NEW MODELS FOR BOREHOLE VALUATION OF NEW MINING OPERATIONS AND ROUTINE VALUATION OF ORE RESERVE BLOCKS

Winfred Assibey-Bonsu

A thesis submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy in Mining Engineering

Johannesburg, 1991

DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

20th day of August 1991

ABSTRACT

This thesis proposes some new valuation models and procedures for global and local ore reserve estimation.

To obtain efficient grade and tonnage estimates for global borehole valuations of new gold mining properties, the appropriate spatial and distributional models for the mineralisation are absolutely essential.

The three parameter lognormal distribution has been practically useful for modelling the gold distributions in most of the Witwatersrand reefs. However, the geological complexity of the type of reefs being mined at present in the Witwatersrand basin demands the need for more applicable distributional models. The thesis places the geology of the Witwatersrand basin in perspective and draws some important conclusions from the genesis of the deposition. Based on these geological models two more general models- the Compound Lognormal and the Log-Generalised Inverse Gaussian distributions are proposed. A detailed, though preliminary comparative study of these two new models with the three parameter lognormal model is done and the efficiencies of estimates and modelling analysed. It is shown that the broad assumption that gold grades follow a three parameter lognormal law is not always the case and the need to test for departures from lognormality should be emphasized so as to ensure the use of appropriate models.

Furthermore, due to the high cost and time delays in drilling deep boreholes, most gold mines are opened on estimates based on small numbers of boreholes. Taking cognisance of the risk which accompanies these capital intensive gold mining investments, the study investigates the possibility of deriving additional information from neighbouring mines based on geological similarities to improve global borehole estimates.

The thesis also introduces a new concept of spatial structures on variances as a means of deriving additional useful information on grade variabilities.

Using a very large number of chip samples from Hartebeestfontein mine and accepting these as equivalent to borehole values - also 1Km x 1Km areas as equivalent to 'mines'- a series of borehole valuation procedures based on the three parameter lognormal model are applied to these 'mines'. Large undeveloped sections of existing mines are also seen as equivalent of such 'mines'. Conclusions are drawn on the relative importance of the various items of additional information used and the procedures followed and on the significance of the advantages gained.

Particulary, it is found that severe conditional biasses can be observed not only at the routine micro (local) block level but also at a mine (global) scale. Macro simple kriging with spatial structures for grade levels and grade variabilities will eliminate these conditional biasses and reduce error variances significantly.

The procedures for the global analyses are also applied (in principle) for face (panel) valuations. It is further shown that unlike global valuation the customary Sichel 't' or t" estimator will not be useful for routine face (panel) valuations.

To my lovely family

" GLORY TO GOD IN THE HIGHEST "

ACKNOWLEDGEMENT

I wish to express my sincere appreciation and thanks to my supervisors Professor D G Krige and Dr W J Kleingeld for their invaluable assistance and encouragement throughout the various stages of the study. I also thank Professor H S Sichel for his assistance in connection with the work on the new distributions and Mr M M Oosterveld for his assistance regarding the geology of the Witwatersrand basin.

I am indebted to the University of the Witwatersrand and the CSIR who provided the financial support for the study over a period of three years. I thank the Head of Department Professor H R Phillips who was always available to assist in many issues. I also appreciate the cooperation I received from all the staff members of the Mining Engineering Department.

The Anglo American/De Beers Ore Evaluation Division deserves a credit for assisting among other things with computer facilities and part of the data base used for the study. The assistance and encouragement I received from this Department served as the major stimulus for the study. I thank Messrs C F Prins and R A Pereira also for their assistance in computer programming.

The cooperation from the Anglo Vaal, JCI, Rand Mines and Gencor Mining Groups is appreciated. The large Hartebeestfontein data base used throughout the study was provided by Anglo Vaal and the efforts of Dr F A Camisani and Mrs G Knox in this regard is much appreciated.

I would like also to express my deep and heartfelt gratitude to all my Christian friends and the Ghanaian community in Johannesburg.
Finally, I thank my wife Joyce and my son Ebenezer for their patience and encouragement.

CONTENTS

CONTENTS Page

CONTENTS | Page

CONTENTS Page

CONTENTS

CONTENTS Page

LIST OF FIGURES

FIGURE | Page

LIST OF TABLES

PREFACE

The Witwatersrand gave birth to geostatistics mainly due to the pioneering work of Professor D G Krige and Professor H S Sichel and the vast amount of data resources available in the Witwatersrand field. The author deems it a great opportunity to be able to contribute to the developments in this relatively new field using a data base from the original source (the Witwatersrand Basin) and working with the pioneers.

Some of this work has already been submitted to SAIMM for publication; i.e.

> *Krige D. G., and Assibey-Bonsu W. (1991). 'New developments in borehole valuations of new gold mines and undeveloped sections of existing mines.'*

> *Sichel H. S. , Kleingeld W. J. and Assibey-Bonsu W. (1991). 'Comparative Study of three distributional models: 3-parameter, Compound Lognormal and Log-generalised Inverse Gaussian distributions.'*

But this thesis presents a logical sequence as well as a more detailed development of the research than in the relevant papers.

CHAPTER 1

1 INTRODUCTION

1.1 Statement of the Problem and the Objective of the Study

Depletion of mineral resources is accompanied by loss of readily accessible reserves resulting in increases in extraction costs over time. Though technological innovations have opposite compensating effects, escalating mining costs, low prices of most minerals and the increasing trend towards the exploitation of lower grade orebodies demand accurate ore reserve estimation. Accurate ore reserve estimates will help to reduce the very high risk involved in most mining operations and also improve overall profitability.

When estimating recoverable ore reserves, two different cases must be distinguished. These are global estimates (e. g., for initial feasibility studies) and local block estimates. Global estimation refers to the whole deposit or usually to the domain covered by the available data. Local estimation however, refers to part of the deposit e. g., face (panel) or block. This thesis addresses these two distinctive features of ore evaluation by proposing some new valuation models and procedures.

In order to provide grade and tonnage estimates for borehole valuation of new mining operations, the efficient estimates of the following elements are essential.

(i) the mean grade
(ii) confidence limits on the mean grade and
(iii) grade tonnage curves

Appropriate spatial and distributional models for the mineralisation are needed for the optimal estimates of these important elements. The 3-parameter lognormal model as suggested by Krige[1] and its t-estimation technique developed by Sichel[2] has been used extensively for modelling the underlying ore value distributions especially in the Witwatersrand basin. Though this model has been practically useful, the application of the model presents some difficulties. First, the estimate of the threshold parameter (the additive constant) is very difficult when small set of borehole values are available. Secondly, the need for more applicable distributional models has been felt due to the geological complexity of the type of reefs being mined at present in the Witwatersrand basin.

The first objective of this study is therefore to research into the possibility of using more appropriate distributional models. The study considers the Compound Lognormal (CLD)[3] and the Log-Generalised Inverse Gaussian (LN-GIG) Distributions proposed by Sichel (personal communication). Compared to the 3-parameter model the new distributions have the advantage of possessing different shapes depending on the Pearson's skewness and kurtosis coefficients ($\sqrt{\beta_1}$ and β_2). If $\sqrt{\beta_1} = 0$ and $\beta_2 = 3$, the new distributions converge to the 2- or 3-parameter model. The 3-parameter model is therefore a limiting case of these more general models. Consequently, the CLD and the LN-GIG models have a greater range of applicability than the 3-parameter model.

Furthermore, due to the high cost and time delays in drilling deep boreholes, most mines are opened on estimates based on small numbers of boreholes. Though deflection of individual boreholes have been very useful, one cannot count these deflections as independent observations due to the high correlation between original and deflected borehole values. In fact, having developed an appropriate distributional model for large sampling data sets, when faced with small number of boreholes from a new property, it is impossible to define precisely the type of distribution the borehole values originate from. We cannot construct any reliable histograms and cumulative frequency plots. It is within this serious information constraint that the second objective of this thesis is defined.

The second objective is to investigate the possibility of deriving information from neighbouring mines based on geological similarities to improve estimates for obvious economic reasons. Such statistical information from any adjacent existing mines, will help to select appropriate distributional models. It will also help to obtain better estimates for statistical parameters (e. g., the threshold parameter for the 3-parameter model). Geostatistical methods cannot be applied when only small sets of borehole values are under consideration since it is impossible to construct variograms. However, where the parameters to be estimated display spatial structures on the relevant scale of support, such spatial information can also be derived from adjacent mines.

Geostatistical techniques have obviously brought significant improvements in ore valuation. However, ore valuation procedures so far derive additional information mainly via spatial structures on grade levels. As an introduction of expanding this important geostatistical technique, the thesis proposes the concept of spatial structures for variances as a means of deriving additional useful information. This concept has been proposed for various statistical and geological reasons. It has been observed practically that variances for global and local valuations (especially for skew orebodies) are badly estimated in most cases. For instance, it is not uncommon that estimates may provide minimum estimation errors on the basis of limited available information. But these estimates, however good, will still be poor estimates of the variability of the 'true' values unless appropriate outside information is brought in to eliminate conditional biasses. In addition, plants designed on feasibility estimates of variability to receive mill feed (even with large superimposed confidence limits of the actual feed), end up in most cases undergoing costly unforseen alterations (See also Dowd[4]).

Furthermore, the geology of the Witwatersrand basin as presented by Minter[5,6], Antrobus[7,8] and Hallbauer[9], and detailed in this thesis (See Chapter 2) gives rise to the following important geological expectations (See also Krige et al[10]):

(i) firstly, that inspite of the differences within and between the reefs, similarities should also be expected.

(ii) secondly, that because of these expected similarities, one will further expect that, within a sedimentary fan system, across adjacent fans or even from one such system to another, similar levels of gold concentrations and variations within these concentrations could arise.

(iii) Thirdly, that conditions within such systems probably changed gradually in space giving rise to spatial structures for grade levels and also for the grade variabilities.

Due to the above geological expectations, considerable information can be derived from variances (of the grades) from macro-mine to micro-panel level if 'variance' can be introduced as a spatial variable. As a result of these geological and statistical reasons, spatial structures not only for grade levels but also for grade variabilities have been introduced in this thesis as a means of deriving more information.

Routine valuation of large undeveloped sections of existing mines for longterm mine planning is usually based on a few borehole values and the known grade pattern in adjacent developed and/or mined out sections. This routine valuation presents a common problem which is in principle the same as that of the valuation of a new mine. Valuation procedures as discussed above will therefore be applicable. Face (panel) valuations can similarly be improved.

At all scales of valuation therefore, from the macro-mine to the micro-panel level, proper valuation procedures and the use of appropriate models will ensure better estimates. At the global scale e.g., better confidence limits can be applied to improved estimates of the mean and the effect of selectivity can be estimated for the relevant scales of operations. These will ensure better economic valuations with significant reduction of the risk which accompanies most mining investments. These better valuation techniques will also lead to proper production planning and help to avoid improper mill feed. They will also assist - some extent in reducing the incidence of unnecessary opening and closing of stope faces.

1.2 Organization of the Thesis

Chapter one defines the problem and the objectives of the study. It also includes a historical review of geostatistics. Chapter two discusses the 3-parameter as well as the new distributional models. This chapter places the geology of the Witwatersrand basin in perspective and draws some important conclusions from the genesis of the deposition which makes the new models more applicable. Mathematical aspects of the distributions including parameter estimates are also given. A comparative study of these distributions is also included. Chapter three presents the study on the new developments in borehole valuations of new gold mines and undeveloped sections of existing mines. Chapter four applies the valuation procedures and models used for the borehole analyses at a local scale for the valuation of faces (panels). The last chapter summarises the important findings and conclusions of the study.

1.3 Methodology

1.3.1 Distributional models

A detailed comparative study of the models was done in chapter 2. Chip sample data from large mined out sections of different reefs within the Witwatersrand basin were used. Computer programs were developed for the various models (for parameter estimates and modelling). Other relevant computer programs for test of goodness of fit for the distributions fitting were also developed. Details of the program listings is found in Appendix C.

1.3.2 Borehole analyses

Chip sample values (total of 72767) from large mined out sections of the Hartebeestfontein gold mine (Vaal Reef) provided a database for the borehole analyses in chapter 3. The chip samples were accepted as equivalent to individual borehole core values. The area was divided into 1 Km x 1 Km blocks to simulate 20 'mines'. From each of these 'mines' 9 values were drawn on a stratified random basis. These were accepted in each case as the equivalent of 9 single boreholes, i.e., without deflections. The process was repeated to yield a total of 60 borehole sets. The sets of simulated borehole values were subjected to a series of estimation procedures outlined in chapter three.

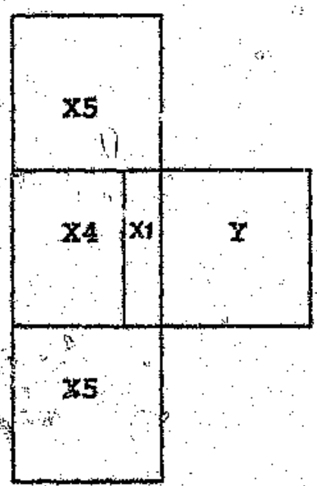
1.3.3 Face (panel) valuations

The same database as used for the borehole analyses in chapter 3 was also used for these analyses in chapter 4. In this case the area was divided into various block sizes and patterns as shown in Figure 1.1. The aim was to estimate 50m x 50m blocks. The estimation procedures adopted are also outlined in chapter four.

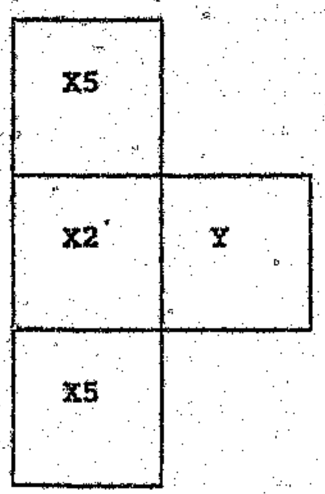
FIGURE 1.1 DATA CONFIGURATIONS FOR FACE (PANEL) VALUATIONS

Y = BLOCK TO BE ESTIMATED, 50 x 50 M
X1,X2,X3,X4,X5 = DATA BLOCKS
X2,X5 = 50 x 50 M
X1,X3 = 10 x 50 M
X4 = 40 x 50 M

DATA SET A



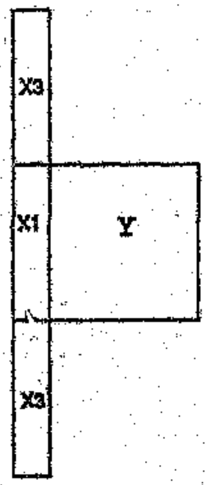
DATA SET B



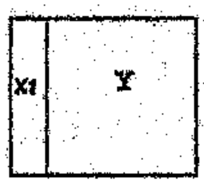
(CONTINUED NEXT PAGE)

FIGURE 1.1 (CONTINUATION)

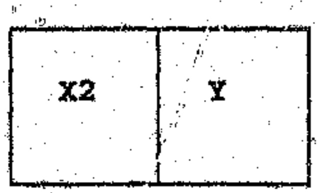
DATA SET C



DATA SET D



DATA SET E



1.4 Historical Background

The first attempt to model the underlying gold distribution in the Witwatersrand basin was made by Hooper and Watermeyer[11] in 1919. Watermeyer proposed a dual modelling of the observed gold values with two normal distributions joined at the mode. He further proposed that, in order to obtain a reliable mean estimate for a limited number of gold values, the observed values should be weighted by the square of the frequency i.e. $(\text{frequency})^2$. Truscott[12] favoured Watermeyer's proposal and made a further suggestion in 1929 that gold values should be weighted not only by $(\text{frequency})^2$, but also by the values themselves. Truscott also observed that when gold values were plotted on a logarithmic scale, the values displayed symmetry about the geometric mean. This observation brought him very close to Sichel's proposed lognormal distribution which emerged two decades later.

Credit for the first significant introduction of mathematical statistics in South African gold ore valuation must however go to Sichel[13] who proposed the lognormal frequency distribution model. Sichel's work in 1947 was followed shortly by Ross[14] and Krige[15]. This led directly to the use of this model for improved estimates via Sichel's 't' estimator (Sichel[16]; Krige[17]). The frequency distribution model was later improved to the 3-parameter lognormal model by Krige[1] which made it more flexible.

Systematic over- and under-valuation of ore block grades was a well known practical problem in early South African gold mines. Ross[14] in 1950, for instance, stressed the importance of compensating for these conditional biasses. It was Krige's[15,18] work in 1951 that first provided an explanation and solution to the problem of conditional biasses through regression analyses. These regression procedures provided the first use of an elementary spatial structure via the correlation of the peripheral chip sample values with those subsequently available from inside the ore blocks. Regression can thus properly be accepted as the first and elementary simple kriging application. These procedures also demonstrated that an ore body could be seen as a series of populations each with its own type and size of support for its members, e.g., chip samples, large ore reserve blocks etc., so that an individual ore block was effectively seen as a member of a population of such blocks in an ore body.

Krige's[17] 1952 borehole analysis of the relationship between logarithmic 'variances' of gold values and the 'size of area' led to the variance additivity relationship known today as 'krige's relationship'. De Wijs[19] and Matheron[20] later provided the theoretical background to Krige's variance size of area relationship.

Krige's work in 1951 and 1952 together with other early work of Sichel triggered the interest of overseas researchers particularly Matheron who developed the general underlying theory of geostatistics applicable to regionalised variables. In his publication in 1960 for instance, Matheron[20] presented outstanding theoretical and practical developments such as the now generally used variogram, the de Wijsian model, the theory of linear equivalents, kriging and lognormal geostatistics.

During this time (i.e., in the 1960s), techniques which were particulary in use in the United States by Whitten, Link, Koch and others were mainly the theory and applications of polynomial trend surface analyses. Though the development of computers at this time was an advantage, the intractable computations involved in these techniques made them practically unattractive; they were also found to be unsuitable for ore block valuations.

Due to the role South Africa had then played in the birth of the subject, Johannesburg was chosen as the venue in 1966 for the 'Symposium on Mathematical and Computer applications in Ore Valuation'. Numerous papers on the subject were presented during the conference. During this conference, Krige's 'moving average technique' and Whitten's polynomial trend surface approach emerged as two severely opposing techniques. Matheron's subsequent investigations however favoured Krige's approach and Matheron's contribution[21] appears to have resolved the controversial argument which emerged during the abovementioned conference. Matheron replaced what Krige originally called the 'weighted moving average' by the terminology kriging, thus acknowledging Krige's intimate association with the birth and development of geostatistics. Generally, geostatistics gained acceptance more readily in French-speaking countries than in the others, especially, the English-speaking countries where it initially encountered some stormy receptions.

Lognormal geostatistics has played an important rôle amongst the various techniques which have so far been applied in the South African gold mines. In his work in 1974 Marechal[22] made a significant contribution in the development of theory of lognormal kriging. Rendu[23] expanded Marachel's results and Dowd[24] in 1982 presented a generalised form of lognormal kriging. Journel and Huijbregts[25] have also given an adequate treatment of the subject.

During the Geostatistics Conference in Italy in 1975 more advances in the theory of regionalised variables was presented by Matheron[26] particulary on non-linear geostatistics. Techniques such as disjunctive kriging, universal kriging, conditional simulation, non-parametric geostatistics etc. have since then appeared in journals and textbooks as a reflection of theoretical developments in the relatively new field of geostatistics. Some of these new techniques have been applied in South Africa such as universal kriging at Prieska Copper Mine (Krige and Rendu[27]) and on the Carbon Leader Reef (Braun[28]). Disjunctive kriging technique has also been applied at Loriane (Krige et al[29]). Kleingeld[30] also applied disjunctive kriging technique to deposits with discrete regionalised variables and this technique is currently being applied for diamond deposits. However, very little of this new work has so far found routine applications in the Witwatersrand deposits.

In the meantime, the problem of improving grade estimates for new South African gold mines based on small number of borehole results did not advance beyond the Sichel's 't' estimator until Krige[31] suggested that a new mining property should, like an ore block, be seen as a member of a population of such units. Where this is feasible and suitable information is available on the average variance within such mines, this can be introduced as *a priori* information for the t" estimator. Where additional information is available on the spatial characteristics of the population of mines, a 'macro' kriging estimate can also be made for the mine (Krige[32a,32b]), similar in principle to a Bayesian approach. It was, however, not until recently that the role of the variance in borehole valuations has been analysed (Krige et al[10,30]).

CHAPTER 2

2 GEOLOGICAL AND DISTRIBUTIONAL MODELS ANALYSES

2.1 Outline

A frequency distribution model plays a fundamental role in ore evaluation especially for the feasibility study of new deep gold mines.

The geological complexity of the type of reefs being mined at present in the Witwatersrand basin poses some difficulties regarding the modelling of the underlying ore value distribution. This geological complexity therefore, demands the need for more applicable distributional models. This chapter places the geology of the Witwatersrand basin in perspective, and draws some important conclusions from the genesis of the deposition. Based on these geological models, it is shown that the enrichment or loss of gold values at the tail and upper end of the distributions are not artifacts but are geologically related. Two more general distributional models -Compound Lognormal (CLD) and Log-Generalised Inverse Gaussian (LN-GIG) distributions are therefore proposed. A detailed, though preliminary comparative study of these two more general models with the 3-parameter Lognormal model is done and efficiencies of estimates and modelling analysed. Mathematical aspects of the new distributions are also presented.

2.2 Introduction

The previous extensive use of the 3-parameter model as suggested by Krige[1] and its associated t-estimation technique developed by Sichel[2] was an indication of its practical usefulness and efficiency for the reefs in which it was being used. Though Sichel[34,35] and Link and Koch[36], commendably accept the practical efficiency of Krige's technique[1], yet they indicate that generally severe biases could result by the application of lognormal theory if in fact the underlying distribution of the variable of interest is not lognormal; and even where it is in fact lognormal a bad estimate of the third parameter can also introd[illegible]sses. They caution that there are a number of theoretical [illegible]piri[illegible]tributions which can mimic the lognormal model in as much as they exhibit unimodality, positive skewness and a long tail to the right. Sichel[illegible] has given detailed theoretical explanations regarding these pseudolognormal models.

It should also be noted that the minimum gold sample value z (cm-g/t) is zero. This implies that, for the 3-parameter lognormal model, the transformed gold value x ($x = \ln(z)$) is bounded by a finite lower size of $\ln(\beta)$, where β is the additive constant. Any value under the mathematical model which is below this finite lower size will result in a negative area and will give negative gold value after back-transformation. Such negative gold values, in reality, do not exist. Sichel[34] has shown that the influence of such a negative area can be significant if it exceeds 2 per cent.

Two more general models - Compound Lognormal (CLD)[3] and Log-Generalised Inverse Gaussian distributions proposed by Sichel (personal communication) have therefore been considered as alternatives. As will be shown later, these new models (CLD and LN-GIG) have a greater range of applicability compared to the lognormal model. This is mainly because the lognormal model is a limiting case of these more general models.

Departures of gold grade distributions from 2- or 3-parameter lognormality can well be observed on the cumulative frequency distribution plot on log-probability paper. The cumulative frequency distribution curve will display distinct curvatures both at the low and high grade categories (See Figure 2.1). In such situations the addition of a constant assists to eliminate the curvature in the lower grade category but the upper grade curvature will persist.

It will be shown that these curvatures are not artifacts but are related to the geological complexity of the type of reefs being mined at present in the Witwatersrand basin. This geological complexity is a motivation for proposing more appropriate distributional models. The geology of the basin is as set out below.

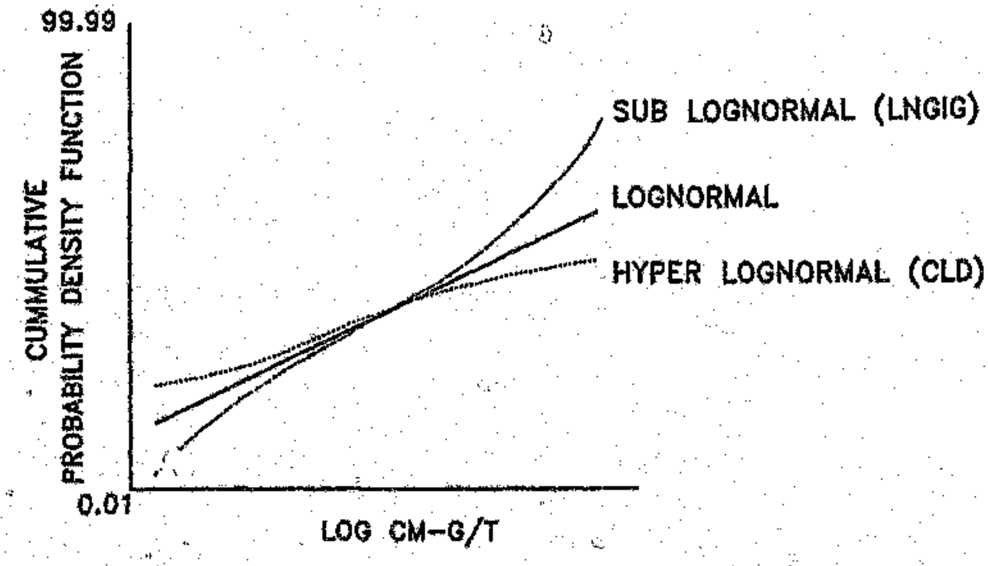


FIGURE 2.1 PROBABILITY DENSITY DISTRIBUTION FUNCTIONS

2.3 Geological Models

Detailed discussion of the geology of the Witwatersrand basin is well presented in Minter[5,6], Antrobus[7,8] and Hallbauer[9]. However for the purpose of this thesis reference should be made also to Krige et al[10].

The gold bearing sedimentary rocks of the Witwatersrand basin were discovered in 1886 and the amount of gold that has so far been produced from the basin makes it unique compared to similar gold formations. Figure 2.2 shows the basin and the major gold producing areas.

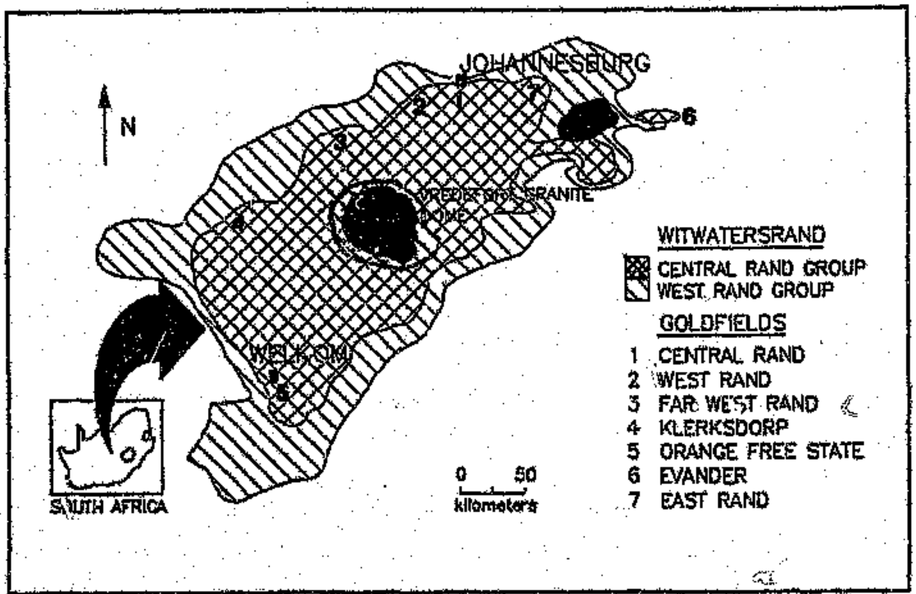


FIGURE 2.2 WITWATERSRAND BASIN AND GOLD FIELDS

In summary from the above mentioned papers it can be concluded that, the gold and uranium bearing sedimentary rocks of the Witwatersrand basin are found in the Dominion Reef Group and the Witwatersrand and Ventersdorp Supergroups. The Witwatersrand Supergroup consists of the (lower) West Rand and the (Upper) Central Rand Groups. The sedimentary rocks which are ancient placers have been deposited by braided rivers on alluvial fans located at the margins of a yoked basin. The sediments in the basin can be subdivided into a number of geological units. These geological units are bounded by unconformities which are a reflection of intermittent sedimentation in the basin. This historical intermittent sedimentation has been attributed to local and regional geological as well as geomorphological factors. In addition, regressive sedimentation and sediment reworking led to varying gold concentration in the different geological units resulting in the development of gravel horizons (reefs) containing relatively high concentrations of gold.

Most of the gold in the Witwatersrand basin is of a detrital nature and the overall gold content in a reef is related to the amount of gold released in the provenance area and to the degree of reworking which has taken place. The distribution of gold within and between reefs shows varying degrees of variability. On a micro scale (as observed in closely spaced borehole deflections) the level of variance is high and this variability increases on macro and mega scales as a result of the inclusion of more diversified features. Furthermore, within a reef the gold content can vary on a large scale due to facies differences related to the positions on the alluvial fans, or to overlapping fans which have different entry points and provenance areas.

The geological setting discussed above gives rise to the following most salient features:

First, the hydro-dynamic circumstances in alluvial environment dictate that the river systems that distributed the placers could transport only certain sizes of sediments at different stages as the river systems gradually reduced their velocities downstream. Hence the heavier mineral particles would be deposited much earlier closer to the source. For instance, Minter[37] demonstrated on the Vaal Reef that wherever an optimum hydraulic energy existed for deposition of particular-sized gold particles available, an anomalously high gold deposition occurred. He observed that these trends could be on ridges, channels sides, or within channels. He concluded that gold content was largely hydraulically controlled (depending on paleosurface conditions) as detrital particles. This is analogous with Sichel's[3] original observation of hydraulic sorting and trapping of diamonds, and is a further confirmation that hydro-dynamic circumstances played an important role in concentrating gold in the placers. One would therefore, expect a relationship (probably linear) to be set between the log-means and log-variances of the gold variable as a function of distance from the stream source. One would further expect that if gold samples are taken from different localities within the same horizon or even from different overlapping reefs within the same horizon, the log-mean and log-variance parameters would be different, resulting in a mixture of lognormal distributions which will no longer follow the lognormal law. The Compound lognormal model(CLD) has been developed to model these hyper-lognormal types of distributions.

Secondly, due to the predominant erosional features of certain types of reefs, the high grade values appear to be partly missing (sub-lognormality). This gives rise to the gold distribution being associated with pockets or trap-sites and as such the probability of finding gold is dependent on the probability of finding gold given a pocket or trap-site as well as the way in which the trap-sites are distributed. In statistical terms we then have:

$$P(r) = P(r \mid \lambda)P(\lambda)$$

where:

$P(r)$ = Probability of finding gold

$P(r \mid \lambda)$ = Probability of finding gold given a trap-site and

$P(\lambda)$ = Probability of finding a trap-site

Such type of genetic models have been previously explored in diamonds[3] and gave rise to the Compound Poisson model, which is a subset of a more general model called the Log-Generalised Inverse Gaussian distribution (LN-GIG) discussed in this thesis. From these two salient geological features it can be contended that the curvatures observed on the cumulative frequency plots are therefore not artifacts, but are depositional features relating to enrichment or leaching.

Unlike the more general models, the lognormal models presume the absence of any linear relationship between Log-mean and Log-variance parameters. As a result of the above discussions, it seems advisable that whenever there are departures from lognormality, one of the more general models should be applied to avoid severe biasses for obvious economic reasons.

This chapter therefore, presents among other aspects, a comparative study of the new and the old gold grade distribution models in the South African context. Data from some of the important reefs of the Witwatersrand basin have been used. The reefs which were considered are: Basal Reef, VCR, Vaal Reef and Kimberley Reef.

When dealing with large sets of samples (as used for this study), the arithmetic mean for the samples is a good estimator of the population mean. For such large samples, the sample arithmetic mean is a good reference point for all practical comparative purposes. The mathematical mean estimates were therefore compared in each case with the respective sample arithmetic mean. Bias error defined as:

$$\left(\frac{\hat{z}}{\bar{z}}-1\right)100\%$$

was used as the relative efficiency measurement of the various mathematical mean estimates as obtained in the different distributional models. Where $\hat{z}$ is the mathematical mean estimate and $\bar{z}$ is the corresponding arithmetic mean of the samples. [illegible] the first time these more general models have been applied to represent gold grade distributions and the preliminary results are very successful.

2.4 The Models in Perspective

2.4.1 The 3-parameter Lognormal Distribution

The 3-parameter density function is defined as follows:

$$f(z) = \frac{1}{\sqrt{2\Pi}\sigma} \frac{1}{(z+\beta)} \exp\left(-\frac{1}{2} \frac{[LN(z+\beta) - \xi]^2}{\sigma^2}\right)$$

where z = observed ore value in cm. g/t $(0 < z \leq \infty)$

ξ , σ and β are parameters to be estimated, β is the additive constant, ξ is the average of the $\ln(z+\beta)$ values and σ^2 is the variance of the corresponding log values.

2.4.2 The Compound Lognormal Distribution (CLD)

This distribution has the following mathematical representation[3]:

$$\Omega(x) = \frac{s(1-\rho)^{\nu+½}}{2^\nu \sqrt{\pi}\,\Gamma(\nu+½)} \exp[\pm\sqrt{\rho}\, s(x-a)]\,(s|x-a|)^\nu K_\nu(s|x-a|)$$

where x = ln(z), $(-\infty < x < +\infty)$, z = observed ore value in cm. g/t and $K_\nu(.)$ is the modified Bessel function of the second kind of order ν.

The four parameters of the distribution are:

a = location parameter, $-\infty < a < +\infty$

s = spread parameter, $s > 0$

c = skewness parameter, $0 \leq c < 1$

v = kurtosis parameter, $v > -1/2$

2.4.3 Estimation of Parameters for the Compound Lognormal Distribution

The distribution of gold in the Witwatersrand basin is very skew. By taking a logarithmic transformation the new distributions are only moderately skew. As a result, the moment estimates of the parameters of ln(cm-g/t) distribution should be fairly efficient. Moment estimates of the parameters have therefore been used for this study. In exceptional cases however, where the kurtosis is very peaked, the fourth moment of the sample will be an unreliable estimate for the population. This can arise because of undue influence of outliers. Such cases require that chi-square minimisation technique should be used together with the moment estimation procedure to ensure more efficient parameter estimates. This will be shown later.

2.4.4 Central Moments of Compound Lognormal Distribution

A detailed procedure for calculating the moments is presented in Appendix B. Summary of the first four Central moments are as follows:

$$\mu_1'(x) = a + \frac{(2v+1)(\pm\sqrt{c})}{s(1-c)}$$

$$\mu_2(x) = \frac{(2\nu + 1)(1 + c)}{s^2(1 - c)^2}$$

$$\mu_3(x) = \frac{2(2\nu + 1)(3 + c)(\pm\sqrt{c})}{s^3(1 - c)^3}$$

$$\mu_4(x) = 3(2\nu + 1)\frac{(2\nu + 3)(1 + c^2) + 2(2\nu + 7)c}{s^4(1 - c)^4}$$

The Pearson's shape coefficients are:

$$\beta_1(x) = \frac{\mu_3^2(x)}{\mu_2^3(x)} = \frac{4c(3 + c)^2}{(2\nu + 1)(1 + c)^3}$$

$$\beta_2(x) = \frac{\mu_4(x)}{\mu_2^2(x)} = 3 + \frac{6(c^2 + 6c + 1)}{(2\nu + 1)(1 + c)^2}$$

Where $\beta_1(x)$ and $\beta_2(x)$ are the skewness and kurtosis coefficients respectively. The CLD is applicable only if the following condition is satisfied (as shown in Appendix B)

$$\beta_2(x) > \frac{1}{2}[3\beta_1(x) + 6]$$

2.4.5 Procedure for estimation of Parameters of the Compound Lognormal Distribution

From the logarithm of the observed gold values (cm-g/t), estimates of the skewness $\beta_1(x)$ and kurtosis $\beta_2(x)$ coefficients are calculated, from which

$$\hat{q} = \frac{3\hat{\beta}_1(x)}{2\hat{\beta}_2(x) - 3\hat{\beta}_1(x) - 6}$$

Parameter c is estimated by iteration from

$$\hat{q} = \hat{c}\left(\frac{3+\hat{c}}{1-\hat{c}}\right)^2$$

Next, we calculate

$$\hat{\nu} = \frac{2\hat{c}(3+\hat{c})^2}{\hat{\beta}_1(1+\hat{c})^3} - \frac{1}{2}$$

and

$$\hat{\delta} = \sqrt{\frac{(2\hat{\nu}+1)(1+\hat{c})}{\hat{\mu}_2(1-\hat{c})^2}}$$

where $\hat{\mu}_2$ is the sample variance of the natural logarithms of the observed gold values (cm-g/t) and $\beta_1 = \beta_1(x)$. The fourth parameter is finally calculated from

$$\hat{a} = \hat{\mu}_1' - \frac{(2\hat{\nu}+1)(\pm\sqrt{\hat{c}})}{\hat{\beta}(1-\hat{c})}$$

where $\hat{\mu}_1'$ is the sample mean of the logarithms of the observed gold values (cm-g/t).

The compound lognormal mean estimate can then be calculated using:

$$\hat{\mu}_1'(z) = \left[\frac{1-\hat{c}}{1-(1/\hat{\beta} \pm \sqrt{\hat{c}})^2}\right]^{(\hat{\nu}+1/2)} \exp(\hat{a})$$

2.4.6 The Log-Generalised Inverse Gaussian Distribution (LN-GIG)

The four parameter Log-generalised Inverse Gaussian Distribution is defined as:

$$\Lambda(x) = \frac{1}{2sK_\gamma(b)} \exp\left[\gamma\left(\frac{x-\xi}{s}\right) - b\cosh\left(\frac{x-\xi}{s}\right)\right]$$

where $x = \ln(z)$, ($-\infty < x < +\infty$) and z is the observed ore value in cm. g/t., $K_\gamma(b)$ is the modified Bessel function of the second kind of order γ and argument b. The four parameters are:

ξ = location parameter ($-\infty < \xi < +\infty$)

s = scale parameter ($s > 0$)

b = kurtosis parameter ($b > 0$)

γ = skewness parameter ($-\infty < \gamma < +\infty$)

2.4.7 Estimation of Parameters of LN-GIG Distribution

The moment generating function for the x-variable ($x = \ln z$) is

$$M_x(t) = e^{\xi t}\frac{K_{\gamma+ts}(b)}{K_\gamma(b)}$$

from which the moments for the x-variable can be generated (See Appendix B). The standardised form is found very convenient in computing the parameters. Make a linear transformation

$$u = \frac{x-\xi}{s} \tag{1}$$

where $-\infty < u < \infty$. It can be shown that the standard form of the LN-GIG is

$$\varphi(u) = \frac{1}{2K_\gamma(b)}e^{\gamma u - b\cosh u}$$

The rth moment about the origin for the standard form is then

$$\mu_r'(u) = \frac{1}{2K_\gamma(b)}\int_{-\infty}^{+\infty} u^r e^{\gamma u - b\cosh u}du \tag{2}$$

The first four moments about the origin of the standard form from (2) are

$$\mu_1'(u) = \frac{1}{K_\gamma(b)}\int_0^\infty u\sinh(\gamma u)\, e^{-b\cosh u}du \tag{3}$$

$$\mu_2'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^2 \cosh(\gamma u)\, e^{-b\cosh u}\, du \tag{4}$$

$$\mu_3'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^3 \sinh(\gamma u)\, e^{-b\cosh u}\, du \tag{5}$$

$$\mu_4'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^4 \cosh(\gamma u)\, e^{-b\cosh u}\, du \tag{6}$$

From which the central moments can be computed. As 'u' is a linear transformation of 'x', we have

$$\sqrt{\beta_1(u)} = \sqrt{\beta_1(x)} = \frac{\mu_3(u)}{[\mu_2(u)]^{3/2}} \tag{7}$$

$$\beta_2(u) = \beta_2(x) = \frac{\mu_4(u)}{[\mu_2(u)]^2} \tag{8}$$

Where $\mu_r(u)$ is the rth central moments of u.

2.4.8 Procedure for estimation of the Parameters of the LN-GIG Distribution

1. From the natural logarithms of the observed gold values (cm-g/t) $\sqrt{\beta_1}$ and β_2 are calculated.
2. Using equations (3) to (8) parameters γ and b are estimated by iteration. The values of γ and b which will give similar values as calculated under 1 above are the required solution.
3. Next, one calculates the other two parameter estimates from

$$\hat{s} = \sqrt{\frac{\hat{\mu}_2(x)}{\hat{\mu}_2(u)}} \tag{9}$$

and

$$\hat{\xi} = \hat{\mu}_1'(x) - \hat{s}\hat{\mu}_1'(u)$$

where $\hat{\mu}_1'(x)$ and $\hat{\mu}_2(x)$ are the mean and variance estimates of the natural logarithms of the observed gold values (cm-g/t); $\hat{\mu}_1'(u)$ is defined in (3) and $\hat{\mu}_2(u)$ is the second central moments of the standard form. To calculate the mean estimate of the untransformed distribution, we use

$$\hat{E}(z) = \frac{e^{\xi}\, K_{\gamma+s}(\hat{b})}{K_{\gamma}(\hat{b})} \tag{10}$$

2.5 Regions of Applicability of the New Models

Pearson's shape Coefficients β_1 and β_2 are used to determine the specific model applicable for a given empirical distribution function as shown in Figure 2.3, where:

$$\sqrt{\beta_1} = \mu_3/\sigma^3 \tag{11}$$

$$\beta_2 = \mu_4/\sigma^4 \tag{12}$$

$\sqrt{\beta_1}$ is the skewness coefficient and β_2 is the kurtosis coefficient, μ_3 and μ_4 are the third and fourth central moments of the transformed variable x respectively, σ is the corresponding standard deviation. The new models do overlap in certain regions as indicated in Figure 2.3 with the hatched area.

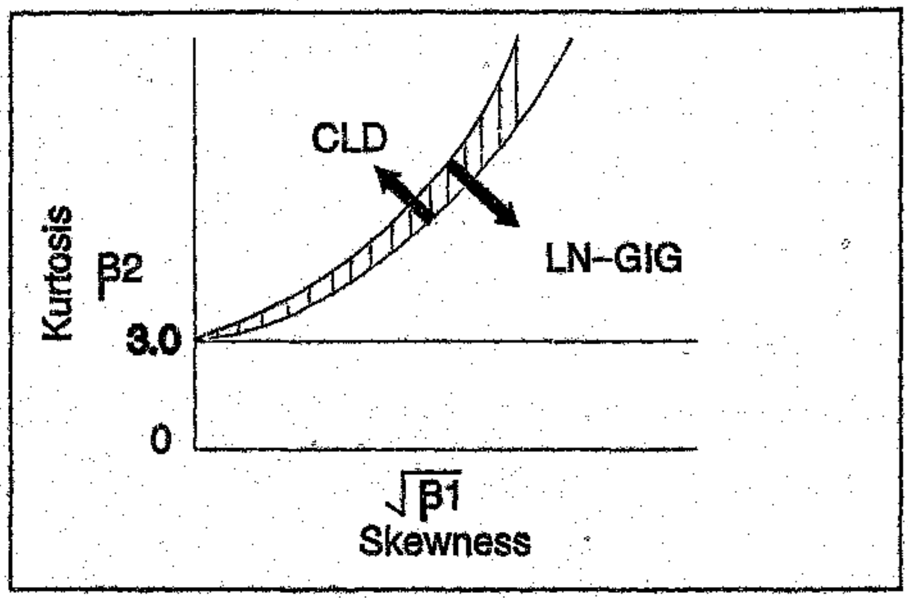


FIGURE 2.3 SHOWING REGIONS OF APPLICABILITY OF THE VARIOUS MODELS

2.6 The Case Study of the Three Distributions

2.6.1 Compound Lognormal and Log-Generalised Inverse Gaussian Distributions

Applying the new distribution density functions (whichever applicable) theoretical models were fitted to the empirical gold data in 21 different cases.

One-Sample Kolmogorov-Smirnov (K-S) test of goodness of fit, based on[38]:

$$n^{\frac{1}{2}} D(n) = n^{\frac{1}{2}} \; Sup \, |F_n(x) - F(x)| \; ,$$

where $F_n(x)$ is the empirical distribution function from a population with continuous distribution function F(x), was used in all the cases. A test of the hypothesis that the sample has arisen from F(x) is provided by rejecting at the P per cent level if:

$$n^{\frac{1}{2}}D(n) \geq d(p)$$

At the 5 per cent level used for the study, d(p) = 1.358 for large n, implying that all calculated d(p) values less than 1.358 pass the K-S test for goodness of fit. Chi-square test of goodness of fit was also used.

2.6.2 Results

Typical results are shown in Figures 2.4 and 2.5 and in Appendix A. In almost all the cases the hypothetical models do describe the empirical data as observed by the 'test of the eye' (cf. Figure 2.4 and 2.5).

Almost all the K-S tests confirmed the above observation 'by the eye test' (cf. Appendix A). However, Chi-square test rejected the test for goodness of fit almost in all the cases. The latter indication is not unexpected due to the rigorous and super-sensitivity of chi-square test especially when dealing with large numbers of observations. Detailed results of the analyses are tabulated in Appendix A.

It was observed in some exceptional cases that the empirical distribution models were more leptokurtic than their corresponding theoretical models (See Figure 2.6). These were the cases where the kurtoses were very peaked. As a result, the respective fourth moments of the samples could not be reliable estimates for the corresponding populations. The study showed that these cases can arise because of the undue influence of outliers.

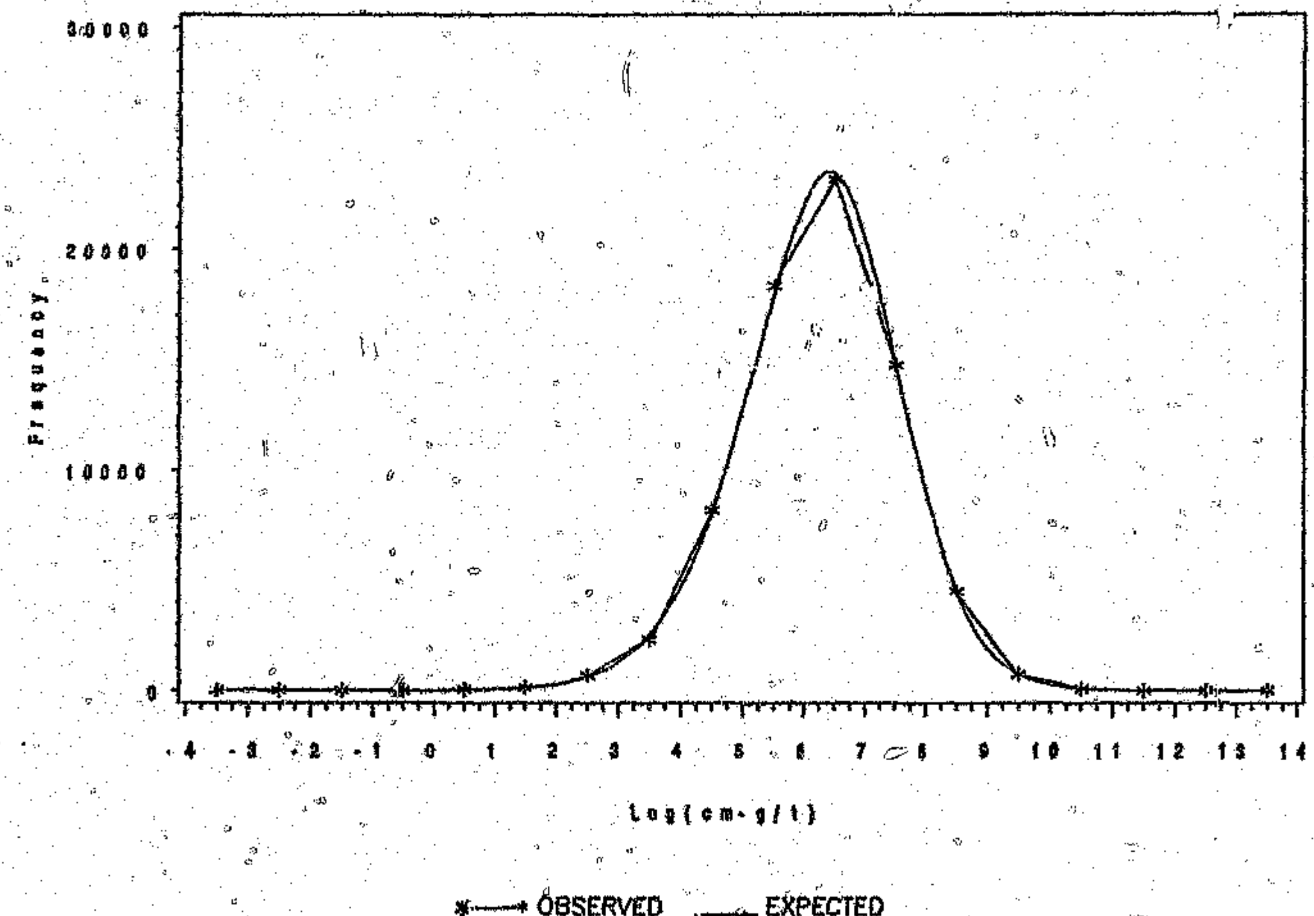


FIGURE 2.4 COMPOUND LOGNORMAL MODEL FITTED TO GOLD DATA
AREA H, N = 72767

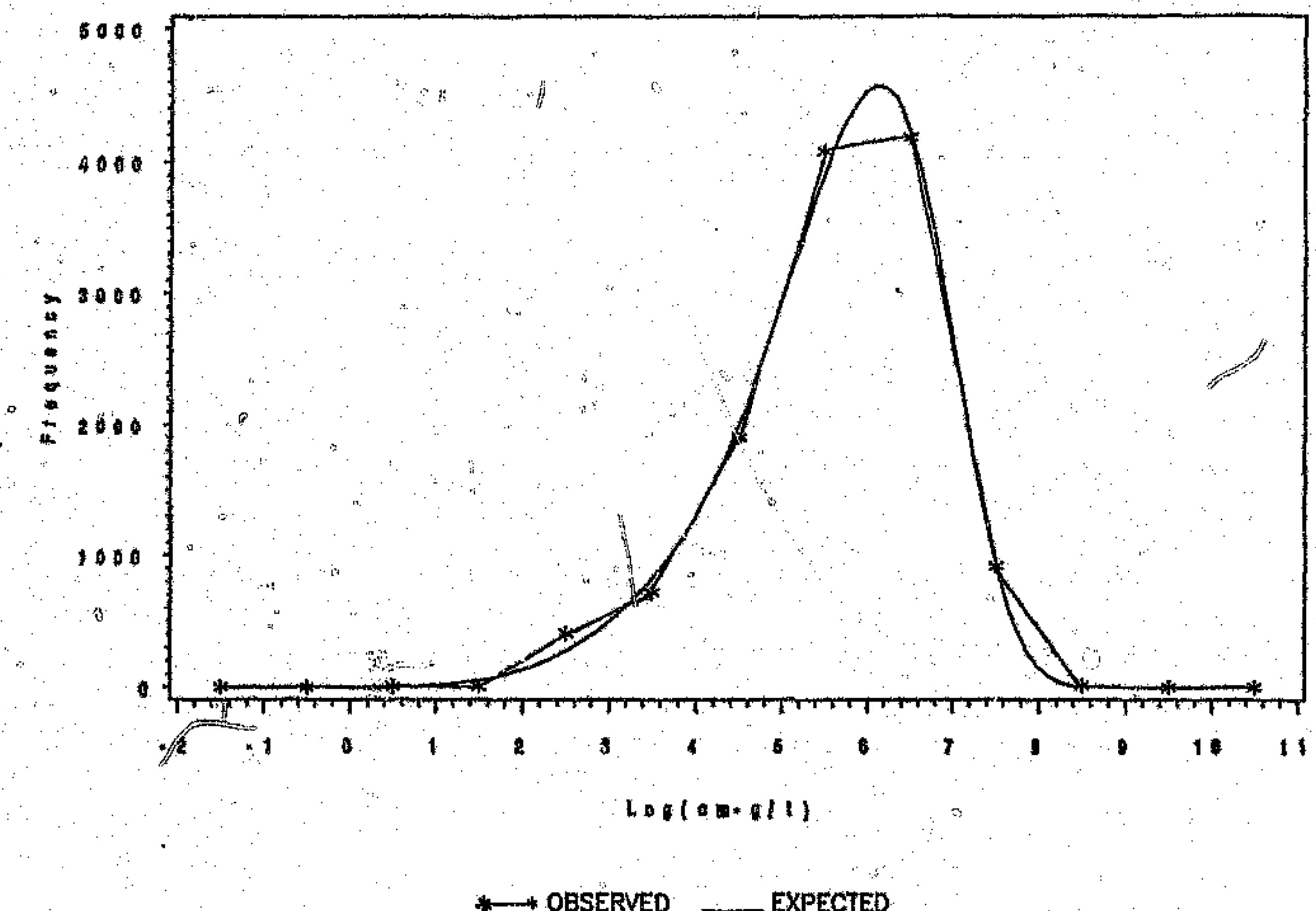
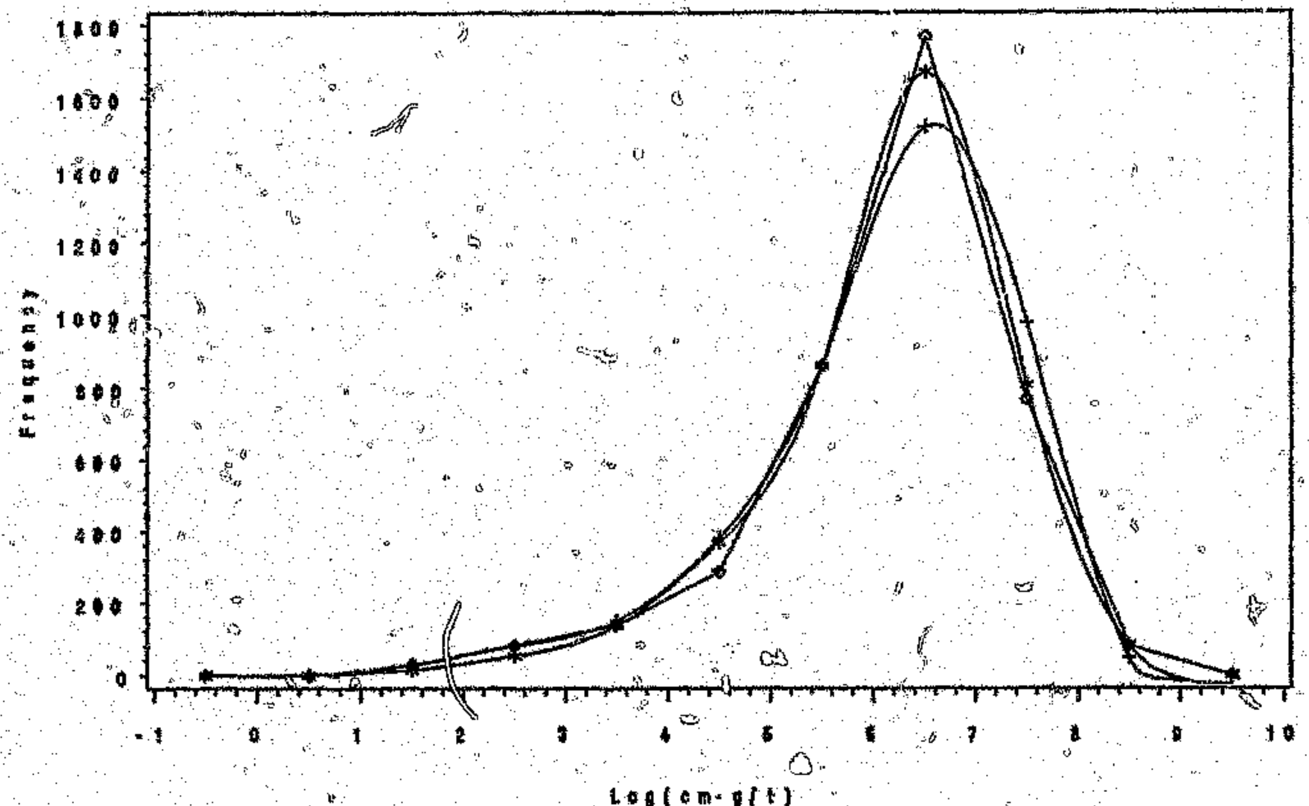


FIGURE 2.5 . LN–GIG MODEL FITTED TO GOLD DATA
AREA J, N = 12241



+ EXPECTED1 * EXPECTED2 o OBSERVED

FIGURE 2.6 COMPOUND LOGNORMAL MODELS FITTED TO GOLD DATA
AREA AA, N = 3962

Such cases required the use of the chi-square minimisation technique as an alternative. It is worth noting that the use of the chi-square minimisation technique for obtaining population parameters is different from the usual chi-square test of goodness of fit. The skewness estimate for the respective populations however, did not suffer from the above limitation. This is due to the 'third power' in the skewness definition (unlike that of the kurtosis) which implies both positive and negative compensating components. Equations 11 and 12 give the definitions of the skewness and kurtosis measures.

The chi-square minimisation technique essentially involves the following. In a particular modelling process, different kurtosis values are used and the distribution is fitted for each of these kurtosis values. In each case the respective chi-square values are also calculated. The resulting optimal kurtosis value corresponding to the minimum chi-square value is finally used to fit the theoretical model (See Figure 2.7). Figure 2.6 shows one of the results and demonstrates a significant improvement in modelling, with a reduction of 70 per cent in chi-square value. A further reduction of bias from almost 5 per cent to 2 per cent was also obtained (See Appendix A).

The analysis of the Kimberley reef data indicated that the distribution falls within the overlap region of CLD and LN-GIG distributions. The two models were therefore used for the analysis independently. The results from the two analyses were very similar. This finding also indicates that in cases where a distribution falls in the overlap region, either of the new distributions can be used and practically similar results will be obtained.

Furthermore, the new distributions have proved to give a more robust estimate of the population mean. Their efficient estimate of the population means are shown in Appendix A.

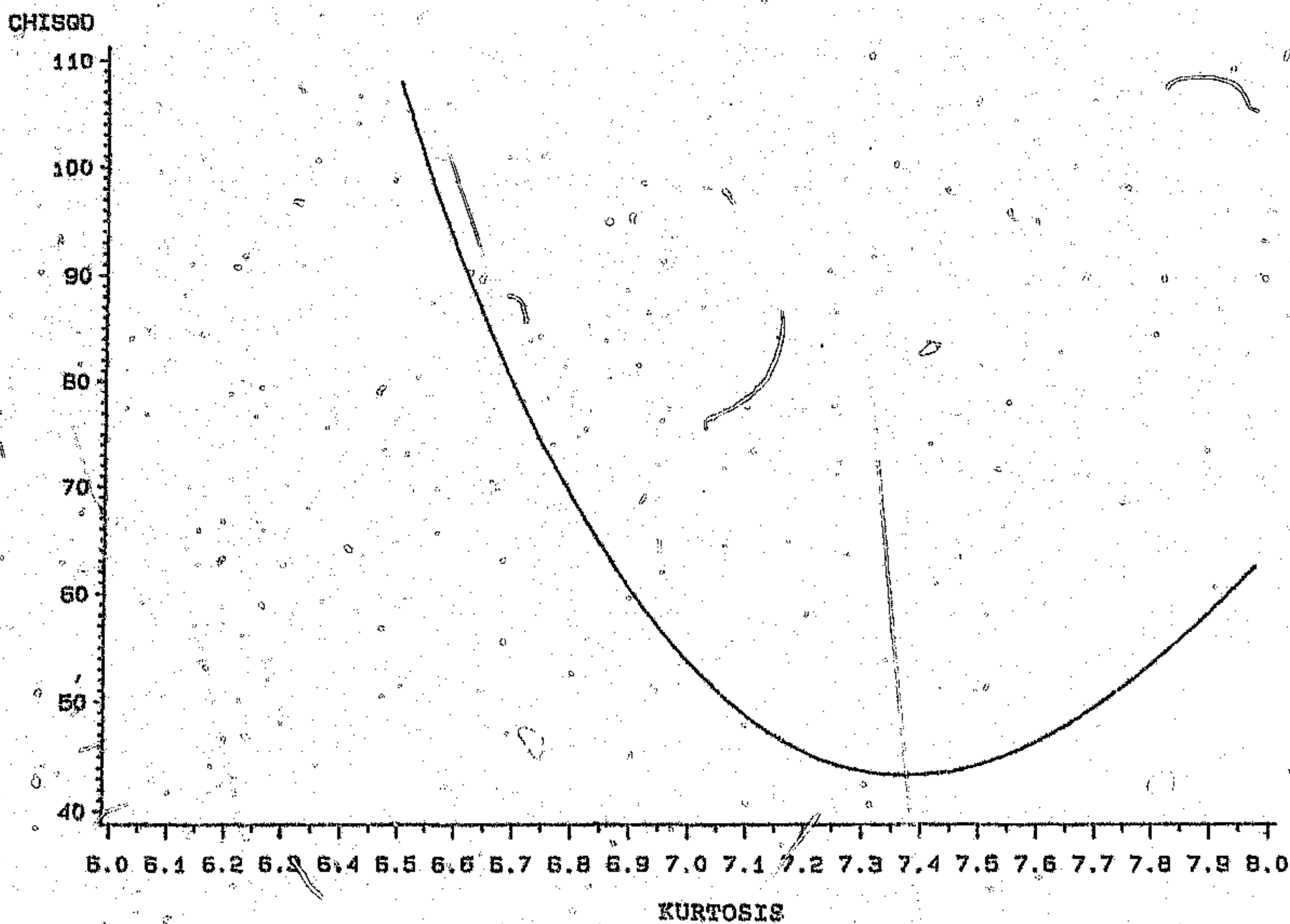


FIGURE 2.7 MINIMISATION OF CHI-SQUARE TO OBTAIN OPTIMUM KURTOSIS VALUE

Combining the chi-square minimisation procedure, not more than 2.5 per cent bias in the population mean estimate was observed. Chi-square minimisation approach was adopted to improve the moment estimates of the parameters in some cases. This approach was not adopted in other cases since it was not going to make much difference. The estimated parameters for the new distributions in the areas concerned are also shown in Appendix A. As indicated in Appendix A, the location parameters of the new models seem to be stable in the respective reefs.

2.6.3 The 3-Parameter Lognormal model Case Study

Applying Krige's technique[1], all the original empirical distributions were also fitted after estimating the respective additive constants.

2.6.4 Results

The analyses indicated a good agreement between the mathematical models with the empirical data only beyond the logarithm of the additive constant value. Plots of observed and expected frequencies are shown in Figures 2.8 and 2.9 and they pass the 'test of goodness of fit by the eye'. K-S test of goodness of fit accepted almost all the goodness of fit test, though chi-square test rejected almost all of them (the reason for this has already been given). In the Klerksdorp area H (Vaal Reef) where 72767 samples were analyzed, the a-priori additive constant of 50 gave a less efficient estimate of the population mean though the bias was practically not significant. The overall analysis using this model showed less than 5 per cent bias in the estimation of the population mean. However, in more than 30 per cent of the cases the negative area under the distributional curve was more than 1 per cent(cf. Appendix A).

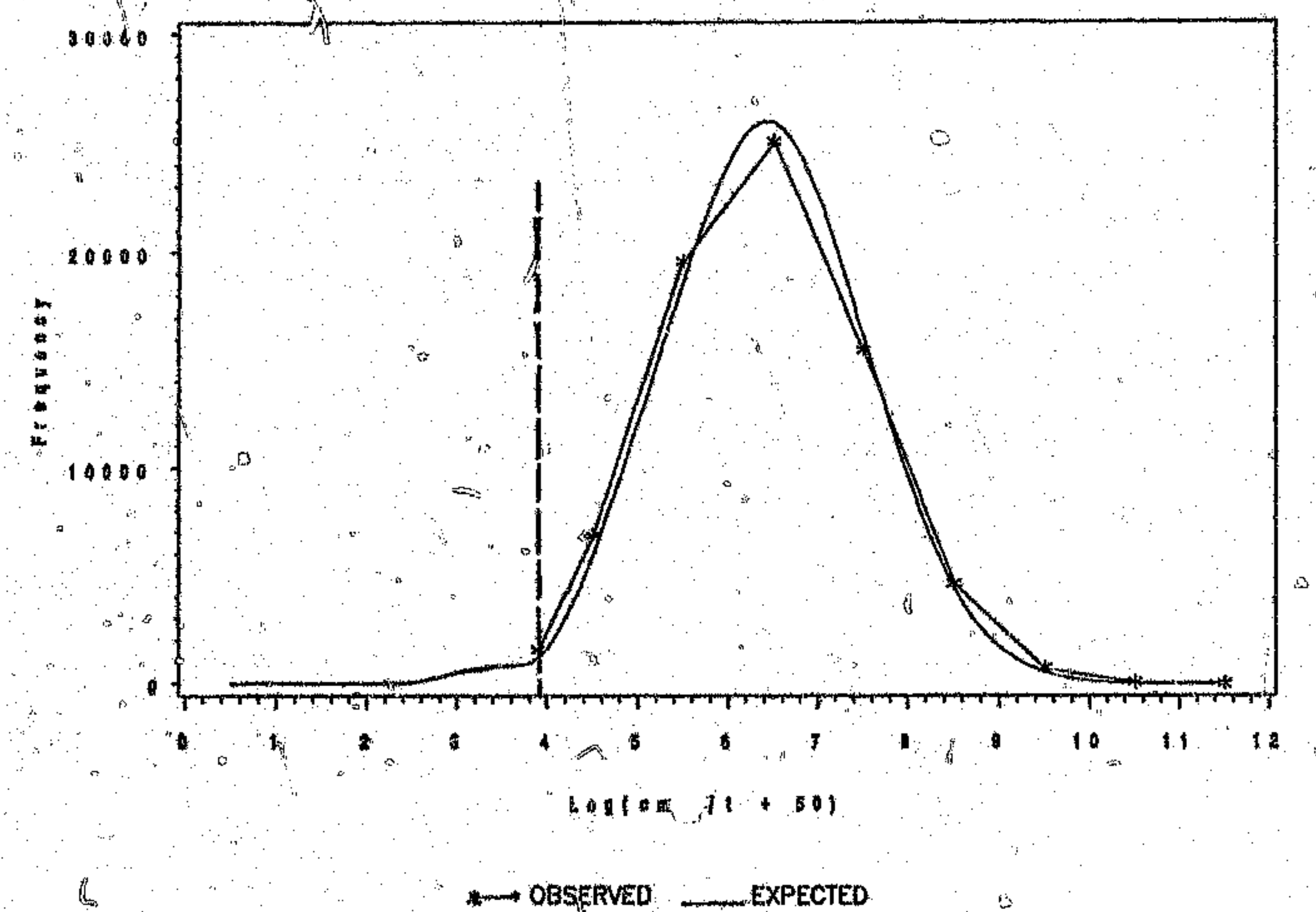
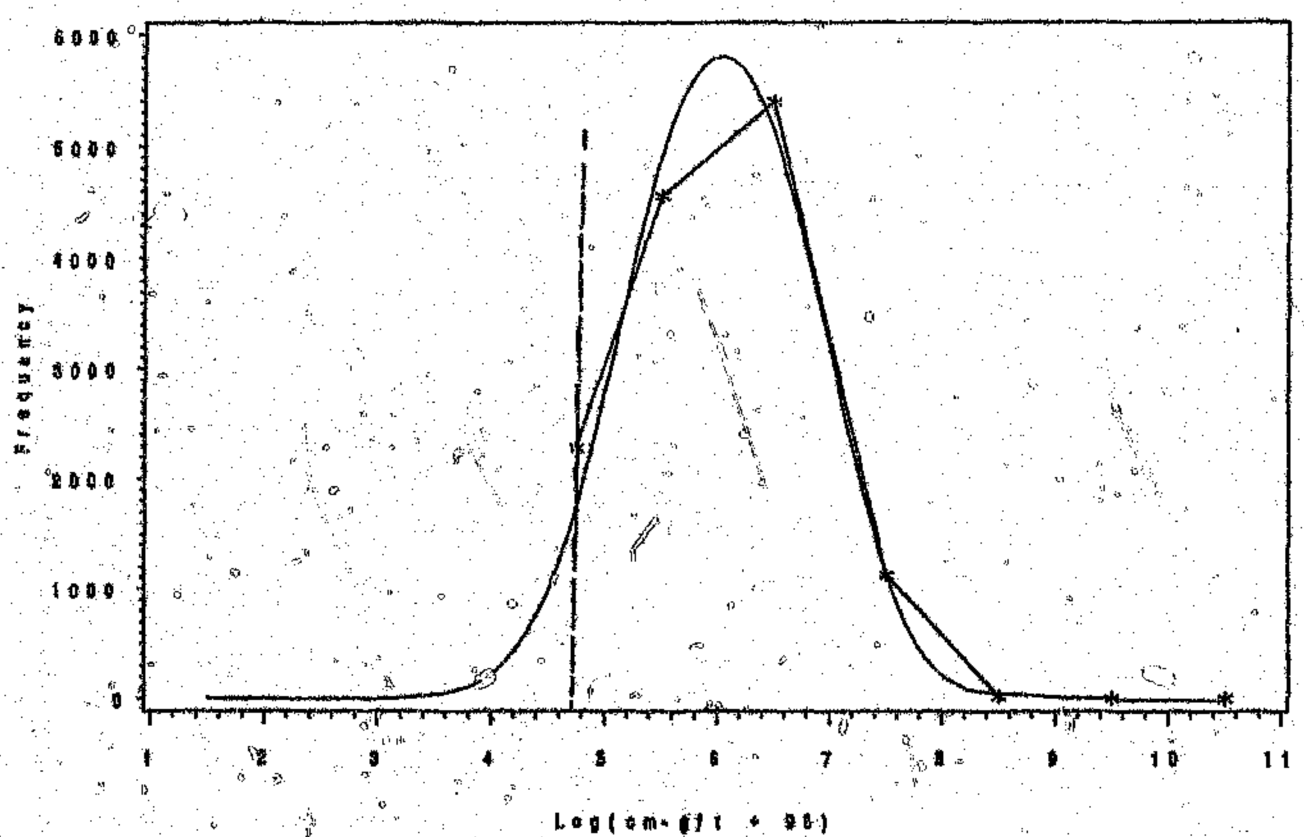


FIGURE 2.8 3—PARAMETER LOGNORMAL MODEL FITTED TO GOLD DATA
AREA H, N = 72767



✱—✱ OBSERVED ——— EXPECTED

FIGURE 2.9 3–PARAMETER LOGNORMAL MODEL FITTED TO GOLD DATA
AREA J, N = 12241

2.6.5 Summary of the overall results of the Case Study

From the above analyses the 3-parameter model for gold grade distribution gave an efficient estimate of the population mean with less than 5 per cent bias. The new distributions showed similar efficient estimates with bias of not more than 2.5 per cent (replacing original results with chi-square minimisation results wherever applicable). The 3-parameter model showed comparable biasses to those using the new models. However, in all the cases where the departure from the 3-parameter model was significant and also where the additive constant was inefficiently estimated, the 3-parameter estimates showed significantly higher biasses.

The theoretical models of the three distributions practically fitted the empirical data though to different extents. It should be emphasized that due to the bounded finite lower size of $\ln(\beta)$ [indicated by the broken lines in Figures 2.8 and 2.9], where β is the additive constant, any value under the mathematical model which is below this finite lower size will result in a negative gold value after back-transformation. Such negative gold values in reality, do not exist. For more than 16 per cent of the cases, the 3-parameter model indicated a negative gold proportion of more than 2 per cent though as suggested by Sichel[34] this area should not be more than 2 per cent.

In the Klerksdorp area H, (Vaal Reef) 72767 gold values were analyzed and the compound lognormal model gave better results than the 3-parameter model. This further confirms that for large mining areas, where we are likely to deal with mixtures of lognormal distributions, the new distributions will give better estimates. However, change of support and small sampling theory is yet to be developed for these new models.

2.7 Summary of Operational Advantages and Disadvantages of the three Models

2.7.1 The 3-Parameter Lognormal Model

ADVANTAGES

- No complex mathematics is required for the estimates of the three parameters and the distributions are simple to handle.
- Generally less computer time is required for the estimates of the three parameters.
- Only three parameters are required to be estimated.
- Working permanence is accepted for change of support calculations.
- Efficient estimate of population mean.
- Third parameter can usually be accepted a-priori from nearby data, if such data exist.

DISADVANTAGES

- Problem of truncation and bimodality, the squashing of the frequency distribution makes it devoid of its naturality.
- Problem of obtaining negative gold values on back-transformation.
- Difficulty in obtaining an estimate for the additive constant especially for small sample sizes.
- Possibility of departure from 3-parameter lognormality.

2.7.2 Compound Lognormal and LN-GIG Models

ADVANTAGES

- The new models have no truncation since the logarithm of the actual values ranges from $-\infty < x < +\infty$. There is therefore no issue of negative gold value results.
- There is no inherent theoretical limitation.
- Since the new distributions deal with the logarithm of the gold values themselves the frequency categories look very natural.
- Can handle mixtures of lognormal distributions to a better extent.
- Efficient estimates of the population mean.

DISADVANTAGES

- The mathematics of the new distributions is more complicated.
- Generally more computer time is required.
- Four parameters are required.
- The problem of change of support and small sampling theory is yet to be developed, and presently this serves as a limitation.

2.8 The Distributional Models and Regularized data

Table 2.1 shows the statistics for 'point' and regularized data for the Vaal Reef (Klerksdorp, Area H) with additive constant of zero. The criteria for applying the CLD is also given (See Section 2.4.4). The Table shows that the distribution of such regularised data have much lower variances and are much less skew than the individual 'point' values. The Table further shows that for large supports the 2-parameter model will even be suitable. Thus, unlike the 'point' data the advantage gained by using the new distributions for the regularized data are not likely to be significant in this case. It is for this reason combined with the on going research for the small sampling theory for the new distributions that the analyses in chapter 3 and 4 were done on the 3-parameter lognormal model.

Table 2.2 also shows the statistics for the same data in Table 2.1 but using an additive constant of 25. The results further emphasizes the need to use a good estimate of a regional *a priori* additive constant. This is essential since a bad estimate of the additive constant can introduce skewness for the regularized data which will result with biasses on the mean estimates.

TABLE 2.1 SHOWING STATISTICS FOR 'POINT' AND REGULARIZED DATA FOR VAAL REEF (ADDITIVE CONSTANT = 0)

BLOCK SIZE(M)	LOGMEAN	LOGVAR-IANCE	SKEWNESS	KURTOSIS	CRITERIA
'POINT'	6.244	1.598	-0.266	3.466	3.035
10x10	6.694	0.814	-0.015	3.243	3.000
20x20	6.676	0.628	-0.052	3.044	3.000
30x30	6.767	0.500	-0.024	3.113	3.000
50x50	6.796	0.436	0.030	2.935	3.000
100x100	6760	0.402	-0.004	2.939	3.000

CRITERIA = 0.5[3(SKEWNESS)^2 + 6]

TABLE 2.2 SHOWING STATISTICS FOR 'POINT' AND REGULARISED DATA FOR VAAL REEF (ADDITIVE CONSTANT = 25)

BLOCK SIZE(M)	LOGMEAN	LOGVAR-IANCE	SKEWNESS	KURTOSIS
'POINT'	6.336	1.314	0.087	2.886
10x10	6.643	0.739	0.120	3.109
20x20	6.718	0.578	0.044	2.942
30x30	6.803	0.465	0.049	3.073
50x50	6.830	0.409	0.087	2.922
100x100	6.794	0.375	0.055	2.901

2.9 Conclusions

The study showed that, though, the 3-parameter model is a practically reasonable model, especially for the estimation of the population mean of the gold distributions, the usual assumption that gold grade distributions follow a 3-parameter law is not always true. As a result, the plot of the empirical cumulative frequency distribution is very necessary to test departures from lognormality.

The study further showed that the new models also provide good estimates of the population means of the gold distributions. Another outstanding feature which makes the new distributions more attractive is the fact that no inherent theoretical problems would be anticipated. The new distributions also seem to give better estimates when larger mining areas are under consideration. It is however worth noting that, change of support and small sampling theory have to be resolved for the new models.

This study further shows that in more than 95 per cent of the cases the compound lognormal model could be used for modelling the gold grade distributions. This indicates that the compound lognormal model is more applicable for modelling the gold grade distributions in the Witwatersrand placers than the LN-GIG model.

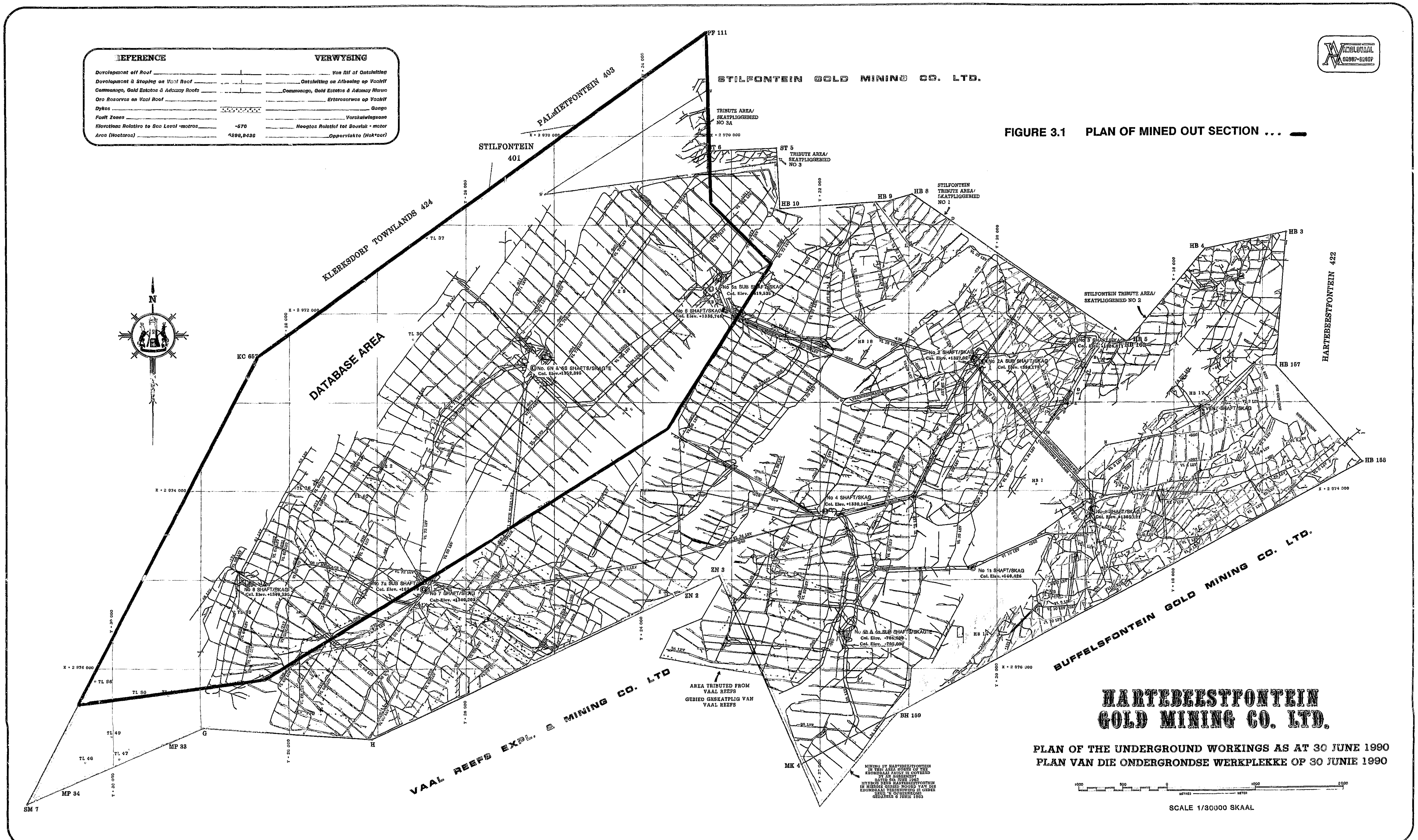
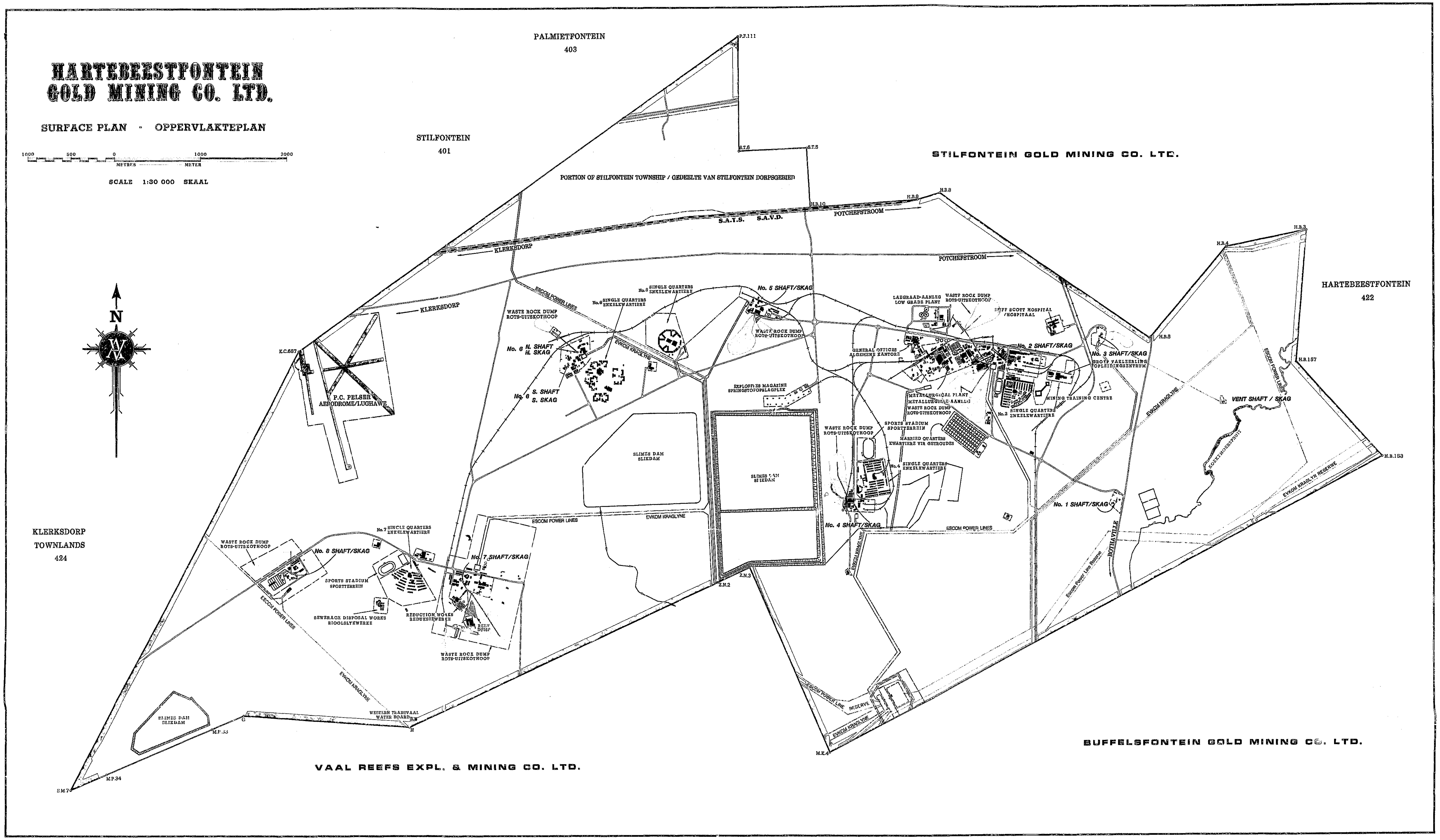


FIGURE 3.1 PLAN OF MINED OUT SECTION ...

PRINTED BY J. C. INCE & SON (PTY.) LIMITED


HARTEBEESTFONTEIN GOLD MINING CO. LTD.
SURFACE PLAN - OPPERVLAKTEPLAN
1000 500 0 1000 2000
METRES METER
SCALE 1:30 000 SKAAL
N
PALMIETFONTEIN 403
STILFONTEIN 401
STILFONTEIN GOLD MINING CO. LTD.
PORTION OF STILFONTEIN TOWNSHIP / GEDEELTE VAN STILFONTEIN DORPSGEBIED
S.A.T.S. S.A.V.D.
POTCHEFSTROOM
KLERKSDORP
HARTEBEESTFONTEIN 422
KLERKSDORP TOWNLANDS 424
P.C. PELSER AERODROME/LUGHAWE
ESCOM POWER LINES
EVKOM KRAGLYNE
WASTE ROCK DUMP ROTS-UITSKOTHOOP
No. 6 N. SHAFT N. SKAG
No. 6 S. SHAFT S. SKAG
SINGLE QUARTERS ENKELKWARTIERE
No. 5 SHAFT/SKAG
EXPLOSIVES MAGAZINE SPRINGSTOFOPSLAGPLEK
SLIMES DAM SLIKDAM
LAEGRAAD-AANLEG LOW GRADE PLANT
GENERAL OFFICES ALGEMENE KANTORE
No. 2 SHAFT/SKAG
No. 3 SHAFT/SKAG
MINING TRAINING CENTRE
METALLURGICAL PLANT
SPORTS STADIUM SPORTTERREIN
MARRIED QUARTERS KWARTIERE VIR GETROUDES
No. 4 SHAFT/SKAG
No. 1 SHAFT/SKAG
VENT SHAFT / SKAG
EVKOM KRAGLYN RESERVE
ESCOM POWER LINE RESERVE
No. 8 SHAFT/SKAG
No. 7 SHAFT/SKAG
SEWERAGE DISPOSAL WORKS RIOOLSUIWERKE
REDUCTION WORKS REDUKSIEWERKE
WESTERN TRANSVAAL WATER BOARD
VAAL REEFS EXPL. & MINING CO. LTD.
BUFFELSFONTEIN GOLD MINING CO. LTD.
ORKNEY

CHAPTER 3

3 BOREHOLE VALUATIONS OF NEW GOLD MINES AND UNDEVELOPED SECTIONS OF EXISTING MINES

3.1 Introduction

Due to the high cost of drilling and time delays in drilling deep boreholes, the feasibility study of new gold mines (especially in South Africa) are based and will continue to be based on limited numbers of boreholes. Sichel[39] confirms the difficulty regarding the use of such limited information:

> When a few borehole values from newly drilled virgin ground have come to hand, the mining engineer is faced with one of the most important and difficult decisions which he is called upon to make during his entire professional career, ... to mine or not to mine, that is the question.

Taking cognisance of the large capital involved in such deep level gold investments particularly in the present difficult period for the gold mining industry, the need for appropriate models and valuation procedures for these capital intensive projects cannot be overemphasized. This chapter considers the possibility of deriving additional information from neighbouring mines based on geological similarities to improve borehole estimates. These valuation procedures should also be applicable to the routine valuation of large undeveloped sections of existing mines which are usually based on similar limited information.

The chapter also introduces the concept of spatial structure for 'variances' as a means of deriving additional useful information.

Chip sample data from a large mined out section of the Hartebeestfontein Gold mine in the Klerksdorp Goldfield (See Figure 2.2, Chapter 2) were used for the series of borehole valuation procedures. Conclusions are drawn regarding the relative advantages derived from such additional information and the various procedures used.

3.2 Geology of the Study Area

Hartebeestfontein mine is based on the Vaal Reef in the Klerksdorp Goldfield. Figure 3.1 shows the area of the mine which provided the database for the study.

The Vaal Reef is the most important reef in the Klerksdorp area and lies stratigraphically near the middle of the Central Rand Group. The Vaal placer covers an area of 260 square km. It consists of a narrow conglomerate seldom exceeding 50 cm in thickness but often 10 cm thick. However, in certain areas the close contact of the hanging wall pebble bed (the Zandpan Marker) can exaggerate the reef thickness. According to Minter[40] the Zandpan Marker generally contains little gold and the effective distortion of the indicated Vaal Reef gold values, is therefore, very small. Gold is usually concentrated at the base of the reef and high gold concentrations are often found associated with thin carbon seams and scattered carbon specks. Carbon in the Vaal Reef is believed to be of algal origin (Pretorius[41]). The general feature of the reef is of an infilled drainage pattern with south-east trending channels. Minter[40] has demonstrated that the Vaal Reef was derived from two sources and deposited as two populations (the Witkop and Stilfontein facies) with limited amount of overlap at the interface. All the data used for this study occur in a geologically homogeneous section of the Stilfontein facies.

3.3 Distributional Analyses

The chip sample data from Hartebeestfontein Gold mine provided the main database for the analyses. The database consisted of a total of 72767 sampling sections on the Vaal Reef measuring some 25 million square meters (See Figure 3.1). The surface borehole data found in the area was also used as will be shown later. The distributional analyses in chapter 2 showed that these gold values from Hartebeestfontein mine can be modelled very well with the compound lognormal distribution. The analyses further showed that the 3-parameter lognormal model can also be used without introducing any significant biasses especially with a very good estimate of the threshold parameter. Figure 3.2 shows the cumulative frequency plots of the gold grades for the 'point' values as well as the 50 x 50 m and 1 Km x 1 Km block supports (with a threshold parameter $\beta = 50$). The figures demonstrate that the 3-parameter model represents the distribution of gold values for these supports sizes. The 50 x 50 m and 1 Km x 1 Km blocks were the blocks used for the face (panel) valuations (in Chapter 4) and the borehole analyses respectively. In addition the distribution of single section values to estimate the average of 9 boreholes was used for the borehole valuations. The 3-parameter model was used for both valuations. The new models were not used for the face and borehole valuations due to the on going research into the small sampling theory for the new distributions. Also, the distribution of such regularized data as shown in chapter 2 have much lower variances and are much less skew than for individual values, thus the new distributions are not likely to show any significant improvement.

3.4 Statistical analyses of the Underground chip Samples and Borehole cores in the Study Area

Some detailed statistical analyses were done using the 72767 underground chip sample values and the surface boreholes found in the study area.

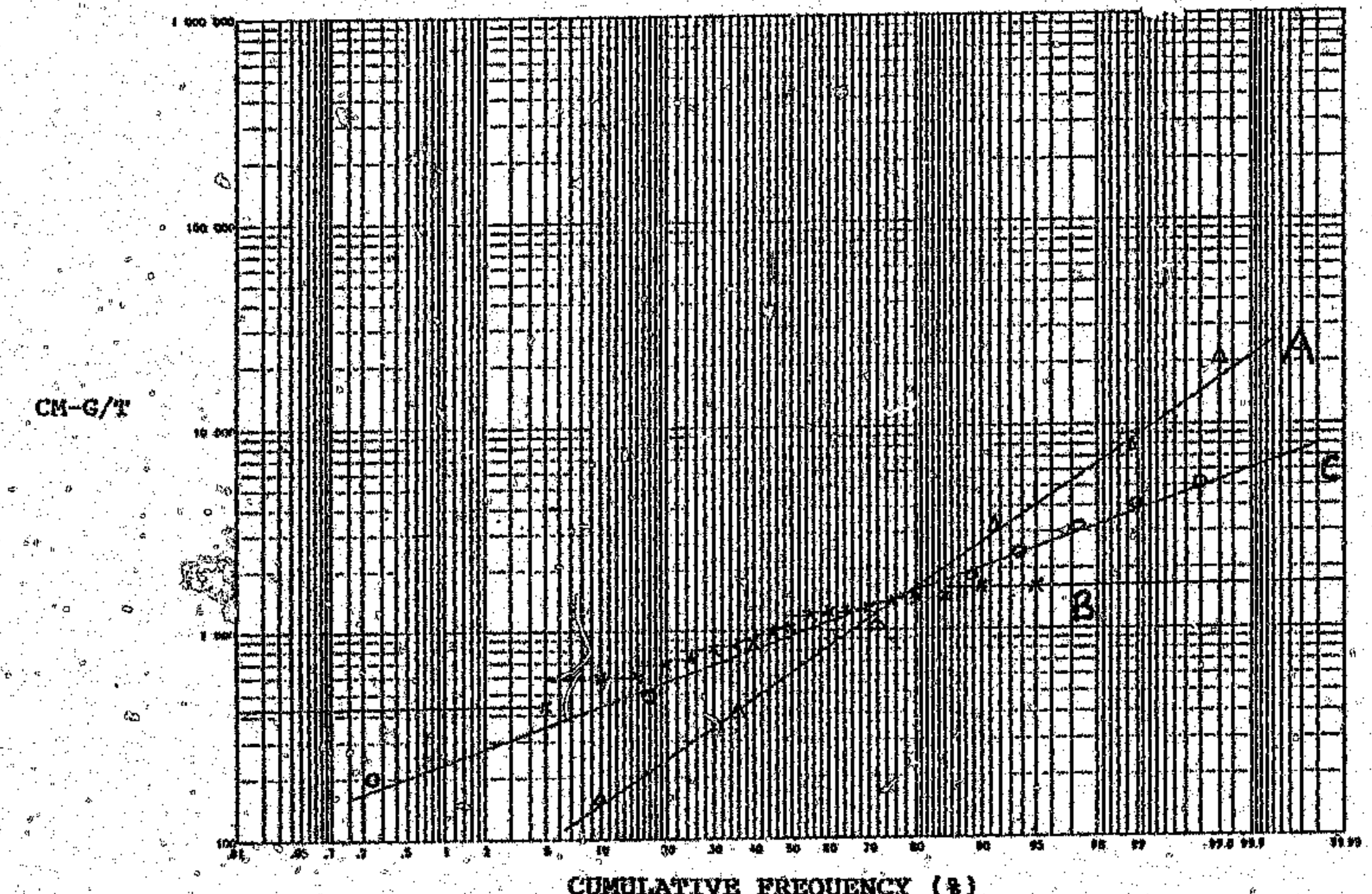


FIGURE 3.2 SHOWING THE CUMULATIVE FREQUENCIES OF THE CM-G/T VALUES (PLUS 50)
A: FOR 72707 INDIVIDUAL SAMPLING SECTIONS ... △
B: FOR 20 ACTUAL MINE MEAN GRADES ... x
C: FOR 50 X 50 M BLOCK MEAN GRADES ... o

The objective was to find out how the statistical parameters of the underground chip samples compare with the surface borehole cores. Results of the analyses are shown in Table 3.1.

TABLE 3.1 STATISTICAL ANALYSES OF CHIP SAMPLES AND BOREHOLE CORES FOR VAAL REEF

CM.G/T VALUES	KLERKSDORP GOLDFIELD - VAAL REEF			
SUPPORT TYPE	UNDERGROUND CHIPS			BORE-HOLE CORES FIELD
	4 MINES	100x100m BLOCKS MINE Y	1x1km BLOCKS MINE Y	
NO OF AREAS	49	3	20	11
SIZES OF AREAS(10^6m^2)	0.4/92.9	1.1/3.8	92.9	28.5/455
NO OF VALUES PER AREA	100/72767	213/245	168/14239	28/149
B VALUES (3rd PAR.)	10/250	14/51	50	0/300
AVERAGE ACCEPTED	50	50	50	50
<u>Ln. VARIANCE (AVER. B):</u>				
RANGE	0.58/1.55	0.99/1.11	0.64/1.43	0.55/1.79
AVERAGE	0.99	1.05	1.00	1.06
VARIANCE	0.06	–	0.0[illegible]	0.26
<u>MEAN GRADES:</u>				
MEAN	1423	–	1002	1013
Ln. VARIANCE	0.18	–	0.13	0.11

Table 3.1 shows that the average variances for the chip samples at 0.99/1.05 compares very well with that of the borehole cores at 1.06. The chip samples however have larger support size (compared to that of the borehole cores) and could be expected to have smaller variances. The observed variances of the chip samples (which are similar to that of the boreholes) can be explained by the accompanying higher nugget effects. The small borehole cores being close to perfect samples have smaller nugget effects but larger variances of the actual grades. The compensating influence between the variances and the nugget effects of the two different data sets explains their comparable average variances. Table 3.1 however shows that the variances of the variances of borehole cores are higher than that of the chip samples. This is due to the limited numbers of borehole samples compared with underground values.

In addition the results in Table 3.1 show that using an *a priori* regional mean value for the additive constant will not result in any significant bias. Similar analyses done on some Witwatersrand reefs (including the Vaal Reef) by Krige et al[10] showed similar results. Based on these findings it was decided to use the chip sample data as the equivalent of borehole values for the analyses in this chapter.

3.5 Spatial Structure Analyses

3.5.1 Introduction

Details on semivariograms are well documented and can be found e.g., in Krige[42], Matheron[20], Journel and Huijbregts[25] and David[43] and will not be discussed here. Studies by, Krige and Magri[44] and Braun[28] on the Witwatersrand reefs have shown that the logtransformed semivariogram has a significant advantage of reducing the effect of outliers. In addition Krige[42] has shown that the logtransformed semivariogram can be validated using the variance-size of area relationship. In all the spatial analyses for this study, the logtransformed semivariograms were used and were validated using the variance-size of area relationship.

3.5.2 Spatial Structure analyses for Grade Levels

Based on the 72767 chip sample data logtransformed semivariograms were calculated on the grade levels (with a threshold parameter of 50). Models were fitted to these experimental semivariograms. Based on this 'point' semivariogram, average semivariograms within larger blocks were computed using numerical integration (descretization). As a check for the semivariograms within the bigger blocks obtained via descretization, regularized logtransformed semivariograms were also computed. The two methods of semivariogram calculations gave similar results as will be shown later. Two sets of logtransformed semivariograms were calculated for the grade levels (with $\beta = 50$):

(i) Semivariograms on the logarithm of the arithmetic mean within the various block supports, and
(ii) semivariograms on the mean of all the logvalues within the various block supports.

The results for the above semivariogram analyses are shown in Figures 3.3 to 3.11. In fitting models to these experimental semivariograms, the intended estimation was kept well in mind since it is no use e.g., to specify structures on the petrographic scale if the objective is to estimate blocks on a kilometre scale. It was observed that in each case a spherical or exponential model with nugget effect can be fitted to the experimental semivariograms. Some weak geometric anisotropies were observed but they were not relevant at the scale of the analyses. Figures 3.3 to 3.11 further demonstrate that from the 'point' values through the micro-panel (50 x 50 m blocks) to the macro scale (1 Km x 1 Km) spatial structures on the grade levels are present.

3.5.3 Spatial Structures on Variances

The whole mined out section was divided into various areas ranging from 10 x 10 m to 1 Km x 1 Km. In each case variances within such areas (blocks) were calculated for the ln(cm-g/t + 50) values. Following a similar procedure as used for calculating semivariograms for grades (Section 3.5.2), semivariogram values were computed for the variances within the different blocks sizes.

Results are shown in Figures 3.12 to 3.17 and they demonstrate the presence of spatial structures on the variances. It was found that spherical or exponential semivariograms as used for the grade levels can

FIGURE 3.3 SEMIVARIOGRAM FOR 'POINT' VALUES

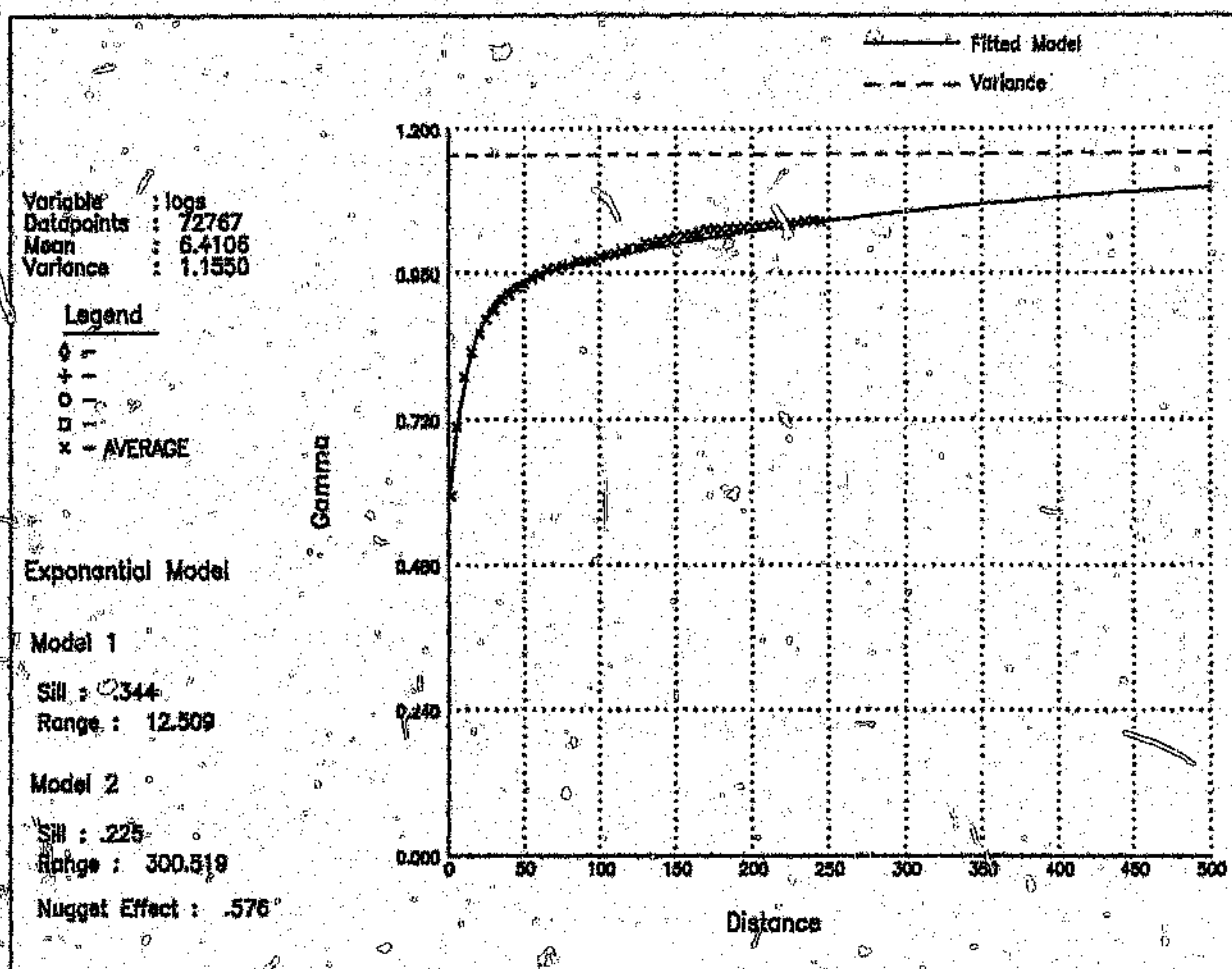


FIGURE 3.4 SEMIVARIOGRAM FOR LOGMEAN VALUES, 50X50 M BLOCKS

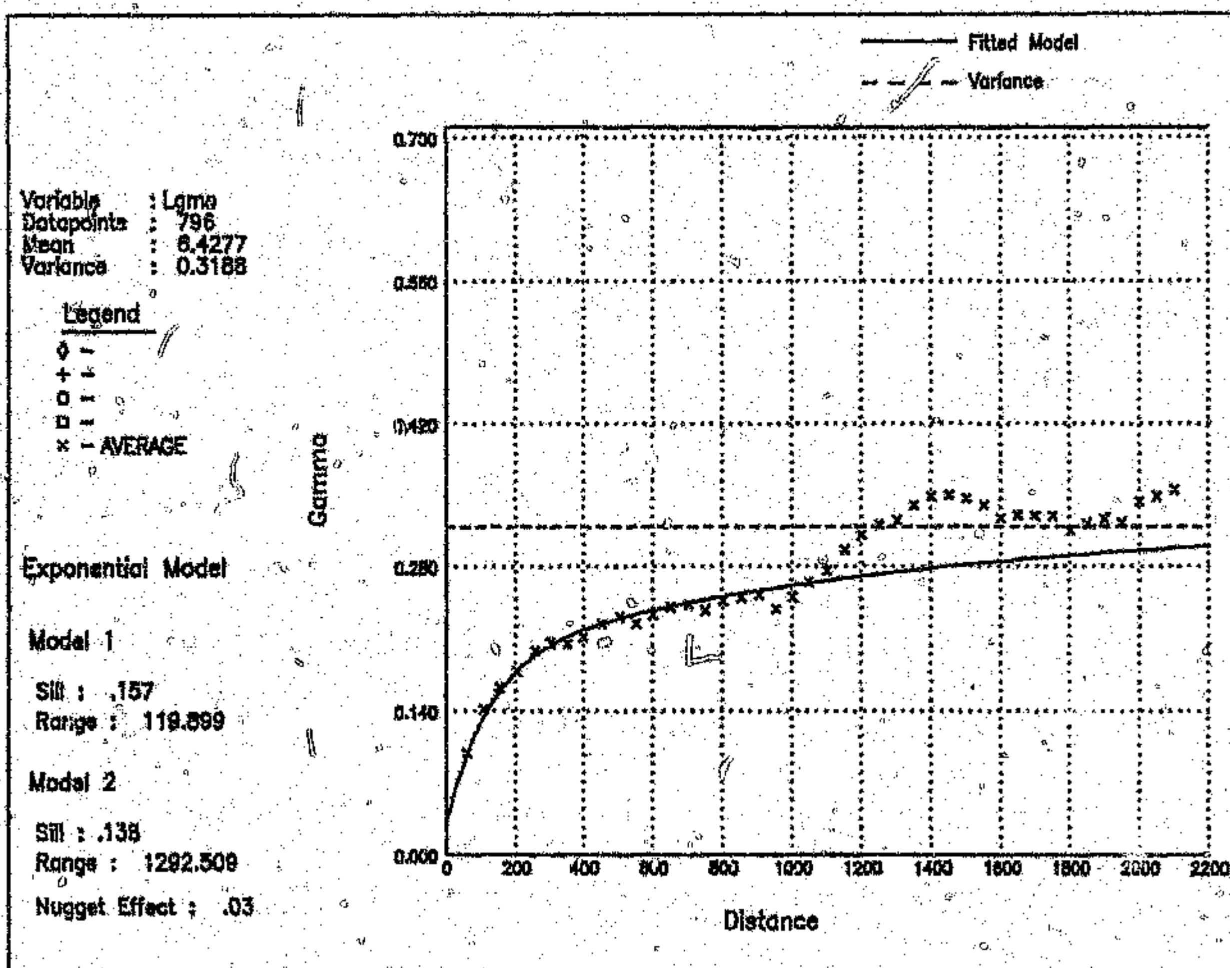


FIGURE 3.5 SEMIVARIOGRAM FOR LOGMEAN VALUES, 50X50 M BLOCKS

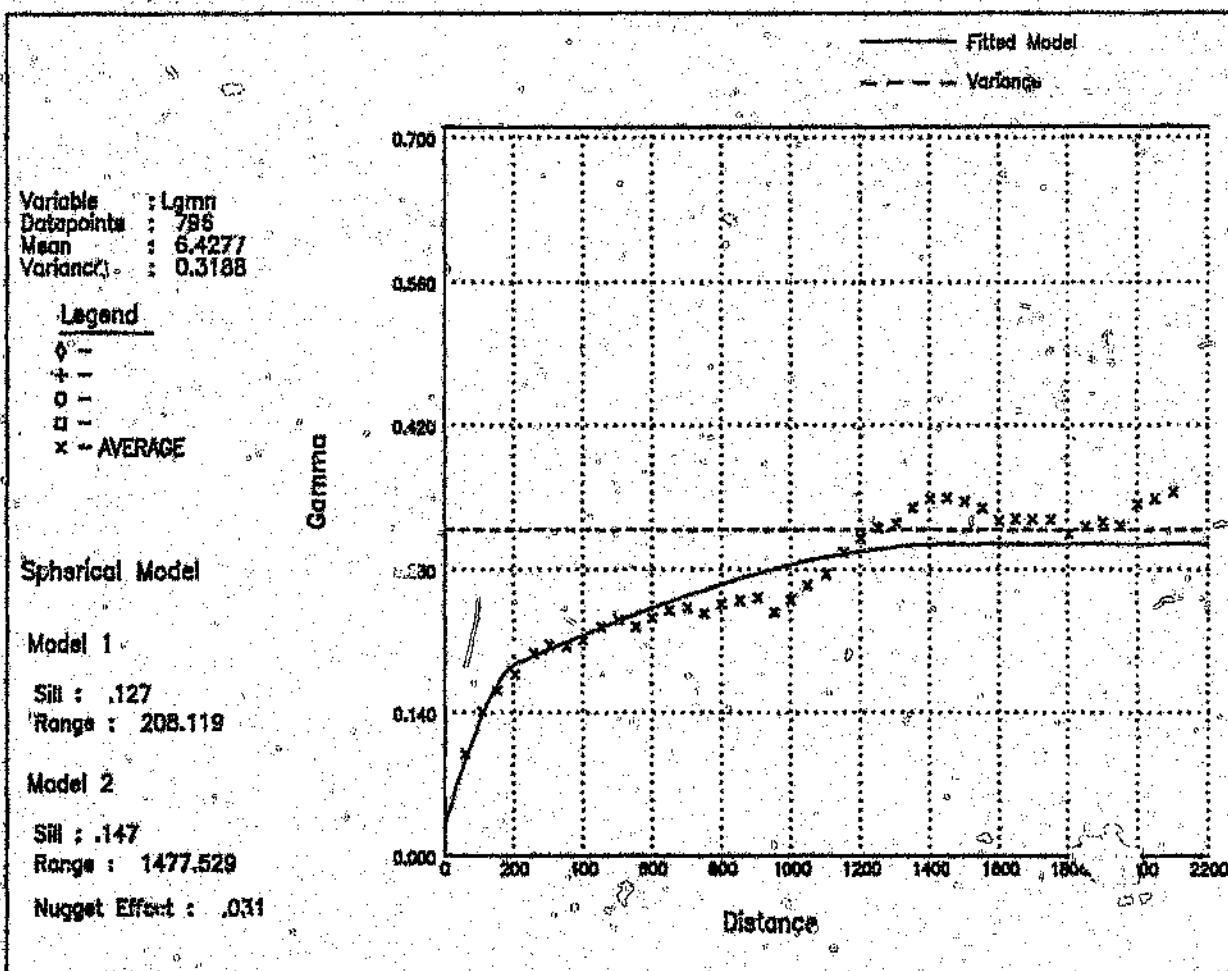


FIGURE 3.6 SEMIVARIOGRAM FOR LOGMEAN VALUES, 200X200 M BLOCKS

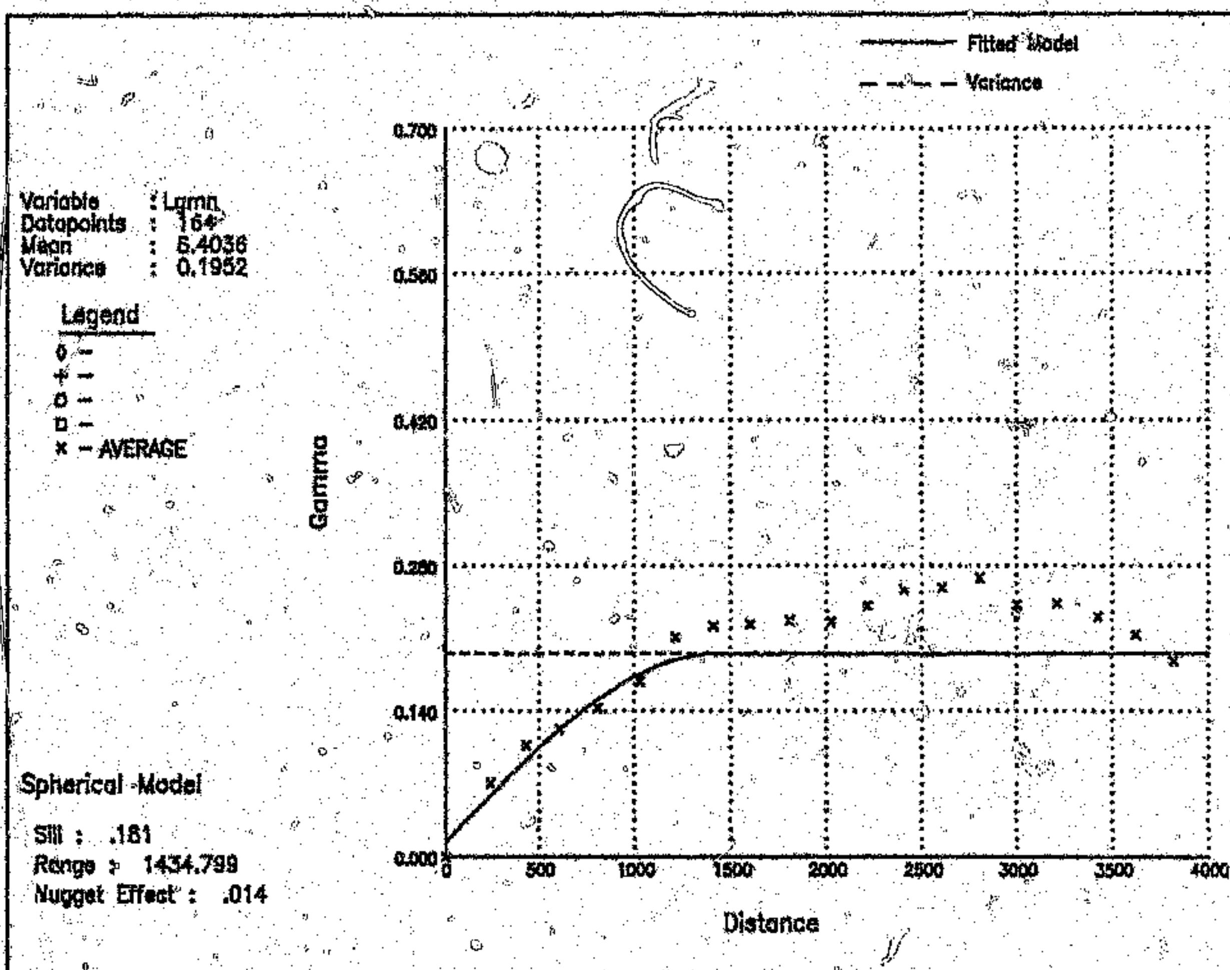


FIGURE 3.7 SEMIVARIOGRAM FOR LOGMEAN VALUES, 500X500 M BLOCKS

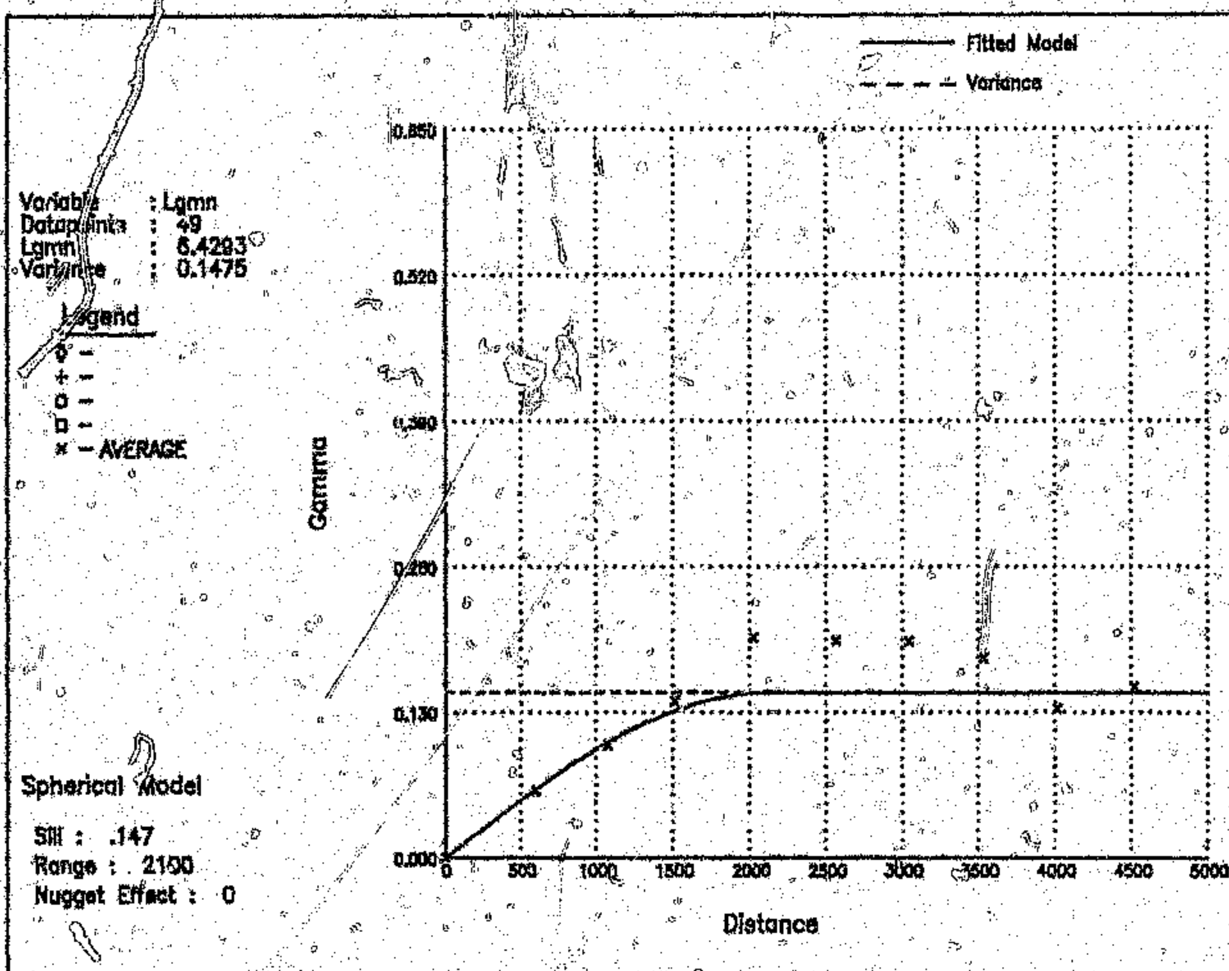


FIGURE 3.8 SEMIVARIOGRAM FOR LOGMEAN VALUES, 1Kmx1Km BLOCKS

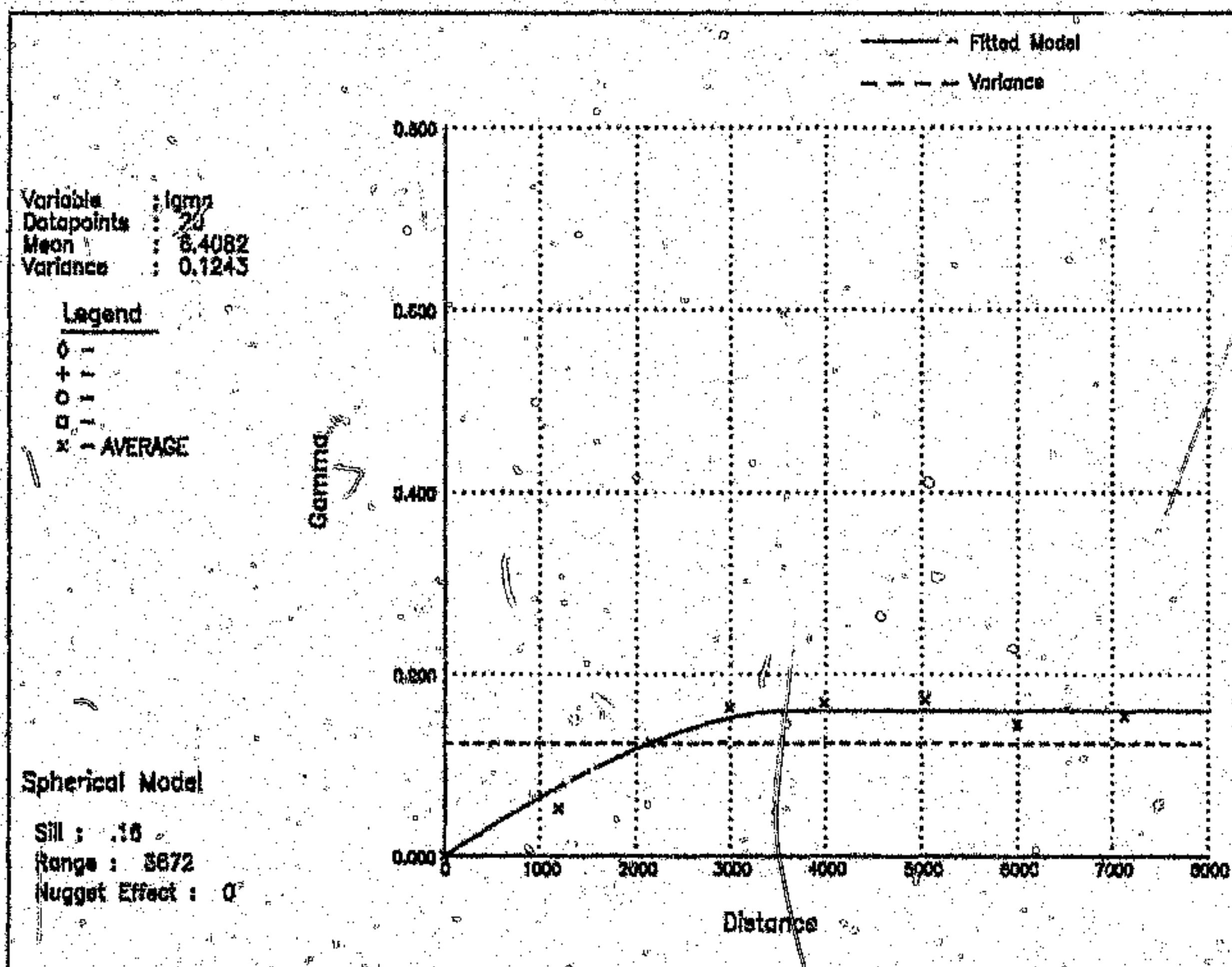


FIGURE 3.9 SEMIVARIOGRAM FOR LOG ARITH. MEANS, 10X10 M BLOCKS

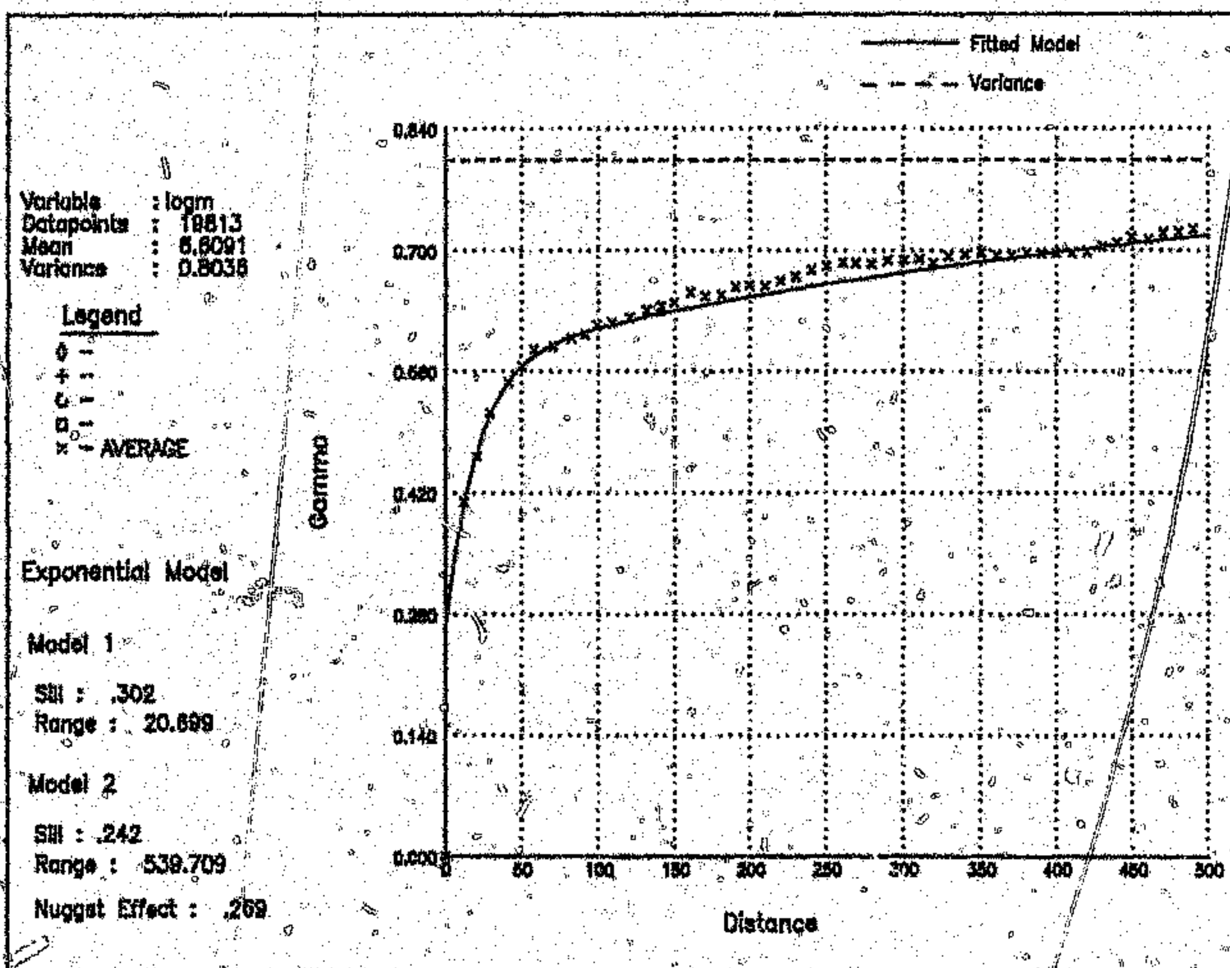


FIGURE 3.10 SEMIVARIOGRAM FOR LOG ARITH. MEANS, 50X50 M BLOCKS

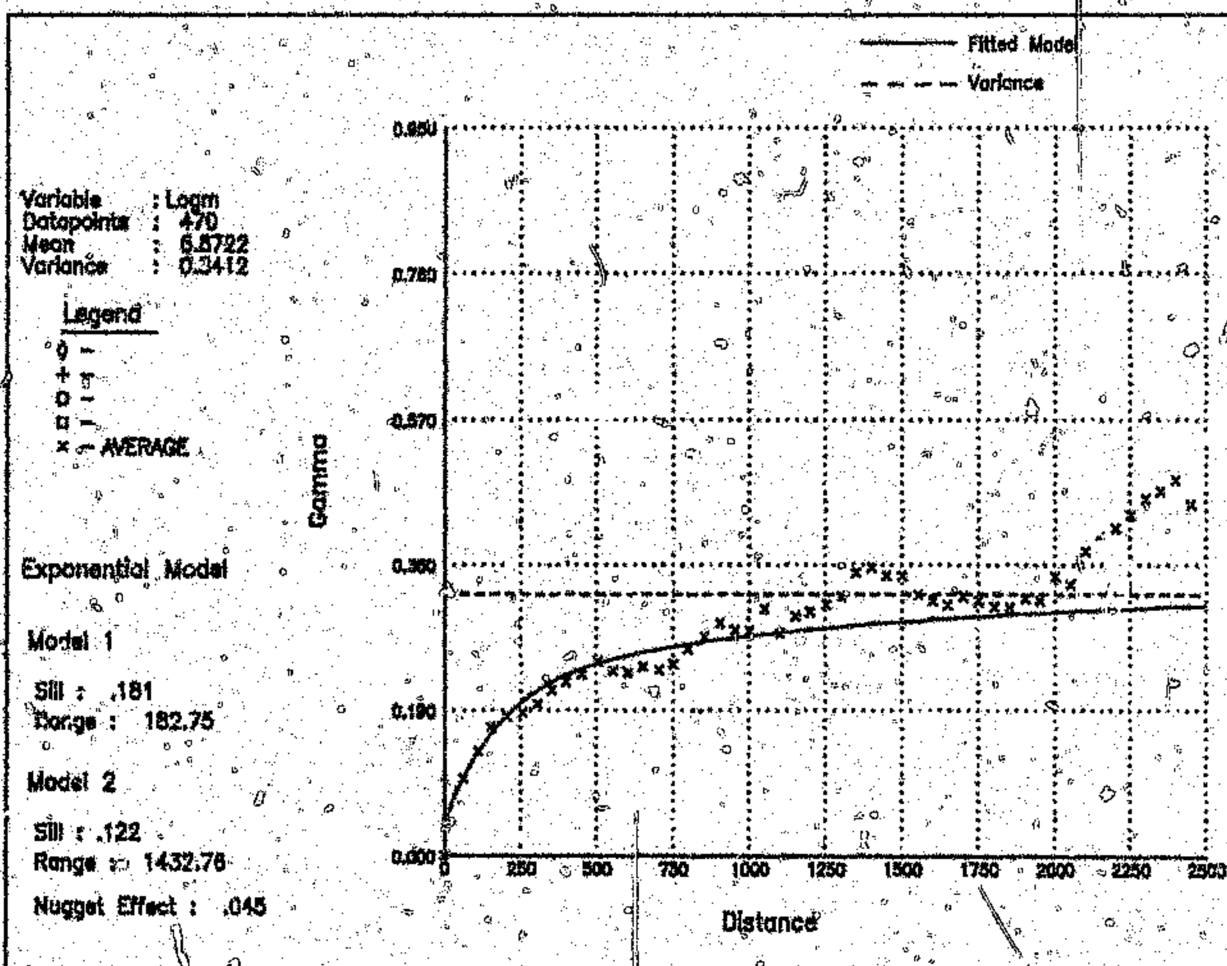


FIGURE 3.11 SEMIVARIOGRAM FOR LOG ARITH. MEANS, 1Km x 1Km BLOCKS

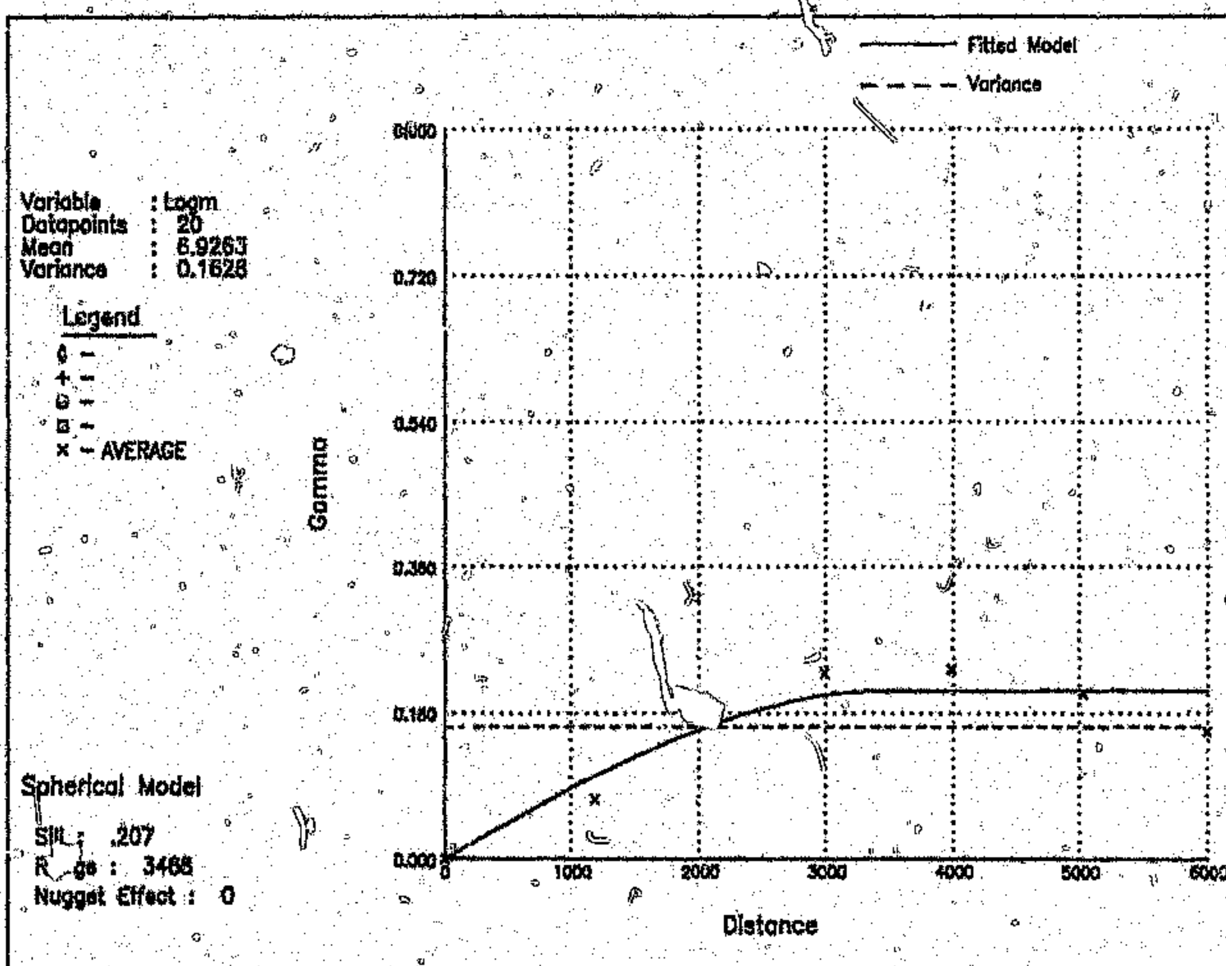


FIGURE 3.12 SEMIVARIOGRAM FOR LOGVARIANCE, 10X10M BLOCKS

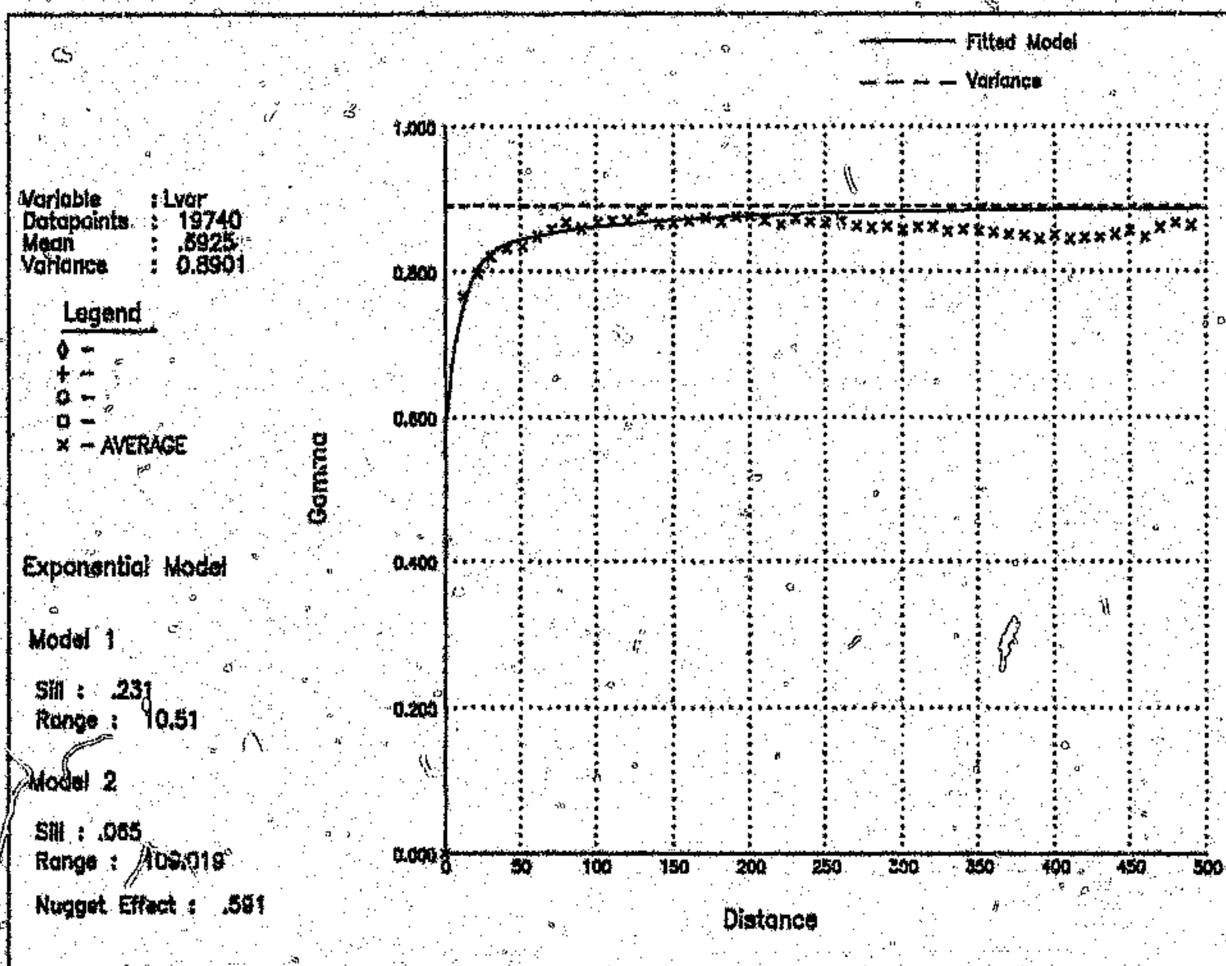


FIGURE 3.13 SEMIVARIOGRAM FOR LOGVARIANCE, 50X50 M BLOCKS

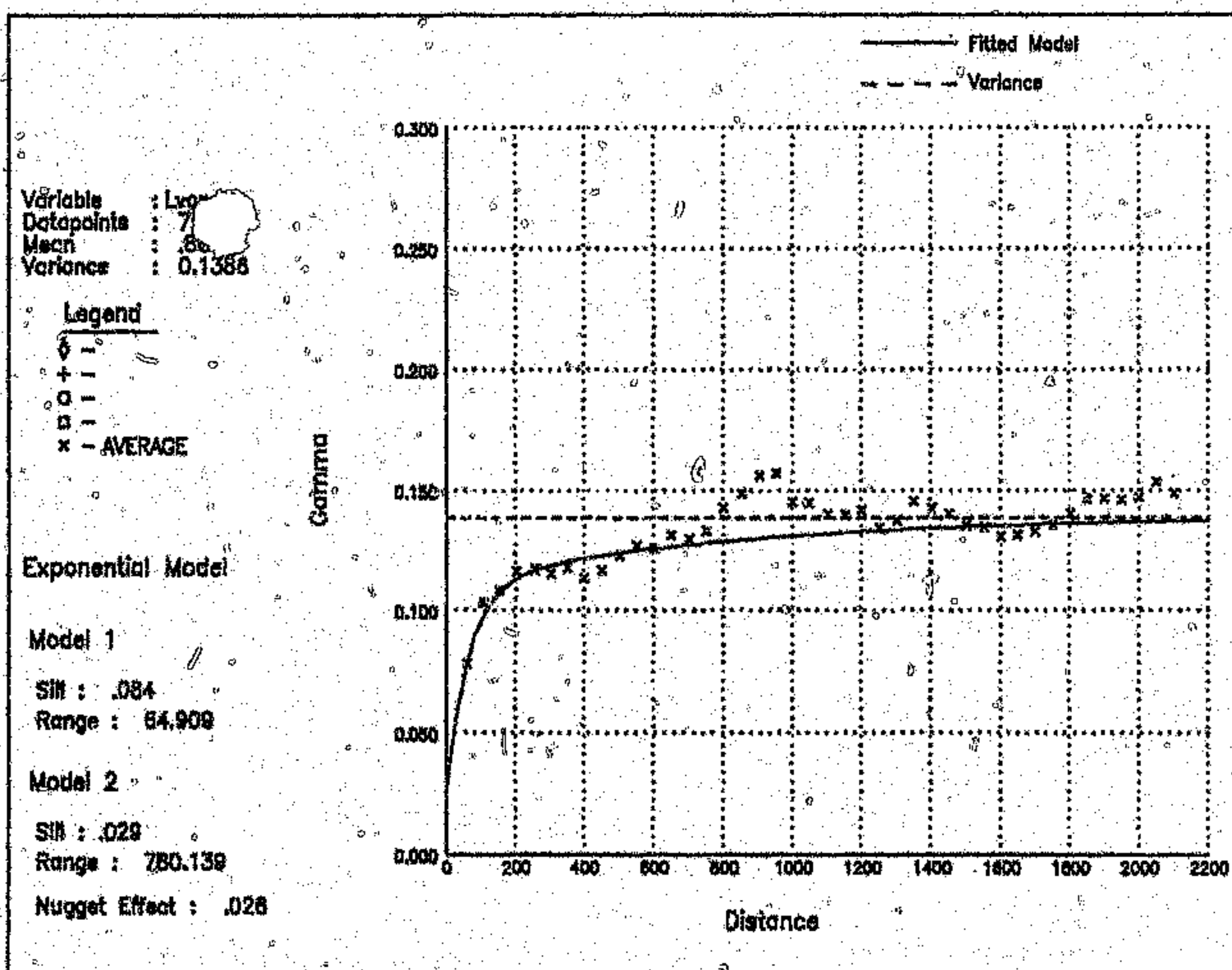


FIGURE 3.14 SEMIVARIOGRAM FOR LOGVARIANCE, 50X50 M BLOCKS

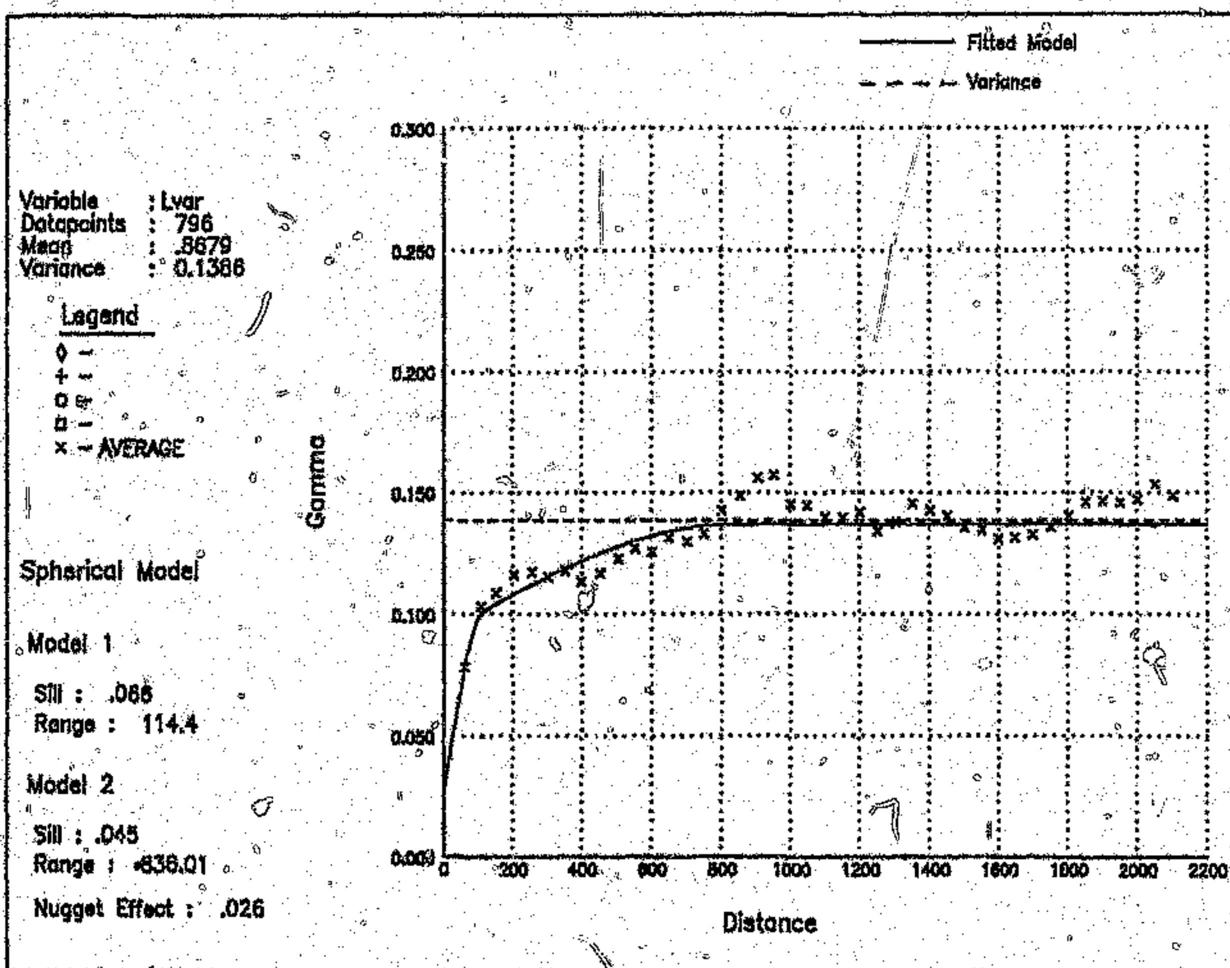


FIGURE 3.15 SEMIVARIOGRAM FOR LOGVARIANCE, 200X200 M BLOCKS

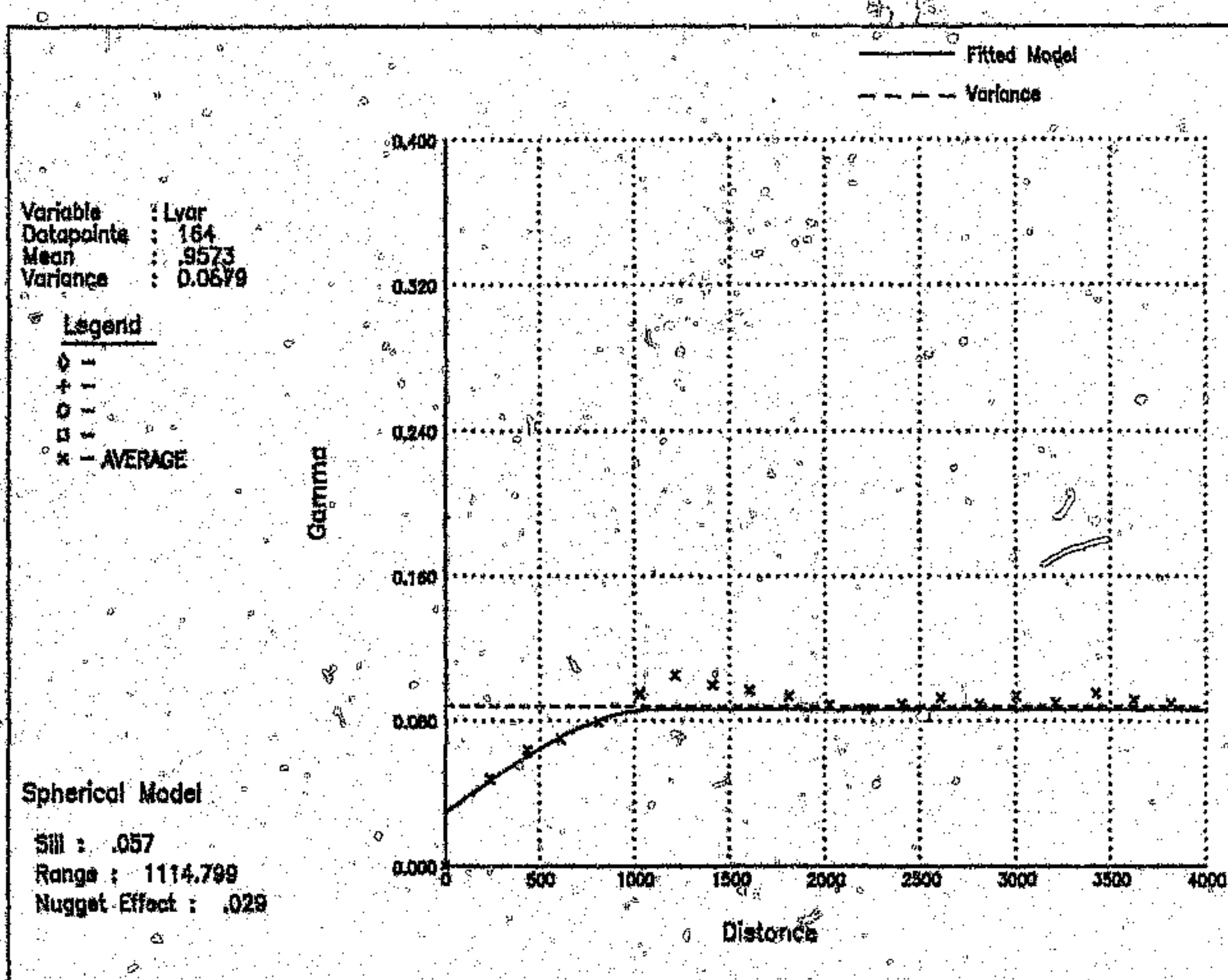


FIGURE 3.16 SEMIVARIOGRAM FOR LOGVARIANCE, 500X500 M BLOCKS

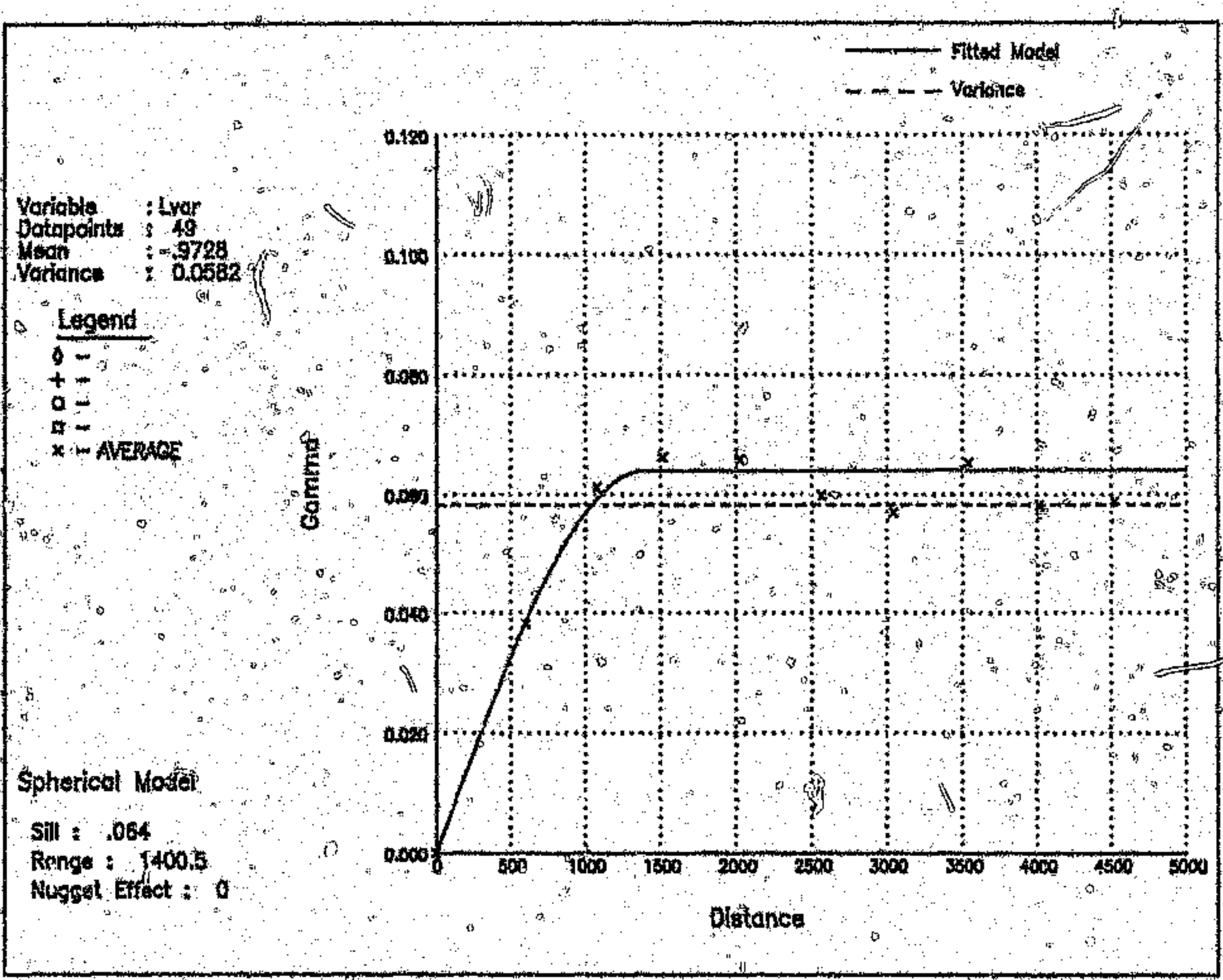
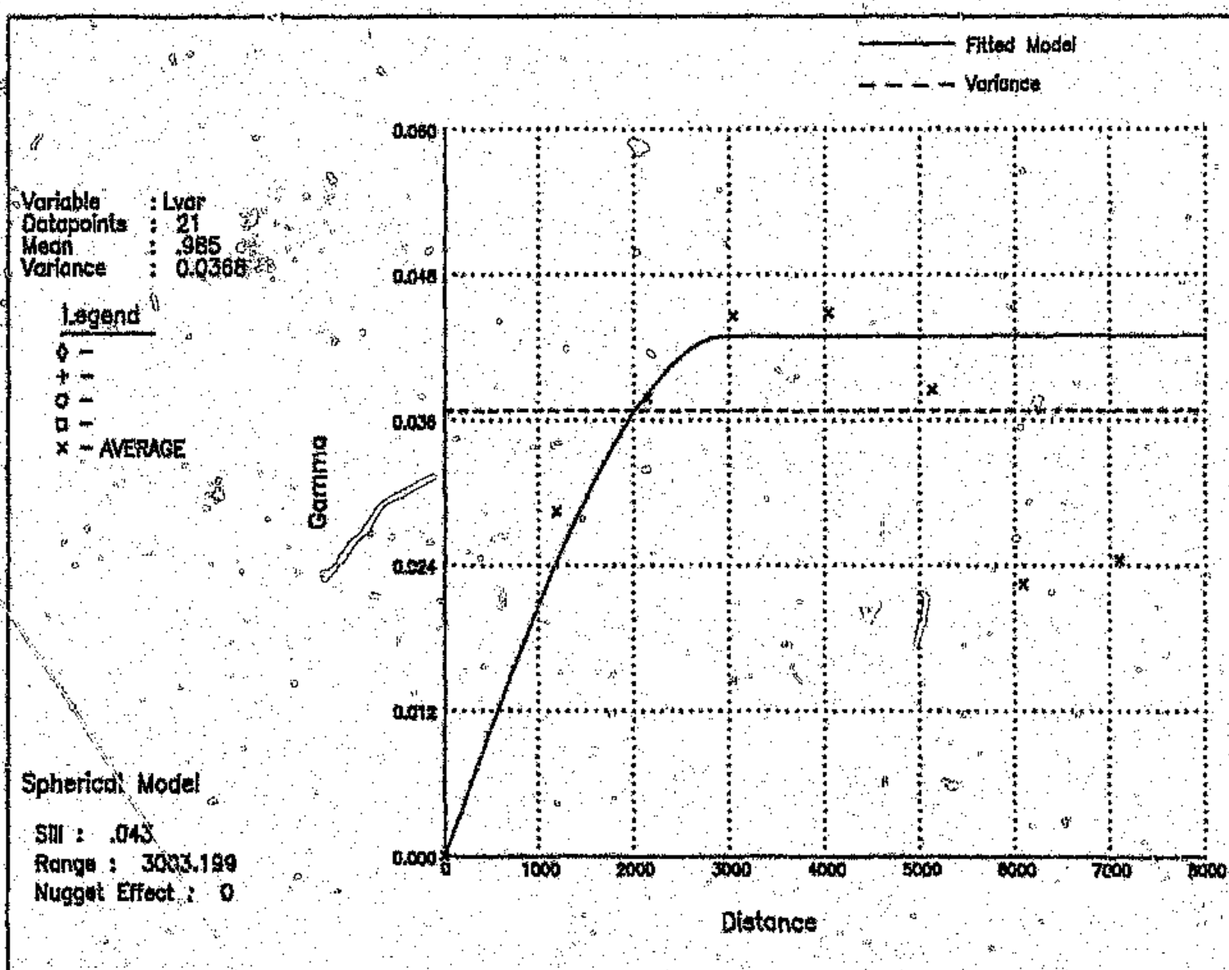


FIGURE 3.17 SEMIVARIOGRAM FOR LOGVARIANCE, 1Km x 1Km BLOCKS



describe the experimental semivariograms on the variances. Figures 3.12 to 3.17 further demonstrate the presence of spatial structures on the variance levels from the micro-panel to macro-panel scale.

The presence of spatial structures on the variances implies that additional information can be derived on the variances to improve ore reserve estimates. It should be emphasized that estimates based on limited amount of information may provide minimum estimation errors, yet these estimates, however good, can still be poor estimates of the variability of the 'true' values unless outside information is brought in to eliminate conditional biasses. In addition, it is a common incidence that plants designed on feasibility estimates of variability to receive mill feed (even with large superimposed confidence limits of the actual feed), end up undergoing costly unforseen alterations. Thus by introducing the concept of spatial structure on the variances, not only information on grade values can be used, but also additional important information on grade variabilities can efficiently be accessed. Such additional information on the variances if efficiently used will provide definite significant improvements in ore reserve valuation.

3.6 Simulated Data for the Borehole Analyses

Based on the findings in Section 3.4, the 72767 individual sampling sections were accepted as equivalent to individual borehole core values. A grid of 1 Km x 1 Km was laid on the whole mined out section in order to simulate 'mines'. On average some 3000 values were available in each of the 20 'mines' and these values provided the follow-up 'actual' mean grade and the logarithmic variance for each 'mine'. From each of these 'mines' 9 values were drawn on a stratified basis. These were accepted in each case as the equivalent of 9 single borehole values, i.e., without deflections. The process was repeated twice to yield a total of 60 borehole sets which were used for the analyses.

3.7 Distributional and Spatial Models used for the Borehole Valuations

In analysing the simulated mine values the 3-parameter model was accepted as a suitable model as was shown in section 3.3. The suitability of this model on a macro 'mine' scale is demonstrated in Figure 3.2 (Section 3.3) which shows that the distribution of the 20 actual mine grades can be accepted as normal. A test for the suitability of this model for the 20 mines showed:

(i) a low correlation between the 'actual' logmeans and logvariances after addition of the threshold parameter of 50 (r^2 = 16%)

(ii) a difference between the overall means for the 20 mines based on

 (a) the arithmetic means, and

 (b) the lognormal mean

 of 1.5 per cent, and a correlation level between the two sets of means of 99.8 per cent.

The presence of spatial structures on the 'mine' scale (i.e, 1 Km x 1 Km areas) for the grade levels and the variances have also been demonstrated in Section 3.5. The sets of the simulated borehole values were subjected to a series of estimation procedures as outlined below.

3.8 Estimation Procedures used

The <u>first</u> procedure is that of the orthodox arithmetic mean, i.e., where the 9 borehole grades are used at face value, without introducing any 'additional' information.

The <u>second</u> procedure introduces the knowledge of the 3-parameter lognormal distribution pattern through the use of Sichel's 't' estimator.

The substitution of the average mine variance for the variances as estimated from the borehole sets, yields the t" estimates as an alternative.

Thirdly, the concept of spatial structure is introduced on the elementary basis of regression. The additional information used is that of a parent distribution of mine grades with known mean and variance plus the knowledge of the error variances of the arithmetic mean, t or t" borehole grade estimates. With this additional information the required correlation models can be used (Krige[42]) to provide regressed estimates for the arithmetic mean, Sichel's 't' and t".

Fourthly, the spatial structures for the population of mine grades (and variances) are introduced using the semivariograms observed. Accepting the lognormal model, the borehole estimates and mine grades can be treated as spatial variables directly, or grade can be seen as a function of the mean of the transformed grades and the corresponding variances.

Kriging, ordinary and simple, can therefore be done on the grades directly or on the transformed grades (ξ) and variances (σ^2) separately to arrive at the corresponding grade estimates via the appropriate function. For a population this function is defined as follows (Krige[42]):

$$\text{Mean Grade} = \text{Exp}(\xi + \sigma^2/2) - \beta$$

When dealing with estimates, this relationship will be approached only as the error variances of the estimates become small. Therefore, the correct relationship for small 'n' values, similar to that for Sichel's 't' estimator, is required. As a first approximation, the concept of the 'equivalent' number (n) of boreholes can be used by determining in each case the estimated (regressed or kriged) error variances for ξ and σ^2 and comparing these with those corresponding to orthodox estimates normally used for Sichel's 't' estimator (see also Kleingeld[30]). However, for the range of estimated variances and the equivalent 'n' values applicable in this investigation, the differences between using the population relationship as distinct from the equivalent 'n' approach, are less than 1 per cent and the population relationship was therefore used. For purposes of estimating confidence limits the equivalent 'n' approach will, of course, be essential.

3.9 Basis for Comparison of different Estimation Procedures

The estimates were statistically analysed and correlated with the 'actual' mine values in order to provide comparisons of the relative improvements shown by the various procedures. Improvements were measured by the observed error variances of the estimates (inclusive of any small global biasses); also by the level of correlation between estimates and actuals and the extent, if any, of the remaining conditional biasses as reflected by the slope of the regression of actuals on the estimates.

Observed error variances give a measure of the deviation of estimates from 'actual' values. Should any estimation procedure give perfect results the observed error variance should ideally be zero. The observed error variances were measured by the variance of:

$$[Ln(\hat{Y}+\beta)-Ln(Y+\beta)]$$

about a mean of zero. Where $\hat{Y}$ = given estimated grade, Y = the follow-up ('actual') grade and β is the additive constant.

Conditional biasses in ore valuation deserve special attention since their presence will always lead to systematic over- and under-valuation in certain grade categories as demonstrated by Krige[15,17]. The slope of the regression of 'actual' on estimated grades provides a measure of conditional biasses. A slope of zero indicates excessive conditional bias whereas a slope close to one will ensure conditional unbiasedness.

Conditional non-bias relation can be defined statistically as:

$$E(Y|\hat{Y}) = \hat{Y} \qquad (1)$$

where $\hat{Y}$ = given estimated grade
Y = 'Actual' grade and
$E(Y|\hat{Y})$ = conditional expectation of Y given $\hat{Y}$

It should be noted that the conditional non-bias condition (equation (1)) is more severe than the global non-bias condition $E(Y-\hat{Y}) = 0$. In addition, any estimation procedure which is conditionally unbiassed will provide a definite advantage in selective mining (Krige[42], Journel and Huijbregts[25]); it will be shown that this also applies to global borehole valuations of new mines.

Conditional biasses were observed from the early years of gold mining in South Africa. The valuation procedures were then based on the arithmetic averages of all values on ore block peripheries. The reason for conditional biasses was first explained by Krige[15] in 1951 as an elementary statistical regression effect. This lead to the use of regression corrections based on weighted averages of the peripheral block grades and the global or local mean value for the orebody. Krige's regression analyses was the first elementary form of kriging which developed into the present sophisticated kriging procedures.

3.10 Empirical Observation of Conditional Biasses in Borehole Valuations thus validating the introduction of Spatial Structures on Variances for Ore Reserve Valuations

A detailed correlation analysis based on the errors of the borehole estimates (without 'outside' information) with the corresponding variance estimates were done.

Conditional biasses such as were observed by Krige[15,17] in 1951/52 were also found on a mine scale for the orthodox arithmetic mean, Sichel's 't', t" and for some kriged estimates (particularly for ordinary kriging). Figure 3.18 shows one of these analyses for the Sichel's 't' estimates. Similar patterns were found for the orthodox mean and t". Figure 3.18 demonstrates clearly that, for estimates based on a small set of values drawn from such skew distributions, even the more sophisticated 't' will result in serious conditional biasses. Where such a set gives a low grade/or variance estimate, the tendency will be to undervalue the mine grade and in the case of high estimates overvaluation will tend to occur. The observed severe conditional biasses of the grade estimates which resulted from strong positive correlation of the grade estimates with the corresponding variance estimates is a clear motivation for introducing 'outside' information on the variance, including the concept of spatial structure for the variance.

3.11 Results

Table 3.2 highlights the main problems underlying the orthodox arithmetic mean approach. It is evident that the variance of the 'actual' mine grades at 0.157 is lower than the error variance of the borehole means at 0.178; thus the regional mean grade, if accepted as an estimate of an individual mine's grade, is in this sense, a slightly better estimate than that provided by the borehole means. However, for the variance the corresponding two figures are 0.052 and 0.226, indicating that the regional mean of the 'actual' within mine variances, is a far superior estimate for a new mine than that shown by the 9 borehole values. Also, on log-transformation, the variance of the actual values reduces somewhat from 0.157 to 0.120 but the error variance of the borehole estimates is nearly halved from 0.178 to 0.095; this is clearly the reason why Sichel's 't' will be an improvement on the arithmetic mean, as will be seen later.

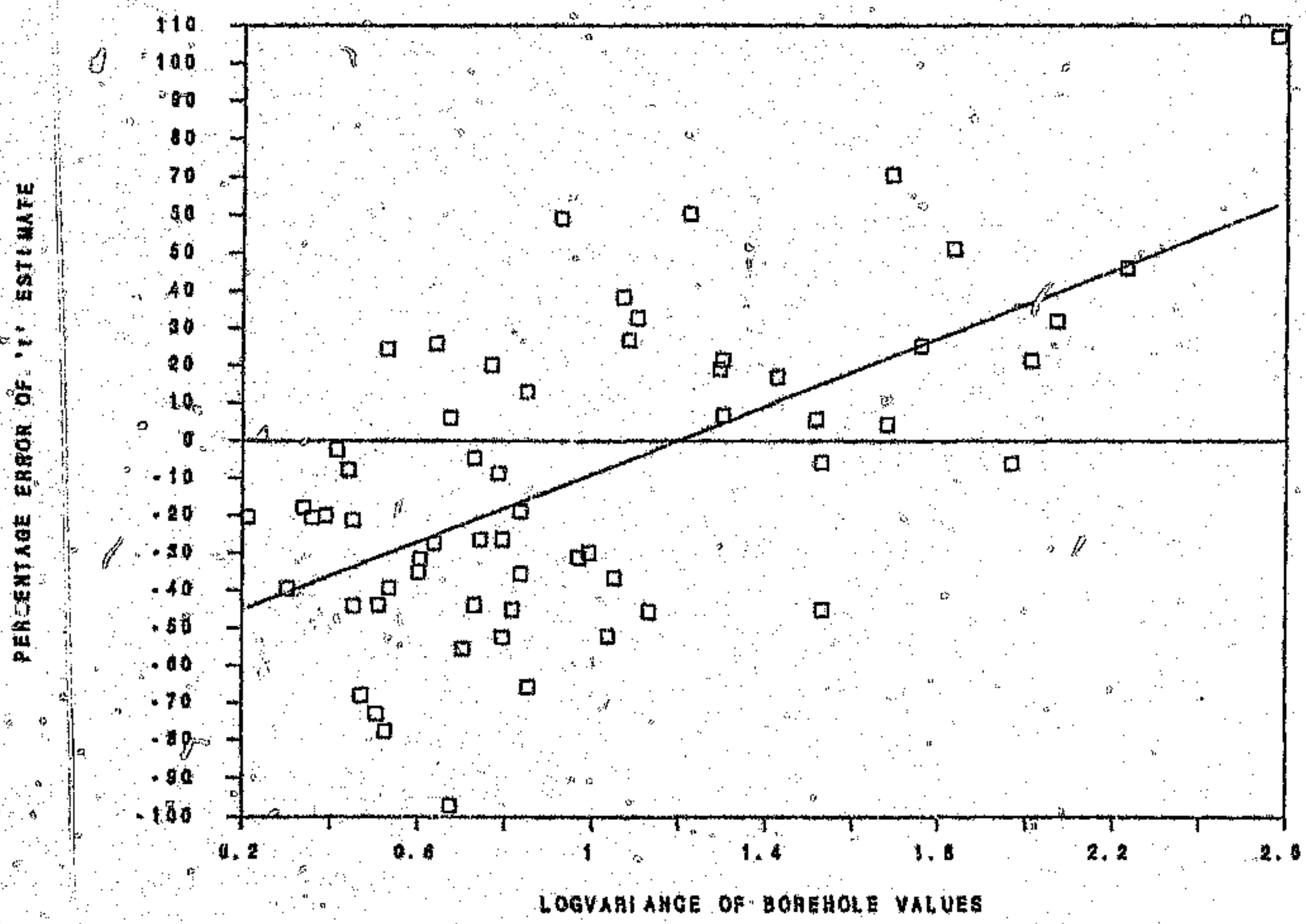


FIGURE 3.18 SHOWING CORRELATION OF ERRORS OF BOREHOLE ESTIMATES WITH VARIANCES OF BOREHOLE VALUES

TABLE 3.2 SHOWING SOME BASIC STATISTICS FOR TWENTY "MINES" ANALYSED

	ACTUAL MINE MEANS	BOREHOLE MEANS	
	LOG VARIANCE	LOG.VAR.	LOG ERROR VARIANCE
Grade - Arithmetic Mean	0.157	0.282	0.178
- Mean of log values	0.120	0.173	0.095
Variances of log values within mines	0.052	0.290	0.226

However, this improvement is severely hampered by the very high error variance of the variance (0.226) as estimated from the 9 boreholes and used in Sichel's 't' estimator together with the mean of the 9 log-grades.

These results already stress the importance of outside information in the form of regional averages for both grade and variance. This was confirmed by the series of estimates carried out on the 60 sets of borehole values, the results of which are tabulated in Table 3.3.

Compared to the arithmetic mean (1 in Table 3.3) Sichel's 't' (No 2) shows an improvement on all 3 measures of correlation with actual grades, conditional biasses (regression slopes) and error variances. The t" estimates (No 3) which are based on the regional average log-variance (and ignore the borehole variance estimate) shows a very substantial further gain in efficiency over the 't' estimator.

Following the introduction of the further information of the parameters for the distribution of the actual mine grades in the region via regression, or elementary simple kriging, further improvements are obtained as shown (No 4/6), mainly because this process, if efficiently applied, will eliminate the conditional biasses and thus reduce the error variances. Up to this stage, the error variance has been reduced progressively from 0.18 (arithmetic mean), to 0.16 ('t'), to 0.11 (t") and 0.08/0.10 (regression).

Further improvements are shown when the macro spatial structure for grades is introduced together with the known grades of existing adjacent mines. The configurations for adjacent mines are shown in Figure 3.19. If the regional mean grade is ignored as in ordinary kriging, and only one outside mine is used, the error variance remains at 0.10 (No 7) and substantial conditional biasses are still present; these disappear only when a substantial number of outside mines are introduced (not shown in Table 3.3), i.e when the regional mean is effectively used, or with simple kriging.

TABLE 3.3 SHOWING RESULTS OF SERIES OF ANALYSES FOR "MINES" ANALYSED

Codes for additional information used:

Three-parameter lognormal model

For borehole values....................1a

For mine grades..........................1b

Parameters for population of mines:

	20 "Mines"	Klerksdorp field
Mean	2a	2b
Mean logvariance within mines	3a	3b
Logvariance of mine mean grades	4a	4b
Spatial Structures:		
Mine mean grades - untransformed	5a	5b
Mine mean grades - transformed	6a	
Within Mine variances - transformed	7a	

(CONTINUED NEXT PAGE)

(TABLE 3.3 CONTINUATION)

TECHNIQUE	Addition. Inform. - Codes	Correlation Coefficient	Reg. Slope Actual/ Estimate	Observed Log error variance of estimate
1. Arithmetic Mean of Boreholes	-	0.56	0.37	0.178
2. Sichel's t	1a	0.65	0.49	0.162
3. t"	1a, 3a	0.69	0.65	0.110
REGRESSION:				
4. Of arithmetic mean	1a, 2a, 4a	0.63	1.00	0.095
5. Of Sichel's t	1a, 1b, 2a, 4a	0.65	1.00	0.091
6. Of t"	1a, 1b, 2a, 3a, 4a	0.69	1.00	0.083
MACRO LOGNORMAL KRIGED ESTIMATES - WITH MEAN GRADE OF ONE ADJACENT MINE:				
7. Ordinary - b/h arithmetic mean	1b, 4a, 5a	0.65	0.81	0.097
8. Simple - b/h arithmetic mean	1b, 2a, 4a, 5a	0.67	1.16	0.088
9. Simple - b/h 't'	1a, 1b, 2a, 4a, 5a	0.69	1.09	0.082
10. Simple - b/h t"	1a, 1b, 2a 3a, 4a, 5a	0.73	1.01	0.067
- WITH MEAN GRADES OF TWO ADJACENT MINES:				
11. Simple - b/h t"	as for 10	0.73	1.01	0.066
MACRO KRIGED ESTIMATES WITH MEAN GRADES OF 3 ADJACENT MINES AND b/h t"				
12. Simple lognormal	as for 10	0.77	1.03	0.061
13. Simple Normal	as for 10	0.77	1.05	0.060
14. Simple Normal(Field Parameters)	1a, 1b, 2b, 3b, 4b, 5b	0.78	1.11	0.059
ESTIMATES VIA MACRO KRIGED LOGGRADES & LOG VARIANCES:				
15. Simple - with 1 Mine	1a, 1b, 3a, 6a, 7a	0.78	0.99	0.070
16. Simple - with 3 Mines	1a, 1b, 3a, 6a, 7a	0.82	1.08	0.049

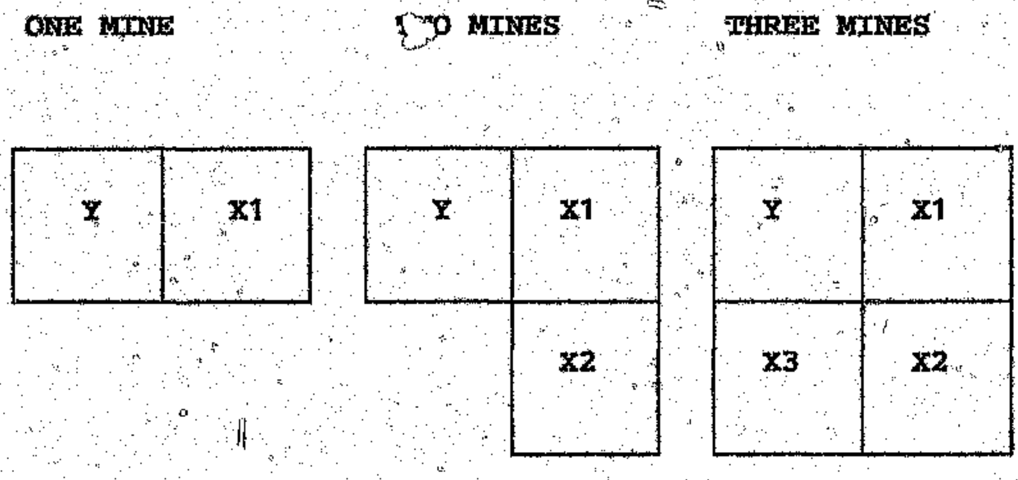
The remaining analyses were, therefore, confined to simple kriging.

Simple kriging with one mine and using sequentially the borehole mean (No 8), 't' (No 9) and t" (No 10) show a progressive increase in correlation, negligible remaining conditional biasses, and a progressive decrease in the error variance from 0.09 (borehole means), through 0.08 (borehole 't',s) to 0.07 (borehole t",s). Introducing two adjacent mines with the borehole t", shows very small further improvements(No 11), but when three mines are used with the borehole t" (No 12), the correlation improves to 0.77 and the error variance to 0.06.

In order to compare the macro kriging procedure on the lognormal basis, (i.e. on transformed values) with the normal approach (i.e. using untransformed values) kriging with the borehole t" and three adjacent mines (No 12 in Table 3.3) was repeated with the same weights on untransformed values (No. 13). Almost identical results were obtained with a correlation coefficient between the two sets of estimates of over 99 per cent. This was to be expected because the semivariogram of the transformed and untransformed grades when scaled in units of the population variance will be identical if the third parameter $\beta = 0$ and almost identical if β is small relative to the population mean.

The robustness of such a macro kriging estimator was also demonstrated by substituting for the parameters of the distribution of '20' mines parameters for the Vaal Reef in the Klerksdorp field obtained from various other analyses (Krige[45]) ; these parameters covered the regional grade distributions and spatial structures for 1 km x 1 km areas. On these regional parameters, the macro simple kriging with the borehole t" and three mines (No 14 in Table 3.3) again showed virtually identical results with the previous two analyses (Nos 12 & 13).

FIGURE 3.19 SHOWING CONFIGURATIONS OF ADJACENT MINES



Y = NEW MINE X1, X2, X3 = ADJACENT MINES

The results of the analyses done on simple macro kriging of the means of the transformed grades and their log-variances separately are shown in Table 3.3 under Nos 15 and 16 for the borehole values with one and three adjacent mines respectively. It is evident that with one adjacent mine there is no significant difference between this approach (No 15) and the t'' approach (No 10). However, with three mines the error variance further improved from 0.06 (No 10) to 0.05 (No 16). This seems logical because the macro kriged variance estimate used in No 16 should be superior to the regional average variance used for the t'' estimate on which No 10 is based.

Figure 3.20 demonstrates graphically the overall advantage gained by estimating the grade of a new mine by macro kriging of the transformed grades and their variances separately (using the borehole values, the values for three adjacent mines, the regional mean values, and the regional spatial structures) as compared with the orthodox borehole arithmetic mean. The two sets of estimates are shown in terms of units of the actual mean grades of the mines and thus provide a visual indication of the narrowing of the observed confidence limits. The central 80 per cent confidence limits are reduced from about -50/+60 per cent to ±30 per cent. In the practical case of a new mine to be opened up, such an improvement in the confidence of the grade estimate will be of substantial significance.

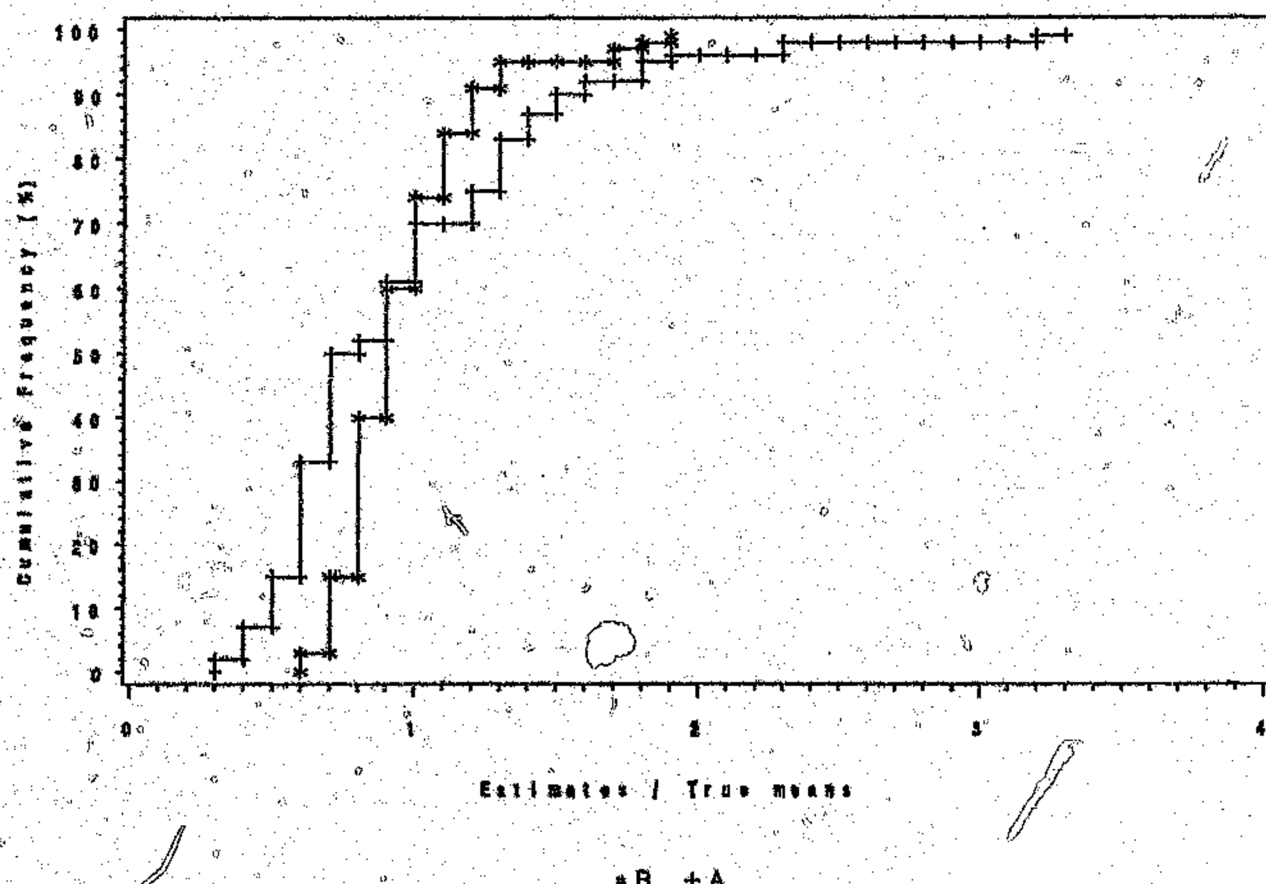


FIGURE 3.20 CUMULATIVE FREQUENCY DISTR. OF BOREHOLE ARITHMETIC MEANS(A) AND MACRO-KRIGED ESTIMATES WITH 3 MINES(B) – NOS 1 & 16 IN TABLE II

3.12 Conclusions

From these results the following conclusions can be drawn for borehole valuations of skew grade data with variabilities such as for the Hartebeestfontein gold mine.

3.12.1 Distribution Model

Wherever applicable the three parameter lognormal model should be used in dealing with individual borehole values. For this purpose the 't' estimator is preferred to the arithmetic mean but, given an average of actual variances within mines in the region the t" estimator can be better. Where the departures from this model are significant the more general sub- or hyper-lognormal model should be considered (Chapter 2). However, small sampling theory for these models is yet to be developed. When dealing with actual mean grades of large mining units, e.g. existing mines, there seems to be little or no advantage in using any of the lognormal models in preference to the Normal, except for the estimation of confidence limits where some skewness is still present.

3.12.2 Macro Kriging

Macro simple kriging shows definite advantages over ordinary kriging. The most efficient method appears to be simple macro kriging of the log-transformed borehole and mine grade and variances separately and to combine these to arrive at the grade estimate for the new mine.

CHAPTER 4

4 ROUTINE FACE (PANEL) VALUATIONS

4.1 Introduction

Face (panel) valuation essentially presents the same problems as the estimation of the grade and tonnage in mineable ore reserve blocks. Accurate face (panel) valuations are therefore essential for optimal selection of payable ore for stoping and assist in reducing wasteful mining of unpayable ore. The results from face (panel) valuations are used in production planning, grade control and for the calculation of various factors (e.g., 'block factors'). The need for employing proper models and valuation procedures to improve local reserve estimates are therefore essential.

This chapter addresses this problem of face (panel) and block valuations by employing the same model and valuation procedures as used for the borehole analyses in chapter 3. Conclusions are drawn on the relative importance of the various items of information accessed and the corresponding advantages gained for the various procedures used.

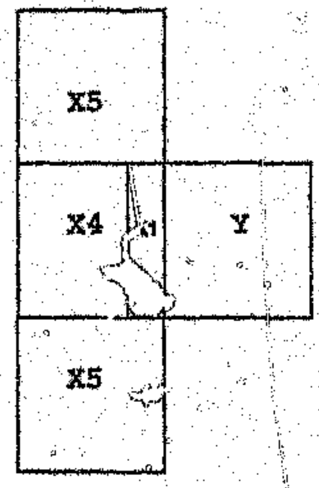
4.2 Database and Analyses

The same database used in chapter 3 consisting of 72767 individual sampling sections from Hartebeestfontein Gold Mine in the Klerksdorp Goldfield was used for this analyses. In this case the area was divided into various block sizes and patterns as shown in Figure 4.1. The aim was to estimate the grades of 50 x 50 m blocks. A total of 97 ore blocks were available for the analysis. In principle the estimation procedures were the same as employed for the borehole analyses in chapter 3 and are summarised below.

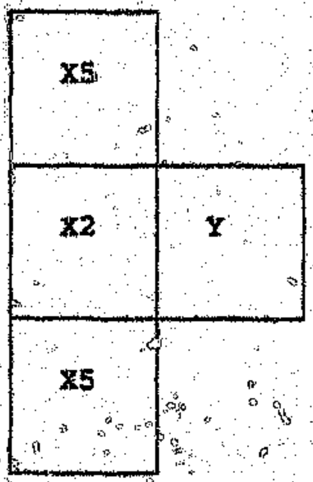
FIGURE 4.1 DATA CONFIGURATIONS FOR FACE (PANEL) VALUATIONS

Y = BLOCK TO BE ESTIMATED, 50 x 50 M
X1,X2,X3,X4,X5 = DATA BLOCKS
X2,X5 = 50 x 50 M
X1,X3 = 10 x 50 M
X4 = 40 x 50 M

DATA SET A



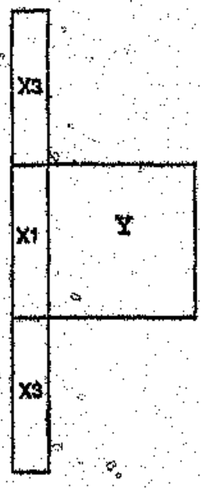
DATA SET B



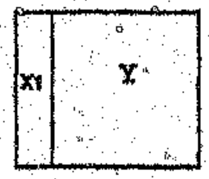
(CONTINUED NEXT PAGE)

FIGURE 4.1 (CONTINUATION)

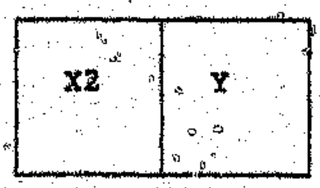
DATA SET C



DATA SET D



DATA SET E



4.3 Estimation Procedures Used

The first procedure is that of the orthodox arithmetic means of available periphery values. In th case, the arithmetic mean of the 'known' values within narrow strips or blocks (e.g. 10 x 50 m blocks) along the periphery of the ore blocks are used as estimates without introducing any 'additional' information. On average some 17 values were available in each of these 10 x 50 m blocks. These values are close to those normally available from a panel sampling.

The second procedure introduces the knowledge of the 3-parameter lognormal distribution through the use of Sichel's 't' and the t" estimator (Section 3.8, Chapter 3).

Thirdly, the concept of spatial structure is introduced on the elementary basis of regression. The additional information used is that of a parent distribution of data block values with known mean and variances. In addition the error variances of the arithmetic mean, Sichel's 't' or t" of the orthodox blocks are used for this procedure.

Fourthly, the spatial structures for the population of data block grades (and variances) are introduced using the semivariograms observed. The lognormal model is used and the orthodox block grades and the other data block grades are treated first as spatial variables directly. The grade is also seen as a function of the mean of the transformed grades and the corresponding variances. Kriging, ordinary and simple, can be done on the grades directly or on the transformed grades (ξ) and variances (σ^2) separately to arrive at the corresponding grade estimates via the appropriate function. For a population this function is defined as (Krige[42]):

$$\text{Mean Grade} = \text{Exp}(\xi + \sigma^2) - \beta \quad \text{--- (1)}$$

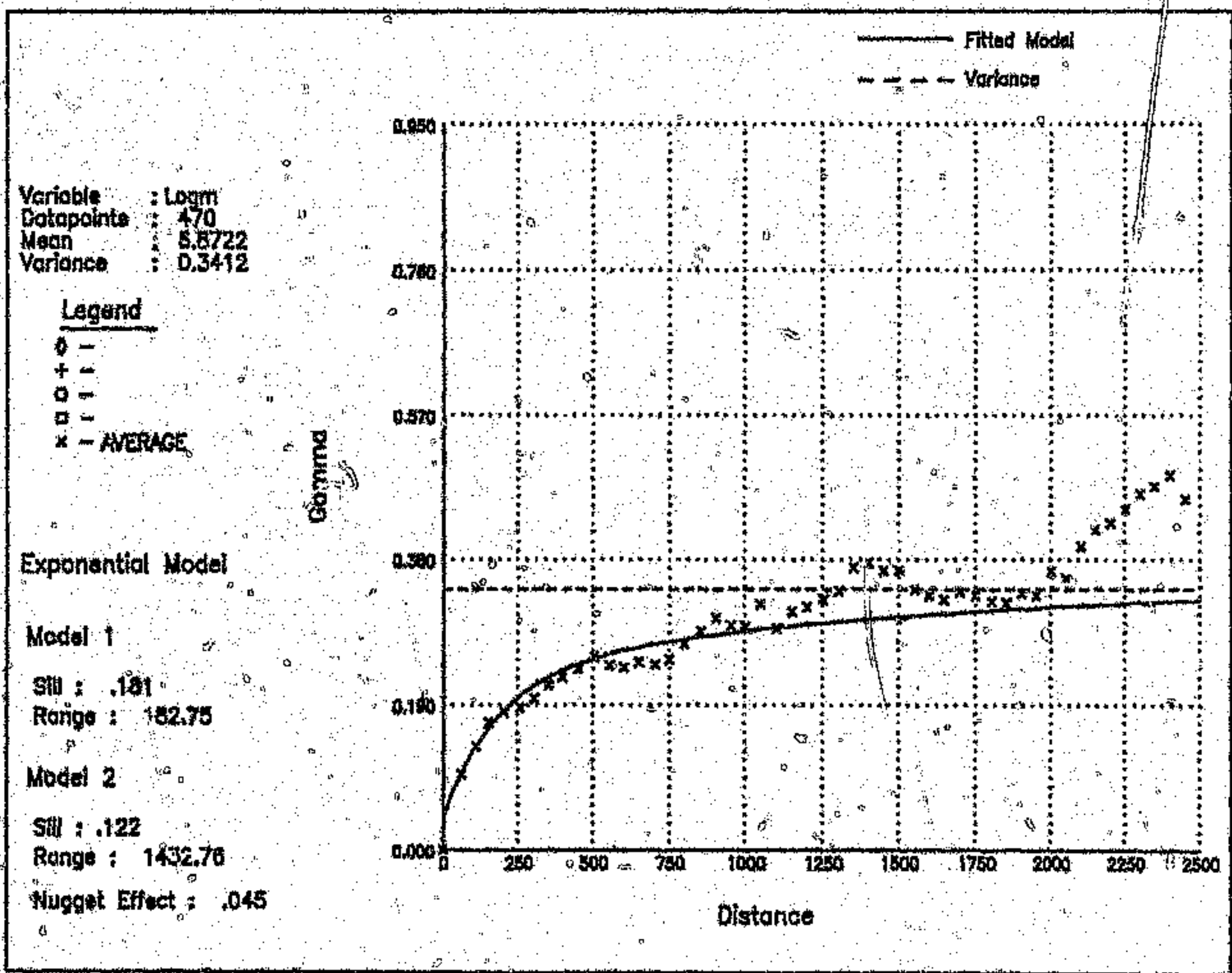
As observed in chapter 3, the range of estimated variances in this case indicated that the population relationship can be used.

Similar to the borehole analyses (Chapter 3), comparisons of various procedures were based on measured observed error variances (inclusive of any small global biasses); by the level of correlation between estimates and 'actuals' and the extent of (any remaining) conditional biasses. In all cases the arithmetic mean of all the samples available in each of the ore blocks were used as follow-up 'actual' mean grade. Based on dispersion variance calculations (Braun[28]) only blocks with a minimum of 50 samples were accepted as ore blocks. Only 97 of these ore blocks could provide the data patterns required for the analysis (See Figure 4.1). On average 70 values were available in each of these ore blocks. The follow-up values themselves will be to some extent in error, due to the limited number of samples available in the ore blocks. This is demonstrated in Figure 4.2 by the accompanying nugget effect on the semivariogram of the follow-up blocks. However, these follow-up values will provide relative error variances of the various estimates used, as the latter will in all cases include the error variance or nugget effect of the follow-up 'actual' values. For all practical comparative purposes therefore, the ore blocks follow-up estimates were accepted as 'actual' mean grades.

4.4 Distributional and Spatial Structure Models Used

Detailed analyses for the distributional and spatial models used for these analyses have been presented in chapter 3 (Sections 3.3 and 3.5). As demonstrated in chapter 3 the 3-parameter model is suitable for modelling the gold values. Figure 4.3 shows the cumulative probability plot for values within the 50 x 50 m blocks and confirms the applicability of the 3-parameter model. Statistics for the 50 x 50 m block values will be shown later (Section 4.5). Spatial structures for both the grade levels and for the variance were used. Figures 4.4 and 4.5 show the

FIGURE 4.2 SEMIVARIOGRAM FOR LOG ARITH. MEANS, 50X50 M BLOCKS



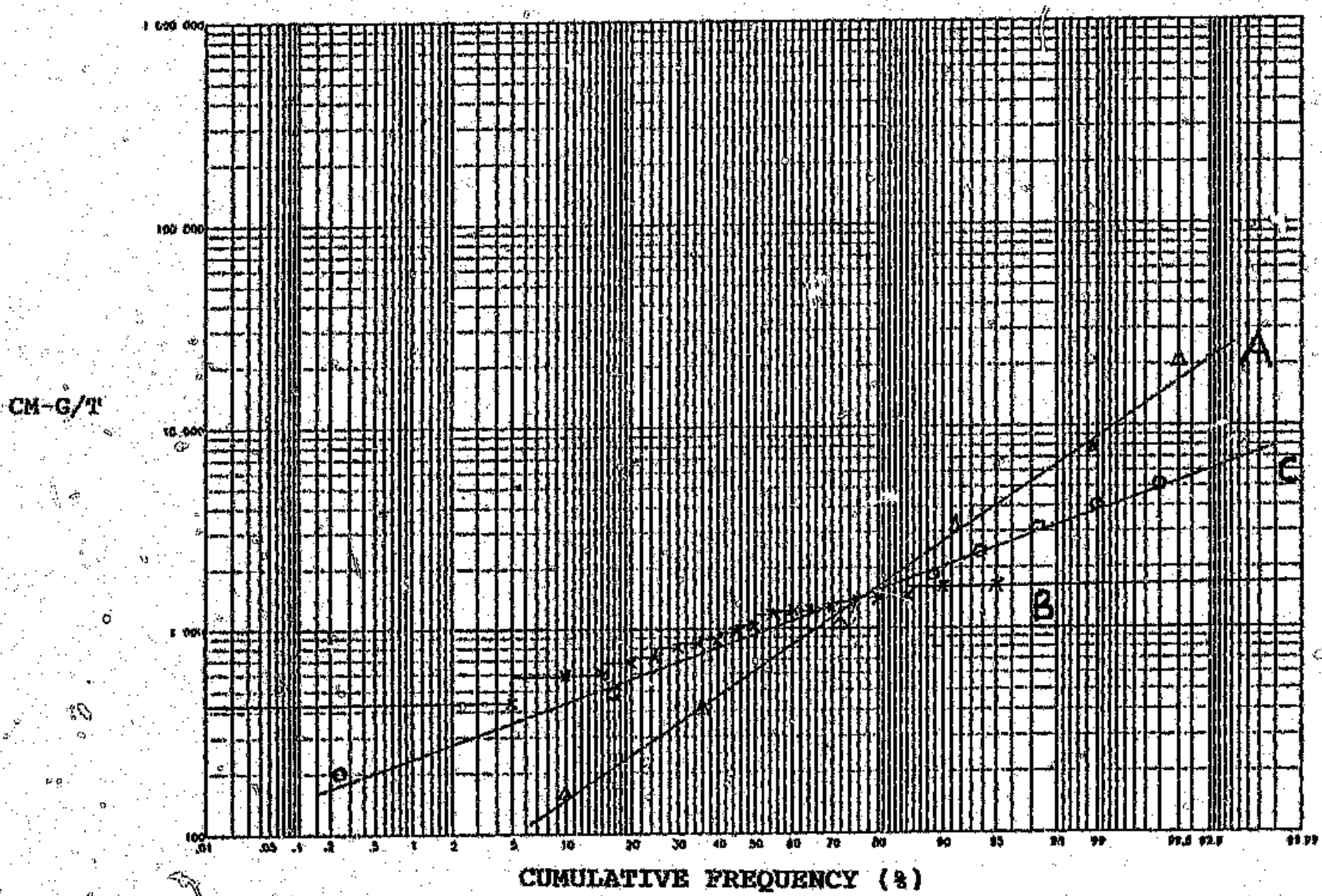


FIGURE 4.3 SHOWING CUMULATIVE FREQUENCIES OF THE CM-G/T VALUES (PLUS 50)

A: FOR 72767 INDIVIDUAL SAMPLING SECTIONS ... Δ

B: FOR 20 ACTUAL MINE MEAN GRADES ... x

C: FOR 50 X 50 M BLOCK MEAN GRADES ... o

FIGURE 4.4 SEMIVARIOGRAM FOR LOGMEAN VALUES, 50X50 M BLOCKS

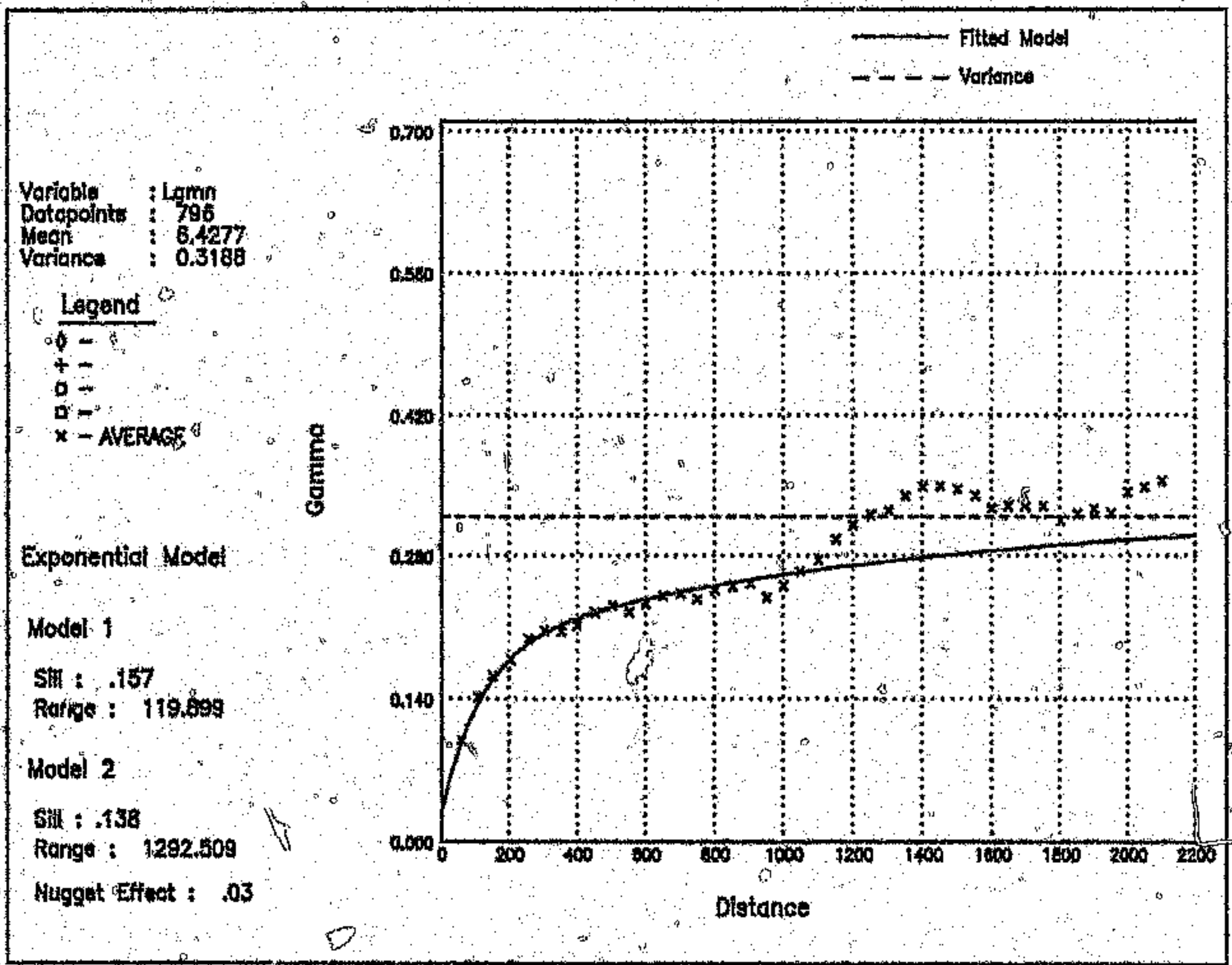
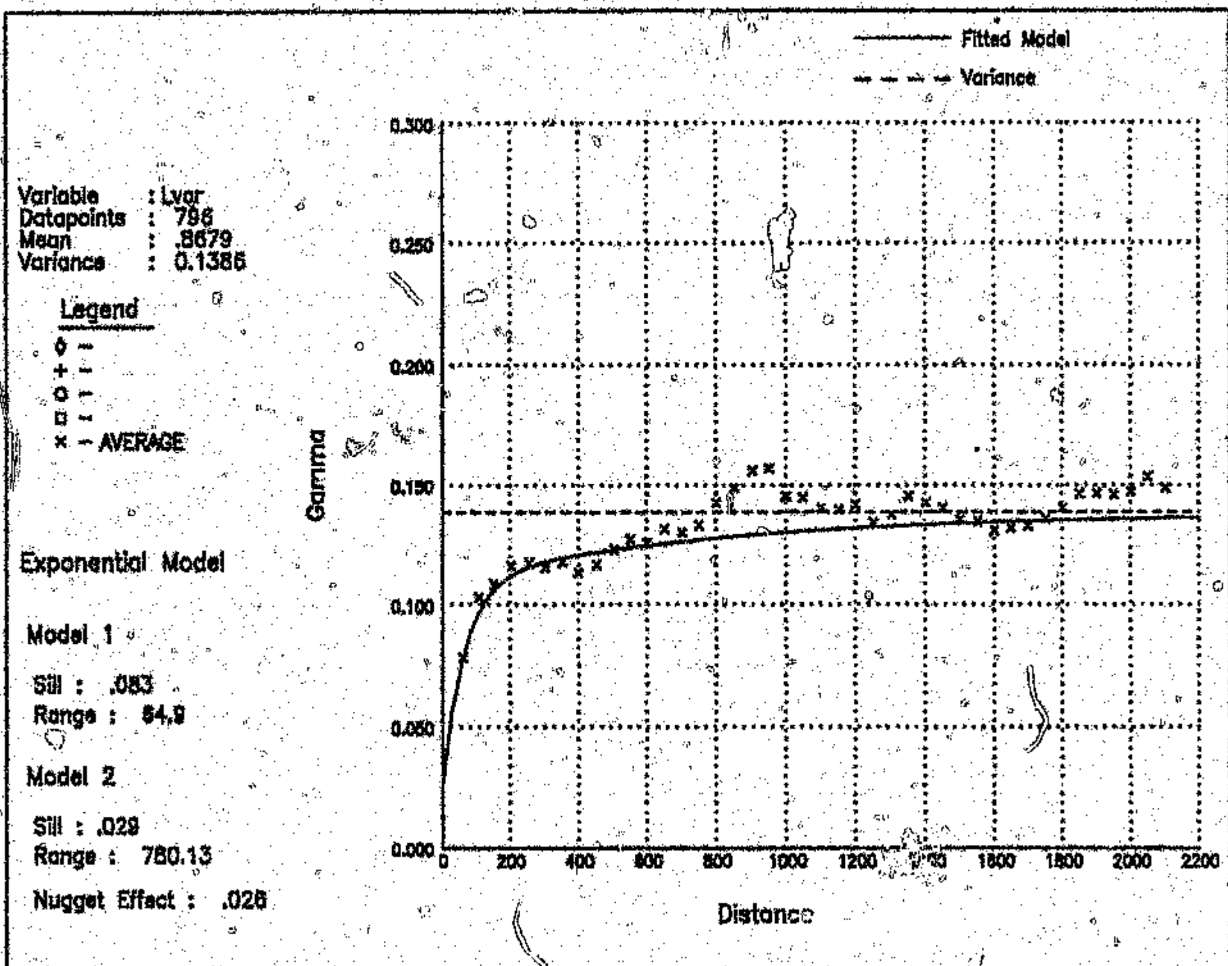


FIGURE 4.5 SEMIVARIOGRAM FOR LOGVARIANCE, 50X50 M BLOCKS



semivariograms for gr... and variances for the 50 x 50 m blocks. Semivariograms based on random selections of blocks in the whole mined out section indicated that, including the follow-up blocks in computing the semivariogram used for the analyses should not make any significant difference to the results. This further confirms the possibility of using semivariograms obtained from another section within a geological homogeneous zone.

4.5 Results

Tables 4.1 to 4.3 show the results obtained for the face (panel) analyses. Table 4.1 demonstrates an important aspect of spatial correlation on micro-panel scale. It throws more light on why the orthodox arithmetic mean was instinctively used as an estimate of individual block grades before the birth of geostatistics. Table 4.1 shows that (unlike the macro-'mine' statistics) the variance of the 'actual' follow-up grades (for 50 x 50 m blocks) at 0.384 is higher than the error variance of the orthodox block means at 0.252 (for 10 x 50 m orthodox blocks) and 0.184 (for 50 x 50 m orthodox block). This shows that the regional mean of the 'family' of 50 x 50 m blocks when accepted as estimate for individual block grades, is in this sense worse. Based purely on experience and common sense, the old miners accepted the orthodox arithmetic mean estimates rather than the regional mean. For the variances the corresponding figures of 0.139 and 0.149 show that the mean of the regional within block variances for 50 x 50 m follow-up blocks do not significantly differ from that of the orthodox blocks. On log-transformation, the variance of the 'actual' values improves from 0.384 to 0.284 but the error variance for the orthodox block estimates reduces only from 0.252 to 0.237. This combined with the slightly higher error variance of the variance at 0.149 as estimated from the 10 x 50 m blocks explains why the customary Sichel's t estimator will not be an improvement (on this micro-panel scale) on the orthodox arithmetic mean. This is shown later in Table 4.2

and 4.3 and emphasizes the need of accessing more information outside the orthodox block if any improvement can be ensured.

Table 4.2 explains the codes used for the results in Table 4.3. Table 4.3 (Nos 1/6) show that compared to the arithmetic mean Sichel's t shows no improvement on all the 3 measures of correlation with the follow-up grades, conditional bias (regression slope) and error variance. The t" (based on regional average variance) also shows no improvement on the orthodox arithmetic mean even when 50 x 50 m block is used as orthodox blocks. This is because in this case orthodox arithmetic mean estimates based on both 10 x 50 m or 50 x 50 m orthodox blocks give better estimates than their corresponding t" estimates (No 1 versus No 4 and No 2 versus No 6 in Table 4.3). These findings show that neither the customary Sichel's 't' nor the t" estimator can provide useful estimates for routine face (panel) valuations in preference to the arithmetic mean. Krige[46] obtained similar results on the B reef at the Loraine Gold Mine. He attributed these inefficiencies of t and t" estimators at the micro-panel scale to a probable departure from strict lognormality in small reef areas.

As a further introduction of information, regression of the block values, which should be seen as the first elementary simple kriging procedure was introduced. Further improvements are obtained through significant reduction of conditional biasses and error variances (Nos 7/11 in Table 4.3). The conditional bias reduction in this sense however refers to the perfect situation, since in all practical situations the actual follow-up values will not be available. Up to this stage therefore, the error variances have been reduced from 0.38 (t") to 0.173/0.205 (regression).

In order to access more information outside the orthodox blocks, the spatial structure for the grade is introduced. If the regional mean is ignored as in ordinary kriging for Data Sets D and E (Figure 4.6), these should be seen as the orthodox arithmetic mean procedure (Nos 1 and 2), since all the weights are given to the orthodox block, and the original substantial conditional biasses and error variances are then maintained (Nos 1 and 2). It is only when the regional mean is effectively used that these disappear for these data configurations (Nos 12 and 13, Data Sets D and E).

Further improvement is obtained if in addition to the spatial structure for grades, adjacent block grades are also introduced. If the ordinary kriging procedure is used and two other 10 x 50 m data blocks are introduced (Data Set C), the correlation and the error variances improve, however substantial conditional biasses are still present (Nos 14 and 16).

Simple kriging with two other 10 x 50 m data blocks (Data Set C), using sequentially 't' estimator (No 17) and orthodox block arithmetic mean (No 15), show progressive increase in the correlation, negligible conditional biasses, as well as a progressive reduction in the error variance. Compared to regression therefore, this procedure provides definite significant improvements in correlation level from 0.63 (regression) to 0.73 (simple kriging) and a reduction of the error variance from 0.173 (regression) to 0.132 (simple kriging). This significant improvement via the simple kriging procedure (with Data Set C) has a very important bearing on block valuations after putting in raises for mine development purposes. These results show that, in such situations, if the spatial structure for the grades together with the regional mean grade, can be used with information obtained from the raises, significant improved estimates can be ensured.

Introduction of three 50 x 50 m data blocks (Data Set B), shows very similar results (compared to Data Set C) inspite of the large information accessed. Data Set A however demonstrates that further improvement can be obtained if one of the three 50 x 50 m blocks (Data Set B) is split as shown in Data Set A (See Figure 4.6). Thus when Data Set A is used (with simple kriging) together with orthodox arithmetic mean (No 32) the correlation improves to 0.75 and the error variance to 0.127. Results based on 50 x 50 m blocks (Data Sets A and B) show that ordinary kriging and simple kriging procedures show similar results only when enough information is accessed.

Data Set A further shows an example of the exercises done as a check for calculating covariances via discretization (numerical integration) and regularization for the different support sizes. Results for Nos 30 and 32 are based on discretized covariances, whereas Nos 31 and 33 are based on regularized covariances. Both procedures use the orthodox arithmetic mean for kriging. The respective ordinary kriging (Nos 30 and 31) and simple kriging (Nos 32 and 33) results from the two methods are practically the same. These results show that covariances calculated via discretization or regularization will give practically the same results.

In order to compare kriging procedures on the lognormal basis (i.e, on transformed values) with the normal approach (i.e, using untransformed values) kriging with the orthodox arithmetic mean (using Data Set A) was repeated with the same kriging weights on untransformed values. The results (Nos 34 and 35) are almost identical with that on transformed basis (Nos 30 and 32). This was not unexpected since the semivariogram of the transformed and untransformed grades when scaled in units of the population variance will be identical if the threshold parameter $\beta = 0$ and very similar if β is relatively small compared to the population mean. Similar results were also obtained using data set C (Nos 20 and 21).

The results of the analyses based on various kriging procedures which introduce 'variance' as a spatial variable are shown in Table 4.3 (Nos 36/45). In this case the means of the transformed grades and their variances are used in the population function as shown in equation (1). Various kriging analyses using Data sets A/E were done. Very similar results as observed for the direct approach (Nos 12/35) were observed. Table 4.3 (Nos 36/45) show that applying this procedure leads to an increase in correlation from 0.63 (orthodox arithmetic mean, No 1) to 0.75 (Nos 42/45), reduction of error variance from 0.30 (t" No 4) to 0.13 (No 44). Similarly, negligible remaining conditional biasses are observed only when the regional transformed means of the log values and their variances are effectively used (i.e with simple kriging). In addition, data Set A shows how practically the same result will be obtained (under this procedure) whether regularized or discretized covariances are used. These results show that the covariances via descretization (Nos 42 and 44) compare very well with that of regularization (Nos 43 and 45) for the respective ordinary and simple kriging procedures.

4.6 Conclusions

4.6.1 Distributional models and Face (Panel) valuations

For the purpose of face (panel) valuations the arithmetic mean is preferred to the Sichel 't' or t" estimator. Thus the customary Sichel 't' or t" estimator will not be useful for routine face (panel) valuations. When using the regularized mean grades as in this study for face (panel) valuation, both the lognormal and Normal models seem to give practically similar results.

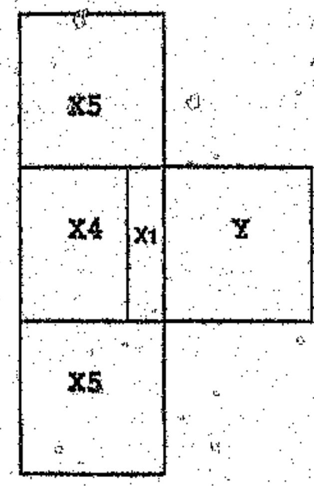
4.6.2 Kriging at Micro-Panel Scale

Simple kriging shows definite advantage over ordinary kriging especially if limited data are available. However, if more information is accessed, very similar results are obtained for both ordinary and simple kriging. Block estimates obtained via the grades directly or using the transformed grades (ξ) and variances (σ^2) separately seem to give the same results.

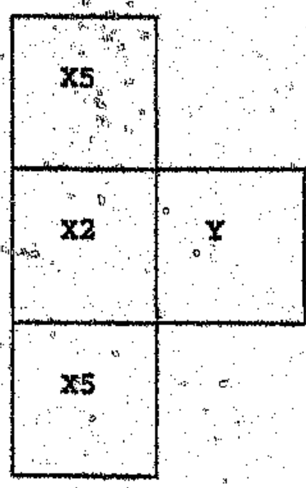
FIGURE 4.6 DATA CONFIGURATIONS FOR FACE (PANEL) VALUATIONS

Y = BLOCK TO BE ESTIMATED, 50 x 50 M
X1,X2,X3,X4,X5 = DATA BLOCKS
X2,X5 = 50 x 50 M
X1,X3 = 10 x 50 M
X4 = 40 x 50 M

DATA SET A



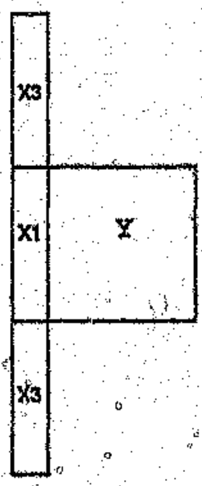
DATA SET B



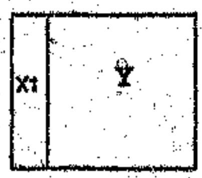
(CONTINUED NEXT PAGE)

FIGURE 4.6 (CONTINUATION)

DATA SET C



DATA SET D



DATA SET E

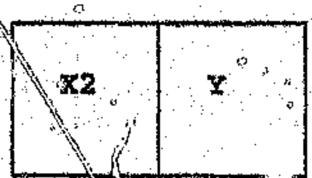


Table 4.1 SHOWING STATISTICS OF ORTHODOX BLOCKS AND 50m x 50m FOLLOW-UP BLOCKS ANALYSED

	50m x 50m Blocks Follow-up	10m x 50m Blocks Orthodox	
	LOG VARIANCE WITHIN MINE SECTION	LOG VAR. WITHIN MINE SECTION	* LOG ERROR VARIANCE
Grade - Arithmetic Mean	0.384	0.459	0.252
- Mean of log values	0.284	0.411	0.237
Variances of log values within the blocks in the mine section	0.139	0.201	0.149
			50m x 50m Blocks as Orthodox
Grade - Arithmetic Mean	0.384	-	0.184
- Mean of log values	0.284	-	0.159
Variance of log values within the blocks in the mine section	0.139	-	0.130

* Error variance using the mean of the available block (10x50m or 50x50m) as an estimate of the adjacent 50 x 50m block.

Table 4.2 SHOWING CODES FOR ADDITIONAL INFORMATION USED IN TABLE 4.3

Codes for additional information used:

Three parameter lognormal model (See Figure 4.1)

X1	For 10m x 50m (orthodox) data block values..........	1a
X2	For 50m x 50m data (used as orthodox) block grades..	1b
X3	For other ?m x 50m data block grades..............	1c
X4	For 40m x 50m data block grades	1d
X5	For other 50m x 50m data blocks	1e

Parameters for the whole mine section

	50 x 50m blocks	10 x 50m blocks
Mean	2a	2b
Mean logvariance within block		3b
Logvariance of block mean grades	4a	4b
Mean log grades for blocks within mine section	5a	5b

Spatial structures (covariances obtained via discretization or regularization)

Mean grades - untransformed (direct)	6a
Mean grades - transformed	7a
Within block variances - transformed	8a

Table 4.3 SHOWING RESULTS OF SERIES OF ANALYSES FOR FACE (PANEL) VALUATIONS

TECHNIQUE	Addition. Inform. -Codes	Correlation Coefficient	Reg.Slope Actual/ Estimate	Observed Log error variance of estimate
DATA SETS D AND E				
1. Orthodox arithmetic mean (10x50m)	-	0.63	0.55	0.252
2. Orthodox arithmetic mean (50x50m)	-	0.64	0.74	0.184
3. Sichel's t	1a	0.62	0.53	0.269
4. t"	1a,3b	0.55	0.48	0.300
5. t"	1a,3a	0.55	0.48	0.299
6. t"	1b,3a	0.61	0.75	0.192
REGRESSION DATA SETS D AND E				
7. Of Orthodox arithmetic mean	1a,2a,4a	0.63	1.00	0.177
8. Of Orthodox arithmetic mean	1b,2a,4a	0.64	1.00	0.173
9. Of Sichel's t	1a,1b,2a, 4a	0.62	1.00	0.178
10. Of t"	1a,1b,2b, 3b,4a	0.55	1.00	0.205
11. Of t"	1b,2a,3a, 4a	0.61	1.00	0.185
LOGNORMAL KRIGED ESTIMATE WITH DATA SET D				
12. Simple - orthodox arithmetic mean	2a,4a,6a	0.63	0.96	0.172
LOGNORMAL KRIGED ESTIMATE WITH DATA SET E				
13. Simple - orthodox arithmetic mean	2a,4a,6a	0.64	1.17	0.174

(Continued next page)

Table 4.3 (Continuation)

TECHNIQUE	Addition Inform. - Codes	Correlation Coefficient	Reg. Slope Actual/ Estimate	Observed Log error variance of estimate
LOGNORMAL KRIGED ESTIMATE WITH DATA SET C				
14. Ordinary - orthodox arithmetic mean	1c,4a,6a	0.73	0.81	0.143
15. Simple - orthodox arithmetic mean	1c,2b,4a, 6a	0.73	1.05	0.132
16. Ordinary - orthodox 't'	1a,1c,4a, 6a	0.73	0.78	0.149
17. Simple - orthodox 't'	1a,1c,2b, 4a,6a	0.72	1.03	0.134
18. Ordinary - orthodox t"	1a,1c,3b, 4a,6a	-	-	-
19. Simple - orthodox t"	1a,1c,2b, 3b,4a,6a	-	-	-
20. Ordinary - Normal (arithmetic mean)	as for 16	0.73	0.80	0.143
21. Simple - Normal (arithmetic mean)	as for 17	0.72	1.08	0.137
LOGNORMAL KRIGED ESTIMATE WITH DATA SET B				
22. Ordinary - orthodox arithmetic mean	1b,4a,6a	0.71	0.93	0.144
23. Simple - orthodox arithmetic mean	1b,2a,4a, 6a	0.71	1.17	0.144
24. Ordinary - t"	1b,3a,4a, 6a	0.71	0.96	0.145
25. Simple - t"	1b,2a,3a, 4a,6a	0.71	1.25	0.148

(Continued next page)

Table 4.3 (Continuation)

TECHNIQUE	Addition. Inform. - Codes	Correlation Coefficient	Reg. Slope Actual/ Estimate	Observed Log error variance of estimate
LOGNORMAL KRIGED ESTIMATED WITH DATA SET A				
26. Ordinary - orthodox t"	1a, 1d, 1e, 3b, 4a, 6a	0.72	0.95	0.142
27. Simple - orthodox t"	1a, 1d, 1e, 2a, 3b, 4a, 6a	0.72	1.19	0.142
28. Ordinary - orthodox 't'	1a, 1d, 1e, 4a, 6a	0.74	0.93	0.129
29. Simple - orthodox 't'	1a, 1d, 1e, 2a, 4a, 6a	0.74	1.12	0.128
30. Ordinary - orthodox arithmetic mean	1d, 1e, 4a 6a	0.75	0.94	0.128
31. Ordinary - orthodox arithmetic mean	as for 30	0.75	0.95	0.124
32. Simple - orthodox arithmetic mean	1b, 1d, 2a, 4a, 6a	0.75	1.13	0.127
33. Simple - orthodox arithmetic mean	as for 32	0.75	1.04	0.123
34. Ordinary - Normal (arithmetic mean)	as for 30	0.74	0.93	0.129
35. Simple - Normal (arithmetic mean)	as for 31	0.73	1.15	0.133

(Continued next page)

Table 4.3 (Continuation)

TECHNIQUE	Addition. Inform. -Codes	Correlation Coefficient	Reg.Slope Actual/ Estimate	Observed Log error variance of estimate
<u>KRIGED ESTIMATES VIA LOG-GRADES AND LOG-VARIANCES</u>				
<u>KRIGED ESTIMATE WITH DATA SET D</u>				
36. Simple	3a,5a,7a 8a	0.61	0.93	0.188
<u>KRIGED ESTIMATE WITH DATA SET E</u>				
37. Simple	3a,5a,7a, 8a	0.66	1.15	0.168
<u>KRIGED ESTIMATES WITH DATA SET C</u>				
38. Ordinary	1c,7a,8a	0.73	0.76	0.151
39. Simple	1c,3b,5b, 7a,8a	0.72	1.01	0.143
<u>KRIGED ESTIMATES WITH DATA SET B</u>				
40. Ordinary	1b,7a,8a	0.72	0.89	0.140
41. Simple	1b,3a,5a, 7a,8a	0.72	1.07	0.141
<u>KRIGED ESTIMATES WITH DATA SET A</u>				
42. Ordinary	1a,1d,1e, 7a,8a	0.75	0.90	0.129
43. Ordinary	as for 42	0.75	0.91	0.127
44. Simple	as for 43+ 3a, 5a	0.75	1.07	0.130
45. Simple	as for 44	0.75	1.03	0.127

CHAPTER 5

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary and Conclusions for Geological and Distributional Analyses from Chapter 2

The study in Chapter 2 showed that, though, the 3-parameter model is a practically reasonable model, especially for the estimation of the population mean of the gold distributions, the usual assumption that gold grade distributions follow a 3-parameter law is not always true. As a result, the plot of the empirical cumulative frequency distribution is very necessary to test departures from lognormality.

The study also showed that the new models also provide good estimates of the population means of the gold distributions. Another outstanding feature which makes the new distributions more attractive is the fact that no inherent theoretical problems would be anticipated. The new distributions also seem to give better estimates when larger mining areas are under consideration.

This study further shows that in more than 95 per cent of the cases the compound lognormal model could be used for modelling the gold grade distributions. This indicates that the compound lognormal model is more applicable for modelling the gold grade distributions in the Witwatersrand placers than the LN-GIG model.

It is however worth noting that, change of support and small sampling theory have to be resolved for the new models. One approach to the change of support problem would be to empirically verify a working hypothesis of permanence of the new distributions as it pertains to the 3-

parameter model. Since the 3-parameter model is a limiting case of the new models it would seem very likely that the permanence hypothesis is an attribute shared by these other family of models. The difference between these family of models at large support sizes (as against the chip sample values on which these results are based) is another interesting area of research. It will also be useful to draw small sample sets from the 72767 Vaal Reef data for small sample analysis using the new models. Variability analysis for the new model parameters can also be of practical use both for borehole valuation as well as local ore reserve estimation analysis. The use of maximum likelihood or frequency moments approach for parameter estimates can also be researched.

5.2 Summary and Conclusions for Global Borehole Analyses from Chapter 3

From the results in chapter 3 the following conclusions can be drawn for borehole valuations of skew grade data with variabilities such as for the Hartebeestfontein gold mine.

5.2.1 Deriving Additional Information

Introducing spatial structures not only on grade levels but also on grade variabilities (variances) will enable useful additional information to be accessed to improve global estimation.

Chip samples can be used as equivalent of borehole values for global borehole analyses.

5.2.2 Conditional Biasses

Conditional biasses as observed by Krige[15,17] on routine block valuations have also been found on a mine scale for the presently used valuation procedures (including the Sichel 't'). Macro kriging procedures should therefore be used to ensure minimum error variances as well as conditional unbiased global estimates.

5.2.3 Distribution Model for Global Valuations

Wherever applicable the three parameter lognormal model should be used in dealing with individual borehole values. For this purpose the 't' estimator is preferred to the arithmetic mean but, given an average of actual variances within mines in the region the t" estimator can be better. Where the departures from this model are significant the more general sub- or hyper-lognormal (LN-GIG or CLD) model should be considered (Chapter 2). However, small sampling theory for these models is yet to be developed. When dealing with actual mean grades of large mining units, e.g. existing mines, there seems to be little or no advantage in using any of the lognormal models in preference to the Normal, except for the estimation of confidence limits where some skewness is still present.

5.2.4 Macro Kriging

Macro simple kriging shows definite advantages over ordinary kriging. The most efficient method appears to be simple macro kriging of the log-transformed borehole and mine grade and variances separately and to combine these to arrive at the grade estimate for the new mine.

5.3 Summary and Conclusions for Routine Face (Panel) Valuations from Chapter 4

5.3.1 Distributional Models in Face (Panel) Valuations

Unlike global analyses, the arithmetic mean is preferred to the Sichel 't' or t" estimator for face (panel) valuations. In addition, when using the regularized mean grades for face (panel) valuation, both the lognormal and Normal models seem to give practically similar results.

5.3.2 Kriging at Micro-Panel Scale

Simple kriging shows definite advantage over ordinary kriging especially if limited data are available. However, if more information is accessed, very similar results are obtained for both ordinary and simple kriging. Block estimates obtained via the grades directly or using the transformed grades (ξ) and variances (σ^2) separately seem to give the same results.

APPENDIX A

RESULTS OF THE CASE STUDY OF THE THREE DISTRIBUTIONAL MODELS: 3-PARAMETER, COMPOUND LOGNORMAL AND LOG-GENERALISED INVERSE GAUSSIAN DISTRIBUTIONS

ANALYSIS OF EMPIRICAL AND THEORETICAL DISTRIBUTIONS

MINE WITH VARIOUS AREAS	REEF	REEF AREA MILLIONS SQ.M	NO. OF SAMPLES	ACTUAL VALUES FROM EMPIRICAL DISTRIBUTION					VALUES ASSUMING COMPOUND LOGNORMAL OR LN-GIG* DISTRIBUTION				VALUES ASSUMING 3-PARAMETER LOGNORMAL DISTRIBUTION									MOMENT ESTIMATE FOR THE FOUR PARAMETERS OF THE NEW DISTRIBUTIONS			
				ARITHMETIC MEAN	LOG. VARIANCE	LOG. MEAN	LOG. SKEWNESS	LOG. KURTOSIS	ESTIMATED MEAN	X² PROBABILITY FOR GOODNESS OF FIT []	K-S TEST VALUE	% BIAS	ADDITIVE CONSTANT	SKEWNESS OF LOG(VALUES + CONSTANT)	KURTOSIS OF LOG(VALUES + CONSTANT)	VARIANCE OF LOG(VALUES + CONSTANT)	ESTIMATED MEAN	X² PROBABILITY FOR GOODNESS OF FIT []	K-S TEST VALUE	% BIAS	% OF AREA IN NEGATIVE GOLD REGION	SKEWNESS PARAMETER	KURTOSIS PARAMETER	SPREAD OR SCALE PARAMETER	LOCATION PARAMETER
ORANGE FREE STATE																									
AREA A ...	Basal	0.72	3962	812.0	1.428	6.154	-1.210	5.418	775.1	<.05%[204]	2.579	-4.60	125	-0.001	3.421	0.601	782.7	<.05%[13]	0.671	-3.60	1.50	0.358852	1.704851	3.189133	7.445670
AREA A1 ...	Basal	0.24	2525	940.2	1.326	6.334	-1.393	6.581	884.5	<.05%[90]	1.997	-4.86	131	0.006	3.649	0.580	893.9	<.05%[14]	0.499	-4.92	1.02	0.239773	0.860618	2.098297	7.168990
AREA A2 ...	Basal	0.48	1433	586.1	1.459	5.838	-1.032	4.384	577.8*	<.05%[18]*	0.920*	-1.42*	115	-.044	3.050	0.579	587.8	0.3%[9]	0.581	0.29	2.28	0.750000*	0.032150*	0.797948*	3.572008*
AREA AA+ ..	Basal	0.72	3962	812.0	1.433	6.154	-1.210	6.000	796.6	<.05%[39]	0.898	-1.90	N/A									0.180932	1.018951	1.931575	6.970482
AREA A11+ .	Basal	0.24	2525	940.2	1.326	6.334	-1.393	7.360	926.1	<.05%[43]	0.703	-1.50	N/A									0.148255	0.497549	1.543289	6.918064
AREA A22+ .	Basal	0.48	1463	586.1	1.459	5.838	-1.032	5.365	600.5	1.4%[13]	0.700	2.46	N/A									0.153785	1.362535	2.027919	6.689711
AREA B ...	Basal	3.92	3295	666.4	1.079	6.057	-0.659	3.892	668.9	0.6%[10]	0.309	0.38	90	-0.043	2.892	0.597	671.5	9%[5]	0.073	0.76	0.89	0.180394	4.604444	4.077832	7.353945
AREA AB ...	Basal	2.64	7257	745.9	1.275	6.110	-1.003	4.937	730.3	<.05%[62]	1.309	-2.10	105	-0.019	3.180	0.617	735.0	<.05%[29]	0.803	-1.46	1.22	0.211816	1.947750	2.733739	7.158718
AREA C ...	Basal	1.44	1418	628.8	1.481	5.735	-0.240	3.343	620.8	40%[1.8]	0.257	-1.27	9	-0.063	3.034	1.355	625.0	26%[4]	0.404	-0.60	0.02	0.034022	9.370583	3.842793	6.715765
AREA D ...	Basal	1.44	1015	1239.9	1.11[illegible]	6.665	-0.757	4.428	1245.6	63%[1.7]	0.125	0.47	130	-0.026	3.002	0.653	1241.1	2%[6]	0.501	0.10	0.62	0.120372	2.406130	2.745898	7.499414
AREA E ...	Basal	3.52	2448	898.1	1.352	6.239	-0.757	4.379	890.3	<.05%[21]	0.660	-0.87	70	0.040	3.225	0.804	882.4	<.05%[15]	0.655	-1.75	0.69	0.128085	2.550879	2.567789	7.207190
AREA F ...	Basal	0.96	2209	1097.5	1.416	6.485	-3.970	4.695	1090.2	<.05%[29]	1.074	-0.67	110	-0.044	2.812	0.767	1101.0	30%[2]	0.378	0.32	1.07	0.258232	2.424435	3.073016	7.767990
AREA G ...	Basal	0.96	2804	1149.3	1.146	6.589	-0.905	4.920	1131.9	<.05%[28]	1.026	0.23	120	-0.030	3.049	0.664	1152.3	6%[5]	0.370	0.26	0.14	0.135728	1.717597	2.423696	7.366757
KLERKSDORP																									
AREA H	V.R.	24.70	72767	1081.0	1.598	6.244	-0.266	3.456	1092.3	<.05%[51]	0.920	1.05	50	0.262	2.834	1.155	1034.7	<.05%[1047]	4.161	-4.30	0.99				
													25	-0.087	2.886	1.314	1066.3	<.05%[191]	1.135	-1.40	0.33	0.020255	6.572023	3.135737	7.064020
STILFONTEIN																									
AREA J	VCR	6.25	12241	447.9	1.268	5.618	-0.784	3.487	447.1*	<.05%[121]*	0.996*	-0.18*	90	-0.068	2.581	0.554	456.4	<.05%[116]	1.453	1.90	2.02	0.702500*	0.092530*	0.784566*	4.037013*
S. ROODEPOORT																									
AREA K	VCR	0.19	1156	641.9	0.765	6.095	-0.385	5.274	557.1	0.07%[13]	0.625	2.37	24	0.010	3.940	0.629	634.1	1%[6]	0.569	-1.22	0.01	0.117279	0.879650	1.930119	6.251569
AREA L	VCR	0.18	1378	687.9	0.880	6.117	-0.241	3.615	693.6	0.25%[9]	0.483	0.83	20	0.024	3.235	0.757	684.7	0.3%[11]	0.399	-0.47	0.01	0.017510	4.711047	3.533014	6.514262
AREA M1	VCR	0.08	1067	635.3	0.702	6.176	-1.069	6.606	648.6	<.05%[39]	1.312	2.09	75	-0.114	3.145	0.429	645.2	12%[2]	0.274	1.56	0.09	0.080660	0.561888	1.966968	6.509271
AREA M2	VCR	0.08	1066	635.9	0.726	6.181	-0.852	5.107	644.6	0.06%[15]	0.658	1.37	N/A									0.092135	1.363942	2.710510	6.640887
WINKELHAAK																									
AREA X	KIMB.	[illegible]	8439	752.1	1.897	5.890	-0.747	3.747	736.6*	<.05%[93]*	1.206*	-2.10*	47	0.036	2.482	1.088	756.7	<.05%[139]	1.929	0.61	2.17	1.286000*	0.223000*	1.374096*	3.055494*
AREA XX+ ...	KIMB.	[illegible]	8439	752.1	1.897	5.890	-0.747	4.000	747.1	<.05%[99]	1.187	-0.66	N/A									0.262266	4.475246	3.487997	7.870077
AREA Y	KIMB.	1.92	8095	532.5	1.449	5.685	-0.663	3.926	528.5	<.05%[36]	0.801	-0.75	40	0.020	2.735	0.897	532.8	<.05%[25]	0.857	0.06	0.96	0.195387	4.511835	3.573866	7.225978

NOTES :
+ RESULTS BASED ON MINIMISATION OF CHI-SQUARE (AREAS: AA+ FOR A; A11+ FOR A1; A22+ FOR A2; XX+ FOR X)
[ ] CHI-SQUARE VALUES
* RESULTS BASED ON LN-GIG ANALYSIS
= ESTIMATES

APPENDIX B

MATHEMATICAL ASPECTS OF THE NEW DISTRIBUTIONS

B1.1 THE COMPOUND LOGNORMAL DISTRIBUTION (CLD)

The Compound Lognormal Distribution has a mixing distribution capable of describing the underlying geological model as described in chapter 2. The distribution is defined as (Sichel[3]):

$$\Omega(x) = \int_0^{\infty} \psi(x/\sigma^2)\,\delta(\sigma^2)\,d\sigma^2. \tag{1}$$

where $\psi(x/\sigma^2)$ is a conditional probability law for lognormal distribution with different logarithmic variances and a linear relationship between the means and variances (σ^2) of the logarithm of the observations (x = ln(cm-g/t). Also $\delta(\sigma^2)$ is the appropriate mixing distribution of the logarithmic variance σ^2. Sichel[3] originally observed this linear relationship between the logmean and the logvariance parameters in his work with diamonds. This linear relationship has also been observed with some of the Witwatersrand reefs. Thus:

$$\psi(x/\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-a-b\sigma^2)^2}{2\sigma^2}\right] \tag{2}$$

and $$\xi = a + b\sigma^2$$

where ξ is the mean of the logarithm of the observations, and a and b are two constants.

If we assume that the mixing distribution of the logarithmic variance σ^2 follows a Pearson Type III function (Gamma):

$$\delta(\sigma^2) = \frac{\rho^\tau}{\Gamma(\tau)}(\sigma^2)^{\tau-1}\exp[-\rho\sigma^2] \tag{3}$$

where $\rho > 0$ and $\tau > 0$ are two parameters and $\sigma^2 \geq 0$. Then the CLD can be obtained from equation (1). By substitution of (2) and (3) into (1) and integrating gives

$$\Omega(x) = C_L \cdot E_R \tag{4}$$

where

$$C_L = \sqrt{\frac{2(2\rho+b^2)}{\pi}}\frac{1}{\Gamma(\tau)}\left(\frac{\rho}{2\rho+b^2}\right)^\tau$$

and

$$E_R = \exp[b(x-a)]\kappa_{\tau-1/2}(\sqrt{2\rho+b^2}\,|x-a|)$$

where x = ln z ($-\infty < x > +\infty$) and ρ, τ, a and b are parameters. $k_{\tau-1/2}(\cdot)$ is the auxiliary modified Bessel function of the second kind of order (τ-1/2). By introducing the following new parameters

$\varphi = b^2/(2\rho + b^2)$, $s = \sqrt{(2\rho + b^2)}$ and $v = \tau - 1/2$,

then

$$1 - c = 2\rho/(2\rho + b^2) \quad \text{and} \pm\sqrt{c}.s = b$$

Equation (4) becomes

$$\Omega(x) = \frac{s(1-c)^{\nu+½}}{2^{\nu}\sqrt{\pi}\,\Gamma(\nu+½)} \exp[\pm\sqrt{c}\,s(x-a)]\,\kappa_{\nu}(s|x-a|) \qquad (5)$$

where $x = \ln(z)$, $(-\infty < x < +\infty)$, z = observed ore value in cm. g/t and

$$\kappa_{\nu}(s|x-a|) = (s|x-a|)^{\nu}\,K_{\nu}(s|x-a|)$$

which is the auxiliary modified Bessel function of the second kind. The four parameters of the distribution are:

a = location parameter, $-\infty < a < +\infty$

s = spread parameter, $s > 0$

c = skewness parameter, $0 \leq c < 1$

ν = kurtosis parameter, $\nu > -1/2$

B1.2 MOMENT ESTIMATION OF PARAMETERS OF COMPOUND LOGNORMAL DISTRIBUTION

In order to obtain the standardised form of the CLD we make a linear transformation

$u = s(x - a)$; $|u| = s|x - a|$; and

$-g = \pm\sqrt{c}$

It follows that

$$\varphi(u) = \Omega(x)\,\frac{dx}{du}$$

Substituting from (5)

$$\varphi(u) = \frac{(1-g^2)^{\nu+1/2}}{2^\nu\sqrt{\pi}\,\Gamma(\nu+1/2)}\, e^{-gu}|u|^\nu \mathrm{K}_\nu(|u|) \tag{6}$$

where $-\infty < u < +\infty$. From (6) it follows that

$$\int_{-\infty}^{+\infty} e^{-gu}|u|^\nu \mathrm{K}_\nu(|u|)\,du = \frac{2^\nu\sqrt{\pi}\,\Gamma(\nu+1/2)}{(1-g^2)^{\nu+1/2}} \tag{7}$$

The moment generating function of the standardized form is

$$\mathrm{M}_u(t) = \int_{-\infty}^{+\infty} e^{tu}\varphi(u)\,du$$

From (6)

$$\mathrm{M}_u(t) = \frac{(1-g^2)^{\nu+1/2}}{2^\nu\sqrt{\pi}\,\Gamma(\nu+1/2)} \int_{-\infty}^{+\infty} e^{-(g-t)u}|u|^\nu \mathrm{K}_\nu(|u|)\,du \tag{8}$$

substituting equation (7) into (8) gives

$$\mathrm{M}_u(t) = \left(\frac{1-g^2}{1-(g-t)^2}\right)^{\nu+1/2} \tag{9}$$

The cumulant generating function using (9) is

$$\lambda_u(t) = (\nu+1/2)\ln(1-g^2) - (\nu+1/2)\ln[(1-(g-t)^2] \tag{10}$$

The first four central moments for the standardised form can then be generated using equation (10) and are given below

$$\mu_1'(u) = \frac{(2\nu+1)(\pm\sqrt{c})}{(1-c)} \tag{11}$$

$$\mu_2(u) = \frac{(2\nu+1)(1+c)}{(1-c)^2} \tag{12}$$

$$\mu_3(u) = \frac{2(2\nu+1)(3+c)(\pm\sqrt{c})}{(1-c)^3}, \tag{13}$$

$$\mu_4(u) = 3(2\nu+1)\frac{(2\nu+3)(1+c^2)+2(2\nu+7)c}{(1-c)^4} \tag{14}$$

The Pearson's shape coefficients are:

$$\beta_1(u) = \beta_1(x) = \frac{\mu_3^2(u)}{\mu_2^3(u)} = \frac{4c(3+c)^2}{(2\nu+1)(1+c)^3} \tag{15}$$

$$\beta_2(u) = \beta_2(x) = \frac{\mu_4(u)}{\mu_2^2(u)} = 3 + \frac{6(c^2+6c+1)}{(2\nu+1)(1+c)^2} \tag{16}$$

Where $\beta_1(x)$ and $\beta_2(x)$ are the skewness and kurtosis coefficients respectively. Note that due to the linear transformation these two coefficients are the same in both 'x' and 'u' units. Since

$$x = u/s + a$$

the moments of the x-variable can then be derived from that of 'u'. Similarly, the first moment estimate about the origin of the untransformed distribution (cm-g/t) can be shown to be

$$\hat{\mu}_1'(z) = \left[\frac{1-\hat{c}}{1-(1/\hat{s}\pm\sqrt{\hat{c}})^2}\right]^{(\hat{v}+1/2)} \exp(\hat{a}) \tag{17}$$

This is the mean estimate of the underlying population of the observed values in a compound lognormal distribution.

B1.3 ESTIMATION OF THE FOUR PARAMETERS OF THE CLD

Using equations (15) and (16) it can be shown that

$$q = \frac{3\beta_1}{2\beta_2-3\beta_1-6} = c\left(\frac{3+c}{1-c}\right)^2 \tag{18}$$

This equation is solved iteratively e.g., by Newton-Raphson technique to obtain the parameter c.

From (15)

$$v = \frac{2c(3+c)^2}{\beta_1(1+c)^3} - \frac{1}{2} \tag{19}$$

which gives the second parameter in terms of the already estimated c parameter.

Also,

$$u = s(x-a), \text{ and } x = u/s + a \tag{20}$$

Thus

$$E(x) = \mu_1'(x) = \frac{1}{s}E(u) + a$$

where $\mu_1'(x)$ is the mean of the logarithm of the observation. Substituting from (11)

$$a = \mu_1'(x) - \frac{(2\nu + 1)(\pm\sqrt{c})}{s(1 - c)} \tag{21}$$

Finally, due to the linear transformation in (20),

$$\mu_2(x) = \left(\frac{\partial x}{\partial u}\right)^2 \mu_2(u) = \frac{1}{s^2}\mu_2(u) \tag{22}$$

substitution of (12) in (22) gives

$$s = \sqrt{\frac{(2\nu + 1)(1 + c)}{\mu_2(1 - c)^2}}$$

B1.4 THE LOGNORMAL DISTRIBUTION AS A LIMITING CASE OF THE CLD

From equation (15) and (16) if c = 0

$$\beta_1(x) = 0$$

and

$$\beta_2(x) = 3 + \frac{6}{2\nu + 1} \tag{23}$$

If c = 0 and $\nu \to \infty$

$$\beta_2(x) = 3$$

Thus

for c = 0 and $\nu \to \infty$, $\beta_1(x) = 0$ and $\beta_2(x) = 3$ which implies a normal distribution for the x-variable (x = lnz).

B1.5 REGION OF APPLICABILITY OF THE CLD

From (15)

$$(2v + 1) = \frac{4c(3+c)^2}{\beta_1(1+c)^3} \tag{24}$$

substitution of (24) in (16) gives

$$\beta_2(x) = 3 + 6(c^2 + 6c + 1)\frac{(1+c)\beta_1}{4c(3+c)^2}$$

If lim c → 1 then

$$\beta_2(x) > \frac{1}{2}[3\beta_1 + 6\ ] \tag{25}$$

Thus the CLD is applicable only if the condition in (25) is satisfied.

B2.1 THE LOG-GENERALISED INVERSE GAUSSIAN DISTRIBUTION (LN-GIG)

B2.2 ESTIMATION OF PARAMETERS OF LN - GIG DISTRIBUTION

The LN-GIG distribution arises from the logarithmic transformation of the Generalised Inverse Gaussian Distribution (See Jorgensen[47]). It is a very flexible distribution and can be both leptokurtic and platykurtic.

The four parameter Log-generalised Inverse Gaussian Distribution is defined as:

$$\Lambda(x) = \frac{1}{2sK_\gamma(b)}\exp\left[\gamma\left(\frac{x-\xi}{s}\right) - b\cosh\left(\frac{x-\xi}{s}\right)\right] \tag{26}$$

where $x = \ln(z)$, $(-\infty < x < +\infty)$ and z is the observed ore value in cm. g/t., $K_\gamma(b)$ is the modified Bessel function of the second kind of order γ and argument b. The four parameters are:

ξ = location parameter $(-\infty < \xi < +\infty)$

s = scale parameter $(s > 0)$

b = kurtosis parameter $(b > 0)$

γ = skewness parameter $(-\infty < \gamma < +\infty)$

Make a linear transformation

$$u = \frac{x-\xi}{s} \tag{27}$$

where $-\infty < u < \infty$. From (26) and (27)

$$\varphi(u) = \Lambda(x)\frac{dx}{du} = \frac{1}{2K_\gamma(b)}e^{\gamma u - b\cosh u} \tag{28}$$

The rth moment about the origin for the standard form from (28) is then

$$\mu'_r(u) = \frac{1}{2K_\gamma(b)}\int_{-\infty}^{+\infty} u^r e^{\gamma u - b\cosh u}du. \tag{29}$$

The first four moments about the origin of the standard form using (29) are

$$\mu'_1(u) = \frac{1}{K_\gamma(b)}\int_0^\infty u\sinh(\gamma u)\,e^{-b\cosh u}du \tag{30}$$

$$\mu_2'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^2 \cosh(\gamma u)\, e^{-b\cosh u}\, du \qquad (31)$$

$$\mu_3'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^3 \sinh(\gamma u)\, e^{-b\cosh u}\, du \qquad (32)$$

$$\mu_4'(u) = \frac{1}{K_\gamma(b)} \int_0^\infty u^4 \cosh(\gamma u)\, e^{-b\cosh u}\, du \qquad (33)$$

From which the central moments can be computed. As 'u' is a linear transformation of 'x', we have

$$\sqrt{\beta_1(u)} = \sqrt{\beta_1(x)} = \frac{\mu_3(u)}{[\mu_2(u)]^{3/2}}$$

$$\beta_2(u) = \beta_2(x) = \frac{\mu_4(u)}{[\mu_2(u)]^2}$$

Where $\mu_r(u)$ is the rth central moments of u.

From (27)

$$x = \ln z = su + \xi$$

from which

$$E(x) = \mu_1'(x) = sE(u) + \xi$$

and

$$\mu_1'(x) = \xi + s\mu_1'(u)$$

and

$$\xi = \mu_1'(x) - s\mu_1'(u)$$

where $\mu_1'(x)$ is the mean of the logarithms of the observed gold values, and $\mu_1'(u)$ is as defined in (30).

Also we have an exact condition for the following due to the linear transformation in (27)

$$\mu_2(u) = \left[\frac{\partial u}{\partial x}\right]^2 \mu_2(x) = \frac{\mu_2(x)}{s^2}$$

Thus

$$\mu_2(x) = s^2 \mu_2(u)$$

From which,

$$s = \sqrt{\frac{\mu_2(x)}{\mu_2(u)}}$$

B2.3 THE LN-GIG MEAN ESTIMATE

Since $x = \ln z$ and $z = e^x$, from (26)

$$\varphi(z) = \Lambda(x)\frac{dx}{dz}$$

and

$$\varphi(z) = \frac{1}{2sK_\gamma(b)z}e^{\gamma\left(\frac{\ln z-\xi}{s}\right)-b\cosh\left(\frac{\ln z-\xi}{s}\right)} \tag{34}$$

where $0 < z < \infty$. From (34) the rth moment about the origin of the z-distribution is given by

$$\mu'_r(z) = \frac{1}{2sK_\gamma(b)}\int_0^\infty z^{r-1}e^{\gamma\left(\frac{\ln z-\xi}{s}\right)-b\cosh\left(\frac{\ln z-\xi}{s}\right)}dz \tag{35}$$

But

$$u = \frac{\ln z-\xi}{s}\ ;\ dz = se^{su+\xi}\,du$$

Therefore (35) becomes

$$\mu'_r(z) = \frac{e^{r\xi}}{2K_\gamma(b)}\int_{-\infty}^{+\infty} e^{(rs+\gamma)u-b\cosh u}du \tag{36}$$

Now, from the standardised distribution function in (28)

$$\int_{-\infty}^{+\infty}\varphi(u)\,du = \frac{1}{2K_\gamma(b)}\int_{-\infty}^{+\infty} e^{\gamma u-b\cosh u}du = 1 \tag{37}$$

from which

$$\int_{-\infty}^{+\infty} e^{\gamma u - b\cosh u}\, du = 2\mathrm{K}_\gamma(b) \tag{38}$$

Hence (36) becomes

$$\mu'_r(z) = e^{r\xi}\frac{\mathrm{K}_{\gamma+rs}(b)}{\mathrm{K}_\gamma(b)}$$

if r = 1

$$\mu'_1(z) = \mathrm{E}(z) = e^{\xi}\frac{\mathrm{K}_{\gamma+s}(b)}{\mathrm{K}_\gamma(b)}$$

This is the mean of the underlying population of observed values using the LN-GIG distribution. Similarly, it can be shown that the moment generating function of the x-variable is

$$\mathrm{M}_x(t) = e^{t\xi}\frac{\mathrm{K}_{\gamma+ts}(b)}{\mathrm{K}_\gamma(b)}$$

B2.4 THE LOGNORMAL DISTRIBUTION AS A LIMITING CASE OF THE LN-GIG DISTRIBUTION

In its standard form LN-GIG distribution is

$$\varphi(u) = \frac{1}{2\mathrm{K}_\gamma(b)}e^{\gamma u - b\cosh u}$$

To be symmetric we have to make $\gamma = 0$

$$\varphi(u|\gamma=0) = \frac{1}{2K_0(b)} e^{-b\cosh u}$$

If b is large (say b > 10)

$$K_0(b) \approx \sqrt{\frac{\pi}{2b}}\, e^{-b}$$

implying

$$\varphi(u) = \frac{e^{b}}{\sqrt{2\pi}\,(1/\sqrt{b})} e^{-b\cosh u}$$

Therefore,

$$\varphi(u) = \frac{1}{\sqrt{2\pi}\,(1/\sqrt{b})} e^{b(1-\cosh u)}$$

But

$$\cosh u = 1 + \frac{u^2}{2!} + \frac{u^4}{4!} + \ldots$$

Thus

$$\varphi(u) = \frac{1}{\sqrt{2\pi}\,(1/\sqrt{b})} e^{b\left(1-1-\frac{u^2}{2!}-\frac{u^4}{4!}-\ldots\right)}$$

But $u = (x - \xi)/s$ is relatively small, being a kind of a standard measure. We can neglect $u^4/4!$ and terms with higher powers of u, compared to $u^2/2!$

Hence

$$\varphi(u) = \frac{1}{\sqrt{2\pi}(1/\sqrt{b})} e^{-\frac{1}{2}\frac{u^2}{(1/b)}}$$

This is a normal distribution with variance $\sigma^2 = 1/b$.

APPENDIX C

PROGRAM LISTING FOR 3-PARAMETER, COMPOUND LOGNORMAL AND LOG-GENERALISED INVERSE GAUSSIAN DISTRIBUTIONS

SYMBOLS AND ABBREVIATIONS FOR PROGRAM LISTING

BETA	Additive constant (or threshold parameter for the 3-parameter lognormal model
BETA1	The square of the skewness coefficient
BETA2	The kurtosis coefficient
CVALUE OR C	Skewness parameter for the Compound lognormal distribution (CLD)
SQRT(C)	Square root of C (Skewness parameter of CLD). It is negative if the distribution of the natural logarithms of the observed values is negatively skewed.
LMEANX	The sample mean of the natural logarithms of the observed values.
LVARX	The sample variance of the natural logarithms of the observed values.
V OR ORDER	Kurtosis parameter for the CLD
S	Spread parameter for the CLD
A	Location parameter for the CLD
B	Kurtosis parameter for the Log-generalised Inverse Gaussian distribution (LN-GIG)
FREQN(N)	Observed frequency
FREQ(N)	Expected frequency
CFREQN(I)	Observed cumulative frequency
CFREQ(I)	Expected cumulative frequency

LEONV

```
C    PROGRAM 1
C    THIS PROGRAM CALCULATES THE NATURAL LOG. OF GIVEN DATA
C    'GOLD' IS THE VARIABLE OF INTEREST

     REAL GOLD(30000),X,Y,BETA,A,B
     INTEGER N,I
     character*80 OUTNAME,INNAME

     WRITE(*,*) '*******************************************'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '* THIS PROGRAM CALCULATES THE NATURAL     *'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '* LOGARITHMS  OF A GIVEN DATA             *'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '*   MINING ENGINEERING DEPT.              *'
     WRITE(*,*) '*   UNIVERSITY OF THE WITWATERSRAND       *'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '*                                         *'
     WRITE(*,*) '*   AUTHOR:  W  Assibey-Bonsu             *'
     WRITE(*,*) '*******************************************'
     WRITE(*,*) ' '
     WRITE(*,*) 'WHAT IS THE NAME OF THE INPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') INNAME
     OPEN(10,file=INNAME,status='OLD')
     WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') OUTNAME
     OPEN(8,file=OUTNAME,status='NEW')
     WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF SAMPLES IN THE INPUT FILE'
     WRITE(*,*) 'N <= 30,000'
     READ(*,*) N
```

```
C    INPUTING BETA VALUE

     WRITE(*,*) 'PLEASE SUPPLY VALUE FOR BETA'
     READ(*,*)  BETA

10   DO 20 I = 1,N
        READ(10,*, END = 20) X,Y,GOLD(I),a,b
        WRITE(8,100) X,Y,ALOG(GOLD(I)+BETA),a,b
100     FORMAT(2F6.0,F10.6,2F6.0)
20   CONTINUE
     CLOSE(8)
     CLOSE(10)
     WRITE(*,*)'GOOD BYE!!!!!!'
     STOP
     END
```

COMPIN

```
C    PROGRAM 2
C    THIS PROGRAM CALCULATES THE UNIVARIATE STATISTICS OF GROUPED
C    LOGARITHMIC DATA (INCLUDING THE FIRST FOUR CENTRAL MOMENTS)
C    FOR EQUAL CLASS INTERVALS OF ONE IN LOG UNITS.  IF DIFFERNT CLASS
C    INTERVALS ARE TO BE USED THEN THE RELEVANT SHEPPARDS
C    CORRECTION SHOULD BE DONE.
C    INPUT FILE SHOULD CONTAIN INDIVIDUAL UPPER CLASS LIMITS AND THEIR
C    RESPECTIVE FREQUENCIES

     REAL*8  XBAR,MID,M2,MU2,MU3,MU4,CM2,CM3,CM4,SB,B1,B2,SD
     REAL ULIM(50),WID1,LLIM,OBS(50),N,WID2,TOT,TEMP,FSTUPL
     INTEGER IPROG,NUMF,I
     character*80 OUTNAME,inname
     DATA N,TOT,MU2,MU3,MU4/5*0/

     WRITE(*,*) '********************************************'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '* THIS PROGRAM CALCULATES THE UNIVARITE    *'
     WRITE(*,*) '* STATISTICS OF GROUPED DATA (IN LOG. UNITS) *'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '*  MINING ENGINEERING DEPT.                *'
     WRITE(*,*) '*  UNIVERSITY OF THE WITWATERSRAND         *'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '*                                          *'
     WRITE(*,*) '*  AUTHOR:  W  Assibey-Bonsu               *'
     WRITE(*,*) '********************************************'
     WRITE(*,*) ' '
     WRITE(*,*) 'WHAT IS THE NAME OF THE INPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') INNAME
     OPEN(10,file=INNAME,status='OLD')
     WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
     WRITE(*,*) ' '
```

```
      READ(*,'(a80)') OUTNAME
      OPEN(8,file=OUTNAME,status='NEW')

      WRITE(*,*) 'ENTER 0 IF EQUAL LENGTH INTERVALS,& 1 OTHERWISE'
      READ(*,*) IPROG
      IF(IPROG.EQ.0) THEN
       WRITE(8,*) 'PROGRAM FOR INTERVALS OF EQUAL LENGTH'
              ELSE
       WRITE(8,*) 'PROGRAM FOR INTERVALS OF UNEQUAL LENGTH'
             ENDIF

C     HEADING

      WRITE(8,101)
101   FORMAT('CLASS INTERVALS',5X,'OBS FREQ')

      WRITE(*,*) 'WHAT IS THE WIDTH OF FIRST CLASS INTERVAL'
      READ(*,*) WID1
      WID2 = WID1/2
      WRITE(*,*) 'ENTER THE TOTAL NUMBER OF CLASSES ',
     &         'ADD ADDITIONAL 2 CLASSES AT EACH END'
     &         'WITH ZERO FREQUENCIES'
      READ(*,*)  NUMF
      WRITE(*,*) 'WHAT IS THE UPPER LIMIT OF THE 1ST CLASS'
      READ(*,*)  FSTUPL
      TEMP = FSTUPL - WID1

C     READING UPPER LIMITS & OBSERVED FREQUENCIES

      DO 10 I = 1,NUMF
      READ(10,*) ULIM(I),OBS(I)

C     WRITING CLASS LIMITS TO OUTPUT FILE

      WRITE(8,103) TEMP,ULIM(I),OBS(I)
103   FORMAT(F6.2,' - ',F6.2,5X,F9.2)
      TEMP = ULIM(I)
```

```
C    ACTUAL ANALYSIS

     N = N + OBS(I)
     IF(I.NE.1) WID2 = (ULIM(I)-LLIM)/2
     MID = ULIM(I) - WID2
     LLIM = ULIM(I)
     M2 = MID*MID
     TOT = TOT +MID*OBS(I)
     MU2 = MU2+M2*OBS(I)
     MU3 = MU3 + M2*MID*OBS(I)
     MU4 = MU4 +M2*M2*OBS(I)
10   CONTINUE
     XBAR = TOT/N
     MU2 = MU2/N
     MU3 = MU3/N
     MU4 = MU4/N
     CM2 = MU2 - XBAR*XBAR
     IF(IPROG.EQ.0) CM2 = CM2 -1./12
     SD = SQRT(CM2)
     CM3 = MU3 - 3*XBAR*MU2 + 2*XBAR**3
     CM4 = MU4 - 4*XBAR*MU3+6*XBAR*XBAR*MU2-3*XBAR**4
     IF(IPROG.EQ.0) CM4 = CM4 -.5*(CM2+1./12) + 7./240
     SB = CM3/CM2**1.5
     B1 = SB*SB
     B2 = CM4/(CM2*CM2)
     GO TO 60

C    WRITING OUTPUT

60   WRITE(8,106)'    TOTAL FREQUENCY  =',N
     WRITE(8,106) '             XBAR =',XBAR
     WRITE(8,106)'2ND CENTRAL MOMENT  =',CM2
     WRITE(8,106)'STANDARD DEVIATION  =',SD
     WRITE(8,106)'3RD CENTRAL MOMENT  =',CM3
     WRITE(8,106)'4TH CENTRAL MOMENT  =',CM4
     WRITE(8,106)'    SQRT(BETA(1))  =',SB
     WRITE(8,106)'          BETA(1)  =',B1
```

```
      WRITE(8,106)'          BETA(2)    =',B2
106   FORMAT(A21,F18.10)
      CLOSE(8)
      STOP
      END
```

```
C     PROGRAM 3
C     PROGRAM TO ESTIMATE C PARAMETER IN THE COMPOUND LOGNORMAL
C     DISTRIBUTION USING NEWTON RAPHSON TECHNIQUE
C     USES INPUT FROM PROGRAM 2

      REAL*8 BETA1,BETA2,Q,C,CNEW,C1
      INTEGER N
      CHARACTER*80 OUTNAME


      WRITE(*,*) '*************************************************************'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '*THIS PROGRAM CALCULATES TH                  PARAMETER,     *'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '* C-VALUE IN THE COMPOUND L          RMAL DISTRIBUTION       *'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '* MINING ENGINEERING DEPT.                                  *'
      WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                           *'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '*                                                           *'
      WRITE(*,*) '* AUTHOR: W Assibey-Bonsu                                   *'
      WRITE(*,*) '*************************************************************'
      WRITE(*,*) ' '

      WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
      WRITE(*,*) ' '
      READ(*,'(a80)') OUTNAME
      OPEN(8,file=outname,status='new')

C     READING BETA1 AND BETA2 VALUES

      WRITE(*,*) 'PLEASE SUPPLY BETA1 AND BETA2  VALUES'
      WRITE(*,*) ' '
      read(*,*) BETA1,BETA2
```

```
C    CHECKING CONDITION FOR THE USE OF COMPOUND LOG. MODEL

25   IF( BETA2 .GT. 0.5*(3.0*BETA1 + 6.0)) GO TO 26
     WRITE(*,*) ' '
     WRITE(*,*) 'CONDITION FOR COMP. LOGNOMALITY NOT MET'
     STOP
26   Q = (3.0 * BETA1)/(2.0 *BETA2 - 3.0* BETA1 6.0)

C    CALCULATION OF C VALUE USING NEWTON RAPHSON TECHNIQUE
C    START ITERATION ARBITRARY AT A REALISTIC APPROXIMATION

     WRITE(*,*) 'WHAT IS THE INITIAL APPROXIMATION ( 0 <= C < 1 )'
     write(*,*) ' '
     read(*,*) C1
     C = C1

C    START ITERATION AT COUNTER AT 1

     N = 1

C    CNEW APPROXIMATION

27   CNEW = C - ((( 1.0-C) * (3.0+C))/ ( 3.0 + 6.0 * C - C * C ))*
    2((C - ( Q * ( 1.0-C )**2) / ( 3.0+C )**2 ))
     WRITE(*,*) ' '
     WRITE(*,*) CNEW
     WRITE(*,*) N
     WRITE(8,*) ' '

C    CHECK FOR CONVERGENCE

     IF(ABS(C - CNEW) .LT. 1.E-15) GO TO 29

C    CHECK COUNTER

     IF(N .GT. 20) GO TO 30
```

```
      N = N+1
      C = CNEW
100   GO TO 27
30    WRITE(*,*) 'FAILS TO CONVERGE AT N = 20 ITERATIONS'
29    WRITE(8,*)  'RESULTS OF THE ROOTS FOR CVALUE'
      WRITE(8,*) ' '
      WRITE(8,*) 'BETA1 =',BETA1
      WRITE(8,*) ' '
      WRITE(8,*) 'BETA2 = ',BETA2
      WRITE(8,*) ' '
      WRITE(8,*) 'Q VALUE = ',Q
      WRITE(8,*) ' '
      WRITE(8,*) 'C VALUE = ',CNEW
      WRITE(*,*) ' '
      WRITE(*,*)  'RESULTS OF  THE ROOTS FOR CVALUE'
      WRITE(*,*) 'BETA1 =',BETA1
      WRITE(*,*) 'BETA2 = ',BETA2
      WRITE(*,*) 'Q VALUE = ',Q
      WRITE(*,*) 'C VALUE = ',CNEW
      CLOSE(8)
      END
```

WCOMP

```
C   PROGRAM 4
C   PROGRAM TO ESTIMATE THE OTHER THREE PARAMETERS (V,S,A) FOR
C   COMPOUND LOGNORMAL DISTRIBUTION AS WELL AS THE COMPOUND
C   LOGNORMAL MEAN ESTIMATE

    REAL*8 BETA1,BETA2,C,CROOT,LMEANX,LVARX,V,S,A,EST1,EST2,ESTIMATE
    REAL*8 V1,V2,A1,A2,NUM,DNUM1,DNUM,S1,S2
    INTEGER  SKEW
    CHARACTER*80 OUTNAME

    WRITE(*,*) '*******************************************************'
    WRITE(*,*) '*                                                     *'
    WRITE(*,*) '* THIS PROGRAM CALCULATES OTHER PARAMETERS          *'
    WRITE(*,*) '*      (V,S,A) AND THE COMPOUND                      *'
    WRITE(*,*) '*  LOGNORMAL DISTRIBUTION MEAN ESTIMATE              *'
    WRITE(*,*) '*                                                     *'
    WRITE(*,*) '*                                                     *'
    WRITE(*,*) '*                                                     *'
    WRITE(*,*) '*    MINING ENGINEERING DEPT.                         *'
    WRITE(*,*) '*    UNIVERSITY OF THE WITWATERSRAND                  *'
    WRITE(*,*) '*                                                     *'
    WRITE(*,*) '*    AUTHOR: W  Assibey-Bonsu                         *'
    WRITE(*,*) '*******************************************************'

    WRITE(*,*) ' '
    WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
    WRITE(*,*) ' '
    READ(*,'(a80)') OUTNAME
    OPEN(8,file=outname,status='new')

C   READING BETA1 AND BETA2 AND OTHER VALUES

    WRITE(*,*) 'INPUT  BETA1,BETA2,C,LMEANX,LVARX,  VALUES'
    WRITE(*,*) ' '
    READ(*,*) BETA1,BETA2,C,LMEANX,LVARX
```

```
C   DETERMINING THE SIGN OF SQRT(C)

    WRITE(*,*) 'WHAT IS THE SKEWNESS OF THE LOG VALUES'
    WRITE(*,*) ' '
    WRITE(*,*) ' 1 = POSITIVE SKEWNESS, 2 = NEGATIVE SKEWNESS'
    WRITE(*,*) ' '
    READ(*,*)  SKEW
10  IF(SKEW .EQ. 2)  GO TO 100
    CROOT = SQRT(C)
    GO TO 11
100 CROOT  = -SQRT(C)

C   CALCULATING V

11  V1 = (2.0*C)*(3.0+C)**2
    V2 = BETA1*(1.0+C)**3
    V =  V1/V2 - 0.5
    WRITE(8,*) ' '
    WRITE(8,*) 'THE VALUE OF V = ',V
    WRITE(*,*) ' '
    WRITE(*,*) 'THE VALUE OF V =', V

C   CALCULATING S

12  S1 = (2.0*V+1.0)*(1.0+C)
    S2 = LVARX*(1.0-C)**2
    S = SQRT(S1/S2)
    WRITE(8,*) ' '
    WRITE(8,*) 'THE VALUE OF S = ',S
    WRITE(*,*) ' '
    WRITE(*,*) 'THE VALUE OF S =', S

C   CALCULATING THE VALUE OF A

13  A1 =  (2.0*V +1.0)*CROOT
    A2 =  S*(1.0-C)
    A =  LMEANX - (A1/A2)
```

```
      WRITE(8,*) ' '
      WRITE(8,*) 'THE VALUE OF A = ',A
      WRITE(*,*) ' '
      WRITE(*,*) 'THE VALUE OF A =', A

C     CALCULATING THE COMPOUND LOGNORMAL ESTIMATE

14    NUM   = ( 1.0-C)
      DNUM1 = (1.0/S+CROOT)**2
      DNUM  = 1.0 - DNUM1
      EST1  = (NUM/DNUM)**(V+0.5)
15    EST2 = EXP(A)
16    ESTIMATE = EST1 * EST2
      WRITE(8,*) ' '
      WRITE(8,*) 'THE COMPOUND LOGNORMAL ESTIMATE = ',ESTIMATE
      WRITE(*,*) ' '
      WRITE(*,*) 'THE COMPOUND LOGNORMAL ESTIMATE = ',ESTIMATE
      CLOSE(8)
      STOP
      END
```

WBESELR

```
C   PR. GRAM 5
C   THIS PROGRAM FITS A THEEORETICAL MODEL FOR THE COMPOUND
C   LOGNORMAL DISTRIBUTION VIA NUMERICAL INTEGRATION

C   THE MAIN PROGRAM

    EXTERNAL HYPB,GAMMA
    REAL*8 INTEG,LOWER,UPPER,V,B,V1,B1,BESSEL,GAMAV,INITIAL,FINAL
    REAL*8 COEFN,COEFD,COEFF,PI,C,VP05,S,CROOT,ORD1,ORD,IN,X,A,VP06
    INTEGER NUM,I,KK
    COMMON V,B,VP06
    CHARACTER*80 OUTNAME

    WRITE(*,*) '***************************************************'
    WRITE(*,*) '*                                                 *'
    WRITE(*,*) '* THIS PROGRAM FITS A THEORETICAL MODEL           *'
    WRITE(*,*) '* FOR THE COMPOUND LOGNORMAL DISTRIBUTION         *'
    WRITE(*,*) '*                                                 *'
    WRITE(*,*) '*                                                 *'
    WRITE(*,*) '*  MINING ENGINEERING DEPT.                       *'
    WRITE(*,*) '*  UNIVERSITY OF THE WITWATERSRAND                *'
    WRITE(*,*) '*                                                 *'
    WRITE(*,*) '*                                                 *'
    WRITE(*,*) '*  AUTHOR: W. Assibey-Bonsu                       *'
    WRITE(*,*) '***************************************************'
    WRITE(*,*) ' '

    WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
    WRITE(*,*) ' '
    READ(*,'(a80)') OUTNAME
    OPEN(8,file=outname,status='new')
    WRITE(8,700)
700 FORMAT(2X,'X',7X,'ARGUMENT(B1)',9X,'GAMMA',10X,'Kv(|B1|)',11X,
   E'ORDINATE'//)
```

```
C     INPUTING ORDER VALUE(V)

      WRITE(*,*) 'PLEASE SUPPLY VALUE FOR ORDER '
      WRITE(*,*) ' '
      READ(*,*)  V1
      V   = V1
      VP05= V1+0.5

C     USING RECURRENCE RELATION REQUIRE GAMMA(VP05+1)

      VP06 = VP05+1.0
      INITIAL = 0.0
      FINAL   = 200.00
      GAMAV = INTEG(INITIAL,FINAL,GAMMA)/VP05

C     CALCULATION OF OTHER PARAMETERS REQUIRED FOR EXPECTED
C     FREQUENCY CALCULATIONS

C     INPUTING VALUE FOR SPREAD PARAMETER S

      WRITE(*,*) 'PLEASE SUPPLY THE VALUE FOR SPREAD PARAMETER(S)'
      WRITE(*,*) ' '
      read(*,*) S

C     INPUTING VALUE FOR PI

      PI = 3.14159265358979323846

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF SAMPLES'
      READ(*,*) NUM
      WRITE(*,*) 'PLEASE SUPPLY C,& SQRT(C) VALUES (SKEWNESS PARAMETER)'
      WRITE(*,*) 'IF LOG DISTRIBUTION IS NEGATIVELY SKEWED SQRT(C) IS',
     &      'NEGATIVE'
      WRITE(*,*) ' '
      READ(*,*) C,CROOT
      COEFN = NUM*(1.0-C)**(V+0.5)
      COEFD = 2.0**V * SQRT(PI) * GAMAV
```

```
      COEFF = S*COEFN/COEFD

      WRITE(*,*) 'COEFFICIENT OF THE DISTRIBUTION FUNCTION=',COEFF

C     CALCULATINNG FINAL ESTIMATE FOR THE ORDINATES OF THE
C     DISTRIBUTION

      LOWER = 0.0
      UPPER = 6.00

C     INPUT VALUE FOR LOCATION PARAMETER(A)

      WRITE(*,*) 'WHAT IS THE VALUE FOR THE LOCATON PARAMETER(A)'
      READ(*,*) A

      WRITE(*,*) ' '
      WRITE(*,*) 'SUPPLY THE INITIAL X (X = LOGe(z)) VALUE AND THE TOTAL',
     2     'NUMBER OF X CLASSES(10 x TOTAL NUMBER OF CLASSES)'

      READ(*,*) IN,KK

      DO 500 I = 1,KK
      X = IN + 0.125*FLOAT(I-1)
      B1 = S*(X-A)
      B = ABS(B1)

      BESSEL = INTEG(LOWER,UPPER,HYPB)

      WRITE(*,*) ' '
      WRITE(*,*) 'ORDER =',V, 'ARGUMENT =',B, 'MODBESSEL =', BESSEL

      ORD1 = EXP(CROOT*B1)*(ABS(B))**V*BESSEL
      ORD = COEFF*ORD1

      WRITE(*,*) 'CORDINATE OF THE DISTRIBUTION FUNCTION=',ORD

500   WRITE(8,222) X,B1,BESSEL,ORD
```

```
222  FORMAT(F7.3,1X,E17.10,2X,E18.10,2X,F15.5)
     CLOSE(8)
     STOP

C    THIS IS THE END OF THE MAIN PROGRAM
     END

C    THIS IS THE SUBPROGRAM TO CALCULATE THE BESSEL INTERGRAND

     FUNCTION HYPB(X)
     REAL*8 X,V,B
     COMMON V, B
     HYPB  =(EXP(V*X) + EXP(-V*X))/2.0*EXP(-B *(EXP(X) +EXP(-X))/2.0)
     RETURN
     END

C    THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES GAMMA
C    FUNCTION(X>0)
     FUNCTION GAMMA(T)
     REAL*8 T, V,B,VP06
     COMMON V,B,VP06
     GAMMA = T**(VP06-1.0)*EXP(-T)
     RETURN
     END

C    SUBPROGRAM FOR INTERGRATION

     REAL*8 FUNCTION INTEG(A, B, FUNC)
     REAL*8 A,B,H,ENDS,TWO,FOUR,OLDINT,T
     INTEGER N,I

C    INITIALIZE THE INTERGRATION PROCESS
     H = (B-A)/2.0
     N = 1
     ENDS = FUNC(A) + FUNC(B)
     TWO = 0.0
     FOUR = FUNC(A + H)
```

```
      OLDINT = H / 3.0 * (ENDS + 4.0 * FOUR)

C     EVALUATION LOOP

25    H = H / 2.0
      N = 2*N
      TWO = TWO + FOUR
      FOUR = 0.0
      T = A + H
      DO 26 I = 1, N
      FOUR = FOUR + FUNC(T)
26    T = T + H + H
      INTEG = H / 3.0 * (ENDS +2.0 * TWO + 4.0 * FOUR)

      WRITE(*,*) ' '
      WRITE(*,*) 'INTERGRAND VALUE AT THIS ITERATION = ',INTEG
      WRITE(*,*) 'TOTAL NUMBER OF DIVISION = ', N

C     CHECK CONVERGENCE FOR EXCESSIVE NUMBER OF ITERATIONS

      IF(ABS(OLDINT-INTEG).LT. 1.0E-20 .OR. N .GT. 10000)  RETURN
      OLDINT = INTEG
      GO TO 25
      END
```

WDFREQ

```
C     PROGRAM 6: USES INPUT FROM PROGRAMS 5,9 OR 10
C     IN EACH CASE OUTPUT 'X  AND 'ORD' ARE USED FOR INPUT FILE
C     PROGRAM TO CALCULATE THE EXPECTED FREQUENCIES
C     ALL THE DISTRIBUTIONS

      REAL*8 ORD(500), EXP(10),VALUE(500),FREQ1,FREQ2,ONE,X(500)
      INTEGER N,I,KK,II,III
      CHARACTER*80 INNAME, OUTNAME

      WRITE(*,*) '*****************************************************      *'
      WRITE(*,*) '*                                                          *'
      WRITE(*,*) '* THIS PROGRAM CALCULATES EXPECTED                         *'
      WRITE(*,*) '*       FREQUENCIES FOR THE                                *'
      WRITE(*,*) '* COMPOUND LOGNORMAL DISTRIBUTION,                         *'
      WRITE(*,*) '*       3-PARAMETER AND LN-GIG                             *'
      WRITE(*,*) '*                                                          *'
      WRITE(*,*) '* MINING ENGINEERING DEPT.                                 *'
      WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                          *'
      WRITE(*,*) '*                                                          *'
      WRITE(*,*) '*                                                          *'
      WRITE(*,*) '*       AUTHOR: W  Assibey-Bonsu                           *'
      WRITE(*,*) '*****************************************************      *'

      WRITE(*,*) ' '
      WRITE(*,*) 'PLEASE SUPPLY NAME FOR THE INPUT FILE'
      WRITE(*,*) ' '
      READ(*,'(a80)') INNAME
      OPEN(10, File = inname, status='old')
      WRITE(*,*) 'PLEASE SUPPLY NAME FOR THE OUTPUT FILE'
      WRITE(*,*) ' '
      READ(*,'(a80)') OUTNAME
      OPEN(8, File = outname, status='new')
```

```
C    PUTTING A COUNTER FOR THE FINAL DO LOOP CALCULATION

     WRITE(*,*) 'WHAT IS THE NUMBER OF VALUES IN THE INPUT FILE'
     READ(*,*) II
     III=II-7
C    READING THE FIRST 9 VALUES FROM THE INPUT FILE

10   DO 20 N = 1,9 ,1
       READ(10,*) X(N),ORD(N)
       EXP(N) = ORD(N)
       WRITE(*,*) X(N),EXP(N)
11     IF(N.EQ.9) GO TO 22
20     CONTINUE

C    KEEPING THE THE FIRST RECURRING ORDINATE IN A VARIABLE CALLED
C    ONE
C    AND  INCREASING N BY 1

22   ONE = ORD(N)
     N   = N+1
     FREQ1 = (0.125/3.0)*(EXP(1)+4*EXP(2)+2*EXP(3)+4*EXP(4)+2*EXP(5)+
    &4*EXP(6)+2*EXP(7)+4*EXP(8)+EXP(9))
     WRITE(8,222) X(5),FREQ1
222  FORMAT(2X,'CLASS MIDPOINT VALUE -- ',2x,F5.2,4X,F15.8)
     WRITE(*,*) X(5),FREQ1

C    CALCULATING THE REMAINIG EXPECTED FREQUENCIES

40   KK= N+7
     DO 30 I = N,N+7
       READ(10,*) X(I),ORD(I)
       VALUE(I) = ORD(I)
       WRITE(*,*) X(I),VALUE(I)
       IF(I.EQ.KK ) GO TO 32
30   CONTINUE
```

```
32  FREQ2 = (0.125/3.0)*(ONE+4*VALUE(I-1)+2*VALUE(I-2)
   &+4*VALUE(I-3)+2*VALUE(I-4)+4*VALUE(I-5)+2*VALUE(I-6)+4*VALUE(I-7)
   &+VALUE(I))
    WRITE(8,333) X(I-4),FREQ2
    WRITE(*,*) X(I-4),FREQ2
333 FORMAT(2X,'CLASS MIDPOINT VALUE -- ',2x,F5.2,4X,F15.8)

C   PUTTING RECURRING VALUE IN 'ONE' AND INCREASING N VALUE BY ONE

    ONE = VALUE(I)
    N  = N + 1
    IF(N.LE.III) GO TO 40
    WRITE(*,*) 'END OF ITERATION'
    CLOSE(8)
    CLOSE(10)
    STOP
    END
```

WDCOUNT

```
C    PROGRAM 7
C    PROGRAM TO SUM THE EXPECTED FREQUENCIES FOR THE
C    DISTRIBUTIONS AS A CHECK AGAINST TH_ OBSERVED
C    FREQUENCIES

     REAL FREQ(500),SUM
     INTEGER N,I
     CHARACTER*80 INNAME, OUTNAME

     WRITE(*,*) '*******************************************************     *'
     WRITE(*,*) '*                                                           *'
     WRITE(*,*) '* THIS PROGRAM SUMS THE EXPECTED                            *'
     WRITE(*,*) '*      FREQUENCIES FOR THE                                  *'
     WRITE(*,*) '*        DISTRIBUTIONS                                      *'
     WRITE(*,*) '*                                                           *'
     WRITE(*,*) '*                                                           *'
     WRITE(*,*) '* MINING ENGINEERING DEPT.                                  *'
     WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                           *'
     WRITE(*,*) '*                                                           *'
     WRITE(*,*) '*                                                           *'
     WRITE(*,*) '* AUTHOR: W  Assibey-Bonsu                                  *'
     WRITE(*,*) '*********************************************************'
     WRITE(*,*) ' '
     WRITE(*,*) 'PLEASE SUPPLY THE NAME FOR THE INPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') INNAME
     OPEN(10, File = inname, status='old')
     WRITE(*,*) 'PLEASE SUPPLY THE NAME FOR THE OUTPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') OUTNAME
     OPEN(8, File = outname, status='new')

C    READING N VALUE
     WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF VALUES IN THE FILE'
     READ(*,*) N
```

```
      SUM = 0
10    DO 20 I = 1,N
        READ(10,*) FREQ(I)
        SUM = SUM +FREQ(I)
20      CONTINUE
      WRITE(8,*) 'THE SUM OF THE EXPECTED FREQUENCIES = ', NINT(SUM)

      CLOSE(10)
      CLOSE(8)
      STOP
      END
```

```
C   PROGRAM 8
C   THIS PROGRAM CALCULATES THE GAMMA AND BETA PARAMETERS FOR
C   FOR THE LN-GIG DISTRIBUTION
C   THE MAIN PROGRAM

    EXTERNAL HYPB,HYPB1,HYPB2,HYPB3,HYPB4
    REAL*8 INTEG,LOWER,UPPER,V,B,V1,B1,BESSEL,BESEL1
    REAL*8 COEFN,COEFF,S,ORD1,ORD,IN,X,A,BESEL2,VARU,MEANU1
    REAL*8 MEANU,VARUOR,MEANX,VARX,EST,VARZ,VARZN,VARZD
    REAL*8 COEFVA,U3PRIM,U4PRIM,U3,U4,SQRTB1,BETA2
    INTEGER NUM,I,KK
    COMMON V,B
    CHARACTER*80 OUTNAME

    WRITE(*,*)'*********************************************************'
    WRITE(*,*) '*                                                       *'
    WRITE(*,*) '* THIS PROGRAM CALCULATES THE GAMMA                     *'
    WRITE(*,*) '*       AND BETA  PARAMETERS                            *'
    WRITE(*,*) '* FOR THE LN-GIG DISTRIBUTION                           *'
    WRITE(*,*) '*                                                       *'
    WRITE(*,*) '*                                                       *'
    WRITE(*,*) '* MINING ENGINEERING DEPT.                              *'
    WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                       *'
    WRITE(*,*) '*                                                       *'
    WRITE(*,*) '*                                                       *'
    WRITE(*,*) '* AUTHOR:  W Assibey-Bonsu                              *'
    WRITE(*,*) '*********************************************************'
    WRITE(*,*) ' '

    WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
    WRITE(*,*) ' '
    READ(*,'(a80)') OUTNAME
    OPEN(8,file=outname,status='new')
```

```
C   INPUTING SKEWNESS PARAMETER

    WRITE(*,*) 'PLEASE SUPPLY APPROX. VALUE FOR SKEWNESS PARAMETER'
    WRITE(*,*) ' '
    READ(*,*)  V1
    V   = V1

C   INPUTING ARGUMENT
    WRITE(*,*) 'PLEASE SUPPLY APPROX. VALUE FOR ARGUMENT(B)'
    WRITE(*,*) ' '
    READ(*,*)  B
    WRITE(8,100) 'GAMMA(V) =',V1
    WRITE(8,100) 'B        =',B

    LOWER = 0.0
    UPPER = 10.00
    BESSEL = INTEG(LOWER,UPPER,HYPB)
    WRITE(*,*) ' '
    WRITE(*,*) ' =', BESSEL
    WRITE(8,*) 'MOD BESSEL =', BESSEL
    WRITE(8,*) ' '

C   CALCULTING MEAN OF U ABOUT ORIGIN I.E U PRIME 1 OF U

    LOWER = 0.0
    UPPER = 10.0
    MEANU1 = INTEG(LOWER,UPPER,HYPB1)
    MEANU  = MEANU1/BESSEL
    WRITE(*,*) ' '
    WRITE(8,*) 'MEAN IN U UNITS MEAN(U1) =', MEANU
    WRITE(8,*) ' '

C   CALCULTING VARIANCE OF U ABOUT ORIGIN i.e U PRIME 1 OF U

    VARUOR = INTEG(LOWER,UPPER,HYPB2)/BESSEL
    WRITE(*,*) ' '
    WRITE(8,100) '2ND MOMENT ABOUT ORIGIN  =', VARUOR
```

```
      WRITE(8,*) ' '

C     CALCULATING SECOND CENTRAL MOMENT IN U UNITS

      VARU = VARUOR - (MEANU)**2
      WRITE(8,100) 'SECOND CENTRAL MOMENTS IN U UNITS =', VARU
      WRITE(8,*) ' '

C     CALCULATING 3RD MOMENT ABOUT THE ORIGIN(U3PRIM)

      U3PRIM = INTEG(LOWER,UPPER,HYPB3)/BESSEL
      WRITE(*,*) ' '
      WRITE(8,100) '3RD MOMENT ABOUT ORIGIN = ', U3PRIM
      WRITE(8,*) ' '

C     CALCULATING 4TH MOMENT ABOUT THE ORIGIN(U3PRIM)

      U4PRIM = INTEG(LOWER,UPPER,HYPB4)/BESSEL
      WRITE(*,*) ' '
      WRITE(8,100) '4TH MOMENT ABOUT ORIGIN =',U4PRIM
      WRITE(8,*) ' '

C     CALCULATING 3RD CENTRAL MOMENT OF U

      U3 = U3PRIM - 3*MEANU*VARUOR + 2*MEANU*MEANU*MEANU

C     CALCULATING 4TH CENTRAL MOMENT OF U

      U4 = U4PRIM-4*MEANU*U3PRIM+6*MEANU*MEANU*VARUOR-3*MEANU**4

C     CALCULATING SQRT BETA-1

      SQRTB1 = U3/((SQRT(VARU))**3))

C     CALCULATING BETA-2

      BETA2 = U4/(VARU*VARU)
```

```
C     WRITING RESULTS
      WRITE(8,100) ' 4TH CENTRAL MOMENT ABOUT ORIGIN    =', U4
      WRITE(8,100) 'SKEWNESS: SQRT(BETA1)              = ', SQRTB1
      WRITE(8,100) 'KURTOSIS: BETA2                    = ', BETA2
100   FORMAT(A24,F16.10)

      CLOSE(8)
      STOP

C     THIS IS THE END OF THE MAIN PROGRAM
      END

C     THIS IS THE SUBPROGRAM TO CALCULATE THE BESSEL INTERGRAND
      FUNCTION HYPB(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB  =(EXP(V*X) + EXP(-V*X))/2.0*EXP(-B *(EXP(X) +EXP(-X))/2.0)
      RETURN
      END

C     THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C     U*SINH(VU)*EXP(-BCOSHU)

      FUNCTION HYPB1(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB1=(X)*(EXP(V*X) - EXP(-V*X))/2.0*EXP(-B *(EXP(X)+EXP(-X))/2.0)
      RETURN
      END

C     THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C     U**2*COSH(VU)*EXP(-BCOSHU)

      FUNCTION HYPB2(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB2 =(X*X)*(EXP(V*X)+EXP(-V*X))/2.0*EXP(-B*(EXP(X)+EXP(-X))/2.0)
```

```
      RETURN
      END

C     THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C     U**3*SINH(VU)*EXP(-BCOSHU)

      FUNCTION HYPB3(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB3=(X**3)*(EXP(V*X)-EXP(-V*X))/2.0*EXP(-B*(EXP(X)+EXP(-X))/2.0)
      RETURN
      END

C     THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C     U**4*COSH(VU)*EXP(-BCOSHU)

      FUNCTION HYPB4(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB4=(X**4)*(EXP(V*X)+EXP(-V*X))/2.0*EXP(-B*(EXP(X)+EXP(-X))/2.0)
      RETURN
      END

C     THIS IS THE FUNCTION SUBPROGRAM THAT INTERGRATES THE ARBITRARY
C     FUNCTION

      REAL*8 FUNCTION INTEG(A, B, FUNC)
      REAL*8 A,B,H,ENDS,TWO,FOUR,OLDINT,T
      INTEGER N,I

C     INITIALIZE THE INTERGRATION PROCESS
      H = (B-A)/2.0
      N = 1
      ENDS = FUNC(A) + FUNC(B)
      TWO = 0.0
      FOUR = FUNC(A + H)
      OLDINT = H / 3.0 * (ENDS + 4.0 * FOUR)
```

```
C     EVALUATION LOOP

25    H = H / 2.0
      N = 2*N
      TWO = TWO + FOUR
      FOUR = 0.0
      T = A + H
      DO 26 I = 1, N
      FOUR = FOUR + FUNC(T)
26    T = T + H + H
      INTEG = H / 3.0 * (ENDS +2.0 * TWO + 4.0 * FOUR)
      WRITE(*,*) ' '
      WRITE(*,*) 'INTERGRAND VALUE AT THIS ITERATION = ',INTEG
      WRITE(*,*) 'TOTAL NUMBER OF DIVISION = ', N

C     CHECK CONVERGENCE FOR EXCESSIVE NUMBER OF ITERATIONS

      IF(ABS(OLDINT-INTEG).LT. 1.0E-20 .OR. N .GT. 10000) RETURN
      OLDINT = INTEG
      GO TO 25
      END
```

LNGIG

```
C    PROGRAM 9
C    THE PROGRAM CALCULATES THE LOCATION AND SCALE PARAMETERS AND
C    FITS A THEORETICAL MODEL FOR THE LN-GIG DISTRIBUTION.
C    TO OBTAIN THE EXPECTED FREQUENCIES THE OUTPUT FROM THIS
C    PROGRAM SHOULD BE RUN IN PROGRAM 6
C    IT ALSO CALCULATES THE LN-GIG MEAN ESTIMATE, VARIANCE AND
C    CORRELATION COEFFICENT

C    THE MAIN PROGRAM

     EXTERNAL HYPB,HYPB1,HYPB2
     REAL*8 INTEG,LOWER,UPPER,V,B,V1,B1,BESSEL,BESEL1
     REAL*8 COEFN,COEFF,S,ORD1,ORD,IN,X,A,BESEL2,VARU,MEANU1
     REAL*8 MEANU,VARUOR,LMEANX,LVARX,EST,VARZ,VARZN,VARZD
     REAL*8 COEFVA
     INTEGER NUM,I,KK
     COMMON V,B
     CHARACTER*80 OUTNAME

     WRITE(*,*) '**********************************************************'
     WRITE(*,*) '*                                                        *'
     WRITE(*,*) '* THIS PROGRAM FITS A THEORETICAL                        *'
     WRITE(*,*) '* MODEL AND CALCULATES THE MEAN                          *'
     WRITE(*,*) '* ESTIMATE FOR THE LNGIG DISTRIBUTION                    *'
     WRITE(*,*) '*                                                        *'
     WRITE(*,*) '*                                                        *'
     WRITE(*,*) '* MINING ENGINEERING DEPT.                               *'
     WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                        *'
     WRITE(*,*) '*                                                        *'
     WRITE(*,*) '*                                                        *'
     WRITE(*,*) '* AUTHOR:  W Assibey-Bonsu                               *'
     WRITE(*,*) '**********************************************************'
     WRITE(*,*) ' '
```

```
      WRITE(*,*) 'WHAT IS THE NAME OF THE OUTPUT FILE'
      WRITE(*,*) ' '
      READ(*,'(a80)') OUTNAME
      OPEN(8,file=outname,status='new')

C     INPUTING ORDER VALUE(V)

      WRITE(*,*) 'PLEASE SUPPLY VALUE FOR SKEWNESS PARAMETER'
      WRITE(*,*) ' '
      READ(*,*)  V1
      V   = V1

C     INPUTING ARGUMENT

      WRITE(*,*) 'PLEASE SUPPLY VALUE FOR ARGUMENT(B)'
      WRITE(*,*) ' '
      READ(*,*)  B

C   FIND THE INTEGRAL OF COSH(VU)EXP(-BCOSHU)--FOR MODBESEL FUNCTION

      LOWER = 0.0
      UPPER = 10.00
      BESSEL = INTEG(LOWER,UPPER,HYPB)
      WRITE(*,*) ' '
      WRITE(*,*) 'ORDER =',V, 'ARGUMENT =',B, 'MODBESSEL =', BESSEL
      WRITE(8,*) 'ORDER =',V, 'ARGUMENT =',B, 'MODBESSEL =', BESSEL
      WRITE(8,*) ' '

C     CALCULTING MEAN OF U ABOUT ORIGIN I.E U PRIME 1 OF U

      LOWER = 0.0
      UPPER = 10.0

      MEANU1 = INTEG(LOWER,UPPER,HYPB1)
      MEANU  = MEANU1/BESSEL
      WRITE(*,*) ' '
      WRITE(8,*) 'ORDER =',V, 'ARGUMENT =',B, 'MEAN(U1) =', MEANU
```

```
      WRITE(8,*) ' '

C     CALCULTING VARIANCE OF U ABOUT ORIGIN i.e U PRIME 1 OF U

      VARUOR = INTEG(LOWER,UPPER,HYPB2)/BESSEL

      WRITE(*,*) ' '
      WRITE(8,*) 'ORDER =',V, 'ARGUMENT =',B, 'MEAN(U1) =', VARUOR
      WRITE(8,*) ' '

C     CALCULATING SECOND CENTRAL MOMENT IN U UNITS

      VARU = VARUOR - (MEANU)**2
      WRITE(8,*) 'SECOND CENTRAL MOMENTS IN U UNITS =', VARU
      WRITE(8,*) ' '

C     READING MEAN AND VARIANCE OF THE LOG OF GOLD VALUES

      WRITE(*,*) 'PLEASE SUPPLY VALUES FOR THE MEAN AND VARIANCE OF',
     &           'THE LOG VALUES(LMEANX,LVARX)'
      READ(*,*) MEANX,VARX

C     CALCULATING SCALE PARAMETER S

      S = SQRT(VARX/VARU)
      WRITE(8,*) 'SCALE PARAMETER =',S
      WRITE(8,*) ' '

C     CALCULATING LOCATION PARAMETER A
      A = MEANX- S*MEANU
      WRITE(8,*) 'LOCATION PARAMETER =',A
      WRITE(8,*) ' '

C     CALCULATING LNGIG MEAN ESTIMATE FOR Cmg/t VALUES

C     CALCULATING V+S FOR Kv+s(B)
      V = V+S
```

```
      UPPER = 10.00
      BESEL1 = INTEG(LOWER,UPPER,HYPB)
      EST = EXP(A)*BESEL1/BESSEL
      WRITE(8,*) 'THE LNGIG ESTIMATE =',EST
      WRITE(8,*) ' '

C     CALCULATING VARIANCE OF Cmg/t VIA LNGIG(P.S V CONTAINS 1*S ALREADY)

      V =  V+S
      BESEL2 = INTEG(LOWER,UPPER,HYPB)
      VARZN  = (BESSEL*BESEL2)
      VARZD  = BESEL1*BESEL1
      VARZ   = (VARZN/VARZD -1.0)*(EST*EST)
      WRITE(8,*) 'THE LNGIG ESTIMATE OF VARIANCE OF (Cmg/t) =',VARZ
      WRITE(8,*) ' '

C     CALCULATING COEFFICIENT OF VARIATION OF Cmg/t VIA LNGIG

      COEFVA = SQRT(VARZ/(EST*EST))
      WRITE(8,*) 'THE LNGIG ESTIMATE OF COEFF. OF VARIATION =',COEFVA
      WRITE(8,*) ' '

C     CALCULATING FINAL ESTIMATE FOR THE ORDINATES OF THE DISTRIBUTION

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF SAMPLES'
      READ(*,*) NUM
      COEFN = NUM/(2*BESSEL)
      COEFF = COEFN/S

      WRITE(*,*) 'COEFFICIENT OF THE DISTRIBUTION FUNCTION=',COEFF

C     USING A DO LOOP TO CALCULATE SERIES OF ARGUMENT VALUES (B1)

      WRITE(8,*) ' '
      WRITE(8,700)
700   FORMAT(2X,'X',7X,'ARGUMENT(B1)',9X,'BESEL2',10X,'Kv(|B1|)',11X,
     E'ORDINATE'//)
```

```
      WRITE(*,*) ' '
      WRITE(*,*) 'SUPPLY THE INITIAL X (X = LOGe(z)) VALUE AND THE TOTAL
     2 NUMBER OF X CLASSES(10 x NUMBER OF CLASSES)'
      READ(*,*) IN,KK

C     OBTAIN ORIGINAL VALUE FOR ORDER

      V = V-2.0*S
      DO 500 I = 1,KK
      X = IN + 0.125*FLOAT(I-1)
      B1 = (X-A)/S

C     READING ORIGINAL V (I.E ORDER) VALUE FOR THE DISTRIBUTION
C     FUNCTION
      ORD1 = EXP(V*B1)*EXP(-B *(EXP(B1) +EXP(-B1))/2.0)
      ORD = COEFF*ORD1

C     WRITE(*,*) 'CORDINATE OF THE DISTRIBUTION FUNCTION=',ORD

500   WRITE(8,222) X,B1,BESEL2,BESSEL,ORD
222   FORMAT(F5.2,2X,E17.5,1X,E15.5,2X,E18.5,2X,F15.5)

      CLOSE(8)
      STOP

C     THIS IS THE END OF THE MAIN PROGRAM
      END

C     THIS IS THE SUBPROGRAM TO CALCULATE THE BESSEL INTERGRAND
      FUNCTION HYPB(X)
      REAL*8 X,V,B
      COMMON V,B
      HYPB =(EXP(V*X) + EXP(-V*X))/2.0*EXP(-B *(EXP(X) +EXP(-X))/2.0)
      RETURN
      END
```

```
C    THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C    U*SINH(VU)*EXP(-BCOSHU)

     FUNCTION HYPB1(X)
     REAL*8 X,V,B
     COMMON V,B
     HYPB1=(X)*(EXP(V*X) - EXP(-V*X))/2.0*EXP(-B *(EXP(X)+EXP(-X))/2.0)
     RETURN
     END

C    THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES
C    U**2*COSH(VU)*EXP(-BCOSHU)

     FUNCTION HYPB2(X)
     REAL*8 X,V,B
     COMMON V,B
     HYPB2 =(X*X)*(EXP(V*X)+EXP(-V*X))/2.0*EXP(-B*(EXP(X)+EXP(-X))/2.0)
     RETURN
     END

C    THIS IS THE FUNCTION SUBPROGRAM THAT INTERGRATES THE ARBITRARY
C    FUNCTION

     REAL*8 FUNCTION INTEG(A, B, FUNC)
     REAL*8 A,B,H,ENDS,TWO,FOUR,OLDINT,T
     INTEGER N,I

C    INITIALIZE THE INTERGRATION PROCESS

     H = (B-A)/2.0
     N = 1
     ENDS = FUNC(A) + FUNC(B)
     TWO = 0.0
     FOUR = FUNC(A + H)
     OLDINT = H / 3.0 * (ENDS + 4.0 * FOUR)
```

```
C   EVALUATION LOOP

25  H = H / 2.0
    N = 2*N
    TWO = TWO + FOUR
    FOUR = 0.0
    T = A + H
    DO 26 I = 1, N
    FOUR = FOUR + FUNC(T)
26  T = T + H + H
    INTEG = H / 3.0 * (ENDS +2.0 * TWO + 4.0 * FOUR)
    WRITE(*,*) ' '
    WRITE(*,*) 'INTERGRAND VALUE AT THIS ITERATION = ',INTEG
    WRITE(*,*) 'TOTAL NUMBER OF DIVISION = ', N

C   CHECK CONVERGENCE FOR EXCESSIVE NUMBER OF ITERATIONS

    IF(ABS(OLDINT-INTEG).LT. 1.0E-20 .OR. N .GT. 10000)  RETURN
    OLDINT = INTEG
    GO TO 25
    END
```

THPAR

```
C   PROGRAM 10
C   CALCULATING EXPECTED FREQ. FOR 3-PARAMETER LOG. DISTR. & ESTIMATE
C   USING THE NORMAL DISTR. FUNCTION AFTER LOG. TRANSFORMATION
C   THE MEAN ESTIMATES ARE GIVEN ONLY FOR LARGE N
C   Z IS THE UNTRANSFORMED GOLD VALUE

      REAL*8 COEFN,COEFF,ORD1,ORD,IN,X,BETA,PI,U
      REAL*8 LMEANX,LVARX,EST,VARZ,STDE
      INTEGER NUM,I,KK
      CHARACTER*80 OUTNAME

      WRITE(*,*) '*****************************************************'
      WRITE(*,*) '*                                                   *'
      WRITE(*,*) '* THIS PROGRAM FITS A THEORETICAL MODEL FOR         *'
      WRITE(*,*) '*                                                   *'
      WRITE(*,*) '* THE 2 OR 3-PARAMETER LOGNORMAL                    *'
      WRITE(*,*) '* DISTRIBUTION                                      *'
      WRITE(*,*) '*                                                   *'
      WRITE(*,*) '* MINING ENGINEERING DEPT.                          *'
      WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                   *'
      WRITE(*,*) '*                                                   *'
      WRITE(*,*) '*                                                   *'
      WRITE(*,*) '* AUTHOR:  W Assibey-Bonsu                          *'
      WRITE(*,*) '*****************************************************'
      WRITE(*,*) ' '

      WRITE(*,*)'WHAT IS THE NAME OF THE OUTPUT FILE'
      WRITE(*,*) ' '
      READ(*,'(a80)') OUTNAME
      OPEN(8,file=outname,status='new')
```

```
C     INPUTING BETA MEAN AND VARIANCE OF X = LOG(Z+B)

      WRITE(*,*) 'PLEASE SUPPLY BETA VALUE',
     &' AS WELL AS LMEANX & LVARX OF LOG(Z+B)'
      WRITE(*,*) ' '
      READ(*,*)  BETA,LMEANX,LVARX
      STDEV = SQRT(VARX)

C     CALCULATING  MEAN ESTIMATE FOR Cmg/t VALUES

      EST = EXP(MEANX+VARX/2) - BETA
      WRITE(8,*) 'THE 3-PARMETER MEAN ESTIMATE =',EST
      WRITE(8,*) ' '

C     CALCULATING VARIANCE OF Z-VALUES VIA 3-PAR

      VARZ = ((MEANX+BETA)**2)*(EXP(VARX)-1.0)
      WRITE(8,*) 'THE 3-PARMETER ESTIMATE OF VARIANCE OF Cmg/t =',VARZ
      WRITE(8,*) ' '

C     CALCULATINNG FINAL ESTIMATE FOR THE ORDINATES OF THE
C     DISTRIBUTION

      PI = 3.14159265358979323846

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF SAMPLES'
      READ(*,*) NUM
      COEFN = NUM/SQRT(2.0*PI)

C     OBTAINING COEFFICIENT IN TERMS OF X = LOG(Z+B) UNITS FROM U UNITS

      COEFF = COEFN/STDEV

      WRITE(*,*) 'COEFFICIENT OF THE DISTRIBUTION FUNCTION=',COEFF

C     USING A DO LOOP TO CALCULATE SERIES OF ARGUMENT VALUES (U)
```

```
      WRITE(8,*) ' '
      WRITE(8,700)
700   FORMAT(2X,'X',7X,'ARGUMENT(U)',9X,'ORDINATE'//)
      WRITE(*,*) ' '
      WRITE(*,*) 'SUPPLY THE INITIAL X (X = LOGe(z)) VALUE AND THE TOTAL
     2 NUMBER OF X CLASSES(10 x TOTAL NUMMBER OF OF CLASSES)'
      READ(*,*) IN,KK

      DO 500 I = 1,KK
      X = IN + 0.125*FLOAT(I-1)
      U =  (X-MEANX)/STDEV

C     CALCULATING THE EXPONENTIAL PART 0F THE DISTRIBUTION FUNCTION
      ORD1 =  EXP(-0.5*U*U)
      ORD  =  COEFF*ORD1

C     WRITE(*,*) 'CORDINATE OF THE DISTRIBUTION FUNCTION=',ORD

500   WRITE(8,222) X,U,ORD
222   FORMAT(F7.3,2X,E17.5,1X,F15.5)

      CLOSE(8)
      STOP
      END
```

CHISQ1

```
C    PROGRAM 11
C    PROGRAM FOR CHI-SQUARE CALCULATION AND GOODNESS OF FIT TEST
C    INPUT FILE SHOULD CONTAIN OBSERVED AND EXPECTED FREQUENCIES
C    RESPECTIVELY.

C    THE MAIN PROGRAM

     EXTERNAL CHISQ,GAMMA
     REAL*8 INTEG,LOWER,UPPER,V,GAMAV,INITIAL,FINAL,ALPHA,DCHISQ
     REAL*8 DGREE,FGAMMA,CHISQV,OBS(500),EXP(500),CALCHISQ,VRCUR

     INTEGER N,I
     COMMON V,VRCUR
     CHARACTER*80 INNAME ,OUTNAME

     WRITE(*,*) '*******************************************************'
     WRITE(*,*) '* PROGRAM FOR CHI-SQUARE CALCULATION                  *'
     WRITE(*,*) '*      AND GOODNESS OF FIT TEST                       *'
     WRITE(*,*) '*                                                     *'
     WRITE(*,*) '*                                                     *'
     WRITE(*,*) '* MINING ENGINEERING DEPT.                            *'
     WRITE(*,*) '* UNIVERSITY OF THE WITWATERSRAND                     *'
     WRITE(*,*) '*                                                     *'
     WRITE(*,*) '*                                                     *'
     WRITE(*,*) '* AUTHOR: W  Assibey-Bonsu                            *'
     WRITE(*,*) '*******************************************************'

     WRITE(*,*) 'PLEASE SUPPLY THE NAME OF THE INPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') INNAME
     OPEN(10,file=INNAME,status='OLD')
     WRITE(*,*) 'PLEASE SUPPLY THE NAME OF THE OUTPUT FILE'
     WRITE(*,*) ' '
     READ(*,'(a80)') OUTNAME
     OPEN(8,file=OUTNAME,status='NEW')
```

```
C    CALCULATIN OF CHI-SQUARE VALUE

     WRITE(*,*) 'PLEASE SUPPLY THE TOTAL NUMBER OF CELLS',
    &          '(EXPECTED FREQUENCIES IN EACH CELL > = 5)'
     WRITE(*,*) ' '
     READ(*,*)  N

     CALCHISQ = 0.0
10   DO 20 I = 1,N
       READ(10,*) OBS(I), EXP(I)
       CALCHISQ = CALCHISQ + (OBS(I)-EXP(I))**2/EXP(I)
20   CONTINUE

     WRITE(*,*) 'THE VALUE OF CHI-SQUARE =', CALCHISQ
     WRITE(8,*) 'THE VALUE OF CHI-SQUARE =', CALCHISQ

C    FIND THE INTEGRAL OF CHI-SQUARE DIST. FUNCTION

     WRITE(*,*) 'PLEASE SUPPLY NUMBER OF DEGREE OF FREEDOM'
     WRITE(*,*) ' '
     READ(*,*)  DGREE
     V      = DGREE/2.0
     CHISQV = CALCHISQ

C    SUPPLYING LOWER AND UPPER BOUNDARY VALUES FOR
C    CHI-SQUARE DISTRIBUTION FUNCTION(LOWER = CALCULATED CHI-SQUARE
C    VALUE, UPPER = INFINITY--A BIG VALUE'

     WRITE(*,*) ' '
     LOWER = CHISQV
     UPPER = 200.00
     DCHISQ = INTEG(LOWER,UPPER,CHISQ)

     WRITE(*,*) ' '
     WRITE(*,*) ' RESULTS FOR INTEGRAL CHISQ =', DCHISQ
```

```
      VRCUR = V+1
      INITIAL = 0.0
      FINAL   = 200.00
      GAMAV = INTEG(INITIAL,FINAL,GAMMA)/V
      WRITE(8,*) ' '

C     CALCULATION OF ALPA VALUE FOR THE CHI-SQ TEXT
C     CALCULATONS

      FGAMMA = (2.0**V)*GAMAV
      ALPHA  = DCHISQ/FGAMMA
      WRITE(8,*) ' '
      WRITE(8,*)  ' THE PROBABILITY VALUE OF THE CHI-SQUARE TEST'
      WRITE(8,*) ' '
      WRITE(8,*) 'P(CHI-SQUARE >= CHISQ) =',ALPHA
      WRITE(*,*) 'P(CHI-SQUARE >= CHISQ) =',ALPHA

      CLOSE(8)
      CLOSE(10)
      WRITE(*,*) 'GOOD BYE!!! HOPE YOUR TEST FITS'
      STOP

C     THIS IS THE END OF THE MAIN PROGRAM
      END

C     THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES GAMMA
C     FUNCTION(X>0)
C     USES THE RECURRENCE RELATION: GAMMA(X+) =  GAMMA(X+1)/X
      FUNCTION GAMMA(T)
      REAL*8 T, V,VRCUR
      COMMON V,VRCUR
      GAMMA = T**(VRCUR-1.0)*EXP(-T)
      RETURN
      END
```

```
THIS IS A FUNCTION SUBPROGRAM THAT INTERGRATES 2ND FUNCTION(X>0)
      FUNCTION CHISQ(X)
      REAL*8 X,V
      COMMON V
      CHISQ = X**(V-1.0)*EXP(-X/2.0)
      RETURN
      END

C     THIS IS THE FUNCTION SUBPROGRAM THAT INTERGRATES THE ARBITRARY
C     FUNCTION

      REAL*8 FUNCTION INTEG(A, B, FUNC)
      REAL*8 A,B,H,ENDS,TWO,FOUR,OLDINT,T
      INTEGER N,I

C     INITIALIZE THE INTERGRATION PROCESS

      H = (B-A)/2.0
      N = 1
      ENDS = FUNC(A) + FUNC(B)
      TWO = 0.0
      FOUR = FUNC(A + H)
      OLDINT = H / 3.0 * (ENDS + 4.0 * FOUR)

C     EVALUATION LOOP

25    H = H / 2.0
      N = 2*N
      TWO = TWO + FOUR
      FOUR = 0.0
      T = A + H
      DO 26 I = 1, N
      FOUR = FOUR + FUNC(T)
26    T = T + H + H
      INTEG = H / 3.0 * (ENDS +2.0 * TWO + 4.0 * FOUR)
```

```
      WRITE(*,*) ' '
      WRITE(*,*) 'INTERGRAND VALUE AT THIS ITERATION = ',INTEG
      WRITE(*,*) 'TOTAL NUMBER OF DIVISION = ', N

C     CHECK CONVERGENCE FOR EXCESSIVE NUMBER OF ITERATIONS

      IF(ABS(OLDINT-INTEG).LT. 1.0E-20 .OR. N .GT. 10000)  RETURN
      OLDINT = INTEG
      GO TO 25
      END
```

CUM3

```
C    PROGRAM 12
C    PROGRAM FOR CALCULATION OF CUMULATIVE FREQUENCIES FOR
C    KOLMOGOROV-SMIRNOV TEST
C    INPUT FILE SHOULD CONTAIN OBSERVED AND EXPECTED FREQUENCIES
C    RESPECTIVELY

     REAL*8 Freq(5000),FREQN(5000), Cumfreq,CUMFREQN
     INTEGER N,I
     CHARACTER*80 INNAME, OUTNAME

     WRITE(*,*) '**********************************************'
     WRITE(*,*) '*  PROGRAM FOR CUMULATIVE FREQUENCIES *'
     WRITE(*,*) '*  ONE-SAMPLE KOLMOGOROV-SMIRNOV TEST *'
     WRITE(*,*) '*         OF GOODNESS OF FIT                  *'
     WRITE(*,*) '*                                             *'
     WRITE(*,*) '*  MINING ENGINEERING DEPT.                  *'
     WRITE(*,*) '*  UNIVERSITY OF THE WITWATERSRAND           *'
     WRITE(*,*) '*                                             *'
     WRITE(*,*) '*      AUTHOR: W  Assibey-Bonsu              *'
     WRITE(*,*) '**********************************************'
     WRITE(*,*) ' '

     WRITE(*,*) 'WHAT IS THE INPUT FILENAME'
     WRITE(*,*) ' '
     READ(*,'(a80)') INNAME
     OPEN(10, File = INNAME, STATUS='OLD')

     WRITE(*,*) 'WHAT IS THE OUTPUT FILENAME'
     WRITE(*,*) ' '
     READ(*,'(a80)') OUTNAME
     OPEN(8, File = OUTNAME, STATUS='NEW')
```

```
C     READING TOTAL NUMBER OF VALUES IN THE INPUT FILE

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF VALUES IN THE INPUT FILE'
      READ(*,*) N

C     INITIALISING

      CUMFREQN = 0
      Cumfreq = 0
      WRITE(8,212)
212   FORMAT(9X,'OBS. FREQ'8X,'OBS. CUM FREQ.'7X,
     &'EXP. FREQ.'10X,'EXP. CUM FREQ.')
      DO 200 I = 1, N
        Read(10,*) FREQN(N),freq(n)
        CUMFREQN = CUMFREQN + FREQN(N)
        CUMFREQ = CUMFREQ + FREQ(n)
      WRITE(*,*) FREQN(N),CUMFREQN,FREQ(n), CUMFREQ
      WRITE(8,222) FREQN(N),CUMFREQN,FREQ(n), CUMFREQ
222   FORMAT(2X,F15.6,4X,F15.6,4X, F15.6,4X,F15.6)
200   CONTINUE
      WRITE(*,*) ' '
      WRITE(*,*) 'GOODBYE'
      CLOSE(8)
      CLOSE(10)
      STOP
      END
```

KSMINOV

```
C     PROGRAM 13
C     PROGRAM FOR ONE SAMPLE KOLMOGOROV-SMIRNOV TEST OF GOODNESS
C     OF FIT
C     INPUT FROM PROGRAM 11; OBSERVED AND EXPECTED CUMULATIVE
C     FREQUENCIES RESPECTIVELY

      REAL*8 CFreq(500),CFREQN(500), DIF(500),DIFF(500),GTABS,DN,RNDN
      REAL*8 CUMN(500), CUM(500)
      INTEGER N,I,M ,N1
      CHARACTER*80 INNAME, OUTNAME

      WRITE(*,*) '*****************************************************************'
      WRITE(*,*) '* PROGRAM FOR ONE-SAMPLE KOLMOGOROV-SMIRNOV        *'
      WRITE(*,*) '*   TEST FOR GOODNESS OF FIT  AT 5% LEVEL          *'
      WRITE(*,*) '*                                                  *'
      WRITE(*,*) '*        MINING ENGINEERING DEPT.                  *'
      WRITE(*,*) '*  UNIVERSITY OF THE WITWATERSRAND                 *'
      WRITE(*,*) '*                                                  *'
      WRITE(*,*) '*  AUTHOR: W Assibey-Bonsu                         *'
      WRITE(*,*) '*****************************************************************'
      WRITE(*,*) ' '

      WRITE(*,*) 'WHAT IS THE INPUT FILENAME'
      WRITE(*,*) ' '
      READ(*,'(a80)') INNAME
      OPEN(10, FILE = INNAME, STATUS='OLD')
      WRITE(*,*) 'WHAT IS THE OUTPUT FILENAME'
      WRITE(*,*) ' '
      READ(*,'(a80)') OUTNAME
      OPEN(8, FILE = OUTNAME, STATUS='NEW')

C     READING TOTAL NUMBER OF SAMPLES(TOTAL EXPECTED FREQUENCY)

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF SAMPLES',
     &'(I.E TOTAL EXPECTED FREQUENCY)'
```

```
      READ(*,*) N
C     INPUTING HEADING
      WRITE(8,111)
111   FORMAT(2X,'OBS. % CUM. FREQ.',3X,'EXP. % CUM. FREQ.',14X, 'DIFF.')

C     READING THE FIRST CUM. FREQ. DIFF. TO HELP TO GET MAX. ABS.
C     DIFFERENCE

      DO 100 I = 1,2
        READ(10,*) CFREQN(I),CFREQ(I)
        CUMN(I) = (100*CFREQN(I))/N
        CUM(I)  = (100*CFREQ(I))/N
        DIF(I) = ABS( CUMN(I) - CUM(I) )
        WRITE(8,222)  CUMN(I),CUM(I),DIF(I)
222      FORMAT(2X, F15.6,2X,F15.6,14X,F12.6)
        IF(I.EQ. 1) GO TO 200
100   CONTINUE

C     PUTTING FIRST % CUM FREQ. IN GTABS

200   GTABS = DIF(I)
      M = 2

C     CONTINUING WITH THE REMAINING DIFFERENCE ESTIMATES
C     READING TOTAL NUMBER OF VALUES IN THE INPUT FILE

      WRITE(*,*) 'WHAT IS THE TOTAL NUMBER OF VALUES IN THE INPUT FILE'
      READ(*,*) N1
      DO 300  I = M,N1
        READ(10,*) CFREQN(I),CFREQ(I)
        CUMN(I) = (100*CFREQN(I))/N
        CUM(I)  = (100*CFREQ(I))/N
        DIFF(I) = ABS( CUMN(I) - CUM(I) )
        WRITE(8,333)  CUMN(I),CUM(I),DIFF(I)
333      FORMAT(2X, F15.6,2X,F15.6,14X,F12.6)
        IF(DIFF(I) .GT. GTABS) GTABS = DIFF(I)
300   CONTINUE
```

```
C    WRITING THE MAX. ABSOLUTE DIFF.

     WRITE(8,*) ' '
     WRITE(8,*) 'MAXIMUM ABSOLUTE CUM. DEVIATION = ',GTABS
     WRITE(8,*) ' '
     WRITE(8,*) ' '

C    OBTAINING ACTUAL PROBABILITY(DIVIDE BY 100.0)

     DN   = GTABS/100.0
     RNDN = DN * SQRT(N)
     WRITE(8,*) ' '
     WRITE(8,*) 'SQRT(N)*D(N) = ',RNDN
     WRITE(8,*) ' '
     WRITE(8,*) ' '

C    COMPARING DN*SQRT(N) WI   THAT OF TABLES AT 5% LEVEL(VALUE=1.358)

     IF(RNDN .LT. 1.358) GO TO 400
     WRITE(8,*) 'REJECT THE HYPOTHESIS THAT THE DISTRIBUTION IS f(x)',
    &' AT 5% LEVEL OF PROBABILITY'
     WRITE(*,*) 'REJECT THE HYPOTHESIS THAT THE DISTRIBUTION IS f(x)',
    &' AT 5% LEVEL OF PROBABILITY'
     GO TO 500
400  WRITE(8,*) 'ACCEPT THE HYPOTHESIS THAT THE DISTRIBUTION IS f(x)',
    &' AT 5% LEVEL OF PROBABILITY'
     WRITE(*,*) 'ACCEPT THE HYPOTHESIS THAT THE DISTRIBUTION IS f(x)',
    &' AT 5% LEVEL OF PROBABILITY'

500  CLOSE(8)
     CLOSE(10)
     STOP
     END
```

REFERENCES

1 KRIGE, D.G. On the departure of ore value distribution from the lognormal model in South African gold mines, *J.S. Afr. Inst. Min. Metall.*, 61, 1960, pp. 231 -244.

2 SICHEL, H.S. The estimation of the means and associated confidence limits for small samples from lognormal populations, In: Symposium on Mathematical statistics and Computer Applications in Ore Valuation, Johannesburg, *S. Afr. Inst. Min. Metall.*, 1966, pp. 106-122.

3 SICHEL, H.S. Statistical valuation of diamondiferous deposits, APCOM 72, Johannesburg 1972, pp. 17-24.

4 DOWD, P.A. Advances in Geostatistics: Numerical methods and their applications, PhD Thesis, University of Leeds, U.K. 1978.

5 MINTER, W.E.L. The Welkom Gold Field exhumed: borehole predictions compared with thirty years of mining results, Proceedings, 12th CMMI Congress Johannesburg, *S. Afr. Inst. Min, Metall. (or Geol. Soc. S. Afr.)*, 1982, pp. 137-144.

6 MINTER, W.E.L. Sedimentology of the Witwatersrand basin, Optima, 30, 1981- 82. pp. 263-272.

7 ANTROBUS, E.S.A. Witwatersrand Gold - 100 years, Geol. Soc. S. Afr. 1986.

8 ANTROBUS, E.S.A. The Klerksdorp Goldfield, Mineral deposits of Southern Africa, Geol. Soc. S. Afr. 1986, pp. 549-596.

9 HALLBAUER, D.K. A Geochemical identification of Provenance-controlled gold distribution patterns in the Ventersdorp Contact Reef placer of the Carletonville Gold Field, Gold 100 Proc. Int. Conf. on Gold. 1, Gold Mining Technology, Johannesburg, SAIMM, 1986, pp. 45-62.

10 KRIGE, D.G., KLEINGELD, W.J. and OOSTERVELD, M.M. A-priori parameter distribution patterns for gold in the Witwatersrand basin to be applied in borehole valuations of potential new mines using Bayesian

Geostatistical techniques, APCOM 90, Berlin, pp. 715-726.
11 WATERMEYER, G.A. Application of the theory of probability in the determination of ore reserves, *J. chem. Metall. Min. Soc. S. Afr.*, 19, 1919, pp. 97-107.
12 TRUSCOTT, S.J. The computation of the probable value of ore reserves from assay results, *Trans. Inst. Min. Metall., Lond.*, 39, 1929, pp. 482-496.
13 SICHEL, H.S. An experimental and theoretical investigation of bias error in mine sampling with special reference to narrow gold reefs, *Trans. Inst. Min. Metall., Lond.*, 56, 1947, pp. 403-473.
14 ROSS, F.W.J. The development and some practical applications of statistical value distribution theory for the Witwatersrand auriferous deposits, MSc (Eng) Thesis, University of the Witwatersrand, Johannesburg, 1950.
15 KRIGE, D.G. A statistical approach to some mine valuation and allied problems on the Witwatersrand, MSc(Eng) Thesis, Univ. of the Witwatersrand, Johannesburg, 1951.
16 SICHEL, H.S. New methods in the statistical evaluation of mine sampling data, *Bull. Inst. Min. Metall., Lond.*, June 1952, pp. 261-288.
17 KRIGE, D.G. A statistical analysis of some of the borehole values in the Orange Free State Goldfield. *J, Chem. Metall. Min. Soc. S. Afr.*, Sept. 1952, pp. 47-64.
18 KRIGE, D.G. A statistical approach to some basic mine valuation problems on the Witwatersrand, *J. Chem. Metall. Min. Soc. S. Afr.*, Dec. 1951, pp. 119-139.
19 DE WIJS, H.J. Statistical ore distribution, *Geologie en Mijnbouw*, Nov. 1951, Jan. 1953.
20 MATHERON, G. *Traité de geostatistique appliquée*, tome .. Memoires de Bureau de Recherches Geologiques et Minières, no. 14. Editions Technip, Paris. English translation: *Treatise on Applied Geostatistics*, Kennecott Copper Corporation, Salt Lake City Utah, 1960.
21 MATHERON, G. Kriging or polynomial interpolation?, *CIMM Transactions*, 70, 1967, pp. 240-244.

22 MARECHAL, A. *Krigeage normal et lognormal*, Centre de Morphologie Mathematique, N376, Ecole Nationale Supérieure des Mines de Paris, France, 1974.

23 RENDU, J.M. Normal and lognormal estimation, *Math. Geol.*, 11, 1979, pp. 407-422.

24 DOWD, P.A. Lognormal kriging - The General Case, *Math. Geol.*, 14, No. 5 1982, pp. 475-499.

25 JOURNEL, A.G. and HUIJBREGTS, C.J. Mining Geostatistics, Academic Press, London, 1978.

26 GUARASCIO, M., DAVID, M. and HUIJBREGTS, C. J. ed. : Advanced Geostatistics in the Mining Industry, NATO Advanced Study Institute Series, D. Reidel Publishing Co., Dordrecht, Holland, 1975.

27 KRIGE, D.G. and RENDU, J.M. The fitting of contour surfaces to hanging and footwall data for an irregular ore body, In: DORSTEWITZ, G. et al ed. *13th Int. Symp. on the Application of Computers and Mathematics for Decision Making in Mineral Industries*, Verlag Gluckauf, Essen, 1975, C-V1-12.

28 BRAUN, R. A study to optimize ore evaluation at Western Deep Levels Gold Mine, Witwatersrand, by using geostatistics and geological subdivision, PhD Thesis, University of the Witwatersrand, Johannesburg, 1988.

29 KRIGE, D.G., GUARASCIO, M. and CAMISANI-CALZOLARI, F.A. Early South African geostatistical technique in Today's perspective, In: ARMSTRONG. M. ed., Proc. of the 3rd International Geostatistics Congress, Kluwer Academic Publishers, Dordrecht, Holland, 1988, pp. 1-19.

30 KLEINGELD, W.J. *La Geostatistique pour des variables discretes*, PhD thesis, L'Ecole Nationale supérieure des mines de Paris. Fontainebleau, France, 1987.

31 KRIGE, D.G. Statistical applications in mine valuation. *J. Inst. Mine Survey. S. Afr.*, 1962, 12(2), pp. 45-84, 12(3), pp.95-136.

32a KRIGE, D.G., Geostatistics and definition of uncertainty. *Trans. Instn.*

Min. Metall. (sect. A: Min. industry), 93, July. 1984, pp. A41-A47.

32b KRIGE, D.G. The use of geostatistics in defining and reducing the uncertainty of grade estimates, *Trans. Geol. Soc. S. Afr.*, 88, 1985, pp. 69-72.

33 KRIGE, D.G. and ASSIBEY-BONSU, W. New developments in borehole valuations of new gold mines and undeveloped sections of existing mines, Paper submitted for publication, *J. S. Afr. Inst. Min. Metall.* 1991.

34 SICHEL, H.S. and LINDER, A. Contribution to a paper given as reference (1). *J. S. Afr. Inst. Min. Metall.* 61, 1961. pp. 333-338.

35 SICHEL, H.S. Some advances in Lognormal theory. APCOM 87, 3, : Geostatistics. Johannesburg, SAIMM, 1987. pp. 3-8.

36 LINK, R.F. and KOCH, G.S. Some consequences of applying Lognormal distributions. *Math. Geol.* 7, 1975. pp. 117-127.

37 MINTER, W.E.L. Sedimentology of the Vaal Reef in the Klerksdorp area, PhD Thesis, University of the Witwatersrand, Johannesburg, 1972.

38 LINDLEY, D.V., and SCOTT, W.F. New Cambridge elementary statistical tables. Cambridge University Press. Cambridge. 1984. pp. 80.

39 SICHEL, H.S. Contribution to a paper on: Developments in the valuation of gold mining properties from borehole results. In: KRIGE, D.G. Applications of statistical techniques to mine valuation problems, DSc Thesis, University of the Witwatersrand, Johannesburg, 1963.

40 MINTER, W.E.L. Detrital gold, uranium and pyrite concentrations related to the sedimentology in the Precambrian Vaal Reef placers, Witwatersrand, *S. Afr. Econ. Geol.*, 71, 1976, pp. 157-176.

41 PRETORIUS, D.A. The depositional environment of the Witwatersrand Goldfields: a chronological review of speculations and observations, In: Some Sedimentary basins and associated ore deposits of South Africa, *Geol. Soc. of S. Afr.*; spec. publ. No. 6, 1979, pp. 33-55.

42 KRIGE, D.G. Lognormal-de Wijsian Geostatistics for Ore Evaluation. SAIMM Monograph series, Geostatistics 1, Johannesburg, 1978.

43 DAVID, M. Handbook of applied advanced geostatistical ore reserve

estimation, Elsevier Scientific Publishing Company, Amsterdam, 1988.

44 KRIGE, D.G. and MAGRI, E.J. Studies of the effects of outliers and data transformation on variogram estimates for a base metal and a gold ore body, Jour. *Math. Geol.*, 14, No.6, 1982, pp. 557-564.

45 KRIGE, D.G. Unpublished Research still in progress. 1990/1991.

46 KRIGE, D.G. Effective pay limits for selective mining: economic aspects of stoping through unpayable ore, *J. S. Afr. Inst. Min. Metall.*, 62, 1962, pp. 364-374.

47 JORGENSEN, B. Statistical properties of the Generalised Inverse Gaussian Distribution, Springer-Verlag, New York Inc., 1982.

Author: Assibey-bonsu Winfred.
Name of thesis: New Models For Borehole Valuation Of New Mining Operations And Routine Valuation Of Ore Reserve Blocks.

PUBLISHER:
University of the Witwatersrand, Johannesburg
©2015

LEGALNOTICES:

Copyright Notice: All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed or otherwise published in any format, without the prior written permission of the copyright owner.

Disclaimer and Terms of Use: Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page)for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.