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The asymmetric reverting property of stock returns on the JSE: An ANAR-GJR GARCH approach

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I, Amber Unterhorst, declare that this research report is my own unaided work. It is submitted in partial fulfilment of the requirements for the degree of Master of Commerce at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in this or any other university.

Signed at the University of the Witwatersrand on 28 August 2014

Amber Unterhorst

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Abstract

This research report examines the asymmetric reverting behaviour of the conditional mean and conditional variance of daily, weekly and monthly nominal stock return observations on the Johannesburg Stock Exchange ("JSE"), across eight value-weighted indices for the sixteen-year period from July 1995 to October 2011.

Utilizing the Asymmetric Nonlinear Autoregressive ("ANAR") model of Nam, Kim and Arize (2006) to model asymmetry in the conditional mean, and the GJR Generalized Autoregressive Conditional Heteroskedasticity ("GJR GARCH") model of Glosten, Jagannathan and Runkle (1993) to model asymmetry in the conditional variance, this research report presents an empirical investigation into the asymmetric reverting behaviour of stock returns upon the JSE.

Asymmetry in the conditional mean and conditional variance manifests in stochastic time-series data by way of a negative return reverting more quickly and with a greater magnitude to a positive return than a positive return reverting to a negative return (Nam, Kim, & Arize, 2006). The results of the empirical investigation of stock returns on the JSE indicate that the uneven reverting qualities are present across seven indices and are more pronounced in the daily and weekly observed time intervals.

Note: Any defined terms are used consistently thought-out this document and will not be redefined.

1. Introduction

Fundamental to the study of finance and capital markets is the understanding of the underlying stochastic process followed by asset returns, as well as, the consideration of the time varying nature of the relationship between risk and return in financial markets (Nam, 2003). Insights into both of these fundamentals hinge on the deeper understanding of how investors view risk and how these views affect their subsequent asset pricing behaviour.

Recent studies by Nam (2003), Nam, Washer & Chu (2005), Nam, Kim & Arize (2006), Kulp-Tag (2007) and Choe, Krausz & Nam (2011) suggest that various types of non-linear dynamics, which have been observed in both the conditional mean and conditional variance, better characterize the data generating process.

This research report investigates the nonlinear time varying pattern of stock returns: specifically the tendency for nominal returns to display asymmetric reverting behaviours whereby negative returns revert more quickly and with a greater magnitude to positive returns than positive returns revert to negative returns.

The ANAR–GJR GARCH model utilized in this research report is a combination of two asymmetric models developed, and used, by Nam, Kim & Arize (2006) to capture asymmetry in the conditional mean, with an application of the GJR GARCH model of Glosten, Jagannathan and Runkle (1993) to capture asymmetry in the conditional variance.

This research report contributes to the existing body of research by expanding the study to a South African context. The ANAR-GJR GARCH model is applied to the daily, weekly and monthly nominal returns of the All Share Index ("ALSI"), the Top 40 Index ("Top40"), the MidCap Index ("MidCap"), the SmallCap Index ("SmallCap"), the Financial and Industrial Index ("FINDI"), the Financial Index ("FINI"), the Resources Index ("RESI") and the Industrial Index ("INDI"). The cross section of data allows this study to detect the presence of asymmetric reverting tendencies across industry, market capitalisation and different time intervals over a period of 16 years from July 1995 to October 2011.

The layout of this research report is as follows. A literature review outlines previous research and findings of, asymmetry in the conditional mean and asymmetry in the conditional variance. International and local studies are discussed and their findings compared. The Nam, Kim and Arize (2006) theoretical framework, altered to include the GJR GARCH model of Glosten, Jagannathan and

Runkle (1993) is presented, followed by a discussion of the empirical results of the study. Lastly, the findings, implications and conclusions of this research are offered.

2. Literature Review

For decade's economists, statisticians and finance scholars have been interested in developing and testing financial models, which effectively capture the behaviour of stock market prices (Fama, 1995). The interest stemmed not only from the desire to achieve abnormal profits, which would be possible through the ability to model stock prices, and by inference returns, with a certain degree of precision, but also from the need to understand investor behaviour and its influence on the stock market (Fama, 1995).

A significant number of studies of financial economics relate the expected return of stocks to some notion of risk (Corhay & Tourani Rad, 1994). These studies commonly model risk as the covariance between a stock's return and one or more variables whilst assuming a linear relationship between risk and return (Corhay & Tourani Rad, 1994). Models such as Sharpe's (1964) Capital Asset Pricing Model relates returns of stocks to their covariance with the market portfolio's returns, the Arbitrage Pricing Model of Ross (1976) relates stock returns to their covariance with several factors, whilst the Consumption Asset Pricing Model of Breeden (1979) draws on the relationship between returns and their covariance with aggregate consumption (Corhay & Tourani Rad, 1994).

Early evidence from authors such as French, Schwert & Stambaugh (1987) and Baillie & De Gennaro (1990) has suggested that the relationship between risk and return is time varying, which has led to the questioning of the implications and relevance of linear risk return models (Corhay & Tourani Rad, 1994). Researchers have since re-examined financial valuation models in the conditional form, which allows the relationship between risk and return to change through time, and simultaneously capture the characteristics of financial time-series. (Corhay & Tourani Rad, 1994).

Various stochastic models have been developed to mimic the behaviour of stock returns and conditional volatility (Corhay & Tourani Rad, 1994). These models are stochastic in nature, such that the randomness quality of return dynamics suggested by the random walk theory is inherent within the modelling. These stochastic processes estimate the mean and variance jointly in order to allow for the conditionality of the mean to the variance and vice versa.

Over the last decade, studies by Nam (2003), Nam, Washer & Chu (2005), Nam, Kim & Arize (2006), Kulp-Tag (2007), Wang & Wang (2010) and Choe, Krausz & Nam (2011) have discovered that not only does the relationship between risk and return vary through time, but also, both the conditional mean and conditional variance behave asymmetrically to positive and negative shocks. These authors have endeavoured to combine the asymmetric time-varying volatility characteristic of the conditional variance with the asymmetric pattern in return reversals, into a seemingly complete model of stock market behaviour capable of capturing the nonlinear features of both the conditional mean and conditional variance (Wang & Wang, 2011).

The write-up to follow begins with the evolution of models to capture the behaviour of stock returns and volatility, before exploring the development of the Autoregressive ("AR"), Autoregressive Conditional Heteroskedasticity ("ARCH") and Generalized Autoregressive Conditional Heteroskedasticity ("GARCH") models, with a specific focus upon the development of extensions to these models to capture asymmetry in the conditional mean and conditional variance.

2.1. Modelling the Behaviour of Stock Returns

The random walk model, developed by Bachelier in 1914, is the oldest and most referenced statistical model to have emanated from early research on stock price modelling. The crux of the random walk theory, states that a series of stock price changes has no memory- historical stock prices, and by inference returns[1], cannot be used to predict future stock prices (returns) in any meaningful way (Fama, 1995). In essence, the future path of a stock price is no more predictable than the path of a series of cumulative random numbers (Fama, 1995).

The random walk theory produced two separate hypotheses: firstly, that successive price changes are independent, and secondly that price changes conform to some probability distribution (Fama, 1965). Researchers have systematically tested both assumptions of the random walk theory over the last few decades.

The independence assumption implied that the price change in one period is autonomous from the sequence of price changes during previous periods. Fama (1965, p.35) commented "...in fact we can

[1] As is standard practice in empirical literature, stock prices are converted to stock returns through the following process: $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$.
Stock prices are said to follow a lognormal distribution, which by inference means stock returns follow a normal distribution. Many studies convert prices to return observations as returns carry more attractive statistical properties on which to perform analysis.
The random walk theory, whilst refers to stock prices, translates to a theory on the randomness of returns as well.

probably never hope to find a time series that is characterized by perfect independence. Thus, strictly speaking the random walk theory cannot be a completely accurate description of reality. For practical purposes, however, we may be willing to accept the independence assumption of the model as long as the dependence in the series of successive price changes is not above some minimum acceptable level".

Many studies from authors such as Bodie (1976), Jaffe & Mandelker (1976), Nelson (1976) Fama and Schwert (1977), Fama (1981), Campbell (1987) French, Schwert & Stambaugh (1987), Jegadeesh (1990), Kim, Nelson & Startz (1991) and Volos & Siokis (2006) have shown that in fact daily return observations tend to have small autocorrelation coefficients, indicating a degree of dependence between observations. Furthermore, squared daily returns tend to exhibit strong autocorrelation, and persistence through periods with similar volatilities, which span several periods – another strong indicator of dependence.

When considering the distribution assumption of stock returns, it is a stylized fact that the distribution differs from normality with the presence of excess kurtosis, also referred to as leptokurtosis (fatter tails and higher peaks than predicted under a normal distribution). The distribution of stock returns is typically negatively skewed (left skewed) than what is predicted under a normal distribution and therefore asset pricing models which rest on the assumption of normality will contain estimation errors (Rachev, 2008).

The above findings are not necessarily at odds with the random walk theory, after all, the reality is that investors cannot consistently generate abnormal profits, which is evidence that stock returns are to some extent random, but at the same time may contain some conditionality to past returns and underlying volatility.

Fama & French (1988) hypothesized that in fact stock returns can be modelled as the sum of a stationary component and a random component, whereby the stationary component produces autocorrelation[2] between return observations and the random component produces "white noise". The model described by Fama and French (1988) is an autoregressive of order 1 ("AR(1)") model and is a natural candidate for conditioning on the past observations of a return series whilst retaining an element of randomness (Rachev, Stoyanov, Biglova, & Fabozzi, 2006). These models are referred to as conditional as there is a time evolution of both the stationary component and the random component, which affects the current return observation.

[2] Also referred to as serial correlation

A simple AR(1)[3] process, as cited by Fama & French (1988) is given by:

$$r_t = \mu + \phi . r_{t-1} + \varepsilon_t \quad (1)$$

Where:

r_t is the return in time "t"

μ is a constant, or intercept of the data generating process

ϕ is the measure of serial correlation with the lagged return

r_{t-1} is the return observation, lagged by one period

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

The stationary component of stock returns is of great interest to academics and investors alike, as the greater the degree of influence autocorrelation has on the future value of stock market returns, the greater the predictability (if any) of future stock returns (Fama & French, 1988). Note that the AR(1) process described is symmetrical, in that r_{t-1} has the same impact on r_t regardless of the sign of the previous periods return.

The AR(1) process fits time series that display weak stationarity whereby, the mean, variance and autocovariance of the series are constant at any point in time[4] (Rachev, 2008)

The "white noise" produced by the random component is a random process with a mean of 0 and a constant variance. This observation is generated from a normal distribution[5] of identically distributed random variables, such that $\varepsilon_t \sim N.I.D\ (0,\ \sigma^2)$ (Rachev, 2008). The constant variance assumption is defined as a homoskedastic assumption. The converse to this assumption is heteroskedasticity, whereby the variance of the residual term is not equal and may be larger during some periods than others. The ability to model volatility under homoskedastic and heteroskedastic assumptions is important as, in addition to excess kurtosis and leptokurtosis, financial time series display volatility clustering whereby periods of high volatility and low volatility tend to group together in time (Rachev, 2008). Non-constant error variance modelling accounts for the heteroskedastic characteristics of financial time series data.

[3] This process is referred to as an AR(1) process as there is one lagged return observation. An AR(p) process can be defined as $r_t = \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \varepsilon_t$

[4] To satisfy the stationary condition of the AR(1) process, $|\phi|$< 1must hold

[5] A student's t-distribution can also be used

Volatility modelling has received a lot of attention over the last few decades, firstly as an input into the autoregressive models for stock returns, through the residual term generation process and secondly because correlation between volatility observations is a stylized fact of financial time series.

The ability to produce estimates of future volatilities is imperative for value-at-risk ("VaR") calculations[6], the valuation of derivative instruments, and portfolio allocation (Anderson & Bollerslev, 1998). Accurate measures and good forecasts of volatility are therefore critical for the implementation and evaluation of asset and derivative pricing theories as well as the implementation and valuation of hedging and trading strategies (Anderson & Bollerslev, 1998).

Through stochastic volatility modelling, researchers can produce near accurate forecasts of future volatility. If one could, with reasonable certainty, predict the *sign* of tomorrow's return, this information would be enough to make a rational and prudent investment decision (Kulp-Tag, 2007).

2.2. Stochastic Volatility Modelling

Volatility refers to the spread of all outcomes of an uncertain variable, and is a quantified measure of market risk (Poon & Granger, 2003). Whilst volatility and risk are related, risk is defined as the uncertainty of a negative outcome of some event, whereas volatility measures the spread of both positive and negative outcomes (Poon & Granger, 2003).

In finance, volatility is used to describe a measure of risk and refers to the standard deviation (σ) or variance (σ^2)[7] measured from a set of observations as:

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_t - \bar{r})^2 \tag{2}$$

[6] VaR is a calculation used by financial institutions in the risk management process. VaR is defined as the minimum expected loss at the 1% or 5% confidence interval for a given time horizon (1 -10 days).
The 1996 and 1999 introduction of the Basel Accords has made it compulsory for financial institutions to calculate their financial risk exposure as an input into the capital reserve requirements governed by Central Banks (Thupayagale, 2010).

[7] Standard deviation or variance is used interchangeably throughout the paper to describe volatility. This is acceptable, as a simple relationship exists between the two measures.

Where, as cited by Poon & Granger (2003):

$\hat{\sigma}^2$ is the sample variance statistic, to be used as the variance forecast

N is the number of observations in the sample

r_t is the return in time "t"

$\bar{r}$ is the mean return for the sample

If the true unconditional variance is constant through time, then utilising a measure of volatility estimated from the historical variance approach of Equation 2 above is relatively straightforward, and the constant sample variance using all available data will provide the best forecast of future volatility (Poon & Granger, 2003).

If, however, volatility varies through time, the constant value of $\hat{\sigma}^2$ will mis-represent the actual risk. Standard volatility models, such as Equation 2, which assume a constant variance, have shown to explain little of the variability in ex-post squared stock returns, as evidenced by Cumby, Figlewski & Hasbrouck (1993), Figlewski (1997) and Jorion (1995) as cited by Poon & Granger (2003).

Campbell, Lo, & MacKinlay (1997, p.481) argued, "it is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time."

Several models have been developed to forecast time-varying volatility using historical data. These models include the Moving Average ("MA"), both the Simple Moving Average ("SMA") and Exponentially Weighted Moving Average ("EWMA") and stochastic volatility models such as ARCH and GARCH.

The SMA measure of volatility uses equally weighted past squared returns over a "moving window" of time such that:

$$\sigma^2_{t+1|t} = \left(\frac{1}{N}\right)\sum_{j=1}^{N}\left(r_{t-j+1} - \bar{r}_t\right)^2 \tag{3}$$

Where, as cited by Johnston, Boyland, Meadows, & Shale (1999),

$\sigma^2_{t+1|t}$ is the volatility forecast for period "t+1" given the information available at time "t"

N is the number of periods in the moving window

r_{t-j+1} is the return in time "t – j+1"

$\bar{r}_t$ is the mean return for the window

The EWMA[8] allows the weights of the past squared returns to decline exponentially (Minkah, 2007). In this approach, each historical return estimate in the moving window is attributed a weight which declines as the window moves forward through time, placing more weight upon more recent observations, such that:

$$\sigma^2_{t+1|t} = (1-\lambda)\sum_{j=1}^{N}\lambda^{j-1}\left(r_{t-j+1} - \bar{r}_t\right)^2 \tag{4}$$

Where, as cited by (Minkah, 2007):

$\sigma^2_{t+1|t}$ is the volatility forecast for period "t+1" given the information available at time "t"

λ is the decay factor between $0 < \lambda < 1$

N is the number of periods in the moving window

r_{t-j+1} is the return in time "t – j+1"

$\bar{r}_t$ is the mean return for the window

The EWMA has two advantages over the SMA. Firstly, the volatility forecast under the EWMA reacts quicker to recent changes in volatility as the recent events are given a higher weighting, and secondly after a shock, the volatility forecast will decline gradually as time passes instead off dropping sharply (Minkah, 2007).

As mentioned, conditional models estimate the mean and variance jointly. The combination of the AR process for return observations of Equation 1 with the SMA or EWMA is referred to as an Autoregressive Moving Average model ("ARMA").

[8] RiskMetrics is the most popular EWMA model, developed and used by JPMorgan Bank

A shortfall of the SMA and EWMA is that the changes in volatility are not regarded as random, in fact in both models; historical return is the only input into the forecast of future volatility. The development of the ARCH, and later the GARCH, family of stochastic volatility models addresses the characteristics of financial time series: namely leptokurtosis, and most importantly, the tendency of volatility to "cluster" through time, an indication of intertemporal dependence, where large changes in stock returns tend to be followed by further large changes, all whilst allowing the changes in volatility to be regarded as random (Karlin & Taylor, 1975).

There are a few suggested reasons for the presence of volatility clustering, most of which relate to investor behaviour. LeBaron (2001) attributes the increase in return variability to market participants with differing investment horizons, which leads to patterns in volatility, and trading volumes. Lux & Marchesi (2000) postulate that volatility clustering arises from the behavioural switching of market participants between fundamentalist and chartist behaviour. Cont (2005, p.9) explains "Fundamentalists expect that the price follows the fundamental value in the long run. Noise traders try to identify price trends, which results in a herding tendency. Agents are allowed to switch between these two behaviours according to the performance of various strategies. Price changes are brought about by a market maker reacting to imbalances between demand and supply. Most of the time a stable and efficient market persists. However the usual tranquil performance is interspersed by sudden transient phases of destabilization brought upon by agents using chartist techniques"

Robert Engle (1982) was the first to develop a stochastic volatility model capable of handling the clustering of volatility. The ARCH stipulates that the variance of the residual term at a particular point in time depends upon the variance of the residual term at previous points in time. In this manner, periods of persistence in high or low volatility will exist (Engle, 1982)

The underlying premise behind Engle's (1982) ARCH model involves defining the specification for the process of describing the residual term ε_t, which factors into the r_t return data generating process such that:

$$\varepsilon_t = V_t \cdot \sqrt{h_t} \tag{5}$$

Where, as cited by Engle (1982):

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

V_t is the "white noise" component or random component

h_t is the conditional variance described by an autoregressive process[9]

The residual term ε_t follows a normal distribution with a mean of 0 and a variance of h_t.

The ARCH(q) process is given by the following:

$$h_t = \propto_0 + \sum_{i=1}^{q} \propto_i \varepsilon_{t-i}^2 \tag{6}$$

Where, as cited by Engle (1982):

h_t is the conditional variance in time “t”

$\propto_0$ is a constant, or intercept of the data generating process

$\propto_i$ is the correlation coefficient of the lagged squared residual term

ε_{t-i}^2 is the squared residual term, lagged by “i” time periods.

In order for the model to hold, $\propto_0 > 0$ to ensure non-negativity of the conditional variance h_t and, $0 < \propto_i < 1$ to prevent the change in ε_{t-i}^2 from mathematically explaining more than 100% of the conditional variance in time “t” (Engle, 1982).

The conditional variance h_t acts like an autoregressive process on the current residual term ε_t in Equation 5, such that the conditional variance is a function of past lagged residuals. The stronger the correlation of the conditional variance to past residual values, measured by $\propto_i$ in Equation 6, the longer the persistence of a shock, which enters into the generation of the current periods return, r_t in Equation 1 through the V_t process. Similarly, the greater the ϕ parameter of Equation 1, the greater the tendency of the r_t series to be displaced (Karlin & Taylor, 1975).

In 1986, Tim Bollerslev expanded upon the existing ARCH literature by constructing an ARCH that incorporates both a moving average component as well as an autoregressive component. The generalized ARCH allows for a longer memory of shocks and more flexible lag structure than the ARCH, after studies by Engle (1982), Engle (1983) and Engle and Kraft (1983) showed that a longer lag in the conditional equation is needed in order to avoid problems with negative variance parameter estimates, as cited by Bollerslev (1986).

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[9] Conditional variance is represented by the symbol h_t instead of σ_t^2 purely to differentiate between the estimation of variance under the MA models and the ARCH/GARCH models however both are forecasts of volatility.

The Bollerslev (1986) standard GARCH (p,q) process is defined below:

$$h_t = \propto_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{7}$$

Where:

h_t is the conditional variance in time "t"

$\propto_0$ is a constant, or intercept of the data generating process

$\propto_i$ is the correlation coefficient of the lagged squared residual term

ε_{t-i}^2 is the squared residual term, lagged by "i" time periods.

β_i is the correlation coefficient of the lagged conditional variance term

h_{t-i} is the conditional variance, lagged by "i" time periods

In order for the model to hold $\propto_0 > 0$ to ensure non-negativity of the conditional variance, $|\beta_i| < 1$ and, $1 - \alpha_i - \beta_i > 1$. These stipulations ensure that the variables included in the model do not mathematically account for more than 100% of the change in the conditional variance at time "t" (Bollerslev, 1986)

The GARCH (p,q) process allows for "q" lagged residual terms and "p" lagged variance terms to play a role in determining the current period volatility, and in doing so, incorporates a moving average component and an autoregressive component. Note that the correlation coefficient, $\propto_i$, is the same for prior positive or negative shocks in the residual term, in other words, the impact the prior residual term has on the current conditional variance is symmetrical.

Whilst the specification of the GARCH (p,q) model allows for an infinite number of lags, most studies have found that low orders of 2 lags or less is sufficient to capture the behaviour of volatility (Bollerslev, Chou, & Kroner, 1992).

ARCH and GARCH models have proven very popular methods for forecasting volatility over the SMA and EWMA models, as discovered by authors such as: Dimson & March (1990), Cumby, Figlewski & Hasbrouck (1993), West & Cho (1995), Brailsford & Faff (1996), Figlewski (1997), Alexander & Leigh (1997), Anderson & Bollerslev (1998) and Alexander (1998).

GARCH models have been applied in a South African context to volatility studies on the JSE by the following authors: Samoulihan (2006), Samoulihan & Shannon (2008), Mangani (2009), Mandimika & Chinzara (2010), Makhwiting, Lesaoana & Sigauke (2012), Niyitegeka & Tewari (2013) and Mzamane (2013) all of which have found stochastic volatility modelling an appealing and accurate description of volatility on the JSE.

The theoretical appeal and empirical success of the traditional ARCH and GARCH model can be traced to not only the ability of these models to effectively capture volatility clustering and other stylized facts but also to its numerous applications to diverse areas such as: testing of asset pricing models like the CAPM, ICAPM and APT models, to development of hedging strategies by measuring the term structure of interest rates, to examine the flow of information across countries, markets and asset classes, to price and model options, to measure inflationary uncertainty and examine the relationship between the exchange rate and trade, to study the effects of central bank interventions and quantify the relationship between the macro-economy and the stock market (Bera & Higgins, 1993).

On the other hand, simplistic ARCH/GARCH models as those found in Equations 6 and 7 contain three vital limitations, as outlined by Nelson (1991): Firstly, early research from Black (1976) and Christie (1982) has indicated that stock returns themselves are negatively correlated to changes in return volatility. ARCH and GARCH models account for the magnitude of the return but not the sign of the previous return in estimating volatility forecasts (Nelson, 1991). If the distribution of the residual term is symmetric this translates to a change in volatility tomorrow which is uncorrelated to the sign of the return today.

Secondly, these traditionall models prevent the conditional variance forecast from being negativie by placing nonnegativity constraints on the estimations of the constant, $\propto_0$ and the correlation coefficient $\propto_i$. These constraints imply that increases in the residual term, ε_t, in any period will increase the conditonal variance for all future periods, ruling out any random oscillation in the conditional variance process (Nelson, 1991). These nonnegativity constraints can cause issues in estimating the GARCH model as experienced by Engle, Lilien & Robins (1987).

The third shortfall refers to the interpretation of the persistence of shocks to the conditional variance. Researchers are intrigued by the persistence of these shocks. If volatility shocks persist indefinitely; it will result in a shift of the term structure of risk premia, which will have a significant impact on longer life capital goods. Under a GARCH (1,1) specification it is difficult to tell whether shocks persist or not, as shocks may present themselves in one norm and die out in another, which leads to the estimation of GARCH (1,1) conditional moments, which may explode even when the process itself is stationary. Furthermore, traditional tests to measure persistence do not agree with these models (Nelson, 1991).

Since the development of the ARCH and GARCH literature in 1982 and 1986, numerous authors have been spurred on by the idea of developing refinements to the current techniques to better forecast the time-varying financial market volatility[10].

2.3. Asymmetry of the Conditional Volatility

Conditional variance asymmetry refers to the occurrence of a negative correlation between returns and volatility (Cox & Ross, 1976). Returns are often found to have an asymmetric impact on volatility (Thupayagale, 2010). In this sense, volatility is frequently reported as being 'directional', in that, volatility responds unevenly to past negative and positive return shocks with negative shocks resulting in larger future volatilities than positive shocks of the same magnitude (Thupayagale, 2010). The result is that volatility is greater in a bear market than in a bull market.

Asymmetry in volatility will result in negative return shocks, as a product of "bad news" exerting a stronger influence on future volatility than a positive return shock. Black (1976), Christie (1982), both reported volatility asymmetry and were the first authors to detect this asymmetry before further studies by French, Schwert & Stambaugh (1987), Schwert (1990), Nelson (1991), Campbell & Hentschel (1992), Cheung & Ng (1992), Glosten, Jagannathan & Runkle (1993), Bae & Karolyi (1994), Braun, Nelson & Sunier (1995), Ng (1996), Koutmos (1999), Bekeart & Wu (2000) and Blasco, Corredor & Santamaria (2002) as cited by Liau & Yang (2008), revealed this phenomenon to be a common characteristic of the relationship between stock returns and volatility.

Among the studies investigating volatility asymmetry, Bekaert & Harvey (1997) provide evidence that the effect is present in 20 emerging markets[11]. Fraser & Power (1997) discover asymmetric volatility is detectable in the United Kingdom, Japan, and Malaysia. Brooks, Faff, McKenzie & Mitchell (2000) find volatility asymmetry is evident in equity indices for ten mature markets, whilst

[10] For reviews of the ARCH/GARCH literature and developments see: Andersen and Bollerslev (1998), Andersen, Bollerslev, Christoffersen and Diebold (2006), Bauwens, Laurent and Rombouts (2006), Bera and Higgins (1993), Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994), Degiannakis and Xekalaki (2004), Diebold (2004), Diebold and Lopez (1995), Engle (2001, 2004), Engle and Patton (2001), Pagan (1996), Palm (1996), and Shephard (1996) as cited by Bollerslev (2009)

[11] These markets include Argentina, Brazil, Chile, Columbia, Greece, India, Indonesia, Jordan, Korea, Malaysia, Mexico, Nigeria, Pakistan, Philippians, Portugal, Taiwan, Thailand, Turkey, Venezuela, Zimbabwe

Talsepp & Rieger (2010) detect the presence of volatility asymmetry across 49 mature and emerging market countries[12], including South Africa. Within a South African context, Samouilhan & Shannon (2008), Cifter (2012), and Makhwiting, Lesaoana & Sigauke (2012) detect the presence of asymmetry in volatility on the JSE.

There have been two theories offered as explanations for the asymmetry inherent in stock return volatility – the leverage effect, and the volatility feedback mechanism (Liau & Yang, 2008).

Black (1976) was the first to propose the 'leverage effect' to explain the apparent asymmetry of volatility. According to this theory, when the price of a stock falls (negative return), its equity value also drops, causing the financial leverage of the firm (debt-to-equity ratio) to increase. When leverage rises, the company is considered more risky and a higher degree of risk is associated with higher volatility. The theory was later supported by Christie (1982), Schwert (1989) and Duffee (2001) whom all detect conditional variance asymmetry at the firm or portfolio level; however the authors acknowledged that the financial leverage effect alone was not empirically sufficient enough to explain the size of the observed asymmetry.

Pindyck (1984) offers an alternative explanation for the asymmetry coined the volatility feedback hypothesis, which suggests that if volatility is priced in a stock return, an anticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline. This theory was supported by the research of French, Schwert & Stambaugh (1987), Campbell & Hentschel (1992) and Bekaert & Wu (2000) whom detect variance asymmetry at market level returns across an index.

One can see that the causality of these two theories is different. The leverage hypothesis states that return shocks further lead to asymmetric changes in volatility whereas the volatility feedback mechanism contends that return shocks are caused by changes in conditional volatility (Bekaert & Wu, 2000). Whilst the determinant of asymmetric volatility remains an open question, the importance of modelling for volatility asymmetry remains applicable in order to avoid mis-specification errors.

[12] These markets include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, and United States. The emerging markets included Argentina, Brazil, Chile, China, Columbia, Czech Republic, Egypt, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, South Africa, Sri Lanka, Taiwan, Thailand, Turkey, and Venezuela.

Exley, Mehta and Smith (2004) believe volatility asymmetry is not contradictory of the efficient market hypothesis. Economic rationale dictates that after the fall of a stock price, volatility will increase along with the market price of risk because investors with a set level of wealth can now afford to lose less with their core consumption at risk. All of these factors are likely to drive up the return in the next period, and therefore allow for negative correlations between returns and volatility. Under these assumptions, a limited degree of volatility asymmetry is consistent with efficient markets and in line with financial theory (Exley, Mehta, & Smith, 2004).

In the absence of a good theoretical model for volatility asymmetry, the GARCH literature has searched for econometric ways of describing the non-linearity. Models such as the Nonlinear ARCH ("NARCH") model of Engle and Bollerslev (1986), the Exponential GARCH ("EGARCH") process introduced by Nelson (1991), the Asymmetric GARCH ("AGARCH") of Engle (1990), the Threshold GARCH ("TGARCH") model of Zakoian (1991), the Nonlinear GARCH ("NGARCH") of Engle and Ng (1993), the GJR GARCH of Glosten, Jagannathan and Runkle (1993) and the Quadratic GARCH ("QGARCH") process of Sentana (1991) are among the popular asymmetric GARCH models as cited by Hentchel (1995).

The ability of these models to capture asymmetry in volatility rests on the introduction of additional terms into the conditional volatility equation, which allows for a different response in volatility, conditional on both the magnitude and the sign of the previous return. Studies by authors such as Pagan & Schwert (1990), Lee (1991), Cao & Tsay (1992), Cumby, Figlewski & Hasbrouck (1993), Brailsford and Faff (1996), Loudon, Watt & Yadav (2000), Bali (2000) and Blair, Poon & Taylor (2001) as cited by Poon & Granger (2003) all discovered that the asymmetric models produced better forecasts of volatility than their symmetric GARCH counterparts.

Engle and Ng (1993) suggested the use of News Impact Curves, developed by Pagan and Schwert (1990), as an indication of which asymmetric model: EGARCH, AGARCH, NGARCH or GJR GARCH, was a better candidate for modelling time varying volatility. The authors fitted the models to daily Japanese stock returns from 1980 to 1988. All of the models indicated that negative shocks introduced more volatility than positive shocks; however, the asymmetry modelled by certain asymmetric models was not accurate. Specifically, the authors concluded that whilst the EGARCH model captured most of the asymmetry, the variability of the conditional variance implied by the EGARCH was too high. Under these circumstances, the level of asymmetry with the use of the

EGARCH is overstated. The diagnostic tests conducted indicated that the most parsimonious model for capturing the asymmetric effect was the GJR GARCH.

Bali (2000) examined the predicative power of the NGARCH, GJR GARCH, TGARCH, AGARCH, QGARCH and GARCH in describing the volatility of United States interest rates by forecasting the 3, 6 and 12 month Treasury-Bill rates using historical rates from 1954 to 1998. The NGARCH and GJR GARCH proved to produce the most accurate asymmetric forecasts.

Loudon, Watt and Yadav (2000) examined the FTSE All Share index for the period 1971-1997 in their study to reveal which of the asymmetric models: the EGARCH, GJR GARCH, TGARCH, NGARCH or GARCH were the best predictor of future volatility. The findings indicated that the EGARCH and GJR GARCH produced forecasts closest to the observed volatility for the forecasting period.

Taylor (2004) executed a similar study to Engle and Ng (1993) by comparing the effectiveness of the GJR GARCH in forecasting volatility of the DAX index, Standard & Poor 500 index, HANG SENG index, FTSE100 index, Amsterdam EOE index, NIKKEI index and the Singapore All Share index over the 7-year period from 1988 to 1995. In each instance, the GJR GARCH was found to produce the most accurate forecasts in out of sample testing.

Jiang (2012) judged the forecasting performance of the GARCH, EGARCH and GJR GARCH across five global stock market indices: NASDAQ's daily index, Standard & Poor's 500 daily index, FTSE100 daily index, HANG SENG daily index and NIKKEI daily index for the period 2007 – 2011. Of the models, the GJR GARCH model was found to be the most parsimonious model with the lowest Root Mean Squared Error ("RMSE") for four of the five indices, being the NASDAQ, the Standard & Poor 500, the FTSE100 and the NIKKEI index.

These international studies have been extended to the JSE by research conducted by Samouilhan (2006) who utilized an EGARCH model to examine the relationship between market returns and asymmetric volatility across the JSE and its sub-indices for the period 1996 to 2004. Samouilhan (2006, p.8) concludes "the estimated coefficients suggest that the volatility of the market reacts far more to negative shocks (bad news) than to positive shocks (good news); negative market returns appear to have a far greater effect on the magnitude of current volatility than do positive past errors"

Samouilhan & Shannon (2008) in their research titled "Forecasting Volatility on the JSE" compared the forecasting ability of the ARCH, GARCH, TGARCH, EWMA and Implied Volatility models to the volatility of the Top40 index for the period 2004-2006. Samouilhan & Shannon (2008 p. 24) noted "The results from fitting the asymmetric TGARCH model show that, in keeping with the international literature, the asymmetrical response of volatility to the direction of the return is found to be significant". Furthermore Samouilhan & Shannon (2008, p. 28) commented "The TGARCH model is judged to be the best specification to model and forecast domestic volatility, being relatively the most accurate and unbiased over both one day and one week ahead forecast horizons. From an investor's perspective, the use of asymmetric models provides better forecasts".

Thupayagale (2010) investigates the use of asymmetric GARCH models in value-at-risk calculations for 10 emerging market economies[13], including South Africa, for the period 1998-2010. Thupayagale (2010) concluded that the results suggest that models with long memory or asymmetric effects are important considerations in the forecasting of future volatility.

Mandimika & Chinzara (2010) utilized a GARCH, GJR GARCH, EGARCH and TGARCH to capture the asymmetric effects of volatility on the JSE at the aggregate, industrial and sectorial level for the period 1995 - 2009. The authors discovered volatility on the JSE to be largely persistent, asymmetric and best captured through the TGARCH model.

Mzamane (2013) makes use of the GARCH, GJR GARCH, EGARCH and Asymmetric Power GARCH ("APARCH") to model the ALSI index of the JSE for the period 1995–2012. The study indicated that the volatility in the residuals and the leverage effect was present in the JSE index returns. The results of the research revealed that negative residuals increase volatility more than positive residuals of the same magnitude – an indication of the volatility asymmetry. The EGARCH model was chosen as the most parsimonious model based on the the Akaike Information Criterion ("AIC") and Bayesian Information Criterion ("BIC"), with the GJR GARCH model following as the second best asymmetric model for describing volatility on the JSE. These results are in line with Cifter (2012).

Chinzara & Slyper (2013) reported that of the three conditional volatility models, GARCH, EGARCH and GJR GARCH, applied to a dataset which included the ALSI index, Industrials index, General Retailers index, Mining index and Financial index of the JSE, for the period 1995–2010, with the

[13] These markets include: Brazil, China, Egypt, India, Kenya, Nigeria, Russia, South Africa, Turkey and the United States. The United States, whilst not an emerging market, was included in the study as a benchmark.

finding that asymmetry in volatility is present across each of the indices, and is best captured by the GJR GARCH model to avoid mis-specification errors.

Given the recall of studies which have detected asymmetry in the conditional volatility of stock returns, it is no wonder that the phenomenon is now regarded as conventional of financial time series (Wang & Wang, 2011). Unlike asymmetry in the conditional variance, asymmetry in the first order moment is still a debated topic of financial theorists.

2.4. Asymmetry in Return Reversals

Whilst sufficient explanations and empirical evidence has been offered to account for the asymmetric characteristics of time-varying volatility, mixed evidence of asymmetry in stock returns themselves has been presented (Wang & Wang, 2011).

Researchers have documented time variation of stock returns, specifically two phenomena involving the asymmetry of return dynamics: firstly, stock returns often do not appear to reflect all associated risks, which are predictable ex ante, particularly those risks which arise from bad news and secondly, negative returns tend to revert more quickly to positive returns following bad news, than positive returns revert to negative returns (Nam, Pyun, & Avard, 2001).

The asymmetric behaviour of the conditional mean[14] of stock returns has been a topic of debate since the early 1980's. This observed behaviour involves the uneven reverting tendencies of the conditional mean, whereby negative returns revert more quickly and with greater magnitude to positive returns than do positive returns revert to negative returns (Nam, 2003). This asymmetry of the return dynamics implies that stock returns, on average, rebound more quickly under a recent decline, than a recent rise (Nam, 2003).

Asymmetry in return reversals was first detected in a widely referenced study by De Bondt and Thaler (1985). Their research explored the phenomenon, now coined the overreaction hypothesis, whereby previous loser stocks (yielding negative returns and deemed to be undervalued) will outperform previous winner stocks (yielding positive returns and deemed to be overvalued). De Bondt and Thaler (1985) selected a 56-year period of the New York Stock Exchange ("NYSE") from

[14] The mean of stock returns is defined as a first order moment, or average that is used to determine the central tendency of the data.

1926–1982 on which to conduct their study, which involved forming non-overlapping portfolios of the fifty most extreme losers and fifty most extreme winners based on the stocks monthly cumulative abnormal returns ("CARS") measured thirty six months prior to portfolio formation. The authors demonstrated that after tracking the portfolios for 36 months after formation, loser portfolios outperform the market portfolio (used as a benchmark) by 20%, whereas winner portfolios underperformed the market portfolio by 5% during the same period (De Bondt & Thaler, 1985). The difference in the over and underperformance was interpreted as evidence of asymmetry in the behaviour of returns whereby negative returns reverted to positive returns at a quicker and greater magnitude than positive returns reverted to negative returns. Post accusations that the observed asymmetry was a result of the "firm size effect[15]" and inherent riskiness of the share, the authors adjusted for these factors in a later study in 1987, but found the asymmetry to be statistically significant and unchanged (De Bondt & Thaler, 1987).

The results presented by De Bondt and Thaler (1985) were reasoned to be linked to biases in investor decision making. Investors making decisions during the course of an altering environment will formulate a provisional decision, gather new information, update their decision and use it when the time comes to act (Page & Way, 1992). The weight placed on a piece of information received by the investor should be proportional to its respective accuracy and credibility (Cubbin, Eidne, Firer & Gilbert, 2006). Griffin and Tversky (1992) distinguish between the strength and weight of a signal, a string of positive earnings announcements has great strength as it is well publicised and reported but little weight as the runs do not guarantee the next announcement will be positive rather than negative.

Overconfidence occurs when investors place too much weight on information they have collected themselves rather than information released to the public, and consequently overestimate the precision of the information (Daniel, Hirshleifer, & Subrahmanyam, 1998). Griffin and Tversky (1992, p. 413) clarify that "if people are highly sensitive to variations in extremeness of evidence and not sufficiently sensitive to variations in its credence or predictive validity, then judgments will be over confident". Individuals have a tendency to filter information to allow them to maintain confidence in their own decisions (Cubbin, Eidne, Firer & Gilbert, 2006). In doing so they underweight or completely ignore information that would or has caused them losses to keep their self-esteem intact

[15] The firm size effect refers to the hypothesis that smaller firms with small market capitalizations achieve higher risk adjusted returns than larger firms. Return reversals attributed to the firm size effect implies that smaller firms are more risky and undervalued and therefore once the risk adjusted return is taken into account, this observed return reversal will disappear (Albert & Henderson, 1995).

(Cubbin, Eidne, Firer & Gilbert, 2006). An example would be the refusal to acknowledge firms as losers and sell their stock, as this would be admitting to an error in judgement (Odean, 1998). Griffin and Tversky (1992) found that experts tend to be more overconfident than inexperienced individuals are. Daniel, Hirshleifer and Subrahmanyam (1998) believe that investor judgement is tainted by overconfidence more so in the event of them analyzing vague or subjective information such as book to market ratios, which are not directly intuitive.

Fama & French (1988) and Poterba & Summers (1988) both interpreted the findings of De Bondt and Thaler (1985) as evidence that the relationship between return observations was not a static one. Both sets of authors examined the autocorrelation coefficients of stock returns to gain insight into the relationship between one return observation, with another through time. Fama & French (1988) put forward their theory that stock returns can be modelled as the sum of a stationary component and a random component[16]. Poterba and Summers (1988), after having conducted a study on the United States and seventeen other countries, discovered the presence of negative autocorrelation. The authors subsequently hypothesized that the stationary component of stock prices, which conveys the relationship between the current period's returns and the previous period's return, advocates the desirability of investment strategies involving the purchase of securities that have recently declined in value.

Chan (1988) provided evidence in support of asymmetry of return reversals over a 53-year period on the NYSE when the author replicated the De Bondt and Thaler (1987) empirical investigation, accounting for the "beta" of the stock, as a measure of risk. Chan (1988) found evidence of asymmetry, albeit not as significant as De Bondt and Thaler (1987). Ball and Kothari (1989) continue in the same vein as Chan (1988) and document that the equity betas of past extreme losers exceed the equity betas of past extreme winners by 0.76 in the period following the formation of loser and winner portfolios, which they suggest accounts for the asymmetry they discovered.

Chopra, Lakonishok and Ritter (1992) controlled for both size and beta of firms on the NYSE and still found evidence of the overreaction effect of losers outperforming winners of between 5-10% per annum during the five years after portfolio formation. Chopra, Lakonishok and Ritter (1992) were the first to suggest that the phenomenon constituted a change in the way we price assets for risk and proposed a revision of the Capital Asset Pricing Model ("CAPM") to incorporate a term to capture the inclination of returns to revert asymmetrically.

[16] Already discussed in detail in Section 2.1

Lakonishok, Shleifer and Vishny (1994) find evidence of asymmetry in return reversals on the NYSE and the American Stock Exchange ("AMEX") even when controlling for firm size and attribute this finding to investor overreaction. Albert and Henderson (1995) illustrated that return reversal of shares cannot be attributed solely to the firm size and is more consistent with the overreaction hypothesis.

Chen and Sauer (1997) extend on the earlier research performed by Chopra, Lakonishok and Ritter (1992) and foundthe existence of return reversals, despite controlling for firm size and equity betas. In addition, Chen and Sauer (1997) suggested that this observed return reversal is stronger during certain periods.

Richards (1997) presents evidence of winner-loser portfolio reversals for 16 markets including Austria, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States over the period 1969–1995. Richards (1997) found that the reversal momentum is the strongest for winner portfolios and found no evidence that loser portfolios are riskier than the winner portfolios after adopting three measures of risk. In addition, Richards (1997) concludes that winner–loser reversals cannot be considered exclusively as a small market phenomenon because they are also observed in large markets.

Otchere and Chan (2003) examined the return reversals of stock returns in the Hong Kong market using data from 1996 -1998, which included the Asian financial crisis. Consistent with prior studies the authors found evidence of overreaction which was more pronounced for winners than losers and present pre and post the financial crisis even after controlling for the bid-ask bounce, firm size effect and the day-of-the-week effect[17].

Chiao and Hueng (2005) prove that firm size and book-to-market ratios cannot fully explain the reversal of returns in Japan. The authors discovered that the overreaction effect is significant and suggest the addition of a new factor, which captures the asymmetry, to the 3-factor model of Fama and French (1993) to improve the performance of the model.

Plaistowe and Knight (1986) were the first authors to translate the De Bondt and Thaler (1985) study to a South African setting. In their research, 35 firms with activities spanning the industrial sector of the JSE were examined for the period 1973-1980. The weekly return for each share was recorded for the period, leading to four hundred and four return observations per share. The authors then ranked

[17] The day of the week effect refers to the phenomenon whereby stock returns appear to trade higher on a Friday and lower on a Monday (Otchere & Chan, 2003).

the shares according to their market to book ratios, as a measure of whether the shares are under or overvalued. All those trading at a premium were placed in a winner portfolio, and all those trading at a discount, were placed in a loser portfolio. Whilst the results of the study showed that the loser portfolio did not give rise to significant abnormal gains relative to the returns of the RDM 100 Index of Industrial Shares (benchmark), the winner portfolios did exhibit statistically significant abnormal losses, which were interpreted as evidence of overreaction.

Page and Way (1992) examined a 15-year period of the JSE for signs of overreaction on the behalf of investors, which could lead to asymmetric return reversals. The winner and loser portfolios were created and their results concluded that the loser portfolio outperformed the winner portfolios by an alarming average of 15%, statistically significant at the 1% level (Page & Way, 1992).

Cubbin, Eidne, Firer and Gilbert (2006) used price-to-earnings ratios to classify the shares on the JSE, over a 15-year period as losers or winners, and then tracked those portfolios 60 months after formation. The results detailed by the authors were consistent with the findings of De Bondt and Thaler (1985) and asymmetric return reversals were found to be present on the JSE.

Hsieh and Hodnett (2011) documented stronger reversals for the winner portfolio than the loser portfolio when the authors examined stock returns on the JSE for the period 1993–2009. The strength of the reversals was found to be cyclical and fluctuates around the South African business cycle. The authors suggested that contrarian investment strategies, which are based on making investments, which are opposite to traditional thought, such as investing in previous loser stocks, could be a safe haven for investors during the financial market turmoil due to their low correlations with the market during the economic downturn (Hsieh & Hodnett, 2011).

The idea and large body of supporting knowledge, that a portfolio of recent loser stocks will outperform a portfolio of recent winner stocks forms the basis of contrarian investment strategies. These trading rules have been developed to exploit the asymmetry in return reversals and are still in use more than 40 years later. In fact, many other investment strategies such as those based on the price to earnings ratio or the market to book ratio are regarded as variants of this strategy (Chan, 1988).

Recent studies in econometrics have suggested that the use of non-linear time series structures can better model the attitude and behaviours of investors towards risk, and models, which capture the conditional mean, need to be appropriately modelled to avoid mis-specification in the conditional variance (Lundbergh & Terasvirta, 1998).

2.5. The Nonlinear Time-Varying Pattern of Stock Returns and Volatilities

Authors have recently endeavoured to combine asymmetric time-varying volatilities and asymmetric reversals of returns into a complete stochastic model of stock market behaviour capable of capturing nonlinear features in the mean and conditional variance (Wang & Wang, 2011). It appears that these endeavours have yielded empirical results, which account for asymmetric patterns in the return reversals and asymmetry in volatilities. Although the authors use various combinations of stochastic models capable of capturing asymmetry[18], all arrive at the same conclusion: both the conditional mean and conditional variance display nonlinear behaviour.

Koutmos (1998) utilised the Asymmetric Autoregressive Threshold GARCH ("AR-TGARCH") model to test for asymmetries in the conditional mean and conditional variance across nine national stock markets of Australia, Belgium, Canada, France, Germany, Italy, Japan, United Kingdom and United States. The empirical evidence presented suggests that both the mean and variance respond asymmetrically to past information. Koutmos (1998, p. 277) summarises his findings as follows, "In agreement with other studies, the conditional variance is an asymmetric function of past innovations rising proportionately more during market declines, the so-called leverage effect. With two exceptions, the conditional mean is also an asymmetric function of past returns. Specifically, positive past returns are twice as persistent as negative past returns of an equal magnitude. This behaviour is consistent with an asymmetric partial adjustment price model where prices incorporate negative returns (bad news) faster than positive returns (good news)".

Nam, Pyun and Avard (2001) examine the reverting pattern of monthly return indices of the NYSE, AMEX and NASDAQ through the utilisation of the Asymmetric Nonlinear Smooth Transition GARCH ("ANST-GARCH"). The authors documented that between 1926 and 1997, not only did negative returns revert to positive returns quicker than positive returns reverted to negative returns, but negative returns in fact experienced reduced risk premiums due to predictable high volatility – empirical support for the overreaction hypothesis.

[18] The AR(1) process of Equation 1 and the conditional volatility processes of Equations 6 & 7 would not be appropriate models to capture asymmetry as the correlation coefficients which capture the relationship with the previous periods returns (ϕ and $\propto_i$) are symmetric in nature, and do not allow for a conditional response to a prior positive or negative return shock.

Nam, Pyun and Kim (2003) utilized the same ANST-GARCH stochastic modelling to capture the asymmetrical behaviour of the first and second order moments of stock returns of the Pacific basin countries: Australia, Hong Kong, Indonesia, Japan, Malaysia, Singapore, South Korea, Thailand and Taiwan. The aim of the research was twofold, firstly to report findings regarding the presence of asymmetrical reverting patterns in the mean and variance as well as identify the dominant source for such asymmetry found in the nine markets using short horizon return series. The findings of Nam, Pyun and Kim (2003, p.483) are as follows: "Negative returns are on average more likely to revert, with a greater reverting magnitude, to positive returns than are positive returns to revert to negative. The observed asymmetry appears systematic in that it is intrinsic to an aggregate time series of returns and investors respond differently in their pricing behaviour to a positive and a negative return shocks".

Chen, So and Gerlach (2005) examined asymmetries of the G7 countries by employing a double threshold GARCH ("DTGARCH") model to the daily stock returns for an 11-year period. Like the authors before them, Chen, So and Gerlach (2005) discovered that both the conditional mean and conditional variance exhibit nonlinearity, responding asymmetrically to prior return shocks across all seven markets.

Doong, Yang and Chiang (2005) examined autocorrelation and cross autocorrelation patterns for selected Asian stock returns with the intention of detecting asymmetry in the first and second order moments via the use of an Asymmetric Threshold GJR GARCH ("AT-GJR GARCH"). The authors considered the response of the Asian markets to news coming out of the United States and discovered that asymmetric effects are found to be present in both mean and variance equations. The results also showed that investors in Asian markets tend to react more significantly to negative stock news originating from US sources than they do to positive news.

Nam, Kim and Arize (2006) examined the daily and weekly returns of the S&P500 and the 30 Dow Jones in their endeavours to test for nonlinearity in the short horizon return dynamics of these markets for the period 1962 – 2003. The authors used a similar stochastic model to the ANST-GARCH of Nam, Pyun and Kim (2003), called the ANAR- EGARCH. The authors discovered that the indexes exhibited strong asymmetric reverting patterns, which intensified after the 1987 October stock market crash in the United States, in support of the leverage effect and overreaction hypothesis. The model developed by Nam, Kim and Arize (2006) has been utilized in this research report, with a GJR GARCH model employed to capture asymmetry in the conditional variance instead of the EGARCH.

The theoretical framework surrounding this model is discussed in Section 3.4, the findings of this research compared with the results of Nam, Kim, and Arize (2006) discussed in Section 5.1.

Kulp-Tag (2007) applied the ANAR – EGARCH of Nam, Kim and Arize (2006) to the Nordic stock market and supported the findings of Nam, Kim and Arize (2006) by detecting statistically significant asymmetry in both the mean and the variance. Liau and Yang (2008) extended the Nam, Pyun and Kim (2003) research by utilizing their ANST-GARCH model to the same dataset, which had been extended by a further 4 years. The results were unchanged, statistically significant asymmetry was detected and interpreted as evidence of investor overreaction.

Ibrahim (2010) describes the asymmetric return patterns of Indonesia, Malaysia, the Philippines, Singapore, Thailand and Vietnam in his study utilizing the Autoregressive Exponential GARCH ("AR-EGARCH") to estimate the daily returns for the period 2000-2010. Ibrahim (2010) discovered that all of the markets had quick reversion speeds but each had distinct patterns of return dynamics. Indonesia and Vietnam displayed the strongest reverting pattern, which Ibrahim (2010) deemed to be the most appropriate markets for investors looking to apply contrarian investment strategies.

Research into the topic of asymmetry in the conditional mean and conditional variance has gained momentum during the past decade. Whilst behavioural finance remains a valuable area of study and understanding, empirical attempts to examine this fundamental topic may provide misleading results if studies ignore the nonlinearity intrinsic in return dynamics and volatilities (Nam, Kim, & Arize, 2006).

2.6. Contribution of this Research

Whilst research into the overreaction hypothesis and volatility asymmetry has been conducted on the JSE in separate studies by authors such as Cubbin, Eidne, Firer & Gilbert (2006) and Chinzara & Slyper (2013) respectfully, no study combines these two asymmetries into a single model, able to detect nonlinear behaviour in both the conditional mean and conditional variance of South African stocks.

The presence of asymmetries in the conditional mean and variance within the realm of international markets, specifically those of developed nations, does not automatically translate to emerging markets and the South African context. Emerging markets in particular have distinctive attributes which set them apart from the developed counterparts. These attributes include, high, non-normally distributed returns and volatility, synchronous movements of individual stocks, low market efficiency and higher capital costs (Marais, 2008). There has been limited empirical testing conducted on the South African market in relation to other international exchanges and therein lies the importance of conducting such research.

In addition to this, the South African economy and capital market is remarkably smaller than other economies and hold certain unique characteristics that may not exist in developed countries (Graham & Uliana, 2001). Some of these characteristics may be a result of the country's historical socio-political situation, ownership concentration and the natural resource supply. A large portion of the listed shares are dominated by resource stocks which are subject to their own exclusive sources of risk, which could affect investor behaviour differently (Auret & Sinclaire, 2006).

Although the model utilized in this research report is that of Nam, Kim and Arize (2006), there are few extensions to the scope of their research. Firstly, this research report not only considers the daily and weekly returns, but also the monthly returns of the JSE in detecting asymmetry. This extension will indicate whether asymmetry is present in short, medium or longer-term horizons. Secondly, whilst this research report considers the value-weighted market portfolio of the ALSI, this research is extended to the value-weighted indices of the FINDI, FINI, INDI and RESI to detect whether asymmetry is present across different industries. Thirdly, this research report considers market capitalisation, and the possible influence this may have on asymmetry, by including the indices of the Top40, MidCap and SmallCap in the testing.

Given the findings of South African authors Mzamane (2013) and Chinzara & Slyper (2013), the GJR GARCH was found to be the most parsimonious in detecting asymmetry of volatility in the context of the JSE. For this reason, the EGARCH of Nam, Kim and Arize (2006) was replaced with the GJR GARCH.

Due to the importance of volatility and the dynamics of returns to domestic financial markets, this study wishes to provide practical information with regards to the correct specification of volatility and returns, and test whether asymmetry occurs in the first and second order moments of this market.

3. Theoretical Framework

The theoretical framework outlined in this research report aims to model, and capture the presence of asymmetry in the conditional mean and conditional variance through a stochastic process, which assumes non-linearity in the first and second order moments of stock returns. The presence of strong asymmetric reverting tendencies of the conditional mean and variance would give rise to a negative return reverting more quickly and with greater magnitude to a positive return, than a positive return reverting to a negative return.

The ANAR–GJR GARCH model utilized in this research report is a combination of two asymmetric models developed and used by Nam, Kim & Arize (2006) to capture asymmetry in the conditional mean, with an application of the GJR GARCH model of Glosten, Jagannathan and Runkle (1993) to capture asymmetry in the conditional variance.

The theoretical frameworks surrounding the two models and that of the combined ANAR–GJR GARCH is presented below to provide an understanding of the models, and reasoning for the selection and subsequent utilization in this research report.

3.1. The Asymmetric Reverting Property of Return Dynamics

The asymmetric reverting properties of return dynamics cannot be captured through traditional autoregressive models as these models are restricted by the assumption of a constant serial correlation coefficient (Nam, 2003). In other words, these models assume the serial correlation between returns remains constant over time (Nam, 2003). In order to capture asymmetry in the mean, the serial correlation between return observations needs to change in response to a prior negative or positive return shock (Nam, 2003).

Nam, Kim & Arize (2006) demonstrated this concept through the following explanation:

Suppose stock returns follow the below non-linear autoregressive process, a small modification to the AR process described in Equation 1:

$$r_t = \mu + \phi^{-} r_{t-1} + \varepsilon_t, \text{ if } r_{t-1} < 0 \tag{8}$$

or

$$r_t = \mu \ + \phi^{+} r_{t-1} + \varepsilon_t, \text{ if } r_{t-1} \geq 0$$

(9)

Where, as cited by Nam, Kim & Arize (2006):

r_t is the return in time "t"

μ is a constant, or intercept of the data generating process

ϕ^{-} is the measure of serial correlation when $r_t < 0$

ϕ^{+} is the measure of serial correlation when $r_t \geq 0$

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

The autoregressive processes depicted by Equations 8 and 9 above are data generating processes for return estimations at time "t". The autoregressive process models the return in time "t", r_t , as a function of the sum of a constant μ, the previous periods return, r_{t-1} , its relationship with the current period return which is allowed to change through time, and a residual term ε_t which captures the "randomness" of stock returns. ε_t follows a normal distribution with a mean of 0 and a variance of h_t (Nam, Kim, & Arize, 2006)

The relationship between r_t and r_{t-1} is captured through the serial correlation coefficient ϕ. The serial correlation between return observations is measured by ϕ^{-} when $r_t < 0$ and ϕ^{+} when $r_t > 0$. This specification allows for a different return generating autoregressive process, and response, of r_t under a prior positive and negative return shock (Nam, Kim, & Arize, 2006).

In order for the stationary condition to hold for the return generating process, r_t , the absolute value of the serial correlation observation has to be less than 1, ie: $|\phi^{-}|$ <1 and $|\phi^{+}|$ <1 (Nam, Kim, & Arize, 2006).

The asymmetric reverting pattern this autoregressive process aims to capture, would imply that ϕ^{+} > ϕ^{-}; in other words the serial correlation under a prior positive return is greater than the serial correlation under a prior negative return[19]. The condition ϕ^{+} > ϕ^{-} has two implications. Firstly,

[19] The reverting magnitude of autoregressive processes with unconditional variance such as the SMA and EWMA assume a constant serial correlation coefficient and therefore prior return shocks, positive or negative, illicit an identical response in r_t. This implies serial correlation in these models are captured by the condition ϕ^{+} = ϕ^{-} (Nam, 2003). See Equation 1

since ϕ^+ and ϕ^- measure the reverting *speed* of r_t, through allowing r_{t-1} to have a great impact on r_t, if $r_{t-1} > 0$ than if $r_{t-1} < 0$, $\phi^+ > \phi^-$ would imply that a negative return reverts on average more quickly to a positive return than a positive return reverts to a negative return with the same magnitude. Secondly, $\phi^+ > \phi^-$ measures the relative reverting *magnitude* of a positive and negative return, whereby the reverting magnitude of negative returns is greater than the reverting magnitude of positive returns (Nam, Kim, & Arize, 2006).

By way of explanation, let $r_{t-1} = \mu = 0$, the autoregressive process will then take on the form: $r_t = \varepsilon_t$. Let ε_t^- denote the magnitude of the negative return shock ($\varepsilon_t < 0$), and ε_t^+ denote the magnitude of the positive return shock ($\varepsilon_t > 0$) at time "t", such that $\varepsilon_t^+ = -\varepsilon_t^-$. Under these assumptions the conditional expected return is denoted as $E(r_{t+1}|\varepsilon_t) = \phi^- \varepsilon_t^-$ if $\varepsilon_t < 0$ or $E(r_{t+1}|\varepsilon_t) = -\phi^+ \varepsilon_t^+$ if $\varepsilon_t > 0$. According to the stability condition of the autoregressive process, the reverting magnitude of the expected return at time t+1 can be measured by the difference between r_t and $E(r_{t+1}|\varepsilon_t)$, which can be denoted as: $r_t - E(r_{t+1}|\varepsilon_t) = (1 - \phi^+).\varepsilon_t^+$ if $\varepsilon_t > 0$ or $r_t - E(r_{t+1}|\varepsilon_t) = (\phi^- - 1).\varepsilon_t^-$ if $\varepsilon_t < 0$. If the reverting magnitude in absolute value of negative returns is greater than the reverting magnitude of positive returns, then the comparison of the reverting magnitude of positive returns and negative return can be expressed as an inequality as such: $|r_t - E(r_{t+1}|\varepsilon_t < 0)| > |r_t - E(r_{t+1}|\varepsilon_t > 0)|$. Since $\{r_t - E(r_{t+1}|\varepsilon_t < 0)\} > 0$ and $\{E(r_{t+1}|\varepsilon_t > 0) - r_t\} > 0$, the inequality can be expressed as $(\phi^- - 1).\varepsilon_t^- > (1 - \phi^+).\varepsilon_t^+$ or $\phi^+ > \phi^-$. Hence by deduction, $\phi^+ > \phi^-$ confirms that the reverting magnitude of a negative return is greater than that of a positive return. Therefore, the condition $\phi^+ > \phi^-$ implies that a negative return reverts more quickly, with greater magnitude to a positive return than a positive return reverts to a negative return (Nam, Kim, & Arize, 2006).

3.2. The Asymmetric Nonlinear Autoregressive Process

Given the rationale and mathematical proof detailed by Nam, Kim & Arize (2006) above, and the empirical evidence of LeBaron (1992) and Sentana & Wadhwani (1992), the ability to capture asymmetry in return dynamics rests on a nonlinear autoregressive process, which allows for the coefficient of the serial correlation term to change in response to a prior positive or negative return shock.

The model used in this research report is the ANAR model of Nam, Kim & Arize (2006). The ANAR model acts to capture the nonlinear dynamics of stock returns, and when modelled in tandem with the GJR GARCH of Glosten, Jagannathan and Runkle (1993), is able to effectively capture nonlinear behaviour in the conditional mean and the conditional variance.

The data generating process for stock returns – the ANAR model- is specified as follows:

$$r_t = \mu + [\phi_1 + \rho_1 D_1(r_{t-1} < 0)] . r_{t-1} + \varepsilon_t \tag{10}$$

Where, as cited by Nam, Kim & Arize (2006):

r_t is the return in time "t"

μ is a constant, or intercept of the data generating process

ϕ_1 is a partial measure of serial correlation

D_1 is an indicator function specified for a dummy variable which takes on the value of 1 if $r_{t-1} < 0$ or 0 otherwise.

ρ_1 is the coefficient of the dummy variable D_1

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

The ANAR model (10) above combines models (8) and (9) into a single autoregressive return process able to capture asymmetry in the response to both a prior positive and negative return shock, by allowing the autocorrelation coefficient of stock returns to vary with the sign of the prior periods return (Nam, Kim, & Arize, 2006).

The factors of influence in determining r_t, those being the constant μ, the previous periods return r_{t-1}, its degree of influence controlled by the serial correlation coefficient, and the residual component which captures "randomness" in the stock return, ε_t, remains the same. ε_t follows a normal distribution with a mean of 0 and a variance of h_t (Nam, Kim, & Arize, 2006).

The return serial correlation of the ANAR model is measured by $\phi_1 + \rho_1 D_1$. If r_{t-1} < 0, D_1 will take on the value of 1 and the serial correlation will be measure through the sum of $\phi_1 + \rho_1$. If r_{t-1} > 0, D_1 will take on the value 0 and the serial correlation will be measured by ϕ_1 (Nam, Kim, & Arize, 2006).

The return serial correlation is restricted by $|\phi_1 + \rho_1 D_1|$ < 1 for the stationarity condition of return dynamics to hold.

The condition of asymmetry in reverting speed and magnitude, ϕ^+ > ϕ^-, derived in Section 3.1 for the asymmetry property is equivalent to ρ_1 < 0 in the ANAR model (10) above. By deduction ϕ^+ > ϕ^- is the equivalent to $\phi_1 > \phi_1 + \rho_1$ under model (10) hence ρ_1 < 0.

The condition ρ_1 < 0 implies an asymmetric reverting pattern of stock returns that a negative return reverts more quickly, and with a greater magnitude than a positive return reverts to a negative return. If $\rho_1 = 0$, then the reverting pattern is symmetrical and a prior positive or negative return has the same effect on the current periods return, which is the case for Equation 1.

3.3. Conditional Heteroskedasticity of Stock Returns

Conditional heteroskedasticity is a stylized fact of stock return observations (Nam, Kim, & Arize, 2006). The family of ARCH and GARCH models were developed to successfully capture heteroskedasticity in the second order moment of return dynamics (Nam, Kim, & Arize, 2006). The early generation models are not well suited to capturing the asymmetric response of volatility present in financial time series data (Nam, Kim, & Arize, 2006). There have been many refinements to the GARCH family of models to incorporate the asymmetric volatility response to the conditional variance of stock return series (Nam, Kim, & Arize, 2006). The refined model used to capture conditional variance in this research report is the GJR GARCH model of Glosten, Jagannathan and Runkle (1993), specified as:

$$h_t = \propto_0 + \propto_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \omega_1 \varepsilon_{t-1}^2 I_{t-1} \tag{11}$$

h_t is the conditional variance in time "t"

$\propto_0$ is a constant, or intercept of the data generating process

$\propto_1$ is the correlation coefficient of the lagged squared residual term

ε_{t-1}^2 is the squared residual term, lagged by one time period.

β_1 is the correlation coefficient of the lagged conditional variance term

h_{t-1} is the conditional variance, lagged by one time period

ω_1 is the asymmetry coefficient

I_{t-1} is an indicator function, where $I_{t-1} = 1$ if ε_{t-1} < 0, or $I_{t-1} = 0$ if ε_{t-1} > 0

The GJR GARCH model differs from the traditional GARCH model of Equation 7 with the addition of the dummy variable term, $\omega \varepsilon_{t-1}^2 I_{t-1}$. The dummy variable, I, allows past residuals to have an asymmetrical effect on the conditional volatility depending on the sign of the shock. If the prior period residual term was negative, then I_{t-1}, takes on the value 1, and the squared residual term is a functional input into the calculation of the conditional variance. If the prior period residual term was positive, then I_{t-1}, takes on the value 0 and eliminates the term from Equation 11 (Glosten, Jagannathan, & Runkle, 1993).

Asymmetry in volatility, caused by the leverage effect would imply that good news, measured by $\varepsilon_{t-1} > 0$, and bad news, measured by $\varepsilon_{t-1} < 0$, will have different impacts on conditional volatility. The GJR GARCH models allows the impact of good news to be captured by $\propto_1$ and the impact of bad news to be captured by $\propto_1 + \omega$. If the asymmetry coefficient, $\omega > 0$, is positive and significant this would imply that bad news increases future volatility, an indication that the leverage effect is at play, however if $\omega \neq 0$ then the impact of news on the conditional volatility is asymmetric but bad news does not necessarily increase future volatility (Mandimika & Chinzara, 2010).

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The conditions which ensure a positive conditional variance estimation are $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$.

3.4. The ANAR-GJR GARCH Model

The ANAR–GJR GARCH specification is a combination of the nonlinear ANAR model of Nam, Kim and Arize (2006) to capture asymmetry in the conditional mean, and the GJR GARCH model of Glosten, Jagannathan and Runkle (1993) to capture asymmetry in the conditional volatility. The combined ANAR GJR GARCH involves the joint estimation of the conditional mean and conditional variance as follows:

$$r_t = \mu + [\phi_1 + \rho_1 D_1(r_{t-1} < 0)] \,.\, r_{t-1} + \varepsilon_t \tag{12}$$

Where, as cited by Nam, Kim & Arize (2006):

r_t is the return in time "t"

μ is a constant, or intercept of the data generating process

ϕ_1 is a partial measure of serial correlation

D_1 is an indicator function specified for a dummy variable which takes on the value of 1 if $r_{t-1} < 0$ or 0 otherwise.

ρ_1 is the coefficient of the dummy variable D_1

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

The return serial correlation is restricted by $|\phi_1 + \rho_1 D_1| < 1$ for the stationarity condition of return dynamics to hold.

The residual term of Equation 12 is captured through the following random process:

$$\varepsilon_t = V_t \,.\, \sqrt{h_t} \tag{13}$$

Whereby:

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

V_t is the "white noise" component or random component

h_t is the conditional variance described by a GJR GARCH autoregressive process

The residual term ε_t follows a normal distribution with a mean of 0 and a variance of h_t.

The stochastic process of the conditional variance follows a GJR GARCH specification of Glosten, Jagannathan and Runkle (1993) as follows:

$$h_t = \propto_0 + \propto_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \omega_1 \varepsilon_{t-1}^2 I_{t-1} \tag{14}$$

h_t is the conditional variance in time "t"

$\propto_0$ is a constant, or intercept of the data generating process

$\propto_1$ is the correlation coefficient of the lagged squared residual term

ε_{t-1}^2 is the squared residual term, lagged by one time period.

β_1 is the correlation coefficient of the lagged conditional variance term

h_{t-1} is the conditional variance, lagged by one time period

ω_1 is the asymmetry coefficient

I_{t-1} is an indicator function, where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, or $I_{t-1} = 0$ if $\varepsilon_{t-1} > 0$

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The conditions which ensure a positive conditional variance estimation are $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$.

4. Research Methodology

4.1. Data Description

In order to test for asymmetry in the conditional mean and conditional variance across an array of factors and industry specific characteristics, the data used in this study comprises of three observed time intervals across four industries and three different market capitalizations obtained from the FTSE/JSE Africa Index Series. The FTSE/JSE Africa Index Series is the result of a joint venture between the JSE Limited and the world leader in the creation and management of indices, FTSE (JSE, FTSE/JSE Africa Index Series: A Comprehensive Guide, 2004).

In keeping with the Nam, Kim & Arize (2006) study, the asymmetry analysis is conducted on value-weighted indices. A value weighted index or free float weighted index uses the freely traded shares to calculate market capitalisation, and based on the capitalisation, determines a weight of that share in the index.

The indices utilised in this study are calculated based on the FTSE/JSE Africa Index Series Calculation Guide which reads as follows, "The price index is the summation of the market values (or capitalisations) of all companies within the index and each constituent company is weighted by its market value (shares-in-issue multiplied by share price multiplied by investability weighting). The investability weighting is also called the free float factor. The price movement of a larger company (say, representing five per cent of the value of the index) will, therefore, have a larger effect on the index than a smaller company (say, representing one per cent of the value of the index)" (JSE, FTSE/JSE Africa Index Series: FTSE Guide to Calculation Methods, 2004, p. 3). One of the advantages of using a value-weighted index for empirical analysis is that these indices automatically adjust for corporate actions[20].

The data for this study was attained from McGregor's BFA and consisted of the daily, weekly and monthly closing price for the ALSI index, Top40 index, MidCap index and the SmallCap index for the period 03/07/1995 to 18/10/2011. These indices represent the market portfolio, and a cross section of firm size on the JSE, as sorted by market capitalisation. In order to examine a cross section of industries, the daily, weekly and monthly closing price data was obtained for the FTSE/JSE Specialist Indices: the FINDI index and the INDI index for the period 03/07/1995 to 18/10/2011 and the FINI index and RESI index for the period 02/03/1998 to 18/10/2011[21]. The dataset for each index comprises of approximately 4072 daily data points. Non-trading days were excluded from the dataset, which is in line with empirical studies.

As is a common practise in financial literature, the daily, weekly and monthly closing prices for each index were converted into a daily, weekly or monthly return series with continuous compounding via the following equation:

$$y_t = (\ln P_t - \ln P_{t-1}) \tag{15}$$

Where, as cited by Mandimika and Chinzara (2010):

y_t is the continuous compounded return in time "t"

P_t is the closing price of the index at time "t"

P_{t-1} is the closing price of the index at time "t-1"

[20] An example of a corporate action would be a stock split. After a stock split the decline in price per share is offset by the increase in the total number of share and weight allocated to that corporate as a measure of market capitalisation.

[21] The FINI and RESI indices were not calculated before March 1998.

4.2. Investigating the Univariate Properties of the Time-Series

Prior to conducting regression analyses or running the ANAR- GJR GARCH model it is imperative that tests are conducted on the index return series to consider the properties and the suitability of the series to stochastic modelling. The tests run on each of the daily, weekly and monthly return series for each index are discussed below.

General descriptive statistical tests are an important part of empirical analysis. These tests, which include plots of the price and return series for each index, and the generation of summary statistics for the series: which contain estimations of the mean, median, standard deviation, variance, kurtosis, skewness and range, provide valuable insight into the series and evidence of the presence of stylized facts of financial time series such as leptokurtosis and stationarity. These insights provide the researcher with information on the distributional properties and unique characteristics of the data at a cursory glance.

In financial statistics, it is important to test whether the assumptions of the model, intended to describe the return series, hold. The assumptions play a role in determining the goodness of fit of the model and therefore whether that model is applicable to the dataset. If a researcher tries to fit a model to a dataset where the assumptions of the model do not hold, incorrect inferences will be drawn from the results of the empirical tests.

In order for Engle's (1982) ARCH model, and all further extensions thereof, to be applicable for modelling financial time series, and producing accurate volatility forecasts, the series in question should show signs of non-normality (kurtosis and skewness) as well as heteroskedasticity (volatility clustering and serial dependence between residual terms). In addition, the return series is to be stationary (constant probability distribution through time).

Normality is a common assumption when applying statistical procedures like regressions. Departures from normality may lead to substantially incorrect statements drawn from economic models, which require normality; therefore, normality testing is crucial (Thadewald & Buning, 2004). The tests conducted for normality in this research include plotting the return distribution, with the superimposed normal distribution curve as well as the Jarque & Bera (1980)[22] ("JB") test, which tests the joint null hypothesis of zero skewness and a kurtosis measure of 3, which you would expect to see in a normally distributed series (Thadewald & Buning, 2004). The alternative hypothesis of this

[22] Other normality tests include the test of Kuiper (1960), Shapiro & Wilk (1965) and Kolmogorov-Smirnov & Cramervon Mises, as cited by Thadewald and Buning (2004).

test is a non-normal distribution, whereby skewness and excess kurtosis is present. The calculation for the test statistic is as follows:

$$\mathrm{JB} = n\,.\left[\frac{S^2}{6} + \frac{(EK)^2}{24}\right]$$

(16)

Where, as cited by Jarque & Bera (1980):

JB is the test statistic measuring normality

n is number of observations

S is the skewness measure

EK is the excess kurtosis or kurtosis measure minus 3

Skewness is a measure of asymmetry of a distribution and is calculated as follows, as demonstrated by Bodie, Kane and Marcus (2008):

$$S = {}^{1}\!/_{n} \frac{\sum_{t=1}^{n}(y_t - \bar{y}\,)^3}{\sigma^3}$$

(17)

Where:

y_t is the return at time “t”

$\bar{y}$ is the mean of the return series

n is the number of return observations

σ is the sample standard deviation

The excess kurtosis is a measure of fat tails and peakedness, which reveal deviations from the normal distribution, as explained by Bodie, Kane and Marcus (2008):

$$EK = \left\{ {}^{1}\!/_{n} \frac{\sum_{t=1}^{n} (y_t - \bar{y}\,)^4}{\sigma^4} \right\} - 3$$

(18)

Where:

y_t is the return at time "t"

$\bar{y}$ is the mean of the return series

n is the number of return observations

σ is the sample standard deviation

The JB test statistic is compared with a chi-squared distribution with two degrees of freedom, if the critical value of the test statistic exceeds the chi-squared distribution[23], one rejects the null hypothesis of normality (Thadewald & Buning, 2004).

The test for independence is the test for serial correlation of the residual terms, ε_t. If the residuals are independent from one another, the autocorrelation and partial autocorrelation between the residual observations should be zero. A portmanteau test is a statistical test where the null hypothesis is specifically defined, but the alternative hypothesis is open ended. The classical portmanteau tests are the Box-Pierce (1970) Ljung-Box (1978) tests. Under these tests, the null hypothesis states that no autocorrelation exists in the residuals, with the alternative hypothesis stating that autocorrelation is present.

The calculation for the Box & Pierce (1970) Q statistic is as follows:

$$Q_{BP} = n \sum_{k=1}^{h} \hat{p}_k^2$$

(19)

Where, as cited by Box & Pierce (1970)

Q_{BP} is the Q statistic measuring serial correlation in the residuals

n is the number of observations

[23] The chi squared distribution with "k" degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables (Thadewald & Buning, 2004).

$\hat{p}_k^2$ is the sample autocorrelation of the residuals at lag "k"

h is the number of lags being tested

The calculation for the Ljung-Box (1978) is similar to that of the Box-Pierce:

$$Q_{LB} = n(n+2)\sum_{k=1}^{h}\frac{\hat{p}_k^2}{n-k} \tag{20}$$

Where, as cited by Ljung & Box (1978):

Q_{LB} is the Q statistic measuring serial correlation in residuals

n is the number of observations

$\hat{p}_k^2$ is the sample autocorrelation of the residuals at lag "k"

h is the number of lags being tested

In both cases, if the Q test statistic is greater than the chi-squared distribution with "h" degrees of freedom, one rejects the null hypothesis of no autocorrelation, and serial correlation is present in the residuals (Aranz, 2005).

Brock, Dechert, Scheinkman and LeBaron (1996) suggest another portmanteau test for time-based dependence in a series, often considered as the non-linear analogue of the Box-Pierce Q statistic or the "BDS test". This test can be used for testing a variety of deviations from independence including linear, nonlinear and chaos. To perform the BDS test one first considers a distance (ε) and constructs two points from the series of observations. If the observations are identically and independently distributed then for any pair of points, the probability of the distance between these two point being less than or equal to ε would be constant. This described probability is denoted by C_1 (ε). Next assume a set of multiple pairs of points, as follows:

$$\{\{y_s, y_t\}, \{y_{s+1}, y_{t+1}\}, \dots, \{y_{s+m}, y_{t+m}\}\} \tag{21}$$

Where, as cited by Brock, Dechert, Scheinkman and LeBaron (1996):

y_s, y_t are a pair of points

m is the number of consecutive points used

The joint probability of every pair of points in the set satisfying the ε condition is denoted by the probability, $C_m(\varepsilon)$.

Under the assumption of independence in this BDS test, the joint probability should be the product of the individual probabilities for each of the pairs. That said if the series under evaluation is independent, then:

$$C_m(\varepsilon) \approx [C_1(\varepsilon)]^m$$

(22)

The BDS test applied in this research report utilize the fraction of pairs method to determine the distance ε. In this fashion ε is computed so as to ensure a certain fraction of the total number of pairs of points within the total sample lie within the distance ε from each other.

The BDS test statistic is calculated as follows:

$$C_{m,T}(\varepsilon) = \frac{2}{(T-m+1)(T-m)} \sum_{t<s} I_\varepsilon(y_t^m, y_s^m)$$

(23)

Where, as cited by Brock, Dechert, Scheinkman and LeBaron (1996):

$C_{m,T}(\varepsilon)$ is the BDS test statistic given “m” number of consecutive pairs and “ε” distance

y_s, y_t are a pair of points

I_ε is an indicator function which takes the value of 1 if $|y_t^m - y_s^m| < \varepsilon$ and zero otherwise

The null hypothesis of the BDS test states that the residual terms of the series are identically and independently distributed, in other words, no serial correlation exists between residuals. The alternative hypothesis implies that serial dependence between residuals is present. By using geometric returns, the linear dependence between residual terms has been eliminated; therefore the BDS test run on the residuals terms is testing for non-linear dependence. A z-statistic is constructed by dividing the BDS statistic by the corresponding standard error. The z-statistic is the result used in the hypothesis test, as it is asymptotically normal. If the probability of the z-statistic is less than 0.05, one rejects the null hypothesis and residuals are not independent.

The tests for stationarity of a series include the sectioning of the series to examine the properties of each section (namely comparing the mean and standard deviation to see if they remain constant) and the Augmented Dickey Fuller test (“ADF test”).

A formal and common test for stationarity is the Augmented Dickey-Fuller test. This is a unit root test, which produces a negative statistic; the more negative the reading, the stronger the rejection of the null hypothesis of a unit root.

The ADF test is carried out by performing a regression as follows:

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \ldots + \xi_k \Delta y_{t-k} + \varepsilon_t \tag{24}$$

Where, as cited by Dickey and Fuller (1979):

Δy_t represents the change in the return series

y_{t-1} is the return in time “t-1”

β is the correlation coefficient

k is the chosen number of lags included

δt is white noise innovation

Testing whether β = 0 is equivalent to testing whether y_t follows a unit root process (non-stationary). The number of lags “k” of this regression is a very important practical issue for the implementation of this test. If the number of lags is too small then the serial correlation in the remaining errors will cause bias in the test. If the number of lags is too large then the power of the test will be diminished. There are a number of Monte Carlo simulations which one can run to determine the ideal number of lags, such as the Akaike Information Criterion, Schwarz-Bayesian Information Criterion and the Hannan-Quinn Criterion. Using an algorithm under the Dickey-Fuller with GLS trending, it is possible to obtain the optimum number of lags.

The ADF test statistic is produced by:

$$ADF = \frac{\hat{\beta}}{SE(\hat{\beta})} \tag{25}$$

If the p-value for the ADF test statistic is less than 0.05, one rejects the null hypothesis of non-stationarity in favour of the alternative hypothesis and the series is stationary.

The test for ARCH effects is a joint test for the characteristics of financial time series the ARCH model was specifically designed to accommodate: namely non-constant volatility and serially correlated residual terms.

Testing for ARCH effects involves running a simple regression and performing Engle's (1982) Lagrange Multiplier test for heteroskedasticity, whereby the null hypothesis is the absence of ARCH effects and the alternative hypothesis is the presence of ARCH effects provided that the test statistic exceeds the chi-squared distribution.

The Breusch & Godfrey (1979) test for higher order serial dependence in residuals, whereby regressors are not exogenous carries a null hypothesis of no serial correlation in residuals up to a certain lag order, and an alternative hypothesis of the presence of serial correlation. Provided that the test statistic is greater than the chi-squared distribution one can reject the null hypothesis in favour of the alternative hypothesis. The Breusch-Godfrey test is applicable for the dataset given that the test does not rely on the assumption of strictly exogenous regressors. The ANAR- GJR GARCH utilises lagged return regressors and therefore would distort the results of tests such as the Durbin & Watson (1950) d statistic for serial correlation in the residual term when all regressors are strictly exogenous.

4.3. The ANAR-GJR GARCH Model

The joint estimation of the ANAR-GJR GARCH model will produce estimates of each of the coefficients of the models, specified in Equation 12, 13 and 14.

Under each of the stochastic processes, one is looking to see whether the estimate of the coefficients is significant and in line with expectations given the tests for non-linearity.

The ANAR stochastic model produces estimates of the following correlation coefficients:

The constant or intercept of the ANAR data generating process, μ, is expected to be positive and significant. The coefficient, which measures partial serial correlation, ϕ_1, is expected to be positive and significant. The coefficient of the dummy variable ρ_1 should be negative and significant (Nam, Kim, & Arize, 2006).

Recall, the return serial correlation of the ANAR model is measured by $\phi_1 + \rho_1 D_1$. If $r_{t-1} < 0$, D_1 will take on the value of 1 and the serial correlation will be measure through the sum of $\phi_1 + \rho_1$. If $r_{t-1} > 0$, D_1 will take on the value 0 and the serial correlation will be measured by ϕ_1 (Nam, Kim, & Arize, 2006). The condition $\rho_1 < 0$ implies an asymmetric reverting pattern of stock returns that a negative return reverts more quickly, and with a greater magnitude than a positive return reverts to a negative return. If $\rho_1 = 0$, then the reverting pattern is symmetrical and a prior positive or negative return has the same effect on the current period's return, which is the case for Equation 1 (Nam, Kim, & Arize, 2006).

In order for the stationarity condition of the return dynamics to hold, the return serial correlation is restricted by $|\phi_1 + \rho_1 D_1| < 1$.

The GJR GARCH stochastic process produces estimates of the following correlation coefficients:

The intercept of the data generating process, α_0 is expected to be positive and significant. The correlation coefficient, $\propto_1$, measuring the relationship between the conditional volatility and the lagged squared residual term is expected to be positive and significant. The correlation coefficient, β_1, measuring the interaction between the conditional variance with the lagged conditional variance is expected to be positive and significant. The asymmetry coefficient, ω_1, can take a positive or negative value, but should be statistically significant (Glosten, Jagannathan, & Runkle, 1993).

Recall that, asymmetry in volatility, caused specifically by the leverage effect would imply that good news, measured by $\varepsilon_{t-1} > 0$, and bad news, measured by $\varepsilon_{t-1} < 0$, will have different impacts on conditional volatility. The GJR GARCH models allows the impact of good news to be captured by $\propto_1$ and the impact of bad news to be captured by $\propto_1 + \omega$. If the asymmetry coefficient, $\omega > 0$, is positive and significant this would imply that bad news increases future volatility, an indication that the leverage effect is at play, however if $\omega \neq 0$ then the impact of news on the conditional volatility is asymmetric but bad news does not necessarily increase future volatility (Mandimika & Chinzara, 2010)

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The conditions which ensure a positive conditional variance estimation are $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$.

As a visual demonstration of the asymmetry in volatility, it is possible to generate Pagan and Schwert's (1990) News Impact Curve. The News Impact Curve demonstrates the degree of asymmetry in volatility to prior positive and negative return shocks, by plotting the next period conditional variance (h_t) that would arise from a combination of positive and negative values of residual terms (ε_{t-1}), given the estimated GJR GARCH model. The curves are generated by using the estimated conditional variance per the GJR GARCH specification, with the given coefficient estimations, and the lagged conditional variance (h_{t-1}) set to the unconditional variance. Successive values of residual terms (ε_{t-1}) in the range of -2 to +2 are then used in the GJR GARCH equation to determine what the corresponding values of the conditional variance (h_t) derived from the model would be. The ANAR-GJR GARCH model should produce a News Impact Curve, which is asymmetrical to positive and negative shocks.

5. Empirical Analysis

5.1. Data Descriptive Statistics

The dataset under empirical investigation consists of three different time intervals, daily, weekly and monthly as well as eight indices, which demonstrate the market portfolio, a cross section of market capitalisation and a cross section of industries on the JSE. The results of the empirical investigation will be presented for each index per time interval.

5.1.1. Daily Return Series

Before examining the daily return dynamics, it is useful to consider the daily closing prices of the indices. It is apparent from Figure 1 that daily closing prices have an upward drift; this observation could be a result of new stocks added to the index, in other words, the number of stocks contained in the index has swollen and this has caused the rise in the index closing price over time. Although this suggestion is reasonable, a likely contributor to the upward trend in the index closing price is the comparable larger gains experienced by the index than losses over the period. The Top40 index would be a good example to test these two theories, as the number of stocks in the index remains constant over time. Stocks in the index are reviewed on a quarterly basis and replaced with those with higher market capitalisations (JSE, FTSE/JSE Africa Index Series: A Comprehensive Guide, 2004). As can be seen in Figure 1, the Top40 index has shown an upward trend despite the number of stocks included in the index calculation remaining constant. These observations point to the persistence of positive gains over negative losses.

The visible upward trend is also an indication of non-stationarity. A stationary series will typically oscillate around a central point and contains more desirable characteristics to perform analysis, than stock price levels.

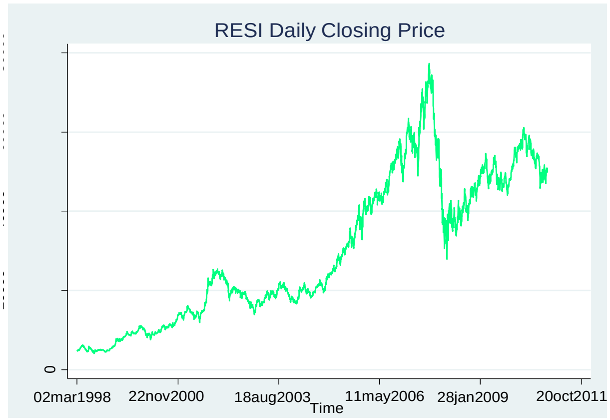
Keeping scale in mind, as the FINI and RESI index is only available from 1998, it is evident that the industry with the highest stock prices is the RESI index which closed out at R498.20 per share in comparison to the FINI which closed out at R78.88 per share on 20/11/2011. This fact is unsurprising given that the resources sector contributes 23% to the JSE's market capitalisation (Borkum, 2014).

Another interesting observation to consider is the shape of the closing prices across indices. The graph for each index has a very similar shape, even when considering scale. This is a reflection of the impact the macro economy has on the stock market as a whole, and the performance of industries in South Africa. Each graph in Figure 1 indicates a steep drop in stock price across industry for the period ending 2006 to the beginning of 2009. The period represents the starting point and beginning point of recovery after the Global Financial Crisis[24], which hit emerging market economies in September 2008 (Bakrania & Lucas, 2009). South Africa was affected by the economic slowdown in our key export markets, coupled with weaker commodity prices and an ebbing of capital flows into the country (Baxter, 2009). These effects can be seen in the steep declines of stock prices in the RESI index. Despite these effects, the country limited the impact of the Global Financial Crisis through various buffers such as low levels of external debt, appropriate fiscal and monetary policies and flexibility in our exchange rate (Baxter, 2009).

The impact of the Global Financial Crisis is a prime highlighter of the volatility inherent in the stock exchange. Volatility speaks to the randomness and uncertainty of the gains to be made on the stock market. Many trading and hedging strategies take advantage of stock market volatilities such as derivative instruments, used to buffer and profit from movements in risk.

[24] The Global Financial Crisis began as the sub-prime credit crisis in the United States and quickly spread to the rest of the world. During this time, the world's top 15 banks saw their market capitalisation fall from USD1.7trillion to USD500billion in a mere 18 months (Baxter, 2009).

Figure 1: Daily Closing Price per Index



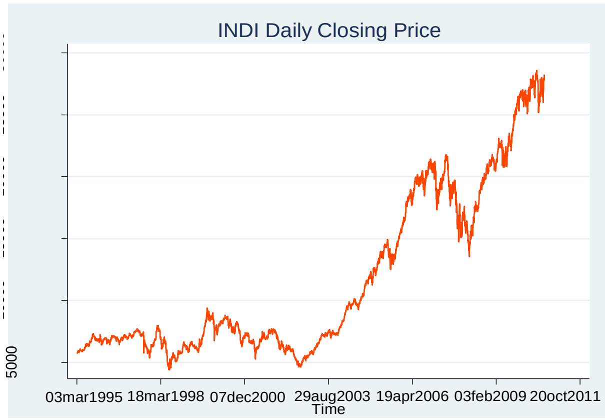


Figure 2 below shows the geometric return series of the eight indices. There are a few noticeable features of the return series. Firstly, the series appears to oscillate around a mean value. If returns follow a normal distribution, this implied mean value is zero, on first glance, this appears to be the case, although later test results presented will confirm or disprove this.

Secondly, given this oscillation, it appears as though this series may be stationary, whereby the series has a constant probability distribution through time. One of the conditions of ARCH/GARCH stochastic modelling requires stationarity of the dataset.

Third, there appears to be clustering of returns, large swings of positive gains and negative losses are clustered together, whilst periods of calm and tranquillity exist between them, a result of volatility clustering – this effect is best seen best in the SmallCap index.

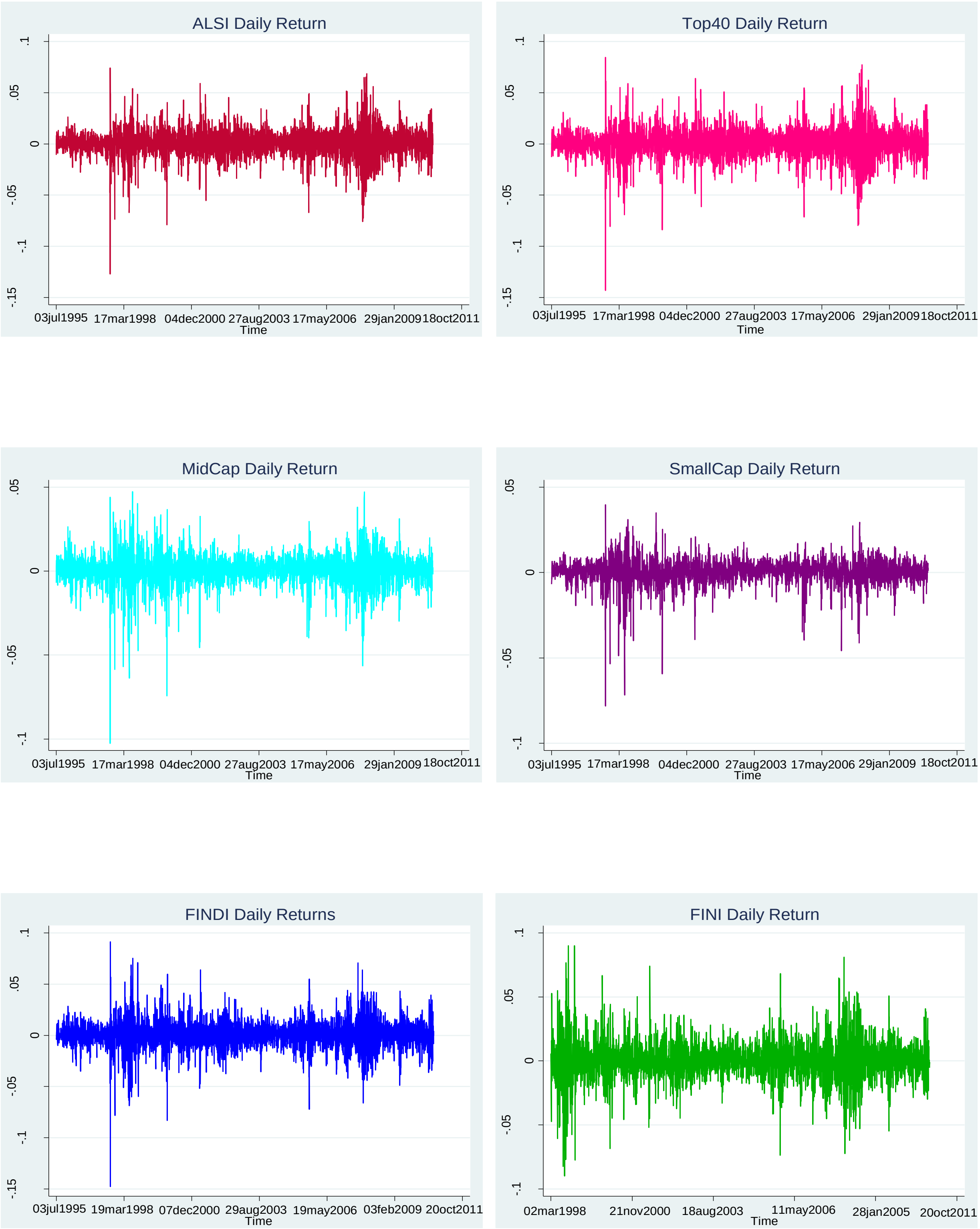
There appears to be two distinct periods of high volatility, the first is in 1997 and the second in 2008. The 1997 spike in volatility can be attributed to the spill over effects of global economic events such as the Asian Financial Crisis[25], the Russian Debt Crisis[26] and the collapse of Long Term Capital Management[27] in the United States (Marais, 2008). For a discussion on contagion between the South African and global economies refer to Collins and Biekpe (2003).

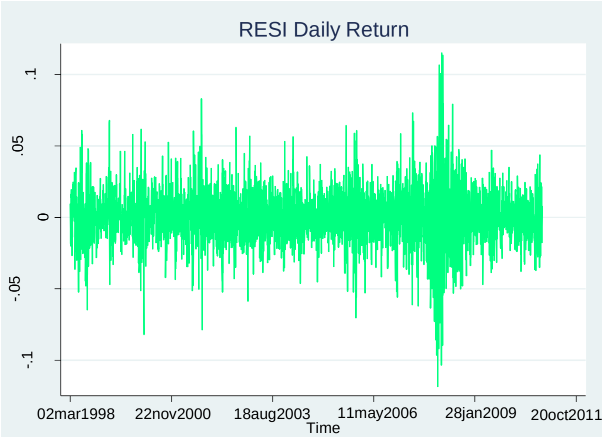
[25] The Asian Financial Crisis of 1997 refers to the failing of a series of currency markets in Asia, beginning with Thailand. The currency declines, caused partly by fault policy interventions of the World Bank and International Monetary Fund caused international stock market declines, reduced import revenues ad government upheavals. Stock market and investor overreaction worsened these effects (Corsetti, Pesenti, & Roubini, 1998).

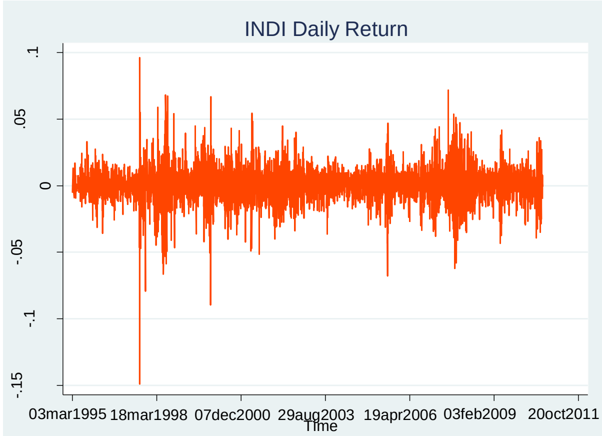
[26] The Russian Debt Crisis of 1998 was of a result of the devaluation of the Russian ruble after speculative trading of the currency began due to the high domestic deficient and low foreign reserves. The substantial devaluation of the currency lead to the Russian default of public and private debt (Chiodo & Owyang, 2002).

[27] Long Term Capital Management was a large hedge fund in the United States which used convergence trades on the bond market to make money. After the Russian Debt Crisis, the fund sustained massive losses and was at risk of defaulting on its loans before it was rescued by the Federal Reserve (Edwards, 1999).

Figure 2: Daily Return per Index







Descriptive statistics of the return series shred greater light on the unique characteristics of the series. The descriptive statistics for the daily return series on each index is presented below.

Table 1: Daily Descriptive Statistics per Index

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Mean	0.00045	0.00044	0.00051	0.00043	0.00036	0.00014	0.00068	0.00039
Standard Error	0.00020	0.00023	0.00015	0.00011	0.00021	0.00027	0.00033	0.00021
Median	0.00085	0.00089	0.00083	0.00088	0.00063	0.00010	0.00074	0.00077
Standard Deviation	0.01306	0.01436	0.00930	0.00711	0.01359	0.01557	0.01918	0.01358
Sample Variance	0.00017	0.00021	0.00009	0.00005	0.00018	0.00024	0.00037	0.00018
Excess Kurtosis	5.91014	5.92030	9.43539	13.64992	7.35875	3.88086	3.49198	7.30798
Skewness	-0.47293	-0.40025	-1.09342	-1.74982	-0.41195	-0.03047	0.01461	-0.45400
Range	0.20113	0.22734	0.14992	0.11780	0.23893	0.17994	0.23315	0.24515
Minimum	-0.12690	-0.14287	-0.10253	-0.07813	-0.14751	-0.08984	-0.11815	-0.14897
Maximum	0.07423	0.08447	0.04738	0.03967	0.09142	0.09010	0.11500	0.09618
Sum	1.84988	1.78151	2.08820	1.77077	1.46298	0.46282	2.32541	1.58147
Count	4072	4072	4072	4072	4074	3408	3408	4074
Confidence Level (95.0%)	0.00040	0.00044	0.00029	0.00022	0.00042	0.00052	0.00064	0.00042

The above reported descriptive statistics across market capitalisation and industry all contain a positive mean estimate, a sign of an overall bullish market for the period under consideration. The marginally positive mean can be interpreted as weak evidence towards the persistence of positive returns.

If risk were a priced factor of returns, one would expect the riskiest industry or firm capitalisation, measured by standard deviation, to be associated with the highest mean return. In this case, the RESI index has the highest level of volatility and this corresponds with the highest return – an indication that risk is a priced factor on the JSE. This observation speaks to the relationship between risk and return.

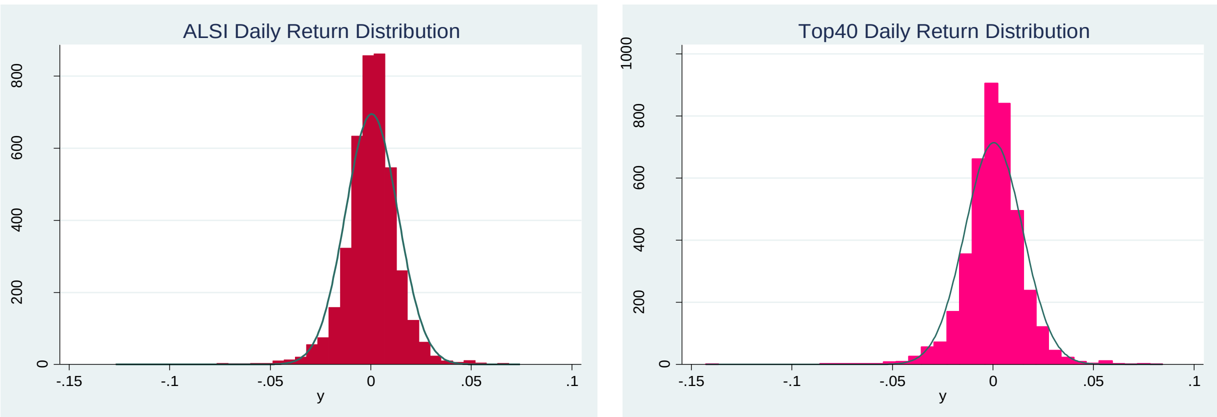
When comparing standard deviation it appears that the Top40 index, containing those firms with the largest market capitalisation is riskier than the MidCap and SmallCap, this is interesting given small capitalisation firms are thought to be the riskier stocks, as evidenced by the small firm effect hypothesis of Albert & Henderson (1995). Of course one needs to examine risk adjusted returns to determine if this is the case.

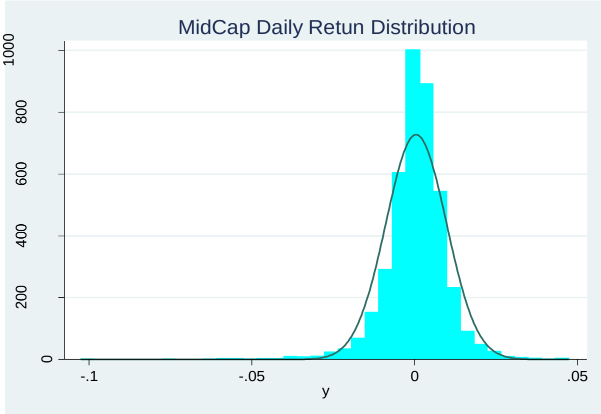
It appears that the INDI index had the largest range of returns with the SmallCap showing the lowest range of returns. Of the industries to invest in, resources carry the most risk.

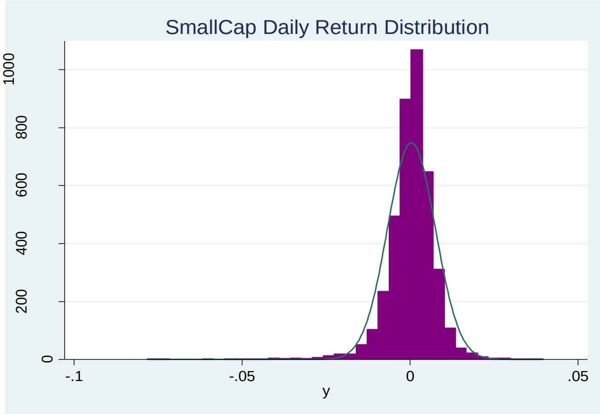
It is apparent from the excess kurtosis measure that each of the series display signs of fat tails and peakedness. The measure of kurtosis under the normal distribution is three. Each of the indices displays significant excess kurtosis. The SmallCap index displays the highest level of excess kurtosis at 13.64. The skewness statistic indicates that each of the indices are negatively or left skewed, with the highest degree of skewness found in the return series of the SmallCap index. Given this information, if one were to assume the returns were normally distributed, the researcher would underestimate the likelihood of negative events. The results indicate that the market portfolio, various market capitalisations and industries, have return distributions which can be considered non-normal.

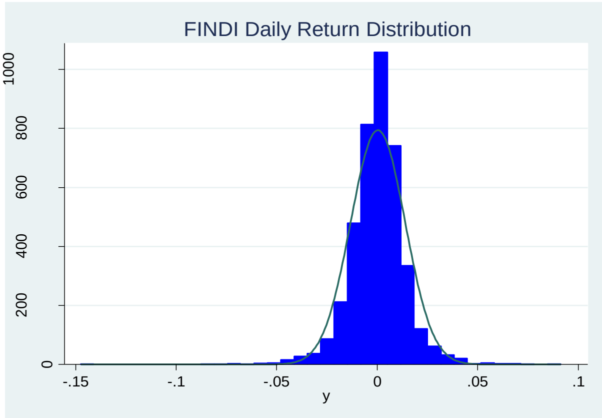
To confirm the above finding, distribution plots for the return series were generated, and these are presented in Figure 3 below:

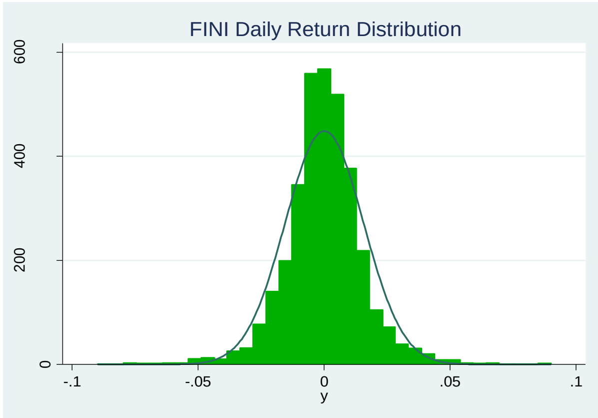
Figure 3: Daily Distribution Plot per Index

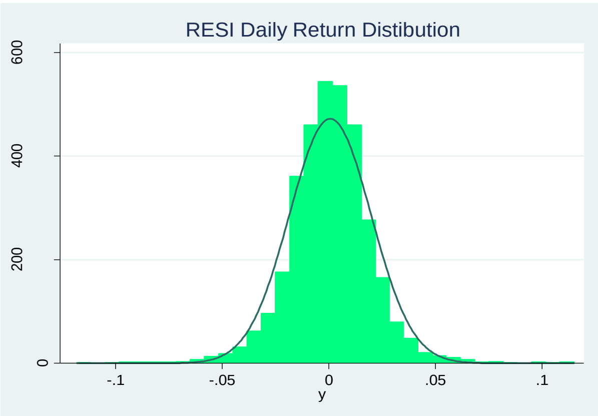


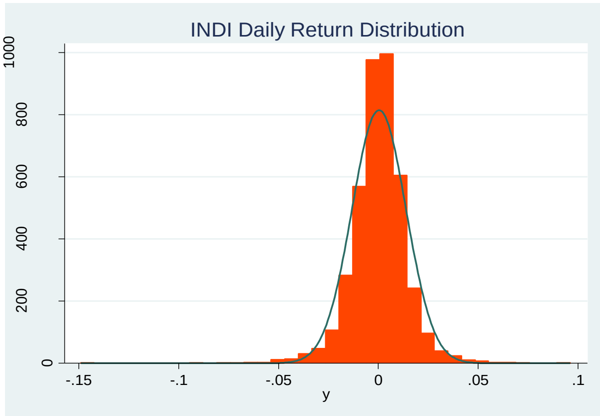












The superimposed normal curve visually illustrates the extent of the leptokurtosis and fat tails.

The Jarque-Bera test was conducted to test the joint null hypothesis of zero skewness and a kurtosis measure of three. The results of the Jarque-Bera test for each index are presented in Figure 4 below. In order for ARCH/GARCH stochastic volatility models to be applied to this dataset, the return series need to show signs of non-normality.

Figure 4: Jarque-Bera Test for Daily Return Indices

ALSI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	0.0000

Top40

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	0.0000

MidCap

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	0.0000

SmallCap

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	.

FINDI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	0.0000

FINI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	3.4e+03	0.4670	0.0000	.	0.0000

RESI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	3.4e+03	0.7272	0.0000	.	0.0000

INDI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	4.1e+03	0.0000	0.0000	.	0.0000

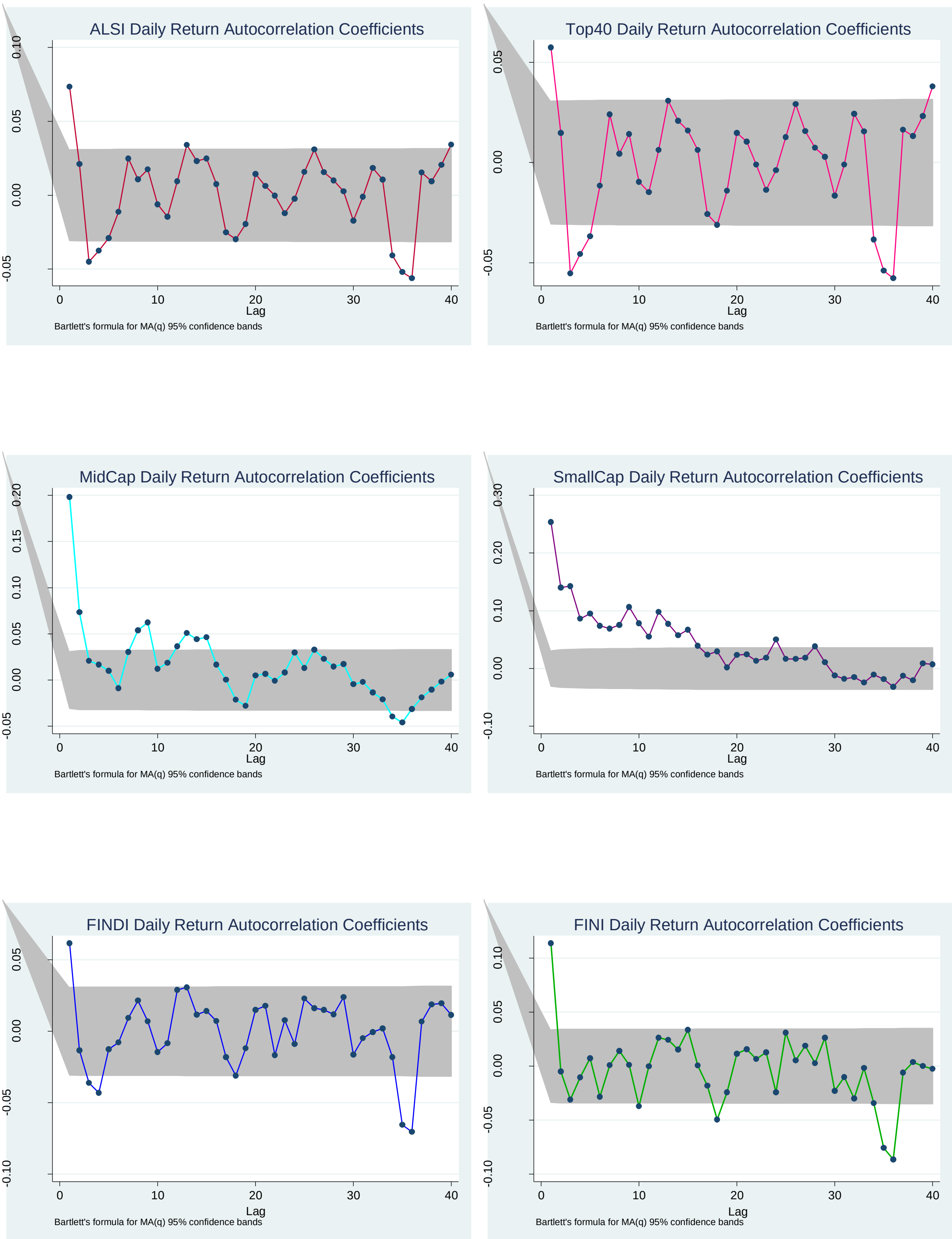
When interpreting the output for the Jarque-Bera test, one considers the probability of the JB test statistic being greater than the chi distribution with two degrees of freedom. The probability in each case is less than 0.05, therefore we can safely reject the null hypothesis of normality in favour of the alternative hypothesis of non-normality at the 95% confidence interval.

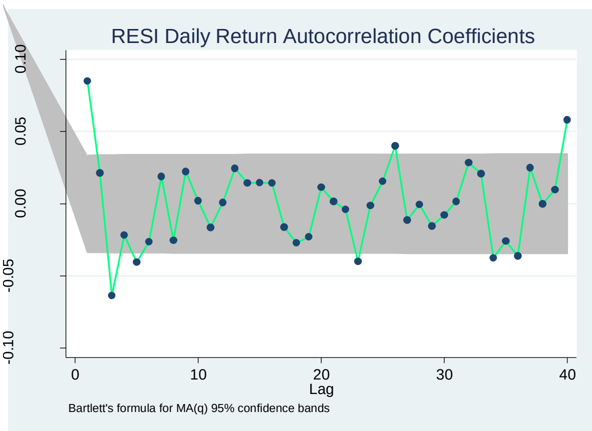
The results of the Jarque-Bera test support the findings of the summary statistics and the distribution plots- each of the indices display elements of non-normality in the form of fatter tails, skewness, and higher peaks.

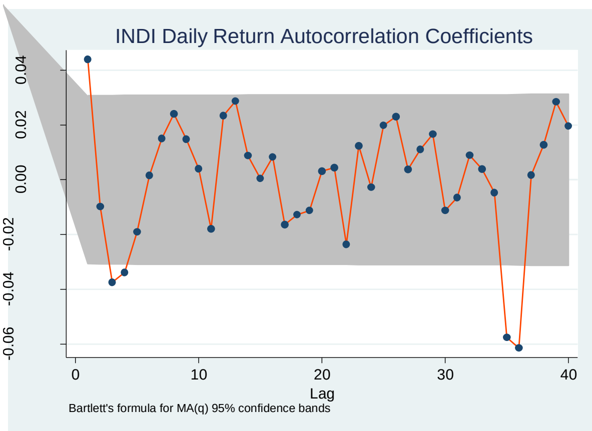
Tests conducted on the series for independence in the residual term, ε_t, include the Portmanteau Q statistic to detect autocorrelation and the BDS test.

Prior to testing for serial correlation in the residuals, it is useful to consider whether serial correlation exists between the return observations. The autocorrelation plots presented in Figure 5 below graph the autocorrelation coefficients or relationship between a return observation with prior returns up to lag 40. The points which lie outside of the grey bar represent the statistically significant autocorrelation between return observations at the 95% confidence interval.

Figure 5: Autocorrelation Coefficients per Daily Return Index







It is interesting to note the sequence of points on the plots. The points whereby the direction changes, represent the negative autocorrelation coefficients. It appears as though the significant points, which lie outside of the 95% confidence interval band, are the negative autocorrelation coefficients, which can be interpreted as weak evidence of asymmetry in return dynamics.

To test for serial correlation of the residual terms, the Portmanteau Q statistic is calculated. The null hypothesis implied by the Portmanteau Q statistic is one of no autocorrelation in the residual terms, and the alternative hypothesis is the presence of autocorrelation of residuals. Note that only the first twenty lagged residuals are shown, however the tests were conducted up to lag 40. Evidence of serial correlation of the residual terms would indicate that the size and magnitude of the residual term has an influence on the following period return observation. Serial correlation of the residual term is an assumption of ARCH/GARCH testing.

Figure 6: Results of the Portmanteau Q statistic for Daily Return Indices

ALSI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0733	0.0733	21.896	0.0000		
2	0.0210	0.0157	23.695	0.0000		
3	-0.0449	-0.0479	31.925	0.0000		
4	-0.0376	-0.0314	37.677	0.0000		
5	-0.0289	-0.0223	41.092	0.0000		
6	-0.0113	-0.0084	41.614	0.0000		
7	0.0249	0.0245	44.137	0.0000		
8	0.0108	0.0043	44.612	0.0000		
9	0.0174	0.0128	45.845	0.0000		
10	-0.0061	-0.0078	45.997	0.0000		
11	-0.0145	-0.0126	46.862	0.0000		
12	0.0095	0.0148	47.227	0.0000		
13	0.0339	0.0344	51.927	0.0000		
14	0.0231	0.0167	54.101	0.0000		
15	0.0248	0.0207	56.616	0.0000		
16	0.0074	0.0056	56.843	0.0000		
17	-0.0252	-0.0233	59.448	0.0000		
18	-0.0297	-0.0213	63.055	0.0000		
19	-0.0195	-0.0116	64.606	0.0000		
20	0.0143	0.0165	65.445	0.0000		

Top40

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0575	0.0575	13.46	0.0002		
2	0.0149	0.0116	14.361	0.0008		
3	-0.0554	-0.0571	26.883	0.0000		
4	-0.0457	-0.0397	35.385	0.0000		
5	-0.0369	-0.0307	40.933	0.0000		
6	-0.0115	-0.0097	41.475	0.0000		
7	0.0241	0.0218	43.837	0.0000		
8	0.0045	-0.0032	43.919	0.0000		
9	0.0143	0.0097	44.752	0.0000		
10	-0.0096	-0.0105	45.126	0.0000		
11	-0.0149	-0.0131	46.029	0.0000		
12	0.0062	0.0109	46.188	0.0000		
13	0.0308	0.0311	50.074	0.0000		
14	0.0209	0.0155	51.857	0.0000		
15	0.0159	0.0126	52.892	0.0000		
16	0.0062	0.0066	53.051	0.0000		
17	-0.0258	-0.0223	55.775	0.0000		
18	-0.0311	-0.0235	59.742	0.0000		
19	-0.0141	-0.0070	60.554	0.0000		
20	0.0148	0.0154	61.456	0.0000		

Midcap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1982	0.1982	160.12	0.0000		
2	0.0735	0.0355	182.11	0.0000		
3	0.0207	-0.0004	183.87	0.0000		
4	0.0167	0.0106	185	0.0000		
5	0.0100	0.0043	185.41	0.0000		
6	-0.0090	-0.0136	185.74	0.0000		
7	0.0305	0.0354	189.54	0.0000		
8	0.0538	0.0443	201.34	0.0000		
9	0.0624	0.0424	217.22	0.0000		
10	0.0121	-0.0132	217.83	0.0000		
11	0.0186	0.0129	219.24	0.0000		
12	0.0364	0.0300	224.66	0.0000		
13	0.0510	0.0382	235.28	0.0000		
14	0.0443	0.0257	243.3	0.0000		
15	0.0464	0.0292	252.08	0.0000		
16	0.0169	-0.0067	253.25	0.0000		
17	0.0004	-0.0113	253.25	0.0000		
18	-0.0213	-0.0242	255.11	0.0000		
19	-0.0280	-0.0217	258.31	0.0000		
20	0.0048	0.0120	258.41	0.0000		

SmallCap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2536	0.2536	262.13	0.0000		
2	0.1402	0.0811	342.21	0.0000		
3	0.1428	0.0964	425.33	0.0000		
4	0.0862	0.0233	455.63	0.0000		
5	0.0951	0.0532	492.48	0.0000		
6	0.0739	0.0225	514.78	0.0000		
7	0.0689	0.0287	534.17	0.0000		
8	0.0754	0.0343	557.38	0.0000		
9	0.1065	0.0678	603.66	0.0000		
10	0.0786	0.0203	628.92	0.0000		
11	0.0550	0.0047	641.28	0.0000		
12	0.0978	0.0587	680.34	0.0000		
13	0.0772	0.0214	704.71	0.0000		
14	0.0576	0.0069	718.27	0.0000		
15	0.0674	0.0225	736.84	0.0000		
16	0.0393	-0.0073	743.14	0.0000		
17	0.0239	-0.0138	745.48	0.0000		
18	0.0295	-0.0009	749.03	0.0000		
19	0.0019	-0.0266	749.05	0.0000		
20	0.0234	0.0112	751.29	0.0000		

FINDI

LAG	AC	PAC	Q	Prob>Q
1	0.0616	0.0616	15.48	0.0001
2	-0.0134	-0.0172	16.21	0.0003
3	-0.0362	-0.0345	21.566	0.0001
4	-0.0431	-0.0392	29.158	0.0000
5	-0.0125	-0.0085	29.795	0.0000
6	-0.0079	-0.0090	30.047	0.0000
7	0.0094	0.0073	30.406	0.0001
8	0.0214	0.0179	32.275	0.0001
9	0.0069	0.0034	32.472	0.0002
10	-0.0146	-0.0150	33.338	0.0002
11	-0.0084	-0.0048	33.628	0.0004
12	0.0290	0.0317	37.077	0.0002
13	0.0308	0.0272	40.956	0.0001
14	0.0117	0.0079	41.512	0.0001
15	0.0141	0.0149	42.329	0.0002
16	0.0071	0.0093	42.533	0.0003
17	-0.0181	-0.0155	43.879	0.0004
18	-0.0311	-0.0259	47.836	0.0002
19	-0.0120	-0.0068	48.423	0.0002
20	0.0150	0.0139	49.347	0.0003

-1 0 1 -1 0 1
[Autocorrelation] [Partial Autocor]

FINI

LAG	AC	PAC	Q	Prob>Q
1	0.1137	0.1137	44.082	0.0000
2	-0.0048	-0.0180	44.163	0.0000
3	-0.0310	-0.0288	47.449	0.0000
4	-0.0104	-0.0038	47.821	0.0000
5	0.0072	0.0084	48	0.0000
6	-0.0283	-0.0315	50.737	0.0000
7	0.0008	0.0075	50.74	0.0000
8	0.0141	0.0135	51.422	0.0000
9	0.0011	-0.0037	51.427	0.0000
10	-0.0371	-0.0375	56.139	0.0000
11	-0.0003	0.0100	56.139	0.0000
12	0.0264	0.0243	58.523	0.0000
13	0.0241	0.0162	60.511	0.0000
14	0.0154	0.0120	61.319	0.0000
15	0.0334	0.0337	65.149	0.0000
16	0.0004	-0.0082	65.15	0.0000
17	-0.0182	-0.0163	66.284	0.0000
18	-0.0494	-0.0421	74.646	0.0000
19	-0.0242	-0.0134	76.66	0.0000
20	0.0114	0.0120	77.108	0.0000

-1 0 1 -1 0 1
[Autocorrelation] [Partial Autocor]

RESI

LAG	AC	PAC	Q	Prob>Q
1	0.0849	0.0849	24.559	0.0000
2	0.0213	0.0142	26.11	0.0000
3	-0.0636	-0.0671	39.918	0.0000
4	-0.0216	-0.0111	41.517	0.0000
5	-0.0406	-0.0356	47.142	0.0000
6	-0.0265	-0.0240	49.539	0.0000
7	0.0190	0.0231	50.769	0.0000
8	-0.0255	-0.0335	52.987	0.0000
9	0.0224	0.0225	54.708	0.0000
10	0.0020	0.0001	54.722	0.0000
11	-0.0163	-0.0229	55.631	0.0000
12	0.0008	0.0073	55.633	0.0000
13	0.0244	0.0245	57.673	0.0000
14	0.0142	0.0073	58.363	0.0000
15	0.0144	0.0144	59.076	0.0000
16	0.0143	0.0117	59.773	0.0000
17	-0.0163	-0.0169	60.68	0.0000
18	-0.0272	-0.0209	63.223	0.0000
19	-0.0230	-0.0160	65.042	0.0000
20	0.0113	0.0157	65.479	0.0000

-1 0 1 -1 0 1
[Autocorrelation] [Partial Autocor]

INDI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0440	0.0440	7.8817	0.0050		
2	-0.0099	-0.0119	8.2808	0.0159		
3	-0.0374	-0.0366	14	0.0029		
4	-0.0338	-0.0307	18.66	0.0009		
5	-0.0191	-0.0171	20.152	0.0012		
6	0.0016	0.0012	20.163	0.0026		
7	0.0151	0.0123	21.089	0.0036		
8	0.0241	0.0208	23.467	0.0028		
9	0.0149	0.0123	24.371	0.0038		
10	0.0040	0.0041	24.438	0.0065		
11	-0.0179	-0.0156	25.745	0.0071		
12	0.0234	0.0279	27.978	0.0056		
13	0.0287	0.0283	31.342	0.0030		
14	0.0087	0.0063	31.654	0.0045		
15	0.0005	0.0008	31.655	0.0072		
16	0.0083	0.0107	31.937	0.0102		
17	-0.0164	-0.0148	33.035	0.0112		
18	-0.0128	-0.0101	33.711	0.0137		
19	-0.0113	-0.0099	34.231	0.0173		
20	0.0031	0.0018	34.271	0.0243		

When interpreting the output for the Portmanteau Q statistic, one considers the probability of the Q test statistic being greater than the chi distribution with two degrees of freedom. The probability in each case is less than 0.05; therefore we can safely reject the null hypothesis of no serial correlation in favour of the alternative hypothesis of serial correlation in residuals at the 95% confidence interval across all eight indices. The existence of autocorrelation of residuals means that the residuals are related to one another either linearly or nonlinearly.

It is interesting to note the estimates of the autocorrelation coefficients. Where the sign is positive, this indicates a positive relationship between the residuals and might be an indication of persistence. Where the sign is negative, this indicates a negative relationship between the residuals, in other words a positive residual is likely to be followed by a negative residual than a positive residual. A negative relationship between residuals is evidence of asymmetry. The SmallCap index is the only test results that do not show signs of possible asymmetry.

The last test conducted for independence is the BDS test. This test is particularly important for the purposes of this study as it directly tests for the presence of non-linear behaviour of the residuals. The null hypothesis of the BDS test states that the residual terms of the series are identically and independently distributed. The alternative hypothesis implies that serial dependence between residuals is present. By using geometric returns, the linear dependence between residual terms has been eliminated; therefore, the BDS test run on the residuals terms is testing for non-linear dependence.

Figure 7: BDS Test of the Daily Return Indices Residuals

ALSI

BDS Test
Sample: 7/03/1995 10/18/2011
Included observations: 4072

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.020885	0.001385	15.07713	0.0000
3	0.045633	0.002197	20.77067	0.0000
4	0.064577	0.002611	24.73193	0.0000
5	0.075288	0.002716	27.71745	0.0000
6	0.080051	0.002615	30.61781	0.0000

Raw epsilon	0.016763		
Pairs within epsilon	11655810	V-Statistic	0.702954
Triples within epsilon	3.63E+10	V-Statistic	0.538335

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4265044.	0.514823	5822398.	0.702807	0.493938
3	3251454.	0.392668	5818933.	0.702734	0.347035
4	2552529.	0.308413	5815845.	0.702707	0.243835
5	2040155.	0.246626	5812936.	0.702701	0.171337
6	1657242.	0.200435	5809960.	0.702686	0.120384

Top40

BDS Test
Sample: 7/03/1995 10/18/2011
Included observations: 4072

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.020615	0.001372	15.02917	0.0000
3	0.044443	0.002176	20.42887	0.0000
4	0.062856	0.002586	24.31085	0.0000
5	0.073214	0.002690	27.22094	0.0000
6	0.077832	0.002589	30.06500	0.0000

Raw epsilon	0.018548		
Pairs within epsilon	11656760	V-Statistic	0.703011
Triples within epsilon	3.63E+10	V-Statistic	0.537984

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4263468.	0.514633	5822866.	0.702864	0.494018
3	3242282.	0.391560	5819389.	0.702790	0.347117
4	2538978.	0.306775	5816342.	0.702767	0.243919
5	2023732.	0.244640	5813539.	0.702774	0.171426
6	1639510.	0.198291	5810559.	0.702759	0.120459

MidCap

BDS Test
Sample: 7/03/1995 10/18/2011
Included observations: 4072

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.024461	0.001441	16.98001	0.0000
3	0.048464	0.002290	21.16405	0.0000
4	0.066009	0.002728	24.19901	0.0000
5	0.076123	0.002844	26.76407	0.0000
6	0.079440	0.002744	28.94960	0.0000

Raw epsilon	0.011508		
Pairs within epsilon	11678210	V-Statistic	0.704305
Triples within epsilon	3.66E+10	V-Statistic	0.542003

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4310529.	0.520313	5833670.	0.704168	0.495853
3	3291621.	0.397519	5830200.	0.704095	0.349055
4	2579528.	0.311675	5826727.	0.704022	0.245665
5	2059665.	0.248984	5823238.	0.703946	0.172861
6	1662602.	0.201084	5820049.	0.703907	0.121644

SmallCap

BDS Test
Sample: 7/03/1995 10/18/2011
Included observations: 4072

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.025060	0.001418	17.67581	0.0000
3	0.047833	0.002255	21.20709	0.0000
4	0.064133	0.002689	23.85096	0.0000
5	0.072056	0.002806	25.67979	0.0000
6	0.074516	0.002709	27.50456	0.0000

Raw epsilon	0.008475		
Pairs within epsilon	11689730	V-Statistic	0.705000
Triples within epsilon	3.66E+10	V-Statistic	0.542254

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4323477.	0.521876	5839337.	0.704852	0.496817
3	3294897.	0.397914	5835911.	0.704785	0.350082
4	2571989.	0.310764	5832440.	0.704712	0.246630
5	2033081.	0.245770	5828979.	0.704640	0.173715
6	1627978.	0.196896	5825904.	0.704615	0.122380

FINDI

BDS Test
Sample: 7/03/1995 10/20/2011
Included observations: 4074

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.021416	0.001411	15.18203	0.0000
3	0.046632	0.002243	20.79081	0.0000
4	0.066272	0.002672	24.79819	0.0000
5	0.078561	0.002787	28.18646	0.0000
6	0.084448	0.002690	31.39728	0.0000

Raw epsilon	0.017325		
Pairs within epsilon	11695250	V-Statistic	0.704640
Triples within epsilon	3.66E+10	V-Statistic	0.541530

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4293896.	0.517797	5842514.	0.704543	0.496381
3	3284333.	0.396249	5839063.	0.704473	0.349617
4	2588678.	0.312473	5835634.	0.704405	0.246201
5	2085773.	0.251892	5832134.	0.704329	0.173331
6	1708699.	0.206456	5828679.	0.704258	0.122008

FINI

BDS Test
Sample: 3/02/1998 10/20/2011
Included observations: 3408

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.022953	0.001549	14.82153	0.0000
3	0.048732	0.002460	19.81278	0.0000
4	0.067394	0.002927	23.02145	0.0000
5	0.078985	0.003050	25.89851	0.0000
6	0.083904	0.002940	28.54040	0.0000

Raw epsilon	0.020174		
Pairs within epsilon	8174246.	V-Statistic	0.703799
Triples within epsilon	2.14E+10	V-Statistic	0.540529

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	3005798.	0.518052	4082560.	0.703632	0.495098
3	2301885.	0.396965	4079636.	0.703541	0.348232
4	1809701.	0.312270	4076740.	0.703455	0.244876
5	1454569.	0.251138	4073844.	0.703368	0.172153
6	1186074.	0.204902	4070949.	0.703282	0.120997

RESI

BDS Test
Sample: 3/02/1998 10/20/2011
Included observations: 3408

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.014359	0.001408	10.19479	0.0000
3	0.030484	0.002232	13.65727	0.0000
4	0.042208	0.002650	15.92508	0.0000
5	0.047928	0.002755	17.39902	0.0000
6	0.049616	0.002649	18.73099	0.0000

Raw epsilon	0.025563		
Pairs within epsilon	8162746.	V-Statistic	0.702809
Triples within epsilon	2.12E+10	V-Statistic	0.535045

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	2948775.	0.508224	4077471.	0.702755	0.493865
3	2188650.	0.377437	4074635.	0.702679	0.346953
4	1656969.	0.285916	4071869.	0.702615	0.243707
5	1268751.	0.219056	4068983.	0.702529	0.171128
6	983441.0	0.169896	4066914.	0.702585	0.120280

INDI

BDS Test
Sample: 7/03/1995 10/20/2011
Included observations: 4074

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.019009	0.001382	13.75758	0.0000
3	0.041369	0.002196	18.83963	0.0000
4	0.058813	0.002615	22.49060	0.0000
5	0.069540	0.002726	25.51218	0.0000
6	0.074564	0.002629	28.36317	0.0000

Raw epsilon	0.017480		
Pairs within epsilon	11691366	V-Statistic	0.704406
Triples within epsilon	3.65E+10	V-Statistic	0.540279

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	4271400.	0.515084	5840712.	0.704326	0.496075
3	3238109.	0.390672	5837312.	0.704262	0.349303
4	2524745.	0.304756	5834103.	0.704220	0.245943
5	2009212.	0.242646	5830616.	0.704145	0.173106
6	1625336.	0.196383	5827173.	0.704076	0.121819

Recall that to perform the BDS, pairs of points were created, and the probability of the distance between these points being equal to a chosen distance, ε, was evaluated. From the results above, across all market capitalisations and industries, as well as, dimensions two though to six, the probabilities are all less than 0.05. Given this result, one can reject the null hypothesis of no serial dependence in favour of the alternative hypothesis of non-linear dependence between residuals at the 95% confidence interval.

The Portmanteau Q statistic and the BDS test indicate that serial dependence exists between the residual terms. Whilst serial dependence causes volatility clustering and is a characteristic needed to be present in the dataset to perform stochastic volatility modelling, the non-linearity exposed by these tests speak to the aim of this research and provide a valuable basis on which to continue with the investigation of asymmetry in the conditional mean and conditional volatility.

Stationarity is a requirement of stochastic volatility modelling, and as such, before the models are fitted to the data, one should test for the presence of stationarity. Recall stationarity refers to a time series whereby the mean and variance do not change over time. The tests conducted for stationarity are the segmentation of the series to examine the properties through time and the Augmented Dickey Fuller test.

The simplest means of testing for stationarity is to divide the time series into sections, run descriptive statistics techniques on each section and compare the mean and variance between sections. If stationarity is present, the mean and variance should not deviate significantly between the sections. Table 2 below illustrates this point.

Table 2: Descriptive Statistics of Sections of Daily Index Returns

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Section 1								
Mean	0.00034	0.00036	0.00018	-0.00001	0.00028	-0.00038	0.00127	0.00020
Standard Error	0.00034	0.00038	0.00030	0.00024	0.00041	0.00053	0.00053	0.00041
Median	0.00075	0.00068	0.00069	0.00067	0.00048	-0.00083	0.00150	0.00044
Standard Deviation	0.01238	0.01385	0.01121	0.00886	0.01517	0.01796	0.01771	0.01521
Sample Variance	0.00015	0.00019	0.00013	0.00008	0.00023	0.00032	0.00031	0.00023
Section 2								
Mean	0.00073	0.00066	0.00100	0.00106	0.00043	0.00095	0.00082	0.00044
Standard Error	0.00030	0.00033	0.00018	0.00015	0.00030	0.00034	0.00048	0.00031
Median	0.00084	0.00083	0.00095	0.00123	0.00066	0.00115	0.00110	0.00101
Standard Deviation	0.01119	0.01223	0.00647	0.00535	0.01095	0.01148	0.01622	0.01154
Sample Variance	0.00013	0.00015	0.00004	0.00003	0.00012	0.00013	0.00026	0.00013
Section 3								
Mean	0.00029	0.00029	0.00036	0.00025	0.00037	-0.00017	-0.00003	0.00052
Standard Error	0.00041	0.00045	0.00026	0.00018	0.00039	0.00049	0.00068	0.00037
Median	0.00101	0.00112	0.00081	0.00080	0.00079	-0.00007	-0.00003	0.00090
Standard Deviation	0.01526	0.01666	0.00958	0.00663	0.01427	0.01652	0.02298	0.01374
Sample Variance	0.00023	0.00028	0.00009	0.00004	0.00020	0.00027	0.00053	0.00019

At first glance, when one compares the mean of each index across the three sections, the variation is not large, typically a change in mean of 0.00005. The standard deviation and sample variances remain consistent. Given that no large inconsistencies are present per index it appears as though the series are stationary.

The Augmented Dickey Fuller test is a commonplace test for stationarity. The Dickey-Fuller with GLS Trending will provide the optimum number of lags for use into the Augmented Dickey Fuller test. Recall that the null hypothesis under the ADF test is one of non-stationary with the alternative hypothesis being stationarity of the series. To determine whether one accepts of rejects the null hypothesis one considers the MacKinnon approximate p-value for the z(t) statistic. If the probability is less than 0.05 one can reject the null hypothesis in favour of the alternative hypothesis.

The Dickey-Fuller with GLS Trending and the ADF test are presented in Figure 8 below.

Figure 8: ADF Test Results per Daily Return Index

ALSI

DF-GLS for y Number of obs = 4041

Maxlag = 30 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-4.562	-3.480	-2.833	-2.546
29	-4.710	-3.480	-2.833	-2.546
28	-4.776	-3.480	-2.834	-2.547
27	-4.914	-3.480	-2.834	-2.547
26	-5.076	-3.480	-2.834	-2.547
25	-5.265	-3.480	-2.835	-2.548
24	-5.584	-3.480	-2.835	-2.548
23	-5.879	-3.480	-2.835	-2.548
22	-6.101	-3.480	-2.836	-2.549
21	-6.246	-3.480	-2.836	-2.549
20	-6.441	-3.480	-2.836	-2.549
19	-6.700	-3.480	-2.837	-2.549
18	-7.103	-3.480	-2.837	-2.550
17	-7.342	-3.480	-2.838	-2.550
16	-7.522	-3.480	-2.838	-2.550
15	-7.693	-3.480	-2.838	-2.551
14	-8.107	-3.480	-2.839	-2.551
13	-8.702	-3.480	-2.839	-2.551
12	-9.353	-3.480	-2.839	-2.552
11	-10.309	-3.480	-2.839	-2.552
10	-11.234	-3.480	-2.840	-2.552
9	-11.985	-3.480	-2.840	-2.552
8	-12.919	-3.480	-2.840	-2.553
7	-14.353	-3.480	-2.841	-2.553
6	-16.008	-3.480	-2.841	-2.553
5	-18.554	-3.480	-2.841	-2.554
4	-21.257	-3.480	-2.842	-2.554
3	-24.584	-3.480	-2.842	-2.554
2	-29.001	-3.480	-2.842	-2.554
1	-35.045	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 30 with RMSE .013146

Min SC = -8.612349 at lag 15 with RMSE .0132652

Min MAIC = -8.591513 at lag 30 with RMSE .013146

Augmented Dickey-Fuller test for unit root Number of obs = 4041

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-10.859	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
y						
L1.	-.9066088	.0834916	-10.86	0.000	-1.070299	-.7429189
LD.	-.0252032	.0822479	-0.31	0.759	-.1864548	.1360484
L2D.	-.0054319	.0810099	-0.07	0.947	-.1642564	.1533925
L3D.	-.0483298	.0797411	-0.61	0.544	-.2046666	.1080071
L4D.	-.075438	.0784396	-0.96	0.336	-.2292232	.0783473
L5D.	-.0972824	.0771006	-1.26	0.207	-.2484424	.0538776
L6D.	-.1089168	.0756555	-1.44	0.150	-.2572437	.0394101
L7D.	-.0857165	.074135	-1.16	0.248	-.2310624	.0596294
L8D.	-.0802613	.0725764	-1.11	0.269	-.2225513	.0620287
L9D.	-.0624329	.0710281	-0.88	0.379	-.2016874	.0768216
L10D.	-.0657437	.0694611	-0.95	0.344	-.201926	.0704386
L11D.	-.0795894	.0678494	-1.17	0.241	-.212612	.0534332
L12D.	-.0685669	.0661458	-1.04	0.300	-.1982495	.0611157
L13D.	-.0382706	.0644425	-0.59	0.553	-.1646136	.0880724
L14D.	-.0250869	.0627673	-0.40	0.689	-.1481458	.0979719
L15D.	-.005004	.061132	-0.08	0.935	-.1248566	.1148486
L16D.	.0026695	.0594215	0.04	0.964	-.1138296	.1191687
L17D.	-.0186846	.0575697	-0.32	0.746	-.1315533	.0941841
L18D.	-.0408392	.0556069	-0.73	0.463	-.1498597	.0681813
L19D.	-.0554274	.0534524	-1.04	0.300	-.1602239	.049369
L20D.	-.0387047	.0511508	-0.76	0.449	-.1389886	.0615792
L21D.	-.0366304	.0487902	-0.75	0.453	-.1322863	.0590255
L22D.	-.0418116	.0463134	-0.90	0.367	-.1326117	.0489884
L23D.	-.0546424	.0436112	-1.25	0.210	-.1401445	.0308597
L24D.	-.0553548	.0407109	-1.36	0.174	-.1351708	.0244611
L25D.	-.0392767	.0374165	-1.05	0.294	-.1126338	.0340804
L26D.	-.0107333	.0338728	-0.32	0.751	-.0771428	.0556762
L27D.	-.0029256	.0300948	-0.10	0.923	-.0619281	.056077
L28D.	.0027107	.0260436	0.10	0.917	-.0483492	.0537706
L29D.	.0057639	.0216838	0.27	0.790	-.0367484	.0482761
L30D.	-.0075684	.0158684	-0.48	0.633	-.0386792	.0235425
_cons	.000407	.0002086	1.95	0.051	-1.95e-06	.0008159

Top40

hosen by Schwert criterion

Number of obs = 4041

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-4.139	-3.480	-2.833	-2.546
29	-4.274	-3.480	-2.833	-2.546
28	-4.346	-3.480	-2.834	-2.547
27	-4.482	-3.480	-2.834	-2.547
26	-4.625	-3.480	-2.834	-2.547
25	-4.808	-3.480	-2.835	-2.548
24	-5.097	-3.480	-2.835	-2.548
23	-5.360	-3.480	-2.835	-2.548
22	-5.568	-3.480	-2.836	-2.549
21	-5.705	-3.480	-2.836	-2.549
20	-5.890	-3.480	-2.836	-2.549
19	-6.164	-3.480	-2.837	-2.549
18	-6.545	-3.480	-2.837	-2.550
17	-6.817	-3.480	-2.838	-2.550
16	-6.994	-3.480	-2.838	-2.550
15	-7.187	-3.480	-2.838	-2.551
14	-7.610	-3.480	-2.839	-2.551
13	-8.137	-3.480	-2.839	-2.551
12	-8.773	-3.480	-2.839	-2.552
11	-9.683	-3.480	-2.839	-2.552
10	-10.569	-3.480	-2.840	-2.552
9	-11.338	-3.480	-2.840	-2.552
8	-12.270	-3.480	-2.840	-2.553
7	-13.691	-3.480	-2.841	-2.553
6	-15.276	-3.480	-2.841	-2.553
5	-17.814	-3.480	-2.841	-2.554
4	-20.613	-3.480	-2.842	-2.554
3	-23.936	-3.480	-2.842	-2.554
2	-28.362	-3.480	-2.842	-2.554
1	-34.374	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 30 with RMSE .0144799
Min SC = -8.416022 at lag 15 with RMSE .0146334
Min MAIC = -8.412507 at lag 30 with RMSE .0144799

Augmented Dickey-Fuller test for unit root   Number of obs   =   4041

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-11.021	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
y						
L1.	-.9799272	.088917	-11.02	0.000	-1.154254	-.8056005
LD.	.0320323	.0875413	0.37	0.714	-.1395974	.2036619
L2D.	.0471462	.0861768	0.55	0.584	-.1218082	.2161007
L3D.	-.0051971	.0847772	-0.06	0.951	-.1714075	.1610133
L4D.	-.040721	.0833405	-0.49	0.625	-.2041147	.1226728
L5D.	-.0710699	.0818556	-0.87	0.385	-.2315524	.0894126
L6D.	-.0834735	.080263	-1.04	0.298	-.2408336	.0738866
L7D.	-.0628234	.0785946	-0.80	0.424	-.2169126	.0912657
L8D.	-.0644291	.0768856	-0.84	0.402	-.2151676	.0863095
L9D.	-.0498885	.075188	-0.66	0.507	-.1972988	.0975217
L10D.	-.055985	.0734713	-0.76	0.446	-.2000295	.0880595
L11D.	-.0691986	.0716911	-0.97	0.334	-.2097529	.0713558
L12D.	-.0613994	.069816	-0.88	0.379	-.1982776	.0754788
L13D.	-.0337269	.0679182	-0.50	0.620	-.1668843	.0994306
L14D.	-.0214706	.0660534	-0.33	0.745	-.1509721	.1080309
L15D.	-.0098875	.0642203	-0.15	0.878	-.1357949	.11602
L16D.	-.0012198	.0622978	-0.02	0.984	-.1233581	.1209185
L17D.	-.0215865	.060255	-0.36	0.720	-.1397197	.0965468
L18D.	-.046035	.0580942	-0.79	0.428	-.159932	.067862
L19D.	-.0558128	.0557399	-1.00	0.317	-.165094	.0534685
L20D.	-.0405136	.0532383	-0.76	0.447	-.1448902	.0638631
L21D.	-.0342817	.0506608	-0.68	0.499	-.1336051	.0650417
L22D.	-.0400774	.0479623	-0.84	0.403	-.1341101	.0539553
L23D.	-.0536767	.0450356	-1.19	0.233	-.1419716	.0346182
L24D.	-.0542496	.0419287	-1.29	0.196	-.1364531	.0279539
L25D.	-.040003	.0384127	-1.04	0.298	-.1153133	.0353073
L26D.	-.0129625	.03462	-0.37	0.708	-.0808369	.054912
L27D.	-.0038355	.0306187	-0.13	0.900	-.0638652	.0561941
L28D.	-.0001443	.0263721	-0.01	0.996	-.0518483	.0515596
L29D.	.0042191	.0218589	0.19	0.847	-.0386366	.0470748
L30D.	-.0071007	.0158697	-0.45	0.655	-.0382142	.0240127
_cons	.0004232	.000229	1.85	0.065	-.0000258	.0008722

MidCap

DF-GLS for y   Number of obs = 4041
Maxlag = 30 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-5.628	-3.480	-2.833	-2.546
29	-5.748	-3.480	-2.833	-2.546
28	-5.812	-3.480	-2.834	-2.547
27	-5.991	-3.480	-2.834	-2.547
26	-6.138	-3.480	-2.834	-2.547
25	-6.347	-3.480	-2.835	-2.548
24	-6.691	-3.480	-2.835	-2.548
23	-6.869	-3.480	-2.835	-2.548
22	-7.226	-3.480	-2.836	-2.549
21	-7.466	-3.480	-2.836	-2.549
20	-7.608	-3.480	-2.836	-2.549
19	-7.843	-3.480	-2.837	-2.549
18	-8.199	-3.480	-2.837	-2.550
17	-8.293	-3.480	-2.838	-2.550
16	-8.364	-3.480	-2.838	-2.550
15	-8.547	-3.480	-2.838	-2.551
14	-8.777	-3.480	-2.839	-2.551
13	-9.359	-3.480	-2.839	-2.551
12	-9.981	-3.480	-2.839	-2.552
11	-10.833	-3.480	-2.839	-2.552
10	-11.732	-3.480	-2.840	-2.552
9	-12.555	-3.480	-2.840	-2.552
8	-13.130	-3.480	-2.840	-2.553
7	-14.634	-3.480	-2.841	-2.553
6	-16.556	-3.480	-2.841	-2.553
5	-18.847	-3.480	-2.841	-2.554
4	-20.688	-3.480	-2.842	-2.554
3	-23.537	-3.480	-2.842	-2.554
2	-27.658	-3.480	-2.842	-2.554
1	-33.482	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 28 with RMSE .0091544
Min SC   = -9.345813 at lag 14 with RMSE .0092021
Min MAIC = -9.289849 at lag 30 with RMSE .0091526

Augmented Dickey-Fuller test for unit root Number of obs = 4043

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-10.168	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.59033	.0580548	-10.17	0.000	-.7041497	-.4765104
LD.	-.224358	.0576402	-3.89	0.000	-.3373648	-.1113511
L2D.	-.1903258	.0569645	-3.34	0.001	-.3020078	-.0786437
L3D.	-.1945501	.0562227	-3.46	0.001	-.3047779	-.0843223
L4D.	-.1870049	.055399	-3.38	0.001	-.2956177	-.0783922
L5D.	-.1815273	.0545725	-3.33	0.001	-.2885198	-.0745348
L6D.	-.2048456	.0536619	-3.82	0.000	-.3100527	-.0996385
L7D.	-.1797255	.0527591	-3.41	0.001	-.2831626	-.0762884
L8D.	-.1418374	.0518609	-2.73	0.006	-.2435135	-.0401613
L9D.	-.0954841	.0509118	-1.88	0.061	-.1952996	.0043313
L10D.	-.1102295	.0498787	-2.21	0.027	-.2080195	-.0124394
L11D.	-.1053517	.0489101	-2.15	0.031	-.2012427	-.0094607
L12D.	-.0851627	.0479889	-1.77	0.076	-.1792476	.0089222
L13D.	-.0541259	.0470662	-1.15	0.250	-.1464017	.0381499
L14D.	-.034202	.046129	-0.74	0.458	-.1246405	.0562366
L15D.	-.005095	.0450692	-0.11	0.910	-.0934557	.0832658
L16D.	-.0102056	.0439085	-0.23	0.816	-.0962907	.0758794
L17D.	-.0180109	.0425981	-0.42	0.672	-.1015267	.065505
L18D.	-.0398572	.04116	-0.97	0.333	-.1205537	.0408393
L19D.	-.0652621	.0396489	-1.65	0.100	-.1429959	.0124717
L20D.	-.0527191	.0381471	-1.38	0.167	-.1275087	.0220704
L21D.	-.0513977	.036356	-1.41	0.158	-.1226756	.0198803
L22D.	-.0647549	.0342802	-1.89	0.059	-.1319632	.0024534
L23D.	-.0652489	.0319223	-2.04	0.041	-.1278344	-.0026635
L24D.	-.0425079	.0295179	-1.44	0.150	-.1003795	.0153636
L25D.	-.0482182	.0268539	-1.80	0.073	-.1008669	.0044304
L26D.	-.02089	.0238362	-0.88	0.381	-.0676222	.0258422
L27D.	-.0104003	.0203975	-0.51	0.610	-.0503908	.0295902
L28D.	-.0097878	.015803	-0.62	0.536	-.0407705	.0211949
_cons	.0002983	.0001463	2.04	0.042	.0000114	.0005851

SmallCap

DF-GLS for y Number of obs = 4041
Maxlag = 30 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-5.668	-3.480	-2.833	-2.546
29	-5.673	-3.480	-2.833	-2.546
28	-5.641	-3.480	-2.834	-2.547
27	-5.671	-3.480	-2.834	-2.547
26	-5.953	-3.480	-2.834	-2.547
25	-6.076	-3.480	-2.835	-2.548
24	-6.217	-3.480	-2.835	-2.548
23	-6.265	-3.480	-2.835	-2.548
22	-6.627	-3.480	-2.836	-2.549
21	-6.804	-3.480	-2.836	-2.549
20	-6.921	-3.480	-2.836	-2.549
19	-7.093	-3.480	-2.837	-2.549
18	-7.359	-3.480	-2.837	-2.550
17	-7.361	-3.480	-2.838	-2.550
16	-7.549	-3.480	-2.838	-2.550
15	-7.650	-3.480	-2.838	-2.551
14	-7.801	-3.480	-2.839	-2.551
13	-8.205	-3.480	-2.839	-2.551
12	-8.514	-3.480	-2.839	-2.552
11	-8.983	-3.480	-2.839	-2.552
10	-9.877	-3.480	-2.840	-2.552
9	-10.334	-3.480	-2.840	-2.552
8	-11.017	-3.480	-2.840	-2.553
7	-12.410	-3.480	-2.841	-2.553
6	-13.643	-3.480	-2.841	-2.553
5	-15.052	-3.480	-2.841	-2.554
4	-16.675	-3.480	-2.842	-2.554
3	-19.395	-3.480	-2.842	-2.554
2	-22.361	-3.480	-2.842	-2.554
1	-29.191	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 27 with RMSE .0068265
Min SC = -9.936247 at lag 11 with RMSE .0068709
Min MAIC = -9.905867 at lag 27 with RMSE .0068265

Augmented Dickey-Fuller test for unit root   Number of obs   =   4044

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
		Interpolated Dickey-Fuller		
Z(t)	-9.697	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
y						
L1.	-.4215743	.0434735	-9.70	0.000	-.5068066	-.3363421
LD.	-.3643401	.0438448	-8.31	0.000	-.4503003	-.2783799
L2D.	-.3181938	.0436577	-7.29	0.000	-.4037871	-.2326005
L3D.	-.2402392	.0433774	-5.54	0.000	-.325283	-.1551953
L4D.	-.2385008	.043013	-5.54	0.000	-.3228302	-.1541714
L5D.	-.1962607	.0425441	-4.61	0.000	-.2796707	-.1128506
L6D.	-.1898961	.0420311	-4.52	0.000	-.2723003	-.1074919
L7D.	-.1743306	.0415277	-4.20	0.000	-.2557479	-.0929132
L8D.	-.1579181	.0410081	-3.85	0.000	-.2383168	-.0775194
L9D.	-.0987257	.0404342	-2.44	0.015	-.1779993	-.0194522
L10D.	-.0827879	.0398922	-2.08	0.038	-.1609987	-.004577
L11D.	-.0926346	.0393235	-2.36	0.019	-.1697304	-.0155387
L12D.	-.0412486	.038787	-1.06	0.288	-.1172927	.0347955
L13D.	-.0208528	.0382303	-0.55	0.585	-.0958053	.0540998
L14D.	-.0174133	.0375996	-0.46	0.643	-.0911294	.0563029
L15D.	.0046343	.0369463	0.13	0.900	-.067801	.0770695
L16D.	.0008179	.0362173	0.02	0.982	-.0701882	.0718239
L17D.	-.0123265	.0352973	-0.35	0.727	-.0815288	.0568758
L18D.	-.0089954	.0343871	-0.26	0.794	-.0764131	.0584224
L19D.	-.0405944	.0333938	-1.22	0.224	-.1060648	.024876
L20D.	-.0290801	.0321201	-0.91	0.365	-.0920534	.0338932
L21D.	-.0314297	.030714	-1.02	0.306	-.0916462	.0287867
L22D.	-.0400052	.0291658	-1.37	0.170	-.0971863	.017176
L23D.	-.0457709	.0274962	-1.66	0.096	-.0996787	.0081369
L24D.	-.0077203	.0254571	-0.30	0.762	-.0576304	.0421897
L25D.	-.0248758	.0232162	-1.07	0.284	-.0703925	.0206409
L26D.	-.0239661	.0200723	-1.19	0.233	-.0633189	.0153866
L27D.	-.0305677	.0157816	-1.94	0.053	-.0615084	.000373
_cons	.0001819	.0001085	1.68	0.094	-.0000309	.0003947

FINDI

DF-GLS for y   Number of obs = 4043
Maxlag = 30 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-7.543	-3.480	-2.833	-2.546
29	-7.724	-3.480	-2.833	-2.546
28	-7.783	-3.480	-2.834	-2.547
27	-8.161	-3.480	-2.834	-2.547
26	-8.427	-3.480	-2.834	-2.547
25	-8.751	-3.480	-2.835	-2.548
24	-9.104	-3.480	-2.835	-2.548
23	-9.594	-3.480	-2.835	-2.548
22	-9.786	-3.480	-2.836	-2.549
21	-10.193	-3.480	-2.836	-2.549
20	-10.292	-3.480	-2.836	-2.549
19	-10.753	-3.480	-2.837	-2.549
18	-11.288	-3.480	-2.837	-2.550
17	-11.627	-3.480	-2.838	-2.550
16	-11.750	-3.480	-2.838	-2.550
15	-12.003	-3.480	-2.838	-2.551
14	-12.590	-3.480	-2.839	-2.551
13	-13.321	-3.480	-2.839	-2.551
12	-14.043	-3.480	-2.839	-2.552
11	-15.175	-3.480	-2.839	-2.552
10	-16.594	-3.480	-2.840	-2.552
9	-17.583	-3.480	-2.840	-2.552
8	-18.517	-3.480	-2.840	-2.553
7	-20.004	-3.480	-2.841	-2.553
6	-22.180	-3.480	-2.841	-2.553
5	-24.680	-3.480	-2.841	-2.554
4	-27.455	-3.480	-2.842	-2.554
3	-31.281	-3.480	-2.842	-2.554
2	-35.487	-3.480	-2.842	-2.554
1	-42.391	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 28 with RMSE .0136303
Min SC   = -8.563534 at lag 1 with RMSE .0137899
Min MAIC = -8.136484 at lag 30 with RMSE .0136284

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Augmented Dickey-Fuller test for unit root         Number of obs   =      4045

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -10.831            -3.430            -2.860            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |  -.9046209   .0835232   -10.83   0.000    -1.068373   -.7408691
         LD. |  -.0368819   .0822364    -0.45   0.654     -.198111    .1243472
        L2D. |   -.052628   .0808979    -0.65   0.515    -.2112329    .1059769
        L3D. |  -.0842309   .0795016    -1.06   0.289    -.2400981    .0716363
        L4D. |  -.1234564   .0780458    -1.58   0.114    -.2764696    .0295567
        L5D. |  -.1302759   .0764965    -1.70   0.089    -.2802514    .0196996
        L6D. |  -.1393372   .0749512    -1.86   0.063    -.2862831    .0076087
        L7D. |  -.1332061   .0733383    -1.82   0.069    -.2769899    .0105778
        L8D. |  -.1149703   .0717567    -1.60   0.109    -.2556532    .0257126
        L9D. |  -.1075273   .0700868    -1.53   0.125    -.2449363    .0298816
       L10D. |  -.1188069   .0683279    -1.74   0.082    -.2527675    .0151537
       L11D. |  -.1238976   .0665582    -1.86   0.063    -.2543886    .0065935
       L12D. |  -.0939058   .0648432    -1.45   0.148    -.2210345    .0332228
       L13D. |  -.0688114    .063117    -1.09   0.276    -.1925557     .054933
       L14D. |   -.064088    .061305    -1.05   0.296    -.1842798    .0561037
       L15D. |  -.0515387   .0593866    -0.87   0.386    -.1679694    .0648921
       L16D. |  -.0423453    .057373    -0.74   0.461    -.1548283    .0701377
       L17D. |  -.0565925   .0551737    -1.03   0.305    -.1647635    .0515785
       L18D. |  -.0828858   .0527454    -1.57   0.116     -.186296    .0205244
       L19D. |  -.0905822   .0502245    -1.80   0.071    -.1890501    .0078857
       L20D. |  -.0778708   .0476227    -1.64   0.102    -.1712378    .0154962
       L21D. |   -.064738   .0448276    -1.44   0.149    -.1526251     .023149
       L22D. |  -.0859268   .0417488    -2.06   0.040    -.1677776   -.0040759
       L23D. |  -.0737531   .0383837    -1.92   0.055    -.1490063    .0015002
       L24D. |  -.0838756   .0347408    -2.41   0.016    -.1519868   -.0157644
       L25D. |  -.0590611   .0307343    -1.92   0.055    -.1193174    .0011952
       L26D. |  -.0455522   .0264824    -1.72   0.085    -.0974723     .006368
       L27D. |  -.0326546   .0217436    -1.50   0.133    -.0752842    .0099749
       L28D. |  -.0256022   .0158342    -1.62   0.106    -.0566461    .0054417
             |
       _cons |   .0003188   .0002156     1.48   0.139    -.0001039    .0007415
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FINI

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DF-GLS for y                                             Number of obs =  3379
Maxlag = 28 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------

    28           -7.306           -3.480            -2.833            -2.546
    27           -7.696           -3.480            -2.833            -2.546
    26           -7.852           -3.480            -2.834            -2.547
    25           -8.189           -3.480            -2.834            -2.547
    24           -8.394           -3.480            -2.834            -2.547
    23           -8.983           -3.480            -2.835            -2.548
    22           -8.982           -3.480            -2.835            -2.548
    21           -9.393           -3.480            -2.836            -2.549
    20           -9.745           -3.480            -2.836            -2.549
    19          -10.207           -3.480            -2.837            -2.549
    18          -10.727           -3.480            -2.837            -2.550
    17          -11.005           -3.480            -2.837            -2.550
    16          -10.935           -3.480            -2.838            -2.550
    15          -11.171           -3.480            -2.838            -2.551
    14          -11.543           -3.480            -2.839            -2.551
    13          -12.455           -3.480            -2.839            -2.551
    12          -13.221           -3.480            -2.839            -2.552
    11          -14.143           -3.480            -2.840            -2.552
    10          -15.305           -3.480            -2.840            -2.553
    9           -16.459           -3.480            -2.841            -2.553
    8           -16.918           -3.480            -2.841            -2.553
    7           -18.082           -3.480            -2.841            -2.554
    6           -19.889           -3.480            -2.842            -2.554
    5           -22.035           -3.480            -2.842            -2.554
    4           -23.651           -3.480            -2.842            -2.555
    3           -27.058           -3.480            -2.843            -2.555
    2           -31.709           -3.480            -2.843            -2.555
    1           -37.696           -3.480            -2.844            -2.556

Opt Lag (Ng-Perron seq t) = 28 with RMSE  .0154663
Min SC   = -8.308444 at lag  1 with RMSE  .0156603
Min MAIC = -7.880739 at lag 28 with RMSE  .0154663
```

Augmented Dickey-Fuller test for unit root   Number of obs   =   3379

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-10.148	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
y						
L1.	-.8690439	.0856355	-10.15	0.000	-1.036947	-.7011408
LD.	-.0169382	.08425	-0.20	0.841	-.1821249	.1482485
L2D.	-.0309058	.0828324	-0.37	0.709	-.1933131	.1315015
L3D.	-.0591368	.0813369	-0.73	0.467	-.2186118	.1003381
L4D.	-.061175	.0798328	-0.77	0.444	-.217701	.0953509
L5D.	-.0439841	.0781591	-0.56	0.574	-.1972285	.1092604
L6D.	-.0778649	.0765643	-1.02	0.309	-.2279825	.0722526
L7D.	-.0718797	.0748829	-0.96	0.337	-.2187006	.0749412
L8D.	-.0596058	.0731496	-0.81	0.415	-.2030282	.0838166
L9D.	-.0569234	.0713299	-0.80	0.425	-.196778	.0829312
L10D.	-.0957255	.069425	-1.38	0.168	-.2318451	.0403941
L11D.	-.0872235	.0675318	-1.29	0.197	-.2196313	.0451842
L12D.	-.0678003	.0657601	-1.03	0.303	-.1967343	.0611337
L13D.	-.052577	.0639745	-0.82	0.411	-.1780099	.072856
L14D.	-.0459558	.0621697	-0.74	0.460	-.1678502	.0759386
L15D.	-.0124095	.0601564	-0.21	0.837	-.1303565	.1055375
L16D.	-.0174231	.0580401	-0.30	0.764	-.1312208	.0963746
L17D.	-.0279524	.0557702	-0.50	0.616	-.1372995	.0813946
L18D.	-.0713307	.0532884	-1.34	0.181	-.1758118	.0331504
L19D.	-.0823927	.0506396	-1.63	0.104	-.1816804	.0168951
L20D.	-.0714903	.048074	-1.49	0.137	-.1657476	.0227671
L21D.	-.060498	.0453352	-1.33	0.182	-.1493854	.0283894
L22D.	-.0574461	.042345	-1.36	0.175	-.1404707	.0255786
L23D.	-.0387643	.0390856	-0.99	0.321	-.1153984	.0378698
L24D.	-.0727927	.0357587	-2.04	0.042	-.1429038	-.0026815
L25D.	-.0337404	.0319619	-1.06	0.291	-.0964071	.0289263
L26D.	-.037095	.0277191	-1.34	0.181	-.0914429	.017253
L27D.	-.0201696	.0230062	-0.88	0.381	-.0652773	.024938
L28D.	-.0281297	.0172327	-1.63	0.103	-.0619174	.0056581
_cons	.000101	.0002656	0.38	0.704	-.0004197	.0006217

RESI

DF-GLS for y   Number of obs = 3379
Maxlag = 28 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
28	-6.803	-3.480	-2.833	-2.546
27	-6.916	-3.480	-2.833	-2.546
26	-7.132	-3.480	-2.834	-2.547
25	-7.206	-3.480	-2.834	-2.547
24	-7.704	-3.480	-2.834	-2.547
23	-8.115	-3.480	-2.835	-2.548
22	-8.475	-3.480	-2.835	-2.548
21	-8.438	-3.480	-2.836	-2.549
20	-8.677	-3.480	-2.836	-2.549
19	-9.008	-3.480	-2.837	-2.549
18	-9.536	-3.480	-2.837	-2.550
17	-9.790	-3.480	-2.837	-2.550
16	-10.006	-3.480	-2.838	-2.550
15	-10.277	-3.480	-2.838	-2.551
14	-10.881	-3.480	-2.839	-2.551
13	-11.598	-3.480	-2.839	-2.551
12	-12.327	-3.480	-2.839	-2.552
11	-13.413	-3.480	-2.840	-2.552
10	-14.423	-3.480	-2.840	-2.553
9	-15.109	-3.480	-2.841	-2.553
8	-16.281	-3.480	-2.841	-2.553
7	-18.118	-3.480	-2.841	-2.554
6	-19.242	-3.480	-2.842	-2.554
5	-21.967	-3.480	-2.842	-2.554
4	-24.241	-3.480	-2.842	-2.555
3	-26.911	-3.480	-2.843	-2.555
2	-31.618	-3.480	-2.843	-2.555
1	-36.268	-3.480	-2.844	-2.556

Opt Lag (Ng-Perron seq t) = 25 with RMSE .0192266
Min SC   = -7.867329 at lag 1 with RMSE .0195248
Min MAIC = -7.547298 at lag 28 with RMSE .0192208

Augmented Dickey-Fuller test for unit root   Number of obs   =   3382

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-11.459	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
y						
L1.	-1.012129	.0883256	-11.46	0.000	-1.185307	-.838952
LD.	.0947062	.08647	1.10	0.273	-.0748331	.2642454
L2D.	.1137304	.0845386	1.35	0.179	-.052022	.2794827
L3D.	.0465959	.08256	0.56	0.573	-.115277	.2084689
L4D.	.0412782	.0806702	0.51	0.609	-.1168896	.199446
L5D.	.0050711	.0787532	0.06	0.949	-.1493379	.1594802
L6D.	-.0219951	.0767802	-0.29	0.775	-.1725358	.1285456
L7D.	.0058875	.0746999	0.08	0.937	-.1405744	.1523494
L8D.	-.0268879	.0726359	-0.37	0.711	-.1693031	.1155273
L9D.	-.0036339	.0706045	-0.05	0.959	-.142066	.1347983
L10D.	.0009257	.0685761	0.01	0.989	-.1335295	.1353808
L11D.	-.0229903	.0664344	-0.35	0.729	-.1532464	.1072658
L12D.	-.0195172	.0641701	-0.30	0.761	-.1453336	.1062992
L13D.	.0031335	.0617916	0.05	0.960	-.1180194	.1242865
L14D.	.0084368	.0592058	0.14	0.887	-.1076463	.1245198
L15D.	.0197227	.0564719	0.35	0.727	-.0910003	.1304456
L16D.	.0320862	.0537384	0.60	0.550	-.0732772	.1374496
L17D.	.0169126	.0508192	0.33	0.739	-.0827271	.1165524
L18D.	-.0037997	.0475609	-0.08	0.936	-.097051	.0894516
L19D.	-.0225707	.044295	-0.51	0.610	-.1094187	.0642773
L20D.	-.0078716	.0405669	-0.19	0.846	-.0874099	.0716668
L21D.	-.0056533	.0366664	-0.15	0.877	-.0775442	.0662375
L22D.	-.0103081	.0326248	-0.32	0.752	-.0742746	.0536585
L23D.	-.0502026	.0280626	-1.79	0.074	-.1052242	.004819
L24D.	-.0467242	.0234109	-2.00	0.046	-.0926253	-.0008232
L25D.	-.0334359	.0172832	-1.93	0.053	-.0673226	.0004508
_cons	.0006562	.0003334	1.97	0.049	2.45e-06	.0013099

INDI

DF-GLS for y   Number of obs = 4043
Maxlag = 30 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
30	-6.890	-3.480	-2.833	-2.546
29	-7.034	-3.480	-2.833	-2.546
28	-7.136	-3.480	-2.834	-2.547
27	-7.446	-3.480	-2.834	-2.547
26	-7.732	-3.480	-2.834	-2.547
25	-7.963	-3.480	-2.835	-2.548
24	-8.363	-3.480	-2.835	-2.548
23	-8.781	-3.480	-2.835	-2.548
22	-9.029	-3.480	-2.836	-2.549
21	-9.464	-3.480	-2.836	-2.549
20	-9.526	-3.480	-2.836	-2.549
19	-9.867	-3.480	-2.837	-2.549
18	-10.251	-3.480	-2.837	-2.550
17	-10.538	-3.480	-2.838	-2.550
16	-10.838	-3.480	-2.838	-2.550
15	-11.104	-3.480	-2.838	-2.551
14	-11.691	-3.480	-2.839	-2.551
13	-12.216	-3.480	-2.839	-2.551
12	-12.874	-3.480	-2.839	-2.552
11	-13.944	-3.480	-2.839	-2.552
10	-15.204	-3.480	-2.840	-2.552
9	-15.934	-3.480	-2.840	-2.552
8	-17.123	-3.480	-2.840	-2.553
7	-18.707	-3.480	-2.841	-2.553
6	-20.866	-3.480	-2.841	-2.553
5	-23.424	-3.480	-2.841	-2.554
4	-26.496	-3.480	-2.842	-2.554
3	-30.086	-3.480	-2.842	-2.554
2	-34.681	-3.480	-2.842	-2.554
1	-41.665	-3.480	-2.843	-2.555

Opt Lag (Ng-Perron seq t) = 28 with RMSE .0136674
Min SC   = -8.549415 at lag 1 with RMSE .0138876
Min MAIC = -8.27186 at lag 30 with RMSE .0136653

Augmented Dickey-Fuller test for unit root   Number of obs   =   4045

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-10.996	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.9184526	.0835275	-11.00	0.000	-1.082213	-.7546924
LD.	-.0404899	.0822155	-0.49	0.622	-.201678	.1206981
L2D.	-.0517609	.080842	-0.64	0.522	-.2102561	.1067344
L3D.	-.087563	.0794395	-1.10	0.270	-.2433085	.0681826
L4D.	-.1170166	.0779525	-1.50	0.133	-.2698468	.0358136
L5D.	-.1331993	.076386	-1.74	0.081	-.2829582	.0165597
L6D.	-.1314744	.0748065	-1.76	0.079	-.2781366	.0151878
L7D.	-.119313	.0731438	-1.63	0.103	-.2627154	.0240893
L8D.	-.0974601	.0715301	-1.36	0.173	-.2376987	.0427785
L9D.	-.0819854	.0698648	-1.17	0.241	-.2189591	.0549884
L10D.	-.0744774	.0681494	-1.09	0.275	-.2080881	.0591333
L11D.	-.0907181	.0664307	-1.37	0.172	-.2209592	.039523
L12D.	-.063554	.0647146	-0.98	0.326	-.1904305	.0633226
L13D.	-.0365973	.0629969	-0.58	0.561	-.1601062	.0869117
L14D.	-.0314332	.0611963	-0.51	0.608	-.151412	.0885456
L15D.	-.032504	.0593468	-0.55	0.584	-.1488567	.0838487
L16D.	-.0230003	.0574065	-0.40	0.689	-.135549	.0895484
L17D.	-.0389316	.0552795	-0.70	0.481	-.1473101	.0694469
L18D.	-.0500562	.0529458	-0.95	0.344	-.1538593	.0537469
L19D.	-.0613957	.0505756	-1.21	0.225	-.1605519	.0377606
L20D.	-.0597667	.0480389	-1.24	0.214	-.1539497	.0344163
L21D.	-.0577298	.0452824	-1.27	0.202	-.1465084	.0310488
L22D.	-.0852979	.0422337	-2.02	0.043	-.1680993	-.0024965
L23D.	-.0702161	.0388692	-1.81	0.071	-.1464212	.005989
L24D.	-.0740338	.0351501	-2.11	0.035	-.1429475	-.0051201
L25D.	-.055298	.0311277	-1.78	0.076	-.1163255	.0057295
L26D.	-.0335092	.0267723	-1.25	0.211	-.0859977	.0189793
L27D.	-.030896	.0219442	-1.41	0.159	-.0739188	.0121269
L28D.	-.0192019	.0158399	-1.21	0.225	-.0502569	.011853
_cons	.0003481	.0002162	1.61	0.107	-.0000757	.0007718

The MacKinnon approximate p-value in each ADF test allows one to reject the null hypothesis in favour of the alternative hypothesis and conclude that of each of the eight indices presented, the series are stationary.

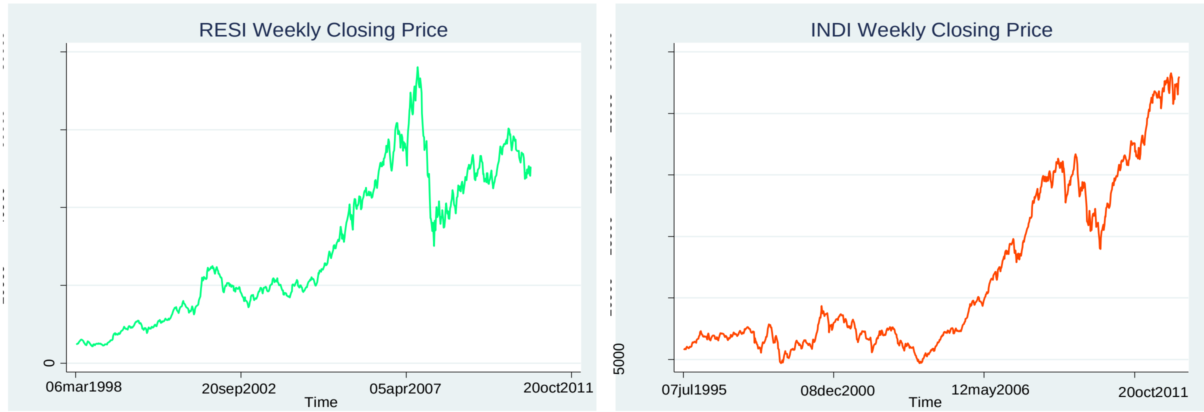
In summary, all eight of the indices presented, which span market capitalisation and industry of the JSE, has offered evidence of non-normality, serial correlation of residuals and stationarity. Various statistical analysis have pointed to evidence of asymmetry in return dynamics and conditional volatility, but these findings are considered inconclusive until further statistical investigation is presented later in the document.

The weekly and monthly return analysis is presented below.

5.1.2. Weekly Return Series

The interpretation of the weekly closing price plots per index is unchanged from that of the daily series. It is evident from the series that the index closing prices are non-stationary, given the upwards drift which could also be interpreted as persistence in positive returns. The shapes of the graphs remain unchanged when sampled at a different time interval and this is attributed to the effects of the macro-economy on all industries and market capitalisations.

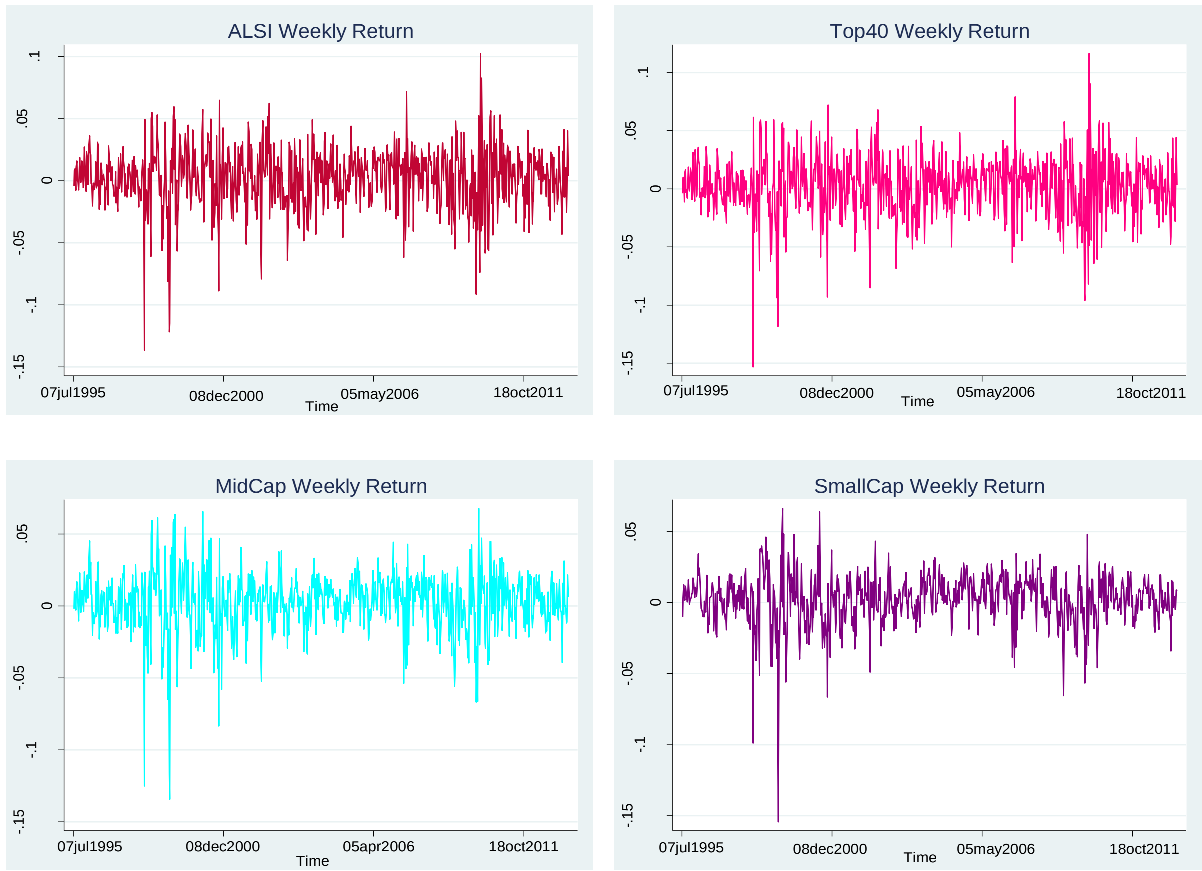
Figure 9: Weekly Closing Price per Index

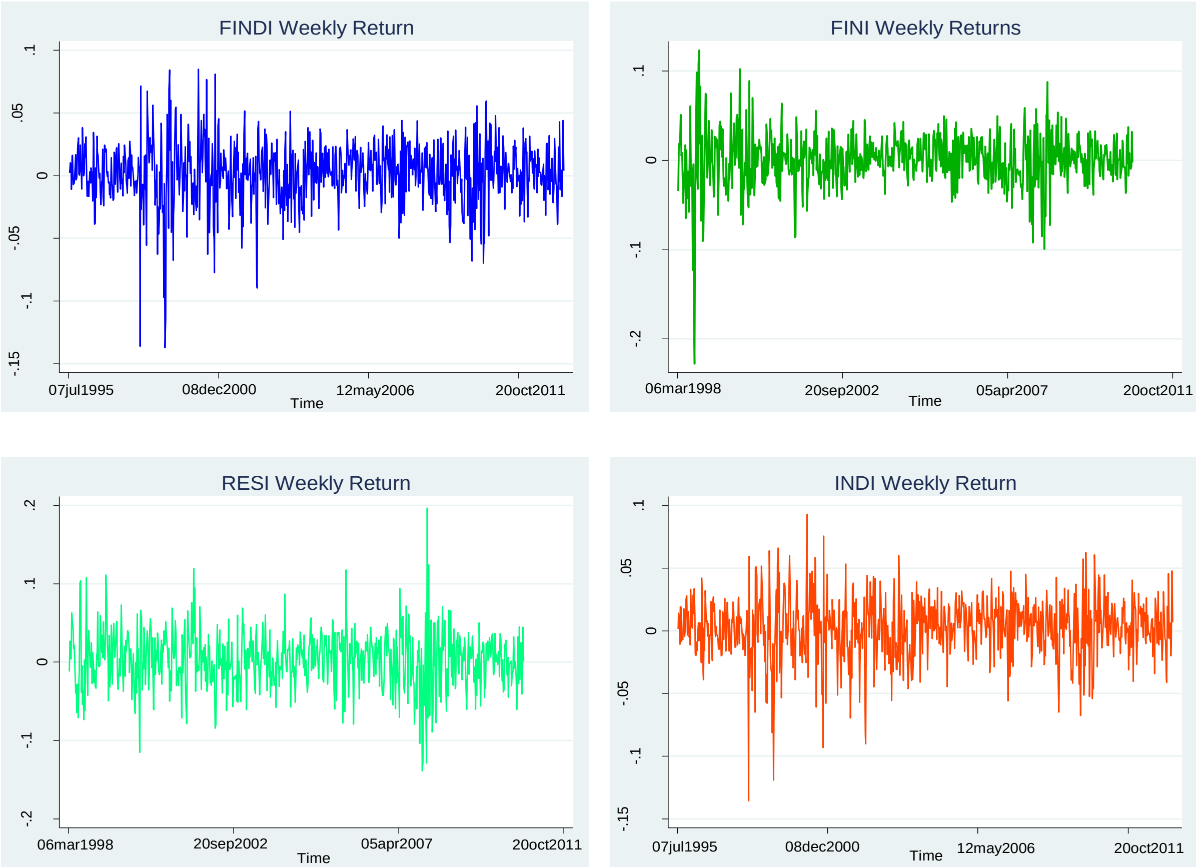


The characteristics of a series of levels is not desired for conducting statistical analysis and therefore the closing prices are converted into geometric returns using Equation 15.

The weekly return indices are presented in Figure 10 below.

Figure 10: Weekly Return per Index





The weekly return series, like that of the daily return series exhibit three distinct characteristics. Firstly the series appears to oscillate around a mean value of zero, secondly given this oscillation the series appears to be stationary and third, periods of high and low returns are bunched together, as a direct result of volatility clustering.

The descriptive statistics presented in Table 3 could assist in identifying distinct qualities of each series.

Table 3: Weekly Descriptive Statistics per Index

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Mean	0.00216	0.00208	0.00244	0.00207	0.00171	0.00065	0.00326	0.00185
Standard Error	0.00084	0.00090	0.00071	0.00062	0.00084	0.00112	0.00135	0.00083
Median	0.00414	0.00354	0.00413	0.00349	0.00259	0.00094	0.00434	0.00327
Standard Deviation	0.02452	0.02623	0.02074	0.01821	0.02452	0.02994	0.03605	0.02425
Sample Variance	0.00060	0.00069	0.00043	0.00033	0.00060	0.00090	0.00130	0.00059
Excess Kurtosis	3.14067	2.87259	5.49114	9.49647	3.49624	7.33849	1.92684	2.62048
Skewness	-0.72646	-0.60012	-1.04820	-1.48940	-0.70492	-0.89269	0.02375	-0.65306
Range	0.23910	0.26937	0.20220	0.22047	0.22207	0.35095	0.33529	0.22838
Minimum	-0.13655	-0.15305	-0.13433	-0.15428	-0.13721	-0.22770	-0.13883	-0.13553
Maximum	0.10255	0.11632	0.06787	0.06619	0.08486	0.12325	0.19647	0.09284
Sum	1.85054	1.78215	2.08919	1.77024	1.46692	0.46843	2.33702	1.58458
Count	857	857	857	857	857	717	717	857
Confidence Level (95.0%)	0.00164	0.00176	0.00139	0.00122	0.00164	0.00220	0.00264	0.00163

The above reported descriptive statistics across market capitalisation and industry all contain a positive mean estimate, a sign of an overall bullish market for the period under consideration- which is interpreted as evidence towards the persistence of positive returns. Risk is a priced factor as the RESI index yields the highest return and highest risk observation.

When comparing standard deviation it appears that the Top40 index, containing those firms with the largest market capitalisation remains riskier than the MidCap and SmallCap.

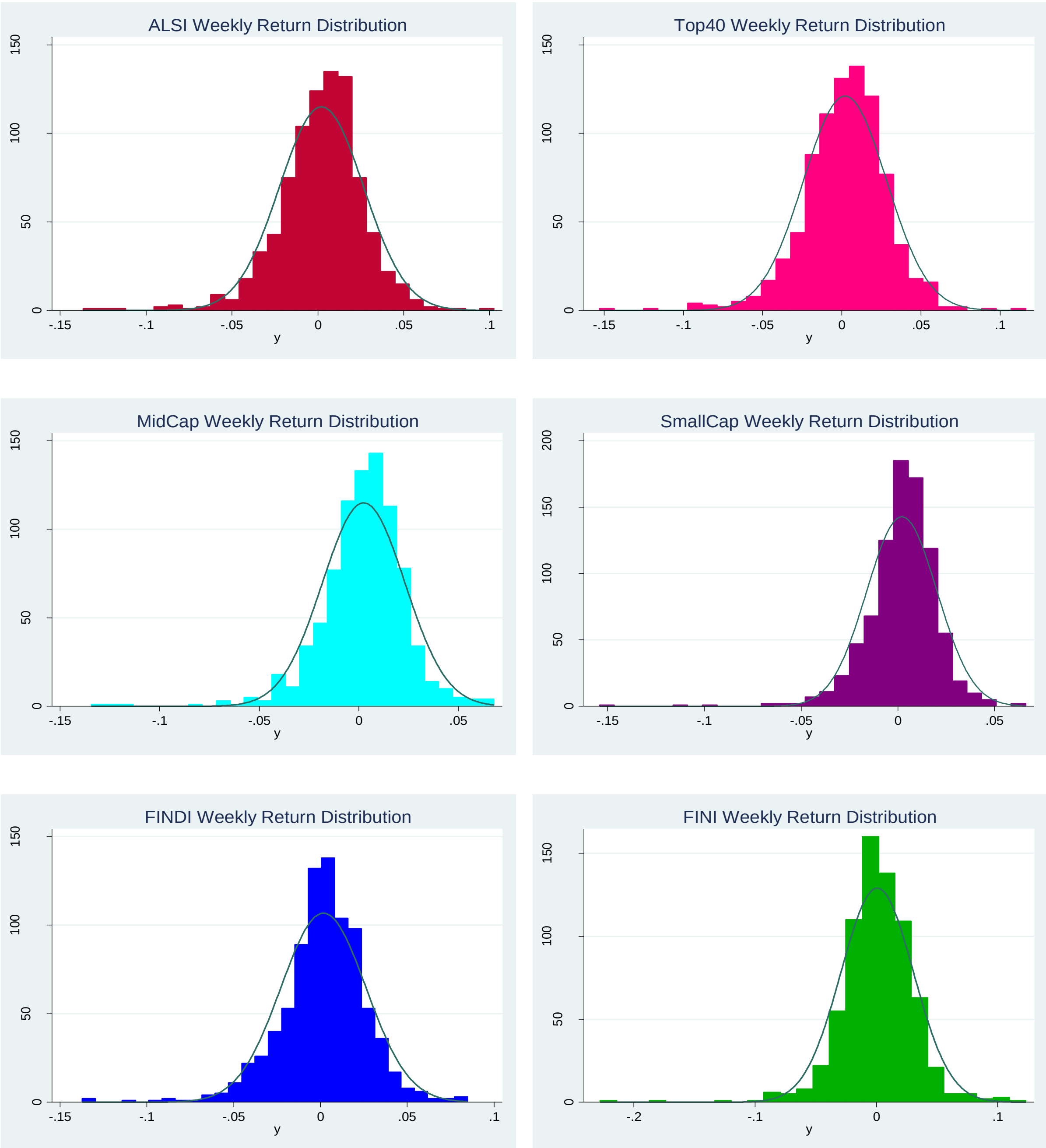
When one looks at daily sampling intervals, it appears that the INDI index had the largest range of returns. The weekly statistics however show that on a weekly basis, the FINI index contains the largest range, with the MidCap showing the lowest range in returns.

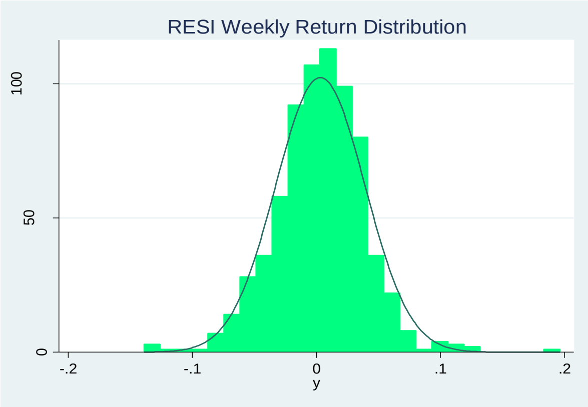
It is apparent from the excess kurtosis measure that each of the series display signs of fat tails and peakedness.

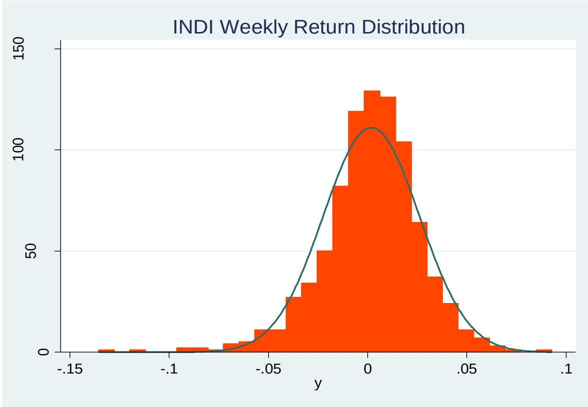
The SmallCap index displays still the highest level of excess kurtosis at 9.49. The skewness statistic indicates that each of the indices are negatively or left skewed, apart from the RESI index which is positively or right skewed. The highest degree of skewness can be found in the return series of the SmallCap index. The results indicate that the market portfolio, various market capitalisations and industries, have return distributions, which are non-normal.

To confirm the above finding, distribution plots for the return series were generated, and these are presented in Figure 11 below:

Figure 11: Distribution Plots of Weekly Returns per Index







The superimposed normal curve visually illustrates the extent of the leptokurtosis and fatter tails in the weekly returns.

The Jarque-Bera test was conducted to test the joint null hypothesis of zero skewness and a kurtosis measure of three. The results of the Jarque-Bera test for each index are presented in Figure 12 below.

Figure 12:Jarque-Bera Test for Weekly Return Indices

ALSI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

Top40

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

MidCap

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

SmallCap

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

FINDI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

FINI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	717	0.0000	0.0000	.	0.0000

RESI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	717	0.7935	0.0000	29.01	0.0000

INDI

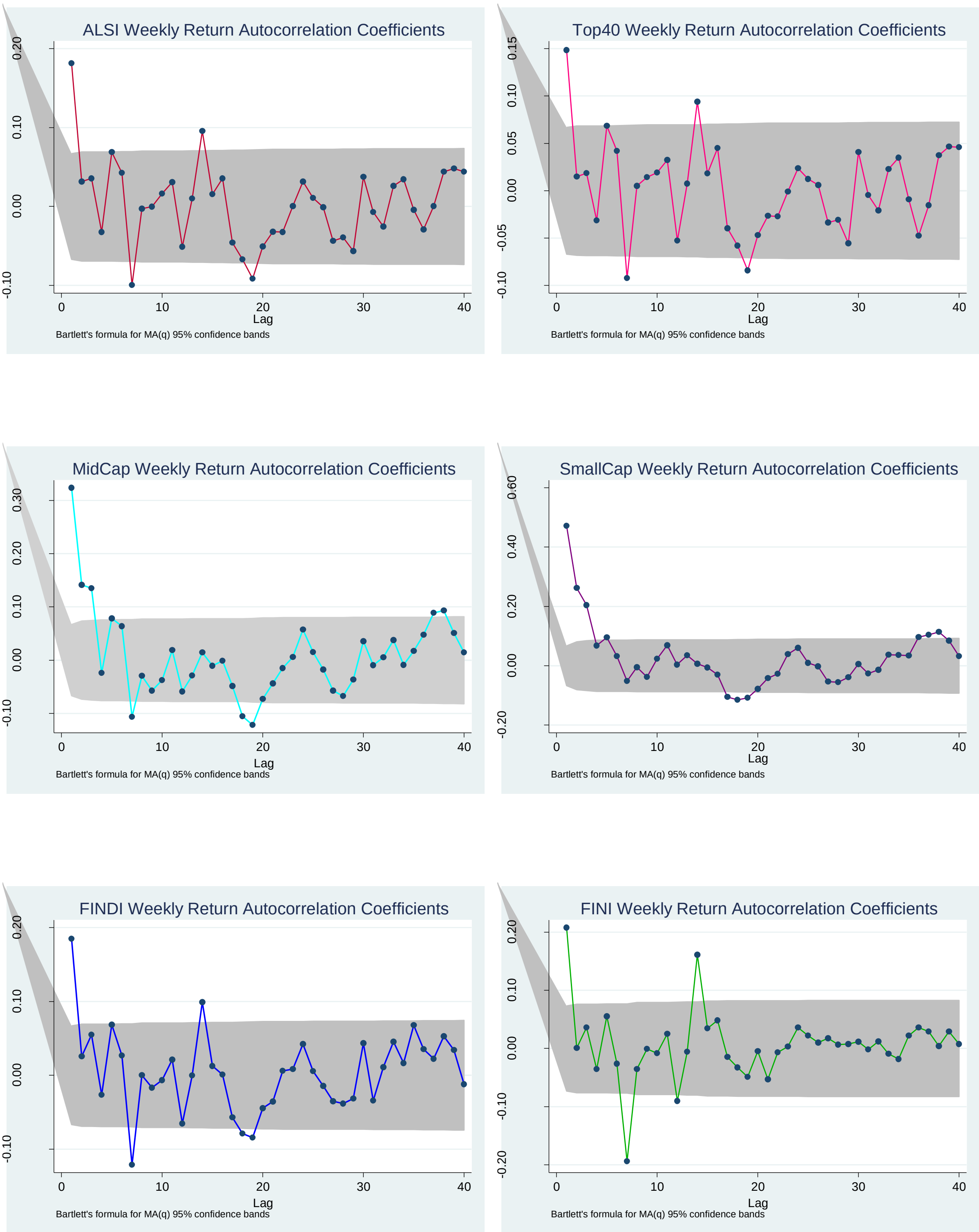
Skewness/Kurtosis tests for Normality

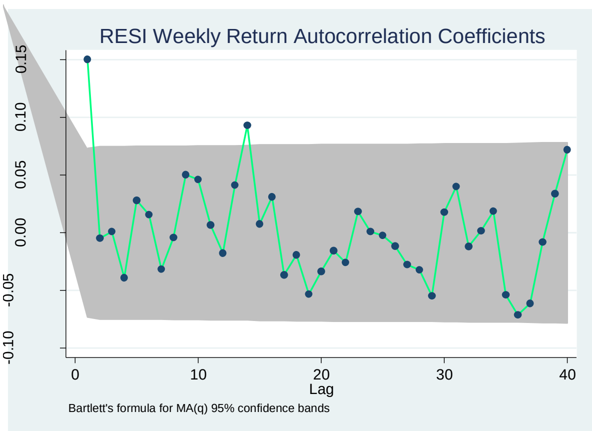
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	857	0.0000	0.0000	.	0.0000

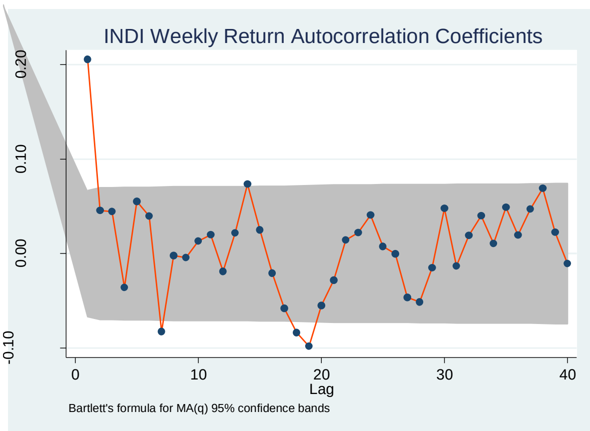
For each Jarque-Bera test the null hypothesis of normality can be rejected in favour of the alternative hypothesis of non-normality at the 95% confidence interval.

The autocorrelation plots presented in Figure 13 below graph the autocorrelation coefficients or relationship between a return observation with prior returns up to lag 40. The points which lie outside of the grey bar represent the statistically significant autocorrelation between return observations at the 95% confidence interval.

Figure 13: Autocorrelation Coefficients per Weekly Return Index







It appears as though the points, which lie outside of the 95% confidence interval band, are the negative autocorrelation coefficients, which can be interpreted as weak evidence of asymmetry in return dynamics. The plots of daily return autocorrelation coefficients and weekly autocorrelation coefficients do have some differences – the statistically significant coefficients are at different lag intervals, which could be an indication that the asymmetric relationship between daily returns is not present when one considers weekly intervals. This observation could affect the degree of asymmetry in the return dynamics across the daily, weekly and monthly time horizon. The first author to suggest that the asymmetric dynamics change across time intervals was Chen & Sauer (1997).

To test for serial correlation in the residual terms, the Portmanteau Q statistic is calculated. Evidence of serial correlation in the residual term would indicate that the size and magnitude of the residual term has an influence on the following period return observation.

Figure 14: Results of the Portmanteau Q statistic for Weekly Returns

ALSI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1813	0.1813	28.28	0.0000		
2	0.0317	-0.0013	29.143	0.0000		
3	0.0355	0.0311	30.229	0.0000		
4	-0.0323	-0.0459	31.13	0.0000		
5	0.0689	0.0855	35.228	0.0000		
6	0.0424	0.0151	36.786	0.0000		
7	-0.0994	-0.1145	45.348	0.0000		
8	-0.0028	0.0308	45.354	0.0000		
9	-0.0003	0.0028	45.354	0.0000		
10	0.0163	0.0210	45.585	0.0000		
11	0.0310	0.0081	46.419	0.0000		
12	-0.0509	-0.0475	48.676	0.0000		
13	0.0102	0.0368	48.767	0.0000		
14	0.0954	0.0822	56.713	0.0000		
15	0.0156	-0.0154	56.925	0.0000		
16	0.0358	0.0241	58.045	0.0000		
17	-0.0460	-0.0560	59.901	0.0000		
18	-0.0669	-0.0383	63.827	0.0000		
19	-0.0914	-0.1070	71.163	0.0000		
20	-0.0506	-0.0122	73.411	0.0000		

Top40

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1485	0.1485	18.961	0.0000		
2	0.0148	-0.0074	19.15	0.0001		
3	0.0184	0.0178	19.442	0.0002		
4	-0.0314	-0.0377	20.294	0.0004		
5	0.0683	0.0808	24.327	0.0002		
6	0.0420	0.0202	25.85	0.0002		
7	-0.0924	-0.1047	33.246	0.0000		
8	0.0051	0.0322	33.268	0.0001		
9	0.0141	0.0150	33.441	0.0001		
10	0.0191	0.0159	33.759	0.0002		
11	0.0326	0.0137	34.685	0.0003		
12	-0.0528	-0.0500	37.115	0.0002		
13	0.0075	0.0312	37.164	0.0004		
14	0.0939	0.0815	44.868	0.0000		
15	0.0182	-0.0071	45.156	0.0001		
16	0.0451	0.0372	46.933	0.0001		
17	-0.0398	-0.0491	48.321	0.0001		
18	-0.0578	-0.0345	51.249	0.0000		
19	-0.0842	-0.1025	57.481	0.0000		
20	-0.0469	-0.0202	59.414	0.0000		

MidCap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.3237	0.3237	90.105	0.0000		
2	0.1415	0.0411	107.35	0.0000		
3	0.1350	0.0871	123.06	0.0000		
4	-0.0238	-0.1079	123.55	0.0000		
5	0.0785	0.1163	128.87	0.0000		
6	0.0638	0.0059	132.39	0.0000		
7	-0.1063	-0.1429	142.17	0.0000		
8	-0.0296	0.0200	142.93	0.0000		
9	-0.0570	-0.0367	145.75	0.0000		
10	-0.0377	0.0234	146.99	0.0000		
11	0.0190	0.0021	147.3	0.0000		
12	-0.0587	-0.0489	150.31	0.0000		
13	-0.0289	0.0157	151.04	0.0000		
14	0.0145	0.0151	151.22	0.0000		
15	-0.0104	0.0002	151.32	0.0000		
16	-0.0013	-0.0254	151.32	0.0000		
17	-0.0485	-0.0512	153.38	0.0000		
18	-0.1052	-0.0679	163.09	0.0000		
19	-0.1218	-0.0922	176.13	0.0000		
20	-0.0728	0.0113	180.78	0.0000		

SmallCap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.4712	0.4712	190.91	0.0000		
2	0.2619	0.0512	249.97	0.0000		
3	0.2042	0.0808	285.91	0.0000		
4	0.0681	-0.0838	289.92	0.0000		
5	0.0950	0.0909	297.71	0.0000		
6	0.0326	-0.0598	298.63	0.0000		
7	-0.0515	-0.0695	300.93	0.0000		
8	-0.0049	0.0419	300.95	0.0000		
9	-0.0378	-0.0357	302.19	0.0000		
10	0.0237	0.0821	302.68	0.0000		
11	0.0695	0.0356	306.89	0.0000		
12	0.0041	-0.0443	306.9	0.0000		
13	0.0350	0.0292	307.97	0.0000		
14	0.0067	-0.0352	308.01	0.0000		
15	-0.0061	0.0066	308.04	0.0000		
16	-0.0301	-0.0677	308.83	0.0000		
17	-0.1055	-0.0708	318.58	0.0000		
18	-0.1149	-0.0381	330.17	0.0000		
19	-0.1077	-0.0235	340.35	0.0000		
20	-0.0782	0.0274	345.74	0.0000		

FINDI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1846	0.1846	29.308	0.0000		
2	0.0256	-0.0087	29.874	0.0000		
3	0.0550	0.0539	32.478	0.0000		
4	-0.0265	-0.0483	33.086	0.0000		
5	0.0686	0.0857	37.148	0.0000		
6	0.0267	-0.0051	37.765	0.0000		
7	-0.1213	-0.1276	50.517	0.0000		
8	0.0000	0.0399	50.517	0.0000		
9	-0.0168	-0.0201	50.762	0.0000		
10	-0.0067	0.0101	50.801	0.0000		
11	0.0213	0.0055	51.194	0.0000		
12	-0.0651	-0.0536	54.89	0.0000		
13	-0.0004	0.0264	54.89	0.0000		
14	0.0990	0.0850	63.452	0.0000		
15	0.0128	-0.0112	63.595	0.0000		
16	0.0007	-0.0141	63.596	0.0000		
17	-0.0572	-0.0605	66.463	0.0000		
18	-0.0791	-0.0481	71.954	0.0000		
19	-0.0844	-0.0967	78.213	0.0000		
20	-0.0447	-0.0064	79.967	0.0000		

FINI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2078	0.2078	31.086	0.0000		
2	0.0007	-0.0448	31.086	0.0000		
3	0.0359	0.0472	32.018	0.0000		
4	-0.0357	-0.0563	32.938	0.0000		
5	0.0553	0.0806	35.151	0.0000		
6	-0.0261	-0.0636	35.644	0.0000		
7	-0.1940	-0.1771	62.97	0.0000		
8	-0.0353	0.0384	63.878	0.0000		
9	-0.0008	-0.0019	63.878	0.0000		
10	-0.0081	0.0011	63.926	0.0000		
11	0.0250	0.0140	64.382	0.0000		
12	-0.0902	-0.0827	70.336	0.0000		
13	-0.0060	0.0297	70.362	0.0000		
14	0.1610	0.1266	89.365	0.0000		
15	0.0347	-0.0193	90.25	0.0000		
16	0.0483	0.0471	91.963	0.0000		
17	-0.0148	-0.0398	92.125	0.0000		
18	-0.0325	0.0007	92.904	0.0000		
19	-0.0489	-0.1067	94.668	0.0000		
20	-0.0046	0.0453	94.684	0.0000		

RESI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1504	0.1504	16.284	0.0001		
2	-0.0048	-0.0282	16.301	0.0003		
3	0.0010	0.0062	16.302	0.0010		
4	-0.0390	-0.0413	17.401	0.0016		
5	0.0283	0.0416	17.979	0.0030		
6	0.0156	0.0037	18.156	0.0059		
7	-0.0316	-0.0343	18.883	0.0086		
8	-0.0041	0.0045	18.895	0.0154		
9	0.0502	0.0538	20.734	0.0139		
10	0.0460	0.0312	22.279	0.0137		
11	0.0067	-0.0075	22.311	0.0221		
12	-0.0176	-0.0156	22.538	0.0319		
13	0.0414	0.0545	23.791	0.0331		
14	0.0930	0.0814	30.138	0.0073		
15	0.0076	-0.0220	30.181	0.0113		
16	0.0313	0.0386	30.9	0.0139		
17	-0.0364	-0.0410	31.876	0.0156		
18	-0.0192	-0.0036	32.147	0.0211		
19	-0.0533	-0.0719	34.247	0.0172		
20	-0.0334	-0.0168	35.073	0.0197		

INDI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2055	0.2055	36.329	0.0000		
2	0.0456	0.0036	38.122	0.0000		
3	0.0443	0.0359	39.811	0.0000		
4	-0.0358	-0.0548	40.919	0.0000		
5	0.0551	0.0757	43.542	0.0000		
6	0.0395	0.0142	44.892	0.0000		
7	-0.0823	-0.0977	50.753	0.0000		
8	-0.0021	0.0281	50.757	0.0000		
9	-0.0043	-0.0014	50.773	0.0000		
10	0.0133	0.0219	50.928	0.0000		
11	0.0198	-0.0023	51.271	0.0000		
12	-0.0186	-0.0136	51.574	0.0000		
13	0.0218	0.0350	51.988	0.0000		
14	0.0734	0.0594	56.693	0.0000		
15	0.0251	-0.0016	57.246	0.0000		
16	-0.0208	-0.0391	57.625	0.0000		
17	-0.0577	-0.0480	60.542	0.0000		
18	-0.0837	-0.0599	66.685	0.0000		
19	-0.0981	-0.0844	75.136	0.0000		
20	-0.0548	-0.0186	77.781	0.0000		

The probability in each case is less than 0.05; therefore we can safely reject the null hypothesis of no serial correlation in favour of the alternative hypothesis of serial correlation in residuals at the 95% confidence interval across all eight indices.

In the Portmanteau Q test on daily return indices, the Small Cap index was the only test results that do not show signs of possible asymmetry; however it appears that given the weekly test results, all of the indices show negative serial correlation in residuals.

The last test conducted for independence is the BDS test. The null hypothesis of the BDS test states that the residual terms of the series are identically and independently distributed. The alternative hypothesis implies that serial dependence between residuals is present. By using geometric returns, the linear dependence between residual terms has been eliminated; therefore, the BDS test run on the residuals terms is testing for non-linear dependence.

Figure 15: BDS Test of the Weekly Return Indices Residuals

ALSI

BDS Test
Sample: 7/07/1995 10/18/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.024430	0.002708	9.021218	0.0000
3	0.043817	0.004298	10.19453	0.0000
4	0.054885	0.005111	10.73836	0.0000
5	0.061955	0.005320	11.64670	0.0000
6	0.063410	0.005123	12.37858	0.0000

Raw epsilon	0.033085		
Pairs within epsilon	516887.0	V-Statistic	0.703775
Triples within epsilon	3.37E+08	V-Statistic	0.534915

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	189829.0	0.518744	257283.0	0.703074	0.494313
3	143211.0	0.392268	256906.0	0.703688	0.348450
4	109186.0	0.299771	256223.0	0.703463	0.244886
5	85270.00	0.234659	255752.0	0.703818	0.172704
6	66931.00	0.184624	255034.0	0.703492	0.121214

Top40

BDS Test
Sample: 7/07/1995 10/18/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.024366	0.002710	8.991261	0.0000
3	0.043586	0.004305	10.12537	0.0000
4	0.054433	0.005123	10.62533	0.0000
5	0.061261	0.005336	11.48037	0.0000
6	0.062621	0.005143	12.17664	0.0000

Raw epsilon	0.035648		
Pairs within epsilon	517307.0	V-Statistic	0.704347
Triples within epsilon	3.37E+08	V-Statistic	0.535748

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	190094.0	0.519468	257488.0	0.703634	0.495101
3	143470.0	0.392977	257137.0	0.704321	0.349391
4	109331.0	0.300169	256445.0	0.704072	0.245736
5	85287.00	0.234706	255971.0	0.704421	0.173445
6	66878.00	0.184478	255259.0	0.704112	0.121857

MidCap

BDS Test
Sample: 7/07/1995 10/18/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.031658	0.002908	10.88642	0.0000
3	0.056046	0.004627	12.11355	0.0000
4	0.069925	0.005516	12.67732	0.0000
5	0.076457	0.005755	13.28441	0.0000
6	0.079818	0.005557	14.36450	0.0000

Raw epsilon	0.027145		
Pairs within epsilon	517861.0	V-Statistic	0.705101
Triples within epsilon	3.40E+08	V-Statistic	0.539709

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	193158.0	0.527841	257769.0	0.704402	0.496183
3	148077.0	0.405596	257176.0	0.704428	0.349550
4	115109.0	0.316033	256542.0	0.704339	0.246108
5	90751.00	0.249743	255924.0	0.704291	0.173285
6	73043.00	0.201483	255192.0	0.703927	0.121665

SmallCap

BDS Test
Sample: 7/07/1995 10/18/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.045978	0.002899	15.85783	0.0000
3	0.081758	0.004605	17.75470	0.0000
4	0.099205	0.005480	18.10297	0.0000
5	0.105807	0.005708	18.53603	0.0000
6	0.106078	0.005501	19.28237	0.0000

Raw epsilon	0.023281		
Pairs within epsilon	516949.0	V-Statistic	0.703860
Triples within epsilon	3.39E+08	V-Statistic	0.537833

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	197793.0	0.540507	257339.0	0.703227	0.494529
3	156659.0	0.429103	256634.0	0.702943	0.347345
4	124941.0	0.343027	255944.0	0.702697	0.243822
5	100602.0	0.276852	255259.0	0.702461	0.171046
6	81882.00	0.225865	254531.0	0.702104	0.119787

FINDI

BDS Test
Sample: 7/07/1995 10/20/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.024795	0.002802	8.848486	0.0000
3	0.046308	0.004448	10.41077	0.0000
4	0.060701	0.005290	11.47418	0.0000
5	0.068809	0.005507	12.49496	0.0000
6	0.070800	0.005304	13.34844	0.0000

Raw epsilon	0.032855		
Pairs within epsilon	516803.0	V-Statistic	0.703661
Triples within epsilon	3.37E+08	V-Statistic	0.536131

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	189905.0	0.518951	257242.0	0.702962	0.494156
3	144187.0	0.394941	256951.0	0.703811	0.348633
4	111374.0	0.305778	256273.0	0.703600	0.245078
5	87649.00	0.241206	255661.0	0.703568	0.172397
6	69507.00	0.191730	254934.0	0.703216	0.120929

FINI

BDS Test
Sample: 3/06/1998 10/20/2011
Included observations: 717

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.019923	0.003053	6.525335	0.0000
3	0.041832	0.004843	8.638032	0.0000
4	0.057899	0.005755	10.05984	0.0000
5	0.068480	0.005987	11.43851	0.0000
6	0.072239	0.005762	12.53730	0.0000

Raw epsilon	0.038385		
Pairs within epsilon	361441.0	V-Statistic	0.703071
Triples within epsilon	1.97E+08	V-Statistic	0.535157

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	131313.0	0.513002	179741.0	0.702196	0.493079
3	99159.00	0.388470	179308.0	0.702466	0.346638
4	76581.00	0.300859	178707.0	0.702076	0.242960
5	60572.00	0.238634	178118.0	0.701727	0.170154
6	48411.00	0.191260	177524.0	0.701354	0.119021

RESI

BDS Test
Sample: 3/06/1998 10/20/2011
Included observations: 717

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.016585	0.002802	5.918281	0.0000
3	0.029036	0.004441	6.538035	0.0000
4	0.035930	0.005273	6.814113	0.0000
5	0.039993	0.005479	7.299049	0.0000
6	0.040583	0.005268	7.703959	0.0000

Raw epsilon	0.050108		
Pairs within epsilon	361369.0	V-Statistic	0.702931
Triples within epsilon	1.96E+08	V-Statistic	0.531604

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	130422.0	0.509521	179715.0	0.702094	0.492936
3	95860.00	0.375546	179286.0	0.702380	0.346510
4	70964.00	0.278792	178689.0	0.702005	0.242862
5	53561.00	0.211013	178299.0	0.702440	0.171020
6	40618.00	0.160472	177739.0	0.702204	0.119889

INDI

BDS Test
Sample: 7/07/1995 10/20/2011
Included observations: 857

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.026846	0.002807	9.563385	0.0000
3	0.047266	0.004458	10.60335	0.0000
4	0.058612	0.005304	11.05090	0.0000
5	0.063722	0.005523	11.53660	0.0000
6	0.064355	0.005322	12.09201	0.0000

Raw epsilon	0.033058		
Pairs within epsilon	517011.0	V-Statistic	0.703944
Triples within epsilon	3.38E+08	V-Statistic	0.536602

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	190810.0	0.521424	257352.0	0.703263	0.494579
3	144772.0	0.396543	257109.0	0.704244	0.349277
4	110863.0	0.304376	256452.0	0.704092	0.245763
5	86074.00	0.236872	255884.0	0.704181	0.173150
6	67404.00	0.185929	255160.0	0.703839	0.121574

In each test the probability of the z-statistic is less than 0.05, therefore one can reject the null hypothesis in favour of the alternative hypothesis and non-linear dependence is present in the residuals at the 95% confidence interval.

The Portmanteau Q statistic and the BDS test indicate that serial dependence exists between the residual terms. The non-linearity exposed by these tests, in both the daily and weekly returns of these indices speaks to the aim of this research.

Stationarity is a requirement of stochastic volatility modelling. The tests conducted for stationarity are the segmentation of the series to examine the properties through time and the Augmented Dickey Fuller test.

The simplest means of testing for stationarity is to divide the time series into sections, run descriptive statistics techniques on each section and compare the mean and variance between sections. If stationarity is present, the mean and variance should not deviate significantly between the sections.

Table 4: Descriptive Statistics of Sections of Weekly Index Returns

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Section 1								
Mean	0.00169	0.00171	0.00094	-0.00003	0.00131	-0.00180	0.00603	0.00099
Standard Error	0.00152	0.00161	0.00155	0.00139	0.00174	0.00258	0.00241	0.00165
Median	0.00230	0.00104	0.00238	0.00223	0.00170	-0.00051	0.00567	0.00067
Standard Deviation	0.02569	0.02721	0.02621	0.02349	0.02934	0.03987	0.03724	0.02794
Sample Variance	0.00066	0.00074	0.00069	0.00055	0.00086	0.00159	0.00139	0.00078
Section 2								
Mean	0.00352	0.00329	0.00455	0.00517	0.00206	0.00450	0.00374	0.00233
Standard Error	0.00127	0.00137	0.00087	0.00077	0.00124	0.00125	0.00192	0.00133
Median	0.00515	0.00559	0.00555	0.00515	0.00400	0.00353	0.00453	0.00464
Standard Deviation	0.02142	0.02309	0.01476	0.01295	0.02114	0.01936	0.02974	0.02239
Sample Variance	0.00046	0.00053	0.00022	0.00017	0.00045	0.00037	0.00088	0.00050
Section 3								
Mean	0.00128	0.00118	0.00196	0.00106	0.00173	-0.00078	0.00001	0.00222
Standard Error	0.00154	0.00167	0.00116	0.00096	0.00133	0.00173	0.00260	0.00130
Median	0.00419	0.00381	0.00395	0.00249	0.00268	-0.00169	0.00270	0.00343
Standard Deviation	0.02617	0.02816	0.01948	0.01623	0.02239	0.02669	0.04025	0.02206
Sample Variance	0.00068	0.00079	0.00038	0.00026	0.00050	0.00071	0.00162	0.00049

When one compares the mean of each index across the three sections, the variation is not large. The standard deviation and sample variances too remain consistent. Given that, no large inconsistencies are present per index it appears as though the series are stationary.

The Dickey-Fuller with GLS Trending and the ADF test are presented in Figure 16 below.

Figure 16: ADF Test on Weekly Return per Index

ALSI

```
DF-GLS for y                                            Number of obs =   836
Maxlag = 20 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------
    20           -6.380           -3.480            -2.826            -2.541
    19           -6.582           -3.480            -2.828            -2.543
    18           -6.745           -3.480            -2.830            -2.545
    17           -6.267           -3.480            -2.832            -2.547
    16           -6.226           -3.480            -2.834            -2.549
    15           -6.063           -3.480            -2.836            -2.551
    14           -6.398           -3.480            -2.838            -2.552
    13           -6.510           -3.480            -2.840            -2.554
    12           -7.344           -3.480            -2.842            -2.556
    11           -7.961           -3.480            -2.844            -2.557
    10           -7.943           -3.480            -2.846            -2.559
    9            -8.408           -3.480            -2.847            -2.561
    8            -9.083           -3.480            -2.849            -2.562
    7            -9.689           -3.480            -2.851            -2.564
    6           -10.738           -3.480            -2.853            -2.565
    5           -10.251           -3.480            -2.854            -2.567
    4           -11.267           -3.480            -2.856            -2.568
    3           -13.692           -3.480            -2.857            -2.570
    2           -14.809           -3.480            -2.859            -2.571
    1           -18.188           -3.480            -2.861            -2.573

Opt Lag (Ng-Perron seq t) = 18 with RMSE   .023828
Min SC   = -7.406334 at lag  1 with RMSE  .0244478
Min MAIC = -6.561931 at lag 15 with RMSE  .0239823

Augmented Dickey-Fuller test for unit root         Number of obs   =       838

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -7.291            -3.430            -2.860            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |  -.8711609   .1194774    -7.29   0.000    -1.105679   -.6366426
         LD. |   .0576406   .1165438     0.49   0.621    -.1711197    .2864008
        L2D. |   .0565668   .1136518     0.50   0.619    -.1665167    .2796502
        L3D. |   .0918458   .1110207     0.83   0.408    -.1260731    .3097648
        L4D. |   .0380316    .108014     0.35   0.725    -.1739856    .2500488
        L5D. |   .1165338    .105134     1.11   0.268    -.0898303    .3228979
        L6D. |   .1591552   .1014405     1.57   0.117    -.0399591    .3582695
        L7D. |   .0486092   .0973723     0.50   0.618    -.1425198    .2397382
        L8D. |   .0685165   .0936427     0.73   0.465    -.1152917    .2523248
        L9D. |   .0705034   .0896118     0.79   0.432    -.1053928    .2463995
       L10D. |   .0883182   .0852148     1.04   0.300    -.0789472    .2555836
       L11D. |   .1004121   .0804531     1.25   0.212    -.0575067    .2583309
       L12D. |   .0417905   .0751886     0.56   0.578    -.1057948    .1893757
       L13D. |   .0647323   .0710017     0.91   0.362    -.0746347    .2040992
       L14D. |   .1577035    .066041     2.39   0.017     .0280736    .2873333
       L15D. |   .1346798   .0595375     2.26   0.024     .0178155    .2515441
       L16D. |   .1726841   .0530867     3.25   0.001     .0684819    .2768862
       L17D. |   .1240818   .0450842     2.75   0.006     .0355874    .2125762
       L18D. |   .1069987   .0351114     3.05   0.002     .0380797    .1759178
             |
       _cons |    .001829   .0008676     2.11   0.035     .0001261    .0035319
------------------------------------------------------------------------------
```

Top40

DF-GLS for y   Number of obs =   836
Maxlag = 20 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
20	-6.352	-3.480	-2.826	-2.541
19	-6.534	-3.480	-2.828	-2.543
18	-6.623	-3.480	-2.830	-2.545
17	-6.164	-3.480	-2.832	-2.547
16	-6.132	-3.480	-2.834	-2.549
15	-6.000	-3.480	-2.836	-2.551
14	-6.406	-3.480	-2.838	-2.552
13	-6.570	-3.480	-2.840	-2.554
12	-7.400	-3.480	-2.842	-2.556
11	-7.965	-3.480	-2.844	-2.557
10	-7.915	-3.480	-2.846	-2.559
9	-8.413	-3.480	-2.847	-2.561
8	-9.023	-3.480	-2.849	-2.562
7	-9.732	-3.480	-2.851	-2.564
6	-10.792	-3.480	-2.853	-2.565
5	-10.412	-3.480	-2.854	-2.567
4	-11.526	-3.480	-2.856	-2.568
3	-13.974	-3.480	-2.857	-2.570
2	-15.335	-3.480	-2.859	-2.571
1	-18.712	-3.480	-2.861	-2.573

Opt Lag (Ng-Perron seq t) = 18 with RMSE .0256669
Min SC   = -7.262626 at lag 1 with RMSE .0262691
Min MAIC = -6.358779 at lag 15 with RMSE .0258191

Augmented Dickey-Fuller test for unit root   Number of obs   =   838

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-7.050	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8720379	.1236921	-7.05	0.000	-1.114829	-.6292466
LD.	.0256563	.1208182	0.21	0.832	-.211494	.2628065
L2D.	.0229709	.1179814	0.19	0.846	-.208611	.2545527
L3D.	.0443823	.1153658	0.38	0.701	-.1820656	.2708303
L4D.	.0007717	.1122967	0.01	0.995	-.219652	.2211953
L5D.	.0735631	.1093218	0.67	0.501	-.1410213	.2881474
L6D.	.1140152	.1055282	1.08	0.280	-.0931227	.3211531
L7D.	.0131654	.1013586	0.13	0.897	-.1857881	.2121189
L8D.	.0354809	.0975831	0.36	0.716	-.1560618	.2270236
L9D.	.0506739	.0934418	0.54	0.588	-.13274	.2340879
L10D.	.0639196	.088985	0.72	0.473	-.1107462	.2385854
L11D.	.0818418	.084008	0.97	0.330	-.0830549	.2467384
L12D.	.02378	.0784813	0.30	0.762	-.1302685	.1778285
L13D.	.0441587	.0739135	0.60	0.550	-.1009238	.1892411
L14D.	.1331232	.0685333	1.94	0.052	-.0013987	.2676451
L15D.	.1178248	.0615712	1.91	0.056	-.0030314	.238681
L16D.	.1640642	.0544238	3.01	0.003	.0572375	.270891
L17D.	.1201974	.0458462	2.62	0.009	.0302074	.2101875
L18D.	.1024879	.0351127	2.92	0.004	.0335663	.1714094
_cons	.0017584	.0009295	1.89	0.059	-.000066	.0035829

MidCap

```
DF-GLS for y                                        Number of obs =   836
Maxlag = 20 chosen by Schwert criterion
```

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
20	-7.237	-3.480	-2.826	-2.541
19	-7.530	-3.480	-2.828	-2.543
18	-7.943	-3.480	-2.830	-2.545
17	-7.532	-3.480	-2.832	-2.547
16	-7.302	-3.480	-2.834	-2.549
15	-7.179	-3.480	-2.836	-2.551
14	-7.242	-3.480	-2.838	-2.552
13	-7.518	-3.480	-2.840	-2.554
12	-7.952	-3.480	-2.842	-2.556
11	-8.441	-3.480	-2.844	-2.557
10	-8.398	-3.480	-2.846	-2.559
9	-8.833	-3.480	-2.847	-2.561
8	-9.587	-3.480	-2.849	-2.562
7	-9.793	-3.480	-2.851	-2.564
6	-10.658	-3.480	-2.853	-2.565
5	-9.789	-3.480	-2.854	-2.567
4	-10.532	-3.480	-2.856	-2.568
3	-13.023	-3.480	-2.857	-2.570
2	-12.789	-3.480	-2.859	-2.571
1	-15.980	-3.480	-2.861	-2.573

```
Opt Lag (Ng-Perron seq t) = 18 with RMSE  .0190983
Min SC   =  -7.84031 at lag  6 with RMSE   .019287
Min MAIC =  -7.12586 at lag  5 with RMSE  .0194772
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       838
```

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-8.204	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8312216	.1013218	-8.20	0.000	-1.030103	-.6323402
LD.	.1547807	.0981603	1.58	0.115	-.0378951	.3474566
L2D.	.1682907	.0952314	1.77	0.078	-.0186359	.3552174
L3D.	.2668362	.0924509	2.89	0.004	.0853672	.4483052
L4D.	.1448164	.0898352	1.61	0.107	-.0315182	.3211511
L5D.	.2420477	.0867705	2.79	0.005	.0717286	.4123668
L6D.	.3066924	.0836756	3.67	0.000	.1424483	.4709366
L7D.	.1617692	.0804976	2.01	0.045	.0037631	.3197753
L8D.	.1829551	.0773407	2.37	0.018	.0311455	.3347647
L9D.	.1396895	.0739353	1.89	0.059	-.0054358	.2848149
L10D.	.1567155	.0701036	2.24	0.026	.0191114	.2943196
L11D.	.1713569	.0664936	2.58	0.010	.0408388	.3018751
L12D.	.1094515	.0624691	1.75	0.080	-.0131672	.2320701
L13D.	.1269907	.0598469	2.12	0.034	.009519	.2444623
L14D.	.1493395	.0564686	2.64	0.008	.038499	.2601799
L15D.	.1549067	.0514098	3.01	0.003	.0539961	.2558173
L16D.	.1568845	.0477826	3.28	0.001	.0630934	.2506755
L17D.	.1287641	.0420354	3.06	0.002	.0462542	.211274
L18D.	.092196	.0350189	2.63	0.009	.0234584	.1609335
_cons	.0019953	.0007099	2.81	0.005	.0006019	.0033887

SmallCap

DF-GLS for y Number of obs = 836
Maxlag = 20 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
20	-5.334	-3.480	-2.826	-2.541
19	-5.525	-3.480	-2.828	-2.543
18	-5.911	-3.480	-2.830	-2.545
17	-6.034	-3.480	-2.832	-2.547
16	-6.075	-3.480	-2.834	-2.549
15	-5.904	-3.480	-2.836	-2.551
14	-5.734	-3.480	-2.838	-2.552
13	-5.988	-3.480	-2.840	-2.554
12	-6.007	-3.480	-2.842	-2.556
11	-6.434	-3.480	-2.844	-2.557
10	-6.410	-3.480	-2.846	-2.559
9	-6.933	-3.480	-2.847	-2.561
8	-7.951	-3.480	-2.849	-2.562
7	-8.144	-3.480	-2.851	-2.564
6	-9.071	-3.480	-2.853	-2.565
5	-9.053	-3.480	-2.854	-2.567
4	-9.123	-3.480	-2.856	-2.568
3	-10.884	-3.480	-2.857	-2.570
2	-10.915	-3.480	-2.859	-2.571
1	-13.273	-3.480	-2.861	-2.573

Opt Lag (Ng-Perron seq t) = 9 with RMSE .0160126
Min SC = -8.20982 at lag 4 with RMSE .016163
Min MAIC = -7.922554 at lag 12 with RMSE .0159681

Augmented Dickey-Fuller test for unit root Number of obs = 847

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-7.963	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.4684328	.0588232	-7.96	0.000	-.5838913	-.3529743
LD.	-.0654848	.0586121	-1.12	0.264	-.180529	.0495593
L2D.	-.0647656	.0562919	-1.15	0.250	-.1752557	.0457246
L3D.	.0504427	.0532192	0.95	0.343	-.0540162	.1549016
L4D.	-.0548609	.0510489	-1.07	0.283	-.1550599	.0453381
L5D.	.0462737	.0489553	0.95	0.345	-.049816	.1423635
L6D.	.0313756	.0455224	0.69	0.491	-.057976	.1207271
L7D.	-.06623	.0431976	-1.53	0.126	-.1510185	.0185585
L8D.	-.0082017	.0391037	-0.21	0.834	-.0849547	.0685513
L9D.	-.0820533	.0345186	-2.38	0.018	-.1498067	-.0143
_cons	.0009661	.0005594	1.73	0.085	-.0001319	.0020641

FINDI

```
DF-GLS for y                                           Number of obs =   836
Maxlag = 20 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value

    20           -7.300           -3.480            -2.826            -2.541
    19           -7.544           -3.480            -2.828            -2.543
    18           -7.773           -3.480            -2.830            -2.545
    17           -7.292           -3.480            -2.832            -2.547
    16           -7.174           -3.480            -2.834            -2.549
    15           -6.955           -3.480            -2.836            -2.551
    14           -7.065           -3.480            -2.838            -2.552
    13           -7.214           -3.480            -2.840            -2.554
    12           -8.163           -3.480            -2.842            -2.556
    11           -8.752           -3.480            -2.844            -2.557
    10           -8.676           -3.480            -2.846            -2.559
    9            -9.162           -3.480            -2.847            -2.561
    8            -9.776           -3.480            -2.849            -2.562
    7           -10.162           -3.480            -2.851            -2.564
    6           -11.362           -3.480            -2.853            -2.565
    5           -10.682           -3.480            -2.854            -2.567
    4           -11.457           -3.480            -2.856            -2.568
    3           -13.834           -3.480            -2.857            -2.570
    2           -14.803           -3.480            -2.859            -2.571
    1           -18.560           -3.480            -2.861            -2.573

Opt Lag (Ng-Perron seq t) = 18 with RMSE  .0236432
Min SC   = -7.416917 at lag  1 with RMSE  .0243188
Min MAIC = -6.263157 at lag  4 with RMSE  .0241687

Augmented Dickey-Fuller test for unit root         Number of obs   =       838

                               ---------- Interpolated Dickey-Fuller ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value

 Z(t)            -7.797           -3.430            -2.860            -2.570

MacKinnon approximate p-value for Z(t) = 0.0000
```

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.9581337	.1228799	-7.80	0.000	-1.199331	-.7169366
LD.	.1482697	.1191515	1.24	0.214	-.085609	.3821483
L2D.	.1330625	.115701	1.15	0.250	-.0940432	.3601683
L3D.	.187668	.1125259	1.67	0.096	-.0332056	.4085415
L4D.	.1392714	.1093364	1.27	0.203	-.0753416	.3538844
L5D.	.2160627	.1061054	2.04	0.042	.0077918	.4243336
L6D.	.2391838	.1020452	2.34	0.019	.0388826	.439485
L7D.	.1164762	.0977177	1.19	0.234	-.0753308	.3082832
L8D.	.15205	.0936286	1.62	0.105	-.0317305	.3358306
L9D.	.1279235	.0892635	1.43	0.152	-.047289	.3031361
L10D.	.1307754	.0845743	1.55	0.122	-.0352328	.2967836
L11D.	.1417283	.0798149	1.78	0.076	-.0149378	.2983943
L12D.	.0788337	.074364	1.06	0.289	-.0671329	.2248004
L13D.	.0896303	.0701327	1.28	0.202	-.0480309	.2272916
L14D.	.1844367	.0654022	2.82	0.005	.0560607	.3128127
L15D.	.1747271	.0592426	2.95	0.003	.0584416	.2910125
L16D.	.1777818	.0531393	3.35	0.001	.0734765	.2820872
L17D.	.1255529	.0449654	2.79	0.005	.0372918	.2138141
L18D.	.0966863	.0351034	2.75	0.006	.027783	.1655897
_cons	.0015479	.0008509	1.82	0.069	-.0001223	.003218

FINI

DF-GLS for y   Number of obs =   697
Maxlag = 19 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
19	-2.610	-3.480	-2.825	-2.541
18	-2.860	-3.480	-2.828	-2.543
17	-2.727	-3.480	-2.830	-2.545
16	-2.855	-3.480	-2.833	-2.548
15	-2.867	-3.480	-2.835	-2.550
14	-3.150	-3.480	-2.837	-2.552
13	-3.244	-3.480	-2.840	-2.554
12	-3.925	-3.480	-2.842	-2.556
11	-4.429	-3.480	-2.844	-2.558
10	-4.416	-3.480	-2.847	-2.560
9	-4.941	-3.480	-2.849	-2.562
8	-5.517	-3.480	-2.851	-2.564
7	-6.127	-3.480	-2.853	-2.566
6	-7.292	-3.480	-2.855	-2.568
5	-6.918	-3.480	-2.857	-2.570
4	-7.295	-3.480	-2.859	-2.572
3	-9.160	-3.480	-2.861	-2.574
2	-10.372	-3.480	-2.863	-2.575
1	-13.718	-3.480	-2.865	-2.577

Opt Lag (Ng-Perron seq t) = 19 with RMSE .0283827
Min SC   = -6.969619 at lag 13 with RMSE .0287086
Min MAIC = -6.991787 at lag 19 with RMSE .0283827

Augmented Dickey-Fuller test for unit root   Number of obs   =   697

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-6.108	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8885733	.1454767	-6.11	0.000	-1.174214	-.6029329
LD.	.1290532	.1409144	0.92	0.360	-.1476292	.4057357
L2D.	.0690117	.1373668	0.50	0.616	-.2007051	.3387286
L3D.	.1251509	.1335194	0.94	0.349	-.1370118	.3873136
L4D.	.0760349	.1300638	0.58	0.559	-.1793426	.3314124
L5D.	.1468474	.1258842	1.17	0.244	-.1003237	.3940185
L6D.	.1218815	.1221563	1.00	0.319	-.1179699	.3617328
L7D.	-.0421387	.1166372	-0.36	0.718	-.2711534	.1868761
L8D.	-.0061295	.1108625	-0.06	0.956	-.2238057	.2115467
L9D.	-.0109445	.1058753	-0.10	0.918	-.2188284	.1969395
L10D.	-.0105372	.1004171	-0.10	0.916	-.2077041	.1866298
L11D.	.0247568	.0945842	0.26	0.794	-.1609573	.210471
L12D.	-.0730473	.0882781	-0.83	0.408	-.2463795	.1002849
L13D.	-.0658609	.0810062	-0.81	0.416	-.2249148	.093193
L14D.	.0760916	.0757035	1.01	0.315	-.0725507	.2247339
L15D.	.0288917	.0704694	0.41	0.682	-.1094734	.1672568
L16D.	.094217	.0633952	1.49	0.138	-.0302581	.2186921
L17D.	.0416005	.0568013	0.73	0.464	-.0699277	.1531286
L18D.	.0689522	.0476456	1.45	0.148	-.024599	.1625035
L19D.	-.0452912	.0379671	-1.19	0.233	-.1198388	.0292565
_cons	.000756	.001073	0.70	0.481	-.0013508	.0028628

RESI

```
DF-GLS for y                                           Number of obs =   697
Maxlag = 19 chosen by Schwert criterion
```

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
19	-4.302	-3.480	-2.825	-2.541
18	-4.379	-3.480	-2.828	-2.543
17	-4.208	-3.480	-2.830	-2.545
16	-4.336	-3.480	-2.833	-2.548
15	-4.290	-3.480	-2.835	-2.550
14	-4.582	-3.480	-2.837	-2.552
13	-4.647	-3.480	-2.840	-2.554
12	-5.243	-3.480	-2.842	-2.556
11	-5.766	-3.480	-2.844	-2.558
10	-5.915	-3.480	-2.847	-2.560
9	-6.162	-3.480	-2.849	-2.562
8	-6.755	-3.480	-2.851	-2.564
7	-7.659	-3.480	-2.853	-2.566
6	-8.434	-3.480	-2.855	-2.568
5	-8.960	-3.480	-2.857	-2.570
4	-10.100	-3.480	-2.859	-2.572
3	-12.087	-3.480	-2.861	-2.574
2	-13.586	-3.480	-2.863	-2.575
1	-16.794	-3.480	-2.865	-2.577

```
Opt Lag (Ng-Perron seq t) = 13 with RMSE   .035268
Min SC   =  -6.63095 at lag  1 with RMSE  .0359773
Min MAIC = -6.236614 at lag 15 with RMSE  .0352227
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       703
```

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-5.861	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.7169795	.122321	-5.86	0.000	-.9571468	-.4768121
LD.	-.1409966	.1189462	-1.19	0.236	-.3745378	.0925446
L2D.	-.1701061	.114657	-1.48	0.138	-.3952258	.0550137
L3D.	-.1617304	.1104476	-1.46	0.144	-.3785852	.0551243
L4D.	-.214854	.105962	-2.03	0.043	-.4229017	-.0068063
L5D.	-.1740093	.1009687	-1.72	0.085	-.372253	.0242344
L6D.	-.1545865	.0950085	-1.63	0.104	-.3411279	.0319549
L7D.	-.1817694	.0887411	-2.05	0.041	-.3560053	-.0075336
L8D.	-.183293	.0827055	-2.22	0.027	-.3456784	-.0209075
L9D.	-.1333947	.0759666	-1.76	0.080	-.2825488	.0157595
L10D.	-.0974414	.0678725	-1.44	0.152	-.2307035	.0358207
L11D.	-.0999871	.0595973	-1.68	0.094	-.2170016	.0170274
L12D.	-.122418	.0496396	-2.47	0.014	-.2198814	-.0249547
L13D.	-.0814327	.038036	-2.14	0.033	-.1561132	-.0067522
_cons	.0023171	.0014013	1.65	0.099	-.0004344	.0050685

INDI

```
DF-GLS for y                                         Number of obs =   836
Maxlag = 20 chosen by Schwert criterion
```

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
20	-7.167	-3.480	-2.826	-2.541
19	-7.447	-3.480	-2.828	-2.543
18	-7.562	-3.480	-2.830	-2.545
17	-7.170	-3.480	-2.832	-2.547
16	-6.957	-3.480	-2.834	-2.549
15	-6.820	-3.480	-2.836	-2.551
14	-6.736	-3.480	-2.838	-2.552
13	-6.930	-3.480	-2.840	-2.554
12	-7.607	-3.480	-2.842	-2.556
11	-8.176	-3.480	-2.844	-2.557
10	-8.407	-3.480	-2.846	-2.559
9	-8.773	-3.480	-2.847	-2.561
8	-9.436	-3.480	-2.849	-2.562
7	-9.968	-3.480	-2.851	-2.564
6	-10.960	-3.480	-2.853	-2.565
5	-10.609	-3.480	-2.854	-2.567
4	-11.628	-3.480	-2.856	-2.568
3	-13.920	-3.480	-2.857	-2.570
2	-14.802	-3.480	-2.859	-2.571
1	-18.090	-3.480	-2.861	-2.573

```
Opt Lag (Ng-Perron seq t) = 18 with RMSE  .0234672
Min SC   = -7.448386 at lag  1 with RMSE  .0239391
Min MAIC = -6.374573 at lag 13 with RMSE  .0236384
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       838
```

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-7.552	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8863205	.1173658	-7.55	0.000	-1.116694	-.6559468
LD.	.0883431	.1140405	0.77	0.439	-.1355034	.3121897
L2D.	.0894837	.1109852	0.81	0.420	-.1283657	.307333
L3D.	.1290747	.108077	1.19	0.233	-.0830663	.3412157
L4D.	.0662757	.1053464	0.63	0.529	-.1405053	.2730568
L5D.	.1373321	.1024931	1.34	0.181	-.0638485	.3385126
L6D.	.179763	.0990972	1.81	0.070	-.0147519	.3742778
L7D.	.0829311	.0953576	0.87	0.385	-.1042433	.2701054
L8D.	.1067961	.0916003	1.17	0.244	-.0730032	.2865954
L9D.	.0971891	.0876972	1.11	0.268	-.0749489	.2693272
L10D.	.118024	.0833396	1.42	0.157	-.0455606	.2816087
L11D.	.1166954	.0787763	1.48	0.139	-.0379321	.2713228
L12D.	.0909517	.0736904	1.23	0.217	-.0536927	.2355962
L13D.	.1174774	.0695693	1.69	0.092	-.0190779	.2540327
L14D.	.1811715	.0647351	2.80	0.005	.0541051	.3082378
L15D.	.1868063	.0584607	3.20	0.001	.0720556	.301557
L16D.	.1618177	.0525177	3.08	0.002	.0587325	.264903
L17D.	.1265305	.0447786	2.83	0.005	.0386361	.2144249
L18D.	.0844193	.0351685	2.40	0.017	.0153881	.1534505
_cons	.001563	.0008467	1.85	0.065	-.000099	.0032249

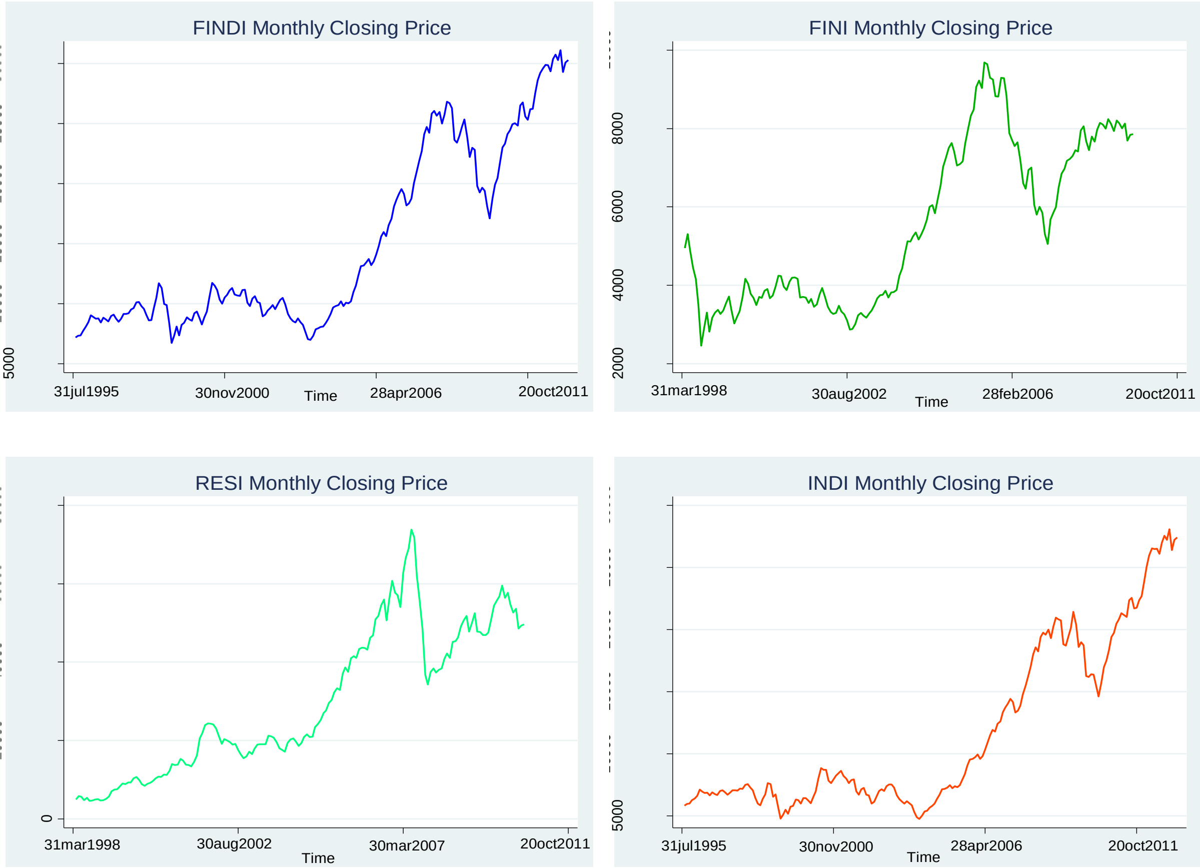
The MacKinnon approximate p-value allows one to reject the null hypothesis in favour of the alternative hypothesis and conclude that each of the eight indices presented, the series are stationary.

In summary, all eight of the indices presented, which span market capitalisation and industry on the JSE, has offered evidence of non-normality, serial correlation in residuals and stationarity using weekly data.

5.1.3. Monthly Return Series

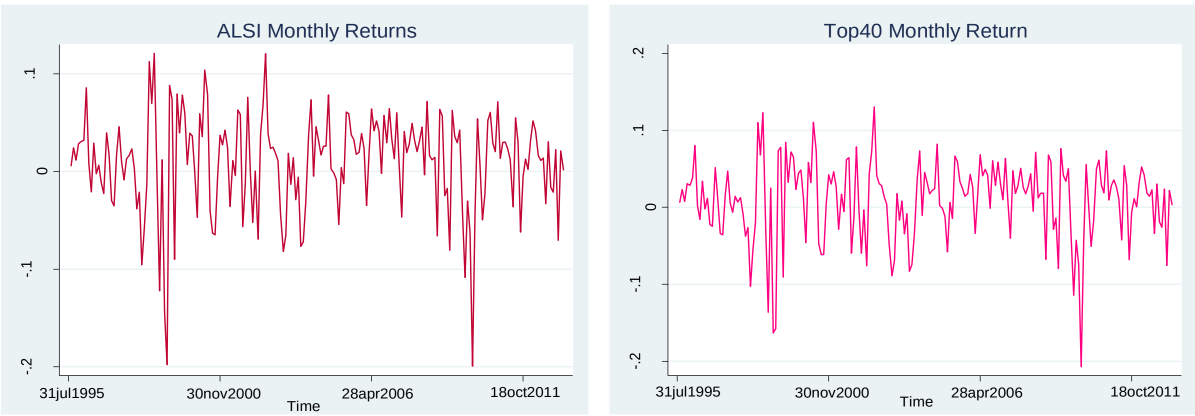
The interpretation of the monthly closing price plots per index is unchanged from that of the daily or weekly series - index closing prices are non-stationary, given the upwards drift which could also be interpreted as persistence in positive returns. The shapes of the graphs remain unchanged when sampled at a different time interval and this is attributed to the effects of the macro-economy on all industries and market capitalisations.

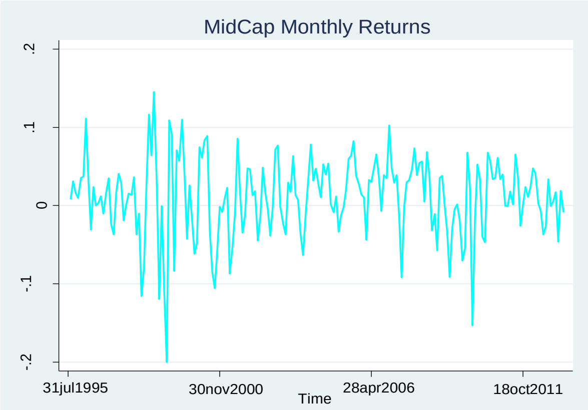
Figure 17: Monthly Closing Prices per Index

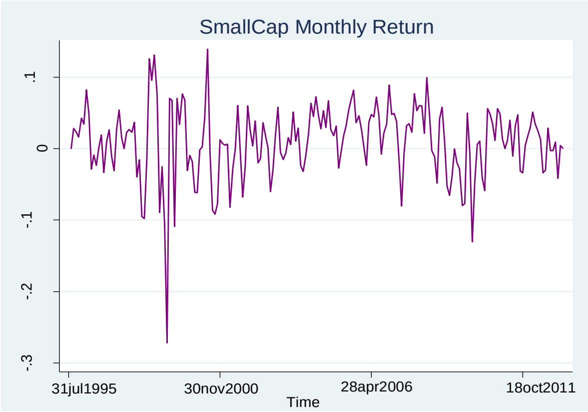


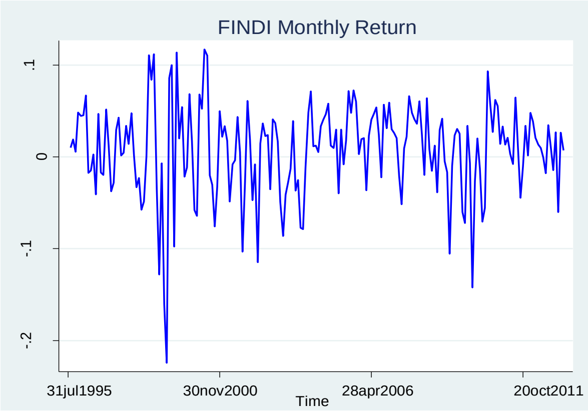
The monthly series, like that of the daily and weekly return series exhibit three distinct characteristics. Firstly the series appears to oscillate around a mean value of zero, secondly given this oscillation the series appears to be stationary and third, periods of high and low returns are bunched together, as a direct result of volatility clustering. These results are presented below.

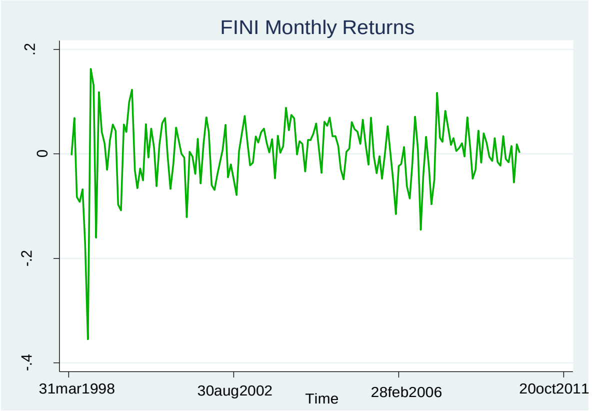
Figure 18: Monthly Returns per Index

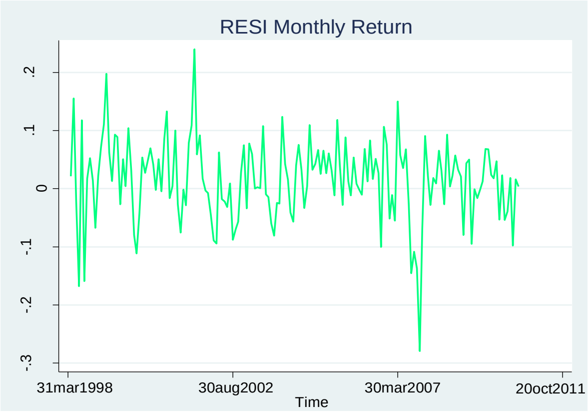


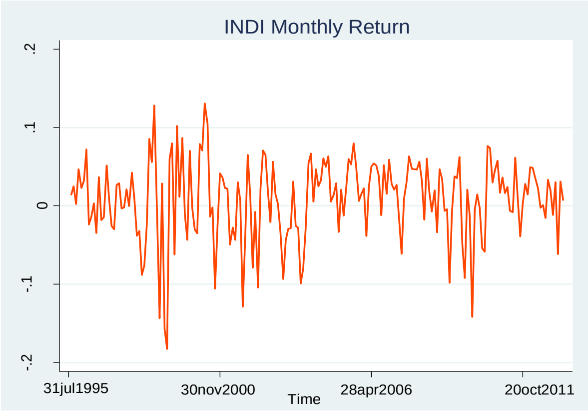

MidCap Monthly Returns
.2
.1
0
-.1
-.2
31jul1995
30nov2000
28apr2006
18oct2011
Time


SmallCap Monthly Return
.1
0
-.1
-.2
-.3
31jul1995
30nov2000
28apr2006
18oct2011
Time


FINDI Monthly Return
.1
0
-.1
-.2
31jul1995
30nov2000
28apr2006
20oct2011
Time


FINI Monthly Returns
.2
0
-.2
-.4
31mar1998
30aug2002
28feb2006
20oct2011
Time


RESI Monthly Return
.2
.1
0
-.1
-.2
-.3
31mar1998
30aug2002
30mar2007
20oct2011
Time


INDI Monthly Return
.2
.1
0
-.1
-.2
31jul1995
30nov2000
28apr2006
20oct2011
Time

The descriptive statistics presented in Table 5 could assist in identifying distinct qualities of each series.

Table 5: Monthly Descriptive Statistics per Index

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Mean	0.00934	0.00899	0.01059	0.00898	0.00739	0.00280	0.01415	0.00797
Standard Error	0.00358	0.00368	0.00358	0.00362	0.00362	0.00480	0.00545	0.00364
Median	0.01692	0.01733	0.01625	0.01284	0.01384	0.01072	0.01738	0.01529
Standard Deviation	0.05011	0.05150	0.05013	0.05063	0.05062	0.06141	0.06975	0.05098
Sample Variance	0.00251	0.00265	0.00251	0.00256	0.00256	0.00377	0.00486	0.00260
Excess Kurtosis	2.48303	1.91428	1.72609	4.41697	2.42329	6.69730	1.95971	1.44727
Skewness	-1.03170	-0.93305	-0.72443	-1.08609	-0.93265	-1.42384	-0.42978	-0.84567
Range	0.32041	0.33778	0.34499	0.41104	0.34131	0.51781	0.51944	0.31344
Minimum	-0.19944	-0.20723	-0.19983	-0.27206	-0.22426	-0.35507	-0.27939	-0.18246
Maximum	0.12097	0.13055	0.14516	0.13898	0.11704	0.16273	0.24005	0.13098
Sum	1.83074	1.76117	2.07495	1.75981	1.44861	0.45873	2.32051	1.56305
Count	196	196	196	196	196	164	164	196
Confidence Level (95.0%)	0.00706	0.00725	0.00706	0.00713	0.00713	0.00947	0.01075	0.00718

The above reported descriptive statistics across market capitalisation and industry all contain a positive mean estimate, a sign of an overall bullish market for the period under consideration and persistence of positive returns. Risk is a priced factor as the RESI index yields the highest return and highest risk observation.

When comparing standard deviation it appears that the Top40 index, containing those firms with the largest market capitalisation remains riskier than the MidCap and SmallCap, as was the case for daily and weekly sampling periods.

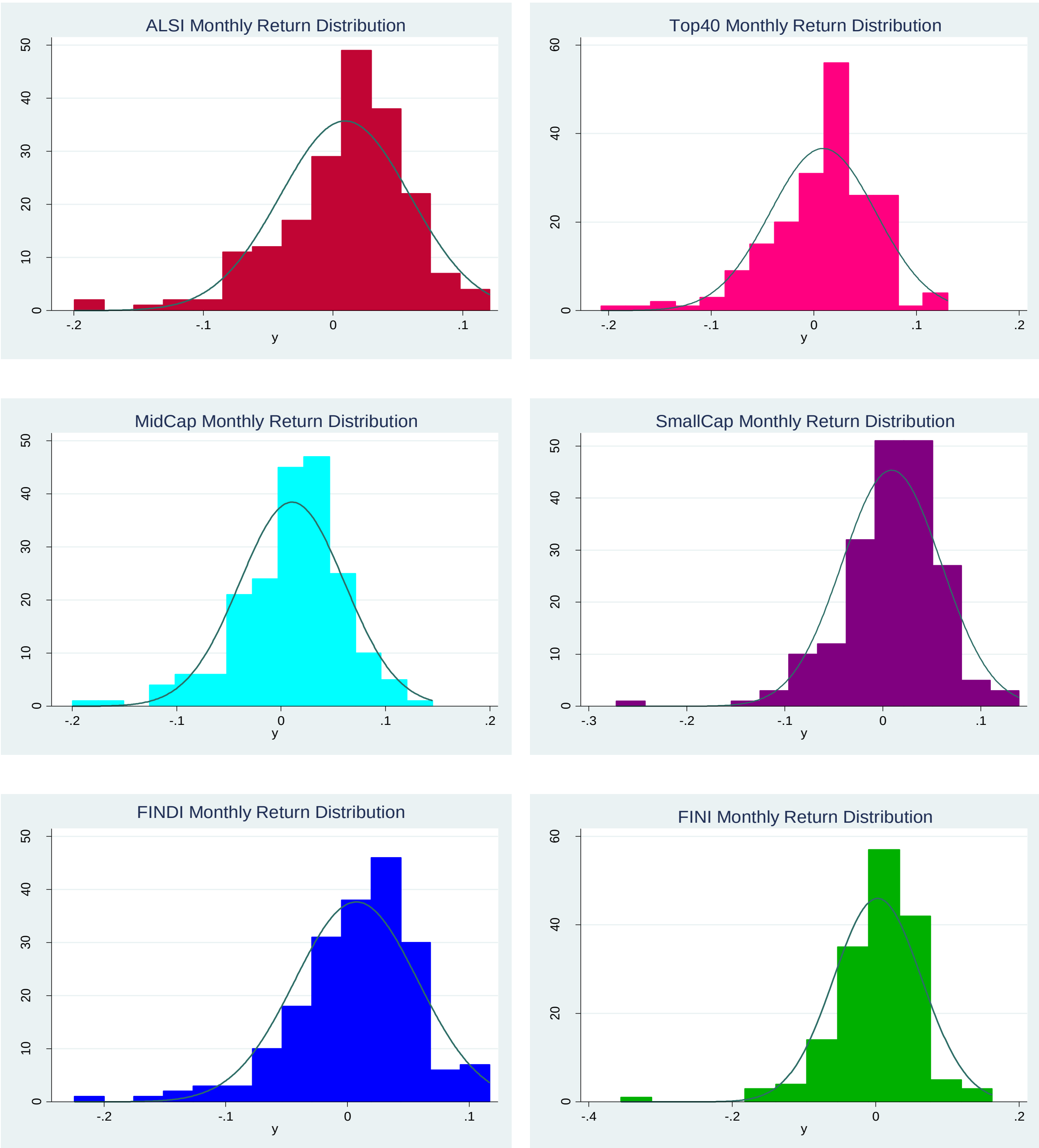
When one looks at daily sampling intervals, it appears that the INDI index had the largest range of returns. The weekly statistics however show that on a weekly basis, the FINI index contains the largest range, and the monthly sampling indicates that the RESI index has the largest range.

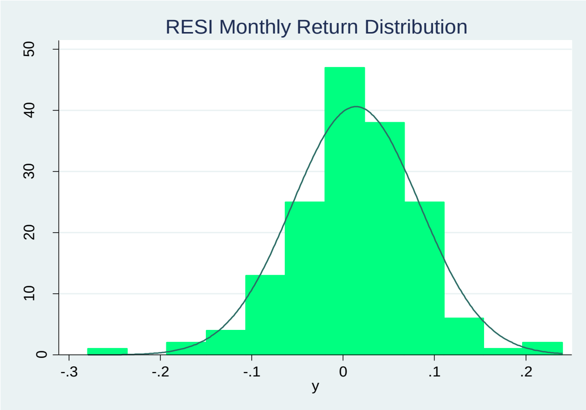
It is apparent from the excess kurtosis measure that each of the series display signs of fat tails and peakedness.

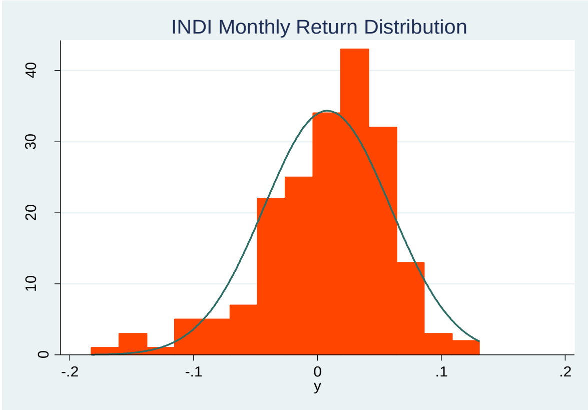
The FINI index displays the highest level of excess kurtosis of 6.69 although in daily and weekly sampling intervals the SmallCap index displayed the highest level of excess kurtosis at 9.49. The skewness statistic indicates that each of the indices are negatively or left skewed. The highest degree of skewness can be found in the return series of the RESI index. The results indicate that the market portfolio, various market capitalisations and industries, have return distributions, which are non-normal.

To confirm the above finding, distribution plots for the return series were generated, and these are presented in Figure 19 below:

Figure 19: Monthly Return Distribution per Index







The superimposed normal curve visually illustrates visually the extent of the leptokurtosis and fatter tails.

The Jarque-Bera test was conducted to test the joint null hypothesis of zero skewness and a kurtosis measure of three. The results of the Jarque-Bera test for each index are presented in Figure 20.

Figure 20: Jarque-Bera Test for Monthly Return Indices

ALSI

Skewness/Kurtosis tests for Normality

				joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
y	196	0.0000	0.0001	32.06	0.0000

Top40

Skewness/Kurtosis tests for Normality

				joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
y	196	0.0000	0.0001	32.06	0.0000

MidCap

Skewness/Kurtosis tests for Normality

				joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
y	196	0.0001	0.0015	20.24	0.0000

SmallCap

Skewness/Kurtosis tests for Normality

				joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
y	196	0.0000	0.0000	41.21	0.0000

FINDI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	196	0.0000	0.0001	29.07	0.0000

FINI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	164	0.0000	0.0000	49.43	0.0000

RESI

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	164	0.0254	0.0014	12.86	0.0016

INDI

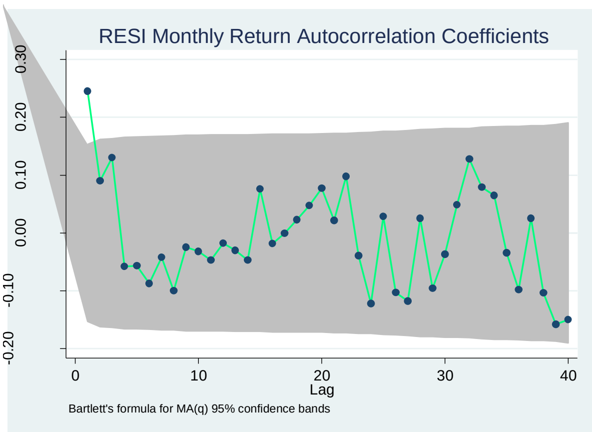
Skewness/Kurtosis tests for Normality

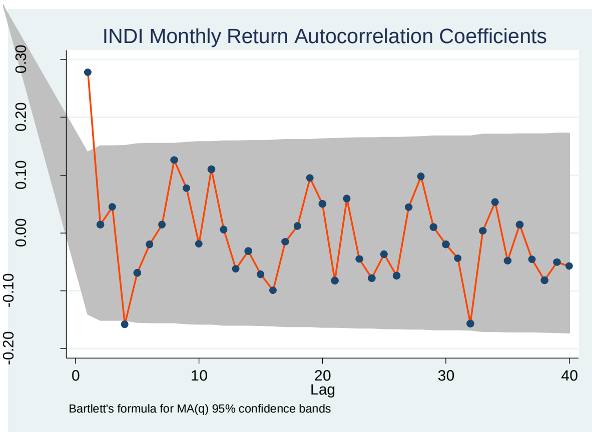
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
y	196	0.0000	0.0042	22.01	0.0000

For each Jarque-Bera test, the null hypothesis of normality can be rejected in favour of the alternative hypothesis of non-normality at the 95% confidence interval.

The autocorrelation plots presented in Figure 13 below graph the autocorrelation coefficients or relationship between a return observation with prior returns up to lag 40. The points which lie outside of the grey bar represent the statistically significant autocorrelation between return observations at the 95% confidence interval.

Figure 21: Autocorrelation Coefficients per Monthly Return Index





The plots of the daily and weekly autocorrelation coefficients display statistically significant negative autocorrelation coefficients; however, none of the autocorrelation coefficients measured on monthly returns are statistically significant. These observations could point to the "dying out" of asymmetry in the return dynamics over time, in other words asymmetry in return dynamics could be present in short horizon intervals only and disappear when one looks at longer horizons such as months or years.

To test for serial correlation in the residual terms, the Portmanteau Q statistic is calculated. The null hypothesis implied by the Portmanteau Q statistic is one of no autocorrelation in the residual terms, and the alternative hypothesis is the presence of autocorrelation in residuals.

Figure 22: Results of the Portmanteau Q statistic for Monthly Return Indices

ALSI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2890	0.2890	16.619	0.0000		
2	-0.0030	-0.0946	16.621	0.0002		
3	0.0767	0.1183	17.806	0.0005		
4	-0.1278	-0.2148	21.108	0.0003		
5	-0.1063	0.0200	23.402	0.0003		
6	-0.0618	-0.0810	24.183	0.0005		
7	-0.0455	0.0281	24.608	0.0009		
8	0.0330	0.0250	24.833	0.0017		
9	0.0606	0.0378	25.594	0.0024		
10	-0.0514	-0.1089	26.145	0.0035		
11	0.0272	0.0849	26.301	0.0059		
12	0.0167	-0.0527	26.359	0.0095		
13	-0.0186	0.0485	26.432	0.0149		
14	0.0041	-0.0408	26.436	0.0228		
15	-0.0044	0.0271	26.44	0.0336		
16	-0.0910	-0.1329	28.227	0.0297		
17	-0.0330	0.0611	28.464	0.0398		
18	-0.0057	-0.0463	28.471	0.0552		
19	0.0435	0.1235	28.886	0.0678		
20	0.0318	-0.0993	29.109	0.0856		
21	-0.0337	0.0218	29.361	0.1056		
22	0.0608	0.0272	30.185	0.1140		
23	-0.0923	-0.1398	32.095	0.0982		
24	-0.1234	-0.0260	35.529	0.0609		
25	0.0034	0.0425	35.531	0.0790		

Top40

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2864	0.2865	16.33	0.0001		
2	0.0278	-0.0593	16.484	0.0003		
3	0.0818	0.1010	17.83	0.0005		
4	-0.1097	-0.1830	20.261	0.0004		
5	-0.0962	-0.0011	22.14	0.0005		
6	-0.0551	-0.0515	22.76	0.0009		
7	-0.0345	0.0215	23.005	0.0017		
8	0.0090	0.0068	23.022	0.0033		
9	0.0462	0.0377	23.466	0.0052		
10	-0.0595	-0.1123	24.205	0.0071		
11	0.0177	0.0779	24.271	0.0116		
12	-0.0022	-0.0608	24.272	0.0187		
13	-0.0067	0.0591	24.282	0.0286		
14	0.0050	-0.0456	24.287	0.0423		
15	0.0071	0.0381	24.298	0.0602		
16	-0.0803	-0.1321	25.686	0.0586		
17	-0.0191	0.0753	25.766	0.0788		
18	0.0043	-0.0423	25.77	0.1051		
19	0.0395	0.1160	26.111	0.1271		
20	0.0257	-0.0933	26.257	0.1575		

MidCap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.3255	0.3258	21.091	0.0000		
2	-0.0878	-0.2173	22.633	0.0000		
3	-0.0463	0.0718	23.065	0.0000		
4	-0.1655	-0.2300	28.602	0.0000		
5	-0.1167	0.0391	31.372	0.0000		
6	-0.0342	-0.0774	31.611	0.0000		
7	-0.0228	0.0031	31.718	0.0000		
8	0.1077	0.1057	34.112	0.0000		
9	0.1200	0.0147	37.098	0.0000		
10	0.0019	-0.0248	37.099	0.0001		
11	0.0667	0.1095	38.031	0.0001		
12	0.0689	0.0172	39.033	0.0001		
13	-0.0105	0.0265	39.056	0.0002		
14	0.0558	0.0888	39.721	0.0003		
15	-0.0114	-0.0569	39.749	0.0005		
16	-0.1475	-0.0992	44.441	0.0002		
17	-0.1314	-0.0796	48.186	0.0001		
18	-0.0471	0.0171	48.669	0.0001		
19	0.0336	0.0113	48.917	0.0002		
20	0.0599	-0.0269	49.708	0.0002		

SmallCap

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.3810	0.3810	28.884	0.0000		
2	0.0143	-0.1533	28.924	0.0000		
3	0.0380	0.1089	29.214	0.0000		
4	-0.1591	-0.2629	34.333	0.0000		
5	-0.0768	0.1405	35.532	0.0000		
6	-0.0156	-0.1078	35.582	0.0000		
7	-0.0382	0.0577	35.881	0.0000		
8	0.1278	0.1108	39.251	0.0000		
9	0.1445	0.0454	43.583	0.0000		
10	0.0612	0.0126	44.365	0.0000		
11	0.1358	0.1201	48.235	0.0000		
12	0.1741	0.1337	54.631	0.0000		
13	0.0045	-0.1059	54.636	0.0000		
14	0.0213	0.1234	54.732	0.0000		
15	0.0164	-0.0530	54.79	0.0000		
16	-0.1145	-0.0554	57.616	0.0000		
17	-0.0785	-0.0647	58.953	0.0000		
18	-0.0490	-0.0061	59.476	0.0000		
19	0.0150	0.0466	59.526	0.0000		
20	0.1037	-0.0250	61.896	0.0000		

FINDI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2710	0.2710	14.611	0.0001		
2	-0.0535	-0.1371	15.183	0.0005		
3	0.0208	0.0833	15.27	0.0016		
4	-0.1522	-0.2149	19.953	0.0005		
5	-0.0816	0.0499	21.307	0.0007		
6	-0.0331	-0.0808	21.531	0.0015		
7	0.0109	0.0734	21.555	0.0030		
8	0.1289	0.0816	24.987	0.0016		
9	0.0644	0.0009	25.848	0.0022		
10	-0.0315	-0.0456	26.054	0.0037		
11	0.1075	0.1536	28.48	0.0027		
12	0.0671	0.0000	29.429	0.0034		
13	-0.0257	0.0138	29.569	0.0054		
14	0.0034	-0.0113	29.571	0.0087		
15	-0.0447	-0.0241	30	0.0119		
16	-0.1118	-0.1071	32.694	0.0081		
17	-0.0450	0.0242	33.134	0.0108		
18	-0.0161	-0.0278	33.19	0.0158		
19	0.0686	0.0734	34.223	0.0173		
20	0.0300	-0.0972	34.422	0.0234		
21	-0.0565	-0.0075	35.131	0.0273		
22	0.0484	0.0395	35.653	0.0331		
23	-0.0530	-0.0955	36.284	0.0386		
24	-0.0792	0.0150	37.7	0.0372		
25	-0.0392	-0.0674	38.048	0.0458		
26	-0.0979	-0.0708	40.235	0.0370		
27	0.0161	0.0788	40.294	0.0481		
28	0.0964	0.0646	42.44	0.0394		
29	0.0349	0.0209	42.724	0.0483		
30	-0.0283	-0.1066	42.911	0.0597		

FINI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.1742	0.1742	5.0667	0.0244		
2	-0.1473	-0.1834	8.7147	0.0128		
3	0.1013	0.1786	10.45	0.0151		
4	-0.0273	-0.1272	10.577	0.0318		
5	-0.0351	0.0606	10.788	0.0558		
6	0.0567	0.0111	11.342	0.0784		
7	0.0477	0.0721	11.735	0.1096		
8	0.0443	0.0142	12.078	0.1478		
9	0.0155	0.0166	12.12	0.2066		
10	-0.0900	-0.0903	13.551	0.1945		
11	0.0639	0.1179	14.279	0.2180		
12	0.1120	0.0319	16.527	0.1683		
13	0.0100	0.0023	16.545	0.2210		
14	0.0674	0.0774	17.369	0.2371		
15	-0.0075	-0.0559	17.379	0.2967		
16	-0.1473	-0.1208	21.371	0.1647		
17	-0.0551	-0.0210	21.934	0.1873		
18	-0.0139	-0.0345	21.97	0.2333		
19	0.0388	0.0423	22.252	0.2719		
20	-0.0300	-0.1180	22.422	0.3180		
21	-0.0849	-0.0065	23.795	0.3031		
22	0.0002	-0.0060	23.795	0.3581		
23	-0.0298	-0.0412	23.966	0.4057		

RESI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2450	0.2450	10.023	0.0015		
2	0.0901	0.0324	11.387	0.0034		
3	0.1301	0.1093	14.252	0.0026		
4	-0.0580	-0.1284	14.825	0.0051		
5	-0.0566	-0.0264	15.374	0.0089		
6	-0.0869	-0.0772	16.675	0.0106		
7	-0.0421	0.0252	16.983	0.0175		
8	-0.1000	-0.0939	18.729	0.0164		
9	-0.0244	0.0476	18.833	0.0266		
10	-0.0316	-0.0559	19.01	0.0401		
11	-0.0468	-0.0109	19.4	0.0543		
12	-0.0179	-0.0274	19.458	0.0781		
13	-0.0297	-0.0034	19.616	0.1052		
14	-0.0473	-0.0473	20.022	0.1295		
15	0.0759	0.1290	21.075	0.1344		
16	-0.0183	-0.0986	21.137	0.1733		
17	-0.0009	0.0419	21.137	0.2202		
18	0.0231	-0.0315	21.236	0.2677		
19	0.0475	0.0907	21.66	0.3015		
20	0.0775	0.0201	22.795	0.2990		
21	0.0219	0.0183	22.886	0.3501		
22	0.0982	0.0689	24.733	0.3100		
23	-0.0394	-0.0935	25.033	0.3486		

INDI

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.2775	0.2775	15.328	0.0001		
2	0.0144	-0.0679	15.37	0.0005		
3	0.0447	0.0657	15.772	0.0013		
4	-0.1581	-0.2106	20.825	0.0003		
5	-0.0692	0.0518	21.797	0.0006		
6	-0.0199	-0.0413	21.878	0.0013		
7	0.0142	0.0649	21.92	0.0026		
8	0.1262	0.0861	25.206	0.0014		
9	0.0773	0.0131	26.447	0.0017		
10	-0.0193	-0.0531	26.525	0.0031		
11	0.1098	0.1484	29.052	0.0022		
12	0.0058	-0.0605	29.059	0.0039		
13	-0.0621	-0.0040	29.878	0.0049		
14	-0.0313	-0.0554	30.087	0.0074		
15	-0.0714	-0.0158	31.179	0.0083		
16	-0.0991	-0.1055	33.298	0.0067		
17	-0.0155	0.0432	33.35	0.0102		
18	0.0119	-0.0109	33.38	0.0150		
19	0.0947	0.0941	35.345	0.0127		
20	0.0505	-0.0686	35.907	0.0158		
21	-0.0823	-0.0496	37.41	0.0151		
22	0.0595	0.0978	38.201	0.0174		
23	-0.0453	-0.0772	38.661	0.0216		
24	-0.0780	0.0155	40.033	0.0212		
25	-0.0369	-0.0715	40.342	0.0269		
26	-0.0737	-0.0273	41.581	0.0271		
27	0.0444	0.0853	42.034	0.0327		
28	0.0976	0.0504	44.234	0.0263		
29	0.0103	-0.0286	44.259	0.0347		
30	-0.0195	-0.0806	44.348	0.0443		
31	-0.0443	-0.0474	44.809	0.0518		

The Portmanteau Q statistic results for the daily and weekly interval sampled indices carried probability values of less than 0.05, resulting in the statistical significance of autocorrelation in the residuals past lag 20. The Portmanteau Q statistics for monthly return observations show that the statistical significance of autocorrelation in residuals is not as prolonged as that of the daily and weekly results.

The ALSI index contains statistically significant autocorrelation in the residuals up to lag 17, the Top40 to lag 15, the MidCap, SmallCap, FINDI and INDI index, to lag 30, the FINI index to lag 4, whilst the RESI index reflects autocorrelation to lag 10.

The reduced level of autocorrelation in residuals could speak to the lack of asymmetry in return dynamics in longer time horizons.

The last test conducted for independence is the BDS test. The null hypothesis of the BDS test states that the residual terms of the series are identically and independently distributed. The alternative hypothesis implies that serial dependence between residuals is present. By using geometric returns, the linear dependence between residual terms has been eliminated; therefore, the BDS test run on the residuals terms is testing for non-linear dependence.

Figure 23: BDS Test of the Monthly Return Indices Residuals

ALSI

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.019393	0.005832	3.325311	0.0009
3	0.041708	0.009301	4.484009	0.0000
4	0.051559	0.011115	4.638501	0.0000
5	0.059107	0.011626	5.083936	0.0000
6	0.058345	0.011251	5.185596	0.0000

Raw epsilon	0.068055		
Pairs within epsilon	27110.00	V-Statistic	0.705696
Triples within epsilon	4056350.	V-Statistic	0.538725

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9706.000	0.513138	13291.00	0.702670	0.493745
3	7232.000	0.386304	13125.00	0.701084	0.344596
4	5539.000	0.298953	13067.00	0.705257	0.247394
5	4249.000	0.231730	12904.00	0.703752	0.172623
6	3250.000	0.179113	12757.00	0.703059	0.120767

Top40

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.021920	0.005771	3.798616	0.0001
3	0.043895	0.009204	4.769302	0.0000
4	0.054949	0.010998	4.996108	0.0000
5	0.064052	0.011503	5.568145	0.0000
6	0.063796	0.011132	5.730831	0.0000

Raw epsilon	0.070714		
Pairs within epsilon	27112.00	V-Statistic	0.705748
Triples within epsilon	4053684.	V-Statistic	0.538371

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9751.000	0.515517	13289.00	0.702564	0.493596
3	7270.000	0.388334	13123.00	0.700978	0.344439
4	5599.000	0.302191	13065.00	0.705149	0.247243
5	4336.000	0.236475	12901.00	0.703589	0.172422
6	3352.000	0.184734	12760.00	0.703224	0.120938

MidCap

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.022814	0.005877	3.881973	0.0001
3	0.049086	0.009347	5.251410	0.0000
4	0.060212	0.011139	5.405327	0.0000
5	0.064643	0.011619	5.563384	0.0000
6	0.068491	0.011214	6.107635	0.0000

Raw epsilon	0.069498		
Pairs within epsilon	27032.00	V-Statistic	0.703665
Triples within epsilon	4037170.	V-Statistic	0.536178

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9730.000	0.514407	13262.00	0.701137	0.491593
3	7326.000	0.391325	13095.00	0.699482	0.342239
4	5595.000	0.301975	12992.00	0.701209	0.241763
5	4256.000	0.232112	12826.00	0.699498	0.167469
6	3337.000	0.183907	12661.00	0.697768	0.115416

SmallCap

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.035596	0.005764	6.175360	0.0000
3	0.061093	0.009177	6.656955	0.0000
4	0.071708	0.010948	6.550111	0.0000
5	0.079467	0.011430	6.952214	0.0000
6	0.085744	0.011042	7.765092	0.0000

Raw epsilon	0.067844		
Pairs within epsilon	27064.00	V-Statistic	0.704498
Triples within epsilon	4040080.	V-Statistic	0.536564

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9983.000	0.527782	13270.00	0.701560	0.492186
3	7564.000	0.404038	13104.00	0.699963	0.342945
4	5808.000	0.313472	12992.00	0.701209	0.241763
5	4529.000	0.247000	12827.00	0.699553	0.167534
6	3658.000	0.201598	12669.00	0.698209	0.115854

FINDI

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.024305	0.005856	4.150775	0.0000
3	0.044816	0.009321	4.808010	0.0000
4	0.052301	0.011118	4.704324	0.0000
5	0.058149	0.011606	5.010079	0.0000
6	0.063310	0.011211	5.647252	0.0000

Raw epsilon	0.068808		
Pairs within epsilon	27056.00	V-Statistic	0.704290
Triples within epsilon	4042672.	V-Statistic	0.536909

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9754.000	0.515675	13259.00	0.700978	0.491370
3	7249.000	0.387212	13097.00	0.699589	0.342396
4	5483.000	0.295930	13017.00	0.702558	0.243629
5	4173.000	0.227585	12856.00	0.701134	0.169436
6	3283.000	0.180931	12701.00	0.699972	0.117621

FINI

BDS Test
Sample: 1998M03 2011M10
Included observations: 164

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.038161	0.006427	5.937694	0.0000
3	0.060305	0.010254	5.881214	0.0000
4	0.069042	0.012258	5.632451	0.0000
5	0.077069	0.012826	6.008898	0.0000
6	0.081275	0.012417	6.545449	0.0000

Raw epsilon	0.079041		
Pairs within epsilon	18976.00	V-Statistic	0.705532
Triples within epsilon	2376626.	V-Statistic	0.538802

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	7004.000	0.530485	9264.000	0.701659	0.492325
3	5257.000	0.403113	9127.000	0.699870	0.342808
4	4001.000	0.310637	9030.000	0.701087	0.241595
5	3105.000	0.244104	8893.000	0.699135	0.167034
6	2468.000	0.196481	8762.000	0.697556	0.115206

RESI

BDS Test
Sample: 1998M03 2011M10
Included observations: 164

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.017936	0.006123	2.929511	0.0034
3	0.033854	0.009765	3.466988	0.0005
4	0.033693	0.011668	2.887587	0.0039
5	0.036924	0.012203	3.025820	0.0025
6	0.035215	0.011808	2.982259	0.0029

Raw epsilon	0.096173		
Pairs within epsilon	18976.00	V-Statistic	0.705532
Triples within epsilon	2368062.	V-Statistic	0.536861

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	6744.000	0.510793	9269.000	0.702037	0.492857
3	4915.000	0.376888	9129.000	0.700023	0.343034
4	3604.000	0.279814	9072.000	0.704348	0.246121
5	2645.000	0.207940	8935.000	0.702437	0.171016
6	1953.000	0.155481	8825.000	0.702571	0.120266

INDI

BDS Test
Sample: 1995M07 2011M10
Included observations: 196

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.015981	0.005936	2.692184	0.0071
3	0.034398	0.009461	3.635768	0.0003
4	0.043782	0.011298	3.875182	0.0001
5	0.054139	0.011809	4.584497	0.0000
6	0.060490	0.011421	5.296564	0.0000

Raw epsilon	0.069604		
Pairs within epsilon	27086.00	V-Statistic	0.705071
Triples within epsilon	4055198.	V-Statistic	0.538572

Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	9619.000	0.508538	13275.00	0.701824	0.492557
3	7076.000	0.377971	13112.00	0.700390	0.343574
4	5346.000	0.288536	13032.00	0.703368	0.244754
5	4114.000	0.224367	12868.00	0.701789	0.170228
6	3247.000	0.178947	12716.00	0.700799	0.118457

In each BDS test the probability of the z-statistic is less than 0.05, therefore one can reject the null hypothesis in favour of the alternative hypothesis and non-linear dependence is present in the residuals at the 95% confidence interval.

The Portmanteau Q statistic and the BDS test indicate that serial dependence exists between the residual terms. The non-linearity exposed by these tests, in daily, weekly and monthly returns of these indices speaks to the aim of this research.

Stationarity is a requirement of stochastic volatility modelling. The tests conducted for stationarity are the segmentation of the series to examine the properties through time and the Augmented Dickey Fuller test.

The descriptive statistics of each index, split into three sections to examine the mean and variance is presented below.

Table 6: Descriptive Statistics of Sections of Monthly Index Returns

Summary Statistics	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	INDI
Section 1								
Mean	0.007714	0.007855	0.004198	0.0001438	0.006261	-0.007436	0.026043	0.005079
Standard Error	0.007274	0.007265	0.00806	0.0082817	0.007944	0.011709	0.010776	0.007638
Median	0.011851	0.012706	0.012104	0.0046962	0.005485	-0.001026	0.02186	0.00319
Standard Deviation	0.058646	0.058576	0.064979	0.0667692	0.064043	0.08604	0.079918	0.061576
Sample Variance	0.003439	0.003431	0.004222	0.0044581	0.004102	0.007403	0.006387	0.003792
Section 2								
Mean	0.014636	0.013585	0.020661	0.0222726	0.009282	0.018904	0.017254	0.009498
Standard Error	0.005348	0.00566	0.004499	0.0041945	0.005357	0.005077	0.006934	0.005863
Median	0.018794	0.017779	0.021222	0.0271692	0.019535	0.021218	0.012856	0.020485
Standard Deviation	0.043115	0.045636	0.036274	0.0338172	0.043189	0.037307	0.051424	0.047269
Sample Variance	0.001859	0.002083	0.001316	0.0011436	0.001865	0.001392	0.002644	0.002234
Section 3								
Mean	0.005727	0.005569	0.008739	0.004587	0.006641	-0.002866	-0.001822	0.010139
Standard Error	0.005869	0.006146	0.005347	0.0053678	0.005233	0.006427	0.010298	0.005314
Median	0.016196	0.018245	0.014147	0.0096475	0.01285	-0.002533	0.0169	0.017892
Standard Deviation	0.047682	0.049933	0.042779	0.0436086	0.04251	0.048092	0.074259	0.042842
Sample Variance	0.002274	0.002493	0.00183	0.0019017	0.001807	0.002313	0.005514	0.001835

When one compares the mean of each index across the three sections, the variation is not large – although, section 2 does carry a slightly high mean value across all indices. The standard deviation and sample variances too remain consistent. Given that, no large inconsistencies are present per index it appears as though the series are stationary.

The Dickey-Fuller with GLS Trending and the ADF test for stationarity are presented in Figure 24 below.

Figure 24: ADF Test on Monthly Return per Index

ALSI

DF-GLS for y Number of obs = 181
Maxlag = 14 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
14	-3.383	-3.465	-2.797	-2.520
13	-3.614	-3.465	-2.810	-2.532
12	-3.615	-3.465	-2.823	-2.544
11	-3.965	-3.465	-2.836	-2.556
10	-3.942	-3.465	-2.849	-2.568
9	-4.565	-3.465	-2.861	-2.579
8	-4.317	-3.465	-2.872	-2.589
7	-4.796	-3.465	-2.883	-2.599
6	-5.305	-3.465	-2.894	-2.609
5	-6.025	-3.465	-2.904	-2.618
4	-6.238	-3.465	-2.914	-2.627
3	-7.227	-3.465	-2.923	-2.635
2	-6.501	-3.465	-2.931	-2.643
1	-8.774	-3.465	-2.939	-2.650

Opt Lag (Ng-Perron seq t) = 3 with RMSE .0475133
Min SC = -5.978605 at lag 3 with RMSE .0475133
Min MAIC = -4.966564 at lag 2 with RMSE .0486177

Augmented Dickey-Fuller test for unit root Number of obs = 192

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-7.420	-3.479	-2.884	-2.574

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8325632	.1122041	-7.42	0.000	-1.053912	-.6112147
LD.	.1861461	.1033441	1.80	0.073	-.017724	.3900161
L2D.	.0234243	.0859771	0.27	0.786	-.1461854	.193034
L3D.	.2148451	.0721426	2.98	0.003	.0725271	.3571631
_cons	.0075952	.0035754	2.12	0.035	.0005419	.0146486

Top40

DF-GLS for y   Number of obs = 181
Maxlag = 14 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
14	-3.386	-3.465	-2.797	-2.520
13	-3.659	-3.465	-2.810	-2.532
12	-3.645	-3.465	-2.823	-2.544
11	-4.042	-3.465	-2.836	-2.556
10	-3.991	-3.465	-2.849	-2.568
9	-4.589	-3.465	-2.861	-2.579
8	-4.321	-3.465	-2.872	-2.589
7	-4.794	-3.465	-2.883	-2.599
6	-5.176	-3.465	-2.894	-2.609
5	-5.815	-3.465	-2.904	-2.618
4	-6.195	-3.465	-2.914	-2.627
3	-6.956	-3.465	-2.923	-2.635
2	-6.457	-3.465	-2.931	-2.643
1	-8.471	-3.465	-2.939	-2.650

Opt Lag (Ng-Perron seq t) = 3 with RMSE .0493769
Min SC = -5.916558 at lag 1 with RMSE .0504385
Min MAIC = -4.935876 at lag 2 with RMSE .0502013

Augmented Dickey-Fuller test for unit root   Number of obs = 192

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-7.144	-3.479	-2.884	-2.574

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8028359	.1123746	-7.14	0.000	-1.024521	-.5811511
LD.	.131457	.1037551	1.27	0.207	-.0732238	.3361379
L2D.	.0229067	.0873937	0.26	0.794	-.1494975	.1953109
L3D.	.182976	.0726231	2.52	0.013	.0397103	.3262418
_cons	.0070392	.0036947	1.91	0.058	-.0002493	.0143278

MidCap

```
DF-GLS for y                                            Number of obs =   181
Maxlag = 14 chosen by Schwert criterion
```

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
14	-2.890	-3.465	-2.797	-2.520
13	-2.804	-3.465	-2.810	-2.532
12	-3.152	-3.465	-2.823	-2.544
11	-3.349	-3.465	-2.836	-2.556
10	-3.544	-3.465	-2.849	-2.568
9	-4.182	-3.465	-2.861	-2.579
8	-4.313	-3.465	-2.872	-2.589
7	-4.667	-3.465	-2.883	-2.599
6	-5.687	-3.465	-2.894	-2.609
5	-6.341	-3.465	-2.904	-2.618
4	-6.656	-3.465	-2.914	-2.627
3	-8.052	-3.465	-2.923	-2.635
2	-7.287	-3.465	-2.931	-2.643
1	-9.706	-3.465	-2.939	-2.650

```
Opt Lag (Ng-Perron seq t) =  3 with RMSE   .045847
Min SC   = -6.050005 at lag  3 with RMSE   .045847
Min MAIC = -4.879915 at lag 13 with RMSE  .0449469
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       192
```

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-8.251	-3.479	-2.884	-2.574

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.9359584	.1134335	-8.25	0.000	-1.159732	-.7121847
LD.	.3656618	.1010077	3.62	0.000	.1664008	.5649229
L2D.	.0616462	.0826068	0.75	0.456	-.1013149	.2246073
L3D.	.2300038	.0716533	3.21	0.002	.0886511	.3713565
_cons	.0097996	.0035262	2.78	0.006	.0028434	.0167558

SmallCap

DF-GLS for y   Number of obs = 181
Maxlag = 14 chosen by Schwert criterion

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
14	-2.497	-3.465	-2.797	-2.520
13	-2.417	-3.465	-2.810	-2.532
12	-2.795	-3.465	-2.823	-2.544
11	-2.571	-3.465	-2.836	-2.556
10	-3.031	-3.465	-2.849	-2.568
9	-3.574	-3.465	-2.861	-2.579
8	-3.777	-3.465	-2.872	-2.589
7	-4.169	-3.465	-2.883	-2.599
6	-5.023	-3.465	-2.894	-2.609
5	-5.824	-3.465	-2.904	-2.618
4	-5.778	-3.465	-2.914	-2.627
3	-7.655	-3.465	-2.923	-2.635
2	-6.520	-3.465	-2.931	-2.643
1	-8.741	-3.465	-2.939	-2.650

Opt Lag (Ng-Perron seq t) = 11 with RMSE .0435892
Min SC = -6.072608 at lag 3 with RMSE .0453318
Min MAIC = -5.56696 at lag 13 with RMSE .0430066

Augmented Dickey-Fuller test for unit root   Number of obs = 184

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-2.606	-3.482	-2.884	-2.574

MacKinnon approximate p-value for Z(t) = 0.0917

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.4594453	.1762884	-2.61	0.010	-.807427	-.1114635
LD.	-.0307171	.1753202	-0.18	0.861	-.3767875	.3153534
L2D.	-.34812	.1682082	-2.07	0.040	-.6801518	-.0160882
L3D.	-.0721736	.1630738	-0.44	0.659	-.3940706	.2497234
L4D.	-.4198223	.15428	-2.72	0.007	-.7243609	-.1152837
L5D.	-.2272595	.1457156	-1.56	0.121	-.5148924	.0603735
L6D.	-.3361226	.1334541	-2.52	0.013	-.5995521	-.0726931
L7D.	-.3157206	.1253828	-2.52	0.013	-.563218	-.0682233
L8D.	-.2128102	.1087359	-1.96	0.052	-.4274478	.0018273
L9D.	-.1746569	.1022518	-1.71	0.089	-.3764952	.0271813
L10D.	-.1828674	.083008	-2.20	0.029	-.3467198	-.019015
L11D.	-.1337253	.0756134	-1.77	0.079	-.2829811	.0155305
_cons	.0034062	.003677	0.93	0.356	-.0038519	.0106643

FINDI

```
DF-GLS for y                                         Number of obs =   181
Maxlag = 14 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------
    14           -2.999           -3.465            -2.797            -2.520
    13           -3.017           -3.465            -2.810            -2.532
    12           -3.077           -3.465            -2.823            -2.544
    11           -3.220           -3.465            -2.836            -2.556
    10           -3.335           -3.465            -2.849            -2.568
    9            -4.106           -3.465            -2.861            -2.579
    8            -4.114           -3.465            -2.872            -2.589
    7            -4.372           -3.465            -2.883            -2.599
    6            -5.101           -3.465            -2.894            -2.609
    5            -6.071           -3.465            -2.904            -2.618
    4            -6.303           -3.465            -2.914            -2.627
    3            -7.600           -3.465            -2.923            -2.635
    2            -7.012           -3.465            -2.931            -2.643
    1            -9.317           -3.465            -2.939            -2.650

Opt Lag (Ng-Perron seq t) = 10 with RMSE  .0468753
Min SC   = -5.968673 at lag  1 with RMSE  .0491412
Min MAIC = -4.832437 at lag 12 with RMSE  .0468714


Augmented Dickey-Fuller test for unit root         Number of obs   =       185

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.403            -3.482            -2.884            -2.574
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0108

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   -.712537   .2093567    -3.40   0.001    -1.125759   -.2993147
         LD. |   .0670193   .1997384     0.34   0.738    -.3272186    .4612572
        L2D. |  -.1573446   .1910439    -0.82   0.411    -.5344216    .2197325
        L3D. |   .0181623   .1808812     0.10   0.920    -.3388559    .3751805
        L4D. |  -.2275721     .16814    -1.35   0.178    -.5594421    .1042978
        L5D. |  -.1308452   .1528592    -0.86   0.393    -.4325544    .1708639
        L6D. |  -.2380119   .1391868    -1.71   0.089    -.5127349     .036711
        L7D. |  -.1463314   .1211654    -1.21   0.229    -.3854843    .0928215
        L8D. |  -.1066296   .1103118    -0.97   0.335    -.3243598    .1111006
        L9D. |   -.051579   .0890297    -0.58   0.563    -.2273032    .1241453
       L10D. |  -.1535883   .0753825    -2.04   0.043     -.302376   -.0048006
             |
       _cons |   .0047567   .0038499     1.24   0.218    -.0028422    .0123556
------------------------------------------------------------------------------
```

FINI

```
DF-GLS for y                                         Number of obs =   150
Maxlag = 13 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------

    13          -2.388            -3.503            -2.795            -2.519
    12          -2.662            -3.503            -2.812            -2.535
    11          -2.725            -3.503            -2.828            -2.550
    10          -2.912            -3.503            -2.844            -2.564
    9           -3.438            -3.503            -2.859            -2.579
    8           -3.228            -3.503            -2.874            -2.592
    7           -3.240            -3.503            -2.888            -2.605
    6           -3.768            -3.503            -2.902            -2.618
    5           -4.153            -3.503            -2.915            -2.630
    4           -4.706            -3.503            -2.927            -2.641
    3           -6.318            -3.503            -2.939            -2.651
    2           -6.923            -3.503            -2.949            -2.661
    1           -9.040            -3.503            -2.959            -2.670

Opt Lag (Ng-Perron seq t) =  4 with RMSE  .0444604
Min SC   = -6.127227 at lag  1 with RMSE  .0451838
Min MAIC =  -5.45408 at lag 13 with RMSE   .043158
```

Augmented Dickey-Fuller test for unit root   Number of obs = 159

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-5.509	-3.490	-2.886	-2.576

MacKinnon approximate p-value for Z(t) = 0.0000

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.8601999	.1561499	-5.51	0.000	-1.168688	-.5517118
LD.	.1231074	.1385949	0.89	0.376	-.1506993	.3969141
L2D.	-.1458972	.1252113	-1.17	0.246	-.3932635	.1014691
L3D.	.0865485	.0976479	0.89	0.377	-.1063637	.2794608
L4D.	-.0606485	.0793537	-0.76	0.446	-.2174189	.0961219
_cons	.0034621	.0046568	0.74	0.458	-.0057378	.0126621

RESI

```
DF-GLS for y                                      Number of obs =   150
Maxlag = 13 chosen by Schwert criterion
```

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
13	-4.017	-3.503	-2.795	-2.519
12	-3.987	-3.503	-2.812	-2.535
11	-4.155	-3.503	-2.828	-2.550
10	-4.139	-3.503	-2.844	-2.564
9	-4.187	-3.503	-2.859	-2.579
8	-4.298	-3.503	-2.874	-2.592
7	-4.866	-3.503	-2.888	-2.605
6	-4.911	-3.503	-2.902	-2.618
5	-5.731	-3.503	-2.915	-2.630
4	-5.493	-3.503	-2.927	-2.641
3	-5.993	-3.503	-2.939	-2.651
2	-5.918	-3.503	-2.949	-2.661
1	-7.513	-3.503	-2.959	-2.670

```
Opt Lag (Ng-Perron seq t) =  5 with RMSE  .0606871
Min SC   = -5.494279 at lag  1 with RMSE  .0620048
Min MAIC = -4.569406 at lag  2 with RMSE  .0617178
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       158
```

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-5.392	-3.491	-2.886	-2.576

```
MacKinnon approximate p-value for Z(t) = 0.0000
```

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.7654547	.1419576	-5.39	0.000	-1.045934	-.4849749
LD.	.0748057	.1318742	0.57	0.571	-.1857511	.3353626
L2D.	.0598088	.1199981	0.50	0.619	-.1772833	.2969009
L3D.	.1654285	.1104015	1.50	0.136	-.0527027	.3835597
L4D.	.0813802	.0948023	0.86	0.392	-.1059302	.2686905
L5D.	.0771665	.0758625	1.02	0.311	-.0727226	.2270556
_cons	.0118589	.0054754	2.17	0.032	.0010406	.0226773

INDI

```
DF-GLS for y                                             Number of obs =   181
Maxlag = 14 chosen by Schwert criterion

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------

    14           -3.189           -3.465            -2.797            -2.520
    13           -3.256           -3.465            -2.810            -2.532
    12           -3.193           -3.465            -2.823            -2.544
    11           -3.291           -3.465            -2.836            -2.556
    10           -3.208           -3.465            -2.849            -2.568
    9            -3.916           -3.465            -2.861            -2.579
    8            -3.885           -3.465            -2.872            -2.589
    7            -4.169           -3.465            -2.883            -2.599
    6            -4.842           -3.465            -2.894            -2.609
    5            -5.671           -3.465            -2.904            -2.618
    4            -6.107           -3.465            -2.914            -2.627
    3            -7.311           -3.465            -2.923            -2.635
    2            -6.747           -3.465            -2.931            -2.643
    1            -8.571           -3.465            -2.939            -2.650

Opt Lag (Ng-Perron seq t) = 10 with RMSE   .047871
Min SC   = -5.937002 at lag  1 with RMSE  .0499256
Min MAIC = -5.043909 at lag 10 with RMSE   .047871
```

```
Augmented Dickey-Fuller test for unit root         Number of obs   =       185

                             ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -3.307            -3.482            -2.884            -2.574
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0146
```

D.y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
y						
L1.	-.6479967	.1959643	-3.31	0.001	-1.034785	-.261208
LD.	-.0185954	.1882101	-0.10	0.921	-.390079	.3528882
L2D.	-.1430139	.1806334	-0.79	0.430	-.4995429	.2135151
L3D.	-.0053513	.1719382	-0.03	0.975	-.3447179	.3340154
L4D.	-.2274004	.1604308	-1.42	0.158	-.5440541	.0892534
L5D.	-.1554953	.1471614	-1.06	0.292	-.4459583	.1349678
L6D.	-.226722	.1344214	-1.69	0.093	-.4920391	.0385951
L7D.	-.1465396	.1181377	-1.24	0.217	-.3797163	.0866372
L8D.	-.0969841	.1082756	-0.90	0.372	-.3106953	.1167271
L9D.	-.0444314	.0902711	-0.49	0.623	-.2226059	.133743
L10D.	-.1483936	.0755628	-1.96	0.051	-.2975372	.0007501
_cons	.0048617	.0039202	1.24	0.217	-.0028758	.0125993

The MacKinnon approximate p-value in each test allows one to reject the null hypothesis in favour of the alternative hypothesis and concluding that each of the eight indices presented, the series are stationary.

Various testing has been conducted on eight indices, over three different sampling intervals to investigate the univariate properties of the time series. Regardless of the frequency of the sampling, daily, weekly or monthly, all eight indices display characteristics typical of financial time series, such as, non-normality, serial correlation in the residual terms and stationarity. These characteristics allow for the use of stochastic volatility models, utilized in the further testing, with the aim of detecting asymmetric properties of the conditional mean and conditional variance.

In addition, weak evidence of persistence in positive returns and asymmetric dynamics in the return series was revealed. This evidence was presented in the form of: the upward trend in the Top40 index, despite a consistent number of shares over the time period; the positive mean value of each index, even when sectioned, significant positive and negative autocorrelation displayed in the coefficient plots, which indicates a relationship between return observations, and lastly, the results of the BDS test – specifically designed to detect non-linearity – displayed the presence of an asymmetric relationship between residuals across each time interval.

The descriptive statistics have shed light on possible indications of asymmetry in the conditional mean and conditional variance. Before presenting the results of the ANAR- GJR GARCH model, some preliminary investigations on this asymmetry were conducted.

5.2. Asymmetry in the Conditional Mean and Conditional Volatility

Recall, asymmetry in the conditional mean refers to the tendency of nominal returns to display nonlinear reverting patterns whereby negative returns revert more quickly and with a greater magnitude to positive returns than positive returns revert to negative returns; this asymmetry would lead to a persistence of positive returns through time (Nam, 2003).

Conditional variance asymmetry refers to the occurrence of a negative correlation between returns and volatility and is considered a stylized fact of financial time series (Cox & Ross, 1976).

In order to investigate the preliminary evidence of asymmetry in return dynamics, the consecutive runs of positive and negative returns are examined per index, following the methodology of Nam, Washer & Chu (2005) and Evans & McMillan (2009).

To demonstrate preliminary evidence of conditional variance asymmetry, one simply needs to investigate whether periods of high volatility are accompanied by a stock price decline (Engle & Ng, 1993).

5.2.1. Daily Return Series

The asymmetric pattern of return dynamics can be observed in the consecutive runs of positive and negative returns of each index.

Table 7 below illustrates the consecutive daily positive and negative returns for each index over the period under consideration.

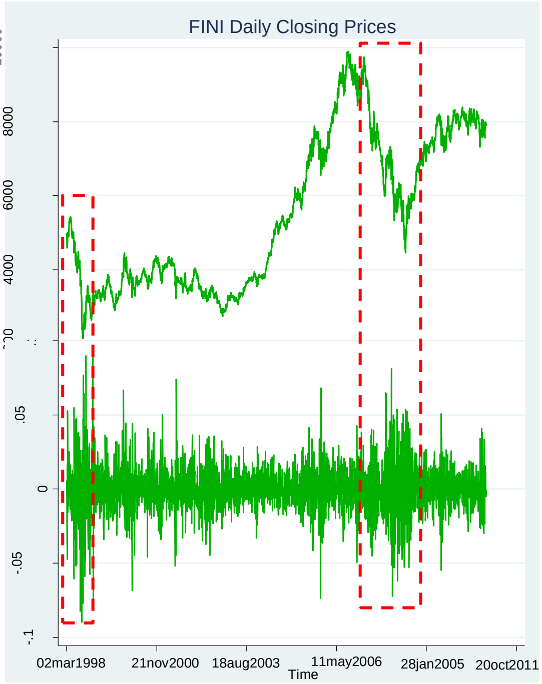
Table 7: Consecutive Daily Returns

Runs of Consecutive Returns	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	*INDI*
Negative Sequences								
1 day	483	478	417	388	468	370	380	468
2 days	211	217	206	184	238	219	199	237
3 days	133	129	102	97	117	109	107	116
4 days	66	71	60	48	65	52	59	64
5 days	27	30	36	26	36	32	23	26
6 days	12	11	18	15	9	11	15	17
7 days	12	12	7	13	12	3	7	14
8 days	4	4	4	6	2	7	3	3
9 days	1	1	4	6	3	3	0	2
Positive Sequences								
1 day	409	422	318	324	417	372	359	405
2 days	234	235	200	165	240	207	184	243
3 days	136	134	135	95	129	106	119	133
4 days	78	70	70	64	69	64	64	69
5 days	40	43	51	39	39	30	36	45
6 days	19	22	32	29	30	12	10	24
7 days	9	11	11	15	14	8	10	10
8 days	14	9	20	10	6	5	6	10
9 days	7	5	4	9	1	4	1	3

The results of the consecutive day patterns illustrate that one-day negative returns are more common than one-day positive returns across all indices. However, as the number of consecutive day returns increased, the number of consecutive negative returns decreased and the number of consecutive positive returns increases, this effect is distinctly identifiable in consecutive 6-day returns and more. These results would indicate that positive returns are more persistent than negative returns, and clearly, the one-day negative returns revert to positive returns more frequently than a positive return reverts to a negative return. The above results could be interpreted as evidence of asymmetry in return dynamics.

The presence of conditional volatility asymmetry is best evidenced graphically in the FINI daily index, presented in Figure 25 below. The presence of periods of increased volatility accompanied by a drop in stock prices is highlighted

Figure 25: FINI Daily Closing Price and Return



5.2.2. Weekly Return Series

Table 8 below illustrates the consecutive weekly positive and negative returns for each index over the period under consideration.

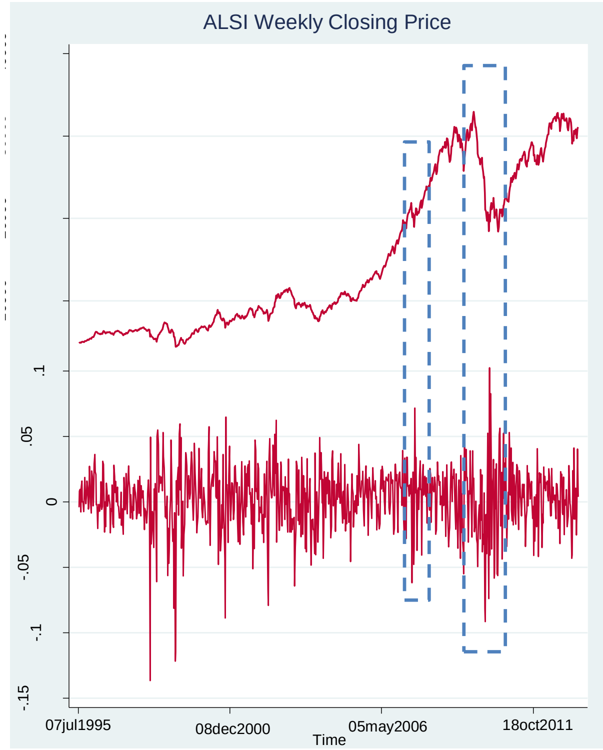
Table 8: Consecutive Weekly Returns

Runs of Consecutive Returns	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	*INDI*
Negative Sequences								
1 week	70	68	68	60	82	66	56	80
2 weeks	46	51	35	23	48	46	41	51
3 weeks	24	22	18	15	23	22	24	16
4 weeks	9	10	11	8	13	12	7	12
5 weeks	7	6	9	7	6	6	7	5
6 weeks	3	3	5	5	1	1	2	2
7 weeks	4	3	4	2	2	1	2	4
8 weeks	1	1	0	2	3	2	2	2
9 weeks	0	1	1	2	0	0	1	1
Positive Sequences								
1 week	57	57	49	36	63	64	58	62
2 weeks	38	39	32	24	44	41	29	39
3 weeks	25	26	20	13	28	19	18	28
4 weeks	15	12	15	12	19	12	11	16
5 weeks	7	9	10	11	7	5	9	12
6 weeks	9	9	4	4	6	10	7	2
7 weeks	3	4	6	8	4	2	3	5
8 weeks	5	5	2	3	5	3	2	5
9 weeks	2	1	6	6	0	0	2	2

Similarly, to the results of the consecutive daily returns, the results of the consecutive weekly patterns illustrate that one-week negative returns are more common than one-week positive returns across all indices. However, as the number of consecutive week returns increased, the number of consecutive negative returns decreased and the number of consecutive positive returns increases, this effect is distinctly identifiable in consecutive 6-week returns and more. These results would indicate that positive returns are more persistent than negative returns, and clearly, the one-week negative returns revert to positive returns more frequently than a positive return reverts to a negative return. The above results could be interpreted as evidence of asymmetry in return dynamics.

The presence of conditional volatility asymmetry is best evidenced graphically in the ALSI weekly index, presented in Figure 26 below. The presence of periods of increased volatility accompanied by a drop in stock prices is highlighted

Figure 26: ALSI Weekly Closing Price and Return



5.2.3. Monthly Return Series

Table 9 below illustrates the consecutive monthly positive and negative returns for each index over the period under consideration.

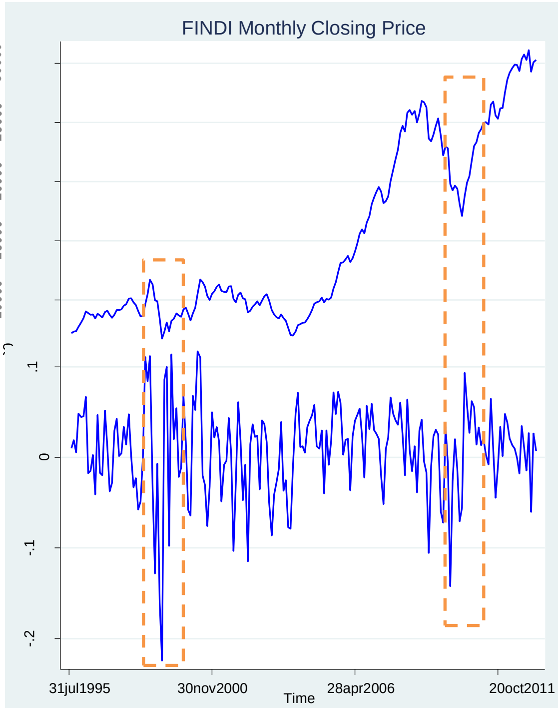
Table 9: Consecutive Monthly Returns

Runs of Consecutive Returns	*ALSI*	*Top40*	*MidCap*	*SmallCap*	*FINDI*	*FINI*	*RESI*	*INDI*
Negative Sequences								
1 month	19	18	12	8	13	14	17	16
2 months	9	8	6	8	10	10	4	11
3 months	3	4	10	8	4	3	4	5
4 months	1	1	2	2	3	3	2	2
5 months	2	1	1	1	3	3	1	3
6 months	1	2	1	1	0	0	2	0
7 months	0	0	0	0	0	0	0	0
8 months	0	0	0	1	0	0	0	0
9 months	0	0	0	0	0	0	0	0
Positive Sequences								
1 month	10	10	5	4	8	12	6	12
2 months	9	8	10	8	10	8	11	11
3 months	2	3	1	3	3	5	3	4
4 months	5	3	6	4	4	2	3	3
5 months	1	2	3	2	1	1	1	1
6 months	0	0	1	1	1	1	5	1
7 months	3	2	2	3	2	2	0	2
8 months	5	6	2	1	3	1	1	2
9 months	0	0	0	1	0	0	0	1

As in Table 7 and 8, the monthly return dynamics show signs of asymmetry particularly when one considers the 6 month consecutive return and greater. Positive returns are evidently more persistent than negative returns, whilst negative returns revert to positive returns more frequently than a positive return reverts to a negative return

Evidence of conditional volatility asymmetry is presented in Figure 26 below through the presence of periods of increased volatility accompanied by a drop in stock prices.

Figure 27: FINDI Monthly Closing Price and Return



5.3. Regression Estimation and Diagnostics

Prior to fitting the ANAR- GJR GARCH to the data, it is crucial to test for ARCH effects. The test for ARCH effects is a joint test for the characteristics of financial time series the ARCH model was specifically designed to accommodate: namely non-constant volatility and serially correlated residual terms.

The test for ARCH effects is conducted on each series, per sampling period. The results of these tests are presented below.

5.3.1. Daily Return Series

Testing for ARCH effects involves first running a simple regression and performing Engle's (1982) Lagrange Multiplier test for heteroskedasticity, whereby the null hypothesis is the absence of ARCH effects and the alternative hypothesis is the presence of ARCH effects provided that the test statistic exceeds the chi-squared distribution.

Figure 28: Lagrange Multiplier Test of Daily Returns per Index

ALSI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	388.425	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

Top40

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	388.510	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

MidCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	566.827	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

SmallCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	592.128	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	491.484	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	175.690	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

RESI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	123.335	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

INDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	504.142	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

The probability of the test statistic is less than 0.05 across all indices; therefore, one can reject the null hypothesis in favour of the alternative hypothesis of heteroskedasticity at the 95% confidence interval.

Recall the Breusch-Godfrey (1979) test for higher order serial dependence in residuals, whereby regressors are not exogenous, carries a null hypothesis of no serial correlation in residuals up to a certain lag order, and an alternative hypothesis of the presence of serial correlation. Provided that the test statistic is greater than the chi-squared distribution one can reject the null hypothesis in favour of the alternative hypothesis.

Figure 29: Breusch-Godfrey Test of Daily Returns per Index

ALSI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	7.711	1	0.0055

H0: no serial correlation

Top40

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	7.837	1	0.0051

H0: no serial correlation

MidCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	4.895	1	0.0269

H0: no serial correlation

SmallCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	26.732	1	0.0000

H0: no serial correlation

FINDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	4.662	1	0.0308

H0: no serial correlation

FINI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	4.931	1	0.0264

H0: no serial correlation

RESI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	8.455	1	0.0036

H0: no serial correlation

INDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	4.740	1	0.0295

H0: no serial correlation

The probability of the test statistic in each case is less than 0.05; therefore one can reject the null hypothesis in favour of the alternative hypothesis of serial correlation in the residuals at the 95% confidence interval across all eight indices.

The results of Engle's (1982) Lagrange Multiplier and the Breusch-Godfrey (1979) test indicate that the daily return series of each of the eight indices exhibits conditional heteroskedasticity and serial correlation in the residual terms. These characteristics allow for the application of the ANAR-GJR GARCH without compromising the power and significance of the results.

5.3.2. Weekly Return Series

The results of Engle's (1982) Lagrange Multiplier test for heteroskedasticity are presented below:

Figure 30: Lagrange Multiplier Test of Weekly Returns per Index

ALSI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	70.185	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

Top40

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	62.244	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

MidCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	77.463	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

SmallCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	141.214	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	59.661	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	85.127	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

RESI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	69.221	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

INDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	39.843	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

The probability of the test statistic is less than 0.05 across all indices; therefore, one can reject the null hypothesis in favour of the alternative hypothesis of heteroskedasticity at the 95% confidence interval.

The results of the Breusch-Godfrey (1979) test are presented below:

Figure 31: Breusch-Godfrey Test of Weekly Returns per Index

ALSI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	6.230	1	0.0126

H0: no serial correlation

Top40

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	5.471	1	0.0193

H0: no serial correlation

MidCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	5.800	1	0.0160

H0: no serial correlation

SmallCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	6.019	1	0.0142

H0: no serial correlation

FINDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	6.601	1	0.0102

H0: no serial correlation

FINI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	2.321	1	0.1277

H0: no serial correlation

RESI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	1.229	1	0.2677

H0: no serial correlation

INDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	2.766	1	0.0963

H0: no serial correlation

The probability of the test statistic for the ALSI, Top40, MidCap, SmallCap and FINDI, is less than 0.05; therefore one can reject the null hypothesis in favour of the alternative hypothesis of serial correlation in the residuals at the 95% confidence. In the case of the INDI, the probability is less than 0.1 and therefore the null hypothesis can be rejected in favour of the alternative hypothesis at the 10% confidence interval. The FINI and RESI index test fails to reject the null hypothesis and no higher order serial correlation is detected in these indices. In this case, it could be possible that there is serial correlation present in the lower lag order, and lower confidence intervals.

The results of Engle's (1982) Lagrange Multiplier and the Breusch-Godfrey (1979) test indicate that the weekly return series of each of the eight indices exhibits conditional heteroskedasticity and the majority of the indices present signs of serial correlation in the residual terms. These characteristics allow for the application of the ANAR-GJR GARCH without compromising the power and significance of the results.

5.3.3. Monthly Return Series

The results of Engle's (1982) Lagrange Multiplier test for heteroskedasticity are presented below:

Figure 32: Lagrange Multiplier Test of Monthly Returns per Index

ALSI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	10.416	1	0.0012

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

Top40

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	4.285	1	0.0385

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

MidCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	11.854	1	0.0006

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

SmallCap

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	22.203	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	26.748	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

FINI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	26.974	1	0.0000

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

RESI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	2.522	1	0.1123

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

INDI

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(*p*)	chi2	df	Prob > chi2
1	7.514	1	0.0061

H0: no ARCH effects *vs.* H1: ARCH(*p*) disturbance

The probability of the test statistic in all but one of the indices is less than 0.05; therefore, one can reject the null hypothesis in favour of the alternative hypothesis of heteroskedasticity at the 95% confidence interval. It appears as though the weekly return series for the RESI index fails to reject null hypothesis and conditional heteroskedasticity is not present in the time series.

The results of the Breusch-Godfrey (1979) test are presented below:

Figure 33: Breusch-Godfrey Test of Monthly Returns per Index

ALSI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	8.806	1	0.0030

H0: no serial correlation

Top40

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	6.484	1	0.0109

H0: no serial correlation

MidCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	10.061	1	0.0015

H0: no serial correlation

SmallCap

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	13.152	1	0.0003

H0: no serial correlation

FINDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	3.936	1	0.0473

H0: no serial correlation

FINI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	0.525	1	0.4689

RESI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	2.576	1	0.1085

H0: no serial correlation

INDI

Breusch-Godfrey LM test for autocorrelation

lags(*p*)	chi2	df	Prob > chi2
1	9.095	1	0.0026

H0: no serial correlation

The probability of the test statistic for all indices except the FINI and RESI index is less than 0.05; therefore one can reject the null hypothesis in favour of the alternative hypothesis of serial correlation in the residuals at the 95% confidence. In the case of the FINI and the RESI index, the probability is greater than 0.05 and therefore one fails to reject the null hypothesis and no higher order serial correlation in detected in these indices. In this case, it could be possible that there is serial correlation present in the lower lag order, and lower confidence levels.

It appears as though the presence of conditional heteroskedasticity and serial correlation in the residual terms is present across the sampling intervals and most of the indices. These findings speak to the assumptions made under an ANAR-GJR GARCH specification and the applicability of the model to the dataset. Given the presence of heteroskedasticity and serial correlation, one can reasonably apply the ANAR-GJR GARCH model to the time series to produce estimates of the coefficients of the model which would reasonably hold statistical significance.

One needs to be cautious of interpreting the results of the ANAR-GJR GARCH specification for particular series in light of the findings of homoskedasticity of the RESI index of monthly returns, the lack of higher order serial correlation in the weekly returns of the FINI and RESI index and the lack of higher order serial correlation in the monthly returns of the FINI and the RESI index.

5.4. The ANAR–GJR GARCH Estimation Results, Interpretations and Diagnostics

Recall, the ANAR–GJR GARCH specification is a combination of the nonlinear ANAR model of Nam, Kim and Arize (2006) to capture asymmetry in the conditional mean, and the GJR GARCH model of Glosten, Jagannathan and Runkle (1993) to capture asymmetry in the conditional volatility.

The combined ANAR-GJR GARCH model involves the joint estimation of the conditional mean and conditional variance as follows, as per Equation 12:

$$r_t = \mu + [\phi_1 + \rho_1 D_1(r_{t-1} < 0)] \,.\, r_{t-1} + \varepsilon_t$$

Where, as cited by Nam, Kim & Arize (2006):

r_t is the return in time "t"

μ is a constant, or intercept of the data generating process

ϕ_1 is a partial measure of serial correlation

D_1 is an indicator function specified for a dummy variable which takes on the value of 1 if $r_{t-1} < 0$ or 0 otherwise.

ρ_1 is the coefficient of the dummy variable D_1

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

The residual term is captured through the following random process, as per Equation 13:

$$\varepsilon_t = V_t \,.\, \sqrt{h_t}$$

Whereby:

ε_t is the residual term or deviation of the return from the estimated mean return at time "t"

V_t is the "white noise" component or random component

h_t is the conditional variance described by a GJR GARCH autoregressive process

The residual term ε_t follows a normal distribution with a mean of 0 and a variance of h_t.

The stochastic process of the conditional variance follows a GJR GARCH specification of Glosten, Jagannathan and Runkle (1993) as follows, as per Equation 14:

$$h_t = \propto_0 + \propto_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \omega_1 \varepsilon_{t-1}^2 I_{t-1}$$

h_t is the conditional variance in time "t"

$\propto_0$ is a constant, or intercept of the data generating process

$\propto_1$ is the correlation coefficient of the lagged squared residual term

ε_{t-1}^2 is the squared residual term, lagged by one time period.

β_1 is the correlation coefficient of the lagged conditional variance term

h_{t-1} is the conditional variance, lagged by one time period

ω_1 is the asymmetry coefficient

I_{t-1} is an indicator function, where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, or $I_{t-1} = 0$ if $\varepsilon_{t-1} > 0$

The regression outputs of the ANAR-GJR GARCH produce estimations of the correlation coefficients described above. The test for non-linearity lies in the significance of these estimates and the corresponding interpretations. The regression outputs are presented per index before the results are discussed.

5.4.1. Daily Return Series

The regression outputs per index are presented in Figure 34 below. The relevant details to consider are the estimates of the coefficients, the respective p values of the coefficients and p-value of the model itself when fitted to the dataset.

Figure 34: ANAR- GJR GARCH Regression Results of Daily Returns

ALSI

ARCH family regression -- AR disturbances

Sample: 03jan1960 - 24feb1971 Number of obs = 4071
Distribution: Gaussian Wald chi2(2) = 45.71
Log likelihood = 12570.44 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1381068	.0328684	-4.20	0.000	-.2025277	-.0736858
_cons	.0005769	.0001859	3.10	0.002	.0002125	.0009414
ARMA						
ar						
L1.	.1640803	.0245224	6.69	0.000	.1160173	.2121432
ARCH						
arch						
L1.	.1762199	.011546	15.26	0.000	.1535901	.1988497
tarch						
L1.	-.1275135	.0118432	-10.77	0.000	-.1507257	-.1043012
garch						
L1.	.8719803	.0069465	125.53	0.000	.8583655	.8855951
_cons	2.96e-06	3.88e-07	7.63	0.000	2.20e-06	3.72e-06

Top40

ARCH family regression -- AR disturbances

Sample: 03jan1960 - 24feb1971 Number of obs = 4071
Distribution: Gaussian Wald chi2(2) = 33.99
Log likelihood = 12158.4 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1314963	.0333635	-3.94	0.000	-.1968875	-.066105
_cons	.000496	.0002004	2.48	0.013	.0001032	.0008887
ARMA						
ar						
L1.	.1428514	.0245842	5.81	0.000	.0946671	.1910356
ARCH						
arch						
L1.	.1741231	.0120905	14.40	0.000	.1504262	.19782
tarch						
L1.	-.1286922	.0123802	-10.39	0.000	-.1529569	-.1044274
garch						
L1.	.8733724	.007369	118.52	0.000	.8589295	.8878153
_cons	3.78e-06	5.17e-07	7.30	0.000	2.76e-06	4.79e-06

MidCap

```
ARCH family regression -- AR disturbances

Sample: 03jan1960 - 24feb1971                    Number of obs   =      4071
Distribution: Gaussian                           Wald chi2(2)    =    245.05
Log likelihood =  14057.67                       Prob > chi2     =    0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
  d1ytminus1 |  -.2121406   .0278954    -7.60   0.000    -.2668147   -.1574666
       _cons |   .0007741   .0001637     4.73   0.000     .0004532    .0010949
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .3256964   .0211332    15.41   0.000      .284276    .3671167
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .2455052   .0131323    18.69   0.000     .2197663     .271244
             |
       tarch |
         L1. |  -.1702421   .0138826   -12.26   0.000    -.1974515   -.1430326
             |
       garch |
         L1. |    .803911   .0103231    77.87   0.000      .783678    .8241439
             |
       _cons |   3.10e-06   2.97e-07    10.44   0.000     2.52e-06    3.68e-06
------------------------------------------------------------------------------
```

SmallCap

```
ARCH family regression -- AR disturbances

Sample: 03jan1960 - 24feb1971                    Number of obs   =      4071
Distribution: Gaussian                           Wald chi2(2)    =    322.52
Log likelihood =  15194.75                       Prob > chi2     =    0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
  d1ytminus1 |  -.2644747   .0262199   -10.09   0.000    -.3158648   -.2130846
       _cons |   .0008793   .0001369     6.42   0.000     .0006109    .0011477
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .3947912   .0220465    17.91   0.000     .3515808    .4380016
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .3387362   .0158478    21.37   0.000      .307675    .3697975
             |
       tarch |
         L1. |  -.2425902   .0152623   -15.89   0.000    -.2725038   -.2126765
             |
       garch |
         L1. |   .7238432   .0114578    63.17   0.000     .7013864    .7463001
             |
       _cons |   3.10e-06   2.10e-07    14.72   0.000     2.68e-06    3.51e-06
------------------------------------------------------------------------------
```

FINDI

ARCH family regression -- AR disturbances

Sample: 03jan1960 - 26feb1971 Number of obs = 4073
Distribution: Gaussian Wald chi2(2) = 32.52
Log likelihood = 12433.55 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1296092	.0325966	-3.98	0.000	-.1934974	-.065721
_cons	.0005045	.000184	2.74	0.006	.0001439	.0008651
ARMA						
ar						
L1.	.134482	.0235903	5.70	0.000	.0882458	.1807183
ARCH						
arch						
L1.	.1828628	.0108079	16.92	0.000	.1616797	.204046
tarch						
L1.	-.1350216	.0115506	-11.69	0.000	-.1576603	-.1123828
garch						
L1.	.8675823	.0059662	145.42	0.000	.8558888	.8792758
_cons	3.49e-06	3.99e-07	8.73	0.000	2.70e-06	4.27e-06

FINI

ARCH family regression -- AR disturbances

Sample: 03jan1960 - 01may1969 Number of obs = 3407
Distribution: Gaussian Wald chi2(2) = 36.32
Log likelihood = 9914.906 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.0766888	.0352505	-2.18	0.030	-.1457786	-.0075991
_cons	.000206	.0002332	0.88	0.377	-.0002511	.0006631
ARMA						
ar						
L1.	.1338148	.0238302	5.62	0.000	.0871085	.1805211
ARCH						
arch						
L1.	.1650999	.012649	13.05	0.000	.1403084	.1898914
tarch						
L1.	-.1150537	.0130851	-8.79	0.000	-.1407001	-.0894073
garch						
L1.	.8777895	.0075144	116.81	0.000	.8630615	.8925175
_cons	4.03e-06	5.95e-07	6.78	0.000	2.87e-06	5.20e-06

RESI

```
ARCH family regression -- AR disturbances

Sample: 03jan1960 - 01may1969                   Number of obs   =      3407
Distribution: Gaussian                          Wald chi2(2)    =     33.97
Log likelihood =  9007.781                      Prob > chi2     =    0.0000
```

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.110828	.0362106	-3.06	0.002	-.1817995	-.0398564
_cons	.0006075	.0003325	1.83	0.068	-.0000442	.0012591
ARMA						
ar						
L1.	.1440276	.02534	5.68	0.000	.0943621	.1936931
ARCH						
arch						
L1.	.1134366	.0137444	8.25	0.000	.0864981	.1403752
tarch						
L1.	-.0750537	.0128845	-5.83	0.000	-.1003068	-.0498006
garch						
L1.	.8928144	.0120623	74.02	0.000	.8691727	.916456
_cons	.0000104	1.76e-06	5.90	0.000	6.91e-06	.0000138

INDI

```
ARCH family regression -- AR disturbances

Sample: 03jan1960 - 26feb1971                   Number of obs   =      4073
Distribution: Gaussian                          Wald chi2(2)    =     21.93
Log likelihood =  12370.85                      Prob > chi2     =    0.0000
```

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1313926	.0331933	-3.96	0.000	-.1964504	-.0663348
_cons	.0006061	.0001843	3.29	0.001	.0002449	.0009674
ARMA						
ar						
L1.	.1134178	.0246547	4.60	0.000	.0650955	.1617401
ARCH						
arch						
L1.	.1657532	.0098795	16.78	0.000	.1463899	.1851166
tarch						
L1.	-.1169931	.0107874	-10.85	0.000	-.138136	-.0958503
garch						
L1.	.8715023	.006598	132.09	0.000	.8585705	.8844341
_cons	3.84e-06	4.62e-07	8.32	0.000	2.94e-06	4.75e-06

A summary of the above regressions is presented for ease of reference in Table 10 below.

Table 10: ANAR-GJR GARCH Results Summary for Daily Returns

	ANAR-GJR GARCH SPECIFICATION FOR NOMINAL DAILY RETURNS							
Coefficients	**ALSI**	**Top40**	**MidCap**	**SmallCap**	**FINDI**	**FINI**	**RESI**	**INDI**
μ	0.0005769	0.000496	0.0007741	0.0008793	0.0005045	0.000206	0.0006075	0.0006061
	(0.002)	*(0.013)*	*(0.000)*	*(0.000)*	*(0.006)*	*(0.377)*	*(0.068)*	*(0.001)*
ϕ	0.1640803	0.1428514	0.3256964	0.3947912	0.134482	0.1338148	0.1440276	0.1134178
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
ρ	-0.1381068	-0.1314963	-0.2121406	-0.2644747	-0.1296091	-0.0766888	-0.110828	-0.1313926
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.030)*	*(0.002)*	*(0.000)*
α_0	0.00000296	0.00000378	0.0000031	0.0000031	0.00000349	0.00000403	0.0000104	0.00000384
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
α_1	0.1762199	0.1741231	0.2455052	0.3387362	0.1828628	0.1650999	0.1134366	0.1657532
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
β_1	0.8719803	0.8733724	0.803911	0.7238432	0.8675823	0.8777895	0.8928144	0.8715023
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
ω_1	-0.1275135	-0.1286922	-0.1702421	-0.2425902	-0.1350216	-0.1150537	-0.0750537	-0.1169931
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*

Prior to examining the statistical relevance of the correlation coefficients, one needs to test whether the stationarity conditions of the model hold, given the estimates produced by the regression.

In order for the stationarity condition of the return dynamics to hold, the return serial correlation, measured by $\phi_1 + \rho_1$ when $r_{t-1} < 0$ and ϕ_1 when $r_{t-1} > 0$, is restricted by $|\phi_1 + \rho_1 D_1| < 1$. The results of this test for each index are as follows: ALSI (0.026), Top40 (0.011), MidCap (0.114), SmallCap (0.130), FINDI (0.005), FINI (0.057) RESI (0.033) and INDI (0.018), as evidence that the stationarity conditions of the model hold across all eight indices.

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The results of this test for stationarity are: ALSI (0.921), Top40 (0.919), MidCap (0.879), SmallCap (0.820), FINDI (0.915), FINI (0.928), RESI (0.931) and INDI (0.920). As these results are all less than 1, the stationarity condition across all eight indices holds.

The conditions which ensure a positive conditional variance estimation are: $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$. Across all eight indices, the $\propto_0$, $\propto_1$, and β_1 estimates are greater than zero at the 95% confidence interval. The results of the condition $(\propto_1 + \omega_1) > 0$ are ALSI (0.049), Top40

(0.045), MidCap (0.075), SmallCap (0.096), FINDI (0.048), FINI (0.050), RESI (0.038) and INDI (0.049), as evidence that the results produce positive conditional variance estimations.

The null hypothesis of the regression estimate is such that the coefficients are all equal to zero, in other words the model holds no statistical power or relevance in explaining the return and variance dynamics. The test for this hypothesis lies in the p-value of the regression. If the p-value is less than 0.05 then one rejects the null hypothesis in favour of the alternative hypothesis that the coefficients produced by the model are statistically significant from zero. The p-value of each of the regressions is less than 0.05, and the estimations of the coefficients are significant at the 95% confidence interval.

Upon examining the coefficients themselves, one notices that the p-values across the majority coefficients for each index are statistically significant at the 95% confidence interval. This allows inferences made on the interpretation of these coefficients to be statistically sound.

The results of the ANAR model to test asymmetry in the mean are discussed first. The constant or intercept of the ANAR data generating process, μ, is expected to be positive and significant. The coefficient, which measures partial serial correlation, ϕ_1, is expected to be positive and significant. The coefficient of the dummy variable ρ_1 should be negative and significant. The condition $\rho_1 < 0$ implies an asymmetric reverting pattern of stock returns that a negative return reverts more quickly, and with a greater magnitude than a positive return reverts to a negative return. If $\rho_1 = 0$, then the reverting pattern is symmetrical and a prior positive or negative return has the same effect on the current periods return[28].

The results of the regression indicate that the intercept of the ANAR process is statistically significant for all of the indices besides the FINI and the RESI index. Non-significance of the intercept can be interpreted as a mean value of zero when all other coefficients are zero. The statistically significant estimates of the partial serial correlation coefficient, ϕ_1, is evidence that positive prior returns are relatively persistent across all indices. The most telling parameter of the regression is the ρ_1 estimate. A ρ_1 of less than zero measures the asymmetric reverting pattern and speed of stock returns. The statistically significant negative p-value of the ρ_1 coefficient coupled with the statistically significant positive ϕ_1 coefficient indicates that the serial correlation between negative returns is smaller than the correlation between positive returns. Furthermore, if $\phi_1 + \rho_1 < \phi_1$ this would imply that positive returns show persistence whilst negative returns show strong reverting tendencies. The results of the above test are as follows: ALSI (0.026 < 0.164), Top40 (0.011 <0.143),

[28] Refer to Section 3.1 and 3.2 for the mathematical proof

MidCap (0.114 < 0.326), SmallCap (0.130 < 0.395), FINDI (0.005 < 0.134), FINI (0.057 <0.134), RESI (0.033 < 0.144) and INDI (-0.018 < 0.113).

The results of the ANAR regression show that asymmetry in daily returns is a statistically significant detectable characteristic across industry and market capitalisation for daily stock returns on the JSE.

The results of the GJR GARCH regression will now be discussed.

Recall, the intercept of the data generating process, α_0 is expected to be positive and significant. The correlation coefficient, $\propto_1$, measuring the relationship between the conditional volatility and the lagged squared residual term is expected to be positive and significant. The correlation coefficient, β_1, measuring the interaction between the conditional variance with the lagged conditional variance is expected to be positive and significant. The asymmetry coefficient, ω_1, can take a positive or negative value, but should be statistically significant.

The constant term α_0 is positive and significant for all indices, the serial correlation coefficient $\propto_1$, and β_1, are too, positive and significant for all indices.

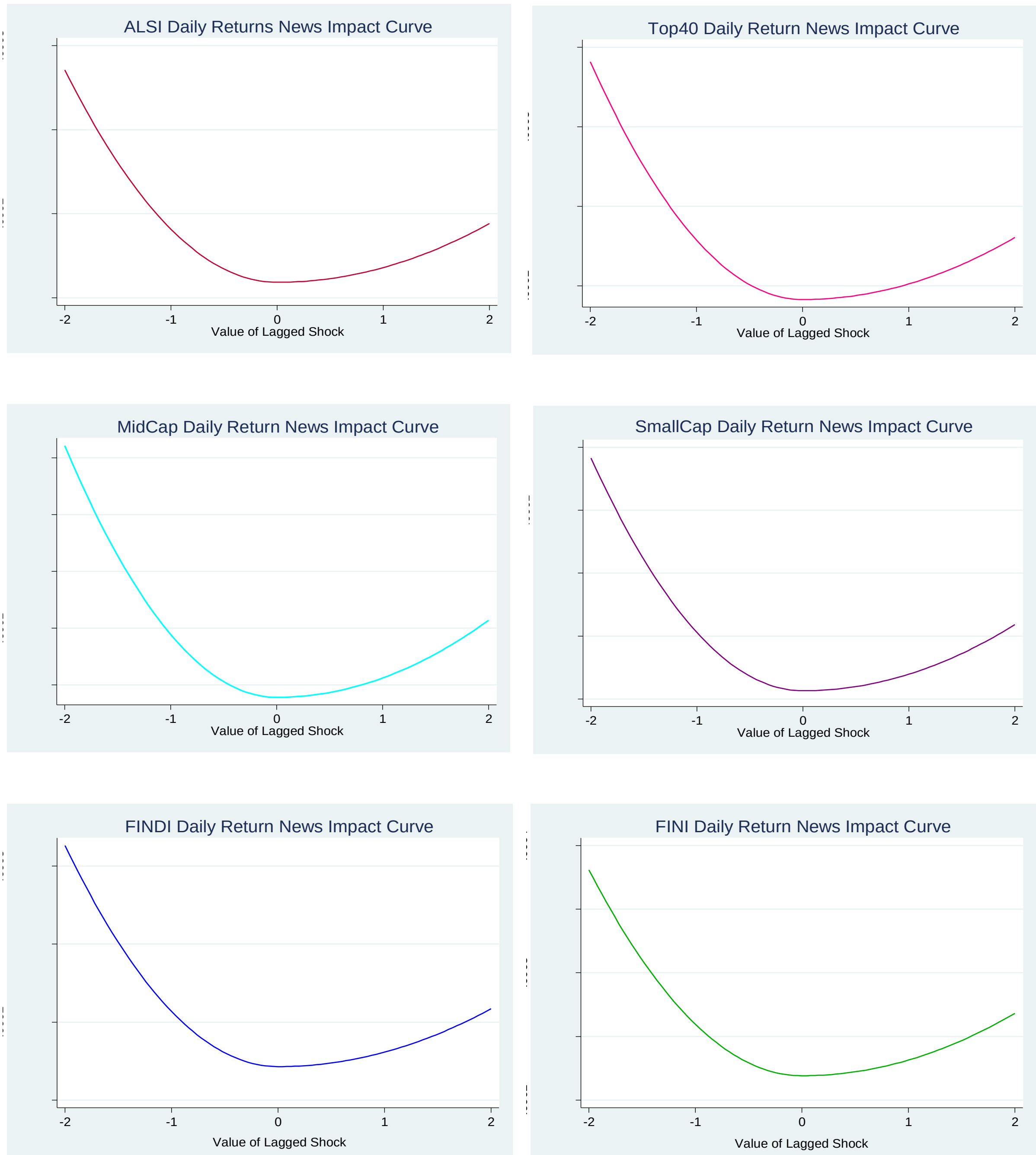
Recall that, asymmetry in volatility, caused specifically by the leverage effect, would imply that good news, measured by $\varepsilon_{t-1} > 0$, and bad news, measured by $\varepsilon_{t-1} < 0$, will have different impacts on conditional volatility. The GJR GARCH models allow the impact of good news to be captured by $\propto_1$ and the impact of bad news to be captured by $\propto_1 + \omega$. If the asymmetry coefficient, $\omega > 0$, is positive and significant this would imply that bad news increases future volatility, an indication that the leverage effect of Black (1976) is at play, however if $\omega \neq 0$ then the impact of news on the conditional volatility is asymmetric but bad news does not necessarily increase future volatility (Mandimika & Chinzara, 2010). The results indicate that the ω statistic is negative and significant for each of the indices, which implies that asymmetric affects are present in the conditional volatility; however bad news does not necessarily increase future volatility. This result, whilst it acknowledges asymmetry is more supportive of the volatility feedback mechanism of Pindyck (1984) than the leverage hypothesis of Black (1976).

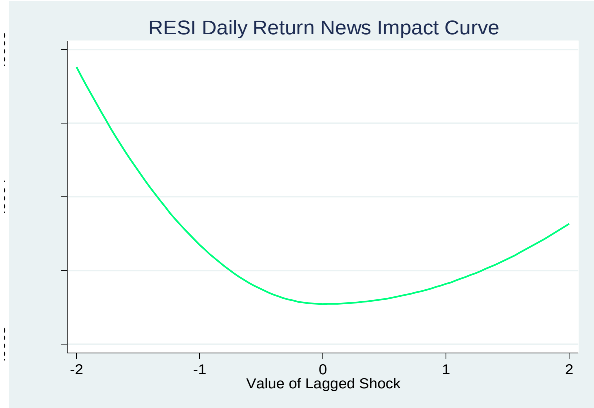
The News Impact Curve developed by Pagan and Schwert (1990) demonstrates the degree of asymmetry in volatility to prior positive and negative return shocks, by plotting the next period conditional variance (h_t) that would arise from a combination of positive and negative values of residual terms (ε_{t-1}), given the estimated GJR GARCH model.

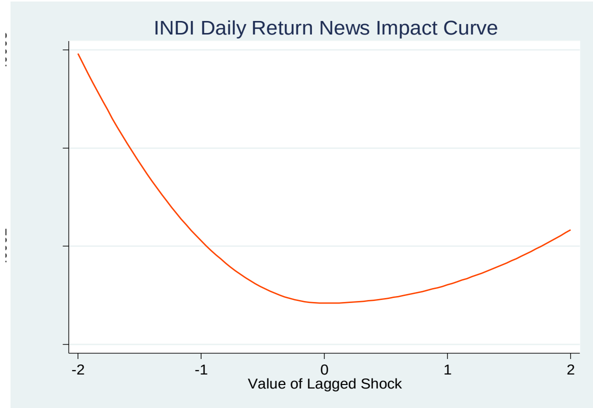
The curves are generated by using the estimated conditional variance per the GJR GARCH specification, with the given coefficient estimations, and the lagged conditional variance (h_{t-1}) set to the unconditional variance. Successive values of residual terms (ε_{t-1}) in the range of -2 to +2 are then used in the GJR GARCH equation to determine what the corresponding values of the conditional variance (h_t) derived from the model would be. The ANAR-GJR GARCH model should produce a News Impact Curve, which is asymmetrical to positive and negative shocks.

The News Impact Curves of the ANAR GJR-GARCH model are presented below:

Figure 35: News Impact Curve of Daily Returns







All of the above News Impact curves demonstrate that negative lagged shocks (negative returns as a result of bad news) have a greater effect on volatility than positive lagged shocks. The asymmetric volatility detected in the regression results of the GJR GARCH is visually depicted in these plots.

In conclusion, the regression results of the ANAR GJR GARCH model prove that statistically significant asymmetries are present in the daily returns and daily volatilities of the stocks of the JSE, despite market capitalisation and industry.

5.4.2. Weekly Return Series

The regression outputs per index are presented in Figure 36 below. The relevant details to consider are the estimates of the coefficients, the respective p-values of the coefficients and p-value of the model itself when fitted to the dataset.

Going into this analysis, one needs to take heed of the fact that in the case of the FINI, RESI and INDI index one failed to reject the null hypothesis of no higher order serial correlation. In this case, it could be possible that there is serial correlation present in the lower lag order, and lower confidence levels.

Figure 36: ANAR-GJR GARCH Regression Results of Weekly Returns

ALSI

ARCH family regression -- AR disturbances

Sample: 1960w3 - 1976w26	Number of obs = 856
Distribution: Gaussian	Wald chi2(2) = 37.82
Log likelihood = 2053.033	Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1801499	.0669464	-2.69	0.007	-.3113625	-.0489374
_cons	.001762	.0010735	1.64	0.101	-.000342	.003866
ARMA						
ar L1.	.2863934	.0482598	5.93	0.000	.191806	.3809808
ARCH						
arch L1.	.2026967	.0392994	5.16	0.000	.1256714	.2797221
tarch L1.	-.2330774	.0408032	-5.71	0.000	-.3130502	-.1531046
garch L1.	.827507	.0307492	26.91	0.000	.7672397	.8877743
_cons	.000046	9.08e-06	5.07	0.000	.0000283	.0000638

Top40

ARCH family regression -- AR disturbances

Sample: 1960w3 - 1976w26	Number of obs = 856
Distribution: Gaussian	Wald chi2(2) = 33.10
Log likelihood = 1987.113	Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1864842	.0678212	-2.75	0.006	-.3194112	-.0535572
_cons	.0015542	.001135	1.37	0.171	-.0006705	.0037788
ARMA						
ar L1.	.2695264	.0480952	5.60	0.000	.1752615	.3637913
ARCH						
arch L1.	.1944118	.0386437	5.03	0.000	.1186715	.270152
tarch L1.	-.2307334	.0408291	-5.65	0.000	-.3107568	-.1507099
garch L1.	.839874	.0282321	29.75	0.000	.7845401	.8952079
_cons	.000051	9.67e-06	5.27	0.000	.000032	.00007

MidCap

```
ARCH family regression -- AR disturbances

Sample: 1960w3 - 1976w26                          Number of obs   =       856
Distribution: Gaussian                            Wald chi2(2)    =    182.16
Log likelihood =  2248.525                        Prob > chi2     =    0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
  d1ytminus1 |  -.3653223    .053689    -6.80   0.000    -.4705507   -.2600938
       _cons |   .0025565   .0011961     2.14   0.033     .0002121    .0049009
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .5034269   .0373179    13.49   0.000     .4302852    .5765686
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .6240216   .0788157     7.92   0.000     .4695456    .7784976
             |
       tarch |
         L1. |  -.6151724   .0798505    -7.70   0.000    -.7716765   -.4586682
             |
       garch |
         L1. |   .5093193   .0445723    11.43   0.000     .4219592    .5966795
             |
       _cons |     .00008   9.84e-06     8.14   0.000     .0000608    .0000993
------------------------------------------------------------------------------
```

SmallCap

```
ARCH family regression -- AR disturbances

Sample: 1960w3 - 1976w26                          Number of obs   =       856
Distribution: Gaussian                            Wald chi2(2)    =    448.66
Log likelihood =  2431.682                        Prob > chi2     =    0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
  d1ytminus1 |   -.289036   .0495015    -5.84   0.000    -.3860571   -.1920148
       _cons |   .0018154   .0012083     1.50   0.133    -.0005529    .0041837
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .6032922   .0285495    21.13   0.000     .5473363    .6592482
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .3222061   .0474735     6.79   0.000     .2291596    .4152525
             |
       tarch |
         L1. |  -.3410701   .0536498    -6.36   0.000    -.4462218   -.2359183
             |
       garch |
         L1. |   .7638413   .0303932    25.13   0.000     .7042716     .823411
             |
       _cons |   .0000226   4.15e-06     5.45   0.000     .0000145    .0000308
------------------------------------------------------------------------------
```

FINDI

ARCH family regression -- AR disturbances

Sample: 1960w4 - 1976w27	Number of obs = 856
Distribution: Gaussian	Wald chi2(2) = 37.76
Log likelihood = 2050.192	Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.3163521	.0676539	-4.68	0.000	-.4489512	-.1837529
_cons	.0015338	.0010622	1.44	0.149	-.0005482	.0036157
ARMA						
ar						
L1.	.3111757	.0513053	6.07	0.000	.2106191	.4117323
ARCH						
arch						
L1.	.4281811	.0871804	4.91	0.000	.2573108	.5990515
tarch						
L1.	-.4264859	.0886435	-4.81	0.000	-.600224	-.2527478
garch						
L1.	.5853156	.0576871	10.15	0.000	.4722509	.6983803
_cons	.0001192	.0000184	6.48	0.000	.0000831	.0001552

FINI

ARCH family regression -- AR disturbances

Sample: 1960w3 - 1973w42	Number of obs = 716
Distribution: Gaussian	Wald chi2(2) = 15.53
Log likelihood = 1622.642	Prob > chi2 = 0.0004

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.139302	.0727507	-1.91	0.056	-.2818907	.0032866
_cons	.0017126	.0011316	1.51	0.130	-.0005052	.0039304
ARMA						
ar						
L1.	.2165242	.0574133	3.77	0.000	.1039962	.3290521
ARCH						
arch						
L1.	.1924376	.0348557	5.52	0.000	.1241217	.2607535
tarch						
L1.	-.1471571	.0303285	-4.85	0.000	-.2065998	-.0877143
garch						
L1.	.8464378	.0248518	34.06	0.000	.7977291	.8951464
_cons	.0000258	8.20e-06	3.15	0.002	9.73e-06	.0000419

RESI

ARCH family regression -- AR disturbances

Sample: 1960w3 - 1973w42 Number of obs = 716
Distribution: Gaussian Wald chi2(2) = 25.60
Log likelihood = 1426.407 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.1269279	.0739559	-1.72	0.086	-.2718788	.018023
_cons	.0027804	.0016065	1.73	0.084	-.0003683	.0059291
ARMA						
ar						
L1.	.2486604	.0535457	4.64	0.000	.1437127	.353608
ARCH						
arch						
L1.	.1489156	.0384734	3.87	0.000	.0735091	.2243221
tarch						
L1.	-.1298424	.0359744	-3.61	0.000	-.200351	-.0593339
garch						
L1.	.8603991	.041952	20.51	0.000	.7781747	.9426235
_cons	.0000665	.0000299	2.23	0.026	7.94e-06	.0001251

INDI

ARCH family regression -- AR disturbances

Sample: 1960w3 - 1976w26 Number of obs = 856
Distribution: Gaussian Wald chi2(2) = 46.97
Log likelihood = 2046.832 Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.275899	.0663907	-4.16	0.000	-.4060224	-.1457756
_cons	.0019342	.0011129	1.74	0.082	-.0002471	.0041155
ARMA						
ar						
L1.	.3233011	.0472609	6.84	0.000	.2306714	.4159307
ARCH						
arch						
L1.	.4381252	.0949061	4.62	0.000	.2521127	.6241377
tarch						
L1.	-.4239587	.0986253	-4.30	0.000	-.6172607	-.2306566
garch						
L1.	.5361062	.0643467	8.33	0.000	.4099891	.6622234
_cons	.0001391	.0000222	6.28	0.000	.0000957	.0001826

A summary of the above regressions is presented for ease of reference in Table 11 below.

Table 11: ANAR GJR GARCH Results Summary for Weekly Returns

	ANAR-GJR GARCH FOR NOMINAL WEEKLY RETURNS							
Coefficients	**ALSI**	**Top40**	**MidCap**	**SmallCap**	**FINDI**	**FINI**	**RESI**	**INDI**
μ	0.001762	0.0015542	0.0025565	0.0018154	0.0015338	0.0017126	0.0027804	0.0019342
	(0.101)	*(0.171)*	*(0.033)*	*(0.133)*	*(0.149)*	*(0.130)*	*(0.084)*	*(0.082)*
ϕ	0.2863934	0.2695264	0.5034269	0.6032922	0.3111757	0.2165242	0.2486604	0.3233011
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
ρ	-0.1801499	-0.1864842	-0.3653223	-0.289036	-0.3163521	-0.139302	-0.1269279	-0.275899
	(0.007)	*(0.006)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.056)*	*(0.086)*	*(0.000)*
α_0	0.000046	0.000051	0.00008	0.0000226	0.0001192	0.0000258	0.0000665	0.0001391
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.026)*	*(0.000)*
α_1	0.2026967	0.1944118	0.6240216	0.3222061	0.4281811	0.1924376	0.1489156	0.4381252
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
β_1	0.827505	0.839874	0.5093193	0.7638413	0.5853156	0.8464378	0.8603991	0.5361062
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
ω_1	-0.2330774	-0.2307334	-0.6151724	-0.3410701	-0.4264859	-0.1471571	-0.1298424	-0.4239587
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*

Prior to examining the statistical relevance of the correlation coefficients, one needs to test whether the stationarity conditions of the model hold, given the estimates produced by the regression.

In order for the stationarity condition of the return dynamics to hold, the return serial correlation, measured by $\phi_1 + \rho_1$ when $r_{t-1} < 0$ and ϕ_1 when $r_{t-1} > 0$, is restricted by $|\phi_1 + \rho_1 D_1| < 1$. The results of this test for each index are as follows: ALSI (0.106), Top40 (0.083), MidCap (0.138), SmallCap (0.314), FINDI (0.005), FINI (0.077) RESI (0.122) and INDI (0.047), as evidence that the stationarity conditions of the model hold across all eight indices.

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The results of this test for stationarity are: ALSI (0.797), Top40 (0.804), MidCap (0.518), SmallCap (0.745), FINDI (0.587), FINI (0.892), RESI (0.879) and INDI (0.550). As these results are all less than 1, the stationarity condition across all eight indices holds.

The conditions which ensure a positive conditional variance estimation are: $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$. Across all eight indices, the $\propto_0$, $\propto_1$, and β_1 estimates are greater than zero at the 95% confidence interval. The results of the condition $(\propto_1 + \omega_1) > 0$ are ALSI (-0.030), Top40 (-0.036), MidCap (0.009), SmallCap (-0.019), FINDI (0.002), FINI (0.045), RESI (0.019) and INDI (0.014).

The results show that the stationary condition of the ALSI, Top40 and SmallCap do not hold. One of the weaknesses of the GJR GARCH model is the condition of positive coefficients to ensure a nonnegative conditional volatility, when in reality this condition is frequently violated (Rodriguez & Ruiz, 2009). The remaining indices' coefficients meet the restrictions of the model.

The null hypothesis of the regression estimate is that the coefficients are all equal to zero. The test for this hypothesis lies in the p-value of the regression. If the p-value is less than 0.05 then one rejects the null hypothesis in favour of the alternative hypothesis that the coefficients produced by the model are statistically significant from zero. The p-value of each of the regressions is less than 0.05, and the estimations of the coefficients are significant at the 95% confidence interval.

Upon examining the coefficients themselves, one notices that the p-values across the majority of coefficients for each index are statistically significant at the 95% confidence interval. This allows inferences made on the interpretation of these coefficients to be statistically sound.

The results of the ANAR model to test asymmetry in the mean are discussed first.

The results of the regression indicate that the intercept of the ANAR process is statistically significant for the MidCap only. Although all indices show a positive coefficient, only the MidCap index demonstrates significance in this value. Non-significance of the intercept can be interpreted as a mean value of zero when all other coefficients are zero.

The statistically significant estimates of the partial serial correlation coefficient, ϕ_1, is evidence that positive prior returns are relatively persistent across all indices.

The most telling parameter of the regression is the ρ_1 estimate. A ρ_1 of less than zero measures the asymmetric reverting pattern and speed of stock returns. All of the indices produce negative ρ_1 estimates; however, the FINI and RESI index ρ_1 estimates are not statistically significant at the 95% confidence interval, only the 90% confidence interval. This result could also explain why the Breusch-Godfrey (1979) test did not detect higher order serial correlation in the residuals of the FINI and RESI index. If the confidence interval for the coefficient, which explains serial correlation between the returns, needed to be increased to detect significance, the significance in serial correlation between residuals was that of a lower order in FINI and RESI index and not detected by the Breusch-Godfrey (1979) test.

A statistically significant negative p-value of the ρ_1 coefficient coupled with the statistically significant positive ϕ_1, coefficient indicates that the serial correlation between negative returns is smaller than the correlation between positive returns. Furthermore, if $\phi_1 + \rho_1 < \phi_1$ this would imply that positive returns show persistence whilst negative returns show strong reverting tendencies. The results of the above test are as follows: ALSI (0.106 < 0.286), Top40 (0.083 < 0.270), MidCap (0.138 <

0.503), SmallCap (0.314 < 0.603), FINDI (-0.005 < 0.311), FINI (0.077 < 0.217), RESI (0.122 < 0.249) and INDI (0.047 < 0.323).

The results of the ANAR regression show that asymmetry in returns is a statistically significant detectable characteristic across industry and market capitalisation for weekly stock returns on the JSE.

The results of the GJR GARCH regression will now be discussed.

Recall, the intercept of the data generating process, α_0 is expected to be positive and significant. The correlation coefficient, $\propto_1$, measuring the relationship between the conditional volatility and the lagged squared residual term is expected to be positive and significant. The correlation coefficient, β_1, measuring the interaction between the conditional variance with the lagged conditional variance is expected to be positive and significant. The asymmetry coefficient, ω_1, can take a positive or negative value, but should be statistically significant.

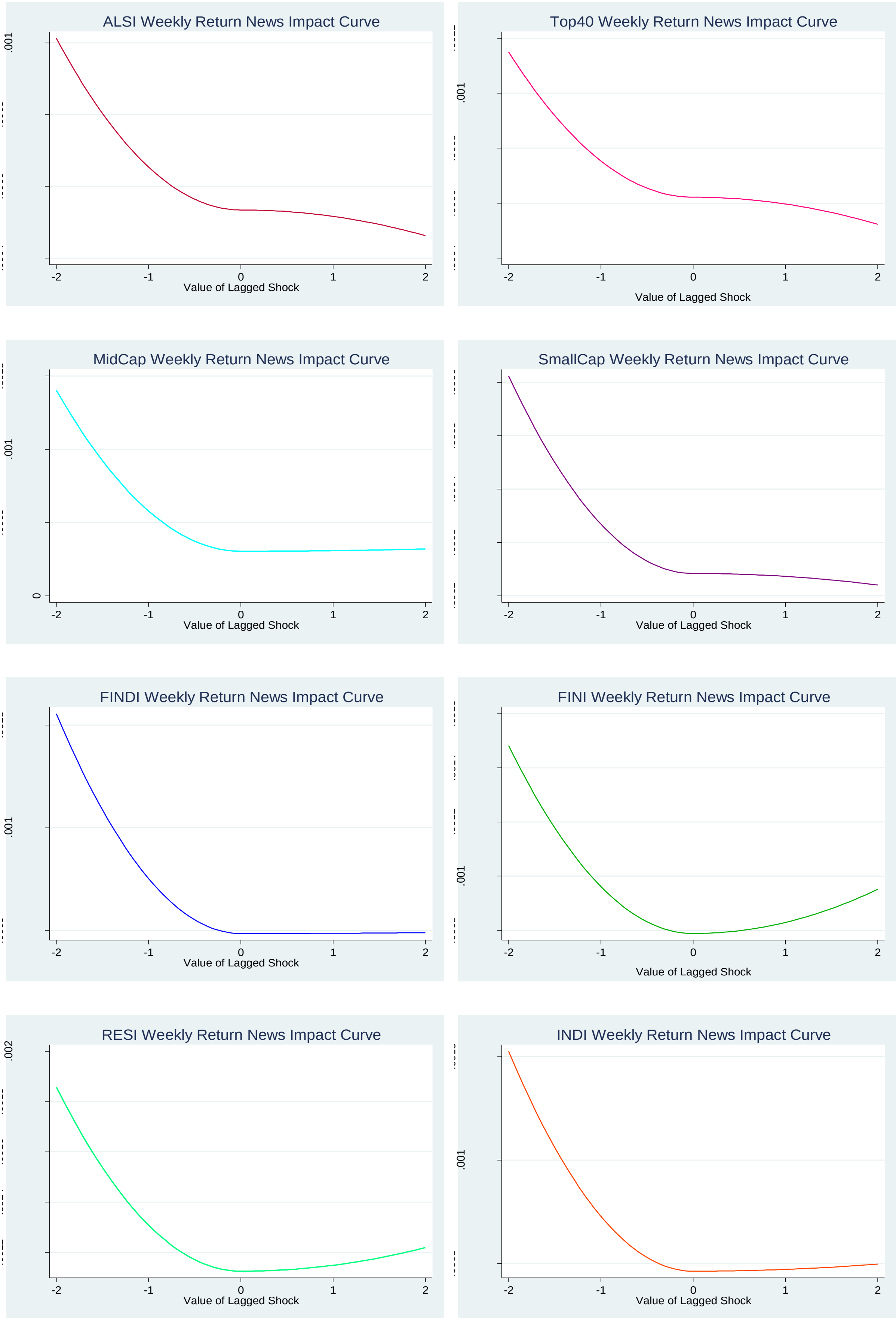
The constant term α_0 is positive and significant for all indices, the serial correlation coefficient $\propto_1$, and β_1, are too, positive and significant for all indices.

Recall that, asymmetry in volatility, caused specifically by the leverage effect would imply that good news, measured by $\varepsilon_{t-1} > 0$, and bad news, measured by $\varepsilon_{t-1} < 0$, will have different impacts on conditional volatility. The GJR GARCH models allows the impact of good news to be captured by $\propto_1$ and the impact of bad news to be captured by $\propto_1 + \omega$. The results indicate that the ω statistic is negative and significant for each of the indices, which implies that asymmetric affects are present in the conditional volatility; however bad news does not necessarily increase future volatility. This result, whilst it acknowledges asymmetry is more supportive of the volatility feedback mechanism of Pindyck (1984) than Black's (1976) leverage hypothesis.

The News Impact Curve developed by Pagan and Schwert (1990) demonstrates the degree of asymmetry in volatility to prior positive and negative return shocks, by plotting the next period conditional variance (h_t) that would arise from a combination of positive and negative values of residual terms (ε_{t-1}), given the estimated GJR GARCH model.

The News Impact Curves of the ANAR GJR-GARCH model are presented below:

Figure 37: News Impact Curve of Weekly Returns



All of the above News Impact curves demonstrate that negative lagged shocks (negative returns as a result of bad news) have a greater effect on volatility than positive lagged shocks. The asymmetric volatility detected in the regression results of the GJR GARCH is visually depicted in these plots. Interestingly the shape of the News Impact Curve has changed from those of the daily return series. The shape of the curve demonstrating the volatility responses to negative shocks has remained the same, however the shape of the curve demonstrating the volatility response to positive shocks appears to have flattened and weakened when measured weekly. This could be an indication that the asymmetric dynamics are fluid and although asymmetry still exists, it is not in the strong form detected in the daily return series. The concept of changing dynamics in the asymmetric relationship was first suggested by Chen & Sauer (1997) and was later supported by Ibrahim (2010) who discovered distinct patterns of asymmetry in different time intervals.

In conclusion, the regression results of the ANAR GJR GARCH model prove that statistically significant asymmetries are present in the weekly returns and weekly volatilities of the stocks of the JSE, despite market capitalisation and industry.

5.4.3. Monthly Return Series

The regression outputs per index are presented in Figure 38 below. The relevant details to consider are the estimates of the coefficients, the respective p-values of the coefficients and p-value of the model itself when fitted to the dataset.

Going into this analysis, one needs to take heed of the fact that in the case of the RESI index, Engle's (1982) Lagrange Multiplier test for heteroskedasticity failed to reject null hypothesis at the 95% confidence interval. This result could allude to homoskedasticity, in which case the GJR GARCH model for this data set is not applicable.

In addition, in the case of the FINI and the RESI index, the probability of the test statistic of the Breusch-Godfrey (1979) test for higher order serial correlation in the residuals is greater than 0.05 and therefore one fails to reject the null hypothesis and no higher order serial correlation is detected in these indices. In this case, it could be possible that there is serial correlation present in the lower lag order, and lower confidence levels.

Figure 38: ANAR- GJR GARCH Regression Results of Monthly Returns

ALSI

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5	Number of obs	=	195
Distribution: Gaussian	Wald chi2(2)	=	16.83
Log likelihood = 335.9143	Prob > chi2	=	0.0002

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.2203685	.1003446	-2.20	0.028	-.4170403	-.0236966
_cons	.009425	.0052577	1.79	0.073	-.0008799	.0197298
ARMA						
ar						
L1.	.3977281	.0987378	4.03	0.000	.2042055	.5912507
ARCH						
arch						
L1.	.6878107	.3134985	2.19	0.028	.073365	1.302256
tarch						
L1.	-.6990733	.3030234	-2.31	0.021	-1.292988	-.1051583
garch						
L1.	.4510188	.1742634	2.59	0.010	.1094688	.7925687
_cons	.0005501	.0002156	2.55	0.011	.0001275	.0009728

Top40

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5	Number of obs	=	195
Distribution: Gaussian	Wald chi2(2)	=	14.26
Log likelihood = 329.2381	Prob > chi2	=	0.0008

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.2753052	.1017917	-2.70	0.007	-.4748132	-.0757972
_cons	.0105942	.0051766	2.05	0.041	.0004483	.0207402
ARMA						
ar						
L1.	.3771125	.0998915	3.78	0.000	.1813286	.5728963
ARCH						
arch						
L1.	.7510077	.2971926	2.53	0.012	.168521	1.333495
tarch						
L1.	-.7541546	.2908648	-2.59	0.010	-1.324239	-.1840702
garch						
L1.	.4304969	.1585346	2.72	0.007	.1197748	.7412189
_cons	.0005799	.0002117	2.74	0.006	.000165	.0009947

MidCap

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5		Number of obs	= 195
Distribution: Gaussian		Wald chi2(2)	= 26.45
Log likelihood =	336.423	Prob > chi2	= 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.2058278	.1172627	-1.76	0.079	-.4356584	.0240028
_cons	.008458	.0057006	1.48	0.138	-.0027151	.019631
ARMA						
ar						
L1.	.4441722	.0874065	5.08	0.000	.2728586	.6154858
ARCH						
arch						
L1.	.3136066	.1377401	2.28	0.023	.0436409	.5835723
tarch						
L1.	-.3279791	.1608916	-2.04	0.041	-.6433208	-.0126373
garch						
L1.	.7531517	.1028795	7.32	0.000	.5515116	.9547918
_cons	.0002164	.0001155	1.87	0.061	-.00001	.0004429

SmallCap

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5		Number of obs	= 195
Distribution: Gaussian		Wald chi2(2)	= 37.97
Log likelihood =	345.2008	Prob > chi2	= 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.0983489	.1221555	-0.81	0.421	-.3377691	.1410714
_cons	.010708	.0056828	1.88	0.060	-.00043	.0218461
ARMA						
ar						
L1.	.4792303	.0914352	5.24	0.000	.3000207	.6584399
ARCH						
arch						
L1.	.2475328	.0867086	2.85	0.004	.0775871	.4174785
tarch						
L1.	-.2045411	.1110801	-1.84	0.066	-.422254	.0131718
garch						
L1.	.771714	.0664812	11.61	0.000	.6414133	.9020148
_cons	.0001535	.0000756	2.03	0.042	5.27e-06	.0003018

FINDI

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5	Number of obs = 195
Distribution: Gaussian	Wald chi2(2) = 25.30
Log likelihood = 332.622	Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.2200423	.1089239	-2.02	0.043	-.4335292	-.0065553
_cons	.0076786	.0052667	1.46	0.145	-.002644	.0180012
ARMA						
ar						
L1.	.4253292	.0887111	4.79	0.000	.2514586	.5991999
ARCH						
arch						
L1.	.4627468	.2202681	2.10	0.036	.0310293	.8944642
tarch						
L1.	-.4656805	.2353992	-1.98	0.048	-.9270545	-.0043065
garch						
L1.	.6393708	.127637	5.01	0.000	.3892068	.8895347
_cons	.0003194	.0001589	2.01	0.044	7.86e-06	.0006309

FINI

ARCH family regression -- AR disturbances

Sample: 1960m3 - 1973m9	Number of obs = 163
Distribution: Gaussian	Wald chi2(2) = 31.32
Log likelihood = 259.6403	Prob > chi2 = 0.0000

y	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf.	Interval]
y						
d1ytminus1	-.2872446	.1068002	-2.69	0.007	-.4965691	-.0779202
_cons	.0010081	.0064714	0.16	0.876	-.0116755	.0136917
ARMA						
ar						
L1.	.4365082	.0780235	5.59	0.000	.2835849	.5894314
ARCH						
arch						
L1.	.309062	.0898437	3.44	0.001	.1329716	.4851524
tarch						
L1.	-.4980001	.1113787	-4.47	0.000	-.7162983	-.279702
garch						
L1.	.8129442	.0675173	12.04	0.000	.6806127	.9452758
_cons	.0003315	.0001206	2.75	0.006	.0000951	.0005678

RESI

The ANAR-GJR GARCH regression was not able to be performed due to a flat log likelihood encountered. This means STATA's maximization procedures failed to converge to a solution and is an indication of the illness of fit of the model to the data.

INDI

```
ARCH family regression -- AR disturbances

Sample: 1960m3 - 1976m5                         Number of obs   =        195
Distribution: Gaussian                          Wald chi2(2)    =      16.59
Log likelihood =  327.0444                      Prob > chi2     =     0.0002

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
  d1ytminus1 |  -.1012245   .1232073    -0.82   0.411    -.3427065    .1402574
       _cons |   .0090139   .0055084     1.64   0.102    -.0017824    .0198102
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |    .382194   .1015616     3.76   0.000     .1831369    .5812512
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .2469229   .1114263     2.22   0.027     .0285314    .4653145
             |
       tarch |
         L1. |   -.205135   .1347249    -1.52   0.128    -.4691908    .0589209
             |
       garch |
         L1. |   .7777401   .0820533     9.48   0.000     .6169186    .9385615
             |
       _cons |   .0001738   .0000995     1.75   0.081    -.0000212    .0003687
------------------------------------------------------------------------------
```

A summary of the above regressions is presented for ease of reference in Table 12 below.

The RESI index did not produce any results as the regression was unable to be performed. This is due to the homoskedastic variance of the model, and the lack of serial correlation in the higher order residuals of the data, as alluded to by Engle's (1982) Lagrange Multiplier and the Breusch-Godfrey (1979) test. In essence, the dataset was not ideal for performing an ANAR GJR-GARCH model as the assumptions of the model[29] were not satisfied.

Prior to examining the statistical relevance of the correlation coefficients, one needs to test whether the stationarity conditions of the model hold, given the estimates produced by the regression.

[29] Namely, non-constant variance and serial correlation in the residuals

Table 12: ANAR-GJR GARCH Results Summary for Monthly Returns

	ANAR-GJR GARCH FOR NOMINAL MONTHLY RETURNS							
Coefficients	**ALSI**	**Top40**	**MidCap**	**SmallCap**	**FINDI**	**FINI**	**RESI**	**INDI**
μ	0.009425	0.0105942	0.008458	0.010708	0.0076786	0.0010081	-	0.0090139
	(0.073)	*(0.041)*	*(0.138)*	*(0.06)*	*(0.145)*	*(0.876)*	-	*(0.102)*
ϕ	0.3977281	0.3771125	0.4441722	0.4792303	0.4253292	0.4365082	-	0.382194
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	-	*(0.000)*
ρ	-0.2203685	-0.2753052	-0.2058278	-0.0983489	-0.2200423	-0.2872446	-	-0.1012245
	(0.028)	*(0.007)*	*(0.079)*	*(0.421)*	*(0.043)*	*(0.007)*	-	*(0.411)*
α_0	0.0005501	0.0005799	0.0002164	0.0001535	0.0003194	0.0003315	-	0.0001738
	(0.011)	*(0.006)*	*(0.061)*	*(0.042)*	*(0.044)*	*(0.006)*	-	*(0.081)*
α_1	0.6878107	0.7510077	0.3136066	0.2475328	0.4627468	0.309062	-	0.2469229
	(0.028)	*(0.012)*	*(0.023)*	*(0.004)*	*(0.036)*	*(0.001)*	-	*(0.027)*
β_1	0.4510188	0.4304969	0.7531517	0.771714	0.6393708	0.8129442	-	0.7777401
	(0.010)	*(0.007)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	-	*(0.000)*
ω_1	-0.6990733	-0.7541546	-0.3279791	-0.2045411	-0.4656805	-0.4980001	-	-0.205135
	(0.021)	*(0.010)*	*(0.041)*	*(0.066)*	*(0.048)*	*(0.000)*	-	*(0.128)*

In order for the stationarity condition of the return dynamics to hold, the return serial correlation, measured by $\phi_1 + \rho_1$ when $r_{t-1} < 0$ and ϕ_1 when $r_{t-1} > 0$, is restricted by $|\phi_1 + \rho_1 D_1| < 1$. The results of this test for each index are as follows: ALSI (0.177), Top40 (0.102), MidCap (0.238), SmallCap (0.381), FINDI (0.205), FINI (0.149) and INDI (0.281), as evidence that the stationarity conditions of the model hold.

In order for the stationarity condition of the GJR GARCH to hold, $(\alpha_1 + \beta_1 + \omega_1) < 1$. The results of this test for stationarity are: ALSI (0.440), Top40 (0.427), MidCap (0.739), SmallCap (0.815), FINDI (0.636), FINI (0.624), and INDI (0.820). As these results are all less than 1, the stationarity condition across all seven indices holds.

The conditions which ensure a positive conditional variance estimation are: $\propto_0 > 0$, $\propto_1 > 0$, $\beta_1 > 0$ and $(\propto_1 + \omega_1) > 0$. Across all seven indices, the $\propto_0$, $\propto_1$, and β_1 estimates are greater than zero at the 90% confidence interval. The results of the condition $(\propto_1 + \omega_1) > 0$ are ALSI (-0.011), Top40 (-0.033), MidCap (-0.014), SmallCap (0.043), FINDI (-0.003), FINI (-0.189) and INDI (0.042).

The results show that the stationary condition of the ALSI, Top40, MidCap, FINDI and FINI do not hold. The SmallCap and INDI index coefficients meet the restrictions of the model.

The results of the ANAR model to test asymmetry in the mean are discussed first.

The results of the regression indicate that the intercept of the ANAR process is statistically significant at the 90% confidence interval for the ALSI, Top40 and SmallCap. Non-significance of the intercept can be interpreted as a mean value of zero when all other coefficients are zero.

The statistically significant estimates of the partial serial correlation coefficient, ϕ_1, is evidence that positive prior returns are relatively persistent across all indices.

The most telling parameter of the regression is the ρ_1 estimate. A ρ_1 of less than zero measures the asymmetric reverting pattern and speed of stock returns. All of the indices produce negative ρ_1 estimates; however, the SmallCap and INDI index ρ_1 estimates are not statistically significant at the 90% confidence interval.

A statistically significant negative p-value of the ρ_1 coefficient coupled with the statistically significant positive ϕ_1, coefficient indicates that the serial correlation between negative returns is smaller than the correlation between positive returns. Furthermore, if ϕ_1 + ρ_1 < ϕ_1 this would imply that positive returns show persistence whilst negative returns show strong reverting tendencies. The results of the above test are as follows: ALSI (0.177 < 0.398), Top40 (0.102 < 0.377), MidCap (0.238 < 0.444), SmallCap (0.381 < 0.479), FINDI (0.205 < 0.425), FINI (0.149 < 0.437) and INDI (0.281 < 0.382).

The results of the ANAR regression show that asymmetry in returns is not a detectable characteristic of the RESI index, however is a statistically significant detectable characteristic of the ALSI, Top40, MidCap, SmallCap, FINFI, FINI and INDI for monthly stock returns on the JSE.

The results of the GJR GARCH regression will now be discussed.

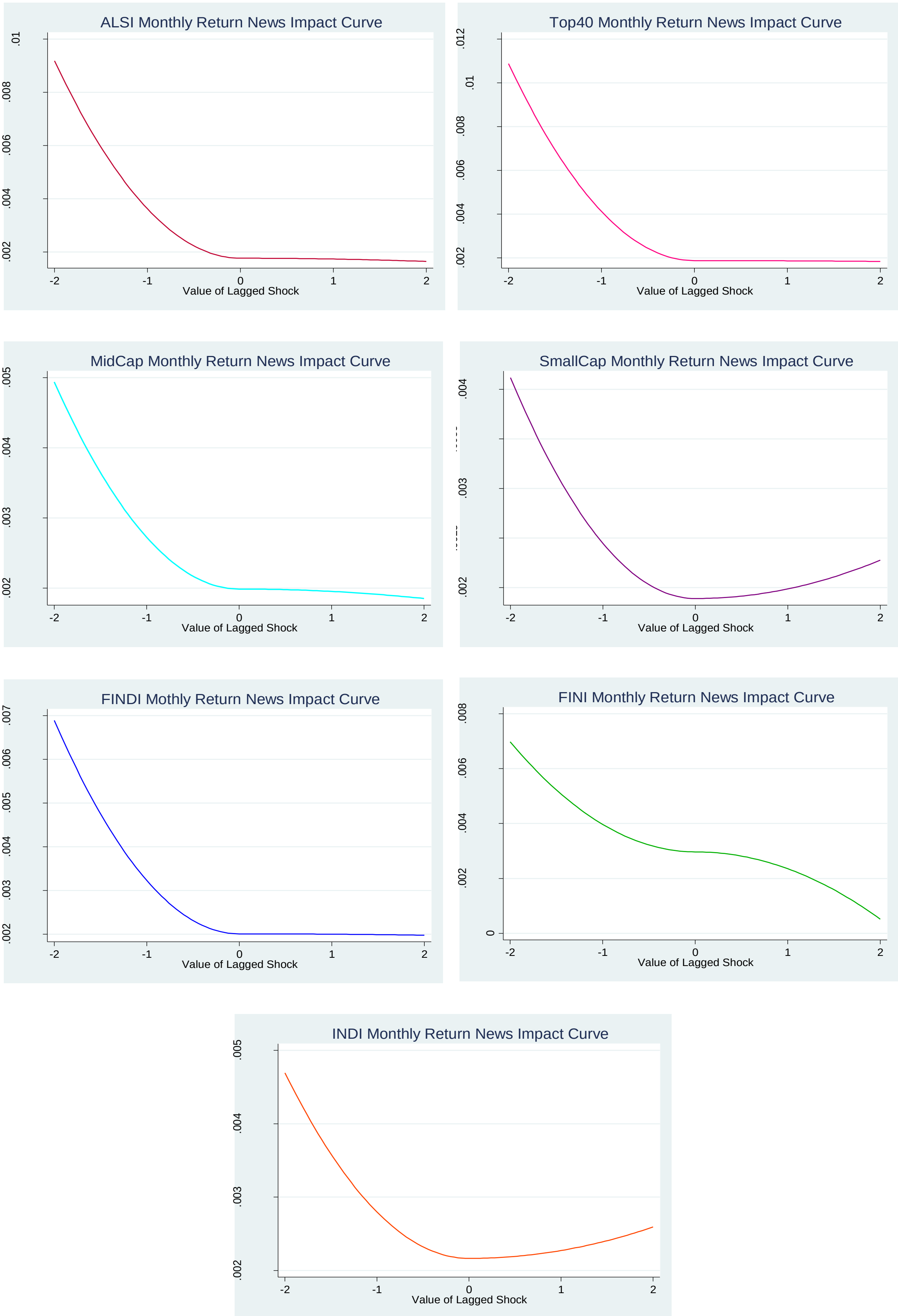
The constant term α_0 is positive and significant for all indices, the serial correlation coefficient $\propto_1$, and β_1, are too, positive and significant for all seven indices.

The results indicate that the ω statistic is negative and significant at the 90% confidence interval for each of the indices, besides the INDI index, which implies that asymmetric affects are present in the conditional volatility; however bad news does not necessarily increase future volatility. This result, whilst it acknowledges asymmetry is more supportive of the volatility feedback mechanism of Pindyck (1976) than the leverage hypothesis of Black (1976).

The News Impact Curve developed by Pagan and Schwert (1990) demonstrates the degree of asymmetry in volatility to prior positive and negative return shocks, by plotting the next period conditional variance (h_t) that would arise from a combination of positive and negative values of residual terms (ε_{t-1}), given the estimated GJR GARCH model.

The News Impact Curves of the ANAR GJR-GARCH model are presented below:

Figure 39: News Impact Curve of Monthly Returns



All of the above News Impact Curves demonstrate that negative lagged shocks have a greater effect on volatility than positive lagged shocks. The asymmetric volatility detected in the regression results of the GJR GARCH is visually depicted in these plots. Interestingly the shape of the News Impact Curve has changed from those of the daily and weekly return series. It appears as though, in the case of the ALSI, Top40, MidCap, Smallcap and FINDI, the conditional volatility does not respond to positive shocks, only negative shocks, given the flatness of the curve. These results were true for these same indices when measured on a weekly interval.

When analysing the regression results on an index basis, it appears as though the ALSI index contains strong evidence that asymmetry in the returns is present in a daily, weekly and monthly time horizon. These statistical results are in line with the basic analysis performed in Section 5.2 on the consecutive runs of positive and negative returns of the daily, weekly and monthly sampling period. This asymmetry is also in keeping with the findings of Plaistowe & Knight (1986) for reversals in weekly returns of the JSE, Page & Way (1992), Cubbin, Eidne,Firer & Gilbert (2006) and Hsieh & Hodnett (2011) for monthly returns of the JSE.

The positive conditional variance condition of the GJR GARCH for the ALSI over weekly and monthly time intervals did not hold, whilst this result is not ideal, this condition is a weakness of the GJR GARCH and often violated in reality (Rodriguez & Ruiz, 2009). The asymmetry in volatility shows signs of a weakening of the impact of positive return residuals on the conditional volatility as evidenced in the shape of the News Impact Curve. Considering these factors, asymmetry in volatility, whilst present across all time horizons appears to be the most significant and strongest in the daily return series. Asymmetry in the market portfolio of daily returns is consistent with the findings of Koutmos (1998), Nam, Pyun & Kim (2003), Doong, Yang & Chiang (2005), Nam, Kim & Arize (2006), Kulp-Tag (2007) and Ibrahim (2010).

In order to examine whether asymmetry in return reversals and conditional volatility was present in specific stocks based on market capitalisation, the Top40, MidCap and SmallCap index were examined. Preliminary examinations on the runs of consecutive positive and negative returns indicated that positive runs were more prevalent than negative runs in a daily, weekly and monthly time interval. The results of the ANAR-GJR GARCH regression indicated that when measured on daily, weekly and monthly time horizons, asymmetry in the mean was present across all three industries for daily and weekly sampling intervals, but on a monthly basis, the estimate of the ρ_1

coefficient was insignificant for the SmallCap index, indicating that the tests for asymmetry in return reversals was inconclusive for this index.

Asymmetry in the conditional variance was present across all capitalisations when measured on a daily, weekly and monthly interval; however the condition of non-negativity of the conditional variance estimate was violated by the Top40 and SmallCap index for monthly returns. The shape of the News Impact Curves point to a change in the dynamic of asymmetry between market capitalisations, which is a finding supported by Ibrahim (2010). Asymmetry at the firm level is a finding supported by Christie (1982), Schwert (1989) and Duffee (2001).

The inclusion of industries examined in this research is intended to reveal whether asymmetry in return dynamics and volatilities is pervasive in specific industries of the JSE. An examination of the positive and negative runs of returns across industries suggested that indeed asymmetric return reversals were present in the daily, weekly and monthly horizons. The regression results indicated that on a daily and weekly return basis, reversals in the mean are statistical significant. Measured on a monthly basis, it appears as though the RESI index does not contain serial correlation and heteroskedasticity and as such no asymmetry was present in the mean or conditional variance. The INDI index appears to produce statistically insignificant ρ_1 coefficient for the ANAR regression, which leads to inconclusive results of asymmetry in the mean for this index.

Asymmetry in the conditional variance was present across the FINDI, FINI and INDI when measured on a daily, weekly and monthly interval; however the condition of non-negativity of the conditional variance estimate was violated by the FINDI and FINI index for monthly returns. Volatility asymmetry across industries is supported by the findings of Mandimika & Chinzara (2010).

5.5. Shortfalls of this Research

Admittedly, there are a number of shortfalls of this research which need to be mentioned.

Firstly, the time period under consideration consisted of 16 years. The Nam, Kim& Arize (2006) academic study this research was modelled on, examined a time period of 41 years. Other authors in the field of asymmetry of the conditional mean and conditional variance, too, had longer time horizons, such as the 20 year dataset of Nam, Pyun & Kim (2003). Whilst the length of the dataset, and more importantly the number of observations considered, depending on the sampling period, play a role in the accuracy of the results, some authors in this field have used smaller datasets and produced statistically significant results, such as Koutmos (1998) who examined a time period of 5 years and Liau & Yang (2008) whose dataset comprised of 11 years.

Secondly, the choice of the GJR GARCH, to replace the EGARCH utilized in the Nam, Kim & Arize (2006) studyto capture asymmetry in the conditional variance does have restrictions. The non-negativity restriction of the model is often violated in reality, and was in certain cases violated by some indices of the dataset of this research report (Rodriguez & Ruiz, 2009). In addition the GJR GARCH model has difficulties in representing high persistence and finite measures of kurtosis and these conditions of the model can act to supress a marginal amount of the leverage effect present in the data (Rodriguez & Ruiz, 2009).

Thirdly, structural breaks in the dataset were ignored. Structural breaks can cause sensitivities in the results. The Nam, Kim & Arize (2006) study accounts for the 1987 October stock market crash in their empirical examination to ensure that the results are jot sensitive to this sample period.

Fourthly, the Nam, Kim & Arize (2006) study extends the ANAR model to contain, two and three consecutive lagged return terms, to confirm the asymmetric reverting pattern across multiple points in time. The extension provided information on the role of the intertemporal relationship between future volatility and expected return. The extension also helped to test whether the asymmetry present in returns was due to the overreaction hypothesis (Nam, Kim, & Arize, 2006).

Lastly, the model was fitted to in-sample data. The forecasting ability of the model for out-of-sampling testing was not conducted.

6. Conclusions

The asymmetric reverting behaviour of the conditional mean and conditional variance of daily, weekly and monthly nominal stock return observations on the JSE, across eight value-weighted indices over a sixteen-year period was explored by this research. Utilizing the ANAR model of Nam, Kim and Arize (2006) to model asymmetry in the conditional mean, and the GJR GARCH model of Glosten, Jagannathan and Runkle (1993) to model asymmetry in the conditional variance, this studypresented an empirical investigation into the asymmetric reverting behaviour of stock returns.

The findings of this research indicate that the strongest asymmetric effects in both the mean and volatility can be seen in short-term horizons, namely daily and weekly time intervals, as supported by Nam, Kim & Arize (2006) and Kulp-Tag (2007). Asymmetry in both the first and second moments is not unique to the market portfolio and found to be statistically significant across market capitalisation and industry during the daily time horizon.

The dynamics of the asymmetry in both the return reversals and conditional volatility change through time. The statistical significance of return reversals on a monthly time horizon weakened and the response of volatility to positive return shocks deteriorates when measured on a monthly basis, as suggested by Ibrahim (2010).

The strong tendency for a quicker reversion of negative returns to positive returns, than positive returns to negative returns, supports the overreaction hypothesis of De Bondt and Thaler (1985) and short-run contrarian profit trading strategies of Nam, Washer and Chu (2005).

Asymmetry detected in conditional volatility was found to support the volatility feedback hypothesis of Pindyck (1984) rather than the leverage hypothesis of Black (1976).

7. Future Research Opportunities

Whilst this research report investigates the nonlinear time varying pattern of stock returns and volatilities it makes no attempt to examine the relationship or link between the observed asymmetry in the conditional mean and asymmetry in the conditional variance.

Few authors have made valuable attempts to classify the causality and connection between the observed asymmetry in the first and second order moments.

Koutmos (1998, p.277) states that “asymmetries in the conditional mean are linked to asymmetries in the conditional variance because the faster adjustment of prices to bad news causes higher volatility during down markets.” The findings of Koutmos (1998) imply that volatility asymmetry increases return reversal asymmetry.

Wang and Wang (2011) on the other hand believe that whilst both asymmetries may originate from similar causes or sources, they do not reinforce each other – one may find strong return asymmetry without strong volatility asymmetry.

There is lacking empirical evidence to support either statement. Whilst many studies have observed asymmetries in both the conditional mean and conditional variance, future research is needed to shed light on the relationship between these two asymmetries, and whether one type causes the other, or whether the nonlinearity can exist independently.

Another suggested avenue of research is the profitability of trading strategies designed to benefit from the asymmetric reverting properties of returns. Nam, Washer and Chu (2005) and Choe, Krausz and Nam (2011) believe that trading strategies based on asymmetry generate a positive return for buy signals, a negative return for sell signals and a positive return for the spread between buy and sell signals. Further research needs to be conducted in this field.

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