TRANSMISSION TRANSFORMER TAP CHANGER OPTIMISATION WHILE MINIMISING SYSTEM LOSSES USING SECURITY UNCONSTRAINED OPTIMAL POWER FLOW (OPF) TECHNIQUES

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A dissertation submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, in fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 2011

DECLARATION

I declare that this dissertation is my own unaided work. It is being submitted for the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

(Signature of Candidate)

08th day of August 2011

Day *month* *year*

ABSTRACT

This dissertation investigates the feasibility of *fixed tap or reduced tap transformers* on the Eskom Transmission system, thereby reducing transformer failures and reducing the cost of new or replacement transformers.

The main analysis uses Optimal Power Flow (OPF) techniques based on a Siemens PSS/E software platform and a minimum system loss operating point or objective function while reducing tap movements and maintaining prudent system limits but allowing shunt VAr devices to freely compensate for the reduced tapping. Various years and system conditions are analysed.

The research shows that:

- 135 Transmission-Distribution transformers are suitable for changing to fixed tap transformers
- 160 Transmission-Distribution transformers are suitable for changing to reduced tap transformers
- 80 Transmission-Distribution transformers should be left as-is
- A 5.7% reduction in system losses from 1118.4 MW to 1054.9 MW (63.5 MW) is possible as a result of an optimised system

Finally it is shown that replacing some Transmission transformers with fixed-tap transformers is a more cost effective solution.

The contribution of this research is two-fold:

1. to show that there is scope for Eskom System Operations to operate its system more efficiently in terms of system losses
2. to show that an OPF methodology is a practical technique to limit transformer tap movements and allow optimal VAr placement while minimising system losses

ACKNOWLEDGEMENTS

This dissertation was conducted while the author was an employee of Parsons Brinckerhoff Africa (Pty) Ltd and is based on a study undertaken for Eskom Transmission Technology.

I would sincerely like to thank:

My supervisor Doctor John van Coller for his guidance and advice.

Gavin Hurford for his early insights into the best way to approach the research.

Roger Cormack whose idea it was to undertake the research.

Gavin Bruce for authorising the use of Eskom system information in the dissertation.

Jarrad Wright and Jane Swarbreck for proof-reading the various drafts of the dissertation and for advice.

The Faculty of Engineering and the Built Environment of the University of the Witwatersrand for giving me the opportunity to carry out my part time research.

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LIST OF SYMBOLS

Complex nodal currents	**I̲**
Complex nodal voltages	**V̲**
Passive components including lines, series elements, transformers and shunts	**Y̲**
Real power or active power in a transmission line or a transformer	*P*
Voltage	*V*
Tap limits	*t*
Phase shift angle of a transformer or voltage angle between nodes	Ɵ
Shunt capacitance or reactance	*s*
Reactive power generation of a PV node	*Q*
MVA flow in transmission line or transformer	*S*
Current magnitude in transmission line or transformer	*I*
Total number of buses (when dealing with Standard Power Flow (SPF) analysis)	N
Net complex power generation at bus i	S_{Gi}
Net complex load at bus i	S_{Li}
Complex power flow on branch connecting buses i and j	s_{ij}
Control Variable	U
State Variable	X
Number of penalised control variables (when dealing with quadratic penalty function)	N
Scalar quadratic penalty weight	ρ
Control variable current value (per unit) (when dealing with quadratic penalty function)	x
Control variable penalty offset (per unit)	x_0
Power system variables (control and dependent, column vector)	x
Objective function (e.g. active power dispatch cost)	f(x)
Equality constraints including bus-power mismatch equations (column vector)	h(x)

Lagrange multiplier variable (column vector)	λ
OPF operating point	x^k

LIST OF ABBREVIATIONS

AC	Alternating current
AGC	Automatic Generator Control
AVG	Average
AVR	Automatic voltage control
Capex	Capital expenditure
DTC	De-energised tap-changers
EHV	Extremely High Voltage
HV	High Voltage
HVDC	High Voltage Direct Current
kV	kilovolt
LMP	Locational Marginal Pricing
LP	Linear Programming
LRMC	Long Run Marginal Cost
LTC	Load Tap Changer
MAT	Maximum Allowable Transfer
MD	Main Distribution
MD transformer	Main Distribution transformer (e.g. 132/88 kV)
MS Excel	Microsoft Excel
MTS	Main Transmission System
MTS transformer	Main Transmission System transformer (e.g. 765/400 kV)
MVA	Megavolt Ampere (apparent power)
MVAr	Megavar (imaginary power)
MW	Megawatt (real power)
OLTC	On load tap changer

Opex	Operational expenditure
OPF	Optimal Power Flow
Pmax	Maximum power output from a generator
Pmin	Minimum power output from a generator
POS	Point of Supply
PQ node	Bus or node where the real power and power factor are fixed and the voltage is allowed to vary
PV node	Bus or node where the real power and voltage are fixed and the power factor is allowed to vary
PSS/E	Siemens/PTI Power System Simulator for Engineers
p.u.	per unit
Qmax	Maximum output VArs from a generator
Qmin	Minimum output VArs from a generator
QP	Quadratic Programming
RTO	Regional Transmission Operator
RTS	Real Time Simulator
SCADA	System Control and Data Acquisition
SCOPF	Security Constrained Optimal Power Flow
SD	Sub-Distribution
SD transformer	Sub-Distribution transformer (e.g. 132/22 kV)
SO	System Operator
SPF	Standard power flow
STD DEV	Standard deviation
SVC	Static VAr Compensator
TCR	Thyristor controlled reactor
TD	Transmission-Distribution
TD transformer	Transmission-Distribution transformer (e.g. 400/132 kV)

TNSP	Transmission Network Service Provider
VAr	Reactive power described as voltage ampere reactive
VCB	Vacuum circuit breaker
VS	Voltage stability
VSAT	Voltage stability simulation tool
ZAR	South African rand
#	Number (usually used in Tables)

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1. Introduction

1.1 Problem statement

This dissertation investigates the feasibility of *fixed-tap* or *reduced-tap transformers* on the Eskom Transmission system, thereby reducing transformer failures and reducing the cost of new transformers or replacement transformers when existing transformers have reached the end of their useful lives or when transformer ratings need to be increased due to stepped or progressive load increases, while minimising system real and reactive losses.

The research uses Optimal Power Flow (OPF) computer simulation techniques based on the non-linear programming (NLP) method consisting of an objective function with a linear combination of equality constraints to form a scalar Lagrangian function and inequality constraints imposed through implicit objective terms expressed as a function of the constrained variable [1], [2].

1.2 Background

On Load Tap Changers (OLTCs) contribute to a large number of transformer failures. This can be explained by the fact that the OLTC is the only transformer component that contains moving parts [3].

OLTC factors responsible for transformer failures include: oil quality (particulate contamination), LTC contact temperature rise, contact coking, carbon film build-up, short circuit mechanical forces and contact wear and arcing [4]. See Figure 1.1 below.

In addition to the above, utility operational control practices, which may cause tap changer hunting and therefore excessive tap changer operations also contribute to tap changer aging and failures [4].

Eskom Transmission has records showing the incidence of transformer failures due to tap-changer failures and how transformers where tapping is seldom performed are often more problematic due to contact coking and high contact resistances causing localised heating [3].

Figure 1.1 – Typical damage to changeover selector contacts [3]

1.3 Background on the Eskom Transmission system

Eskom Transmission is a division of Eskom Holdings in South Africa.

Eskom Transmission transmits electricity via a Transmission network which supplies electricity at high voltages (HV) and extremely high voltages (EHV) to a number of key customers and distributors.

The Transmission license is held by Eskom Transmission, the Transmission network service provider (TNSP). There is one non-Eskom Transmission network and license in South Africa and this is the Motraco 400 kV Transmission line linking Camden Power Station in South Africa to Swaziland and Mozambique.

Planning the Transmission network is the responsibility of the Grid Planning Department in the System Operations and Planning Division in MegaWatt Park, Johannesburg.

The operation of the Transmission network is the responsibility of System Operations (SO) in the System Operations and Planning Division in Simmerpan, Johannesburg.

Transmission voltages in Eskom and South Africa are defined as follows:

- 765 kV
- 533 kV (HVDC)
- 400 kV
- 275 kV
- 220 kV

The Eskom Transmission network comprises 130 Transmission substations and 400 Transmission transformers.

Figure 1.2 shows a geographical representation of the South African Transmission network.

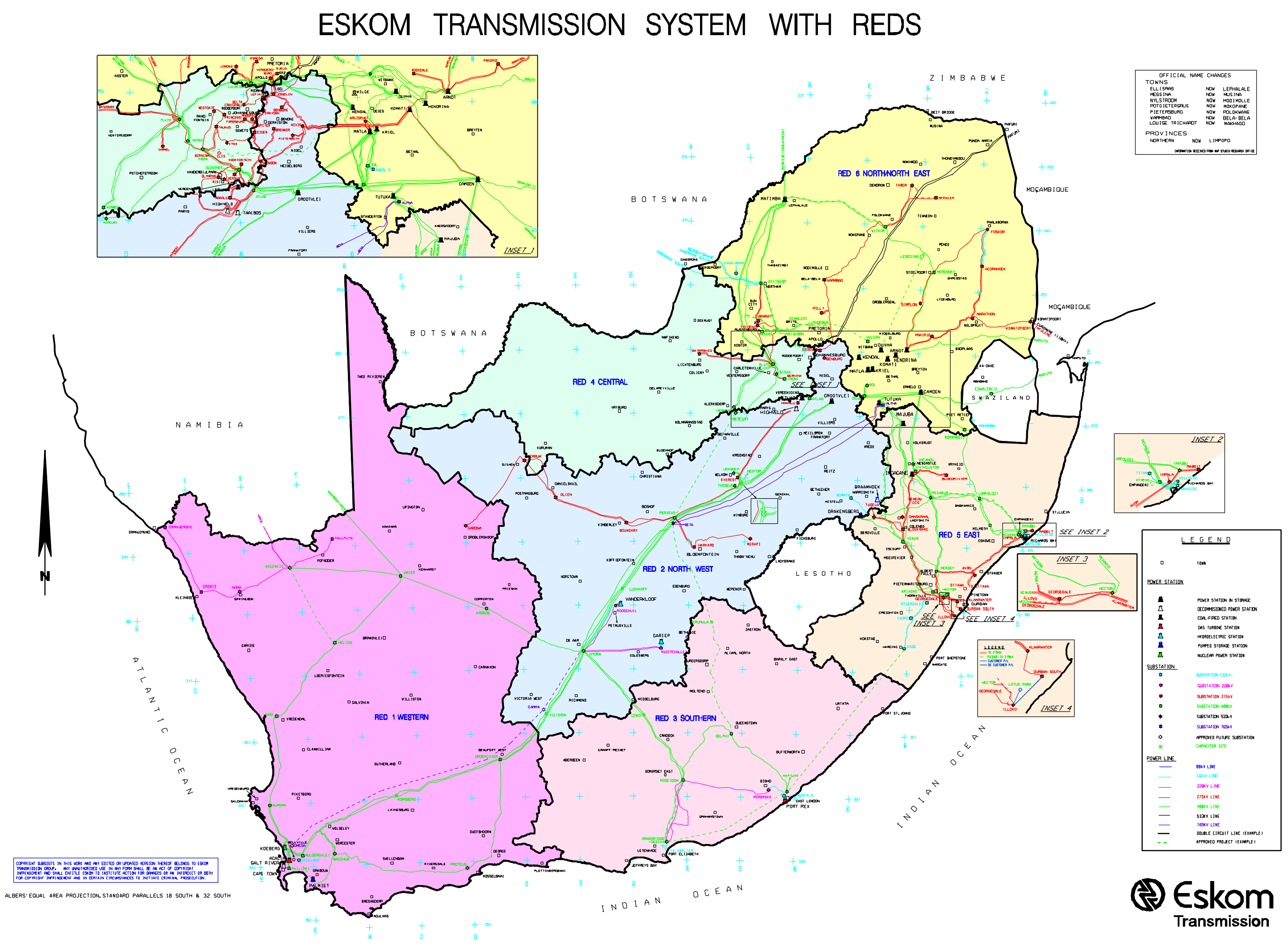

Figure 1.2: Eskom geographical Transmission diagram [Source: Eskom Grid Planning, 2010]

In Figure 1.2, the main 400 kV (green) Transmission backbone can be seen traversing the main regions of the country. 765 kV (purple) lines can be seen linking the Mpumalanga generation area to the Bloemfontein area. 275 kV (red) lines can be seen in the Johannesburg, Natal, and Northern areas as well as linking the Bloemfontein area to the Northern Cape. 220 kV (light purple or pink) lines can be seen in the East London and Port Elizabeth areas, in the North Western Cape area (just below the Namibian border) and south of Bloemfontein, connecting Gariep and Vanderkloof hydro power stations to the main Transmission grid. 533 kV High Voltage Direct Current (HVDC) lines (black lines) can be seen entering South Africa in the North East of the country and traversing the Northern Province towards Apollo substation between Pretoria and Johannesburg.

It is the responsibility of the SO to ensure that the power system is operated efficiently and reliably, ensuring that least-cost generation is dispatched, real and reactive power flows on the Transmission network are managed in the most efficient manner, that all loads are supplied adequately and that all generators have access to the transmission network.

1.4 Validation and justification

This dissertation uses Optimal Power Flow (OPF) techniques based on a Siemens PSS/E software platform [1] ensuring a minimum system loss operating point (commonly referred to as the objective function) [5], [6], [7], [8] to establish whether existing or planned Transmission tapped transformers can be replaced with fixed-tap or reduced-tap transformers while utilising mechanically-switched or thyristor-controlled VAr devices to mitigate the removal of the tapping function from the Transmission transformers [9].

The reasons for choosing the *explicit objective function* as *minimising real and reactive system losses* [1], [2] are as follows:

- The main objective of the System Operator (SO) is to operate the system in the most cost efficient way while maintaining adequate system security. It makes little sense for the SO to operate the least efficient generators (if there is no operational reason to do so e.g. at system peak) and operate at high system losses, even though system security may be achieved
- The PSS/E OPF software does not allow the choosing of *minimising transformer tapping* as an *explicit objective function*, however, the setting of a *quadratic penalty coefficient* [1] in PSS/E enables the tapping of transformers to be reduced or fixed as an *implicit objective function* [1]

- Choosing fixing of taps as the *explicit objective function* at the cost of system efficiency was considered unfeasible considering the SO's responsibility to operate the Transmission system optimally
- No optimisation on system dispatch or system cost is undertaken in the analysis as the system PSS/E simulation snapshot files are taken directly from the system operator (SO) Real Time Simulator (RTS) system. The Eskom SO uses an optimised Automatic Generator Control (AGC) system to optimise generator dispatch based on real generator costs and it is assumed to operate correctly in this research. Adding an optimise-system-dispatch *explicit objective function* is a possible area for future research however a detailed generator cost analysis would need to be carried out on dispatchable generation connected to the Eskom Transmission system as these costs are not currently updated at the SO

Recently, innovative techniques are being used to improve the reliability of tap changing units e.g. vacuum enclosed diverter switches and thyristor controlled tapping [10], [11]. Innovative tap-changer solutions come at a cost and any tap changer adds from 5% to 20% to the cost of a transformer [12]. This research attempts to show that the Eskom Transmission system can function optimally from a system losses, cost and voltage control point of view without the need for transformer tapping or with reduced transformer tapping on a subset of the Transmission transformers.

Although the intention of this research is to investigate reduced tapping on Transmission transformers, one of the benefits resulting from the conclusions of the research is to encourage Eskom System Operations to review their operating practices in terms of system losses and optimal system voltages.

1.5 Status of current work on the subject

OPF has been a key part of power system control since the early 1970s [13], [14], [15].

Initial implementations of OPF were oriented towards the solution of economic dispatch with Transmission constraints. Other implementations included modelling of the power system network including real power and reactive power constraints, and the modelling of reserve constraints. Later implementations included the modelling of security constraints, otherwise known as Security Constrained Optimal Power Flow (SCOPF) and the introduction of preventive, corrective, and preventive-corrective modes of operation [13].

Recent developments include the use of OPF as a starting point for the calculation of Locational Marginal Prices (LMPs) in recently developed electricity markets and the auctioning of financial Transmission rights with both obligations and options. Other recent implementations include the use of OPF techniques to manage congestion across neighbouring Regional Transmission Operators (RTOs) [13].

The history of OPF research can be characterised by the application of increasingly powerful digital computer optimisation tools to a problem which has been well-defined since the early 1960's [16], [17]. The OPF constitutes a static non-linear optimisation problem which computes optimal settings for electrical variables in a power network, for given settings of loads and system parameters. Progress has manifested itself with solutions of larger and more complex problems in a suitable computation time frame [13].

Historically, the solution of the economic dispatch by the Equal Incremental Cost method (EICC) was a precursor of the OPF. The arrival of the OPF marked the end of the "classical" period of economic dispatch, which had developed for almost 30 years [18]. The OPF was a radical departure from dispatching, although now those dispatching algorithms can be seen as crude simplifications of the OPF. Both are optimisation problems, with the same minimum cost objective. Economic dispatch, however, only considers real power generation and represents the electrical network by a single equality constraint, the power balance equation [13].

Reactive power and voltage are often dispatched separately from real power using the remaining degrees of freedom offered by the network. The benefits of this are reduced production costs, unloading of equipment and improved voltage profile [19]. Typically some norm of reactive power deviations or a closely related function such as real power losses or dependent *slack* generation is minimised, subject to voltage, reactive power, shunt and tap constraints [13].

Pandya and Joshi [20] list the following OPF analysis techniques:

- Linear Programming (LP) Method
- Newton-Raphson (NR) Method
- Quadratic Programming (QP) Method
- Nonlinear Programming (NLP) Method
- Interior Point (IP) Method
- Artificial Intelligence (AI) Methods
    - Artificial Neural Network (ANN)
    - Fuzzy Logic (FL) Methods
    - Genetic Algorithm (GA) Methods
    - Miscellaneous AI Methods
    - Evolutionary Programming (EP)
    - Ant Colony Optimisation (ACO)

- Particle Swarm Optimisation (PSO)

Pandya and Joshi conclude their paper with a summary of the methods listed above. They conclude that the NLP, QP, IP and AI methods are the most accurate OPF analytical techniques for reactive power optimisation and optimal location of FACTS devices [20].

Later in Section 2 of this dissertation, it can be seen that the method used by the software platform used in this research (PSS/E [1]) is the Nonlinear Programming (NLP) method.

The purpose of this dissertation however is not to analyse the advantages and disadvantages of the many different OPF techniques nor is it to develop a new mathematical computational technique to minimise Transmission transformer tapping or optimising costs and losses on a power system.

As mentioned previously, the objective of this dissertation is to investigate whether it is feasible to reduce or eliminate Transmission transformer tapping on the Eskom Transmission network. Therefore, the author has used a commercial power system analysis tool in PSS/E [1] for the OPF analysis and the author has assumed that the simulation tool has been tested and verified accordingly to result in feasible OPF solutions.

A verification or validation of the Siemens PSS/E software package is therefore not within the scope of this research. PSS/E is a widely used and validated power system analysis software package internationally (http://www.energy.siemens.com/us/en/services/power-transmission-distribution/power-technologies-international/software-solutions/pss-e.htm). PSS/E has been in existence as a top power system analysis tool since the 1980's.

The author is not aware of any similar studies undertaken on a large Transmission system with the specific objective of *minimising Transmission transformer tapping using OPF*, although there are many examples of studies attempting to minimise system real and reactive system losses [6], [13], [14], [16] [20].

1.6 Main research tasks

The main tasks in the research comprise the following:

- Undertake an analysis of MTS and TD transformer tapping frequency and ranges utilising available historical Eskom SCADA and operational recorded data
- Determine the optimised tapping range of the MTS and TD transformers using the OPF module in PSS/E
- Conduct the above studies for a range of generating pattern, Transmission configuration and loading scenarios

- Advise on the preferred tap range for TD transformers based on the SCADA and operational data and the optimisation analysis
- Determine the pervasiveness of fixed-tap or reduced-tap Transmission transformers in other utilities (benchmark research)
- Obtain the cost comparison of reduced-tap TD transformers versus full-tapped transformers and make an estimate of the capex saving if the reduced-tap transformer standard were adopted

1.7 Sources of data

The author held a meeting with Eskom System Operations staff at National Control, Simmerpan and collected the necessary research data.

The OPF modelling uses the listed generator base cases, Transmission contingencies and declared voltages outlined in the following sections 1.7.1 to 1.7.4.

1.7.1 Generator dispatch base files

The following PSS/E generator dispatch base cases are used in the studies, eight in total, as shown in Table 1-1. The major difference between Generation Dispatch Scenario One (G1) and Generation Dispatch Scenario Two (G2) is that G1 has both Koeberg units in service whereas G2 only has 1 Koeberg unit in service. Koeberg is the base-load 2 x 900 MW nuclear power station in Cape Town.

Table 1-1: Generator dispatch scenarios

	WINTER MAX LOADING				SUMMER MIN LOADING			
Research Years	Gen Dispatch 1	Case	Gen Dispatch 2	Case	Gen Dispatch 1	Case	Gen Dispatch 2	Case
2009	G1	1	G2	5	G1	2	G2	7
2015	G1	3	G2	6	G1	4	G2	8

Table 1-2 shows total system generation, load and losses for the base-winter-peak and base-summer-minimum system load files.

In Table 1-2, Drakensburg and Palmiet pumped-storage power stations are pumping (and not generating) at minimum system load, total generating VArs are importing at minimum system load (due to lightly loaded capacitive lines) and it can be seen how dramatically total system MVAr losses drop from the peak-load system case to the low-load system case.

Table 1-2: Generator dispatch (MW)

Generation Dispatch, Total Load and Total System Losses in MW (unless where otherwise stated)			
Power Station	**Type of Energy**	**System Winter Peak File 2009**	**System Summer Minimum File 2009**
Camden	Coal-fired	1 075	745
Grootvlei	Coal-fired	309	328
Komati	Coal-fired	168	61
Drakensburg	Pumped storage	-	-
Acacia	Gas turbines	-	-
Port Rex	Gas turbines	-	-
Matimba	Coal-fired	3 043	2 349
Arnot	Coal-fired	1 694	1 453
Hendrina	Coal-fired	1 456	869
Kendal	Coal-fired	3 741	1 944
Duvha	Coal-fired	3 270	2 527
Matla	Coal-fired	3 277	2 084
Kriel	Coal-fired	2 342	1 787
Tutuka	Coal-fired	3 193	1 298
Majuba	Coal-fired	3 768	1 281
Drakensburg	Pumped storage	999	-927
Van der Kloof	Hydro	239	-
Koeberg	Nuclear	1 739	1 747
Palmiet	Pumped storage	397	-394
Gariep	Hydro	88	-
Lethabo	Coal-fired	3 484	1 054
Total Generation		**34 452**	**18 341**
Total System Load		**33 333**	**17 861**
Total System VAr Load		**9 547**	**5 653**
Total System Generator Vars		**5 370**	**-2 252**
Total System MW Losses		**1 118**	**480**
Total System MVAr Losses		**17 847**	**7 809**

1.7.2 Transmission contingencies

Transmission line (N-1) contingencies are as shown in Table 1-3 for all the above generator base cases. *Contingencies* is a generic term for describing unplanned or disruptive system incidents e.g. the faulting and tripping of important buses, lines or transformers.

The Eskom Transmission system is generally planned and operated as an (N-1) system i.e. the system should still be able to operate correctly if one line or one transformer on a particular sub-system trips or becomes out of service.

In some instances, systems are planned to operate even after (N-2) contingencies (two items of plant become out-of-service on a particular sub-system) however only (N-1) contingencies are considered in this research.

Table 1-3: Transmission contingencies

Transmission Contingency Number	Description of Transmission Contingency
1	Arnot – Merensky 400 kV
2	Jupiter – Prospect 275 kV
3	Spitskop – Ararat 275 kV
4	Beta – Hydra 765 kV
5	Beta – Delphi 765 kV

The above contingencies are chosen due to the large impact these line outages have on their local Transmission systems and on the overall Transmission system. The most heavily loaded lines between two substations or the most heavily loaded transformers in a substation are typically the most onerous branch elements to trip as the tripping of these lines or transformers cause the biggest increment of change to normal power transfer.

1.7.3 Declared or contracted voltages

A *Declared Voltages* or *Contracted Voltages* file was received from System Operations.

In most cases Transmission and Distribution substation bus voltages lie within 1 p.u. and 1.05 p.u. voltage under normal conditions. Stressed systems are allowed to operate at 0.95 p.u. voltage under normal conditions and as low as 0.90 p.u. voltage under system contingencies.

Some Distribution Points of Supply (POS) are allowed to operate marginally above 1.05 p.u. voltage where long Distribution and reticulation system are connected to the ends of these points of supply. This is to assist these Distribution and reticulation systems to provide at least 0.95 p.u. voltage at the end of their loaded networks under normal system conditions.

In some instances, these >1.05 p.u. POS voltages are agreed with distributors or large customers and are even included in their contracts with Eskom Transmission. This is what is meant by the term: *Declared Voltages* or *Contracted Voltages*.

1.7.4 Tap-change statistics and SCADA data

Historical tap-changer frequency and range statistics were received from Eskom System Operations.

Tap positions from the SCADA data are compared with tap positions from the Standard Power Flow (SPF) file and the values derived in the OPF analysis.

Table 1-4 shows historically how many transformers utilized their tap ranges and by how much. SCADA data shows that over the 3 years studied about 65 TD transformers on the Eskom Transmission system used only a single tap, about 100 transformers kept within a tap range of only 3 taps, and the remaining approximately 180 transformers tapped over a wide tap range of >3 taps.

Table 1-4: Number of transformers which utilized 1, 2, 3 or >3 taps for 95% of the time (historical data)

Tap Range Used	2002	2003	2008
1 tap	69	66	55
2-3 taps	115	100	76
>3 taps	158	177	213
Total number of transformers	342	343	344

1.8 Transformer description definitions

In this research, Main Transmission System (MTS) transformers are defined as transformers with the following voltage ratios:

- 765/400 kV
- 400/275 kV
- 400/220 kV

Transmission/Distribution (TD) transformers are defined as transformers with the following voltage ratios:

- 400/132 kV
- 275/132 kV
- 400/88 kV
- 275/88 kV

1.9 Structure of the dissertation

This section comprises the introduction to the dissertation.

Section 2 provides some background theory to SPF and OPF analysis in power systems.

Section 3 describes the theory of Standard Power Flow (SPF) and Optimal Power flow (OPF), discusses the PSS/E OPF tool in detail and provides brief extracts from the OPF analysis in an attempt to explain the OPF research process.

Section 4 describes the benchmark analysis carried out on other utilities to provide an indication on international trends in the area of Transmission transformer tapping.

Section 5 discusses the techno-financial analysis, comparing the cost of tapped transformers to non-tapped transformers together with the cost of reactive power mitigating devices e.g. shunt capacitors and inductors.

Section 6 ends the main body of the dissertation with conclusions.

A full list of references used in the research and dissertation is provided after the conclusions.

The dissertation ends with appendices which include detailed data tables and analysis relevant to the research.

2. Standard Power Flow (SPF) and Optimal Power Flow (OPF) - theory

The purpose of this research is to use a recognised, commercially available, Optimal Power Flow (OPF) tool to investigate the necessity or not of Transmission transformer tapping on the Eskom Transmission system, while minimising overall system losses.

The purpose of this research was not to evaluate OPF methodologies or come up with new OPF modelling or analytical techniques [2], [13], [20].

However, in this section, by way of background, a brief description of SPF and OPF modelling techniques is presented, especially as used by the PSS/E power system simulation tool [1] to provide some background and understanding of the research undertaken.

2.1 Conventional power flow analysis or SPF

The SPF specifies loads in megawatts and megavars to be supplied at certain nodes or busbars of a Transmission or Distribution system and generated megawatts and megavars and the voltage magnitudes at the remaining nodes with a complete topological description of the system including impedances [2].

The objective of the SPF is to determine the complex nodal voltages from which all other quantities like line flows, currents and losses are derived. The model of the Transmission system is given in complex quantities since an alternating current system is assumed to generate and supply the powers and loads [2].

The relation between complex nodal voltages $\underline{\mathbf{V}}$ and complex nodal currents $\underline{\mathbf{I}}$ of the Transmission network, composed of passive impedance components, transmission lines, series elements, transformers and shunts is:

$$\underline{\mathbf{I}} = \underline{\mathbf{Y}} \,.\, \underline{\mathbf{V}} \qquad (2.1)$$

In computer based power simulation programs, voltages, currents and impedances are modelled as matrices.

The result of a SPF simulation or analysis shows how lines in the system are loaded, what voltages exist at the various buses, how much of the generated power is lost as system losses and where limits are exceeded. Exceeded limits could include a bus voltage exceeding 1.1 p.u. voltage or a generator at maximum megavar output operating at 1.1 p.u. megawatt output, i.e. the generator operating outside its capability curve [2].

In an SPF, loads and generator megawatts (except for the slack generator bus) are normally fixed and form the starting point of the loadflow analysis.

The conventional power flow problem or SPF in a typical power system simulation package, e.g. PSS/E, solves a series of simultaneous nonlinear equations which ensure that the net complex power injection at every bus is equivalent to the sum of complex power flows on each connected branch [1], [2].

$$S_{Gi} - S_{Li} = \sum_{j=i}^{N} s_{ij} \qquad (2.2)$$

Where:

N	=	Total number of buses
S_{Gi}	=	Net complex power generation at bus i
S_{Li}	=	Net complex load at bus i
s_{ij}	=	Complex power flow on branch connecting buses i and j

Bus power injections and branch power flows are expressed as functions of complex bus voltage. Through the iterative solution process, a bus voltage vector is determined which satisfies these equality constraints to within a mismatch tolerance [1], [2].

A desired solution might require that a dependent variable set, such as a group of bus voltage magnitudes and branch flows, must satisfy maximum and minimum limits. The power flow may offer a set of automatic controls for which the values vary independently as functions of local objectives. For example, generator reactive power varies continuously to control the machine's terminal bus or some remote bus voltage magnitude. Other controlling equipment may include transformer tap ratios, transformer phase shift angles, susceptive bus shunts and dc converter control angles [1].

Control variable range is limited and may be insufficient to satisfy a local objective. It is necessary, therefore, for the analyst to manually vary either the control values or their local objectives to attempt to achieve a good solution. By observing the impact of parametric variations, the analyst develops an intuition of the power system model [1].

This is normally a long and tedious process.

2.2 Economic operation of power systems

Economic operation is important for a power system to return a profit on the capital invested or at least realise an acceptable rate of return based on the asset owners' cost of capital.

Maximum efficiency minimises the cost of power to the consumer and the cost to the utility or power company of delivering the power in the face of constantly changing variable costs including fuel, labour, supplies and maintenance.

Operational economics involving power generation and delivery can be subdivided into two parts:

- Minimum cost of power production called economic dispatch, and
- Minimum-loss delivery of the generated power to the loads.

For any specified load condition, economic dispatch determines the power output of each plant (and each generating unit within each plant) which will minimise the overall cost to serve the system load. Therefore economic dispatch focuses upon coordinating the production costs at all power plants operating on the system.

The minimum-loss problem can assume many forms depending on how control of the power flow in the system is exercised. The economic dispatch problem and the minimum-loss problem can be solved by means of OPF techniques [20].

2.3 Optimal Power Flow (OPF)

The OPF calculation can be viewed as a sequence of conventional Newton-Raphson power-flow calculations in which certain controllable parameters are automatically adjusted to satisfy the network constraints while minimising a specified objective function [20].

The OPF is distinguished from the traditional power flow or SPF in that it solves an optimisation problem consisting of an objective function augmented by equality and inequality constraints. The OPF algorithms solve a nonlinear problem of the form:

Minimise f(x,y)

Subject to: equality constraints and inequality constraints

Where:

- The minimisation objective may consist of one or more functions such as generator cost functions or Transmission losses
- The equality constraints constitute such items as the power flow equations presented in Equation 2.2 which ensure that the net power injection at each bus equals the sum of the power flows on the connected branches.
- The inequality constraints consist of variables such as the bus voltage magnitudes and angles or active power output which can be varied between a minimum and maximum value

Power system performance is achieved by satisfying both the global objective and the constraints. By adjusting control variables, the solution process determines conditions that simultaneously satisfy the constraint equations and minimise the objective function [1].

2.4 Principles of OPF

The PSS/E OPF application complements the main PSS/E power flow program or SPF program. A valid power flow data model must be present in the current PSS/E power flow working case before any OPF activities can be successfully executed. However, the power flow case does not necessarily need to be solved [1], [2].

To put the power of the OPF in perspective, it may be easiest to contrast it with the traditional SPF. With the SPF, a significant amount of time is often spent trying to achieve a good solution. Multiple iterations are required in which solution executions are performed, results analysed and new estimates of the control values are determined for use in the next solution.

Time is spent trying to parametrically determine what values of the controls will provide a feasible solution. If estimates prove to be unacceptable, as may result in violation of the operating criteria, new estimates must be provided and the procedure started again. This can often prove to be a very costly process/cycle [1].

The OPF, on the other hand, provides a completely analytic model, one which automatically changes certain control variables to arrive at the best solution with respect to a stated quantitative performance measure (i.e. objective function). It efficiently achieves this result by formulating and solving an optimisation problem, defining the goal as a combination of objective functions and a set of variable constraints to satisfy. After the problem statement is set up, the OPF often requires much less user intervention than the SPF [1].

PSS/E OPF formulates and solves the optimisation problem within constraints, and automatically updates any power flow data values affected by the solution. Upon completion of the solution, results are presented in one or more formatted reports and output files [1]. A useful paper explaining SPF and OPF can be found in Appendix G.

2.4.1 Objective functions

Objective functions are expressions of cost in terms of the power system variables. For example, the fuel cost incurred to produce power is a function of the active power generation among participating machines. The OPF automatically adjusts the participating machines' active power generation, within capability limits, to reduce the total fuel cost [1].

Objective functions have both *explicit* and *implicit* components.

Explicit objective components are generally functions expressed in terms of the optimised subsystem variable set.

Implicit objective functions may be introduced as approximations for discrete or discontinuous control actions. These implicit objectives are weighted quadratic penalties applied to the excursion of a variable from an offset [1].

The quadratic penalty function is discussed further in Section 2.4.7 below.

2.4.2 Constraints and controls

If the objective of the OPF is to reduce cost, reducing participating active power generation to zero will certainly minimise fuel cost, but it does not provide for the customer power demand.

It is necessary, therefore, to augment the objective function with constraint equations, including both equality and inequality constraints. Inequality constraints define upper and lower bounds on a variable. For the fuel cost dispatch example, these constraint equations will ensure that the active power generation is sufficient to provide for the customer demand and power system Transmission losses [1].

The complex power flow equations of the conventional power flow or SPF problem are always included. Controls, such as active power generation, generator voltage magnitude and transformer tap ratio, may be assigned a fixed value or assigned upper and lower limits. Certain dependent variables, such as load bus voltage magnitudes and branch flows, are assigned upper and lower bounds. It is possible however, for the entire constraint set to not be simultaneously satisfied. This situation results in an infeasible termination [1].

The solution process adjusts each control to find the setting that satisfies all stipulated constraints, and further minimises the objective function.

Two variable types exist in the model: control variables (also referred to as independent decision variables) and dependent variables (also referred to as state variables) [1].

2.4.3 Control variables (or independent variables)

Control variables are all real world quantities which can be modified to satisfy the load-generation balance under consideration of the operational system limits. In power simulation literature, control variables are often represented by the vector U [2].

A typical set of control variables of an OPF problem can include inter alia:

- Active power of a PV node
- Tap position of a transformer
- Shunt capacitance or reactance

2.4.4 State variables (or dependent variables)

State variables include all variables which can describe any unique state of the power system. State variable are often represented by the vector X [2].

A typical set of state variables of an OPF problem can include inter alia:

- Voltage magnitude at a PQ node
- Power flow (MVA, MW, MVAr)
- Reactive generation at PV node

State or dependent variables (as their names would suggest) normally change their values as the control variables change to achieve a certain objective.

2.4.5 Operational limits

In power systems and in power system simulations, many of the variables are limited and may not be exceeded without damaging equipment or bringing the network into unstable operating states.

The following is a listing of limits normally imposed on variables:

Limits on active power of a (generator) PV node:

$$P_{low_i} \leq P_{PV_i} \leq P_{high_i} \quad (2.3)$$

Limits on voltage of a PV or PQ node:

$$|V|_{low_i} \leq |V|_i \leq |V|_{high_i} \quad (2.4)$$

Limits on tap positions of a transformer:

$$t_{low_i} \leq t_i \leq t_{high_i} \quad (2.5)$$

Limits on phase shift angles of a transformer:

$$\Theta_{low_i} \leq \Theta_{PV_i} \leq \Theta_{high_i} \quad (2.6)$$

Limits on shunt capacitances or reactances:

$$s_{low_i} \leq s_i \leq s_{high_i} \quad (2.7)$$

Limits on reactive power generation of a PV node:

$$Q_{low_i} \leq Q_{PV_i} \leq Q_{high_i} \quad (2.8)$$

Upper limits on active power flow in transmission lines or transformers:

$$P_{ij} \leq P_{high_{ij}} \quad (2.9)$$

Upper limits on MVA flows in transmission lines or transformers:

$$P_{ij}^2 + Q_{ij}^2 \leq S_{high_{ij}}^2 \quad (2.10)$$

Upper limits on current magnitudes in transmission lines or transformers:

$$|I|_{ij} \leq |IV|_{high_{ij}} \tag{2.11}$$

Limits on voltage angles between nodes:

$$\Theta_{low_{ij}} \leq \Theta_i - \Theta_j \leq \Theta_{high_{ij}} \tag{2.12}$$

2.4.6 Sensitivities

Each variable, both independent and dependent, has a sensitivity associated with it.

Sensitivity values quantify the expected change in the objective function value in response to a change in the variable. A negative sensitivity indicates that an increase in the variable's value will decrease the objective function value. The optimal setting for any variable is one which results in a sensitivity of zero. For some variables, the optimal setting exists outside of the variable's limits. In this case, the OPF drives the variable to the limit and reports the sensitivity value.

The relative size of the sensitivity magnitude directs attention to the constraints or fixed controls that have the most influence on the objective [1].

2.4.7 Quadratic penalties

A weighted quadratic penalty functions as an *implicit objective function* and is inherently applied to four power flow controls in the PSS/E OPF software as follows:

- Generator bus voltage magnitude
- Transformer tap ratio
- Transformer phase shift angle
- Switched shunt admittance

The OPF adjusts these controls to achieve a feasible solution and balance the penalty against the other selected objective components [1].

Global quadratic penalties take the form:

$$\rho \sum_{i=1}^{N} (\mathrm{x_i} - \mathrm{x_{i0}})^2 \tag{2.13}$$

where:

N = number of penalised control variables

ρ = scalar quadratic penalty weight

x = control variable current value (per unit)

x_0 = control variable penalty offset (per unit)

By default, the non-optimised generator bus voltage magnitudes are penalised with a default weight of one hundred (100) and the quadratic penalty weight for the transformer and switched shunt controls is zero (0). These values are globally applied to all non-optimised controls and may be altered by modifying the *Penalty* for fixed voltage value and the *Quadratic penalty coefficient* value, respectively, in the OPF change parameters window [1], [2].

The different set-up windows in PSS/E are discussed in more detail in Section 3, below.

2.4.8 Bus data

By default, all bus voltage limits in the PSS/E case file are set to -9999 and 9999 p.u. The wide limits on the range of voltages have the potential of contributing to an infeasible solution. It is usual that the voltage limit ranges on all buses in the OPF are set to more constraining values before commencing the OPF simulation [1].

2.4.9 Load data

Load data from the PSS/E case file is utilised in the OPF simulation and there are no additional factors to be considered.

2.4.10 Fixed shunt data

Fixed shunt MVAr values are included directly into the OPF simulation.

There is a relationship between *fixed shunts* and *adjustable shunts*. If a corresponding bus and bus shunt identifier is found within the fixed shunt data, then the MVAr value of the Fixed Shunt is updated or adjusted with the new OPF solution value added in, otherwise a new bus shunt (adjustable shunt) is added to the power flow network data [1].

2.4.11 Generator data

Active power and reactive power generation and reactive power generation limits are initialised from the SPF data model. Generator bus voltage magnitudes are control variables that are varied by the OPF to optimally settle on a value to respect objectives and constraints [1]. In this research, generator active power is determined by the SO system snapshot simulation file and is fixed. As mentioned, optimising generation dispatch is not analysed in this research.

2.4.12 Non-transformer branch data (line or cable data)

Resistance, reactance and charging values of lines and cables are all obtained from the branch data of the SPF.

2.4.13 Transformer branch data

Upper and lower limits in the tap ratio or phase shift angle of transformers and tap ratio size are set up in the SPF model.

2.4.14 Switched shunt data

Switched shunt data in the SPF is recognised in the OPF. The OPF switches the switched shunts in and out to achieve the objective of the OPF simulation.

2.4.15 Adjustable shunt data

A subset of buses may be identified as MVAr bus adjustment candidates for additional shunt compensation. Adjustable bus shunts are employed in addition to switched shunts and on buses where switched shunts may not exist to minimise the total system bus shunt component to achieve the objective function for e.g. minimise system real and reactive losses [1]. Fixed shunt values may be increased with adjustable shunts in the OPF analysis.

2.4.16 The nonlinear problem

PSS/E OPF solves a nonlinear problem consisting of an objective function and a linear combination of equality constraints to form a scalar Lagrangian function of the following form:

$$L\,(x,\lambda) = f(x) + [\lambda]^t[h(x)] \qquad (2.14)$$

And

$h(x) = 0$

$x_{min} \leq x \leq x_{max}$

Where:

x = power system variables (control and dependent, column vector)

f(x) = objective function (e.g. active power dispatch cost)

h(x) = equality constraints including bus-power mismatch equations (column vector)

λ = Lagrange multiplier variable (column vector)

The Lagrangian function is constructed in terms of both the power system variables and the Lagrange multipliers, thereby resulting in a dual variable problem [1], [2].

The equality constrained optimal solution is a Lagrangian stationary point, and is determined by equating the function's gradient to zero and solving for x and λ. This is the Kuhn-Tucker (KT) optimality condition [21], and the resulting set of simultaneous equations is referred to as the KT formulation. Solving the KT problem produces a set of optimally defined power system variables (x), along with the objective function sensitivities to changes in the constraints (λ) [1], [2].

The OPF formulation is always nonlinear in x because the constraints include the power flow mismatch equations. The objective function is also likely to be nonlinear in x either through explicitly selected components, like active power loss, or the implicit introduction of quadratic penalties. Therefore, an iterative solution technique which will recognize the limits in x is required.

A solution to the nonlinear problem results from solving a series of simpler sub problems, wherein the inequality constraints are satisfied at every sub problem [1].

In this research, the objective function attempts to minimise real and reactive losses via allowing VAr devices to adjust (the generator dispatch is fixed).

2.4.17 Inequality constraints

Inequality constraints have two types: *hard limits* and *soft limits*.

Hard limits introduce objective function terms that are asymptotic to the variables limits (i.e. barrier terms), while *soft limits* introduce objective function terms which are defined in the infeasible region and grow in magnitude as the variable value departs further from its violated limit (i.e. penalty terms).

Limits imposed on the power system control variables, such as transformer tap settings, are considered extremes in the equipment's physical range, and thus are always treated as hard limits. This treatment is in addition to the global quadratic penalty that may optionally be imposed on power flow control values as they move away from their initial values.

This quadratic penalty (discussed above) is imposed on all optimised transformers and switched shunts, provided that the quadratic penalty coefficient is greater than zero.

Limits applied to power system dependent variables, including voltage magnitude, branch flows, and interface flows, represent operating criteria and may receive either hard or soft limit treatment [1].

2.4.18 Class A and class B OPF algorithms

Glavitsh and Bacher [2] have divided OPF algorithms into two main classes:

- Class A
- Class B

Class A algorithms are based on the iterative and separate use of the SPF to solve for a given operating point and use Linear Programming (LP) or Quadratic Programming (QP) for the optimisation problem around the power flow solution.

The power flow part of this Class A OPF algorithm is the SPF. Features e.g. PV-PQ node type switching and local tap control can be handled by the power flow.

The classical LP and QP algorithms as described in mathematical text books are often quite slow for the OPF analysis.

The only necessary link between the SPF and the OPF is the transfer of the operating point x^k, representing the OPF variables. The power flow solution is transferred to the OPF to be used as the solution around which the approximations are made. The LP or QP algorithm is then solved. The OPF solution is transferred back to the SPF and represents another power flow input data set. The power flow corrects the approximations made in the preceding LP or QP optimisation. Thus the power flow adapts the nodal voltages and the slack power such that the mismatches are below predefined, small tolerance values. By executing this procedure several times the power flow solution point tends towards the optimum, i.e. the result of the very last LP or QP solution should be identical (within a certain tolerance) to the preceding power flow solution. At this point the optimal solution is reached [2].

Class B algorithms solve iteratively for the Kuhn-Tucker conditions explicitly using a conventional power flow. All equality constraints, inequality constraints, the objective function and the OPF variable movements are handled simultaneously. The OPF class B can be compared with the SPF solved with the Newton-Raphson method [2].

The PSS/E OPF uses the Class B algorithm and this is why the SPF does not have to be a solved case in order for the OPF to solve and find an optimal point according to the objective function.

3. OPF Analysis on the Eskom Transmission system

As mentioned, Alternating Current (AC) power flow based optimisation analysis in this research is performed with the primary objective of determining the minimum number of tapping positions or tapping range on Transmission transformers, while maintaining minimum MW and MVAr losses.

3.1 Stage 1 – Minimise system losses while constraining transformer tapping

The analysis is performed using the PSS/E software according to the flowchart in Figure 3.1.

The analysis starts with a PSS/E file for the first year (2009). The AC OPF analysis runs for a number of generation dispatches, (N-1) generation and Transmission contingencies and load values as discussed in Section 1.

In Stage 1, the explicit objective function minimises MW losses and MVAr losses, with constraints including busbar voltage limits, and generator reactive limits.

The implicit objective function is achieved using quadratic penalties on transformer tap positions, switched shunts and generator bus voltage magnitude.

Contracted voltages limits are included in the research and a list of contracted voltages is included in Appendix A.

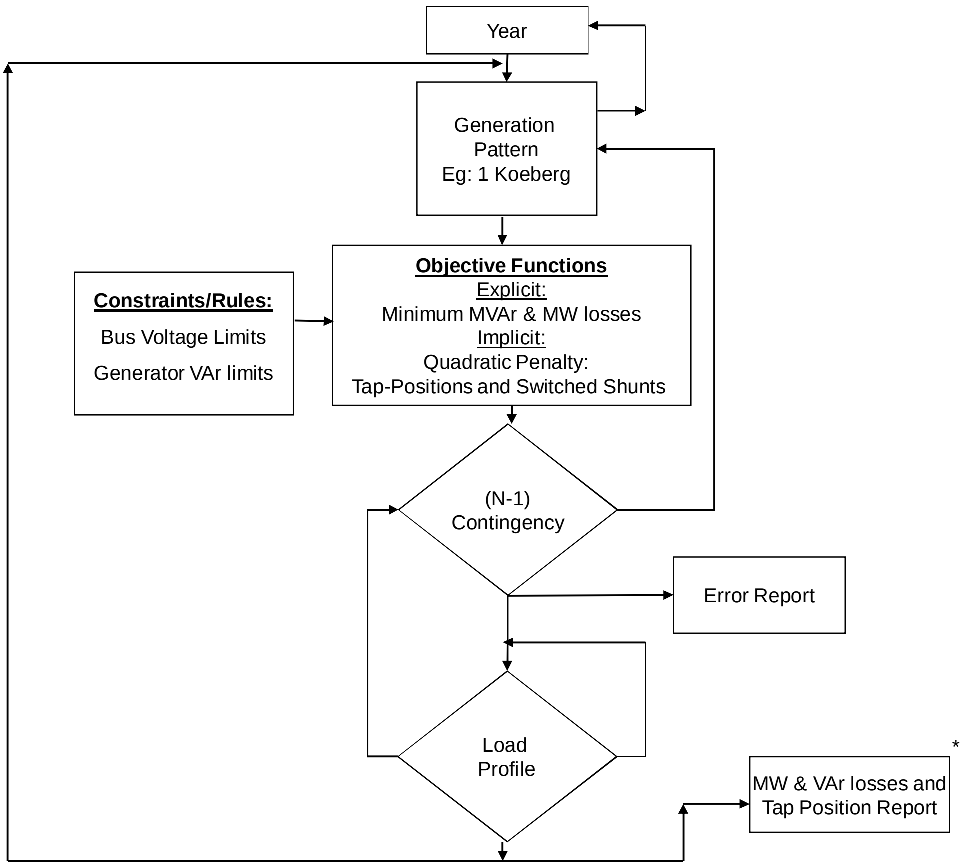

Figure 3.1: Stage 1 of the OPF process – minimise MW and MVAr Losses

The initial output from Stage 1 is a tap position and MW and MVAr loss report for convergence.

As mentioned, no optimisation on system dispatch or system cost is undertaken in the analysis as the system PSS/E 2009 snapshot files are taken directly from the system operator (SO) RTS system. The Eskom SO uses an optimised AGC system to optimise generator dispatch based on real generator costs and it is assumed to operate correctly in this research. In addition, the objective of the research is to investigate the possibility of reducing tapping on the Transmission system while minimising system losses and not to optimise system dispatch. Dispatch optimisation for the 2015 files is also not undertaken for similar reasons.

Figure 3.2 below shows a typical output from the OPF Results Window. In the figure, as an example, overall system MW losses are reduced by 21.6 MW and overall system MVAr losses are reduced by 340.1 MVAr after an OPF solve.

```
All data appears to be okay.

Optimal Solution Found.
Minimum  R   loss  objective:     21.683472
Minimum  X   loss  objective:    340.112391
Quadratic Penalty  objective:      0.033925
```

Figure 3.2: OPF solve summary example

Figure 3.3 below shows the OPF *Change Parameters* Dialog – *Objectives*, with the minimising of the system losses options ticked or checked.

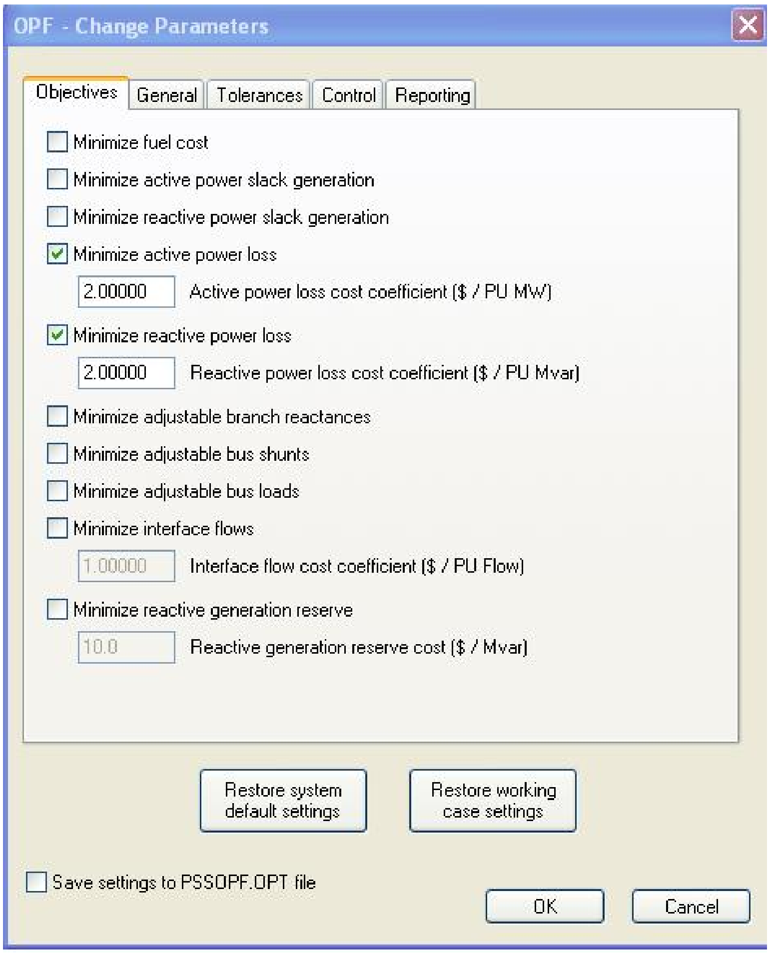

Figure 3.3: OPF – Change parameters dialog - objectives

The analysis is then repeated for each of the years discussed in Section 1 above and for the generation patterns, contingencies and load patterns as included in Section 1.

3.1.1 Constraints

A number of constraints are used in the research as follows:

- Bus voltage limits
- SVC voltage set-point limits
- SVC maximum and minimum real and reactive power limits (Pmax, Pmin, Qmax, Qmin)
- Machine maximum and minimum real and reactive power limits (Pmax, Pmin, Qmax, Qmin)
- Branch thermal limits (transformers and lines)
- Transformer max and min tap limits

Of the above constraints, all except *bus voltage limits* are already included in the SPF set up as a normal part of the loadflow file. Bus voltage limits are set in the OPF Bus Voltage data table as shown in Figure 3.4.

Bus Number	Bus Name	Normal Volt Limit Max(pu)	Normal Volt Limit Min(pu)	Adjust Limits	Emergency Volt Limit Max(pu)	Emergency Volt Limit Min(pu)	Limit Type	Soft Limit Penalty
1090	ESSLN2 275.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1091	CAMDN88 88.000	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1093	KOMATI1 132.00	1.0740	0.9720	No change	1.0740	0.9720	Hard limit	1.0000
1098	KLIPCOL1 132.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1099	SOL1N 132.00	1.0740	0.9720	No change	1.0740	0.9720	Hard limit	1.0000
1100	SOL1S 132.00	1.0740	0.9720	No change	1.0740	0.9720	Hard limit	1.0000
1102	VULCN1 132.00	1.0900	0.9860	No change	1.0900	0.9860	Hard limit	1.0000
1120	MINRV2 275.00	1.1000	0.9950	No change	1.1000	0.9950	Hard limit	1.0000
1121	VERDN88 88.000	1.0630	0.9650	No change	1.0630	0.9650	Hard limit	1.0000
1125	KWAGG2 275.00	1.1000	0.9650	No change	1.1000	0.9650	Hard limit	1.0000
1135	PROSP2 275.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1140	FORDS2 275.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1144	LETHB2A 275.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000
1145	BERNN2 275.00	1.0500	0.9500	No change	1.0500	0.9500	Hard limit	1.0000

Bus Voltage | Bus Voltage / Sbs | Adj. Bus Shunt | Adj. Bus Shunt / Sbs | Adj. Branch Reac. | Adj. Branch Reac. / Sbs | Branch Flow | Brar

Figure 3.4: Bus voltage limits in the OPF data tables set up

It can be seen that not all the normal and emergency voltage limits are between 0.95 p.u. and 1.05 p.u. as shown in Figure 3.4 above. The reason for this is because the contracted voltages as included in Appendix A, have been included. (See also Section 1.7.3).

It can be noted that while there may be operational reasons to run parts of the network in a certain way e.g. run the Cape system to provide extra dynamic SVC capacitive VAr support and generator VAr support for e.g. the loss of a Koeberg unit, the OPF as used in this research (especially using adjustable shunts [see Stage 2]) always finds a solution which strives to attain the objective function and ***does*** override operational requirements i.e. the OPF is not a Security Constrained OPF (SCOPF) [22].

The author is of the opinion that for the OPF analysis as undertaken in this research, i.e. not a SCOPF, the OPF should be allowed to find an optimal solution notwithstanding the operational constraints. This provides a minimum-losses and optimal-VAr device starting point for operators. Further research could be undertaken to enhance the analysis to incorporate SCOPF.

Two example 400 MVAr shunt reactors, one at Hydra and the other at Perseus are inserted into the model.

Table 3-1 below shows total system losses in MWs and MVArs after an SPF solve, after an OPF solve and after another OPF solve with example 400 MVAr shunt reactors installed at Hydra and Perseus substations.

Table 3-1: Loss values in the SPF, OPF and OPF after reactor placement at Hydra and Perseus

	Total System Losses (MW)	Total System Losses (MVAr)
Standard Power Flow (SPF) Solve	1118	17848
OPF Solve	1055	16534
2 x 400 MVAr Reactors Installed after OPF	1075	16741

As can be seen from Table 3-1, above, optimised solutions provide significant improvements to system losses and the case where 400 MVAr shunt reactors are installed is still optimised, but less so than the case where they are not installed.

A feasible optimisation methodology for Eskom in the absence of SCOPF could be to run an OPF to achieve an objective (e.g. reduce taps and losses) and overlay this OPF solution with known operational requirements to achieve both objectives of loss savings and operational requirements. This is a methodology that a number of SCOPF techniques use except in a more automated and integrated way [22].

3.1.2 Tolerances

Figure 3.5 shows the Tolerance values in the PSS/E OPF – *Parameters – Tolerances* window. It can be seen that the Quadratic Penalty Coefficient is set to ten (10) [1].

As mentioned, a weighted quadratic penalty is inherently applied to four power controls as follows:

- Generator bus voltage magnitude
- Transformer tap ratio
- Transformer phase shift angle
- Switched shunt admittance

As mentioned, the quadratic penalty is an *implicit objective function* [1], [2].

The OPF will adjust these controls to achieve a feasible solution and to balance the penalty against the other selected objective components.

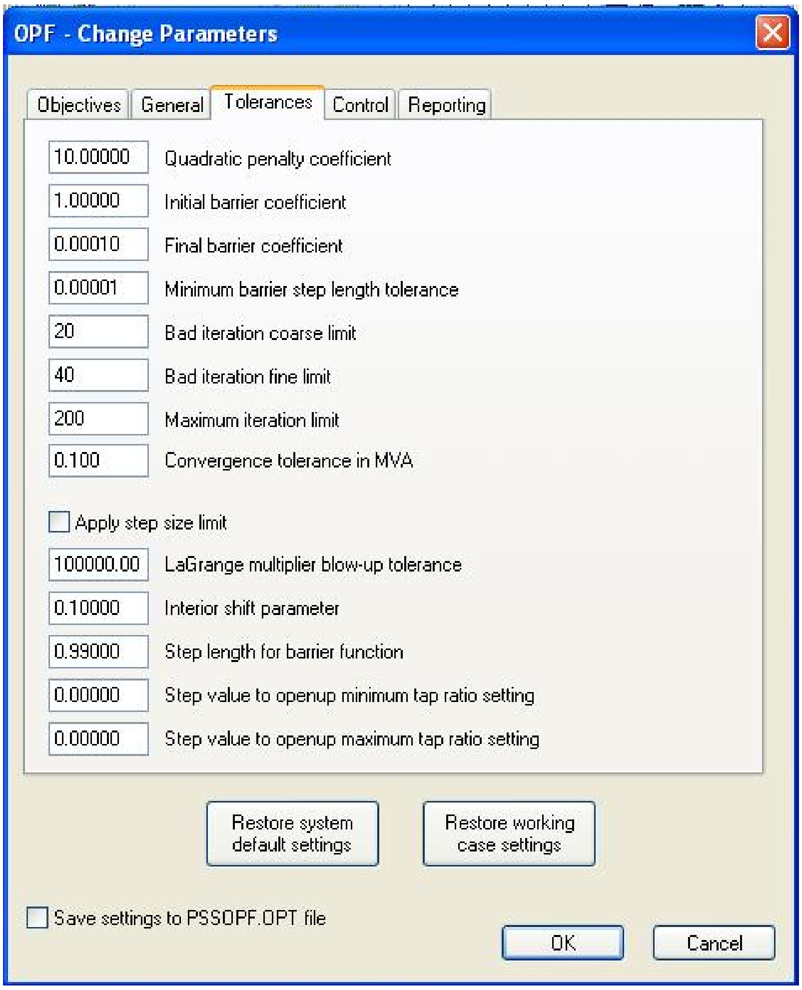

Figure 3.5: Tolerance values in the OPF parameters window

Quadratic penalty coefficients are explained in Section 2.4.7.

In stage 1 of the optimisation process, setting the quadratic penalty to 10 allows the real and reactive power losses to be minimised whilst partially penalising the movement of taps and switched shunts. This is assumed to simulate the behaviour of system operators who are disinclined to undertake unnecessary switching and control actions while controlling the system.

A quadratic penalty of one hundred (100) is high and has the effect of virtually fixing tap positions and preventing switched shunts to switch.

A quadratic penalty of zero (0) is low and allows transformer taps and switched shunts to change freely.

(In Stage 2 of the research, the quadratic penalty is set high; however adjustable shunts (as opposed to switched shunts) are added to the simulation and allowed to compensate for the restriction on tap and switchable shunt movement).

Most of the other settings in Figure 3.5 relate to the numerical calculation method e.g. if an OPF solve is not reaching a minimum or optimal point after a maximum iteration limit of 20, this limit can be increased to say 200, as shown in Figure 3.5.

3.1.3 General OPF settings

Figure 3.6 shows the *General* set up in the OPF module. The only item ticked or checked in this module is the *Round switched shunt VARs* parameter.

The reason this is done is so that fractional values of switched shunts are not applied to obtain an optimal solution but instead the full switched shunt value is applied. For example, if a switched shunt has a magnitude of -100 MVAr (switched shunt reactor), if the *Round* feature were not ticked, the OPF may decide that -72.3 MVAr is required at this particular bus to achieve the global OPF objectives. This does not reflect the installed equipment power rating on the ground.

Ticking the *Fix Transformer Tap Ratios* box would contradict the purpose of Stage 1 of the research which is to determine the range of transformer tapping under a range of system scenarios and then decide on “average” tap setting for Stage 2 of the analysis.

Later, in Stage 2, when adjustable shunts are used to optimise VAr devices to make up for the deficiency in transformer tapping and switchable shunts, the adjustable shunts can be any value, to achieve the global OPF objectives.

As mentioned, adjustable shunts are additional shunts, and are allowed to be set freely over and above the existing fixed and switched shunts.

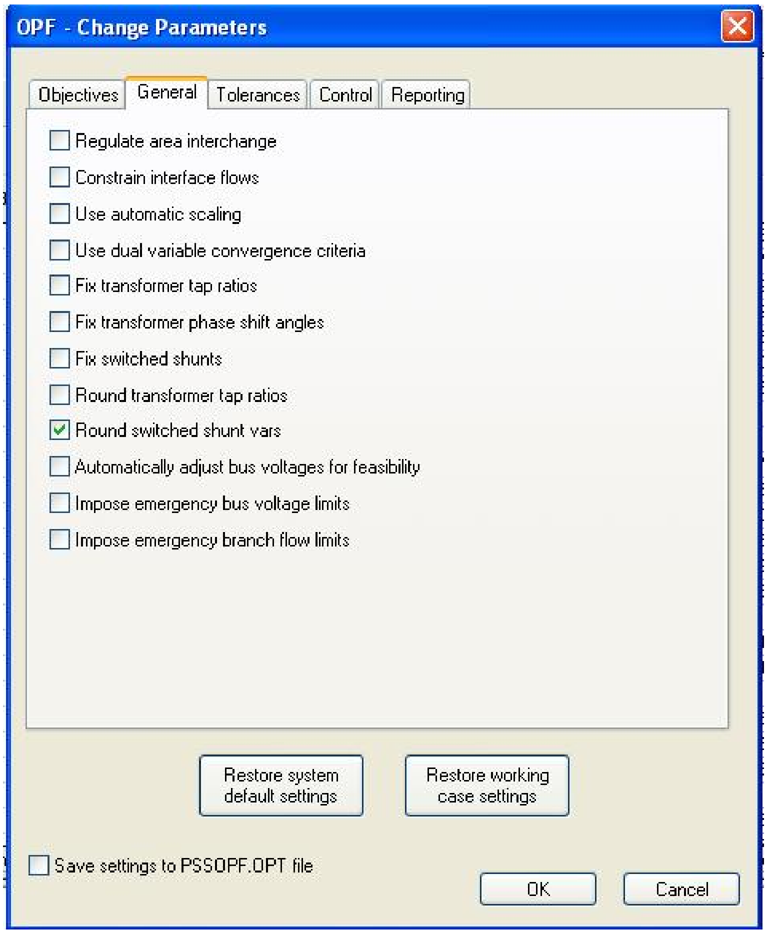

Figure 3.6: Rounding switched shunts in general OPF window

3.1.4 Control OPF settings

Nothing is ticked in the *Control* OPF settings page as shown in Figure 3.7.

In terms of the *Penalty for fixed voltage* box, quadratic terms are introduced into the objective function to penalise the excursion of regulated voltage magnitudes from their offsets. The value in the box (100 in this case) indicates the extent to which excursions are penalised. This is a default value and is left unchanged and is assumed to represent the reticence of power station controllers to be continuously changing their regulated voltage magnitudes.

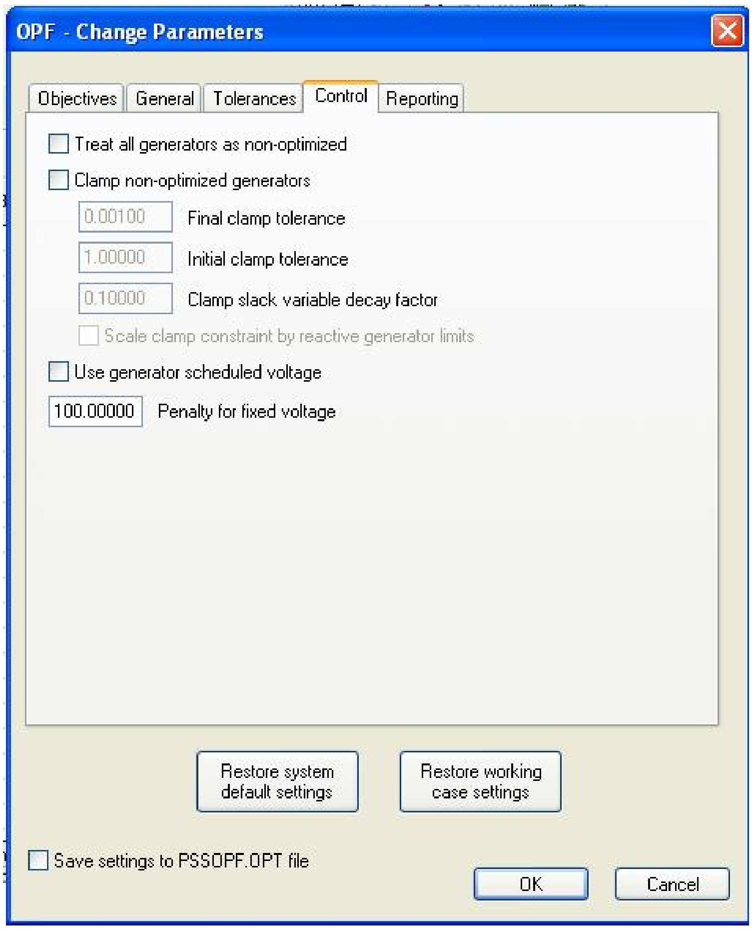

Figure 3.7: Control settings in the OPF parameters window

3.1.5 Reporting OPF settings

OPF output files can be titled and saved in a specific area by using the *Reporting* page in the OPF options (see Figure 3.8).

When an OPF case is solved, the progress dialogue also includes useful information which can be cut and pasted into spreadsheet formats and further analysed.

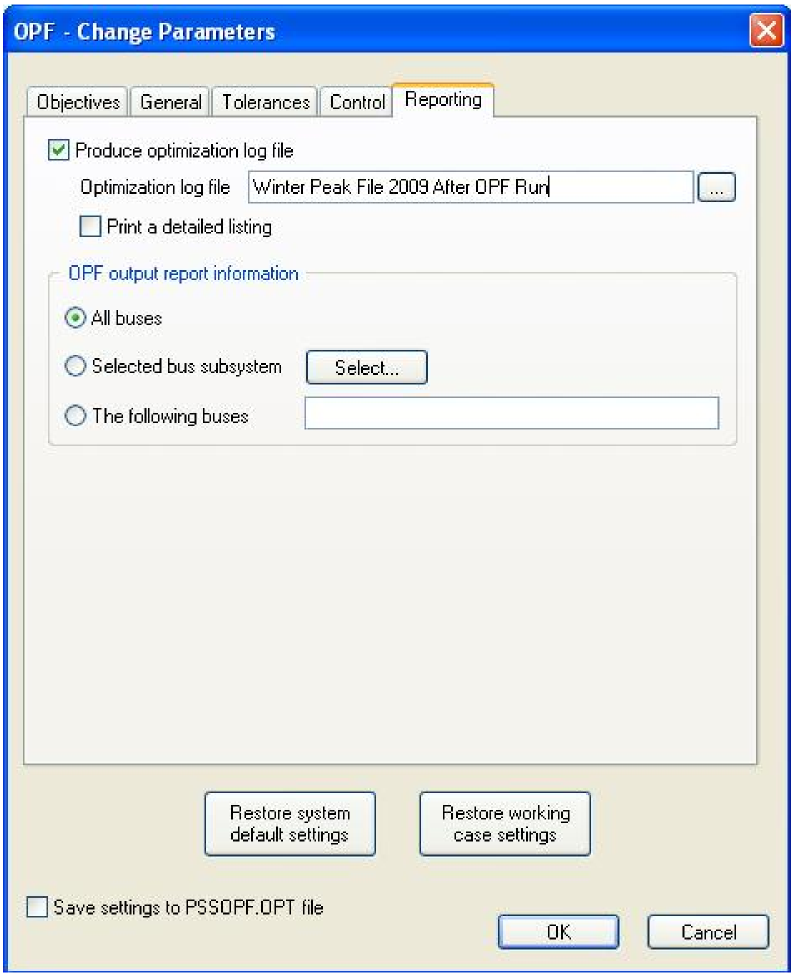

Figure 3.8: OPF reporting settings in the OPF parameters window

3.1.6 OPF solve

When all the OPF settings are complete, it is then possible to solve the OPF by pressing the *GO* in the OPF solve window (see Figure 3.9). The solve window also allows the user to reconsider the parameter settings and to choose all the system busses or a subset of the system busses.

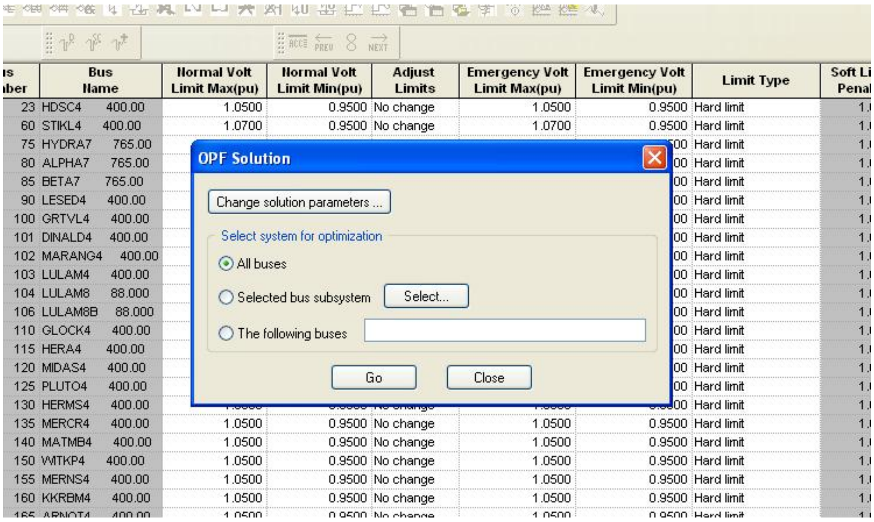

Figure 3.9: OPF solve window

In the OPF analysis in this dissertation, all the busses are chosen.

Figure 3.2, earlier above, shows a typical output dialogue heading stating that all the data appears to be "ok" and that an optimal solution is found. If the output dialogue after a solve says that the solve is infeasible, in most cases this is due to the iteration number (Figure 3.5) not being high enough.

The OPF almost always finds a solution (often when an SPF solve does not find a solution) due to the robustness of the limits e.g. bus voltages are not allowed to approach voltage collapse levels due to the strict 0.95 and 1.05 p.u. and other constraints imposed on the solution. In fact in many instances, planners start with the OPF with defined limits and work back to a SPF solve, when they know the system is potentially unstable. This is especially true on the Eskom Transmission PSS/E model where in most cases the power flow SPF does not solve if the switchable shunts are not locked. In the OPF, the switchable shunts are allowed to switch, but because of other sensible constraints and limits, the OPF solves.

3.2 Stage 1.1 – Statistical analysis of tap positions

The next stage (Stage 1.1) of the optimisation process conducts a statistical analysis of tap positions to determine the most common tap position or most common tap ranges.

The simple mean of the tap positions can be used in this analysis. The block diagram for Stage 1.1 is shown in Figure 3.10.

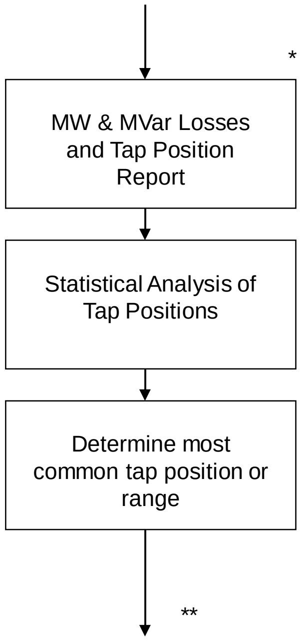

Figure 3.10: Stage 1.1 of the OPF process – statistical analysis of tap positions

The simple mean of the tap positions for a range of historical and analysis scenarios is applied to the Transmission transformers in Stage 2 of the OPF analysis, below. The spreadsheet analysis for the simple mean calculation can be found in Appendix B.

3.3 Stage 2 – Minimising losses while allowing adjustable shunts to switch

Stage 2 of the optimisation process described in Figure 3.11 optimises VAr device placement with the system constraints of busbar voltage limits, generator VAr limits and *new tap range limits* (derived from Stage 1 and Stage 1.1, above (See Figure 3.1 and Figure 3.10 above).

It should be noted that the original explicit objective function of minimising real and reactive power losses is maintained in Stage 2, as it should be a requirement for all VAr devices to be optimised within an overall system that is less lossy.

The final output of the third optimisation phase or Stage 2 of the entire optimisation process is a VAr placement report, a VAr loss report and a tap range report.

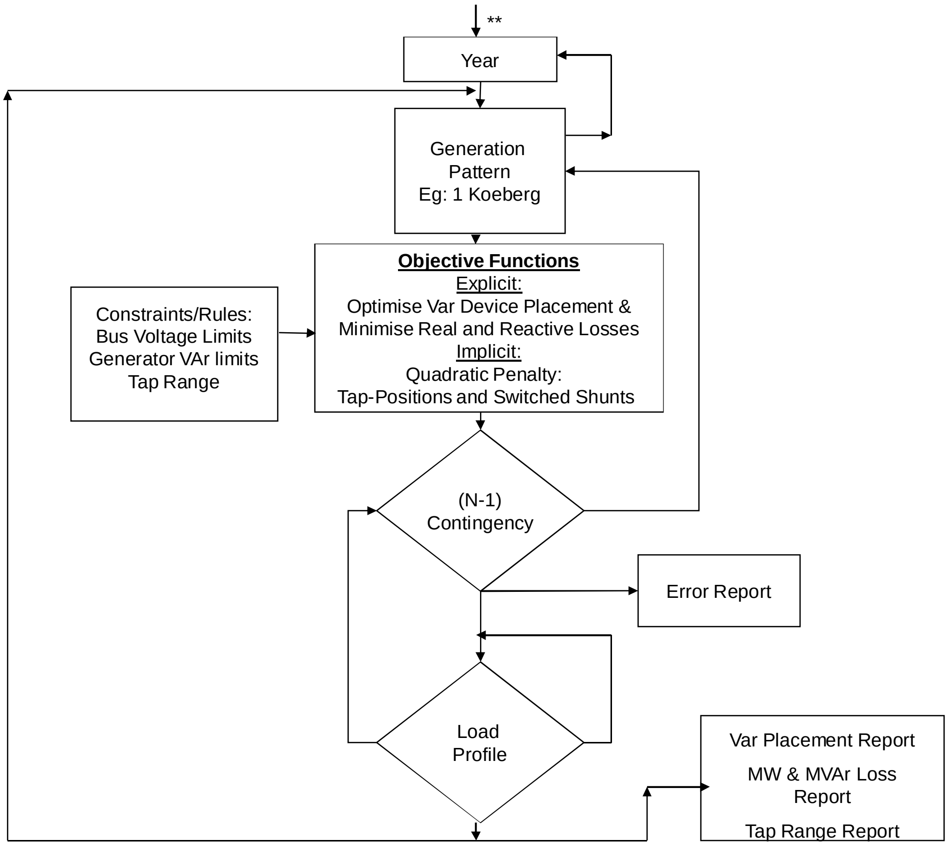

Figure 3.11: Stage 2 of the OPF process – optimise VAr device placement

The tap position and range conclusions, the MW and MVAr losses and the VAr device conclusions are discussed and included in the financial analysis which is discussed in Section 5.

The optimised file should be checked for voltage stability using a voltage stability (VS) simulation tool (Eskom's VSAT simulation tool would be sufficient) to check whether VS limits are still maintained [23]. This could be an area for further research.

3.3.1 Manipulation of output files

After solving the OPF for the range of generation dispatch scenarios and a number of Transmission contingencies, as discussed in Section 1, above, the output files are arranged as follows in Table 3-2:

Table 3-2: Reporting settings in the OPF

SPF Solve	OPF Solve
Tap Positions	Tap Positions
MW/MVAr Losses	MW/MVAr Losses

Bus Voltages	Bus Voltages
Switched Shunts	Switched Shunts
Adjustable Shunts	Adjustable Shunts
Generator VArs	Generator VArs
Voltage Limit Report	Voltage Limit Report
Branch or Thermal Limit Report	Branch or Thermal Limit Report

The example tables in the sections 3.3.1.1 - 3.3.1.6 show example output value extracts before and after OPF analysis.

The same methodology is applied to the other generation dispatch scenarios and years.

3.3.1.1 MW/MVAr losses

Table 3-3 shows an example of the total system MW and MVAr losses from the SPF simulation, after both stages of the OPF process for a single scenario. A full list of results for all the years, load patterns and generation dispatch patterns can be found in Appendix B.

Table 3-3: MW and MVAr outputs before and after OPF analysis – (base case)

OPF	Losses	Unit	Base	Contingency				
				1	2	3	4	5
Standard power flow	Total	MW	1118.4					
		MVAr	17847.5					
Phase 1 (minimising losses, no adjustable shunts)	Total	MW	1077.7	1104.3	1083.3	1078.4	1140.1	1116.1
		MVAr	16933.6	17126.4	16960.2	16918.5	17279.4	17240.4
	Difference from base case	MW	-40.7	-14.1	-35.1	-40	21.7	-2.3
		MVAr	-913.9	-721.1	-887.3	-929	-568.1	-607.1
Phase 2 (fixed tap positions, adjustable shunts allowed)	Total	MW	1054.9	1084	1060.3	1057.6	1117.8	1095.9
		MVAr	16534	16765.2	16606.3	16555.1	16908.1	16890.1
	Difference from base case	MW	-63.5	-34.4	-58.1	-60.8	-0.6	-22.5
		MVAr	-1313.5	-1082.3	-1241.2	-1292.4	-939.4	-957.4

It can be seen that real and reactive total system losses decrease substantially after each stage of the optimisation process and decrease by 63.5 MW and 1 313.5 MVAr after both stages of the OPF process which amounts to a 5.7% real system losses reduction.

3.3.1.2 Tap positions

Table 3-4 shows an example extract from the tap positions output tables. An algorithm is used to convert the tap positions from a per unit (p.u.) winding ratio (e.g. 0.8755) as used in PSS/E to a tap position (e.g. tap 11). A full list of results for all the years, load patterns, generation dispatch patterns and Transmission contingencies can be found in Appendix B.

The table also includes historical tap position information and SPF tap position information to be compared with tap position information after the first OPF stage.

The standard deviations show the variability of the historical, the SPF, the base OPF and Transmission contingency OPF tap positions.

The average tap positions are calculated using the historical, the SPF, the base OPF and Transmission contingency OPF tap positions as the data set.

The mode tap value is the statistically most common tap. It is considered better to do the rounding or "moding" in the spreadsheet analysis rather than in the OPF, so that the OPF can obtain optimised solutions without rounding or moding constraints.

Standard deviation colours in the last column become redder as the standard deviation values increase.

If no data exists for certain busses in the historical data column, this is because no data is available for this bus.

Table 3-4: Tap positions outputs before and after OPF and after contingencies – (base case)

			Historical (most		SPF	OPF stg 1:Q = 10, min loss 2$/MVAR						OPF stg 2:Q = 100, min loss 2$/MVAR									
From Bus #	From Name	To Bus#	%	Pos	Base	Base	Contingency 1	2	3	4	5	Base	Contingency 1	2	3	4	5	std dev1	std dev2	Avg tap	Mode tap
60	STIKL4	1345			1	5	5	5	5	5	5	5	5	5	5	5	5	0.08	1.47	4.34	5
60	STIKL4	1345			1	5	5	5	5	5	5	5	5	5	5	5	5	0.08	1.47	4.34	5
75	HYDRA7	375			17	17	17	17	17	17	17	17	17	17	17	17	17	0.00	0.00	17.00	17
80	ALPHA7	221	24%	15	16	16	16	16	16	16	16	16	16	16	16	16	16	0.00	0.35	15.99	16
80	ALPHA7	221	24%	15	16	16	16	16	16	16	16	16	16	16	16	16	16	0.00	0.35	15.99	16
80	ALPHA7	221	100%	9	16	16	16	16	16	16	16	16	16	16	16	16	16	0.00	2.47	15.99	16
85	BETA7	337	56%	18	17	17	17	17	17	17	17	17	17	17	17	17	17	0.00	0.35	17.00	17
85	BETA7	337	57%	18	17	17	17	17	17	17	17	17	17	17	17	17	17	0.00	0.35	17.00	17
90	LESED4	71992			2	4	4	4	4	4	4	4	4	4	4	4	4	0.22	0.74	3.62	4

Note: std dev 1 = the std deviation of the base and Transmission contingencies (after OPF)

Std dev 2 = std deviation of the OPF base and Transmission contingencies as well as standard power flow (before OPF) and historical data

3.3.1.3 Bus voltages

Table 3-5 shows an extract from the bus voltage output tables. A full list of results for all the different research scenarios can be found in Appendix B.

The bus voltages are shown for the SPF case and both OPF stages. The standard deviation is calculated on the SPF and OPF values. Standard deviation colours in the last column become redder as the standard deviation values increase.

In some instances final voltages are outside the 1.05 p.u. range, however these are busses where contracted voltages have been applied (see Section 1.7.3).

Table 3-5: Bus voltage outputs before and after OPF and after contingencies – (base case)

Voltages (p.u.)			SPF	OPF phase 1							OPF phase 2							std dev (b/n avg and SPF)
Bus			Base	Base	Contingency					Average	Base	Contingency					Average	
Bus#	Name	kV			1	2	3	4	5			1	2	3	4	5		
23	HDSC4	400	0.9587	0.9837	0.984	0.9847	0.9838	0.9705	0.9788	0.9809	0.9765	0.9767	0.9771	0.9771	0.9653	0.9718	0.9741	0.0129
60	STIKL4	400	1.0198	0.9963	0.9955	0.9939	0.9958	0.9964	0.9964	0.9957	1.0052	1.005	1.0045	1.0045	1.0044	1.0049	1.0048	0.0119
75	HYDRA7	765	0.9815	0.9933	0.9934	0.9936	0.9934	0.9736	0.9894	0.9895	1.001	1.0009	1.0008	1.0008	0.975	0.9972	0.9960	0.0098
80	ALPHA7	765	0.9767	0.9967	0.9979	0.998	0.9977	0.9952	0.9977	0.9972	1.014	1.0139	1.0139	1.0139	1.014	1.0142	1.0140	0.0187
85	BETA7	765	0.9693	0.9871	0.9873	0.9877	0.9874	0.9713	0.9868	0.9846	1.0075	1.0072	1.0071	1.007	1.0124	1.0078	1.0082	0.0191
90	LESED4	400	1.006		0.9934	1.0085	1.0054	1.0074	1.0065	1.0042	0.9974	0.9841	0.9976	0.9976	0.9974	0.9974	0.9953	0.0061
100	GRTVL4	400	1.0101	1.0277	1.0287	1.0352	1.0292	1.0346	1.029	1.0307	1.0393	1.0393	1.0388	1.0385	1.0401	1.0393	1.0392	0.0147
101	DINALD4	400	0.9921	1.0027	1.0032	1.0067	1.0032	1.0048	1.0042	1.0041	1.0079	1.0075	1.0072	1.0069	1.008	1.0079	1.0076	0.0081
102	MARANG4	400	1.0096	1.0122	1.0161	1.016	1.0124	1.0143	1.0137	1.0141	1.0158	1.0165	1.0156	1.0153	1.016	1.0158	1.0158	0.0031
103	LULAM4	400	0.9671	0.991	0.9928	0.9871	0.9919	0.9943	0.9929	0.9917	0.9985	0.9985	0.9987	0.9985	0.9985	0.9985	0.9985	0.0164
104	LULAM8	88	1.0286	1.0362	1.0364	1.0362	1.0364	1.0365	1.0364	1.0364	1.0342	1.0343	1.0347	1.0348	1.0342	1.0342	1.0344	0.0039
106	LULAM8B	88	1.0081	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.0499	1.0499	1.0499	1.0499	1.0499	1.0499	1.0499	0.0242
110	GLOCK4	400	1.0118	1.0292	1.0303	1.0325	1.03	1.0328	1.0302	1.0308	1.0322	1.0324	1.0319	1.032	1.0325	1.0323	1.0322	0.0110

3.3.1.4 Switched shunts

Table 3-6 shows an extract from the switched shunt output tables. A full list of results for all the different study scenarios can be found in Appendix B.

The columns show the following:

- What switched shunts are in service in the SPF file
- What switched shunts are in service after the Base OPF stages
- What switched shunts are in service after the Transmission contingency OPF runs

The last column before the standard deviation (coloured) column rounds the real switched shunt value to an integer number according to the incremental MVAr size of the specific switched shunt.

The standard deviation column shows the variability of the switched shunt sizes between the base SPF case and the Stage 1 OPF runs. Standard deviation colours become redder as the standard deviation values increase.

In some cases, a row is highlighted in grey. The reason for this is to highlight specific switched shunts (e.g. a 50 MVAr switched shunt) where it is difficult to decide whether the rounded integer value should be 0 MVAr or 50 MVAr. A mean or average is taken of the values and the decision is taken on the result of the mean e.g. for bus 280 NORMN4, a switchable shunt value of 0 MVAr is chosen even though contingency 2 required a value of - 45.44 MVAr.

Table 3-6: Switched shunts outputs before and after OPF and after contingencies – (base case)

		SPF	OPF phase 1: Switched shunts adjusted in order to minimise losses								OPF phase 2: Switched shunts adjusted in order to					
Bus#	Name	Binit (Mvar)								Std dev b/n avg and step	Binit (Mvar)					
		Base	Base	Contingency					Avg(rounded to step)		Base	Contingency				
				1	2	3	4	5				1	2	3	4	5
80	ALPHA7	0	-0.06	-0.05	-0.06	-0.06	-0.04	-0.06	0	0.06	-0.23	-0.2	-0.18	-0.18	-0.23	-0.22
85	BETA7	0	-0.14	-0.14	-0.15	-0.14	-0.02	-0.11	0	0.12	-0.23	-0.21	-0.19	-0.19	-0.22	-0.23
135	MERCR4	100	98.51	96.57	97.26	98.09	95.91	97.35	100	2.72	100	100	100	100	100	100
160	KKRBM4	-200	-201.81	-203.54	-206.77	-202.7	-203.47	-203.06	-200	3.56	-200	-200	-200	-200	-200	-200
260	PEGSS4	0	-1.43	-2.23	-2.67	-1.88	-2.69	-2.2	0	2.18	-0.23	-0.21	-0.19	-0.19	-0.24	-0.23
265	UMFLZ4	0	-2.77	-5.79	-10.9	-4.32	-8.71	-5.73	0	6.37	-0.24	-0.22	-0.19	-0.2	-0.24	-0.24
267	ATHEN4	150	149.22	148.54	147.54	148.86	147.94	148.55	150	1.56	149.76	149.78	149.81	149.8	149.76	149.76
280	NORMN4	0	-10.34	-22.66	-45.44	-16.55	-35.14	-22.44	0	25.43	-0.24	-0.22	-0.19	-0.2	-0.24	-0.24
335	PERSS4	-200	-200	-200	-200	-200	-200	-200	-200	0.00	-200	-200	-200	-200	-200	-200

3.3.1.5 Fixed (adjustable) shunts

Table 3-7 shows an extract from the fixed shunts output table. A full list of results for all research scenarios can be found in Appendix B.

The columns show the following:

- Fixed shunts in the SPF case file and the OPF base case
- Changes to the fixed shunts after Stage 2 of the OPF process (if adjustable shunts are assigned to the same bus and ID as fixed shunts, the OPF adjusts the fixed shunt value. If adjustable shunts are assigned their own ID, these additional shunts are assigned a value, independent of the fixed shunts, depending on the requirements of the OPF)
- Mean and standard deviations of the fixed (adjustable) shunt values

Standard deviation colours become redder as the standard deviation values increase.

Table 3-7: Fixed shunts outputs before and after OPF and after contingencies – (base case)

			B-Shunt (Mvar)							
			Base	Contingency						
Bus #	Name	Id		1	2	3	4	5	Trim mean	Std Dev
80	ALPHA7	1	606.65	606.59	610.36	607.03	541.23	619.55	607.03	31.58
85	BETA7	1	450.33	461.12	460.65	453.25	818.57	533.81	485.19	156.15
135	MERCR4	1	-127.32	-124.56	-115.81	-124.33	-124.11	-126.92	-124.33	4.25
160	KKRBM4	1	-78.69	-78.68	-78.64	-78.64	-78.74	-78.7	-78.67	0.04
260	PEGSS4	1	292.55	292.88	297.94	298.19	293.19	291.98	294.67	2.98
265	UMFLZ4	1	76.83	77.11	76.76	76.78	76.47	77.08	76.87	0.26
267	ATHEN4	1	69.8	69.77	69.83	69.84	69.93	69.81	69.83	0.06
280	NORMN4	1	-196.27	-194.63	-188.57	-188.09	-197.85	-196.2	-193.13	4.48
281	CAMDN_1	1	0	0	0	0	0	0	0.00	0.00
281	CAMDN_1	2	17.51	16.99	16.38	16.38	17.63	17.59	16.99	0.62

Note: the rows highlighted in olive green indicate the fixed shunts with their own ID (in standard power flow and stage 1 of OPF)

All the other shunts are the shunts that have been added in stage 2 of the OPF using the adjustable bus shunt tool

The fixed/adjustable shunt values are important in the techno-financial analysis, undertaken later in Section 5 of the dissertation. The cost of the optimised shunt devices versus the reduced costs of transformers without tap-changers is compared.

3.3.1.6 Generator VArs

Table 3-8 shows an extract from the generator VArs output table. A full list of results for all research scenarios can be found in Appendix B.

The variation of generator VAr outputs between the SPF and OPF simulations is captured and measured using the standard deviation in the last column.

Table 3-8: Generator VArs

						SPF	OPF phase 1 Qgen (MVARs)						OPF phase 2 Qgen (Mvars)						std dev
bus				VSched (pu)	Pgen (MW)	Qgen (Mvars)	Base	Contingency					Base	Contingency					
#	Name	kV	Id					1	2	3	4	5		1	2	3	4	5	
281	CAMDN_1	16.5	1	1.0158	3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
282	CAMDN_2	16.5	2	1.0342	147.43	7.22	-41.30	-37.07	-19.88	-38.55	-32.24	-37.62	19.18	19.34	19.53	19.52	19.18	19.18	27.65
283	CAMDN_3	16.5	3	1.036	154.43	7.91	-40.89	-37.56	-19.88	-38.71	-32.27	-38.18	19.17	19.33	19.51	19.51	19.17	19.17	27.74
284	CAMDN_4	16.5	4	1.0411	172.43	9.84	-39.78	-38.13	-19.88	-40.23	-32.30	-38.68	19.43	19.68	19.97	19.97	19.43	19.43	28.13
286	CAMDN_5	16.5	5	1.0362	121.43	41.47	-43.52	-37.09	-19.92	-40.30	-32.26	-37.61	10.72	7.97	4.86	4.90	10.82	10.79	27.13
287	CAMDN_6	16.5	6	1.0433	154.43	44.53	-41.73	-38.49	-19.91	-41.60	-32.35	-39.03	8.18	4.55	0.25	0.28	8.28	8.25	26.84
288	CAMDN_7	16.5	7	1.0452	162.43	45.38	-41.27	-38.64	-19.91	-42.16	-32.36	-39.19	8.29	4.69	0.44	0.47	8.39	8.37	27.07
289	CAMDN_8	16.5	8	1.0452	162.43	45.38	-41.27	-38.64	-19.91	-42.16	-32.36	-39.19	8.29	4.69	0.44	0.47	8.39	8.37	27.07
381	POSSVC4	30	1	1.0238	0.00	-200.00	-194.65	-195.08	-193.31	-196.65	-101.92	-39.42	-99.94	-100.14	-100.51	-100.51	-99.91	-101.91	54.70
801	GRTVL_G1	16.5	1	1.0329	155.00	-11.54	16.53	10.35	49.45	20.57	53.84	12.85	18.69	18.76	18.81	18.79	18.73	18.69	16.26
802	GRTVL_G2	16.5	2	1.0325	154.00	-11.64	16.44	10.51	49.45	20.54	53.85	12.99	18.69	18.76	18.81	18.79	18.74	18.70	16.26
803	GRTVL_G3	16.5	3	1.0072	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

A more detailed discussion of OPF results can be found in Appendix F.

4. Benchmark research

In this section, reduced Transmission transformer tapping or fixed tapping from other utilities across the world is investigated. Senior Transmission planning and operations staff from utilities from a wide range of countries were requested to complete a short questionnaire. Engineers from the following countries responded to the questionnaire:

- Chile
- Singapore
- Spain
- Romania
- Canada
- USA
- UK
- India

The terms MTS and TD were defined as per Section 1 of this dissertation report and the following questions were included in the questionnaire:

- Do MTS transformers on your power system include tap-changers with a wide tap range (e.g. 1-17 taps)?
- Do TD transformers on your power system include tap-changers with a wide tap range (e.g. 1-17 taps)?
- Do MTS transformers on your power system include tap changers with a reduced tap range (e.g. 1-3 taps)?
- Do TD transformers on your power system include tap changers with a reduced tap range (e.g. 1-3 taps)?
- Does the Transmission system include fixed-tap (no tap-changer) MTS transformers?
- Does the Transmission system include fixed-tap (no tap-changer) TD transformers?
- Are MTS transformers automatically tapped (via a substation voltage regulator and tap-change panel) or tapped from national control?
- Are TD transformers automatically tapped (via a substation voltage regulator and tap-change panel) or tapped from national control?
- Does the system operator generally use VAr device switching more regularly to control system VArs as opposed to continually changing MTS transformer taps?
- Does the system operator generally use VAr device switching more regularly to control system VArs as opposed to continually changing TD transformer taps?

A summary of the replies from various utilities can be found in Table 4-1. The collated replies and comments received from the Utilities can be found in Appendix D.

Some of the main findings from the multi-country survey are as follows:

- It is evident that all the utilities surveyed (with the exception of Southernco in the US) use transformers with a wide tap range on their MTS and TD transformers, with no MTS and TD transformers having a reduced tap range. Only National Grid (NGC) in the United Kingdom (UK) includes fixed tap changers on their MTS transformers. All utilities in the survey control their MTS transformers from National Control whereas there is no discernable trend regarding where TD transformers are controlled from. <u>Most of the findings appear to contradict the objective of this study i.e. to eliminate or reduce tapping. NGC does use fixed tap transformers on their MTS transformers which supports the objectives of this study.</u>
- In France, the utility (RTE) states that all transformers are equipped with tap changers and this trend is set to continue as the RTE grid is operated closer and closer to its technical limits. <u>This appears to contradict the objective of this study i.e. to eliminate or reduce tapping.</u>
- In Canada, Manitoba Hydro states that the use of tap changers which contain vacuum diverter switches virtually eliminates tap-changer maintenance costs and eliminates heavy coking of diverter contacts. <u>Reduced tapping is not recommended and alternative technologies to overcome tapping failures are used.</u>
- The strategies used by system operators to control system VArs is consistent across MTS and TD transformers for each utility. Most utilities use VAr device switching in conjunction with MTS and TD tapping to control system VArs. For large adjustments, VAr device switching is used, whereas for more fine-grain control transformer tap-changing is used. Of the seven respondents, only Romania admitted to using tapping of transformers more than VAr switching to control system VArs. <u>This appears to contradict the objective of this study i.e. to eliminate or reduce tapping. Tapping is used extensively on MTS and TD transformers to control system voltages and VAr flows.</u>
- Some utilities such as National Grid (UK) use transformers with a wider range of tap positions compared to Eskom. <u>This appears to contradict the objective of this study i.e. reduce tapping.</u>

Notwithstanding the extensive use of transformer tapping on the canvassed utilities (with the exception of the NGC in the UK), the author is of the opinion that the conclusions of this study are compelling (significantly reduced real and reactive system losses, while eliminating the need for transformer tapping on the majority of Eskom Transmission's transformers), and that Eskom Transmission should pursue the idea of eliminating tapped transformers using the following methodology:

- Identification of a pilot site where system performance can be monitored
- Use of non-tapped transformers at identified pilot substations where historical and future tapping is low
- Installation of non-tapped transformers at pilot sites where transformers have reached the end of their useful lives and require replacement
- Installation of non-tapped transformers at pilot sites where connected loads have outgrown the firm capacity of transformers and where the transformers require replacement
- Installation of non-tapped transformers at pilot sites where nearby SVCs or mechanically switched shunt VAr devices can be used to assist with voltage and voltage control in the event that the subtraction of tapped transformers from the system may cause system voltage and VAr problems

The use of pilot sites could be used to confirm or disprove the conclusions of this research.

Eskom can also investigate better tap-change technologies e.g. vacuum tap-changers for reduced maintenance on tap-changers (see Appendix D – sub-section 6 - Canada).

The full comments made by respondents can be found in Appendix D.

Table 4-1: Summary of replies from utilities

Question:	Do MTS have wide tap range?	Do TD have wide tap range?	Do MTS have reduced tap range?	Do TD have reduced tap range?	Do MTS include fixed tap changer?	Do TD include fixed tap changer?	Do MTS auto/national tapped?	Do TD auto/national tapped?	Is VAr switching more common than MTS tapping?	Is VAr switching more common than TD tapping?
Chile	Yes	Yes	No	No	No	No	National	Both	Yes	Yes
Spain - REE	Yes	Yes	No	No	No	No	National	National	Both	Both
Romania	Yes	Yes	No	No	No	No	National	Both	No	No
Singapore	Yes	Yes	No	No	No	No	National	National	Yes	Yes
Canada-Manitoba	Yes	Yes	No	No	No	Yes	National	Automatic	Both	Both
US - TVA	Yes	Yes	No	Yes	No	Yes	National	Both	Yes	Both
US - Southernco	No	Yes	No	No	No	No	National	Automatic	Yes	Yes
India	Yes	Yes	No	No	No	No	National	National	No	No
UK - National Grid	Yes	Yes	No	No	Yes	No	Unknown	Automatic	Unknown	Unknown

5. Cost comparison between reduced-tapped and fully-tapped Transformers

In this section, an algorithm is used to determine the suitability of Transmission transformers for fixed, reduced tapping or full tapping and the cost comparison of fixed or reduced-tap TD transformers including mitigating VAr devices versus fully-tapped transformers is made.

5.1 Methodology

5.1.1 Transformer score

The Eskom TD transformers are ranked in order of their suitability for conversion to fixed tapping based on a score. The score is calculated such that the difference between the maximum and minimum loading OPF tap position has the largest weighting. The difference between the maximum and minimum loading SPF tap is weighted the next highest and then the difference between the SPF and OPF average tap for both minimum and maximum loading are weighted the least (see Figure 1.1).

A score close to zero (0) indicates a high level of suitability (no tap movement). As the score increases, the suitability for conversion of the transformer to fixed tap decreases.

The formula (as extracted directly from the MS Excel score worksheet) for calculating the suitability of TD transformers for reduced, fixed or full tapping is shown in Figure 5.1.

```
Score =  Difference between SPF average tap position and
         Average OPF position (max load) +
         Difference between SPF average tap position
         and average OPF position (min load) +
         2*difference between
         maximum and minimum average SPF tap position +
         3*difference between
         maximum and minimum average
         OPF tap position                            (6.1)
```

Figure 5.1: Formula for calculating suitability of transformers for fixed or reduced tapping (extracted from the MS Excel spreadsheet included in Appendix E)

The score is normalised to a value between zero (0) and one (1); 0 representing the least amount of tap movement and 1 representing the most amount of tapping.

5.1.2 Corresponding shunt analysis

Adjustable or increased fixed shunts at the corresponding LV or HV buses of the transformer (after phase 2 of the OPF) are analysed. Adjustable or fixed shunts are not present on all the transformer buses. The positions of adjustable shunts were chosen based on the position of existing system shunts. The buses with adjustable shunts compensate for the reactive power requirements of other buses without existing shunts.

The maximum amount of capacitance and inductance required under all contingencies and load scenarios is determined and evaluated against existing reactive devices on the respective busbars.

The OPF has a significant impact on overall system losses. Therefore all the shunt requirements cannot be assigned to dealing with the fixed tap positions of the transformers alone. Put another way: in addition to compensating for fixing the tap positions, the shunts also contribute to decreasing the losses (the explicit objective of the OPF).

For this reason, the required shunts are de-rated by a constant (between 0 and 1) depending on their transformers' suitability for conversion to fixed tapping. The relationship between suitability for conversion to fixed tap transformer score and shunt allocation is illustrated in the graph in Figure 5.2.

The "0.1" offset in this graph compensates for the case where a transformer tap position may be on its limit for all contingencies and load scenarios because it requires more reactive power. This transformer would erroneously score very well because its tap sits persistently on its limit and doesn't move. Such a transformer still gets allocated some reactive power in a form of a shunt as a result of the offset.

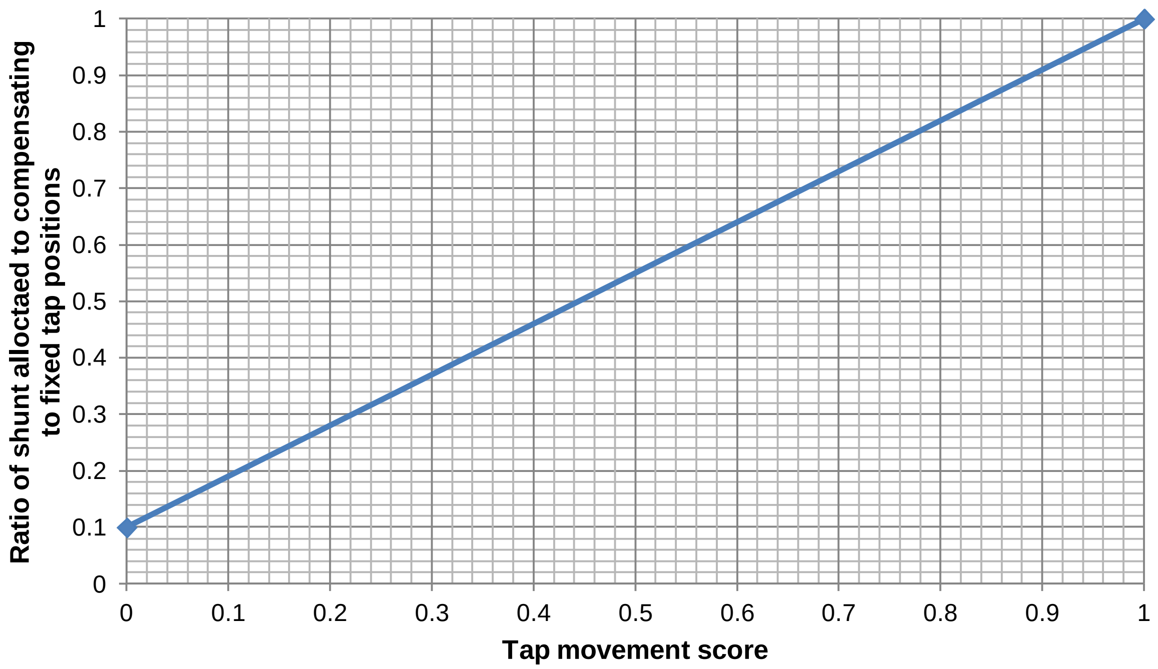

Figure 5.2: Transformer fixed tap suitability score versus compensating shunt ratio

To explain further, in Figure 5.2, for a transformer that taps excessively (1 or the right hand side of the x – axis of the figure), 100% of any OPF allocated adjustable shunt MVArs is attributed to compensating for tap-movements.

In the same figure, a transformer that has a tap-changer that never moves, will have virtually no adjustable shunt MVArs allocated to compensating for tap movements.

Some transformers that are lying on their tap extreme position (due to some system constraint) may not move at all. It is not fair to allocate 100% of adjustable shunt MVArs to loss reduction in this situation, as the lack of shunt movement is not a result of a voltage stable network, but is the result of the tap-changer being on its extreme limit, and not being able to move further.

5.1.3 Shunt and transformer prices

Standard sizes of capacitors and reactors are used for the cost analysis. These sizes and their estimated costs are listed in Table 5-1 and Table 5-2. Cost data was supplied by Eskom Transmission Grid Planning. The relationship between size and cost for capacitors and inductors is illustrated in Figure 5.3 and Figure 5.4.

Table 5-1: Standard capacitor sizes and costs used for the research

Bus voltage (kV)	Cap size (MVAr)	cost (ZAR million)
88 or 132	18	3.8
88 or 132	36	7.6
88 or 132	48	9.7
88 or 132	72	10.2
88 or 132	96	17
88 or 132	144	23
275	50	9.9
275 or 400	100	20.1
275 or 400	150	25.7
400	200	30.5
400	300	42.5

Table 5-2: Standard inductor sizes and costs used for the research

Bus voltage (kV)	Inductor size (MVAr)	cost (ZAR million)
88 or 132	30	18.7
88 or 132	60	50
275	50	45
275 or 400	100	59.4
275 or 400	200	118
400	400	165.2

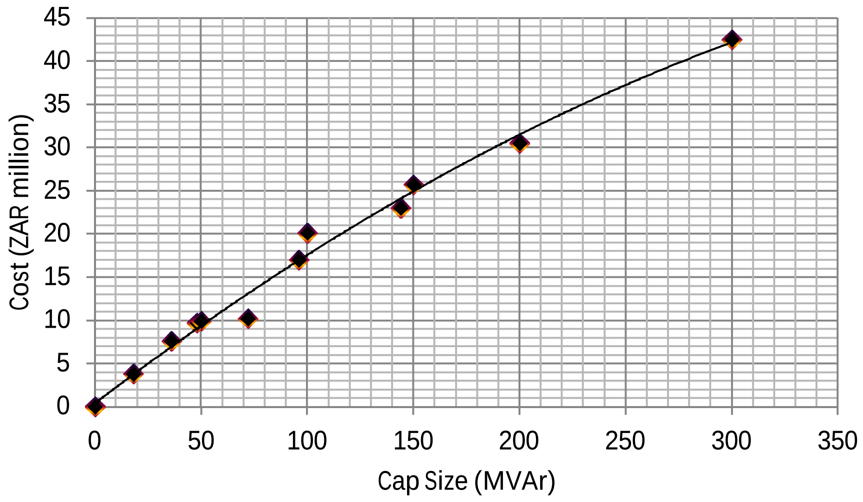

Figure 5.3: Capacitor size and cost relationship used for costing analysis

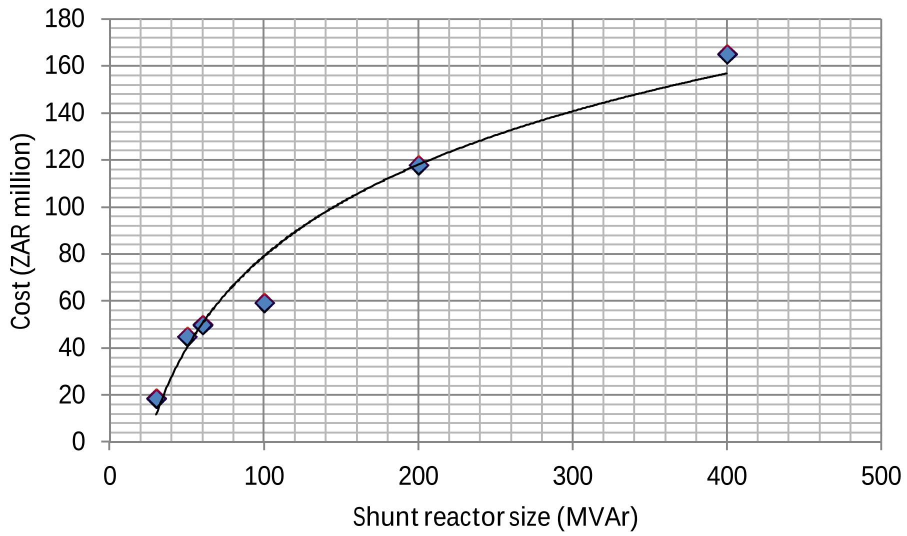

Figure 5.4: Inductor size and cost relationship used for costing analysis

The Eskom standard transformer sizes and estimated costs are listed in Table 5-3. Where transformers are currently not standard sizes, they are costed as standard sized transformers as it is assumed that they will be replaced with standard transformers when necessary.

Table 5-3: Transformer standard sizes and costs (Eskom Grid Planning 2010)

HV side (kV)	LV side (kV)	MVA rating	Cost (ZAR million)
275	132	250	46
275	88	315	60
400	132	250	51
400	88	315	90
275	132	500	86
400	132	500	77
400	275	800	108
765	400	2000	223

A representative from ABB confirmed that of the total cost of a transformer the tap changer contributes between 5 - 20% depending on the MVA rating of the transformer [12].

5.2 Outcomes of cost analysis

Three normalised score categories are set up as shown in **Table 5-4**.

Table 5-4: Normalised transformer scores

Category	Normalised Transformer Score	Outcome
A	0 – 0.25	Suitable for conversion to fixed tap
B	0.25 – 0.5	Suitable for conversion to reduced tap
C	0.5 - 1	Transformer to be left as is

Transformers that do not tap much for a range of years and generation and Transmission scenarios get low movement or deviation scores in the range of 0 to 0.25 (Category A). These transformers are suitable for conversion to fixed tap transformers.

Transformers with some tapping activity or that are in the range of 0.25 to 0.5 in the spreadsheet analysis are suitable for reduced tapping (Category B).

Transformers which tap frequently or are in the range of 0.5 to 1 should be left unchanged or if they need to be replaced, should be replaced with tappable transformers (Category C).

The outcome of the cost comparison research is shown in Table 5-5.

Table 5-5: The results of the cost comparison research

Normalised Transformer Score	Outcome	Total number of TD transformers	Asset value of these transformers with tap changer (billion ZARs)	Cost as a percentage cost of transformer with tap changer	Cost of associated VAr devices (million ZAR)
0 – 0.25	Suitable for conversion to fixed tap	135	8.237	80 - 95%	372.5
0.25 – 0.5	Suitable for conversion to reduced tap	160	9.247	99%	960.8
0.5 - 1	Transformer to be left as is	80	5.007	100%	N/A

Table 5-6 shows the difference in total savings in costs if the transformers in Category A are replaced by fixed transformers and suggested VAr devices are installed. The table also shows the difference in transformer costs for different cost contributions of the tap-changer to the overall transformer cost.

Table 5-6: Difference in costs between status quo and fixed tapping

Tap changer cost (% of total cost of	Cost of replacing all TD transformers (billion ZARs) for Category A (0 – 0.25)	Cost of replacing all TD transformers (no tap changers (billion ZARs)	Cost of all the new reactive devices required (million ZAR) for Category A (0 – 0.25)	Cost of fixed tap transformers and reactive devices	Total Savings (ZAR billion)
5	8.237	7.825	372.5	8.198	0.039
10		7.413		7.786	0.451
20		6.590		6.962	1.275

If Eskom were to replace the suggested selected TD transformers with fixed tap transformers, reactive devices as presented by Table 5-7 would need to be installed for optimal system performance.

In addition if Eskom were to replace the suggested selected TD transformers with reduced tap transformers, reactive devices as presented by Table 5-8: would need to be installed for optimal system performance.

Table 5-7: Additional shunts required if TD transformers are converted to fixed tap

Bus #	Bus	Voltage (kV)	Capacitance (MVAr)	Inductor (MVAr)
135	MERCR4	400	100	100
267	ATHEN4	400	50	0
445	AUROR4	400	50	0
460	MULDR4	400	0	100
1038	AUROR1A	132	36	0
1102	VULCN1	132	36	30
1440	MERSY2	275	0	100
1450	IMPAL2	275	50	0
2065	BERNN1	132	0	60
2790	LNDER1	132	0	30
2855	MERNS1	132	36	0

4011	ARART8	88	96	0
6054	KKFN88	88	48	0
6081	TRIDN88	88	18	0
71348	VULCN12N	132	18	0

Table 5-8: Additional shunts required if suggested TD transformers are converted to reduced tap

Bus #	Bus	Voltage (kV)	Capacitance (MVAr)	Inductor (MVAr)
375	HYDRA4	400	0	400
465	BACCH4	400	0	100
505	DROER4	400	200	50
590	GRASS4	400	100	0
595	DEDISA4	400	100	0
630	FERRM4	400	150	0
630	FERRM4	400	150	50
710	DELPH4	400	0	200
1015	BIGHR2	275	0	100
1082	PROTS4	400	0	100
1090	ESSLN2	275	150	0
1299	MERAP1	132	0	30
1339	PROTS1	132	144	0
1425	ILLOV2	275	100	0
2853	MERC132	132	0	30
2872	MIDAS1	132	36	60
3190	SPITS1	132	96	0
3445	WITKP1	132	48	0
3500	SIMPLN1	132	48	0
6017	BACCH1	132	96	0
6056	MARANG8	88	144	30
6070	PRINC88	88	0	30
6072	PTRBT8	88	48	0

The theoretical cost analysis (capital expenditure only) of replacing the selected TD transformers with fixed tap transformers and installing new reactive devices is shown in Table 5-9.

Table 5-9: Cost of transformers and VAr devices for different assumed tap-changer costs

Assuming tap changer is 20% of the total cost of transformer	
Description of Cost Category	**Cost in ZAR billions**
Cost of replacing all TD transformers	**8.24**
Cost of replacing all TD transformers (no tap changers)	6.59
Cost of all the new reactive devices required	0.37
Cost of fixed tap transformers and reactive devices	6.96
Total saving	**1.28**
Assuming tap changer is 10% of the total cost of transformer	
Cost of replacing all TD transformers	**8.24**
Cost of replacing all TD transformers (no tap changers)	7.41
Cost of all the new reactive devices required	0.37
Cost of fixed tap transformers and reactive devices	7.79
Total saving	**0.45**
Assuming tap changer is 5% of the total cost of transformer	
Cost of replacing all TD transformers	**8.24**
Cost of replacing all TD transformers (no tap changers)	7.83
Cost of all the new reactive devices required	0.37
Cost of fixed tap transformers and reactive devices	8.20
Total saving	**0.04**

The complete re-installation of all Eskom's TD transformers is clearly not practical but the total cost of replacement is used for comparison purposes.

In practice, transformers are likely to be replaced on unit failure, end of useful life or for incremental load or step load increases.

For parallel transformer arrangements, fixed tap transformers would never be installed in parallel with tapped transformers for single transformer failures as circulating VArs and other undesirable conditions would arise.

The cost of a tap changer with a reduced number of taps (i.e. 3 instead of the standard 17) does not offer a great capital expenditure saving and in addition there are unlikely to be benefits from an operational and maintenance point of view. The cost of associated VAr devices for the reduced tap case is significant. This cost far outweighs the cost saving benefit of reducing the total number of taps.

For this reason, only transformers in the category 0 – 0.25 (Category A) are recommended to be investigated for changing to fixed tap transformers.

It would need to be investigated whether the reduced maintenance of fixed tap transformers would outweigh the increased maintenance of the mitigating VAr devices.

This research has considered capital expenditure only. In terms of life-cycle losses, although total losses on the system improve after the OPF, there is difficulty in ascribing what portion of this is due to fixing the tap positions or the minimise loss explicit objective function. Therefore the cost benefits of reduced life-cycle losses have not been included in the cost-benefit analysis.

In conclusion, the research shows that replacing the selected Category-A TD transformers (135 in total) with fixed tap transformers (and VAr devices) can be a cost effective solution. The extent of the total capital saving is dependent on the percentage that the tap changer device contributes to the total cost of the transformer.

The research shows that replacing the selected Category-B TD transformers (160 in total) with reduced tap transformers (and VAr devices) is not a cost effective solution. This is because reducing the number of taps does not significantly reduce the capital cost of the transformer and the shunt devices required to maintain voltage stability under these conditions are comparatively high.

Transformers with tap movement scores in the range of 0.5 to 1, (Category C), should be left unchanged until operational experience confirms or not the above analysis. If a tap changer fails with a score of less than 0.25 (Category A), Eskom should consider not replacing the tap changer.

Appendix C includes a spreadsheet with all the recommended TD tap positions.

Appendix E includes a full cost analysis spreadsheet.

6. Conclusions

This dissertation investigates the feasibility of *fixed-tap* or *reduced-tap transformers* on the Eskom Transmission system, thereby reducing transformer failures and reducing the cost of new transformers or replacement transformers when existing transformers have reached the end of their useful lives or when transformer ratings need to be increased due to stepped or progressive load increases, while minimising system real and reactive losses.

The research uses Optimal Power Flow (OPF) computer simulation techniques based on the nonlinear programming (NLP) method with the attendant objective functions and constraints.

Research is undertaken on 2009 and 2015 Eskom Operations and Planning PSS/E simulation files for a range of system loadings and system contingencies. Changes to the Eskom Transmission system beyond 2015 are not considered in this research.

While the OPF is run on all Transmission transformers on the Eskom system, only Transmission-Distribution (TD) transformers are considered for possible changing to fixed tapping and only TD transformers are analysed in the techno-financial section (Section 5) of the dissertation.

<u>Tap positions</u>

Tap positions of the MTS and TD transformers for the 2009 and 2015 case files before and after the OPF are analysed in the research.

There is relatively little movement of the Transmission transformer tap positions between different loading and generator patterns.

The movement in tap positions between the years 2009 and 2015 is more significant. This can be partly attributed to a significant step in system strengthening e.g. Medupi, Kusile and Ingula power stations and significant Transmission upgrading including 765 kV Transmission overlay to Richards Bay, Durban, Cape Town and Port Elizabeth and a large number of new 400 kV lines connecting new power stations, new loads and improving Transmission system reliability.

<u>Loss savings</u>

Minimizing losses is the main objective function or the explicit objective of the OPF.

The losses are significantly greater in the 2015 files and the percentage loss saving after the OPF is less in 2015 than in 2009. This can be attributed to the system "working harder" in 2015 as a result of more generation and load.

From the results, the first stage of the OPF improves the losses by about 4% across the whole range of contingencies and the second stage by about 5.7%.

The PSSE OPF that is run on the PSS/E case files is security unconstrained, without for example enforced spare generating VAr capacity for system contingencies or for improved transient system transient stability.

A feasible optimisation methodology for Eskom in the absence of SCOPF could be to run an OPF to achieve specified objectives (e.g. reduce taps and losses) and overlay this OPF solution with known operational requirements to achieve both objectives of loss savings/reduced taps and operational requirements.

Notwithstanding the operational restrictions from running a less lossy system according to OPF results, 63.5 MW of savings is a significant result and has the potential to save large operational costs for Eskom.

As an indication, 63.5 MW savings in losses has the potential to save Eskom ZAR 334 million per year (assuming an estimated load factor of 0.6 and a long run marginal cost of generation of ZAR1/kWh)).

Switched shunts

Switched shunts on the Eskom Transmission system are in general correctly sized however there are some instances where increments of switched shunt sizes are required from the OPF analysis.

The sizing and application of switched shunts is largely academic in Phase 2 of the studies where adjustable shunt devices are applied in order to achieve the research explicit objective of minimising system losses.

Fixed and adjustable shunts

There are significant changes in the adjustable shunts between different loading and generation patterns because the OPF cost setting associated with the adjustable shunts is zero. The adjustable shunts move without restriction while the taps are kept relatively constant in stage 2 of the OPF analysis.

Some of these shunts swing from being a large capacitor to a large inductor between maximum and minimum loading. This is acceptable as the OPF attempts to find the best MW and MVAr loss solution.

Bus voltages

The second phase of the OPF works to increase system voltages above their SPF nominal values or towards their upper limits (normally 1.05 p.u. unless specified by a contract voltage).

Generator MVAr outputs

In general, generator VArs do not deviate significantly from their SPF levels in the research.

Benchmark analysis

Notwithstanding the extensive use of transformer tapping on the canvassed utilities (with the exception of the NGC in the UK), the author is of the opinion that the conclusions of this study are compelling (significantly reduced real and reactive system losses, while eliminating the need for transformer tapping on the majority of Eskom Transmission's transformers), and that Eskom Transmission should pursue the idea of eliminating tapped Transmission transformers using the following methodology:

- Identification of a pilot site where system performance can be monitored
- Use of non-tapped transformers at identified pilot substations where historical and future tapping is low
- Installation of non-tapped transformers at pilot sites where transformers have reached the end of their useful lives and require replacement
- Installation of non-tapped transformers at pilot sites where connected loads have outgrown the firm capacity of transformers and where the transformers require replacement
- Installation of non-tapped transformers at pilot sites where nearby SVCs or mechanically switched shunt VAr devices can be used to assist with voltage and voltage control in the event that the subtraction of tapped transformers from the system may cause system voltage and VAr problems

The use of pilot sites could be used to confirm or disprove the conclusions of this research due to cost benefits.

Cost Comparison of Fixed or Reduced Tap Transformers with Tapped Transformers

The outcomes of the techno-financial analysis show that:

- 135 TD transformers are suitable for changing to fixed-tap transformers
- 160 TD transformers are suitable for changing to reduced-tap transformers
- 80 TD transformers should be left as-is

The research shows that replacing the selected Category-A TD transformers (135 in total) with fixed-tap transformers (and VAr devices) can be a cost effective solution.

The research shows that replacing the selected Category-B TD transformers (160 in total) with reduced-tap transformers (and VAr devices) is not a cost effective solution. This is because reducing the number of taps does not significantly reduce the capital cost of the transformer and the shunt devices required to maintain voltage stability under these conditions are comparatively high.

The 80 Category C transformers with tap movement scores in the range of 0.5 to 1 (i.e. with widely varying tapping), should be left as-is until operational experience confirms or not the conclusions from this research.

If a tap changer fails or faults on the network with a score of less than 0.25, Eskom should consider not replacing the tap changer.

General Conclusions

Although the intention of this research is to investigate reducing tapping on Transmission transformers, one of the benefits resulting from the conclusions of the research is to encourage Eskom System Operations to review their operating practices in terms of system losses and optimal system voltages.

Recently, innovative techniques are being used to improve the reliability of tap changing units employing methods e.g. vacuum enclosed diverter switches and thyristor controlled tapping. Innovative tap-changer solutions come at a cost and this research attempts to show that the Eskom Transmission system can function optimally from a system losses, cost and voltage control point of view without the need for transformer tapping or with reduced transformer tapping on a subset of the Transmission transformers.

Recommendations for future work

Security Constrained Optimal Power Flow (SCOPF)

The research carried out in this dissertation does not analyse Security Constrained Optimal Power Flow (SCOPF) or take into account transient stability limits in conjunction with the OPF analysis or the Maximum Allowable Transfer (MAT) approach [22]. Enlarging the research to incorporate SCOPF is a possible area for future work.

While there are limitations to the conclusions of this research as a result of a non-SCOPF analysis being undertaken, verbal and written feedback received from senior Eskom SO staff has confirmed that many of the conclusions and adjusted operating settings “make sense” and in fact this research has encouraged the Eskom SO to allocate more resources to OPF and indeed SCOPF studies on their network.

Stability aspects

While the analysis ensures that busbar voltage limits are maintained and that generator VAr limits are maintained, this dissertation does not analyse Voltage Stability (VS) or loadability [24], [25] and [26] in detail. The optimised file should be checked for voltage stability using VS simulation tools [23]. This is a possible avenue for future work.

Generator transformer tapping

Although there is much research on the tapping of *generator* transformers, the analysis of generator transformer tapping is excluded in this dissertation [27].

At present Eskom utilises tapped generator transformers on all its generator transformers including its large 600 MW – 700 MW coal-fired power station units. Tap-changer costs on these extremely large transformers are high and other options could be considered e.g.:

- Non-tapped generator transformers with tapped auxiliary transformers and an AVR controlled HV bus
- Non-tapped generator transformers with tapped auxiliary transformers from the HV bus and not the unit board bus and an AVR controlled HV bus

<u>Distributed generation</u>

The analysis of the effects of embedded or distributed generation on Distribution and Transmission transformer tapping is excluded from this dissertation as well as the impact on Distribution networks resulting from the fixing of Transmission transformer taps [28].

<u>Generator VAr limits</u>

Ideally, generators should operate far away from their MVAr import/export limits (or close to zero MVArs export/import). In the ideal case, the reactive reserve of the generators is maximized leading to a more secure system that can quickly respond to increased/ decreased MVAr demands following outages on the system.

Maximizing generator reactive reserve, by keeping the generator MVAr import/export level as far from its limits as possible under normal operating conditions, would be a good additional operational objective for the OPF function. However limitations in the PSS/E software make this unfeasible at present. Currently this objective in PSS/E - *minimise reactive generation reserve* [1], takes the MVAr outputs of the generators close to their minimum limit, and so is not used or is not appropriate for this research.

Maximising generator reactive reserve could provide an opportunity for future research.

<u>Operations and maintenance costs</u>

Operations and maintenance (O&M) costs-benefits and life-cycle losses cost-benefits using for example time-value-of-money and discounted cash flows are excluded from the research due to the difficulty in obtaining reliable O&M data from Eskom and the difficulty in attributing what portion of system loss benefits after the OPF are due to the fixing of taps or due to the loss maximising objective of the OPF. Assessing the capital and life-cycle benefits of fixed tap transformers versus the status quo is a possible area for further research.

Extension to further planning horizon

Research is undertaken on 2009 and 2015 Eskom Operations and Planning PSS/E simulation files only for a range of system scenarios. Impacts on the research from changes to the Eskom Transmission system beyond 2015 are not captured. Incorporating more future years in the analysis e.g. 2020, 2025 and 2030 using reliable Eskom Planning files is a possible area for further research.

Extension of the study to include MTS transformers for changing to fixed tapped transformers

While the OPF is run on all Transmission transformers on the Eskom system, only Transmission-Distribution (TD) transformers are considered for possible changing to fixed tapping and only TD transformers are analysed in the techno-financial (Section 5) of the dissertation. The Eskom SO utilises the tapping capability of the MTS transformers frequently to control system VAr flows, especially between Mpumalanga and the Cape so the likelihood of these transformers ever having fixed tapping is not high. Therefore including the analysis of MTS transformers in further research could be considered, taking the aforementioned comments into account.

Inclusion of generator cost functions

Inclusion of generator cost functions and the accurate determination of these costs from historical deterministic data and possible future trends of these costs to allow for a fully inclusive OPF including generator dispatch especially considering prospective IPPs coming into South Africa's generation mix would be a significant area for future work.

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APPENDIX A: Contracted voltages

See attached spreadsheet.

APPENDIX B: PSS/E OPF output data

See attached spreadsheet.

APPENDIX C: Recommended TD tap positions

Fixed tap transformers

Transformer						Suitability score for conversion to fixed tap (0 – 1, 0 being most suitable)	Recommended Tap
From bus			To bus				
#	Name	kV	#	Name	kV		
280	NORMN4	400	91567	NORMN_D4	88	0.00	1
1080	SEBENZA2	275	92988	SBNZA_D1	88	0.00	5
1080	SEBENZA2	275	92990	SBNZA_D2	88	0.00	5
1650	SOWETO2	275	92958	SOWET_D1	88	0.00	5
1650	SOWETO2	275	92959	SOWET_D2	88	0.00	5
1745	ORNJM2	220	91569	ORNJM_D1	66	0.00	5
1745	ORNJM2	220	91570	ORNJM_D2	66	0.00	5
1750	GROMS2	220	91485	GROMS_D1	66	0.00	5
1750	GROMS2	220	91486	GROMS_D2	66	0.00	5
1755	NAMA2	220	91555	NAMA_D1	66	0.00	5
1755	NAMA2	220	91556	NAMA_D2	66	0.00	5
1760	AGGNS2	220	91618	AGGNS_D3	66	0.00	5
1760	AGGNS2	220	91619	AGGNS_D4	66	0.00	5
92889	MATOLA2	275	92891	MATOLA_D	66	0.00	5
1255	SCAFL2	275	41083	SCFLDS1	132	0.00	8
1254	RIGI2	275	6075	RIGI8	88	0.00	3
1618	EROS1	132	1620	EROS4	400	0.00	7
280	NORMN4	400	2930	NORMN1	132	0.00	3
1093	KOMATI1	132	1232	KOMAT2	275	0.01	4
1093	KOMATI1	132	1232	KOMAT2	275	0.01	5
225	MAJUB4	400	6237	MAJUB8	88	0.01	1
225	MAJUB4	400	6237	MAJUB8	88	0.01	1
1185	KRSPN2	275	2740	KRSPN1	132	0.02	4
268	MELMOTH4	400	92951	MELMT_D1	132	0.02	1
268	MELMOTH4	400	92953	MELMT_D2	132	0.02	1
1185	KRSPN2	275	2740	KRSPN1	132	0.03	3
1185	KRSPN2	275	2740	KRSPN1	132	0.03	3
91977	DELTA88	88	91982	DELTA_2B	275	0.03	5
265	UMFLZ4	400	6083	UMFLZ88	88	0.04	3

1152	VERDN2B	275	9169	VERDN_D2	88	0.05	6
175	VULCN4	400	1102	VULCN1	132	0.07	3
5946	MAPUT4	400	91972	MAPUT_D3	132	0.07	3
5946	MAPUT4	400	91971	MAPUT_D2	132	0.07	3
1282	POSDN2	220	3038	POSDN1	132	0.09	7
1282	POSDN2	220	3038	POSDN1	132	0.09	7
265	UMFLZ4	400	6083	UMFLZ88	88	0.09	3
130	HERMS4	400	2540	HERMS1	132	0.09	2
267	ATHEN4	400	1532	ATHEN1	132	0.09	2
1151	VERDN2A	275	9168	VERDN_D1	88	0.10	5
1232	KOMAT2	275	91514	KOMAT_D1	132	0.10	4
1232	KOMAT2	275	91516	KOMAT_D3	88	0.10	12
1185	KRSPN2	275	2740	KRSPN1	132	0.10	6
1240	MARTH2	275	2845	MARTH1	132	0.10	9
280	NORMN4	400	6066	NORMN88	88	0.10	4
560	ARDN4	400	2013	ARDN1	132	0.10	5
1005	ARART2	275	4011	ARART8	88	0.11	6
1091	CAMDN88	88	1195	CAMDN2	275	0.11	5
170	HENDR4	400	2531	HENDR1B	132	0.11	2
1225	FOSKR2	275	2435	FOSKR1	132	0.11	4
1225	FOSKR2	275	2435	FOSKR1	132	0.11	4
155	MERNS4	400	2855	MERNS1	132	0.12	4
175	VULCN4	400	71348	VULCN12N	132	0.12	2
1145	BERNN2	275	2065	BERNN1	132	0.12	5
267	ATHEN4	400	1532	ATHEN1	132	0.12	2
5946	MAPUT4	400	91970	MAPUT_D1	132	0.12	2
267	ATHEN4	400	1532	ATHEN1	132	0.12	2
267	ATHEN4	400	1532	ATHEN1	132	0.12	2
90	LESED4	400	71992	LESED1	132	0.14	3
90	LESED4	400	71992	LESED1	132	0.14	3
445	AUROR4	400	1787	AUROR1B	132	0.14	3
210	SOL4	400	1100	SOL1S	132	0.14	4
7002	OTTAWA	275	7015	OTTD1	132	0.14	6
7002	OTTAWA	275	7016	OTTD2	132	0.14	6
7060	DNORTH	275	7063	DND1	132	0.14	6
7061	DNORTH	275	7064	DND2	132	0.14	6
210	SOL4	400	1100	SOL1S	132	0.14	4
170	HENDR4	400	2530	HENDR1A	132	0.14	2
1232	KOMAT2	275	91515	KOMAT_D2	132	0.14	5
330	THESS4	400	2873	THESS1	132	0.14	2
210	SOL4	400	1099	SOL1N	132	0.15	3
210	SOL4	400	1099	SOL1N	132	0.15	3

140	MATMB4	400	2852	MATMB1	132	0.15	2
140	MATMB4	400	2852	MATMB1	132	0.15	2
1440	MERSY2	275	2860	MERSY1	132	0.15	5
1149	KKFN2B	275	6054	KKFN88	88	0.15	5
1196	RDEKL2	220	3085	RDEKL1	132	0.16	6
1445	AVON2	275	2030	AVON1	132	0.16	6
1445	AVON2	275	2030	AVON1	132	0.16	6
1121	VERDN88	88	1151	VERDN2A	275	0.16	4
1121	VERDN88	88	1152	VERDN2B	275	0.16	4
435	JUNO4	400	60231	JUNO1	132	0.16	7
1005	ARART2	275	4011	ARART8	88	0.16	6
1195	CAMDN2	275	9035	CAMDN_D1	88	0.17	6
1195	CAMDN2	275	9036	CAMDN_D2	88	0.17	6
1195	CAMDN2	275	91472	CAMDN_D5	88	0.17	6
91925	DELTA_2A	275	91977	DELTA88	88	0.17	6
1450	IMPAL2	275	2620	IMPAL1	132	0.18	4
330	THESS4	400	2873	THESS1	132	0.18	3
1265	TRIDN2	275	6081	TRIDN88	88	0.18	7
1265	TRIDN2	275	6081	TRIDN88	88	0.18	7
1395	DANSK2	275	2250	DANSK1	132	0.18	4
1395	DANSK2	275	2250	DANSK1	132	0.18	4
1330	KKFN2A	275	6054	KKFN88	88	0.19	4
445	AUROR4	400	1787	AUROR1B	132	0.19	5
1170	BENBG2	275	92918	BENBG_D3	132	0.19	7
175	VULCN4	400	71348	VULCN12N	132	0.19	3
130	HERMS4	400	2540	HERMS1	132	0.19	2
1440	MERSY2	275	2860	MERSY1	132	0.19	5
255	INVUB4	400	92925	INVUB_D4	132	0.19	2
255	INVUB4	400	92927	INVUB_D5	132	0.19	2
255	INVUB4	400	93002	INVUB_D6	132	0.19	2
280	NORMN4	400	6067	NORMN88B	88	0.19	3
175	VULCN4	400	1102	VULCN1	132	0.19	2
560	ARDN4	400	2013	ARDN1	132	0.19	4
100	GRTVL4	400	6049	GRTVL8	88	0.20	3
100	GRTVL4	400	6049	GRTVL8	88	0.20	3
1091	CAMDN88	88	1195	CAMDN2	275	0.20	5
445	AUROR4	400	1787	AUROR1B	132	0.20	5
445	AUROR4	400	1038	AUROR1A	132	0.21	5
1005	ARART2	275	4011	ARART8	88	0.21	7
450	KOEBG4	400	2705	KOEBG1	132	0.21	2
450	KOEBG4	400	2705	KOEBG1	132	0.21	2
130	HERMS4	400	2540	HERMS1	132	0.22	3

1242	MERNS2	275	2855	MERNS1	132	0.22	5
1242	MERNS2	275	2855	MERNS1	132	0.22	5
1011	RGTVL2A	220	3095	RGTVL1	132	0.23	4
1078	RGTVL2B	220	3095	RGTVL1	132	0.23	4
103	LULAM4	400	106	LULAM8B	88	0.23	7
325	LNDER4	400	2790	LNDER1	132	0.23	2
275	INCND4	400	2625	INCND1	132	0.23	2
275	INCND4	400	2625	INCND1	132	0.23	2
1263	SPITS2	275	6077	SPITS8	88	0.23	4
460	MULDR4	400	6024	MULDR_1S	132	0.23	3
7003	DSOUTH	275	7006	DSD1	132	0.24	4
7003	DSOUTH	275	7008	DSD3	132	0.24	4
7004	DSOUTH	275	7007	DSD2	132	0.24	4
7004	DSOUTH	275	7009	DSD4	132	0.24	4
7030	LOTUSP	275	7034	LPD1	132	0.24	4
7031	LOTUSP	275	7035	LPD2	132	0.24	4
1470	LOMND2	275	6058	LOMND88	88	0.24	6
1470	LOMND2	275	6058	LOMND88	88	0.24	6
1450	IMPAL2	275	2620	IMPAL1	132	0.24	4
103	LULAM4	400	92965	LULAM_D4	88	0.24	5
1243	KOMTP2	275	2715	KOMTP1	132	0.24	6
1243	KOMTP2	275	2715	KOMTP1	132	0.24	6

Reduced tap transformers

Transformer						Suitability score for conversion to fixed tap (0 – 1, 0 being most suitable)	Avg tap	Recommend tap range	
From bus			To bus						
#	Name	kV	#	Name	kV			Min	Max
135	MERCR4	400	92996	MERCR_D4	132	0.25	4	2	5
435	JUNO4	400	60231	JUNO1	132	0.25	6	4	8
1450	IMPAL2	275	2620	IMPAL1	132	0.26	4	3	5
103	LULAM4	400	106	LULAM8B	88	0.26	7	6	8
1056	DURBS2A	275	91914	DURBS_D1	132	0.26	5	4	6
1056	DURBS2A	275	91915	DURBS_D2	132	0.26	5	4	6
1215	SIMPLN2	275	3500	SIMPLN1	132	0.27	6	4	8
1215	SIMPLN2	275	3500	SIMPLN1	132	0.27	6	4	8
455	ACAC4	400	2000	ACAC1	132	0.27	6	5	8
135	MERCR4	400	2853	MERC132	132	0.27	3	2	5
135	MERCR4	400	2853	MERC132	132	0.27	3	2	5
455	ACAC4	400	2000	ACAC1	132	0.27	6	5	8
1440	MERSY2	275	92970	MERSY_D6	132	0.27	5	4	6
1156	SNOWD2	275	1280	SNOWD8_S	88	0.28	12	10	13
1156	SNOWD2	275	1281	SNOWD8_N	88	0.28	12	10	13
1156	SNOWD2	275	1280	SNOWD8_S	88	0.28	12	10	13
1015	BIGHR2	275	1343	BIGHR88	88	0.28	8	6	11
1260	PRAIR2	275	3070	PRAIR1	132	0.28	8	5	12
1156	SNOWD2	275	1281	SNOWD8_N	88	0.28	12	10	13
1260	PRAIR2	275	3070	PRAIR1	132	0.28	8	5	12
1227	LEPIN2	275	4156	LEPIN88	88	0.28	7	6	9
1227	LEPIN2	275	4156	LEPIN88	88	0.28	7	6	9
1015	BIGHR2	275	1343	BIGHR88	88	0.28	8	6	11
1015	BIGHR2	275	1343	BIGHR88	88	0.28	8	6	11
1528	LOTPR2A	275	9001	LOTPR_D1	132	0.29	6	4	7
1529	LOTPR2B	275	9002	LOTPR_D2	132	0.29	6	4	7

1620	EROS4	400	91945	EROS_D1	132	0.29	3	2	5
1620	EROS4	400	92906	EROSD_2	132	0.29	3	2	5
60992	PAULP220	220	92005	PAULP_D1	132	0.29	5	3	6
60992	PAULP220	220	92935	PAULP_D2	132	0.29	5	3	6
60	STIKL4	400	1345	STIKL1	132	0.29	3	1	4
60	STIKL4	400	1345	STIKL1	132	0.29	3	1	4
1450	IMPAL2	275	2620	IMPAL1	132	0.29	4	3	5
1410	GRGDL2	275	2490	GRGDL1	132	0.29	4	3	6
1240	MARTH2	275	2845	MARTH1	132	0.29	7	4	9
1240	MARTH2	275	2845	MARTH1	132	0.29	7	4	9
1445	AVON2	275	92916	AVON_D3	132	0.30	5	4	7
455	ACAC4	400	92051	ACAC_D3	132	0.30	6	4	8
1410	GRGDL2	275	2490	GRGDL1	132	0.30	5	3	6
1253	PRINC2	275	6070	PRINC88	88	0.30	5	4	6
1170	BENBG2	275	2055	BENBG1	132	0.30	6	4	8
1170	BENBG2	275	2055	BENBG1	132	0.30	6	4	8
595	DEDISA4	400	2485	DEDISA1	132	0.30	2	0	5
1410	GRGDL2	275	2490	GRGDL1	132	0.31	4	3	6
103	LULAM4	400	92885	LULAM_D2	88	0.31	8	6	9
1048	KLRWT2	275	7021	KLAD1	132	0.31	5	3	6
1048	KLRWT2	275	7022	KLAD2	132	0.31	5	3	6
1048	KLRWT2	275	7023	KLAD3	132	0.31	5	3	6
1048	KLRWT2	275	7024	KLAD4	132	0.31	5	3	6
175	VULCN4	400	71348	VULCN12N	132	0.31	3	1	5
1186	CARML2A	275	2150	CARML1	132	0.32	4	2	7
1031	CARML2B	275	2150	CARML1	132	0.32	4	2	7
1245	MAKAL2	275	6059	MAKAL88	88	0.32	6	4	7
1245	MAKAL2	275	6059	MAKAL88	88	0.32	6	4	7
1245	MAKAL2	275	6060	MAKAL8B	88	0.32	6	4	7
1245	MAKAL2	275	6060	MAKAL8B	88	0.32	6	4	7
630	FERRM4	400	92057	FERRM_D3	132	0.32	3	2	5

630	FERRM4	400	93000	FERRM_D4	132	0.32	3	2	5
540	PHILP4	400	9142	PHILP_D2	132	0.32	7	6	9
540	PHILP4	400	92937	PHILP_D3	132	0.32	7	6	9
595	DEDISA4	400	2485	DEDISA1	132	0.33	2	0	5
385	NEPTN4	400	2902	NEPTN1	132	0.33	5	1	8
385	NEPTN4	400	2902	NEPTN1	132	0.33	5	1	8
1252	PELLY2	275	2992	PELLY1	132	0.33	6	4	7
1252	PELLY2	275	2992	PELLY1	132	0.33	6	4	7
1150	EIGER2	275	35181	EIGER88N	88	0.33	6	4	7
1150	EIGER2	275	6041	EIGER8SB	88	0.33	6	4	7
1155	PTRTBT2	275	6072	PTRBT8	88	0.33	5	4	7
1710	GRASS2	220	2480	GRASS1	132	0.33	4	2	7
1710	GRASS2	220	2480	GRASS1	132	0.33	4	2	7
590	GRASS4	400	2480	GRASS1	132	0.33	3	1	5
590	GRASS4	400	2480	GRASS1	132	0.33	3	1	5
1202	ROCKD2_1	275	3105	ROCKD1	132	0.33	6	4	8
91981	DELTA_D2	88	91982	DELTA_2B	275	0.33	6	5	8
460	MULDR4	400	6024	MULDR_1S	132	0.33	3	1	4
505	DROER4	400	2305	DROER1	132	0.33	13	10	16
1425	ILLOV2	275	2615	ILLOV1	132	0.33	4	2	5
168	PRAIR_B4	400	92923	PRAIR_D3	132	0.33	2	1	4
168	PRAIR_B4	400	93004	PRAIR_D4	132	0.33	2	1	4
1425	ILLOV2	275	2615	ILLOV1	132	0.34	4	2	5
1385	BLDR2	275	6034	BLDR88	88	0.34	5	4	7
710	DELPH4	400	3535	DELPH1	132	0.34	4	0	8
710	DELPH4	400	3535	DELPH1	132	0.34	4	0	8
1145	BERNN2	275	2065	BERNN1	132	0.35	4	2	6
1145	BERNN2	275	2065	BERNN1	132	0.35	4	2	6
1257	WGATE2	275	3415	WGATE1	132	0.35	4	2	7
1057	DURBS2B	275	91916	DURBS_D3	132	0.36	5	4	7
1057	DURBS2B	275	91917	DURBS_D4	132	0.36	5	4	7

275	INCND4	400	93010	INCND_D3	132	0.36	6	5	8
109	KYALAMI4	400	92992	KYALM_D1	132	0.36	7	5	8
109	KYALAMI4	400	92994	KYALM_D2	132	0.36	7	5	8
660	UMTATA4	400	90034	UMTT_D1	132	0.36	3	1	4
660	UMTATA4	400	90044	UMTT_D2	132	0.36	3	1	4
1079	VERWR2A	275	9171	VERWR_D2	132	0.36	4	2	5
1079	VERWR2A	275	92943	VERWR_D3	132	0.36	4	2	5
1311	OLYMP2A	275	2940	OLYMP1	132	0.36	5	3	7
1310	OLYMP2	275	2940	OLYMP1	132	0.36	5	3	7
1385	BLDR2	275	6034	BLDR88	88	0.37	5	4	7
1266	WARMB2	275	3400	WARMB1	132	0.37	6	4	8
3105	ROCKD1	132	92010	ROCKD2_2	275	0.37	15	11	19
102	MARANG4	400	6056	MARANG8	88	0.37	5	2	8
102	MARANG4	400	6056	MARANG8	88	0.37	5	2	8
102	MARANG4	400	6056	MARANG8	88	0.37	5	2	8
1091	CAMDN88	88	1195	CAMDN2	275	0.37	3	2	5
1145	BERNN2	275	2065	BERNN1	132	0.37	5	3	7
150	WITKP4	400	3445	WITKP1	132	0.37	4	1	6
465	BACCH4	400	6017	BACCH1	132	0.38	3	1	5
1299	MERAP1	132	1363	MERAP2	275	0.38	5	3	7
1266	WARMB2	275	3400	WARMB1	132	0.38	6	4	8
1227	LEPIN2	275	92911	LEPIN_D3	88	0.38	7	5	9
1227	LEPIN2	275	92913	LEPIND_4	88	0.38	7	5	9
1299	MERAP1	132	1363	MERAP2	275	0.38	5	3	7
1079	VERWR2A	275	1296	VERWR1	132	0.40	5	3	7
1006	PEMBR66	66	1194	PEMBR2	220	0.40	12	10	14
1006	PEMBR66	66	1194	PEMBR2	220	0.40	12	10	14
1194	PEMBR2	220	91660	PEMBR_D3	66	0.40	7	6	9
1194	PEMBR2	220	91661	PEMBR_D4	66	0.40	7	6	9
585	SPITS4	400	3190	SPITS1	132	0.41	4	1	7
120	MIDAS4	400	2872	MIDAS1	132	0.41	4	2	6

120	MIDAS4	400	2872	MIDAS1	132	0.41	4	2	6
325	LNDER4	400	2790	LNDER1	132	0.41	5	2	7
1082	PROTS4	400	1339	PROTS1	132	0.41	4	2	6
1082	PROTS4	400	1339	PROTS1	132	0.41	4	2	6
625	VRYBURG4	400	92945	VRYBG_D1	132	0.42	3	1	5
625	VRYBURG4	400	92947	VRYBG_D2	132	0.42	3	1	5
1073	CRGHL2	275	6039	CRGHL8	88	0.42	7	6	9
1073	CRGHL2	275	6039	CRGHL8	88	0.42	7	6	9
1254	RIGI2	275	6075	RIGI8	88	0.43	4	2	6
1254	RIGI2	275	6075	RIGI8	88	0.43	4	2	6
1251	NEVIS2	275	1481	NEVIS1A	132	0.43	7	4	9
1296	VERWR1	132	1725	VERWR2B	275	0.45	6	4	7
505	DROER4	400	2305	DROER1	132	0.45	12	10	14
1310	OLYMP2	275	90004	OLYMP_D3	132	0.45	6	4	8
9118	ROCKD_D2	132	92010	ROCKD2_2	275	0.45	8	6	9
1223	TUGEL2	275	92955	TUGEL_D3	132	0.45	4	2	6
375	HYDRA4	400	2610	HYDRA1	132	0.46	9	3	15
1410	GRGDL2	275	92949	GRGDL_D7	132	0.46	4	2	6
1390	INGGN2	275	55131	INGGN88	88	0.46	11	7	14
1701	OTTAWA2A	275	91572	OTTAW_D1	132	0.46	7	5	9
1702	OTTAWA2B	275	91573	OTTAW_D2	132	0.46	7	5	9
1223	TUGEL2	275	3295	TUGEL1	132	0.46	3	1	5
1223	TUGEL2	275	3295	TUGEL1	132	0.46	3	1	5
540	PHILP4	400	1042	PHILP1	132	0.47	7	5	10
465	BACCH4	400	6017	BACCH1	132	0.47	3	1	6
1090	ESSLN2	275	92013	ESSLN_D7	88	0.47	4	2	6
107	MOGWASE4	400	92975	MOGWS_D1	132	0.48	12	9	14
107	MOGWASE4	400	92976	MOGWS_D2	132	0.48	12	9	14
95	TABOR4	400	90121	TABOR_D3	132	0.48	3	1	5
1251	NEVIS2	275	1480	NEVIS1	132	0.48	7	5	10
143	MOKOPN4	400	92979	MOKOP_D1	132	0.49	7	4	9

143	MOKOPN4	400	92980	MOKOP_D2	132	0.49	7	4	9
1090	ESSLN2	275	1656	ESSLN8B	88	0.49	5	3	7
1400	BLKRN2	275	2080	BLKRN1	132	0.50	4	2	6
1400	BLKRN2	275	2080	BLKRN1	132	0.50	4	2	6
1282	POSDN2	220	91662	POSDN_D3	132	0.50	9	6	12
1282	POSDN2	220	91663	POSDN_D4	132	0.50	9	6	12
1238	MALELAN2	275	92915	MLELN_D1	132	0.50	5	3	7
1235	TABOR2	275	3475	TABOR1	132	0.50	7	4	10
1235	TABOR2	275	3475	TABOR1	132	0.50	7	4	10
1253	PRINC2	275	6070	PRINC88	88	0.50	6	4	8
1253	PRINC2	275	6070	PRINC88	88	0.50	6	4	8

Transformers to leave unchanged

Transformer						Suitability score for conversion to fixed tap (0 – 1, 0 being most suitable)
From bus			To bus			
#	Name	kV	#	Name	kV	
1257	WGATE2	275	3415	WGATE1	132	0.51
1155	PTRTBT2	275	6072	PTRBT8	88	0.52
1085	CROYD2	275	90006	CROYD_D3	132	0.52
375	HYDRA4	400	2610	HYDRA1	132	0.53
1263	SPITS2	275	6077	SPITS8	88	0.53
1085	CROYD2	275	2225	CROYD1	132	0.54
1230	ACRNH2	275	2005	ACRNH1	132	0.54
1230	ACRNH2	275	2005	ACRNH1	132	0.55
590	GRASS4	400	92067	GRASS_D8	132	0.55
1330	KKFN2A	275	9092	KKFN_D2	88	0.55
1330	KKFN2A	275	92963	KKFN_D3	88	0.55
150	WITKP4	400	3445	WITKP1	132	0.55
150	WITKP4	400	3445	WITKP1	132	0.55
1390	INGGN2	275	55131	INGGN88	88	0.55

120	MIDAS4	400	92929	MIDAS_D3	132	0.56
1657	ETNA2	275	4070	ETNA8	88	0.56
1657	ETNA2	275	4070	ETNA8	88	0.56
585	SPITS4	400	3190	SPITS1	132	0.57
1085	CROYD2	275	2225	CROYD1	132	0.57
1350	EVRST2	275	2390	EVRST1	132	0.57
1350	EVRST2	275	2390	EVRST1	132	0.57
101	DINALD4	400	71236	DINALD1	132	0.57
101	DINALD4	400	71236	DINALD1	132	0.57
1194	PEMBR2	220	2995	PEMBR1	132	0.57
1194	PEMBR2	220	2995	PEMBR1	132	0.57
460	MULDR4	400	2900	MULDR_1N	132	0.58
1165	BRENN2	275	4015	BRENN88	88	0.58
1092	NRAND2	275	92932	NRAND_D1	132	0.58
1092	NRAND2	275	92933	NRAND_D2	132	0.58
1165	BRENN2	275	4015	BRENN88	88	0.59
1360	HARVR2	275	2575	HARVR1	132	0.59
1360	HARVR2	275	2575	HARVR1	132	0.59
1090	ESSLN2	275	2365	ESSLN1	132	0.59
1075	JUPTR2	275	92986	JUPTR_D4	88	0.60
1715	SPENC2	275	3443	SPENC132	132	0.60
1256	TNUS2	275	3245	TNUS1	132	0.60
1256	TNUS2	275	3245	TNUS1	132	0.60
1256	TNUS2	275	3245	TNUS1	132	0.60
1090	ESSLN2	275	2365	ESSLN1	132	0.61
1058	DURBN2A	275	92003	DURBN_D1	132	0.61
1059	DURBN2B	275	92004	DURBN_D2	132	0.61
1042	PHILP1	132	9008	PHILP4	400	0.61
1075	JUPTR2	275	9085	JUPTR_D1	88	0.62
112	DEMITER4	400	93006	DMTER_D1	132	0.62
112	DEMITER4	400	93008	DMTER_D2	132	0.62

1270	BNDRY1	132	1365	BNDRY2	275	0.63
1270	BNDRY1	132	1365	BNDRY2	275	0.63
1165	BRENN2	275	4015	BRENN88	88	0.64
103	LULAM4	400	104	LULAM8	88	0.65
1093	KOMATI1	132	1232	KOMAT2	275	0.67
1150	EIGER2	275	6040	EIGER88S	88	0.69
1204	OLIEN2	275	91900	OLIEN_D2	132	0.71
460	MULDR4	400	2900	MULDR_1N	132	0.71
1140	FORDS2	275	6042	FORDS88	88	0.71
1140	FORDS2	275	6042	FORDS88	88	0.71
1140	FORDS2	275	6042	FORDS88	88	0.71
1140	FORDS2	275	6042	FORDS88	88	0.71
1075	JUPTR2	275	6053	JUPTR88	88	0.72
1390	INGGN2	275	55131	INGGN88	88	0.75
1255	SCAFL2	275	41083	SCFLDS1	132	0.76
1090	ESSLN2	275	2365	ESSLN1	132	0.77
1135	PROSP2	275	6044	PROSP88	88	0.77
1135	PROSP2	275	6044	PROSP88	88	0.77
1135	PROSP2	275	6044	PROSP88	88	0.77
1135	PROSP2	275	6044	PROSP88	88	0.77
1255	SCAFL2	275	41083	SCFLDS1	132	0.78
585	SPITS4	400	91674	SPITS_D6	132	0.79
1075	JUPTR2	275	6053	JUPTR88	88	0.79
1365	BNDRY2	275	9029	BNDRY_D1	132	0.85
1380	GARN2	275	2441	GARN1	132	0.87
1375	FERRM2	275	91990	FERRM_D1	132	0.88
1715	SPENC2	275	92687	SPENC_D2	132	0.89
1267	WATRS2	275	6091	WATRS8B	88	0.90
1075	JUPTR2	275	6053	JUPTR88	88	0.91
1363	MERAP2	275	91541	MERAP_D2	132	0.92
1363	MERAP2	275	9109	MERAP_D1	132	0.92

1375	FERRM2	275	91991	FERRM_D2	132	0.92
1267	WATRS2	275	6090	WATRS8A	88	0.93
108	PHOEBUS4	400	92941	PHOBS_D2	132	0.94
108	PHOEBUS4	400	93012	PHOBS_D3	132	0.94
1365	BNDRY2	275	9030	BNDRY_D2	132	0.99
1019	OLIEN_D1	132	1204	OLIEN2	275	

APPENDIX D: Benchmark questionnaire feedback

D.1 Chile

Country:		Chile
Company/Utility:		Transelec
Contact Person:		Juan Carlos Araneda
Email:		jcaraneda@transelec.cl
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National Control
8	Are TD auto/national control tapped?	Both
9	VAr switching more common than MTS tapping?	Yes
10	VAr switching more common than TD tapping?	Yes

D.2 United States

Country:		United States
Company/Utility:		Southern Company
Contact Person:		Steve R. Holsomback
Email:		srholsom@southernco.com
Question		Answer
1	Do MTS have wide tap range?	No
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National
8	Are TD auto/national control tapped?	Auto
9	VAr switching more common than MTS tapping?	Yes
10	VAr switching more common than TD tapping?	Yes

Comments:

MTS and TD transformers are always replaced with tapchangers that have five taps. MTS transformers are tapped using a selectable tap changer set on a fixed position.

Contact has also been made with Transformer Specialists in the United States. These specialists say that several of the large power companies in the US do not use load tap changers on HV autotransformers or other HV Transmission transformers. Most of the power companies use de-energized taps in a +-5% range. Steve Holsomback of Southern Company confirmed that they do not buy any network auto transformer with a load tap changer. They do have several capacitor banks, some SVCs and shunt reactors installed.

Country:		United States
Company/Utility:		Tennessee Valley Authority
Contact Person:		Ian S Grant
Email:		isgrant@tva.gov
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	Yes
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	Yes
7	Are MTS auto/national control tapped?	National
8	Are TD auto/national control tapped?	Both
9	VAr switching more common than MTS tapping?	Yes
10	VAr switching more common than TD tapping?	Both

Comments to above questions:

1. Yes in general. All 500/161 kV and most 230/161 kV
2. Some, where the distributor doesn't have any downstream regulation in place
4. Yes. Some distributor owned banks with downstream regulators already in place
5. Yes. Some distributor owned banks with downstream regulators already in place
8. Some or both, depending on the distributor of power.
9. Yes, typically operate shunt capacitors and reactors before tapping the transformers
10. No, usually on Distribution level systems, the capacitors and tap-changers are coordinated to work together.

In response to your questions:

TVA has three primary BES voltages, 500 kV , 230 kV and 161 kV. We also have some 345 kV, 138 kV and 115 kV transformers. So according the definitions mentioned, we don't own or operate any MTS transformers. However, to allow delineation of 500/161 kV and 230/161 kV transformers as being BES transformers, I've answered the questions as if these transformers are MTS transformers, and 230/115 kV and 161/69 kV transformers are TD. TVA is not a fully integrated company, therefore we do not always own down to the Distribution system. So most (80% or more) of the Distribution transformers are owned by the local distributor.

TVA answered:

MTS = 500/161 kV and 230/161 kV

TD = 230/115 kV, 161/69 kV, 161/46 kV

Main Transmission system (MTS) transformers: 765/400 kV, 400/275 kV, 400/220 kV, 500/275 kV etc.

Transmission/Distribution transformers (TD): 400/132 kV, 275/88 kV, 400/88 kV, 275/132 kV, 220/132 kV, 500/150 kV etc.

D.3 France

Country:		France
Company/Utility:		RTE
Contact Person:		Patrick Lhuillier
Email:		patrick.lhuillier@RTE-FRANCE.COM
Question		**Answer**
1	Do MTS have wide tap range?	?
2	Do TD have wide tap range?	?
3	Do MTS have reduced tap range?	?
4	Do TD have reduced tap range?	?
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No

7	Are MTS auto/national control tapped?	?
8	Are TD auto/national control tapped?	?
9	VAr switching more common than MTS tapping?	?
10	VAr switching more common than TD tapping?	?

RTE in France have OCTC (off circuit tap changer) transformers on their autotransformers (400/225 kV). This decision was made before the split from EDF. It was motivated by economic and technical reasons in order to optimize the size of generators and their capacity range, output transformers (transformers just after generators) and autotransformers (400/225 kV). No changes have been made since the separation between EDF and RTE (2000). From his point of view, Jean-Pierre (of RTE) explains that it would be easier to regulate the 225 kV level with automatic tap-changing (A – OLTC) on RTE autotransformers rather than to operate on EDF generators.

Furthermore, all of RTE transformers (400/90 kV, 400/63 kV, 225/90 kV or 225/63 kV) are equipped with OLTC. No study is presently being carried out to modify this in the near future. As the RTE grid is operated closer and closer to the limits the trend is likely to continue.

D.4 Spain

Country:		Spain
Company/Utility:		REE
Contact Person:		Juan Fran. Alonso Llorente
Email:		jfalonso@ree.es
	Question	**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No

5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National Control
8	Are TD auto/national control tapped?	National Control
9	VAr switching more common than MTS tapping?	Both
10	VAr switching more common than TD tapping?	Both

Comments: A few old MTS and TD transformers include tap changers with a reduced tap range that are used in a phase-shifting function. These are rarely used. Tap changing and VAr devices are both used depending on the magnitude of the required control. Reactive compensation elements provide "gross regulation" and tap-changing provides smaller regulation.

Country:		Spain
Company/Utility:		Iberdrola
Contact Person:		Joaquín Cabetas
Email:		Joaquin.cabetas@iberdrola.es
	Question	**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	Yes
6	Do TD include fixed tap changer?	Yes
7	Are MTS auto/national control tapped?	National Control

8	Are TD auto/national control tapped?	National Control
9	VAr switching more common than MTS tapping?	Both
10	VAr switching more common than TD tapping?	Both

Comments:

5. When they don't have LTC
6. When they don't have LTC
9. First VArs are connected to compensate reactive power as needed , then the voltage adjustment between voltage levels and substations are adjusted with LTC
10. In Spain the distributors are forced to achieve an accurate compensation of VArs

Received comments:

Our company sold the Transmission network to the Spanish system operator (REE), so I would recommend you contacting them (see www.ree.es) to get further information. However, Iberdrola still owns and operates sub-Transmission networks (132 kV down to LV) and we do have some data and experience we can share with you (although for MTS it can be somehow outdated).

Most of the Transmission (400/220 kV) and Transmission/sub Transmission transformers (400/132 kV, 220/132 kV, 220/66 kV) have LTC, always manually operated. When they don't have LTC, at least they have some taps (usually 3, adjusted off-load) to adjust in some degree the secondary voltage. In Spain we also have MTS/MV transformer (220/20 kV) that always have LTC with automatic adjustment (because 20 kV is radially operated, there are no issues with VAr loops). Until we sold the Transmission network to REE, our dispatch operators had an OPF that advised the operator of the optimal setting of both LTC point and VAR switching; however, we didn't have an integral program that minimized total cost including losses, wearing of LTC, etc.

At a system wide perspective, the Spanish network usually has large distances between generation and load sites, so you can't do large VAr transportation between generator and loads, and you have to compensate VAr consumption close to 1 power factor at the interfaces between MTS and TD. In Spain the distributors compensate most of VArs at the MTS-TD interfaces, so VAR compensation during the day is usually achieved (at night, urban areas have an excess of VArs because of cable shunt capacitance).

I suppose that it will depend on the topology, but in Spain transformers on MTS and TD transformers without LTC have been a nightmare to operate. On our network it is difficult to keep voltages on every voltage level between margins. Trying to do it with VARs is difficult because you don't have individual flow control (in a meshed system) and the interactions between different voltage levels (you can compensate on MV, and the effects goes up to the MTS), and also because of the voltage steps caused which are a source of complaints from clients. This last problem could also happen if you increase the voltage step on transformers, as you suggest.

D.5 Romania

Country:		Romania
Company/Utility:		Transelectrica SA
Contact Person:		Traian Chiulan
Email:		Traian.Chiulan@transelectrica.ro
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National Control
8	Are TD auto/national control tapped?	Both

9	VAr switching more common than MTS tapping?	No
10	VAr switching more common than TD tapping?	No

D.6 Canada

Country:		Canada
Company/Utility:		Manitoba Hydro
Contact Person:		David Jacobson
Email:		dajacobson@hydro.mb.ca
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	Yes
7	Are MTS auto/national control tapped?	National Control
8	Are TD auto/national control tapped?	Auto
9	VAr switching more common than MTS tapping?	VAr for large adjustments
10	VAr switching more common than TD tapping?	VAr for large adjustments

Comments:

1. Yes, Dorsey BK51 and BK52 500/230 kV Banks include tap-changers with a wide tap range (-10 to +10).

2. Yes, a large number of Manitoba Hydro's TD transformers have tap-changers with a wide range (at least 1-17 taps, usually 21-33 taps). There are approximately 88 in the above mentioned voltages.
3. No, Manitoba Hydro's only two Main system transformers of this voltage level do not have a reduced tap range.
4. No, Manitoba Hydro's TD transformers do not include tap-changers with a reduced tap range. Although, the answer is Yes, assuming you mean de-energized tap changers (DTC). We have a few in this category.
5. No, Manitoba Hydro's only two MTS transformers of this voltage level do not have fixed taps.
6. Yes, a number of Manitoba Hydro's TD transformers are of the fixed tap type. There are approximately 62 in the above mentioned voltages. They are eventually being phased out and replaced by transformers with on-load taps.
7. Manitoba Hydro's MTS transformers (500 kV range) are tapped by system control operators from the system control centre
8. Manitoba Hydro's TD transformers are in large party automatically tapped by a voltage regulating device (approx. 70 in total). However there are approx. 18 that are manually operated by System Control Operators.

Received comments:

Your questions triggered some additional thinking here. Here's some follow-up information you might find useful and interesting.

Today's best load tap changers (LTC) in Manitoba contain vacuum diverter switches (Reinhausen tap changers) which virtually eliminate tapchanger maintenance costs, and eliminates heavy coking of diverter contacts.

We currently only have a few transformers with vacuum diverters at the moment. But it has now become our preferred LTC on new equipment orders.

We used to use ABB tapchangers where practical (7.5/10 MVA prior to 2006, and 95 MVA 230-66 kV), and when desired by the transformer manufacturer. However, essentially all of our newer transformers use vacuum tap-changers from Reinhausen. For the 7.5/10 MVA transformers we need a larger current rating for the required overloading so we use Reinhausen RMV reactor-type vacuum tap changer. For 230/66 kV, we now use the Reinhausen VR resistor-type vacuum tap changer.

Reinhausen has also released a replacement vacuum diverter for non-vacuum model-M tapchangers. It is a direct, simple swap.

Presently the tapchangers that give us trouble are Federal Pioneers of any year with coking issues and ABB UZE model types built between 1980 and 1995 have coking issues due to a component design. However we are replacing the components at inspections with better designed ones.

For 138 kV and below, the Reinhausen tapchangers are 11% of the total and Transmission 230 kV auto transformers are 65%. We've had no problems with ABB UC type models and the older Reinhausen D, E and F cover mount type tapchangers to date. The Reinhausen M type has been a costly diverter due to continuous upgrade of components with a few minor component failures. ABB and Reinhausen both make a vacuum-type cover-mounted tapchanger but we elected to go with Reinhausen.

The number of tapchangers for 230 kV, 66 kV, 24 kV and 12 kV voltages - 17 ABB, 20 Reinhausen, 4 FPE, 4 Ferranti Packard.

The reason the total number is only 20% is because we have so many Federal Pioneer tapchangers in Distribution. In Transmission, most of the old 230-66 have ABB gears, as well as most of the auto-transformers from CGE and ABB.

D.7 UK

Country:		UK
Company/Utility:		National Grid
Contact Person:		John Douglas
Email:		douglasjak@pbworld.com
Question		**Answer**
1	Do MTS have wide tap range?	No taps
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	Yes
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	No taps

8	Are TD auto/national control tapped?	Both
9	VAr switching more common than MTS tapping?	Yes
10	VAr switching more common than TD tapping?	No

Comments:

Licensee	Voltage (in kV)	Rating (in MVA)	Reactance (% on Rating Base)	Tapping Limit	Cyclic Load Factor
NGET	400/275	1000	16	Non-Tapped	1.3
NGET	400/275	750	20	Non-Tapped	1.3
NGET	400/275	750	12	Non-Tapped	1.3
NGET	400/275	500	12	Non-Tapped	1.3
NGET	400/132	240	20	+15% to -5%	1.2
NGET	275/132	240	20	+15% to -15%	1.2
NGET	275/132	180	15	+15% to -15%	1.2
NGET	275/132	120	15	+15% to -15%	1.2
NGET	275/66	180	20	+15% to -15%	1.3
NGET	275/66	120	20	+15% to -15%	1.3
NGET	275/33	120	24-36	+10% to -20%	1.3
NGET	275/33	100	20-30	+10% to -20%	1.3
SHETL	275/132	240	21.6	+18% to -13%	
SHETL	132/33	60	16.2	+10% to -20%	
SHETL	132/11	30	15.0	+10% to -20%	
SPT	400/275	1000	16.0	Fixed	
SPT	400/132	240	19.9	+15% to -15%	
SPT	275/132	240	20.4	+15% to -15%	
SPT	275/33	120	24.0	+10% to -20%	
SPT	132/33	60	15.0	+10% to -20%	
SPT	132/11	30	23.7	+10% to -10%	

D.8 Singapore

Country:		Singapore
Company/Utility:		PB Singapore
Contact Person:		Keehan Chan
Email:		Chan.keehan@pbworld.com
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	No
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National
8	Are TD auto/national control tapped?	National
9	VAr switching more common than MTS tapping?	Yes
10	VAr switching more common than TD tapping?	Yes

D.9 India

Country:		India
Company/Utility:		
Contact Person:		MataPrasad
Email:		Matap6@gmail.com
Question		**Answer**
1	Do MTS have wide tap range?	Yes
2	Do TD have wide tap range?	Yes
3	Do MTS have reduced tap range?	No
4	Do TD have reduced tap range?	
5	Do MTS include fixed tap changer?	No
6	Do TD include fixed tap changer?	No
7	Are MTS auto/national control tapped?	National
8	Are TD auto/national control tapped?	National
9	VAr switching more common than MTS tapping?	No
10	VAr switching more common than TD tapping?	No

Comments:

1. MTS Transformers in India at 400 kV and 800 kV include tap changers with 1-17 taps for +/_ 10% range
2. TD Transformers in India still have normally +/- 10% OLTC with tapping range but some are with other ratio like +7.5% and -12.5% as well
3. We have so far not used any reduced tap range (1-3 Taps). For the reduced tapping range(1-3 taps) this could become an economic choice, however the MTS transformers with +/- 10% OLTC are now being provided with shunt reactors and capacitors where needed on the tertiary). An earlier proposal to. have dynamic compensation of 35 MVAr Inductive /70 MVAr capacitive connected to 33 kV tertiary

windings of 400/220 kV MTS transformers to provide smooth control of voltage profile and reactive power flow but this project was apparently aborted. After that we installed an SVC of rating +/- 140 MVA at one 400 kV substation for system stabilization after fault of 400 kV line or HVDC pole outage

5. So far but some thought is being given to using a reduced tapping range and even tap-less transformers with added dynamic and passive reactive power support to control the voltage profile not only on TD but also for generator transformers specially in Hydro Plants backed-up by good dynamic compensation. In generator transformers the tap-less transformer could still work with better performance if the dynamic compensation has been provided and the excitation system augmented
6. Despite the reluctance of operators to use the on-load tap-changer to control the voltage, still the OLTC is being provided on TD transformers. It could still be economically better to have tap-less transformer installed along with a liberal static VAr system
7. We had provided AVR for the OLTC but kept out due to frequent operation of OLTC more than 50-60 times a day. The OLTC is now controlled manually and presently not automatically from load dispatch centres but in future this will be implemented
8. See point 7.
9. At present the VAr devices are not installed except on the 400 kV system. However the capacity is 2 x +/- 140 MVAr is insufficient due to acute shortage of reactive power in the system. The OLTC is changed manually by operators but above SVCs run in susceptance mode
10. No operation of VAr devices except a few 33 kV 25 MVAr shunt reactors provided on tertiary terminals are used for manual switching

Further comments from the respondent:

For the voltage regulation under all operating conditions the tap changing is the most widely used method all over the world. The selection criteria is based on important issues of transformer reliability as also the life, besides affecting the downtime due to maintenance of OLTC more frequently than the transformer itself that is definitely affected by providing the OLTC. It must always be appreciated that a transformer without OLTC is cheaper and more reliable than the one with OLTC. But the requirement for voltage control of downstream system is the overriding issue. The psychological fear of failure of OLTC on generator or MTS or TD affecting the system operation is quite widespread. You will recall that the development of VCB and Vacuum switching for tap changing to make it most reliable has been brought only to mitigate such fear.

For tap changing gear on transformers we have following choices:

1. Use Off-circuit taps, +/- 5% each tap for 2.5% (normally on generator transformers)

2. OLTC taps on MTS and TD usually with 17 taps each of 1.25% value with provision of VAr Support (TCR or Statcom) facilitating for control of voltage profile. This is no doubt costly.

3. OLTC with only 3 taps on TD with liberal VAr Support with fast control for regulation and control of voltage profile. This appears to be giving good solution and economical transformer. The cost benefit analysis of the composite transformer with 3 step OLTC and VAr support will however show the real economics of the provision

4. Static tap-changers requiring only 3 or 4 tappings. This has been used in rectifier plants, in Aluminum smelters and other similar factories needing rectifiers where tap changing is required to maintain the current through potline required to maintain the current levels in the potline. The high cost of such static OLTC is offset by smooth operation less down time. No MTS or TD transformers in India have been reported using this type of OLTC

In the following paragraphs some information is given as practiced in India just to apprise you the background of transformers with OLTC or with off-circuit tap changers.

The power transformers in category of MTS and TD in India are provided with OLTC usually with 1-17 taps. Provision of OLTC on Generator transformers was started in late seventies for two projects (6 transformers). However some new 500 MW or 600 MW projects by private sector units are again asking for OLTC on generator transformers.

The technical specifications (for MTS and TD) 400 kV system transformers in India had large international inputs from Sweden, Germany, UK, France Canada, USA, Brazil and Venezuela and above all from ESKOM in 1973-74 period. The contribution from Mr. Sollegren, Mr. Bertil Thoren, Mr. Gunnar Jancke, from Sweden Claude Gary from France and Mr. Bruce Norman from ESKOM were highly useful that helped us in standardizing the parameters and technical specifications of the 400 kV Transformers. The ratings selected were 315 MVA and 500 MVA. The standard parameters for 400 kV transformers are still valid except for some minor topical add-ons in the technical specifications. All these transformers are provided with OLTC with 1-17 taps. The location of all the four windings viz. series, common regulating and tertiary were standardized and so arranged to give natural high impedance between primary and tertiary and secondary to tertiary.

Further comments from the respondent:

For Indian MTS and TD Transformers we have OLTC taps 1-17 as confirmed to you. For generator transformers after dropping OLTC we had opted for off-circuit taps with +/- 5% with tap no 3 as normal tap and each step was generally of 2.5%

In our planning departments in collaboration with Central Electricity Authority under Ministry of Power, Government of India, there is now a general feeling that instead of having such a large range of taps on power transformers that increase the cost and reduces the reliability, some better alternative of voltage regulation be visualized that should be robust, cost effective and reliable.

In this connection we considered use of variable static VAr sources to be effectively installed at sensitive nodes in the grid to take care of voltage profile and also improve the overall stability margin. The use of static tap-changers in the power network, although attractive, its cost was not be justified due to relatively smaller amount of tap changes duty in power system but could be effectively applied in rectifier-transformers of aluminum smelters and similar processes.

In another attempt we did try to use a concept with a view to minimize the cost of the product as static tap-changer but still we are waiting for its success

In your proposal for reduced taps 1-3 that will be totally off -circuit with normal tap at 1 and one +/- tap on its each side. Such off- circuit taps on Distribution transformers (11 kV /415 kV) have been used, where tap change is done after switching of the transformer for a short time.

APPENDIX E: Costing analysis

Please see attached spreadsheet.

APPENDIX F: OPF Results

OPF analysis is performed on 2009 and 2015 case files received from Eskom as mentioned in the introduction to this dissertation. This includes summer light loading and winter maximum loading for both generator dispatch patterns (2009 and 2015), (8 case files in total).

The Transmission contingencies chosen, (Table F-1), are used to demonstrate the robustness of the solution when the network is under different types of stress.

For Stage 2 of the OPF simulations, the tap positions are held constant for all the system conditions while the bus voltages, generator VAR outputs, additional adjustable shunts and improvement in losses are evaluated.

Table F-1: Transmission contingencies chosen for investigation

Contingency Number	Description of Transmission Contingency
1	Arnot – Merensky 400 kV
2	Jupiter – Prospect 275 kV
3	Spitskop – Ararat 275 kV
4	Beta – Hydra 765 kV
5	Beta – Delphi 765 kV

F.1 Tap positions

The tap positions of a selection of the MTS transformers for all the 2009 and 2015 case files before and after the OPF are shown in Table F-2.

In Table F-2 the tap positions under the SPF column are the tap positions in the base SPF case. The tap positions under the OPF column represent the mode or the most common tap for the specific transformer under all the base and contingency conditions specified for investigation. The value in the OPF column is the value that the tap position is set at after Stage 1 of the OPF and thereafter a high penalty is placed on this value, thereby holding it generally constant in the second phase of the OPF.

The table also compares the tap positions under maximum load for both generator dispatches and the tap positions under minimum load for the dispatch patterns. A standard deviation between the tap positions for all the case files is also calculated for ease of comparison.

Standard deviations in general are low which implies a small movement in tap positions

The tap positions for the Transmission and generator transformers can be found in similar tables in the Appendix B. A shortened list of TD transformers selected for their suitability for conversion to fixed tap is presented in Table F-3 (a full list of suitable transformers is given in Appendix C). The transformers are ranked in order of their suitability based on a score calculated where the difference between the max and min loading OPF tap position has the highest weighting, the difference between the max and min loading SPF tap is weighted the next highest and the difference between the SPF and OPF average tap for both min and max loading are weighted the least.

A score close to zero (0) indicates a high level of suitability. As the score increases, the suitability for conversion of the transformer to fixed tapping decreases.

This ranking is used in Section 5 in the cost analysis section.

As can be seen from Table F-2 , there is relatively little movement of the tap positions between different loading and generator patterns.

The movement in tap positions between the years 2009 and 2015 is more significant. This can be partly attributed to a significant step in system strengthening e.g. Medupi, Kusile and Ingula power stations and significant Transmission upgrading including 765 kV Transmission overlay to Richards Bay, Durban, Cape Town and Port Elizabeth and a large number of new 400 kV lines connecting new power stations, new loads and improving Transmission system reliability.

Table F-2: Tap positions of a selection of MTS transformers in the Eskom system

						WINTER MAX LOAD								SUMMER MIN LOAD							
						2009				2015				2009				2015			
						g1		g2		g1		g2		g1		g2		g1		g2	
From bus			To bus			SPF	OPF	SPF	OPF	SPF	OPF	SPF	OPF	SPF	OPF	SPF	OPF	SPF	OPF	SPF	OPF
#	Name	kV	#	Name	kV	Base	Mode	Base	Mode	Base	Mode	Base	Mode	Base	Mode	Base	Mode	Base	Mode	Base	Mode
80	ALPHA7	765	221	ALPHA4	400	16	16	16	16	18	18	18	18	18	18	18	18	18	18	18	18
80	ALPHA7	765	221	ALPHA4	400	16	16	16	16	18	18	18	18	18	18	18	18	18	18	18	18
80	ALPHA7	765	221	ALPHA4	400	16	16	16	16	18	18	18	18	18	18	18	18	18	18	18	18
85	BETA7	765	337	BETA4	400	17	17	17	17	10	10	10	10	11	11	11	11	10	10	10	10
85	BETA7	765	337	BETA4	400	17	17	17	17	10	10	10	10	11	11	11	11	10	10	10	10
110	GLOCK4	400	1335	GLOCK2	275	4	4	4	4	3	3	3	3	1	1	1	1	3	3	3	3
110	GLOCK4	400	1335	GLOCK2	275	3	3	3	3	3	3	3	3	1	1	1	1	3	3	3	3
115	HERA4	400	1157	HERA2	275	7	7	7	7	9	9	9	9	3	3	3	3	9	9	9	9
115	HERA4	400	1157	HERA2	275	7	7	7	7	9	9	9	9	4	4	4	4	9	9	9	9
125	PLUTO4	400	1020	PLUTO2	275	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
125	PLUTO4	400	1020	PLUTO2	275	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
150	WITKP4	400	1244	WITKP2	275	8	8	5	8	5	5	5	5	1	1	2	2	5	5	5	5
150	WITKP4	400	1244	WITKP2	275	8	8	1	1	5	5	5	5	1	1	1	1	5	5	5	5
155	MERNS4	400	1242	MERNS2	275	3	3	6	3	6	6	6	6	6	6	6	5	6	6	6	6
155	MERNS4	400	1242	MERNS2	275	3	3	3	3	5	5	5	5	4	4	6	6	5	5	5	5
158	MARTH4	400	92968	MARTH_D8	275					4	4	4	4					4	4	4	4
160	KKRBM4	400	1159	KKRBM2	220	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
160	KKRBM4	400	1159	KKRBM2	220	4	4	4	4	5	5	5	5	6	6	6	6	5	5	5	5
165	ARNOT4	400	1200	ARNOT2	275	4	3	4	2	1	1	1	1	4	4	4	4	1	1	1	1
165	ARNOT4	400	1200	ARNOT2	275	7	7	7	7	1	1	1	1	5	5	5	5	1	1	1	1

Table F-3: Selection of TD transformers for conversion to fixed tap transformers (0 being the most suitable)

					WINTER MAX LOAD								SUMMER MIN LOAD				Statistical analysis - Max load			Statistical analysis - Min load			Diff btn max & min SPF	Diff btn max & min OPF	Suitability for conversion to fixed tap
					2009				2015				2009		2015										
					g1		g2		g1		g1				g2										
From bus			To bus																						
#	Name	kV	Name	kV	SPF Base	OPF Mode	SPF Base	OPF Mode	SPF Base	OPF Mode	SPF Base	OPF Mode	OPF Mode	OPF Mode	OPF Mode	OPF Mode	Avg SPF base tap	Avg OPF tap	Diff btn SPF & OPF avgs	Avg SPF base tap	Avg OPF tap	Diff btn SPF & OPF avgs			
1255	SCAFL2	275	SCFLDS1	##	8	8	8	8	9	9	9	9	8	8	9	9	8	8	0	8	9	0	0	0	0.00
1254	RIGI2	275	RIGI8	88	1	1	1	1	5	5	5	5	1	1	5	5	3	3	0	3	3	0	0	0	0.00
1618	EROS1	132	EROS4	##	7	7	7	7					7	7			7	7	0	7	7	0	0	0	0.00
280	NORMN4	400	NORMN1	##	5	5	5	5	1	1	1	1	4	6	1	1	3	3	0	3	3	0	0	0	0.00
225	MAJUB4	400	MAJUB8	88	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0.01
225	MAJUB4	400	MAJUB8	88	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0.01
1185	KRSPN2	275	KRSPN1	##	3	3	3	3	4	4	4	4	4	4	3	3	4	4	0	4	4	0	0	0	0.02
1185	KRSPN2	275	KRSPN1	##	3	3	3	3	3	4	3	4	3	5	3	3	3	4	1	3	4	1	0	0	0.03
1185	KRSPN2	275	KRSPN1	##	3	3	3	3	3	4	3	4	3	5	3	3	3	4	1	3	4	1	0	0	0.03
91977	DELTA88	88	DELTA_2B	##	5	5	5	5					5	5			5	5	0	5	5	0	0	0	0.03
265	UMFLZ4	400	UMFLZ88	88	4	4	4	4	1	2	1	2	4	4	2	2	3	3	1	2	3	1	0	0	0.04
1152	VERDN2B	275	VERDN_D2	88					5	6	5	6			6	6	5	6	1	5	6	1	0	0	0.05
175	VULCN4	400	VULCN1	##	4	4	4	4	2	3	2	3	4	4	3	3	3	4	1	2	4	1	1	0	0.07
5946	MAPUT4	400	MAPUT_D2	##	1	3	1	2	3	3	3	3	3	3	3	3	2	3	1	2	3	1	0	0	0.07
5946	MAPUT4	400	MAPUT_D3	##	1	3	1	2	3	3	3	3	3	3	3	3	2	3	1	2	3	1	0	0	0.07
1282	POSDN2	220	POSDN1	##	7	7	8	7	5	7	5	6	6	9	5	5	6	7	1	7	6	1	1	1	0.09

F.2 Loss savings

As mentioned in the Introduction and elsewhere, minimizing losses is the key objective function (or explicit objective function) of the OPF for this research.

The loss savings from the OPF analysis compared to the SPF for the 2009 and 2015 case files are summarized in Table F-4. This table records only the loss savings for the base cases under normal operating conditions; when there are no contingencies, for all the 2009 and 2015 case files. Loss analysis data under emergency or contingency conditions is listed in Appendix B.

The losses are significantly greater in the 2015 files and the percentage loss saving after the OPF is less in 2015 than in 2009. This can be attributed to the system "working harder" in 2015 as a result of more generation and load

Table F-4: Summary of loss reduction of OPF versus SPF (2009 and 2015 winter loading cases)

Base			2009				2015			
PF	Losses	Unit	g1		g2		g1		g2	
			winter	summer	winter	summer	winter	summer	winter	summer
Standard power flow	Total	MW	1118.4	479.9	1333.4	603.8	1751.2	908.6	1589.5	810.7
		MVAr	17847.5	7809	19721	8853.2	28512.7	16022.1	26693.4	14943.4
Phase 1 (minimizing losses, no adjustable shunts)	Total	MW	1077.7	461.2	1281.5	579.5	1729.9	939.3	1569.6	836.2
		MVAr	16933.6	7528	18749	8509.9	27459.8	15948.3	25723	14802.2
	Difference from spf	MW	-3.64%	-3.90%	-3.89%	-4.02%	-1.22%	3.38%	-1.25%	3.15%
		MVAr	-5.12%	-3.60%	-4.93%	-3.88%	-3.69%	-0.46%	-3.64%	-0.94%
Phase 2 (fixed tap positions, adjustable shunts allowed)	Total	MW	1054.9	447.1	1248.5	566.8	1683.8	907.3	1534.1	809.3
		MVAr	16534	7217	18279	8257.2	26957.3	15591.6	25301.3	14470.7
	Difference from spf	MW	-5.68%	-6.83%	-6.37%	-6.13%	-3.85%	-0.14%	-3.49%	-0.17%
		MVAr	-7.36%	-7.58%	-7.31%	-6.73%	-5.46%	-2.69%	-5.22%	-3.16%

The total system losses data before and after the OPF are shown in Table F-5 for a single case file (winter max load 2009 – Generation Dispatch pattern 2). From this data the first phase of the OPF improves the losses by about 4% across the whole range of contingencies and the second phase by about 5.7%.

This is a significant loss saving and amounts to a saving of ZAR 334 million per year (assuming a Long Run Marginal Cost (LRMC) of generation of ZAR1/kWh at an estimated loss load factor of 0.6).

Real and reactive loss savings reduce after Transmission contingencies however the OPF is still able to find a lower loss solution during these contingencies. In addition, Transmission contingencies are temporary phenomena and should not necessarily determine optimal system set up under normal conditions

This is the best case loss reduction but practically speaking there are other operational factors which may contribute to losses which are not modelled in PSS/E or taken into account [6], [13], [14].

Furthermore the PSS/E OPF that is run on the case file is security unconstrained, without for example enforced spare generating VAr capacity for system contingencies and the maintenance of system stability after a system event. If a SCOPF is run on the system, it has been shown that there would still be a significant loss saving although decreased [22].

A feasible optimisation methodology for Eskom in the absence of SCOPF could be to run an OPF to achieve specified objectives (e.g. reduce taps and losses) and overlay this OPF solution with known operational requirements to achieve both objectives of loss savings/reduced taps and operational requirements.

The loss reduction data tables for the other 2009 and 2015 scenarios can be found in Appendix B.

Table F-5: Winter max loading case (2009) generation pattern 2: losses before and after OPF

OPF			Standard power flow	Phase 1 (minimizing losses, no adjustable shunts)			Phase 2 (fixed tap positions, adjustable shunts allowed)		
Losses		Unit	Total	Total	Difference from base case		Total	Difference from base case	
Base		MW	1333.4	1281.5	-51.9	-3.89%	1248.5	-84.9	-6.37%
		MVAr	19720.5	18749	-971.7	-4.93%	18279	-1441.2	-7.31%
Contingency	1	MW		1305.3	-28.1	-2.11%	1272.4	-61	-4.57%
		MVAr		18953	-767.3	-3.89%	18478	-1242.8	-6.30%
	2	MW		1291.9	-41.5	-3.11%	1253.6	-79.8	-5.98%
		MVAr		18863	-857.2	-4.35%	18358	-1362.1	-6.91%
	3	MW		1284.6	-48.8	-3.66%	1251.7	-81.7	-6.13%
		MVAr		18769	-951.7	-4.83%	18300	-1420.2	-7.20%
	4	MW		1421.2	87.8	6.58%	1369.7	36.3	2.72%

		MVAr		19643	-77.8	-0.39%	18990	-730.1	-3.70%
	5	MW		1337.9	4.5	0.34%	1301.1	-32.3	-2.42%
		MVAr		19234	-486.5	-2.47%	18732	-988.6	-5.01%

F.3 Switched shunts

Table F-6 records a selection of the switchable shunt values for the 2009 and 2015 case files. A full table can be found in Appendix B.

For each file, the table records the base value and the standard deviation between this base value and the average step value after the first stage of the OPF (for all the base and contingency situations chosen for investigation).

From this table, the values in red indicate shunts where the inductance/capacitance values required and implemented in the OPF are far from the base value or next shunt step on that bus. It is at these buses where an additional shunt or shunt step size could be advisable.

Switched shunts on the Eskom Transmission system are in general correctly sized however there are some instances where increments of switched shunt sizes are required from the OPF analysis.

The sizing and application of switched shunts is largely academic in Phase 2 of the studies where adjustable shunt devices are applied in order to achieve the research explicit objective of minimising system losses.

Table F-6: Switched shunt values for 2009 and 2015

		2009								2015							
		g1				g2				g1				g2			
		winter		summer		winter		summer		winter		summer		winter		summer	
		Binit (MVAr)				Binit (MVAr)				Binit (MVAr)				Binit (MVAr)			
Bus#	Name	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step	Base	Std dev b/n avg and step
265	UMFLZ4	0	4.50	0	6.48	0	3.69	0	4.95	0	0.04	-100	0.98	0	0.25	-100	1.01
267	ATHEN4	150	1.10	150	0.70	150	0.91	150	0.45	150	0.05	0	0.58	150	0.30	0	0.53
280	NORMN4	0	17.98	0	51.02	0	13.83	0	43.43	0	0.04	0	0.11	0	0.28	-100	1.44
710	DELPH4	0	0.06	-100	0.12	0	0.03	-100	0.22	-400	10.54	-400	0.24	-400	4.60	-400	0.21
850	PHOKJ4	-100	1.64	-100	0.17	-100	1.16	-100	0.12	0	0.02	0	0.08	0	2.69	0	0.08
1158	AGGNS22A	-30	0.00	-30	0.00	-30	2.04	-30	1.28								
1241	SENAKNG2	0		0		0		0		72	0.05	0	0.57	72	3.67	0	0.64
1275	KLAARW	144	1.40	144	62.33	144	1.99	144	14.92	144	0.04	144	1.03	144	0.51	144	1.13
1425	ILLOV2	100	0.59	0	0.01	100	0.38	0	0.00	100	0.09	0	0.18	100	0.60	0	0.18
1440	MERSY2	300	21.19	0	0.01	300	18.01	0	0.00	300	0.06	0	0.16	300	0.32	0	0.15
1445	AVON2	0		0		0		0		150	0.05	0	0.15	150	0.35	0	0.15
2480	GRASS1	144	0.08	-30	0.16	144	7.92	-30	0.23	180	4.37	180	1.04	180	19.27	180	1.10
2490	GRGDL1	0	0.04	0	0.01	0	0.04	0	0.00	66	0.19	66	1.05	66	0.65	66	0.96
2790	LNDER1	0	1.00	0	0.01	0	9.89	0	0.01	0	2.12	0	0.06	0	3.10	0	0.04
2853	MERC132	0	0.22	0	0.02	0	0.36	0	0.02	0	10.39	0	0.03	0	22.51	0	0.03
71154	LOUISTR1	12	3.73	12	1.48	12	4.41	12	0.56								
71172	MAPOCH1	18	0.03	18	0.10	18	0.07	18	0.34	18	0.05	18	0.37	18	1.50	18	0.34
71188	MESSINA1	36	1.02	0	0.22	36	3.24	0	0.25	18	0.01	18	0.73	18	0.37	18	0.61

F.4 Fixed and adjustable shunts

A selection of the fixed and adjustable shunts in the 2009 and 2015 case files (for both loading and generation patterns) is listed in Table F-7. A full list is given in Appendix B.

The initial values of the shunts as well as the mean values of the adjustable shunts after Stage 2 of the OPF simulation are recorded in this table. The mean value is calculated without the statistical outliers for the base case and all the contingencies chosen for investigation.

There are significant changes in the adjustable shunts between different loading and generation patterns because the OPF cost setting associated with the adjustable shunts is set to zero (0) [1]. Adjustable shunts move without restriction while the taps are kept relatively constant.

Some of these shunts swing from being a large capacitor to a large inductor between maximum and minimum loading. This is considered acceptable as the OPF attempts to find the best MW and MVAr loss solution.

As mentioned in Section F.2, the OPF simulation has no security constraints so will differ in places to the system real-time snapshot.

However, as mentioned in the introduction to this dissertation and elsewhere, a cost-efficient optimal operational system configuration is required as a basis for the research. In addition, a wide range of system conditions are simulated which attempt to cover the security constrained condition. The author is of the opinion that the results from the OPF analysis are still valid and that the adjustable shunt conclusions could be very similar to conclusions obtained from a SCOPF [22].

A standard deviation is calculated between the additional VARs required by the OPF for the base case and all the contingency cases. The standard deviation data is formatted conditionally so that the shunts which are affected directly by the contingencies are coloured red and easily visible.

Table F-9lists a selection of busbars that requires significantly increased reactive device sizes. This conclusion is drawn from the OPF process. Further, Table F-8 lists the shunts required by the OPF that have significant fluctuations in their required value for different loading values and generation dispatches.

Table F-7: A selection of adjustable and fixed shunts for 2009 and 2015

			2009								2015							
			g1				g2				g1				g2			
Bus #	Name	kV	winter		summer		winter		summer		winter		summer		winter		summer	
			SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean
335	PERSS4	400	-200	-548.52	-300	-46.53	-200	-450.7133	-300	144.46	-300	214.46667	-300	-29.27	-300	97.583333	-300	46.68
337	BETA4	400	0	138.77667	0	-26.68	200	14.883333	0	30.98	0	-399.17	0	-598.35	0	-399.3	0	-595.54
375	HYDRA4	400	-200	-513.6433	-200	-622.86	-100	-789.5967	-200	-802.35	100	-395.37	0	-341.70	-300	-3.033333	-300	-91.20
380	POSDN4	400									0	-72.92667	0	-191.72	0	-65.84	0	-190.99
381	POSSVC4	30	100	-60.86667	0	-32.16	100	-33.80333	0	-30.77	0	35.326667		77.08	0	33.253333	0	79.77
381	POSSVC4	30		50		17.84		16.2		19.23	0	85.326667	0	27.08	0	83.253333		29.77
400	KRONS4	400	-100	174.68	-100	127.35	-100	257.51333	-100	197.76	0	-36.46	-100	42.76	0	-41.77	-100	31.25
410	ARIES4	400	-200	4.3433333	-200	1.82	-100	-68.25667	-100	-85.22	0	-193.8733	-100	-194.55	-200	-52.99667	-200	-139.87
425	HELS4	400	-200	54.613333	-100	-57.88	-200	79.49	-100	-48.39								
435	JUNO4	400	0	-156.8467	-100	-70.01	0	-126.4933	-100	-59.96	-100	-68.42	-200	-12.96	-200	4.71	-200	-37.74
445	AUROR4	400	0	94.956667	0	135.13	0	116.72333	0	110.68	0	132.30667	-110	155.58	0	144.83667	-110	185.69
460	MULDR4	400	0	18.696667	0	-274.59	0	-28.06667	0	-269.62	100	21.39	100	-64.46	0	100.33333	0	33.58
465	BACCH4	400	0	-140.7233	-100	-196.84	0	-123.9167	-100	-166.32	-100	-61.77	-100	-70.11	-100	-85.52333	-100	-87.39
505	DROER4	400	0	68.926667	-200	-2.25	0	521.15667	-200	251.92	-100	-128.9767	-200	-75.77	-100	-169.9167	-200	-112.09
850	PHOKJ4	400	-100	64.186667	-100	0.62	-100	72.803333	-100	-2.77	0	289.04	0	114.84	0	289.47	0	114.87
1005	ARART2	275	150	-107.1433	0	2.92	150	-113.8	0	-5.25	150	-42.67	0	-28.47	150	-48.76	0	-28.24
1015	BIGHR2	275	150	27.423333	0	108	150	37.89	0	124.8	150	-185.86	0	-157.42	150	-183.8767	0	-163.70
1020	PLUTO2	275	300	-180.5667	0	-153.08	300	-193.1167	0	-134.63	156	-36.16333	0	-73.61	156	-43.79667	0	-76.04

Table F-8: Extract of additional shunts required by the OPF that have large fluctuations

			2009								2015							
			g1				g2				g1				g2			
Bus																		
#	Name	kV	winter		summer		winter		summer		winter		summer		winter		summer	
			SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean
85	BETA7	765	0.00	485.19	0.00	-595.95	0.00	1024.89	0.00	-336.61	0.00	196.44	-400.00	325.80	0.00	144.75	-400.00	326.01
80	ALPHA7	765	0.00	607.03	0.00	-177.40	0.00	785.92	0.00	-114.87	0.00	391.54	0.00	125.04	0.00	353.96	0.00	78.87
55	GAMMA7	765									-800.00	242.52	-800.00	-269.84	-800.00	-2.21	-800.00	-441.64
337	BETA4	400	0.00	138.78	0.00	-26.68	200.00	14.88	0.00	30.98	0.00	-399.17	0.00	-598.35	0.00	-399.30	0.00	-595.54
375	HYDRA4	400	-200.00	-513.64	- 200.00	-622.86	-100.00	-789.60	- 200.00	-802.35	100.00	-395.37	0.00	-341.70	-300.00	-3.03	-300.00	-91.20
335	PERSS4	400	-200.00	-548.52	- 300.00	-46.53	-200.00	-450.71	- 300.00	144.46	-300.00	214.47	-300.00	-29.27	-300.00	97.58	-300.00	46.68
505	DROER4	400	0.00	68.93	- 200.00	-2.25	0.00	521.16	- 200.00	251.92	-100.00	-128.98	-200.00	-75.77	-100.00	-169.92	-200.00	-112.09
630	FERRM4	400									0.00	300.00		-55.49	0.00	300.00		-116.45
1440	MERSY2	275	300.00	-390.59	0.00	-187.84	300.00	-392.04	0.00	-187.80	300.00	37.71	0.00	58.53	300.00	15.85	0.00	50.56
70	OMEGA7	765									0.00	-184.83	-400.00	122.07	0.00	-262.40	-400.00	80.47
48	ARDN7	765									0.00	85.06	0.00	-236.69	0.00	57.09	0.00	-235.10
1075	JUPTR2	275	300.00	321.66	0.00	39.01	300.00	299.10	0.00	59.57	300.00	349.02	0.00	495.13	300.00	370.76	0.00	480.80
135	MERCR4	400	100.00	-124.33	0.00	-160.58	100.00	-111.61	0.00	-146.49	200.00	111.41	100.00	-318.23	200.00	52.66	100.00	-350.75
280	NORMN4	400	0.00	-193.13	0.00	-147.51	0.00	-205.78	0.00	-128.83	0.00	114.09	0.00	72.98	0.00	102.70	-100.00	169.74

Table F-9: Extract of busbars that need extra VARs (2009 and 2015)

			2009								2015							
			g1				g2				g1				g2			
Bus #	Name	kV	winter		summer		winter		summer		winter		summer		winter		summer	
			SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean	SPF Base	OPF additional VARS - Trim mean
375	HYDRA4	400	-200.00	-513.64	-200.00	-622.86	-100.00	-789.60	-200.00	-802.35	100.00	-395.37	0.00	-341.70	-300.00	-3.03	-300.00	-91.20
85	BETA7	765	0.00	485.19	0.00	-595.95	0.00	1024.89	0.00	-336.61	0.00	196.44	-400.00	325.80	0.00	144.75	-400.00	326.01
80	ALPHA7	765	0.00	607.03	0.00	-177.40	0.00	785.92	0.00	-114.87	0.00	391.54	0.00	125.04	0.00	353.96	0.00	78.87
1075	JUPTR2	275	300.00	321.66	0.00	39.01	300.00	299.10	0.00	59.57	300.00	349.02	0.00	495.13	300.00	370.76	0.00	480.80
337	BETA4	400	0.00	138.78	0.00	-26.68	200.00	14.88	0.00	30.98	0.00	-399.17	0.00	-598.35	0.00	-399.30	0.00	-595.54
1676	APOCN2	275	1168.00	-428.36	996.00	-255.99	1168.00	-410.23	996.00	-157.81	1449.00	-95.77	1113.00	-256.81	1449.00	-96.97	1044.00	-213.08
65	ZEUS7	765									-400.00	399.77	0.00	596.04	-400.00	399.65	0.00	415.22
335	PERSS4	400	-200.00	-548.52	-300.00	-46.53	-200.00	-450.71	-300.00	144.46	-300.00	214.47	-300.00	-29.27	-300.00	97.58	-300.00	46.68
40	GRASS7	765									0.00	277.81	-400.00	472.88	0.00	283.93	-400.00	481.22
260	PEGSS4	400	0.00	294.67	-100.00	254.02	0.00	274.93	-100.00	250.64	0.00	64.70	-100.00	97.67	0.00	83.25	-100.00	94.01
135	MERCR4	400	100.00	-124.33	0.00	-160.58	100.00	-111.61	0.00	-146.49	200.00	111.41	100.00	-318.23	200.00	52.66	100.00	-350.75
2065	BERNN1	132	72.00	-13.63	72.00	-50.01	72.00	-26.42	72.00	-24.69	72.00	-244.40	72.00	-383.97	72.00	-238.26	72.00	-352.00
505	DROER4	400	0.00	68.93	-200.00	-2.25	0.00	521.16	-200.00	251.92	-100.00	-128.98	-200.00	-75.77	-100.00	-169.92	-200.00	-112.09
1440	MERSY2	275	300.00	-390.59	0.00	-187.84	300.00	-392.04	0.00	-187.80	300.00	37.71	0.00	58.53	300.00	15.85	0.00	50.56

F.5 Bus voltages

Table F-10 lists the per unit voltage levels on a selection of the 400 kV buses in the system. In this table, the voltages on the buses before and after the OPF runs are listed for the 2009 and 2015 case files. A full list of buses can be found in the Appendix B.

The voltages after the first stage of the OPF are also listed in this table as well as basic statistical analysis for ease of identification of the buses which have experienced the most significant voltage magnitude shift.

From Table F-10 it is evident that the second phase of the OPF works to increase the voltage above its nominal value or towards its upper limit (normally 1.05 p.u. unless specified by a contract voltage).

The load is modelled as constant power which explains the relationship between increased voltages and decreased losses. The load, as seen by the Transmission system is constant power despite the Distribution load having a large proportion of constant impedance characteristics because this load is modelled beyond the Transmission transformers [26].

While the research ensures that busbar voltage limits are maintained and that generator VAr limits are maintained, this dissertation does not analyse voltage stability (VS) or loadability in detail. The optimised file should be checked for voltage stability using VS simulation tools [23]. This is a possible avenue for future work.

Table F-10: A selection of the 765 kV and 400 kV bus voltages before and after the OPF analysis

			2009								2015							
			G1				g2				G1				g2			
			Max 2009		Min 2009		Max 2009		Min 2009		Max 2015		Min 2015		Max 2015		Min 2015	
Voltages (p.u.)			SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2	SPF	OPF phase 2
Bus			Base	Avg	Base	Avg	Base	Avg	Base	Avg	Base	Avg	Base	Avg	Base	Avg	Base	Avg
Bus#	Name	kV																
75	HYDRA7	765	0.98	1.00	1.01	1.00	0.96	0.99	0.99	1.00	1.03	1.03	1.02	1.02	1.03	1.03	1.03	1.02
80	ALPHA7	765	0.98	1.01	0.97	1.00	0.97	1.02	0.96	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.01
85	BETA7	765	0.97	1.01	1.02	1.01	0.95	1.02	1.00	1.02	1.01	1.01	1.02	1.01	1.02	1.01	1.03	1.01
23	HDSC4	400	0.96	0.97	1.03	1.03	0.96	0.96	1.02	1.03	0.98	0.97	0.99	0.98	0.97	0.98	0.99	0.98
60	STIKL4	400	1.02	1.00	1.01	1.02	1.02	0.99	1.01	1.01	1.03	1.02	1.03	1.03	1.02	1.02	1.03	1.02
90	LESED4	400	1.01	1.00	0.99	1.01	1.01	1.00	0.99	1.01	1.00	1.01	1.02	1.01	1.00	1.01	1.02	1.01
100	GRTVL4	400	1.01	1.04	1.03	1.05	1.01	1.04	1.02	1.05	1.02	1.03	1.05	1.03	1.03	1.03	1.05	1.03
101	DINALD4	400	0.99	1.01	1.00	1.03	1.00	1.01	1.00	1.03	0.94	1.00	1.04	1.00	0.94	1.00	1.04	1.00
102	MARANG4	400	1.01	1.02	1.01	1.03	1.01	1.02	1.01	1.04	0.93	0.98	1.01	0.98	0.93	0.98	1.01	0.98
103	LULAM4	400	0.97	1.00	0.99	1.03	0.97	1.00	0.99	1.03	0.99	1.01	1.01	1.01	0.99	1.01	1.01	1.01
110	GLOCK4	400	1.01	1.03	1.03	1.05	1.02	1.03	1.03	1.05	1.01	1.00	1.03	1.00	1.01	1.00	1.03	1.00
115	HERA4	400	0.98	1.00	1.01	1.03	0.98	1.00	1.01	1.03	0.97	0.97	1.01	0.97	0.97	0.97	1.01	0.97
120	MIDAS4	400	0.99	1.01	1.01	1.03	0.99	1.01	1.01	1.03	0.97	0.99	1.02	0.98	0.98	0.99	1.02	0.98
125	PLUTO4	400	1.00	1.01	1.01	1.03	1.00	1.01	1.01	1.04	0.98	1.00	1.03	1.00	0.98	1.00	1.03	1.00
130	HERMS4	400	1.00	1.01	1.01	1.03	1.00	1.01	1.00	1.03	0.99	1.01	1.04	0.99	1.00	1.01	1.04	0.99
135	MERCR4	400	1.00	1.01	1.01	1.03	1.00	1.01	1.00	1.02	1.00	1.01	1.04	0.99	1.00	1.01	1.04	0.99
140	MATMB4	400	1.03	1.01	1.02	1.05	1.03	1.02	1.02	1.05	1.02	1.03	1.03	1.01	1.02	1.03	1.03	1.01
150	WITKP4	400	0.99	0.97	1.01	1.03	0.99	0.98	1.01	1.04	0.98	1.00	1.02	1.00	0.98	1.00	1.02	1.00
155	MERNS4	400	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.01	1.00	1.00	1.02	1.01	1.00	1.00	1.02	1.01

F.6 Generator MVAr outputs

A selection of the generator MVAr outputs are recorded in Table F-11. A full list is found in Appendix B. The average MVAr output of the base case and all the contingencies after the OPF are listed for each generator. The detailed table in Appendix B shows, in addition, the MVAr output after phase 1 of the OPF.

Ideally the generators should operate far away from their MVAr import/export limits (or close to zero MVArs export/import). In the ideal case, the reactive reserve of the generators is maximized leading to a more secure system that can quickly respond to increased/ decreased MVAr demands following outages on the system.

Maximizing generator reactive reserve, by keeping the generator MVAr import/export level as far from its limits as possible under different operating conditions, would be a good operational objective for the OPF function. However limitations in the PSS/E software make this unfeasible at present.

Currently this objective in PSS/E - *minimise reactive generation reserve* [1], takes the MVAr outputs of the generators close to their minimum limit, and so is not used or is not appropriate for this research.

Maximising generator reactive reserve could provide an opportunity for future research.

In general, generator VArs do not deviate significantly from their SPF levels in the research.

Table F-11: Extract of generator MVAr output (2009 and 2015) before and after OPF

bus			2009								2015							
			g1				g2				g1				g2			
			winter		summer		winter		summer		winter		summer		winter		summer	
			SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.	SPF Qgen (MVAr)	OPF phase 2 Qgen (MVAr) Avg.
#	Name	kV																
9530	MATMB_ 1	20	97.57	35.67	0.00	0.00	98.93	61.60	0.00	0.00	299.85	189.50	299.86	299.86	294.11	184.66	294.11	294.11
9535	MATMB_ 2	20	98.11	36.66	1.81	6.62	99.51	63.26	-0.44	-4.27	305.58	193.31	95.02	76.57	299.70	187.78	95.63	77.11
9540	MATMB_ 3	20	98.38	36.99	1.23	5.76	99.79	63.82	-0.95	-5.10	307.51	194.51	95.27	77.23	301.58	188.84	95.88	77.77
9545	MATMB_ 4	20	1.00	1.00	1.30	5.67	98.72	63.38	-0.90	-5.12	305.58	193.31	95.02	76.57	299.70	187.78	95.63	77.11
9550	MATMB_ 5	20	98.76	36.68	1.22	5.52	100.16	63.18	-0.97	-5.26	305.58	193.31	95.02	76.57	299.70	187.78	95.63	77.11
9555	MATMB_ 6	20	98.17	36.99	1.41	6.09	99.58	63.85	-0.79	-4.81	307.51	194.51	95.27	77.23	301.58	188.84	95.88	77.77
9560	ARNOT_1	15	34.98	40.84	40.11	-34.58	-4.11	23.99	46.05	-55.51	77.77	51.22	-1.05	21.25	78.57	48.86	5.04	22.96
9565	ARNOT_2	15	38.26	41.02	40.01	-34.74	-0.88	25.81	45.95	-55.68	92.38	46.74	92.38	92.38	93.17	47.75	93.17	93.17
9570	ARNOT_3	15	135.08	60.82	35.91	-4.86	123.20	59.75	41.06	-7.77	68.40	21.98	-3.35	-1.34	65.56	65.56	-32.27	-0.88

APPENDIX G: Explanation of SPF and OPF [2]

OPTIMAL POWER FLOW ALGORITHMS

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1 PROBLEM DEFINITION

1.1 Optimal power flow problem

1.1.1 The ordinary power flow

The ordinary power flow or load flow problem is stated by specifying the loads in megawatts and megavars to be supplied at certain nodes or busbars of a transmission system and by the generated powers and the voltage magnitudes at the remaining nodes of this system together with a complete topological description of the system including its impedances. The objective is to determine the complex nodal voltages from which all other quantities like line flows, currents and losses can be derived. The model of the transmission system is given in complex quantities since an alternating current system is assumed to generate and supply the powers and loads.

In mathematical terms the problem can be reduced to a set of nonlinear equations where the real and imaginary components of the nodal voltages are the variables. The number of equations equals twice the number of nodes. The nonlinearities can roughly be classified being of a quadratic nature. Gradient and relaxation techniques are the only methods for the solution of these systems.

The result of a power flow problem tells the operator or a planner of a system in which way the lines in the system are loaded, what the voltages at the various buses are, how much of the generated power is lost and where limits are exceeded.

The power flow problem is one of the basic problems in which both load powers and generator powers are given or fixed. Today, this basic problem can be efficiently handled on the computer for practically any size system.

1.1.2 The optimal power flow

For the planner and operator fixed generation corresponds to a snapshot only. Planning and operating requirements very often ask for an adjustment of the generated powers according to certain criteria. One of the obvious ones is the minimum of the generating cost. The application of such a criterion immediately assumes variable input powers and bus voltages which have to be determined in such a way that a minimum of the cost of generating these powers is achieved.

At this point it is not only the voltages at nodes where the loads are supplied but also the input powers together with the corresponding voltages at the generator nodes which have to be determined. The degree of freedom for the choice of inputs seems to be exceedingly large, but due to the presence of an objective, namely to reach the minimum of the generating cost the problem is well defined. Of course the mathematics become more demanding as compared to the original power flow problem, however, the aim still being the same, i.e. the determination of the nodal voltages in the system. They play the role of state variables from which all other quantities can be derived.

It turns out that the extended problem requires a more detailed definition and different methods of solution.

The problem can be generalized by attaching different objectives to the original power flow problem. As long as the power flow model stays the same it is considered the optimal power flow problem where the objective is a scalar function of the state variables. In essence, any optimal power flow problem can be reduced to such a form.

Now, practical requirements ask for a more realistic definition, the main addition being the statement of constraints. In the real world any variable in the system will be limited which changes the mathematical nature of the problem drastically. Whenever a variable reaches its upper or lower limit it becomes a fixed quantity and the method of solution has to recognize it as such and be sure that the fixed quantity is optimal.

Fortunately, the theory developed by Kuhn and Tucker [1] is able to provide the optimality conditions which guarantee the correctness of the result in the end. However, the optimality conditions do not offer a solution method.

Present requirements are aimed at solution methods suitable for computer implementations which are easy to handle, capable of large systems, have good convergence and are fast. Experience shows that the performance of solution methods in the power system analysis area are dependent on the nature of the system model, on the type of nonlinearities, on the type of constraints, on the number of constraints, etc.

Thus, the basic theory of optimization contribute a small part to the success of a solution method only. It is the genius of the system analyst and of the computer scientist which becomes the key factor for the success of a method.

Optimal power flow algorithms are the outcome of development work of this kind and are determinant for the performance of whole classes of programs. Hence it is worthwhile and quite rewarding to engage in the investigation of algorithms within this problem class.

Scanning through the literature [2], [3], [4], [5], [7], [9] it will be observed that there are many attempts to describe, define, formulate and solve the optimal power flow problem. However, it seems that successful solutions emerged only at the point where proven schemes of optimization such as linear and quadratic programming could be applied to this very problem [8], [10], [11]. This late development was supported by other techniques which proved useful in the area of the ordinary power flow such as the exploitation of sparsity and Newtons's method.

Thus, in the subsequent sections great emphasis will be placed on a thorough formulation of the optimal power flow problem and on techniques which lend themselves to an application of proven optimization methods.

1.2 Power flow simulation of an electrical power transmission system

This subsection discusses briefly the basics for the simulation of an electrical power transmission system on a digital computer. More information can be obtained from many textbooks which discuss the basic power flow problem in more detail.

1.2.1 Nodal current - nodal voltage relationship

The relation between the complex nodal voltages $\underline{\mathbf{V}}$ and the complex nodal currents $\underline{\mathbf{I}}$ of the transmission network, composed of the passive components, transmission lines, series elements, transformers and shunts is:

$$\underline{\mathbf{I}} = \underline{\mathbf{Y}} \cdot \underline{\mathbf{V}} \tag{1}$$

Every complex nodal current $\underline{I}_i$ can be formulated in rectangular coordinates:

$$\underline{I}_i = I_{e_i} + j \cdot I_{f_i} \ \ ; \text{i} = 1...N; \ N = \text{number of electrical nodes} \tag{2}$$

For every complex nodal voltage $\underline{V}_i$, the following is valid in rectangular coordinates for the complex nodal voltage:

$$\underline{V}_i = e_i + j \cdot f_i \ \ ; \text{i} = 1...N \tag{3}$$

Note that usually at one node the angle of the complex voltage is held constant. Thus the following relationship must be valid for this one node, called the slack node:

$$\frac{f_{slack}}{e_{slack}} = k_{slack} = constant \tag{4}$$

Note that very often this constant value k_{slack} is assumed to be zero, i.e. the voltage angle at this node is assumed to be zero. However, in this paper the general case of (4) is assumed to be valid.

The complex elements at row i and column j of the matrix $\underline{\mathbf{Y}}$ are as follows:

$$\underline{Y}_{ij} = g_{ij} + j \cdot b_{ij} \tag{5}$$

or in polar form

$$\underline{Y}_{ij} = y_{ij} \cdot (cos\theta_{ij} + j \cdot sin\theta_{ij}) \tag{6}$$

It follows from (1), (2) and (5)

$$I_{e_i} = \sum_{j=1}^{N} (e_j g_{ij} - f_j b_{ij}) \ \ ; \text{i} = 1...N \tag{7}$$

$$I_{f_i} = \sum_{j=1}^{N} (e_j b_{ij} + f_j g_{ij}) \ \ ; \text{i} = 1...N \tag{8}$$

In polar coordinates the complex voltages $\underline{V}_i$ are defined as follows:

$$\underline{V}_i = |V|_i \cdot (cos\Theta + j \cdot sin\Theta) \ \ ; \text{i} = 1...N \tag{9}$$

As defined in (4), the voltage angle at the so-called slack node is kept fixed:

$$\Theta_{slack} = \arctan(k_{slack}) = constant \tag{10}$$

It should be noted that other network components like DC-transmission lines are not included in this paper. Balanced three-phase network operation is assumed.

1.2.2 Nodal power nodal voltage - nodal current relationship

In this paper in order to make certain derivations easier to understand, the following assumptions are made with respect to node numbering:

- The network has a total of N electrical nodes
- The l load PQ-nodes are numbered $1...l$
- The m generator PV-nodes are numbered $(l+1)...(l+m)$
- $l+m$=N
- The last generator node is called the slack node (i.e. the slack node number is N).

Note that the above mentioned slack node is usually treated as a normal PV-generator bus with the additional constraint of a fixed voltage angle (see (4) and (10)).

The active and reactive powers of all l PQ-load-nodes must be computed by the following relationship:

$$P_i = Real\{\underline{V}_i \cdot \underline{I}_i^*\} \;\; ; \text{i} = 1...l \tag{11}$$

$$Q_i = Imag\{\underline{V}_i \cdot \underline{I}_i^*\} \;\; ; \text{i} = 1...l \tag{12}$$

(11), (12) formulated in **rectangular coordinates**:
For all l PQ-nodes:

$$P_i = e_i I_{e_i} + f_i I_{f_i} \;\; ; \text{i} = 1...l \tag{13}$$

$$Q_i = f_i I_{e_i} - e_i I_{f_i} \;\; ; \text{i} = 1...l \tag{14}$$

For all m PV-nodes:

$$P_i = e_i I_{e_i} + f_i I_{f_i} \;\; ; \text{i} = l+1...N \tag{15}$$

$$|V|_i^2 = e_i^2 + f_i^2 \;\; ; \text{i} = l + 1 ... N \tag{16}$$

Inserting (7) and (8) into (13) and (14) yields:

$$P_i = \sum_{j=1}^{N} (e_i(e_j g_{ij} - f_j b_{ij}) + f_i(f_j g_{ij} + e_j b_{ij})) \;\; ; \text{i} = 1 ... l \tag{17}$$

$$Q_i = \sum_{j=1}^{N} (f_i(e_j g_{ij} - f_j b_{ij}) - e_i(f_j g_{ij} + e_j b_{ij})) \;\; ; \text{i} = 1 ... l \tag{18}$$

For the generator PV-nodes the active power P and the voltage magnitude are computed as follows:

$$P_i = \sum_{j=1}^{N} (e_i(e_j g_{ij} - f_j b_{ij}) + f_i(f_j g_{ij} + e_j b_{ij})) \;\; ; \text{i} = l + 1 ... N \tag{19}$$

$$|V|_i^2 = e_i^2 + f_i^2 \;\; ; \text{i} = l + 1 ... N \tag{20}$$

(11), (12) formulated in **polar coordinates**:
For all l PQ nodes:

$$P_i = \sum_{j=1}^{N} (V_i V_j y_{ij} cos(\Theta_i - \Theta_j - \theta_{ij})) \;\; ; \text{i} = 1 ... l \tag{21}$$

$$Q_i = \sum_{j=1}^{N} (V_i V_j y_{ij} sin(\Theta_i - \Theta_j - \theta_{ij})) \;\; ; \text{i} = 1 ... l \tag{22}$$

For all m PV nodes (inclusive slack node):

$$P_i = \sum_{j=1}^{N} (V_i V_j y_{ij} cos(\Theta_i - \Theta_j - \theta_{ij})) \;\; ; \text{i = l+1 ... N} \tag{23}$$

$$|V|_i = V_i \;\; ; \text{i = l+1 ... N} \tag{24}$$

Note that (24) is trivial and in principle not necessary. The equations of (24) are omitted in the following derivations when using the polar coordinate system.

1.2.3 Operational limits

In the real power system many of the variables used in the above equations are limited and may not be exceeded without damaging equipment or bringing the network into unstable, insecure operating states:

- Limits on active power of a (generator) PV node:

$$P_{low_i} \leq P_{PV_i} \leq P_{high_i} \tag{25}$$

- Limits on voltage of a PV or PQ node:

$$|V|_{low_i} \leq |V|_i \leq |V|_{high_i} \tag{26}$$

- Limits on tap positions of a transformer

$$t_{low_i} \leq t_i \leq t_{high_i} \tag{27}$$

- Limits on phase shift angles of a transformer

$$\theta_{low_i} \leq \theta_i \leq \theta_{high_i} \tag{28}$$

- Limits on shunt capacitances or reactances

$$s_{low_i} \leq s_i \leq s_{high_i} \tag{29}$$

- Limits on reactive power generation of a PV node

$$Q_{low_i} \leq Q_{PV_i} \leq Q_{high_i} \tag{30}$$

  In reality the reactive limits on a generator are complex and usually state dependent. (30) is a simplification of the limits, however, by adapting the actual limit values during the optimization, the real-world limits can be simulated with sufficient accuracy.

- Upper limits on active power flow in transmission lines or transformers:

$$P_{ij} \leq P_{high_{ij}} \tag{31}$$

- Upper limits on MVA flows in transmission lines or transformers

$$P_{ij}^2 + Q_{ij}^2 \leq S_{high_{ij}}^2 \tag{32}$$

- Upper limits on current magnitudes in transmission lines or transformers

$$|I|_{ij} \leq |I|_{high_{ij}} \tag{33}$$

- Limits on voltage angles between nodes:

$$\Theta_{low_{ij}} \leq \Theta_i - \Theta_j \leq \Theta_{high_{ij}} \tag{34}$$

- Limits on total flows between areas

  These inequality constraints can be formulated for MVA-, and MW-values as follows:

  - Limits on active power area flows

$$P_{low_{area_a}} \leq \sum_{\text{a to b}} P_{ab} \leq P_{high_{area_a}} \tag{35}$$

  - Limits on MVA area power flows

$$S_{low_{area_a}}^2 \leq \sum_{\text{a to b}} (P_{ab}^2 + Q_{ab}^2) \leq S_{high_{area_a}}^2 \tag{36}$$

1.2.4 Summary

It is an essential goal of the network operator to have all of above mentioned inequality constraints, representing real world operating limits, under control. The power demand which must be in balance with the generation is automatically considered in the real system. Any simulation, i.e. also the OPF, must consider this equality constraint unconditionally in order to simulate the real power system correctly.

It must be noted that not in all networks all these constraints have the same degree of importance. However, in general, and this is assumed in the formulations of this paper, all these constraints have to be satisfied. Thus, any electrical network simulation result, also the one of an OPF simulation, should observe the above operational limits in its final result.

The mathematical model must always consider the equations (1), (11) and (12), i.e. the relation between nodal voltages, currents and nodal powers must be considered correctly.

It is the goal of the OPF to simulate the state of the real power system which satisfies all of the above constraints and at the same time minimizes a given objective, e.g. network losses or generation cost.

1.3 Formulation of OPF constraints

1.3.1 Variable classification

The process of solving the (optimal) power flow problem is easier to understand if the variables are classified in several categories. They are shown in the following.

- Demand variables: They include the variables representing constant values. Demand variables are represented by the vector $\mathcal{P}$. The final simulation result must leave these variables unmodified. Typical demand variables:
  - Active power at load nodes
  - Reactive power at load nodes
  - In general all those variables which could be control variables (see below) but are not allowed to move (for operational or other reasons). Example: Voltage magnitude of a PV node where the voltage is not allowed to move
- Control variables: All real world quantities which can be modified to satisfy the load - generation balance under consideration of the operational system limits (see previous subsection). Since, especially when using the rectangular coordinate system, not all these quantities can be modelled directly, they have to be transformed into variables with purely mathematical meaning. After the computation these variables can, however, be transformed back into the real world quantities. Control variables are represented by the vector $\mathcal{U}$.

  A typical set of control variables of an OPF problem can include:
  - Rectangular Coordinates:
    * Active power of a PV node
    * Reactive power generation at a PV node (sometimes used)
    * Tap position of a transformer
    * Shunt capacitance or reactance
    * Real part of complex tap position (only if the transformer has both taps and phase shift, otherwise the tap is a real number and thus usually a control variable)
    * Imaginary part of complex tap position (see remark above) (This and the previous item are transformed back to the real

world quantities tap and phase shift of the transformer after the OPF computation)

  - Polar Coordinates
    * Active power of a PV node
    * Voltage magnitude of a PV node
    * Tap position of a transformer
    * Phase shift angle of a phase shift transformer
    * Shunt capacitance or reactance

- State variables: This set includes all the variables which can describe any unique state of the power system. State variables are represented by the vector $\mathcal{X}$.

  Examples for state variables:

  - Rectangular Coordinates:
    * Real part of complex voltage at all nodes
    * Imaginary part of complex voltage at all nodes (This and the previous item are transformed back into the real world quantities voltage magnitude and angle after the OPF computation)
  - Polar Coordinates:
    * Voltage magnitude at all nodes
    * Voltage angle at all node

- Output variables: All other variables; they must be expressed as (nonlinear) functions of the control and state variables.

  Examples:

  - Rectangular Coordinates:
    * Voltage magnitude at PQ and PV node
    * Voltage angle at PQ and PV node
    * Tap magnitude of phase shift transformer
    * Tap angle of phase shift transformer
    * Power flow (MVA, MW, MVAr, A) in the line from i to j
    * Reactive generation at PV node
  - Polar Coordinates:
    * Power flow (MVA, MW, MVAr, A) in the line from i to j

* Reactive generation at PV node

Most variables are continuous, however some, like the transformer tap or the status of shunts are discrete. In this paper all variables are assumed to be continuous. The discrete variables are assumed to be set to their nearest discrete value after the optimization has been done. This does not guarantee optimality, however, results have shown that this approach leads to practically acceptable results.

1.3.2 Equality constraints - power flow equations

As discussed in the subsection above the power flow equations have to be satisfied to achieve a valid power system simulation result. Thus, in summary, the following sets have to be satisfied unconditionally:

SET A: Nodal currents not eliminated, rectangular coordinates

- (7), (8), (13), (14), (15), (16) and (4) (i.e. $4N + 1$ equations)
- This set A includes
  - $2N$ current related variables (I_{e_i}, I_{f_i}, i = 1...N)
  - $2N$ voltage related variables (e_i, f_i, i=1...N)
  - $2l$ PQ-node power related variables (P_i, Q_i, i=1...l)
  - m PV-node active power related variables (P_i, i=$l + 1$...N)
  - m PV-node voltage magnitude related variables ($|V|_i^2$, i = $l + 1$...N)
- **For these 6N variables, 4N+1 equality constraints are given**.

SET B: Nodal currents eliminated, rectangular coordinates

- (17), (18), (19), (20) and (4) (i.e. $2N + 1$ equations)
- This set B includes
  - $2N$ voltage related variables (e_i, f_i, i = 1...N)
  - $2l$ PQ-node power related variables (P_i, Q_i, i=1...l)
  - m PV-node power related variables (P_i, i = $l + 1$...N)
  - m voltage magnitude related variables ($|V|_i^2$, i=$l + 1$...N).

- **For these 4N variables, 2N+1 equality constraints are given**.

SET C: Polar coordinates

- (21), (22), (23) and (10) (i.e. $2N-m+1$ equations). This set C includes
  - $2N$ voltage related variables (V_i, Θ_i, i = 1...N)
  - $2l$ PQ-node power related variables (P_i, Q_i, i = 1...l)
  - m PV-node power related variables (P_i, i = $l+1...N$).
- **For these** $2N-m$ **variables,** $2N-m+1$ **equality constraints are given**.

Note, that in the actual implementation, only one of these sets A, B or C will actually be chosen. If one is satisfied, the other two are also satisfied. Also note that set C has fewer variables and equations than sets A and B. However, this does not mean that set C and as a consequence the polar coordinate system should always be preferred for power system modelling.

The complex tap of a transformer is also a variable which should be included in the above sets A, B or C. However, since they do not change the principles of the following derivations and also for space reasons, they are omitted in the subsequent sections.

1.3.3 Equality constraints - demand variables

For every demand variable an additional equality constraint has to be formulated. The loads in a power system are usually assumed to have a constant active part P and a constant reactive part Q. These two values usually cannot be changed by the operator (not taking into consideration load management) and must not be modified by the normal OPF computation. Thus for every load node where the load cannot be controlled, the two following equality constraints must be valid:

$$P_{scheduled_{PQ_i}} - P_i = 0 \tag{37}$$

$$Q_{scheduled_{PQ_i}} - Q_i = 0 \tag{38}$$

An additional demand variable is the voltage magnitude of a generator PV node where the voltage is not allowed to move. This is represented in the following simple equation with **polar** coordinates:

$$V_{scheduled_{PV_i}} - V_i = 0 \tag{39}$$

In **rectangular** coordinates this is:

$$V^2_{scheduled_{PV_i}} - e_i^2 - f_i^2 = 0 \tag{40}$$

For other demand variables (and fixed control variables) similar equality constraints can be formulated.

1.3.4 Summary - equality constraints

The equations for those equality constraints which have to be satisfied unconditionally can be summarized in general form as follows:

$$\mathbf{g}(\mathcal{X},\mathcal{U},\mathcal{P}) = \mathbf{0} \tag{41}$$

In (41), $\mathbf{g}(\mathcal{X},\mathcal{U},\mathcal{P})$ represents either the equality constraints of sets A, B or C and also those for all demand variables. The variables of the vectors $\mathcal{X}$, $\mathcal{U}$ and $\mathcal{P}$ are either all rectangular coordinates or all polar coordinates.

1.3.5 Inequality constraints

As shown in a previous subsection, many operational values must be limited in the real power system. These limits must be modelled correctly in the OPF simulation in order to have valid simulation results. Mathematically they are formulated as inequality constraints.

The inequality constraints (25) ... (36) can be used in the OPF formulation directly only if they represent bounds on OPF control or state variables or functions of OPF control or state variables. E.g. (31) where the active flow between nodes i and j is limited, cannot be taken directly in the OPF formulation since the variable P_{ij} is an output variable and must be expressed as a function of the control and state variables.

The active and reactive flows P_{ij} and Q_{ij} are computed with the state and control variables in rectangular coordinates as follows:

$$P_{ij} = (e_i f_j - e_j f_i)B_{ij} + (e_i^2 + f_i^2 - e_i e_j - f_i f_j)G_{ij} \tag{42}$$

$$Q_{ij} = (-e_i^2 - f_i^2 + e_i e_j + f_i f_j)B_{ij} + (e_i f_j - e_j f_i)G_{ij} - (e_i^2 + f_i^2)B_{i_o} \tag{43}$$

In polar coordinates this is:

$$P_{ij} = V_i^2 y_{ij} cos\theta_{ij} - V_i V_j y_{ij} cos(\Theta_i - \Theta_j - \theta_{ij}) \tag{44}$$

$$Q_{ij} = V_i^2(-B_{i_o} - y_{ij} sin\theta_{ij}) - V_i V_j y_{ij} sin(\Theta_i - \Theta_j - \theta_{ij}) \tag{45}$$

(42) and (44) will result in the OPF inequality constraints for pure active (MW) -flow limits:

$$P_{ij} \leq P_{high_{ij}} \tag{46}$$

For MVA-flow limits the following inequality constraints are valid:

$$P_{ij}^2 + Q_{ij}^2 \leq S_{high_{ij}}^2 \tag{47}$$

Depending on the choice of the coordinate system either (42) and (43) or (44) and (45) have to be substituted into (47).

The rule that all inequality constraints are either written in polar or all in rectangular coordinates is also valid here.

All inequality constraints must be expressed as functions of the vectors $\mathcal{U}$ and $\mathcal{X}$ which contain all the control and state variables. The general formulation for all these inequality constraints is as follows:

$$\mathbf{h}(\mathcal{X}, \mathcal{U}) \leq \mathbf{0} \tag{48}$$

In (48) every function $h_i(\mathcal{X}, \mathcal{U})$ represents one of the above inequality constraints. The actual limit values are put to the left hand side of the equation in order to have a vector $\mathbf{0}$ at the right hand side of (48).

1.3.6 Summary - OPF constraints

The constraints of the OPF problem can be split into two parts: The equality constraints, representing the power flow equations and the demand variables and the inequality constraint set, representing all the operational constraints. The following is the general mathematical expression for these two sets:

$$\mathbf{g}(\mathcal{X}, \mathcal{U}, \mathcal{P}) = \mathbf{0} \tag{49}$$

$$\mathbf{h}(\mathcal{X}, \mathcal{U}) \leq \mathbf{0}$$

Every OPF algorithm must try to satisfy (49). Only then will the result simulate the real power system correctly and show a practically useful result.

In the subsequent mathematical treatment of the OPF, it is usually not important to make a distinction between the various types of variables. Thus (49) can be formulated with general OPF variables $\mathbf{x}$:

$$\boxed{\begin{aligned} \mathbf{g}(\mathbf{x}) &= \mathbf{0} \\ \mathbf{h}(\mathbf{x}) &\leq \mathbf{0} \end{aligned} \qquad (50)}$$

1.4 Objective functions

1.4.1 Introduction

The formulation of equality and inequality constraints to model the power system and its operational constraints correctly has been discussed in the preceding subsections. These mathematical constraints, however, do not specify one unique network state. An enormous number of power system states can be computed when taking these constraints into account only. Thus the choice of an objective to simulate special, maybe extreme or optimal power system states follows naturally.

There are mainly two objectives which present-day electric utilities try to achieve beside the consideration of the operational constraints:

- Reduction of the total cost of the generated power: Although the switching in and out of generating units (with consideration of operational constraints like minimum down time, etc.) should also be considered this is usually not part of the OPF computation and handled outside by special unit commitment algorithms. Unit commitment algorithms consider the network only as a set of point sources and loads with predicted changes over time and do not take into account constraints like maximum branch flows and voltage limits. Thus today the scope of the OPF is limited to short term (i.e. approx. 15min. - 1h) network optimization with a given and fixed set of on-line generating units. This is also assumed in this paper.

- Reduction of active transmission losses in the whole or parts of the network: This is a common goal of utilities since the reduction of active power losses saves both generating cost (economic reasons) and creates at the same time higher generating reserves (security reasons).

The operator at a utility has to decide which goals are most important. Often the type of utility and its network, generation and load characteristics (e.g. predominant hydro power against predominant thermal power, a network with many long lines with few meshes against a highly meshed network, etc.) determines the main goals of a utility.

1.4.2 Objective function A: minimization of total generating cost

Usually generator cost curves, i.e. the relationship between generated power and the cost for this generated power is given in piecewise linear **incremental** cost. This has an origin in the simplification of piecewise concave cost curves with the valve-points as cost curve breakpoints. Since concave objective functions are very hard to optimize they were made piecewise quadratic which again corresponds to piecewise linear incremental cost curves. This type of objective function could be used in the simple so-called Lambda-Dispatch (Economic Dispatch, ED) where the set of optimal unit base can be determined easily by graphic methods with the consideration of generating unit upper and lower active power limits only.

Piecewise linear incremental cost curves (incremental cost usually monotonically increasing with increasing power) correspond to piecewise quadratic cost curves by doing an integration of the incremental cost curves. This type of cost curve with smooth transition in the cost curve breakpoints (i.e. same first derivative of cost curve segment at left and right hand side of the cost curve break points) can be approximated with very high accuracy by one convex non-linear function.

Although specialized algorithms can use the fact that the cost curves are piecewise quadratic it is assumed in this paper that the cost curves are of general nature with the only condition of being convex and monotonically increasing.

Generation cost curve objective functions are usually functions of their own generated power and not the power of another generating unit j.

Thus for the following derivations the total cost C of all generated powers to be optimized can be written as follows in function of the generated powers:

$$\text{Minimize } \mathcal{F}_{cost} = \sum_{i=l+1}^{N} \mathcal{F}_{cost_i}(P_i) \tag{51}$$

$m = N - l =$ number of generating units to be optimized

$l =$ number of fixed load PQ-nodes

Note that the power generated at the slack node N has also a cost function. This must be considered in the cost objective function of (51).

Also note that in many algorithms the cost curves $\mathcal{F}_{cost_i}$ are assumed to be quadratic or piecewise quadratic.

1.4.3 Objective function B: minimization of active transmission losses

The active transmission losses can be expressed in different ways: a) By a summation of the branch losses of all branches to be considered or b) by a summation of the active nodal powers over all nodes of the network.

a) Losses: computed over branches The total losses are the sum of the losses of all branches and transformers in the area of the network (or the whole network) where the losses are to be minimized:

$$\mathcal{F}_{Loss} = \sum_{i=1}^{NB} \mathcal{F}_{Loss_i} \tag{52}$$

NB = Number of branches of optimized area

where

$$\mathcal{F}_{Loss_i} = P_{km} + P_{mk} \;\; ; \text{branch } i \text{ lies between nodes } k \text{ and } m \tag{53}$$

In (53) the flows between nodes k and m can be replaced by the equations (42) and (43) for rectangular coordinates respectively (44) and (45) for polar coordinates:

In rectangular coordinates the following results:

$$\mathcal{F}_{Loss_i} = G_{mk}((e_m - e_k)^2 + (f_m - f_k)^2) \tag{54}$$

In polar coordinates the following results:

$$\begin{aligned} \mathcal{F}_{Loss_i} &= (V_k^2 - V_m^2) y_{mk} cos\theta_{mk} \\ &+ V_m V_k y_{km} \left(cos(\Theta_m - \Theta_k - \theta_{km}) - cos(\Theta_k - \Theta_m - \theta_{km}) \right) \end{aligned} \tag{55}$$

b) Losses: computed over nodes In this case only the losses of the whole network can be computed and not those of a subnetwork. The computation of the total losses is very similar to the computation of the total cost: The total network losses are given when all active nodal powers are added.

The total active losses are computed as follows:

$$\mathcal{F}_{Loss} = \sum_{i=1}^{N} P_i \;\; ; N = \text{Number of network nodes} \tag{56}$$

The slack node is always included in the total loss objective function.

1.4.4 Discussion

As has been shown in the preceding two subsections the losses can be formulated in two different ways, one going over branches the other over the nodes. Method a (branches) is more flexible since it allows to formulate the losses for only parts of a network. This corresponds often to a practical case where each utility models its own network and also those of neighbouring utilities (for reasons of the accuracy of the result) but it can optimize and control its own area only.

Method b on the other side has certain advantages since it allows a rather simple formulation for the total network losses which again allows the use of specialized algorithms for their solution as will be shown in the next section.

For the following derivations both objective functions are assumed to be of general nature and can be formulated as follows.

$$\text{Minimize } \mathcal{F}(\mathcal{X},\mathcal{U}) = \sum_{i \in EL} \mathcal{F}_i(\mathcal{X}_i, \mathcal{U}_i, \mathcal{X}_j, \mathcal{U}_j ...) = \sum_{i \in EL} \mathcal{F}_i(\mathcal{X},\mathcal{U}) \tag{57}$$

where EL = set containing either

a) m generator nodes (cost optimization) or

b) N network nodes (total network loss minimization) or

c) NB area branches (partial network area loss minimization).

Since the OPF does not need a distinction between control ($\mathcal{X}$) and state variables ($\mathcal{U}$) the general objective function formulation in OPF variables is as follows.

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = \sum_{i \in EL} \mathcal{F}_i(\mathbf{x}) \tag{58}$$

This general formulation covers both the losses and also the cost objective functions.

1.5 Optimality conditions

In this subsection the conditions which have to be satisfied in the optimal solution are discussed. The way how to reach the solution where these optimality conditions must be satisfied is not discussed here. The subsequent sections discuss how to reach the optimum.

The general OPF problem formulation is summarized as follows:

$$
\boxed{\begin{array}{lll} \text{Minimize} & \mathcal{F}(\mathbf{x}) & \\ \text{subject to} & \mathbf{g}(\mathbf{x}) & = \mathbf{0} \\ \text{and} & \mathbf{h}(\mathbf{x}) & \leq \mathbf{0} \end{array} \qquad (59)}
$$

The optimality conditions for (59) can be derived by formulating the Lagrange function $\mathcal{L}$:

$$
\mathcal{L} = \mathcal{F}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x}) + \boldsymbol{\mu}^T \mathbf{h}(\mathbf{x}) \tag{60}
$$

The Kuhn-Tucker theorem [1] says that if $\widehat{\mathbf{x}}$ is the relative extremum of $\mathcal{F}(\mathbf{x})$ which satisfies at the same time all constraints of (59), vectors $\widehat{\boldsymbol{\lambda}}, \widehat{\boldsymbol{\mu}}$ must exist which satisfy the following equation system:

$$
\begin{array}{rcrcl}
\dfrac{\partial \mathcal{L}}{\partial \mathbf{x}} & = & \dfrac{\partial}{\partial \mathbf{x}} \Big(\mathcal{F}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x}) + \boldsymbol{\mu}^T \mathbf{h}(\mathbf{x}) \Big) \,|_{\widehat{\mathbf{x}}, \widehat{\boldsymbol{\lambda}}, \widehat{\boldsymbol{\mu}}} & = & \mathbf{0} \\
\dfrac{\partial \mathcal{L}}{\partial \boldsymbol{\lambda}} & = & \mathbf{g}(\mathbf{x}) \,|_{\widehat{\mathbf{x}}} & = & \mathbf{0} \\
\mathbf{diag}\{\boldsymbol{\mu}\} \dfrac{\partial \mathcal{L}}{\partial \boldsymbol{\mu}} & = & \mathbf{diag}\{\boldsymbol{\mu}\}\ \mathbf{h}(\mathbf{x}) \,|_{\widehat{\mathbf{x}}, \widehat{\boldsymbol{\mu}}} & = & \mathbf{0} \\
 & & \widehat{\boldsymbol{\mu}} & \geq & \mathbf{0}
\end{array} \tag{61}
$$

The third constraint set together with the last set means that an inequality constraint is only active when $\mu_i > 0$.

It is the goal of the OPF algorithms to find a solution point $\widehat{\mathbf{x}}$ and corresponding vector $\widehat{\boldsymbol{\lambda}}, \widehat{\boldsymbol{\mu}}$ which satisfy the above conditions.

If this solution is found there is no guarantee that the global optimum is found. The Kuhn-Tucker conditions guarantee a local or relative optimum only. However, although no formal proof is possible, usually only one optimum (i.e. the global optimum) exists for practical OPF problem formulations. æ

2 HISTORICAL REVIEW OF OPF DEVELOPMENT

2.1 The early period up to 1979

The development of an optimal solution to network problems was initiated by the desire to find the minimum of the operating cost for the supply of electric power to a given load [2], [3]. The problem evolved as the socalled dispatch problem. The principle of equal incremental cost to be achieved for each of the control variables or controllers has already been realized in the pre-computer era when slide rules and the like were applied.

A major step in encompassing not only the cost characteristics but also the influence of the network, in particular the losses was the formation of an approximate quadratic function of the network losses expressed by the active injections [2]. Its core was the B-matrix which was derived from a load flow and was easily combined with the principle of equal incremental cost thus modifying the dispatched powers by loss factors. The method has lent itself to analog computer solutions in the online operation of systems. At this point, however, no constraints could be considered.

In the following period the development has mainly emphasized the formulation of a more complete optimal power flow towards the inclusion of the entire AC network [4], [5], [7], [9], [10]. The necessity to consider independent and dependent variables has led to a considerable increase of the system of equations which where nonlinear and thus difficult to handle. The formulation of the problem must be considered as a remarkable improvement as shown by Squires, Carpentier, however, still there was no effective algorithm available. At that time the ordinary load flow made considerable progress [6], [12] and the capabilities of computers showed promising aspects. Hence, the analysts were intrigued by the possibilities in the area of the load flow and tried to incorporate this success in the area of the optimal power flow.

A remarkable conceptual progress was made by Dommel, Tinney [7] when they formulated the exact optimality conditions for an AC based OPF which allowed the use of the solution of an ordinary load flow. By eliminating the dependent variable with the help of a solved load flow iteration a gradient method was designed which led to a true optimal solution of a dispatch problem including the detailed effects of the AC network. This step marks an important step in the development of the OPF since there was an algorithm which had several ramifications (reduced gradients, etc.) and it considered already constraints of variables. The technique employed was based on penalty functions which could easily be attached to the Lagrangian function of the basic method. The gradient or reduced gradient included derivatives of the quadratic penalty functions also which by their character had quite different magnitudes as compared to the gradients of the objective functions. As a consequence the parameter which determined the step length in the direction of the gradient was not able to confine the solution sufficiently close. The result was that the convergence of the whole approach was quite poor. In particular, maintaining constraints by taking in and releasing constrained variables was not satisfactory. Programming packages were developed but required detailed tuning and turned out not to be applicable to general problems. A quite complete overview of these developments is given in [17].

2.2 Recent developments since 1979

Since the gradient concept did not turn out to be successful, also from the point of view of treating constraints several other concepts were pursued. One line was the application of linear programming which offered a clear approach to handling constraints [15], [16]. Another direction was the use of quadratic programming whereby standard quadratic routines were used [14], [20]. A different approach led to exploiting the optimality conditions in the form of Newton's method.

The first two methods are characterized by the use of a solved load flow which yields a feasible starting point. Newton's method led to iterative solution steps which approach the optimal result in a global way [19].

Each of these approaches showed considerable progress over gradient methods both as far as convergence is concerned and with regard to treating constraints.

Linear programming methods showed a first success in the area of dispatching generator outputs whereby cost curves have been represented by linear segments and the load flow was incorporated in a linearized fashion (Stott, [15]). This line has been further refined recently such that an AC model of the network could be treated as well and a reactive dispatch for the purposes of minimizing losses was made possible.

Quadratic programming followed [18] more closely the facts of the system model which shows piecewise quadratic cost curves, a quadratic behavior of losses and of powers in general, e.g. the slack power. Since the quadratic behavior is sufficiently accurate for small deviations only the quadratic approach is also iterative whereby standard quadratic programming routines were applied. The general observation was that convergence of these methods was extremely good, however, the formation of quadratic forms, of loss formulae and other conversions require a considerable effort which turned out to be a drawback as far as the overall performance was concerned.

For both linear and quadratic methods the load flow solution has to be converted to a compact form or the socalled incremental power flow which can be extended to a quadratic form. It was instrumental for the application of these methods and still is for the most recent forms of the OPF.

The development of the Newton approach for the purposes of the OPF is a consequence of the success of the techniques derived for the ordinary power flow [19]. Sparsity techniques, ordering, decoupling methods, etc. have suggested to maintain and keep the original optimality conditions derived from the Lagrangian and to treat the large system of equations as if it would be a power flow problem which nowadays can be solved for thousands of nodes. The formulation and the solution of the problem is easy for the unconstrained case. Constraints had to be treated by penalty functions, however, no straightforward routine could be devised which leads to active constraints. The method remains with heuristic steps which take in and release constraints which requires updating steps of the factorized system matrix. Although Newton's method was considered as the only approach to treat the loss minimization problem effectively some time ago this image is fading somewhat and is giving way to methods which incorporate linear programming routines for reasons of performance, uniqueness of approach and use of proven routines.

In a broader perspective the optimal power flow is becoming the main tool for the assessment and enhancement of the security of the system [22], [23]. The objective function may have a direct relation to security, e.g. in the case where the deviation from a desirable voltage profile is to be minimized. Otherwise it is the tool to achieve a well defined solution, with an economic benefit, as given by minimum losses.

Security, however, is a problem where constraints are to be maintained or where excess variables are corrected. A modern OPF lends itself to the treatment of these requirements and the recent efforts in improving the methods, in particular, as far as constraints are concerned, prove the great interest in this aspect of the OPF.

3 CLASSIFICATION OF ALGORITHMS TO ACHIEVE OPF OPTIMALITY CONDITIONS

3.1 Practical constraints and desirable features of the algorithms

It has been shown in the preceding sections that the OPF problems can be defined in different ways. The determination of an optimal, steady state network operation is the general goal. Utilities are interested in achieving this goal for both network planning studies and also in real-time operation.

In **planning studies** the utility wants to know how to expand or change its network in order to achieve e.g. minimum losses under a variety of load scenarios. Another problem is the minimization of cost of future planned generation. The OPF is used to propose to the utility where to put what generator capacity in the present or future network to achieve minimum cost operation. It is obvious that statistical values for load changes or approximations for the expected cost of new generators will have to be considered and thus make the result of the OPF subject to many assumptions, predictions and uncertainties. The OPF algorithm used for planning studies should be able to handle this data which is usually based on statistics.

Another important area where the OPF is and will be applied, is the **real-time OPF**, i.e. the use of the OPF result for the actual network operation. The goal is here to take the OPF result and try to realize the computed values in the actual, real-time network. This real-time network optimization is usually done under operator control, i.e. the computed optimal values are read by the operator who changes the actual controls to achieve the same network state as obtained in the OPF simulation. A closed loop OPF, i.e. the automatic realization of the optimal computed solution in the real network, is - at least within the near future - not realistic, but may be approached by a close interaction between the operator and the simulated OPF result, maybe with expert system guidance. The practical aspects of the OPF implementation are key to the real-time use of the OPF. In this application of the OPF the algorithms are useless if their output does not conform to practical aspects. Under the assumption that the operator tries to achieve the optimal solution some practical constraint considerations are critical to the application of the OPF:

- **Computational speed**: The OPF result must be obtained within a reasonable timeframe, starting at the time when the real-time data is obtained from the network. Since state estimation algorithms usually take the raw data before being used by the OPF another time delay exists. Both state estimation and succeeding OPF computations must be fast enough to be practically applicable. The realization of speed is a combination of fast hardware and fast algorithms.

- The **hardware** must be fast, but must be in the right price range and computer class used in the energy management systems at utilities. A practical solution to this constraint is today, with the systems offered by the energy management system vendors, often quite difficult to achieve. New technologies, fully applied to the energy management systems, should help to solve this problem in the near future.
- The **software** must be such that it can compute OPF problems with network sizes of thousands of electrical nodes within a reasonable (wall-clock) time. Speed can mainly be achieved by translating the physically given special characteristics of the electrical network in special OPF algorithms. An example is given by the loosely connected network topology which is translated into a special sparsity storage scheme in the computer which again makes fast iterations possible (only non-zero value arithmetic operations). Another typical electrical behavior is the locality of network state changes, e.g. the effect of changing the voltage at a generator node remains in the local vicinity of the changed generator and does not spread over the whole network. This is translated into algorithms which use the localized behavior of the network and speed up computation by not having to compute all network variables but only the local ones. Also the fact that not many branch limits will be active at the optimal solution can be used by the OPF algorithm and computational speed will be improved by doing so. Consideration of data uncertainties can be used to speed up the algorithmic solution: E.g. if the accuracy of a large generator output power measurement is about five MW, making a computation with an accuracy of one MW is useless and consumes unnecessary computing time.

- **Robustness**: The OPF may not, under any circumstances, diverge or even crash. Fast and straight-forward convergence is important to acceptance and real-time application of the OPF result. Even in cases where there is no optimal solution with consideration of all constraints the OPF must tell the user that there is no solution and output a near-optimal solution which satisfies most of the constraints. Operator or expert system involvement in these difficult to solve cases is desirable to achieve a practically useful OPF solution.

- **Controller movements**: The OPF assigns an optimal value to each possible control variable. Assuming that there is a large number of possible control variables the OPF algorithm would move most of them from the actual state to the optimal state. However, a practical real-time realization of this optimal state is not possible since the operator cannot have e.g. hundred generator voltages be moved to different settings within a reasonable time. Only the most effective subset should be moved, which means that within the OPF the algorithmic problem of moving the minimum number of controllers with maximum effect has to be solved. Another problem with the movement of controllers is the distance it has to move from the actual to the desired, OPF computed optimal value. Time constraints like maximum controller movement per minute must be considered to achieve practical OPF use. This again leads to another critical OPF point: When talking about time aspects of movement the load changes within pre-determined time frames should also be considered. As an example, when the load changes very rapidly within the next fifteen minutes the generation should be optimized with consideration of the actual and the expected load in fifteen minutes. The OPF can result in different optimum solution points depending on the constraints considered in the optimum.

- **Local controls**: Tap changers are usually used to regulate voltages locally to scheduled values. These scheduled voltage values can be more desirable than any optimal voltages computed by the OPF. A localized, not optimal control might practically be preferred to the solution for this control obtained by the OPF. If this is the case the OPF algorithm has to handle this situation.

Some of these practical constraints can be incorporated into the classical OPF formulation shown in preceding sections. Where possible this is done in the inequality constraint set. However, some practical constraints like the local control discussed above is usually taken out of the optimization algorithm. These constraints are taken into account separately as part of an overall OPF solution, where one part is the optimization algorithm and the other is an algorithm based usually on heuristics and algorithmic application of special characteristics of the electrical network. This separation will be discussed in the next sections.

The solution of the classical OPF problem formulation (see section 1), the practical aspects discussed above and the mathematically known algorithms lead to OPF classifications which are discussed in the next subsection.

3.2 Classification of OPF algorithms

3.2.1 Distinction of two classes

The separation of OPF algorithms into classes is mainly governed by the fact that very powerful methods exist for the ordinary load flow which provide an easy access to intermediate solutions in the course of an iterative process. Further, it can be observed that the optimum solution is usually near an existing load flow solution and hence sensitivity relations lead the way to the optimum. Hence, one class exists which relies on a solved load flow and on tools provided by the load flow.

The second class originates from a rigorous formulation of the OPF problem, employs the exact optimality conditions and uses techniques to fulfill the latter. In this case a solved load flow is not a prerequiste. The preferred method for reaching the optimality conditions is Newton's method.

There are advantages and disadvantages in both methods which have a certain bearing depending on the objective, the size of the problem and the envisaged application.

Hence, optimal power flow algorithms will be discussed in two classes:

- Class A: Methods whereby the optimization starts from a solved load flow. The Jacobian and other sensitivity relations are used in the optimizing process. The process as a whole is iterative. After each iteration the load flow is solved anew.

- Class B: Methods relying on the exact optimality conditions whereby the load flow relations are attached as equality constraints. There is no prior knowledge of a load flow solution. The process is iterative and each intermediate solution approaches the load flow solution.

3.2.2 Discussion of class A algorithms

When the load flow is solved in the known way the following information is available or can be extracted.

- the set of nodal voltages (complex or amplitude/angle)

- the Jacobian matrix either original or in factorized form

- the incremental power flow either in linearized form or with a quadratic extension

The dependent and independent variables fulfill the load flow equations and are consistent. The variables are within limits or not too far off. Hence the Jacobian and any derived functions may be used as sensitivity relations.

The actual optimizing process is separate whereby sensitivity relations of the load flow are incorporated. Constraints are introduced at this stage. In some cases dependent variables are eliminated before the actual solution process in order to arrive at smaller size matrices, tableaus, etc.

An examples for class A methods is given by Dommel [7].

The choice of class A methods can be appreciated when performance aspects and certain limitations are considered.

One outstanding advantage is the clear and systematic treatment of constraints when linear and quadratic programming methods are employed in the optimization part. The load flow supplies sensitivity relations which are quite often extractable in a reduced form, e.g. linear incremental power flow which is a scalar relation. Constraints are formulated in terms of the set of remaining variables (when a subset of variables has been eliminated). The active power dispatch is an excellent example of a class A method. The Hessian matrix derived from the quadratic cost functions is diagonal and the incremental power flow is a scalar.

In Stott [15] cost curves are approximated by straight lines. Hence the optimization is done on the basis of linear programming.

Class A methods have been applied to loss minimization but in this case the quadratic form has to be derived from the load flow (extended incremental power flow). The computational effort in forming the quadratic form and its treatment within the quadratic programming routine limits the application of class A methods for loss minimization. The observation is that systems above 300 nodes require comparatively large computing times.

There is however one aspect of class A methods, namely the use of approximations in the formation of the Hessian or the use of linear approximations. It turns out that it is the linear relations of the load flow (incremental power flow) which determine the exact optimum. Quadratic relations and their approximations determine the speed of convergence, they limit step length etc. If suitable approximations to the Hessian can be found, quadratic and linear methods within class A can be quite powerful.

3.2.3 Discussion of class B algorithms

Class B algorithms start from the optimality conditions evolving from a Lagrangian function. The optimality conditions comprise derivatives of the objective functions and equality constraints. It is to be remembered that they are conditions and give little indications as to their fulfillment. Class B methods aim at the satisfaction of the optimality conditions in a direct way whereby inequality constraints usually are treated in a special form.

There are two approaches which fall into this category. It is Newton's method which allows to meet the optimality conditions as long as they are differentiable. A second method is available if the Lagrangian is quadratic which results in linear optimality conditions. Constraints can be treated by linear programming as will be shown later. As a matter of fact Newton's method and this quadratic approach merge into one single method when the Lagrangian is quadratic or when the first derivatives of the Lagrangian are kept constant (quasi- Newton).

The advantages of class B methods lie in the fact that the Hessian is very sparse or remains constant or can be inserted in approximate terms. It is a non-compact method which does not result in a progressive increase in computation time for the formation of the Hessian or for the solution of the optimization part. The overall system of equations can be very large in dimension but it is very sparse. Large numbers of nodes can be handled. In case of Newton's methods the coefficients of the matrices need not be precise since the accuracy of the solution is guaranteed by the mismatches (right hand sides), e.g. decoupled loadflow methods can be employed.

As it stands now class B methods have difficulties in handling constraints. The standard approach at the moment seems to be to treat constraints by penalty terms whereby active constraints are determined by heuristic methods. The consequence is that the system of equations needs updating and refactorization which in the end deteriorates the performance.

The quadratic method mentioned above avoids this problem and is able to treat constraints in a systematic fashion.

The recent development has favored class B methods for large systems, in particular when losses are to be minimized.

4 OPF CLASS A: POWER FLOW SOLVED SEPARATELY FROM OPTIMIZATION ALGORITHM

4.1 Introduction

In this section the OPF formulation is solved by a class of algorithms where the power flow is used in the conventional way to solve the power flow problem for a given set of control and demand variables with fixed values. This solution is then taken to be the starting point for an optimization. The optimization is thus separated from the conventional power flow solution algorithm. Since as will be shown in the next subsection the optimization represents only an approximation to the original OPF problem, its solution may not be the final one and so the optimized OPF variables are transferred back to the power flow problem which is solved again. The result of the optimization is thus taken as the input for the power flow which is solved, this result is again taken as input for the optimization problem, etc. All OPF Class A algorithms have this procedure in common.

The power flow is not discussed in this paper and is assumed to be known. Extensive literature can be found in papers and student text books. However, the optimization part where several algorithms can be used is discussed in the following subsections.

Thus the various OPF class A algorithms show differences mainly in the optimization part. One of two algorithms is usually used for the optimization part: Either a linear programming (LP) or a quadratic programming (QP) based algorithm. Both algorithms can solve their respective optimization problem with straightforward procedures and no heuristics are needed. The main difference between both optimization problem definitions can be found in the objective function formulation: The LP can handle only linear objective functions,

$$\text{LP: Minimize } \mathcal{F}(\mathbf{x}) = \mathbf{c}^T\mathbf{x} \tag{62}$$

and the QP handles quadratic objective functions:

$$\text{QP: Minimize } \mathcal{F}(\mathbf{x}) = \mathbf{c}^T\mathbf{x} + \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} \tag{63}$$

Both optimizations are restricted to consider linear equality and inequality constraints:

$$\mathbf{J}\mathbf{x} = \mathbf{b_1} \tag{64}$$

and

$$\mathbf{Ax} \leq \mathbf{b_2} \tag{65}$$

The LP objective function can be seen as a simplification of the QP objective function by neglecting the quadratic objective function terms as represented in the matrix $\mathbf{Q}$. From this point of view any QP formulation can easily be transformed into an LP formulation.

Note, however, that the actual solution processes for both LP and QP are distinctly different.

Both LP and QP solution algorithms are described in textbooks and mathematical details of how to get the iterative optimal LP or QP solution are briefly discussed in the appendix section A.1 (LP) and A.2 (QP) of this paper. However, in section 4.5 of this paper, an engineered LP version is mentioned which goes beyond the conventional LP linear objective: This LP-based algorithm is tuned to the typical OPF problem objective functions and can solve general separable, convex objective functions. In addition, in the appendix A.2 a QP-algorithm is described which works with well known LP tools. It is important to note, that independent of the engineered modifications to the original LP or QP algorithms, the basic principles of the chosen LP or QP optimization remain always valid.

In the OPF class A approaches the general OPF problem formulation is approximated around an operating point vector x^k. The index ^{k} means that this operating point will vary during the OPF class A solution process where k is incremented by 1 from one iteration to the next. The OPF problem is formulated in a quadratic approximation around this operating point x^k for the objective function $\mathcal{F}$, however in linearized form for the equality and inequality constraints. The linearization of the constraints is justified by the fact that both LP and QP algorithms can handle linearized constraints only. Thus the problem formulation is adapted to the mathematical problem formulation, which then leads to a straightforward optimization solution.

Approximations to both the objective functions and to the constraints lead to inaccuracies which must be corrected by some means. In OPF class A algorithms this is done by solving an exact AC power flow once an optimized solution (which is optimal only with respect to the approximated problem formulation) has been obtained. The repetitive execution of power flow and LP, respective QP optimization must lead to better, more accurate approximations, as more power flow-LP or QP optimizations are executed. The solution to the problem of getting this iterative process to converge is critical. Note, since the power flow has no degree of freedom and thus no ability to influence the overall convergence process, the iterative LP or QP optimization steps alone are responsible for obtaining convergence. In order to clarify this point, an example is given: In order to justify the approximations it might be necessary to restrict the movement of certain variables x from the starting point x^k to its optimum x^k_{opt}. No straighforward mathematical algorithm exists which tells, how far the variables are allowed to move within the optimization algorithm. Thus, since approximations are valid only for small deviations from an operating point, the definition of what small means can be critical to the overall convergence.

In the following subsection a derivation is given of how to get an LP or QP problem formulation, starting from the general OPF problem formulation.

4.2 OPF class A optimization problem formulation

The original OPF problem formulation as given in (59) is taken as starting point for an approximated optimization problem. In the following, a special formulation with an approximation of the quadratic objective function with second and first order approximated equality constraints and linearized inequality constraints is derived. This formulation is needed to derive a QP formulation which can be solved by the algorithms given from the mathematicians. The LP formulation can easily be derived from the QP by neglecting the quadratic terms of the objective function. Note that an LP can always be derived from a QP. However, it is not evident that the LP algorithms for the LP problem formulation (even if derived from the original QP problem) converge in a comparable way to QP algorithms for the QP problem formulation.

The following general derivations are made such that in a later subsection the different LP and QP optimization problem formulations for the cost and the loss optimization are easy to understand.

In the following formulas the OPF variable vector $\mathbf{x}$ is split into several subvectors:

$$\mathbf{x}^T = (\mathbf{x_1^T x_2^T x_3^T x_4^T}) \tag{66}$$

where

- $\mathbf{x_1}$: All active power variables P_i at generator PV nodes (dimension: m)
- $\mathbf{x_2}$: All active power variables P_i at load PQ nodes (dimension: l)
- $\mathbf{x_3}$: Vector containing the subvectors $\mathbf{x_{31}}$ and $\mathbf{x_{32}}$:
  - $\mathbf{x_{31}}$: All reactive power variables Q_i of all PQ-load nodes (dimension: l)

- $\mathbf{x_{32}}$: All voltage magnitude variables $|V|_i^2$ of all generator PV nodes (dimension: m) (only when taking rectangular coordinates; when using polar coordinates, the vector $\mathbf{x}_{32}$ does not exist)

- $\mathbf{x_4}$: Either all real and imaginary parts of voltage variables e_i, f_i (dimension: 2N) (when taking rectangular coordinates) or all voltage magnitudes and all voltage angle variables V_i, θ_i (dimension: 2N) (when taking polar coordinates)

The equality constraint set is also split into several subsets. Note that the subset B, as explained in subsection 1.3.2 of this paper, is taken in the following derivations. For the other sets, similar derivations can be made.

$$\mathbf{g}^T = (\mathbf{g_1^T} \mathbf{g_2^T} \mathbf{g_3^T} \mathbf{g_4^T}) \tag{67}$$

where

- $\mathbf{g}_1$: Load flow equations representing the active powers at all PV nodes (number of equality constraints of type $\mathbf{g_1}$: m).

- $\mathbf{g}_2$: Load flow equations representing the active powers at all PQ nodes (number of equality constraints of type $\mathbf{g_2}$: l).

- $\mathbf{g}_3$: Load flow equations representing the other non-active-power variables like voltage magnitude at PV nodes, reactive power at PQ nodes and the equality constraint for the fixed slack-node angle (number of equality constraints of type $\mathbf{g_3}$: $N+1$).

- $\mathbf{g}_4$: Demand variable related equality constraints: Fixed active and reactive loads at some PQ nodes, fixed voltage at some PV nodes, etc. (number of equality constraints of type $\mathbf{g_4}$: d; note that the number cannot be given in function of network nodes or other typical network parameters; the actual number, assumed to be d, depends on the available choice of demand variables of the network).

The approximated optimization problem is now as follows: Minimize either the total generation cost

$$\mathcal{F}_{\mathbf{cost}}(\mathbf{x_1}) = \mathcal{F}_{\mathbf{cost}}(\mathbf{x_1^k}) + \mathbf{c}^{T^k}\Delta\mathbf{x_1} + \frac{1}{2}\Delta\mathbf{x_1}^T\mathbf{Q}^k\Delta\mathbf{x_1} \tag{68}$$

or minimize the total network losses:

$$\mathcal{F}_{\mathbf{loss}}(\mathbf{x_1}, \mathbf{x_2}) = \mathcal{F}_{\mathbf{loss}}(\mathbf{x_1^k}, \mathbf{x_2^k}) + \mathbf{1}^T\Delta\mathbf{x_1} + \mathbf{1}^T\Delta\mathbf{x_2} \tag{69}$$

(In this paper only the loss objective function of (56) is used for further derivations. Similar derivations are possible for the other loss objective function (52).)

subject to the equality constraints (quadratic approximation for all equality constraints $\mathbf{g_1}$, $\mathbf{g_2}$ and $\mathbf{g_3}$):

$$\mathbf{g_1}(\mathbf{x_1^k}, \mathbf{x_4^k}) + \Delta\mathbf{x_1} + \mathbf{J_{14}}^k \Delta\mathbf{x_4} + \frac{1}{2} \Delta\mathbf{x_4}^T \mathbf{M_{14}}^k \Delta\mathbf{x_4} = \mathbf{0} \tag{70}$$

$$\mathbf{g_2}(\mathbf{x_2^k}, \mathbf{x_4^k}) + \Delta\mathbf{x_2} + \mathbf{J_{24}}^k \Delta\mathbf{x_4} + \frac{1}{2} \Delta\mathbf{x_4}^T \mathbf{M_{24}}^k \Delta\mathbf{x_4} = \mathbf{0} \tag{71}$$

$$\mathbf{g_3}(\mathbf{x_3^k}, \mathbf{x_4^k}) + \Delta\mathbf{x_3} + \mathbf{J_{34}}^k \Delta\mathbf{x_4} + \frac{1}{2} \Delta\mathbf{x_4}^T \mathbf{M_{34}}^k \Delta\mathbf{x_4} = \mathbf{0} \tag{72}$$

For the equality constraint set $\mathbf{g_4}$ only a linearized approximation is used:

$$\mathbf{g_4}(\mathbf{x_1^k}, \mathbf{x_2^k}, \mathbf{x_3^k}, \mathbf{x_4^k}) + \sum_{i=1}^{4} \mathbf{J_{4i}}^k \Delta\mathbf{x_i} = \mathbf{0} \tag{73}$$

The same holds for the inequality constraint set $\mathbf{h}$:

$$\mathbf{h}(\mathbf{x_1^k}, \mathbf{x_2^k}, \mathbf{x_3^k}, \mathbf{x_4^k}) + \sum_{i=1}^{4} \mathbf{A_i}^k \Delta\mathbf{x_i} \leq \mathbf{0} \tag{74}$$

In (68) ... (74) some abbreviations have been used:

$$\mathbf{c}^k = \left.\frac{\partial \mathcal{F}_{\mathbf{cost}}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x^k}} \quad ; \mathbf{Q}^k = \left.\frac{\partial^2 \mathcal{F}_{\mathbf{cost}}}{\partial \mathbf{x^2}}\right|_{\mathbf{x}=\mathbf{x^k}}$$

$$\mathbf{J}_{ij}^k = \left.\frac{\partial \mathbf{g}_i}{\partial \mathbf{x}_j}\right|_{\mathbf{x}=\mathbf{x^k}} \quad ; \mathbf{M}_{ij}^k = \left.\frac{\partial^2 \mathbf{g_i}}{\partial \mathbf{x_j^2}}\right|_{\mathbf{x}=\mathbf{x^k}}$$

$$\mathbf{A}_i^k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_i}\right|_{\mathbf{x}=\mathbf{x^k}}$$

Note that index ^{k} means that these variables, vectors and matrices are state dependent and can vary from one state to the other (or from iteration to iteration).

Assume that a power flow has been solved for this operating point, thus the equality constraints $\mathbf{g}(\mathbf{x^k}) = \mathbf{0}$ are satisfied:

$$\mathbf{g}(\mathbf{x}^k) = \mathbf{0} \tag{75}$$

The optimization problem defined with (68) ... (74) is not a classic QP formulation because quadratic equality constraints exist. Now, different steps can be undertaken for cost and loss optimization in order to derive QP or LP formulations.

Because of their different nature, different assumptions can be made when setting up the above optimization problem for the cost and the loss minimization OPF problem. Both derivations are given in the following two subsections.

4.3 Total generation cost as objective function in OPF class A formulations

4.3.1 Sparse, non-compact QP cost optimization problem

After the general derivation of the previous subsection the total generation cost as OPF objective function is discussed in this subsection.

Since the cost of each generator active power is not dependent on the cost of another generator the second derivatives of the cost function with respect to the active power variables of all generators ($\mathbf{x}_1$) lead to a diagonal matrix:

$$\mathbf{Q}^k = \mathbf{diag}(q_i^k) \tag{76}$$

with

$$q_i^k = \left.\frac{\partial^2 \mathcal{F}_i}{\partial x_{1i}^2}\right|_{x_i = x_i^k} \tag{77}$$

and x_{1i}: active power of the generator i; $\mathcal{F}_i$: cost of generator i in function of its active power.

Note that when assuming quadratic cost curves these factors q_i^k are constant, i.e. not state dependent.

When optimizing cost, all quadratic terms of the optimization problem exclusive the one of the objective function are usually neglected. This is possible because the cost curves are already of a (near) quadratic nature and turn out to be dominant. Thus the cost optimization problem is as follows:

$$\text{Minimize } \mathcal{F}_{cost} = \mathcal{F}_{cost}(\mathbf{x_1}^k) + \mathbf{c}^{Tk}\Delta\mathbf{x_1} + \frac{1}{2}\Delta\mathbf{x_1}^T\mathbf{diag}(q_i^k)\Delta\mathbf{x_1} \tag{78}$$

subject to

$$\mathbf{g}(\mathbf{x^k}) + \mathbf{J}^k\Delta\mathbf{x} = \mathbf{0} \tag{79}$$

and

$$\mathbf{h}(\mathbf{x^k}) + \mathbf{A}^k\Delta\mathbf{x} \leq \mathbf{0} \tag{80}$$

with

$$\mathbf{J}^k = \begin{bmatrix} \mathbf{U} & \mathbf{0} & \mathbf{0} & \mathbf{J_{14}}^k \\ \mathbf{0} & \mathbf{U} & \mathbf{0} & \mathbf{J_{24}}^k \\ \mathbf{0} & \mathbf{0} & \mathbf{U} & \mathbf{J_{34}}^k \\ \mathbf{J_{41}}^k & \mathbf{J_{42}}^k & \mathbf{J_{43}}^k & \mathbf{J_{44}}^k \end{bmatrix} ; \mathbf{g}(\mathbf{x^k}) = \begin{bmatrix} \mathbf{g_1}(\mathbf{x^k}) \\ \mathbf{g_2}(\mathbf{x^k}) \\ \mathbf{g_3}(\mathbf{x^k}) \\ \mathbf{g_4}(\mathbf{x^k}) \end{bmatrix} ; \tag{81}$$

$$\mathbf{A}^k = \begin{bmatrix} \mathbf{A_1^k A_2^k A_3^k A_4^k} \end{bmatrix} ; \boldsymbol{\Delta}\mathbf{x}^T = \begin{bmatrix} \boldsymbol{\Delta}\mathbf{x_1^T} \boldsymbol{\Delta}\mathbf{x_2^T} \boldsymbol{\Delta}\mathbf{x_3^T} \boldsymbol{\Delta}\mathbf{x_4^T} \end{bmatrix} ;$$

The resulting problem is now a classic QP problem. Note that in this formulation the problem is very sparse. This sparsity must be considered when applying the QP algorithms to this problem. In [21] sparsity techniques are discussed in detail.

4.3.2 Compact, non-sparse QP cost optimization problem (Linear incremental power flow)

The cost optimization problem has been formulated as a QP with sparse matrices. However, the number of variables is very high and thus many variable related operations will result. In the following a derivation of the cost optimization problem is given where on one side the number of variables is reduced to a much smaller set, however, on the other side the sparsity of the matrices gets lost.

In order to achieve this compact QP formulation variables have to be eliminated from the equality constraint set $\mathbf{g}(\mathbf{x^k}) + \mathbf{J}^k \Delta\mathbf{x} = \mathbf{0}$ (79).

Note that this equality constraint set contains $2N + 1 + d$ equality constraints. The variable vector $\boldsymbol{\Delta x}$ contains $4N$ variables.

This set can be reduced to one equation with $4N - (2N + 1 + d) + 1 = 2N - d$ variables. This means that from the total of $4N$ variables, $2N + d$ variables must be eliminated. Note that $d \leq 2N$.

In order to achieve a compact formulation, the variables of the vector $\Delta\mathbf{x_4}$ (without the real and imaginary slack node voltage variable) (i.e. $2N - 2$ variables) are eliminated. From the vectors $\Delta\mathbf{x_2}$ and $\Delta\mathbf{x_3}$, $d + 2$ variables have to be eliminated: The rule is to eliminate first the variables of $\Delta\mathbf{x_2}$ for which demand variable constraints exist (formulated in the equality constraint set $\mathbf{g_4}$). Doing this will eliminate all active reactive power variables of non-manageable load PQ nodes. The remaining variables to be eliminated are taken from the vector $\Delta\mathbf{x_3}$.

Eliminating the variables accordingly in the inequality constraint set $\mathbf{h}(\mathbf{x^k})+\mathbf{A}^k\Delta\mathbf{x} \leq \mathbf{0}$ reduces the optimization problem to $2N-d$ non-eliminated variables.

Two voltage variables at the slack node are not eliminated. This comes from the fact that there is a chance of having singularity or linearly dependent equality constraints among the equality constraints of the set $\mathbf{g}$. Linear dependence can lead to zero pivots during factorization. A division by a zero pivot can usually be avoided if the real and imaginary part of the slack node voltage are not eliminated.

Since the variable set $\Delta\mathbf{x_1}$ is not eliminated the objective function is unchanged.

The optimization problem is now as follows:

$$\text{Minimize } \mathcal{F}_{cost} = \mathcal{F}_{cost}(\mathbf{x_1}^k) + \mathbf{c}^{T^k}\Delta\mathbf{x_1} + \frac{1}{2}\Delta\mathbf{x_1}^T\mathbf{diag}(q_i^k)\Delta\mathbf{x_1} \tag{82}$$

subject to

$$\boldsymbol{\alpha_1}^{T^k}\Delta\mathbf{x_1} + \boldsymbol{\alpha_2}^{T^k}\Delta\mathbf{x_2'} = \alpha_0 \tag{83}$$

((83) is called the **linear incremental power flow** equation.)
and

$$\mathbf{h}(\mathbf{x_1^k}, \mathbf{x_2^k}) + \mathbf{A_1'}^k\Delta\mathbf{x_1} + \mathbf{A_2'}^k\Delta\mathbf{x_2'} \leq \mathbf{0} \tag{84}$$

with

- $\Delta\mathbf{x_2'}$ including all non-eliminated variables excluding $\Delta\mathbf{x_1}$ (see paragraph above for what variable types are included).

- $\mathbf{h}(\mathbf{x_1^k}, \mathbf{x_2^k})$ representing the inequality constraint set values at the operating point $\mathbf{x}^k$

- $\mathbf{A_1'}^k$ representing the sensitivities of the inequality constraints with respect to $\Delta\mathbf{x_1}$ at the operating point $\mathbf{x}^k$.

- $\mathbf{A_2'}^k$ representing the sensitivities of the inequality constraints with respect to $\Delta\mathbf{x_2'}$ at the operating point $\mathbf{x}^k$.

Note that all these matrices can be derived by simple variable elimination in all equality (79) and inequality constraints(80).

4.4 Total network losses as objective function in OPF class A formulations

The formulation of the loss QP optimization problem must be derived differently than the cost QP optimization problem. The main reason comes from the fact that the loss objective function as shown in (69) is linear when using the active powers of all nodes as a subset of the OPF variables.

Several QP derivations are possible. Two of them are shown in the following two subsections.

4.4.1 Sparse, non-compact QP loss optimization problem

The basic idea of this optimization problem formulation is the elimination of the variables of the vectors $\boldsymbol{\Delta x_1}$, $\boldsymbol{\Delta x_2}$, $\boldsymbol{\Delta x_3}$ from the optimization problem as formulated with (69) ... (74). Thus the goal is to formulate the optimization problem only in variables of the vector $\boldsymbol{\Delta x_4}$ ($\boldsymbol{\Delta x_4}$ represents the complex nodal voltages). The loss optimization problem is now as follows:

$$\text{Minimize } \mathcal{F}_{loss} = \mathcal{F}_{loss}^{\prime k} + \mathbf{c}^{\prime T^k} \Delta\mathbf{x_4} + \frac{1}{2}\Delta\mathbf{x_4}^T \mathbf{M_4}^{\prime k} \Delta\mathbf{x_4} \tag{85}$$

subject to the equality constraints $\mathbf{g_4}$ (Note that a quadratic approximation is used for the variables $\boldsymbol{\Delta x_1}$, $\boldsymbol{\Delta x_2}$, $\boldsymbol{\Delta x_3}$ for the substitution in the loss-objective function, however, a linearized approximation is used for the variables $\boldsymbol{\Delta x_1}$, $\boldsymbol{\Delta}\mathbf{x_2}$, $\boldsymbol{\Delta x_3}$) in the constraint sets:

$$\mathbf{g}_4^{\prime k} + \mathbf{J_4}^{\prime k} \Delta\mathbf{x_4} = \mathbf{0} \tag{86}$$

The same holds for the inequality constraint set $\mathbf{h}$:

$$\mathbf{h}(\mathbf{x^k}) + \mathbf{A_4}^{\prime k} \Delta\mathbf{x} \leq \mathbf{0} \tag{87}$$

with

$$\mathcal{F}_{loss}^{\prime k} = \mathcal{F}_{loss}(\mathbf{x}_1^k, \mathbf{x}_2^k) - \mathbf{1}^T \mathbf{g_1}(\mathbf{x_1^k}, \mathbf{x_4^k}) - \mathbf{1}^T \mathbf{g_2}(\mathbf{x_2^k}, \mathbf{x_4^k})$$

$$\mathbf{c}^{\prime T^k} = -\mathbf{1}^T \mathbf{J_{14}}^k - \mathbf{1}^T \mathbf{J_{24}}^k \;\; ; \mathbf{M_4}^{\prime k} = -\sum_{i=1}^{m} \mathbf{M_{14}}_i^k - \sum_{i=1}^{l} \mathbf{M_{24}}_i^k$$

$$\mathbf{J_4}^{\prime k} = \mathbf{J}_{44}^k - \sum_{i=1}^{3} \left(\mathbf{J}_{4i}^k \mathbf{J}_{i4}^k\right) \;\; ; \mathbf{g}_4^{\prime k} = \mathbf{g}_4(\mathbf{x}^k) - \sum_{i=1}^{3} \mathbf{J}_{4i}^k \mathbf{g}_i(\mathbf{x}^k)$$

$$\mathbf{A_4}^{\prime k} = \mathbf{A}_4^k - \sum_{i=1}^{3} \left(\mathbf{J}_{i4}^k \mathbf{A}_i^k\right)$$

Assuming that a power flow has been calculated with high accuracy for the solution point $\mathbf{x^k}$, the following is valid: $\mathbf{g}(\mathbf{x}^k) = \mathbf{0}$. This leads to some simplifications in the above formulas:

$\mathcal{F}'^{k}_{loss} = \mathcal{F}_{loss}(\mathbf{x}_1^k, \mathbf{x}_2^k) \ \ ; \mathbf{g}_4'^{k} = \mathbf{0}$

The optimization problem formulated with (85) ... (87) is a QP formulation. Note that the matrices are still sparse. The optimization problem is now stated with the variables of the vector $\mathbf{\Delta x_4}$, i.e. the nodal voltage related variables.

Solving this problem with a standard QP program is possible, however, due to the large dimension of the problem ($2N$ variables), the number of non-zero matrix and vector elements gets very large, as long as no sparsity techniques are applied during the QP solution process.

In the following a derivation is given where the number of OPF variables is again reduced to a much smaller set. It must be noted, however, that sparsity is lost by doing the following steps.

4.4.2 Compact, non-sparse QP loss optimization problem (Quadratic incremental power flow)

The goal of this OPF loss formulation is to reduce the variable set to the same set as used for the compact QP cost optimization formulation as shown in the previous subsection 4.3.2. There are several ways to derive compact, non-sparse loss QP-optimization problem formulations. All these derivations have in common that at some point linearizations have to be applied to the original quadratic approximations of the equality constraints.

Without showing the derivations the compact loss optimization problem formulation is as follows:

$$\text{Minimize } \mathcal{F}_{loss} = \mathcal{F}_{loss}(\mathbf{x_1^k}, \mathbf{x_2^k}) + \Delta x_{1_N} + \mathbf{1}^T \Delta\mathbf{x}_1' + \begin{bmatrix} \mathbf{1}^T & \mathbf{0}^T \end{bmatrix} \Delta\mathbf{x}_2' \tag{88}$$

subject to

$$\begin{aligned} \alpha_{1_N} \Delta x_{1_N} \quad & + \boldsymbol{\alpha_1}'^{T^k} \Delta\mathbf{x}_1' + \boldsymbol{\alpha_2}^{T^k} \Delta\mathbf{x}_2' \\ & + \frac{1}{2} \left(\begin{bmatrix} \Delta\mathbf{x}_1'^T \Delta\mathbf{x}_2'^T \end{bmatrix} \mathbf{Q_{Loss}}^k \begin{bmatrix} \mathbf{\Delta x}_1' \\ \mathbf{\Delta x}_2' \end{bmatrix} \right) = \alpha_0 \end{aligned} \tag{89}$$

((89) is an extension to (83) and is called the **quadratic incremental power flow** equation.)
and

$$\mathbf{h}'(\mathbf{x_1^k}, \mathbf{x_2'^k}) + \mathbf{A}_1'^k \Delta\mathbf{x}_1' + \mathbf{A}_2'^k \Delta\mathbf{x}_2' \leq \mathbf{0} \tag{90}$$

Note that

- $\Delta \mathbf{x_1}^T = \left[\Delta \mathbf{x_1'}^T \Delta x_{1_N} \right]$. The separation of this vector into two parts is only needed for conceptual reasons.

- $\left[\mathbf{1}^T \;\; \mathbf{0}^T \right]$: This has to be represented in such a way, since the losses are, in the reduced variable set form, a linear function of the active power variables of PQ load nodes with manageable active load which represent only a subset of the vector $\Delta \mathbf{x_2}'$.

- $\Delta \mathbf{x_2}'$: This is the same vector of non-eliminated variables as in the compact cost optimization problem.

- in (89) the same variables appear as in the compact cost optimization problem (82) ... (84).

- one variable (Δx_{1_N}) does not show a quadratic extension in the equality constraint formulation of (89).

- the inequality constraints formulation of (90) is identical to the one of (84). However, it is assumed that no limits will be active for functions of the variable Δx_{1_N}. This can be justified by using an active power of a generator as this variable which is far away from its limits and/or which is not sensitive to optimum solution movements for different OPF problem conditions. This is important because this variable will be eliminated, as discussed below and it should not create any quadratic terms in the (linear) QP inequality constraint set. This assumption can be justified since usually no functions of this variable (it is an active generation variable) are used for inequality or equality constraints formulations. Only the variable itself (i.e. the corresponding active generation) can in principle be limited. In the actual OPF implementation care has to be taken that this variable should not be limited at the OPF optimum.

The OPF problem of (88) ... (90) can be transformed into a classical QP formulation by eliminating the variable Δx_{1_N}, i.e. replacing it in the objective function by the other non-eliminated variables of (89):

$$
\begin{aligned}
\text{Minimize } \mathcal{F}_{loss} \quad &= \mathcal{F}_{loss}(\mathbf{x_1^k}, \mathbf{x_2^k}) + \left(\mathbf{1} - \frac{\boldsymbol{\alpha_1}'}{\alpha_{1_N}} \right)^{T^k} \Delta \mathbf{x}_1' \\
&+ \left(\begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} - \frac{\boldsymbol{\alpha_2}'}{\alpha_{1_N}} \right)^{T^k} \Delta \mathbf{x}_2' \\
&- \frac{1}{2\alpha_{1_N}} \left(\left[\Delta \mathbf{x_1'}^T \Delta \mathbf{x_2'}^T \right] \mathbf{Q_{Loss}}^k \begin{bmatrix} \boldsymbol{\Delta} \mathbf{x}_1' \\ \boldsymbol{\Delta} \mathbf{x}_2' \end{bmatrix} \right) + \frac{\alpha_0}{\alpha_{1_N}}
\end{aligned}
\tag{91}
$$

subject to

$$\mathbf{h}'(\mathbf{x_1^k}, \mathbf{x_2'^k}) + \mathbf{A_1'}^k \Delta\mathbf{x_1'} + \mathbf{A_2'}^k \Delta\mathbf{x_2'} \leq \mathbf{0} \tag{92}$$

The exact derivation of the matrix $\mathbf{Q_{Loss}}^k$ cannot be given in this paper due to space limitations. Note however, that several derivations are possible. The problem to be solved is always to find the point at which during the derivations the quadratic terms are to be neglected or replaced by a linear approximation.

Note that an exact computation of this matrix $\mathbf{Q_{Loss}}^k$ can be very CPU time consuming and is usually not worth the effort [18]. The key in this OPF method is the right approximation of the quadratic terms by the right variable. It has been shown with prototypes that even a diagonal matrix approximation for the matrix $\mathbf{Q_{Loss}}^k$ can lead to good and fast convergence. However in any case, care has to be exercised by these approximations: They are the driving values for the optimization, i.e. they determine how fast the variables move towards the optimum, how much they move during the intermediate QP steps. Research is still going on in this area of OPF problem formulation and solutions look quite promising.

4.5 Class A algorithms: Linear programming (LP)

4.5.1 LP formulation

In the following formulations will be given which lead to practical applications of linear programming and finally to efficient programs.

According to class A a basic requirement is the derivation of linearized relations for the load flow. This can be either in the form of the Jacobian

$$\mathbf{J}\Delta\mathbf{x} = \mathbf{0} \tag{93}$$

or in the form of the incremental power flow

$$\boldsymbol{\alpha}_1^T \Delta\mathbf{x}' + \Delta x_{1N} = 0 \tag{94}$$

Note that in (94), as compared to (83), the equality constraint has been normalized in such a way that the factor associated with the variable x_{1_N} is 1. For both (93) and (94) it is assumed that a power flow has been solved with high accuracy around the operating point, leading to a right hand side value of $\mathbf{0}$.

Both forms (93), (94) can be readily incorporated in an LP-tableau.

Since these forms are equality constraints a part of the variables may be eliminated according to the requirements of the LP-algorithm.

A delicate problem is the formation of a linearized objective function. Thereby it is to be observed that the LP-algorithm requires a separable objective function

$$\text{Minimize} \quad \mathcal{F} = \mathbf{c}^T \mathbf{x} \tag{95}$$

Cost curves are a good example of separable objective functions. Quadratic cost curves for each generator are assumed to be the true cost curves for the following derivations. Note that general smooth, convex cost curves could also be taken and similar derivations could be made.

With quadratic cost curves the optimization problem is as follows:

$$\text{Minimize} \quad \mathcal{F}_{cost} = \sum C_i \tag{96}$$

$$\text{where} \quad C_i = \frac{1}{2} q_i P_i^2 + c_i P_i + \mathcal{C}_{i_o}$$

(separable quadratic cost functions)

In order to use an LP algorithm for the solution of this optimization problem a further approximate step must be considered, namely the conversion of real cost curves to piece-wise linear curves which can be done to any desired accuracy, see schematic sketch in Figure 1.

Abbildung 1: Cost curves (piece-wise linear)

An analytic expression for the approximation for the generating cost of one generating unit is

$$\begin{aligned} C_i &\geq d_{o_{1_i}} + d_{1_i} P_i \\ C_i &\geq d_{o_{2_i}} + d_{2_i} P_i \\ C_i &\geq d_{o_{3_i}} + d_{3_i} P_i \end{aligned} \tag{97}$$

Thereby the expressions $d_{o_{j_i}} + d_{j_i} P_i$ represent the straight lines which form the approximation to the quadratic cost curve.

For the purposes of the class A algorithm this model has to be converted to an incremental form whereby both costs and generating powers appear as variables.

$$C_{i_o} + \Delta C_i \geq d_{o_{j_i}} + d_{j_i}(P_{i_o} + \Delta P_i) \tag{98}$$

The vectors $\Delta\mathbf{C}$ and $\Delta\mathbf{P}$ may be replaced by general x_i- variables:

$$\begin{aligned} &\Delta\mathbf{P} \quad ...\Delta\mathbf{x}_1 \\ &\Delta\mathbf{C} \quad ...\Delta\mathbf{x}_5 \end{aligned} \tag{99}$$

Then

$$\mathcal{F}(\mathbf{x}) = C_o + [1,1,1,...1]\,\Delta\mathbf{x}_5 \tag{100}$$

subject to

$$\mathbf{diag}\,(\mathbf{d}_j)_i\,\Delta\mathbf{x}_1 - \Delta\mathbf{x}_{5_i} \leq \mathbf{C}_{o_i} - \mathbf{d}_{o_{j_i}}\mathbf{P}_{i_o} \tag{101}$$

(i = 1, 2, ... m (m: Number of generators to be optimized); j = 1, 2, ... S (S: number of straight line sections per generator))

Here it becomes obvious that the formulation of the cost function leads to numerous entries in the LP-tableau. At this point a relatively small number of straight line sections for generators is considered only so as to limit the size of the LP-tableau.

There are further relations in the form of inequality constraints to be considered for the tableau, namely limits on the control variables and functional constraints.

Again, the reasons of keeping the tableau small, generating powers P_i are considered as control variables only.

Hence, limits and functional constraints are given by

$$\begin{aligned} &+-^{**}\,\Delta\mathbf{x}_1^{'} &&\leq \mathbf{b}_v &&\text{(variable limits)} \\ &\mathbf{A}^{'}\Delta\mathbf{x}_1^{'} &&\leq \mathbf{b}_{fc}^{'} &&\text{(functional constraints)} \end{aligned} \tag{102}$$

(**: meaning that both the upper and lower limits of the variables must be considered)
where $\mathbf{A}^{'}$ can be full.

Beyond that there is the incremental power flow which is taken as the scalar equality constraint. It must be incorporated in the tableau. This is done by eliminating one of the control variables.

Thus the LP problem is given by

$$\text{M}inimize\;\; \mathcal{F}_{cost} = [1,1,1,...]\,\Delta\mathbf{x}_5 \tag{103}$$

subject to

$$
\begin{aligned}
&\mathbf{diag}\,(\mathbf{d}_j)_i\,\Delta\mathbf{x}_1^{'} && -\Delta\mathbf{x}_{5_i} && \leq \mathbf{C}_{i_o} - \mathbf{d}_{o_{j_i}}\mathbf{P}_{i_o} \;\; \text{(i = 1, 2, ... m-1)} \\
&+ - \Delta\mathbf{x}_1^{'} && && \leq \mathbf{b}_v \\
&\mathbf{A}^{'}\Delta\mathbf{x}_1^{'} && && \leq \mathbf{b}_{fc}^{'} \\
&\mathbf{D}\Delta\mathbf{x}_1^{'} && -\Delta\mathbf{x}_{5_m} && \leq \mathbf{C}_{m_o} - \mathbf{d}_{o_{jm}} \;\; \text{(only } m^{th} \text{ variable)}
\end{aligned}
\tag{104}
$$

The last entry is due to the elimination of the equality constraint. Hence $\Delta\mathbf{x}_1^{'}$ comprises $m-1$ variables only (m = number of generating powers to be optimized).

It must also been observed that $\mathbf{x}_5$ is not constrained.

As the set of relations above stands it is quite sparse which may be an advantage depending on the method of solution to be chosen.

If a small number of variables is desired the variables of the vector $\mathbf{x}_5$ can be eliminated and expressed by components of $\mathbf{x}_1$ which leads to a tableau whose variables are control variables only (generating powers).

This general approach to the use of LP within class A algorithm may be extended to other OPF-problems as long as a separable cost function can be formulated.

A most recent application of this kind is loss minimization (Stott, [26]) whereby losses are approximated by linearized relations in terms of active and reactive injections. A basic requirement in this approach is an exact representation of the linear incremental power flow. The segments to the left and the right of the operating point need not be accurate.

The problem of choosing the right approximation is pronounced in the case of loss minimization by reactive injections only. As long as there is no technical constraint on reactive injections the straight line subsections are the only means for the limitation of the variables. The subsections must be made artificially smaller in the course of the iterations (e.g. dichotomy).

4.5.2 LP-solution

For purposes of illustration this particular method of solution is dispensed in more detail thereby referring to the standard LP method in appendix A.1.

The starting point is an operating point of the power system given by a load flow solution. This solution is designated by the vector of P_{o_i}'s around which an improved solution is sought. According to the linearized model the individual P_{o_i}'s are located at the breakpoints of the straight line sections (besides one variable). The situation for one generator is depicted by the sketch in Fig. 2.

Abbildung 2: Change of segment in piece-wise linear cost curves

Since an incremental model is used it is to be observed that the increments must be feasible

$$\Delta P_i \geq 0 \ \text{ or } \ \Delta x_1^{'} \geq 0 \tag{105}$$

For this purpose each generator power variation is to be modeled by two LP-variables as indicated in Fig. 2.

At this point it is assumed that the vector $\mathbf{\Delta x}_5$ is eliminated and substituted by $\mathbf{\Delta x}_1^{'}$. As a consequence the cost function is modified and will consist of $\mathbf{c}^T \mathbf{\Delta x}_1^{'}$ whereby the c's are the result of a transformation.

$$\text{Minimize } \ \mathcal{F}_{cost} = \mathbf{c}^T \mathbf{\Delta x}_1^{'} \tag{106}$$

Since the starting point was a solution to the load flow and, of course, to a previous LP step the vector $\mathbf{\Delta x}_1^{'}$ is zero and can be considered the non basic vector of the LP tableau (see appendix A.1). Thereby it is taken for granted that at this point no control variables are exceeded. Functional constraints are not considered at the moment.

Thus, a classical LP tableau can be established whereby the vector $\mathbf{\Delta x}_1^{'}$ corresponds to the non-basic solution $\mathbf{x}_D$ of the tableau. The slack variables (Luenberger [8]) are the basic variables.

The relative cost vector will indicate which variable will have to become a basic variable.

The LP-tableau is exactly the one in Luenberger [8].

A change of base may be caused by one of the following items:

- due to the change of α's for the new load flow solution a cost coefficient has changed sign
- the straight line approximation to the quadratic cost curve of a generator has been changed.

These items are assumed to be of such a nature that a cost coefficient has changed its sign.

Beyond that there are indications that constraints and limits have been exceeded. These may be due to

- a change in the straight line approximations of the cost curves
- functional constraints which have not been considered so far

- consequences of an updated load flow solution, e.g. the m^{th} control variable not explicit in the tableau has exceeded its limits

These constraint violations require another type of change of base as explained in appendix A.1.2.

The necessary changeover to a feasible solution may be performed step-by-step, i.e. constraint by constraint in order to keep the tableau small.

The computational effort in using the linear programming method depends on

- the number of update operations for the incremental power flow

- the number of update operations for the inequalities

Updating on the right hand side is not very demanding. Updating the coefficient of the tableau results in a complete recalculation of the partially inverted tableau. In the iterative process updating is necessary whenever a new load flow solution becomes available.

It is obvious that the overall effort depends on the dimension of the tableau which can be kept to a minimum if the cost curves are modelled by small number of segments (straight line approximations). However, in order to achieve the required accuracy the lengths of the segments have to be reduced as the number of iterations increases. This process is called segment refinement.

The idea of segment refinement is to keep the number of segments in the tableau fixed and to reduce the lengths of the segments.

One possible procedure is the following: The tableau always comprises a fixed number of segments which cannot be less than two, if limits (artificial or real) are applied on the outside of the segments or four, if the limits are located at a distance of the operating region.

Whenever an optimal solution for a given segmentation is found the lengths of the segments are reduced thereby changing the coefficients of the rows in the tableau corresponding to the representation of the cost curves. If at this point the solution turns out to be infeasible a change of base has to be performed as outlined in the appendix A.1. (problem a).

From here on the refinement process can be continued or a new load flow solution can be asked. The decision will depend on the segment size, the relative improvement of the objective function and the mismatches at the iteration where the optimization is performed.

The overall effort depends on the dimension of the tableau which can be kept to a minimum if the cost curves are modelled by a small number of segments only, namely in an adaptive way in the vicinity of the solution point (segment refinement). However, adapting the segments also requires updating of the tableau.

Finally, the various steps in the course of one iteration will be as follows:

- 1. solve an ordinary load flow
- 2. extract Jacobian or incremental power flow
- 3. create or update segments of cost function, form functional constraints
- 4. generate LP-tableau
- 5. solve LP
- 6. check: size of segments; active limits; size of corrections resulting from LP
- 7. if corrections, steps etc. small enough stop, otherwise go to 1.

The effectiveness of linear programming in class A methods will depend on the programming skill, in particular in handling the tableau, base change operations, updating and segment refinement.

4.6 Class A algorithms: Quadratic programming (QP)

4.6.1 QP formulation

As under 4.5.1 a basic requirement is the derivation of linearized relations for the load flow. Again this can be done by taking the Jacobian (93) or by working out the incremental power flow (94). Either form will be needed in the formulation of the Lagrangian which plays a central role in QP.

The objective function can either be quadratic (cost) or linear (losses) as given by the relations (68) and (69).

The quadratic function describing operating cost consists of a quadratic form having a diagonal matrix only (separable functions) as given by (78).

$$\text{Minimize } \mathcal{F}_{cost} = \mathcal{F}_{cost}(\mathbf{x_1}^k) + \mathbf{c}^{T\,k}\Delta\mathbf{x_1} + \frac{1}{2}\Delta\mathbf{x_1}^T\mathbf{diag}(q_i^k)\Delta\mathbf{x_1} \qquad (107)$$

In order to convert the loss minimization problem to a quadratic one the incremental power flow is extended as explained in chapter 4.4. Thereby a number of variables is eliminated and the incremental power flow is incorporated in the objective function yielding the relation (91). This can be done if the slack power can be expressed by other non-eliminated variables, i.e. active power, voltage magnitude or reactive injections variables.

The problem is thus brought into a form where a quadratic objective function is left without the need to consider an equality constraint any further. This can also be understood by the fact that the n^{th} reactive injection need not be considered since there is no cost attached to it.

In the cost minimization problem (MW dispatch) the equality constraint cannot be eliminated because all control variables have a quadratic or in general convex, non-linear cost function.

Thus, the general QP-problem is formulated as follows

$$\text{Minimize } \mathcal{F} = \mathcal{F}^k + \mathbf{c}^T \Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^T \mathbf{Q} \Delta\mathbf{x} \tag{108}$$

subject to

$$\mathbf{g}(\mathbf{x}^k) + \mathbf{J}\Delta\mathbf{x} = \mathbf{0} \tag{109}$$

As outlined above the equality constraint disappears when a compact loss minimization problem with a reduced variable set is considered.

Beyond that variable and functional constraints have to be attached which in general will be given by

$$\mathbf{h}(\mathbf{x}^k) + \mathbf{A}\Delta\mathbf{x} \leq \mathbf{0} \tag{110}$$

Here $\Delta\mathbf{x}$ is understood as the deviation of the control variable from its operating point as determined by the power flow.

At this point the Lagrangian in terms of the deviations can be formulated as

$$\begin{aligned} \mathcal{L} &= \mathbf{c}^T \Delta\mathbf{x} + \tfrac{1}{2}\Delta\mathbf{x}^T \mathbf{Q} \Delta\mathbf{x} \\ &+ \lambda^T(\mathbf{g}(\mathbf{x}^k) + \mathbf{J}\Delta\mathbf{x}) + \mu^T(\mathbf{h}(\mathbf{x}^k) + \mathbf{A}\Delta\mathbf{x}) \Rightarrow min. \end{aligned} \tag{111}$$

Since the Lagrangian in this form is quadratic one of the QP- algorithms may be applied for the solution of the QP-problem.

4.6.2 QP solution

The Lagrangian above or its components are suitable for a direct application of a QP-algorithm.

One example is the use of the Beale algorithm which is successful for networks up to about 250 - 300 nodes and to 50 - 80 control variables.

For larger networks other methods have to be used.

For the dispatch problem (MW-Dispatch), i.e. cost minimization the method outlined under A.2 is quite suitable. The important feature of the dispatch problem is the fact that $\mathbf{Q}$ is a diagonal matrix and the equality constraint is a scalar only.

The system to be treated for the unconstrained solution is extremely sparse as shown below

$$\mathbf{M}\mathbf{u_o} = \begin{bmatrix} -\mathbf{c} \\ \mathbf{b}_1 \end{bmatrix} \tag{112}$$

Due to the sparsity of the matrix M the formation of

$$-\begin{bmatrix} \mathbf{A} & \mathbf{0} \end{bmatrix} \mathbf{M}^{-1} \begin{bmatrix} \mathbf{A}^T \\ \mathbf{0} \end{bmatrix} \tag{113}$$

will benefit considerably from various sparsity techniques.

As explained under A.2 the further steps are LP-like and in the end the final solution is obtained by superposition.

$$\mathbf{u}_c = \mathbf{u}_o + \Delta\mathbf{u} \tag{114}$$

Working with this method will show that it is advisable to add constraints step by step, in particular functional constraints in order to maintain a small tableau.

The interesting feature of the lastmentioned algorithm is that it is fully based on linear methods. In a first step the unconstrained problem is linear. The superimposed corrections are determined by linear programming methods. The linear methods are fully effective if the sparsity of the system can be exploited.

In summary, the various steps in the course of one iteration will be as follows:

- 1. solve an ordinary load flow
- 2. extract Jacobian or incremental power flow
- (2.a. extract extended incremental power flow for loss minimization)
- 3. setup sparse system which determines the unconstrained solution
- 4. generate LP-tableau
- 5. solve LP
- 6. determine superimposed solution and update
- 7. if corrections, steps, etc. small enough stop, otherwise go to 1.

4.7 Summary

In summary the class A OPF algorithms are based on the iterative and separate use of the power flow to solve for a given operating point and a LP or QP for the optimization problem around the power flow solution.

The power flow part of these class A OPF algorithms is the conventional power flow as known from student text books. All special features like PV-PQ node type switching, local tap control can be handled by the power flow.

The classical LP and QP algorithms as described in mathematical text books are often quite slow for the solution of the OPF optimization problem. In the appendix some points are discussed about efficient handling of the LP and QP algorithms considering the special features of the OPF.

In principle the only necessary link between the power flow part and the optimization part is the transfer of the operating point x^k, representing the OPF variables: The power flow solution is transferred to the OPF to be used as the solution around which the approximations are made. Then the LP or QP algorithm is solved. The optimal solution (note: optimality is valid only with respect to the approximations around the previous power flow solution) is transferred back to the power flow and represents another power flow input data set. The power flow corrects the approximations made in the preceding LP or QP optimization. Thus the power flow adapts the nodal voltages and the slack power such that the mismatches are below predefined, small tolerance values. By executing this procedure several times the power solution point tends to go toward the optimum, i.e. the result of the very last LP or QP solution should be identical (within a certain tolerance) to the preceding power flow solution. At this point the optimal solution is reached.

5 OPF CLASS B. POWER FLOW INTEGRATED IN OPTIMIZATION ALGORITHM

5.1 Introduction

In this section the OPF formulation is solved by an integrated method as compared to the OPF formulation of the Class A where the power flow is separated from the optimization part.

First the easiest case is discussed: The solution of the OPF problem with a given set of equality constraints only. Although this certainly does not satisfy the real-world constraints (which would include inequality constraints), it is discussed here in order to show the principles of the Newton-Raphson based approach which are also used in the following sections. There the more realistic OPF problem is solved with consideration of both equality and inequality constraints.

The objective function will usually be formulated as a general function $\mathcal{F}(\mathbf{x})$, however, where the OPF algorithm results in special cases for either cost or loss objective functions special discussion is given.

The same holds for the inequality constraints $\mathbf{h}(\mathbf{x})$: When any special derivation results this is discussed.

5.2 Solution of OPF with equality constraints only

The problem is as follows:

$$\begin{aligned} &\text{Minimize} \quad \mathcal{F}(\mathbf{x}) \\ &\text{subject to} \;\; \mathbf{g}(\mathbf{x}) = \mathbf{0} \end{aligned} \tag{115}$$

The solution is based on the Lagrange formulation (the index $_{eq}$ refers to the **equality** constrained OPF problem):

$$\mathcal{L}_{eq} = \mathcal{F}(\mathbf{x}) + \lambda^T \mathbf{g}(\mathbf{x}) \tag{116}$$

The optimality conditions for (116) are:

$$\begin{aligned} \frac{\partial \mathcal{L}_{eq}}{\partial \mathbf{x}} &= \frac{\partial}{\partial \mathbf{x}} \left(\mathcal{F}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x}) \right)\Big|_{\mathbf{x}=\widehat{\mathbf{x}},\lambda=\widehat{\lambda}} &= \mathbf{0} \\ \frac{\partial \mathcal{L}_{eq}}{\partial \boldsymbol{\lambda}} &= \mathbf{g}(\mathbf{x})\Big|_{\mathbf{x}=\widehat{\mathbf{x}},\lambda=\widehat{\lambda}} &= \mathbf{0} \end{aligned} \tag{117}$$

In (117) the following substitutions can be made; $\mathbf{J}$ is the Jacobian matrix:

$$\mathbf{J} = \frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}} \tag{118}$$

Thus the following system has to be solved to achieve these optimality conditions:

$$\begin{aligned} \tfrac{\partial \mathcal{F}(\mathbf{x})}{\partial \mathbf{x}} + \mathbf{J}^T \boldsymbol{\lambda} &= \mathbf{0} \\ \mathbf{g}(\mathbf{x}) &= \mathbf{0} \end{aligned} \tag{119}$$

(119) can be summarized as one non-linear system:

$$\mathbf{W}(\mathbf{x}, \boldsymbol{\lambda}) = \mathbf{0} \tag{120}$$

This non-linear system must be solved by any efficient method. General mathematical methods for solving non-linear systems can be used. However, the solution based on the Newton approach is most often employed.

5.2.1 Newton based solution

(119) or (120) can be solved by the iterative Newton-Raphson approach which leads to the following linear system for the solution of (120) (the index ^{k} refers to the value of the associated variable at iteration k):

$$\mathbf{W}(\mathbf{x^k}, \lambda^{\mathbf{k}}) + \left.\frac{\partial \boldsymbol{W}}{\partial \boldsymbol{x}}\right|_{\mathbf{x}=\boldsymbol{x}^k, \boldsymbol{\lambda}=\boldsymbol{\lambda}^k} \Delta \boldsymbol{x}^{\boldsymbol{k}} + \left.\frac{\partial \boldsymbol{W}}{\partial \boldsymbol{\lambda}}\right|_{\mathbf{x}=\boldsymbol{x}^k, \boldsymbol{\lambda}=\boldsymbol{\lambda}^k} \Delta \boldsymbol{\lambda}^{\boldsymbol{k}} = \mathbf{0} \tag{121}$$

Now, the linear system which must be solved iteratively, takes the form:

$$\begin{bmatrix} \mathbf{H} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta} \mathbf{x^k} \\ \boldsymbol{\Delta \lambda}^{\mathbf{k}} \end{bmatrix} = \begin{bmatrix} \mathbf{r}^k \\ \mathbf{g}^k \end{bmatrix} \tag{122}$$

with

$$\mathbf{H} = \mathbf{H_{eq}} = \frac{\partial \mathcal{F}^2(\boldsymbol{x})}{\partial \boldsymbol{x}^2} + \mathrm{diag}(\boldsymbol{\lambda}) \frac{\partial \boldsymbol{g}^2(\boldsymbol{x})}{\partial \boldsymbol{x}^2} \tag{123}$$

and

$$\begin{bmatrix} \mathbf{r}^k \\ \mathbf{g}^k \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{eq^k} \\ \mathbf{g}_{eq^k} \end{bmatrix} = \begin{bmatrix} -\left.\left(\tfrac{\partial \mathcal{F}(\mathbf{x})}{\partial \mathbf{x}} + \mathbf{J^T} \lambda\right)\right|_{\mathbf{x^k}, \lambda^{\mathbf{k}}} \\ -\mathbf{g}(\mathbf{x^k}) \end{bmatrix} \tag{124}$$

(122) is solved iteratively, i.e. the values for $\mathbf{x}$ and λ from the previous iteration are inserted into $\mathbf{H}$ and $\mathbf{J}$ and the right hand side of (122). Then (122) is solved for $\Delta\mathbf{x}$ and $\Delta\lambda$ which again are used to update the values for $\mathbf{x}$ and λ as follows:

$$\mathbf{x^{k+1}} = \mathbf{x^k} + \Delta\mathbf{x^k} \tag{125}$$

$$\lambda^{\mathbf{k+1}} = \lambda^{\mathbf{k}} + \Delta\lambda^{\mathbf{k}}$$

Doing this for some iterations will usually result in a convergent solution. This solution is the optimum for the OPF equality constrained problem as given in (115), i.e. the resulting values for $\mathbf{x}$ and λ are the values where the objective function $\mathcal{F}(\mathbf{x})$ is minimal and where all equality constraints $\mathrm{g}(\mathbf{x})$ are satisfied.

(122) is a linear system which, in principle, can be solved by any linear equation solving algorithm. Note, however, that the matrices can be very sparse and thus specialized sparsity algorithms must be applied to solve the system efficiently [21].

Decoupling principles as used in the decoupled power flow could be used if polar coordinates are chosen. However, experience has shown that for the OPF decoupling can have drawbacks when looking at overall robustness. However, in general, most algorithms which have been developed for power flows, can be applied to the equality constrained OPF problem with little modifications.

The conclusion from this subsection is, that whenever the equality constraint for an OPF problem is given the solution is not more difficult than the solution of an ordinary power flow problem. The problem, however, are the inequality constraints. If one would know beforehand which inequality constraints will be active, i.e. limited in the OPF optimum, one could include these constraints as equality constraints from the beginning of the optimization and solve with the procedure discussed above.

The active set of inequality constraints, however, is not known in advance and thus special algorithms have to be found to determine whether to make an inequality constraint active or not. This is discussed in the next subsection.

5.3 OPF solution with consideration of inequality constraints

5.3.1 Introduction

The Kuhn-Tucker conditions (see (61)) determine if at any solution point a relative optimum has been found, i.e. for all inequality constraints which has been included in the active constraint set, the Lagrange multiplier μ must be positive in order to justify the inclusion of the corresponding inequality constraint in the active set. This active set includes all inequality constraints being binding at their respective limits. In the OPF class B, discussed in this section, two approaches are used to solve the inequality constraints problem: The handling of inequality constraints by penalty techniques, mainly used for variable related limits and the explicit modelling of functional inequality constraints as functional equality constraints, once they become active at their limits. Note that active functional constraints can also be modelled by the penalty approach.

The penalty based approach leads to an extension of the equality constrained OPF problem as discussed in the previous subsection, i.e. the possible inequality constraints are handled in a quadratic form as extensions to the original objective function. By using small or large weights (penalties) for these additional quadratic objective functions terms, the equality constrained OPF problem is forced to a solution which is optimal with respect to the equality constraint set, but in addition to that, considers the inequality constraints. Those with a large weighting factor, will have the effect of being binding, i.e. limited, those with small weighting factors will be free, i.e., these inequality constraints will not be binding at their limits in the OPF optimum. In summary, this penalty technique based approach can be seen as an equality constrained OPF problem with an artificially extended objective function.

This approach has one problem: When should an inequality constraint be held at its limit and when should it be freed.

It must be noted that there are no penalty based approaches known today, for solving the Kuhn-Tucker conditions with straightforward solution processes. Today, in order to improve speed, convergence and robustness, trial passes, heuristics or other similar measures are used in this approach. The use of very fast sparsity routines for updating factorized matrices, to add or remove rows and columns is usually the selling point for the penalty based methods for the OPF problems. Without them this approach would not make much sense, since very quickly they would become slow and the use of some heuristics or trial iterations for the determination of the correct constraint set could not be justified any more.

5.3.2 Penalty term approaches for handling inequality constraints

When using the penalty term approach two main categories of inequality constraints can be distinguished.

- Limits on OPF variables

- Limits on output variables, i.e. non-linear or linear functions of OPF variables

The distinction is done because these two types can be handled with different efficiency in the penalty term based OPF algorithms. Among the various constraints of these categories most can be treated in the same way in the algorithms. However, there are distinct differences between the implementations of these types.

In the following subsections the penalty term approaches for the two inequality constraint types are discussed.

Limits on OPF variables The general idea of the penalty term techniques is to add an additional quadratic function for every inequality constraint to the original objective function. By using large weights for these quadratic functions, the optimization algorithm is forced to move constraint values, which are thus made artificially expensive, to desired limit values. The effect of this penalty term technique corresponds to including the violated constraint into the active set.

The function added to the original objective function looks as follows with limited OPF variables x_i:

$$\mathcal{L} = \mathcal{L}_{eq} + \sum \left(\frac{1}{2} \mathcal{W}_i (x_i - x_{i_{Lim}})^2 \right) \tag{126}$$

In (126), the Lagrangian $\mathcal{L}_{eq}$ corresponds to the Lagrangian as given in (116) of the equality constrained OPF problem. The $\sum$ goes over all control variables x which could become limited at the OPF optimum.

The Lagrange optimality conditions are derived in exactly the same way as in (119). The main difference lies in the derivatives of $\mathcal{L}$ with respect to the variables $\mathbf{x}$:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \frac{\partial \mathcal{L}_{eq}}{\partial \mathbf{x}} + \operatorname{diag}(\mathcal{W})(\mathbf{x} - \mathbf{x}_{Lim}) \tag{127}$$

Making now the same derivation as for the equality constrained OPF problem, i.e. solve the optimality conditions by an iterative Newton solution, the matrices $\mathbf{H}$ and the right hand side of the equation (122) must be adapted:

$$\mathbf{H} = \mathbf{H}_{eq} + \operatorname{diag}(\mathcal{W}) \tag{128}$$

$$\mathbf{r}^k = \mathbf{r}^k_{eq} - \operatorname{diag}(\mathcal{W})(\mathbf{x^k} - \mathbf{x}_{Lim}) \tag{129}$$

Adding the terms for a possibly binding OPF variable i to the original objective function with a large value for $\mathcal{W}_i$ will force the variable x_i within ϵ to its limit value x_{iLim}. The rule is that for larger $\mathcal{W}_i$ smaller ϵ values will result. Note that by adding this term to the objective function, i.e. also to the Lagrangian, the optimality conditions and also the subsequent Newton-based solution matrices are changed. This is shown in the above equations (126) ... (129). In (122) only diagonal terms and the right hand side are changed (see (128)) with this type of constraint which means that a fast factor update technique can be used to update the factorized matrix. A large value $\mathcal{W}_i$ is used to enforce the constraint, a small value $\mathcal{W}_i$ is used to relax the constraint. The sparsity schemes, i.e. the fill-in patterns are not affected whether this constraint is activated or not during the iterations.

Other techniques can be used to speed up this process: Assuming that a variable x_i violates its limit by $+\Delta x_i$ in the present iteration the limit value $x_{i_{Lim}}$ can be shifted by $-\Delta x_i$ so that in the next iteration the variable x_i will be forced near its real limits. Doing this iteratively has the advantage that only the right hand side of the iterative solution process has to be changed and not the matrix factors. However, the speed gain could be offset by less accuracy in the limit enforcement.

The question when to enforce a limit is usually quite simple, i.e. whenever it violates its limit. However, the problem when to relax a variable during the solution process, i.e. when to use small $\mathcal{W}_i$ values, is not as clear. The use of quadratic penalty terms in second order methods, however, tells, if an enforced, highly penalized variable is truly binding or not: If the variable is on the violated side by a value ϵ it can be assumed that the variable is actually binding. If this is not the case, the variable should be freed, i.e. the weight variable must be reduced to a small, non-penalizing value.

Another method is the usage of soft constraints, i.e. the enforcing of an inequality constraint i with a value for $\mathcal{W}_i$ being finite and much less than the maximum value needed for complete inequality constraint enforcement. By doing this an intermediate solution can be obtained which can show which of the variables tend to go their respective limits and which ones not.

It is obvious that the chance of finding the active inequality constraints immediately is quite low. Thus trial iterations can be employed to find a better set of binding inequality constraints. This is usually done by holding the matrices involved constant, i.e. no refactorization in done. Only the set of possibly binding constraints is changed from trial iteration to the next. Note, that for this reason, trial iterations can be much faster than the normal Newton-based iterations.

Limits on output variables Output variables are represented by functions of OPF variables. Branch flow or voltage magnitude (only when using rectangular coordinates) constraints are typical examples for this constraint type. Two different ways to implement them are possible. One method is to use the same technique as for state variables, i.e. the addition of quadratic penalty terms for each potentially binding output variable. In the other method those inequality constraints which have been determined by some heuristic method to become active are explicitely added as equality constraints, i.e. they are treated in exactly the same manner as equality constraints.

The treatment of equality constraints has been discussed in the previous subsection. Note, however, that adding or removing equality constraints must be done with consideration of sparsity techniques in order to maintain overall speed. Further a Lagrangian multiplier has to be used whose sign indicates if the constraint should be active or not. This method of handling inequality constraints is not discussed further in this text.

When adding a functional inequality constraint $h_i(x)$ in penalty form, the general form for the Lagrangian function looks as follows :

$$\mathcal{L} = \mathcal{L}_{eq} + \sum_{i=1}^{H} \frac{1}{2} \mathcal{W}_i \left(h_i(\mathbf{x}) - h_{i_{Lim}} \right)^2 \tag{130}$$

(H=number of output variable constraints)

The optimality conditions (first order derivations) and the necessary matrices and right hand sides for the Newton based solution process (second order derivations) are not given here for space reasons. Their derivations, however, are straightforward.

The constraints would be enforced by either changing the weighting factor $\mathcal{W}_i$ or by moving the limits in order to enforce or relax the inequality constraint i. This penalty approach for output variables is, mathematically seen, possible, however, new terms will be created in the optimality condition matrices and its subsequent Newton-based solution process which will need sophisticated matrix-factor updating algorithms in order to maintain a fast solution process. However, the usual output variable inequality constraints do not destroy the general sparse structure of the Newton based OPF solution process and in principle do allow sparsity storage and matrix factor techniques.

The rules to enforce and to relax a variable by changing the weight h_i are in analogy to the procedure for handling limits on OPF variables by penalty techniques. Thus trial iterations, soft limit enforcement and other heuristic techniques can be applied.

However, note, when using penalty techniques, no systematic algorithm exists to determine which inequality constraints should be relaxed and which should be enforced at any stage during the Newton solution process.

Thus, convergence problems are quite common when the network is not tuned to this penalty based approach. Tuned penalty based algorithms for OPF problems can converge well and fast, however, one tuning set might only be valid for a small load variation and must be adapted to other load conditions.

5.4 Summary

The OPF class B algorithms solve iteratively for the Kuhn-Tucker conditions without explicitely using a conventional power flow. Thus in this class B of OPF algorithms all active constraints, i.e. all power flow equality constraints and all binding inequality constraints, the objective function reduction and the OPF variable movements are handled simultaneously. The OPF class B can be compared with the conventional power flow solved with the Newton-Raphson method. The main problem of the OPF class B algorithms lies in the handling of inequality constraints, i.e. the determination of the set of binding inequality constraints. This is done with heuristic methods which include mainly trial iterations and soft limit enforcement.

6 FINAL EVALUATION OF THE METHODS

As with the ordinary power flow OPF methods are judged by their performance with respect to speed, versatility and robustness. At this point in time, however, there is no single OPF method which meets all requirements satisfactorily.

Class A and class B methods have their relative merits and perform well for one or the other particular application. In any one problem, however, a method could show poor performance.

LP methods in class A have the advantage of treating constraints in a systematic and efficient way. However, cost minimization and loss minimization, although being treated by this approach are not equally efficient. Constraints can be treated well in both cases whereas the exact extremum of the objective function can be reached in case of cost minimization only. The loss minimum is approximated.

When applying QP methods in class A both abovementioned problems can be handled accurately. Cost minimization is at least as efficient as with LP. Loss minimization is hampered by the cumbersome quadratic form specifying the objective function and its treatment by the QP algorithm. The experience is, however, that a few iterations are needed only.

Class A methods are also attractive because the starting point is a solved load flow which in most cases represents a feasible solution for the optimization problem. Quite often the iterative solutions in the beginning need not be very accurate. So the total number of load flow iterations is not considerably larger than for an ordinary load flow, e.g. twice as high.

Class B methods are attractive at a first glance . They solve the problem, i.e. they meet the optimality conditions in a global way. Convergence in the Newton approach is very good. However, when considering the way in which constraints have to be handled its attractiveness is moderated. Heuristics and tuning are needed which is somewhat compensated by the advantage that sparsity techniques can be employed, refactorization of the Hessian is avoided and well-known techniques of the ordinary load flow are applicable.

At the moment it seems that class A methods are taking the lead and this will be even more so when LP- and QP-methods are being further improved.

A APPENDIX

A.1 Linear programming (LP) algorithms

A.1.1 The basic linear programming method (Simplex)

In the following a series of LP-methods and -algorithms is presented which follows closely Luenberger [8]. The nomenclature and definitions are taken from there.

The standard linear programming problem is defined as

$$\mathcal{F} = \mathbf{c}^T\mathbf{x} \quad \Rightarrow \quad min \tag{131}$$

subject to:

$$\begin{aligned} \mathbf{Ax} &= \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned} \tag{132}$$

where

- $\mathbf{x}$ is the vector of unknowns ($\mathbf{x}$ comprises both original and LP-slack variables), dim $\mathbf{x}$ = n
- $\mathbf{c}$ is the vector of cost coefficients
- $\mathbf{A}$ is an m x n matrix
- $\mathbf{b}$ is the vector specifying the constraints, dim $\mathbf{b}$ = m

By partitioning the matrix $\mathbf{A}$ into $\mathbf{B}$ (m x m) and $\mathbf{D}$ (m x n-m), the vector $\mathbf{x}$ into $\mathbf{x_B}$ and $\mathbf{x_D}$ the problem is formulated as

$$\mathcal{F} = \mathbf{c_B}^T\mathbf{x_B} + \mathbf{c_D}^T\mathbf{x_D} \quad \Rightarrow \quad min \tag{133}$$

subject to

$$\begin{array}{llll} \mathbf{B} & \mathbf{x_B} + \mathbf{D} & \mathbf{x_D} & = \mathbf{b} \\ & \mathbf{x_B} & & \geq \mathbf{0} \\ & & \mathbf{x_D} & \geq \mathbf{0} \end{array} \tag{134}$$

where

- $\mathbf{B}$ is the basis,

- $\mathbf{x_B}$ is the basic solution and
- $\mathbf{x_D}$ is the non-basic solution.

Since it is known that the optimum solution will be found at one of the feasible basic solutions, the latter are checked only.

At the start it is assumed that a feasible basic solution is available, i.e. $\mathbf{x_B} \geq \mathbf{0}$, $\mathbf{x_D} \geq \mathbf{0}$. Methods will be shown later which allow to find a feasible solution if such one is not given. Then

$$\mathbf{x_B} = \mathbf{B}^{-1}\mathbf{b} \tag{135}$$

or

$$\mathbf{x_B} = \mathbf{B}^{-1}\mathbf{x_B} - \mathbf{B}^{-1}\mathbf{D}\mathbf{x_D} \tag{136}$$

The cost function z is given by

$$\begin{aligned} z &= \mathbf{c_B}^T(\mathbf{B}^{-1}\mathbf{b} - \mathbf{B}^{-1}\mathbf{D}\mathbf{x_D}) + \mathbf{c_D}^T\mathbf{x_D} = \\ &= \mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{b} + (\mathbf{c_D}^T - \mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{D})\mathbf{x_D} \end{aligned} \tag{137}$$

The last term is called the relative cost vector consisting of relative cost coefficients

$$\mathbf{r}^T = \mathbf{c_D}^T - \mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{D} \tag{138}$$

These relations are put in a frame which is called the tableau

$$\mathcal{T} = \begin{bmatrix} \mathbf{U} & \mathbf{B}^{-1}\mathbf{D} & \mathbf{B}^{-1}\mathbf{b} \\ \mathbf{0} & \mathbf{c_B}^T - \mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{D} & -\mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{b} \end{bmatrix} \tag{139}$$

whereby the left side matrix $\begin{bmatrix} \mathbf{U} \\ \mathbf{0} \end{bmatrix}$ is superfluous and need not be stored or manipulated ($\mathbf{U}$ is a unity matrix).

The tableau contains the following important information.

- $-\mathbf{c_B}^T\mathbf{b}^{-1}$ is the negative value of the cost function of the current base
- $\mathbf{B}^{-1}\mathbf{b}$ is the base vector (current)
- $\mathbf{c_B}^T - \mathbf{c_D}^T\mathbf{B}^{-1}\mathbf{D}$ is a row vector whose elements indicate by their sign if the cost function can be further decreased

A negative sign of an element of the relative cost vector says that a further decrease of the objective function is possible. The change of the corresponding non-basic variable in $\mathbf{x_D}$ against a basic variable in $\mathbf{x_B}$ will yield this decrease. The basic variable is located by checking the ratios y_{i_o}/y_{ij} and taking the smallest positive values (y_{i_o} = value of $\mathbf{x_B}$ in row i, y_{ij} = coefficient in column j which has the negative cost coefficient).

The base change is executed by manipulating all elements of $\mathcal{T}$. The minimum of the objective function is found when all cost coefficients are positive (=optimum feasible basic solution).

A.1.2 Changeover from a non-feasible to a feasible solution

Problem statement In LP- and QP-problems there are situations or starting solutions which are not feasible, i.e. $\mathbf{x_B} < \mathbf{0}$. This means that the base point is outside the feasible region.

If a feasible region exists a feasible basic solution can be reached by one or several base change operations. The operations will depend on the specific problem. In the OPF- algorithms two kinds of problems are encountered, namely

- Problem a.: A constraint is added to the tableau which generates a negative slack variable when the current basis solution is inserted
- Problem b.: The basic solution is not feasible right from the beginning

Problem a. is faced in LP-based OPF methods, e.g. after completing a load flow or after segment refinement. The cost coefficients may be the same or may have changed also. A change of base is necessary. The question is how to perform the base change operation.

Problem b. is found in the QP-method which treats constraints by LP-steps, see appendix A.2. In this particular case base change operations are confined to the row with the negative base value and the column where i=j (diagonal).

Solution of problem a. For the explanation of the algorithm the LP-tableau is extended the following way:

$$\mathcal{T} = \begin{bmatrix} \mathbf{U} & \mathbf{0} & \mathbf{B}^{-1}\mathbf{D} & \mathbf{B}^{-1}\mathbf{b} \\ \mathbf{d}^T & 1 & \mathbf{0}^T & b_A \\ \mathbf{0} & \mathbf{0} & \mathbf{c_B}^T - \mathbf{c_B}^T\mathbf{B}^{-1}\mathbf{D} & -\mathbf{c_B}\mathbf{B}^{-1}\mathbf{b} \end{bmatrix} \tag{140}$$

where $\mathbf{d}^T\mathbf{x_B} \geq b_A$ is the violated constraint. ($\mathbf{d}$ is a row vector, b_A is a scalar).

In a first step the elements of $\mathbf{d}^T$ are eliminated by adding rows appropriately scaled to the last row such that the elements of the row disappear (**LU** factorization). The result is a standard tableau with the only difference that the values of the last element of the base vector will be negative $y_{i_o} < 0$.

It is now obvious that the last basic variable has to leave the base and the non-basic variable showing the smallest positive value of y_{i_o}/y_{ij} has to enter the base. After the change of base the basic solution is feasible but not necessarily optimum. However, the subsequent base change operation is standard.

Solution of problem b. In this problem the tableau contains $\mathbf{B^{-1}D}$ and $\mathbf{B^{-1}b}$ only. There is no relative cost vector nor is there a cost function, see appendix A 2.

The objective of the base change operation is to achieve a feasible basic solution subject to the condition that the operation is pivoted around the diagonal of $\mathbf{B^{-1}D}$. This is a condition of the QP-algorithm.

The algorithm starts with one or more elements of $\mathbf{B^{-1}b}$ being negative. The pivot element is the diagonal element of this particular row. Hence the base change operation is straight forward. If there are further negative elements in the base the process is continued.

The process stops when all elements of the base vector are positive. There is just one solution to the problem (for a convex QP-problem).

A.2 Quadratic Programming

The classic objective function of a QP problem is as follows:

$$\mathcal{F} = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x} \quad \Rightarrow \quad min \tag{141}$$

subject to linearized equality and inequality constraints:

$$\begin{aligned} \mathbf{Jx} - \mathbf{b}_1 &\quad = \mathbf{0} \\ \mathbf{Ax} - \mathbf{b}_2 &\quad \leq \mathbf{0} \end{aligned} \tag{142}$$

The matrices $\mathbf{Q}, \mathbf{J}, \mathbf{A}$ are of general nature. Depending on the OPF QP-variable choice they can be either sparse, constant or also non-sparse.

In the following the QP will be transformed into an unconstrained QP optimization problem whose solution is trivial. In order to achieve the QP solution

with consideration of the inequality constraints a superposition is applied. The resulting optimization problem is a Linear Programming based optimization problem ([24], [25]). This derivation is briefly shown in the following.
The Lagrange function with consideration of equality constraints only and the corresponding optimality conditions are as follows:

$$\mathcal{L} = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x} + \boldsymbol{\lambda}^T(\mathbf{J}\mathbf{x} - \mathbf{b}_1) \tag{143}$$

$$\begin{bmatrix} \mathbf{Q} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{bmatrix} \mathbf{u}_0 = \mathbf{M}\mathbf{u}_0 = \begin{bmatrix} -\mathbf{c} \\ \mathbf{b}_1 \end{bmatrix} \tag{144}$$

with

$$\mathbf{u}_0 = \begin{bmatrix} \mathbf{x}_0 \\ \boldsymbol{\lambda}_0 \end{bmatrix} \tag{145}$$

The Lagrangian for the problem with inequality constraints and its optimality solutions is as follows:

$$\mathcal{L} = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x} + \boldsymbol{\lambda}^T(\mathbf{J}\mathbf{x} - \mathbf{b}_1) + \boldsymbol{\mu}^T(\mathbf{A}\mathbf{x} - \mathbf{b}_2) \tag{146}$$

$$\begin{aligned} \mathbf{M}\mathbf{u}_c + \begin{bmatrix} \mathbf{A}^T \\ 0 \end{bmatrix} \boldsymbol{\mu}_c &= \begin{bmatrix} -\mathbf{b}_0 \\ \mathbf{b}_1 \end{bmatrix} \\ \mathbf{A}\mathbf{x}_c &\leq \mathbf{b}_2 \\ \boldsymbol{\mu}_c &\geq \mathbf{0} \end{aligned} \tag{147}$$

The solution of this inequality constrained problem is now split into the equality constraint solution and a superposition:

$$\mathbf{u}_c = \mathbf{u}_0 + \Delta\mathbf{u} \tag{148}$$

It follows for the optimality conditions for the inequality constrainted OPF problem:

$$\begin{aligned} \mathbf{M}\Delta\mathbf{u} + \begin{bmatrix} \mathbf{A}^T \\ 0 \end{bmatrix} \boldsymbol{\mu}_c &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \mathbf{A}(\mathbf{x}_0 + \Delta\mathbf{x}) &\leq \mathbf{b}_2 \end{aligned} \tag{149}$$

Since the vector $\mathbf{x}$ is a subvector of the vector $\mathbf{u}$ the inequality constraints can be rewritten. If substituting also the change of the variables $\mathbf{\Delta u}$ the following inequality constraint set results:

$$-\left[\mathbf{A}, \mathbf{0}\right] \mathbf{M}^{-1} \begin{bmatrix} \mathbf{A}^T \\ 0 \end{bmatrix} \boldsymbol{\mu}_c \leq \mathbf{b}_2 - \mathbf{A}\mathbf{x}_0 \tag{150}$$

This inequality constraint system corresponds conceptually to the following problem:

$$\mathbf{T}\boldsymbol{\mu}_c \leq \mathbf{b} \qquad \boldsymbol{\mu}_c \geq \mathbf{0} \ , \ \ \mathbf{b} \geq \mathbf{0} \tag{151}$$

The problem is to find a vector $\boldsymbol{\mu}_c$ which satisfies the above inequality constraints. Conventional LP techniques can be applied to do this.

After having found the feasible point for the above inequality constraint problem the other (eliminated) variables can be found be replacing the values for $\boldsymbol{\mu}_c$ into the relevant equations:

$$\Delta\mathbf{u} = -\mathbf{M}^{-1} \begin{bmatrix} \mathbf{A}^T \\ 0 \end{bmatrix} \boldsymbol{\mu}_c \tag{152}$$

Of course the inversion of the matrix $\mathbf{M}$ is not actually done in a computer implementation. A forward and backward solution is executed with the factors of the matrix $\mathbf{M}$.

As derived above the solution must be found for the following inequality constrained system:

$$\mathbf{T}\boldsymbol{\mu}_c \leq \mathbf{b} \tag{153}$$

This is in principle a classical LP problem. Several solution methods can be found in literature. In this appendix one possible solution is briefly discussed.

A vector of slack variables $\mathbf{x_B}$ is introduced. They can be seen as a set of base variables. $\mathbf{U}$ is a unity matrix.

$$\mathbf{T}\boldsymbol{\mu}_c + \mathbf{U}\,\mathbf{x_B} = \mathbf{b} \tag{154}$$

The base variables of non-satisfied inequality constraints are negative. In the optimum all variables of the LP problem must be positive. By choosing a negative pivot in the row a negative base variable it can be made positive. The principle is to make base changes such that all base variables are finally

positive. If all base variables are positive a feasible solution for the inequality is found.
In this special case of inequality consideration a special choice for the pivot is necessary: If an inequality constraint i becomes active, i.e. binding at its limit, the associated base variable $x_{B_i} = 0$ becomes zero. At the same time the associated variable $\mu_i \neq 0$, i.e. each equality constraint or binding inequality constraint must have an associated Lagrange multiplier with a value $\neq 0$. This means that for every set of associated variables (x_{B_i}, μ_i) one and only one of them must be exactly zero. This means that in the LP tableau of the inequality constraints the pivot for base changes can only be a diagonal element.

Without giving a proof in this paper, it can be shown that the solution for the problem, if it exists, is unique.

It can be also be shown that the actual implementation of this LP-optimization can be done with clever and fast updating techniques when the size of the inequality constraint set changes. However, due to space reasons this is not done in this paper.

A.3 Symbols

The following notations are used througout this text:

- Symbols representing complex variables are underlined.
- Matrices are shown in capital boldface letters.
- Vectors are shown in small boldface letters.

A.3.1 Symbols used in the power flow

The following symbols are used in the conventional Power Flow equations.
j: complex multiplier (for imaginary part of complex variable)
*: conjugate complex operator
^{k}: associated variable or expression is state (or iteration) dependent
$_{opt}$: associated variable is optimum variable
$Real$: Real part of following complex expression
$Imag$: Imaginary part of following complex expression
^{T}: Transposed - operator
$_{low}$: low limit of a variable
$_{high}$: upper (high) limit of a variable
$_{scheduled}$: related to variable with scheduled, predetermined value
Δ: change operator for variables, matrices, vectors

∂: derivative operator
N: total number of electrical nodes
m: total number of generator PV nodes
l: total number of load PQ nodes
EL: number of elements in loss objective summation function
$_{slack}$: slack node index
k_{slack}: constant slack node voltage ratio
P_i: active power at node i
Q_i: reactive power at node i
$P_{scheduled_{PQ}{}_i}$: scheduled active power at PQ node i
$Q_{scheduled_{PQ}{}_i}$: scheduled reactive power at PQ node i
$\underline{\mathbf{V}}$: vector of complex voltages
$\underline{V}_i$: complex voltage at node i
V_i: voltage magnitude at node i
e_i: real part of $\underline{V}_i$
f_i: imaginary part of $\underline{V}_i$
Θ_i: voltage angle at bus i : $arctan\frac{f_i}{e_i}$
e_{slack}: real part of $\underline{V}_i$, i: slack node
f_{slack}: imaginary part of $\underline{V}_i$, i: slack node
$V_{scheduled_{PV}{}_i}$: scheduled voltage magnitude at PV node i
$\underline{\mathbf{I}}$: vector of complex currents
$\underline{I}_i$: complex current at node i
I_{e_i}: real part of $\underline{I}_i$
I_{f_i}: imaginary part of $\underline{I}_i$
P_{ij}: active power flow in the branch from node i to node j
Q_{ij}: reactive power flow in the branch from node i to node j
$P_{high_{ij}}$: upper MW flow limit in the branch from node i to node j
$S_{high_{ij}}$: upper MVA flow limit in the branch from node i to node j
Q_{ij}: reactive power flow in the branch from node i to node j
$\underline{\mathbf{Y}}$: complex nodal admittance matrix
$\underline{Y}_{ij}$: complex element of $\underline{\mathbf{Y}}$-matrix at row i and column j
y_{ij}: absolute value of $\underline{Y}_{ij}$
g_{ij}: real part of $\underline{Y}_{ij}$
b_{ij}: imaginary part of $\underline{Y}_{ij}$
G_{ij}: real part of admittance of a π - element between nodes i and j
B_{ij}: imaginary part of admittance of a π - element between nodes i and j
θ_{ij}: angle of admittance $g_{ij} + jb_{ij}$: $arctan\frac{b_{ij}}{g_{ij}}$
B_{i_o}: charging/2 (purely capacitive) of line from i to j measured at node i
t_{ij}: tap of transformer between nodes i and j

A.3.2 Symbols used in optimal power flow optimization algorithm

The following symbols are used only in connection with the OPF.

^{k}: index referring to state and iteration dependent matrices, vectors

diag: representing a diagonal matrix

U: identity (unity) matrix

$\mathcal{X}$: vector of control variables

$\mathcal{U}$: vector of state variables

$\mathcal{P}$: vector of demand variables

x_i: OPF variable i

$\mathbf{x}$: vector of OPF variables

$\mathbf{x_i}$: subset i (i = 1 ... 4) of vector $\mathbf{x}$

$\mathbf{x_{3j}}$: subset j (j = 1 or 2) of vector $\mathbf{x_3}$

$\mathcal{F}$: objective function

$\mathcal{F}_{cost}$: total cost objective function

$\mathcal{F}_{cost_i}$: cost function of generator i

$\mathcal{F}_{loss}$: total loss objective function

$\mathcal{F}_{loss_i}$: losses related to branch i

$\mathbf{g}$: set of OPF equality constraints

$\mathbf{g_i}$: subset i (i = 1 ... 4) of OPF equality constraints $\mathbf{g}$

$\mathbf{h}$: set of inequality constraints

λ_i: Lagrange function multiplier for equality constraint i

μ_i: Lagrange function multiplier for inequality constraint i

$\boldsymbol{\lambda}$: vector of all λ_i

$\boldsymbol{\mu}$: vector of all μ_i

$\mathcal{L}$: Lagrange function

$\mathbf{H}$: Hessian matrix

$\mathbf{Q}$: quadratic cost coefficient matrix of quadratic objective function

$\mathbf{Q_{Loss}}$: quadratic loss coefficient matrix of quadratic loss objective function

q_i: quadratic cost coefficient of variable active generator power i

$\mathbf{c}$: vector of linear cost coefficients of objective function

$\mathbf{A}$: sensitivity matrix for inequality constraints in linearized form

$\mathbf{A_i}$: submatrix i (i = 1 ... 4) of $\mathbf{A}$

$\mathbf{M}$: matrix representing second derivatives of the power flow equations

$\mathbf{M}_{ij}$: submatrix of $\mathbf{M}$

$\mathbf{A_i}$: submatrix i (i = 1 ... 4) of $\mathbf{A}$

$\mathbf{b_1}$: right hand side values of linearized equality constraints

$\mathbf{b_2}$: right hand side limit values of linearized inequality constraints

$\mathbf{J}$: Jacobian matrix (first derivatives of power flow equations)

$\mathbf{J}_{ij}$: submatrix of $\mathbf{J}$
$\boldsymbol{\alpha}$: vector of linear incremental power flow equality constraint
$\boldsymbol{\alpha}_i$: subvector i (i = 1 or 2) of $\boldsymbol{\alpha}$
$\mathbf{W}$: non-linear system representing optimality conditions (OPF class B)
$\mathbf{H}$: matrix representing second derivatives of the Lagrangian
$_{eq}$: index associated with a variable of the equality constrained OPF æ

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