As a result, the measurement system was applied to ascertain the real time motion, and hence the frequency content of the motion, as a function of shaft depth, based on a single or dual camera measurement. This information could then be employed to identify the natural frequencies of the catenary, and the location of maximum modal response relative to shaft depth.

I.5 Commissioning Tests

During the commissioning tests, a total of three test measurements were performed at Elandsrand Gold Mine. The purpose of the measurements was primarily directed at commissioning and evaluating the performance and versatility of the video system. The first two tests were performed on the Siemens Rock Vent Shaft catenaries, whilst due to availability problems, the third test was performed on one of the adjacent rock winders. The test configurations are listed below.

I.5.1 Measurements Performed

(i) Test 1 - This consisted of simultaneously measuring the overlay and underlay motion of the catenaries on the Siemens RV winder. The cameras were positioned vertically below each catenary.

(ii) Test 2 - This consisted of focussing one camera on the underlay catenary of the Siemens RV winder, whilst the other camera was focussed on its corresponding vertical rope at the shaft collar.

(iii) Test 3 - The purpose of this test was to evaluate the feasibility of monitoring the 2D motion of the rope by locating each camera at a base distance from the vertical plane of the catenary, and aiming the cameras at the same portion of rope, as illustrated in figure 1. The cameras were inclined at approximately 40° from the horizontal plane. The test was performed on an adjacent rock winder as the Siemens RV winder had stopped winding due to a bin blockage.

I.5.2 Measurement Results

The results of the measurements for each test are presented graphically in figures I.3,I.4,I.7. These figures present the displacement time traces, which clearly illustrate the gross motion across the drum, followed by a layer change at the end of a traverse. The vertical scale on all plots represents pixels; One rope diameter is approximately 20 pixels. Thus, the system measures to well within one rope diameter of motion. In addition, the higher frequency components of the motion lie within the bandwidth of the system[1]. Figure I.9(a,b) represents a section of the data trace extracted from figure I.7(a), and illustrates the consistency of the measured data.

The time traces presented in figure I.3 for test 1, indicate a difference in the dynamic behaviour of the underlay and overlay catenaries. This arises as a result of the different catenary lengths of the underlay and overlay ropes. The overlay rope exhibits a lower overall amplitude of vibration during the winding cycle, whereas the underlay rope clearly experiences dynamic amplification. Consequently this measurement illustrates the sensitivity of the system to small changes in physical parameters. This behaviour complicates the process of defining the optimum hoisting speed, such that low amplitude vibration is achieved for both the overlay and underlay catenaries during ascent and descent.

Cascaded autospectra of the data captured in test 2 was constructed for the catenary and vertical rope, during the ascending and descending cycle. The cascade was constructed by applying a moving data window with a 50% overlap, and calculating the autospectrum for each window sequentially. This data is presented in contour form in figures I.5,I.6. It is evident that the natural frequencies of the catenary increase when the skip is descending, and decrease on ascent. This is to be expected, as the natural frequencies are directly related to the rope tension in the catenary, which increases with depth due to the increasing rope mass.

In order to construct the in and out-of-plane motion as measured in test 3, a transformation was applied to the measured data[2], to obtain data in a cartesian co-ordinate system, aligned with the vertical and horizontal axes. At the same time the data was calibrated to physical dimensions by measuring the rope diameter in each frame for each camera independently[3]. By assuming an average rope diameter of 48mm, the motion was converted to a displacement measurement. The physical motion of the rope in the in-plane (vertical) and out-of-plane (horizontal) directions is presented in figure I.8. A number of observations can be made concerning the transformed data. During an ascending

[1]On typical mine hoisting installations, the first catenary mode occurs at approximately 1 Hz. Thus the measurement system should track up to the twentieth mode without aliasing problems.

[2]Each camera was inclined at an angle of 40° from the horizontal plane.

[3]This process accounted directly for the differing amounts of zoom on each camera.

cycle the rope tension decreases with shaft depth due to the changing mass of the vertical rope. Consequently the catenary sag increases, which would represent a downward drift of the mean position of the catenary. The opposite would occur during a descending cycle. This behaviour is clearly evident in figure I.8(a,b). It would also be expected that due to rope curvature, the rope would rise as it approached the maximum motion in the out-of-plane direction. An expanded plot of the in plane and out-of-plane motion as presented in figure I.8, presented in figure I.9(c,d) confirms this behaviour. The change in mean position at the end of the descending cycle and at the beginning of the ascending cycle, as illustrated in I.8(b), occurs due to loading the conveyance.

I.6 Discussion

The data obtained during test 2 was considered the most pertinent for discussion, since it contains information concerning the catenary and vertical rope motion, and reflects the coupling between the two systems. Figures I.5,I.6 present the frequency content versus depth records of the catenary and vertical rope respectively.

I.6.1 Catenary Response

Test 2 consisted of monitoring the catenary motion with a full skip ascending, and subsequently an empty skip descending. A number of features are evident in figure I.5, and are noted below.

(i) On ascending, the catenary frequencies decrease as the catenary tension decreases.

(ii) On reaching the shaft collar, the skip is emptied, thus the catenary tension decreases over a short time interval; consequently a discontinuity is evident between the corresponding harmonics.

(iii) The natural frequency lines exhibit a higher slope on decent than on ascent.

(iv) Closer scrutiny of the initial and final ascent and descent regions, where the skip is accelerating to constant speed or decelerating from the winding speed until it is stationary, reveals that a local change in slope of the frequency lines occur. This is consistent with an increase/decrease in catenary tension due to inertial loading during acceleration and deceleration respectively.

(v) The Lebus coil-cross over excitation frequency is evident at approximately 1.8 Hz. This compares favourably with the calculated value of 1.73 Hz.

(vi) Harmonics of the Lebus coil-cross over excitation frequency are evident at various stages of the wind. For instance the second harmonic appears continuously, whilst the fourth and fifth harmonics of the coil cross over frequency are evident in the first phase of ascent.

Accurate data pertaining to the winder and catenary configuration was not available, and correlation of the natural frequencies with analytically calculated values was not attempted. However, excellent correlation between measured and calculated natural frequencies was achieved at Wildebees South 9 shaft, where the physical parameters of the system were accurately measured.

I.6.2 Lateral Response of the Vertical Rope

A cascaded spectral contour plot of the lateral response of the vertical rope is presented in figure I.6. The behaviour of the vertical rope, as reflected in figure I.6 is related to the motion of the catenary.

Possible sources of excitation to the vertical rope consist of:

- Parametric longitudinal excitation due to the coil-cross over excitation at the winder drum.
- Autoparametric excitation due to the lateral response of the catenary.
- Lateral excitation at the skip due to guide misalignment at the buntons.
- Lateral response of the sheave and supporting structure due to the lateral forces applied by the catenary.

On examination of figure I.6, the following observations are made:

- Vibration levels are higher on ascent, where the lateral motion of the vertical rope at the shaft collar rises to an amplitude of approximately 0.25 m peak to peak.

- Frequency trends are apparent which correlate with the catenary response. Namely, on ascent, the contour plot reflects a band of response spanning the second and third catenary natural frequencies, and narrower bands corresponding to the fourth and sixth catenary natural frequencies.

- There is no clear indication of the Lebus coil-cross over excitation frequency, or the bunton passing frequency.

Dimitriou and Whillier[1973] proposed that parametric response of the vertical rope would be induced by lateral motion in the catenary. The existence of lateral response at frequencies related to the natural frequencies of the catenary tends to support such a relationship. However, it is also possible that this may be caused by lateral response of the sheave and its supporting structure to the catenary forces, and hence direct excitation of the vertical rope. Thus it is probable that both parametric and direct excitation apply simultaneously.

Pursuing the hypothesis of Dimitriou and Whillier[1973], the lateral response of the vertical rope spanning the second and third catenary frequencies would represent main parametric resonance of the vertical rope due to the lateral motion of the catenary in the second and third mode. Since response of the fourth catenary mode is not evident, the response of the vertical rope at a frequency related to the fourth catenary mode may represent a secondary parametric resonance region where the lateral natural frequency of the vertical rope tunes to the parametric excitation induced by lateral response of the second lateral catenary mode. This argument may be applied to explain the response at the sixth catenary mode. For instance if f_p represents the parametric excitation frequency induced by the catenary motion, the following relationships may hold.

$$f_p = 2f_2^{catenary} = f_4^{catenary} = f_n^{vertical}$$

$$f_p = 2f_3^{catenary} = f_6^{catenary} = f_n^{vertical}$$

Where response occurs at the n^{th} lateral natural frequency of the vertical rope $f_n^{vertical}$.

In fact the presence of the fourth catenary mode in the lateral response of the vertical rope, presents the most convincing evidence of autoparametric response. Since the lateral motion of the fourth catenary mode is not evident, the support structure cannot be excited at this frequency, and thereby provide direct excitation to induce lateral response of the vertical rope. Since no other explanation describes the presence of the fourth mode in the lateral response of the vertical rope, it is attributed to autoparametric excitation by the second catenary mode, which excites the second region of main parametric resonance, as described above.

I.7 Summary

The results obtained whilst commissioning the system confirmed the success of the measuring technique. The system was passive and did not interfere with production, and its resolution was well within one rope diameter. Although two dimensional measurement of the rope was not considered practical for highly accurate measurements, the transformed time traces presented in figure I.8 indicates that the approximate motion of the response can be successfully extracted. Two cameras were applied to capture the motion of two ropes simultaneously. This provided evidence of autoparametric excitation of the longitudinal rope due to lateral response of the sheave, thus emphasising the coupling between the catenary and vertical rope. The simultaneous measurement of the underlay and overlay catenaries indicated that small differences in catenary length can result in appreciable differences in the response.

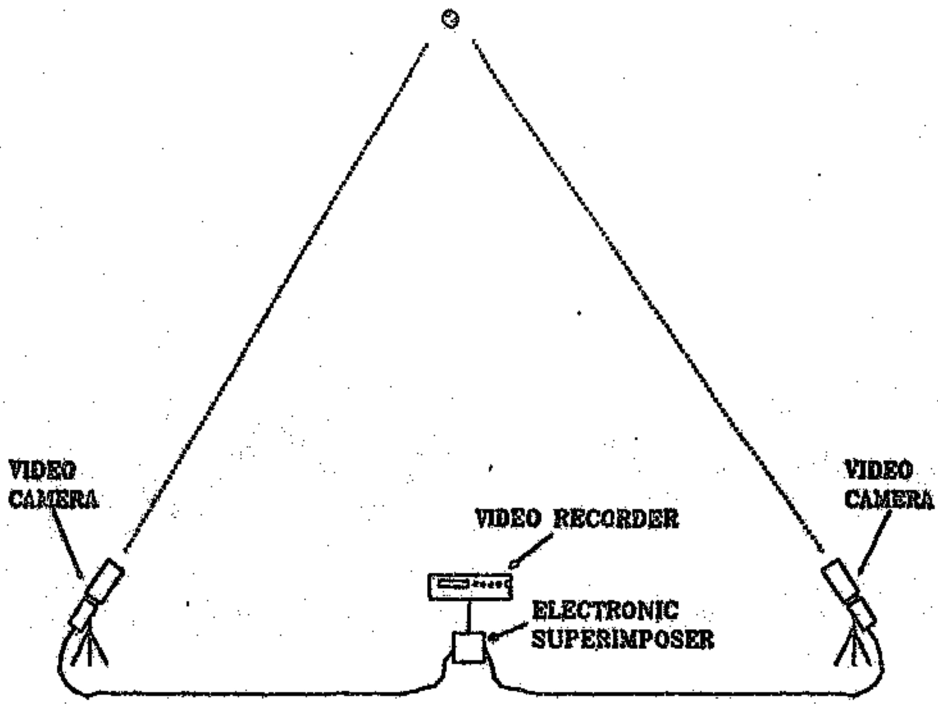

Figure I.1: Video camera configuration

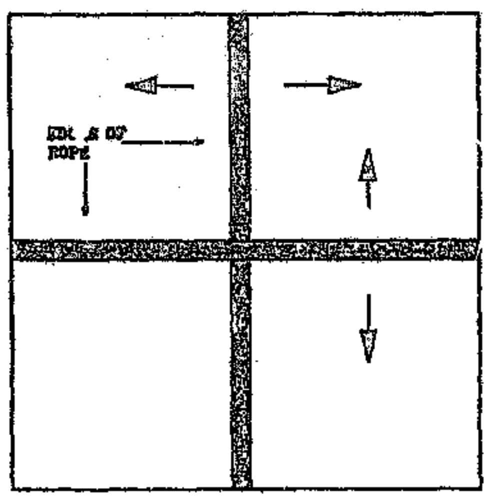

Figure I.2: Mixed video image of the rope

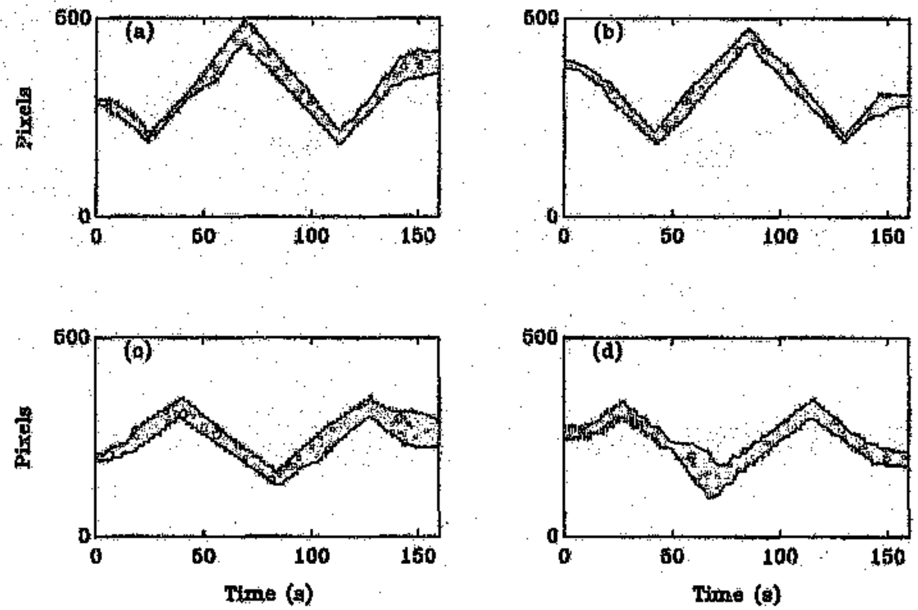

Figure I.3: Test 1: Time trace

Siemens RV winder, Elandsrand Gold Mine
(a) Overlay Catenary - Descending Conveyance. (b) Overlay Catenary - Ascending Conveyance.
(c) Underlay Catenary - Ascending Conveyance. (d) Underlay Catenary - Descending Conveyance.

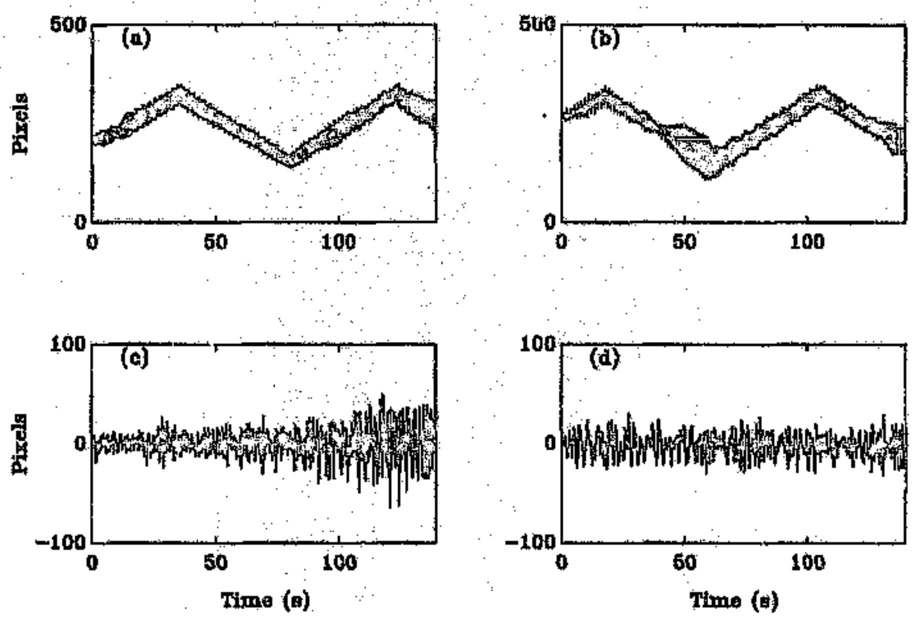

Figure I.4: Test 2: Time trace

Siemens RV winder, Elandsrand Gold Mine
(a) Underlay Catenary - Ascending Conveyance. (b) Underlay Catenary - Descending Conveyance.
(c) Vertical Rope - Ascending Conveyance. (d) Vertical Rope Catenary - Descending Conveyance.

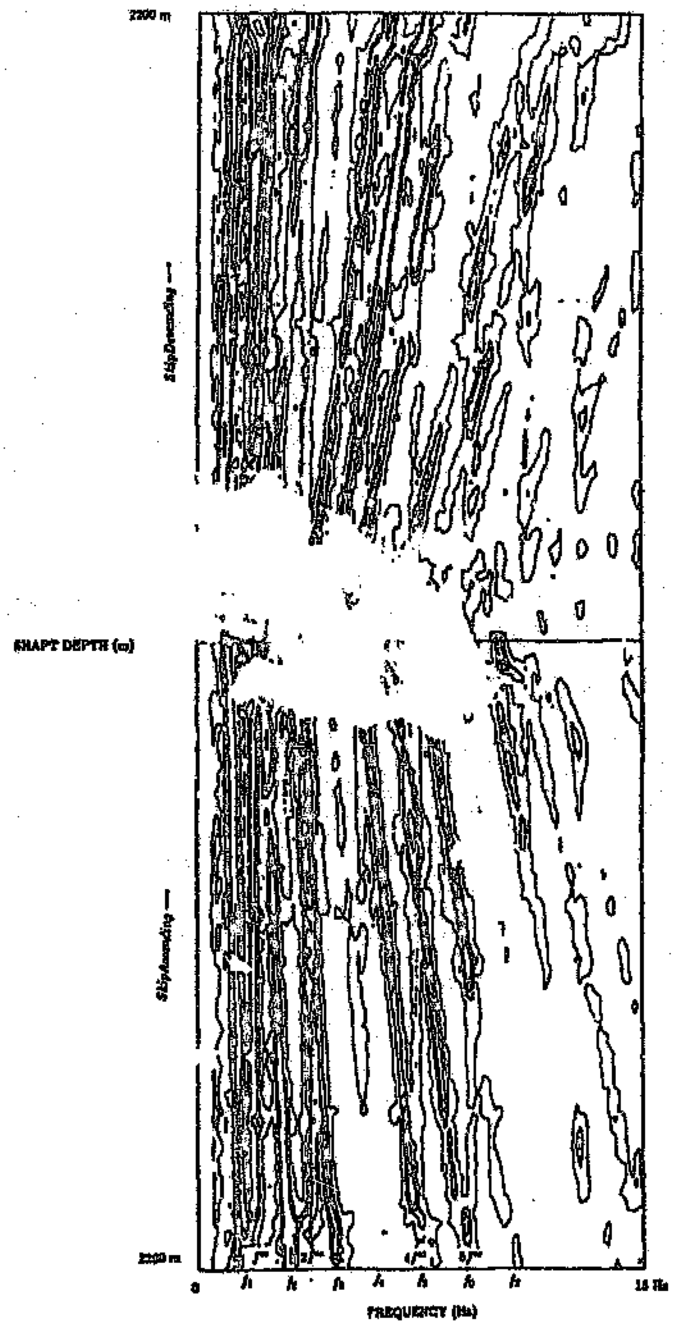

Figure I.5: Cascaded spectral contour plot

Siemens RV Winder Underlay Catenary, Elandsrand Gold Mine
f_n - n^{th} lateral natural frequency of the catenary.
nf^{cc} - n^{th} coil cross-over excitation frequency.

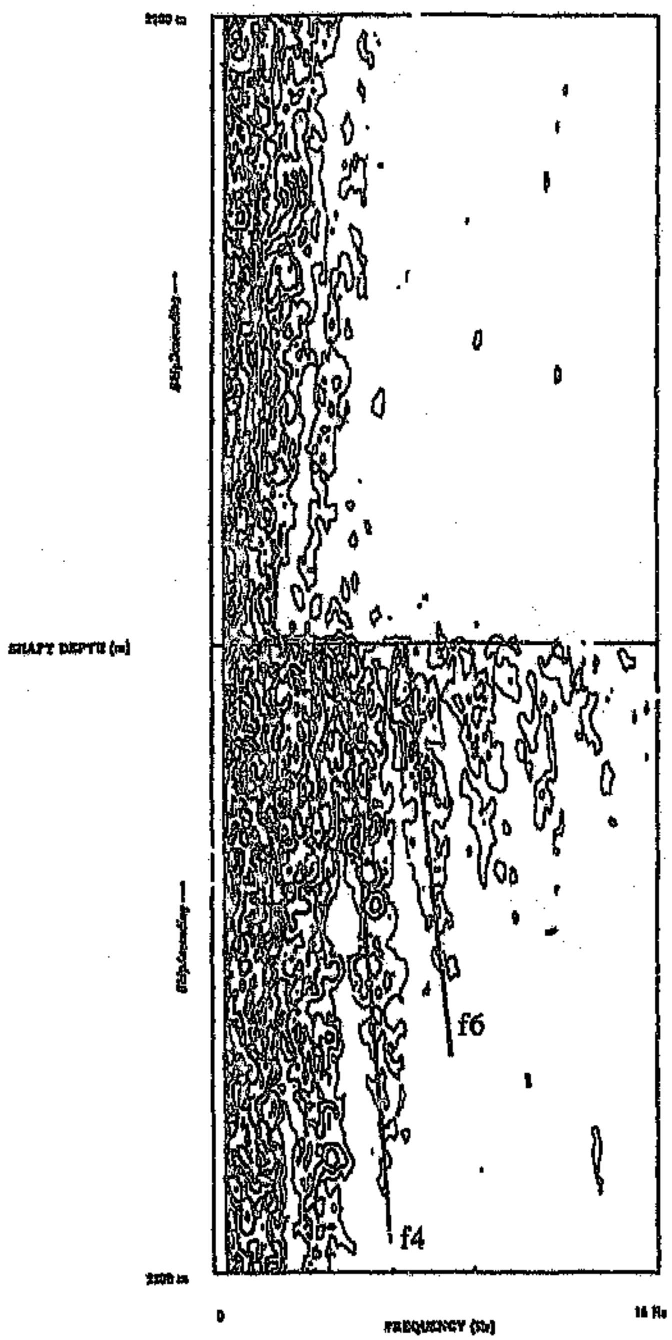

Figure I.6: Cascaded spectral contour plot

Siemens RV Winder, Vertical Rope at Shaft Collar, Elandsrand Gold Mine

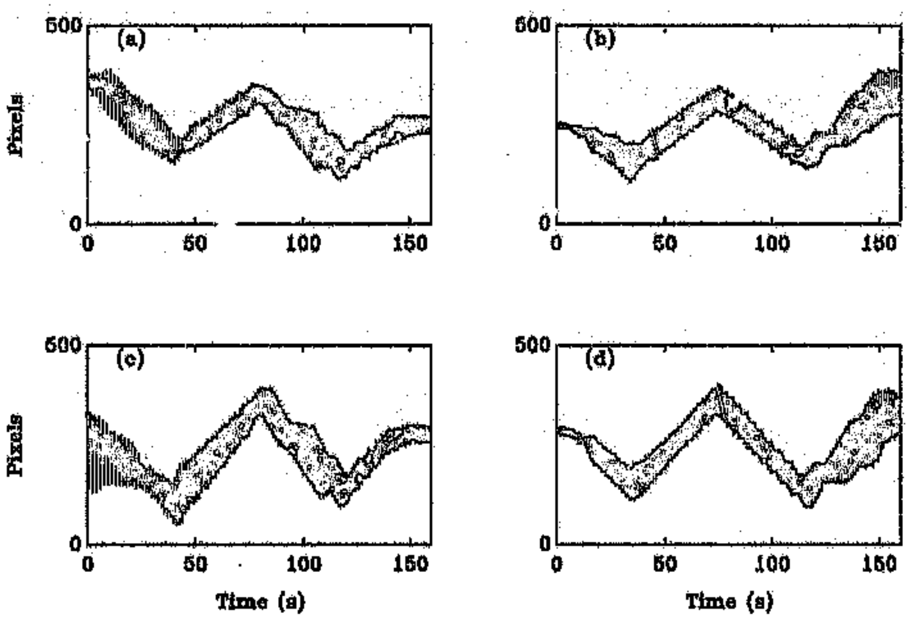

Figure I.7: Test 3: Time trace

Rock winder, Elandsrand Gold Mine
(a) Camera 1 - Ascending Conveyance. (b) Camera 1 - Descending Conveyance.
(c) Camera 2 - Ascending Conveyance. (d) Camera 2 - Descending Conveyance.

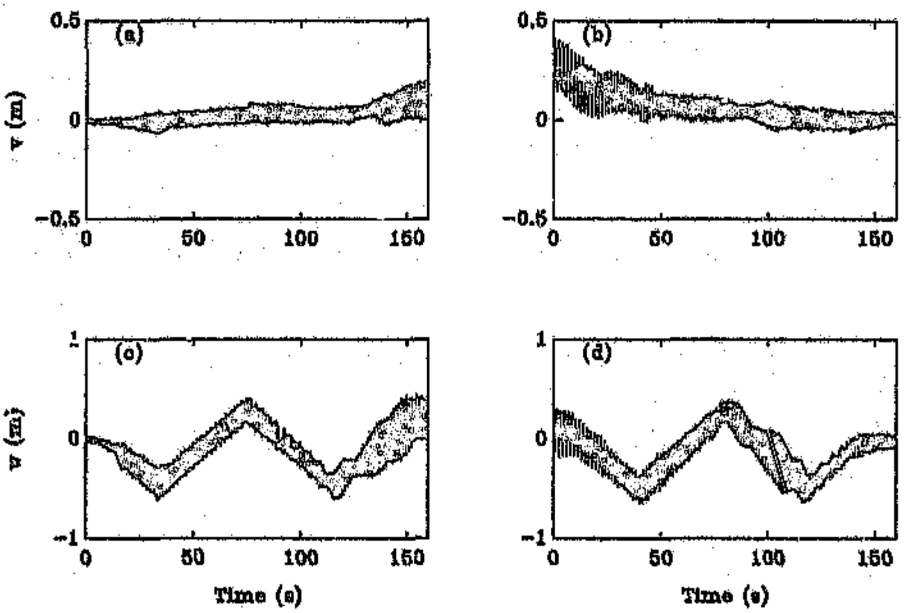

Figure I.8: Test 3: Transformed time trace

Rock winder, Elandsrand Gold Mine

(a,c) Descending Conveyance. (b,d) Ascending Conveyance.
(a) In-plane Motion. (b) In-plane Motion.
(c) Out-of-plane Motion. (d) Out-of-plane Motion.

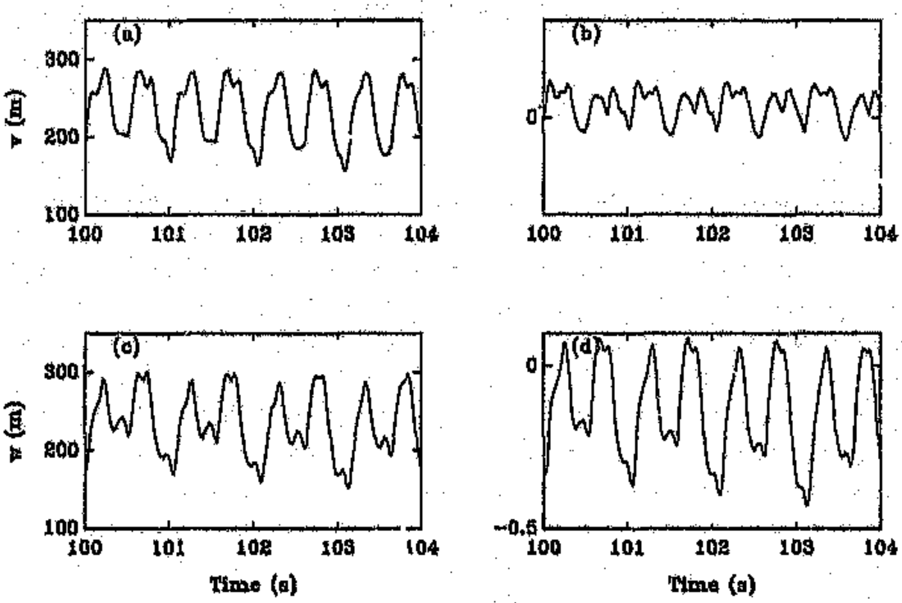

Figure I.9: Expanded time trace: Test 3

Rock winder, Elandsrand Gold Mine
(a) Camera 1 - Ascending Conveyance. (b) In-plane Motion - Ascending Conveyance.
(c) Camera 2 - Ascending Conveyance. (d) Out-of-plane Motion - Ascending Conveyance.

Appendix J

Stationary Stability Analysis - Further Considerations

In Chapter 4 the stability of the first approximation to the planar steady state motion was examined. The stability of the the motion was investigated as a criterion to identify areas where nonlinear coupling could be expected. The analysis confirmed the existence of regions of simple and combination parametric resonance related to the lateral catenary and longitudinal system modes. It is important to note that the steady state solution which was applied as the basis for the stability analysis, represents a linear solution. This solution is only valid away from regions of primary external resonance of the catenary. Normally, the steady state solution is approximated via a harmonic balance or perturbation technique, and results in an approximate nonlinear description of the response, which may be multi-valued. The stability of the approximate nonlinear steady state solution then dictates which branch of the solution is physically realisable. This leads to the identification of the jump phenomenon, and subcritical stability where more than one stable solution occurs simultaneously. Although this aspect was appreciated, the linear steady state solution was purposely applied as a datum for the stability analysis. The rational for this was dictated by the need for a practical design tool in industry, and it was considered that a nonlinear steady state solution would introduce complexities beyond the scope of a typical design team. It is also likely that a condition of external resonance of the catenary will occur during the winding cycle. The system parameters can be chosen to shift the condition of resonance with respect to shaft depth, and thereby change the tuning between the lateral and longitudinal modes. Thus by utilising the linear solution as a datum for the stability analysis, the eigenvalue exponent provides a direct indicator of the severity of the region, reflecting the compliance of the lateral to longitudinal system tuning.

Although the approach adopted in Chapter 4 is practical for industrial needs, and this constitutes the primary motivation for its adoption, a more conventional approach where the nonlinear steady state solution is constructed could provide further insight into the system behaviour. This appendix presents a discussion regarding the potential nonlinear behaviour of the system. This is achieved by considering studies of similar nonlinear multi-degree-of-freedom systems presented in the literature. In the process, techniques applied in nonlinear studies are briefly discussed. Finally, by applying the method of multiple scales, system tuning which may induce conditions of secondary resonance and internal resonance in the system, are identified. Such conditions confirm the results of Chapter 4 concerning simple and combination resonance. However, the steady state analysis is not attempted. Such an analysis would require the study of a series of tuning conditions, the development of which is presented as an incentive for further study and research.

J.1 Nonlinear Equations of Motion

The nonlinear partial differential equations of motion for the mine hoist system were developed in Chapter 3. These equations were simplified in Chapter 4, where the linear steady state stationary nature of the system was examined, in the absence of catenary curvature and transport velocity. In Chapter 5 the nonlinear partial differential equations of motion of the system, as developed in Chapter 3, were transformed into nonlinear ordinary differential equations via a normal mode expansion. A numerical simulation was applied to account for the non-stationary nature of the system as well as steady state and transient excitation. The nonlinear ordinary differential equations of motion considered in Chapter 5 are reproduced below:

$$\ddot{p}_i + 2\zeta_u\omega_i\dot{p}_i + \omega_i^2 p_i + (\frac{v(0,t)}{l_c})U_{ij}^u r_j + K_{ij}^u q_j + \Gamma_{ijk}[r_j r_k + q_j q_k] = P_i(t) \tag{J.1}$$

$$\begin{aligned}\ddot{q}_i + 2(\zeta_w\overline{\omega}_i + V\mathcal{V}_{ij})\dot{q}_i + \overline{\omega}_i^2 q_i &+ \Delta_{ijk}p_j q_k \\ +\beta_{ijkl}(\frac{3}{2}q_j q_k q_l + \frac{1}{2}r_j r_k q_l + r_j q_k r_l) &+ \gamma_{ijk}q_j q_k + \\ \epsilon_{ij}p_j + \eta_{ijk}(q_j q_k - r_j r_k) - \kappa K_{ij}q_j &+ \frac{v(0,t)}{l_c}(\zeta_{ijk}r_j q_k + K_{ij}^w r_j) = Q_i(t)\end{aligned} \tag{J.2}$$

$$\begin{aligned}\ddot{r}_i + 2(\zeta_v\overline{\omega}_i + V\mathcal{V}_{ij})\dot{r}_i + \overline{\omega}_i^2(1 + (\frac{c}{c_l})^2(\frac{v(0,t)}{l_c})^2)r_i &+ \Delta_{ijk}p_j r_k \\ +\beta_{ijkl}(\frac{3}{2}r_j r_k r_l + \frac{1}{2}q_j q_k r_l + q_j r_k q_l) &+ \alpha_{ijk}q_j r_k + \\ \frac{v(0,t)}{l_c}\left[U_{ij}^v p_j + \delta_{ijk}(3r_j r_k + q_j q_k) + K_{ij}q_j\right] &= R_i(t)\end{aligned} \tag{J.3}$$

where $p_i(t)$, $q_i(t)$, $r_i(t)$ represent the i^{th} principal co-ordinate in the longitudinal, in-plane and out-of-plane lateral directions respectively. T. coupling coefficients contained in these equations are described in Chapter 5.

Although it may be possible to conside equations by means of analytical techniques, the purpose of this appen better served by simplifying the equations at hand. Firstly the analysis presented will consider the planar stationary system in the absence of rope curvature. In this regard only the $p_i(t)$, $r_i(t)$ co-ordinates will be considered, and in the absence of curvature and rope velocity the equations of motion reduce to:

$$\ddot{p}_i + 2\zeta_u\omega_i\dot{p}_i + \omega_i^2 p_i + (\frac{v(0,t)}{l_c})U_{ij}^u r_j + \Gamma_{ijk}r_j r_k = P_i(t) \tag{J.4}$$

$$\begin{aligned}\ddot{r}_i + 2\zeta_v\overline{\omega}_i\dot{r}_i + \overline{\omega}_i^2(1 + (\frac{c}{c_l})^2(\frac{v(0,t)}{l_c})^2)r_i + \frac{v(0,t)}{l_c}U_{ij}^v p_j \\ +3\frac{v(0,t)}{l_c}\delta_{ijk}r_j r_k + \Delta_{ijk}p_j r_k + \frac{3}{2}\beta_{ijkl}r_j r_k r_l &= R_i(t)\end{aligned} \tag{J.5}$$

These equations form the basis for the ensuing discussion, and will be considered further with regard to the possibility of secondary resonances, and conditions of internal resonance.

J.2 Nonlinear Methods of Analysis

The purpose of the analysis presented is to extract salient features of the equations of motion developed. Existing analyses in the literature are discussed where pertinent. A brief discussion concerning a few of the available strategies for analysing weakly nonlinear systems are presented. These strategies can be found in detail in Nayfeh and Mook[1983] , Nayfeh[1973b], Schmidt & Tondl[1986], Hagedorn[1988].

Weakly nonlinear systems are characterised by periodic oscillations occurring in the region of a stable focus or centre. In essence perturbation methods attempt to approximate the nonlinear solution by perturbing its linear counterpart, and thus periodic expansions are developed about a centre or equilibrium position to approximate the nonlinear oscillations occurring in the neighbourhood of the centre. Current methods comprise of the straight forward expansion, the Lindstedt-Poincare' method, the method of multiple scales, the method of harmonic balance, and averaging methods. It is established that the straight forward expansion is not uniformly valid, however the latter procedures can be designed to overcome this problem. Nayfeh and Mook[1983], briefly discuss these methods with regard to the nonlinear oscillator:

$$\ddot{x} + \sum_{n=1}^{N} \alpha_n x^n = 0$$

<u>The straight forward expansion</u>

In this method, the solution is sought as an expansion with respect to a small parameter ϵ in the neighbourhood of the linearised solution x_1.

$$x(t,\epsilon) = \epsilon x_1(t) + \epsilon^2 x_1(t) + \epsilon^3 x_1(t) + \cdots$$

Substituting the expansion into the equation of motion and equating powers of ϵ, leads to a sequence of linear equations, each containing terms from the preceding order. The method successively solves the equations to obtain the solution to order ϵ^n. However, the expansion may not be uniformly valid, since as the expansion progresses, secular terms appear in higher order equations These terms result in a non-periodic solution, as well as being unbounded with respect to time. Thus the situation arises whereby higher order terms x_n contribute to the solution more significantly than lower order terms.

The Lindstedt-Poincare' Method

In this method, it is assumed that the frequency and amplitude of the motion interact, a phenomenon commonly associated with nonlinear systems. By introducing a new independent variable $\tau = \omega t$, where ω is an unspecified function of ϵ, an expansion for ω can be devised such that the secular terms are eliminated. By assuming expansions of the form:

$$\omega = \omega_o + \epsilon\omega_1 + \epsilon^2\omega_2 + \cdots$$

$$x(t,\epsilon) = \epsilon x_1(\tau) + \epsilon^2 x_1(\tau) + \epsilon^3 x_1(\tau) + \cdots$$

$$\tau = \omega t$$

and ensuring that the secular terms vanish, a uniformly valid expansion can be achieved. This method results in periodic expressions for the motion, as well as a frequency amplitude relationship, higher order harmonics in the higher order expansions, and drift or streaming terms.

The Method of Multiple Scales

Nayfeh[1965] proposed the method of multiple time scales as an approach to analyse the periodic solutions of weakly nonlinear systems. In this method, the solution is considered to be a function with multiple independent variables, or time scales, where the number of scales is determined by the order of the expansion required. Thus in general the expansion is sought as:

$$T_n = \epsilon^n t$$

$$x(t,\epsilon) = \epsilon x_1(T_o, T_1, T_2, \cdots) + \epsilon^2 x_1(T_o, T_1, T_2, \cdots) + \epsilon^3 x_1(T_o, T_1, T_2, \cdots) + \cdots$$

It follows that:

$$\frac{d}{dt} = \frac{dT_o}{dt}\frac{\partial}{\partial T_o} + \frac{dT_1}{dt}\frac{\partial}{\partial T_1} + \cdots = D_o + \epsilon D_1 + \cdots$$

$$\frac{d^2}{dt^2} = D_o^2 + 2\epsilon D_o D_1 + \epsilon^2(D_1^2 + 2D_o D_2) + \cdots$$

Back substitution into the equation of motion, and equating powers of ϵ leads to the definition of solvability conditions, dependent on eliminating secular terms from the higher order equations. Satisfaction of these conditions leads to the approximate nonlinear solution.

The Method of Harmonic Balance

Unlike the previous methods, the method of harmonic balance does not depend on the concept of a small parameter. In this method the periodic steady state nonlinear solution is assumed to be represented by a trigonometric expansion of the form:

$$x = \sum_{m=0}^{M} A_M cos(m\omega + \beta_m)$$

The trigonometric expansion is substituted into the equation of motion, and the harmonics are equated to zero, resulting in a set of nonlinear algebraic equations in terms of the amplitudes A_n , and frequency ω. Thus a frequency amplitude relationship is implicitly contained in the method. Nayfeh and Mook[1983] show that unless a sufficient expansion is assumed in the initial solution, erroneous results may result. Thus either adequate knowledge of the solution is required a priori, or sufficient terms are required in the expansion for the method to work adequately. Schmidt and Tondl[1986] support this view, however consider that if the method is correctly applied, superior results may be achieved. Mickens[1984, 1986] discusses the application of the method, noting that providing certain conditions are satisfied, the method gives superior results. Mickens notes that if multiple equilibrium states exist, the method should be applied in the neighbourhood of each state separately. Handy[1985] generalises the method, by combining the harmonic balance method with the method of slowly varying phase and amplitude (Kryloff Bogoliuboff), broadening the range of the harmonic balance method to include transient response.

Methods of Averaging

The methods of averaging are based on the concept of slowly varying phase and amplitude. Thus the solution is assumed to be:

$$x = a(t)cos(\omega_o t + \phi(t)) = a(t)cos\Phi(t)$$

Since the phase and amplitude are assumed to vary slowly:

$$\frac{dx}{dt} = -asin(\Phi)$$

Thus the condition of slowly varying phase and amplitude requires that:

$$a'cos\Phi - a\Phi' sin\Phi = 0$$

By applying these relations, the equation of motion is transformed into differential equations in terms of a, Φ, t. Since the phase and amplitude vary slowly with respect to the period, they are assumed to be constant over a period. By averaging the equations over a period, the variable t is eliminated, resulting in linear autonomous equations defining the steady state amplitude and phase.

Other methods such as the integral equation, and the Van der Pohl method are discussed briefly by Schmidt and Tondl[1986]. It appears that a popular method currently applied to analyse weakly nonlinear systems is that of multiple scales. This method results in uniform expansions, and will be applied for the purpose of discussing the nonlinear equations of motion developed.

J.3 The Analysis of Nonlinear Systems with Quadratic and Cubic Nonlinearities.

The equations of motion for the mine hoist system result in a set of coupled nonlinear equations with quadratic and cubic nonlinearities, subject to multi-frequency parametric and external excitation. A general analysis of the ensuing motion will require the study of a series of special cases. Analyses exist in the literature, but usually restrict attention to the interaction between two modes, or specific cases of multi-degree-of-freedom systems. In the mine hoist system, the natural frequencies of the lateral modes are integer multiples of the fundamental, and repeated frequencies occur with respect to the in and out of plane lateral modes. Longitudinal modes may tune to promote internal resonance, a condition which may occur simultaneously with secondary resonance regions involving the longitudinal and lateral modes, or with external resonances, or combinations of these[1]. Note that any number of these conditions may occur simultaneously. Thus where a description of the amplitude of the motion is required, an intensive study would be necessary to examine these possibilities for a particular hoist configuration. Although such an approach would be beyond the capabilities of design teams in industry, an appreciation of the potential interaction in a region where nonlinear effects are dominant is essential. With this in mind, it is considered important to examine the behaviour of nonlinear systems presented in the literature, as a datum for discussion, prior to any examination of the equations at hand.

Although the subsequent section will examine existing analyses for nonlinear systems with finite degrees-of-freedom, many analyses examining the behaviour of single degree of freedom systems exist in the literature. Efstathiades[1974] examined the occurrence of combination tones generated in a system with cubic nonlinearities, as a consequence of multi-frequency excitation. It was shown that it was possible to generate single mode response at combination tones of the excitation frequencies of a two degree of freedom system. Nayfeh[1983d] examined the response of a single degree of freedom system to subharmonic excitation, which identified regions of subcritical stability dependent on the excitation amplitude, where both trivial and finite amplitude steady state motion ensues depending on the initial conditions. Nayfeh[1984b] re-examined combination tones in the response of SDOF systems with quadratic and cubic nonlinearities, due to dual frequency external excitation. Applying the

[1] Internal resonance refers to tuning between the natural modes of the system, such as $\omega_2 = 2\omega_1$, external resonance refers to linear resonance of the system which occurs when the frequency of an external excitation coincides with the linear frequency of a mode. Secondary resonance regions refer to regions where a combination of the linear frequencies tune to the frequency of the external excitation ie $\Omega = \omega_2 \pm \omega_1$ etc.

method of multiple scales, the solution to second order was developed, showing that the combination tones arise in the solution to both first and second order, and that the natural frequency of the system changes, and thus a jump phenomenon may arise. Nayfeh[1984c] examined the interaction between a fundamental parametric resonance and a subharmonic resonance of order one half, and defined conditions leading to quenching or enhancement of the motion. Nayfeh[1984a, 1984d] furthered this analysis by considering the quenching of a primary resonance by a superhamonic resonance and a combination resonance of additive or difference type. Zavodney and Nayfeh[1988] examined the response of a SDOF system with quadratic and cubic nonlinearities to a fundamental parametric resonance. The aspects of subcritical stability, combination tones in the case of dual-frequency external excitation, and quenching of a fundamental parametric resonance by a subharmonic resonance induced by external excitation, represent a few features extracted from weakly nonlinear SDOF systems by applying a perturbation method such as the method of multiple scales. These analyses clearly demonstrate the complexity of the potential response, as well as the consistent and systematic application of the method of multiple scales. Numerous analyses exist, which examine different orders of subharmonic and superharmonic resonance. These will not be discussed, as the primary interest with respect to the mine hoist system lies in analyses dealing with multi-degree-of-freedom nonlinear systems, where in addition to the features mentioned, a condition of internal resonance may arise when the natural frequencies of the system are commensurable, or nearly so. In the absence of internal resonance, where intermodal coupling does not arise, the results of the SDOF analyses are pertinent.

Nayfeh and Mook[1983] discuss the analysis of a two degree of freedom system of equations with quadratic nonlinearities, which are similar to those applied by Nayfeh, Mook and Marshall[1973a] in a study of ship roll pitch motion. Since these equations are similar in form to those of the mine hoist system, the behaviour of system is discussed below. The equations discussed by Nayfeh and Mook[1983] are reproduced below:

$$\ddot{u}_1 + 2\mu_1\dot{u}_1 + \omega_1^2 u_1 + \alpha_1 u_1 u_2 = f_1 cos(\Omega t + \tau_1)$$

$$\ddot{u}_2 + 2\mu_2\dot{u}_2 + \omega_2^2 u_2 + \alpha_2 u_1^2 = f_2 cos(\Omega t + \tau_2)$$

The free and forced response of this system is considered, where the behaviour of the system is studied under various conditions of external, internal, and combination resonance.

This system is similar to a truncated form of equations (J.4,J.5) which represent the planar stationary mine hoist system, where the parametric excitation and the cubic nonlinearity in the lateral equation of motion has been neglected. In this context u_1, u_2 represent the lateral $(r_i(t))$and longitudinal $(p_i(t))$ modes respectively. The salient features of Nayfeh and Mook's[1983] discussion of this set of equations is presented, to illustrate the potential behaviour of the system.

With regard to the free oscillations of the system, Nayfeh and Mook show that behaviour of systems with quadratic nonlinearities is the same as the linear system to second order, unless an internal resonance exists, coupling the modes. In the absence of an internal resonance the frequencies of free vibration are independent of amplitude, and the modes are uncoupled. When an internal resonance arises such that $\omega_i = 2\omega_j, \omega_n = \omega_m \pm \omega_k$, the modes are coupled through the internal resonance, and the first order approximation differs from the linear solution, in that a continual exchange, and dissipation of energy results between the modes. Figure J.1 illustrates the results of the free oscillation, for an internal resonance $\omega_2 = 2\omega_1$, where the first mode has been excited.

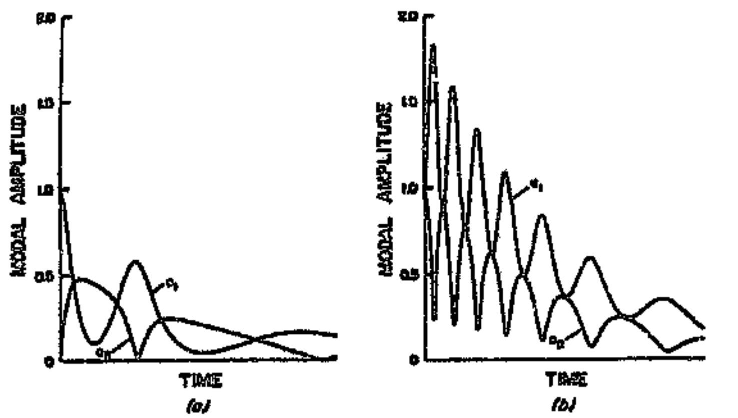

Figure 6-8. Free-oscillation amplitudes of a damped two-degree-of-freedom system with quadratic nonlinearities; $\omega_2 \approx 2\omega_1$: (a) $a_1(0) = 1, a_2(0) = 0$; (b) $a_1(0) = a_2(0) = 1$.

Figure J.1: Nayfeh & Mook (1983): Figure 6.8 p 386

The forced response of the system, due to single frequency excitation, is considered for a number of cases. Firstly the case were Ω is near to ω_2 is considered. This would be analogous to the situation where a longitudinal mode is resonant, whilst the lateral mode is detuned from the excitation. It is shown that in the absence of an internal resonance, the first approximation is not influenced by the nonlinear terms. However when a condition of internal resonance exists, $\omega_2 = 2\omega_1$, dramatic differences in the behaviour arise. A similar situation occurred on the upwind of the Kloof Mine, where the second harmonic excites the fourth longitudinal resonance, which is tuned to approximately twice the second lateral mode of the catenary. The behaviour of the Kloof system is far more complex however. On further examination of the Kloof system, it is evident that the first harmonic of the excitation tunes closely to the second lateral mode and the second longitudinal mode, as well as being twice that of the first lateral mode. Although the system discussed by Nayfeh and Mook[1983] is a substantial simplification of a realistic hoist system, the discussion presented is instructive. Nayfeh, Mook and Marshall[1973a] illustrated the character of the ship roll-pitch response by arbitrarily choosing parameters. Figures J.2,J.3 are reproduced from Nayfeh and Mook[1983]. In these figures the steady state modal amplitudes a_1, a_2 represent the u_1, u_2 degrees of freedom, or in the context of the mine hoist system, the lateral and longitudinal modal amplitudes respectively[2]. In order to assess the behaviour according to the degree of tuning between the external excitation frequency Ω and the condition of internal resonance in the system, detuning parameters σ_1, σ_2 are introduced as:

$$\Omega = \omega_2 + \epsilon\sigma_1$$

$$\omega_2 = 2\omega_1 + \epsilon\sigma_2$$

In figures J.2,J.3 the amplitude of the excitation applied to the longitudinal mode u_2 is varied, and the steady state modal response of the system is constructed. In both figures the internal resonance is perfectly tuned, ie $\sigma_2 = 0$. Figure J.2 represents the case where the external resonance is not perfectly tuned, ie $\sigma_1 \neq 0$, whilst figure J.3 represents perfect tuning of the external resonance, ie $\sigma_1 = 0$. It is evident in both figures that a saturation phenomenon arises in the u_2 (longitudinal) mode, whereby increasing the magnitude of the excitation induces no further increase in amplitude. At the point of saturation, the trivial solution for the u_1 (lateral) mode becomes unstable, nonlinear effects become dominant and thus the lateral mode increases in amplitude. As noted, the longitudinal mode u_2 provides parametric excitation for the u_1 mode, which becomes unstable once the amplitude of u_2 exceeds a threshold value. Figure J.2 illustrates the presence of a jump phenomenon, where multiple stable solutions co-exist for the lateral mode when $\zeta_1 < f_2 < \zeta_2$. The arrows guide the progression of the response as f_2 is slowly increased from a low level to a high level an visa versa. If a sufficiently strong disturbance is applied to the system in the region $\zeta_1 < f_2 < \zeta_2$, then the jump phenomenon may be

[2]These amplitudes relate to the modal system response at frequencies ω_1, ω_2 respectively. In the absence of nonlinearity, such response would only occur as transient motion, or due to linear resonance.

induced resulting in a sudden increase in the modal amplitudes, even though a stable trivial solution may have existed prior to the disturbance. This is pertinent to the mine oist system, where a substantial disturbance occurs at each layer cross-over. The large amplitude whip observed in the Kloof system was preceded by such a layer change.

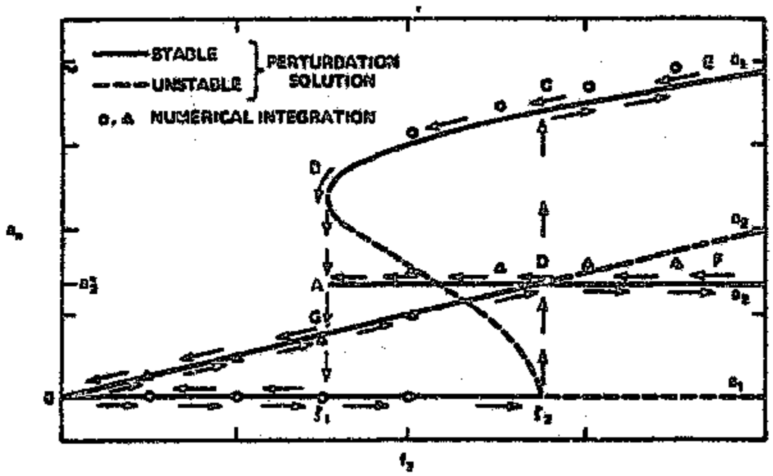

Figure 6-13. Amplitudes of the response as functions of the amplitude of the excitation; $\Gamma_1 < 0$; $\Omega \approx \omega_2$.

Figure J.2: Nayfeh & Mook (1983): Figure 6.13 p 406

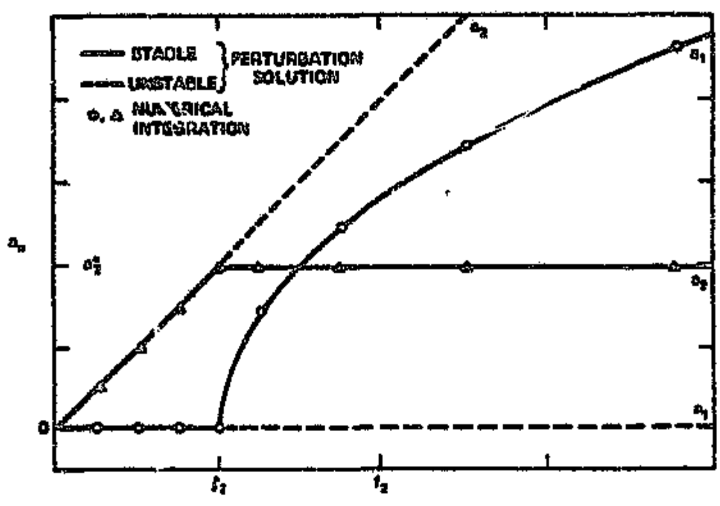

Figure 6-14. Amplitudes of the response as functions of the amplitude of the excitation; $\sigma_1 = \sigma_2 = 0$; $\Omega = \omega_2$.

Figure J.3: Nayfeh & Mook (1983): Figure 6.14 p 407

The case where Ω is near to ω_1 is also considered. In the absence of an internal resonance, the solution corresponds closely to that of the linear solution. Once again the condition of internal resonance is examined where $\omega_2 = 2\omega_1$. This condition relates to the situation in a mine hoist system, where the lateral mode is in resonance, whilst a longitudinal mode is tuned to twice the lateral mode. Once again this applies to the condition arising in the Kloof system, where the second lateral mode was in resonance due to the excitation of the first harmonic of the coil cross-over frequency, whilst the fourth longitudinal mode was tuned to twice the second lateral mode. Under these conditions, a saturation phenomenon does not arise. Figures J.4,J.5 are reproduced from Nayfeh and Mook[1983]. In figure J.4, the external resonance is perfectly tuned, $\sigma_1 = 0$, whilst the tuning of the internal resonance is varied. In figure J.5 the internal resonance is not perfectly tuned, $\sigma_1 > 0$, whilst the external resonance is varied.

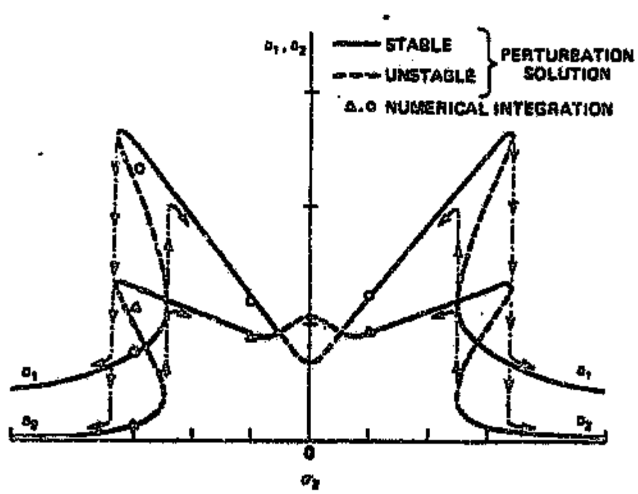

Figure 6-17. Frequency-response curves; $\sigma_1 = 0$, $\Omega = \omega_1$.

Figure J.4: Nayfeh & Mook (1983): Figure 6.17 p 410

A jump phenomenon is evident in both these figures. Also, a region exists where no stable steady state solution occurs. In this region, a continuous shuttling of energy occurs between the modes.

The non-resonant case is also examined. In this case, the excitation frequency does not coincide with a linear resonance frequency. Nayfeh and Mook[1983] show that secondary resonance regions exist where $\Omega \approx 2\omega_1, \Omega \approx \frac{1}{2}\omega_1, \Omega \approx \frac{1}{2}\omega_2, \Omega \approx \omega_2 \pm \omega_1$ as well as an internal resonance when $\omega_2 = 2\omega_1$. Nayfeh and Mook[1983] discuss the second and fourth cases.

In the case of $2\Omega \approx \omega_1$, where the system does not possess an internal resonance, the modes are uncoupled and a superharmonic response occurs for the

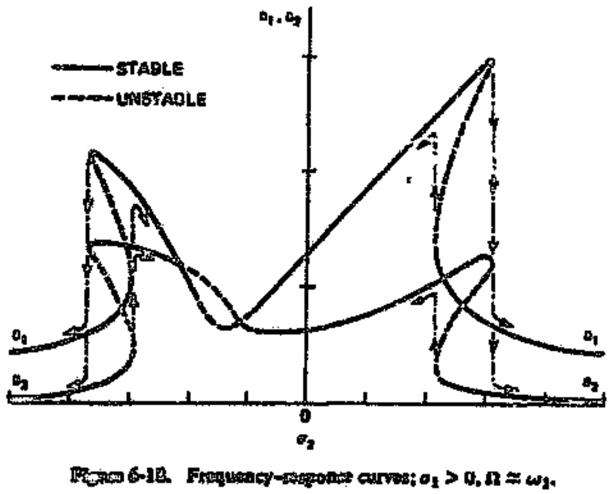

Figure J.5: Nayf‑h & Mook (1983): Figure 6.18 p 411

u_1 mode alone[3]. When the system possesses an internal resonance $\omega_2 \approx 2\omega_1$, Nayfeh and Mook[1983] show that the response is of a similar form to that illustrated in figures J.4,J.5. Consequently for some parameters, regions occur where steady state periodic motion does not exist.

In the case of $\Omega \approx \omega_2 \pm \omega_1$, in the absence of internal resonance, Nayfeh and Mook[1983] show that the solution to first order is either stable and equivalent to that of the linear system, or unstable in the homogeneous part of the first order solution, and consequently the response grows without bound. In the latter case, in reality the response does not grow without bound, but rather the system is no longer weakly nonlinear, and the analysis is no longer valid. However, Nayfeh and Mook point out that one could expect to observe a marked change in the response behaviour. A laboratory model confirmed the existence of this region. It was evident that the system could be tuned to a condition of neutral stability, by adjusting the excitation level. A disturbance resulted in growth in response at the natural frequencies of the respective modes. This response represents the homogeneous term in Nayfeh and Mook's analysis[4].

[3]Thus although the catenary is excited at a frequency at half its linear resonant frequency, response occurs at both the excitation frequency and close to its resonant frequency ω_1 or 2Ω, which is a superharmonic of the excitation.

[4]In the laboratory model, the tuning $\omega_1 \approx 35Hz, \omega_2 \approx 20Hz, \Omega \approx 55Hz$ existed. Thus at low excitation, response occurred at $55Hz, 110Hz$ due to the forced excitation, whilst beyond the point of neutral stability, the dominant response occurred at $20, 35Hz$ in the longitudinal and lateral modes respectively.

When an internal resonance $\omega_2 \approx 2\omega_1$ co-exists with a combination resonance $\Omega \approx \omega_2 \pm \omega_1$, the first order solution can be either: the equivalent linear response, or may differ from the linear solution due to the existence of fractional harmonic pairs. In this case Nayfeh and Mook[1983] give the solution to second order as[5]:

$$u_1 = \frac{F_1}{\omega_1^2 - \Omega^2} cos(\Omega t + \tau_1) + \epsilon a_1 cos[\frac{1}{3}(\Omega t \tau_1 - \gamma_1 - \gamma_2)] + O(\epsilon^3)$$

$$u_2 = \frac{F_2}{\omega_2^2 - \Omega^2} cos(\Omega t + \tau_2) + \epsilon a_2 cos[\frac{2}{3}(\Omega t \tau_1 + \frac{1}{2}\gamma_2 - \gamma_1)] + O(\epsilon^2)$$

Since $\omega_2 \approx 2\omega_1$ and $\Omega \approx \omega_2 \pm \omega_1$, $\frac{1}{3}\Omega, \frac{2}{3}\Omega$ are equivalent to ω_1, ω_2 respectively. Thus forced response will result at the excitation frequency, combined with response frequencies close to the linear natural frequencies of the system. The modal amplitudes a_1, a_2 are dependent on the modal damping factors, the system tuning, and the excitation amplitude F_1. Figure J.6 depicts the modal amplitudes as a function of the excitation amplitude $F_1 = \epsilon f_1$. It is evident that a critical excitation amplitude exists beyond which a_1, a_2 differ from zero. A form of saturation occurs with respect to the longitudinal mode, whilst the lateral mode increases with increasing excitation. At a sufficiently high amplitude, steady state motion no longer occurs and energy may be shuttled continuously between the modes. Note that although a critical excitation amplitude exists, progression to non-trivial modal amplitudes is not as a consequence of multiple solutions, and thus cannot be induced via a disturbance. Furthermore, beyond the critical point, the response amplitudes are related to the excitation level. This result contrasts with that obtained in the absence of an internal resonance, where the first order approximation grew without bound for certain system parameters.

According to the equations of motion derived, the mine hoist system contains external and parametric excitation, as well as quadratic and cubic nonlinearities. If one considers the linearised problem, where the parametric excitation is retained, then in regions of parametric resonance, the trivial solution grows without bound. Hsu[1963] analysed the regions of parametric resonance of linear systems with multiple degrees of freedom and multi-frequency excitation, obtaining expressions for the transition curves between stable and unstable motion. To achieve this, Hsu[1963] applied a straight forward expansion, and a method of averaging. Hsu[1963] identified that in addition to the normal sum and difference parametric resonance regions, anomalous

[5] Fractional harmonic pairs were discussed qualitatively by Eller[1972].

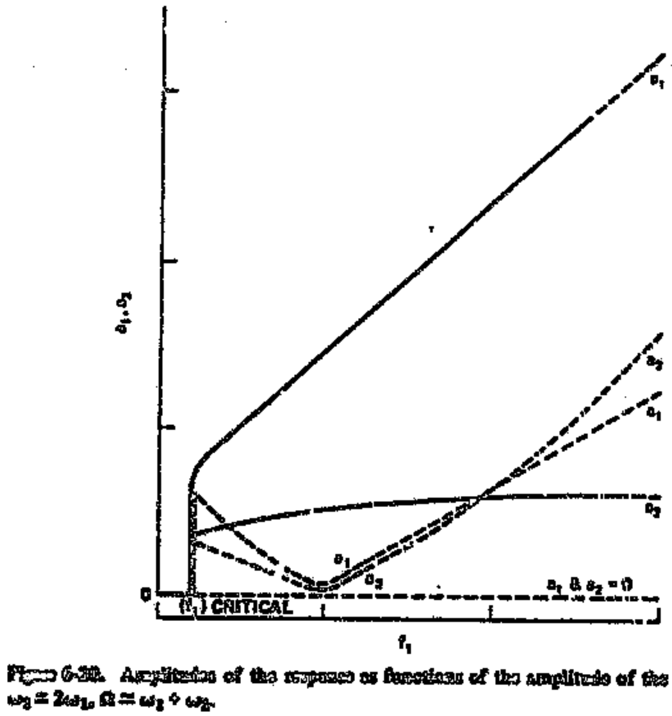

Figure J.6: Nayfeh & Mook (1983): Figure 6.20 p 416

regions may exist where these resonances interact simultaneously, a feature which may easily arise in the mine hoist system. Nayfeh and Mook[1977] applied the method of multiple scales to investigate the regions of parametric resonance of linear systems with many degrees of freedom. Their analysis confirmed that the method of multiple scales correctly predicted established parametric resonance regions and expressions for the transition curves, ie $p\Omega \approx \omega_n \pm \omega_n$ where n, m are integers. The anomalous cases mentioned by Hsu[1963] were analysed, ie simultaneous combination sum and difference resonance $\Omega \approx \omega_n \pm \omega_n \approx \omega_s \pm \omega_r$ with m, n being different from r, s. Nayfeh[1983a] extended this analysis by focussing on the aspect of simultaneous resonance in the response of a two degree of freedom linear system to multi-frequency parametric excitation. Nayfeh[1977] considered the case of two simultaneous parametric resonances, namely simultaneous resonance of the sum and difference type ie $\Omega \approx \omega_2 - \omega_1 \approx \omega_2 + \omega_1$, and simultaneous main resonance and sum or difference combination resonance ie $\Omega \approx 2\omega_1 \approx \omega_2 - \omega_1$, and $\Omega \approx 2\omega_1 \approx \omega_2 + \omega_1$. In these cases, the equations describing the modulation of the phase and amplitude could be transformed into four real first order autonomous equations. By solving these equations, the characteristic exponent of the system could be defined, and hence where this exponent becomes positive, the transition curve of stability is attained. Nayfeh[1983a] concludes that when three or more simultaneous resonances exist, it is not possible in general to transform the the modulation equations into an autonomous form, and thus numerical analysis may be required .

Nayfeh[1983c] considers the response of a two degree of freedom system with quadratic nonlinearities to a parametric excitation. The behaviour of the system is examined in the presence of an internal resonance $\omega_2 \approx 2\omega_1$. Two conditions of tuning to the parametric excitation are considered, namely: $\Omega = 2\omega_1$, a fundamental parametric resonance of the first mode; $\Omega = 2\omega_2$, a fundamental parametric resonance of the second mode. The analysis demonstrates the phenomena of saturation in a directly excited mode, as well as the existence of subcritical stability. In the case, $\Omega = 2\omega_2$, Nayfeh[1983c] shows that a saturation phenomenon arises in the higher mode which is directly excited, where the amplitude of the response is independent of the level of excitation[6], whereas the lower mode which is indirectly excited through the internal resonance, increases in amplitude with increasing excitation. In the case where $\Omega = 2\omega_1$, a saturation phenomenon does not arise, and beyond a threshold excitation level, both modes increase in amplitude with an increasing amplitude of excitation. In both cases, depending on the system parameters, a subcritical stability may arise, where below a certain excitation amplitude $F < \zeta_1$, the trivial motion is stable, and disturbances decay, $F > \zeta_2$ the trivial solution is unstable and the motion grows in both modes to a steady state amplitude. When $\zeta_1 < F < \zeta_2$, the trivial solution is stable and the motion decays to zero according to the linear analysis, whilst according to the nonlinear analysis, the solution may either be trivial or achieve a nontrivial stable steady state motion depending on the initial conditions. This condition is referred to as subcritical stability. Nayfeh[1983e] extends this analysis in a later publication to examine the response of a multi-degree-of-freedom system with quadratic nonlinearities to a harmonic parametric resonance. In this case, an internal resonance of the combination type is examined ie $\omega_3 = \omega_1 + \omega_2$. Under this condition of tuning, the response to a fundamental parametric resonance of the third mode is examined ie $\Omega \approx 2\omega_3$. Nayfeh[1983e] shows that the third mode exhibits a saturation with respect to the excitation, whilst the first and second modes grow in amplitude. The system also exhibits subcritical resonance. When the first or second mode is excited by a fundamental parametric resonance, ie $\Omega \approx 2\omega_1$, or $\Omega \approx 2\omega_2$, and when $F < \zeta_2$, the trivial response is the only steady state condition. When $F > \zeta_2$ the trivial solution is unstable resulting in amplitude growth according to the linear analysis, however according to the nonlinear analysis the motion is non-periodic. Plaut et al.[1983] examine a similar system under two frequency excitation, where a natural frequency is close to the sum of the two excitation frequencies ie. $\omega_n = \Omega_1 + \Omega_2$,

[6]The response is similar to that described previously, where the longitudinal mode was externally excited at resonance.

whilst an internal resonance such as $\omega_n = 2\omega_j$ tunes that frequency to another frequency. It is shown that a saturation phenomenon arises in the directly excited mode, when the natural frequency of that mode ω_n is higher than the other mode coupled through the internal resonance ω_j. Mook et al.[1985] consider an analysis of a multi-degree-of-freedom system with quadratic and cubic nonlinearities, with a fundamental parametric resonance $\Omega \approx 2\omega_2$ as well as internal resonance $\omega_2 \approx 2\omega_1$. The analysis demonstrated that the internal resonance gives rise to saturation in the directly excited mode, whilst the indirectly exci d mode behaves in a manner similar to that of the directly excited mode in the absence of internal resonance, and thus a behaviour described as *mode reversal* occurs. Since the amplitudes may be significantly reduced in the presence of an internal resonance, it is suggested that this may lead to a passive vibration control strategy. The analysis also demonstrates that under the condition of internal resonance, in certain regions of the parameter space, interaction occurs between the modes, resulting in a non-periodic solution, and a continual intermodal exchange of energy. Plaut et al.[1986] consider a series of cases involving two simultaneous external resonances of a system subjected to two-frequency excitation. The cases presented consider two simultaneous conditions of combination, subharmonic, superharmonic and primary external resonances. It is demonstrated that quenching can occur for certain combinations of the resonances. Nayfeh[1987] examined the parametric excitation of a double pendulum, under the condition of internal resonance and a fundamental parametric resonance. Saturation was observed when the higher mode was directly excited by the parametric excitation $\Omega \approx 2\omega_2$, whereas the lower mode which was indirectly excited through the internal resonance increased in amplitude. Due to the nature of the equations analysed, it was found that in certain regions of the parameter space, both Poincare'[7] and Hopf bifurcations existed. When the lower mode was excited by a principal parametric resonance, the periodic solutions exhibited period doubling, leading to chaos.

Tezak, Nayfeh and Mook[1982] considered the case of a parametrically excited nonlinear multi-degree-of-freedom system with repeated natural frequencies. A study of panel flutter illustrated the nature of the solutions. Nayfeh[1983b] considered the linear parametric resonance of a multi-degree-of-freedom system with three repeated natural frequencies, for the case of fundamental resonance of the modes with the repeated frequency, and combination resonance involving the repeated modes and one other. Conditions of stability of the motion were then derived for these cases.

Bux and Roberts[1986] examined the vibratory interactions in systems of coupled beams. The method of multiple scales was successfully applied to examine

[7] A Poincare' bifurcation arises where the number of fixed points increases as a control parameter changes.

the system response in the case of three mode interaction, where a combination internal resonance and an external resonance of one of the modes existed simultaneously ie $\Omega \approx \omega_2 \approx \omega_B+\omega_T$. This is similar to the analysis by Nayfeh[1983e], where the system was tuned to a combination internal resonance and a fundamental parametric resonance simultaneously. Bux and Roberts[1986] obtained stationary solutions for this condition, showing that multi-modal response is possible even though the modes may not be directly excited. The four mode interaction was investigated, where in addition to the tuning $\Omega \approx \omega_2 \approx \omega_B + \omega_T$, an additional internal resonance $\omega_1 \approx 2\omega_B$ occurs. Stationary solutions were not obtained from the perturbation analysis, however experimental results presented illustrated violent interactions between all four modes. Ashworth and Barr[1987] extended Bux and Roberts[1986] analysis in an investigation of the resonances of structures with quadratic inertial nonlinearities under direct and parametric harmonic excitation. A combination of an harmonic balance and perturbation method was successfully applied. This analysis discusses the concept of cascading behaviour of a multi-mode structure, where either the response of the structure to the external excitation or the parametric excitation, autoparametrically induces further excitation of other modes of the structure through internal resonance, thus achieving a multi-modal response. Ashworth and Barr[1987] demonstrate this behaviour on a four mode beam structure, where one parametrically excited combination resonance interacts through a combination of internal resonances providing autoparametric excitation of the two remaining modes. Consequently modal response is obtained in all four modes due to a single parametric excitation frequency. Cartmell and Roberts[1988] consider a similar problem of interacting beams, in the vicinity of a simultaneous combination sum and difference internal resonance, where one mode is externally excited at resonance. In this analysis the method of multiple scales was applied, and a set of autonomous first order differential equations defined the modulation of the modal amplitudes and phases. The resulting equations for the steady state amplitudes could not be solved analytically, and consequently an integration procedure was attempted. Experimental results were employed to demonstrate the multi-modal response of the system. The difficulty encountered in analysing three or more simultaneous resonances was considered by Nayfeh[1983a], where it was shown that in the general case, it may not be possible to reduce the equations to an autonomous form, and numerical analysis may be required. Multi-Modal interaction, and simultaneous resonance as discussed above, is pertinent to the mine hoist system and clearly defines the potential complexity of a thorough dynamic analysis of a mine hoist system. Note that in these analyses, a single excitation frequency was applied, resulting in a multi-frequency response. In the mine hoist system, multi-frequency external excitation co-exists with multi-frequency parametric excitation, and simultaneous parametric and external resonances may arise in the presence of simultaneous internal resonance between various modes of the system, complicating the potential response further. HaQuang et al.[1987b] examine the response of a single degree of freedom system with quadratic and cubic nonlinearities to primary external and secondary parametric excitation. Applying the method of multiple scales, asymptotic solutions for small motions were developed. The steady state amplitudes were investigated with increasing amplitude of the parametric excitation, and different parameters dictating the strength of the quadratic and cubic nonlinearities. It was shown that as the parametric excitation increases, multiple solution branches develop. The asymptotic solutions were compared with numerically integrated solutions, illustrating that additional large amplitude solutions exist, which lie beyond the realm of approximation of an asymptotic method. These large amplitude solutions exhibit period doubling leading to chaos. HaQuang et al.[1987a] extend this analysis to a multi-degree-of-freedom system where one mode is involved in the external resonance, and demonstrate that multi-modal response can be generated in the first order solution, even in the absence of internal resonance. Plaut et al.[1990a] considered the resonances in a multi-degree-of freedom system with quadratic and cubic nonlinearities subjected to external excitation at frequencies Ω_1, Ω_2, in the absence of internal resonance. When there is more than one excitation frequency, response may arise at combination tones of the excitation frequency, which may be further complicated by simultaneous resonances. The analysis considers conditions which arise at $2\omega_n \approx \Omega_1 \pm \Omega_2$ and $\omega_n \approx |2\Omega_1 \pm \Omega_2|$. Plaut et al.[1990b] extended the previous analysis to include multi-frequency parametric excitation in addition to multi-frequency external excitation. The analysis examined the condition of resonance involving both the parametric and external excitation frequency ie $|\Omega_i^p \pm \Omega_j^e \pm \Omega_k^e| \approx= \omega_q$, and $|\Omega_i^e \pm \Omega_j^p \pm \Omega_k^p| \approx= \omega_q$, where superscripts e, p refer to external and parametric excitation, and ω_q refers to the natural frequency of the q^{th} mode. The analysis considered these conditions in the absence of any other resonance. Thus the more pertinent case of simultaneous resonance involving other modes, or internal resonance was not considered, and consequently a single mode response occurred.

Nayfeh and Asfar[1988] consider the case of a single degree of freedom system with cubic nonlinearities to a non-stationary principal parametric excitation. In this instance the system natural frequency remains constant, whereas either the frequency of the parametric excitation, or the amplitude of the parametric excitation was allowed to change slowly with respect to time ie either the parametric excitation frequency was assumed to be $\Omega = \Omega_o + rT_1$,or the amplitude of the parametric excitation was assumed as $f = f_o + rT_1$, where r represents the sweep rate, and T_1 the slow time scale. The mine hoist system could be considered to be the inverse of this case, where the natural frequencies of the system change with time, whilst the excitation frequency and amplitude remain constant. Research to date has treated the mine hoist system in a quasi static form, where the axial velocity and hence the rates at which the natural frequencies vary have been neglected. The primary motivation for this assumption is two fold, namely the analysis would become protracted and is viewed as a future research incentive, and generally the rate of change of the natural frequencies becomes significant close to the shaft head, where the longitudinal modes change rapidly with respect to depth. In practice whip has been observed at greater depth. Nayfeh and Asfar[1988] show that the rate of change of the parametric excitation frequency or amplitude can significantly alter the response, in that the nonlinear resonance curves for the stationary case are penetrated and become smudged as the rate increases. Thus if the excitation is initiated at a frequency where the stationary trivial response is stable, and progresses into a region where this response is unstable, then the depth of penetration into the unstable region is related to this rate; this penetration is then followed by a jump to the nonlinear stable solution, which may result in overshoot and oscillation. Miyasar and Barr[1988] consider the response of a linear oscillator under parametric excitation with fluctuating frequency. In their analysis, the parametric excitation is assumed to vary about a mean ie. $\cos[2\Omega t + \alpha cos\beta t]$. Although this study is of interest, it is not particularly relevant to the mine hoist system.

It is evident that the mine hoist system represents an extremely complex dynamic system. A thorough understanding of the behaviour would prescribe a consistent study of the possible mechanisms of interaction. These interactions will be parameter dependent, and thus may vary in severity for different installations. Although the industry currently attempts to ensure the dynamic integrity of a system by simply avoiding regions of external resonance of the lateral catenary modes, this is clearly not sufficient. The neglect of the longitudinal interaction in current design procedures is clearly highlighted by the foregoing discussion. The dynamic integrity of the system with respect to the catenary response has not been considered significant with respect to the fatigue life of the rope. This assumption has no theoretical basis, and one surmises that it is linked to the neglect of longitudinal and lateral coupling between the lateral and longitudinal motion in the linear wave equations. Thus the behaviour of mine hoist dynamics as perceived by the industry is severely limited by linear assumptions.

As it is the purpose of this study to consider the potential dynamic behaviour associated with mine hoist systems, it is pertinent to examine conditions which may lead to resonant conditions other than those predicted by a linear analysis. Thus if the system is correctly designed to avoid conditions leading to external resonance of the catenary, the dynamic integrity of the system may be compromised by regions of secondary resonance. Although the method presented in Chapter 4, where the stability of the steady state planar motion is examined, leads to a design methodology capable of avoiding regions of nonlinear interaction, a more direct approach is desirable to confirm that such regions exist, as well as to lay the foundation and flavour of future research incentives. A perturbation analysis utilising the method of multiples scales is presented in the following section. The conditions necessary to induce secondary resonance are determined. The steady state perturbation solution for the condition of secondary resonance is not developed further, as this is viewed as a future research incentive.

J.4 Secondary Resonance Conditions

The intention of the following analysis is to demonstrate the potential existence of secondary regions of resonance by applying the method of multiple scales. Secondary resonance regions are identified by the occurrence of secular terms in the perturbation expansion, which would induce unbounded growth in that component of the solutions expansion. This leads to the formulation solvability conditions, ensuring that the secular terms in the expansion are identically zero. These solvability equations define the steady state response of the system. Multiple solutions may exist, requiring further analysis to determine the stability of the various solutions.

In the following analysis, the method of multiple scales is applied up to second order expansion in ϵ^2, where small response of the order of ϵ is assumed. ie:

$$
\begin{aligned}
p_i(t,\epsilon) &= \epsilon p_{io} + \epsilon^2 p_{i1} \\
r_i(t,\epsilon) &= \epsilon r_{io} + \epsilon^2 r_{i1} \\
\frac{\partial}{\partial t} &= D_o + \epsilon D_1 \\
\frac{\partial^2}{\partial t^2} &= D_o^2 + 2\epsilon D_o D_1
\end{aligned} \tag{J.6}
$$

Where ϵ introduces the normal notion of a small parameter about which the solution is expanded. The equations are ordered by ensuring that the damping, parametric and nonlinear terms appear in the second order expansion. Equations (J.4,J.5), may be presented as[8]:

$$
\ddot{p}_i + 2\epsilon\zeta_i\omega_i\dot{p}_i + \omega_i^2 p_i + \epsilon\{\sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n)\}U_{ij}^u r_j + \Gamma_{ijk} r_j r_k = \epsilon P_i(t) \tag{J.7}
$$

$$
\begin{aligned}
&\ddot{r}_i + 2\epsilon\overline{\zeta}_i\overline{\omega}_i\dot{r}_i + \overline{\omega}_i^2 r_i + \epsilon\{\sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n) U_{ij}^u p_j + \\
&\quad \sum_{n=0}^{4} \overline{V}_n cos(n\Omega t + \overline{\beta}_n)\} V_{ij} r_j + \Delta_{ijk} p_j r_k + \{\sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n)\}\mathcal{E}_{ijk} r_j r_k \\
&\qquad + \beta_{ijkl} r_j r_k r_l = \epsilon R_i(t)
\end{aligned} \tag{J.8}
$$

[8]The constants in equations (J.4,J.5) have been factored into the coefficients of equations (J.7,J.8). The lateral excitation at the drum is represented as: $\frac{v_a}{l_a} = \sum_{n=1}^{2} V_n cos(n\Omega t+\beta_n) = \sum_{n=1}^{2} \frac{1}{2} V_n e^{in\Omega T_0} + cc$, whilst $(\frac{v_a}{l_a})^2 = \sum_{n=0}^{4} \overline{V}_n cos(n\Omega t + \overline{\beta}_n) = \sum_{n=0}^{4} \frac{1}{2}\overline{V}_n e^{in\Omega T_0} + cc.$

By substituting relationships (J.6) into the equations of motion (J.7,J.8), the first and second order expansions are obtained as:

First Order : ϵ

$$D_o^2 p_{io} + \omega_i^2 p_{io} = \sum_{n=1}^{4} P_n cos(n\Omega t)$$

$$D_o^2 r_{io} + \omega_i^2 r_{io} = \sum_{n=1}^{2} R_n cos(n\Omega t)$$

The solution to these equations is:

$$p_{io} = A_i(T_1)e^{i\omega_i T_o} + \sum_{n=1}^{4} \Lambda_{in} e^{in\Omega T_o} + cc \tag{J.9}$$

$$r_{io} = B_i(T_1)e^{i\overline{\omega}_i T_o} + \sum_{n=1}^{2} \chi_{in} e^{in\Omega T_o} + cc \tag{J.10}$$

where cc represents the complex conjugate, and:

$$\Lambda_{in} = \frac{1}{2}\frac{P_n}{(\omega_i^2 - n^2\Omega^2)}$$

$$\chi_{in} = \frac{1}{2}\frac{R_n}{(\overline{\omega}_i^2 - n^2\Omega^2)}$$

The second order expansion of the equations of motion are:

<u>Second Order : ϵ^2</u>

$$\begin{aligned} D_o^2 p_{i1} + \omega_i^2 p_{i1} = -[2\zeta_i \omega_i D_o + 2D_o D_1] p_{io} - \sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n) U_{ij}^u r_{jo} \\ -\Gamma_{ijk} r_{jo} r_{ko} = 0 \end{aligned} \tag{J.11}$$

$$\begin{aligned} D_o^2 r_{i1} + \overline{\omega}_i^2 r_{i1} = \; & -[2\overline{\zeta}_i \overline{\omega}_i D_o + 2D_o D_1] r_{io} - \textstyle\sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n) U_{ij}^v p_{jo} \\ & - \textstyle\sum_{n=0}^{4} \overline{V}_n cos(n\Omega t + \overline{\beta}_n) V_{ij} r_{jo} - \Lambda_{ijk} p_{jo} r_{ko} + \\ & \textstyle\sum_{n=1}^{2} V_n cos(n\Omega t + \beta_n) \delta_{ijk} r_{jo} r_{ko} = 0 \end{aligned} \tag{J.12}$$

substituting p_{io}, r_{io} from equations (J.9,J.10) into equations (J.11,J.12), and employing complex algebra, where $*$ represents the complex conjugate, leads to:

$$\begin{aligned} D_o^2 p_{i1} + \omega_i^2 p_{i1} = -2i\omega_i [A_i' + \zeta_i \omega_i A_i] e^{i\omega_i T_o} - 2i\zeta_i \omega_i \sum_{n=1}^{4} n\Omega \Lambda_{in} e^{in\Omega T_o} \\ -\frac{1}{2} \sum_{n=1}^{2} \left\{ V_n U_{ij}^u B_j e^{i(\overline{\omega}_j + n\Omega) T_o} + V_n^* U_{ij} B_j e^{i(\overline{\omega}_i - n\Omega) T_o} \right\} \\ -\frac{1}{2} \sum_{n=1}^{2} \left\{ V_n U_{ij} \sum_{m=1}^{2} \chi_{jm} e^{i(m+n)\Omega T_o} + V_n^* U_{ij} \sum_{m=1}^{2} \chi_{jm} e^{i(n-m)\Omega T_o} \right\} \\ -\Gamma_{ijk} \left\{ B_j B_k e^{i(\overline{\omega}_j + \overline{\omega}_k) T_o} + B_j B_k^* e^{i(\overline{\omega}_j - \overline{\omega}_k) T_o} + \sum_{n=1}^{2} [B_j \chi_{kn} e^{i(\overline{\omega}_j + n\Omega) T_o} + \right. \\ B_j \chi_{kn}^* e^{i(\overline{\omega}_j - n\Omega) T_o} + B_k \chi_{jn} e^{i(\overline{\omega}_k + n\Omega) T_o} + B_k^* \chi_{jn} e^{i(n\Omega - \overline{\omega}_k) T_o} + \\ \left. \chi_{jn} \sum_{m=1}^{2} \{ \chi_{km} e^{i(m+n)\Omega T_o} + \chi_{km}^* e^{i(n-m)\Omega T_o} \}] \right\} + cc = 0 \end{aligned} \tag{J.13}$$

$$
\begin{aligned}
D_o^2 r_{i1} + \omega_i^2 r_{i1} = & -2i\overline{\omega}_i[B_i' + \overline{\zeta}_i\overline{\omega}_i B_i]e^{i\overline{\omega}_i T_o} - 2i\overline{\zeta}_i\overline{\omega}_i \sum_{n=1}^2 n\Omega\chi_{in}e^{in\Omega T_o} \\
& -\tfrac{1}{2}\sum_{n=1}^2 \Big\{V_n U_{ij}^u[A_j e^{i(\omega_j+n\Omega)T_o} + \sum_{m=1}^4 \Lambda_{jm} e^{i(n+m)\Omega T_o}]\Big\} \\
& -\tfrac{1}{2}\sum_{n=1}^2 \Big\{V_n^* U_{ij}^u[A_j e^{i(\omega_j-n\Omega)T_o} + \sum_{m=1}^4 \Lambda_{jm} e^{i(m-n)\Omega T_o}]\Big\} \\
& -\tfrac{1}{2}\sum_{n=0}^4 \Big\{\overline{V}_n V_{ij}[B_j e^{i(\overline{\omega}_j+n\Omega)T_o} + \sum_{m=1}^2 \chi_{jm} e^{i(n+m)\Omega T_o}] + \\
& \quad \overline{V}_n^* V_{ij}[B_j e^{i(\overline{\omega}_j-n\Omega)T_o} + \sum_{m=1}^2 \chi_{jm} e^{i(m-n)\Omega T_o}]\Big\} \\
& \quad -\Delta_{ijk}\Big\{A_j B_k e^{i(\omega_j+\overline{\omega}_k)T_o} + A_j B_k^* e^{i(\omega_j-\overline{\omega}_k)T_o} \\
& \quad +B_k \sum_{n=1}^4[\Lambda_{jn} e^{i(\overline{\omega}_k+n\Omega)T_o} + \Lambda_{jn}^* e^{i(\overline{\omega}_k-n\Omega)T_o}] \\
& \quad +A_j \sum_{n=1}^2[\chi_{kn} e^{i(\omega_j+n\Omega)T_o} + \chi_{kn}^* e^{i(\omega_j-n\Omega)T_o}] \\
& +\sum_{n=1}^2\sum_{m=1}^4 \chi_{kn}[\Lambda_{jm} e^{i(n+m)\Omega T_o} + \Lambda_{jm}^* e^{i(n-m)\Omega T_o}]\Big\} + \\
& \tfrac{1}{2}\delta_{ijk}\Big[\sum_{n=1}^2 V_n \Big\{B_j B_k e^{i(\overline{\omega}_j+\overline{\omega}_k+n\Omega)T_o} + B_j B_k^* e^{i(\overline{\omega}_j-\overline{\omega}_k+n\Omega)T_o} + \\
& \sum_{m=1}^2 \big[B_j \chi_{km} e^{i(\overline{\omega}_j+(n+m)\Omega)T_o} + B_j \chi_{km}^* e^{i(\overline{\omega}_j-(m-n)\Omega)T_o} + \\
& \quad B_k \chi_{jm} e^{i(\overline{\omega}_k+(n+m)\Omega)T_o} + B_k^* \chi_{jm} e^{i((n+m)\Omega-\overline{\omega}_k)T_o} + \\
& \quad + \chi_{jm}\sum_{l=1}^2[\chi_{kl} e^{i(l+n)\Omega T_o} + \chi_{kl}^* e^{i(n-l)\Omega T_o}]\big]\Big\} + \\
& \sum_{n=1}^2 V_n^* \Big\{B_j B_k e^{i(\overline{\omega}_j+\overline{\omega}_k-n\Omega)T_o} + B_j B_k^* e^{i(\overline{\omega}_j-\overline{\omega}_k-n\Omega)T_o} + \\
& \sum_{m=1}^2 \big[B_j \chi_{km} e^{i(\overline{\omega}_j+(m-n)\Omega)T_o} + B_j \chi_{km}^* e^{i(\overline{\omega}_j-(m+n)\Omega)T_o} + \\
& \quad B_k \chi_{jm} e^{i(\overline{\omega}_k+(m-n)\Omega)T_o} + B_k^* \chi_{jm} e^{i((m-n)\Omega-\overline{\omega}_k)T_o} + \\
& + \chi_{jm}\sum_{l=1}^2[\chi_{kl} e^{i(l-n)\Omega T_o} + \chi_{kl}^* e^{-i(n+l)\Omega T_o}]\big]\Big\}\Big] + cc = 0 \qquad \text{(J.14)}
\end{aligned}
$$

It is clear from the preceding equations, that secular[9] terms may arise if any of the following conditions are satisfied.

$$
\begin{aligned}
\omega_i &= n\Omega & n &= 1,6 \\
\overline{\omega}_i &= n\Omega & n &= 1,6 \\
n\Omega &= \overline{\omega}_i \pm \omega_j & n &= 1,4 \\
n\Omega &= \overline{\omega}_i \pm \overline{\omega_j} & n &= 1,6 \\
\omega_i &= \overline{\omega}_j \pm \overline{\omega}_k \\
\overline{\omega_i} &= \overline{\omega}_j \pm \overline{\omega}_k
\end{aligned}
\qquad \text{(J.15)}
$$

[9]ie If any complex exponent on the right hand side of the equality is equal to the natural frequency of that equation, the corresponding term is secular, as it induces a resonant condition.

The first two conditions represent the case of external resonance, the second two conditions represent secondary resonance conditions arising at the sum and difference frequencies between the lateral and longitudinal modes, whilst the remaining conditions represent the case of internal resonance between the longitudinal and lateral modes. This analysis was primarily concerned with identifying regions of secondary resonance, and thus the first two conditions are excluded. Note that the latter conditions may be satisfied simultaneously for a number of combinations of the modes. Also, since the internal resonance arises at sum and difference combinations of the lateral modes, and because the lateral modes are integer multiples of the fundamental, a number of internal resonances arise simultaneously. This gives rise to additional coupling between the modes, which generally results in multi-modal response. The analysis required to determine the steady state response will be significantly more complex, to the extent where it may not be possible to determine the steady state solution analytically.

In the absence of secular terms, the solvability conditions for the amplitudes A_i, B_i reduce to:

$$A_i' + \zeta_i \omega_i A_i = 0$$

$$B_i' + \overline{\zeta}_i \overline{\omega}_i B_i = 0$$

Since these equations are damped first order equations, the steady state amplitudes A_i, B_i, C_i are trivial, and consequently the solution is governed by the first order solutions p_{io}, r_{io}.

J.5 Conclusion

Secondary resonance regions may arise when the excitation frequency tunes to a sum or difference combination of the lateral and longitudinal modal frequencies. In addition, it has been demonstrated that the condition of internal resonance arises when a longitudinal mode is tuned to a sum or difference combination of the lateral natural frequencies. Since the lateral frequencies are related by integer numbers, the difference condition introduces an internal resonance when a longitudinal natural frequency is equal to a lateral natural frequency. Also, the system is always internally resonant with respect to the lateral modes. By accounting for the in-plane equation of motion, it can be shown that similar conditions of secondary resonance exist with respect to the longitudinal and in-plane lateral modes. An additional condition of internal resonance however couples the in and out-of-plane lateral modes. Since the in-plane and out-of-plane linear natural frequencies are nearly identical, this condition is always approximately satisfied. In this analysis, excitation frequencies were assumed to be harmonic, and the first and second harmonic of the Lebus drum frequency was accounted for. As a result of the nonlinear coupling between the lateral and longitudinal modes, external excitation was induced to excite the system up to the sixth harmonic of the Lebus drum frequency.

The general nonlinear analysis of the mine hoist system by means of an analytical technique is probably an intractable mathematical problem. The studies presented regarding nonlinear multi-degree-of-freedom systems illustrate the potential response of the system. Namely, the possibility of a quenched response and mode reversal, non-periodic solutions, subcritical stability, and the effect of internal resonance. Although the inherent features of the mine hoist system dictate to a large extent the need for a direct numerical simulation of the nonlinear equations of motion, the theoretical discussion presented provides a necessary basis to interpret the numerical results.

Appendix K

Stability of Liner ODE's with Periodic Coefficients

K.1 Parametrically Exited Systems

In the linear analysis of mechanical vibration, it is usual to arrive at a causal relationship between the system response and excitation, where this relationship may be expressed as a set of non-homogeneous differential equations with constant coefficients. The systems behaviour reflects the paradigm of a single degree-of-freedom oscillator, and thus the concept of free and forced response, and resonance is introduced. When considering realistic conditions, where positive damping exists, the response is always bounded and stable.

Parametrically excited systems are described by differential equations with time dependent coefficients. In this context the system *parameters* such as mass, stiffness and damping which are normally deemed constant, vary with time, and thus the term parametric excitation refers to the *excitation* of these parameters, rather than the existence of an external forced excitation[1]. The term excitation implies that a system response can be achieved in the absence of the normal notion of an external excitation. This concept is well established, where the salient feature of a parametrically excited system, namely instability,

[1]Although parametric excitation is induced by a forcing function, as in the case of the lateral parametric resonance of a string as presented in Appendix B, the resulting equation of motion is homogeneous, and devoid of an external excitation.

is reflected by the behaviour of the single degree of freedom Mathieu equation.

$$\ddot{x} + \omega^2(1 + \alpha Cos\Omega t)x = 0$$

The behaviour and analysis of the Mathieu equation is presented in many sources, such as Stoker [1950], Bolotin [1964], Mclachlan [1947], Nayfeh and Mook [1983], Newland [1989]. In essence thi· quation exhibits regions of instability for certain combinations of the natu .l frequency ω, the parametric excitation frequency Ω, and the parametric excitation amplitude α. It is important to note that the equation is linear and homogeneous. Unlike systems with constant coefficients, where a causal relationship exists between the external forcing function and the amplitude, in the case of a parametrically excited system, the .mplitude grows in an unbounded fashion as a result of dynamic instability. The growth of the motion thus depends only on the existence of an infinitesimal disturbance or initial condition. Although damping generally exerts a stabilising influence, instability may still arise for a sufficiently large parametric excitation. With respect to the Mathieu equation, regions of instability occur in the neighbourhood of the ratio of the natural to the parametric excitation frequency such that $\Omega = \frac{2\omega}{n}$, where the span of the region is dependent on the degree of damping and the amplitude of parametric excitation α. The primary or principal parametric region occurs when $n = 1$, with a sequence of higher order regions existing for $n = 2$ These regions become successively narrower, and more difficult to excit Newland[1989] considers the response of the Mathieu equation in a region of instability, and demonstrates that on the boundary of stability, the dominant response frequency is always close to the linear natural frequency of the system. The primary region of parametric resonance, where $\Omega = 2\omega$, is thus often termed subharmonic resonance, since the dominant response occurs at half of the parametric excitation frequency. The shaded areas in Figure K.1 represent regions of instability related to simple parametric *resonance* of the Mathieu equation, where the response grows in an unbounded manner. This figure reflects the fact that on the boundary of regions of instability, the solution is periodic of period T or 2T, where the period T refers to the period of the parametric excitation.

In reality, the growth in a region of instability is contained by nonlinear restoring forces, and is ultimately bounded. Thus although the initial phase of growth within a region of parametric resonance may be described by a linear differential equation, ultimately nonlinear effects must be included to obtain a description of the final motion. In nonlinear dynamic studies, the linear notion of single valued response and of the principle of superposition no longer applies, and thus multiple dynamic states may arise. Thus a nonlinear solution

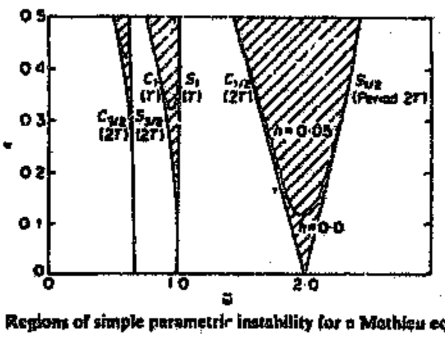

Figure K.1: Takahashi (1982): Figure 1 - Regions of simple parametric resonance

where $\bar{\omega} = \theta/\omega$.

requires that a stability analysis is performed for each solution branch, in order to determine whether that branch is physically realisable. By considering small perturbations to the nonlinear steady state motion, the stability of the motion may be defined in terms of a variational equation. Linearisation of this equation generally results in a differential equation with periodic coefficients, or a parametrically excited system. If the variational equation is stable, then a perturbation or small disturbance applied to the motion results in a transient motion, which dissipates if damping is present. The converse indicates divergence of the motion to an alternative solution. This is commonly associated with the jump phenomenon in a nonlinear system with cubic stiffness. However, this technique is limited to determining the stability of the motion to small disturbances, and consequently fails to indicate the dependence of the final steady state motion, where multiple stable steady state solutions exist, on large disturbances. This reflects the *global* stability of the system, where large disturbances or initial conditions may result in the system response being *attracted* from one stable solution branch to another. The determination of the influence of initial conditions on the final steady state motion results in the definition of domains of attraction of the fixed points in the phase plane. Guttalu and Hsu[1984] discuss three techniques to accomplish a global study, and present the cell mapping technique as a computationally economic alternative. This technique is applied to determine the domain of attraction of asymptotically stable periodic solutions of an impulsively excited pendulum.

This thesis is concerned with the stability of linear parametrically excited systems, or conversely the stability of the nonlinear steady state motion subjected to small disturbances. Particular attention is directed to multi-degree-of-freedom systems with periodic coefficients. Thus, consideration is given to a set of equations of the form:

$$\ddot{x}_i + 2\zeta_i\omega_i\dot{x}_i + \omega_i^2 x_i + \sum_{n=1}^{m} P_{i,j_n} cos(n\Omega t + \phi_n)x_j = 0$$

where P_{i,j_n} represents the parametric excitation coefficient of the n^{th} harmonic of the parametric excitation frequency Ω. Such a system was examined by Hsu[1963], by applying a perturbation technique. Hsu[1963] demonstrated that in addition to regions of simple parametric resonance at $\Omega = 2\frac{\omega_i}{n}$, regions of combination resonance of the sum and difference type could arise at $\Omega = \frac{\omega_i \pm \omega_j}{n}$. The existence of the sum type combination resonance is dependent on the magnitude of the terms P_{i,j_n}, P_{j,i_n} and on $P_{i,j_n} \times P_{j,i_n} > 0$. Difference type combination resonances exist when $P_{i,j_n} \times P_{j,i_n} < 0$. The technique applied by Hsu[1963] is examined in detail in Appendix B. Hsu's analysis considers the primary parametric regions for each harmonic of the parametric excitation, neglecting higher order regions and anomalous regions, where conditions of simultaneous parametric resonance may arise. The method is also restricted to the concept of a small parameter, and hence small parametric excitation. Nayfeh and Mook[1977] examined the same system, confirming Hsu's[1963] results by applying the method of multiple scales. Nayfeh and Mook[1977] also considered a number of anomalous regions. In the context of the mine hoist system, it was considered that a more general approach was required which accounted for all regions directly, without requiring the study of special cases. Consequently techniques involving the harmonic balance method were considered.

Szemplinska-Stupnika[1978] considered the application of the harmonic balance method to determine regions of combination parametric resonance in which the notion of a small parameter was not introduced. It was demonstrated that although perturbation techniques can be applied to examine regions of combination resonance for small parametric excitation, they inherently constrain the motion to the normal co-ordinates involved in that region, and therefore neglect all other coupling coefficients. Bolotin[1964] applied the harmonic balance method to examine simple regions of parametric resonance of multi-degree-of-freedom systems. Szemplinska-Stupnika[1978] notes that in this solution, the direct and indirect parametric coupling terms of the parametric excitation are inherently accounted for, and thus unlike the perturbation techniques, the solution is not constrained to the normal co-ordinate involved in the resonance. Bolotin's method can be applied successfully to examine regions of simple parametric resonance, since the solution can be shown to be of period T or 2T on the boundary of stability, where T is the period of the parametric excitation. This method cannot be applied to regions of combination resonance, since on the boundary of stability of these regions, the system is not periodic. It is almost periodic consisting in the first approximation of two harmonic components, with uncommensurable frequencies (Szemplinska-Stuprika[1978]). Since the Fourier expansion applied by Bolotin[1964] cannot accommodate this requirement it is not applicable to regions of combination resonance. Szemplinska-Stupnika[1978] developed a method utilising the method of harmonic balance, and imposed the condition that on the boundary of instability of a primary combination resonance, $\omega_i + \omega_j = 2\theta$. Ultimately the method led to a system of nonlinear algebraic equations for the definition of the boundary of stability. Also the method did not account for higher order regions of combination resonance which may be of period T rather than 2T.

Takahashi[1981a, 1981b, 1982] proposed an alternative implementation of the method of harmonic balance, and considered a system of equations of the form:

$$[\mathbf{C}]\{\ddot{f}\} + 2[\mathbf{C}][\mathbf{H}]\{\dot{f}\} + ([\mathbf{I}] - \alpha[\mathbf{A}])\{f\} - \beta[\mathbf{B}]cos\theta t\{f\} = 0 \qquad (K.1)$$

where $[\mathbf{C}] = diag(\frac{1}{\omega_i})^2$, $[\mathbf{H}] = diag(\zeta_i)$, $[\mathbf{I}] = diag(1)$, $[\mathbf{A}]$ represents an initial stress matrix, and $[\mathbf{B}]$ represents the parametric coupling matrix, α is the non-dimensional initial force, β is the non-dimensional parametric excitation, and θ is the parametric excitation frequency. The solution to this equation was sought as:

$$\{f(t)\} = e^{\lambda t}\{\frac{1}{2}\mathbf{b}_o + \sum_{n=1}^{\infty}(\mathbf{a}_n sinn\theta t + \mathbf{b}_n cosn\theta t)\} \qquad (K.2)$$

Direct substitution of equation (K.2) into equation(K.1), and applying the harmonic balance method leads to a relationship defining the response of the system in terms of the exponent λ, and the coefficients $\mathbf{b}_o, \mathbf{a}_n, \mathbf{b}_n$. Takahashi demonstrated that this relationship could be formulated in matrix notation as:

$$([\mathbf{M}_o] - \lambda[\mathbf{M}_1] - \lambda^2[\mathbf{M}_2])\{\mathbf{X}\} = \{0\} \qquad (K.3)$$

where $[\mathbf{M}_o], [\mathbf{M}_1], [\mathbf{M}_2]$ represent coefficient matrices of the powers of λ, and $\{\mathbf{X}\}$ represents a column vector containing the coefficients of the expansion $\mathbf{b}_o, \mathbf{a}_n, \mathbf{b}_n$. Equation (K.3) may be transformed into an eigenvalue equation in λ by introducing the transformation $\{Y\} = \lambda\{X\}$. Thus equation (K.3) becomes:

$$\begin{bmatrix} 0 & I \\ M_2^{-1}M_o & M_2^{-1}M_1 \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} = \lambda \begin{Bmatrix} X \\ Y \end{Bmatrix} \tag{K.4}$$

Equation (K.4) represents an eigenvalue extraction for the exponent λ. The stability of the system is thus dependent simply on $real(\lambda) < 0$.

It is important to note that the expansion assumed in equation (K.2) is of period T. As Takahashi[1982] demonstrates, the requirement for the solution to accommodate solutions of period T and 2T and thus higher order regions of instability, is satisfied by accounting for the imaginary part of the exponent λ in the expansion. Since λ is a natural consequence of the solution process, it enables the solution to assume a periodicity of either T or 2T on the boundary of stability. With regard to regions of combination resonance, the imaginary component of the exponent introduces side-band [illegible]cies in the expansion such that additive or difference combinations of these components equate to the parametric excitation frequency. Thus the method satisfies the conditions required on the boundary of simple and combination parametric resonance. Takahashi[1981a] demonstrates that this technique yields similar results to those obtained by Szemplinska-Stupnika[1978] for a two degree of freedom parametrically excited oscillator. Takahashi[1982] also demonstrates that the method accounts for the potential destabilising influence of non-uniform damping on regions of combination resonance. This is illustrated in figure K.2, which is reproduced from Takahashi[1982], for three different cases of damping.

The method is easily to coded, and applies a general approach to the problem without introducing the concept of a small parameter; most importantly the method does not require the analyst to identify special or anomalous cases as in the perturbation techniques. The disadvantage of the technique is that it is computationally intensive, however in light of the rapid developments in computational resources, this is not viewed as a serious limitation. With regard to this study, the technique was coded using Matlab, and accounted for systems of the form:

$$[\mathbf{C}]\{\ddot{f}\} + 2[\mathbf{C}][\mathbf{H}]\{\dot{f}\} + ([\mathbf{I}] - \alpha[\mathbf{A}])\{f\} - \sum_{n=1}^{2} \{[\mathbf{S}_n] sinn\theta t + [\mathbf{C}_n] cosn\theta t\}\{f\} = 0$$

The parametric coupling matrices are defined for the mine hoist system in appendix F. During the eigenvalue extraction process, the real part of the exponent was stored, enabling the regions of instability to be complimented by

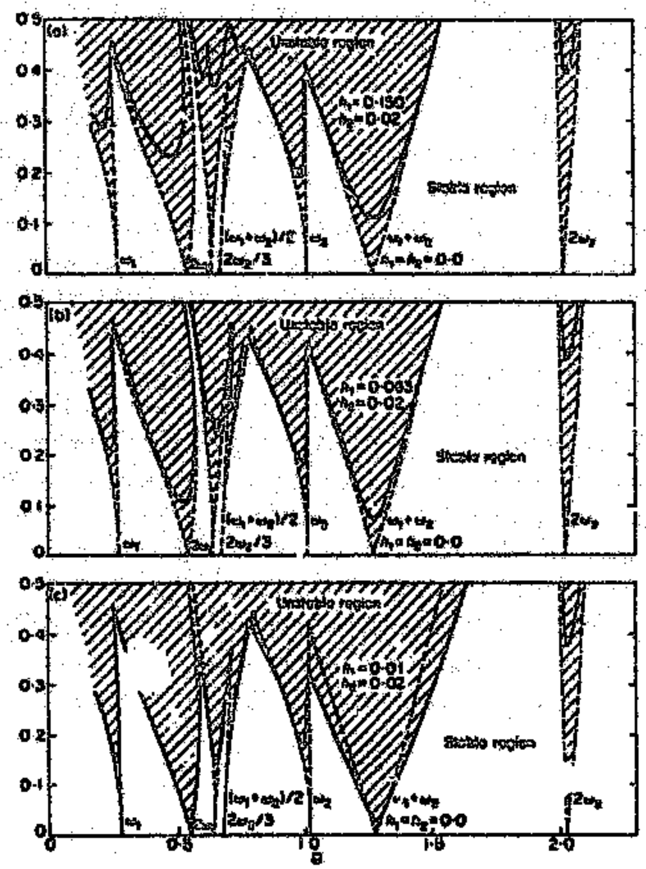

Figure 2. Regions of parametric instability for coupled Mathieu equations. (a) $h_1 = 0{\cdot}150$, $h_2 = 0{\cdot}02$; (b) $h_1 = 0{\cdot}063$, $h_2 = 0{\cdot}02$; (c) $h_1 = 0{\cdot}01$, $h_2 = 0{\cdot}02$.

Figure K.2: Takahashi (1982): Figure 2 - Regions of parametric instability for coupled Mathieu equations.
(a) $h_1 = 0.15$, $h_2 = 0.02$; (b) $h_1 = 0.063$, $h_2 = 0.02$; (c) $h_1 = 0.01$, $h_2 = 0.02$;

contour plots of the maximum exponent. This exponent indicates qualitatively the degree of severity of the region, as it is indicative of the exponential growth of a disturbance. In the context of the mine hoist system, if it is necessary to traverse a region of instability, then it would be desirable to traverse a region where this exponent is minimal. It is also important to consider that the stability analysis of the mine hoist system as presented in chapter 4, only considers stationary steady state response. Thus the influence of the transport speed which induces non-stationary system behaviour is not accounted for. This limits the applicability of the analysis, since in the case of catenary resonance, the maximum steady state amplitude will not be reached, and furthermore it will be delayed with respect to the resonant condition. In addition, once the system passes through a region of instability, the response may be substantially different from the steady state condition. Thus although the steady state stability analysis may indicate that the motion may be stable in the context of the stationary steady state response, transient or residual response induced by a layer change or by the earlier traverse of a region of instability may induce further regions of instability, and hence a bifurcation to alternative dynamic state. It is emphasised that in the context of this study, the purpose of the stability analysis was two fold. Firstly, to demonstrate a mechanism describing the potential nonlinear interaction inherent in the system, and confirmation of this mechanism with respect to a laboratory experiment. Secondly, to provide an approximate method to asses the dynamic characteristics of the system for the purpose of avoiding regions of nonlinear interaction, prior to a more sophisticated nonlinear numerical simulation of the system. In this context the objective of the analysis was fulfilled; clearly, substantial scope exists for refinement of the stability analysis to accommodate the non-stationary nature of the system.

Appendix L

Numerical Sensitivity Study

Since the system parameters on a realistic hoisting system will not be known precisely, and may change during the life of the rope, the quasi-static simulation was applied to investigate the sensitivity of the system. In this study, the winding velocity was changed from 14 m/s to 16 m/s in 0.2 m/s increments. This was conducted for the ascending and descending cycle of the Kloof hoist system. The numerical results are presented in figures (L.1 - L.22). Each figure is headed by the linear system characteristics, where the first four longitudinal (dotted lines) and lateral modes (solid lines) are presented, and the first two Lebus groove excitation harmonics (dashed lines). The layer-change locations are represented by vertical lines.

Since the lateral frequency lines have a low slope, a small change in the winder velocity can shift the resonant location. Since the layer layer change locations remain fixed relative to the shaft depth, the effect of a change in winding velocity is to shift the resonant locations relative to the layer change location. This is an important effect, which was evident in the simulations. As the lateral excitation changes phase after a layer change to accommodate the change in the traverse direction across the drum, the location of the layer change relative to a resonant condition changes. In this regard a layer change may be located such that the motion which has increased on approaching resonance, is attenuated by the change in phase of the excitation. Since the in-plane motion is dependent on the amplitude of the out-of-plane motion, such an attenuation can retard the growth of the in-plane motion. This effect is evident in the ascending and descending cycle. Consider the difference in the response on the ascending cycle at 14 m/s and 14.2 m/s, illustrated in figures (L.1,L.3) respectively. In the latter condition, the layer change is located close to the linear resonance. This location attenuates the growth in the out-of-plane motion, and as a result the in-plane motion does not grow to the degree that it would at 14m/s or 14.4 m/s. This pattern is repeated with respect to figures (L.15,L.17). Similar trends are evident in the results of the descending simulation, for instance in figures (L.2L.4L.16L.18).

The results of this study confirm that the system is sensitive to its overall state of tuning. It is also evident that in the Kloof mine hoist system, the response is caused by primary external resonance of the catenary. With regard to the descending cycle, the natural frequencies are lower, and thus as the winding speed increases, higher lateral modes are excited. For instance above a winding velocity of 15.4 m/s the third lateral mode is excited by the first harmonic of the Lebus excitation. Since the higher modes are stiffer and more difficult to excite, the response and tension fluctuations are lower. This tends to indicate that an appropriate technique to design the winder would be to locate resonances on the descending cycle, and to increase the winding so as to avoid the resonance on the ascending cycle. Although winding speeds of 18 m/s are employed, this approach is counter-intuitive, and usually the winding speed is lowered. In the case of Kloof, the speed was lowered to below 14 m/s to overcome rope whip.

On the Kloof winder, no clear indication exists of secondary regions of resonance, where large amplitude motion is observed in the absence of external resonance of the catenary. Further simulations on different installations, combined with experimental correlation is required to ascertain if this observation is generally consistent.

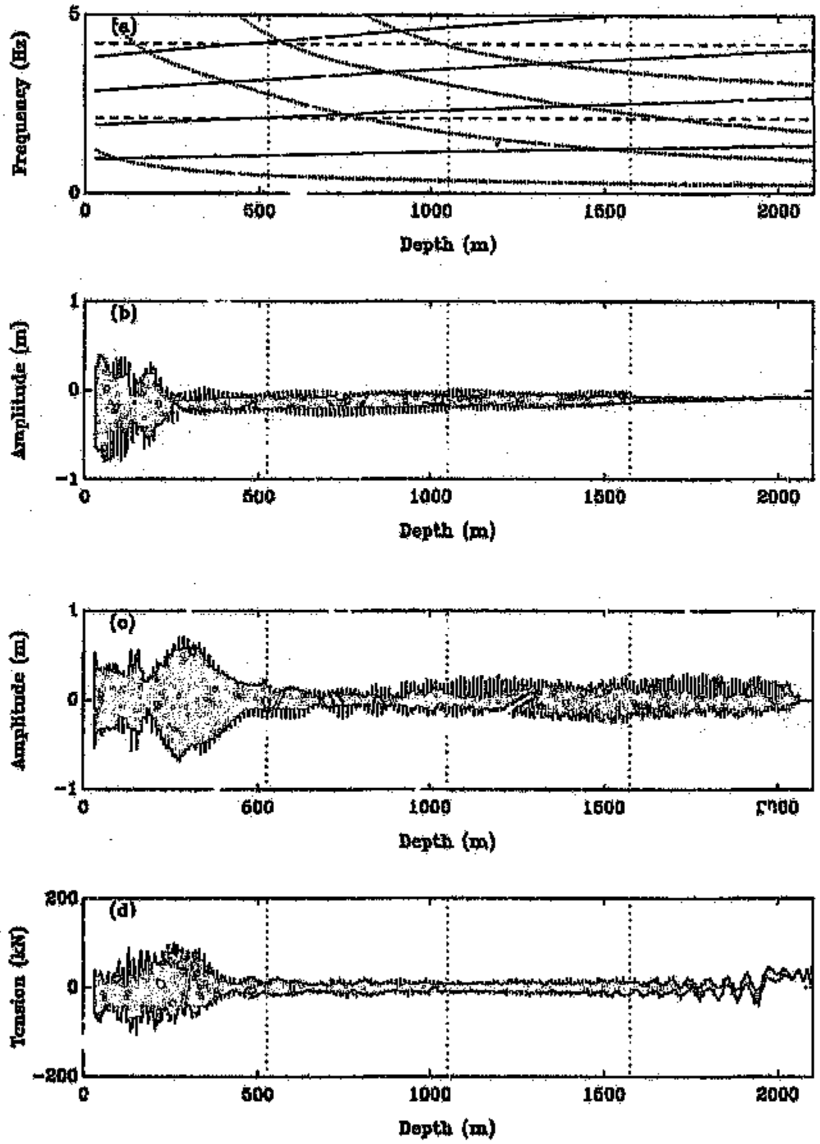

Figure L.1: Ascending cycle - Kloof Mine hoist system: 14 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

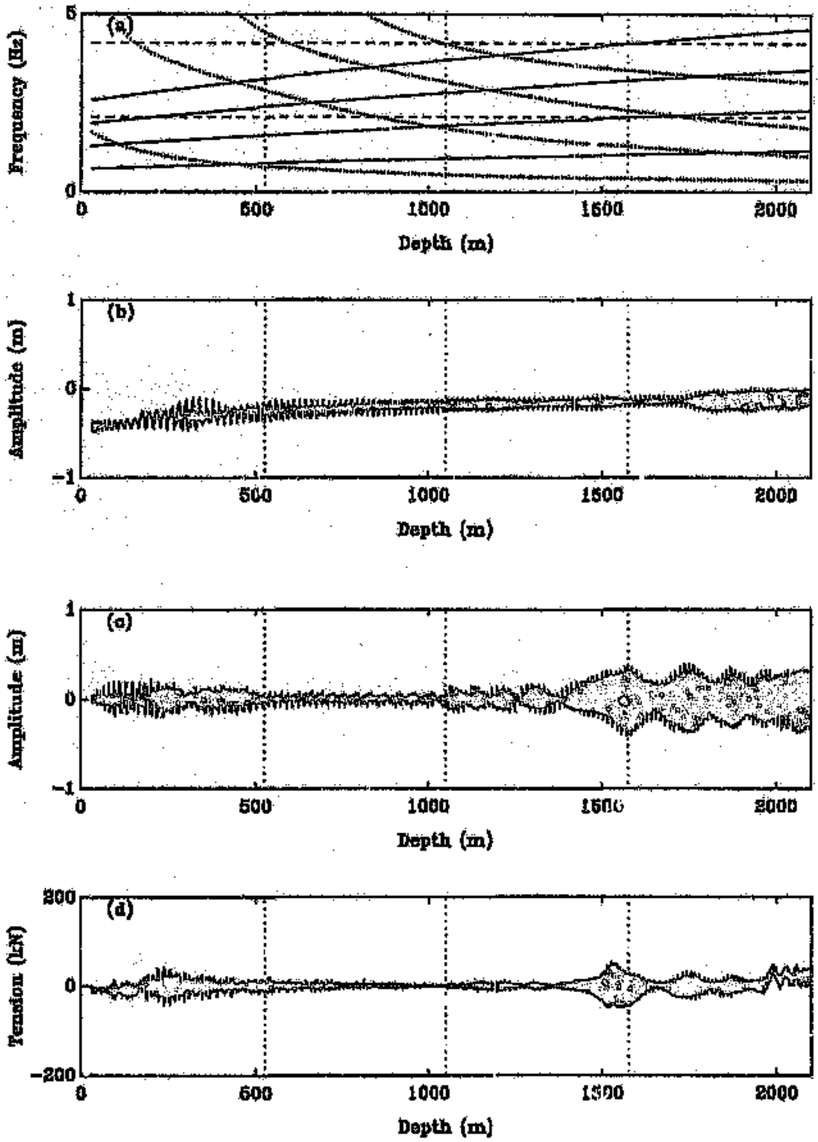

Figure L.2: Descending cycle - Kloof Mine hoist system: 14 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

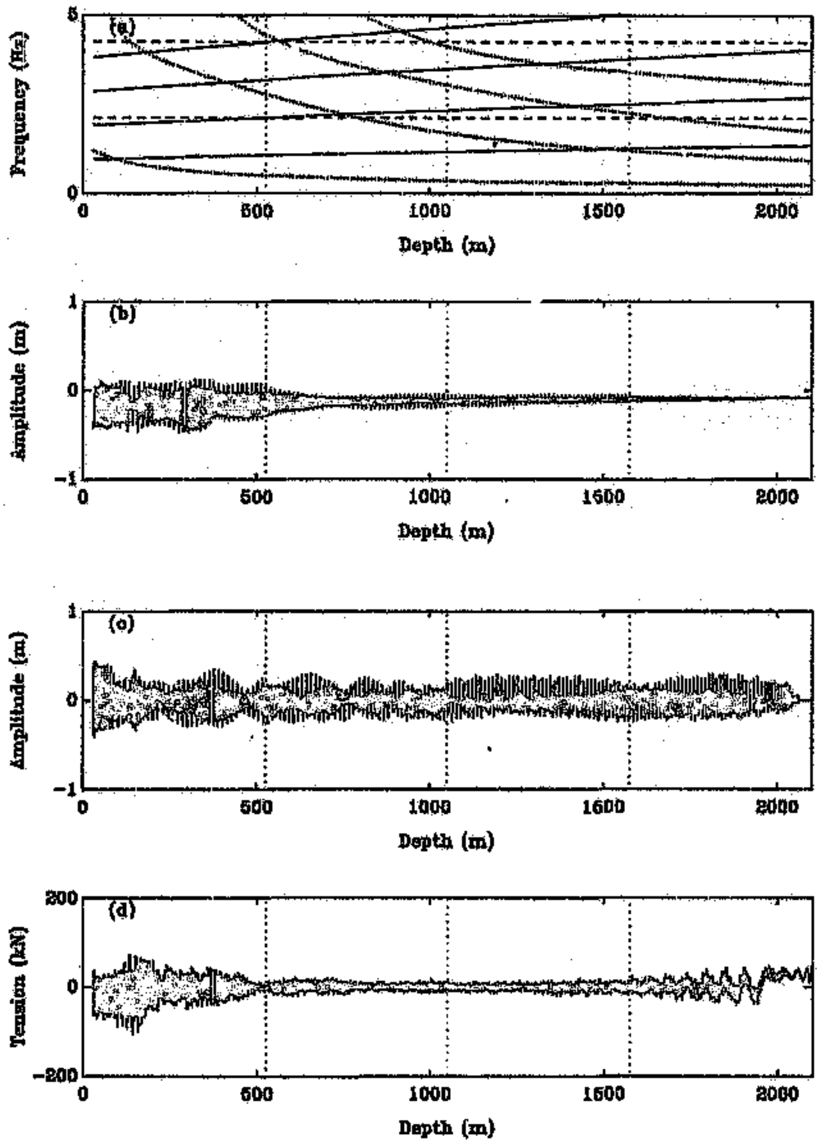

Figure L.3: Ascending cycle - Kloof Mine hoist system: 14.2 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

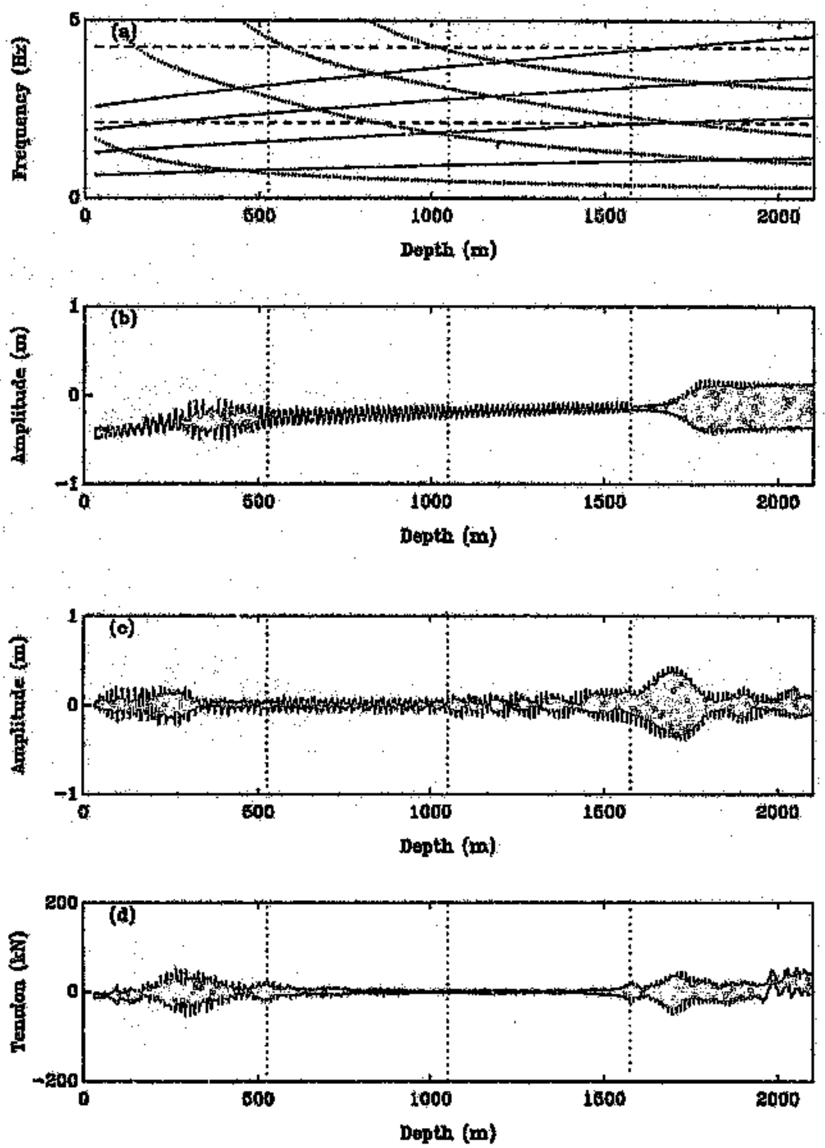

Figure L.4: Descending cycle - Kloof Mine hoist system: 14.2 m/s

a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

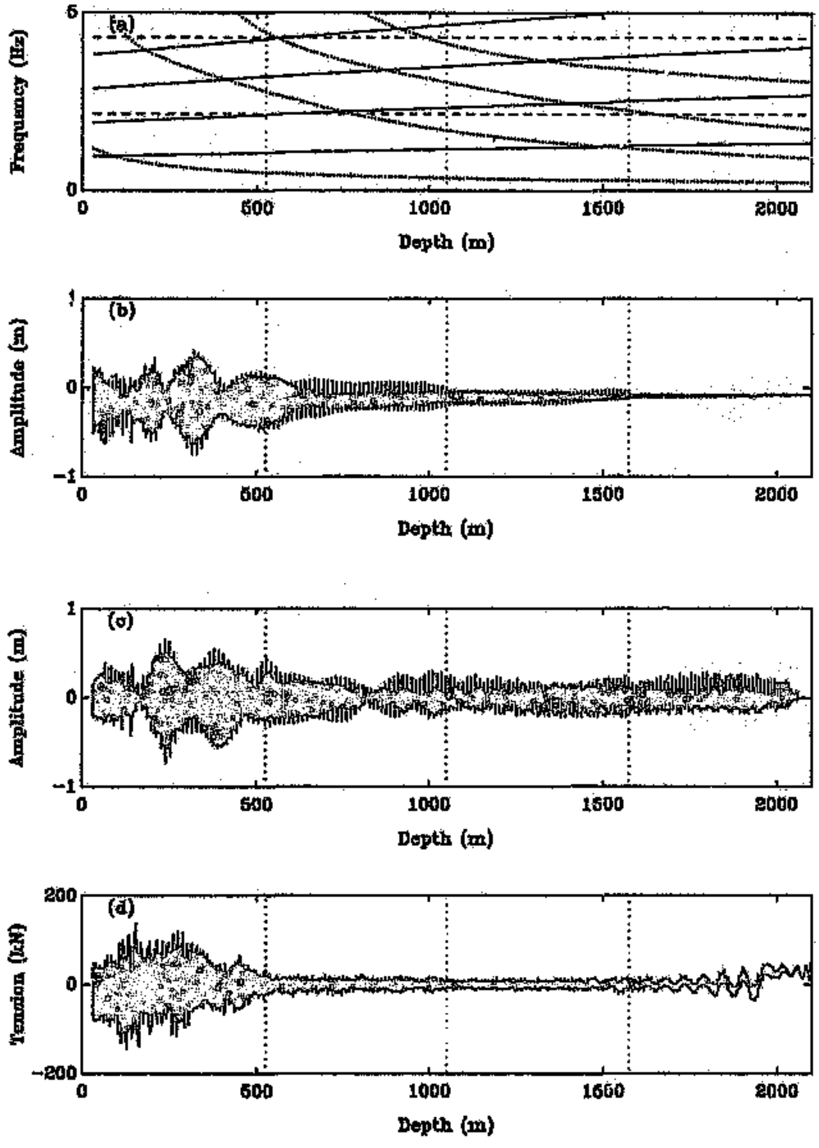

Figure L.5: Ascending cycle - Kloof Mine hoist system: 14.4 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

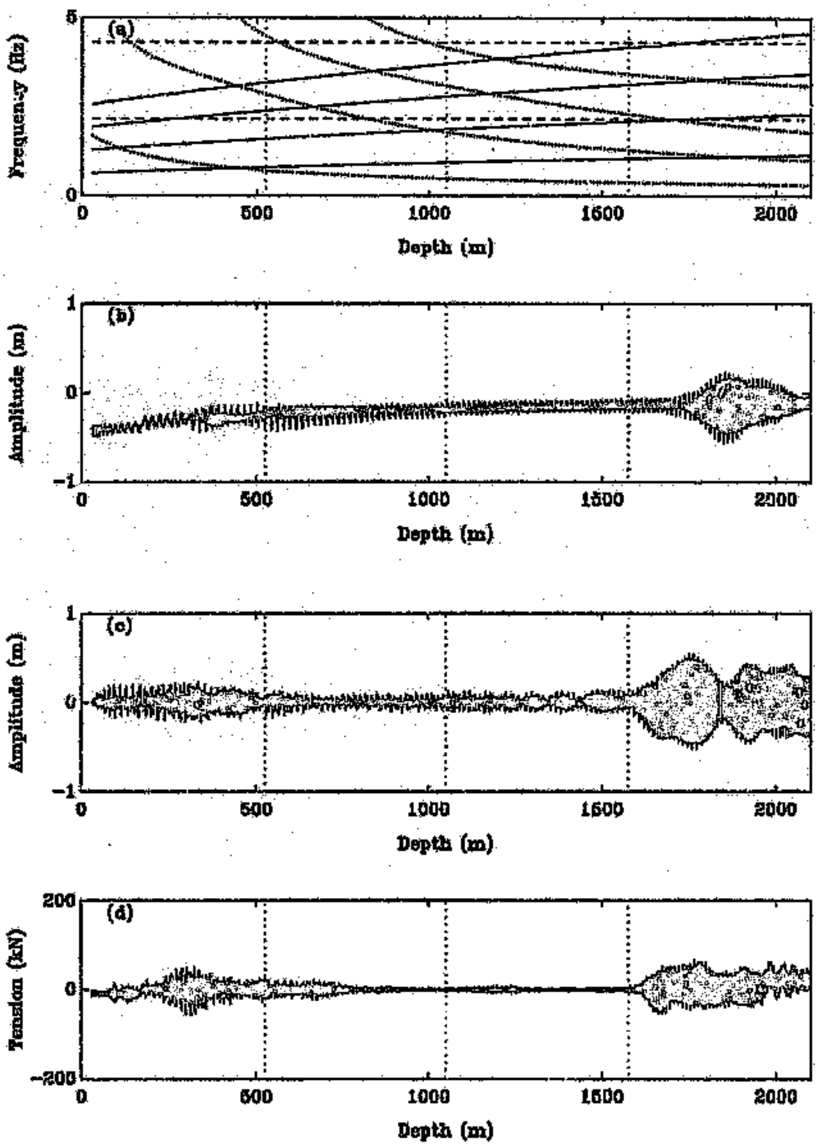

Figure L.6: Descending cycle - Kloof Mine hoist system: 14.4 m/s

a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

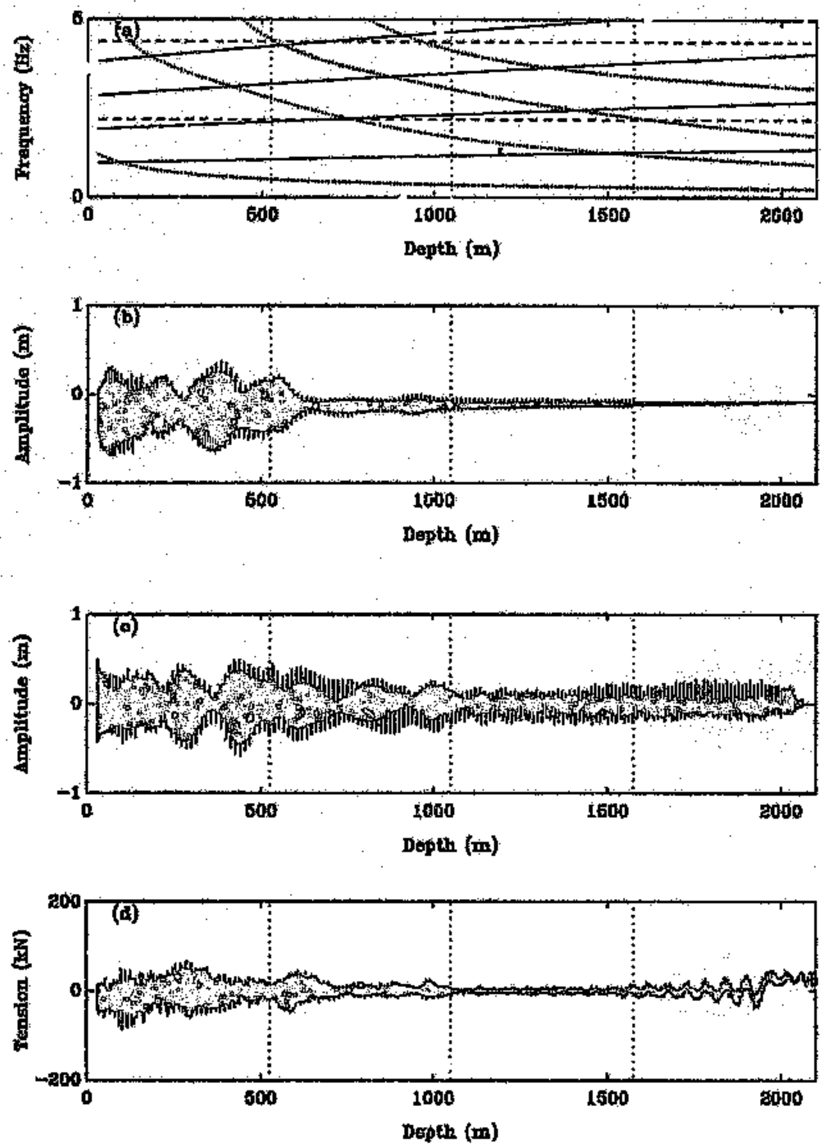

Figure L.7: Ascending cycle - Kloof Mine hoist system: 14.6 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

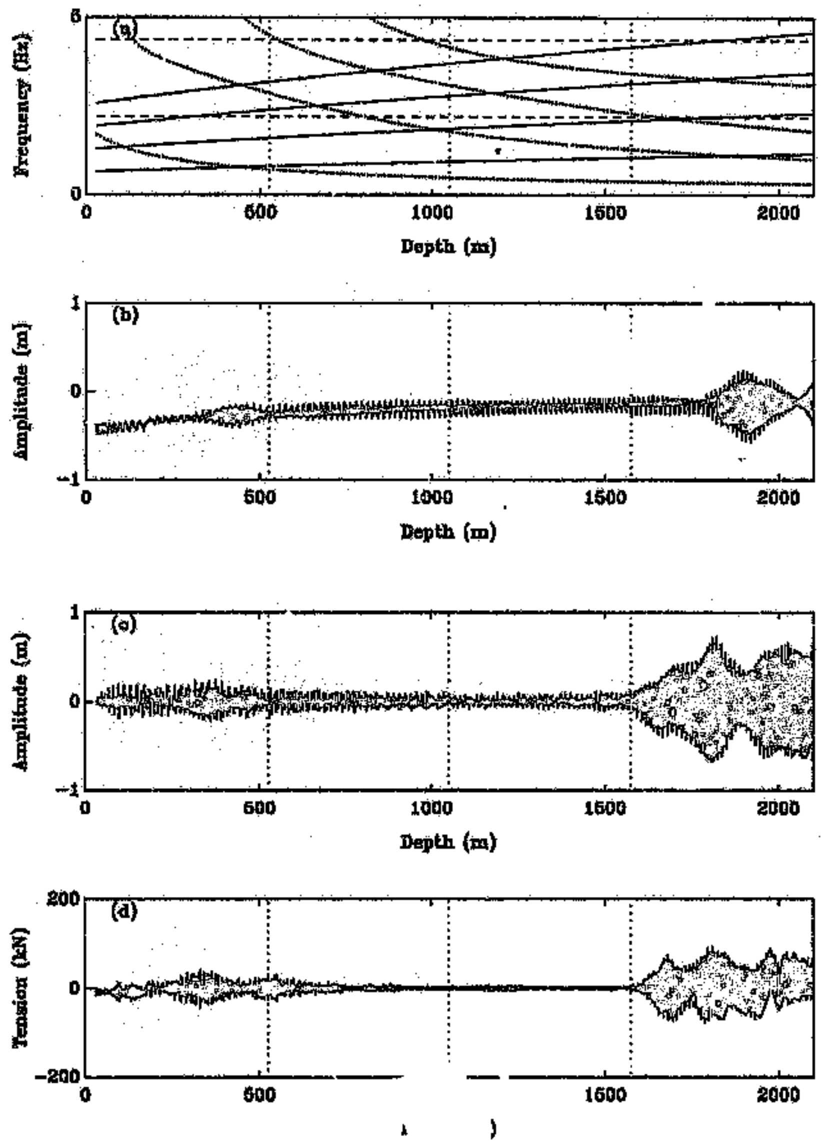

Figure L.8: Descending cycle - Kloof Mine hoist system: 14.6 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

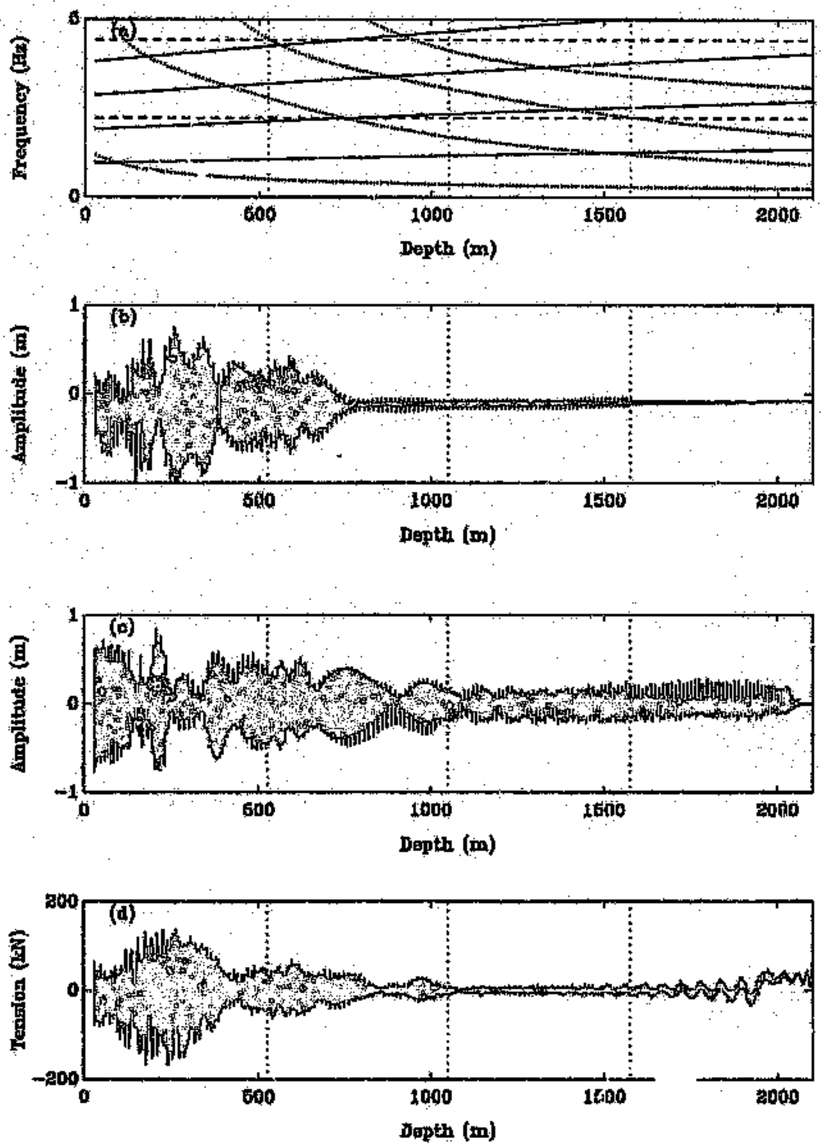

Figure L.9: Ascending cycle - Kloof Mine hoist system: 14.8 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

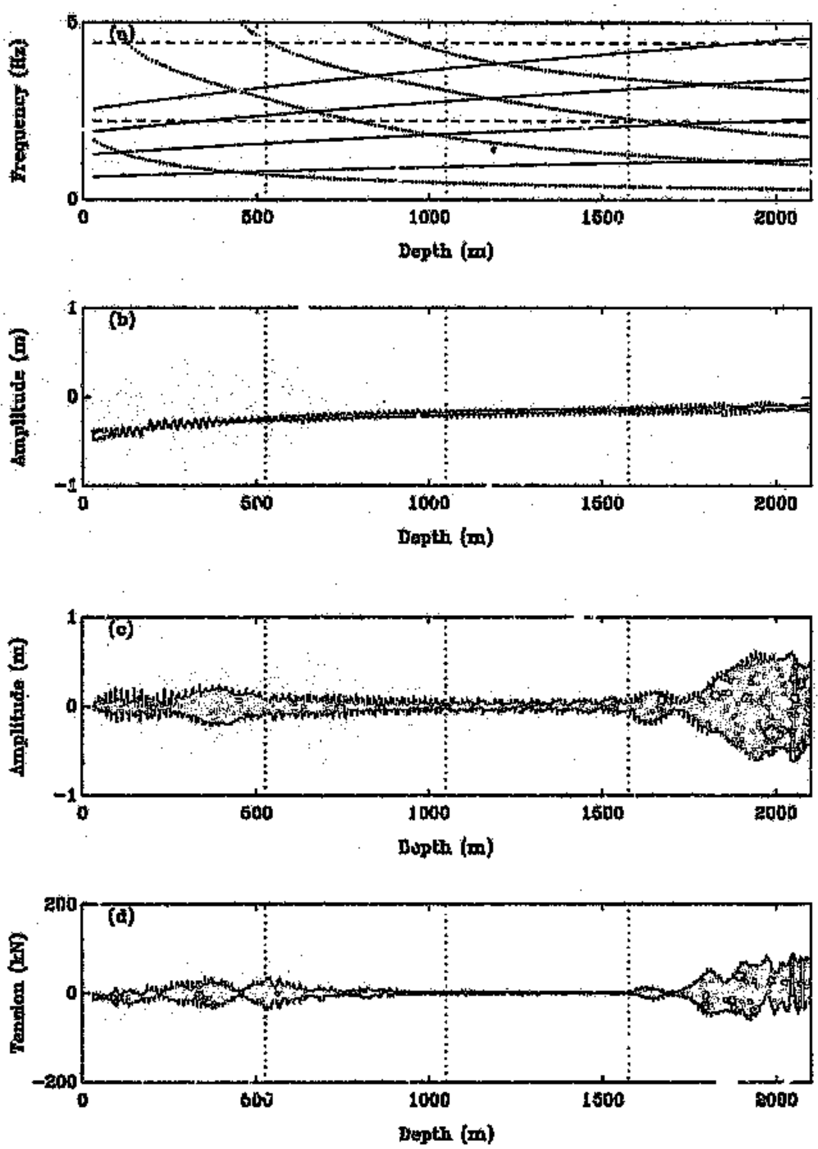

Figure L.10: Descending cycle - Kloof Mine hoist system: 14.8 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

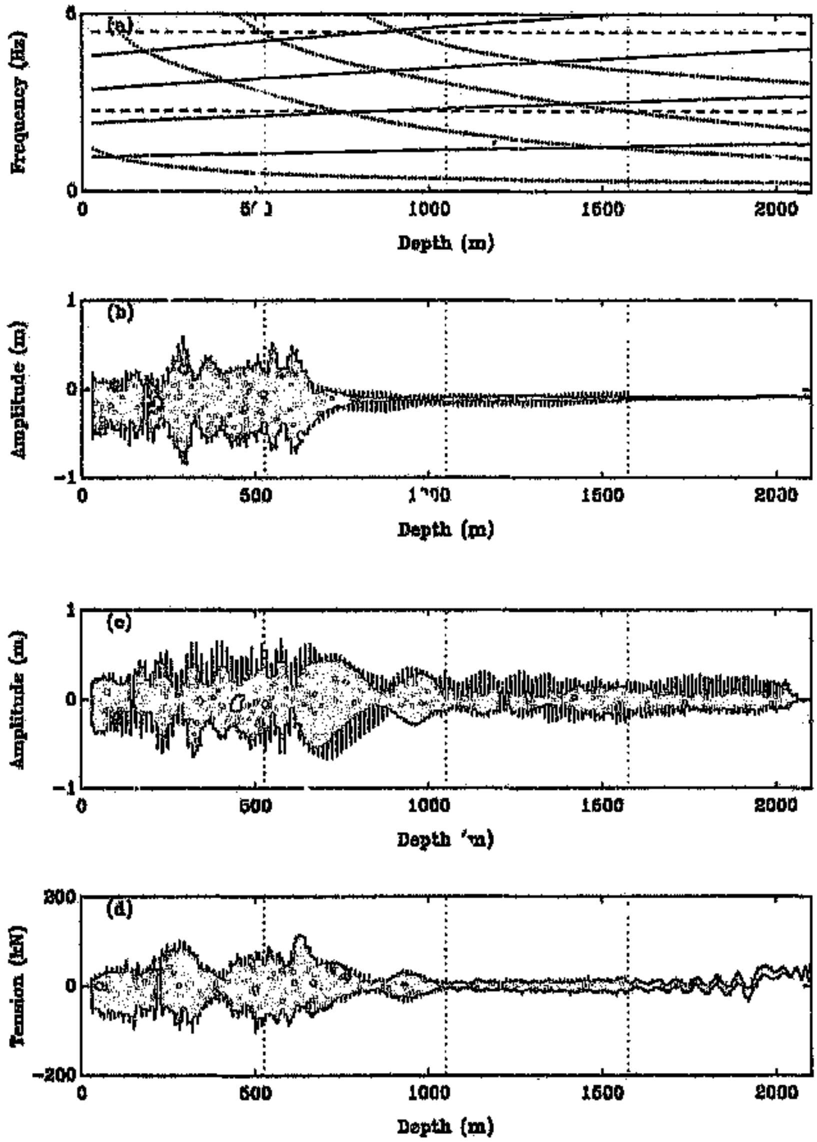

Figure L.11: Ascending cycle - Kloof Mine hoist system: 15 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

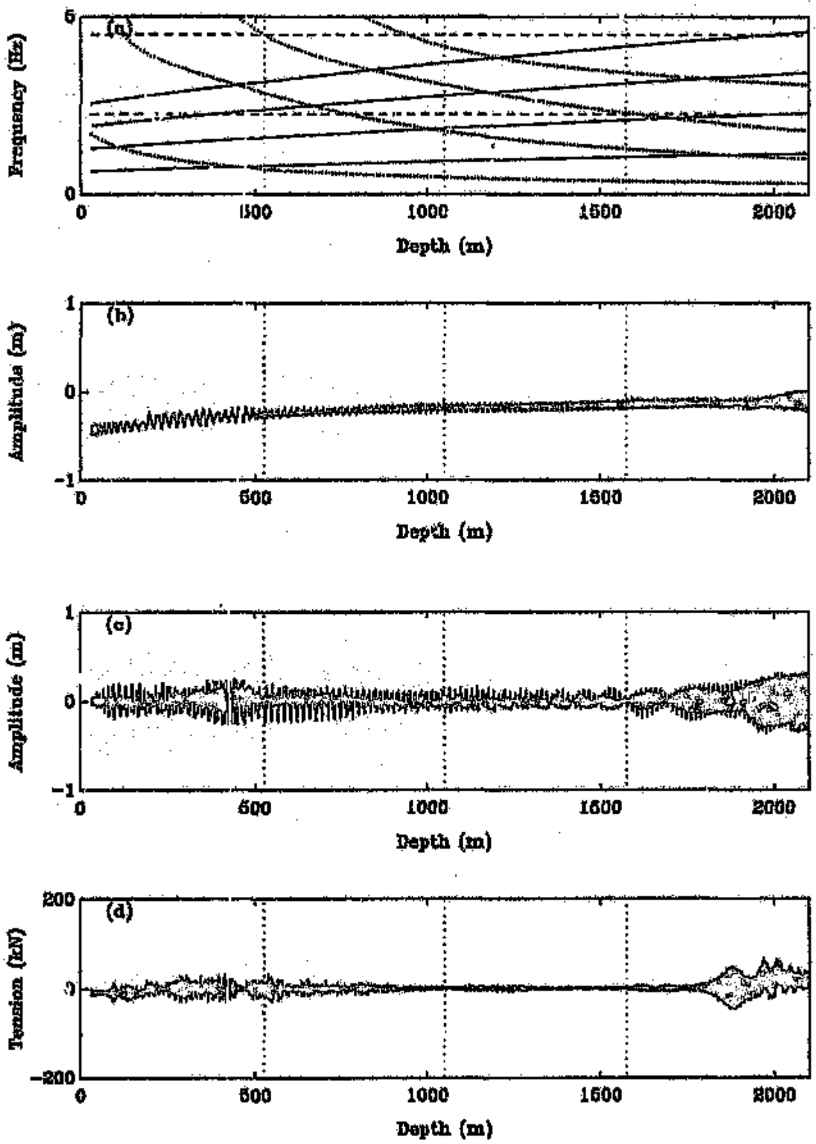

Figure L.12: Descending cycle - Kloof Mine hoist system: 15 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

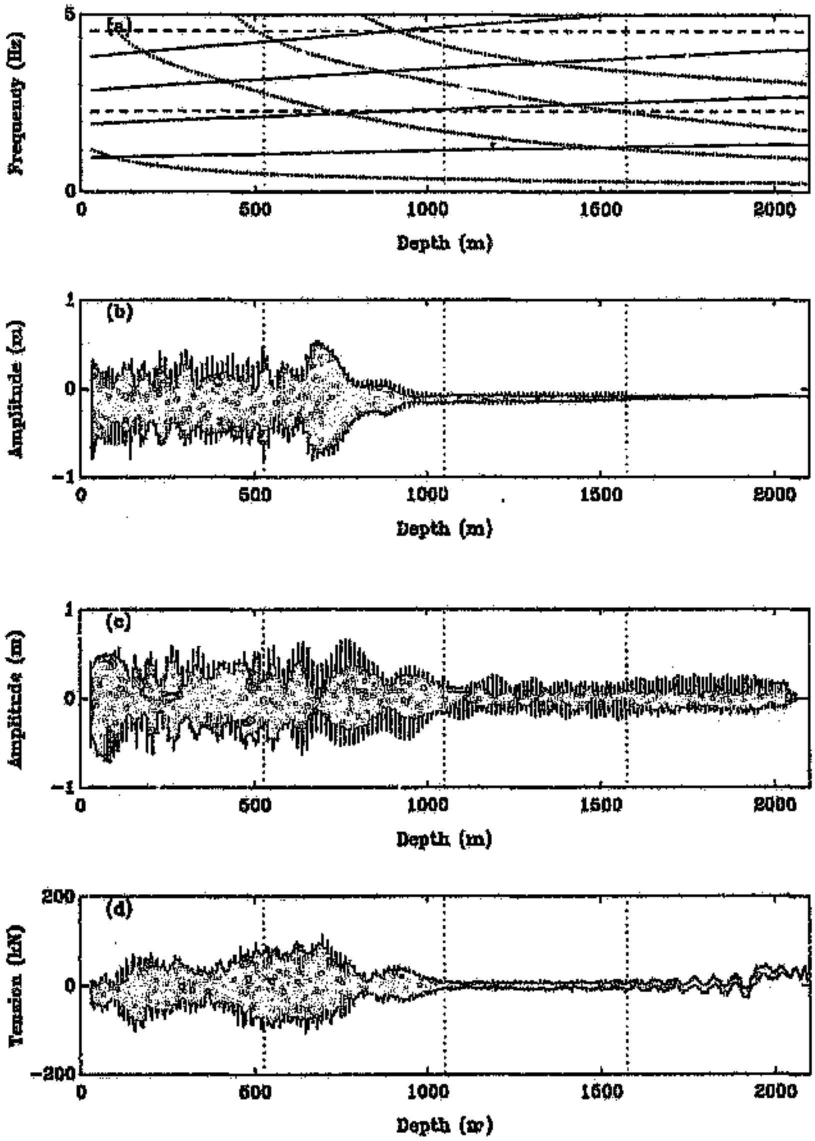

Figure L.13: Ascending cycle - Kloof Mine hoist system: 15.2 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

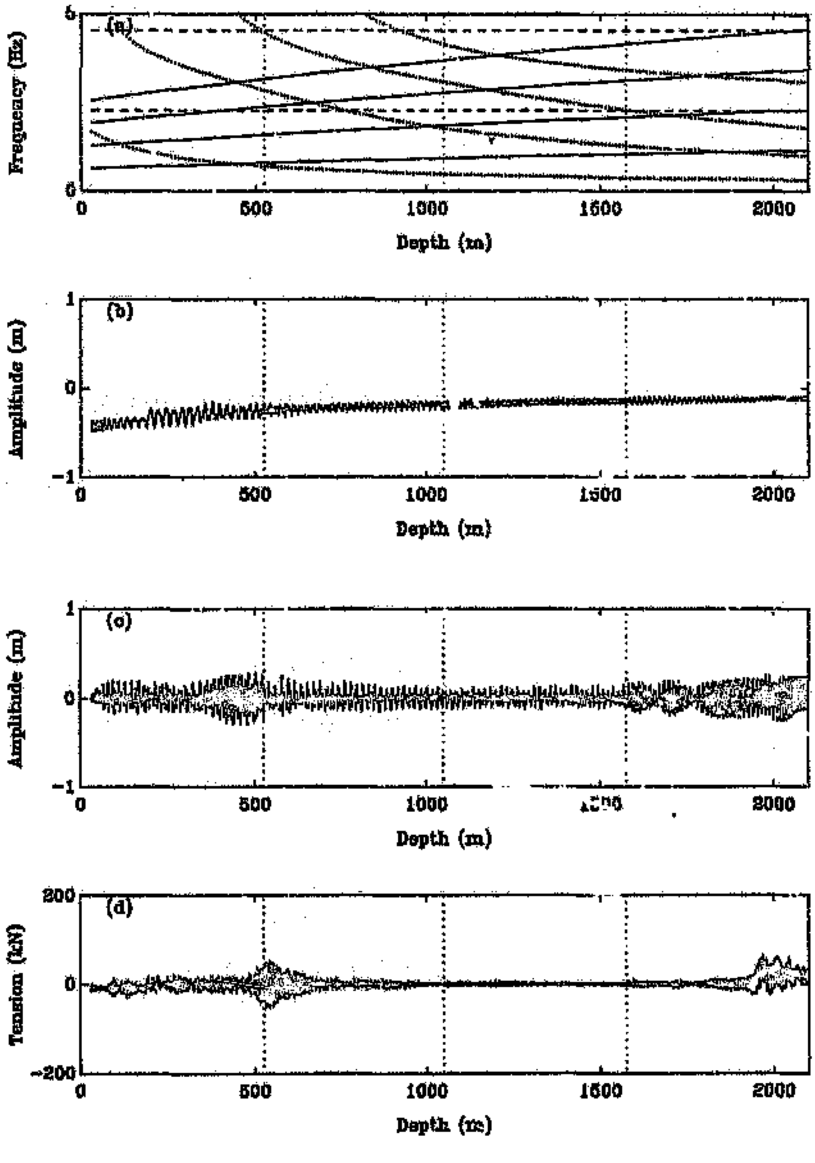

Figure L.14: Descending cycle - Kloof Mine hoist system: 15.2 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

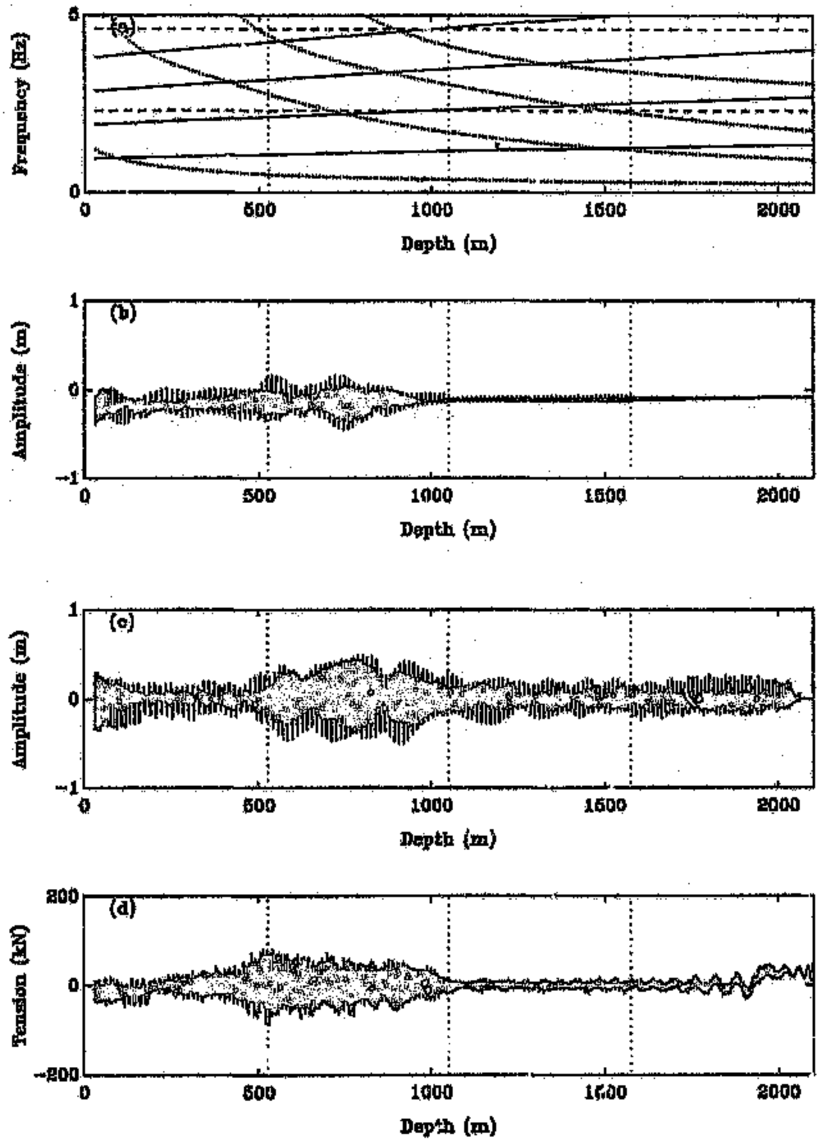

Figure L.15: Ascending cycle - Kloof Mine hoist system: 15.4 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

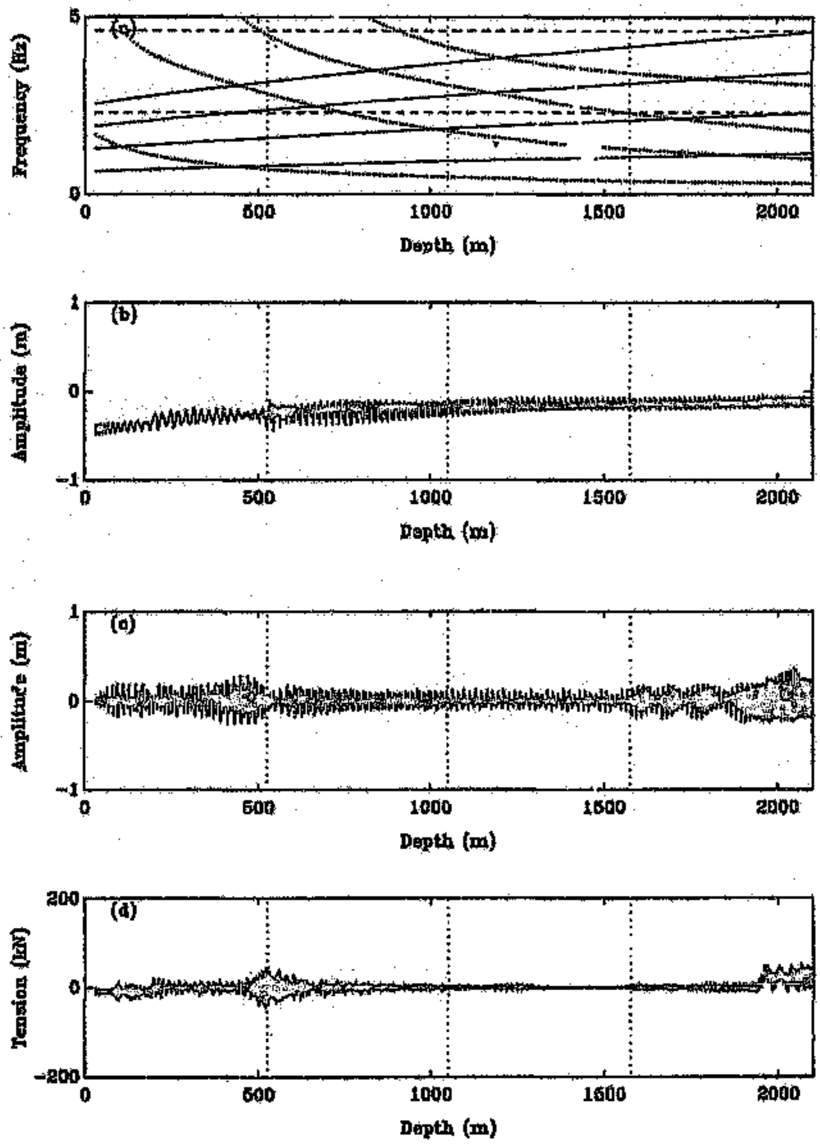

Figure L.16: Descending cycle - Kloof Mine hoist system: 15.4 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

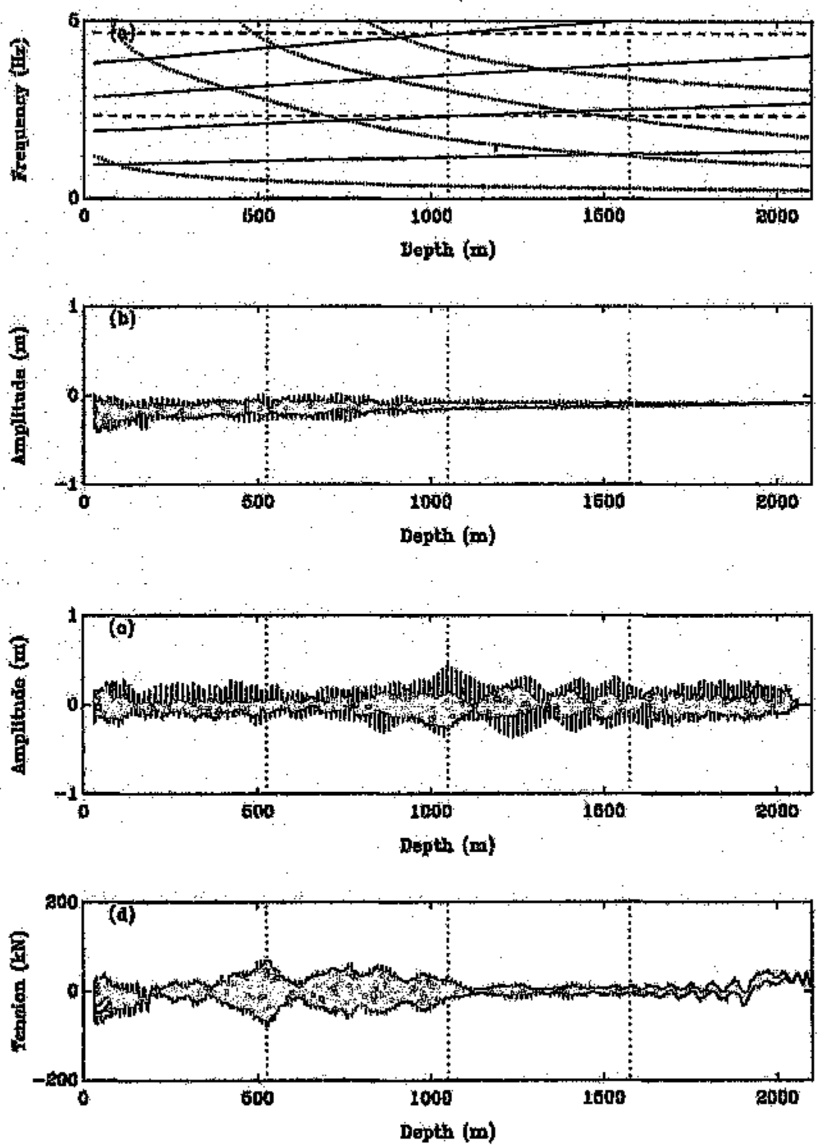

Figure L.17: Ascending cycle - Kloof Mine hoist system: 15.6 m/s

a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

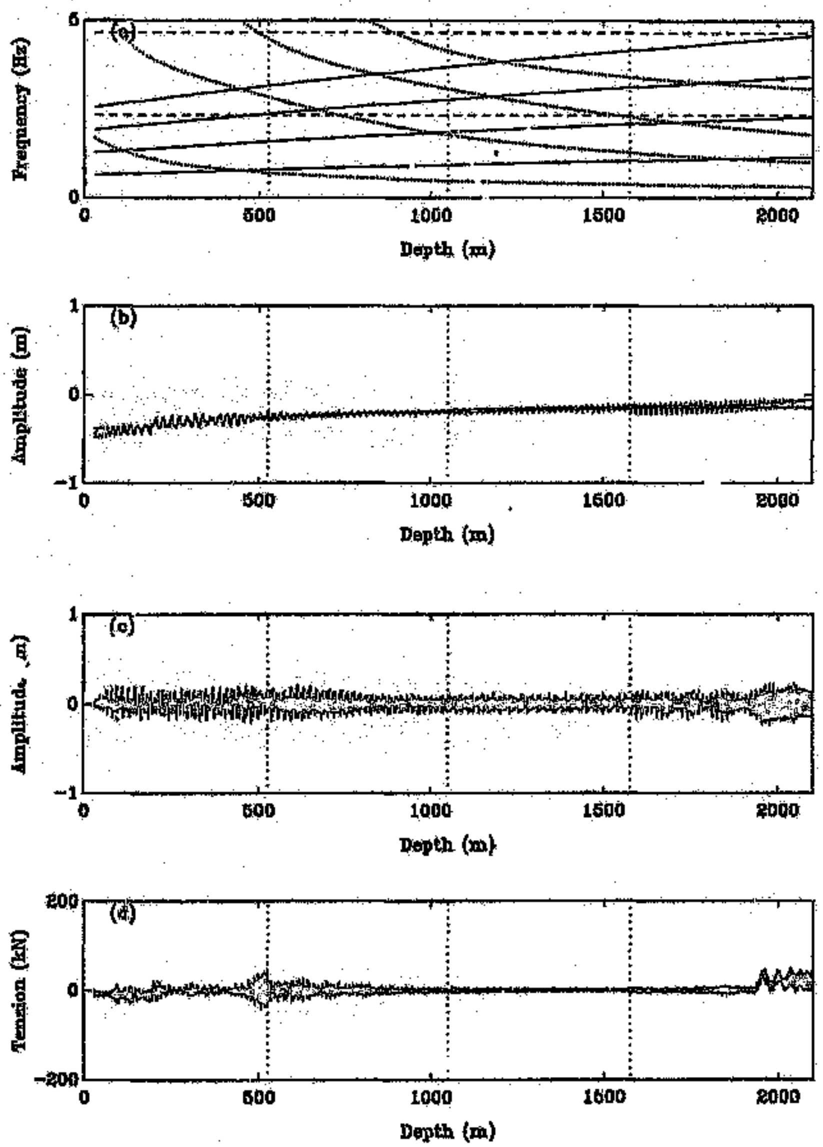

Figure L.18: Descending cycle - Kloof Mine hoist system: 15.6 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

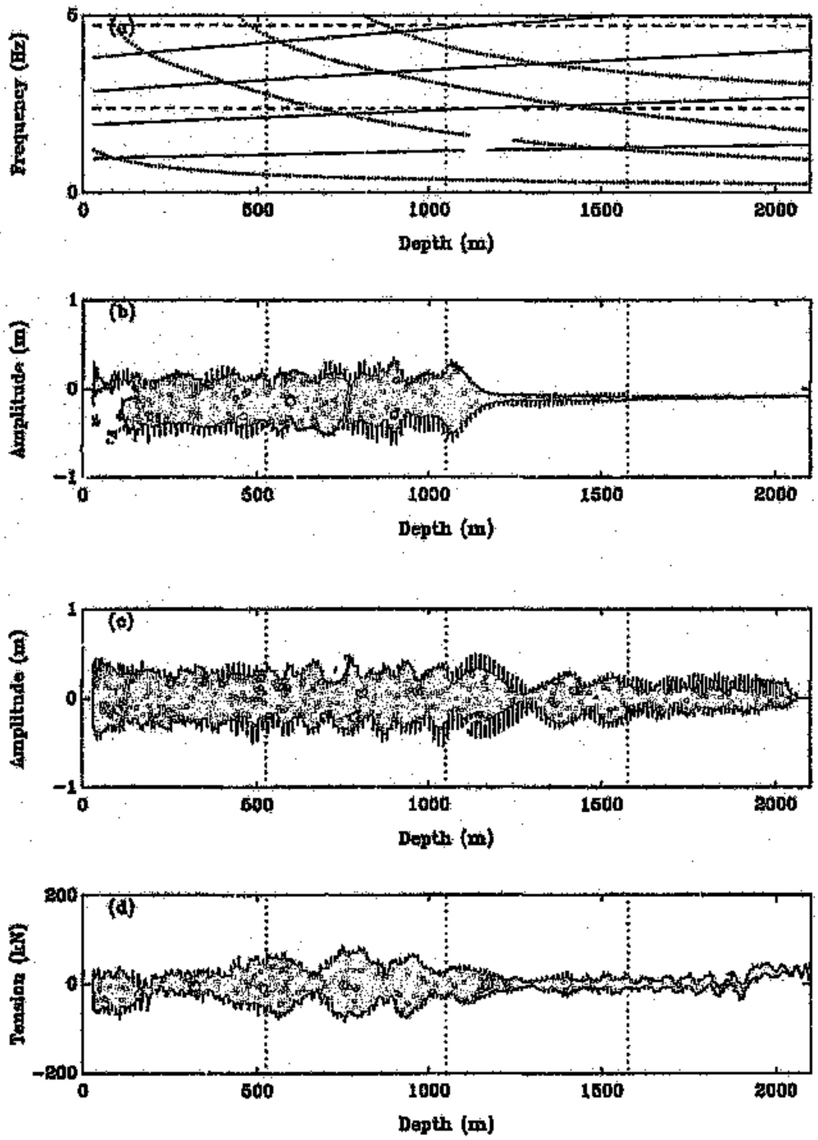

Figure L.19: Ascending cycle - Kloof Mine hoist system: 15.8 m/s

a) Linear Frequency Map.

b) In-Plane Motion - $s = l_c/4$.

c) Out-of-Plane Motion - $s = l_c/4$.

d) Dynamic Catenary Tension.

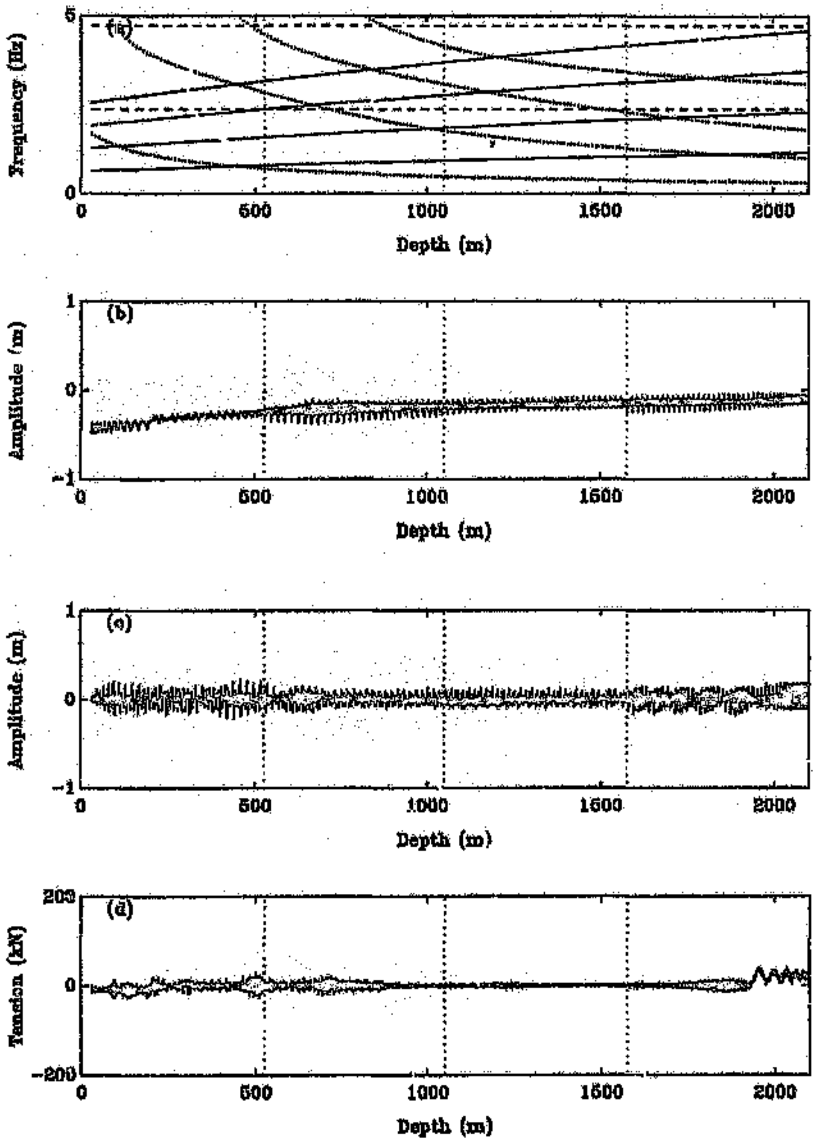

Figure L.20: Descending cycle - Kloof Mine hoist system: 15.8 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

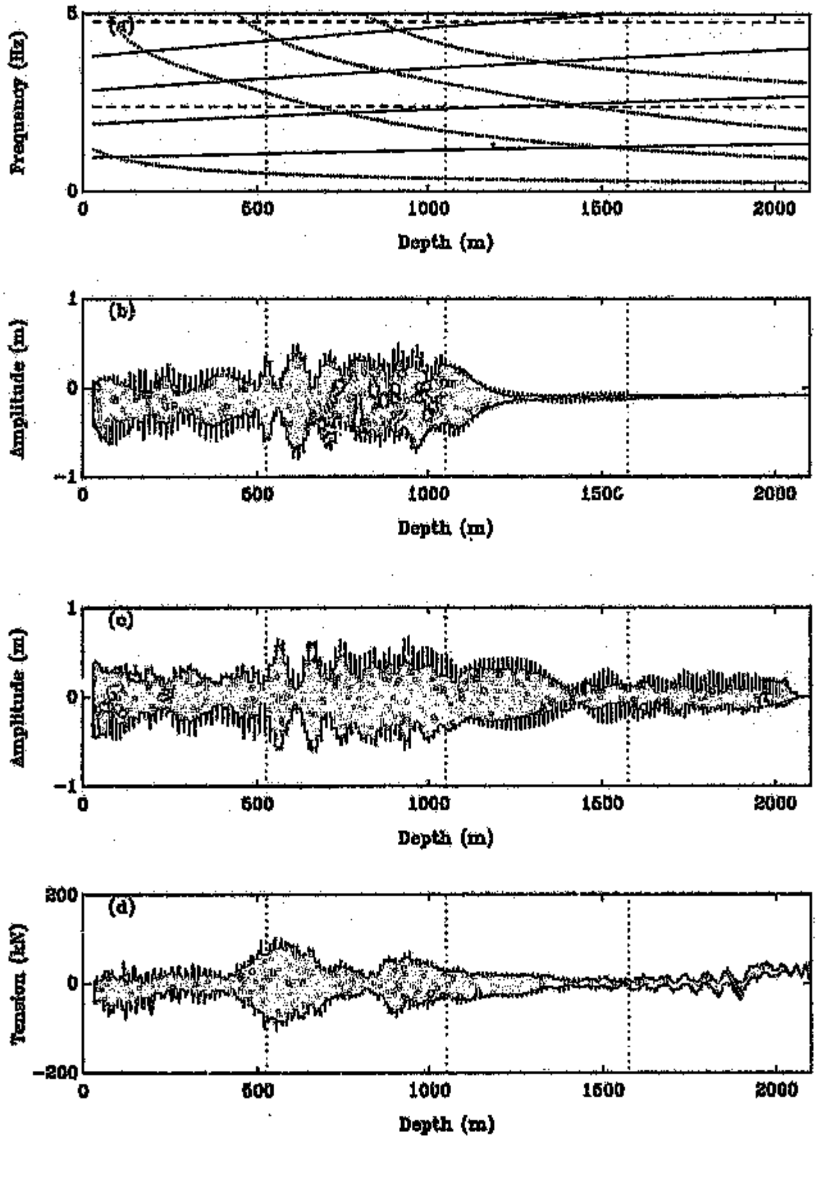

Figure L.21: Ascending cycle - Kloof Mine hoist system: 16 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

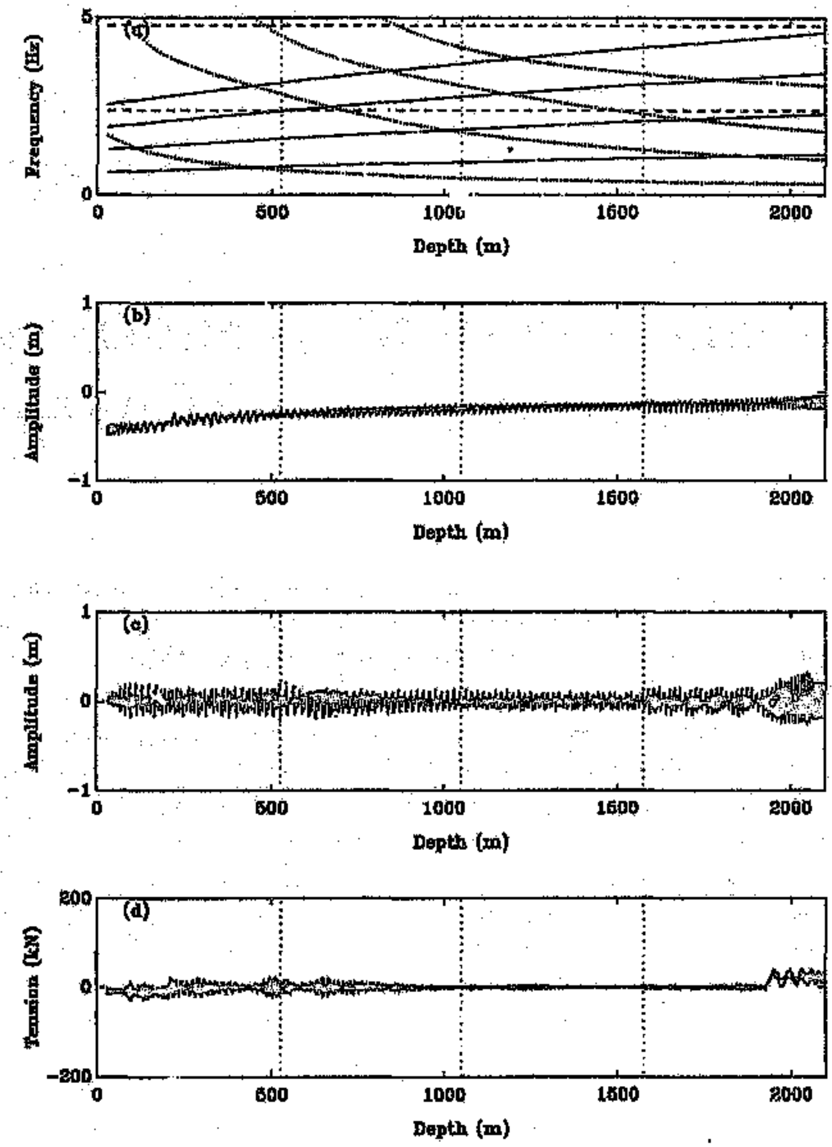

Figure L.22: Descending cycle - Kloof Mine hoist system: 16 m/s
a) Linear Frequency Map.
b) In-Plane Motion - $s = l_c/4$.
c) Out-of-Plane Motion - $s = l_c/4$.
d) Dynamic Catenary Tension.

Appendix M

Equations of Motion Including Sheave Wheel Constraints

The equations of [illegible] apter 3 apply if the lateral motion of the rope at the she [illegible] is constrained. This boundary condition is satisfied if a friction [illegible] which is aligned to the tangent of the equilibrium profile o [illegible] he sheave end. The rope is attached to this guide, such that mo [illegible] angential direction is admitted, whilst lateral motion is restrained. This constraint is a realistic approximation, since small longitudinal motion is generated at the sheave for realistic lateral deflections. Consequently the lateral movement which does exist in reality is not considered significant. However, the case where the geometry of the sheave is accounted for, and such an assumption is not made, is presented in this appendix. Since the curvature of the catenary, and the transport velocity of the rope add unnecessary complexity, they are neglected in the subsequent analysis. To further simplify the analysis, only the planar system is considered.

In order to develop the equations of motion without prescriptive boundary conditions at the sheave, the kinematic relationship between the movement of the rope and sheave requires consideration. Figure M.1 illustrates the equilibrium configuration of the rope, and the final configuration due to lateral and longitudinal motion of the station which was initially in contact with the sheave. Figure M.2 presents the geometric relationship between the rope and sheave rotation. A massless chord passes over the sheave wheel and attaches to the vertical rope. Thus the catenary, sheave and vertical rope are considered as separate sub-systems, which are combined via constraint equations developed to account for the kinematic relationships between the three subsystems.

Since the geometric relationship defining the interface between the end of the rope and sheave requires the definition of the displacements $u(l,t), v(l,t)$, the spatial derivative $v_{,x}(l,t)$, and the sheave rotation, three independent kinematic constraint equations involving these variables are required. An additional constraint relationship is required to couple the vertical subsystem to the sheave rotation. The constraint equations necessary to account for the sheave geometry are thus:

$$
\begin{aligned}
f_1 &= v_1 - (R tan\frac{v_{x_1}}{2} + u_1) tan v_{x_1} \\
f_2 &= R\theta - (u_1^2 + v_1^2)^{1/2} \\
f_3 &= \int_0^{l_1} (1 + u_x + \frac{1}{2}v_x^2) sin(v_x - v_{x_1}) dx + (l_1 - R tan\frac{v_{x_1}}{2}) sin v_{x_1} \\
f_4 &= \overline{u}_o - R\theta
\end{aligned}
\tag{M.1}
$$

where u_1, v_1 represent the displacement at the interface between the rope and chord. v_{x_1} represents the slope of the rope at the interface between the rope and chord. θ, $\overline{u}_o$ represent the sheave rotation, and the displacement of upper boundary of the vertical system.

The first equation defines the constraint relationship ensuring that the chord between the end point of the rope and the point of contact with the sheave, is tangential to the rope and the sheave. The second equation relates the total displacement of the end point of the rope to the sheave rotation. The third equation requires that the profile of the catenary is geometrically related to the distance between the drum and sheave, l_1, and the slope v_{x_1}. The fourth equation couples the sheave rotation and the vertical system.

The equations of motion may be obtained by introducing Lagrange multipliers λ_i and applying Hamilton's principle[1]. Following Craig[1981], the potential energy V is modified to account for the constraint forces by utilising V^* where:

$$V^* = V - \sum_{i=1}^{C} \lambda_i f_i$$

and C represents the number of constraint equations f_i.

[1]The constraint equations are holonomic and thus the motion at t0 and t1 is considered defined, hence the variation of the motion at these times is zero (Meirovitch[1970]).

Following the development in chapter 3, and accounting the constraint via the potential energy of the rope, the Lagrangian is defined as:

$$\begin{aligned}
\mathcal{L} &= \int_0^{l_1} \left\{ \frac{1}{2}m(\dot{u}^2+\dot{v}^2) - (P_1 + \frac{1}{2}AE\epsilon)\epsilon + \lambda_3 \overline{f}_3 \right\} dx \\
&+ \int_0^{l_2} \left\{ \frac{1}{2}m\dot{\overline{u}}^2 - (\overline{P} + \frac{1}{2}AE\overline{\epsilon})\overline{\epsilon} + mg\overline{u} \right\} dy \\
&+ \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}M\dot{\overline{u}}_2^2 + Mg\overline{u}_2 \\
&+ \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 \overline{\overline{f}}_3 + \lambda_4 f_4
\end{aligned}$$

where x, y refer to the spatial domain of the catenary and vertical rope; P_1, $\overline{P}(y)$ represent the static tension in the catenary (constant), and in the vertical rope respectively:

$$\overline{f}_3 = (1 + u_x + \frac{1}{2}v_x^2)sin(v_x - v_{x_1})$$

$$\overline{\overline{f}}_3 = (l_1 - Rtan\frac{v_{x1}}{2})sinv_{x_1}$$

$$\epsilon = u_x + \frac{1}{2}v_x^2$$

$$\overline{\epsilon} = \overline{u}_y$$

Hamilton's principle requires stationarity of the action integral, for arbitrary variations in the generalised co-ordinates, vanishing at the limits t_o, t_1.

$$\delta I = \delta \int_{t_0}^{t_1} \mathcal{L} dt = 0$$

The Lagrangian function contains continuous and discrete terms. Through appropriate integration by parts, and accounting for the equilibrium configuration, the resulting equations of motion are:

$$u_{,tt} = c^2 u_{,xx} + c^2\{v_{,x} v_{,xx}\} - \frac{1}{\rho A}\{\lambda_3 \frac{\partial}{\partial x}(\frac{\partial \overline{f}_3}{\partial u_x})\} \tag{M.2}$$

$$v_{,tt} = \overline{c}^2 v_{,xx} + c^2\{(u_{,x} v_{,x})_{,x} + \frac{3}{2} v_{,x}^2 v_{,xx}\} - \frac{1}{\rho A}\lambda_3 \frac{\partial}{\partial x}(\frac{\partial \overline{f}_3}{\partial v_x}) \tag{M.3}$$

$$I\ddot{\theta} = \lambda_2 \frac{\partial \overline{f}_2}{\partial \theta} + \lambda_4 \frac{\partial \overline{f}_4}{\partial \theta} \tag{M.4}$$

$$\overline{u}_{tt} = c^2 \overline{u}_{,yy} \tag{M.5}$$

$$M\ddot{\overline{u}}_2 = -AE\frac{\partial \overline{u}}{\partial y}|_{(l_2,t)} \tag{M.6}$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_2}{\partial u_1} & \frac{\partial \overline{f}_3}{\partial u}|_{(l_1,t)} & 0 \\ \frac{\partial f_1}{\partial v_1} & \frac{\partial f_2}{\partial v_1} & \frac{\partial \overline{f}_3}{\partial v_x}|_{(l_1,t)} & 0 \\ \frac{\partial f_1}{\partial v_{x_1}} & \frac{\partial f_2}{\partial v_{x_1}} & \frac{\partial \overline{f}_3}{\partial v_{x_1}} & 0 \\ 0 & 0 & 0 & \frac{\partial f_4}{\partial \overline{u}_0} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{Bmatrix} = \begin{Bmatrix} AE\epsilon_1 \\ (P_1 + AE\epsilon_1)v_{x_1} \\ 0 \\ AE\overline{\epsilon}_0 \end{Bmatrix} \tag{M.7}$$

Equations (M.7) are functions of the unknown variables $u(l,t), v(l,t), v_x(l,t)$. Thus equations (M.2)-(M.7) represent eight equations in twelve unknowns ie the variables $u(x,t), v(x,t), u(l,t), v(l,t), v_x(l,t), \theta, \overline{u}(0,t), \overline{u}(y,t), \lambda_1, \lambda_2, \lambda_3, \lambda_4$. The four additional equations required to define the system are provided by the four equations of constraint M.1. This results in a set of mixed ordinary and continuous differential and algebraic equations, which must be satisfied simultaneously. In addition, the constraints may be redundant at various times during the motion. The solution of these equations will present a difficult task. It may be possible to transform the equations into a mixed set of ordinary differential equations and nonlinear algebraic equations. This would require the application of an expansion for the continuous variables. If such a transformation is possible then standard numerical techniques exist to integrate the equations with embedded nonlinear algebraic equations of constraint. Such methods are discussed by Amirouche[1992], Nikravesh[1988]. Further consideration was not given to the equations presented in this appendix, since the boundary condition assumed at the interface between the catenary and sheave, as implemented in chapter 3, was considered sufficiently accurate. An accurate representation of the sheave interface would be necessary if very large catenary motion and sheave rotation occurred. In the context of the mine hoist system the rotation of the sheave is of the order of $2v^2/l_cR \approx 0.01$.

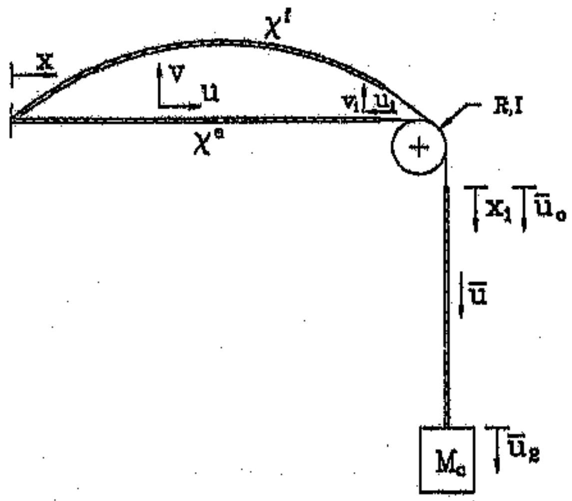

Figure M.1: System configuration with geometric constraints

χ^e Equilibrium configuration
χ^f Final configuration

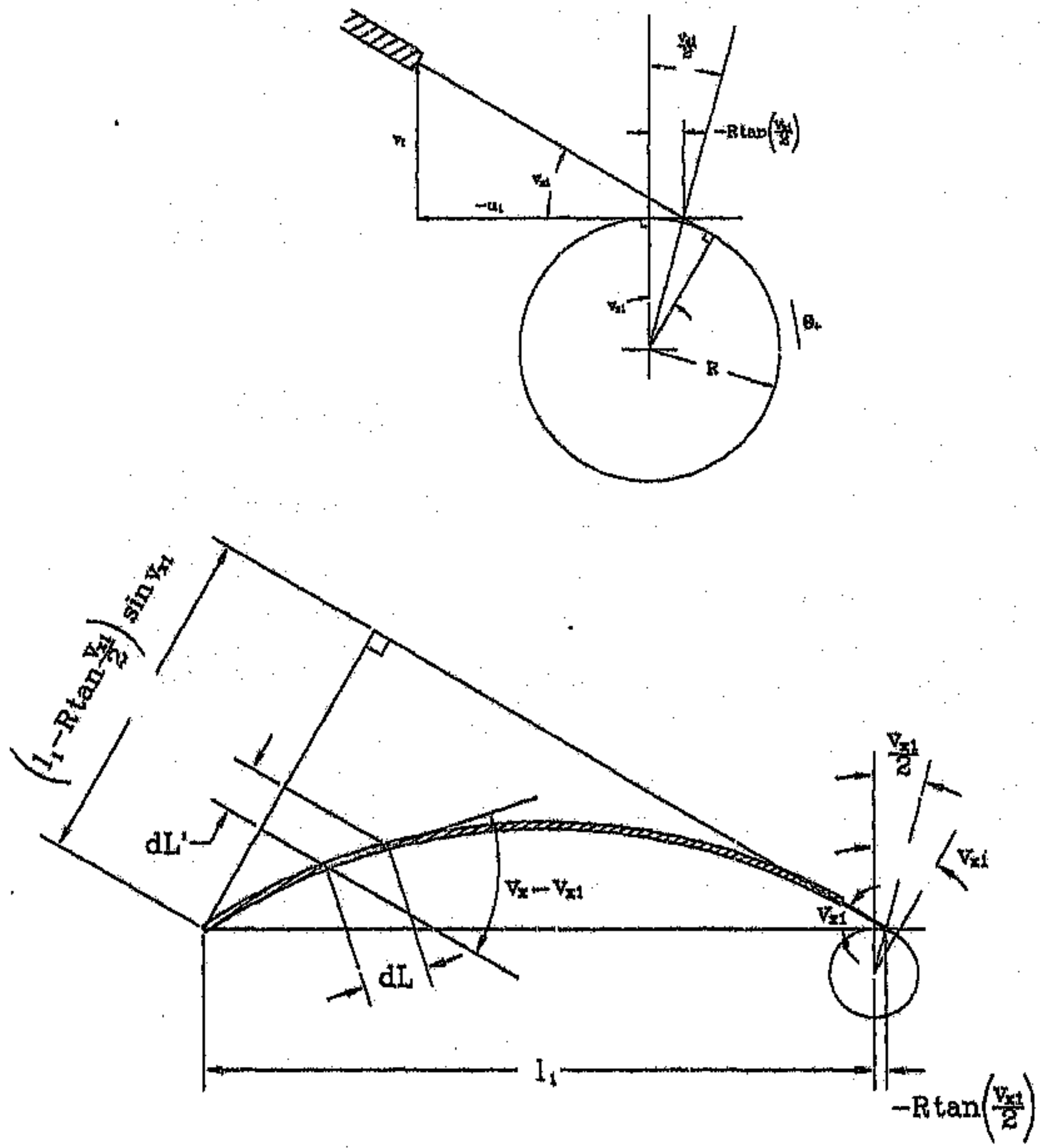

Figure M.2: Geometric constraint relationships

$$dL = \{1 + u_x + \frac{1}{2}v_x^2\}dx, \qquad dL' = dL\sin(v_x - v_{x_1}) \qquad \int_0^{l_1} dL' dx = (l_1 - R\tan\frac{v_{x_1}}{2})\sin v_{x_1}$$

References

[1993] Aligianis G., *"Non-linear Dynamics of a Mine Hoist Model"*, M.Sc. Dissertation in preparation, University of the Witwatersrand, 1993.

[1985] Al-Noury S. I., Ali S. A., *"Large Amplitude Vibrations of Parabolic Cables"*, Journal of Sound and Vibration (1985) 101(4), 451-462.

[1968] Ames W. F., Lee S. Y., Zaiser J. N., *"Nonlinear Vibrations of a Travelling Threadline"*, Int. J. Nonlinear Mech., 3 : 449-469, 1968.

[1992] Amirouche F. M. L., *"Computational Methods in Multibody Dynamics"* Prentice Hall, 1992.

[1966] Anand G. V., *"Nonlinear Resonance in Stretched Strings with Viscous Damping"*, J. Acoust. Soc. Am., 40 (6) :1517 - 1527, 1966.

[1969a] Anand G. V., *"Large Amplitude Free Vibration of a Stretched String"*, J. Am. Acoust. Soc., 45 (5) : 342 - 353, 1969.

[1969b] Anand G. V., *"Stability of Nonlinear Oscillations of Stretched Strings"*, J. Am. Acoust. Soc., 46 (3), Part 2 :667 - 677, 1969.

[1987] Ashworth R. P., Barr A. D., *"The Resonances of Structures with Quadratic Inertial Non-Linearity Under Direct and Parametric Harmonic Excitation"* Journal of Sound and Vibration (1987), 118(1), 47-68.

[1982] Asfar K. R., Nayfeh A. H., Mook D. T., *"Response of Self-Excited Two-Degree-of-Freedom Systems to Multifrequency Excitations"* Journal of Sound and Vibration (1982), 84(2), 199-221.

[1984] Backeberg R. A., *"AN EXPERIMENTAL INVESTIGATION OF NON-LINEAR VIBRATIONS OCCURRING IN MINE WINDING ROPES"*, M.Sc. Dissertation, University of the Witwatersrand, 1984.

[1990] Backeberg R. A., *" A Review of the Shafts Programme*, Chamber of Mines, Internal Report, September 1990.

[1967] Bapat V. A., Srinivasan P., *"Nonlinear Transverse Oscillations in Travelling Strings by the Method of Harmonic Ballance"*, J. Appl. Mech., Trans. ASME, 34 :775 - 777, 1967.

[1987] Benedettini F., Rega G. *"Non-Linear Dynamics of an Elastic Cable Under Planar Excitation"* Int. J. Non-Linear Mechanics, Vol. 22, No. 6, pp 497-509, 1987.

[1961] Bergeron L. *"Water Hammer in Hydraulics and Wave Surges in Electricity"*, John Wilev and Sons, Inc., New York and London, 1961.

[1964] Bolotin V. V., *"The Dynamic Stability of Elastic Systems."*, San Francisco: Holden-Day,Inc. 1964.

[1977] Boshoff G. J., *Rope Whip Calculations for Winders"*, November 1977, Design Services Department, Anglo American Corporation.

[1988] Burgess J. J., Triantafyllou M. S., *"The Elastic Frequencies of Cables"*, Journal of Sound and Vibration (1988), 120(1),153-165.

[1981] Butson G. J., *"Static and Dynamic Analysis of axially loaded wire ropes"*, Ph.D. Thesis, University of Illinois at Urbana Champaign, 1981.

[1986] Bux S. L., R[illegible] J. W., *"Non-Linear Vibratory Interactions in Systems of Coupled Beams"* Journal of Sound and Vibration (1986), 104(3), 497-520.

[1945] Carrier G. F., *"On The Non-Linear Vibration Problem of The Elastic String"*, Quart J. Appl. Math., 3 : 157-165, 1945.

[1988] Cartmell M. P., Roberts J. W., *"Simultaneous Combination Resonances in an Autoparametrically Resonant System"* Journal of Sound and Vibration (1988), 123(1), 81-101.

[1989] Chaplin C. R., *" Drum Winding Rope Deterioration Study"*, Summary Report August 1989, Chamber of Mines Research Organisation. Contract GW2R05.

[1992] Chaplin R, *"Damping"*, communication between R A Backeberg (COMRO) and R Chaplin (University of Reading), 20 March 1992.

[1991] Cheng S. P., Perkins N. C., *"The Vibration and Stability of a Friction Guided, Translating String"*, Journal of Sound and Vibration (1991), 144(2), 281-292.

[1986] Chonan S., *"Steady State Response of an Axially Moving Strip Subjected to a Stationary Load"*, Journal of Sound and Vibration (1986), 107(1),155-165.

[1990] COMRO, *"The Design Guidelines For The Dynamic Performance of Shaft Steelwork and Conveyances"*, COMRO User Guide No 21, November 1990.

[1992] Constancon C. P., *"Damping Measurements - Elandsrand Gold Mine"*, School of Mechanical Engineering, University of the Witwatersrand, 1992.

[1983] Costello G. A., Philips J. W.,*"Stress Analysis of Wire Hoist Rope"*, U.S. Bureau of Mines Report No. Bu Mines OFR 50-84, September 1983.

[1981] Craig R. R. Jr., *"Structural Dynamics"* Wiley, 1981.

[1966] Dimeff J., Lane J. W., Gardner M. S., *"Large Amplitude Linear Vibrations of Tensioned Strings"*, J. Acoust. Soc. Am., 39 (5) . 849 - 855, 1966.

[1973] Dimitriou C., Whillier A.,*" Vibrations in Winding Ropes"*, 1973, Hoisting Conference SAIMechE.

[1972] Eller A. I., *"Driven Nonlinear Oscillations of a String"*, J. Am. Acoust. Soc., 51 (3) Part 2 : 960 - 966, 1972.

[1973] Eller A. I., *"Fractional-harmonic frequency pairs in nonlinear systems"* J. Acoust. Soc. Am., Vol. 53, No.3, 1973.

[1974] Efstathiades G. J., *"Combination Tones in Single Mode Motion of a Class of Non-Linear Systems with Two Degrees of Freedom"* Journal of Sound and Vibration (1974), 34(3), 379-397.

[1984] Guttalu R. S., Hsu C. S., *"A Global Analysis of a Non-Linear System under Parametric Excitation"*, Journal of Sound and Vibration (1984), 97(3),399-427.

[1961] Goodey W. J., *"On the Natural Modes and Frequencies of a Suspended Cable"*, Quart. J. Mech. Appl. Maths., 14 : 118-127, 1961.

[1984] Gough C., *"The nonlinear free vibration of a damped elastic string"*, J. Acoust. Soc. Am., 75 (6) : 1170 - 1776, 1984.

[1989] Greenway M. E., *" Dynamic Loads in Winding Rope. An Analytical Approach"*, Internal Report, Anglo American Corporation, February 1989.

[1990a] Greenway M. E., " *An Engineering Evaluation of the Limits to Hoisting From Great Depth*", Internal Report No. 90-T-08, Anglo American Corporation, June 1990.

[1990b] Greenway M. E., " *A Simple Static and Dynamic Analysis of Coupled Extensional-Torsional Behaviour of Winding Rope*", Internal Report No. 90-T-22, Anglo American Corporation, November 1990.

[1993] Greenway M. E., " *Estimation of Proportional Damping Constants from Measured Damping Data - Winding Dynamics*", Anglo American Corporation, Private communication, 8 March 1993.

[1985] Handy C. R., "*Combining The Methods of Harmonic Balance and Kryloff-Bogoliuboff*" Journal of Sound and Vibration (1985), 102(2), 243-246.

[1965] Harvey T., Laubscher P. S., "*Escort - a winder braking system*". S.African Inst. Elec. Eng. vol 56 Part 2, p37-57, February 1965.

[1973] Harvey T. "*The Calculation of Rope Sizes and Power Requirement of Winders*". International Conference on hoisting men, materials and minerals. J.S.African Inst. Mech. Engineers, October 1973.

[1988] Hagedorn P., "*Non-Linear Oscillations*" Claredon Press - Oxford, 1988.

[1990] Haggie Rand Limited, "*STEEL WIRE ROPES for Cranes & General Engineering*"

[1987a] HaQuang N., Mook D. T., Plaut R. H., "*Non-Linear Structural Vibrations under Combined Parametric and External Excitations*" Journal of Sound and Vibration (1987), 118(2), 291-306.

[1987b] HaQuang N., Mook D. T., Plaut R. H., "*A Non-Linear Analysis of the Interactions between Parametric and External Excitations*" Journal of Sound and Vibration (1987), 118(3), 425-439.

[1963] Hsu C. S., "*On the Parametric Excitation of a Dynamic System having Multiple Degrees of Freedom*" Journal of Applied Mechanics, Vol 30, Trans. ASME. June 1963, 367-372.

[1988] Inyengar R. N., Rao G. V., "*Free Vibrations and Parametric Instability of a Laterally Loaded Cable*, Journal of Sound and Vibration (1988), 127(2), 231-243.

[1981] Irvine H. M., 1981 <u>Cable Structures</u>, Cambridge, Massachusetts: MIT Press.

[1974] Irvine H. M., Caughey T. K., *"The Linear Theory of Free Vibrations of a Suspended Cable"*, Proceedings of the Royal Society London, 1974, A341, pp 299-315.

[1976] Irvine H. M., Sinclair G. B., *"The Suspended Elastic Cable Under the Action of Concentrated Vertical Loads"*, Int. J. Solids Structures, 1976, Vol. 12, pp 309-317.

[1993] Kaczmarczyk S., *"Non-linear Dynamics of Mine Hoist Systems"*, Ph.D. registration, University of Natal, 1993.

[1972] Kim Y., Tabarrok B., *"On the Nonlinear Vibration of Travelling Strings"*, Journal of The Franklin Institute, Vol. 293,No. 6 pp. 381-399, 1972.

[1959] Laasonen P., *"Eigen Oscillations of an Elastic Cable"*, Q.A. Math. 17, 147-154, 1959.

[1992] Lebus Engineering Works, Wadeville, South Africa, 1992. - meeting with Works manager P. Brokenshire.

[1984] Legge K. A., Fletcher N. H., *"Nonlinear generation of missing modes on a vibrating string"*, J. Acoust. Soc. Am., 76 (1) : 5-12, 1984.

[1943] Lubkin S., Stoker J. J., *"Stability of Columns and Strings Under Periodically Varying Forces"*, Quart. J. Appl. Math., 1 : 215 -225, 1943.

[1982] Luogno A., Rega G., Vestroni F., *"Monofrequent Oscillations of a Non-Linear Model of a Suspended Cable"*, Journal of Sound and Vibration (1982) 82(2), 247-259.

[1984] Luogno A., Rega G., Vestroni F., *"Planar Non-Linear Free Vibrations of an Elastic Cable"*, Int. J. Non-Linear Mechanics, Vol. 19, No 1, pp 39-52, 1984.

[1982] Mankowski R. R., *" A STUDY OF NONLINEAR VIBRATIONS OCCURRING IN MINE HOISTING CABLES"*, Ph.D. Thesis, University of the Witwatersrand, 1982.

[1986] Mankowski R. R., *"Response of mine hoisting cables to longitudinal shock loads"* J. S. Afr. Inst. Min. Metall., vol. 86, no. 2. Feb. 1986. pp. 51-60.

[1988] Mankowski R. R., *"Internal damping characteristics of a mine hoist cable undergoing non-planar transverse vibration"* J. S. Afr. Inst. Min. Metall., vol. 88, no. 12. Dec. 1988. pp. 401-410.

[1990] Mankowski R. R., *"Internal power losses occurring at the wavefront of travelling transverse disturbances on mine hoisting cable"* J. S. Afr. Inst. Min. Metall., vol. 90, no. 12. Dec. 1990. pp. 339-344.

[1974] Mankowski R. R., Whillier A., Louw J. H., *" A computer program for Plotting Natural Frequency Curves of Wire Ropes Used on Drum Winders"*. Chamber of Mines, Project No. 105/71, Research Report 28/74, June 1974.

[1947] McLachlan N. W., *Theory and Application of Mathieu Functions*, Claredon Press, Oxford, 1947.

[1970] Meirovitch L., *"Methods of Analytical Dynamics"*, McGraw-Hill, 1970.

[1984] Mickens R. E., *"Comments on The Method of Harmonic Balance"* Letters to the Editor, Journal of Sound and Vibration (1984), 94(3), 456-460.

[1986] Mickens R. E., *"A Generalisation of The Method of Harmonic Balance"* Letters to the Editor, Journal of Sound and Vib (1986), 111(3), 515-518.

[1965] Miles J. W., *"Stability of Forced Oscillations of Vibrating String"*, J. Soc. Acoust. Am., 37 :885 -861, 1965.

[1965] Mitropolskii Y. A., *"Problems of the Asymptotic Theory of Nonstationary Vibrations"* Israel program for Scientific Translations, 1965.

[1988] Miyasar A. M., Barr A. D. S., *"The Linear Oscillator Under Parametric Excitati with Fluctuating Frequency "* Journal of Sound and Vibration (1988), 124(1), 79-89.

[1990] Molteno T., Tufillaro N. B., *"Torus Doubling and Chaotic String Vibration"*, Letters To The Editor, Journal of Sound and Vibration (1990), 137(2), pp 327-330.

[1985] Mook D. T., Plaut R. H., HaQuang N., *"The Influence of an Internal Resonance on Non-Linear Structural Vibrations Under Subharmonic Resonance Conditions"* Journal of Sound and Vibration (1985), 102(4), 473-492.

[1966a] Mote C. D., Naguleswaran S., *"Theoretical and Experimental Band Saw Vibration."*, Transactions of ASME, Series B, Journal of Engineering for Industry 88, 151-156, 1966.

[1966b] Mote C. D., *"On the Nonlinear Oscillation of an Axially Moving String"*, J. App. Mech., Trans. ASME, 33 : 463-464, 1966.

[1985] Mote C. D., Wu W. Z., *"Vibration Coupling in Continuous Belt and Band Systems"*, Journal of Sound and Vibration (1985), 102(1), 1-9.

[1965] Murthy G. S. S., Ramakrishna B. S., J. Acoust. Soc. Am. 38, 855 - 861, 1965.

[1968] Naguleswaran S., Williams C. J. H., *"Lateral Vibrations of Band-Saw Blades, Pulley Belts and the Like"*, Int. J. Mech. Sci., Vol. 10, pp. 239-250, 1968.

[1973a] Nayfeh A. H., Mook D. T., Marshall L. R., *"Nonlinear coupling of pitch and roll modes in ship motion"*, Journal of [illegible]nautics 7, 1973, 145-152.

[1973b] Nayfeh A. H., *"Perturbation Methods"* John Wiley & Sons, 1973.

[1977] Nayfeh A. H., Mook D. T., *"Parametric excitations of linear systems having many degrees of freedom"*, J. Acoust. Soc. Am., Vol. 62, No.2, August 1977.

[1983] Nayfeh A. H., Mook D. T., *"Non-Linear Oscillations"*, Journal of Sound and Vibration (1983), 88(1), 1-10.

[1965] Nayfeh A. H., *"A Perturbation Method for Treating Non-Linear Oscillation Problems"*, Journal of Mathematics and Physics (1965), 44, 368-374.

[1983a] Nayfeh A. H., *"Response of Two Degree-of-Freedom Systems To Multifrequency Parametric Excitations"*, Journal of Sound and Vibration (1983), 88(1), 1-10.

[1983b] Nayfeh A. H., *"Parametrically Excited Multidegree-of-Freedom Systems with Repeated Frequencies"*, Journal of Sound and Vibration (1983), 88(2), 145-150.

[1983c] Nayfeh A. H., *"The Response of Two Degree-of-Freedom Systems with Quadratic Non-Linearities To a Parametric Excitation"*, Journal of Sound and Vibration (1983), 88(4), 547-557.

[1983d] Nayfeh A. H., *"The Response of Single Degree-of-Freedom Systems with Quadratic and Cubic Non-Linearities to a Subharmonic Excitation"*, Journal of Sound and Vibration (1983), 89(4), 457-470.

[1983e] Nayfeh A. H., *"The Response of MultiDegree-of-Freedom Systems with Quadratic Non-Linearities to a Harmonic Parametric Resonance"*, Journal of Sound and Vibration (1983), 90(2), 237-244.

[1984a] Nayfeh A. H., *"Quenching of a Primary Resonance by a Superharmonic Resonance"*, Journal of Sound and Vibration (1984), 92(3), 363-377.

[1984b] Nayfeh A. H., *"Combination Tones in the Response of Single Degree-of-Freedom Systems with Quadratic and Cubic Non-Linearities"*, Journal of Sound and Vibration (1984), 92(3), 379-386.

[1984c] Nayfeh A. H., *"Interaction of Fundamental Parametric Resonances with Subharmonic Resonances of Order One Half"*, Journal of Sound and Vibration (1984), 96(3), 333-340.

[1984d] Nayfeh A. H., *"Quenching of a Primary Resonance by a Combination Resonance of the Additive or Difference Type"*, Journal of Sound and Vibration (1984), 97(1), 65-73.

[1987] Nayfeh A. H., *"Parametric Excitation of Two Internally Resonant Oscillators"*, Journal of Sound and Vibration (1987), 119(1), 95-109.

[1988] Nayfeh A. H., Asfar K. R., *"Nonstationary Parametric Oscillations"*, Journal of Sound and Vibration (1988), 24(3), 529-537.

[1992] Nayfeh A H, Nayfeh J F, Mook D T, *"On Methods for Continuous Systems with Quadratic and Cubic Nonlinearities"*, J. Nonlinear Dynamics 3: 145-162. 1992.

[1990] Neal H. L., Nayfeh A H, *"Response of a Single-Degree-of-Freedom System to a Non-Stationary Principal Parametric Excitation"*, Int.J. Nonlinear Mechanics Vol 25, No. 2/3, pp 275-284. 1990.

[1989] Newland D. E., *Mechanical Vibration Analysis and Computation*, Longman Scientific & Technical, 1989.

[1988] Nikravesh P. E., *"Computer Aided Analysis of Mechanical Structures"*, Prentice Hall, 1988.

[1960] Oplinger D. W., *"Frequency Response of a Nonlinear Stretched String"*, J. Am. Acoust. Soc., 32 (12) : 1529 - 1538, 1960.

[1992] O'Reilly O., Holmes P. J., *"Non-Linear, Non-Planar and Non-Periodic Vibrations of a String"*, Journal of Sound and Vibration (1992), 153(3), 413-435.

[1992] Page J. l., *"An Investigation into the Levelock Cage Holding Device"*, Anglo American Corporation, Mechanical Engineering Department, Report No 92M33, November 1992.

[1986] Perkins N. C., Mote C. D.,*"Comments on curve veering in Eigenvalue Problems"*, Journal of Sound and Vibration (1986), 106(3),451-463.

[1987] Perkins N .C Mote C. D.,*"Three-dimensional vibration of travelling elastic cables"*, Journal of Sound and Vibration (1987), 114(1),325-341.

[1989] Perkins N. C., Mote C. D., *"Theoretical and Experimental Stability of Two Translating Cable Equilibria"*, Journal of Sound and Vibration (1989) 128(3), 397-410.

[1992a] Perkins N. C., *"Modal Interactions in the Non-Linear Response of Elastic Cables Under ParametricExternal Excitation"*, Int. J. Non-Linear Mechanics, Vol. 27, No. 2, pp. 233-250, 1992.

[1992b] Perkins N. C., *Discussion of "Dynamic Stability of Cables Subjected to an Axial Periodic Load"* , Letter to the Editor, Journal of Sound and Vibration (1992) 156(2), 361-365.

[1906] Perry J. F., *"Winding Ropes in Mines"*, Phil. Mag., Series 6, 1906, vol. ii, p 107.

[1932] Perry J. F., Smith D. M.,*"Mechanical Braking and its influence on winding equipment"*,Proc.Inst. Mech. Engrs., 1932, vol.123, p 537.

[1983] Plaut R. H., HaQuang N., Mook D. T., *"The Influence of Internal Resonance on Non-Linear Structural Vibrations Under Two-Frequency Excitation"*, Journal of Sound and Vibration (1983), 107(2), 309-319.

[1986] Plaut R. H., HaQuang N., Mook D. T., *"Simultaneous Resonances in Non-Linear Structural Vibrations under Two Frequency Excitation"*, Journal of Sound and Vibration (1986), 106(3), 361-376.

[1990a] Plaut R. H., Gentry J.J, Mook D. T., *"Resonances in Non-Linear Structural Vibrations Involving Two External Periodic Excitations"*, Journal of Sound and Vibration (1990), 140(3), 371-379.

[1990b] Plaut R. H., Gentry J.J, Mook D. T., *"Non-Linear Structural Vibrations under Combined Multi-Frequency Parametric and External Excitations"*, Journal of Sound and Vibration (1990), 140(3), 381-390.

[1951] Pollock P. J., Alexander G. W.,*"Dynamic Stresses in Wire Ropes for use on Vertical Hoists"*, Paper No.12, Wire Ropes in Mines, Inst. Mining and Metalurgy, 1951.

[1949] Pugsley A. G., *"On the Natural Frequencies of Suspension Chains"*, Quart. J. Mech. Appl. Math., 2, Part 4 : 412-418, 1949.

[1983] Pugsley A. G., *"The Nonlinear Behaviour of a Suspended Cable"*, Quart. J. Mech. Appl. Math., Vol. 36, Part 2, pp 157-162, 1983.

[1964] Quick W. H., *"Theory of the Vibrating String as an Angular Motion Sensor"*, J. Appl. Mech. 31, 523-534 (1964).

[1991] Rao G. V., Inyengar R. N., *"Internal Resonance and Non-Linear Response of a Cable under Periodic Excitation"*, Journal of Sound and Vibration (1991), 149(1), 25-41.

[1945] Rayleigh J. W. S., *"The Theory of Sound"*, Vol. 1, Introduction : 1-31, Dover Publications, New York, 1945.

[1989a] Rega G., Benedettini F., *"Planar Non-Linear Oscillations of Elastic Cables Under Superharmonic Resonance Conditions"*, Journal of Sound and Vibration (1989) 132(3), 353-366.

[1989b] Rega G., Benedettini F., *"Planar Non-Linear Oscillations of Elastic Cables Under Subharmonic Resonance Conditions"*, Journal of Sound and Vibration (1989) 132(3), 367-381.

[1984] Rega G., Vestroni F.,Benedettini F., *"Parametric Analysis of Large Amplitude Free Vibrations of a Suspended cable"*, Int. J. Solids Structures, Vol. 20, No. 2, pp. 95-105, 1984.

[1976] Rodeman R., Longcope D. B., Shampine L. F., *"Response of a String to an Accelerating Mass"*, ASME J. Appl. Mech, December 1976, 675-680.

[1851] Rohrs J. H., *"On the Oscillations of a Suspended Chain"*, Trans. Camb. Phil. Soc., 9, Part 3 : 379-398, 1851.

[1981] Rosenthal F., *"Vibrations of Slack Cables with Discrete Masses"*, Journal of Sound and Vibration (1981), 78, 573-583.

[1884] Routh E. J., *"Dynamics of a System of Rigid Bodies"*, MacMillan and Co., 1884.

[1954] Sack R. A., *"Transverse oscillations in travelling strings"*, Br. J. Appl. Phys., 5 : 224-226, 1954.

[1975] Sagartz M. J., Forrestal M. J., *"Motion of a Stretched String Loaded by an Accelerating Force"*, ASME J. Appl. Mech. 43, 505-506 (1975).

[1953] Saxon D. S., Cahn A. S., *"Modes of Vibration of a Suspended Chain"*, Quart. J. Mech. Appl. Math., 6, Part 3 : 273-285, 1953.

[1986] Schmidt G., Tondl A., *"Non-Linear Vibrations"*, Cambridge University Press,1986, p 270-277.

[1968] Schutz A., *"Large Dynamic Deformations caused by a Force Travelling on an Extensible String"*, Int. J. Solids Structures, 1968, Vol. 4, pp. 799-809.

[1971] Shih L. Y., *"Three-Dimensional Non-Linear Vibration of a Travelling String"*, Int. J. Non-Linear Mechanics, Vol. 6, pp. 427-434, 1971.

[1975] Shih L. Y., *"Motion of Eliptic Ballooning for a Travelling String"*, Int. J. Non-Linear Mechanics, Vol. 10, pp. 183-191, 1975.

[1972] Simpson A., *"On The Oscillatory Motions of Translating Elastic Cables"*, Journal of Sound and Vibration (1972), 20(2),177-189.

[1979] Sinha S. C., C C Chou C. C., Denman H. H., *"Stability Analysis of Systems with Periodic Coefficients: An Approximate Approach"* , Journal of Sound and Vibration (1979) 64(4),515-527.

[1897] Skutsch R., Ann. Phys. Chem., 61, pp. 190, 1897.

[1964] Smith C. E., *"Motions of a Stretched String Carrying a Moving Mass Particle"*, ASME J. Appl. Mech. 33, 29-37 (1964).

[1970] Soler A. I., *"Dynamic Response of Single Cables with Initial Sag"*, J Franklin Inst., 290 (4) : 377-387, 1970.

[1974] Srinivasan T., Sankar T. S., *"A Study of Linear Systems with Time Varying Coefficients By the Methods of Bessel and Weber Functions"* , Journal of Sound and Vibration (1974) 37(3),513-519.

[1950] Stoker J. J., *Nonlinear Vibrations in Mechanical and Electrical Systems*, Interscience, New York, 1950.

[1988] Streit D. A., Bajaj A. K., Krousgrill C. M., *"Combination Parametric Resonance Leading To Periodic and Chaotic Response In Two Degree-of-Freedom Systems with Quadratic Non-Linearities"*, Journal of Sound and Vibration (1988) 124(2),297-314.

[1979] Szemplinska-Stupnika W., *"The Modified Single Mode Method in the Investigations of the Resonant Vibrations of Non-Linear Systems"*, Journal of Sound and Vibration (1979) 63(4), 475-489.

[1983] Szemplinska-Stupnika W., *"Non-Linear Normal Modes and the Generalised Ritz Method In the Problems of Vibrations of Non-Linear Elastic Continuous Systems*, Int J of Non-Linear Mech v 18 n 2 June 1983, p 149-165.

[1978] Szemplinska-Stupnika W., *"The Generalised Harmonic Balance Method For Determining The Combination Resonance in The Parametric Dynamic Systems"*, Journal of Sound and Vibration (1978), 58(3),347-361.

[1977] Tagata G., *"Harmonically Forced Finite Amplitude Vibration of a String"*, Journal of Sound and Vibration (1977), 51(4),483-492.

[1983] Tagata G., *"A Parametrically Driven Harmonic Analysis of a Non-Linear Stretched String with Time Varying Length"*, Journal of Sound and Vibration (1983), 87(3),493-511.

[1981a] Takahashi K., *"An Approach To Investigate The Instability of The Multiple-Degree-Of-Freedom Parametric Dynamic Systems"*, Journal of Sound and Vibration (1981) 78(4),519-529.

[1981b] Takahashi K., *"Stability of Hill's equation with three independent parameters"*, Journal of Sound and Vibration (1981), Letters to the Editor, 79(4),605-608.

[1982] Takahashi K., *"Instability of Parametric Dynamic Systems with Non-Uniform Damping"*, Journal of Sound and Vibration (1982) 85(2), 257-262.

[1987a] Takahashi K., Konishi Y., *"Non-Linear Vibrations of Cables in Three Dimensions, Part I: Non-Linear Free Vibrations"*, Journal of Sound and Vibration (1987) 118(1), 69-84.

[1987b] Takahashi K., Konishi Y., *"Non-Linear Vibrations of Cables in Three Dimensions, Part II: Out-of-Plane Vibrations Under In-Plane Sinusoidally Time-Varying Load"*, Journal of Sound and Vibration (1987) 118(1), 85-97.

[1991] Takahashi K., *"Dynamic Stability of Cables Subjected to an Axial Periodic Load"*, Journal of Sound and Vibration (1991) 144(2), 323-330.

[1982] Tezak E. G. , Nayfeh A. H., Mook D. T., *"Parametrically Excited Non-Linear Multidegree-of-Freedom Systems with Repeated Natural Frequencies"*, Journal of Sound and Vibration (1982), 85(4), 459-472.

[1987] Thomas G. R., Brillhart R. D., *"An Investigation into the Dynamic Rope Loads in Drum Winder Systems"*, Final report by SDRC(Inc.), Sandiego California, for COMRO RSA, October 1987.

[1985] Triantafyllou M. S., *"The Dynamics of Translating Cables"*, Journal of Sound and Vibration (1985), 103(1),171-182.

[1978] Tse F. S., Morse I. E., Hinkle R. T., *"Mechanical Vibrations, Theory and Practice"* Second Edition, Allyn and Bacon.

[1973] Vanderveldt H. H., Chung B. S., Reader W. T., *"Some dynamic properties of axially loaded wire ropes"*, Exp. Mech., vol. 13. Jan. 1973. pp. 14-30.

[1903] Vaughan J. A., *"An investigation regarding the effect of Kinetic Shocks on Winding Ropes in Vertical Shafts"*, Proc. South African Assoc. Engrs., 1903-4,vol.ix.

[1917] Vaughan J. A., *"The Factor of Safety of Wire Ropes used for Winding in Mine Shafts"*, Journal, South African Inst. Engrs.,1917, vol. 16, p94.

[1986] Wang K. W., Mote C. D., *"Vibration Coupling Analysis of Band/Wheel Mechanical Systems"*, Journal of Sound and Vibration (1986) 109(2), 237-258.

[1987] Wang K. W., Mote C D., *"Band/Wheel System Vibration Under Impulsive Boundary Excitation"*, Journal of Sound and Vibration (1987) 115(2), 203-216.

[1992] Watzky A., *"Non-Linear Three Dimensional Large-Amplitude Damped Free Vibration of a Stiff Elastic Stretched String*, Journal of Sound and Vibration (1992) 153(1), 125-142.

[1988] Wickert J. A., Mote C. D., *"Linear transverse vibration of an axially moving string-particle system"*, J. Acoust. Soc. Am., 84(3) :963 - 969, 1988.

[1986] Wu W. Z., Mote C. D., *"Parametric Excitation of an Axially Moving Band by Periodic Edge Loading"*, Journal of Sound and Vibration (1986), 110(1), 27-39.

[1952] Yu A., *"Vibration damping of stranded cable"*, SESA Proceedings, vol 9, no. 2. 1952. pp. 141-158.

[1988] Zavodney L. D., Nayfeh A. H., *"The Response of a Single-Degree-of-Freedom System with Quadratic and Cubic Non-Linearities To a Fundamental Parametric Resonance"*, Journal of Sound and Vibration (1988), 120(1), 63-93.

[1981] Zajaczkowski J., *"An Approximate Method of Analysis of Parametric Vibration"*, Journal of Sound and Vibration (1981), 79(4), 581-588.