The Relationship between Textbook Affordances and Mathematics' Teachers' Pedagogical Design Capacity (PDC)

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DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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15th April, 2015

ABSTRACT

Mathematics textbooks are ubiquitous in classrooms but research on how teachers use this resource which is a major resource and often the only resource that teachers have access to, is “fledging” (Remillard, Herbel-Eisenmann, & Lloyd, 2009, p. xiv). The study investigates the teacher-textbook relationship between the affordances (J. J. Gibson, 1977) of the textbook and teachers’ capacity to perceive and mobilise these affordances for productive mediation of the object of learning, that is, teachers’ pedagogical design capacity (PDC) (Brown, 2002, 2009).

Theoretically grounded in socio-cultural theory wherein all humans are inherently social beings and grow from and through the use of tools (Vygotsky, 1978), the study aligns itself with a conception of textbook use as “participation with the text” (Remillard, 2005, p. 221) and where teaching is a design activity (Brown, 2009) to develop conceptual frameworks for determining the affordances of the textbook, as well as teachers’ mobilization of these affordances.

The analysis of the textbook produces two major affordances for the teachers’ practice as the *mathematical content* **and** the *approach to the teaching and learning* of this content. The analysis of teachers’ lessons on the other hand shows that teachers make *injections* to the textbook content, some of which are robust while others are distractive. However, an important result of the analysis of the lessons is that the teacher-textbook relationship is a function of the *critical omissions* from the textbook that the teacher makes.

The key findings of the study are that for the teachers in the study, their textbook use is generally *tacit* and *not deliberate;* their relationships with their textbooks are *not intimate*, resulting in generally *low* PDCs. Thus, the study warns against notions that making textbooks available implies deliberate use; and argues that textbook use needs to be mediated in order to be deliberate.

In memory of my parents (‘M’e ‘Mampho and Oom Sye) who believed in the power of education;

my darling brother, Eddie;

my father-in-law (Ntate Joe), whom I wish could have waited a bit longer!

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"It always seems impossible until its done". Nelson Mandela

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LIST OF ABBREVIATIONS

CA	Content Area
CAPS	Curriculum and Assessment Policy Statement
CFP	Content Focused Project
DBE	Department of Basic Education
FRF	FirstRand Foundation
HCI	Human-Computer Interaction
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
PDC	Pedagogical Design Capacity
PEI	President's Education Initiative
PISA	Programme for International Student Assessment
POET	Psychology of Everyday Things
SMK	Subject Matter Knowledge
TIMSS	Trends in International Mathematics and Science Study
WMCS	Wits Maths Connect Secondary

CHAPTER 1
Introduction to Study

1.1 Why Textbooks

During familiarisation visits to the ten schools participating in the Wits Maths Connect Secondary[1] project (WMCS), my interest in how teachers use textbooks[2] in their classrooms was piqued by my observations of how two of the teachers in those schools interacted with their textbooks:

Teacher A:

After correcting the previous day's homework on the chalkboard together with learners, the teacher continued with the introduction of a concept which derived from the homework they had just corrected. After a few remarks learners were directed to a set of 'exercises'[3] in their textbooks and instructed to do all the exercises in the order that they appeared in the textbook as the teacher moved around the classroom checking learners' work. After a few rounds the teacher called learners' attention and guided learners through the exercises. Then, they came to one particular question which the teacher told learners to skip and moved onto the next one after that.

I started wondering what made the teacher decide to omit that particular question and about the consequences of such an omission for the general coherence between the activities learners had been engaged in; and teacher's goals for the lesson. In other words, what exactly did teacher want learners to learn in that lesson, and how was that similar or different from the intentions of the textbook author?

Just to give a little background of this class. This was a Grade 9 class and in the previous lesson learners had done exercises on the expansion of factors. For example, expanding the factors of the type $(a+3)(a-2)$ to obtain the expression, a^2+a-6. On this particular day they were provided with a mix of integer values for t and required to generate separate tables of values for the expressions: $H=(t-3)(t-3)$ and $K=t^2-6t+9$, using the same t values. My observation was that learners spent a lot of time generating the tables. For the values of H, instead of substituting a given value of t and evaluating the values directly, learners would firstly multiply out the brackets and then simplify the expression. For example, for $t=5$, learners would multiply out $(5-3)(5-3)$ to obtain the expression $25-15-15+9$ and then simplify the sum to obtain 4. They did not translate the two brackets as $2\times 2=4$.

The question learners were asked to skip by the teacher dealt with the comparison of the two expressions, to show that the two expressions H and K were equivalent. As far as I am concerned, this was the most important question of this exercise. Teacher A had the

[1] See page 3 for the description of this project
[2] In this study I refer to textbooks as prescribed books which have been written for use in classrooms especially by learners and teachers.
[3] By exercises I refer to anything learners have to work on

opportunity to make learners see that the expressions were equivalent and to start to introduce them to transformational algebra, but did not take up this opportunity. However, for the learners, the exercise ended up being about expanding the factors for H and a substitution exercise for K, which I do not believe was the intention of the textbook author for this particular exercise. In other words, the particular omission that the teacher made detracted from the intention of the textbook.

Teacher B:

The teacher introduced a new topic (Variance in Data Handling) to a Grade 11 class by asking learners to calculate variance from a given set of data which the teacher wrote on the chalkboard for learners. Learners asked the teacher what "variance" was as they had not seen it before. To answer the learners, the teacher wrote a formula on the chalkboard, $\text{Variance} = \sum\frac{(x_i-\bar{x})^2}{n}$ →(formula 1), and told them that the formula was the definition of "variance". Learners started working on the problem by applying the given formula. The Teacher commented that learners were taking too long to calculate variance, and learners responded that the procedure was too long, and wondered if there was a shorter procedure. The teacher then suggested that there was a shorter method and started looking for it in the closed cupboard in the front of the classroom. The teacher brought out a workbook[4] different from what learners were using, then wrote the 'shorter' method (formula) on the chalkboard.. The new formula was given as: $\text{Variance} = \sum\frac{{x_i}^2}{n} - \bar{x}^2$→(formula 2). While these two formulae look different they are actually both correct and are used to calculate variance.

Towards the end of the lesson when teacher asked for answers, learners called out different answers, a fact which seemed to surprise learners, and to my surprise, the teacher as well; who wondered what could have happened. Some learners had used the first formula while others used the second formula (formula 2 above). One important observation for me here pertained to the two formulae being used; one from the textbook (formula 1 above) and the other from the workbook (formula 2). Through some algebraic manipulation, formula 2 can be derived from formula 1; but teacher B does not explain this to learners nor show them why the two formulae look quite different but are actually the same. Learners had to take it on faith because the second formula came from some form of a 'textbook' which was presented by their teacher.

As with teacher A above, I wondered about the kind of relationship I was observing between the teacher and his textbook in this lesson. Teacher and textbook were working together until the point where teacher had to change the formula and depart from the approach to the calculation of variance that was advanced by the textbook. The insertion of a different formula (formula 2) instead of enhancing the calculation of variance actually detracted from the intention of the lesson for learners to be able to calculate variance; because at the end of

[4] Taylor and Vinjevold (1999) distinguish between textbooks and workbooks by defining textbooks as providing a systematic learning programme while workbooks provided supplemental and revision materials in support of the textbooks

this lesson, both the teacher and learners were wondering about what went wrong in the two formulae they had used, not certain that they had correctly calculated variance.

The two scenarios above give a glimpse of important issues about how teachers interact with textbooks in their classrooms. Teachers make decisions about the use of textbooks in the course of their lessons; some of which disrupt coherence as we see in the two lessons above. The two teachers used their textbooks differently, but this we know; teachers use textbooks in different and often distinct ways. Moreover, both teachers did not appear to use the textbooks in a systematic way. Teacher A suddenly omits a critical question from the textbook, while Teacher B also suddenly inserts an alternative method of calculating variance which he had not planned for. In this study, I seek to understand more about the decisions that teachers make with regard to their textbooks, how they make them, why they make them, and what the implications of these decisions are with respect to the opportunities opened up in the classroom for learning.

1.2 Textbooks in South Africa

In 2012 in the South African media, textbooks took centre stage in what came to be known as the 'Limpopo saga', when it was reported in the media that six months into the school year, textbooks had not been delivered to schools in the Limpopo Province in South Africa. This caused a national uproar from all stakeholders: teachers, learners, parents, and NGOs, to the extent that the Minister responsible for basic education was called on to resign by the largest teacher union in South Africa. A parliamentary commission had to be set up to investigate the matter; the human rights movement conducted its own investigations and reported this to parliament; and one NGO actually took government to court over the matter. Textbook delivery has since become a burning issue in South Africa, with government called upon to account for this matter continuously.

This incidence shows how highly and importantly the textbook is regarded as a useful resource for learning at school. However, it also highlights underlying assumptions which equate availability of textbooks in schools with appropriate use and consequently to effective mediation in the classroom. The two examples of the teachers I mention at the beginning of this chapter show that this is not always the case, and suggest that there is need for the strengthening of these relationships between availability, use, and effectiveness in mediation. As I embark on this study, I am cognizant of this: our focus in this study is to explore these relationships in order to understand better how they work.

This study is situated within the Wits Maths Connect Secondary Project, a research and development programme funded by the FirstRand Foundation (FRF), and the Department of Science and Technology. The project offers a school-based professional development programme to teachers, in response to a call by the FRF for researchers to find possible solutions to the problem of high failure rates in mathematics in South Africa. I have chosen to study the appropriate use of textbooks by teachers as one such possible solution, for the reasons I have so far advanced. Moreover, as the next sections shall show, mathematics textbooks remain the major resource for teaching mathematics internationally and therefore understanding how teachers utilise them in their practice and how their use can be improved for a better quality education is highly desirable.

In the next sections, I show the status of the field on textbook research in general and in South Africa in particular; which indicates that this is a very young field which still has a long way to go to establish itself theoretically and empirically.

1.3 Research on Textbooks

1.3.1 The Ubiquitous Nature of Mathematics Textbooks

The presence of mathematics textbooks in the classroom dates as far back as 300 BC with Euclid's, *The Elements* (Fan, Zhu, & Miao, 2013). For as long as many of us can remember, and as far back as our primary education, the mathematics textbook has been a very critical feature of our learning. As Remillard (2005) notes, mathematics has long been associated with textbooks and curriculum materials, and has a long history of being driven by the textbook moreso than other literacy-based subjects. Despite the availability of other curricular resources including internet and digital technologies, the mathematics textbook remains the major classroom resource for teaching and learning mathematics in developing and developed countries worldwide (Haggarty & Pepin, 2002; Johansson, 2006; Nicol & Crespo, 2006; Vincent & Stacey, 2008); as well as for high performing countries in international assessments such as PISA and TIMSS[5] (Askew, Hodgen, Hossain, & Bretscher, 2010).

From the TIMSS Video study (Hiebert et al., 2003), it was observed that at least 90% of the lessons in all the seven countries participating in the video lessons made use of a textbook or some form of a worksheet. Hodgen and team (Hodgen, Kuchemann, & Brown, 2010) reckon that textbooks account for 96% usage by teachers at Grade 8 internationally, with Brown & Edelson (2003) positing that

> of all the different instruments for conveying educational policies, [curriculum materials including textbooks] exert perhaps the most direct influence on the tasks that teachers actually do with their learners each day in the classroom (p.1)

As such, textbooks matter in the mathematics classroom and studies which investigate what happens when teachers use textbooks are hence highly desirable; especially in situations where the prescribed textbook is the major and often only resource that the mathematics teacher has access to. I am talking here about contexts of 'poverty' mostly found in developing countries around the world. As Valverde and team (2002) point out, "perhaps only students and teachers themselves are a more ubiquitous element of schooling than textbooks" (Valverde et al., 2002, p. 1); and as such and because of this centrality of textbooks to schooling, "understanding textbooks is essential to understanding the learning opportunities provided in educational systems around the world" (ibid).

However, despite their ubiquity in the classroom, there seems to have been very little interest for research on textbooks. Apple (1986) comments that legitimate knowledge is made

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[5] PISA is the acronym for (Programme for International Student Assessment), while TIMSS stands for (Trends in International Mathematics and Science Study)

available to schools "through something to which we have paid too little attention – the textbook…Yet even given the ubiquitous character of textbooks, they are one of the things we know least about" (Apple, 1986, p. 85). Some researchers believe that the research on textbooks that Cronbach (1955) termed as "scattered, inconclusive, and often trivial" (1955, p. 4), has not changed that much, especially with respect to the use of textbooks by teachers. Sosniak and Stodolsky (1993) comment that "systematic attention to textbooks and their use by teachers and students is long overdue"(p.249), adding that Cronbach's (1955) call to rectify this situation has gone unheeded for over three decades.

Hodgen et al. (2010) note a complete absence of chapters on textbooks or curriculum materials in the *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992), and only one chapter by Stein et al. (2007) in the second edition of the *Second Handbook on Mathematics and Learning* (Lester, 2007). I also note that there is only one chapter on textbooks in the *International Handbook of Mathematics* Education (Bishop, Clements, Keitel, Kilpatrick, & Laborde, 1997) and none on the *Second International Handbook of Mathematics Education* (Bishop, Clements, Keitel, Kilpatrick, & Leung, 2003).

1.3.2 Research on Textbooks in General

Fan and colleagues (Fan et al., 2013) in their compilation of textbook research in mathematics from 1980 to 2012, show that research on textbooks divides into four distinguishable categories: *role of textbooks,* which is mostly philosophical and non-empirical; *textbook analysis and comparison*, concerned with the analysis of the content and comparison between textbooks; *textbook use,* which entails how textbooks are used by teachers and students, and how the use shapes the teaching and learning; and *other areas* involving aspects such as electronic textbooks and student achievement in relation to textbooks. Figure 1 shows the distribution of the research for three categories excluding the role of textbook and indicates that *textbook analysis and comparison* makes up some 63% of individual studies (textbook analysis at 34% and comparison at 29%); *textbook use* is at 25%; and *other areas* of research make up 12% of the total research.

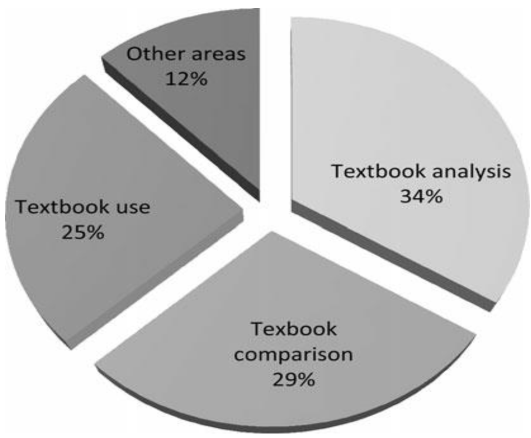

Fig 1.1 Distribution of empirical studies on mathematics textbooks surveyed in different focus areas (n = 100) (from (Fan et al., 2013, p. 635))

1.3.2.1 Roles of Textbooks

The section on the ubiquity of the textbook in the mathematics classrooms is self-explanatory with respect to *the role of the textbook* in the teaching and learning of mathematics. That the textbook is an “arbiter of the curriculum of a school system”(Talmage, 1972, p. 21); or that the role of new technology “pales into insignificance when compared with that of textbooks and other written materials” (Howson, 1995, p. 21); or that “ textbooks are commonly charged precisely with the role of translating policy into pedagogy”(Valverde et al., 2002, p. viii), is undisputed in the mathematics education community. Valverde et al. go further to portray textbooks as mediators between intended curriculum and the implemented curriculum, referring to them as ‘potentially implemented’ curriculum (p.13), as Figure 2 below shows:

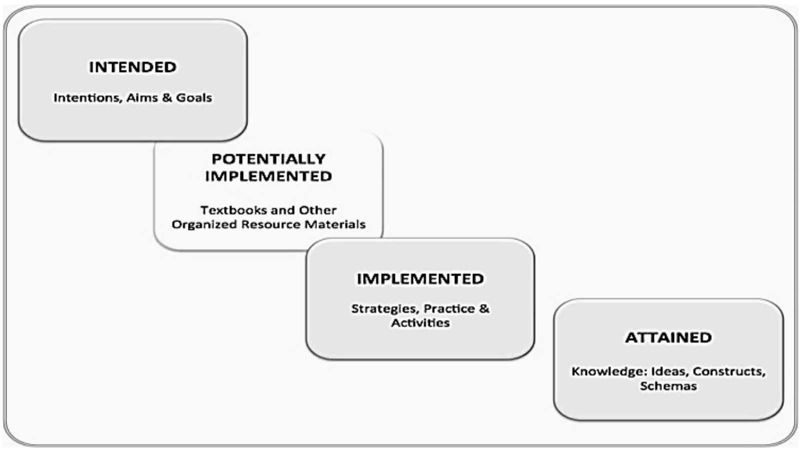

Fig 1.2 Textbooks as potentially implemented curriculum (Valverde et al., 2002, p. 13)

Almost all researchers who study textbooks allude to the key role in teaching and learning that textbooks play, and this extends to the inclusion of sociological issues such as gender and class (Apple, 1986; Dowling, 2002) and cultural values (Leung, Graf, & Lopez-Real, 2006), and their effects.

1.3.2.2 Content Analysis and Comparison

Here again, the 1995 TIMSS study provides the “largest cross-national study of textbooks carried out to date” which includes “a detailed, page-by-page inventory of the mathematics and science content, pedagogy, and other characteristics collected from hundreds of textbooks in over forty countries” (Valverde et al., 2002, back cover). There had been earlier studies on content analysis (for example, Dormolen, 1986; Flanders, 1987; Freeman et al., 1983), however, many of these studies would be recorded later after the TIMSS study (for example, Reys, 1996; Stylianides, 2007, 2009; Vincent & Stacey, 2008).

There are many more studies on cross-national textbook comparison. For example, the study of textbooks and teachers in England, France and Germany conducted by Pepin and Haggarty (Haggarty & Pepin, 2002; Pepin, 2011; Pepin & Haggarty, 2001); between France and Norway (Pepin, Gueudet, & Trouche, 2013a) Zhu and Fan (2006) looked at textbooks for Mainland China and the US; Zhongjun and team compared textbooks for China and Australia (Zhongjun, Tiong, & Bishop, 2006); Charalambous and team for Cyprus, Ireland and Taiwan

(Charalambous, Delaney, Hsu, & Mesa, 2010); and, Hong and Choi (2014) for Korea and the US; Breakell did an analysis of textbooks in England and Wales (Breakell, 2001); and Mayer and team compared problem solving in textbooks in Japan and the United States (Mayer, Sims, & Tajika, 1995), just to mention a few.

1.3.2.3 Textbook Use

As Fan et al. (2013) point out, research on textbook use accounts for only 25% of all research between 1980 and 2012. Attention here is on how teachers and learners use textbooks for teaching and learning. Freeman & Porter (1989) begin to dispute a popular notion then that teachers strictly teach to the textbook. Stodolsky (1989) supports Freeman and Porter when she concludes that in their study of four elementary teachers, they find no evidence that the teachers are strictly following the textbook in their teaching.

It is around this time that the *Standards (National Council of Teachers of Mathematics [NCTM], 1989)* were published in the United States and caused what Remillard and team call an "explosion of curriculum development projects" (Remillard et al., 2009, p. xv) which resulted in the production of "newly designed curriculum materials"(Remillard, 2005, p. 211) and a boost to research on how teachers used these *standards*-based curriculum materials, and whether and how these materials influenced classroom practices and teaching in general. Earlier studies on the *standards*-based curriculum materials focused on whether the materials worked and found these new curricula as effective as the old in promoting learners' procedural fluency but more effective with enhancing learners' conceptual understanding and problem-solving.

However, recent studies have shifted focus from "examining whether these curricula work to exploring *how* they might work" (Charalambous & Hill, 2012, p. 448). Studies by Tarr et al.(2006; 2008) for example, showed that instructional quality in the classroom did not improve just by inserting the *standards*-based curricula, but was dependent on how the curricula was enacted, thus reflecting a shift to a more relational view of textbook use. More detailed studies on curricula use and the influence teachers have on the effectiveness of curricular resources will be elaborated on in the next chapter, as this is the basis of the current study.

The present study falls within the studies on *textbook use* in which textbook use is viewed as relational. The study could have opted to focus on either one of the categories mentioned above, but chooses the category of *textbook use* because it seeks to understand the relationship between teachers and their textbooks; it seeks to understand how teachers use their textbooks in practice and why they do what they do.

1.3.3 Research on Textbooks in South Africa

Out of South Africa itself, I have found five studies on textbooks: The "President's Education Initiative Report (PEI)" (Taylor & Vinjevold, 1999), which discussed research on the availability and use of textbooks in primary schools in general and not particularly on mathematics textbooks. The study reports that the textbooks were available even though the quantities were not sufficient for all learners, but they were not used in systematic ways.

Moreover, teachers were reluctant to use them in their classrooms for various reasons including lack of content knowledge; poor quality and inappropriateness of the textbooks according to the teachers; and, some disparaging remarks about self-reflecting teachers not using “packaged-information”(1999, p. 177).

Adler and team (Adler, Dickson, Mofolo, & Sethole, 2001) had some issues with the PEI report and felt that the PEI tended to overgeneralise on some aspects pertaining to the use of textbooks in the schools. They hence engaged in a focussed study on the availability and use of textbooks and other written materials in Grades 7 and 9, and concluded through classroom observations, teacher and learner interviews, and other instruments, that contrary to the findings of the PEI report, textbooks were positively valued as classroom resources by both teachers and learners, and extensively used in teachers’ classrooms.

Following on the study above, is one by Ensor and team (Ensor et al., 2002) which explored the impact of a textbook *Maths for all,* in primary schools. Using a sociological perspective, the study went further than Adler’s by characterising textbook use with respect to both its mathematics content and pedagogic approach. They undertook a content analysis of some aspects of the textbook together with an analysis of how teachers were using the textbook in their classrooms, including the impact of the textbook on learner achievement. Some key findings from this study include the following:

i) The textbook worked largely *inductively*, that is, “introduces a topic by engaging students in a range of activities that can be regarded as instances of the concepts which students are to master” (Ensor et al., 2002, p. 23), which then leads to definitions and then practice exercises.
ii) The textbook projected an “ideal classroom” (p.26) in how it structured and linked mathematical tasks together for purposes of instruction, and through the guidance it provided to teachers through the teachers’ guides.
iii) However, researchers found that most teachers preferred a “*deductive*” (p.23) style of teaching, as opposed to the textbook’s inductive style. In this style teachers focussed primarily on teaching procedures, giving examples and then practice exercises to learners, thus subjecting the textbook to a resource that a teacher could ‘act selectively upon’ (p.28).
iv) With respect to the impact of the textbook on learners’ performance, researchers could not make hard claims as the achievement of learners with a textbook was also very low.

Following on Fan et al.’s categorisation of international research, these three studies were all concerned with use, albeit in different ways. Adler’s and Ensor’s studies both begin opening windows into what teachers do with the textbook in the classroom. Ensor et al. include analysis of the content and approach in the textbook, with more emphasis on the pedagogical approach. Interestingly, there are no apparent studies of teachers’ use of textbooks in South Africa following on from these two studies.

Two studies which come some ten years later, look at different aspects of the textbook. The next study is by Fleisch and team (Fleisch, Taylor, Herholdt, & Sapire, 2011) and compares the learning gains of primary school learners using a conventional textbook against those using a customised workbook, thus backgrounding the content and approaches in these resources, focusing instead on their learning effects. The randomised control trial study (McMillan & Schumacher, 2010) was undertaken in response to a new policy on learning resources which saw the Department of Basic Education (DBE) beginning to supply learners in Grades 1 to 9 in public schools with workbooks in numeracy/mathematics, and other selected subjects, in 2011. They found that the customised workbook showed no advantage over a conventional textbook, and yet was more expensive than the conventional textbook, thus questioning this move that seems more costly and yet not better than what was in existence already.

The last study is a doctoral project by Bowie (2013) and is a comparison of how geometry is constituted in the national curriculum statement and in two popular textbook series. She shows through analysis of the curriculum in the first instance that there are inherent tensions in curriculum prescriptions, and then traces how interpretation across two different textbooks impacts on the constitution of geometry in each. A particular tension is around inductive pedagogies, and the deductive nature of much of mathematics – and so confusing messages being conveyed. She also shows how incorporating 'socially relevant contexts' into the school curriculum which do not fit easily with the mathematical ideas, creates tensions in the geometry produced in the textbooks; "the artefacts that teachers would need to work with" (p.3).

Five research studies on textbooks from South Africa is a relatively small number considering how importantly the textbook is regarded in South Africa as a resource in teaching and learning. All the researchers mentioned in this section confirm the high regard for textbooks in their studies: The PEI report states that

> all indications are that the availability of sufficient textbooks and stationery is one of the most important factors in improving learning. It is imperative therefore, that provincial deficits be eliminated and the balance between salary and the other costs be restored so as to free up funds for the provision of books and stationery (Taylor & Vinjevold, 1999, p. 184)

The authors further recommend that teachers should be assisted in using the textbooks or a systematic learning programme through teacher development programmes. Thus, it is not only about the availability of the textbook, but also about teachers knowing what to do with them.

The realisation of the PEI report's wish to free up funds for provision of books, happened in 2011 with the allocation of a large budget for the supply of customised workbooks for learners in all eleven languages due to "the Department of Basic Education (DBE), recognising the centrality of learning support materials in the learning process"(Fleisch et al., 2011, p. 488) and also that "[G]iven the scale and cost of this intervention, the provision of workbooks is clearly central to the South African government's strategy to improve learning outcomes"(ibid).

Adler and team write that "[i]n Grade 9 where textbooks are not available, learners are actively denied access to mathematics"(Adler et al., 2001, p. 52); Bowie (2013) as already mentioned, considers textbooks as key artefacts for the teacher and writes that "one of the key questions for me became how this mathematics came to be constituted in two key artefacts that teachers would need to work with: the curriculum and textbooks" (2013, p. 3). A similar sentiment is echoed by the team led by Ensor about the assumptions of their study:

> textbooks potentially impact on learning and therefore learner performance in two ways: indirectly, by influencing teachers and their teaching styles and thereby affecting learning outcomes, and directly, by influencing learning through giving learners access to textbooks (Ensor et al., 2002, p. 22)

What is also seen across these studies is how the particular problem in focus foregrounds and backgrounds various elements of the textbook. With focus on use and then learning gains in the first four studies discussed, analysis of the actual content and its affordances and constraints is backgrounded. Bowie (2013), in contrast, brings this latter into focus in her study, but then backgrounds use. As I argue in Chapter 2, textbook use is relational, requiring a study to bring both the content of the textbook, **and** teachers' use of this resource into view. Hence my study, underpinned as it is by a relational view, and building from these studies, particularly Ensor et al. (2002), is a welcome addition and extension to this small and yet critical field of research on textbooks in South Africa.

1.4 Research Questions and Outline of Chapters

As I have indicated in my opening tale of two teachers using their textbooks in their classrooms, my objective is to understand better the relationship between the teacher and her textbook. From the narration of the status of research on textbooks, there is a dearth of research on textbook use (Love & Pimm, 1996). While over the past decade or so the situation has changed and this part of textbook research has "grown tremendously"(Lloyd, Remillard, & Herbel-Eisenmann, 2009), there is still a long way to go in order to understand the teacher-textbook relationship. Remillard (2005) points out that this body of research "rests on underdeveloped theoretical ground"(p.212), and lacks a much needed theoretical and conceptual base (Remillard, 2009). This is evidenced by the fact that, the compilation of research studies on teachers' use of curriculum materials by Remillard et al. (2009), marks the first book of its kind since 1990. It is hence my wish to make a contribution to the growth of the research on teacher-textbook relationships.

1.4.1 Research Questions

The objective of the study is to investigate the relationship between the affordances (J. J. Gibson, 1977) of a textbook, that is, those features which afford teachers' practice; teachers' pedagogical design capacity (PDC), defined by Brown (2002, 2009) as the teachers' capacity to perceive and mobilise existing resources in order to craft classroom episodes; and their mediation of the object of learning (Marton, Runesson, & Tsui, 2004) referring to the 'that' which learners have to learn, in the mathematics classroom.

In order to achieve this, the main question is broken down into four critical questions as follows:

a) What are the affordances of the prescribed textbook to the teachers' practice?

b) How do teachers mobilise the affordances of the prescribed textbook?

c) To what extent are teachers aware of the affordances and constraints of the textbook in their practice?

d) What is the relationship between the affordances of the prescribed textbook and teachers' pedagogical design capacity ? And how might this be explained?

1.4.2 Outline of Chapters

The study is divided into nine (9) chapters as outlined and described below:

1.4.2.1 Chapter 1 – Introduction to Study

Chapter 1 introduces the study and its rationale, touching on the importance of the textbook to the mathematics teacher's practice, and the high regard for textbooks in South Africa as useful resources for the promotion of quality education. The chapter highlights the status of research on textbooks in general on the international arena, before zooming in on available textbook research in South Africa, and the implications for strengthening the teacher-textbook relationships.

1.4.2.2 Chapter 2 – Literature Review

Chapter 2 outlines available research on the teacher-textbook relationship and what this research helps us to understand about this relationship. I expand on key constructs in the study:

a) conceptions of 'use'
b) conceptualisations of teaching as a design process
c) conceptions of curricular materials
d) affordances
e) Pedagogical Design Capacity (PDC)

Research on the different constructs mentioned above informs and shapes the understanding of the teacher-textbook relationship that this study seeks.

1.4.2.3 Chapter 3 – The Theoretical Grounding

In chapter 3, the teacher-textbook interaction is theorised: firstly as grounded in Vygotsky's (1978) socio-cultural theory, wherein all humans are inherently social beings and grow from and through the use of tools. Secondly, the teacher-textbook interaction is characterised as a dynamic interrelationship in which the teacher and the textbook, each shapes the other and together they shape instructional outcomes (Stein & Kim, 2009), through their respective features or resources.

Hence in this chapter, conceptual tools for analysing the prescribed textbook for its affordances; for determining teacher features for mobilising the textbook affordances; and, for characterising teachers' PDC, are developed.

1.4.2.4 Chapter 4 – Research Design and Methodology

Chapter 4 narrates the design of the study to include issues pertaining to sampling of participants; data collection instruments; ethical considerations; and, the actual data collection process. Data collection for the study began in the wake of a new curriculum in South Africa and subsequently new textbooks to be effected at Grade 10 in the following year. During this period there were uncertainties about whether current textbooks would still be part of the recommended prescribed textbooks for schools for the following year, and these had to be filtered into the plan for data collection and subsequent analysis of the textbook.

1.4.2.5 Chapter 5 – Determining the Affordances of the Prescribed Textbook

In order to determine how teachers used their textbooks in practice, the analysis of the textbook itself had to be undertaken to determine what it could afford teacher's practice. Thus, this chapter constitutes the analysis of the affordances of the textbook to teacher's practice

1.4.2.6 Chapter 6 – Teachers' Mobilisation of the Textbook: The Case of Teacher A1

In this chapter, the three lessons for teacher A1 are used to exemplify the process of analysis of the lessons. The lessons are analysed for how teacher appropriates the affordances of the textbook determined in the analysis of the textbook in chapter 5, thereby helping with developing analytical tools for investigating how teachers mobilise the textbook.

1.4.2.7 Chapter 7 – Teachers' Pedagogical Design Capacity

Using the analytical tools developed in chapter 6, in chapter 7 the rest of the seventeen lessons for the remaining six teachers are analysed for how teachers mobilise the textbook. Conclusions about teachers' PDC, and therefore about the relationship between the teacher and the textbook are drawn.

1.4.2.8 Chapter 8 – Teachers' Awareness of Textbook Affordances

Chapter 8 explores teachers' views about textbook use in general and specifically for each teacher, thus helping with the triangulation of results from chapter 7 as well as informing the study about how aware of the affordances of the prescribed textbook to teachers' practice the teachers are. From the results of chapter 7 and the present chapter, the study draws conclusions about teachers' capacity to firstly perceive the affordances, and then, to mobilise these affordances: the teachers' capacity for pedagogical designs (PDC).

1.4.2.9 Chapter 9 – Conclusions and Recommendations

I use chapter 9 to reflect on the PhD journey as a whole; discussing both the findings of the study and their implications; and also the study's limitations and their implications. Finally, I make recommendations for further research and to different stakeholders on the use of textbooks for the teaching and learning of mathematics.

1.5 Conclusion

In this first chapter of the study, I have shown how key textbooks are to the teaching and learning of mathematics, and how highly regarded they are internationally and in South Africa in particular. I have also shown that while there is an impressive growth in research on textbooks, there is very little research on textbooks coming out of South Africa itself; and most importantly, there is still a long way to go for research that involves the teacher-textbook relationship generally, in order for us to understand this relationship and how it can be strengthened. It is my wish that this study should make both theoretical and empirical contributions to the field of research on the teacher-textbook relationship; and provide part of the solutions being sought for the development of quality education in mathematics in South Africa.

CHAPTER 2
Literature Review

> Just as modern music has come to rely on sheet music as a representational medium for conveying musical concepts, forms, and practices…, classroom instruction has come to rely on curriculum materials as tools to convey and reproduce curricular concepts, forms, and practices. Musicians interpret musical notations in order to bring the intended song to life; similarly, teachers interpret the various words and representations in curriculum materials to enact curriculum. In both cases, no two renditions of practice are exactly alike (Brown, 2009, p. 17)

2.1 Introduction

In chapter 1, I showed that this study falls under the research category of 'textbook use' which investigates how teachers use their textbooks and how the use shapes teaching. As previously mentioned, most of the studies used the "standards-based" curriculum materials which may not be the same as the textbook in their construction. However both the curriculum materials and the textbook serve the same purpose of being used by teachers in the teaching of mathematics in their classrooms. Moreover, while it is appreciated that learners also use textbooks and can use the textbook without the teacher, and that the end-result of the teacher-text interactions is the enactment in the classroom, learners are out of view in this study and therefore reference is made to the opportunities for mediation in the classroom and not to the actual mediation of the object of learning.

The studies in the curriculum/textbook use category frame use as relational, that is, teachers and textbooks when they interact form a relationship which Remillard (2005) set out to investigate in her synthesis of over 25 years of research on curriculum use in mathematics. For consistency, the relationship between teacher and curricular materials shall be referred to as the teacher-text relationship throughout the study, and shall include textbooks and all types of curriculum materials[6].

In this chapter, the factors influencing the use of curricular materials are reviewed as a prelude to the investigation of the teacher-text relationship in the study. The chapter begins with conceptualisations of use from which "the constructions of curriculum materials, teachers, and the teaching–curriculum relationship implicit in these conceptualizations, as well as their underlying theoretical influences" (Remillard, 2005, p. 213) can be identified. Remillard (ibid) posits that while the conceptualisations of use "overlapped to some extent and were not always mutually exclusive, each could be associated with a particular theoretical or epistemological perspective on human activity, material use, or meaning making" (p. 215). The four conceptualisations involve 'use' as *following or subverting* the text; 'use' as *drawing on* the text; 'use' as *interpretation of* the text; and finally, 'use' as *participation with* the text. I discuss each one in turn.

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[6] Remillard uses 'curriculum materials', 'curriculum', and 'textbooks' to refer "to printed, often published resources designed for use by teachers and students during instruction"(Remillard, 2005, p. 213). We adopt the same definition throughout the study for consistency.

2.2 'Use' and its Conceptualisation

Curriculum 'use' is defined as "how individual teachers interact with, draw on, refer to, and are influenced by material resources designed to guide instruction" (Remillard, 2005, p. 212); and is differentiated from 'implement' because implementing diminishes "the importance of considering the activity of the teacher and the influence of the classroom in this process" (Lloyd et al., 2009, p. 7). The four major conceptions of curriculum 'use' under which most but not all research on textbook use can be placed according to Remillard are: 'use' as *following or subverting* the text; 'use' as *drawing on* the text; 'use' as *interpretation of* the text; and finally, 'use' as *participation with* the text (Remillard, 2005). A summary of each conceptualisation of 'use' is provided below.

2.2.1 'Use' as following or subverting the text

Studies in this category place emphasis on fidelity to the text, regarding the text as the primary tool for structuring learning opportunities for learners, and viewing *teachers as enactors of planned curriculum* (for example, Chval, Chavez, Reys, & Tarr, 2009; Freeman & Porter, 1989; Stodolsky, 1989). As McClain et al. point out, "a fidelity approach to implementation gives authority for both the mathematics that is to be taught and the sequencing and presentation of that content to the text, and places strict adherence to it as the goal of teaching" (McClain, Zhao, Visnovska, & Bowen, 2009, p. 56).

However, while most of the studies are concerned with the extent of use of the text by teachers, thus relegating teacher decision-making to just following scripted procedures (McClain et al., 2009), Chval et al. (2009) offer a different focus on fidelity which they call *textbook integrity*, to measure the extent to which the teacher draws from the textbook as a 'primary guide' for planning and enacting their lessons. This focus emphasises the "process of using, rather than following, a curriculum resource" (Remillard, 2009).

In this stance, teachers are viewed as mere conduits of curricular materials and the teacher-text relationship is one that gives agency to the textbook.

2.2.2 'Use' as drawing on the text

The emphasis here is on the agency (McClain et al., 2009) of the teacher, where the teacher is viewed as having authority over the sequencing and presentation of content; and texts are viewed as just one of the many resources that teachers use in constructing the enacted curriculum. However, the textbook does not shape teacher's activity (for example, Freeman & Porter, 1989; Sosniak & Stodolsky, 1993), and so, distinct from the process of using the textbook as a 'primary guide'.

In this stance, while teacher is viewed as a designer of curriculum and not just a user, fidelity to the text is still possible.

2.2.3 'Use' as interpretation of text

In this stance the teacher is framed as an interpreter of the written curriculum such that fidelity then cannot be considered an option, as teachers' personal beliefs and convictions guide their experience of the curriculum. Here teachers are viewed as creating their own meaning and therefore interpreting the intentions of the authors. Examples of studies here include Ben-Peretz (1990) and Chavez (2003). This perspective is different from the two already outlined as here the relationships forged between teachers and curricular materials, together with their effect in the classroom, are explored.

2.2.4 'Use' as participation with the text

This view seems a "less common perspective taken by researchers studying teachers and curriculum materials" (Remillard, 2005, p. 221). Researchers here seek to understand how teachers interact and use their curriculum materials, and regard use as a collaboration between the materials and teachers. While in the 'use as interpretation' stance above the teacher is regarded as a sole meaning maker, in this stance the teacher and the textbook become collaborators in the design of the enacted curriculum. The relationship between teacher and textbook is 'participatory', meaning that teacher and curriculum materials are regarded as active participants and each bring own resources in the interactions, engaging in a "dynamic interrelationship that involves participation on the parts of both the teacher and the text" (Remillard, 2005, p. 221); and as Stein and Kim (2009) assert, in this dynamic interrelationship "each participant (teacher and text) shapes the other; together they shape instruction" (p. 39).

Understanding the teacher-text relationship entails analyses of individual teacher's features, features of the curriculum, and the interactions that occur between them. Furthermore, curricular materials in this stance are viewed as artefacts or tools (Vygotsky, 1978), and products of socio-cultural evolution (Wertsch, 1998). Remillard asserts that it is the studies which view 'use' as participation with text and which examine how teachers actively collaborate with curricular resources, that bring the teacher-text relationship to the forefront.

The present study hence aligns itself with this view of textbook use as 'participation with the text'. It is the view of this study that the interaction between the teacher and her textbook involves and should involve a 'conversation' or collaboration between these two parties (teacher and textbook), as the two are influential in how the curriculum is eventually enacted. Chapter 1 of this study showed that the studies on textbook use in South Africa are not underpinned by a relational view of textbook use. It is the goal of this study to bring to the forefront, both the features of the textbook and how teachers interact with them.

A relational view, of course then requires consideration of what each of the texts and the teachers bring to the interaction. In the next section, the study explores the resources which the textbook brings into the teacher-text interactions.

2.3 What the Textbook brings to the Teacher-Text Interactions

2.3.1 The Notion of Affordances

The notion of *affordances* was coined by conceptual psychologist James Gibson (J. J. Gibson, 1977), to describe how the environment affords the animals living in it: “The *affordances* of the environment are what it *offers* the animal, what it *provides* or *furnishes*, either for good or ill” (p. 127). An affordance, this seems to imply, is a resource that the environment offers any animal that has the capabilities to perceive and use it. As such, affordances are meaningful to animals: they provide opportunity for particular kinds of behaviour. Thus, affordances are properties of the environment but taken relative to an animal (Chemero, 2003, p. 1).

The Gibsonian notion of affordance has found its way into different fields which have transformed it and given it different interpretations and definitions according to the needs of each individual field. Chemero (2003) points to three different theories of affordances as: one that views affordances as relations between the environment and animals (for example, Chemero, 2003; Heft, 1989, 2001; Stoffregen, 2003; Turvey, 1992). However, one popular definition of affordance is one used in the field of Human-Computer Interaction (HCI) which was introduced by Norman (1988) in the context of design in his book “The Psychology of Everyday Things (POET)”. In this book Norman defines an affordance as

> the perceived and actual properties of the thing, primarily those fundamental properties that determine just how the thing could possibly be used. […] Affordances provide strong clues to the operations of things. Plates are for pushing. Knobs are for turning. Slots are for inserting things into. Balls are for throwing or bouncing. When affordances are taken advantage of, the user knows what to do just by looking: no picture, label, or instruction needed (Norman, 1988, p. 9)

While the field would be grateful to Norman for the introduction of the term, this came with confusion and misunderstanding, as Torenvliet (2003) posits:

> It sprang up everywhere, moving from dusty journals to magazines and newspapers, and from there to the conceptual prime time of your local Starbucks. Somewhere on the way from academia to Starbucks, however, something happened. The meaning of affordance became distorted and confused. At first it was subtle, but by now its meaning has bifurcated wildly (Torenvliet, 2003, pp. 12-13)

Norman accepted the blame for the confusion but also clarified that

> the concept (of affordance) has caught on, but not always with complete understanding. My fault: I was really talking about perceived affordances, which are not at all the same as real ones. POET was about “perceived affordance”…. The designer cares more about what actions the user perceives to be possible than what is true. (Norman, 1999, p. 39)

However, the definition had ‘gone viral’ as they say in social media, resulting in the Normanian definition being the most popular. For the present study, it is important to note the difference between the Gibsonian and Normanian definitions according to McGrenere and Ho (2000), who point out that the Gibsonian affordance points to “an action possibility available in the environment to an individual, independent of the individual’s ability to perceive this possibility”(p. 1); while for Norman an affordance is both an action possibility together with the way the action possibility is made visible to the actor (Norman, 1988).

In other words, for Norman, affordances need to be perceived before they are of any use to the user. Thus, Norman focusses more on the usability than the usefulness (McGrenere & Ho, 2000) of an affordance. Moreover, Norman referring to Gibson’s definition of an affordance mentions that “…the physical objects conveyed important information about how people could interact with them, a property he named “affordance”.(Norman, 1988, p. 28).

For the present study, making the design rationale of the author(s) of the textbook visible to the teacher is important in perceiving the affordances of the textbook (Ball & Cohen, 1996; Davis & Krajcik, 2005; Stein & Kim, 2009). I certainly agree with Norman that unperceived affordances of the textbook are useless for the teacher: for the teacher to be able to utilise the affordances, they have to be able to firstly perceive them.

A further important aspect that Norman(1999) incorporates is the notion of cultural *constraints* or *conventions*, which he terms as a powerful design tool. As he stipulates “….cultural constraints are conventions shared by a cultural group…A convention is a constraint in that it prohibits some activities and encourages others” (Norman, 1999, p. 40). Thus, it is not enough to talk about affordances alone because this only provides a partial picture that “overlooks a countervailing, though equally inherent, characteristic of mediational means – namely, they can constrain or limit the forms of action we undertake”(Wertsch, 1998, p. 39). Wertsch describes authors who approach mediation from how it empowers users as working from a “half-full” perspective; while the “half-empty” perspective focuses on the constraints that mediation imposes, meaning that constraints should not be considered only as hindrances, but “can be interpreted in terms of how they define the nature of the task and how they provide clear boundaries that define activity”(Brown, 2009, p. 20).

In mathematics education research, the concept of *affordances* and *constraints* has been used in studies which explore the relationship between teachers and curricular resources that they use (Brown, 2002; Gueudet, Pepin, & Trouche, 2012; Stein & Kim, 2009). However, in many instances, the term *affordances* is used loosely, and perhaps as Torenvleit (2003) asserts, has lost its meaning, becoming a common sense notion rather than a technical term systematically employed in the field. As example, and for the fact that this particular book has been published quite recently, my search for the use of *affordances* and/or *constraints* in Gueudet et al.’s book on mathematics curriculum resources and their use by teachers (Gueudet et al., 2012), came up with ten (10) uses of *affordances,* about five (5) uses of *affordances and constraints,* and some over twenty five (25) uses of *constraints.* While the authors of the book define the affordances of a resource as “the attributes and characteristics of the resource which provide potential for its use with peers/colleagues and students/pupils in the course of teachers’ work” (Gueudet et al., 2012, p. 354), the other authors do not define what they mean by affordances. This is the same with constraints: the term is used to refer to hindrances in a colloquial way, and thus opens to multiple interpretations, for example, time constraints.

My point here is that *an affordance* of a curricular resource is an important construct in the teacher-text relationship, with theoretical origins from Gibson (1977), and therefore needs to be recognised as such in order to address part of the theoretical grounding which Remillard (2005, 2009) bemoans a lack of for this particular field. In this study, the affordances of the textbook are used as a construct as intended by Gibson and expanded on by Norman. They shall be used to define both the existing affordances that the textbook can afford the teacher's practice, *and* what teachers actually perceive as affordances of the textbook in their practice. Potential constraints as in both boundaries that the resource maintains on teachers' practice shall also be investigated; and as potential hindrances to the teachers' practice as well. The next section explores the affordances and constraints of curriculum resources to the teacher's practice.

2.3.2 The Affordances of Curricular Resources

Remillard (2012) claims that curriculum materials position teachers in particular ways as readers of the materials through their *modes of address*. These modes are reflected and reinforced through their *forms of address*, that is,

> the physical, visual, and substantive forms it takes up, the nature and presentation of its contents, the means through which it addresses teachers. The form of address is what teachers actually see, examine, and interact with when using a curriculum resource (p.108)

She identifies five such forms of address, as the *structure*, the *look*, the *voice*, the *medium*, and the *genre*. The *structure* refers to what is contained and how this is packaged in the resource, and is undoubtedly a major affordance of any curriculum resource for the teacher's practice. Due to the centrality of the mathematics content to the teacher's practice, the *structure* is reviewed in more detail in the next section. The *look* is about the visual aspects of the resource; for example, whether the paper used in the production is glossy or whether the photographs are black and white and so forth.

The *voice* of the resource addresses how the resource communicates to the teacher, and what it communicates about (Herbel-Eisenmann, 2007; Love & Pimm, 1996; Remillard, 2012; Remillard et al., 2011); whether it is talking *through* (Remillard, 2000) the teacher when for example, the resource gives guidance to the teacher through teacher guides; or talking *to* the teacher, when communicating to teachers about central ideas in the text. Kim & Remillard (2011) also identify the voice of the text as key to how teachers are positioned in relation to the materials. Some of the ways that resources can talk to the teachers offered by Davis and Krajcik (2005) include: to

> a) make the developers' design decisions visible, b) help the teacher anticipate and interpret what learners may think or do, c) support teachers in learning more about the content, or d) support the teacher in making decisions while enacting the curriculum (see Remillard et al., 2011, p. 3)

The *medium* is whether the resource is print or/and digital: and, the *genre* pertains to the type of material in the larger classifications of materials, for example, whether it is a textbook in a traditional sense, or a curriculum material. Remillard (2012) claims that the genre of the curriculum resource triggers a 'zone of expectation'(Ongstad, 2006) from the teacher.

The notion of the forms of address is cognisant with Otte's (1986) view of mathematical textbooks which he suggests should be regarded in two ways as "objectively given structure of information" and as a "subjective scheme"

> something additional happens upon reading and interpreting texts, especially in mathematical textbooks. Reading is not automatized like breathing, walking, or seeing. Hence, the problem of the interaction between text as a subjective scheme and text as an objectively given structure of information stands as a permanent problem not to be solved once and for all (Otte, 1986, p. 175)

The objectively given structure of information refers to "what can be seen when looking at such materials" (Love & Pimm, 1996, p. 379), while the subjective scheme pertains to how the materials are perceived and understood. From the five forms of address above, the *structure,* the *look*, the *voice* and the *medium* speak to the textbook as an objectively given structure; it is what teachers see when they interact with the textbook. The fifth form of address, the *genre*, on the other hand, is more about what teachers expect of a curricular resource, that is, their perceptions, than what they see. Of the five modes of address, the study chooses to focus only on the *structure* of the textbook, mainly due to its centrality to the mathematical content and how it is organised.

2.3.2.1 The Structure of Curriculum Resources

There are variations in the literature in how the structure of a textbook is constituted. For example, Remillard's (2012) composition of the structure of a curriculum resource is a description of "the nature and organization of the content of the curriculum, the particular mathematical concepts and goals, and the underlying pedagogical assumptions" (p.110); and its analysis is usually about

> how the various components are organized, the mathematics content included or excluded through the representations, and the valence or emphasis of the content, including how the content is represented (Remillard, 2012, p. 111)

For Valverde et al. (2002), the structure and form are characterised by "how they incorporate content, performance expectations, and presentation formats into pedagogical structures" (p. 54). Performance expectations include actions expected of learners such as 'representing', or performing routine procedures, while presentation formats include how the mathematics is conveyed, for example, through narratives, or exercise sets, or worked examples.

These authors also distinguish between two structural levels in textbooks as macrostructures and microstructures:

> There are differences in sequence and complexity specific to particular parts of textbooks designed for a small number of class periods. There are also structural features that cut across the entire book. These more pervasive features represent an important aspect of textbooks that seems likely to influence the learning opportunities the textbooks are intended to promote throughout an entire school year. We will term these more pervasive

> features 'macro' structures. This is in contrast to the structures associated with specific lessons intended for use in a small number of classroom instructional sessions that we term 'micro' structures. (p.21)

They also point out that it is from the macro structures that the vision of mathematics for a particular textbook is built, while the micro structures embody pedagogical intentions for a single lesson.

Brown (2009) 'sees' the content and structure in three facets of a curriculum resource as, "(a) physical objects and representations of physical objects, (b) representations of tasks (procedures), and (c) representations of concepts (domain representations)" (2009, p. 26); using 'representations' to include physical objects which may be recommended for use but not supplied with the resource; or recommendations for structuring a lesson, and particular sequencing of concepts. For Pepin and Haggarty (2001), textbooks influence the questions, issues and topics covered and discussed in classrooms, and therefore they need to be analysed in terms of their content and structure, not leaving out their use in classrooms by teachers and learners. They point out four main areas according to which textbooks have been analysed in terms of their content and structure as "the mathematical intentions of textbooks; pedagogical intentions of textbooks; sociological contexts of textbooks; and the cultural traditions represented in textbooks." (p.160). The mathematical intentions include the mathematics represented in textbooks, the implicit beliefs about the nature of mathematics, as well as the presentation of mathematical knowledge.

Other facets of structure include the *pedagogical orientation* of the textbook to refer to "pedagogical practices consistent with those suggested in the teacher's guides" (Chval et al., 2009, p. 76); *instructional tasks* for learners which Stein & Kim (2009) show that they can be of high cognitive demand which requires greater capacity from the teacher as they are more intellectually and conceptually challenging, than low-demand tasks. Kim & Remillard (2011) on the other hand summarise the mathematical and pedagogical demands that a curriculum places on the teacher as:

> a) the mathematical emphasis, or focus; b) the cognitive demand of the central tasks of the lesson, c) the primary instructional representations used to communicate mathematical ideas, and d) the instructional approach, which includes the teacher's and students' roles throughout the lesson, the role of the text, and expectations of how learning takes place (p.3)

These dimensions show that the structure of a curriculum is highly influential in how the teacher interacts with the curriculum, and even determines the kind of role the teacher plays. For example, Kim & Remillard (2011) find that the teacher's role in programs consisting mainly of high demand tasks, is more of a facilitator or an orchestrator, than one of telling, showing, directing or guiding learners.

Ensor et al. (2002) bring in a different angle to the structure of the textbook that specifically comments about the approach of the textbook. While they concur with other authors that "any school mathematics textbook constitutes a particular prioritising of mathematics content and the ways in which this content should be sequenced, exemplified and taught" (p.22), they stipulate that

> Two dominant pedagogic approaches are discernible in mathematics textbooks and in mathematics classrooms today that are of interest to us here. The first approach is *deductive*. Here, the teacher (or textbook) initially states appropriate definitions or concepts, which are then exemplified and followed by exercises for students to practice. …The second pedagogic approach by contrast is largely *inductive*. Rather than starting with definitions, the teacher (or textbook) introduces a topic by engaging students in a range of activities that can be regarded as instances of the concepts which students are to master. Activities lead to definitions and from this point opportunities may be provided for students to practice. Framing relations are relatively weak. (Ensor et al., 2002, pp. 22-23)

Ensor et al. thus distinguish between two pedagogic approaches in a mathematics textbook; one in which concepts are initially stated to learners and then exemplified; and one in which learners initially engage with concepts.

This section on the structure of the textbook shows that the analysis of a textbook entails analysing its structure; the structure illuminates the *mathematical content*, as well as the *pedagogical approach* to the teaching and learning of the content privileged in the textbook. For this study I conclude that the textbook affords the teachers' practice with the mathematics content as well as the approach to the teaching and learning of this content. Hence, in the analysis of the textbook in forthcoming chapter 5, these affordances are illumined through the delineation of the structure of the textbook.

2.4 What the Teacher brings to the Teacher-Text Interactions

Studies in curriculum use have shown that teachers bring different features to the teacher-text interactions. For example, research shows that teachers' subject matter knowledge, beliefs and goals, and their stance towards curricular materials (Brown, 2009; Kim, 2007; Remillard & Bryans, 2004; Schnepp, 2009; Spillane, 1999) shape the enacted curriculum in the classroom. Remillard and Bryans (2004) introduce the notion of teachers' *orientation towards the curriculum* which they describe as "a set of perspectives and dispositions about mathematics, teaching, learning, and curriculum that together influence how a teacher engages and interacts with a particular set of curriculum materials" (p. 364). The orientation towards the curriculum plays a role in shaping the curriculum that is enacted in the classroom as well.

McClain et al. (2009) include teachers' instructional reality, agency, and teachers' professional status in the features that influence teachers in the teacher-text interactions. Other features include teachers' human and social capital (Stein & Kim, 2009) which focus on the social relations and networks that influence teachers' interactions with curricular materials. They also look at ways in which institutional context play a part in the teacher-text relationship.

However, a critical question being posed by this study is how teachers draw from curricular materials to design classroom episodes. This is discussed in the next section.

2.4.1 How Teachers Draw from Curricular Resources for Classroom Enactment

Remillard (1999) develops *arenas of curriculum development* in Fig 2.1 below which show the activities teachers engage with as they interact with curricular materials. There are three arenas: the *design arena* in which teachers select and design tasks; the *construction arena* involving enactment of the tasks in the classroom as well as responding to learners; and the *mapping arena*, where choices about the organisation and the content that go into the classroom are made. Remillard shows that in these arenas, teachers' decisions about what tasks to select for the classroom, are influenced by "their ideas about mathematics, students, and their learning, as well as by the teaching context and available resources" (p. 323).

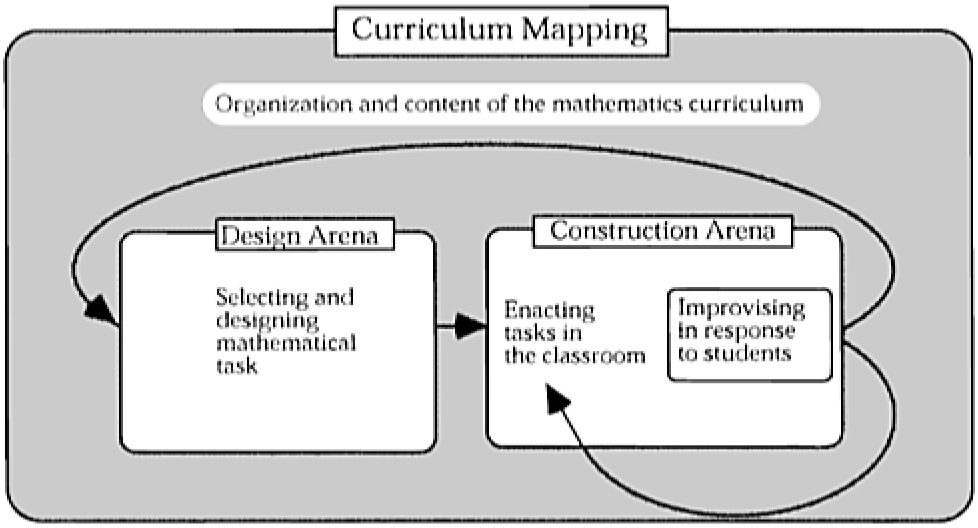


Fig 2.1 Remillard's Arenas of Curriculum Development (taken from (Remillard, 1999, p. 322)

Sherin and Drake (2009) develop the *curriculum strategy framework* which consists of three activities of *reading, evaluating* and *adapting*, and three time periods of *before, during* and *after instruction.* In this framework, the authors show that teachers have general approaches to adapting the curriculum which lie on a continuum as shown below

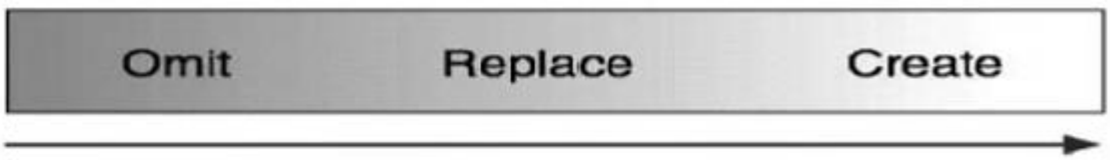


Fig 2.2 The curriculum adaptation spectrum (taken from (Sherin & Drake, 2009, p. 487)

According to Sherin and Drake (ibid), teachers either *omit* components of a lesson, or *replace* one component with another, or completely *create* new components during the adaptation process.

Brown (2009) argues that teachers' interactions with curricular materials lies on a continuum as well, with different "degrees of artefact appropriation" (p. 24). He argues that teachers offload a large degree of agency for guiding instruction to curriculum materials at one extreme, while on the other improvising strategies of their own with minimal reliance on the materials, thus shifting the agency to a larger part to the teacher. Teachers adapt when they share equally the agency for instruction with curriculum materials. This is where the notion of teachers' *pedagogical design capacity* (PDC) is derived. This notion is discussed in the next section.

The study is interested in how teachers use their textbook in ways that lead to productive mediation of the object of learning in the classroom. Specifically, the study seeks to explore the notion of teachers' PDC and how it relates to the affordances of the textbook.

2.4.2 Teachers' Pedagogical Design Capacity (PDC)

The notion of a teacher's pedagogical design capacity (PDC), that is, a teacher's "capacity to perceive and mobilise existing resources in order to craft instructional episodes" (Brown, 2002, 2009; Brown & Edelson, 2003), originates from Brown's assertion of teachers' use of curriculum resources as a process of design, arguing that in order to understand how teachers use curriculum materials to craft instruction, we must be

> explicit about the representations curriculum materials use to communicate concepts and actions…attentive to the ways in which teachers perceive and interpret these representations, and understanding how these representations can constrain and afford teacher practice (2009, p. 18)

Thus, the processes of perception and mobilization are highly influenced by the available personal and external resources. To be able to perceive classroom needs and opportunities, a teacher would need to draw from their knowledge and to be able to generate and apply appropriate responses to the curriculum is dependent on available personal and external resources. Brown describes a teacher's PDC as a particular design skill that a teacher enacts to put various pieces into play, hence PDC is not just about the teacher's knowledge or goals, but about their ability to accomplish new things with that knowledge (Ball & Cohen, 1999); it is about the "manner and degree to which teachers create deliberate, productive designs" (Brown, 2009, p. 29). As Brown stipulates

> To a certain extent, the elegance of a teacher's design is a subjective manner. Yet the fact remains that not all designs are equally effective at helping teachers reach their goals, not all designs reflect the same responsiveness to the needs of a particular setting, not all designs are purposeful, and not all designs embody the same degree of utility. PDC is not simply an indicator of whether a teacher will be likely to design something for the classroom; it is an indicator of whether the teacher's designs are pedagogically beneficial (Brown, 2009, p. 31)

Consequently, teachers with different personal resources can share similarities in the way they craft instruction; while teachers with seemingly similar personal resources may produce quite different instructional episodes, because they exhibit different degrees of PDC. The degree of PDC being referred to here, hence implies that PDC is evaluative: thus, teachers can "possess high levels of pedagogical design capacities" (Brown, 2009, p. 31), that is, those who are able to "deconstruct curriculum materials, recognize their essential elements, and reconstruct them in order to suit their own needs" (ibid). An important lesson that the notion of PDC brings to the study of resources is that merely availing a teacher does not necessarily mean the teacher is able to perceive and mobilise their affordances in ways that support their goals. As Adler (2000) argues, availability does not imply use.

Larson comments that "PDC appears to be an overlooked component of effective teaching that is not attended to during the implementation process in most districts" (Larson, 2009). I concur with Larson. My review of literature on PDC produced only three studies exploring the notion of teachers' PDC; one in science (Beyer, 2009; Beyer & Davis, 2012; Davis, Beyer, Forbes, & Stevens, 2007), exploring how to develop PDC for pre-service teachers; another in mathematics (Kim & Remillard, 2011; Remillard et al., 2011), seeking to study and conceptualise PDC, and a doctoral study investigating how PDC helps elementary mathematics teachers to design a context in which the National Council of Teachers of Mathematics (NCTM) vision is put into practice (Land, 2011). Thus, this study should be a welcome addition to studies that try to delineate the nature of the work that teachers do with curriculum materials (Remillard, 2009).

This study investigates how teachers perceive the affordances of the textbook; how they actually mobilise these affordances; and how all these manifest in the classroom in an effort to explore

> how teachers develop pedagogical design capacity, the relationship between perception and mobilization of pedagogical affordances, and the degree to which other personal resources influence the emergence of PDC (Brown, 2009, p. 31)

Choppin opins that the outcome of the development of teachers' PDC is their ability to design *learned adaptations* (Choppin, 2011) which requires teachers to have "both an understanding of the design rationale and empirically developed knowledge of how that rationale plays out in practice"(p. 351). Teachers' development of their PDC is therefore an integral and critical part of their interactions with the textbook.

As mentioned in the introductory section in this chapter, the study has chosen to work with the mathematics topic of *functions* for reasons which will be explained in the chapter on Methodology, as the object of learning for the study (Marton et al., 2004). I recruit the notion of 'object of learning' from Marton et al. as they foreground the importance of what it is that is to be learned in pedagogical encounters, and particularly school learning. This *what*, however, is not simply content per se, but also the *capability* (Marton et al., 2004, p. 4) desired of the learner with respect to the content. Hence, the next section reviewing pertinent literature on 'functions' also focuses on its teaching and learning, especially the tasks.

The study recruits the notion of object of learning from Marton et al., but not the theory of variation that informs their study of such objects. With my interest in text, I select instead to focus on tasks – for which there is extensive research, a review of which goes beyond the scope of the study. A key text on tasks is Stein et al.(2000), which alerts to what we know well, not all tasks make the same cognitive demand on learners, and provide analytic tools for distinguishing between low and high cognitive demand. They also provide an interesting account of how task demands are not simply given by the task, but are relative to learners on the one hand and can be transformed in teaching from higher to lower and the reverse on the other. The textbook used in this study prides itself with a large collection of exercises which shall be analysed for performance expectations or the actions on tasks expected of learners. The performance expectations should illumine the *capacity* learners are supposed to learn.

In the next section of this chapter, which marks the end of the review of literature on curriculum use, I provide a review of the aspects of the teaching and learning of functions, especially at school level which are essential to the study.

2.5 Teaching and Learning Functions

The centrality of the concept of function to undergraduate mathematics, and therefore it being foundational to modern mathematics and other related areas (Carlson, Oehrtman, & Engelke, 2010; Leinhardt, Zaslavsky, & Stein, 1990; Oehrtman, Carlson, & Thompson, 2008; Zaskis, Liljedahl, & Gadowsky, 2003) is undisputed in research. While there are interesting explorations being done on aspects of functions, of interest to this study are two aspects of research on functions: i) issues related to the teaching and learning of functions at school level; and, ii) ways of approaching the function concept.

2.5.1 The Learning of Functions at School Level

Leinhardt et al. (1990) in their synthesis on research on the teaching and learning of functions and graphs show that there are three perspectives from which the literature on these topics may be considered, including: a consideration of tasks given to learners; the development of understanding in learners; and, how functions and graphs are taught.

Among aspects to consider on the task given to learners, Leinhardt et al. mention the *action* of the learner on the task. As they say, "the student learns the domain through his or her engagement in tasks" (1990, p. 5). Most of these actions are either interpretation actions where learners are expected to gain meaning, and/or construction actions such as plotting a graph from a table of values. However, the actions can be point-wise or global depending on the features that are being attended to.

Even (1990, 1998) focuses on the issue of point-wise and global actions and points out that because of the different uses of functions, it sometimes becomes necessary to deal with functions point-wise, that is, doing actions that attend to points such as plotting, or reading off a value. On the other hand, when one needs to look at the behavior of functions, for example in expressions of sketching graphs of functions, a more global approach needs to be adopted. An important point that Even makes is that "each one of the alternative ways of approaching functions is different from the others and neither one of them is appropriate for all situations" (Even, 1990, p. 534).

Research also shows that many learners are able to deal with functions point-wise only, and are not able to work around functions globally. For example, Monk (1994) in his study of beginning calculus students who were given problems which tested for a 'Pointwise' understanding and 'Across-Time' understanding shows that 85% of students got the Pointwise questions correct when only 53% got the Across-Time questions correct, and concludes from this study that "Pointwise understanding of graphs is prerequisite to Across-Time understanding, but the jump from the one to the other is a considerable one for students"(1994, p. 25). Monk goes further to show that

> at least in simple situations, students have a confident and secure Pointwise understanding of function, but even at the end of the first of second quarter of calculus – when we tend to assume they have already acquired this ability – they are still struggling to see functions in an Across-Time manner. (p.21)

More recently, Ronda (2009) also found that, "[I]t seemed a big step for the majority of the students working with the equation representation of function to move from individual point interpretation to a more holistic interpretation" (Ronda, 2009, p. 47). This claim emerges from a study in which Ronda develops a framework for analysing and monitoring learners' understanding of functions in equation form; the results of which produced four growth points as:

> GP 1: Equations are formulas or procedures for generating values
> GP 2: Equations are representations of relationships
> GP 3: Equations represent properties of relationships
> GP 4: Functions are objects that can be manipulated or transformed (ibid)

Each growth point describes a particular stage of understanding that learners reach, which provides teachers with "a structure for assessing students and designing classroom instruction that would facilitate students' attainment of object conception of function" (ibid).

These remarks hence suggest that the point-wise and global actions demand different kinds of 'facility' from a learner, point-wise seemingly more easy for learners to deal with than global, so that teachers need to attend to how to assist learners to move from the point-wise to the global and then flexibly between.

2.5.2 The Teaching of Functions at School Level

The previous section highlights some of the important aspects which teachers have to be aware of that impact learners' understanding of the function concept. For example, Even (1990) suggests that teachers will not be able to help learners to flexibly make choices between point-wise and global approaches if they cannot move flexibly between pointwise and global actions themselves.

However, there are a few more aspects that address the teacher specifically with regards to the teaching of functions, for example, issues pertaining to; a) entry to the topic; b) sequencing; c) explanations; d) example; and e) representations (Leinhardt et al., 1990). These researchers suggest that these are key issues in the teaching and learning of functions which teachers should primarily focus on, where "the object is to craft the introduction, and later sequencing, in ways that enhance the early understanding and limit the misunderstandings that may have developed" (1990, p. 47).

Lloyd and Wilson (1998) contend that "teachers' comprehensive and well-organised conceptions contribute to instruction characterised by emphases on conceptual connections, powerful representations, and meaningful discussions" (1998, p. 270), versus a teacher with limited views of function that lead to a "narrow instruction marked by missed opportunities" (ibid). They talk about teachers having a 'basic repertoire' that consists of competence with the families of functions that are encountered at school; access to a wide range of examples that also reflect problem situations and shifting representations.

In the teaching of functions therefore, teachers should be able to sequence the content in ways that enhance the understanding of the concept; and approach the teaching in ways that provide access to powerful representations. In other words, the teacher's approach to the teaching of functions is critical in how learners eventually learn the topic.

2.5.3 Covariation, Variance and Invariance

Selden and Selden (1992) indicate that functions can be viewed in terms of *action* or *process* conceptions, or as objects. Dubinsky and Harel (1992) define the action and process views of functions as follows:

About the action view, they write that

> Such a conception of function would involve, for example, the ability to plug numbers into an algebraic expression and calculate. It is a static conception in that the subject will tend to think about it one step at a time (e.g., one evaluation of an expression). (Dubinsky & Harel, 1992, p. 85)

And for the *process* view, that:

> The subject is able to think about the transformation as a complete activity beginning with objects of some kind, doing something to these objects, and obtaining new objects as a result of what was done (ibid)

Carlson et al. (2010) show that students with an action view of actions have 'an impoverished' function view of "weak understandings of function" (p. 116) which are highly procedural. Students in this view rely solely on computational reasoning, and view a formula such as $f(x) = 2x^2 + 1$ as set of instructions instead of regarding it as a "means of mapping any input value of the function represented by x to an output value represented by $f(x)$" (p.116), which points to a process view of function. Thus in the process view, learners have moved from a procedural view of function to a more dynamic view where "they can begin to imagine a continuum of input values in the domain of a function producing a continuum of output values" (ibid).

The process view of function is thus essential for imagining and describing how two quantities covary, that is covariational reasoning (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002), which is crucial in further studies in courses such as calculus. Covariational reasoning is described as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson et al., 2002, p. 354). Researchers emphasise that covariation and not correspondence should be the way that the function concept is introduced to learners as covariational reasoning "has been documented to be essential for representing and interpreting the changing nature of quantities in a wide array of function situations and for understanding major concepts in beginning calculus" (Carlson et al., 2010, p. 115).

In functions, two things are varying at the same time, that is, the input and the output and the teaching of functions therefore has to reflect this variation and how to attend to it. Watson and Mason (2006) show that it is the variance and invariance that opportunity for learning is created. The authors show that an aspect (that is present) is more likely to be discerned if its variation is foregrounded against relative invariance of other features"; and therefore by asking the highly mathematical question of " 'what changes and what stays the same?' and by examining the nature of the changes offered, one can be precise about what an exercise together with an established way of working affords the learner and with what constraints" (Watson & Mason, 2006, p. 103).

It would be important therefore to pay attention to the notion of covariation, variance and invariance in the teaching of functions from the textbook in chapter 5; as well as what the teachers do with these notions in the lessons.

2.6 Conclusion

Chapter 2 has narrated relevant literature on the notion of textbook use by teachers, and establishes that the teacher-text interactions are not unidirectional, but are influenced by both the design of curriculum resources with respect to what they can afford teachers' practice and at times constrain it; as well as how teachers perceive and mobilise these 'offerings' of the textbook. The chapter also demonstrates the complexities of teaching and learning the topic of *functions*, and how some of these may be overcome. The next chapter grounds the study theoretically, providing a lens through which the research questions shall be investigated and answered.

CHAPTER 3
The Theoretical Grounding

> Theory is vital to a body of research because it frames the questions asked and the way they are asked. It also helps us understand and explain what we see.
> (Remillard, 2009, p. 85)

3.1 Introduction

The previous chapters illustrate that the teacher-text relationship ultimately involves “bi-directional influences” (Brown, 2009, p. 23): curriculum materials through their affordances and constraints influencing teachers, and teachers through their perceptions and decisions, mobilising the curriculum resources. This bi-directionality is captured in the conceptual frameworks offered by Remillard (2005, 2009) and Brown’s (2009) Design Capacity for Enactment (DCE) framework, which are discussed later on in this chapter The study adopts and adapts these conceptual frameworks in developing frameworks for analysing how teachers perceive the affordances and constraints of the textbook and mobilise the textbook with the objective of mediating a particular object of learning in the classroom. Teachers’ use of the textbook thus points to the notion of mediated action with origins in socio-cultural theory (Vygotsky, 1978). In this chapter thus, Vygotsky’s socio-cultural theory and the use of tools are discussed to ground the study theoretically. Secondly, the chapter develops conceptual tools for analysing the affordances and constraints of the textbook in the study; and for analysing how teachers perceive and mobilise the textbook in their practice.

3.2 Vygotsky and the Socio- Cultural Theory

The study situates itself within sociocultural theory as its overarching theoretical framework, wherein all humans are inherently social beings and grow from and through the use of tools (Vygotsky, 1978). In this framework, the relationship between the subject and object is mediated through artefacts, or cultural tools which mediate action (Wertsch, 1998) as shown in the model below.

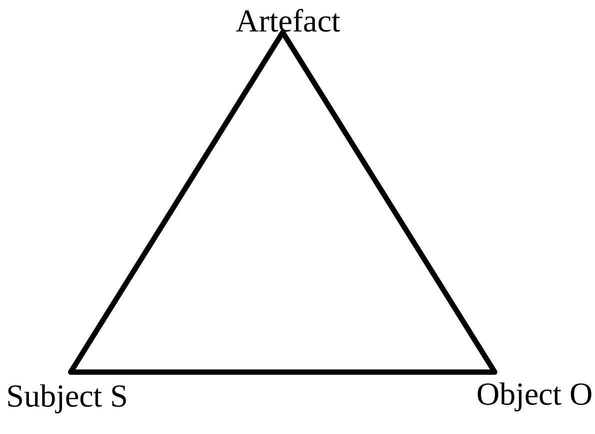


Fig 3.1 The basic triangular representation of mediation (Daniels, 2001)

According to Vygotsky (1978), a function of a cultural tool

> is to serve as the conductor of human influence on the object of activity; it is *externally* oriented; it must lead to changes in objects. It is a means by which human external activity is aimed at mastering, and triumphing over, nature. (p.55)

Wertsch (1991) argues that the cultural tools that are employed in mediated action are the key to understanding the relationship between sociocultural settings and human action. Hence, appreciating this "agent-acting-with-mediational-means" (Wertsch, 1998, p. 24), forces us to "go beyond the individual agent when trying to understand the forces that shape human action" (ibid) and to focus on the agent-instrument dialectic.

Wertsch (1998) makes some claims about mediated action which are appropriate for the current study. Firstly, that there is an irreducible tension between agent and mediational means, such that understanding the agent-tool relationship involves "the isolation and recombination of the elements of agent and mediational means" (p.30). This means that there are times when separate characteristics of each are analysed singly, but the end-result in the action is a consequence of the two acting together. This point is important in this study as it points to the need to analyze the affordances and constraints of the textbook, and the individual characteristics of the teacher. However, to understand their relationship, these separate features need to be brought together. The second claim is that mediational means constrain as well as enable action, a point which has already been discussed in the previous chapter. This says to be aware that while the textbook enables some aspects to the teacher's practice, it also restrains the practice in other ways.

A third claim is that the introduction of new mediational means transforms mediated action, to show that sometimes agents may demonstrate outstanding skills when working with one cultural tool but the skills could be different or even average when functioning with other tools. The interest for the study here lies in the fact that during data collection of the study, a new textbook was being introduced into the schools, for reasons which shall be explained in the next chapter; and therefore two different editions of the textbooks were available to the teacher. The question would be whether the introduction of the new textbooks shaped different enactments by teachers.

For the last claim, I have combined two claims from Wertsch which indicate that the relationship of agents towards mediational means can be characterized "in terms of mastery" or "in terms of appropriation" (Wertsch, 1998, p. 25). Wertsch mentions that the processes of 'mastery' and 'appropriation' are intertwined, but analytically and empirically, they can be distinct. Wertsch defines *appropriation* from translating the Russian language; as a process of 'taking something that belongs to others and making it one's own" (Wertsch, 1998, p. 53). Attached to this meaning lies a notion of resistance such that, while it is expected that the agent shall use the mediational means, it has to be noted that the agent also has power over whether the mediational means are embraced fully or resisted strongly. This implies that mastery and appropriation may not always be correlated, for example, in cases where an agent uses the tools with feelings of strong resistance towards the cultural tools. In such cases, mastery may not be expected even though there is still some appropriation of the cultural tool. To this effect, Brown (2009) characterizes how teachers use curricular resources as a continuum that illustrates the degree of appropriation. These degrees of appropriation are important to the study as they illustrate teachers' differential ways of using curriculum materials, and are discussed in subsequent sections.

Socio-cultural theory and mediated action point to the importance of attending to the context in which the teacher-text interactions occur. The study is cognizant of the tension between enculturation which carries connotations of imposition; and participation that points to teachers' agency. As Daniels (2001) points out "all thinking is inextricably bound up with context and that to speak of the individual development is inappropriate" (p.39). Notwithstanding, it is the view of the study that the mediational model of the socio-cultural theory entails "the mutual influence of individual and supra-individual factors" and acknowledges that human beings "also actively shape the very forces that are active in shaping them" (Daniels, 2001, p. 1). This is the stance of the "interpretation of and participation with resources" (Pepin, Gueudet, & Trouche, 2013b) theoretical perspective represented by the frameworks by Remillard (2005) and Brown (2002, 2009) in this chapter; and consequently the stance of the present study. As explained in chapter 1, the study aligns itself with the view that the teacher-textbook interactions are dynamic interrelationships in which the teacher and the textbook both shape each other and together they shape instruction (Remillard, 2005; Stein & Kim, 2009). Chapter 1 of the study highlights that making textbooks available to teachers does not necessarily imply they are used effectively, and therefore the theoretical stance of a participatory relationship between the teacher and the textbook gives flesh to this metaphoric sense of textbook use.

In the next sections, a conceptual framework for analyzing the affordances of the textbook to the teacher's practice as well as tools for analyzing teachers' lessons are developed

3.3 Conceptual Framing of the Study

Two frameworks, Brown's (2002, 2009) *Design Capacity for Enactment* (DCE) *framework* and Remillard's (2005) *Framework of components of teacher–curriculum relationship* have been influential in developing a conceptual framework for analysing the textbook for its affordances and constraints, and then for how teachers appropriate the affordances of the textbook I have to mention that the study recognises the *documentational genesis* framework (Gueudet & Trouche, 2009)which focuses on teachers' documentation work, and its likeness to the two frameworks mentioned above, on resources shaping teacher activity and teacher shaping the resource. However, for the purposes of this study, I have chosen to work only with Remillard's and Brown's frameworks, as they are closely linked and serve my purposes better. Each framework is discussed separately beginning with the DCE framework.

3.3.1 The Design Capacity for Enactment Framework

Brown's framework in Fig 3.2 is illustrative of the factors which influence the teacher-text relationship and has been shown in previous chapters and sections to consist of the interaction between features of teacher resources and those of curriculum resources. The framework is based on the assumption that teaching is a design activity, and therefore as teachers interact with the textbook, they are engaged in design (Brown, 2009). According to the DCE framework, the textbook brings its *domain representations, procedures* and *physical objects* to the interactions while the teacher brings her *subject matter knowledge, pedagogical content knowledge, goals* and *beliefs* to the interactions.

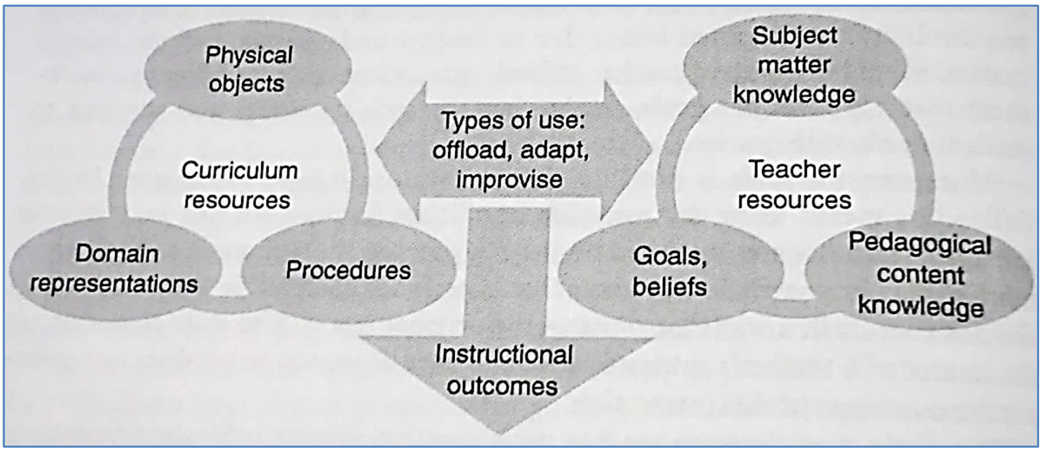


Fig 3.2 The Design Capacity for Enactment framework (taken from Brown (2009, p. 26))

The double-arrow in the middle of the two rings representing the curriculum features on the one side and teacher resources on the other, shows three types of use of curriculum materials by teachers as *offloading*, *adapting*, and *improvising*. According to Brown, teachers *appropriate* (Wertsch, 1998) the affordances of curriculum in three different ways which may be depicted as a continuum with *offloading* and *improvising* at the two extremes of the continuum and *adapting* lying in the middle. The three types of uses depict the degree of appropriation of the affordances of the curriculum resource, with offloading referring to teachers relying to a very large extent on the curriculum resource for the delivery of the lesson. Improvising refers to when the teacher improvises own materials in the lesson and uses the curriculum resource quite minimally; while adapting implies relying equally on the curriculum resource and own resources to deliver the lesson.

Brown's scale of curriculum appropriation shares some similarities with the *curriculum strategy* framework (Sherin & Drake, 2009), while there are differences between them as well. As mentioned in chapter 2, the *curriculum strategy* is "the stance that the teacher adopts as he or she plays th[e] role of interpreter" (Sherin & Drake, 2009, p. 472); and consists of three 'interpretive' activities of: *reading, evaluating,* and then *adapting* the materials. In the first instance, the *curriculum strategy* framework takes a view of curriculum use as 'interpretation of text' (Remillard, 2005) where the teacher's role is to make meaning of the text; as opposed to that of a "collaborator with materials to design enacted curriculum" (p. 217) in Brown's framework.

By *adapting* in the curriculum strategy the authors refer to "significant changes that teachers make in the intended curriculum e.g. changes in the structure of a lesson, in the activities that comprise the lesson, or in the purpose of the lesson" (Sherin & Drake, 2009, p. 486). Therefore, the curriculum strategy complements the appropriation framework: *omitting* or deleting parts of the materials without replacing them; or *replacing* them with a different component; or *creating* new materials altogether; all form part of the adapting and improvising in the appropriation framework, and therefore confirm Brown's assertions. Additionally, placing the activities on a continuum adds another layer to how Brown describes the process of appropriation because it distinguishes the time periods in which the different activities take place. For example, Sherin and Drake point out that no teacher would omit components of materials at one time period and create another one at another time period; the processes remained consistent throughout the instructional process. In the current study, the view of textbook use as "participation with the text" (Remillard, 2005, p. 221) that the study aligns with requires not only an analysis of how the teacher adapts the textbook but what the textbook furnishes the teacher's practice as well.

An important observation from Sherin and Drake pertains to their finding that teachers would only adapt the mathematics component of instruction during the lesson or after the lesson. At this point I wish to point out that this is one of the differences between Sherin and Drake's study and the present study. In this study, the focus is on the mathematics, the object of learning, and what the teacher-textbook interactions do to the opportunities for the mediation of the mathematics. Non-mathematical activities in the lesson are not considered as a result.

3.3.2 The Framework of components of teacher–curriculum relationship

The framework by Remillard shown in Fig 3.3 utilises Brown's DCE framework to conceptualise the teacher-curriculum relationship as being shaped by what the teacher and the textbook bring into the teacher-curriculum interactions.

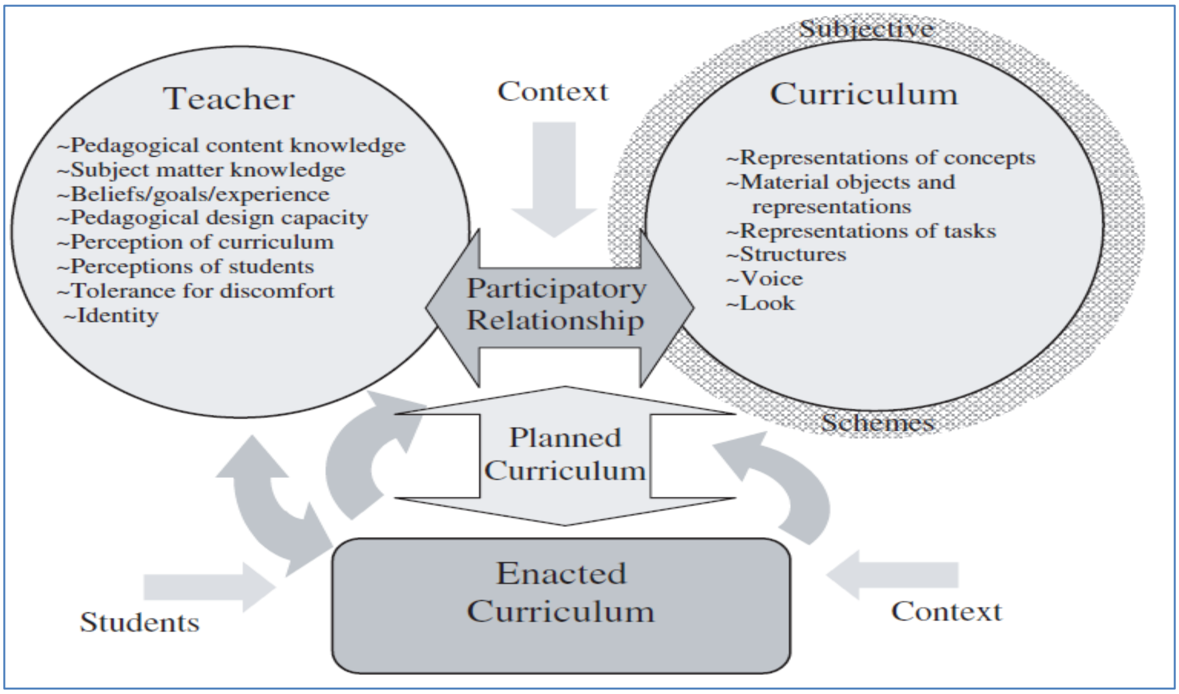


Fig 3.3 Framework of components of teacher-curriculum relationship
(taken from Remillard (2005, p. 235))

She highlights the four principal constructs of the framework that help to understand the teacher-curriculum relationship as "(a) the teacher, (b) the curriculum, (c) the participatory relationship between them, and (d) the resulting planned and enacted curricula" (Remillard, 2005, p. 236).

Important additions to the DCE framework include:

a) The influence of context on the participatory relationship, as well as on enactment in the classroom.
b) Learners are an integral part of the teacher-text interactions
c) A direct interaction between teacher and the enacted curriculum, but none between the curriculum and the enacted curriculum
d) The cyclic nature of the participatory relationship where the enacted curriculum is a result of the participatory relationship, while the enacted curriculum informs the participatory relationship as well.

It is noted that while Remillard includes teachers' pedagogical design capacity (PDC) in the ring for teacher resources, Brown does not include it in the DCE framework. However, Brown explicitly declares that teachers' PDC deserves a place in the DCE framework:

> this skill (meaning PDC) represents yet another characteristic that teachers bring to their interactions with curriculum materials. Thus, PDC itself ultimately warrants inclusion within the DCE framework (Brown, 2009, p. 28)

The study adopts and adapts these two frameworks to develop a framework for the study shown below and underpinned by the socio-cultural theory.

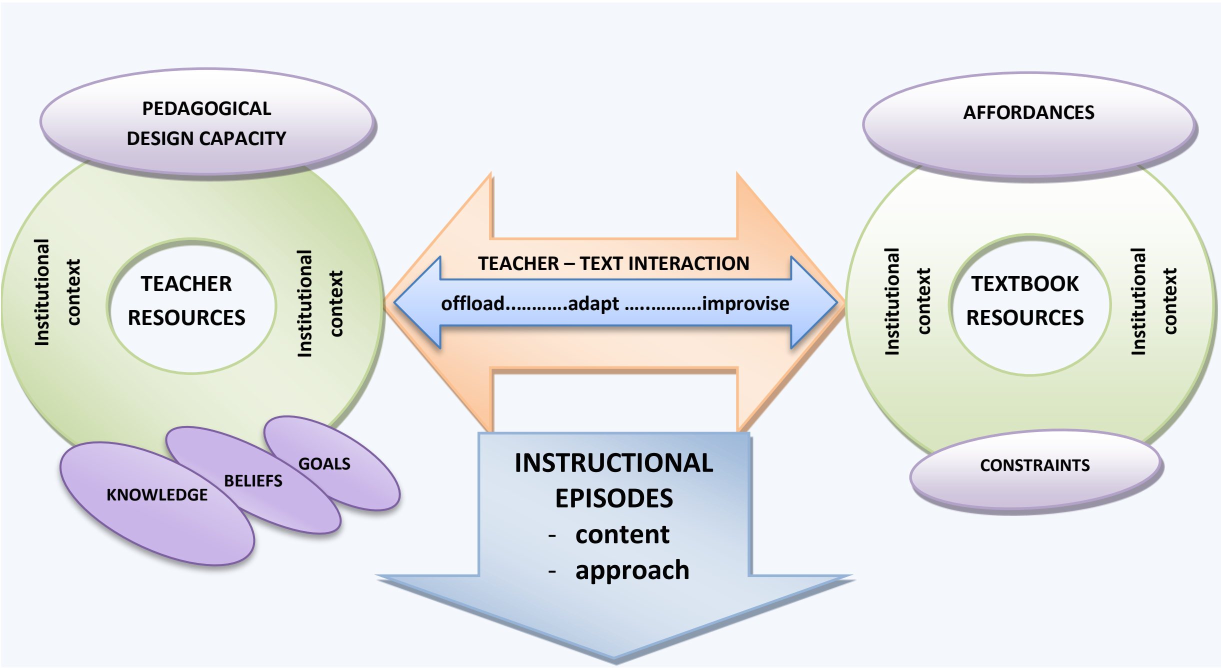


Fig 3.4 A Conceptual Framework for the Teacher-Textbook Relationship

My question explores the relationship between the affordances of the textbook and teachers' pedagogical design capacity in the mediation of the object of learning, hence the two rings for the teacher resources and the textbook resources stay the same. In the ring for the textbook resource is included captions for *affordances* and *constraints* as the components of the textbook which need to be determined. The discussions on the notion of affordances by Gibson and Norman from chapter 2, together with Remillard's and Brown's frameworks discussed herein, suggest a definition of affordances for the study that encompasses the resources that the textbook furnishes the teacher, as well as their enabling and constraining characteristics. As Gibson points out, "what they [affordances] afford the observer, after all depends on their properties"(quoted from (E. J. Gibson, 2001, p. 88)). From the literature narrated in the previous chapter, the intended and enacted curricula in form of instructional episodes include elements of the *mathematical content* together with the *approach to the teaching and learning of the content*. In the ring for the textbook resources, there is the PDC itself together with three features that research illustrates are prominent in influencing teachers' PDC: teachers' knowledge in all its facets, their beliefs and their goals. For example, Schoenfeld((2011) in his research on decision making which includes teacher decision making, found that knowledge, goals, orientations, including beliefs and dispositions, were all influential factors.

> what people do is a function of their resources (their knowledge, in the context of available material and other resources), goals (the conscious or unconscious aims they are trying to achieve), and orientations (their beliefs, values, biases, dispositions, etc.). …if enough is known, in detail, about a person's orientations, goals, and resources, that person's actions can be explained at both macro and micro levels. That is, they can be explained not only in broad terms but also on a moment-by-moment basis" (p.26)

Both the teacher resources and the textbook are influenced by institutional context which is reflected in Remillard's (2005) framework as an influential factor for the teacher-textbook interactions. However in this study that is situated in particular school context, context itself is not in focus. Lerman (2000) points out that while the challenge the 'social turn' presents for research in mathematics education is "to develop accounts that bring together agency, individual trajectories…, and the cultural, historical, and social origins of the ways people think, behave, reason, and understand the world" (p. 36); it is a necessity of research in mathematics education that cannot be ignored. He proposes the metaphor of a zoom lens in which what a researcher chooses to focus on can be seen as "a particular moment in the zoom of a lens" (Lerman, 2001, p. 87) in which

> one might envisage a researcher choosing what to focus on in research through zooming in and out in a classroom, as with a video camera, and selecting a place to stop. Whilst the particular focus creates the object of research, the researcher must take into account how the object is constituted in its relations to the wider macro-situation and the micro-situations. (p. 90)

In this study hence, the context is in the background. However in the conclusion of the study, I return to the issue of context and some discussion on how context influences the teacher-textbook relationship.

I bring *offloading, adapting,* and *improvising* as characteristic of how teachers appropriate the affordances of the textbook. The two important aspects of the mediation of the object of learning in the classroom shall be about the content of the lesson as well as how the content is taught.

As Baxter & Jack (2008) opine that a conceptual framework serves as an anchor for a study, I am suggesting here an anchor that would guide the study moving forward. I expect that after the interrogation of the data collected for the study during data analysis, the framework will be revisited and refined.

3.4 Conclusion

In this chapter, I have grounded the study theoretically, and suggested a conceptual framework for guiding the study towards a better understanding of the relationships formed by the teacher and the prescribed textbook. Thus relevant literature and theory pertaining to this study have now been established through the two chapters, namely 2 and 3. In the next chapter, I discuss the research design and methodology of the study.

CHAPTER 4
Research Design and Methodology

4.1 Research Design

This chapter outlines the study's research design and methodology informed by research questions outlined in chapter 1, the literature review in chapter 2, and the theoretical underpinnings discussed in chapter 3. To reiterate the purpose of the study: the study seeks to understand better the relationship between the teacher and the prescribed textbook in the mediation of the object of learning in the classroom. The research is thus qualitative and the study adopts qualitative research methods to answer its research questions (McMillan & Schumacher, 2010). These authors outline the research purpose for qualitative research as "understanding a social situation from participants' perspectives" (p.12), which this study sets out to do. Thus in keeping with qualitative research approaches, the research design and methodology reported here takes the following aspects into consideration: the selection and description of the research site including the overall context of the study; my role as a researcher; selection and description of participants including ethical consideration pertaining to this selection; the duration of the study; its limitations; data collection and analysis strategies; and validity, reliability and generalizability issues.

Because of the nature of my research questions, which seek answers to the "how" and "why" of textbook use by teachers in their practice; I have approached the study as a case study (Yin, 2009) research, which I describe below.

4.2 Case Study Research

Yin (2009) posits that case study methods work "when a "how" and "why" question is being asked about a contemporary set of events, over which the investigator has little or no control" (p.13). McMillan & Schumacher (2010) define a case study as follows:

> a case study examines a *bounded system*, or a case, over time in depth, employing multiple sources of data found in the setting. The case may be a program, an event, an activity, or a set of individuals bounded in time and place. The researcher defines the case and its boundary. A case can be selected because of its uniqueness or used to illustrate an issue. The focus may be one entity (within-site study) or several entities (multisite study) (p.24)

This definition hence demands one to be explicit about the nature of the case to be investigated, and its boundaries in the design of the study. Another key feature of a case study is about data sources in case study research. Baxter & Jack (2008) point out that "a hallmark of case study research is the multiple data sources, a strategy which also enhances data credibility" (p.554); a point that Yin concurs with:

> The case study inquiry
>
> - Copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result

- Relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
- Benefits from the prior development of theoretical propositions to guide data collection and analysis.

(Yin, 2009, p. 18)

According to Yin, there are two variations of case study designs, namely, single-case designs where a single case is being studied, or multiple-case designs. Within each variant, there could also be a unitary unit of analysis, which Yin refers to as holistic, or an embedded case study where there are multiple units of analysis. The categorisations are illustrated in Fig 4.1 below.

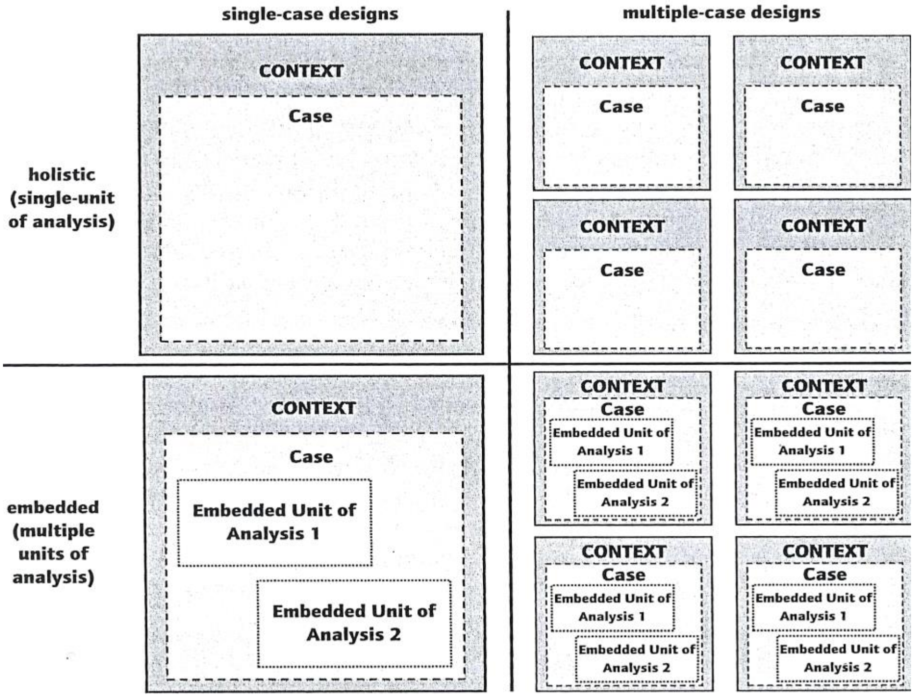


Fig 4.1 Categories of Case Studies (Source: (Yin, 2009, p. 46))

The categorization is very critical in the research design as it focuses what the study is about and what it is not about, hence forcing one to be explicit about the unit of analysis in the study. As Hatch (2002) asserts, "defining the boundaries, or specifying the *unit of analysis* is the key decision point in case study design" (p.30). Stake (1995) also categorises case studies into two: an intrinsic case study when the focus is on the case itself, and an instrumental case study when there is need for an in-depth understanding of an issue and the case study is used to elucidate that issue.

The present study is hence an instrumental case study if we were to adopt Stake's categories. The study seeks an in-depth understanding of how teachers mobilise the affordances of the prescribed textbook by investigating teachers in one cluster of schools in the WMCS project schools. The interest is on the process of mobilization, how teachers mobilise the textbooks and what factors influence their decisions and actions. Thus, the focal point in the study is the process of mobilizing the textbook, and not the individual teachers: teachers are a means to understanding the process of mobilization. In the analysis of lessons in chapters 6 and 7, the emphasis is on the processes by which teachers appropriate the affordances and the opportunities for mediation these processes open up, and therefore in the study the emphasis is on common features of PDC across teachers and not for within the teacher for so that generalisations about teachers' PDC can be drawn.

In the next sections, I elaborate on design issues of the study, beginning with the context.

4.3 The Context

As discussed in chapter 2, institutional context is critical in the teacher-text interactions. The context that I am providing here pertains to the schools and also the context of the larger project within which the study is taking place. The intention here is placing boundaries around the study, a requirement of case study designs mentioned above depicting case study research as a bounded system. Suggestions for binding a case study include: by time and place, or activity, (Stake, 1995) or by definition and context (Miles & Huberman, 1994). This section shows how the present study is bounded by the context, including time, place and activity.

4.3.1 The Context of the Schools

I have already alluded to the fact that this study is situated within the larger project, the Wits Maths Connect Secondary Project (WMCS) which works with teachers in ten schools within one district in the province of Gauteng in South Africa. The teachers participating in the study are located in these schools. For purposes of working in the schools, the WMCS had initially divided the schools into four clusters based on geographical location, and I found myself working in one particular cluster regularly which became my logical choice of cluster for my study. This cluster is made up of three secondary schools with learners from Grade 8 (ages 13 – 14) to Grade 12 (the last year of secondary schooling in South Africa). Two of the schools are located very close together, about 5 minutes of walking distance, while the third school is situated some one kilometre away from the other schools, and the schools serve a common community of learners: in a township setting with low socio-economic status and 100% 'black[7]'. The teachers in these schools are all black and mostly South African nationals but in the subjects of mathematics and science, there are a number of foreign teachers especially from Zimbabwe, as is the case with all the other project schools. All the teachers participating in the study were active participants in project activities before data collection started.

With regard to access to textbooks, all teachers have a copy of the prescribed textbook and other textbooks that they use, however, not all learners have textbooks. In school A for example, where there are about 60 to 70 learners in a classroom in the lower grades, a textbook is shared by between 6 to 10 learners. As discussed in chapter 1 with the 'Limpopo textbook saga', there is a problem with access to textbooks by learners due to administrative challenges by government to deliver the textbooks in a timely fashion to learners. Thus while the textbook is the sole resource for learners it is not fully accessible to them.

[7] In post-apartheid South Africa, 'black' is still commonly used for people of African descent and 'white' for people of Western descent. There is only one 'white' teacher in these schools, teaching music at one school

4.3.2 The Context of the Larger Project

In order to focus the study and its scope, I chose to work on the mathematical topic of *functions* at Grade 10 level, and this decision was influenced by the focus and objectives of the larger project. The mathematical focus of the larger project was established as the topics of *algebra* and *functions* due to their importance in the curriculum in South Africa particularly, and in mathematics generally. At the same time, the larger project had decided that the Grade 9/10 transition in mathematics constituted a leakage in the pipeline in the teaching and learning of mathematics in South Africa, and therefore that would be one strategic point where the project would focus its attention. The other reason for choosing *functions* is about my own personal inclination towards the topic: I have taught the topic at first year undergraduate level for over a decade and found that while it is a very important aspect of mathematics for further studies, it is also one of the topics in mathematics which students find quite difficult. Working with *functions* hence would also provide me a space of personal learning about the challenges of teaching and learning this topic at secondary level. Furthermore, *functions* as a topic is formally introduced at Grade 10 in the South African curriculum, hence my reasons for choosing to work with Grade 10 *functions*.

It is important to note here that the topic of *functions* is only a means to study the relationship between the teacher and their textbook. I could have chosen any other topic.

4.4 Participants in the Study

The teachers participating in the study together with the textbooks they use in their practice form the two types of participants that I am reporting on below.

4.4.1 Teacher Participants

The sampling adopted in the study is purposive but opportunistic as well (Cohen, Manion, & Morrison, 2000). The teachers whom I worked with formed part of a larger project made of four different clusters, as mentioned in earlier sections, hence the opportunity. Furthermore, the larger project would hold cluster workshops which made it convenient for me to work in one cluster, instead of different ones. My intention had been to work with two Grade 10 teachers from each school. However, when inviting the teachers to participate, all Grade 10 teachers in the schools showed interest to participate: four from School A; two in School B, and one School C, thus a total of seven teachers. Prior to the study and as demanded by the institution, an ethical clearance for working with teachers in their schools was sought and granted for the study. Furthermore, consent for participation in the study was sought from each teacher, each learner in each teacher's chosen class, and from the parents of all these learners (consent letters appended to thesis as Appendix A1 and A2, respectively). I assured teachers, learners and their parents of confidentiality and anonymity, together with the fact that participation in the study was voluntary, and learners and teachers could stop participating at any time they wished. I also pledged that the information gathered in the study would be used solely for the purposes of the study. Hence, in keeping with these assurances, all teachers are referred to anonymously without disclosing their schools, nor their cluster. I have given each teacher a lettered name based on their school. For example, the four teachers in School A are referred to as teachers A1, A2, A3, and A4, and so forth.

Teacher		School	Qualification	Additional qualification	Experience in teaching by 2011 (years)	Experience in grades taught by 2011 (mathematics)
CC	A1	A	3-year Teachers' Diploma/National Professional Diploma in Education		2	Grade 8 - 10
RS	A2	A	3-year Teachers' Diploma/National Professional Diploma in Education		1	Grade 8 - 10
TM	A3	A	Four-year Higher Diploma in Education	Advanced Certificate in Education (ACE)	15	All Grades
EM	A4	A	3-year Teachers' Diploma/National Professional Diploma in Education		1	Grade 9 – 10
ZS	B1	B	3-year Teachers' Diploma/National Professional Diploma in Education STD	Advanced Certificate in Education (ACE)	8	All Grades
OM	B2	B	3-year Teachers' Diploma/National Professional Diploma in Education	Advanced Certificate in Education (ACE)	12	All Grades
CCH	C1	C	Four-year Higher Diploma in Education (Accounting)		1	Grades 8 and 10

Table 4.1 Participant Teacher Information

As it happened, Table 4.1 above shows that all teachers in the study except for teacher C1 are qualified mathematics teachers. Teacher C1's school had been experiencing a shortage of qualified teachers in mathematics and teacher C1 filled in on some mathematics teaching. Three teachers have an additional Advanced Certificate in Education (ACE) qualification which was introduced in South Africa post-apartheid, as a formal upgrading programme for teachers who only had access to a three-year or four-year college diploma in education. Thus, while the teachers are all qualified, there is a difference between the three-year diploma, four-year diploma, and the three-year plus additional ACE diploma.

With respect to experience, two teachers have teaching experience of more than ten years each, and have taught all Grades; one teacher has taught all grades with an experience of eight years of teaching, and the rest of the teachers have taught up to Grade 10 with experiences ranging from one to two years. Teachers' experiences will be an important aspect in the analysis of data, as the previous chapters show that the experience plays an important role in how teachers interact with their curricular resources.

4.4.2 Textbooks as Participants

The prescribed textbook at Grade 10 for the three schools participating in the study is Classroom Mathematics (Laridon et al., 2008), which I shall refer to simply as CM. Bowie (2013) (see Chapter 2) analyzes two textbooks in her study and chooses CM as one of the two textbooks. I quote below her reasons for the choice of textbooks and in particular her views about this particular textbook

> However I did want to choose textbooks that were published by well-known publishers, written by respected authors and intended for the mass market. This was to ensure the textbooks were not unusual cases (e.g. written for schools with access to technology or aimed at high achievers) and that they had credibility. For these reasons I chose the book Classroom Mathematics Grade 10 (which henceforth I refer to as CM) (Laridon et al., 2004) as the first book to analyze. This book was the bestselling grade 10 mathematics textbook in the country at the time I embarked on my study and was written by a team of authors who are (or were) schoolteachers with extensive classroom experience. A number of authors of this textbook were also involved in the curriculum process and have been in leadership positions in mathematics education organisations. The Classroom Mathematics brand is well known in South Africa. The coordinator of the team of authors as well as a number of members of the team had been involved in writing previous editions of the book. (Bowie, 2013, pp. 62-63)

Thus, CM is a very popular textbook in South Africa, and I gather from interactions with teachers from some private schools that it is a textbook of choice with some prestigious private schools as well.

The present study commenced at an interesting time in South Africa with respect to textbooks due to a phasing out of the old curriculum and an inception of a new one. In 2011 when data collection began for the study, South African schools followed two national curricula, the *Revised National Curriculum Statement Grades R-9* and the *National Curriculum Statement (NCS) Grades 10-12* (2002). A process of revision that started in 2009 culminated in a joint curriculum statement, the *National Curriculum Statement for Grades R-12* (2012). The new curriculum statement is said to have been built on and updated the previous curricula with the aims "to provide clearer specification of what is to be taught and learnt on a term-by-term basis" (NSC 2012, p.4) and includes the following documents

(a) Curriculum and Assessment Policy statements (CAPs) for all approved subjects;
(b) *National policy* for programme *and* promotion requirements of the *National Curriculum Statement Grades R-12*; and
(c) *National Protocol for Assessment Grades R-12"*

Thus, CAPS is not a complete reform of curriculum but provides clearer specification.

The textbooks used in the South African schools are aligned to the curriculum statement and undergo an approval process facilitated by the Department of Basic Education (DBE), to become prescribed textbooks. With the looming change in the curriculum for inception at Grade 10 in 2012, textbook authors and publishers began the revision of their textbooks to align with the new CAPS curriculum, CM included. When data collection began for the study, the new CM edition was in draft form. While the new edition would still have to be subjected to the approval process, teachers were convinced that it would pass; possibly for the reasons outlined above by Bowie. Through my professional contacts, I was then able to make arrangements for the draft copy of the new CM to be availed to the participating teachers, such that there were two versions of the textbook in place for the teachers, which I shall distinguish by referring to the old version as the *Pre-CAPS textbook*, and the new one as the *CAPS textbook*. However, it is also important to note that learners had copies of the Pre-CAPS textbook, and as mentioned in previous sections, except in School B where all learners each had their own copy of the textbook, in Schools A and B, learners were sharing a textbook, and at times, there were six or ten learners to one copy. The larger project hence decided to provide textbooks to School A and B so that each learner in all participating classrooms in this study had their own copy of the Pre-CAPS textbook. In this way, lack of textbooks for learners could no longer be used as an excuse for non-utilisation of the textbook.

Guided by teachers' assumptions and convictions that the new CM textbook (CAPS textbook) would still be the prescribed textbook for 2012; that CM seemed to be the most popular among the seven teachers participating in the study as the teacher's book; and the fact that the new chapters were availed to the teachers to utilise, I used both textbooks in determining the affordances of the textbook in chapter 5. In fact, the CAPS textbook, *'Classroom Mathematics' Grade 10 Learner's Book* by Pike et al. (2011) has indeed passed the approval process as one of the recommended textbooks for the CAPS curriculum, and in two schools (Schools A and B) has remained as the prescribed textbook for Grade 10. In School C, another textbook series has been recommended as the prescribed textbook and this is discussed in later chapters.

This means that during data collection and for class observations, the teachers in the schools had the two editions of the textbook to work with but learners only had the Pre-CAPS textbook, the fact that was quite obvious in the observations.

4.5 Data Collection

Two important events took place prior to the data collection process and during the process which have a bearing on the process. Firstly, prior to the data collection process, and based on literature regarding the need for making the designers' rationale visible to the teacher (Ball & Cohen, 1996; Davis & Krajcik, 2005; Stein & Kim, 2009) when using textbooks, I invited the author of the topic on *functions* in the CAPS textbook, who happened to be the same author in the Pre-CAPS textbook, to talk to the participating teachers about the design rationale of the textbooks. The intention again was not for the author to tell the teachers what to do, but to 'visibilise' her design rationales. Secondly, during data collection in the schools, a professional development theme on Grade 10 *functions* organised by the larger project was being conducted in the schools and the seven teachers participating in the study attended both events. I have to emphasise that the purpose of the workshop was not on textbook use at all but on opportunity to enhance teachers' mathematics on functions.

4.5.1 Making the Author's Design Rationale Visible

The meeting between the author and the participating teachers was scheduled for two hours, and it is here that the author availed the draft copies of the chapters on *functions* in the CAPS textbook to the teachers, for teachers to use as they saw fit. The meeting was also an opportunity for teachers to engage with the author on the author's perspectives of the major changes in the new *functions* chapters. The author went through the two chapters on *functions* in the textbook, demonstrating how they would use the textbook for teaching, which included the amount of time they would allocate for each section, and fundamentally why the author presented the textbook in the way she did. Two important observations I made in this meeting were that: a) the author explicitly indicated to teachers that she regarded the CAPS textbook as an improvement on the Pre-CAPS textbook and it was her wish that teachers would use the CAPS textbook for teaching *functions* right away even though it was still in draft form; and b) while the author had this wish, it was also made clear to teachers that the decision on how to use the textbook and which textbook to use rest solely with the teacher.

In all fairness, the meeting could have served to sway teachers towards a certain orientation of using the textbook, but I felt that it was a necessary meeting for teachers to have as it furnished them with the knowledge about their prescribed textbook which they could not have obtained elsewhere. I return to this in the conclusion chapter.

4.5.2 Professional Development on the Teaching of Functions

The seven teacher participants as mentioned are part of the programme for the larger project whose Content Focused Project (CFP) arm aimed to address issues of teachers' mathematical knowledge for teaching, and the teaching of *functions* had been earmarked as one of the topics to be addressed in 2011 in the project plans. Data collection for the study coincided with this programme which was addressing the teaching of *functions* at Grade 10. The programme which was run in collaboration with partners from the United Kingdom consisted of once-a-week workshops held at one school in each cluster, and continued for four weeks. The programme mostly attended to at least two aspects of the teacher-text interaction aimed at supporting teachers in effective use of curricular resources, namely

- identifying critical features of the topic of *functions*, and objects within this
- making visible the authors' and developers' pedagogic intentions as has already been alluded to.

All the teachers participating in the study attended these workshops and it is important to note here the possibilities that professional development programmes such as the one mentioned above indeed hope for, such as

- learning to craft customised solutions that meet their own teaching goals and their students' needs
- exploring which resources to use and how to use them, in addition to support for their subject matter knowledge and ways of teaching the content
- design skills required by teachers for effective use of curricular resources.

Thus the study took up data collection after and amongst these teacher development issues. However, it is important here to clarify that the study is not concerned with the impact of professional development, but to describe and explain the relationship between teachers and textbooks in the mediation of the object of learning, so as to enable reflection back on maximising the use of this key resource as a route to a possible solution in enhancing teaching and learning.

4.5.3 Data Collection Instruments and units of analysis

The next step was for me to determine the appropriate instruments for the kinds of questions I wanted to explore. In Table 4.2 below, I have captured each research question and how I collected the data for it, including the textbook itself as the source of data for the first question which seeks to determine how the CAPS textbook affords and constrains the teacher's practice.

Question	Data collection instrument	Unit of analysis
a) What are the affordances of the prescribed textbook to the teacher's practice?	Artefact (textbook), literature	Units in textbook
b) How do teachers mobilise the prescribed textbook?	Lesson observation (video) Interviews (audio) Field notes	A lesson and its episodes
c) To what extent are teachers aware of the affordances and constraints of the textbook in their practice?	Lesson observation (video) Interviews (audio) Field notes	Teacher utterances
d) What is the relationship between the textbook's affordances, teachers' pedagogical design capacity in the mediation of the object of learning? And how might this be explained?	Analysis and synthesis of above	

Table 4.2 Data Collection Instruments

For the first question the textbooks (Pre-CAPS and CAPS textbooks) were used as data for determining the affordances of the textbook, and the unit of analysis in this case was the predetermined unit in the textbooks. For teachers' mobilisation of the textbook, video-recording of lesson observations was used for data collection, and the unit of analysis became the lesson itself chunked into analysable episodes. Interviews were used as a source of data for determining the extent of teachers' awareness of the textbook, and here the utterances were used as units of analysis. Thus in each level of analysis, a different unit of analysis was used following the case study design. In the next sections, I relate how the processes of data collection unfolded in the study.

4.5.3.1 Lesson Observations

My intention had been to observe and video-tape three successive lessons (that is, lessons that follow each other directly or with no other lesson in between) on functions for each participating teacher. In my mind, this ideal situation would provide me with a deeper sense of the processes which take place as teacher plans for the next lesson, and how each lesson builds on from the previous lesson. I soon learned that, such plans were not easily realised in practice. Things went according to plan on the first day only. After that, the timetable had to be rescheduled almost daily for various reasons, including change of timetable, or teacher being absent from school, or teacher not being ready for me at all. However, at the end of the four weeks of lesson observations, I had video recordings of three non- successive lessons for five teachers. I had only two lessons observed for one teacher which were successive and due to timetable changes, it was not possible to reschedule the third and last lesson observation. I observed and recorded four lessons for the seventh teacher but only three are used for data as the teacher had to be called away on civic duty before one of the lessons was completed.

Table 4.3 below shows that a total of 20[8] lessons were observed, totalling roughly 17 hours of video time. As noted, one lesson was not used because it was deemed incomplete for the reasons advanced above. However, while many teacher lessons were not successive as planned, there were a few which still occurred one after the other. For example, teacher B1 had two successive lessons, one on a Friday, and the next one on the following Monday. If things had gone according to plan, the next lesson would have been the third of his lessons, but the timetable changed and it was not possible to get back to his classroom again. Teacher C1 has the first two lessons, lesson C11 and C12 successive even though there was a gap of a week between the two lessons when C1 was away from school for a week. Teacher A3 also has two of his three lessons successive, lessons A32 and A33.

	Teacher	Lesson	Date	Duration	Comment
1	A1	A11	09/05/2011	0:47:47	3 non-successive lessons
2	A1	A12	13/05/2011	0:48:17	
3	A1	A13	27/05/2011	0:51:53	
4a	A2	A21	12/05/2011	0:19:40	3 non-successive lessons
4b	A2		12/05/2011	0:18:27	
4c	A2		12/05/2011	0:08:23	
5	A2	A22	17/05/2011	0:51:43	
6	A2	A23	26/05/2011	0:45:43	
7	A3	A32	23/05/2011	0:51:23	3 lessons; lessons 1 and 2 successive
8	A3	A33	24/05/2011	0:49:18	
9	A3	A34	27/05/2011	1:00:04	
10	A4	A41	09/05/2011	0:47:43	3 non- successive lessons
11	A4	A42	23/05/2011	0:57:26	
12	A4	A43	26/05/2011	0:53:41	
13	B1	B11	13/05/2011	0:50:10	2 successive lessons; lesson 3 impossible to arrange
14	B1	B12	16/05/2011	0:33:47	
15	B2	B21	12/05/2011	0:45:58	3 non- successive lessons
16	B2	B22	16/05/2011	0:32:06	
17	B2	B23	01/06/2011	0:40:00	

[8] As noted, 21 lessons were observed and recorded but one lesson was omitted in the analysis for reasons provided

	Teacher	Lesson	Date	Duration	Comment
18	C1	C11	12/05/2011	0:58:09	3 lessons; lessons 1 and 2 successive even though spaced by a week
19	C1	C12	20/05/2011	1:07:22	
20	C1	C13	24/05/2011	1:03:41	
			TOTAL	**17:14:27**	

Table 4.3 A list of video recordings for the study

Besides the rescheduling of lessons which I have already mentioned, other challenges for the lesson observations included the setting up of the video cameras in some of the classrooms. Some of the classrooms were so tightly packed such that the available space was at the front of the class too near the chalkboard or near the door resulting in very poor lighting. Another incidence was when teacher used a purple-coloured chalk which the camera could not pick up on the chalkboard. The last challenge involved my experiences behind the video camera where at times I would find myself too immersed in the teaching and forgetting to move the camera accordingly. While this did not happen more than once, I nevertheless lost some important data which I did not capture on the camera due to the lapse in concentration. This brings me to an important consideration in this process about my role as a researcher and my behaviour which I elaborate below in section 4.5.3.3.

4.5.3.2 Interviews

While the lesson observations provided me with the information I needed in order to understand the teacher-text relationship, I supplemented these with individual interviews with teachers as they could serve to clarify and explain some of the questions emerging from the lesson observations, for example, teachers' motives and their feelings about the textbook. My original intention had been to study the transcribed video records of lessons in order to formulate ideas for which I needed clarification or explanation from teachers, and also what other aspects of their use of textbooks I felt needed to be probed further. I had envisaged the process of transcribing the videos and the initial analysis of the transcripts would take about six months to complete, but it took a lot more than that. The pilot of the interviews alerted me to the fact that I needed to have some insights into teachers' lessons before conducting the interviews with them. I ran a second pilot some twelve months after lesson observations after which I arranged for individual interviews with the teachers and travelled to the schools.

A year later after lesson observations, all the seven teachers in the three schools were still available and still willing to participate. I sent each teacher a copy of the semi-structured interview protocol (copy attached as Appendix B) a week prior to the scheduled date of the interview in order to prepare themselves for the interview, appreciating that a long time had lapsed between lesson observations and the interviews. Participants had also been requested to bring with them to the interview site all the different textbooks which they used for planning and teaching. When I got to the schools, I found that three of the seven teachers were no longer teaching Grade 10, but were teaching Grade 11. However, they were still willing to continue with the interviews. The interviews were set up to probe three important aspects in the teacher-text relationship:

i) teachers' familiarity or knowledge of their textbooks with respect to how the two CM textbooks compared. The interview took place after the topic of *functions* had been taught, and the CAPS textbook was then the prescribed textbook, at least in two of the three schools
ii) teachers' familiarity or knowledge of the curriculum requirements with respect to *functions* at both Grade 10 and Grade 11
iii) teachers' views about the impact of textbooks to teaching and learning in the context of the 'Limpopo textbook saga' wherein non-delivery of textbooks in schools in the Limpopo province in South Africa caused an uproar among stakeholders in the country.

I had asked all teachers to bring to the interview site all textbooks which they used in their practice for ease of reference, and I brought the Pre-CAPS and CAPS textbooks with me. However, only one teacher brought a textbook to the interview site, but then it was a teacher's guide which did not help as it only contained answers to questions. The interviews themselves were not easy to conduct as I found teachers talking quite generally about their textbooks; and their responses were also quite brief for some of the questions where I had hoped to elicit more information. As a result, I initially felt that the interviews did not serve the purpose of supplementing the data from lesson observations as I had intended them to, and thought they would not be usable in the study after all. However, towards the end of the analysis of the lessons, I found that the interviews could serve another purpose. I could use them to establish the extent of teachers' conscious awareness of the affordances of the textbook. In other words, what I first saw as absence (poor interviews) became a new presence for the study (Adler, in discussion).

4.5.3.3 My Role as a Researcher

Merriam (1998) mentions that a researcher can "assume one of several stances while collecting information as an observer; stances range from being a full participant – the investigator is a member of the group being observed – to being a spectator" (p.100). While I wanted my role to be one of a complete observer, it was not very easy to accomplish this due to the relationship already formed through interactions in professional development activities of the larger project. For over a year I had been interacting with the participating teachers in a role of a professional developer per se, which involved classroom observations and talking to teachers about their practice. Hence during data collection, while I was firmly planted behind a camera, on numerous occasions, teachers would request assistance with a mathematical concept for example, or I would have to take over a lesson when a teacher was called away in the middle of the lesson, as in the case of the lesson A31; and I would have to respond positively to those requests. Sometimes, teachers would request that I say something about their teaching at the end of a lesson, or say something to the learners as well.

In other words, while I had defined my role clearly to myself, teachers saw that role differently due to prior experiences and interactions, thus drawing me into a participant researcher. Although I feel that this had not affected the data collection process as such, it certainly influenced some of the ways with which I have interpreted the data, because I already knew so much about a particular teacher, which could not be completely ignored. I fetl some kind of tension as I considered this as a limitation on the study to some extent, even though on the other hand it was an advantage as I understood better the context of the schools and the teachers, an important influence on the study. Adler in (Adler, Ball, Krainer, Lin, & Novotna, 2005) cautions that it can be difficult to take a sceptical stance when one has an investment in a situation such as this, and argues for developing "strong and effective theoretical languages that enable us to create a distance between us and what we are looking at" (p.372). This allows researchers to navigate the tension of 'nearness and distance', hence a need for strong analytical tools and theoretical language which I develop for this study in the next section.

4.6 Analyzing the Data

There are various sets of data which are being analysed in the study, namely: the prescribed textbook, lesson observations; and individual interviews with teachers. The prescribed textbook was analysed for its affordances to the teacher's practice (*chapter 5*). The objective of the analysis was to determine what the textbook furnished the teacher with before embarking on how teachers appropriated these affordances in their practice (*chapters 6* and *7*). In *chapter 8*, the individual interviews with teachers were analysed in order to determine teachers' awareness of the affordances.

4.6.1 Determining the Affordances of the Textbook

The first analysis of the study in *chapter 5* involved the analysis of the structure (Brown, 2009; Remillard, 2012; Valverde et al., 2002) of the textbook in terms of the mathematical content and instructional approach of the textbook. The notion of the structure of the textbook has already been discussed in chapter 2. As has already been alluded to, the objective of this analysis was to establish resources which constituted as affordances of the textbook to the teachers' practice.

The analysis began with the development of a conceptual framework that would be used to assess the textbook resources. With respect to the content, the framework looked at what content was covered and how it was organized. Recruiting from Valverde et al.(2002), the constructs of *presentation formats* (partitioning blocks of content) and *performance expectations* (actions on function tasks) were adopted to describe how the content in the textbook was organized.

For describing the instructional approach built into the textbook, the notions of *quasi-deductive* and *quasi-inductive* approaches were coined to distinguish between pedagogical approaches that are didactic and investigative, respectively. The terms were derived from inductive and deductive pedagogic approaches in textbooks from the study by Ensor et al (2002).

The conceptual framework thus developed was used to analyze the chapters on *functions* in the Pre-CAPS and CAPS textbooks to determine the kind of mathematical content privileged in the textbooks with respect to the content areas covered and how these were sequenced in both textbooks. For the approach to the teaching and learning of the content, both textbooks were utilized to determine the presentation formats and their sequencing in order to determine if the approach was quasi-deductive or quasi-inductive. The CAPS textbook was further utilized to analyze for the performance expectations of the learners and how these were sequenced in order to establish whether the approach to Functions was pointwise or global (Even, 1998). Here each one of the tasks in the textbook (from worked examples, practice exercises, activities, and end of chapter exercises) was coded for a particular performance expectation, and the results determined groundedly (Strauss & Corbin, 1990). These produced five main performance expectations, namely: *substitute actions, plot and draw actions, read off actions, generalize properties actions, and interpret properties actions.* These codes were used to describe the content-specific approach to teaching the topic of Functions in the textbook.

4.6.2 Investigating how Teachers Mobilise the Textbook

The next stage of analysis in *chapters 6* and *7* involved an in-depth analysis of the transcripts of the video-taped lesson observations which began with the analysis of lessons for one teacher A1 in *chapter 6*. The objective was to use teacher A1's lessons to develop a framework and approach for analyzing the rest of the teachers' lessons in *chapter 7*

The process began with the development of conceptual tools for analyzing teacher A1's appropriation of the affordances of the textbook. This was based on the theoretical framework from *chapter 3* and the conceptual framework for analyzing the affordances of the textbook in *chapter 5*...

The analytical approach which I followed consisted of analyzing each lesson for mobilisation of the content and mobilisation of the approach. Each lesson was chunked into episodes which were coded for presentation formats, performance expectations, and comments about what content was taken from the textbook, inserted from external resources, or omitted from the textbook. This information helped in determining the extent of use of the textbook based on three aspects;

a) the coverage of content and how it was organized;

b) the degree to which the teacher offloaded, adapted, or improvised the content of the textbook;

c) the *opportunities for mediation* the teacher created for her learners in the classroom based on the content she injected into the lesson from external resources; the content from the textbook which she omitted in her lessons; and errors she committed in the lessons.

From these analyses, I was able to differentiate between *injections* of content which enhanced opportunities for the mediation of the object which I called *robust injections* and those which detracted from the opportunities for mediation which I termed *distractive injections.* Similarly, I differentiated between *productive omissions* which did not detract from the opportunities for mediation and *critical omissions* of content which I considered to be critical to the mediation of the object. I used the results of the mobilization of content to describe teacher A1's use of the textbook as *tacit* (Polanyi, 1967), as opposed to deliberate due to the presence of critical omissions in her lessons.

For the mobilization of the approach, the presentation formats in each episode and each lesson were sequenced in order of appearance and helped to determine whether the approach teacher A1 followed was *quasi-deductive* (or teacher-led and beginning with worked examples or explanatory talk by teacher) or *quasi-inductive* (or investigative and beginning with activities for learners). Furthermore, the performance expectations in each episode and each lesson were also sequenced to determine whether the approach to teaching and learning functions was pointwise or global or a progression from pointwise to global. From the manner in which teacher A1 mobilised the approach of the textbook, I was able to form an opinion about whether or not she perceived the approach of the textbook as an affordance in her practice.

The combined results of the mobilization of the content and approach were used to decide about teacher A1's relationship with her textbook which while I found to be strong, I also found not participatory, and therefore referred to it as being, *not intimate*. At this stage I could then infer about teacher A1's PDC, her capacity for pedagogical design when utilizing existing resources, as being, *not high*. I avoided the term 'low' on purpose until I had done similar analyses for the rest of the six teachers.

I had thus developed analytical tools which I could use in *chapter* 7 to explore the mobilization of the affordances of the textbook for the rest of the teachers' lessons, and to establish their levels of PDC.

Fig 4.2 below illustrates an overview of the conceptual framing for analyzing teachers' mobilization of the textbook

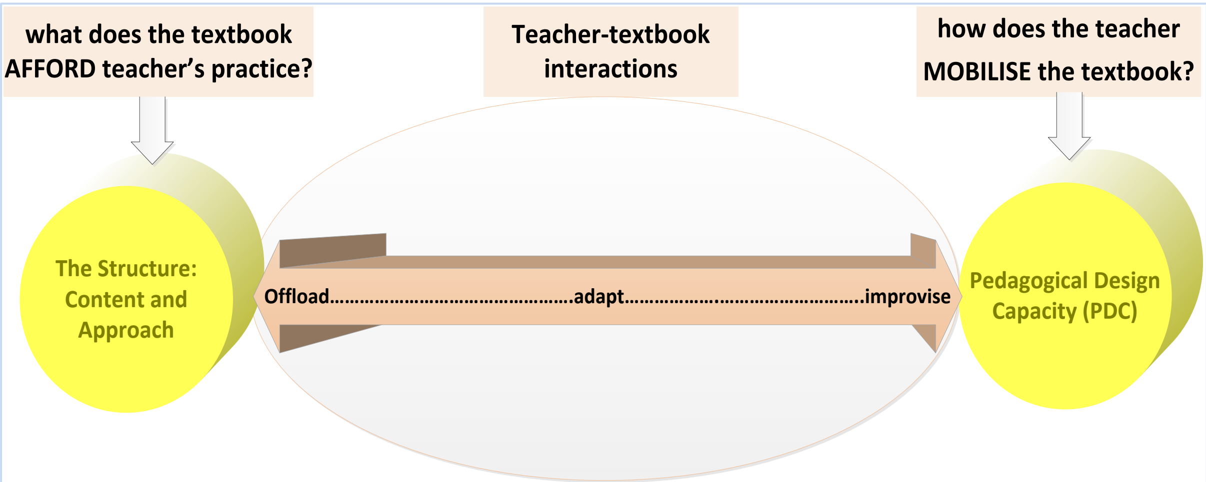


Fig 4.2 Conceptual Framing for Teachers' Mobilisation of the Textbook

The teacher-textbook interaction in Fig 4.2 involves a participatory collaboration between the textbook affordances and teachers' PDC. The textbook affords the teachers' practice with its resources determined from its structure (Brown, 2009; Remillard, 2012; Valverde et al., 2002), the question for *chapter 5* of the study. According to Brown (2009), teachers' appropriation of the affordances may be represented on a continuum that indicates the contribution of resources from the teacher and the textbook: the teacher *offloads* when she relies wholly on the textbook for the delivery of the lesson, *improvises* if she relies on her own resources or external resources for the delivery of the lesson, and *adapts* when the resources from the textbook and teacher are shared equally. Brown (2009) has defined PDC as the capacity for teachers to perceive and mobilise existing resources to craft productive instructional episodes, and therefore how teachers appropriate the affordances in their interactions with their textbook determine their level of PDC.

4.6.3 Determining Teachers Pedagogical Design Capacity

In *chapter 7* of the study, the analyses of the rest of the teachers' lessons were undertaken. The results of *chapter 6* had provided analytical tools for analyzing the lessons for the mobilization of the content; the mobilization of the approach; determining whether teachers' textbook use was deliberate or tacit; determining whether the teacher-textbook relationships were participatory or intimate; and consequently deciding whether the levels of PDC were high or low.

The analysis was undertaken for all twenty (20) lessons including lessons for teacher A1 analysed in *chapter 6*. This was due to the decision that I took to aggregate the lessons and look for patterns across all lessons regardless of the teacher, in order to be able to generalize about the teachers' use of textbooks instead of a looking at a particular teacher and what she did in her mobilization of the textbook.

In the first stage of analysis for mobilizing the content, each one of the twenty lessons was chunked into analyzable episodes and analysed for *coverage* of content; *degree of appropriation* of content; and *for opportunities for mediation,* as described in the previous section. Then a summary of each lesson followed with comments particularly on the resource used (if obvious), the *injections, omissions,* and *errors* made or occurring in each lesson.

The analysis led to the delineation of teachers' offloads and improvisations in order to explore the type of content teachers depended on the textbook for (offloaded) and what content they inserted from external resources (improvisations). The analysis also helped to differentiate between *improvisations* to lessons, as content that is available in the textbooks that teachers brought in from other resources; and *injections of content* that was not required by the relevant grade level but which teachers brought in anyway. Opportunities for mediation were determined through the explorations and categorisations of all *injections*, *omissions,* and *errors* occurring in lessons.

For the mobilization of the approach, all lessons were analysed for the degree of appropriation of the approach which illustrated whether the approach adopted in a lesson was quasi-deductive or quasi-inductive and whether each approach in the lesson was adopted from the textbook or not. Similarly for the content-specific approach on the teaching of functions, each lesson was analysed for whether the approach to functions was pointwise, global or a progression from pointwise to global strategies.

The results of the analyses of the mobilization of content and approach once again helped me to make claims about: a) whether teachers' textbook use was deliberate or tacit; b) whether the teacher-textbook relationships were participatory or intimate; and c) whether teachers'

PDC was high or low. A number of analytical terms were introduced in this and previous section which I present in Table 4.4 for ease of reference even for subsequent chapters.

Term	Description
Quasi-deductive approach	Investigative approach where learners engage with the concept before it is taught or told
Quasi-inductive approach	A didactic approach that is teacher-led in lessons or begins with worked examples or definitions illustrating a procedure
Injections	Content teachers bring into lessons that is not yet required by the curriculum at the current grade level
Robust injections	Injections of content which enhance opportunities for mediation of the object of learning
Distractive injections	Injections of content which detract from opportunities for mediation
Omissions	Content that is available in the textbook which teacher omits in the lesson
Productive omissions	Content omitted which does not detract from opportunities for mediation
Critical omissions	Content omitted from lessons that is critical to opportunities for mediation
Tacit textbook use	Teacher's use of textbook that is not deliberate characterized by distractive injections and critical omissions
Strong teacher-text relationship	A relationship between teacher and textbook where teacher mobilises both the content and the approach of the textbook
Intimate teacher-text relationship	Teacher-text relationship that is participatory in nature and which does not include critical omissions

Table 4.4 A list of analytical terms for the mobilization of the textbook

4.6.4 Establishing the Extent of Teachers' Awareness of the Affordances of the Textbook

Teachers' PDC requires that teachers should firstly perceive the affordances of the textbook and then mobilise them in productive ways in the classroom. A separate analysis of the teachers' interviews was conducted for each individual teacher in order to establish how aware teachers were of the affordances and possible constraints of the textbook to their practice, in light of the results obtained from chapters 6 and 7 about how teachers mobilised the textbook. Baxter and Jack (2008) caution that

> one danger associated with the analysis phase is that each data source would be treated independently and the findings reported separately. This is not the purpose of a case study. Rather, the researcher must ensure that the data are converged in an attempt to understand the overall case, not the various parts of the case, or the contributing factors that influence the case (p.555)

The interviews in the study were used to confirm and substantiate the claims the study made about how teachers appropriated the affordances. Furthermore, the chapter was not meant to be an in-depth analysis of the interviews but an attempt to explain why things happened the way they did in the lessons with regard to teachers' use of the textbook.

Each teacher's response to the questions in the interview schedule (Appendix B) according to the following categories was noted:

a) Curriculum and textbook: included questions on a) what teachers viewed as 'non-negotiables' for learners leaving grade 10 to grade 11; and b) teachers' views on textbook support for the 'non-negotiables'

b) Knowing my textbook: included questions on a) teachers' opinion on the differences between the CAPS and the Pre-CAPS textbooks; and b) teachers' use of the prescribed textbooks in planning of lessons

c) Limpopo[9] crisis: included questions on a) teachers' views on textbook's general support for teachers; and b) teachers' views on textbook support for learners.

Teachers' responses were tabulated and conclusions drawn about their awareness of the affordances of the textbook in practice.

4.7 Reporting the Study

The different analyses and findings of the study are reported in separate chapters of the study as shown below; and the analytical tools employed are also discussed in each chapter.

Chapter 5:	Affordances of the textbook
Chapter 6:	Teachers' Mobilisation of the Textbook: The Case of Teacher A1
Chapter 7:	Teachers' Pedagogical Design Capacity
Chapter 8	Teachers' Awareness of Textbook Affordances
Chapter 9:	Conclusions and Recommendations

4.8 Validity, Reliability and Generalizability

4.8.1 Validity

Maxwell (2005) defines the notion of validity in what he terms 'commonsense ways' as referring to "the correctness or credibility of a description, conclusion, explanation, interpretation, or other sort of account" (p.106). He argues that this commonsense use does not "pose any serious philosophical problems"(ibid) as it does not imply "the existence of any 'objective truth' to which an account can be compared"(ibid). Cohen et al.(2011) echo Maxwell arguing that validity in qualitative research "attaches to accounts, not to data methods… it is the meaning that subjects give to data and inferences drawn from the data that are important" (p.181). For Maxwell, what is key to validity is to show how one deals with validity threats in their research. He mentions researcher 'bias', or subjectivity of the researcher with respect to the selection of data that fits the researcher's preconceptions, or data that 'stand out' to the researcher as the most important validity threats that researchers should deal with.

[9] Limpopo is a province in South Africa where it was reported that textbooks had not been delivered to the schools well into six months of the school year

Cohen et al. (2011) show that

> it is very easy to slip into invalidity; it is both insidious and pernicious as it can enter at every stage of a piece of research. The attempt to build out invalidity is essential if the researcher is to be able to have confidence in the elements of the research plan, data acquisition, data processing analysis, interpretation and its ensuing judgement (p.198)

Therefore after identifying where invalidity lurks, researchers can then take steps to minimalize it as best as they can. McMillan and Schumacher (2010) suggest strategies to enhance validity including: use of multimethod strategies, for example, triangulation in data collection and data analysis; and mechanically recorded data.

4.8.2 Reliability

Cohen et al. (2011) describe reliability in general as

> essentially a synonym for dependability, consistency and replicability over time, over instruments and over groups of respondents. It is concerned with precision and accuracy…For research to be reliable if must demonstrate that if it were to be carried out on a similar group of respondents in a similar context (however defined), then similar results would be found (p.199)

However, as the authors point out, in qualitative research the suitability of the word 'reliability' is contested, which has to do with notions such as replication which could be viewed as distorting the natural occurrence of situations in qualitative research. As such, they note that other researchers replace reliability with other notions. For example, Lincoln and Guba (1985) (quoted in Cohen et al., 2011) replace reliability with 'credibility', 'neutrality', applicability', 'trustworthiness' and 'transferability', while showing more preference for 'dependability'. Dependability is also preferred by Brock-Utne (1996) also quoted in Cohen et al. In qualitative research therefore, reliability is regarded as "a fit between what researches record as data and what actually occurs in the natural setting that is being researched, that is, a degree of accuracy and comprehensiveness of coverage" (Cohen et al., 2011, p. 202). This means that as with validity, researchers need to address how dependable their research is.

In the study, a number of measures were put in place to address the issues of validity and reliability. The major source of data for determining teachers' pedagogical design capacity (PDC) has been the lesson observations. However, individual interviews with teachers were held as a way of triangulating the data. All the lesson observations were video recorded and interviews audio-taped for accuracy in recording what teachers were doing and saying in the classroom; a move for ensuring both validity and reliability. Furthermore, the analysis has been conducted very rigorously and systematically, and the work-in-process has been presented to colleagues at seminars, local, regional and international conferences for scrutiny and feedback. The coding of textbook exercises was done more than once and the last time it was done with a lapse of a year in between; similarly for codes for how teachers interacted with their textbooks. All these were done in order to ensure both credibility and dependability. Lastly, the theoretical underpinnings of the study resonate with those of other studies in the field of teacher-curricular materials relationship, and some of the results confirm results and findings of other studies in the field, thus a mark of reliability.

4.8.3 Generalizability

Generalizability in qualitative studies “is interpreted as generalizability to identifiable, specific settings and subjects rather than universally” (Cohen et al., 2011, p. 220). For example, in ethnographic research, generalizability is interpreted in two ways as ‘comparability’ and ‘translatability’ (LeCompte & Preissle, 1993). The characteristics of the group being studied should be explicitly described so that comparisons may be made to similar or dissimilar groups by readers, for comparability.

For translatability, the characteristics to be made explicit are those of analytic tools used. Maxwell (2005) distinguishes between internal and external generalizability, arguing that in the process of data analysis, it should be pointed out to whom the conclusions drawn are generalizable. The internal generalizability refers to “the generalizability of a conclusion *within* the setting or group studied” (p.115); while external generalizability, which Maxwell points out that it is “often not a crucial issue for qualitative studies” (p.115), refers to ability to generalise beyond the group being studied.

For comparability, the characteristics of the group that is being studied need to be made explicit so that readers can compare them with other similar or dissimilar groups. For translatability the analytic categories used in the research as well as the characteristics of the groups are made explicit so that meaningful comparisons can be made to other groups and disciplines.

Larsson (2009) argues that on the whole qualitative studies have difficulties in avoiding making claims about generalization since it would “reduce the interest in many qualitative studies to practically nothing” (p.31), and goes on to point out that

> if someone has made a study of a classroom in the spring of 2005, it is difficult to take seriously if there are no ambitions to say something that can be of use outside this situation in time and space and the persons involved (Larsson, 2009, p. 31)

Thus echoing Wolcott’s (1994) view that there would be no point conducting qualitative research if there is no *capacity* for generalisation. Larsson argues for three lines of reasoning that would enhance generalisation potential as: a) covering more of the variation in qualitative different views; b) generalization through context similarity between the researcher and the wider contexts; and c) generalization through recognition of patterns that are similar between research and other contexts.

In the earlier sections of this chapter, I have provided explicit descriptions of the teachers and their school and local contexts. This takes care of the comparability that might be needed for generalisations. Additionally, the analytic tools which I have used in the study have been developed from other studies, and from the data itself, to take care of the issue of translatability in the study. The study makes generalisations that are internal (Maxwell, 2005) and within similar contexts, while at the same time providing implications for other contexts.

4.8 Limitations

As with all case studies, this study is bounded by limitations. One of them is that learners are out of view as noted. Other limitations emerge in chapters 5 through to chapter 8, and are described in the appropriate chapters as they emerge.

4.9 Conclusion

The chapter shows the complexity of the teacher-textbook relationship being investigated in the study, with changing units of analysis that are apparent on different levels of analysis. This makes the study complex but also interesting to see how the different units of analysis link together to make us understand better what happens when teachers and textbooks interact. I would like to also point to an important feature in the study which is implicit in the chapter; that the learners are in the background. The study does not analyze the mediation of the object of learning per se, but the opportunities for mediation that the teacher-textbook interactions open up in the classroom. The analyses commence in the next chapter beginning with determining the affordances of the textbook.

CHAPTER 5
Affordances of the Textbook

> Even when the goals and their specificity are the same across countries or within the same country, the precise manner in which these goals are sequenced in textbooks will likely vary. This is the "signature" of a textbook, its own particular interpretation of the best structure by which to present the prescribed content.
> (Valverde et al., 2002, p. 53)

5.1 Introduction

The purpose of this study is to investigate teachers' relationship with their textbooks in their practice. In other words, the study seeks to explore how teachers appropriate (Brown, 2009; Wertsch, 1998) the affordances of the textbooks, which begs the question, what these affordances to the teacher's practice are. This question guides the exploration in the present chapter, in which the textbook is analysed for its affordances to the teacher's practice. My conjecture is that teachers 'worry' about two things when they go into the classroom to teach: the content that is supposed to be taught as well as how to teach that content. These are inevitably the components of the structure of the textbook, and therefore I argue in this study that, analysing the textbook for its affordances to the teacher's practice is tantamount to analysing the structure of the textbook.

From chapter 2 of this study, the structure of the textbook which describes how the textbook is organised and what is contained in the textbook, is said to communicate particular mathematics content, plus its sequencing and exemplification together with a particular orientation towards that content the textbook wants to convey (Brown, 2009; Ensor et al., 2002; Remillard, 2012); which Valverde et al. (2002) refer to as advancing "a distinct pedagogical model" (p.54). Therefore, analysing the structure of the textbook implies analysing the particular mathematics content in the textbook, as well as the textbook's particular approach towards the teaching and learning of that content. Thus, I argue further that the textbook furnishes two distinct affordances to the teacher' practice, namely, the *mathematical content* as well as the *approach to the teaching and learning of the content,* referred to henceforth as the *content* and the *approach.*

I argue that analysing the affordances of the textbook to the teacher's practice in this study therefore constitutes analysis of the *content* and the *approach* of the textbook, for which a framework for analysis is needed. However, I have to declare upfront that what the analysis here does not do is to make value judgements about the textbook itself in terms of how 'good' or not the textbook is; the analysis is solely an investigation of the perceived and actual characteristics of the textbook for teachers' practice.

5.2 A Framework for Textbook Analysis

In this section, I develop a conceptual framework for analysing the *content* and the *approach* of the textbook.

5.2.1 The Mathematical Content

The *mathematical content* consists of the *nature of the content* which describes the *content areas* that comprise the topic on functions at grade 10 level in each textbook. The *nature of content* also depicts the ordering of the content areas in the textbooks. Hence the *nature of content* is about which content areas under the topic are emphasised by the textbook and in what order.

5.2.2 The Approach

The *approach* of the textbook is described through how the *content* areas are partitioned, as well as through the kinds of actions the textbook developers expect of learners when dealing with the content. Valverde et al. (2002) refer to the partitioning blocks of content as *presentation formats*, and the actions expected of learners as *performance expectations*. The *presentation formats* include aspects such as definitions, worked examples, practice exercises, and so forth. The *performance expectations* in the study represent the actions on function tasks (Leinhardt et al., 1990) expected of learners, for example, substituting, drawing graphs using point-by-point plotting and so forth. Of critical importance to the approach in the textbook is the sequencing of these *presentation formats* and the *performance expectations*. The sequencing of the *presentation formats* points to particular approaches to the teaching and learning of the content, referred to as the [10]*quasi-deductive* and the *quasi-inductive approaches*. The sequencing of *performance expectations* on the other hand, conveys a particular approach to the teaching and learning of functions, the *pointwise approach* or the *global approach* to the teaching of functions (Even, 1998). The different approaches are described in detail in the next sections.

5.2.2.1 The Quasi-Deductive and Quasi-Inductive Approaches

As the analysis shall show, the introduction of some *content areas* begins with some form of explanatory text, that is, a narrative or definition, followed by practice exercises, while in others, the introduction is done through activities that give learners opportunity to firstly engage with the concepts and draw conclusions before worked examples or summaries are provided in the textbook. These two examples illustrate two different *approaches* to the teaching and learning of the *content*.

In chapter 2 and my review of the literature, I discussed Ensor et al.'s (2002) identification of two dominant *pedagogic approaches* in mathematics textbooks as the *deductive* and the *inductive pedagogic approaches*, respectively. In the *deductive approach*, the textbook states

[10] The study notes the existing tension between inductive/deductive reasoning in mathematics which distinguish between abstraction and generality, versus inductive/deductive pedagogy which distinguish between didactical and investigative approaches in teaching

definitions, which are then exemplified, and then learners would be given opportunity to practice. The *inductive approach* on the other hand starts with learners engaging in a range of activities relating to the concept to be discussed, the engagement which leads to definitions, and possibly to practice exercises as well. Recruiting from these authors, the study coins the *quasi-deductive* and *quasi-inductive approaches,* to describe the sequencing of the *presentation formats* in the textbooks. In this study hence, the *approach* in which a content area begins with an introductory exercise meant to engage learners in some form of discovery, or a 'hands-on' engagement per se, is referred to as a *quasi-inductive approach.* On the other hand, in instances where the content area commences with an explanatory text or worked examples followed by practice exercises, that is, where a concept is learned through worked examples, then this is termed a *quasi-deductive approach.* The *quasi-deductive* and *quasi-inductive approaches* form part of the framework for analysing the textbook.

5.2.2.2 Pointwise and Global Approaches to Functions

The content areas in the textbooks identified in this chapter show an underlying linking of representations in the teaching and learning of functions. For example, given an equation of a function, it can be represented as a table of values, or as a graph; or the graph of an equation can be sketched by interpreting the properties of the function without having to draw a table of values. This illustrates the difference between the pointwise approach and the global approach to functions (Even, 1990, 1998), which were discussed in chapter 2. When the graphing of functions from their equations is achieved through point-by-point plotting of points, or when values are read from a given graph, this is approaching a function *pointwise*.

On the other hand, sketching of functions from their graphs by looking at the function in a more holistic and interpretive manner and such examples which consider the behaviour of the function holistically, are examples of approaching functions *globally*. The pointwise and global approaches to functions depend on the kinds of actions on functions task, and therefore the performance expectations of the learners. Thus, depending on the sequencing of the presentation formats in the content area, the conception of function conveyed in the textbook could be strictly pointwise; strictly global; a progression from a pointwise to a global approach; or it could depict a flexibility of movement between the pointwise and the global.

The pointwise and global approaches to the teaching of functions are included in the framework and they point to the approach to the teaching of functions that the textbook affords to the teacher's practice.

5.2.3 The Framework

Drawing from the literature mentioned above and from chapter 2 of this study, the framework for analysing the affordances of the textbook is suggested as shown in Fig 5.1.

In the framework in Fig 5.1, the structure of the textbook divides into two main categories as discussed: the *mathematics content*, and the *approach* to the teaching and learning of this content. The *mathematics content* depicts the *nature of content* from which is determined the *content areas* and their *ordering.* The *approach* divides into the *organisation of content* which yields the kinds of *presentation formats* and *performance expectations* featured in the textbook; the sequencing of the *presentation formats* which points to whether the *approach* adopted is *quasi-deductive* or *quasi-inductive*; as well as to the sequencing of *performance expectations* that determines the conception of function conveyed with respect to whether the expected *actions on function tasks* employ pointwise or more interpretive global strategies.

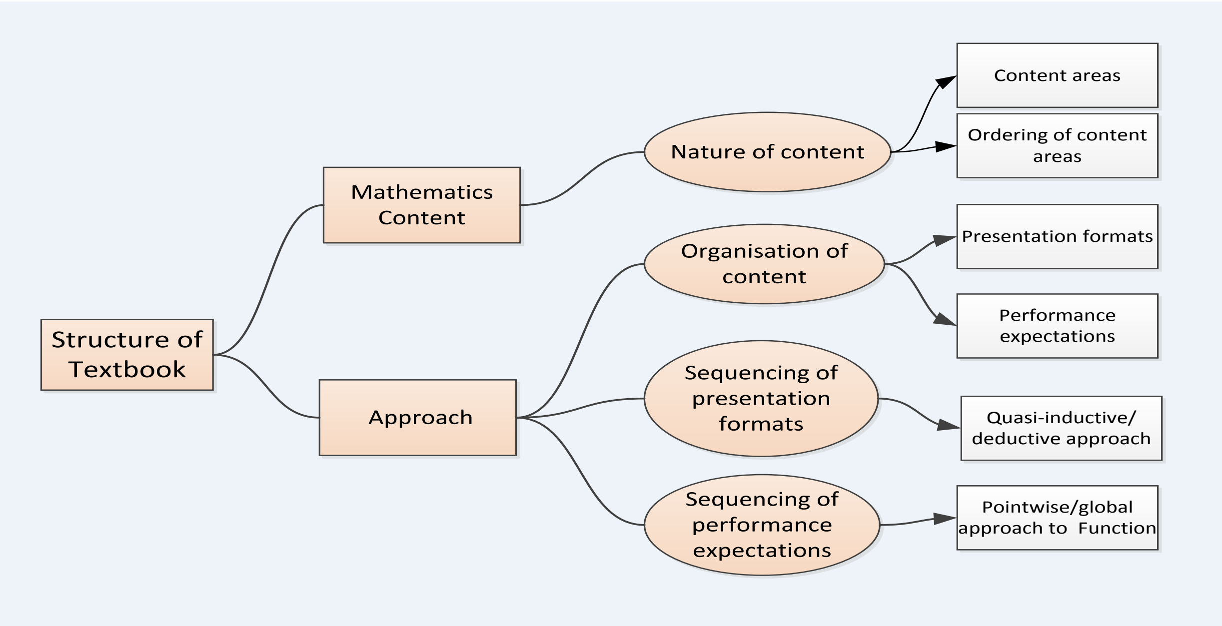


Fig 5.1 A Framework for Analysing the Textbook

In the next sections, the analysis of the textbook is undertaken starting with the *content* and then followed by the *approach*. As stipulated in chapter 4, there are two editions of the textbook, the Pre-CAPS and the CAPS textbooks, respectively, under consideration in the study for the reasons already outlined, and since teachers utilise these two textbooks interchangeably, the analyses are performed on both of them unless otherwise stated.

5.3 Analysis of the Mathematical Content

The two textbooks under consideration in the study namely, the Pre-CAPS and the CAPS textbooks, feature similar mathematical content since the curriculum requirements from the NCS[11] to the CAPS curriculum did not change for the topic of functions at Grade 10. However, there is one major difference in the composition of the chapters dealing with Functions involving trigonometric functions. In the Pre-CAPS textbook, a chapter is devoted to trigonometry and trigonometric functions, where trigonometry is introduced for the first time to learners. This leads to the study of trigonometric functions. In the CAPS textbook, trigonometry is done separately and introduced prior to the introduction of functions. Trigonometric functions are then incorporated in the chapter featuring transformations and interpretation of function properties. Furthermore, in the lesson observations, there are very few occasions where trigonometric functions feature. As a result of these discrepancies,

[11] The NCS (National Curriculum Statement) refers to the curriculum statement in place prior to the CAPS (Curriculum and Assessment Policy Statement) which was implemented at Grade 10 in 2012

trigonometry and trigonometric functions have been excluded from the analyses of the textbooks. However, there are lessons in which content on trigonometric functions appears, but these shall be dealt with as they arise.

The analysis of the *content* entails identifying the content areas featuring in the Pre-CAPS and the CAPS textbooks (excluding trigonometric functions), and then indicating how they are ordered. An indepth analysis of the content in each textbook was undertaken and the results compared to compile a summary of the critical content areas featured in both textbooks as illustrated in Fig 5.2.

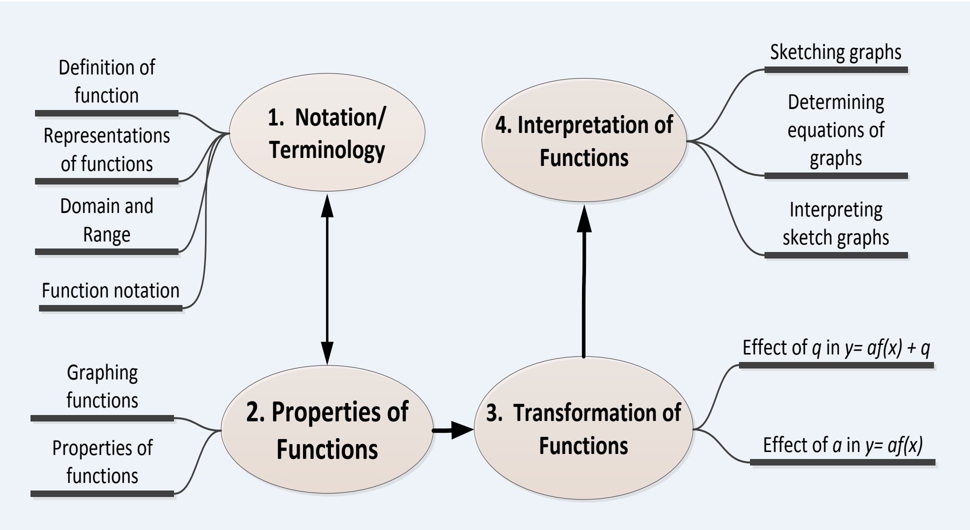


Fig 5.2 Content Areas and their Ordering in the Pre-CAPS and the CAPS Textbooks

Fig 5.2 shows four major *content areas* (CA) featuring in the Pre-CAPS and the CAPS textbooks. These include: *Notation/Terminology* which consists of introductory aspects of the topic of function, for example, the definition of function, representations of functions, domain and range of a function, and illustrating function notation. *Properties of functions*, forms the second content area and comprises the graphing of the function classes and determining their properties. The double arrow between *Notation/Terminology* and *Properties of Functions* illustrates that the units between the two content areas can be interchangeable in the two textbooks. For example, in the Pre-CAPS textbook, the graphing of functions is included in both the first and the second content areas. *Transformation of functions* forms the third content area under which the effect of the parameters a and q in functions of the form $y = af(x) + q$, are explored. This is followed by the last content area, *Interpretation of functions*, which involves sketching graphs given equations by interpreting the properties of the given equation, determining equations of given functions, and interpreting sketch graphs.

Thus, the two textbooks afford the teacher with the aforementioned content areas in which issues of notation and terminology are dealt with first, followed by the graphing of the quadratic function, the hyperbola, and the exponential functions, together with the identification of their properties. The vertical transformations of the functions, reflections over the x-axis, and the vertical stretch of these functions follow the identification of their properties. The last aspect that the textbooks expect to be taught and learned is the capability

to interpret functional properties; to be able to determine an equation of a given graph through interpreting its properties, and to be able to sketch a graph of a function given its equation without point-by-point plotting, but by interpreting the properties of the given equation.
The description of the content areas above corroborates the curriculum requirements for grade 10 functions which state that learners should be able to

> discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable)…investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$ ….sketch graphs, find the equation of given graphs and interpret graphs (CAPS, 2011, p. 24)

The content areas in the Pre-CAPS and the CAPS textbooks cover the objectives of the curriculum statement above on grade 10 functions, thus confirming that one of the affordances of the textbook to the teacher's practice is to interpret the curriculum statement and act as the 'potentially implemented curriculum' (Valverde et al., 2002).

The findings in the analysis of the *content* above do not necessarily mean that all the content in the two textbooks is similar. There are some differences in the nature of the content but which do not pertain to the content areas. However, it is beyond the scope of the present study to delve into those differences. All that the findings are pointing to are the major content areas and how they are ordered in the textbooks.

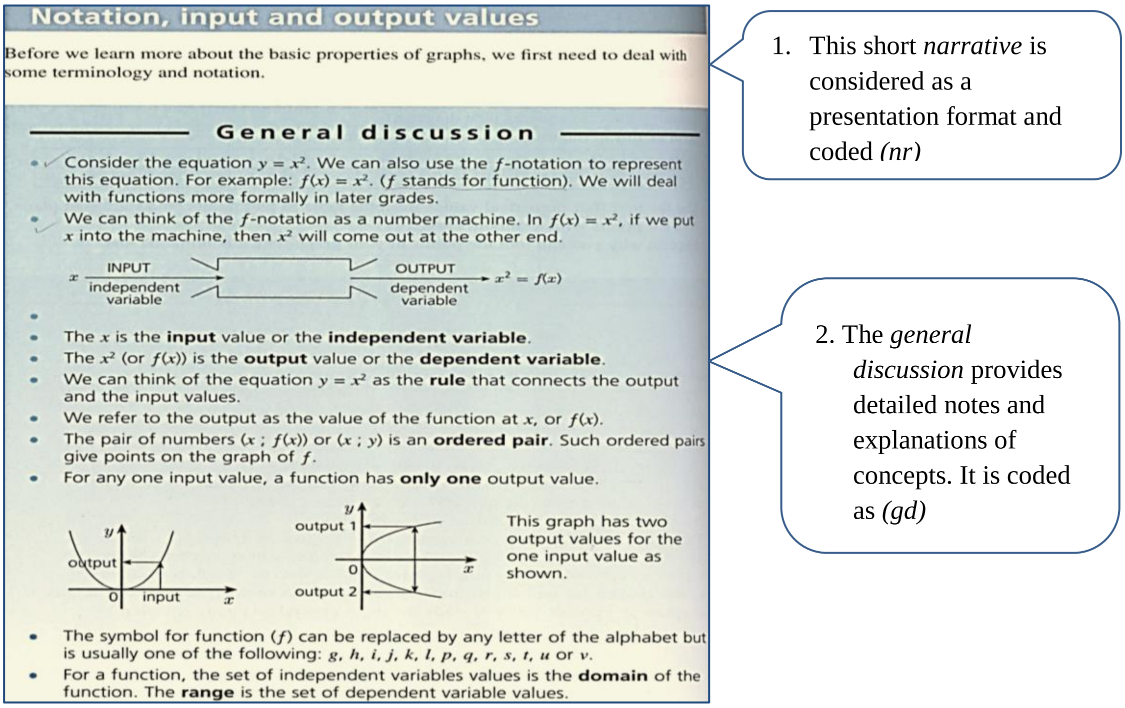
In the next section, the analysis of the *approach* is undertaken.

5.4 Analysis of the General Approach to the Teaching and Learning of the Content

The analysis of the approach of the textbook was conducted in two stages. Firstly, the presentation formats in each textbook were identified and their sequencing determined, to guide whether the approach to the teaching and learning of content was *quasi-deductive* or *quasi-inductive*. Each textbook was analysed separately and then the results compared. The second stage was the identification of performance expectations in order to determine the approach to the teaching of functions through the sequencing of the performance expectations.

5.4.1 Presentation Formats in the Pre-CAPS Textbook

The analysis began with identifying the different partitioning blocks of content in the textbook and coding them. These constituted the *presentation formats* in the chapters dealing with functions. For example, Fig 5.3 shows an extract taken from the Pre-CAPS textbook, to illustrate how the process of analysis was conducted. In the extract, different presentation formats are numbered, and then coded.


Notation, input and output values

Before we learn more about the basic properties of graphs, we first need to deal with some terminology and notation.

General discussion

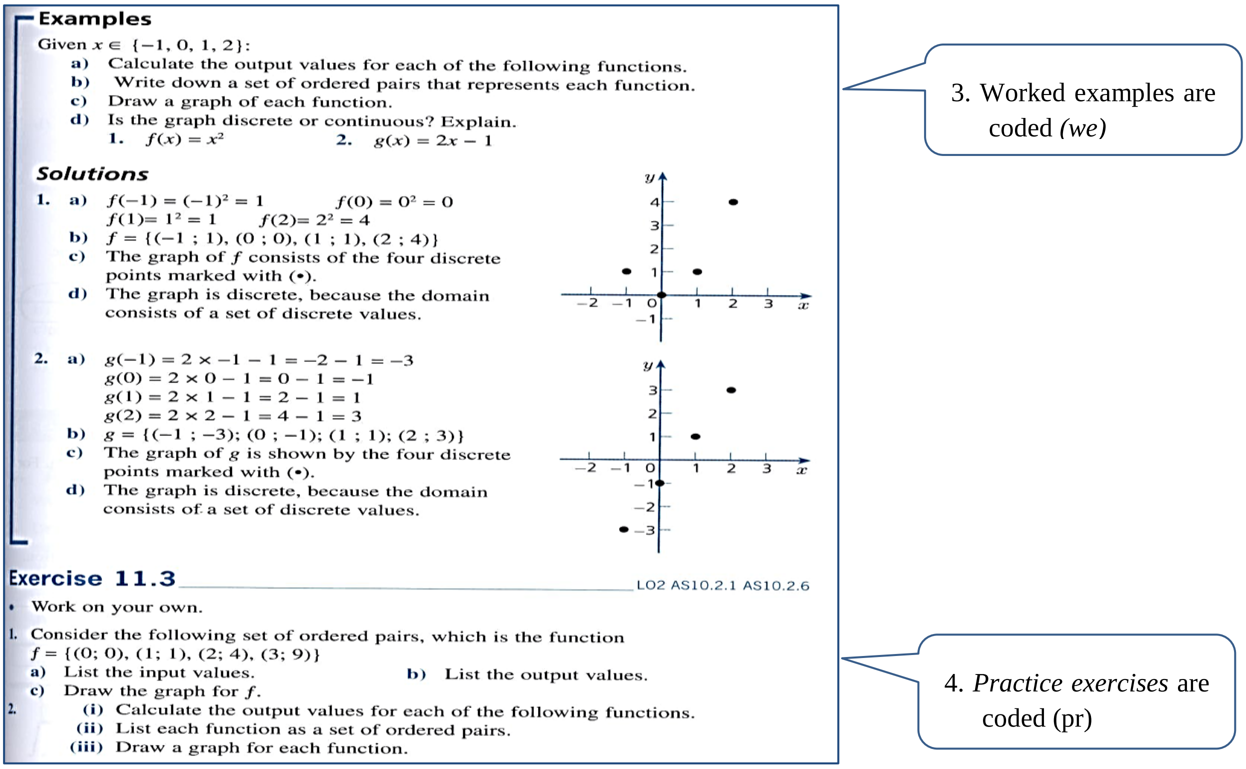
- Consider the equation $y = x^2$. We can also use the f-notation to represent this equation. For example: $f(x) = x^2$. (f stands for function). We will deal with functions more formally in later grades.
- We can think of the f-notation as a number machine. In $f(x) = x^2$, if we put x into the machine, then x^2 will come out at the other end.

- The x is the **input** value or the **independent variable**.
- The x^2 (or $f(x)$) is the **output** value or the **dependent variable**.
- We can think of the equation $y = x^2$ as the **rule** that connects the output and the input values.
- We refer to the output as the value of the function at x, or $f(x)$.
- The pair of numbers (x ; $f(x)$) or (x ; y) is an **ordered pair**. Such ordered pairs give points on the graph of f.
- For any one input value, a function has **only one** output value.

- The symbol for function (f) can be replaced by any letter of the alphabet but is usually one of the following: $g, h, i, j, k, l, p, q, r, s, t, u$ or v.
- For a function, the set of independent variables values is the **domain** of the function. The **range** is the set of dependent variable values.

Fig 5.3 A narrative and a general discussion from the Pre-CAPS textbook (Laridon et al., 2008, pp. 264-265)

In Fig 5.3, two presentation formats are identified, namely a short *narrative*, which is coded as (*nr*) that introduces the section, and a *general discussion*, coded as (*gd*) which provides detailed notes, illustrations and explanations of the elements of the content area. A page that follows in the textbook shown in Fig 5.4 shows two more presentation formats, namely, *worked examples* which are coded as (*we*), and then *practice exercises* given the code of (*pr*).


Examples

Given $x \in \{-1, 0, 1, 2\}$:

a) Calculate the output values for each of the following functions.
b) Write down a set of ordered pairs that represents each function.
c) Draw a graph of each function.
d) Is the graph discrete or continuous? Explain.

1. $f(x) = x^2$ 2. $g(x) = 2x - 1$

Solutions

1. a) $f(-1) = (-1)^2 = 1$ $f(0) = 0^2 = 0$
$f(1) = 1^2 = 1$ $f(2) = 2^2 = 4$
b) $f = \{(-1 ; 1), (0 ; 0), (1 ; 1), (2 ; 4)\}$
c) The graph of f consists of the four discrete points marked with (•).
d) The graph is discrete, because the domain consists of a set of discrete values.

2. a) $g(-1) = 2 \times -1 - 1 = -2 - 1 = -3$
$g(0) = 2 \times 0 - 1 = 0 - 1 = -1$
$g(1) = 2 \times 1 - 1 = 2 - 1 = 1$
$g(2) = 2 \times 2 - 1 = 4 - 1 = 3$
b) $g = \{(-1 ; -3); (0 ; -1); (1 ; 1); (2 ; 3)\}$
c) The graph of g is shown by the four discrete points marked with (•).
d) The graph is discrete, because the domain consists of a set of discrete values.

Exercise 11.3 LO2 AS10.2.1 AS10.2.6

- Work on your own.

1. Consider the following set of ordered pairs, which is the function $f = \{(0; 0), (1; 1), (2; 4), (3; 9)\}$
a) List the input values. b) List the output values.
c) Draw the graph for f.
2. (i) Calculate the output values for each of the following functions.
(ii) List each function as a set of ordered pairs.
(iii) Draw a graph for each function.

Fig 5.4 Worked Examples and a Practice Exercise from the Pre-CAPS textbook (Laridon et al., 2008, pp. 264-265)

The practice exercises in Fig 5.4 are not necessarily similar to the worked examples which precede them. For example, there are two worked examples in the section which involve evaluating function values from a set of ordered pairs, and then drawing the graphs which are both discrete. The practice exercise shown includes more questions which are not shown in Fig 5.4 due to lack of space, but which include: evaluating function values for various functions given in functional notation; evaluating function values with a variable as input instead of an integer, for example, evaluating $f(a), f(-a)$ and $f(a+1)$, given $f(x) = -\frac{x^2}{2}$. Other questions include finding the input value if given the output for a specified function. The practice exercise includes contextual questions as well. In other words, the practice exercises are not necessarily a replica of the worked examples, but feature extra questions which demand more skills than those demonstrated in the worked examples.

There are four more presentation formats which are different from those already shown in this textbook. They include a set of questions referred to as *activity*, coded as (*act*); and three types of end of chapter exercises called *check your skills* given a code (*ch*), *apply your skills* (*ap*), and *problem solving* (*ps*) exercises. An example of such an *activity* is shown in Fig 5.5

Activity 13.3 LO2 AS10.2.1 AS10.2.2 AS10.2.3

- Work in groups of six.

1. a) Assign to each member of the group a different function from the list below:
   (i) $y = ax$
   (ii) $y = ax^2$
   (iii) $y = \frac{a}{x}$
   (iv) $y = a.2^x$
   (v) $y = a.\sin x$
   (vi) $y = a.\cos x$
   b) Each member must plot graphs of their function for the following values of a: $1, 2, 3, -1, -2, -3, \frac{1}{2}$ and $-\frac{1}{2}$.
2. Once each group member has plotted graphs of his/her assigned function he/she must meet with members from other groups, who have been assigned the same function, to discuss observations about the effect of different values of a on:
   a) average gradient over various intervals
   b) intercepts on the axes (if there are any)
   c) turning points (if any)
   d) any other features that were noticed.
3. Learners then return to their original groups and
   a) identify effects that are the same for all functions
   b) write a group report on these general observations.
4. For assessment purposes each group is then to submit:
   a) all graphs drawn by each member
   b) the observations made for each function
   c) the group report on general observations.

Fig 5.5 An example of an *activity* from the Pre-Caps Textbook (Laridon et al., 2008, p. 323)

The extract shown in Fig 5.5 is an *activity* on investigating the effect of the parameter a in the equations of the form, $y = af(x)$. This particular *activity* is not preceded by another presentation format, but is followed by a section on 'finding the equations of functions given sufficient information' which begins with two *worked examples* (we) followed by a set of *practice exercises*. The *activity* thus gives opportunity for learners to engage with the functions of the form $y = af(x)$ and be able to draw conclusions about the effect of the parameter a (and its negation, $-a$) on the function $y = f(x)$ before worked examples which utilise the generalisations drawn from the *activity* are demonstrated.

With respect to the presentation formats in the Pre-CAPS textbook, a total of eight were identified as shown in Fig 5.6. However, since there was only one narrative, I grouped it together with *general discussions* to form a presentation format which I call, *explanatory texts* (*exp*). The *worked examples* and *practice exercises* remain distinct presentation formats because they are not the same, but I have grouped the three end-of-chapter exercises into a category of presentation formats which I call the *assessment exercises* (*as*). A list of all presentation formats in the Pre-CAPS textbook is presented in Table 5.1 below.

Identified presentation formats		Final categories of presentation formats	
Narrative	nr	Explanatory text	exp
General discussion	gd		
Worked examples	we	Worked examples	we
Practice exercise	pr	Practice exercise	pr
Activity	act	Activity	act
Check your skills exercise	ch	Assessment exercise	as
Apply your skills exercise	ap		
Problem solving exercise	ps		

Table 5.1 Presentation formats in the Pre-CAPS textbook

Table 5.1 shows that the Pre-CAPS textbook yielded a total of five presentation formats, namely, *explanatory texts (exp); worked examples (we); practice exercise (pr); activity (act);* and *assessment exercises (as).* In order to determine the approach in the Pre-CAPS textbook, the sequencing of these presentation formats was explored and this is reported in the next section.

5.4.2 The General Approach in the Pre-CAPS Textbook

A process of analysis for the approach in the Pre-CAPS textbook is presented in Table 5.2. To reiterate, these results include the two chapters dealing with functions in the Pre-CAPS textbook excluding the chapter on trigonometry and trigonometric functions (see section 5.3 for details).

In the analysis, the subheadings in the chapters dealing with functions were listed in the same order that they appeared in the textbook. The content area, abbreviated, CA, under which each subheading featured was then identified as follows: The first content area was referred to as CA1, the second content area as CA2, the third as CA3, and the fourth content area as CA4. Then the sequence which the presentation formats followed was determined by indicating with a number how they followed each other. For example, if worked examples came first under a particular subheading, this was indicated with the number, i. The next presentation format was numbered ii, and so forth. The final decision on whether the approach was quasi-deductive or quasi-inductive was based on whether the sequencing of presentation formats in a subheading began with an *activity* or the *explanatory text/worked example* combination. If the sequence of presentation formats began with an *activity (act)*, then the *approach* was *quasi-inductive*, and *quasi-deductive* if the sequence began with either an *explanatory text (exp)* or *worked examples (we)*

Subheadings	Content area	Presentation Formats					Sequencing of Presentation Formats					The Approach
		Explanatory text (exp)	Worked example (we)	Practice exercise (pr)	Activity (act)	Assessment exercises (as)	**direction of progression** →					
Drawing graphs of functions	Notation and Terminology (CA1)	ii, iv			i, iii		act	exp	act	exp		quasi-inductive
Notation, input and output values		ii	iii	iv			exp	we	pr			quasi-deductive
Features of curves	Properties of functions (CA2)	ii	iii	iv	i		act	exp	we	pr		quasi-inductive
Mathematical modelling		i			ii		exp	act				quasi-deductive
	Assessment exercises					ch, ap, ps	as					
Symmetry (reflections)	Transformations of functions (CA3)	ii			i		act	exp				quasi-inductive
Average gradient	Properties of functions (CA3)	i	i	ii			exp/we	pr				quasi-deductive
Functions of the form $y = a.f(x)$	Transformations of functions (CA3)				i		act					quasi-inductive
Finding the equation of a graph, given sufficient information	Interpretation of functions (CA4)		i, ii	iii			we	we	pr			quasi-deductive
Sketching graphs, given the defining equation			i, ii, iii	iv			we	we	we	pr		quasi-deductive
Functions of the form $y = a.f(x) +$	Transformation of functions (CA3)				i		act					quasi-inductive
Drawing and interpreting sketch graphs	Interpretation of functions (CA4)		i, ii, iii, iv	v			we	we	we	we	pr	quasi-deductive
Focus on gradient (modelling)	Application of function properties (CA4)				i		act					quasi-inductive
	Assessment exercises					ch, ap	as					

Table 5.2 Determining the Presentation Formats in the Pre-CAPS Textbook

Table 5.2 shows some patterns in the sequencing of presentation formats. All subheadings begin with an *activity*, an *explanatory text* or a *worked example*. For the subheadings beginning with an *activity*, either the *activity* is on its own and the next subheading follows immediately, or the next presentation format is an *explanatory text*. For the *activities* followed by another subheading, those subheadings all begin with worked examples. This means that the quasi-inductive approach in this textbook involves sequences which begin with an *activity* followed by an *explanatory text* or *worked examples*. Table 5.3 provides a list of all subheadings which advance the quasi-inductive approach in the Pre-CAPS textbook.

Subheadings advancing the quasi-inductive approach	Content area
Drawing graphs of functions	CA1
Features of curves	CA2
Symmetry (reflections)	CA3
Functions of the form $y = a.f(x)$	CA3
Functions of the form $y = a.f(x) + q$	CA3
Focus on gradient (modelling)	CA4

Table 5.3 Subheadings advancing the quasi-inductive approach in the Pre-CAPS Textbook

All the subheadings in Table 5.3, except for the 'focus in gradient' which is a mathematical modelling exercise, belong to the first three content areas (CA1, CA2, CA3) and involve generalisations of the properties of the function classes investigated and their transformations.

Going back to Table 5.2, the subheadings which begin with an *explanatory text* and/or *worked examples* reflect some patterns in their sequencing of presentation formats as well. They fall into two groups: those which involve *explanatory texts* and those which do not include it. Table 5.4 lists these subheadings and the content areas in which they belong.

Subheadings beginning with 'explanatory text' or 'worked examples'	Content area
Notation, input and output values	CA1
Mathematical modelling (property of functions)	CA2
Average gradient	CA3
Finding the equation of the graph given sufficient information	CA4
Sketching graphs, given the defining equation	CA4
Drawing and interpreting sketch graphs	CA4

Table 5.4 Subheadings advancing the quasi-deductive approach in the Pre-CAPS Textbook

The first three subheadings in Table 5.4 involve definitions or explanations and descriptions of terminology, and therefore begin with an *explanatory text* which is then followed with a *worked example(s)* and *practice exercises*. The exception is the subheading on 'mathematical modelling' which involves an *activity* after the *explanatory text*. For the last three subheadings their sequences involve *worked examples* followed by *practice exercises*. Thus, the *quasi-deductive approach* in the Pre-CAPS textbook is advanced in subheadings where terminology and definitions have to be provided as well as in subheadings under the CA4 on interpreting and applying functional properties.

To conclude, the Pre-CAPS textbook displays a mix of quasi-deductive and quasi-inductive approaches and therefore cannot be labelled as privileging one over the other, but does so in particular content areas. For CA2 and CA3 where the properties of functions and their transformations are introduced, the textbook advances a quasi-inductive approach that is investigative and encourages engagement with the concepts before explanations and worked examples can be offered. On the other hand, for CA1 which involves the introduction of notation and terminology for functions, together with CA4 where the functional properties are interpreted, the Pre-CAPS textbook privileges a quasi-deductive approach in which concepts are introduced through worked examples.

5.4.3 Presentation Formats in the CAPS Textbook

A similar procedure to that utilised in determining presentation formats and the approach in the Pre-CAPS textbook was adopted herein. The two chapters that deal with the topic of functions in the CAPS textbook were analysed for blocks that partition *content*. The first level of analysis yielded ten (10) different types of partitioning blocks whose descriptions are provided in Table 5.5: a *narrative*; an *explanatory text*; a *note box*; a *graphic*; *worked examples*; *practice exercises*; an *introductory exercise*; *summary tables*; and two end of chapter exercises, namely, *check your skills* and *extend your skills* exercises.

Presentation format	Description
Narrative	A general statement about the chapter or subheading
Explanatory text	A brief explanation of the aspects being dealt with, including definitions. Not as detailed as the 'general discussion' in the Pre-CAPS but serving similar purpose
Note-box	'boxed' information that is deemed important for each worked example or question to which it is attached.
Graphic	A photograph related to content being dealt with
Worked example	An example of an aspect under discussion usually used to illustrate a procedure for tackling similar questions. In these chapters it usually precedes a practice exercise
Practice exercise	A set of exercises giving learners an opportunity to practice aspects being illustrated in the worked examples
Introductory exercise	A set of highly scaffolded investigative exercises intended for learners to probe a particular aspect or aspects being dealt with under the particular subheading. Similar to 'activity' in Pre-CAPS textbook
Summary tables	Provides a summary of the properties of each function class
Check your skills exercise	Given at end of chapter as revision exercises for the content dealt it in the chapter
Extend your skills exercise	Extends the revision with more challenging questions and real life application of content. Follows after the check your skills exercises

Table 5.5 Descriptions of Presentation Formats in the CAPS Textbook

Of the ten presentation formats in Table 5.5, the *note-box*, *graphic* and *summary tables* are new to the CAPS textbook, meaning that that they do not feature in the Pre-CAPS textbook. The *introductory exercise* is similar to the *activity* in the Pre-CAPS textbook. Examples of a *note-box*, and a *summary table* in the CAPS textbook are shown in Fig 5.6 and Fig 5.7, respectively.

Example

Given that $f(x) = 2x - 3$, determine the values of:

a) $f(3)$ b) $f(-3)$

c) $f\left(-\frac{3}{2}\right)$

Solution

a) $f(3) = 2(3) - 3$
$= 3$

b) $f(-3) = 2(-3) - 3$
$= -9$

c) $f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right) - 3$
$= -6$

Note
Although the letter f is often used in functional notation, other letters can also be used.

Fig 5.6 A *note-box* in the CAPS Textbook (Pike et al., 2011, p. 141)

The *note-boxes* in the CAPS textbook similar to the one shown in Fig 5.6 above are attached where appropriate to worked examples, or questions in introductory exercises and provide an explanation, a clarification or a description relevant to the example or question to which they are attached. While they serve a similar purpose to explanatory texts, they are placed strategically. The worked examples and questions where note boxes are attached shall be identified with a letter *t*.

The parabola $y = f(x) = x^2$

The function $y = f(x) = x^2$ is a quadratic function.

- The graph of any quadratic function is called a *parabola*.
- The domain of $y = f(x) = x^2$ is the set of real numbers, $\mathbb{R} = (-\infty; \infty)$.
- The range of $y = f(x) = x^2$ is $\{y \,/\, y > 0\}$ or the closed / open interval $[0; \infty)$.
- The line $x = 0$ is the axis of symmetry of $y = f(x) = x^2$.
- As the values of x *increase* from $-\infty$ to 0 the values of $y = f(x)$ *decrease* and as the values of x *increase* from 0 to ∞, the values of $y = f(x)$ *increase*.
- A sketch graph is drawn to show the general shape of a curve, any intercepts on the axes and any asymptotes or axes of symmetry. This is a sketch of $y = f(x) = x^2$:

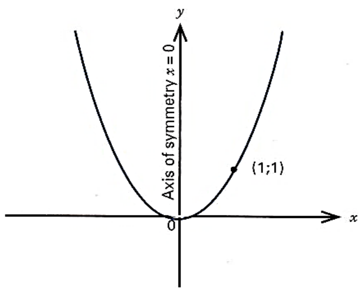


Fig 5.7 A *summary table* for the Parabola in the CAPS Textbook (Pike et al., 2011, p. 153)

Fig 5.7 shows a *summary table* for the parabola, $y = x^2$ in the CAPS textbook. All *summary tables* in the CAPS textbook are preceded by the investigative *introductory exercises* in determining the properties of the function classes. Each *summary table* provides a summary of properties for each function class investigated. As with the *note-boxes*¸ *summary tables* also serve a similar purpose to *explanatory texts*, and so are coded as *exp.*

A list of final codes for the presentation formats in the CAPS textbook are listed in Table 5.6

Presentation format	Coding	Comment
Explanatory text	exp	Includes narratives and summary tables
Worked example	we	
Practice exercise	pr	
Introductory exercise	act	
Assessment exercises	as	Includes end of chapter exercises: check your skills exercises and extend your skills exercises
Graphic	gr	
Note- box	t	

Table 5.6 Presentation Formats in the CAPS Textbook

Thus there were seven different presentation formats partitioning content in the CAPS textbook. Five of these presentation formats were similar to those in the Pre-CAPS textbook, namely, *explanatory text, worked examples, practice exercise, introductory exercise/activity and assessment exercises.* The *note-boxes* and *graphic* were the only ones unique to the CAPS textbook.

In the next section, the sequencing of these presentation formats is explored in order to determine the approach to the teaching and learning that the CAPS textbook advocates.

5.4.4 The General Approach in the CAPS Textbook

A similar process of analysis as followed in the Pre-CAPS textbook has been followed to determine whether the approach in a particular subheading and content area was quasi-deductive or quasi-inductive. The only difference is that since both the *note boxes* and the *graphics* are attached to another presentation format, in the analysis they were shown as such and did not appear on their own. For example, an entry under *worked examples* shown as ii(t) in the analysis table means that the *worked examples* were a second presentation format under the subheading and had a *note-box* attached to it; while a ii(g) means the second presentation format with a graphic attached. The results of the analysis for the CAPS textbook are presented in Table 5.7.

Subheadings	Content area	Presentation Formats					Sequencing of Presentation Formats							The Approach
		Explanatory text (exp)	Worked example (we)	Practice exercise (pr)	Introductory Exercise (act)	Assessment (as)	**direction of progression** →							
The relationship between two variables	Notation and Terminology (CA1)	i, iii	ii				exp	we	exp					quasi-deductive
Domain and range		i	ii(t)	iii			exp	we(t)	pr					quasi-deductive
Function notation		i	ii, iii(t), iv, v(t)	vi			exp	we	we(t)	we	we(t)	pr		quasi-deductive
Functions and graphs (linear functions)	Interpretation CA4	i, iii, iv	ii(t), v	vi			exp	we(t)	exp	exp	we	pr		quasi-deductive
Sketch graphs, restricted domains and finding the equations of straight lines			i, ii	iii			we	we	pr					quasi-deductive
Functions and graphs—(parabola, hyperbola, exponential function)	Properties of functions (CA2)	iv, v, vi, vii			i, ii(t), iii		act	act(t)	act	exp	exp	exp	exp	quasi-inductive
	Assessment exercises						as							
Investigating Functions	(CA3)	i(gr)					exp(g)							
Average gradient		i	ii, iii				exp	we	we					quasi-deductive
The effect of a in $y = a.f(x)$ on the graph of $y = f(x)$	Transformation of functions (CA3)				i, ii, iii		act	act	act					quasi-inductive
The effect of q in $y = a.f(x) + q$ on the graph of $y = af(x)$		iv, v, vi, vii			i, ii, iii		act	act	act	exp	exp	exp	exp	quasi-inductive
Sketch graphs	Interpretation of functions (CA4)	i	ii(g), iii, iv(g)	v			exp	we(g)	we	we(g)	pr			quasi-deductive
Determining the equation of a graph		i	ii(t), iii	iv			exp	we(t)	we	pr				quasi-deductive
Interpretation of sketch graphs			i, ii(t),	iii			we	we(t)	pr					quasi-deductive
	Assessment exercises					ch ext	as							

Table 5.7 Determining the Presentation Formats in the CAPS Textbook

As with the Pre-CAPS textbook, the sequencing of the presentation formats showed two distinct patterns: where the sequence began with an *explanatory text* or *worked examples* and therefore exhibiting a *quasi-deductive approach*; or where the sequence began with the *introductory exercise/activity* and hence a quasi-inductive approach. Furthermore, as in the Pre-CAPS textbook, particular subheadings and content areas depicted a particular approach: all of the subheadings under CA1 (notation and terminology) and CA4 (interpreting and applying functional properties) adopted a *quasi-deductive approach*; while for CA2 (properties of functions) and CA3 (transformations of functions) the approach was *quasi-inductive*.

Additionally, the study of linear functions which only features in practice exercises in the Pre-CAPS textbook is included in the CAPS textbook. However, since the assumption is that linear graphs have been introduced already in prior grades, here they are brought in, in a more interpretive manner, unlike the other function classes (quadratic, exponential and hyperbola functions) which are being introduced for the first time. As such, the subheading featuring linear functions featured under CA4. Thus, the CAPS textbook as well, like the Pre-CAPS textbook, did not privilege one approach over the other, but adopted the *quasi-inductive approach* in the content areas where generalisation of properties was required; and a *quasi-deductive approach* where procedures were exemplified.

The next section examines the performance expectations of the textbook and the approach to the teaching and learning of functions advocated

5.5 Analysis of the Content-Specific Approach

In the framework for analysing the textbook developed in earlier sections of this chapter, it was from the sequencing of performance expectations in the textbook that the approach to the teaching of functions could be determined in the textbook. In this section, the performance expectations of the textbook with respect to the expected actions of learners on function tasks were identified and their sequencing examined in order to determine whether the approach to the teaching of functions in the textbook was pointwise or global. It is important to clarify that this section does not delve into an indepth analysis of the mediation of functions as in the conception of functions that learners have, for example, the action and process conceptions of functions mentioned in chapter 2.

In chapter 2, the study mentioned the action and process conceptions as conceptions of functions that learners have, and that in order for learners to develop covariational reasoning, they need to move from action conception to process conception of functions. However, to study these conceptions and how they are manifested in the textbook, learners would have to be brought in, which as mentioned already, is beyond the scope of this study. Instead, this section is an investigation of the approach to functions only with respect to the actions on function tasks *expected* of learners from the textbook. In this way, what learners do in the classroom does not have to be studied in order to determine what actions are expected of them.

It is also important to clarify that for this particular analysis, only one textbook, the CAPS textbook was utilised for the following reasons: Firstly, as has been illustrated in previous sections, the Pre-CAPS and the CAPS textbooks feature similar *content areas* which have also been ordered similarly. It is the assumption of the study that the expected *actions on function tasks* would therefore be similar in both textbooks, prompting the study to analyze these in depth from only one textbook. Secondly, the analysis of performance expectations in the study was used mostly to determine the approach to the teaching of functions with respect to the pointwise or global conception of function portrayed. Prior analysis of both textbooks showed that while there were differences with which aspects of function were varied and which were kept constant, the Pre-CAPS and the CAPS textbooks both portrayed a similar conception of function in these respects.

Thirdly, performance expectations were determined from the tasks in the textbook, which derived from worked examples, practice exercises, activities and end of chapter exercises; and these tasks accounted for more than 80% of the textbook *content*. The analysis herein entailed going through each and every task in the textbook on functions and as such, this would be a highly tedious process if both textbooks were used, hence the decision to use only one textbook. Lastly, the CAPS textbook is the substantive prescribed textbook moving forward with the new curriculum, and therefore a logical choice to exemplify this part of the analysis between the two textbooks.

The analysis in this section seeks to investigate the nature of *actions on function tasks* expected of learners by the textbook. As Valverde et al. (2002) posit

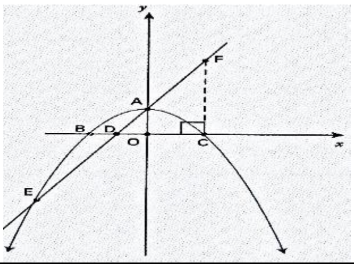
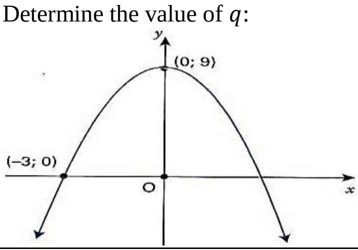
> textbooks not only put forward the content students are to learn but they also advocate what students should be able to do with that content. Textbook developers do not intend to simply convey information, but to encourage behavio[u]rs[12] on the part of students. (p.125)

5.5.1 Performance Expectations in the CAPS Textbook

Using my own mathematics experience and also guided by the worked examples where appropriate, each task in the two chapters on functions was translated into expected or anticipated actions on the task. These actions were then coded and the final categories of performance expectations determined. The process was approached in a grounded manner (Strauss & Corbin, 1990), beginning with the translation of the tasks into actions; and then forming the actions into categories. In order to maintain consistency and coherence, two iterations of the coding process were undertaken with a difference of a year between them, producing five main performance expectations which included: *Substitute actions, Plot and draw actions, Read off actions, Generalise properties actions* and *Interpret properties actions.*

[12] 'behaviours' in the study shall be used as Valverde et al. use it, that is, as actions on tasks or capabilities expected of learners

In Table 5.8 below, the five performance expectations determined in the CAPS textbook are presented and then discussed.

Example(s) of Task	Expected action(s) on Task	Performance expectation	Description
Calculate $g(x)$ for $x = -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 2$ Determine the coordinates of B 	Substitute the x values in the equation of $g(x)$ and calculate the output values equate the equation of the parabola to 0 and solve for x (equating the equation to 0 regarded as a 'substitution')	Substitute	Involves any form of a substitution of one value in order to calculate another value
Plot the points $\left(\frac{1}{4}, g\left(\frac{1}{4}\right)\right), \left(\frac{1}{2}, g\left(\frac{1}{2}\right)\right), (1, g(1))$, and $(2, g(2))$ and join then to form a smooth curve	Plot points on the Cartesian plane and join them	Plot and draw	Plotting points on the coordinate system in the graphing
For which value(s) of x is $f(x) = \frac{9}{4}$? Read from the graphs the solution of the inequality: (i) $2x > x^2 - 8$; Determine the value of q: 	Read off the values of x corresponding to $y = \frac{9}{4}$ on the graph Identify intervals for which the y values for the straight line lie higher than the corresponding values for the parabola Read off the y value of the turning point	Read off	Reading off values from graphs Reading off intervals from the graph Reading off features of functions from graphs
The straight line $y = x$ is an axis of symmetry of the hyperbolas sketched. Write the equation of another axis of symmetry Choose the correct option: the larger the value of $a > 0$, the (further from/closer to) the origion the graph of $y = \frac{a}{x}$ is Two hot air balloons are observed. A blue one is 150 metres above the ground and is descending at a constant rate of 20 metres per minute…. Draw graphs to show the heights of the two balloons from the time they were observed unit the time the blue balloon hits the ground	Determine the second axis of symmetry for the functions from their graphs given that $y = x$ is another one Compare the distances from the origin of the graphs and generalise about the size of a and how far from the origin the graph would be Construct equations of graphs from the information given and use them to draw the graphs	Generalize properties	Determining properties of functions from graphs or equations of functions (initially) Choosing correct options in order to generalise Actions in which generalisation of properties is expected

Example(s) of Task	Expected action(s) on Task	Performance expectation	Description
2. Which functions $y = f(x) = x, y = g(x) = x^2, y = h(x) = \frac{1}{x}$ for $x > 0, y = j(x) = 3^x$ and $y = k(x) = \left(\frac{1}{4}\right)^x$ has: a) y-values which increase as x increases b) y-values which decrease as x increases c) y-values which decrease over one subinterval of the domain and increase over another subinterval.	Match a function with an appropriate property	Interpret properties	Actions involving interpreting properties of functions such as: matching properties to functions;
Sketch the graph of $y = x^2 - 9$	Identify the properties of the function necessary for sketching the graph eg intercepts, shape of graph, turning point		sketching graphs of functions by applying the properties of functions;
Determine the equation of an exponential function of the form $y = a.k^x + q$ given that it passes through the points $(1; 0)$ and $(2; -2)$ and has a horizontal asymptote $y = 2$	Identify the value of q in the equation as 2 and use the other coordinates to find the values of a and k		using properties of functions for problem solving

Table 5.8 Descriptions of Performance Expectations in the CAPS Textbook

In the *substitute actions* learners were expected to perform some form of substitution, for example, substituting input values to evaluate output values. These actions also included performing some algebraic manipulation related to inputs and outputs, for example, evaluating an input value given the output. The *plot and draw actions* involved plotting of points on the Cartesian plane and joining the points to draw a graph. In tasks where learners were expected to read off values from graphs, the performance expectation was referred to as *read off*. The *generalise properties actions* included all actions which involved determining properties of functions, or choosing a correct option from a number of options in order to generalise some properties. Lastly, the *interpret properties actions* involved actions in which interpretation of properties was expected, for example, in sketching graphs of functions, or determining equations of functions from their graphs.

However, there were incidences when an action would produce a combination of codes, that is, where a task involved more than one action. For example, in order to draw a graph of a function using point-by-point plotting, a learner might have to populate coordinate pairs by substituting the inputs into the equation to produce corresponding output values. This was coded as *substitute*, while the subsequent action of plotting the points on the Cartesian plane and joining them was coded as *plot and draw*. A final code allocated to such actions was the one that corresponded to the ultimate task, in this case, *plot and draw*.

5.5.2 The Approach to Teaching and Learning Functions

Having identified the five performance expectations, the next step was to decide on the nature of each action category: whether the actions were pointwise or global in nature. The *Substitute*, *Plot and draw*, and *Read off* categories focus on discrete point-by-point strategies and have therefore been identified as pointwise actions. On the other hand, the other two categories: *Generalise properties* and *Interpret properties* focus on the interpretive and holistic behaviour of functions, and therefore are global in nature. All the actions were hence categorised as either point-wise or global, or a combination of pointwise and global where appropriate.

Table 5.9 illustrates only a part of the results of the analysis. A full report is appended to the thesis as Appendix C. The first two columns of Table 5.9 represent the content area and a subheading (see previous sections in this chapter) under which the particular task fell. Column 3 represents the task taken 'as is' from the textbook. The tasks were ordered in exactly the same way they appeared in their respective presentation formats in the textbook. This helped in studying the sequencing of performance expectations throughout the subheadings, content area, and chapters. The anticipated actions on task in the fifth column represent my interpretation of the expected or anticipated performance expectations in each task. For example, consider task 1a) in Table 5.9 about matching the graphs with their respective equations.

Using my mathematics knowledge and experience I concluded that in order to match the graphs with their respective equations, a learner would use their knowledge of the graph of $y = x^2$ to match it with its graph, which required interpretation of properties of the particular parabola. Secondly, at some point, whether as the first step or the second step, the learner would have to substitute some points in the equations to confirm which graph belonged to which equation; and lastly, the matching itself was some form of generalisation, since a choice had to be made based on some properties. Thus, in general, I proposed that there were three actions on this particular task alone in the following order: *interpret properties* →*substitute* → *generalise properties*. Since the final step in the task was the matching of the graphs and the equations, which is a global action, the actions were classified as global.

In some cases, for example, in task 1d), there were two distinct tasks: calculating the average gradient, which required substitution of values into the formula. The second task was to compare the steepness of the graphs with the average gradients. This task required a learner to make judgements or generalise about steepness of graphs and average gradients, hence the task had two distinct actions, *substitute* and *generalise properties* which stood on their own. In this case, the actions were classified as point-wise/global to indicate a mix of actions in this task. In the first example above in contrast, the substitution was a means to get to the matching and did not stand on its own. The last column concerns all comments about the different performance expectations evident in each task; and about the progression of performance expectations in a particular task, or through a group of tasks.

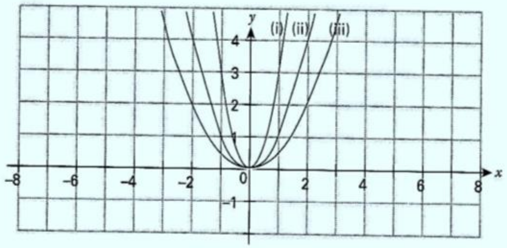
Content area	Subheading	Presentation format	Task	Anticipated Action on task	Performance expectation	Nature of action	comment
Transformation of functions	The effect of a in $y = af(x)$	Introductory exercise	**1.** The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$.  **a)** Match the number **(i)**, **(ii)** or **(iii)** on each graph with its defining equation. **b)** Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. **c)** Write the range of each of the three quadratic functions that have been drawn. **d)** Calculate the average gradient of each graph between the points where $x = 0$ and $x = 1$. Compare the steepness of the graphs with the average gradients between those points. **e)** Repeat **d)** above for $x = -1$ and $x = 0$. **2.** Investigate the graphs of $y = p(x) = -x^2$ and $y = q(x) = -3x^2$ as well as $y = r(x) = -\frac{x^2}{2}$. **a)** Draw the graphs for $x \in [-2; 2]$. **b)** Describe how the graphs of these parabolas compare with those drawn in question **1.** **c)** Write the range of the given functions for $x \in \mathbb{R}$. **d)** Write the equation of the axis of symmetry of the graphs of the given functions.	1: Match a graph with its equation: decide which graph matches with the parent function; substitute values to check out points on other graphs, and match the graphs	Interpret properties/ substitute/ generalise properties	Global	All performance expectations, except for *read off* are evident in this introductory exercise
				Describe how the graphs of the form $f(x) = ax^2, a > 0$ differ from the graph of $y = x^2$	Generalise properties	Global	Investigation of the effect of a in $y = ax^2$ on the graph of $y = x^2$ to be conducted in a global manner
				Determine the range of each of the drawn graphs	Generalise properties	Global	
				Calculate the average gradient between two given points on each graph and compare these with the steepness of the graphs	Substitute/ generalise properties	Pointwise/ global	
				2: Draw the graphs of the form $f(x) = ax^2, a < 0$; for given values of a	Substitute/ plot and draw	Pointwise	
				Compare these graphs above with those with $a > 0$ and describe differences and similarities	Generalise propertes	Global	
				Determine the range of each function	Generalise properties	Global	
				Determine the axis of symmetry of the graphs and write its equation	Generalise properties	Global	

Table 5.9 Determining the Performance Expectations from the CAPS Textbook

Table 5.9 shows an example of an introductory activity under the content area, CA3 (transformations of functions) where the effect of the parameter 'a' on the quadratic graph $f(x) = x^2$ was investigated. A total of eight (8) tasks and therefore eight expected actions on these tasks, that is, eight performance expectations, were identified in this *activity*. In the first performance expectation, learners were expected to: interpret properties, substitute values and generalise properties. Since the main actions here were interpretative, the actions were categorised as global rather than pointwise, even though there was substitution involved. On the whole in this exercise, there were six (6) actions that were global, one pointwise action and one combination action that started as pointwise and ended as global. Hence, the approach in this 'activity' was determined to be global. This means that the textbook advances a global approach to the investigations of the effect of the parameter 'a' on the graph of $f(x) = x^2$. This process was repeated for each and every task on functions in the CAPS textbook, and the results of this analysis are discussed in the next section.

The results of the analysis of the performance expectations of the CAPS textbook are discussed in the next section. Since all performance expectations can be classified into whether they are pointwise or global, the distribution of these performance expectations across content areas is important for determining the approach to the teaching and learning of functions. This shall be reported first followed by a discussion of the patterns of sequencing of the performance expectations observed across content areas.

5.5.2.1 Distribution of Performance Expectations

Table 5.10 below provides a summary of the distribution of performance expectations across the four content areas; *notation/terminology (CA1), properties of functions (CA2), transformation of functions (CA3),* and *interpretation of functions (CA4).*

Performance expectation	CA1	CA2	CA3	CA4				totals	
	Notation	properties of functions	transformations	linear functions	sketching graphs	determining equations of graphs	interpreting sketch graphs		
Substitute	27	7	7	13	5	7	9	75 (30%)	Pointwise actions (43%)
Plot and draw	0	7	1	6	0	0	0	14 (6%)	
Read off	0	3	8	7	0	0	1	19 (8%)	
Generalise properties	0	19	20	13	0	0	1	53 (21%)	Global actions (57%)
Interpret properties	0	2	8	5	11	7	57	90 (36%)	
Totals	27	38	44	44	16	14	68	251	

Table 5.10 Distribution of Performance Expectations in the CAPS Textbook

Looking horizontally across Table 5.10, it is quite notable that the *plot and draw* actions are the least occurring (6%) of the five presentation formats, followed by *read off* actions (8%). This suggests that while this is a study of functions and their graphs, the focus of the textbook is not on the graphs per se, but on the properties of the functions in the graphs: graphs are used to explicate functions (Leinhardt et al., 1990). On the other hand, the most frequent of the presentation formats are the *interpret properties* actions (36%) followed by *substitute* (30%) and then *generalise properties* actions at 21%. Thus altogether, there are more global actions (57%) than pointwise actions (43%) which suggests an approach to functions that is more interpretive and towards the global.

The vertical interpretation of Table 5.10 on the other hand points to the fact that the approach to functions in the textbook is rather a progression from pointwise to global actions, as presented in Fig 5.8

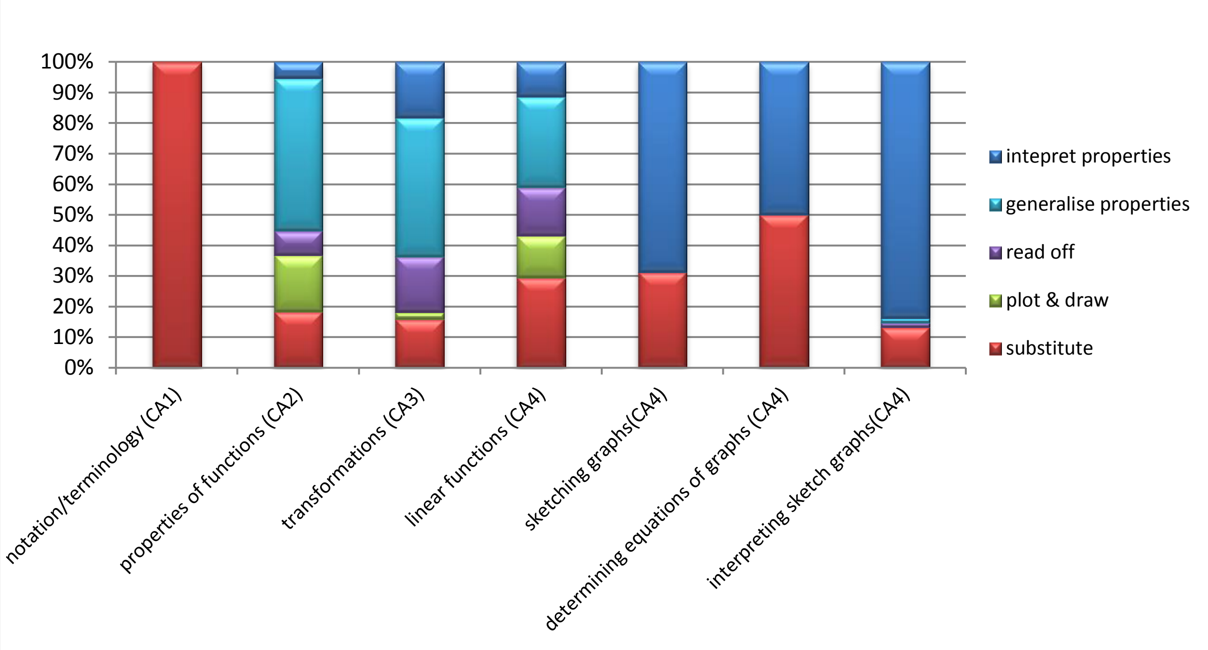


Fig 5.8 Distribution of Performance Expectations across Content Areas

Two extremes are observed in Fig 5.8, namely that *notation and terminology (CA1)* consists only of *substitute* actions while *interpretation of functions (CA4)* is made up of close to 90% of *interpret properties* actions. While the case for CA1 is understandable since in CA1 the notion of function notation and calculating function values through substitution is only being introduced, the textbook expects that by the end of CA4 learners should have experienced the functions, learned about their properties, and can now look at functions in a more holistic manner through their properties. The experiencing of the properties of functions is seen at CA2, CA3 and the first three subheadings under CA4, where *generalise properties* actions are introduced in CA2 and dominate in both CA2 and CA3, dwindle in linear functions (CA4) and become quite infinitesimal to non-existent in CA4. Similarly for the *plot and draw* actions: they start out in CA2, and become minimal in CA3 and in linear functions while disappearing completely in the rest of CA4.

The trend shows a decreasing number of pointwise actions (substitute, read off and plot and draw) from CA1 to CA4, while global actions (generalise properties and especially interpret properties) increase from CA1 to CA4. Additionally, while *substitute* actions decrease quite remarkably from CA1 to CA3, they also seem to be on a rise in CA4 though not to the same extent as in CA1. These *substitute* actions involve calculations needed to evaluate coordinate pairs or features of an equation. The textbook thus moves from pointwise actions to global actions thus reinforcing Monk's (1994) argument that the pointwise actions are a prerequisite to global actions. Furthermore, this reinforces the power of global actions over pointwise actions. Of course as literature points out, this does not mean that the global approach is always 'better' than the pointwise approach (Even, 1998).

5.5.3 The Differences in the Approach of the Textbooks

Once again, it is important to explain what the study is not saying with respect to the findings in this section. It is not that there are no differences in the teaching approaches of the two textbooks. In fact, the most glaring difference between the two textbooks occurs in CA2 where the properties of functions are determined. In the Pre-CAPS textbook, all functions are dealt with at the same time. Fig 5.9 exemplifies how the two textbooks deal with the functions: in the Pre-CAPS textbook, all the functions are dealt with together around a particular property. For example, the extract in Fig 5.9 shows that in the Pre-CAPS textbook, in order to investigate the properties of the functions, the three function classes are drawn together in the first question, and in the second question, the property of symmetry is investigated for all functions, and similarly for the end-behaviour in question 3. Secondly, the function of a hyperbola used is not a parent function, that is, the most simplest function, $f(x) = \frac{1}{x}$.

Pre-CAPS

Work in groups.

1. Draw the graphs of the following on separate axes. Use the table method. Include both positive and negative values for the independent variable.
   a) $g(x) = x^2$ b) $f(x) = 2^x$ c) $h(x) = \frac{8}{x}$
2. Look up the meaning of the word "symmetry" in a dictionary (or refer to Chapter 9 for a definition).
   a) Which of the curves you have drawn has symmetry?
   b) Identify the line(s) of symmetry in each case.
3. a) Investigate what happens to the function values for each function when the x values become:
   (i) bigger and positive (for example use $x = 10; 100; 1\,000$)
   (ii) smaller and negative (for example use $x = -10; -100; -1\,000$)
   (iii) closer to zero, either positive or negative (for example use $x = 0{,}1; 0{,}01; 0{,}001; -0{,}1; -0{,}01; -0{,}001$).
   What effects do these values have on each graph?

CAPS

1. The function $y = f(x) = x^2$ is called a **quadratic function**.
   a) Calculate the values of $f(-3), f(-2), f(-1), f(0), f(1), f(2)$, and $f(3)$, and plot the points $(-3; f(-3)), (-2; f(-2)), (-1; f(-1)), (0; f(0)), (1; f(1)), (2; f(2))$, and $(3; f(3))$. Join these points to form a smooth curve, called a **parabola** (there must be no sharp points).
   b) If the domain of f is the set of real numbers, what is the range of f?
   c) The graph of the given quadratic function is symmetrical about a straight line called an **axis of symmetry**. What is the equation of this straight line?
   d) For which value(s) of x is $f(x) = \frac{9}{4}$? Confirm your answer(s) graphically and algebraically.
2. The graph of the function $y = g(x) = \frac{1}{x}$ is called a **rectangular hyperbola**.
   a) Calculate $g(x)$ for $x = -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 2$.
   b) Explain why $g(0)$ does not exist.
   c) Explain why there is no value of x for which $g(x) = 0$.
   d) Plot the points $(-2; g(-2)), (-1; g(-1)), (-\frac{1}{2}; g(-\frac{1}{2}))$

Fig 5.9 Determining Properties of the Parabola in the Pre-CAPS and the CAPS Textbooks

In the CAPS textbook (graph on the right in Fig 5.9), the approach is different. A graph of a particular function class is dealt with on its own. In question 1 in Fig 5.9, the quadratic function is investigated for the domain and range, as well as for its axis of symmetry. A part of question 2 shown investigates the rectangular hyperbola in its simplest form, that is, its parent function.

That is, in dealing with properties of functions and their transformations, the Pre-CAPS textbook varies the function while keeping the property being investigated constant, with the reverse occurring in the CAPS textbook. Watson and Mason (2006) show that it is in the midst of variance and invariance that a feature is discerned, and therefore learning possible. Varying functions around the property foregrounds the property itself and what it does to a function that features such a property; while varying properties around one function focusses on properties that a particular function possesses or does not possess. Thus the variance and invariance open up different opportunities for the mediation of Functions in the classroom, and therefore enable and constrain different actions for the teacher

The difference in variance and invariance in the approach to functions in the two textbooks highlights that there are other aspects of the textbook that are relevant to the relationship between teachers and their textbooks which the study does not deal with; not because they are insignificant, but due to how much the present study can focus on. This is one limitation of the study, and one of those aspects which the study recommends as part of further research. The analysis in this chapter highlights the resources that the textbook furnishes the teacher's practice with by examining the structure of the textbook. The examination looks at the constitution and organisation of the mathematical content, as well as the instructional approach embedded in the textbook. The results of these analyses are used to make claims about the affordances of the textbook to the teachers' practice that include not only the existing resources but their enabling and constraining characteristics. According to Gibson and Norman in chapter 2 of this study, an affordance defines not only the resource that exits in the text but also the capacity of the resource to enable as well as constrain action on the part of the user. The conclusions are discussed in detail in the next section.

5.6 Conclusion

In this chapter I set out to determine the affordances (J. J. Gibson, 1977; Norman, 1988) of the textbook to the teacher's practice as constituted in the structure of the textbook (Brown, 2009; Remillard, 2012; Valverde et al., 2002). The analysis of the content in both the Pre-CAPS and the CAPS textbooks shows that the textbook provides the teacher's practice with particular *content areas (CA)* to be taught and learned in a particular order. For the specific topic of Functions, the textbooks furnish the teacher with four content areas of *notation and terminology (CA1); properties of functions (CA2); transformations of functions (CA3);* and *interpretation of functions (CA4)* respectively, to be taught and learned in the order outlined.

The affordances with respect to the approach have been considered in two ways: firstly in general (*general approach)* and then specific to the content of the topic of Functions (*content-specific approach)*. The general approach determined through the sequencing of the *presentation formats* in both textbooks points to two approaches to the teaching and learning of specific content areas. The *quasi-deductive approach* that is more didactic is used in content areas introducing the notation and terminology, as well as those dealing with interpretation; while the *quasi-inductive approach* which is a more investigative approach is used when determining properties of functions and their transformations. Thus, with respect to the general approach, the textbook offers the teacher a specific approach for specific content.

With respect to the content-specific approach to the teaching and learning of Functions on the other hand, the textbook presents an *interpretive approach* constituting of a progression from pointwise actions to global actions. The sequencing of learners' *performance expectations* in the CAPS textbook reflects an approach to the teaching of Functions that begins with point-by-point strategies in content areas taught at the beginning of the topic; followed by a mixture of pointwise and global actions where properties of Functions and their transformations are determined, and then becoming completely global at the end of the topic where functional properties are being interpreted. This content-specific approach to the teaching of Functions constitutes yet another affordance of the textbook to the teacher's practice.

In the next chapter, the study explores how teachers appropriate these affordances in their practice, especially in the classroom. In other words, the next chapter shall explore how teachers present the content and the sequencing of this content in the classroom; how they approach the teaching and learning of the content in general, that is, whether the approach is didactic (quasi-deductive) or investigative (quasi-inductive); but also how they approach the teaching of Functions in particular with respect to pointwise and global strategies.

Some limitations have also been expressed in the present chapter which the study recommends as part of further research. The study does not delve into the mediation of the object of learning in the classroom as well as the nature of the mathematics on offer in the textbooks. . In other words, I have not explored whether and how these affordances in the textbook are productive to learners, and as such, I am not in a position to comment about the effect of the constraints exerted by the textbook with respect to the content as well as the approach.

CHAPTER 6
Teachers' Mobilisation of the Textbook: The Case of Teacher A1

> Teachers act as mediators of the text. Teachers decide which textbooks to use; when and where the textbook is to be used; which sections of the textbook to use; the sequencing of topics in the textbook; the ways in which pupils engage with the text; the level and type of teacher intervention between pupil and text; and so on.
> (Pepin & Haggarty, 2001)

6.1 Introduction

The opening quote above encapsulate what Pepin and Haggarty (2001) term teachers' mediatory role with respect to textbooks in instruction. The present chapter and the one that follows it seek to explore how the teachers participating in this study mediate the textbook in the classroom, and what opportunities are opened up in the processes by which teachers mediate the textbook. The exploration is carried out with a specific focus on the affordances of the textbook to the teachers' practice which were established in chapter 5, namely, the *mathematical content* and the *approach to the teaching and learning of the content*. The question is how do teachers mobilise the affordances of the textbook in their practice?

There are twenty (20) lessons between the seven (7) teachers participating in the study to analyze for how teachers mobilise the affordances of the textbook. In the present chapter, I have chosen teacher A1 and analyze her three (3) lessons to study the mobilisation of the *content* and the *approach* of the textbook. The analysis for teacher A1's lessons is intended firstly to exemplify the process of analysis, and then to assist in developing analytical tools for the analysis of the rest of the lessons in the next chapter. I have to mention that the choice of teacher A1 has been for no other reason other than that she was the first teacher whom I observed when collecting data for the study, and her lessons became the first lessons which I started analysing from the onset. I was therefore satisfied that her varied ways of using the textbook would provide ample tools to carry the analysis into the next phase.

The analysis commences with a process of developing conceptual tools for teacher A1's appropriation of the affordances of the textbook.

6.2 Conceptual Tools for the Appropriation of the Affordances

In chapter 3, a *teacher-textbook interaction framework* for the study, shown below in Fig 6.1 was proposed. The framework has been developed from Brown's (2009) *design capacity for enactment framework* (DCE) and Remillard's (2005) *framework of components of teacher-curriculum relationship* discussed in chapter 3, and depicts the interaction between teacher resources and textbook resources.

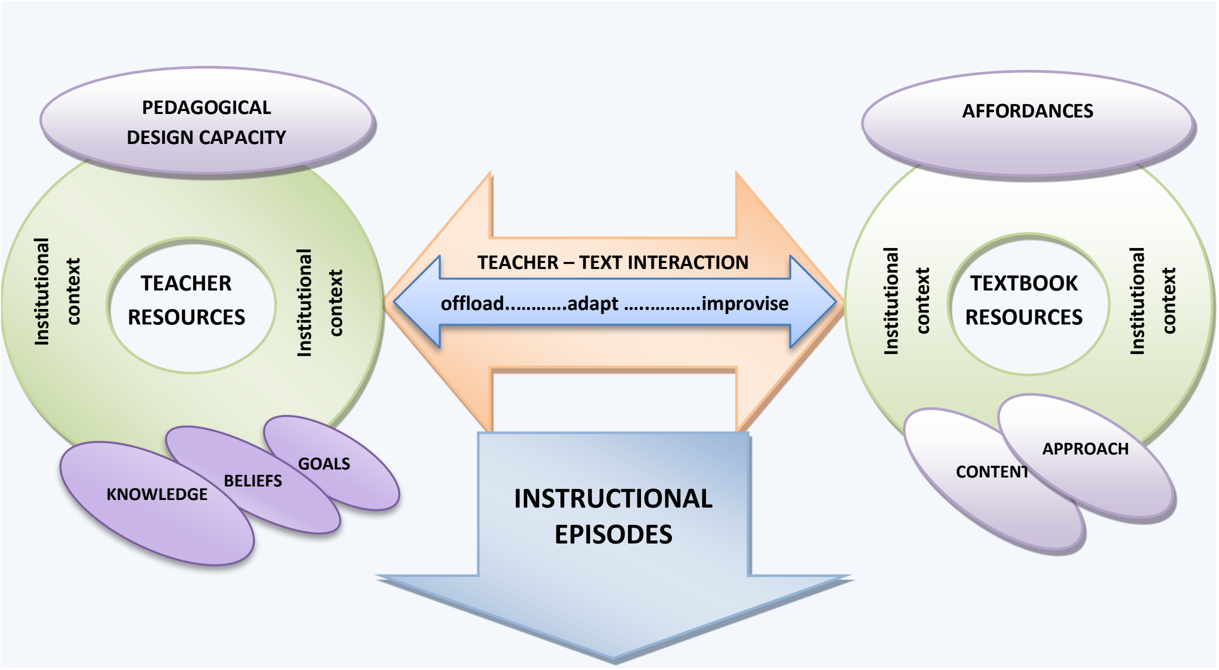


Fig 6.1 The proposed *Teacher-Textbook Interaction Framework*

After establishing the *content* and the *approach* as the affordances of the textbook to the teachers' practice in chapter 5, they are placed under textbook resources in the framework. The interaction between the teacher and textbook which is the focus of the present chapter is represented by a double arrow joining the teacher resources and the textbook resources. Brown (2009) has proposed three processes of *offloading,* that is, relying mostly on the textbook for the delivery of the lesson; *adapting,* which indicates an equally-shared responsibility for the delivery of the lesson between teacher and textbook; and *improvising*, where the teacher relies mostly on external and own resources for delivering the lesson, that lie on a continuum to characterise teachers' appropriation of the textbook affordances. In the analysis, the three processes are explored in depth in order to understand better what teachers do with both the *content* and the *approach* of the textbook in the classroom, and the opportunities for mediation thus created.

Since the analysis of the lessons describes how teachers appropriate the affordances of the textbook, the conceptual framework developed in chapter 5 is adapted for the process of analysis of the lessons in the present chapter as shown in Fig 6.2.

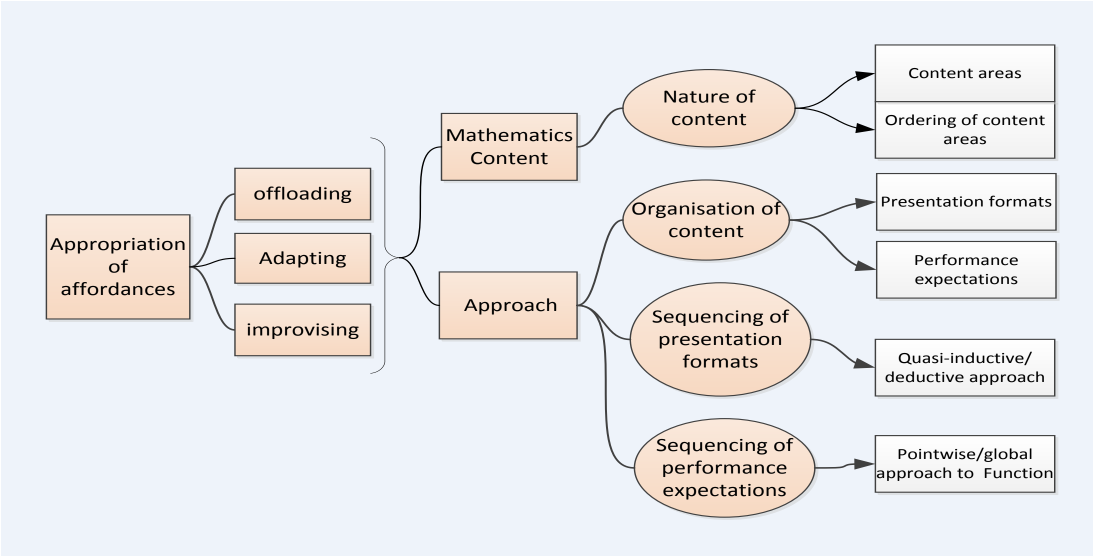


Fig 6.2 A Framework for Teachers' Appropriation of the Affordances

As Fig 6.2 illustrates, how teachers appropriate the content of the textbook is determined through the nature of the content areas and their ordering in teachers' lessons, that is, the *coverage* of content. The sequencing of the presentation formats identifies the general approach with respect to the *quasi-deductive* and *quasi-inductive approaches*; while the sequencing of performance expectations points to whether the approach to the teaching and learning of functions in particular in the lessons is *pointwise* or *global*.

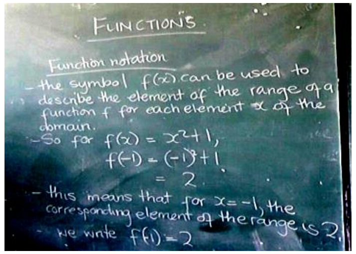
6.3 Analysis of Teacher A1's Mobilisation of the Textbook

Teacher A1 was observed teaching in three lessons which I have named, A11, A12, and A13, respectively. Lessons A11 and A12 took place within the first week of introducing the topic of Functions: A11 on a Monday, and A12 on a Friday of the same week. Lesson A13 however took place exactly two weeks after A12. This implies that for teacher A1, I observed her lessons quite at the beginning of the topic on functions and the end of the topic since there were four weeks allocated to the teaching of functions. The duration of the lessons was fifty (50) minutes for each lesson. Lesson A11, was on *function notation* and the evaluation of function values for a given function; A12 was on *transformations of functions*; and in A13, teacher A1 and learners went over homework questions on *determining equations of graphs*, which was followed by the introduction of the section on *sketching of functions.*

The analysis of each lesson begins with chunking it into units which are determined by a shift in focus in the content during the lesson. The units are used as episodes for the analysis of the appropriation of both the content and the approach of the textbook.

The process of analysis of lesson A11 is presented in Table 6.1. A complete transcript of the lesson is shown and alongside it, textboxes are used to describe the chunking; to identify the presentation formats and performance expectations; and to provide additional comments pertaining to the appropriation of the affordances of the textbook.

Table 6.1 Analysing the transcript for lesson A11

Episode		Transcript
Episode 1 definition of function notation	 Tr[13] Tr	*A1 is writing the following on the chalkboard, copying it from a textbook*  Okay let's stop writing. Okay, we are looking at the functional notation, the notation that is used to write or to in short, to write functions. So the symbol f of x can be used to describe the element of the range of a function f for each element x of the domain. Remember I say that the domain are the input values or the values of x in most cases. Then the range is the, the output value or the value of y, or in other words is the dependent value that you get from an independent value which is the input value. Now for, so for f of x equal to x squared *(points to board)* plus one, negative one, f negative one is equal to where there is x you substitute with negative one because here you are now having negative one *(Learners chorus "negative one" with Teacher)* and you are going to use the function that you have been given here, which is x squared plus one. So where there is x, you are now putting the value negative one. So here, this is your negative one and your x is squared so your negative one should be squared plus one and you get two. Now this means that for x is equals negative one the corresponding element of the range is two, I write f negative one is equal to two. In other words, this is your input value *(points at (-1))*, so if you are having a table of values you are going to be having negative one on top for x then at the bottom for the value of y you are going to have two *(learners chorus "2" with teacher). (Teacher bends down to pick up book and paper off floor and places these on a learner's desk.)*

<u>Episode 1: Chunking</u>

A1 defined functional notation and explained its relationship to the range of a function. The definition as written on the chalkboard includes an example which illustrates what $f(-1)$ would mean if given a function $f(x) = x^2 + 1$, showing learners how the value of the function is evaluated. A1 then worked out an example to illustrate how to evaluate function values

<u>Episode 1</u>

<u>Presentation formats</u>

- Teacher announced focus of lesson as function notation (exp)
- Teacher explained the definition of function notation written on the board (exp)
- Teacher reminded learners about definition of a domain and a range of a function (exp)
- Teacher demonstrated how substitution is used to evaluate function values for a given function (exp)

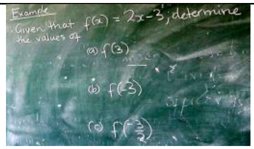
<u>Performance expectations</u>:

- Learners expected to follow the teacher's actions: substitute -1 for x in the function $f(x) = x^2 + 1$ (substitute)

<u>Comment:</u>

all content derives from the CAPS textbook

[13] Tr represents Teacher and Lnr represents Learner

Episode 2 Worked examples		
	Tr	Now I put an example, given that fx is equal to two x minus three, determine the values of; the first one is for f of three. Now you are using which function? *(Pauses)* Which function have you been given here?
	Lnrs	fx *(Teacher points to 2x written on board.)*
	Tr	Is two x minus three *(learners chorus "three" with teacher)*. So where there is x you are going to put three *(teacher points to "three" on board and learners chorus "three" with her)* for the first one, okay. So this is going to be equal to? Nelson?
	lnr	I say where there is x, I put three.
	Tr	Yes, so it's going to be? *(points at 2)*
	Lnr	Okay. Open bracket then.. close bracket minus three. *(Teacher writes 2 (3) – 3)*
	Tr	Okay. So in other words you are saying two times three, because this two x is two times x, okay, so what do I get here?
	Lnrs	six
	Tr	Minus three *(teacher writes this on board and learners chorus "minus three" with her)*
	Lnr	Equals three *(teacher writes 3 on board)*
	Tr	So this is the first one; and the second one, what are we going to have? *(Pauses a while.)* What are we going to have? Yes? *(Points to learner.)*
	lnrs	two open brackets….negative three minus three *(Teacher writes on board = 2 (-3) – 3.)*
	Tr	Okay. And what do you get 2 times negative three?
	Lnr	Negative six *(Teacher writes on board – 6 – 3)*
	Tr	Okay and this will give you …?
	Lnr	Negative nine *(Teacher cleans portion of board and writes – 6, erases the 6 with her hand, and replaces with 9. Learners chorus nine" as she writes. Teacher writes (c)* $f\left(-\frac{3}{2}\right) =$)
	Tr	And the last one? (*Pauses*) The last one. (*Pauses again*) Elijah? Where there is x what are you supposed to do now? What is the value of x? (*Teacher points to board*) Yes?
	Lnr	I'm going to say … um … two and then open bracket and negative three over two close bracket minus three. (*Teacher writes as he is talking* $2\left(-\frac{3}{2}\right) - 3$)
	tr	And what do you get here? (*Points at* $2\left(-\frac{3}{2}\right) - 3$). This is the fraction, ne? Your denominator is two.
	lnrs	Yes.
	tr	So you are going to cancel two into two (*teacher marks this on board and learners chorus one"*) into this two, (*teacher crosses out as she speaks and learners chorus "one"*) and you are going to remain with what?
	lnrs	Negative one.
	tr	Negative what? one times negative three what do you get?
	lnrs	Negative three. (*Teacher writes = -3.*)
	tr	Negative three minus three (*Teacher writes this on board and learners chorus "minus 3" with her*).

Episode 2: Chunking

The worked examples involve evaluating $f(3), f(-3)$, and $f(-\frac{3}{2})$ given the function $(x) = 2x - 3$.

Teacher guided learners through the substitution procedure for evaluating the function values above. Episode 2 ended when the teacher moved on to introduce the notion of the vertical line test.

Episode 2

Presentation formats

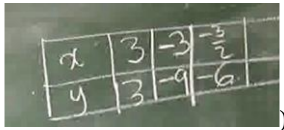
Teacher exemplified function notation by working out three examples for evaluating function values: (3), $f(-3)$, and $f(-\frac{3}{2})$ for the function $f(x) = 2x + 3$ (we)

Performance expectations

Learners expected to substitute $3, -3$, and $-\frac{3}{2}$ for x in $f(x) = 2x + 3$ and evaluate (substitute)

Comment:

The worked examples have been selected from the textbook. However, the teacher used two out of four worked examples in the textbook, thus omitting some worked examples. I look at the omitted examples and what this could mean for the opportunities for the mediation of the object of learning in the classroom.

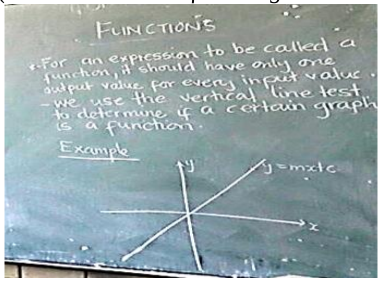
	tr	And what is your final answer?
	lnrs	Negative six (*teacher writes this on board*).
	tr	So this is basically what we are saying. Now we are saying if you have got a table of values, suppose you we are having a table of values, (*draws this table of values on the chalkboard* ) your x values and your y values (*writes x and y on the table in 1st column*). So where the x is equal to three, your output value is going to be three. So it will be here (*places 3 under 3 on the table in 2nd column*). When x is equal to negative three, your output value is negative nine (*learners chorus "negative nine" with teacher*), so you are going to have negative three here and negative nine here (*learners chorus "negative nine" with teacher as she writes -3 and -9 in the 3rd column*). When x is negative nine over two (*writes this on top line of 4th column*) your output value is going to be negative six (*learners chorus "negative axis" with teacher as she inserts this into bottom line of 4th column*.) So this is basically what I are saying,
	tr	okay? (*the table of values now looks like this* )
	lnrs	Yes.
	tr	We are not finished yet?
	lnrs	Yes. (*Learners write notes from the board into their books. Sound of aeroplane overhead.*) (*Teacher is reading a book standing at a learner's desk and occasionally looks up to watch learners.)*
	tr	I think you are finished here (*points to section of blackboard*).
	lnrs	Yes (*Teacher erases some text on left side of blackboard) (Teacher begins to write on board. Text is not immediately visible due to camera angle.)*
	tr	This side? (*Points to right hand side of blackboard.) (Some learners say "yes" others say "no" or "I'm not done".)*

<u>Presentation formats</u>

Teacher demonstrated how an input and its corresponding output may be represented as an (x;y) pair on a table of values, with the input as an x value and the corresponding output as a y value (demonstration). This is similar to worked examples and so I included it as part of the we presentation formats

<u>Performance expectations</u>

Learners expected to know how to represent an input/output pair as an x/y pair on a table of values. Since this action is similar to actions in the textbook where leaners were expected to express contents of a table of values as an ordered pair, and these were coded as *substitute* actions; this action was also coded as a *substitute* action.

Episode 3 The vertical line test of functions		*(Teacher writes the following on the chalkboard while learners continue writing in their books)* 
	Tr	Okay, can you please stop writing? Now remember I said that a function should be having only one output value for every input value that you have. So if you put in a certain value for an expression that you have been given, you should only get one output value for it to qualify as a function. Now if you get two values or more, it means it's not a function. Okay. So I use the vertical line test to determine if a certain graph is a function. Now I am going to show you how you use the vertical test. When I say vertical what do I mean? Which direction or…? Vertical. *(Various learners make gestures with their hands.)* I don't hear you, I just see this *(makes a gesture with her hand, followed by laughter from learners)*. Which one is vertical?
	Lnr	The *y*-axis
	Tr	The …?
	Lnrs	Y-axis
	Tr	The y-axis is vertical. Okay, that's an example of a vertical line. (*Points to the straight line drawn on the board*) Eh. Now so what you do is, if suppose you pretend (*picks up a ruler from a learner's desk*) that you are having maybe a ruler and you move your ruler (*places ruler on blackboard next to the straight line*) you are having your ruler vertically like this. (*Starts moving a ruler across the axis as shown on the Figure.*) 
	Tr	If you move your ruler throughout your Cartesian plane where you have this graph; and it only cuts your graph at the same point once, it means this is a function. Like you can see that this ruler, if I draw a straight line, this straight line is going to cut the graph only once here. Okay.
	Lnrs	Yes.
	Tr	Even if I move to this direction (*moves ruler to the right*) it's going to cut only once. If I move to this direction (*moves ruler diagonally right and up*) it's only going to cut only once. So it means this graph is a function (*learners chorus "function" with her)* because you are only having one value for y (*learners chorus "y" with her as she points to the y axes*) for every value of x. Okay?
	lnr	Yes.

<u>Episode 3</u>
<u>Chunking:</u>

Teacher described the vertical line test, demonstrated how it was used to distinguish between functions and non-functions and invited a learner to demonstrate the procedure on the chalkboard. This episode ended when teacher assigned a class activity.

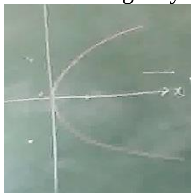
<u>Episode 3</u>

<u>Presentation formats</u>

- Teacher reminded learners about the definition of a function: one input produces exactly one output (exp)
- Teacher described the vertical line test for determining whether a given graph is a function or not; if the vertical line cuts the graphs more than once, the graph does not represent a function (exp)
- Teacher demonstrated the vertical line test using a ruler on a straight line graph and determined that the straight line graph was a function;(we)

<u>Performance expectations:</u>

To draw a vertical line that cuts through the given graph to determine the number of times the vertical line cuts through the graph. This action required applying a particular procedure on functions and it was new. I coded this action as *apply procedure* action. I also argued that it was not a global action as it did not specifically consider the behaviour of a function; hence it fell under point-wise actions.

	Tr	Do we understand each other? (*Most learners say "yes" and a few say "no".*) I see some people are (*shakes her head*). Yes, Nelson?
	Lnr	Please repeat ma'am…
	Tr	I repeat. Okay. I am saying if you take a ruler (*places ruler on diagram on board*) and you, you put it vertically on your Cartesian plane, you are having this graph. I want to determine if this graph is a function. I say a function is an expression where you put one value you get one value out, your input value will get you only one output value. Are we together?
	Lnr	Yes.
	Tr Tr	Now if I move this ruler across the …Cartesian plane like this (*moves ruler to the right*), it is cutting my graph only at one point. When it's here, I've got a certain value for x here. (*Teacher picks up chalk and marks where ruler intersects x axis and one y axis.*) There is a certain value for x here and one value for y (*learners chorus "y" with her*). Can you see that?
	lnrs	Yes.
	Tr	If I come to this point I only have one value for x and one value for y (*learners chorus "y" as she marks the values on the axes*). So I would say it is a function.
	Tr	Now let me give you another example. (*Teacher draws the following graph on the chalkboard*. 
	Tr	Can someone come and determine if this is a function or not, using the vertical line tests? (*After quite a long silence.*)
	Tr	Who wants to come and try? Yes? (*Teacher points to a learner, who comes to the board. She places ruler on board which touches the graph.*) If you draw a straight line there. (*Learner moves ruler along the graph and marks where*  *the ruler intersects the graph ass shown on the graph*)
	Tr	Can you tell us what is happening? (*Learner remains quiet and seems embarrassed.*)
	Tr	Okay, how many, how many output values are you having, or values of y are you having? How many? How many are you having?
	Lnr	Two.
	Tr	Two. Which one and which one? (*Learner points to the two y values on the graph*)
	Tr	Yes, in the y-axis. Okay. And how many values for x are you having? Two (*learner shows two fingers*).
		So here is your x. How many are you having? (*Teacher marks one place on x axis. She then speaks in an African*

Episode 3 continued:

Presentation format

- Teacher provided a different example and invited a learner to the chalkboard to demonstrate the vertical line test on graph that is a non-function. Teacher guided the learner through the process and determined that the graph was a non-function since a vertical line cut it more than once (we)

Performance expectation

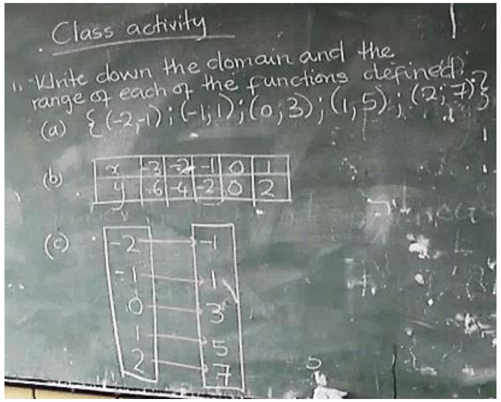
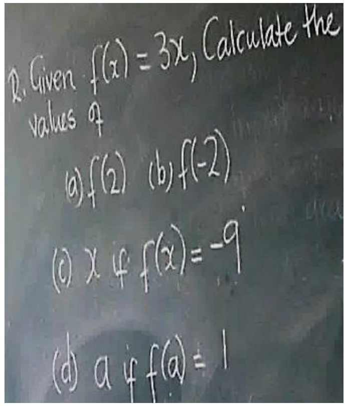
- Learners expected to draw a vertical line through a graph and to count the number of times the line cuts the graph, as teacher had demonstrated (apply procedure)

		language and says the word "one".)
	Lnr	One.
	Tr	Okay. You go and sit.
	Tr	Okay, so for this graph you can see that if you are going to move across your Cartesian plane like this, (*moves ruler over the graph*) every time your vertical line is cutting your graph how many times? Two times (*learners chorus with teacher "two times"*). Okay.
	Lnrs	Yes.
	Tr	So it means that this is the value of x (*learners chorus "x" with her*). So what it means for each input value or for each x value you are getting how many values of y? Two (*learners chorus "two" with her*), this one and this one, so it means this is not a function. I said a function is only one input value and one output value (*learners chorus "input value" and "output value" with teacher*). So if you see that you are having two values of y it means this is not a function. Do we understand each other?
	Lnrs	Yes.
	Tr	Okay can you please copy that (*points to the notes on the board*).
	Tr	So I say that this is a function (*points to 1st graph*) and you write the reason why it's a function. This one is not a function (*points to 2nd graph*), you write the reason why it's not a function. (*Learners begin to write and teacher walks around classroom*.) (*Learners continue writing as teacher cleans part of board. Teacher has a conversation with somebody out of sight of camera. I did not transcribe this conversation as it appeared to be "off the record".*)

Comment

Since the vertical line test does not feature in either the Pre-CAPS or the CAPS textbook, it was considered as an *injection* to the content of the lesson.

In the discussion, I note the different injections and explore what they make available to learn in the lesson

Episode 4 Learners working on practice exercises individually		*Teacher is copying writes the following on chalkboard as learners continue to work in their books.* 
	Tr	Okay after you have finished copying can you please do this work? Write down the domain and the range of each of the functions defined. So you are going to state, or you list, from what value to what value, for the domain and range
	Lnrs	Hmm?
		(a) and (b)
	Tr	Hmm?
		what is the domain of the function?
	Lnrs	The x value.
	Tr	It's the … ?
	lnrs	x value.
	Tr	Yes. So you are going to list down the domain of the function and the range. Okay. So here you have been given co-ordinates of different points. You are supposed to check the domain and the range, the domain and the range. So you are going to list the domain first, of all these co-ordinates; and the range of all these *(points to board).* The same for (b), (c). For question two the first one and the second one I think they are straight forward.

<u>Episode 4: Chunking</u>

This episode continued to the end of lesson. Learners were working individually on some practice questions as teacher moved around the classroom assisting them and checking their work.

<u>Presentation formats</u>

- Teacher wrote the class activity on the board consisting of questions from the textbook (pr)
- Teacher explained what each question required of learners, giving learners hints and tips where necessary (exp)

<u>Performance expectations</u>

- 1.a) – c): list elements of the domain and the range separately (substitute)
- 2. a) & b): substitute given values for x in $f(x) = 3x$ and evaluate their function values (substitute)
- 2 c): equate the given function to -9 and solve for x (substitute)
- 2.d): equate the given equation to 1 and solve for a (substitute)

<u>Comment:</u>

All questions in the practice exercise given in class had been selected from different sets of exercises in the CAPS textbook. The teacher made some omissions of the questions. In the discussion below I look at the role of these *omissions* on the opportunities for the mediation of the object of learning.

6.3.1 Appropriation of Affordances in lesson A11

The results of the analysis of lesson A11 are presented in Table 6.2 and their findings discussed. The five columns in Table 6.2 represent: the *episode;* the *degree of appropriation* which indicates whether an episode was offloaded, adapted, or improvised; the *presentation formats* identified in each episode; the *performance expectations* on function tasks; and general comments relevant to the teacher's appropriation of the affordances. The last row on Table 6.2 summarises the columns.

Episode	Degree of Appropriation	Presentation formats	Performance expectations	Comments
1 (6 min) Function notation	Offloaded	Explanatory talk (exp)	Substitute (pointwise)	A1 uses the CAPS textbook for definition
2 (6 min) Worked examples	Offloaded	Worked examples (we) Demonstration (we)	Substitute (pointwise)	An omission: Two other worked examples from textbook omitted in the lesson
3 (10 min) The vertical line test for functions	not in textbooks	Definition (exp) Explanatory talk (exp) Demonstrations (we) Worked example (we)	Apply Procedure (pointwise)	Injection: The vertical line test for distinguishing between functions and non-functions inserted into the lesson
4 (25 minutes) Practice exercises	Offloaded	Practice exercises (pr) Explanatory talk (exp)	Substitute (pointwise)	Practice exercises do not include questions on the vertical line test
Content coverage: CA1 on notation and terminology	Offloaded $(O^{+}I^{-})$	quasi-deductive approach	Pointwise actions	Injection enhances opportunities for mediation Omissions do not detract from opportunities for mediation

Table 6.2 Summary of Analysis for Lesson A11

In the four episodes in this lesson, A1 offloaded the content from the textbook in three of them, 1, 2 and 4; and injected content in episode 3. In other words, the teacher relied on the textbook for the definition and description of function notation (episode 1); for the worked examples (episode 2); and the practice exercises (episode 4) which learners worked on individually until the end of the lesson. As a result, the lesson was classified as *offloaded* with a code $(O^{+}I^{-})$ to indicate that the lesson was predominantly offloaded, but that there had been some injecting (episode 3) of external content as well. Hence the following codes shall be applied for the rest of the lessons: O = offloaded; $I =$ improvising; and $A =$ adapting; Inj= injection of external content; Om=omission of content from the textbook. The content covered in lesson A11 corresponds to CA1 in the textbooks on notation and terminology; and except for the vertical line test, all definitions, worked examples, and practice exercises derived from the textbook.

The presentation format *explanatory text* became *explanatory talk* since in the lesson it was mostly through talk that the teacher explained even though the definitions were written down. However, the *explanatory talk* was used to include both talk and text. A new presentation format was added to those emerging from the textbook: *demonstration*. A demonstration occurred in episode 3 where the teacher demonstrated the vertical line test to learners. Due to the similarity of the demonstration to the worked example, since this was where the teacher showed learners how to do something, I put the demonstration under the worked examples and with the same code, (*we)*, as well. The sequencing of presentation formats showed that *explanatory talk/text* including definitions, descriptions, and demonstrations, preceded worked examples throughout the lesson, and then at the end of the lesson, learners had the opportunity to practice individually. Thus the object of learning was made available through worked examples hence, a *quasi-deductive approach* to teaching.

With respect to performance expectations, *apply procedure*, was added to the textbook list of performance expectations. I regarded *apply procedure* as a form of practice for learners: the teacher had shown them how to apply the vertical line test, and then learners applied the procedure. I also considered it as an action that did not require global properties of functions, since all one needed to do to decide whether a given graph represented a function or not was to look at specific points as the vertical line moved along the graph. Hence I placed *apply procedure* under *pointwise* category of actions on function tasks. Since all actions on function tasks in the lesson were pointwise, I concluded that the lesson conveyed a pointwise conception of function. This was the same conception conveyed by the textbook the teacher used.

The analysis of the approach of the textbook in chapter 5 indicated that the textbooks adopted a quasi-deductive approach for the content area on notation and terminology, the content area in which the present lesson falls. Furthermore, the performance expectations for the same content area in the textbook were pointwise too just like in the present lesson. This implies then that in this first lesson, teacher A11 offloaded both affordances of the textbook; the content and the approach.

Looking back at Table 6.2 again, it shows an *injection* of the vertical line test in episode 3. The vertical line test is referred to as an *injection* in the lesson because firstly, it does not form part of the content in either the Pre-CAPS or the CAPS textbooks; and secondly because it has not been specified as part of the grade 10 curriculum. This second feature of *injections* distinguishes them from *improvised* content in the lesson: the teacher *improvises* if she brings into the lesson content that is available in the textbook but decides to bring it from external resources. Furthermore, the vertical line test is used as a visual means to distinguish between graphs which represent functions and those which do not, and is regarded as an important aspect of the teaching and learning of functions which is featured in many textbooks on functions. In fact, it was featured in the workshop on functions organised and conducted by the WMCS for teachers in the project schools which teacher A1 also attended. Thus the vertical line test is an *injection* that enhances opportunities for the mediation of the object of learning. Such *injections* are referred to as *robust injections* in the study, in contrast to those which detract from the opportunities for mediation, which are called *distractive injections.*

On the other hand, while teacher A1 offloaded most of the content of the present lesson from the textbook, she also omitted some parts of the content. The study refers to the omitted parts as *omissions*. For example, she utilised only two worked examples in the lesson out of the four provided by the textbook. One of those worked examples she did not utilise featured a contextual question involving temperature. Another worked example not utilised in the lesson is shown in Fig 6.3.

Example

Given the function $g(x) = 1 - 2x$, evaluate:

a) $2g(x)$
b) $g(2x)$
c) $g(x) + 2$
d) $g(x + 2)$

Solution

a) $2g(x) = 2(1 - 2x)$
$= 2 - 4x$
b) $g(2x) = 1 - 2(2x)$
$= 1 - 4x$
c) $g(x) + 2 = 1 - 2x + 2$
$= 3 - 2x$
d) $g(x + 2) = 1 - 2(x + 2)$
$= 1 - 2x - 4$
$= -3 - 2x$

Fig 6.3 An example of an omitted worked example from the textbook

In this worked example, the function $g(x)$ is regarded firstly as an object that can be operated on in its own right, for example, as in adding a number 2 to the object, or multiplying the object by 2. Secondly, the input of the function changes from an integer, as in $g(-1)$ to an expression, for example, evaluating $g(2x)$ given that $g(x) = 1 - 2x$. Teacher A1 omitted this worked example in episode 2, and all similar questions in the practice exercises. However, of significance is that A1 did not completely ignore these kinds of questions because she dealt with them in the second lesson, lesson A12 under transformations of functions.

The significance of the *omissions* made in this lesson lies in their nature: in the example in Fig 6.3, examples of its kind were picked up later in other lessons, and therefore the *omission* did not detract from opportunities for mediation in the classroom; similarly for the contextual questions. I refer to these kinds of *omissions* in the study as *productive omissions*; their non-selection by the teacher in the lesson did not compromise the opportunities for mediation. In the next chapter on the analysis of the rest of the lessons, a different category of *omissions* is observed which results in detracting from opportunities for mediation. These I refer to as *critical omissions:* those aspects of the object of learning that are critical to its mediation that teachers omit from the lessons.

In lesson A11 therefore, teacher A1 offloaded the content and the approach of the textbook while making *productive omissions* of the textbook content and at the same time enhanced the lesson with *robust injections* which did not feature in the textbook. She utilised the textbook for all aspects of the lesson: definitions, worked examples and practice questions. All these suggest that A1 *has a participatory and therefore a strong relationship* with her textbook in this lesson.

In the next sections, I present the analyses of teacher A1's other two lessons, A12 and A13. Following a similar process to that in lesson A11, the lessons were chunked into units that served as episodes for analysis. The analysis of each episode followed four steps: a) a description of the episode; b) identifying presentation formats in the episode; c) identifying performance expectations; and, d) identifying omissions and injections, where applicable. At the end of the analyses of the four episodes, the results are summarised and discussed.

6.3.2 Appropriation of Affordances in Lesson A12

Lesson A12, took place four days after lesson A11, and derived from a workshop on functions conducted by the WMCS[14] on the teaching of functions. Hence, it was a completely improvised lesson. The content covered in the lesson corresponded to CA3 on transformations of functions in the textbook. The full transcript for lesson A12 showing the episodes is appended to the thesis (Appendix D). The objective of the lesson was for learners to investigate the effect of the parameter 'a' on the function $f(x) = x^2$ when the position of 'a' was varied around $f(x)$, for example when 'a' was added to the function as in $f(x) + a$; and when 'a' was part of the argument of the function, as in, $f(x + a)$. Learners had been given homework to complete the tables of values for the functions: $f(x-1), f(x) - 1, f(x+1), f(x) + 1, 2f(x), f(2x),$ and $f(\frac{1}{2}x)$. In the lesson, A1 and learners were completing the values on the board and drew the graphs of the functions. A1 guided learners to formulate generalisations about the transformations of the parabola.

The transcript of lesson A12 has been chunked into the following three episodes: In Episode 1, a table of values for the functions, $f(x) = x^2$, $f(x+1)$ and $f(x) + 1$ was populated, their graphs drawn on the same set of axes, and generalisation made about the effect of adding 1 inside the bracket versus adding 1 outside the bracket. In Episode 2, the effects of the functions, $f(x-1)$ and $f(x) - 1$ were generalised from observing the behaviour of functions in Episode 1, and their graphs sketched without plotting the points. Episode 3 entailed a class activity in which learners were expected to complete a table of values for the function, $f(\frac{1}{2}x)$, draw the graphs of the functions, $f(2x), 2f(x)$ and $f\left(\frac{1}{2}x\right)$; and generalise about their effect on the function, $f(x) = x^2$.

6.3.2.1 Drawing the graphs of the functions $f(x+1)$ and $f(x) + 1$: Episode 1

The lesson began with A1 drawing the following table of values for different functions on the board:

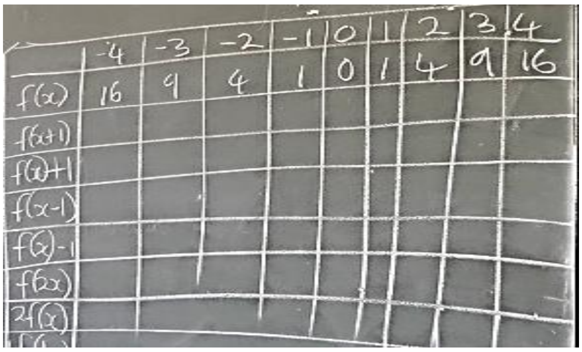


Fig 6.4a Table of values to be completed in lesson A12

This table had been given as homework for learners to complete and to draw the graphs of these functions. A learner, whom I refer to as Lnr1, completed values for the function

[14] This workshop was run by Bernard Murphy of the Mathematics in Education and Industry (MEI), UK

$f(x+1)$ and drew its graph on the same set of axes with the graph of $f(x) = x^2$ as shown below.

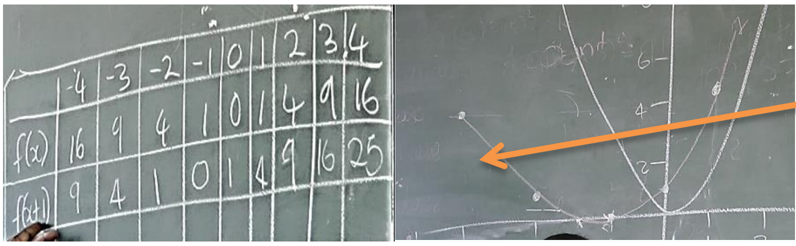


Fig 6.4b A table of values and a graph of $f(x+1)$

A1 then guided learners to generalise about the effect on the function $f(x)$, of adding 1 inside the bracket from the tables through comparing function values for the same input value in each function. A1 showed learners that function values for the function $f(x+1)$ shifted the values for $f(x)$ one unit to the left. For example, $f(-4)$ for $f(x+1)$ which is the value 9 is the same as $f(-3)$ for $f(x)$ and so forth, as the arrows below indicate.

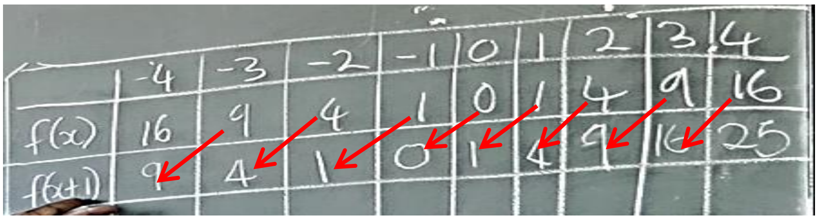


Fig 6.4c A table of values showing a shift of values for $(x+1)$

Learners concluded hence that $f(x+1)$ had shifted the graph of $f(x)$ horizontally to the left by one unit; and that the turning point for the graph of $f(x+1)$ was the point $(-1; 0)$.

For the function, $f(x) + 1$, learners called out its function values as A1 completed its table of values

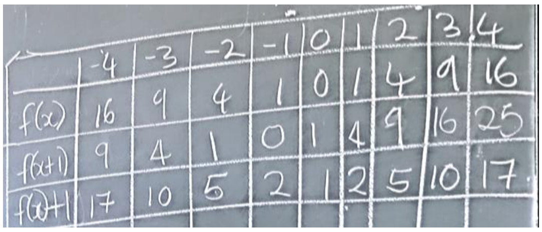


Fig 6.4d A table of values for $f(x) + 1$ added

Learners concluded with the help of A1 that $f(x) + 1$ added 1 to function values for $f(x)$, as shown in Fig 6.4d above; and generalised that $f(x) + 1$ shifted the graph of $f(x)$ one unit upwards, thus shifting the turning point from $(0; 0)$ to the point $(0; 1)$. A1 drew the graph of $f(x) + 1$ on the same set of axes with the other graphs as shown below

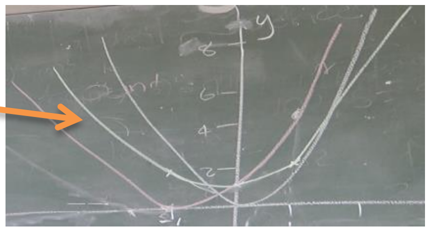


Fig 6.4e A sketch of the function $f(x) + 1$

The results for the rest of the analysis of episode 1 are presented in Table 6.3 below.

Presentation formats	Performance expectations	Injections and omissions	Comments
The presentation formats in this episode are an *activity (act)* because learners were provided opportunity to engage with the functions first; next was the *explanatory talk (exp)* by the teacher mostly when she stepped in with guidance for learners to generalise about the effects of $f(x+1)$ and $f(x)+1$ on the function $f(x)$. The *explanatory talk* included learners' responses to teacher's questions and prompts. This episode hence adopted a *quasi-inductive approach* to teaching and learning.	Learners were firstly expected to *substitute* given input values into the function and evaluate function values; *plot* the points on the Cartesian plane; *draw* the graphs by joining the points; and then to *generalise* about the effects of the two functions on the function, $f(x)$. Thus the episode consists of three performance expectations in the order: *substitute, plot and draw*, and *generalise properties* and an approach towards teaching functions that *progresses from from pointwise to global* strategies	The function $f(x+1)$ which depicts a horizontal shift of the function, $f(x)$, does not feature in the Grade 10 curriculum and therefore constitutes an *injection* to the content. $f(x+1)$ was brought in to contrast with $f(x)+1$. Hence, its inclusion enhances opportunities for mediation, therefore a *robust injection.*	Episode completely improvised: *improvised* *(I^{+})* Content coverage: CA3 on transformations

Table 6. 3 Analysing Episode 1 of Lesson A12

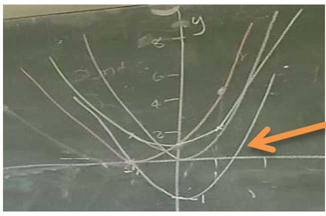
In this episode, there are two presentation formats evident, namely, *activity (act)* followed by the *explanatory text (exp)*, thus a *quasi-inductive approach.* Three performance expectations have been identified beginning with *substitute* actions which are pointwise, then *plot and draw* actions which are also pointwise, followed by *generalise properties* actions that are global. Therefore, the conception of function in this episode reflects a *progression from pointwise to global* actions. The inclusion of the function $f(x+1)$ is considered as a *robust injection* as it stands in contrast to the function $f(x)+1$ and therefore illuminates the difference in shifts effected by "adding 1 in inside the bracket" versus "adding 1 outside the bracket" as the teacher refered to the transformations:

Tr	Okay, so this is a shift in the y-axis and remember here we added one inside and now this one is being added to the function *(points to $f(x)+1$))*. Okay?
Lnrs	Yes.
Tr	So, when you add one inside of the bracket, the movement is being affected in which axis? (*Pointing to* $f(x+1)$). In the x-axis. Now we added outside the function, in the function plus one the movement is going to be affecting the y-axis. Okay?
Lnrs	Yes.

As a result therefore, the inclusion of the function $f(x+1)$ is considered as a *robust injection:* it enhances opportunities for mediation.

6.3.2.2 Sketching the graphs of $f(x-1)$ and $f(x)-1$: Episode 2

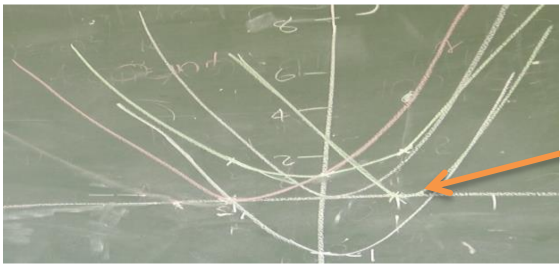
In this second episode, A1 asked learners what they thought the effect of the function, $f(x)-1$ on the graph of $f(x)$ would be, based on the conclusions they had made about the effect of the functions, $f(x)+1$ and $f(x+1)$ on the function $f(x)$ in episode 1. Learners concluded that the graph of $f(x)$ would shift downwards by one unit, and the turning point would now be the point $(0;-1)$. A1 then drew the sketch of the function, $f(x)-1$ on the same set of axes with the other graphs from episode 1 as follows:



Graph of $f(x) - 1$

Fig 6.5a A sketch of the graph of $f(x) - 1$

A1 again asked learners the same question about the function $f(x-1)$ and what they thought its effect on $f(x)$ would be. Learners concluded that the function would shift the graph of $f(x)$ one unit to the right, with a new turning point at $(1; 0)$, which teacher drew on the same set of axes with all the other graphs.



Graph of $f(x-1)$

Fig 6.5b A sketch of the graph of $f(x-1)$

The results of the rest of the analysis for episode 2 are presented in Table 6.4

Presentation formats	Performance expectations	Injections and omissions	Comments
In this episode, A1 led a discussion with learners from which learners generalised the effect of the two functions, $f(x-1)$ and $f(x)-1$ on the function $f(x) = x^2$. While it was A1 who drew the graphs, she was directed by learners. Hence in this episode, the presentation format has not changed from an *activity (act).*	Learners were required to generalise the effect of the two functions on the function $f(x)$, and to sketch their graphs based on the generalisations they make. Hence the two performance expectations on the learners are: *generalise properties* and *interpret properties*	The function $f(x-1)$ also is not in the curriculum requirements for Grade 10, hence it is an *injection* to the content. Like $f(x+1)$ in previous episode, it enhances the opportunities for mediation, therefore a *robust injection*	Episode completely *improvised* (I^+) Content coverage: CA3 on transformations

Table 6.4 Analysing Episode 2 of Lesson A12

The only presentation format in this episode is an *Activity (act)*, rendering the episode a *quasi-inductive approach.* The two performance expectations of *generalise properties* and *interpret properties* feature only global actions on function tasks and therefore the conception of function is interpretive and *global*. The inclusion of the function $f(x-1)$ is a *robust injection* for similar reasons as advanced in the previous episode for $f(x+1)$; that they enhance the opportunities for mediation.

6.3.2.3 Drawing the graphs of $f(2x)$, $2f(x)$, and $f(\frac{1}{2}x)$: Episode 3

In this episode 3, A1 completed the rest of the table of values except for the function $f(\frac{1}{2}x)$ which she asked learners to do, as shown below

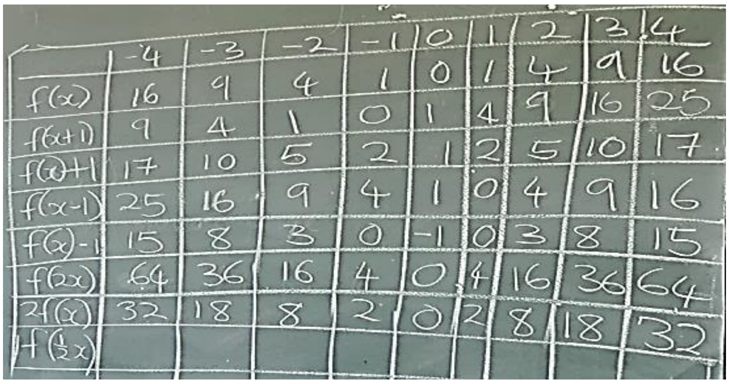


Fig 6.6 A completed table of values for all functions

She then instructed learners to draw the graphs of $f(2x), 2f(x)$ and $f(\frac{1}{2}x)$ on the same set of axes, and provided a scale for learners to use for graphing. For the rest of the episode, learners worked in their books individually while teacher walked around the classroom assisting them.

The rest of the analysis of this episode is presented in Table 6.5

Presentation formats	Performance expectations	Injections and omissions	Comments
This episode is a continuation of the activity to complete the table of values and to draw the graphs of the remaining functions. Hence, the presentation format illustrated is still an *activity (act)* and therefore a *quasi-inductive approach*	Learners were expected to *substitute* input values and *evaluate* the function values for the function $f\left(\frac{1}{2}x\right)$; *plot the points* for the three functions mentioned, and *draw their graphs* on the same set of axes. Finally, they were required to *generalise* about the effect of the functions $f(2x), 2f(x)$ and $f(\frac{1}{2}x)$. Hence, the three performance expectations in this episode are sequenced in the order: *substitute, plot and draw* and *generalise properties* which	Among the three functions drawn in this episode, two of them, namely, $f(2x)$ and $f(\frac{1}{2}x)$ do not feature in the textbooks as they are also not required in the Grade 10 curriculum, hence they constitute *robust injections* to the content.	Episode fully *improvised* (I^+) Content coverage: CA3 (transformations)

Table 6.5 Analysis of Episode 3 of Lesson A12

Similar to episode 2 above, an *activity (act)* is the only presentation format in this episode, and therefore the *approach* is *quasi-inductive.* The episode features three performance expectations: *substitute* (pointwise)*, plot and draw* (pointwise) and *generalise properties* (global). Thus, the conception of function in this episode depicts a progression from *pointwise to global* strategies. Once again, the variance and invariance notion brought in with the functions $f(2x)$ and $f(\frac{1}{2}x)$ renders these *robust injections* to the content.

6.3.2.4 Mobilising the Textbook in Lesson A12 – Synthesis across episodes

Lesson A12 covers content on transformations of functions that corresponds to CA3 in the textbooks. In this second lesson, teacher A1 did not utilise content from the textbook, but from external resources. Lesson A12 is therefore a completely improvised lesson with no offloading, and therefore classified as *improvised* (I^+). In all three episodes, teacher A1 adopted a *quasi-inductive approach* to the teaching and learning of functions. Learners engaged initially with evaluating function values for the given transformations, drawing their graphs, and then making generalisations about their properties. They were then expected to sketch the next set of functions in the next episode not through point by point plotting of points, but by interpreting the behaviour of the functions drawn in the first episode. Thus the first two episodes started with actions on function tasks expected of learners which were completely pointwise, and then progressed towards completely global actions. The same progression from pointwise to global actions was expected of learners in the last episode. Thus, the conception of function conveyed in lesson A12 is one that depicts a progression from a pointwise to a global outlook.

However, more than this, the approach also featured the notion of variance and invariance (Watson & Mason, 2006) in its execution, where learning of aspects of function is done through varying aspects to be learned, and as Watson and Mason posit, an aspect is "more likely to be discerned if its variation is foregrounded against relative invariance of other features"(p. 98). They encourage that it is not through individual items in an exercise that this is accomplished but through the structure of the exercise as a whole. I point out that this is what this entire lesson does: it is not based on individual exercises but on one single table of values where the next exercise follows the previous and the aspects are varied in a particular way. The contrast to the position of the parameter in the function provided opportunity for learners to differentiate between transformations that produced vertical shifts and those which produced horizontal shifts to the parent function. Since the injection of the horizontal shifts, eg $f(x-1)$and $f(x+1)$, and horizontal stretches and compressions eg $f(2x)$ and $f\left(\frac{1}{2}x\right)$, in this lesson enhances the possibilities for mediation, these functions are considered as *robust injections.*

Brown and Edelson (2003) found that improvisations of curriculum materials is generally deliberate as teachers depart from the materials and pursue their own path. This is the same characteristic observed in lesson A12. Teacher A1 has improvised from another resource in order to achieve her instructional goals of helping learners to learn concepts through contrasting horizontal and vertical shifts which the textbook does not employ. This suggests some deliberateness to the departure from the textbook, especially when the injections she made to the lesson are robust and enhance opportunities for mediation. The improvisation of lesson A12 thus confirms that teacher A1 *has a relationship* with her textbook.

6.3.3 Appropriation of Affordances in Lesson A13

The third lesson for teacher A1, lesson A13 took place two weeks after the second lesson. This had been in the last week of the four-week duration for teaching functions. The full transcript of lesson A13 is appended to the thesis as Appendix E. The lesson covers content in CA4 on interpretation of functional properties in the textbook. Lesson A13 started with correction of homework where A1 and learners were going over the questions. Learners had been given homework from the Pre-CAPS textbook on determining equations of functions from graphs. After correcting the homework, A1 introduced the unit on sketching of functions, also based on the Pre-CAPS textbook.

Applying the same procedure as in the two previous lessons, lesson A13 was divided into four main episodes analysed individually below. In episode 1, a graph of a function showing one or two points through which it passes was drawn and its equation presented in the form $y = af(x)$. The task here was to determine the equation of each graph by evaluating the value of 'a'. Episode 2 included the same task as in episode 1, but additional to episode 2 was that five different equations in the form $y = af(x)$ were presented, and before the value of 'a' was evaluated, the equations had to be firstly matched with appropriate graphs. Episode 3 involved the sketching of the function $y = \left(-\frac{3}{2}\right)x^2$; while the last episode was a class activity where learners were assigned some functions to sketch.

6.3.3.1 Evaluating the value of 'a' in functions of the form $y = af(x)$ given $f(x)$: Episode 1

In this first episode of lesson A13, there were two graphs for which the equations had to be determined namely, the graphs of $y = ax^2$ and $y = a.2^x$, respectively. Teacher A1 began with the graph of $y = ax^2$ by substituting the point $(-2; 2)$ into the equation, and solving for 'a' for which the value obtained was $\frac{1}{2}$. She then repeated the same process on the same function with the other point $(2; 2)$ to show learners that they would obtain the same answer when substituting the other value. The question and the solutions are shown in Fig 6.7a

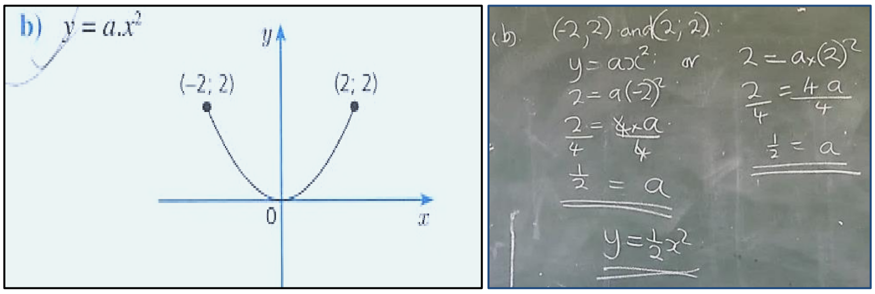


Fig 6.7a Finding the equation of the graph of $y = ax^2$

The second function was an exponential function and A1 substituted the point on the graph, $(0; 3)$ in the equation $y = a.2^x$ to obtain the value of 'a' as 3 and the equation of the graph in $y = 3.2^x$ as shown in Fig 6.7b

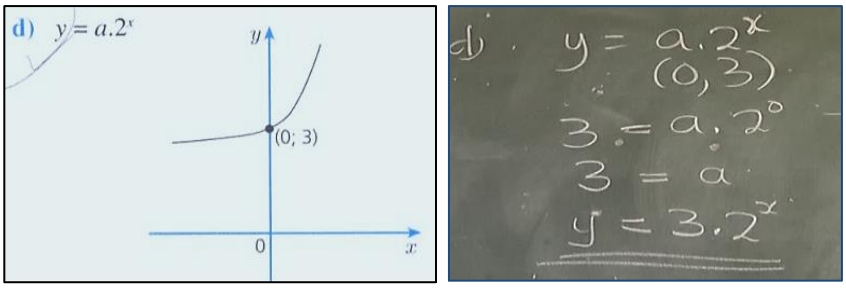


Fig 6.7b Finding the equation of the graph of $y = a.2^x$

Table 6.6 represents the rest of the analysis of episode 1

Presentation formats	Performance expectations	Injections and omissions	Comments
A1 worked out the questions on the board and showed learners how they should have answered them. I regard these as *worked examples* because of the fact that the teacher was showing learners how to answer the questions. The teacher also offered explanations and clarifications where appropriate, thus *explanatory talk (exp)*.	Learners were expected to follow what the teacher did: *substitute* values into the given equation to evaluate 'a', and then *substitute* this value of 'a' in the general equation to determine the equation of the given graph. These are all pointwise actions, and therefore the approach to functions in this episode is *pointwise*	In the textbook exercise from which the questions were selected, there were six questions from which A1 selected the two, therefore there are *omissions* of content which A1 makes. However these are omissions which do not detract from the object and therefore *productive omissions*	The questions have been selected from a textbook exercise in the Pre-CAPS textbook. Therefore *offloaded* (O^{+}) content

Table 6.6 Analysing Episode 1 of Lesson A13

Since A1 does all the work on the board offering explanations and clarifications and asking for responses from learners, the presentation format is classified as *explanatory talk*. Since itis the only presentation format, then the general approach to teaching and learning of the content is *quasi-deductive*. For the performance expectations, there is only one, the *substitute actions*: learners are expected to substitute the points through which the graphs pass into the general equations and calculate the value of 'a'. The processes of substituting, calculating or evaluating the value of 'a' all fall under the *substitute* category of performance expectations in which all the actions on function tasks are pointwise. Hence the conception of function in this episode is *pointwise*.

The two questions have been selected from a textbook exercise which contains six questions. Thus the episode is offloaded completely from the textbook and therefore carries the code *offloaded* (O^{+}) which indicates that there is only offloading in the episode. Furthermore, the selection of only two questions out of six questions in the exercise does not diminish opportunities for mediation as the process of substitution which teacher A1 has demonstrated to learners applies to the rest of the questions. Thus these are *productive omissions (om^{+})* as far as the study is concerned.

6.3.3.2 Determining the equations of graphs: Episode 2

In this episode, the task was to match a graph with its appropriate general function, from the functions: $y = ax$; $y = ax^2$; $y = \frac{a}{x}$; $y = a.b^x$; $y = a \sin x$; $y = a \cos x$; and $y = a \tan x$. The next task was to determine the equation of the graph. There were four different function classes involved in this episode, namely, exponential, quadratic, trigonometric[15], and a hyperbola.

Teacher A1 began with the exponential graph asking learners to identify the type of function and to match it to the appropriate equation. Learners identified the function as an exponential

[15] This is one lesson in which trigonometric functions feature and teacher selects them

function and took some time to match it with the function, $y = a.b^x$. The next question that A1 asked learners was what they knew about the parameter 'b' and its effect on the graph of the exponential function, which led to a long discussion about the effect of 'b' and the direction of the exponential function. Teacher explained that the value of 'b' determines the direction of the exponential graph and illustrated this with the following examples.

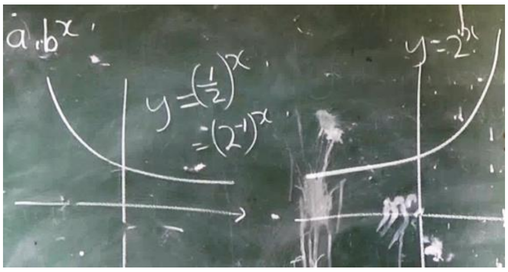


Fig 6.8a Graphs for explaining the difference between exponential functions

The teacher's explanation of the direction of the exponential function continued as follows:

A1	When you have got an exponential graph and when it's facing this side (*pointing to the graph on the left*) it means your 'b' here is a fraction. You are having a fraction here or you are having a number, or your exponent is having a power which is negative, okay?
Learners	Yes.
A1	So this is the same as two to the power negative one (*teacher writes $y = (2^{-1})^x$ on the graph on the left*). And this one is two to the power, one (*pointing to the graph on the right*)
Learners	One.
A1	A positive one *(learners chorus 'YES')* to the power x. That's why they are different, the other one is facing this side (*points to the left)* and the other one is facing this side (*points to the right*). Now, (*indicating to the graph on the left*) but this can be written as a fraction. So when this exponential graph is facing in this direction (*pointing to graph on left sloping downwards from the upper left)* it means you are having a fraction inside here (*pointing to the $\frac{1}{2}$ on $y = \left(\frac{1}{2}\right)^x$*). And when it's going in this direction (*indicating with her arms from upper right sloping downwards, for the graph on the right*), it means you are having a whole number here. (*pointing to the 2 on $y = 2^x$)* Okay?
Learners	Yes.

The teacher then substituted the two points, $(-1; 2)$ and $(0; 1)$ that the graph passes through to evaluate the values of 'a' ($a = 1$) and 'b' ($b = \frac{1}{2}$). The equation of the graph was then determined as the function $y = \left(\frac{1}{2}\right)^x$.

A1 then used the similar procedure for the quadratic graph, that is, asking learners to identify the type of function and what they knew about 'a' and its effect. Learners were able to say that the graph was a parabola and to match it with the equation $y = ax^2$. But teacher explained before learners could offer a reason for their answer and drew the two parabolas as shown and asked learners what the difference between them was as shown in the extract below?

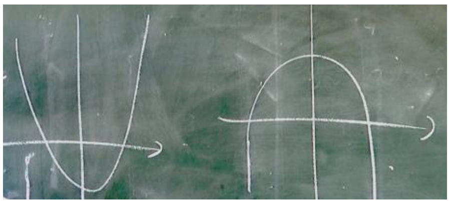


Fig 6.8b Graphs for explaining the difference between quadratic functions

A1	What's the difference between these two graphs? *(Pause – learners talk, teacher points to a learner.)* Yes? *(The learner's response is inaudible)*
A1	What is negative? *(begins writing on the board above the graph on the right)* Give me a general formula? *(Turns and looks at learner.)* Where do I put the negative? *(Pause – teacher turns back to the board and begins writing).* It's negative?
Learners	x squared.
A1	x squared *(writes $y = -x^2$ near the graph on the right then moves to the graph on the left and begins to write next to it).* And this one is?
Learners	Positive x squared.
A1	Okay, so just by looking at the graph it's facing? *(pointing to the graph on the left.)*
Learners	Downwards.
A1	Downwards. It means you're 'a' is going to be what?
	(There is a pause while the teacher waits for a response, she moves right and points to the graph on the right.)
Learners	Negative. *(some learners are heard saying fraction)*
A1	It's not going to be a fraction, it's going to be?
Learners	Negative.

Teacher A1 then substituted the point(s) on the graph, that is, $(-1;-1)$ and $(1;-1)$ to evaluate the parameter 'a' ($a = -1$), and the equation of the graph as $y = -x^2$. The next graph that teacher A1 dealt with was a trigonometric graph. As I mentioned in chapter 5, even though trigonometric functions were not used as part of the textbook analysis, and as very few teachers dealt with them during the classroom observations, where they did feature in the lessons, I treated them like all other data, as in this case. Teacher A1 asked learners to identify the type of graph which learners said was a trigonometric function and paired it with the function, $y = a\tan x$. Teacher then substituted the point $(45°; 0)$ in the equation and obtained, $a = 2$, and the equation of the graph as $y = 2\tan x$. There was a resounding "yes" from learners when A1 wrote down the equation of the graph.

The last graph that A1 and learners dealt with was a graph of a hyperbola. Learners identified this graph quickly and matched it with the graph of $y = \frac{a}{x}$. A1 then substituted the point $(4;-2)$ which the graph passes through in the general equation to evaluate the value of 'a' as -8, and the equation of the graph as $y = -\frac{8}{x}$.

Table 6.7 presents the rest of the analysis for episode 2

Presentation formats	Performance expectations	Injections and omissions	Comments
The questions in this episode were given as homework, implying that they were practice questions for learners. However, in class, the teacher led the discussion of the questions and worked them out on the board. Hence, I regard them as *worked examples* instead of practice exercises. The two presentation formats in the episodes are the *explanatory talk (exp)* and *worked examples (we)* and	From the guidance by the teacher, learners were expected to firstly determine what type of function the graph is and then match it with the appropriate equation. Learners did this through *interpreting* the properties of these functions. Learners were then expected to perform some *substitution* to obtain values of the coefficient 'a' in the equation and for determining the actual equation of the given graph. Thus the approach to functions is *global*. The substitution done	The questions in this episode were selected from among 18 in the textbook exercise; spread over four different function classes. therefore *productive omission*	Error: Teacher mistakenly labelled the two parabolas in as $y = -x^2$ and $y = x^2$ even though they do not turn at the origin when illustrating the effect of the sign of the parameter 'a' in $y = ax^2$ Content *offloaded* (O^+)

therefore a *quasi-deductive* approach.	at the end is the result of the global consideration of the properties of the function		

Table 6.7 Appropriation of affordances in episode 2 of lesson A13

The two presentation formats for the whole episode as Table 6.7 shows are the *explanatory talk (exp)* since the entire episode features teacher explanations, illustrations and learner responses to teacher prompts; followed by *worked examples (we).* Teacher A1 works on the board showing learners how to answer questions. These suggest a *quasi-deductive approach* for the current episode. Identifying the function type for all the graphs involves interpreting the features of the graphs in order to associate them with a particular function. This is a global consideration of the function. Similarly, matching a graph with its general equation involves global actions. This means that the conception of function in the episode is global, even though the substituting of points that lie on the graphs in order to calculate the value of the parameters in the general equation is pointwise. The substitution is a product of a global consideration and is a technique that is used to obtain the parameters.

There are no apparent *injections* to content; the examples for illustration that teacher A1 brought in were well within the curriculum of grade 10. However, the four homework questions were selected from about twenty questions, meaning that teacher omitted some of these questions. What is noticeable here is that A1 chose questions that depict different function classes, thus giving learners the opportunity to experience interpreting the features of all the functions. The omissions are therefore *productive* as the selection of homework questions seems not to detract from the opportunities for mediation.

An important comment from Table 6.7 concerns an error that teacher A1 made when she named the two graphs she used to illustrate the effect of the parameter 'a' in $y = ax^2$. She labelled the two graphs as $y = -x^2$ and $y = x^2$, respectively in Fig 6.8b even though the graphs do not turn at the origin. This is an error. While this is a serious error as the episode or more generally the lesson as a whole was about interpreting and applying functional properties; it is noted that the focus of the illustrations was on the direction of the parabolas, that is, if they face down or up, and not on the general properties of the graph, and therefore it is possible that the wrong message of the labelling of the graphs did not reach the learners.

6.3.3.3 Sketching graphs of functions: Episode 3

The episode began with A1 informing learners that the objective for the lesson was to sketch graphs when given defining equations. She invited learners to turn to a practice exercise in the Pre-CAPS textbook where the textbook outlines the particular features which learners should show when sketching graphs of functions as:

- any intercepts on the axes
- the coordinates of one other point on the graphs (the turning point or vertex where applicable)
- asymptotes and/or axes of symmetry should be shown where appropriate
- for trigonometric graphs, show at least two periods (Laridon et al., 2008, p. 330)

A1 then illustrated how to show periods when working with trigonometric functions. She chose the first question, $y = 2\sin x$ to work with, but learners opted for a different question

$y = \left(-\frac{3}{2}\right)x^2$ instead. The first question the teacher put to learners was to name the type of function the equation represented and it took some coaxing from the teacher for learners to answer that the function would be a parabola facing downwards. The first properties of the function that A1 and learners investigated were the x- and the y-intercepts of the function. Teacher began by asking learners if the graph of the function would cut the y-axis as illustrated below.

A1	And is it going to cut the y axis? (*pause*) Is it going to cut the y axis?
learners	No, no.
A1	It's not going to cut the y axis? What is the value of x in the y axis?
learners	Zero.
A1	If we substitute here, what are we going to get? (*pointing to the equation* $y = \left(-\frac{3}{2}\right)x^2$). When x is equal to zero what are we going to get for y?
Learners	Zero.
A1	It's a zero also?
Learners	Yes.
A1	(*points at the origin*) So it means this graph is passing through this point. (*learners chorus 'yes'*). Do you agree? (*learners chorus 'yes'*)

For the y-intercept, teacher followed the same procedure urging learners to substitute the value $x = 0$ into the equation to show that the y value would be equal to 0 as well.

A1 then chose x-values of 1 and -1 to evaluate their function values and plotted the points $\left(1; -\frac{3}{2}\right)$ and $\left(-1; -\frac{3}{2}\right)$ on the set of axes as points where the graph would pass through, and remarked to learners that choosing any values of x would yield smaller values of y that are negative. A1 drew the following graph as the sketch of the function $y = -\frac{3}{2}x^2$

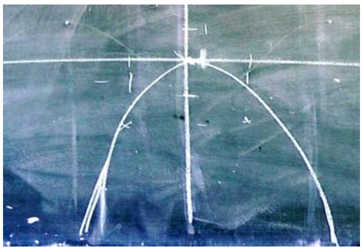


Fig 6.9 A sketch of the function $y = -\frac{3}{2}x^2$

One learner went to the board and wrote the function $y = \left(-\frac{3}{2}\right)^2 x^2$ asking about its sketch. Teacher asked learners what kind of graph would emerge and learners indicated that it would be a parabola because it has x squared. The teacher then simplified the equation to $y = \frac{9}{4}x^2$ asking learners about the properties of its graph. Learners indicated that the graph would be a parabola facing upwards because the coefficient of x^2 in the equation was positive.

A1 then summarised the sketching of graphs of functions by referring learners to the work they did in earlier lessons when they completed a table of values for the function $f(x) = x^2$ and its transformations. The teacher reminded learners that in that work, they concluded that adding a parameter 'a' inside the bracket, as in $f(x \pm a)$ and $f(ax)$ would affect the x-axis while adding the parameter outside the bracket, as in $f(x) \pm a$ and $af(x)$ would affect the y-axis. Teacher A1 then pointed out to learners that when sketching graphs of functions, they needed to decide whether the transformation affected the y values or the x values, by checking on whether the parameter is added inside the bracket or outside the bracket.

The rest of the analysis is presented in Table 6.8

Presentation formats	Performance expectations	Injections and omissions	Summary/comments
This episode began with the teacher reading out the procedure to follow for sketching functions from the textbook, and explaining the procedure before guiding learners through the process using an example. This is therefore an *explanatory text/text (exp)*, and it is followed by *worked examples (we);* hence a *quasi-deductive approach*	Various actions on function tasks expected of learners in this episode included: deciding firstly what type of graph the function would be which is *interpretation of properties*; *substituting* $x = 0$ and $y = 0$ in the given equation in determining the intercepts of the graph; *substituting* $x = 1$ *and* $x = -1$ into the equation to determine two more points that lie on the graph; *plotting* the crucial points of the function and *drawing* the graph. Thus the performance expectations are sequenced in the order: *Interpret properties; Substitute* and *plot and draw* The approach to the teaching of functions therefore is *global* but utilises pointwise actions to sketch the graphs	Teacher did not determine the symmetry, nor talk about the turning point of the parabola. The textbook mentions these properties in the instructions for sketching functions. Hence *critical omissions*	The Pre-CAPS textbook introduces the sketching of graphs with three worked examples, and these are followed by the practice exercises from which the example in this episode has been selected. It is noted that teacher did not use the worked examples to introduce this unit of the lesson but used a question from the practice exercises as a worked example. This is an example of instances when the sequencing in the content of the textbooks is not followed by the teacher

Table 6.8 Appropriation of affordances in episode 3 of lesson A13

The content is offloaded from the Pre-CAPS textbook with no improvisation and therefore carries the code *offloaded* (O^{+}). However, the sequencing of content does not follow that of the textbook. In this episode, the general approach to the teaching and learning of content is *quasi-deductive* as explained in Table 6.8. The approach to the teaching of functions is global as the main performance expectation in the episode is to be able to identify the different properties of the function and interpret them. This exercise involves a more global outlook to functions. The pointwise actions of substituting and then finally drawing the graph result become necessary processes for identifying critical points on the graph.

A1 introduced the sketching of functions through an exercise in the textbook and read out the procedure to follow from the textbook. However, two of the properties pertaining to quadratic functions which are listed in the textbook among those which should be determined include; symmetry and the turning point of the function. A1 neither explored the symmetry of the graph nor discussed the turning point of the function. I consider this is as a *critical omission* on the side of the teacher, as there are implications for opportunities for mediation.

6.3.3.4 Class activity on sketching graphs of functions: Episode 4

In this episode, A1 assigned a class activity of selected questions from the same exercise in episode 3 from which the worked example was taken. Learners were expected to work on their own as practice exercises. The questions involved sketching graphs of the following functions: $y = 2\sin x$; $y = -\frac{x}{2}$; $y = -3.2^{x}$; and, $y = -3^{x}$.

Teacher then walked around the classroom assisting learners and doing some administrative work until the end of the lesson. In Table 6.9, the rest of the analysis is presented.

Presentation formats	Performance expectations	Injections and omissions	Comments
Learners worked on *practice exercises* for the rest of the period with minimum input from the teacher. Therefore there is only one presentation format, namely *practice exercises (pr).* The practice exercise follows from the worked example in the previous episode and therefore the approach is still *quasi-deductive*	Learners were expected to follow the steps which teacher demonstrated in the previous episode on sketching graphs of functions. These included the performance expectations: *Interpret properties, Substitute,* and *Plot and draw.* For similar reasons advanced in the previous episode, the approach to functions in this episode is global.	Apparent omissions in this episode involve the selection of questions. The teacher selected four out of seven questions in the textbook exercise. Once more, this is not an omission that is likely to detract from the opportunities for mediation, therefore it is a *productive omission*	For the class activity A1 reverts to using the practice exercises for learners to practice on, meaning that teacher maintains the content and its sequencing in this episode.

Table 6.9 Appropriation of affordances in episode 3 of lesson A13

For the class activity, all the questions were selected from a practice exercise in the textbook, which means that the content is fully *offloaded* (O^+). The activity resulted from a discussion of a procedure in the previous episode, and therefore was not a first encounter with sketching functions for learners, hence a *quasi-deductive* approach. Furthermore, since the questions selected came from the same practice exercise from which the worked example was chosen, learners were expected to follow the procedure that the teacher demonstrated in the previous episode on sketching graphs of functions; hence a *global approach* to functions. With respect to omissions, for the reasons advanced in previous episodes, the omissions in this episode are *productive*.

6.3.3.5 Mobilising the Textbook in Lesson A13 – Synthesis across episodes

In the third lesson, teacher A1 offloaded all content from the Pre-CAPS textbook (*offloaded* (O^+). The content falls under the fourth content area of the textbook (CA4) on interpreting functional properties, and except in one incidence where the teacher used a question from practice exercises as a worked example for introducing the sketching of graphs, the ordering of the content followed that of the textbook. However, A1 omitted parts of the content: in three of the four episodes in the lesson, but the omissions are *productive* as a result of selection of questions which do not detract from opportunities for mediation. In one episode, there are *critical omissions* where the teacher did not discuss two of the critical properties of the quadratic function specified by the textbook as part of the procedure for sketching graphs of functions; and this has implications for opportunities for mediation.

With respect to the approach, A1 adopts a general approach that is *quasi-deductive*; and a conception of function that progresses from pointwise in the first episode to global for the remaining episodes.

In this particular lesson, A1's relationship with the textbook cannot be described as participatory and therefore not a strong teacher-textbook relationship. She offloads all content and the approach from the textbook, but omits critical aspects of the content which suggests that while there is a relationship, the use of the textbook seems not to be deliberate. In fact, A1's participation with the textbook is uneven in nature: at one level, she relies on the textbook for the procedure for sketching graphs of functions, while omitting critical elements of the procedure.

6.4 Teacher A1's Mobilisation of the textbook

This section draws together the results from the analyses of the three lessons, A11, A12, and A13, of teacher A1 in order to construct a fuller picture of how teacher A1 mobilises the prescribed textbook in terms of its affordances, that is, the *content* and the *approach* to the teaching and learning of this content. The results are presented in Table 6.10 .

Lesson	Mobilisation of the Content			Mobilisation of the Approach	
	Coverage	Degree of appropriation	Opportunities for mediation	Degree of appropriation	Conception of function
A11 (week 1 beginning)	CA1	$O^+\mathrm{I}^-$	inj^+ om^+	QD^{a+}	Pointwise (a^+)
A12 (week 1 end)	CA3	I^+	inj^+	QI^{a+}	Pointwise to global (a^+)
A13 (week 3 end)	CA4	O^+	om^- om^+ err	QD^{a+}	Global (a^+)

Table 6.10 A summary of A1's mobilisation of the textbook

Table 6.10 shows each lesson and the *mobilisation of content* as well as the *mobilisation of the approach* in each lesson. These two categories are discussed separately below.

6.4.1 Mobilisation of the Content

Mobilisation of the content in Table 6.10 is indicated by: *coverage*; *degree of appropriation*; and *opportunities for mediation*. Each one of these indicators is discussed individually with respect to teacher A1 and her lessons.

6.4.1.1 Coverage

Coverage determines the content area (CA) under which each lesson falls according to the four content areas established in the textbook analysis in chapter 5. It also establishes how the ordering of units in the lesson correlates with that of the textbook. The three lessons for teacher A1 covered three of the four content areas as determined in the textbook analysis in chapter 5, namely, CA1, CA3 and CA4. However, from lesson A12, learners answered questions about turning points of functions from the teacher, which was an indication that they knew about the turning point of a function. This suggests that CA2 had been dealt with in this particular class. Thus in terms of coverage, in the three weeks of lesson observations for teacher A1, her lessons suggest that she had covered all the content areas. Furthermore, except for the incidence in lesson A13 where she chose to use a question from the practice exercise of the textbook for a worked example, the ordering of the content areas in her lessons, was commensurate with that of the Pre-CAPS and the CAPS textbook.

Teacher A1 therefore perceives and mobilises the coverage of content of the textbook as an affordance in her practice.

6.4.1.2 Appropriation of Content

Under the *degree of appropriation*, the extent to which content is offloaded, adapted or improvised is indicated. As previously explained, the following codes apply with respect to the appropriation of content:

Code	Description
O^+	Content fully offloaded from textbook
O^+I^-	More offloading and less improvising
$O\ I$	Content adapted from textbook and external resources
I^+O^-	More improvising and less offloading
I^+	Content completely improvised

Table 6.11 A list of codes in analysing the appropriation of content

From Table 6.10, lesson A11 carries the (O^+I^-) code to show that there has been much more offloading from the textbook than improvisations. A13 is completely offloaded with no improvisations, while A12 is completely improvised with no offloading. This implies that A1 shows more reliance on the textbook for delivering her lessons than on other external resources; thus suggesting an existence of a relationship between the teacher and the textbook. However, without the evidence of opportunities for mediation that she provides in her lessons, it is not yet possible to determine how productive for the classroom this relationship is. Brown and Edelson (2003) point out that the degree of appropriation is 'value-neutral' as it neither illuminates the quality of the improvisations, offloads and adaptations, nor "qualify the extent to which each example supports or departs from the curricular goals carried by teachers or materials"(p.7).

6.4.1.3 Opportunities for Mediation

Opportunities for mediation relate to the existence of *injections, omissions* and *errors* in the lessons. . Since *errors* (*err*) have implications for the opportunities for mediation, they are an important aspect of the analysis.

In two of A1's lessons, there are *robust injections* (inj^+), which means that A1 brings additional content that is not available in the textbooks to the lessons to enhance the opportunities for mediation. The *robust injections* in the lessons suggest that teachers are able to identify constraints to content of the textbook and remedy these constraints; which teacher A1 does.

In two lessons again, A11 and A13, which are both mostly offloaded lessons, A1 commits two types of *omission:* the *productive omission* (om^+) and *critical omission* (om^-). The *productive omission* in both lessons relates to the selection of worked examples and practice exercises from the textbook, which has been shown not to detract from the opportunities for mediation, and which therefore suggests deliberateness on the side of the teacher. On the other hand the *critical omission* in lesson A13 where teacher leaves out discussion of critical properties of a function in a lesson where interpretation of these properties is crucial suggests that the teacher's use of the textbook is not deliberate. This creates a contradiction in A1's textbook use that is crucial to opportunities for mediation in her lessons. I refer to this contradictory use of the textbook as *tacit* (Polanyi, 1967) use.

Lastly, the error in A13 has already been acknowledged and while it is potentially distractive to the opportunities for mediation, in this case it is not related to the textbook and therefore does not play a part in the teacher-text interactions.

In summary therefore, A1 mobilises the content of the textbook as an affordance in her practice, and displays an existence of a relationship with her textbook judging by the *productive omissions* and *robust injections* in her lessons. However, the existence of *critical omissions* in her lessons suggests in general, that her textbook use is not deliberate.

6.4.2 Mobilisation of the Approach

The *mobilisation of the approach* to the teaching and learning of content has two indicators. Firstly, it is the general approach to the teaching and learning of the content; and then the approach to the teaching of functions from which a conception of function portrayed in a lesson is determined.

6.4.2.1 Degree of Appropriation of the Approach

The general approach to the teaching and learning of content considers whether the approach is a function of teacher directed pedagogy, a *quasi-deductive approach (QD*), or if there is initial engagement with concepts by learners, *quasi-inductive approach (QI*). However in order to determine the degree of appropriation of the approach, it is necessary to determine whether the approach has been adopted (a^+) from that of the textbook or not (a^-).

In the three lessons of teacher A1, the two offloaded lessons carry a QD^{a+} code to indicate that in these lessons A1 adopts a general approach to teaching and learning that is *quasi-deductive*, and that this is the same approach that is advanced by the textbooks. For the improvised lesson which deals with CA3 (transformations), the approach is *quasi-inductive* and also carries the adoptive code. This means that the approach in the lesson is adopted from the approach of the external resource from which the content has been improvised; in this case the WMCS workshop. Teacher A1 therefore perceives and mobilises the approach of the textbook (and resources she improvises from) as an affordance in her practice.

6.4.2.2 Conception of Function

The second aspect of the mobilisation of the approach considers the approach that is specific to the teaching of functions; whether the conception of function advocated in the lesson is *pointwise* or *global*. For this aspect too, it is necessary to determine whether the conception of function in the same content area matches that of the textbook (a^+) or not (a^-).

In lesson A11, the conception of function conveyed in the textbook is pointwise, just like in the textbook on the same content area. In lesson A12, the approach to functions progresses from the pointwise to the global just like in the WMCS workshop materials; and global in the lesson A13. It is additionally important to note that the approach to functions progresses from pointwise actions in lesson A11 to global actions by the time the content in lesson A13 is taught; and that this is the progression in the textbook as well from the first content area CA1 to the last content area, CA4.

In both facets of the approach to the teaching and learning of the content in A1's lessons, she perceives and mobilises the approach of the textbook as an affordance in her practice. This is a confirmation of the existence of a strong relationship between teacher and textbook: the recognition that the textbook affords her not only the content as an affordance to her practice, but the approach as well.

The analyses of teacher A1's mobilisation of the content and approach of the textbook in this chapter provide critical information about teacher A1's extent of use of the textbook content and approach in her lessons. These analyses provide information about the extent to which A1 offloads, adapts, and improvises content; the extent to which she perceives and mobilises the instructional approach of the textbook; and opportunities for learning she creates in her lessons. The results of these analyses are used to reach conclusions pertaining to: a) the deliberateness or non-deliberateness of the use of textbooks by A1; b) the level of her PDC; and c) the type of teacher-textbook relationship that exists between A1 and her textbook.

6.5 Conclusion

In this chapter, the interactions between teacher A1 and available resources were explored, in order to understand better how teacher A1 mobilises the affordances of the textbook in her practice. The process has identified three indicators for mobilising the content of the textbook as: the *extent of coverage of content,* the *degree of appropriation of the content,* and *opportunities for mediation* that teacher creates in the classroom. On the other hand, the analysis provides the indicators for the mobilisation of the approach as the *degree of appropriation of the general approach* to the teaching and learning of the content, together with the specific approach to the teaching and learning of functions that indicates the *conception of function* conveyed in the lesson.

The investigation of teacher A1 using these indicators has shown that while the different indicators together point to the existence of a strong relationship between the teacher and the textbook, it is mostly in the *opportunities for mediation,* that is, in the kinds of *injections, omissions and errors,* that the teacher makes in the classroom where the deliberateness of textbook; the kind of relationship that exists between the teacher and her textbook; and consequently the teacher's design capacity, may be determined. The existence of *critical omissions* and *distractive injections* in the teacher's lessons point to: i) textbook use that is *not deliberate and tacit;* ii) PDC that is *not high*; and therefore iii) a teacher-textbook relationship between the teacher and her textbook that while it may be strong, does not reflect the participatory nature. I refer to this type of teacher-textbook relationship in the study as being not *intimate*. In other words, A1 utilises the content and the approach of the textbook as affordances in her lessons, and this suggests a strong relationship between her and her textbook. However, because she omits critical aspects of the object of learning from the textbook which detract from opportunities for mediation, her PDC is not high, and her relationship with the textbook is *not intimate*.

Thus, the analysis of teacher A1's lessons furnishes the study with analytical tools for investigating teachers' capacity for pedagogical design (PDC) for the rest of the seventeen lessons in the next chapter. How teachers perceive and mobilise the textbook affordances and the opportunities for mediation they open up in the classroom should help in determining the teacher-textbook relationship for the teachers participating in this study.

In the next chapter, the study poses the following questions for the teachers participating in this study with respect to their mobilisation of the textbook

a) Is textbook use among teachers deliberate or tacit?

b) Are teachers' design capacities high or low in their interactions with their textbooks?

c) Are the teacher-textbook relationships participatory and intimate?

CHAPTER 7
Teachers' Pedagogical Design Capacity

> It is the skill in weaving various modes of use together and in arranging the various pieces of the classroom setting that is the mark of teacher with high PDC, not whether they happen to be offloading, adapting, or improvising at any given moment
> (Brown, 2009, p. 29)

7.1 Introduction

The objective of the current study is to investigate the relationship between teachers' pedagogical design capacity (PDC) and the affordances of the textbook in the mediation of the object of learning. PDC describes the capacity of teachers to perceive and mobilise existing resources in creating deliberate and productive instructional episodes (Brown, 2009). Thus, PDC involves the processes of perceiving and mobilising the textbook affordances. However, as Brown stipulates,

> PDC is not simply an indicator of whether a teacher will be likely to design something for the classroom; it is an indicator of whether the teacher's designs are pedagogically beneficial (p. 31).

To this effect, the processes by which teacher A1 mobilises the affordances of existing resources identified in chapter 6, provide this chapter with analytical tools for illuminating teachers' PDC, that is, how teachers craft "instructionally meaningful episodes" (Brown, 2002, p. 84) in their interactions with the textbook. The investigation of teacher A1's mobilisation of the affordances of the textbook does not end at identifying the different modes of textbook use that teacher A1 employs, for example, offloading, adapting, and improvising the content and approach; but goes further to examine the opportunities for mediation embedded in teacher A1's lessons. This is a measure of teacher's PDC: the "manner and degree to which teachers create deliberate, productive designs that help accomplish their instructional goals" (Brown, 2009, p. 29).

The present chapter undertakes the analyses of the seventeen (17) lessons of the remaining six teachers participating in the study to determine how they perceive and mobilise the content and the approach of the textbook; and what opportunities for mediation they provide in their lessons. The analyses utilise the indicators for mobilising the affordances of the textbook as determined in chapter 6 and presented in Fig 7.1.

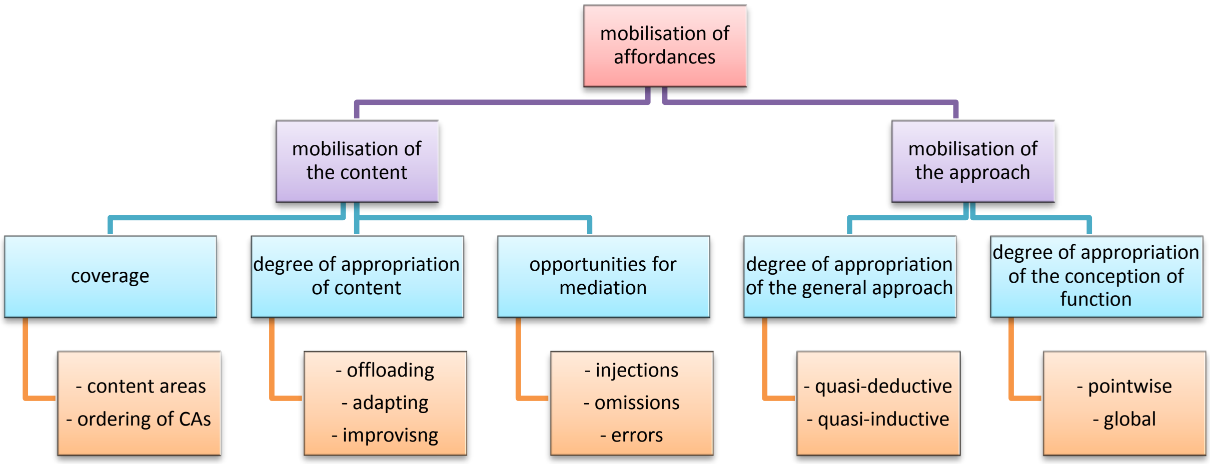


Fig 7.1 Indicators for Mobilising the Affordances of the Textbook

The indicators for the teacher mobilising the content include: *coverage*, *degree of appropriation*, and, *opportunities for mediation* that the teacher provides in the lesson. The indicators for the mobilisation of the approach on the other hand consist of the *degree of appropriation of the general approach* to the teaching and learning of the content, and the *degree of appropriation of the approach to functions*.

7.2 Determining Teachers' PDC

The analyses for teachers' mobilisation of the affordances of the textbook for the remaining six teachers are undertaken to investigate teachers' mobilisation of the content as well as the approach of the textbook in their practice. The results of the analysis of teacher A1's lessons are utilised as analytical tools for the mobilisation of the content and the approach. Teachers' mobilisation of the content is explored and discussed separately from the mobilisation of the approach

7.2.1 Teachers' Appropriation of the Content

Table 7.2 on the next page presents the results of the analysis of teachers' mobilisation of the content. These include the results on: the *coverage* which illustrates the content area(s) covered in each lesson; the *degree of appropriation of content* showing how teachers offload, adapt or improvise in lessons; and the *opportunities for mediation* which illustrate the *injections*, *omissions* and *errors* in teachers' lessons. Together, these indicators show the extent to which teachers' mobilisation of the content opens up opportunities for mediation in the classroom, thus illuminating elements of teachers' PDC. I include a reminder of the codes used in the analysis below:

Code	Description	Code	description
O^{+}	Content fully offloaded from textbook	inj^{+}	Robust injection
$O^{+}I^{-}$	More offloading and less improvising	inj^{-}	Distractive injection
$O\ I$	Content adapted from textbook and external resources	om^{+}	Productive omission
$I^{+}O^{-}$	More improvising and less offloading	om^{-}	Critical omission
I^{+}	Content completely improvised		

Table 7.1 Codes used in the analysis of the mobilisation of the content

Table 7.2 Teachers' Mobilisation of Textbook Content

Teacher	Lesson	Appropriation of Content			Comments
		Coverage	Degree of appropriation	Opportunities for mediation	
Teacher A1	A11 (week 1) Function notation Vertical line test	CA1	O^+I^- O^+ I	inj^+ om^+	CAPS textbook used Offloaded definitions, exercises Injected the vertical line test (*robust)* Omitted some worked examples and exercises (*productive)*
	A12 (week 1) Transformations of the quadratic function	CA3	I^+	inj^+	WMCS materials used Injected horizontal shifts/stretch/compressions (*robust)*
	A13 (week 3) Determining equations of graphs of functions Sketching graphs of functions	CA4	O^+	om^- om^+ err	Pre-CAPS textbook used Offloaded homework exercises, procedure for sketching functions, practice exercises Omitted discussion of some properties when sketching graphs (*critical)* Omitted some exercises and worked examples (*productive)* Error unrelated to textbook:
Teacher A2	A21 (week 1) Introducing functions Drawing graphs of functions	CA1	$O\ \ I$	inj^- err	Pre-CAPS textbook used Offloaded class activity Injected error with mapping diagrams (*distractive)* confused 'one-to-many' with 'many-to-one' (injected content)
	A22 (week 2) Properties of functions	CA2	O^+	om^- err	Pre-CAPS textbook used Offloaded homework exercises Omitted questions dealing with all other properties of functions except symmetry (*critical*) Did not recognise the line $y = x$ as an axis of symmetry for $y = \frac{8}{x}$ despite the textbook explicitly showing it
	A23 (week 3) Properties of the hyperbola	CA2	I^+	inj^- err	'Blue Book': workbook from the RADMASTE project used Injected functions dealing with horizontal shifts but through point-by-point plotting (*distractive)* Did not 'appreciate' the vertical asymptotes of hyperbolas

Teacher	Lesson	Appropriation of Content			Comments
		Coverage	Degree of appropriation	Opportunities for mediation	
Teacher A3	A32 (week 3) Transformations of the parabola Sketching the parabola	 CA3 CA4	 I^+	 inj^+ om^-	Omitted other values of a in determining the effect of a in $y = ax^2$, eg $a = 3\ or\ \frac{1}{2}\ etc$ instead only used $a = -1$ (*critical*) Injected the vertical line test and other methods for distinguishing between functions and non-functions (*robust*)
	A33 (week 3) Sketching the parabola Properties of functions Interpreting graphs	 CA4 CA2 CA4	I^+O^- I^+ I^+ O^+	 inj^+	Pre-CAPS textbook used for a worked example on interpreting functions Injected cubic functions and other functions which are not part of the grade 10 syllabus for illustrating domain and range (*robust*)
	A34 (week 3) Properties of functions and determining equations of graphs Interpreting graphs	 CA2 CA4	 O^+ O^+	 inj^+ err (in textbook)	Pre-CAPS textbook used for homework questions However worked examples and homework for next lesson assigned from the CAPS textbook (both textbooks have similar questions) Injected a function depicting a horizontal shift in order to illustrate how the domain of a hyperbola function is related to its asymptote (*robust*) Solutions from textbook contained errors which caused a heavy disagreement between teacher and learners; (*constraint for textbook*) and raises issues of 'authority' in the classroom
Teacher A4	A41(week 1) Introducing functions	CA1	I^+O^-		Pre-CAPS textbook used for one question for class activity Drawing the graph of $y = x + 1$ alone takes 45 minutes: lesson characterised by periods of long unfocussed discussions
	A42 (week 3) Properties of functions	 CA2	 I^+O^-	 err	Pre-CAPS textbook used for class activity and homework for next lesson Homework questions discussed in class include $y = 3x - 1, f(x) = 3x^2 - 3$, $g(x) = \frac{3}{x} + 3, h(x) = 3^x$, and $t(x) = -2^x$ depicting variance and invariance: 3 is kept constant in almost all functions as the functions vary. However, all graphs were drawn using point-by-point plotting despite the fact that they included transformations. In the lesson the transformations were not discussed (*a mismatch in the sequencing of units with the textbook*) the asymptotic behaviour of the hyperbola was not reflected on the graph (*critical omission*)
	A43(week 3) Properties of quadratic functions	 CA2	 I^+	 err	Another lesson dominated by unfocussed long discussions and debates for 50 minutes Error on restricted domains is made by the teacher in her explanations even though the textbook contains a section of restricted domain

Teacher	Lesson	Appropriation of Content			Comments
		Coverage	Degree of appropriation	Opportunities for mediation	
Teacher B1	B11(week 1) Transformation of the parabola (vertical shifts)	CA3	I^+	inj^+	No obvious resource used Injected horizontal shifts to contrast vertical shifts (*robust*)
	B12(week 2) Transformation of the parabola (horizontal shifts) Interpreting graphs	CA3 CA4	I^+	inj^+ inj^+	Resource used not obvious+ Injected horizontal shifts to contrast vertical shifts (*robust*)
Teacher B2	B21(week 1) Terminology on functions Properties of linear functions	CA1 CA2	I^+		Resource used not obvious Mediation broke down: object of learning not in focus for learners resulting in confusion
	B22 (week 2) Properties of the exponential function	CA2	I^+O^-	err om^-	Pre-CAPS textbook Class activity offloaded Used $y = -2^x$ as an example for functions with $a < 1$ in $y = a^x$ resulting in erroneous mediation. Leaners had suggested $y = \left(\frac{1}{2}\right)^x$ Omitted other questions relating to the exponential function during class activity and assigned questions on the quadratic function instead (*critical*)
	B23 (week 4) Sketching graphs of functions	CA4	I^+		Improvised content is available in Pre-CAPS and CAPS textbook
Teacher C1	C11 (week 1) Properties of the hyperbola Transformations of functions	CA2	I^+		Teacher prepared handwritten tasks on transformations of functions. Questions innovative and quite different from textbooks tasks Homework questions taken from a 'commercial workbook' and included similar content to that of the textbook

Teacher	Lesson	Appropriation of Content			Comments
		Coverage	Degree of appropriation	Opportunities for mediation	
	C12 (week 2) Properties of functions	CA2	$I^{+}O^{-}$	inj^{+} om^{-} err (on handout)	A handout from a 'commercial workbook' used the discussion of the properties of functions Homework questions offloaded from Pre-CAPS textbook at end of lesson Injected functions depicting horizontal shifts in explaining the relationship between the asymptote of a hyperbola function and its domain and range in $y = \frac{a}{x+p} + q$ (*robust)* The handout contained two errors: shows only the line $y = -x$ as the line of symmetry for $y = \frac{1}{x}$ (teacher did not correct this); showed the range of $y = b^{x}$ as $\mathbb{R}$ instead of $x > 0$ (teacher corrected this error); Omits the following key features of functions from discussions: symmetry for $y = x^2$; increasing/decreasing intervals for $y = \frac{1}{x}$; Teacher extended a textbook exercise which changed the sense of the question from sketching graphs of questions to determining their properties
	C13(week 3) Properties of functions Past examination questions (interpretation)	CA2 CA4	$O \quad I$		Pre-CAPS textbook used for homework questions Past examinations papers for class activity exercises Going over homework, learners drew the graphs of functions using point-by-point plotting instead of sketching the graphs as per the textbook instructions Learners disregarded the domain $[0; 2; 4; 6\}$ provided for the function $y = \frac{1}{2}x^2$ and drew the graph as if the domain is the set of real numbers (teacher did not pick up on this)

The discussion of Table 7.2 begins with the appropriation of the content and its implications for teachers' PDC. This is followed by a discussion and implications for teachers' PDC of how teachers appropriate the approach of the textbook. There are two ways that the results in Table 7.2 can be interpreted as discussed in chapter 4: with the teacher as the unit of analysis from which we can say something about the individual teacher's PDC; or the lesson as the unit of analysis. The latter enables the study to generalise about the processes by which teachers mobilise the affordances. The study therefore considers the lesson as the unit of analysis and the interpretation of the results that follows considers the twenty lessons in the study (including teacher A1) and attempts generalisations about teachers' processes of perceiving and mobilising the affordances for productive instructional episodes.

In this section, the issues of coverage, degree of appropriation of the content and the opportunities for mediation in the twenty lessons are discussed.

7.2.1.1 Coverage

In terms of coverage, the results in Table 7.2 show that among the seven teachers participating in the study, five of them, namely A1, A3, B1, B2 and C1 covered all four content areas (CA1 to CA4) as determined in the textbook within the four week period allocated for the teaching of functions. Furthermore, except for teacher A3, the teachers sequenced their content areas in the same way as the textbook. Thus, these teachers perceive and mobilise the coverage of content as an affordance in their lessons. Teacher A3 on the other hand weaved the content areas together to produce instructional episodes which responded to her goals. For example, in her first two lessons A32 and A33 which came one after another (with a space of one day in between), the study of transformations of the parabola (CA3) led to its sketching (CA4) which led to the discussion about the domain and range of functions (CA2) and then to interpreting the functional properties (CA4).

While the ordering of the content areas did not resemble that of the textbook, as a whole her designs for the classroom lay within the range of the content areas in the textbook. For two teachers A2 and A4 however, they covered CA1 and CA2 over the four weeks, and they did not deal with CA3 (transformations) and CA4 (interpretation of functions). In their case then, the teachers did not perceive and mobilise the content coverage of the textbook as an affordance in their lessons.

At a very basic level therefore, the inability to perceive and mobilise the content coverage on the part of teachers A2 and A4 is indicative of a weak relationship with the textbook. If they do not cover two of four content areas in the textbook, it suggests a non-collaborative/participatory use of the textbook on their part, a point that has implications for teachers' PDC. On the other hand, for the other five teachers, there is an indication of a participatory relationship with the textbook. However, this is not enough yet to make suggestions about the teachers' PDC.

In the next sections, more indicators on the appropriation of the content of the textbook are explored in order to shed more light on the teacher-textbook interactions in these lessons

7.2.1.2 Degree of Appropriation of Content

The degree of appropriation of content is the next indicator discussed. It defines the extent of the reliance on the textbook of the teachers for the delivery of lessons. From Table 7.2, 19% of the lessons were fully offloaded (O^{+}), 5% were offloaded with some improvising ($O^{+}I^{-}$), 9% adapted (O I), 43% were improvised (I^{+}), and 24% were largely improvised but with some offloading ($I^{+}O^{-}$). This information is presented pictorially in Fig 7.2.

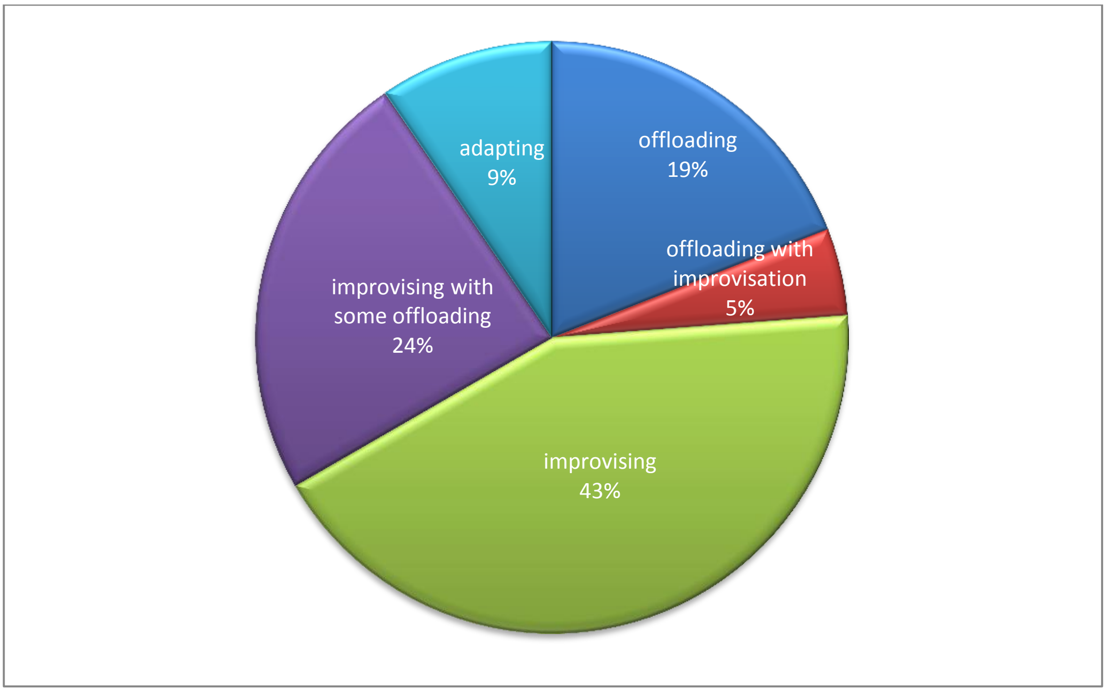


Fig 7.2 Degree of Appropriation of Content

On one level, Fig 7.2 shows that the teachers in the study improvise much more than they offload, meaning that they rely on their personal and external resources more than they rely on the textbook for the delivery of their lessons. In roughly half of the lessons, teachers use external resources exclusively for delivering the lesson against only a fifth of the lessons in which they exclusively use the textbook. On another level however, offloading of the content of the textbook occurs in more than 50% of the lessons, if taking ‘adapting’ and ‘improvising with some offloading’ into consideration. What this point highlights is that not offloading from the textbook does not necessarily imply that the textbook is not being used. In fact, if one considers the individual teachers, it is in teacher B1’s lessons only that the results in Table 7.2 do not reflect offloading of some form from the textbook. For all the other six teachers, their appropriation in lessons includes some degree of offloading. Thus, the teachers in the study use the prescribed textbook; admittedly with varying degrees of appropriation, but they use their textbook.

This declaration however does not tell us anything more about the teachers’ relationship with the textbook than we know so far. For example, we already know from the previous section that teacher A2 has not completed the curriculum requirements for functions and therefore her relationship with the textbook is being questioned; yet she is one of three teachers who offload content in the study. On the other hand, teacher A1 and A3, who mobilise the coverage of content in their lessons and therefore have some relationship with the textbook, also offload some of their lessons to the textbook. Brown and Edelson (2003) confirm that the degree of appropriation does not address the quality of appropriation but only the quantity.

> This scale of offloads, adaptations and improvisations provides a means to classify the nature of the teachers' partnerships with curriculum materials by identifying differential contributions of instructional resources and distributions of design responsibility. In doing so, the scale is value-neutral, for it says nothing of the quality or effectiveness of each case, nor does it qualify the extent to which each example supports or departs from the curricular goals carried by teachers or materials. (p.7)

In other words, offloading and similarly improvising are done by teachers with or without a relationship with the textbook, and therefore more delineation of the elements at play in the offloading and improvising of the content is needed in order to understand better how these elements play out and their implication to the teacher-textbook relationship. With respect to the teachers' PDC therefore, whether they offload or improvise does not provide insights into their PDC, a point asserted by Brown (2009) in the opening quotation of this chapter. The next two sections delineate the processes of offloading and improvising by the teachers.

7.2.1.2.1 Delineation of Teachers' Offloads

In this section, the study explores the lessons for what aspects of the content teachers offload from the textbook. All lessons which involved offloading, that is, those carrying the codes: (O^+), ($O\ I$) and (I^+O^-) were analysed for the presentation formats which teachers offloaded from the textbook. Table 7.3 presents the results of this analysis.

Lessons	Presentation formats offloaded	Comments
A11 (O^+)	Explanatory talk Worked examples Practice exercises	All presentation formats offloaded
A13 (O^+)	Practice exercises (Homework) Explanatory talk Worked examples Homework exercise for next lesson	All presentation formats offloaded
A22 (O^+)	Practice exercises (Homework) Practice exercises	Only presentation formats in the lesson
A34 (O^+)	Practice exercises (Homework) Worked examples Practice exercises	Only presentation formats in the lesson
A21 ($O\ I$)	Practice exercise	Explanatory talk, worked examples improvised
A33 (I^+O^-)	Worked example	Everything else improvised
A41 (I^+O^-)	Practice exercise	Everything else improvised
A42 (I^+O^-)	Practice exercise Homework exercise for next lesson	Everything else improvised
B22 (I+O-)	Practice exercise	Everything else improvised
C12 (I+O-)	Homework exercise for next lesson	Everything else improvised
C13(O I)	Homework exercise	Everything else improvised

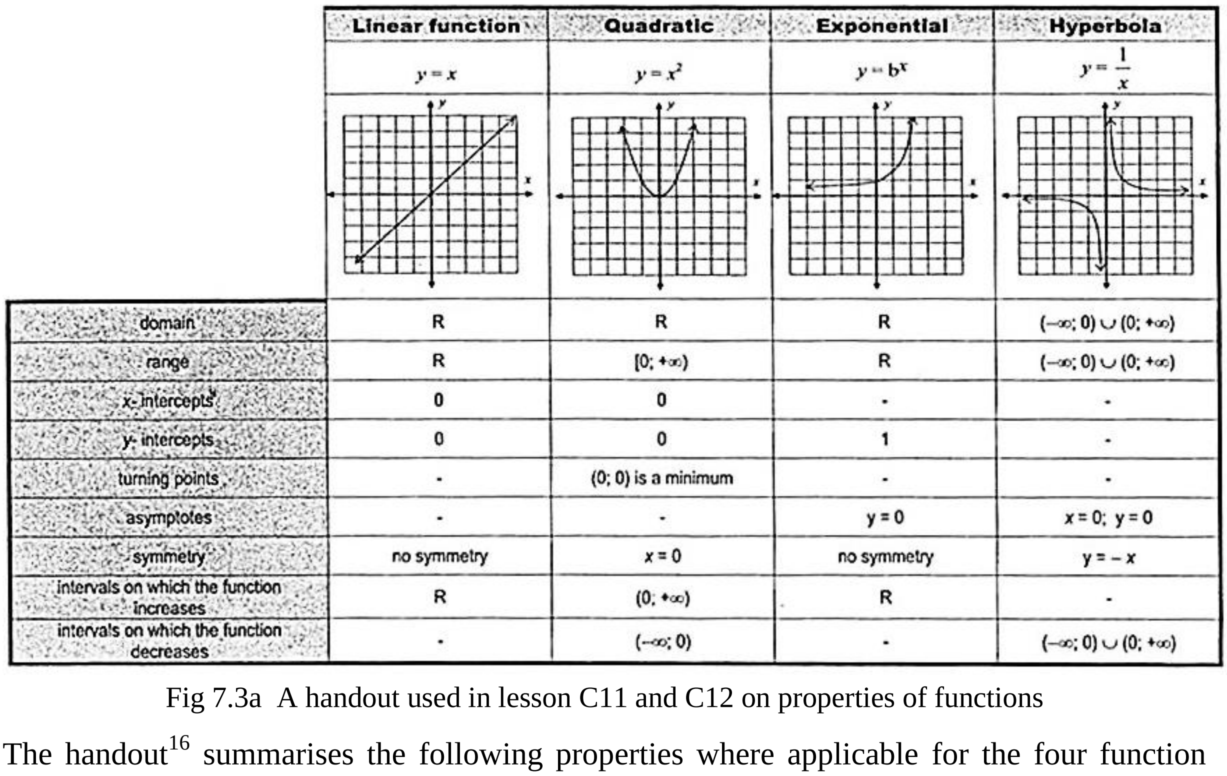
Table 7.3 Offloaded Content in the lessons

In two of the four lessons which are completely offloaded (O^+), the teacher offloads exercises including the explanatory talk. In lesson A11, all definitions and descriptions are based on the content in the textbook, together with the worked examples and practice exercises in terms of a class activity. In A13, the explanatory talk offloaded involves the procedure outlined by the textbook for sketching graphs of functions which she adopts and explains to learners. The other two lessons only include exercises and no other presentation formats. In the rest of the lessons involving offloading, teachers offload some form of exercise; worked examples, homework exercises and class activities. Thus, *the teachers in the study mostly value the textbook as a source of exercises.*

7.2.1.2.2 Delineation of Teachers' Improvisations

On the other hand, a closer look at the content teachers improvise in lessons reflects three main observations. Firstly, there are representations of functions not privileged by the textbook which teachers bring into the lessons. For example, teacher A2 represents functions as mapping diagrams; teacher A1 and A3 bring in the vertical line test for distinguishing between functions and non-functions. These kinds of additional content are discussed in later sections under opportunities for mediation.

Secondly, teachers utilise external resources other than the prescribed textbook for explanations and/or exercises for content that is quite similar to what is available in the textbook. For example, in lesson C11, teacher C1 structures the lesson around a handout extracted from a commercial workbook on properties of different function classes shown in Fig 7.3a.



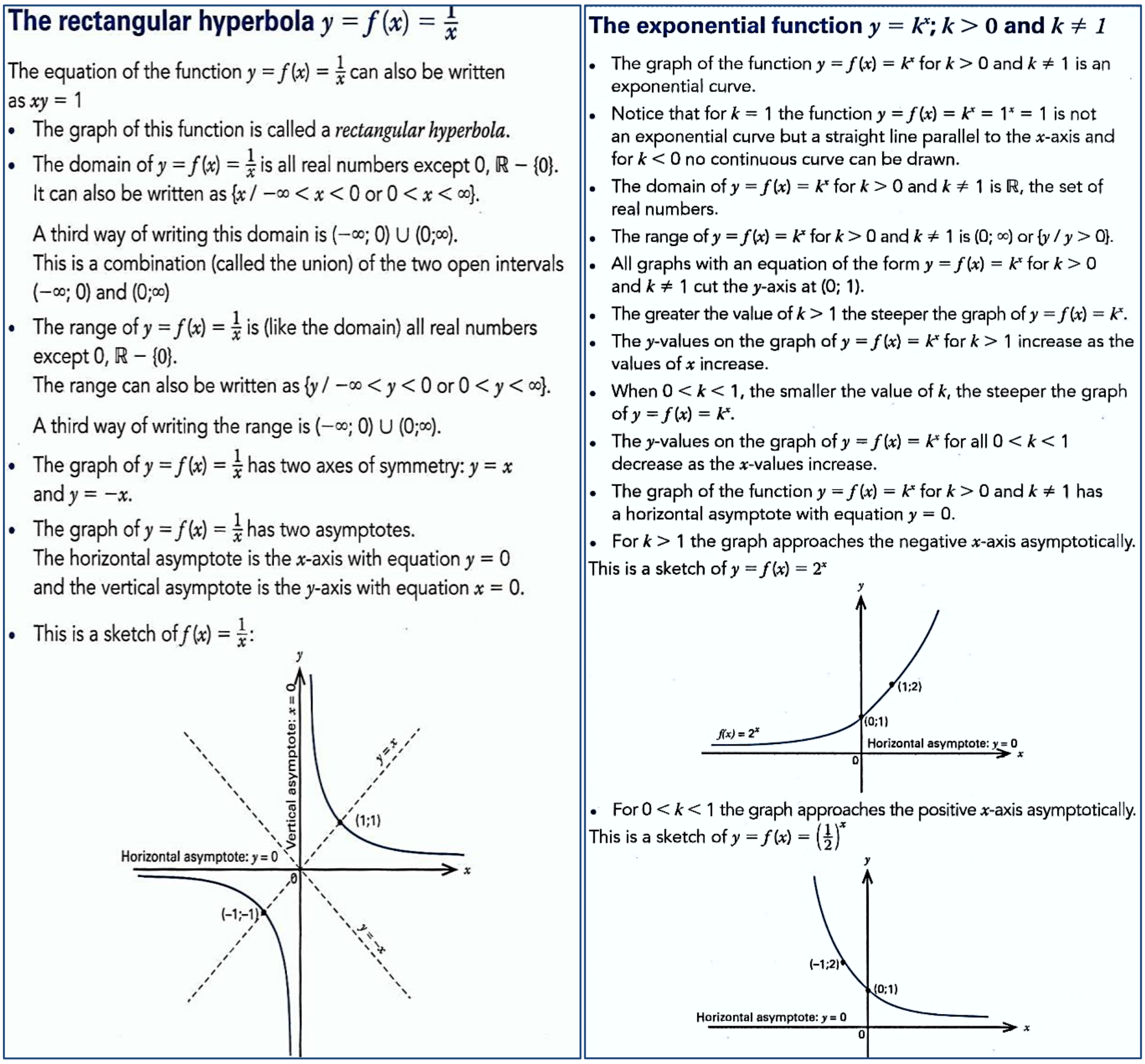
	Linear function	Quadratic	Exponential	Hyperbola
	$y = x$	$y = x^2$	$y = b^x$	$y = \frac{1}{x}$
domain	R	R	R	$(-\infty; 0) \cup (0; +\infty)$
range	R	$[0; +\infty)$	R	$(-\infty; 0) \cup (0; +\infty)$
x-intercepts	0	0	-	-
y-intercepts	0	0	1	-
turning points	-	(0; 0) is a minimum	-	-
asymptotes	-	-	$y = 0$	$x = 0$; $y = 0$
symmetry	no symmetry	$x = 0$	no symmetry	$y = -x$
intervals on which the function increases	R	$(0; +\infty)$	R	-
intervals on which the function decreases	-	$(-\infty; 0)$	-	$(-\infty; 0) \cup (0; +\infty)$

Fig 7.3a A handout used in lesson C11 and C12 on properties of functions

The handout[16] summarises the following properties where applicable for the four function classes taught at grade 10 level: domain, range, x- and y-intercepts, turning points, asymptotes, symmetry, and intervals on which the function increases or decreases. Similar

[16]The handout contains errors as well which I come back to in later sections

information is available in the CAPS textbook where the summaries of properties of functions are provided as shown in Fig 7.3b. While the handout has all properties on one page, the textbook provides a more detailed summary of the properties of each function over several pages as each function occupies its own page. Fig 7.3b shows the summary tables of two functions in the CAPS textbook.



The rectangular hyperbola $y = f(x) = \frac{1}{x}$

The equation of the function $y = f(x) = \frac{1}{x}$ can also be written as $xy = 1$

- The graph of this function is called a *rectangular hyperbola*.
- The domain of $y = f(x) = \frac{1}{x}$ is all real numbers except 0, $\mathbb{R} - \{0\}$. It can also be written as $\{x / -\infty < x < 0 \text{ or } 0 < x < \infty\}$.

  A third way of writing this domain is $(-\infty; 0) \cup (0;\infty)$.
  This is a combination (called the union) of the two open intervals $(-\infty; 0)$ and $(0;\infty)$
- The range of $y = f(x) = \frac{1}{x}$ is (like the domain) all real numbers except 0, $\mathbb{R} - \{0\}$.
  The range can also be written as $\{y / -\infty < y < 0 \text{ or } 0 < y < \infty\}$.

  A third way of writing the range is $(-\infty; 0) \cup (0;\infty)$.
- The graph of $y = f(x) = \frac{1}{x}$ has two axes of symmetry: $y = x$ and $y = -x$.
- The graph of $y = f(x) = \frac{1}{x}$ has two asymptotes.
  The horizontal asymptote is the x-axis with equation $y = 0$ and the vertical asymptote is the y-axis with equation $x = 0$.
- This is a sketch of $f(x) = \frac{1}{x}$:

The exponential function $y = k^x$; $k > 0$ and $k \neq 1$

- The graph of the function $y = f(x) = k^x$ for $k > 0$ and $k \neq 1$ is an exponential curve.
- Notice that for $k = 1$ the function $y = f(x) = k^x = 1^x = 1$ is not an exponential curve but a straight line parallel to the x-axis and for $k < 0$ no continuous curve can be drawn.
- The domain of $y = f(x) = k^x$ for $k > 0$ and $k \neq 1$ is $\mathbb{R}$, the set of real numbers.
- The range of $y = f(x) = k^x$ for $k > 0$ and $k \neq 1$ is $(0; \infty)$ or $\{y / y > 0\}$.
- All graphs with an equation of the form $y = f(x) = k^x$ for $k > 0$ and $k \neq 1$ cut the y-axis at $(0; 1)$.
- The greater the value of $k > 1$ the steeper the graph of $y = f(x) = k^x$.
- The y-values on the graph of $y = f(x) = k^x$ for $k > 1$ increase as the values of x increase.
- When $0 < k < 1$, the smaller the value of k, the steeper the graph of $y = f(x) = k^x$.
- The y-values on the graph of $y = f(x) = k^x$ for all $0 < k < 1$ decrease as the x-values increase.
- The graph of the function $y = f(x) = k^x$ for $k > 0$ and $k \neq 1$ has a horizontal asymptote with equation $y = 0$.
- For $k > 1$ the graph approaches the negative x-axis asymptotically.

This is a sketch of $y = f(x) = 2^x$

- For $0 < k < 1$ the graph approaches the positive x-axis asymptotically.

This is a sketch of $y = f(x) = \left(\frac{1}{2}\right)^x$

Fig 7.3b Summary tables for the hyperbola and the exponential function from the CAPS textbook (Pike et al., 2011, pp. 154-155)

A comparison of the handout versus the summary tables from the CAPS textbook also shows mathematical errors in the handout the teacher uses in these two lessons:

The range of the function $y = b^x$ in the handout is incorrect (it should be $(0; +\infty)$) and the handout does not specify the restrictions on the base 'b' which the textbook does.

The handout shows only one line of symmetry for $y = \frac{1}{x}$, the line $y = -x$ when there should be two of them; the textbook shows the other line of symmetry, $y = x$ as well.

The point being made in this example about teacher C1 is that while it is the teacher's prerogative to use other resources other than the prescribed textbook (CAPS or Pre-CAPS textbook), when they choose resources with less information and which contain errors such as the handout in teacher C1's lesson, while the textbook contains the same information and is error-free, this suggests a weak relationship with the textbook.

There is further evidence of the relationship with the textbook that could be described as weak also provided by teacher C1, and that applies to other teachers in the study. In the lesson C12, teacher C1 produced her own practice exercise for learners on transformation of functions shown in Fig 7.3c

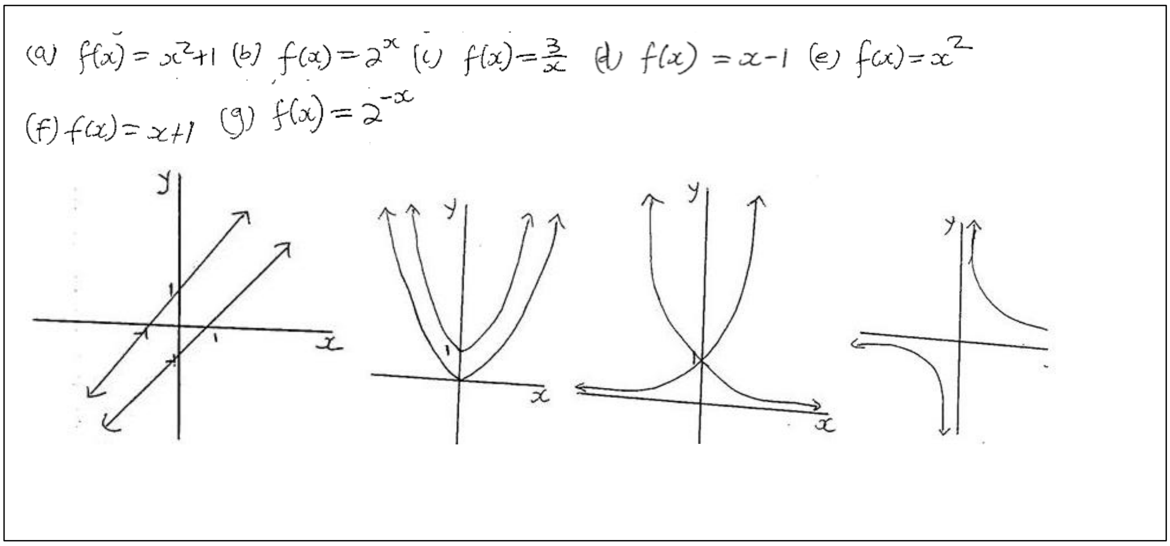


Fig 7.3c Teacher C1's practice exercise on transformations of functions

In the exercise in Fig 7.3c the task is to match a graph with its appropriate function. It is noted that the CAPS textbook utilises the same matching strategy for determining properties of the transformations of functions for all function classes as illustrated in Fig 7.3d.

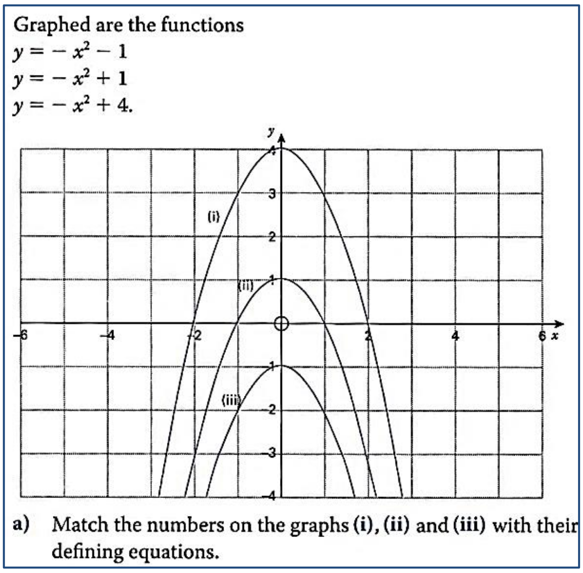


Fig 7.3d A matching question from the CAPS textbook (Pike et al., 2011, p. 162)

In other words, teacher C1 improvised content that is very similar to the textbook, begging a question why the teacher did not just use the textbook instead. While here I can only speculate (C1 does not get to explain this decision), this example and others like it suggest a *tacit* use of the textbook by teacher. The question arises why spend lots of time preparing the same thing that is available in the textbook, photocopy it, as teacher did when learners all have a textbook, if it is not about use that is not deliberate?

The last observation for the improvised lessons pertains to cases where a deliberate choice of external resources or approach to teaching and learning is made by the teacher. As Brown and Edelson (2003) point out, improvisations would generally occur "when a teacher recognizes additional opportunity in a classroom situation and possesses the necessary knowledge and skill to depart on a new instructional path" (p.7). Two teachers fall into this category in the study. In lesson A12 of teacher A1, she structured the whole lesson around a worksheet from the WMCS workshop on functions which approached the transformation of the parabola through the parent function approach whereby functions of the form $y = f(x + a)$ and $y = f(x) + a$, were contrasted around the parent function $y = f(x)$ in order to determine their effects on the parent function. The worksheet is an activity that the teacher used throughout the lesson. Teacher B1 adopted the same parent function strategy, except that in her case, the resource she was using was not visible.

These two examples point to a different type of improvising which brings content that is not yet required at the particular grade level, but which teachers need to use in order to achieve their instructional goals. The textbook is not being used in a physical sense, but there is a certain deliberateness to the decision not to use the textbook because the textbook in these cases does not serve teachers' instructional goals; some other personal or external resource does. These kinds of improvisations are discussed in later sections but they begin to point to the aspect of deliberateness in the mobilisation of the resources which is a characteristic of teachers' PDC. They also indicate the centrality of the approach to the teaching and learning of content in the teacher-textbook interactions.

The delineation of the offloads and improvisations in the study shows that teachers in the study mostly utilise the textbook for exercises. However, when it comes to other aspects of the lesson they either use their personal resources or external resources even though some of the content that they improvise is available in the textbook. This suggests textbook use that is not deliberate and contrasts with Sherin and Drake's (2009) findings in their study where teachers' use of curriculum materials was not haphazard. Thus the teachers in my study do not fully perceive and mobilise the content of the textbook as an affordance in their practice; pointing to a relationship that may not be described as a close relationship. The fact that the improvised content could be mathematically erroneous confirms these initial impressions of the teacher-textbook relationship and begins to suggest therefore a *tacit* use of the textbook.

The next section provides the last indicator of the appropriation of the content of the textbook, the opportunities for mediation that teachers provide in the lessons.

7.2.1.3 Opportunities for Mediation in the Mobilisation of Content

The opportunities for mediation in the study are characterised by three constructs of *injections*, *omissions*, and *errors* (refer to Table 4.4 for a description of these analytical constructs)). The analysis in this section derives from Table 7.2 in which the results of the analysis for each lesson are presented. These results are discussed separately starting with *injections.*

7.2.1.3.1 Injections

With respect to injections, two kinds of analyses are reported in this section. Firstly, the study reports on the disbursement of the injections throughout the lessons, presented in Fig 7.4; followed by the nature of the injections, which is presented in Table 7.4. To recap, robust injections (inj^+) enhance opportunities for mediation, and distractive injections (inj^-) detract from the opportunities for mediation.

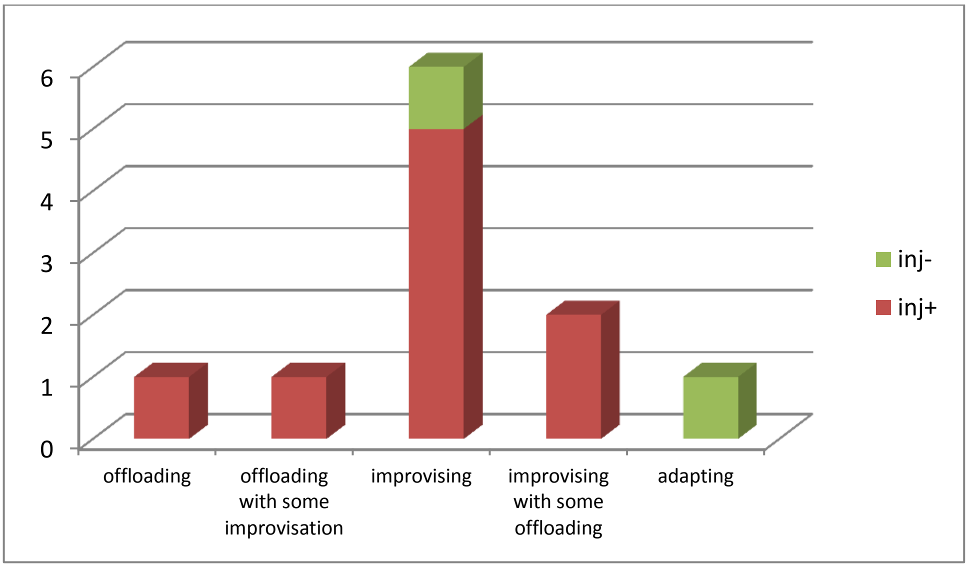


Fig 7.4 Injection of content in lessons

With respect to the disbursement of the injections throughout the lessons, Fig 7.4 shows that there are more *robust injections* (82%) in the lessons than *distractive injections* (18%). In other words, teachers brought into the lessons more content that enhanced the opportunities for mediation than content that detracted from opportunities for mediation. Another observation is that *robust injections* are occurring in all types of lessons except in adapted lessons, reinforcing results from chapter 6 that teachers inject content that enhances the lessons whether they are offloading or improvising. The *distractive injections* occur in adapted lessons and improvised lessons. It is noted here that the distractive omissions are made by the same teacher, which could be a function of the teacher's subject matter knowledge. I come back to this point later in this section.

Table 7.4 describes the different injections which teachers make in the lessons populated from the analysis of the lessons in Table 7.2. The exploration here is about the kind of content that teachers inject into the lessons..

Lesson	Robust injection	Description	Comments
C12	Functions depicting a horizontal shift	Injected in order to explain the relationship between the asymptote of a parabola and its domain and range	Robust injection
B12	Horizontal shifts	Content not yet required at grade 10 and injected in order to provide contrast to the teaching of vertical shifts	Robust injection
A34	A function depicting a horizontal shift	Injected to illustrate the link between domain and range and the asymptote of the hyperbola function	Robust injection
A33	Cubic functions and other functions not in grade 10	Injected for illustrating how to obtain the domain and range of a function from its graph	Robust injection

	syllabus		
A32	The vertical line test	The test is used for distinguishing functions and no-functions	Robust injection
A23	Functions depicting a horizontal shift	Function drawn using point-by-point plotting and not through sketching	Distractive injection
A21	Mapping diagrams representation of functions	Teacher interchanges 'one-to-many' with 'many-to-one' mappings resulting in erroneous mediation	Distractive injection
A12	Horizontal shifts/stretches/compressions	Injected to contrast vertical shifts etc in the transformation of a parabola	Robust injection
A11	The vertical line test	Injected to distinguish between functions and non-functions	Robust injection

Table 7.4 Teachers' injections of content in the lessons

Table 7.4 shows four aspects of the content that teachers inject into the lessons as:

- horizontal shifts which are required at grade 11;
- cubic functions and other functions which learners will encounter in subsequent grades;
- the vertical line test for distinguishing between functions and non-functions; and
- mapping diagrams.

The vertical line test and mapping diagrams may be introduced at grade 10, but the textbook does not privilege them, hence why they have been categorised as injections instead of improvisations.

The horizontal shifts were introduced in five of nine lessons in which injections are made and became the most popular injection in the study. In two of these lessons, the purpose of injecting the horizontal shifts was to help learners learn about the vertical transformations by experiencing the difference with horizontal transformations; learning through variance and invariance as Watson and Mason (2006) call it. This injection therefore enhances opportunities for mediation just like in lesson A12 of teacher A1 described in chapter 6.

Another lesson that brings this contrast in is lesson B12. Teacher B1 adopted a parent function approach to transformations of the quadratic function, $f(x) = x^2$. After the exploration of the effect of the functions of the form $f(x) + a$ which depict a vertical shift she asked learners about what they thought the effect of placing the 'a' inside the bracket would be:

Teacher	This is what we have for y equals to x (*teacher has drawn the graph of $y = x^2$ on the board*) So, before you continue, I want to find out, because we did something last week that in actual fact you still remember that thing that we did? What happened now when we say y is equals to minus x squared (*writes $y = -x^2$*). What's going to happen to this particular graph? …
Learner 1	The graph will go one unit down
Teacher	One unit down. The graph will go one unit down, isn't it?
Learner 2	The graph will face downwards.
Teacher	The graph will face downwards, anyone? Oh the graph will face downwards?
Learners	Yes.
Teacher	(*goes back to the board and begins writing*) Okay, so what about if you get something like this one, x squared plus one? (*writes $\boldsymbol{y = x^2 + 1}$ on the board*). We did this particular one, uh, yes? (*Points to a learner.*)
Learner	The graph will go one unit up.
Teacher	The graph will go one unit upwards, hey? In other words the number that you get here (*points to*

	the number 1 on $y = x^2 + 1$) is the shift, ja? So, if you get, if you have y equals to x squared, um, minus one. Or minus ten. Let me redraw this one, minus four (*writes* $y = x^2 - 4$ *on the board*) yes?
…	...
Teacher	What's going to happen to our graph if you see *x plus one*? (*writes* $y = (x+1)^2$) So, what's going to happen to that particular, can you get the co-ordinate? I give you half a minute to get the co-ordinate out of this particular one, ne? (Points to the board.)
…	
Teacher	(*begins joining all the points*) So, it says that our graph will be going this way, ne? Oh it's turning here and then it goes up like that. So, when you compare with the original one, tell me what happened to this particular graph, when you compare with the original one?
Learner	It's shifted to the left
Teacher	It's shifted to the left. So what is the meaning of that particular one? (*writes* $y = (x-2)^2$ *on the board*). Can you quickly now without the waste of time, 'cause we don't have enough time today.

After the examples of the form $y = f(x+a)$ teacher wrote examples of functions which depicted both the vertical and the horizontal shift, for example, $y = (x+2)^2 - 3$, for which learners had to decide on the turning points of the functions drawing from the conclusions they had just made. The CAPS textbooks adopts a parent function approach but does not use the contrasting of the vertical and horizontal shifts as horizontal shifts feature from grade 11 only. From teacher B1's approach, learners have an opportunity to learn about the form of functions depicting either a vertical or horizontal shift through the varying of the position of 'a' as in $f(x) = x^2 + a$ or $f(x) = (x+a)^2$. This is another example of a robust injection.

In two other lessons, C12 and A34, the functions depicting horizontal shifts were injected in order to illustrate a relationship between the domain and range of a function and its asymptotes. For example, in lesson C12, teacher C1 injected functions such as $y = \frac{2}{x+1} + 1$ and $y = \frac{3}{x-2} - 5$ shown in Fig 7.5 to show that for a function expressed in the form, $y = \frac{a}{x-p} + q$, then the vertical asymptote of such a function equals the value of 'p' and the horizontal asymptote equals the value of 'q' while the domain and range of the function are determined as $x \neq p$ and $y > q$, respectively.

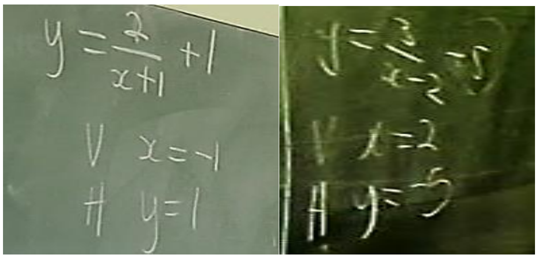


Fig 7.5 Examples of injected functions from lesson C12

These injections enhance opportunities for mediation as well.

The injection of functions depicting a horizontal shift has had an effect of detracting from opportunities for mediation too. In lesson A23, teacher A2 injected the functions $y = \frac{2}{x} - 2$ and $y = \frac{1}{x-2} + 1$, for learners to draw their graphs using the 'table method' which could be described as a signature method in all her classes. According to the curriculum and hence the textbook, functions which are not parent functions should be sketched through interpreting their properties and not by point-point plotting. Teacher A2 did not expose learners in her lessons to the interpretation of properties techniques. A graph to be drawn in all her lessons was drawn using the 'table method' which she went over with the learners in the first lesson. For example, in lesson A23 when learners were not able to tell the differences between the graphs of $y = 3^x$ and $y = (x + 3)^2 + 1$, teacher A2 told learners to draw the two graphs in order to point out the differences in their respective graphs. This move goes against the approach in the textbook to be able to describe the functions in terms of their properties, hence a distracting rather than a robust injection. These distractive injections suggest a 'miscommunication' with the textbook from the teacher's side which implies a low PDC and therefore a weak relationship with the textbook. In other words, the injections are much more than the content the teachers inject into the lesson; they are also about the opportunities for mediation that teachers open up in the classroom.

The vertical line test for distinguishing between functions and non-functions in studies of functions has been one robust injection made by more than one teacher in the study. The enhancing effect of the vertical line test has already been discussed in chapter 6.

Another injection made in the lessons is the representation of functions as mapping diagrams. In lesson A23 however, where mapping diagrams are injected in the lesson, while they are an enhancement to opportunities for mediation, teacher A2 confused the 'one-to-many' mappings with the 'many-to-one' mappings and therefore mistakenly referred to the 'one-to-many' mappings as representing functions when it is the other way round. An excerpt from lesson A23 below begins where teacher and learners were going over function values for $f(x) = x^2$ for input values 2 , -2 , 1 and -1.

Teacher:	Minus two gave me?
Learners:	Four.
Teacher:	And two gave me?
Learners:	Four.
Teacher:	Minus one gave me?
Learners:	One.
Teacher:	And one gave me?
Learners:	One.
Teacher:	Now these ones, I said one input is to one output (*referring to values for the function* $y = 2x + 1$). What about this one? (*pointing to the current function* $y = x^2$) Yes?
Learners:	One input to two outputs.
Teacher:	One input is to two?
Learners:	Two outputs.
Teacher:	So I can have one, one input is to many, okay?
Learners:	Yes.
Teacher:	It's more than two ne?
Learners:	Yes.
Teacher:	So I can have one input is to?
All together:	Many.
Teacher:	Okay?
Learners:	Yes.
Teacher:	So those, those are characteristics of a function ne?
Learners:	Yes.
Teacher:	So those are the characteristics of a?
All together:	Function.

Teacher A2's declaration that 'one-to-many' representations are characteristics of a function is mathematically incorrect and has implications for opportunities for mediation. In lesson A23, as teacher A2 summarises the topic on functions, she affirms the same mathematically incorrect statement as illustrated in the following excerpt.

Teacher:	And then I showed the relationship that we were talking about between our input value and our output value. I said I have a function whereby the output that I get I say I had one is to one. What did that mean? What did one is to one mean? Remember when I started this I did the input value and output value.
Teacher:	And I said I have one is to one and sometimes I have? One is to many. (*Learners chorus with teacher*.) What does that mean? (*Learners talk among themselves*.) What does that mean? What does it mean? Hmm? What is one is to one? Yes, try.
Learner:	The number that is in the input is related to one number in the output.
Teacher:	She says that the number that is in the input is related to one number that is in the output. I said one is to one is when the input value gives you one output value, ne?
Learners:	Yes.
Teacher:	For this one input value there's this one output value, ne?
Learners:	Yes.
Teacher:	And you also did one is to many. What did I say when I did one is to many? Yes? That one input value can give two or more output values. Isn't it?
Learners:	Yes.
Teacher:	Okay, so I spoke about one is to many. Then I had our functions, ne? And then I started drawing our graphs, ne?
Learners:	Yes.

The representation of functions as mappings diagrams is used in many other classrooms by teachers and is illustrated in many other textbooks, and it could therefore be an enhancing injection to the content. In this case however, it brings with it a mathematical error that is distractive and therefore detracts from opportunities for mediation of functions. While distractive injections were far fewer than the robust injections in the lessons, their very existence is a cause for concern for opportunities for mediation. In this particular case, the issue is of incorrect mathematics which could be an indication of the teacher's subject matter knowledge (SMK) (Shulman, 1986).

In relation to the teacher's PDC however, the example points to the influence of SMK in the PDC, a point which has not been investigated in the study but which literature alludes to, as demonstrated in chapter 2. For example, SMK serves as one of three types of resources that teachers bring to their interactions with curriculum materials in Brown's Design Capacity for Enactment (DCE) framework (Brown, 2009); and features in Remillard's Framework of components of teacher–curriculum relationship (Remillard, 2005) as a teacher resource. Thus, literature points to the influential nature of teachers' SMK in the teacher-text relationship and therefore a need for further study.

The robust injections in this study point to the supplemental nature of content that teachers bring into the lessons. These point to teachers' capacities to perceive what the textbook affords and also what it constrains in their practice. Thus robust injections may be characterised as aspects of content which teachers perceive as constraints to their practice, and are therefore indicative of teachers' PDC; a capacity not just to be able to identify the affordances and constraints but to be competent in the use of the materials (Choppin, 2011). On the other hand, the study shows that injections to content can be distractive in two ways: firstly when they diverge from the common objectives of teaching and learning the topic, and then when they introduce mathematical errors. In both cases, the injections detract from opportunities for mediation, and become a reflection of a weak relationship between teacher and textbook on the one hand, and of issues involving teacher's SMK.

7.2.1.3.2 Omissions

A graphical presentation of the omitted content from the textbook in lessons from Table 7.2 is made in Fig 7.6. As with injections, the results show that the omissions occur whether teachers are offloading or improvising. In other words, while fully relying on the textbook for the delivery of the content, teachers may still omit some aspects of the content in the textbook. Similarly when improvising. The study disaggregates the different kinds of omissions made by teachers and discusses their implications to the teacher-textbook relationship.

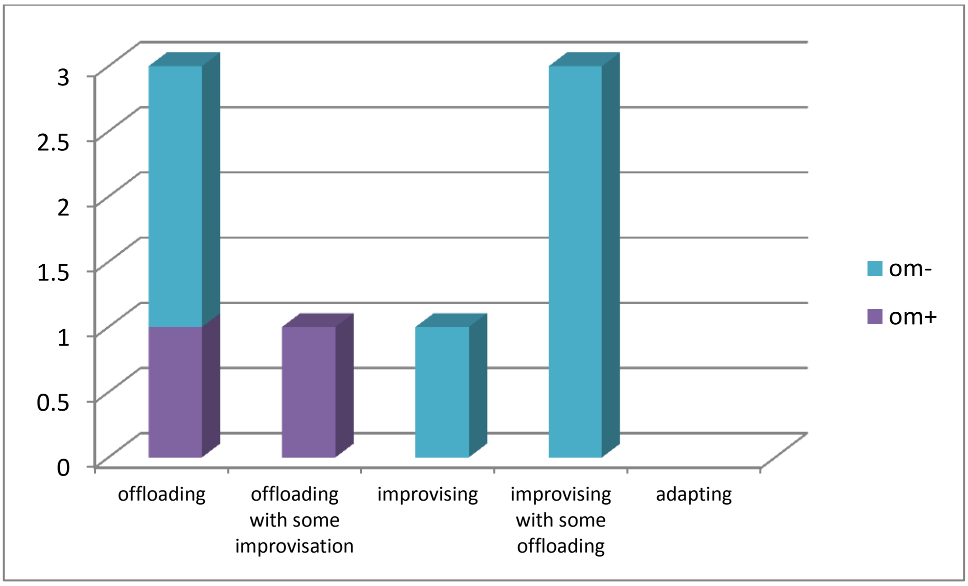


Fig 7.6 Omission of content in lessons

Fig 7.6 shows that there are more critical omissions (Om^{-}) in the lessons than the productive omissions (Om^{+}), another cause for concern with respect to the teacher-textbook relationship.

Productive omissions

From Table 7.2, the productive omissions mostly involve leaving out some worked examples, or practice exercises from the textbook when assigning class activity or homework, as observed in teacher A1's lessons in chapter 6. The productive omissions are ineffectual in the teacher-textbook relationship.

Critical omissions

Table 7.5 presents the critical omissions observed in teachers' lessons compiled from Table 7.2

Lesson	Degree of appropriation	Description of the Critical omission	Comment
A13	O^{+}	Discussion of some properties when sketching graphs of functions	Properties
A22	O^{+}	Questions dealing with properties of functions except symmetry	Properties

A32	I^+	Other values of 'a' in determining the effect of 'a' in $y = ax^2$. Uses only $a = 1$ and $a = -1$	Properties
B22	I^+O^-	Questions relating to the exponential function after discussing its properties. Instead assigns one question on the exponential function for class activity and the rest of the questions are on the quadratic function	Selection
C12	I^+O^-	A discussion on the properties of symmetry and increasing/decreasing interval when discussing the properties of functions	Properties

Table 7.5 Critical Omissions in teachers' lessons

Except for lesson B22 in which the omission involves the selection of content, all the other critical omissions involve the properties of functions. In B22, the lesson was on determining the properties of the exponential function. However, at the end of the lesson, B2 assigned a class activity from the textbook where she selected one question that dealt with exponential functions and two on quadratic functions. These are questions a), e) and g) in the first set in Fig 7.7a.

Draw sketch graphs of each of the following on separate axes:

a) $y = \left(\frac{1}{2}\right)^x$ b) $y = \left(\frac{1}{2}\right)^{-x}$ c) $y = x + 2$

d) $y = -2x + 1$ e) $y = 2x^2$ f) $y = -2x^2$

g) $f(x) = \frac{1}{2}x^2$ with domain $\{0; 2; 4; 6\}$ h) $y = \frac{4}{x}$

i) $y = -\frac{4}{x}$

Draw sketch graphs of each of the following on separate axes:

a) $y = 3^x$ b) $y = 3^{-x}$ c) $y = \left(\frac{2}{5}\right)^x$

d) $y = \frac{2}{x}$ e) $y = \frac{x}{2}$ f) $y = \left(\frac{5}{2}\right)^x$

g) $f(x) = \frac{1}{3}x^2$ h) $h(x) = -\frac{1}{3}x^2$ on domain $\{0; 3; 6\}$

i) $g(x) = -\frac{9}{x}$

Fig 7.7a A practice exercise from which B2 selects questions for class activity

As Fig 7.7a illustrates, there are six questions on exponential functions which the teacher could have chosen from in this practice exercise: a), b) in the first set, and a), b), c) and f) in the second set. One comment in Table 7.2 in the analysis of lesson B22 that the study made was about erroneous mediation that B2 introduced during this lesson (discussed later under errors). Thus, the class activity could have been an opportunity for B2 to afford learners practice exercises with exponential functions. In the light of the erroneous mediation therefore, the study considers this as a critical omission as it detracts from opportunities for mediation of the exponential function.

In lessons A13, A32 and C12, teachers omit discussion of some of the properties of functions when dealing with these functions. For example, A32 in discussing the effect of the parameter 'a' in $y = ax^2$, only uses the values of $a = 1$ and $a = -1$ for this investigation; while the textbook introduces different values of 'a' as per the syllabus requirement. This implies that learners in this class only get exposure to the reflection of $y = x^2$ and not to other effects such as vertical stretch and compression which are required by the syllabus.

This is thus a critical omission on the side of teacher A32. A similar incidence occurs in lesson C12 where teacher C1 skips over ‘symmetry’ and ‘intervals on which the function increases/decreases’ when she discusses the properties of functions. As with teacher A3, learners in C1’s class are denied the opportunity to engage with some of the key features of functions and yet the properties are the basis for the teaching of this topic.

Perhaps the most ‘classic’ case of the critical omissions occurs in lesson A22. Fig 7.7b shows a textbook exercise from which teacher A2 assigned questions 1 and 2 as homework. These questions involve drawing the graphs of the three functions and investigating the property of symmetry for these functions. In lesson A22, she went over the homework questions with the learners and when that was done, one would have expected that questions 3 and 4 would follow, but that was not to be the case.

1. Draw the graphs of the following on separate axes. Use the table method. Include both positive and negative values for the independent variable.
   a) $g(x) = x^2$ b) $f(x) = 2^x$ c) $h(x) = \frac{8}{x}$
2. Look up the meaning of the word “symmetry” in a dictionary (or refer to Chapter 9 for a definition).
   a) Which of the curves you have drawn has symmetry?
   b) Identify the line(s) of symmetry in each case.
3. a) Investigate what happens to the function values for each function when the x values become:
   (i) bigger and positive (for example use $x = 10$; 100; 1 000)
   (ii) smaller and negative (for example use $x = -10$; -100; -1 000)
   (iii) closer to zero, either positive or negative (for example use $x = 0{,}1$; 0,01; 0,001; –0,1; –0,01; –0,001).
   What effects do these values have on each graph?

   b) (i) Which of these curves never cut (intersects)the x-axis?
   (ii) Which of these curves never intersect the y-axis?
   (iii) Explain your answers to (i) and (ii).
4. Look at the dependent variables of each function. For each function, state whethe the dependent variable has:
   a) a maximum value, or
   b) a minimum value.

Fig 7.7b Textbook exercise from which teacher A2 selects homework

As Fig 7.7b shows, question 3 in the textbook deals with the asymptotic properties including end-behaviour of functions, and question 4 with local extrema. These properties are key features of the hyperbola, the exponential function and the parabola, and form the background for the study of these functions. Instead, after discussing the property of symmetry, teacher A2 chose questions from a different exercise (Fig 7.7c below).

- Work on your own.
- Draw the graphs in this exercise by plotting points if necessary.

1. Draw the graphs of $t(x) = \left(\frac{3}{2}\right)^x$ and $p(x) = \left(\frac{2}{3}\right)^x$ on the same set of axes.

2. **a)** On the same set of axes draw the graphs of $y = b^x$, on the domain $[-3; 3]$ for:

(i) $b = 2$; $h(x) = 2^x$

(ii) $b = \frac{1}{2}$; $g(x) = \left(\frac{1}{2}\right)^x$

b) Describe the features of the graphs of h and g under the following headings:

(i) symmetry **(ii)** maxima or minima

(iii) asymptotes **(iv)** range.

6. On the same set of axes:

a) Draw the graphs of $h(x) = -\frac{12}{x}$ and $g(x) = -\frac{16}{x}$.

b) Discuss the features of each curve.

8. The temperature readings on the Celsius (C) and Fahrenheit (F) scales are linked by the formula: $F = \frac{9}{5}C + 32$.

a) Draw a graph of F against C, for $C \in [-20°; 100°]$.

b) Use your graph to determine:

(i) Normal body temperature (37 °C) in Fahrenheit.

(ii) The freezing point of water in Fahrenheit.

(iii) The boiling point of water in Fahrenheit.

(iv) Express C in terms of F (i.e. make C the subject of the formula $F = \frac{9}{5}C + 32$).

Fig 7.7c An extract from a prescribed textbook

In fact, in questions 2 and 6 of Fig 7.7c the textbook expects that learners have done the investigation into the properties in Fig 7.7b as the exercise in Fig 7.7c comes quite later than Fig 7.7b. The omission of questions 3 and 4 in the previous exercise hence disregards an investigation of very critical properties of the functions for the function classes being studied, and therefore denies learners the exposure to these critical properties of functions. The implications for this omission are serious with respect to the opportunities for learning which are afforded to the learners. Additionally, this and the other critical omissions thus far mentioned, suggest a teacher-textbook relationship that is not only weak for some teachers, but that cannot be characterised as an intimate relationship. So far the teachers involved in making these critical omissions, A1, A3, and C1 at least, are the teachers who have shown that they have a relationship with the textbook from the way they mobilise the content coverage of the textbook; to the robust injections that they bring to the content (except for teacher A2). Their relationship with the textbook therefore cannot be classified as weak, but as lacking the closeness that is required to be intimate, hence why I characterise it in this manner. Additionally, this is an example of textbook use by teachers which I have called tacit.

With respect to teachers' PDC, the critical omissions do not illuminate teachers with high levels of PDC. If PDC is about the "capacity to use the resources in ways that enhanced students' opportunities to learn" (Choppin, 2011, p. 350) then the teachers have not displayed this capacity with the critical omissions, suggesting a low PDC.

7.2.1.3.3 Errors

Mathematical errors obviously detract from opportunities for mediation and therefore it is reasonable to have a discussion about them in this section. Table 7.6 presents a summary of all errors found in teachers' lessons, and shows two types of mathematical errors: those found in the resources and those made by teachers.

Lesson	Description of errors	Comments
A13	Teacher mistakenly labels two parabolas as $y = -x^2$ and $y = x^2$ even though they do not turn at the origin when illustrating the effect of the sign of the parameter 'a' in $y = ax^2$	Error unrelated to textbook
A21	confuses 'one-to-many' with 'many-to-one'	Error repeated over two lessons
A22	Does not recognise the line $y = x$ as an axis of symmetry for $y = \frac{8}{x}$ despite the textbook explicitly showing it	Teacher rejects a learner's suggestion of this line being a line of symmetry for the function without justifying why not $y = x$ shown as a line of symmetry in the textbook
A23	Does not 'appreciate' the vertical asymptotes of hyperbolas	Indicate horizontal asymptotes only on all hyperbolas
A34	Solutions from textbook contain errors which caused a heavy disagreement between teacher and learners; and raises issues of 'authority' in the classroom	Error in the Pre-CAPS textbook solutions
A42	the asymptotic behaviour of the hyperbola is not reflected on the graph	
A43	Error on restricted domains is made by the teacher in her explanations even though the textbook contains a section of restricted domain	Mathematics explicitly explained in textbook
B22	Uses $y = -2^x$ as an example for functions with $a < 1$ in $y = a^x$ resulting in erroneous mediation. Leaners had suggested $y = \left(\frac{1}{2}\right)^x$	Textbook explains this quite explicitly
C12	The handout contains two errors: shows only the line $y = -x$ as the line of symmetry for $y = \frac{1}{x}$ (teacher does not correct this); shows the range of $y = b^x$ as $\mathbb{R}$ instead of $x > 0$	Teacher corrects one error but not the other

Table 7.6 Errors in lessons

The error in lesson A13 has already been discussed in chapter 6 and is not related to the textbook, therefore I will not discuss it further here. Similarly, the error in lesson A21 about teacher A2 confusing 'one-to-many' and 'many-to-one' mapping diagrams has already been discussed under injections in this chapter. This leaves two types of errors, namely errors found in curricular resources, and those made by teachers, for discussion.

Errors in curricular resources

There are two errors in this category, one found in the Pre-CAPS textbook, and another on a handout that teacher C1 used in lesson C12. I would like to clarify that the study has not undertaken to look for errors in the textbook, but the errors discussed here emerged in the analysis of the lessons and therefore could not be ignored.

In the lesson A34, the error involves an exercise in which learners had to determine whether or not the given graphs represented functions (Fig 7.8a). The textbook classified the graphs a) and b) as *not* representing graphs of functions, but c) as representing a function

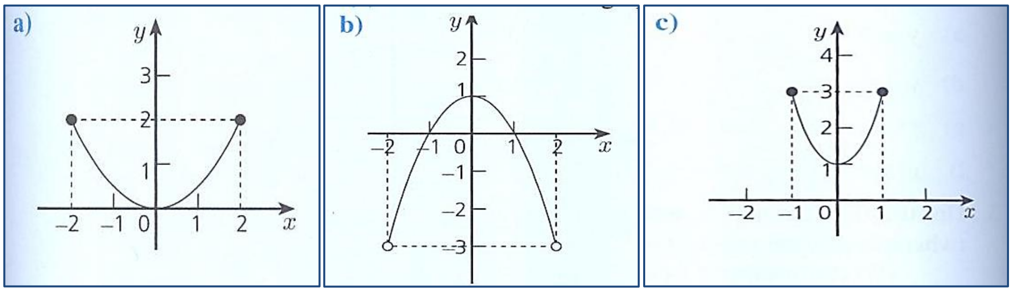


Fig 7.8a Exercises from the prescribed textbook

Teacher A3 and other learners disagreed with the textbook that $a)$ and $b)$ are not functions and their argument was based on the fact that since $a)$ and $c)$ are not different, and $c)$ represents a function according to the textbook, then $a)$ should also be a function. They considered the dotted lines as demarcations for restricted domains and not as part of the graph. They used the vertical line test and other techniques to show that in these functions, one input value corresponds to exactly one output value. However, another group of learners disagreed with teacher for the mere reason that if the textbook says graphs $a)$ and $b)$ are not functions, then they are not functions. No amount of justification or substantiation from teacher would dissuade this group.

This incidence points to two important consideration in the teacher-textbook relationship. Firstly, an error such as this one in the textbook constrains teaching and learning in a negative way as seen in this case where the teacher could not persuade learners who believe that the textbook is never wrong. Issues of 'authority' in the classroom have been documented in literature (Love & Pimm, 1996) but beyond the scope of this study. Secondly, it highlights the importance of the collaboration or the participatory relationship between teacher and textbook in which each shapes the other: teacher's subject matter knowledge must be in place in order to deal with situations such as this, even though it was not enough to persuade learners in this case.

Other errors from curriculum resources came from a commercial workbook that teacher C1 used in his lessons when determining properties of functions. The handout teacher used in class is shown previously in Figure 7.3a. In this handout on summarising properties of the function $y = b^x$, firstly, the handout does not specify the restrictions on the values of the base b; which the prescribed textbook explicitly specifies as $b > 0$ and $b \neq 1$. In this lesson, teacher did not pay attention to this omission from the textbook. Secondly, the range of the function $y = b^x$ is also erroneously written as the set of real numbers, (R); but teacher attended to this error and helped learners to determine the correct range as $(0; \infty)$. Thirdly, for the function $y = \frac{1}{x}$, the handout only reflects one axis of symmetry as the line $y = -x$, but does not regard the line $y = x$ as another line of symmetry for the hyperbola. Again, the teacher did not deal with this particular error. The implications for the opportunities for mediation here are grave: the errors thus mentioned are critical in the mediation of functions, and so if they go uncorrected as teacher does with some of them; they leave learners with incorrect mathematics that learners carry with them as they progress to Grade 11.

This case points to the relationship between teacher C1 and the curriculum resources that she chooses to utilise in practice. As I have pointed out in previous sections in the present chapter, the CAPS textbook features summary tables for all functions as well. These summaries are error free to start with. However, C1 chooses to work with a resource that contains errors. This suggests a weak relationship between the teacher and the prescribed textbook: the teacher was not aware firstly that the prescribed textbook featured exactly the same content; and also that this content in the prescribed textbook was error free and even more detailed. Thus, this case here points to a low design capacity for C1.

Errors made by the teacher

Three teachers commit mathematical errors in their lessons: all of teacher A2's lessons have mathematical errors; two out of three of teacher A4's lessons show errors; and one of teacher B2's lessons.

In lessons A23 and A42 the error had to do with asymptotes of the hyperbola. As Fig 7.8b shows for lesson A23, teacher A2 did not seem to appreciate that the hyperbola has both horizontal and vertical asymptotes. When drawing the graphs of the functions $y = \frac{2}{x} - 2$ and $y = \frac{1}{x-2} + 1$, despite showing that the first function is undefined at $x = 0$ and the second one is undefined at $x = 2$ as shown in the table of values in Fig 7.8b, A2 only showed the horizontal asymptotes of these functions with dotted lines, and not their vertical asymptotes.

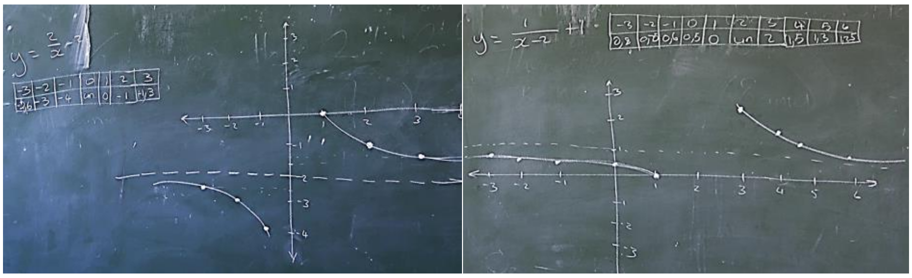


Fig 7.8b Graphs of the functions $y = \frac{2}{x} - 2$ and $y = \frac{1}{x-2} + 1$ drawn on the board

For teacher A4 in lesson A42, the error is more error by omission because in this lesson, learners were assigned functions of different function classes to draw including the function, $y = \frac{3}{x} + 3$. A learner drawing this hyperbola on the board shown in Fig 7.8c did not reflect its asymptotic behaviour;nor does the teacher when she corrected it.

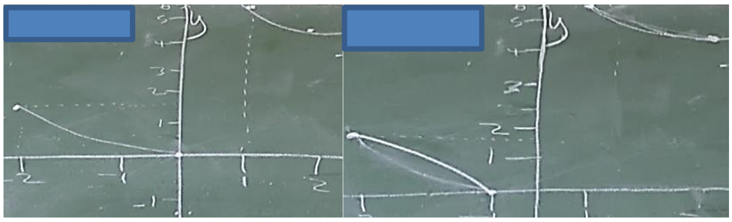


Fig 7.8c A learner's graph of $y = \frac{3}{x} + 3$ and a teacher's corrected graph

The graph on the left was drawn by a learner and that on the right is the teacher's corrected version of the same graph. The learner's graph is incorrect, but the teacher version does not indicate the asymptotic behaviour of this function towards the lines $y = 3$ and $x = 0$.

Teacher A4 makes another error on restricted domains in lesson A43. Drawing the graph shown in Fig 7.8d with the range [– 2; 6], she let learners guess its domain to be the interval

[0.5; 3.5]. I refer to this as guessing since the equation of the graph was not provided and so learners were guessing the x values corresponding with $y = 6$ on the graph.

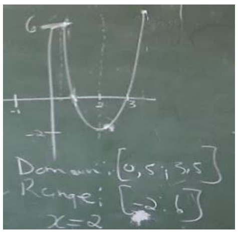
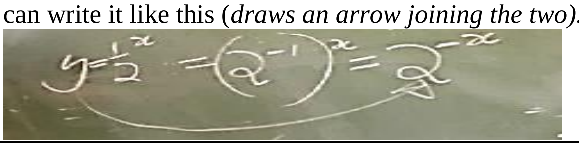


Fig 7.8d An error made by teacher in the classroom

In order to determine the x values corresponding to $y = 6$ on the parabola drawn in Fig 7.8d, one has to consider the properties of the graph: its turning point $(2; -2)$, the x-intercepts, 1 and 3 and so forth. To let learners guess the value without considering its properties is mathematically incorrect. My point here is that the graph drawn on the board above is a graphical representation of a particular equation of a function for which the values of x and y which determine its domain and range should satisfy this equation. However, in this lesson, the choice of the x values is arbitrary, and teacher accepts any value which learners thought could be the x value corresponding to the y-value, 6. This implies that the graphical representation does not correspond with its algebraic representation which is mathematically incorrect as one should be representing the other. The CAPS textbook features a section that deals with restricted domains, and so it is not as if the teacher did not have somewhere to refer to for this topic. This is yet another example of a weak teacher-textbook relationship.

In lesson A22, teacher had invited learners to suggest the axes of symmetry for the function $h(x) = \frac{8}{x}$ and one learner went to the board and demonstrated on the graph of this function that the line represented by the equation $y = x$ was an axis of symmetry for this function; but teacher rejected the learner's suggestion without explanation, despite the fact that the textbook shows that $y = x$ is an axis of symmetry for the function $h(x) = \frac{8}{x}$. This is yet another mathematical error which is committed by the teacher on an aspect that the textbook deals with quite explicitly.

A similar error is committed by teacher B2 in her lesson on determining the effect of the parameter 'a' in $y = ax^2$ in exponential functions, lesson B22. For exploring the effect of 'a' values less than 1 on the exponential function, B2 had asked learners to suggest a value of 'a' less than 1 that they would like to investigate, and learners had suggested $a = \frac{1}{2}$, that is, the function $y = \left(\frac{1}{2}\right)^x$. However, while teacher accepted this suggestion from learners, she nevertheless decided to use her own example and chose the function $y = -2^x$ instead. This was obviously an incorrect choice for $a < 1$ in $y = ax^2$. The graphs of the two functions $y = -2^x$ and $y = \left(\frac{1}{2}\right)^x$ however did not behave the same way causing a major dilemma for B2 as she could not then form a uniform conclusion about the behaviour of functions where 'a' is less than 1. In trying to get out of the situation, teacher made another error by convincing learners that in the example they had suggested, that is, $y = \left(\frac{1}{2}\right)^x$, the value of '$a$' is not actually less than 1, but greater than 1

Teacher	…Somebody said I must do this one (*points to* $y = \left(\frac{1}{2}\right)^x$), ne?
Learners	Yes.
Teacher	It is y equals to the half of? x (*chorus)*
Teacher	Ne. This half of x is giving us another shape. Ne? (*Learners agree.*) But they all cut at the same point, ne? So what is happening? Is 'a' in this one less than or greater than one? From that one over two. Is a greater than or less than one? And if you check there (*points to the equation on the board written as* $y = \frac{1}{2}^x = 2^{-x}$). Is '$a$' greater than or less than one? Oh you can't just see, ne? Who can try? Remember we try is nice, ne? Yes, who can try?
Teacher	What is happening with the value of 'a' from that one? Hmm? …
Learners	No.
Teacher	Let me go back. I want to ask to see this one, ne? Remember you said half, yes this half is less than one, ne?
Learners	Yes.
Teacher	… What is going to happen? You change half ne? Into exponent. Which is going to be two to the exponent of one. Ne? This is to x. Remember you are going to say ...... minus one times x, ne. And still it's going to give us two negative x, no? If you check this function now (*points to what he has just written on the board shown below*) is not ending this side, ne? This function, this equation you can write it like this (*draws an arrow joining the two).* 
Teacher	Ne? So you tell me now what is our 'a'? Is 'a' less than or greater than one?
Learners	Greater.
Teacher	Our 'a' is greater than? One (*chorus)*

A point to make about all these errors except for the mapping diagrams is that the textbook addresses all these aspects. Yet teachers still make them. For example, the error in A22 with the axes of symmetry, the Pre-CAPS textbook has drawn the graph with both axes of symmetry but teacher still rejects the other axis. For the error in B22, the CAPS textbook outlines all the different possibilities of the base and deals explicitly with functions where the base is negative, ie $y = (-2)^x$ and reflections such as $y = -2^x$.

All the errors outlined in this section detract from opportunities for mediation. They also point to a weak relationship between the teachers and the textbook. The relationship is weak because the textbook has availed the information that teachers need for mediation, but which teachers do not utilise despite its availability and end up with mathematical errors[17] which could have been avoided had the teachers collaborated with their textbooks on those issues. A weak teacher-textbook relationship has implications for teachers' PDC since it is evident that teachers are not mobilising the affordances of the textbook adequately.

7.2.1.4 Discussion on the Appropriation of the Content

In this section, I discuss the implications for textbook use as well as the teacher-textbook relationship of the processes of mobilising the content of the textbook which teachers

[17] The errors could also be a function of teachers' SMK

participating in the study adopt. In Table 7.7 I present a summary of each teacher's mobilisation of content on aspects which have a bearing on the teacher-textbook relationship such as whether the teacher perceives and mobilises the affordances and/or constraints of the textbook. This is indicated through the appropriation of content coverage and the robust injections, respectively. Secondly, the existence of critical omissions in teachers' lessons indicate to either a tacit use of the textbook or non-deliberate use depending on whether teachers mobilise the affordances and/or constraints, or not.

Teacher	Mobilisation of Content	The Teacher-Textbook Relationship
A1	Perceives and mobilises affordances and constraints but omits critical aspects	Textbook use tacit Teacher-textbook relationship not intimate
A2	Limited coverage of topic, lessons full of errors some of which textbook deals explicitly with	Textbook use haphazard and not deliberate Teacher-textbook relationship very weak
A3	Perceives and mobilises affordances and constraints but omits critical aspects	Textbook use tacit Teacher-textbook relationship not intimate
A4	Limited coverage; while no injections and no omissions, commits errors on aspect dealt with explicitly in the textbook	Textbook use haphazard and not deliberate Teacher-textbook relationship very weak
B1	Partially perceives and mobilises affordances and constraints	Did not use a textbook in his lessons, therefore undecided on teacher-textbook relationship
B2	Perceives and mobilises affordances but omits critical aspects and commits error on aspect dealt with explicitly in textbook	Textbook use tacit Teacher-textbook relationship not intimate
C1	Perceives and mobilises affordances and constraints but omits critical aspects and choice of external resource introduces error	Textbook use tacit Teacher-textbook relationship not intimate

Table 7.7 The teacher-textbook relationship with respect to mobilisation of content

Table 7.7 indicates that there is indecision on textbook use and consequently the teacher-textbook relationship of one teacher (B1) among the seven teachers participating in the study. This is due to the fact that teacher B1 departs from the textbook's approach to the teaching and learning of the content in very distinct ways as we shall see in the next section. For two teachers other teachers, A2 and A4, textbook use has been described as haphazard and not deliberate, to reflect that with these two teachers' there is more than just non-deliberateness in the use of the textbook which results in a teacher-textbook relationship that is very weak. For the rest of the teachers, the use of textbook has been described as being tacit, mainly to reflect that there is obvious use in every way other than when teacher does not collaborate fully with the textbook that results in teacher omitting very critical elements which have the potential to disrupt the opportunities for the mediation of the object of learning.

Sherin and Drake (2009) using their curriculum strategy framework, in which their teachers' patterns of adapting curriculum materials lie on a continuum from *omitting* on one end, to *creating* at another with *replacing* lying in the middle, conclude that

> While we might know that a teacher tends to create new materials, we have no indication of whether those creations focus on substantive or superficial aspects of a lesson, whether they are aligned with the goals of the curriculum, or whether they subvert the intended purpose of a lesson (Sherin & Drake, 2009, p. 490)

In the present study, the notions of *injections* and *omissions* have provided analytical tools that address this concern from Sherin and Drake (ibid). It has been illustrated in the study how teachers make *robust injections* that *enhance the opportunities for mediation* or *distractive injections* that have a *potential to detract from the opportunities for mediation*. In studying teachers' *omissions*, the study has revealed that the *critical omissions* are *disruptive to opportunities for mediation* and the *productive omissions* are not disruptive. Hence the framework of injections and omissions starts to open up the teacher-textbook relationship and the opportunities for mediation in the classroom which the curriculum strategy framework does not deal with.

Additionally, the framework of injections and omissions point to a change in agency - from shared agency which results in robust injections; to where agency lies with the teacher and she omits critical aspects of the topic. As this happens, the use of textbook is no longer participatory (Remillard, 2005) and the dynamic interrelationship where each shapes the other ceases.

7.2.2 Teachers' Appropriation of the Approach

The analysis of the approach to the teaching and learning for teacher A1 in chapter 6 shows that the appropriation of the approach to the teaching and learning of the content in the study is determined from: a) the degree of appropriation of the general approach to the teaching and learning of the content and b) the conception of function conveyed in teachers' lessons. The appropriation of the general approach indicates whether the approach in the lesson has been quasi-deductive ('teacher led') or quasi-inductive (initial learner engagement with content); while the conception of function points to whether the approach to functions is pointwise, global or a progression from pointwise to global strategies.

The results for the twenty lessons in the study on teachers' appropriation of the approach are presented in Table 7.8To recap, the degree of appropriation of the approach carries the codes QD^{a+} or QD^{a-} to show whether the quasi-deductive approach in the lesson adopts that of the textbook; or QI^{a+} and QI^{a-} for the quasi-inductive approach adopted from the textbook or not adopted, respectively. Similarly for the approach to functions, the codes *pointwise (a^+), global (a^+)* and *pointwise-global (a^+)* point to the different conceptions of functions adopted from the textbook or resource, and *pointwise (a^-), global (a^-)* and *pointwise-global (a^-)* point to the conception of functions which has not been adopted from the textbook or resource. The results pertaining to the degree of the appropriation of the general approach are considered firstly.

Teacher	Lesson	Appropriation of Approach		Comments
		Degree of appropriation	Conception of function	
Teacher A1	A11 (week 1) Function notation Vertical line test	QD^{a+}	Pointwise (a^+)	Teacher adopts the approach of the textbook Teacher adopts the approach to function
	A12 (week 1) Transformations of the quadratic function	QI^{a+}	Pointwise to global (a^+)	Teacher adopts the approach of the WMCS materials Teacher adopts the approach to function
	A13 (week 3) Determining equations of graphs of functions Sketching graphs of functions	QD^{a+}	Pointwise to global (a^+) Global (a^+)	Teacher adopts the approach of the textbook Teacher adopts the approach to function
Teacher A2	A21 (week 1) Introducing functions Drawing graphs of functions	QD^{a+}	Point-wise (a^+)	Teacher adopts the approach of the textbook Teacher adopts the approach to function
	A22 (week 2) Properties of functions	QD^{a-}	Point-wise (a^-)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher does not adopt the global approach to functions
	A23 (week 3) Properties of the hyperbola	QD^{a-}	Point-wise (a^-)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher does not adopt the global approach to functions
Teacher A3	A32 (week 3) Transformations of the parabola Sketching the parabola	QD^{a-} QD^{a-}	Global (a^+) Pointwise to global (a^+)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the approach to functions
	A33 (week 3) Sketching the parabola Properties of functions Interpreting graphs	QD^{a+} QD^{a-} QD^{a+}	Global (a^+)	Teacher does not adopt the quasi-inductive approach to properties of the textbook Teacher adopts the global approach to functions
	A34 (week 3) Properties of functions and determining equations of graphs Interpreting graphs	QD^{a-} QD^{a+}	Global (a^+)	Teacher does not adopt the quasi-inductive approach of the textbook for properties of functions Teacher adopts the approach to functions
Teacher A4	A41(week 1) Introducing functions	QD^{a-}	Point-wise (a^+)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the pointwise approach to functions
	A42 (week 3) Properties of functions	QD^{a-}	Pointwise to global (a^+)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the approach to functions
	A43(week 3) Properties of quadratic functions	QD^{a-}	Global (a^+)	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopt the global approach to functions

Teacher	Lesson	Appropriation of Approach		Comments
		Degree of appropriation	Conception of function	
Teacher B1	B11(week 1) Transformation of the parabola (vertical shifts)	QI^{a+}	Pointwise to global (a^{+})	Teacher adopts the quasi-inductive approach of the textbook Teacher also adopts the approach to functions
	B12(week 2) Transformation of the parabola (horizontal shifts) Interpreting graphs	QI^{+} QI^{a-}	Pointwise to global (a^{+})	Teacher adopts the quasi-inductive approach of the textbook for transformations but not for interpretations Teacher adopts the approach to functions
Teacher B2	B21(week 1) Terminology on functions Properties of linear functions	QI^{a+} QI^{a+}	Point-wise (a^{+}) Point wise (a^{-})	Teacher adopts the quasi-inductive approach of the textbook Teacher does not adopt the approach to functions fully (mediation breaks down)
	B22 (week 2) Properties of the exponential function	QD^{a-}	Pointwise to global but dominated by point-wise (a^{+})	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the approach to functions
	B23 (week 4) Sketching graphs of functions	QD^{a+}	Point-wise (a^{+})	Teacher adopts the approach of the textbook Teacher also adopts the approach to functions
Teacher C1	C11 (week 1) Properties of the hyperbola Transformations of functions	QD^{a-}	Pointwise to global (a^{+})	Teacher does not adopt the approach of the textbook to properties and transformations Teacher adopts the approach to functions
	C12 (week 2) Properties of functions	QD^{a-}	Global (a^{+})	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the approach to functions Teacher extends a textbook exercise which changes the sense of the question from sketching graphs of questions to determining their properties
	C13(week 3) Properties of functions Past examination questions (interpretation)	QD^{a-}	Global (a^{+})	Teacher does not adopt the quasi-inductive approach of the textbook Teacher adopts the approach to functions Going over homework, however learners draw the graphs of functions using point-by-point plotting instead of sketching the graphs as the textbook demands

Table 7.8 The results of the Analysis of the Appropriation of the Approach

The results for the degree of appropriation of the general approach and of the conception of function are discussed separately below.

7.2.2.1 Degree of Appropriation of the General Approach

The results from Table 7.8 are presented pictorially in Fig 7.9a and Fig 7.9b

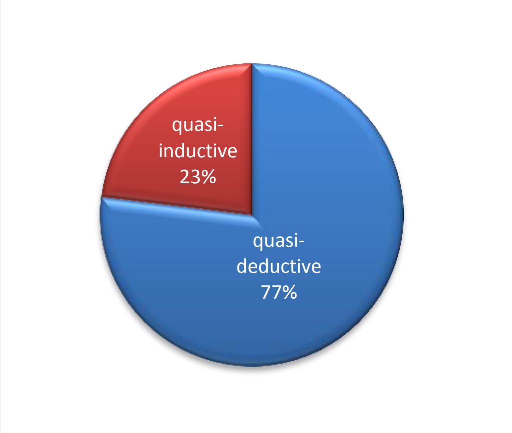


Fig 7.9a Degree of Appropriation of Approach

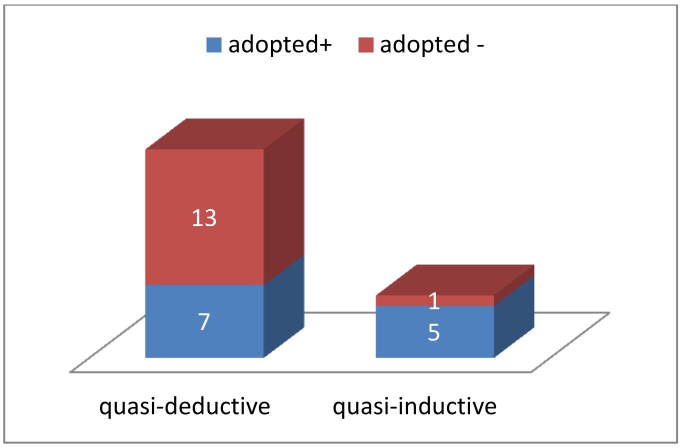


Fig 7.9b Rate of adoption of the approach

Fig 7.9a shows that there is an 80% prevalence for the *quasi-deductive* approach against 20% of the *quasi-inductive* approach in the lessons. Further analysis in Fig 7.9b which illustrates how the approach matches with that of the textbook, reveals that the quasi-deductive approach in the lessons matches with that of the textbook (QD^{a+}) in only 7 out of the 20 occurrences. This amounts to 35% adoption of the approach of the textbook in lessons, which implies that there is a 65% mismatch of the approach between the lessons and the textbook. In other words, in 65% of the occurrences where the approach is quasi-deductive in the lessons, the approach is actually quasi-inductive in the textbook.

For the quasi-inductive approach the adoption rate exceeds 80%, meaning that in lessons where the approach is quasi-inductive then it is highly likely that the approach is also quasi-inductive in the textbook or resource being utilised.

The implication of Fig 7.9a and Fig 7.9b is that the quasi-deductive approach is the teachers' preferred or privileged way of teaching and it is not influenced by the approach of the textbook. Ensor et al.(2002) point out in their studies that there is a mismatch between the teachers' approach which tends to be more deductive and the textbook's that is inductive. The results of this study confirm this assertion, and therefore in this study the conclusion is that the approach of the textbook does not influence the approach the teacher adopts in her practice. Put in another way, the study argues that the teachers do not necessarily appropriate the approach of the textbook in their lessons. This assertion is illustrated with some examples below.

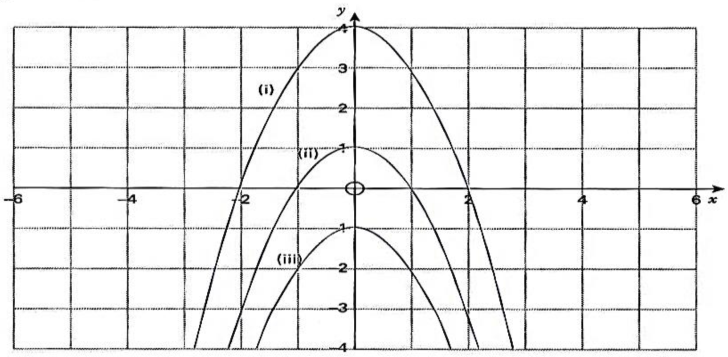
Lesson A32 is one example of a lesson where there is a mismatch between a quasi-deductive approach by the teacher, and a quasi-inductive approach of the textbook. The lesson was about transformations of the parabola looking at the effects of the parameters 'a' and 'q' in $y = ax^2 + q$. The following excerpt shows how the teacher introduced this concept:

teacher	If I say y is equals to ax squared plus q, ax squared plus q (teacher writes on the board $y = ax^2 + q$). This q simply means what movement? What does this represent? Which movement? Which movement does this represent? Vertical movement or horizontal movement?

learners	Horizontal
teacher	So you don't know the difference between vertical and horizontal?
learners	Yes, no.
teacher	Eh? Which one, which movements we say, do we say is horizontal movement? (learners talk at once gesturing)
teacher	Oh, horizontal movements, which ones?... Just come and demonstrate the horizontal. Horizontal. Yes (pointing to a student). Oh it's when you are moving to the left and the right. Vertical movement it's up and down (together with students).
teacher	So remember when we are doing a straight line, it's a y is equals to x and that line is passes through the origin (together with students). So we say y is equals to x plus one. What happened to the graph? (Teacher writes on the board y = x y = x + 1.)
learners	It shifted up.
teacher	It's going to shift one unit up (throw his hands into the air to show upward movement).
learners	Yes, yes.
teacher	So the same applies with this graph here. This graph is called a parabola (teacher writes parabola on the board). This is called a parabola. Then if you add something here you are shifting your graph up or down, depending on the number that you're adding. If the number that you're adding is negative, then it means you are not going to shift down. If it's positive, it means it's going to shift up. (Something happens, or is said, to make students laugh.) So y is equals to ax squared plus q (teacher points to board). Sharp, sharp. Y is equal to ax squared plus q. Then it means your graph is going to shift. If it's plus q, it means your graph's gonna shift two units up. If it's minus q, it means your graph gonna shift two units down (students chorus with teacher).

The teacher basically 'tells' learners what the effect of 'q' on the graph of $y = x^2$ would be. The textbook on the other hand, introduces this concept as a matching activity in which learners match the given graphs with their equations:

Graphed are the functions
$y = -x^2 - 1$
$y = -x^2 + 1$
$y = -x^2 + 4.$



a) Match the numbers on the graphs (i), (ii) and (iii) with their defining equations.

Fig 7.9c Introducing the effect of q in $y = af(x) + q$ from the CAPS Textbook

The approach in the textbook as shown above is quasi-inductive: learners engage with the activity of matching the graphs with their equations beforehand; unlike in the lesson where teacher 'tells' it to them. Hence, the two approaches are different.

Teacher B1 provides another example of a mismatch in the approach. In lesson B12 whose excerpt is illustrated below, the teacher adopted a quasi-inductive approach to sketching the graphs of functions, while the textbook's approach is quasi-deductive.

Teacher	(*writes* $y = (x + 1)^2 + 1$ *on the board*)…. Right, so when you do shifting. So what graph am I going to get? (*Learners call out "left and up"*.) Yes,[*calls a learner's name*] left and up. Where is the graph going to turn? Is it shifting left and up, where is the graph going to turn? Before I draw the graph? If you put a rough sketch tell me where is the graph going to turn? Who, yes my boy?
Learner	Minus one and positive one.
Teacher	Minus one and?
Learners	Positive one.
Teacher	It's going to turn at, let's find out (*points to the board*). It's going to get minus one here, and a positive one. So it means now our graph is going to turn somewhere here, if I put it at that particular, I don't know about the other points. You going to get that particular one, then you put it in a graph, hey? But it's telling us to shift to the left and upwards we are going to get our turning point for this particular one. Right, if this, the very same graph that I have, we shifting and shifting, hey? According to you, you have shifted this particular graph uh, one unit backwards, huh? One unit away from the positives. According to you, ne? *(Teacher then writes the equation* $y = (x + 1)^2 - 2$ *and asks learners again where this particular graph would turn)*
Teacher	Oh explain my sis, tell us why the graph is going to turn here, tell them please. (*Points to the board.*) Tell them why the graph going to turn here. Tell them.
Learner	It's going to turn at minus two because it's going to move two units downwards.
Teacher	It's moving two units down, that's the answer. Are you happy now? It's moving two units downwards. Yes? Yes.
Learner	Sir, can I disagree with that?
Teacher	Before you help me, can I get some help from [*calls another learner's name*]?
Learner	It's going to turn at minus one.
Teacher	Minus one? *(Points to the graph*) Here, here? is going to turn here? Why are you saying we are going to turn here?
Learner	Because Sir you minus one from two
Teacher	You minus one from two?
Learner	Yes.
Teacher	Oh? [*calls first learner again*]? Yes? (*Points to a learner*.)
Learner	Sir, I say minus one and, I mean minus one and minus two.
Teacher	Minus one and minus two?
Learner	Yes, so move one unit to the left and two units downwards.
Teacher	Minus one and minus two, here hey? Can you tell why it's moving that particular way, can you explain? Yes?
Learner	Sir, it is because sir in the brackets when you get plus one it means that it's going to move to the left hand side (*pause)* So now that it has moved to the left hand side we are going to move it two units downwards at minus two.

In the extract, teacher B1 was engaging learners to use the properties of functions to determine the turning point. B1 did not 'tell' learners but used probing questions to assist learners in making conjectures about the horizontal movement of the parabola.

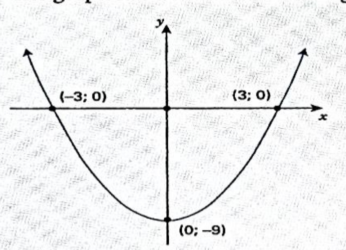
The following worked example shows how the textbook introduces the sketching of the graph of the function $y = x^2 + 9$.

Example

Sketch the graph of $y = x^2 - 9$.

Solution

- Since the coefficient of x^2 is positive, the parabola is 'cup' shaped ∪.
- To find the y intercept put $x = 0$
  $y = 0^2 - 9 = -9$
  y intercept: $(0; -9)$
- To find the x intercepts put $y = 0$:
  $0 = x^2 - 9$
  $0 = (x + 3)(x - 3)$
  $x = -3$ or $x = 3$
  x intercepts: $(-3; 0)$ and $(3; 0)$
- The graph has a minimum turning point on the y-axis at $(0; -9)$



- This is the graph of $y = x^2$ translated (shifted) downwards by 9 units.

Fig 7.9d Sketching of graphs from the prescribed textbook (Pike et al., 2011, p. 165)

In this example, the values $y = 0$ and $x = 0$ are substituted into the equation in order to calculate the intercepts. After the calculation of the intercepts, it is stated that the graph has a minimum point on the y-axis at $(0; -9)$. However, how these values have been obtained is not indicated in the textbook; so that what other teachers begin to do is to tell learners that in a function $y = ax^2 + q$, the coordinates $(0; q)$ provide the turning point, without any form of substantiation. As the example shows, the textbook prescribes a procedure for sketching the function, while teacher B1 uses a strategy that engages learners and where learners are encouraged to apply the properties of functions.

In summary, with respect to the approach to the teaching of functions, the finding under the degree of appropriation of the general approach to teaching and learning is that, the quasi-deductive approach is preferred over the quasi-inductive approach (80 % : 20%). However, only 35% of the quasi-deductive approach in lessons is adopted from the approach of the textbook. The teachers in this study in general do not mobilise the approach of the textbook as an affordance in their practice.

7.2.2.2 Conception of Function in Lessons

The results from Table 7.8 on the conception of function conveyed in teachers' lessons are represented pictorially in Fig 7.10a and Fig 7.10b

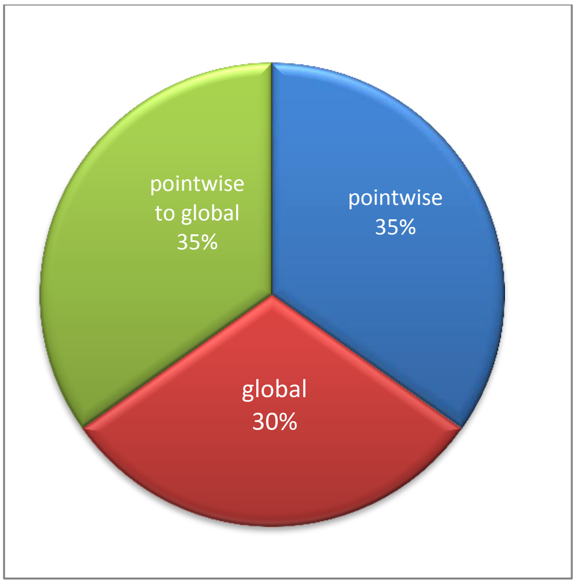


Fig 7.10a Conception of function

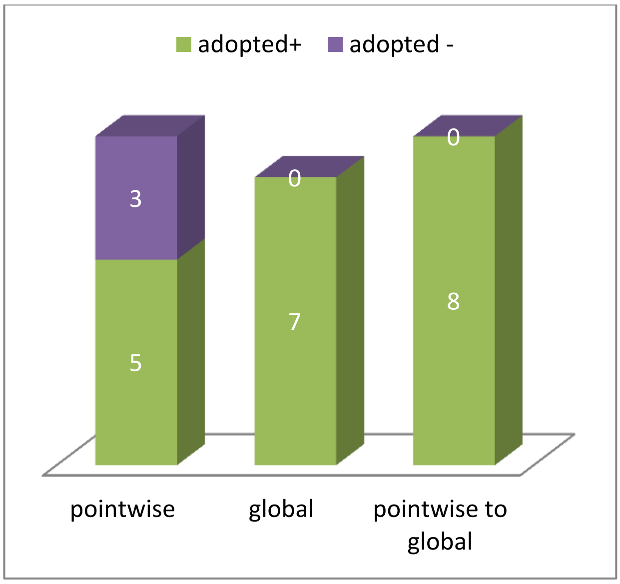


Fig 7.10b Degree of adoption

From Fig 7.10a, 35% of the performance expectations are pointwise actions; 35% progress from pointwise to the global; and 30% of actions on function tasks are global. Thus, the conception of function across the lessons conveys progression from pointwise to global actions on function tasks, which implies a more interpretive conception of function. This matches the approach of the textbook with respect to the conception of function, a fact reflected in Fig 7.10b which shows that both the global approach and the progression from pointwise to global approach have been adopted (they carry a (a^{+}) code) from the textbook or resource used. Furthermore, there are only three lessons out of eight (about 40%) that convey a pointwise approach to function which do not match the approach to function of the textbook. One of these lessons belongs to teacher B2 (lesson B21), and the other two to teacher A2 (lessons A22 and A23).

In B21, the teacher was investigating the effect of the gradient on a graph of a linear function, using the gradient formula to calculate the gradient of different graphs and then making a comparison. The difference with the approach in the textbook lies in the type of actions on function tasks from the teacher and the textbook when the straight line graphs have been drawn and their gradients determined. The actions on function tasks in the textbook include both point-wise and global strategies, while in B21 learners spend most of the time using the gradient formula to calculate gradients, thus remaining at pointwise level throughout the lesson.

A22 is a lesson in which teacher A1 made *critical* omissions when she left out the questions dealing with end-behaviour of functions and maxima/minima properties. Learners in the lesson were hence only using point-by-point plotting of points and did not tackle these questions which required global actions on function tasks.

In A23, when the teacher was recapping the topic on functions, and learners could not give correct responses to the teacher question about what the graph of the function $y = (x+3)^2 + 1$ would be, A2 directed learners to use the point-by-point plotting methods to resolve an issue for which learners could have been requested to interpret the properties of this function.

In the excerpt below, A2 had already asked learners what the graph of equation $y = 2x + 3$ would be, and learners had called out that it would be a straight line; and that the graph of $y = \frac{3}{x} + 4$ would be a hyperbola. The extract begins after teacher had written the equation $y = (x+3)^2 + 1$ on the board and asked learners what type of graph it would produce

Learners:	Exponential.
Teacher:	What would that one be?
Learners:	Exponential graph.
Teacher:	Exponential?
Learners:	Yes.
Learners:	Parabola.
Teacher:	(*pointing to the board*) This one is parabola?
Learners:	Exponential, exponential. Trig.
Teacher:	Which one is this? Which one is this? (teacher writes $y = 3^x$ on the board) (*Long pause as the learners decide which graph is which)* Hmm?
Learners:	exponential
Teacher:	(*pointing to* $y = 3^x$) Which one is this? This one is an exponential?
Learners:	Yes.
Teacher:	And this one? (*Points to* $y = (x+3)^2 + 1$)
Learners:	Exponential.
Teacher:	(*pointing to both functions*) These two are both exponential?
Learners:	Yes.
Teacher:	Okay my angels I want you to draw these two graphs ($y = 3^x$ and $y = (x+3)^2 + 1$) Okay?
Learners:	Yes.

The point I wish to make here is that the textbook would have handled the two functions $y = 3^x$ and $y = (x+3)^2 + 1$ differently. For both functions, actions on function tasks would have proceeded beyond point-wise actions; by the end of the topic on functions, the textbook would have expected learners to distinguish between the two functions through interpreting their properties, thus considering them globally, instead of drawing the functions from scratch using point-by-point plotting which is completely pointwise as teacher A2 did in her lesson. This is an example of a teacher for whom the design rationale is not visible (Davis & Krajcik, 2005) and therefore at the end of the topic there is a mismatch between the potentially implemented curriculum and the implemented curriculum (Valverde et al., 2002).

In conclusion therefore, except for very few cases, the study finds that teachers in the study mobilise the approach of the textbook with respect to the conception of function as an affordance in their practice. However, this is done more through a teacher-led quasi-deductive approach and not through the ‘investigative’ quasi-inductive approach recommended by the textbook, and therefore creating a mismatch between the approach of the textbook and that of teachers in the lessons.

7.3 Discussion and Conclusion

In the opening quote of this chapter Brown (2009) posits that a teacher with high PDC is marked by the skill with which she weaves the various pieces and how she puts them into play. Brown goes further to point out that teachers’ PDC “describes the manner and degree to which teachers create deliberate, productive designs that help accomplish their instructional goals" (p.29); that although

> the elegance of a teacher's design is a subjective determination… not all designs are equally effective at helping teachers reach their goals, not all designs reflect the same responsiveness to the needs of a particular setting, not all designs are purposeful, and not all designs embody the same degree of utility (p. 31)

In the current chapter, the study has described the processes by which the participating teachers have perceived and mobilised both the content and the approach to the teaching and learning of this content. It is through these processes that conclusions are reached about: a) teachers' textbook use; b) teacher-textbook relationships; and consequently c) teachers' PDC, that is, the degree to which they 'create deliberate productive designs' for the classroom.

Under the mobilisation of content, the chapter examines the nature of teachers' offloads and improvisations to conclude that teachers use the textbook mostly for exercises and improvise on other aspects of the content in the classroom, thus suggesting a generally non-deliberate, tacit use of the textbook by teachers. The examination of the opportunities for mediation reveals the existence of distractive injections, critical omissions, and most importantly errors. This is yet another confirmation of the tacit use of the textbook by teachers. Furthermore, the critical omissions and errors point to non-intimate teacher-textbook relationships in the study.

From the mobilisation of the approach, the analysis finds that teachers generally do not perceive and mobilise the approach of the textbook in their practice; showing preference for a more quasi-deductive approach that is not informed by the textbook. This observation is another confirmation of the relationship between teachers and textbooks that is not intimate. The results of the mobilisations of the content and the approach in this chapter lead to the conclusion that teachers' PDC in this study is generally low. The results of the analyses are discussed in detail below. From the way teachers perceive and mobilise the content (see Section 7.2.1.4), they generally reflect textbook use that has been described as tacit and seemingly non-deliberate. This is use that is characterised by a) utilising 'other' resources when the textbook is stronger; b) omitting critical components of content from the textbook which define the goals for teaching and learning the topic; and c) injecting mathematical error which in most cases could have been avoided through closer collaboration with the textbook. The tacit use of the textbook has been confirmed by the teachers' non-mobilisation of the approach of the textbook.

The implication for PDC in this case is that teachers' PDC which is a function of teachers' deliberate use of the textbook, can therefore not be high in general. It is a fact that the teachers in the study are not operating at the same level, and therefore their individual PDCs were not all the same, however, the general finding that teachers' PDC is low stands in the light of the generally tacit use of the textbook by all teachers in the study.

Teachers' PDC has implications for the teacher-textbook relationship while at the same time it is a function of the teacher-textbook relationship. Section 7.2.1 shows that the teacher-textbook relationship with respect to teachers' appropriation of the content can best be described generally as not 'intimate' and weak. A form of relationship definitely exists between the teacher and the textbook: they utilise the exercises from the textbook in more than half of the lessons; most teachers make robust injections to the content; which are signs of this relationship. However, the tacit use of the textbook together with the evident non-mobilisation of the general approach, show that this relationship lacks 'intimacy'; a 'taken-for-granted' kind of relationship.

Thus, the chapter makes three critical findings of the study, that: textbook use by teachers is generally tacit; the teacher-textbook relationship generally not intimate and weak; and consequently, teachers' PDC is generally low.

In the next chapter, the teachers' interviews are explored; firstly as a form of triangulation to confirm the conclusions drawn in the current chapter; but also to obtain the teachers' views about the use of the prescribed textbook in their practice. One of the questions emerging from this analysis of the teachers' lessons is why do teachers do what they do with their prescribed textbook in their practice, especially about the tacit use of the textbook? If PDC is the teachers' capacity to firstly perceive the affordances of the textbook before they can mobilise them, how do teachers view these affordances? This is the question the study explores in the next chapter.

CHAPTER 8
Teachers' Awareness of Textbook Affordances

> If we are regularly remiss in not teaching pedagogical knowledge to our students in teacher education programs, we are even more delinquent with respect to the third category of content knowledge, *curricular knowledge* (Shulman, 1986)

8.1 Introduction

The main objective of this present chapter is to explore teachers' responses to interview questions on their use of the prescribed textbook, so as to probe further the inference in Chapter 7, that, teachers' use of the prescribed textbook appears tacit and not deliberate, reflecting low PDCs. I draw from the interviews to illuminate teachers' awareness of the affordances of the textbook as they use the textbook and will argue that this data serves to confirm the inference made in chapter 7.

I have to declare upfront that this chapter is in no way an in-depth analysis of the interviews, but an elaboration of teachers' views on the use of the textbook in an effort to understand better their observed actions with the textbook from Chapter 7, and as an attempt towards a formulation of an explanation about why some things happened the way they did. I use the results of the interviews to describe what I perceive as teachers' awareness of both the content of the textbook and the textbook's approach as affordances for their practice.

Fig 8.1 summarises the results from chapter 5 on the analysis of the affordances of the textbook to teachers' practice, and Chapters 7, on the analyses of teachers' lessons.

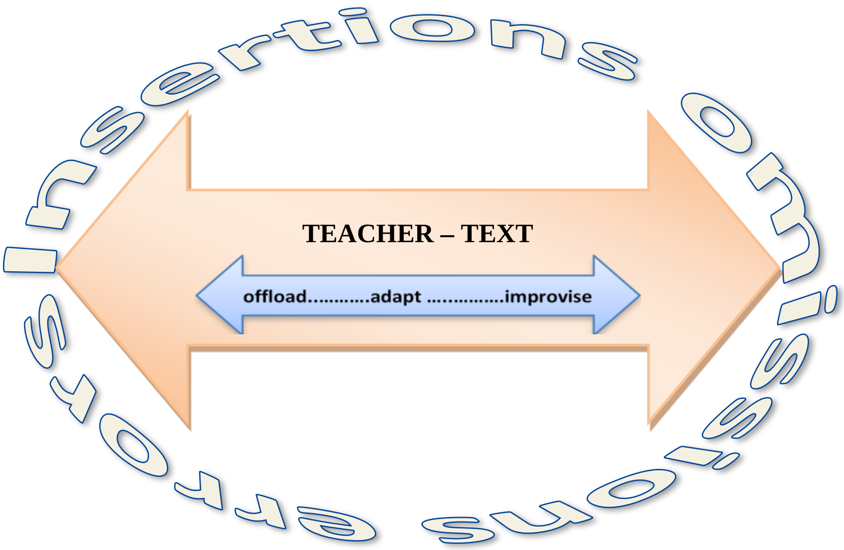


Fig 8.1 A summary of teacher-textbook interactions

The teacher-text interactions in Fig 8.1 occur within injections and omissions by teachers which reflect contradictions in the ways in which teachers interact with their textbooks. Teachers have been found to be making robust injections to the existing content that enhance the opportunities for mediation while at the same time omitting critical aspects of the object of learning in their teaching which detract from opportunities for mediation. Some teachers who used 'other' resources other than the textbook have introduced mathematical errors in the lessons, because those resources contained errors. Yet the same content was available in the textbook and error-free, while an obvious explanation for "errors" might lie in gaps in SMK, this does not seem be the full story as the textbook could be the source of the error. These contradictions in the appropriation of the content of the textbook have let to inferences of tacit use of the textbook. In the midst of all these, it was found that teachers actually improvised (bring in content from external resources) much more than they offloaded from the textbook; and that they utilised the textbook mainly for exercises and not much else. The question in this chapter is why is this so?

With respect to the appropriation of the approach the study found that teachers mobilise the approach to teaching and learning functions in so far as this conveys progression from pointwise actions on function tasks to more interpretive global actions. However, teachers do this contrary to the textbook recommendation of an 'investigative' quasi-inductive approach to the teaching and learning of the content, as they privilege the teacher 'led' quasi-deductive approach. Teachers' mobilisation of the approach of the textbook together with the way they mobilise the content have led to the study arguing that teachers do not display high levels of capabilities to craft productive and deliberate instructional episodes (PDC) using their textbooks; and inference that the teacher-text relationship between teachers and their textbooks in this study is not intimate. I suggest that teachers are not aware of the affordances of the prescribed textbook in their practice.

In the present chapter, I use examples from the interview sessions that I conducted with individual teachers to further illumine teachers' awareness of the affordances of the textbook and to explore whether the inference about teachers' low PDCs and tacit use of the textbook are confirmed or not. To achieve this objective, I interrogate teachers' responses to the individual interviews which I held with each teacher about a year after lesson observations. Some changes had happened by then in the schools which I summarise in Table 8.1 below.

Teacher	Grade 10 teaching	Grade 11 teaching	Prescribed Textbook(s)
A1	✓	ML[18]	Same
A2	✓	ML	Same
A3	✗	M	Same
A4	✗	M	Same
B1	✓	✗	Same
B2	✗	M	Same
C1	✓	✗	Changed

Table 8.1 Teacher and textbook status a year after classroom observations

[18] ML is Mathematical Literacy, intended for all learners from Grade 11 who do not take Mathematics (M) as an option

Table 8.1 above show that three of the seven participating teachers in the study were no longer teaching Grade 10 a year after classroom observations, but were all teaching Grade 11 Mathematics (M). This shows that these teachers at least have experience of the curriculum requirements for both grades. I have mentioned in previous chapters that data collection for the study occurred in an interesting period when the curriculum was changing and therefore necessitating a change of textbooks as well. A year later when I came back for the individual interviews with teachers, the CAPS textbook had become the prescribed textbook in two of the three schools in the study replacing the Pre-CAPS textbook. A reminder that these are two editions of the same textbook series. In the third school, the prescribed textbook had changed to a different series altogether. This background information is important in the light of the questions which teachers were asked in the interviews.

8.2 The Interviews

I have mentioned in Chapter 4 that the interviews were not easy to conduct: a copy of the interview protocol was given to teachers a week before the interviews so that they could prepare for the interviews. I had also requested teachers to bring with them to the interview all the other resources (especially textbooks and other curricular materials) that they would normally use for planning and teaching. However, only one teacher brought one other textbook to the interview, interestingly, this was the teacher's guide containing solutions to practice exercises only and not the learners' book which the teacher used for planning and teaching. I had brought both editions of the prescribed textbook with me, but throughout the interviews I struggled to get teachers to open the textbooks and refer to any part of it to show me more directly what they might be talking about.

My sense of the interviews was that despite the time teachers were afforded to prepare for them, teachers did not find it easy to talk about their use of textbooks. Teachers' responses to the interview questions were by and large general or superficial, and not specific either to the topic of functions or the use: their responses could have been about any textbook, any topic, or even any subject for that matter. As the analysis of the responses will show in subsequent sections, teachers spoke about the textbook affording them with examples and exercises, but they did not elaborate the nature of these examples and exercises, nor the mathematics that the textbook affords through these examples and exercises.

I have also mentioned in Chapter 3 that had I conducted the interviews after the analyses in Chapter 6 had been completed, I would have done it differently, and been more proactive in my probing in some of the responses teachers gave. This, as I have alluded to already is one limitation of the methodology in the study. However, notwithstanding this limitation, I shall show in this current chapter that the teachers' responses serve to confirm that teachers' use of their textbook was tacit, and their relationship with their textbook was not intimate

The interview protocol used with participating teachers is appended to the thesis (Appendix B) and includes three running themes: (a) the 'curriculum and textbooks'; (b) 'knowing my textbook'; and (c) a 'scenario: 'Limpopo Textbooks Saga[19]', which I describe separately below.

8.2.1 Curriculum and Textbooks

Textbooks speak to the curriculum and they are described as the potentially implemented curriculum (Valverde et al., 2002). In South Africa, they are actually grade-specific and aligned with the official teaching schedule. Hence, being conversant with the curriculum requirements for a specific grade would imply that teachers would perceive the affordances and constraints of their textbooks more readily than if they were not conversant with these requirements. In this part of the interview, teachers were requested to provide what I referred to as 'non-negotiables' for learners to proceed from Grade 10 to Grade 11 in the topic of functions; that is, the aspects of the topic which teachers deemed could not be omitted for any reason; they have to be in place for learners who transit from Grade 10 to Grade 11; together with their views on whether they felt the prescribed textbook was helpful for them in achieving their goals. In some ways, this question illuminates individual teacher's curricular knowledge (Shulman, 1986), that is, "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (p.10); and their priorities in teaching Grade 10 functions. The assumption at work was that if the teacher was not conversant with curriculum requirements, then they would not perceive any of the affordances of the textbook.

This chapter seeks to explore teachers' awareness of the affordances of the textbook, and therefore, I argue that teachers' curricular knowledge is a factor in teachers' use of the textbook; teachers need to know what they are looking for in the textbook as they use it.

8.2.2 Knowing my textbook

There were two questions in this section; one about the differences in the two editions of the prescribed textbook, which I have referred to as the Pre-CAPS and CAPS textbooks; and another on how teachers utilised the prescribed textbook and other resources in the planning of their lessons. As I have elaborated in Chapter 5, in the two textbooks the content for the topic on functions is similar; the difference is in how each deals with the different function classes, that is, the quadratic function, the exponential function, and the rectangular hyperbola function. To determine the properties of the different function classes, in the Pre-CAPS textbook a specific property is explored in each one of the function classes to determine which functions exhibit that feature; while in the CAPS textbook, each function class is investigated for different features to determine which ones the particular function possesses. Similarly for the investigation of the effects of the parameters a and q on the function $y = f(x)$ in $y = af(x) + q$.

Other pertinent differences between the two textbooks relate to the presentation formats: the Pre-CAPS textbook offers detailed explanations and notes, while the explanatory text in the

[19] The Limpopo Textbooks Saga has already been described in chapter 1 of this thesis. It is the province where the textbooks had not been delivered well into six months of the school year.

CAPS textbook is short and supplemented with note boxes and graphics where necessary. With respect to the exercises, it was notable that the CAPS textbook does not use "real life" questions, while the Pre-CAPS textbooks contains the *'Apply your skills'* exercises which depict real life situations.

For the planning of lessons, I wanted to understand the role that the prescribed textbook played versus the other resources, and so of the value that teachers place on their prescribed textbooks. For the present chapter, being able to point to similarities and differences between the two textbooks is an indication of an intimate relationship between teacher and textbook.

8.2.3 The "Limpopo Textbook Saga" Scenario

The description of this 'textbook saga' has been provided in detail in Chapter 1. The 'saga' stemmed from the department of education failing to deliver textbooks to learners in one province well into six months of the start of the school year which resulted on some NGO's taking government to court over this issue. In the interview, I used the scenario as a means to solicit teachers' views on the impact on teaching and learning of the absence of the textbook. The scenario also probed on teachers' views about the department of education's proposed 'catch-up' plan in response to the public outcry about its failure to provide the textbooks to the learners; and what this 'catch-up' could entail. Teachers' responses in this regard would be an indicator of teachers' regard for the textbook, and therefore their awareness of what the textbook affords their practice

The results of the analysis of the teachers' interviews are presented in Table 8.2 and discussed in detail in subsequent sections.

8.3 Results of the Analysis of Teachers' Interviews

Table 8.2 below depicts responses by individual teachers according to the three headings of the interview protocol as described in section 8.2 above. Under 'curriculum and textbook' are teachers' lists of non-negotiables as well as their views on if and how they perceive the textbook to be supporting their objectives with respect to the non-negotiables from Grade 10 to Grade 11. Since the non-negotiables are tied to the actual content on functions, teachers' responses shall be categorised according to the four content areas (CAs) from the textbook, namely, *introduction and terminology* (CA1), *properties of functions* (CA2), *transformation of functions* (CA3) and *interpretation of functions* (CA4).

Under 'knowing my textbook' are responses to the differences between the two versions of the prescribed textbook, the Pre-CAPS textbook and the new edition, the CAPS textbook, as I have previously referred to them, as well as how teachers use the prescribed textbook in the planning of their lessons. The 'Limpopo crisis' includes teachers' general views on textbook's support for teachers as well as for learners. The last column on general comments includes comments pertaining to the content of the textbook and its approach, which I use to draw conclusions and comment on teachers' awareness of these affordances of the textbook.

Teacher	Curriculum and textbook		Knowing my textbook		Limpopo crisis		Comments
	Teachers lists of ‘non-negotiables’	Textbook support for ‘non-negotiables’	Difference between Pre-CAPS and CAPS textbooks	Use of prescribed textbook in planning of lessons	Views on support for teacher	Views on support for learners	Awareness of content Awareness of approach
A1	Multiple representations (CA2) Interpreting functions (CA4) Understanding gradient (CA4)	exercises for learners to work on their own	not compared the textbooks	Uses other textbooks and resources in conjunction with the prescribed textbook Injections to textbook content	Consults textbook most of the time for explanations Textbook supplements teacher’s work Ready-made questions Sequencing and coverage of topic For emphasis on topics	Exercises for learners Revision After school work	Does not like solutions provided at end of textbook ‘Type’ of learners determine resources to use (teacher gets additional stuff for quick learners) talk about textbook use generalised nothing about the approach of the textbook
A3	What is a function? CA1) Sketching graphs of the four function classes using their properties (CA4) Identifying properties of functions (CA2)	Learners able to check that no aspects to be learned have been left out in lessons by teacher	CAPS textbook more user friendly than the Pre-CAPS (explanations more detailed)	Uses other textbooks as well but prefers a particular textbook which focuses on questions similar to the examination questions	Aligned to the curriculum, therefore constrains teachers to grade specifications	Provides a form of checking for learners that teacher is not skipping topics	Talk about textbook generalised All teachers need a textbook for preparing lessons: whether experienced or not Nothing about the approach of the textbook
B1	Linking functions with solving equations Different shapes of the function classes (CA2) The effect of a on the parabola (if a is negative graph faces downwards etc) (CA3) Properties of functions (CA2/4)	not enough on its own for his classroom goals (different textbooks provide different things eg complex problems than in another textbook)	CAPS has more explanations than Pre-CAPS textbook More mistakes in CAPS textbook than Pre-CAPS Different method of teaching for CAPS textbook (in maths x is x, but methods of teaching change)	Uses about three different textbooks for planning Prefers prescribed textbook to others because of its large collection of activities Prefers different examples in class than the ones in the textbook so that learners have different examples to work with	Provides ready-made questions for teacher which have been tested Saves teacher on time	Activities for practice	Talk less generalised but still not specific Learners are different and depending on the learners decides which textbook(s) to use Some indication about the approach
C1	Identify different types of graphs (CA2) Know features of the functions and be able to	Exercises more user friendly in CAPS Has got lots of exercises	CAPS has a different approach to teaching from the Pre-CAPS textbook		Uses different textbooks/resources for planning her lessons utilises past examination	Learners know what they need to learn and what follows next (self-	Prescribed textbook changed Talk about textbook generalised

Teacher	Curriculum and textbook		Knowing my textbook		Limpopo crisis		Comments
	Teachers lists of ‘non-negotiables’	Textbook support for ‘non-negotiables’	Difference between Pre-CAPS and CAPS textbooks	Use of prescribed textbook in planning of lessons	Views on support for teacher	Views on support for learners	Awareness of content Awareness of approach
	differentiate them (CA2) Be able to interpret the graph (CA4)		CAPS is aligned with the work schedule		papers to expose learners to the structure of questioning in examinations textbook aligns with work schedule (coverage and pacing of syllabus) Saves time for teacher	learning)	Textbook lacks structure of questioning for Grade 12 examinations Some indication of the approach
A2	relationship between inputs and outputs (CA1) function notation and evaluating function values (CA1) Graphical representation of functions (CA1) Obtain an equation from its graph (moving back) (CA4)	“it is all we have… it can be enhanced. It is sufficient, but it can be better”	not sat with two textbooks to check their differences believes that a new edition must be an improvement on the old one	Uses other textbooks and workshop materials from WMCS	Textbook is important, but everything starts with the teacher	Use at home	talk about textbook use generalised Believes in the authority of the teacher in the classroom Nothing on the approach
A4	Inputs and outputs (CA1) Completing a table of values and drawing a graph (CA1) How to interpret a function by getting output from input (CA1)		CAPS explains steps for answering questions more than the Pre-CAPS textbook More worked examples in CAPS than in Pre-CAPS	Uses about three different textbooks and teacher guides for planning her lessons (depended on how quick or slow learners are)	Teacher is the source in the classroom and the textbook is only the surplus	Self- learning without teacher (if learners had textbooks, there would not be this low pass rate)	Believes in the authority of teacher in classroom talk about textbook use generalised Her goal is to prepare learners for Grade 12 examinations already from Grade 10 Nothing on the approach
B2	Understand the Cartesian plane (GR 9) Be able to draw graphs of functions (CA1) Find the equation and the distance of the two functions (eg parabola and straight line) (GR?)	Prescribed textbook would not be enough on its own: because does not elaborate on finding the distance between two points Prescribed textbook not aligned to curriculum Has lots of activities but not explanations		Uses four different textbooks for planning lessons	Mentions a specific textbook she prefers because it elaborates, and provides explanations, while the prescribed textbook does not do so Prescribed textbook assumes teacher is good in mathematics because it does not provide explanations		Has major misgivings towards the prescribed textbook, that it has been there for just too long Talk generalised but a bit more specific Believes that all textbooks are the same

Table 8.2 Teachers’ Views on Textbook Use

Three common threads emerging from Table 8.2 include: a) the generalised way in which teachers talk about their use of the textbook, and not only the prescribed textbook; b) the absence of talk that identified similarities and differences between the two editions, the Pre-CAPS and the CAPS textbooks; and c) two distinct groups into which the teachers fell with respect to aspects of function which teachers regard as absolute non-negotiables for learners' transition from Grade 10 to Grad 11. I discuss each one of these three threads separately below.

8.3.1 Teachers' Generalised Talk about the Textbook

All the seven teachers in Table 8.2 above talk in a generalised way about their textbooks and use of them and they did not talk specifically to the teaching of functions. Teacher B2, whom I suggest is more specific than others in her talk about the textbook (though still rather general), responds thus to a question about whether she found the prescribed textbook supportive in delivering her list of non-negotiables.

Mon [20]	Ok thank you very much. Then the other thing, on the textbook itself with the work that you are doing at grade eleven and what you were doing at grade ten last year, because you are saying this year you are not teaching grade ten. Now the grade ten textbook which we used last year was the NCS classroom maths. Were you finding it useful for your needs in terms of the things that you want students to know when they get to grade eleven?
B2	Due for the different types of textbooks we are using, ja they are useful, though some of the books they can't indicate to a child how to find the distance but because of the materials we are using, ja they are all useful from the grade ten to eleven. They do assist in that regard.

From what B2 says above, it is not possible to say whether or not the prescribed textbook is among those that do not indicate how to find the distance between two points. Other examples of generalised talk are discussed below:

For teacher C1, the CAPS textbook in her view is more supportive than the Pre-CAPS textbook: This is the teacher in whose school the prescribed textbook has changed to a new textbook series on the inception of the new CAPS curriculum.

C1	Ja.. but I've seen, I've gone through it, it's also user friendly... and I think this one [*CAPS textbook*[21]] it's more clearer than ... than this one [*Pre-CAPS textbook*] the third one..ja
Mon	Oh ok, so the new one is clearer than the old one?
C1	Ja the new one [*CAPS textbook]* I've seen, it's more clearer
Mon	How do you compare *the other textbook series* with this old one [*Pre-CAPS textbook]*
C1	Um... but what I have seen actually we should have ordered more of the *CAPS textbook*, this new textbook.
Mon	Than the *other series*?
C1	Than the *other series*
Mon	Ok?
C1	Because this new textbook [*CAPS textbook*] uh when I paged through I have seen that it is more important than the *other series*, all the examples uh, all the exercises that are there, ja, they are more user friendly

[20] Mon is the researcher
[21] *Italics* used to obscure the names of the other textbooks that C1 mentions in the extract

C1 uses expressions such as 'user friendly', 'more clearer', and 'more important', to express how she views her prescribed textbook; expressions which do not illuminate what the textbook affords her practice.

From teacher B1 on the question of how she utilises the different textbooks when planning her lessons: She had just responded that she usually utilises different textbooks when planning her lesson. She named some of the books, including the prescribed textbook; and commented that she found the major difference between the textbooks as being the way they introduce the lessons:

Mon	Yes, so which one do you prefer when you come to introducing your lesson?
B1	The books, normally I don't really say which book, uh depending on
Mon	You would go for what you are looking for?
B1	Ja, depending on the learners that I am having. Ja, because sometimes the learners are different. Sometimes you have got bright learners, slow learners and that. So sometime if you have got brighter learners you take the book, the book eh cover to master the slow learners or cover too much the brighter learners. Sometime you need to think as well as look at the book. You can think as what you say 'eish maar'[dialect] this topic how can I approach it? The textbook is in this way I can, I think this way it can help them to understand, ja.

The statement above says more about the learners than about the textbooks the teacher uses, but does point to how topics are approached

iii) From teacher A1 on if she found the prescribed textbook supportive in her practice:

A1	Yes I, I think [the prescribed textbook] it's helping ja especially the exercises like when you teach them you give them to work on their own, I think it's helping yes

And finally,

iv) From teacher A2 on the support the prescribed textbook offers to her and her learners

Mon	And it is the recommended textbook for the students here? Now how does it help, do you think this book helps you in terms of these key things that you want the students to learn? Do you find it sufficient, is it really helping you?
A2	Eh ma'am, it is all we have, if I can put it that way. Maybe there are better books out there but this is all we have and we've studied in different areas.[*goes on to explain how teachers may have not studied some topics like geometry at school but still have to teach it*]. So, and also given like for instance me, I studied long time ago some of the things; it depends where you are in life. I worked in a different sector whereby maths was not really a focus. Now I came back it is a focus and some of the things I have to re-teach myself as well. So I think it would be better if, even if it's just for teachers to have as much material that they can work on than just using the textbook. Going to WITS[22] I've learnt about websites that we can use to get more information and it makes you realise that there are easier ways of doing things compared to this one bible that you are holding to class every day. So it can be enhanced. It is sufficient, but it can be better

[22] WITS is an acronym for the University of the Witwatersrand. Some of the workshops for teachers participating in the WMCS project were held at the university, hence A2 mentioning going to WITS

Teacher A2 certainly expressed her view that there could be more to enhance the textbook so that it could serve her objectives and other teachers' objectives more adequately. However, she did not specify which aspects she found lacking or what she would like to see enhanced; hence another indication of what I call generalised talk about the textbook.

8.3.2 Teachers' Views on the Similarities and Differences between the Pre-CAPS and the CAPS textbooks

Two teachers confessed that they had not taken time to study these differences. One of these two teachers stated a view that a new textbook, or more specifically, a new edition of a textbook, must be better than an earlier edition. As a result she does not see a need to compare the different editions. In her own words, she says:

A2	…Um! I've found that the academics who sat and planned the new textbook, I would like to believe that they, they did thoroughly search to see what is it that they can introduce that will make this topic more easier for learners to understand. That's why I come in with that belief that this makes it easier, it explains
…	…
Mon	So you'll use one that is there?
A2	Yes
Mon	So this one comes, I use it?
A2	I use that one, yes

For other teachers, they saw the differences with respect to detailed explanations (A3) in the CAPS textbook; or more detailed steps in working out examples (A4); or more mistakes in the CAPS textbook than the Pre-CAPS textbook (B1). However, more importantly was the persistent nature of teachers' talk that was once more generalised and not particular. For example, teachers did not mention where the detailed explanations were; nor did they provide details of the particular content that contained the mistakes. However, two teachers, namely, B2 and C1 did recognise that there were differences in the approach to teaching between the two textbooks. I return to this below as it points to teachers' awareness of the approach as an affordance in their practice.

Thus, with respect to the differences between the two editions of the prescribed textbook, it was either that teachers admitted they had not done the comparison; or for those who had done the comparison, that they articulated them in very general terms. This observation is critical for the study because it provides a confirmation of the non-deliberate use of the textbook and a teacher-textbook relationship that is not intimate as concluded in the previous chapter. In other words, notwithstanding limitations and the setting of the interviews, this observation strengthens the claim that teachers' textbook use is tacit. It is noteworthy that teachers could not articulate the differences between the two textbooks, which include the most basic of actions that teachers perform in teaching properties of functions and dealing with each function separately. However, what this result strongly suggests is that the approach of the textbook in not on the radar when teachers use the textbooks; that thinking about the approach of the textbook as an affordance is not as obvious for teachers as thinking about the exercises, for example.

8.3.3 Teachers' Views on the Non-negotiable Aspects of Teaching Grade 10 Functions

With respect to the non-negotiable aspects of functions from Grade 10 to Grade 11 from Table 8.2 above, teachers' responses were categorised into two groups as shown in Table 8.3 below.

Group	Teachers	Content areas emphasised			
I	A1		CA2		CA4
	A3	CA1	CA2		
	B1		CA2	CA3	CA4
	C1		CA2		CA4
	B2	CA1			CA4
II	A2	CA1			CA4?
	A4	CA1			

Table 8.3 Grouping teachers' responses to non-negotiables

Five teachers make up the first group whose responses include two or more content areas from the textbook/curriculum statement. Four of the teachers in this group mentioned CA2 on determining properties of functions as a non-negotiable aspect of Grade 10 functions. Furthermore, four of the five teachers also mentioned elements of CA4 that is, using the properties of different function classes to interpret their graphs and for problem solving, as non-negotiable. One teacher, B1, included CA3 on transformations of functions in her list.

The second group comprising two teachers emphasised elements of CA1, mainly on point-by-point plotting of functions, as non-negotiables from Grade 10 to Grade 11. The question mark (?) near CA4 for teacher A2 indicates that while she included determining the equation of a function from its graph which is an aspect from CA4 in her list, she did not view this (determining the equation of a function from its graph) as part of a larger process of interpreting function properties, but as an isolated case of something that learners needed to know, as the excerpt from her interview below shows:

A2	Ok, um the one thing that I think um that they should know, they should know the relationship between, we will start with the inputs and the outputs... They should know how to do these two if, how do they relate when we are talking about the input and the output. Eh, they should also know what does that, when we say the function of, what does that mean? What is inside the bracket? So if they understand what does that mean this is a function of this, if it's one number this a function of two. If it's an expression, this is a function of x plus two, um, they should also understand the relationship. Like with functions we also use them with graphs, to also understand if they are looking at a certain function already have a graphical representation of what this looks like. And again looking at graphs because what I've learnt is that at first I didn't, I couldn't move from graph to equations myself. I only knew one direction. So learning that the moving back as well, looking at the graph and being able to use the graph to determine the function of that graph, that also helps because it makes a lot of things that are being added to be easier for me if you understand these basics.

From the excerpt above, A2 advanced the process of 'moving back' as something learners needed to learn because she did not know it before and had found it useful, which is different from how teachers in the first group described the aspects from CA4, as a curriculum requirement. For example, the following excerpt shows how teacher C1, who is in the first group, described elements of CA4 which learners must be conversant with in order to proceed to Grade 11 in the topic of functions. The part pertaining to CA4 is underlined.

C1	Ja I think learners should able to identify the different types of graphs of functions because what I've seen if they cannot identify them, then they cannot be able to go with the ...**inaudible.**.. and then they should able to know their features and then they should be able also to differentiate them in terms of saying if the feature of this one is like this and the feature of the quadratic function it's like this, the feature of the exponential function it's like this – in that manner. Um, and <u>the other part that they should be able to identify, they should be able to read through the graphs because that is the most important part where they should be able to interpret the graph, able to understand how a graph it looks like, how they can answer such types of question when they appear, because I've seen it's a little bit of a challenge to them. So but I think this is non-negotiable. If they can able to interpret graphs it's more important</u>...ummmm!

As a reminder, the curriculum statement for Grade 10 functions stipulates that learners should be able to

> discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable)…investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$ ….sketch graphs, find the equation of given graphs and interpret graphs (CAPS, 2011, p. 24)

Looking at the two groups of teachers above, it is evident that the first group reflects views on non-negotiable aspects of Grade 10 functions which are closer to the curriculum statement, than the second group. The second group emphasises aspects which are introductory and which would not advance learners beyond point-by-point plotting of graphs of functions. Thus, the categorisation highlights what each teacher or group of teachers considers as important and what then they would emphasise in their teaching. This has implications for the utilisation of the textbook: I would expect that teachers in the first group would be looking for content and strategies that would support the investigation of features of the functions as well as for supporting the other aspects which they consider as non-negotiable.

The analyses of the interviews reported in this section reinforce inferences of generally tacit use of textbooks by teachers and teacher-textbook relationships that are not intimate. In the next section, the study explores the teachers' awareness of each affordance (the content and approach) separately, using the results of the interviews for the five teachers in the first group namely, teachers A1, A3, B1, B2 and C1.

8.4 Awareness of Content as an Affordance to Teachers' Practice

In the interviews of teachers A1, A3, B1, B2, and C1, there were significant aspects with respect to the content of the textbook that teachers talked about: These included i) talk about exercises, examples and activities in the textbook; ii) talk about the explanations and definitions; iii) the sequencing and pacing of content in the textbook; iv) the style of questioning in the textbook. Other illuminating aspects of the teacher-textbook relationship included Teacher A1 explaining why she made injections of content not required by curriculum at grade 10, while teacher B1, pointed out that there were mistakes in the CAPS textbook; and teacher B2 perceiving the textbook as having more constraints than affordances.

8.4.1 Examples, Exercises and Activities

With respect to the content of the prescribed textbook, as would expected, teachers talked more about examples, exercises, and activities for learners either in class, or for homework and self-study. The key feature for the prescribed textbook among teachers was the fact that it had lots of activities or exercises for learners, unlike in the other textbooks. For example, teacher B1 explained why she would recommend the prescribed textbook over other textbooks:

Mon	What’s better do you think with this prescribed textbook than the others which are not recommended? If you were the one recommending the different textbooks that you were using would you have chosen this textbook?
B1	Yes
Mon	As the, as the prescribed one, among the others that you, you have used?
B1	Yes.
Mon	Why?
B1	The reason that I indicated they have got a lot of activity, ja, for learners to practice
Mon	For learners, ok
B1	Ja because sometimes a textbook like, I won’t mention the name of that book, you find they've got five they've done one example, some of the learners didn’t understand the example. You have to take out of five, you take maybe two, you are left with three. They are going to do the three. So this one have got a lot of, even if you take the first two as an example, the remaining, you still have a lot

For some teachers, the number of activities and exercises featured in the textbook made the prescribed textbook the preferred textbook as the following examples show.

Mon	So in your opinion how does this book [*prescribed textbook*], help you achieve this thing [*her objectives*] for yourself and maybe for the students?
C1	Ja it’s user friendly because it has got some examples on it, though sometimes some of the stuff it looks like it’s more complicated. But it’s user friendly and it has got lot of exercises that could help them to understand. That’s why we love this textbook.

Teacher C1 whose school changed the prescribed to another textbook series in the wake of the inception of the CAPS curriculum opined that her school should have obtained more copies of the CAPS textbook than of the new textbook series:

Mon	How do you compare *the other textbook series* with this old one [*Pre-CAPS textbook]*
C1	Um... but what I have seen actually we should have ordered more of the *CAPS textbook*, this new textbook.
Mon	Than the *other series*?
C1	Than the *other series*
Mon	Ok?
C1	Because this new textbook [*CAPS textbook*] uh when I paged through I have seen that it is more important than the *other series*, all the examples uh, all the exercises that are there, ja, they are more user friendly

Similarly, for teacher B1, the prescribed textbook became the textbook of choice because of its large collection of activities. While she admitted that she used different textbooks for choosing the exercises and examples, her preferred textbook was the prescribed textbook because of its large collection of activities:

Mon	Ok, yes, so if we talk about those books and the prescribed textbook, especially the prescribed textbook, is it helping you to achieve the goals for the students? Or as you say you use many different books, so if you are going to be planning your lesson it's really just from the classroom maths but using many textbooks?
B1	Ja I use different textbooks, especially if I want to choose eh examples
Mon	Yes?
B1	Ja they come from different textbooks
Mon	Ok?
B1	These examples is little bit is different than the other examples
Mon	Than the other ones yes?
B1	Ja I use even the exercise, but the *prescribed textbook* one have got a lot of activities ja
Mon	Okay?
B1	The *prescribed textbook* has a lot of those, that's why I prefer that book. The other textbooks have got a few, they have got ten exercise, some have got five, this one have got a lot
Mon	Okay, so that's why you prefer it
B1	Ja for learners because if they…for practice purposes they use the book to prepare to practice more and more with a lot of eh activities in the book

However, not all teachers regarded the prescribed textbook as their preferred textbook as teacher A3 pointed out below:

Mon	Unfortunately this part is where I had asked you to bring all the books that you would use but we can talk through that. So what I really would like to say because my work is to understand what the teachers do when they have a textbook and they use this textbook for planning and for teaching. So if you were to have these textbooks, the *Pre-CAPS* and the *CAPS textbooks*, now - if you were to have these textbooks and all other textbooks that you are using; what other textbooks would you use more? Uh, on top of the *prescribed textbooks* which ones do you normally use?
A3	There's another book by, I don't know one of these professors, [*mentions a name*] is one of the authors
Mon	Ok!
A3	It's... Ja! Ja! The information that is there, it is excellent to be honest. A learner can study on his or her own without the help of the teacher and still do well
Mon	Ok! So this works for the learner. How about for yourself? If you have to compare it with classroom maths?
A3	Ja! It is better than the *prescribed textbook(s)*. Yes! Because they are concentrating much on the questions that you would get from the examination
Mon	Ok!
A3	Ja! So the questions are how to answer the questions. So it becomes easy and it is also helpful to the new teachers who are not very clear on the subject because they can see question and the answer, ja, how to answer those questions

Teacher B2 shared the sentiments of teacher A3 about the prescribed textbook not being her preferred textbook. However, as Table 8.3 shows, teacher B2 did not have a good word for the prescribed textbook, showing her preference for another textbook series. Her major issue with the prescribed textbook concerned what she called lack of explanations from the textbook; that while it had lots of activities and exercises, it did not explain the concepts. However, B2 was not able to show me what she meant because she had only brought the teacher's guide to the interview site and not the learner book she used for planning and teaching.

The two teachers, A3 and B2 highlighted an important factor about their use of the textbook. They outlined their perceived constraints of the textbook; which hindered them from utilising the textbook. As Norman (1988) points out, the affordances of a tool, and hence their constraints, need not be real; they can also be perceived affordances/constraints. What is important is that these perceived constraints have implications for textbook use. B1 had also observed that the CAPS textbook came with many mistakes so she had to check all exercises carefully before assigning them, which was also another constraint of the textbook.

Another significant observation from teacher A3's non-preference of the textbook was that A3 was also the head of the mathematics department in her school. From what she was saying, it sounded like someone else had control over what textbook was recommended as the prescribed and not the head of the department. In that case then, if the choice was made by members who were not from the mathematics department, the decision would not be about the quality of the mathematics in the textbooks. However, this notwithstanding; from the teachers' responses, it did not look as if even if it were them making such a decision, it would be based on the mathematics the textbooks afford, but on the general aspects such as the number of exercises; orientation towards examination, and so forth, and not on the ideas and mathematical message the textbook advances. Teachers also mentioned the fact that the exercises in the textbook were ready-made and tested, and so they did not have to generate their own, and this saved them time.

What teachers say is well and good and definitely confirms that exercises and activities are the most obvious or prominent affordances to the teacher's practice from the textbook. However, what emerges is that their preference of the prescribed textbook is described in terms of the quantity of exercises and activities, and not in terms of the quality of mathematics the many exercises afford with regards to teachers' objectives. In the analysis of the textbook affordances in Chapter 5, I pointed out the performance expectations or actions on function tasks expected from learners. Teachers did not get to this kind of depth in their consideration of the use of their textbooks. I showed how these panned out in teachers' lessons in chapter 6 and 7, and how teachers' performance expectations of learners differed from those of the textbook. Why is this? Could it be that these are things that are not on teachers' radar? Could it be that these are not part of the social constructions around textbook use? Hence my argument that indepth analysis of the textbook affordances is not a spontaneous or natural act for teachers, and there needs to be a way to bring them to teachers' immediate awareness.

8.4.2 Explanations and Definitions

Teachers confirm that they consulted the textbook for explanations and definitions, and some teachers felt that the CAPS textbook had more explanations and better still, more detailed explanations than the Pre-CAPS textbook. These came from teachers A3 and B1.

Mon	So, I'm going to go further now into the textbooks where, for now at least, you are saying you are no longer teaching grade ten, but now we have the new textbook of grade ten. Have you had a chance to look at it?
A3	Yes
Mon	The grade ten and how, what do you feel about it comparing it to the last year's one?
A3	Ja! I think this one [*CAPS textbook*] is much better than the first one [*Pre-CAPS textbook*] because is in especially when you looking at the functions, the explanations is more detailed than in the previous one. Ja it's user friendly to both teachers and the learners, that's what I've seen... ja

Similar sentiments were echoed by teacher B1

Mon	Can you say if you thought of the differences between them, between the new one [*CAPS textbook*] and the old one [*pre-CAPS textbook*] for this year when you used this? Where did you notice any differences, eh from this one compared to this old one? Because this is the old one from last year
A3	Ja this one [*CAPS textbook*] have got a lot of explanation ja and examples and a different way of doing it

For C1, the CAPS textbook was 'more broader' than the other textbooks with regard to definitions. In the excerpt below C1 was responding to the question about how she chose which textbook to use when planning her lessons

C1	So I check from the different textbooks, and see how do they cover some definitions. Some they don't do definitions, some they do define. So I just take from there and there *[meaning different textbooks]* and bring them together.
Mon	How do you feel about this one here [*CAPS textbook*]? How do you feel about this one here in terms of definitions and explanations?
C1	Mmm, that's why I said if we should have seen it from the beginning, we should have uh bought more of the *prescribed textbook*
Mon	Mmm
C1	Because then we only saw from the *newly prescribed textbook* that it's having more of the exercises, And without seeing the work coverage as to how is it defining, how is it outlining
Mon	Ok
C1	But as I've gone through the *CAPS textbook* I have seen that it is more broader than the *newly prescribed textbook*, ja...

These are the utterances from teachers about explanations from the textbook. I do not wish to debate whether teachers are right or not about the explanations in the CAPS textbook being more detailed than those in the Pre-CAPS textbook in the light of the analysis in Chapter 5, in which I show that the Pre-CAPS textbook contains more detailed and elaborate notes than the CAPS textbook. The point I wish to reiterate is that teachers' talk is generalised, and as I have expressed previously, teachers do not articulate what the nature of the explanations are; what is being 'more broader' with explanations in the CAPS textbook as C1 observes than less broad in other textbooks?

8.4.3 Sequencing, Coverage and Pacing of Content

The textbook affords teachers' practice with sequencing and pacing of content and one of the reasons stipulated by teachers was that this was the case because the textbook was aligned to the official work schedule. Teacher C1 was the most elaborate of all teachers on this issue. She saw this is an affordance of the CAPS textbook and all new textbooks under the new

CAPS curriculum as she believed that the textbooks under the previous curriculum were not aligned to the official work schedule.

Mon	One of the questions I was going to ask was about… um, that's why I had asked you if you could bring the textbooks, to take me through the planning of your lesson... It's for example... it's time to be teaching functions again this year, so what do you do? So can you take me through how, what would you feel are important for you to use? What would you do? How does this one feature, the *prescribed textbook*, how does it feature in the work that you do?
C1	Ok when I do the planning, I check from the work schedule. I check what is it that I have to cover and then I try to check from different textbooks. Unfortunately because these textbooks, the *CAPS textbook*, and the *other textbook series* and all those other new books that come with the CAPS [*new curriculum*] they follow what is on the work schedule. … It's not the same with the NCS [*old curriculum*] documents whereby you have to check the other pages...these follow, they follow nicely.

Commenting about the Limpopo textbook saga on the non-deliverance of the textbooks and what this meant to her with respect to teaching and learning of mathematics, C1 commented thus:

C1	Without textbooks, it is difficult for and especially because of the work schedule they do have to do. We do have in most cases when they [*officials from the department of* education] come they want to see us covering much of the work. So if we don't have textbooks we tend to cover the work late because most of the learners can't get exercises from anywhere because the textbook in most cases is the one that covers, and then, even if we do have different kind of materials like... questions papers and like they won't, they won't cover to that extent
Mon	Mmm ...
C1	So the textbook is very much important, the learner will not be in a position to know what is next. If they have textbooks they know some of the most intelligent learners they go ahead of the teachers eh. Sometimes when you come into the class and start teaching or introducing the lesson they already know because they have covered if they have textbooks. So I see this as a challenge because of the grade nines we don't have textbooks. All of the learners they don't have textbooks so it's uh very big challenge. That's what we are seeing. We don't cover the work schedule on time cause learners they don't know what to do, they rely solely on us as educators, so it's more of a challenge if textbooks are not there.

It is interesting to note that while teacher C1 and other teacher felt that the prescribed textbook aligned with the official work schedule from the department of education, teacher B2 did not share these views. In the excerpt below I asked her about the differences between the Pre-CAPS and the CAPS textbooks:

Mo	…how is it [*CAPS textbook]* comparable to the old one [*Pre-CAPS textbook]*?
M	I do check these books. The difference is that some of the topics are out and somewhere with the work schedule the *prescribed textbook* is not retained …… It's not on par with the given work schedule and somewhere again in the *prescribed textbook* there are some topics of which they are not being elaborated clearly hence I'm using those different books

It is noteworthy too to mention that most of the teachers including teacher B2 mobilised the coverage and sequencing of the textbook as an affordance in their lessons in chapter 7.

The last question I wish to explore on teachers' awareness of the content concerns teachers' choice of the textbook for use in the planning of the lessons.

8.4.4 The Choice of Curricular Resources in Teachers' Practice

All teachers confirmed that they used different textbooks and resources for planning their lessons. So, what is the basis of teachers' choices? And why is this significant in this chapter? I have shown in previous sections that teacher A3's preferred choice of a prescribed textbook would be one which is oriented towards examinations. She was not alone in this; while C1 did not go that far about being making examination-type questions the main factor in her choice of textbook, she nonetheless pointed out that the lack of examination type questioning in the prescribed textbook was a factor in the decision about what other resources she used:

C1	So but I think it would work hand in hand with the textbook if we had worksheets and we would be able to see the structure of questioning . So but we cannot rely solely on the textbook because some of the questions that they ask you see from grade twelve question paper they are far from what they are asking everyone in the textbook. In most cases some of the learners they ask us, they say: "why sir, when you give us work from the textbook it seems easier, but when now we have to write in common exams it becomes more difficult"? So I tell them sometimes I say the textbook is the one that guides you towards answering the question paper and then we give you some work sheets and so that you can see how do they ask you in an exam. So you need to combine the textbook and some of the worksheets that I give you

Teacher A1 pointed to learners being influential in the decision about injecting content not required by the curriculum at Grade 10:

Mon	Yes so you bring in the horizontal shifts which I thought were not in the, in the syllabus for grade 10, would you care to say why?
A1	Ok eh I, I think I sometimes the learners are different. Sometimes you use a certain approach to, or a certain method to teach them. Some do not understand and just because it's in grade ten doesn't mean that they won't understand what you are trying to. So like last year I was having grade ten A, they were very intelligent, quick eh learners, so they could understand some of the concepts even if they were going to do them in grade eleven. So basically that's what I was trying to, so that I was trying to do to give them to them many options. Yes some of them might be having problems with the plotting of points, finding the y value. There are some who struggle with that, but if they've got an idea to say ok if my graph is like this, my function is like this, it means the graph is going to be this way, yes.

In other words, her perception of whether a particular group of learners in a class was bright or slow would influence her decision for injecting content into the lessons. For teacher C1, it was just a matter of preference, depending also on how many exercises she could keep in her head to reproduce in class.

Mon	Ok, um what else? What I saw as I was seeing in your videos, the two videos that we had and among all these seven teachers that I'm working with, you were the only one that I actually took a lesson today and then the following day. So it, it was a continuation which I looked at. So what I saw was that you were not using any examples at that time from the textbook. I didn't see any examples from this textbook [*prescribed textbook*], even as you were teaching. I didn't see you referring to a textbook at all in the classroom. How do you normally work with textbook especially in terms of in the classroom?
B1	Mmm, normally they got one, it's a simple one here…Ja the graphs are simple ones. The equation I normally use the different textbook when I prepare the class. I say oh when I use this example maybe I've taken some example from another textbook or other sources I use. When I use this is going to be

	easier for them or maybe according to me. It's easy for me to memorise this, like how many eh five or six eh equations that you need to use. Unlike the other thing that you, you, six equations in my head is this, and I gave them this exercise is going to be easier for them, ja that's how I normally do it

As she said, her preference was to use different examples from what was contained in the prescribed textbook so that learners had a mix of different questions to work with. The choice was clearly not about the mathematics these examples afforded but about the quantity of questions. In Chapter 7 on the analyses of the lessons, teachers were found to be improvising content more than they were offloading from the prescribed textbook. As I have explained in that chapter, *improvising* content and *injecting* content are two different processes: improvising entails bringing in content that is similar to what the prescribed textbook also contains or features, while injections are the aspects of content not required by the curriculum at the particular grade but which teachers bring in especially to enhance the object of learning, the *robust injections.*

What is clear from this section is that teachers' choices to improvise or inject content are not linked to teachers' perception of the affordances of the textbook, but on other factors external to what the textbook can afford teachers' practice. This is thus another indication of a tacit use of the textbook among the teachers in the study.

8.5 Awareness of the Approach as an Affordance in Teachers' Practice

With respect to the approach of the textbook to the teaching and learning of content only two teachers, namely, B1 and C1, indicated that the CAPS textbook had a different way of teaching from the Pre-CAPS textbook. There were utterances from these teachers in the interviews which made me believe that teachers were talking about the approach of the textbook to teaching and learning of content. The other three teachers did not talk about the approach.

When responding to the question about the media reports on that in the 'Limpopo textbook saga', the Minister of Basic Education had commented that she did not understand why teachers did not use the previous years' textbooks if the current textbooks had not been delivered yet, this is what teacher C1 had to say:

Mon	So the idea that the minister said 'well I don't see what the big issue is, why don't they use last year's textbooks?'
C1	Yes! Like as I said, last year's textbooks for grade ten perhaps I don't think we can use it because some of the stuff are already changed. Ja, some of them when we go for CAPS training they are telling us that we have to teach like this and some of which on the NCS [old curriculum], sometimes you find that it's not there…It's not covered there, but with the new textbook you find that everything is covered. So if we still using the NCS, the older textbook, somewhere somehow we won't cover a whole lot of the whole work

The second teacher who talked about the approach is B1. Again, the utterance is subtle but I also believe that it signalled recognition of the change in the approach from the Pre-CAPS to the CAPS textbook. The utterance was a response to the question about the differences between the two textbooks.

Mon	How, can you say if you, you thought of the differences between them, between the new one and the old one for this year when you used this? Where did you notice any differences, eh from this one [*CAPS textbook*] compared to this old one [*Pre-CAPS textbook*]? Because this is the old one from last year
B1	Ja this one [*CAPS textbook*] have got a lot of explanation ja and the example and a different way of doing things
Mon	The CAPS one ?
B1	The CAPS one, ja
Mon	Ok, ok

While I am convinced that the two teachers were referring to the approach to the teaching and learning of the content in their utterances, I am not able to substantiate this claim with further evidence from the interviews.

This is significant because it highlights some of the important findings in the study about the approach of the textbook as an affordance to teachers' practice. Firstly, it is another confirmation of the teacher-textbook relationship that is not intimate, where there is no indepth consideration of what the textbook has to offer the teacher in terms of the approach, especially with conspicuous differences in the approach between the Pre-CAPS and the CAPS textbooks as discussed in Chapter 5. One of these differences is one that is quite close to teachers: while the strategy for determining properties of functions in the Pre-CAPS textbook involved keeping a specific property invariant and varying the function classes, the CAPS textbook instead, keeps the function itself constant and varies the properties. The latter strategy is what most teachers do and one would have expected that that would have been one of the first differences in approach which teachers would notice.

Secondly, most teachers when selecting exercises from the textbook would normally pick and choose them because the textbooks usually provides for that possibility. However, this is not the case in the CAPS textbook, again on determining properties of functions; the activities are designed such that one activity follows from the one before it, hence taking away the 'picking and choosing' latitude.

With respect to the approach to the teaching and learning of content as an affordance to the teachers' practice hence, my conclusion is that it is not in the conscious awareness of the teachers in this study. This confirms the conclusion from Chapter 7 on the analyses of the lessons where it was found that there has been very little adoption of the approach of the textbook in the teachers' lessons where in only 35% of the lessons, there was a perfect match in the approach of the textbook and that of the teacher in the classroom; and only in the quasi-deductive approach.

8.6 Conclusions

I set out in this chapter to explore further the conclusions from chapter 7 that teachers' use of the prescribed textbook comes across generally as tacit, and not in teachers' conscious awareness. The analysis of teachers' responses in their interviews provides further support that teachers are generally clear about what they need to teach to learners in preparation for grade 11 with respect to the topic of functions. I would go further and say that their curricular knowledge (Shulman, 1986) is in place. However, they do not appear to be consciously aware of the affordances of the prescribed textbooks and tools offered to deliver their specific classroom objectives.

a) On teachers' awareness of the content
Teachers are aware of the aspects of content: exercises, sequencing, explanations and definitions; but generally not as affordances in their practice: that is, not in terms of the authors' intentions; not in terms of the nature of the mathematics these afford to the teacher with respect to the structure of the textbook; and mostly not in the ideas conveyed by this content with respect to functions. Teachers' talk about the content is highly generalised. And as I have pointed out in the introduction if this chapter, teachers' talk could have been about any textbook, and any topic, and in many cases even on any subject for that matter.

b) On teachers' awareness of the approach
With respect to the approach of the textbook, there is little evidence that teachers are aware of the basic features of the approach of the textbook. Two teachers hint at the different ways of doing things, and that is the extent of the awareness of the approach of the textbook as an affordance to the teachers in the study. In other words, the awareness in respect of the approach appears minimal.

Could this mean that teachers do not find the textbook supportive or helpful in their practice? The answer is most certainly not. All teachers pointed to the importance of the textbook and all the advantages of having the textbooks, and not only for themselves, but especially for the learners. What this chapter illuminates is that, the affordances of the textbook are much more than a list of aspects and components of the textbook that teachers may utilise; mobilisation requires perceiving the structure of the textbook in its entirety, and delineating the mathematical ideas advocated in the different presentations of the content and in the approach that the author(s) advance. This is not to suggest that teachers should give up their right to select the materials or resources that they want in order to deliver their lessons; but that their decisions could be informed by a deliberate process that involves a full engagement of the affordances and constraints of the textbook.

In chapter 4 on methodology, I pointed to how difficult it was to conduct the interviews with teachers: despite requesting them to bring the textbooks which they use to the interview site, teachers did not heed my call; during the interviews teachers would not open the textbooks even though I had them with me: they would point at them as in 'that one' or 'this one', but not open them to validate their views. I believe that this chapter has in a way shed light to the fact that the difficulty lay mostly in the fact that teachers' awareness of the affordances is not spontaneous: teachers were saying what they know and believe, and I wanted to hear something else, and I did not push and probe further. Hence, the level of awareness of the affordances of the textbook points to teachers' capacities to firstly perceive these affordances in order to mobilise them effectively. In the case of the teachers in this study, that level of awareness of the affordances is low, suggesting that their PDCs, that is, their capacities to perceive and mobilise the affordances are generally low.

In a context where resources are scarce and the textbook becomes the only available resource for teachers, the study is important as it shows that putting textbooks in the hands of teachers does not necessarily mean the textbooks are used deliberately and for productive and beneficial mediation in the classroom. The results are surprising because in South Africa the states' response to the issue of textbooks is to 'provide' them, but the results of this study suggest that this will not necessarily turn into productive and deliberate use.

South Africa emerges from a legacy of apartheid that denied 'solid' content in mathematics for teachers in the schools such as those participating in the study, and other contextual factors. This means that deliberate use of the textbook could be a productive professional development (PD) work that would enable teachers to identify both the affordances of the textbook as well as the constraints of the textbook. Brown (2009) suggests that PDC may emerge over time with increased familiarity with the resources. I agree with this assertion, but I also assert that the familiarity should be based on deliberate use.

I now turn to the last chapter in which I consolidate the study and provide my final reflections on this journey.

CHAPTER 9
Conclusions and Recommendations

9.1 Introduction

The central thesis to emerge from this study is that teachers' pedagogical design capacity (PDC) was generally low, resulting in a teacher-textbook relationship that can best be described as not intimate. While all the teachers attest to the importance of the textbook, evidence from lesson observations in particular pointed to *omissions* of critical elements of mathematics yet these were afforded in the textbook. Teachers opted for additional resources that were weaker mathematically than what was offered in the textbook; and made errors even when correct versions were available in the textbook. Not only does availability not imply use in a general sense, but the mobilisation of the affordances was tacit, with cause for concern for the opportunity for mediation.

While all these suggest a relationship to the strength of teachers' own mathematics, however, these kinds of weak use occurred even when errors or omissions were not present; thus the selective use of the textbook appeared to be tacit and not accompanied by conscious reflection even among teachers with stronger subject matter knowledge. Thus, deliberate use is not simply a function of SMK.

In addition to the generally weak relationship with the textbook as defined in this study, mobilisation of affordances varied considerably across teachers. This is obvious at one level, teachers mobilise the textbook in varied ways; at another it is a reminder that a policy that expects less divergence from a one size fits all with respect to textbook use is problematic. Thus, putting textbooks in the hands of teachers on its own is not a guarantee of deliberate and productive mediation; deliberate use of textbooks needs to be mediated.

These findings emerged from the investigations into: i) the affordances of the prescribed textbook to the teacher's practice; ii) the mobilisation of these affordances by the teacher; and iii) teachers' reflection of their use of the prescribed textbook. In this chapter I expound on these findings and reflect on the conceptual advances made in the study as a whole.

9.2 The Affordances of the Prescribed Textbook to Teachers' Practice

The study has revealed two distinct affordances of the textbook to the teacher's practice, namely, the *content* itself and then the *approach* to the teaching and learning of this content. These are not new as the literature review in chapter 2 shows. However, studying them together as this study has done is new. The simultaneous consideration of the *what* and *how* of textbook affordances as depicted through the *content* and *approach* to the teaching and learning of the content, provides useful insights into what the textbook affords to the teacher in its totality. The comparison of the Pre-CAPS and CAPS editions of the prescribed textbook in chapter 5 showed that the mathematical content of the textbooks with respect to their content areas and presentation formats were the same in both textbooks. However, the textbooks differed in the elements of the *approach* to the teaching and learning of the content, specifically in the variance and invariance in the aspects of functions each textbook adopted.

This study argues hence that the totality of the affordances of the textbook is determined through consideration of both its content and its approach to the teaching and learning of this content; that the approach of the textbook is its 'signature' (Valverde et al., 2002), and therefore any consideration of affordances which excludes the approach might be regarded as incomplete.

9.3 Teachers' Mobilisation of the Affordances of the Textbook is Tacit

Different teachers mobilise the affordances of the textbook in varied ways. This is well-known. It is also well-known that most teachers utilise the textbook more for exercises than any other presentation format. This has been confirmed too by this study in chapter 7. Furthermore, the study has also confirmed that the teachers' mobilisation of the affordances of the textbook are characterised by offloading both *content* and *approach* from the textbook 'as is'; and improvising from personal and external resources while adapting from both personal and external resources when necessary (Brown, 2009). The study has shown that with respect to the content, teachers in the study showed preference for improvising from external resources, than for offloading or adapting the *content*. With respect to the *approach*, the conclusion is that teachers in this study do not mobilise the *approach* of the textbook as an affordance in their practice.

However, the study has made significant contributions with respect to the mobilisation of the affordances of the textbook in terms of the *omissions* from and *injections* to the textbook which teachers make while offloading, adapting or improvising.

9.3.1 Teachers' Omissions and Injections on the Textbook Content

Two different types of *omissions* were observed from teachers' interactions with the textbook. There were *productive omissions* which were deliberate intentions to leave out parts of the content from the textbook. These *omissions* did not disrupt opportunities for mediation. However, there were also the *critical omissions* which detracted from the opportunities for the mediation of the object of learning. Furthermore, teachers would *inject* content that is not featured in the textbooks into the lessons. The *robust injections* serve to enhance opportunities for mediation, while other *injections* of content were found to be *distractive* leading to erroneous mediation.

Looking at mobilisation of affordances through the *critical omissions* from the textbook and *distractive injections* has provided an alternative analytical lens to describing the teacher-textbook interactions which shape the teacher-textbook relationship. This is one significant theoretical contribution of the study to the research on textbook use, as the nature of *omissions* and *injections* that teachers make in mediation are not a focus in the textbook use literature. Sherin and Drake (2009) mention that teachers sometimes omit or replace aspects of the resource as part of adapting the resource in their 'curriculum strategy'. However, these are related as descriptions of actions such as "unintentionally omits or replaces activities" (p.479) or "adds new examples and new contexts for problems" (p.480), without an indication of the effect on the opportunities for mediation of such actions. Thus, this particular analytical lens of *critical omissions* and *distractive injections* extends Brown's

(2009) work on teachers' scale of appropriation of curricular resources in delineating further on the nature of the appropriations.

The *critical omissions* and *distractive injections* described above point to a relationship between the teacher and the textbook that is not intimate: otherwise teachers would not omit the critical mathematics elements made available in the textbook, or introduce error when the textbook has clearly taken care of that, if the teacher-text relationship was strong.

9.4 Teachers' Awareness of the Affordances

Brown's (2009) description of teachers' PDC includes teachers capacity to perceive the affordances of the curricular resource. The conclusion of the study from teachers' reflection of their use of textbooks is that teachers' awareness of the affordances of the textbook is not conscious. The study mentions that teachers' talk about the affordances was mostly generalised and could have been about any other textbook or any other topic other than functions. This is hence confirmation that teachers' PDC in the study was generally low: if PDC is firstly about perceiving the affordances, and the teachers in the study do not seem to be consciously aware of these affordances of the textbook, it is logical to conclude that their PDC would be low.

Remillard (2009) argues that the verbs "perceive and mobilise" in the definition of PDC "have implications for the nature of the work teachers do with curriculum resources, but much of this process has yet to be delineated"(p. 90). I argue here that in embarking on the analytical process of determining teachers' awareness of the affordances of the textbook, the study has contributed theoretically in the process of delineating the verbs in the definition of PDC. My argument is that teachers' ability to perceive the affordances of the textbook begins with a conscious awareness of such affordances, and therefore this conscious awareness of the affordances is an important factor in the teacher-textbook relationship and deserves a place under the teacher resources in the conceptual framework for the teacher-curriculum interactions (Remillard, 2005, 2009).

9.5 Factors in the Relationship between Affordances and Teachers' PDC

This study begins in chapter 1 with stories of two teachers who prompted me to embark on this investigation of the relationship between the teachers and their prescribed textbooks. As I come to the end of the PhD journey, I can certainly claim that I understand better why those two teachers did what they did with their textbooks. Textbook use is certainly complex and is about the social practice: just making the textbooks available to teachers will not provide answers to the problems of weak performance in mathematics at school. The study evidences that deliberate use of the textbook is not a spontaneous act and needs to be mediated to teachers of mathematics before it can be realised. This brings me to another issue of context which I did not set out to investigate in the study, but which has been illuminated as an integral aspect of the relationship between the teacher and the textbook.

The schools participating in the study are what are referred to as "under the radar schools" and working in these 'poor schools' brings particular demands (Shalem & Hoadley, 2009), but also particular ways of working with the textbook. I explained in earlier chapters that in two of the schools including the school with the largest number of learners and participating teachers, the WMCS had to supplement the learners with textbooks at the beginning of the study. Thus, for many of the participating teachers, this was the first time in a very long time that they worked with learners who had a copy of the prescribed textbook. The lack of textbooks for learners certainly influences how teachers use their prescribed textbooks in the practice, and therefore their relationship with the textbook.

Notwithstanding, as I conclude this daunting task, I believe that I have accomplished what I set out to do at the beginning of this study, but not without limitations, which I shall outline in the next sections. At the same time, the study has provoked some recommendations which I wish to make with respect to further research as well as recommendations to the field. On a personal note, my own relationship with textbooks has changed from what it was at the beginning of the study. I no longer pick up a textbook and just 'pick and choose' examples or exercises as I used to: I now look at textbooks with the eyes of affordances in terms of what they offer with respect to the mathematical content as well as the approach. I have to confess that it is not easy but I also believe it has enriched my relationship with textbooks.

9.6 Limitations of the Study

A study of this nature and scale is bound to feature some limitations, some which might be inconsequential but others which bear mightily on the outcomes of the study. Some of the practical challenges which I faced included:

a) A changing textbook in the middle of the study

I have already alluded to this change of the prescribed textbook in previous chapters, but here I wish to acknowledge that it became quite a challenge to keep focus in the study, as teachers were using the Pre-CAPS and the CAPS textbooks interchangeably. When I had thought that teachers would be using the CAPS textbook after it had been confirmed as the newly prescribed textbook, it was actually not to be the case. Most teachers in the study still utilised the Pre-CAPS textbook; and yet the investigation on the performance expectations and their sequencing were based on the CAPS textbook. While I believe that this did not influence the outcome of the investigations because as the study showed, many teachers improvised from external resources, it could have created a major upset in the interpretation of the findings.

Another consequence of this limitation was the failure of the study to critique the constraints of the prescribed textbook, a point which I come back to in the next section.

b) Participation of teachers

Due to logistical and organisational issues in the schools, collection of video data did not take place as had originally been intended. The original plan was to have three lessons for each other, one at the beginning when functions were being introduced, another in the middle of the topic, and the last one at the end of the topic. However, this was not to be the case for many of the participating teachers as at any time a lesson could be cancelled or changed, or the school timetable would change suddenly, so that lots of readjustments to the schedule were required and there was no longer a pattern to the collection of the video data.

The implications here were that I could no longer compare how all the participating teachers introduced the topic on functions and compare that with the introduction of functions from the textbook; or the middle and the end of the topic. This has been one of the reasons that the teaching of functions itself was pushed to the background in the study, and therefore an obliteration of the opportunities for the mediation of the function concept in the study.

c) A delay in the individual interviews with teachers

As I have already elaborated in chapter 3 and 8, the individual interviews with teachers only took place about a year after I had collected data for classroom observations. The major reason for this delay had been methodological: having to develop an interview protocol after a preliminary analysis of classroom observation in order to follow up with teachers on what they were doing in the lessons; having to conduct a complete analysis of the textbook analysis in order to determine the affordances of the textbook to teachers' practice. Yet even a year later, when analysing the interviews, I still felt that there were questions which I could have asked to some of the teachers.

The interviews were also quite challenging to conduct with teachers and so I had concluded that they would not form part of the study any longer. It was only after completing the analysis of the classroom observations when the findings pointed to a tacit and not deliberate mobilisation of affordances by teachers that I brought back the interviews to seek teachers' reflections on the use of their textbooks. While the interviews provided the information the study sought, it was not the original intention of the interviews. I have to confess that I found the process of developing the interview protocol and conducting the interviews quite a challenge in this PhD journey. Even as I wrap up the study, I still feel that the interviews could have been better.

d) Learners' use of textbooks out of view

In the study, the learners' use of textbooks was out of view. It was not possible to collect learner books as had been intended in order to explore about the mediation of the object of learning. Secondly, my research design did not include an exploration of the mediation of the object of learning in the classroom. As a result, the study could only speculate about the opportunities for mediation instead of the actual mediation. This is one area that I strongly feel needs to be explored further. I come back to this in the next section.

e) Mediation of the object of learning not in focus

The teaching of functions itself in the study was in the periphery, which I believe was a design issue. However, the study provided a glimpse into the different approaches teachers adopted when teaching functions, together with conceptions of function the teachers and the textbook portrayed. However, it was not possible in this present study to explore these further.

9.7 Areas for Further Development and Research

The research on textbook use accounts for only 25% of all textbook research according to Fan et al. (2013). The research on textbooks on the whole is minimal in South Africa and less so regionally, yet as the teachers participating in the study have confirmed, many teachers and classrooms use the mathematics textbooks as a major resource. I regard this study as only the beginning for me in researching textbook use and plan to continue research in this field as I continue with my mathematics education career. However, from the study itself, I outline below my recommendations for further development and research in the field.

a) Firstly, the study has not been able to interrogate the constraints of the prescribed textbook fully, and focussed more on the affordances to the teachers' practice. Constraints are an important part of the textbook and a more focussed effort is needed in order to establish them alongside the affordances; otherwise the research is not complete.

b) Secondly, as the study reaches its culmination, I can attest to the fact that the teacher-textbook relationship is complex, and as such needs a concerted and deliberate programme specifically designed to address issues of textbook use with teachers. I believe that in order for teachers to appreciate the affordances of the textbook, they need to experience their interactions with their textbooks in different ways. Firstly, there should be opportunities created for this through professional development (PD) programmes on textbook use for in-service teachers; and specialised courses on textbook use for pre-service teachers. These programmes would contribute to developing teachers pedagogical design capacities, and researching them would benefit research on teacher-textbook relationships.

c) Thirdly, learners were not part of this study, and therefore with respect to them, the study could only speculate on their performance expectations and not on what they actually do. Research on how learners use their textbooks with respect to their awareness of the textbook affordances and their appropriation of these affordances would be of great benefit to mathematics education. As mentioned in this study, for the past four to five years in South Africa issues of non-deliverance of textbooks to learners by government are at centre stage and even in the courts of law. The question that remains unanswered is what do learners do with these textbooks when they are in their possession? In the light of the conclusions of the study that the teachers' mobilisation of the textbook affordances was tacit and not deliberate, it would be beneficial to investigate what learners themselves do with their textbooks, and if the whole exercise of provision of the textbook is worth the cost and the energy.

d) Lastly, as already mentioned, the teaching of functions itself in the study was in the periphery. Further research on what the textbook makes available for the mediation of function and how teachers appropriate this and their implications to the nature of the function concept mediated by the teacher-textbook interactions would benefit the field of teaching functions at school level.

9.8 Possible Implications for Practice

As a teacher of mathematics, I have always used a textbook for definitions, proofs, exercises, and to look for explanations for some of the content which I did not understand well; and I would go to the relevant pages in the textbook. If I did not find what I was looking for, I would seek another textbook and so forth. Secondly, the approach of the textbook would not have featured as an aspect to look out for in the textbooks which I used. I could say that my relationship with textbooks has been as I have described in this study: not intimate and textbook use, tacit. However, from working with the textbooks and the teacher-textbook relationship in this study, there are some things which I believe mathematics education practice can benefit from this study.

The notion of pedagogical design capacity (PDC) is critical in what teachers do with their textbooks. The study has shown that PDC is not a function of use alone, but of a deliberate, pedagogically beneficial use. As such, PDC starts with teachers knowing what their textbooks afford or constrain their practice. In other words, as teachers we should be aware that there is more to the textbook than the exercises, for example, the approach to the teaching and learning of the content. The study has shown how two editions of the same textbook series could differ in how the author approaches the same topic from one edition to another. This implies that the author has engaged in some revision of the approach to teaching the content, which suggests that our teaching need not remain static, and the textbooks are there to support the variety that teachers need.

However, we need to forge intimate relationships with our textbooks, and the starting point is to familiarise ourselves with the author's design intentions of the textbook. Thus, for teachers in the schools, the choice of the prescribed textbook should be based on the textbook's affordances as well as its constraints more than by its marketing from the publishers. As such the textbooks produced would be those informed by teachers' needs instead of the other way round.

Additionally, the approval of a textbook as a prescribed textbook by government departments should also be a reflection of its greater affordances against minimal constraints to the teachers' practice. In other words, the decisions about what textbooks to approve or not as prescribed textbooks should be informed by teachers who are able to say what the particular textbook affords or constrains their practice. This is important in contexts of 'poverty' where the textbooks are costly and therefore compete with other resource provision.

9.9 Concluding the Study

This study was set up to investigate the relationship between the affordances of the prescribed textbook and teachers' PDC in the mediation of the object of learning. I feel that as I conclude the study, I have achieved what I set out to do. This has been both an empirical and theoretical journey and a venture into the unknown for me. While literature on textbook use is not as abundant as that on use of curriculum materials, it nevertheless guided me into developing conceptual frameworks for analysing both the affordances of the textbook for teachers' practice, as well as the mobilisation of these affordances by the teachers in their practice. In this study I have succeeded to work with both the content and the approach to the teaching and learning of the content, which in South Africa, have been dealt with separately. Ensor et al.'s (2002) study foregrounds the approach and backgrounds the content, while Bowie's (2013) does the opposite of foregrounding the content and its affordances while backgrounding the approach. I consider this a significant contribution to research on textbooks specifically in South Africa.

The theoretical grounding of the study, from Vygotsky's social cultural theory and the notion of mediated action, to the conception of textbook use as a participatory relationship between the teacher and the textbook, illumined an important conceptual element in the consideration of teachers' PDC, that the was no single 'unit' of analysis in the study. As a case study, the unit of study was the person using tools. However, the study had multiple data sources and therefore multiple 'units' of analysis which kept changing depending on the focus of analysis. This complexity of the PDC points to the complex nature of the teacher-textbook relationship and a need for further research if we are to better understand this complex construct. However, I feel that this study has achieved theoretical strides by extending: a) Remillard's framework of the teacher-curriculum relationship; b) Brown's Design Capacity for Enactment Framework; and c) the use of the concept of 'affordances' from earlier notions in the analysis of mathematics textbooks.

Finally, the PhD ends here, but the research continues.

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LIST OF APPENDICES

Appendix A1: Teacher Consent Letter

February, 2011

Dear ……………………

INVITATION TO PARTICIPATE IN MY PHD STUDY

As part of the research area of the Wits Maths Connect Secondary Project, I am conducting an in-depth PhD study on how teachers use textbooks. My study will look at how teachers use their textbooks for preparation, for teaching in the classroom, and for assigning homework. We are interested in whether and how the textbooks you have support you in your teaching.

I would like to invite you to participate in this study with me. In order to contain the study so that it is not too big, I have decided to work with the teaching of functions, at Grade 10 and with the three Ivory Park schools in the project. Functions as we know are important and permeate almost all areas of mathematics.

Your participation in the study, if you agree to be part of it will involve the following:

a) **Meeting with textbook writers**: Research shows that curriculum developers' (including textbook writers) rationales for designing materials are often implicit, and it is usually left to the individual teacher to decide how they adapt them for their own teaching purposes. As a result, research suggests that making these rationales visible to the teacher enhances the teacher's ability to draw from personal resources and from textbooks to make necessary adjustments to their classroom situation.

   We would like therefore to invite authors of the chapter on functions in Classroom Mathematics Grade 10, and of Grade 10 Focus on Mathematics to talk to the teachers participating in this study, to enlighten us of what goes on in the production of these textbooks materials, what they view as most important in their chapters, and how if they were teaching, they would use the textbook.

   The meeting(s) will take place at Ivory Park during the months of March or April, before the topic on Functions is taught in the schools.

b) **Classroom observations**: In order to report on and investigate how the textbook is used, I will be visiting the classrooms of each of the teachers participating in the study observing their lessons for the duration of the teaching of Functions.

To capture the exactness of the classroom observations, it will be necessary for me to **video-tape** some of the lessons, but not all. During the classroom visits I will also take **notes** of details that I consider useful to the study and follow these up with **audio-taped interviews** with the individual teachers.

In 2009, a consent form for **classroom observations**, **video-taping** of lessons and **audio-taping** of interviews was sent to each teacher in the Wits Maths Connect Secondary Project, and we are grateful that you have given consent for us to work with you.

c) **Looking at teachers' preparation books**: Together with classroom observations, I will also need to look at the preparation books of all the teachers participating in the study. The idea here is that it is only through looking at the teachers' preparation books and interviews that I can find out how teachers use the textbooks for preparation, which is one of the questions in my study.

If you agree to participate in this study, please sign the consent for me to look at your preparation books which is attached to this letter.

d) **Looking at your students' classbooks**: It will also be necessary to look at students' books in order to explore how much of the textbook has been used in the examples given to students in the classroom, and also in giving students homework.

The consent from the parents/guardians of your students will be requested in due course if you agree to be part of the study.

c) **Team meetings**: We also envisage that there will be meetings where all the participants discuss issues pertaining to the research. These will be agreed with the participants beforehand. These will be opportunities for the whole research group of teachers to interact and reflect on their textbooks, whether and how they are using them;

I look forward to working with you.

Yours sincerely

<u>Moneoang Leshota</u>

Appendix A2: Parent Consent Letter

April, 2011

INFORMATION SHEET AND CONSENT LETTERS: PARENTS/GUARDIAN

Dear parent/guardian

Your child ……………………………………………is invited to be part of a PhD Study

'Investigating the Relationship Between Mathematics Teachers' Pedagogical Design Capacity and the Mediation of the Object of Learning'

Undertaken as part of the First Rand Foundation Project and directed by Professor Jill Adler, and researched by Mrs Moneoang Leshota, University of the Witwatersrand

The research is funded by the First Rand Foundation, Department of Science and Technology, and managed by the National Research Foundation. Project funding is from 2010 to 2014.

The research is being conducted across ten schools in Gauteng. The project has the support of the Gauteng Department of Education, and the Johannesburg East District Office. Your child's school is participating in this project from 2010.

This letter requests your consent to allow the researchers for this PhD study to look at your child's mathematics classwork book(s) as part of the research, and to watch some of their classes and take notes.

If you are happy to allow the researchers to look at your child's books please sign below.

I am happy to allow researchers to look at my child's mathematics classwork book(s) and to watch some of the child's classes and make notes as part of the research

Signed ……………………………………………

Date ……………………………………………

Name of Parent/guardian…………………………

Name of learner …………………………………

You may keep the information sheet on the next page.

For more information speak to your child's mathematics teacher

WHAT WILL THE RESEARCHERS DO?

The researchers want to find ways and means of improving the teaching and learning of mathematics in your child's school, and other schools.

The researchers from the Wits Maths Connect Secondary project at the University of the Witwatersrand request your consent to

- Look at the mathematics exercise books of selected learners
- Watch some of your child's classes and make notes

HOW WILL THE INFORMATION BE USED

Researchers will use the information from your child's books as part of the PhD study, to help them work with your child's teachers, and to study the progress of the work done with the teachers. The researchers will write a report which will be discussed at conferences and in journal articles. At the end of the project, a book resulting from the research is planned for.

The data from your child's books and from the notes the teachers have taken from their class will be used for the duration of the project and stored for a further five years. Thereafter they will be destroyed.

YOUR RIGHTS AND THE RIGHTS OF YOUR CHILD

We will not use your child's name in any reports or articles.

The research is completely separate from your child's school work. All information obtained for research purposes will not affect your child's assessment in school.

There will also be no problem if you do not want your child to take part in the research. If you choose that your child does not participate, this will not affect your child in any way.

If you decide that your child should no longer continue participating in the study, you are free to withdraw this consent at any time. You should then inform your child's mathematics teacher who will inform the researchers.

Appendix B: Interview Protocol

HOW DO TEACHERS USE THEIR MATHEMATICS TEXTBOOKS AND HOW DO THE WAYS TEACHERS USE THE TEXTBOOKS MEDIATE THE OBJECT OF LEARNING IN THE CLASSROOM?

A. CURRICULUM AND TEXTBOOKS

1. In your opinion, what four key things are absolute non negotiables from grade 10 to move into grade 11 with regard to the learning of functions?

   In other words what are the four key things that learners coming into grade 11 should bring with them with regard to the learning of functions?

   (for teachers not taking grade 11)

   In other words, if you were teaching grade 11, what four key things would you want learners coming from grade 10 to be bringing with them into grade 11 with regard to the learning of functions?

2. How does the prescribed grade 10 textbook support you in this regard?

B. KNOWING MY TEXTBOOK

3. What textbooks do students in your class(es) have? And, do they all have the textbooks?
4. I recall that last year when we videotaped the grade 10 lessons, the new CAPS textbook was still being developed, but you were given a copy of the two chapters on functions to use if you wanted to.
   a) What is your opinion on the differences and similarities of the NCS edition and the CAPS edition?
   b) Between the two editions, which one can you say you find easier to work with, and why?
5. When we spoke last year about which textbooks you used in planning your grade 10 lessons on functions, you said that you used different textbooks and other materials on top of the prescribed textbook. Thank you for bringing these textbooks and materials for this interview. For purposes of recording, please tell me what the textbooks and the materials are.

   Thank you. Now, as I explained in my communication to you about this interview, I asked you to bring all these materials so that you can help me understand better how you would use them in the planning of your lessons. We have the textbooks and materials in front of us: let us suppose that you were preparing your lessons for teaching grade 10 functions, could I please ask you to take me through the process of how you would go about it?

C. SCENARIO: LIMPOPO TEXTBOOKS SAGA

In the South African media recently, there is this big issue about the fact that textbooks were not delivered to Limpopo schools well into six months of the start of the academic year. We hear that there is a catch-up plan that has been proposed which “is meant to mitigate the impact of the textbook crisis on Limpopo pupils" (Mail &Guardian, August, 2012). There is also talk of an assessment that was carried out on the impact of the lack of textbooks on learning and teaching in Limpopo. On the other hand, the Hon Minister of Basic Education has been reported as saying she does not see what the big issue is; why weren't last year's textbooks used?

As a teacher of mathematics, especially at grade 10, what would be your views on:

a) the catch-up plan

b) the impact of the lack of textbooks on learning and teaching

c) the statement that last year's textbooks could have been used?

d) any other views on this textbook crisis?

Appendix C: Analysis of Performance Expectations

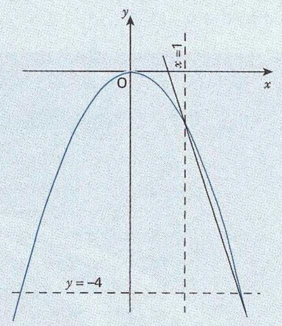
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
2. Properties of Functions	The quadratic function	**1.** The function $y = f(x) = x^2$ is called a **quadratic function.** **a)** Calculate the values of $f(-3), f(-2), f(-1), f(0), f(1), f(2)$, and $f(3)$, and plot the points $(-3; f(-3)), (-2; f(-2)), (-1; f(-1)), (0; f(0)), (1; f(1)), (2; f(2))$, and $(3; f(3))$. Join these points to form a smooth curve, called a **parabola** (there must be no sharp points). **b)** If the domain of f is the set of real numbers, what is the range of f? **c)** The graph of the given quadratic function is symmetrical about a straight line called an **axis of symmetry.** What is the equation of this straight line? **d)** For which value(s) of x is $f(x) = \frac{9}{4}$? Confirm your answer(s) graphically and algebraically.	1a): - Calculate function values of given domain values by substitution; - plot points and join them with a smooth curve to obtain a parabola 1b): Determine the range of the quadratic function for the domain of real numbers from the graph 1c): Determine the equation of the axis of symmetry of the function from its graph 1d) - Read off the values of x corresponding to $y = \frac{9}{4}$ on the graph - Equate x^2 to 9 and solve for x	Substitute Plot & draw generalise properties generalise properties Read off substitute	Pointwise Pointwise Global Global Pointwise pointwise	

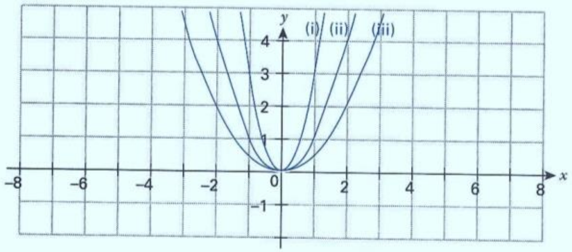
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
2. Properties of Functions	The rectangular hyperbola	**2.** The graph of the function $y = g(x) = \frac{1}{x}$ is called a **rectangular hyperbola.** **a)** Calculate $g(x)$ for $x = -2, -1, -\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 2$. **b)** Explain why $g(0)$ does not exist. **c)** Explain why there is no value of x for which $g(x) = 0$. **d)** Plot the points $(-2; g(-2)), (-1; g(-1)), \left(-\frac{1}{2}; g\left(-\frac{1}{2}\right)\right)$ and $\left(-\frac{1}{4}; g\left(-\frac{1}{4}\right)\right)$. Join these points to form a smooth curve. **e)** Plot the points $\left(\frac{1}{4}, g\left(\frac{1}{4}\right)\right), \left(\frac{1}{2}, g\left(\frac{1}{2}\right)\right), (1, g(1))$ and $(2, g(2))$ and join them to form a smooth curve. **f)** Complete the statements, choosing the correct options: **(i)** If more points were plotted for values of $x < -2$, and joined to the first four points plotted, the graph would approach, but never reach the negative/positive x/y-axis. **(ii)** We could say that for $x < -2$, the curve approaches the line $x = 0$ / $y = 0$ asymptotically. **(iii)** If more points were plotted for $x > 2$, and joined to the other four points, the graph would approach but never reach the negative/positive x/y-axis. **(iv)** For $x > 2$ the curve approaches the line $x = 0$ / $y = 0$ asymptotically. **(v)** The x/y-axis with equation $x = 0$ / $y = 0$ is called a horizontal asymptote to the curve. **(vi)** If more points were plotted for $-\frac{1}{4} < x < 0$ and joined to the first four points plotted, the curve would approach the negative/positive x/y-axis asymptotically. **(vii)** If points were plotted for $0 < x < \frac{1}{4}$ and joined to the points $\left(\frac{1}{4}, g\left(\frac{1}{4}\right)\right), \left(\frac{1}{2}, g\left(\frac{1}{2}\right)\right), (1, g(1))$ and $(2, g(2))$ the curve would approach the negative/positive x/y-axis asymptotically. **(viii)** The x/y-axis with equation $x = 0/y = 0$ is a vertical asymptote to the graph of $y = g(x) = \frac{1}{x}$. **g)** The hyperbola $y = g(x) = \frac{1}{x}$ has two axes of symmetry. This means there are two straight lines about which one half of the hyperbola is a mirror image of the other half. Draw the two axes of symmetry on your graph and label each line with its equation. **Note** When a curve approaches but never actually reaches a straight line, this straight line is called an **asymptote** to the curve. We say that a curve which gets closer and closer to a straight line but never actually cuts or reaches the straight line approaches the straight line **asymptotically.**	2: - Calculate function values for given domain by substitution - explain why $g(0)$ does not exist and why there is no value of x for which $g(x) = 0$ plot points for $x < 0$ and join them with a smooth curve plot points for $x > 0$ of the domain and join them with a smooth curve extend the domain and graph for $x < -2$ and $x > 2$; and for extend the domain and graph for $0 < x <$ ¼ and $-$¼ $< x < 0$ - determine the asymptotic behaviour of the graph by choosing correct options - determine the vertical and horizontal asymptotes for the function - identify the two axes of symmetry for the graph, draw them and label each of them with its equation	Substitute substitute Plot & draw Plot & draw Substitute and Plot & draw Generalise properties Generalise properties Generalise properties	Pointwise Pointwise Pointwise Pointwise Pointwise Global Global Global	

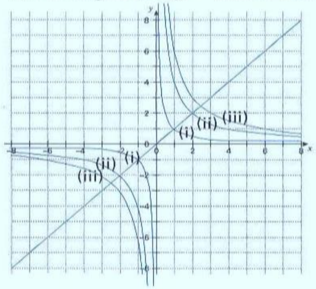
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
2. Properties of Functions	The exponential function	**3.** The function defined by the equation $y = h(x) = k^x$ with $k > 0$ and $k \neq 1$ is called an **exponential function.** **a)** Draw graphs of $y = p(x) = 2^x$, $y = q(x) = 3^x$ and $y = r(x) = 4^x$ on the same system of axes. Place the axis close to the bottom of your page and use $x \in \{-2; -1; 0; 1; 2\}$. **b)** Each graph you have drawn passes through a single common point. What is this point? **c)** Choose the correct option: In each of the graphs drawn, as x increases, so y increases/decreases. **d)** Choose the correct option: Each of the graphs drawn is of the form $y = h(x) = k^x$ for $k > 1$. The larger the base, k, the steeper/less steep the graph $y = h(x) = k^x$. **e)** If the domain of $y = h(x) = k^x$ is the set of real numbers, what is the range? **f)** Choose the correct option: All the curves drawn, approach the negative/positive x/y-axis asymptotically. **g)** Choose the correct option: the graphs drawn have a horizontal/vertical asymptote and the equation of the asymptote is $x = 0$ / $y = 0$, $y = x$ / $y = -x$.	3: - Evaluate function values by substitution for the given domain for the graphs inidcated - Draw the graphs using point by point plotting - Determine the common y-intercept for the functions from their graphs - Generalise about the behaviour of the y values as x values increase for $k > 1$ by choosing the correct option - Generalise about steepness of the graphs from the size of k for $k > 1$ by choosing the correct option. - Determine the range for the domain of real numbers from the graphs - Determinie the asymptotic behavoiour of the graphs by choosig a correct option - Determinge the asymptotes for the graphs by choosing the correct option	Substitute plot & draw read off generalise properties generalise properties generalise properties generalise properties generalise properties	Pointwise Pointwise Pointwise Global Global Global Global Global	

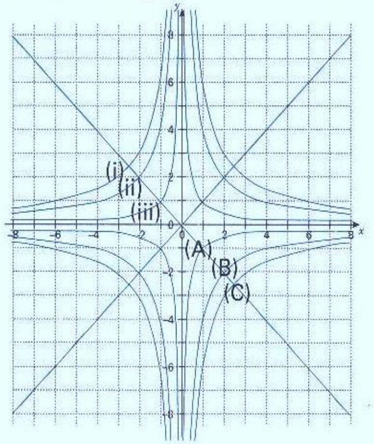
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
2. Properties of Functions	The Exponential Function continued	**4. a)** Draw graphs of $y = p(x) = \left(\frac{1}{2}\right)^x$, $y = q(x) = \left(\frac{1}{3}\right)^x$ and $y = r(x) = \left(\frac{1}{4}\right)^x$ on the same system of axes. Place the x-axis close to the bottom of your page and again use $x \in \{-2; -1; 0; 1; 2\}$. **b)** Each graph you have drawn passes through a common point. What is this point? **c)** Choose the correct option: In each of the graphs drawn, as x increases, y, the dependent variable increases/decreases. **d)** Choose the correct option: Each of the graphs drawn is of the form $y = h(x) = k^x$ for $0 < k < 1$. The larger the base, k, the steeper/less steep the curve. **e)** If the domain of $y = h(x) = k^x$ for $0 < k < 1$ for is the set of real numbers, what is the range? **f)** Complete the following statement, choosing the correct options: All the curves drawn approach the negative/positive x/y-axis asymptotically. **g)** Complete the following statement, choosing the correct options: The graphs drawn have a horizontal/vertical asymptote and the equation of the asymptote is $x = 0$ / $y = 0$ / $y = x$ / $y = -x$. **5.** Consider the function $y = j(x) = (-2)^x$ for $x \in \{-3; -2; -1; 0; 1; 2; 3\}$. **a)** For the given domain, write down the seven numbers that make up the range (the set of all y-values) and plot the seven points on a set of axes. **b)** Explain why $j\left(\frac{1}{2}\right)$ does not exist (remember that $k^{\frac{1}{2}} = \sqrt{k}$). **c)** Write another value of x for which $j(x)$ does not exist. **d)** The points you have plotted cannot be joined to form a continuous curve. The y-values only exist for $x \in \mathbb{Z}$. Write another value of k for which the graph $y = j(x) = k^x$ is not a continuous curve.	4:			
			- Evaluate function values by substitution for the given domain for the graphs inidcated	Substitute	Pointwise	
			- Draw the graphs using point by point plotting	Plot & draw	Pointwise	
			- Determine the common y-intercept for the functions from their graphs	Read off	Pointwise	
			- Generalise about the behaviour of the y values as x values increase for $0 < k < 1$ by choosing the correct option	Generalise properties	Global	
			- Generalise about steepness of the graphs from the size of k for $0 < k < 1$ by choosing the correct option.	Generalise properties	Global	
			- Determine the range for the domain of real numbers from the graphs	Generalise properties	Global	
			- Determine the asymptotic behavoiour of the graphs by choosing a correct option	Generalise properties	Global	
			5.			
			• Calculate the function values for the function $y = (-2)^x$ by substitution for the given domain	Substitute	Pointwise	
			• plot the points on the cartesian plane	Plot & draw	Pointwise	
			• Explain why the function value for $x = \frac{1}{2}$ does not exist	Generalise properties/ substitute	Global	
			• Suggest other values of x for			

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			which $(-2)^x$ does not exist • Show that the function $y=(-2)^x$ cannot form a continuous curve, and suggest other values of k for which the graph of $y = k^x$ is not a continuous curve	generalise properties Generalise properties	Global Global	
PRACTICE EXERCISES ON PROPERTIES OF FUNCTIONS	Check your skills exercises	1. Which functions $y=f(x)=x, y=g(x)=x^2, y=h(x)=\frac{1}{x}, y=j(x)=3^x$ and $y=k(x)=\left(\frac{1}{4}\right)^x$ has/have: a) a domain $=\mathbb{R}$; b) a domain $=\mathbb{R}-\{0\}$; c) a range $=\mathbb{R}$; d) a range $=\mathbb{R}-\{0\}$; e) a range $=[0;\infty)$; f) a range $=(0;\infty)$; g) a horizontal asymptote $y=0$; h) a vertical asymptote $x=0$; i) an axis of symmetry $y=x$; j) an axis of symmetry $y=-x$; k) an axis of symmetry $x=0$. 2. Which functions $y=f(x)=x, y=g(x)=x^2, y=h(x)=\frac{1}{x}$ for $x>0$, $y=j(x)=3^x$ and $y=k(x)=\left(\frac{1}{4}\right)^x$ has: a) y-values which increase as x increases b) y-values which decrease as x increases c) y-values which decrease over one subinterval of the domain and increase over another subinterval. 3. a) Draw sketch graphs showing the coordinates of the intercept of each graph with the y-ax and the coordinates of one other point on each graph. $y=3^x$ and $y=\left(\frac{1}{3}\right)^x$ on the same system of axes. b) $y=(\frac{2}{3})^x$ and $y=(\frac{3}{2})^x$ on the same system of axes. c) Using the sketch graphs drawn decide whether the following statement is true or false: The graphs of $y=k^x$ and $y=\left(\frac{1}{k}\right)^x$ for $k>0$ are mirror images of each other about the y-axi 4. a) Draw each pair of graphs of $y=f(x)$ and $y=-f(x)$ on the same system of axes where: (i) $f(x)=x$ (ii) $f(x)=\frac{1}{x}$ (iiii) $f(x)=x^2$ (iv) $f(x)=2^x$ b) for all cases (i) to (iv) state whether $y=-f(x)$ is a mirror image of $y=f(x)$ about: (i) the x-axis (ii) the y-axis.	1.& 2 allocate properties to functions 3. • sketch the graphs of the indicated functions by obtaining coordinates of one point of the graph, plotting the point and the y intercept; then generalise about the relationship with respect to reflection between the pairs of exponential functions of the type $y=\left(\frac{a}{b}\right)^x$ and $y=\left(\frac{b}{a}\right)^x$ 4. draw pairs of graphs of the functions $f(x)$ and $-f(x)$ for the indicated functions and generalise about reflections about the x and the y-axes	Interpret properties Interpret properties/ substitue/ plot & draw Generalise properties Sustitute/ plot & draw/ Generalise properties	Global Global/ pointwise Global Global	

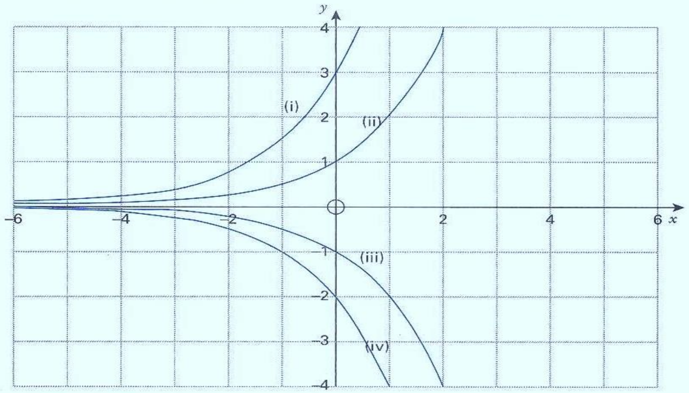
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
Investigating Functions	Average gradient	**Example** Determine the average gradient of the graph of $y = f(x) = -x^2$ between the points where $x = 1$ and $x = 2$ (that is between $(1; f(1))$ and $(2; f(2))$) **Solution** Average gradient $= \frac{f(2) - f(1)}{2 - 1}$ $= \frac{-4 - (-1)}{2 - 1}$ $= -3$  	calculate average gradient between two points on a given curve	substitution	pointwise	
	Average gradient	**Example** Determine the average gradient of the graph of $y = f(x) = -x^2$ between the points where $x = -3$ and $x = 0$ (that is between $(-3; f(-3))$ and $(0; f(0))$) **Solution** Average gradient $= \frac{f(0) - f(-3)}{0 - (-3)}$ $= \frac{0 - (-9)}{0 - (-3)}$ $= 3$	calculate average gradient given two points through which a graph of an indicated function passes	substitution	pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
3. Transformation of functions	The effect of a in $y = af(x)$ (The quadratic function)	**1.** The graphs of three parabolas are given: $y = f(x) = x^2$ $y = g(x) = 3x^2$ $y = h(x) = \frac{1}{2}x^2$.  **a)** Match the number **(i)**, **(ii)** or **(iii)** on each graph with its defining equation. **b)** Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$. **c)** Write the range of each of the three quadratic functions that have been drawn. **d)** Calculate the average gradient of each graph between the points where $x = 0$ and $x = 1$. Compare the steepness of the graphs with the average gradients between those points. **e)** Repeat **d)** above for $x = -1$ and $x = 0$. **2.** Investigate the graphs of $y = p(x) = -x^2$ and $y = q(x) = -3x^2$ as well as $y = r(x) = -\frac{x^2}{2}$. **a)** Draw the graphs for $x \in [-2; 2]$. **b)** Describe how the graphs of these parabolas compare with those drawn in question **1**. **c)** Write the range of the given functions for $x \in \mathbb{R}$. **d)** Write the equation of the axis of symmetry of the graphs of the given functions.	1: • Match a graph with its equation: decide which graph matches with the parent function; substitute values to check out points on other graphs, and match the graphs • Describe how the graphs of the form $f(x) = ax^2, a > 0$ differ from the graph of $y = x^2$ • Determine the range of each of the drawn graphs • Calculate the average gradient between two given points on each graph and compare these with the steepness of the graphs 2: • Draw the graphs of the form $f(x) = ax^2, a < 0$; for given values of a • Compare these graphs above with those with $a > 0$ and describe differences and similarities • Determine the range of each function • Determine the axis of symmetry of the graphs and write its equation	Interpret properties/ sustitute/ generalise properties Generalise properties Generalise properties Substitution/ generalise properties Substitute/ plot & draw Generalise propertes Generalise properties Generalise properties	Global Global Global Pointwise/ global Pointwise Global Global Global	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
3. Transformation of Functions	The effect of a in $y = af(x)$ (The Rectangular Hyperbola)	**3.** The graphs of $y = f(x) = \frac{1}{x}$; $y = g(x) = \frac{4}{x}$ and $y = h(x) = \frac{6}{x}$ have been drawn on the same system of axes. The straight line $y = x$ has also been drawn.  **a)** The graphs have been labelled **(i)**, **(ii)** and **(iii)**. Match these numbers with the defining equations. **b)** Read from the graphs the images: **(i)** $g(4)$ **(ii)** $h(-1)$. Check your answers algebraically. **c)** The straight line $y = x$ is an axis of symmetry of the hyperbolas sketched. Write the equation of another axis of symmetry. **d)** Write the equations of the horizontal and vertical asymptotes of each hyperbola. **e)** Write down the domain and the range of each function graphed. **f)** Read the coordinates of the points of intersection of the straight line $y = x$ with the two hyperbolas that are closest to the origin (labelled **(i)** and **(ii)**) from the graph. **g)** Gumani says the coordinates of the points where the graph labelled **(iii)** cuts the line $y = x$ are $(\sqrt{6}; \sqrt{6})$ and $(-\sqrt{6}; -\sqrt{6})$. Is he correct? Explain. **h)** Choose the correct option: the larger the value of $a > 0$, the (further from / closer to) the origin the graph of $y = \frac{a}{x}$ is.	3: • Match a graph with its equation: decide which graph matches with the parent function; substitute values to check out points on other graphs, and match the graphs • Read off from the graphs the y values corresponding to indicated x values; confirm by substituting the indicated values in the equations and evaluating the values • Determine the second axis of symmetry for the functions given that the line $y = x$ is another axis of symmetry • Identify the horizontal and vertical asymptotes, and the domain and range of the functions • Read off points of intersection of the graphs and the line $y = x$ • Prove that the coordinates of the point of intersection of the graph $h(x) = \frac{6}{x}$ and the line $y = x$ are $(\sqrt{6}; \sqrt{6})$ and $(-\sqrt{6}; -\sqrt{6})$ by equating the axes to the function and solving for x • Compare the distance from the	Interpret properties/ substitute/ generalise properties Read off Generalise properites Generalise properties Read off Substitute	Global/ pointwise Pointwise Global Global Pointwise Global	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			origin of the graphs and generalise about size of a in $y=\frac{a}{x}$, $a>0$ and how far from the origin the graph would be by choosing the correct option	Generalise properties	global	
3. Transformation of Functions	The effect of a in $y=af(x)$ (The Rectangular Hyperbola)	**4.** Added to the initial graphs are the graphs of $y=-\frac{1}{x}$, $y=-\frac{4}{x}$ and $y=-\frac{6}{x}$.  **a)** Link the number of each graph **(i)**, **(ii)** and **(iii)** with its defining equation. **b)** Determine the coordinates of the points A, B and C, the points at which the graphs of $y=-\frac{1}{x}$, $y=-\frac{4}{x}$ and $y=-\frac{6}{x}$ cut the line $y=-x$ **c)** In which two quadrants does the graph of $y=\frac{a}{x}$ lie if $a<0$?	4: • Compare the new graphs with the old ones and match do the matching • Read off points of intersection of the graphs and the given axis of symmetry form the graphs • Generalise about the quadrants in which the graphs of the form $y=\frac{a}{x}$, $a<0$ lie	Interpret properties/ substitute/ generalise propreties Read off Generalise properties	Global Pointwise global	

<table>
<tr><th>Content Area</th><th>Section</th><th>Example/Exercise/Activity</th><th>Tasks/Actions</th><th>Code</th><th>Category</th><th>Comments</th></tr>
<tr>
<td>3. Transformation of Functions</td>
<td>The effect of a in $y = af(x)$

(The Exponential function)</td>
<td>**5.** The graphs of
$y = f(x) = 2^x$,
$y = g(x) = 3.2^x$,
$y = h(x) = -2.2^x$
$y = i(x) = -2^x$ have been drawn.


a) Link the graphs labelled **(i)** to **(iv)** with their defining equations.
b) **(i)** List the y intercepts of the graphs
$y = 2^x$
$y = 3.2^x$
$y = -2.2^x$
$y = -2^x$
(ii) In general, where will the graph $y = a.2^x$, $a \neq 0$ cut the y-axis?
c) Determine the average gradient on each of the graphs between the points where $x = 0$ and $x = 1$.
d) Does $y = 3.2^x$ have a greater average gradient than $y = 2^x$ between $x = -1$ and $x = 0$?
Justify your answer by referring to the graph.
e) Describe the relationship between the shape and positions of the graphs of $y = 2^x$ and $y = -2^x$.
f) The graphs of $y = f(x) = 2^x$,
$y = g(x) = 3.2^x$,
$y = h(x) = -2.2^x$
and $y = i(x) = -2^x$ all approach the same line asymptotically. Describe the line and write down its equation.</td>
<td>5:
• Match a graph with its equation: decide which graph matches with the parent function; substitute values to check out points on other graphs, and match the graphs

• Read off y-intercepts of the different graphs

• generalise about the y-intercept of the graph of the form $f(x) = a.2^x$, $a \neq 0$

• Calculate the average gradients between $x = 0$ and $x = 1$ for all graphs

• Compare average gradients of $y = 3.2^x$ and $y = 2^x$ between $x = -1$ and $x = 0$

• Compare the graphs of $y = 2^x$ and $y = -2^x$ with respect to the shape and position and describe this relationship

• determine the asymptote for the graphs and write down its equation</td>
<td>Interpret properties/ substitute/ generalise properties

Read off

Generalise properties

definition

generalise properties

generalise properties

generalise properties</td>
<td>Global

Pointwise

Global

Pointwise

Global

Global

Global</td>
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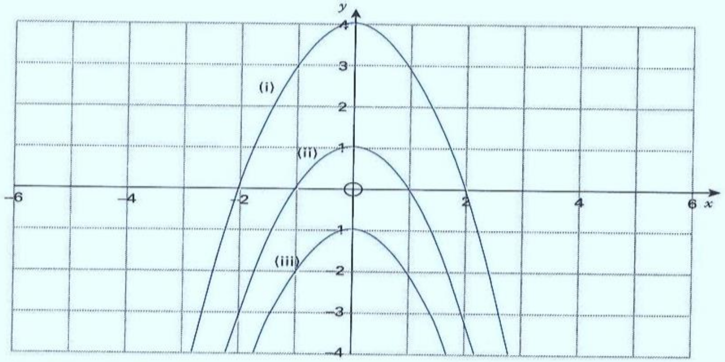
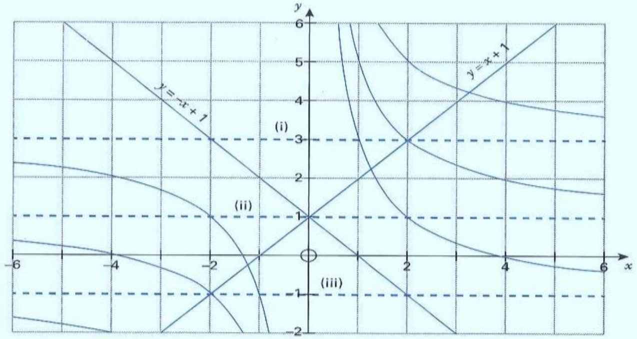
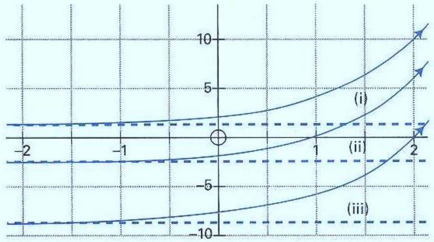
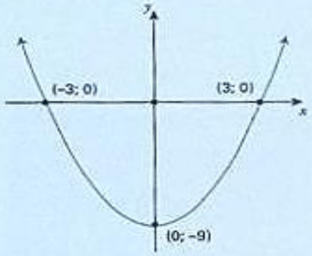
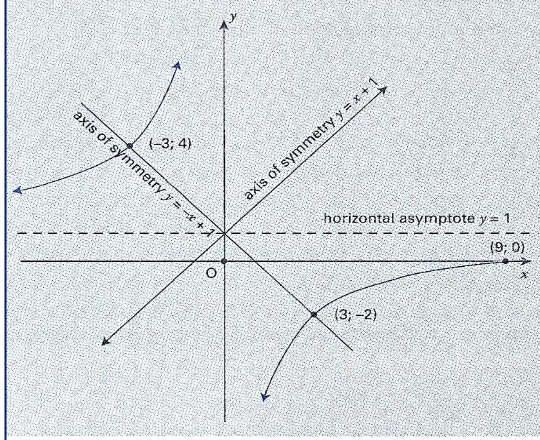
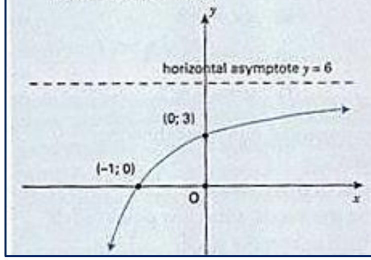
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
3. Transformation of Functions	The Effect q in $y = a.f(x) + q$ (The Quadratic Function)	**1.** Graphed are the functions $y = -x^2 - 1$ $y = -x^2 + 1$ $y = -x^2 + 4$.  **a)** Match the numbers on the graphs **(i)**, **(ii)** and **(iii)** with their defining equations. **b)** List the range of the functions defined by: **(i)** $y = -x^2 - 1$ **(ii)** $y = -x^2 + 1$ **(iii)** $y = -x^2 + 4$ **c)** Complete the statement, choosing the correct option: The graphs of $y = -x^2 + q$ all have the same shape as $y = -x^2$ but are (vertical/horizontal) translations of $y = -x^2$ by q units: up if $q > 0$ and down if $q < 0$. **d)** Read from the graphs solutions (if they exist) of the quadratic equations: **(i)** $-x^2 - 1 = 0$ **(ii)** $-x^2 + 1 = 0$ **(iii)** $-x^2 + 4 = 0$ **e)** Check your solutions algebraically to the equations in the previous question.	1: • Match a graph with its equation: decide which graph matches with the parent function; substitute values to check out points on other graphs, and match the graphs • Determine the range for each function • Describe the differences and similarities between the graphs and the graph of $y = -x^2$ by choosing the correct option • read off the x intercepts of the graphs where possible • Equate the functions to 0 and solve the equations algebraically	Interpret properties/ substitute/ generalise properties Generalise properties Generalise properties Read off Subsitute	Global/ pointwise Global Global Pointwise Pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
3. Transformation of Functions	The Effect q in $y = a.f(x) + q$ (The Rectangular Hyperbola)	**2.** Drawn are graphs of $y = \frac{4}{x} - 1$, $y = \frac{4}{x} + 1$ and $y = \frac{4}{x} + 3$:  **a)** Write the equations of the horizontal straight lines labelled **(i)**, **(ii)** and **(iii)**. **b)** Each of the horizontal straight lines is a horizontal asymptote to one of the hyperbolas sketched. Match the horizontal asymptotes to the defining equations of the hyperbolas. **c)** Write the range of the functions defined by: **(i)** $y = \frac{4}{x} - 1$ **(ii)** $y = \frac{4}{x} + 1$ **(iii)** $y = \frac{4}{x} + 3$ **d)** Write the domain of the functions defined by: **(i)** $y = \frac{4}{x} - 1$ **(ii)** $y = \frac{4}{x} + 1$ **(iii)** $y = \frac{4}{x} + 3$ **e)** Read from your graphs the solution of: **(i)** $\frac{4}{x} - 1 = 0$ **(ii)** $\frac{4}{x} + 1 = 0$ **f)** Check algebraically the solutions to the equations in the previous question. **g)** The straight lines $y = x + 1$ and $y = -x + 1$ have been drawn and labelled on the graph. They are axes of symmetry of the hyperbola $y = \frac{4}{x} + 1$. Write the equations of the axes of symmetry of **(i)** $y = \frac{4}{x} - 1$ **(ii)** $y = \frac{4}{x} + 3$ **h)** Read from the graph the solutions (if they exist) of the equations: **(i)** $\frac{4}{x} + 1 = x + 1$ **(ii)** $\frac{4}{x} = -x$	2: • Write equations of horizontal straight lines • Match a horizontal asymptote with its corresponding hyperbola • Determine the domain and range of the hyperbolas • read off the x intercepts of the graphs • Equate the functions to 0 and solve the equations algebraically • Determine the axes of symmetry for the other graphs, given the equations of the axes of symmetry for the other ones • Read off points of intersections between the graphs and their axes of symmetry	Interpret properties Interpret properties/ substitute/ generalise properties Genealise properties Read off Substitute Generalise properties Read off	Global Global/ pointwise Global Pointwise Pointwise Global Pointwise	

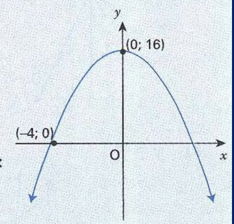
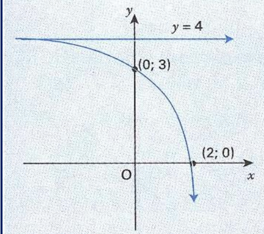
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
3. Transformation of Functions	The Effect q in $y = a.f(x) + q$ (The Exponential Function)	**3.** Graphed are the functions $y = f(x) = 3^x - 9$, $y = g(x) = 3^x - 3$ $y = h(x) = 3^x + 1$  **a)** The straight lines labelled **(i)**, **(ii)** and **(iii)** are horizontal asymptotes of the exponential functions graphed. Match each defining equation to its horizontal asymptote. **b)** List the range of the functions defined by : **(i)** $y = f(x) = 3^x - 9$ **(ii)** $y = g(x) = 3^x - 3$ **(iii)** $y = h(x) = 3^x + 1$ **c)** Read from the graphs the solutions to the equations (if they exist): **(i)** $3^x - 9 = 0$ **(ii)** $3^x = 3$ **(iii)** $3^x + 1 = 0$ **d)** Check algebraically the solutions to the equations read from the graphs in the previous question. **e)** Show that the average gradient of each of the exponential functions above is the same between $x = 0$ and $x = 1$.	3: • Identify the horozintal asymptote for each graph from the graphs, and match each equation with its asymptote • Determine the range of each function • Read off the x intercepts of the graphs if they exist • Equate the functions to 0 and solve the equations algebraically • Calculate average gradient between $x = 0$ and $x = 1$ for each graph and show that it is the same for all graphs	Interpret properties/generalise properties Generalise properties Read off substitute substitute/ generalise properties	Global Global Pointwise pointwise Pointwise/ global	

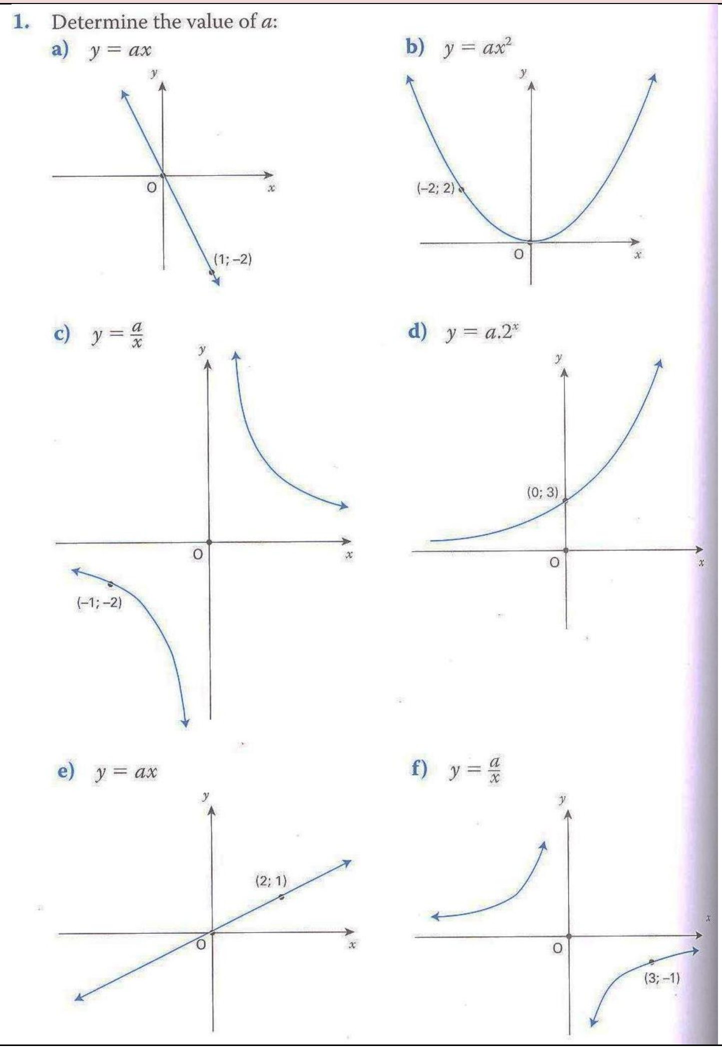
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	Sketching graphs of Functions (the quadratic function)	**Example** Sketch the graph of $y = x^2 - 9$. **Solution** • Since the coefficient of x^2 is positive, the parabola is 'cup' shaped ∪. • To find the y intercept put $x = 0$ $y = 0^2 - 9 = -9$ y intercept: $(0; -9)$ • To find the x intercepts put $y = 0$: $0 = x^2 - 9$ $0 = (x + 3)(x - 3)$ $x = -3$ or $x = 3$ x intercepts: $(-3; 0)$ and $(3; 0)$ • The graph has a minimum turning point on the y-axis at $(0; -9)$  • This is the graph of $y = x^2$ translated (shifted) downwards by 9 unit	Sketch the parabola $y = x^2 + 9$ using its properties: • Find the y-intercept by putting $x = 0$ • Find the x-intercepts by putting $y = 0$ and solving the quadratic equation • Show that the function has a minimum turning point at $(0; -9)$ because it is a downward translation of the graph of $y = x^2$ by 9 units	Substitute Substitute Interpret properties	Pointwise Pointwise global	

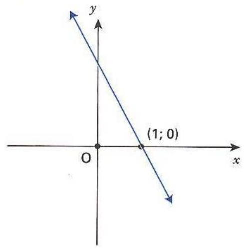
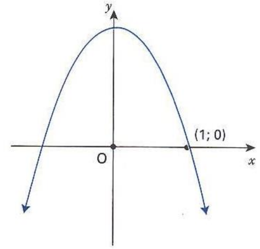
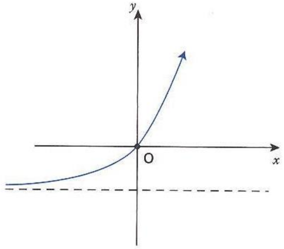
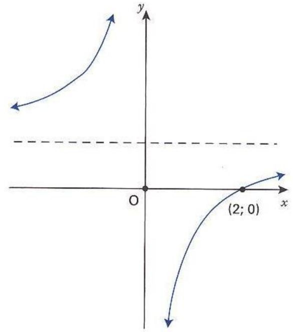
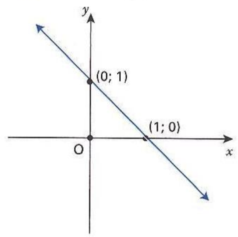
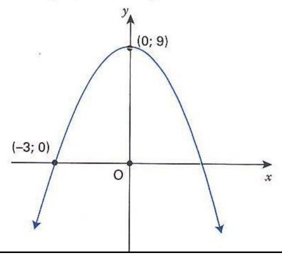
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	Sketching graphs of Functions (the rectangular hyperbola)	**Example** Sketch the graph of $y = -\frac{9}{x} + 1$ showing asymptotes, axes of symmetry and at least one point on the graph. **Solution** • This is a translation of the hyperbola $y = -\frac{9}{x}$ (which lies in the second and fourth quadrants) upwards by 1 unit. • To find the intercept on the x-axis we put $y = 0$: $-\frac{9}{x} + 1 = 0$ $x = 9$ • The horizontal asymptote is the line $y = 1$ and the vertical asymptote is the y-axis with equation $x = 0$. • One axis of symmetry of the hyperbola is $y = -x + 1$. The other is $y = x + 1$. • To find where the axis of symmetry $y = -x + 1$ intersects the hyperbola put: $-\frac{9}{x} + 1 = -x + 1$ $x^2 = 9$ $x = \pm 3$ When $x = -3$ then $y = 4$. When $x = 3$ then $y = -2$. Points of intersection: $(-3; 4)$ and $(3; -2)$.  	Sketch the hyperbola $y = -\frac{9}{x} +$ 1 + using its properties: • Show that the graph is a translation of the hyperbola $y = -\frac{9}{x}$ by 1 unit upwards • Find the x intercept by putting $y = 0$ and solving for x • Find the horizontal and the vertical asymptotes of the function and show them on the graph • Find and show the two axes of symmetry • Find the point of intersection for the function and its graph by solving the equation: $-\frac{9}{x} + 1 = -x + 1$	Interpret properties Substitute Interpret properties Interpret properties Interpret properties	Global Pointwise Global Global Global	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	Sketching graphs of Functions (the exponential function)	**Example** Sketch $y = -3.2^{-x} + 6$. **Solution** • The asymptote is $y = 6$ • The coefficient of 2^{-x} is $-3 < 0$ so the graph lies below its horizontal asymptote. • To find the intercept on the y-axis, put $x = 0$: $y = -3 \times 2^0 + 6$ $= +3$ y intercept: (0; 3) • To find the intercept on the x-axis put $y = 0$: $0 = -3 \times 2^{-x} + 6$ Solving the exponential equation to get: $3 \times 2^{-x} = 6$ $2^{-x} = 2^1$ $x = -1$ x intercept: $(-1; 0)$ 	Sketch the exponential graph $y = -3.2^{-x} + 6$ using its properties: • Show that the horizontal asymptote of the graph is the line $y = 6$ • Show that the graph lies below the horizontal asymptote because the coefficient of 2^{-x} is negative • Find the y intercept of the function by putting $x = 0$ • Find the x intercept by putting $y = 0$ and solving for x	Interpret properties Interpret properties Substitute Substitute	Global Global Pointwise Pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	Effect of a and q in $y = af(x) +$	**1.** Draw sketch graphs of the parabolas showing: • the intercepts on the axes (if there are any) • the equation of the axis of symmetry • the coordinates of the turning point • where the parabola does not cut the x-axis, show one point (other than the turning point). a) $y = 2x^2$ b) $y = x^2 - 4$ c) $y = -x^2 + 1$ d) $y = -x^2 - 1$ e) $y = 2x^2 - 18$ f) $y = -\frac{x^2}{2} + 8$ **2.** Draw sketch graphs of the hyperbolas showing: • the asymptotes and any axes of symmetry with their equations • the coordinates of any intercepts with the axes • if there is no intercept with the axes, show the coordinates of at least one point on the graph. a) $y = \frac{12}{x}$ b) $xy = -8$ c) $y = f(x) = \frac{6}{x} + 1$ d) $y = -\frac{2}{x} + 4$ e) $y = g(x) = \frac{10}{x} - 5$ f) $y = \frac{1}{2x} + 1$ **3.** Draw sketch graphs of the exponential functions showing: • the intercept on the y-axis • the intercept on the x-axis (if there is one) • if there is no intercept on the x-axis, show any point (other than the y-intercept) which lies on the graph • the horizontal asymptote and its equation. a) $y = 2^x - 1$ b) $y = 3.2^x$ c) $y = -3^x$ d) $y = 2.3^x - 18$ e) $y = -2.\left(\frac{1}{3}\right)^x + 6$ f) $y = 5.5^{-x} - 1$ **4.** Sketch the graphs of the functions, showing the coordinates of significant points and the equations of any asymptotes and axes of symmetry: a) $y = x^2$ b) $y = \frac{x}{2}$ c) $y = \frac{2}{x}$ d) $y = 2 + x$ e) $y = 2^x$ f) $y = 2x$ g) $y = -\frac{x^2}{3}$ h) $y = -2^x$ i) $2x - y = 6$ j) $y = 9 - x^2$ k) $xy + 2 = 0$ l) $y = -2^{-x} + 2$ m) $y = \frac{1}{2}(x^2 - 4)$ n) $y = -\frac{12}{x} + 4$	1. sketch the parabolas by using the propeties of the quadratic function as shown 2. sketch the hyperbolas by using the properties of the hyperbola functions as shown 3. sketch the exponential graphs using the properties of the exponential function as shown 4. sketch the different graphs using their properties	Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute	Global/ pointwise Global/ pointwise Global/ pointwise Global/ pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	Determining the equation of a graph (the quadratic function)	**Example** Determine the values of a and q in the parabola $y = ax^2 + q$. **Solution** There are two unknowns (a and q). The two pieces of information are the two points $(-4; 0)$ and $(0; 16)$. The value of q is 16, so the equation is $y = ax^2 + 16$. Substitute $x = -4$ and $y = 0$ into this equation: $0 = a(-4)^2 + 16$ $0 = 16a + 16$ $a = -1$ So $a = -1$ and $q = 16$  Note Before we calculated the value of a we knew it was a negative number because	• Identify the value of q as 16 from the graph (y intercept) Substitute the point $(-4; 0)$ in the general equation with $q = 16$ and solve for a	Interpret properties substitute	Global pointwise	
	Determining the equation of a graph (the quadratic function)	**Example** Determine the values of a, k and q if the equation of the graph sketched is $y = a.k^x + q$. The horizontal asymptote is the line $y = 4$.  **Solution** To determine three unknowns we have been given three pieces of information. The value of q is 4, so the equation is $y = a.k^x + 4$. Substituting $x = 0$ and $y = 3$: $3 = a.k^0 + 4$ $3 = a + 4$ $-1 = a$ So the equation is $y = -k^x + 4$ Substituting $x = 2$ and $y = 0$: $0 = -k^2 + 4$ Solving this equation gives $k = \pm 2$ But we know that $k > 0$, so $k = 2$ $a = -1, k = 2$ and $q = 4$	• Identify the value of q from the graph as 4 (the horizontal asymptote is $y = 4$) • Substitute the point $(0; 3)$ in the general equation with $q = 4$ and solve for a • Substitute the point $(2; 0)$ and solve for k • Disqualify a value of $k < 0$ because k should always be greater than zero by definition	Interpret properties Substitute Substitute Interpret properties	Global Pointwise Pointwise global	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	Determining equations of graphs	 **1.** Determine the value of a: **a)** $y = ax$ (1; –2) **b)** $y = ax^2$ (–2; 2) **c)** $y = \frac{a}{x}$ (–1; –2) **d)** $y = a.2^x$ (0; 3) **e)** $y = ax$ (2; 1) **f)** $y = \frac{a}{x}$ (3; –1)	1 a) – f): Substitute a given point(s) in the general equation and solve for a	Substitute	pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	Determining equations of graphs	**2.** Determine the value of q: **a)** $y = -2x + q$  **b)** $y = -x^2 + q$  **c)** $y = 3^x + q$  **d)** $y = -\frac{2}{x} + q$  **3.** Determine the values of a and q: **a)** $y = ax + q$  **b)** $y = ax^2 + q$ 	2a) – d): substitute the given point in the general equation to solve for the value of q 3a) & b): • identify the value of q from the graphs (y intercept of the graph) • then substitute the other given point in the eaution and solve for x	substitute Interpret properties substitute	Pointwise Global Pointwise	

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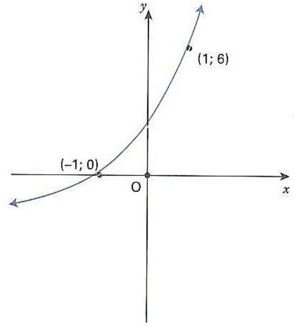
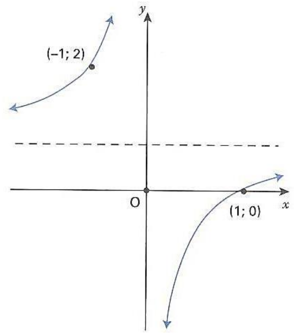
<td>PRACTICE EXERCISES</td>
<td>Determining equations of graphs</td>
<td>

c) $y = a.2^x + q$

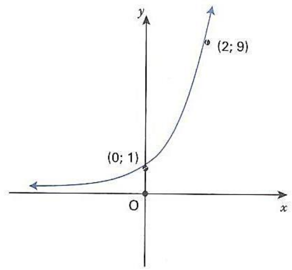


d) $y = \frac{a}{x} + q$

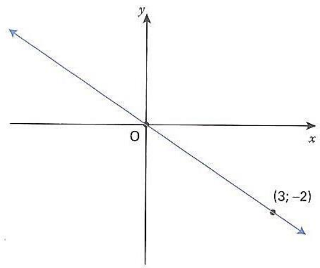


4. Determine the equation of the graphs.
Each one is of the form $y = ax + q$, $y = ax^2 + q$, $y = \frac{a}{x} + q$ or $y = a.k^x + q$.

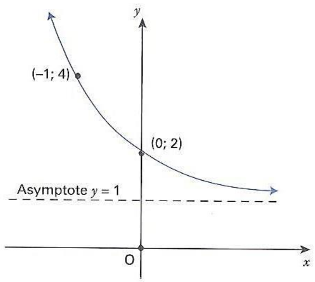
a)



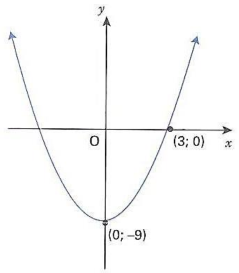
b)



c)



d)



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<td>3c) & d): substitute the two points into the general equation to form two equations and solve them simultaneously for a and q

4 a) – h):
• Match graphs with corresponding general equations

• Identify the value of q from the graphs where possible

• Substitute given points in the general equations to solve for the other unknowns</td>
<td>Interpret properties /substitute

Interpret properties

Interpret properties

substitute</td>
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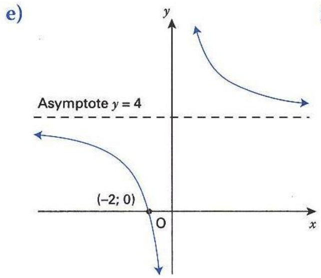
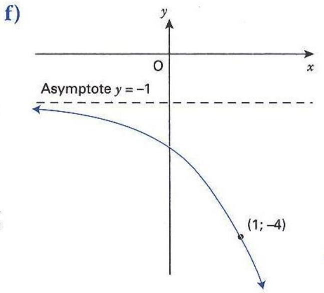
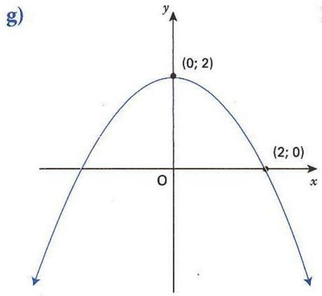
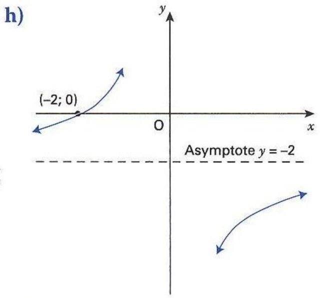
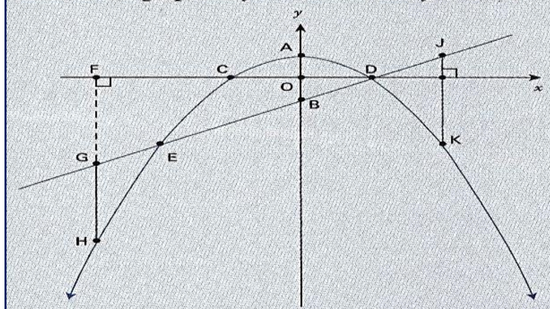
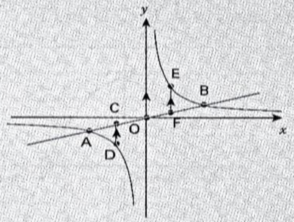
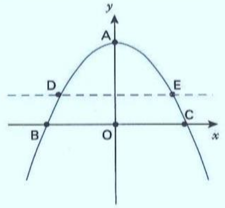
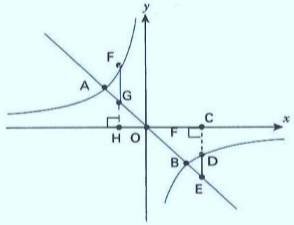
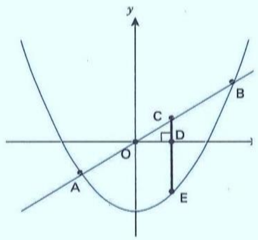
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	Determining equations of graphs	e)  f)  g)  h) 				
4. Interpretation of Functions	interpreting sketch graphs	**Example** Look at the graphs of $y = -x^2 + 1$ and $y = x - 1$  Find: **a)** the coordinates of A, B, C, D and E; **b)** the distance AB; **c)** the distance GH if the distance OF = 3 units; **d)** the coordinates of J and K if the distance JK = 4 units.	a) - use the fact that A is the turning point of the parabola whose y value is the q in the general equation, $y = -ax^2 + q$ - use the fact that B is the y intercept of the linear function, which is given by q in $y = ax + q$ - C and D are the x intercepts of the parabola: equate the equation to 0 and solve for x - equate the parabola to the linear function and solve for x:	Interpret properties Interpret properties Interpret properties/ substitute	Global Global Global/ pointwise	

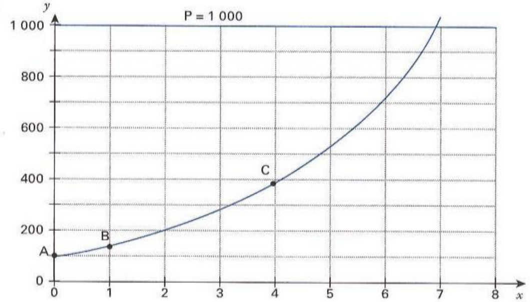
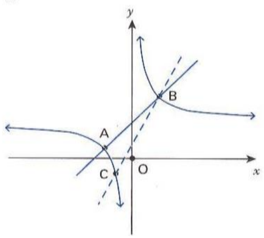
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
		Solution **a)** Since A is the turning point of the parabola A is $(0; 1)$ B is the y-intercept of the straight line, so B is $(0; -1)$ C and D are the intercepts of the parabola on the x-axis. Putting $y = 0$: $-x^2 + 1 = 0$ $x^2 - 1 = 0$ $(x + 1)(x - 1) = 0$ $x = -1$ or $x = 1$ So C is the point $(-1; 0)$ and D is the point $(1; 0)$ At E (and at D) the parabola cuts the straight line: $-x^2 + 1 = x - 1$ $0 = x^2 + x - 2$ $0 = (x + 2)(x - 1)$ $x = -2$ or $x = 1$ So E is the point $(-2; k)$. To find k we substitute $x = -2$ either into the equation of the parabola or into the equation of the straight line. Choosing the simpler option: $y = -2 - 1 = -3$ So E is the point $(-2; -3)$	one of the values represents E and the other D	Interpret properties/ substitute	Global/ pointwise	

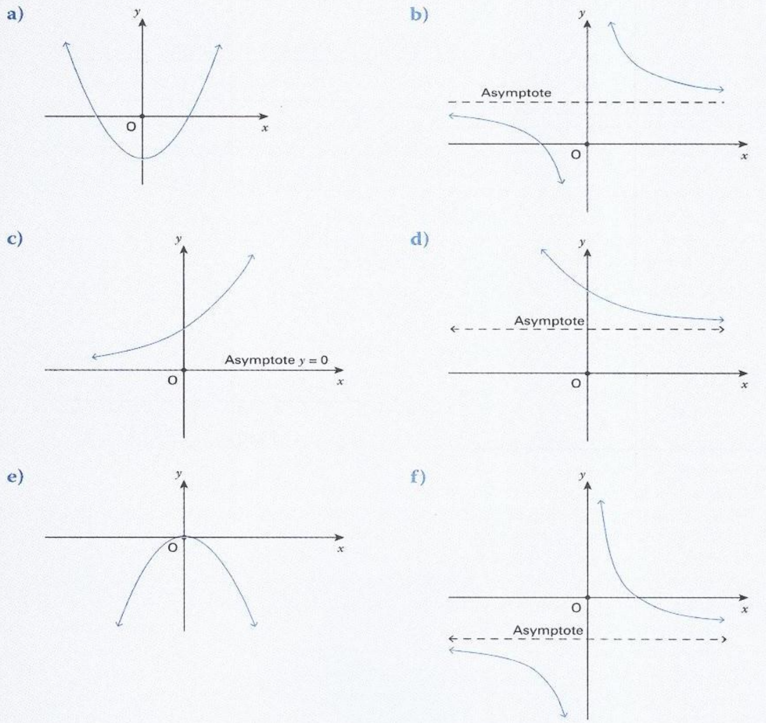
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	interpreting sketch graphs	**b)** The distance AB is the difference between the y-value at A and the y-value at B (always top value − bottom value): AB = 1 − (−1) = 2 units **c)** The distance OF = 3 units. This means that at F, G and H, $x = -3$ (as these points lie to the left of the origin). G lies on the straight line, so $y = -3 - 1 = -4$ H lies on the parabola, so $y = -(-3)^2 + 1 = -8$ and the distance GH = −4 − (−8) = 4 units **d)** JK = (y-value on the straight line) − (y-value on the parabola) $= (x - 1) - (-x^2 + 1)$ $= x^2 + x - 2$ But JK = 4 units, so $x^2 + x - 2 = 4$ $x^2 + x - 6 = 0$ $(x + 3)(x - 2) = 0$ $x = -3$ or $x = 2$ At J and K $x = 2$ as these points lie to the right of the origin At J on the straight line $y = 2 - 1 = 1$. So J is the point (2; 1) At K on the parabola $y = -(2)^2 + 1 = -3$. So K is the point (2; −3)	b): the distance AB is the difference between the y values at A and B c): - use the fact that since G lies on the straight line, it satisfies the equation $y = x - 1$ where $x = -3$ and calculate the value of y; - use the fact that H lies on the parabola and therefore satisfies the equation $y = -x^2 + 1$ and calculate the value of y; - subtract the y value for G from the y value for H and obtain the distance GH d) the y value at J satisfies the equation $y = x - 1$ and that at K satisfies $y = -x^2 + 1$: subtract $-x^2 + 1$ from $x - 1$; equate the result to 4 ans folve for x	Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Substitute Interpret properties substitute	Global/ Pointwise Global/ pointwise Global/ pointwise Pointwise Global Pointwise	

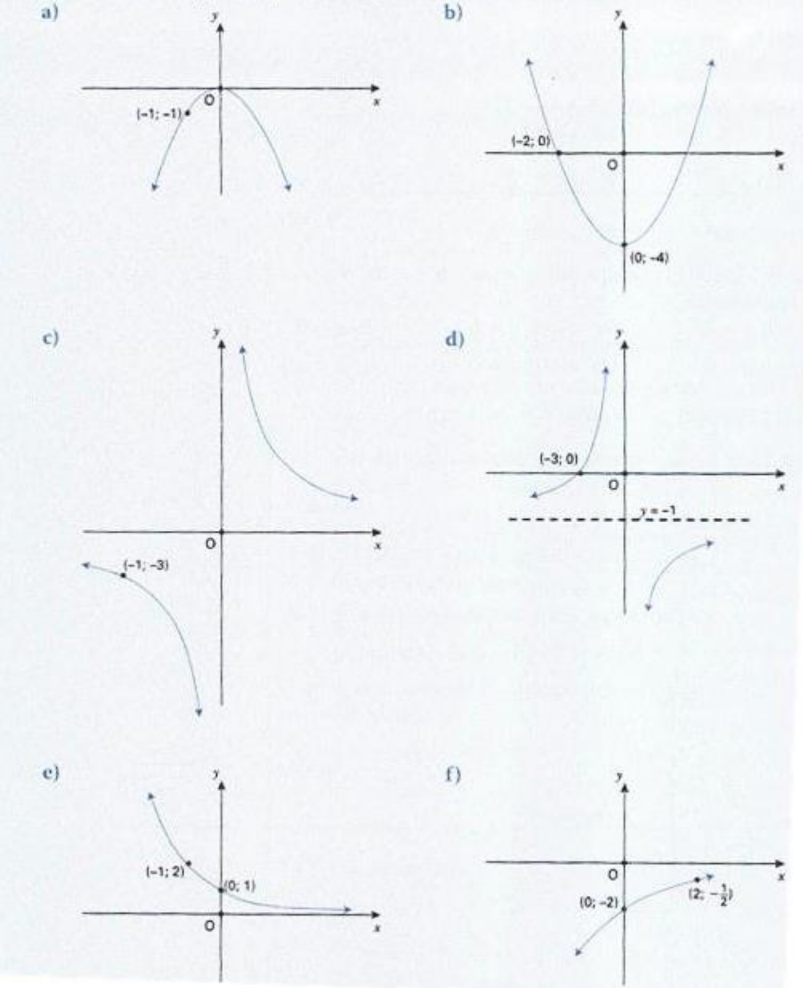
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
4. Interpretation of Functions	interpreting sketch graphs	**Example** Sketched are the graphs of $y = f(x) = \frac{x}{4}$ and $y = g(x) = \frac{4}{x}$ CD and EF are parallel to the y-axis.  **a)** Determine the coordinates of A and B, the points of intersection of the graphs of the two functions. **b)** Calculate CD $= f(-2) - g(-2)$. **c)** Calculate the value of k if it is given that EF $= g(k) - f(k) = \frac{3}{2}$ **d)** Calculate, with the aid of the sketch graph, the solution of the inequality $\frac{x}{4} > \frac{4}{x}$ **Solution** **a)** At A and B $\frac{x}{4} = \frac{4}{x}$ Multiply both sides of the equation by $4x$: $x^2 = 16$ So $x = -4$ or $x = 4$ At A, $y = \frac{-4}{4} = -1$ and at B $y = \frac{4}{4} = 1$ So A is the point $(-4; -1)$ and B is the point $(4; 1)$ **b)** CD $= -\frac{2}{4} - \frac{4}{-2}$ $= -\frac{1}{2} + 2$ $= 1\frac{1}{2}$ (or $\frac{3}{2}$) **c)** $\frac{4}{k} - \frac{k}{4} = \frac{3}{2}$ Multiply both sides of the equation by $4k$: $k^2 + 6k - 16 = 0$ $(k + 8)(k - 2) = 0$ Choose $k = 2$ since E and F are to the right of the y-axis. **d)** We are looking for the x-values where the y-values on the straight line are greater than (higher up than) the corresponding y-values on the hyperbola. The straight line is above the hyperbola between A and the y-axis and to the right of point B. The required solution is $-4 < x < 0$ or $x > 4$. **Note** $x = 0$ is not part of the solution because 0 is not part of the domain of the hyperbola. The solution can also be written in interval notatic $x \in (-4; 0) \cup (4; \infty)$	a): equate $\frac{4}{x}$ to $\frac{x}{4}$ and solve for x; then substitute the values in the equations to obtain corresponding y values b): substitute -2 in both functions and calculate the values c): substitute k for x in both equations; simplify $g(k) - f(k)$ and equate it to $\frac{3}{2}$ and solve for k d): use the x coordinates for A and B calculated in a) above to look for the intervals where the y for the straight line lie higher up than the correspoinding y values for the hyperbola	Substitute/ interpret properties Substitute Substitute Substitute/ interpret properties	Pointwise/ global Pointwise Pointwise Ppointwise/ global	

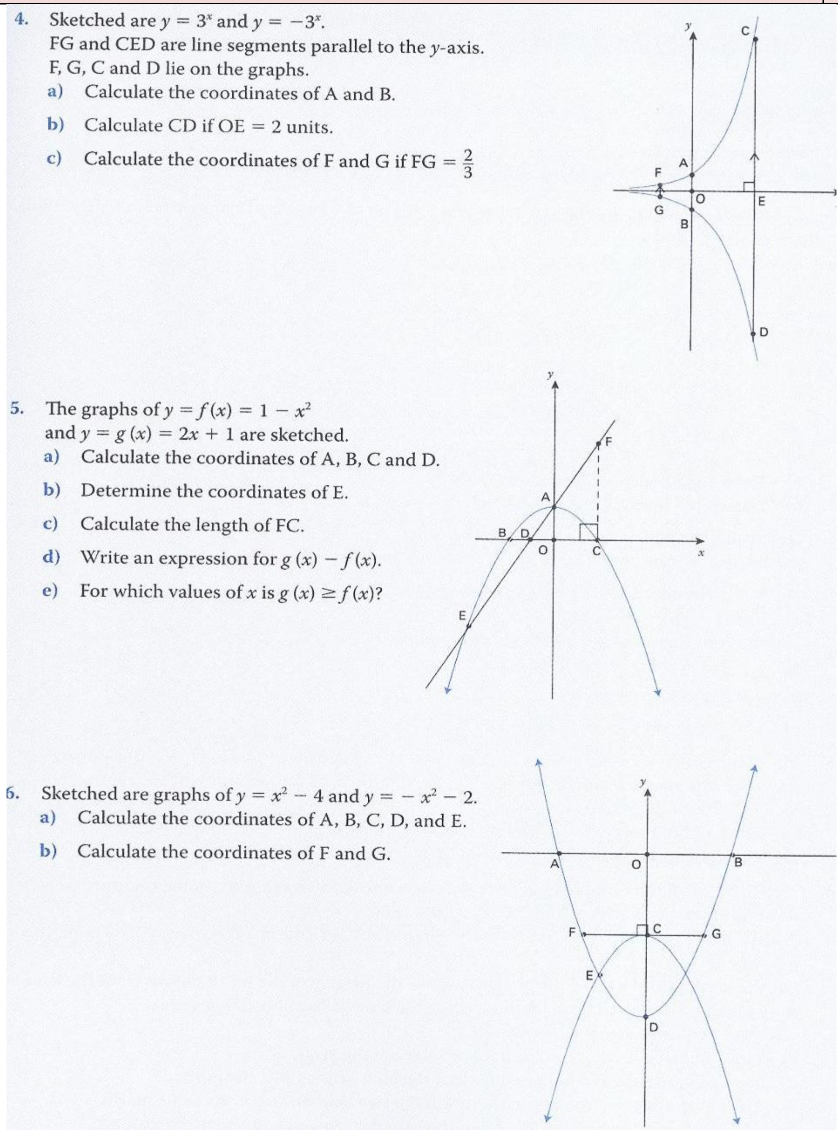
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	interpreting sketch graphs	**1.** The sketches show the graphs of $y = -x^2 + 16$ and $y = 7$. Calculate the coordinates of A, B, C, D and E.  **2.** The sketches show the graphs of $y = -\frac{6}{x}$ and $y = -x$. **a)** Determine the coordinates of A and B. **b)** Determine DE if OC = 3 units. **c)** Determine FG if OH = 1 unit (that means H is the point $(-1; 0)$)  **3.** The sketches show the graphs of $y = x^2 - 8$ and $y = 2x$. CDE is perpendicular to the x-axis. **a)** Determine the coordinates of A and B. **b)** Determine the coordinates of C and E if CE = 9. **c)** Read from the graphs the solution of the inequality: **(i)** $2x > x^2 - 8$ **(ii)** $2x \leq x^2 - 8$ 	1: • To find A: identify the y value of A as 16 from the equation of the parabola • To find B and C: equate $-x^2 + 16$ to 0 and solve for x • To find D and E: equate $-x^2 + 16$ to 7 and solve for x 2: a) equate $-\frac{6}{x}$ to $-x$ and solve for x b) substitute 3 for x in $-\frac{6}{x}$ to obtain the value of y for D; substitute 3 for x in $-x$ to obtain the value of y for E and then subtract the y value of D from that of E c):substitute -1 for x in $-\frac{6}{x}$ and $-x$ respectively to caluclate the y values for F and G respectivelyg and subtract the y value for G from that of F 3: a) equate $x^2 - 8$ to $2x$ and solve for x b) subtract $x^2 - 8$ from $2x$ and equate the result to 9 and solve for x. Use the $x$value to calculate the y values	Interpret properties Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute Interpret properties/ substitute	Global Global/ pointwise Global/ pointwise Global/ pointwise Global/ pointwise Global/ pointwise Global/ pointwise Global/ pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			c)i) identify the interval(s) for which the y values of the straight line lie higehr than the corresponding values for the parabola	Read off/ interpret properties	Global/ pointwise	
			ii) identify the interval(s) for which the y values for the parabola lie higher than the corresponding values for the straight line, including the values for which they are the same	Read off/ interpret properties	Global/ pointwise	

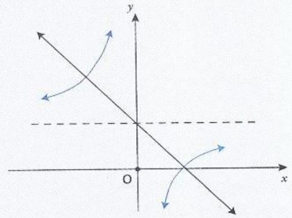
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
PRACTICE EXERCISES	interpreting sketch graphs	**4.** A school opened in 2011 and 100 pupils were enrolled. The rate of growth has increased by the same percentage each year. The increase in enrolment is modelled by $P = p(t) = 100(1,4)^t$ where t is the number of years since the school started.  **a)** Calculate the value of $P = p(1)$ and explain what your answer means. **b)** Calculate the school enrolment 4 years after the school opened. **c)** Should the answer to **b)** be read at A, B or C? **d)** Calculate the approximate average increase per year in enrolment between the first and fourth years. **e)** The school is built to accommodate up to 1 000 pupils. When will the district need to build a new school or provide additional classrooms? An answer correct to the nearest year is adequate. **5.** The sketches show the graphs of $y = \frac{4}{x} + 3$, the line AB, which is an axis of symmetry of the hyperbola, and the line $y = 2x + q$ which passes through the point B. **a)** Determine the equation of the line through A and B. **b)** Calculate the coordinates of A and B. **c)** Determine the value of q in the equation of the line CB. **d)** Calculate the coordinates of C. **e)** Use the sketch to help you find the solution of the inequality $\frac{4}{x} > x$ 	4: a) substitute 1 for t in the equation and calculate: explain that this is the number of pupils in the school 1 year after it opened b) substitute 4 for t in the equation and calculate c) show that the answer should be read at C where t represents the number of years after school opened d) subtract the value calculated in a) from that calculated in b) above and divide the result by 3 (4 – 1) e) read off the value of x corresponding to $y = 1000$ on the graph. 5: a) identify the line as the line $y = x + 3$ because the axes of symmetry for the hyperbola $y = \frac{a}{x} + q$ are given by the lines $y = x + q$ and $y = -x + q$ b) equate $\frac{4}{x} + 3$ to $x + 3$ and solve for x c) use the fact that the point B with coordinates $(2; 5)$ lies on the graph and substitute this point in the equation $y = 2x + q$ to find the value of q	Subtitute/ interpret properties Substitute Interpret properties/ substitute Substitute/ interpret porperties Read off interpret properties Substitute Interpret properties/ substitute	Pointwise/ Global Pointwise Global/ pointwise Pointwise/ global Pointwise Global Pointwise Global/ pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
CHECK YOUR SKILLS EXERCISES	Sketching graphs of functions Properties of functions	**1.** Draw sketch graphs of the functions: **a)** $xy = -12$ **b)** $y = \left(\frac{1}{5}\right)^x$ **c)** $2x - y = 4$ **d)** $y = x^2 - 1$ **e)** $y = -\frac{2}{x} + 4$ **f)** $y = 2^x - 1$ **g)** $y = 9 - 4x^2$ **h)** $y = 9 - 3^x$ **2.** Match each graph to one equation: $y = ax + q$, $y = ax^2 + q$, $y = \frac{a}{x} + q$ or $y = a.k^x + q$ and one condition: **(i)** $a > 0, q > 0$ **(ii)** $a > 0, q = 0$ **(iii)** $a > 0, q < 0$ **(iv)** $a < 0, q > 0$ **(v)** $a < 0, q = 0$ **(vi)** $a < 0, q < 0$ 	1. sketch graphs using properties of particular questions: for a), b), g) and h), the equations have to be rearranged to standard form first 2. use the properites of fucntions to match each graph to one genral equation and one condition each	Interpret properties/ substitute Interpret properties/ substitute	Global/ pointwise Global/ pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
CHECK YOUR SKILLS EXERCISES	Determining equations of graphs	3. Determine the equation of each graph sketched, given that it is a function of the form $y = ax^2 + q$; $y = \frac{a}{x} + q$ or $y = ak^x + q$.  	3. decide the general form of the equation of each graph - use the properties of functions to identify the value of q where possible - substitute the points in the general equation to solve for the values of unknown variables	Interpret properties/read off Substitute	Global/ pointwise Pointwise	

Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
CHECK YOUR SKILLS EXERCISES	Interpreting sketch graphs	 4. Sketched are $y = 3^x$ and $y = -3^x$. FG and CED are line segments parallel to the y-axis. F, G, C and D lie on the graphs. a) Calculate the coordinates of A and B. b) Calculate CD if OE = 2 units. c) Calculate the coordinates of F and G if FG = $\frac{2}{3}$ 5. The graphs of $y = f(x) = 1 - x^2$ and $y = g(x) = 2x + 1$ are sketched. a) Calculate the coordinates of A, B, C and D. b) Determine the coordinates of E. c) Calculate the length of FC. d) Write an expression for $g(x) - f(x)$. e) For which values of x is $g(x) \geq f(x)$? 6. Sketched are graphs of $y = x^2 - 4$ and $y = -x^2 - 2$. a) Calculate the coordinates of A, B, C, D, and E. b) Calculate the coordinates of F and G.	4: a) substitute 0 in the two equations to find the y values for the points A and B b) subtitute $x = 2$ in both equations to find the y values for C and D; and then subtract the value for D from that of E c) subtract (-3^x) from (3^x) and equate the result to $\frac{2}{3}$ and solve for x 5a) • identify the y value of A as 1 from the graph • equate $1 - x^2$ to 0 and solve for x to obtain x values for B and C • equate $2x + 1$ to 0 and solve for x to obtain the x value for D 5b) equate $1 - x^2$ to $2x + 1$ and solve for x 5c) substitute 1 (x value for C in $2x + 1$ to find the y value for F 5d) simplify $(2x + 1) - (1 - x^2)$ 5e) identify interval(s) on the graph for which the values for	Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties Interpret properties Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties	Pointwise/ global Pointwise/ global Pointwise/ global global Pointwise/ global Pointwise/ global Pointwise/ global Pointwise/ global Pointwise/ global Pointwise/ global	Questions similar to those in practice exercises of the same content area

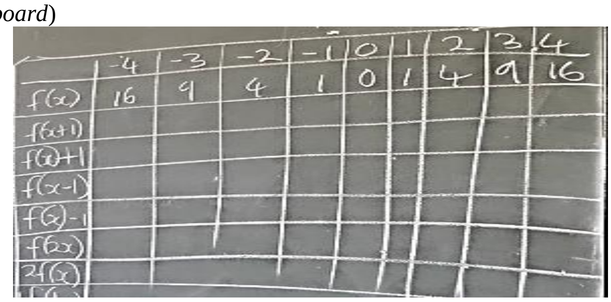
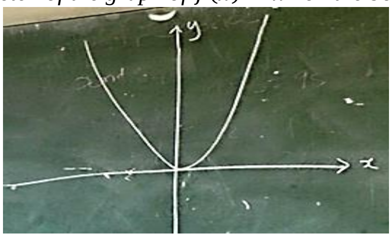
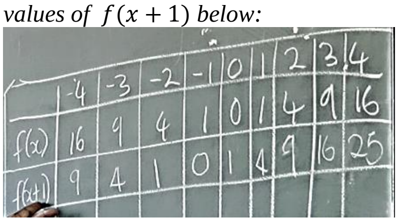
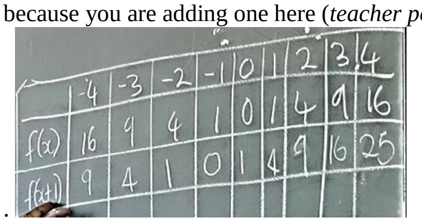
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			the straight line lie higher than the corresponding values for the parabola including the values for which they are the same	Read off/ interpret properties	Pointwise/ global	
			6a) - for A and B: equate $x^2 - 4$ to 0 and solve for x			
			- For C: identify the y value for C as -2 from the graphs (the q value in $y = -x^2 - 2$)	Substitute	Pointwise	
			- For D: identify the y value for D as -4 from the graphs (the q value in $y = x^2 - 4$)	interpret properties/ Substitute	global / Pointwise	
			- For E: equate $-x^2 - 2$ to $x^2 - 4$ and solve for x	interpret properties	Global	
			6a) substitute $y = -2$ in in $y = x^2 - 4$ and solve for x	Substitute/ interpret properties	Pointwise/ global	
				Substitute/ interpret properties	Pointwise/ global	

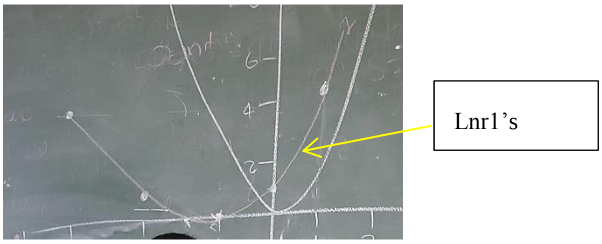
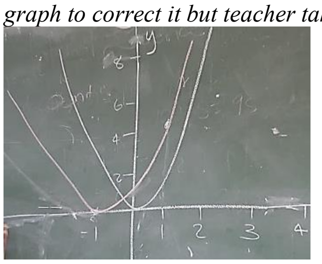
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
EXTEND YOUR SKILLS EXERCISES		1. Draw sketch graphs of: a) $2y + x^2 = 1$ b) $y = \frac{x-1}{x}$ c) $y = -2^{x-1} - 1$ 2. The sketch shows the graphs of a hyperbola $y = f(x)$ and an axis of symmetry of the hyperbola, $y = g(x) = -x + 1$  a) Determine the equation defining $f(x)$. b) Determine the values of x for which $f(x) \leq g(x)$. 3. Determine the equation of an exponential function of the form $y = a.k^x + q$ given that it passes through the points (1; 0) and (2; −2) and has a horizontal asymptote $y = 2$. 4. For $a > 0$, the coordinates of the two points of intersection of $y = \frac{a}{x}$ and the straight line $y = x$ are $(-\sqrt{a}; -\sqrt{a})$ and $(\sqrt{a}; \sqrt{a})$. List the coordinates of the points of intersection of: a) $y = \frac{a}{x} + q$ and $y = x + q$ for $a > 0$ b) $y = \frac{a}{x}$ and $y = -x$ for $a < 0$ c) $y = \frac{a}{x} + q$ and $y = -x + q$ for $a < 0$. 5. The graphs of $A = 1\,000\,(1{,}11)^n$ and $A = 1\,000\,(1 + 0{,}11n)$ intersect at (1; 1 110) showing that after one year, the simple interest and the compound interest, compounded annually, on R1 000 is R110 in each case. For which (if any) values of n is the compound interest less than the simple interest? Draw sketch graphs of the two functions to illustrate your answer. 6. Two hot air balloons are being observed. A blue one is 150 metres above the ground and is descending at a constant rate of 20 metres per minute. A red balloon is only 10 metres above the ground and is rising at a constant rate of 15 metres per minute. On the same pair of axes, draw graphs to show the heights of the two balloons from the time they were first observed until the time when the blue balloon hits the ground. Use your graphs to answer the questions: a) How long will it take the blue balloon to reach the ground? b) How high will the red balloon be when the blue balloon hits the ground? c) How long after the first observation will the two balloons be at the same height? d) How high will the balloons be when they are the same height above the ground?	1. rearrange equations to standard form and use the properties of functions to sketch the graphs 2a) • use the fact that the line $y = -x + 1$ is the axis of symmetry to conclude about the q in $f(x) = \frac{a}{x} + q$; • Use the fact that the original graph lies in quadrants II and IV to conclude that the sign of a is negative; • Find the x intercept for the straight line and since this is where the two graphs intersect, use it to calculate the value of a for the hyperbola 2b) identify the interval(s) for which the values of the straight line are equal to or lie higher than the correspoding values on the hyperbola 3. identify the value of q in the equation as 2 and substitute the two points in the equation $y = a.k^x$ to solve simultaneously for a and k	interpret properties interpret properties interpret properties substitute/ interpret properties read off/ interpret properties interpret properties/ substitute	Global Global Global Pointwise/ global Pointwise/ global Global/ interpret properties	

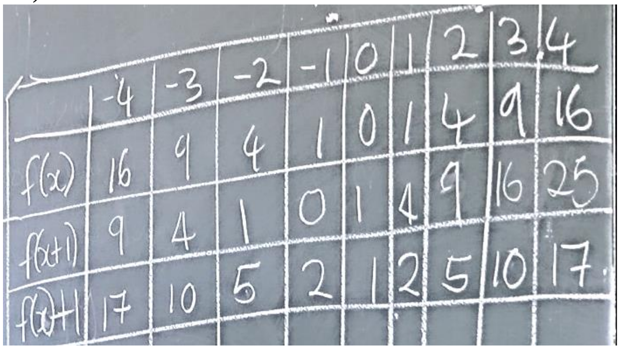
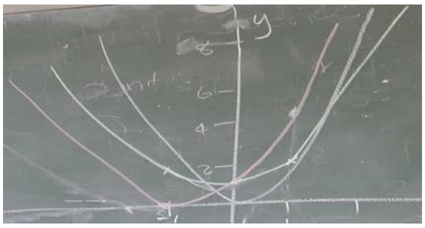
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			4a) add q to the y values of the coordinates	Interpret properties	Global	
			4b) the coordinates of the points of intersection become the reflections of the original coordinates about the x axis	Interpret properties	Global	
			4c) the coordinates become the reflections of the coordinates in 4b) above about the $x$axis	Interpret properties	Global	
			5. - sketch an exponential function starting at the point (0;1000) and increasing from left to right:	Interpret properties	Global	
			- sketch a straight line graph starting at $(0; 1000)$ and intersecting with the exponential graph at the point $1; 1110)$	Interpret properties	Global	
			- Between $n = 0$ and $n = 1$, the straight line lies above the exponential graph, and after $n = 1$ the exponential lies above the straight line	Interpret propeties	Global	
			6. draw two graphs of the equations: $y = 150 - 20x$ and $y = 10 + 15x$	Generalise properties/	Global/ pointwise	
			6a) solve $150 - 20x = 0$	Interpret properties/ substitute	Global/ pointwise	

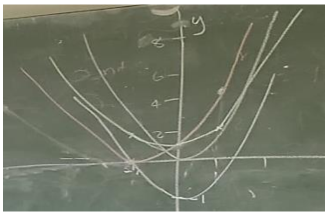
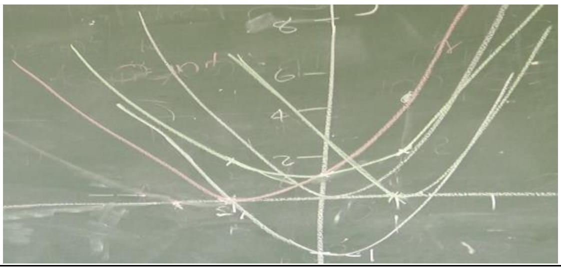
Content Area	Section	Example/Exercise/Activity	Tasks/Actions	Code	Category	Comments
			6b) substitute the x value obtained in 6a) in $y = 10 + 15x$ 6c) equate the two equations and solve for x: the x values gives the answer d) substitute the x value from 6c) above into any one of the two equations to calculate the height above ground	Substitute/ interpret properties Substitute/ interpret properties Substitute/ interpret properties	Pointwise/ global Global/ pointwise Pointwise/ global	

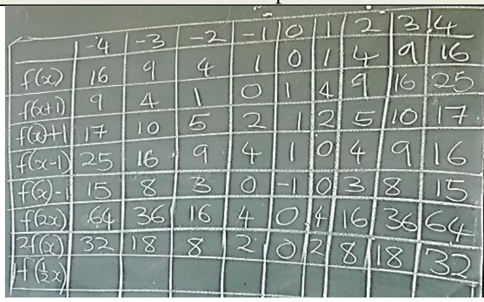
Appendix D: A Full Transcript of Lesson A12

Episode	Tr	Transcript
Episode 1 Populating tables of values for $f(x+1)$ and $f(x)+1$ and drawing their graphs-		(*Teacher is busy writing on the board. Learners are chatting to each other*.)
	Tr	Okay, okay. Let me check out our graphs; the ones that you were supposed to plot. (*teacher has drawn the following table on the board*)  (*Teacher moves away from board and picks up a pile of text books and puts them on her desk*.)
	Tr	If you are having a text book don't take. (*Learners continue to talk to each other while teacher moves more text books to her desk*.)
	Tr	Now can we have a volunteer to come and do the first graph? Let's have someone to do the first graph. (*Teacher draws a graph a rough sketch of the graph of* $f(x) = x^2$ *on the board*) 
	Tr	Can we have someone to come and do the first one? (*Teacher takes text books out of the box and hands them to learners*)
	Tr	Yes [*calls out a learner's name: I shall call her lnr1*]? (*Teacher continues to hand out textbooks while learners talk to each other*)
	Tr	Please be quiet. (*lnr 1 approaches the board*).
	Tr	Let's be quiet please. (*learners shush each other.* *(lnr 1 opens the book she is holding and studies the graph on the board and then begins to fill the table for values of* $f(x+1)$ *below:* 
	Tr	Let's be quiet please. (*lnr 1 looks at the teacher on her left and says something I cannot hear*)
	Tr	Yes (*lnr 1 moves to the right of the board where the graph of* $f(x) = x^2$ *is drawn. Teacher talks to other learners while lnr 1 draws a graph of* $f(x+1)$)
	Tr	Do you all agree? (*Learners chorus YES!*)
	Tr	So here, because you are now adding one, the outcome for negative four is now the outcome for negative three because you are adding one here (*teacher points to* $f(x+1)$)  .
	Tr	Okay? So it's like all these numbers shifted one unit going to the left (*pointing to the row for* $f(x)$). Let's see what our graph is going to look like. (*Teacher looks across to Lnr1, pauses and goes over to the learner*) (*Lnr1 has labelled the* x*-axis from* -4 to 4)
	Tr	Where is your graph going to turn now? (*Teacher points to the graph on the board and points to* -1 *on the table of value*) Where's it going to turn?

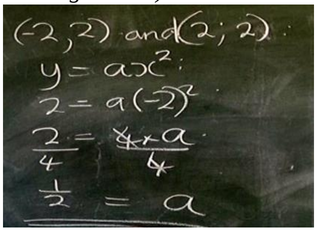
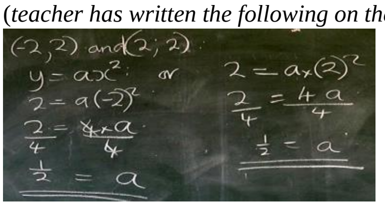
Episode	Tr	Transcript
	Lnr	It's going to turn… (*Lnr1's answer inaudible*)
		(L*nr1 labels the y-axis in 2's up to 8*)
	Tr	At which point?
	Lnr	uhh (*Lnr1 looks at her book in her hand, and says something inaudible to teacher*)
	Tr	Yes. (Lnr1 labels the y-axis in 2's up to 8 because she cannot reach the top of the graph and makes for a chair)
	Tr	No it's fine. No, you don't need it, it's fine like this (*Lnr1leaves the chair and returns to the board*). Just put the numbers that you can get.
	Tr	(coughs) Okay when x is negative four what is your value for y? (*Teacher asks Lnr1*)
	Lnr1	It's nine.
	Tr	It's nine. You are not having nine (*pointing to the board*). Okay, when x is negative three what is your value for y?
	Lnr1	It's four (*points to four on the graph*).
	Tr	It's four, plot that one (*Lnr 1 plots the point* $(-3;4)$ *on the graph*).
	Tr	When x is negative two what is y?
	Lnr1	One (*plotting the point* $(-2;1)$*on the graph*).
	Tr	And when x is negative one what is y?
	Lnr1	Zero (*plots* $(-1;0)$ *on the graph*).
	Tr	When x is zero what is y? (Camera zooms in on the board so I can see what the learner has done on the graph.)
	Lnr1	One. (*plots* $(1;1)$ *on the graph*).
	Tr	Teacher: When x is one, y is equal to what?
	Lnr1	(*Learners chorus, four!*) Four (*plots* $(1;4)$ *on the graph*).
	Tr	Use a different colour (*Lnr1 joins the points and draws the following graph*) 
	Tr	Lnr1, where's your turning point? (*Points to the point* $(-1;0)$ *on the graph, and Lnr1 rubs off part of the graph to correct it but teacher takes the chalk from her and changes Lnr1's graph to the following graph*) 
		(*Teacher and Lnr1 then rub off the labels on the* x*-axis*)
	Tr	(*addresses the whole class*) Okay so what happened to the graph that we started with, function of x is equal to x squared? (*Teacher moves to the graph drawn on the left of the board and then moves to the graph on the right.*) This is the one that we started with. (*Pointing to one of the graphs on the board.*) When we added a one inside of the front, what happened? (*Pauses*) This is the graph that we got (*pointing to the board*) so what happened here? (*Pauses – waiting for learners to respond.*)
	Tr	Teacher: Yes?
	Lnr	shifted
	Tr	It has shifted to, going to which direction?
	Lnr	To the negative side.
	Tr	To the negative side or to the left hand side. But when you are looking at this*)* plus one (*points to* $f(x+1)$ you would think it's going to shift going to which direction?
	lnrs	positive
	Tr	To the positive (*learners chorus with teacher*) side. Okay, so it shifted one unit to the left (*learners chorus "left"*). Now your point which was your turning point zero zero (*pointing at the origin*) is now negative one and zero. Okay?
		(Teacher moves to the table of values on the left hand side of the board)
	Tr	Now! Function of x plus one (*points to where it is written* $f(x)+1$ *on the table of values*). Function of x plus one, what is the co-ordinate for y when x is negative four? (*Pointing to the slot below* -4 *on the table next to* $f(x)+1$)
	Lnrs	Seventeen.
	Tr	Seventeen (*writes* 17 *in the slot next to* $f(x)+1$) is the co-ordinate for x when y is negative four.

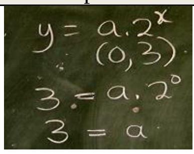
Episode	Tr	Transcript
	Tr	for y when x is negative 3?
	Lnrs	Ten.
	Tr	Ten (*writes 10 next to 17 and indicates for the class to call out what she must write next, as they call out the number she writes it in the graph along the line* $f(x)+1$).)
	Lnrs	Five, two, one, two, five, ten seventeen.
	Tr	(*points to the line she has just written on the board*) Okay, so what is happening here? *(points to values of* $f(x)$ *on the table of values*)
		
		Your y output,or your y value, was sixteen here (*pointing to 16 below – 4 on the table*), now it's seventeen (*pointing to 17 on the graph*) it means you added?
	Lnrs	One.
	Tr	So what is going to happen to our graph? (*Pauses, learners talking*). What is going to happen to our graph? (*Pauses*) What is going to happen to our graph? Let's plot the points. When x is negative one, what is y? (*teacher moves to right side of board where the graphs have been drawn*). When x is negative one, what is the value of y here?
	Lnrs	Two.
	Tr	It's two (*plots the point* $(-1;2)$ *on the graph where the other graphs have been drawn*). When x is negative three, what is our y?
	Lnrs	Five.
	Tr	Five. And when x is zero what is our y?
	Lnrs	One.
	Tr	One (*plots the point* $(0;1)$ *on the graph*). When x is one what is our y?
	Lnrs	Two.
	Tr	Two (*plots the point* $(1;2)$ *on the graph*). Are we having another point which is less than one (*points to the graph on the board*) on the y axis? (*Turns and points to the table of values on the left side*)
	Lnrs	No.
	Tr	Do we have a point which is less than one?
	Lnrs	(*big chorus*) No.
	Tr	So which one is now our turning point?
	Lnrs	One. (*Teacher gives questioning look at the class*)
	Lnrs	One, two
	Tr	Which one is the turning point? (*moves to the graph on the left and points to the point* $(0;1)$ *on the graph*)
	Lnrs	One, zero one, zero and one ...
	Tr	Okay, so that if I'm going to draw my graph *(joins the points plotted together forming another graph turning at the point* $(0;1)$).
		
	Tr	What happened to the original graph? What happened to the original graph? Now it's here (*points to the graph just drawn*). What happened when we added one? It shifted?
	Lnrs	Up.
	Tr	In the?
	Lnrs	y axis.
	Tr	In the y axis (*learners chorus with teacher*). Okay, so this is a shift in the y axis (*points to graph on board*) and remember here we added one (*teacher not in view of camera*)inside and now this one is being added to the function $f(x)+1$, okay?
	Lnrs	Yes.

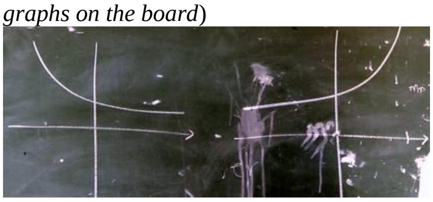
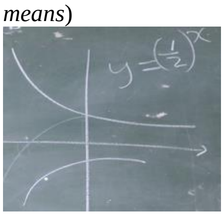
Episode	Tr	Transcript
	Tr	So, when you add one inside of the bracket, the movement is being affected in which axis? (*Pointing to* $f(x+1)$)
	Tr	In the x axis. Now we added outside the function, in the function plus one the movement is going to be affecting the y axis. Okay?
	Lnrs	Yes.
	Tr	So your graph is moving in the y axis.
Episode 2 Sketching the graphs of $f(x-1)$ and $f(x)-1$	Tr	What do you think is going to happen to this graph? (*Marks next to the function (f(x)-1) and pauses.*) What do you think is going to happen to this graph? (*Points to the function* $f(x)-1$.) We were having f of x plus one. Now we are having f of x minus one (*points to the function, pauses while learners talk).* What do you think is going to happen? (*calls out a learner whom I shall call Lnr2)*
	Tr	Is going to?
	Lnr2	Shift down.
	Tr	Is going to shift downwards (*learners chorus: downwards*) How many units?
	Lnrs	One.
	Tr	One unit. So where will our turning point be at?
	Lnrs	One, negative one.
	Lnr	Zero and negative one.
	Tr	Zero and negative one (*learners chorus with teacher*). Okay, so this is where you are having negative one. (*Camera moves back to teacher who is finishing drawing the graph of* $f(x)-1$ *on the board*) 
	Tr	Now if I subtract one inside the bracket here (*points to the function,* $f(x-1)$ *on the left of the board*) ...What do you think is going to happen to the original graph?
	Tr	If I subtract (*points to the function, f(x-1)*) one inside the bracket?
	Lnrs	It's going to shift the x axis.
	Tr	It's going to shift in the x axis (*chorused by learners*) to which direction? (*Learners respond but their answer is not clear*.)
	Tr	To which direction?
	Lnrs	Right hand side.
	Tr	To the right hand side. So if I subtract one my turning point will be at? (*points to a point on at* $(1;0)$)
	Lnrs	Zero, one.
	Tr	Is it zero and one?
	Lnr	One and zero
	Tr	One and zero, (*learners chorus "zero"*). So it's going to be here somewhere here (*plots the point* $(1;0)$*on the graph*). Okay?
	Lnrs	Yes. 
Episode 3 Class activity: drawing the graphs of $f(2x)$, $2f(x)$, and $f(\frac{1}{2}x)$	Tr	So today we are going to finish the other graphs. We did our main graphs, four graphs (*pointing to graphs on*
	Tr	*the board*).
	Lnrs	Five.
	Tr	Five, okay. So I am going to give you some graph papers to finish the other graphs (*moves to desk and gets the graph paper, learners talk as the teacher hands out graph paper*). (*Another female adult appears on the right of the screen and discusses something which I cannot hear with the teacher and then moves out of shot. Teacher discusses something with a few of the learners*.) (*Teacher starts writing on the left hand graph on the board, while learners are passing the graph paper around and talking. Camera zooms in on the board and teacher has completed the table of values on the board except for the function* $f(\frac{1}{2}x)$*, as shown below*)

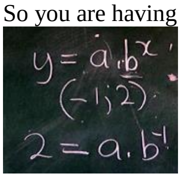
Episode	Tr	Transcript
		
	Tr	Okay can you please listen. Now we are starting from where we finished yesterday. You did the graphs until f of x negative one. Now you are going to do the graph and on that Cartesian plane remember you are going to start with the f of x is equal to x squared. First do that graph, then we are going to do the graph of f of two x, two f of x, to f of half x. Okay?
	Lnrs	Yes.
	Tr	On the other graph and you are using different colours here, like yesterday.
	Lnrs	Yes.
	Tr	And remember the other graph you are going to paste in your book.
		(*Teacher pauses, learners talking in background and then teacher moves around classroom*)
		(*Teacher moves to the board and writes the following on the board: Scale 2cm : 2 units For both axes*)
		(*Teacher moves away from the board and out of the camera view*.)
	Tr	The same scale, two centimetres is equal to two units for both axes (*learners talking loudly in the background and it is difficult to hear what the teacher is saying*). (*teacher is moving around the classroom and discussing with learners and occasionally addressing all learners*)
	Tr	(*claps her hands to get learners' attention*) Guys can you please listen for a second. (*teacher holds up a piece of paper*.) Last time you saw that both of the values of y that we want are positive. So try to put your x axis at the bottom ne. (*Points to the bottom of the paper she is holding up*.)
	Lnrs	Yes.
	Tr	Because most of the negative but you are not using them for y. So put your x axis down there (*pointing to the piece of paper she is holding*), so that it gives you more values for y.
	Tr	(*after about 10 minutes)* Please if you don't know how to scale your, your tables, your Cartesian plane just (*inaudible*) because some of you don't know what you are doing. (*teacher goes back to moving around the classroom assisting learners*)
	Tr	(*teacher addresses whole class after 14 minutes*) Okay, can you please listen ... those who have been plotting these graphs. Can you please make a conclusion from what you are observing on your graphs.
	Lnr	Yes.
	Tr	What is happening when you put a number inside the bracket and when there is the number outside?
	Lnr	Outside the bracket.
	Tr	Please write a conclusion (*moves away from the board*).
	Tr	Those who didn't do the graphs that you were supposed to do yesterday, please come to my office lunch time with those graphs. Do we understand each other?
	Lnrs	Yes.
	Tr	I want those graphs in different colours.
	Lnrs	Yes.
	Tr	Those who did not do them for yesterday.
	Lnr	Learner: What time?
	Lnrs	Yes.

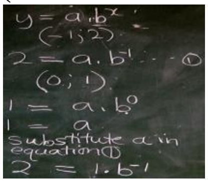
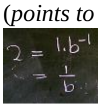
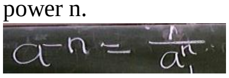
Appendix E: A Full Transcript of Lesson A13

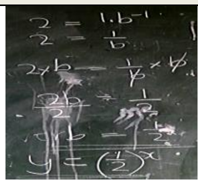
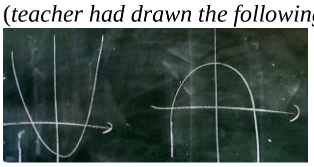
Episode		Transcript
Episode 1		(*Teacher is busy writing on the board.*)
	Tr	Now you divide throughout by? Four. (*Teacher and learners chorus together*)
Evaluating the value of a in $y = af(x)$ provided with $f(x)$		(*Teacher continues writing on the board, camera zooms in on the board and we can see the following written)*
		
	Tr	Here you get?
	Lnrs	One over two
	Tr	One over two. Those who used this point (*points to the top of the board*). Did you get half?
	Lnrs	Yes.
	Tr	Okay, let's see if you are going to get the same answer. (*Teacher begins writing on the board.*) So here your, your y value is also two. And your x value is?
	Lnr	Two.
	Tr	(*continues writing*) A positive two.
	Lnr	Squared.
	Tr	(*pointing to the board*) Squared. So positive two squared (*teacher writes on the board*)?
	Lnr	Four.
	Lnrs	Four times a?
	Lnrs	a.
	Tr	(*continues writing on the board*): You divide four by four; a is equal to half.
	lnrs	Half. (*teacher has written the following on the board*)
		
	Tr	So you were still going to get the same answer. Now when they found the value of y you come back to your equation you substitute for a
	Lnrs	x squared (*learners say aloud what teacher is writing*)
	Tr	x squared. Is this the equation that you were looking for? (*teacher has written $y = \frac{1}{2}x^2$) on the board*)
	Lnrs	Yes.
	Tr	Okay! d: y is equal to a times two to x. (*writes $y = a.2^x$ on the board*). What type of graph will this be?
	Tr	It's a? (*Still pointing to the board*) it's a?
	Lnrs	Exponential.
	Tr	It's an exponential graph.
	Lnrs	Yes.
	Tr	Because we are having to the power x (*pointing to the 2 on $y = a.2^x$*). Now which co-ordinates were you given?
	Lnrs	Zero and three.
	Tr	Which one is the y co-ordinate?
	Lnrs	Three.
	Tr	Three, and the x co-ordinate?
	Lnrs	Zero.
	Tr	So this becomes a times two to the power?
	Lnrs	Zero.
	Tr	And what is two to the power zero?
	Lnrs	One.
	Tr	It is one. So you are having one a, this side you are having three.
	Lnrs	Three, yes. (*Teacher has written the following on the board*)

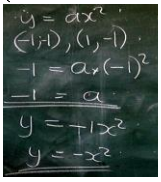
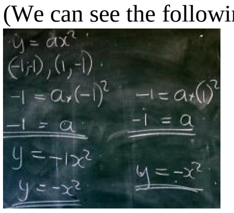
Episode		Transcript
		
	Tr	Can we go further than this? (*points to the board*). Can we go further than this?
	Lnrs	No.
	Tr	(*points to one of the learners*) Yes.
	Lnr	No Mam.
	Tr	No! So, this is the value of a?
	Lnrs	Yes.
	Tr	Okay, so remember anything multiplied by one it doesn't change its value. So this is still one a (*points to* a *in* $3 = a$).
	Lnrs	Yes.
	Tr	So you go back to your equation y. Where there is a, you put?
	Lnrs	Three.
	Tr	Three times two to the power x? (*writes* $y = 3.2^x$ *below* $3 = a$)
	Lnrs	x.
	Tr	It was supposed to be in your calculations.
	Lnrs	Yes.
Episode 2. Determining equations of graphs	Tr	Now let's go to question two? For question two you were first supposed to identify the type of graph that you are having and go back to the expressions that you were given or the factors that you were given in the question and you match the graph to one of those. Okay?
	Lnrs	Yes.
	Tr	So, c!
	Lnrs	Yes.
	Tr	2c, what type of graph is that one?
	Lnrs	Exponential.
	Tr	It's an?
	Lnrs	Exponential.
	tr	And, what type of exponential is that one? (*pauses*) So we are going to use the equation y is equal to?
	Lnrs	ax.
	Tr	ax? (*Learners seem unsure of what the answer is*)
	Tr	ax squared?
	Lnrs	No, no, a over x.
	Tr	a over x?
	Lnrs	Yes.
		(*teacher written* $y = \frac{a}{x}$ *on the board*)
	Tr	(*pointing to the* $y = \frac{a}{x}$) Is this the equation for an exponential?
	Lnrs	No.
	Tr	You said this is an exponential? (*pointing to the function* $y = a.2^x$)
	Lnrs	Yes.
	Tr	Does this look like that one? (*pointing to* $y = \frac{a}{x}$)
	Lnrs	No.
	Tr	So which equation were you supposed to use? (*Learners mumble*)
	Tr	Which equation were you supposed to use? (*Learners reply but it is unclear what is being said. Teacher is looking at papers in front of her.)*
	Tr	Which equation were you supposed to use? (*calls out a learner's name*)
	Lnr	a b to the power x.
	tr	a b to the power?
	Lnrs	x.
	Tr	Okay, so you supposed to use, it was a times b. y is equal to a times b to the power? (*teacher writes* $y = a.b^x$ *on the board*).
	Lnrs	x.
	Tr	Now what do we know about a before we find it? (*pause*). What do we know about a?(*pause*). So what do you know about b (*teacher the b in the function and waits for learners to respond: Learners are talking in the background)*
	Tr	(*teacher calls a learner's name*) What do you know about b?
	Tr	Our graph is facing which direction?
	Tr	(*calls another learner)* What do we know about b? (*pause*) What do we know about b? *(calls another learner's name)*

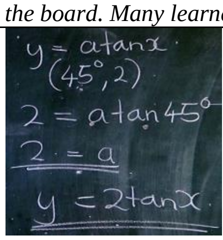
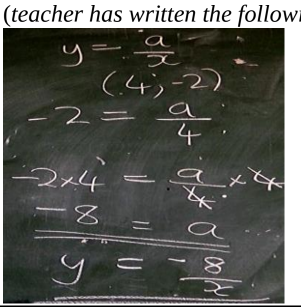
Episode		Transcript
	Tr	Your graph, is like this. I also know a graph to face in this direction. (*teacher has drawn the two graphs on the board*) 
	Tr	What's the difference between these graphs? Are they all exponential graphs?
	Lnrs	No.
	Tr	They are not exponential graphs? (*Waits for learners to respond.*)
		(The learners respond but we cannot hear what is being said clearly)
	Tr	Are they exponential graphs?
	Lnrs	Yes.
	Tr	So why is the other one facing the other direction?
		(*Learners are talking in the background. A learner sitting in front of the teacher says something which we cannot hear to the teacher and the teacher responds, this we also cannot hear*.)
	Tr	Okay, *[he]* is just telling me something. You are saying this one, which is facing this side (*graph on the right)*, it's y equal to two to the power x. What about this one? (*points to the graph on the left*). It's y is equal to?
		(*a learner responds but we cannot hear what he has says.*)
	Tr	(*writes* $y = \left(\frac{1}{2}\right)^x$ *near the graph on the left*) Do we agree?
	Lnrs	Yes.
	Tr	Do we agree?
	Lnrs	Yes.
	Tr	(*points to a learner in the class and says name which I can't hear*) you don't agree, why?
		(*There is a pause while the teacher listens to what the learner is saying, but we cannot hear what is said*.)
	Tr	Yes.
	Lnr	The function y is equal to half. .... it's supposed to face downwards. …
	Tr	It's supposed to face downwards?
	Lnr	Yes.
	Tr	Like how? Like this? (*teacher starts to draw but learner comes to the board and draws what he means*) 
	Tr	Okay, that's what he thinks.
	Tr	We all agree? (*Learners respond quietly*)
	Tr	We don't?
	Lnrs	Yes.
	Tr	Okay, there are some thinks that this one is supposed to face downwards if it's half. What do you think?(*calls a learner's name*)?
	Tr	What do you think? You just know that these two (pointing to the two graphs) are exponentials, is that correct?
	Lnrs	Yes.
	Tr	(*pointing to the two graphs*) You are saying these are exponential graphs?
	Lnrs	Yes.
	Tr	They are all exponential graphs?
	Lnrs	Yes.
	Tr	So why is the other one facing this side and why is the other one facing that side (*Learner responds but we cannot hear what he is saying*)
		(*The learners are talking in the background and the camera moves to where the teacher is. The teacher is standing at a learner's desk looking in her work book.*)
	Tr	Okay, can you go back to the work that you did on the nineteenth of May, where you were sketching graphs. (*Learners start to shuffle through their books and papers*)
	Tr	Teacher: You are having the first graph, how does it look? Is it this one or this one? (*pointing to the two graphs on the board*)
	Lnrs	This one (*pointing to the graph on the left*).

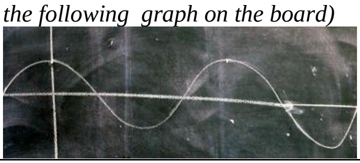
Episode		Transcript
	Tr	It’s this one?
	Lnrs	Yes.
	Tr	So what is the equation there?
	Lnrs	y is equals to half, brackets ....
	Tr	Half to the power x?
	Lnrs	Yes.
	Tr	So still you can’t see why is facing this side and why this one is facing this side? (*learners talk in the background*.) No [*calls a learner’s* name]? You are still stuck? Okay!
	Tr	When you have got an exponential graph and when it’s facing this side (*pointing to the graph of* $y = \left(\frac{1}{2}\right)^x$ *on the left*) it means your b here is a fraction. You are having a fraction here or you are having a number or your equation is having a power which is negative, okay?
	Lnrs	Yes.
	Tr	So this is the same as two to the power negative one (*teacher writes* $y = (2^{-1})^x$ *below* $y = \left(\frac{1}{2}\right)^x$) And this one is two to the power, one (*pointing to the 2 on* $y = 2^x$*)*
	Lnrs	One.
	Tr	A positive one.
	Lnrs	Yes.
	Tr	To the power x. That’s why they are different, the other one is facing this side and the other one is facing this side (*indicating with her hands in opposite motions*). Now, (*pointing to the graph of* $y = \left(\frac{1}{2}\right)^x$) but this can, but this can be written as a fraction. So when this exponential graph is facing in this direction (pointing to graph on left sloping downwards from the upper left) it means you are having a fraction inside here (*pointing to:* ½). And when it’s going in this direction (pointing to the graph of $y = 2^x$, it means you are having a whole number here (*pointing to* 2) Okay?
	Lnrs	Yes.
	Tr	So the graph that we are having on c, two c, where is it facing? That direction (*pointing to the graph of* $y = \left(\frac{1}{2}\right)^x$)
	Lnrs	Yes.
	Tr	So it means b, (*underlines the b on the function* $y = a.b^x$. When you see that your graph is facing in that direction and you have identified that it’s an exponential graph it means b is a?
	Lnrs	Fraction.
	Tr	Okay?
	Lnrs	Yes.
	Tr	So let’s go and substitute. Which point were you given? Which point were you given?
	Lnrs	Negative one and two.
	Tr	(*begins writing on the board*) Negative one and?
	Lnrs	And two.
	Tr	Which one is y?
	Lnrs	Two.
	Tr	(*writes on the board*) And which one is x?
	Lnrs	Negative one.
	Tr	Negative one.
	Lnr	Yes.
	Tr	So you are having how many unknowns? (*teacher has written on the board*) 
	Lnrs	Two.
	Tr	Two unknowns. Do you have another point?
	Lnrs	No.
	Tr	You don’t have another point? (*Learners respond but we can hear what they say*)
	Tr	Which point is that one?
	Lnrs	Zero and one.
	Tr	Zero and one. Which one is x and which one is y?
	Lnrs	One is y, x is zero.
	Tr	b to the power?
	Lnrs	Zero.
	Tr	Now you know that any number to the power zero gives us?
	Lnrs	One.

Episode		Transcript
	Tr	One, so you are going to have one and one this side *(of the equation on the board*). It means a is equal to one.
	Tr	You go back to your equation, the first part and you substitute for, a?
	Lnrs	a.
	Tr	Okay, you want to find the value of?
	Lnrs	a.
	Tr	We want to find the value of b. We are now having a, we want to find the value of b (*learners chorus in*). Okay, so you are going to use this one ($2 = a.b^{-1}$). Let's substitute on our first equation
	Tr	Equation one. So you are going to add two is equal to one times b to the power negative?
	Lnrs	One.
	Tr	One.
		(*teacher has written the following on the board*)
		
	Tr	(*points to* b^{-1}) So this will give me one over b.
		
		(*Learners say something but we cannot hear what is being said.*)
	Tr	Where's the negative one?
	Lnrs	Yes.
	Tr	You go back to your laws of indices (*teacher writes on the top of the board)* a to the power negative n is equal to what? (*learners chorus with teacher as she is writing*) One over a to the power n.
		
	Tr	Teacher: So this is why my b is.
	Lnrs	Yes.
	Tr	Okay, so I want to remove this b from this side (*pointing to the one over b and writes 2 on the left of it*). It's dividing here, how do I remove it? I am going to multiply everything by b (*b is chorused by learners*) - one over b times?
	Lnrs	b.
	Tr	This side you are having b and b will cancel, you remain with?
	Lnrs	One.
	Tr	This side you are having two?
	Lnrs	Two b.
	Tr	I don't want the value of two b, I want the value of what?
	Lnrs	b.
	Tr	So I am going to remove the two by?
	Lnrs	Two.
	Tr	Dividing by two. So b is equal to one over two (*chorused together with learners*).
	Tr	So this is the value for b. And also a fraction.
	Lnrs	Fraction.
	Tr	So you go back to your question (points to the question 2c. and then begins writing below what she has just completed).
	Tr	You, you are going to substitute. You know that a is one?
	Lnrs	Yes.
	Tr	You can write the one or you cannot write the one it's still fine because we are saying one multiplied by anything doesn't change its value.
	Tr	So you are going to have one over?
	Lnrs	Two.
	Tr	To the power?
	Lnrs	x.
		(*teacher has written the following on the board)*

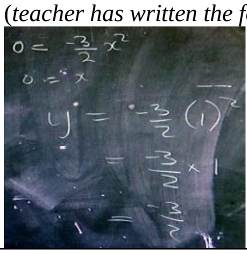
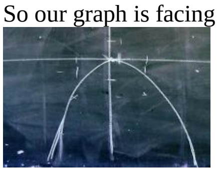
Episode		Transcript
		
	Tr	How many people got that one correct?
	Lnrs	No-one, no-one.
	Tr	So again that's why they gave you two points. You were supposed to have two equations. (*Teacher refers to something on her desk and is discussing something with the learner sitting in front of her, we cannot hear clearly what is being discussed*.)
	Tr	Okay, let's go to d.
	Tr	What type of graph is that one? d? It's a?
	Lnrs	Parabola.
	Tr	And which general equation were you supposed to use?
	Lnrs	ax squared.
	Tr	y is equal to?
	Lnr	x.
	Tr	Y is equal to what?
		(*Learners respond but I cannot hear what is said*.)
	Tr	a times?
	Lnrs	x squared.
	Tr	a times x squared. (*Teacher writes that on the board*.) Okay, what do you know about your value for a? What do you know about the value for a? Remember we've got two parabola. (Teacher *draws two graphs next to each other*.)
	Tr	Teacher: One which is facing downwards (*draws one facing down*). And the other one which is (*draws one facing up*).
	Lnr	Facing upwards.
	Tr	Upwards.
		(*teacher had drawn the following graphs on the board*) 
	Tr	Teacher: (points to the graphs) What's the difference between these two graphs? (*learners talk, teacher points to a learner*.) Yes? (*The learner responds but I cannot hear what is said. The teacher moves to the graph on the right.*)
	Tr	What is negative? Where do I put the negative? It's negative?
	Lnrs	x squared.
	tr	x squared (*writes* $y = -x^2$ *on the graph facing down*). And this one is?
	Lnrs	Positive x squared.
	Tr	Okay, so just by looking at the graph it's facing?
	Lnrs	Downwards.
	Tr	Downwards on d. It means your a is going to be what?
	Lnrs	Negative.
	Tr	It's not going to be a fraction, it's going to be?
	Lnrs	Negative.
	Tr	Okay.
	Lnrs	Yes. *(Teacher cleans the board.)*
	Tr	So if you got a positive a, there was a mistake somewhere. Now you were given two Negative one and?
	Lnrs	Negative one.
	Tr	And negative one?
	Lnrs	Yes.
	Tr	Negative one and?
	Lnrs	Negative one.
	Tr	Okay. And the other one?
	Lnrs	One and negative one.
	Tr	One and negative one
	Tr	Okay, so these are the points that you were given. Let's start with this one (*points to* $(-1,-1)$). What is the value of y here?
	Lnrs	Negative one.

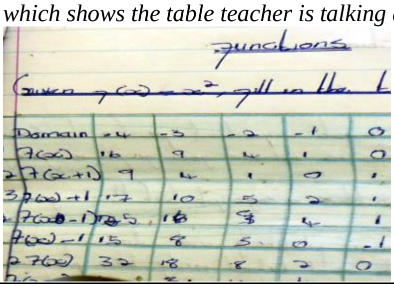
Episode		Transcript
	Tr	Negative one; a times negative one (*learners chorus what she is saying*) to the power?
	Lnrs	Two.
	Tr	And what is negative one to the power two? (*Turns and looks at the learners. Pauses while she waits for learners to respond. Learners seem unsure.*) Negative one to the power two. Is two?
	Lnrs	Yes.
	Lnrs	One, one.
		(*Teacher writes 2 on the board and turns and looks at the class.)*
	Tr	It's one?
	Lnrs	Yes.
	Tr	It's a positive one?
	Lnrs	Yes.
	Tr	It's a positive one?
	Lnrs	Yes.
		(*Teacher erases the 2 off the board.*)
	tr	Okay, (*begins writing on the board*) negative one squared is the same as negative one times negative one (*learners chorus in, there is a loud siren in the background*). So it gives you a positive?
	Lnrs	One.
	Tr	So you are going to have one a is equal to negative one.
	Tr	So this gives you y is equal to negative one x squared, which is the same as y equal to negative x squared.
	Lnrs	Yes..
		(*teacher has written the following on the board*)
		
	Tr	(underlines the final answer, then turns and points to (1,-1)) Did someone use this points? (pause) No-one used these points?
	Lnrs	Yes.
	Tr	(moves to the right of what she had previously written and begins writing) Okay, let's see if we are still going to get the same answer. The value of y here? (pointing to the board) Is what?
	Lnrs	Negative one.
	Tr	(writes on board as she speaks) Negative one, a times in brackets a positive one squared. One squared gives us what?
	Lnrs	One.
	Tr	(continues writing) One, one times a is a minus one. (Teacher underlines the final answer.)
	Tr	(continues writing) So you still get the same answer.
	Lnrs	Yes.
		(We can see the following which has been written on the board:
		
	Tr	(moves away from the board) So this is the equation that you were looking for. (*Teacher cleans part of the board, talks to some learners and starts writing*)
	Tr	Okay, what type of graph is that one?
	Lnrs	Tan.
	Tr	It's the graph for tan?
	Lnrs	Yes.
	Tr	So you use which equation? Y is equal to? (*Learners respond all together but it is not clear what they say*)
	Tr	$a\ tan\ x$ (*learners chorus with teacher*) and which co-ordinates were you given?
	Lnrs	Forty five degrees and two.
	Tr	Forty five degrees and?
	Lnrs	Two.
	Tr	Which one is the y co-ordinate?
	Lnrs	Two.
	Tr	It's two, so you put it here. a times tan. Our angle is?

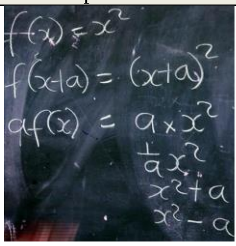
Episode		Transcript
	Lnrs	Forty five degrees. (*learners heard saying: yes*!).
	Tr	And what is tan forty five degrees? Forty five degrees? Is one point six?
	Lnrs	one, one.
	Tr	Tan forty five degrees?
	Lnrs	One.
	Tr	One, so you are going to have one times a, it gives you?
	Lnrs	a.
	Tr	So it means a is equal to two. You go back to your formula y is equal to two (*learners chorus in*) tan x. This is the one that you were looking for. (*teacher has written the following on the board. Many learners are heard saying YES!*)
		
		(*Teacher talks to a learner sitting in front of her desk but we are unable to hear what they are discussing*.)
	Tr	Please if you use another method like what that I am seeing here (*pointing downwards to the pupil in front of her, learners giggle and teacher turns back to the board*); I know you have got answers in the back of your text books.
	Lnrs	Yes.
	Tr	You just put this answer, two tan forty five. Please write corrections.
	Tr	Okay, what type of graph is e? (*Learners respond but we cannot hear what they say*.)
	Tr	It's a hyperbola?
	Lnrs	Yes.
	Tr	(moves to the right side of the board and begins writing) So which equation were you supposed to use?
	Lnrs	Y equals a over x. (*teacher writes* $y = \frac{a}{x}$ *on the board*)
	Tr	(*writes what the learners have called out*) a over x.
	Tr	(*turns and looks over her book*) Which points were you given?
	Lnrs	Four and negative two.
	Tr	Four and negative two.
	Lnrs	Yes.
	Tr	Okay. so which one is the y co-ordinate?
	Lnrs	Negative two.
	Tr	Negative two then a over four.
	Lnrs	Four.
	Tr	(*pointing at* $-2 = \frac{a}{4}$) Now how do we remove this four here? We are going to?
	Lnrs	Multiply.
	Tr	Everything by?
	Lnrs	Four.
	Tr	And to this side what do you get? (*left hand side)* (*Learners respond with "negative" but seem unsure.)*
	Tr	Negative?
	Lnrs	Six, eight.
	Tr	Negative?
	Lnrs	Eight.
	Tr	Negative eight.
	Tr	So you go back to substitute y is equal to negative (*learners chorus with teacher*) eight over x.
		(*teacher has written the following on the board)*
		

Episode		Transcript
Episode 3 Sketching graphs of functions: Example of $y = \left(-\frac{3}{2}\right)x^2$	Tr	Okay, so today we want to look at sketching graphs given the defining equations. You are still going to do, what we did in the other exercise, find the co-ordinates of, find the value of the x intercept and the y intercept; where your graph is going to cut the y axis and the x axis. And if it's having a turning point, we also want the co-ordinates of that turning point, okay?
	Lnrs	Yes.
	Tr	If there is an asymptote, we want you to show the asymptote. Can you please go to exercise thirteen point five. Exercise thirteen point five. It's page three twenty, I think the other one is three ten or three thirty.
	Lnrs	Three thirty.
	Tr	Three thirty, okay. Okay (*teacher looks down and starts reading from a text book).* When an individual draws sketch graphs of the following functions showing clearly any intercepts on the axis, the co-ordinates of one other point on the graph, the turning point or vertex where applicable. Asymptote and / or axis of symmetry should be shown where appropriate. For trig graphs show at least two periods . (*Teacher looks up at learners.)* What do they mean by that? Show at least two periods. Remember for trig graphs there is a certain… So when they say for two periods: Let's say you are starting from the y axis here. (*teacher demonstrates the periods on the board*) We want you to show this pattern where it's going to repeat itself. So you are supposed to show when it repeats itself twice. So if you start from here, you go in this direction. When you get to this point you are now starting to repeat the same movement. From this point, okay?
	Lnrs	Yes.
	Tr	So at least you draw this (*indicating to what she has drawn on the board*) and another one. That's two periods
	Lnrs	Oh.
	Tr	Okay, when the pattern starts to repeat itself (starts demonstrating on the "wave" pattern she has drawn on the board). From here you go down, you go up. When you start going down you are now starting, you are now repeating the same movement that you started with here, when you follow this. Okay, so we want to see that. So you are going to have a period twice. (*Teacher has the following graph on the board)* 
	Tr	We are going to do the first one together. You want us to do number eight?
	Lnrs	Yes, yes.
	Tr	You want number eight?
	Lnrs	Yes.
	Tr	You think it's difficult?
	Lnrs	Yes.
	Tr	That's very easy. Okay, y is equal to? (*Learners and teacher chorus together*) Negative three over two.
	Lnrs	x squared.
	Tr	This negative, what does it say, okay before we do anything, what type of graph is this? (Learners seem unsure.) What type of graph? (Pause) What type of graph is this? (long silence) I think one day, I think towards when you are having a period, towards lunch. I will stand by the door until you tell me the type, the different types of graphs (Learners laugh). Because we cannot be repeating this. What type of graph is this?
	Lnrs	Parabola.
	Tr	It's a parabola. Because you are having?
	Lnrs	x squared.
	Tr	And because there's a negative in, where is it going to be facing?
	Lnrs	Downwards.
	Tr	Downwards. That's the two things that you know about your graph already.
	Lnrs	Yes.
	Tr	Okay
	Tr	And is it going to cut the y axis? (*pause*) Is it going to cut the y axis?
	Lnrs	No, no.
	Tr	It's not going to cut the y axis? What is the value of x in the y axis?
	Lnrs	Zero.
	Tr	If we substitute here (*pointing to x in* $-\frac{3}{2}x^2$), what are we going to get?
	Tr	When x is equal to zero what are we going to get for y?
	Lnrs	Zero.
	Tr	It's a zero also?

Episode		Transcript
	Lnrs	Yes.
	Tr	(*plots the point* $(0;0)$ *on the graph*) So it means this graph is passing through this point (*the origin)* (*Learners mumble a response and the teacher gives a questioning look.*)
	Lnrs	Yes.
	Tr	Do you agree?
	Lnrs	Yes.
	Tr	What about in the x axis what is the value of y? In the x axis? What is the value of y in the x axis? (*Pause while she waits for learners to respond*) Hei guys, every day? What is the value of y in the x axis?
	Lnrs	One.
	Tr	It's one? (*Learners seem unsure. Some say "zero". Teacher turns and points to the board*.)
	Tr	When you are numbering your y axis, you start from what here? (*marking a point on the y-axis on the graph*)
	Lnrs	one
	Tr	What about the number here? (*points to a point below the origin*)
	Lnrs	Negative one
	Tr	So what is in between?
	Lnrs	Zero.
	Tr	So where is the zero? (*looks at a learner*) Huh?
	Lnr	At the origin.
	Tr	At the origin? (*Points to the origin)* So what is the value of y in the x axis?
	Lnr	One.
	Tr	It's one? (*she marks a 1 on the y axis*). So then here we are also having one? (Marks another point on the y axis.)
	Lnrs	No ... yes.
	Tr	Why? That's what you are saying.
	Lnrs	Zero.
	Tr	It's what?
	Lnrs	Zero.
	Tr	You guys, every day, hai! So in the x axis y is equal to?
	Lnrs	Zero.
	Tr	So you are going to substitute here. We want to find the value of x. It's still going to give us what? Zero. X is also zero (*she points to the graph on the left*). This is the point where it's going to cut the axis. (*Teacher marks a point at the origin*.) Let's substitute when x is one. When x is one, what is the value of y? (*Learners mumble and seem unsure*)
	Tr	(*looking at one learner*) Huh?
	Lnr	Zero.
	Tr	When x is one, y is zero?
	Lnrs	No, it's one.
	Tr	It's?
	Lnrs	One.
	Tr	It's one? (*smiling*)
	Lnrs	Yes.
	Tr	When x is one, y is one? (*Pauses, then points to the x squared on the board.)* Let's substitute. We are going to have one squared times three, negative three over two. What do you get? (Teacher finishes writing on the board, turns and faces the class.)
	Tr	What do you get? (*Pause, learners respond but we cannot hear*) Negative?
	Lnr	One.
	Tr	Negative one and a half?
	Lnrs	Yes.
	Tr	So it's still this one? (*pointing to a point on the graph*)
	Lnrs	Yes.
	Tr	It's negative, so negative is somewhere here?
	Lnrs	Yes.
	Tr	(*still pointing at the graph on the left side)* Let's take the negative value here. What do we get? What do you get? Substitute with one here (*points to the board, pauses*). Negative one. (*pause*). Negative one squared gives us what? *(pause*) Negative one squared?
	Lnrs	Negative one.
	Tr	Negative one?
	Lnrs	Yes. (*teacher writes the following on the board,* $-1\ \times -1$ *and then looks at the learners.*)
	Lrn	That's a positive.
	Lnrs	Positive one.
	Tr	It's a positive one?

Episode		Transcript
	Lnrs	Yes.
	Tr	Positive one times this? We are still going to get negative three over?
	Lnrs	Two. (*teacher has written the following on the board*) 
	Tr	Which is somewhere here (*points a point on the graph below the x-axis*). So already you can see where your graph is going.
	Lnrs	Yes.
	Tr	The more we take smaller values here (*indicating on the left of the x axis*) and bigger values here (*indicating to right of the x axis)* we are still going to get smaller values of y (*indicating downwards of the y axis*).
	Lnrs	Y
	Tr	Because we are having a negative, okay?
	Lnrs	Yes.
	Tr	So our graph is facing downwards. (*draws the following graphs*) 
	Tr	And it's facing downwards because we already know that we are having a negative here (*points to -3 on $y = -\frac{3}{2}x^2$*). We are having a negative here. This is where its, it turns at the origin. And it's facing (*indicating a downward motion with her hands*)?
	Lnrs	Downwards.
	Tr	(*looking at a learner*) Ja? (*Learner says something but we cannot hear what is said*.)
	Tr	Yes?
	Lnr	.... a question.
	Tr	You have got a question?
	Lnr	Ja.
	Tr	Yes? (*Learner says something but we cannot hear clearly*)
	Tr	You want to write it down?
	Lnr	Yes, yes.
	Tr	Okay come here (*teacher moves away from the board, the learners are talking amongst themselves).*
	Tr	Okay so all of these sketching, it comes down to what we were doing (*male learner walks to the front of the class, the camera then moves right to where the teacher is standing*) in that table where we were moving the parabola y is equal to x squared. (*Pauses, waiting for the learner to finish at the board. Some of the learners begin to giggle and the teacher is also giggling*.) Ha, what type of graph is that one? (*learners respond but what they say is not clear)*
	Tr	(*smiling*) Hey? What type of graph is that one? What type of graph is that one? (*the learner ha written $y = \left(-\frac{3}{2}\right)^2 x^2$ on the board*)
	Tr	It's a?
	Lnrs	Parabola.
	Tr	It's still a parabola?
	Lnrs	Yes.
	Tr	It's still a parabola. You are having?
	Lnrs	x squared.
	Tr	It's still a?
	Lnrs	Parabola.
	Tr	Now the difference is here you are now having one. (*Begins writing on the board*.) This is nine over?
	Lnrs	Four.
	Tr	x squared. (*teacher has written the function as $y = \frac{9}{4}x^2$*)
	Tr	But this is still a parabola and this one is facing? (*Indicating an upward motion with her hands*.)
	Lnrs	Upwards.

Episode		Transcript
	Tr	Because?
	Lnrs	There is no negative.
	Tr	It's positive.
	Tr	Okay, so I wanted to see if you can identify... Remember on that table, let's go back to that table: the one that we did on y is equal to x squared when you were adding different functions. Are we there?
	Lnr	Yes.
	Tr	Now the different graphs or the different functions that we know are the core of how you are going to sketch the graph. Now we have been given minus three over two x squared, and we said this is a parabola. We know the shape of the parabola? (*camera zooms in on a learner's book which shows the table teacher is talking about*) 
	Lnrs	Yes.
	Tr	Okay. This is the one that we know?
	Lnrs	Yes.
	Tr	Now if you go to that table, whenever we added and this is the same as saying function of x is equal to x squared.
	Lnrs	x squared.
	Tr	When we added a inside of the bracket (*writes* $f(x+a)$*below* $f(x)=x^2$*)* Which axis did this a effect?
	Tr	Did it affect the y values or the x values? (*Learners respond we can't hear clearly what they said*.)
	Tr	It affected the x values. So when it's inside here it affects the x values? And how do you see that it's inside your bracket? Now here you'll be having x plus a to the power?
	Lnrs	Two. (*teacher writes* $f(x+a)=(x+a)^2$)
	Tr	When I had a f of x (*writes* $af(x)$ *below* $f(x+a)$) Which values did it affect?
	Lnrs	y, the y.
	Tr	The y values?
	Lnrs	Yes.
	Tr	Because you are having a times x to the power two.
	Lnrs	Two.
	Tr	Now, which bring us back to this one. (*pointing to* $y=-\frac{3}{2}x^2$). We are saying this is function of x, so in other words we are saying negative three over two function of x *(writes* $-\frac{3}{2}f(x)=-\frac{3}{2}x^2$). This is what we are saying. (*points to what she has just written*) So just by looking at this, it's affecting which values?
	Lnrs	y values.
	Tr	The y values.
	Lnrs	Yes.
	Tr	It means all the y values for this parabola [$f(x)=x^2$] which is our core are going to be multiplied by negative three over?
	Lnrs	Two.
	tr	Only the y values (*chorused together with learners*). The x values are meant to remain the same. But the y values are going to be affected, we are going to multiply every y value by negative three over two (*chorused together by learners*). So this is basically what we are saying, so now if you know the type of graph that you are having from the expression that they have given you, you can now go to your expression and try to make sense what happened here. Is this change or is this addition affecting the x value or the y value? Is it affecting the x value or is it affecting the out, the y value? If it's affecting the y value it will be outside. It's multiplying your function, it's dividing your function, it's an addition to your function or it's a subtraction to your function. Which is outside? Okay?
	Lnrs	Yes. (*teacher has written the following on the board*)

Episode		Transcript
		
	Tr	All this is affecting your (*learners chorus in*) your y values. So this is basically what you should look for when you are given an expression. Is it affecting the x value or the y value? Then you start from there. Okay?
	Lnrs	Yes.
Episode 4 Class activity on sketching graphs of functions	Tr	Can you please do exercise (*teacher pauses while writing on the board looking in the textbook*).
	Lnrs	Thirteen point five.
	tr	Thirteen point five: number one *(turns and refers to her book, continues writing)* number two (*pause*) number four and number seven. (*teacher starts walking around the classroom while the learners are doing the work*)
	Tr	Is everyone having a pencil?
	Lnrs	Yes.
	Tr	I want to see them. (*learners are working and talking while the teacher walks around the classroom*)
	Tr	(*looking at learners*) Are these people absent or outside?
	Lnrs	Absent
	Tr	All of these people?
	Lnrs	Yes.
	Tr	Ha (*shakes her head, and the turns to help a learner*).
		(*learners continue to talk, work and pass books around. Teacher is walking around the class assisting learners*.)
	Tr	Okay, on Monday can you please make sure you are going to do a .... of this during the weekend, okay?
	Lnrs	Yes.
	Tr	(*holding up a piece of paper*) We will discuss it on Monday. I think everyone is having this sheet?
	Lnrs	No.
	Tr	Who does not have it? How? Why? (*learners respond*) I gave everyone.
	Lnrs	No. (*Teacher turns and listens to what some of the learners are saying*)
	Tr	How many people are not having this? (*Learners respond, teacher points to one learner.)* Even I gave you. (*She shakes her head says something I cannot hear.)* I remember I gave you. (*Looks at another learner and begins counting.*)
	Tr	Please raise your hand, I want to count how many of you. (*Learners raise their hands. Teacher counts and then moves to her desk at the front of the classroom*.)
	tr	(*starts walking around the classroom*) And remember it's not a true graph, it's a sketch. We are sketching.
	Lnrs	Yes. (*Teacher continues to move around the class and assist learners and see what they are doing. Siren goes off in the background. Teacher moves to front of class after some time)*
	Tr	Okay, guys. (*The learners are making a lot of noise*.) Hey, can we please go to the staff now?
	Lnrs	Yes.
	Tr	Let me go with and collect the other papers for those who don't have. And please give them to the people who are not having. Yesterday I left some, some papers here. (*She looks down at the learner sitting in front of her and they discuss something which we cannot hear.*)
	Tr	(*looks up smiling*) Okay, I will give you two on Monday.