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SMALL-GROUP WORK AND OWNERSHIP OF MATHEMATICAL KNOWLEDGE

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Degree awarded with distinction on 28 June 1995

A research report submitted to the Faculty of Education, University of the Witwatersrand, in partial fulfilment of requirements for the degree of Master of Education by coursework and research report.

Johannesburg, November 1994

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Education by coursework at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.



Karin Brodie

1 November 1994

ACKNOWLEDGEMENTS

I would like to thank the following people for their contributions to this work:

My supervisor, Jill Adler, for her guidance and support, and for so skilfully mediating my ownership of this work.

The principal and staff of St Barnabas College, Johannesburg, for allowing me to undertake this study there. In particular, the Std 7 mathematics teachers, who gave of their time to discuss the project with me, Mellony Graven, who gave up a week of her teaching schedule to allow her pupils to work on the tasks I provided, and the pupils who allowed me into their classroom to observe their work and interaction. Although they do not know it, the three pupils whose work forms a focus of this study, have greatly enriched my life. They remain annonymous - their names have been changed in this report.

Muzi Mavuso and Joyce Ndlovu for spending hours with me translating parts of the tapes into English.

My sister Susan for hours spent painstakingly proofreading the text, and my sister Ros for her help with the layout of the text.

My parents, for their unstinting support of all my intellectual efforts, and for their love and support in all other areas of my life.

Dawn, for her love and friendship.

The financial assistance of the Centre for Science Development (HSRC South Africa) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the Centre for Science Development.

ABSTRACT

This study investigates the relationships between small-group work and ownership of mathematical knowledge. In order to explicate these relationships, an account of the interactions of a group of three Std 7 pupils working together for a week on mathematical tasks in their mathematics classroom is presented. The notion of ownership is used to encompass both the elements of democracy and the development of knowledge which are expected to be produced by small-group work. The notion of ownership is elaborated through the research process. The study finds both beneficial and problematic aspects in the interactions of the group. Ownership is seen to be established, but in different ways for each pupil, because of inequalities within the group. A serious mathematical mistake is made and perpetuated throughout the week. It is argued that direct teacher mediation is appropriate to deal with some of the difficulties that occur in the group, and that teaching needs to be conceptualised as integral to learning in small groups.

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CHAPTER 1

INTRODUCTION

The process of reconstructing education in South Africa must include the transformation of classroom practices. The twin goals of developing individual and social potential, and the realisation of democracy (ANC, 1994:3) require approaches to learning and teaching which are "learner-centred and non-authoritarian and encourage the active participation of students in the learning process" (ANC, 1994:69).

Mathematics is a subject which has been particularly characterised by authoritarian "chalk and talk" teaching and the training of pupils in specific procedures for solving a limited range of problems. This has resulted in widespread failure of mathematics education to provide the majority of learners with the knowledge and qualifications to enter scientific and technological careers (NEPI, 1992) or to prepare learners for critical participation in democratic societies (Skovsmose, 1990; Abraham and Bibby, 1988).

Small-group work is one strategy which aims to promote learner-centred and democratic pedagogies, and is beginning to be used more widely in mathematics classrooms in South Africa. Current mathematics curriculum initiatives in South Africa rely strongly on peer-interaction in groups (Murray et al, 1994, 1992; Olivier et al, 1992), and many individual teachers are developing ways of working with small groups in their classrooms (Setati, 1994; McLachlan and Ryan, 1994; Brombacher, 1993).

1. QUESTIONS FROM THE CLASSROOM

I used peer-group work extensively in my secondary mathematics classrooms. From a teaching perspective, I found it challenging and exciting. I was able to gain access to pupils' thinking which stimulated me to reflect on different ways of learning and teaching mathematics. I learned to value pupils' knowledge and ways of expressing themselves, and learned more about mathematics myself in the process. It was liberating for me as a teacher to feel that I was not totally responsible for my pupils' learning, that pupils could and should assume some of the responsibility.

Small-group work also seemed to be beneficial for my pupils. There was usually a productive-sounding, dynamic buzz during these lessons, and pupils seemed to be more actively engaged in learning. However, as I became more experienced and skilled at working with pupils in groups, questions began to arise which warranted more detailed investigation. These can be expressed in four overriding questions:

1. To what extent, and in what ways, does small-group work facilitate better mathematics learning among secondary school pupils?
2. To what extent does small-group work enable democratic ways of working in classrooms?
3. How do pupils work together for extended periods of time without the teacher?
4. How do teachers mediate groupwork?

These are the questions from practice which underpin my study.

2. DEVELOPING A RESEARCH QUESTION

Research in mathematics education argues convincingly that small-group work can help foster mathematical development. Small groups create time and space for individual pupil participation in discussion and activities and hence opportunities for the construction of knowledge. Interaction with different points of view increases possibilities for conceptual development (Hoyles, Sutherland and Healy, 1991; Wood and Yackel, 1990; Hoyles, 1985). Encouraging pupils to talk stimulates reflection and the refining of ideas (Pimm, 1991, 1987; Barnes and Todd, 1977; Barnes, 1976; 1969).

More generally, classroom-based research has shown that teachers' control over knowledge and discourse in classrooms can be detrimental to learning (Cazden, 1988; Edwards and Mercer, 1987; Barnes, 1969). Peer groups are expected to provide more equality in interaction and to allow pupils more control over the learning situation and the knowledge developed. Small groups facilitate the opening up of mathematics classrooms to different points of view, the acknowledging and valuing of pupils' knowledge, and provide pupils with opportunities to learn to value their peers' ideas and experience. This is particularly important in building a unified and democratic society given the increasingly diverse backgrounds of children in the same classroom.

Democratic ideals need to be carefully considered within a South African context. Peer interaction is often presented unproblematically as supportive. The complex social, emotional, and political relationships between pupils are not dealt with. These will be present in any classroom. In addition, most South African pupils' experiences of life outside the

classroom are of a conflictual and violent society pervaded with discrimination. These experiences will influence interactions in the classroom. Moreover, secondary school pupils have expectations of classroom practice and activities and have developed ways of learning mathematics based on their previous experiences of schooling. For most pupils in South Africa, this experience is of under-resourced and authoritarian schools. If classrooms are to become environments where small-group work does facilitate conceptual development and democracy, these constraints need to be carefully considered.

In structuring a research question which would enable investigation of both conceptual development and democracy, I have used the notion of "ownership". This word is used loosely in the literature to signify the control that an individual asserts over learning and the knowledge developed. I will use "ownership" in preference to "learning" or "understanding", because it is able to capture the cognitive, emotional and political processes that are part of interactions in small groups both with and without the teacher, and which are crucial to the development of knowledge and democracy in the classroom.

So my research investigates the relationships between interactions in small groups and ownership of mathematical knowledge. The notion of ownership has been elaborated in the research process, through the empirical data and the theoretical concepts used to analyse and interpret the data.

3. THE STUDY

An appropriate way to begin to answer the questions raised above is an in-depth investigation of the interactions of pupils engaged in small-group work. My study focuses on the talk of one group of standard 7 pupils as they work together on a mathematical task in their mathematics classroom. My analysis of their talk attempts to understand the ways in which their working together facilitates ownership in their learning. The ways in which the teacher works with the group is also analysed, as a crucial aspect of group interaction and ownership.

4. THE REPORT

This report begins with the establishment of a theoretical framework for the analysis, drawing on Vygotsky's socio-cultural theory of development, learning and teaching. It then relates the research questions to the existing literature on peer-interaction and mathematics learning. The research methodology and design are discussed, followed by an account of the pupils' talk and interaction. The account focuses on ownership in the learning process, namely, learner control, democracy and the development of mathematical knowledge, and simultaneously elaborates the notion of ownership. It looks at the conflicts generated within the group and the consequences for ownership. It then examines the role of the teacher in mediating the groupwork and the development of mathematical knowledge. The report ends with a discussion of implications for classroom practice.

CHAPTER 2

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This research is informed by a range of theoretical and empirical work concerned with cognitive development, classroom interaction, and the learning and teaching of mathematics. In this chapter I will discuss Vygotsky's theory of sociocultural development, and review the literature on peer interaction in classrooms. This will establish a framework within which I will begin to elaborate the notion of ownership and analyse the relationships between interaction in a small group and ownership of mathematical knowledge.

1. VYGOTSKY'S SOCIOCULTURAL THEORY

Vygotsky posits a theory of psychological development of the individual through social interaction within cultural-historical contexts. One of the most important of these contexts is schooling. Thus his theory provides a useful theoretical framework for a study of small-group interaction and ownership in a mathematics classroom in South Africa. Furthermore, Vygotsky emphasises the interrelatedness of developmental processes and products, which will be important for understanding the relationships between teaching and ownership.

1.1. SOCIAL INTERACTION AND HIGHER PSYCHOLOGICAL PROCESSES

The central tenet of Vygotsky's theory is that the higher psychological processes (voluntary memory and attention, abstract thinking and logical reasoning) are formed through social

interaction - social processes are a source of the higher mental functions (Vygotsky, 1991, 1978; Wertsch, 1991; Hickmann, 1985). Vygotsky states: "all the higher functions originate as actual relations between human individuals" (1978: 57) and formulated the genetic law of cultural development:

> "An interpersonal process is transformed into an intrapersonal one. Every function in the child's cultural development appears twice: first on the social level, and later on the psychological, first between people (interpsychological) and then inside the child (intrapsychological)." (1978:57)

Central to this development of the higher psychological processes is mediation by psychological tools, or signs. These include counting systems, mnemonic devices, symbol systems (e.g. algebra), writing, diagrams, maps, but most particularly speech (Wertsch, 1991). Speech mediates both social interaction and individual thinking processes. By using language for communication the individual internalises it for use as an intrapsychological tool which mediates thinking.

The signs which mediate interaction and thinking are socioculturally situated (Wertsch, 1991, 1985), as they are historical products of the cultural development of the species in general, and of particular social contexts. Hence, even when speech is internalised and used by the individual on her own, for her own purposes, it continues to connect her thinking to her sociocultural contexts.

Although Vygotsky writes primarily about social interaction between children and adults, small groups of children interacting with each other provide opportunities for verbal communication within a school context, and hence the potential for internalising speech as a tool for thinking.

1.2. SCHOOLING AND THE ZONE OF PROXIMAL DEVELOPMENT

An important function of schooling is to induct children into the structured sign and knowledge systems of their society[1]. This occurs through instruction, conceptualised by Vygotsky as a relationship between a learner and a more knowledgeable other. Vygotsky formalised this relationship with his construct of the zone of proximal development (ZPD) which he defined as:

> "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." (1978:86)

The notion of the ZPD has been elaborated conceptually and empirically, primarily in situations of adult-child interaction outside of school (Tharp and Gallimore, 1991; Wertsch and Hickmann, 1987; Wertsch, 1984; Greenfield, 1984). While it is not obvious how to translate successful out-of-school teaching and learning practices into the very different context of the classroom (Tharp and Gallimore, 1991; Zuell, 1986), some important aspects of the learning process, which can help us to understand classroom processes, have been illuminated.

Wertsch (1984) elaborates on the negotiation of intersubjectivity between the adult and the child. At the beginning of a problem solving or instructional interaction, the teacher and child

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[1] Other functions of schooling include the reproduction of social relations and the maintenance of social cohesiveness and control (see for example Apple, 1985). These considerations are beyond the scope of this study.

have different intrapsychological situation definitions - they understand the task to be achieved differently. Through the interaction, particularly through the adult's talk (verbal mediation) in relation to the child's actions, they reach a mutual, intersubjective situation definition. This represents learning on the part of the child, a redefinition of the task in a way that more closely resembles the adult's definition.

For my purposes, it will be important to consider the extension of these ideas in relation to: 1. the negotiation of intersubjectivity among peers in small groups; 2. the development of mathematical concepts in the classroom; and 3. the teacher's interaction with the learners as mediator of group processes and mathematical knowledge.

1.2.1. *Peer interaction and the ZPD*

According to Wertsch (1984), understanding the different situation definitions of the teacher and the learner requires analyses of the task that each is attempting in terms of their goal-directed actions. In a group of peers we can expect a number of competing goals. These need to be negotiated and co-ordinated. However, in many peer-interaction arrangements (for example collaborative learning, which will be discussed below), none of the participants may have the necessary metaknowledge of the task and the others' task definitions, which would enable them to play a teaching role in negotiating intersubjectivity. Moreover, the adult in Wertsch's analysis represents the sociocultural knowledge, and mediates this in interaction. Unless the situation is specially structured (for example, in peer-tutoring arrangements), pupils will not have the mathematical knowledge required to mediate in this way.

At the same time, there is an assumption that the adult or teacher has the necessary access to the mathematical knowledge and to the pupils' situation definitions. This is not always the case. How the teacher relates her (and society's) mathematical knowledge to that generated within the group, is an important consideration in small-group work.

1.2.2. *Spontaneous and systematic mathematical knowledge*

Central to Vygotsky's conception of schooling and the ZPD is his distinction between spontaneous and scientific (or systematised) concepts (Vygotsky, 1986). Spontaneous concepts are developed in everyday interaction and thus are imbued with meaning derived from the life of the child. Scientific concepts develop through formal instruction and form part of a knowledge system. Their meaning derives from being part of this system as well as from instantiation in everyday concepts. Systematised concepts permit flexibility of thinking and generalisation of meaning and provide control and structure for unsystematised everyday concepts.

The relationship between spontaneous and scientific concepts is to be found in the ZPD. Vygotsky writes:

> "These two conceptual systems, developing "from above" (scientific) and "from below" (spontaneous), reveal their real nature in the interrelations between actual development and the zone of proximal development. Spontaneous concepts that confront a deficit of control find this control in the zone of proximal development, in the co-operation of the child with adults."
> (1986:194, quotes in original, my brackets)

The synthesis of spontaneous and scientific concepts forms true concepts (Vygotsky, 1986), which are flexible, and meaningful in relation to individual experience and generalised systems of knowledge.

It is clear that mathematical knowledge forms a system of scientific concepts[2]. However, children develop many spontaneous mathematical concepts in everyday life. Research into everyday concepts (Saxe et al, 1993; Carraher and Schliemann, 1985; Carraher, Carraher and Schliemann, 1985) has shown that they remain unseen, unheard, and even blocked by school mathematics, and that a synthesis into true concepts rarely occurs successfully in schools.

Small-group discussion is one way of bringing spontaneous concepts into the classroom. Being removed for periods of time from the teacher's situation definitions and mathematical concepts, gives children the space to express their own ideas in their own words. However, their ideas need to be brought into connection with scientific concepts. This suggests a further role for teaching: not only is it the mediation of the scientific concepts of society, it also facilitates interaction between spontaneous and scientific concepts.

[2] Exactly what constitutes the scientific concepts of mathematics is contested. A detailed discussion of the debates is beyond the scope of this report (see Ernest, 1991; Bishop, 1988; Davis and Hersh, 1981 for such discussions). For the purposes of this project, I assume that mathematics is a socially constructed body of conceptual knowledge, open to debate and interpretation. The purpose of teaching mathematics is to make the concepts, together with the debates and different interpretations, accessible to pupils.

1.2.3. *Teaching, learning and development*

The centrality of teaching in this regard is evident in Vygotsky's assertion of the close relationship between learning and development (Vygotsky, 1978). Vygotsky posited the ZPD as a challenge to the perspective, prevalent at his time (and still today), that real learning can occur only after the prerequisite development has taken place. It is a view made prominent in education by Piagetian psychology (Edwards and Mercer, 1987), and suggests that teaching is either irrelevant to development, or worse still, may inhibit it.

Vygotsky (1978) considered instruction (a teaching-learning relationship) to be essential for development. The notion of levels of potential development, represented by the teacher's situation definitions, and reached and maintained with her assistance, asserts that learning can and should precede development, and that teaching is central to learning. He argues that if we teach to current developmental levels, we condemn the child to remain at these levels. If we want the child to grow, we need to be constantly in advance of her development.

This reflects the interdependence between process and product in Vygotsky's theory. The Piagetian view presumes that if we understand the processes of learning and development and work in concordance with them, the products (knowledge) will follow unproblematically. We cannot teach the processes nor the products, but can only facilitate their development by providing appropriate experience (Miller, 1989). In contrast, Vygotsky (1978) asserts that the development process and its products (concepts and the higher psychological processes) are intimately related - the products depend on the nature of the process. Development, and hence the higher psychological functions and concepts, can be lead by instruction within the ZPD.

So if the aim of schooling is the development of the higher psychological processes and true concepts, and peer interaction in small groups is a strategy to achieve this, the interaction of the teacher with the groups must be a central consideration.

1.3. INTERNALISATION

The development of the higher psychological processes and true concepts through mediated social interaction requires internalisation. Internalisation is the mechanism through which the ZPD operates. The process of internalisation is a complex one, and does not consist of mere imitation of the external (Cazden, 1988). Vygotsky states:

> "adults, through their verbal communication with the child, are able to predetermine the path of the development of generalisations and its final point - a fully formed concept. But the adult cannot pass on to the child his mode of thinking." (1986:120)

Internal processes do not mirror external ones. Leont'ev writes:

> "the process of internalisation is not transferal of all activity to a pre-existing plane of consciousness; it is the process in which this internal plane is formed." (1981:57, quoted in Cazden, 1988)

So internalisation involves a transformation of the external. It is the mechanism through which social relations and interpsychological knowledge are transformed to become intrapsychological thinking processes and knowledge.

1.4, SUMMARY

The development of intrapsychological thinking processes and true concepts are important goals of mathematics education in schools. A Vygotskian framework asserts that mediation (particularly verbal mediation) through teaching-learning relationships is central to this process. In many schooling contexts, particularly in South Africa, this relationship has become an overly authoritarian one, and learners' spontaneous concepts have been excluded from classrooms. Small-group work should facilitate both the presence and the development of spontaneous concepts, while their connections with scientific concepts and the subsequent development of true concepts need to be mediated by the teacher. The goal-directed processes involved in peer interaction, with and without the teacher, are also transformed and internalised as mediators of the higher psychological functions.

I have shown how a Vygotskian framework provides for an analysis of interaction between peers and the teacher in realization of the goal of the development of mathematical knowledge. In the next section I will present a review of empirical work on interaction between peers, with specific reference to learning mathematics.

2. PEER INTERACTION IN SMALL GROUPS

There is a substantial body of research into peer interaction and learning. Most researchers agree that social interaction, beyond mere teacher talk and pupil listening, is important for learning and development, for more open classrooms, and for increased pupil control over learning and knowledge. However, the research is informed by diverse theoretical and

methodological underpinnings, and attempts to answer a variety of questions regarding learning, development and classroom practice. Thus only particular aspects will be useful for my purposes. A survey of this body of research, in relation to perspectives on: 1. cognition; 2. conflict, power and equality; and 3. the role of the teacher, will serve to locate within it, my questions regarding the development of knowledge and democracy in small groups with and without the teacher.

2.1. PEER INTERACTION AND COGNITION

Two well-known approaches to studies of peer interaction are the co-operative and collaborative approaches (Saxe et al, 1993; Damon, 1990; Damon and Phelps, 1989). The notions of cognition and learning which underpin these two approaches determine the kinds of tasks employed and the nature of the interaction that is expected to occur.

The motivation for co-operative learning is to move away from the competitiveness and individualisation of classroom life. Pupils are encouraged to teach each other, and incentives are usually provided in such a way that it is the group's responsibility to ensure that all members understand all aspects of the task, and are able to pass tests or complete individual tasks at a later point. The tasks tend to be procedural in nature, and closely connected to the school syllabus (Saxe et al, 1993; Slavin, 1991). The strength and appeal of this work is that it is classroom-based. However, although it seems to have a social emphasis, the implicit notion of learning is that it is primarily an individual activity and amounts to the acquisition of skills and content.

This research provides some evidence for positive effects on school achievement, motivation and social relationships, including interracial relationships, resulting from co-operation between pupils (Davidson and Kroll, 1991; Slavin, 1991). Important factors have been identified as implicated in this success. These include the goal and accountability structures (Slavin, 1991), the composition of groups, and the amount of help given and received (Webb, 1991, 1989). I have included these considerations in my sampling procedures and analysis (see chapter 3). However, the lack of a social-cognitive theory of learning and development underpinning most of this work, and the emphasis on procedural tasks, limits its usefulness for my purposes.

Collaborative learning usually refers to situations where children interact on a task which none would be able to solve alone, but on which they make substantial progress together. Children usually work in pairs with an apparatus which forces them to work together, even though some division of labour is possible (Forman, 1989; Glachan and Light, 1982). It is expected that children will generate problem-solving strategies rather than learn pre-existing material. The research is primarily "laboratory" based, conducted outside of the classroom, under different conditions, and the tasks used are different from what pupils would generally do in class.

Collaborative learning has its roots in Piagetian theory, which asserts that conflict between learners' existing ideas and new information that they come into contact with is necessary for cognitive development. Social interaction can create such conflict, but is considered no more likely to do so than other factors, particularly individual action on the environment. Thus the source of knowledge and development is still seen as primarily individual.

More recent work on collaboration has drawn on a Vygotskian framework (Forman, 1992; 1989; Forman and Cazden, 1985). This research emphasises a co-operative approach, rather than a contradictory or conflictual one, as more prominent and effective in peer interaction. (Ellerton and Clements, 1991; Forman and Cazden, 1985). Learners are seen to incorporate aspects of their partners' situation definitions and ideas. Instead of merely restructuring individual ideas, a "temporarily shared social world is co-constructed" (Forman, 1989:59), through the negotiation of goals and the achievement of intersubjectivity.

Research that integrates Piagetian and Vygotskian theoretical concepts argues that a number of processes will be important in peer groups. These include the resolution of conflicting points of view (Hoyles et al, 1991; Wood and Yackel, 1990), together with verbalising ideas for others which provides a critical distance from which ideas can be reconsidered (Hoyles et al, 1991; Hoyles, 1985; Barnes and Todd, 1977). This potentially contributes to the development of verbally-mediated thinking.

These collaborative and interactive processes will inform my analysis of a particular small group at work. However, the collaborative approaches to peer interaction can be described as process - rather than product - oriented. A full account of the efficacy of peer interaction in schools must also focus on the conceptual products of the learning process, the mathematical knowledge that is developed.

One cognitively informed research project that does focus on the mathematical content of schooling is that of Pirie and her partners (Pirie, 1991; Pirie, Newman and Schwarzenberger, 1989; Pirie and Schwarzenberger, 1988). Their long-term project is to establish a "causal

relationship between mathematical discussion and mathematical understanding". Pirie argues that mathematical understanding is multi-dimensional, and related to the processes through which it is achieved. Thus an apparently instrumental response, when viewed as the automation of previously relational conceptualisations, is an important part of mathematical understanding (Pirie, 1988). This resonates with Vygotsky's claim that to understand the higher psychological processes, we have to understand how they came to be achieved (Vygotsky, 1978). However, Pirie's work departs from a Vygotskian framework when it separates effective use of language from understanding, the claim being that the pupils' language is evidence of understanding rather than part of it (Pirie and Schwarzenberger, 1988).

In my study, the integral nature of process and product in mathematical understanding will be brought to bear on the development of relationships between scientific and spontaneous concepts, and the ultimate formation of true concepts.

However, there is an important aspect of my study which all of the research discussed above ignores. Power relations between pupils are excluded, for example, in Pirie's work, by explicitly excluding from the analysis discussions where some pupils remain passive (Pirie and Schwarzenberger, 1988). Thus my notion of ownership, which will be developed in the final section of this chapter, will include, but go beyond notions of mathematical understanding. It will also include emotional aspects of learning, and power relationships between pupils. These will be discussed in the following section.

2.2. EQUALITY AND CONFLICT IN PEER INTERACTION

Teaching approaches which put children in situations where conflictual or co-operative behaviour is expected and promoted, need to consider how this behaviour might affect and be affected by pupil goals, beliefs and relationships with each other. Saxe et al (1993) state: "we would imagine that peers have varying beliefs about conflict and varying investments in resolution that emerge in the activity setting" (p. 120).

Most of the research on co-operative learning does not explicitly take such positioning into account, primarily because the reward structures are designed to encourage co-operation rather than competition and conflict[3]. Collaborative learning, on the other hand, assumes that children will pursue cognitive goals while ignoring social-relational or emotional situations. Moss (1992) discloses this problem:

> "the adult in ... the adult-child dyad as a problem solving system could be, for all intents and purposes, mother, father, teacher, or detested great aunt whereas the peer in studies of peer influences could be one's best friend, worst enemy, or the girl in front whose pigtails one regularly pulls ... It is evident that different relationship contexts fundamentally will affect collaborative cognitive endeavours with respect to the participants' motivations for engaging in joint activity and how responsibility is shared during interaction." (1992:118)

There is an approach to classroom research which does focus on power and control in the classroom. This research can broadly be termed sociolinguistic (see, for example, Campbell,

[3] An important part of the research on co-operative learning is the assessment of interracial relations among group members. Slavin (1991) summarises research results which show that co-operative learning arrangements positively influence children's friendship choices across ethnic groups. No mention of conflict is made.

1989; Cazden, 1988; Voigt, 1985; Wilkinson, 1982; Mehan, 1979; Stubbs and Delamont, 1976). Much of the emphasis of this research is on teacher-pupil interaction as this is where power relationships are most evident. Most of the research is not informed by cognitive theory (Cazden, 1988, is an exception), so its usefulness for my purposes is in raising questions of power and control, and pointing to the ways in which these relationships are established and maintained in classroom discourse.

Studies that attempt to integrate cognitive perspectives with a sociolinguistic approach provide evidence that many pupils can and do interact in ways which enhance their learning, while others often struggle with the social relationships involved, and are less successful (Cooper, Marquis and Ayers-Lopez, 1982; Barnes and Todd, 1977). Different ways of talking and interacting are seen either to enable or to constrain learning (Phillips, 1985; Barnes and Todd, 1977).

In a study of pairs of children working on primary school mathematics, Cobb (1994) argues that learning opportunities are more frequent for pairs where a taken-as-shared basis for mathematical communication is established and neither child is established as the mathematical authority of the group. He distinguishes mathematical authority, when one child's mathematical knowledge dominates, from social authority, where one child regulates the kinds of interaction that occur. Social authority is seen to be important in establishing the taken-as-shared basis for communication, i.e. intersubjectivity, whereas mathematical authority directly influences possibilities for learning. The relationships between mathematical and social authority are not analysed.

Forman (1989) integrates both social and mathematical authority. Working from a Vygotskian perspective, she analyses the interactions of a pair of pupils working on a task. Each partner in the pair takes responsibility for different aspects of the task, which reflects their individual situation definitions. However, during the time in which they worked together "each subject's explanations incorporated characteristics of her partner's stance" (p. 66) and "both girls took over more and more responsibility for the role that had originally been assumed by the other ... they became more involved in both roles" (p. 67). Thus she shows that a measure of intersubjectivity can be achieved. However, she warns that:

> "In other collaborative relationships where power and knowledge are not equally distributed, an asymmetric division of labour may be seen. Some of these situations may result in a form of peer-tutoring if the more powerful is also the more knowledgeable. For some dyads, however, the more powerful is not the more knowledgeable. In these situations, the educational potential of collaborative learning is greatly diminished." (1989:68)

The above two studies reflect an increasing concern to understand what actually happens when children interact on mathematical tasks, rather than merely to record outcomes, as the co-operative and collaborative approaches (discussed in the previous section) have tended to do. More detailed, "dynamic" (Forman, 1989) analyses do access the different positions that children establish in small-group interaction.

However, none of the research discussed in this section documents the conflicts that arise as a result of the dominance of some knowledge and agendas and the exclusion of others when peers interact, nor the emotional and political consequences of such conflict in addition to cognitive ones. An important reason for the exclusion of this dimension may be the assumption that the most obvious and damaging power-relationships in classrooms exist

between the teacher and the pupils, and not between the pupils themselves. Peer interaction is seen as an alternative to teacher domination.

In this study, I will explore an important counter to this view, that there is substantial inequality between peers and this can have serious ramifications for control, and hence ownership of knowledge. In particular, consequences for the development of true mathematical concepts will be considered. This re-asserts the teacher's role as mediator of both interactive processes and mathematical development. I now turn to more detailed considerations of teaching and peer interaction.

2.3. PEER INTERACTION AND THE TEACHER

Most of the research on peer interaction cited above does not include considerations of the teacher's interactions with the groups (for example: Forman, 1992; Hoyles et al, 1991; Phillips, 1985; Forman and Cazden, 1985; Glachan and Light, 1982; Barnes and Todd, 1977). The ways in which teacher-pupil relationships are established and maintained have been documented by the sociolinguistic research referred to in the previous section, and this research has highlighted the potentially damaging nature of these relationships (Edwards and Mercer, 1987; Barnes, 1969).

A South African research project (RUMEUS) which is implementing and researching a new curriculum in primary schools argues very strongly that teacher intervention in children's discussions should be limited (Murray et al, 1992), because of the beliefs that exist among both pupils and teachers that the teacher's knowledge is unquestionable and should be imitated. Peer interaction is then seen as an alternative to teacher support:

> "... confronting pupils with difficult problems that they are expected to solve without help from the teacher must necessarily create tension, and tension is not conducive to optimal cognitive functioning. Establishing alternative support systems in these classrooms is therefore essential for the learning process, and takes the form of peer interaction ... the less teacher *direction* there is in setting up peer interaction groups or in the questions to be discussed, the higher the quality of the discourse among these young pupils."
> (Murray et al, 1994:126, emphasis in original)

And further:

> "Students, even very young students, should not only be trusted to construct their own mathematical knowledge, but should also be trusted to evolve the social structures that they need for optimal learning."
> (Murray et al, 1992:10)

The two questions raised in the previous section need to be considered here. Firstly, is peer interaction so unproblematically supportive? Secondly, how do children develop responsibility for their own learning? Responsibility requires the metacognitive processes of reflection and control over knowledge, and thus the achievement of responsibility is itself a developmental process, with all the partial achievements and setbacks that development entails. It also needs to be engaged within the zone of proximal development. Thus teacher interaction is crucial in this regard, and should not be seen as an inhibiting factor.

How the teacher balances the two aims of helping to develop responsibility and control, and mathematical knowledge, is an important issue for small-group work. A United States research program (Cobb, Yackel and Wood, 1992; Wood and Yackel, 1990) begins to point to the dynamics of this role:

> "the teacher is the only member of the classroom community who can assess which of the students' contributions constitute a productive basis for further learning. One facet of the teacher's active and demanding role is therefore to facilitate mathematical discussions between students while at the same time acting as a participant who can legitimise certain aspects of their activity and sanction others."
> (1992:102)

Facilitating the discussion involves the teacher modelling aspects of the discourse for the pupils, for example expressing disagreement and explaining and justifying solutions (Wood and Yackel, 1990; Cobb, Wood, Yackel and McNeal, 1992). Many of the transcripts from this research show the teacher repeating comments of the children, or using other means to imply that further response is needed, without actually contributing to the mathematics. However, the research in the previous section suggests that greater mediation of the group processes may be necessary.

Legitimising the mathematics is less clearly elaborated, other than that it involves the "explicit negotiation of mathematical meanings" (Cobb, Yackel and Wood, 1992:102). The teacher tries to understand the constructions of the children and acts accordingly, providing the appropriate challenges and support.

A danger, which both the South African and American research projects want to avoid, is that the teacher's mathematisations come to be accepted as the most (or only) valid ones, by virtue of her authority. She treads a fine line between using the power that her superior knowledge brings, to guide the development of mathematical knowledge in pupils, and creating or perpetuating destructive power relations, and hence limited ownership[4].

How the teacher might achieve both the development of responsibility in the pupils and the development of true mathematical concepts is a further focus of this study. The aim of developing responsibility and control has been encapsulated as part of a notion of ownership on the part of the pupils. In the next section I will begin to elaborate on this notion.

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[4] These power relations are not only perpetuated within the classroom. It is a question as to what extent changes in teacher-pupil power relationships can occur without corresponding societal changes in adult-child relationships.

3. THE NOTION OF OWNERSHIP

Psychologically oriented research on social interaction considers the relationships between talk and interaction; and learning, development or understanding (e.g., Hoyles et al, 1991; Pirie et al, 1989, 1988; Barnes, 1976). I have pointed to some of the complexities of these relationships, particularly when teaching is considered. Sociological research adds the dimension of power relations in the classroom (e.g., Cazden, 1988; Edwards and Mercer, 1987), a dimension which cannot be separated from psychological factors if we are to understand fully small-group work in mathematics classrooms. So, in trying to establish a framework from which to examine small-group work, I found the psychological constructs of learning and understanding to be limited, in that they did not capture the unity of the social, the political, and the psychological.

I therefore chose to work with a notion of "ownership", in an attempt to bring together the notions of development and democracy in the classroom, and to include cognitive, political and emotional aspects of learning and interacting. This notion brings its own complications, in that it is usually used loosely in the literature and needs to be more clearly elaborated for my purposes. However, attempts to elaborate it too carefully before the data analysis created a situation where a theoretical construct might distort the actual evidence obtained. Thus much of my original conceptualising was discarded in favour of a concept grounded in the data. This section outlines my notion of ownership, in relation to current usage. The analysis that gave rise to it will be discussed in detail in chapter 4.

The word ownership is often used in the literature, although it is not clearly elaborated. Children owning mathematical knowledge is presented as an alternative to mere imitation of

the teacher's or text's mathematisations, and suggests a change in the locus of control and responsibility for the acquisition of knowledge.

Cognitive processes are seen to be an important part of ownership. Among such processes I include: pupils' posing problems which they are interested in solving, making their own decisions about how to solve them, and evaluating their own solutions and problem-solving processes.

Emotions are another important aspect of ownership. Ellerton and Clements (1991) speak of appreciation, satisfaction and pride which participants in a workshop (teachers) felt when they had developed their own problem-solving strategies. Crawford (1992) comments on the written responses of pupils to a relatively unstructured project which they found difficult at first, but on which they made substantial progress: "most felt secure and proud of the work they had done ... the quality of their responses reflected their ownership of the activities they were undertaking and a clearer understanding of their goals". One student wrote, "I felt fabulous and proud that we made this creation" (pp. 21-22). This is to be expected, as a full account of learning and development cannot ignore emotions (Salzberger-Wittenberg et al, 1983). Negative, as well as positive emotions accompany any attempts to learn, and small-group learning involves these emotions in addition to the emotions that accompany social interactions.

In the mathematics education literature, ownership is discussed most explicitly by Ellerton and Clements (1991). They identify ownership with the construction of knowledge according to a radical constructivist, neo-Piagetian, paradigm. The active meaning- and connection-making of the learner is crucial. Ownership is evident when the learner makes comments

such as "I *know* this because *I* worked it out *myself*" (p. 56, emphasis in original). The assumption that direct teaching cannot facilitate this process and in fact might hinder it has been discussed previously.

In investigating the notion of teacher control and the subsequent tensions that arise in classrooms between ownership and content acquisition, Ellerton (1991) sets up a continuum according to the amount of initiative the pupil is allowed in choosing her own content and ways of learning (Ellerton, 1991; Ellerton and Clements, 1991). Such initiative is seen as an important index of ownership. Classrooms where there is less teacher control are expected to promote more self-motivated, self-directed and self-regulated learning on the part of the pupils. In one particular classroom children bore total responsibility for posing mathematical problems and deciding how to investigate them and with whom. The teachers (there were two) in this classroom defined their roles as doing their own mathematics and in this way modelling for the children what being a mathematician is.

After surveying evidence from this and six other classrooms where, according to their criteria, different levels of teacher control are evident, Ellerton and Clements conclude that there is no simple relationship between the amount of control that pupils have and the quality of the learning environment. They show that classrooms which are teacher-directed, but leave space for group discussion and activity by the pupils, can and do lead to understanding of the mathematical content.

These results suggests that there are two problems with this notion of ownership. The first is in the identification of ownership with the construction of knowledge alone, or with cognitive processes such as problem-solving, decision-making and evaluation. This is

accompanied by the assumption that the construction of scientific concepts and more importantly, true concepts, will follow unproblematically when learners control the learning process. Thus, I will argue that while these processes are necessary for ownership, they are not sufficient. The development of true concepts, through internalisation of scientific concepts and making connections with spontaneous concepts is also necessary for ownership of mathematical knowledge.

The second problem with this notion of ownership is its close association with the absence of the teacher. From the evidence obtained in this study, I will argue that ownership can be achieved with direct teacher guidance, and often is not achieved when the teacher is not there. In addition, the arguments about the development of true concepts as being integral to ownership suggest a crucial instructional role for the teacher.

Thus ownership captures both the processes and products of learning mathematics. It does not exist as a fixed quality or characteristic of a learner. Rather, it is subject to the same processes as development or learning - partial achievements, setbacks, and fundamental transformations - and these occur between learners as well as "within" them. Ownership cannot be directly measured, just like understanding, learning or development (Pirie, 1988), but can be inferred from learner talk and interaction. The political and emotional aspects of such interaction are crucial.

The notion of ownership, as a unity of cognitive, political and emotional relationships with knowledge, will be used to interpret the interactions of one peer group, engaged in a collaborative mathematical task, and will be elaborated through these interpretations. From the framework developed in this chapter, I will look at the relationships between the pupils

in the interaction and how they influence and are influenced by ownership. I will also consider the crucial role of the teacher, in relation to pupil interaction, ownership and the development of mathematical knowledge. Finally I will draw out some implications for pedagogy.

CHAPTER 3

METHODOLOGY AND RESEARCH DESIGN

In the previous chapter I discussed different approaches to peer interaction from the point of view of their theoretical underpinnings, particularly regarding cognition, power relationships and teaching. The methodologies of the various approaches also differ as these are closely connected with the assumptions made and questions asked. I will briefly review the different methodologies in order to provide a background for the methodological choices I have made.

1. METHODOLOGICAL APPROACHES TO CLASSROOM INTERACTION

Most research into co-operative and collaborative approaches to peer interaction has been conducted in a positivist paradigm. This assumes that important factors can be neatly separated into independent and dependent variables, and one-way, cause-effect relationships established between them (Miller, 1987). Within the co-operative school, variables such as incentive structure, group arrangements and task specialisation (Saxe et al, 1993; Slavin, 1991) are manipulated and outcomes such as academic achievement are measured. Constructs such as inter-group relations, self esteem and attitudes towards school are operationalised and measures obtained (Slavin, 1991). Control groups are used, and statistical analysis is the method whereby a cause-effect relationship is established between the various co-operative arrangements and the outcome variables.

Many collaborative learning studies also work within a positivist paradigm. Most use pretest-posttest research designs with control groups, and show gains in problem solving strategies (Forman and Cazden, 1985; Glachan and Light, 1982) or achievement on tests (Cloutier and Goldschmid, 1978).

However, recent research in this tradition has found such an approach insufficient to capture important aspects of peer interaction. Forman (1989), having initially reported a pretest-posttest design, states: "What this static, decontextualised assessment of cognitive growth cannot reveal is the interpersonal and cognitive dynamics of their collaboration" (p. 64). So she supplements her quantitative methods with a "dynamic" qualitative analysis which is able to capture changes in the subjects' task definitions, reasoning, and assumption of responsibility for various aspects of the task.

Such a dynamic approach necessitates concentrating on small samples, which in turn changes the relationships between evidence and theory generation. Cobb, Yackel and Wood (1992) elaborate these relationships in the case of research in classrooms:

> "The goals of making sense of particular events in one classroom and of developing a more general orienting framework are interdependent in that the analysis of particulars constitutes occasions to reconsider what needs to be explained and to revise general explanatory constructs. Conversely, the segmentation of classroom life into potentially understandable [illegible] reflects one's theoretical orientation. Thus particular events [illegible] ground theoretical constructs, and theoretical constructs inform the interpretation of particular events." (1992:100)

This perspective has a long tradition in sociolinguistic approaches to classroom interaction, which encompass qualitative research methodologies such as ethnography, constitutive

ethnography and ethnomethodology (Edwards and Mercer, 1987; Edwards and Westgate, 1987; Burgess, 1985a; 1985b; Bogdan and Biklen, 1982). These methodologies regard interactions as constituting the important events to be explained. These are observed in great detail, and empirical analysis and the development of explanatory concepts progress interdependently. There are a number of important differences in the above methodologies which I will not address here, since my approach does not fit neatly into any of these categories. They have in common a phenomological orientation (Bogdan and Biklen, 1982), which asserts the primacy of subjectivity and meaning, and which attempts to generate grounded theory (Henwood and Pidgeon, 1992; Wood, 1985). My methodology is informed by this perspective.

A recent methodological development in sociolinguistics is discourse analysis. Discourse analysis is seen by some as a systematic approach to the classification of classroom talk in terms of its forms and functions. Proponents of discourse analysis (Stubbs, 1981; Sinclair and Coulthard, 1975) claim that unless due attention is paid to linguistic form and function as well as meaning, the research will lack rigour, and will not access the fundamental structuring nature of the talk. Discourse analysis, while still working with notions of grounded theory, is an attempt to formalise the analysis of data, making it less dependent on researcher subjectivity. However, Edwards and Mercer (1987) point out that no analysis can claim to be totally researcher-independent since even though formal categories are created for the classification of discourse, decisions about which categories utterances belong to require researcher interpretation. Moreover, paying such particular attention to the forms and function of language may obscure, rather than illuminate educational questions. So, educational research into classroom interaction, which needs to focus on the content and

meaning of talk together with learning and development, requires less formal methodologies. My research aims to understand the relationships between interaction in small groups of junior high school mathematics students and their ownership of mathematical knowledge. These two variables cannot be separated into independent and dependent variables, and therefore a positivist paradigm is not appropriate (Miller, 1987). Rather, a Vygotskian orientation views them as constantly interrelated, with certain ways of interacting producing ownership, which in turn produces particular interaction patterns. Interaction and ownership combine to produce conceptual development, or lack of it. Thus an in-depth, qualitative and interpretive study, which captures changing relationships between the phenomena being studied, is appropriate.

2. AN INTERPRETIVE RESEARCH METHODOLOGY

Bogdan and Biklen (1982) present five characteristics of qualitative research:

1. It takes place in a natural setting and is fundamentally concerned with context.
2. It is descriptive, with emphasis on details rather than generalities.
3. It is concerned with process rather than outcomes.
4. The data is analysed inductively, with the aim being to develop grounded theory, rather than to prove or disprove hypotheses.
5. The meanings that the research subjects attribute to the events and actions being studied is of central concern.

I will discuss each of these in relation to my study.

My research is classroom-based and attempts to understand interaction and ownership within the constraints of the classroom context. Newman et al (1984) have shown that there are differences in children's understandings of and responses to tasks, when studies are conducted in laboratory settings on the one hand, and classrooms on the other. Thus, if educational research wants to illuminate classroom occurrences and provide possibilities for teacher action, it should be conducted in classrooms, and needs to take account of the contextual constraints in classrooms.

My analysis will be descriptive and will focus on the interactions of one group of three pupils for the duration of a week. It will employ Geertz' (1975) notion of "thick description", in which the intentions and meanings of the participants form an integral part of the analysis.

My concern with the interrelatedness of interaction and ownership necessitates a process approach. I am, however, concerned with outcomes to the extent that teaching and learning is a goal-directed process, the aim being the development of knowledge in the learners. However, from a Vygotskian framework, these outcomes cannot be observed outside of the processes that produce them (Vygotsky, 1978). Pirie (1988) points to dangers in working with preconceived notions of an outcome such as understanding. An approach is required which allows contextual interpretations of the evidence, and which can help avoid such inflexibility.

An analysis which is grounded in the data requires that the researcher's perspectives and the evidence are constantly challenging and influencing each other. The researcher is not neutral, but comes to the data with a theoretical framework, and particular experiences. In my case,

I had conceptions of what might constitute ownership, interaction, and mathematical knowledge. However, the evidence suggests that only some of these are appropriate. Thus doing the research has refined and changed some of the founding conceptions of the research.

Finally, I attempt to give an account of the events that occurred from the points of view of the participants. Their meanings and intentions are the focus of my analysis. But Bogdan and Biklen (1982) point to the problems associated with this approach. Any analysis necessarily distorts some aspects of the subjects' experience, as the researcher brings her own conceptual frameworks and understandings to bear on the data. Thus "the process of doing qualitative research reflects a kind of dialogue or interplay between researchers and their subjects since researchers do not approach their subjects neutrally" (p. 33). However, remaining self-conscious about reflecting participants' perspectives can enable researchers to remain as faithful as possible to these perspectives.

A final characteristic of interpretive research which the above discussion assumes, is that the researcher is the primary research "instrument". Ownership and interaction cannot be meaningfully measured with conventional research instruments. They can, however, be observed, interpreted, understood and explained by a researcher in relation to a dynamic theoretical framework.

3. RESEARCH DESIGN AND METHODS

3.1. *The context*

My research was conducted in October 1993, in one of four Std 7 (14-15 year olds) classes at a non-racial, co-educational, independent secondary school in Johannesburg. Students at

the school come from a variety of socio-economic, linguistic and educational backgrounds. In this sense they represent the broader population of South African pupils. Educationally, however, the school differs from most in South Africa in that schooling has proceeded uninterrupted for the past five years, with only minor interruptions before that. Teachers are all well qualified[1], classes are small (there were 17 children in this particular class), and there is no lack of material resources. The medium of instruction is English, but almost all South African languages are represented among the pupils.

The mathematics teachers at the school view mathematics as a dynamic, socially constructed activity with an interesting and important history. They try to convey some of these aspects to the pupils, within the constraints of the school curriculum and national examinations. They encourage pupil talk and verbalisation of ideas both in small groups and in whole class discussion. Most children develop confidence and competence in using English to express their opinions, in other subjects and in mathematics. So, while the student population is representative of the broader population, there are significant differences between the classroom I observed and most South African mathematics classrooms. Therefore, my observations and explanations may not be immediately transferable to other contexts (see section 4 for a discussion on generalisability in qualitative research). However, seeing what is possible and what is problematic in more fortunate and "progressive" environments will have implications for our understandings of what might occur in other situations (Edwards and Mercer, 1987).

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[1] Almost all have degrees and many have post-graduate degrees.

3.2. *The tasks*

In the three months prior to the data-collection phase of the project, I visited the school on a number of occasions to discuss the project with the teachers. Regarding the activities that the pupils would be involved in, the teachers suggested a worksheet which would help the pupils move from work they had been doing on Euclidean geometry to work they would be doing on transformation geometry. Hence the idea of the geoboard, so that the pupils could begin to gain an intuitive feel for working with shapes and co-ordinates together. At the same time, the teachers felt that an opportunity to spend time working on the geoboard could help to develop a spatial understanding of area, perimeter and the relationships between them.

For my purposes, the tasks should encourage interaction between pupils and the generation and expression of different points of view. There should also be opportunities for children to engage in investigation and the associated processes, such as posing problems, making decisions as to solution strategies and evaluating progress (see chapter 2, section 3). Finally, there should be opportunities for connections to be made between spontaneous and scientific concepts (chapter 2, section 1.2.2).

I set up a worksheet of related tasks in line with both the teachers' requests, and my research needs (see Appendix A)[2]. Question 1 was intended to be an easy introduction to the worksheet and to provide a period of adjustment to the tape recorder. However, this task did not turn out to be as easy for the pupils as their teachers originally envisaged, and in fact provided the focus of most of my analysis. The purpose of question 2 was to move the pupils

[2] Some of the ideas for the worksheet come from Massyn (1991) and MEP (undated).

away from the notion that a square must be "straight" on the geoboard and that only perfect square numbers could be the area of a square. There was also the possibility of a geometrical interpretation of irrational numbers. Each part of the question has fixed answers, but the tasks require more self-definition and self-evaluation of progress than question 1.

Question 3 aimed to develop understandings of possible relationships between changing perimeter and changing area. Such understandings could have been intuitive, but hopefully would develop into more formalised concepts. The task is open-ended, and the requirements were left vague specifically to allow for more interactive and investigative processes in defining and completing the task. Question 4 provided for verbalisation and articulation of ideas developed in the other three tasks. This was to provide an opportunity for reflection on the part of the pupils. The original worksheet had a fifth question, but the pupils only spent a few minutes during the last lesson on this question, so it has been excluded from the analysis.

The worksheet provided for one week of work for the pupils. It formed part of the teacher's plan of work for the year, and was presented to the students as part of their usual classroom activities. The three classes which did not form part of the study also used the worksheet. Each group in the class was provided with one 5x5 geoboard and coloured elastic bands.

3.3. *The sample*

Sampling in qualitative research differs from quantitative sampling, in that it is not random. Theoretical sampling aims to ensure that variables which will illuminate the issues under

investigation are present in the sample (Maxwell, 1992). My choice of school, class and group were based, firstly, on having pupils with some experience of group work, who were easily able to engage with each other and the work, and secondly, who reflected diversity, specifically in gender and home languages.

On one of my visits to the school, I observed all four of the Std 7 classes and spoke to the teachers about the extent to which their classes had experienced small-group work. On the basis of the teachers' responses and my observations, I chose one class which I believed had had more experience than the others. This class was considered to be a "difficult" class in that there was an underlying dynamic of resistance to teacher control. However, I decided that such attitudes might not be as predominant in peer interaction as they are in whole-class interaction, and if they were, they would enhance rather than detract from the issues with which my study is concerned.

I explained the purposes and procedures of the research to the class, and asked their permission to record their interactions. I stressed that the recordings would be heard only by myself, their teacher, and possibly a translator. There was one pupil who implicitly expressed objections to being recorded, and his group was excluded from the study. In addition to this, one group whose members constantly and self-consciously spoke into the tape recorder was excluded from the beginning.

The pupils chose their own groups and I spent one lesson observing the groups to decide which would be the subjects of my study. I chose a group of three students, two girls and a boy, all second-language speakers of English. This group also displayed, at least from the

outside, considerable engagement with each other and with the tasks. It seemed that their interactions reflected equality and mutuality of engagement (Damon and Phelps, 1989). They all contributed to the discussion, listened to and took account of what the others were saying.

This group was not aware that it was the only group on which I would focus. I tape-recorded all the groups in the classroom so that this group would not feel unduly self-conscious.

3.4. *The data*

The main sources of evidence are the transcripts of audio tape recordings of the talk of the group while they worked on the tasks (Appendix B). Edwards and Westgate (1987) raise a number of issues regarding the investigation and analysis of classroom talk, the most important one being the question of "how transparently does language carry meaning?" (p. 56). They contrast o assumptions: 1. that meaning and the reality of classroom life is simply reflected in the talk; and 2. that classroom activity and events are constituted, created and recreated in the discourse, and that the talk is part of the reality that is constantly being created. The use of coding schemes such as those of Flanders (1970) reflects the first assumption, since they presume that the function of an utterance is immediately evident to an observer, and that utterances can be unambiguously assigned to particular categories. The use of recordings reflects the second assumption because they allow the words to be listened to repeatedly, and heard in the context of subsequent utterances and events as well as preceding ones. Only such an approach can begin to capture the meanings constituted in the interaction.

However, even recordings do not give a complete picture. They do not always convey the histories of the participants, the shared history of the class (Walker and Adelman, 1976), the importance of certain contextual features or the norms, values and assumptions of the participants. Other evidence might be needed to provide such information. However, even with other evidence, the researcher must make inferences, decide on the relative importance of parts of the data and their connections with each other. Edwards and Westgate assert:

> "The analyst's task is to infer what was meant from what was said. ... discourse and its organisation represent the best available evidence. Despite all the problems of recreating the relevant context, the linguistic record is held to be the necessary starting point." (1987:137)

Video recordings can provide added information such as gestures or body language, which would help to understand the discourse. In my case it also would have provided essential information on the pupils' actions on the blackboard, and hence the meaning of their talk. However, the benefits gained would not have outweighed the self-consciousness created by the intrusion of the camera focusing on a small group. The pupils quickly forgot about the small tape recorder on the desk, and thus the recordings provide a reflection of a situation which is as naturalistic as possible.

The transcripts are supplemented by the pupils' written work while working on the task, a final, individual version handed to the teacher for assessment at the end of the week (Appendix C), my own observations in the classroom during the week, and the tape recordings of an oral presentation made to the class at the end of the week (Appendix B). This presentation was the teacher's initiative; I had not suggested it as part of my original design.

No tests were conducted, as they would not give any additional information on the processes engaged in during the interaction, nor on the construct of ownership. The pupils' work handed in to the teacher was considered to provide sufficient information about what they had learned during the week, and more importantly, about the relationships between the processes and the observed outcomes. Due to time constraints, and the small scope of this project, I was not able to conduct interviews with the pupils, in order to gain insi[illegible] into their individual prior or subsequent understandings of area and perimeter. However, I am able to make inferences from the transcripts as to the pupils' understandings of these concepts while they collaborated on the worksheets.

3.5. *The transcripts*

The transcripts were difficult to construct and interpret. The noise from the other groups in the classroom was picked up by the tape-recorder and obscured some of the quieter talk of the group under study. A mix of languages also makes the talk difficult to hear. While the children speak English most of the time, there are also long sections of Sotho; isolated Sotho, Zulu, Xhosa and Afrikaans words; "street language"; and "school expressions"; the last two being colloquial combinations of and variations on all the languages spoken by the pupils. Two translators helped me with these parts. In the transcript I have recorded the language used by the pupils, with English translations in brackets.

For these reasons, the transcript can only be a partial reflection of the group's conversation. In addition to difficulties in capturing all of the pupils' talk, the extent to which other details such as exact lengths of pauses, intonation, or pitch are provided, will determine the

correspondence of the transcript with the recording (Edwards and Westgate, 1987). However, in providing such detail, the content and meaning of the talk may be lost (Edwards and Mercer, 1987), and researchers using talk as evidence have to make choices as to what will be retained, and what lost. I have not reflected many conversational devices in my transcript (see Appendix B for those that I have retained), preferring to concentrate on the content and meaning of the talk. Where such details, such as tone of voice, are necessary to understand the meaning, I have provided these as footnotes to the analysis rather than in the initial transcript.

Due to the length of the transcript I was forced to exclude parts of it from the analysis. I did not consider the off-task dialogue, the speech which accompanied the making of shapes on the geoboard or the work on perimeter. These sections are marked off on the transcript. Thus the analysis focuses only on the work on area, and in particular, on one aspect of this work, a method which the pupils developed to calculate area.

Interpretation of the transcripts was complicated by the fact that the pupils use a concrete apparatus, the geoboard. A lot of the talk consists of pronouns such as "this", "that", "its" etc, whose referents are shapes on the geoboard to which the pupils are pointing. In many cases, through using my observation notes and the pupils' written work, I know, or am able to infer, what the pupils are speaking about, but some extracts remain uninterpretable.

3.6. *Analysing the transcripts*

I have chosen to focus on the pupils' interactions, their meanings, and the development of

their mathematical knowledge, rather than on the specifics of their language. There are many interesting linguistic aspects to the discourse that could have been taken up, particularly since the pupils are second-language speakers of English. These include code-switching, the creative use of English by the pupils, and their difficulties in expressing their thinking. However, as discussed above and in the previous chapter, this is not a focus of the study.

The transcripts were difficult to analyse on two levels. Firstly, there were difficulties in making sense of the content of the pupils' talk. Which question they were working on, which shape they were discussing, what numbers and pronouns refer to, and incomplete sentences, made substantial cross referencing, inferences and "detective work" necessary.

On a second level, looking for regularities in the talk, interaction and ownership across the lessons proved unproductive. Therefore, I did not attempt to generate categories to describe the data. I was not looking for similarities or differences between specific instances of talk, nor was I interested in quantifying any aspects of the interaction. I decided rather to trace the development of the pupils' mathematical knowledge in relation to their interactions. To maintain a rigorous analysis, I went through the transcripts thoroughly a number of times, while listening to the tapes, interpreting them from the points of view of each of the participants, and from my own point of view as an outsider. I also checked them with my observation notes and the pupils' written work. Any discrepancies were examined again, to make sure that all possible interpretations had been taken into account.

My initial analysis focused on an unexpected method which the group generated and developed to calculate area. I charted the progress of the method through the transcript, to

try to understand how it was produced and maintained, and the effects that it created in the interaction and for the pupils' mathematical knowledge. I then used the pupils' relationships with the method to illuminate, and elaborate, aspects of ownership. Some of these aspects had formed part of my initial concepts of ownership, while others were generated directly from the data and elaborated through the research process.

An initial draft of the analysis was given to the teacher, to obtain a measure of reliability for my interpretations, and to make sure that I had not unintentionally misrepresented her or the school.

3.7. *The report*

In qualitative research, writing the report is an integral part of the research process (Woods, 1985). It is in this process that the relationships between the data analysis and the theoretical framework in which it takes place are clarified. This is an essential part of an interpretive process which grounds explanation in the data. The theory informs the data analysis, which in turn informs the theory. The process is dialectical in a way which cannot be accurately represented in a linear report.

The report presents the elaborated framework, followed by an account of what happens in the group, drawing on the theoretical concepts. Examples from the transcript and the pupils' written work are presented and analysed as part of the account. The examples that are chosen reflect important points in the development of the method and the pupils' knowledge, or exemplify key explanatory concepts.

During the analysis and writing up, the key issues of reliability and validity of the research are continually confronted. In qualitative research these are not issues that can be addressed once and for all by the research design (Maxwell, 1992), but need to be constantly assessed in relation to the interpretations being made and the concepts being used to explain the data.

4. VALIDITY AND RELIABILITY IN QUALITATIVE RESEARCH

Validity and reliability are constructs that originated in experimental, hence positivistic research (Maxwell, 1992; Hammersley, 1987; Bogdan and Biklen, 1982), their purpose being to ensure the objectivity[3] of the research. However, the terms are used in a variety of ways, with little conceptual clarity as to what they refer to or how they are being used (Hammersley, 1987).

Qualitative researchers' use of the terms are even more divergent with some claiming they have very little applicability (Bogdan and Biklen, 1982), since qualitative research views subjectivity as primary. Others argue that they can and should be redefined for qualitative research, especially to avoid extreme relativism (Maxwell, 1992).

Maxwell argues that validity in qualitative research can be usefully viewed as a "relationship between an account and something outside of that account, whether this something is construed as objective reality, the construction of actors, or a variety of other possible interpretations" (p. 283). Validity is a characteristic of accounts or data, and can only be

[3] Objectivity refers to the discovery of facts about people, which are assumed to remain unchanged by a valid and reliable research process.

assessed in relation to the aims and purposes of the research. Validity cannot be guaranteed from the outset in qualitative research, but a researcher continually faces potential threats to validity which need to be dealt with.

Maxwell elaborates five categories of validity which, he argues, many qualitative researchers use implicitly. They are: descriptive validity, interpretive validity, theoretical validity, generalisability, and evaluative validity. He advises they be used as a framework for thinking about the kinds of threat to validity that have arisen in the research, and ways of dealing with them.

I have already discussed various aspects of validity pertaining to my research in previous sections. The degree to which the transcripts accurately reflect what was said by the pupils refers to descriptive validity. I have identified in the previous discussion some of the problems I encountered, including my inability to hear certain utterances, my inability to see the geoboard at all times, and the omission of certain vocal and physical details from the transcript. For my purposes, which is to gain access to the content and meaning of the talk, the third issue is not an important threat to descriptive validity. The first two are, however, but could only have been avoided by removing the group from the classroom, and using a video camera. This would have detracted from the naturalistic setting. Thus a gain in descriptive validity would have created a loss in generalisability (see later discussion). The teacher was able to be of some help in this regard, identifying shapes that she worked on with the group, which I had not seen.

At the level of interpretation, I am primarily concerned with the question posed previously (section 3.5) concerning the extent to which the meanings and thinking of the pupils can be inferred from their talk. This is where concerns can be raised about the researcher being seen as the primary research "instrument", as her interpretations may be idiosyncratic. However, the interpretations are shaped by an elaborated theoretical framework, which ensures contact with established theory, as well as by her experience and knowledge of classrooms, which ensures contact with established practices. When practical and theoretical knowledge are constantly related to each other, better interpretive validity can be obtained.

An often-proposed strategy to obtain more interpretive validity is for the researcher to check her accounts with the participants. While this reflects the importance attributed to remaining faithful to the participants' perspectives and meanings, it can be problematic in relation to validity, since people's own accounts of their actions and experiences might be obscured by subsequent events, or by the simple fact that they are being asked to recount them for a researcher. In particular, in relation to talk, Barnes and Todd (1977) point out that:

> "even for the participants in a conversation, meaning is not static. Reflecting upon a conversation which is past, where one can stop the tape to check back on unclear bits, is not the same as taking part in a conversation.... (even) for the participants in a conversation, their retrospective account of what happened will be different from their on-going account of what is happening (while taking part in the conversation)." (p. 18)

In my case the participants are the three pupils and the teacher. Because of the time constraints of the project, I was not able to speak to the pupils until three months after I had collected the data. At this point the new year had begun, and I felt that the pupils were too far removed from the events being discussed for their accounts to be useful. However, I

decided to corroborate my accounts with the teacher, both to ascertain whether I was reflecting her perspective adequately and also to obtain some measure of reliability for my own interpretations of what the pupils are doing. As an actor in the research situation, who is also in a privileged position to interpret the ideas of the other actors, her perspectives provide a crucial test for interpretive validity, as well as reliability.

Validity and reliability are closely related in qualitative research (Maxwell, 1992; Bogdan and Biklen, 1982). In quantitative research, reliability is usually defined as the replicability of the results, by a different researcher, using the same instruments (Hammersley, 1987). If the primary instrument of qualitative research is the researcher, then this becomes meaningless, especially because different researchers' theoretical orientations will usually not be identical (Bogdan and Biklen, 1982). However, if different readers of a transcript do see similar events occurring, this helps to remove concerns about idiosyncratic interpretations. In this way, reliability can be seen to increase validity (Maxwell, 1992). In addition to verifying my interpretations with the teacher, I also presented part of the transcript to a group of colleagues. My research supervisor also provided a consistent other "eye" in on the transcripts. Many of my interpretations were corroborated, and I was prompted to reconsider some, thus adding both to the reliability and validity of the account.

Theoretical validity, in Maxwell's typology, concerns the theoretical concepts used to explain the data, and the relationships between them. Theoretical validity depends upon agreement within the research community as to the validity of the concepts and terms used. Difficulties can arise where concepts from one discipline, for example "conflict" and "power" from sociology are employed together with those from another, for example, "learning" and

"development" from psychology. I have attempted to bring these together in a notion of ownership. By grounding this notion in the data, and by showing how it relates to the frameworks that I use from both disciplines, some theoretical validity is obtained.

Generalisability in qualitative research relates to the extent to which the explanations can help illuminate other similar situations, rather than whether similar processes would be observed in a wider population (Maxwell, 1992). Contextual and sampling decisions are important here. I investigated a group who had had some experience of groupwork, in order to try to gain a clearer understanding of some of the processes involved. Explanations of such processes should be beneficial to teachers who are starting out with groupwork, or those whose classes are more experienced. On the other hand, I chose to work in a classroom situation, because explanations of out-of-classroom practices do not help to illuminate what happens in classrooms. I also chose a group of mixed gender, and second-language speakers because research on small groups is often conducted in homogenous groups, and heterogenous groups may well require different explanatory concepts.

Maxwell's final category is evaluative validity. This relates to judgements made about what is observed and explained. The evaluations that I make, concerning the pupils' progress on the tasks, their learning and interactions are essential in answering the research questions about the extent to which small-group work facilitates ownership of mathematics. The judgements are made in relation to the teachers' and my (as a teacher) understandings of the goals of the worksheet in particular and mathematics education in general.

In confronting the issues of validity and reliability, I hope to have made my account as valid, and hence as useful (Maxwell, 1992) as possible. However, it remains an account, based on

a particular theoretical orientation, and admits the possibility of other accounts of the same events. In the end, its usefulness will be determined by the addition of evidence to existing knowledge about peer interaction, its contribution to discussion and debate on the topic and to the generation of further research in this area, and most importantly the implications that other teachers might draw from this research, to help them understand and improve their own practice.

CHAPTER 4

OWNERSHIP: A METHOD TO CALCULATE AREA

In general the tapes present a picture of a lively group, who work well together on certain of the tasks, but manage to do very little on others. The pupils' talk is creative and witty, and they intersperse their work with chatting, joking and singing, which enhances their interaction and their work. However, at times their interaction degenerates into social conflict and fighting, which inhibits their work.

The worksheet aimed to develop concepts of area and perimeter and the relationships between them. The tasks ranged from convergent to divergent: each shape in question 1 required only one answer (for each of area and perimeter), whereas question 3 was totally open-ended and required the pupils to define and frame the tasks themselves. Unexpectedly, the original, convergent task provided more opportunities for sustained interaction among the pupils, and for engaging in the mathematical processes, such as problem-posing, and making decisions about and evaluating solutions, that I identified as an important part of ownership (chapter 2, section 3).

While working on question 1, the group developed a particularly innovative, yet problematic, method to calculate area. This method forms a focus for their work for most of the week, and their ownership of the method is crucial to the ongoing interactions of the group and the development of concepts of area[1]. This chapter will outline the method, and elaborate the notion of ownership through analysing the group's relationship with the method.

[1] My analysis will focus on their method, and will consider other aspects of their work on area where appropriate.

1. "USING HOOKS TO FIND THE AREA"

The method, in its most general form, can be described as follows (see fig 4.1):

For a "complicated shape", i.e., not a rectangle

1. Divide the shape up into squares or rectangles and triangles.
2. Calculate the areas of the rectangles.
3. Count the hooks in the triangles, including those on the perimeter and inside the triangle, and make a rectangle with the same number of hooks, also on the perimeter and inside the rectangle.
4. Calculate the area of the rectangle formed in this way. It will have the same area as the original triangle.
5. Add up the areas found, to get the area of the original shape.

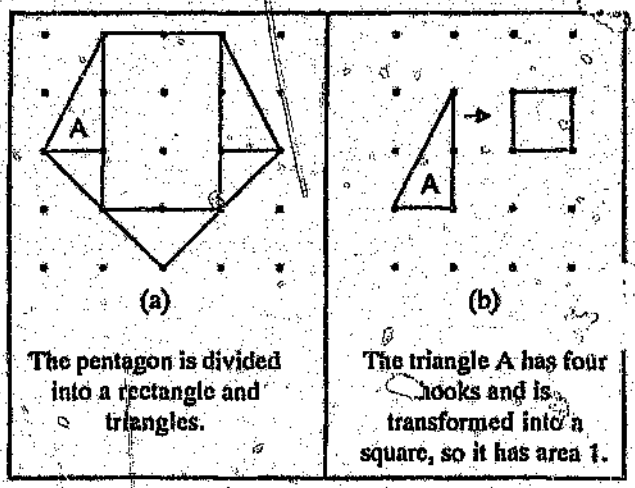


Figure 4.1

The method is articulated most clearly by Thabo in the following excerpt[2]:

[2] Each extract is preceded by a square bracket which lists: 1. the number of the extract in this report; 2. the day on which it occurred; 3. the question of the worksheet being discussed; 4. the page number of the extract in Appendix B and 5. the line numbers of the extract in Appendix B.

[extract 4.1; day 4; question 4a; p. 162; lines 2020-2039]

Thabo: *How to work out a complicated area, ne, first you find, uh, you try and divide your things i...o triangles, and sq, rectangles or squares, ne, are you writing Nandipha*
Nandipha: *Yes*
Thabo: *Where*
Nandipha: *On the sheet, on the sheet, mm, and then*
Thabo: *And then, mm, you work out your, area, of, for the squares, or the rectangles, ja, to work out the area of the, of the, to work out the, the, the area of, of a triangle, you, no yes, cause it will be easy to work out, work out the area of the square and this thing ne, to work out the area, the area of the triangle you count the hooks which the elastic is*
Nandipha: *Hooked to*
Thabo: *Ja, that includes the middle ones, and then*
Nandipha: *Ah, wait, Thabo*
Thabo: *And then*
Nandipha: *Yes*
Thabo: *e.g*
someone: *Sh*
Nandipha: *To (laughs), and then, and then O.K. e.g*
Thabo: *e.g, if the triangle's got 6 hooks*
Nandipha: *The triangle's got 6 hooks, ne*
Thabo: *Ja, you try to make a square that has, or a triangle[3] that has 6 hooks[4],*

There are a number of variations to the method. Step 4 of the method (as I have articulated it above) is the most contentious, as it is based on a mistaken notion of conservation of area (figure 4.2a). For most of the time the group works with a restricted version of this idea, that triangles containing four hooks can be transformed into a square and thus have an area of 1 (see for example figure 4.1). This version is found to work in almost all cases (figure 4.2b). The limitation that the hooks inside the shape are included is important here. The triangle in figure 4.2c does not have an area of 1 because it has 5 hooks, not 4.

A further variation is developed towards the end of the week, to deal with triangles with a number of hooks (most odd numbers) that cannot be converted into a rectangle. In these cases, a triangle (half a square) is added on to the rectangle. In the final presentation to the class, Lerato articulates this version as follows:

[3] He means rectangle

[4] See figure 4.2a

[extract 4.2; day 6; presentation; p. 176; lines 2775-2782]

Lerato: *O.K., I'm going to show you a method like using hooks to find the area, O.K. jaa, I'm going to start with 3 hooks because 1 hook and 2 hooks don't work, so I'm gonna start with 3 hooks. It's 1, 2, 3 and then you join them up, and then you know that this is going to be ½ of a, of a square, so the area of this is ½[5], and then you use 4 hooks, 1, 2, 3, 4, and then the area is going to be 1, because if we draw it like this it's going to be ½ plus ½, 1[6], and then I'm jumping to 7, 7, 7 hooks, 1, 2, 3, 4, 5, 6, 7 (whispers to others) ja, so it's 1, 2, 3, 4, 5, 6, 7, join them up, and then you get 1, 2, and then ½[7]. So my conclusion is that whenever you add a dot, a hook, uh the, the, the area of the, the shape increases with ½*

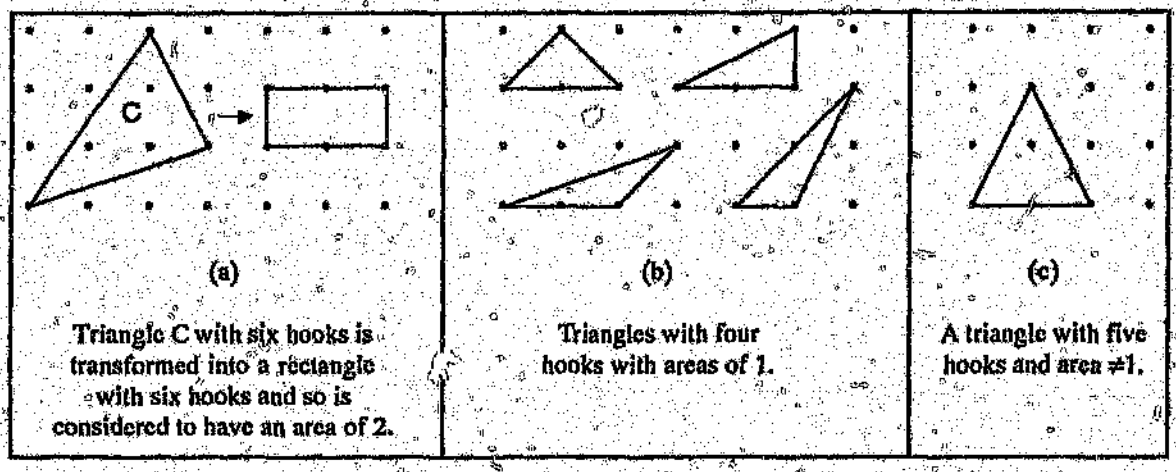


Figure 4.2

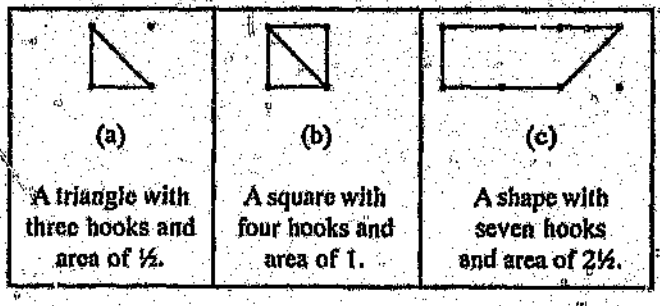


Figure 4.3

[5] See figure 4.3a

[6] See figure 4.3b

[7] See figure 4.3c

The latter formulation was first articulated by Thabo in the last few minutes of the last lesson, as an attempt to deal with the problematic part of the method (step 4 above). At this point he gives up the general form of the method, that any triangle can be converted to a rectangle with the same area, but tries to maintain the four-hook part. He then proceeds to make the generalisation expressed by Lerato above, but does not relate it back to the areas of triangles:

[extract 4.3; day 5; question 1a; p. 174; lines 2704-2708]

Thabo: *O.K., O.K. we always, let's, let I give you, let me tell you, we always count to, to the things, 4, always 4, ish, the triangle, we always make the things to this, we always divide it into thing, ne, to get to the, let's forget the hooks, let's forget the hooks, let's forget the hooks, but then it works with the square, ne, so then from there, if you if the triangle is bigger than that, if it needs 5 hooks ne, then 5 hooks is 1½*

This is the final version of the method presented to the class at the end of the week. However, the development of the method during the course of the week is a complex, nuanced process. The pupils struggle with the mathematical concepts and reasoning involved, as well as the group interactions. Mistakes are made and repeated, ideas are returned to, modified, rejected or maintained. The interaction in the group will be discussed in more detail in the following chapters. For the rest of this chapter, I will discuss emotional and cognitive aspects of ownership as they relate to the method, to begin to articulate the notion of ownership.

2. OWNING THE METHOD

It is reasonable to assume that the pupils would not have developed the crucial transformation part of the method (step 4) if they had worked on the same exercise in a whole class situation

with teacher direction. In this sense the method can be said to be the group's "own". From the pupils' perspectives, they identify the method as theirs by constantly referring to it as "*our* method" (pp. 129, 157, 164, 176), "*our* solution" (p. 131), and "*our* invention" (pp. 163). The pupils identify the method as their own, something that they have initiated as a solution to a problem that they need to solve, are working with, and are constantly evaluating. These are some of the process elements of ownership, discussed in chapter 2.

There is also evidence in the discourse of emotional aspects of ownership. Both positive and negative emotions are expressed. When the method is found to work we hear excitement: "it works, it works, it works" (p. 132) and

[extract 4.4; day 1; question 1a; p. 133; lines 420-426]

Nandipha: *Voila*
Thabo: *And then here, then here again it's gonna give us 1, so that's 1, 2, 3, 4, 5, 6, [7,*
8, 9, 10
Lerato: *[7,*
8, 9, 10
Thabo: *No, yes*
Lerato: *Mm, 10*
Thabo: *10, It works, so ev, for every shape we get we divide it into squares and triangles, and then we find our thing, superb, thank you, (laughter) that was B*[8]

They also express appreciation of their work. While working on question 2, Thabo and Nandipha make the square in fig 4.4. This is Nandipha's assessment of their work:

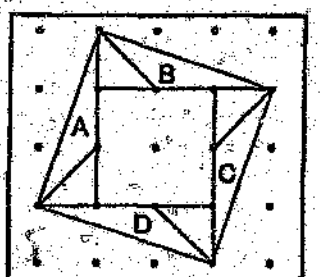
[extract 4.5; day 2; question 2b; p. 145; line 1115]

"but this is an excellent one, look at the shapes in the four quarters"

[8] They are working on the pentagon in question 1 (shape B), see figure 4.1.

[9] She probably means corners

They go on to calculate the area of this square, using from their method the fact that triangles with four hooks have an area of one, to calculate the areas of the four bigger triangles (A,B,C,D in fig 4.4).



A square of area 10.
The four triangles
A,B,C,D have four hooks
and areas = 1.

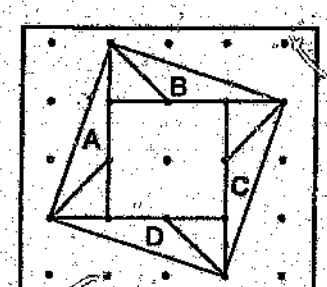
Figure 4.4

Negative emotions, such as frustration and disappointment, are also expressed, particularly when the method seems to be failing (pp. 164-165). In the excerpt below, Thabo is almost inconsolable, although Nandipha tries to keep them going and Lerato suggests that they might be able to salvage part of the method:

[extract 4.6; day 4; question 4a; p. 164; lines 2131-2160]

Lerato: *The method doesn't work*
Thabo: *Our method doesn't work anymore, let's check, let's go back*
Nandipha: *(sings)*
Thabo: *A flop, before the due date (laughs), we flopped me*
Nandipha: *So, we want to work it out, find and divide into triangles and squares. Work out the area of squares and triangles, count how many hooks the elastic, is hooked on to and the inside, e.g*
Thabo: *Ish, we flopped*
Nandipha: *So what method can we use*
Thabo: *We flopped*
Nandipha: *If we have*
Thabo: *We flopped*
Nandipha: *If we have an elastic that's big we get a [big triangle*
Lerato: *[Come, come guys, let's leave it, tabatsa*

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A square of area 10. The four triangles A,B,C,D have four hooks and areas = 1.

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Thabo: *Ish, we flopped*
Nandipha: *So what method can we use*
Thabo: *We flopped*
Nandipha: *If we have*
Thabo: *We flopped*
Nandipha: *If we have an elastic that's big we get a [big triangle*
Lerato: *[Come, come guys, let's leave it, tabatsa*

{it's no use} here we must do a, a square, or what like, understand, I.e count the hooks, and then you do a square like

Thabo: We already flopped for our presentation

Nandipha: No, we can always do this that we know already

Thabo: Which one, ish,

Nandipha: We got this

Thabo: It doesn't work

Nandipha: Eh, Lerato

Thabo: We flopped, but howcome, it works with the oth, it worked with the others, maybe the geoboard is not balanced

Nandipha: Thabo, stop smiling

Thabo: We flopped (sings) we flopped, ja, nee, how can we flop like this, but see with some things it worked, ne, check ne

Nandipha: (laughs) Some days it were our day, today uh-uh, wait, we had some, what did we discover last Friday

Thabo: We discovered this thing, ne, I.e, all the triangle what, what, {now it doesn't want to work anymore

Although some of the dialogue in the above extract is light-hearted, the pupils, in particular Thabo, are upset about the problem with the method. Having to present the method to the class in two days time adds to these feelings.

The process of developing the method and maintaining confidence in it is a difficult one for all three members of the group. Nandipha encapsulates this in her final written version handed in to the teacher[10]:

> "The discovery made our work lighter even though sometimes, when a teacher came to argue or discuss with us, we sometimes wanted to undo our discovery!"

From a cognitive perspective, the group members defend their method against challenges, modify, adapt or qualify it to incorporate difficulties, but remain with it as a useful method to calculate area. The following example show Nandipha and Thabo extending the theory to include the number of hooks inside the triangle while interacting with the teacher:

[10] See Appendix C

[extract 4.7; day 2; question 2b; p. 146; lines 1141-1147]

Teacher: *O.K. but I'm gonna disagree with you now*
Nandipha: *Why*
Teacher: *because I'm gonna say that this one here*[11] *with 4 hooks gives me an area of 1½, not 1*
Thabo: *4 hooks, which one*
Nandipha: *Miss, this is 5 hooks, 1,2,3,4,5*
Teacher: *Oh I see, O.K,*
Nandipha: *Ah-hah miss ah-hah*

and

[extract 4.8; day 2; question 2b; pp. 146-147; lines 1178-1185]

Thabo: *...and we found the area of the thing without having to use this triangle, all you count the hooks here, and then see there.*
Teacher: *But this one here's*[12] *got 4 hooks*
Thabo: *No, 1,2,3,4,5,6*
Teacher: *Oh I see, you [count,*
Thabo: *[you count there*
Teacher: *aah, even the ones on the inside*
Nandipha: *Yes*

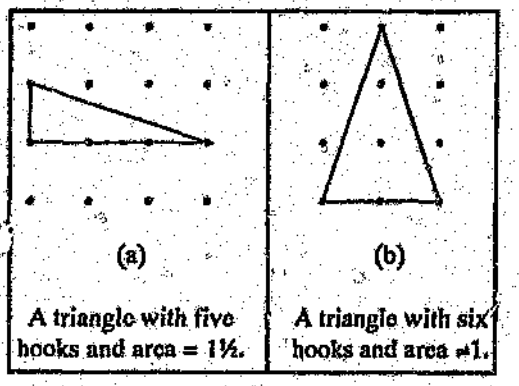
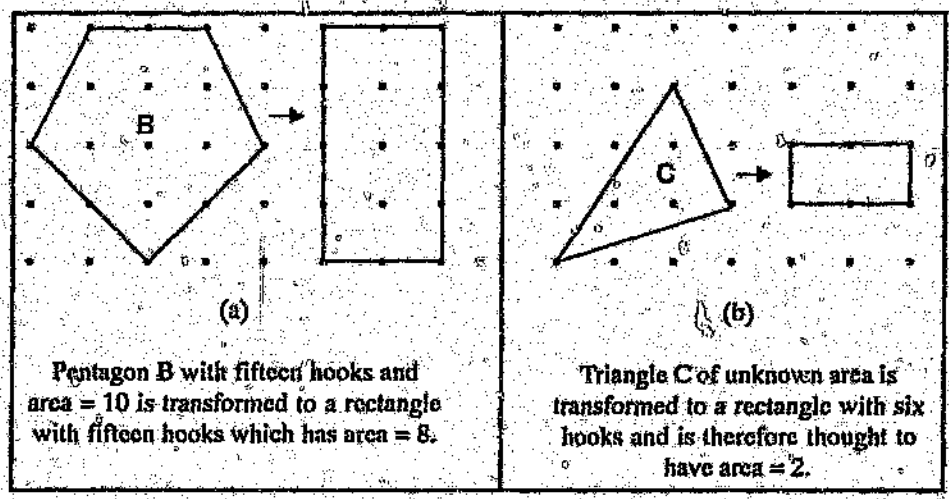


Figure 4.5

[11] See figure 4.5a. At this point the teacher counts the hooks incorrectly. It is in the following episode that she really challenges them.

[12] I am unable to tell exactly what triangle they are talking about, but it is probably the one in figure 4.5b. According to their method, this triangle should have an area of 2, although its area is actually 3. The pupils did not consider this, and they had only explained the 4 hook version of the method to the teacher, so she could not engage them on this inconsistency at this point.

Modifications are made to the method when it is found not to work, or to give an answer different from that which is expected. Early on, when testing the method on shape B in question 1, they discover that it does not work (fig 4.6a), but are still convinced that it works for shape C (fig 4.6b).



Pentagon B with fifteen hooks and area = 10 is transformed to a rectangle with fifteen hooks which has area = 8.

Triangle C of unknown area is transformed to a rectangle with six hooks and is therefore thought to have area = 2.

Figure 4.6

Thabo then concludes that it works with triangles, and moreover that since you can divide all shapes into rectangles and triangles, it remains a useful method:

[extract 4.9; day 1; question 1a; p. 132; lines 384-390]

Thabo: *Ja, this is a it works with triangles, that's one of ours the presentation, it works, it works*

Nandipha: *But it didn't work with other shapes*

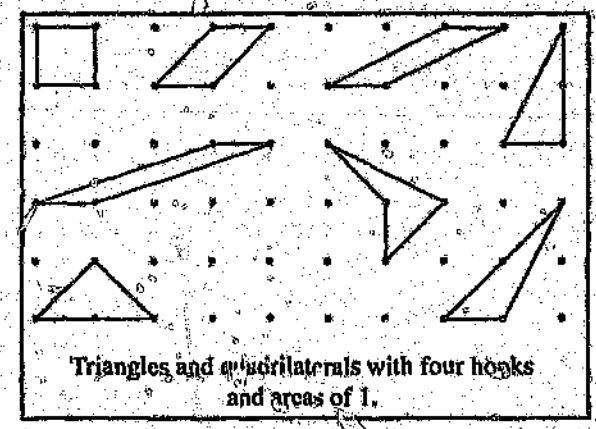
Thabo: *Yes, but it works with triangles*

(silence)

Thabo: *Yes, yes N it works with triangles, it works, it works, look at this, look at this ne, all the shapes here we can divide them into, into, into, into, into, we can somehow cut them into uh, squares and triangles ne, let's, let's continue and see*

Their being ready to defend and modify the method suggests that the pupils have a certain investment in it. This is a further indication of ownership, since it is reasonable to assume that if they did not experience some ownership of the method, they would give it up more quickly, especially when two teachers[13] showed disbelief, and explicitly challenged them. The pupils are also making decisions about solving their problems, and evaluating these decisions when considering changes to the method.

The pupils use the method to help them with other tasks on the worksheet. For example, for question 2 they make the square in fig 4.4 and use the four-hook part of the method to calculate the area of the triangles A, B, C and D. In this case it proves correct and invaluable. They are also able to use the four-hook conjecture to begin question 3a, which is causing them difficulties. In this case Thabo suggests that they choose an area of 1, because they already know that triangles with four hooks have area 1. While doing this task they extend the theory slightly to include quadrilaterals with four hooks having area 1. Thus they can generate many shapes (fig 4.7) with area 1.



Triangles and quadrilaterals with four hooks and areas of 1.

Figure 4.7

[13] There is evidence in the transcripts that the pupils considered me to be a teacher. Both their teacher and I constantly challenged the method and asked for justification of their ideas.

Applying or transferring knowledge is traditionally associated with understanding. In the case of this group it also displays ownership, in that they are able to use the method for their own purposes. Ownership and use are dialectically related: because the pupils own the method they are able to use it, and using it strengthens their ownership of it.

To summarise, the emotional and cognitive process aspects of ownership discussed in chapter 2 are evident from what the pupils do with the method and say about it. They develop it as a strategy to solve problems, they talk about it as their own, they express positive and negative emotions in relation to the method, and they constantly evaluate it. I have also shown that the group develops the method through defending it against challenges, modifying it to incorporate new evidence, and using it to help with other tasks. I have argued that these also constitute aspects of ownership, and I classify them together with other cognitive process aspects, such as posing and solving problems (chapter 2, section 3). They are crucial to this discussion since the pupils' use of these processes should form an important part of their mathematical development.

If we consider the mathematical development of the three pupils in this group, some serious concerns can be raised, particularly in relation to the mathematically invalid part of their method. To explain how this mistake was produced and sustained, the following chapters will focus firstly on the interactions among the members of the group, and secondly on the development of mathematical concepts through their interactions. Regarding the interactions, it will be seen that the group's engagement with the above processes was not sustained throughout the week, and the processes were never developed further. The pupils often ignored crucial evidence, did not relate new conjectures to old ideas and evidence (see figure

4.5b and footnote 12, this chapter), and remained with the method even when it became clear that it did not work. Regarding conceptual development, no members of the group made progress in developing a more systematic concept of area, and their spontaneous concepts were not related to broader mathematical knowledge.

These concerns support the arguments in chapter 2 that there is a crucial role for teaching in learning and development and, of particular concern for this study, in ownership of mathematical knowledge. These issues will be discussed in chapter 6. The following chapter will analyse the relationships between interactions in the group and ownership of the method, with a focus on power relations between the pupils, and the political aspects of interaction and ownership.

CHAPTER 5

INEQUALITY IN PEER-INTERACTION: WHO OWNS THE METHOD?

Proponents of peer-interaction advance a variety of reasons and objectives for small-group work in classrooms (see chapter 2). With regard to ownership, absence of the teacher is expected to enable pupils to gain control over the learning process and become more self-directed in their learning. The peer group then provides the social interaction which is necessary for learning and development. Peer interactions are expected to be more equal than teacher-pupil relationships and to provide opportunities for pupils to speak, thus allowing spontaneous concepts to be heard, facilitating the integration of different ideas, and enabling the development of language as a mediator of thinking.

This chapter will focus on an important question: To what extent does the small-group situation provide equality of interaction among the pupils, and how does this relate to ownership? The ways in which the teacher's interventions enable or constrain equality in the interaction will be briefly touched upon, but will be dealt with in more detail in the next chapter.

1. PASSIVITY IN THE INTERACTION

Damon and Phelps (1989) define equality of engagement in peer interaction as follows:

> "Equality means that both parties in an engagement take direction from one another rather than one party submitting to a unilateral flow of direction from the other" (1989:10)

This resonates with Pirie and Schwarzenberger's (1988) insistence that discussion consists of "genuine pupil contributions and interaction which move the talk or thinking forward". One person talking at another, and passive responses or carrying out of instructions do not count as genuine discussion.

In this particular group there are many examples of such passive responses. In the following extract, Nandipha twice discards her value for the area in favour of Thabo's or Lerato's, there is no discussion about the different values and only one initial attempt by Thabo to justify his method for obtaining area:

[extract 5.1; day 3; question 3a; p. 153; lines 1530-1545]

Thabo: *Area, O.K. we find a square, ne, we know the sides are going to be, no it's, we we find a square here, we find the, the the area of this, and then we find the area of this divide by 2*[1]
Nandipha: *So that's 4*
Thabo: *What*
Nandipha: *The area of this square*
Thabo: *It's 2½*
Nandipha: *O.K. it's [2½*
Thabo: *[Lets ask miss, ask miss, phela, this, this also can [be, I think, oh, O.K.*
Nandipha: *[My throat Thabo*
Thabo: *Oh, O.K.*
Nandipha: *?*
Thabo: *This has got 5 sides*
Lerato: *Isn't this 1½*
Thabo: *Ja, this is 1½*
Nandipha: *Oh ja ubuwanete {he's telling the truth}*
Thabo: *Aah, you can also work*

Thabo's final remark shows that he regards Nandipha's passivity in a negative light, he has an expectation that all should contribute. However he is not able to set up a more equal relationship, neither at this point nor at other times when similar incidents occur. This will be discussed in more detail later in this chapter (section 6).

[1] I do not have records of the shapes they are talking about.

Nandipha also takes on a very passive role at points where ideas are articulated and summarised, merely repeating and writing what Thabo says;

[extract 5.2; day 5; question 4a; p. 175; lines 2710-2731]

Thabo: O.K. here's the pattern, if it's a square, ne, 1 unit squared[2]
Nandipha: If it's a square
Thabo: it's 4 hooks
Nandipha: Ja
Thabo: if it's 1½, it's 5
Nandipha: If it's 1½
Thabo: It's 5. Everytime, everytime the number increase, the thing increase by ½
Nandipha: Oh, O.K. O.K
Thabo: Everytime, everytime the [ar, the area increase [by half
Nandipha: [Ja, [by half
Thabo: Write it down
Lerato: Wut
Thabo: Everytime the
Nandipha: Anytime the area increases
Thabo: Um
Nandipha: Ne, oh everytime the hooks increases, the area increases by half, O.K.
Thabo: Write it down, Nandipha
Nandipha: I have written it down
Thabo: Where
Nandipha: Here
Thabo: Are you going to remember that
Nandipha: Yes, ummmm, hew, where'll I write

Passivity and inequality of contributions are clearly evident in the above examples. However, most of the group's discussion appears, on the surface, to be more equal, with all participating and genuine interaction occurring. Neither the teacher nor the pupils recognise that, even in these cases, certain ideas dominate, and there is inequality in the interaction. Some of these exchanges occur at critical moments when the method is being constructed, and involve the more contentious part (what I called step 4 of the method in chapter 4). I will show that although the pupils seemed to be interacting productively, they did not attempt to

[2] This extract refers to the final version of the method described in extract 4.2 and fig 4.2, chapter 4.

negotiate meaning, and did not manage to achieve intersubjectivity. Because of misunderstandings, mistakes were made and misconceptions perpetuated.

2. SITUATION DEFINITIONS: WHAT ROLE FOR A METHOD?

Question 1 on the worksheet did not explicitly ask the pupils to articulate a method. The fact that the pupils tried to develop a general method for calculating area early on in the week, means that they had to bring to the situation the knowledge that a method would be appropriate. This would come from their previous experience of similar activities[3], and possibly from question 4 on the worksheet if they had read ahead.

However, they each have different ideas about the nature and purpose of a general method. The following extract (from the discussion about shape B) reveals each pupil's ideas:

[extract 5.3; day 1; question 1a; pp. 127-128; lines 118-163]

Lerato: *This thing is (laughing) this, hae, man come, this thing is five-sided ne and there are 1,2,3,4,5 ne, eh, eh there are four spaces, Thabo, there are*[4]
Thabo: *No, no*
Nandipha: *Come on, come on, come on, can I please see it*
Thabo: *Ca, ca*
Nandipha: *O.K, this one will give you 1, ne this one [will give you 1, ne*
Thabo: *[will give you 1, no, no, no, no, no, it will gi, it will give us, ntho {something} 1½*
Nandipha: *ena {this one}*
Thabo: *Eh, eh, eh*
Nandipha: *Oh, [O.K, O.K O.K*
Thabo: *[this 1½*
Nandipha: *Oh, O.K, O.K this is 1½, this is 1½, so that's 3*
Thabo: *Jaaa*
Nandipha: *Ne, sharp, this one is a ½*
Thabo: *It's a 1*

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[3] These pupils have previously done work in which they would be expected to generalise and verbalise relationships from a number of examples. They have also worked with formulae for calculating area.

[4] See figure 5.1a for her meaning

Nandipha: it's 1
Thabo: so it's [4
Nandipha: [it's 1 so it's 4, [5,[6,7,8,9,10[5]
Thabo: [6,7,8,9,10, lets find a way to find, to, to, to
Lerato: to work this out,
Thabo: Eh, so we'll say, mm, no, I know, I know, you count, you count, mm, 1,2,3,4,5,6,7,8, no, no, no, no, no
Lerato: No, there's 1,2,3,4,5,6,7,8,9,
Nandipha: and then, and then
Thabo: No, I know, I know, [you count
Nandipha: [Thabo, you always here
Thabo: You count down from here, ne, 1,2,3,4,5, ne, and then you say by 2, multiply by 2, there 2 spaces here[6]
Lerato: uh-uh, it doesn't work
Thabo: It will work, it will work
Nandipha: [What
Lerato: [What if it was like this here Thabo
Thabo: Oh, come on, the sides must be the same size, no
Lerato: No, wena you just assuming here {that} the answer is 10
Thabo: eh, it is 10 but then we can find, a method
Lerato: It's not
Thabo: O.K
Nandipha: What is it, Lerato
Lerato: I don't know, we all don't
Nandipha: No, you know what, I think
Thabo: No, no, no, no, no, no
Nandipha: Ish, mmm, maara waitse wena kobotsitse {but I told you} (?) 1,2,3,4,5,6, 7,8,9, it's 10
Thabo: It's te, eh ja, we, we, know Nandipha here {that} it's 10, but then we find, how we found it, in eh, in eh
Nandipha: Ag, just cut it up man it's supposed to be

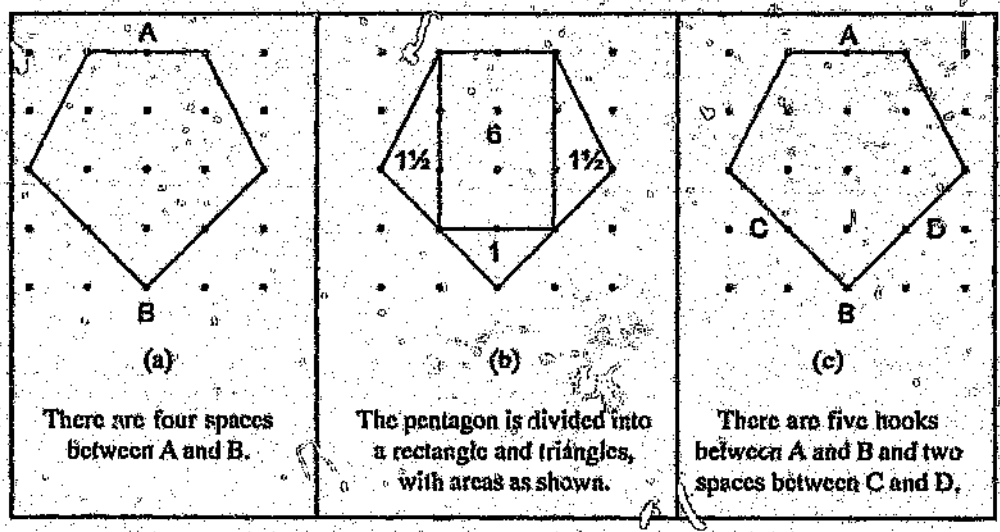


Figure 5.1

[5] See figure 5.1b for their working

[6] See figure 5.1c for his meaning

Lerato tries to adapt an implicit formula which they had used for shape A (see extract 5.10 this chapter). It involves counting the spaces between the hooks, to obtain the base and height of the parallelogram A. Her understanding seems to be that a formula is validated on one example and then used relatively unproblematically for the rest.

Nandipha and Thabo ignore her efforts and divide the pentagon into a rectangle and triangles (figure 5.1b) to obtain the area 10. Lerato does not participate at all in this discussion. Having found the answer, Thabo suggests that they find a more general method and tries to construct one. In this he is supported by Lerato.

He suggests 2 multiplied by 5 (figure 5.1c), because it gives the answer 10. As with Lerato's suggestion, Thabo's is also related to the formula they had used for parallelogram A. However, he brings in the hooks as well as the spaces in order to get the answer 10. Lerato evaluates his suggestion in terms of the shape, arguing that it will not work if the shape is changed slightly. She is not persuaded by the fact that his method gives the answer 10, since she does not agree that the area is 10. This suggests that she did not accept their division of the shape into a rectangle and triangles because she is working with the idea that they must get the area from a predetermined formula.

Thabo, on the other hand, has an understanding of a method as a formalisation of their more intuitive way of finding particular areas. He accepts the answer 10, but attempts to articulate how they found it.

Nandipha becomes impatient with both Thabo and Lerato, with Lerato because she won't accept the answer 10 and with Thabo's insistence on finding a method. She does not see a need for a generalisation at all, she is content to work out the area for each shape by counting blocks and piecing parts of blocks together.

Thus each pupil sees the role of the method or formula differently, and therefore they have different situation or task definitions (Wertsch, 1984).

3. DEALING WITH DISAGREEMENT: TEACHER INTERVENTION

Thabo and Lerato both attempt to justify their positions in the above extract. Nandipha, however, does not, and becomes impatient[7]. After a few more lines of talk about dividing up the pentagon, Thabo reconsiders and discards his desire for a method, thus moving towards Nandipha's situation definition. Lerato disagrees and they decide to call the teacher:

[extract 5.4; day 1; question 1a; pp. 128-129; lines 188-200]

Thabo: *O.K. le, lets just, how, no, mm-mm, nee they said we don't have, we don't have to have a,a,a particular method, how to work them out, all you need to do is just to write them down*
Lerato: *Uh-uh*
Thabo: *Ask miss, ask miss, ask miss*
Lerato: *Nandipha, please call miss*
Thabo: *Miss*
Teacher: *Ja*
Thabo: *are we supposed to have a*
Nandipha: *method*
Thabo: *a method of how we found this thing*
Teacher: *Ja, I mean you should have some kinda method, otherwise how you gonna do it*
Thabo: *Our, our, our method is there, but then, it's not in eh, in written form*
Teacher: *O.K. explain to me your method, O.K. nevermind written form but then, it must be explainable*

[7] This is evident in her tone when she says "Ag just cut it up man its supposed to be".

The teacher endorses Thabo's approach, suggesting that while a method needn't be formally articulated at this stage, there should be one and it should be "explainable" ... some point. Lerato is quiet during this interaction (and most others with the teacher) and her situation definition is not discussed. Thus the teacher does not get a chance to help arbitrate between all the different points of view within the group, nor to facilitate more fruitful negotiation among the pupils.

Immediately after this interchange Thabo verbalises the way he and Nandipha calculated the area for shape B (figure 5.1b) to the teacher and she validates it. The group then moves on to shape C and continues trying to develop a method. They work with Thabo's notion of what a method is and how it is generated. Lerato's ideas have not been taken into account and Nandipha's have been dismissed, with the help of the teacher. This enables the group to move on with the task as if they have shared understandings of what they are doing. However, as will become evident in the next section, they do not.

It would be wrong to suggest that it is only the intervention of the teacher at this point that produced an assumption of intersubjectivity when in fact there was none. In the above interaction with the teacher, only Thabo and Nandipha's disagreement on the importance of a method is discussed. Lerato does not bring up her different understanding of what a method might be, and thus the teacher does not get a chance to interact with her situation definition. It may be that Lerato is silenced by the fact that the teacher validates Thabo and Nandipha's answer for B. She also may be unable to articulate clearly her disagreement with Thabo and her own situation definition. In fact, she and Thabo had hardly begun to talk through their differences when the teacher was called. There are many other instances where such

negotiations do not happen and the teacher is not available for help. In the following section I will show that in these situations intersubjectivity is also not achieved, and this has serious implications, particularly for the development of the method.

4. INTERSUBJECTIVITY AND THE METHOD

The mathematically invalid part of the method, that if a triangle is transformed into a rectangle with the same number of hooks, then it will have the same area as the rectangle (see figure 4.2a, chapter 4), is initiated by Thabo when they are working on triangle C[8]. They cannot work out the area for shape C in the same way as they did for shapes A (parallelogram formula) and B (dividing up the shape and counting the blocks)[9]. So Thabo suggests counting the hooks and the following discussion ensues:

[extract 5.5; day 1; question 1a, pp. 130-131, lines 307-328]

Thabo: *O.K, a solution ne, we count the strings, not the strings, the sticks ne, no, what, what, the things, the hooks, 1,2,3,4,5,6, yes*
Nandipha: *Uh*
Thabo: *So, O.K, is it 6*
Lerato: *seven*
Thabo: *seven*
Nandipha: *uuhh*
Thabo: *It's 6, so we put*
Lerato: *Thabo, it's [seven*

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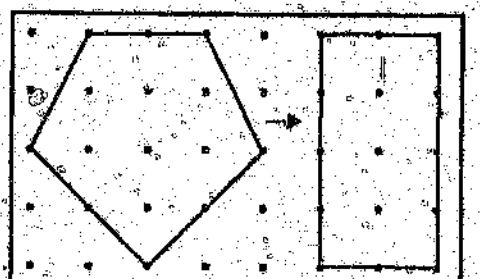
[8] The idea that the hooks are important had by this point already been expressed a number of times. It can be traced back to a remark made by Nandipha in the discussion on shape A (see extract 5.10). Thus it is not only Thabo's idea. However, he formalises it into the method.

[9] Both of these, however, contribute to the development of the method. Shape A brings up the issue of a formula (Lerato's situation definition) and the hooks (see extract 5.10 and footnote 8 above). Dividing the shape into squares and triangles, as they did for shape B (extract 5.3, fig 5.1b) later becomes an important part of the method.

Thabo: *[We put a thing on 6 like this, and then*
Lerato: *Thabo, it's seven*
Thabo: *How, how*
Lerato: *Sheba {look}, 1,2,3,4, (laughs) no man look, 1,2 3,4, [5,6*
Thabo: *[5,6, so, so akere {that is}*
got [6, 6 things
Nandipha: *[ha ha, yobaba {oh father (exclamation)}*
Thabo: *So, here's our solution, lets say*
Nandipha: *So it's a rectangle*
Thabo: *Yes, so it's 2 by 1*
Nandipha: *If we convert it*
Thabo: *Lets, lets try do the thing*
Lerato: *It's 2 by 1*[10]

Aside from a brief disagreement about how many hooks there are in triangle C, which is quickly sorted out, both Nandipha and Lerato seem happy to go along with Thabo's idea. Nandipha confirms: "So it's a rectangle" and Lerato repeats: "It's 2 by 1".

Following this articulation of the method, Thabo decides to test it on shape B, and Nandipha and Lerato engage fully with him in trying to do so (pp. 131-132). They count 15 hooks in the pentagon and make a rectangle with 15 hooks which gives an area of 8 (see figure 5.2).



The pentagon with fifteen hooks and area of 10 is transformed to a rectangle with fifteen hooks and area of 8.

Figure 5.2

[10] See fig 4.2a, chapter 4

So the method gives an answer which is inconsistent with the area for B obtained previously. The way in which the group deals with this problem, shown in the following extract, is critical:

[extract 5.6; day 1; question 1a; p. 132; lines 368-391]

Thabo: *It doesn't work*
Lerato: *If it doesn't work then it means that number, our number B is wrong*
Thabo: *number B, hae number C, we haven't done number C ne*
Lerato: *number C it's, 3 x 1*[11]*,*
Nandipha: *Miss*
Lerato: *2 x 1 I mean*
(silence)
Thabo: *It works, it works, it works*
Lerato: *What*
Thabo: *It works with triangles, it works with triangles, but no other ang,*
Nandipha: *With no other kind of quadrilateral*
Thabo: *Ja, but check, check, it works with triangles because look this one's going to give us 1 ne, ne, and this one, 1 also ne, this ne, no, this piece is going to fit here to make it how much, to make it how much, ish it works with triangles*
Nandipha: *miss, miss ?*
(silence)
Thabo: *Ja, this is a it works with triangles, that's one of ours the presentation, it works, it works*
Nandipha: *But it didn't work with other shapes*
Thabo: *Yes, but it works with triangles*
(silence)
Thabo: *Yes, yes Nandipha it works with triangles, it works, it works, look at this, look at this ne, all the shapes here we can divide them into, into, into, into, into, we can somehow cut them into uh, squares and triangles ne, lets, lets continue and see, lets see wabona {can you see} here, here is 2 x 1,*

Thabo concludes that the current conjecture doesn't work for shape B. Lerato uses this to support her original contention (extract 5.3) that their answer for B was wrong. This suggests that she believes that the method is correct, and so she is still working with her original notion of a method as something that gives the answers rather than being derived from them. So even though the teacher's intervention allowed Thabo's situation definition, and Thabo

[11] She makes a mistake which she quickly corrects in her next utterance, Her mistake suggests that she might have been thinking about the hooks in the 2 x 1 rectangle.

and Nandipha's answer for B, to prevail publicly, privately Lerato maintains and works with her previous understandings.

Lerato's challenge leads Thabo to reconsider triangle C. Lerato, who is happy with the method, then states emphatically that the answer for C is "2 x 1". She is not asked by either of her groupmates to explain further. Nandipha is too concerned with calling the teacher (to ask for a break in the double lesson). Thabo on the other hand takes on Lerato's confidence in the answer for C and continues to entrench his conjecture. He briefly attempts to work out the area of C by finding squares inside it ("this one's going to give us 1 ne, and this one, 1, also, ne ..."), and quickly concludes that his method will give the same answer, and is therefore correct. Lerato does not challenge this, because she is happy with the answer for C and because she does not see the need to test the method by finding the area in another way.

So Thabo qualifies his original idea by saying that the method works with triangles although not with other shapes (this excludes the pentagon B). Moreover, this will help them because all shapes (particularly the pentagon B) can be divided into squares and triangles. Thabo has listened to Lerato, and has taken on her belief that the method gives the correct answer for triangle C. However, this reflects a lack of, rather than real communication between Thabo and Lerato.

This lack of communication stems from their different situation definitions, and their inability to negotiate their different understandings and attempt to attain some measure of intersubjectivity. They do not realise that they are working from different frameworks. This

would require a metacognitive capacity which they clearly do not display. Instead they work alongside each other, each with her/his own understanding of what she/he is doing. However they do listen to each other, and at certain points take up on ideas that the other expresses. In this case, however, the ideas are taken on inappropriately because the framework which generates them (Lerato's understanding) is different from the one which interprets them (Thabo's understanding).

The lack of intersubjectivity and communication enabled the articulation of invalid mathematics. This might have presented an opportunity for learning, but this did not occur. The group sustained their belief in the validity of the method for most of the week. Reasons for this will be seen to relate to pupils' ownership of the method, which severely constrained further learning and had serious consequences for the continued interaction of the group.

5. OWNERSHIP OF THE METHOD REVISITED

In extract 5.6, Lerato becomes quiet after her question: "what", because she still is not understanding Thabo's notion of a method, and hence his reasoning. Nandipha challenges Thabo twice[12], but not very persuasively (she is more interested in having a break than in thinking about their problem). This is significant because she is excluded (or excludes herself) from the process at this point.

[12] Her tone of voice indicates that both her utterances in extract 5.6 are challenges.

Neither girl is as excited as Thabo about the method and they do not display the expressive and emotional characteristics of ownership that he does. Many of the references to "our method", "our solution", which were seen to be expressions of ownership (chapter 4, section 2) are made by Thabo. He even talks about "my method" (pp. 132, 137). Neither of the other two refers to it as her own method. Nandipha uses "an invention" (p. 166), and Lerato speaks of "the method" (pp. 164,165) and even "your method" (p. 172).

Thabo, however, is extremely excited with his discovery, he replies to Nandipha, ignores Lerato, and articulates the more general form of the method, apparently to Nandipha (extract 5.6). However, as they continue with their work (after the break), Nandipha and Lerato become concerned as to the validity of his ideas. The ways in which their concerns are expressed and ignored have further implications for ownership of the method.

The group checks the restated method together on shape B, by dividing it into a rectangle and triangles with three or four hooks (see figure 5.3). They find that it works:

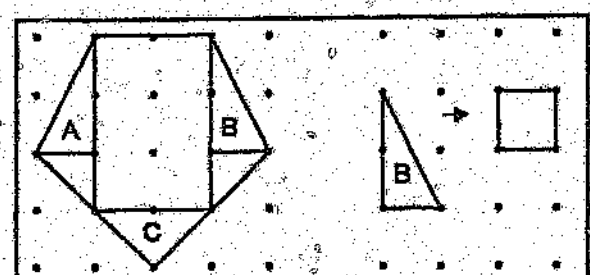
[extract 5.7; day 1; question 1a; pp. 132-133; lines 409-426]

Thabo: *? wabo {can you see}, it works with triangles, bona {look}, lets see, wobona {can you see} number B, ne,*
Nandipha: *Mm*
Thabo: *akere we found our square already ne*[13]*,*
Lerato: *Ja, [and*
Thabo: *[We found our square*
Lerato: *[and these are triangles*
Nandipha: *[Mm*
Thabo: *and you've got 4 strings*[14] *here*
Lerato: *[Ja*

[13] See figure 5.3. He means rectangle when he says square.

[14] He means hooks.

Thabo: *[you can move them and put it here[15] it's gonna give us 1, and here gonna give us 1, ne*
Nandipha: *Voila*
Thabo: *And then here, then here again it's gonna give us 1, so that's 1, 2, 3, 4, 5, 6, [7, 8, 9, 10*
Lerato: *[7, 8, 9, 10*
Thabo: *No, yes*
Lerato: *Mm, 10*
Thabo: *10, It works, so ev, for every shape we get we divide it into squares and triangles, and then we find our thing, superb, thank you, (laughter) that was B*



The pentagon is divided into a rectangle and triangles and the areas of triangles A,B and C are calculated to be 1 because they have four hooks and be transformed into a square.

Figure 5.3

Thabo generalises from triangles with four hooks to all triangles and thus confirms his answer for C and his conjecture. It is at this point that Nandipha and Lerato begin to express serious doubts about this part of the method. First with regard to shape C:

[extract 5.8; day 1; question 1a; p. 133; lines 428-432]

Thabo: *So number C, number C's going to be*
Lerato: *It's going to be 2 x 1*
Thabo: *2 x 1*
Nandipha: *Honeng recho {How many times have you been saying that}, I heard*
Lerato: *Thabo, write there 2 x 1*

[15] He means transform the triangle into a square to get an area of 1.

Lerato and Thabo agree that the area of triangle C is 2 x 1. Nandipha does not express agreement, in fact her "Hoheng recho" is a veiled challenge to their agreement. However, they ignore this and continue, with her collusion, with shape D, where the following crucial interaction occurs:

[extract 5.9; day 1; question 1a, pp. 133-134 lines 461-477]

Thabo: *So lets see if it works, this time 1, 2, 3, 4, [5, 6*
Nandipha: *[O.K, this converting to a*
Lerato: *This on[e's*
Thabo: *Wait, lets try to, to see, so lets shading, shading, shading[16], this is going to give us a half, ja man, to make it for 2, yes, 6, 6, there's our [thing*
Lerato: *[So, they the same, as long as they shaded*
Nandipha: *Howcome*
Lerato: *(sings) "Oh, na"*
Nandipha: *(laughs)*
Thabo: *there, 6, so it's 2 by 1, 2 by 1[17]*
Nandipha: *Hape {again}*
Thabo: *eh*
Lerato: *2 by 1*
Thabo: *Yeah*
Nandipha: *Mm-mm, 20 by 30*
Thabo: *It's your turn*
Nandipha: *Buddy it's your choice*

This time Nandipha is definitely not convinced, asking "Howcome" and "Hape"[18]. Lerato is also not sure[19], yet Thabo insists "yeah" and ignores both of them. He suggests they continue with the next shape by saying "it's your turn"[20]. Nandipha's final remarks in the

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[16] It is not clear to me what he means by "shading".

[17] Again he converts the triangle to a rectangle with 6 hooks.

[18] Both of these are uttered as questions.

[19] This is evident when the girls' tones of voice are heard. "Hape" is expressed in a tone of disbelief, and "2x1" as a question, with doubt in Lerato's voice.

[20] The pupils took turns to make the shapes on the geoboard. The statement "its your turn" signals that it's her turn to make the next shape; he is ready to move on.

above extract are significant. She makes an outlandish suggestion, "20 by 30" which suggests that she thinks 2 x 1 is equally outlandish. She also cynically tells Thabo that it's his choice. She might mean that the method is his choice and not hers, or that to continue while they're not sure is a choice that he is making without herself and Lerato. Even though these doubts are expressed, Lerato and Nandipha submit to Thabo's wishes and they continue with shape E[21].

We have seen that Thabo takes most of the initiative in developing the method. The girls do make contributions. Nandipha originated the idea that the hooks are important. Lerato participated in confirming the validity of transforming triangles to rectangles while conserving area. Thabo integrates their ideas with his own to formulate the method in his way. He is able to internalise parts of the discussion for his own purposes, and they become his. He has developed ownership of the method, although not all the ideas that make it up were his.

On the other hand, the two girls do not take on his ideas as readily. They do not agree with his formulation of the method at this point, and they attempt to challenge it, specifically with regard to shapes C and D. Their challenges are not taken up by Thabo. In fact he does not even hear them, perhaps precisely because he has developed such ownership of the method, and is excited by the aspects that do work. They do not insist on being heard or taken seriously, but, certainly in Nandipha's case, abdicate responsibility for the method.

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[21] Their work on shapes E,F and G does not involve the contentious part of the method, and so is not analysed here.

Thus the three pupils own the method differently, and each owns different aspects of the method. This is evident in their written work (Appendix C).

Nandipha writes:

> "We started out with 4 and we only stayed at for (sic) because we made an important discovery with 4. We found out that if for instance you put an elastic (on geoboard) and make a square with 4 hooks, you can also make a triangle with 4 hooks and they both have the same area...you could use our method of working out. If it worked on the complicated shapes that we had it could work on anything."

This reflects the fact that the four-hook part of the method does seem to work, particularly when they use it for question 2 (chapter 4, figure 4.4). She glosses over the more general formulation of transforming any triangle into a rectangle with the same area, although she claims that the method works for any shape.

Lerato writes:

> "Divide your polygon into triangles and squares then find the areas of both the squares and triangles then add them."

This is very different from the version that Lerato presents carefully and articulately to the class at the end of the week (chapter 4, extract 4.2). This shows that although she could present the ideas to the class, she does not own the more contentious parts of the method. The only part with which she is comfortable is the division into squares and triangles and adding the areas to obtain the area of the whole shape. It must be pointed out that this does represent learning on her part - originally she disagreed that this was an acceptable method for calculating area (extract 5.3, this chapter).

Thabo writes:

> "Area of a complicated shape is always divided into rectangles and triangles with one side equals one unit. The pattern: starting from 3 dots, the area is equals to ½ units². 4 dots is equals to 1 unit². So everytime you add 1 dot the (incomplete)."

Thabo is the only one of the three who presents the final version of the method, suggesting that he alone holds on to some notion of transformation. Even he, however, does not write down the initial formulation of transforming triangles to rectangles, and thus has realised that it is problematic (see also extracts 6.1 and 6.2, chapter 6).

From their written work, we see that both Lerato and Nandipha have moved from their original situation definitions (section 2, this chapter), and come to own aspects of Thabo's ideas. Nandipha has accepted the idea of a method, and Lerato writes about the first part of Thabo's method. They have internalised some of his ideas, although not the main one about transforming areas. This is because they think it's wrong[22]. Their disagreements have not been fully incorporated into the method, and so they have distanced themselves from it. Thabo, on the other hand, owns the method in its entirety, having taken on some of their ideas, but only as they fit into his original definition of a method. In all three cases we see that internalising the ideas of others can lead to ownership, but the ways in which each individual internalises the different ideas available will produce differential ownership.

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[22] Neither Nandipha nor Lerato has an answer written down while they were working for the areas of C and D. Only Thabo has written down 2 for each. This adds to the argument that they did not think that the method was correct.

From this chapter we have also seen that ownership requires more than understanding. Lerato understood enough of the method to be able to verbalise it to the class, because it was important to present a group effort. Nandipha also articulates the method to the teacher on a number of occasions (see for example pp. 146 and 165). However, they do not choose to report on the whole method individually to the teacher. Thus ownership entails a level of choice, such that when you are removed from classroom or group constraints, you will still find the knowledge important, talk about it, and use it if appropriate.

Having worked collaboratively for a week in developing a method which is peculiarly theirs, the group has not emerged with joint ownership of the main ideas generated. While it is not problematic for each pupil to have gained something different from the groupwork, it is problematic that one pupil's ideas predominate to such an extent. Thus far in this chapter I have presented many examples of interactions where Thabo's ideas dominate, and have argued that this affects and is affected by the group's inability to negotiate their different situation definitions and to achieve some form of intersubjectivity. Consequently, differential ownership of the method is produced.

More seriously, this differential ownership then sets up further inequality in the interaction, and in fact leads to the degeneration of the group's work later in the week. The following section will expand on this.

6. INEQUALITY AND CONFLICT

I chose to focus on this group because on the face of it they seemed to display a high degree of equality and mutuality (Damon and Phelps, 1989) in their interactions. They did not require as much teacher intervention as some of the other groups to ensure that they interacted and worked productively.

There is a lot of evidence of positive relations between the pupils. They quickly set up co-operative ways of working, for example, taking turns at making the shapes on the geoboard. There is also good-natured chatting and joking, and they establish a number of recurring comments and jokes which help to maintain cohesiveness and group relations (see for example the phrase "genius at work", pp. 134, 135, 137, 144, 145, 158, 159, 172, which is sometimes said sarcastically, but often appreciatively when one of the three completes a task successfully).

The first two days in particular are characterised by positive social relations, as well as a high degree of collaboration, engagement, motivation and interest. However, from day 3 onward, there is more evidence of fighting (socially and over the work), social chat, and repeated attempts to work, with little success. This change might be related to the changing nature of the tasks (see chapter 6, section 1). Another important reason, related to the current discussion, is Nandipha's and Lerato's exclusion from ownership of the method, and the dominance of Thabo's ideas.

Forman (1989) suggests that in asymmetric peer interactions, as this one has been shown to be, the relationship between the power and knowledge of the participants is important. A more powerful and knowledgeable peer might be able to teach the others, while if the more powerful pupil has less knowledge, the beneficial aspects of peer collaboration might not be realised. In the case of this group, the relationships between power and knowledge are seen to be complex.

Nandipha is powerful in the group interactions. She often takes the lead in reading out the tasks from the worksheet, and in trying to get the group to work. She is quick to contribute, quick to take offence if challenged, and is often resistant, initiating social conflict and fights. The following extract comes from the first few minutes of the first lesson, and shows some of her ways of interacting. They are discussing the area of shape A:

[extract 5.10; day 1; question 1a; p. 126; lines 37-55]

Lerato: *So, lets work out*
Thabo: *The area*
Lerato: *The area*
Nandipha: *It's πr^2*
Lerato: *πr^2, phela {but} this is a quadrilateral, so you can say*
Thabo: *2 times 3, 6*
Nandipha: *? that's 4*[23]
Thabo: *eh*
Nandipha: *that's 4*
Thabo: *No, man, you cou, you don't count the things, the things that are separated, [you count the*
Nandipha: *[you count the spaces*
Thabo: *spaces, so that's 1,2 times 3*
Lerato: *Why the spaces man*
Thabo: *Ow, it's always been like {that*
Lerato: *[Thabo, these are the points phela {but}*
Thabo: *[No*

[23] She is probably referring to the 4 hooks on the base of the parallelogram.

Lerato: *[It's just, wait, lets pretend here {lets say} this thing has got blocks, ja, 1,2,3 maara {but}*
Thabo: *1,2,3, 6*
Lerato: *It's 2 times 3 it's 6*

Nandipha jumps in immediately with "πr^2" as a formula to calculate area. This is challenged as inappropriate, and a few lines later she jumps in with the number 4, probably referring to the hooks. She is challenged again, and does not contribute further to this discussion. While both Lerato and Thabo accept Thabo's answer, there is no indication that Nandipha does.

Nandipha's knowledge does not stand up to the others' challenges, and so while she is an enthusiastic contributor, her ideas are not often taken up. Her passivity, demonstrated in section 1 of this chapter (extracts 5.1 and 5.2), and her abdication of responsibility for the method, can be seen as resistance to this state of affairs. Her resistance grows as the week progresses, suggesting that she is even more unhappy at not being taken seriously. When she is challenged, or when Thabo does try to include her (extract 5.1), she uses excuses, or threatens conflict to silence him (for example, pp. 135, 153, 155).

On the other hand, Thabo's knowledge dominates, but he is no more powerful socially in the group than either of the other two. He is certainly not strong enough to set up a situation where real negotiating can take place, and in which the others can participate. In extract 5.1 above we see him realise that Nandipha is not pulling her weight, and yet he cannot do anything about it. In extract 5.10 above he neither explains, nor justifies his ideas, resorting to "Ow, it's always been like that".

It should be possible for the two girls to support each other in challenging Thabo, but they do not. It is beyond the scope of this project to investigate the reasons for this. There are probably gender factors involved, together with the relationships between each pair of individuals. The fact that they do not challenge him has serious consequences for their work. It allows the development of a faulty method, and the degeneration of their further work. They do not return to a serious reconsideration of the method until day 4, when they have to articulate it in response to question 4. At this point it is too late for them to make substantial changes. Moreover, they are not able to, because they are not working well together.

In summary, differential ownership of the method has both produced and been produced by conflict and asymmetries in the power relationships between the pupils. Because of the relationships between power, knowledge, and ways of interacting, none of the pupils is able to take a lead in facilitating negotiations of meanings or intersubjectivity. The teacher, who should be able to play this role, is also hampered, because she cannot hear everything that happens in the group, and when the group reports back to her, only some voices are heard.

This suggests that an important role for the teacher interacting with small groups is consciously to attempt to ensure that all ideas are reported on, and all voices heard. The teacher can use her power to ensure a more democratic division of power within the group. This will be discussed in more detail in chapter 7. There is also a crucial role for teaching in the development of mathematical knowledge among the pupils. The following chapter takes this up.

CHAPTER 6

THE DEVELOPMENT OF MATHEMATICAL KNOWLEDGE

In the previous chapter I examined the nature of the pupils' interaction, particularly its inequality and the pupils' consequent inability to achieve intersubjectivity. I argued that this produced and was produced by a mistake in the method, and differential ownership of the method. However, there were positive aspects to the interaction. In chapter 4, I showed that the small-group work provided opportunities for the pupils to engage in mathematical processes such as defending ideas, modifying them to incorporate new evidence, and transferring them to solve new problems.

These process elements of mathematics are important for learning, doing and owning mathematics. However, the fact that the method contains a fundamental flaw, relating to conservation of area, and that this was not dealt with in any substantial way throughout the week, raises two serious questions about the pupils' learning. Firstly, did the pupils begin to develop more systematic conceptions of area (see chapter 2, section 1.2.2)? Secondly, were the pupils' abilities to engage in the above processes extended in any way?

These questions raise the issue of the relationships between mathematical processes and products. One of two assumptions often underlies process-oriented pedagogies, among which many approaches to peer interaction and small group work can be included. These assumptions are that: 1. processes are more important than products, or 2. if we concentrate on the processes, and provide the correct tasks, the products, ie: conceptual development, will follow automatically.

In answering the two questions raised above, I will challenge both of these assumptions and will suggest a crucial role for teaching, both in developing processes and bringing together process and product in true concepts (chapter 2), which are essential in achieving ownership of mathematical knowledge.

1. DEVELOPING A CONCEPTION OF AREA - THE TASKS

The worksheet aimed to draw on the pupils' current knowledge and spontaneous concepts, and to build a more systematic notion of area and perimeter and the relationships between them (see chapter 3, section 3.2 for a more detailed discussion). The initial tasks were more tightly framed, and as the worksheet progressed the tasks became more divergent.

The group's responses to the tasks were unexpected. On the more convergent first task they generated a particularly original idea, their method. However, as the tasks became more divergent, they struggled to engage fully with them.

For question 2a, their initial reaction is that it is impossible, until the teacher intervenes and prompts them to see a "diagonal" square (p. 143). Her intervention enables them to make progress. They continue, and decide that 1,2,4,8,9 and 10 could be areas of squares[1]. However they soon decide that they have enough squares and have completed the task

[1] Nandipha even tries to find a method for generating these numbers (p. 147). Although her method is inappropriate, she has obviously been convinced of the importance of a method, which suggests a change in her situation definition. However, her ideas in this regard are not taken up in the group and do not develop further.

(p. 147). They do not consider a way of evaluating whether they have all the squares[2]. In this case interest in and ownership of their method detracted attention from the task at hand, and contributed to a lack of further thinking about it.

They find question 3 even more difficult. Again, they call the teacher, and she gives them an example of how they might work with a particular area. They then struggle to choose their own area, not doing more than suggesting and rejecting numbers haphazardly (p. 155). The only possibility that they engage with for an extended period of time is 1, because, from their method, they know of many shapes that have an area of 1 (figure 4.7, chapter 4). However, they do not delve very deeply, and do not consider whether they have all the shapes with area 1 before continuing. Lerato's criterion is that they have "a lot already" (p. 158), while Thabo, more wedded to the method, insists on continuing with the area 1. However, even he soon loses interest and they move on to question 3b, where the same problems arise (p. 158-159). There is no evaluation of what they have done, no thought that they might try to work out whether they had all shapes of area 1, and no move towards a more systematic generation of areas and shapes. If they had been more systematic at this point, they may have found some shapes which disprove their method.

In both cases (questions 2 and 3), strong ties to the method prevented the pupils from moving to other ways of thinking. In question 2b, they are praised for having made an interesting square (figure 4.4, chapter 4). In discussing the square, the teacher engages them on the four-hook part of the method and asks them to test it some more. They get involved with the method again, and do not progress with the task at hand. In question 3, they are unable to

[2] At least one other group in the class did this.

move beyond generating shapes using their method. So while the method provides an initial point of entry, it constrains further thinking.

Moreover, the possibilities for divergence in the open-ended tasks meant that the pupils had to do more interactive work in trying to define what they should be doing. As I have shown, they were not very good at negotiating situation definitions, even on the convergent tasks. This was exacerbated later during the week, by their differential ownership of the method and the accompanying conflicts (see chapter 5).

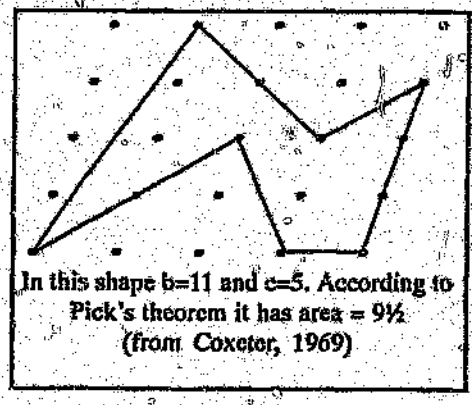
Engagement with these tasks as a group, together with some teacher intervention, was not sufficient to enable the development of a systematised conception of area, nor were connections made between the pupils' spontaneous knowledge and more systematic knowledge.

2. SPONTANEOUS AND SYSTEMATIC AREA CONCEPTS

The spontaneous area concepts of the pupils that are evident in this transcript are the division of shapes into smaller areas, the relationship between hooks and area, and their fundamental misconception: that a shape can be transformed while conserving the area, as long as the number of hooks remains the same. In order to develop more systematised concepts, these ideas would need to be related to each other more carefully, and brought into contact with other ideas, such as using formulae to calculate area.

There is a mathematical theorem, known as Pick's theorem, which relates the area of a shape on a lattice to the lattice points on the boundary and inside the shape (Coxeter, 1969). It states:

> The area of any simple polygon whose vertices are lattice points is given by the formula ½b + c - 1 where b is the number of lattice points on the boundary while c is the number of lattice points inside.



In this shape b=11 and c=5. According to Pick's theorem it has area = 9½
(from Coxeter, 1969)

Figure 6.1

The proof of this theorem is reproduced in appendix D. It breaks down shapes into "unit cells" of parallelograms with 4 vertices and no lattice points on the sides of, or inside, the parallelogram[3], or triangles with only 3 lattice points (half a parallelogram). The areas of these are then added to prove the theorem.

This theorem, as part of socially constructed objective mathematical knowledge (Ernest, 1991) could provide an exciting point of contact between the pupils' method and the

[3] This resonates with the pupils' idea that the hooks inside must be counted.

systematised body of mathematical knowledge[4]. However, given the current mathematics curriculum, very few teachers would have seen this theorem. Neither the teacher nor I had come across it[5].

If the teacher had known about it, she may have decided to introduce it to the pupils' discussion. However, she would have had to teach explicitly the relevant parts of the theorem, as the children could not be expected to be able to read the theorem on their own. There are very few secondary classrooms where the teacher would have enough time to spend with one group going into the details of this theorem. If she were to introduce such a discussion to the whole class, she would have to be convinced that all groups, even those who did not focus on the hooks in their work, would find it worthwhile, and that it would be a useful activity for all.

Moreover, she still would have had to deal with the group's mistaken notion of conservation of area. An earlier intervention, giving credence to Nandipha's and Lerato's discomfort (chapter 5, section 5) might have been sufficient for this. However, this did not happen, and was unlikely to, since Lerato never made her views known to the teacher, and it took most of the week for the pupils to discover their mistake on their own. A different approach might have been to deal with the pupils' focus on the hooks early on, even to pre-empt it, by talking first about the relationships between hooks and spaces on the geoboard. This kind of framing would have curtailed the likelihood of such an innovative invention, but it might have allowed for more fruitful development of the area concept.

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[4] It also raises questions as to just how idiosyncratic their method is.

[5] Thanks to Chris Breen for referring me to this theorem. There is also an investigation developed by the Shell Centre for Mathematics Education, University of Nottingham (1984) based on this theorem.

Arguments that open-ended tasks are more facilitative of different points of view, and a more democratic approach to mathematics learning, stress the constraints imposed by strict framing. I have shown previously that less specific task definitions do not easily facilitate interactive processes. The above discussion argues that they also are not always conducive to the development of true mathematical concepts. Teacher mediation is necessary to achieve better interaction and learning on these tasks. In the following section I will argue that teaching is also necessary to enable the development of the thinking processes that the pupils do engage in, if they are to result in the development of true concepts.

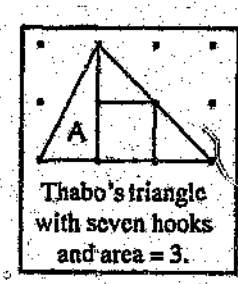
3. MATHEMATICAL PROCESSES

In chapter 5, I pointed out that the group lost many opportunities to reconsider their method, and hence many possibilities for learning, about area and about doing mathematics and developing ideas. They do engage in mathematical processes, such as generalising, specialising, checking theories against evidence and resolving contradictions. These processes could be labelled spontaneous processes, in that they have not been explicitly and formally taught. These spontaneous processes are not developed nor systematised during the week. The following examples show that there were points in the discussions about the method which did present opportunities for such development. These opportunities were not, however, taken up.



A serious challenge is posed to the method when values obtained for the area by dividing it up into smaller pieces do not agree with the areas predicted by the more general form of the method. This is glossed over early in the week when the method is first proposed. However, later on it becomes clear even to Thabo that the method gives incorrect answers. When they

are required to articulate the method in response to question 4a, they try to find examples other than those with 4 or 6 hooks. They are unable to make triangles with 7 hooks that have areas of 2½ units² as predicted by the method[6]. The triangle most evident in Thabo's rough work is the one in figure 6.2 which has an area of 3 and to which he refers in the extract below:



Thabo's triangle with seven hooks and area = 3.

Figure 6.2

[extract 6.1; day 4; question 4a; p. 162-163; lines 2049-2062]

Nandipha: *O.K, O.K, O.K relax, 1,2,3,4,5,6,7*[7]
Thabo: *Let's slow down, this is 1, ne*
Nandipha: *That's 1*
Thabo: *That's 1*
Nandipha: *That's ½*[8]
Lerato: *And this, what is this*
Thabo: *Oh, no this, ish, it's not gonna work*
Nandipha: *Ag*
Thabo: *It's not gonna work*
Nandipha: *So, it doesn't make a ½, [?*
Thabo: *[You know why, you know why, our thing's not gonna work, our invention, cause cava hier, ne cavé hier {look here}, our, we supposed to get 2½ ne, ne, the, the, the area, the here's it's gonna to be 1,2, it's 2 already, ne, so you can't tell me this is ½ cause you've got 4 things, give us 3*[9]

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[6] At this point the method is extended slightly to deal with shapes that have a number of hooks which cannot easily be made into a rectangle, i.e., most odd numbers. A rectangle with half a triangle on the end can be used (see figure 4.3c).

[7] He is counting the hooks.

[8] They have divided the triangle into a square and smaller triangles (figure 6.2) and are adding the areas of the smaller shapes.

[9] He means that the right angled triangle A (figure 6.2) has 4 hooks and so an area of 1, which, when added to the 2 already obtained, gives 3.

Thabo divides the triangle into squares and triangles, and uses the four-hook part of the method to find the area of one of the triangles. This gives a (correct) value of 3, which disproves the more general form of the method. He is faced with a conflict: the general form of the method has been disproved, but the four-hook part seems to work. Therefore he is reluctant to give it up. His reluctance can be seen in the following extract from the final lesson, when they are writing up their work:

[extract 6.2; day 5; question 1a; p. 174; lines 2678-2708]

Lerato: *Kanna {by the way} how did you guys work out the area of number C*[10]
Nandipha: *Number C*
Lerato: *Ja, this one*
Nandipha: *Count it, how many hooks you had*
Thabo: *There it goes again*[11]
...
Lerato: *Hae, the area of this thing is not 2*
Thabo: *Which one*
...
Thabo: *Ja man, it's not, it's not*
Lerato: *?*
Nandipha: *ehhh*
Thabo: *It's it's it's 4*
Nandipha: *Yes that is 1, that has to be 1*
Thabo: *O.K., O.K. we always, let's, let I give you, let me tell you, we always count to, to the things, 4, always 4, ish, the triangle, we always make the things to this, we always divide it into thing, ne, to get to the, let's forget the hooks, let's forget the hooks, let's forget the hooks, but then it works with the square, ne, so then from there, if you if the triangle is bigger than that, if it needs 5 hooks ne, then 5 hooks 1½*

In this extract, both Lerato and Thabo realise that the area of C is not 2. They begin to try to work out what it might be, with Nandipha still wanting to use the hooks. Thabo wants to discard the method of counting hooks, but he is aware that it works for four hooks, and wants to try to keep that aspect. As a compromise, he articulates the pattern presented by Lerato to the class (extract 4.2, chapter 4).

[10] Shape C, question 1 (see figure 4.2a, chapter 4).

[11] His tone indicates that he now has substantial doubts about the method.

The above examples show how the two main ideas used by the pupils, dividing up the shape into smaller pieces, and using the hooks, come into contact and conflict. They can co-exist when the dividing up method is used to begin with, leaving only triangles with four hooks, as in figure 6.2; in shape B in question 1 (figure 4.1, chapter 4); or in the "diagonal" square (figure 4.4, chapter 4). In these cases the four-hook part of the method is reinforced. They contradict each other when the more general form of the method is required for shapes where dividing up does not work so easily, for example shapes C and D in question 1, and when the two methods are used to check each other. The value of such contradictory evidence, the relationships between general and more specific theories, the need for justification, and the need and role for a method, could all usefully have been discussed with this group. Such discussions could have helped to develop more facility in generating and developing ideas. Through this process, a more systematic concept of area could have been developed, and brought into contact with the pupils' spontaneous ideas, for the formation of a true mathematical concept.

The teacher does take one opportunity to talk to the pupils about generating mathematical ideas. This is when they are upset that their method is not working:

[extract 6.3; day 4; question 4a; p. 166; lines 2223-2230]

Teacher: *O.K. now what I want you to do, and this is something that's valuable to learn, when you discover an invention, and it flops, it's still a valuable process, the fact that you thought you came up with something, and then you disproved it, mathematicians do it all the time, they come up with theories, they prove it, and then some guy comes a hundred years later and he disproves it, and both were valid, the one guy tried, and the other guy disproved it, that is what maths is about, O.K. so don't feel that it was a flop, you failed, you didn't fail, O.K. the fact that you found your error in your theory means that you succeeded, it's fine, O.K, but now you have to give me another way that you actually found these, if your method flopped, how else did you find all of these areas.*

She is pointing to the fact that most of the time the method works, and that they should be able to articulate the differences between the cases when it works and those when it does not. She begins to help them to do this, and they are led back to a discussion of the validity of the four-hook part of the method. While some progress is made on justifying the method, they do not engage with the question about the relationships between the general and more specific methods, and the contradictory answers obtained.

So the pupils struggle with both interactive and mathematical processes. There is no doubt that they are able to interact and think mathematically, and they do so as best they can. However, their spontaneous thinking is not systematised, nor developed further. I have shown that there is scope for such further development. This should be a fundamental aspect of teaching in more process-oriented classrooms.

In summary of the argument so far: The worksheet tasks and the group interactions do not on their own enable connections to be made between scientific and spontaneous concepts. Teacher mediation is essential, for helping to frame open-ended tasks, for refocusing pupil attention on certain aspects of the apparatus and tasks (e.g., spaces instead of hooks), and for developing the mathematical processes that the pupils engage in. In the case of this group, such teaching could have helped to deal with a misconception concerning conservation of area, and in so doing enabled connections to be made between scientific and spontaneous concepts, and the development of true concepts. The work of this group presented opportunities for the development of true concepts, which, as I argued in chapter 2, are necessary for ownership of mathematical knowledge.

However, arguments have been presented that direct teaching can inhibit ownership of mathematics (see chapter 2). An analysis of the teacher's interventions in the work of this small group will challenge this view, and help illuminate the important relationship between teaching and learning, and teaching and ownership.

4. TEACHING AND OWNERSHIP

In their discussion of ownership (see chapter 2), Ellerton and Clements (1991) see teacher control (or lack of it) as an index of ownership, setting up a continuum of ownership that reflects "the extent to which the learner's thinking was teacher directed" (Ellerton, 1991: 23). This emanates from the radical constructivist view that teacher direction inhibits learning. Ellerton and Clements challenge this view, arguing that classrooms where teachers maintain control, and impose structure, can be successful learning environments. From the evidence and the conception of ownership developed in this study, I will argue even further, that it is inappropriate to associate even ownership so strongly with lack of teacher direction.

Firstly, there is evidence in the discourse of pupil ownership of the knowledge when the pupils are directly guided by the teacher, and evidence of no ownership when the teacher is not there.

For example, in question 2a, the teacher elicits from Thabo and Nandipha the idea that a square need not be "straight" on the geoboard, i.e., the sides of the square can join the hooks on the geoboard diagonally. She states explicitly for them:

"as long as its still got 90 degree angles, ... rotate your square ... instead of you working with normal straight squares, where it's straight on your geoboard, you have made the side a diagonal of your geoboard" (p. 142-143)

They are then able to move on and construct more of their own "diagonal" squares, including the one in figure 4.4 (chapter 4). Nandipha's statement:

"but this is an excellent one, look at the shapes in the four quarters" (p. 145)

displays pride and ownership of this shape even though its generation can be seen to be directly enabled by the teacher's input. The pupils worked confidently with this shape, using their method to help calculate its area, and developing their method further while working with it.

The pupils' learning in this situation should be distinguished from imitation. They did not passively do what the teacher told them. Rather, they were able to take the teacher's suggestion and develop it for their own purposes. The nature of the task together with their understandings of their method enabled them to do this. In this case, the teacher's direct intervention refocused a situation where the pupils were experiencing difficulties in making progress.

In addition to this, there is evidence that for long periods of time when the teacher is absent (i.e., working with other groups), this group does not engage meaningfully with the work, and makes little progress towards ownership. Particularly when they work on question 3 (days 3 and 4), they do not engage in any of the processes, evidenced in chapter 4, that

characterise their ownership of the method: they do not discuss, defend, modify, adapt or transfer their ideas, and they show little emotional investment in the work. The work that they do is dispersed and not very fruitful as discussed previously (section 1, this chapter).

So, in terms of the notion of ownership developed in chapter 4, these three pupils did develop ownership as a result of direct teacher intervention, and often did not come close to achieving ownership, or even engagement with the work, when the teacher was not there.

The above analysis contrasts direct teaching with no teacher intervention at all in relation to pupil ownership. Most of the time, however, the teacher's mediation strategies could be characterised as trying to facilitate thinking in the pupils, rather than giving them direct guidance. This approach will be seen to have considerable influence on the development of the method and the pupils' consequent misconception about conservation of area.

5. TEACHING AS "FACILITATING LEARNING"

While the pupils were developing their strange method, both the teacher and I[12] were intrigued as to its apparent validity in some cases because we knew that it was not mathematically correct, and could not be generally true. We both tried to engage the pupils in thinking about the general validity of their theory, by providing counter-examples and

[12] I am including my own interactions with the pupils in the following discussion. I became involved while asking the pupils for clarification about certain shapes they had made and drawn, as part of my observation. In some instances I reverted to a teaching role, although I had not planned to. It is also clear from the transcripts that the pupils viewed me as an extra teacher in the classroom, and sometimes sought my help if their teacher was busy with another group.

asking for justification of their ideas. We were not willing to tell them that part of their method was wrong. The following extracts exemplify the interactions:

[extract 6.4; day 2; question 2b; pp. 146-147; lines 1171-1190]

Teacher: *Hey, this, this, this 4 hook theory's quite interesting, [does it, are you, have you tested it, thoroughly?*
Nandipha: *[1 plus*
Thabo: *Yes, we tested it here, [here it worked, it worked with all the triangles*
Nandipha: *[2 plus 4 plus 8 plus 10, hu-uuh*
Teacher: *You must write that one down hey*
Thabo: *No, no, no, no, not that it, it, before, not that it worked with all the triangles we've given to, we counted the hooks, then if we keep all the hooks like on one square, and we found the area of the thing without having to use this triangle, all you count the hooks here, and then see there.*
Teacher: *But this one here's got 4 hooks*
Thabo: *No, 1,2,3,4,5,6*
Teacher: *Oh I see, you [count,*
Thabo: *[you count there*
Teacher: *aah, even the ones on the inside*
Nandipha: *Yes*
Teacher: *O.K. You must write it down, cause that'll be nice for your presentation, but I'm gonna look for a, for a counter-example, I dunno if there is one, oh I see because the minute you counting your inside ones, but I would've liked to say, that's got 4 hooks, but no its got 5, you see, and it doesn't have an area of, you saying 4 hooks has an area of 1, you must write down your conjecture, O.K. (walks away)*

The teacher suggests that the idea needs to be tested thoroughly, and provides a counter-example to the theory. Thabo deals with the challenge which satisfies the teacher in that case. However, she reamins unsure about the theory, but encourages the pupils to write down and think about their idea, saying that she will come back with more challenges. She gives them time and space to think about their ideas.

Soon after this, I go over to the group to observe what they are doing and engage in the following discussion:

[extract 6.5; day 2; question 2b; pp. 147-148; lines 1224-1267]

Thabo: *it was here, we say you count 4 hooks, and then you say, since it's 4 hooks, we put, count any 4 hooks and put it there it'll give us our 1 squared, we do it like, let's use eh, ja let's use this, this here, that, that's 1 ne*[13],

Karin: *Ja, that's 1*

Thabo: *so then*

Karin: *but I dunno about this one*[14]

Thabo: *That one, it is also cause it's 1,2,3,4*

Karin: *But why, how d'you know that the area there is 1*

Thabo: *The area*

Nandipha: *Because its got 4 hooks miss*

Thabo: *4 hooks*

Karin: *O.K. but I, I mean I, I agree with you that with those 4 hooks the area [will be 1*

Thabo: *[yes*

Karin: *And with those 4 hooks the area will be one, but what about these, its not, there's no law that says 4 hooks*

Teacher: *No, no, no it works, it does work*

Karin: *But why, why is this area 1*

Thabo: *Miss, I, eh*

Nandipha: *(laughs)*

Thabo: *O.K. I can't explain but miss the, the, the, the working out that we've been doing, the,*

Karin: *But what is it, tell me, cause I'm not quite sure? I mean, I don't understand, if I tried to divide this up, I can't see that I'm getting 1, like, I dunno what that is, that's not a ½ O.K, that would change the area hey*

Nandipha: *Miss, isn't it that if you, O.K., let's say for instance if you cut up these, these pieces in- no*

Thabo: *No, no, no, here, here, here I'll prove it to you, uh, let's say, we, we'll use this*

Nandipha: *That's [5 hooks*

Karin: *[ja, so that's, that's ja*

Teacher: *You've got it*

Karin: *just explain it*

Thabo: *O.K. then, no, this one's supposed to be like this, supposed to like this, here, ish, O.K. here it is, here miss, it, it doesn't matter whether, as long as its, its uh, it doesn't matter if the, if the things, the angles, are, different, if, as long as the thing, the shape has got 4 sides, the, the area will always be the same, let's say there it was*

Karin: *O.K. I, I'll go with that, so what's the area of this one*

Thabo: *So the area of this one's going to be 2 units squared, so half of it which is the line that's cutting here in half*[15]

Karin: *is 1*

Thabo: *is 1*

Karin: *O.K. now that I'll accept*

Nandipha: *So d'you agree that it works*

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[13] See fig 6.3a

[14] see fig 6.3b

[15] see fig 6.3c

Karin: *For this, these 4*
Thabo: *Yes*
Karin: *So I agree that it works for a square, that triangle, and this triangle*
Thabo: *There, can we show more, can we show you more with the 6 hooks*

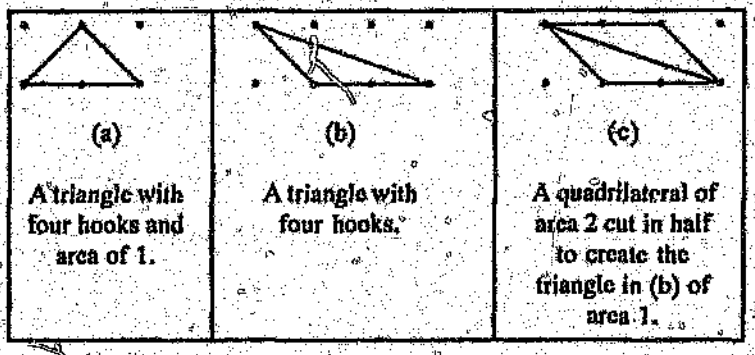


Figure 6.3

I insist that the pupils explain their theory to me, firstly to try to understand what they are thinking, and secondly, hoping that by verbalising their ideas they may come to see the shortcomings in them. Thabo develops a valid justification for the area of one of the more difficult triangles. I leave them saying that I'm satisfied that their theory works for the particular shapes they've shown me. I do not, however, push them to consider the general validity of their ideas.

In the final sentence of the above extract, Thabo wants to show me the more general case. However, I preferred to leave at this point because I felt that I had already intruded for too long. If I had stayed, I may have been able to make a crucial difference to the development

of the method and their (mis)conception of area[16]. There were a few other occasions when the pupils mentioned the more general version of the method. However, neither the teacher nor I understood the full extent of the method, for two reasons: firstly because it was so different from anything we expected and secondly because the pupils struggled to verbalise it. This points to a need for teachers to be able to listen more carefully to what pupils are saying, and for pupils to be able to explain more assertively.

The teacher's and my interventions could be described as trying to create cognitive conflict and a need for verbalisation and justification on the part of the pupils, hoping that they might develop their ideas further, or change them. We did not want to tell them that they were wrong, or suggest ways out of the difficulty. Our behaviour suggests that we wanted them to discover and deal with their own problem; we were not prepared to be more directive.

Different teacher intervention strategies may have critically affected the development of the method and had positive effects on the later work of the pupils. The above interactions occurred when the pupils were working on question 2. This is the point where they started experiencing difficulties in sustaining their work and interaction. More direct challenging of the method, and its location in the mistaken notion of conservation of area, would have brought in scientific concepts, and formed a basis for the formation of true concepts. If we had known more about the nature of the pupils' method, we could have made the contradictions between the different versions explicit, and enabled ownership, both of the

[16] My decision to leave at this point was influenced by my understanding of my own position in the classroom, as an observer, rather than a teacher. However, had my primary role been as a teacher, I may have made the same decision for a number of reasons, including: giving the pupils more time on their own, or needing to move to another group.

processes of developing the method, and of the product: a better method and better conceptions of area.

A final point about guidance and ownership needs to be made, which is that there are constraints other than the teacher's ideas, that influence children's thinking and the development of knowledge in classrooms. The next section deals with this.

6. OTHER CONTEXTUAL CONSTRAINTS

The ownership of the method that was established by the pupils can be seen to be constrained by other contextual factors, which are indirectly related to teacher control.

Firstly, the decision to find a method, and the particular method that was developed, was not initiated by the teacher. It was also not suggested by the worksheet. So it can be seen to be a problem that the pupils set up for themselves. I have identified problem-posing as an important aspect of ownership.

The pupils see the generation of a method as an appropriate problem because of their previous experiences of mathematics at the school. On the one hand, they have used formulae as general methods to solve problems, particularly area problems. So Lerato's insistence on a formula is understandable. On the other hand, the mathematics teachers regularly encourage pupils to explain and articulate their ideas and thinking processes. So these pupils would have encountered activities which required them to generalise across specific cases and to verbalise methods that they had used to solve problems. So while neither the teacher nor the

worksheet specifically asked for a method to be articulated, the mathematical history of the pupils would indicate it as an appropriate activity.

Secondly, the notification by the teacher that there would be a presentation to the class at the end of the week would also strongly suggest that something more than answers should be found. The teacher states explicitly that they would have to "make some kind of deduction or presentation on the board" (p. 128). Again, their previous experience would have involved making and listening to such presentations, and they would know that what it entails is explanation and justification of ideas. The presentation serves as a constant goal, toward which much of the group's work and emotions are directed.

Finally, the particular method can be seen to have come from a focus on the hooks of the geoboard. It seems that the hooks provide a way of framing the task for the pupils. They were unable to perceive and gain access to other aspects of the concept of area because of their ties to the hooks and the method. Even the teacher's introduction of the diagonal square did not detract from the pupils' emphasis of the importance of the hooks.

So there are always constraints acting on pupils working in a small group, and while they might be removed from the teacher's mathematisations at particular points in their work, they are never free from other contextual influences. While I am not able to identify the origins of the idea that counting hooks can help to find the area, constructivist epistemologies suggest that it must make sense in terms of the pupils' current knowledge. That knowledge is constructed within social contexts and influences. Thus to suggest that an index of ownership is teacher involvement alone is to ignore other social constraints acting on the pupils and their construction of knowledge.

This brings us back to the Vygotskian conception of knowledge as being socioculturally situated, with the social context encompassing the learners, teacher, classroom, and artifacts or tools (Saxe et al, 1993; Mellin-Olsen, 1987), and the relationships between them. Certain aspects, for example the tasks or thinking processes, cannot be considered in isolation from the others.

I have argued that the teaching-learning relationship requires special consideration. An important beginning to understanding this relationship is to shift from viewing the teacher as only a constraining influence in achieving ownership. The teacher can and should be a powerful enabling influence, mediating both interaction and conceptual development in small-group work.

A role for teaching in the development of ownership exists in developing further spontaneous thinking processes and concepts, in mediating scientific concepts, possibly directly, and in leading the development of connections between the two, to form true concepts.

CHAPTER 7

CONCLUSIONS AND IMPLICATIONS FOR PRACTICE

1. SUMMARY OF FINDINGS

Through a detailed analysis of a small group working together in a mathematics classroom, and a simultaneous elaboration of the notion of ownership, this report has shown that there are both beneficial and problematic aspects in small-group work relating to ownership.

Working in a small group enabled the pupils to pose their own problems, work out ways of solving them, and make evaluations and decisions as to what were appropriate solutions and ways of working. They also engaged in a number of mathematical processes, such as making conjectures, adapting and modifying their ideas in the face of evidence or arguments, generalising and specialising, and transferring knowledge to new situations. The small group enabled the pupils to express their spontaneous concepts, to work with them, and to develop ownership of a method which they generated and developed during the week.

The problematic aspects relate to the interaction within the group and the development of mathematical concepts. Although on the surface there was apparent equality and mutuality of engagement in the group, at a deeper level certain ideas were seen to predominate. This inequality produced and was produced by the inability of the group members to negotiate meanings, in particular their different situation definitions, and to achieve a level of intersubjectivity. The dominance and lack of intersubjectivity in turn produced and was

produced by differential ownership of the method, and an abdication of responsibility on the part of two of the students for some of the ideas generated in the group. Thus, even in a group which appeared to be able to work democratically, difficulties were experienced.

The difficulties in achieving intersubjectivity and shared responsibility for the method resulted in the perpetuation of a serious mathematical misconception. This was not dealt with in any way during the week, neither by the group, nor by the teacher. This has serious consequences for the development of true concepts and ownership of mathematical knowledge.

The teacher's role is seen to be crucial to this development. A particular approach to teaching, that of facilitating learning with a reluctance to intervene directly, created a situation where firstly, the thinking processes that the pupils did engage in were not developed in any way, and secondly, the pupils' mistake was not dealt with. Thus, contrary to perspectives which maintain that ownership can be better achieved without the presence of a teacher, this research has argued that direct teacher mediation is necessary to promote ownership. A re-conceptualisation of teaching is needed. Firstly, teaching can and should promote democracy in groups, and secondly, teaching should be viewed as integral to learning, rather than as a disruption of an otherwise spontaneous process.

2. IMPLICATIONS FOR PEDAGOGY

The questions from practice which motivated this research project (chapter 1) concern the ways in which small-group work can facilitate mathematics learning and democracy in

classrooms, and the nature of the teacher's contribution in this regard. The ultimate success of a research project such as this will be in the ways in which answers to these questions, generated from the study of one particular group, are able to inform other teachers' practices. There can be no easy or direct transfer of the ideas generated by research into "rules" for practice. The general arguments presented here, seen in the light of one group at work, can only hope to illuminate some of the complexities that teachers and learners face everyday in their classrooms, and in so doing provide ways of thinking about classrooms which will enable better practice.

2.1 *Direct teaching*

I have argued strongly that teaching must be seen as integral to learning, and that there are situations in classrooms where it is appropriate for teachers to mediate directly and to teach their pupils. While it may seem redundant to make this point, it is made against a background (in South Africa) of directives to teachers not to "interfere" with pupils' constructions, and to "facilitate" learning rather than to teach[1]. While facilitating discussion and learning is part of a teacher's obligation to her pupils, so too are other activities, such as framing tasks, and direct teaching. We can emphasise teaching and be learner-centred at the same time.

[1] I am arguing for teachers to reclaim the word "teach". It does not have to be associated with a transmission paradigm, but can encompass all aspects of the relationship between a teacher and a learner. We need to broaden our conceptions of teaching rather than replace the word.

The evidence presented in this report highlights many possibilities for direct teaching in the case of this particular group. The most important of these should have been direct engagement with the pupils' mistaken notion of conservation of area. Making mistakes and constructing misconceptions is a central part of learning. But their centrality lies in overcoming the misconceptions, and moving on to the construction of new knowledge. While the process is important, the products are equally so, and, as we have seen, do not follow automatically from the process.

This group had a week to deal with their misconception, and could not do so alone, nor with the kind of mediation provided by the teachers. The group only began to consider their mistake when faced with the external requirements of the presentation, when a product was required. However at that point they did not have enough time to deal with their mistake. An earlier intervention into the process would have ensured a better product.

There are a number of possibilities for different interventions. Whole class activities could be structured where the ideas from the groups are made more public, debated, and mediated by the teacher. Presentations need not be seen only as a concluding activity, but part of the ongoing learning process. They should be structured so that the same purpose is served, i.e., a need for a product.

In light of the observation that the more tightly framed tasks were more productive for the group, a teaching contribution to the small-group work could have been to frame the tasks more carefully in the beginning (i.e., on the worksheet). In addition to this, the teacher could

have spent some time relating area to the hooks and spaces on the geoboard. As the week progressed, other important ideas, such as the formulae for area (that the group had seen before), could have been brought in. Thus the teacher's knowledge can and should be brought in to help the development of the pupils' knowledge.

2.2 Inequality in group

The composition of groups is often seen to be an important factor in small-group work. There is much debate over whether groups whose members have similar mathematical competence will enable better discussion and learning than those with a wider range of competencies. This research did not set out to answer these questions, and the design did not allow for comparisons to be made. However, the fact that a group whose members seemed to be relatively equal in their interaction displayed a substantial amount of inequality suggests a different perspective on this issue.

No matter how much equality we try to create within groups (if this is seen to be desirable), there will always be areas of conflict, and dominance of some views over others. Thus the teacher needs to be aware of these possibilities and mediate them. As teachers we can promote democracy in classrooms, by using our more powerful positions and knowledge to ensure that all voices are heard.

In order to do this (or work towards achieving it), we ourselves need to be able to hear all voices. However, this is not an easy task. I have shown that a skilled and confident teacher,

who is aware of many of the difficulties inherent in peer interaction, was not able to understand everything that happened in the group. This detracted from her ability to draw fully on the ideas of the pupils, and to develop true concepts. One pupil, Lerato, did focus on the spaces, rather than the hooks of the geoboard, early on in the week. However, she did not communicate with the teacher. If the teacher had known about Lerato's ideas, she could have drawn them in and helped with the pupils' mistake.

To do this, she would have had to realise that she was not hearing the whole story from the group. Lerato's silence could have alerted her to this. However, even the more vocal Nandipha's views were not always heard. This points to the fact that we should assume possible inequalities in groups that may not be obvious from the outside, and take seriously the difficult task of trying to hear every voice in the group.

However, we too are constrained by our own histories and we hear our students from within our own understandings of mathematics, learning and development, and democracy. Nevertheless, given our constraints, we can try to develop strategies and techniques to become better at hearing different points of view in our classrooms. Part of this process could be to try to develop in our pupils the confidence and language competence (Adler, 1994) necessary to make their voices heard. Of course this development will require teaching.

3. CONCLUSION

I have argued that two seemingly incompatible teaching approaches are in fact complementary, and should be used to enhance the achievement of ownership through small-

ork. They are direct teaching, and careful listening and soliciting of pupil ideas. Each teacher needs to make decisions regarding the groups she is working with as to which is appropriate in different situations.

M shers are good at direct teaching - we have developed the skills as part of our spontaneous conceptions of teaching. However, we have not developed strategies for helping pupils to talk and voice their ideas, and most importantly for listening to them. If these can be considered part of a systematic notion of teaching, developed through research in constructivist paradigms, then we need to begin to integrate the two to develop true conceptions of teaching. An implication for teacher education is that it needs to lead this development.

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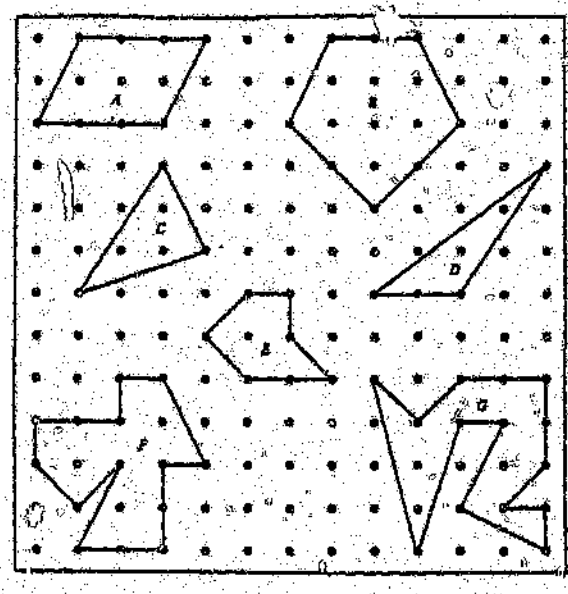
APPENDIX A: THE WORKSHEET

Form II **Geoboard Worksheet**

Work on this worksheet in groups of 3 or 4. Use the geoboards and the dotted paper provided, and do the necessary working in your books. When the worksheet is finished, you will have to hand in the answers to this worksheet on a piece of paper. The final product should be your own work.

1. a) Work out the area of each of the shapes below.

   b) Work out the perimeter of each of the shapes below. (without measuring it)



2. a) On your geoboard, make a square with an area of 2 units^2, and draw it on your dotted paper.
   b) What possible areas can squares have on your geoboard? Draw these on your dotted paper, and show any working that you do.
   c) What possible perimeters can squares have on your geoboard? Show any working that you do.

3. a) Choose any area. Find as many different polygons as you can which have this area.
   b) Choose any perimeter. Find as many different polygons as you can which have this perimeter.

4. a) Discuss with your partner and then write down how you would explain to a Form I student how to work out the area of complicated shapes on a geoboard.

   b) Write down how you would explain to a Form I student how to work out the perimeter of complicated shapes on a geoboard (without measuring it).

APPENDIX B: THE TRANSCRIPT

Note: The names of the pupils have been changed in this transcript.

Transcription conventions used:

[- simultaneous utterances
[

? - inaudible talk

{} - brackets contain English translation of preceding words

() - brackets contain contextual information

Day 1: Thursday 14th October 1993
(two lessons, work on the worksheet only started towards the end of the first lesson)

Teacher: *Did you hear what I was saying to that group*
Lerato: *No,[No*
Nandipha: *[No*
Thabo: *No, No*
Teacher: *O.K. these worksheets we gonna work on them for about a week O.K, they basically self-explanatory, for today I want you to work on question 1, in your groups, O.K. I'm also, you'll see this thing here ? (walks away), like this*
Lerato: *Miss must we leave number 15 now*
Teacher: *Leave number 15 for now, that was just something for you to think about before we did this O.K, don't mark it now, O.K. I'm gonna give you some elastics if you want to represent, this, on here concretely, to explain something to him, say for instance, he doesn't believe that the area is 4, then you can show him, say but look, if I do this, look, count the area, O.K, or whatever, O.K. so its just a, its a nice way nice way of representing it, but you don't have to use it if you don't want to, O.K its called a geoboard, O.K. just on question 1 today, don't rush through this worksheet, see.*
Lerato: *question 1 is ?*
Nandipha: *(reads from worksheet) Work on this worksheet in groups of 3 or 4. Use the geoboards and the dotted paper provided, and do the necessary working in your books. Work out the area of each of the shapes below.*
Thabo: *?*
Nandipha: *Yes genius*
Thabo: *No I'm just clearing the place*
Nandipha: *So you gonna put it on*
Lerato: *Come, Nandipha, lets do it*
(silence)
Nandipha: *O.K. here ? 2, that one's 2 ne, then this across 4, 4 ne, and its, ne, ne, hey man di*
Lerato: *Yes*
Thabo: *No, no, its a 3*
Nandipha: *its 1,2,3,4*
Thabo: *its a 3*
Nandipha: *its 1,2, [3,4,4*
Lerato: *[No*
Thabo: *But there are three [hooks in the middle*
Nandipha: *[its down three, its down three*

34 *Thabo:* *Take a bigger one*
35 *Nandipha:* *Ohhh like so*
36 *Thabo:* *Yes, ye*
37 *Lerato:* *So, lets work out*
38 *Thabo:* *The area*
39 *Lerato:* *The area*
40 *Nandipha:* *Its* πr^2
41 *Lerato:* πr^2*, phela {but} this is a quadrilateral, so you can say*
42 *Thabo:* *2 times 3, 6*
43 *Nandipha:* *? that's 4*
44 *Thabo:* *eh*
45 *Nandipha:* *that's 4*
46 *Thabo:* *No, man, you cou, you don't count the things, the things that are separated, [you count the*
47 *Nandipha:* *[you count the spaces*
48 *Thabo:* *spaces, so that's 1,2 times 3*
49 *Lerato:* *Why the spaces man*
50 *Thabo:* *Ow, its always been like [that*
51 *Lerato:* *[Thabo,these are the points phela {but}*
52 *Thabo:* *[No*
53 *Lerato:* *[its just, wait, lets pretend here {lets say} this thing has got blocks, ja, 1,2,3 maara {but}*
54 *Thabo:* *1,2,3, 6*
55 *Lerato:* *Its 2 times 3 its 6*
56 *Thabo:* *6, lets see if it works like this*
57 *Nandipha:* *?*
58 *Thabo:* *How many is it here, 4*
59 *Lerato:* *What*
60 *Thabo:* *yeah, there it works, its 6 it, it does work*
61 *Nandipha:* *ja, ja, ja, ? 4*
62 *Thabo:* *its*
63 *Lerato:* *triangle 1*
64 *Thabo:* *2 times 2 ish equals to 6*
65 *Lerato:* *Eh, not triangle, quadrilateral*
66 *Nandipha:* *O.K.*
67 *Thabo:* *its not your turn, its her turn*
68 *Nandipha:* *O.K., O.K. its her turn*
69 *Thabo:* *(laughs)*
70 *Lerato:* *Wait*
71 *Teacher:* *(to whole class) sorry, who's been camping recently,*
72 *Nandipha:* *camping*
73 *someone:* *camping*
74 *Teacher:* *camping in a tent. I know Miss Brodie loves camping, doesn't she (laughs), who does, O.K when*
75 *you have a guy rope, is the guy rope just the one side or do you consider the guy rope being the*
76 *rope going up and the rope going down, is it one continuous rope or is it just one d'you see what*
77 *I mean, do you say this is two guy ropes or do you say its one guy rope, I think its [two*
78 *Lerato:* *[one*
79 *Thabo:* *its two, its two miss, [its two*
80 *Nandipha:* *[its one*
81 *Teacher:* *O.K. whichever one, basically whichever one you decide, if you decide its two, then you have to take*
82 *it into account when you do your question, because working out your guy rope with the Pythagoras*
83 *one, you only work out one side of the guy rope, if you decide that you've got a guy rope which is*
84 *only the one side then I have your answer as it is*
85 *Someone:* *Uh*
86 *Teacher:* *and if you decide that your guy rope was a continuous one and you had to consider the other side*
87 *as well, then you have to obviously double it, O.K., so thats a, its a good point that, its a good*
88 *question to think [about*

89 *Lerato:* [there you go
90 *Thabo:* I think its wrong, I [think its right
91 *Nandipha:* [Its wrong, its
92 *Lerato:* [This its right
93 *Thabo:* look at it
94 *Nandipha:* its wrong
95 *Thabo:* No wait, wait its right
96 *Lerato:* No, it still has to be pointed
97 *Nandipha:* No-o-o, its wrong
98 *Thabo:* No, its wrong yes, yep
99 *Nandipha:* ?
100 *Thabo:* Its at a distance
101 *Lerato:* What d'you mean, like how distance
102 *Thabo:* 1,2,3,4,5,6 lo, lets, lets say it, its gonna give us a ½
103 *Lerato:* No
104 *Thabo:* Uh-huh, and this
105 *Lerato:* This can't be a half
106 *Thabo:* No, no, no this, is going to give us a full, full,
107 *Lerato:* eh yes
108 *Thabo:* a full side, ne and this
109 *Lerato:* ja
110 *Thabo:* and this 2, and this 1, so that means its 1, 2, 3,4,5,6,7,8,9,10, O.K, and then
111 *Nandipha:* ?
112 *Lerato:* Sheba {look} this is a hanna {by the way} a five sided thing, what d'you call it
113 *Thabo:* Uh, Uh-uh, fivetagon
114 *Lerato:* uh (laughs) uh, this things is five-sided ne
115 *Thabo:* Ja
116 *Lerato:* This thing, hae man, stop laughing,
117 *Nandipha:* Hae wena but its funny
118 *Lerato:* This thing is (laughing) this, hae, man come, this thing is five-sided ne and there are 1,2,3,4,5 ne,
119 eh, eh there are four spaces, Thabo,there are
120 *Thabo:* No, no
121 *Nandipha:* Come on, come on, come on, can I please see it
122 *Thabo:* Ca, ca
123 *Nandipha:* O.K, this one will give you 1, ne this one [will give you 1, ne
124 *Thabo:* [will give you 1, no, no, no, no, no, it will gi, it will give
125 us, ntho {something} 1½
126 *Nandipha:* ena {this one}
127 *Thabo:* Eh, eh, eh
128 *Nandipha:* Oh, [O.K, O.K O.K
129 *Thabo:* [this 1½
130 *Nandipha:* Oh, O.K, O.K this is 1½, this is 1½, so that's 3
131 *Thabo:* Jaaa
132 *Nandipha:* Ne, sharp, this one is a ½
133 *Thabo:* Its a 1
134 *Nandipha:* its 1
135 *Thabo:* so its [4
136 *Nandipha:* [its 1 so its 4, i'5,[6,7,8,9,10
137 *Thabo:* [6,7,8,9,10, lets find a way to find, to, to, to
138 *Lerato:* to work this out,
139 *Thabo:* Eh, so we'll say, mm, no, I know, I know, you count, you count, mm, 1,2,3,4,5,6,7,8, no, no, no,
140 no, no
141 *Lerato:* No, there's 1,2,3,4,5,6,7,8,9,
142 *Nandipha:* and then, and then
143 *Thabo:* No, I know, I know, [you count

144 *Nandipha:* *[Who you always here*
145 *Thabo:* *You count down from here, ne, 1,2,3,4,5, ne, and then you say by 2, multiply by 2, there 2 spaces*
146 *here*
147 *Lerato:* *uh-uh, it doesn't work*
148 *Thabo:* *it will work, it will work*
149 *Nandipha:* *[What*
150 *Lerato:* *[What if it was like this here Thabo*
151 *Thabo:* *Oh, come on, the sides must be the same size, no*
152 *Lerato:* *No, wena you just assuming hore {that} the answer is 10*
153 *Thabo:* *eh, it is 10 but then we can find, a method*
154 *Lerato:* *Its not*
155 *Thabo:* *O.K*
156 *Nandipha:* *What is it, Lerato*
157 *Lerato:* *I don't know, we all don't*
158 *Nandipha:* *No, you know what, I think*
159 *Thabo:* *No, no, no, no, no, no*
160 *Nandipha:* *Ish, mmm, maara waitse wena kobotsitse {but I told you} ? 1,2,3,4,5,6, 7,8,9, its 10*
161 *Thabo:* *Its te, eh ja, we, we, know Nandipha hore {that} its 10, but then we find, how we found it, in eh,*
162 *in eh*
163 *Nandipha:* *Ag, just cut it up man its supposed to be*
164 *Thabo:* *Cut it out*
165 *Nandipha:* *I mean like those out here,*
166 *Thabo:* *[Uh-huh*
167 *Nandipha:* *[This is 1*
168 *Thabo:* *Then, [you ge, you get your 6*
169 *Nandipha:* *[thanks*
170 *Lerato:* *and then there's 6 there*
171 *Nandipha:* *What 6*
172 *Thabo:* *That's 6*
173 *Lerato:* *The ones in the middle*
174 *Nandipha:* *Ja that's 6 there, if we do this*
175 *Thabo:* *That is 1,1*
176 *Nandipha:* *[1,2,3,4,5*
177 *Teacher:* *Sorry, can I just make one [announcement, everyone, at the end of the week, O.K, or, at the end*
178 *of this worksheet which will probably be, middle of next week sometime, I'm gonna get your groups*
179 *to present something on this, O.K, it might just be presenting a question, it might be giving you*
180 *another question in relation to this that I'm gonna expect you to present, O.K, but you gonna have*
181 *to work as a group so that at the end of this you can come together as a group and make some kind*
182 *of deduction or presentation on the board, O.K.*
183 *Nandipha:* *Must we go and research and all of that*
184 *Teacher:* *Pardon*
185 *Nandipha:* *Must we go and research and everything like that*
186 *Teacher:* *No, no, no, no, I'll, I'll tell you later, all I'm saying is that, somewhere along the line you going*
187 *to give a group presentation on this work, O.K, and that'll count towards your bookmark, O.K*
188 *Thabo:* *O.K. le, lets just, how, no, mm-mm, nee they said we don't have, we don't have to have a,a,a*
189 *particular method, how to work them out, all you need to do is just to write them down*
190 *Lerato:* *Uh-uh*
191 *Thabo:* *Ask miss, ask miss, ask miss*
192 *Lerato:* *Nandipha, please call miss*
193 *Thabo:* *Miss*
194 *Teacher:* *Ja*
195 *Thabo:* *are we supposed to have a*
196 *Nandipha:* *method*
197 *Thabo:* *a method of how we found this thing*
198 *Teacher:* *Ja, I mean you should have some kinda method, otherwise how you gonna do it*

199 Thabo: Our, our, our method is there, but then, its not is ,h, in written form
200 Teacher: O.K. explain to me your method, O.K. nevermind written form but then, it must be explainable
201 Thabo: O.K. we find the squares, which we find in here
202 Teacher: [Ah
203 Thabo: [and then we find the area of the square,
204 Teacher: O.K. so where's [your square, show me
205 Thabo: [Oh, no, no, not, not the square,the
206 Nandipha: the rectangle
207 Thabo: quad, [quad
208 Teacher: [the rectangle, O.K, so you finding the area of, this,
209 Thabo: Yes
210 Teacher: That one
211 Thabo: Yes
212 Teacher: O.K.
213 Thabo: And then, from there
214 Nandipha: Where we co, we co, can I say we concluded that this gives us 1½, right
215 Teacher: How
216 Thabo: How, because this
217 Nandipha: Is our
218 Thabo: This together will give us [1
219 Nandipha: [1
220 Thabo: And
221 Nandipha: So this, [together
222 Thabo: [this [will give us 1
223 Teacher: [Aaah
224 Nandipha: right
225 Thabo: [and that will give us 1, and that will give us 1
226 Nandipha: [and that will give us 1, and that will give us 1
227 Thabo: but then
228 Teacher: This will give you 1 because its a ½ and a ½, that's fine
229 Thabo: And here its, three quarters and a quarter
230 Teacher: O.K., So what you actually doing is you combining pieces together, O.K. you're saying that's a half
231 and that's a half, and but here how did you do it, are you saying that's because of symmetry they
232 make a whole
233 Thabo: Yes
234 Teacher: O.K., I see, so you matching up halves
235 Thabo: Yes
236 Nandipha: What are we supposed to do here
237 (silence)
238 Teacher: O.K. you cut it like that
239 Thabo: Yes
240 Teacher: So simply show me, show me how you cut it, which one, is it this one
241 Thabo: Yes
242 Lerato: Yes miss
243 Teacher: You can show me then, how you cut it, O.K.
244 Thabo: Yes
245 Teacher: Good
246 Lerato: Thabo, you musn't be too sure
247 Thabo: No I wasn't being too sure, uh for the ? (sings)
248 Nandipha: Aren't you out of your function
249 Thabo: ?
250 Nandipha: Ei, what you writing, ? rubbish (laughs)
251 Lerato: Miss who
252 Nandipha: Ms C, do you think ? millionaires
253 Thabo: Ai, ?

254 *Lerato:* *? Nandipha*
255 *Nandipha:* *Ja*
256 *(silence)*
257 *Thabo:* *How we gonna write it*
258 *Nand'pha:* *Next one*
259 *Thabo:* *Ha, so how we gonna write it*
260 *Nandipha:* *Eh*
261 *Thabo:* *No, you just write um, 2 plu, 2 times 3, plus*
262 *Nandipha:* *Its 10, can I*
263 *Lerato:* *10*
264 *Thabo:* *[one-and-a-half plus [one-and-a-half plus*
265 *Nandipha:* *[its 10*
266 *Lerato:* *[Here its your turn*
267 *Nandipha:* *Its your choice, like [this*
268 *Lerato:* *[Ouch*
269 *Nandipha:* *[½*
270 *Thabo:* *[½ plus, 1 plus 1, equals to 10*
271 *(inaudible)*
272 *Thabo:* *6*
273 *Nandipha:* *Its my turn*
274 *Thabo:* *no my turn, its my turn, aah*
275 *Teacher:* *If any group wants more elastics or you want different colours say, if if you've only got yellow and*
276 *you want other colours [there are more*
277 *Nandipha:* *[? red*
278 *Lerato:* *Tula ?*
279 *Thabo:* *No*
280 *Nandipha:* *Uh-uh*
281 *Thabo:* *No*
282 *Nandipha:* *Its not, its no its this*
283 *Thabo:* *No, its*
284 *Lerato:* *It is*
285 *Thabo:* *You can't do that*
286 *Nandipha:* *O.K. its this*
287 *Thabo:* *No, na, its this*
288 *Lerato:* *Which one are you doing ne*
289 *Nandipha:* *No, that's a, its not that*
290 *Lerato:* *Thabo, its like this let the girls do it Thabo*
291 *Thabo:* *Its my turn this,*
292 *Lerato:* *Its 3, Nandipha*
293 *Thabo:* *you can always find a, find a way to*
294 *Nandipha:* *(laughs)*
295 *Lerato:* *No*
296 *Thabo:* *Come give me a chance, its my turn, thank you, ne lets say its, its, its ,its, no*
297 *Lerato:* *6 squares*
298 *Thabo:* *Ja, its 4,4,4, by, 4 by 4, ne, 4 by 4*
299 *Nandipha:* *3 by 3 ek se {I say}*
300 *Thabo:* *yeah, no, ja, yep, ne 4, its 4 by 4 ne*
301 *Nandipha:* *? its not it ?*
302 *Thabo:* *yes, yes, yepity, yepitty yo, so*
303 *Nandipha:* *(makes funny noise and sings)*
304 *Thabo:* *I found a solution ne*
305 *Lerato:* *Nandipha why do you like fighting with her*
306 *Nandipha:* *With who, ag, I just like it, oska tloikesa ncono {don't act smart}*
307 *Thabo:* *O.K, a solution ne, we count the strings, not the strings, the sticks ne, no, what, what, the things,*
308 *the hooks, 1,2,3,4,5,6, yes*

309 *Nandipha:* Uh
310 *Thabo:* So, O.K, is it 6
311 *Lerato:* 7
312 *Thabo:* 7
313 *Nandipha:* uuhh
314 *Thabo:* Its 6, so we put
315 *Lerato:* Thabo, its [seven
316 *Thabo:* [We put a thing on 6 like this, and then
317 *Lerato:* Thabo, its seven
318 *Thabo:* How, how
319 *Lerato:* Sheba {look}, 1,2,3,4, (laughs) no man look, 1,2 3,4, [5,6
320 *Thabo:* [5,6, so, so akere {that is} got [6, 6 things
321 *Nandipha:* [ha ha,
322 yobaba {oh father (exclamation)}
323 *Thabo:* So, here's our solution, lets say
324 *Nandipha:* So its a rectangle
325 *Thabo:* Yes, so its 2 by 1
326 *Nandipha:* If we convert it
327 *Thabo:* Lets, lets try do the thing
328 *Lerato:* Its 2 by 1
329 *Thabo:* Come, what was E, it was like this, B was like this ne
330 *Nandipha:* B
331 *Thabo:* Lets see if it workss
332 *Lerato:* ?
333 *Thabo:* [Ja
334 *Nandipha:* [Ja
335 *Lerato:* [Ja, ja
336 *Thabo:* 1, 1,2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
337 *Lerato:* Thabo, oballa jwang {how do you count} 1,2,3, Thabo, ish, [7, 8, 9, 10,
338 *Thabo:* [7, 8, 9 10, 11, 12, 13, 14, 15, so
339 15, 3 by 5, ne
340 *Nandipha:* Ja
341 *Lerato:* Ja 3 by [5
342 *Nandipha:* [Ja 3, 3, 3 by [f-i-v-e
343 *Thabo:* [O.K
344 *Lerato:* You getting clever now
345 *Thabo:* 1, 2, 3, 4, 5, 6, 7, 8, no, this not ntho {it}
346 *Lerato:* 1, 2, 3, [4, 5, 6, 7,
347 *Nandipha:* [It can't work on this board
348 *Thabo:* No it will work,[it will work
349 *Nandipha:* [On this board, its too small
350 *Thabo:* We must find 15, so its 1, 2, 3, 4, no 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
351 *Nandipha:* You counted this and thought you didn't
352 (bell goes)
353 *Thabo:* Here
354 *Nandipha:* Let me take a break please
355 *Thabo:* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, hey it doesn't work, cause here we find 8 ne, it
356 doesn't work
357 *Nandipha:* I told you, miss,
358 *Lerato:* Nandipha
359 *Nandipha:* We need a break (sings)
360 *Lerato:* dikae jwanong {how many are there}
361 *Thabo:* Eighteen
362 *Lerato:* 3, 4, 5, 6, 7, 8, 9, ke badile jwang {how did I count} 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
363 15, Uh-uh do, don't count this

Thabo: *(sings) So*
Nandipha: *Ke batla {I want} break*
Thabo: *No, it, my method doesn't work, lets go on*
Nandipha: *(laughs)*
Thabo: *It doesn't work*
Lerato: *If it doesn't work then it means that number, our number B is wrong*
Thabo: *number B, hae number C, we haven't done number C ne*
Lerato: *number C its, 3 x 1,*
Nandipha: *Miss*
Lerato: *2 x 1 I mean*
(silence)
Thabo: *It works, it works, it works*
Lerato: *What*
Thabo: *It works with triangles, it works with triangles, but no other ang,*
Nandipha: *With no other kind of quadrilateral*
Thabo: *Ja but check, check, it works with triangles because look this one's going to give us 1 ne, ne, and this one, 1 also ne, this ne, no, this piece is going to fit here to make it how much, to make it how much, ish it works with triangles*
Nandipha: *miss, miss ?*
(silence)
Thabo: *Ja, this is a it works with triangles, that's one of ours the presentation, it works, it works*
Nandipha: *But it didn't work with other shapes*
Thabo: *Yes, but it works with triangles*
(silence)
Thabo: *Yes, yes Nandipha it works with triangles, it works, it works, look at this, look at this ne, all the shapes here we can divide them into, into, into, into, into, we can somehow cut them into uh, squares and triangles ne, lets, lets continue and see, lets see wabona {can you see} here, here is 2 x 1,*
Nandipha: *Can we take a break miss*
Thabo: *Hae :s*
Teacher: *O.K did you all want your one minute break or whatever, between your lesson*
Chorus: *Yes*
Teacher: *O.K, one minute and don't go out the class, don't go out the class*

END OF SIDE 1, TAPE 1

Day 1: discussion resumes

Nandipha: *Ohh, that*
Thabo: *Did you hear*
Nandipha: *Oh ?, can you imagine ?, come on*
Thabo: *ohh, de gulf war*
Nandipha: *(laughter)*
Thabo: *O.K it works with triangles ne*
Nandipha: *Uh-huh*
Thabo: *? wabo {can you see}, it works with triangles, bona{look}, lets see, wobona {can you see} number B, ne,*
Nandipha: *Mm*
Thabo: *akere we found our square already ne,*
Lerato: *Ja, [and*
Thabo: *[We found our square*
Lerato: *[and these are triangles*
Nandipha: *[Mm*
Thabo: *and you've got 4 strings here*
Lerato: *[Ja*

364 *Thabo:* *(sings) So*
365 *Nandipha:* *Ke batla {I want} break*
366 *Thabo:* *No, it, my method doesn't work, lets go on*
367 *Nandipha:* *(laughs)*
368 *Thabo:* *It doesn't work*
369 *Lerato:* *If it doesn't work then it means that number, our number B is wrong*
370 *Thabo:* *number B, hae number C, we haven't done number C ne*
371 *Lerato:* *number C its, 3 x 1,*
372 *Nandipha:* *Miss*
373 *Lerato:* *2 x 1 I mean*
374 *(silence)*
375 *Thabo:* *It works, it works, it works*
376 *Lerato:* *What*
377 *Thabo:* *It works with triangles, it works with triangles, but no other ang,*
378 *Nandipha:* *With no other kind of quadrilateral*
379 *Thabo:* *Ja but check, check, it works with triangles because look this one's going to give us 1 ne, ne, and*
380 *this one, 1 also ne, this ne, no, this piece is going to fit here to make it how much, to make it how*
381 *much, ish it works with triangles*
382 *Nandipha:* *miss, miss ?*
383 *(silence)*
384 *Thabo:* *Ja, this is a it works with triangles, that's one of ours the presentation, it works, it works*
385 *Nandipha:* *But it didn't work with other shapes*
386 *Thabo:* *Yes, but it works with triangles*
387 *(silence)*
388 *Thabo:* *Yes, yes Nandipha it works with triangles, it works, it works, look at this, look at this ne, all the*
389 *shapes here we can divide them into, into, into, into, into, we can somehow cut them into uh,*
390 *squares and triangles ne, lets, lets continue and see, lets see wabona {can you see} here, here is*
391 *2 x 1,*
392 *Nandipha:* *Can we take a break miss*
393 *Thabo:* *Hae its*
394 *Teacher:* *O.K did you all want your one minute break or whatever, between your lesson*
395 *Chorus:* *Yes*
396 *Teacher:* *O.K, one minute and don't go out the class, don't go out the class*
397
398 END OF SIDE 1, TAPE 1
399
400 Day 1: discussion resumes
401
402 *Nandipha:* *Ohhh, that*
403 *Thabo:* *Did you hear*
404 *Nandipha:* *Oh ?, can you imagine ?, come on*
405 *Thabo:* *ohh, de gulf war*
406 *Nandipha:* *(laughter)*
407 *Thabo:* *O.K it works with triangles ne*
408 *Nandipha:* *Uh-huh*
409 *Thabo:* *? wabo {can you see}, it works with triangles, bona{look}, lets see, wobona {can you see} number*
410 *B, ne,*
411 *Nandipha:* *Mm*
412 *Thabo:* *akere we found our square already ne,*
413 *Lerato:* *Ja, [and*
414 *Thabo:* *[We found our square*
415 *Lerato:* *[and these are triangles*
416 *Nandipha:* *[Mm*
417 *Thabo:* *and you've got 4 strings here*
418 *Lerato:* *[Ja*

419 *Thabo:* *[you can move them and put it here its gonna give us 1, andhere gonna give us 1, ne*
420 *Nandipha:* *Voila*
421 *Thabo:* *And then here, then here again its gonna give us 1, so that's 1, 2, 3, 4, 5, 6, [7, 8, 9, 10*
422 *Lerato:* *[7, 8, 9, 10*
423 *Thabo:* *Na, yes*
424 *Lerato:* *Mm, 10*
425 *Thabo:* *10, It works, so ev, for every shape we get we divide it into squares and triangles, and then we find*
426 *our thing, superb, thank you, (laughter) that was B*
427 *Nandipha:* *?*
428 *Thabo:* *So number C, number C's going to be*
429 *Lerato:* *Its going to be 2 x 1*
430 *Thabo:* *2 x 1*
431 *Nandipha:* *Honeng rxcho {How many times have you been saying that}, I heard*
432 *Lerato:* *Thabo,write there 2 x 1*
433 *Nandipha:* *What's the next one*
434 *Thabo:* *2 times, its your turn now*
435 *Nandipha:* *I'm doing number e, oh number d first O.K its 1 here, then its 1,2,3, here, then its 2 here, then no*
436 *ways, O.K. thank youuuu*
437 *Thabo:* *No, its wrong*
438 *Nandipha:* *[U-u*
439 *Lerato:* *[its right*
440 *Nandipha:* *eh, [eh its wrong, its wrong, [its wrong,*
441 *Lerato:* *[no, it isn't [its wrong*
442 *Nandipha:* *It is right, it is right*
443 *Thabo:* *Its wrong [no*
444 *Nandipha:* *[Its right*
445 *Thabo:* *Look, look at the [middle here*
446 *Lerato:* *[Hold it like this*
447 *Nandipha:* *O.K, O.K, O.K, O.K, O.K, lets move these two here, ne*
448 *Lerato:* *Nandipha it was right, I swear*
449 *Thabo:* *No Nandipha here, here, no, this one over there*
450 *Nandipha:* *There, ena {this one}*
451 *Thabo:* *Ja*
452 *Lerato:* *Nandipha its right, [the first time*
453 *Thabo:* *[There, its right, this one's right, this one's right,*
454 *Nandipha:* *Ja, this one is*
455 *Thabo:* *right*
456 *(laughter)*
457 *Thabo:* *?*
458 *Thabo:* *[O.K*
459 *Nandipha:* *[O.K.*
460 *Lerato:* *This is better than a half*
461 *Thabo:* *So lets see if it works, this time 1, 2, 3, 4, [5, 6*
462 *Nandipha:* *[O.K this converting to a*
463 *Lerato:* *This one's*
464 *Thabo:* *Wait, lets try to, to see, so lets shading, shading, shading, this is going to give us a half, ja man,*
465 *to make it for 2, yes, 6, 6, there's our [thing*
466 *Lerato:* *[So, they the same, as long as they shaded*
467 *Nandipha:* *Howcome*
468 *Lerato:* *(sings) "Oh, na"*
469 *Nandipha:* *(laughs)*
470 *Thabo:* *there, 6, so its 2 by 1, 2 by 1*
471 *Nandipha:* *Hape {again}*
472 *Thabo:* *eh*
473 *Lerato:* *2 by 1*

474 Thabo: Yeah
475 Nandipha: Mm-mm, 20 by 30
476 Thabo: Its your turn
477 Nandipha: Buddy its your choice
478 Lerato: (sings)
479 (whispers and laughter)
480 Nandipha: Ish, I need a tissue, I do
481 Lerato: You from the toilet but you didn't get one
482 Nandipha: I did, I go to the toilet, I did, I go there
483 Thabo: O.K, lets divide our thing into squares
484 Nandipha: [This is 1
485 Lerato: [There's the first one
486 Thabo: There's the, this is the first [square
487 Lerato: [1,2
488 Nandipha: [1
489 Thabo: [That's 1, [2
490 Nandipha: [2, 3½
491 Lerato: Sheba {look}, [its 1,2,3
492 Nandipha: [which one, this is 1, this is 1,2,3 ja
493 Thabo: its 3½
494 Nandipha: Thank ya
495 Thabo: So there's our triangle, there's our triangle, and [there's our rectangle
496 Nandipha: [this is our rectangle, or two squares, so its got
497 3, [ne
498 Lerato: [3½
499 Nandipha: 3½
500 Thabo: Its my turn, is it
501 Lerato: Yes
502 Thabo: Yes the most complicated one
503 Nandipha: Its your chance
504 Lerato: Moona that's easy, if you can't do it, ae
505 Nandipha: U'stupid
506 Lerato: He must be dof
507 Nandipha: But[though
508 Lerato: [first from the top wena Thabo
509 Thabo: No
510 Nandipha: (sings)
511 Lerato: Lets see the genius
512 Thabo: Genius at work
513 Nandipha: You've gotta be kidding
514 Thabo: Genius at work, Nandipha, ?
515 Nandipha: You've gotta be kidding, ?
516 Thabo: Genius
517 Nandipha: ? (laughs), Lucky
518 Thabo: Genius at work, genius at work
519 Lerato: There, Nandipha
520 Teacher: (to whole class) Once, once you as a group have worked out all the areas, it might be a good idea
521 for one person from your group to go to one person from another group and compare
522 Nandipha: What if you know, you know you absolutely right
523 Teacher: You know, how d'you know that, hey
524 Nandipha: I just, just know miss
525 Teacher: You just know
526 Nandipha: Thank you
527 Teacher: Ultra ?
528 Nandipha: (sings)

529 Lerato: Nandipha, ?
530 (inaudible talk)
531 Thabo: A triangle, this is also a triangle
532 Lerato: This and this goes together
533 Thabo: Its 2, Lerato
534 Lerato: its 1, its 1
535 Thabo: This, this is going to be 1 and 1
536 Nandipha: Lets take a red
537 Thabo: 3 plus 6
538 Nandipha: Eh, not there, the genius must, where's my pencil, Hae maru {but} you zapped my pencil just when
539 I wax gone
540 Lerato: Uh-uh
541 Nandipha: Where's my pencil
542 Lerato: here
543 Thabo: So this piece, this piece is
544 Nandipha: Lerato, nkudeme {lend me} sharpener tu {please}
545 (sings)
546 Thabo: This piece comes here, so you see, 1,2,3
547 Nandipha: Mm, not bad
548 Thabo: No, 1, aah, 1, 2, 3, 4, 5, 6, 7, 8, 9
549 Lerato: What about this one
550 Thabo: Its 1, its 1, its
551 Nandipha: Wait its 1,2
552 Thabo: 5,6,7,8 and 9
553 Nandipha: O.K emapele {wait a minute} [I'm gonna get it just now
554 Thabo: [lets take, lets take this piece, this piece, we put it here, ne, we
555 [need a square
556 Nandipha: [uh, ja
557 Thabo: We going to need a square grand, see Nandipha, we going to need, a piece grand
558 Lerato: Where does this piece come from
559 Nandipha: So its 1
560 Thabo: Its from here, there, this is 4, so its 1
561 Nandipha: Oh its 1, 2, 3, 4, 5, [6, 7, 8, 9, its 9
562 Thabo: [Then this piece goes here, it is 9
563 Nandipha: Thank you, 9, ke'chance ya Lerato {its Lerato's turn}
564 Lerato: Its your turn
565 Nandipha: Its mine, the genius is at work, once again
566 (silence)
567 Lerato: this ½
568 Thabo: And the other ½
569 Lerato: Guys, this ½ ?
570 Nandipha: Empha {give me}
571 Lerato: Ne
572 Nandipha: Nee
573 Lerato: Lets check this so its 1,2,3 ish 1, 2, 3, 4, 5, 6, 7, 8
574 (Nandipha is singing)
575 Thabo: 9
576 Lerato: 9
577 Nandipha: Oh my goodness
578 Thabo: Hah
579 Nandipha: (laughs) [Wanhluwela {leave me alone}
580 Thabo: [Nandipha, Nandipha, don't, Nandipha
581 Nandipha: No, this one is too small
582 Thabo: No, its not too small
583 Nandipha: It is, it is, and I'm gonna get it right, in less than 5 minutes

584 Thabo: Take this one
585 Nandipha: O.K
586 Thabo: take this one
587 Nandipha: ug its ugly
588 Thabo: Nandipha, start from the thing, put it like this, start from there, don't start its going to be difficult,
589 start it from there
590 Nandipha: O.K.
591 Thabo: You know, where, what's happening
592 Nandipha: [Eh
593 Thabo: [Ja, like that
594 Nandipha: O.K, and then
595 Thabo: Right to the corner
596 Nandipha: And then, hey I've got this feeling that this thing's just gonna burst or whatever
597 Lerato: Ja, you should have taken that
598 Nandipha: And then, heh, and now, O.K, no, uh-uh this one is small, its te klein
599 Thabo: No its not small
600 Nandipha: Ja it is
601 Thabo: You only putting a, a small on top
602 Nandipha: Uh-uh wena
603 Thabo: for piece of mind hey
604 Nandipha: O.K., eh, its like that, O.K, O.K, let me start this side
605 Lerato: which way are you starting
606 Nandipha: then, its here, O.K, its here, no its here, and then, its here, right to the corner ne
607 Lerato: Mm
608 Nandipha: O.K, then bring it down 2, 3, ne, and then, it goes here ne, and then
609 Lerato: Where are you now
610 Thabo: Wrong
611 Lerato: Yo, come and get help
612 Nandipha: Uh, I can do it, I must get this right,
613 Lerato: Come and try together, because I haven't got any idea
614 Nandipha: O.K., ne
615 Thabo: No Nandipha you wrong there
616 Nandipha: Why
617 Lerato: It comes like [this
618 Thabo: [this, there
619 Lerato: Sorry, sorry, sorry, sorry [?
620 Thabo: [Don't keepa {put it} there its the wrong string
621 Nandipha: O.K.
622 Thabo: and then, there it is there, here, and then
623 Lerato: there
624 Thabo: [No
625 Lerato: [No, Thabo, doesn't it come like
626 Thabo: here, here, here
627 Nandipha: Emapele, emapele, emapele {just wait}, ja, it goes like so ne, and then, since its [coming
628 Thabo: [yes, [yes
629 Nandipha: [ja
630 I've got this, I've got this, it comes [down like here
631 Thabo: [It Nandipha this part, this part, this part
632 Nandipha: Oh, it goes down further, this thing is te klein
633 Thabo: Too small, its not, press here, press here, let me pull this one
634 Nandipha: Now with this thing le dingas {these things} in my a, on my precious face, I swear
635 Thabo: Press there, press there wena {but}
636 Nandipha: O.K.
637 Thabo: No, not there,
638 Nandipha: Oh, sorry

639 *Thabo:* *Is this here, is this here*
640 *Nandipha:* *Hey, ja, ja*
641 *Lerato:* *Thabo*
642 *(silence, while Nandipha works on geoboard)*
643 *Nandipha:* *"Ooooh, ooh, ooh ooh" (sings)*
644 *(silence)*
645 *Thabo:* *Genius at work*
646 *Nandipha:* *Thank you*
647 *Thabo:* *Genius at work*
648 *Nandipha:* *O.K. nkademe Prit {Lend me some Prit}*
649 *Thabo:* *?*
650 *Lerato:* *Come I count tu {please}, ka wobane wena, wetsa {because you are doing something else}*
651 *Thabo:* *"Al, yeah, yeah yeah" (sings)*
652 *Nandipha:* *its all yours*
653 *Thabo:* *(sings)*
654 *Nandipha:* *O.K*
655 *Thabo:* *? no, (sings)*
656 *Lerato:* *I know what I'm doing*
657 *Thabo:* *You kind of must ?*
658 *Nandipha:* *(laughs) Its 1,2, ae*
659 *Lerato:* *? trap*
660 *Nandipha:* *Yoohoo*
661 *Thabo:* *Trap who till today*
662 *Nandipha:* *what am I doing, bravo (sings) O.K, this is 1, ne, akere, 1*
663 *Thabo:* *This is 2 so lets make this*
664 *Nandipha:* *Ena, {this one}*
665 *Lerato:* *Which one*
666 *Nandipha:* *its 1, so that's 1, that's 2, ne, that's 3, O.K sharp, that's 4, that's 5, that's 6, ooh, 7,8,9, its 9, oh*
667 *10, eh,eh*
668 *Thabo:* *Just wait*
669 *Lerato:* *this is 1*
670 *Thabo:* *Ja, it, its 1, and this one is 1 also, cause its 4, 1,2,3,4 hooks, ne, and then*
671 *Nandipha:* *emapele {wait a minute}*
672 *Thabo:* *And then, we've still got this here*
673 *Nandipha:* *This is 3*
674 *Thabo:* *This is ½*
675 *Nandipha:* *here alone, [this is 4,*
676 *Lerato:* *[Thabo, this can't be 1*
677 *Nandipha:* *5, 6, 7, 8, 9,[10*
678 *Thabo:* *[Nine, ne*
679 *Nandipha:* *Its 10, people, d'you know why, O.K let me show from my book*
680 *Thabo:* *Lerato, Lerato*
681 *Nandipha:* *Let me show you, let me show you from my book, come I show you ne, this is one, akere, ne, this*
682 *is one ne, these things are confusing me, this is one, this corner here, this one here,*
683 *Lerato:* *it [can be 5*
684 *Nandipha:* *[its 1, this is 2, [this is 3*
685 *Lerato:* *[these [things are not equal*
686 *Thabo:* *[No, Nandipha it can, it can,these things are not equal [check,*
687 *Lerato:* *[they are not*
688 *equal*
689 *Thabo:* *this things not here, its wrong, bye, so, no my me, my method there ne, this, here, lets forget about*
690 *this, so,this is going to give us 1, this, this is going to give us 1, and this is a ½ you can see ne,*
691 *and that's going to give us a 1*
692 *Nandipha:* *Ah [ah ah*
693 *Thabo:* *[O.K, so this is 2 [2½, somewhere here*

694 *Nandipha: It, it fits somewhere here*
695 *Thabo: And then this is ½*
696 *Nandipha: Mara {but}, that this give's us one, people*
697 *Thabo: [No,*
698 *Lerato: [It doesn't, [cause*
699 *Thabo: [The thing has to be in this side, this side*
706 *Lerato: eh, ja hore {lets say}, it needs to be this side*
701 *Nandipha: Then you match up, Oh, ja*
702 *Thabo: So, so 4,4,4,4,4, 1,2,3,4, this is 1, this is 1, this is 1,and that is 1, that is 1*
703 *Nandipha: 1*
704 *Thabo: Yeah*
705 *Nandipha: So, that's 1*
706 *Lerato: I know, I [kn*
707 *Nandipha: [Is there, emapele, emapele, {wait, wait} ja*
708 *Thabo: That'll be ½*
709 *Nandipha: ½, ½ this*
710 *Thabo: y-es*
711 *Nandipha: So it means that if you add that and that its 1*
712 *Thabo: Um*
713 *Nandipha: So that's another one, ne, this is*
714 *Lerato: Sheba {look} ne Thabo,its like this ne, and this is, lets say, this is a triangle, this is 1 triangle*
715 *Nandipha: Ja*
716 *Lerato: Sa-a-a, you, you, you ,you, you, you, you, you, you, you, Nandipha*
717 *Nandipha: Keng, keng, keng, keng {what what}, wait*
718 *(silence, singing)*
719 *Thabo: Unwanted part anymore 3, 3½ ne, this ones going to give us 1*
720 *Lerato: [Ja*
721 *Thabo: [Surely this one is, its also 1, so that's 2,3,4,5 plus ½, plus this ½'s gonna give us*
722 *Lerato: [6*
723 *Nandipha: [1, 6*
724 *Thabo: And then*
725 *Nandipha: Hae look, look, look, look, this one, plus that there's gonna give us another one, akere, 7*
726 *Thabo: This, this*
727 *Nandipha: Mm*
728 *Thabo: Plus this*
729 *Nandipha: Mm*
730 *Thabo: Corner*
731 *Lerato: Look*
732 *Nandipha: Corner 7*
733 *Lerato: Ai [sheba {look} ne*
734 *Thabo: [7, 8, ni-i-ine*
735 *Nandipha: and I told you there's nine in the beginning,*
736 *Lerato: Look and then you bring this one, like this, how, selentse {you've made} confusion*
737 *Nandipha: E'e man umametse {are you listening}*
738 *Lerato: But anyway, you bring this one here ne, and then this one here and then this is a good square like*
739 *this one, this will be a good square*
740 *Thabo: Ja*
741 *Lerato: Excluding this, you exclude this, so you've got 1,2,3,4 squares now*
742 *Thabo: 5*
743 *Lerato: Ja 5*
744 *Thabo: 5, wh, this ½ plus this ½*
745 *Lerato: [Ja*
746 *Thabo: [is 6*
747 *Lerato: is 6*
748 *Thabo: plus this thing is*

749 Lerato: How much is that
750 Thabo: 7, this is another ? ne, then and this plus [this
751 Lerato: [that
752 Thabo: is another half, and this plus this is 2, so 9
753 Lerato: 9
754 Nandipha: Rubbing out and ? to it, before they just gave birth
755 Thabo: You must take it through, you must take it through
756 Nandipha: They said its, what's the perimeter guys, the area
757 Lerato: The perimeter its
758 Thabo: Ohhh
759 Nandipha: Area around
760 Lerato: plus the length, le rubbers
761 Nandipha: (sings) Ae, O.K
762 Thabo: Read, mara {but}, b
763 Nandipha: (reads) Find the perimeter of each of the shapes below without measuring it
764 Lerato: Where are you
765 Thabo: Number b
766 Nandipha: Number b
767 Lerato: Oh
768 Thabo: (sings softly)
769 Nandipha: Ae, ke batla {I want} hey guys, I need a Lip-Ice, ?
770 Lerato: Don't forget it ?
771 Nandipha: ?
772 Thabo: Hey, here's the square
773 (sing)
774 Teacher: And this, you learning about rotation here, hey
775 Nandipha: (laughs)
776 Thabo: Konje, lebitsa eng {By the way, what do you call this}
777 Lerato: The perimeter of the first one, its 4 plus 4 plus
778 Nandipha: Why did my pen, pencil become to from there
779 Thabo: No 3 plus 3 plus 2, 2, when its squared
780 (silence)
781 Nandipha: Ohh
782 Lerato: Thabo its 4, [plus
783 Nandipha: [Eh the first one
784 Thabo: We don't count the dots, we count the...
785 Lerato: Perimeter, Thabo the perimeter you count outside ne
786 Thabo: Ja exactly, exactly, its [3 its 3,3,2,3,2
787 Nandipha: [not again
788 Lerato: Somethings ?
789 Nandipha: So, you count all of them 3 2 3 2, 12, ne 12, I already lost my ?
790 Lerato: 12
791 Thabo: 12 metres
792 Lerato: Nna aketlhanye ke sebetsa kele one {I'm not mad, I can't work alone}, Thabo onkile ruler yaka {did
793 you take my ruler}
794 Thabo: ja
795 Lerato: ?
796 Nandipha: Keakopa
797 Lerato: Kekopa leseka la cabanela ruler {please guys, don't fight over a ruler}
798 Nandipha: (laughs)
799 Thabo: No, no, wait wait a minute, waitse keng {you know what}, Ngoako unkile beke ya sokolo aba keepa
800 dilo tsaka abadisiya mompetong gonna daar {Ngoako took my school bag and left things over there}
801 Lerato: Hae utloilatiha {you going to loose it}
802 Nandipha: ? and Magogodi
803 Lerato: ?

804 *Thabo:* *Two hundred and fifty rands*
805 *Lerato:* *?*
806 *Thabo:* *(sings) perimeter, ?*
807 *(silence)*
808 *Lerato:* *Come lets find the perimeter,*
809 *Nandipha:* *Mara I swear, waitse lona leabora nkabelenya lana ?, Thabokabe unyala Lerato {You guys are*
810 *boring, Thabo, you must marry Lerato}*
811 *Thabo:* *?*
812 *Lerato:* *?*
813 *Nandipha:* *Ma, nkabelenya lana, maar haletsa so letlo re nxabanya {when you do this, you'll cause a fight}*
814 *Lerato:* *Kemang orileng keo orata {Who said I love you}*
815 *Thabo:* *Keatseba hore haon rate, onahana hore nkarata osanrate {I know that you don't love me, do you*
816 *think that I can love you while you don't love me}*
817 *Lerato:* *Le, Lindiwe hao rate {Even Lindiwe doesn't love you}*
818 *Thabo:* *Keatseba {I know}*
819 *Nandipha:* *Umaha oarata {you lying, she loves you}*
820 *Lerato:* *?*
821 *Nandipha:* *Uh-uh, uh-uh keahana {I refuse}*
822 *Thabo:* *(laughs) its work*
823 *Nandipha:* *?*
824 *Thabo:* *?*
825 *Nandipha:* *?*
826 *Lerato:* *?*
827 *Thabo:* *?*
828 *Thabo:* *Lets work, lets work*
829 *Nandipha:* *Amaholo {the big ones}*
830 *Lerato:* *We can go on now Nandipha*
831 *Nandipha:* *Uh-uh, uh-uh keboa le Thabo, oska intefere {I'm talking to Thabo, don't intefere}Uh-uh Mm-Mm*
832 *Lerato:* *{its over, the joke*
833 *its over*
834 *Nandipha:* *How, was there a joke*
835 *Lerato:* *Vele {of course} its a joke*
836 *Nandipha:* *Keng joke {what joke}*
837 *Lerato:* *Ontse otsheha {you were laughing} if there wasn't a joke*
838 *(silence)*
839 *Thabo:* *I Thabo, ?*
840 *Nandipha:* *Eh, next one*
841 *(sings)*
842 *Nandipha:* *Guys, bare file {they gave us} a group effort, kekupa reese {lets do} maths as a group effort*
843 *Lerato:* *(inaudible)*
844 *Nandipha:* *Lerato, try the next one, the perimeter, Lerato,*
845 *Lerato:* *Wena osebetsa haobatla {You work when you want}*
846 *Nandipha:* *Jwanong ... hy ompoca nna okatse niks, uh-uh ke motho ontse arguer lena uskansa le boring mood*
847 *ya hao mona {Why are you being sarcastic to me, I didn't do anything to you, its the person that*
848 *you're arguing with, don't take out your boring mood on me}*
849 *Lerato:* *Am I in a bad mood*
850 *Nandipha:* *Ake tsebe keng, ake tsebe keng {I don't know}*
851 *Lerato:* *Am I in a bad mood*
852 *Nandipha:* *Kema*
853 *Lerato:* *(inaudible)*
854 *Nandipha:* *Sho, so,*
855 *Lerato:* *?*
856 *Nandipha:* *?*
857 *Lerato:* *?*
858 *Nandipha:* *?*

859 *(silence)*
860 *Nandipha: (hums)*
861 *(another conversion, Lerato and Nandipha fighting, Lerato asks Thabo about perimeter)*
862 *Teacher: O.K you gonna need to start finishing off now, you not gonna get any homework for today so you*
863 *should be happy, do your homework of your other subjects, Um please do not lose your worksheets,*
864 *O.K. all your working out for this I want you to do in your books, any rough work, any methods*
865 *you come up with that you think are good and you wanna remember them in case you wanna include*
866 *them in your presentation write them in your books, any discoveries that you come up with that's*
867 *not part of the worksheet, write them in your books, O.K, so all your working out, I want these*
868 *pasted in your books, and all your working out into your books, O.K. please don't tell me you've*
869 *lost these worksheets, O.K. and bring them tomorrow. O.K please put the elastics onto the*
870 *geoboard, all your elastics just put them onto the geoboard and then I'm gonna come around and*
871 *collect them.*

END OF SIDE 2, TAPE 1

Day 2: Friday 15th October 1993 (Lerato absent)

872 *Teacher: Get into your groups please, take out your work that you've been doing, O.K. today we're going*
873 *to carry on with that, like you were doing yesterday*
874 *(discussion in Sotho between Nandipha, Thabo and another student about who smells)*
875 *Teacher: Uh-uh, come on*
876 *Nandipha: You smell so bad that you think its other people*
877 *Teacher: Can I also just say, everyone,*
878 *someone: (shrieks)*
879 *Teacher: uh-uh, come on, you supposed to have started lon, ago*
880 *Thabo: She's filling with you= blank page ?*
881 *Nandipha: (laughs)*
882 *Thabo: Look at them*
883 *Nandipha: Miss, miss*
884 *Teacher: Stop playing with the girls*
885 *Thabo: ? girls*
886 *Nandipha: (laughing)*
887 *Teacher: O.K. listen class, if you want any more of these sheets, O.K. in other words if you've filled up your*
888 *one and you want to show or you want to, demonstrate something, and you need more of these, I've*
889 *got, you simply have to ask, O.K, I'm gonna bring round your geoboards and then you can carry*
890 *on, O.K, whose was this, does anybody want their [specific artwork back O.K.*
891 *Nandipha: [lets see, lets see, I think that's ours*
892 *Thabo: (laughs) its not*
893 *Nandipha: Oh its not miss, its not*
894 *Thabo: (laughs)*
895 *Nandipha: (sings)*
896 *Thabo: (sings) "You know the truth ha" (laughs)*
897 *Nandipha: Keng wena {what's wrong with you}*
898 *Thabo: ? girls*
899 *Nandipha: (laughs) ? girls*
900 *Teacher: Is she absent*
901 *Thabo: Yes*
902 *Nandipha: Yes*
903 *Nandipha: (sings)*
904 *(silence)*
905 *Thabo: (sings) "Lets get going baby, and I'm gonna work with this baby", number b, what no, no, no*
906 *Nandipha: Reentse {we have done} number b, oh number 2, number 2 akere, hey, hullo, earth calling Thabo*
907 *Thabo: No, no, its not, its impossible to make, Mm*
908 *Nandipha: (reads) On your geoboard, what is this thing*

Thabo: Its not possible
Nandipha: (reads) Make a square with an area of two units squared, and draw it on your dotted paper, how d'we do that
Thabo: Uh I'm also wondering
Nandipha: (laughs)
Thabo: What times by, by, by something gives you two, two, hey miss, [miss
Nandipha: [times one
Thabo: Miss, like the, the first one we don't think its possible to, to, do
Teacher: O.K. let me have a look, ?
(silence)
Teacher: O.K lets see see
Thabo: [Like
Nandipha: [number b, number 2
Teacher: Work out the perimeter
Nandipha: No, [miss
Teacher: [Oh here number two, so you've done these, hey
Nandipha: Ja,
Teacher: You've done them for all of them, O.K, (reads) on your geoboard make a square with an area of 2 units squared, and draw it on your dotted paper, O.K so lets try, you want an area of
Thabo: 2 units squared
Teacher: O.K. but that's not too difficult, come
Nandipha: How d'we do that
Teacher: Here, you need a small elastic, you can have any perimeter, do they, do they say you can have any perimeter
Thabo: They say a square
Teacher: Oh, a square, lets see, on your geoboard make a square, O.K. so a rectangle's easy, so you wanna make a square, a rectangle
Thabo: And we did [that
Teacher: [that would be easy, so what you gonna do for a square
Thabo: That means a square can be a rectangle
Teacher: No, no, no, [no, no, no, this is not
Thabo: [but not
Teacher: correct, I'm just showing you that its easy to do it with a rectangle, now with a square, if we do that
Thabo: Then, then its not 2 units squared
Teacher: Then its one
Thabo: Yes.
Teacher: So, what else can we try
Thabo: Um
Teacher: If we do a bigger square
Nandipha: Then its 6
Thabo: No it, [its 4
Nandipha: [I mean its 4
Teacher: O.K. so now what, what else can we try, I mean I've been doing um, a very ordinary square, like that
Nandipha: There miss, uh you can do this miss, um, look at this, you make you, uh must you make a square
Teacher: Ja, it must be a square, but I mean does a square always have to be like that
Thabo: No, it can be, like almost in a position of a diamond isn't it
Nandipha: Like ? like this
Thabo: As, long as the
Teacher: Yes, O.K it can be like that, yes, as long as its still got, 90 degree angles, O.K. so try'n, try'n move this around, but still have 90 degree angles
Nandipha: Hae aicona
Thabo: No miss, but we, we've tried here
Teacher: Come try move it around, lift it up, and move it, rotate your square, you had your square there, rotate it, move it around somewhere else, try

964 Nandipha: Here, its still the same thing
965 Teacher: O.K there its still the same
966 (silence)
967 Teacher: But you don't have to use the 1 square, show me, all the different places you can make a square on
968 this board, aah, now check, w'how much is that
969 Thabo: It is 2
970 Nandipha: Its 2
971 Teacher: O.K. so what did you do, what did you do that makes that a square
972 Nandipha: Because, um, you can put this, thi towards the edge miss, and then you put the one in the middle,
973 so it divides into 4 quarters
974 Teacher: O.K. so you didn't stick to your usual geoboard
975 Thabo: [yes
976 Teacher: [the sides of your square is not the normal 1, what's the sides of your square here, its basically the
977 diagonals of your geoboard, d'you see that
978 Thabo: Ja, ja
979 Teacher: I'm just saying that instead of you working with normal straight squares, where its straight on your
980 geoboard, you have made the side a diagonal of your geoboard
981 Nandipha: Oh, O.K.
982 Teacher: Instead of just being a straight, now you can keep going with a straight, but then you only ever
983 gonna go 1,4, the next one, 9, until you go skew, O.K, now, now have a look, hold that on here
984 (silence)
985 Thabo: 1 squared, thank you miss
986 Teacher: O.K., no you got it
987 Nandipha: (reads) What possible areas can squares have on your geoboard
988 Thabo: Lerato, Lets say, so we found 1 is obviously possible ne,
989 Nandipha: Uh-huh
990 Thabo: 2, no, ja 2, is 3 possible
991 Nandipha: Like when you put
992 Thabo: No, 3 squared
993 Nandipha: It is, here
994 Thabo: No, that is 4
995 Nandipha: Oh, ooh, O.K (sings)
996 Thabo: But its hot in here
997 Nandipha: Its hot
998 (silence)
999 Nandipha: That's the same thing
1000 Thabo: (sings) "Canna get you, out of this ? shew she says"
1001 (chatting/singing)
1002 Nandipha: "Oooh-oh-oh"
1003 Thabo: (sings) "It is Jackie"
1004 Nandipha: Which one, Mofokeng, but which is which
1005 Thabo: Mm
1006 Nandipha: The new miss South Africa, not bad ne
1007 Thabo: (sings) Yes ma - [a-a-n
1008 Nandipha: [Lets go this way
1009 Thabo: What is it, that's 90 degrees
1010 Nandipha: O.K. O.K. O.K, how many this
1011 Thabo: Its 1
1012 Nandipha: 1, this is 1, ne
1013 Thabo: This is 2 [3
1014 Nandipha: [This is 2, sorry
1015 Thabo: noo
1016 Nandipha: So its not possible, 1,2, 4, O.K., 1, its 1, ae, the next one is
1017 Thabo: 4, 4 is possible
1018 Nandipha: 8

1019 Thabo: 2 [is possible
1020 Nandipha: [Mm-Mm, plus 1 ne
1021 Thabo: No,
1022 Nandipha: Mm, plus[2
1023 Thabo: [1 is possible
1024 Nandipha: Mmm, 2 plus 2
1025 Thabo: 2 is possible, ne
1026 Nandipha: so its 4 plus 3, 7, lets try out 7, I think a 7 is possible
1027 Thabo: Is does 8 work
1028 Nandipha: I don't know, I think I've found a method, the genius at work
1029 (silence)
1030 Nandipha: Mm, what you doing, that's gonna be too much ne
1031 Thabo: This is 9, this is 9, is this the area or, there its 9, its 9, its, its, there'sss 1,2,3, ohh, you know
1032 what, that means that 9 is, we dom, we dom, its going to be, like, 1,2, 1,2, 1,2,4, is 8 possible
1033 Nandipha: I think it is
1034 Thabo: So its going to
1035 Nandipha: Uh, pull, pull, pull
1036 Thabo: Pull, (sings) Now lets see is 8 possible
1037 Nandipha: (sings)
1038 Thabo: (whistles) gewaar, serves her right, ish
1039 (both laugh, while teacher talks to Lerato who has just entered the classroom)
1040 Thabo: 1,2, [3,4
1041 Nandipha: [?, hey but ae
1042 Thabo: Yes, 8 is possible ne, [8 is possible
1043 Nandipha: [But ja but, (to Lerato) ja my iron's under my bed (laughs), mi's
1044 Thabo: Go to reception
1045 Nandipha: Wait, this is 1, this is 2, (laughs) 1,2,3,4 how lets see how many things are here, yeah
1046 Thabo: Its 4 in the middle ne
1047 Nandipha: Ja
1048 Thabo: So this is 1, no,no,no
1049 (silence)
1050 Nandipha: It is 4
1051 Thabo: No, let me do it on the paper, should we, 8 is possible, Nandipha; 8 is possible
1052 Nandipha: Why
1053 Thabo: Uh, here, square, its possible, this is
1054 Nandipha: how will I do that
1055 Thabo: 1
1056 Nandipha: Nkadenie ntwena {lend me this}
1057 Thabo: 1,2
1058 Nandipha: (sings)
1059 Thabo: this is 9 man
1060 Nandipha: How do you do this hey, how do I do this,
1061 Thabo: So this is 4, 5 is not possible
1062 Nandipha: Obviously not
1063 Thabo: I know how it works, ne, its going to be 1, [2,4,8,9
1064 Nandipha: [2,4,8,9, ae, ae
1065 Thabo: Lets check 12, 12 come
1066 Nandipha: 8;8, [9
1067 Thabo: [What's this
1068 Nandipha: Its 4 again, is it 8, check
1069 Thabo: Its 8, is she still alive
1070 Nandipha: (laughs) ?
1071 (laughter)
1072 Nandipha: ? How many things are here, ish
1073 (laughter)

1074 *Nandipha: Miss, miss can you come and help us, oh*
1075 *(wait and work separately – Nandipha talks softly)*
1076 *Nandipha: I can do this, its 1, then you pass those ones, and you get here, ja, ja, I've got it, then you pass*
1077 *these two, uh-uh, wait, wait, the genius is at work, once again*
1078 *Thabo: Oh-oh, uh-uh, (laughs) is she still alive*
1079 *Nandipha: (laughs) ? toilet*
1080 *Nandipha: O.K. that's that one, is this a square*
1081 *Thabo: No*
1082 *Nandipha: (laughs) O.K. then we pass these 3 here, then we get that ne, then you pass, those ones then you*
1083 *get there, you pass those ones, then you get there, I've got it, so, its 1,*
1084 *(both sing and laugh)*
1085 *Thabo: Ae, it doesn't work, I don't know what it is, I don't know what it is, what it is*
1086 *Nandipha: (sings) "I don't know what it is" (laughs)*
1087 *Thabo: ?*
1088 *Nandipha: ?*
1089 *Thabo: ?*
1090 *Nandipha: (laughs) uh-uh man, uh-uh, she's your friend, what, what*
1091 *Thabo: Al, she, she, she's, ah forget it*
1092 *Nandipha: (laughs) miss, miss*
1093 *Teacher: O.K. hold on see*
1094 *Nandipha: I, I've discovered something here and I don't know how many squares there are*
1095 *Thabo: (laughs) Nandipha, we, no, no, lets use the things, why d'you think these things are here*
1096 *Nandipha: O.K., sharp that's*
1097 *Thabo: That's a ½, so you know 1,2,3,4 will give us 1 ne, 1 more*
1098 *Nandipha: Mm*
1099 *Thabo: So we know from yesterday*
1100 *Nandipha: This is 1 plus, [the 4 in the middle*
1101 *Thabo: [this ½*
1102 *Thabo: Keepa ding regular, just the small ones, chill out with the, there's more, more in the middle*
1103 *Nandipha: (sings) "donna, donna" donna keitse {here they are} ?*
1104 *Thabo: ?*
1105 *Nandipha: (laughs)*
1106 *Thabo: 1,1, ne*
1107 *Nandipha: yeah, 2*
1108 *Thabo: so*
1109 *Nandipha: You a genius, [its 1,2,3,4,5,6*
1110 *Thabo: [I am Nandipha*
1111 *Nandipha: So 6 is possible*
1112 *Thabo: Is it 6, its 1,2,3,4, 8 man*
1113 *Nandipha: Ohhhhh*
1114 *Thabo: ? Nandipha I got 8 ? Nandipha, no but this is different man, si, 8*
1115 *Nandipha: But this is a, this is an excellent one, look at the shapes in the 4 quarters*
1116 *Thabo: So, 7's not possible, 8's possible*
1117 *Nandipha: 1 plus 2 plus*
1118 *Teacher: Yes, ah nice, ooh*
1119 *Nandipha: Thank you miss, Thank you miss, is this, is this the um 8*
1120 *Thabo: It is cause yesterday we worked out if*
1121 *Nandipha: its 1 or 2, 3, 4 right, there's 1*
1122 *Teacher: Uh-uh, hold on, hold on, I would just, ja O.K 1½, this whole thing*
1123 *Nandipha: Mm-hm*
1124 *Thabo: No, its 1, we worked out yesterday that when you get 4 hooks like this,*
1125 *Teacher: Yes*
1126 *Thabo: They, they gi, they always give you 1, with the triangles*
1127 *Teacher: Which one gives you 1*
1128 *Thabo: A, a triang, if you get a triangle with, uh the [elastic band on the 4 hooks ne*

1129 *Nandipha:* [4 hooks
1130 *Teacher:* On 4 hooks
1131 *Thabo:* Yes
1132 *Teacher:* O.K., O.K
1133 *Nandipha:* O.K, [so if
1134 *Teacher:* [lets see, is that always true
1135 *Thabo:* Yes, [miss
1136 *Teacher:* [An elastic band on four hooks will [always give you 1
1137 *Thabo:* [Yes miss, yes miss, yes
1138 *Teacher:* O.K. that's an interesting thing then, well you must write it down and test it
1139 *Nandipha:* O.K. miss, so, this one gives you 1, this one gives you 1, this one gives you 1, this one gives you
1140 1
1141 *Teacher:* O.K. but I'm gonna disagree with you now
1142 *Nandipha:* Why
1143 *Teacher:* because I'm gonna say that this one here with 4 hooks gives me an area of 1½, not 1
1144 *Thabo:* 4 hooks, which one
1145 *Nandipha:* Miss, this is 5 hooks, 1,2,3,4,5
1146 *Teacher:* Oh I see, O.K,
1147 *Nandipha:* Ah-hah miss ah-hah
1148 *Teacher:* so where's your 1, oh I see, this one gives you 1
1149 *Thabo:* Yes
1150 *Nandipha:* Yes
1151 *Teacher:* Aah, O.K. you must write down that conjecture, hey
1152 *Nandipha:* O.K, so it's 1,2,3,4 right
1153 *Teacher:* Yes
1154 *Nandipha:* And this is, 1,
1155 *Teacher:* yes
1156 *Nandipha:* so that's 5
1157 *Thabo:* Yeah, oh
1158 *Nandipha:* This is 6
1159 *Teacher:* 6, O.K
1160 *Nandipha:* this is 7, 8, 9 (laughs) 10, this is 10
1161 *Teacher:* perfect
1162 (they laugh)
1163 *Teacher:* So what's wrong
1164 *Nandipha:* Miss, we thought it was 8
1165 *Teacher:* You thought it was 8
1166 *Thabo:* Oh, yes, we've been counting this as ½'s
1167 *Teacher:* Which ones as ½'s
1168 *Thabo:* Here
1169 *Teacher:* Aah, O.K, instead of counting it as 1
1170 *Nandipha:* Yes
1171 *Teacher:* Hey, this, this, this 4 hook theory's quite interesting, [does it, are you, have you tested it,
1172 thoroughly?
1173 *Nandipha:* [1 plus
1174 *Thabo:* Yes, we tested it here, [here it worked, it worked with all the triangles
1175 *Nandipha:* [2 plus 4 plus 8 plus 10, hu-uuh
1176 *Teacher:* You must write that one down hey
1177 *Thabo:* No, no, no, no, not that it, it, before, not that it worked with all the triangles we've given to, we
1178 counted the hooks, then if we keep all the hooks like on one square, and we found the area of the
1179 thing without having to use this triangle, all you count the hooks here, and then see there
1180 *Teacher:* But this one here's got 4 hooks
1181 *Thabo:* No, 1,2,3,4,5,6
1182 *Teacher:* Oh I see, you [count,
1183 *Thabo:* [you count there

1184 Teacher: aah, even the ones on the inside
1185 Nandipha: Yes
1186 Teacher: O.K. You must write it down, cause that'll be nice for your presentation, but I'm gonna look for a,
1187 for a counter-example, I dunno if there is one, oh I see because the minute you counting your inside
1188 ones, but I would've liked to say, that's got 4 hooks, but no its got 5, you see, and it doesn't have
1189 an area of, you saying 4 hooks has an area of 1, you must write down your conjecture, O.K. (walks
1190 away)
1191 Nandipha: Next one
1192 Teacher: (to other group) nice, very nice, very nice
1193 Nandipha: Can I see please
1194 Zandile: Its for number 2
1195 Teacher: No, don't tell her which number, they've still gotta see if, figure it out for themselves
1196 Nandipha: 10, this is 10, I didn't draw 1 ne, aah, what am I doing, aah doesn't matter, O.K. which one
1197 Thabo, are we finished with 10 now, is 10 enough
1198 Thabo: Yeah lets get up to 10
1199 Nandipha: Ja, we finished
1200 Thabo: (reads) Draw, on your, on this dotted paper, show any working out, ja we have, we have drawn,
1201 you have
1202 Nandipha: Mm
1203 Thabo: Why you scrape it off, how are we gonna
1204 Nandipha: O.K. I'll put it back, I'll put it back
1205 Thabo: Don't, don't think you famous don't
1206 Nandipha: But I am you see
1207 Karin Miss Graven says you must explain to me what you
1208 Nandipha: O.K miss, so let me just put back this square there, right, ja, O.K, so miss, this one's a 6, ne, these
1209 are 6 squares, I mean 4, 4, 4, 4 squares right, and then like, this is one ¼, this is one ¼
1210 Karin One ¼
1211 Nandipha: Yes, I mean a ½, ½ of a square, right and then you find out that when you, when you get 4 hooks,
1212 what have I done, ohja, when you get 4 hooks, miss, ne, they give you a square, of a triangle, if,
1213 if your triangle has got 4 hooks right,
1214 Karin ja
1215 Nandipha: it always gives you a square
1216 Karin You mean an area of 1
1217 Nandipha: Yes
1218 Thabo: How [we
1219 Karin [Always
1220 Thabo: Look here lets say, lets say [this was here
1221 Karin [How d'you know
Thabo: This was here
arin ja
Thabo: it was here, we say you count 4 hooks, and then you say, since its 4 hooks, we put, count any 4
hooks and put it there it'll give us our 1 squared, we do it like, lets use eh, ja lets use this, this
26 here, that, that's 1 ne,
1227 Karin Ja, that's 1
1228 Thabo: so then
1229 Karin but I dunno about this one
1230 Thabo: That one, it is also cause its 1,2,3,4
1231 Karin But why, how d'you know that the area there is 1
1232 Thabo: The area
1233 Nandipha: Because its got 4 hooks miss
1234 Thabo: 4 hooks
1235 Karin O.K. but I, I mean I, I agree with you that with those 4 hooks the area [will be 1
1236 Thabo: [yes
1237 Karin And with those 4 hooks the area will be one, but what about these, its not, there's no law that says
1238 4 hooks

1239 Teacher: No, no, no it works, it does work
1240 Karin But why, why is this area 1
1241 Thabo: Miss, 1, eh
1242 Nandipha: (laughs)
1243 Thabo: O.K. I can't explain but miss the, the, the, the working out that we've been doing, the,
1244 Karin But what is it, tell me, cause I'm not quite sure? I mean, I don't understand, if I tried to divide this
1245 up, I can't see that I'm getting 1, like, I dunno what that is, that's not a ½ O.K, that would change
1246 the area hey
1247 Nandipha: Miss, isn't it that if you, O.K., lets say for instance if you cut up these, these pieces into, no
1248 Thabo: No, no, no, here, here, here I'll prove it to you, uh, lets say, we, we'll use this
1249 Nandipha: That's [5 hooks
1250 Karin [ja, so that's, that's ja
1251 Teacher: You've got it
1252 Karin just explain it
1253 Thabo: O.K. then, no, this one's supposed to be like this, supposed to be like this, here, ish, O.K. here it
1254 is, here miss, it, it doesn't matter whether, as long as its, its uh, it doesn't matter if the, if the
1255 things, the angles, are, different, if, as long as the thing, the shape has got 4 sides, the, the area
1256 will always be the same, lets say there it was
1257 Karin O.K. I, I'll go with that, so what's the area of this one
1258 Thabo: So the area of this one's going to be 2 units squared, so half of it which is the line that's cutting
1259 here in half
1260 Karin is 1
1261 Thabo: is 1
1262 Karin O.K. now that I'll accept
1263 Nandipha: So d'you agree that it works
1264 Karin For this, these 4
1265 Thabo: Yes
1266 Karin So I agree that it works for a square, that triangle, and this triangle
1267 Thabo: There, can we show more, can we show you more with the 6 hooks
1268 Karin Ja, but try and write it down, cause I don't wanna sit here all the time, I want to go back there, so
1269 try and write it down and then Ms G will mark it, d'you see that you could have explained it to me
1270 this way, what you started, you said there's this big one which is what, what's the area of this big
1271 one
1272 Nandipha: Mm, its um 10 miss
1273 Karin 10, and this one is
1274 Nandipha: its 4, which one, which one, which one
1275 Karin This big one
1276 Nandipha: ja its 4, yeah
1277 Karin So that leaves you with 6 on the outside
1278 Thabo: Yes
1279 Nandipha: Uh
1280 Karin And then these is each ½, so its 2, so that leaves you with 4 on the outside
1281 Nandipha: Ohhh
1282 Thabo: What did she say
1283 Nandipha: (laughs)
1284 Thabo: Why, don't laugh
1285 Nandipha: (laughs) O.K. ne, since this thing has got an area of 10 ne, we've got 4 in the inside here ne, and
1286 these are ½'s which give us another 4 ne
1287 Thabo: Mm, 2
1288 Nandipha: I mean another 2 ne, so that's 6, so we left with four,1,2,3,4, so each one of these must equal to
1289 1
1290 Thabo: Which one of these
1291 Nandipha: These, these ones with the four hooks must equals to one cause that's the 4 that's 4 left, just wait,
1292 we've got, there's this thing with an area of 10 ne, ne, this is, the red one is an area of 4 ne, this
1293 one's give an area of 2, so that's 6, so we've got 4 left, so each one, this one, must equals to 1,

1294 that one equals to 1
1295 Thabo: Exactly, but we weren't sure before we proved it, that this is equals to 1, and this is ?
1296 Nandipha: (laughs) I don't believe you didn't hear what she said
1297 Thabo: No, I wasn't concentrating, no I, I wasn't concentrating
1298 Nandipha: No wait
1299 Thabo: Here, here ?, Zandile
1300 Zandile: ?
1301 Thabo: ?
1302 Zandile: ?
1303 Nandipha: (reads) O.K, What possible perimeters can squares have on your geoboard, show any working out
1304 that you do
1305 Thabo: What possible, perimeters, O.K. [can squares
1306 Nandipha: [can squares have
1307 Thabo: Keepa everything, so here, one, that is write down, write down
1308 Nandipha: Mm-hm
1309 Thabo: That is, that is, um that is, 4,4,4,4, I don't have a pen, can you believe it
1310 Nandipha: Which one 1,2,3,4,5, akere 1,2,3,4,5
1311 Thabo: No not 4,5, 4
1312 Nandipha: it is, 1,2,3 ohh, thank you, Mm, do I have a pencil, d'you have a pencil ?
1313 Thabo: It is mine ?
1314 Nandipha: Lets see 4
1315 Thabo: O.K. Mm-hm, those
1316 Nandipha: That's 1, 2 ne and also something else, and also
1317 Thabo: No, Nandipha its the same
1318 Nandipha: So you can't add it
1319 Thabo: No you don't add it
1320 Nandipha: O.K. I did something wrong here, its supposed to be like this, this one like this, a, and then the last
1321 corner one, then akere say yes the perimeters
1322 Thabo: No, no, no, there's still more, there's still more, phela {but}, what about that, this is equals to 2
1323 squared
1324 Nandipha: Ohhh
1325 Thabo: And Zandile, they didn't get it
1326 Nandipha: Ag man O.K, 2, this is 2 squared akere, can I see something, can I
1327 Thabo: Do our one of 10, per
1328 Nandipha: O.K Um
1329 Thabo: Pe, there, there, there, there, isn't it, ja
1330 Nandipha: is it here, O.K.
1331 Thabo: Aah
1332 Nandipha: And
1333 Thabo: (sings) "Lets make a move"
1334 Nandipha: Wait man, we did something wrong here, oh I didn't, there's that one, Mm-hm
1335 (both hum)
1336 Thabo: No, something
1337 Nandipha: So you think
1338 Thabo: No, no, no, no, no, this [illegible] find the area
1339 Nandipha: Mm-mm, is, it is not po[illegible]e akere, isn't Lerato coming back to class, miss is getting worried
1340 Thabo: ? Lerato ?
1341 Nandipha: It is stinking so much in this class
1342 Thabo: (sings)
1343 Nandipha: We finished miss
1344 Thabo: Hey Zandile, wabona, wabona yours, wabona yours {let me see yours}, oh, they found it, they found
1345 it, we just want
1346 Zandile: de seven mohoe {they are seven there}
1347 Thabo: 1,2,3,4,
1348 Zandile: tsarona de seven {we have seven}

1349 Thabo: 5,6, no, 1,2,3,4,5,6 tsarona de 7 are counteng {we also have 7, lets count}
1350 Zandile: 1,2,3, ne 4,5,6,7
1351 Nandipha: [1,2,3,4,5,6,7,8
1352 Thabo: [Check 1,2,3
1353 (laughter)
1354 Zandile: its 1,2 [3,4, [5,6,
1355 Nandipha: [3,4, [5,6, 7
1356 Thabo: [5
1357 Zandile:
1358 Nandipha: Ebathung {oh my goodness}
1359 Thabo: No its 1,2,3,4, [5, 6,7
1360 Nandipha: [No you counting
1361 Zandile: No you counted, its a one
1362 Nandipha: Ja its 7, 7
1363 Thabo: Its 1
1364 Zandile: Uh-uh its 6, wena 1, [2,3,4,5
1365 Thabo: [2,3,4,5
1366 Nandipha: Thabo, u'countile hape {you counted again}
1367 Zandile: So its its 1, [3,4
1368 Nandipha: [Oh no, oh no, ja
1369 Nandipha: Uh-uh emapele {just wait} its 1 guys, are'counting {lets count it} its 1,2,3,4,5, [6
1370 Zandile: [6, so its 6
1371 Thabo: How come you had a 7
1372 Zabdile: Ntaldbone, linali, lina ne {let me see what you have}
1373 Thabo: (sings) go-on
1374 Zandile belina le {you had}, alina yona {you haven't got}
1375 Thabo: (sings) go-on
1376 Zandile a, a, alina yona {you haven't got}, Oh kee {here it is}
1377 Nandipha: Uh-uh
1378 Zandile O.K belina {you had..} leya le lina la 4, le lina la 5 {you've got 4, you've got 5}
1379 Thabo: Le lina le, la 10 {we've got 10}, so counting, wrong
1380 Zandile Uh-uh, its 1, look sheba
1381 Nandipha: emapele {just wait} ne ne, na I'm gonna count it over, you repeated this, oh
1382 Zandile No I think 1,2, [3
1383 Nandipha: [3
1384 Thabo: No count it from here, 1, [2,3
1385 Nandipha: [2,3 ne
1386 Thabo: 4
1387 Nandipha: 5
1388 Thabo: Hey they had 2 here man, they had 2
1389 Zandile No, we have 2 what
1390 Nandipha: Why did you put 2 there
1391 Thabo: No, no, no Zandile, Zandile
1392 Zandile Its one, ne tlaki count {let me count} rensitse {we subtracted} 1 ke 1 ne
1393 Nandipha: 3,
1394 Zandile ke 3, [4,
1395 Nandipha: [4, 5, 6,
1396 Zandile 7 kare rensitse of {maybe we've subtracted it}
1397 Nandipha: Because that thing was wrong, you probably counted the sameone
1398 Zandile Ikaa {where's} the same one ke eseyong keye ere yetseseng {this is not there, here, the one we
1399 subtracted} keye ereyetseseng
1400 Nandipha: Ohhh askuus
1401 Thabo: No, man how come, how come
1402 Zandile But
1403 Nandipha: Thanks [bell goes]

1349 Thabo: 5,6, no, 1,2,3,4,5,6 tsarona de 7 are counteng {we also have 7, lets cour:}
1350 Zandile: 1,2,3, ne 4,5,6,7
1351 Nandipha: [1,2,3,4,5,6,7,8
1352 Thabo: [Check 1,2,3
1353 (laughter)
1354 Zandile: its 1,2 [3,4, [5,6,
1355 Nandipha: [3,4, [5,6, 7
1356 Thabo: [5
1357 Zandile: 6
1358 Nandipha: yebathung {oh my goodness}
1359 Thabo: No its 1,2,3,4, [5, 6,7
1360 Nandipha: [No you counting
1361 Zandile: No you counted, its a one
1362 Nandipha: Ja its 7, 7
1363 Thabo: Its 1
1364 Zandile: Uh-uh its 6, wena 1, [2,3,4,5
1365 Thabo: [2,3,4,5
1366 Nandipha: Thabo, u'countile hape {you counted again}
1367 Zandile: So its its 1,2, [3,4
1368 Nandipha: [Oh no, oh no, ja
1369 Nandipha: Uh-uh emapele {just wait} its 1 guys, are'counting {lets count it} its 1,2,3,4,5, [6
1370 Zandile: [6, so its 6
1371 Thabo: How come you had a 7
1372 Zabdile: Tlakibone, linali, lina ne {let me see what you have}
1373 Thabo: (sings) go-on
1374 Zandile belina le {you had}, alina yona {you haven't got}
1375 Thabo: (sings) go-on
1376 Zandile a, a, alina yona {you haven't got}, Oh kee {here it is}
1377 Nandipha: Uh-uh
1378 Zandile O.K belina {you had..} leya le lina la 4, le lina la 5 {you've got 4, you've got 5}
1379 Thabo: Le lina le, la 10 {we've got 10}, so counting, wrong
1380 Zandile Uh-uh, its 1, look sheba
1381 Nandipha: emapele {just wait} ne ne, na I'm gonna count it over, you repeated this, oh
1382 Zandile No I think 1,2, [3
1383 Nandipha: [3
1384 Thabo: No count it from here, 1, [2,3
1385 Nandipha: [2,3 ne
1386 Thabo: 4
1387 Nandipha: 5
1388 Thabo: Hey they had 2 here man, they had 2
1389 Zandile No, we have 2 what
1390 Nandipha: Why did you put 2 there
1391 Thabo: No, no, no Zandile, Zandile
1392 Zandile Its one, ne tlakl count {let me count} rensitse {we subtracted} 1 ke 1 ne
1393 Nandipha: 3,
1394 Zandile Re 3, [4,
1395 Nandipha: [4, 5, 6,
1396 Zandile 7 kare rensitse of {maybe we've subtracted it}
1397 Nandipha: Because that thing was wrong, you probably counted the sameone
1398 Zandile Ikaa {where's} the same one ke eseyong keye ere yetseseng {this is not there, here, the one we
1399 subtracted} keye ereyetseseng
1400 Nandipha: Ohhh askuus
1401 Thabo: No, man how come, how come
1402 Zandile But
1403 Nandipha: Thanks [bell goes]

1404 Zandile: *Hey but there's still one, one more, one more er, eresaltseng {which we didn't make} which we*
1405 *didn't make it*
1406 Thabo: *Which one*
1407 Zandile: *Ish a, aruna aresofetse ema ke'1, lina, lina aketsebe horeki etseseng {we didn't finish it, wait its 1,*
1408 *I don't know why I didn't do it}*
1409 Thabo: *Oh, never mind, we know it, come back, come back ?*
1410 Nandipha: *I knew it ?*

END OF SIDE 1, TAPE 2

Day 3: Monday 18th October 1993

1411 Teacher: *Come on, take out your worksheets, Can I just ask, which number are most of you on, what number*
1412 *are you busy working on now?*
1413 Someone: *4*
1414 Someone: *3*
1415 Someone: *4*
1416 Someone: *1*
1417 Someone: *number 2*
1418 Someone: *(laughs)*
1419 Teacher: *No, 2, 3, Excuse me Nandipha but don't come into my class, firstly you come in late, and secondly*
1420 *you come in and you stand there and you (inaudible)*
1421 Nandipha: *Sorry miss*
1422 Teacher: *O.K*
1423 Thabo: *(sings)*
1424 Nandipha: *(laughs)*
1425 Teacher: *This group's on 3, your group*
1426 someone: *2*
1427 Teacher: *2, 3, (to this group) what number is your group on now*
1428 Thabo: *Uuuum, 2*
1429 Teacher: *O.K, O.K. that's fine, O.K. I'm handing out the geoboards again, (t[illegible]other group) no but yours*
1430 *has changed because we handed them in, ? the other groups could use them*
1431 Thabo: *This is ours*
1432 Nandipha: *That's definitely not ours, its an observation I made*
1433 Thabo: *(laughs)*
1434 Teacher: *If you need more elastics, you can get from me, O.K here's more elastics*
1435 Thabo: *Lets check the square, Nandipha, ne mm, check*
1436 Nandipha: *Is that a square, Thabo*
1437 Thabo: *(inaudible)*
1438 someone: *Lerato ukaa {where's Lerato}*
1439 Thabo: *Huh*
1440 someone: *(inaudible)*
1441 Thabo: *Why d'you ask me*
1442 Nandipha: *Because you would know, ish*
1443 Thabo: *I know why, who d'you ask*
1444 Nandipha: *You*
1445 Thabo: *Why d'you ask me*
1446 Nandipha: *Where are we*
1447 Thabo: *Oh I thought you said to the new guy*
1448 Nandipha: *(laughs) oooh but then you should know ?*
1449 Thabo: *?*
1450 Nandipha: *Na aketsebe {I don't kno.}*
1451 Thabo: *Akere specialise idaar {I don't specialise there}*
1452 Nandipha: *(reads) choose any, number 3*
1453 Thabo: *?*

1454 Nandipha: ?
1455 Thabo: I-ca introduce myself, ? Kgomotso ?
1456 (silence)
1457 Nandipha: Eh, ja, choose any area, find as many different polygons as you can (laughs) which, idiot (laughs)
1458 Thabo: plus, she fined me, I said
1459 Nandipha: O.K. actually I've got this lump in my throat and it won't go away
1460 Thabo: What has lump got to do with maths
1461 Nandipha: I can't speak properly, ? to find a pen, so what's a polygon
1462 Lerato: Its a five-sided figure
1463 Nandipha: Bustutlala tu {please don't joke}
1464 Lerato: Come now
1465 Thabo: You can choose, we don't have to make all those
1466 (silence)
1467 Nandipha: Thabo
1468 (silence)
1469 Thabo: Did you see how many squares here
1470 Nandipha: Please can I have a ?
1471 Thabo: O.K lets put it here, O.K lets start, number 3
1472 Nandipha: (reads) choose any area, [find
1473 Thabo: [Nooo, kanna {by the way} we are doing, we got 7 ne, they got 9, oh
1474 Baby
1475 Nandipha: They, we got, we got 6, they got 7
1476 Thabo: And Zandile said, no, its impossible, lets try again
1477 (work on geoboard, silence)
1478 Lerato: what number
1479 Nandipha: Eh, number c
1480 Thabo: (sings and works on geoboard) this is ?
1481 Nandipha: (clears throat) Man, yissis man, this throat, I swear I can't even talk
1482 (silence)
1483 Nandipha: There's something missing there, wait, I've got it on this board
1484 Thabo: No, there's nothing missing
1485 Nandipha: There is, there is, there is, come bring
1486 Thabo: This is only 4, Nandipha
1487 Nandipha: Come bring, exactly, I've got the rest of the ?
1488 Lerato: ?
1489 Thabo: You know ?
1490 Nandipha: The other ones, and then li ee {this one too}
1491 Thabo: there's more
1492 Nandipha: No not there
1493 Thabo: It doesn't matter Nandipha, where
1494 Nandipha: Ja
1495 Thabo: So lets find the 6th, 7th one
1496 Nandipha: Eh
1497 Thabo: No, it, its, a genius
1498 Nandipha: And you there, there's it lesta number mang {which number are we on}, 3 O.K. (laughs)
1499 Thabo: Come, stupid one at work (laughs) give, give here, Nandipha no its 6
1500 Nandipha: 6
1501 Thabo: Mm
1502 Nandipha: ? (laughs) Mara it would be, I don't know what my, what I, what am I doing ?
1503 Lerato: Ja
1504 Nandipha: ?
1505 Lerato: Mm
1506 Nandipha: Sharp, that's it, choose an area, O.K. polygon, are we doing number 3 guys (sings) Lerato,
1507 Lerato: Ja
1508 Nandipha: utlaentsa jwang {How are you going to take it away}

1509 Lerato: Ish
1510 Thabo: (laughs) Nandipha, don't forget, the the the sides there don't have to be with one
1511 Nandipha: first one
1512 Lerato: That's too small
1513 Nandipha: A five-sided figure akere
1514 Lerato: Mm, elastic bands
1515 Nandipha: 1,2,3,4 (laughs) O.K, O.K guys I'm, I'm sorry, bona keisang {look what I'm doing} Oh, it is too
1516 small dunkie, and then, no, ? 1,2, 1, 2, threeee, 4
1517 Thabo: No
1518 Lerato: Ah-eh {it 4 sides
1519 Thabo: {No its, no its 4, not 5 hooks, 4 hooks
1520 Nandipha: keeo {there it is} its {5,
1521 Thabo: {It's 5
1522 Nandipha: another {ne
1523 Lerato: So what's the area of the thing
1524 Nandipha: Its 5 (laughs)
1525 Lerato: Area
1526 Nandipha: It is, its 5
1527 Lerato: Moona {hey man} its 5-sided
1528 Thabo: Area, not the perimeter
1529 Nandipha: Ohhhhh, area, how d'we do area again
1530 Thabo: Area, O.K. we find a square, ne, we know the sides are going to be, no its, we we find a square
1531 here, we find the, the the area of this, and then we find the area of this divide by 2
1532 Nandipha: So that's 4
1533 Thabo: What
1534 Nandipha: The area of this square
1535 Thabo: Its 2½
1536 Nandipha: O.K. its {2½
1537 Thabo: {Lets ask miss, ask miss, phela, this, this also can {be, I think, oh, O.K.
1538 Nandipha: {My throat Thabo
1539 Thabo: Oh, O.K.
1540 Nandipha: ?
1541 Thabo: This has got 5 sides
1542 Lerato: Isn't this 1½
1543 Thabo: Ja, this is 1½
1544 Nandipha: Oh ja ubuwanete {he's telling the truth}
1545 Thabo: Aah, you can also work
1546 Nandipha: (laughs)
1547 Lerato: ?
1548 Nandipha: Hey people, my throat, my throat is, its killing me now (sings) ?
1549 Lerato: This shape is like this ne
1550 Nandipha: Ohh, {no
1551 Lerato: {I'm right ne
1552 Nandipha: nthwe ejwalo {something like that}
1553 Lerato: I'm right
1554 Nandipha: Hae, na aketsebe Lerato {I don't know}
1555 Teacher: Does any other group need more dotted paper, I can bring you each, around, anybody else need
1556 more of this, or think they will need more of this
1557 Lerato: ?
1558 Nandipha: Ja jwanong aotola {now when you find} any, anything what d'you do
1559 Thabo: Eh
1560 Nandipha: (reads) Choose any per, find as many different polygons as
1561 Thabo: Nandipha, you not looking for the perimeter, we looking for the area
1562 Nandipha: Oh, I must say I'm sorry ne
1563 Lerato: Hae, 1, 2, 3, aah we've done this one

1564 *Nandipha: Ke pencil yamang eh {whose pencil is this}*
1565 *Thabo: Its mine, I got it on Friday*
1566 *Nandipha: (laughs) its yours, O.K.*
1567 *Thabo: It was given to me, guess by who*
1568 *Nandipha: Who*
1569 *Lerato: Why*
1570 *Thabo: Mr C (laughs)*
1571 *Nandipha: You've gotta be kidding, Mm-mm Mm-mm, yeah I've got it ? aspro clear (laughs), uh-uh ? its 1,2,3*
1572 *(silence)*
1573 *Nandipha: ? are you gonna talk here or you just gonna laugh at each other*
1574 *Lerato: We talking about ?*
1575 *Nandipha: That's not it, no, that's the [same thing*
1576 *Lerato: [What's the area of that thing*
1577 *Nandipha: Miss, I*
1578 *Teacher: What*
1579 *Nandipha: What do we have to do here*
1580 *Teacher: Where, Nandipha*
1581 *Nandipha: 3, number 3 ne*
1582 *Teacher: Show me*
1583 *Nandipha: number 3 miss*
1584 *Teacher: Show me, what part of number 3*
1585 *Nandipha: The whole of number 3*
1586 *Teacher: O.K. read it for me*
1587 *Nandipha: O.K. (reads) Choose an area, find a, as many as you can which have this area*
1588 *Teacher: O.K. so pick an area, how much d'you wanna pick*
1589 *Nandipha: 5*
1590 *Teacher: 5, fine, O.K. find me a shape that has an area of 5*
1591 *Thabo: There*
1592 *Nandipha: There*
1593 *Teacher: Fine, then there's one shape with an area of 5, draw it onto your dotted paper, find me another one*
1594 *with an area of 5, now that's what they asking, O.K.*
1595 *Thabo: Nandipha, you nee, you needed*
1596 *Teacher: D'you see what they asking*
1597 *Thabo: Professional help bo*
1598 *Nandipha: O.K, O.K*
1599 *Thabo: Cause we've been telling you that's what*
1600 *Teacher: Have they been explaining it to you*
1601 *Thabo: Yes*
1602 *Teacher: An you didn't wanna listen to them*
1603 *Nandipha: No miss, I,I*
1604 *Thabo: She needed professional help*
1605 *Teacher: That doesn't have an area of 5, its got a area of 3*
1606 *Nandipha: Ohh, O.K, O.K*
1607 *Teacher: So maybe, maybe, so keep your 3, keep your 3, now you wanna add another 2 onto it, so just go*
1608 *down another, 2, keep your basic shape, there, and now keep going, find as many as you can with*
1609 *an area of 5*
1610 *Thabo: Oh, area of 5*
1611 *Teacher: But that's the area she chose, O.K. I dunno, maybe you wanna decide as a group*
1612 *Thabo: Yes we*
1613 *Teacher: What area, maybe you don't wanna choose 5, maybe you wanna discuss that*
1614 *Thabo: Lets choose 2*
1615 *Teacher: O.K. but it would be nice if you, if your group chooses the same area because then you can check*
1616 *that you've covered all the shapes, O.K.*
1617 *Thabo: Not the sides, the area, (laughs)*
1618 *Nandipha: Please leave me alone*

1619 *Thabo:* *(laughs)*
1620 *Nandipha:* *(sings) "come on leave me alone, leave me alone, come on baby", I must go drink water, and now*
1621 *Thabo:* *Lets make more*
1622 *Nandipha:* *Yes*
1623 *Lerato:* *Choose a number*
1624 *Nandipha:* *Hae*
1625 *Lerato:* *kaeng {with what} 4,5, 2,1*
1626 *Thabo:* *2, O.K.*
1627 *Nandipha:* *What about nought (laughs)*
1628 *Thabo:* *Eh come, lets choose 1, there's 1, ne, and there's 1, here's the first one*
1629 *Nandipha:* *Its said polygon, polygon, [polygon*
1630 *Lerato:* *[Polygon is any sides*
1631 *Thabo:* *Any sides man*
1632 *Nandipha:* *Oh*
1633 *Thabo:* *(laughs) hey Nandipha*
1634 *Lerato:* *A five-sided figure is a pentagon*
1635 *Thabo:* *(laughs)*
1636 *Nandipha:* *(laughs) O.K., O.K., O.K*
1637 *Thabo:* *Did she have a miscarriage or*
1638 *Nandipha:* *(laughs) O.K. but its funny*
1639 *Lerato:* *Ja*
1640 *(silence)*
1641 *Lerato:* *How d'you draw this*
1642 *(silence)*
1643 *Thabo:* *Nandipha see 9*
1644 *Nandipha:* *Oh, O.K. (laughs), so, I, O.K. there's one like this akere ushebileng wena {what are you looking*
1645 *at} oh (laughs)*
1646 *Thabo:* *Lets share*
1647 *Nandipha:* *Well I did write here, another one*
1648 *Lerato:* *Why*
1649 *(silence)*
1650 *Lerato:* *Ja I, now I've got it, ?*
1651 *Nandipha:* *? (laughs) mara its not funny, Hah-ah ?*
1652 *Lerato:* *?*
1653 *Thabo:* *Nandipha, a, a triangle*
1654 *Nandipha:* *There, ohh, It doesn't look like a triangle to me (laughs) that's not I*
1655 *Lerato:* *Its I, ? lets first try*
1656 *Thabo:* *No, its never gonna work (sings)*
1657 *Lerato:* *(sings)*
1658 *Nandipha:* *How do you do this then, ? sh (laughs) O.K., (sings) du,du,du,du, isn't it boring, O.K*
1659 *(silence)*
1660 *Thabo:* *Ek is moeg {I am tired}*
1661 *Nandipha:* *Ek is lazy {I am lazy}*
1662 *Lerato:* *Nandipha do something its your turn*
1663 *Nandipha:* *Wentseng {what did you do}*
1664 *Thabo:* *The first one*
1665 *Nandipha:* *Ohh*
1666 *Thabo:* *The square (laughs)*
1667 *(silence)*
1668 *Thabo:* *Wetsang {what are you doing} Nandipha no, it doesn't work, that's going to give us different areas*
1669 *Nandipha:* *No it doesn't, sheba, it's one*
1670 *Thabo:* *No, its not*
1671 *Nandipha:* *Oh, oh ja it isn't*
1672 *Thabo:* *Its ½*
1673 *Nandipha:* *Its ½, but then you see, you gonna do the other way*

1674 *Thabo:* *Nandipha, Nandipha, [are you*
1675 *Nandipha:* *[No ema, ema, ema, ema {wait, wait, wait, wait}*
1676 *Thabo:* *using formula or what, if you do it the other way you gonna get this, a square*
1677 *Nandipha:* *Which is one, it is true, but you know what, ne, its just that I divided is into four parts, like so*
1678 *Thabo:* *And what does it give you*
1679 *Nandipha:* *1*
1680 *Thabo:* *A square*
1681 *Nandipha:* *Which is 1*
1682 *Thabo:* *Exactly, but then what does it give you, we not gonna count this, the middle,*
1683 *Nandipha:* *I know, [I know, I know*
1684 *Thabo:* *[we gonna count this*
1685 *(silence)*
1686 *Nandipha:* *Sere yedetse akere {you have made it}*
1687 *(silence)*
1688 *Lerato:* *Hae, Nandipha*
1689 *Thabo:* *(laughs)*
1690 *Nandipha:* *But we taking turns*
1691 *Lerato:* *I don't know why, ?*
1692 *Nandipha:* *Ja ?*
1693 *Lerato:* *My grandmother used to say that whenever we've got a ?*
1694 *Nandipha:* *(laughs) uh-uh that's not 1, that's not 1, that's 1*
1695 *Thabo:* *this is 1, [this is 1*
1696 *Nandipha:* *[Ja*
1697 *Thabo:* *No its not, its 1½*
1698 *Lerato:* *But we've got it [its 1, oh*
1699 *Thabo:* *[its 1 and a, this is 1, ja, a boomerang*
1700 *Nandipha:* *This is one, no its not, is that 1*
1701 *Thabo:* *No, 2*
1702 *Nandipha:* *oho, ukeobutse {I must tell you} hore ke 1 of wat {is this 1 or what?}*
1703 *Thabo:* *no, Nandipha, this is 2*
1704 *Nandipha:* *O.K. its 1, there, across a daar, ne, ne there mona etlamo {it comes here} ne*
1705 *Thabo:* *Its wrong already*
1706 *Nandipha:* *No*
1707 *Thabo:* *Its wrong already*
1708 *Nandipha:* *Its right wena ubowakang {what are you talking about}*
1709 *Thabo:* *O.K listen, any 4 hooks meeting, that's any 4 hooks*
1710 *Nandipha:* *It is right*
1711 *Thabo:* *Its supposed to make a sh, a, V-shape*
1712 *Nandipha:* *A what*
1713 *Teacher:* *(to class) Listen carefully everyone, what you cannot make with elastics on your geoboard you can't*
1714 *actually include, so like for example David has got one where the square's corner is in the middle*
1715 *of the four dots, how you gonna, make that there, O.K. its a problem, O.K. you, you've got certain*
1716 *limitations, [that's why we've given you a geoboard, O.K don't exceed them too much*
1717 *Thabo:* *[You go up, then, no, the other side*
1718 *Teacher:* *For sure if you see that you could do one by exceeding it, write that down, but don't actually*
1719 *[exceed the limitations, O.K.*
1720 *Nandipha:* *[Oh, O.K, ja I understand, ne, swop*
1721 *Thabo:* *Nandipha,I used to*
1722 *Nandipha:* *e Thabo O.K my[brain is not quite working proply {properly} today*
1723 *Lerato:* *[Like this ne*
1724 *Thabo:* *Ja*
1725 *Nandipha:* *Jwang (laughs) etlakebone {How, let me see} Ohh, oh was it that simple, wetse so, utle so, utle so,*
1726 *utle so {first do it like this, then like this, then like this, then like this}*
1727 *Thabo:* *Thank you*
1728 *Nandipha:* *Hey, nee man how can you do those two ?*

1729 Thabo: I show you why
1730 Nandipha: Lets
1731 Lerato: Another thing please Nandipha
1732 Nandipha: Lerato akesobone wetsa e one {I haven't seen you doing one}
1733 Thabo: I did, [I did all, the sheets
1734 Lerato: [I did do it
1735 Thabo: You couldn't even do the first one, on the square, like ?, we did do it, Nandipha, come on, the
1736 elastic ca, can't cross
1737 Nandipha: Oh yeah
1738 Thabo: They shouldn't be able to cross
1739 Lerato: Come lets try another number
1740 Thabo: Le, lets lets lets use our thing, our thing, our, our, our method
1741 Nandipha: [This will give you 1
1742 Thabo: [lets see, 1, 2, 1, 2
1743 Lerato: Hae, Nandipha
1744 Thabo: No, a straight line, ja
1745 Nandipha: Ag nee, I don't know
1746 (silence)
1747 Nandipha: (laughs) ?
1748 (sing and talk to selves)
1749 Thabo: Oh, oh we didn't get this one
1750 Nandipha: We did
1751 Thabo: That, but we didn't get this one, yep, yep, yep
1752 Nandipha: We did
1753 Thabo: Is the same thing
1754 Nandipha: Yes, darling
1755 Thabo: O.K. then we didn't get this one
1756 Nandipha: We did
1757 Thabo: Nandipha, look at my side
1758 Nandipha: Oh my goodness, [Thabo, if you got that
1759 Thabo: [No, we got this, we got this, but we haven't got this, we haven't got this
1760 Nandipha: No wait what's going on here, its like that
1761 Thabo: Le, lets put it this way, (sings) Nandipha here, here, here, here, here the other the other thing, here,
1762 this time its, ish
1763 Lerato: Jaaa
1764 Karin Which one did you have before this one, which is the one before this, and the new one
1765 Thabo: The new one
1766 Nandipha: Come on
1767 Karin The one you just did now, that I just interrupted you
1768 Thabo: No, Nandipha
1769 Nandipha: No, wait, wait, wait, wait, wait, wait, wait
1770 Thabo: No
1771 Nandipha: Oh no, 3 ne
1772 Thabo: [no
1773 Nandipha: [no
1774 Karin You must put numbers on your worksheets cause, so that you'll know afterwards
1775 Nandipha: O.K. miss, here's number 2 here
1776 Lerato: ?
1777 Nandipha: miss, miss,
1778 Lerato: So
1779 Nandipha: ? its like this ? and there, this one goes here akere ja, there, that one goes, [there
1780 Thabo: [No, Nandipha there's
1781 no space over there
1782 Nandipha: Oh, O.K I get you now
1783 Thabo: Shew

1784 *Nandipha: Guys I need to go drink water*
1785 *Thabo: We didn't do the one right angle*
1786 *Nandipha: We did*
1787 *Thabo: Right angle, we didn't, yasamin, 7*
1788 *(silence)*
1789 *Nandipha: Aa-aa-ah what am I doing, what am I doing O.K rubber, rubber*
1790 *Lerato: Like this, ne, 6 this is 6*
1791 *(Thabo sings, someone whistles, silence, Nandipha sings)*
1792 *Nandipha: I've got one, Thabo, tlesa {give it to me}*
1793 *Thabo: You lying*
1794 *Lerato: Lets see yours, Nandipha!*
1795 *Nandipha: (laughs), Please do not disturb the genius at work*
1796 *Thabo: Its wrong, I can tell you now, Nandipha it doesn't work*
1797 *Nandipha: It doesn't give us 1 ne, yeah*
1798 *Thabo: So*
1799 *Nandipha: There*
1800 *Thabo: So the genius thinks*
1801 *Lerato: Come lets change the number*
1802 *Thabo: [illegible] we doing 1*
1803 *Lerato: There's a lot already*
1804 *Nandipha: Awe hape, ? reetse utwena {we must do this again}*
1805 *Thabo: {number b*
1806 *Lerato: {its just that ae? ? there's a lot*
1807 *Thabo: No*
1808 *Nandipha: On the geoboard*
1809 *Lerato: This is 1, this, this, this is 1.*
1810 *(silence)*
1811 *Nandipha: Its wrong*
1812 *Lerato: Where's the big elastic*
1813 *Nandipha: ? stronger*
1814 *(silence)*
1815 *Thabo: Lets choose ano thing the perimeter this time, we looking for the perimeter*
1816 *Nandipha: Miss*
1817 *Teacher: Yes*
1818 *Nandipha: Please can I go to drink water I've got this big lump in my throat*
1819 *Teacher: Pardon, yes O.K*
1820 *Nandipha: Its been there for the whole, long, time*
1821 *Lerato: What about this*
1822 *Thabo: You, you ate the whole chicken again*
1823 *Nandipha: (laughs)*
1824 *Lerato: Its 1*
1825 *(talk to selves, sing)*
1826 *Lerato: How bout this one Thabo*
1827 *Thabo: Perimeter, choose a 2*
1828 *Lerato: 2*
1829 *Thabo: A perimeter of 2, choose*
1830 *Lerato: Or 3*
1831 *Thabo: (reads) choose any perimeter*
1832 *Teacher: O.K. you need to start finishing off because the bell's going to go, so you've got about another four*
1833 *minutes, I'm gonna come around and take in your geoboards, can I just ask you, um take the*
1834 *elastics off your geoboard, O.K, cause what I saw of the last one was some of you had the squares*
1835 *done very nicely, and when I came to give them out to the next group their answers were kind of*
1836 *almost there, O.K, so put the elastics on but don't put it in any kind of nice pattern because*
1837 *otherwise you giving away a pattern for the next group, O.K*
1838 *Thabo: Are we packing up already*



1839 *Teacher:* *O.K. you've got about another two minutes for the geoboard and then I'm taking them in*
1840 *Thabo:* *Here, here, here, here, no, no, no, no*
1841 *Lerato:* *Come lets choose a number for the perimeter and do it*
1842 *Nandipha:* *(yawns) I don't know, miss, [please can I have more dotted paper*
1843 *Lerato:* *[What about 6, or 10, Nandipha, what about 10*
1844 *Teacher:* *Ja, sure*
1845 *Thabo:* *Here's the last one*
1846 *Teacher:* *Here we are, put, pop your elastics on your geoboard please*
1847 *Nandipha:* *? Let me not go away, what you got with that ?, like this, no like this*
1848 *Teacher:* *Are you finished with your geoboard, pop the elastics on please, I think I should get you guys to*
1849 *design some Logo's, you've got some nice designs on these board*
1850 *Nandipha:* *Ne miss, we did*
1851 *Teacher:* *Hey*
1852 *Nandipha:* *We designed for the Std 5's and 6's*
1853 *Thabo:* *Sheba*
1854 *Nandipha:* *Logo*
1855 *Lerato:* *?*
1856 *Nandipha:* *? genius at work, Lerato d'you know that I discovered this one kabonste {let me show you}*
1857 *Thabo:* *Is it, um can you believe it I, I did all the shapes around*
1858 *Nandipha:* *Its a once in a lifetime opportunity*
1859 *Teacher:* *Do I have all the geoboards*
1860 *Lerato:* *? geoboard*
1861 *Nandipha:* *It is, do you know hore I discovered this, hey*
1862 *Lerato:* *(sings)*
1863 *Nandipha:* *Hae man what di {did} I do*
1864 *Lerato:* *(sings)*
1865 *Nandipha:* *Ai my nerves*

END OF SIDE 2, TAPE 2

Day 4: Tuesday 19th October 1993

1866 *Karin* *O.K. people sit down, get your work, and start working*
1867 *(settling down)*
1868 *Nandipha:* *Once again*
1869 *(general chatter)*
1870 *someone:* *Thabo ? compass*
1871 *Thabo:* *Compass*
1872 *someone:* *Compass*
1873 *Karin* *Nandipha, Thabo, sit down and start working*
1874 *Thabo:* *(sings)*
1875 *Nandipha:* *Kademe thwewu e'pen {lend me a pen}, any pen oska {you not} using right now, so I can use this*
1876 *one*
1877 *Karin* *Does anyone need more elastic bands*
1878 *Thabo:* *Yes*
1879 *Nandipha:* *No, these two are there*
1880 *Lerato:* *?*
1881 *Nandipha:* *Don't worry sis, sing man ? (sings) sommer ne genius at work, now genius at work*
1882 *Thabo:* *Snappy genius at work*
1883 *Nandipha:* *That's her*
1884 *Thabo:* *Aaah*
1885 *Nandipha:* *O.K.*
1886 *Thabo:* *Where were we, where were we*
1887 *Nandipha:* *We gwalu number mang {what number are you writing}*
1888 *Thabo:* *we clubbed those*

1889 *Nandipha: (sings)*
1890 *someone: Shhhhh*
1891 *Nandipha: Let's see we've seen those 1,2,3, serious*
1892 *Thabo: You gonna die soon*
1893 *Nandipha: I wish I would like crazy, just on Friday expect a writing*
1894 *(muttering)*
1895 *Thabo: Choose a perimeter*
1896 *Lerato: 5, 5*
1897 *Nandipha: No, I'm not gonna choose that ?*
1898 *Lerato: Choose a number*
1899 *Thabo: 16, and then its easy 16 ne, perimeter, retsa once fela {we'll do it only once}*
1900 *Nandipha: Let me see this*
1901 *Thabo: No*
1902 *Lerato: 16*
1903 *Thabo: (sings), Nee, aretse {lets use} 10*
1904 *Lerato: 10*
1905 *Thabo: 10*
1906 *(silence)*
1907 *Lerato: 9*
1908 *(silence)*
1909 *Nandipha: 1, {no its*
1910 *Thabo: {no its the perimeter*
1911 *Nandipha: O.K. O.K. O.K.*
1912 *Lerato: 4, 5, {6,*
1913 *Thabo: {6,7, {8*
1914 *Nandipha: {8,9,10,11*
1915 *Thabo: Lets do it again*
1916 *Nandipha: 7*
1917 *Thabo: 8, 9, 10, 11, 12*
1918 *Lerato: How this is ?*
1919 *Thabo: this is going to be 4, 4*
1920 *Nandipha: Oh, Lerato, what are you doing*
1921 *Thabo: So, 2,4,6,8 ngiyabonga, ngiyabonga {thank you, thank you} (talks funny, laughs)*
1922 *Nandipha: (laughs)*
1923 *Thabo: aretse 4, aretse 4 {lets use 4}, ne its 4*
1924 *Lerato: 4*
1925 *Nandipha: Lerato lets not draw*
1926 *Lerato: No, we no longer doing that*
1927 *Thabo: What were you drawing (laughs) hey Ngoma*
1928 *Lerato: 4*
1929 *Nandipha: I thought it was one*
1930 *Thabo: 4*
1931 *Lerato: 4, Nandipha, 4*
1932 *Nandipha: ?*
1933 *Lerato: ? on Friday*
1934 *Nandipha: That's 4*
1935 *Lerato: Nandipha, Nandipha,it can't be 4*
1936 *Thabo: Nandipha, its not 4, its 5*
1937 *Teacher: O.K don't forget that you gonna present your findings, O.K when we finished these and we all*
1938 *finished the, worksheets you actually gonna present what it is that you found, how you approached*
1939 *the questions, what you learned etc.. etc...*
1940 *Nandipha: When miss by*
1941 *Teacher: O.K so make sure that the way in which you writing your answers, you gonna be able to re-interpret*
1942 *what you wrote, cause some of you have just got drawings all over the show, when you look at them*
1943 *in two days time you not gonna know what those drawings were about, you need to label your work,*

1944 O.K., here's paper if anybody needs and here's more elastics as well
1945 Lerato: This is 1, its 3 ne, 1, 2
1946 Thabo: Its 5 isn't it, its 3
1947 (silence)
1948 Lerato: A diamond, yes a diamond, 4, its 4, 1,2,3,4
1949 (silence)
1950 Thabo: Mm its ½
1951 Lerato: 1½
1952 Thabo: Mm, this going to be 2
1953 Nandipha: Which one
1954 Thabo: There, this, this, ther, ther, there, no come on eh, lets skip it, the, what, the, the thing, the thing,
1955 the, the hypotenuse is going to be greater than thi, ó
1956 (Nandipha & Lerato talk ?)
1957 Nandipha: Oh how boring
1958 Lerato: Why d'you think like that
1959 Thabo: ish
1960 Lerato: Hae, lets choose another number, 4
1961 (Nandipha & Lerato ?)
1962 Lerato: Ohhh, na, na, na, na
1963 Nandipha: Ja, this is 2 so it must go down
1964 Lerato: ?
1965 Nandipha: Show me a na,
1966 Lerato: ja
1967 Nandipha: Show me a na
1968 Lerato: I thought we doing another thing
1969 Nandipha: Ke la debona {they'll see for themselves}
1970 Thabo: Ne, ne, that thing 5
1971 Lerato: what, ish
1972 Thabo: ...ve did 4 man, so we find, found 2, 4 point, 4 point ai
1973 Lerato: ...is
1974 Nandipha: ...this one ne, this is 4, this is 4, aretse {lets use} 5 here, 5
1975 Thabo: No, come
1976 Nandipha: Which one
1977 Thabo: We are finished with that
1978 Lerato: ?
1979 Teacher: (to class) What number are most of you on please, what, O.K. what number is each group on, your
1980 group
1981 Nandipha: Oh no
1982 Lerato: Number 4
1983 Thabo: We starting 4
1984 Teacher: (to this group) Number
1985 Lerato: 4
1986 Teacher: 4
1987 (teacher asks other groups)
1988 Lerato: How would you explain ?
1989 Teacher: O.K. listen I think what I want to ask you to do is that Thursday we actually start presenting these
1990 things O.K. because the presentations of them will probably take about two periods, O.K. we'll
1991 probably only get two groups done, in a lesson,
1992 Nandipha: [Where miss
1993 Teacher: [O.K. and I don't wanna extend this right into next week
1994 Nandipha: We having a double lesson on Thursday
1995 Thabo: Exactly
1996 Teacher: O.K. well then Thursday's fine to start presenting, it'll be perfect, O.K. if you do not finish, I don't
1997 want you to rush one problem, if you finding a lot on one problem, you finding nice patterns, don't
1998 rush it to go onto the next question, if your group so happens not to finish the last question, I don't

1999 *thiu² its the end of the world, is that O.K., hey, O.K.*
2000 *(turn pages)*
2001 *Nandipha: What are you doing*
2002 *Thabo: Lerato*
2003 *Lerato: Its a quadrilateral*
2004 *Thabo: ? u'4*
2005 *Nandipha: How would you explain to a form 1 student*
2006 *Lerato: Hah*
2007 *Thabo: Nandipha*
2008 *Lerato: But then, you said its wrong, ?*
2009 *Nandipha: It is*
2010 *Thabo: Ja, but here you can see 1*
2011 *Lerato: This is 1*
2012 *Nandipha: This is 1, ne*
2013 *Lerato: its ½, its ½, O.K. akegane {I don't disagree} but if you said something like this, 1, 2, 3, 4, 5, 4,*
2014 *5*
2015 *(silence)*
2016 *Nandipha: That's the last one we did*
2017 *Lerato: But it's wrong*
2018 *(silence)*
2019 *Lerato: Which one is which*
2020 *Thabo: How to work out a complicated area, ne, first you find, u² you try and divide your things into*
2021 *triangles, and sq, rectang,s or squares, ne, are you writing Nandipha*
2022 *Nandipha: Yes*
2023 *Thabo: Where*
2024 *Nandipha: On the sheet, on the sheet, mm, and then*
2025 *Thabo: And then, mm, you work out your, area, of, for the squares, or the rectangles, ja, to work out the*
2026 *area of the, of the, to work out the, the, the area of, of a triangle, you, no yes, cause it will be easy*
2027 *to work out, work out the area of the square and this thing ne, to work out the area, the area of the*
2028 *triangle you count the hooks which the elastic is*
2029 *Nandipha: Hooked to*
2030 *Thabo: Ja, that includes the middle ones, and then*
2031 *Nandipha: Ah, wait, Thabo*
2032 *Thabo: And then*
2033 *Nandipha: Yes*
2034 *Thabo: e.g*
2035 *someone: Sh*
2036 *Nandipha: To (laughs) and then, and then O.K. e.g*
2037 *Thabo: e.g, if the triangle's got 6 hooks*
2038 *Nandipha: The triangle's got 6 hooks, ne*
2039 *Thabo: Ja, you try to make a square that has, or a triangle that has 6 hooks, let, let's see, it works with*
2040 *7, this is ½, this is our 6 ne, this is our 6 ne, its 7, actually, its 7, ne, its 7, we know 6, ja, its 7,*
2041 *so that means its 2, 2uni, 2½ units squared ne, so that's how to make a triangle of 7*
2042 *Nandipha: Come, come, come, come I get it please, please can I do it, please can I do it Thabo, keaokopa {I'm*
2043 *begging you}*
2044 *Thabo: A triangle, Nandipha*
2045 *Nandipha: Yes, yes, yes, wait, that one*
2046 *Thabo: That's 4*
2047 *Nandipha: That's 5, oh, O.K.*
2048 *Thabo: That's not a triangle, Nandipha, that's not a triangle, no, that's not a triangle, that's a*
2049 *Nandipha: O.K, O.K, O.K relax, 1,2,3,4,5,6,7*
2050 *Thabo: Lets slow down, this is 1, ne*
2051 *Nandipha: That's 1*
2052 *Thabo: That's 1*
2053 *Nandipha: That's ½*

2054 Lerato: And this, what is this
2055 Thabo: Oh, no this, ish, its not gonna work
2056 Nandipha: Ag
2057 Thabo: Its not gonna work
2058 Nandipha: So, it doesn't make a ½, [?
2059 Thabo: [You know why, you know why, our thing's not gonna work, our invention,
2060 cause cava hier, ne cava hier {look here}, our, we supposed to get 2½ ne, ne, the, the, the area,
2061 the here's its going to be 1,2, its 2 already, ne, so you can't tell me this is ½ cause you've got 4
2062 things, give us 3
2063 Nandipha: So come we'll make another one
2064 Thabo: Let's see
2065 Nandipha: So, use the example of
2066 Thabo: Lets say, it'll work
2067 Nandipha: 8
2068 Thabo: Let's say it'll work
2069 Lerato: There's a ½, 2½
2070 Thabo: But, but, but, but cav {but look}
2071 Lerato: This is 1 ne
2072 Thabo: Lerato let's go back to our thing, you said count the hooks ne, so that means we gonna have to put
2073 [this
2074 Lerato: [Ja, we must ?
2075 Nandipha: This is 1, ne, this one, O.K., look this and that are 1
2076 Thabo: Its ½
2077 Nandipha: O.K, [but then
2078 Karin [What number you on
2079 Thabo: number, number 4
2080 Nandipha: That's ½ ne
2081 Thabo: That is ½,
2082 Lerato: Miss
2083 Thabo: that so that's add up to 2
2084 Nandipha: And then what's this one
2085 Thabo: Yeow, it is ½, it is ½
2086 Nandipha: Sharp, O.K., yes people
2087 Lerato: Sorry, sorry, sorry ?
2088 Nandipha: That's a quarter, ja
2089 Lerato: Ja, and then you add this, this one this part here miss,
2090 Karin No you, I'm, um listening
2091 Lerato: this part here, here gure etloyetse {so as to make it straight} e straight there
2092 Thabo: So its 5,
2093 Lerato: plus it comes here
2094 Thabo: plus we still got this
2095 Nandipha: So its 2½
2096 Thabo: 2, 2 mona, mona {here} 2 ish
2097 Nandipha: How much are we getting
2098 Thabo: Ish lets say it don't work with
2099 Nandipha: How much are we getting
2100 Thabo: Ish and we already there man, aekona {oh no} a flop hey
2101 (silence)
2102 Nandipha: Rectangle, make a rectangle
2103 Thabo: 8
2104 Nandipha: Ja, there its made
2105 Thabo: This
2106 (silence)
2107 Nandipha: How much is that
2108 Thabo: 7, is that 7, let's see, let's see let's lets count again

2109 *Nandipha:* O.K.
2110 *Thabo:* Let, let's count 7 again, these are different angles, lets count 7 again and see if it works, this is 1,
2111 this is ½,
2112 *Nandipha:* That's 6
2113 *Thabo:* it doesn't work
2114 *Lerato:* This is a ½
2115 *Thabo:* It doesn't work no, this time it will give us 3, ish, kawu {china} shucks, is that 8
2116 *Nandipha:* Yes
2117 *Thabo:* Nandipha its 9
2118 *Nandipha:* 1,2,3,4,5 O.K. here its 1,2,3,4,5,6,7,8, it is 8 people, 1,2 there's 5, 3, 4, 8
2119 *Thabo:* What about this
2120 *Lerato:* ?
2121 *(silence)*
2122 *Thabo:* How much was the area of 8
2123 *Nandipha:* It is, it is, 8 again, 1,2,3,4 [its 8
2124 *Thabo:* [It was
2125 *Nandipha:* Its not, ? 3, this is 1, ne, this is, uh
2126 *Teacher:* O.K. listen everyone, I need to also tell you, that on Friday you going to be writing a maths test,
2127 O.K., your maths test is going to be on all the algebra we've done so far, it's just very similar to
2128 what we've, everything we've covered so far, O.K., factorisation, geometry, just a, test on
2129 everything again
2130 *Nandipha:* Mmm
2131 *Lerato:* The method doesn't work
2132 *Thabo:* Our method doesn't work anymore, let's check, lets go back
2133 *Nandipha:* (sings)
2134 *Thabo:* A flop, before the due date (laughs), we flopped me
2135 *Nandipha:* So, we want to work it out, find and divide into triangles and squares. Work out the area of squares
2136 and triangles, count how many hooks the elastic, is hooked on to and the inside, e.g.
2137 *Thabo:* Ish, we flopped
2138 *Nandipha:* So what method can we use
2139 *Thabo:* We flopped
2140 *Nandipha:* If we have
2141 *Thabo:* We flopped
2142 *Nandipha:* If we have an elastic that's big we get a [big triangle
2143 *Lerato:* [Come, come guys, lets leave it, tabatsa {its no use} here
2144 we must do a, a square, or what like, understand, like count the hooks, and then you do a square
2145 like
2146 *Thabo:* We already flopped for our presentation
2147 *Nandipha:* No, we can always do this that we know already
2148 *Thabo:* Which one, ish,
2149 *Nandipha:* We got this
2150 *Thabo:* It doesn't work
2151 *Nandipha:* Eh, Lerato
2152 *Thabo:* We flopped, but howcome, it works with the oth, it worked with the others, maybe the geoboard is
2153 not balanced
2154 *Nandipha:* Thabo, stop smiling
2155 *Thabo:* We flopped (sings) we flopped, ja, nee, how can we flop like this, bit see with some things it
2156 worked, ne, check ne
2157 *Nandipha:* (laughs) Some days it were our day, today uh-uh, wait, we had some, what did we discover last
2158 Friday
2159 *Thabo:* We discovered this thing, ne, the, all the triangle what, what, [now it doesn't want to work
2160 anymore
2161 *Teacher:* [O.K. listen for today I'm gonna give
2162 you a little bit of homework, [O.K
2163 *Nandipha:* [Ah miss

2164 Teacher: I want you to go on, on your own, and think about, the next few questions, whatever you haven't
2165 done, [O.K.
2166 Nandipha: [Think about them
2167 Teacher: Think about them, jot down some ideas, and then when we come together tomorrow you can solidify
2168 them as a group, O.K. so I want a little bit of individual work at home, O.K. cause we not,
2169 otherwise we not going to get through this worksheet, and I think its a shame because the last
2170 question is quite nice, O.K.
2171 (general comments)
2172 Teacher: Yes so you've got homework, first time this week, don't moan about it, O.K.
2173 (silence)
2174 Thabo: Flop magazi {brothers} so-o-o
2175 Nandipha: Ah man what did we discover last Friday
2176 Teacher: Come, which number are you on
2177 Nandipha: Number 4 miss
2178 Teacher: Which one, b
2179 Nandipha: number a
2180 Teacher: a, (reads) discuss with your partner and write down how you would explain to a Form 1 student how
2181 to work out the area, have you written it down in your book
2182 Thabo: Ja
2183 Nandipha: Yes here
2184 Teacher: Oh, O.K
2185 Thabo: We have, but the thing, eh, our
2186 Lerato: Its the method
2187 Thabo: The method doesn't work anymore
2188 Teacher: The method doesn't work
2189 Thabo: Yes, the method we showed you
2190 Teacher: What method did you say
2191 Thabo: Of triangles and
2192 Teacher: Explain the method to me
2193 Thabo: You know, we shown you one, how to find, [uh
2194 Nandipha: A shape
2195 Thabo: A area of a thing, of a triangle, so but now it doesn't work anymore
2196 Teacher: What doesn't work, how did you say that they must do it
2197 Nandipha: We said ne, eh, first you divide it miss into triangles, for instance ja, if we got 6 hooks ne, try to
2198 make a square with 6 hooks, so you've got 6 hooks on a triangle, and so you've got 6 hooks on a
2199 triangle
2200 Teacher: Oh, O.K. you still using that
2201 Nandipha: And then, then
2202 Thabo: But it flopped
2203 Nandipha: But it doesn't work
2204 Teacher: It flopped, well then write, where, on, on, on what, when did you find out that it flopped, and how
2205 did you find out that it flopped, write that, its a va, its a valuable finding, O.K, and then write to
2206 me an alternative method for the students
2207 Nandipha: We can't find an alternative
2208 Thabo: (laughs)
2209 Teacher: Come on, you've answered all the questions beforehand, [don't tell me you can't find an
2210 Thabo: [yeah, we have, we
2211 Teacher: alternative method you've, you've answered all the [questions and you did it correctly
2212 Thabo: [we
2213 Teacher: Look here, don't tell me you didn't have a method to do all of this
2214 Nandipha: [But miss, we
2215 Thabo: [We did, [but, but
2216 Teacher: [Its perfect
2217 Thabo: now when we want to conclude, the thing doesn't work anymore
2218 Nandipha: And you can't [explain

2219 *Teacher:* *[Your conclusion*
2220 *Thabo:* *[Yes*
2221 *Nandipha:* *[to a Form 2 student that you akere discovered an invention which was a flop so you can't work it*
2222 *out*
2223 *Teacher:* *O.K. now what I want you to do, and this is something that's valuable to learn, when you discover*
2224 *an invention, and it flops, its still a valuable process, the fact that you thought you came up with*
2225 *something, and then you disproved it, mathematicians do it all the time, they come up with theories,*
2226 *they prove it, and then some guy comes a hundred years later and he disproves it, and both were*
2227 *valid, the one guy tried, and the other guy disproved it, that is what maths is about, O.K. so don't*
2228 *feel that it was a flop, you failed, you didn't fail, O.K. the fact that you found your error in your*
2229 *theory means that you succeeded, its fine, O.K, but now you have to give me another way that you*
2230 *actually found these, if your method flopped, how else did you find all of these areas*
2231 *Nandipha:* *You know why miss, because these one's were simpler than the other ones, you just look and then*
2232 *you know, O.K, I divide this into this and this and this*
2233 *Teacher:* *But write that, write that*
2234 *Nandipha:* *But miss uh, uh, you can't*
2235 *Teacher:* *How did you do this one, this one wasn't so simple*
2236 *Nandipha:* *But miss you can't teach your form 2, form 1 student look divide like this, like this, like this and you*
2237 *should get your area, fine*
2238 *Teacher:* *Why can't you*
2239 *Nandipha:* *Because miss, they won't understand, they, they won't, I'm telling you they won't*
2240 *Teacher:* *So make it simpler, explain it simpler*
2241 *(Thabo laughs)*
2242 *Nandipha:* *(laughs) he's laughing, O.K. you can say, miss I don't know*
2243 *Thabo:* *You can say*
2244 *Teacher:* *No, but you not trying now, but you do know, you know very well how to do it*
2245 *Nandipha:* *But miss, you*
2246 *Teacher:* *Cause I've seen you do it, and I've seen you explaining perfectly to me*
2247 *Thabo:* *O.K let lets do it like this*
2248 *Nandipha:* *O.K., miss, but you know what miss, its not, its not a simple process like the one's that we had here*
2249 *Teacher:* *O.K. what was the process for those one's*
2250 *Nandipha:* *We did that*
2251 *Teacher:* *You didn't do that, did you add these two blocks together, Hm, how did you do this one, did you*
2252 *use a subtraction method*
2253 *Nandipha:* *No*
2254 *Thabo:* *No*
2255 *Teacher:* *No, how did you do it*
2256 *(brief shuffle)*
2257 *Thabo:* *O.K*
2258 *Nandipha:* *O.K*
2259 *Thabo:* *Wow*
2260 *Nandipha:* *There, its right*
2261 *Thabo:* *No*
2262 *Teacher:* *No*
2263 *Nandipha:* *This one was 1 miss*
2264 *Teacher:* *I don't see any full block, not one*
2265 *Thabo:* *O.K*
2266 *Nandipha:* *Oh, O.K, O.K*
2267 *Thabo:* *N. Nandipha, co come here, this ½*
2268 *Nandipha:* *Miss we divided into halves*
2269 *Thabo:* *That's ½*
2270 *Nandipha:* *That's ½*
2271 *Thabo:* *And then 4,4,4*
2272 *Teacher:* *I don't see any halves either*
2273 *Thabo:* *That's a ½ miss, that's a ½*

2274 *Nandipha:* That's a, but miss
2275 *Teacher:* That's a ½, I said I don't see any other half
2276 *Thabo:* Yes, Nandipha, give, give
2277 *Nandipha:* O.K, O.K, O.K, O.K, O.K,
2278 *Thabo:* Eh, si, since we know that uh, if, uh, if we have to put a triangle like that, the area is going to be
2279 4, its going to be I mean 1, as you can see
2280 *Teacher:* I, no, I don't agree that the area's 1
2281 *That :* You ca, you don't,
2282 *Teacher:* No
2283 *Thabo:* Look here, it is, it is miss because
2284 *Teacher:* Oh yes, O.K
2285 *Thabo:* Its going to be ½ and ½
2286 *Nandipha:* Yes
2287 *Thabo:* You see
2288 *Teacher:* Yes, yes, quite right, yes, you are right there
2289 *Teacher:* ha-ha
2290 *Thabo:* O.K there, there, there, did you see we used 4 hooks, there you go again, there, there, so this is
2291 our 1
2292 *Nandipha:* Ohhhh,
2293 *Teacher:* You, so you still on your theory that if its got 4 hooks
2294 *Thabo:* ja
2295 *Teacher:* Then its 1, but now you've just told me that your theory flopped
2296 *Thabo:* Yes, well its that, you know, like, ai, like, miss we just showing how we got the area of this, so
2297 *Teacher:* O.K but I want you to prove to me, I want you to prove to me that this one that you've just shown
2298 me is 1, I want you to prove to me that the area of 4 hooks is 1, here I can see the area of 4 hooks
2299 is 1, O.K, here I can see that the area of, no O.K. that would be 5 hooks, uuuh, I want you to
2300 prove to me that the area of those 4 hooks is 1
2301 *Nandipha:* Miss, can I prove it to you
2302 *Teacher:* (to someone else) O.K. I'm coming in a sec
2303 *Nandipha:* O.K. this is definitely ½ right
2304 *Teacher:* Yes
2305 *Nandipha:* We know this is ½, so, so you left with 2 ¼'s, 2 ⅛'s, this is a ⅛, here, because
2306 *Teacher:* No, I don't agree
2307 *Nandipha:* O.K, Thabo, ja
2308 *Thabo:* But as, as you can see its, its less than 1, O.K. this
2309 *Teacher:* Ja, I can see its less than 1, its less that ½ yes,
2310 *Thabo:* Less than ½
2311 *Teacher:* We don know how much less, it could be a third, it could be a fifth, it could be a tenth, three tenths
2312 *Nandipha:* It could be 0
2313 *Teacher:* or a fifteenth
2314 *Lerato:* So miss
2315 *Thabo:* So, from here, from here, here it is, there then, we can see its ½ because the area
2316 *Teacher:* Aaah very good, now we getting somewhere, you've proven to me that that parallelogram is [1,
2317 *Thabo:* [], yes
2318 *Teacher:* And now we know that the diagonal of a parallelogram cuts it in half, O.K. now I'm convinced, so
2319 now I agree with you that that is 2
2320 *Nandipha:* Yes
2321 *Teacher:* Good
2322 *Nandipha:* That's 1 miss, not 2, 1, you said 2
2323 *Teacher:* Ja, that there, is a whole thing
2324 *Nandipha:* Its 1
2325 *Teacher:* Is 1, yes
2326 *Nandipha:* Yes
2327 *Thabo:* Yeah
2328 *Teacher:* Yes, ½ and ½ is 1, and now is this congruent to this

2329 *Nandipha: Yes*
2330 *Teacher: I dunno, they don't look [congruent to me*
2331 *Thabo: [n, n, not not congruent, but,*
2332 *Teacher: So now I'm still, O.K, I'm convinced that, the area of these 4 together is 1, 4 hooks is 1, I'm*
2333 *convinced here, I'm convinced that these 4 hooks is 1, but I'm still not convinced [by this one*
2334 *Thabo: [O.K, O.K*
2335 *Teacher: Cause this is not the same as either of these shapes*
2336 *Thabo: Look at this miss, th, this, let's say this angle is this angle, and then those angles are, lets put like*
2337 *this, ne, this angle is this angle, and this angle is this angle*
2338 *Teacher: But this isn't the same, because this here was*
2339 *Thabo: No, lets say, lets say, [O.K, this points, O.K the points*
2340 *Teacher: [3*
2341 *Thabo: Lets say the points, no, its*
2342 *Teacher: Yes, yes, yes, now that looks congruent, yes, yes, yes, yes, now I'm happy with congruent, now I*
2343 *want you to show me that the area of this one here is also 1*
2344 *Thabo: O.K*
2345 *Teacher: O.K think abou, I'm gonna go and help that other group, I'm gonna come back*
2346 *Nandipha: How*
2347 *Thabo: Come on, come I help, eow, O.K here*
2348 *(inaudible, laughing)*
2349 *Thabo: O.K. cava, verstaana hier ne {look, understand here}*
2350 *Lerato: Lets say this one ne, and this we bring here, and this we bring it here, and this [bell goes]*
2351 *Thabo: We writing a test*
2352 *Lerato: We writing a test ne,*
2353 *Thabo: We*
2354 *Nandipha: We must hurry*
2355 *Teacher: O.K listen I need your geoboards please*
2356 *Nandipha: We writing a test miss*
2357 *Thabo: Nandipha, take it there*

END OF SIDE 1, TAPE 3

Day 5: Wednesday 20th October 1993

2358 *Teacher: O.K everyone, sit down, take out your books we've got a lot of work to do today*
2359 *(general chatter)*
2360 *Nandipha: ? 4 by 4 (laughs)*
2361 *Thabo: Funny*
2362 *Lerato: Don't be a stomach ache*
2363 *Nandipha: Initially*
2364 *Lerato: Ke osi Emily {its Emily}*
2365 *Nandipha: Aah-aah leya mhluwela {leave her alone}, it wasn't her*
2366 *Teacher: Listen everyone, sit down, Nandipha*
2367 *Nandipha: Sorry miss*
2368 *Teacher: There two teachers, so you supposed to have gone through the rest of the worksheet on your own*
2369 *at home and thought about it*
2370 *Thabo: Mm*
2371 *Nandipha: Yes*
2372 *Teacher: O.K. what you supposed to do now is trying to finish the worksheet, O.K, if you don't get finished*
2373 *its not the end of the world but you should at least try, O.K. you also have to decide how you are*
2374 *going to do your presentation tomorrow, O.K. When you are finished your presentation, you are*
2375 *going to hand in your books but I'm only gonna take them in on Friday because you have a test, so*
2376 *you'll need to keep your maths books*
2377 *Nandipha: Miss*
2378 *Teacher: On Friday, when you write your test you are gonna hand your books in, at the start, so as you come*

2379 in to class you gonna put your books down here, and then I'm gonna hand out the test, on Friday
2380 someone: Which books
2381 Teacher: These books with these in it, O.K.
2382 someone: For a book mark
2383 Teacher: It will count as a book mark, but I'm going to be marking this specifically, O.K, I will page back
2384 to check that you've done other work, but I will be marking this specifically, you'll be getting a mark
2385 out of, say 15 or 20, for this, O.K so today you have to basically finish off, you have to tie up all
2386 your loose ends, you have to give your books headings, etc., you have to decide in your group how
2387 you going to be presenting, O.K., so don't waste time today, you don't really have the time to waste
2388 Nandipha: O.K.
2389 Thabo: Where were we, O.K. ish, our triangle, no its works, come leave the boarding house kos alone, its
2390 healthy, why d'you think I'm so big
2391 Lerato: ?
2392 Nandipha: ne lenama {it was the meat} ?
2393 Thabo: ? she can't cook
2394 Lerato: What's there
2395 Nandipha: The triangle, where's my pens, I told you hore
2396 Thabo: Mm-mm
2397 Nandipha: (laughs) who's got, kabe ke kudesa {I should have pretended that I'm sick}
2398 Thabo: Na nona {hey man} I don't want to drink anything medicine
2399 Nandipha: Kodisa {lend me}, batloetsa {are they going to do it} 1,2,3,4, 6,7
2400 Thabo: This 7, 1 ne
2401 Nandipha: No, I'm counting the hooks
2402 Thabo: Its 6, Nandipha
2403 Nandipha: Hey, the hooks
2404 Teacher: (to another group) what's going on here, hey, excuse me, we not discussing what's going on in the
2405 boarding house now
2406 Thabo: (laughs) tell them miss
2407 Nandipha: Boarding house kos is kicking them like stones
2408 Thabo: 1,2,3,4, this is 2, so it works, it works
2409 Nandipha: It works, the method yamaobane {of yesterday}
2410 Lerato: Oh, so, so, so, if you make a triangle here, and then draw another
2411 Thabo: Yeah
2412 Lerato: Another, congruent here
2413 Nandipha: Eh
2414 Thabo: Ja
2415 Nandipha: Eh ?, what next
2416 Thabo: So, so
2417 Lerato: someone utswere kemala {someone's got a stomach ache}
2418 Nandipha: (laughs) another one ek se, aahh
2419 Lerato: Wastha {she's burning}
2420 Nandipha: Shrr
2421 Thabo: She went to the wrong Miss \ (laughs)
2422 Nandipha: They dying, ne, they dying there
2423 Lerato: Maobane {yesterday} ?
2424 Nandipha: Mara watsi ba tshilo hore {but they said that - } ?
2425 Lerato: O.K, accident (?)
2426 Thabo: Did you go to the doctor in the afternoon
2427 Nandipha: Who me
2428 Thabo: So Mr F went around looking for people who are sick
2429 Nandipha: I don't know
2430 Lerato: ?
2431 Nandipha: Aah I'm not coming back to the boarding house next yearorata di'bacon {I like bacon}
2432 Lerato: Li Lizo wakula {Lizo is sick}
2433 Nandipha: Wetsang {what is he doing}

2434 Lerato: Otswere kenala, {He's sick in the stomach}, u'weaki ne, Aah shame
2435 Nandipha: Wabona hala {You can see it}
2436 Lerato: Wabona hala hore u'weak hatsebe ? {you can see that he's feeling weak}
2437 Teacher: You feeling O.K. are you feeling O.K
2438 Lerato: ?
2439 Thabo: Nandipha, make me a grave (laughs)
2440 Lerato: After we find the perimeter
2441 Thabo; The perimeter
2442 Teacher: O.K. listen carefully, I also just wanna tell, for tomorrow, what I'll try and do is I'll organise an
2443 overhead and I'll photocopy the dots onto it, so that when you do your presentation, you can
2444 Nandipha: [Draw
2445 Teacher: [Show, so you will have a dotted, like just like your dotted paper, O.K. on an overhead, and I'll
2446 give you overhead pens
2447 Lerato: How d'you find the perimeter,1,2,3,4
2448 Nandipha: Guys, akebone niks {I don't see anything} ne unless [this is found
2449 Thabo: [O.K,O.K, lets mona, they are difficult de papiri tseho {these worksheets are difficult} this can't be
2450 the same as this, check, ooh, ooh, yeah
2451 Lerato: Its, its, this one's like this one
2452 Thabo: Yes, this
2453 Nandipha: Eh, eh
2454 Thabo: That means that the perimeter's tsahona di wrong {the perimeters are wrong}
2455 Nandipha: Uh-uh Thabo
2456 Thabo: One
2457 Nandipha: Which means hore retswa netse hogotlela ko number one {we must go back to number one}
2458 Thabo: No, No, O.K, O.K so then, otloreng {what are you going to say}
2459 Nandipha: Ai Thabo otsore kimala hape {he's sick in the stomach again}
2460 (all laugh)
2461 Lerato: Ja, you can giggle
2462 Thabo: O.K. ja, there
2463 Lerato: And we, plus we know that, that this length ne, the square what, this length squared plus this length
2464 squared is equal to this length, the square root of it, so
2465 Nandipha: ?
2466 Thabo: Ooh then, then, that means we don't have to work out this one, we just say 1 squared plus 4
2467 squared, its, its 17, 17, ne
2468 Lerato: Ja
2469 Nandipha: Ja
2470 Thabo: 17, this is 17 squared
2471 Nandipha: Ah, ja the square
2472 Thabo: 17 [squared ne, 17
2473 Lerato: [The square root of 17
2474 Nandipha: How
2475 Thabo: I mean the square root of 17
2476 Nandipha: ?
2477 Thabo: Nandipha
2478 Lerato: Nandipha, guys, kitlo kupa hore llntsware kenala lelona {I'm asking you also to have stomach
2479 aches} ?
2480 Nandipha: serious ko boarding house
2481 Lerato: ?
2482 Thabo: Oh, so, so, so, so, so, only it works with right angle uh [triangles
2483 Nandipha: [triangles
2484 Thabo: So but complicated, complicated, the perimeters, cause
2485 Lerato: Not only triangles, lets do something else, lets try something else
2486 Thabo: Eh, oh perimeter complicated, complicated shape
2487 Nandipha: ?
2488 Thabo: O.K. so, so, so, so, so, so lets say, lets say

2489 *Lerato:* This is right
2490 *Thabo:* So, so
2491 *Lerato:* And this is a right angle, right too, so its
2492 *Thabo:* Ja, you can always convert it
2493 *Nandipha:* What are you talking about
2494 *Thabo:* Nandipha, stop sleeping, Lerato here, here it is no, ne, and then you say, ne, you find the perimeter,
2495 its 6, 4, its 4 ne, and then you this is 3, 2, that's, its 2 right, its 2,2, so its
2496 *Nandipha:* Wait, 1, 1, I also know how it is, [its 1
2497 *Thabo:* [Its 4, its 4,4 eh, it equals to 8 squared, this is 8 squared, we
2498 always divide our thing to, if its complicated we can always try to divide it into squares and right
2499 angled triangles
2500 *Nandipha:* Ja, ja, that I understand you know
2501 *Thabo:* So, then this is gonna be 1 plus 1 squared, that means that this going to be 1, 2 squared, ne, plus
2502 1 squared, actually why, why
2503 *Lerato:* This one's the hypotenuse, akere
2504 *Thabo:* Yeah
2505 *Lerato:* This is 1
2506 *Thabo:* Ja
2507 *Lerato:* So its 1,1, so, so the length of this one is the square root of [2
2508 *Thabo:* [2 ja, so Nandipha,like,
2509 *Nandipha:* ? this one is 2 like, I dunno kitope lekae hore {where I got it} so what do I write
2510 *Thabo:* Em
2511 *(silence)*
2512 *Thabo:* Nandipha, why are you waiting for this exercise
2513 *Lerato:* Write here, obalala {write here}
2514 *Nandipha:* What am I supposed to write guys
2515 *Thabo:* Can, ja, complicated
2516 *Lerato:* perimeters
2517 *Thabo:* perimeters, no, ja, shapes
2518 *Nandipha:* What about that
2519 *Teacher:* (to another group) O.K. think about this when you hand this in, and I mark it, if its not pasted in,
2520 I'm not gonna know what it is, O.K. so [paste it in, put headings, ...
2521 *Lerato:* [You divide it into
2522 *Thabo:* Right angle triangles
2523 *Lerato:* Right angle triangles, [and squares
2524 *Thabo:* [and squares
2525 *Nandipha:* (writes) Divide in, Uh-huh
2526 *Lerato:* And then, work out the, the, the length of the hypotenuse, and you know, you know that the
2527 perimeter of the square is 1,1,1,1
2528 *Nandipha:* Nuse, nuse, nuse
2529 *Lerato:* What's nuse
2530 *Nandipha:* Hypotenuse
2531 *Thabo:* O.K. we already on number 5 you know, (reads) if you have fixed a perimeter (sic), which shape
2532 give you the bigggest area
2533 *Nandipha:* (continues reading) For this question you do not need to only consider shapes on your geoboard,
2534 you may use any instruments and equipment you like, huh
2535 *Thabo:* If you have a fixed perimeter, which shapes give you the biggest area, the, the thing
2536 *Nandipha:* The triangles, the triangles, wait a minute
2537 *Thabo:* The squares
2538 *Nandipha:* The squares, the squares
2539 *Thabo:* Because, then you use the whole thing like this
2540 *Lerato:* And here, and here
2541 *Nandipha:* Ja, and here, ja, like that
2542 *(silence)*
2543 *Lerato:* Nagaletse hobona ? {I want to see ?}

2489 Lerato: This is right
2490 Thabo: So, so
2491 Lerato: And this is a right angle, right too, so i-
2492 Thabo: Ja, you can always convert it
2493 Nandipha: What are you talking about
2494 Thabo: Nandipha, stop sleeping, Lerato here, here it is ne, ne, and then you say, ne, you find the perimeter,
2495 its 6, 4, its 4 ne, and then you this is 2, 2, that's, its 2 right, its 2,2, so its
2496 Nandipha: Wait, I, I, I also know how it is, [its 1
2497 Thabo: [its 4, its 4,4 eh, it equals to 8 squared, this is 8 squared, we
2498 always divide our thing to, if its complicated we can always try to divide it into squares and right
2499 angled triangles
2500 Nandipha: Ja, ja, that I understand you know
2501 Thabo: So, then this is gonna be 1 plus 1 squared, that means that this going to be 1, 2 squared, ne, plus
2502 1 squared, actually why, why
2503 Lerato: This one's the hypotenuse, akere
2504 Thabo: Yeah
2505 Lerato: This is 1
2506 Thabo: Ja
2507 Lerato: So its 1,1, so, so the length of this one is the square root of [2
2508 Thabo: [2 ja, so Nandi[,s,like,
2509 Nandipha: ? this one is 2 like, I dunno kitope lekae lore {where I got it} so what do I write
2510 Thabo: Em
2511 (silence)
2512 Thabo: Nandipha, why are you waiting for this exercise
2513 Lerato: Write here, obalala {write here}
2514 Nandipha: What am I supposed to write guys
2515 Thabo: Can, ja, complicated
2516 Lerato: perimeters
2517 Thabo: perimeters, no, ja, shapes
2518 Nandipha: What about that
2519 Teacher: (to another group) O.K. think about this when you hand this in, and I mark it, if its not pasted in,
2520 I'm not gonna know what it is, O.K. so [paste it in, put headings, ...
2521 Lerato: [You divide it into
2522 Thabo: Right angle triangles
2523 Lerato: Right angle triangles, [and squares
2524 Thabo: [and squares
2525 Nandipha: (writes) Divide in, Uh-huh
2526 Lerato: And then, work out the, the, the length of the hypotenuse, and you know, you know that the
2527 perimeter of the square is 1,1,1,1
2528 Nandipha: Nuse, nuse, nuse
2529 Lerato: What's nuse
2530 Nandipha: Hypotenuse
2531 Thabo: O.K. we already on number 5 you know, (reads) if you have fixed a perimeter (sic), which shape
2532 give you the bigggest area
2533 Nandipha: (continues reading) For this question you do not need to only consider shapes on your geoboard,
2534 you may use any instruments and equipment you like, huh
2535 Thabo: If you have a fixed perimeter, which shapes give you the biggest area, the, the thing
2536 Nandipha: The triangles, the triangles, wait a minute
2537 Thabo: The squares
2538 Nandipha: The squares, the squares
2539 Thabo: Because, then you use the whole thing like this
2540 Lerato: And here, and here
2541 Nandipha: Ja, and here, ja, like that
2542 (silence)
2543 Lerato: Nagaletse hobona ? {I want to see ?}

2544 *Nandipha: Aah utlomona uskawara {you'll see him, don't worry}*
2545 *Thabo: You need also*
2546 *Nandipha: Waitse, {you know} (laughs)*
2547 *Lerato: Lebone ? ko chapel hore waphegela {he's defending himself} ?*
2548 *Nandipha: I think we should boycott, plus the ? we got to this morning for breakfast, uh-uh, boycott, boycott*
2549 *Lerato: ?*
2550 *Nandipha: ?*
2551 *Lerato: Nandipha, uya post office {you must go to the post office} ?*
2552 *Nandipha: I need someone to take me there you see*
2553 *Thabo: Why don't you ask Mr F*
2554 *Nandipha: But I should*
2555 *Thabo: I us, I used always to go to Mr B ne, ? now I go to Mr F*
2556 *Teacher: O.K listen everyone, what you need is you need, is you need, for your handing in O.K, when you*
2557 *hand in your books,O.K, if you've done some stuff roughly, that's fine, O.K. you must keep your*
2558 *rough work, but if you writing up for your presentation, O.K. you need to basically structure for me,*
2559 *question 1, O.K., what did you discover, what was your answer, what are the pictures which relate*
2560 *to that answer, O.K., you could even refer to your rough worksheet there, O.K., but say for instance*
2561 *you did twenty rough work drawings, O.K. and actually out of those you saw that there five of them*
2562 *gave you a pattern and the rest were irrelevant then you can redo those five, you might need some*
2563 *more dotted paper, O.K but I basically need each and every question, answered, O.K. in how, what*
2564 *you discovered, what you did, and I need that done individually, O.K. so what I'm saying is, it can't*
2565 *just be your rough work, O.K. that you've given headings to, I mean I think that's fine, but I need*
2566 *a proper written out question 1, question 2, question 3, question 4, question 5, O.K., so we need*
2567 *it done a little bit more formally, O.K., You gonna have to that for homework today, you not gonna*
2568 *get it done in class, you can start, but I don't know, its gonna take you quite a bit of time, so I think*
2569 *you should get started*
2570 *Nandipha: Oooh*
2571 *Lerato: We klaar ne*
2572 *Nandipha: Yep*
2573 *Thabo: (sings)*
2574 *Nandipha: Finally*
2575 *Thabo: (sings)*
2576 *Lerato: how, your method, its right, ne*
2577 *Thabo: (sings) yeah*
2578 *Nandipha: (sings) "Yeaho baby", O.K., O.K., O.K*
2579 *Thabo: Okidok*
2580 *Nandipha: Ja, {ets see if this works, ja*
2581 *Thabo: Genius at work*
2582 *Nandipha: Yes, yes, yes, thank you.*
2583 *Thabo: I mean {genius*
2584 *Nandipha: [Autographs later*
2585 *Thabo: Nandipha genius, ne, Nandipha genius*
2586 *Nandipha: (laughs)*
2587 *Thabo: Hae man Nandipha, check*
2588 *Lerato: Huh, can I see those things ?*
2589 *Nandipha: What things, you also genius*
2590 *Thabo: Say you mean it*
2591 *Nandipha: I mean it ?*
2592 *Thabo: Gimme that one (sings)*
2593 *Nandipha: First question*
2594 *Thabo: (sings)*
2595 *Nandipha: (laughs) Hey you know I want to sneeze*
2596 *Thabo: Come, come get out*
2597 *Teacher: How many more of you need dotted paper*
2598 *Thabo: Come now you*

2599 Nandipha: (sneezes)
2600 Lerato: bless you, say thank you
2601 Nandipha: Did you say bless you
2602 Lerato: Yes
2603 Nandipha: Thank you, I must have something to clean my nose
2604 Thabo: (laughs) Its the boarding house coffee
2605 Nandipha: (laughs), (sneezes again)
2606 Lerato: Bless you
2607 Nandipha: Thank you, is there pollen around here or something
2608 Thabo: (sings)
2609 Teacher: Who alse needs more, cause then I need to copy, this is my last one
2610 Nandipha: Miss I think I also need more
2611 Thabo: Ai, [Nandipha, what for
2612 Lerato: [Oh Nandipha
2613 Teacher: Its one per person for your neat one, because on your neat one you not gonna be copying out every
2614 drawing that you ever made on your geoboard, you gonna be copying out the relevant ones, O.K.
2615 you've done a lot of rough work, and now what you gonna do is from that rough work you gonna
2616 extract what was relevant, what, what, the drawings which made you come up with your discoveries,
2617 O.K., you not gonna show me everything you ever made on your geoboard
2618 s!heone: Miss, so must we have the neat work for Friday
2619 Teacher: Yes, yes, that's right, that's gonna be handed in on Friday, O.K, but I suggest you do it today,
2620 because you gonna need to do it anyway for your presentation, it'll help you structure it, O.K, I just
2621 wanna photocopy more dotted paper, then I can give you each one
2622 (inaudible conversation in Sotho)
2623 Nandipha: (reads softly) draw the, draw these on your dotted paper and show any working out that you do,
2624 ohhh, now I see, where's my other dotted paper,
2625 Thabo: (sings) Eich (ooh
2626 Nandipha: [I, ouch sorry, I
2627 Lerato: Nandipha, akena ? paper
2628 Nandipha: Heh
2629 Lerato: Akena ? paper {I don't have the paper}
2630 Nandipha: Is she gonna kill us, I
2631 Thabo: eh mona, eh, eh, eh
2632 Nandipha: I need a ruler, miss
2633 Thabo: Here, here's a ruler, is this your ruler
2634 Nandipha: Achoo, oh if we had a bank, we could get, ? but ne, it was too dirty
2635 Thabo: No, it is too dirty
2636 Nandipha: (laughing) It is
2637 Thabo: Its truly, its truly from the drawing
2638 Lerato: ?
2639 Nandipha: No, ooh no
2640 Lerato: You must ?
2641 Nandipha: No its not that I've got a stomach ache, but my stomach, ish, how, you can't make slides, ubona
2642 it has to be a brooch
2643 (inaudible - singing, writing, talking to selves)
2644 Teacher: O.K. who wants some more dotted paper
2645 Nandipha: (sings) Its threeee, 1,2,3 1,2,threee, 1,2,threee, 1,2,threee, 1,2,threee
2646 (continue singing)
2647 Thabo: What went wrong with the song in chapel today
2648 Nandipha: I don't know
2649 Thabo: Was it up
2650 Nandipha: What did she say, she said some stale joke
2651 Thabo: It was not from Sou A, the new South Africa
2652 (laugh, and continue singing)
2653 Thabo: Mona {hey man} I swear to you, you know what, my stomach, Nandipha

2654 *Lerato:* *Did you eat supper*
2655 *Thabo:* *And guess what I showered afterwards, that's why it diluted the poison*
2656 *Nandipha:* *I remember, I remember that you, that you were eating apples last week, and I ate an apple before*
2657 *I ate supper, and then gatsha {I ate} another one afterwards, uh-huh, and prep, uh-huh, I had to*
2658 *go to the toilet for about twenty minutes*
2659 *Thabo:* *You know, its usually you go for thirty, did they know*
2660 *(singing, talking to selves)*
2661 *Thabo:* *I'm waiting for my ruler, are you klaar,*
2662 *Nandipha:* *Uh-huh*
2663 *Thabo:* *Is jy klaar*
2664 *Nandipha:* *Klaar, I'm not begging, akoweteng ya ntse {I'm not begging}*
2665 *Thabo:* *Aoweteng whe {don't you beg me}, Nandipha, I think its wrong*
2666 *Nandipha:* *What*
2667 *Thabo:* *Oh, you just doing a example*
2668 *Nandipha:* *Yes (sings)*
2669 *Thabo:* *Oh, Nandipha, you such a show-off*
2670 *Nandipha:* *Why, what did I do now*
2671 *(singing, talking to selves)*
2672 *Nandipha:* *Now, lets put this, I'm in for big trouble*
2673 *(sing)*
2674 *Lerato:* *Keng wena, serious {what's wrong with you, I'm serious}*
2675 *Nandipha:* *If you still around, she'll speak with you, and now, ? is crying, ?*
2676 *Lerato:* *ruler he {give me a ruler}*
2677 *Nandipha:* *Wait, now, O.K*
2678 *Lerato:* *Kanna {by the way} how did you guys work out the area of number C*
2679 *Nandipha:* *Number C*
2680 *Lerato:* *Ja, this one*
2681 *Nandipha:* *Count it, how many hooks you had*
2682 *Thabo:* *There it goes again*
2683 *Nandipha:* *(laughs), 10, Lerato kekopa unkademe {lend me the} geoboard ne, how does it ?*
2684 *Thabo:* *No, no, no,*
2685 *Nandipha:* *(sings) O.K. na, oh, oh, oh, lets get aids*
2686 *Thabo:* *Wrong stuff, wrong stuff, wrong stuff*
2687 *Nandipha:* *The basic thing is to, is to measure le shaba ? {are you afraid of}*
2688 *Thabo:* *Mr F is*
2689 *Nandipha:* *Hah, he better be, he better be, ? of my face*
2690 *Thabo:* *No, have you seen Mr F eating*
2691 *Nandipha:* *No*
2692 *Thabo:* *Have you seen*
2693 *Lerato:* *Hae, the area of this thing is not 2*
2694 *Thabo:* *Which one*
2695 *Nandipha:* *(starts singing loudly)*
2696 *Teacher:* *You know I never thought of that, you'll also be able to use the geoboard in your presentation*
2697 *Nandipha:* *Oh yes*
2698 *Teacher:* *O.K. or the overhead*
2699 *Thabo:* *Ja man, its not, its not*
2700 *Lerato:* *?*
2701 *Nandipha:* *ehhh*
2702 *Thabo:* *Its its its 4*
2703 *Nandipha:* *Yes that is 1, that has to be 1*
2704 *Thabo:* *O.K., O.K. we always, lets, let I give you, let me tell you, we always count to, to the things, 4,*
2705 *always 4, ish, the triangle, we always make the things to this, we always divide it into thing, ne,*
2706 *to get to the, lets forget the hooks, lets forget the hooks, lets forget the hooks, but then it works with*
2707 *the square, ne, so then from there, if you if the triangle is bigger than that, if it needs 5 hooks ne,*
2708 *then 5 hooks is 1½*

2709 *Nandipha: It is*
2710 *Thabo: O.K. here's the pattern, if its a square, ne, 1 unit squared*
2711 *Nandipha: If its a square*
2712 *Thabo: its 4 hooks*
2713 *Nandipha: Ja*
2714 *Thabo: if its 1½, its 5*
2715 *Nandipha: If its 1½*
2716 *Thabo: Its 5. Everytime, everytime the number increase, the thing increase by ½*
2717 *Nandipha: Oh, O.K. O.K*
2718 *Thabo: Everytime, everytime the [ar, the area increase [by half*
2719 *Nandipha: [Ja, [by half*
2720 *Thabo: Write it down*
2721 *Lerato: What*
2722 *Thabo: Everytime the*
2723 *Nandipha: Anytime the area increases*
2724 *Thabo: Mm*
2725 *Nandipha: Ne, oh everytime the hooks increases, the area increases by half, O.K.*
2726 *Thabo: Write it down, Nandipha*
2727 *Nandipha: I have written it down*
2728 *Thabo: Where*
2729 *Nandipha: Here*
2730 *Thabo: Are you going to remember that*
2731 *Nandipha: Yes, ummmm, hew, where'll I write*
2732 *Thabo: ?*
2733 *Nandipha: ? there to there ?*
2734 *Thabo: Complicated*
2735 *Nandipha: (to self) (inaudible) yes, yes, yes, yes, so this ? is 8, kay, this is 1, so I've discovered I, O.K, there*
2736 *its done, now*
2737 *Lerato: (sings while Nandipha talks)*
2738 *Thabo: Ketswere ke makaka {I want to go to the toilet}*
2739 *Nandipha: (laughs)*
2740 *(singing)*
2741 *Nandipha: Boetie {brother}, you know what, you know what group I like, Lerato*
2742 *Lerato: Mm*
2743 *Nandipha: Wilson Phillips, don't you like them*
2744 *(singing)*
2745 *Nandipha: ?*
2746 *Lerato: finished*
2747 *Nandipha: I need to borrow the song and dance*
2748 *(singing)*
2749 *Nandipha: So there's 1,2,3,4 things left*
2750 *Thabo: ?*
2751 *(singing)*
2752 *Nandipha: 1,2,3,4, 1,2,3,4,5,6,7,8 okidaisy*
2753 *(silence)*
2754 *Lerato: Guys, ? see your number 2 please*
2755 *Nandipha: number 2*
2756 *Lerato: yes*
2757 *[bell goes]*
2758 *Teacher: Can I have your geoboards please, don't forget to write up your um your findings, answer them*
2759 *question 1, question 2 etc...*
2760 *Nandipha: Eh ja, ja sharp*

END OF SIDE 2, TAPE 3

Day 6: Thursday 21st Thabo, Nandipha and Lerato's presentation

2761 *Thabo:* *O.K uh, miss, firstly we had to find a method to, how to work out the areas, the complicated areas*
2762 *of certain whatever, eh, eh yes and then uh, ja, so we used this, this, the first uh thing we're given,*
2763 *(puts elastics on geoboard - quadrilateral 1A) ? we had 12 dots, 12 dots, so then we, we tried*
2764 *triangles like the other groups did and so we found the area of, of that, 4, and then, so, this equals*
2765 *to 1½, the area, this equals to 1½, this equals to 4, which give us 7*
2766 *Teacher:* *Sorry how did you get half*
2767 *Thabo:* *No, I mean 1, miss*
2768 *Teacher:* *O.K. good, why 1*
2769 *Thabo:* *Why 1, because the, the the whole area's going to give us 2, (someone coughs) ?*
2770 *Teacher:* *O.K*
2771 *Thabo:* *And then half of that is 1 and then its 2 by 2 which is 4, the area is equal to 6,*
2772 *Teacher:* *O.K*
2773 *Thabo:* *So Lerato is going to show us our method*
2774 *(mutter and work on geoboard)*
2775 *Lerato:* *O.K., I'm going to show you a method like using hooks to find the area , O.K. jaa, I'm going to*
2776 *start with 3 hooks because 1 hook and 2 hooks don't work, so I'm gonna start with 3 hooks. Its 1,*
2777 *2, 3 and then you join them up, and then you know that this is going to be ½ of a, of a square, so*
2778 *the area of this is ½, and then you use 4 hooks, 1, 2, 3, 4, and then the area is going to be 1,*
2779 *because if we draw it like this its going to be ½ plus ½,1, and then I'm jumping to 7, 7, 7 hooks,*
2780 *1, 2, 3, 4, 5, 6,7 (whispers to others) ja, so its 1, 2, 3, 4, 5, 6, 7, join them up, and then you get*
2781 *1, 2, and then ½. So my conclusion is that whenever you add a dot, a hook, uh the, the, the area*
2782 *of the, the shape increases with ½*
2783 *Teacher:* *Naah that's interesting*
2784 *Nandipha:* *And also miss, can I, can I add on, um if if, [if you have right*
2785 *Teacher:* *[Ummm I'm not sure, O.K*
2786 *Nandipha:* *If you have um lets say eh if you have 4 hooks ne, 4 hooks, that is equal to 1, so we can prove that*
2787 *when you have 4 hooks on a geoboard, it, it, is will always equal to 1.*
2788 *Teacher:* *O.K. show us some other shapes that give you 1 for 4 hooks, show us all the shapes for 1 that give*
2789 *you 4 hooks*
2790 *Thabo:* *Lets, lets use (trails off)*
2791 *Teacher:* *But that one's not 4 hooks, hey, did you rub out now what you were, had before*
2792 *someone:* *yes*
2793 *Teacher:* *shame*
2794 *(work on geoboard)*
2795 *Thabo:* *Then, oh, this is the, miss, this easy one, there's our, there, there, one of the tri, the triangle that*
2796 *we did with 3 hooks will give you ½, the area of ½ a square, so here just draw a line perpendicular*
2797 *to that, to that edge, then you see its ½ plus ½*
2798 *Teacher:* *Ah*
2799 *Thabo:* *which gives you 1, cause they are ?*
2800 *Teacher:* *O.K*
2801 *Thabo:* *And the other one gives, no matter how you stretch your, your thing, the area you will always be*
2802 *one, um*
2803 *Teacher:* *If it encloses 4 hooks, you were saying, hey*
2804 *Thabo:* *Yes, here's our, as you can notice, the thing, the, the length of the base is still always the same,*
2805 *but then the angles decrease, so then you can, you can always go like that, draw a, bring a line to*
2806 *the thing, to the dot, to the hook here*
2807 *someone:* *it works*
2808 *Thabo:* *and then uh a straight line like this, like that, like, like this, and all the triangles here are, are*
2809 *thing,*
2810 *someone:* *congruent*
2811 *Thabo:* *congru, congruent, yes and, and, and thing and therefore, therefore we say, if we draw the triangle*
2812 *here, this, this is equal to this, they equal, and therefore, these 2 triangles are equal, ne miss, so*
2813 *therefore we've got these 2 triangles here, and then we find the length of this, lets say its here and*

2814 then we draw a this triangle here, and this one's gone, we find this one, here its gone, we got 2,2
2815 triangles left, therefore they all equal.
2816 Teacher: O.K., also you could have used the subtraction method, that like Tefo's and, and other groups were
2817 using, where you said the whole rectangle is 1, 2, 3, 4, units and then subtract the triangles on
2818 either side, you see what I'm saying, you right, its a nice, very nice intuitive method, but if you want
2819 to be exact, their method was if you complete your rectangle, and you subtract these parts, O.K.
2820 do you, do you understand what they were explaining, that would also work for you
2821 Thabo: And, the, the other part that Lerato said about the thing, about, uh, every, every, every time the dots
2822 increase by one the area of the thing increase by ½, but it won't work with lets say you use 7 dots,
2823 it won't work, here 7 dots
2824 (talk softly to each other)
2825 Teacher: It doesn't even work with, does it work with 4 dots even
2826 Thabo: Yes, it does work, yes it works, it works
2827 Teacher: And what if I stretch this, has it increased by half
2828 Thabo: [Yes, it has
2829 Nandipha: [Yes miss, it has
2830 Thabo: So there, there, there, there's our thing again ne
2831 someone: No, but what about that, what about the ½ there
2832 Lerato: The triangle here must be right angled
2833 Teacher: Oh, O.K.
2834 Thabo: No, it still works miss, we got 5 things ne and therefore, there's our thing, this is ½ and this is ½
2835 someone: ?
2836 Teacher: But that's not ½
2837 Thabo: The area, its ½
2838 Teacher: The skew one here
2839 Thabo: Here
2840 Teacher: Ja, you told me that when you had a skew
2841 Thabo: Miss, let me, that's a that's a 4, 4 hooks, 4 hooks, therefore its 1
2842 Nandipha: Therefore its one, this is ½
2843 Thabo: This is ½, so therefore 5 dots makes [?
2844 Teacher: [O.K., O.K.,
2845 Thabo: You see what we doing miss, and it will work
2846 someone: Wait, what's the thing there
2847 Thabo: Here,
2848 Teacher: Wait, just just just Ol
2849 Thabo: You see what Lerato did ne, she increase the dots, she increase the dots, ne to see if our method
2850 works, it still works because, and we know this area is 1, its 1 and then we find, we, its 1, and then
2851 we find the thing, the congruent triangle to this one, ne, so it can give us a ?, so this is ½
2852 Teacher: Uh
2853 (laughter and some talking)
2854 Thabo: Ja, so this is ½ and this is also ½, this area here it equals to 2, and we said our 5 dots will give
2855 us, 2
2856 someone: No, no, no, that's not it
2857 Zandile No miss
2858 Nandipha: 1½
2859 Zandile Miss
2860 Thabo: 1½ ja
2861 Nandipha: 1½
2862 Teacher: But now I've inc, I've increased it by 1 dot and I've increased the area by 1, not ½
2863 someone: yes, yes, yes
2864 someone: Ja, by 1
2865 Teacher: D'you see that, [one dot has given me an extra ½ and ½
2866 Thabo: [O.K. O.K. can I, can I show you how it works, I was going to explain how it
2867 works.
2868 (laughter)

2869 Thabo: You must always have a, a, a rectangle
2870 Te... What you were saying here, O.K.
2871 Th... A rectangle, like, lets say, like that, this, this side must always be 1 unit squared
2872 Tea... O.K.
2873 Thabo: But then, once you've got your, your 4 thing, the triangle can be, uh, an anyhow, it can be like
2874 that, or like that
2875 Zandile Miss, [miss
2876 Thabo: [And that's going to give us 1½
2877 Lerato: Ask us, not miss
2878 Zandile Miss
2879 Teacher: Yes
2880 Zand... Miss, they only increased, the, ru, the, the, the rule, I don't think it works miss, cause it now the
2881 the one hook is not connected with the other its in the middle, that's why you increasing [it by one
2882 Teacher: [aah, you
2883 right, you right, you quite right, I've in fact increased your drawing by 2 dots,
2884 Thabo: ja
2885 Lerato: ja
2886 Teacher: Maybe your theory still works, [d'you see what she's saying
2887 Zandile [yes miss because that one
2888 Teacher: Cause remember when you told me that the inner ones also count, remember when you said 4 hooks,
2889 the inner ones you said also counted, you right
2890 Zandile Yes because there its 5 hooks there altogether
2891 Teacher: You right, well done, well done
2892 Nandipha: Yes
2893 Teacher: O.K. I'm, I'm practically convinced about your theory
2894 Thabo: ja, so the, the side, the thing must always have, be one, this must, the uh ? must be 1, but, and then
2895 you can have your triangles anyhow, anyhow but then you'll find the thing, our method works
2896 Teacher: [O.K
2897 Thabo: [Of 3 hooks [gives us ½
2898 Teacher: [O.K., did you get to the last one, O.K
2899 Nandipha: Ja, we've just corrected number 4 (mutter) of, how do you prove to a Form 1 student how to work
2900 it out, you can like
2901 Teacher: O.K. quickly, see
2902 Nandipha: Ja I'm just gonna sit down, you can divide your, your shape into a right angled triangles and work
2903 out the hypotenuse al, the, [yes
2904 Lerato: [ja
2905 Nandipha: with Pythagoras theorem her
2906 Teacher: For the area
2907 Nandipha: yes
2908 Teacher: Pythagoras for the area
2909 Nandipha: I mean, hypote... miss
2910 Teacher: yes, what abou... hypotenuse
2911 Nandipha: Miss like
2912 Thabo: Perimeter
2913 Nandipha: Ye
2914 Teacher: For the perimeter
2915 Nandipha: Ja
2916 Teacher: O.K. did you get to the last question
2917 Nandipha: No miss
2918 Teacher: No
2919 Lerato: What's the last question
2920 Thabo: What's the last question
2921 Teacher: O.K. never mind, thank you
[bell goes]
END OF TAPE

APPENDIX C: THE PUPILS WRITTEN WORK

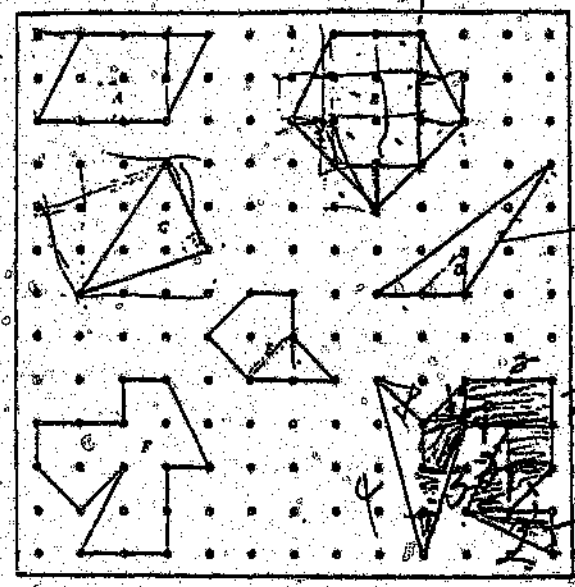
Thabo: Rough work ①

Form II Geoboard Worksheet

Work on this worksheet in groups of 3 or 4. Use the geoboards and the dotted paper provided, and do the necessary working in your books. When the worksheet is finished, you will have to hand in the answers to this worksheet on a piece of paper. The final product should be your own work.

1. a) Work out the area of each of the shapes below.

   b) Work out the perimeter of each of the shapes below. (without measuring it)



Hooks. Give us squa

2. a) On your geoboard, make a square with an area of 3 units², and draw it on your dotted paper.
   b) What possible areas can squares have on your geoboard? Draw these on your dotted paper, and show any working that you do.
   c) What possible perimeters can squares have on your geoboard? Show any working that you do.

3. a) Choose any area. Find as many different polygons as you can which have this area.
   b) Choose any perimeter. Find as many different polygons as you can which have this perimeter.

4. a) Discuss with your partner and then write down how you would explain to a Form I student how to work out the area of complicated shapes on a geoboard.

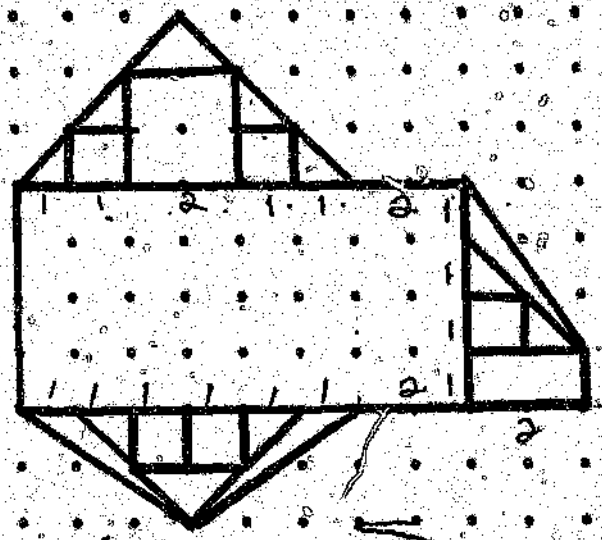
   b) Write down how you would explain to a Form I student how to work out the perimeter of complicated shapes on a geoboard (without measuring it).

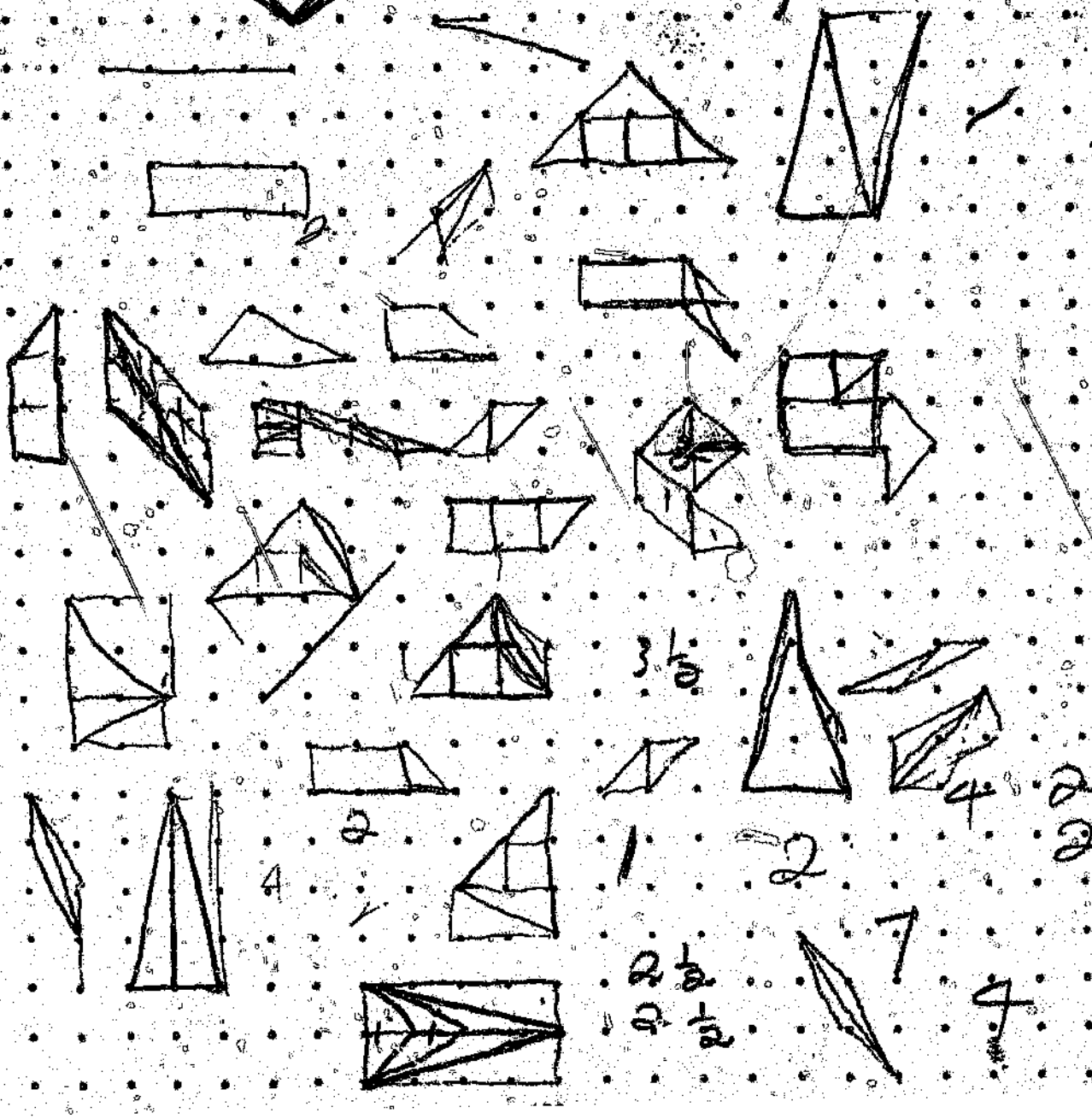
5. If you have a fixed perimeter, which shape gives you the biggest area. For this question you do not need to only consider shapes on your geoboard, and you may use any instruments and equipment you like.

$\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 $2\frac{1}{2}$ 3
$3\frac{1}{2}$ 4 5 6 7 8

9 Maths: rough work ②

Dotter Paper





Key 207 dots = $2\frac{1}{2}$ rectangles

1.(a) $3 \times 2 = 6$

(b) $2 \times 3 + 1,5 + 1,5 + 1 + 1 = 10$

(c) $2 \times 1 = 2$.

(d) 2×1

(e) $3\frac{1}{2}$.

(f) 9. ⑨ 9

B. $3+3+2+2 = 10$

(b) $2+2+2+1+1+2 = 10$

(c) $3+2+3 = 8$

(d) $2+2+3 = 8$

(e) $2+1+1+1+1+1 = 7$

(f) $(2)6 + 4 = 16$

(g) $3+3+2(4)+(1)6 = 24$

2

Conclusions of the presentation.

Area of a complicated ~~shape~~ ~~area~~ is always divided into rectangles and triangles with one side equals 1 unit

Perimeter of a complicated shape divide you shape into right angle triangles so that you can find the length of the hypotenuse and also divided into rectangles which are easy of course

The pattern: Every starting from 3 dots the area is equals to $\frac{1}{2}$ units^2, for 4 dots is equals to 1 unit^2 ... 1/4 ... 1/4 ...

Thabo: Work handed in ①

AREA

1a. $(2\times2)+(\frac{1}{2}\times2\times2)+1+2$
$= 6$ units²

$2+(2^2+1^2)2$
$6+\sqrt{5}\times2$

b. $(2\times3)+1+(1\times\frac{1}{2})+(2\times\frac{1}{2})$
$6+1+1+2$
$= 10$ units²

$2+(2^2+1^2)2+(2^2+2^2)2$
$= 2+\sqrt{5}\times2+4\sqrt{8}\times2$

c. $1+1+\frac{1}{2}+1 = 3\frac{1}{2}$ units²

$(2^2+1^2)+(2^2+2^2)+(3^2+1^2)$
$5+\sqrt{8}+\sqrt{10}$

d. $1+1+1 = 3$ units²

$(2^2+3^2)+(4^2+3^2)$
$\sqrt{13}+\sqrt{25} = 7.6$

e. $2+1+\frac{1}{2} = 3\frac{1}{2}$ units²

$1+2+\sqrt{(1^2+2^2)}+3$

f. $4+2+1+1+1 = 9$ units²

$9+(1^2+2^2)2+(1^2+1^2)$
$9+\sqrt{5}\times2+\sqrt{2}$

g. $2+\frac{1}{2}+1+1+\frac{1}{2} = 4+1+1\frac{1}{2}$
$= 6\frac{1}{2}$ units²

$7+(1^2+2^2)2+(1^2+2^2)$
$(1^2+1^2)3$
$7+\sqrt{5}\times2+\sqrt{2}+\sqrt{2}\times3$

b) Perimeter

2 a) c

c) Many

3 a) 1 unit² Many of my dotted paper

b) Perimeter 2

4 a) F

← Turn back ... 4 centimeters.

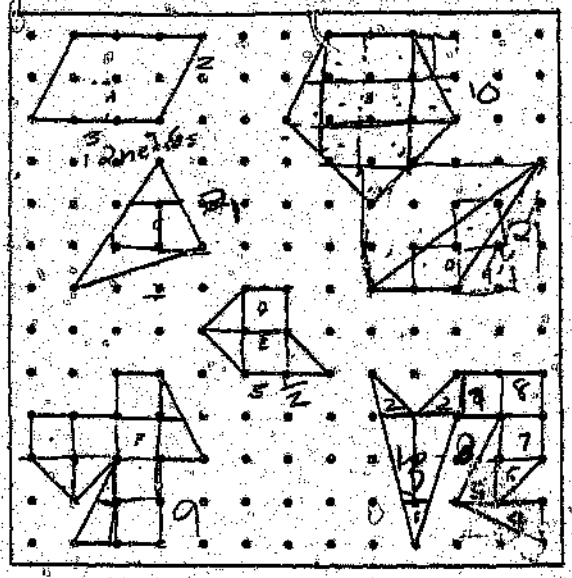
Nandipha: Rough work ①

Form II **Geoboard Worksheet**

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1. a) Work out the area of each of the shapes below.

   b) Work out the perimeter of each of the shapes below. (without measuring it)



2. a) On your geoboard, make a square with an area of 2 units², and draw it on your dotted paper.
   b) What possible areas can squares have on your geoboard? Draw these on your dotted paper, and show any working that you do.
   c) What possible perimeters can squares have on your geoboard? Show any working that you do.

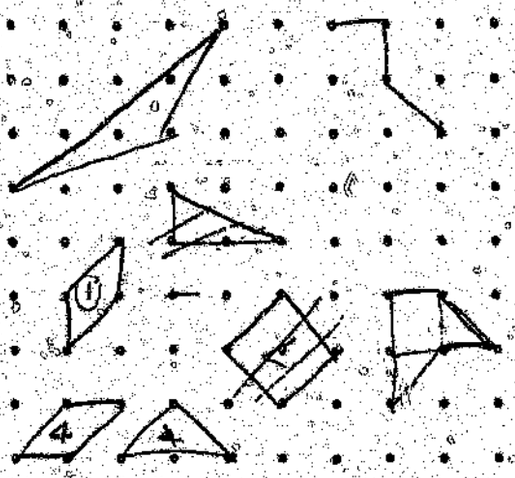
3. a) Choose any area. Find as many different polygons as you can which have this area.
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5. If you have a fixed perimeter, which shape gives you the biggest area. For this question you do not need to only consider shapes on your geoboard, and you may use any instruments and equipment you like.

Dotter Paper



1. Tried divide → into triangles and squares
→ Work out area of squares or rectangles
to work out area of triangle if you count
how many hooks the elastic is hooked to & the inside
(eg: 5 hooks → try make a square or Δ with 5 hooks

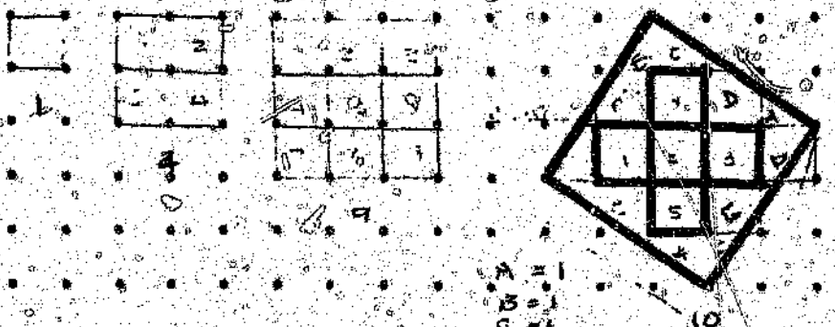
8

Nandipha: rough work ③

(she does this on Wed)

Dotter Paper

6 Question



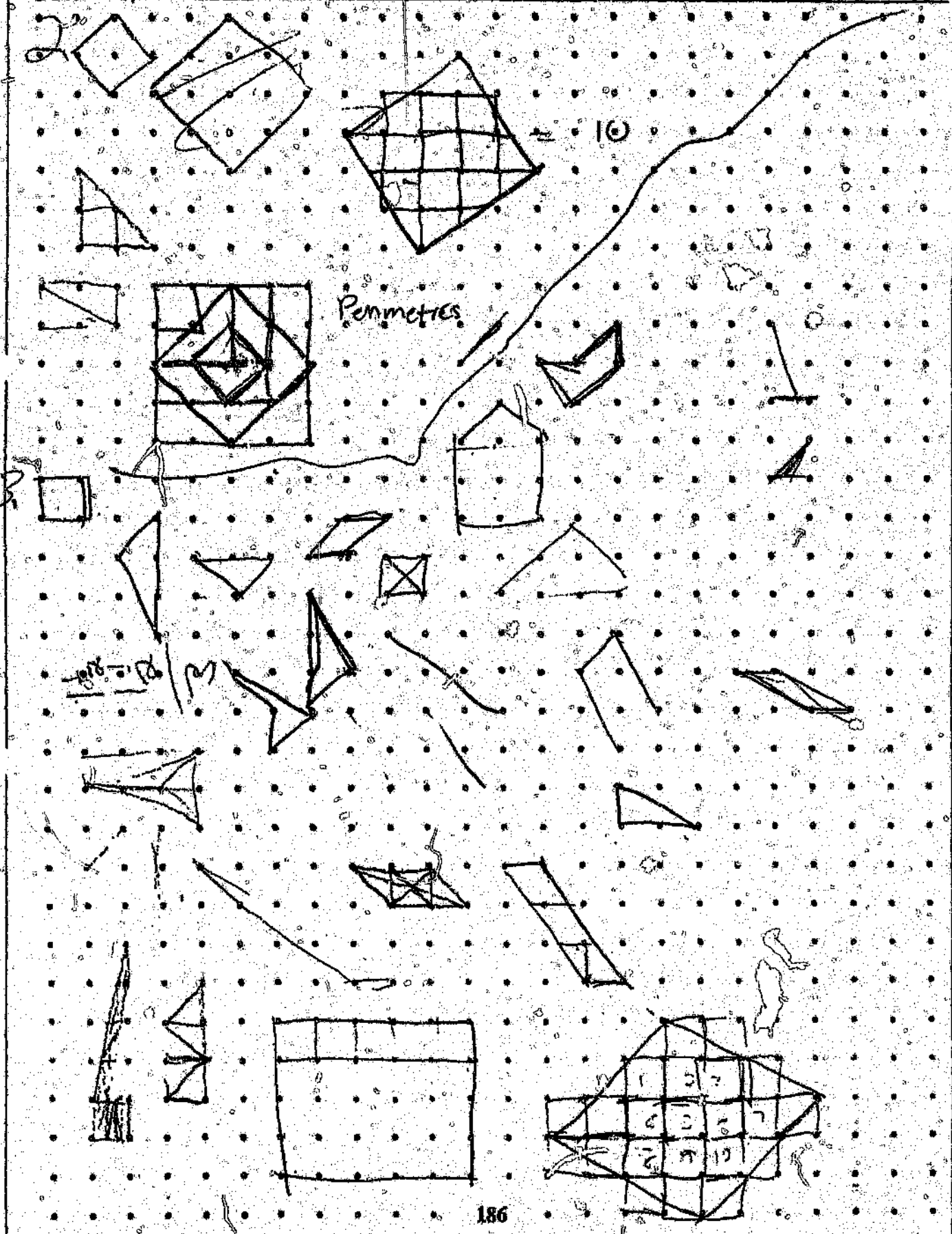
Nandipha's Rough work @

8 9 3

+ = 1+2+4+8+10

1+4 1 1+2 +

Dotter Paper



Nandipha: Work awarded in ①

Geoboard Worksheet

1a) Area: ✓

1a) 6 b) 9

c) $2\frac{1}{2}$ d) 3

e) $3\frac{1}{2}$ ✓ f) 9 ✓ g) ± 9 ✓

b) a) 3 x 2 ✗ b) ✗

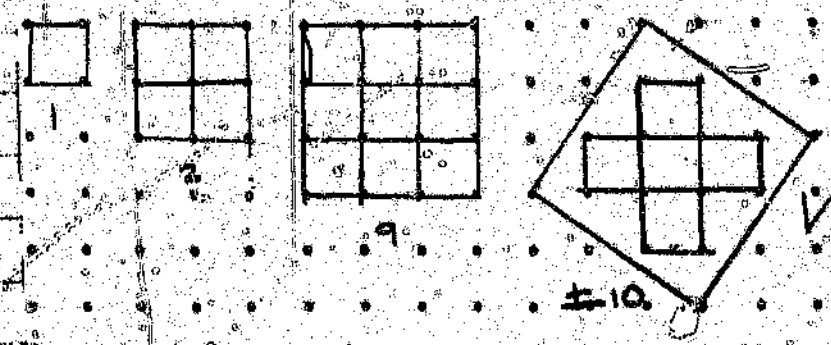
c) 1 x 2 ✗ d) 3 x 2 ✗

e) 1 x 2 ✗ f) ✗

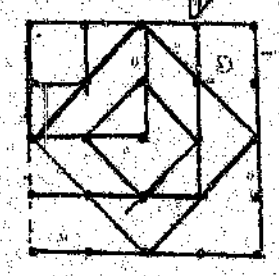
g) ✗

2a? ?

2a) These are the possible squares one could have:



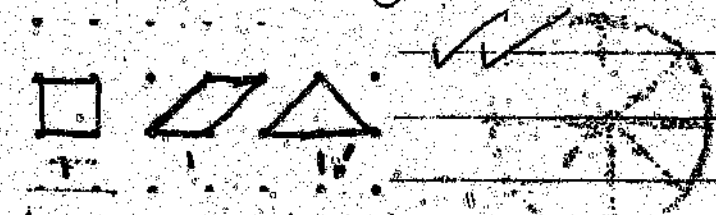
c) This was 6 perimetres ? ✓ ? possible squares ✓



3a) We started out with 4 and we only stayed at 4 because we made an important discovery with it. We found out that if for instance you put on elastic (on pinboard) and make a

Nandipha: Work handed in ②

Square with 4 sides, you can also make a triangle with 4 sides and they both have the same area. Note:



This discovery made our work lighter even though sometimes, when a teacher came to argue or discuss with us, we sometimes wanted to undo our discovery! ✓

3b → ?

4(a) Well, for this question, I figured you could use your method of working out. If it worked on the complicated shapes once we had it could work on anything ✓

b) For complicated shapes all you do is you divide your shape into RA and then you work out the hypotenuse.

5. Its a circle, — why? explain! show!

We also discovered once when you increase a shape by ½! ✓ good hack it increases area by ½

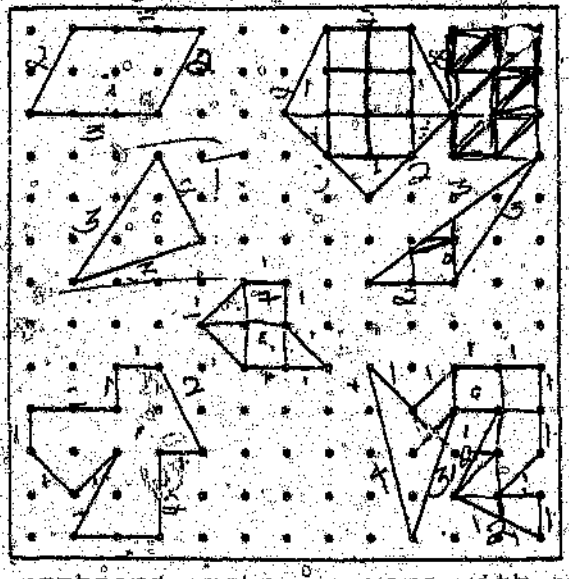
Lerato: rough work ①

Form II — Geoboard Worksheet

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   b) Write down how you would explain to a Form I student how to work out the perimeter of complicated shapes on a geoboard (without measuring it).

5. If you have a fixed perimeter, which shape gives you the biggest area. For this question you do not need to only consider shapes on your geoboard, and you may use any instruments and equipment you like.

Whiteboard Worksheet

1a. 6

b. 10

c.

d.

e. $5\frac{1}{3}$

f. 9

g.

2a) 10

b) 5

c) 8

d) 7

e) 70

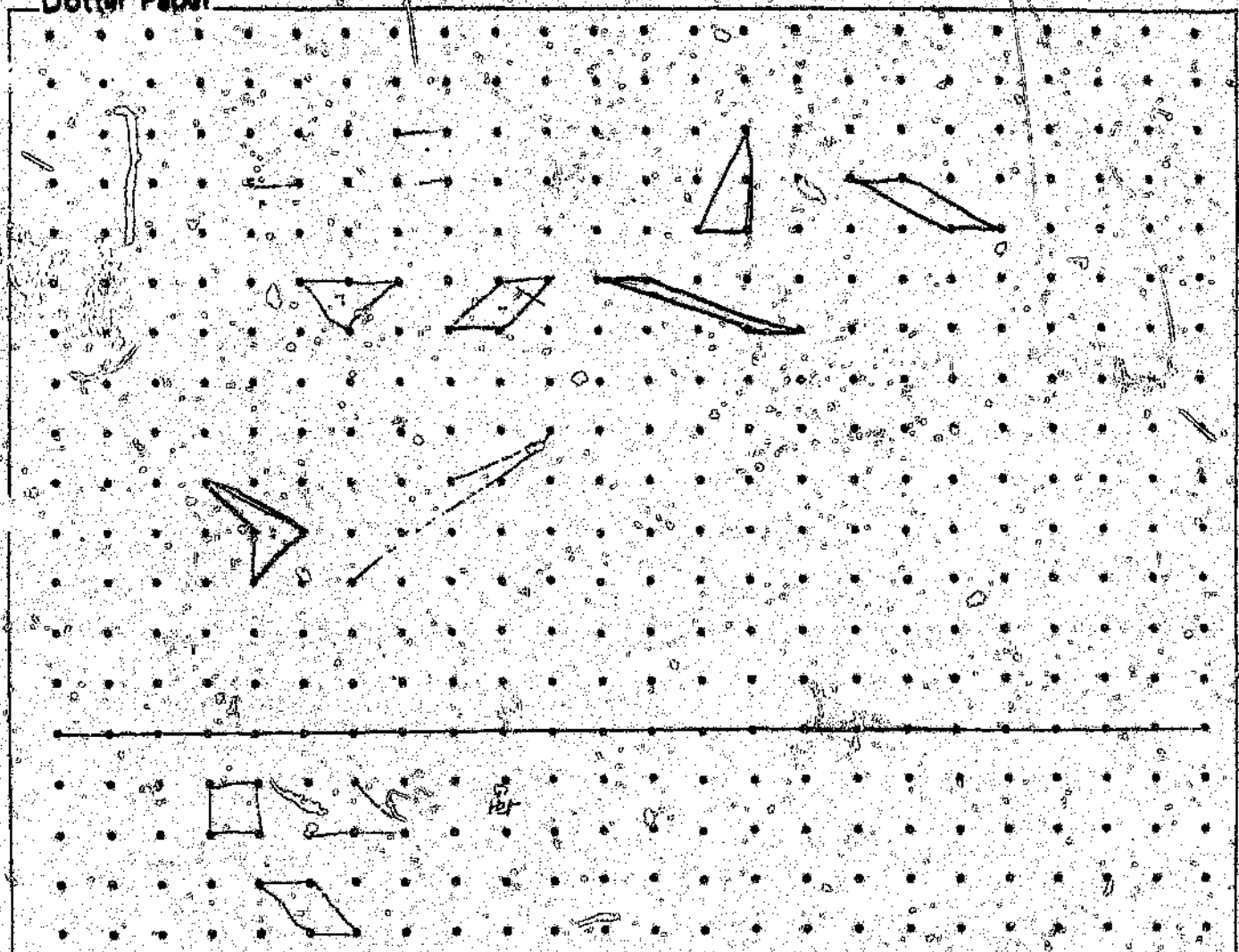
f) 16

g) 81.

Perimeter = 10

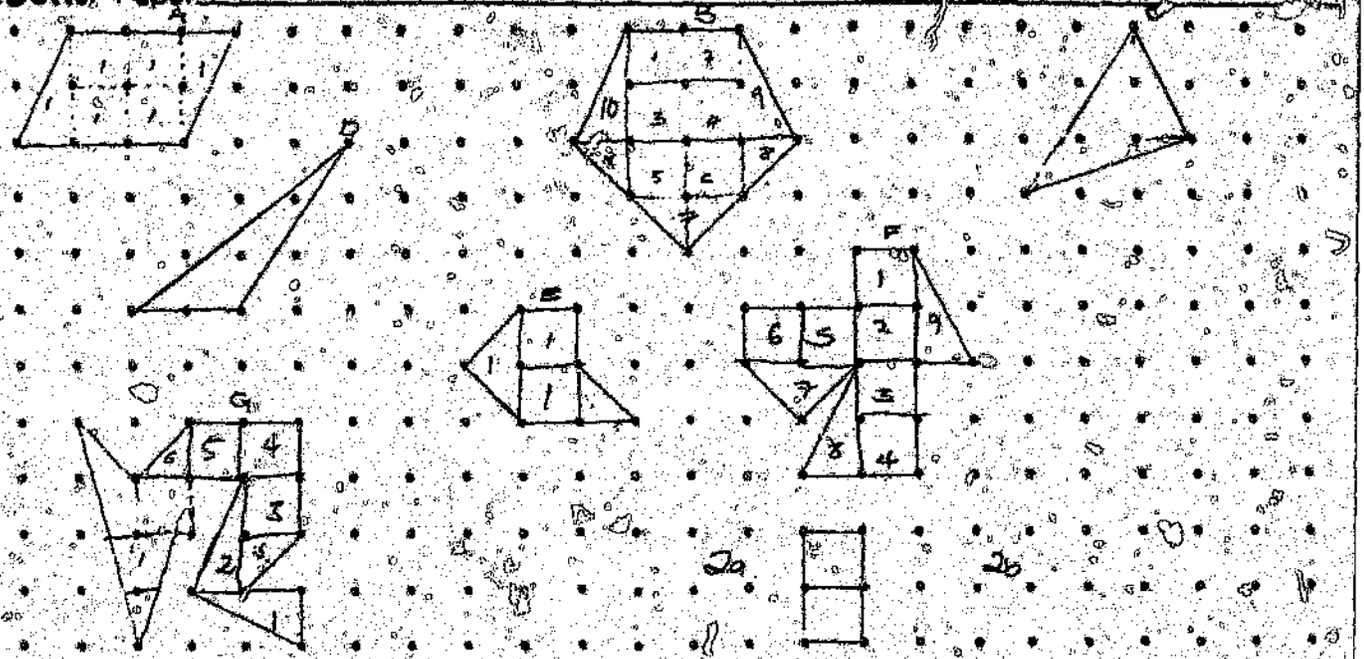
Levato : rough work (3)

Dotter Paper



Herato : rough work ④

Dotter Paper



S

Kerala: worksheet handed in ①

Geogard worksheet

6

a 10

a 2

d 5 2

= 3½

f 9

3 7

b 10

c 10

c 8

d 8

3 7

4 16

9 21

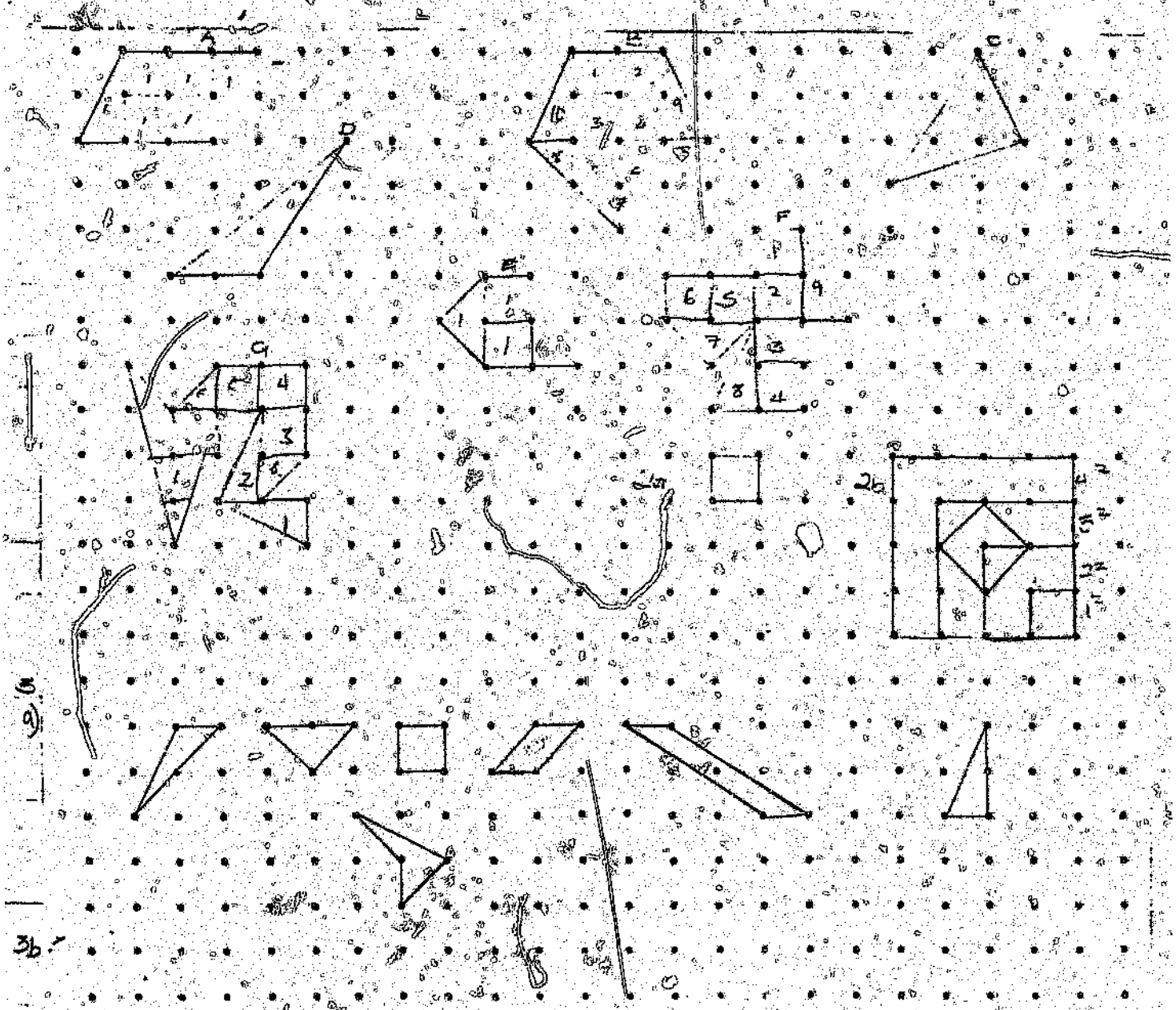
4. Divide your polygon into triangles and squares then find the areas of both the squares and triangles then add them.

b) Also divide into squares & triangles

If it is squares measure the lenght of the outer side then use Pythegorases theorem to find the lenghts of the hypoturus. Then add all the lengths using this method.

Perimeter = l + l etc. depending on how much sides there are.

Lerarp: work handed in (2



APPENDIX D: PICK'S THEOREM

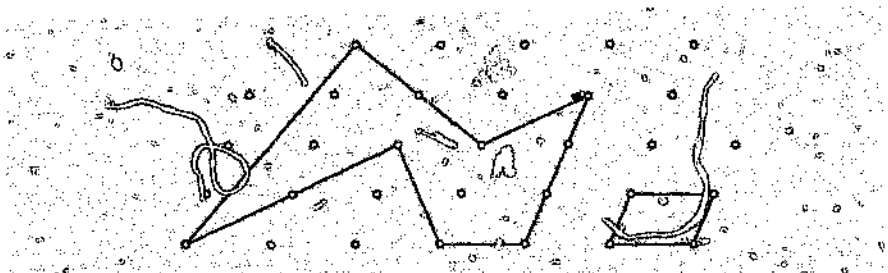


Figure 13.5c

13.51 *The area of any simple polygon whose vertices are lattice points is given by the formula*

$$\tfrac{1}{2}b + c - 1,$$

where b is the number of lattice points on the boundary while c is the number of lattice points inside.

(By a "simple" polygon we mean one whose sides do not cross one another. Figure 13.5*a* shows an example in which $b = 11$, $c = 3$.)

Proof. We first observe that the expression $\frac{1}{2}b + c - 1$ is additive when two polygons are juxtaposed. In fact, if two polygons, involving $b_1 + c_1$ and $b_2 + c_2$ lattice points respectively, have a common side containing n ($\geqslant 0$) lattice points in addition to the two vertices at its ends, then the values of b and c for the combined polygon are

$$b = b_1 + b_2 - 2n - 2, \qquad c = c_1 + c_2 + n,$$

so that

$$\tfrac{1}{2}b + c - 1 = (\tfrac{1}{2}b_1 + c_1 - 1) + (\tfrac{1}{2}b_2 + c_2 - 1).$$

Next, the formula holds for a parallelogram having no lattice points on its sides (so that $b = 4$ and the expression reduces to $c + 1$). For, when N such parallelograms are fitted together, four at each vertex, to fill a large region, the number of lattice points involved (apart from a negligible peripheral error) is $N(c + 1)$, and this must be the same as the number of unit cells needed to fill the same region.

Splitting the parallelogram into two congruent triangles by means of a diagonal, we see that the formula holds also for a triangle having no lattice points on its sides. A triangle that does have lattice points on a side can be dealt with by joining such points to the opposite vertex so as to split the triangle into smaller triangles. This procedure may have to be repeated, but obviously only a finite number of times. Finally, as we remarked on page 204, any given polygon can be dissected into triangles; then the expressions for those pieces can be added to give the desired result.

Author: Brodie K
Name of thesis: Small-group work and ownership of mathematical knowledge

PUBLISHER:
University of the Witwatersrand, Johannesburg
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