Re_H	regenerator heat transfer Reynolds number (Equation (4.81))
St	Stanton number (St ≡ Nu/Pe)

Other Symbols (x used to denote arbitrary symbol)

$\langle x \rangle$	time averaged value
$[x]$	vector quantity (where applicable)
(x)	scaler quantity (where applicable)
$\lvert x \rvert$	absolute value; phasor magnitude (or amplitude)
$\bar{x}$	spatially averaged value
x'	(prime) value in terms of displacer and piston motions; conditional value
x^{*}	non-dimensional quantity
x^*	complex conjugate; referenced to axis fixed to displacer
$\hat{x}$	Laplace transformed variable
$\dot{x}$	first derivative with respect to time
$\ddot{x}$	second derivative with respect to time
∇	$[\partial/\partial x \quad \partial/\partial y \quad \partial/\partial z]$
∇^2	$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$
D/Dt	$(\partial/\partial t + u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)$
$O[x]$	order of magnitude
bold	bold type indicates vector or tensor quantity
$\emptyset$	shape factor

1 INTRODUCTION

1.1 Background

The uncertainty of energy costs and supplies and the increasing awareness of pollution and noise has made it important to investigate alternative prime movers. Major requirements for these devices are that they be efficient, non-polluting, reliable, economical and socially acceptable.

The development and utilization of such systems are important not only for first world countries, but possibly more so for third world and developing countries. Widespread use of efficient prime movers can have a considerable effect on the balance of payments for countries that import energy supplies [Fin77]. While the high cost of fuel is particularly stifling to countries which are on the verge of industrialisation.

The Stirling engine has aroused much interest in recent times because of its many favorable characteristics. These include:

i) Minimal pollution. The exhaust gases are comparatively clean and cool.

ii) Silent and practically vibrationless operation in some configurations.

iii) Potential for low fuel consumption. The maximum attainable efficiency for any heat engine operating between the same temperature differential.

iv) Multi-fuel capability. The energy source may be almost of any form whatever, so long as it is available at a sufficiently high temperature. Stirling engines have been run on solar energy and a variety of liquid and solid fuels.

v) In many instances it is possible to hermetically seal the engine thus eliminating problems arising from dirt ingress. Some of these configurations have demonstrated operating lives exceeding 10 years.

vi) Reasonable specific power (currently between 0.067 kW(e)/kg for higher power machines to 0.033 kW(e)/kg for lower power machines).

vii) Reversible operation which allows the same device to be used as a refrigerator or heat pump. This feature also introduces the possibilities of regenerative braking .

viii) Favourable torque characteristics for transportation applications. This leads to simpler transmission designs.

ix) Mechanical simplicity.

x) The highest specific work output of any closed cycle.

1.2 The Stirling Cycle

The Carnot theorem (*Reflexions sur la Puissance Matrice du feu, et sur les Machines Propres a Developer cette Puissance. Par S Carnot, 1824*) states that the efficiency of all reversible engines operating between the same two temperatures is the same, and no irreversible engine working between these two temperatures can have a higher efficiency.

This statement was later shown by Clausius and Kelvin to be a necessary consequence of the Second Law of Thermodynamics. The process during which heat is transferred must be done isothermally. Thus the two temperatures referred to by Carnot are the higher temperature at which heat is added reversibly, and the lower temperature at which heat is extracted reversibly. The processes connecting the heat addition to the heat extraction must then be externally adiabatic when taken together, and must obviously also be reversible to achieve a cycle which is completely reversible.

The cycle suggested by either James or Robert Stirling (there appears to be some contention here, see for example [Wo1894] and [Ko72]) in 1816 was originally envisaged as a perpetual motion engine of the second kind, that is, all the heat supplied would be converted to work. This misconception of the possibility of perpetual motion is borne out by the fact that the first Stirling patent of 1816 did not include a cooler --- only a heater was indicated (Fig 1.1). The omission was, however, soon corrected in later patent disclosures.

The Stirling cycle is a closed cycle in which a fixed mass of gas is alternately expanded, cooled, compressed and reheated to the begining of the expansion phase. Figure 1.2 shows the two most common mechanical configurations for realizing the Stirling cycle, known respectively as the dual piston and piston-displacer arrangements, or more commonly as the alpha and beta arrangements. The processes of heating and cooling are appreciably augmented by what Stirling called an *economiser*. This device, which he was the first to use, functions on a principle that is now known as regeneration. The economiser, or regenerator, is an energy store; it generally consists of a matrix of fine wires, porous metal or sometimes simply an annular gap. It serves to store unspent heat from the expanding gas as the gas leaves the hot section and to transfer this heat back to the

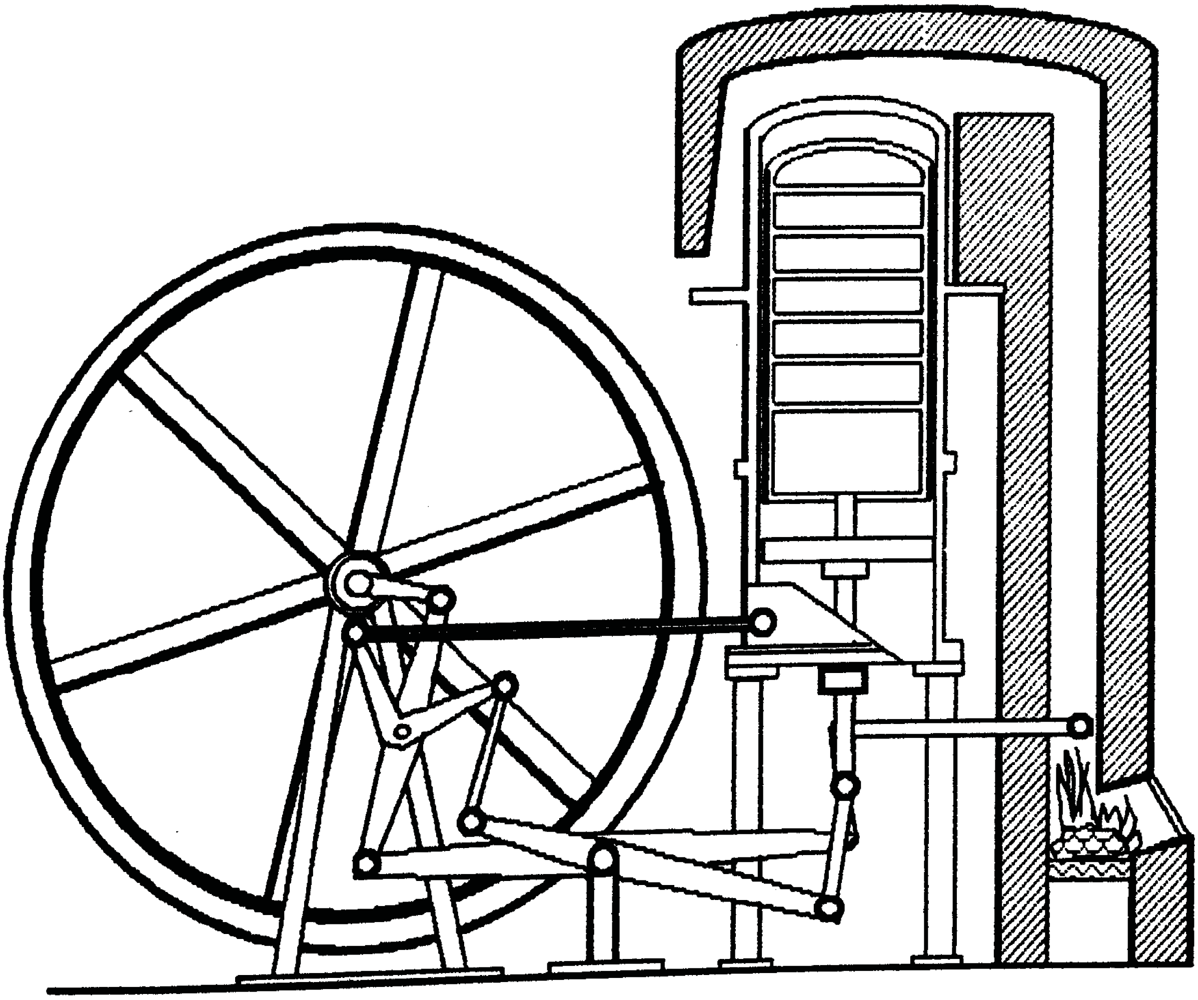


Figure 1.1 The Original Stirling Engine Patent of 1816 (after Walker [Wa73]). Note no distinct cooler is indicated

compressed gas as the gas returns from the cold end. The regenerator substantially improves the efficiency of the cycle. Following the process in Figure 1.2 it can be seen that if the expansion and compression spaces can be made isothermal at their respective temperatures and the regeneration perfect, then the Stirling engine fulfills Carnot's requirements for maximum attainable efficiency.

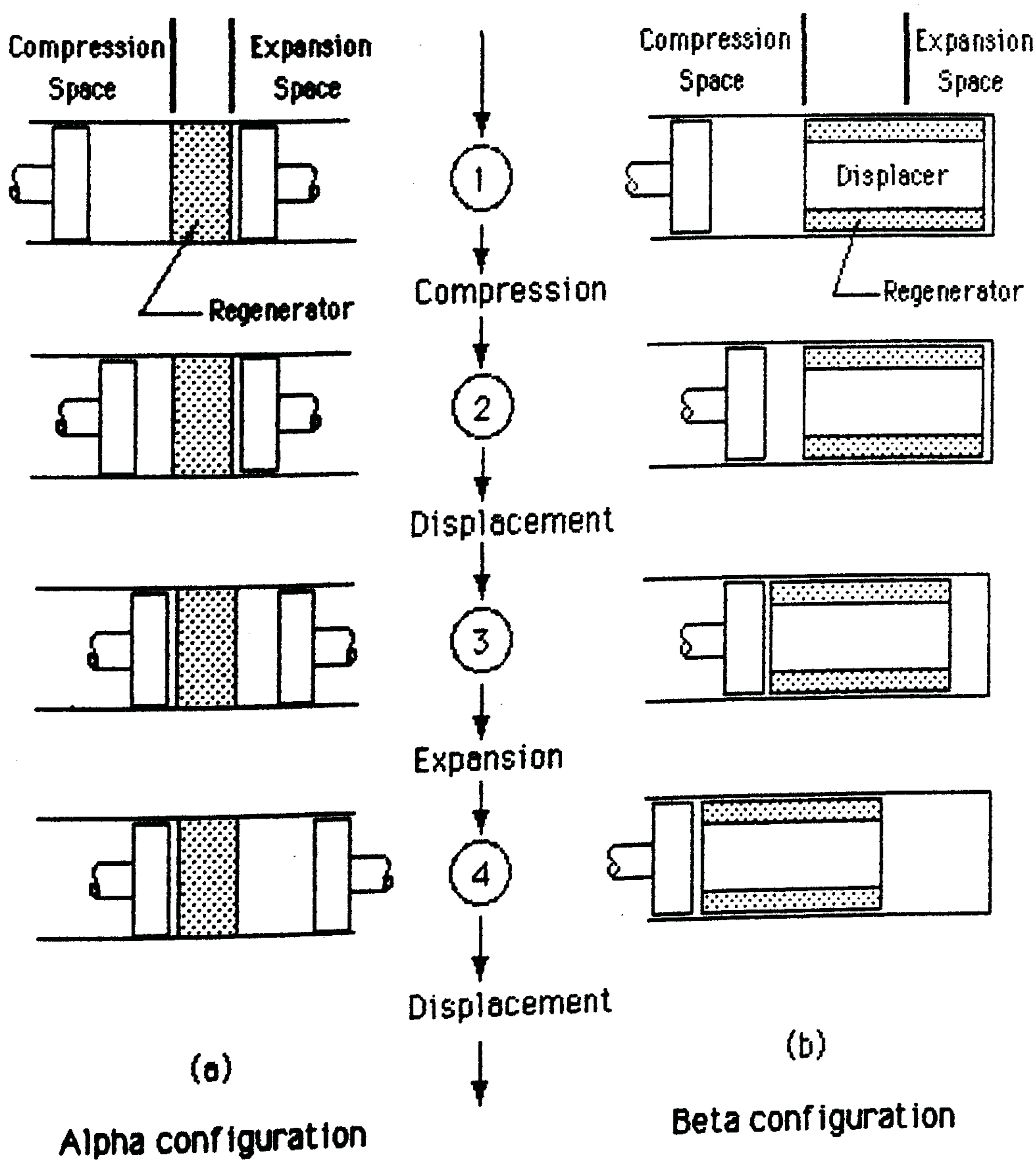


Figure 1.2 Piston and Displacer Motions

Whereas it is difficult to even approach isothermal conditions in the working spaces with conventional heat exchanger technology, regenerators, on the other hand, regularly obtain efficiencies greater than 98%. Therefore, and especially in machines of 0.5kW and above, separate heat exchangers for the heater and cooler are introduced as shown in Figure 1.3. It is more plausible then, to imagine that the ideal cycle would have isothermal heaters and coolers while the working spaces would be adiabatic. This configuration has been referred to as the pseudo-Stirling cycle [RU77] or sometimes as the adiabatic Stirling cycle, and is generally considered to be more representative of what may be understood by the ideal cycle *yardstick*.

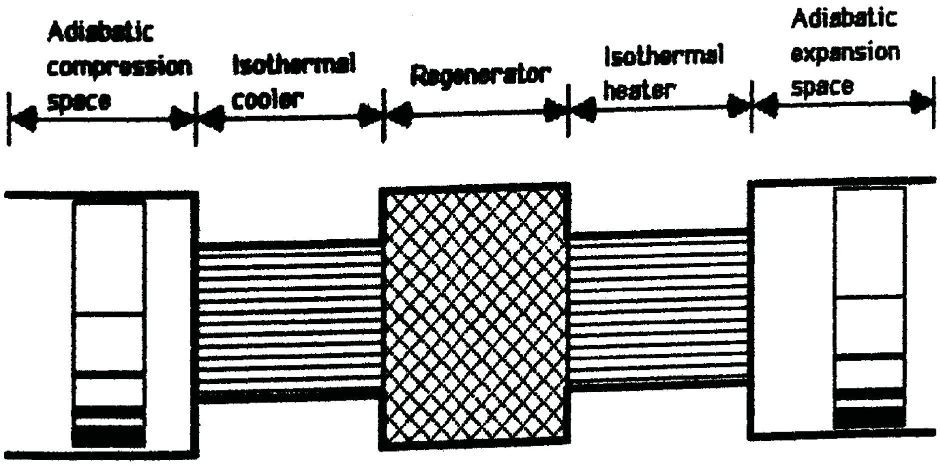


Figure 1.3 The Pseudo-Stirling Cycle Engine

However, despite this argument, it is still held by many that the ideal cycle representing all Stirling engines is the one with isothermal working spaces and ideal heat exchangers.

1.3 Mechanical Configurations

The mechanical configurations of Stirling engines are generally divided into three groups known as the alpha, beta and gamma arrangements. Alpha engines have two pistons in separate cylinders which are connected by a cooler, regenerator and heater. Both beta and gamma engines use displacer-piston arrangements, the beta engine having both the displacer and piston in the same cylinder whilst the gamma engine has separate cylinders for the displacer and piston. The alpha and beta arrangements are indicated in Figure 1.2 and the gamma arrangement in Figure 1.4.

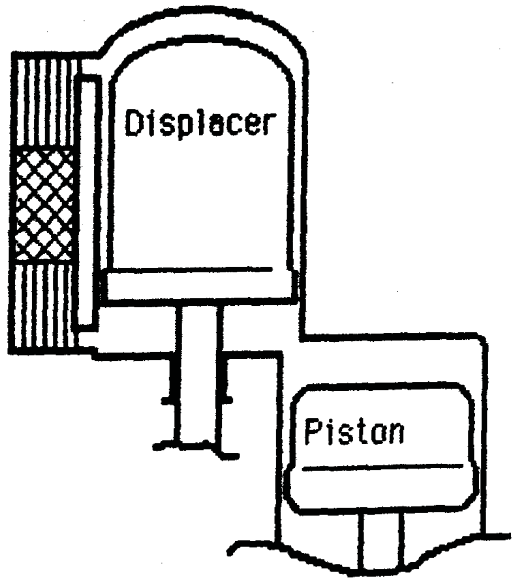


Figure 1.4 The Gamma Arrangement

Drive methods may be broadly divided into two groups, namely the kinematic and free-piston drives. Kinematic drives may be defined as a series of mechanical elements such as cranks, connecting rods and flywheels which move together so as to vary the volumes of the working spaces in a prescribed manner. This may be considered as the conventional method by which reciprocating heat engines deliver their thermodynamic work to an output shaft. Free-piston drives, on the other hand, use the working gas pressure variations to move the reciprocating elements. In this case the work is transferred by a linear alternator, hydraulic pump or a second free-piston Stirling heat pump. From an analysis standpoint, it is important to distinguish between free-piston and kinematic engines in that the mechanical dynamics and thermodynamics are strongly coupled in free-piston engines. In kinematic engines, for given operating conditions, only the way the volumes change need be defined in order to determine the engine's performance.

Other workers, notably Professor Graham Walker, have further subdivided the mechanical configurations on the basis of regenerator location [Wa73, Wa80]. For the purposes of this work, the regenerator location has no effect on the methods of analysis and therefore this distinction is not relevant here.

Alpha configurations have been pursued mainly by the proponents of automotive Stirling engines, of which the best examples are the Philips-Ford 4-215 swash-plate drive engine [BK77] and the USS-MTI Mod 1 crank drive engine [CM85] shown in Figures 1.5 and 1.6 respectively. Figure 1.7 indicates the simple way in which the alpha arrangement is compounded into multiple cylinder configurations. A much older and rather successful alpha arrangement is the Rider engine of the late nineteenth century shown in Figure 1.8.

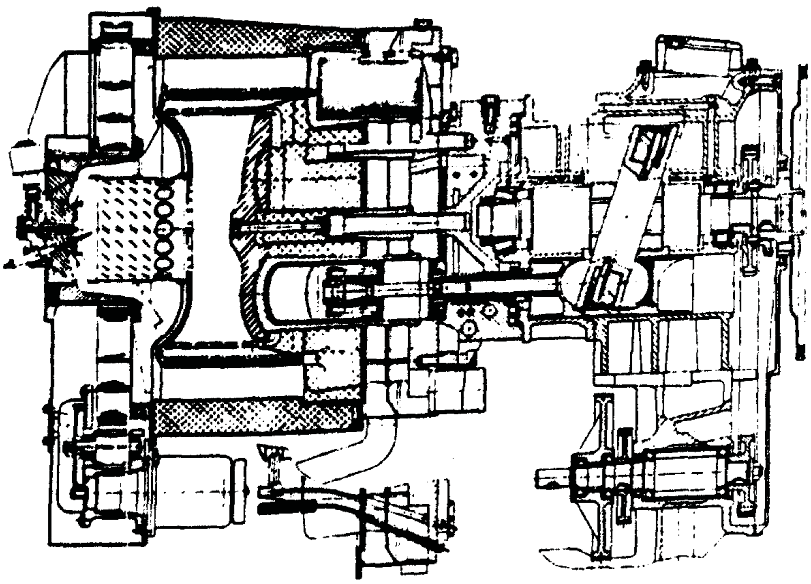


Figure 1.5 Philips-Ford 4-215

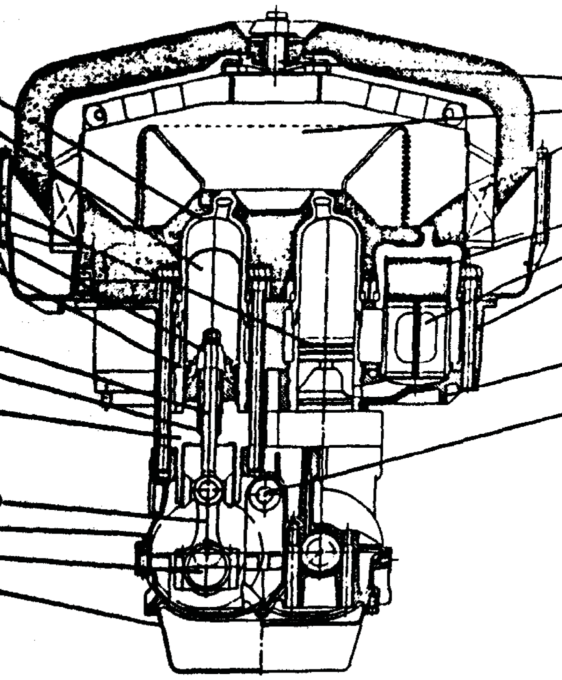


Figure 1.6 USS-MTI Mod 1 (United Stirling of Sweden and Mechanical Technology Incorporated joint effort)

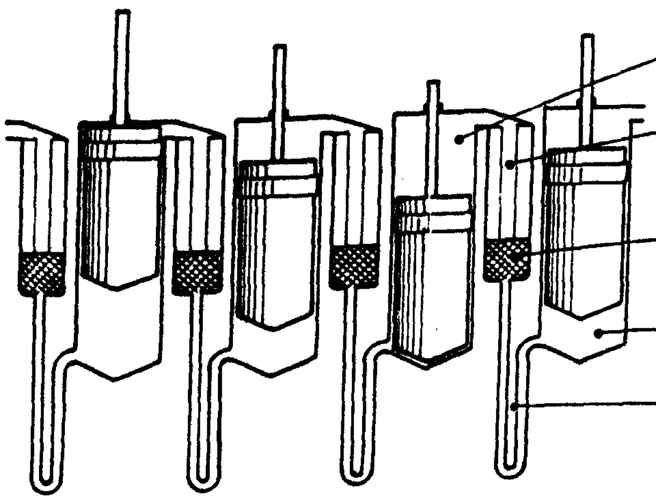
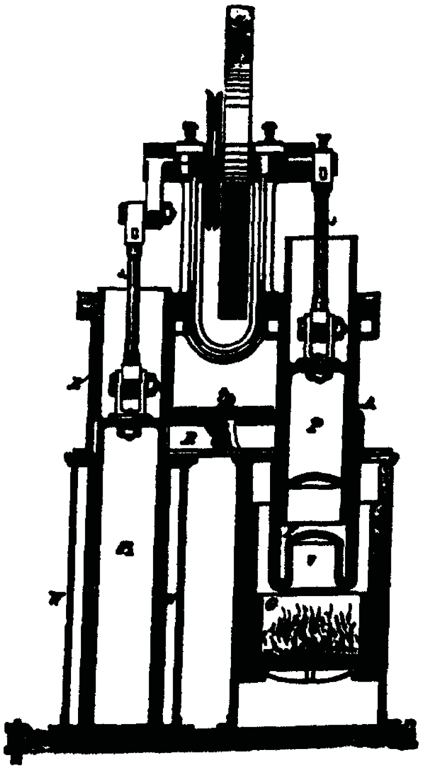


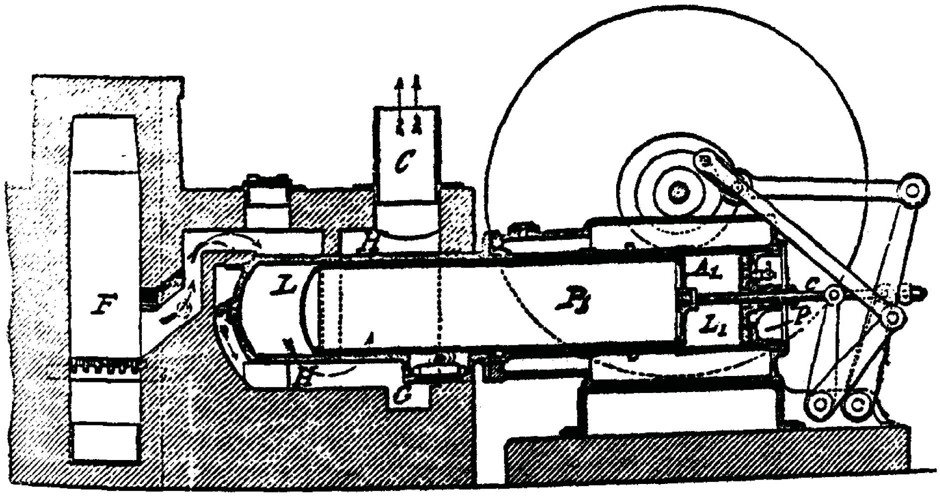
Figure 1.7 Alpha Compounding



Key	
A	Hot cylinder
B	Cold cylinder
F	Heater
G	Fire box
J	Connecting rod to hot piston
J_1	Connecting rod to cold piston
P	Hot piston
P_1	Cold piston
R	Regenerator
W	Water jacket

Figure 1.8 Rider Engine

Beta engines are by far the most common arrangement for single cylinder type engines. To obtain higher power densities, beta engines are often placed in banks of two or more engines which are geared to a common shaft. Most of the original Stirling ideas were beta arrangements, for example Figure 1.1. Another important early beta engine is Lehmann's machine on which Professor Gustav Schmidt did the first reasonable analysis of the Stirling cycle in 1871 (Figure 1.9). Philips derived their famous vibrationless rhombic drive for beta machines in the early nineteen sixties (Figure 1.10). Probably the most ingenious Stirling machines yet devised are the free-piston engines and heat pumps invented by Professor William Beale at Ohio University in the mid nineteen sixties. His machines are generally beta arrangements and employ no mechanical linkage system. An example of a machine developed by his company, Sunpower Incorporated, is shown in Figures 1.11.



Key	
A_1	Displacer base
C	Chimney
c	Sealing collar
F	Fire box
G	Plenum chamber
H	Hot section
L	Expansion space
L_1	Compression space
P	Piston
P_1	Displacer

Figure 1.9 Lehmann's Machine

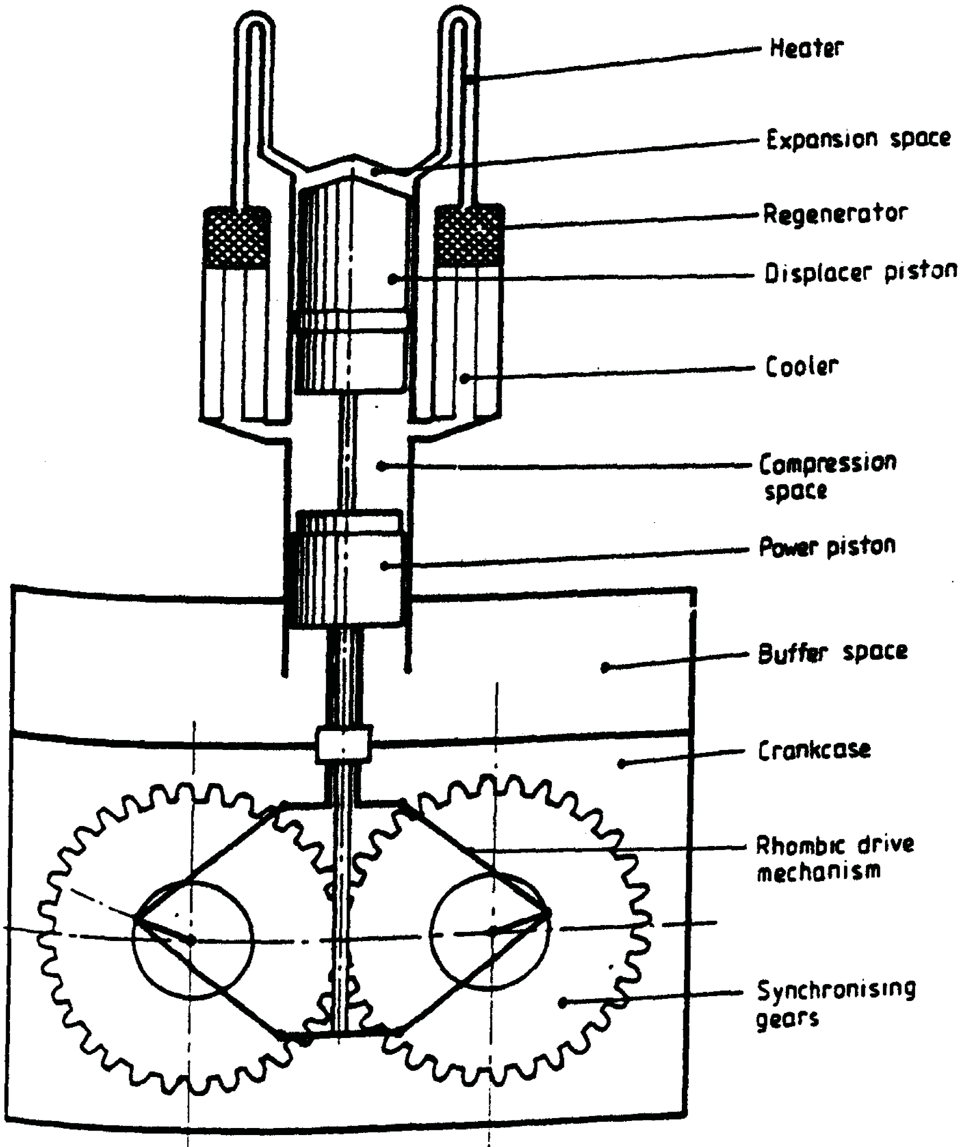


Figure 1.10 Philips Rhombic Drive

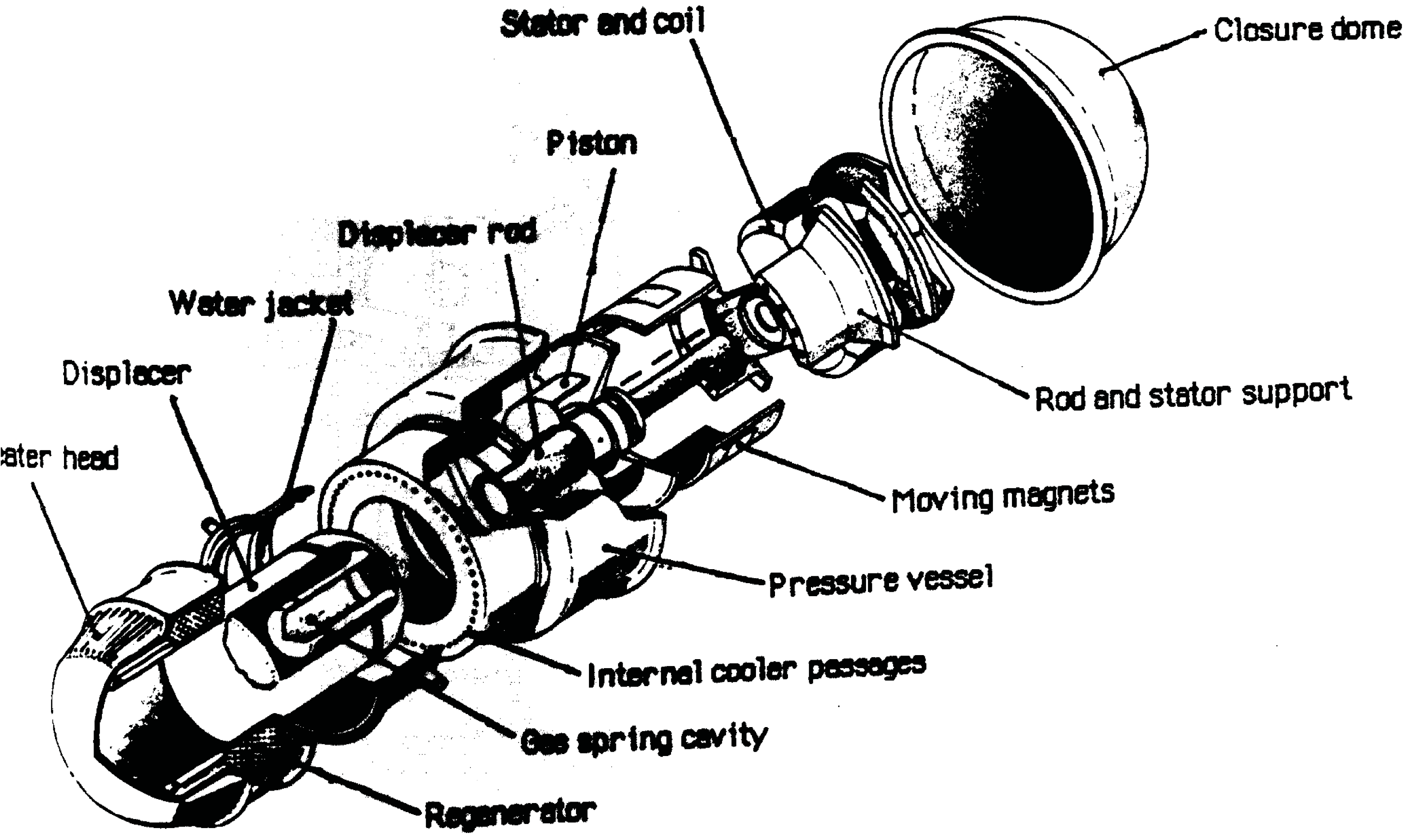


Figure 1.11 Beale Free-Piston Engine

Gamma engines tend to have somewhat larger dead (or unswept) volumes than either the alpha or beta engines. This often leads to a reduction in specific power. Gamma engines are therefore used when the advantages of having separate cylinders outweigh the power disadvantage. An example of this engine is the Robinson, which in Figure 1.12 employs a travelling regenerator in the displacer. One of the original engines built by the Stirling brothers around 1830 was also a gamma engine (Figure 1.13).

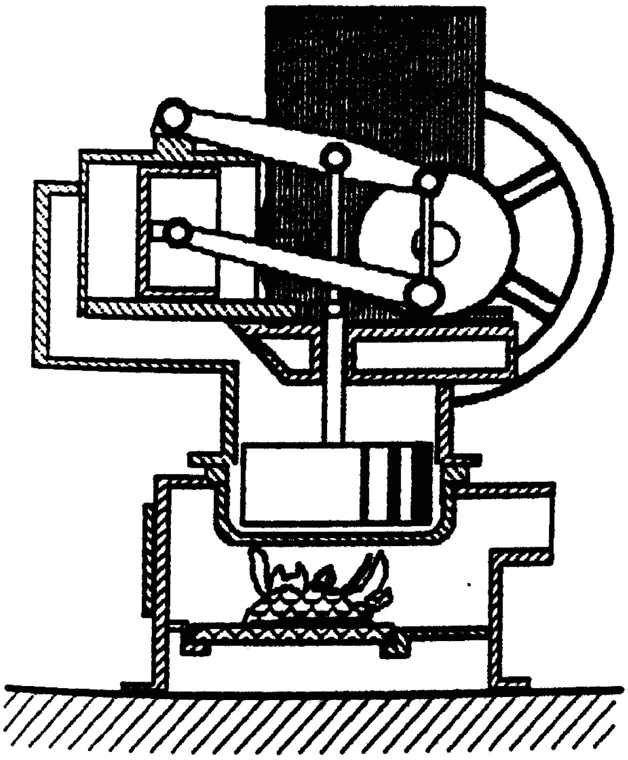
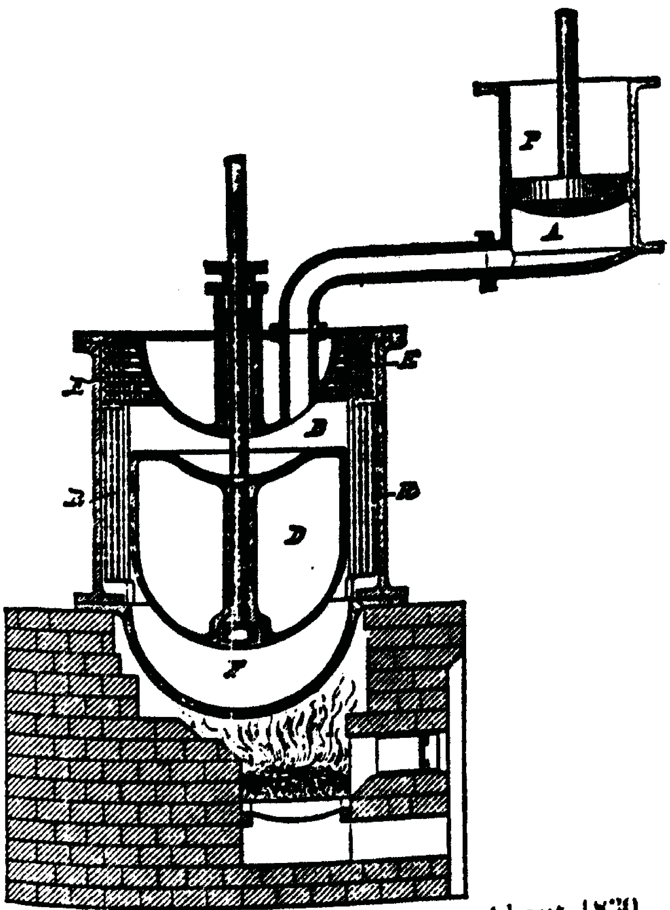


Figure 1.12 Robinson Engine



First Stirling Engine. About 1830.

Figure 1.13 Original Stirling Engine of about 1830

Key	
A	Compression space
B	Cold space
D	Displacer
E	Cooling tubes
F	Expansion space
P	Piston
R	Regenerator

1.4 Applications

Stirling cycle engines were in wide use up to the beginning of the First World War. The most attractive features in those days were the inherent simplicity of the machine, its ease of maintenance and its safety. Safety was an important consideration since the early steam engines could compete favorably on almost any other grounds but had unreliable boilers which had an alarming tendency to blow up. Surprising as it may seem, efficiency was also a consideration. Coal, the chief energy source had been climbing in price ever since the beginning of the Industrial Revolution. Typically an overall efficiency of 7% could be obtained with a Stirling engine [Be84] and this was considered quite competitive, indeed, even today this would be respectable in many applications. Table 1.1 lists the performance of some early Stirling engines.

Table 1.1 Air Engine Trials (taken from [Do1896])

Air engine	Bore (mm)	Stroke (mm)	Speed (RPM)	Indicated power (kW)	Brake power (kW)	Consumption of coal per hour (kg)	Mechanical efficiency (%)	Overall efficiency (%)*
Buckett	610	406	61	15.06	10.73	16.59	71	7
Bailey	372	175	106	1.77	0.98	4.52	55	2
Bénier	340	350	117.6	4.36	3.00	6.59	69	5
Stirling	406	1219	-	30.00	-	45.40	-	7(ind)
Ericsson	4.3m	-	-	239.00	-	262.30	-	10(ind)

*Based on average heating value of 32.5MJ/kg

In 1876 Otto presented his now famous Otto cycle engine. These engines were safe, reliable and, very importantly, had excellent specific power ratings which allowed them to be easily incorporated in motor driven transport. To further complicate matters for the Stirling engine, the Otto engines were cheap to run owing to their higher efficiencies and cheaper fuels. Broadly speaking, an Otto cycle's efficiency is dependent on its compression ratio [RM72], while that of the Stirling cycle is dependent on the temperature ratio of the hot and cold ends. Owing to material limitations, Stirling engines could not operate at the high temperatures necessary to ensure good efficiency. Furthermore, early Stirling engines did not often employ preheaters (also called recuperators), resulting in exhaust temperatures that were unnecessarily high with a consequent further reduction in overall efficiency. On the other hand, it was a fairly simple matter to design an Otto engine such that it operated with a compression ratio commensurate with higher efficiency. Thus the rising successes of the early Otto engines and the introduction of boiler codes which made steam engines safe soon overwhelmed the technologically retarded Stirling engine. However, even in these circumstances, in the remoter areas of the world, the Stirling engine enjoyed a longer success*. Here the Stirling's ability to burn almost any fuel helped grant the engine a longer stay.

The recent interest in the Stirling engine's high theoretical efficiency has been encouraged by high inflation rates in the cost of primary energy sources, particularly in the early ninteen seventies. Many governments and organisations have committed vast sums of money to research the possibility of the Stirling engine replacing the Otto and Diesel engines as

*For example, Stirling engines were used in India until recently - and it is still possible to purchase old engines there.

the prime mover of the world's automobiles. The fact that it should be so considered is not altogether clear. It is true that of all practical engines the Stirling has the highest theoretical efficiency for a particular source to sink temperature differential. Unfortunately, the cold end is limited by the ambient temperature and the hot end is limited by the strength of the materials that can be used in the manufacture of the heater. At the moment, ignoring future possibilities with ceramics [MC85], the maximum temperature that can be tolerated is about 1000K owing to the high pressures which the heater must contain. This gives an ideal cycle efficiency of about 67% assuming a cooler temperature of 300K (See §2). For typical compression ratios, the ideal cycle efficiencies of the Otto and Diesel are around 60% and 63% respectively. Since internal combustion engines currently operate at around 30% brake efficiency, there is a lot of room for improvement and it would appear that, efficiency wise, the Stirling is only marginally competitive if at all. However, should there be a major breakthrough in materials technology which would allow heaters to be designed to safely tolerate considerably higher temperatures at pressures of around 160bar, the automotive Stirling would then have an advantage in ideal efficiency (eg, for a source temperature of 1600K and a sink temperature of 300K the ideal efficiency would be 81%). Under such circumstances the Stirling could possibly provide an attractive alternate automotive power plant.

Perhaps another way in which the automotive Stirling may be successful would be to utilise the engine's potential for regenerative braking. To this end a thermal energy store would have to be attached to the hot end to provide an energy store to which heat would be pumped by reversing the thermodynamic cycle. During braking, the engine would be so designed that reversing the thermodynamic cycle would brake the vehicle and at the same time regenerate the thermal store. By effectively using the braking energy normally wasted during braking, a considerable overall energy saving could be made. This may even allow the engine to operate at a lower thermal efficiency than its internal combustion counterparts and yet maintain an overall energy usage advantage. The design requirements of the engine may even be a good deal less stringent than by competing directly on an engine efficiency basis. Various mechanical systems which offer this flexibility to the Stirling engine have been patented by the author [Be81.1, Be81.2]. One example of a mechanism which allows the stroke and phase angle between the pistons to be independently varied is shown in Figure 1.14. By varying the stroke and phase in this manner the power of the engine may be continuously changed to meet the demands of the circumstance while maintaining high efficiency. The phase angles can be changed sufficiently to reverse the thermodynamic process and convert the engine into a heat pump. In this configuration the engine brakes the vehicle by converting kinetic energy into thermal energy stored locally in the heater section of the engine. This energy would then be available for immediate reuse if needed. Though the engine mechanism is more complicated for such a device, the transmission and gear box is greatly simplified and is, therefore, of much less mass. The overall system mass may therefore not be much different from the more conventional method of using mean pressure control on the engine and a standard gear box which, of course, is not a regenerative braking system. Finally, two other important advantages are the much higher response speed inherent in a phase-stroke control system [Wa80] and a good part-load engine efficiency curve. Figure 1.15 shows how indicated power and efficiency vary with phase angle; at zero phase the losses are predominantly gas hysteresis losses while at 180° the losses are mainly viscous dissipation losses. By including stroke control both these losses are minimised and consequently the efficiency curve is even flatter.

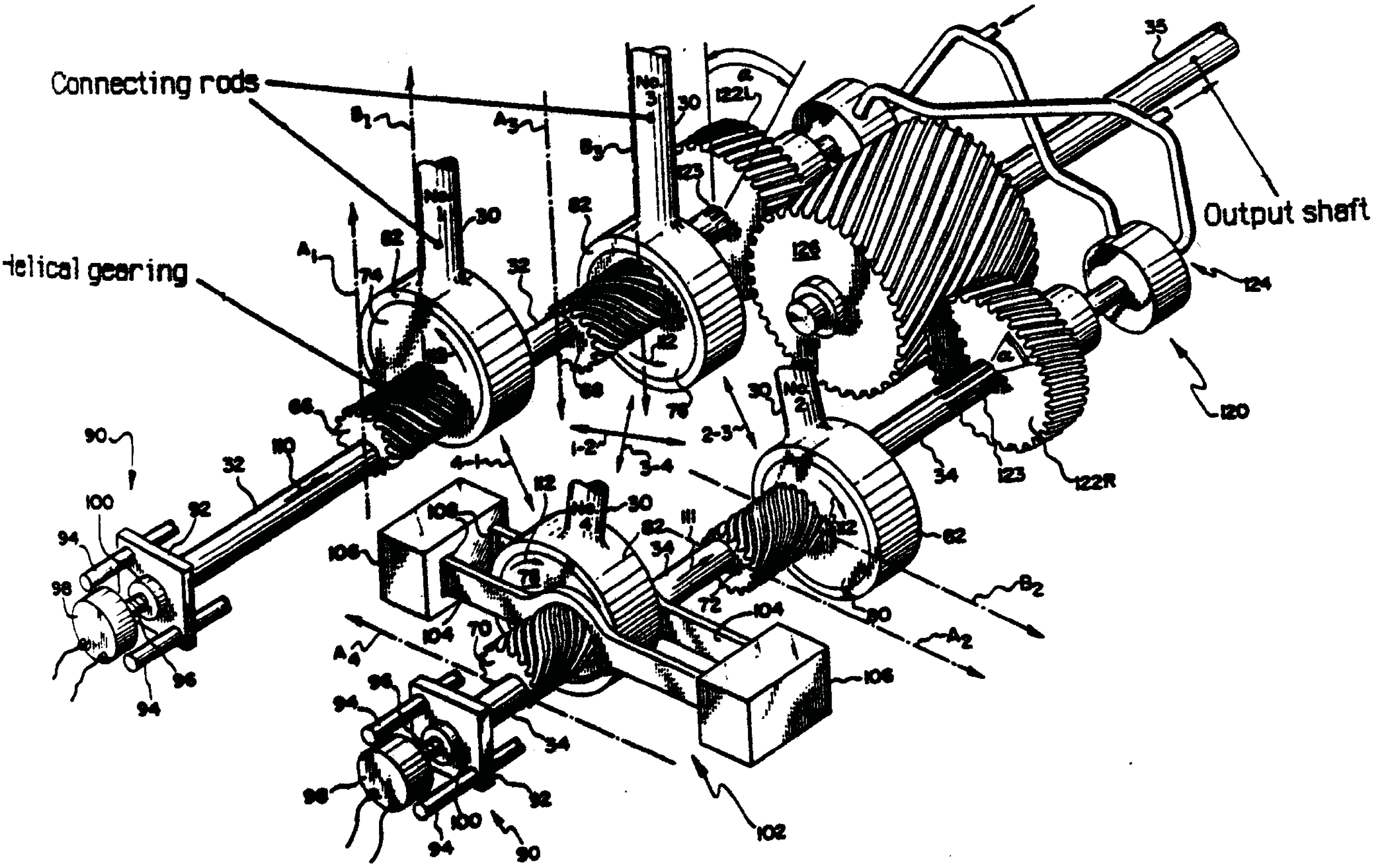


Figure 1.14 Phase-Stroke Control for Double Acting Alpha Engines

Notwithstanding the possibilities of regenerative braking and though the Stirling has been revived on the basis of efficiency, it seems that the engine's other virtues will be the reason for its ultimate success as an important future engine. As it now stands, the most significant advantages are its multifuel capability (in particular, its ability to burn solid fuels), its potential for ultra high reliability and low emissions.

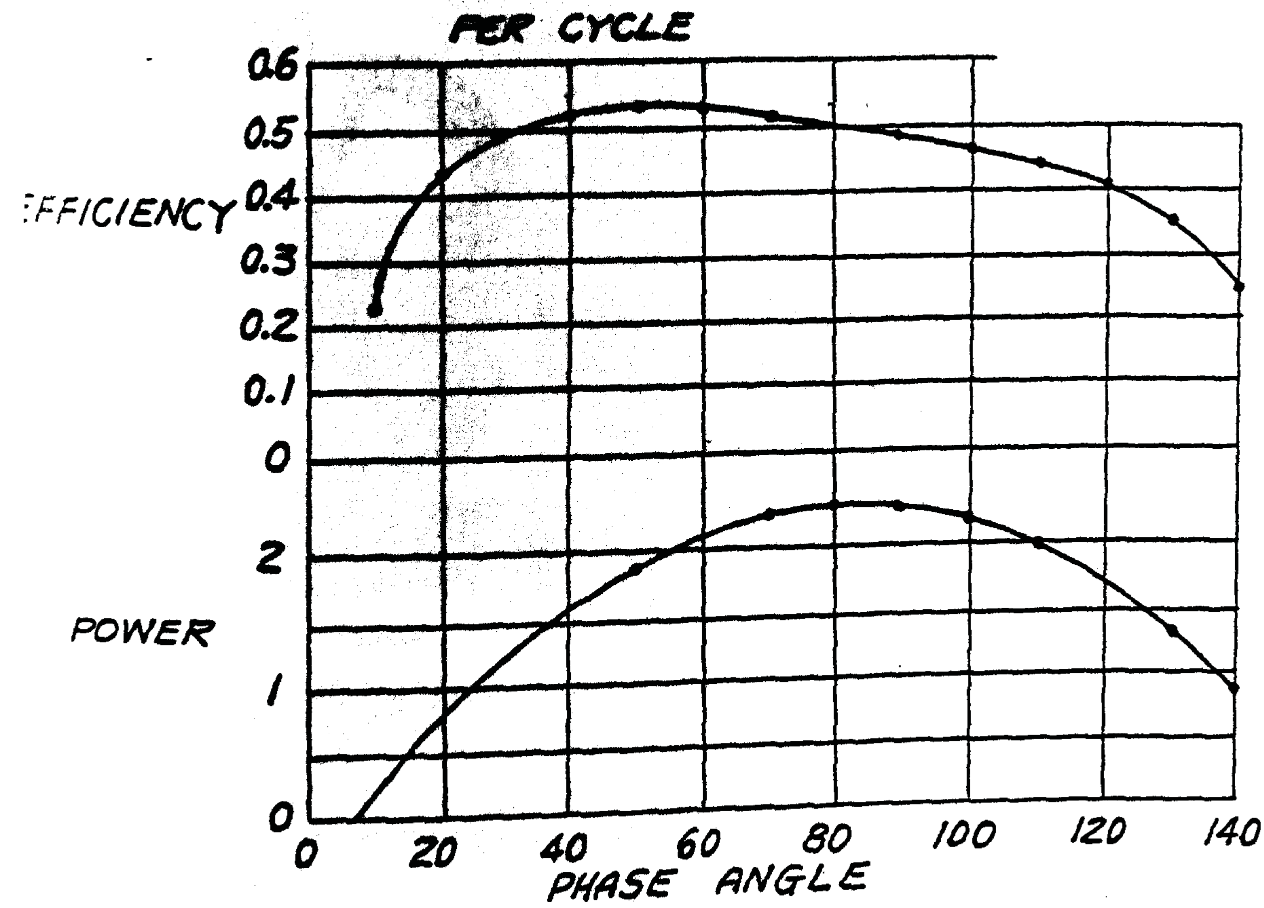


Figure 1.15 Indicated Efficiency and Power vs Phase Angle

In many parts of the world, particularly the Third World countries, the only meaningful energy sources are usually solar and one or more of the solid fuels, for example, coal, animal dung, rice husks, etc. In these situations, the Stirling engine has no rival. Its ease of maintenance, reliability and simplicity help avoid the alienation that more sophisticated machines tend to create. Such an engine which was designed by Sunpower Inc is shown in Figure 1.16 using rice husks as fuel. The prototype was capable of 5kW brake power at 20% brake efficiency (excluding combustor efficiency). This engine has been developed to higher performance and is currently in production in India under the auspices of Stirling Technology Inc.

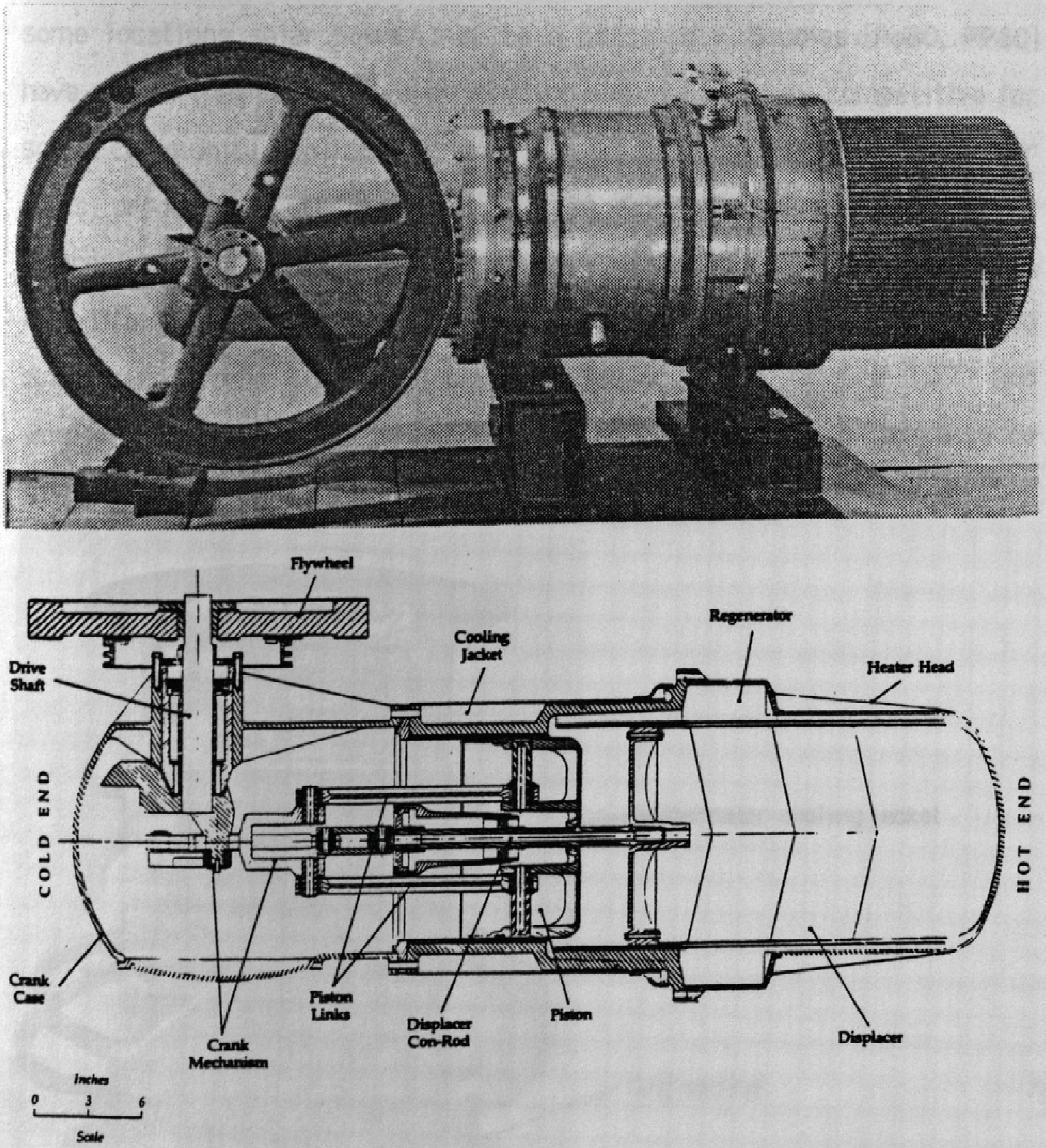


Figure 1.16 Third World Engine (Sunpower Inc and Stirling Technology Inc)

As unattended remote power generators for such applications as repeater stations, the Stirling has excellent potential. One such unit, designed by members of the Harwell laboratories of the United Kingdom Atomic Energy Research Authority (UKAERA), has operated for at least ten years without any significant maintenance other than renewing its fuel [CF74, Co77]. In some locations solar power may be a possibility. Studies [Po80, PP80] have shown that solar powered Stirling engines are very competitive for small community applications. In 1978 US dollars, the capital cost for mass produced systems comes out at 90¢/Watt. Free-piston linear alternator combinations are particularly attractive in solar and quiet long-life electric generator applications. Sunpower Inc has designed and built a prototype 10kW(e) free-piston solar engine (Figure 1.17) and smaller electric generator engines, one of which has been delivered to Kawasaki of Japan for evaluation as a gas-fired electric generator.

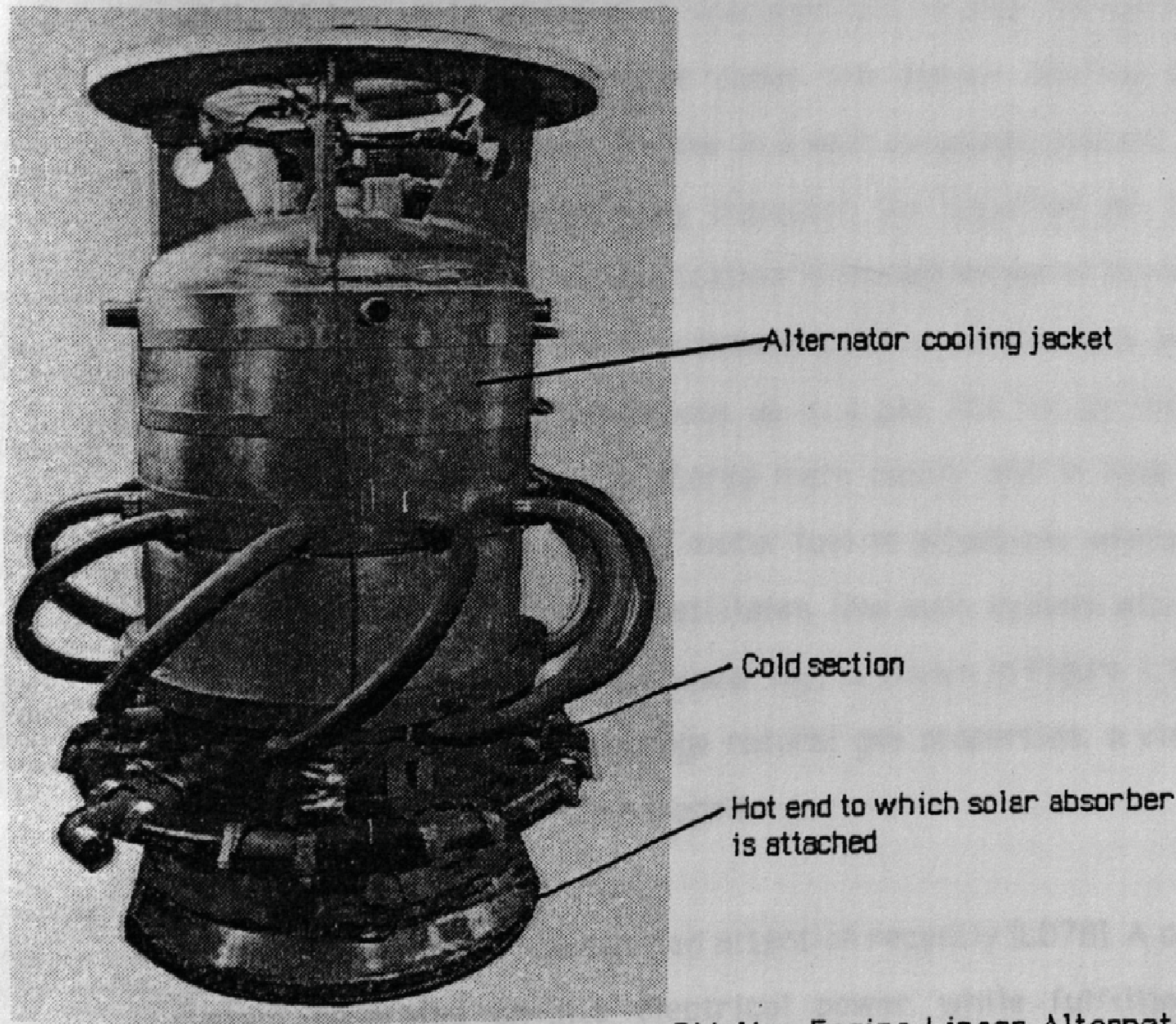


Figure 1.17 10kW(e) Solar Free-Piston Stirling Engine Linear Alternator Combination.

Recently a new field of applications has been opened by a practical demonstration of the duplex Stirling configuration. The duplex Stirling system involves two free-piston Stirling machines in a back-to-back arrangement (Figure 1.18). One machine operates as a heat engine and drives the other which then operates as a heat pump. This idea was first suggested by Professor Beale at Ohio University in 1965 [Bea69]. Immediate applications include gas fired food freezers, domestic gas fired heat pumps and natural gas liquefaction [PU84, Be82]. Of these the more intriguing is that of natural gas liquefaction. There are many remote gas wells of low yield where it is uneconomical to pipe the gas to a central distribution network. In these cases the duplex Stirling enables the natural gas to be liquefied on site in a self-contained system. A container vehicle would then periodically transport the liquefied gas to a central distribution point. Another application is the advantage of having a ready supply of liquid fuel --- either domestically or in the work place. Here a duplex Stirling system would hook up to a gas line for on-line liquid gas production. Liquefied gas is stored more easily and in less space than pressurized gas and is an ideal motor fuel in situations where its cost is competitive with petroleum distillates. One such system which has been undergoing development at Sunpower Inc, is shown in Figure 1.19. From the energy balance and given average natural gas properties, a yield of 3.3kg liquefied to 1kg burned is envisaged.

Finally, cogeneration has elicited attention recently [LD78]. A cogeneration system generates on-site electrical power while fulfilling the site demands for heating from the waste heat generated by the prime mover. Silent operation and long life are important features in this application.

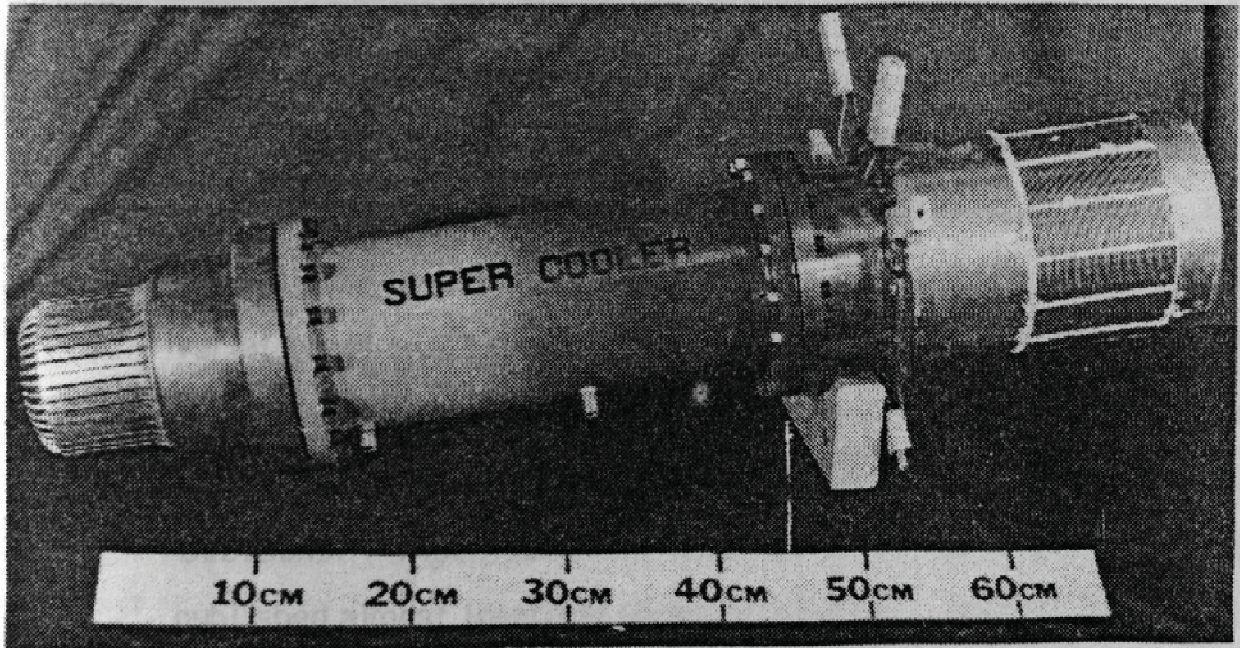


Figure 1.18 Free-Piston Duplex Stirling Machine

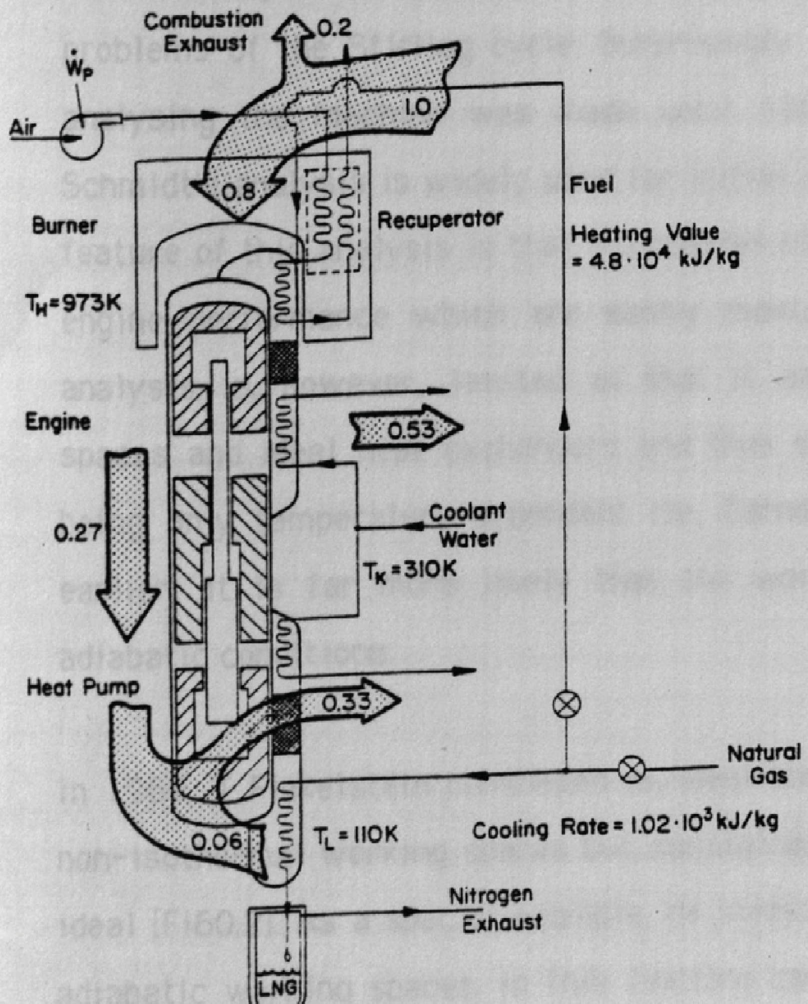


Figure 1.19 Yield of Natural Gas Liquefier

1.5 Analytical Research

Compared with the analysis of the Otto or Diesel cycles, the Stirling cycle presents considerable difficulties. Chiefly responsible for these difficulties is that the working gas is at any instant in a continuous variation of state from one end of the machine to the other. The first effort to address this issue was made by Professor Gustav Schmidt of the German Polytechnic Institute in Prague and published in *Zeitshrift fur Deutscher Ingenieure* in 1871 [Sc1871]. More recently, others have published almost identical forms of Schmidt's analysis [RD46, Fi53, De53, KJ54, Fi60.1, Kö60, Ki62, Wa62.1, Ki63, Me68, UR76, Ur77, Be78, Go78, Wa80, RH83, UB84]. Schmidt was certainly an extraordinary individual as indicated by his appreciation of the thermodynamics and practical problems of the Stirling cycle. Surprisingly, very little improvement in analysing the machine was made until 1960, and indeed, even today Schmidt's analysis is widely used for initial sizing of engines. A valuable feature of this analysis is that it produces closed-form solutions for the engine performance which are easily manipulated by a designer. The analysis is, however, limited in that it assumes isothermal working spaces and ideal heat exchangers and thus calculates the efficiency as being only temperature dependent (ie, Carnot efficiency). As explained earlier, it is far more likely that the working spaces will approach adiabatic conditions.

In 1960, T Finkelstein completed an ideal analysis in which he assumed non-isothermal working spaces but maintained the heat exchangers to be ideal [Fi60.2]. As a special example, he investigated the limiting case of adiabatic working spaces. In this limiting case, Finkelstein was able to show that the ideal efficiency was not equal to Carnot. This analysis has now become the ideal by which the ultimate performance of a Stirling engine may be judged. Unfortunately, Finkelstein was not able to obtain closed-form solutions and therefore resorted to numerical techniques for solution. Walker and Kahn [WK65] optimised Finkelstein's analysis for the special case of adiabatic working spaces.

Both the ideal analyses of Schmidt and Finkelstein are highly optimistic. They only give a reasonable indication of engine performance at the point of maximum efficiency since at this point irreversibilities (or non-ideal behaviour) must be at a minimum. Off design characteristics, which are also important, are thus more difficult to estimate.

Recent Stirling cycle research has been evolving along two distinct paths, namely: a) Techniques by which the irreversibilities are treated in a decoupled manner from the basic ideal cycle; the overall performance then being determined by calculating the reduction of the ideal performance due to the irreversibilities. b) Numerical simulation of the complete cycle without any attempt to decouple any of the processes.

In the first group, Creswick [Cr65] was the first to decouple the irreversibilities from the ideal cycle. The thrust of Creswick's analysis was to obtain simple relationships that could be used as a guide in design. His ideal cycle was that of Schmidt which he used to determine heat rates and mass flows. The losses were evaluated in a qualitative way in that proportionalities were derived rather than absolute quantities.

Qvale, under Professor J Smith at MIT was the first to decouple the irreversibilities from the ideal cycle in an extensive analysis [Qv67]. Much of this work was later published with Smith [QS68]. Qvale's ideal cycle consisted of adiabatic working spaces varying sinusoidally and ideal heat exchangers. Qvale and Smith published an analysis of the regenerator in which the concept of enthalpy flux was established [QS69]. Rios, also working under Professor J Smith, [Ri69, RS70] extended and refined the approach adopted by Qvale. The relative effects of decoupling the system were examined in greater detail, particularly the losses due to pressure drop and non-sinusoidal motion of the moving parts. Good correlation was found between Rios's analysis and the experimental results from a Stirling refrigerator. Rios also analysed the effects of incidental irreversibilities such as the losses associated with the clearance gap around the displacer [Ri71]. The most recent contribution to the decoupled approach from MIT is due to Lee and Smith [LS80]. They investigated the specific problems associated with heat transfer in the working spaces of Stirling engines. Later Lee developed his own version of the Rios-Smith analysis [Le81], again using the ideal adiabatic analysis as the basic cycle. The major advantage of the decoupled approach is that a detailed accounting of the losses is produced which is very useful as an aid in making modifications or design changes.

Qvale, Rios and Lee's basic cycle theory, though derived slightly differently from Finkelstein's ideal cycle, is essentially the same. Also the method of solution is the same. That is, numerically solving the derived set of differential equations by time stepping integration. Even for this simple model, solutions of this type are impractical on small computers in that the calculation times are very long. And simple, approximate hand calculations are not possible. This is a limitation in preliminary design in that even for very simple initial studies, a large computer and the appropriate code would be required.

Martini [Ma78] was the first to derive a simplified decoupled analysis which gave a reasonably good indication of how a Stirling cycle device might perform. He relied as far as possible on closed-form solutions. The only use of numerical integration was for cases in which the motions of the moving parts were not sinusoidal. Since his basic cycle was the Schmidt cycle, this only involved integrating the work and heat equations. Convergence of the analysis required only a few iterations and the entire analysis could easily fit into the larger programmable hand caculators. Though Martini's analysis is limited by the isothermal basic cycle, he has shown [Ma80] that his simple analysis will predict the performance of real engines with comparable and possibly better accuracy than the analysis by Rios. Martini has developed an adiabatic version for improved accuracy which has not yet been published. His adiabatic version excludes the effect of gas enthalpy entering and leaving the working spaces which is an inaccurate assumption (this point will be elaborated on later).

Another approach to the decoupled analysis has been that of Godin [Go78], Rauch [Ra80] and Chen *et al* [CG83, CG84, CG85.1, CG85.2, CG86]. In these analyses the describing equations are linearized which allows them to be solved much more rapidly, even on personal computers. Linearization assumes that all cyclic variations are dominated by the first harmonic (or fundamental component). The errors introduced by linearization are small for a large majority of practical engines. For some low pressure crank engines the error in linearization is unacceptably large. Godin's analysis is limited by assuming isothermal working spaces and Rauch's analysis, though it includes adiabatic working spaces, is limited by exclusion of the gas enthalpy entering and leaving the working spaces. This exclusion is not made in the work of Chen *et al*. Not all of the irreversibilities are decoupled in Chen *et al*'s analysis, some losses are included directly in the analysis as in the third order approach. In their most recent work (authored only by Chen and Griffin) [CG86], the analysis is extended to include the dynamics of free-piston machines. Though Chen and Griffin mention that their analysis is semi closed-form and point out the inadequacies of the typical time-stepping explicit simulation methods, their solution is still essentially numerical in that a sizeable matrix needs to be inverted in an iterative process. However, the describing equations are manipulated to a point where they do begin to offer some physical insight into the thermodynamic and gas dynamic processes occuring in the engine. The dynamic solutions presented in [CG86] follow standard numerical solution procedures for linear systems and do not answer basic design questions about the dynamics of free-piston engines. Both Rauch and Chen *et al* did derive some basic relationships that would be useful in preliminary design but they did not particularly address the issues of design and optimisation.

The second path of analysis, ie, the method of numerical simulation, has mainly been argued along the lines that the Stirling cycle consists of a number of interrelated processes and cannot be decoupled in a simple manner [Ur77]. Though this point may be true in the pursuit of highly accurate calculations, it does not appear to be a limitation for reasonably accurate approximate solutions.

Urieli [Ur77] has made an interesting distinction between numerical simulation and the traditional methods of analysis, namely theoretical and experimental analysis. He points out that numerical (or computer) simulation is different from theoretical and experimental analysis but has aspects of both. Numerical simulation does not have the rigour of theoretical (or mathematical) analysis but relies upon fundamental mathematics to set up the equations for solution. On the other hand, it is much closer to experimental analysis and retains many of its disadvantages. The analyst 'turns on' the computer and waits to see what happens, just as the experimenter does. A numerical 'experiment' is run with a particular configuration of machine and the effects of varying the different parameters on the performance can only be determined after an exhaustive set of trials. Simulation has the advantage over experiment in that the configuration of the system can be varied at will. For example, the effects may be examined of replacing the working gas, say air, with helium or hydrogen without involving additional expensive equipment and safety measures. Tests of the sensitivity of performance to independent theoretical approximations may be made. Furthermore, there is no concern with the accuracy of the instruments, or whether or not they interfere with or modify the parameter that is being measured; there is continuous access to all parameters at even the most inaccessible locations in the machine. The list is extensive.

A note of caution is, however, in order. A simulation model will always be