

A GEOSTATISTICAL STUDY OF THE COPPER OREBODY
WITHIN THE PHALABORWA COMPLEX

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ABSTRACT

This Report makes a geostatistical comparison between Benches 10 and 20 of the Palabora open pit and presents a suite of geostatistics programmes developed for the Sinclair Spectrum personal computer.

It has always been assumed for planning and grade control purposes, that values in one bench can be extrapolated into the bench below and this is an attempt to validate the assumption.

Assays are done on drill chip samples from every blasthole drilled in copper bearing composites; the values are then recorded by position on 1:500 scale Bench Plans. These data are regularised by averaging values within 10m x 10m blocks which are then used in the geostatistical analysis. A three parameter lognormal model was used to represent the distribution; the two benches were compared with each other in the analysis through to Kriging of each central area.

Although the benches were found not to have the same distribution there is a high degree of similarity and the use of geostatistics would improve the accuracy of grade predictions.

DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

Adrian Patrick Jiro

27th day of June, 1984

To my Daughter
Alison

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PREFACE

This study was undertaken at the suggestion of the Chief Geologist of Palabora Mining Company Ltd, to test the regularity of the deposit with depth by geostatistically comparing two separate benches. An assumption that the deposit is vertically consistent is made for planning purposes.

Due to the limited scope, time and resources available for the study, the benches were considered in two dimensions only; vertically in line central areas 500m x 500m were selected on benches 10 and 20 and divided into 10m x 10m blocks. An analysis was then made of the contained blasthole assay data and a comparison made of each bench by distribution parameters, semi-variograms and the Kriging of a smaller area.

The study also required the development of a full suite of geostatistics programmes which were written in BASIC for the Sinclair Spectrum personal computer.

I am indebted to the management of Palabora Mining Company Ltd. for permission to use the geological data and to Professor D.G. Krige for his supervision and guidance.

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1 INTRODUCTION

1.1 Statement and analysis of the problem

The Palabora Mining Company operates a low grade +327 000 ton per day open pit copper mine in the Northern Transvaal adjacent to the Kruger National Park. The copper orebody, which is part of the much larger Phalaborwa Complex, is a nearly vertical intrusive deposit which is mined in a series of horizontal slices or benches by conventional truck and shovel methods.

As there are distinct geological boundaries in the pit between the copper bearing carbonatites, dolerite dykes, fenite and micaceous pyroxenite, the benches are further subdivided into composites which are mined separately. Grades are calculated from assays of drill chippings taken from blastholes during drilling; the results are recorded on bench plans against the accurately plotted blasthole.

For medium term planning purposes, unsampled composites are valued by projecting down values from directly above which implies an assumption about the vertical regularity of grade within the deposit. By the time a composite has been blasted, all the blasthole assays are available and a more accurate valuation of the composite can be made which is essential for daily grade control.

As has been previously suggested, an assumption is made that grade distribution is consistent from one bench to another and at the suggestion of the Chief Geologist it was decided to test this assumption by making a geostatistical analysis of two widely separated benches; the most suitable were found to be benches 10 and 20 which are separated by a vertical distance of 118.82 m. The results of this study have implications for long and

medium term planning purposes as well as ore reserve estimates at depth.

1.2 Scope and objective of the study

An examination of the two sets of bench plans revealed that each bench had approximately 37 000 blasthole assay values available for analysis - a total of 74 000 separate data points which would require storing in a computer and manipulating with various programmes. This amount of data was felt to be outside the scope of this study and so an area 500m x 500m was chosen in the main copper mineralised area of each bench, the one vertically above the other (see Fig.1).

These areas were subdivided into 10m x 10m blocks and by manual calculation, values falling within these blocks were averaged to give a central value; this gave 2500 point values for each bench which was more manageable.

As it was not possible to gain access to a main frame computer, a Sinclair Spectrum 48K personal computer was obtained which had sufficient memory capacity to handle the data. A suite of geostatistics programmes was written in BASIC for this machine and is presented at the end of the dissertation. The data were entered into the computer and stored on magnetic tape in a 50 x 50 matrix form for later analysis.

The analysis consists of comparisons of the two benches through the following procedures:-

- sorting into frequency by range interval
- histograms of unadjusted values
- evaluation of additive constants
- calculation of mean and variance
- calculation of semivariograms
- calculation of anisotropy functions
- Kriging of 200m x 200m area

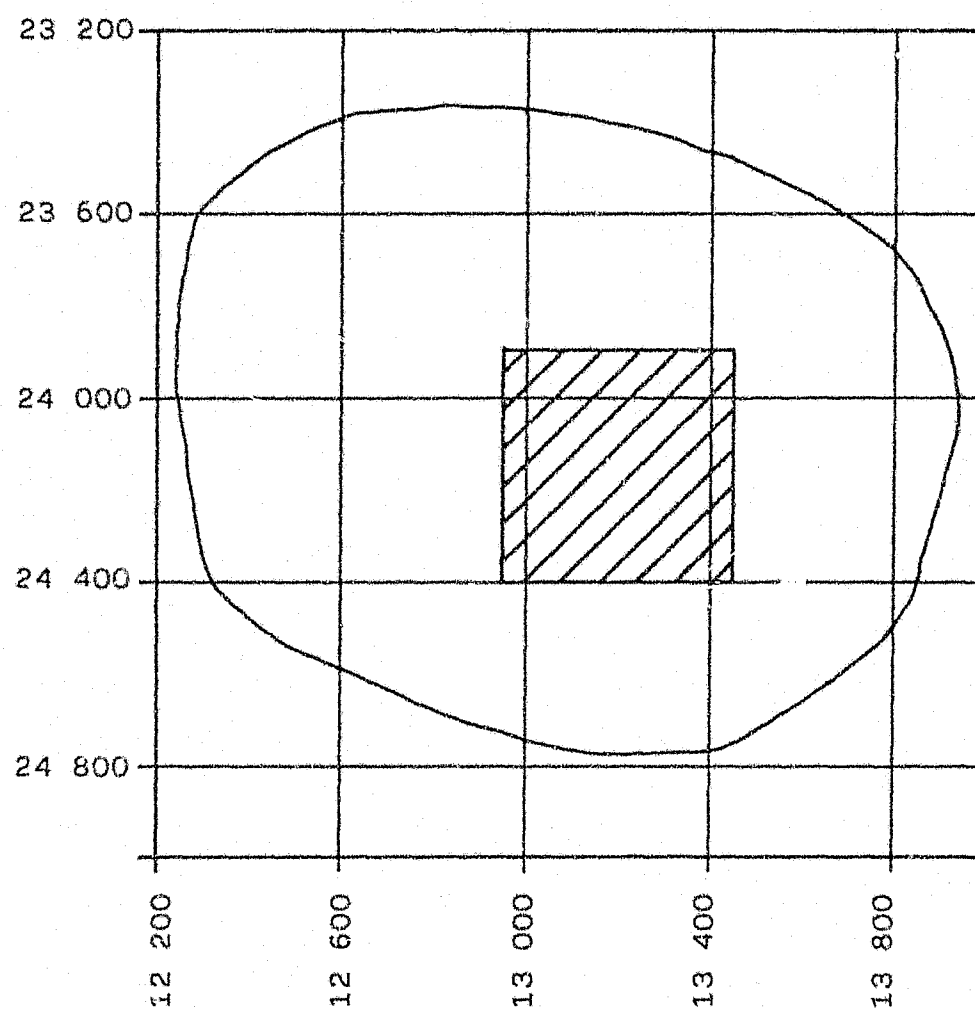
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The early stages of the analysis showed that with the application of a third parameter, the distribution was lognormal; consequently Lognormal-de Wijsian methods have been applied to the study throughout.

Semivariograms were calculated in the four principal directions and checked for anisotropy; as this was present it was taken into account in calculating the Kriging weights.

Kriging was carried out on a smaller 200m x 200m block within the larger area under investigation; diagrams were prepared of blocks greater than or equal to 1% and 0.7% Cu content.

Fig. 1

AREA UNDER INVESTIGATION WITHIN OPEN PIT

2 THE PALABORA OPEN PIT MINE

2.1 Geology (5)

The Palabora Complex is a large pipe approximately eight kilometres long by three kilometres wide, vertically invading the old Precambrian granitic basement rocks. Copper, apatite and vermiculite are mined from subsidiary carbonatite pipes and their associated rocks within the much older Palabora Complex. The Complex is mainly pyroxenite, a diopside-phlogopite-apatite rock in which apatite constitutes about 15 percent of the rock.

A mafic pegmatitic rock composed largely of serpentine and hydrated phlogopite (vermiculite) crops out in the northern part of the complex and is being mined for vermiculite. In the west - central part is a nearly vertical pipe of carbonatite and phoscorite, a rock composed of partly serpentinised olivine, magnetite, apatite and accessory mica. Minor amounts of baddeleyite (ZrO_2) are also present.

Carbonatite is said to have been intruded in two stages. The older is a medium to coarse grained rock with conspicuous banding roughly parallel in the walls of the pipe. This carbonatite, as well as much of the younger material, consists of calcite intergrown with dolomite and containing accessory chondrodite, olivine, phlogopite and biotite. Banding results from the alignment of magnetite, which makes up as much as 25 percent of the rock, and silicate grains. Contacts between the older carbonatite and phoscorite are commonly gradational; towards the borders of the pipe the two rocks are intimately interlayered. Micaceous pyroxenite, a mica rich phlogopite-biotite rock, lies close to the outer border of the phoscorite.

The younger carbonatite, generally called transgressive carbonatite because it cross-cuts other rocks, is especially abundant near the centre of the pipe and tails out, east and west. Part of the younger carbonatite is fine grained and is banded locally like the older rock, thus making it difficult to distinguish the two.

Dolerite dykes, which are apparently not related to the economic mineralisation and which range in width from less than a centimetre to more than fifty metres, trend northeastwards in a swarm (see Fig.2). The dykes appear to be late in geological history; much of the pipe shows tight, closely spaced shearing, especially near the centre, where the principal elongated pipe-like body of transgressive carbonatite was emplaced. The shearing trends generally to the north and is considered postcarbonatite.

Copper mineralisation spreads throughout the carbonatites, mostly in the transgressive carbonatite near the core of the pipe. This is attributed to the localisation of the younger carbonatite where the rocks were repeatedly fractured. Both the older carbonatite and the phoscorite contain an earlier carbonatite-copper mineralisation. Blebs in the phoscorite seem to have resulted from replacement of interstitial carbonate grains not related to shearing. Much of the older carbonatite is below ore grade, except in conjunction with the younger carbonatite.

Within the copper orebody, (about 1200m x 600m in plan view), chalcopyrite is the principal sulphide, though bornite, chalcocite and locally minor amounts of other copper minerals are present. Magnetite is a major constituent of the ore and much of it contains titanium. The transgressive carbonatite in the centre of the plug contains only minor amounts of apatite, but elsewhere, especially in the phoscorite, apatite is abundant.

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2.2 Mining Methods

The open pit is mined in a series of horizontal benches by conventional truck and shovel methods, moving up to 350 000 tons of ore and waste every working day.

The first stage in the process is the drilling of blastholes by electric rotary drills; they are then filled with a sensitised ammonium nitrate based slurry explosive. The blastholes are drilled to a regular pattern which varies according to the rock type to be blasted; after the blast, the muckpile is accurately demarcated to indicate to the shovel operator the destination of the material (i.e. ore to crushers, waste to dumps).

A variety of electric face shovels are in operation which load into 150 ton haul trucks; a unique feature of these is the electric trolley assist system which was installed to counteract the effects of increases in the cost of diesel fuel.

Benches are currently mined to a height of 15,2 metres, however due to the scale of earlier equipment being smaller, benches down to bench 16 were mined to a height of 12,2 metres; this means that Bench 10 is 12,2 m high while Bench 20 is 15,2 high.

2.3 Sampling

As the mine is a high tonnage, low grade operation, it is essential that accurate up to date assays are available. Sampling trays are placed adjacent to blastholes being drilled and as the chippings are flushed away from the tricone bits up the hole by compressed air, some of them fall into the trays. By the time a hole is completed the tray should contain a random sample of chippings from the full length of the hole.

The samples are analysed on the mine and the results recorded against the hole positions on bench plans. The grade of a composite can then be calculated before it is blasted which is vital for planning an average daily grade.

FIG. 2

PEP GEOLOGY OF THE 122M LEVEL

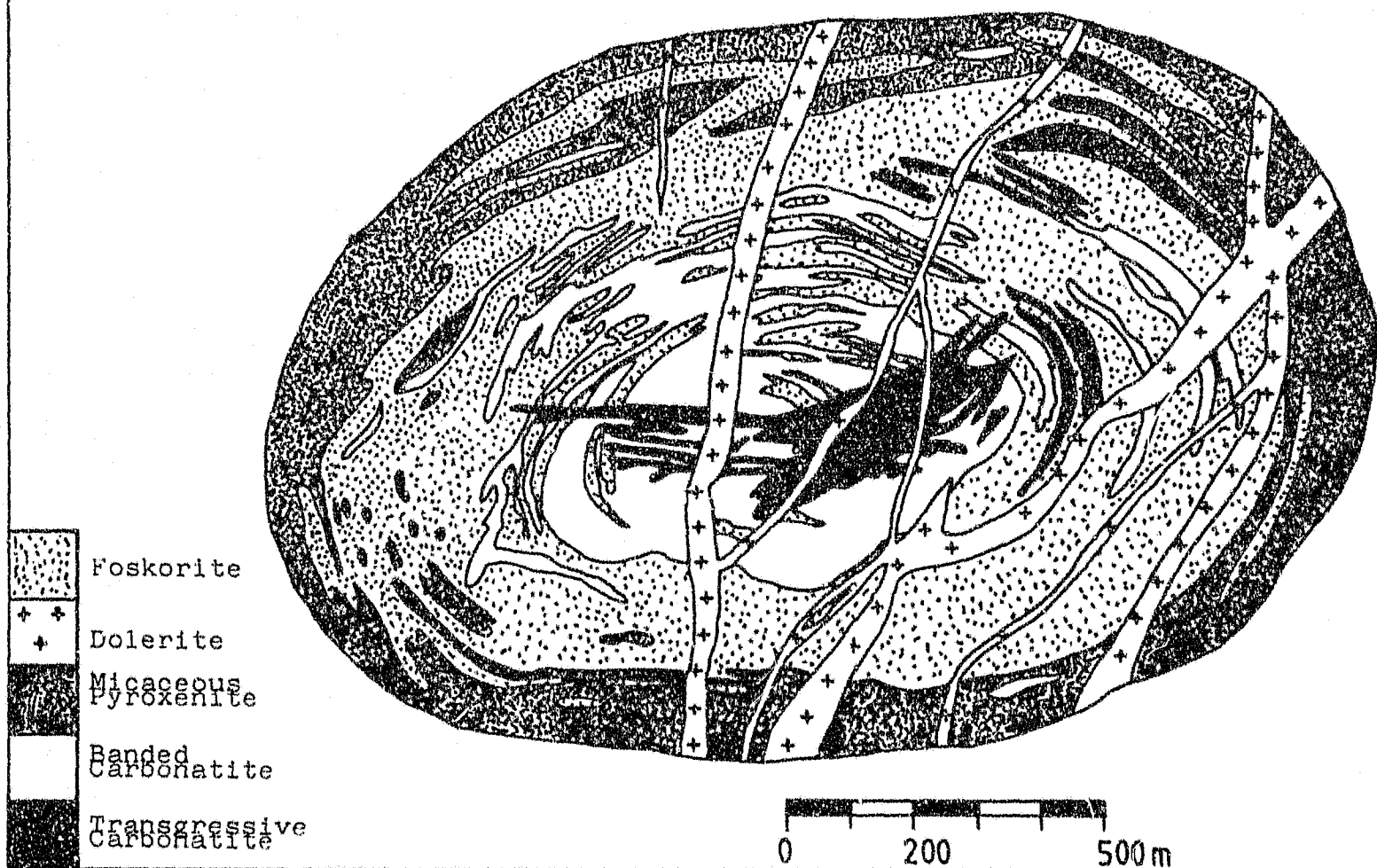
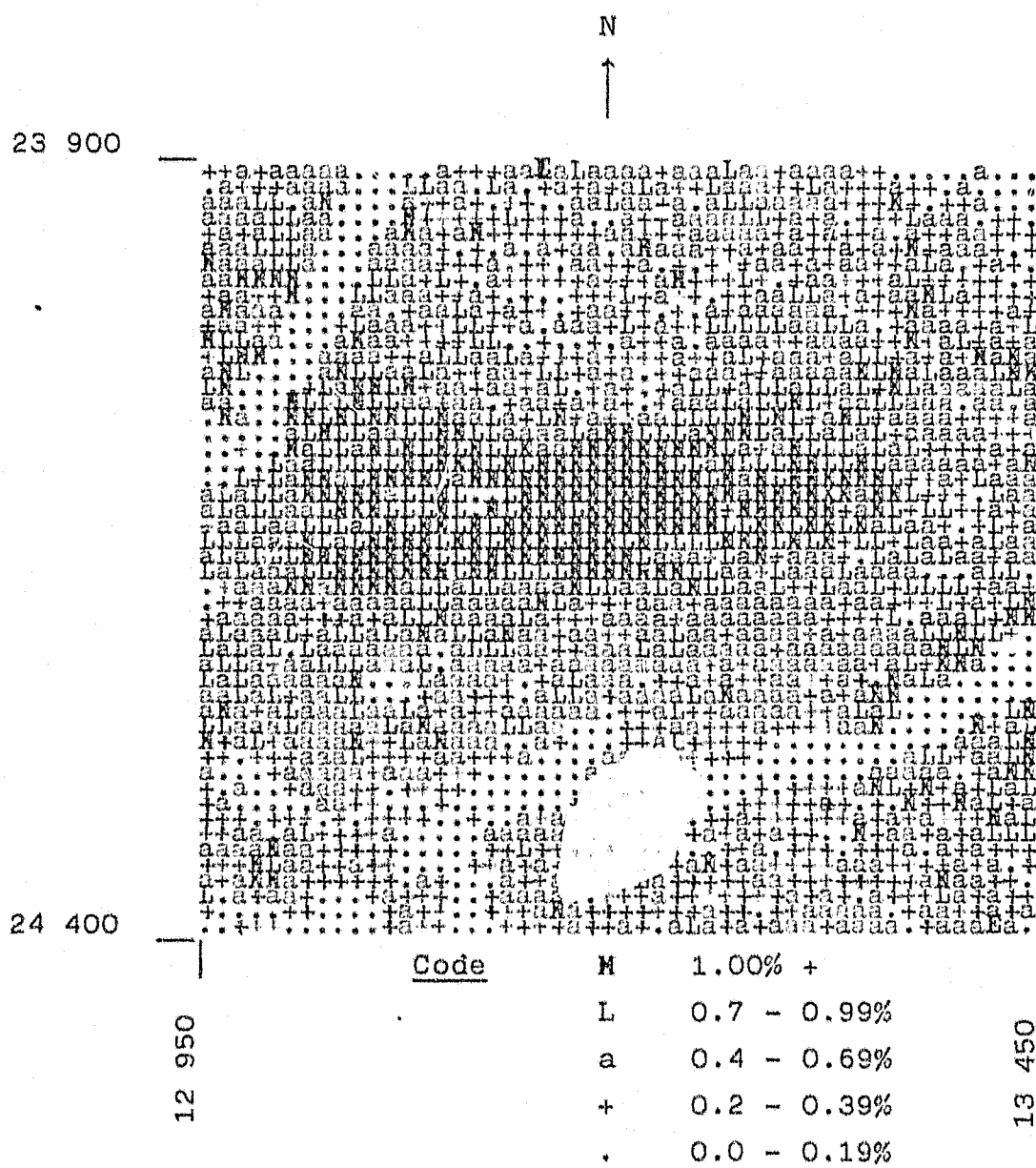


Fig. 3

ALPHAGRAPHIC REPRESENTATION OF UNADJUSTED VALUES:BENCH 10

3 THE GEOSTATISTICAL ANALYSIS

3.1 Population distributions

Figs. 3 and 4 show an alphagraphic representation of 2500 values on each bench; these values were run through GEOSTATS-2 which ignores zeros (representing dyke material) and gives the frequencies of values falling within 0.05% categories from 0.0% to 3.0%; the results are given in Tables 1 to 4. The cumulative frequency percentages were plotted on logarithmic probability paper (Figs. 7 and 8) and for both benches, an additive constant of 0.1 was found to give the distributions a good approximation to lognormality.

The values were then run through GEOSTATS-3 which also ignores the dyke material, and gives the sum of the logs of the values plus the additive constant, the sum of the logs of the values plus the additive constant squared and the number of values.

The population means and variances were calculated as follows:

BENCH 10

number of values	- n	2283
$\sum \ln(x_i + b)$	- (a)	-1159.0292
mean (a)		-0.5077
geometric mean		0.6019
$\sum [\ln(x_i + b)]^2$	- (b)	1155.1315
mean (b)		0.5060
$[\text{mean}(a)]^2$		<u>0.2578</u>
variance (unadjusted by $(n/n-1)$)		0.2482
s^2		0.2483

$$\begin{aligned}
 \text{Pop. mean} &= (\text{geom. mean} \times e^{s^2/2}) - \beta \quad (\text{for } n \text{ large}) \\
 &= 0.6019 \times 1.132 - 0.1 \\
 &= \underline{0.5814}
 \end{aligned}$$

BENCH 20

number of values - n	2285
$\sum \ln(x_i + \beta)$ (a)	-1307.8097
mean (a)	-0.5723
geometric mean	0.5642
$\sum [\ln(x_i + \beta)]^2$ (b)	1329.1812
mean (b)	0.5817
$[\text{mean}(a)]^2$	<u>0.3275</u>
variance (unadjusted by $(n/n-1)$)	0.2542
s^2	0.2543

$$\begin{aligned}
 \text{Pop. mean} &= (\text{geom. mean} \times e^{s^2/2}) - \beta \quad (\text{for } n \text{ large}) \\
 &= 0.5642 \times 1.136 - 0.1 \\
 &= \underline{0.5409}
 \end{aligned}$$

Table 1
DISTRIBUTION OF VALUES: BENCH 10

Range	Freq.	Cum. Freq.	Cum. Freq. %
0.0 - 0.049	11	11	0.48
0.05 - 0.099	22	33	1.44
0.1 - 0.149	51	84	3.67
0.15 - 0.199	82	166	7.26
0.2 - 0.249	109	275	13.02
0.25 - 0.299	170	445	19.46
0.3 - 0.349	192	637	27.85
0.35 - 0.399	175	812	35.51
0.4 - 0.449	178	990	43.29
0.45 - 0.499	166	1156	50.55
0.5 - 0.549	167	1323	57.85
0.55 - 0.599	147	1470	64.28
0.6 - 0.649	108	1578	69.00
0.65 - 0.699	73	1651	72.19
0.7 - 0.749	85	1736	75.91
0.75 - 0.799	64	1800	78.71
0.8 - 0.849	66	1866	81.59
0.85 - 0.899	47	1913	83.65
0.9 - 0.949	46	1959	85.66
0.95 - 0.999	46	2005	87.67
1.0 - 1.049	35	2040	89.20
1.05 - 1.099	28	2068	90.42
1.1 - 1.149	39	2107	92.13
1.15 - 1.199	25	2132	93.22
1.2 - 1.249	25	2157	94.32
1.25 - 1.299	25	2182	95.41
1.3 - 1.349	11	2193	95.89
1.35 - 1.399	19	2212	96.72
1.4 - 1.449	10	2222	97.16
1.45 - 1.499	14	2236	97.77

Table 1 (cont.)

DISTRIBUTION OF VALUES: BENCH 10 (cont.)

Range	Freq.	Cum. Freq.	Cum. Freq. %
1.5 - 1.549	6	2242	98.03
1.55 - 1.599	4	2246	98.21
1.6 - 1.649	6	2252	98.47
1.65 - 1.699	5	2257	98.69
1.7 - 1.749	4	2261	98.86
1.75 - 1.799	5	2266	99.08
1.8 - 1.849	7	2273	99.39
1.85 - 1.899	2	2275	99.48
1.9 - 1.949	2	2277	99.56
1.95 - 1.999	3	2280	99.69
2.0 - 2.049	2	2282	99.78
2.05 - 2.099	0	2282	99.78
2.1 - 2.149	0	2282	99.78
2.15 - 2.199	0	2282	99.78
2.2 - 2.249	0	2282	99.78
2.25 - 2.299	0	2282	99.78
2.3 - 2.349	1	2283	99.83
2.35 - 2.399	0	2283	99.83
2.4 - 2.449	1	2284	99.87
2.45 - 2.499	2	2286	99.96
2.5 - 2.549	1	2287	100.00
2.55 - 2.599	0	2287	100.00
2.6 - 2.649	0	2287	100.00
2.65 - 2.699	0	2287	100.00
2.7 - 2.749	0	2287	100.00
2.75 - 2.799	0	2287	100.00
2.8 - 2.849	0	2287	100.00
2.85 - 2.899	0	2287	100.00
2.9 - 2.949	0	2287	100.00
2.95 - 2.999	0	2287	100.00

Table 2

DISTRIBUTION OF VALUES: BENCH 20

Range	Freq.	Cum. Freq.	Cum. Freq. %
0.0 - 0.049	7	7	0.31
0.05 - 0.099	47	54	2.36
0.1 - 0.149	82	136	5.95
0.15 - 0.199	82	218	9.54
0.2 - 0.249	133	351	15.37
0.25 - 0.299	168	519	22.72
0.3 - 0.349	202	721	31.57
0.35 - 0.399	180	901	39.45
0.4 - 0.449	160	1061	46.45
0.45 - 0.499	172	1233	53.98
0.5 - 0.549	153	1386	60.68
0.55 - 0.599	145	1531	67.03
0.6 - 0.649	112	1643	71.94
0.65 - 0.699	80	1723	75.44
0.7 - 0.749	77	1800	78.81
0.75 - 0.799	66	1866	81.70
0.8 - 0.849	60	1926	84.33
0.85 - 0.899	36	1962	85.90
0.9 - 0.949	54	2016	88.27
0.95 - 0.999	49	2065	90.41
1.0 - 1.049	40	2105	92.16
1.05 - 1.099	26	2131	93.30
1.1 - 1.149	25	2156	94.40
1.15 - 1.199	19	2175	95.23
1.2 - 1.249	20	2195	96.10
1.25 - 1.299	22	2217	97.07
1.3 - 1.349	9	2226	97.46
1.35 - 1.399	10	2236	97.90
1.4 - 1.449	8	2244	98.25
1.45 - 1.499	6	2250	98.51

Table 2 (cont.)

DISTRIBUTION OF VALUES: BENCH 20 (cont.)

Range	Freq.	Cum. Freq.	Cum. Freq. %
1.5 - 1.549	9	2259	98.91
1.55 - 1.599	4	2263	99.08
1.6 - 1.649	3	2266	99.21
1.65 - 1.699	4	2270	99.39
1.7 - 1.749	0	2270	99.39
1.75 - 1.799	2	2272	99.47
1.8 - 1.849	0	2272	99.47
1.85 - 1.899	0	2272	99.47
1.9 - 1.949	1	2273	99.52
1.95 - 1.999	5	2278	99.74
2.0 - 2.049	0	2278	99.74
2.05 - 2.099	1	2279	99.78
2.1 - 2.149	0	2279	99.78
2.15 - 2.199	0	2279	99.78
2.2 - 2.249	0	2279	99.78
2.25 - 2.299	0	2279	99.78
2.3 - 2.349	0	2279	99.78
2.35 - 2.399	0	2279	99.78
2.4 - 2.449	0	2279	99.78
2.45 - 2.499	0	2279	99.78
2.5 - 2.549	1	2280	99.82
2.55 - 2.599	0	2280	99.82
2.6 - 2.649	0	2280	99.82
2.65 - 2.699	2	2282	99.91
2.7 - 2.749	1	2283	99.96
2.75 - 2.799	1	2284	100.00
2.8 - 2.849	0	2284	100.00
2.85 - 2.899	0	2284	100.00
2.9 - 2.949	0	2284	100.00
2.95 - 2.999	0	2284	100.00

Fig. 5
HISTOGRAM OF VALUES AND FREQUENCY CURVE: BENCH 10
CURVE FITTED MANUALLY

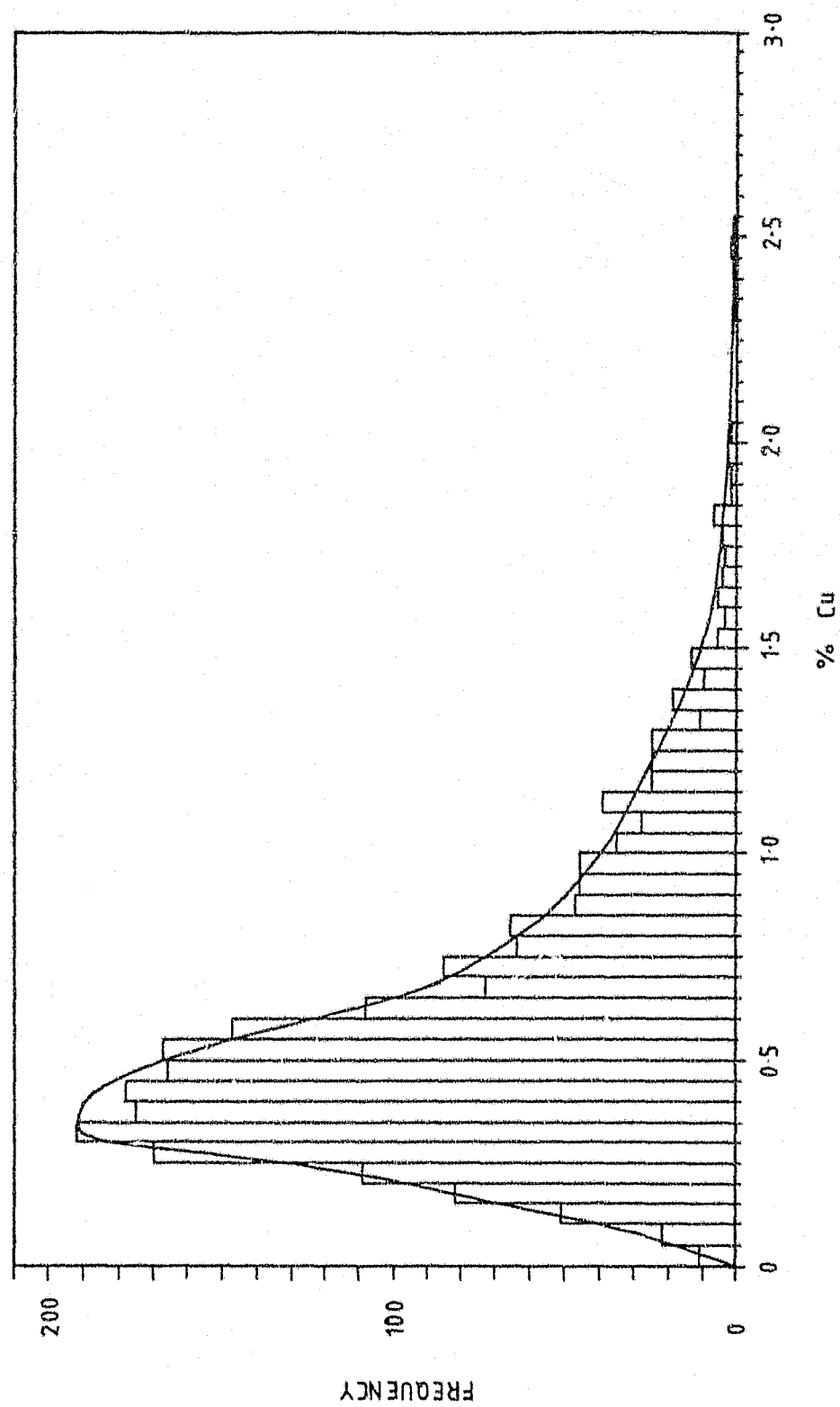


Fig. 6

HISTOGRAM OF VALUES AND FREQUENCY CURVE: BENCH 20

CURVE FITTED MANUALLY

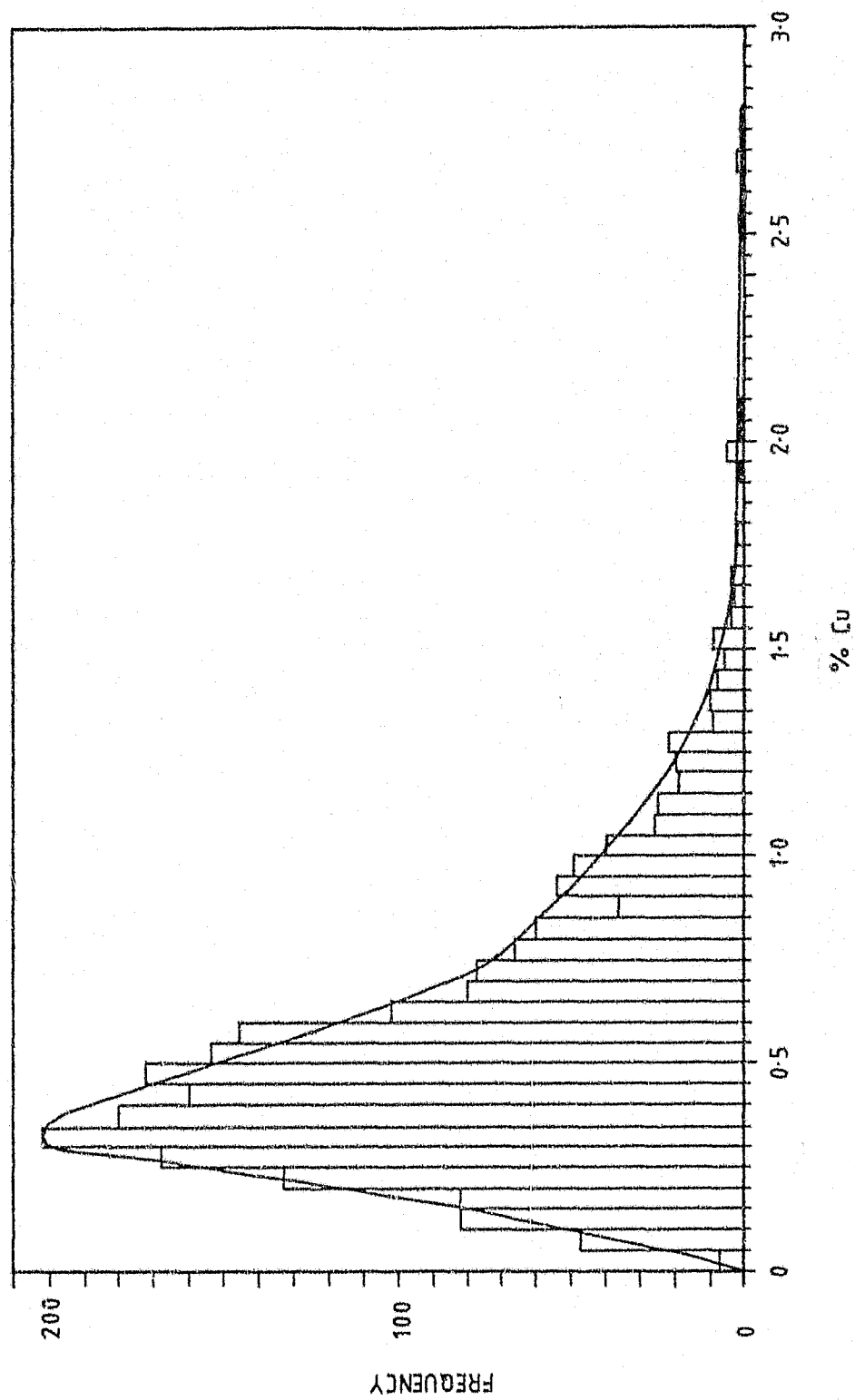


Fig. 6

HISTOGRAM OF VALUES AND FREQUENCY CURVE: BENCH 20

CURVE FITTED MANUALLY

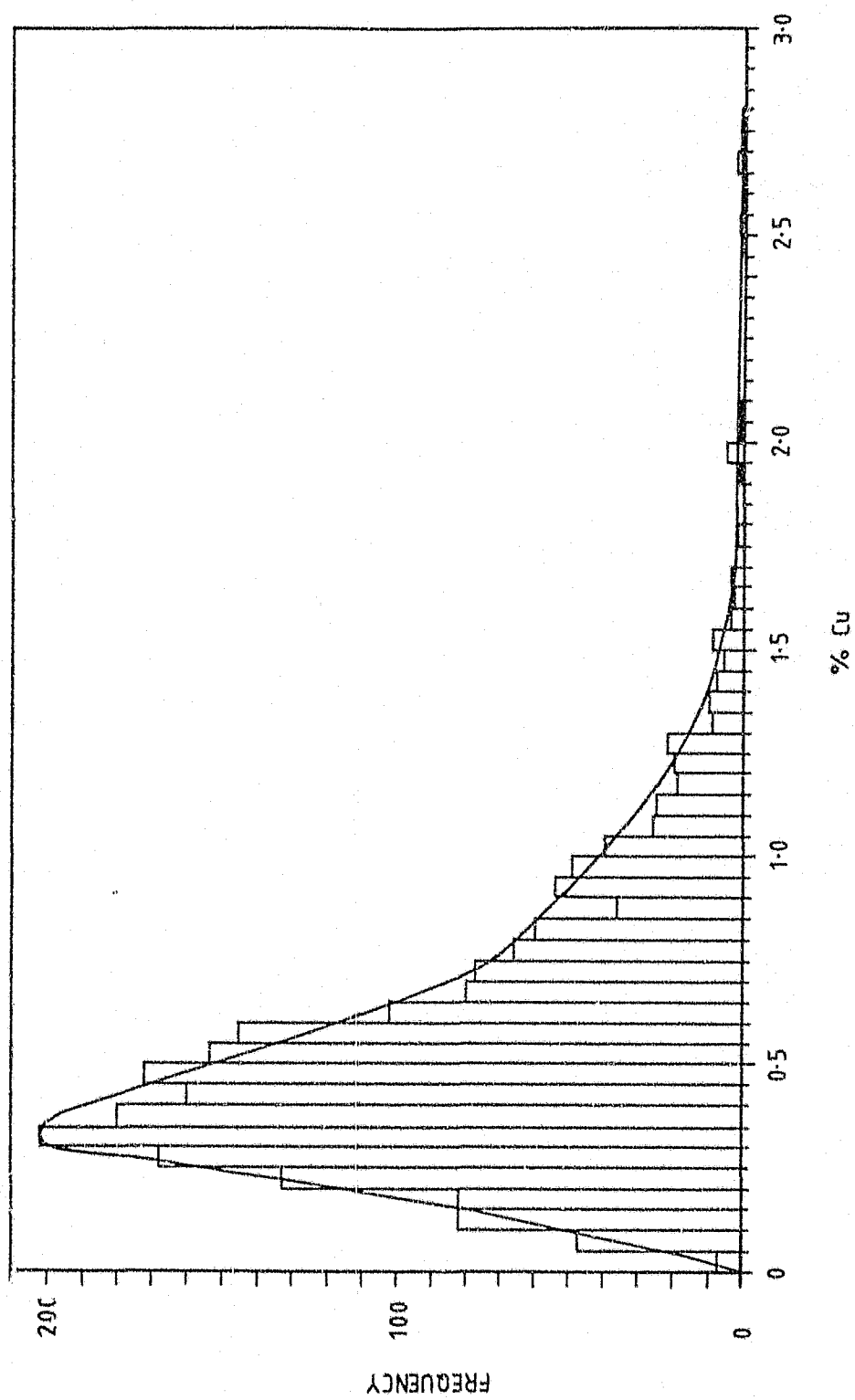


Fig.7 GRAPHICAL ESTIMATION OF ADDITIVE CONSTANT
WITH FITTED THREE PARAMETER LOGNORMAL MODEL: BENCH 10

Vertical scale ± 100

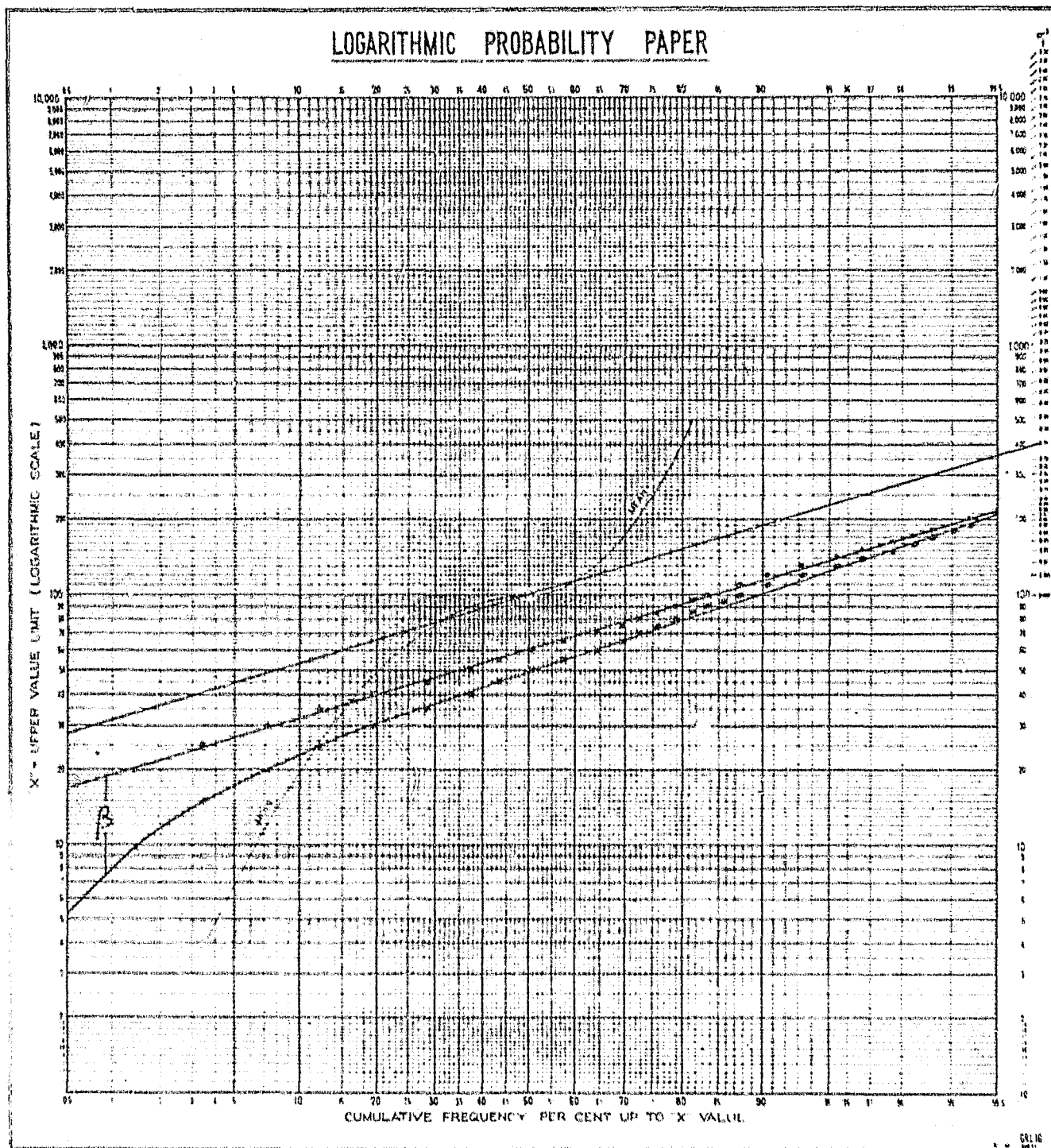
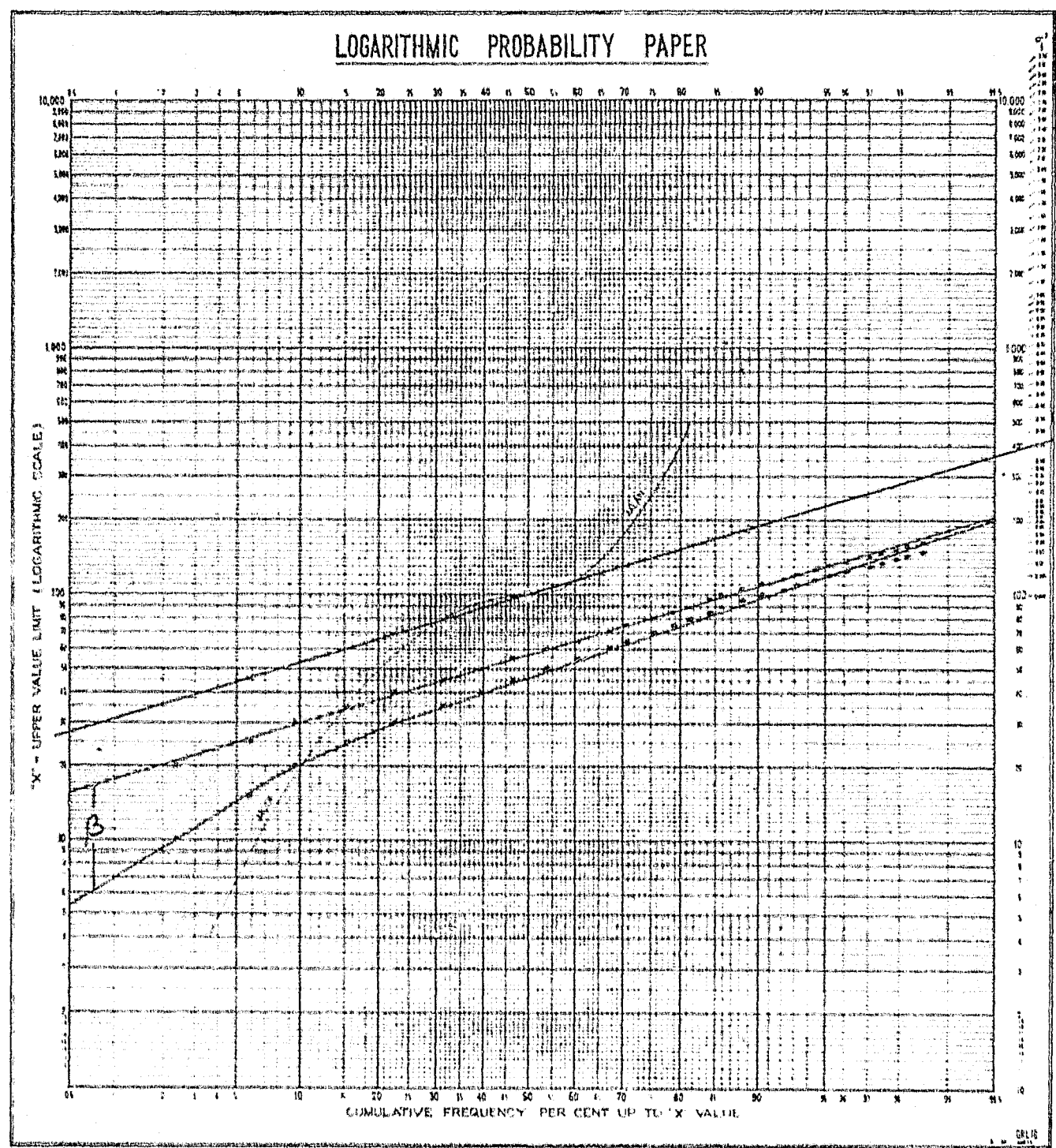


Fig.8 GRAPHICAL ESTIMATION OF ADDITIVE CONSTANT
WITH FITTED THREE PARAMETER LOGNORMAL MODEL: BENCH 20

Vertical scale +100



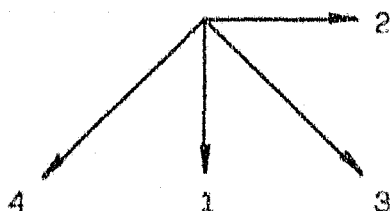
3.2 The semivariograms

The semivariogram function is given as follows: (4)

$$\gamma(h) = \frac{1}{2n(h)} \sum [x(z_i) - x(z_i+h)]^2$$

In the three parameter lognormal model used in this study, the $x(z_i)$ function is the natural log of the value plus the additive constant. This formulation forms the basis of the programme GEOSTATS-4.

Given that the data used is a 50 x 50 array, $\gamma(h)$ was calculated up to a maximum of lag 30 in four principal directions as shown below:



The results are set out in Tables 3 and 4.

Figs.9 and 10 show the individual semivariograms for each bench plotted on a log scale; as will be seen there is anisotropy present and this is dealt with later. Figs.11 and 12 show the combined semivariograms for each bench and show a remarkable degree of similarity.

As the format used later for Kriging only requires the calculation of covariance to a maximum lag of 5.66, the semivariogram up to this value follows the following de Wijsian model: (4)

$$\gamma(h) = N + 3\alpha \log \frac{h}{0.446}$$

Table 3
SEMIVARIOGRAM VALUES: BENCH 10

Lag	Dir. 1		Dir. 2		Dir. 3		Dir. 4	
	$\gamma(h)$	n	$\gamma(h)$	n	$\gamma(h)$	n	$\gamma(h)$	n
1	0.1229	2165	0.1169	2174	0.1276	2085	0.1285	2132
2	0.1494	2074	0.1292	2087	0.1518	1934	0.1500	2017
3	0.1587	1996	0.1431	2008	0.1571	1830	0.1672	1914
4	0.1789	1927	0.1524	1946	0.1727	1751	0.1805	1818
5	0.1919	1869	0.1533	1890	0.1824	1682	0.1958	1728
6	0.2010	1817	0.1569	1846	0.2025	1614	0.2110	1640
7	0.2148	1771	0.1579	1805	0.2197	1548	0.2199	1561
8	0.2322	1732	0.1570	1767	0.2288	1479	0.2347	1491
9	0.2485	1696	0.1613	1728	0.2392	1414	0.2555	1431
10	0.2621	1663	0.1655	1695	0.2555	1354	0.2671	1369
11	0.2724	1630	0.1764	1658	0.2583	1293	0.2816	1313
12	0.2919	1593	0.1838	1620	0.2627	1230	0.2978	1261
13	0.2911	1558	0.1855	1579	0.2680	1165	0.3089	1206
14	0.2922	1519	0.1893	1540	0.2644	1098	0.3190	1147
15	0.2901	1480	0.1837	1494	0.2710	1032	0.3215	1089
16	0.2934	1444	0.1874	1451	0.2826	970	0.3325	1035
17	0.3052	1406	0.1920	1407	0.2884	914	0.3462	979
18	0.3133	1366	0.2002	1365	0.3189	856	0.3528	926
19	0.3066	1325	0.2072	1321	0.3296	803	0.3544	871
20	0.3166	1280	0.2011	1273	0.3311	750	0.3491	817
21	0.3134	1230	0.2054	1226	0.3352	695	0.3441	761
22	0.3166	1183	0.2032	1177	0.3497	636	0.3429	708
23	0.3206	1131	0.2080	1129	0.3683	580	0.3418	653
24	0.3075	1084	0.2114	1082	0.3470	531	0.3318	604
25	0.3057	1033	0.2085	1034	0.3271	484	0.3230	553
26	0.2984	983	0.2049	986	0.3159	439	0.2970	506
27	0.2807	937	0.1929	939	0.3183	399	0.2585	462
28	0.2809	887	0.1957	894	0.2905	352	0.2505	421
29	0.2682	840	0.1937	850	0.2666	314	0.2550	382
30	0.2436	793	0.1891	807	0.2809	278	0.2472	345

Table 4

SEMIVARIOGRAM VALUES: BENCH 20

Lag	Dir. 1		Dir. 2		Dir. 3		Dir. 4	
	$\gamma(h)$	n	$\gamma(h)$	n	$\gamma(h)$	n	$\gamma(h)$	n
1	0.1249	2175	0.1106	2184	0.1323	2098	0.1287	2144
2	0.1517	2082	0.1376	2102	0.1532	1941	0.1540	2022
3	0.1716	2002	0.1523	2030	0.1697	1828	0.1816	1911
4	0.1868	1934	0.1577	1966	0.1804	1751	0.1969	1807
5	0.1986	1886	0.1588	1909	0.1772	1681	0.2100	1713
6	0.2113	1843	0.1616	1854	0.1919	1611	0.2297	1625
7	0.2207	1805	0.1680	1805	0.2034	1541	0.2348	1550
8	0.2310	1776	0.1673	1761	0.2115	1477	0.2564	1480
9	0.2415	1745	0.1759	1724	0.2258	1428	0.2616	1428
10	0.2579	1718	0.1836	1691	0.2385	1385	0.2817	1387
11	0.2654	1684	0.1861	1656	0.2355	1322	0.2895	1336
12	0.2784	1642	0.1875	1617	0.2412	1251	0.2978	1281
13	0.2844	1597	0.1944	1580	0.2446	1180	0.3138	1226
14	0.2882	1554	0.2002	1543	0.2386	1111	0.3256	1171
15	0.2946	1511	0.2085	1501	0.2449	1043	0.3422	1114
16	0.2971	1468	0.2105	1456	0.2460	978	0.3624	1056
17	0.2954	1424	0.2141	1408	0.2629	913	0.3728	997
18	0.3077	1379	0.2247	1360	0.2728	852	0.3761	939
19	0.3047	1333	0.2247	1311	0.2698	794	0.3782	879
20	0.3120	1284	0.2135	1263	0.2796	738	0.3757	822
21	0.3202	1234	0.2135	1216	0.2876	682	0.3815	766
22	0.3244	1184	0.2103	1172	0.2917	627	0.3787	713
23	0.3201	1134	0.2099	1126	0.3087	575	0.3717	660
24	0.3191	1085	0.2162	1080	0.2998	527	0.3744	609
25	0.3150	1035	0.2159	1035	0.2853	480	0.3620	560
26	0.2976	986	0.2078	990	0.2713	433	0.3257	514
27	0.2818	938	0.2095	944	0.2611	390	0.3202	470
28	0.2650	890	0.2100	899	0.2355	350	0.3240	428
29	0.2686	842	0.2091	857	0.2455	311	0.3215	388
30	0.2686	842	0.2091	857	0.2455	311	0.3215	388

Fig. 9
SEMIVARIOGRAMS IN FOUR PRINCIPAL DIRECTIONS: BENCH 10

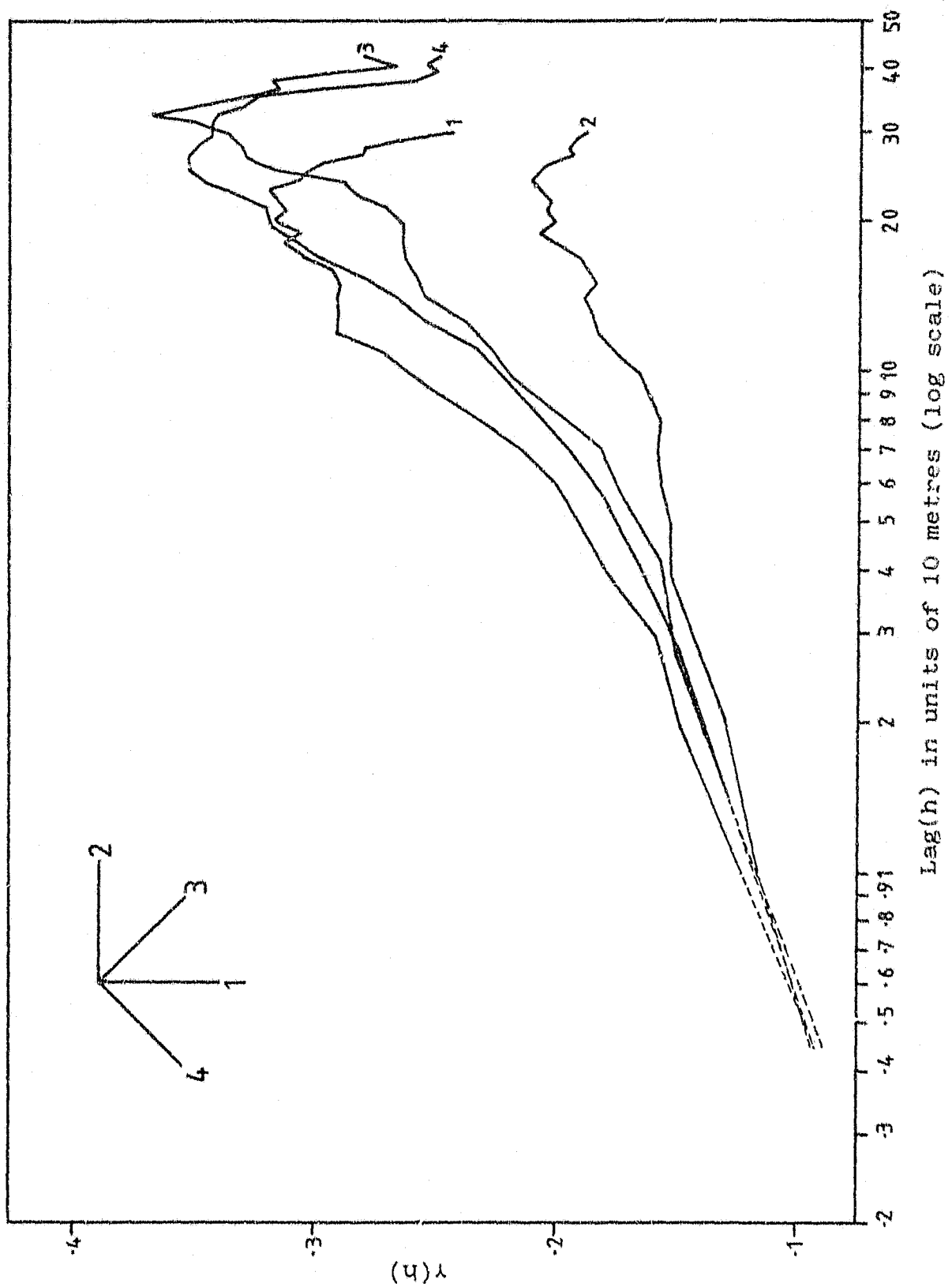


Fig. 10

SEMIVARIOGRAMS IN FOUR PRINCIPAL DIRECTIONS: BENCH 20

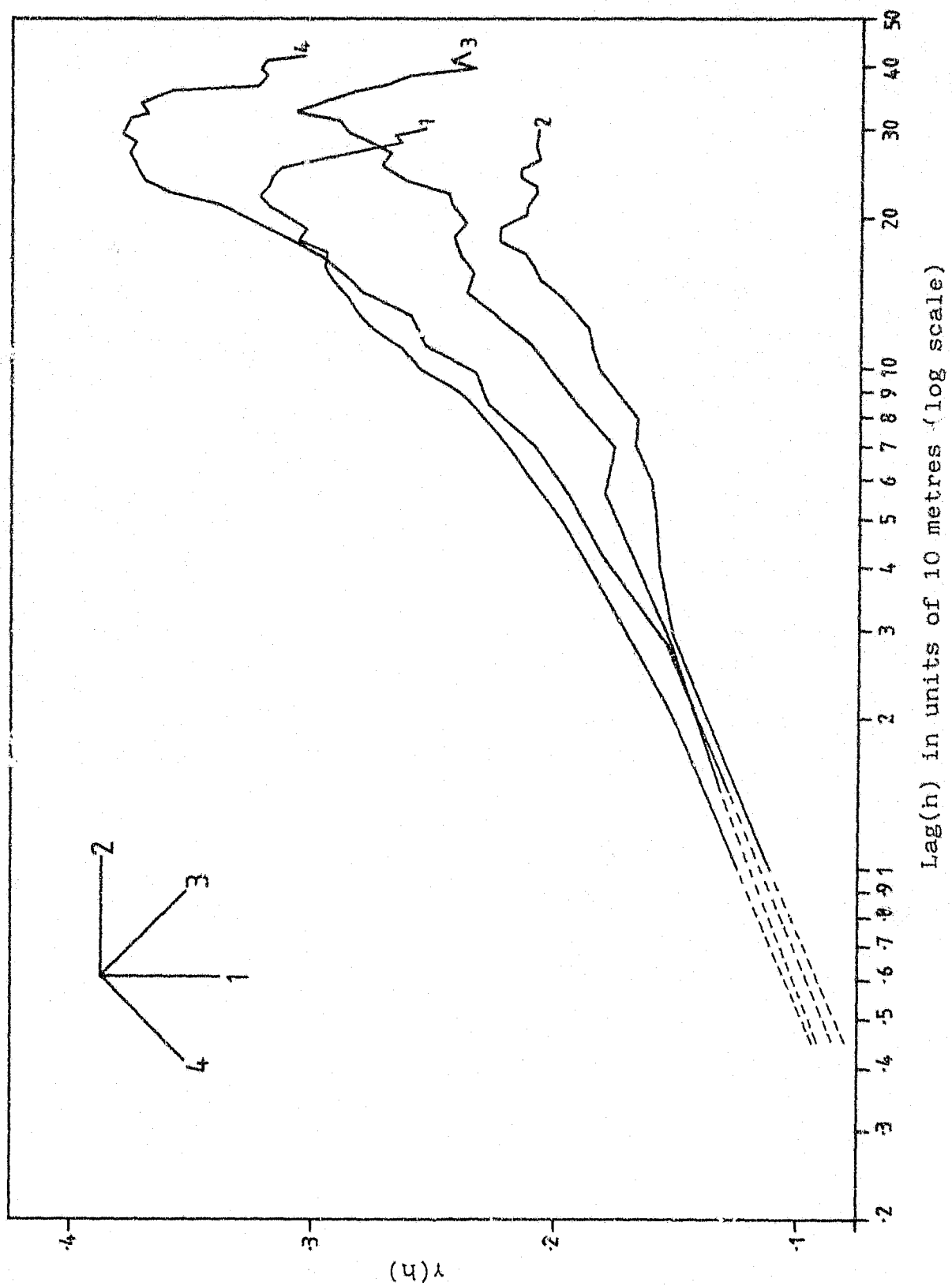


Fig. 11

COMBINED SEMIVARIOGRAM: BENCH 10

Showing anisotropy over part of model used for Kriging

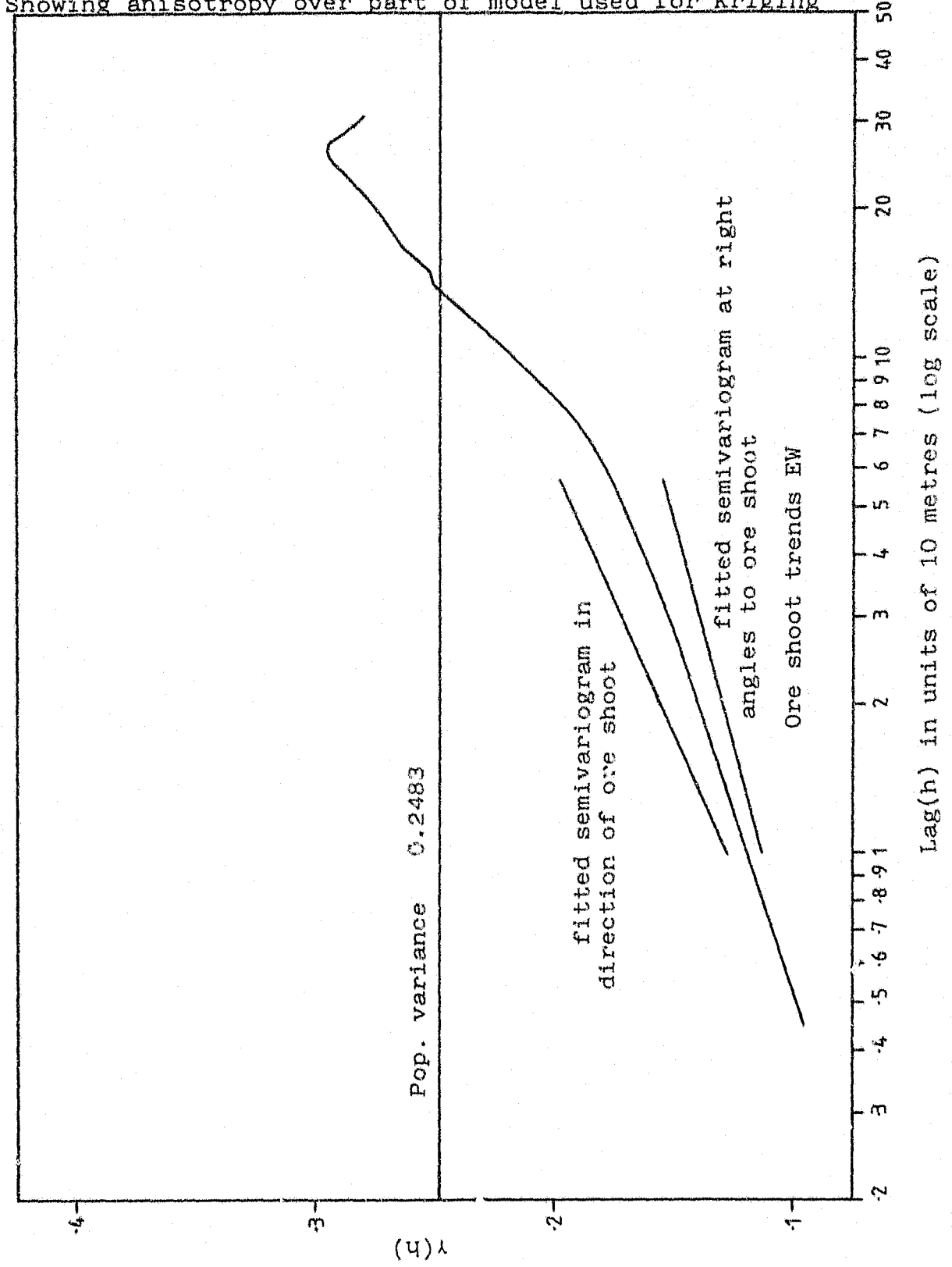
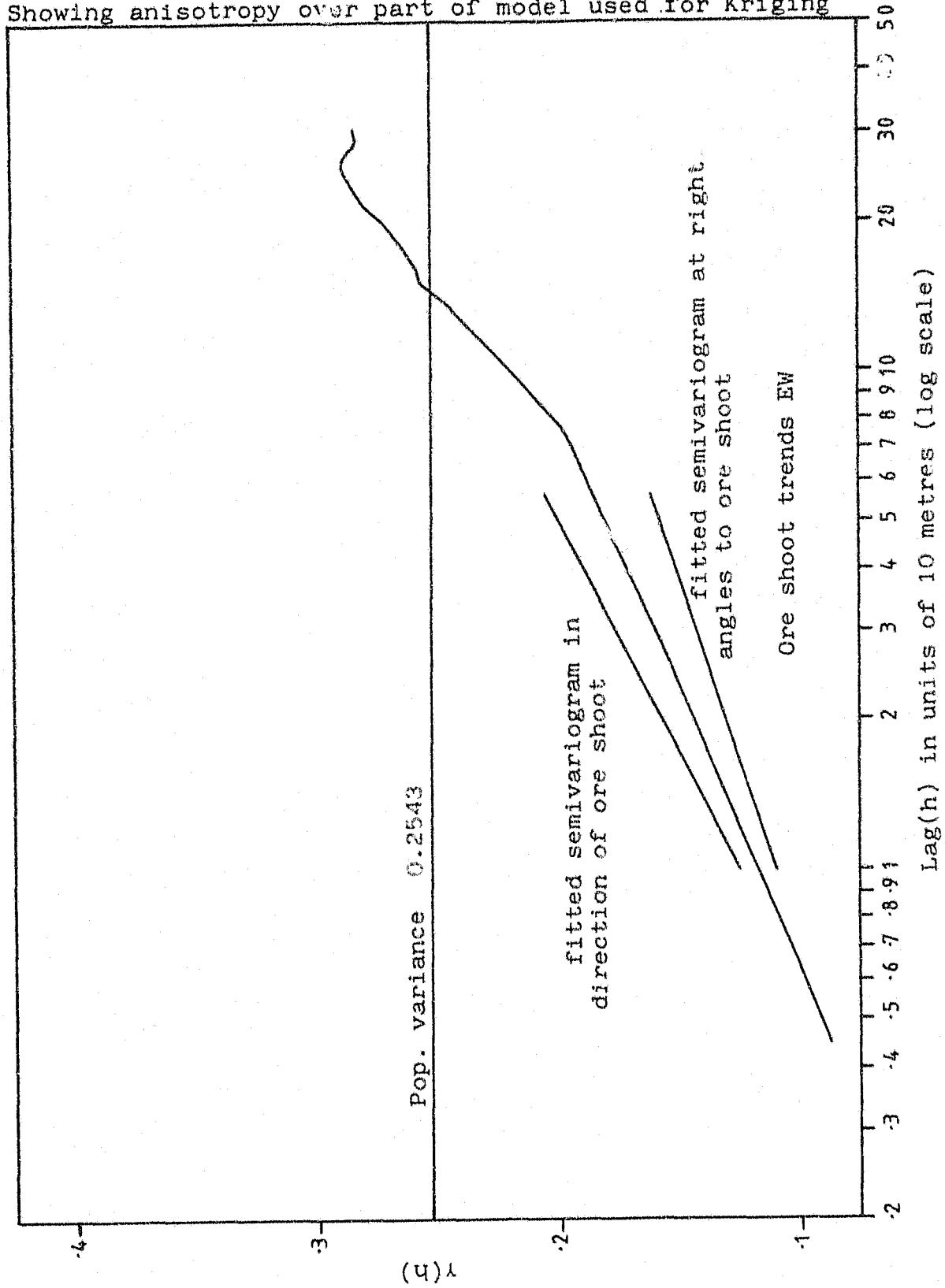


Fig. 12
 COMBINED SEMIVARIOGRAM: BENCH 20

Showing anisotropy over part of model used for Kriging



3.3 Calculation of anisotropy function

The anisotropy function is given by (3):

$$\gamma(h) = N + 3\alpha \left(\frac{a^2 b^2}{(b^2 \cos^2(\beta) + a^2 \sin^2(\beta))} \right)^{\frac{1}{2}} \log \frac{h}{0.446}$$

where β = angle between lag direction and direction at right angles to 'ore shoot'.

The values were calculated for both benches as follows:

BENCH 10

$$\gamma(h) = 0.093 + 0.035 \left(\frac{0.657}{(0.468 \cos^2(\beta) + 1.403 \sin^2(\beta))} \right)^{\frac{1}{2}} \log \frac{h}{0.446}$$

BENCH 20

$$\gamma(h) = 0.086 + 0.040 \left(\frac{0.783}{(0.559 \cos^2(\beta) + 1.4 \sin^2(\beta))} \right)^{\frac{1}{2}} \log \frac{h}{0.446}$$

The values of the above functions when subtracted from the population variance, give the covariances used in the simultaneous equations which are solved to find the Kriging weights.

3.4 Kriging

The Kriging process involves the solving of a series of simultaneous equations to calculate a set of weights. The number and magnitude of the equations depends on the format of the data pattern used to value a block; the format is shown below and represents 10m x 10m blocks:

x_1	x_2	x_3	x_4	x_5
x_6	x_7	x_8	x_9	x_{10}
x_{11}	x_{12}	Y	x_{13}	x_{14}
x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
x_{20}	x_{21}	x_{22}	x_{23}	x_{24}

Note: x_1 to x_{24} are the 24 data blocks used in arriving at a Kriged estimate of block Y. In Sect.4.2 the Kriged values are compared with the available data values for blocks Y.

The series of simultaneous equations takes the following form for Kriging with population mean known:

$$\begin{aligned} \sigma_1^2 a_1 + \sigma_{12} a_2 + \dots + \sigma_{124} a_{24} &= \sigma_{y1} \\ \sigma_{21} a_1 + \sigma_2^2 a_2 & \\ \cdot & \\ \cdot & \\ \sigma_{241} a_1 + \sigma_{242} a_2 + \dots + \sigma_{24}^2 a_{24} &= \sigma_{y24} \end{aligned}$$

The covariances were derived by calculating the $\gamma(h)$ function from the foregoing anisotropy function and subtracting from the relevant population variance. These values were calculated with a programmable calculator and are shown as a series of coefficients in Tables 5 and 6.

The equations are solved as an augmented matrix using Gaussian elimination and back substitution; the procedure is fully outlined in the description of the computer programmes. The method is incorporated into GEOSTATS-6 and the results (i.e. the Kriging weights) are shown in Table 7.

The Kriging equation used to value a block from surrounding data is given as follows:- (4)

$$\log(\mu_k + \beta) = (1 - \sum a_i) \log(M + \beta) + \sum a_i \log(x_i + \beta) + \frac{1}{2} \sum a_i (\sigma_{xi}^2 - \sigma_{yi})$$

The error variance is given by: (4)

$$\sigma_k^2 = \sigma_y^2 - \sum a_i \sigma_{yi}$$

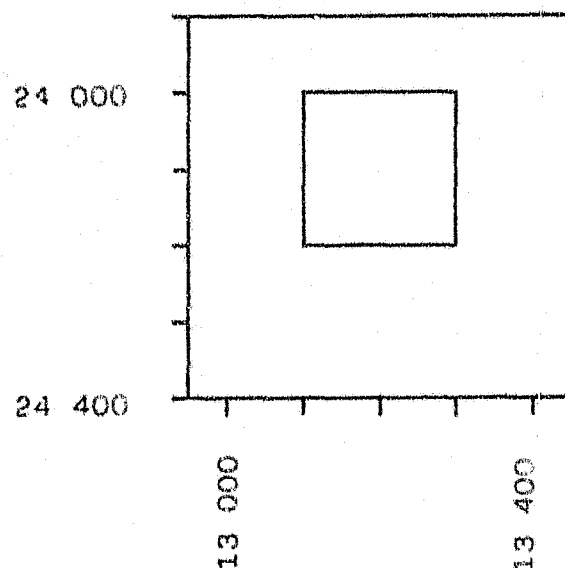
The Kriging equation is incorporated into GEOSTATS-7 and was used to value 400 blocks on each bench on a 20 x 20 grid. As the blocks were all of uniform size, the error variance was the same for each block on the same

bench. The variances were calculated manually and are given below.

	<u>BENCH 10</u>	<u>BENCH 20</u>
error variance	0.1241	0.1229

The Kriged values of the 400 blocks on each bench are given in Tables 8 and 9. From these results diagrams were prepared showing the areas on each bench greater than or equal to 0.7% and 1.0% respectively (Figs 13 to 16)

Diagram of Kriged area within larger area under investigation



Various comparisons were drawn from the results and are shown below:-

	<u>BENCH 10</u>	<u>BENCH 20</u>
Average value of blocks	0.81%	0.76%
% of blocks 0.7% or greater	73.5%	69.5%
% of blocks 1.0% or greater	30.5%	25.25%

Simple Kriging was chosen because:

- (i) ample data was available for estimating the mean and variance of the population and where the population is known this type of Kriging is the most efficient.

(IAMG 14, Dec.82. p.547)

- (ii) universal Kriging was not attempted, being outside the scope of this project.

Table 5

COEFFICIENTS FOR MATRIX INVERSION: BENCH 10

All coefficients x 10^{-4}

2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	776	824	883	904	908	659	695	759	592	521	1021
1364	2482	1364	1201	1105	1221	1225	1221	1146	1076	1019	942	998	1002	979	824	776	824	883	904	695	695	759	802	998
1201	1364	2482	1364	1201	1146	1221	1225	1221	1146	1076	1019	942	998	1002	824	824	824	883	904	695	695	759	942	
1105	1201	1364	2482	1364	1076	1146	1221	1225	1221	1146	1076	1019	942	998	904	883	776	824	883	759	695	695	998	
1037	1105	1201	1364	2482	1019	1076	1146	1221	1225	1221	1146	1076	1019	942	904	883	824	824	824	759	695	695	1021	
1225	1221	1146	1076	1019	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	776	824	883	904	1146	
1221	1146	1076	1019	2482	1364	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	776	824	883	1221	
1146	1221	1225	1221	1146	1076	1019	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	776	824	1225	
1076	1146	1221	1225	1221	1146	1076	1019	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	776	1221	
1019	1076	1146	1221	1225	1221	1146	1076	1019	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	1146	
942	998	1002	1037	1225	1221	1146	1076	1019	2482	1364	1201	1105	1037	1225	1221	1146	1076	1019	942	998	1002	979	1201	
998	942	998	1021	1002	1221	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1364	
1002	1021	998	942	938	1076	1146	1221	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1019	942	1364	
979	1032	1021	998	942	1019	1076	1146	1221	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1019	1201	
776	824	883	904	908	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1019	1146	
824	776	824	883	904	998	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1221	
883	824	776	824	883	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1146	1076	1019	1225	
904	883	824	776	824	1002	1021	998	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1037	1225	1221	1201	
900	904	883	824	776	979	1032	1021	998	942	1019	1076	1146	1221	1225	1221	1146	1076	1019	942	998	1002	1037	1146	
659	695	759	802	821	776	824	883	904	908	942	998	1002	1037	1225	1221	1146	1076	1019	942	998	1002	1037	1021	
695	559	695	759	802	824	776	824	883	904	998	942	1021	1002	1221	1225	1221	1146	1076	1019	942	998	1002	998	
759	695	659	695	759	883	824	776	824	883	904	998	942	1021	1002	1221	1225	1221	1146	1076	1019	942	998	942	
802	759	695	659	695	904	883	824	776	824	883	904	998	942	1021	1002	1221	1225	1221	1146	1076	1019	942	998	
821	802	759	695	659	904	883	824	776	824	883	904	998	942	1021	1002	1221	1225	1221	1146	1076	1019	942	1021	

Table 6

COEFFICIENTS FOR MATRIX INVERSION: BENCH 20

All coefficients $\times 10^{-4}$

2542	1443	1238	1118	1032	1304	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028
1443	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028
1238	1443	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	683	735	778	763
1118	1238	1443	2542	1443	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028
1032	1118	1238	1443	2542	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028
1304	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874	1028
1273	1304	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885	874
1173	1273	1304	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872	885
1084	1173	1273	1304	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826	872
1011	1084	1173	1273	1304	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789	826
979	1029	1028	979	954	1032	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789
1028	979	1029	1028	979	954	1032	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954
979	1028	1029	979	954	1032	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954	789
954	979	1028	1029	979	954	1032	1273	1173	1084	1011	2542	1443	1238	1118	1032	1273	1173	1084	1011	979	1029	937	954
789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028
826	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763
872	826	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778
885	872	826	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735
874	885	872	826	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683
683	874	885	872	826	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874
654	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028	979	1029	937	954
735	683	654	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028	979	1029
770	735	683	654	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028	979
783	770	735	683	654	683	735	778	763	1028	979	1029	937	954	789	826	872	885	874	683	735	778	763	1028

Table 7

KRIGING WEIGHTS: BENCHES 10 AND 20

BENCH	10	20
weight no.	weight	weight
a ₁	0.0293	0.0214
a ₂	0.0081	0.0055
a ₃	-0.0521	-0.0429
a ₄	0.0081	0.0055
a ₅	0.0293	0.0214
a ₆	0.0380	0.0294
a ₇	0.0753	0.0748
a ₈	0.0569	0.0801
a ₉	0.0753	0.0748
a ₁₀	0.0380	0.0294
a ₁₁	0.0450	0.0349
a ₁₂	0.1480	0.1638
a ₁₃	0.1480	0.1638
a ₁₄	0.0450	0.0349
a ₁₅	0.0380	0.0294
a ₁₆	0.0753	0.0748
a ₁₇	0.0569	0.0801
a ₁₈	0.0753	0.0748
a ₁₉	0.0380	0.0294
a ₂₀	0.0293	0.0214
a ₂₁	0.0081	0.0055
a ₂₂	-0.0521	-0.0429
a ₂₃	0.0081	0.0055
a ₂₄	0.0293	0.0214
mean	0.0016	0.0038

Table 8

400 KRIGED BLOCKS: BENCH 10

All percentages $\times 10^{-2}$

53	46	50	34	24	32	31	32	32	47	31	41	35	34	47	46	53	61	56	60
59	56	50	41	33	24	32	34	31	41	32	40	40	34	51	43	52	53	51	51
54	56	55	46	42	36	34	37	33	37	31	37	38	31	44	41	45	48	44	41
53	48	50	50	56	43	43	48	38	40	31	39	32	37	49	54	48	46	51	50
47	46	42	41	53	49	40	53	41	35	35	36	34	43	56	62	67	66	68	65
54	48	51	46	50	49	51	50	51	34	35	43	44	56	69	72	80	83	94	89
75	61	57	51	63	59	57	70	55	50	55	58	63	68	76	82	94	90	96	90
92	82	82	72	68	70	76	76	83	76	86	93	90	90	97	98	87	88	80	85
124	94	92	90	100	92	88	100	108	123	128	114	113	109	102	101	89	80	78	77
103	129	139	101	101	110	125	138	138	143	149	133	137	126	111	105	87	94	78	80
95	83	83	88	118	129	143	154	156	155	178	136	151	115	108	124	101	102	90	83
63	69	72	73	104	124	136	145	156	158	174	140	151	133	106	97	94	100	101	111
73	76	59	87	110	129	130	141	140	151	165	139	150	127	93	99	87	107	113	141
101	109	120	105	113	132	135	137	144	140	138	113	131	114	95	91	94	105	112	137
115	125	121	127	140	138	159	142	139	132	126	101	98	84	81	101	93	106	77	103
127	116	128	134	128	138	132	131	130	128	118	95	98	83	76	78	71	83	57	76
116	111	104	115	100	97	110	106	112	104	104	93	91	87	84	71	69	71	59	60
102	89	76	85	74	78	82	76	81	72	72	75	77	85	75	75	68	66	61	57
80	70	62	64	68	64	64	57	55	51	55	57	56	55	64	62	65	62	55	55
80	65	66	68	62	60	55	45	44	44	46	51	51	53	54	52	56	55	55	55

Table 9

400 KRIGED BLOCKS: BENCH 20

All percentages $\times 10^{-2}$

36	33	37	27	20	29	28	30	41	41	37	51	42	40	47	44	53	59	53	55
50	50	37	37	37	33	37	33	38	35	34	38	39	34	48	43	57	59	49	50
47	48	43	43	48	46	46	31	36	28	28	34	39	33	45	41	48	44	46	40
62	52	65	62	71	54	44	34	37	29	36	33	36	38	44	51	48	49	48	48
48	77	62	78	64	52	43	36	35	39	32	45	35	43	48	54	59	61	65	69
55	68	78	73	66	51	42	44	42	43	47	47	48	54	55	59	63	75	80	100
71	75	64	68	60	56	49	46	49	56	46	61	69	69	68	73	77	77	83	84
73	69	72	69	71	56	61	63	55	70	64	83	89	93	83	80	80	70	68	81
87	89	78	79	90	81	94	82	86	95	83	86	93	89	99	85	95	64	77	75
98	97	102	96	100	91	116	111	106	110	101	92	101	103	93	92	92	78	87	87
112	133	106	104	106	89	99	115	109	107	98	99	105	106	103	106	109	102	90	89
113	116	112	122	106	88	86	77	75	113	102	127	135	119	110	107	107	103	104	94
106	129	114	123	111	103	71	86	75	105	122	127	144	125	126	118	113	114	106	107
118	108	108	121	122	110	94	90	94	120	125	151	145	129	123	108	96	103	109	115
94	101	105	103	110	122	129	134	133	144	124	148	142	120	114	104	95	94	89	92
88	83	93	115	118	136	111	141	159	120	125	107	106	98	97	99	78	81	67	76
67	76	93	96	90	121	110	116	115	106	83	76	71	60	77	89	84	75	66	52
54	58	64	80	78	93	65	72	89	54	66	47	50	54	55	69	63	62	63	53
56	53	69	67	61	70	59	59	51	50	49	51	45	46	48	57	50	56	58	55
57	62	65	61	61	55	47	47	53	47	44	46	47	48	53	55	42	43	48	50

Fig. 13

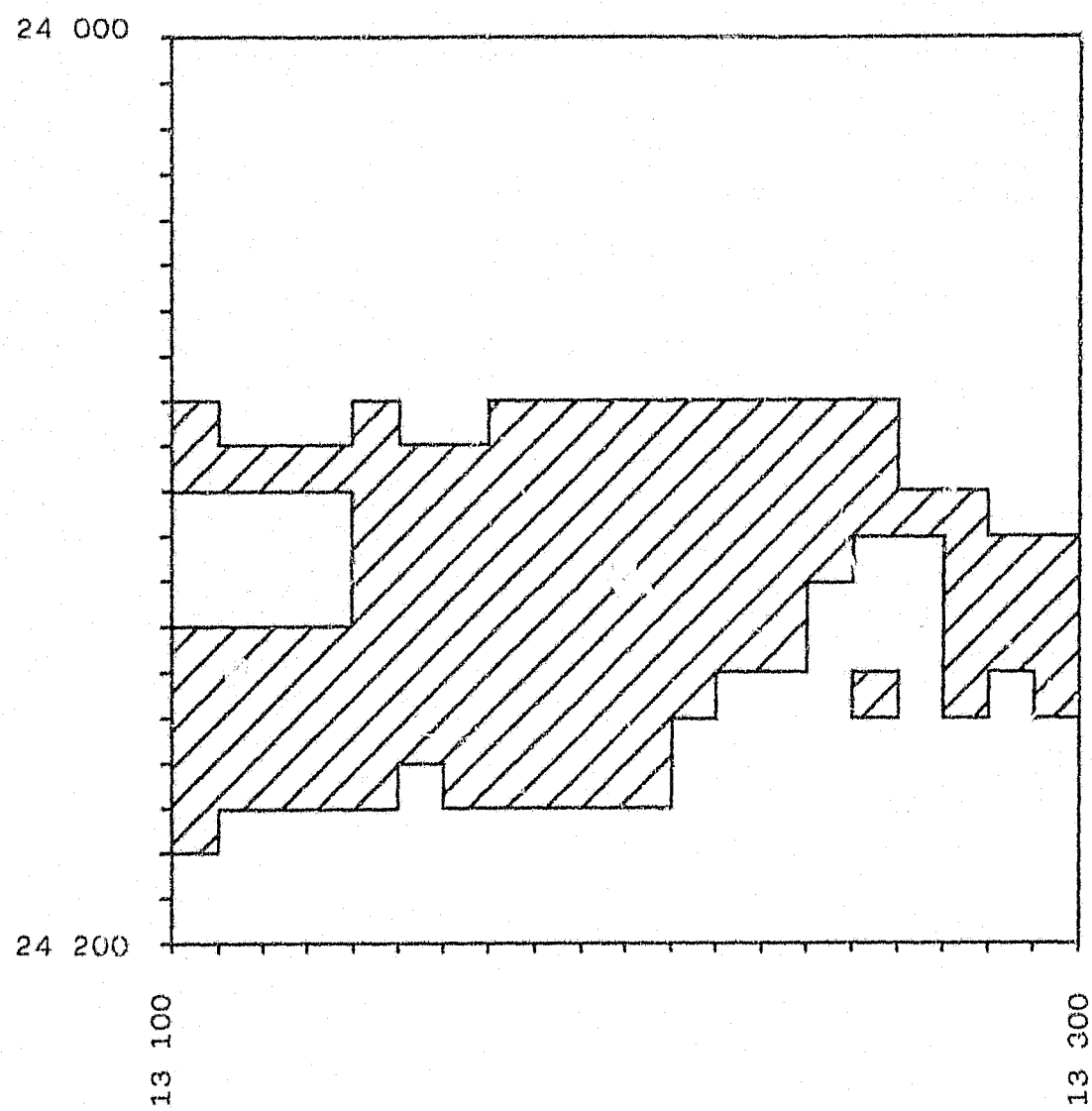
BLOCKS KRIGED AT 1.0% OR GREATER: BENCH 10

Fig. 14

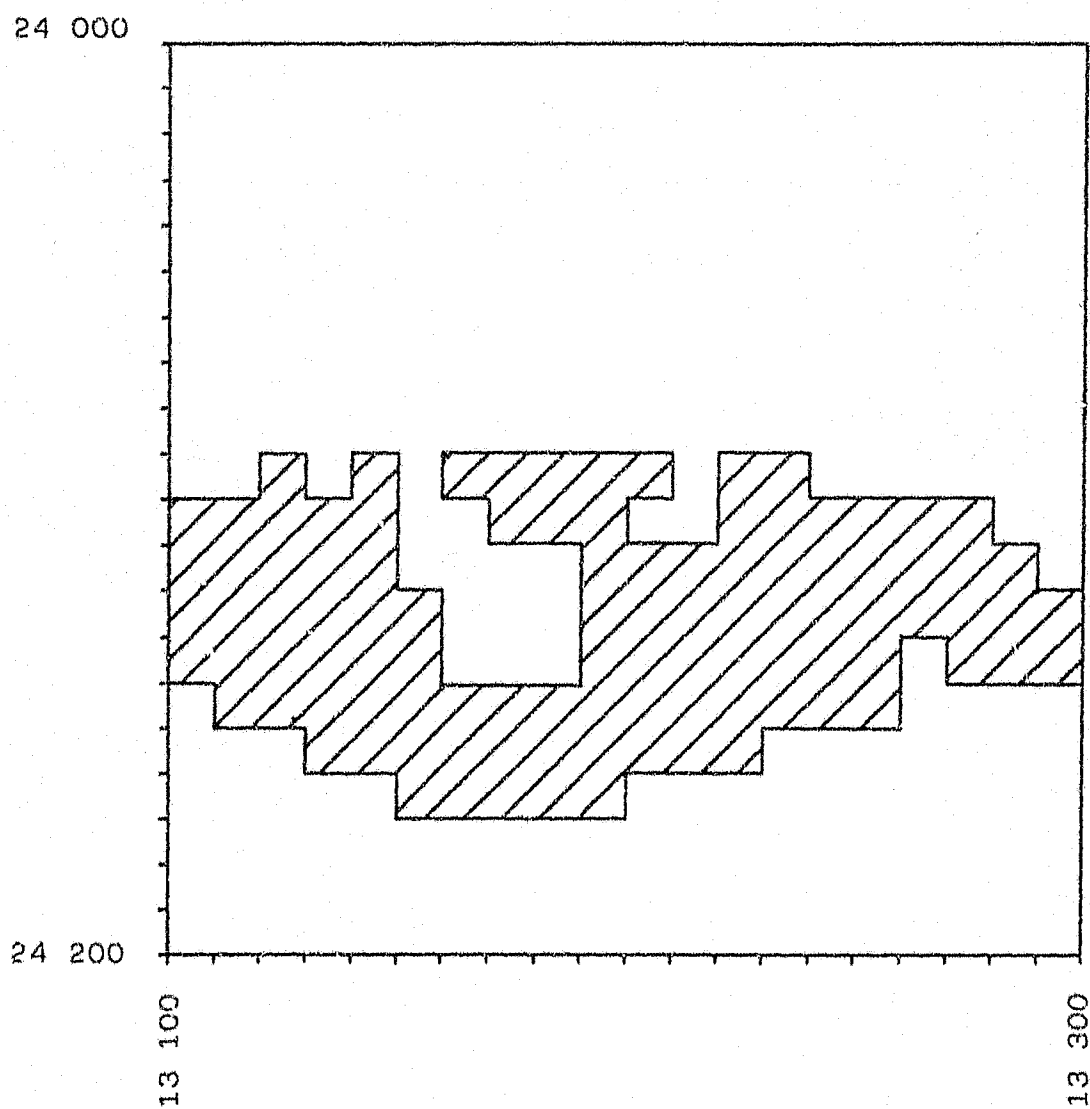
BLOCKS KRIGED AT 1.0% OR GREATER: BENCH 20

Fig. 15

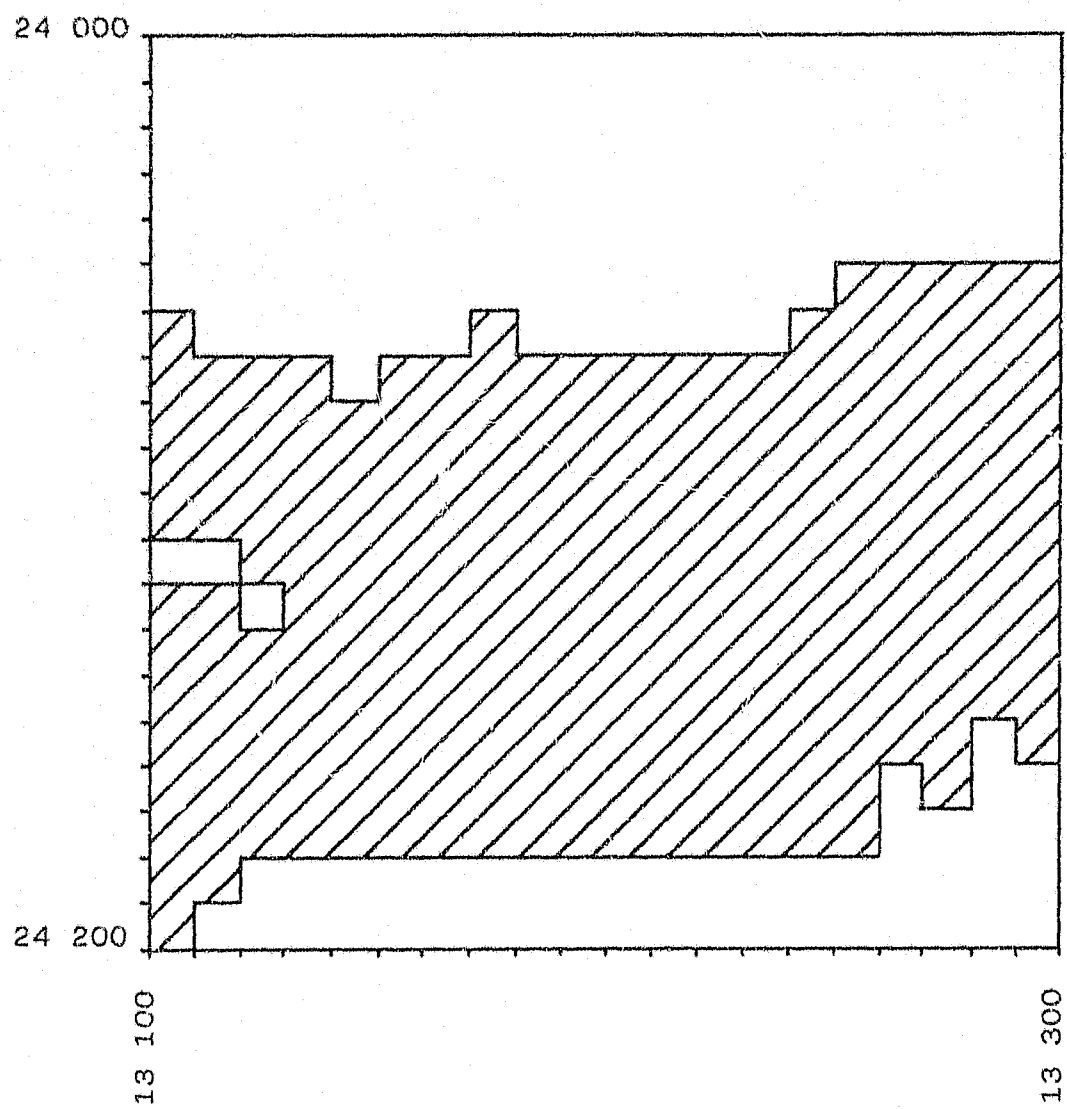
BLOCKS KRIGED AT 0.7% OR GREATER: BENCH 10

Fig. 16

BLOCKS KRIGED AT 0.7% OR GREATER; BENCH 20

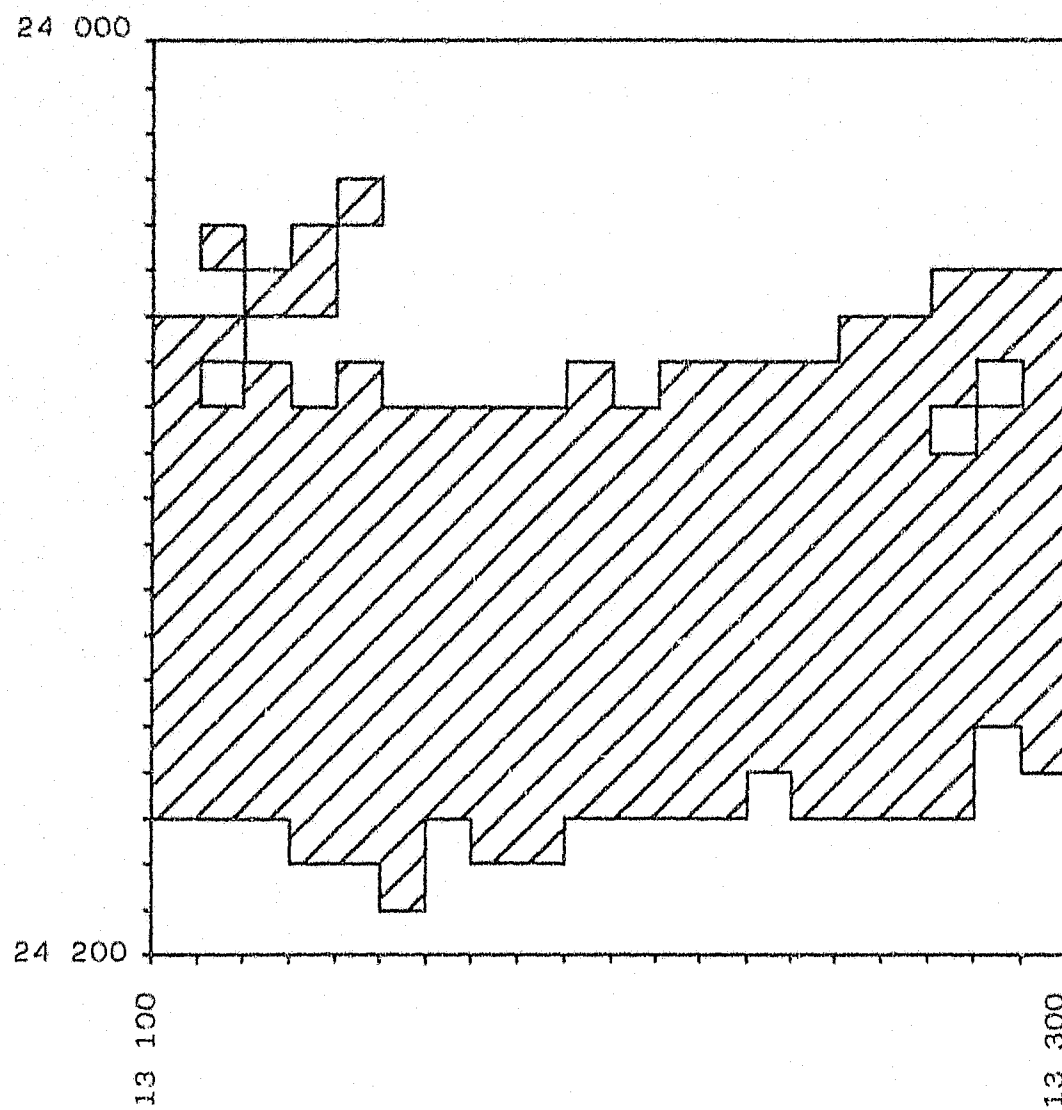


Fig.17

KRIGED V. SAMPLE VALUES: BENCH 10

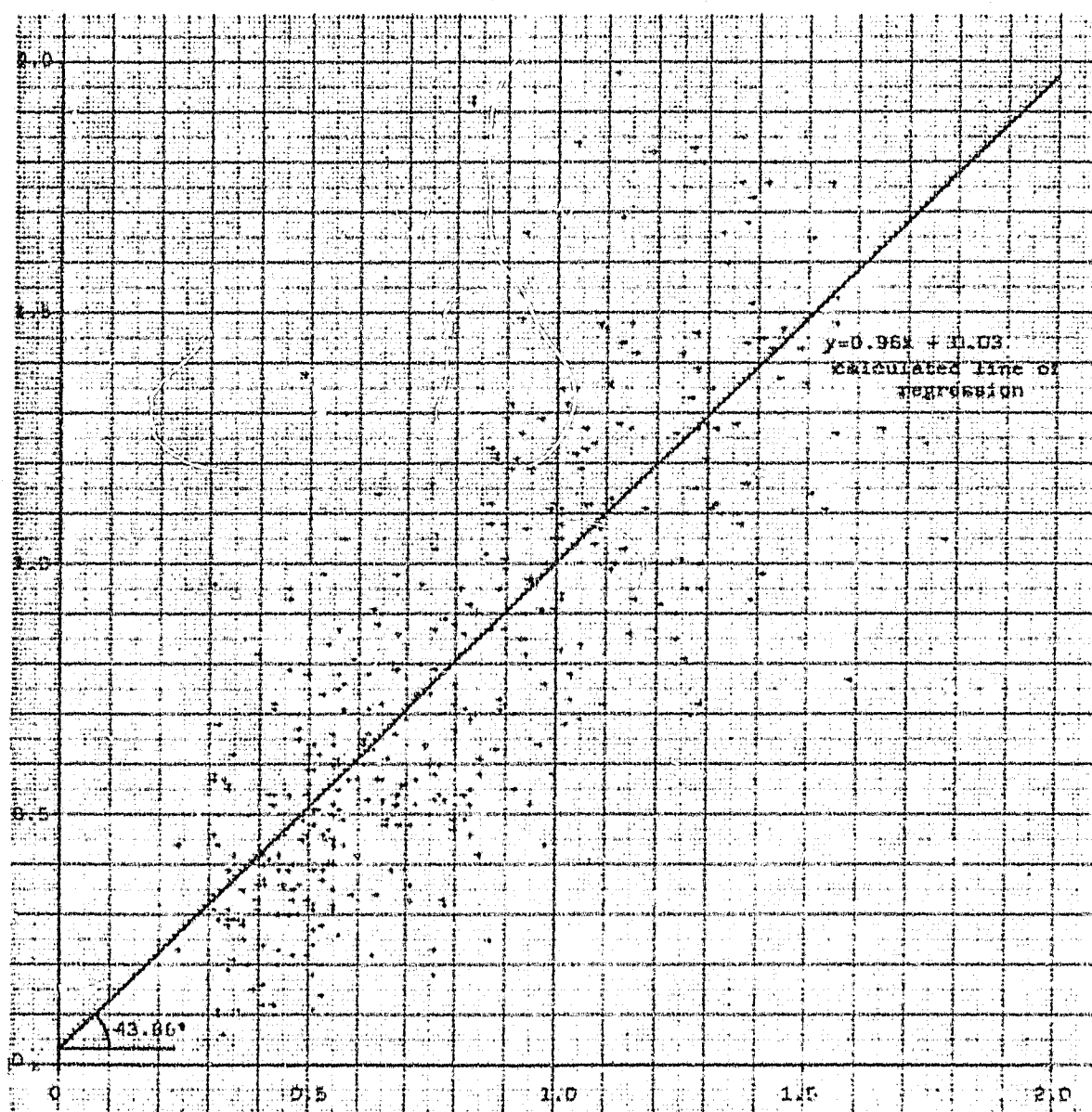
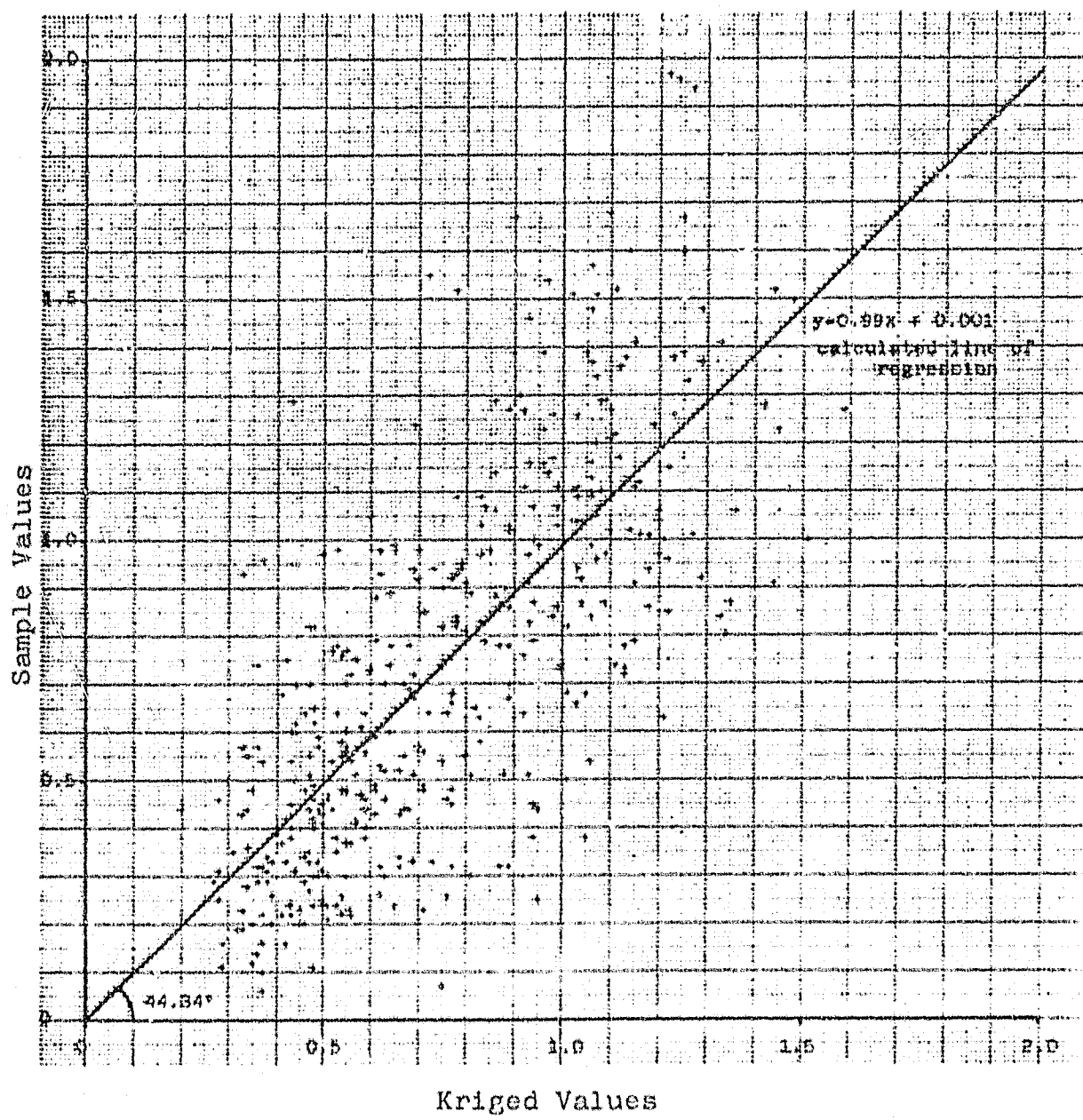


Fig.18

KRIGED V. SAMPLE VALUES: BENCH 20

4 COMPARISON AND DISCUSSION

4.1 Comparison

A statistical test was made to see whether the two transformed population means were the same within 95% confidence limits i.e. the null hypothesis is that the means are the same and the alternative hypothesis is that the mean of Bench 10 is greater.

Expressed formally:

$$1. H_0: \mu_1 = \mu_2 = \mu_0$$

$$2. H_a: \mu_1 > \mu_2$$

$$3. \alpha = 5\%$$

4. TESTSTATISTIC

$$Z_{\bar{x}_1 - \bar{x}_2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\bar{x}_1, \bar{x}_2$ = transformed sample means of samples 1 and 2
 n_1, n_2 = sample sizes
 σ_1^2, σ_2^2 = variances of populations 1 and 2

Substituting the values into the equation:

$$Z_{\bar{x}_1 - \bar{x}_2} = 4.3548$$

5. Rejection area:

reject H_0 in favour of H_a when greater than

1.645

i.e. within 95% confidence limits the means are not the same and the mean of Bench 10 is greater than that of Bench 20.

It must be stressed however that the foregoing test is somewhat lenient because the data block values form a systematic and not a 'random' set.

4.2 Kriged vs. Known Values

The 400 Kriged values were plotted against the known values of the Kriged blocks for both benches (Figs. 17 and 18); regression lines were calculated and as can be seen the intercepts on the y axis are close to zero and the slopes very close to 45°. This indicates that the Kriging process gives conditionally unbiased results, i.e. there is no tendency to exaggerate high or low values.

The exercise was also used to test the following relationship, in particular the correctness of the estimate of the Kriging variance:

$$\sigma^2_{ye} = N + \sigma^2_k$$

where σ^2_{ye} = variance of difference between the sample values and the corresponding Kriged values, i.e. the conditional y variance on Figs. 17 and 18.

N = nugget effect

σ^2_k = Kriging variance

This gave the following results: (all log. variances - see sections 3.3 and 3.4)

BENCH 10

$$\begin{aligned} 0.1980 &= 0.0925 + 0.1241 \\ &= 0.2166 \end{aligned}$$

BENCH 20

$$\begin{aligned} 0.1830 &= 0.086 + 0.1229 \\ &= 0.2089 \end{aligned}$$

Although the equalities do not balance, they are sufficiently close to give confidence to the estimate of the Kriging variance.

4.3 Discussion

Initial analysis of the 2500 values on each bench showed that BENCH 10 was richer than BENCH 20 as indicated by the higher mean; a somewhat lenient statistical test showed that even within the given variances, the means were not the same and supported the hypothesis that BENCH 10 was of a higher average grade.

The semivariogram analysis shows a remarkable similarity between the two combined semivariograms indicating similar spatial structures within the two benches; the semivariograms for the individual directions are also similar for each bench. It was difficult to decide on a suitable model for the combined semivariograms due to the sudden upturning of the graph beyond about Lag 8; a parabolic model was considered but was not suitable for the function values below Lag 8. The form is probably due to 'drift' in the grades - high in the centre of the area and lower towards the outside. This would require Universal Kriging for proper modelling. As the format for Kriging only required calculation of the semivariogram function below Lag 8, the Lognormal de Wijsian model was used.

The results from Kriging of 400 blocks on each bench, while emphasising the continuity of the central higher grade structure, reinforce the hypothesis that BENCH 10 is of higher grade than BENCH 20.

With reference to the diagrams showing the Kriged blocks at different cut-offs, it should be noted that due to the variance of the estimates, the edges of the areas are not hard and fast and more consensus between the benches is possible than the diagrams seem to indicate.

5 CONCLUSION

The analysis shows that there could be a declining grade with depth over the area under investigation. The scope of this study is insufficient to put any meaningful value to any such drift. This would require a much more extensive study over the whole of the mined out areas of the pit, both horizontally and vertically, including a complete three dimensional semi-variogram analysis.

However considering the vertical distance separating the two benches, the differences in grade distributions are not considerable and, assuming the vertical drift is regular, the values on one bench will give a very good indication of the values in the bench directly below.

As far as valuation practice is concerned, numerous studies on other deposits have proved that Kriging gives the best estimate of blocks, or in Palabora's case - composites; and this has repercussions in reconciling mined grade with mill yield. As Kriging methods require the use of computers, one advantage is that composite grade estimates can be prepared very quickly once sampling values have been entered into the data base.

6 COMPUTER PROGRAMMES

6.1 Description of the programmes

The following programmes are written in BASIC for the Sinclair Spectrum 48K personal computer, data being labelled and stored on cassette tape.

GEOSTATS-1

Collects 2500 data points and stores them in a 50 x 50 matrix form for use in the following programmes. Each set of points was labelled either BENCH10 or BENCH20 and stored on tape.

GEOSTATS-2

Takes data and sorts them into category frequency with a range of 0 to 3% in steps of 0.05%. The results are used to construct a histogram.

GEOSTATS-3

Takes data, adds constant, converts to natural log and calculates the sum of log values and the sum of $(\log \text{ values})^2$ used to calculate the mean and variance.

GEOSTATS-4

Calculates semivariograms in four principal directions, gives semivariogram function for a given lag.

GEOSTATS-5

Collects input for a 24 x 25 matrix ready for inversion.

GEOSTATS-6

Takes the data collected by GEOSTATS-5 and solves the augmented matrix by Gaussian elimination. The procedure is as follows:

1. Take the first entry in the first row and make it equal to +1 by dividing the row by the first entry.
2. Make all the entries in the column below the +1 equal to 0 by subtracting suitable multiples of the first row from the remaining rows.
3. Disregard the top row and repeat the procedure with the remaining rows of the matrix.

When the procedure terminates, the original matrix has been converted into row reduced or echelon form. The system of equations is solved by back substitution and gives the weights used in the Kriging process.

GEOSTATS-7

Kriges 10m x 10m blocks from surrounding twenty four 10m x 10m blocks.

GEOSTATS-1

```
10  PRINT  "GEOSTATS-1"
20  PRINT  "point data file"
30  DIM  a(50,50)
40  PRINT
50  FOR  y=1 TO 50
60  FOR  x=1 TO 50
70  PRINT  "          a(";x;"",";y;")=";
80  INPUT  c
90  LET  a(x,y)=c
100 PRINT  ,c
110 NEXT  x
120 NEXT  y
130 STOP
```

GEOSTATS-2

```
10  PRINT  "GEOSTATS-2"
20  LET   z=0
30  PRINT  "HISTOGRAM OF POINT DATA"
40  PRINT  "LOAD DATA"
50  LOAD  "POINT DATA"  DATA a()
60  PRINT
70  PRINT  "RANGE","VALUES"
80  DIM   a(50,50)
90  FOR   d=0 TO 300 STEP 5
100 LET   e=d/100
110 LET   z=0
120 FOR   y=1 TO 50
130 FOR   x=1 TO 50
140 IF   a(x,y)=e THEN GOTO 170
150 IF   a(x,y)>e THEN GOTO 170
160 IF   a(x,y)<e THEN GOTO 180
170 IF   a(x,y)<(e+0.049) THEN LET z=z+1
180 NEXT  x
190 NEXT  y
200 PRINT  e;"-";" ";e+0.049,z
210 NEXT  d
220 STOP
```

GEOSTATS-3

```
10  PRINT  "CALCULATION OF MEAN AND VARIANCE"
20  DIM  a(50,50)
30  PRINT  "LOAD DATA"
40  LOAD  "BENCH10" DATA a()
50  PRINT  "ENTER ADDITIVE CONSTANT";
60  INPUT  b
70  PRINT  b
80  LET  m=0
90  LET  q=0
100  FOR  y=1 TO 50
110  FOR  x=1 TO 50
120  IF  a(x,y)=0 THEN GOTO 170
130  LET  n=LN(a(x,y)+b)
140  LET  m=m+n
150  LET  p=(n)+2
160  LET  q=q+p
170  NEXT  x
180  NEXT  y
190  PRINT  "SUM OF LN="; m
200  PRINT  "SUM OF LN+2="; q
210  STOP
```

GEOSTATS-4

```
10  PRINT "SEMIVARIOGRAMS - 4 DIRECTIONS"
20  DIM a(50,50)
30  PRINT "LOAD DATA"
40  LOAD "BENCH10" DATA a()
50  INPUT "ENTER MAX. LAG", z
60  INPUT "FOR x BETWEEN", v
70  INPUT "          AND", u
80  INPUT "ENTER 3rd PARAMETER", w
90  CLS
100 PRINT "DIRECTION 1"
110 FOR h=1 TO z
120 LET l=0
130 LET n=0
140 FOR y=1 TO 50
150 FOR x=v TO (u-h)
151 IF a(x,y)=0 THEN GOTO 220
152 IF a(x+h,y)=0 THEN GOTO 220
160 LET b=LN(a(x,y)+w)
170 LET c=LN(a(x+h,y)+w)
180 LET m=b-c
190 LET k=m*m
200 LET l=l+k
210 LET n=n+1
220 NEXT x
230 NEXT y
240 LET g=1/(2*n)
250 PRINT "LAG";h;"=";g,"n=";n
260 NEXT h
270 PRINT "FOR NEW DIRECTION PRESS CONT"
280 STOP
290 CLS
300 PRINT "DIRECTION 2"
310 FOR h=1 TO z
320 LET l=0
```



```
330 LET n=0
340 FOR x=v TO u
350 FOR y=1 TO (50-h)
351 IF a(x,y)=0 THEN GOTO 420
352 IF a(x,y+h)=0 THEN GOTO 420
360 LET b=LN(a(x,y)+w)
370 LET c=LN(a(x,y+h)+w)
380 LET m=b-c
390 LET k=m*m
400 LET l=1+k
410 LET n=n+1
420 NEXT y
430 NEXT x
440 LET g=1/(2*n)
450 PRINT "LAG";h;"=";g,"n=";n
460 NEXT h
470 PRINT "FOR NEW DIRECTION PRESS CONT"
480 STOP
490 CLS
500 PRINT "DIRECTION 3"
510 FOR h=1 TO z
520 LET l=0
530 LET n=0
540 FOR x=v TO (u-h)
545 LET r=x
550 LET y=1
551 IF a(x,y)=0 THEN GOTO 620
552 IF a(x+h,y+h)=0 THEN GOTO 620
560 LET b=LN(a(x,y)+w)
570 LET c=LN(a(x+h,y+h)+w)
580 LET m=b-c
590 LET k=m*m
600 LET l=1+k
610 LET n=n+1
620 IF x=u-h THEN GOTO 655
630 LET y=y+1
640 LET x=x+1
```

```
650 GOTO 551
655 LET x=r
660 NEXT x
670 FOR y=2 TO (50-h)
675 LET r=y
680 LET x=1
681 IF a(x,y)=0 THEN GOTO 750
682 IF a(x+h,y+h)=0 THEN GOTO 750
690 LET b=LN(a(x,y)+w)
700 LET c=LN(a(x+h,y+h)+w)
710 LET m=b-c
720 LET k=m*m
730 LET l=1+k
740 LET n=n+1
750 IF y=50-h THEN GOTO 785
760 LET y=y+1
770 LET x=x+1
780 GOTO 681
785 LET y=r
790 NEXT y
800 LET g=1/(2*n)
810 PRINT "LAG";h;"=";g,"n=";n
820 NEXT h
830 PRINT "FOR NEW DIRECTION PRESS CONT"
840 STOP
850 CLS
860 PRINT "DIRECTION 4"
870 FOR h=1 TO z
880 LET n=0
890 LET l=0
900 FOR x=v TO (u-h)
905 LET r=x
910 LET y=50
911 IF a(x,y)=0 THEN GOTO 980
912 IF a(x+h,y-h)=0 THEN GOTO 980
920 LET b=LN(a(x,y)+w)
930 LET c=LN(a(x+h,y-h)+w)
```

```
940 LET m=b-c
950 LET k=m*m
960 LET l=l+k
970 LET r=n+1
980 IF x=u-h THEN GOTO 1015
990 LET x=x+1
1000 LET y=y-1
1010 GOTO 911
1015 LET x=r
1020 NEXT x
1030 FOR y=49 TO (h+1) STEP -1
1035 LET r=y
1040 LET x=1
1041 IF a(x,y)=0 THEN GOTO 1110
1042 IF a(x+h,y-h)=0 THEN GOTO 1110
1050 LET b=LN(a(x,y)+w
1060 LET c=LN(a(x+h,y-h)+w)
1070 LET m=b-c
1080 LET k=m*m
1090 LET l=l+k
1100 LET n=n+1
1110 IF y=h+1 THEN GOTO 1145
1120 LET x=x+1
1130 LET y=y-1
1140 GOTO 1041
1145 LET y=r
1150 NEXT y
1160 LET g=1/(2*n)
1170 PRINT "LAG";h;"=";g,"n=";n
1180 NEXT h
1190 STOP
```

GEOSTATS-5

```
10  PRINT  "GEOSTATS-5"
20  PRINT  "MATRIX DATA"
30  DIM  a(24,25)
40  PRINT
50  FOR  y=1 TO 25
60  FOR  x=1 TO 24
70  PRINT  "a(";x;",";y;")=";
80  INPUT  c
90  LET  a(x,y)=c
100 PRINT, c
110 NEXT  x
120 NEXT  y
130 STOP
```

GEOSTATS-6

```
10  PRINT  "GEOSTATS-6"
20  PRINT  "MATRIX INVERSION"
30  DIM  a(24,25)
40  LOAD  "BENCH10"  DATA a()
50  LET  d=1
60  PRINT
70  FOR x=d TO 24
80  FOR y=25 TO d STEP -1
90  LET  c=a(x,d)
100 LET  a(x,y)=a(x,y)/c
110 NEXT y
120 NEXT x
130 FOR x=(d+1) TO 24
140 FOR y=d TO 25
150 LET  a(x,y)=a(x,y)-a(d,y)
160 NEXT y
170 NEXT x
180 IF  d=23 THEN GOTO 210
190 LET  d=d+1
200 GOTO 60
210 DIM  w(24)
220 FOR  z=1 TO 24
230 LET  w(z)=0
240 NEXT z
250 FOR  z=24 TO 1 STEP -1
260 LET  w(z)=(a(z,25)-w(24)*a(z,24)-w(23)*a(z,23)
-w(22)*a(z,22)-w(21)*a(z,21)-w(20)*a(z,20)
-w(19)*a(z,19)-w(18)*a(z,18)-w(17)*a(z,17)
-w(16)*a(z,16)-w(15)*a(z,15)-w(14)*a(z,14)
-w(13)*a(z,13)-w(12)*a(z,12)-w(11)*a(z,11)
-w(10)*a(z,10)-w(9)*a(z,9)-w(8)*a(z,8)
-w(7)*a(z,7)-w(6)*a(z,6)-w(5)*a(z,5)-w(4)*a(z,4)
-w(3)*a(z,3)-w(2)*a(z,2)-w(1)*a(z,1)/a(z,z)
270 NEXT z
```

```
280  FOR  z=1 TO 24
290  PRINT  "w(";z;")","=";w(z)
300  NEXT  z
310  STOP
```

GEOSTATS-7

```
10  PRINT  "KRIGED BLOCKS"
20  PRINT  "ENTER WEIGHTS"
30  LET  h=0
40  DIM  w(25)
50  FOR  z=1 TO 25
60  PRINT  "w(";z;")=";
70  INPUT  b
80  LET  w(z)=b
90  PRINT  b
100 LET  h=h+b
110 NEXT  z
120 INPUT  "ENTER ADDITIVE CONSTANT", c
130 INPUT  "ENTER MEAN", d
140 PRINT  "ENTER COVARIANCE OF X TO Y"
150 DIM  p(25)
160 FOR  x=1 TO 25
170 PRINT  "SIGMA y(";x;")=";
180 INPUT  f
190 LET  p(x)=f
200 PRINT  f
210 NEXT  x
220 INPUT  "ENTER POPULATION VARIANCE", e
230 DIM  a(50,50)
240 PRINT  "ENTER BENCH DATA"
250 LOAD  "BENCH10" DATA a()
260 LET  i=(1-h)*LN(d+c)
270 LET  z=0
280 LET  r=0
290 FOR  x=1 TO 25
300 LET  z=z+1
310 LET  q=w(z)*(e-p(x))
320 LET  r=r+q
330 NEXT  x
340 LET  s=r/2
```

```
350 FOR y=16 TO 35
360 FOR x=11 TO 30
370 LET z=0
380 LET u=0
390 FOR k=(x-2) TO (x+2)
400 FOR l=(y-2) TO (y+2)
410 LET z=z+1
420 LET m=w(z)*LN(a(k,l)+c)
430 LET u=u+m
440 NEXT l
450 NEXT k
460 LET t=EXP(i+u+s)-c
470 PRINT "a(";x;",";y;")=";
480 PRINT t
490 NEXT x
500 NEXT y
510 STOP
```


LIST OF REFERENCES

- (1) David, M. Geostatistical Ore Reserve Estimation. Elsevier, Amsterdam. 1977
- (2) Journel and Huijbregts. Mining Geostatistics. Centre de Geostatistique, Fontainebleau, France, 1978
- (3) Krige, D.G. Lognormal - de Wijsian Geostatistics for Ore Evaluation. South African Institute of Mining and Metallurgy, Johannesburg, 1981
- (4) Krige, D.G. Geostatistical Methods in Mineral Evaluation. Course Notes, (1983)
- (5) Park, C.F. and MacDiarmid, R.A. Ore Deposits. W.H. Freeman & Co., San Francisco, 1975
- (6) Rendu, J.-M. An Introduction to Geostatistical Methods of Mineral Evaluation. South African Institute of Mining and Metallurgy, Johannesburg, 1978
- (7) Royle, A.G. Practical Geostatistics. Course Notes, (1979)

LIST OF REFERENCES

- (1) David, M. Geostatistical Ore Reserve Estimation. Elsevier, Amsterdam. 1977
- (2) Journel and Huijbregts. Mining Geostatistics. Centre de Geostatistique, Fontainebleau, France, 1978
- (3) Krige, D.G. Lognormal - de Wijsian Geostatistics for Ore Evaluation. South African Institute of Mining and Metallurgy, Johannesburg, 1981
- (4) Krige, D.G. Geostatistical Methods in Mineral Evaluation. Course Notes, (1983)
- (5) Park, C.F. and MacDiarmid, R.A. Ore Deposits. W.H. Freeman & Co., San Fransisco, 1975
- (6) Rendu, J.-M. An Introduction to Geostatistical Methods of Mineral Evaluation. South African Institute of Mining and Metallurgy, Johannesburg, 1978
- (7) Royle, A.G. Practical Geostatistics. Course Notes, (1979)

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