Third Thoughts on the New Mathematics

by
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SINCE I was in Johannesburg in July 1964, the mathematical revolution has accelerated so that much that was new and relevant then is now in some danger of rejection either as novelty for novelty's sake or as something by-passed by more recent thought and experiment. The effort to be fully 'with it' is a considerable one. A few years ago the educational vogue word was 'new': we had new this and new that, but now the password is 'creative'. I wonder how many of the users of 'creative' really know what it means, or whether they realise that the majority of people never will be creative, except in a very limited sense of the term. Will an attempt to make poets or artists of us all have any beneficial effect? Maybe such an attempt will have the unhappy effect of making us dislike these delights.

Few people now dispute the inadequacy of traditional mathematics teaching, and many are the panaceas offered, from apparatus of a bewildering variety to experiment, exploration and discovery and a search for pattern and the structure of mathematics. Certainly, the rule of dictatorship by the teacher, the teaching by authoritarian prescription, is on its way out. No longer must a teacher ask a child, 'Don't you remember the rule?' but rather he must persuasively and gently ask, 'Do you see the pattern?' Gone are the days when the child was faced with a tedious succession of pedestrian tasks requiring him to do just what he was told, when he was told and in the way he had been told. It is the structure, the pattern, that is the elusive object of the search, elusive indeed because it is a structure of relationships between concepts, themselves abstractions having existence only in the mind of man. Whitehead said in 1925, 'The science of pure mathematics may claim to be the most original creation of the human spirit.' It is this sublime creation that we ask our children to appreciate and understand.

Mathematics is an adventure in ideas, and the ideas are the abstractions. In broad outline, the progression in this adventure must be:

1. The experience of the ideas.
2. Experiment in and with the ideas.
3. Discussion and creation of further experiment for enrichment of the emerging concept.

The first stage takes a stand on the necessity for experience as the only satisfactory foundation on which concepts can be built. This, of course, can be argued, but it is not my intention to do that here. In the provision of this experience the teacher seeks for the child the best experiences with mathematics which can be provided in his environment. The child is brought face to face with the mathematics itself, the role of the teacher being to contrive the confrontation and then step back and allow the mathematics to communicate directly with the child. In this the teacher has the difficult task of guidance and stimulation which he can only do effectively through his professional skill and through his knowledge of the child. To know the child he must, above all else, listen to him. He must be a good and constant listener as this is his one sure means of relating the material to the readiness and to the individuality of the child. It is when talking, more than at any other time, that there is close communication between teacher and child.

The emphasis, then, is on experience. It is the purpose of this article to examine the place of experience, and the sort of experience that is likely to lead to learning. Let Carlyle have the first word, 'Experience is the best of schoolmasters but the fees are heavy.' Not bad for the middle 19th century! If this whole question of the provision of experience is considered then further elaboration is needed or it might be interpreted as just continuously doing things, an inadequate and misleading interpretation. It is necessary to break down the types of experience needed in a child's mathematical development into categories. There are really only three types — doing, talking and listening — and each is valuable. For all of these, certain obvious criteria for their introduction and use are desirable:

(*a*) a reasonable degree of readiness.

(*b*) an active participation by the child.

(*c*) a clear appreciation by the teacher of the fundamental mathematical ideas involved.

As a rider to these, it is well to remember that a child may be 'actively participating' when sitting perfectly still and quiet. An absorbing and professionally skilful class lesson can be a most stimulating and valuable experience. Other such experiences are listening to the radio, reading, looking at films or television, in all of which a child is in close communion with another mind.

A few words, in parenthesis, about the teacher-given class lesson, a topic of controversy, some holding that it is outmoded as a teaching medium, others feeling that, come-what-may, it will always have a part to play. Generally speaking, the users of structured apparatus and of experimental methods tend to play down the class lesson, the believers in a rather more comprehensive experiential basis retaining it. In all cases, however, it is less prominent than hitherto and this shift of emphasis is natural as an expression of the powerful reaction to traditional methods of chalk and talk. The writer recently attended a teachers' course in England, at a Teacher Training College. In the room allotted there was no wall blackboard, and a blackboard and easel were obtained only after much searching, it being said, 'Lecturing with blackboard is discouraged now!' An odd state of affairs, surely, as the blackboard must remain an essential piece of equipment. It is one of the readiest means of contact a teacher has with his pupils.

Certain mathematical ideas can obviously only adequately be gained through doing. The child, by his own efforts, has to learn how to learn and in so doing will make many mistakes. Praise be that the value of error is now recognised and that the traditional right/wrong criterion in mathematics is suspect. Removing the *correctness* of arithmetic from the authority of the teacher has greatly increased child motivation; no longer does the tick tyrannise. Jerrold Zacharias writes, 'Science is a game played against nature, it is not a game played against the teacher.' Another American, a leader in the move towards commonsense in mathematics and a pioneer of new methods, says, 'Every child gives the right answer to every question as seen from his point of view.' A case, it seems, of the truth as far as in me lies!

In his experimenting the greater the child's freedom to choose his own method and to decide what is for him an acceptable answer the more will he take his work seriously. But it should not be forgotten that learning by doing is not always good; it is only good if the doing itself is good; if it is bad then there is little learning and, maybe, much waste of time and effort. It is here that the teacher has an essential task, that of directing the child towards good doing, towards practical and experimental work well saturated mathematically and of a degree of difficulty suitable to the child. There is a vast field of exploration and discovery in a child's environment and in science. Many books are now available for those wishing to encourage mathematical experiment. Among these must list high the H.M.S.O. pamphlet *Mathematics in the Primary School*, a veritable feast of experiment and a most useful source book.

The second kind of experience, talking, follows naturally from the first. Doing leads to discussion, the child wanting to pass on his discoveries to his classmates. This, indeed, is one of the rewards of the experimental method; the child wants to communicate and, at the same time, clarify his own thinking. 'How do I know what I think until I hear what I say?' In this talking the child is the true son of his parents, both of whom, in their different ways, indulge from time to time in fruitful discussion. This necessity for talking stresses the important part played by language in mathematics as in all studies. Much of the early work must have as an aim the acquisition and enlargement of vocabulary. This is accepted. Quite soon in his development, however, the child finds the need for symbols, the true mathematical language. Useful though it is, English is limited, it lacks the precision and the power of the symbol, which perfectly fit the ideas. Consequently, as an integral part of vocabulary training, and as a valuable mathematical tool for later use, the child must be introduced to such symbols as:

$=$ (equal to),

$\neq$ (not equal to),

$\simeq$ (approximately equal to),

$>$ (greater than),

$<$ (less than) and the easier set theory symbols

$\in$ (belongs to), $\cap$ (intersection), $\cup$ (union).

How much simpler it is to write $8 \times 7 = 56$ or $19 \times 21 = 20^2$ than the longhand form of these! A useful type of exercise in this language work and training is the translation from word to symbol and vice versa e.g. Write in mathematical form, using no words:

(*a*) Two and three make five.

(*b*) Three tens are greater than four sevens.

(*c*) The difference between three nines and five sevens is equal to the sum of the square of two and one fifth of twenty.

Such exercises are easily graded from the simplest to post graduate level. They can be either spoken or written if the translation is from symbol to common tongue.

The third experiential type is that of listening. This must be taken to include reading and looking. A book that interests is a rich experience; a lesson that absorbs a child is a memorable experience as we all know. A film broadcast or television programme can have equal virtue. All add to and enrich the background of the child. All take their part in creating right attitudes, a very important goal of early mathematics teaching and learning. The Nuffield Project in operation now in England, takes as its main criteria the 3 P's, Pleasure, Profit and Purpose, and good criteria they are.

The vogue words in education today are *discovery* and *creative*. What they will be in 1976 is an intriguing speculation. It is worth examining the claims and virtues of the so-called discovery way of learning. Is it new or is it a rehash of what was called, 60 years ago, the heuristic method? What exactly do we mean by discovery? Is it as important as is claimed? These are questions to be answered, bearing in mind, of course, that the answers will depend largely on the objectives of the course of work. If the traditional stress is on arithmetic as a complex of manipulative, computational skills, then discovery methods have little merit. It is when an understanding of mathematical ideas, concepts and such is made a main objective that discovery becomes a force to be reckoned with. These ideas can only be acquired, in Piaget's words, through the 'thoughtwork on the experience', a phrase aptly underlining the essential aspects of learning. When we add to this his definition of thought as 'internalised action' then the experiential aspect is reinforced.

The young child lives in a world of constant exploration and discovery; his environment is an exciting challenge. When he reaches school age the urge is still very strong and the art of the Infant School is to harness and control it without suppressing it. As he grows older the excitement of discovery often dies and the eagerness and enthusiasm die with it. How often do we hear a teacher saying how a child has lost most of his creative talent by 11 or 12, especially in art, music and drama, but little less so in mathematics. One of the reasons for this is the dogmatic nature of much of our teaching to young children. It may well be that learning by exploration and discovery will keep alive that zest, that willingness to have a go, that courage of his convictions so desperately lacking in many children. It is astonishing how few children will hold fast to their views when they are in any way challenged. Lambourn wrote thirty years ago in his *Reason in Arithmetic* of his way of testing new entrants. 'When I admit a new boy to my school I usually say to him, "Three eights?" The answer almost always comes promptly or not at all — it is very rare for a boy to reckon the answer if he fails to remember it. If he says, "Twenty four!", I ask, "How do you know?" and he generally replies, "They told me at the other school." Now you must tell a child that there are three persons in the Trinity, because that is a dogma: but it is a sin against a child's mind to tell him what three eights are. When a boy says they told him that three eights are twenty four I usually reply, "Well here we say three eights are something else". It is rarely indeed that he raises any objections: he is usually quite prepared to believe that three eights is whatever the local Head Teacher chooses to make it.' All teachers of mathematics in the Bad Old Days were not lacking in skill and originality! The point he so brilliantly makes is surely a relevant one to us today.

It may help to draw attention to some of the guiding principles and objectives behind this new way of learning, insofar as it is new. In brief, first there is guiding the child to discover pattern in the world around him, to extend through experiment the *open-ended* mathematical situations around him, to understand fundamental mathematical laws and to master certain arithmetical techniques. This implies the belief that mathematics can be discovered, that children have the ability to discover it and that mathematics is exciting and surprisingly unpredictable in so much as it is not solely an intellectual pursuit, but rather that it gives considerable scope for enlightened intuition. What a thrill it is when we suddenly *see* a mathematical pattern, something clicks and we see the rule governing an aspect of mathematics. As teachers we must do all we can to *arrange* these intuitive flashes and, for the moment at least, forget all about rigorous proof. That can await its proper time and place.

There is a good case for learning through discovery. Attitudes can improve astonishingly and thought takes its rightful place in the work. I shall close with a quotation from an unlikely source. D. H. Lawrence once wrote, "Man is a Thought Adventurer. But by thought we mean, of course, discovery. We don't mean this telling himself of stale facts and drawing false deductions, which usually passes for thought. Thought is an adventure, not a trick." Or in the words of G. K. Chesterton: "Merely having an open mind is nothing; the object of opening the mind, as of opening the mouth, is to shut it again on something solid." Let the last word be with these two.