mean of Group	
	CON	EXP	CONTROL	EXPERIMENTAL
PRETEST	88	123	20.3068182	22.8130081
POSTTEST	85	117	27.2235294	31.8547009
CHANGE	85	117	7.1764706	8.8974359
ATTITUDE	82	93	72.9756098	73.1290323

Table 4.3 : Comparison of the mean scores achieved by the girls in the experimental and control groups.

These tables clearly indicate that the scores of the pupils of different genders, in both the experimental and the control group improved appreciably. However the performance scores by the pupils in the experimental sample on the pre-test were marginally better. The fact that the pretest means of the two groups were slightly different, indicated the possibility that the preknowledge of the pupils in the two samples was not the same. Since this extraneous variable could not be controlled experimentally, it was necessary to implement an indirect control. Thus in order to eliminate the effects of student preknowledge a statistical procedure which provides an avenue for adjusting the posttest score in terms of the student's pretest score was necessary. If the analysis of covariance is applied to intact groups, then the precision of the experiment is actually increased by removing the bias that may result due to unmatched characteristics (Wildt and Ahtola, 1987). Thus this procedure was adopted in order to test the null hypotheses that were associated with the principal and its subsidiary questions.

4.2 The principal research question

The null hypotheses associated with the principal research question with the posttest as the dependent variable were:

H_0) There is no difference in the posttest performance of the students, who experienced different teaching methodologies, taking the pretest as a covariate.

H_0) There is no difference in the post test performance of the genders, taking the pretest as a covariate.

The results of the analysis of covariance are reflected in table 4.4

SOURCE	DF	F VALUE	P
GROUP	1 ; 326	20.69	0.0001
GENDER	1 ; 326	2.20	0.1393
GROUP*GENDER	1 ; 326	1.88	0.1714
PRETEST	1 ; 326	478.40	0.0001

Table 4.4 : Analysis of covariance with posttest as dependent variable.

The analysis of covariance which effectively eliminated the disparity shown up by the pretest clearly rejected the first hypothesis. Although both groups improved (See table 4.3), table 4.4 clearly shows that the improvement for the experimental group was greater and statistically significantly so, for the experimental group. ($F_{1.326}$= 20.69, $p < 0.0001$)

Table 4.4 also shows that the second hypothesis was not rejected by the method of analysis. Both genders actually gained a better understanding of the concept being tested and there was no statistical significance in the differences in performance on the posttest between the genders. These results are in line with the sentiments that were expressed after the Potsdam Mathematics Project. (See 1.3.2)

The null hypotheses associated with the principal research question with attitude as the dependent variable were:

H_0) There is no difference in the attitudes of the pupils, who experienced different teaching methodologies, taking the pretest as a covariate.

H_0) There is no difference in the attitude of the genders, taking the pretest as a covariate.

The results of the analysis of covariance are reflected in table 4.5

SOURCE	DF	F VALUE	P
GROUP	1 ; 280	0.20	0.6526
GENDER	1 ; 280	0.01	0.9422
GROUP*GENDER	1 ; 280	1.41	0.2358
PRETEST	1 ; 280	15.53	0.0001

Table 4.5: Analysis of covariance with attitude as dependent variable

Table 4.5 shows that both hypotheses were not rejected, that is, there was no difference between the comparison groups, the genders, or the interaction of gender on the treatment. This clearly indicated that the attitude of the pupils to mathematics was not related to either the treatment or the genders. These results confirm the expectations of the researcher as stated in 3.3.2.

The results depicted above indicated that the understanding of the variable concept as portrayed by the scores attained on the CDMTA was significantly better for the experimental group as opposed to the control group on the posttest. Furthermore the pedagogic approach of the constructivists was statistically more successful with both genders. This in addition was highlighted by the fact that there was no significant difference in the achievements of the different gender groups.

4.3 Exploration of the frequency of misconceptions.

In an attempt to resolve the research question as stated in 1.5.2, the researcher decided to analyse the results of a number of items of the CDMTA.

Question 4b	Number Before	Number After	Number Resolved
Experimental (140) Control (149)	56 41	40 31	16 10
Experimental % Control %	40.0 27.5	28.6 20.8	11.4 6.7

Table 4.6: No of pupils resolving the misconception in item 4b

In question 4b students were told that 4 added to n can be writtten as n + 4. They were then asked to add 4 onto n + 5. A number of pupils gave the answer 9. In this instance the variable was entirely ignored. The results, as depicted in the table 4.6, show that 6.7% of the pupils who experienced this misconception in the control group were able to resolve it, whereas 11.4% of the experimental group were able to do so.

In question 4f the pupils were asked to multiply 3n by 4 if n multiplied by 4 can be written as 4n. In this instance the pupils often gave 12 as the solution. The results are shown in table 4.7.

Question 4f	Nmber Before	Number After	Number Resolved
Experimental (140)	54	32	22
Control (149)	50	38	12
Experimental %	38.6	22.8	15.8
Control %	33.6	25.5	8.1

Table 4.7: No of pupils resolving the misconception in item 4f

The misconception in this instance is basically the same as that in question 4b as the variable is being ignored. The experimental group experienced a 15.8% resolution of the misconception as opposed to the control group's 8.1%. The results of these fairly elementary questions strongly indicated that the constructivist pedagogy as stated in 1.4, 1.6, 2.2, 2.3 and 2.4 could be very beneficial in overcoming this type of misconception.

In question 4c students were told that 4 added to n can be writtten as n + 4. They were then asked to add 4 onto 3n. A number of pupils gave the answer 7 or 7n. In this instance the variable is either ignored or no meaning is associated with it. The results, as depicted in table 4.8, show that 3.4% of the pupils who experienced this misconception in the control group were able to resolve it, whereas only 2,9% of the experimental group were able to do so.

Question 4c	Number Before	Number After	Number Resolved
Experimental (140) Control (149)	63 66	59 61	4 5
Experimental % Control %	45 44.3	42.1 40.9	2.9 3.4

Table 4.8: No of pupils resolving the misconception in item 4c

In question 4e the pupils were asked to multiply n+5 by 4 if n multiplied by 4 can be written as 4n. In this instance the pupils often gave n+20 or simply 20 as the solution. The results are shown in table 4.9. The misconception in this instance is basically the same as that in question 4c. The experimental group experienced a 3.5% resolution of the misconception as opposed to the control group's 2.7%. Clearly these results were inconclusive and the misconception proved to be persistent. This possibility was mentioned in 2.6.

Question 4e	Number Before	Number After	Number Resolved
Experimental (140) Control (149)	72 66	67 62	5 4
Experimental % Control %	51.4 44.3	47.9 41.6	3.5 2.7

Table 4.9: No of pupils resolving the misconception in item 4e

In question 5c the pupils were asked to find the value of e+f+g if e + f = 8. A large number of respondents evaluated the variable and gave the solution as 15. Table 4.10 shows that 20% of the pupils in the experimental group, who experienced this particlar misconception were able to resolve it as opposed to 15.4% of the control group. This clearly indicates that the constructivist approach to teaching and learning algebra as advocated in 1.6, 2.2.2, 2.3 and 2.4.2 could play a role in reducing this type of misconception.

Question 5c	Number Before	Number After	Number Resolved
Experimental (140) Control (149)	71 75	43 52	28 23
Experimental % Control %	50.7 50.3	30.7 34.9	20.0 15.4

Table 4.10: No of pupils resolving the misconception in item 5c

In question 9 the pupils were told that if the length of one of the sides of a square was g then the perimeter of the square could be written as P = 4g

In question 9b the pupils were asked to find the perimeter of a five sided figure. Four of the five sides have a length of h and one side a length of t. A number of students gave the ambiguous answer of 4ht as the solution as a result of premature closure. Table 4.11 indicates the results. This malady was also prevalent in questions 4c and 4f. 20% of the pupils in the experimental group, who experienced this misconception were able to resolve it as opposed to 14,8% of the control group. Once again the type of instruction appears to play an important role in the resolution of this misconception.

Question 9b	Number Before	Number After	Number Resolved
Experimental (140) Control (149)	58 69	30 47	28 22
Experimental % Control %	41.4 46.3	21.4 31.5	20.0 14.8

Table 4.11: No of pupils resolving the misconception in item 9b

In question 9c the pupils were asked to find the perimeter of five sided figure with sides of length u, u, 5, 5 and 6. The result of premature closure as well as the tendency to ignore the variable were illustrated by answers such as 2u556 or 16. Table 4.12 illustrates this phenomenon. 18,6% of the pupils in the experimental group who experienced this misconception were able to resolve their difficulty whereas only 14% of the control group were able to do so.

Question 9c	Number Before	Number After	Number Resolved
Experimental (140)	63	37	26
Control (149)	78	57	21
Experimental %	45	26.4	18.6
Control %	52.3	38.3	14.0

Table 4.12: No of pupils resolving the misconception in item 9c

The statistical work done certainly does not indicate whether the outcomes were statistically significant or not. However, the results to the above mentioned items seem to indicate that the constructivist pedagogy as mentioned in 1.4, 1.6, 2.2, 2.3 and 2.4 could provide a fruitful way of overcoming a number of misconceptions that currently beset a number of our students.

THE CONCLUSION

5.1 Conclusions

5.1.1 Methodology

The absolutists' approach to the teaching and learning of algebra was elaborated in sections 1.2.1; 2.2.1; 2.3 and 2.4.1. The results attained in 4.1 and 4.2 clearly indicate how ineffective this type of pedagogy was. The creative teaching and learning approach to algebra, that was advocated in sections 1.2.2; 2.2.2; 2.3 and 2.4.2 of this report encouraged teachers of algebra to make their classrooms

> "places where originality, independent and creative thinking and imagination are valued." (Isaacson, 1990 : 226)

The nature of the constructivist lesson induces the pupil to think in a creative way, thereby making him more receptive to the subject (Van den Berg and De Vaal, 1977) and increases the likelihood that the student will experience a greater in-depth knowledge of algebra. The significance of the pedagogy was illustrated by the results of the experiment in 4.1 and 4.2 which clearly showed that the students who experienced the constructivist approach to teaching and learning algebra improved statistically significantly more ($p < 0.0001$) than those students who had been exposed to the more traditional style of teaching and learning algebra.

The results that were obtained in chapter 4 appear to vindicate the researcher's belief that a philosophy of mathematics that is based on a fallibillist epistemology will enable students to obtain a better understanding of the concept of variable, thereby providing them with a better foundation or framework from which they can develop the more advanced concepts of algebra. The results in chapter 4 clearly indicate that the processes of verbalisation, social negotiation and the various roles adopted by the teachers (See 2.2, 2.3 and 2.4) contributed significantly to enhancing the pupils' conceptual understanding of algebra.

5.1.2 Misconceptions

The constructivist approach to teaching and learning algebra, that was advocated in sections 1.2.2; 2.2.2; 2.3 and 2.4.2 of this report, and in particular, the emphasis that the social constructivists place on social interaction and the fact that conflicting points of view cause pupils to reflect and re-examine their previosly held beliefs could have played an important part in reducing the number of misconceptions (See 2.6) that are experienced by pupils. This is substantiated by the frequency analysis in 4.3 which indicates that the constructivist approach to teaching and learning algebra will lead to fewer pupils experiencing misconceptions.

5.2 Recommendations

The importance of the results in 4.1 and 4.2 is highlighted by the fact that the recent information and technological explosion has reduced the significance of recall, which was the main focus of past examinations, and emphasised the importance of a pupil's ability to reason, to solve problems, and the need to successfully demonstrate both practical and oral skills (Roby, 1990). Thus, in future the ability to locate and utilise information in order to be able to solve problems will be vital (Glencross, 1991). These emerging needs of a rapidly changing society imply that it is:

1) **essential to broaden and redefine the aims and objectives of the mathematics curriculum** (see 1.3.2.1 and 2. .1) in order to accommodate the ever expanding frontiers of knowledge and the associated advances in technology (Glencross, 1991).

2) crucial to produce flexible, critical thinkers. The fact that past examinations (See 1.3) have stressed a student's ability to learn and to recall facts to the detriment of critical thinking and problem-solving skills (Riding, 1991) means that the **adoption of a constructivist philosophy must inevitably be accompanied by a form of assessment that leads to curriculum development that promotes the needs of all groups as well as enlightened teaching and learning strategies.**

3) essential to nurture and develop the skills of the teacher. This is vital because the teacher is the most flexible learning resource of all (Little, 1988). The critical shortage of teachers and the social constraints that they are subjected to (See 2.5.3 and 2.5.4), means that it is imperative to **implement in-service courses that boost the expertise, self belief and confidence of the teacher.** These courses and the interaction that the teachers have with their colleagues and pupils should **generate a positive attitude towards the constructivist pedagogy.** This is crucial to the ultimate success of the constructivists in their approach to teaching and learning algebra.

4) **important to create a culture of awareness amongst teachers.** Inservice courses, social interaction and a willingness to read educational literature could help to keep the enthusiastic teacher informed about the latest innovations that are taking place in the field of teaching and learning algebra. This process could counteract the restrictive influence of a number of mathematical text

books.

5) **important to reduce the emphasis on content.** It is also necessary to ensure that the content becomes more relevant to the daily lives of the students and that it also meets the needs of our emerging nation. The focus of education should shift from manipulation to mathematical modelling (see 2.4.1) and interpretation skills. This could help to maximise the the creative potential of both the student and the teacher by stressing the importance of human ingenuity and understanding.

5.3 Directions for future research

Ouche's reservations, as stated in 2.5.4, about the appropriateness of an open-investigative approach to teaching and learning algebra in those societies that frown on children asking questions is pertinent to South Africa. This implies that the present study should be extended to those cultural groups in South Africa where that trait exists.

The results of this report could be supplemented and enhanced by means of a qualitative study. This type of study could be used to pinpoint the mechanisms that pupils use to understand the different concepts of algebra as well as highlighting those approaches which enable pupils to forge effective links between their informal and formal knowledge.

A constructivist approach to the teaching and learning of other sections of the curriculum, such as geometry and trigonometry, could also be studied by either a qualitative or quantitative approach. These studies could provide teachers with a more substantial basis for making decisions about the most effective way of assisting pupils to realise their full potential in the sphere of mathematics.

A qualitative approach to misconceptions could help to pinpoint the conceptual reasoning processes that are actually used by an individual to rectify or resolve the misconception. This could provide educators with valuable insight.

The constructivist approach to teaching and learning algebra emphasises the important role that language plays in the formation of concepts. This report could be effectively extended to the numerous multi-lingual, multi-cultural classrooms that prevail in South Africa. Furthermore it could be illuminating to find out, how effective the constructivist approach is in a classroom that is being taught by a teacher using his second or third language.

Appendix A: Pretest

1. Fill in the gaps:

$x \longrightarrow x + 2$ $x \longrightarrow 4x$

$6 \longrightarrow$ .... $3 \longrightarrow$ ....

$r \longrightarrow$ ....

2. Write down the smallest and the largest of these: smallest largest

$n + 1,$ $n + 4,$ $n - 3,$ $n,$ $n - 7$ ....

3. Which is the larger, $2n$ or $n + 2$? .... 3(

Explain: ....

4. 4 added to n can be written as $n + 4$. Add 4 onto each of these:

8 $n + 5$ $3n$ 4(c)

.... 4(e

n multiplied by 4 can be written as $4n$. Multiply each of these by 4:

8 $n + 5$ $3n$

....

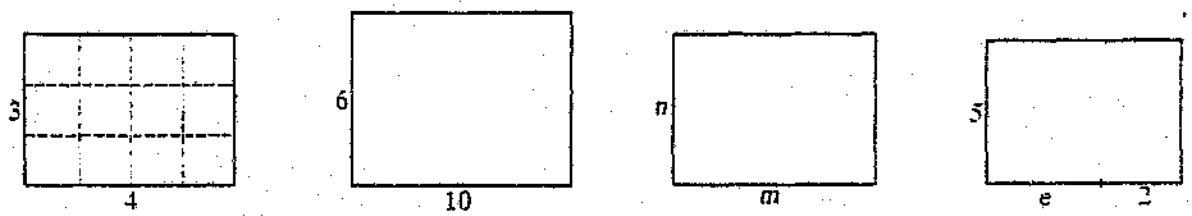
5. If $a + b = 43$ If $n - 246 = 762$ If $e + f = 8$ 5(a

$a + b + 2 =$ .... $n - 247 =$ .... $e + f + g =$ .... 5(c)

6. What can you say about a if $a + 5 = 8$ .... 6(a

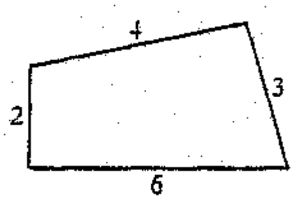
What can you say about b if $b + 2$ is equal to $2b$ ....

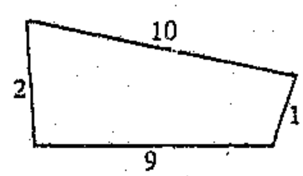
7. What are the areas of these shapes?



A = ... A = ... A = ... A = ...

7(a 7(b 7(c

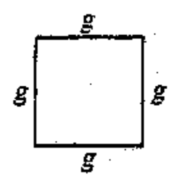




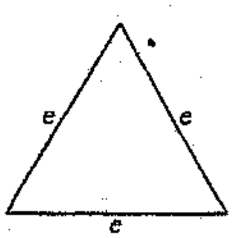
8. The perimeter of this shape is equal to 6 + 3 + 4 + 2, which equals 15.

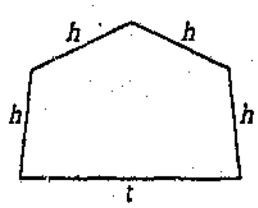
Work out the perimeter of this shape. P = ...

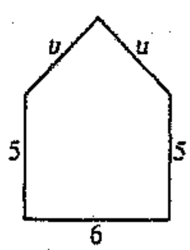
9. This square has sides of length g. So, for its perimeter, we can write $P = 4g$.

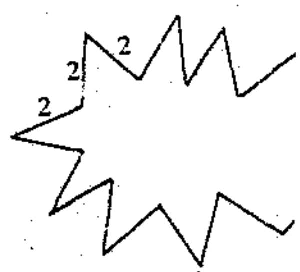


What can we write for the perimeter of each of these shapes?









Part of this figure is not drawn. There are n sides altogether, all of length 2.

P = ... P = ... P = ... P = ...

9(a 9(b 9(c 9(d

10. Cabbages cost 8 pence each and turnips cost 6 pence each.

If c stands for the number of cabbages bought
and t stands for the number of turnips bought,
what does $8c + 6t$ stand for? ....

What is the total number of vegetables bought? ....

11. What can you say about u if $u = v + 3$
and $v = 1$ .... 11(a

What can you say about m if $m = 3n + 1$
and $n = 4$ .... 11(b

12. If John has J marbles and Peter has P marbles, what could you write for the number of marbles they have altogether? ....

13. $a + 3a$ can be written more simply as $4a$.

Write these more simply, where possible:

$2a + 5a =$ .... 13(a

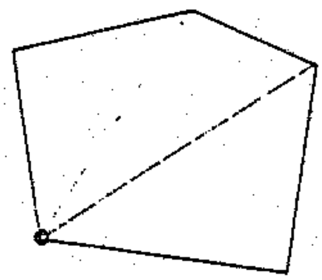
$2a + 5b =$ .... $3a - (b + a) =$ .... 13(b

$(a + b) + a =$ .... $a + 4 + a - 4 =$ .... 13(d

$2a + 5b + a =$ .... $3a - b + a =$ .... 13(e

$(a - b) + b =$ .... $(a + b) + (a - b) =$ .... 13(h

14. What can you say about r if $r = s + t$
and $r + s + t = 30$ 1



15. In a shape like this
you can work out the number of diagonals by taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals:

a shape with 57 sides has .... diagonals: 15(a)

a shape with k sides has .... diagonals. 15(b)

16. What can you say about c if $c + d = 10$
and c is less than d 16

17. Mary's basic wage is £20 per week.
She is also paid another £2 for each hour of overtime that she works.

If h stands for the number of hours of overtime that she works, and if W stands for her total wage (in £s), write down an equation connecting W and h: .... 17(a)

What would Mary's total wage be if she worked 4 hours of overtime? ....

18. When are the following true – always, never, or sometimes?
Underline the correct answer:

$A + B + C = C + A + B$ Always Never Sometimes, when ....

$L + M + N = L + P + N$ Always Never Sometimes, when .... 18(b)

19. $a = b + 3$. What happens to a if b is increased by 2? ....

$f = 3g + 1$. What happens to f if g is increased by 2? ....

20. Cakes cost c pence each and buns cost b pence each.
If I buy 4 cakes and 3 buns,
what doe $4c + 3b$ stand for? ....

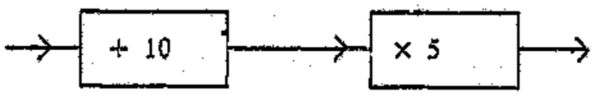
21. If this equation is true when $x = 6$, $(x + 1)^3 + x = 349$

then
what value of x
will make this equation true? $(5x + 1)^3 + 5x = 349$

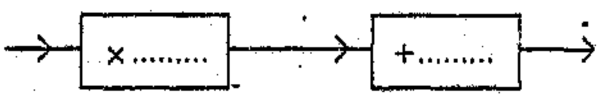
x = ...

22. Blue pencils cost 5 pence each and red pencils cost 6 penc, each.
I buy some blue and some red pencils and altogether it costs me 90 pence.
If b is the number of blue pencils bought, and
if r is the number of red pencils bought,
what can you write down about b and r? ....

23. You can feed any number into this machine:



Can you find another machine that has the same overall effect?



Practice Item 1

1. What number does $a + 4$ stand for if $a = 2$ ....
   if $a = 5$ ....

   What number does $4a$ stand for if $a = 2$ ....
   if $a = 5$ ....

Practice Item 2

2. Fill in the gaps:
   Work down the page

$x \longrightarrow 3x$	$x \longrightarrow x + 3$	$x \longrightarrow 7x$	$x \longrightarrow x + 8$
$2 \longrightarrow 6$	$5 \longrightarrow 8$	$2 \longrightarrow$	$3 \longrightarrow$ ....
$5 \longrightarrow$	$4 \longrightarrow$		
	$n \longrightarrow$		

Appendix B: Posttest

---

1. Fill in the gaps:

x ---->	x + 7	x ---->	3x
2 ---->	......	4 ---->	......
r ---->	......		

---

2. Write down the smallest and the largest of these:

	smallest	largest
n + 1 , n + 5 , n - 3 , n , n - 8	....	....

---

3. Which is the larger, 2n or n + 2 ? ....

Explain:..

....

---

4. **4 added to n** can be written as n + 4.

Add 4 onto each of these: 9 n + 3 5n

....

n multiplied by 4 can be written as 4n.

Multiply each of these by 4: 9 n + 3 5n

....

---

5. If a + b = 23 If n - 246 = 962 If f + g = 10

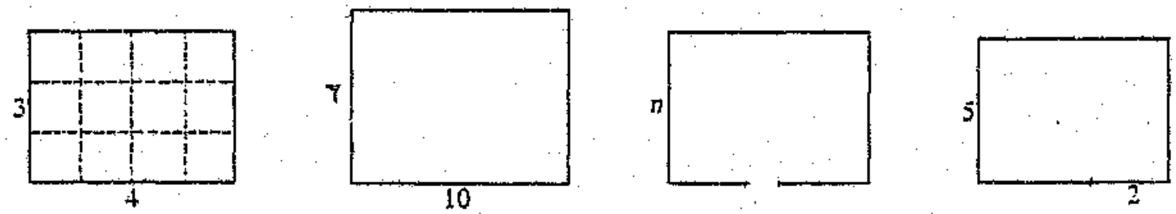
a + b + 2 = ... n - 247 = ... f + g + h = ...

---

6. What can you say about a if a + 4 = 9 ....

What can you say about b if b + 3 = 3b...............

7. What are the areas of these shapes?

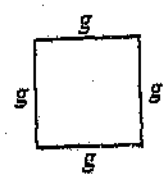


A = ... A = ... A = ... A = ...

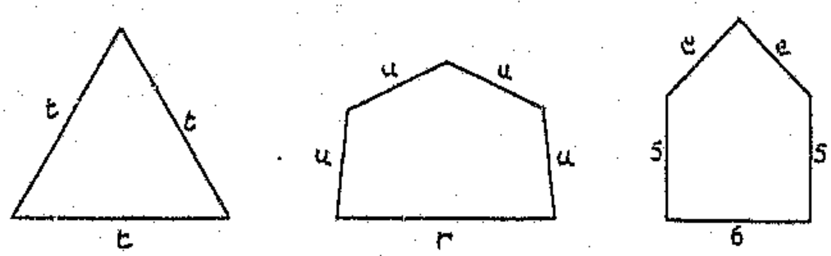
8. The perimeter of this shape is equal to 6 + 3 + 4 + 2, which equals 15.

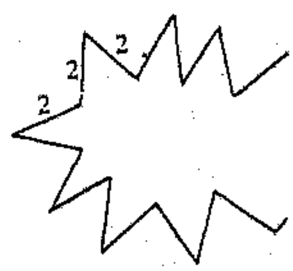
Work out the perimeter of this shape. P =.....

9. This square has sides of length g.
So, for its perimeter, we can write P = 4g.



What can we write for the perimeter of each of these shapes?





Part of this figure is not drawn. There are n sides altogether, all of length 2.

P = ... P = ... P = ... P = ...

10. Cabbages cost 6 cents each and turnips cost 8 cents each.

If c stands for the **number** of cabbages bought
and t stands for the **number** of turnips bought,
what does 6c + 8t stand for? ....

What is the total number of vegetables bought?................

---

11. What can you say about u if u = v + 8
and v = 1 ....

What can you say about m if m = 3n + 2
and n = 4 ....

---

12. If John has K marbles and Peter has H marbles, what could you write for the number of marbles they have altogether?

....

---

13. a + 3a can be written more simply as 4a.
Write these more simply, where possible:

3a + 5a = ...

3a + 5b = ...   2a − (b + a) = ...

(a + b) + a = ...   a + 6 + a − 6 = ...

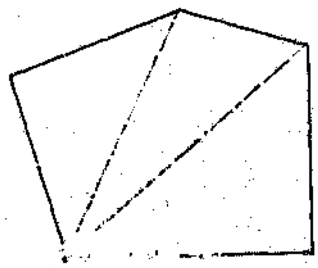
3a + 5b + a = ...   2a − b + a = ...

(a − b) + b = ...   (a + b) + (a − b) = ...

---

14. What can you say about r if r = s + t
and r + s + t = 50

....



15. In a shape like this
you can work out the number of diagonals by
taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals;

a shape with 54 sides has .... diagonals;

a shape with h sides has .... diagonals.

---

16. What can you say about c if c + d = 12
and c is less than d ....

---

17. Mary's basic wage is R30 per week.
She is also paid another R3 for each hour of overtime that she works.

If h stands for the number of hours of overtime that she works, and
if W stands for her **total** wages (in Rs),
write down an equation connecting W and h:........................

What would Mary's total wage be if she
worked 4 hours of overtime? ....

---

18. When are the following true - always, never, or sometimes?
Underline the correct answer:

L + M + N = N + L + M Always Never Sometimes, when.........

A + B + C = A + D + C Always Never Sometimes, when.........

19. $a = b + 2$. What happens to a if b is increased by 3?

....

$f = 2g + 3$. What happens to f if g is increased by 3?

....

---

20. Cakes cost c cents each and buns cost b cents each.
If I buy 5 cakes and 4 buns,
what does $5c + 4b$ stand for? ....

---

21. If this equation $(x + 1)^3 + x = 349$
is true when $x = 6$,
then
what values of x
will make this equation $(5x + 1)^3 + 5x = 349$
true?

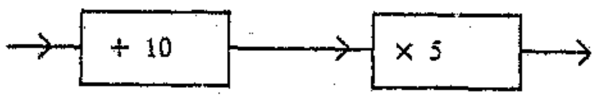
$x =$ ....

---

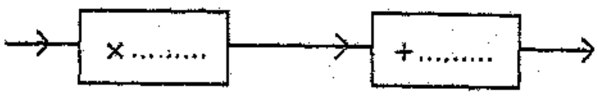
22. Blue pencils cost 5 cents each and red pencils cost 8 cents each.
I buy some blue and some red pencils and altogether it costs me 90 cents.
If b is the number of blue pencils bought, and
if r is the number of red pencils bought,
what can you write down about b and r ? ....

---

23. You can feed any number into this machine:



Can you find another machine that has the same overall effect?



Appendix C: Attitude Test

A SCALE TO MEASURE ATTITUDE TOWARD SCHOOL SUBJECT

NAME ________________ **DATE** ________________

AGE ________________ **SEX** (circle one) M F

GRADE ________________

Directions: Following is a list of statements about Mathematics. Put a tick (✓) before each statement with which you agree.

	1. No matter what happens, Mathematics always comes first.	103
	2. Mathematics has an irresistible attraction for me.	96
	3. Mathematics is profitable to everybody who takes it.	92
	4. Any student who takes Mathematics is bound to benefit.	89
	5. Mathematics is a good subject.	85
	6. All lessons and all methods used in Mathematics are clear and definite.	81
	7. I am willing to spend my time studying Mathematics.	77
	8. Mathematics is a good pastime.	65
	9. I don't believe Mathematics will do anybody any harm.	60
	10. I haven't any definite likes or dislikes for Mathematics	55
	11. Mathematics will benefit only the brighter students.	47
	12. My parents never did Mathematics, so I see no merit in it.	36
	13. I am not interested in Mathematics.	31
	14. Mathematics reminds me of Shakespeare's play - "Much Ado about Nothing."	26
	15. I would not advise anyone to take Mathematics.	22
	16. Mathematics is a waste of time.	16
	17. I look forward to Mathematics with horror.	10

Adapted from the questionnaire compiled by H H Remmers that appears in Tuckman (1978).

The scale values that are reflected in the extreme right hand column do not appear on the respondents form. Remmer used a 10 point scale. I have multiplied those values by 10. To score the responses of the individual student, simply average the scale values of all the items with which the student agrees.

Appendix D: Algebra marking key

Chelsea Diagnostic Mathematics Test: Algebra: Marking Key

Q	Solution: Pretest	Solution: Posttest	Mark
1a	8	9	1
1b	r+2	r+7	1
1c	12	12	1
2a	n-7	n-8	1
2b	n+4	n+5	1
3	If n=2 or 1 or 0 then n+2 > 2n	If n=2 or 1 or 0 then n+2 > 2n	2
4a	12	13	1
4b	n+9	n+7	1
4c	3n+4	5n+4	2
4d	32	36	1
4e	4n+20 or 4(n+5)	4n+12 or 4(n+3)	2
4f	12n	20n	1
5a	45	25	1
5b	761	961	1
5c	8+g	10+h	2
6a	3	5	1
6b	2	3/2	1
7a	12	12	1
7b	60	70	1
7c	nm	nm	1
7d	5e+10 or 5(e+2)	5c+10 or 5(c+2)	2
8	22	22	1
9a	3e or e+e+e	3t or t+t+t	1
9b	4h+t or h+h+h+h+t	4u+r or u+u+u+u+r	1
9c	2u+16 or 2.u+2.5+6	2e+16 or 2.e+2.5+6	1
9d	2n	2n	2
10a	cost (of c cabbages and turnips)	cost of the vegetables	1
10b	c+t	c+t	1

Q	Solution: Pretest	Solution: Posttest	Mark
11a	4	9	1
11b	13	14	1
12	J+P	K+H	1
13a	7a	8a	1
13b	2a+5b	3a+5b	2
13c	2a+b	2a+b	1
13d	3a+5b	4a+5b	1
13e	a	a	2
13f	2a-b	a-b	1
13g	2a	2a	1
13h	4a-b	3a-b	2
13i	2a or a+a	2a or a+a	1
14	15 or r = 30-(s+t)	25 or r = 50-(s+t)	2
15a	54	51	1
15b	k-3	h-3	2
16	c<5	c<6	2
17a	W = 20+2h	W = 30+3h	2
17b	28	42	1
18a	always	always	1
18b	sometimes, when M=P	sometimes, when B=D	2
19a	a increases by 2	a increases by 3	1
19b	f increases by 6	f increases by 6	1
20	the cost	the cost	2
21	6/5 or 1,2	6/5 or 1,2	2
22	5h + 6r = 90	5b + 8r = 90	2
23	x 5 + 50	x 5 + 50	2

Total Number of Marks: 72

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