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Natural supersymmetry and unification in five dimensions

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Abstract: We explore unification and natural supersymmetry in a five dimensional extension of the standard model in which the extra dimension may be large, of the order of 1–10 TeV. Power law running generates a TeV scale A_t term allowing for the observed 125 GeV Higgs and allowing for stop masses below 2 TeV, compatible with a natural SUSY spectrum. We supply the full one-loop RGEs for various models and use metastability to give a prediction that the gluino mass should be lighter than 3.5 TeV for $A_t \geq -2.5$ TeV, for such a compactification scale, with brane localised 3rd generation matter. We also discuss models in which only the 1st and 2nd generation of matter fields are located in the bulk. We also look at electroweak symmetry breaking in these models.

Keywords: Beyond Standard Model, Higgs Physics, Supersymmetric Standard Model

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Contents

1 Introduction

In the context of supersymmetry and through the prism of the naturalness aesthetic, the discovery of a Standard Model-like scalar particle of mass $m_h \sim 125\,\text{GeV}$ [1, 2], and no direct evidence so far of superparticles has motivated renewed interest in non-minimal extensions of the Supersymmetric Standard Model (SSM) that can help to compellingly explain such results. Within the Minimal-SSM (MSSM), for the lightest CP even charge neutral scalar to be the discovered scalar then requires either multi-TeV stops, which is disfavoured from naturalness, an enhancement to the tree-level Higgs mass such as for example [3–6], or a near maximal mixing scenario whereby $|A_t(M_z)| \gtrsim 1\,\text{TeV}$. There are few models that compellingly achieve a large enough A_t if one first assumes A_t to vanish at some initial supersymmetry breaking scale. Even if one obtains such a large A_t, one must still explain why stops are lighter than their first and second generation counterpart squarks, consistent with collider bounds [7–17]. One such framework that can address both problems is a *five dimensional*-SSM.

In five dimensional (5D) SSMs, power law running for a sufficiently low compactification radius R, generates at low energies a large enough A_t to explain the observed Higgs mass [18]. Furthermore, through spatially localising different generations along the extra dimension(s), one can explain geometrically why the third generation can be consistently lighter than its first and second generation counterparts [18].

This framework is sufficiently compelling that it should understandably endure further scrutiny. In particular, five dimensional theories are effective field theories with a cutoff and are (often over-dramatically) defined as non-renormalisable, as many parameters such as gauge couplings can be sensitive to this UV scale. It is therefore important to confirm that results and conclusions made at one loop that are sensitive to this scale are still consistent and under control at two (and higher) loops. For instance one might be concerned that one loop linear sensitivity to the cutoff behaving as ΛR do not result in terms of the form $(\Lambda R)^2$ at two-loop, which would then indicate a break-down of perturbation theory at renormalisation scales of the order of the compactification radius [19]. Whilst this might be of concern to non-supersymmetric theories, the five dimensional SSM is reinterpreted in the language of $\mathcal{N} = 2$ four dimensional supersymmetry. This additional supersymmetry and the protection it affords, helps to reduce such terms [20, 21], at least for gauge couplings. The effect remains but has opposite sign for both Yukawa couplings and their soft breaking trilinear counterparts, and so is still under complete control. For the case of bulk matter and in particular the top Yukawa in the bulk, a Landau pole appears and one must then seriously consider that either perturbation theory is problematic for these models (just

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as one would in any four dimensional theory with a Landau pole), or that a compelling explanation of how the top Yukawa may arise must be found such that this pathology may be avoided. Another issue which is a general one in these effective theories, is that in order to be sure that no unwanted operators are generated one should consider the possible UV completions. This point is discussed in the literature, and is typically a difficult model building effort, but goes beyond the scope of the present work which is limited to the investigation of the effective theory. Furthermore, possible issues related to proton decay shall be briefly mentioned in section 2.1.

There are further criteria for our model to be truly compelling: we require that it is supersymmetric and that supersymmetry is softly broken, that the superpotential is renormalisable and that the theory's gauge couplings *unify* in the five dimensional description with a large enough extra dimensional scale as to make the extra dimensional features practically relevant to the phenomenology of the model. In other words we require a $1/R \sim 1$ to 10^3 TeV scale extra dimension and not simply an (almost) GUT scale extra dimension. Such a criteria is useful to rule out certain models, for instance by this criteria one can straightforwardly rule out flat extra dimensional models in which the 1st and 2nd generation are in the bulk, with the 3rd generation either in the bulk or on a brane, as such a model can only unify with an extra dimensional scale of the order of the GUT scale, a topic we discuss in more detail later.

The outline of the paper is as follows: in section 2 we describe the models in detail and discuss unification. In section 3 we describe our boundary conditions and how the four dimensional (4D) and 5D renormalisation group equations (RGEs) are matched and solved. We discuss the various energy scales of the model and then look at the running of various parameters including the gaugino mass spectrum and trilinear soft breaking terms. In section 4 we explore how to obtain the correct 125 GeV Higgs mass, with stops lighter than 2 TeV. In section 5 we give our conclusions. We also include two detailed appendices, appendix A including all the one-loop and two-loop RGEs of the four dimensional low energy model, of which we used the one-loop RGEs in the plots, and appendix B includes the one-loop RGEs for the five dimensional models 1 and 2 of the main paper. The conventions and notation of this paper follow closely that of [18], which are based on conventions found in [22–26].

2 5D-SSM with additional states: unification

A TeV scale SSM in which the gauge coupling is precisely unified is proposed in [27]. The key idea is to add two new hypermultiplets $F^{\pm}$ which are singlets under $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L$ and charged under $\mathrm{U}(1)_Y$ with $Y_{F^{\pm}} = \pm 1$. The SSM chiral fermions are located on a boundary and in the 5D picture do not have Kaluza-Klein (KK) modes. The SSM Higgs chiral multiplets live in the bulk and we embed them as hypermultiplets in 5D. The gauge fields and the additional states also live in the bulk as listed in table 1: we call this *model 1*. We will also explore our own model in which the third generation of superfields lives in the bulk, as in table 2: we call this *model 2* and this too may unify. We compute and collate all supersymmetric and soft-term RGEs. These new states modify the beta

Superfields	Brane	Bulk	$\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_c$
$\hat{q}^f$	✓	—	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{d}^f$	✓	—	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}^f$	✓	—	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{l}^f$	✓	—	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{e}^f$	✓	—	$(1, \mathbf{1}, \mathbf{1})$
$\hat{H}_d$	—	✓	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	—	✓	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{F}_-$	—	✓	$(-1, \mathbf{1}, \mathbf{1})$
$\hat{F}_+$	—	✓	$(1, \mathbf{1}, \mathbf{1})$
$\hat{B}_V$	—	✓	$(0, \mathbf{1}, \mathbf{1})$
$\hat{W}_V$	—	✓	$(0, \mathbf{3}, \mathbf{1})$
$\hat{G}_V$	—	✓	$(0, \mathbf{1}, \mathbf{8})$

Table 1. The matter content of *model 1*. All superfields of chiral fermions live on a brane and all Higgs-type superfields and gauge vector fields live in the bulk. The superscript $f = 1, 2, 3$ denotes the generations. Neutrino superfields may be included straightforwardly. The gauge couplings of this model unify as in figure 1 (top left).

function coefficient b_1 and lead to precision unification at one-loop. The superpotential for both models is given by

$$W = Y_u\, \hat{u}\, \epsilon_{ij} \hat{q}^i\, \hat{H}_u^j - Y_d\, \hat{d}\, \epsilon_{ij} \hat{q}^i\, \hat{H}_d^j - Y_e\, \hat{e}\, \epsilon_{ij} \hat{l}^i\, \hat{H}_d^j + \mu H_u H_d + \acute{\mu} F^- F^+ \,. \tag{2.1}$$

2.1 Gauge coupling unification

A sufficient condition for unification in a five dimensional model is [28, 29] that

$$R_{ij} = \frac{b_i^{(5D)} - b_j^{(5D)}}{b_i^{\mathrm{SSM}} - b_j^{\mathrm{SSM}}} \tag{2.2}$$

does not depend on (i, j), where b_i^{5D} are the five dimensional beta function coefficients, at one-loop. The β-function of an $\mathrm{SU}(N)$ gauge theory at one-loop is

$$\beta_g = \mu \frac{dg}{d\mu} = -\frac{g^3}{16\pi^2} \left(\frac{11}{3} T(Adj) - \frac{2}{3} T_{\mathrm{fer}}(R) - \frac{1}{3} T_{sc}(R) \right) = \frac{b_g^{(\text{1-loop})}\, g^3}{(2\pi)^4} \tag{2.3}$$

for gauge fields, Weyl fermions and complex scalars respectively. R is the representation and in particular $T(\mathrm{Ad}) = N$ and $T(\Box) = \frac{1}{2}$.

For a $\mathrm{U}(1)$ theory [30] the gauge field is uncharged, there is also an overall normalisation constant which can be fixed to embed the particular $\mathrm{U}(1)$ in a larger group. Such that focusing on the $\mathrm{U}(1)$ of the SSM one finds

$$b_1 = \frac{3}{5} \left(\frac{2}{3} \sum_f Y_f^2 + \frac{1}{3} \sum_s Y_s^2 \right) \qquad \text{or} \qquad b_1 = \frac{3}{5} \left(\sum_\Phi Y_\Phi^2 \right) , \tag{2.4}$$

Superfields	Brane	Bulk	$\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_c$
$\hat{q}^{1,2}$	✓	—	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{d}^{1,2}$	✓	—	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}^{1,2}$	✓	—	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{l}^{1,2}$	✓	—	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{e}^{1,2}$	✓	—	$(1, \mathbf{1}, \mathbf{1})$
$\hat{q}^{3}$	—	✓	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{d}^{3}$	—	✓	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}^{3}$	—	✓	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{l}^{3}$	—	✓	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{e}^{3}$	—	✓	$(1, \mathbf{1}, \mathbf{1})$
$\hat{H}_d$	—	✓	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	—	✓	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{F}_-$	—	✓	$(-1, \mathbf{1}, \mathbf{1})$
$\hat{F}_+$	—	✓	$(1, \mathbf{1}, \mathbf{1})$
$\hat{B}_V$	—	✓	$(0, \mathbf{1}, \mathbf{1})$
$\hat{W}_V$	—	✓	$(0, \mathbf{3}, \mathbf{1})$
$\hat{G}_V$	—	✓	$(0, \mathbf{1}, \mathbf{8})$

Table 2. The matter content of *model 2*. In this case the third generation also lives in the bulk. The gauge couplings of this model unify as in figure 1 (top right).

the latter is for chiral superfields, and the Y's are hypercharges, where the hypercharge is rescaled by $g_1 \equiv \sqrt{5/3}g'$ as usual in unified models [31]. The results for various models may be found in table 4, where we note that unification scales of the order of 10 TeV can still satisfy proton decay constraints, this conclusion being applicable to a wide range of gauge groups or extra-dimensional models [32]. In a number of these scenarios additional matter is required to obtain unification, or indeed the extra dimensional scale $1/R > 10^{10}$ GeV, which for phenomenological purposes is essentially four dimensional and so not of interest.

A useful comment is appropriate here that the additional matter of the $5D$ MSSM-UED scenario means that all beta function coefficients are positive. This forces $1/R \gtrsim 10^{10}$ GeV for unification to still be possible [35]. Low scale (supersymmetric) extra dimensions therefore require that most of the MSSM matter does not live in the bulk. Our preferred scenarios are therefore ones in which the matter multiplets all live on a brane (*model 1*) or where the 1st and 2nd generation live on an opposite brane to the 3rd generation, or where only the third generation lives in the bulk (*model 2*), or where only the third generation lives in the brane (*model 3*, see table 3 for the matter content in this case). In either case the Higgses can live in the bulk or on a brane. Additional fields may be added to accomplish precision unification at low scales [39]. This leads to three options: the models 1, 2 and 3 that we consider in this paper (plotted in figure 1) and one might also be able to combine a

Superfields	Brane	Bulk	$U(1)_Y \times SU(2)_L \times SU(3)_c$
$\hat{q}^{1,2}$	—	✓	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{d}^{1,2}$	—	✓	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}^{1,2}$	—	✓	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{l}^{1,2}$	—	✓	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{e}^{1,2}$	—	✓	$(1, \mathbf{1}, \mathbf{1})$
$\hat{q}^{3}$	✓	—	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{d}^{3}$	✓	—	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}^{3}$	✓	—	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{l}^{3}$	✓	—	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{e}^{3}$	✓	—	$(1, \mathbf{1}, \mathbf{1})$
$\hat{H}_d$	—	✓	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	—	✓	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{F}_-$	—	✓	$(-1, \mathbf{1}, \mathbf{1})$
$\hat{F}_+$	—	✓	$(1, \mathbf{1}, \mathbf{1})$
$\hat{B}_V$	—	✓	$(0, \mathbf{1}, \mathbf{1})$
$\hat{W}_V$	—	✓	$(0, \mathbf{3}, \mathbf{1})$
$\hat{G}_V$	—	✓	$(0, \mathbf{1}, \mathbf{8})$

Table 3. The matter content of *model 3*. In this case the 1st and 2nd generation live in the bulk. The gauge couplings of this scenario do not unify, as in figure 1 (bottom).

4D M-Dirac-SSM [38] with a maximal super Yang-Mills theory only in the bulk [26], rather remarkably, to achieve unification for any and all sizes of inverse radius. In this theory the gauge couplings only run in the four dimensional theory as the beta functions for the gauge couplings vanish exactly to all orders in perturbation theory in the maximal super Yang-Mills theory. As a result there are no power law contributions for gauge couplings (but there may be for the Yukawas and soft terms) and an inverse radius of a few TeV is possible with gauge coupling unification at 10^{17} GeV, which is very counter-intuitive. The effective cutoff of a five dimensional theory is essentially defined as the scale at which some parameter, such as the gauge couplings, hit a Landau pole: as no Landau pole arises this allows for the range of validity of this theory to extend further.

3 Exploring the models

In this section we explore the typical scales of the models, we describe how we solve the various RGEs and the boundary conditions that we use and then look at many of the running parameters of the model, such as trilinear soft breaking parameters and the gaugino mass spectrum.

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Scenario	(b_1, b_2, b_3)	Refs:	$1/R$-GUT
$4D$ SM	$(\frac{41}{10}, -19/6, -7)$		—
$4D$ MSSM	$(\frac{33}{5}, 1, -3)$	[33, 34]	—
$5D$ MSSM: Chiral Higgses in the bulk	$(\frac{3}{5}, -3, -6)$	[20, 29]	$\star$(does not exist)
$5D$ MSSM: Hyper Higgses in the bulk	$(\frac{6}{5}, -2, -6)$	[18, 27]	$\sim 10^{10}$ GeV
$5D$ MSSM-UED	$(\frac{66}{5}, 10, 6)$	[35, 36]	$\geq 5 \times 10^{10}$ GeV
$5D$ 3rd Gen & Hyper Higgses in the bulk	$(\frac{26}{5}, 2, -2)$	[37]	$\sim 10^{10}$ GeV
$5D$ 1st,2nd Gen & Hyper Higgses in the bulk	$(\frac{40}{5}, 4, 2)$		✗
$5D$ Gauge only in the bulk	$(0, -4, -6)$		$\sim 10^{10}$ GeV
$4D$ $SSMF^{\pm}$	$(\frac{39}{5}, 1, -3)$		—
$5DSSMF^{\pm}$:Hyper Higgses in bulk	$(\frac{18}{5}, -2, -6)$	*model 1*	≥ 1 TeV
$5DSSMF^{\pm}$:3rd Gen & Hyper Higgses in bulk	$(\frac{38}{5}, 2, -2)$	*model 2*	≥ 1 TeV
$5DSSMF^{\pm}$:1st,2nd Gen & Hyper Higgses in bulk	$(\frac{58}{5}, 6, 2)$	*model 3*	≥ 1 TeV
$4D$ MSSM+Dirac	$(\frac{33}{5}, -1, 0)$		—
$4D$ M-Dirac-SSM	$(\frac{48}{5}, 4, 0)$	[38]	—
$5D$ MSYM only in the bulk	$(0, 0, 0)$	[26]	any
$5D$ MSYM Hyper-Higgs in the bulk	$(\frac{6}{5}, 2, 0)$		✗

Table 4. The one-loop beta function coefficients of the gauge couplings for various scenarios. Requiring gauge coupling unification puts a bound on the inverse radius of the extra dimension in five dimensional models, which is estimated in the right-most column.

3.1 Typical scales of the models

It is useful to set the mass and energy scales in which we wish to consider these models. We wish for a large extra dimension, which then leads us to fix the gauge coupling unification scale and the scale of the cut off, where the gauge couplings hit a Landau pole (see figure 1):

$$\frac{1}{R} \sim 10\,\text{TeV}\,, \qquad M_{\text{GUT}} \sim 300\,\text{TeV}\,, \qquad \Lambda \sim 1{,}000\,\text{TeV}. \tag{3.1}$$

Although they differ in magnitude, this is natural in that fixing any one of these determines the other two. Next we wish for a gluino mass above collider exclusions and to determine the Higgs mass correctly to be $m_h = 125\,\text{GeV}$ from a sizeable A_t. We find (see for instance figure 3)

$$M_3 = 900\,\text{GeV leads to } A_t \sim -700\,\text{GeV} \;, \qquad M_3 = 1700\,\text{GeV leads to } A_t \sim -1250\,\text{GeV}. \tag{3.2}$$

Strong exclusion limits on the gluino arise from ATLAS and CMS null searches for jets plus missing energy, for example $m_{\tilde{g}} > 1600\,\text{GeV}$ for $m_{\tilde{q}_{1,2}} > 2000\,\text{GeV}$ [40, 41], although this can be lowered if one wishes to also include R-parity violation with our models, hence the $M_3 = 900\,\text{GeV}$ case. Conversely, allowing for an upper bound on the top trilinear coupling, from considering metastability of the electroweak vacuum,

$$A_t = -2\,\text{TeV leads to } M_3 \sim 2.77\,\text{TeV} \qquad \text{and} \qquad A_t = -2.5\,\text{TeV leads to } M_3 \sim 3.5\,\text{TeV}. \tag{3.3}$$

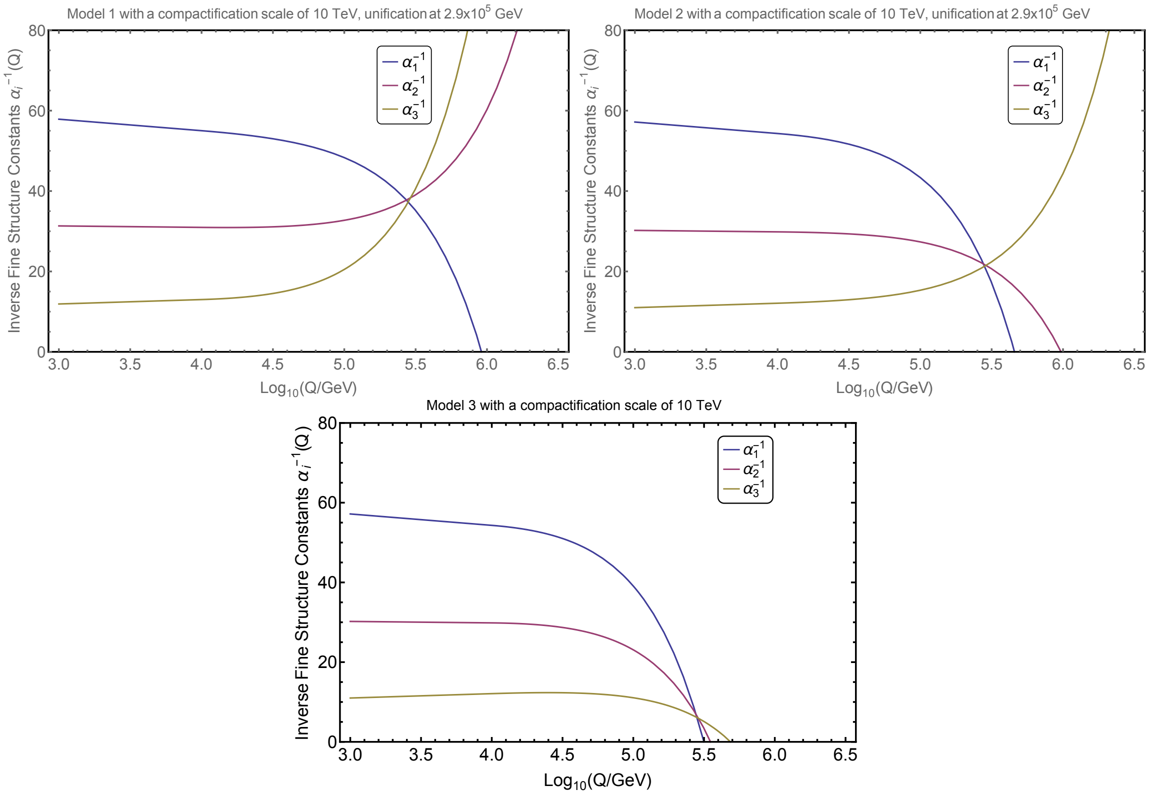

Figure 1. Running of the inverse fine structure constants $\alpha_i^{-1}(Q)$, for three different models with compactification scales 10 TeV as a function of $\mathrm{Log}_{10}(\mathrm{Q/GeV})$.

To allow for the correct Higgs mass $m_h = 125\,\mathrm{GeV}$, the electroweak parameters should be in the range

$$\tan\beta \subset (5, 30), \qquad \mu \leq 1\mathrm{TeV}, \tag{3.4}$$

represented in figure 6. We do not expect $\tan\beta$ to be much larger, due to $B_s \to X_s\gamma$ flavour constraints and μ is bounded by naturalness considerations of the renormalisation group effects on the Higgs tadpole equations (minimisation of the scalar potential).

3.2 Implementation and results

To obtain our results we computed by hand the various RGEs of the four dimensional (zero mode) description that both *model 1* and *2* (tables 1 and 2) reduce to at low energies. We then confirmed these with the output of an implementation of the four dimensional regime in `SARAH` [42–45]. We then computed, by hand only, the one-loop RGEs for each of *model 1*, *2* and *3*, including all the additional fields of the KK sector. Using `MATHEMATICA` we solve the combined set of RGEs and match the four and five dimensional RGEs at the matching compactification scale such that at low energies the theory is described by the four dimensional RGEs only.

Once we have a combined set of RGEs, we must specify a set of boundary conditions. In this case we must simply specify all boundary conditions at the same scale (rather than, for example, having a set of boundary conditions at both the GUT/SUSY-breaking scale and at the electroweak scale), which we took to be $t = 3$, or $Q = 10^3\,\mathrm{GeV}$ (where we define

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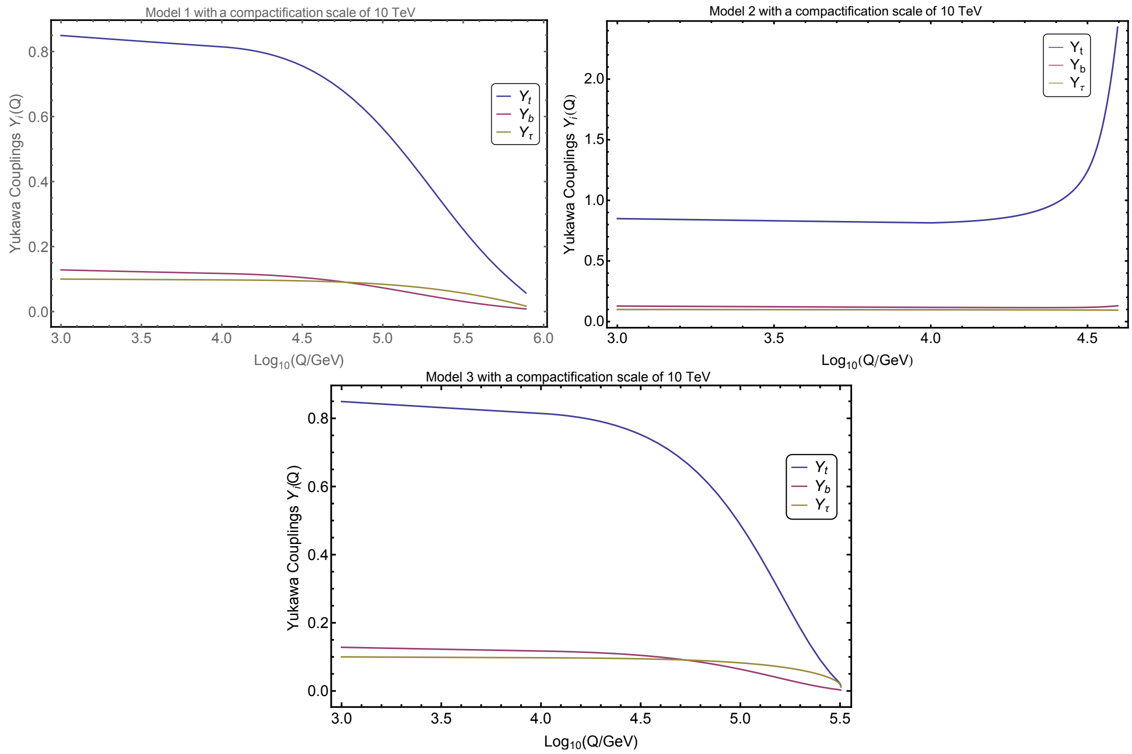

Figure 2. Running of the Yukawa couplings $Y_i(Q)$, for thee different models with compactification scales 10 TeV as a function of $\text{Log}_{10}(\text{Q/GeV})$. The top Yukawa coupling typically hits a Landau pole before the GUT scale when the 3rd generation matter is located in the bulk (right).

Parameter	Value	Name
Q_0	1000	(SUSY Scale)
$g_1(Q_0)$	0.360945804	g1
$g_2(Q_0)$	0.633371083	g2
$g_3(Q_0)$	1.02739852	g3
$\tan\beta$	10	(Tan beta)
$Y_t(Q_0)$	0.849348847	(Top Yukawa)
$Y_b(Q_0)$	0.128188819	(Bottom Yukawa)
$Y_\tau(Q_0)$	0.0999653768	(Tau Yukawa)

Table 5. A table of the boundary conditions used in our study.

$t = \text{Log}_{10} Q$). The gauge couplings and Yukawa couplings are easily obtained by running up from m_Z and are listed in table 5, for example in figure 2 for $\tan\beta \sim 10$. Regarding the soft breaking terms we made some specific choices which we enforce by choosing a low-scale boundary value such that it holds true once we run up to the high scale. We also make the assumption that the SUSY breaking scale is equal to the GUT scale, but of course other scenarios should be considered. For *model 1* and *3*:

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- We assume supersymmetry breaking occurs at the unification scale, which is found by finding the scale at which $g_1 = g_2 = g_3$, which is lowered compared to the 4D MSSM, by the effects of the compactification. This is pictured in figure 1 (top left).

- We specify the value of the gluino mass, $M_3(Q)$, at $Q = 10^3$ GeV. We then find the bino and wino soft masses M_1 and M_2 such that all gaugino masses $M_1 = M_2 = M_3$ at the GUT scale. This is pictured in figure 3.

- We take the trilinear soft breaking terms, $A_{u/d/e}$, to vanish at the unification scale, also pictured in figure 3.

- We take $\mu(t = 3) \sim 500$ GeV and $B_\mu(M_{\text{GUT}}) = 0$, as pictured in figure 5 (left).

The results are rather different for *model 2*:

- We found the scale at which $g_1 = g_2 = g_3$, which is lowered compared to the 4D MSSM, pictured in figure 1 (top right).

- The top Yukawa coupling hits a Landau pole just after $t = 4.595$, as pictured in figure 2 (right).

The result was that we could not set the supersymmetry breaking scale at M_{GUT} and instead chose the supersymmetry breaking scale to occur below the top Yukawa Landau pole, at $t = 4.4$. We then chose for the plots in *model 2*:

- We choose the gaugino masses to unify $M_3(t = 4.4) = M_2(t = 4.4) = M_1(t = 4.4)$ and let $M_3(t = 3) = 1700$ GeV.

- $A_{u/d/e}(t = 4.4)$ are set to vanish and this model does not develop a TeV scale $A_t(t = 3)$, as pictured in figure 3 (right).

- Whilst electroweak symmetry breaking is possible starting from the condition $m^2_{H_d} = m^2_{H_u}$, it does not automatically arise from using $(m_0^2 + \mu^2)^{1/2}$, where m_0^2 would set the scalar soft mass boundary condition. This is pictured in figure 4 (right), where a representative case is given that achieves the correct Higgs mass.

- We take $\mu(t = 3) \sim 500$ GeV and $B_\mu(M_{\text{GUT}}) = 0$, as pictured in figure 5 (left).

3.3 Two ways to accommodate natural supersymmetry

The two models we explore in this paper can accommodate a natural spectrum of sparticles in two very different ways, whilst still obtaining the correctly observed Higgs mass:

In *model 2* the third generation are located in the bulk and feel the effects of supersymmetry more indirectly than the first and second generation. This will allow for a spectrum of light stops with a heavier first and second generation, above present collider exclusions. One may use the NMSSM or D-terms to lift the Higgs mass to its correct value.

In *model 1* the Higgs mass is obtained through a TeV scale A_t term that is generated entirely through RGE evolution, allowing for the correct Higgs mass with stops much below

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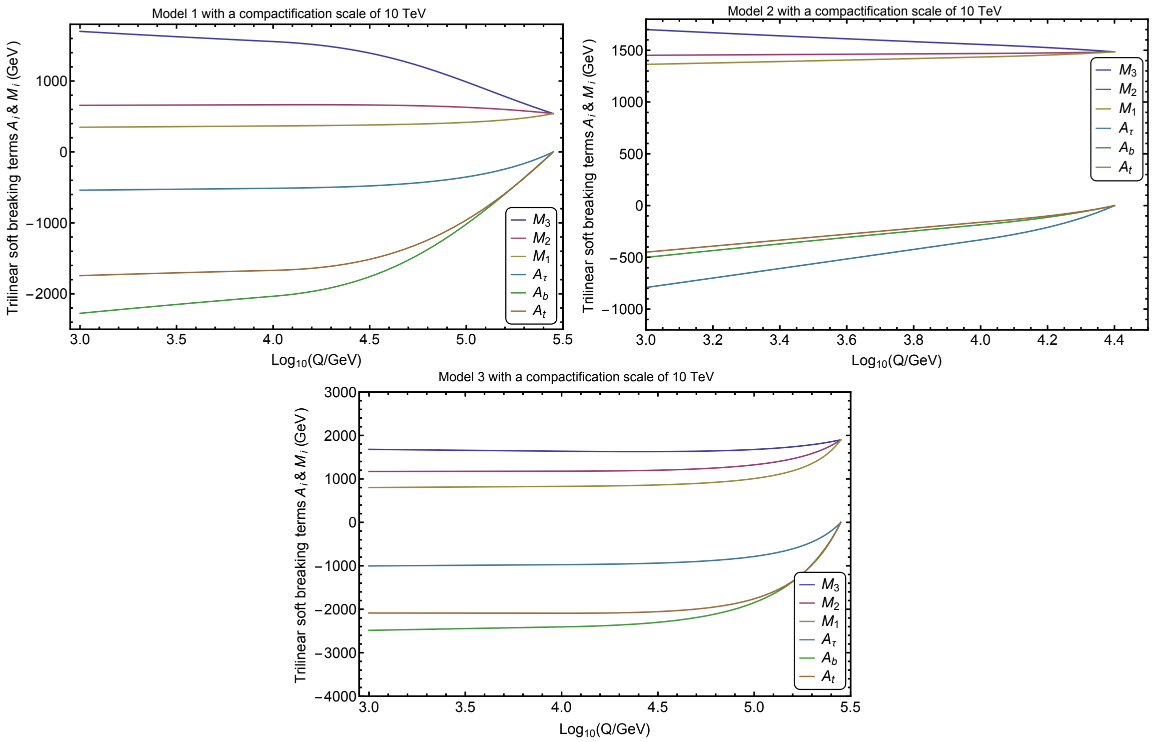

Figure 3. Running of the gaugino masses and trilinear couplings $M_i(Q)$ and $A_i(Q)$, for the two different models with compactification scales 10 TeV, as a function of $\mathrm{Log}_{10}(\mathrm{Q/GeV})$.

2 TeV even within an MSSM-like Higgs sector, but does not yet explain any hierarchy between the generation of squarks. In this subsection we explain these details of each model further.

3.3.1 The third generation in the bulk

Exclusions on first and second generation squarks are presently nearing 2 TeV [46–53], while the aesthetic of naturalness for the Higgs sector (and much weaker bounds on 3rd generation squarks of around 300–400 GeV [7–17] from direct searches) favour a 3rd generation below a TeV. In order for this hierarchy to emerge at low scales it is likely to be imprinted in the soft SUSY breaking terms and not simply a renormalisation group effect. At the supersymmetry breaking scale this might imply that the soft terms, in the flavour basis, take the form,

$$m_{\tilde{f}}^2 \sim \Lambda^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \ldots \tag{3.5}$$

or indeed

$$m_{\tilde{f}}^2 \sim \Lambda^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\varepsilon \end{pmatrix} + \ldots \tag{3.6}$$

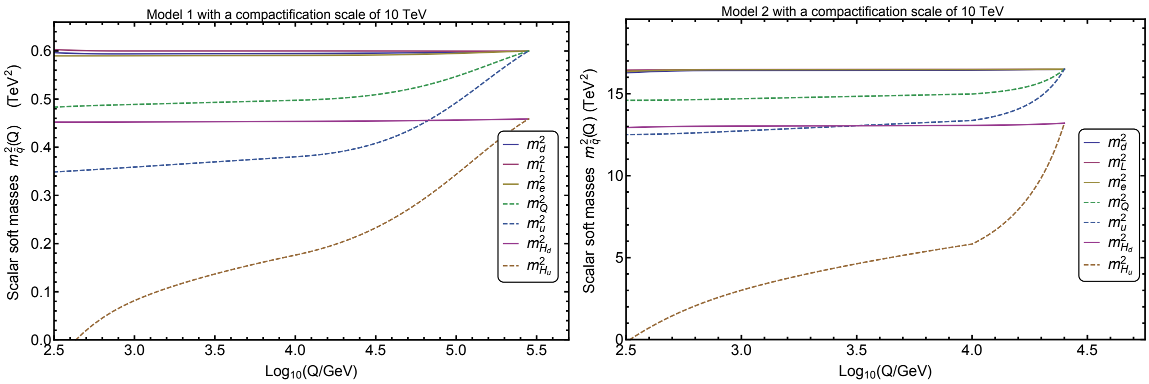

Figure 4. Running of the various soft masses for the two models.

the ε denoting any subleading effects, as one does not require exactly zero entries. Some ideas have been put forward to explain such a heirarchy, see for instance [5, 18, 54, 55], and we wish to advance the argument that a five dimensional model with the *3rd generation in the bulk*, i.e our *model 2*, explains such a hierarchy.

We put forward the idea that the first and second generation of squarks live on the *same brane* as the source of supersymmetry breaking. They will feel directly the effect of supersymmetry breaking and generate large soft breaking terms. The 3rd generation is, however, located in the bulk and will feel the supersymmetry breaking indirectly through either gravity or gauge mediation. This will lead to the boundary conditions in eq. (3.5). For a calculation of gauge mediated soft terms from a *brane to a bulk field* see [24, 56], for *brane to other brane* see [23, 24, 56]. Such an effect is still felt directly by the gauginos (and the gravitino) and they will also have a large SUSY breaking soft mass, which have important RGE effects as discussed in this paper.

3.3.2 A large A_t term

Our *model 1* does not geometrically explain why the first and second generation might be much heavier than the 3rd, but it does allow for a large A_t term generated entirely through RGE evolution, and this can still allow for stops much below 2 TeV and still obtain the correct Higgs mass from the usual MSSM Higgs sector. Therefore for *model 1*, we do not yet offer an explanation of the source of supersymmetry breaking. We discuss obtaining the correct Higgs mass in *model 1* in the next section.

4 The Higgs mass

The Higgs mass is a sensitive parameter in supersymmetric theories and its experimental value at 125 GeV restricts the available parameter space for these models. A realistic and precise calculation of the Higgs mass in supersymmetric models requires the inclusion of two-loop contributions. In the following we shall use the numerical values of the program FeynHiggs 2.11.3 [57, 58] for the Higgs mass at two-loops and interface it with our numerical code for the models 1 and 2 we discussed above. As the Higgs mass is a low energy

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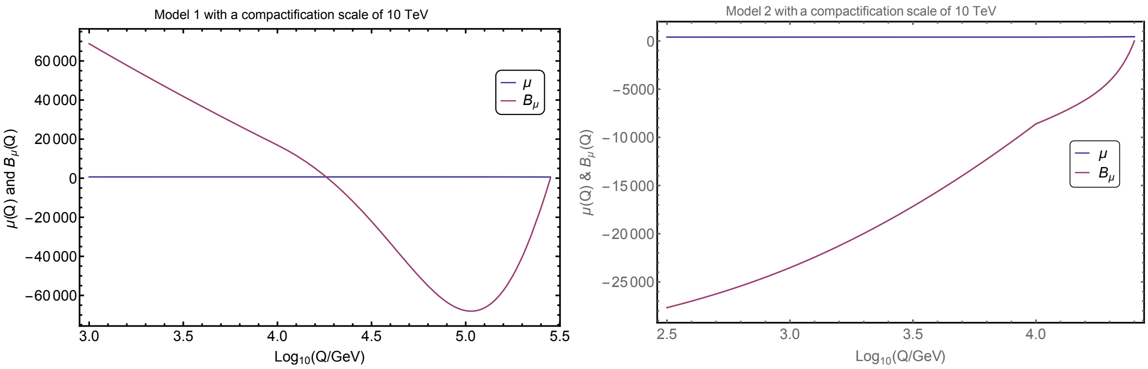

Figure 5. The running of μ and B_μ for the two models.

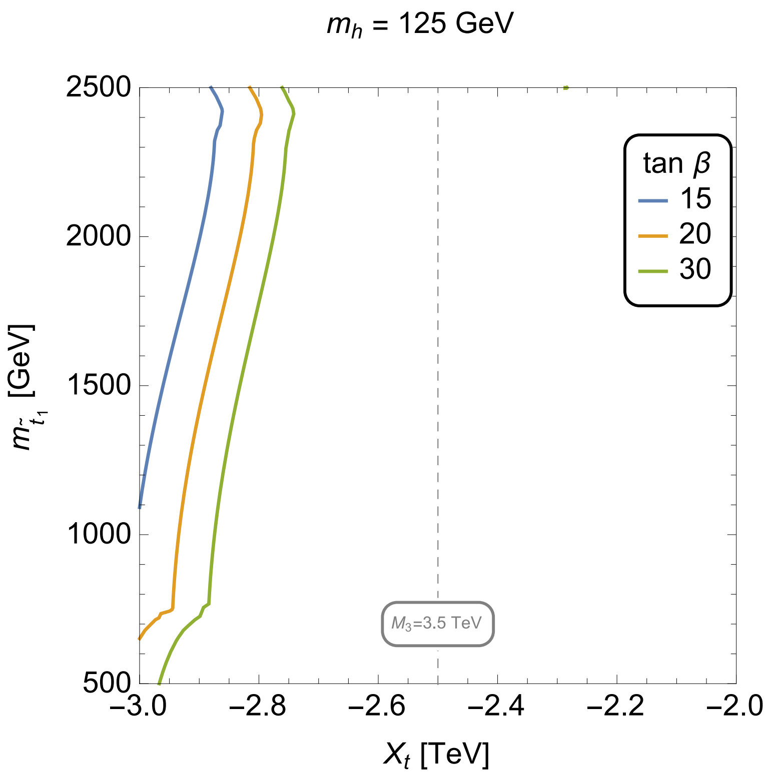

Figure 6. Contours of the lightest Higgs mass $m_h = 125\,\mathrm{GeV}$ in the plane $(m_{\tilde{t}_1}, X_t)$ for various values of $\tan\beta$. The dashed gray line represents a sample gluino mass for the corresponding value of X_t. Stop masses below 2 TeV can be obtained in our model due to the TeV-scale A_t term.

parameter with respect to the extra-dimensional contributions, the corrections due to the extra-dimensional structure are very small in the plot in figure 6. In order to have a rough physical intuition concerning the effect of the large A_t value from our setup, we provide here the leading one-loop self-energy contributions to the lightest CP even Higgs mass [59–63]

$$m_{h,1}^2 \simeq m_z^2 \cos^2 2\beta + \frac{3}{4\pi^2}\frac{m_t^4}{v_{\mathrm{ew}}^2}\left[\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right)\right], \tag{4.1}$$

where $v_{\rm ew}$ is the electroweak Higgs vev, $X_t = A_t - \mu \cot\beta$ and $M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$. If μ is a few 100 GeV and $A_t \gg \mu$ then $X_t \sim A_t$. The result of the one-loop formula allows to understand why a model with large A_t values can help increasing the value of the Higgs mass. Note however that the one-loop formula is not precise enough to for a realistic comparison with data and typically allows a larger parameter space that what really available including the two-loop formulas. We plot, in figure 6, the Higgs mass for representative values of A_t in our setup using the FeynHiggs program at two-loops. This allows for a prediction of $\tan\beta$ and the stop squark masses which can be below 2 TeV. One could also lift the tree-level Higgs mass with the NMSSM + $F^{\pm}$ or else through non decoupling D-terms (see for example [5, 6]) which would require introducing an additional U(1) or SU(2) or both. Such an additional feature would be necessary for *model 2* as a large A_t does not arise in this case.

5 Conclusions

In this paper we explored various five dimensional extensions of the SSM that unify, with an inverse radius of the extra dimension of roughly a 10 TeV scale. Such models are compelling extensions of the MSSM in that they may achieve interesting phenomenological features such as additional Z', W' and G' bosons in the 1–10 TeV range and achieve the correct 125 GeV Higgs mass and a relatively natural sparticle spectrum for model 1, while for model 2 this spectrum is heavier, without sacrificing unification of gauge couplings. Such models achieve a natural spectrum by generating a TeV scale A_t term from "power-law" running and unification of gauge couplings through the addition of two charged superfields $F^{\pm}$ in the bulk.

In particular we look at two models that can achieve unification, either all chiral matter superfields on the boundaries, or just the third generation in the bulk and the first two on a boundary. In either case the Higgs doublet superfields H_u, H_d and $F^{\pm}$ are located in the bulk along with all three gauge groups $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. We also point out that five dimensional models in which the 1st and 2nd generation are located in the bulk cannot possibly achieve unification unless the inverse radius of the extra dimension is essentially at the GUT scale (and in any case not with this matter content), and so are entirely four dimensional from a phenomenological perspective.

This paper can be extended in a number of ways and we discuss just a few. In many models of supersymmetry breaking, electroweak symmetry breaking is not optimal both in terms of fine tuning and in obtaining electroweak breaking from a given parameterisation of soft breaking terms at the high scale. These remain an interesting open question, and may benefit from further discoveries or exclusions in the Higgs sector, at the LHC13/14. Our results are representative only, and clearly a more dedicated spectrum generator built using the RGEs and including threshold corrections will give more precise results, and we provide in this paper a concrete set of RGEs from which this spectrum generator can be constructed. Different supersymmetry breaking parameterisations and how flavour arises is also an interesting further direction to consider.

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A Renormalisation group equations for 4D-SSM+$F^{\pm}$

In this appendix we document the one- and two-loop RGEs for the four dimensional low energy model for which the 5D models 1, 2 and 3 are completions. Recall that the output of our implementation in the four dimensional regime was done using `SARAH` [42–45], as such we have used the same conventions and notations, where $T_i = A_i y_i$ ($i = t, b, \tau$ etc.) in these appendices.

A.1 Anomalous dimensions

$$\gamma_{\hat{q}}^{(1)} = -\frac{1}{30}\left(45g_2^2 + 80g_3^2 + g_1^2\right)\mathbf{1} + Y_d^\dagger Y_d + Y_u^\dagger Y_u \tag{A.1}$$

$$\begin{aligned}\gamma_{\hat{q}}^{(2)} &= +\frac{1}{180}\left(2g_1^2\left(16g_3^2 + 9g_2^2\right) + 47g_1^4 + 5\left(135g_2^4 + 288g_2^2g_3^2 - 32g_3^4\right)\right)\mathbf{1} \\ &\quad + \frac{4}{5}g_1^2 Y_u^\dagger Y_u - 2Y_d^\dagger Y_d Y_d^\dagger Y_d - 2Y_u^\dagger Y_u Y_u^\dagger Y_u \\ &\quad + Y_d^\dagger Y_d\left(-3\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \frac{2}{5}g_1^2 - \mathrm{Tr}\left(Y_e Y_e^\dagger\right)\right) - 3Y_u^\dagger Y_u \mathrm{Tr}\left(Y_u Y_u^\dagger\right)\end{aligned} \tag{A.2}$$

$$\gamma_{\hat{l}}^{(1)} = -\frac{3}{10}\left(5g_2^2 + g_1^2\right)\mathbf{1} + Y_e^\dagger Y_e \tag{A.3}$$

$$\begin{aligned}\gamma_{\hat{l}}^{(2)} &= -2Y_e^\dagger Y_e Y_e^\dagger Y_e + \frac{3}{100}\left(125g_2^4 + 30g_1^2g_2^2 + 81g_1^4\right)\mathbf{1} \\ &\quad + Y_e^\dagger Y_e\left(-3\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \frac{6}{5}g_1^2 - \mathrm{Tr}\left(Y_e Y_e^\dagger\right)\right)\end{aligned} \tag{A.4}$$

$$\gamma_{\hat{H}_d}^{(1)} = 3\mathrm{Tr}\left(Y_d Y_d^\dagger\right) - \frac{3}{10}\left(5g_2^2 + g_1^2\right) + \mathrm{Tr}\left(Y_e Y_e^\dagger\right) \tag{A.5}$$

$$\begin{aligned}\gamma_{\hat{H}_d}^{(2)} &= +\frac{243}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{15}{4}g_2^4 - \frac{2}{5}\left(-40g_3^2 + g_1^2\right)\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \frac{6}{5}g_1^2\mathrm{Tr}\left(Y_e Y_e^\dagger\right) \\ &\quad - 9\mathrm{Tr}\left(Y_d Y_d^\dagger Y_d Y_d^\dagger\right) - 3\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - 3\mathrm{Tr}\left(Y_e Y_e^\dagger Y_e Y_e^\dagger\right)\end{aligned} \tag{A.6}$$

$$\gamma_{\hat{H}_u}^{(1)} = -\frac{3}{10}\left(-10\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + 5g_2^2 + g_1^2\right) \tag{A.7}$$

$$\gamma_{\hat{H}_u}^{(2)} = -3\mathrm{Tr}\left(Y_d Y_u^\dagger Y_u Y_d^\dagger\right) - 9\mathrm{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right) + \frac{15}{4} g_2^4 + \frac{243}{100} g_1^4 + \frac{4}{5}\left(20 g_3^2 + g_1^2\right)\mathrm{Tr}\left(Y_u Y_u^\dagger\right) + \frac{9}{10} g_1^2 g_2^2 \tag{A.8}$$

$$\gamma_{\hat{d}}^{(1)} = 2 Y_d^* Y_d^T - \frac{2}{15}\left(20 g_3^2 + g_1^2\right)\mathbf{1} \tag{A.9}$$

$$\gamma_{\hat{d}}^{(2)} = +\frac{2}{225}\left(-100 g_3^4 + 119 g_1^4 + 80 g_1^2 g_3^2\right)\mathbf{1} - 2\left(Y_d^* Y_d^T Y_d^* Y_d^T + Y_d^* Y_u^T Y_u^* Y_d^T\right) + Y_d^* Y_d^T\left(-2\mathrm{Tr}\left(Y_e Y_e^\dagger\right) + 6 g_2^2 - 6\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \frac{2}{5} g_1^2\right) \tag{A.10}$$

$$\gamma_{\hat{u}}^{(1)} = 2 Y_u^* Y_u^T - \frac{8}{15}\left(5 g_3^2 + g_1^2\right)\mathbf{1} \tag{A.11}$$

$$\gamma_{\hat{u}}^{(2)} = +\frac{8}{45}\left(16 g_1^2 g_3^2 + 25 g_1^4 - 5 g_3^4\right)\mathbf{1} - 2\left(Y_u^* Y_d^T Y_d^* Y_u^T + Y_u^* Y_u^T Y_u^* Y_u^T\right) + Y_u^* Y_u^T\left(6 g_2^2 - 6\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - \frac{2}{5} g_1^2\right) \tag{A.12}$$

$$\gamma_{\hat{e}}^{(1)} = 2 Y_e^* Y_e^T - \frac{6}{5} g_1^2 \mathbf{1} \tag{A.13}$$

$$\gamma_{\hat{e}}^{(2)} = -2 Y_e^* Y_e^T Y_e^* Y_e^T + \frac{54}{5} g_1^4 \mathbf{1} + Y_e^* Y_e^T\left(-2\mathrm{Tr}\left(Y_e Y_e^\dagger\right) + 6 g_2^2 - 6\mathrm{Tr}\left(Y_d Y_d^\dagger\right) - \frac{6}{5} g_1^2\right) \tag{A.14}$$

$$\gamma_{\phi_F^+}^{(1)} = -\frac{6}{5} g_1^2\,, \qquad \gamma_{\phi_F^+}^{(2)} = \frac{54}{5} g_1^4\,, \qquad \gamma_{\phi_F^-}^{(1)} = -\frac{6}{5} g_1^2\,, \qquad \gamma_{\phi_F^-}^{(2)} = \frac{54}{5} g_1^4. \tag{A.15}$$

A.2 Gauge couplings

$$\beta_{g_1}^{(1)} = \frac{39}{5} g_1^3\,, \qquad \beta_{g_2}^{(1)} = g_2^3 \quad , \quad \beta_{g_3}^{(1)} = -3 g_3^3 \tag{A.16}$$

$$\beta_{g_1}^{(2)} = \frac{1}{25} g_1^3\Big(-130\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 135 g_2^2 + 271 g_1^2 + 440 g_3^2 - 70\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 90\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\Big) \tag{A.17}$$

$$\beta_{g_2}^{(2)} = \frac{1}{5} g_2^3\Big(-10\mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 120 g_3^2 + 125 g_2^2 - 30\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 30\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 9 g_1^2\Big) \tag{A.18}$$

$$\beta_{g_3}^{(2)} = \frac{1}{5} g_3^3\Big(11 g_1^2 - 20\mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 20\mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 45 g_2^2 + 70 g_3^2\Big) \tag{A.19}$$

A.3 Gaugino mass parameters

$$\beta_{M_1}^{(1)} = \frac{78}{5} g_1^2 M_1, \qquad \beta_{M_2}^{(1)} = 2 g_2^2 M_2, \qquad \beta_{M_3}^{(1)} = -6 g_3^2 M_3 \tag{A.20}$$

$$\beta_{M_1}^{(2)} = \frac{2}{25} g_1^2\Big(542 g_1^2 M_1 + 135 g_2^2 M_1 + 440 g_3^2 M_1 + 440 g_3^2 M_3 + 135 g_2^2 M_2 - 70 M_1 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) - 90 M_1 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 130 M_1 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 70\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 90\mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 130\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{A.21}$$

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$$\beta_{M_2}^{(2)}=\frac{2}{5}g_2^2\Big(9g_1^2M_1+120g_3^2M_3+9g_1^2M_2+250g_2^2M_2+120g_3^2M_2-30M_2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$-10M_2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-30M_2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+30\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+10\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)+30\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{A.22}$$

$$\beta_{M_3}^{(2)}=\frac{2}{5}g_3^2\Big(11g_1^2M_1+11g_1^2M_3+45g_2^2M_3+140g_3^2M_3+45g_2^2M_2-20M_3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$-20M_3\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+20\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+20\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\Big) \tag{A.23}$$

A.4 Trilinear superpotential parameters

$$\beta_{Y_d}^{(1)}=3Y_dY_d^\dagger Y_d+Y_d\Big(-3g_2^2+3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-\frac{16}{3}g_3^2-\frac{7}{15}g_1^2+\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big)+Y_dY_u^\dagger Y_u \tag{A.24}$$

$$\beta_{Y_d}^{(2)}=+\frac{4}{5}g_1^2Y_dY_u^\dagger Y_u-4Y_dY_d^\dagger Y_dY_d^\dagger Y_d-2Y_dY_u^\dagger Y_uY_d^\dagger Y_d-2Y_dY_u^\dagger Y_uY_u^\dagger Y_u$$
$$+Y_dY_d^\dagger Y_d\Big(-3\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+6g_2^2-9\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+\frac{4}{5}g_1^2\Big)-3Y_dY_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+Y_d\Big(\frac{1687}{450}g_1^4+g_1^2g_2^2+\frac{15}{2}g_2^4+\frac{8}{9}g_1^2g_3^2+8g_2^2g_3^2-\frac{16}{9}g_3^4-\frac{2}{5}(-40g_3^2+g_1^2)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+\frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)-3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)-3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\Big) \tag{A.25}$$

$$\beta_{Y_e}^{(1)}=3Y_eY_e^\dagger Y_e+Y_e\Big(-3g_2^2+3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-\frac{9}{5}g_1^2+\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big) \tag{A.26}$$

$$\beta_{Y_e}^{(2)}=-4Y_eY_e^\dagger Y_eY_e^\dagger Y_e+Y_eY_e^\dagger Y_e\Big(-3\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+6g_2^2-9\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\Big)$$
$$+Y_e\Big(\frac{783}{50}g_1^4+\frac{9}{5}g_1^2g_2^2+\frac{15}{2}g_2^4-\frac{2}{5}(-40g_3^2+g_1^2)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+\frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)$$
$$-9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)-3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)-3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\Big) \tag{A.27}$$

$$\beta_{Y_u}^{(1)}=3Y_uY_u^\dagger Y_u-\frac{1}{15}Y_u\Big(13g_1^2+45g_2^2-45\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+80g_3^2\Big)+Y_uY_d^\dagger Y_d \tag{A.28}$$

$$\beta_{Y_u}^{(2)}=+\frac{2}{5}g_1^2Y_uY_u^\dagger Y_u+6g_2^2Y_uY_u^\dagger Y_u-2Y_uY_d^\dagger Y_dY_d^\dagger Y_d-2Y_uY_d^\dagger Y_dY_u^\dagger Y_u$$
$$-4Y_uY_u^\dagger Y_uY_u^\dagger Y_u+Y_uY_d^\dagger Y_d\Big(-3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+\frac{2}{5}g_1^2-\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big)-9Y_uY_u^\dagger Y_u\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+Y_u\Big(\frac{3211}{450}g_1^4+g_1^2g_2^2+\frac{15}{2}g_2^4+\frac{136}{45}g_1^2g_3^2+8g_2^2g_3^2-\frac{16}{9}g_3^4+\frac{4}{5}(20g_3^2+g_1^2)\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$-3\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)-9\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big) \tag{A.29}$$

A.5 Bilinear superpotential parameters

$$\beta_{\mu}^{(1)} = 3\mu\mathrm{Tr}\Big(Y_d Y_d^{\dagger}\Big) - \frac{3}{5}\mu\Big(5g_2^2 - 5\mathrm{Tr}\Big(Y_u Y_u^{\dagger}\Big) + g_1^2\Big) + \mu\mathrm{Tr}\Big(Y_e Y_e^{\dagger}\Big) \tag{A.30}$$

$$\begin{aligned}\beta_{\mu}^{(2)} = {} & \frac{1}{50}\mu\Big(243g_1^4 + 90g_1^2g_2^2 + 375g_2^4 - 20\Big(-40g_3^2 + g_1^2\Big)\mathrm{Tr}\Big(Y_d Y_d^{\dagger}\Big) + 60g_1^2\mathrm{Tr}\Big(Y_e Y_e^{\dagger}\Big) \\ & + 40g_1^2\mathrm{Tr}\Big(Y_u Y_u^{\dagger}\Big) + 800g_3^2\mathrm{Tr}\Big(Y_u Y_u^{\dagger}\Big) - 450\mathrm{Tr}\Big(Y_d Y_d^{\dagger} Y_d Y_d^{\dagger}\Big) - 300\mathrm{Tr}\Big(Y_d Y_u^{\dagger} Y_u Y_d^{\dagger}\Big) \\ & - 150\mathrm{Tr}\Big(Y_e Y_e^{\dagger} Y_e Y_e^{\dagger}\Big) - 450\mathrm{Tr}\Big(Y_u Y_u^{\dagger} Y_u Y_u^{\dagger}\Big)\Big)\end{aligned} \tag{A.31}$$

$$\beta_{\nu}^{(1)} = -\frac{12}{5}g_1^2\nu\,, \qquad \beta_{\nu}^{(2)} = \frac{108}{5}g_1^4\nu \tag{A.32}$$

A.6 Trilinear soft-breaking parameters

$$\begin{aligned}\beta_{T_d}^{(1)} = {} & +4Y_dY_d^{\dagger}T_d+2Y_dY_u^{\dagger}T_u+5T_dY_d^{\dagger}Y_d+T_dY_u^{\dagger}Y_u-\frac{7}{15}g_1^2T_d-3g_2^2T_d-\frac{16}{3}g_3^2T_d+3T_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big) \\ & +T_d\mathrm{Tr}\Big(Y_eY_e^{\dagger}\Big)+Y_d\Big(2\mathrm{Tr}\Big(Y_e^{\dagger}T_e\Big)+6g_2^2M_2+6\mathrm{Tr}\Big(Y_d^{\dagger}T_d\Big)+\frac{14}{15}g_1^2M_1+\frac{32}{3}g_3^2M_3\Big)\end{aligned} \tag{A.33}$$

$$\begin{aligned}\beta_{T_d}^{(2)} = {} & +\frac{6}{5}g_1^2Y_dY_d^{\dagger}T_d+6g_2^2Y_dY_d^{\dagger}T_d-\frac{8}{5}g_1^2M_1Y_dY_u^{\dagger}Y_u+\frac{8}{5}g_1^2Y_dY_u^{\dagger}T_u \\ & +\frac{6}{5}g_1^2T_dY_d^{\dagger}Y_d+12g_2^2T_dY_d^{\dagger}Y_d+\frac{4}{5}g_1^2T_dY_u^{\dagger}Y_u-6Y_dY_d^{\dagger}Y_dY_d^{\dagger}T_d \\ & -8Y_dY_d^{\dagger}T_dY_d^{\dagger}Y_d-2Y_dY_u^{\dagger}Y_uY_d^{\dagger}T_d-4Y_dY_u^{\dagger}Y_uY_u^{\dagger}T_u-4Y_dY_u^{\dagger}T_uY_d^{\dagger}Y_d \\ & -4Y_dY_u^{\dagger}T_uY_u^{\dagger}Y_u-6T_dY_d^{\dagger}Y_dY_d^{\dagger}Y_d-4T_dY_u^{\dagger}Y_uY_d^{\dagger}Y_d-2T_dY_u^{\dagger}Y_uY_u^{\dagger}Y_u \\ & +\frac{1687}{450}g_1^4T_d+g_1^2g_2^2T_d+\frac{15}{2}g_2^4T_d+\frac{8}{9}g_1^2g_3^2T_d+8g_2^2g_3^2T_d-\frac{16}{9}g_3^4T_d \\ & -12Y_dY_d^{\dagger}T_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big)-15T_dY_d^{\dagger}Y_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big)-\frac{2}{5}g_1^2T_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big) \\ & +16g_3^2T_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big)-4Y_dY_d^{\dagger}T_d\mathrm{Tr}\Big(Y_eY_e^{\dagger}\Big)-5T_dY_d^{\dagger}Y_d\mathrm{Tr}\Big(Y_eY_e^{\dagger}\Big) \\ & +\frac{6}{5}g_1^2T_d\mathrm{Tr}\Big(Y_eY_e^{\dagger}\Big)-6Y_dY_u^{\dagger}T_u\mathrm{Tr}\Big(Y_uY_u^{\dagger}\Big)-3T_dY_u^{\dagger}Y_u\mathrm{Tr}\Big(Y_uY_u^{\dagger}\Big) \\ & -\frac{2}{5}Y_dY_d^{\dagger}Y_d\Big(15\mathrm{Tr}\Big(Y_e^{\dagger}T_e\Big)+30g_2^2M_2+45\mathrm{Tr}\Big(Y_d^{\dagger}T_d\Big)+4g_1^2M_1\Big)-6Y_dY_u^{\dagger}Y_u\mathrm{Tr}\Big(Y_u^{\dagger}T_u\Big) \\ & -9T_d\mathrm{Tr}\Big(Y_dY_d^{\dagger}Y_dY_d^{\dagger}\Big)-3T_d\mathrm{Tr}\Big(Y_dY_u^{\dagger}Y_uY_d^{\dagger}\Big)-3T_d\mathrm{Tr}\Big(Y_eY_e^{\dagger}Y_eY_e^{\dagger}\Big) \\ & +Y_d\Big(-\frac{3374}{225}g_1^4M_1-2g_1^2g_2^2M_1-\frac{16}{9}g_1^2g_3^2M_1-\frac{16}{9}g_1^2g_3^2M_3-16g_2^2g_3^2M_3+\frac{64}{9}g_3^4M_3 \\ & -2g_1^2g_2^2M_2-30g_2^4M_2-16g_2^2g_3^2M_2+\frac{4}{5}\Big(-40g_3^2M_3+g_1^2M_1\Big)\mathrm{Tr}\Big(Y_dY_d^{\dagger}\Big) \\ & -\frac{12}{5}g_1^2M_1\mathrm{Tr}\Big(Y_eY_e^{\dagger}\Big)-\frac{4}{5}g_1^2\mathrm{Tr}\Big(Y_d^{\dagger}T_d\Big)+32g_3^2\mathrm{Tr}\Big(Y_d^{\dagger}T_d\Big)+\frac{12}{5}g_1^2\mathrm{Tr}\Big(Y_e^{\dagger}T_e\Big) \\ & -36\mathrm{Tr}\Big(Y_dY_d^{\dagger}T_dY_d^{\dagger}\Big)-6\mathrm{Tr}\Big(Y_dY_u^{\dagger}T_uY_d^{\dagger}\Big)-12\mathrm{Tr}\Big(Y_eY_e^{\dagger}T_eY_e^{\dagger}\Big)-6\mathrm{Tr}\Big(Y_uY_d^{\dagger}T_dY_u^{\dagger}\Big)\Big)\end{aligned} \tag{A.34}$$

$$\beta_{T_e}^{(1)} = +4Y_eY_e^\dagger T_e + 5T_eY_e^\dagger Y_e - \frac{9}{5}g_1^2 T_e - 3g_2^2 T_e + 3T_e \mathrm{Tr}\Big(Y_dY_d^\dagger\Big) + T_e \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)$$
$$+Y_e\Big(2\mathrm{Tr}\Big(Y_e^\dagger T_e\Big) + 6g_2^2 M_2 + 6\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + \frac{18}{5}g_1^2 M_1\Big) \qquad \text{(A.35)}$$

$$\beta_{T_e}^{(2)} = +\frac{6}{5}g_1^2 Y_eY_e^\dagger T_e + 6g_2^2 Y_eY_e^\dagger T_e - \frac{6}{5}g_1^2 T_eY_e^\dagger Y_e + 12g_2^2 T_eY_e^\dagger Y_e$$
$$-6Y_eY_e^\dagger Y_eY_e^\dagger T_e - 8Y_eY_e^\dagger T_eY_e^\dagger Y_e - 6T_eY_e^\dagger Y_eY_e^\dagger Y_e + \frac{783}{50}g_1^4 T_e + \frac{9}{5}g_1^2 g_2^2 T_e + \frac{15}{2}g_2^4 T_e$$
$$-12Y_eY_e^\dagger T_e \mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 15T_eY_e^\dagger Y_e \mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - \frac{2}{5}g_1^2 T_e \mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+16g_3^2 T_e \mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 4Y_eY_e^\dagger T_e \mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 5T_eY_e^\dagger Y_e \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)$$
$$+\frac{6}{5}g_1^2 T_e \mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - 6Y_eY_e^\dagger Y_e\Big(2g_2^2 M_2 + 3\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big) - 9T_e \mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)$$
$$-3T_e \mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - 3T_e \mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)$$
$$-\frac{2}{25}Y_e\Big(783g_1^4 M_1 + 45g_1^2 g_2^2 M_1 + 45g_1^2 g_2^2 M_2 + 375g_2^4 M_2 - 10\Big(-40g_3^2 M_3 + g_1^2 M_1\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+30g_1^2 M_1 \mathrm{Tr}\Big(Y_eY_e^\dagger\Big) + 10g_1^2 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) - 400g_3^2 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) - 30g_1^2 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)$$
$$+450\mathrm{Tr}\Big(Y_dY_d^\dagger T_dY_d^\dagger\Big) + 75\mathrm{Tr}\Big(Y_dY_u^\dagger T_uY_d^\dagger\Big) + 150\mathrm{Tr}\Big(Y_eY_e^\dagger T_eY_e^\dagger\Big) + 75\mathrm{Tr}\Big(Y_uY_d^\dagger T_dY_u^\dagger\Big)\Big)$$
$$\text{(A.36)}$$

$$\beta_{T_u}^{(1)} = +2Y_uY_d^\dagger T_d + 4Y_uY_u^\dagger T_u + T_uY_d^\dagger Y_d + 5T_uY_u^\dagger Y_u - \frac{13}{15}g_1^2 T_u - 3g_2^2 T_u - \frac{16}{3}g_3^2 T_u$$
$$+3T_u \mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + Y_u\Big(6g_2^2 M_2 + 6\mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + \frac{26}{15}g_1^2 M_1 + \frac{32}{3}g_3^2 M_3\Big) \qquad \text{(A.37)}$$

$$\beta_{T_u}^{(2)} = +\frac{4}{5}g_1^2 Y_uY_d^\dagger T_d - \frac{4}{5}g_1^2 M_1 Y_uY_u^\dagger Y_u - 12g_2^2 M_2 Y_uY_u^\dagger Y_u + \frac{6}{5}g_1^2 Y_uY_u^\dagger T_u$$
$$+6g_2^2 Y_uY_u^\dagger T_u + \frac{2}{5}g_1^2 T_uY_d^\dagger Y_d + 12g_2^2 T_uY_u^\dagger Y_u - 4Y_uY_d^\dagger Y_dY_d^\dagger T_d$$
$$-2Y_uY_d^\dagger Y_dY_u^\dagger T_u - 4Y_uY_d^\dagger T_dY_d^\dagger Y_d - 4Y_uY_d^\dagger T_dY_u^\dagger Y_u - 6Y_uY_u^\dagger Y_uY_u^\dagger T_u$$
$$-8Y_uY_u^\dagger T_uY_u^\dagger Y_u - 2T_uY_d^\dagger Y_dY_d^\dagger Y_d - 4T_uY_d^\dagger Y_dY_u^\dagger Y_u - 6T_uY_u^\dagger Y_uY_u^\dagger Y_u + \frac{3211}{450}g_1^4 T_u$$
$$+g_1^2 g_2^2 T_u + \frac{15}{2}g_2^4 T_u + \frac{136}{45}g_1^2 g_3^2 T_u + 8g_2^2 g_3^2 T_u - \frac{16}{9}g_3^4 T_u - 6Y_uY_d^\dagger T_d \mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$-3T_uY_d^\dagger Y_d \mathrm{Tr}\Big(Y_dY_d^\dagger\Big) - 2Y_uY_d^\dagger T_d \mathrm{Tr}\Big(Y_eY_e^\dagger\Big) - T_uY_d^\dagger Y_d \mathrm{Tr}\Big(Y_eY_e^\dagger\Big)$$
$$-12Y_uY_u^\dagger T_u \mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - 15T_uY_u^\dagger Y_u \mathrm{Tr}\Big(Y_uY_u^\dagger\Big) + \frac{4}{5}g_1^2 T_u \mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+16g_3^2 T_u \mathrm{Tr}\Big(Y_uY_u^\dagger\Big) - \frac{2}{5}Y_uY_d^\dagger Y_d\Big(15\mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 2g_1^2 M_1 + 5\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)$$
$$-18Y_uY_u^\dagger Y_u \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) - 3T_u \mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big) - 9T_u \mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)$$
$$-\frac{2}{225}Y_u\Big(3211g_1^4 M_1 + 225g_1^2 g_2^2 M_1 + 680g_1^2 g_3^2 M_1 + 680g_1^2 g_3^2 M_3 + 1800g_2^2 g_3^2 M_3 - 800g_3^4 M_3$$

$$+225g_1^2g_2^2M_2+3375g_2^4M_2+1800g_2^2g_3^2M_2+180\Big(20g_3^2M_3+g_1^2M_1\Big)\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$-180\Big(20g_3^2+g_1^2\Big)\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)+675\mathrm{Tr}\Big(Y_dY_u^\dagger T_uY_d^\dagger\Big)+675\mathrm{Tr}\Big(Y_uY_d^\dagger T_dY_u^\dagger\Big)$$
$$+4050\mathrm{Tr}\Big(Y_uY_u^\dagger T_uY_u^\dagger\Big)\Big) \tag{A.38}$$

A.7 Vacuum expectation values

$$\beta_{v_d}^{(1)} = \frac{1}{20}v_d\Big(-20\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+3\Big(5g_2^2+g_1^2\Big)\Big(1+\xi\Big)-60\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\Big) \tag{A.39}$$

$$\beta_{v_d}^{(2)} = \frac{1}{400}v_d\Big(-486g_1^4-180g_1^2g_2^2-1200g_2^4-9g_1^4\xi-90g_1^2g_2^2\xi+875g_2^4\xi+9g_1^4\xi^2+90g_1^2g_2^2\xi^2$$
$$-225g_2^4\xi^2-40\Big(5\Big(32g_3^2+9g_2^2\xi\Big)+g_1^2\Big(9\xi-4\Big)\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$-120\Big(5g_2^2\xi+g_1^2\Big(4+\xi\Big)\Big)\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)$$
$$+3600\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)+1200\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)+1200\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)\Big) \tag{A.40}$$

$$\beta_{v_u}^{(1)} = \frac{3}{20}v_u\Big(-20\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+\Big(5g_2^2+g_1^2\Big)\Big(1+\xi\Big)\Big) \tag{A.41}$$

$$\beta_{v_u}^{(2)} = \frac{1}{400}v_u\Big(-486g_1^4-180g_1^2g_2^2-1200g_2^4-9g_1^4\xi-90g_1^2g_2^2\xi+875g_2^4\xi+9g_1^4\xi^2+90g_1^2g_2^2\xi^2$$
$$-225g_2^4\xi^2-40\Big(5\Big(32g_3^2+9g_2^2\xi\Big)+g_1^2\Big(9\xi+8\Big)\Big)\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+1200\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)+3600\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big) \tag{A.42}$$

Note that ξ is the gauge-fixing parameter, where we are using the R_ξ gauge.

A.8 Bilinear soft-breaking parameters

$$\beta_{B_\mu}^{(1)}=+\frac{6}{5}g_1^2M_1\mu+6g_2^2M_2\mu+B_\mu\Big(-3g_2^2+3\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+3\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-\frac{3}{5}g_1^2+\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\Big)$$
$$+6\mu\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)+2\mu\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)+6\mu\mathrm{Tr}\Big(Y_u^\dagger T_u\Big) \tag{A.43}$$

$$\beta_{B_\mu}^{(2)}=+B_\mu\Big(\frac{243}{50}g_1^4+\frac{9}{5}g_1^2g_2^2+\frac{15}{2}g_2^4-\frac{2}{5}\Big(-40g_3^2+g_1^2\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)+\frac{6}{5}g_1^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+\frac{4}{5}g_1^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)$$
$$+16g_3^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-9\mathrm{Tr}\Big(Y_dY_d^\dagger Y_dY_d^\dagger\Big)-6\mathrm{Tr}\Big(Y_dY_u^\dagger Y_uY_d^\dagger\Big)-3\mathrm{Tr}\Big(Y_eY_e^\dagger Y_eY_e^\dagger\Big)-9\mathrm{Tr}\Big(Y_uY_u^\dagger Y_uY_u^\dagger\Big)\Big)$$
$$-\frac{2}{25}\mu\Big(243g_1^4M_1+45g_1^2g_2^2M_1+45g_1^2g_2^2M_2+375g_2^4M_2-10\Big(-40g_3^2M_3+g_1^2M_1\Big)\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+30g_1^2M_1\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)+20g_1^2M_1\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+400g_3^2M_3\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)+10g_1^2\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)$$
$$-400g_3^2\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-30g_1^2\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)-20g_1^2\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)-400g_3^2\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)$$

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$$+450\mathrm{Tr}\Big(Y_dY_d^\dagger T_dY_d^\dagger\Big)+150\mathrm{Tr}\Big(Y_dY_u^\dagger T_uY_d^\dagger\Big)+150\mathrm{Tr}\Big(Y_eY_e^\dagger T_eY_e^\dagger\Big)+150\mathrm{Tr}\Big(Y_uY_d^\dagger T_dY_u^\dagger\Big)$$
$$+450\mathrm{Tr}\Big(Y_uY_u^\dagger T_uY_u^\dagger\Big)\Big) \tag{A.44}$$
$$\beta_{B_\nu}^{(1)}=\frac{12}{5}g_1^2\Big(2M_1\nu-B_\nu\Big),\qquad \beta_{B_\nu}^{(2)}=-\frac{108}{5}g_1^4\Big(4M_1\nu-B_\nu\Big). \tag{A.45}$$

A.9 Soft-breaking scalar masses

$$\sigma_{1,1}=\sqrt{\frac{3}{5}}g_1\Big(-2\mathrm{Tr}\Big(m_u^2\Big)-\mathrm{Tr}\Big(m_l^2\Big)-m_{H_d}^2-m_{F_-}^2+m_{H_u}^2+m_{F_+}^2+\mathrm{Tr}\Big(m_d^2\Big)$$
$$+\mathrm{Tr}\Big(m_e^2\Big)+\mathrm{Tr}\Big(m_q^2\Big)\Big) \tag{A.46}$$
$$\sigma_{2,11}=\frac{1}{10}g_1^2\Big(2\mathrm{Tr}\Big(m_d^2\Big)+3\mathrm{Tr}\Big(m_l^2\Big)+3m_{H_d}^2+3m_{H_u}^2+6\mathrm{Tr}\Big(m_e^2\Big)+6m_{F-}^2+6m_{F+}^2$$
$$+8\mathrm{Tr}\Big(m_u^2\Big)+\mathrm{Tr}\Big(m_q^2\Big)\Big) \tag{A.47}$$
$$\sigma_{3,1}=\frac{1}{20}\frac{1}{\sqrt{15}}g_1\Big(-9g_1^2m_{H_d}^2-45g_2^2m_{H_d}^2+9g_1^2m_{H_u}^2+45g_2^2m_{H_u}^2-36g_1^2m_{F-}^2+36g_1^2m_{F+}^2$$
$$+4\Big(20g_3^2+g_1^2\Big)\mathrm{Tr}\Big(m_d^2\Big)+36g_1^2\mathrm{Tr}\Big(m_e^2\Big)-9g_1^2\mathrm{Tr}\Big(m_l^2\Big)-45g_2^2\mathrm{Tr}\Big(m_l^2\Big)+g_1^2\mathrm{Tr}\Big(m_q^2\Big)$$
$$+45g_2^2\mathrm{Tr}\Big(m_q^2\Big)+80g_3^2\mathrm{Tr}\Big(m_q^2\Big)-32g_1^2\mathrm{Tr}\Big(m_u^2\Big)-160g_3^2\mathrm{Tr}\Big(m_u^2\Big)+90m_{H_d}^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)$$
$$+30m_{H_d}^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-90m_{H_u}^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-60\mathrm{Tr}\Big(Y_dY_d^\dagger m_d^{2*}\Big)-30\mathrm{Tr}\Big(Y_dm_q^{2*}Y_d^\dagger\Big)$$
$$-60\mathrm{Tr}\Big(Y_eY_e^\dagger m_e^{2*}\Big)+30\mathrm{Tr}\Big(Y_em_l^{2*}Y_e^\dagger\Big)+120\mathrm{Tr}\Big(Y_uY_u^\dagger m_u^{2*}\Big)-30\mathrm{Tr}\Big(Y_um_q^{2*}Y_u^\dagger\Big)\Big) \tag{A.48}$$
$$\sigma_{2,2}=\frac{1}{2}\Big(3\mathrm{Tr}\Big(m_q^2\Big)+m_{H_d}^2+m_{H_u}^2+\mathrm{Tr}\Big(m_l^2\Big)\Big) \tag{A.49}$$
$$\sigma_{2,3}=\frac{1}{2}\Big(2\mathrm{Tr}\Big(m_q^2\Big)+\mathrm{Tr}\Big(m_d^2\Big)+\mathrm{Tr}\Big(m_u^2\Big)\Big) \tag{A.50}$$
$$\beta_{m_q^2}^{(1)}=-\frac{2}{15}g_1^2\mathbf{1}|M_1|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2-6g_2^2\mathbf{1}|M_2|^2+2m_{H_d}^2Y_d^\dagger Y_d+2m_{H_u}^2Y_u^\dagger Y_u+2T_d^\dagger T_d$$
$$+2T_u^\dagger T_u+m_q^2Y_d^\dagger Y_d+m_q^2Y_u^\dagger Y_u+2Y_d^\dagger m_d^2Y_d+Y_d^\dagger Y_dm_q^2+2Y_u^\dagger m_u^2Y_u$$
$$+Y_u^\dagger Y_um_q^2+\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1} \tag{A.51}$$
$$\beta_{m_q^2}^{(2)}=+\frac{2}{5}g_1^2g_2^2\mathbf{1}|M_2|^2+33g_2^4\mathbf{1}|M_2|^2+32g_2^2g_3^2\mathbf{1}|M_2|^2$$
$$+\frac{16}{45}g_3^2\Big(15\Big(3g_2^2\Big(2M_3+M_2\Big)-8g_3^2M_3\Big)+g_1^2\Big(2M_3+M_1\Big)\Big)\mathbf{1}M_3^*$$
$$+\frac{1}{5}g_1^2g_2^2M_1\mathbf{1}M_2^*+16g_2^2g_3^2M_3\mathbf{1}M_2^*+\frac{4}{5}g_1^2m_{H_d}^2Y_d^\dagger Y_d+\frac{8}{5}g_1^2m_{H_u}^2Y_u^\dagger Y_u$$
$$+\frac{1}{45}g_1^2M_1^*\Big(\Big(141g_1^2M_1+16g_3^2\Big(2M_1+M_3\Big)+9g_2^2\Big(2M_1+M_2\Big)\Big)\mathbf{1}$$
$$+36\Big(2M_1Y_d^\dagger Y_d-2Y_u^\dagger T_u+4M_1Y_u^\dagger Y_u-Y_d^\dagger T_d\Big)\Big)$$
$$-\frac{4}{5}g_1^2M_1T_d^\dagger Y_d+\frac{4}{5}g_1^2T_d^\dagger T_d-\frac{8}{5}g_1^2M_1T_u^\dagger Y_u+\frac{8}{5}g_1^2T_u^\dagger T_u$$
$$+\frac{2}{5}g_1^2m_q^2Y_d^\dagger Y_d+\frac{4}{5}g_1^2m_q^2Y_u^\dagger Y_u+\frac{4}{5}g_1^2Y_d^\dagger m_d^2Y_d+\frac{2}{5}g_1^2Y_d^\dagger Y_dm_q^2$$
$$+\frac{8}{5}g_1^2Y_u^\dagger m_u^2Y_u+\frac{4}{5}g_1^2Y_u^\dagger Y_um_q^2-8m_{H_d}^2Y_d^\dagger Y_dY_d^\dagger Y_d-4Y_d^\dagger Y_dT_d^\dagger T_d$$

$$
\begin{aligned}
&-4Y_d^\dagger T_d T_d^\dagger Y_d-8m_{H_u}^2 Y_u^\dagger Y_u Y_u^\dagger Y_u-4Y_u^\dagger Y_u T_u^\dagger T_u-4Y_u^\dagger T_u T_u^\dagger Y_u\\
&-4T_d^\dagger Y_d Y_d^\dagger T_d-4T_d^\dagger T_d Y_d^\dagger Y_d-4T_u^\dagger Y_u Y_u^\dagger T_u-4T_u^\dagger T_u Y_u^\dagger Y_u\\
&-2m_q^2 Y_d^\dagger Y_d Y_d^\dagger Y_d-2m_q^2 Y_u^\dagger Y_u Y_u^\dagger Y_u-4Y_d^\dagger m_d^2 Y_d Y_d^\dagger Y_d-4Y_d^\dagger Y_d m_q^2 Y_d^\dagger Y_d\\
&-4Y_d^\dagger Y_d Y_d^\dagger m_d^2 Y_d-2Y_d^\dagger Y_d Y_d^\dagger Y_d m_q^2-4Y_u^\dagger m_u^2 Y_u Y_u^\dagger Y_u-4Y_u^\dagger Y_u m_q^2 Y_u^\dagger Y_u\\
&-4Y_u^\dagger Y_u Y_u^\dagger m_u^2 Y_u-2Y_u^\dagger Y_u Y_u^\dagger Y_u m_q^2+6g_2^4\mathbf{1}\sigma_{2,2}+\frac{32}{3}g_3^4\mathbf{1}\sigma_{2,3}+\frac{2}{15}g_1^2\mathbf{1}\sigma_{2,11}+4\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{3,1}\\
&-12m_{H_d}^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-6T_d^\dagger T_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-3m_q^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)\\
&-6Y_d^\dagger m_d^2 Y_d \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-3Y_d^\dagger Y_d m_q^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-4m_{H_d}^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\\
&-2T_d^\dagger T_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-m_q^2 Y_d^\dagger Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-2Y_d^\dagger m_d^2 Y_d \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\\
&-Y_d^\dagger Y_d m_q^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-12m_{H_u}^2 Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)-6T_u^\dagger T_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)\\
&-3m_q^2 Y_u^\dagger Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)-6Y_u^\dagger m_u^2 Y_u \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)-3Y_u^\dagger Y_u m_q^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)\\
&-6T_d^\dagger Y_d \mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-2T_d^\dagger Y_d \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)-6T_u^\dagger Y_u \mathrm{Tr}\Big(Y_u^\dagger T_u\Big)\\
&-6Y_d^\dagger T_d \mathrm{Tr}\Big(T_d^* Y_d^T\Big)-6Y_d^\dagger Y_d \mathrm{Tr}\Big(T_d^* T_d^T\Big)-2Y_d^\dagger T_d \mathrm{Tr}\Big(T_e^* Y_e^T\Big)\\
&-2Y_d^\dagger Y_d \mathrm{Tr}\Big(T_e^* T_e^T\Big)-6Y_u^\dagger T_u \mathrm{Tr}\Big(T_u^* Y_u^T\Big)-6Y_u^\dagger Y_u \mathrm{Tr}\Big(T_u^* T_u^T\Big)\\
&-6Y_d^\dagger Y_d \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big)-2Y_d^\dagger Y_d \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big)-2Y_d^\dagger Y_d \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big)\\
&-6Y_d^\dagger Y_d \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big)-6Y_u^\dagger Y_u \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big)-6Y_u^\dagger Y_u \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big) \qquad \text{(A.52)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_l^2}^{(1)}=&-\frac{6}{5}g_1^2\mathbf{1}|M_1|^2-6g_2^2\mathbf{1}|M_2|^2+2m_{H_d}^2 Y_e^\dagger Y_e+2T_e^\dagger T_e+m_l^2 Y_e^\dagger Y_e+2Y_e^\dagger m_e^2 Y_e\\
&+Y_e^\dagger Y_e m_l^2-\sqrt{\frac{3}{5}}g_1\mathbf{1}\sigma_{1,1} \qquad \text{(A.53)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_l^2}^{(2)}=&+\frac{3}{5}g_2^2\Big(3g_1^2\Big(2M_2+M_1\Big)+55g_2^2 M_2\Big)\mathbf{1}M_2^*+\frac{12}{5}g_1^2 m_{H_d}^2 Y_e^\dagger Y_e\\
&+\frac{3}{25}g_1^2 M_1^*\Big(-20Y_e^\dagger T_e+3\Big(5g_2^2\Big(2M_1+M_2\Big)+81g_1^2 M_1\Big)\mathbf{1}+40M_1 Y_e^\dagger Y_e\Big)-\frac{12}{5}g_1^2 M_1 T_e^\dagger Y_e\\
&+\frac{12}{5}g_1^2 T_e^\dagger T_e+\frac{6}{5}g_1^2 m_l^2 Y_e^\dagger Y_e+\frac{12}{5}g_1^2 Y_e^\dagger m_e^2 Y_e+\frac{6}{5}g_1^2 Y_e^\dagger Y_e m_l^2\\
&-8m_{H_d}^2 Y_e^\dagger Y_e Y_e^\dagger Y_e-4Y_e^\dagger Y_e T_e^\dagger T_e-4Y_e^\dagger T_e T_e^\dagger Y_e-4T_e^\dagger Y_e Y_e^\dagger T_e\\
&-4T_e^\dagger T_e Y_e^\dagger Y_e-2m_l^2 Y_e^\dagger Y_e Y_e^\dagger Y_e-4Y_e^\dagger m_e^2 Y_e Y_e^\dagger Y_e-4Y_e^\dagger Y_e m_l^2 Y_e^\dagger Y_e\\
&-4Y_e^\dagger Y_e Y_e^\dagger m_e^2 Y_e-2Y_e^\dagger Y_e Y_e^\dagger Y_e m_l^2+6g_2^4\mathbf{1}\sigma_{2,2}+\frac{6}{5}g_1^2\mathbf{1}\sigma_{2,11}-4\sqrt{\frac{3}{5}}g_1\mathbf{1}\sigma_{3,1}\\
&-12m_{H_d}^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-6T_e^\dagger T_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-3m_l^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)\\
&-6Y_e^\dagger m_e^2 Y_e \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-3Y_e^\dagger Y_e m_l^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)-4m_{H_d}^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\\
&-2T_e^\dagger T_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-m_l^2 Y_e^\dagger Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-2Y_e^\dagger m_e^2 Y_e \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)\\
&-Y_e^\dagger Y_e m_l^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big)-6T_e^\dagger Y_e \mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-2T_e^\dagger Y_e \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)
\end{aligned}
$$

$$-6Y_e^\dagger T_e \mathrm{Tr}\Big(T_d^* Y_d^T\Big) -6Y_e^\dagger Y_e \mathrm{Tr}\Big(T_d^* T_d^T\Big) -2Y_e^\dagger T_e \mathrm{Tr}\Big(T_e^* Y_e^T\Big)$$
$$-2Y_e^\dagger Y_e \mathrm{Tr}\Big(T_e^* T_e^T\Big) -6Y_e^\dagger Y_e \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) -2Y_e^\dagger Y_e \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big)$$
$$-2Y_e^\dagger Y_e \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) -6Y_e^\dagger Y_e \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) \tag{A.54}$$

$$\beta_{m_{H_d}^2}^{(1)} = -\frac{6}{5} g_1^2 |M_1|^2 -6 g_2^2 |M_2|^2 -\sqrt{\frac{3}{5}} g_1 \sigma_{1,1} +6 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) +2 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) +6 \mathrm{Tr}\Big(T_d^* T_d^T\Big)$$
$$+2 \mathrm{Tr}\Big(T_e^* T_e^T\Big) +6 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) +2 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) +2 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big) +6 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) \tag{A.55}$$

$$\beta_{m_{H_d}^2}^{(2)} = \frac{1}{25}\Big(15 g_2^2 \Big(3 g_1^2 \Big(2M_2 + M_1\Big) + 55 g_2^2 M_2\Big) M_2^*$$
$$+ g_1^2 M_1^* \Big(729 g_1^2 M_1 + 90 g_2^2 M_1 + 45 g_2^2 M_2 - 40 M_1 \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big) + 120 M_1 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) + 20 \mathrm{Tr}\Big(Y_d^\dagger T_d\Big)$$
$$-60 \mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\Big)$$
$$+10\Big(15 g_2^4 \sigma_{2,2} + 3 g_1^2 \sigma_{2,11} - 2\sqrt{15} g_1 \sigma_{3,1} + \Big(160 g_3^2 |M_3|^2 - 2 g_1^2 m_{H_d}^2 + 80 g_3^2 m_{H_d}^2\Big) \mathrm{Tr}\Big(Y_d Y_d^\dagger\Big)$$
$$+6 g_1^2 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger\Big) - 80 g_3^2 M_3^* \mathrm{Tr}\Big(Y_d^\dagger T_d\Big) + 2 g_1^2 M_1 \mathrm{Tr}\Big(T_d^* Y_d^T\Big) - 80 g_3^2 M_3 \mathrm{Tr}\Big(T_d^* Y_d^T\Big)$$
$$-2 g_1^2 \mathrm{Tr}\Big(T_d^* T_d^T\Big) + 80 g_3^2 \mathrm{Tr}\Big(T_d^* T_d^T\Big) - 6 g_1^2 M_1 \mathrm{Tr}\Big(T_e^* Y_e^T\Big) + 6 g_1^2 \mathrm{Tr}\Big(T_e^* T_e^T\Big)$$
$$-2 g_1^2 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) + 80 g_3^2 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger\Big) + 6 g_1^2 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger\Big) + 6 g_1^2 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e\Big)$$
$$-2 g_1^2 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) + 80 g_3^2 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d\Big) - 90 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_d^\dagger Y_d Y_d^\dagger\Big) - 90 \mathrm{Tr}\Big(Y_d Y_d^\dagger T_d T_d^\dagger\Big)$$
$$-15 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 15 m_{H_u}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 15 \mathrm{Tr}\Big(Y_d Y_u^\dagger T_u T_d^\dagger\Big)$$
$$-90 \mathrm{Tr}\Big(Y_d T_d^\dagger T_d Y_d^\dagger\Big) - 15 \mathrm{Tr}\Big(Y_d T_u^\dagger T_u Y_d^\dagger\Big) - 30 m_{H_d}^2 \mathrm{Tr}\Big(Y_e Y_e^\dagger Y_e Y_e^\dagger\Big) - 30 \mathrm{Tr}\Big(Y_e Y_e^\dagger T_e T_e^\dagger\Big)$$
$$-30 \mathrm{Tr}\Big(Y_e T_e^\dagger T_e Y_e^\dagger\Big) - 15 \mathrm{Tr}\Big(Y_u Y_d^\dagger T_d T_u^\dagger\Big) - 15 \mathrm{Tr}\Big(Y_u T_d^\dagger T_d Y_u^\dagger\Big) - 90 \mathrm{Tr}\Big(m_d^2 Y_d Y_d^\dagger Y_d Y_d^\dagger\Big)$$
$$-15 \mathrm{Tr}\Big(m_d^2 Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 30 \mathrm{Tr}\Big(m_e^2 Y_e Y_e^\dagger Y_e Y_e^\dagger\Big) - 30 \mathrm{Tr}\Big(m_l^2 Y_e^\dagger Y_e Y_e^\dagger Y_e\Big) - 90 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d Y_d^\dagger Y_d\Big)$$
$$-15 \mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d Y_u^\dagger Y_u\Big) - 15 \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u Y_d^\dagger Y_d\Big) - 15 \mathrm{Tr}\Big(m_u^2 Y_u Y_d^\dagger Y_d Y_u^\dagger\Big)\Big)\Big) \tag{A.56}$$

$$\beta_{m_{H_u}^2}^{(1)} = -\frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 + \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} + 6 m_{H_u}^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 6 \mathrm{Tr}\Big(T_u^* T_u^T\Big) + 6 \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big)$$
$$+6 \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big) \tag{A.57}$$

$$\beta_{m_{H_u}^2}^{(2)} = +\frac{3}{5} g_2^2 \Big(3 g_1^2 \Big(2M_2 + M_1\Big) + 55 g_2^2 M_2\Big) M_2^* + 6 g_2^4 \sigma_{2,2} + \frac{6}{5} g_1^2 \sigma_{2,11} + 4\sqrt{\frac{3}{5}} g_1 \sigma_{3,1}$$
$$+\frac{8}{5} g_1^2 m_{H_u}^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 32 g_3^2 m_{H_u}^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 64 g_3^2 |M_3|^2 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big)$$
$$+\frac{1}{25} g_1^2 M_1^* \Big(-40 \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) + 45 g_2^2 M_2 + 729 g_1^2 M_1 + 80 M_1 \mathrm{Tr}\Big(Y_u Y_u^\dagger\Big) + 90 g_2^2 M_1\Big)$$
$$-32 g_3^2 M_3^* \mathrm{Tr}\Big(Y_u^\dagger T_u\Big) - \frac{8}{5} g_1^2 M_1 \mathrm{Tr}\Big(T_u^* Y_u^T\Big) - 32 g_3^2 M_3 \mathrm{Tr}\Big(T_u^* Y_u^T\Big) + \frac{8}{5} g_1^2 \mathrm{Tr}\Big(T_u^* T_u^T\Big)$$
$$+32 g_3^2 \mathrm{Tr}\Big(T_u^* T_u^T\Big) + \frac{8}{5} g_1^2 \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) + 32 g_3^2 \mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u\Big) + \frac{8}{5} g_1^2 \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big)$$
$$+32 g_3^2 \mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger\Big) - 6 m_{H_d}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big) - 6 m_{H_u}^2 \mathrm{Tr}\Big(Y_d Y_u^\dagger Y_u Y_d^\dagger\Big)$$

$$
\begin{aligned}
&-6\mathrm{Tr}\Big(Y_d Y_u^\dagger T_u T_d^\dagger\Big)-6\mathrm{Tr}\Big(Y_d T_u^\dagger T_u Y_d^\dagger\Big)-6\mathrm{Tr}\Big(Y_u Y_d^\dagger T_d T_u^\dagger\Big)-36m_{H_u}^2\mathrm{Tr}\Big(Y_u Y_u^\dagger Y_u Y_u^\dagger\Big)\\
&-36\mathrm{Tr}\Big(Y_u Y_u^\dagger T_u T_u^\dagger\Big)-6\mathrm{Tr}\Big(Y_u T_d^\dagger T_d Y_u^\dagger\Big)-36\mathrm{Tr}\Big(Y_u T_u^\dagger T_u Y_u^\dagger\Big)\\
&-6\mathrm{Tr}\Big(m_d^2 Y_d Y_u^\dagger Y_u Y_d^\dagger\Big)-6\mathrm{Tr}\Big(m_q^2 Y_d^\dagger Y_d Y_u^\dagger Y_u\Big)-6\mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u Y_d^\dagger Y_d\Big)\\
&-36\mathrm{Tr}\Big(m_q^2 Y_u^\dagger Y_u Y_u^\dagger Y_u\Big)-6\mathrm{Tr}\Big(m_u^2 Y_u Y_d^\dagger Y_d Y_u^\dagger\Big)-36\mathrm{Tr}\Big(m_u^2 Y_u Y_u^\dagger Y_u Y_u^\dagger\Big) \qquad \text{(A.58)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_d^2}^{(1)}=&-\frac{8}{15}g_1^2\mathbf{1}|M_1|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2+4m_{H_d}^2 Y_d Y_d^\dagger+4T_d T_d^\dagger+2m_d^2 Y_d Y_d^\dagger+4Y_d m_q^2 Y_d^\dagger\\
&+2Y_d Y_d^\dagger m_d^2+2\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1} \qquad \text{(A.59)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_d^2}^{(2)}=&+\frac{64}{45}g_3^2\Big(-30g_3^2M_3+g_1^2\Big(2M_3+M_1\Big)\Big)\mathbf{1}M_3^*+\frac{4}{5}g_1^2m_{H_d}^2Y_dY_d^\dagger+12g_2^2m_{H_d}^2Y_dY_d^\dagger\\
&+24g_2^2|M_2|^2Y_dY_d^\dagger-\frac{4}{5}g_1^2M_1Y_dT_d^\dagger-12g_2^2M_2Y_dT_d^\dagger\\
&+\frac{4}{225}g_1^2M_1^*\Big(2\Big(357g_1^2M_1+40g_3^2\Big(2M_1+M_3\Big)\Big)\mathbf{1}-45T_dY_d^\dagger+90M_1Y_dY_d^\dagger\Big)-12g_2^2M_2^*T_dY_d^\dagger\\
&+\frac{4}{5}g_1^2T_dT_d^\dagger+12g_2^2T_dT_d^\dagger+\frac{2}{5}g_1^2m_d^2Y_dY_d^\dagger+6g_2^2m_d^2Y_dY_d^\dagger\\
&+\frac{4}{5}g_1^2Y_dm_q^2Y_d^\dagger+12g_2^2Y_dm_q^2Y_d^\dagger+\frac{2}{5}g_1^2Y_dY_d^\dagger m_d^2+6g_2^2Y_dY_d^\dagger m_d^2\\
&-8m_{H_d}^2Y_dY_d^\dagger Y_dY_d^\dagger-4Y_dY_d^\dagger T_dT_d^\dagger-4m_{H_d}^2Y_dY_u^\dagger Y_uY_d^\dagger\\
&-4m_{H_u}^2Y_dY_u^\dagger Y_uY_d^\dagger-4Y_dY_u^\dagger T_uT_d^\dagger-4Y_dT_d^\dagger T_dY_d^\dagger-4Y_dT_u^\dagger T_uY_d^\dagger\\
&-4T_dY_d^\dagger Y_dT_d^\dagger-4T_dY_u^\dagger Y_uT_d^\dagger-4T_dT_d^\dagger Y_dY_d^\dagger-4T_dT_u^\dagger Y_uY_d^\dagger\\
&-2m_d^2Y_dY_d^\dagger Y_dY_d^\dagger-2m_d^2Y_dY_u^\dagger Y_uY_d^\dagger-4Y_dm_q^2Y_d^\dagger Y_dY_d^\dagger-4Y_dm_q^2Y_u^\dagger Y_uY_d^\dagger\\
&-4Y_dY_d^\dagger m_d^2Y_dY_d^\dagger-4Y_dY_d^\dagger Y_dm_q^2Y_d^\dagger-2Y_dY_d^\dagger Y_dY_d^\dagger m_d^2-4Y_dY_u^\dagger m_u^2Y_uY_d^\dagger\\
&-4Y_dY_u^\dagger Y_um_q^2Y_d^\dagger-2Y_dY_u^\dagger Y_uY_d^\dagger m_d^2+\frac{32}{3}g_3^4\mathbf{1}\sigma_{2,3}+\frac{8}{15}g_1^2\mathbf{1}\sigma_{2,11}+8\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{3,1}\\
&-24m_{H_d}^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-12T_dT_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-6m_d^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-12Y_dm_q^2Y_d^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-6Y_dY_d^\dagger m_d^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-8m_{H_d}^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-4T_dT_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-2m_d^2Y_dY_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-4Y_dm_q^2Y_d^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-2Y_dY_d^\dagger m_d^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-12Y_dT_d^\dagger\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-4Y_dT_d^\dagger\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)\\
&-12T_dY_d^\dagger\mathrm{Tr}\Big(T_d^*Y_d^T\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(T_d^*T_d^T\Big)-4T_dY_d^\dagger\mathrm{Tr}\Big(T_e^*Y_e^T\Big)\\
&-4Y_dY_d^\dagger\mathrm{Tr}\Big(T_e^*T_e^T\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(m_d^2Y_dY_d^\dagger\Big)-4Y_dY_d^\dagger\mathrm{Tr}\Big(m_e^2Y_eY_e^\dagger\Big)\\
&-4Y_dY_d^\dagger\mathrm{Tr}\Big(m_l^2Y_e^\dagger Y_e\Big)-12Y_dY_d^\dagger\mathrm{Tr}\Big(m_q^2Y_d^\dagger Y_d\Big) \qquad \text{(A.60)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_u^2}^{(1)}=&-\frac{32}{15}g_1^2\mathbf{1}|M_1|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2+4m_{H_u}^2Y_uY_u^\dagger+4T_uT_u^\dagger+2m_u^2Y_uY_u^\dagger+4Y_um_q^2Y_u^\dagger\\
&+2Y_uY_u^\dagger m_u^2-4\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1} \qquad \text{(A.61)}
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{m_u^2}=&-\frac{128}{45}g_3^2\Big(15g_3^2M_3-2g_1^2\Big(2M_3+M_1\Big)\Big)\mathbf{1}M_3^*-\frac{4}{5}g_1^2m_{H_u}^2Y_uY_u^\dagger+12g_2^2m_{H_u}^2Y_uY_u^\dagger\\
&+24g_2^2|M_2|^2Y_uY_u^\dagger+\frac{4}{5}g_1^2M_1Y_uT_u^\dagger-12g_2^2M_2Y_uT_u^\dagger-12g_2^2M_2^*T_uY_u^\dagger\\
&+\frac{4}{45}g_1^2M_1^*\Big(8\Big(75g_1^2M_1+8g_3^2\Big(2M_1+M_3\Big)\Big)\mathbf{1}+9\Big(-2M_1Y_uY_u^\dagger+T_uY_u^\dagger\Big)\Big)-\frac{4}{5}g_1^2T_uT_u^\dagger\\
&+12g_2^2T_uT_u^\dagger-\frac{2}{5}g_1^2m_u^2Y_uY_u^\dagger+6g_2^2m_u^2Y_uY_u^\dagger-\frac{4}{5}g_1^2Y_um_q^2Y_u^\dagger\\
&+12g_2^2Y_um_q^2Y_u^\dagger-\frac{2}{5}g_1^2Y_uY_u^\dagger m_u^2+6g_2^2Y_uY_u^\dagger m_u^2-4m_{H_d}^2Y_uY_d^\dagger Y_dY_u^\dagger\\
&-4m_{H_u}^2Y_uY_d^\dagger Y_dY_u^\dagger-4Y_uY_d^\dagger T_dT_u^\dagger-8m_{H_u}^2Y_uY_u^\dagger Y_uY_u^\dagger-4Y_uY_u^\dagger T_uT_u^\dagger\\
&-4Y_uT_d^\dagger T_dY_u^\dagger-4Y_uT_u^\dagger T_uY_u^\dagger-4T_uY_d^\dagger Y_dT_u^\dagger-4T_uY_u^\dagger Y_uT_u^\dagger\\
&-4T_uT_d^\dagger Y_dY_u^\dagger-4T_uT_u^\dagger Y_uY_u^\dagger-2m_u^2Y_uY_d^\dagger Y_dY_u^\dagger-2m_u^2Y_uY_u^\dagger Y_uY_u^\dagger\\
&-4Y_um_q^2Y_d^\dagger Y_dY_u^\dagger-4Y_um_q^2Y_u^\dagger Y_uY_u^\dagger-4Y_uY_d^\dagger m_d^2Y_dY_u^\dagger\\
&-4Y_uY_d^\dagger Y_dm_q^2Y_u^\dagger-2Y_uY_d^\dagger Y_dY_u^\dagger m_u^2-4Y_uY_u^\dagger m_u^2Y_uY_u^\dagger-4Y_uY_u^\dagger Y_um_q^2Y_u^\dagger\\
&-2Y_uY_u^\dagger Y_uY_u^\dagger m_u^2+\frac{32}{3}g_3^4\mathbf{1}\sigma_{2,3}+\frac{32}{15}g_1^2\mathbf{1}\sigma_{2,11}-16\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{3,1}-24m_{H_u}^2Y_uY_u^\dagger\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\\
&-12T_uT_u^\dagger\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-6m_u^2Y_uY_u^\dagger\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-12Y_um_q^2Y_u^\dagger\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)\\
&-6Y_uY_u^\dagger m_u^2\mathrm{Tr}\Big(Y_uY_u^\dagger\Big)-12Y_uT_u^\dagger\mathrm{Tr}\Big(Y_u^\dagger T_u\Big)-12T_uY_u^\dagger\mathrm{Tr}\Big(T_u^*Y_u^T\Big)\\
&-12Y_uY_u^\dagger\mathrm{Tr}\Big(T_u^*T_u^T\Big)-12Y_uY_u^\dagger\mathrm{Tr}\Big(m_q^2Y_u^\dagger Y_u\Big)-12Y_uY_u^\dagger\mathrm{Tr}\Big(m_u^2Y_uY_u^\dagger\Big)
\end{aligned}
\tag{A.62}
$$

$$
\begin{aligned}
\beta^{(1)}_{m_e^2}=&-\frac{24}{5}g_1^2\mathbf{1}|M_1|^2+2\Big(2m_{H_d}^2Y_eY_e^\dagger+2T_eT_e^\dagger+2Y_em_l^2Y_e^\dagger+m_e^2Y_eY_e^\dagger+Y_eY_e^\dagger m_e^2\Big)\\
&+2\sqrt{\frac{3}{5}}g_1\mathbf{1}\sigma_{1,1}
\end{aligned}
\tag{A.63}
$$

$$
\begin{aligned}
\beta^{(2)}_{m_e^2}=&\frac{2}{5}\Big(-6g_1^2m_{H_d}^2Y_eY_e^\dagger+30g_2^2m_{H_d}^2Y_eY_e^\dagger+60g_2^2|M_2|^2Y_eY_e^\dagger+6g_1^2M_1Y_eT_e^\dagger\\
&-30g_2^2M_2Y_eT_e^\dagger-30g_2^2M_2^*T_eY_e^\dagger+6g_1^2M_1^*\Big(-2M_1Y_eY_e^\dagger+54g_1^2M_1\mathbf{1}+T_eY_e^\dagger\Big)\\
&-6g_1^2T_eT_e^\dagger+30g_2^2T_eT_e^\dagger-3g_1^2m_e^2Y_eY_e^\dagger+15g_2^2m_e^2Y_eY_e^\dagger\\
&-6g_1^2Y_em_l^2Y_e^\dagger+30g_2^2Y_em_l^2Y_e^\dagger-3g_1^2Y_eY_e^\dagger m_e^2+15g_2^2Y_eY_e^\dagger m_e^2\\
&-20m_{H_d}^2Y_eY_e^\dagger Y_eY_e^\dagger-10Y_eY_e^\dagger T_eT_e^\dagger-10Y_eT_e^\dagger T_eY_e^\dagger-10T_eY_e^\dagger Y_eT_e^\dagger\\
&-10T_eT_e^\dagger Y_eY_e^\dagger-5m_e^2Y_eY_e^\dagger Y_eY_e^\dagger-10Y_em_l^2Y_e^\dagger Y_eY_e^\dagger-10Y_eY_e^\dagger m_e^2Y_eY_e^\dagger\\
&-10Y_eY_e^\dagger Y_em_l^2Y_e^\dagger-5Y_eY_e^\dagger Y_eY_e^\dagger m_e^2+4g_1\mathbf{1}\Big(3g_1\sigma_{2,11}+\sqrt{15}\sigma_{3,1}\Big)\\
&-60m_{H_d}^2Y_eY_e^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-30T_eT_e^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-15m_e^2Y_eY_e^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)\\
&-30Y_em_l^2Y_e^\dagger\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-15Y_eY_e^\dagger m_e^2\mathrm{Tr}\Big(Y_dY_d^\dagger\Big)-20m_{H_d}^2Y_eY_e^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-10T_eT_e^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-5m_e^2Y_eY_e^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-10Y_em_l^2Y_e^\dagger\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)\\
&-5Y_eY_e^\dagger m_e^2\mathrm{Tr}\Big(Y_eY_e^\dagger\Big)-30Y_eT_e^\dagger\mathrm{Tr}\Big(Y_d^\dagger T_d\Big)-10Y_eT_e^\dagger\mathrm{Tr}\Big(Y_e^\dagger T_e\Big)
\end{aligned}
$$

$$-30T_eY_e^\dagger\text{Tr}\Big(T_d^*Y_d^T\Big)-30Y_eY_e^\dagger\text{Tr}\Big(T_d^*T_d^T\Big)-10T_eY_e^\dagger\text{Tr}\Big(T_e^*Y_e^T\Big)$$

$$-10Y_eY_e^\dagger\text{Tr}\Big(T_e^*T_e^T\Big)-30Y_eY_e^\dagger\text{Tr}\Big(m_d^2Y_dY_d^\dagger\Big)-10Y_eY_e^\dagger\text{Tr}\Big(m_e^2Y_eY_e^\dagger\Big)$$

$$-10Y_eY_e^\dagger\text{Tr}\Big(m_l^2Y_e^\dagger Y_e\Big)-30Y_eY_e^\dagger\text{Tr}\Big(m_q^2Y_d^\dagger Y_d\Big)\Big) \tag{A.64}$$

$$\beta^{(1)}_{m^2_{F_+}}=\frac{2}{5}g_1\Big(-12g_1|M_1|^2+\sqrt{15}\sigma_{1,1}\Big) \tag{A.65}$$

$$\beta^{(2)}_{m^2_{F_+}}=\frac{8}{5}g_1\Big(3g_1\sigma_{2,11}+81g_1^3|M_1|^2+\sqrt{15}\sigma_{3,1}\Big) \tag{A.66}$$

$$\beta^{(1)}_{m^2_{F_-}}=-\frac{2}{5}g_1\Big(12g_1|M_1|^2+\sqrt{15}\sigma_{1,1}\Big) \tag{A.67}$$

$$\beta^{(2)}_{m^2_{F_-}}=\frac{8}{5}g_1\Big(3g_1\sigma_{2,11}+81g_1^3|M_1|^2-\sqrt{15}\sigma_{3,1}\Big) \tag{A.68}$$

B Renormalisation group equations for the 5D-SSM+$\boldsymbol{F}^\pm$

In this appendix we supply the one-loop beta functions used in the main paper for the five dimensional model 1, model 2 and model 3, including the five dimensional Kaluza-Klein states and extra fields. Note that the RGEs for model 3 can be read off from model 1 and 2 as in every model the RGEs for fields in the bulk is similar to model 2 RGEs, and the RGEs for fields on the brane is similar to RGEs of model 1. The Higgs sector RGEs for model 3 are always in the bulk in both model 1 and 2. We define $t = \text{Log}_{10} Q$ and $\beta_A = 16\pi^2 dA/dt$. It is useful to also define the power law contribution, which may be written equivalently as

$$(QR)^d = 10^t R\,. \tag{B.1}$$

B.1 Gauge couplings

The one-loop beta function for the gauge couplings if $t > \ln(1/R)/\ln(10)$ are given by

$$16\pi^2\frac{dg_i(t)}{dt} = b^i_{\text{MSSM}}g_i^3(t) + b^i_{5D}g_i^3(t)(S(t)-1), \tag{B.2}$$

where $i = 1, 2, 3$ and $S(t) = R10^t$, the power law contribution. For the $4DSSM + F^\pm$, $b^i = (39/5, 1, -3)$ and for five dimensions $b^i_{5D} = (18/5, -2, -6)+4\eta$, where η is the number of fermion generation in the bulk. The fine structure constants may be defined from $\alpha_i = g_i^2/4\pi$.

B.2 Yukawa couplings

The beta functions for the Yukawa couplings may be related to the matrices of anomalous dimensions

$$\beta^{ijk}_Y = \gamma^i_n Y^{njk} + \gamma^i_n Y^{ink} + \gamma^k_n Y^{ijn}. \tag{B.3}$$

B.2.1 Anomalous dimensions for model 1

$$\gamma_{\tilde{H}_u} = 3\mathrm{Tr}\left(Y_u Y_u^\dagger\right) - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right) S(t) \tag{B.4}$$

$$\gamma_{\tilde{H}_d} = 3\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \mathrm{Tr}\left(Y_e Y_e^\dagger\right) - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right) S(t) \tag{B.5}$$

$$\gamma_{\tilde{F}^\pm} = Y_F Y_F^\dagger - \frac{12}{10}g_1^2 S(t) \tag{B.6}$$

$$\gamma_{\tilde{q}} = \left(2\left(Y_u Y_u^\dagger + Y_d Y_d^\dagger\right) - \left(\frac{1}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right) S(t) \tag{B.7}$$

$$\gamma_{\tilde{u}} = \left(4Y_u Y_u^\dagger - \left(\frac{16}{15}g_1^2 + \frac{16}{3}g_3^2\right)\right) S(t) \tag{B.8}$$

$$\gamma_{\tilde{d}} = \left(4Y_d Y_d^\dagger - \left(\frac{4}{15}g_1^2 + \frac{16}{3}g_3^2\right)\right) S(t) \tag{B.9}$$

$$\gamma_{\tilde{l}} = \left(2Y_e Y_e^\dagger - \left(\frac{3}{5}g_1^2 + 3g_2^2\right)\right) S(t) \tag{B.10}$$

$$\gamma_{\tilde{e}} = \left(4Y_e Y_e^\dagger - \frac{12}{5}g_1^2\right) S(t). \tag{B.11}$$

B.2.2 Anomalous dimensions for model 2

$$\gamma_{\tilde{H}_u} = 3\mathrm{Tr}\left(Y_u Y_u^\dagger\right)\pi S(t)^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right) S(t) \tag{B.12}$$

$$\gamma_{\tilde{H}_d} = \left(3\mathrm{Tr}\left(Y_d Y_d^\dagger\right) + \mathrm{Tr}\left(Y_e Y_e^\dagger\right)\right)\pi S(t)^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right) S(t) \tag{B.13}$$

$$\gamma_{\tilde{F}^\pm} = Y_F Y_F^\dagger \pi S(t)^2 - \frac{12}{10}g_1^2 S(t) \tag{B.14}$$

$$\gamma_{\tilde{q}^3} = \left(Y_t Y_t^\dagger + Y_b Y_b^\dagger\right)\pi S(t)^2 - \left(\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2\right) S(t) \tag{B.15}$$

$$\gamma_{\tilde{u}^3} = 2Y_t Y_t^\dagger \pi S(t)^2 - \left(\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2\right) S(t) \tag{B.16}$$

$$\gamma_{\tilde{d}^3} = 2Y_b Y_b^\dagger \pi S(t)^2 - \left(\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2\right) S(t) \tag{B.17}$$

$$\gamma_{\tilde{l}^3} = Y_\tau Y_\tau^\dagger \pi S(t)^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right) S(t) \tag{B.18}$$

$$\gamma_{\tilde{e}^3} = 2Y_\tau Y_\tau^\dagger \pi S(t)^2 - \frac{6}{5}g_1^2 S(t). \tag{B.19}$$

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B.2.3 Anomalous dimensions for model 3

$$\gamma_{\tilde{H}_u} = 3\mathrm{Tr}\left(Y_t Y_t^\dagger\right)\pi S(t)^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)S(t) \tag{B.20}$$

$$\gamma_{\tilde{H}_d} = \left(3\mathrm{Tr}\left(Y_b Y_b^\dagger\right) + \mathrm{Tr}\left(Y_\tau Y_\tau^\dagger\right)\right)\pi S(t)^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)S(t) \tag{B.21}$$

$$\gamma_{\tilde{F}^\pm} = Y_F Y_F^\dagger \pi S(t)^2 - \frac{12}{10}g_1^2 S(t) \tag{B.22}$$

$$\gamma_{\tilde{q}^3} = 2\left(Y_t Y_t^\dagger + Y_b Y_b^\dagger\right)S(t)^2 - \left(\frac{1}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t) \tag{B.23}$$

$$\gamma_{\tilde{u}^3} = 4Y_t Y_t^\dagger S(t) - \left(\frac{16}{15}g_1^2 + \frac{16}{3}g_3^2\right)S(t) \tag{B.24}$$

$$\gamma_{\tilde{d}^3} = 4Y_b Y_b^\dagger S(t) - \left(\frac{4}{15}g_1^2 + \frac{16}{3}g_3^2\right)S(t) \tag{B.25}$$

$$\gamma_{\tilde{l}^3} = 2Y_\tau Y_\tau^\dagger S(t) - \left(\frac{3}{5}g_1^2 + 3g_2^2\right)S(t) \tag{B.26}$$

$$\gamma_{\tilde{e}^3} = 4Y_\tau Y_\tau^\dagger S(t) - \frac{12}{5}g_1^2 S(t). \tag{B.27}$$

B.2.4 Yukawa coupling RGEs for model 1

The five dimensional contributions for model 1 are given by

$$\beta^{(1)}_{(5D)Y_u} = Y_u\left(\left(6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d + 2Y_F^\dagger Y_F\right) - \left(\frac{79}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t) \tag{B.28}$$

$$\beta^{(1)}_{(5D)Y_d} = Y_d\left(\left(6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u + 2Y_F^\dagger Y_F\right) - \left(\frac{55}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t) \tag{B.29}$$

$$\beta^{(1)}_{(5D)Y_e} = Y_e\left(6Y_e^\dagger Y_e + 2Y_F^\dagger Y_F - \left(\frac{9}{2}g_1^2 + \frac{9}{2}g_2^2\right)\right)S(t) \tag{B.30}$$

$$\beta^{(1)}_{(5D)Y_F} = Y_F\left(4Y_F^\dagger Y_F - \left(3g_1^2 + 3g_2^2\right)\right)S(t). \tag{B.31}$$

B.2.5 Yukawa coupling RGEs for model 2

The five dimensional contributions for model 2 are given by

$$\begin{aligned}\beta^{(1)}_{(5D)Y_t} &= Y_t\left(3\mathrm{Tr}\left(Y_t^\dagger Y_t\right) + 3Y_t^\dagger Y_t + Y_b^\dagger Y_b + Y_F^\dagger Y_F\right)\pi S(t)^2 \\ &\quad - Y_t\left(\frac{31}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t)\end{aligned} \tag{B.32}$$

$$\begin{aligned}\beta^{(1)}_{(5D)Y_b} &= Y_b\left(3\mathrm{Tr}\left(Y_b^\dagger Y_b\right) + \mathrm{Tr}\left(Y_\tau^\dagger Y_\tau\right) + 3Y_b^\dagger Y_b + Y_t^\dagger Y_t + Y_F^\dagger Y_F\right)\pi S(t)^2 \\ &\quad - Y_b\left(\frac{25}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t)\end{aligned} \tag{B.33}$$

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$$\beta^{(1)}_{(5D)Y_\tau} = Y_\tau\left(3\text{Tr}\left(Y_b^\dagger Y_b\right) + \text{Tr}\left(Y_\tau^\dagger Y_\tau\right) + 3Y_\tau^\dagger Y_\tau + Y_F^\dagger Y_F\right)\pi S(t)^2$$
$$-Y_\tau\left(\frac{15}{5}g_1^2 + 3g_2^2\right)S(t) \tag{B.34}$$

$$\beta^{(1)}_{(5D)Y_F} = Y_F\left(2Y_F^\dagger Y_F + 3\text{Tr}\left(Y_b^\dagger Y_b\right) + 3\text{Tr}\left(Y_t^\dagger Y_t\right) + \text{Tr}\left(Y_\tau^\dagger Y_\tau\right)\right)\pi S(t)^2$$
$$-\left(3g_1^2 + 3g_2^2\right)S(t). \tag{B.35}$$

Note that the evolution equations for $Y_{u,c}$, $Y_{d,s}$ and $Y_{e,\mu}$ can be read from eq. (B.28), since the first and second generation live on the brane.

B.2.6 Yukawa coupling RGEs for model 3

The five dimensional contributions for model 3 are given by

$$\beta^{(1)}_{(5D)Y_t} = Y_t\left(\left(6Y_t^\dagger Y_t + 2Y_b^\dagger Y_b + 2Y_F^\dagger Y_F\right) - \left(\frac{79}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t) \tag{B.36}$$

$$\beta^{(1)}_{(5D)Y_b} = Y_b\left(\left(6Y_b^\dagger Y_b + 2Y_t^\dagger Y_t + 2Y_F^\dagger Y_F\right) - \left(\frac{55}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t) \tag{B.37}$$

$$\beta^{(1)}_{(5D)Y_e} = Y_\tau\left(6Y_\tau^\dagger Y_\tau + 2Y_F^\dagger Y_F - \left(\frac{9}{2}g_1^2 + \frac{9}{2}g_2^2\right)\right)S(t) \tag{B.38}$$

$$\beta^{(1)}_{(5D)Y_F} = Y_F\left(4Y_F^\dagger Y_F - \left(3g_1^2 + 3g_2^2\right)\right)S(t). \tag{B.39}$$

B.3 Trilinear soft breaking parameters

B.3.1 Trilinear soft breaking parameters for model 1

$$\beta^{(1)}_{(5D)T_u} = T_u\left(\left(18Y_u^\dagger Y_u + 2Y_d^\dagger Y_d + 2Y_F^\dagger Y_F\right) - \left(\frac{79}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t)$$
$$+Y_u\left(4T_dY_d^\dagger + 4Y_F^\dagger T_F + \frac{79}{15}g_1^2M_1 + 9g_2^2M_2 + \frac{64}{3}g_3^2M_3\right)S(t) \tag{B.40}$$

$$\beta^{(1)}_{(5D)T_d} = T_d\left(\left(18Y_d^\dagger Y_d + 2Y_u^\dagger Y_u + 2Y_F^\dagger Y_F\right) - \left(\frac{55}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right)S(t) \tag{B.41}$$
$$+Y_d\left(4Y_F^\dagger T_F + 4T_uY_u^\dagger + 2T_eY_e^\dagger + \frac{55}{15}g_1^2M_1 + 9g_2^2M_2 + \frac{64}{3}g_3^2M_3\right)S(t)$$

$$\beta^{(1)}_{(5D)T_e} = T_e\left(18Y_e^\dagger Y_e + 2Y_F^\dagger Y_F - \left(\frac{9}{2}g_1^2 + \frac{9}{2}g_2^2\right)\right)S(t)$$
$$+Y_e\left(6T_dY_d^\dagger + 4Y_F^\dagger T_F + \frac{18}{2}g_1^2M_1 + 9g_2^2M_2\right)S(t) \tag{B.42}$$

$$\beta^{(1)}_{(5D)T_F} = T_F\left(12Y_F^\dagger Y_F - \left(3g_1^2 + 3g_2^2\right)\right)S(t) + Y_F\left(6g_1^2M_1 + 6g_2^2M_2\right)S(t). \tag{B.43}$$

B.3.2 Trilinear soft breaking parameters for model 2

$$\begin{aligned}
\beta^{(1)}_{(5D)T_t} &= -T_t\left(\frac{31}{15}g_1^2+3g_2^2+\frac{16}{3}g_3^2\right)S(t)+Y_t\left(\frac{62}{15}g_1^2M_1+6g_2^2M_2+\frac{32}{3}g_3^2M_3\right)S(t)\\
&\quad +Y_t\left(6\mathrm{Tr}\left(Y_t^\dagger T_t\right)+6Y_t^\dagger T_t+2Y_b^\dagger T_b+2Y_F^\dagger T_F\right)\pi S(t)^2\\
&\quad +T_t\left(3\mathrm{Tr}\left(Y_t^\dagger Y_t\right)+3Y_t^\dagger Y_t+Y_b^\dagger Y_b+Y_F^\dagger Y_F\right)\pi S(t)^2 \qquad \text{(B.44)}\\
\beta^{(1)}_{(5D)T_b} &= -T_b\left(\frac{25}{15}g_1^2+3g_2^2+\frac{16}{3}g_3^2\right)S(t)+Y_b\left(\frac{50}{15}g_1^2M_1+6g_2^2M_2+\frac{32}{3}g_3^2M_3\right)S(t)\\
&\quad +Y_b\left(6\mathrm{Tr}\left(Y_b^\dagger T_b\right)+2\mathrm{Tr}\left(Y_\tau^\dagger T_\tau\right)+6Y_b^\dagger T_b+2Y_t^\dagger T_t+2Y_F^\dagger T_F\right)\pi S(t)^2\\
&\quad +T_b\left(3\mathrm{Tr}\left(Y_b^\dagger Y_b\right)+\mathrm{Tr}\left(Y_\tau^\dagger Y_\tau\right)+3Y_b^\dagger Y_b+Y_t^\dagger Y_t+Y_F^\dagger Y_F\right)\pi S(t)^2 \qquad \text{(B.45)}\\
\beta^{(1)}_{(5D)T_\tau} &= -T_\tau\left(3g_1^2+3g_2^2\right)S(t)+Y_\tau\left(6g_1^2M_1+6g_2^2M_2\right)S(t)\\
&\quad +Y_\tau\left(6\mathrm{Tr}\left(Y_b^\dagger T_b\right)+2\mathrm{Tr}\left(Y_\tau^\dagger T_\tau\right)+6Y_\tau^\dagger T_\tau+2Y_F^\dagger T_F\right)\pi S(t)^2\\
&\quad +T_\tau\left(3\mathrm{Tr}\left(Y_b^\dagger Y_b\right)+\mathrm{Tr}\left(Y_\tau^\dagger Y_\tau\right)+3Y_\tau^\dagger Y_\tau+Y_F^\dagger Y_F\right)\pi S(t)^2 \qquad \text{(B.46)}\\
\beta^{(1)}_{(5D)T_F} &= -T_F\left(3g_1^2+3g_2^2\right)S(t)+Y_F\left(6g_1^2M_1+6g_2^2M_2\right)S(t)\\
&\quad +Y_F\left(6\mathrm{Tr}\left(Y_b^\dagger T_b\right)+6\mathrm{Tr}\left(Y_t^\dagger T_t\right)+2\mathrm{Tr}\left(Y_\tau^\dagger T_\tau\right)+4Y_F^\dagger T_F\right)\pi S(t)^2\\
&\quad +T_F\left(3\mathrm{Tr}\left(Y_b^\dagger Y_b\right)+3\mathrm{Tr}\left(Y_t^\dagger Y_t\right)+\mathrm{Tr}\left(Y_\tau^\dagger Y_\tau\right)+2Y_F^\dagger Y_F\right)\pi S(t)^2. \qquad \text{(B.47)}
\end{aligned}$$

B.4 Soft mass parameters

B.4.1 Gaugino soft mass parameters

The gaugino soft masses in 5D run following

$$\beta^{(1)}_{M_i} = 2b^i g_i^2 M_i + 2(S(t)-1)b^i_{5D}g_i^2M_i. \qquad \text{(B.48)}$$

B.4.2 Scalar soft mass parameters for model 1

$$\begin{aligned}
\beta^{(1)}_{m_q^2} &= \left(-\frac{8}{15}g_1^2\mathbf{1}|M_1|^2-\frac{64}{3}g_3^2\mathbf{1}|M_3|^2-12g_2^2\mathbf{1}|M_2|^2+4m_{H_d}^2Y_d^\dagger Y_d+4m_{H_u}^2Y_u^\dagger Y_u\right)S(t)\\
&\quad +\left(4T_d^\dagger T_d+4T_u^\dagger T_u+2m_q^2Y_d^\dagger Y_d+2m_q^2Y_u^\dagger Y_u+4Y_d^\dagger m_d^2Y_d+2Y_d^\dagger Y_dm_q^2\right)S(t)\\
&\quad +\left(4Y_u^\dagger m_u^2Y_u+2Y_u^\dagger Y_um_q^2+\frac{2}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1}\right)S(t) \qquad \text{(B.49)}\\
\beta^{(1)}_{m_u^2} &= \left(-\frac{64}{15}g_1^2\mathbf{1}|M_1|^2-\frac{64}{3}g_3^2\mathbf{1}|M_3|^2+8m_{H_u}^2Y_uY_u^\dagger+8T_uT_u^\dagger+4m_u^2Y_uY_u^\dagger\right)S(t)\\
&\quad +\left(8Y_um_q^2Y_u^\dagger+4Y_uY_u^\dagger m_u^2-4\sqrt{\frac{2}{15}}g_1\mathbf{1}\sigma_{1,1}\right)S(t) \qquad \text{(B.50)}
\end{aligned}$$

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$$\beta^{(1)}_{m_d^2} = \left(-\frac{16}{15}g_1^2\mathbf{1}|M_1|^2 - \frac{64}{3}g_3^2\mathbf{1}|M_3|^2 + 8m_{H_d}^2 Y_d Y_d^\dagger + 8T_d T_d^\dagger + 4m_d^2 Y_d Y_d^\dagger\right) S(t)$$
$$+ \left(8Y_d m_q^2 Y_d^\dagger + 4Y_d Y_d^\dagger m_d^2 + 2\sqrt{\frac{2}{15}}g_1\mathbf{1}\sigma_{1,1}\right) S(t) \qquad \text{(B.51)}$$

$$\beta^{(1)}_{m_l^2} = \left(-\frac{12}{5}g_1^2\mathbf{1}|M_1|^2 - 12g_2^2\mathbf{1}|M_2|^2 + 4m_{H_d}^2 Y_e^\dagger Y_e + 4T_e^\dagger T_e + 2m_l^2 Y_e^\dagger Y_e\right) S(t)$$
$$+ \left(4Y_e^\dagger m_e^2 Y_e + 2Y_e^\dagger Y_e m_l^2 - \sqrt{\frac{6}{5}}g_1\mathbf{1}\sigma_{1,1}\right) S(t) \qquad \text{(B.52)}$$

$$\beta^{(1)}_{m_e^2} = 2\left(4m_{H_d}^2 Y_e Y_e^\dagger + 4T_e T_e^\dagger + 4Y_e m_l^2 Y_e^\dagger + 2m_e^2 Y_e Y_e^\dagger + 2Y_e Y_e^\dagger m_e^2\right) S(t)$$
$$+ \left(2\sqrt{\frac{6}{5}}g_1\mathbf{1}\sigma_{1,1} - \frac{48}{5}g_1^2\mathbf{1}|M_1|^2\right) S(t) \qquad \text{(B.53)}$$

In model 1 the two Higgs doublet soft masses obey the RGE's

$$\beta^{(1)}_{m_{H_d}^2} = \left(-\frac{6}{5}g_1^2|M_1|^2 - 6g_2^2|M_2|^2 - \sqrt{\frac{3}{5}}g_1\sigma_{1,1} + 6m_{H_d}^2 \mathrm{Tr}\left(Y_d Y_d^\dagger\right)\right) S(t)$$
$$+ \left(2m_{H_d}^2 \mathrm{Tr}\left(Y_e Y_e^\dagger\right) + 6\mathrm{Tr}\left(T_d^* T_d^T\right) + 2\mathrm{Tr}\left(T_e^* T_e^T\right) + 6\mathrm{Tr}\left(m_d^2 Y_d Y_d^\dagger\right)\right) S(t)$$
$$+ \left(2\mathrm{Tr}\left(m_e^2 Y_e Y_e^\dagger\right) + 2\mathrm{Tr}\left(m_l^2 Y_e^\dagger Y_e\right) + 6\mathrm{Tr}\left(m_q^2 Y_d^\dagger Y_d\right)\right) S(t) \qquad \text{(B.54)}$$

$$\beta^{(1)}_{m_{H_u}^2} = \left(-\frac{6}{5}g_1^2|M_1|^2 - 6g_2^2|M_2|^2 + \sqrt{\frac{3}{5}}g_1\sigma_{1,1} + 6m_{H_u}^2 \mathrm{Tr}\left(Y_u Y_u^\dagger\right)\right) S(t)$$
$$+ \left(6\mathrm{Tr}\left(T_u^* T_u^T\right) + 6\mathrm{Tr}\left(m_q^2 Y_u^\dagger Y_u\right) + 6\mathrm{Tr}\left(m_u^2 Y_u Y_u^\dagger\right)\right) S(t) \qquad \text{(B.55)}$$

$$\beta^{(1)}_{m_{F^\pm}^2} = \left(-\frac{24}{5}g_1^2|M_1|^2 + 2m_{H_{u,d}}^2 Y_F^\dagger Y_F + 2T_F^\dagger T_F + m_{F^\pm}^2 Y_F^\dagger Y_F\right) S(t)$$
$$+ \left(Y_F^\dagger Y_F m_{F^\pm}^2\right) S(t) \qquad \text{(B.56)}$$

B.4.3 Scalar soft mass parameters for model 2

$$\beta^{(1)}_{m_{q_3}^2} = \left(-\frac{2}{15}g_1^2\mathbf{1}|M_1|^2 - \frac{32}{3}g_3^2\mathbf{1}|M_3|^2 - 6g_2^2\mathbf{1}|M_2|^2\right) S(t) + \left(2m_{H_d}^2 Y_b^\dagger Y_b + 2m_{H_u}^2 Y_t^\dagger Y_t\right)\pi S(t)^2$$
$$+ \left(2T_b^\dagger T_b + 2T_t^\dagger T_t + m_{q_3}^2 Y_b^\dagger Y_b + m_{q_3}^2 Y_t^\dagger Y_t + 2Y_b^\dagger m_{d_3}^2 Y_b + Y_b^\dagger Y_b m_{q_3}^2\right)\pi S(t)^2$$
$$+ \left(2Y_t^\dagger m_{u_3}^2 Y_t + Y_t^\dagger Y_t m_{q_3}^2\right)\pi S(t)^2 + \left(\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1}\right) S(t) \qquad \text{(B.57)}$$

$$\beta^{(1)}_{m_{u_3}^2} = \left(-\frac{32}{15}g_1^2\mathbf{1}|M_1|^2 - \frac{32}{3}g_3^2\mathbf{1}|M_3|^2\right) S(t) + \left(4m_{H_u}^2 Y_t Y_t^\dagger + 4T_t T_t^\dagger + 2m_{u_3}^2 Y_t Y_t^\dagger\right)\pi S(t)^2$$
$$+ \left(4Y_t m_{q_3}^2 Y_t^\dagger + 2Y_t Y_t^\dagger m_{u_3}^2\right)\pi S(t)^2 - \left(4\frac{1}{\sqrt{15}}g_1\mathbf{1}\sigma_{1,1}\right) S(t) \qquad \text{(B.58)}$$

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$$\beta^{(1)}_{m^2_{d_3}} = \left(-\frac{8}{15}g_1^2\mathbf{1}|M_1|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2\right)S(t)+\left(4m^2_{H_d}Y_bY_b^\dagger+4T_bT_b^\dagger+2m^2_{d_3}Y_bY_b^\dagger\right)\pi S(t)^2$$
$$+\left(4Y_bm^2_{q_3}Y_b^\dagger+2Y_bY_b^\dagger m^2_{d_3}\right)\pi S(t)^2+\left(2\sqrt{\frac{1}{15}}g_1\mathbf{1}\sigma_{1,1}\right)S(t) \tag{B.59}$$

$$\beta^{(1)}_{m^2_{l_3}} = \left(-\frac{6}{5}g_1^2\mathbf{1}|M_1|^2-6g_2^2\mathbf{1}|M_2|^2\right)S(t)+\left(4m^2_{H_d}Y_\tau^\dagger Y_\tau+4T_\tau^\dagger T_\tau+2m^2_{l_3}Y_\tau^\dagger Y_\tau\right)\pi S(t)^2$$
$$+\left(2Y_\tau^\dagger m^2_{e_3}Y_\tau+Y_\tau^\dagger Y_\tau m^2_{l_3}\right)\pi S(t)^2-\left(\sqrt{\frac{3}{5}}g_1\mathbf{1}\sigma_{1,1}\right)S(t) \tag{B.60}$$

$$\beta^{(1)}_{m^2_{e_3}} = 2\left(2m^2_{H_d}Y_\tau Y_\tau^\dagger+2T_\tau T_\tau^\dagger+2Y_\tau m^2_{l_3}Y_\tau^\dagger+m^2_{e_3}Y_\tau Y_\tau^\dagger+Y_\tau Y_\tau^\dagger m^2_{e_3}\right)\pi S(t)^2$$
$$+\left(2\sqrt{\frac{3}{5}}g_1\mathbf{1}\sigma_{1,1}-\frac{24}{5}g_1^2\mathbf{1}|M_1|^2\right)S(t) \tag{B.61}$$

In model 2 the two Higgs doublet soft masses obey the RGE's

$$\beta^{(1)}_{m^2_{H_d}} = \left(-\frac{6}{5}g_1^2|M_1|^2-6g_2^2|M_2|^2-\sqrt{\frac{3}{5}}g_1\sigma_{1,1}\right)S(t)+\left(6m^2_{H_d}\mathrm{Tr}\left(Y_bY_b^\dagger\right)\right)\pi S(t)^2$$
$$+\left(2m^2_{H_d}\mathrm{Tr}\left(Y_\tau Y_\tau^\dagger\right)+6\mathrm{Tr}\left(T_b^*T_b^T\right)+2\mathrm{Tr}\left(T_\tau^*T_\tau^T\right)+6\mathrm{Tr}\left(m^2_{d_3}Y_bY_b^\dagger\right)\right)\pi S(t)^2$$
$$+\left(2\mathrm{Tr}\left(m^2_{e_3}Y_\tau Y_\tau^\dagger\right)+2\mathrm{Tr}\left(m^2_{l_3}Y_\tau^\dagger Y_\tau\right)+6\mathrm{Tr}\left(m^2_{q_3}Y_b^\dagger Y_b\right)\right)\pi S(t)^2 \tag{B.62}$$

$$\beta^{(1)}_{m^2_{H_u}} = \left(-\frac{6}{5}g_1^2|M_1|^2-6g_2^2|M_2|^2+\sqrt{\frac{3}{5}}g_1\sigma_{1,1}\right)S(t)+\left(6m^2_{H_u}\mathrm{Tr}\left(Y_tY_t^\dagger\right)\right)\pi S(t)^2$$
$$+\left(6\mathrm{Tr}\left(T_t^*T_t^T\right)+6\mathrm{Tr}\left(m^2_{q_3}Y_t^\dagger Y_t\right)+6\mathrm{Tr}\left(m^2_{u_3}Y_tY_t^\dagger\right)\right)\pi S(t)^2 \tag{B.63}$$

$$\beta^{(1)}_{m^2_{F^\pm}} = -\frac{24}{5}g_1^2|M_1|^2S(t)+\left(2m^2_{H_{u,d}}Y_F^\dagger Y_F+2T_F^\dagger T_F+m^2_{F^\pm}Y_F^\dagger Y_F\right)\pi S(t)^2$$
$$+\left(Y_F^\dagger Y_F m^2_{F^\pm}\right)\pi S(t)^2 \tag{B.64}$$

B.5 Bilinear parameters μ and B_μ

In 5D these are given by:

$$\beta^{(1)}_\mu = \mu\left(3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)+\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-\frac{3}{5}g_1^2-3g_2^2\right)S(t) \tag{B.65}$$

$$\beta^{(1)}_{\acute\mu} = \left(2\acute\mu\left(Y_FY_F^\dagger\right)-\frac{12}{5}\acute\mu g_1^2\right)S(t) \tag{B.66}$$

$$\beta^{(1)}_{B_\mu} = B_\mu\left(-3g_2^2-\frac{3}{5}g_1^2+3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)+\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\right)S(t)$$
$$+\mu\left(6g_2^2M_2+\frac{6}{5}g_1^2M_1+6\mathrm{Tr}\left(Y_u^\dagger T_u\right)+6\mathrm{Tr}\left(Y_d^\dagger T_d\right)+2\mathrm{Tr}\left(Y_e^\dagger T_e\right)\right)S(t) \tag{B.67}$$

$$\beta^{(1)}_{B_{\acute\mu}} = \left(-\frac{12}{5}B_{\acute\mu}g_1^2+\frac{24}{5}\acute\mu g_1^2M_1+2B_{\acute\mu}Y_F^\dagger Y_F+4\acute\mu Y_F^\dagger Y_F\right)S(t). \tag{B.68}$$

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