# THE INFLUENCE OF USING COMPUTERS TO REMEDY LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11


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**ABSTRACT**

The aim of the study was to investigate what influence the use of ICT teaching and learning such as Geogebra software and you tube videos had on grade 11 South African learners' understanding of functions. The theoretical underpinnings informing this study were constructivism, socio-cultural learning theories, variation theory and the Technological Pedagogic Content Knowledge. A mixed method approach was employed using a Quasi-experiment design. Two grade 11 classes from a South African township school in Soweto were used in the study; one as the control group and the other as the experimental group. These samples were purposefully and conveniently selected for the study. The researcher administered a pre-test to both groups followed by a teaching intervention where the control group was taught using traditional methods while the experimental group was taught using computers. Semi structured interviews were conducted to establish reasons for learner errors and misconceptions after marking the pretest while a semi structured interview guided by specific questions was used to establish the effects using computers with the experimental group after the posttest. After the intervention, a posttest was administered to both groups. Data was analyzed from the test scripts for all the learners to establish the misconceptions that learners in this study have on functions and later on from the test scripts of learners separately in their respective groups to establish the influence of using ICT in the intervention. Interviews on the errors identified from the pretest were conducted from a sample of learners from either group to explain the reason for their errors while at a later stage, learners interviewed were selected from the experimental group to give insight into the influence of using computers. Data was analyzed qualitatively and quantitatively. The results indicated that the using learner errors and misconceptions as a resource for intervention had positive impact on learner performance for both groups. However, there was better improvement from those learners who used computers in their intervention than those students who used traditional methods. The use of computers in the mathematics classroom yielded a positive attitude towards mathematics. With an improved attitude, there was also an improvement in the learner performance, hence proving that using computers yields better results.It is also hoped that the study will inform policy developers in mathematics education on how technology can be incorporated into mathematics classrooms to enhance learning.

**DECLARATION**

I declare that this research report, titled:

***The influence of using computers to remedy learner errors and misconceptions in functions at grade 11,***

Is my own work and all sources that I have used or quoted have been indicated and acknowledged by means of complete references.

It is being submitted for the degree of Masters of Science at the University of the Witwatersrand, Johannesburg, South Africa. It has not been submitted before for any degree or examination purposes at any other university.

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This chapter introduces the study. Firstly, it discusses the context of the study. Then it explores literature in mathematics education to locate the gap in research which can be filled by this research. Further, the research aims, research questions and significance of the study are presented.

## 1.1 Introduction

South Africa is a well-developed African country, but it struggles from underperformance in mathematics and science as compared to other African countries (Reddy, 2006). Curriculum reform in South Africa has been characterized by radical school curriculum changes in a bid to improve the education system as observed from the implementations of different curriculums for various reasons. There have been three curriculum changes from 1994 to 2012 in South Africa, these are Curriculum 2005 (C2005), Revised National Curriculum Statement (RNCS), and Curriculum and Assessment Policy Statements (CAPS). (Jansen, 1998).

Curriculum 2005(2005) designers in line with one understanding of Outcome Based Education (OBE) philosophy took excessive care not to prescribe content. This curriculum promoted curriculum and assessment based on constructivism approach, encouraged learners to create their own knowledge, and discouraged traditional education approaches, which were based on direct facts and standard methods (Donnelly, 2002). Teachers were expected to generate content on their own; hence, this compromised the range, depth and the quality of learning in Mathematics. With C2005's lack of content specifications, some teachers could have missed on the key content and this imparted negatively on learner performance in mathematics. The teacher's lack of a rich base in the understanding of the subject matter, may also have limited their ability to design and use higher order thinking skills to probe learner's understanding needed by constructivist approach in the teaching of mathematics. With C2005 failing to yield the desired outcomes, a review committee was established in 2000 to deal with many factors that affected the educational system (Chisholm, 2003). Among the many reasons, the review committee emphasized that the implementation of Curriculum 2005 was compounded by learning support materials that were variable in quality, often unavailable and not sufficiently used in the classroom. With these and many other recommendations from the review Committee, a Revised National Curriculum Statement was developed and established.

The Revised National Curriculum Statement (RNCS) replaced C2005 but was still an outcome-based education; it was just a streamlining version of C2005.The Revised National Curriculum Statement (RNCS) was to promote conceptual coherence, have a clear structure and be written in clear, understandable language (Chisholm, 2003). The RNCS's focus was on addressing the composite nature of Curriculum 2005 by emphasizing on basic skills, content knowledge and grade progression as well as simplifying the outcome statements (DOE, 2011a).This left the teachers with the responsibility of implementing the new changes in the curriculum. The Implemented RNCS did not give the expected results, hence, it was replaced by the Curriculum and Assessment Policy Statement (CAPS) in 2012.Removed from RNCS were all the OBE policy terminology, critical and development outcomes as well as the assessment standards and learning outcomes. These were replaced by the general aims and specific aims in the Curriculum and Assessment Policy Statement. At the writing of this research project, this is the running curriculum.

In the recent policy Curriculum and Assessment Policy Statement (CAPS) document, mathematics in the Further Education and Training(FET)Phase (Grade 10 – 12) covers ten Content Areas. These are Functions; Number Patterns, Sequences and Series; Finance, growth and decay, Algebra; Differential Calculus; Probability; Euclidean Geometry and Measurement; Analytic Geometry; Trigonometry; and Statistics (DOE, 2011b, p.9). Citing the changes that took place in the CAPS document, one would anticipate that a different change in the curriculum would impart positively in the teaching and learning of Mathematics in South African schools and have a positive outcome in learner performance in mathematics examinations. In the FET Phase, the CAPS document stresses that the curriculum exposes learners to a variety of important mathematical experiences that give them the opportunities to develop their mathematical reasoning, as well as to prepare them for more abstract mathematics in higher tertiary education institutions (DOE, 2011b p 10).

Despite these curriculum changes the current CAPS curriculum reduces the teacher's responsibility to interpret the curriculum outcomes, but it still leaves the teacher with the responsibility of ensuring that learning takes place in the mathematics classroom. For

Sfard(1997), reform in mathematics education revolves around mathematics teaching hence for this reason; it is quite possible to make use of Information Communication Technologies in mathematics classrooms in ways that help students to access mathematical concepts. Competency in mathematics is regarded as beneficial to both the individual learner and the society. South Africa, just like other developing countries has a shortage of doctors, engineers and mathematics and science teachers as already noticed from their hiring of foreign nationals to fill in that need. Competency in mathematics and science will help alleviate these challenges. If learners are competent in mathematics, then more learners will be able to take medicine, engineering and other science related courses at university to fill this need. Errors and misconceptions in mathematics constitute challenges to reaching curriculum outcomes as this hinder performance in subjects like mathematics that are required as a prior condition in helping alleviate the skills shortage in the country. Teachers need to understand the errors and misconceptions that learners have in order to design appropriate strategies needed in the classroom to help learners understand mathematics better.

Research has long seen the value of studying learner errors and misconceptions in mathematics. The errors that the learners make in the mathematics classroom may be used as resources for teaching and learning rather than be seen as problems that need to be eradicated. If the learners' reasoning underlying their errors are identified and understood, then these maybe used to create teaching and learning opportunities when explored in the classroom (Ben Zeev, 1996; Nesher, 1987; Erlwanger, 1973). If teachers can understand the errors and misconceptions that learners have in mathematics, then they can use these errors to design teaching strategies that enhances mathematical proficiency (Kilpatrick, Swafford and Findell, 1998). With this move, South Africa will be able to address its manpower skills shortages in the fields of medicine, engineering and other science related jobs as more learners will be able to study for these degrees at universities. Research studies have shown that the misconceptions behind the errors learners make are persistent, similar across context and are independent of curricula or teaching methods, and thus, they are seen as normal and possibly necessary steps in the development of mature concepts when used properly in the mathematics classroom (Smith, diSessa & Roschelle, 1993).

With curriculum reform in South Africa turning towards a paperless classroom, where the emphasis is on using ICT (Integrated Communication Technologies) in the learning and teaching of their curriculum, the use of ICT can be incorporated into the teacher's understanding of the misconceptions that learners have to help learners to learn better.

## 1.2 Why ICT?

Knowledge is no longer organized in a chain form whereby the one who knows should teach the one who does not know in the classroom, but, it is now a network process (Spector, (2004)). Everyone, the teachers, the learners, examiners, the community and curriculum developers are equally involved in that network. ICT is an umbrella term that includes any communication device or application, encompassing: radio, internet, computer hardware and soft wares as well as the various services and applications associated with them. ICT in education deals with using computer technologies and the internet as tools for developing collective intelligence (Spector, 2004).Knowledge is available everywhere, but in an uncontrolled form. It is the role of the teacher to help pupils distinguish accurate and valid knowledge, sort it out and organize it for use in the classroom to enhance learning. The teacher also has to design the instruction, using the new tools and resources available through ICT. ICT can be integrated into the whole set of activities of the teaching profession: designing lessons and performing teaching, as well as a new way of communicating and cooperating with others (Spector, 2004).

Using ICT in the learning and teaching of mathematics can help students in a variety of ways, computers can help the teacher provide fast and give reliable feedback to learners that is non-judgmental and impartial in a mathematics classroom. Students using ICT can receive feedback faster than in a classroom where traditional methods are used. When using computers, students can be encouraged to test and modify their ideas as they create their own conjectures. The speed of computers and calculators can enable learners to work with many examples as they can explore different mathematical problems; this can support their observation of patterns and enable them to make and justify generalizations of the patterns they observe on the computer. The use of ICT like the computer in the learning of some mathematical topics, like functions, enables learners to link the formulae to the tables of values as well as to the graph of the function without much difficulty. The computer's ability to change from one representation to the other enables learners to see and understand the different connections of a function. In mathematics, using computers can afford students the opportunity to manipulate diagrams by observing the effects of different transformations diagrammatically and algebraically. Thus, the use of ICT in the learning and teaching of mathematics can help learners to understand mathematical information represented in a variety of ways.

The use of ICT has already been well established in many branches of industry and commerce. In South Africa, there is need to see further development in how ICT can be implemented in the learning and teaching of mathematics. Using ICT in mathematics topics like functions can enable students to be able to interpret and analyze different functional representations with the purpose of helping students to understand them. Hence, ICT can be incorporated into research and into mathematics as a problem-solving tool that can be used in the classroom.

## 1.3 Errors and misconceptions

### 1.3.1Errors

For Harper, (2010), an error as a “deviation from accuracy or correctness, a mistake as in action or speech, belief in something untrue; the holding of mistaken opinions”. An error is thus, a mistake or inaccuracy or it is a can be viewed as condition of deviating from accuracy. An error is referred to as a mistake, slip, blunder, oversight, fault, transgression and misdeed. When learners make errors, they fail to achieve what is intended. However, some of the errors students make result from ignorance or a deficiency in conceptual knowledge, while some of the errors may just result from mere carelessness.

In mathematics, errors are mistakes made by learners because they lack the relevant experience and knowledge related to a topic in mathematical contexts. Thus, errors in mathematics may result from deviating from a correct solution in problem solving. These errors are the mistakes one makes in the process of solving mathematical problems and they are evident in wrongly answered problems that have flaws in the processes that could have generated a right or wrong answer (Young & O’Shea, 1981). Nesher (1987) asserts that errors are “systematic, persistent and pervasive mistakes formed by learners across a range of contexts”. These errors may reveal an inadequacy of knowledge or may result from carelessness due to lack of concentration. Riccomini, (2005), differentiates two types of errors; errors are either systematic or unsystematic.

Non-systematic errors are unintended; these are non-recurring wrong answers that learners can readily correct by themselves without external help (Riccomini, 2005). These non-systematic errors are inconsistent, less predictable and the mistakes are easily corrected when pointed out (Olivier, 1989). On the other hand, systematic errors are persistent and often feature an incorrect routine in a correct method. Systematic errors reveal a faulty line of thinking; and these systematic errors are thus, referred to as misconceptions (Green, Piel & Flowers, 2008; Riccomini, 2005).

### 1.3.2Misconceptions

A misconception is a display of an already acquired system of concepts and algorithms that has been wrongly applied (Nesher, 1987). Leinhardt, Zaslavsky & Stein (1990) define a misconception as an incorrect feature of learners' knowledge that is explicit and often repeatable. These misconceptions often re -appear whenever the same type of problem is presented to the learner in different contexts. They result from the learner's attempt to construct their own knowledge as they attempt to understand the new information/knowledge in their experimental world (Jaworski, 1994). Constructivists acknowledge that misconceptions arise "when a relatively stable and functional set of beliefs held by an individual comes into conflict with an alternative position held by the community of scholars, experts and teachers as a whole" (Confrey, 1987 p 96). In other words, a misconception is a conceptual structure constructed by the learner in an attempt to make sense of the presented knowledge and it makes sense in line with the learner's current knowledge. This new perception by the learner, although incorrect, does not align with conventional mathematical knowledge and thus is referred to as misconception.(Smith, DiSessa & Roschelle, 1993).

When teachers ignore learner's misconceptions, they affect student's success in problem solving. These misconceptions hinder progress in mathematics, and they negatively affect the learning of new mathematical knowledge (Knuth et.al, 2005According to the constructivist perspective, making errors is part of learning hence instruction channeled towards resolving pupil's misconception can further enhance learning. Misconceptions persist even after typical classroom instruction; they can be resilient to instruction designed to address them directly in the classroom (Smith, DiSessa & Roschelle, 1993).

## 1.4 Link between errors and misconceptions

Confrey, (1990) defines misconceptions as a line of thinking that causes some errors. Despite the differences between errors and misconceptions, these two can be linked. Errors usually signify that learners have misconceptions. While errors may be visible in learner's written artifacts, misconceptions are often hidden from the unsuspecting observer (Luneta and Makonye, 2010). There is need for educators to follow up learners' errors to try to understand why they make them, hence in this way educators may establish the misconceptions that learners have. There is a link between the errors learners make to the misconceptions that learners have, as most researchers have already established that misconceptions result from the errors demonstrated by learners (Riccominni, 2005). If educators find ways to understand why learners make mistakes, then they may find ways to engage learner's mistakes to create new knowledge in a more productive way. Learners often make mistakes in mathematics, and judging from their poor performance in examinations learners make lots of mistakes in mathematics. As educators if we understand why learners make errors, then we may be able to design an effective re-teaching plan that can benefit our learners and our society.

## 1.5 The function concept and functions for Further Education and Training (FET) phase in South Africa

Functions have been defined as a relationship between two variables where a change in one variable resultsin a change in the other variable as well (Insook, 1999). Laridon (2007) also states that a function is a relationship between two sets, say C and D such that every element of C (the input set domain) is mapped to only one element of D (output set- range). According to Sfard (1991) a quantity is called a function only if "it depends on another quantity in such a way that the latter is changed when the former undergoes change". From these definitions of functions, it can be asserted that the study of the function concept can thus be regarded as the study of relationships between the given sets as well as the properties of those sets. This means that learners can experience the function concept whenever they consider how a change in one variable affects the change in the corresponding other variable. Although learners first need to focus on identifying the change and what changes (Sierpinska, 1992); learners also need to recognize change within a variable first and then later they need to observe change between variables while at the same time connecting the changes and looking for relationships (Sfard, 1991).

In the South African curriculum for mathematics at the Further Education and Training phases (Grade 10 – Grade 12), learners are required to understand various types of functions: these are quadratic, exponential, hyperbolic as well as inverse functions. They are also expected to be able to be able to find values of the dependent and the independent variable of these functions by solving equations. Learners are also expected to be able to describe and use function values, derive the equation of a function and be able to transform equations of functions to equivalent forms through the manipulation of given functional equations. In this curriculum (CAPS), functions are grouped together with algebra and equations. For instance, in quadratic functions, learners are expected to factorize and complete the square to transform a quadratic equation to its standard form and into its different forms. This aspect of factorizing and completing the square is the algebraic manipulation required for learners to understand quadratic equations representing a function.

Learners are also expected to generate graphs, identify the properties of functions including the domain, range, turning points, maximum points, minimum points, average gradient, intervals on which the function decreases /or increases as well as be able to distinguish between the discrete and continuous nature of functions.

## 1.6 Learner errors and misconceptions in Mathematics

Although errors and misconceptions are related, they are different as already have been pointed out in the above paragraphs. Some studies in mathematics research have revealed that students have many misconceptions about mathematics that interfere with their learning (Posamentier, 1998). Although most of the times misconceptions lead to errors in mathematics, at times misconceptions are hidden in correct answers (Smith et al, 1993). Causes of errors in mathematics can be established by examining the different mechanisms used by learners in obtaining, processing, retaining and reproducing the information in a mathematical task. Various researchers have studied, classified and coded learner errors to understand their nature and causes.

### 1.6.1 Functions and misconceptions

While it is acknowledged that research on mathematics learning and teaching has focused on the very earliest levels of mathematics content, functions and their graphs on the other hand is a topic that generally has not appealed to the research community until later on (Leinhardt, Zaslavsky& Stein, 1990).Functions represent an aspect in mathematics at which a student uses one symbolic system to understand forms of representations. (Algebraic functions & their graphs). Functional relationships are recognized as important in the development of abstract mathematical knowledge. However, the function concept has not been of interest the educational community until recently (Leinhardt et al., 1990). Due to the complexity of the topic, students often misunderstand the variable concept; and how variables enable them to construct mathematical meanings even when dealing with functions (Graham & Thomas, 2000). One of the aspects that has drawn me to have an interest in functions and their graphs is the relationship between its algebraic and graphical representations. These two representations are different symbol and yet they jointly define and describe holistically the mathematical concept of a function. Algebraic representations involve variables. While variables are important to the graphical representations and the relationships in functions(Leinhardt, et al, 1990), learners struggle with understanding functions and their graphs as abstractions. Neither the algebraic nor the graphical representations can be treated as isolated concepts, hence functions together with their graphs is a topic in which two symbolic systems are used to help understand each different form.

Misconceptions are features of a student's knowledge about a specific piece of mathematics knowledge that may or may not have been instructed. A misconception may occur due to over-generalizing correct information learnt, or may be due to the interference that occurs between the subject content knowledge with the learners' everyday knowledge. Several studies have found that students' have inaccurate ideas of what graphs of functions look like (Vinner& Dreyfus, 1989). Outlined below are some of the difficulties that learners encounter when dealing with functions.

Students' difficulties in interpreting graphs of functions have been noted by Elia &Spyrou, (2006). By interpretation, I am referring to the action by which a learner makes sense or gains meaning from a graph/ or a portion of a graph, a functional equation, or a situation. There are many global features of a graph that learners need to be able to interpret; these include the shape of the graph, the turning points and the intercepts. Zaslavsky (1987) suggests that graphs of a quadratic function may seem as though they are only the part that is visible on the drawing, but in fact, a function represents an infinite domain including those parts that are not visible to the naked eye. This can cause confusion to the students, as they tend to believe that only the points they see marked on a graph represent the given function. In a study conducted by Kerslake (1981) he presented learners with a set of ordered pairs and asked them to plot the points and connect them with a straight line. When asked about points that lay on the graph but were unmarked several students believed that there were no points between two marked points on a graph. Students often interpret points as dots than abstract entities (Mansfield, 1985). Hence, they will tend to believe that dimensions of size and space must bound the number of points that would fit between two marked points, and this may lead to misconceptions.

Another difficulty that students encounter with functions is in making connections between different representations that exist between functions. Graphs, table of values and algebraic equations including verbal descriptions of relationships may be used to display a function. In learning about functions, learners should be able to move from one representation of functions to the other, flexibly and accurately. Moving from graphs to their equations would be the more difficult task because it involves pattern detection, while graphing an equation involves, by comparison straightforward steps like, generating ordered pairs, plotting them on the Cartesian plane, and connecting the points with a line. Empirical evidence from studies by Markovitset al, (1986) supports this notion that movement from a graph to its equation is more difficult task when they conducted research with different age groups.

In addition to the studies cited above, although it is possible to examine student's ability to perform translations between graphical and algebraic representations using pen and paper, it is easier to examine theses translations with the aid of computer technologies. With the use of computers, graphs can be generated quickly and accurately, freeing the student of the burden of calculating and drawing. The computer when used in the classroom is a powerful tool that can provide the student is provided with an opportunity to draw many graphs and to view their corresponding equations while at the same time examining the relationships between the graph and its algebraic representation. In another study, Yerushalmy (1997) exposed his learners to computer-assisted lessons over some period of time emphasized the connection between the algebraic and graphical representations of functions and found an average of 88% on the matching task he conducted on his learners. It is with these views that this research intends to establish the nature of errors and misconceptions learners have on functions and to use developed Geogebra software together with u tube videos to enhance understanding on this topic.

The concept of a function is a mathematical representation of many input-output situations found in the real world and it plays a central role in mathematics (NCTM, 1989). This notion of functions as a topic is highly emphasized in secondary school mathematics as evidenced in the CAPS document for Further Education and Training Phase (DBE, 2011d). Functions and Graphs as a topic is introduced to South African learners at the FET phase (Grade 10, 11 & 12). In the National School Certificate mathematics examinations, Functions and Graphs are awarded 35 marks out of 150 marks in paper 1(DBE, 2011 p 9). This means that this topic, Functions, carries 35 marks out of 300 marks, for both paper 1 and paper 2 that constitute the mathematics examination. This topic carries about 11.7% of the marks for the whole examination.

Functions and their graphs are quite significant in the South African Mathematics examinations as they contribute towards the acquisition of the ability to communicate appropriately by using descriptions in words, graph, symbols, tables and diagrams, as outlined in the specific aims of the CAPS document (DBE, 2011a). It is of interest to the researcher that she focuses on the errors and misconceptions that learners make in this topic for it is through the analysis, diagnosis and the reconciliation of these that competence in functions by learners may be achieved. The errors and misconceptions that learners make, can then be used to enhance their understanding before competence can be achieved (Davis, 1984), and this can impart positively on the performance of learners in examinations like the Matric.

## 1.7 Using Information Communication Technology (ICT) in the learning and teaching of Functions

In most mathematics classrooms, calculators as tools used in the classroom have provided learners with a conducive environment in which they gain necessary experiences to construct mathematical concepts. In a similar way, computers could also provide learners with that opportunity. In a technological rich class environment, instruction can change from the teacher being the conveyer of mathematical knowledge to the learners' being involved in their own concept development and problem solving since the use of computers and/or calculators can minimize the burden of lengthy time-consuming procedures that gives room for the teacher to spend most of the time explaining to the learners. (Hennessy et al, 2001)

With regard to functions, the use of computers will allow learners to manipulate functions and their graphs more easily, quickly and accurately than when using pen and paper methods. With the use of computers, sound, pictures, videos and animations if brought into the classroom, makes mathematics lessons more exciting for the learners. Using mathematics software in computers can motivate both the teachers and learners, leading to a deeper understanding of the subject matter and enhancing learning opportunities. The ability to draw many functions on a computer can enable both learners and teachers to study together or separately the influence of certain aspects imposed on different graphs.(Ponte, 1984).Used in the classroom, ICT can be used to reward learners while at the same time allowing them to demonstrate their abilities (Richardson, 2002).

With the increased realization that technology, particularly computers, may help students in learning mathematics and thus help in minimizing the errors and misconceptions that students have on functions, there is need for research to seek ways of teaching and learning mathematics by integrating new or already existing teaching strategies with technology mediated instruction.

The focus of this study is on how the use of Information Communication Technology (ICT) can help minimize learner errors and misconceptions in the learning and teaching of functions in Grade 11.

### 1.8 Statement of the problem

The nature of the function concept with its different definitions and representations present some challenges for learners when they attempt to learn and understand the topic. Research on learners' understanding of functions has shown that it is one topic in which learners face challenges when they attempt to understand it.(For example Tall, 1996; Markovits, Eylon &

Bruckheimer, 1988) Most of the studies done concur that learners have misconceptions and difficulties when learning this topic.

The concept of function is one of the most important in all mathematics as functions constitute a fundamental concept in secondary school mathematics (Klein, 1945; Ponte, 1984).With recent curriculum orientations clearly emphasizing the importance of functions, (National Council of Teachers of Mathematics, 1989), it is important to find ways of helping learners to have a better understanding of this topic.

Most students arrive at secondary school with many difficulties in abstract thinking. Dealing with graphs and algebraic expressions is not an easy task for many learners. Some of the problems cited by my colleagues on challenges that learners face in Functions is not only about sketching graphs, but also on the fact that learners struggle with questions that involve the interpretation of those function graphs. My interest is on the topic functions, where I believe that if learners have a better understanding this can impart positively in learner performance in national examinations like the matric. Poor performance could be due to learners not understanding concepts from the topics in the curriculum (including functions), or errors resulting from the misconceptions they might have concerning a particular topic. Sfard (1992) observed that learners had difficulties connecting different representations of functions and hence these difficulties were also documented in a research by Knuth(2000). It is when learners have difficulties in understanding a particular topic that may lead into them having misconceptions. These misconceptions impart negatively on learner performance in mathematics.

Sajka (2005) views understanding of the functions as having knowledge of its definition, having knowledge of its origin, as well as having knowledge of its basic properties including the different languages and representations related to the concept. This view by Sajka (2005) is in line with the requirements from the South African CAPS document that stipulates what learners should be able to do in line with this topic in particular. Among the many reasons cited in the South African Curriculum and Assessment Policy Statement (CAPS)document), the requirements are that the learners should demonstrate an ability to work with various types of functions and relations by converting flexibly between numerical, graphical, verbal and symbolic representations (DBE, 2011).

My choice of the function topic is influenced by the fact that functions, is a concept important in secondary school mathematics and is central to university mathematics. Internationally, there is a gap to research on learner understanding of functions and in South Africa; there is a need to focus on how teachers can teach the concepts of functions competently. In South Africa, the Grade 10 – 12 phases are known as the Further Education and Training Phase (FET). Mathematics in the FET phase covers ten main content areas one of which is functions (DBE, 2011d). The notion of function is greatly emphasized in the South African CAPS curriculum as observed from the high weighting of the content area: Functions and graphs across Grade 10, 11 and 12 in Paper 1 in comparison to the other content areas allocated for Paper 1. Hence, a good understanding of the Function concept can affect positively on learner performance in Grade 12.

Recently, Gauteng’s Education MEC, Panyaza Lesufi issued most of Grade 12 learners in Gauteng’s’ township schools with tablets, provided their educators with laptops and installed smart boards in their schools (SABC News, 2016). With this move, is the realization that technology, in particular computers, allow for better mediation in the learning and teaching of all learning areas including mathematics. South Africa, with its available resources, can provide their students with opportunities to explore concepts like functions using technology in a more productive way. These technological tools, wisely used in the learning of functions, may help learners to develop a deeper mathematical understanding that would facilitate the process of conjecturing, testing and generalizing. The use of ICT in the mathematics classroom, may also give learners the necessary power to solve problems that are more difficult and suggest multiple links with other domains in mathematics like geometry, algebra, statistics and their corresponding mathematical models. All this can impart positively on the overall performance by learners in mathematics examinations like the Matric in South Africa.

## 1.9 Aim of the study

Problems concerning the teaching and learning of functions are often confronted but often, few teachers know how learners come to understand functions (Mann, 2000; Yoon, 2007). This study seeks to address the difficulties that learners have on functions in mathematics, by using their errors and misconceptions to adequately address their difficulties in the topic. The study explores the nature of the errors and misconceptions learners in Grade 11 have on functions. It investigates the influence of using ICT in redressing the errors and misconceptions that Grade 11 students show onthe functions topic to help learners have a better understanding of the topic. Further, it explores teacher knowledge for using ICT technologies in teaching the functions topic as a way of encouraging teachers to engage learners and their ways of reasoning in addressing their difficulties.

### 1.10 Research Questions

This study seeks to answer the following questions:

- What errors and misconceptions are elicited by grade 11 learners when they engage the topic of functions?

- What is the influence of using computers to address learner errors and misconceptions in functions at Grade 11?

- What teacher knowledge may be drawn from the use of computer technologies in mediating the learning of functions, through learners' errors?

### 1.11 Significance of the study

The Senior Certificate (SC) now National School Certificate (NCS) examinations commonly referred to as "matric" has become an annual event for public significance in South Africa (NCS Examination Report, 2015). Performance in school exit examinations like these, find South African learners' achievement in mathematics a cause for concern.

The aim of implementing the new changes in the curriculum for South Africa was to identify the challenges and pressure points that imparted negatively on the quality of teaching mathematics in schools. This was done to propose new strategies that would address the problems (DOE, 2011). However, change has to be understood because the change in the curriculum must also for the good bring a change to those who are part of the change – Learners. If learners do still underperform, then it is important that as educators, we try to understand why learners underperform in mathematics.

Poor performance in the examinations may be attributed to the errors and misconceptions that learners have in mathematics. When students learn mathematical concepts without understanding them, they tend to make errors due to some misconceptions. Learner errors and misconceptions in mathematics affect learner performance in general and this can influence negatively on their mathematics results. However, the mathematical errors of learners reveal important hints that could help educators establish the nature of learners' cognition as they try to understand mathematics in their own inappropriate way. Learners in South Africa often manifest many difficulties as evidenced by low achievement in both national examinations as well as in international comparison tests (Howie, 2001). In relation to the learning of mathematics, the National Council of Teachers of Mathematics (2011), emphasizes technology as an essential tool and important component of a high-quality mathematics education, this can provide access to mathematics for all students. However, there is an increased realization that technology, particularly using computers, may help students in learning mathematics and thus help in minimizing the errors and misconceptions that students have in mathematics. There is need for research to seek better ways of teaching and learning mathematics by integrating teaching in the mathematics classroom with strategies that make use of technology. South Africa, has just realized the need to incorporate technology into their education system. With this need comes the challenge of how this technology can be incorporated into teaching without further compromising the standards of teaching and learning. For these reasons, the researcher is selecting errors and misconceptions as a window to help resolve mathematics-learning problems in functions at grade 11 with the help of computer mediation.

Research on mathematical errors and misconceptions has been done on various mathematical topics like focusing on arithmetic operations on whole numbers (Brown & Burton, 1978), algebra (Quinn & Brown, 2006), Calculus (Monson & Kevin, 2001) and Differential Calculus (Makonye, 2011). This evidence show that researchers have long recognized the value of examining learner's errors and misconceptions but further research needs to examine the errors learners make so that they can be used in their classroom as a way of trying to help learners in mathematical proficiency. Hence, for this study, I choose to focus on a mathematics topic: functions, in Grade 11.

## 1.14 Layout of the study

Chapter 1 introduces the study by describing its background. The concepts used in this report writing are explained, these are errors, misconceptions and ICT.The function concept dealt with in this study is also explained together with functions for Grade 11 in the South African curriculum. A brief overview of the learner errors and misconceptions in mathematics is discussed together with a discussion on learner errors and misconceptions in the functions topic. The importance of using ICT in the learning of mathematics is highlighted in general and also included in the discussion is why it is important to use computers in the learning and teaching of functions. The statement of the problem is outlined. This is followed by explaining the aims of the research from which the research questions of the study are formulated and then last but not least, the significance of this study is outlined.

Chapter 2 explains the theories that inform this study in detail. These are Constructivism, socio-cultural learning theory, Variation theory and the framework of Technology, Pedagogy, and Content Knowledge (TPACK). Also outlined in the literature review is how different researchers have classified errors. The role that technology plays in the teaching and learning of mathematics is discussed, together with its impact in mathematics education.

Chapter 3 describes the underlying epistemological assumptions for this study, study site and setting, the sample and sampling techniques, ethical issues, research design, data collection and data analysis methods.

Chapter 4 presents and discusses the findings of this study within the context of the literature review and theoretical framework that guides this study. Patterns or themes emerging from the results are discussed for their relevance to the research questions.

In chapter 5, the researcher will give the conclusion of the study by reflecting on the research design, and methodology; revisit the data collected and draw conclusions based on the findings of this investigation. Based on this study, the researcher will make recommendations for classroom practice and future research.

## 1.13 Conclusion

There are many different approaches can be used to help learners to develop a better understanding of functions and to help overcome different known learner difficulties in the topic. For this study, ICT is to be incorporated with the learner errors and misconceptions identified in the study to see the extent to which it can help eradicate these to enhance mathematical proficiency. This chapter has presented the reader with an overview of the study. What follows seeks to provide in detail each stage of the journey for this investigation. The chapter that follows will examine the theories and concepts in which this study will be framed as well as literature related to the use of errors and misconception in the teaching of functions using Information Communication Technologies.

# CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

## 2.1 Introduction

This study seeks to investigate errors and misconceptions learners have on Functions in secondary school mathematics with a view to explore the nature and origin of these as well as to use Information Communication Technology (ICT) to try to remedy them. The focus will be on learner errors and misconceptions on functions and on the effectiveness of using computers as a technological tool to try to minimize them. Reviewing the literature and situating my research problem in the wider research literature is an important component for my research. The theories informing this study will be discussed first in detail and then will be narrowed to show exactly the aspects of the theories to be incorporated into this investigation. Firstly, to give the reader the purpose of this chapter, I shall define what a literature review is.

## 2.2 What is a Literature review and what is its purpose?

Fraenkel, Wallen and Hyun (1993) define a literature review as a critical analysis of identified documents containing information related to a specific research study. The literature reviewed is identified and analyzed from documents like articles, books, conference proceedings and journals to provide the researcher with a description, summaries and descriptions of what other researchers have done in line with the research problem that needs to be investigated. The purpose of reading this literature is for the researcher to see what has been done, what needs to be done and what gaps needs to be filled in line with what the researcher intends to research on. By reviewing the literature, the researcher is able to identify new and similar ways and to shed light on any gaps or similarities from what other researchers have already done. A good literature review will enable the researcher to support and justify the reasons for studying a particular area using the literature that is already available in research education.

For these reasons, a literature review of studies related to learner errors and misconceptions will help me review and relate these to my research since the area of focus is also on learner errors and misconceptions. Literature on the use and the influence of use of ICT in the learning and teaching of mathematics will also be reviewed and reported on to provide a link and a justification on why ICT should be used to address learner errors and misconceptions in the mathematics classroom.

In reviewing the literature, it is important that in this research I establish the concepts and the theories that supports and inform my study, hence a conceptual framework and a theoretical framework influencing my study will thus be outlined. Concepts and theories influencing my choice of research will be outlined as they help show the readers how I, as the researcher choose to see the world in general and then more specifically, how I choose to view the research problem I intend to investigate. Liehr and Smith (1999), define a theory as "a set of interrelated concepts, which structure a systematic view of phenomena" p.8. Thus a theory describes some observations on the basis of some prescribed model. In other words, a theory will give a clear picture of the events it seeks to explain within a given frame. Chinn and Krammer(1999) see concepts as those parts of a theory. This then means that a theoretical framework is the frame the researchers chooses to guide them researches, while a conceptual framework allows the researcher to choose those parts in the theory that should guide one's research (Imenda, 2014). For Evans (2007), there is no difference between a conceptual and a theoretical framework, as these two serve the same purpose. They both serve guide the researcher by showing how a research fits into what has already been established and to show how the research will contribute to the topic at large respectively (Maxwell, 2005),

My intention is to find out learner's reasoning for their errors and misconceptions on Functions; hence, my focus will be on individuals' construction of knowledge. I will enquire into learner's current thinking processes based on what they would have constructed through their experiences in the classroom and through other learning processes. Although there might be some uniqueness in the learners' individual constructions, it is also very possible for learners to share the same modes of thought. For these reasons, constructivism for me, will provide me with the most appropriate and compatible ways to study learner cognition under these conditions. For the purpose of this study, I will define the theories that guide my study, but in so doing, I will also specify the particular concepts within those theories that will shape my investigation.

This literature review, will "serve as the driving force and the jumping off point" for my own research investigation (Ridley, 2008). I will use the literature to further supports why the problem I have identified is an important problem in mathematics education to be investigated. Iwill also illustrate using literature, the gap that needs to be filled from previous research and to justify why there is need for that gap to be filled. It is important in my literature review to look at research on errors and misconceptions in mathematics in general, before coming down to algebra and ultimately functions. I will also review literature on teaching interventions in general as well as interventions using ICT. In so doing, the learning theories will provide for me the lens that I will use to explain and understand my literature in the context of this study. However, it is possible for the lenses to draw certain areas closer to the eye while ignoring other aspects, hence, for this reason it is might not be easy to find a theory that accommodates every aspect about learning functions in this study (Makonye, 2010). My investigation will thus be informed by the following theoretical frameworks: constructivism, socio-cultural theories, variation theory and the technological pedagogic content knowledge theory; the concepts in those theories that direct my study will also be differentiated within the theories as now outlined below.

## 2.3 Constructivist Perspective on Teaching and Learning

Constructivism as a theory, applies both to learning (how people learn) and to epistemology (the nature of knowledge). The guiding principle for this research's enquiry rests on individual construction of knowledge; therefore, I will set forth a definition that will serve as a foundation of how students make mathematical misconceptions. The concept of constructivism that I align my investigation with is that by Piaget (1968) that view learning as occurring in the mind of a learner, because it is through this domain that errors and misconceptions are created by the learner in the classroom.

The essential core of constructivism is that knowledge is actively constructed by individuals as they make sense of the world based on their experiences (Piaget, 1968). These misconceptions emerge from the learner's prior knowledge in the mathematics class or as the learner interact with the environment outside the mathematics classroom (Smith, diSSesa & Roschell, 1993). Constructivism rejects the idea that knowledge is passively received by the individual from one person to other (Gray, 2007) but sees the learner as a responsible participant in the creation of his/her own knowledge. One of the attributes of a constructivist educator is that s/he is able to embrace each learner's unexpected response with a genuine interest of wanting to learn its origin, and its implication (Confrey, 1990). It is within this constructivist paradigm that this investigation seeks to understand the misconceptions that learners have when learning the functions topic.

The general principles of constructivism that inform this research are based largely on Piaget's processes of assimilation and accommodation. For Piaget (1970), the process of constructing knowledge has to undergo to main stages- assimilation and accommodation Assimilation occurs when a person perceives a new object in line with their already existing knowledge. This happens when a learner actively incorporates his/her new experience to the awareness that already exists for that learner in his/her environment. Accommodation on the other hand, occurs the learner tries to modify the existing cognitive structures to make room for the new information

As a person interacts with the environment,(say learning a new concept in the mathematics classroom)and mental structures(with their thinking), one of two things can happen. Either the perception of the environment is changed in order for new information to be matched within the existing structures through assimilation, or the cognitive structures themselves can change because of the interaction through accommodation. Assimilation occurs as the child actively incorporates new experience into a presentation already available yet when it happens, misconceptions can occur due to over generalizing. Constructivism, derived as a part from the work of Piaget asserts that each person based on his or her own experience must construct conceptual knowledge.

The constructivist perspective of learning postulates a learner's mind as the primary unit of learning (Piaget, 1968; Siegler, 1995; von Glasserfeld, 1989). For von Glasserfeld (1989), knowledge cannot be transferred as it is from one person to another, but, knowledge is actively constructed and mediated by a thinking agent and the major role of the agent is to adapt and reorganize the experienced world in and with his/her mind. Misconceptions in learners usually originate as learners incorrectly generate that prior knowledge to overcome the difficulties they encounter with new tasks in a learning environment (Nesher, 1987). Before the teacher teaches learners new concepts, these learners already have their own concepts different from the concepts accepted in mathematics (Smith et al,1993). The role of the teacher is thus, to assist the learner by providing physical or mental models on which the learner can impose and abstract mathematical meaning by him/herself. These misconceptions are characteristic that the learner's existing knowledge is inadequate and supports only partial understandings (Smith, et.al., 1993). It is when learners display error that a teacher can seek to understand the misconceptions so that these students are helped to learn by transforming and refining that incorrect knowledge into more sophisticated forms of understanding mathematical concepts.

The theory of constructivism maintains that learners are active meaning makers who continually construct their own meanings from ideas communicated to them by negotiating connections between new information and their own existing knowledge base. For constructivism, errors and misconceptions are perceived as natural outcomes of individual learner's cognitive efforts to make sense of mathematical challenges. It is through this constructivist perspective that the researcher may interpret and examine how and why learners construct errors and misconceptions on the functions topic. If students are allowed to do their own thinking, although their first attempts are usually ineffective, with time, they invent increasingly efficient procedures just as the generations before our time have done.

## 2.4 The socio- cultural theory

Socio-cultural theories are based on the social constructivist paradigm which considers that knowledge is constructed socially through interaction and shared by individuals (Bryman, 2001). Learning and development are embedded within social events as learning occurs as the learner interacts with others in a collaborative learning environment (Vygotsky, 1978)

While the researcher presumes that cognitive constructivism, helps explain the learning and forming of misconceptions in mathematics, socio-cultural learning theories can help explain that learning cannot be separated from the environment in which the learning takes place and also explains how learning is assisted or mediated by a more knowledgeable other through signs, tools and cultural artifacts such as language to create uniquely forms of higher level of thinking. ( Vygotsky, 1978).

Vygotsky suggests that knowledge is constructed in the midst of our interactions with others and is shaped by the skills and abilities valued in a particular culture. The socio-cultural theory has explored four aspects of human cognitive development: the mind, tools, ZPD (zone of proximal development) and community of practice (Wertsch, 1979).

According to Vygotsky (1978), the mind is socially distributed and it operates beyond people. Its functioning depends on our interaction and negotiation with others, and is affected by factors like the environment, context and history. The socio-cultural theory of learning, considers learning to be a social activity which is very dependable on tools and interaction (Wertsch, 1991, Cole, 1996). Such tools include language, various systems of counting, mnemonic techniques, algebraic symbol systems, diagrams, maps and mechanical drawing (Vygotsky, 1981c). Human understanding depends on the tools in use as mediating artifacts. The tools mediate the reality and help us interpret and construct an understanding. Although language is a dominant tool, a cultural tool that is shaped in social interaction; the cultural signs and tools a child makes use of do not only affect the content of thinking, but the structure and system of thinking (Vygotsky, 1978). In other words, these tools help people develop their minds to a higher level of thinking. It is with this view that this research seeks to use computers as a tool to help minimizing learner errors and misconceptions in functions. The tools mediate the reality and help us interpret and construct an understanding. These tools help people develop their minds to a higher level of thinking. The computer will expose learners to a graphing software Geogebra and some functions lesson videos on you tube.

Vygotsky (1978) invented the term ZPD (zone of proximal development) as an approach to the problem that learning need to be matched in the same manner with the child's level of development. According to Vygotsky (1978) the ZPD is "the distance between the actual development of the learner as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with a more capable peer" (p.86). Through the assistance of a more knowledgeable other, a child is able to learn skills that go beyond what a child can learn on his/her own. This "zone" is the area of exploration for which the student is cognitively prepared but requires help and social interaction to develop fully (Bruner, 1966). It is within this ZPD where errors and misconceptions might occur and this is where assistance or mediation by a competent teacher (knowledgeable other) is essential. Through the assistance of a more capable person, a child is able to learn skills or aspects of a skill that go beyond what a child can accomplish on his/her own (the child's actual development level). If teachers assess learner's errors correctly, then teachers will be able to determine teaching techniques that effectively facilitate learning. It is with this aspect of the socio-cultural perspective that the researcher will be able to infer into why learners have misconceptions in functions and to use a semiotic tool (computers) to help learners reach their ZPD.

However, if teachers fail to assess learner errors and difficulties, they constrain the learning for the child. It is only when the teacher has knowledge about his/her learners' thinking and is knowledgeable in the content that she/he can facilitate learning of that content. It is with this socio-cultural perspective that the researcher will be able to infer into why learners have misconceptions in functions and to use semiotic tool to help learners reach their ZPD.

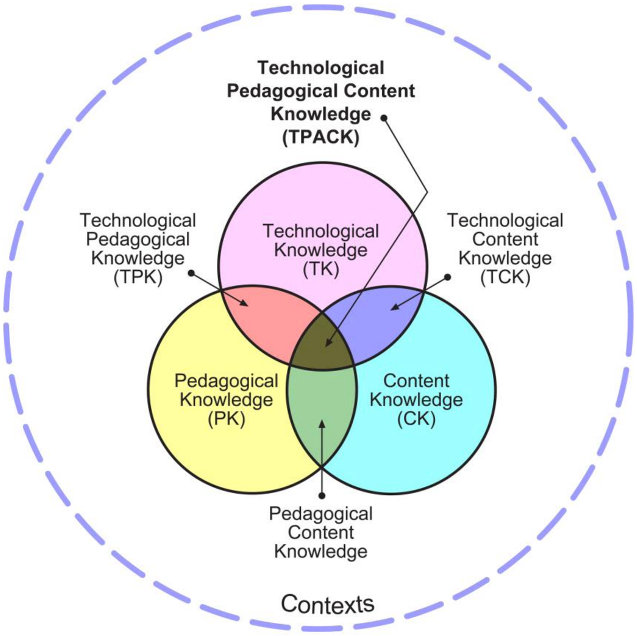
While people can develop their cognition with the tools and reach their ZPD, their learning takes place with the help of collaborative learning. A community of practice, say is a group of people who are recognized as having a special expertise in some area of significant cultural practice (Nut hall, 1997). This community of practice would regard learning mathematics as a process of becoming a member in the community of mathematics (Mason, 2007). Vygotsky (1978) believed that everything is learnt on two levels: first the interaction with other people using signs and tools, and then later integrated into the individual's mental structures. The socio-cultural theory's major theoretical framework is that social interaction plays a fundamental role in the development of cognition (Vygotsky, 1978). It is for this reason that in this research, after identifying errors and misconceptions learners have on function, a community of learners interacting with computers is established to explore computer software: Geogebra and some u tube videos on functions as a way to enhance understanding of functions.

In order to align mathematics classes that use technology, a focus on the effective use of ICT will enable both the teacher and the learners to be equally involved in the network process of teaching and learning mathematics.

### 2.5 Technology, Pedagogy, and Content Knowledge (TPACK) Framework

The theoretical perspective that underlies this work is based on a view of learning as a social process in which tools are seen as critical, mediating elements that have a direct effect on classroom practices (Cobb and Yackel, 1992). TPACK was called TPCK until 2008 when some researcher proposed using the term TPACK which is easy to pronounce (Thompson, 2008).

**Figure 1: The Technology, Pedagogy and Content Knowledge (TPACK) framework and its components**




(*Adopted from Koehler & Mishra, 2008)*

TPACK as a framework is an understanding that emerges from interaction among content, pedagogy and technology knowledge. The relationship that encompasses technology, pedagogy and the content knowledge in this framework goes beyond the understanding of each component separately, but first I will explain each of the components of the framework as shown in figure 1 in the paragraphs below.

Technology Knowledge (TK) refers to the knowledge about various technologies, ranging from pencil and paper to digital technologies such as the internet, interactive white boards and software programs. Koehler& Mishra (2008) do not distinguish the type of technologies encompassed within this framework and as such, they include all technologies including pens and chalkboard. For this reason, in the classroom situation for teaching to occur, one has to use one or more of the mentioned tool. As such for this research, the tool I want to utilize are computer programming software (geogebra) and u tube videos on the internet in the learning of functions.

Content knowledge (CK) refers to the teacher's knowledge about the subject matter to be learned or taught (Mishra & Koehler, 2006). This knowledge includes the knowledge that the teacher has on the subject matter to be taught or learnt including the knowledge of using well developed approaches to convey this knowledge to the learners (Shulman, 1986).

Pedagogical Knowledge (PK) refers to the teachers' knowledge about the effective methods of teaching to be used in the classroom. Teachers with an understanding of pedagogical knowledge, understand how learners construct knowledge and as such plan their lessons to address the needs of their learners in the classroom (Mishra & Koehler, 2006).

Pedagogic Content Knowledge (PCK) refers to the content knowledge that is applicable to the teaching of a specific content subject (Shulman, 1986). Pedagogical Content knowledge blends both content and pedagogy with the goal being to develop better teaching practices in the content areas, for example, the teacher's awareness of the errors and misconceptions learners in their classroom have shown that the teacher has good PCK and in turn this teacher tries to find ways of addressing them

Technological Content Knowledge (TCK) refers to the understanding how knowledge of technology impacts the practices and the knowledge of a particular learning area. (Shin et al, 2009). It is an understanding of how technology and content can influence or constrain one another in any given classroom situation. There is need for teachers to have a deep understanding of how teaching in the classroom can be changed by using different technologies. By using certain specific technologies in the classroom, teachers can constrain or enhance learning for the student; this knowledge of how technology affects the content is very important, they can change the way learners see and understand certain concepts in a specific content area.

Technological Pedagogic Knowledge (TPK) is the understanding of how teaching and learning can change when particular technologies are used for particular subject areas in the classroom (Koehler & Mishra, 2009). The knowledge that each form of technology requires a different way of teaching in the classroom is very crucial for the effectiveness of learning and teaching in the mathematics classroom.(Shin et al, 2009).

Thus, Technological Pedagogic Content Knowledge (TPACK) refers to the knowledge required by teachers for integrating technology into their teaching in any content area. This includes the knowledge of how technology can help redress the errors and misconceptions learners have in various topic in mathematics and how technology can be used to address these (as is the case with this study) to strengthen learner understanding in mathematics. The TPACK framework, offers several possibilities for teacher's use of technology. In this research study, it offers options for integrating two complex phenomena: redressing learner errors and misconceptions with technology in open ways. This framework allows both teachers and learners to move beyond assumptions that treat technology as an add-on but instead to focus on the connections among technology, content and pedagogy as they unfold in the classrooms.

With computer technology, computers can help students develop algebraic thinking by allowing them to explore variables and relationships among variables (Pugalee, 2001). For these reasons, in this study, technology will be incorporated in the learning of functions as a means to try and redress the errors and misconceptions that student have in the topic. It is with the TPACK framework that technology will be used to support the teaching of functions, with the intention of helping students to gain a deeper understanding of the topic.

### 2.6 Variation theory

After identifying learner errors and misconceptions on functions, there is need for intervention to bring about a new experience of functions in a bid to curb those misconceptions. The variation theory of learning espouses that an effective way of seeing a phenomenon by learners is that they must experience important variations of that phenomenon (Marton, 1998). In order for learners to develop a deeper or a new understanding of functions, they should experience important variations of the function concept. Variation is about changing some aspects of the concept while at some time also not changing anything so that learners can make sense of a concept in the learning environment (Leung, 2012).

According to Oliver and Trigwell (2005), for learning to occur, *the learner must experience variation*. If there is no variation, then there is no discernment. Normally one does not attend to a thing that is always the same. Although one cannot guarantee that learning will occur in any given situation, but, one can judge whether or not the conditions for learning were made available for the learners in the classroom (Runesson, 2005) A mathematics pedagogy that is rooted in variation is one that purposefully provides learners with the means to experience learning that is directed at something (e.g. a phenomenon, object, skills, or a quadratic graph), and that learning must result in a qualitative change in the way of seeing something."Powerful ways of acting originate from powerful ways of seeing" (Marton and Tsui, 2004, p. 7), and as such , a learner seeing something in a new way is likely understands and deals with that thing or concept)in a more powerful way. This thing or concept at which learning is directed is called the object of learning (Olteanue & Olteanue, 2012).

In order to bring about learning, it is necessary for teachers to create specific patterns of variation and let students experience the variation (Pang, 2002). As a pedagogical approach, a pattern of variation is a useful tool for structuring teaching to make the learning of the object of learning possible.

There is no learning without the object of learning (Olteanue & Olteanue, 2012). Whether students learn or they don't, and how students learn depends on the "lived object of learning" (Marton et al, 2004).Thus, the teaching and learning relationship depends on what is made possible in the lesson (by the teacher, and sometimes by other students as well) and what possibilities are actually made use of by the students themselves in their learning environment. "No conditions of learning ever cause learning. They only make it possible for learners to learn certain things" (Marton et al, 2004, pp. 22-3).The object of learning is the focus on a teaching situation.

In variation theory the object of learning is examined from three different perspectives: the intended, the enacted and the lived object of learning. The intended object of learning refers to the capabilities the teacher wants to develop a learning environment (Runesson, 2005) it is that part of the content treated in the classroom that the teacher addresses for students to learn. Central to this position is that the learner and not the teacher is the one who should experience that which needs to be learnt in the classroom. Although this is a description of what the teacher wants the students to learn it does not guarantee that the student will experience it in order to learn it. In this study, after identifying the errors and misconceptions that learners have on functions from the pretest, the errors identified will serve as a guide on how the teacher uses intervention, not only to address these but also to help learners have a better understanding of functions.

The “enacted object of learning” is what is afforded to the learners; it refers to what is possible for learners to experience within a learning environment. This is a description of how the teacher structures classroom experiences to make it possible for the object of learning (in this case functions) and its critical features to become more visible. In the classroom, it is very important that the teacher bring out the critical features of the object of learning into the students’ focal awareness. In this investigation, the errors and misconceptions identified will guide the researcher as to which videos to use in the classroom from the internet to make it possible for learners to rectify their misconceptions. It is those selected videos from the researcher’s viewpoint that will make it possible for learners to learn about more about functions and to rectify their misconceptions. Because variation theory states that critical features can only be discerned when learners experience variation in their different dimensions, these dimensions of variation will be further elaborated on when the researcher explains how these dimensions are to be experienced by the learners in functions.

The lived object of learning is what was learned from the learner’s perspective (Runesson & Marton, 2002). The students’ initial level of capability to the appropriate object of learning as well as the way in which students understand the object of learning is the lived object of learning. Variation theory seeks to account for differences in learning and describes the conditions that are made necessary for learning to take place.

This study, in its teaching intervention applies the key features of variance and invariance by making use of the patterns of variation. Variance involves explaining the same concept in different ways while invariance involves contrasting what is an object is and what it is not(Marton & Pang, 2006). Marton, (2009), identified four patterns of variation; these are *contrast*, *generalization*, *separation* and *fusion*. After identifying the errors and misconceptions that learners have on functions, these patterns of variation will be incorporated into the teaching intervention as now discussed in more detail in the paragraphs that follow.

### 2.6.1 Contrast

It has been asserted by Marton et al, (2014) that-

*‘‘... in order to experience something, a person must experience something else to compare it with.’’* (p. 16)

In other words, varying forms of the same aspect produce this pattern of variation. The forms of quadratic functions that are in contrast to one another are:

$f(x) = ax^2 + bx + c$ $g(x) = a(x-p)^2 + q$ $h(x) = a(x-x_1)(x-x_2)$

In the teaching intervention for this study, using this dimension of variation exposes learners to the different forms of quadratic functions. Learners need to able to distinguish from the three forms and distinguish the features of the quadratic function conveyed by each form.

### 2.6.2 Generalization

Marton et al, (2014) state that''… in order to fully understand what ''three'' is, we must also experience varying appearances of ''three''... (p. 16)

With reference to functions, the generalization of the standard form of the quadratic function is due to the various operations performed on the function. The function in this form:
$f(x) = ax^2 + bx + c$ can result in $f(x) = a(x-x_1)(x-x_2)$ after completing the square.

When given critical points and when required to find the equation of a function, it is important that learners chose an appropriate generalized form to find the equation of the required function. This dimension will be implemented in this study.

### 2.6.3 Separation

*''In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant''* (Marton et al, 2004 p. 16)

In order to experience certain aspects of something and to separate it from other aspects, the aspect to be experienced must vary while other aspects remain invariant. With reference to quadratic functions, I cite the examples below:

$f(x) = x^2 - 5x + 6$ $g(x) = x^2 - 5x + 4$ $h(x) = x^2 - 5x - 6$

Separation is a premised on the view that everything has a multitude of features, each of which give rise to a different understanding of that thing. This dimension of variation will also be incorporated in the teaching interventions to redress the errors and misconceptions that learners have in functions.

### 2.6.4 Fusion

‘‘If there are several critical aspects that the learner has to take into consideration
at the same time, they must all be experienced simultaneously.’’ (Marton et al, 2004 p. 16)

Fusion, thus, integrates critical features of variation into a whole under simultaneous co-variation. Although it is important that learners at times focus on different aspects of an object when learning at the same time, at times is also important that learners connect various variation experiences gained in previous and present interactions. These varying critical aspects of the object (function in this case) have to be experienced by the learners simultaneously as outlined in this example of quadratic functions where more than one parameter is varied at the same time:
$f(x) = 2(x-1)^2 + 4$ $g(x) = (x - 1)^2 - 4$ $h(x) = - (x - 1)^2 + 4$

Olteanu (2007) identified two ways to open up the patterns of variation; convergent and divergent. With a convergent variation, different aspects are directed to the whole of the object of learning. These aspects consist of the object’s parts and the relationships between them. According to Olteanu & Olteanu (2012), this variation leads to a positive development in students’ learning. However, in this study due to the short time frame of the intervention lessons, this might not be possible as this will require more time. A divergent variation means that the whole of the object of learning is presented first and afterwards the parts that constitute it, without first discerning the parts in question.

With the application of the theory of variation, the four different patterns of variation and invariance will be created in classroom settings using divergent variation, to bring about a holistic understand of functions as learner errors and misconception will be addressed. To bring out these dimensions in the teaching intervention, the theory of variation will thus be incorporated with the use of computers.

## 2.7 Identifying learner errors from the different classification of errors

Research in mathematics education has been carried out to try to establish the types of errors learners have in mathematics. From literature, the researcher cites some examples of how researchers have coded and classified errors from the research conducted to help establish learner misconceptions and to map ways to help proficiency in mathematics.

Radatz, (1979) classified errors as a part of misconceptions into five main categories:

**Table 1: Radatz's classification of errors**

| **Error Number** | **Explanation** |
|---|---|
| 1 | Language difficulties |
| 2 | difficulties in obtaining spatial configuration |
| 3 | deficient mastery of prerequisite facts and concepts |
| 4 | Incorrect associations or rigidity of thinking |
| 5 | Application of irrelevant rules or strategies |

Watson (1980) conducted research using the Newman model, which stated that all errors could be placed in one of the following categories:

**Table 2: Watson's classification of errors (Watson, 1980)**

| **Category** | **Type of errors** | **Explanation** |
|---|---|---|
| 1 | Reading ability | Can the pupil read the question |
| 2 | Comprehension | Can the pupil understand the question |
| 3 | Transformation | Can the pupil select the mathematical process, which is required to obtain the solution |
| 4 | Process Skills | Can the pupil perform the mathematical operations necessary for the task |
| 5 | Encoding | Can the pupil write the answer in an acceptable way |
| 6 | Motivation | The pupil could have solved the problem had he/she tried |
| 7 | Carelessness | The pupil could do all the steps but made a careless error, which is unlikely to be repeated |
| 8 | Question Form | The pupil makes an error because of the way the problem has been presented |

Luneta and Makonye (2010) also categorized errors into five codes as outlined in the table below:

**Table 3: Luneta and Makonye's(2010) coding and categorizing of errors**

| Code | Description with examples chosen from this study |
|---|---|
| 0 | Non - systematic errors. These are slips, lapses or unintended mistakes. |
| 1 | Generalization or transfer errors. These refer to extension of previously available strategies in new situations where they do not apply |
| 2 | Ignorance of rule restrictions or symbolism. Applying rules to contexts, they do not apply. Failure to understand the bounds where a rule applies. |
| 3 | Incomplete application of rules |
| 4 | False concepts hypothesized to form new concept. |

Coding and categorizing errors helps educators to identify the type of errors held by their learners. Hence, this strategy will be used in this study to identify the nature of the errors and misconceptions learners have in the topic functions. Barrera et al, (2004) reported that errors caused by a lack of meaning could be differentiated into three stages: algebra errors originating in arithmetic; use of formulas or procedural rules inadequately (procedural errors) and errors due to the properties themselves of algebraic language. Many misconceptions apparent in algebra are rooted in misconceptions of arithmetic. Several studies have shown that students as well as teachers have problems understanding numbers or discriminating numbers leading to misconceptions about numbers (Abdullah & Saleh, 2005).

Both the number concept and the algebraic concept are linked to functions. The learning of algebra has received more attention at the middle school level where the transition from arithmetic to algebra occurs. Compared with the goal of arithmetic, which is to find the answer, the focus of algebra is to find the general method and use algebraic symbols to express these in a general form (Booth, 1988). The reasons for difficulties during the transition were investigated from the viewpoints of cognitive development (Hart, 1981), the use of algebra notations (Booth, 1984, 1988), and understanding of fundamental concepts like variable and function (Usiskin, 1988). Specifically, for algebra, there is little firm evidence to support students' errors caused by their mental representations or misconceptions. Much less, known is information about students' errors on specific and fundamental mathematics concepts, especially variable, equation, and function. Most researchers and even textbooks (Graham & Thomas, 2000; Schoenfeld, 1985& Arcavi, 1988) overlook the concept of variable, which is a foundation of advanced mathematics and a basis for the transition from numbers to algebra. McNeill and Alibali (2005) found that the mechanisms underlying children's difficulties with equations and the ultimate emergence of correct strategies were not well documented.

Likewise, although some researchers (Sfard, 1992; Dubinsky & Harel, 1992) have conducted research on functions, few studies about using ICT to address learner errors and misconceptions in high school level have been conducted. Given that middle school students are in the critical stage of transition from arithmetic to algebra, it is important to know the difficulties, the errors or misconceptions that middle school students harbor. If research findings show that students have misconceptions in these topics, then there is reason, enough to believe that exploring learner errors and misconception in functions can help teachers understand error patterns in arithmetic and algebra

In this study, in trying to understand the types of errors learners have in functions, connections will be made to literature already available. As a novice researcher, I have the responsibility of making use of what is already available to try to make sense out of the learner errors so as to establish learner misconceptions. Hence, in data analysis, reference to the types of errors already identified will be referred to.

## 2.8The role of Technology in Teaching and Learning Mathematics

The first uses of technology use of computers relied on their role as aids in numerical calculations, however in this 21st century most of the development has been channeled towards using computers as tools for manipulating graphic images. In this section I outline the role of technology with reference to the topic of my research, which are function, to outline the possibilities that can be enabled by the use of computers in a mathematics classroom.

The use of computers in a mathematics classroom makes it possible to graph different functions defined by different algebraic rules at once. In this regard, computers bring the potential of enabling learners to see visual representations for abstract mathematics ideas, thus, helping make sense of the abstract mathematics in an alternative way. There is hope that access to graphs on software programs like Geogebra will enrich learners' understanding of the algebraic form of the function, while at the same time it gives the visual images of the symbolic information (Fey, 1989).

Geogebra software and other function graphing software enable learners and teachers to enter rules for one or more functions, change the color of the different functions, chose the domain and the range and watch as the high resolution graphs are plotted by the computer (Sutherland, 1990). This can be exciting to the learners in a usually boring mathematics traditional mathematics classroom. This alternative visual method in mathematics problem solving can enhance students understand. In this computer rich environment, the role of both the learner and the teacher can shift.

The teacher's role changes from being the conveyer of information, attempting at most of the times to demonstrate to the learners how the graphs of functions are drawn to discussing questions with the learners on the algebraic situation given by a particular graph or any of the features of the equations of functions in their different forms. Alternatively, the learner's role in the classroom can also shift from plotting points and joining the point to form the graphs, to writing and discussing about the key points or the global features of the graphs drawn by the computers. Kulik (1994) drew conclusions from the studies he carried out and found that students learn more when they receive computer based instruction. When using a function graphing software one can vary the parameters in the function rule and observe the resulting transformations of the graph. The numeric, graphic and symbolic manipulation tool by the computers each offer unique kinds of insight in the leaning and teaching of functions (Fey, 1989).

Although textbooks and the chalkboard do also rely on graphs to illuminate the properties of algebraic expressions and functions in mathematics, the computer makes representations of these mathematical ideas and procedures in a more dynamic way no textbook or chalkboard diagram can do. With computers, one is able to change the parameters in a function rule and simultaneously be able to observe the change in the properties of the graph, something that is not easy to display with the chalkboard (Fey, 1989)

It has been argued that using computers plays a role in helping move students from concrete thinking about and idea or procedure to an ultimately powerful abstract form.

In this 21st century, where learners have access to ICTs at home, bringing ICT into the classroom makes the classroom environment a familiar lovable place for the learners. Most of the learners now have cellphones and have access to the internet outside of the classroom; it is possible that if learners enjoy computers then there is a possibility that they will enjoy the mathematics and this in turn can impart positively on the mathematics results. The richness that technology brings, if brought into the classroom and linked to a content subject area can make learning exciting for learners. The sound, the animation, the pictures and videos will allow learners to visualize mathematics in a positive way. Using computer technology, pupils can transform a graph and watch the algebraic symbolism change or alternatively manipulate the symbolism and watch the graphical representation changing  Using technology in the classroom provides students with access to a variety of sources of information, instead of the learner being just dependent on the teacher and the textbook, the learner can also have different views from the internet and it is possible that when s/he does not understand from one source, the other source can clarify. Computers and mathematical software help allow the learner to visualize phenomena formerly invisible and also helps clarify difficult information.

Technology allows students to explore relevant mathematical ideas through constructivist methods (Pugalee, 2001). As learners explore mathematics using technology as a learning tool; they construct their own knowledge as technology allows students to actively participate and be responsible for their own learning. Technology supports exploration; it allows learners to set achievable objectives and allows them to makes discoveries on their own (Collins, 1991). In an environment where technologies are available, learners are free to exchange ideas with other learners, comparing and verifying their conjectures with one another (Heid, 1998). The flexible interactive characteristics of computer technologies are supportive of outcome –oriented and self-regulated learning which is in line with constructivism. Thus, the availability of technologies in school mathematics may allow students to explore mathematics on their own while at the same time enabling learners to be free to share ideas.

Using ICT allows learners to form multiple-representations hence it allows learners to approach problems from a multi- representational perspective (Hennessy et al., 2001). Computer technology will thus offer students the opportunity to explore the concepts and notion of functions in multi-representational (symbolic, numeric, tabular and graphic) modes The use of multiple representations, interpretation from one representation to another, and the analysis requiring interplay between graphic, numeric and symbolic information are key to understanding functions. A student who makes connections between mathematical ideas creates a deeper understanding of those ideas and different representations of a problem allow a student to represent the problem in a way that best makes sense to the student (NCTM, 2000).

One aspect of technology-mediated instruction is interactivity (Kaput & Thompson, 1994). When doing mathematics in an environment in which traditional methods are used; you write something down in a book, it does not change! Unless one person disagrees with what you wrote. With technology use in the classroom environment, the teacher can encompass interactive technology and cater for different learning styles; in a way through computers the teacher can make the mathematics environment more collaborative and more engaging for all the participants in the classroom. With technology, both teachers and learners can impose constraints on a graph to aid problem changing.

Technology also allow for connectivity (Kaput et al, 1994). Through the internet learners and teachers can access videos from the internet where their different learning needs are meet. Through the internet, technology can link teachers to other teachers, students to other students, and also students to teachers, this can impact positively in the development of learning through the use of technology if these links are used productively for the enhancement of learning mathematics in the classroom.

In summary, technology can play a prominent role in today's mathematics instruction. The availability of such tools to school mathematics education might make possible the teaching and learning of mathematics through constructivist methods. It facilitates group work, interaction, self-regulation (self-observation, self-evaluation and self-reaction), the use of various linked approaches to the same problem situation and the use of the real world problems which are relevant to student experiences.

## 2.9Conclusion

This chapter has tried to marry together the theories informing my study and to bring about those aspects in the theories that are applicable to this study. The role of technology in the teaching and learning of mathematics justifies why this study employs the use of ICT.; the impact that technology has in mathematics also supports why as teachers there is need for us to move away from the old ways of teaching mathematics. If history can prove that indeed evolution in mankind does happen, then it is also our duty as mathematics educators to bring about these changes so that as teachers we also evolve into new and productive ways of teachingmathematics.

# CHAPTER 3: RESEACH DESIGN AND METHODOLOGY

## 3.1 Introduction

This chapter describes the research design and methodology for answering the research questions for this study. First, the researcher will outline the research paradigm that influence this study followed by the research designs to be employed are discussed. This will be followed by a brief discussion on how intervention with technology was done, how participants were sampled and how data collection instruments were used to collect the data. The validity and reliability of the instruments used will be justified and how data analysis procedures were used to answer the research question. Finally, yet importantly, ethical considerations were highlighted to together with how the researcher gained access into the research site. But first, I shall start by explaining the philosophical assumptions that I brought into this study.

## 3.2Research Paradigm

One's worldview on how things work is referred to as a paradigm. For Taylor, Kermode and Roberts (2007) a paradigm is a "broad view or perspective of something" (p.5). The paradigm, which others have termed the "worldview" is "a basic set of beliefs that guide action" (Guba, 1990, p.17). From the four main paradigms in educational research, I these are positivist, post-positivist, interpretive and critical social theory (Weaver & Olson, 2006).

Quantitative purists (Ayer, 1959; Maxwell & Delaney, 2004; Popper, 1959; Schrag, 1992 cited in Johnson &Onwuegbuzie, 2004) articulate assumptions that are consistent with what is commonly known as a positivist philosophy. Quantitative purists or positivists believe that social observations should be treated as entities in much the same way that physical scientists treat physical phenomena. According to the positivists, educational researchers should eliminate their biases, remain emotionally detached and uninvolved with the objects of study, and test or empirically justify their stated hypotheses.

Qualitative purists, also called constructivists or interpretivists reject what they call positivism. These purists contend that multiple constructed realities exist and that time and context free generalizations are neither desirable nor possible (Johnson et al, 2004). They also argue that research is value bound, that it is impossible to fully differentiate causes and effects. These constructivists/interpretivists insist that logic flows from the specific to the general (inductionists) that is; explanations are generated inductively from the data. Interpretive/ constructivist paradigms study individuals with their many characteristics, different human behaviors, opinions and attitudes (Cohen, Manion& Morrison, 2007). It emphasizes the ability of the individual to construct meaning (Ernest, 1994). They argue that the knower and the known cannot be separated because the subjective knower is the only source of reality (Guba, 1994).

For the objectives of the study to be achieved, the researcher, although already suspecting from the poor performance of learners in mathematics had to prove that indeed learners do have problems in the topic of investigation. which are functions, and to do this, the positivist perspective of proving this theory was needed. If this theory that learners in grade 11 indeed had problems in functions was proven, then the researcher would have to go to the second stage of trying to understand the errors and misconceptions that these learners have; this had to be done from an interpretivists' perspective, again using test scripts and interviews. Part of the goal for this research to establish the misconceptions needed the researcher to rely as much as possible on the participant's views of the situation under investigation: learner errors; these errors would then guide the researcher to establish the misconceptions learners in the study have on function with the purpose of designing an intervention that would address them. When learners solve mathematical problems, they display errors, not misconceptions. The researcher's intention was to interpret the meanings of these errors displayed in order to establish the misconceptions. Questions like: what might be the causes of these errors and what are the reasons learners have when making them, were to be asked so that the researcher would have a deeper understanding,understanding why learners make errors would help the researcher to establish the misconceptions that learners have when learning functions and to plan intervention that seeks to address these misconceptions and hence in the end enable the researcher to answer the research questions identified for the study. For these reasons, this research used both qualitative and quantitative research methods.

### 3.3Quantitative and Qualitative Research Designs

Quantitative research is an approach for testing objectives theories by examining the examining the relationship among variables. It is an approach for testing objective theories by examining the relationship among variables. These variables are measured typically on instruments, so that numbered data can be analyzed using statistical procedures. Quantitative research relies upon measurement and various scales to generate numbers that can be analyzed using descriptive and inferential statistics (Bless &Highson- Smith, 2000). Quantitative research is deductive, uses probability sampling and aims to prove or to disprove theory. This type of research aims mainly to: measure the social world objectively, to test hypothesis and to predict and control human behavior (De Vos, 2002). Quantitative methods are a good fit for deductive approaches in which a theory or hypothesis justifies the variables, the purpose, statement and the direction of the narrowly defined research questions. Data is often collected through surveys administered to a sample or subset of the entire population and this allows the researcher to make inferences.

In this study, data was collected from the pretest, lesson observations and interviews. The data from the tests were analyzed qualitatively and quantitatively. The pretest was marked and the types of errors made by learners were categorized into different coding done by previous researchers as cited in chapter 2 of this project. The identification of the codes was done qualitatively using samples from the learners' solutions with errors in the pretest. In identifying the errors to establish the misconceptions, the researcher used scripts from both groups. This was done to give the researcher a picture of those errors common to the group of learners so that these could be used to inform the intervention that was used to address them. The type of errors committed by learners were noted for analysis to help the researcher to understand those errors that were more common among the learners in functions so as to inform the aspects of the function that would be incorporated into the intervention lessons. Learner's marks from the pretest were also analyzed quantitatively to verify if indeed learners had problems dealing with the function concept. After the intervention learner's marksfor the pretest and posttest were compared quantitatively to establish if there is an improvement in the learner performance after the intervention. The differences between the marks in the pretest and posttest for the control group and the experimental group were then analyzed to see the extent of each intervention to each group of learners. After the posttest, it is only the learners' scripts in the experimental group that would be analyzed qualitatively to establish the relevance and importance of using computer mediated instruction in the intervention. It is to these aspects of the tests that qualitative and quantitative research was used in the pretest and posttest of this study.

Using quantitative research may have higher credibility with many people in power for instance, the curriculum developers in education and the funders of this course. However, one major disadvantage associated with using only this type of research for my study would be that the researcher may miss out on phenomena occurring because of the focus on theory. (Johnson et, al, 2004). Hence for this reason, this research will also implement qualitative research.

Qualitative research is the research design of the interpretivist paradigm. It is an approach for exploring and understands the meaning individuals or groups ascribe to a social or human problem. Qualitative research is empathetic, striving to capture phenomena as experienced by the research participants themselves (Creswell, 2007), The process of research involves merging questions and procedures relevant to answering the research questions, data is typically collected in the participants' setting while data analysis is inductively built from particular to general themes. At all times throughout the process of data collection, the researcher makes interpretations of the meaning of the data.

A qualitative methodology also suited this study as it concerns with in depth descriptions of processes in explaining how learners reason to form errors and misconceptions in functions. (Merriam, 1992). Qualitative research was suitable for this study because it is concerned with the meanings that learners conceived or misconceived about function concepts. To understand how learners, think and reason, their answers to functions were analyzed qualitatively as explained in the paragraphs above. Solutions that display errors were studied deeply to establish the misconception behind the error. Interviews were conducted to those selected few learners to establish the reasons that learners have for making the errors identified in their solutions in the pretest. Interview data was transcribed for, analysis in trying to establish why learners demonstrated a particular error. These interviews were analyzed qualitatively together with the learner's pretest. Knowing the reasons why learners made particular errors or did not attempt to answer certain questions helped the researcher plan for the intervention to address these errors and misconceptions in the lesson interventions. In this study, qualitative methods help the researcher to understand what learners mean when their productions display errors and misconceptions in functions.

One of the weaknesses in using qualitative research in this study is that the findings may be unique to the relatively few people included in the research study and from the finding; it is difficult make quantitative predictions. For these reasons, it may have lower credibility with some administrators and commissioners of programs. Thus for this study, both qualitative and quantitative research will be implemented.

**3.4Mixed methods**

The choice of using both qualitative and quantitative research designs will provide this study with the strengths that offset the weaknesses of both qualitative and quantitative research as already explained in the paragraphs above. A mixed method design is a procedure for mixing both qualitative and quantitative data in the collection and/or in the analysis data in one study. Mixing these two research designs helped the researcher to get a fuller/ complete picture of the errors and the misconceptions behind those errors that learners make in this study displayed in functions.

In this study, analyzing the same data qualitatively and the quantitatively helped the researcher to answer the research questions without any bias. By mixing both quantitative and qualitative research and data, the researcher was able to gain a deeper understanding in breadth and depth of the errors learners displayed and to be able to compare the performance of learners in each group to establish a better method of intervention. By mixing these two methods, the researcher, as an educator was be better informed in answering the research questions because what one method missed or overlooked, was addressed by the other method. This study adopted the quasi-experimental design, where for a variety of reasons, the researcher did not have full control over the allocation of participants to experimental conditions as is required in true experiments. Analyzing data quantitatively helped to show differences in performances between the two groups (Control and experimental) but it is with qualitative data analysis that those differences (minimal or huge) were elaborated on to give a full picture of what happened hence, a mixed method approach was implemented. Outlined below is a description of a quasi-experimental design and how it was implemented in this study.

### 3.5The Quasi-experimental design

The study employed a quasi-experimental research project conducted to investigate the influence of using computers to remedy learner errors and misconceptions on functions in Grade 11. Quasi experimental designs involved selecting groups, upon which a variable is tested, that is, the influence of using ICT in the intervention for one group versus, the influence of not using ICT in the intervention for the other group. Two sample grade 11 classes were conveniently selected for this research due to their availability and the feasibility to the researcher to be able to carry out this study in this type of investigation. According to Ary et al, (2002), quasi-experimental designs that do not include randomization and are used where true experimental research is not possible to conduct. In a school situation it was not possible for the researcher to choose any class of grade 11 learners at random, because, there were only 4 grade 11 classes at the school at which the investigation was conducted, doing mathematics. The researcher was only teaching one of the four classes, hence there was need to include this class so that I would minimize the challenges faced in conducting this investigation therefore it was not possible to carry out a true experimental research which would involve choosing the participants at random. The quasi-experimental design was instead adapted for this other reason. Apart from the reason cited above, in a school situation where learners are already grouped according to the streams that they are studying (for example, science stream, accounting stream) it is also very difficult to reorganize classes to accommodate a randomized controlled trail that would involve choosing learners at random into the two groups needed for the experiment. Doing so would bring disorder into the smooth running of the school that functions with timetables and have specific teachers allocated to specific learners at different times of the day. Thus, it was, therefore, necessary to use intact classes including one of the classes that I was teaching mathematics. One class, the experimental group used, was a class that I taught mathematics at a township school in Soweto, Gauteng. The other class, the control group, was a grade 11 class from the same school taught by another educator. The selection of the two sample groups was convenient for the researcher as now explained. Using the class that I taught would make it easy for me to collect the data and to do the intervention with ICT, therefore the class that I taught was chosen to be the experimental group. The other teacher, who agreed to avail herself and her class to be used as a control group, is also a mathematics teacher from the school where the research was conducted. This teacher was also studying with some local university where her area of interest was to study the errors learners make when doing mathematics (error analysis). She availed herself and her class for my study because in so doing she would also gain insight into her area of interest for future research. For these reasons, that is how these two classes were selected for this investigation.

After this selection, the experiment proceeded in a very similar way to any other experiment, with a variable being compared between different groups, or over a period. The cross-sectional presentation of the research design, which is non-randomized control group, pre-test post-test, was as follows:

| Group | Pre-test | Independent Variable | Post-test |
|---|---|---|---|
| Experimental | Y1 | X | Y2 |
| Control | Y1 | -- | Y2 |

In this study, the two groups of learners wrote the same test on functions. After the pretest, results for both groups of learners were analyzed by the researcher to establish the errors and misconceptions learners displayed on functions. These common errors and misconceptions identified from both groups, then were used to design the content that would be taught during the intervention. The teacher teaching the control group (the group that would not use ICT in its intervention) was informed on the findings of the pretest and we both discussed the challenges that were to be addressed during the intervention. Both groups of learners received a treatment after the pretest in the form of an intervention to address the errors identified in the pretest. However, the only difference in the form of the treatment was that while the experimental group made use of ICT in its intervention, learners in the control group were doing their intervention without using computer, by using traditional methods of learning and teaching (pen, paper, textbook, chalkboard etc.). After the lesson interventions, both groups again wrote a posttest. The posttest administered to the control and experimental groups was the same.

Ary et al. (2002) stated that the use of pre-test enables one to check on the equivalence of the groups on the dependent variable before the experiment begins. If there are no significant differences in the pre-test, then selection as a threat to internal validity is eliminated and therefore one can proceed with the study. What this means is this: for this investigation to be able to judge the influence of using different treatments in the intervention, at least the performance of learners in the control group should be similar to the performance of the learners in the experimental group before the intervention. If it is like that, then it is much easier to make a judgement on the group that makes a better improvement than the other after the post test. Is there are unacceptable differences between the capabilities of the two groups, then, statistical measures would have to be used in consideration to these differences so that a fair judgement is reached at the end of the experiment. However, due the nature of the classes at which the experiment was conducted, it was hoped that the results of the pretest will not show much difference between the performances of the two groups because, learners at the school where the research was conducted are not streamed according to their capabilities, but according to their subject choices. These two groups used were learners doing mathematics, both in the science stream.

An advantage of using this design was that it was much easier to select the groups for the study than in true experiments. Using a quasi-experimental design saved time as there was no extensive selecting process used to choose the participants. The other advantage was that results from this type of study, the trends and patterns on how learner errors influence their misconception can be generalized into other topics of mathematics.

However, without proper randomization, one of the disadvantages for using this design would be that statistical tests could be meaningless. For example, in this study, in trying to establish the influence of using computers to remedy learner errors and misconceptions, a quasi-experiment might reveal results that show that one program on two groups of children is more effective than the other. Although there could be other factors that could have contributed to the success of one program over another apart from the type of intervention used, it is very difficult for the researcher to control those other factors. Hence, the results will not stand up to rigorous statistical scrutiny. It is for this reason that this rigor of qualitative research involving hypothesis testing was not used in great detail; hence, data was analyzed qualitatively and quantitatively as outlined in the sections above.

### 3.6 The intervention lessons

The intervention lessons concentrated on redressing the errors and misconceptions that learners have on functions as observed from the pre-test scripts, see (Appendix 1) for the pretest.

Variation theory was used as guiding theory in creating an environment in which learners would learn from their mistakes in this study. The socio cultural learning theories together with the TPACK framework were in cooperated into the lesson interventions for the learners in the experimental group, as this was the group using ICT. The features of functions where learners displayed having misconceptions were the focus of this intervention. The features were varied using variation theory in order to afford learners an opportunity to learn from their errors and misconceptions. For the experimental group, the TPACK framework guided how ICT especially computers and You-Tube videos were to be used in the classroom while the socio-cultural learning theory informed the researcher on the possibilities of how learners working with the computers as a tool could be helped to reach a better understanding of mathematics in an established community of practice within their classroom.

The two groups of grade 11 learners used in this study both received teaching intervention to address the errors and misconceptions identified from the pre-test. For one group, taught by the researcher their intervention lessons were implemented using ICT. While for the other group, taught by a colleague from the same school that the study is conducted, their intervention lessons were implemented using the traditional method.

## 3.7 Sampling

### 3.7.1 Sampling of participants

Sampling refers to the process of selecting the sample from a population to obtain information regarding a phenomenon in a way that ensures that the population will be well represented (Brink, 1991). A population in research refers to those elements that make up the focus of the study that fit fixed criteria (LoBiondo-Wood & Haber, 2010). The population in this study would be all the Grade 11 mathematics learners at a particular school in which the study was conducted, in Soweto, South Africa. The samples used were the two classes chosen and used for the purposes of this study from the same school.

The classes chosen for this study were chosen because they would serve the purpose for the study while at the same time making it easy for the researcher to conduct the study. In other words, the researcher used what iscalled a combination of purposive and convenience sampling strategies. Convenience sampling involves choosing the nearest individuals to serve as respondents (Cohen & Marion, 1995). There are many high schools in the township where the researcher, works as an educator and it would have been possible to conduct this study from any other school as long as the learners were in grade 11 and doing mathematics. However, many factors were considered. Choosing learners from another school would mean that the researcher was going to need money to cater for transport to conduct the research, more time again would have been needed to work with learners from another school as schools already have their programs in place so if I were to conduct my research there I would have to negotiate time according to their availability which could have been difficult coming from a different school. Therefore, my choice was also influenced by the convenience of having to save transport money, working with learners that I am already familiar with as well as working at an environment whose systems I am accustomed to, to reduce the stress and strain in carrying out the investigation.

Purposive sampling involves deliberately selecting participants that meet the criteria relevant to the research study. Purposive sampling techniques are typically used in qualitative studies and may be defined as selecting units; these may be individuals or groups of individuals based on the purposes associated with answering one's research study questions. This study uses a quasi-experiment, to compare as well as to establish the influence of using computers to understand learner errors and misconceptions on functions. Purposive sampling was the means for selecting individuals who will have knowledge of the phenomena studied or deemed potential rich cases (Mapp, 2008).

The participating learners were chosen on the bases that they are taking mathematics as a learning subject, and as a participant researcher, I extended the invitation to all grade 11 learners at the school where the study was conducted. Regarding sampling, Merriam (1998) states that once the problem of research has been identified, the next task is to select the unit of analysis. In this study, learner errors and misconceptions in functions and the effectiveness of using computer software to remedy these, are the units of analysis, hence the two groups were selected for the study are one group taught by the researcher and the other, by a colleague.

A quasi-experiment was conducted on two groups of grade 11 learners where one group (control) and the other group (experimental group) are from two different classes taught by two different teachers from the same school. A pretest was administered to both groups. The researcher marked all the pre-test and identified the errors and misconceptions that the learners have. These wererevealed to the teacher for the control group. Thereafter both teachers separately prepared and administered intervention to redress these. The teacher for the control group used the traditional methods: while the researcher used computers in the intervention for a week. The intervention was meant to last for about three weeks but due to other constraints, this was not possible. Thereafter a posttest was administered to both groups.

Although the researcher could have divided the learners in her class into the two groups, ethical issues were considered. It would have been considered unethical to expose some of my learners to the use of computers while depriving others to that privilege. Hence, after some probing on the methods of teaching used to teach functions by the two other teachers in grade 11, teacher X was chosen on the basis that she uses traditional methods and does not make use of computers.

The two sample groups were from a township High school in Soweto. The current set up in most South African township government schools is that there are more learners per class. The control group has about 45 learners; 25 girls and 20 boys while the experiment group has about 40 learners; 19 girls and 21 boys. However, only 25 learners from each group availed themselves for the intervention and it is from these learner's scripts that data collected was analyzed. The classes in this school are of mixed ability with no streaming involved, therefore it can be said that the classes are heterogeneous. For this study, two sample groups both science classes were chosen with the assumption that there is no group having an advantage over the other.

### 3.8 Data Collection Instruments and Procedures

There are several methods for data collection especially when one considers collecting both qualitative and quantitative data. In this research, the researcher used a pre-test, post-test, systematic observations and follow up interviews as data collection methods in this particular study as now explained.

#### 3.8.1 Pre-test and Post-test

In this study, a pre-test (Appendix 1) and a post-test (Appendix 2) was administered to the sample groups (both the experimental and the control groups). In order to understand the learners' misconceptions and resulting errors, it was necessary for the researcher to gather in-depth information from learners' written work. A pre-test was administered firstly to identify the errors and misconceptions that learners have on the functions topic, quadratic equations in particular. Secondly, using the learner's responses to the pretest, the researcher then identified the learners to be interviewed to try to establish the reasons for the errors to determine the misconceptions. Thirdly from the pretest using the identified errors and misconceptions, an intervention was planned to address these misconceptions. According to Polkinghorne (1989), the first step in accomplishing some phenomena is to gather information from the participants, the second is to analyze the data for any common patterns and the third is to write a report articulating essential themes or patterns identified. Learners had already covered the section of quadratic equations in grade 10; however, after some revision of this section of functions on quadratic equations was done again in grade 11, to remind learners of some basic concepts on the quadratic function, the sample groups were given a pre-test. The pre-test results were analyzed qualitatively and quantitatively. Errors made by learners were analyzed by coding and categorizing to identify the errors and misconceptions that students have on functions. This was then followed by a teaching intervention to address these common errors identified on the concepts of functions, particularly quadratic functions, to the groups for one week (approximately 5 hours). For the control group, its teaching intervention was done using traditional methods while the experimental group's intervention incorporated using ICT in the learning of functions to address the challenges cited in the pretest. For the experimental group, the intervention lessons were informed by variation theory, TPACK and the socio-cultural learning theories.

### 3.8.2 Interviews

For Maree (2007), an interview is "as a two-way conversation in which the interviewer asks the participant questions to collect data and to learn about the ideas, beliefs, views, opinions and behavior of the participant" (p.87). An interview as a method in which the researcher asks questions to the participant to get clarity of some issues in the study. For example, the researcher may ask respondents questions to gain insight into understanding how they reason to help explain the errors they display in their test scripts.

In this research, the researcher was interested in the resulting errors and misconceptions on functions, which are deeply buried in the minds of the participants. The interviews were used as a follow up to learners' responses to the pre-test and also as a follow up to the experimental group after the posttest. The participant learners' interview was to, "go deeper into the motivations of respondents and their reasons for responding as they did", (Cohen &Manion, 1995). The interview was first conducted on selected learners who displayed some errors to help get an understanding of the misconceptions they had. When the pre-test was marked, learner errors were coded and categorized to the various categories as identified from previous research in chapter 2 of the literature review for this investigation. Learners were chosen to help bring to light how they were thinking so that the researcher would be able to establish the misconception behind the errors displayed. For each category identified, some learners were chosen to explain the reasoning behind the misconception displayed. The aim for understanding was to help the researcher design teaching instruction that addressed them. After the posttest, selected learners in the experimental group were interviewed to give insight into how the use of computers influenced their performance in the posttest.

Semi-structured interviews were used after the pretest for individual face-to-face interviews with participants (Appendix 4). The interview was semi-structured in that the interview instruments had pre- planned questions to start the interview and keep it flowing.  Chamarz (2006) suggests that semi structured interviews are flexible as they allow the interviewer to probe respondents for more details and they assist in clearing any misunderstandings and thus, help establish rapport between the researcher and the respondents. Although the interviews conducted after the posttest were also semi structured in nature, these were guided by some questions which were then used as themes to answer the research question when establishing the influence of ICT on the experimental group. also conducted after the posttest (Appendix 9). These interviews with the selected learners were recorded, transcribed and then were interpreted to answer the research questions.

### 3.8.3 Lesson Observations

In trying to understand and to answer the research questions for this study, it was significant for this study to understand how learning and instruction work, hence lessons were observed (Appendix 3).  It is the lessons that the researcher conducted with the use of technology that were observed in which the investigator was a participant observer and not the lessons from the other teacher using traditional methods. The focus of this lesson observation was not on the teacher but on the learners. For this reason, the researcher was a participant observer merely observing how the learners were learning in an environment using ICTs. Three frameworks guiding this research investigation and informing this lesson observation method were variation theory, TPACK and the Socio-cultural learning theories. While variation theory and TPACK will inform the investigation on how to structure and implement activities during the teaching intervention with computers, it is the socio-cultural learning theory that seeks to understand how learners are helped to reach their ZPD in this classroom.

## 3.9 Validity and Reliability

Validity is an essential criterion for evaluating the quality and acceptability of research (Burns, 2000). Researchers use various instrument to collect data, hence, the quality of these instruments is critical because "the conclusions researchers draw are based on the information they obtain using these instruments" (p.160). Thus, it is important that the data and the instruments used to be validated.

The term validity refers to the extent to which an instrument measures what it intends to measure. Validity addresses these two questions: What does the research instrument measure? What do the results mean (De Vos, 2002). In this study, the following process will be implemented to ensure validity of the research instruments.

Before administering the tests used in this study, experienced mathematics high school teachers together with the curriculum CAPS document were used to verify them. Three experienced mathematics educators were consulted to check on the content of the questions and the time required answering the questions. However, since the aim was to investigate how learners answered the questions in the test, the researcher was lenient on the time allowing all those writing the test more time to finish answering all questions to the best of their abilities. Based on the comments and suggestions from the consultations, the pre-and post- test were amended appropriately. After this consultation with mathematics experts, and having incorporating their opinions and suggestions into these instruments, it may be ascertained that these instruments were able to assess all the aspects of the content required for learners to learn quadratic functions in grade 11 as prescribed in the curriculum document. If the same research instruments are to be used in a different setting for the same purpose, the same results will prevail.

Reliability is synonymous with the consistency of a test, observation or other measuring device. A measure is said to have a high reliability if it produces similar results under consistent conditions. Obtaining the same results when the instruments are administered again in a stable condition guarantees that the instrument is reliable. (De Vos, 2002).In this study, the errors that learners were showing on functions were analyzed and compared to those that were found in literature. The errors that learner made were categorized according to the categories already identified by other researchers. This helped to balance the findings of this study with those that have been found by other researchers earlier and hence, ensure that the findings are reliable.

### 3.10 Data Analysis Procedures

When analyzing data, the main objective was to answer the research questions for this study. In outlining the data analysis procedures that were used in this study, I will first refer the reader to the research question and then subsequently outline the data analysis procedures that were employed.

*What errors and misconceptions are elicited by grade 11 learners when they engage the topic of functions?*

The errors identified from the learner's scripts were classified and coded to understand their nature and their causes. Various researchers have classified errors into various categories as part of misconceptions. Based on studying the errors made by the learners in the pre-test and based on the learner's explanations from the interview, learner errors were coded and categorized. In coding and categorizing these errors, the researcher did not come up with her own table of classification but used these the categories already identified in the literature reviewto make reference to the errors that were common among the learners. The researcher used some of the classifications of errors as mentioned in the literature review by the following classifiers: Radatz, 1979; Watson, 1980; Movshovitz-Hadar, Zaslavsky, and Inbar, 1987; Elbrink, 2008; Luneta& Makonye 2010. This coding and categorizing of errors helped this research study understand the misconceptions learners had when dealing with functions and to planning teaching instruction to prevent as many of them as possible in future.

*What is the influence of using computers to address learner errors and misconceptions in functions at Grade 11?*

The tests written by the learners in the experimental group were compared qualitatively to establish improvement, if any, on the errors and misconceptions once identified in the pre-test. The test results for all the learners were also be also analyzed quantitatively; then the qualitative and quantitative analysis will then be incorporated together in the findings. The post- test as a data collection strategy consists of learners writing a post-test to ascertain whether the intervention lessons were effective or not. Comparison of the Pre-test and Post-test results from the control and the experiment group will be analyzed separately to try to establish to what extent using computers can influence conception of functions. Lesson observation schedules will also be analyzed qualitatively together with learner interviews on the posttest to establish the behavior of learners in a computer rich learning environment to establish the influence of using computers in learning functions for the experimental group.

*What teacher knowledge may be drawn from the use of ICT technologies in mediating the learning of functions, through learners' errors?*

Learner's posttest script for the experimental group were analyzed to establish the influence ofusing technology in functions. Selected learners from the experimental group were sampled and interviewed on how the use of technology has influenced their performance in the post test

### 3.11 Ethical Considerations

Before embarking on this study, I first submitted my research proposal to the faculty of science to get approval to conduct the research. When my research proposal was approved (Appendix 18), I then applied for the ethics clearance with the Wits University Ethics committee. After the approval of my ethics (Appendix 16), I then requested permission from Gauteng Department of Education head offices in Johannesburg to get permission to conduct my study at the public school I had chosen. When I was issued with the letter of approval (Appendix, 17), I then requested permission from the principal and the School Governing Board representative of the school where my project was conducted.

Since I am an educator at the school where my research was conducted, I did not face any challenges in persuading both the SGB member and the principal to grant me permission to carry out the investigation. I had letters outlining my intentions, (Appendix 12), I handed out the letters but I also took the opportunity to tell them the purpose of the study, its aims and the research questions for the study. I then told then in brief how and when I intended to carry out the research and with whom. The principal and the SGB representative verbally agreed that I may proceed with my study but on condition that the participants (Learners and the teacher involved) also agree. I then gave the principal a consent form (Appendix 19) so that together with the SGB representative they may put their agreement in writing.

Before talking to the teacher, I also felt it was important that I inform the schools' Head of Department for Mathematics on my intended study. When I got his blessing, I then formally approached the teacher who had agreed to teach learners in the control group. Although the information letters outline the whole procedures with my intentions, I also explained to her verbally my aim, the purpose and the research questions together with how the study was to be conducted. I also gave her information letter (Appendix 15) to read, and a consent form (Appendix30) for her to sign if she was still willing to participate in the study. I requested that she could respond to me the following day after carefully reading the information letter I had given her and signed consent form if she was in. This time was granted to her so that she could get back to me for clarity on any issues in the letters that I had given her. The following day, the teacher verbally agreed after going through the letter that she was going to take part in the study, and she also submitted back to me her signed consent form showing her willingness and dedication to the investigation. But she asked me to make the request to the learners in her class since I was the one conducting the study although it was her who was going to conduct the intervention lessons with them.We (myself and the participant teacher), then set up a meeting with learners from both our classes and I briefed them about my research, and I gave each learner the information letter (Appendix 13) that summarized my intentions. Most of the learners agreed to participate in the study. I gave the learners consent forms to sign (Appendices 20, 21, 22, 23and 24). Each consent form for the learners was stipulating the expectations of the study. Not only were learners to agree to participate in the study, but they also needed to show if they were agreeing or not agreeing to the following: writing the pretest and posttest; to be interviewed; to be audio recorded during the interviews; to be observed in class by the researcher during the lesson interventions and to give permission to the researcher to use their data from the interviews and tests for data analysis. Learners who were below 18 years old were also given information letters and consent forms to give to their parents, because the learners are considered to be minors and they also need permission from their parents: information letter (Appendix 14); consent forms for the parent/guardian (Appendices 25, 26, 27, 28 and 29). Learners willing to participate in the study were requested to bring the consent forms the following day after careful consideration was done on their part in consultation with their parents if they were under the age of 18 years old. A timetable was set up as to when we can start our lessons for the study.

### 3.12 Access to the research site

First, permission was sought from the Gauteng Department of Education under whose jurisdiction the school falls. A letter of approval to conduct the research was issued after submission of the application form to the Gauteng Department of Education head offices in Johannesburg. After approval of my ethics application by the Wits University Ethics committee, I then request permission from the principal of the targeted study school to conduct the study at the school.

After a brief discussion of the study, learners were given letters requesting their voluntary participation in the study. These letters requested learners to respond to the tests, to participate in the follow up interview after the test has been marked, and to be audio taped during the interviews. Learners below the age of 18 years who showed interest to participate in the study were given information sheets and consent forms for their parents to grant permission for them to participate in responding to the tests and follow up interviews and for them to be audiotaped during the interview. Learners above 18 years of age signed consent forms agreeing to participate in responding to the tests and follow up interview and to be audiotaped.

Informed consent to participate in this study was sought from all participants including the teacher who agreed to teach the control group. Informed consent contain a brief description of the purpose and procedure of the research, a statement of any risks or discomfort associated with participation, a guarantee of anonymity and the confidentiality of records. This informed consent stated that participation was voluntary and could be terminated at any time without penalty. In addition, to protect the identity of the participants, codes or pseudo names were used.

## 3.13 Conclusion

In this chapter, the research design and the methodology for answering the research questions for this study were described together with the research paradigm that influences this study. The study site and setting, the sample and the sampling techniques and ethical issues were explained and justified. The data collection and analysis methods for this study were also described and justified. The next chapter summarizes the findings that came from analyzing learner’s responses fromthe pretest, posttest and interviews.

# CHAPTER 4: DATA ANALYSIS

## 4.1 Introduction

Data analysis is a process of inspecting, cleansing, transforming and modeling data with the goal of discovering useful information, suggesting conclusions and supporting decision making. For Mouton (2001), this analysis involves breaking up the data into manageable themes, relationships, trends and patterns. Luneta (2013) also defines “data analysis” as a process of observing and extracting patterns in data. With qualitative data, the researcher collects data from a small number of selected participants and makes use of an interpretive approach to provide narrative descriptions of interviewees, and how they see their contexts. This narrative description can also be given based on the participant’s solutions to questions in a test based on some patterns or trends already identified in literature by other researchers. The purpose of analyzing data is to answer the research questions for the investigation. When analyzing this data, the researcher should try to interpret the data collected with respect to the purpose of the research study while at the same time respecting the anonymity of the participants (Cohen, Manion & Morrison, 2007).

This chapter aims to first establish if indeed grade 11 learners have problems with functions. The chapter then aims to determine the errors and misconceptions that learners display in functions at Grade 11and to use these errors and misconceptions to plan remedial lessons that aim at addressing the problems cited. The research design and methodology that was followed in conducting the study will now be discussed. A pre-test, interview schedule, lesson intervention and a posttest were used. The results of the investigation are now presented, analyzed and interpreted to answer the research questions.

Outlined below is the data analysis for each of the instruments used in this investigation to collect data.

## 4.2Analysis of the pre-test

A pre- test was administered all the grade 11 learners in this study. The participants had been asked to use their class list numbers as well as their class as their name. For instance, someone in

11G appearing first on the list would use the name G1. This was done to enable the researcher to be able to identify those students that would need to be interviewed based on what they had written in the pretest. After marking the pretest scripts, interviews were conducted and then the researcher randomly coded the scripts, the scripts were coded A1 up to the last number of scripts for the control group and from B1 to the last number of script in the experimental group respectively. The purpose of the pretest was to establish if indeed learners do have problems answering functions questions and to establish the nature of the errors and misconceptions that they have on this topic. The test consisted of 11 items, marked out of 40. Data from the pre-test was analyzed qualitatively as well as quantitatively to suit a mixed method approach employed for this study.

### 4.3 Quantitative Data Analysis of the Pre-test

Table 4 **b**elow, is a presentation of the results of the test as observed from this pre –test for the control group

**Table 4: Pretest marks for learners in the control group**

| Name | Raw Mark | Percentage Mark | | Name | Raw Mark | Percentage Mark | | Name | Raw Mark | Percentage mark |
|---|---|---|---|---|---|---|---|---|---|---|
| A1 | 11 | 28 | | A10 | 8 | 20 | | A19 | 0 | 0 |
| A2 | 13 | 33 | | A11 | 7 | 18 | | A20 | 0 | 0 |
| A3 | 10 | 25 | | A12 | 5 | 13 | | A21 | 0 | 0 |
| A4 | 11 | 28 | | A13 | 7 | 18 | | A22 | 24 | 60 |
| A5 | 14 | 35 | | A14 | 2 | 5 | | A23 | 17 | 43 |
| A6 | 14 | 35 | | A15 | 3 | 8 | | A24 | 21 | 53 |
| A7 | 10 | 25 | | A16 | 2 | 5 | | A25 | 24 | 60 |
| A8 | 7 | 18 | | A17 | 1 | 3 | | | | |
| A9 | 5 | 15 | | A18 | 1 | 3 | | | | |

The mean score of the pretest for the control group (Group A) is 22.04%. With an acceptable pass mark of 30% in mathematics for the FET phase in South Africa, an analysis of the results for the learners in the control group shows that only 28% of the learners in the control group obtained a pass mark of ≥ 30%. Overall, the learners' performance is poor with a corresponding failure rate of 72%.

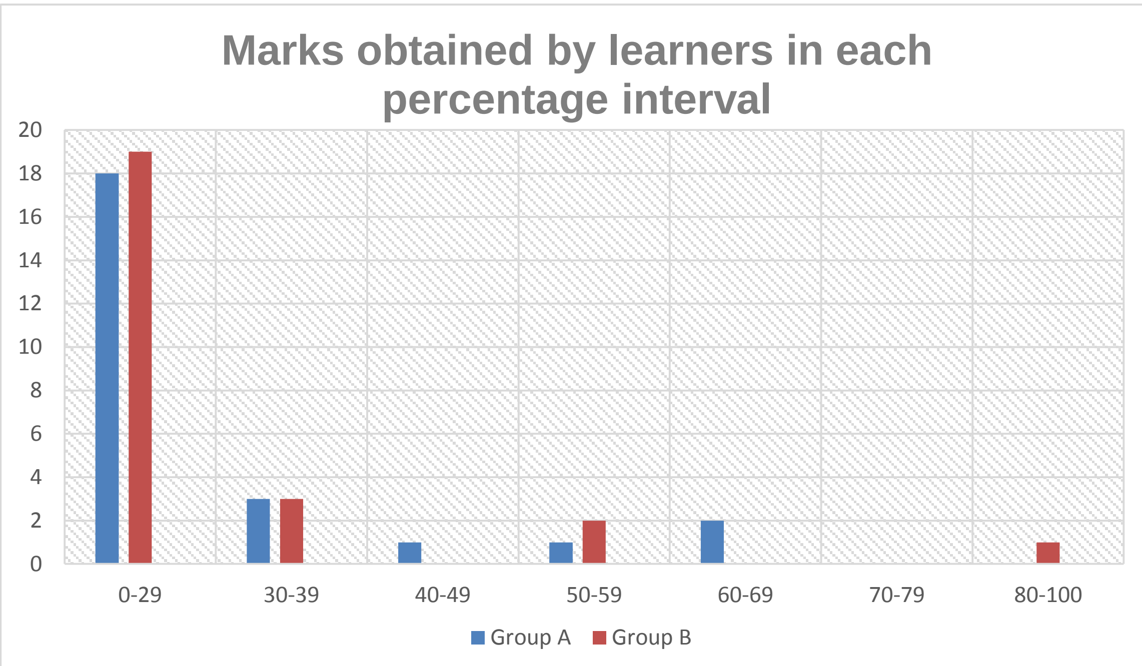
Table 5 **b**elow, is a presentation of the results of the test as observed from this pre –test for the experimental group

**Table 5: Pretest marks for learners in the experimental group**

| **Name** | **Raw Mark** | **Percentage Mark** | | **Name** | **Raw Mark** | **Percentage Mark** | | **Name** | **Raw Mark** | **Percentage Mark** |
|---|---|---|---|---|---|---|---|---|---|---|
| B1 | 14 | 35 | | B10 | 0 | 0 | | B19 | 7 | 18 |
| B2 | 6 | 15 | | B11 | 2 | 5 | | B20 | 15 | 38 |
| B3 | 3 | 8 | | B12 | 3 | 8 | | B21 | 12 | 30 |
| B4 | 20 | 50 | | B13 | 5 | 13 | | B22 | 10 | 28 |
| B5 | 33 | 83 | | B14 | 3 | 8 | | B23 | 8 | 25 |
| B6 | 20 | 50 | | B15 | 7 | 18 | | B24 | 10 | 20 |
| B7 | 3 | 8 | | B16 | 8 | 20 | | B25 | 10 | 25 |
| B8 | 0 | 0 | | B17 | 7 | 18 | | | | |
| B9 | 0 | 0 | | B18 | 6 | 15 | | | | |

The mean score (Group B) for the experimental group is 21.52%. Analyses of the results for the learners in the experimental group show that only 24% of the learners in the control group obtained a pass mark of ≥30%. Overall, the learners' performance is poor with a corresponding failure rate of 76%. Although the control group has a higher mean compared to the experimental group, there is not so much difference in the performance of learners from the two groups. The test scores are presented separately to enable the researcher later on to compare the impact of the different interventions administered on the performance of the two groups of learners when they write a posttest. However, for the purposes of establishing whether learners do have problems with the topic on functions and to establish the nature of errors and misconceptions that learners have, the results from the two groups were combined as now presented in the graph below.

**Figure 2: Marks obtained by learners in the Pre-test**



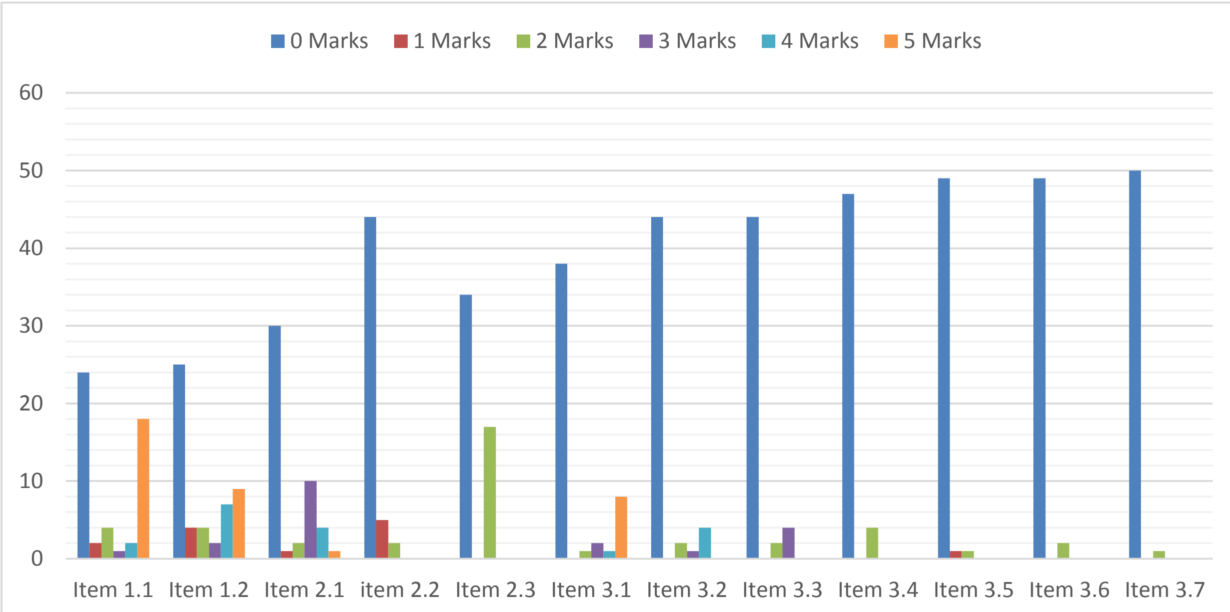

As observed from the graph, the overall performance for both sample groups in the pre – test was very poor. Most learners from both groups are in the range 0 – 29 %, an indication that indeed learners do have problems with the topic on functions. Although there were 11 items from the three questions in the pre-test each addressing a different aspect of the functions topic, it is also necessary to analyze the performance of the learners in each item to see where the learner's performance was weak and to establish the nature of errors that were prevalent for the aspect of function addressed. In analyzing each item, the performance of learners from these groups is now combined so as to come up with an intervention that addresses all learners' misconceptions. The pre-test was to be done in an hour, but learners were given ample time to do this task until they were satisfied that they were done. If a question/ item was not attempted by the learner, this is considered as a zero. However, the highest numbers of zero scores were recorded in Question 3(Item 3.1 to 3.7) where most learners did not attempt answering the questions, the learners' reasons to this shall be solicited from the interviews conducted and these will be outlined and discussed as the question by question analysis unfolds in the paragraphs below. First, the table below shows the marks attained by the learners in this investigation for all questions/ items separately together with a graph that represents learner scores and frequencies. For each item, there were different marks allocated to the question and the possible marks for each item are also indicated in the table below.

**Table 6: Learner's marks and frequencies for each item in the Pretest**

| | **Marks obtained per item** | | | | | | |
|---|---|---|---|---|---|---|---|
| | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
| Item 1.1 (Possible Marks: 5) | 24 | 2 | 4 | 1 | 2 | 17 | |
| Item 1.2 (Possible Marks: 5) | 25 | 4 | 4 | 2 | 7 | 8 | |
| Item 2.1 (Possible Marks 6) | 30 | 1 | 2 | 10 | 4 | 0 | 3 |
| Item 2.2 (Possible Mark 2) | 44 | 5 | 1 | | | | |
| Item 2.3 (Possible Marks: 2) | 34 | 0 | 16 | | | | |
| Item 3.1(Possible Mark: 5) | 38 | 0 | 1 | 2 | 1 | 8 | |
| Item 3.2(Possible Mark:4) | 44 | 0 | 2 | 1 | 4 | | |
| Item 3.3(Possible Mark : 3) | 44 | 0 | 2 | 4 | | | |
| Item 3.4(Possible Mark: 2) | 47 | 0 | 4 | | | | |
| Item 3.5(Possible Mark: 2) | 49 | 1 | 2 | | | | |
| Item 3.6(Possible Mark:2) | 49 | 0 | 2 | | | | |
| Item 3.7 (Possible Mark: 2) | 50 | 0 | 1 | | | | |

The graph below shows learner scores item by item as they wrote the pre-test

**Figure 3: Learner scores, item by item in the pretest**




As the graph clearly shows, most learners had problems dealing with items in the pre-test, hence the high frequency of zero scores. Almost in all the items, there is evidence that these functional concepts are particularly problematic to learners. This means that learners had many errors and misconceptions that prevented them from scoring high marks. Since this investigation employs a quasi-experimental design it would also have been proper to statistically analyze the differences or similarities in performance for the two groups so as to be able to measure accurately the comparison of the two groups' performance after the different interventions, but due to the small number of participants in both group, this is not feasible.

In trying to establish the nature of errors and misconceptions that learners displayed in the pre-test, the researcher needs to know how those students who attempted these questions committed errors in answering the given questions. While examining these vignettes, the researcher will refer to various coding cited in the literature review of this research investigation (Chapter 2) in trying to establish the nature of errors and misconceptions displayed by this group of learners.

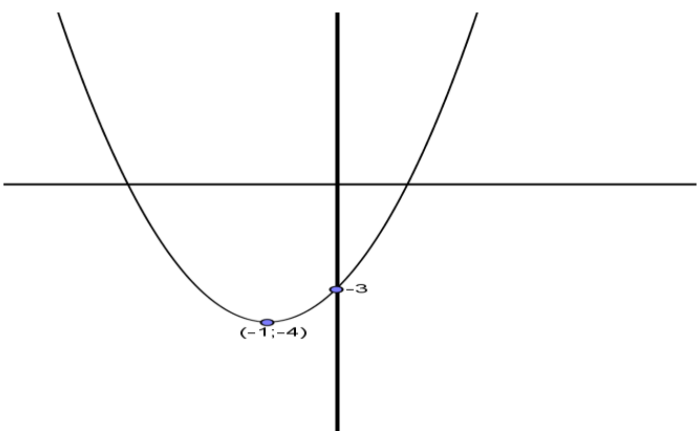
In the following sections, the discussion of the concepts and procedures needed to successfully perform on each item are discussed and then the learners' work is presented to analyze how their errors and misconceptions were expressed. Although vignettes are rich pockets of especially representative data, there is need for the researcher to be careful not to exaggerate when reporting about them (Miles &Huberman, 2004). However, where clarity is sought from the learner, the researcher also interviewed some of the learners to establish the reasons for the errors identified in the pre-test. In this investigation, the purpose of establishing the reasons for the errors is for proper planning for intervention to address these errors in the intervention lessons that follow.

### 4.4: Qualitative Data Analysis of the Pre-test

As with the quantitative data analysis, there were three Questions but each question had some parts to it. These questions will again be analyzed part by part (item by item) for all the learners combined so that the researcher can be able to establish the errors prevalent that will inform the intervention

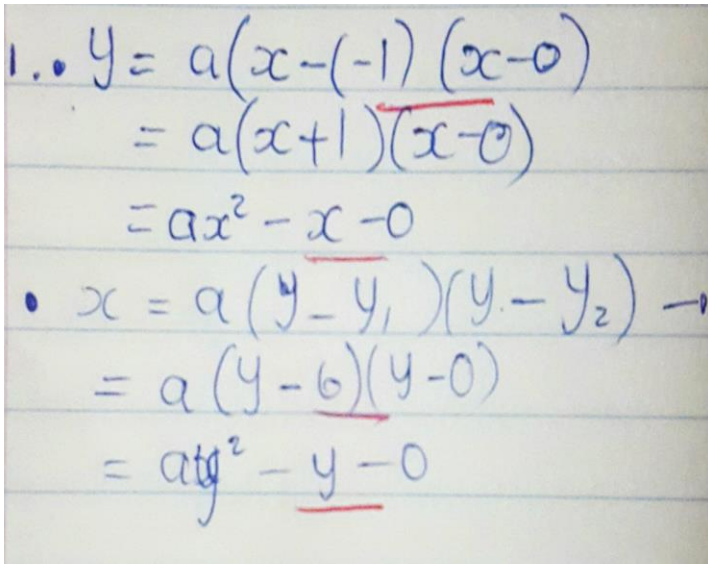
#### 4.4.1: Analyzing learner responses to item 1.1in question 1 for the pretest

In question 1.1, the learners were required to find the equation of the parabolic graph where the turning point of the parabola and another point on the graph was given. Learners were required to interpret the graphical representation of a function to the equation form of a parabola. Learners should have been able to identify key features of the parabolic function and substitute the given points on the parabola in the appropriate form of the parabolic equation.



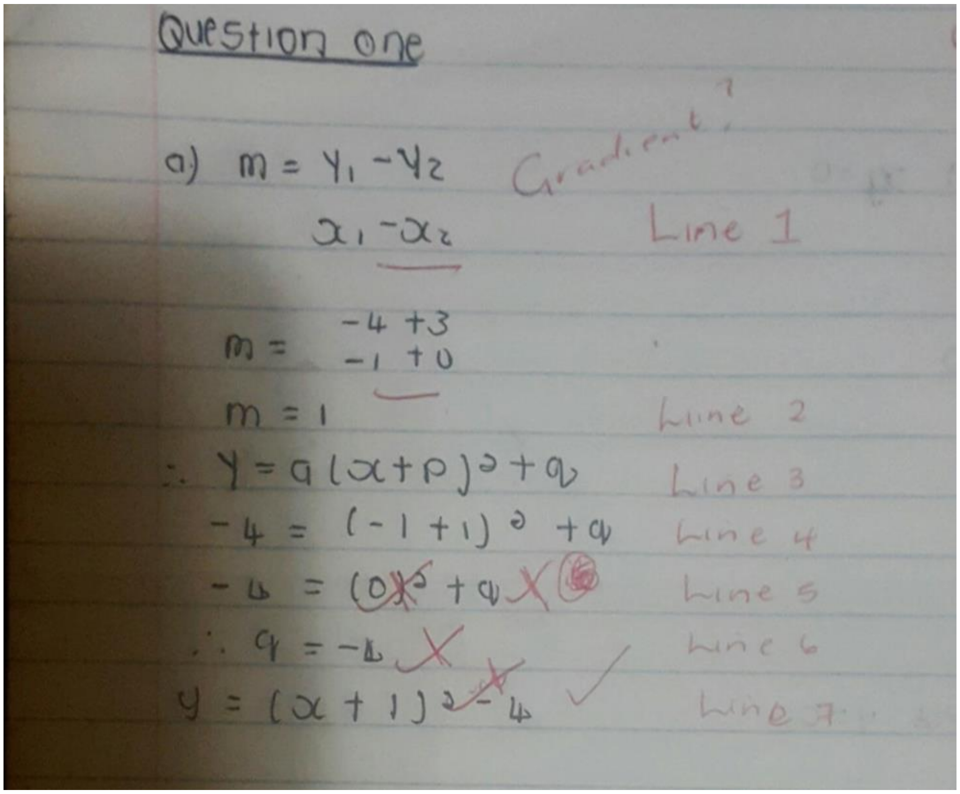

Forty-eight percent (48%) of the learners failed to choose the correct form of the parabolic equation to enable them to answer this question. Although 52% were able to choose the appropriate form of quadratic equation to use in this question, 34% of these learners were still not able to answer this question correctly as observed from these sample vignettes:

**Figure 4: Vignette 1 for item 1.1 (Question 1)**




Evidence from this script (vignette 1) shows that the learner had no knowledge of the turning point and hence failed to choose the appropriate quadratic formula to use. There is also incorrect substitution of values and incorrect simplification leading to the learner having difficulties in finding the appropriate equation for the given graph. According to Radatz's (1979) classification of errors, the errors identified can be classified as error number 3, 4, or 5 where the explanations are that learners could have deficient mastery of prerequisite facts and concepts, or these errors resulted from incorrect association or rigidity of thinking or they resulted from application of irrelevant rules or strategies respectively. The response given in vignette 1 shows that the learner does not understand whether the equation is given as a function of x or as a function of y. The misconceptions revealed from this learners' script were deficient mastery of prerequisite facts and concepts (Radatz, 1979); Incorrect associations or rigidity of thinking (Radatz, 1979); Transformation errors- where the pupil cannot select the mathematical process which is required to obtain the solution (Watson, 1980); Ignorance of rule restrictions or symbolism. Applying rules to contexts, they do not apply. (Luneta& Makonye, 2010)

**Figure 5: Vignette 2 for item 1.1 (Question 1)**




Other learners like B 21 started by first finding the gradient then substituted wrong values but managed to get the answer right. The errors identified from this learner's script are as follows:

- the learner failed to identify the turning point
- substitution of wrong values into the correct form of quadratic equation
- solving for q, the turning point given on the graph instead of solving for a

When interviewed on how he arrived at the correct solution, this is what the learner said:

*Researcher: Please clarify for me, B21, why did you first decide to find m?*

*B21: Eeh…. There I need to find the gradient so that I can substitute it into this equation*

{Pointing at the equation $y = a (x + p)^2 + q$}

*Researcher: Okay, so the value of m represents the gradient? So where exactly did you substitute it into that equation?*

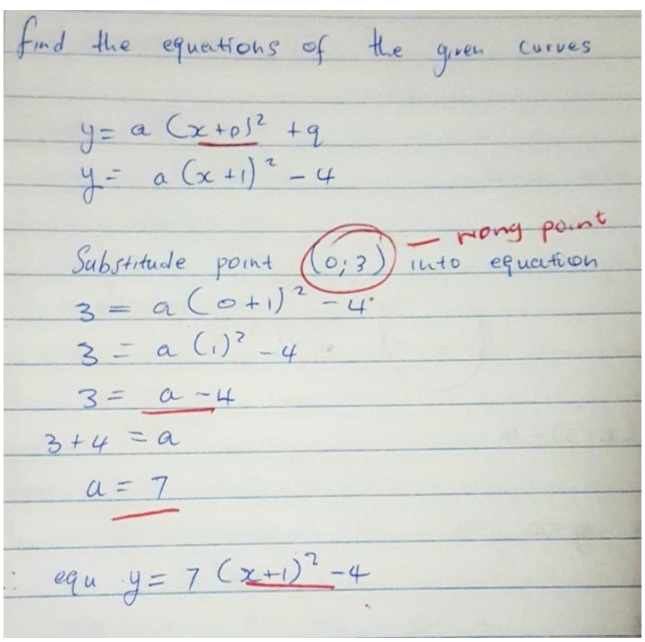
*B21: Here… the value of p in this equation represents the gradient and there at y, I put -4 and at x, I put -1, which is this point on the graph.*

*Researcher: Okay… what about the value of a, because I can see it in your equation. Why did you not obtain it?*

*B21: Which a? Oh…eeh, this one? I think I forgot, but you marked me right…*

From the interview above, although this learner was able to use the value (-1,-4) and substitute it into the equation, this error displays a misconception that could be associated with incorrect association and application of irrelevant rules as identified previously by Radatz (1979). This learner's failure to select the correct mathematical process that is required to obtain the solution is classified by Watson (1980) as a transformation type of error and by Radatz (1979) as error number 3- deficient mastery of prerequisite facts and concepts. As evidenced from the above vignette, some learners were able to use the appropriate form of the parabolic equation, were able to identify the point (-1; -4) from the graph but they failed to identify that this was the turning point and where the turning point was to be substituted into the equation. The misconception that this learner displayed was that he had false concepts hypothesized to form new concepts (Luneta & Makonye, 2010) when he started by calculate the gradient.

**Figure 6Vignette 3 for item 1.1 (Question 1)**




The above-mentioned errors could have hindered the learner from finding the correct equation. According to Luneta and Makonye's (2010) coding and categorizing of errors, the error of mistaking (3;0) as (0;3) can be coded as 0 meaning that this type of error is non- systematic.

These types of errors are slips, lapses or unintended mistakes as now verified from what transpired during the interview.

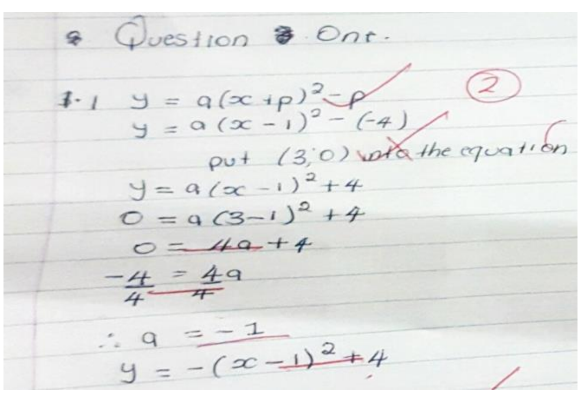
*Researcher: I can see that you were able to use the correct form of the quadratic equation, but please look carefully at the point that you substituted (0; 3) …. Why do you say that is (0;3)?*

*B24: I took it from the graph. Yes, it is (0; 3), no its (3; 0). Oh, I made a mistake…*

The learner thought that he had made a mistake, but, again the learner is wrong, the point is (-3; 0) not (3; 0). However, another research conducted by Graham and Thomas, (2000) suggests that students often misunderstood the variable concept hence this confusion of (3; 0) as (0; 3) as different variables can hinder learners from constructing good mathematical meanings.

Several studies have shown that students as well as teachers have problems understanding numbers or discriminating numbers leading to misconceptions about numbers (Vello, Khrishnasamy & Abdulla, 2015). Hence, as in this case, both the number concept and the algebraic concepts are linked to functions. If students fail to recognize variables correctly, this could also lead to problems in functions. The possible reason for the error might be due to carelessness, (Watson, 1980) or the learner might be lacking the strategy to read the coordinate from the graph.

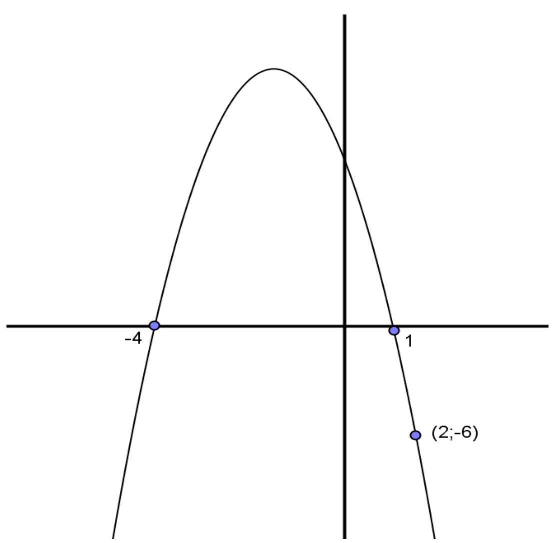
***Figure 7:Vignette 4 for item 1.1 (Question 1)***




In addition to the problems cited from the learner's scripts in vignette1, 2 and 3; this leaners' script in vignette 4 wrote the equation form as $y = a (x + p)^2 – p$ instead of $y = a (x + p)^2 +p$. The four responses from the four learners sampled for question 1.1 represent what the majority of learners did when answering question 1.1. The majority of the learners used the incorrect form of the parabolic equation and some of the learners who used the appropriate form struggled with the substitution of the correct points into the equation. The errors that were prevalent were the wrong choice of the incorrect equation of a parabolic function, substitution of incorrect points, together with the incorrect manipulation of the chosen equations. The errors displayed could be due to inadequate conceptual knowledge in the learning of functions or alternatively it could be due to carelessness (Watson, 1980).

### 4.4.2: Analyzing learner responses to item 1.2 in question 1 of the pretest

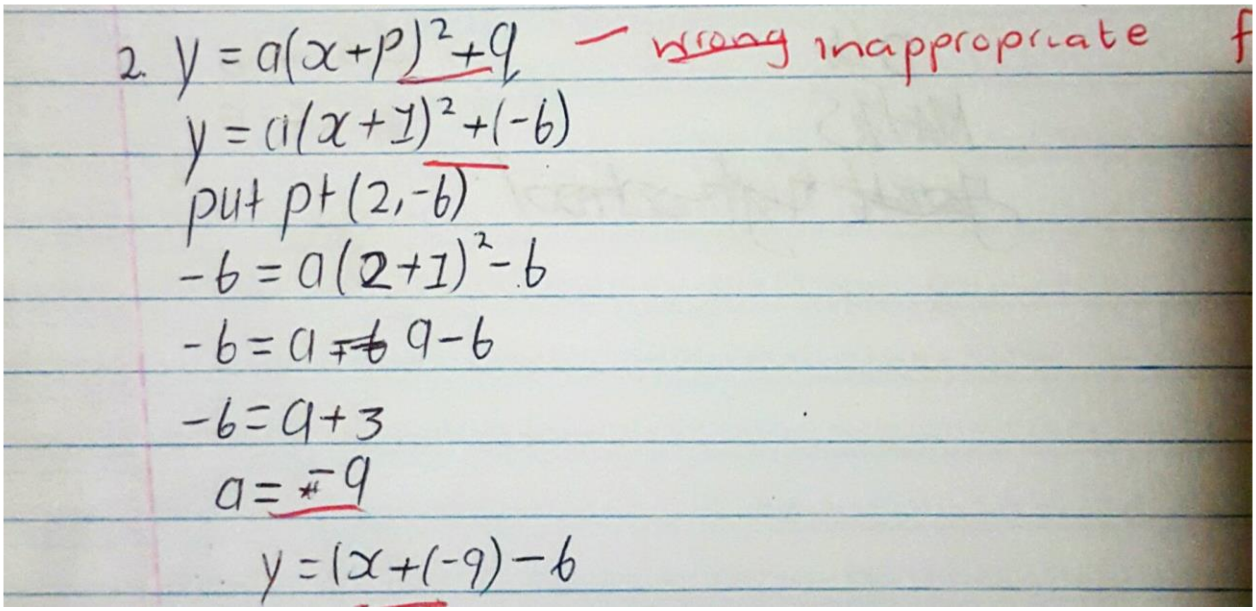
In question 1.2, the learners were required to find the equation of the parabolic graph where the -intercepts of the parabola and another point on the graph was given. Similar to question 1.1, this question required learners to interpret the graphical representation of a parabola and be able to move from the graphical representation of a function to the equation form of the parabola. In order for the learners to be able to do this, they had to be able to recognize the given key features of the parabolic function and substitute the given points on the parabola in the appropriate form of the parabolic equation.




Fifty percent (50%) of the learners could not use the correct quadratic form that would have enabled them to answer this question. Of the fifty percent that was able to use the correct quadratic form, 68% of these learners were still not able to correctly answer the question.

Examples of some of the extracts from the responses of learners for question 1.2 are shown below.

**Figure 8: Vignette 5 for item 1.2 (Question 1)**

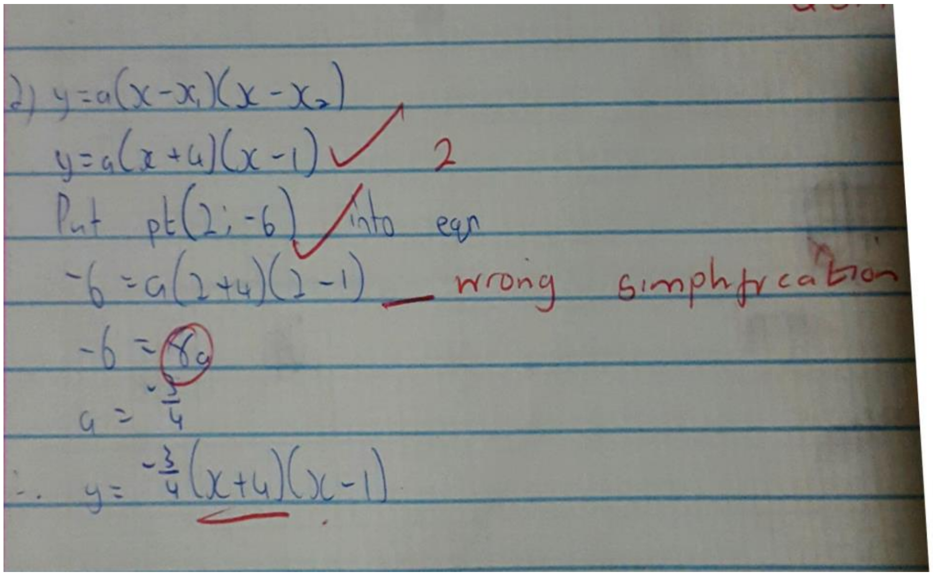



Learner A13

The Learner whose script is shown in figure 8:

- This learner lacks conceptual understanding as they failed to identify the key points on the given parabola hence due to the lack of conceptual knowledge the learner used the wrong equation form, picked a stray point, (2; 6) not significant at all in answering the question and wrongly substituted the point into their equation. This Learner clearly has no idea of the turning point as observed from the substitution of (-1;-6) a point not given on the graph From the literature review of this research report cited in chapter 2, the errors noted for this learner can be categorized as follows: Application of irrelevant rules or strategies (Radatz, 1979); Process skills errors- where the pupils cannot perform the mathematical operations necessary for the task (Watson, 1980)– Response A17 &A8 & A13

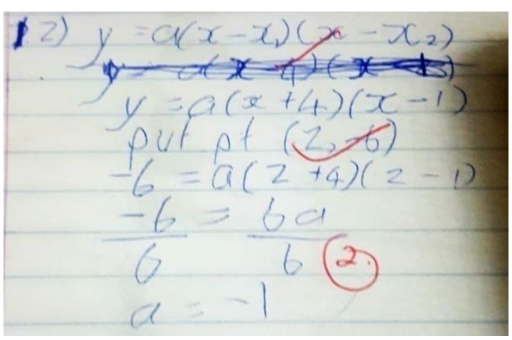
**Figure 9: Vignette 6 for item 1.2 (Question 1)**




Learner A8

The Learner whose script is shown in figure 9 was able to identify the key features from the graph, used the correct quadratic form, was able to identify correct point to substitute into the equation but did wrong simplification to obtain the equation of the graph. What is interesting is how the learner simplified in line 4. Instead of saying (2 + 4) = 6 (simple addition), this learner multiplied 2×4 to get 8; this is recognized from the study done by Makonye and Hantibi, 2014) when they identified this as a misconception of interference, the + interfered with the × in the mind of the learner.From the literature review of this research report cited in chapter 2, the errors noted for this learner can be categorized as follows: Application of irrelevant rules or strategies (Radatz, 1979); Process skills errors- where the pupils cannot perform the mathematical operations necessary for the task (Watson, 1980)– Response A17 &A8 & A13

**Figure 10: Vignette 7 for item 1.2 (Question 1)**




Learner A6

The learner whose script is shown in figure 10 was able to carry out the first steps accurately. However, the possible reason could have been that the learner failed to correctly identify the correct values from the graph required to get the equation. Or the learner thought by finding a they have found the equation of the parabola

The majority of learners made the errors that these extracts show and this could be attributed to the fact that the majority of learners could not interpret the given information from the graphical form of the parabolic function as evidenced by the incorrect form of the parabolic function. Those that were able to identify the correct quadratic form either substituted wrong points while those that substituted correct points struggled simplifying the equation correctly to solve for a. From the literature review of this research report cited in chapter 2, the errors noted for this learner can be categorized as incomplete application of rules (Luneta & Makonye, 2010).

**Question 2**

2.1 On the **DIAGRAM SHEET,** on the same system of axes, draw the graphs of $f(x) = (x+3)^2$ and $g(x) =(x+3)^2 - 4$

Clearly show the coordinates of the intercepts (y intercept and the x intercepts) (5)

2.2 state the domain of *f* and the range of *g* (1)

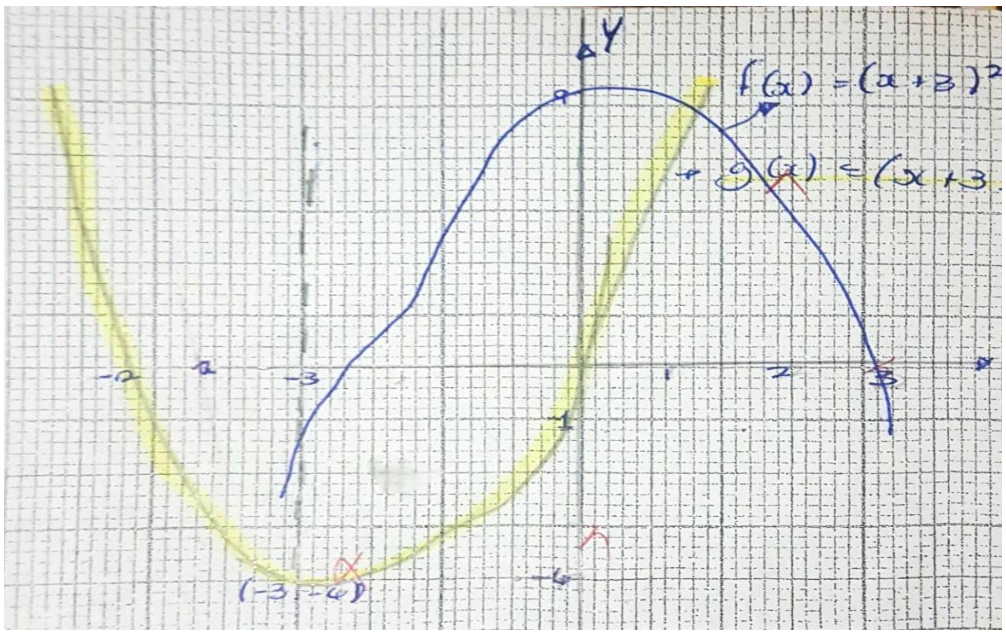
2.3 Explain what has happened to the graph of*f* to obtain the graph $g(x)=(x+3)^2 – 4$ (2)

**4.4.3: Analyzing item 2.1 for question 2 in the pretest**

Question 2.1 required learners to be able to produce sketches of the quadratic functions, f(x) and g(x). In question 2.2, the learners were expected to state the domain and range of one of the functions. In question 2.3 the learners were required to interpret the relationship between the graph of f(x) and g(x) is in terms of the shift that would have occurred. Some of the extracts from the learner responses for question 2.1 of the pre-test from the sample are shown in the extracts below.

As in the other items for this question, a large percentage (90%) of learners was not able to sketch the graphs of f(x) correctly and about 94% failed to sketch the graph of g(x). Although some learners were able to see that these graphs were quadratic, some failed to show the intercepts and the turning point as requested in the question (Example – vignette 8). Leinhardt, Zaslavsky &Stein (1990), suggests that learners should be able to apply interpretation of local processes (regarding point-to-point attention) and as evidenced from some of the learner's transcript the majority of them had problems.

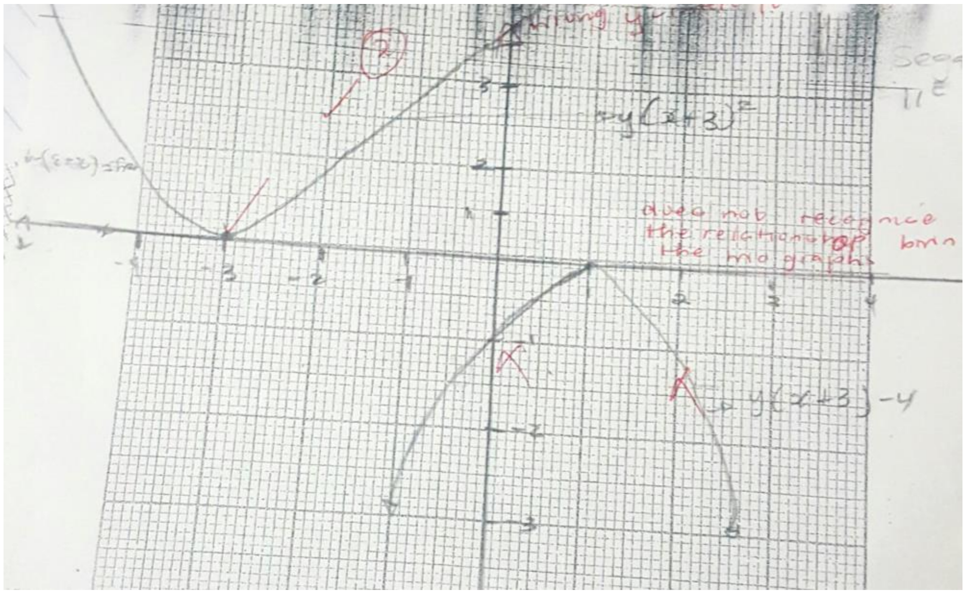
**Figure 11: Vignette 8 for item 2.1 (Question 2)**




From the script above, it is observed that this learner had an idea that the equation of the given function f is quadratic as observed from the shape of the drawn graph, However, this learner like many other learners did not show the correct turning point and the correct y intercepts. Instead, the learner shows the x intercepts to be -2 and 0, while for this graph there are no x intercepts.

However, there were some learners who were able to draw the graph of f(x), but were not able to draw the graph of g(x). An example of such learners is illustrated below:

**Figure 12: Vignette 9 for item 2.1 (Question 2)**



The interpretation of transformation was notproperly applied by the learner with the above transcript (vignette 9) in the sketching of the given function of g. This learner, like the majority of the learners in this study was not able to see the relationship between the two functions f and g and this could also have attributed to him/her failing to draw the graph of g with respect to f.

### 4.4.4 Analyzing Item 2.2 for question 2 in the pretest

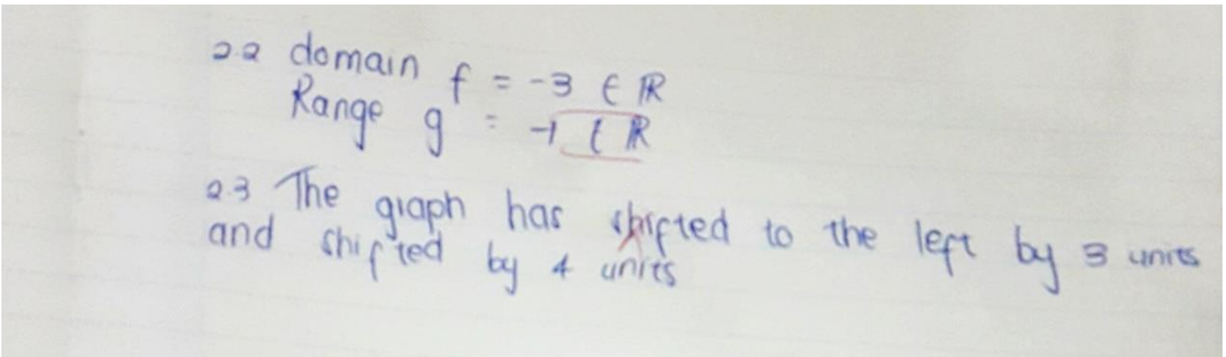
A very large percentage of learners (88%) were not able to state the domain of the function f or the range of the function g. Leinhardt, Zaslavsky & Stein (1990) notes that global interpretation processes (detecting trends), general interpretations (what happens to as increases), and continuation (interpolation/extrapolation) are important attributes when interpreting functions. Although most of the graphs of the learners were not correct, learners still failed to state the domain and the range for their drawn graphs. However, when some learners were interviewed to find out if they understood the concept of the domain or the range, this is what they said:

*Researcher: What do you understand by the domain and the range of a function?*

*Learner A6: The domain is the x values and the range is the y values*

Clearly, like the learner cited in the above interview transcript, most of the learners were able to define their understanding of a range and a domain, but learners failed to link that understanding with respect to the graphs that they had sketched to answer item 2.2 as now observed from the sampled vignettes below:

**Figure 13: Vignette 10 for item 2.2 (Question 2)**




### 4.4.5 Analyzing Item 2.3 for question 2 in the pretest

In the sketching of the given functions in item 2.1, learners did not properly apply the interpretation of transformation.

Hence, most learners (68%) failed to explain what the effects of changing the function g in question 2.3 would do to the parent graph of the function f.

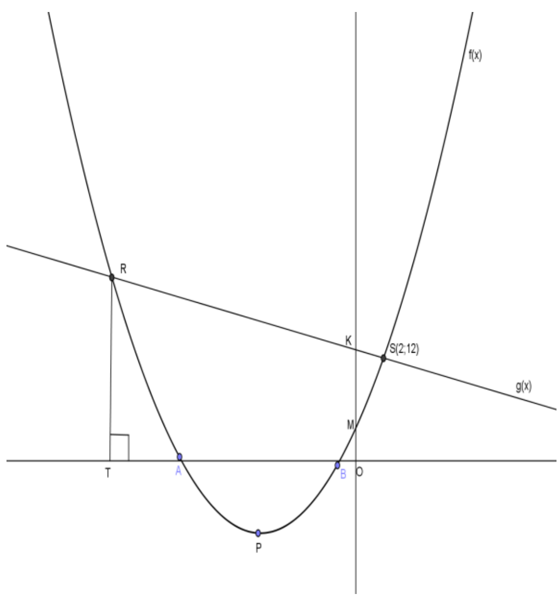
From the literature review of this research report cited in chapter 2, the errors noted in this question (question 2) can be categorized as follows:

- Rigidity of thinking (Radatz, 1979).
- Ignorance of rule restrictions. Learners failed to understand the bound where a rule applies (Luneta & Makonye, 2010).
- Transformation error – the learners failed to select the mathematical processes, which is required to obtain the solution (Watson, 1980).
- Incomplete application of rules (Luneta & Makonye, 2010)

**Question 3**

A parabola $f(x) = ax^2 + bx + c$ with turning point $(-\frac{3}{2}; -\frac{1}{4})$ and a straight-line $g(x) = -x + 14$ intersect at the point S (2; 12). The two graphs not drawn to scale are drawn below. A and B are the x intercepts of the parabola. K is the y intercept of f. RT is a straight line parallel to the y-axis.

.




3.1 Show that a = 1, b = 3 and c = 2. (5)

3.2 Calculate the distance between A and B. (4)

3.3 Determine the length of KM, the distance between the two y intercepts. (3)

3.4 For which values of x is f a decreasing function (2)

3.5 Determine for which values of x is $f(x) \geq g(x)$, where $x \geq 0$ (2)

3.6 Determine the length of RT (3)

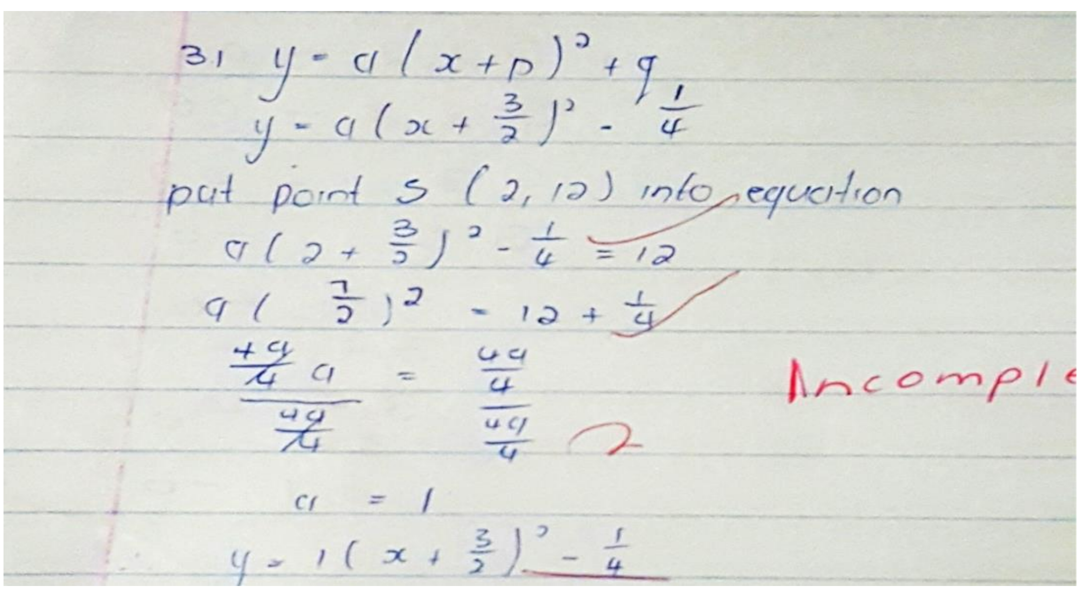
3.7 Determine the value of $g(x^2) + g(\frac{1}{x}) - 28$ (3)

### 4.4.6 Analyzing Question 3 (all items) in the pretest

Most of the learners did not attempt to answer this question. This higher order question required learners to first have a basic understanding of the functions topic so that they could be able to apply that knowledge to answer this question. Failure to attempt or even answer would mean that learners lacked the motivation as they had problems with the basic concepts involving functions. From those few that made an attempt at 3.1, learners' transcripts show that learners had problems choosing the right information from the graph to use to find a, b and c, from the given the information on the graph.

The inability of learners to answer this question could have been due to their struggle to move between representations with respect to the information given in the graph as explained in item 1.1 and 1.2. Learners failed to realize that factors involved in linking representation of functions together as a unit depend on the context and information provided for the function. In this particular question, depending on the information extracted from the graph given, learners could have used any of the three forms of the quadratic equations: $y = ax^2 + bx + c$ or $y = a(x-p)^2 + q$ or $y = (x - x_1)(x - x_2)$to find the values of a, b and c.

**Figure 14:Vignette 11 for item 3.1 (Question 3)**



Although this learner (Vignette 11) was able to use one of the quadratic forms to get the equation of the graph, she/ he could have manipulated this solution and written it in the form $y = ax^2 + bx + c$ to get the values of a, b and c.

Items 3.2 to 3.7 were notanalyzed as the majority of learners did not answer these questions. Most learners scored a zero mark because these were left unanswered. However, from how learners answered question 3.1, the most common error identified is this question was that learners struggledto move between representations in the topic functions hence they failed to answer the application questions.

### 4.5 Analyzing learner interviews after the pretest

The purpose of learner interviews in this investigation was to try to establish the misconception behind the error identified in the pretest. In trying to establish the misconceptions, the researcher marked the pretest gave the scripts back to the learners and asked some of the learners who had problems in answering the items to shade light as to their challenges as now outlined below.

#### 4.5.1 Interview responses for question 1 after the pretest

Respondents were asked to explain why they think they did not get the correct solution to question 1.1 and these are some of their responses:

*Response1: I could not come up with the correct quadratic form to use*

*Response2: I used the correct quadratic form but when I substituted (0; 3) into the equation insteadof (0; -3)*

*Response 3: I first calculated the gradient and I used the correct form of the quadratic equation ... and you marked me right.*

*Response 4: I was not sure which form of quadratic equation to use*

From the responses above, the reasons why some learners could not answer this question is because they could not come up with the correct quadratic form to use, or they failed to read the point to be substituted correct or/ and they failed to simplify the algebraic equation correctly to find the value of *a*. These reasons are also applicable to item 1.2 as these questions were similar but required the learners to use a different quadratic form.

### 4.5.2 Interview responses for question 2 after the pretest

The responses to question 2.1 indicated that most pupils had an idea that $f(x) = (x + 3)^2$ is a parabola although some learners failed to show the turning point and the y intercept on their sketches. However, the challenge was to see the connection between $f(x) = (x + 3)^2$ and $g(x) =(x + 3)^2 – 4$. Among those that were able to sketch the graph of f(x), some learners could not sketch the graph of g(x). When asked to explain why they were able to sketch the graph of f(x) but failed to sketch the graph of g(x), these are some of their responses:

*Response 1: the graph of f(x) has shifted 4 units down* {although the sketched graph does not reflect what the learner is saying}

*Response 2 : the graph g(x) has shifted down* {again the graph drawn by the learner does not reflect what the learner is saying)

Although learners know that the graph g(x) is a transformation of f(x), they do not have a good understanding of the transformation. If they do, that is not reflected in how they have sketched the graph meaning that learner have problems with this aspect of sketching graphs that have undergone some transformation in functions.

### 4.5.3 Interview responses for question 3 after the pretest

Most learners did not attempt this question. When asked why, these were some of their responses:

*Response 1: I could not answer 3.1 so I felt that I cannot answer the rest of the questions…*

*Response 2: … Question 3 was difficult ma'am….*

*Response 3: Aaah, it was challenging*

When asked why they found this question challenging, this is what one participant said:

*Response 4: Maybe because I have problems with simple things in functions therefore I cannot answer this type of question.*

### 4.6 Discussion of the Pretest and the Interview as data collecting instruments

In Question 1.1; 48 % of the learners failed to recognizes the appropriate equation to use while in Question 1.2; 50% of the learners could not recognize the appropriate quadratic form to use. For these two items, not only was the learners' ability to use the appropriate generalized form of the parabolic equation by identifying the features that were given from the parabolic graphs of utmost importance but also for the learners to be able to correctly use the equations. Although 52% of the learners were able to use the appropriate quadratic form in question 1.1, it is only 34% of the learners that was able to answer the question correctly. Of the 50% of the learners that were able to use an appropriate form of quadratic equation in question 1.2; it was only 16% that was able to find the equation of the parabola. In this question (Question 1), learners were supposed to identify the features represented in the structure (the graphical form), choose the appropriate form of the quadratic equation into which these critical features (points, values) can be substituted, perform algorithms and solve the equation and end up with the equation of the parabola. From the interviews conducted and from the literature cited in the literature review to find out the reasons for learners' inability to answer the questions, the following problems were revealed. Learners were not able to use the appropriate form due to failing to recognize the key features of the parabola. In addition, learners were not able to simplify correctly; this according to some of the interviews conducted could be due to (i) ignorance of rule restrictions or symbolism (ii) learners struggling with the variable concept and (iii) carelessness.

Most learners failed to correctly answer these questions because they failed to recognize that a parabolic function exists both as a process and as an object. According to Sfard (1991), the dual nature of a function consists of the structural nature (as an object) and the operational nature (as a process). In these two items (1.1 and 1.2), the object is given as a graph of a parabola and represented as a structure and the operational nature is the computational processes involved to obtain the equations of the given parabolas.

Judging from the poor performance by learners, the researcher concluded that learners have problems moving from one representation of the parabolic function to the other. Hence, this aspect of the dual nature of functions was used to inform the lesson intervention using ICT. Since the lesson intervention is informed by variation theory, learners should be able to flexibly move between different representations of the quadratic graph to the quadratic equation and also to move within the different forms of the quadratic equations. Even (1998) asserts to this by saying that the ability to identify and represent the same thing in different representations and flexibility in moving from one representation to another, allows one to see rich representations, develop a better conceptual understanding and strengthens one's ability to solve problems.

In Question two, starting with item 2.1. Only 6% of the learners were able to draw the required graphs and to show clearly the intercepts. In 2.2 about 88% of the learners failed to answer the question correct which required learners to state the range and the function of the graphs they would have drawn in 2.1. To try to understand if learners understood the definition of the domain and the range, from the interview results it is clear to the researcher that learners know the definition. Again, to gain an insight when marking the solution, the researcher deviated from the expectations of the question that required learners to state the domain and the range of the correct graphs of the functions f and g by trying to look for how they described them using the graphs they had drawn in 2.1. Only 12% of the learners were able to define the range and the domain with reference to their graphs. In Question 2.3, 68 % of the learners were not able to explain the transformation that had happened from f to g. Reasons gathered from the different categories used by other researchers to code the errors identified as well as the interviews conducted revealed that the following as reasons for their misconceptions. Learners are rigid in their thinking (Radatz, 1979); they stick to the definition when asked about the range and the domain instead of reflecting on the definition to answer questions. Luneta & Makonye, (2010) who note that learners fail to understand the bound where a rule applies endorse this. Learners also failed to select processes that would have helped them show the vertex and the intercept on the graphs drawn- referred to as a transformation error by Watson, 1980 hence they failed to answer the questions correctly.

Question 3 had seven items, and of all the questions this was the question which showed the worst performance by learners. This was an applications problem and it required that learners apply their knowledge of the quadratic functions to other concepts in mathematics. Only 8% of the learners were able to answer 3.1 up to 3.4. And only 4% could answer items 3.5 to 3.7.

The results of the pre-test then informed the object of learning in the intervention lessons that would be conducted. The lessons are you tube videos on functions taken from the internet as well as one lesson where learners explore geo gebra- a software used to draw and explore graphs of functions. The analysis of the pretest gives an answer to the first research question: The errors identified, and the misconceptions established informed the lesson intervention as discussed in the paragraphs below.

**4.7Analysis of the lesson interventions**

The objects of learning which informed the planning of the intervention lessons were chosen from the most common errors displayed from the result of the pre-test. Lesson interventions were administered to the two groups of grade 11, one using technology and the other using traditional methods. This question-by-question analysis of the pretest was compiled for the other teacher to inform his intervention with his class while the researcher also used this information to plan for the intervention using technology. The lesson interventions for the experimental group were informed by the errors identified in the pretest using variation theory as explained in Chapter 2 and Chapter 3. Five one-hour lessons were conducted with the experimental group. The intervention lessons were observed for the experimental group and data was collected and summarized using appendix C. From the total allocated time of 300 minutes, the data that was collected revealed that the following percentage of time was spent on these learner activities and teacher Activities as shown in Table 7.

In this study, during the intervention, a different approach to the teaching and learning of the concept of functions was attempted with the help of technology to the experimental group. The aspect of functions that learners found to be most problematic was the use of the appropriate form of the quadratic function in the derivation of the equation from the given graphical representation of a parabola. The approach to this identified problematic area during the intervention was informed by the theory of variation, which was explained in chapter 2. Through the lessons, various aspects of the theory of variation were used in the intervention to address as well as to try to rectify the errors displayed by the learners in the pretest due to their misconceptions using technology for the experimental group. For learners to be able to correctly answer questions on algebraic functions, they should be able to flexibly move between the graphical representation and the equation form of these algebraic functions. The lessons administered using technology were sourced from the internet (U- tube videos on functions) together with the use of Geogebra software installed on the learner's computers for exploring the drawing graphs and their equations.

**Table 7: Lesson intervention observation sheet (300 minutes)**

| Activities | Actual Time Spent during the duration of the intervention (Minutes) | | Percentage of time spent on activity (%) |
|---|---|---|---|
| Watching U tube videos on functions | 100 | | 33,33% |
| Learners exploring Geo gebra using computers | 60 | | 20% |
| Learners exploring function problems using technology | Individually | 40 | 40% |
| | Pairs | 40 | |
| | Groups | 40 | |
| Teacher explaining concepts using Technology | 20 | | 6.67% |
| Teacher explaining concepts using the chalk board | 0 | | 0% |

From the table above learners interacted with technology (93.33%) most of time. It was only when learners needed clarity that the teacher tried to clarify concepts with the aid of technology (smart board and Geogebra) to enhance understanding in learners. Since this investigation is on learners, more time was given to them so that they could interact with technology. The teacher's role was to facilitate that learning.

### 4.7.1 Overview of the Lesson interventions

After the intervention lessons, one conducted by the researcher using technology to the experimental group, and another conducted by another educator using traditional methods, learners then wrote a test (posttest) to assess the effects of the two different interventions. The lesson interventions were selected videos from you tube.

From the school where the researcher teaches, 226 Grade 12 learners received tablets from Gauteng Department of Education at the beginning of the year in 2016. From the learners that I taught in grade 12, I borrowed 25 tablets that I made use of during the intervention with the grade 11s. Each learner had a tablet and using Bluetooth, each day, I sent the videos that I needed learners to watch for each lesson intervention. Sending the videos from my laptop took less than 5 minutes. Each learner was requested to bring headphones so that they could be able to watch the videos individually without any disruptions from one another. For about 30 minutes, learners watched the videos individually, and then in groups of five, they discussed about the main features of the functions they had learnt from the videos, then the last five minutes was set aside for class discussions with the teacher if learners needed any clarity. Outlined below are the videos that were used during the intervention and the time of the video.

### 4.7.2Lesson Intervention 1

The aim of the lesson was to help learners experience what a function is? To experience a function is to experience both its meaning, its structure (composition) and how these two mutually constitute each other. If these two aspects are not focused on in a teaching situation, they remain critical in the students' learning (Olteanue&Olteanue, 2010, 2011). It was with this view that the researcher chose the videos from u tube to introduce learners to what a function is. As an introduction to the intervention, 3 videos were chosen from u tube.

a) Khan Academy: Topic: What is a function? This video was posted on the 6thJune 2003, Time 7minutes 56 second.
b) Maths : Topic : What is a function: y = f(x) This video was posted on 15 July 2011) Time 12 minutes 54 seconds
c) Khan Academy: Topic Functions part 2. This video was posted on the 18 March 2007), Time 9minutes 54 seconds.

### 4.7.3 Lesson intervention 2

a) Khan Academy: Topic: Introduction to the quadratic equation (Posted 28 January 2007) Time 9minutes 15 minutes
a) Alg II- Quadratics- Find the Equation Given Graph. This video was posted on 23 February 2012, Time 5minutes 33 seconds

b) How to get the Equation of a Parabola given its intercepts and a point. This video was uploaded on 12 September 2011, Time 5minutes 54 seconds.

### 4.7.4 Lesson Intervention 3

a) Mark Lund – Equation of parabolas given 3 points. This video was posted on 5 October ,2011, Time 14minutes 58 seconds

b)PausMath: Topic: Algebra 2 – writing Quadratic Functions Given 3 points. This video was posted on 1 December, 2012, Time 10 minutes 20 seconds

### 4.7.5 Lesson intervention 4

Learners were using computer in our school computer lab exploring Geogebra software to draw different graphs of quadratic functions. The researcher took about 15 minutes to demonstrate using the smart box how learners could explore different graphs using Geogebra. For 45 minutes' learners explored different quadratic graphs using Geogebra, helping one another in the process.

### 4.7.6 Lesson intervention 5

Learners were given an activity in which they could refer to the u tube videos as well as the Geogebra software to answer the questions individually. This was then followed by a class discussion on how learners were using ICT to check if their solution was correct. The researcher gave the platform to the learners who were discussing their solutions amongst each other for about 1 hour.

## 4.8Quantitative Analysis for the Post test

**Table 8: Posttest Marks for learners in the control group**

| Name | Mark | Percentage | | Name | Mark | Percentage | | Name | Mark | Percentage |
|---|---|---|---|---|---|---|---|---|---|---|
| A1 | 15 | 50 | | A10 | 14 | 47 | | A19 | 6 | 20 |
| A2 | 18 | 60 | | A11 | 15 | 50 | | A20 | 12 | 40 |
| A3 | 12 | 40 | | A12 | 12 | 40 | | A21 | 10 | 33 |
| A4 | 14 | 47 | | A13 | 10 | 33 | | A22 | 20 | 67 |
| A5 | 14 | 47 | | A14 | 8 | 27 | | A23 | 15 | 50 |
| A6 | 17 | 57 | | A15 | 8 | 27 | | A24 | 118 | 60 |
| A7 | 16 | 53 | | A16 | 6 | 20 | | A25 | 22 | 73 |
| A8 | 12 | 40 | | A17 | 8 | 27 | | | | |
| A9 | 10 | 33 | | A18 | 8 | 27 | | | | |

Given above, in table 8, is a table showing the results of the learners' marks for the posttest for the control group after the intervention lessons. The mean score of the post test for the control group is 42,72%.

For the researcher to be able to analyze the performance of learners in the posttest, the marks for the two groups were presented separately to enable quantitative analysis in comparing each groups' performance with respect to their pretest score as well as to compare the two groups together. Table 9, below shows the marks of learners for the experimental group in the posttest.

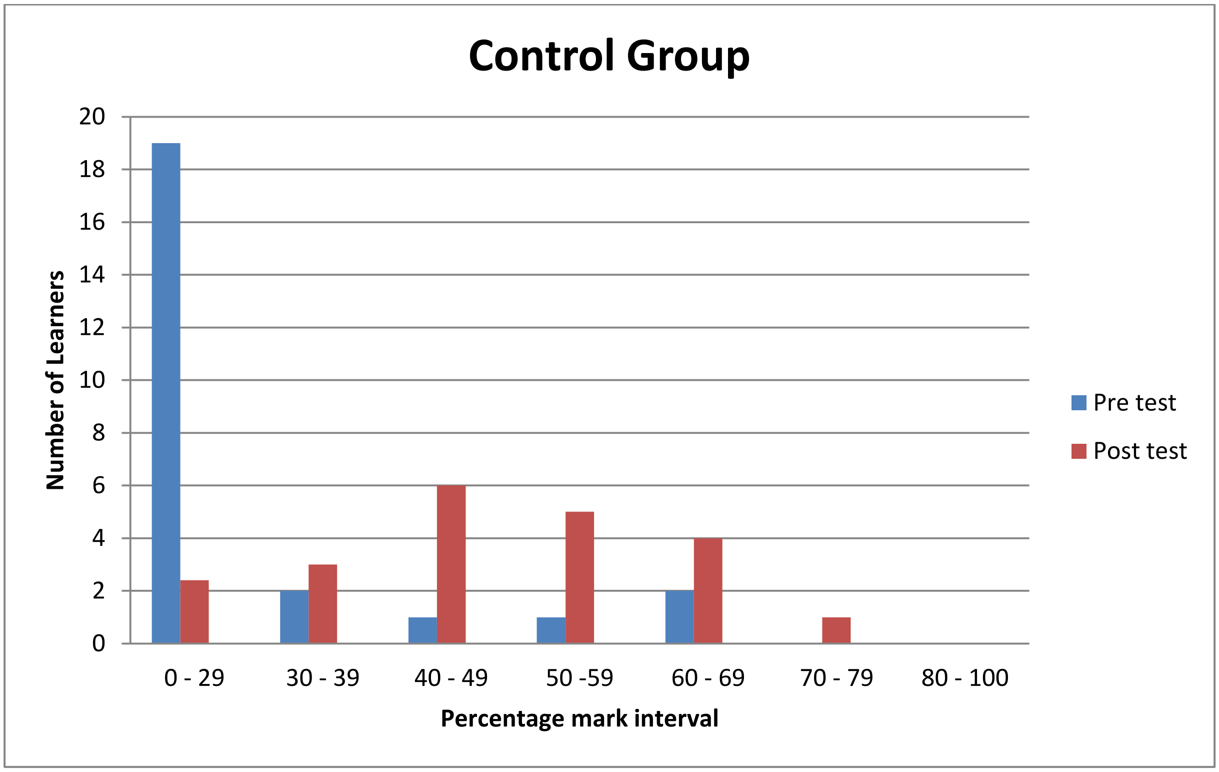
**Table 9: Posttest Marks for learners in the experimental group**

| Name | Mark | Percentage | | Name | Mark | Percentage | | Name | Mark | Percentage |
|---|---|---|---|---|---|---|---|---|---|---|
| B1 | 15 | 50 | | B10 | 16 | 53 | | B19 | 15 | 50 |
| B2 | 10 | 33 | | B11 | 18 | 60 | | B20 | 15 | 50 |
| B3 | 10 | 33 | | B12 | 18 | 60 | | B21 | 18 | 60 |
| B4 | 20 | 60 | | B13 | 14 | 35 | | B22 | 14 | 47 |
| B5 | 26 | 87 | | B14 | 12 | 40 | | B23 | 15 | 50 |
| B6 | 18 | 60 | | B15 | 16 | 53 | | B24 | 10 | 33 |
| B7 | 14 | 47 | | B16 | 20 | 67 | | B25 | 12 | 40 |
| B8 | 12 | 40 | | B17 | 18 | 60 | | | | |
| B9 | 14 | 35 | | B18 | 16 | 53 | | | | |

The mean score for the experimental group is 50,25%. The results of the groups show that both groups improved from their previous results of the pretest. To gain insight to to extent of the improvement, two separate graph will now be presented for each group to show a comparison on their performance in the pretest to their performance in the posttest.

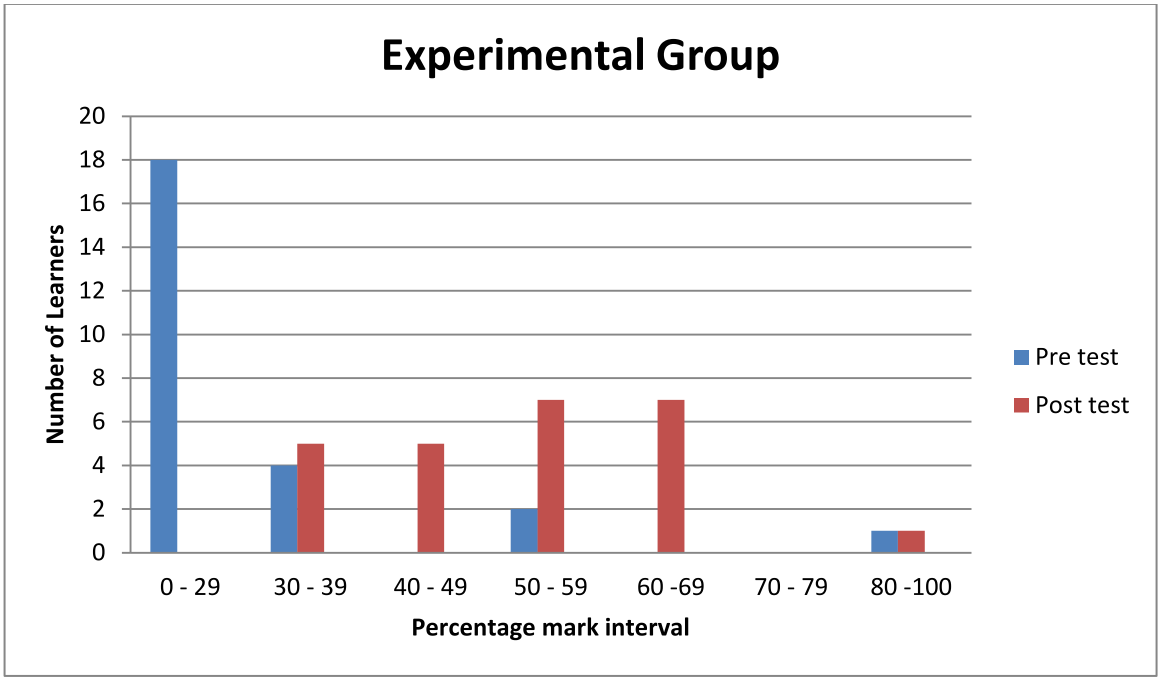
The graphs below, figure 15 and figure 16, show a comparison of each group's performance on the pretest to their performance in the posttest based on the individual percentage scores for learners in the group.

**Figure 15: Comparison of the pretest and the post test scores for the control group**




The mean score for learners in the pretest for the control group was 22, 04% and this increased to 42.72% in the posttest. An increase of 20.68% can be attributed to the lesson interventions that addressed the learner errors and misconceptions identified in the pretest. Considering that in mathematics at F.E.T phase, a pass mark in mathematics is 30%; based on the results of the pretest, only 24% of the learners passed this test and a larger percentage 76% failed to meet the minimum pass requirement. However, in the posttest76% of the learners were able to meet, the pass requirement while it only 24% of the learners that failed; a notable improvement from what had occurred before learners were exposed to an intervention that addressed their errors and misconceptions.

**Figure 16:Comparison of the pretest and the post test scores for the experimental group**



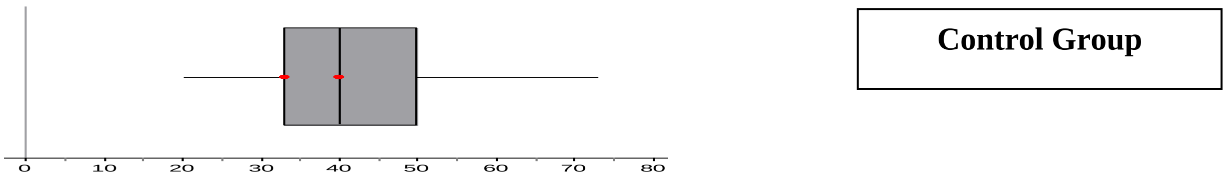

There is an almost normal distribution of the marks of learners for the posttest compared to the learners' marks that were previously skewed to the left of the mark interval in the pretest.The mean mark score for learners in the experimental group improved from 21.52% in the pretest to 50.25% in the posttest. Again, considering the pass mark in mathematics; 72% of the learners failed the pretest while only 28% passed the pretest. However, after the intervention, the results from the posttest reveal that all the learners (100%) passed the test, a massive improvement from what had been previously noted in the pretest.

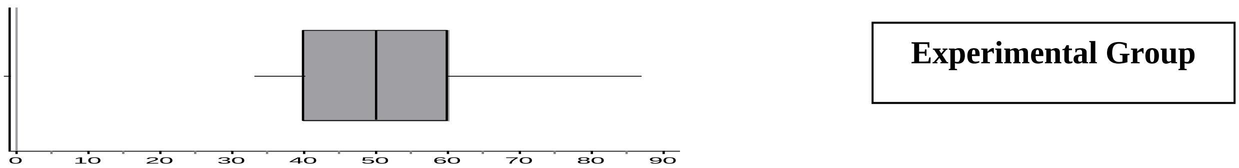
**4.9 Comparing the post test results for the control group and the experimental group**

To help compare and analyze the results from the two groups of learners, the learners' results from both groups separately, are used to draw two box and whisker diagrams as now presented.

Figure 17 below, shows a comparison of the posttest marks for learners in the control group to learners' marks in the experimental group. Although both group's marks improved, analyzing the marks using a box and whisker will help establish which of the two groups performed better than the other.

**Figure 17: A comparison of the pretest and posttest scores for the experimental group**




The lower performing 25% from the control group got marks ranging from 20 to 33%, while the lower 25% from the experimental group got between 33 and 40%. Judging from this, the lower 25% from the experimental group performed better than the lower 25% from the control group. In addition, 50% of the learners from the control group got 40% and above while 50% of the learners from the experimental group got 50% and above. Again, learners from the experimental group performed better that those in the control group. The 25% top performers in the control group got between 50 and 73% while those in the experimental group got between 60 and 87%; again learners from the experimental group performed better. Moreover, all learners from the experimental group passed the posttest scoring more than 30% while there are some learners in the control group who could not achieve a minimum pass mark of 30%. From the box and whisker diagrams above, the researcher can conclude that the experimental group performed better than the control group.

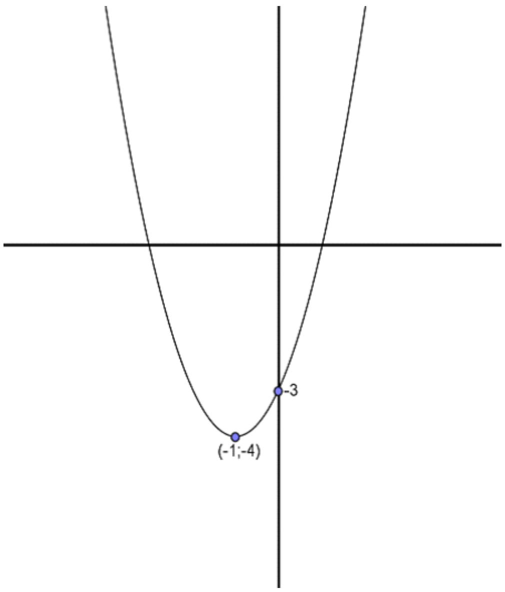
### 4.10Qualitative Analysis of the post-test

The post-test consisted of three questions. The first question of the post-test was similar to the

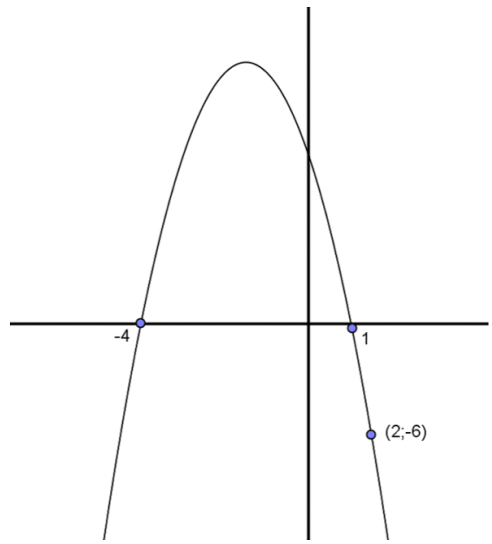
**Question 1**

Learners were required to find the equation of each given graph

1.1




1.2

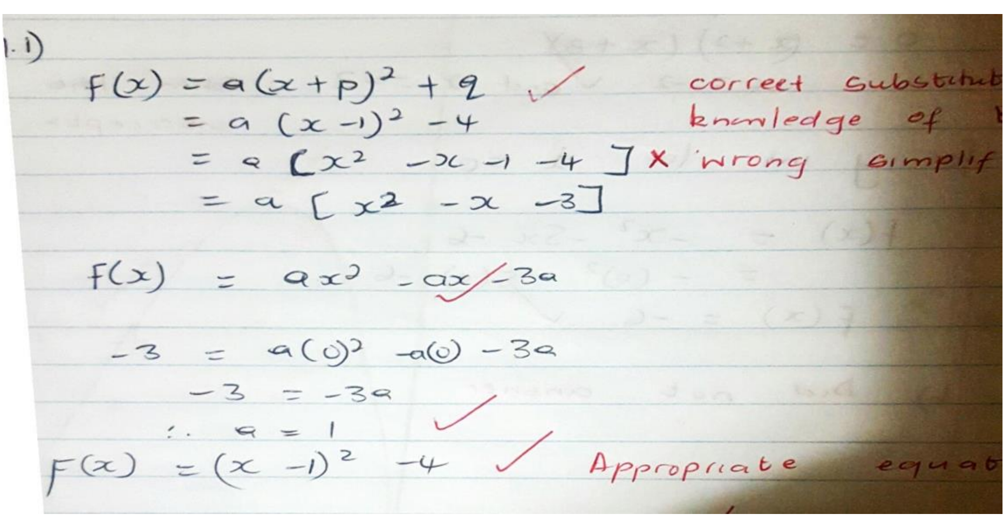



Question 1.1 and question 1.2 were similar to the questions asked in the pretest. After the intervention lesson using computers, there was an improvement on how learners answered these questions as observed from the vignettes below.

### 4.10.1 Analyzing learner responses for question 1.1 in the posttest

The following are extracts of some of the learner's responses to question 1.1 for the post test.

**Figure 17:Vignette 12 for item 1.1 in the posttest (Question 1)**



1.1)

$f(x) = a(x+p)^2 + q$ ✓ correct substitut

$= a(x-1)^2 - 4$ knowledge of

$= a[x^2 - x - 1 - 4]$ × wrong simplif

$= a[x^2 - x - 3]$

$f(x) = ax^2 - ax - 3a$ ✓

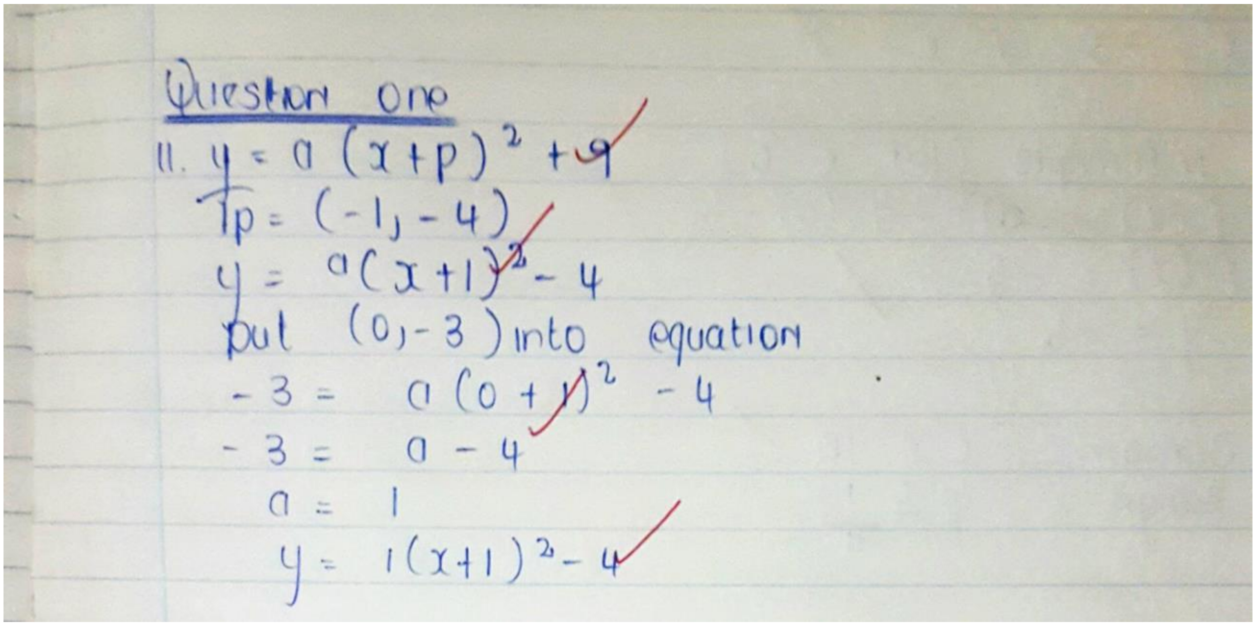
$-3 = a(0)^2 - a(0) - 3a$

$-3 = -3a$

$\therefore a = 1$ ✓

$F(x) = (x-1)^2 - 4$ ✓ Appropriate equat

**Figure 18:Vignette 13 for item 1.1 in the posttest (Question 1)**




Most of the learners were able to recognize the turning point hence they were able to recognizes the appropriate form of the quadratic equation to use as evidenced in the sampled scripts. With regard to substitution of the turning point, again most learners were able to substitute into the equation as they were also able to recognize where into the equation they were to substitute the turning point. Although other learners were able to carry out the steps correctly like the learner in vignette 13, other learners like the learners with vignette 12 and vignette14 had challenges in simplifying algebraic expressions. For example, the learner with vignette 12, Line 1 and Line 2 are correct, but the solution in line 3 implies that this leaner: $(x - 1)^2 = x^2 - x - 1$ which is not correct;

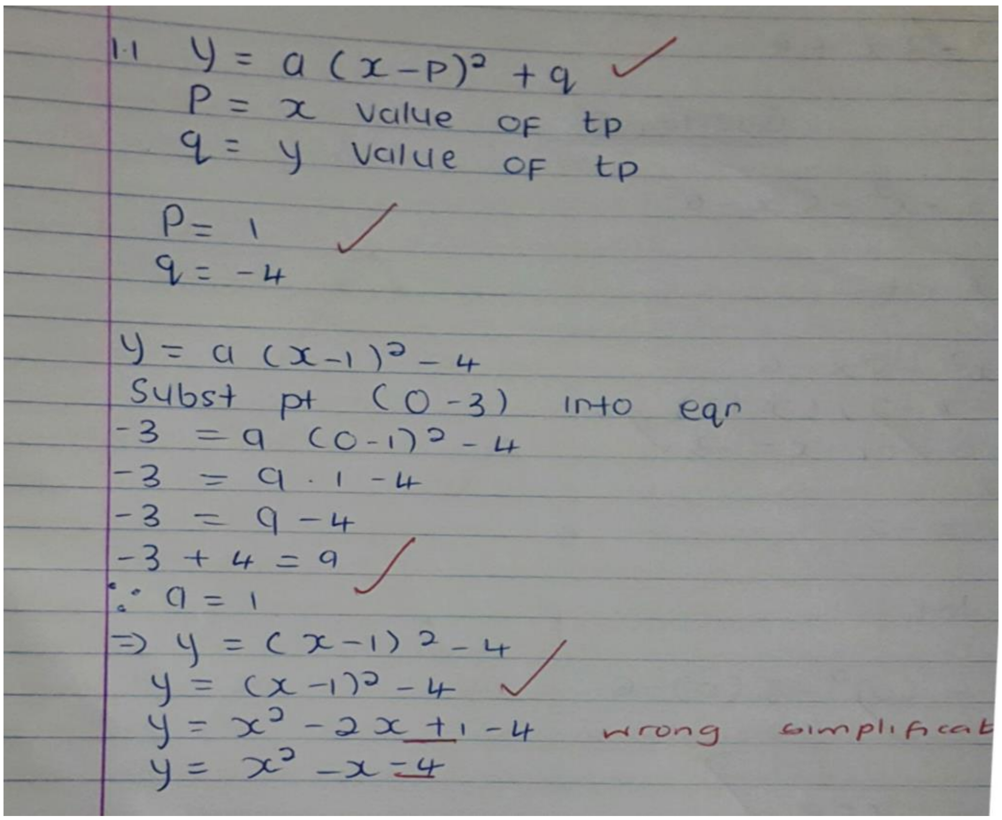
again looking at Line 3 and line 4 what is shown here is that:

$x^2 - x - 1 - 4 = x^2 - x - 3$ meaning that for this learner:

$-1 - 4 = -3$ again this is not correct.

Although the final answer is correct, this learner arrived at the correct answer but demonstrations in the solution that the learner has problems with simplifying algebraic expressions and also struggles with the topic on addition of integers involving negative numbers. The problems cited from this learner (vignette 12)were also observed from other learners and the example cited is shown on vignette 14 below.

***Figure 19:Vignette 14 for item 1.1 in the posttest (Question 1)***


1.1 $y = a(x-p)^2 + q$

$p = x$ value of tp

$q = y$ value of tp

$p = 1$

$q = -4$

$y = a(x-1)^2 - 4$

Subst pt (0 -3) into eqn

$-3 = a(0-1)^2 - 4$

$-3 = a \cdot 1 - 4$

$-3 = a - 4$

$-3 + 4 = a$

$\therefore a = 1$

$\Rightarrow y = (x-1)2 - 4$

$y = (x-1)^2 - 4$

$y = x^2 - 2x + 1 - 4$ wrong simplificat

$y = x^2 - x - 4$

For this leaner (vignette 14), like other learners in the investigation, although she has mastered the concept of finding the equation of the graph given, there is still some challenges in simplifying algebraic expressions. Looking at the last two lines on the above vignette, the learner correctly obtained the equation of the graph, but in trying to simplify it by removing brackets, this is what happened: $y = (x - 1)^2 - 4$ (This is correct)

$y = x^2 - 2x + 1 - 4$ (This is correct)

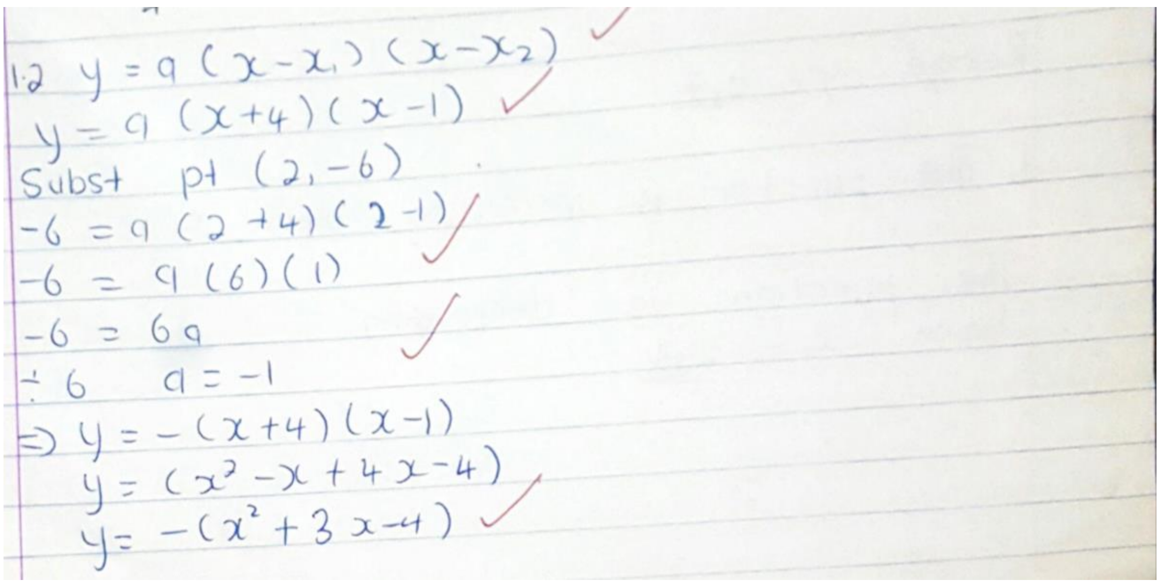
$y = x^{2-}\ x - 4$ (This is NOT correct)

Most of the learners in the experimental group, left their answer in the form y = $(x - 1)^2 - 4$; there is a possibility that they left their answer in that form to avoid simplifying as they have some challenges.

### 4.10.2 Analyzing learner responses for question 1.2 in the posttest

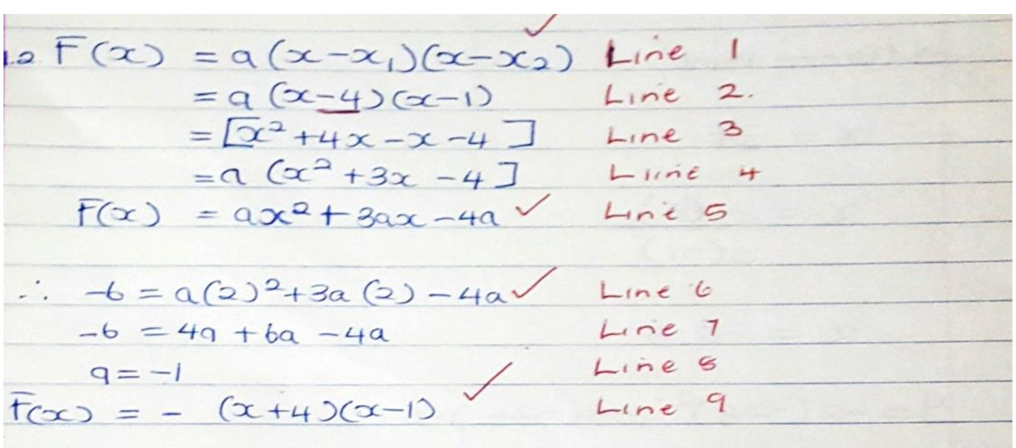
Most learners were able to identify the x intercepts from the graph and a majority of them were able to use the appropriate form of the quadratic equations to answer this question as evidenced from their sampled solutions.

**Figure 20:Vignette 15 for item 1.2 in the posttest (Question 1)**



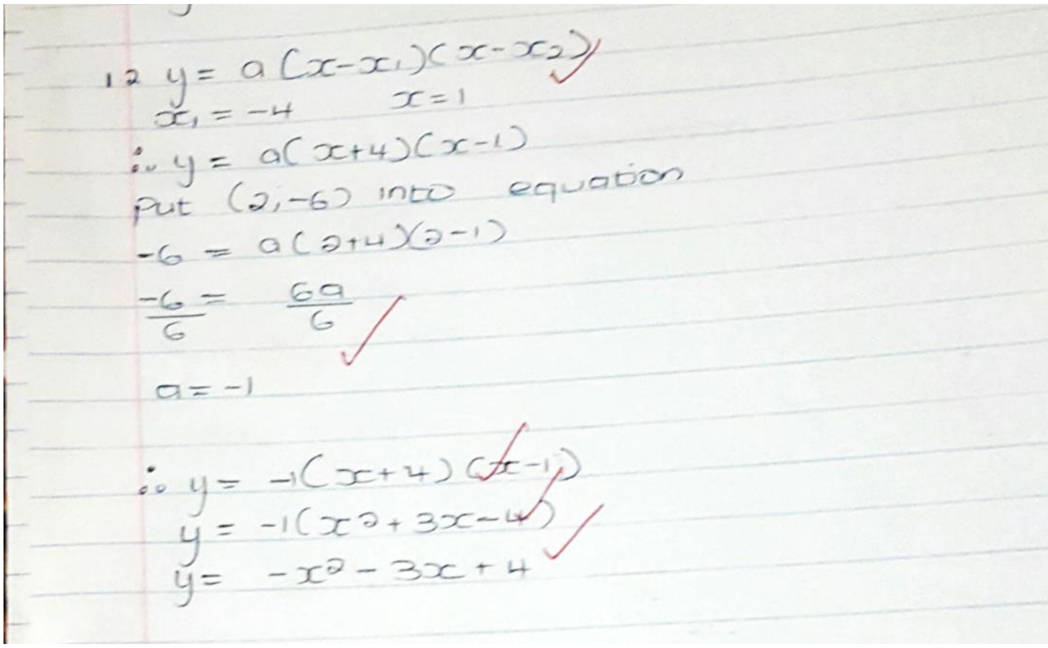
1.2 $y = a(x - x_1)(x - x_2)$ ✓
$y = a(x+4)(x-1)$ ✓
Subst pt $(2, -6)$
$-6 = a(2+4)(2-1)$ ✓
$-6 = a(6)(1)$
$-6 = 6a$ ✓
$\div 6 \quad a = -1$
$\Rightarrow y = -(x+4)(x-1)$
$y = (x^2 - x + 4x - 4)$
$y = -(x^2 + 3x - 4)$ ✓

***Figure 21:Vignette 16 for item 1.2 in the posttest (Question 1)***



1.2 $F(x) = a(x - x_1)(x - x_2)$ ✓ Line 1
$= a(x-4)(x-1)$ Line 2.
$= [x^2 + 4x - x - 4]$ Line 3
$= a(x^2 + 3x - 4]$ Line 4
$F(x) = ax^2 + 3ax - 4a$ ✓ Line 5
$\therefore -6 = a(2)^2 + 3a(2) - 4a$ ✓ Line 6
$-6 = 4a + 6a - 4a$ Line 7
$a = -1$ Line 8
$F(x) = -(x+4)(x-1)$ ✓ Line 9

**Figure 22:Vignette 17 for item 1.2 in the posttest (Question 1)**




Although some learners were able to carry out correctly the steps required to answer this question and simplify their answers like learner with vignette 17, some learners left their equations in not simplified like the script for the learner with vignette 15 and vignette 16. Analyzing the script for the learner whose solution is displayed in vignette 16in detail, this learner was able to use the correct form of the quadratic equation required to answer the question. However, in Line 2, substitution was done wrongly. Again, looking at what this learner wrote in Line 2 and connecting it to line 3, that is : $(x - 4)(x - 1) = (x^2 + 4x - x - 4)$

Although Line 3 is correct according to the solution to this question, what Line 2 implies in Line 3 is not correct. Again, a sign that there could be problems involving simplifying algebraic expressions among some learners. This problem cited from this learner's script could also be the reason why some learners did not simplify their answers (for example, vignettes 3 and 4), but as it is, it was difficult to conclusively suggest that since the questions did not specify that learners should leave their answers in the most simplified form.

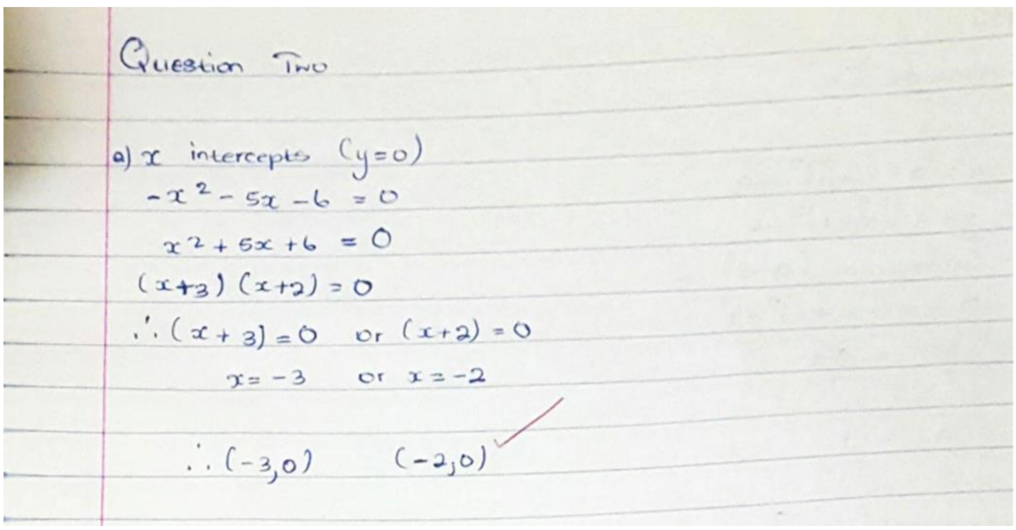
### 4.10.3 Analyzing learner responses for question 2 in the posttest

*For the following quadratic, function:* $f(x) = -x^2 - 5x - 6$

*2.1 Find the x and y - intercepts of the function. (2)*

Most of the learners were able do the calculations involved in answering this question; however, the challenge was that, instead of giving their answers in coordinate form, like in Response 7. Most learners just left their answers as solutions to equations like in Response 8without giving the coordinates of the intercepts.

**Figure 23:Vignette 18 for item 2.1 in the posttest (Question 2)**


Question Two

a) $x$ intercepts $(y=0)$

$-x^2 - 5x - 6 = 0$

$x^2 + 5x + 6 = 0$

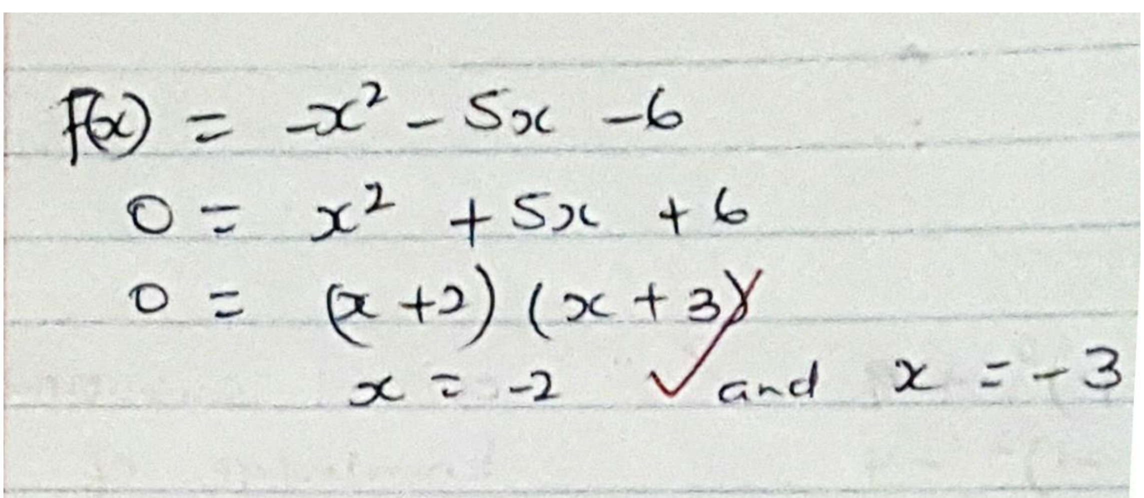
$(x+3)(x+2) = 0$

$\therefore (x+3) = 0$ or $(x+2) = 0$

$x = -3$ or $x = -2$

$\therefore (-3,0)$ $(-2,0)$

**Figure 24:Vignette 19 for item 2.1 in the posttest (Question 2)**


$f(x) = -x^2 - 5x - 6$

$0 = x^2 + 5x + 6$

$0 = (x+2)(x+3)$

$x = -2$ and $x = -3$

*2.2 Find the domain and range of the function. (2)*

This question is no longer a challenge to most learners. Most learners were able to find the domain and the range of the function by making connections with the functions graph they had drawn.

*2.3 Sketch the graphs of the function. (4)*

Using the intercepts, they had calculated, most learners were now able to draw the graph for this function, showing clearly the intercepts on the graph and coming up with the correct shape of the function.However, most learners still have problems answering questions like the one below when they have to refer to their graph:

*2.4 For what values of x is the function increasing? And for what value of x is the function decreasing? (2)*

Below are some of their responses to this question:

**Figure 25: Vignette 20 for item 2.4 in the posttest (Question 2)**

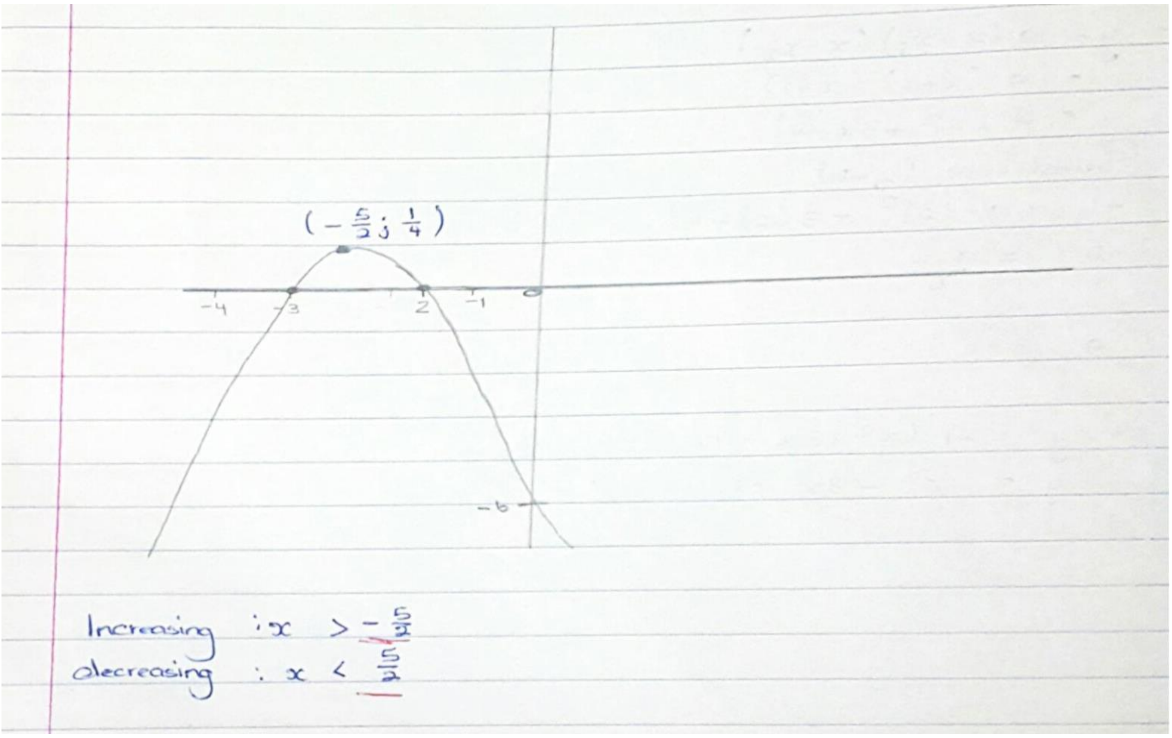
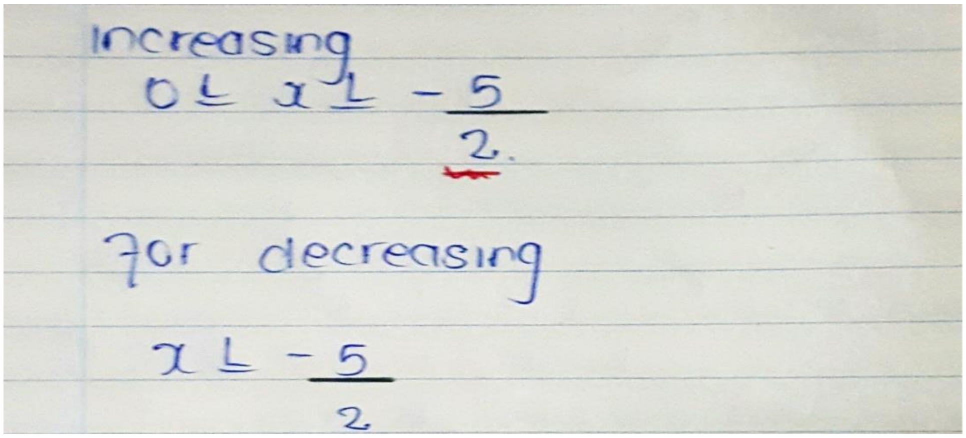


**Figure 26:Vignette 21 for item 2.4 in the posttest (Question 2)**



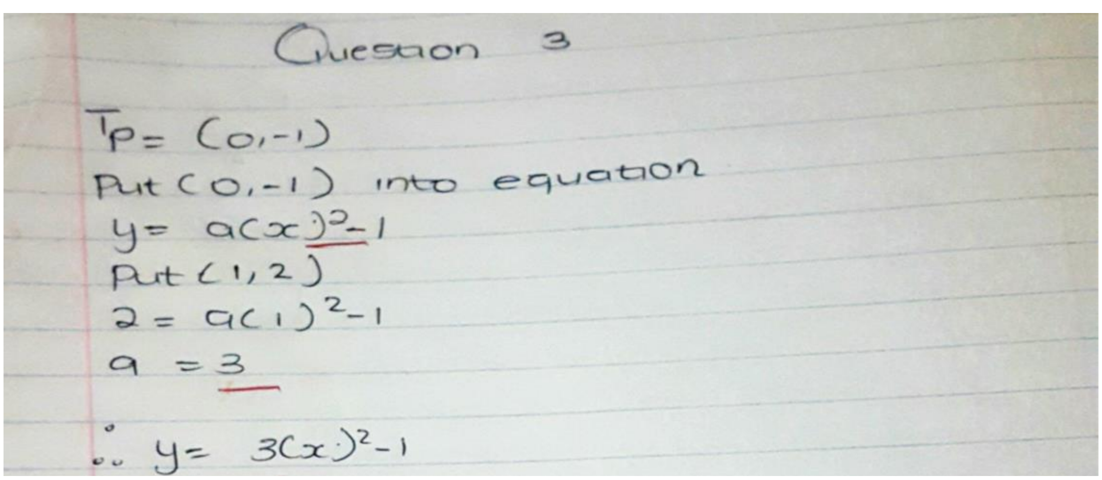

From learners' responses (vignettes 19-21), this concept is still a challenge. Learners may still be having some misconceptions pertaining to this aspect of graphs especially when it is taught in isolation to other aspects. With learners now being able to give the range and domain of the function, it was the researcher's assumption that they would be able to answer this question. However, their solutions show that there are problems.

### 4.10.4 Analyzing learner responses to question 3 in the post test

*Generate the equation of a parabola passing through these given points: A (0; -1); B (1; 2) and C (-2; 5)*

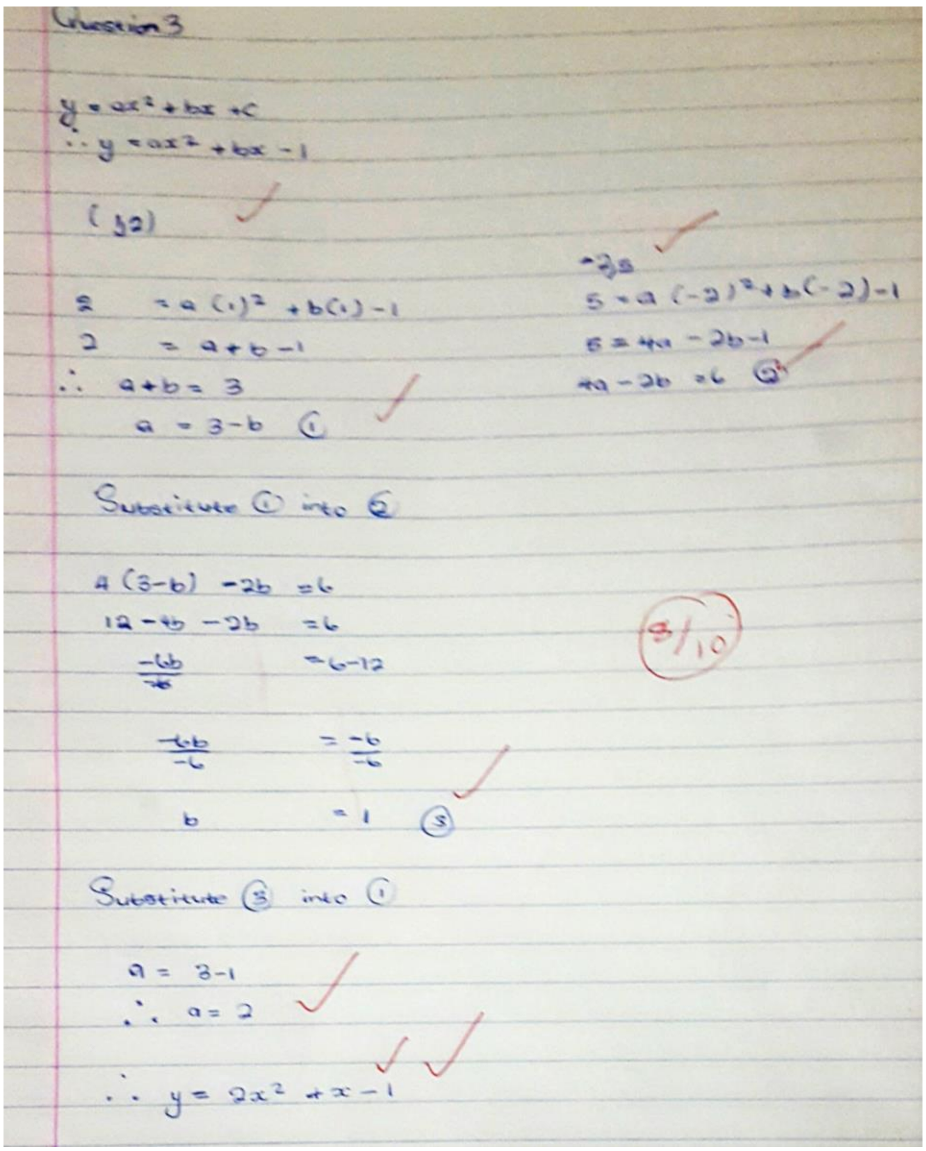
Shown below are examples how learners answered this question

**Figure 27: Vignette 22 for question 3 in the posttest**



Learners (for example vignette 22)had the challenge of coming up with the right equation form to use to substitute the given points. In this situation they were to use $y = ax^2 + bx + c$ and substitute each of the three given points into this equation. This problem proved to be a challenge to a number of learners. Some learners thought they could draw the sketch of the graph using the given points and then use the form: $y = a(x - p) + q$, but they did not realize that it was not possible for them to read the turning point from their sketches where they used and pencil and paper.

**Figure 28: Vignette 23 for question 3 in the posttest**


Question 3

$y = ax^2 + bx + c$

$\therefore y = ax^2 + bx - 1$

$(1;2)$

$2 = a(1)^2 + b(1) - 1$

$2 = a + b - 1$

$\therefore a + b = 3$

$a = 3 - b$ (1)

$-2;5$

$5 = a(-2)^2 + b(-2) - 1$

$6 = 4a - 2b - 1$

$4a - 2b = 6$ (2)

Substitute (1) into (2)

$4(3 - b) - 2b = 6$

$12 - 4b - 2b = 6$

$-6b = -6 - 12$

$\frac{-6b}{-6} = \frac{-6}{-6}$

$b = 1$ (3)

Substitute (3) into (1)

$a = 3 - 1$

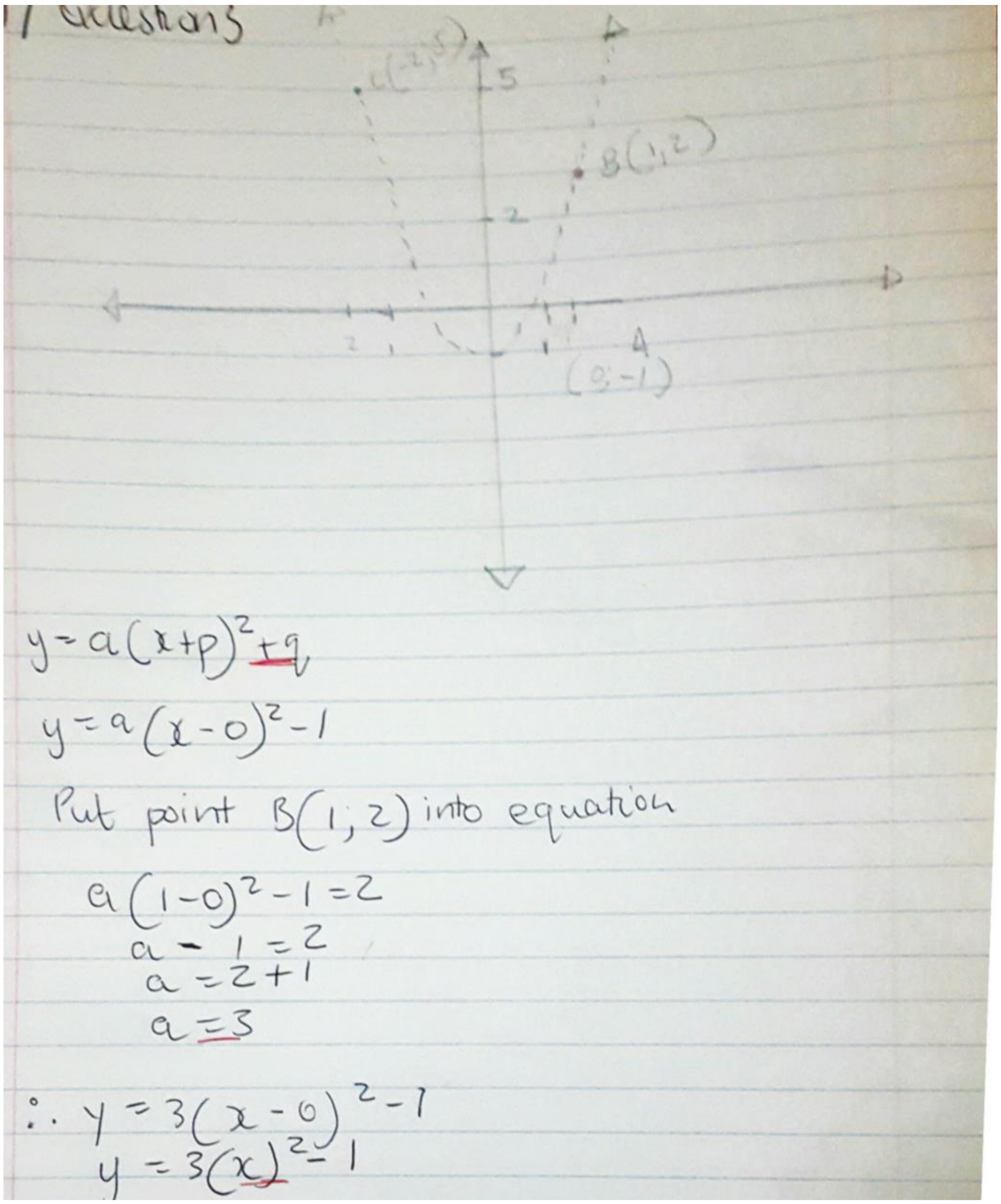
$\therefore a = 2$

$\therefore y = 2x^2 + x - 1$

3/10

However, some learners were able to answer this question correctly, but as is the case with learner with vignette 23, this learner was penalized for not showing how they obtained the value of c from their script, otherwise, her solution to the problem is correct

***Figure 29:Vignette 24 for question 3 in the posttest***




Some learners had the challenge of coming up with the right equation form to use to substitute the given points. In this situation they were to use $y = ax^2 + bx + c$ and substitute each of the three given points into this equation. This problem proved to be common among those learners who were not able to answer this question. These learners thought they could draw the sketch of the graph using the given the three points, but it was not possible for them to do so in this case since when they were writing they only had access to their calculator and then use the form: y = a (x – p) + q, but they did not realize that it was not possible for them to read the turning point from their sketches where they used and pencil and paper.

To further gain insight into the influence of using computers in learning quadratic equations, the researcher selected a few learners from the experimental group to ask them about their experiences using ICT in the learning of functions.

## 4.11 Analyzing learner interview after the lesson intervention using ICT and the posttest

Although the interview was semi structured in nature, it was guided by questions indicated on the post schedule appendix. In analyzing the interview, the analysis will be guided by the themes indicated as questions asked to the learners in the appendix

### 4.11.1 Theme 1

***What was your experience in learning about quadratic functions using computers?***

The following are responses from the learners to the above mentioned question in theme 1:

Learner A: *I think it was more fun to learn from the computer because if I did not understand, I had the opportunity to play the video again until I understand.*

*Learner A: When I was learning from computers, I was concentrating more, because everyone was busy with their computer. It was more interesting because I love computers. And when we were discussing in groups, everyone was contributing because we were all paying attention to what we were seeing. It was like we were watching a movie in class.*

Learner B: *I think with computers, learning quadratic functions was good... we should use computers in mathematics, they make it a little bit easier.*

*Learner C: Me I enjoyed…. It was interesting*

*Learner D:* *I was enjoying*

Learner responses indicate that learners enjoyed using computers. These learners are among the group of people that mostly engages in using computers and the internet at home hence, they embrace easily embrace the use of ICT in the learning of functions in the mathematics classroom. The responses also indicate that using ICT in the learning of quadratic functions can give learners the opportunity to engage with, the nature and properties of quadratic functions and their graphs actively and facilitate learners to develop self-regulation (self- observation, self-evaluation and self-reaction). From these responses, learners also show that they were motivated to learn due to the use of computers as shown by their positive attitude.

### 4.11.2 Theme 2

***What were your experiences in learning about quadratic functions before you made use of computers?***

Below are the responses given by learners to the above question.

*Learner A:* *I did not feel comfortable asking the teacher over and over if I do not understand, but with the computer it is different, you can replay until you understand.*

*Learner B* *Our teacher told us some formulas and showed us some examples. After that she did some few examples from the book on the board and then gave us homework.*

*Learner B:* *If I had problems, I would get help from the teacher, but with computers most of the time when I was comparing my answers with some of the learners, we had similar answers*

The responses by the learners indicate that learners were passive in their learning of mathematics but now with computers, learners were able to explore mathematical ideas through social interaction with other learners as well. These learner responses also indicate that the use of ICT can encourage and motivate learners to learn quadratic functions, produce a positive attitude towards functions and mathematics in general and facilitate student's group work and participation, cooperation and discussion among themselves and between the learners.

#### 4.11.3 Theme 3

***Do you think you had enough time to explore functions using computers?***

These are some of the responses from the interviewed learners:

*Learner C:* *...more time is needed so that we can master Geogebra alone*

*Learner D* *we needed more time so that we can understand more*

The responses from the learners indicate that although computers can be integrated in the learning and teaching of mathematics, they require more time to help learners to master not only how to use computers but the mathematical content as well.

#### 4.11.4 Theme 4

***Did you find Geogebra useful***?

Given below are some of the sampled responses to the above question.

*Learner A:* *Yes. It was easy to draw graphs of many quadratic functions within a short space of time. And it was easier to see the properties of the function from its graph quickly and easily, like the turning point and the intercepts.*

*Learner B:* *Yes… you are able to do many problems within a short space of time, but sometimes you don't understand how the computer has drawn the graph, so the teacher must also explain.*

The responses from these learners acknowledge that computer help the drawing of quadratic graphs and with visualizing the different aspects of the quadratic function, but the suggestion that the teacher must also explain indicates that the computer only may not have helped the learners with conceptual understanding.

#### 4.11.5 Theme 5

***Did the use of computers enhance your understand of functions?***

These are some of the responses to from the learners in an attempt to answer the question above:

*Learner A:* *I was able to do the given exercises faster using the computer and if someone did not understand something it was easier for me to explain because I would have seen it on the computer.*

*Learner D:* *when you punched in an equation like $f(x) = (x-2)(x-3)$ it was easy to see from the computer that 2 and 3 are the y intercepts.*

*Learner C:* *I thought it is increasing when it is more than -5/2 because the graph is above the y axis and then below the y axis the graph is decreasing, that's what I thought but you marked me wrong.*

The response from learner A acknowledges that computers enables one to work faster especially when drawing graphs but when the learner says I can explain because I have seen it, the question one may ask is: Is seeing the same as understand? Learner D was able to see the intercepts as 2 and 3 on the graph, and when he got a similar question it the test to give the intercepts, he gave it as $x = 2$ and $x = 3$. Can we say his solution fully shows us what the intercepts are? Also, the response by learner C is influenced by what he sees, so Is seeing understand? When learner B previously suggested that

…" *sometimes you don't understand how the computer has drawn the graph, so the teacher must also explain*"

Is he also suggesting that the use of ICT alone does not enhance conceptual understand?

### 4.12 Discussion of the data analysis

The theoretical frameworks that guided this research were constructivism, socio cultural learning theory, variation theory and the Technological pedagogic content knowledge (TPACK) frameworks.

The pretest scripts for learners were analyzed and the focus was on the errors and misconceptions displayed by learners as well as the general performance of the learners in the pretest. Constructivism is a view of learning based on the belief that knowledge is not a thing that can be passed on by the teacher in front of the classroom to the learners seated at their desks (Gray, 2007). The errors and misconceptions displayed by learners result from their effort to give meaning to information presented to them in the classroom. These errors and misconceptions can be noted from the answers that learners give verbally or in written activities like tests. A quantitative data analysis was employed to see the general performance of how learners performed in answering each question and in the test as a whole while on the same pretest again a qualitative analysis was also done to establish the nature and the misconceptions that learners displayed for each item in the test. The theory that guided this analysis of establishing the nature and type of errors made by learners was constructivism. Constructivism has a direct bearing when analyzing learner's errors and misconceptions as it enables the researcher to understand how the learner is able to construct his/her knowledge and the challenges faced by the learner to influence the misconceptions that the learner has in committing the errors identified. The constructivist perspective was embraced in this study as it regards making errors as part of the learning process. These errors and misconceptions are seen as the natural effort by learners to construct knowledge in the learning environment and these constructions can be based on correct or incomplete previous knowledge.

Davis (1984) identified two types of errors learners make in mathematics. These are errors common to different learners and those errors that are specific to individual learners. Due to the nature of the investigation that seek to identify the errors and misconceptions that learners have on the topic on quadratic functions with the aim of implementing an intervention to address them, it is the common errors across the different learners that were of interest to the researcher. Errors patterns that are common among different learners suggest that learners possess similar concept images with respect to certain mathematical concepts and procedures and it is these types of errors that were addressed in a classroom situation for the benefit of the majority of the learners in the intervention lessons.

The intervention lessons in this investigation were then guided by the socio cultural learning theory, variation theory and the Technology, Pedagogy and Content Knowledge(TPACK) as now discussed.

In the lesson intervention, the aspect of the socio cultural learning theory that guided learning for the learners is that which considers learning to be a social activity which is very dependable on tools and interaction (Wertsch,1991).The knowledge of using various technologies (you tube videos and Geogebra) to teach , represent and facilitate the creation of the function concept in the classroom is what the TPACK framework advocates(Koehler & Mishra, 2006) .Knowledge about how to use these videos from the internet as a communication tool to enhance collaborative leaning in the mathematics classroom brings together the socio cultural theory and the TPACK into this investigation.  It is with the belief that human understanding depends on tools in use as mediating artifacts and these tools help people develop to higher level of thinking that the researcher brings together these two seemingly different perspectives, that complement one another in shaping the direction for this study. Exposing learners to a graphing software Geogebra and some u tube videos on functions from the internet was meant to help learners construct a better understanding of the functions topic. While learners were using ICT in learning functions and also discussing with other learners they were to reach what Vygotsky termed the zone of proximal development (Vygotsky,1978). This zone, is the area of exploration for which through the use of computers, the learner is now cognitively prepared but still requires social interaction to develop fully (Bruner, 1966). Hence after watching some videos from u tube, learners were then put in groups to discuss the key features of the function concept that they would have learnt and to further explore the concepts of the function concept with the assistance of the more knowledgeable other(other learners), if they still had problems.

The lesson videos that were chosen from the internet were not only guided by the errors identified from the learner's pretest but were also guided by variation theory. The variation theory of learning espouses that an effective way of seeing a phenomenon by learners is that they must experience important variations of that phenomenon (Marton, 1998). In order for learners to develop a deeper understanding of those areas of the function that were identified to be problematic for the learner, they had to experience important variations of the function concept through activities that explained the same concept in different ways and also through some activities that contrasted those function concepts in different ways. The object of learning in the intervention were the features of the quadratic function displayed by its three different equation forms, namely $f(x)= ax^2 +bx+c$; $f(x) = (x-x_1) (x-x_{2)}$ and $f(x) = a(x-p)^2 + p$.

In variation theory the object of learning is the focus on a teaching situation, and this was examined is examined from three different perspectives: the intended, the enacted and the lived object of learning as now outlined in the paragraphs that follow.

To bring out the critical features of the object of learning into the learner's focal awareness, certain videos as outlined in the lesson intervention section were chosen from the internet. Because variation theory states that critical features can only be discerned when learners experience variation in their different dimensions, it was not possible to find videos that incorporated all the four patterns of variation (contrast, generalization, separation and fusion) as different parts into the functions lessons. This according to Olteanue (2007) is convergent variation; where different aspects are directed to the whole of the object of learning and these aspects consist of the object's parts and relationships between them and leads to positive development in student's leaning. Instead, a divergent means of variation was implemented due to the nature of the videos available from the internet. A divergent variation is when the whole of the object of learning is presented without first discerning the parts in question. It was anticipated by the researcher that afterwards when learners were discussing the videos they would then discern of discussing the whole that they would have watched on the videos presented to them.

In selecting the videos used in the lesson intervention as well as to prepare for the lesson intervention using Geogebra software, the TPACK framework offered the researcher with the possibility to connect two complex phenomena: Learner errors and misconceptions with technology. Although it was not possible to find videos from the internet that would directly address the errors and misconceptions that were identified from this study, this framework was thus used beyond treating technology as an add on, but instead used to enhance the connections between technology, content and pedagogy.

While analyzing the learner's errors and misconceptions various coding of errors and misconceptions by different researchers like Radatz (1979), Watson (1980), Luneta and Makonye (2010) helped to understand the errors displayed by the majority of the grade 11 learners in this investigation. Although the codes cited in the literature review of this investigation were used to understand the learner errors shown in the various topic in order to establish the misconceptions. These codes were adapted and where possible modified to suit the topic under study: functions. The main features of this research study is to see the effects of the different interventions on the learner errors and misconceptions, thus the errors identified in this investigation saved as a guide to help understand the misconceptions that learners in this investigation have on functions so that an intervention can be implemented to address them. The errors identified, helped establish the misconceptions to enable the researcher to design an intervention using ICT. Thus the errors for this investigation were not coded, but were identified using codes in literature so that they can serve as a guide for the intervention. To help understand the misconceptions that learners have when they displayed certain errors, interviews after the pretest were also used.

The findings of this research are also informed by the results of the posttest in comparison to the results of the pretest and also in the background of the above mentioned theoretical frameworks. With regard to variation theory, in the findings, the lived object of learning is measured from the outcome of the posttest in. The students' initial level of capability to the appropriate object of learning (before the intervention) as well as the way in which the learners understand the object of learning (after the intervention) is the lived object of learning. This, together with the aims of the study will help answer the research questions as now outlined in the chapter 5.

### 4.13 Conclusion

This chapter began with the quantitative analysis of the pretest script for learners in this investigation. This was then followed by a question by question analysis of the functions items in the pretest to identify the errors that learners displayed in the topic. Vignettes were used to illustrate the different type of errors learners were having in the pretest. To help understand the misconceptions behind the errors displayed in the pretest, the different coding by various researchers cited in this chapter were used, together with the learner interviews. The lesson intervention that was implemented and the videos that were used are then outlined. This is then followed by an analysis of how learners performed in the posttest. A comparison of the results of the pretest and posttest for the two groups in the study is outlined, but furthermore, vignettes where the intervention used computers are shown to try to establish the influence of using ICT in the intervention. To gain insight into the influence of using computers, learners whose intervention used computers were interviewed and the interview data is also presented with their posttest data.

Discussions on how data was analyzed with respect to the theoretical framework guiding this study is then outlined to give the reader the background of where and how the findings of this research are drawn from in order to answer the research questions.

# CHAPTER 5: CONCLUSION, FINDINGS AND RECOMMENDATIONS

## 5.1 Introduction

This study was investigating the influence of using computers to remedy learner errors and misconceptions in functions at grade 11. This research aimed to find the errors and misconceptions learners in grade 11 have on functions with the purpose of investigating the influence of using computers in redressing these errors. In so doing, the research further aimed to explore teacher knowledge for using computers in teaching functions as a way of encouraging teachers to engage their learners and their learners' way of reasoning in addressing the difficulties learners have in the topic. The research questions were proposed in line with the aim of the research, and thus three research questions emerged: 1) What errors and misconceptions are elicited by grade 11 learners when they engage the topic of functions?2) What is the influence of using computers to address learner errors and misconceptions in functions at grade 11?3) What teacher knowledge may be drawn from the use of computer technologies in mediating the learning of functions, through learners' errors? The theoretical lenses used for this study were: constructivism, socio-cultural learning theory, variation theory and the Technological Pedagogical Content Knowledge (TPACK) framework. The research methodology in this study used both qualitative and quantitative research methods. With respect to the research questions for this study, I now present the findings of the study.

## 5.2 Research findings

I now present the research findings for this study. To answer the research questions, the research questions for this study will be used as themes under which the findings of the study will be outlined.

### 5.2.1 Research Question 1: What errors and misconceptions are elicited by grade 11 learners when they engage the topic of functions?

To answer this research question, questions in the pretest for this research were meant to expose student's reasoning in different areas related to the features of the quadratic function. I will elaborate those errors under each conceptual area by relating it to various existing literature and where possible, I will explain how the learner's errors were used to determine their misconceptions as now outlined below, but first I outline the feature of the quadratic equation that was being tested.

*i) Given a parabola, what errors did learners show to interpret and get an appropriate equation?*

One of the errors identified that resulted in learners failing to interpret given graphs to get an appropriate equation was that learners failed to use the appropriate form of the quadratic equation. Most of the learners have no idea that the features given in the graph determine the form of the quadratic equation to be used. In situations where learners were able to choose the appropriate quadratic form required in obtaining the equation, learners had no idea as to which coordinates should be substituted where, hence they made incorrect substitution of values into the quadratic form. Even when learners substituted into the equation, they failed to correctly simplify expressions when they were removing brackets. The reason could be cited from Radatz (1979) who cites that this could be as a result of a deficiency in the mastery of the prerequisite facts and concepts on the function concept by learners. In other words, learners have insufficient conceptual knowledge related to the function concept.

Some of the learners in this study were wrongly linking previous knowledge (of finding the gradient) to the new knowledge (finding the equation of the parabola). For this reason, they demonstrated that they had no idea what the variables in the quadratic form represents, again evidence that learners lack sufficient conceptual knowledge. Graham and Thomas (2000) also suggest that students often misunderstood the variable concept hence; this confusion resulted in the wrong identification of a coordinate from the graph as was observed from some of the learner's scripts in this investigation. Learners in this study failed to correctly identify the point in the given graph, for example, some would substitute (3; 0) or (0; 3) instead of (-3; 0) into their correct quadratic form. Several studies have shown that students as well as teachers have problems understanding numbers or discriminating numbers leading to misconceptions about numbers (Khrishnasamy & Abdulla, 2015).

Some learners' errors resulted from incorrect memorization of the quadratic form to be used. Those that were able to substitute the correct values into the correct quadratic form, failed to correctly simplify the expression to get the required equation. For example: $-6 = a\ (2+4)\ (2-1)$ was simplified as $-6 = 8a$ and $a(x+1)\ (x-0)$ was simplified as $ax^2-x-0$; evidence that learners also had problems with carrying out mathematical procedures accurately. The errors noted from this study were that learners failed to simplify algebraic expressions or they failed to simplify integers.

Both the number concept and the algebraic concepts are linked to functions. Evident from this research is that if learners have problems simplifying integers or/ algebraic expressions, this could also lead to learners having problems with functions as they failed to get the required equation. Some learners got the appropriate equation, but in analyzing their solutions, they had obtained a correct solution by incorrect mathematical processes. This again showed a deficiency in the mastery of prerequisite fact and concepts related to the function concept as observed from some of the learners' scripts who first calculated the gradient and then substituted the value for the gradient into the quadratic form. For Luneta and Makonye (2010) it is possible that learners have false concepts hypothesized to form new concepts and this, like the example cited above about the gradient could also lead to learners having problems with functions. This aspect of applying rules where they do not apply clearly shows that the learner has misconceptions pertaining to the function concept as shown by their ignorance of the restrictions that apply to how one should move between one different functional representations to the other (Luneta & Makonye, 2010). This also could have resulted from overgeneralization of previous learnt mathematics context into a new context in which the rule does not apply.

ii) *Given equations what errors did learner have in drawing the graphs of the given equations?*

Although most of the learners were able to see from the nature of the equation that the graph is that of a parabola, most of the learners were not able to show the correct turning point and the correct intercepts on their graph. The possible reasons for the errors made by these learners is that learners have incomplete knowledge on what key features affect the plotting of a graph, these are the turning point, the intercepts as well as the shape. From the given equation, they failed to use their knowledge of getting the turning points as well as the intercepts to plot the graph of f(x). This according to Watson (1980) is because most learners failed to select mathematical processes that would enable them to get the turning point and the intercept of the graph; hence this resulted in them not showing these important features on their graphs. From the errors identified in this investigation, the errors could have resulted from insufficient conceptual knowledge, as well as the inability to formulate strategies that would enable them to draw the graphs (Kilpatrick, 2001).Failure to recognize that given an algebraic equation of a function, one should can carry out mathematical processes that enables one to get the features of the parabola may hinder learners to move from one different representation to the other. This proved to be a challenge for most learners; hence learners were not able to move from the general given algebraic form to the graphical representation of the function due to incomplete applications of rules as sighted by Luneta and Makonye (2010).

There is evidence from the learners' scripts that although some learners know what the range and the domain are, but due to their rigidity in thinking they were not able to explain these with respect to the graphs that they had drawn. Isolating this knowledge from their graphs, lead to learners not being able to give the range and the domain of the required functions, again, showing some inadequate conceptual knowledge. Other learners although they were able to draw the "mother graph", f(x), they failed to draw the transformed graph, g(x), as learners failed to see the connection between the two given functions. This could be due to learners lacking the strategies needed to draw the graphs so that they could establish the connection. Although some learners were able to see that there was some transformation from the first graph, f(x) to the second graph g(x), their solutions show that learners were not clear as to the type of transformation that had occurred. Instead of translating the graph, some learners were rotating the graph of f(x) to get the graph of g(x), but still, learners could not explain the transformation that had occurred- learners were not able to reflect and explain the transformation that had occurred, hence this inability to reflect also lead to wrong explanations.

*iii)Given the general quadratic equation, what errors did learners have in using their algebraic knowledge to answer questions?*

Learners lack the knowledge required to carry out mathematical procedures that would allow them to flexibly move between different functional relationships. The reason why learners were not able to come up with correct equations given the graph were that the learners failed to identify the features that were given in the graph and to link these features to an appropriate formula. Sfard (1991) observed that, the understanding of a concept should precede the structural conception. In other words, for learners to understand the function concept, learners have to understand the function as a graph, but also the operational stages of the concept of a function and the processes involved in the formation of the function.

Some questions in the pretest learners required a well-defined method to identify the features represented in the structure (the graphs provided), learners also had to choose the appropriate form of the quadratic equation into which these critical features (turning points, y intercept or the x intercept values) where to be substituted. After substituting, learners were to perform algorithms that required them to have knowledge of manipulating algebraic expressions (adding, subtracting and multiplying to simplify algebraic expressions) as well as manipulating (adding, subtracting and multiplying to simplify) integers to end up with the equation of the parabola. The poor performance by learners in answering these questions concurs with what Sfard (1991) observed, learners had problems with the dual nature of functions and thus, this aspect was identified as the object of learning into the lesson interventions.

Other questions (Like question 2) required learners to represent the given algebraic form of the equation into its graphical representation. Again most learners had problems regarding the point to point attention alluded to by Zaslavsky & Stein (1990) when plotting the required graphs. When learners fail to move from the algebraic form to the graphical form their lack of understanding also affects their judgment when they fail to see the translations and the trends of what happens in their graphs. This can lead to learners who are not capable of access knowledge in functions or to transfer that knowledge on functions to other contextual situations in mathematics as observed in the learner performance in Question 3.

The analysis of the pretest gives an answer to the first research question. In summary, learners in this study lacked the strands identified by Kilpatrick, Swafford and Findel (2001) for mathematical proficiency. As observed from this study, where learners were able to use say, the correct quadratic form, but failed to carry out the algorithms correctly in simplifying expressions, they had challenges in getting the correct solution. In other questions like question 3, learners lacked the confidence to attempt this question; hence this resulted in them not being able to answer the question correctly. These challenges identified from the pretest gave rise to the object of learning in the intervention, where the use of computers tried to help develop all the problems cited in the pretest in the learning of functions to eradicate the errors.

### 5.2.2 Research Question 2: What is the influence of using computers to address learner errors and misconceptions in functions at grade 11?

From the comparison of the analysis of the pretest results and the post test results for both the control group and the experimental group, although both groups showed an improvement after the intervention, a better improvement was observed from the experimental group, the group that used ICT in its teaching intervention. The results show that the group that used ICT in its intervention had a better improvement that the group that used traditional methods in learning functions. From the findings of this research investigation, it may be concluded that the use of computers especially Geogebra software, helped students reach a higher level of conceptual knowledge in functions that those students that did not use Geogebra. As already observed from other research studies, this finding of noting an improvement when learner errors and misconceptions are used to support learning concurs with other findings from mathematics research. But to get a picture as to the extent of using ICT in addressing learner errors and misconceptions, learner scripts from the experimental groups were analyzed qualitatively and some few learners where also interviewed to gather further insight as to the extent that computer technologies have in addressing learner errors and misconceptions as now outlined in the paragraphs below.

The object of learning identified the same concepts tested as those tested in the pretest as being problematic to learners. To gain insight into how effective the use of the intervention was, discussions of the posttest findings together with the learner interviews after the posttest were incorporated to find out if there were any changes in learner performance in the posttest as now outlined.

Most of the learners were now able to recognize the critical points shown on the quadratic graph and hence were able to choose an appropriate form of quadratic formula form to use to obtain the required equations. The reasons cited from the learner interviews were that these learners are among the generation that makes use of technology (for example cellphones) in their everyday life almost every day. Bringing the computer into the mathematics classroom made it more fun for them as they were actively involved in the learning and teaching. In other words, using computers, motivated learners to want to learn mathematics. This shows that if students have a positive attitude towards the use of technology this in turn will make them have a positive attitude towards mathematics when using computers. Thus this positive attitude in technology leads to positive attitude in mathematics if mathematics is incorporated with technology.

The learners interviewed in this study also felt that the use of technology made mathematics easier and this could have imparted in students being able to relate to graphs to their equations. From this study, learners who used computers were now able to identify the key features required for them to choose the form of the quadratic equation to use, not only were they able to identify, but they were also able to substitute the correct values and simplify the expression correctly to get the equation. These learners showed a better improvement in their ability to relate the given graphs to the equations they obtained by making connections between the graph, the key features and the equation.

Responses from learners in this study also reveal that computers enabled learners to draw graphs easily. However, it is important to note that during the posttest learners were not using geogebra software to draw the graph or to obtain the equation of the given graph. So what one might ask is how then were they able to draw these graphs accurately using pencil and paper in the classroom during the posttest? Technology allows learners to graph functions more easily as it offers them the opportunity to explore the concepts in multi-representational modes – symbolic, visual and graphic), (Confrey, 1995; Heid, 1998; Fey, 1989). The use of computers in the mathematics classroom allows learners to connect their everyday world (using computers) to their mathematical world, and thus can make mathematics to be fun (Heid, 1998). As the learners were playing with computers doing mathematics, they were learning more about mathematics. If the learners saw how easy it was to draw the graph using the computer, then in their mind, they also believed that drawing graphs was easy and hence could have concentrated more on trying to understand how the computer wad doing it. In that way, they were exploring through "playing" how a quadratic graph is drawn from its equation.

From the interviews conducted, learners endorsed that using computer technologies allowed them to learn independently and endorsed their cognitive development. Watching the videos allowed learners to self-regulate as they interacted with mathematical knowledge and this affects their attitude positively. Using computers can shift the role of learners from being passive recipients of information to becoming more involved in investigating, consulting with technology as well as becoming more involved in group discussions as was observed in this investigation.

Although the learners interviewed in this study felt that the use of technology made mathematics to be easier (Functions in this instance), when analyzing the learners' scripts from the posttest, some of the errorslike in simplifying algebraic expressions were evident in those learners who tried to simplify their answers by removing brackets. From the learner interviews for the study, some of the learners interviewed highlighted that although the computer helped them with visualizing the different aspects of the quadratic function, they pointed out that they did not understand how the computer calculated the intercepts, hence they wished that the teacher would elaborate. From this statement, there is reason to believe that the computer on its own important as it is in the mathematics classroom may not fully help learners with conceptual understanding. The computer, for example the computer software used to draw graphs in mathematics, may provide the learner with correct and complete information, how this information is interpreted by the observer draws a thin line between a wrong answer and a correct answer as explained in the paragraph below.

In this study, during an interview, one of the learners said that the x intercept for the given function: $f(x) = (x + 2)(x + 3)$ was at -2 and at -3, because he could see this on the computer. This observation by the learner was correct, but if left like that it would lead to some errors when learners are asked to give the x intercept in some mathematics test or activity and they give their answers as -2 and -3. It then becomes the teacher's responsibility to give to the learner the acceptable mathematical way of writing the intercept as (-2; 0) and (-3; 0). Learners need the mathematical language and knowledge of being able to interpret the intercepts they see on the computer in a mathematical way acceptable in the community of practice for mathematicians. Hence, as the more knowledgeable other, it is the duty of the teacher to give the learners the vocabulary and the way of "seeing and interpreting" when they analyze graphs and their functions on the computer to help minimize errors. Some students were able to use their mathematical knowledge to interpret the graph on the screen, but some students were not.

As in this study, when some students were asked on the usefulness of using computers they also acknowledged that they did not understand how the computer calculated some values, again showing that the teacher should play a mediating role even where some computer technologies are used in the effective learning and teaching of mathematics. Although computers are powerful tools to use in the class, the role of the teacher as a mediator should not be underestimated.

### 5.2.3 Research Question 3: What teacher knowledge may be drawn from the use of computer technologies in mediating the learning of functions through learners' errors?

Students in this study revealed that they enjoyed using computers, so it is our duty as educators to help bring those computers into our mathematics classroom for the benefit of our learners. Using computers in the investigation created a conducive environment as there were no disturbances during the lessons. All the students in this study were totally and fully engaged when they were watching u tube videos on their laptops or working with Geogebra. There was no time wasted, no noise as learners were attentively engaged in working with computers. In most South African township schools where we have large numbers of learners, using technology into the classroom can help maintain order and discipline and thus facilitate syllabus completion. Without technology, some of the time in the classroom is lost when the teacher tries to instill order and discipline among learners who will be making noise and this can impart negatively on the performance of learners in examinations especially when the syllabus is not completed on time.

The results of the study also show that if used in the mathematics classroom, computers can help improve mathematics results. With the curriculum in South Africa being result oriented, incorporating technology into this study yielded better results, this shows that if ICT is to be in cooperated into the learning and teaching of mathematics, we might be able to bring those much needed good results in mathematics. Learners in this study were excited to learn using computers, they showed enthusiasm, excited to work with computers and this in turn impacted positively into their performance. Using computers can boost the morale of the learners and this in turn can boost their confidence. When learners are excited about learning mathematics, results show indeed that they do learn, hence a remarkable improvement was achieved by learners who used computers.

Using ICT in this investigation enabled the researcher to carter for all the needs of the learners at once. In heterogeneous classrooms, like the ones dominating South African township schools, learners are not streamed according to their capabilities, hence in one class, a teacher might have those learners who grasp concepts faster, those who are average, and those who need more time to grasp concepts. Using computer allows each learner to work at their own pace; the fast learners can move on to more problem solving activities, while those who grasp concepts slow can also take their time until they understand the concept. The end product was pleasing as in the end when learners understood functions, it will not matter who was fast in grasping concepts and who was slow.

Using the Geogebra software in this study enabled learners to explore many function graphs at one. The software enabled them to draw many graphs with different colors in one Cartesian plane, and thus, learners were able to learn and make connections about the functions they had drawn and to be able to explain the effects of the transformed graphs to their equations in relation to each other.

Students from this study also revealed that they felt that using computers made mathematics easier; this could have been due to their increased motivation when they were using technology tools in the learning of mathematics. However, as educators we need to be aware of the different needs and the different methods that students conceptualize different mathematics concepts so that we can address these ways through different teaching techniques. Using ICT in the mathematics classroom brings in that variety, and as such we are able to maintain the enthusiasm that is required in our mathematics classroom so that we have learners who are ready to learn for learning to take place.

Learners in this study enjoyed lessons because they experienced variation in the way lessons were conducted. The better results for the learners who used ICT show that, the conditions for learning were made available for learners by the use of computers. Learners were able to see different graphs and this resulted in a deeper way of perceiving functions. Seeing is more powerful than hearing.

### 5.3 Limitations to this study

This study used a quasi-experimental design, where at the beginning of the study, 43 learners from the experimental group and 41 learners from the control group had indicated willingness to participate in the study. The sample size was reduced to 25 in both groups as the other learners felt that they were not ready for the pretest and they felt that they wanted to use their study time to prepare for other formal tests during the course of the week. A larger sample size could have enabled the researcher to statistically analyze if indeed the use of computers had an acceptable better effect of learner performance after the intervention. Also, during the course of the study, at the school where the investigation was to be conducted, there was a break in into the computer lab, and 40 computers were stolen. For this reason, the researcher and the participants were relying on borrowed tablets (from the grade 12). Having to rely on borrowed tablets meant that the access time was reduced, hence instead of doing the intervention for two weeks, the intervention was only done with computers in one week. Some learners in the study completely did not attempt to answer questions that they were not comfortable with, for example Question 3, in the pretest. For this reason, it was not easy for the researcher to find the errors made so as to establish the misconceptions that learners had in answering that question. Hence, only a few learner's scripts were used and these errors might not have been representative of what the majority of the learners in the group had to establish the misconception. The intervention using traditional teaching methods by the other teacher was not observed to establish how the intervention was done. For this reason, the improvement in the control group could have been due to many other unforeseen factors, maybe, the teacher was teaching concepts in the post test as this was made available to him on his request, although emphasis was made to him not to teach according to the test but according to the errors identified in the pretest.

## 5.4Recommendations

From this study, recommendations are suggested with respect to practice, theory, policymakers as well as recommendations with respect to further study as now outlined in the paragraphs below.

### 5.4.1 Recommendations for Practice

In this day and era where most of our students have access to the cellphone and even computers at home, as teachers, we should not distinguish students' home environment to their classroom environment. Students in this investigation enjoyed working with computers and this in turn imparted positively towards their performance. Using technology in our classrooms can help us with the challenge we face where most of our students do not like mathematics, and this is reflected to their poor performance in the subject. As educators, if we incorporate technology into our teaching, we can integrate technology with the curriculum and this can have a positive outcome on the results in mathematics

When learners use Geogebra software to practice mathematics or any other tool, students have to build uses and representations linking the tool and mathematics. As observed from this investigation, sometimes the students make no difference between the technology truths they see on the computer and the mathematics. Geogebra should be used after basic pen and paper techniques of drawing graphs have been done, as an intervention. If used as a teaching mechanism, then the teacher should always intervene, mediating between the computer and the learners. With technology use, I do not think that pen and paper techniques should be completely abandoned as these also serve as objects that enhance both procedural and conceptual understanding. However, I recommend that the 21$^{st}$ century classroom should be more technologically inclined to motivate our learners into learning mathematics successfully.

### 5.4.2 Recommendations for theory

The functions topic is linked to other mathematical topics, like algebra, integers and the variable concept. When teaching the topic on functions, integration with these topics should be done so that the errors and misconceptions that learners have in those topics can also be identified and addressed. The understanding of the functions topic in mathematics is also influenced by the learner's understanding of directed numbers, algebra as well as the variable concept, just like the understanding of functions in mathematics will also affect learner understanding in topics like calculus. Further study into understanding learner errors and misconceptions, should try to link all those topics that affect each other, and use the errors established as a resource to redress the misconceptions that learners could have in various topics.

### 5.4.3 Recommendations for policy makers

The extent to which the use of ICT can help learning is also influenced by the teacher's beliefs and representations of mathematics and ICT. The teacher is the main "actor" in the integration of technology in his/her classroom. Thus the belief that there is enough information about technology integration in the learning and teaching by researchers for teachers to implement technology may not be feasible. Research has to study the teacher in light of his/her beliefs as well as the challenges he/she has in implementing technology in their classroom and the policymakers in turn should work with the teachers in order to come up with better ways of implementing technology in the classroom.

### 5.4.4 Recommendations for further research

Although there are many videos on the internet that help address different topics in mathematics, most of the videos are not informed by any theoretical framework. Research in integrating ICT should try to upload videos informed by various theoretical frameworks, for example, constructivism, and socio cultural or situated learning theories, to accommodate different beliefs held by teachers.

In this investigation, findings reveal that when both the traditional method of teaching and the use of computers were used to address learner errors and misconceptions, both groups of learners improved. With the learners having used technology showing a better improvement that the group whose teacher used traditional methods. However, the findings reveal many advantages associated with computer use, but the computer being a tool, it has its possibilities and constrains. Further research should document the possibilities of using computers in the mathematics classroom as well as the constraints so that teacher can intelligently incorporate computer technology, where possible, for effective learning and teaching

## 5.5Conclusion

This chapter concluded this investigation by answering the research questions in the study with respect to the data that was collected in Chapter 4 by linking it to the theoretical frameworks that informed this study. The findings revealed that there are more benefits for both the teacher and the learner in using ICT in addressing the errors that learners have in learning functions.

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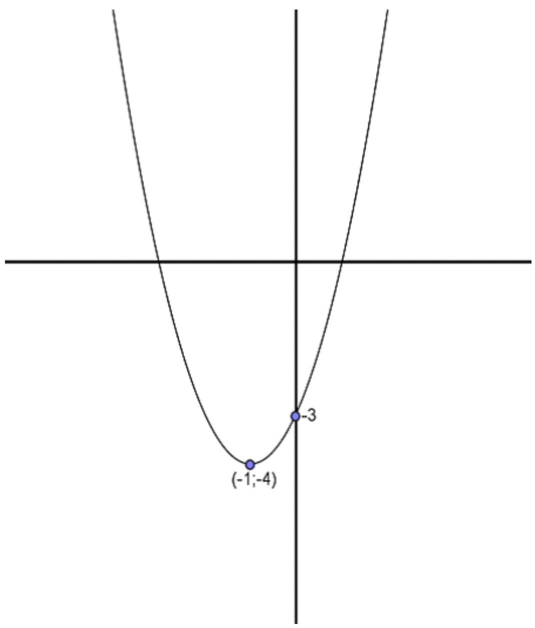
**PRETEST ON FUNCTIONS** **DATE: June 2016**

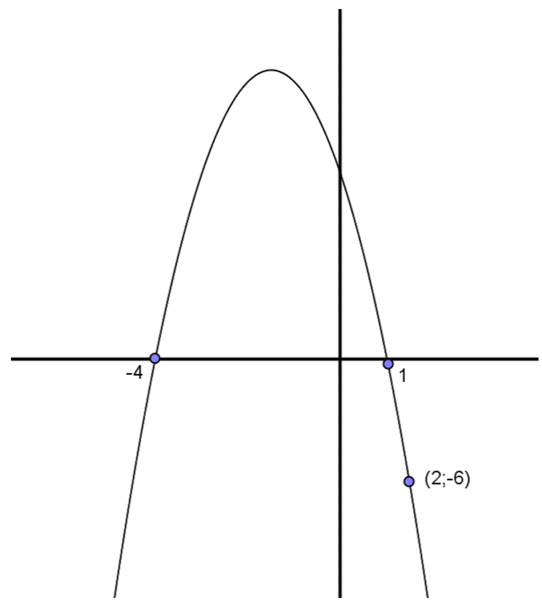
**TOTAL: 40** **TIME: 1H30**

# APPENDIX 1

## QUESTION ONE

Find the equations of the given curves




(5) (5)

## QUESTION TWO

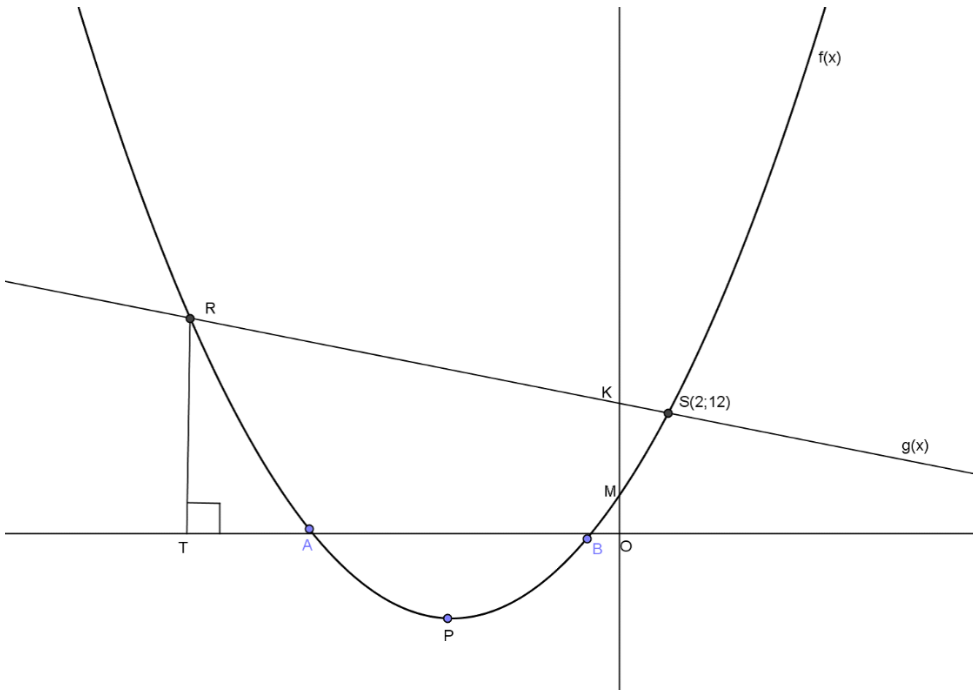
2.1 On the **DIAGRAM SHEET,** on the same system of axes, draw the graphs of $f(x) = (x+3)^2$ and $g(x) =(x+3) - 4$.Clearly show the coordinates of the intercepts (y intercept and the x intercepts)

2.2 state the domain of *f* and the range of *g* (1)

2.3 Explain what has happened to the graph of *f(x)* to obtain the graph $g(x) = (x+3)^2 - 4$ (2)

**QUESTION THREE**

A parabola $f(x) = ax^2 + bx + c$ with turning point $(-\frac{3}{2}; -\frac{1}{4})$ and a straight-line g(x) = -x +1 4 intersect at the point S (2; 12). The two graphs not drawn to scale are drawn below. A and B are the x intercepts of the parabola. K is the y intercept of *f*. RT is a straight line parallel to the y-axis.




3.1 Show that a = 1, b = 3 and c = 2. (5)

3.2 Calculate the distance between A and B. (4)

3.3 Determine the length of KM, the distance between the two y intercepts. (3)

3.4 For which values of x is f a decreasing function (2)

3.5 Determine for which values of x is $f(x) \geq g(x)$, where $x \geq 0$ (2)

3.6 Determine the length of RT (3)

3.7 Determine the value of $g(x^2) + g\left(\frac{1}{x}\right) - 28$ (3)

**APPENDIX 2**

**POST TEST ON FUNCTIONS** **DATE:**

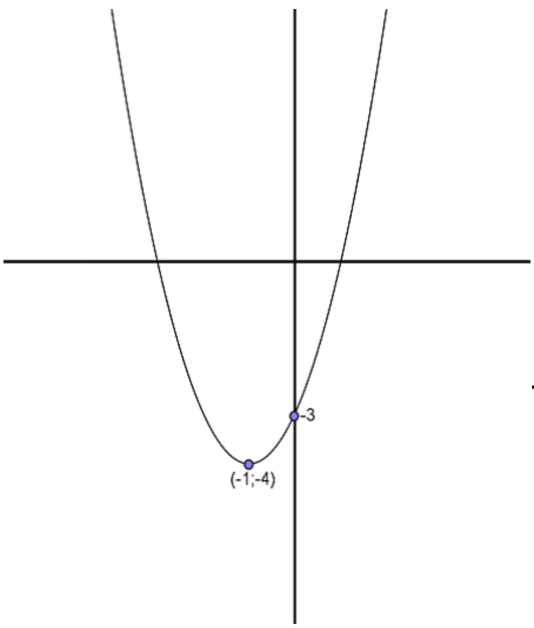
**TOTAL: 30** **TIME: 1 HOUR**

Write yours answers on the answer sheet provided

**QUESTION ONE**

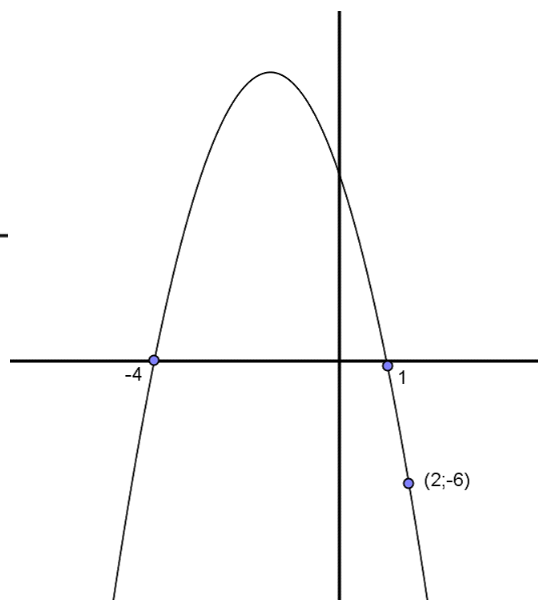
Find the equations of the following curves

1.1




(5)

1.2




(5)

**QUESTION TWO**

For the following quadratic, function: $f(x) = -x^2 - 5x - 6$

a) Find the x and y - intercepts of the function. (2)

b) Find the domain and range of the function. (2)

c) Sketch the graphs of the function. (4)

d) For what values of x are the functions increasing? And for what value of x are the functions decreasing? (2)

**QUESTION THREE**

Generate the equation of a parabola passing through these given points:

A (0;-1); B (1; 2) and C (-2; 5) (10)

**APPENDIX 3**

**INTERVENTION LESSON OBSERVATION GUIDE PER LESSON OF 60 MINUTES**

**TOPIC……………………………………………………………………………………………**

**DATE ……………………………………………………………………………………………**

| | **ACTIVITIES** | **DURATION IN MINUTES** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | **5** | | **10** | | **15** | | **20** | |
| **LEARNER ACTIVITIES** | Watching a U tube video on Functions | | | | | | | | | |
| | Working with Computers | Individually | | | | | | | | |
| | | Pairs | | | | | | | | |
| | | Groups | | | | | | | | |
| | Listening? | | | | | | | | | |
| | Copying from the board/ Screen | | | | | | | | | |
| | Asking Questions? | | | | | | | | | |
| | Solving a Problem/ Exploring Function Problems | Individually | | | | | | | | |
| | | Pairs | | | | | | | | |
| | | Groups | | | | | | | | |
| | Writing their solutions | Individually | | | | | | | | |
| | | Pairs | | | | | | | | |
| | | Groups | | | | | | | | |
| | Discussing their thinking | Pairs | | | | | | | | |
| | | Groups | | | | | | | | |
| | Explaining their thinking | Individually | | | | | | | | |
| | | Pairs | | | | | | | | |
| | | Groups | | | | | | | | |

## APPENDIX 4

## (PRETEST)INTERVIEW GUIDE

After the pretest is marked, learners who displayed errors will be selected to elaborate give reasons why they made those errors.

Questions like these will be asked…

Teacher: I need your help to find out more about … and what you are going to tell me will help. Looking at your solution to this question on the pre-test, what are your reasons for…?

Learner…

Teacher: That is interesting, tell me more …

Learner: …

Teacher: Ok, now looking at a similar problem would you still apply the same strategies/ Tactics/principles?

Learner: …

Teacher: Why?

Learner: ….

Teacher: Go on, I am listening…

## APPENDIX 5

## PRETEST INTERVIEW 1

Researcher: Good Afternoon B21

B21: Good afternoon ma'am

Researcher: Okay … I have some questions for you with reference to your pre-test. Please feel free to answer the questions I have for you as honestly as possible.

B21: Okay ma'am.

Researcher: Good, please have a look at your script. I am going to start by asking questions pertaining to question 1. Please have a look at the question paper as well. I will start by asking about 1.1.

B21: Yes.

Researcher: Please clarify for me. Why did you first decide to find m?

B21: Eeeh…there I needed to find the gradient so that I can substitute it into this equation (pointing to the equation $y = a(x + p)^2 + q$ }

Researcher: Yes, please go on.

B21: I first calculated the gradient and I used this equation …

Researcher: Okay, so the value of m represents the gradient?

B21: Yes.

Researcher: So where exactly into the equation did you substitute the gradient?

B21: Here at p and there at y I put -4 and at x I put -1 which is this point on the graph.

Researcher: Okay, I see. What about the value of a because I can see it in your equation? Why did you not obtain it?

B21: Which a? Oh … ehh, this one? I think I forgot but you marked me right.

Researcher: Okay. I can see that you were able to answer 1.2 very well without any challenges. Please have a look at question 2. Let us start with question 2.1.

B21 Okay ma'am.

Researcher: You were able to draw the graph of f(x) but you failed to draw the graph of g(x) what were the challenges?

B21: Eish… I knew that the graph of g(x) is the graph of f(x) shifted. But I was confused whether to shift the graph up, down, on the left or on the right.

Researcher: Okay. But are you aware how many units the graph was to shift.

B21: Yes. 4 units.

Researcher: Okay. Do you know what is meant by the domain and range of a function?

B21: Yes. The domain are the x values and the range are the y values.

Researcher: Good but then how come you were not able to give the domain of f and the range of g in relation to the graphs that you had drawn?

B21: Heish… I was confused on which values to take.

Researcher: Alright. And then question 3? Why did you not answer it?

B21: Aah...ma'am. For me it was difficult.

Researcher: Thank you very much for your responses and your time in answering the regarding your pretest. You have helped me understand why you were answering the way you did.

B21: It's a pleasure.

## APPENDIX 6

## PRETEST INTERVIEW 2

Researcher: Good Afternoon B24

B24: Good afternoon ma'am

Researcher: Okay … I have some questions for you with reference to your pre-test. Please feel free to answer the questions I have for you as honestly as possible.

B24: Okay ma'am.

Researcher: Good, please have a look at your script. I am going to start by asking questions pertaining to question 1. Please have a look at the question paper as well. I will start by asking about 1.1.

B24: Okay.

Researcher: With reference to question 1.1, I can see that you were able to use the correct form of the quadratic equation, but please look carefully at the point that you substituted (0; 3) … Why do you say that point is (0;3)?

B24: I took it from the graph. Yes, it is (0; 3) … no! It is (3; 0). Oh! I made a mistake.

Researcher: Alright. So why do you think you were not able to get the correct equation for this graph in question 1.1? What were your challenges?

B24: I used the correct quadratic form but then I substituted (0; 3) into the equation instead of (0; -3)

Researcher: I can see that in question 1.2 you tried again to use the equation $y = a(x + p)^2 + q$

B24: Yes

Researcher: so what were the challenges then?

B24: I did not see the y intercept and the turning point from the graph so that I can be able to substitute it into the equation.

Researcher: Okay. Is there no other quadratic form that you could have used so that you could be able to substitute the points that you were seeing on the graph?

B24: I think there is, but I had forgotten it.

Researcher: Okay. In question 2. You were able to draw the correct shape of the graph of f(x) but you did not show the intercepts on your solution. Why?

| | |
|---|---|
| B24: | I did not see that the question wanted me to show them I thought it was okay just to draw the graph. |
| Researcher: | And the graph of g(x). Why is it facing down? |
| B24: | It is the same as the graph of f(x) but has shifted down. That is what I thought but you marked it wrong. |
| Researcher: | Okay. I see. And what can you say about question 3 |
| B24: | Yooo… for me it was challenging ma’am. |
| Researcher: | Thank you very much B24 for answering the questions I had for you. |
| B24 | Okay ma’am. |

**APPENDIX 7**
**PRETEST INTERVIEW 3**

Researcher: Good Afternoon learners (B2 and B13)

B13 / B2: Good afternoon ma'am

Researcher: Okay … I have some questions for you with reference to your pre-test. Please feel free to answer the questions I have for you as honestly as possible.

B13: Okay ma'am.

Researcher: Good, please have a look at your scripts. I am going to start by asking questions pertaining to question 1. Please have a look at the question paper as well. I will start by asking about 1.1.

I will start with you B2. Why do you think you failed to answer question 1?

B2: I forgot which quadratic form of equation to use

Researcher: And you B13?

B13: Me too.

Researcher: Okay, so if I were to provide you with the correct quadratic form to use say for question 1.1 which is $y = a (x + p)^2 + q$. How then will you use that form to find the equation of the graph drawn in question 1.1? B2?

B2: I am not sure ma'am.

Researcher: And B13?

B13: I will substitute the points on the graph into the equation.

Researcher: Which points? where? Please be specific.

B13: Heish…. I am not sure ma'am

Researcher: How do you find the topic on quadratic functions. Can you say it is an easy topic or you find it difficult.

B2: When the teacher explains in class and gives an example, I find it easier because I can do the homework, but if it's a test heish… I forget what the teacher said.

Researcher: And B13 what can you say? How do you find the topic on quadratic functions?

B13: Challenging especially when you do not know which quadratic form to use and why a particular form works and for what situations. I get confused.

Researcher: Thank you B2 and B13 for helping shed some light into why you find this topic difficult.

## APPENDIX 8

## POST TEST INTERVIEW GUIDE

### Questions for the interview

1. What were your experiences in learning about quadratic functions when you were watching you tube videos?
2. Did you find using Geogebra useful? What did you learn?
3. Do you think you had enough time to explore the topic using computers?
4. Tell me about your experiences in learning about quadratic functions before you made use of computers?
5. What did you enjoy most when you were working with computers?

## APPENDIX 9

## POST TEST INTERVIEW 1

Researcher: Good afternoon.

Learner A: Good afternoon ma'am.

Researcher: I would like to ask you about your experiences as a learner and also as someone who had the opportunity to learn using computers, please feel free to answer any of my questions. If any of the questions that I ask you make you uncomfortable, please you are free to say that you cannot answer, but I would really be grateful is I could get your honest opinions on the questions that I have for you.

Learner A: Okay ma'am.

Researcher: Are you ready?

Learner A: Yes, I am ready.

Researcher: Good. What can you say were your experiences in learning about quadratic functions when you were watching you tube videos?

Learner A: I think it was more fun to learn from the computer because If I did not understand, I had the opportunity to re play the video again until I understand.

Researcher: Okay, but in the classroom if the teacher is explaining on the board, if you do not understand, can you not raise your hand and ask the teacher to explain to you again?

Learner A: You may, but you don't want to disturb the teacher, so if other learners are quite, then it means they understand. I do not feel comfortable asking the teacher over and over if I do not understand, but with the computer it is different you can replay until you understand.

Researcher: Okay, you are talking about the videos here. How about Geogebra. Did you find it useful?

Learner: Yes, very useful ma'am. It was easy to draw graphs of many quadratic functions within a short space of time. And it was easier to see the properties of the function from its graph quickly and easily, like the turning point and the intercepts.

Researcher: So can you say plotting your graphs using computers makes you understand better that using pen and paper?

Learner A: It is faster, but sometimes you cannot see how the computer has done it because it does not show you how it found the intercepts, so I was still using my pencil and paper to try to understand how the computer plotted the points.

Researcher: Tell me about me about your experiences in learning about quadratic functions before you made use of computers?

Learner A: When I was learning from computers, I was concentrating more, because everyone was busy with their computer. It was more interesting because I love computers. And when we were discussing in groups, everyone was contributing because we were all paying attention to what we were seeing. It was like watching a movie in class (laughing).

Researcher: Okay, I see. So what did you enjoy most when you were working with computers?

Learner A: I was enjoying working so that I can see what else the computer can do. I was able to do the given exercises faster using the computer and if I someone did not understand something; it was easier for me to explain because I would have seen it on the computer.

Researcher: Thank you very much for your responses, I really appreciate your honesty in answering the questions I had for you.

Learner A: Thank you ma'am.

## APPENDIX 10

## POST TEST INTERVIEW

Researcher: Good Afternoon, I would like to ask you about your experiences as a learner and also as someone who had the opportunity to learn using computers, please feel free to answer any of my questions. If any of the questions that I ask you make you uncomfortable, please you are free to say that you cannot answer, but I would really be grateful is I could get your honest opinions on the questions that I have for you.

Learner B: Yes, teacher.

Researcher: Before using computers, how were you taught functions by your teacher?

Learner B: But,ma'am you are our teacher?

Researcher: Yes, I am and I would like you to answer that question as honestly as you can remember.

Learner B: Our teacher told us some formulas and showed us some examples. After that she did some few examples from the book on the board, and then gives us home work.

Researcher: Yes. go on.

Learner B: And then the next day she explains to us the corrections.

Researcher: So for you, which method did you enjoy more: learning functions using computers or learning functions from your teacher showing you examples?

Learner B: I think with computers, learning quadratic functions was good. If I had problems I would get help from the teacher, but most of the times when I was comparing my answers with some of the learners were had similar answers.

Researcher: Do you have any additional views or suggestions with regard to learning mathematics using computers?

Learner B: Yes, I think we should use computers in mathematics, they make it a little bit easier. You are able to do many problems within a short space of time and but sometimes you don't understand how the computer has drawn the graph, so the teacher must also explain.

**APPENDIX 11**

**POST TEST INTERVIEW 3**

Researcher: Good afternoon learners

Learner C: Good Afternoon.

Learner D: Good Afternoon ma'am.

Researcher: Alright, I hope you do not mind if I interview both of you at the same time. But before I start, if I may ask, is any one of you uncomfortable with doing this interview together?

Learners: It's okay ma'am.

Researcher: Good. May I call you Learner C and you will be Learner D. Looking at your scripts for the post test, I can see that both your performance improved from how you had performed in the post test. Learners, please help me understand. What could have been the reason?

Learner D: I think it's because the first time I did not understand and when we did functions again, now I understand better.

Researcher: Okay, what could be the reason for you Learner C?

Learner C: Yaa, I think maybe because now we watched some videos explaining for us those things that we did not understand.

Researcher: Alright, what exactly did you enjoy about learning using the computer?

Learner D: I think for me I enjoyed because it was my first time learning by watching videos. With the headphones to my ears I was only paying attention to what that teacher in the video was saying, and there were no disturbances. I like it when I have headphones ma'am (laughing) its like I'm listening to music, I was enjoying.

Researcher: Alright and your learner C what did you enjoy most?

Learner C: Me I enjoyed…. What do you call it ma'am? That software that we were using to draw graphs…Yes Geogebra. It was interesting to see how a computer can easily draw graphs like that. It was interesting.

Researcher: Alright would you like to share with me just one thing that you learnt using Geogebra?

Learner D: Yes, when you punched in an equation like f(x) = (x -2) (x-3) it was easy to see from the computer that 2 and 3 are the y intercepts.

Researcher: Alright, thank you , for that Learner D, please refer to your solution in the post test at Question 2 a. The question required you to find the x and y intercepts of the given equation. May you please explain to me how do you get the y intercept.

Learner D: Okay ma'am. When we were working with computers we saw that the x intercept is when y = 0. That's why I substituted here and got (x + 2) (x + 3) = 0, therefore the y intercept is at x=-2 and at x=-3.

Researcher: Okay, and if you use the computer to plot that graph are you able to see those intercepts?

Learner D: Yes,ma'am at x= -2 and at x=-3 the graph will cut the y axis.

Researcher: But when the question asks you to find the intercepts, don't you think you have to write your answer as a coordinate, say ( -2;0) and (-3;0)? (writing on a piece of paper)

Learner D: It's the same ma'am, I can just say x=-2 and x=-3, as long as I have substituted 0 there (pointing to the equation).

Researcher:Alright, Learner C. I would like you to look at your solution to Question 2d (Response 10 on the posttest analysis). I Can see that you were able to draw the graph perfectly for question 2c, but please clarify for me, why do you say it is increasing when x is greater than $-\frac{5}{2}$ and decreasing when x is less than $-\frac{5}{2}$?

Learner C: I thought it is increasing when it is more that -5/2 because the graph is above the y axis and then below the y axis the graph is decreasing, that's what I thought but you marked me wrong.

Researcher: Okay, one last question for you learners. Do you think you had enough time to explore the topic on functions using computers?

Learner C: No, more time is needed so that one can master how to use Geogebra alone.

Learner D: I agree we needed more time so that we can understand more.

Researcher: Thank you very much learners for your time.

**APPENDIX 12**

**INFORMATION SHEET TO THE PRINCIPAL**

Wits School of Education

University of Witwatersrand

Private Bag 3

2050

The Principal

Forte High School

P.O Box 206

Dobsonville

1863

2 August 2016

Dear Sir/ Madam

I, Tsitsi Mable Mugwagwa request your permission that Grade 11 mathematics learners at your school take part in the research entitled "To what extent does the use of computers have on remedying learner errors and misconceptions in functions at grade 11". I am taking this research study as a partial fulfillment of Master of Science Education degree at the University of Witwatersrand in Johannesburg.

I am doing a research on how the use of technology, in particular computers can help remedy the errors and misconceptions that learners have in functions and for this particular research myfocus is two grade 11 groups, one to be used as the control group and the other as an experimental group.

The research instruments used in this study are a pretest and a posttest (copies attached). Both groups will write the pretest and each test will be written at a maximum of 1 hour by the learners. From the pretest, the errors and misconceptions that learners have on functions will be noted and then some learners will be interviewed to establish the reasons for these errors and misconceptions in the test. The interviews will be audio taped and later transcribed for data analysis. After the pretest, a teaching intervention will be administered to both groups, with one group using computers and the other using traditional methods to try to remedy the errors and misconceptions identified in the pretest. After the intervention. a post test will be administered to establish the effects of the interventions.

The participants' participation is voluntary. The research participants will not be disadvantaged or advantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no for seeable risks in participating in this study.

Anonymity and confidentiality of the responses to the tests and interview will be ensured. No names will be used on the test to identify learners. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study.

The research data will be used for academic purposes and possibly shared at a conference presentation and/ or published as a general paper in a journal. The research data will be kept safely and destroyed5 years after completion of the study. The results and findings of the study will be presented as a research report to Wits School of Education.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely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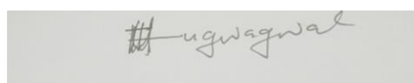
| SIGNATURE |  |
| NAME | TSITSI MABLE MUGWAGWA |
| ADDRESS | NUMBER, 11, 8TH AVENUE, ROODEPORT,1724 |
| E MAIL | tsitsimabel@yahoo.com |
| TELEPHONE/CELLPHONE NUMBER | 011 989 1809/078 012 9993 |
| NAME OF SUPERVISOR | DOCTOR JUDAH MAKONYE |
| E MAIL | judahmakonye@wits.ac.za |

**APPENDIX 13**

**INFORMATION SHEET FOR LEARNERS**

Wits School of Education

University of Witwatersrand

Private Bag 3

2050

The Principal

Forte High School

P.O Box 206

Dobsonville

1863

2 August 2016

Dear Learner

My name is Tsitsi Mable Mugwagwa and I am a Masters student in the School of Education at the University of the Witwatersrand. I request your permission to take part in my research entitled "To what extent does the use of computers have on remedying learner errors and misconceptions in functions at grade 11". I am taking this research study as a partial fulfillment of Master of Science Education degree at the University of Witwatersrand in Johannesburg.

In this research, I want to investigate why learners make mistakes and how learners misunderstand functions at grade 11. For me to be able to identify these mistakes, I will request that you write a test on functions with a maximum duration of 1 hour. Remember, this is not for marks, it is for me to be able to identify the difficulties that you might be having on the topic, functions. After identifying these errors, I may then request that I interview you. The reason for the interview is to find out from you the reasons for some of your answers to the test. With your permission, the interview will be audio recorded for analysis so that I may be able to plan some intervention lessons to address the problems identified in the test (pretest). After the pretest, you will be requested to attend 10 (1-hour) sessions to address the difficulties identified in the pretest. After this intervention, I will then request that you write another test (Posttest), to check the effectiveness of the intervention. Remember, the tests and the interview is voluntary, which means that you do not have to do them. In addition, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writings about this study. Also, all collected information will be stored safely and destroyed after 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent forms, but at the end of the day it is your decision to join us in his study.

I look forward to working with you! Please feel free to contact me if you have any questions.

Thank you.

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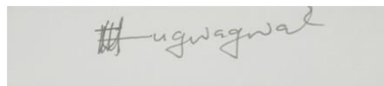
| SIGNATURE |  |
| NAME | TSITSI MABLE MUGWAGWA |
| ADDRESS | NUMBER, 11, 8$^{TH}$ AVENUE, ROODEPORT,1724 |
| E MAIL | tsitsimabel@yahoo.com |
| TELEPHONE/CELLPHONE NUMBER | 011 989 1809/078 012 9993 |
| NAME OF SUPERVISOR | DOCTOR JUDAH MAKONYE |
| E MAIL | judahmakonye@wits.ac.za |

**APPENDIX 14**

**INFORMATION SHEET TO PARENTS**

Wits School of Education

University of Witwatersrand

Private Bag 3

2050

The Principal

Forte High School

P.O Box 206

Dobsonville

1863

2 August 2016

Dear Parent/Guardian

My name is Tsitsi Mable Mugwagwa and I am a Masters student in the School of Education at the University of Witwatersrand.

I am doing a research on functions, a mathematics topic in the Further Education and Training phase (FET grade 10 to 12). My research is “To what extent does the use of computers have on remedying learner errors and misconceptions in functions at grade 11”. I am taking this research study as a partial fulfillment of Master of Science Education degree at the University of Witwatersrand in Johannesburg.

I am doing a research on how the use of technology, in particular computers can help remedy the errors and misconceptions that learners have in functions and for this particular research my focus is grade 11 learners.

The research instruments used in this study are a pretest (to identify the errors and misconceptions that learners have on functions) and a posttest (to establish the influence of the intervention used to address these errors and misconceptions). The duration of each test is a maximum time of 1 hour. From the pretest, the errors and the misconceptions that learners have on functions will be noted and then some learners will be interviewed to establish the reasons for these errors in the test. The interviews will be audio taped and later transcribed for data analysis. The results from the pretest and the interview will be used to plan an intervention to remedy the errors and misconceptions that learners would have demonstrated on functions.

The reason why I have chosen you child’s class is that, learner errors and misconceptions hinder successful learning in mathematics. If a learner has a good understanding of functions, then he/she stands a better chance of doing well in mathematics examinations. Would you mind if your child were to take part in this study? If you agree, then your child is expected to write a pretest and a posttest. She/he may also be interviewed and this interview will be audio taped. Your child will also be expected to attend intervention lessons that will try to address the problems identified in the pre-test. If you agree, your child will have to give up 10 sessions of his/her study time (0700-0800 Monday to Friday) to attend these intervention lessons.

Your child will not be advantaged, or disadvantaged in any way. S/he will be reassured that s/he can withdraw his/her permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child’s name and identity will be kept confidential at all times and in all academic writings about this study. His/her individual privacy will be maintained in all publications, resulting from this study.

All research data will be destroyed 3-5 years after completing this project. Please let me know if you require any further information. Thank you very much for your help.

Yours sincerely

| | |
|---|---|
| SIGNATURE |  |
| NAME | TSITSI MABLE MUGWAGWA |
| ADDRESS | NUMBER, 11, 8TH AVENUE, ROODEPORT,1724 |
| E MAIL | tsitsimabel@yahoo.com |
| TELEPHONE/CELLPHONE NUMBER | 011 989 1809/078 012 9993 |
| NAME OF SUPERVISOR | DOCTOR JUDAH MAKONYE |
| E MAIL | judahmakonye@wits.ac.za |

**APPENDIX 15**

**INFORMATION LETTER TO THE TEACHER**

Wits School of Education

University of Witwatersrand

Private Bag 3

2050

The Principal

Forte High School

P.O Box 206

Dobsonville

1863

2 August 2016

Dear Teacher

I, Tsitsi Mable Mugwagwa request your permission that Grade 11 mathematics learners at your school take part in the research entitled “To what extent does the use of computers have on remedying learner errors and misconceptions in functions at grade 11”. I am taking this research study as a partial fulfillment of Master of Science Education degree at the University of Witwatersrand in Johannesburg.

I am doing a research on how the use of technology, in particular computers can help remedy the errors and misconceptions that learners have in functions and for this particular research my focus is two grade 11 groups, one to be used as the control group and the other as an experimental group. The research design for this study requires that I make use of two grade 11 groups. Since you are also teaching another grade 11 group, I hope you do not mind me asking for your participation in your class to help me with this study. Although the focus of this study are learners, I need your help in doing an intervention lessons in your class using traditional methods on the topic of research as outlined in the paragraphs that follow.

The research instruments used in this study are a pretest and a posttest (copies attached). Both groups will write the pretest and each test will be written at a maximum of 1 hour. From the pretest, the errors and misconceptions that learners have on functions will be noted and then some learners will be interviewed to establish the reasons for these errors and misconceptions in the test. The interviews will be audio taped and later transcribed for data analysis. Findings from the pretest and the interviews on the learner errors and misconceptions will be communicated to you so as to inform your teaching intervention. After the pretest, a teaching intervention will be administered to both groups, with my group using computers and the other group using traditional methods to try to remedy the errors and misconceptions identified in the pretest. After the intervention. a post test will be administered to establish the effects of the interventions.

The participants' participation is voluntary. The research participants will not be disadvantaged or advantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no for seeable risks in participating in this study.

Anonymity and confidentiality of the responses to the tests and interview will be ensured. No names will be used on the test to identify learners or the teacher. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study.

The research data will be used for academic purposes and possibly shared at a conference presentation and/ or published as a general paper in a journal. The research data will be kept safely and destroyed5 years after completion of the study. The results and findings of the study will be presented as a research report to Wits School of Education.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely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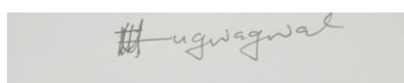
| SIGNATURE |  |
| NAME | TSITSI MABLE MUGWAGWA |
| ADDRESS | NUMBER, 11, 8TH AVENUE, ROODEPORT,1724 |
| E MAIL | tsitsimabel@yahoo.com |
| TELEPHONE/CELLPHONE NUMBER | 011 989 1809/078 012 9993 |
| NAME OF SUPERVISOR | DOCTOR JUDAH MAKONYE |
| E MAIL | judahmakonye@wits.ac.za |

## APPENDIX 16

## ETHICS APPROVAL LETTER


**Wits School of Education**

WITS UNIVERSITY

**27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za**


**04 November 2016**

**Student Number: 729619**

**Protocol Number: 2016ECE060M**

**Dear Tsitsi Mable Mugwagwa**

**Application for Ethics Clearance: Master of Science**

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

**To what extent does the use of computers have on remedying learner errors and misconceptions in functions at Grade 11**

The committee recently met and I am pleased to inform you that **clearance was granted**. However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

**The following comments were made:**

- Stipulate where and how interviews will be held.
- There is insufficient information regarding transcription of interview data and how it will be stored and destroyed afterwards.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

**The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.**

All the best with your research project.

Yours sincerely,

MMaseko

Wits School of Education

## APPENDIX 17

## GDE APPROVAL LETTER

**Reference no: D2017 / 324**
enquiries: 011 843 6503

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

### GDE RESEARCH APPROVAL LETTER

| | |
|---|---|
| Date: | 8 November 2016 |
| Validity of Research Approval: | 6 February 2017 to 29 September 2017 |
| Name of Researcher: | Mugwagwa T.M. |
| Address of Researcher: | Number 11, 8th Avenue; Roodepoort; 1724 |
| Telephone / Fax Number/s: | 078 012 9993; 011 989 1801 |
| Email address: | tsitsimabel@yahoo.com |
| Research Topic: | The influence of using computers to remedy learner errors and misconceptions in functions at Grade 11. |
| Number and type of schools: | ONE Secondary School |
| District/s/HO | Johannesburg West |

***Re: Approval in Respect of Request to Conduct Research***

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted.

*CONDITIONS FOR CONDUCTING RESEARCH IN GDE*

1. *The District/Head Office Senior Manager/s concerned, the Principal/s and the chairperson/s of the School Governing Body (SGB) must be presented with a copy of this letter.*
2. *The Researcher will make every effort to obtain the goodwill and co-operation of the GDE District officials, principals, SGBs, teachers, parents and learners involved. Participation is voluntary and additional remuneration will not be paid.*

*Making education a societal priority*

**Office of the Director: Education Research and Knowledge Management ER&KM)**

**APPENDIX 18**

**APPROVAL LETTER TO CONDUCT RESEARCH**

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

**Private Bag 3 Wits, 2050**
**Fax: 02711 7176029**
**Tel: 02711 7176006**

Reference: **Ms Mpumi Mnqapu**
E-mail: #Fac-Science-PG@wits.ac.za

15 February 2017
Person No: 729619
PAG

Miss TM Mugwagwa
Number 295 Sparrowgate
1 Lark Street
Meredale
2091
South Africa

Dear Miss Mugwagwa

**Master of Science (Coursework and Research Report): Approval of Title**

We have pleasure in advising that your proposal entitled *The Influence of Using Computers to Remedy Learner Errors and Misconceptions in Functions at Grade 11* has been approved. Please note that any amendments to this title have to be endorsed by the Faculty's higher degrees committee and formally approved.

Yours sincerely

Ms René Vosloo
Faculty Registrar
Faculty of Science

+

**APPENDIX 19**

CONSENT LETTER FOR THE PRINCIPAL

Please fill in and return the reply slip below indicating your willingness to allow learners at your school to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I …………………………………...........the Principal of ………………………………..

| **Permission to conduct a research study** | **circle one** |
|---|---|
| I agree that two grade 11 mathematics classes at my school can be invited to participate in this study | YES / NO |
| I agree that the computers at this school can be used for this study | YES / NO |

**Informed Consent**

I understand that

- my learners' names and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- learners do not have to answer every question and can withdraw from the study at any time
- learners can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….(Principal) Date……………….

Sign…………………………………………….(S.G.B member) Date……………….

Sign……………………………………………(H.O.D Computers) Date………………

**APPENDIX 20**

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

My name is …………………………………………………………………………..

| **Permission for test** | **circle one** |
| --- | --- |
| I agree to fill in a question and answer sheet or write a test for this study | YES / NO |

**Informed Consent**

I understand that

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this study

Sign……………………………………

Date …………………………………..

**APPENDIX 21**

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

My name is ………………………………………………………………………………..

| **Permission to be interviewed** | **circle one** |
|---|---|
| I would like to be interviewed for this study | YES / NO |
| I know that I can stop the interview at any time and do not have to answer all the questions asked | YES / NO |

**Informed Consent**

I understand that

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this study

Sign……………………………………

Date …………………………………..

**APPENDIX 22**

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

My name is ………………………………………………………………………………..

| **Permission to be audio taped** | **circle one** |
|---|---|
| I agree to be audio taped during the interview or observation lesson | YES / NO |
| I know that the audio tapes will be used for this project only | YES / NO |

**Informed Consent**

I understand that

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this study

Sign……………………………………

Date …………………………………..

**APPENDIX 23**

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

My name is ………………………………………………………………………………..

| **Permission to observe you in class** | **circle one** |
|---|---|
| I agree to be observed in class | YES / NO |

**Informed Consent**

I understand that

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this study

Sign……………………………………

Date …………………………………..

**APPENDIX 24**

LEARNER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called:

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

My name is …………………………………………………………………………..

| **Permission to review/ collect documents/ artifacts** | **circle one** |
|---|---|
| I agree that the tests (Pre and post- test) can be used for this study only | YES / NO |

**Informed Consent**

I understand that

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this study

Sign……………………………………

Date …………………………………..

**APPENDIX 25**

PARENT/ GUARDIAN CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I ……………………………….........the parent/guardian of ………………………………..

| **Permission for test** | **circle one** |
|---|---|
| I agree that my child may write a test for this study | YES / NO |

**Informed Consent**

I understand that

- my child's name and information will be kept confidential and safe and that my name and the name of my child's school will not be revealed
- he/she does not have to answer every question and can withdraw from the study at any time
- he/ she can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….

Date……………………………………………..

**APPENDIX 26**

PARENT/ GUARDIAN CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I …………………………………...........the parent/guardian of ………………………………..

| **Permission to review/ collect documents/ artifacts** | **circle one** |
|---|---|
| I agree that my child's tests can be used for this study only | YES / NO |

**Informed Consent**

I understand that

- my child's name and information will be kept confidential and safe and that my name and the name of my child's school will not be revealed
- he/she does not have to answer every question and can withdraw from the study at any time
- he/ she can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….

Date……………………………………………..

**APPENDIX 27**

PARENT/ GUARDIAN CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I …………………………………...........the parent/guardian of ………………………………..

| **Permission to observe my child in class** | **circle one** |
|---|---|
| I agree that my child may be observed in class | YES / NO |

**Informed Consent**

I understand that

- my child's name and information will be kept confidential and safe and that my name and the name of my child's school will not be revealed
- he/she does not have to answer every question and can withdraw from the study at any time
- he/ she can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….

Date……………………………………………..

**APPENDIX 28**

PARENT/ GUARDIAN CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I …………………………………...........the parent/guardian of ………………………………..

| **Permission to be audio taped** | **circle one** |
|---|---|
| I agree that my child may be audio taped during interview | YES / NO |
| I know that the audio tapes will be used for this study only | YES / NO |

**Informed Consent**

I understand that

- my child's name and information will be kept confidential and safe and that my name and the name of my child's school will not be revealed
- he/she does not have to answer every question and can withdraw from the study at any time
- he/ she can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….

Date……………………………………………..

**APPENDIX 29**

PARENT/ GUARDIAN CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I …………………………………...........the parent/guardian of ………………………………..

| **Permission to be interviewed** | **circle one** |
|---|---|
| I agree that my child may be interviewed for this study | YES / NO |
| I know that he/ she can stop the interview at any time and does not have to answer all the questions asked | YES / NO |

**Informed Consent**

I understand that

- my child's name and information will be kept confidential and safe and that my name and the name of my child's school will not be revealed
- he/she does not have to answer every question and can withdraw from the study at any time
- he/ she can ask not to be audio taped
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………………….

Date……………………………………………..

**APPENDIX 30**

TEACHER CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called

TO WHAT EXTENT DOES THE USE OF COMPUTERS HAVE ON REMEDYING LEARNER ERRORS AND MISCONCEPTIONS IN FUNCTIONS AT GRADE 11

I ……………………………….............................. ………………………………..

| **Permission to take part in the study** | **circle one** |
|---|---|
| I agree that I will conduct a teaching intervention addressing learner errors and Misconceptions as advised by the researcher. | YES/NO |
| I agree that in my teaching intervention, I will use traditional ways of teaching . | YES/NO |

**Informed Consent**

I understand that

- name and information will be kept confidential and safe and that my name and the name of my school will not be revealed
- I do not have to answer every question and can withdraw from the study at any time
- I will not to be audio taped or observed during the intervention
- all the data collected during this study will be destroyed within 3-5 years after completion of this project

Sign…………………………………..

Date………………………………….