Understanding the transformation of knowledge-building during online lessons: An analysis of online teaching material related to Grade 12 Financial Mathematics using Legitimation Code Theory.

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Abstract:

Education institutions in South Africa and around the world had to close their doors during the lock down in response to Covid-19. As a result of this, many educational institutions proactively engaged in remote teaching to promote a culture of teaching and learning even though learners were not in a physical classroom space. This study aims to comprehend the formation and variation of cumulative knowledge that may occur during distance teaching. To accomplish this, a series of online lessons on the topic of annuities is analysed focusing exclusively on semantic density dimension of Legitimation Code Theory (LCT).

LCT is a sociological framework developed by professors Karl Maton and Susan Hood. It aims to understand and analyse the ways in which knowledge is legitimized within different social fields, such as education. Semantic Density within LCT refers to one of the key dimensions used to analyse knowledge practices. It focuses on the degree of complexity and abstractness in the language and symbols used to convey knowledge within a particular field.

The online teacher (presenter) focuses on annuities in financial mathematics through six lessons where the knowledge is revised and applied in different contexts. This study focused on the analysis of the online lesson transcripts, using a translation device that focuses on the semantic density of the series of six open-source online revision sessions. The outcome of the analysis is plotted in a graphical representation that visually describes the cumulative knowledge building during each lesson.

The semantic profiles for each video of the online lessons illustrates the transformation of cumulative knowledge building that has been achieved. The analysis of the online lessons using semantic density have further indicated that the complexity of lessons does not necessarily decrease or limit the understanding of the pedagogy, in this case, financial mathematics. The examples in the online videos moved from simple to complex and demonstrated a link from one example to the next example.

Declaration:

I, **Anashree Naidoo**, hereby declare that this research report is my own work. It has been submitted exclusively to the University of the Witwatersrand in fulfilment of the requirements for the Master of Education Degree.

The school of the education ethics committee, constituted under the university human research ethics committee (non-medical), has approved this research project unconditionally on the issuing of a clearance of certificate, protocol number 2020ECE073H.

Signature: ____________________

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Acronyms:

A – Accumulated value

CAPS – Curriculum Assessment Policy Statement

GET – General Education and Training

FET – Further Education and Training

FV – Future Value

KCS – Knowledge of Content and Students

KCT – Knowledge of Content and Teaching

LCT – Legitimation Code Theory

MKT – Mathematics Knowledge for Teaching

NSC - National Senior Certificate

NTTL – Non-traditional teaching and learning

P – Principal value

PCK – Pedagogical Content Knowledge

PTSs - Professional Teaching Standards

PV – Present Value

SACE – South African Council of Educators

SD – Semantic Density

SG – Semantic Gravity

SMK – Subject Matter Knowledge

Acknowledgement:

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A special thank you is expressed to the WITS LCT Knowledge Building Research Team that is led by Professor Lee Rusznyak. LCT is indeed a new concept and I have learnt a lot in the initial stages of understanding.

To my wonderful husband, Thivash Naidoo, you are blessing for always wanting me to be the best version of myself. Thank you for all your hard work and sacrifice as this was being done.

To my amazing family, friends, and colleagues, I dedicate this to you, especially to my dad, Dhanpal Govind Singh. My parents always stressed the importance of education. It saddens me that my dad is no longer here to witness this accomplishment.

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Chapter 1: Introduction

As educators our key objective is to build on the cumulative knowledge of any mathematical concept or topic to build sound learner understanding and enable them to engage in reflection and thorough analysis of key concepts they engage with. The primary focus of this research was to analyse knowledge building that occurs through an in-depth analysis of a series of open-source online lessons focused on financial mathematics. This is a section in the CAPS (2010) curriculum that teachers initially found challenging to teach conceptually (Makonye, Weitz, & Parshotam, 2014). Understanding difficult conceptual structures in mathematics is of paramount importance as it forms the foundation for critical analysis and further problem-solving (Du Plessis, 2018). Du Plessis argues that mathematics is not just about memorizing formulas and performing calculations; it is about comprehending the underlying principles and structures that govern the subject.

Legitimation Code Theory (LCT) is an analytical framework that offers valuable insights into how knowledge and practices are legitimized within different social contexts (Maton, 2016). LCT therefore provides a lens through which researchers and educators can critically analyse educational practices and teaching methods, whilst creating more inclusive and equitable learning environments for learners.

In this research, LCT was used for the analysis with specific focus on semantic density as an analytical framework. The semantic density delves into the intricacies of knowledge representation and communication within educational contexts. It explores how the density of meaning is structured and valued. Semantic density therefore refers to the richness and complexity of concepts and language used within a particular field of knowledge (Maton 2014). By analysing semantic density, LCT enables educators to understand the demands and challenges students face when engaging with disciplinary knowledge. It helps teachers identify areas of high and low density within their curriculum and teaching practices, allowing them to make informed decisions about how to scaffold and support student learning. Thus, educators can design instructional strategies that help students navigate and comprehend the complex concepts, specialized vocabulary, and disciplinary ways of thinking that are integral to a particular subject (Maton, 2013, 2017). A semantic density focus provides a valuable framework for analysing teaching and enhancing students' understanding and engagement with disciplinary knowledge.

Semantic density focuses on the transformation of knowledge that moves between the complex and simple. The analysis will provide insight into the ways educators make use of abstract concepts in the classroom (in this case online) and how these are unpacked (or not) to build understanding and connect complex mathematical ideas to learners' everyday lives and possible real-world scenarios.

The building of cumulative knowledge by focusing on semantic density was graphically represented in the formation of semantic waves that provide insight into the practice of the teacher.

1.1. Background

This qualitative research study centres on the analysis of an open-sourced online video lesson series that focuses on the revision of the Grade 12 financial mathematics as prescribed in the CAPS (2010) document. The topic under consideration is annuities. The study aims to understand the transformation of knowledge development using Legitimation Code Theory (LCT) as an analytical framework with specific focus on semantic density as described in the work of Karl Maton (2013, 2020). A detailed analysis has been utilised to evaluate and determine the development of the transformation of cumulative knowledge in an online video series on Grade 12 financial mathematics by focusing particularly on analysing the weakening and strengthening of semantic density in the series of online revision lessons. For this purpose, a translation device has been developed that encapsulates the semantics of teaching financial mathematics. This study has analysed in detail the transcript, making use of the translation device that was developed for this purpose. This translation device was fully described in the methodology chapter of this research report. Below, the author discusses the details of this research by referring to the focus and purpose of the research, the role of the teacher in the online teaching and learning of annuities in financial mathematics.

1.2 The South African Council of Educators standards for teaching

South Africa does not have specific formal standards for online teaching at a national level, and therefore it is up to developers of online teaching platforms to set the standards. Maintaining the standards of teaching and learning is extremely important in the online environment. Effective online teaching should ideally include clear communication, engaging methods, and proficient use of technology. First and foremost, clear communication is essential, well-structured instructions, and explicit learning objectives. Engaging and interactive teaching methods using multimedia presentations foster active student participation and collaboration. Effective online teachers employ technology proficiently, ensuring smooth navigation of learning platforms and utilizing suitable digital tools to support understanding of conceptual material (Tularam, 2018).

The South African Council of Educators (SACE, 2018) formulated the professional teaching standards (PTSs) that aim at advancing the professional teaching practice and culture in South Africa. The PTS document aims to standardise the professional teaching standard for all educators teaching in South Africa. PTSs apply to both the traditional face-to-face teaching and learning of mathematics as well online teaching, which is becoming increasingly needed especially in times of crises when contact sessions are a challenge. Ten PTSs guide teachers in what is expected in the profession.

For this study PTS five and PTS seven are foregrounded:

5. Teaching is fundamentally connected to teachers' understanding of the subjects they teach
7. Teachers (must) understand that language plays an important role in teaching and learning.

These two PTSs are important when working with the complexities of financial mathematics and is reflective on the pedagogies used, as well as the technological tools that is used in the teaching and learning environment. In the absence of formal standards that determines the standards for online teaching and learning, the SACE standards should give the direction needed to maintain an acceptable level of professional programmes that uses the online platform.

1.3. The focus and purpose of the research

The objective of this research was to investigate the process of transformative knowledge-building (TKB) which goes beyond the traditional model of knowledge acquisition and memorization and instead encourages students to engage in deep and meaningful learning experiences. The researcher has investigated this through focusing on the semantic density that the teacher uses to support cumulative knowledge building. This was done through the interpretation and analysis of the transcripts of a series of online lessons through the lens of semantic density by using the translation device which will be discussed later in Chapter 4.

The series of online lessons deals with the topic of annuities in a series or revision lessons in preparation for the examination. The researcher has analysed the semantic density of the lesson by describing the complexity (strengthening) or simplicity (weakening) of the of the subject content that was being dealt with. This has given the researcher a good indication of the semantic density of the online lesson.

The outcome of the analysis has been plotted in a graphical representation that has given the researcher a tool to describe the cumulative knowledge building that was made possible by the online teacher. This graph can take on several forms, as described in Maton (2014), and elaborated on in Chapter 4. This has provided the researcher with a good idea of how online teaching shapes or constricts cumulative knowledge building in this series of online revision lessons.

The researcher was particularly interested in understanding the semantic density of each online lesson, and the overall level of knowledge building in the lesson.

1.4 The research questions.

This study focused on the analysis of the online lesson transcript, using a translation device that focuses on the semantic density of a series of online revision sessions related to the Grade 12 financial mathematics curriculum.. It is important to mention that these lessons are an open-source series of six online revision videos to aid Grade 12 learners in their preparation for the final matric examinations..

This research report will look to answer the following research questions:

a) Is the transformation of cumulative knowledge building achieved during the online lessons on financial mathematics, and if so, how?

b) What does the analysis of the online lesson series reveal about the semantic density of the lesson pedagogy?

c) How do the exercises used in the series of lessons, as well as the teaching, support cumulative knowledge building, if at all?

I will now focus on the literature that relates to the concepts that are being researched in this report.

Chapter 2: Conceptual Framework

2.1. Building conceptual understanding through focusing on relational understanding

Over the years, many frameworks have been introduced to the discipline of mathematics. Of these, the most widely adopted approaches to the teaching of mathematics came from seminal works like that of Skemp (1976) which proposed the notions of relational and instrumental understanding, and Kilpatrick, Swafford & Findel (2001), who formulated five the strands of mathematical proficiency. These works have shaped and re-shaped the way we look at mathematical teaching and learning.

2.1.1 Relational and instrumental understanding

There is a revisit to Richard Skemp's work, as it clearly connects with current theories in the field of mathematics learning and teaching. Instrumental understanding focuses on the application of rules and procedures without necessarily understanding their underlying meaning (Skemp, 1976) and their connection to broader concepts in mathematics. Relational understanding emphasizes the ability to make connections, recognize patterns, and grasp the underlying principles of mathematics. It involves a deeper comprehension of why mathematical procedures work and how they apply across conceptual structures in mathematics, including the advanced ideas in mathematics. Skemp's approach to the understanding of mathematics highlights the importance of promoting relational understanding to enhance students' mathematical proficiency and problem-solving skills.

Relational understanding and learning, together with the five strands of mathematical proficiency, provide a means for any teacher to work conceptually and relationally with mathematical ideas.

2.1.2 Conceptual understanding and Procedural fluency

Conceptual knowledge is the knowledge that is complex in its connections and understanding. It is a web of interrelated knowledge and discrete bits of information that inform and map the conceptual field within which the discipline operates. Procedural knowledge comes to the foreground when a learner is successful in learning and appropriately applying a procedure (Kilpatrick et. al., 2001). The two knowledges do not stand in opposition to one another; rather, they support each other. Without a good grasp of the procedures that are used in a specific area of mathematics, one cannot make conceptual advances. If a learner is unable to assimilate new mathematical ideas, a learning gap may arise and possibly lead to errors and misconceptions (Sarwadi and Shahrill, 2014).

The five strands that are part of Kilpatrick and colleagues work are graphically displayed in figure 1 below.

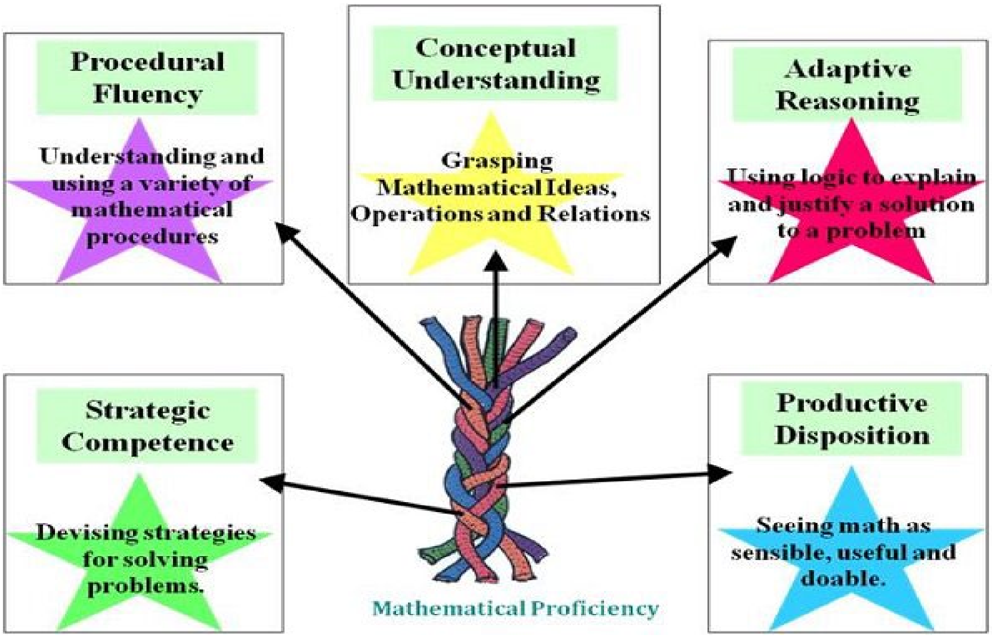


Figure 1: Five strands of mathematical proficiency (Kilpatrick et. al., 2001)

These strands are interwoven and work in support of one another. They are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Kilpatrick et al. (2001) shaped policy around the world and led to the reforms that took place in mathematics education and policy. The two strands that enjoy prominence in this study are procedural fluency and conceptual understanding.

The five strands together provide the ability to make sense of mathematics and to have the certainty to respond to problems that are raised in the mathematics domain (Sahin et al., 2015). This study will include these lenses in the translation device that will be designed and applied in the analysis process.

2.2 Knowledge and Learning

Development of the understanding in mathematics is improved as learners continue to apply conceptual understandings of algorithms to solving different problems. Learning of mathematics is often restricted to learning one concept at a time, with no connection being formulated between mathematical concepts. Due to this lack of interrelatedness, many mathematical concepts may be viewed and practiced in isolation. Relational understanding and learning focus on the learners' knowing what to do and understanding why the approach is significant. Learners develop an in-depth (and thus flexible) knowledge of the concepts in mathematics. A resilient relationship develops between the different contexts in mathematics and other, more applied approaches, such as what is learnt in financial mathematics.

Maton and Moore (2010) maintain that there are two dimensions of learning. The first is that learning has an *objective dimension* that focuses on knowledge itself. The second is that learning has a *subjective dimension* and focuses on those that make and retain that knowledge. Objective knowledge refers to knowledge that is widely accepted, recognized, and validated within a specific field or community. It is considered more authoritative and shared among experts in the field. Objective knowledge is more highly valued in educational and institutional settings, as it carries a higher level of legitimation and is often associated with expertise (Maton & Moore, 2010). Subjective knowledge, on the other hand, pertains to individual perspectives, experiences, and interpretations of knowledge. It is influenced by personal beliefs, values, and social contexts.

If we only see knowledge as that which is created in the minds of knowers, rather than as having its own objective properties, we may risk obscuring important differences between common sense and analytical knowledge (Wheelahan, 2009). Hoadley (2018) argued that the way in which knowledge is delivered would determine the way in which knowledge is acquired and understood by the learner.

2.3. Cumulative knowledge building in mathematics

Cumulative knowledge building highlights the interconnectedness and progression of knowledge over time (Maton 2013). Cumulative knowledge building explores how knowledge is accumulated and developed within specific fields or disciplines. Maton (2013) emphasizes the role of social and institutional contexts in shaping the legitimation and advancement of knowledge (Maton, 2013). These ideas highlight the importance of recognizing and integrating existing knowledge to facilitate deeper learning and contribute to the ongoing development of knowledge within a given domain. By embracing cumulative knowledge building, individuals can deepen their understanding, challenge assumptions, and actively participate in the construction of new knowledge (Maton, 2013).

Maton (2020) suggests that segmental learning can be overcome by the cumulative process of knowledge building. It is the ability of the teacher to build on skills and knowledge that have been obtained previously and which can now be transferred to new situations through the continuous accumulation and integration of new insights, ideas, and discoveries. Muller (2007), as cited in Maton (2010, p. 162), states that "hierarchical knowledge structures are explicit, coherent, systematically principled and hierarchical organizations of knowledge which develop through the integration and subsumption of knowledge". In mathematics education especially, cumulative knowledge building underscores the importance of building on prior knowledge to scaffold learning and facilitate deeper understanding. This would affirm that the hierarchy of financial concepts has the capacity to generate an elevated level of understanding in financial mathematics.

Sarwadi and Shahrill (2014) argue that, to understand mathematics, concepts would need to be formed from previous knowledge and applied to new or current information or knowledge. The primary aim of understanding is achieved when learners develop a productive disposition (Killpatrick et. al., 2001) towards mathematics. Developing an understanding of mathematics follows when relationships are established between the cognitive representations of a mathematical theory and the development of a network of representations that are related to that mathematical theory (Barmby, Harries, Higgins & Suggate, 2007).

Ball, Thames, and Phelps (2008) refers to this as mathematical knowledge for teaching which highlights the integrated knowledge that is required to effectively teach mathematics. It is key that the teacher possess the content knowledge and the pedagogical knowledge to unpack conceptually rich ideas in mathematics. This defines the professional knowledge that exists in teachers specialised mathematical content knowledge (Ball, Thames, and Phelps, 2008), and the specialist ability to sequence complex mathematical ideas into an understandable body of knowledge.

2.4. Legitimation Code Theory and the lens provided by semantic density.

Legitimation Code Theory (LCT) is an analytical tool with which to analyse the development of knowledge in practices such as teaching (Maton, 2013). In this study, LCT is used as a rigorous lens to analyse the building of knowledge through a focus on semantic density.

LCT permits the expansion and integration of practice. It allows for a breakdown analysis of lessons into its different time intervals. These are referred to as events in this study, and they collectively form a combined framework that can be analysed further. LCT defines the different dimensions of specialisation, semantics, and autonomy that are each delineated by four codes that are referred to as the languages of legitimation or legitimation codes. These codes clarify what is valued, what is not valued, the struggles for legitimation, and knowledge development over time (Maton, 2020). The Semantics Dimension refers to the meaning that can be provided through ideas, symbols, words, and images. It consists of two relations of varying strengths, from strong (+) to weak (-). The semantic plane has four modalities which are illustrated further in Figure 2.1 (page 17). These are rhizomatic codes (SG -, SD+); prosaic codes (SG +, SD-); rarefied codes (SG-, SD-); and worldly codes (SG+, SD+).

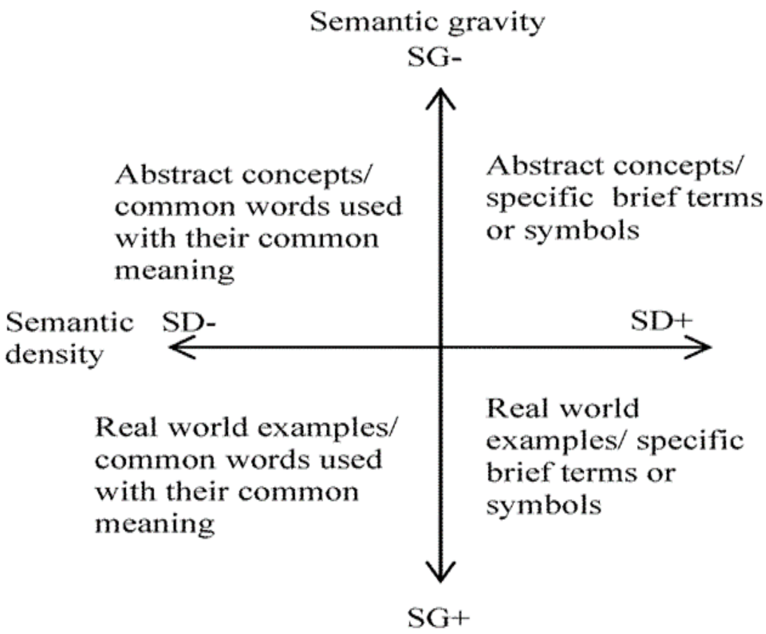


Figure 2.1. Semantic plane reflecting the
Semantic Dimensions of LCT.
(Maton, 2014).

Semantic Gravity (SG) reflects how context bound or abstract an interpretation can be. The semantic gravity is stronger when an interpretation is extremely specific, grounded in context, and linked to a personal experience or example. The semantic gravity is weaker when the interpretation is more general and is related to a rule, a principle, or a procedure. This study will not look at the semantic gravity of the online lesson series.

Semantic Density (SD) is how condensed or complex a concept is. The semantic density is weaker when the interpretation is simple, without specialist terminology, concepts, relationships, or connections. The semantic density is stronger when the interpretation is more complex, such as specialist concepts, terminology, and analytical connections. A differentiation between the different strengths of how complexity is concentrated within semantic density can be achieved with the use of a translation device that would reflect how empirical data will be coded (Maton, 2020). Figures 2.1 and 2.2 (on pages 17 and 18) further illustrate semantic density and semantic gravity. The research study focuses on the analysis of the transcripts of a series of online videos on annuities in financial mathematics through the lens of LCT with a focus on the semantic density of the lessons.

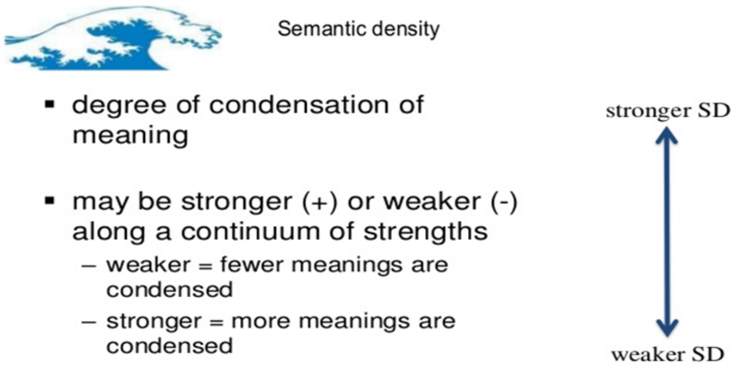


Figure 2.2: Semantic density – a semantic dimension of LCT.

This study will seek to understand what cognition occurs during the revision of Grade 12 financial mathematics through analysis of the transcripts of a series of online videos, using a translation device that was formulated for the purpose of identifying the semantic density of the online lessons. This translation device will be developed from the analytical framework and the literature that is available in the field related to the teaching and learning of financial mathematics. It is discussed fully in Chapter 4.

Chapter 3: Review of the literature

In this review of the literature, I will look at what has been established through research in the field related to the teaching and learning of financial mathematics and knowledge building in the mathematics classroom through the lens of LCT. I start by focusing on the analytical underpinnings of teaching mathematics and move to discuss the traditional and not so traditional practices of teaching and learning, followed by a discussion related to the nature of financial mathematics as well as the knowledge clusters in financial mathematics. This is followed by a focus on the teaching and learning of annuities in financial mathematics.

3.1. The analytical underpinnings of teaching mathematics

The pedagogy of mathematics explores the relationship that exists between the teacher, the learner, and mathematical content knowledge (Lo & Hsueh-I Martin, 2006). However, teachers and learners relate differently to mathematical content knowledge (Jaffer, 2020). A teacher's mathematical knowledge must be both manifold and concept specific (Pournara, 2014).

3.1.1 Sound Pedagogical content knowledge

Lee Shulman (1986) cited by Makonye et al. (2014) argued that Pedagogical Content Knowledge (PCK) should influence the specialised knowledge of educators. PCK is a framework that highlights the unique knowledge and skills required by teachers to effectively teach specific content areas. It recognizes that subject matter expertise alone is insufficient for effective teaching. PCK recognizes that effective teaching involves making content accessible and meaningful to students (Schulman, 1986). The teacher will thus identify and focus on the appropriate remediation to undertake when dealing with errors and misconceptions that learners produce in mathematical and related contexts.

Adler (2017) alluded to the specialisation of mathematical knowledge that should be a pre-requisite for all educators that may require pedagogical training to improve the teaching and learning of mathematics. Research has proven that if an educator's mathematical knowledge exceeds the minimum requirements of the speciality, there are positive changes to the achievement of mathematical learning. There is a consensus that mathematics educators at all levels of schooling should attend a variety of mathematics and pedagogic training that would aim to enhance the teaching and learning of mathematics in the classroom (Adler, 2017).

Educators may be able to teach the procedures of mathematical concepts and the various calculations; however, they may not be knowledgeable on the fundamental theories of these practices in the classroom, or how to communicate conceptually rich relations effectively to learners. Financial mathematics is an interrelated concept that would require the appropriate specialisation for teaching and learning to be successfully achieved in the classroom.

3.1.2 Revisiting mathematical content knowledge

Pournara and Adler (2014) viewed revisiting as the re-establishment of the mathematical content to improve teaching and learning in the classroom. This allows the mathematical knowledge of the educators to broaden as connections are deepened. This solidifies the knowledge of the mathematical concepts that can improve teaching and learning in the mathematics classroom. Pournara and Adler (2014) mention the uncertainty that exists in compound growth questions, as the increase is not specified as being at the beginning or the end of the first year. Revisiting a compound question as an example would allow the educator to consider the learners' conceptions and interpretations of such questions. Educators would also be able to identify and rectify procedures that are not being formulated in the appropriate manner. There was further acknowledgement that the essential feature of revisiting a concept such as compound growth would be to be particular on references to time. Financial mathematics is the contextual knowledge of finance (Pournara and Adler, 2014), and it is therefore key to support teachers in their quest to broaden their relational understanding of the structures and concepts in financial mathematics.

3.2. Traditional and Non-traditional teaching and learning practices.

Traditional teaching and learning practices have been the focus of academic research. These methods use more teacher-centred approaches to teaching and learning mathematics (Tularam, 2018). Online teaching and learning – or non-traditional teaching and learning (NTTL) – has more recently become a necessity for education under COVID-19 conditions. The more common terminology used is e-learning; however, for the purposes of COVID-19 it was Emergency Remote Teaching (ERT). ERT was an unforeseeable situation that arose from learners not being present in the classroom. To prevent the loss of teaching and learning, schools moved to ERT. Whether it be traditional, fully online, or a hybrid learning environment, it would require provision, outlining, design, and implementation to ensure that an effective teaching and learning environment is achievable for both the educators and the learners. Instead, ERT created chaotic learning environments (Richard, Schultz, and Michael, 2020).

Online learning uses more student-centered approaches to the teaching and learning of mathematics (Tularam, 2018). Baran, Correia and Thompson (2011) argue that the pedagogy for traditional teaching and learning should not be used in NTTL (such as e-learning). A pedagogy for NTTL needs to be developed. Together with this, there is a need for professional development programs that assist in the competency levels of teachers using different approaches to teaching and learning to ensure effective, continuous learner engagement. The pedagogy should focus on inquiry-based learning, problem-centred learning, and integrative learning (Richard et. al., 2020, p. 4).

3.2.1 Competency levels for online teaching

The competency levels that have been identified for online teaching include "technology-related competencies, communication competencies and assessment-related competencies" (Baran et al., 2011, p. 23). These competencies are important for this study, as the changes in education platforms demand that teachers can make the shift to an online platform. Richard et al. (2020) states that, for online learning to be a successful platform for both teacher and learner, there needs to be attainable goals for both. For a learner, there needs to be an active role in taking charge of all aspects of reaching their learning goals. For a teacher, there needs to be professional development in being a facilitator in an ever-changing teacher and learning environment that would use different pedagogies and technological aids. It is important to mention that the series of online videos analysed for the purpose of this study was available for online usage prior to COVID-19.

3.3 The nature of financial mathematics

Mathematics and Financial Mathematics are interrelated even though their pedagogical content knowledge is different (Makonye, Weitz and Parshotam, 2014). Mathematics is a dynamic specialty that is associated with investigating problems, pursuing solutions, conceiving ideas, making hypotheses, and reasoning cautiously. This differs from a static specialty consisting only of an organised system of information, strategy, and theory to be retained or knowledge acquired through repetition (Shoenfeld, 1992; Hiebert et al., 1996 cited in Warshauer, 2015).

This integrated learning process of mathematics is clearly depicted in Figure 3 (page 22) below. Financial mathematics is an applied mathematics that can be linked to everyday life. Figure 3 depicts the integrated learning process of mathematics. Working mathematically is depicted as six intersecting cognitive activities which consists of fluency in the base processes and ideas in mathematics, connecting and extending concepts to develop ideas, justification of processes used in problem solving, reasoning, and making sense of the outcomes reached and communicating outcomes effectively. These learning processes form an interconnected whole which develops in support of one another. A weak link in the process will destabilise the understanding in mathematics and create misconceptions which cannot go unaddressed.

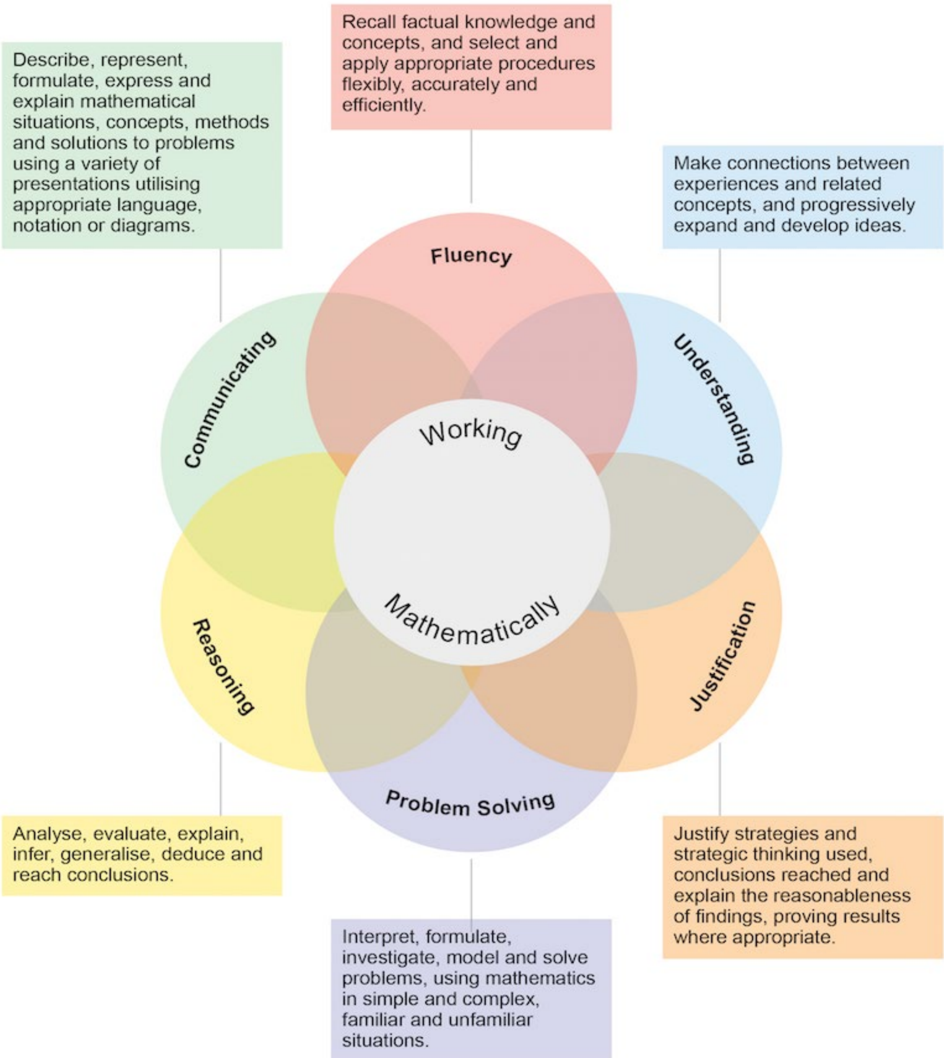


Figure 3: Integrated learning process of mathematics
(https://educationstandards.nsw.edu.au).

This argument supports the argument made by Makonye et al. (2014), which is that errors and misconceptions formulated during the learning of mathematics impact the learning of financial mathematics. Pournara (2013), views the teaching of financial mathematics as incorporating "mathematical knowledge, pedagogical knowledge, and contextual knowledge of financial mathematics". He argues that teachers require specialised knowledge to teach financial mathematics. Pournara (2013) posits the view that to teach annuities, the primary focus is on the "time value of money, compounding, discounting, present value, future value and unit growth factor" (Pournara, 2009, p3).

3.4 The knowledge clusters of financial mathematics

Pournara (2013) suggests three knowledge clusters for the context of mathematical knowledge in the teaching of financial mathematics. These include aspects that are mathematical, aspects that are pedagogical, and the aspects of the contextual knowledge of finance. Knowledge clusters aim to further prove the integration of algebra, calculus, geometry, and many other mathematical concepts such as the integrated learning processes of mathematics that are important in the teaching of annuities. Financial mathematics relies on a strong foundation in mathematical concepts and techniques, such as the structures of sequences and the conceptual shifts that are made when adjusting these structures. These mathematical structures are used to model and solve financial problems. By developing conceptual and relational understanding in these knowledge clusters, individuals can effectively analyze financial information, make informed decisions when problem solving and work with more complex problems where this information is applied.

3.5. The Teaching and Learning of Financial mathematics: A focus on annuities.

The Grade 12 Mathematics curriculum allocates 7% of the total contact time to the teaching of Grade 12 financial mathematics. It also represents 6% of the work that is assessed in the National Senior Certificate (NSC) final examinations. Due to limited time, many teachers do not spend adequate time on the teaching of financial mathematics, especially the complex topic of annuities. The misconception is that the primary constituents of annuities are algebra, rates of change, and the application of geometric series. To understand annuities and its related concepts, a hierarchy of this conceptual network was proposed. This development was formulated from the curriculum. To ensure that the learning process is relevant to learners, teachers must have the appropriate and relevant conceptual knowledge to teach financial mathematics (Pournara, 2009 and 2013).

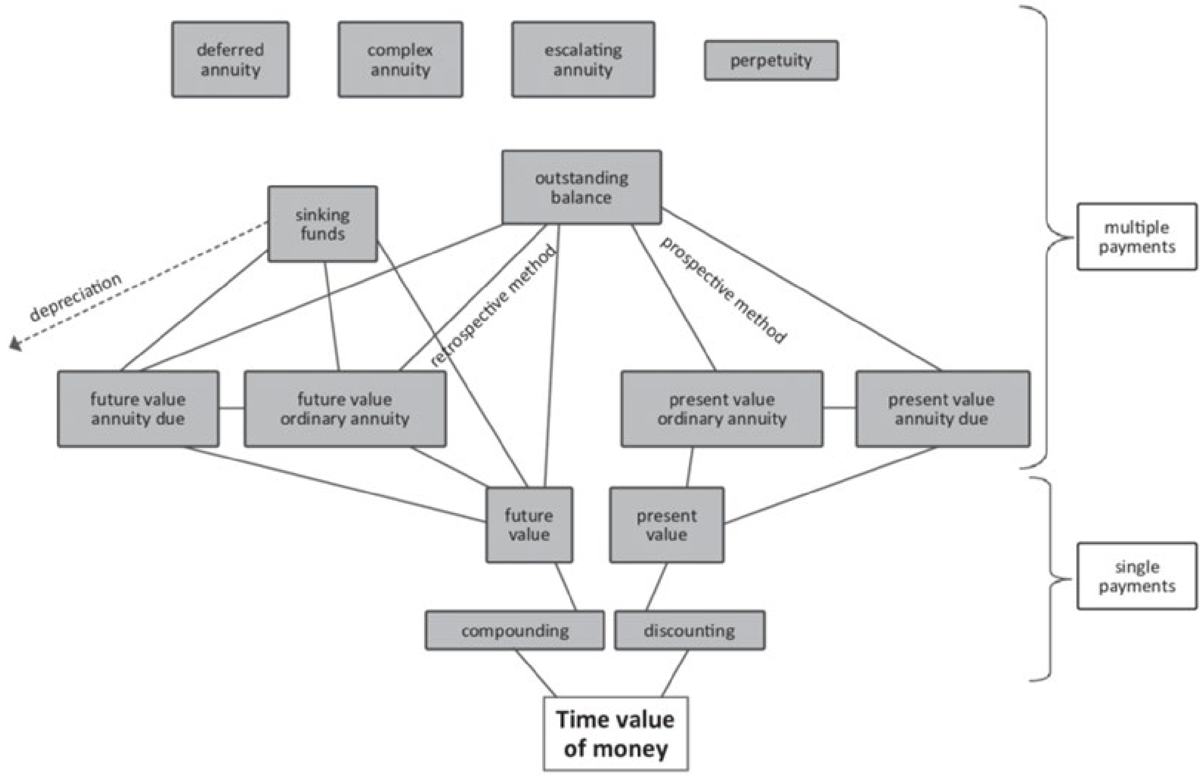


Figure 4: A hierarchy of annuities concepts (Pournara, 2013)

In Figure 4 the hierarchical structure proposes that those concepts at the top of the diagram depend on the concepts that follow lower on. An example of its implications would be that, to learn compounding and discounting, learners would need knowledge that has been developed on the time value of money (Pournara, 2013). Time value of money is the knowledge and understanding that the time value of money at present day is worth more than that same value of money at some time in the future, due to the possibility of interest and inflation being added on to the present value (Pournara, 2009 and 2013). Financial mathematics deals with the use of mathematical methods to solve financial problems.

3.5.1 The formulae used for introductory financial mathematics in the CAPS document.

Learners in South Africa are exposed to Financial Mathematics in Grade 8, when they focus on 'simple interest'. Simple interest is taught as interest that is calculated on the principal or original value of an investment or loan. In most textbooks and according to the CAPS document, learners in Grade 8 are not exposed to the formula $A = P(1+i.n)$ (where A is the accumulated amount, P is the principal amount, i is the interest rate per annum converted to a decimal and n the period in year(s). Learners are introduced to the formula to $I = P \times r \times n$ or $I = P \times i \times n$ and $A = P + I$, where r is the annual rate of interest and $i = \frac{r}{100}$. These formulas are used to find the accumulated amount of money (A) if a principal amount of money (P) is invested for n years at a simple interest rate of r % per annum.

According to the CAPS (2010) General Education and Training (GET) mathematics curriculum, the compound interest formula $A = P(1+i)^n$ is introduced to Grade 9 learners as a calculation of the accumulated value (A) from the principal value (P) for a number of years (n) at an annual rate of r % per annum, compounded annually. Here the i is calculated in years as simply $i = \frac{r}{100}$. Grade 9 learners are not introduced to the time value of money.

The introduction of the compound formula serves as an introduction to financial mathematics, but in no way equips learners to develop an understanding of annuities (Pournara, 2009 and 2013). Pournara (2013) places further emphasis on the need to reinforce the concept of the time value of money by linking accumulated value (A) to the future value of money (F_V) and the principal value (P) to the present value of money (P_V). The general compound formula suggested is therefore $F_V = P_V\left(1+i\right)^n$.

The formulae for annuities are complex and challenging to understand. The present value annuities formula is $P_V = x\left[\frac{1-\left(1+i\right)^{-n}}{i}\right]$ and the future value annuities formula is $F_V = x\left[\frac{\left(1+i\right)^n - 1}{i}\right]$. Both these formulae will be discussed in more detail during the analysis of the series of online lessons. An annuity takes the form of a fixed payment made at regular fixed intervals for a specified period. Most examples of annuities are linked to investments or loans. The financial problems now move away from only focusing on annual compounding to quarterly, monthly, semi-annually, etc. The focus shifts to the time value of money as the operations of compound interest are extended to the concept of an annuity.

Chapter 4: Methodology

Research methodology is a process whereby information is collected with the purpose of further analysis that leads to a solution of the research question (Kothari, 2004). The author will discuss the use of LCT and semantic density, semantic waving, and the translation device that will be used for coding the data.

4.1. The research approaches.

This qualitative research study will make use of a meticulously designed translation device (Maton, 2014) to code the information, and this will be presented as a descriptive case study. The data in this study consists of six open-source online videos that are available on the YouTube platform and hence in the public domain. The online videos focus on the revision of Grade 12 financial mathematics. Each video was transcribed, taking particular care in recording every action and on-screen movement and signal that the educator makes. This method of transcription allows for the comprehension on whether the educator is promoting knowledge building. This will be analysed through the interpretation of the semantic wave (Maton, 2014) that is mapped out by the teaching. These video transcriptions are assessed using semantic density.

4.2. The Three Semantic Profiles

Through semantic profiling, with the aid of this translation device, the strengths of the semantics density on the series of online lessons will be analysed. The translation device focuses on concepts that have an elevated level of abstraction or condensation (Maton, 2013). A semantic profile is therefore a line graph that can document the strengths and weaknesses of semantic density over a specified period. Figure 5 (on page 27) illustrates the three semantic profiles that are possible through analysis of the ranges in the semantic density of a lesson. These reflect the semantic ranges of a high semantic flatline (A), a low semantic flatline (B), and a semantic wave (C) that can exist through semantic profiling. A high semantic flatline illustrates where knowledge in a lesson is constantly at the abstract and complex level. A low semantic flatline illustrates where knowledge in a lesson is constantly communicated through simple and everyday language. A semantic wave would illustrate that the knowledge in the lesson moves in between the semantic planes of simple and everyday language to the abstract and complex, and back again when needed. The analysis of the semantic profile provides the lens to view the transformation of knowledge that may occur in a practice such as teaching (Maton, 2013). The construction of a sequential transformation of knowledge will be discussed using semantic waving as proposed by Maton (2020).

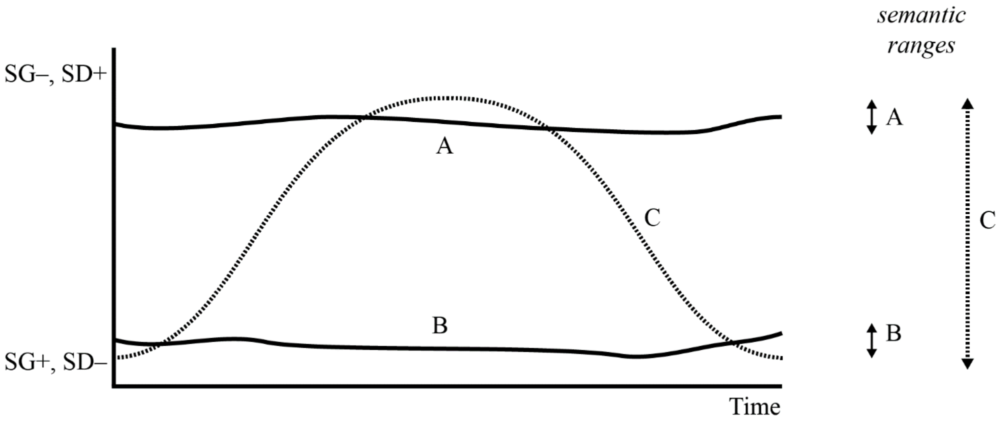


Figure 5: The Three Semantic Profiles (Maton, 2013).

There are, however, more profiles that might appear in the analysis process. A downward escalator is in reference to semantic density as a shift from an overly complex or condensed meaning to a less condensed or simpler meaning. This shift occurs by unpacking the concept being taught with everyday words and ideas, but never repacks the concept to a higher level of density. An upward escalator refers to semantic density as a shift from a weaker level of complexity or simpler meaning to an overly complex or condensed meaning through repacking concepts into complex knowledge (Maton, 2014). An upward escalator also does not return to the everyday simpler application.   A downward escalator is illustrated in Figure 6 below.

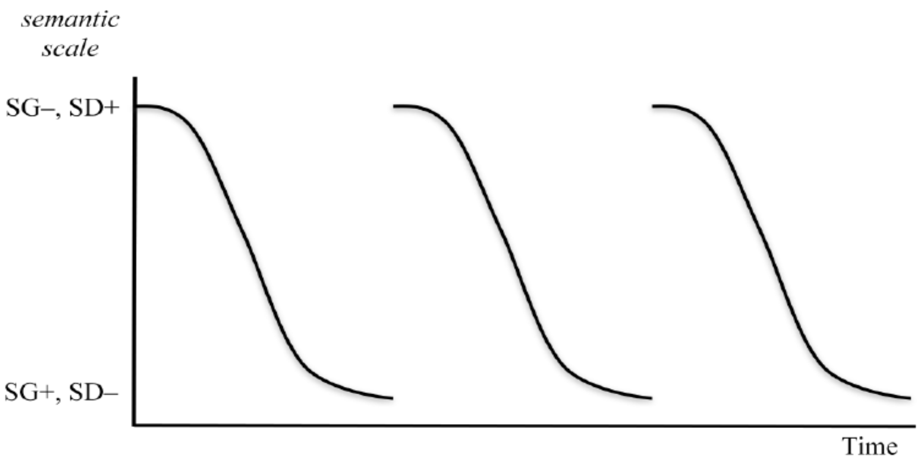


Figure 6: A 'down-escalator' semantic profile (Maton, 2014).

4.3. The Translation Device

The information collected will be analysed with the use of a translation device (Table 1: page 29) that has been designed and populated during the initial phases of coding the data. Maton (2014) argues that a translation device provides a varying classification on the variations of semantic density with its relation to its condensation, complexity, or simplicity.

Code	Index	Description	Example
SD +++	The concept represented by the educator requires specialised knowledge to understand. It is conceptually rich, complex and encapsulates multiple ideas.	The meaning is condensed.	A formula An equation A graphic representation
SD ++	Integrated connections are relationally linked to objects by the educator that would form part of a network of conceptual knowledge and further understanding.	Linking mathematics to financial mathematics.	A process of classification The impact of connections
SD +	The technical terminology for a concept or subject is used by the educator with further explanation that promotes instrumental understanding through its characteristics and properties.	Terminology for financial mathematics.	Present value Future value
SD -	The meaning of concepts or words are presented in such a way by the educator that a non-subject specialist would be able to understand.	Use of alternative language to explain mathematics and financial mathematics.	Value now Value later
SD – -	The educator can convey meaning and understanding with the use of non-specialist terminology in that concept or subject.	Use of everyday scenarios to explain a concept or idea.	Student loan repayment

Table 1: Translation device formulated for the profiling of semantic density.

The translation device reflects the relationship of semantic density with the interrelated concepts in financial mathematics. To identify complex and simple semantic density, the semantic density codes that were created are used in the analysis of the information that has been accumulated and formulated from the transcripts of the lesson series.

This translation device includes five semantic density codes. These are SD +++, SD ++, SD +, SD, – and SD --. Each code has been further described in Table 1 (page 29) under the headings of code, index, description, and examples.

SD +++ is meaning that is most complex and condensed. SD - - is meaning that is simplest and least condensed.

SD +++, in terms of this translation device, relates to meanings that were overly complex and condensed. This would refer to a situation where the formulas were presented at the start of the lesson, and then followed by respective calculations thereof.

SD ++ is the interrelatedness between mathematics and financial mathematics. Examples of such would be converting a value to a decimal. The concepts of logarithms and cross-multiplication are connected due to their impact on the formula, and thus are designated as SD+++.

SD + is the reference to technical terminology such as present value and future value, but there are times when the presenter interlinks these terms with the alternative terms such as value now and value later. Even though alternative word usage is SD-, it depends on the context in which the presenter makes use of a lower register, and hence the classification may vary between SD + and SD -. These two codes are closely related but different.

SD - denotes the use of non-mathematical terminology to explain a mathematical concept in a non-mathematical way. Therefore, it moves the concept away from its specialisation.

SD-- is the use of everyday scenarios to explain the concept, but when the scenario is being used as a basis of a word problem that requires a solution, the semantic density increases. When the presenter narrates the meaning but moves away from the specialisation, it is designated as SD --.

The translation device has been used to translate the transcripts according to events. Each event indicates a semantic code according to its semantic density. There are times in the online videos where the presenter addresses her audience on matters that are not linked to the lesson. In these instances, no semantic code has been allocated.. During the transcripts, the researcher has also added comments in italics to draw attention to a particular point that would need to be further analysed.

In the following chapter, I proceed to present the transcripts for each video analysed against the translation device. Each transcript has a corresponding graph that illustrates the downward and upward escalation of semantic density.

Chapter 5: Data Analysis and Discussion of Findings

Before the data analysis and discussion of findings, it is important to mention that these videos may be accessed online as individual videos. However, there was continuity in the videos, and this has also been demonstrated through the analysis. It is this continuity that provided the motivation in analysing the videos together. The videos range from eight to 13 minutes. It is important to mention that the presenter was very colloquial in her language and tends to repeat herself quite frequently. At the end of the individual analysis of the videos, further findings and analysis has been undertaken. The translation device (as discussed in the previous chapter) has been utilised in the analysing of the online lessons by focusing on the various levels of condensation, complexity, and similarity. The researcher has used events to identify the changes in the semantic density of the online lessons. It is important to note that when these videos were live, there was learners in the studio and there were learners from different schools tuning in. Both sets of learners were sending SMSs during the broadcast.

5.1. Video 1: Simple Interest

At the onset the presenter uses the simple interest formula $A = P(1 + in)$ but refers to A as future value rather than accumulated value and P as present value rather than principal or original value. There is nothing wrong in this assessment or the use of these financial terms. However, to create uniformity for learners who may be watching live, or for future learners who may view the recorded lessons online, a correlation or explanation should have been provided to prevent further confusion. The presenter could have stated that she is aware that most learners are used to the terms 'accumulated value' and 'principal value', but for the purposes of this online Grade 12 session she will be referring to the concepts of accumulated value as 'future value' and the concepts of the principle or original value as the 'present value'.

Another point of clarity should have been the meaning of i in the formula and its difference to r. When one communicates about interest, it is said as, for example, 15%. Fifteen percent means $\frac{15}{100}$, and this value differs for r and for i. r is 15 but $i = \frac{15}{100}$. Rate of interest is the actual value of 15 while i is fifteen divided by one hundred. Many learners are taught in this way and the presenter should have addressed the difference so as not to create confusion later.

The transcripts indicate that the presenter uses simple language with the use of relatable exercises to connect with her learners.

In the video the following example is used:

R4000 is invested in a savings account for 8 years. Calculate how much money will be in the account at a simple interest of 10% p.a.

Financial mathematics is a complex concept where meanings of certain terminologies in the concept can either aid or hinder the understanding of the concept being taught. The presenter's register was colloquial, but the financial and mathematical terminologies are always used with detailed explanation. This was clearly demonstrated in the way that she uses an example to explain the simple interest formula and the substitution for each part of the simple interest formula. Her body language was visible, and she makes continuous use of pointing to appropriate parts of the formula as she explains. She also explains more than once to ensure that the students understand why she is substituting in this way.

The detailed analysis of the semantic profiling for video 1 was the following:

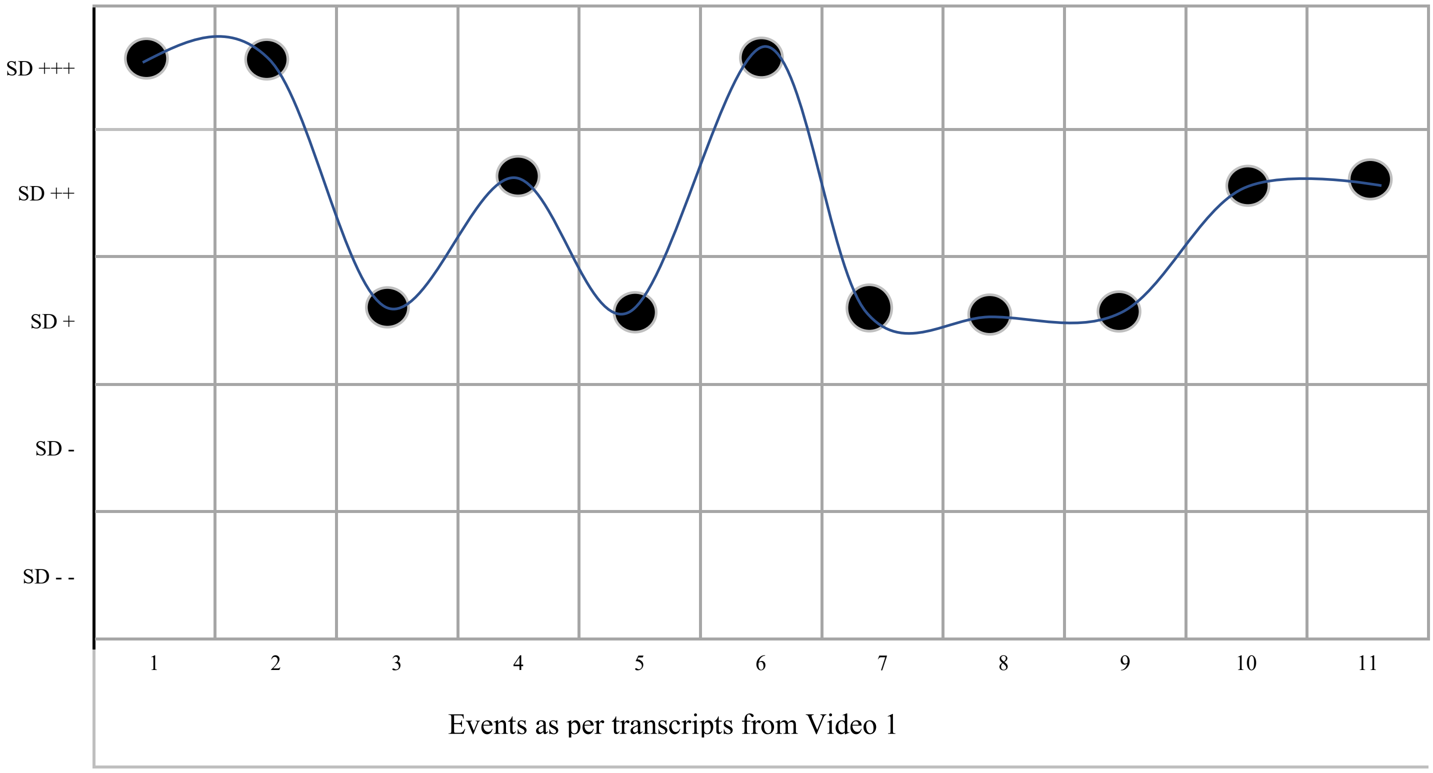
From event 1 to event 2, there was a high flatlining at SD +++ . From event 2 to event 3 there was a downward escalation as there was a movement to a lower semantic coding. The teacher has moved from SD ++ to SD + as she moves from the formula to explain the terminology of financial mathematics, in this case, it was present value. There was then upward escalation, a downward escalation, and another upward escalation to a downward escalation. From event 7 to event 9 there was a high flatline that moves to an upward escalation and again there was high flatlining at SD +++.

At no time during this video does she move down to a SD – or SD - -. Even though she uses everyday examples she still maintains the financial mathematics terminology and hence its relatedness to the concept. Below is the transcript for video 1 as well as the semantic profile.

Event	Video 1: Simple Interest	Code	Time
	Good afternoon learners and welcome to this afternoon's transmission on financial mathematics for Grade 12. This is always a very daunting question in the paper. People struggle with it, so this afternoon I'm going to ask you to pay careful attention and to send me your questions if you are struggling, and to participate in this lesson as far as you possibly can. I have prepared some slides for you to make it easier and you know there's so much reading involved in financial mathematics so I want you to read carefully with me, look at the formulas that I'm going to use, and if you should have any questions, please send them through to me. Just before I start a very, very warm welcome to Breechedin secondary school, Modder Dam City Mozilla. I have seen another high school Atlanta Secondary school, Ela High school. Welcome learners. I am so proud of you for coming on a Sunday afternoon to come and listen, to come and learn, because you want to reach success in life. So, without finding any further ado let us start with our financial mathematics grade 12. I'm going to do a little bit of revision of simple interest and then I'm going to go to compound interest then we're going to do future value and then we're going to do present value. So, I want to look at four things this afternoon. It's quite a lot but I'm going to see if we can get through everything. Remembers to participate, have your calculators out, have your pens your papers in front of you and we're ready to go. So, the first thing we're going to look at this afternoon - I'm going to go to my first slide that is on simple interest.		0.00 – 2.03
1	If you look at the simple interest formula you will see that it is equal to *P* open bracket... (*teacher interrupts herself)* What does all these letters stand for? (*Referring to the variables in the formula)*	SD +++	2.03 -2.14
2	I think you know already but just to let you see again… (*The formula appears on the screen)* $A = P(1 + in)$	SD +++	2.14 – 2.18
3	The *P* is the present value, Present value is what you are going to start with. Your *F*uture value is what you're going to end off with or you're waiting for. *i* is your interest rate and *n* are your number of years. So, everybody that's your simple interest formula and we immediately going to go to our first example. *[Comment: It is important to note that the presenter does not refer to the A in the formula as accumulated value but only as future value and does not clarify from the onset that i is the rate of interest divided by 100. Furthermore, in simple interest n can be referred to as number of years but in compound interest it will be the number of terms]*	SD +	2.18 -2.42
	Now remember I said have your pens ready, have your calculators ready because you are going to do the first few examples with me as it is old work that we've done maybe in grade 10 and grade 11 so you're going to do these examples with me		2.42 -2.50
4	And here comes our first example. Our first interest, simple interest example says *(The slide reflects the question on the screen)* *R4000 is invested in a savings account for 8 years. Calculate how much money will be in the account at a simple interest of 10% p.a.*	SD ++	2.50 -3.00
5	Remember very carefully you have your formula…R4,000 is invested in a savings account. For how many years?... For eight years. So first, calculate how much money will be in the account if you're… if you're going to get simple interest at ten percent per annum. You saw the word simple interest.	SD +	3.00-3.36
6	You immediately write down your simple interest formula which says it is equal to P	SD +++	3.36-3.40

	open bracket 1 plus *i* times… $A = P(1 + in)$ *(On the slide)* Now, everybody as you are writing this formula down remember you are looking for this.		
7	I calculate how much money?... So, you're looking for *P*? What is *P*? Everybody, please, R4,000 you substitute R4,000 into the place of *P*?	SD +	3.40-3.57
8	What is my interest rate? My interest rate is 10%. If you are going to put 10 in this formula you are going to make a huge mistake. Remember it is 10%. So, it is 10 divided by a hundred. That is going to be your *i*. So, 10 divided by a hundred gives me zero comma 10. Divided by a hundred, everybody, on the calculators, and it gives me zero comma one. $i = \frac{10}{100}$ $= 0,1$ *(On the slide)*	SD +	3.57-4.31
9	The *n* we said at the beginning is for your number of years and as you can see that I'm investing this money. For how many years? For 8 years.	SD +	4.31-4.42
10	Put it in a little bracket if you do this on your calculator. $A = 4000(1 + 0,1(8))$ *(On the slide)*	SD ++	4.42-4.50
	Everybody, send through your answers to me quickly. Your answer should be? Let me just see… let's see.		4.50-5.00
11	$A = R7,200$ *(On the slide)*	SD ++	5.00-5.02
	I got some beautiful answers. They're fantastic. Nikita Walters, you got 7,200. That's fantastic. Lola, you got 7,200. I'm getting so many R7,200. So simple interest seems to be no problem. Remember simple interest when you are going to use the simple interest formula. When you see the word simple interest you don't see compounded you just see simple interest, you write your formula down and then you substitute and get your answer. Everybody R7200. Joshua just makes sure quickly you're going to answer eight thousand five hundred and seventy-four. You way off by a thousand, three hundred and, seventy-four and 36 cents. Just check that you have punched in your values correctly. Please, Joshua, and that you have used your brackets. Okay, so that is simple interest.		5.02-5.56

Video 2: Compound Interest



5.2. Video 2: Compound Interest

The detailed analysis of the semantic profiling for video 2 (events 1 to 11) was the following:

From event 1 to event 3, there was a downward escalation from SD +++ to SD +. Thereafter, there was an upward escalation which leads to a high flatlining between event 4 and event 5 at SD +++. There was then a downward escalation that leads to an upward escalation to a high flatling at SD ++. From here there was a downward escalation. The semantic graph indicates that, even though the presenter uses language that the learners can understand, she never moves away from the financial mathematics terminology. As a result, her constant focus was on mathematics and financial mathematics. Therefore, the semantic density of the online lessons remained overly complex with condensed meaning.

Event		Code	Time stamp
	So, let's look at the next slide which is compound interest.		0:00-0:07
1	$A = P(1+i)^n$ *(Appears on the slide)*	SD +++	0:07-0:11
2	I have *A* that is equal to *P* plus I to the power of *n.* *P* is your starting value, *I* is my interest rate, *n* is for the number of years.	SD ++	0:11-0:58
	I'm asking everybody listening to this broadcast to focus. Do not talk to your friend. Do not think what you're going to do this after the session. Just focus on this first example. Read it with me. Somebody's asking? Ma'am, which book are you using? Grade 12's, I brought my own examples. That's why I've made my own slides. So, that's why I'm saying write the example down quickly and read it with me quickly.		0:58-1:23
3	Let's see example number one for compound interest says. *R5000 is invested in a savings account for 8 years.* *Calculate how much money will be in the account if the interest rate is 7,5% per annum, here it comes, compounded quarterly. Use your compound interest formula* *(Appears on the slide under the compound formula).*	SD +	1:23-2:00
4	Says: R5,000 is invested in a savings account. For how many years? For eight years. Again, calculate how much money will be in the account if the interest rate is 7,5% per annum. Here it comes - compounded quarterly. Look, at this word, here - compounded quarterly. Immediately it tells me to what? - use the compound interest formula.	SD +++	2:00-2:27
5	So, everybody I'm expecting you as you're sitting there, by your tables, at your desk, to write down the following formula. $A = P(1+i)^n$ *(Appears on the slide again)*	SD +++	2:27-2:58
6	*A* - my future value, *P* - the money that I am investing, *i* -my interest rate, and *n* - is my number of years. Am I looking for *A*? *(She points to each letter of the formula on the slide as she speaks)* Absolutely because the question is to calculate how much money? So, wait everybody, you're looking for A. *(She points to a on the slide)* So, you put your *A* down. It is what you are looking for. It is your future value after you have deposited R5000. Okay, so I'm going to put R5000 into the place of *P*. *(She points to P on the formula)* Remember, you are starting with this value, you want to know how much you're going to have in the account after eight years? *[Comment: once again the presenter does not refer to A, which is used in the formula not as accumulated value but as future value.]*	SD +	2:58-3:30
7	So put the one down now be very careful I specifically choose 7,5% why did I choose that because it is going to be 0,075. Great, well, where did I get that 0,075? I said 7,5 divided by 100 and if you say 7,5 divided by 100. $\frac{7,5}{100}$ *(Appears on the right-hand slide as the presenter goes on with the explanation.)* Do that on your calculator then you substitute it in there but now very important your interest rate is 7,5%. *[Comment: the presenter does not state that i = r ÷100…which means the rates of interest divided by 100…]*	SD ++	3:30-4:06
8	Everybody, gets the 0,075 but this interest rate is compounded, look, at the slide it is compounded quarterly. *(She points to the slide to where the word appears in the question)*	SD ++	4:06-5:00

	Quarterly means they are going to do this interest rate four times a year. So, it is compounded quarterly. Can you see? It's over, for everybody, they're going do it for eight years. So, it's eight but you are doing it quarterly. So, it's 8 × 4. *8 × 4 (the presenter writes this on the side hand slide under the i calculations. She further writes and underlines the quarterly with an arrow from the 4)* I hope that you understand if you are dividing your interest rate by 4 you take the number for *years* and you multiply it by four – so, this is quarterly – right. *[Comment: the presenter does not emphasise that n is the number of terms in compound interest. In compound interest n = number of years × the compound.]*		
9	I'm going to write here quarterly. If they said semi-annually or half yearly – listen, to those words semi-annually or half-yearly you would take your interest rate divided by 2. Take your number of years and × by two. If they said monthly, you would take your interest rate divided by 12 and times by 12. *[Comment: the presenter does not emphasise that in compound interest* $i = \frac{r \div 100}{c}$*, where c refers to the compound]*	SD ++	5:00-5:21
	Okay, so everybody on your calculators, if you message your answers through to me.		05:21-5:22
10	$A = 5000(1 + \frac{0{,}075}{4})^{8 \times 4}$ *(The presenter substitutes the values in the formula and leaves it on the slide for learners to calculate)*	SD +++	05:22-05:23
	Please – so, everybody on your calculators I'm going to give you a minute quickly to punch this in on your calculators. I'm not going to write the answer down there I'm waiting for you to SMS the answer through to me for this specific example.		05:23-6:00
11	All you do is R5,000 - open your bracket - one plus 0,075. Let's discuss the 4 again - it is quarterly. Take your years times by four as well. So, if you do this correctly on your calculator - what are you supposed to get as your final answer?	SD ++	06:00-6:10
	A = R9060,12 *(The final answer is written on the slide.)*		06:10-6:35
12	You are going to have R9060,12 in your account. This is going to be your future value after you deposited R5000 in an account at 0,075 quarterly and *n* is 32.	SD +	
13	I hope everybody understands that it is going to be 32. You divide by four, you times by four, so after eight years you are going to have R9060,12 – since, what was your interest i?	SD +	06:35-6:47
14	How much more money did you have in your account? What was your interest? So, the interest that you scored is R9060,12 – R5000.	SD ++	06:47-6:57
15	*R9060,12 – R5000 = R4060,12* *(The presenter writes this on the slide to further illustrate the interest that has been accumulated.)*	SD ++	06:57-7:05
	So, after eight years grade 12s you got that much more money. Isn't it fantastic if you're saving then you can get R4060,12 more than just spending the money? So, invest everybody and save.		7:05-7:10
	Okay, I hope you all understand that compound interest.		07:10-7:11
16	Just quickly quarterly - what did I do with my interest rate? I divided by four.	SD ++	07:11-7:15
17	What did I do with my years? I multiplied it by four.	SD ++	07:15-7:30
18	If it wasn't quarterly and it was monthly - divide by twelve - take your years times by 12.	SD ++	07:30-7:35
19	Semi-annually another word for semi-annually: half-yearly. Take your interest rate divided by two and at your years - multiplied by two.	SD ++	07:35-7:54
	Now we're going to come to a very popular sum. It is still compound interest. Grade 12 but in the section, they are asking you to find the number of years. So, what are you looking for - you're looking for the number of years?		07:54-8:09
	So, let's go to example number 2. It's still on the slide. Here everybody - read slowly with me.		8:09-8:20

20	*Example 2:* *R1570 invested at 12% per annum compound interest. After how many years will the investment be worth R23000?*	SD +++	8:20-8:31
21	R1570 is invested at 12% per annum compound interest. Let's read it again: R1570 Rand is invested at 12% per annum compound interest. After how many years - there is the word that you're looking for - after how many years will the investment be worth R23000? Do you all agree with me that it's my compound interest formula?	SD ++	08:31-8:50
22	$A = P(1 + i)^n$ *(The presenter writes the formula on the slide)*	SD +++	08:50-8:51
23	So, I am going to: *A* is equal to *P* 1 plus *i* to the power of *n*. Everybody agrees with me it's my compound interest formula but now I have my investment that will be worth R23000,00. So, I am putting R23000 into the place of *A*. What value goes into the place of *P*? R1570 *(She points to the respective letters in the formula as she confirms the values that would need to be substitutes)*	SD ++	08:51-9:31
24	What is my interest rate? Grade 12s read with me – 12% per annum. 12% per annum compounded. Okay, let's just go back there quickly. It's 12% per annum - so they did compound yearly. Did they do compound quarterly? No, it is just 12%. So, it is going to remain 0,12. *[Comment: As this is a series of online videos, it needs mentioning that it was aired in one session as a result, if she has explained a calculation earlier, she does not re-explain. Here she doe does not explain that interest i =* $\frac{r \div 100}{c}$*]*	SD ++	9:31-10:20
	I'm going do this calculation with you very slowly. I have to use logs.		10:20-10:35
25	Okay, so the first thing I'm going do I'm going to divide here by the R1570 and I'm going to divide this side by R1570 so that its cancels. *(She points to the slide on the screen)*	SD ++	10:35-10:51
26	When you divide side by R1570 it gives you 14,64968.	SD ++	10:51-11:05
27	Why am I not rounding that off to two decimal places? It's not my final answer.	SD ++	11:05-11:28
	If you add that together it gives you 1,12. In grade 12 I am going to use logs.		11:28-11:40
28	So, I am going say n is equal to the log of 14,64968 divided by the log of 1,12.	SD ++	11:40-12:00
	I always teach my students that number which is raised to the power of *n* always goes down to the bottom.		12:00-12:08
	Will you remember that? Don't get confused - don't now say - it's the log of 1,12 divided by the log of 14,64968.		12:08-12:11
29	$\frac{23000}{1570} = \frac{1570}{1570}(1 + 0{,}12)^n$ $14{,}64968 = (1{,}12)^2$ $\frac{\log 14{,}64968}{\log 1{,}12} = n$ *(This appears on the slide as she substitutes)*	SD +++	12:11-12:35
30	Now what you're going to remember is that the number that is connected to the end goes down to my bottom. Now you take your log function on your calculator. So, it's log 14,64968. So, just look with me I have my log function on the calculator. I press log 14.64968 divided by the log 1,12. *(The presenter places her calculator on the slide and demonstrates how to enter the log function on the calculator)*	SD ++	12:35-12:50
31	Grade 12s, you should get an answer of 23,69 correct to two decimal places. $23{,}69 = n$ *(This answer is written under the calculation.)*	SD ++	12:50-12:59

32	And that is the value of *n* if you should round this up to your nearest year, you're going to say *n* is approximately 24 years. So, how long did it take for R1570 to grow to R23000,00 with an interest rate of 12% per year? It wasn't compounded half-yearly or monthly it was just 12%. I got 23,69 to the nearest year it is 24 years and grade 12s, I hope you understood the first three examples.	SD ++	12:59-13:50
	I'm struggling to see anybody sending me some answers. I don't know if the computer is a little bit stuck today, but it doesn't matter if we have done three examples and you understood it and we are now going to go to the grade 12 financial mathematics which is future value.		13:50-14:00

In the second example:

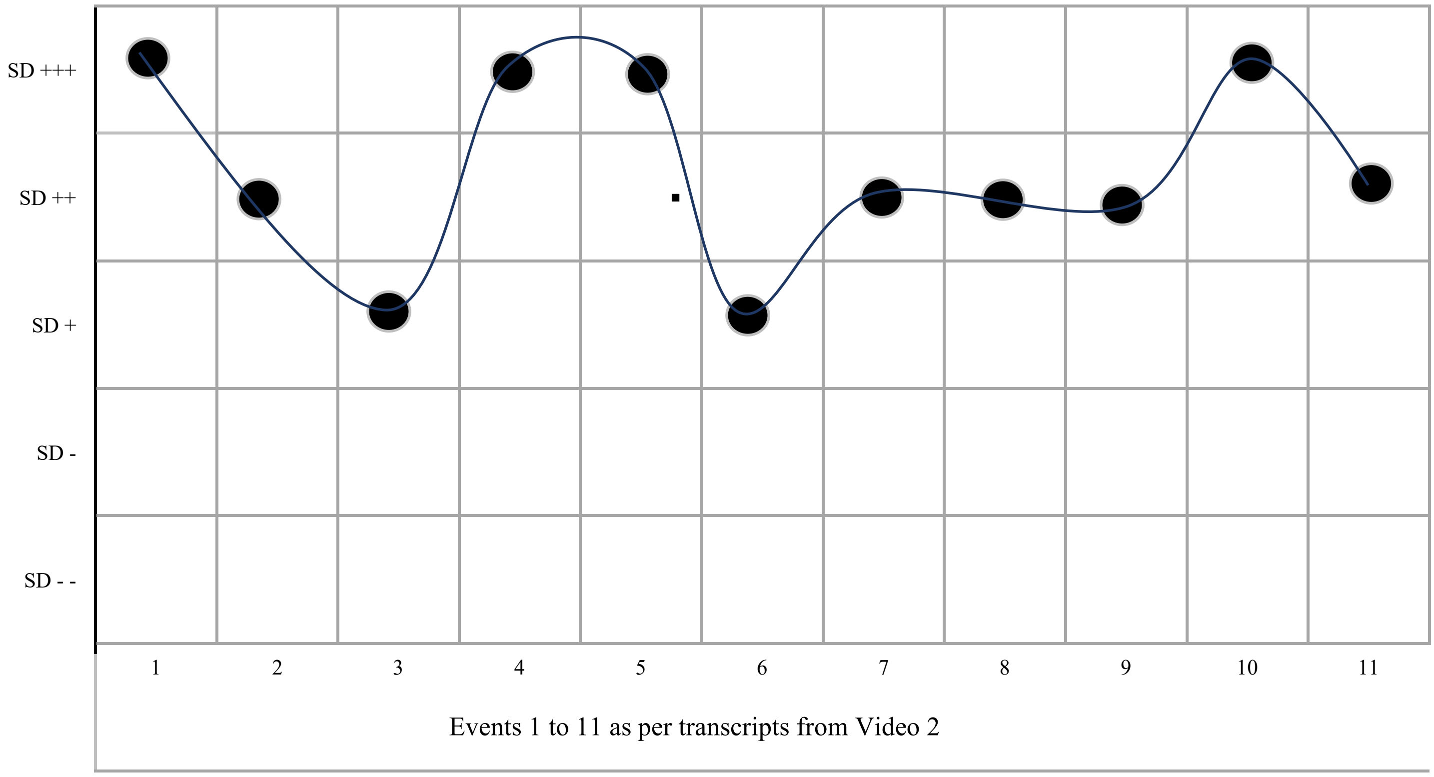
Example 2:R1570 invested at 12% per annum compound interest. After how many years will the investment be worth R23000?

It was in this example that the semantic density has a remarkably high condensation, and this was evident when the graph from event 16 until the end was only SD +++ or SD ++. The reason for this was that the complex financial mathematics was now linked to logs, which are also overly complex. The presenter has done her best to explain logs in this context; however, another example or even a separate video on this combination of financial mathematics and logs may have been helpful.

The detailed analysis of the semantic profiling for video 2 (events 11 to 21) was as follows:

From event 11 to event 12 there was a downward escalation followed by an upward escalation. From event 13 to event 19 there was a high flatline at SD ++. Thereafter, there was an upward escalation followed by a downward escalation. The detailed analysis of the semantic profiling for video 2 (events 21 to 31) was as follows: From event 21 to event 22 there was an upward escalation followed by a downward escalation. From event 23 to event 28 there was a high flatline at SD ++. Thereafter, there was an upward escalation followed by a high flatline between event 30 and event 31 at SD ++.

In video 2, the presenter has once again demonstrated that the use of simple language together with the specific terminologies of financial mathematics and logs can still provide a lesson that does not move away from the complexity or condensation of the concepts, and which allows for the different paths of knowledge that can be transformed by the learner so that understanding maybe achieved.


SD +++
SD ++
SD +
SD -
SD - -
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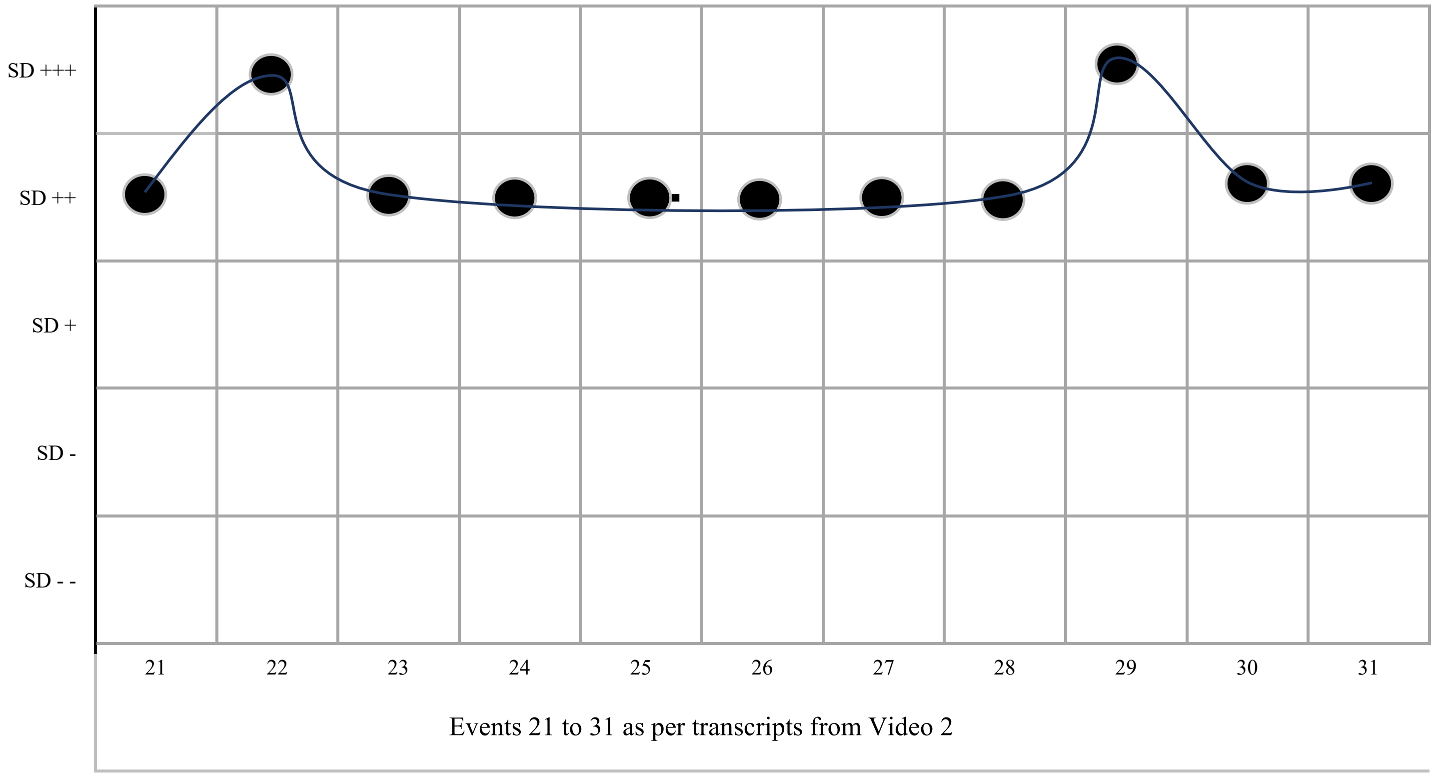
Events 1 to 11 as per transcripts from Video 2


SD +++
SD ++
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Events 21 to 31 as per transcripts from Video 2

5.3. Video 3: Future Value – Example 1

The term 'annuity' can be referred to as a sequence of equal payments that are either paid or received. This occurs during a fixed duration of time with a given amount.

Video 3: Future Value – Example 1			
Event		Code	Time Stamp
1	Now can I explain Future Value to you. Future value is the following: I want to save for the future, I either want to save for my daughter's education, or I want to save for a computer, or I maybe want to become a DJ one day, so I want to buy those sound boxes, and I don't know what else you want to buy but you need to save for it. So that is a future value. Some people save into a pension fund. When I grow older, I don't want to struggle. I want to have money in my account, so I want to save for the future.	SD -	0:00-0:52
2	So, whenever you see a sum were you saving for the future, you take your formula sheet and there is a formula on your formula sheet that I've highlighted there for you. Look at the slide where F is equal to X open the bracket 1 plus *i* to the power of n minus 1 divided by *i*. $F = \frac{x\,[\,(1+i)^n - 1]}{i}$ *(The formula is written on the slide)*	SD +++	0:52-1:23
3	Let's discuss this formula the F as I said is for the future value. You will see this on your formula sheet. What does the x stand for? x is your monthly payments I am going save every single month for my pension fund, I'm going put money into the bank every single month if I want to save for that DJ boxes and the systems that I want so that is my x.	SD +	1:23-1:46
	Once again, my interest rate is *i* just like in the previous examples, my *n* is for the number of years and *i* is also for the interest rate.		1:46-1:59
	Now we can read our first example and then when we done with the first example, I will explain to you why you could not use your compound interest formula. Please you must stay with me. This is not so easy but I'm going to see if I can make it as clear as possible. So, we go to our sum.		1:59-2:17
4	*Example 1:* *Leon deposits R1200 at the end of every month into an account to save for his pension. Calculate how much money he will have in his account after 25 years if the interest rate is 10,2% p.a. compounded monthly.* *(This example appears on the slide)*	SD +++	2:17-2:31
5	Listen very carefully, he deposits R1200 at the end of every month into an account to save for his pension. Can you see everybody? He's saving for something. So, because he's saving for something it is a future value, so you immediately know to get my future value formula from my formula sheet. Calculate how much money he will have in his account after the 25 years if the interest is 10,2% percent per annum compounded monthly. Okay, let's listen very carefully. *(As she reads and discusses the questions, she points to the formula)*	SD ++	2:31-3:07
6	Why can you not use your compound interest formula think quickly, because you're making monthly instalments, monthly payments with a compound interest formula and with a simple interest formula, it was a once-off amount that you put into the bank and that money grew but with a future value you're going to have to remember: number one, future I'm saving for the future I'm saving for something.	SD ++	3:07-3:43
7	Why is it a future value formula and not compound interest because you're making monthly instalments, monthly deposits into your bank so that is why you cannot have the compound interest formula.	SD +++	3:43-3:58
	Whenever you see he is making monthly payments then you know it's a future value		3:58-4:05

	formula. I'm saving for the future.		
8	So, take up your pens and now we're going to fill in our values into our formula. Okay, am I looking for F absolutely because it asked me to calculate how much money he will have in his account of the 25 years? So, I'm looking for F. Okay, everybody put your F down. Do I know what x is? Remember I said x is your monthly payment. Let's go read our sum Leon deposits R1200 and so my sum, my R1200 goes into the place of x. *(She points to the formula as she explains)*	SD ++	4:05-4:39
9	Fantastic, open your block bracket, open your round brackets. Okay, what is my interest rate? 10,2 % compounded monthly. Right, let's go: 10,2 % is 0,102. Everybody how did I get that 0,102? Remember I divided by a hundred. It is compounded how grade 12? It is compounded monthly, remember, divide by 12. Now look here quickly for me, he is going to put that money in the account for 25 years so can I put 25 years? I bet you all saying, no of course, it is no because it's 25 multiplied by 12 if you divide by 12 you multiply by 12. *(She fills in the values as shown in event 12)*	SD ++	4:39-5:22
	Fantastic – 1 close your bracket. What is your interest rate *i*, remember your interest rate that *i* and this *i* is the same and in your sum: R1200 is my monthly instalment, that is my interest rate *I* - 10, 2% compounded monthly for 25 years? *(She fills in the values as shown in the event 12)*		5:22-5:44
	Everybody said ma'am, can't have 25 that's 25 times 12 minus 1 is part of the formula. That interest rate *i* remains the same okay.		5:44-5:46
10	$F = \frac{1200[\left(1+\frac{0{,}102}{12}\right)^{25\times 12}-1]}{\frac{0{,}102}{12}}$ *(After the explanation and substitution, this appears on the slide)*	SD +++	5:46-5:58
	Let me just see if I have some other questions with the previous questions and then you are going to punch this into your calculator. I'm looking for good answers you guys. Remember, future value I'm saving for the future, monthly instalments that is my x, *n* cannot be 25 - it's 25 times 12. Right, I'm waiting for some answers to come through.		5:58-6:23
	I've got some beautiful answers from James school 85, he said *n* is 24 in the previous I have somebody asking me, mam, do we always round up to the nearest year. You know you can leave your answer as 23,69 years. They won't mark it wrong but remember that if they say to the nearest year, you go to 24 years.		6:23-7:05
11	Okay. So, everybody has punched this example into our calculators, and I bet you have this answer. Your answer should be R1 647 505,32. See if your answer compares to mine. I R1 647 505,32 - after I've punched all of that into my calculator. It's quite a big number to punch into your calculators. Please everybody there you see the brackets - you use the brackets. F = R 1 647 505,32	SD +	7:05-7:39
	You can even put this into a little bracket just to make sure if you're not using your fraction button on your calculator and that is your first future value example, I hope that you all understood this example.		7:39-7:47

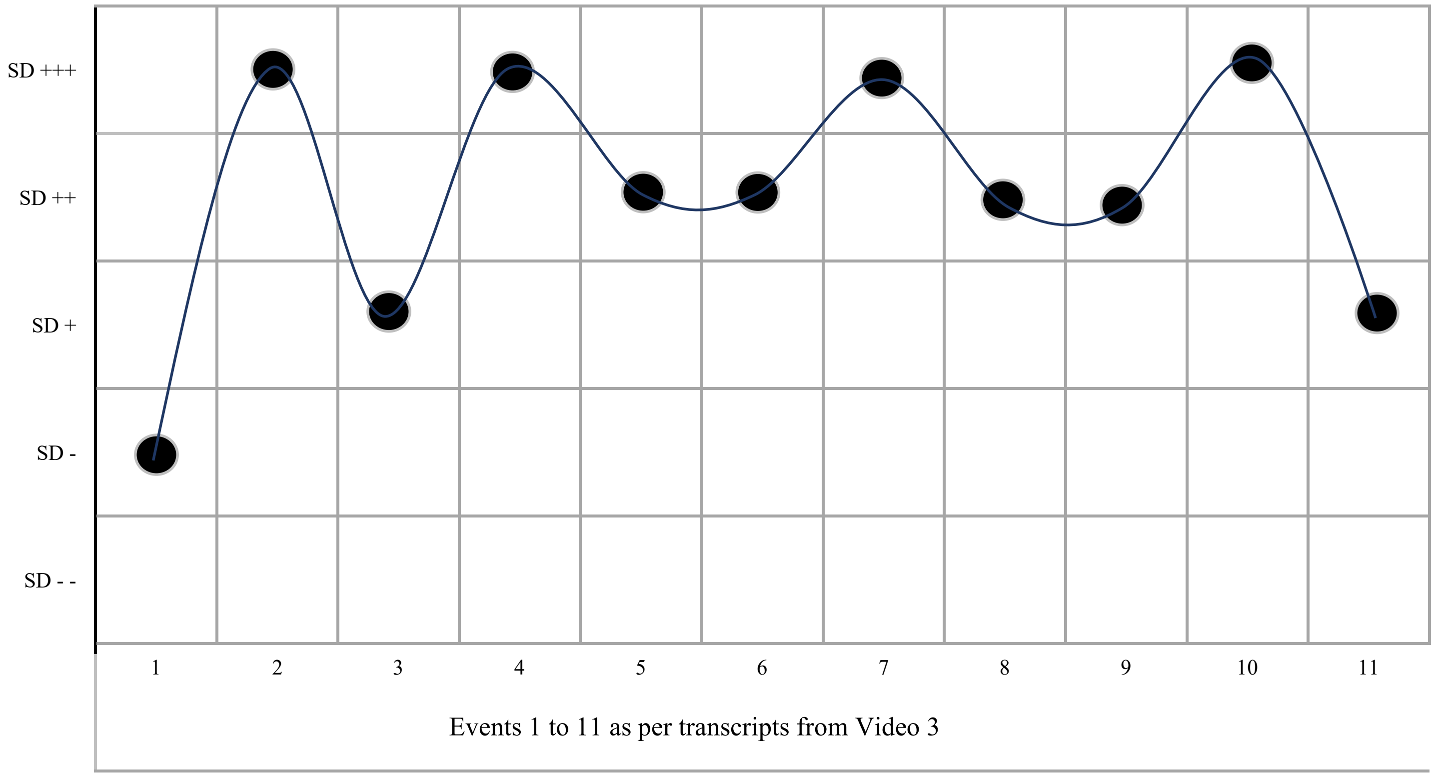
Video 3 has a short transcript, but she was trying to explain an important topic of annuities with which many learners struggle. Initially, the presenter starts this video at a SD –. The reason it was SD - was that, according to the translation device that was created, SD – would be for when words such as ‘value now’ or ‘value then’ could be used. She does mention present value and future value but not as frequently as value now and value then.

It was the first time, since Video 1, that she drops to an SD -, clearly indicating that she had to resort to the everyday language of the learner to explain future value annuity. It does not take her long before she undertakes an upward escalation from SD – to a SD +++, and she then remains in the overly complex or condensed meaning.

The example used:

Example 1:

Leon deposits R1200 at the end of every month into an account to save for his pension. Calculate how much money he will have in his account after 25 years if the interest rate is 10,2% p.a. compounded monthly.



The presenter works hard to explain each aspect of the question and its link to the formula:

$$F = \frac{x\,[\,(1+i)^n - 1]}{i}$$

She continuously repeats herself when she was substituting the values into the equation. Throughout this explanation process, she does not move away from the meanings of the formula and its substitutions thereof.

The detailed analysis of the semantic profiling for Video 3 (event 1 to 11) was the following:

It starts at SD – and there was an upward escalation to SD +++. There was then a downward and another upward escalation. From event 4 there was downward escalation that leads to a high flatline of SD ++ between event 5 and event 6. From event 6 to event 8 there was an upward escalation which leads to a downward escalation to another SD ++ flatline between 8 and 9, followed by an upward and downward escalation between event 10 and event 11.

Once again, the presenter demonstrates that, using appropriate examples, detailed explanations, and the correct financial mathematical terminology in the correct financial mathematical formula, cumulative knowledge was achievable without moving away from the complexity or condensation of mathematical and financial mathematical concepts.

5.4. Video 4: Future Value - Example 2

In this video, the future value of annuities was still being explained but using another example.

Video 4: Future Value Example 2			
Event		Code	Time Stamp
	Let's go to our second example on future values. I'm going to go to the next slide and there is my second example for the future value formula.		0:00- 0:18
1	*Example 2:* *Edwin decides to save a constant amount at the end of each month. He opens a savings account at the end of the month he receives his first salary. The bank offers an interest rate of 4,7% p.a. compounded monthly.* *(The question is on the slide)*	SD +++	0:18- 0:27
	Okay, let's see what it says and also see if you understand why I'm going to use a future value formula. Okay, let's read it.		0:27- 0:37
	Edwin decides to save a constant amount at the end of each month. He opens a saving account at the end of the month. He receives his first salary. This is quite a disciplined guy because he is immediately after he gets his first salary is starting to save. The bank offers an interest rate of 4,75% p.a. compounded monthly.		0:37- 1:04
	2.1. Determine the amount he has to save monthly to have R40 000 on his savings account at the end of 6 years.		1:04- 1:05
2	Right, let's look at our first question for future value. Okay, 2.1. - let's see if we can do the second example. Edwin decides to save. He decides to save. So, what are you immediately knowing - what do you immediately know? It's a future value formula. Immediately grab your formula sheet and write down the future value formula: $F_v = \frac{x[(1+i)^n-1]}{i}$ *(She writes the formula on the slide and frequently points to it as she continues to explain the question and the substitution that follows in the next events)*	SD+++	1:05- 1:34
3	It's x, open up the bracket. 1 plus *i* to the power of *n* minus 1 over *i*. Hi, everybody you wrote down that formula, you've got the example written down - it says he decides to save a constant amount at the end of each month, he opens up a savings account, and he gets 4,75% p.a. but interest rate is compounded monthly. Now, look - determine the amount he has to save monthly - determine the amount that Edwin has to save monthly to have R40,000 in his savings account at the end of six years.	SD ++	1:34-2:16
4	Let's read that slowly - Edwin wants to save an amount every month, he wants to know - how much must I save every month? He's looking for *x*, you are looking for *x* - you do not know what *x* is - how much must I save every month? I'm looking for *x*. *(She continuously points to x as she further explains)*	SD ++	2:16-2:37
5	Okay, how much money does he want in his account, grade 12s, and read, determine the amount he has to save monthly to have R40,000 in his savings account - so his future value that he wants in his account is R40,000 - so the R40,000 goes into the place of *F*, you are looking for *x* let's just see why you're looking for *x* again. It says how much money must he save every month to have this R40,000 in his savings account. *(She points to the formula as she explains)*	SD ++	2:37-3:09
6	Okay, so I do not know what *x* is but it's one plus *i* - okay let's go look at our interest rate - what's my interest rate? So, 0,047 goes into the place of i - how is this interest rate compounded it is compounded monthly, so you are dividing by 12 - close your bracket with me grade 12 – look, at that quickly. I'm going write it in a different colour for you - let's use a bright red - 6 years. Can I just write that 6? Can I just write 6 - of course you can't it must be 6 × 12 - so I	SD +++	3:09-4:18

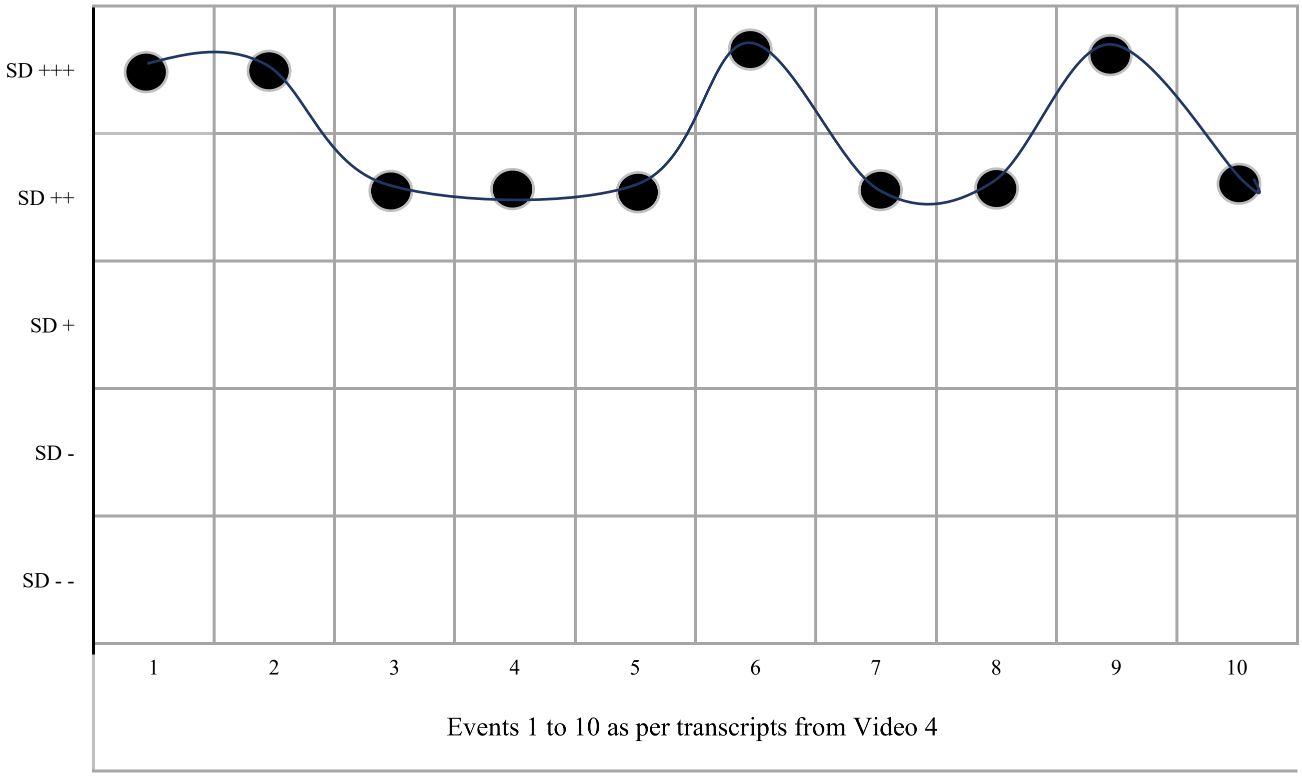
	multiply by 12 - fantastic, minus 1 close your bracket? There, and we are dividing by i. I said that this *i* is the same as this one - so my interest rate is 0,047 ÷ 12. *(As in event 9, she points, explains, and substitutes the values into the value)*		
7	Okay, everybody my unknown in the sum is *x* - I do not know what *x* is – remember, Edwin wants to know how much money must he save every month so he's looking for *x* so that he can have R40,000 in his account after how many years? 6 years. Again, why did I say 6 × 12 - I said 6 times 12 because my interest rate is compounded monthly. Okay, now everybody just a little bit of algebra - here you are looking for *x* - I'm going to give you it to you the best way I think it is to get *x*. *(As in event 9, she points, explains, and substitutes the values into the value)*	SD ++	4:18-5:04
8	Because you're looking for *x* - write *x* alone - *x* is equal to 40 000 × *i* divided by that bracket (she *points to this in the slide).* I'm going to say this again you ever can cross multiply. So, it is 40,000 multiplied by 0,047 over 12 and you divided by the big bracket. There everybody - 1 + 0,047 / 12 to the power of 72 minus 1. *(As in event 9, she points, explains, and substitutes the values into the value)*	SD ++	5:04-5:36
9	$40000 = \frac{x[\left(1+\frac{0,047}{12}\right)^{6\times 12}-1]}{\frac{0,047}{12}}$ $x = \frac{40000(\frac{0,047}{12})}{[\left(1+\frac{0,047}{12}\right)^{72}-1]}$ *(The discussion is done as the computations are written on the slide. She does pause to allow learners to do the calculations)*	SD +++	5:36-5:48
	There you go – so you have to now put all of this in your calculator, and you must now see what answer you get.		5:48-5:59
	I just want to say that Calum – you are from school 9060 – you already SMSed through the answer of R481,90 and that is perfect. This is so so good. Calum, I am very proud of you from Bridgetown High, and you need to have R481,98. *(She writes down the answer on the slide)*		5:59-6:12
10	Let's discuss this value. This is the value Edwin needs to make as saving into his account every month – this – R481,98. (*She points to the answer as she further explains).* Edwin needs to deposit this into his account every month so at the end of 6 years he will have R40000.	SD ++	6:12-6:15

The example used here is:

Example 2:

Edwin decides to save a constant amount at the end of each month. He opens a savings account at the end of the month he receives his first salary. The bank offers an interest rate of 4,7% p.a. compounded monthly.

2.1. Determine the amount he must save monthly to have R40 000 on his savings account at the end of 6 years.



The future value formula is used:

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$40000 = \frac{x[\left(1+\frac{0,047}{12}\right)^{6 \times 12} - 1]}{\frac{0,047}{12}}$$

$$x = \frac{40000(\frac{0,047}{12})}{[\left(1+\frac{0,047}{12}\right)^{72} - 1]}$$

$$x = R481,98$$

At event 9 (which was the substitution of the values in the formula and, thereafter, the cross multiplication), she could have spent more time in this explanation before writing the formula after the cross multiplication.

The semantic profiling for this video was between SD +++ or SD++, and the reason for this was the complexity and substitution of the formula.

The detailed analysis of the semantic profiling for Video 4 (events 1 to 11) was as follows:

It starts with a high flatline between the first two event at SD +++. There was then a downward escalation to a high flatline between event 3 to event 5 at SD ++. An upward escalation was followed by a downward escalation that goes to a high flatline between event 8 and event 9 at SD++. An upward escalation then proceeds to a downward escalation.

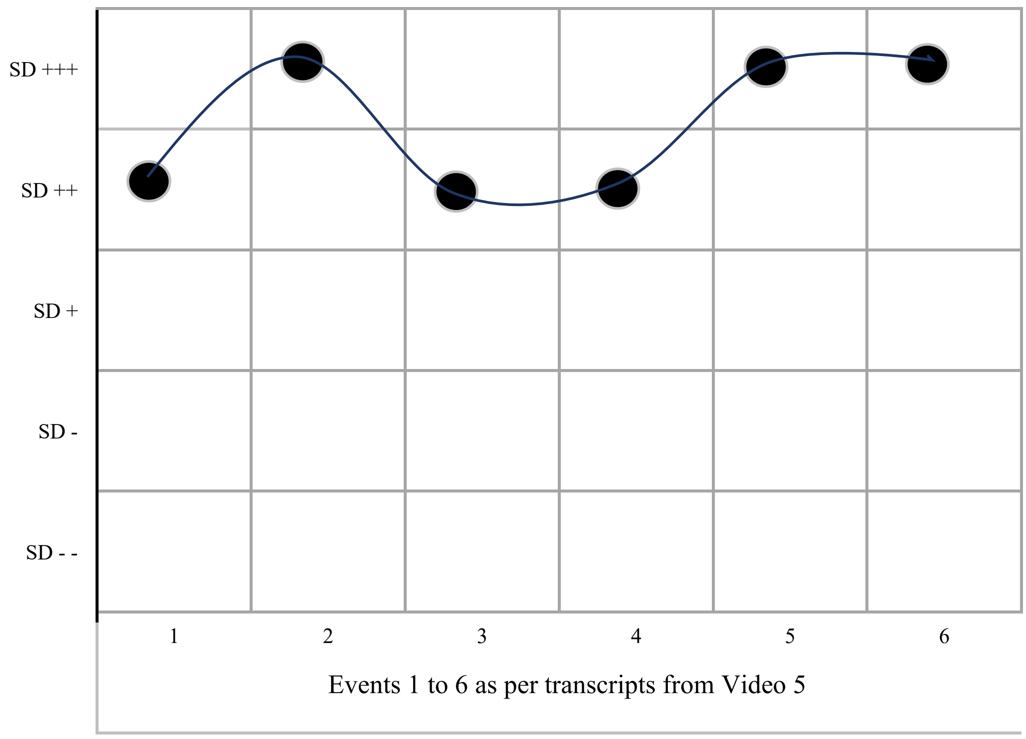
Even though the example used reflects a common scenario, the terminology used never departs from financial mathematics or mathematics, and the explanation and substitution into the formula can be seen as specialised knowledge.

5.5. Video 5: Present Value

The present value of an annuity can be referred to as to the value of that future value now that changes due to the time value of money have occurred. It can be a sequence of equal amounts that occurs during a specific period. It can occur annually, semi-annually, quarterly, or monthly.

Video 5: Present Value			
Event		Code	Time Stamp
	Now, I am going to go – when do I use my present value formula? What is the difference between present value and future value? Listen, very carefully.		0:00-0:26
1	Future Value – I am saving for the future, and it is the F formula on my formula sheet. Now present value – present value means present – I want the money now. I need the money now. Why do you need the money now? Sometimes you need to buy a car now and you do not have the money, so you are going to make a loan. If you go to the bank and ask the bank – Can I make a loan? That means that they are going to give you the money. You will have the money so that is present value. You want to go and study - you may need to make a loan. You can't start saving now to go and study. If you want to study next year you needed to save a few years ago. Do you understand the difference between present value and future value? Let's go to our next slide which is present value.	SD ++	0:26-2:25
2	Okay, so I put the present value formula down on the slide and the present formula looks like this: $P = \frac{x[1-(1+i)^{-n}]}{i}$ That P – this is quite a long formula: x, open up your bracket, 1 – in another bracket 1 + i to the power of negative n. *(She writes the formula on the slide, begins to explain, and keeps pointing to the slide)*	SD +++	2:25-2::40
3	That's the big difference in present value and future value. In your future value it is to the power of positive n and in present value it is to the power of negative n.	SD ++	2:40-5:21

	Let's look at our first example: *Example 1:* *Clyde wants to become an engineer and applies for a student loan of R200 000 to cover the costs of his college studies. The is approved at an interest rate of 10,25% p.a. compounded monthly. He prefers to pay the loan in 48 equal monthly payments. These payments start 1 month after receiving the loan.* *1.1. Calculate his monthly repayments.* *1.2. Calculate the outstanding balance immediately after his 16th payment has been made.* *(The example appears on the screen and remains as she continuous to go into a detailed explanation and refers to the formula as well)* Clyde wants to become an engineer- guys, we don't have a lot of engineers in our country, so, if your maths marks and your science marks are good, you can study engineering. Clyde wants to become an engineer. Alright, and applies for a student loan of R200 000 to cover the cost of his college studies. Remember, he's making a loan – just look at that word – loan – when you see that word loan – it's a present value formula. The loan is approved at an interest rate of 10,25 % p.a. compounded monthly. He prefers to pay the loan in 48 equal monthly payments. These payments start one month after receiving the loan. Grade 12s, please look at that word – one month after receiving the loan – you don't do anything – I am going to explain to you exactly what you're going to do. Calculate these monthly payments. I'd love to do both sums and I wish that I am going to have enough time because this is such an important question. I've seen it in almost every paper.		
4	Okay, so he needs how much money – go slowly with me – he needs R200 000 – R200 000 is his loan. You do not have *x* because Clyde wants to know how much he needs to pay month. So, you do not have *x*, he wants to know how much he needs to repay every month.	SD ++	5:21-5:43
5	*200 000* = $\frac{x[1-\left(1+\frac{0,1025}{12}\right)^{-(48\times12)}]}{\frac{0,1025}{12}}$ Now Clyde makes a loan of R200 000, the interest rate is 10,25 so it will be 0,1025 and is compounded monthly so that is divided by 12. My *n* is 48 multiplied by 12 and the interest rate here is there same as on top. *(As she explains she substitutes the necessary values in the formula)*	SD +++	05:43-6:15
6	Look at the sum quickly. Clyde wants to become an engineer and applies for a student loan of R200 000 - there is my R200000 (*she points to the slide with the formula).* What is x? I do not know. There is my interest rate. You need to make 48 equal payments. You need to cross multiply, and you will get this: X = $\frac{200000(\frac{0,1025}{12})}{[1-(1+\frac{0,1025}{12})^{-48}]}$ X is going to be R200 000 times 0,1025 divided by 12, all over 1 minus, in brackets, 0,1025 divided by 12 to the power of negative 48.	SD +++	6:15-8:37
	I am going to give you a few minutes to do that quickly on your calculators. Punch it in. *(She places her calculator on the slide as she demonstrates the entering of the information in the calculator)* The answer is: (*she writes it on the slide)* *X = R5096,96* I trust that all of you understand the sum and I am going to look at some questions while you guys are working, and you have got some brilliant, brilliant answers. Well done to Marco from Plutus Ville High School. Well done to Stefan from Titusville High School. I have Charmaine as well. Welcome Lonnie – you are from Wellington Secondary. Pulane please remember to always round off your final answers. It is beautiful that you guys understand.		8:37-8:54



The following example was used:

Clyde wants to become an engineer and applies for a student loan of R200 000 to cover the costs of his college studies. The is approved at an interest rate of 10,25% p.a. compounded monthly. He prefers to pay the loan in 48 equal monthly payments. These payments start 1 month after receiving the loan.

1.1. Calculate his monthly repayments.

The following equation was used:

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

A thorough and detailed explanation was given and repeated before she substitutes the values in the equation:

$$200\ 000 = \frac{x[1-\left(1+\frac{0,1025}{12}\right)^{-(48\times12)}]}{\frac{0,1025}{12}}$$

She uses cross multiplication to solve for x. However, there could have been some emphasis placed on this step as in the previous video she does not place emphasis on cross multiplication. If she had placed some emphasis in the previous video and then stated here that cross multiplication was as per the previous video, it may have been easier for the learners to understand. Sometimes, as educators, we assume that learners know because it should be a pre-requisite.

After the cross multiplication:

$$X = \frac{200000(\frac{0,1025}{12})}{[1-(1+\frac{0,1025}{12})^{-48}]}$$

$$X = R5096,96$$

The detailed analysis of the semantic profiling for Video 5 (event 1 to 6) was as follows:

It starts with a SD ++ with an upward escalation to SD +++ that moves downwards towards a SD ++ and high flatlines with event 4 at SD ++. From event 4 to event 5 there was an upward escalation to SD +++ that remains as a high flatline in event 6. The reason for ending the video as SD+++ was that it concludes with the entering of a condensed formula onto a calculator as a definitive answer.

The condensation of the formula never weakens the semantic density to lower than SD ++. This section of financial mathematics was overly complex and very condensed. It therefore remains in the upper levels of the semantic density profile.

This does not mean that the transformation of knowledge did not occur. The use of the formula with the correct financial mathematical terminology and an appropriate example ensures that the condensation of the financial mathematical concept was maintained, and that cumulative knowledge was possible even at elevated levels of complexity.

5.6. Video 6: Outstanding balance

Outstanding balance, with reference to a loan, is the total amount that is still being owed. This video is linked to Video 5. It uses the same example but focuses on the second part of the question, which was:

Example 1:

Clyde wants to become an engineer and applies for a student loan of R200 000 to cover the costs of his college studies. The is approved at an interest rate of 10,25% p.a. compounded monthly. He prefers to pay the loan in 48 equal monthly payments. These payments start 1 month after receiving the loan.

1.1. Calculate his monthly repayments.

1.2. Calculate the outstanding balance immediately after his 16th payment has been made.

She uses the same formula as in the previous video:

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

She takes a lot of time to explain that the value of n was 48 -16 =32. She repeats this explanation a few times.

She uses the answer from 1.1. as x and does that first as a substitution in the equation:

$$P = \frac{5096{,}96[1-(1+i)^{-n}]}{i}$$

She normally fills in the substitution all at once but because she was focusing on the value of n, she substitutes the value of x first, which was calculated at the end of Video 5.

$$P = \frac{5096{,}96[1-\left(1+\frac{0{,}1025}{12}\right)^{-32}]}{\frac{0{,}1025}{12}}$$

She still does not do all the substitution at once. She once again explains the value of n before it is substituted it into the formula as shown above.

The definitive answer is:

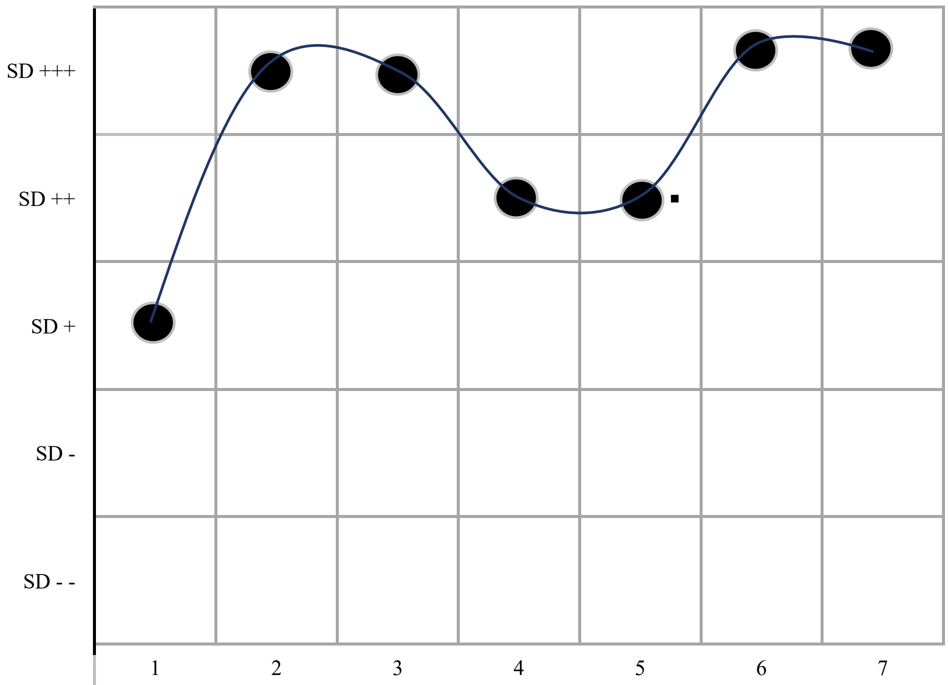
P = R142184,12

The detailed analysis of the semantic density profile for Video 6 (events 1 to 7) was as follows:

It starts with an SD + with an upward escalation to SD +++ that remains as a high flatline with event 3. There was a downward escalation towards SD ++ that has a high flatline during event 5, which proceeds to an upward escalation to event 6 that has a high flatline during event 7 at SD +++. The semantic density profile of this video starts from SD +, as she uses financial mathematical terminology with examples for learners to ensure that understanding was achievable. Thereafter, her constant use of the formula and the appropriate terminologies linked to the formula during the substitution process keeps the semantic profile at the upper levels of semantic density.

Video 6: Outstanding Balance			
Event		Code	Time Stamp
	Our last question before we end off our session is question 1.2. I'm going to keep this here. I looked through a lot of past year papers when I prepared this lesson and I have seen this question in almost four or five papers.		0:00-0:33
1	Let's look at this question - remember now: Clyde is going to take out R200 000. He is going to make 48 equal payments and his payment every time will have to be R5096,96. But listen very carefully he can go to the bank, and he can tell them after a certain time - you know what - I want to know what the balance on my loan is? How much do I still owe you? Because maybe he got some money from a relative and the relative says 'Clyde, don't worry- don't worry, go pay everything at once,' so he wants to settle his loan. Do you understand what I'm trying to say - he doesn't want to continue paying 48 equal payments - he wants to maybe settle his loan before the time - all he wants to do is go to the bank and ask how much must I still pay? What is the balance on my loan so look at 1.2. guys I've seen this in a lot of papers, and I want you to get it right if you see it again.	SD +	0:33-1:40
2	*Example 1:* *Clyde wants to become an engineer and applies for a student loan of R200 000 to cover the costs of his college studies. The is approved at an interest rate of 10,25% p.a. compounded monthly. He prefers to pay the loan in 48 equal monthly payments. These payments start 1 month after receiving the loan.* *1.1. Calculate his monthly repayments.* *1.2. Calculate the outstanding balance immediately after his 16th payment has been made.* In 1.2. it says calculate the outstanding balance - calculate the outstanding balance - immediately after his 16th payment has been made. When you see outstanding balance, you again use your present value formula.	SD+++	1:40-2:00
3	$P = \frac{x[1-(1+i)^{-n}]}{i}$ X open bracket, 1 minus 1 plus I to the power of negative n. *(She writes down the present value formula and explains the value of each part of the formula as she writes it down)*	SD+++	2:00 -2:13
4	And now remember everybody you want to know what - listen - calculate the outstanding balance? So, you are looking for P. So, Clyde is paying R5096.96 every month and after his 16th payment he wants to know how much does he still owe? Now you are looking for P because you want to know how much he owes. That R5096.96 goes into the place of X and here comes the most important thing - you have to remember - what are you going to put into the place of n?	SD++	2:13-3:01
5	Please everybody just looks – you had 48 payments right - you had 48 payments before the loan would have been settled. But he made 16 payments. So how many months would he have left? It's 48 minus 16 is 32 months. *(She writes 48 – 16 = 32 on the right-hand side of the slide)* Do you understand that 32 payments is what you put into the place of n? So, it's going to be n is 32, I'm going to put the interest rate in. Now first I want you to understand if you get these 32 payments - a lot of teachers teach this using a future value and present value at the same time or compound interest formula. Teachers and students, if you want to see what the balance on his loan is take the number of months and just subtract the months that he has paid already. So, he would have made 48 payments, but he wants to know if 16 payments have been made - 48 minus 16 which is 32 - and you merely take that 32 because that is the amount of payments, he would have had left, and you substitute that into your formula.	SD++ SD+++	3:01-4:43 4:43-4:50
6	So, we can now substitute this into our formula X is R5096,96, I - remember our interest rate is still the same - do you agree with me - it is 10,25%. How is it compounded? Everybody it is compounded – monthly. Fantastic - what do I put into the place of n - think with me - 48 you paid 16, this is 32 payments left. Divided - remember your interest rate remains exactly the same and you put that into your calculator.		4:50-4:53

	$P = \frac{5096,96[1-(1+i)^{-n}]}{i}$ $P = \frac{5096,96[1-\left(1+\frac{0,1025}{12}\right)^{-32}]}{\frac{0,1025}{12}}$ *(As she was explaining, she was pointing to the formula and substituting the values in.)*		
	I want everybody to put that that into their calculators quickly and SMS your answers through to me. I hope you all understand the 32 - 48 payments, 16 he made so there is 32 payments left. I want everybody to put that into their calculators and to see what they get. All right it's quite a long calculation so you use your fraction button on your calculator. I am waiting for your answers. Guys, everybody on the calculators. I'm getting some fantastic answers. There some of you - out by a few cents - maybe you all are rounding off too much in your sum.		4:53-5:36
7	I have Nikita with says R142 184,12 and Dale also R142 184,12. Tina - just check your answers you are out by a few rands and everybody I want some more answers coming through and the answer to the sum is indeed R142 184,12. *(She writes the answer on the slide)*	SD+++	5:36-6:53
	Everybody there marks the end - almost the end of our lesson. Remember, quickly compound interest – once-off value, simple interest -once-off value, once off future value. I'm saving for the future then you use the F-formula on your formula sheet. What is the difference between present and future - think with me present I need to need to make a loan I need the money now - future value I'm saving for the future that's the different between future and present value?		6:53-7:28
	I hope that this lesson this afternoon has helped you. I've seen such a lot of students who says that they do understand. I want to wish you well for your prelim exams I want to say that it's almost 3 weeks - just 3 weeks then you're writing these prelims exams. Everything of the best - study hard - no party - now sitting with your books every evening, every day as much as you possibly can, download lots of papers from the internet if you want to and if you have the facilities available and everybody remember financial mathematics is not an easy question in the paper, but I hope that this afternoon's lesson has helped. Thank you everybody for tuning in and for helping me with my answers and working through the sums with me - have a great great afternoon- bye-bye		7:31-8.21



Events 1 to 7 as per transcripts from Video 6

All six videos are online lessons. The presenter has limited direct contact with the learners, but she does have online participants who send SMSs while she presents. Therefore, these online lessons were live and can now be viewed by anyone on YouTube. The presenter is therefore unable to observe learners under normal classroom interaction and is not truly able to assess if knowledge building has occurred during the online lessons. Figure 1 (page 14) describes learning and the integrated process that can occur in mathematics. Through the transcripts, the process of understanding and problem solving stand out. Problem solving involves interpretation, formulation, investigation, and modelling to solve problems such as word problems using mathematics in complex and simple situations. The process of understanding occurs when connections are made between examples, experiences, and concepts.

Pournara (2009) identified how important it is that educators who teach financial mathematics should have specialised knowledge. This was evident through the transcripts since financial mathematics is complex and contains highly condensed concepts. The semantic graphs for all six videos therefore demonstrated a stronger semantic density. The specialised knowledge, apart from finance, is inclusive of the knowledge on concepts such as substitution of values into the formulas, working with decimals, cross-multiplications, and functions such as logarithms. The interrelatedness of mathematics and financial mathematics is further emphasised by the five strands of mathematical proficiency (Kilpatrick et al., 2001). The conceptual knowledge that was evident in the transcript clearly demonstrates that, if a learner does not understand logarithms, then the use of that disciplinary knowledge will impair their ability to solve the appropriate annuity problems in financial mathematics. Procedural knowledge is being able to identify the problem that is being presented, choosing the correct financial mathematical formula, and following the appropriate process before substituting the value into the formula. Similarly, the other three strands of strategic competence, adaptive reasoning, and productive disposition (Figure 1 on page 14) are never examined in isolation but as interrelated, interwoven building blocks for current and future development of knowledge that promotes the variation of knowledge that can occur in the classroom. Adaptive reasoning would use rational thinking to determine and confirm the route that can be taken to solve a particular problem. The five strands of mathematical proficiency were included and considered in the design of the translation device. It also plays a vital role in the consequent analysis of the transcripts that were carried out against the same translation device. The strands of conceptual and procedural understanding are interrelated to relational understanding in mathematics (Skemp, 1976). From the transcripts, it was evident that relational understanding is a vital component of financial mathematics. The complexity of the concept that is interrelated with concepts in mathematics requires applied focus for the transformation of knowledge to occur in the lesson (Skemp, 1976). If a learner did not understand logarithms, it would therefore follow that the learner would struggle with logarithm-based questions in annuities in financial mathematics due to an existing weakened procedural fluency. If a learner is unable to understand, through interpretation, that a particular financial mathematical problem requires the use of a special formula, a learning gap is created and will widen as the learner not be able link new financial mathematical ideas due to their previous errors or misconception (Sarwadi et al., 2014).

Makonye et al. (2014) emphasises the mathematical concepts that underpin financial mathematics and therefore their connections. These concepts include percentages, time, geometric sequences including the nth term, and the sum of the geometric sequences. Furthermore, even if learners do understand these concepts, they may struggle with their practical applications in financial mathematics. In the transcript of the online lessons, the presenter does make the occasional reference to errors or misconceptions that learners make in the calculation of financial mathematics. As mathematical educators, we need to be mindful that errors and misconception can occur during the teaching and learning process either in the field of mathematics or financial mathematics. This could therefore appear in the learners' responses that could be related to application or understanding of meaning. Pournara (2013) suggests that a general formula for compound interest should be used in the curriculum:

$$F_V = PV(1+i)^n$$

If the presenter had used this in her slides, together with the reference of Future Value (FV) and Present Value (PV), then it could have given more substance to her presentation. This formula calculates the future value of money. A close relationship exists between *n* which shows the specified period that needs to be used in the calculation and is the same specified period that will be used in the linkage to the interest rate. The present value annuities formula is $Pv = \frac{x[1-(1+i)^{-n}]}{i}$ and the future value annuities formula is $Fv = \frac{x[(1+i)^{n}-1]}{i}$. The future value formula is very condensed as it consists of the repeated use of the formula FV =.$PV(1+i)^n$ over specified periods that are consecutive. The future value is formulated by the process of a geometric series in which the constant ratio is greater than one. This formula would only be right if there is no open period at the beginning or at the end of the timeline, which would write down that payments would start at once and end on the last day. The present value formula is very condensed as it is founded on repeated use of the formula: Pv = $F\mathrm{v}(1+i)^{-n}$ over specified periods that are consecutive. The future value is formulated by the process of a geometric series in which the constant ratio is less than 1. This formula would only be proper if there is one period open before the first payment is to occur.

The value of i must be formulated on the effective rate per period. The number of payments in the annuity and not the specified period of the loan stands for n. The specified payments paid at the specified periods stands for x. The outstanding balance can be calculated after the last payment has been made. In financial mathematics, there are two types of fixed payment annuities. In the first type, the payments are fixed. Therefore, the period of the loan becomes variable, and the use of logarithms is needed in the calculations. In the second type, the period is fixed. Therefore, the payments become the variable over n, which is the period, and a final payment at $n + 1$, which would be less than the regular payment. A relationship exists between present value with compound interest and the future value with compound interest. Pournara (2013) felt that awareness needed to be practiced in the classroom that would prove this relationship of compound interest to annuities. The condensation of the annuity formula is a constellation of knowledge that is interrelated. Four interrelated ideas were created when Pournara and Adler (2014) focused on revisiting mathematics for mathematical educators. These were the content, goals/purposes, tasks and activity, and resources. These ideas were clearly reflective in the analysis of the transcripts of all six videos. The understanding of the above concepts played a vital role in the creation of the translation device and, therefore, the way the transcripts were analysed against the translation device (Table 1 on page 28).

The semantic profiling as explained focuses on the unpacking and repacking of knowledge building, which is illustrated in Figure 5 (page 27) , Figure 6 (page 27), and Figure 7 (page 61). Semantic density in the semantic profiling focuses on the intricacies of knowledge. It provides valuable insight into positive and meaningful learning experiences in the classroom, as it illustrates the semantic density of the knowledge being interpreted in the classroom over a specified period.

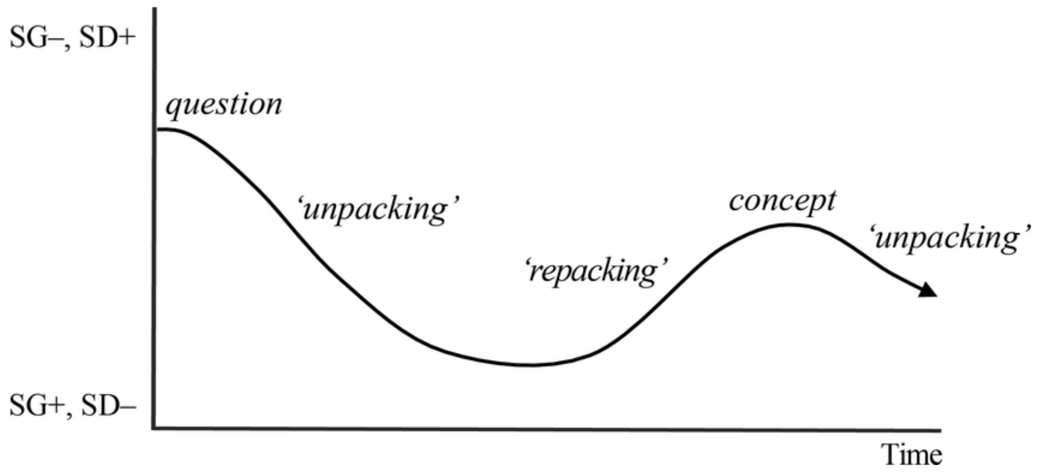


Figure 7: Semantic profiling (Maton, 2020).

This was demonstrated through the transcripts that have produced a semantic wave. A semantic wave in context of the presenter and the online lessons was viewed as a planned effort to connect the complex to the simple by trying to connect the condensation of financial mathematics with practical examples. Maton (2014) alludes to the view that if there is only a downward escalation or an upward escalation it may be interpreted as a breakdown in knowledge building. The condensation and complexity of financial mathematics can also be influenced through the examples that the educator uses in the classroom to enhance the understanding of knowledge building. This is evident in the SACE PTSs six, seven, and eight (SACE, 2018). The presenter uses her own examples to enhance the online-teaching experience. The examples were clear and precise and did not mislead the learners in the knowledge-building process. It is also important to note that the educator carefully considered the choice of questions created to enable the ''learning gains for all learners'' (SACE, 2018). The presenter constantly repeats her explanations as a means of affirming her meaning to promote knowledge building. The codes created for semantic density (LCT) through the translation device have allowed for the transformation and deviation of the knowledge building to be conceptualised (Maton, 2014).

6. Conclusion:

The lens of LCT allows the teaching practice to be documented and measured. Semantic profiling, with reference to semantic density, enables the researcher to observe the extent of the condensation of knowledge. Semantic profiling is important because semantic waves can demonstrate the transformation in the transmission of knowledge. A semantic wave is a semantic profile that demonstrates the transformation of cumulative knowledge that is possible in a practice such as teaching and enables one to observe how this contributes to knowledge building (Maton, 2014).

A downward escalation occurs when the lesson starts with a complex meaning or condensed concept. The educator can unpack the concept being taught through explanations, using simpler language. It is also at this level that the connection of other concepts can be made. For example, the connection of financial mathematics and the calculation of logarithms. Upward escalations occur as learners work with these connections and repacking takes place, which allows for a better understanding of these connected concepts. The upward escalation creates condensation of the concepts being taught and the creation and sharing of specialised knowledge occurs between educator and learners. It does not mean that because there are upward escalations with high flatlines that there was no transformation of knowledge. The significant factor to consider is that the semantic wave created is continuous rather than broken, which reflects the linkage between stronger semantic density and the lower semantic density. A high semantic flat-line could be an indication that knowledge-building was being opened within this context and therefore positive knowledge-building was being achieved. It could therefore imply that a low semantic flat-line limits knowledge-building and can pose difficulties in understanding context. A broken semantic wave would be indicative of knowledge that has not been connected. With the use of the translation device (Table 1 on page 28) the transcripts of the series of videos were analysed and profiled on a semantic graph. Each of the graphs for each of the videos have indicated a semantic wave (figure 5, page 27), which would indicate that there has been a continuous and positive transformation of cumulative knowledge in the online lesson. The presenter in the video could be seen as a specialist in the topic, as she was able to move between simple knowledge and more abstract, complex knowledge and vice versa with relative ease and continuity in the lesson. Maton (2020) did recognise that more research is being done in education using the dimension of semantics in LCT to analyse the ‘semantic waves’ that can promote more success in education.

The complexity and condensation of the concept of financial mathematics can be taught successfully by using simple language with financial mathematical terminology and formulae. This was demonstrated by the teacher’s ability to take overly complex concepts, with a strong semantic density, and impart them in ways that simplify and unpack the meaning through a weakening of the semantic density. However, due to the complex and manifold nature of financial mathematics, which draws on sophisticated mathematical concepts, the semantic density remains strong. This has implications for both learners and teachers since effective knowledge building requires learners to master numerous foundational skills and be equipped with specialist mathematical knowledge. Without these foundations in place, cumulative knowledge building is impaired.

The limitation would therefore be that if knowledge remains condensed and overly complex, over time it may negatively impact on learners in that those learners who do not understand will not understand. It would therefore imply that the educator or expert in the knowledge would need to constantly reaffirm current knowledge before the educator can connect and build new knowledge to prior knowledge. A teaching and learning process that occurs in any lesson, whether it happens in the classroom or in an online lesson, is deemed successful if the knowledge transmitted by the educator is interpreted and transformed by the learner whereby the understanding of the knowledge is achieved as intended. However, as educators or any expert of knowledge, a deeper understanding of the intricacy of knowledge needs to exist. It would include the understanding of the knowledge and integrating that into the application and evaluation of that knowledge. Once that analysis occurs a determination can be made about whether the positive transformation of cumulative knowledge building has been achieved.

This research study has shown that financial mathematics by its curriculum design can be overly complex and has a deep constellation of condensations, as it integrates other mathematical concepts and procedures such as logarithms, geometric sequences, and many more. The utilisation of the financial mathematics and financial mathematical terminology, in collaboration with the appropriate activities, enables a specialist educator to achieve a positive transformation in cumulative knowledge building. LCT as an analytical toolkit was used to measure the practice of the online lessons and has proven that financial mathematics has overly complex content with deep condensation in meaning. This was evident in the analyses of the transcripts that used the translation device that was created for this purpose. It was further evident in the events graph for all six videos that the semantic profiling indicates that the lessons remained in the higher semantic density. It indicated that cumulative knowledge building did transform in a positive way but, as educators, constant monitoring of learners needs to be undertaken to ensure that learners understand a concept such as compound interest, which is the underlying principle of annuities.

In the last few years, the education environment has undergone many changes where the traditional methods of teaching and learning are being re-evaluated. Similarly, the way in which teaching and learning was being evaluated also needs to evolve to include different analytical tools such as LCT. LCT could be utilised to investigate the lens of specialisation in mathematics and financial mathematics through semantic profiling. There are many aspects of LCT, and as it becomes more widely used in academic research in education (especially mathematics and financial mathematics) it would create a greater dialogue in the academic research, which can only aim to benefit the teaching and learning environment for both the teacher and the learner.

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