Optimal Asset Allocation Using Markowitz Portfolio Selection Model in South Africa

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ABSTRACT

The main purpose of this study is to construct and determine the performance of the optimal asset allocation portfolio in South Africa by applying Markowitz portfolio optimization model. To further test whether the optimal asset allocations provide superior returns to risk compared to active multi-asset management. A portfolio is constructed using South African equities, bonds and cash, as well as global equities, global bonds and gold. The Markowitz portfolio optimization model formulates an efficient set, selects portfolio having excess return to standard deviation ratio, while satisfying several constraints to construct the efficient frontier. The yearly weightings are constructed starting from 1998 until 2015, and backtested to determine performance. The Markowitz Portfolio selection efficient frontier and portfolios were developed using the following four strategies: the optimum mean-variance portfolio, a portfolio targeting a return of inflation plus 3 percent, a portfolio targeting a return of inflation plus 4 percent, and a portfolio targeting a return of inflation plus 5 percent. The research found, weightings of the optimum mean-variance portfolio averaged 4.5 percent in SA equities, 5.7 percent in SA bonds, 83.7 percent in SA cash, 1.5 percent in global equities, 1.8 percent in global bonds and 2.9 percent in gold. The optimisations done, show that the optimum mean-variance portfolio underperformed on a return basis but performed exceptionally on a sharpe ratio measure. The inflation targeting strategies outperformed the average South African balanced fund, but underperformed the equally weighted and generic portfolios. The Markowitz strategies does not outperform consistently over multiple periods, as indicated by the short to medium term underperformance to the average South African balanced fund. The results of the optimal portfolios show that the Markowitz optimal portfolio can be used as an investment strategy and as a benchmark to measure against active asset allocation performance.

Keywords - Return, Standard Deviation, Sharpe Ratio, Efficient Frontier, Optimum Portfolio, Markowitz Model, Asset Allocation.

DECLARATION

I, George M. Petje, declare that this research report is my own work except as indicated in the references and acknowledgements. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration in the Wits Business School, Johannesburg. It has not been submitted before for any degree or examination in this or any other academic institution.

George M. Petje

Signed at ……

On the …….. day of …… 2016

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CHAPTER 1. INTRODUCTION

1.1 Purpose of the study

The purpose of this research is to analyse and identify the optimal South African asset allocation over a long-term investment period, using data from as early as 1988 to 2015, based on historical performance of assets. To identify the optimal combination of asset classes, using Markowitz (1952) portfolio selection model, creating the optimal portfolio on a 10 year rolling data set. Determine how the optimal portfolio would have performed, with back testing, and then analyse whether the optimal portfolio would have outperformed or underperformed the average South African active balanced fund over similar periods.

1.2 Context of the study

Asset allocation is the process of deciding how to distribute an investor's wealth across different major asset classes for investment purposes (Sharpe, 1992). Determining the allocation to each asset class is a focus of every multi-asset class manager. It is considered that asset allocation is a considerable contributor to the returns of an investment portfolio, and this is especially true if the portfolio is invested in a number of varying assets (Sharpe, 1992).

The purpose of this research is to find a portfolio, which best meets the expected returns of a multi-asset class investor while minimising the risk of the portfolio. Markowitz in his 1952 paper is considered the father of diversification and portfolio selection. Markowitz (1959) considers two main objectives for investors. Firstly, investors want their returns to be as high as possible, and secondly investors want the returns to be consistent, dependable and as expected, therefore uncertainty is a risk they require to be reduced. The optimal portfolio for an investor lies in the relationship between expected return and

risk. The trade-off between return and risk is different for each investor, suggesting investors with a greater risk appetite can expect higher returns and the converse for investors who choose not to bear a lot of risk.

Pension funds are managed by investment managers, and are allocated into various investment asset classes to create a diversified portfolio. Multi-asset class managers are employed with a mandate of preserving and growing the wealth of individuals through active asset allocation. The portfolio is expected to gain value over the working lifetime to be recouped once a member retires.

Multi-asset class managers allocate funds into asset classes such as equities, bonds, cash, property and commodities. In addition, with global assets, they are blended into a diversified portfolio, as can be seen in Table 1. These funds represent the savings vehicles in the country (Brooke, 2014).

The general public and wealth managers need more knowledge about the impacts their investment decisions will have in their portfolios. The importance of this point is based on the necessity of retiring with a pension that can sustain every individual's life while they cannot earn an income. By testing a portfolio selection model, varying allocation weightings and back testing over a historical period this research can determine how various investment allocations would have performed. Back testing is a method of quantitatively testing to examine the composition and returns of portfolios based on historical data to determine if the selected strategy would have worked in the past (Sharpe, 1992). Back-testing creates an optimal portfolio without a hindsight bias, this allows the results of the research to be more practical to fund managers, wealth managers and the general public. The investment community can use the practical results as benchmarks towards investment pension funds and creating future balanced fund passive quant portfolios.

The expected return of an asset and the variance are the cornerstone of the ideas behind portfolio diversification and selection. Markowitz gave the world the mathematical formula of reducing variance through diversification. He postulates that an investor should maximize expected portfolio return, while minimizing the portfolio variance of

return (Rubinstein, 2002). Together with William Sharpe and Merlton Miller, Harry Markowitz was awarded the Nobel laureates for economics in 1990. Portfolio construction has been a subject of importance for investment managers, as investment managers seek to optimally construct their portfolios' based on the level of risk and required returns. Markowitz (1952) pioneered efforts of dealing with portfolio selection from a quantitative perspective, which stems the basis of this research paper to determine a quantitative balanced fund/index.

By applying Markowitz (1952) portfolio selection model, the optimal portfolio selection can be determined using data from 1988 until 1998, and then creating portfolios for each year based on 10-year rolling data. The research uses the asset allocation ratios to determine how the optimal portfolio selection would have performed from 1998. This research then attempt to compare the performance of the optimal portfolio calculated using Markowitz portfolio theory and the performance of a typical actively managed portfolio over the past decades. Performance is measured using relatively standard investment performance measures, mainly annualised returns, standard deviation and Sharpe ratio (Antolin, 2008). The optimal allocation mix can be used as a benchmark or an investment index tracker fund for the financial industry.

1.3 Problem statement

1.3.1 *Main problem*

The purpose of this research is to determine the optimal asset allocation in South Africa over a long-term investment period, based on past risk-adjusted performance, using Markowitz portfolio selection model, and to compare the performance of the optimal portfolio to the average South African balanced fund.

1.3.2 *Sub-problems*

The first sub-problem: To determine the optimal asset allocation weights based on Markowitz portfolio selection model.

The second sub-problem: The optimal portfolio selection performance on a back-tested risk-adjusted basis over the long-term period in comparison to South African balanced funds.

1.4 Significance of the study

Current methods of evaluating the performance of asset allocators and the value they have created is through a CPI (consumer price index) plus a determined percentage. Other methods are simply comparing multi-asset managers' performances over other multi-manager fund performances. Equity managers are also measured by the same methods, but they are further evaluated by comparing their performance to an index, such as the JSE Top 40 or the SWIX (JSE Share weighted Index). This last measure used by equity managers provides a possible gap in the investment research market, to provide a reliable and back tested asset allocation index to benchmark performance. The research may provide guidance to various areas of the financial industry. Investment managers may use it as a benchmark to relate their own portfolios, they may be able to compare whether their funds have outperformed or underperformed and as a standard in analysis of asset allocation performance.

Clients of investment managers, whose mandate is to choose investment managers on behalf of pension fund members, currently create mandates based on CPI (consumer price index) plus a certain percentage and just compare investment managers to each other in order to determine the adequate investment. The limitations of this method of comparison is that it ignores risk and encourages risk taking strategies that aim to only outperform on a return basis. By conducting this research, clients may be able use the conclusions as a benchmark in allocating investment mandates.

The potential opportunity may be able to create an asset allocation index tracker. Index trackers are investment vehicles that are used to replicate allocations or percentage weights of an Index (e.g. the JSE Top 40). These investment vehicles are used when an investor believes that the index usually outperforms the active decisions of an investment manager. Therefore, by removing the intelligence of the investment manager, the investment vehicle becomes less costly and also reduces the risk of poor investment manager decisions. By conducting this research, an optimal asset allocation could be found that may be used as an index tracker fund or a purely quantitative passive balanced fund, which reduces the cost of fund management to the investor.

1.5 Delimitations of the study

This study attempts to establish the performance of asset classes over a period and to determine the optimal asset allocation weightings throughout that period. The period will be subject to data availability. The best data is from Inet BFA and a multi-asset class manager that can provide data on assets from 1988. Pre-1988 data may be likely unavailable or unreliable for all the asset classes. The longer the historical data, the greater the ability to use statistics to create reliable predictions.

The study focuses entirely on assets a South African asset manager could purchase on behalf of their clients. Foreign assets pre-1994 could not be purchased, therefore, will be excluded from the study pre-1994 (Leape & Thomas, 2011). Currently, the foreign asset allocation holding is limited to 25 percent for South African funds. This limitation is applied throughout the analysis period, in order to create a portfolio that can be extrapolated to indicate future expected returns.

The number of asset classes will be limited to listed assets. This is because investment managers on behalf of their clients can purchase these assets with reasonable liquidity and long-term data on these assets can be attained with accuracy. Private equity, which is an option for some multi-asset managers, will not be considered in this research.

The study will focus on major asset classes that have traditionally been accepted as the lion share of portfolios' allocation by South African managers (Brooke, 2014), which are the following: South African equities, South African bonds, South African cash, world equity index, world bonds, and gold. Platinum and palladium ETF's will be excluded, as these assets are new in terms of investment ability, accessibility and liquidity and will therefore not provide a fair result of allocation weightings.

Chapter 3 will provide further guidance into the methods of solutions and describe how the research will overcome any shortcomings to provide a detailed study and valuable contribution.

1.6 Definition of terms

Asset allocation – is the process of deciding how to distribute an investor's wealth across different asset classes for investment purposes (Sharpe, 1992).

Back testing – a method of quantitatively testing to examine the composition and returns of portfolios based on historical data to determine if the selected strategy would have worked in the past (Sharpe, 1992).

CPI (consumer price index) – "the CPI is a current social and economic indicator that is constructed to measure changes over time in the general level of prices of consumer goods and services that households acquire, use, or pay for. The index aims to measure the change in consumer prices over time (Statssa, 2013; pg. 1)."

Liquidity – ability of an investor to buy and sell assets in a required period. The quicker the investor can complete the trade, the more liquid the asset is considered.

Nominal return – is calculated by dividing the capital gain/loss and income earned by the value of the investment at the beginning of the period without any other adjustments (Feibel, 2003).

Risk-adjusted return – is calculated by dividing the capital gain/loss and income earned by the value of the investment at the beginning of the period and then adjusting it by a

chosen risk measure throughout that period. The Return considers the risk each asset class poses, providing a comparable return across assets (Feibel, 2003).

Real return – is calculated by dividing the capital gain/loss and income earned by the value of the investment at the beginning of the period and then adjusting it inflation throughout that period. The Return eliminates the inflation effect, leaving the only real value created (Feibel, 2003).

1.7 Assumptions

The study attempts to determine the returns of specific asset classes that are commonly held as a part of the asset allocation of a balanced portfolio and determine the optimum weightings based on past risk and return.

By using data that tracks as far back as 1988, it is reasonably assumed that the period is adequate to determine long-term returns that are unlikely to deviate extensively. The longer the period and the lower the covariance between assets, the more reliable the return in determining the future value creation and applying the recommended asset allocation in future benchmarking or index creation (Brooke, 2014).

It is assumed that the quantitative data received through available sources will be a true reflection of the actual indices movements throughout the period. That even when double-checked and crosschecked to other data sources, the data represent the true investable prices and subsequent calculated returns.

The research has assumed that since South African asset managers could only invest in offshore markets post-1994, the research will still use data pre-1994, from 1988 to 2015, creating and measuring the optimal asset allocation using Markowitz portfolio selection

theory. The reason for this is to create a comparable optimal allocation index that can be used in measuring and determining future allocations. A more realistic picture of the past will result in a more realistic determination of the future, therefore in order to recommend future allocation weightings and benchmark we need to calculate it, assuming that the South African fund could always invest offshore, (Leape & Thomas, 2011).

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

The literature review aims to discuss the research relating to portfolio selection within investment markets. The review will look into Markowitz portfolio selection (1952), then how it is related to index tracking funds or passive investment. The historical performance of a general portfolio is discussed to provide contexts of the South African market. The optimal portfolio selection theory is discussed with back testing to determine the optimal portfolio selection.

2.2 Definition of topic or background discussion

A portfolio is a combination of securities that offer the investor with protection and sufficient returns. It is the balance of this portfolio that allows the investor to reduce risk. The purpose of this research is to find a portfolio which best needs the expected returns and minimised risk of a multi-asset class investor.

Markowitz in his 1952 paper is considered to be the father of diversification and portfolio selection. However, the benefits of diversification have been an idea long considered in many a scholars mind: William Shakespeare in The Merchant of Venice, Act I, Scene I, has Antonio say:

“ *. . . I thank my fortune for it, My ventures are not in one bottom trusted, Nor to one place; nor is my whole estate Upon the fortune of this present year . . .*”

The expected return of an asset and the variance which is the economic risk, are the cornerstone of the ideas behind portfolio diversification and selection. Markowitz gave the world the mathematical formula of reducing variance through diversification.

Markowitz postulates that an investor should maximize expected portfolio return, while minimizing the portfolio variance of return (Rubinstein, 2002). Together with William Sharpe and Merlton Miller, Markowitz was awarded the Nobel Prize for economics in 1990. His groundbreaking work on portfolio selection labelled him the father of modern portfolio theory.

The tools used by Markowitz can be summarised by best evaluating the value of an asset to an investor is derived by evaluating the mean, correlation and standard deviation of assets relative to each other (Darko, 2012). This is in contrast to active multi-asset managers because Markowitz ignores the fundamentals that drive assets, such as politics, economics, social, technology, environment and legalities. Markowitz merely applies a few statistical measurements to determine a portfolio, this is why his work was ground breaking and is still commonly used by investment managers.

This research therefore aims to apply Markowitz portfolio selection theory in the South African context. It is applied through the perspective of a multi-asset manager, by eventually creating the optimal asset allocation and diversified portfolio.

2.3 Diversification

Markowitz (1959) considers two main objectives for investors. Firstly, investors want their returns to be as high as possible, and secondly investors want the returns to be consistent, dependable and as expected, therefore uncertainty is a risk they require to be reduced. The optimal portfolio for an investor lies in the relationship between expected return and risk. The trade-off between return and risk is different for each investor, suggesting investors with a greater risk appetite can expect higher returns and conversely for investors who cannot bear a lot of risk. The total risk of a portfolio can be defined into two categories, systematic and unsystematic risk. Systematic risk is the risk inherent in investing in the market, which are political, economic, social, technological,

environmental and legal. Systematic risk cannot be eliminated, but unsystematic risk, which is specific to the assets, can be eliminated through diversification.

Diversification has long been a key topic in investment literature identified to reduce the risk of a portfolio. Correlations among securities influence on the risk of the portfolio. If security returns were perfectly negatively correlated, the risk of the portfolio could be completely eliminated (Markowits, 1959). On the other hand, if correlations for securities were perfect the portfolio will be subjected to high levels of risk, hence diversification into these assets will not help in reducing risk. Therefore, in determining a portfolio it is important to determine the covariance of the assets chosen by the investor. Determining the covariance between assets is useful in identifying diversification and riskiness the portfolio. This will lead to better understanding of the portfolio and portfolio selection, which is an integral part of applying Markowitz (1952) portfolio selection theory.

2.4 Portfolio selection

Markowitz, is considered by many to be the father of Modern Portfolio Theory (MPT). Markowitz (1952) considers a portfolio is selected based on two criteria's: firstly the experience and observations, then secondly, the expected future returns of each asset class. The experience and observation is the determination of the expected returns and variance of each security considered by the investor. The theory behind the selection, in relation to this research, is on the portfolio selection based on the expected returns and variance. Markowitz (1952) accepts the rule that investors expect a return and consider the variance of such return to be undesirable. Sharpe (1964) adds, investors prefer a higher return to a lower return, while they also exhibit risk-aversion choosing a security with a lower value of standard deviation. Therefore, investors will choose among various combinations along a risk-return band (Markowitz, 1952). Therefore, mathematically reducing risk should be an aim to determining a portfolio. Markowitz (1952) developed mathematical formulas to determine the 'critical line' or the 'efficient frontier'. This algorithm identifies the most optimal selection to maximise the returns and minimise the

risk of the portfolio. Determining the efficient frontier line requires the expected return of the assets, the variance of the assets and the correlation of the assets relative to other assets. This line simply indicates investor with a more risk appetite will invest a diversified portfolio on the higher end of the frontier line, while a lower risk profile will choose a portfolio on the lower end of the line. The optimal asset allocation is considered to be somewhere in the middle of the frontier line.

According Markowitz (1959), portfolio selection requires certain conditions to be met. Firstly, an investor must consider liquid assets; liquid assets can be sold and bought at a particular required time and such price is equal to the bought price and sell price. Secondly, the investor aims to maximise value. Thirdly, the distribution of returns and risk of the market remains the same throughout the investment period.

This research assumes that the investor places all funds in various securities with a discounted value implied. Meaning, all assets with the same value will be considered equal. If an investor knew the expected returns for all securities, he would have the choice to distribute all funds to the security with the highest expected return; otherwise, he would remain in a diversified portfolio. The research, as with Markowitz (1952), assumes the investor should invest in a diversified portfolio. To this, Markowitz (1952) proposes there is a combination of securities that offers the maximum expected returns and the least variance. This relationship between returns and variance requires one to subsidise the other (an indirect relationship). Given the relationship, Markowitz (1952), describes the investors choice between various assets to fall under the combinations as expressed in figure 1 below. The key area for Markowitz (1952) portfolio selection is to arrive at an expected return of the investor and the variance of each security. This rule implies diversification, not only the right kind of diversification, but for the right reasons. For an adequate study (the right reasons), the rule requires uncorrelated assets in the portfolio.

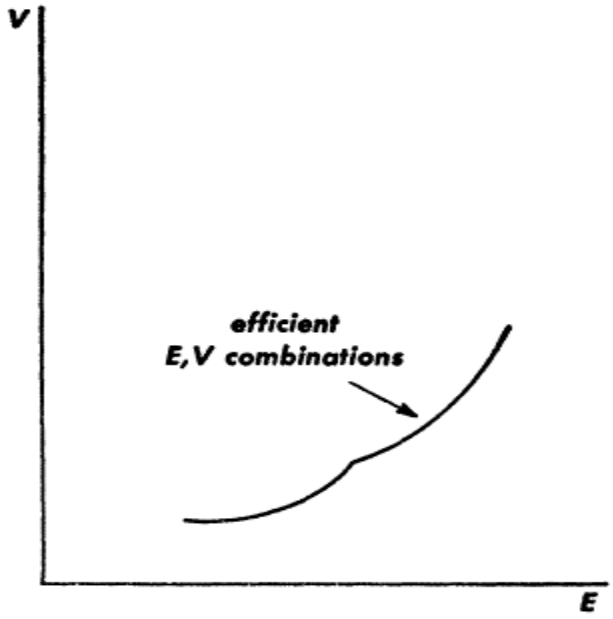


Figure 1: **Markowitz efficient frontier, (Markowitz, 1952)**

Markowitz (1952), states in order to reduce variance it is not enough to invest in many securities, but it is necessary to invest across industries because firms in different industries have lower covariance than firms in the same industry. The relationship between assets, based on the correlation coefficient of returns of a number of assets will determine the risk of the portfolio. This is relevant in our study to consider multiple uncorrelated assets, and assets in different geographies so to insure a lower covariance. The assessment of the desirability of a security is hard to measure due to the difficulty in the effect of the overall portfolio being dependent on not only its expected return and risk, but also its correlations with the other asset classes (Sharpe, 1964). Rubinstein (2002), in relation to the Markowitz portfolio selection formula, reiterates the decision for an investor to hold assets depends on which assets the investor desires to hold and not only on the expected returns and the variance. The decision, in which assets are considered, influences the covariance of the assets to each other and therefore the portfolio.

Markowitz in his 1959 book on portfolio selection advises that investors choose their portfolio to maximize expected returns in accordance with the Savage (1954) methods,

which compare alternative measures of risk: standard deviation, semi-variance, expected value of loss, expected absolute deviation, probability of loss and maximum loss.

2.5 Application of Markowitz portfolio selection theory

In applying the model, Markowitz has the following assumptions: firstly, an investor is concerned with return distribution over a single period, meaning the investment decision is made at the beginning of the period and return is realised at the end of the period. Secondly, an investor seeks to maximize the expected return of the portfolio. Thirdly, all investors are risk-averse; meaning the trade-off for higher risk should be compensated with higher expected return. Fourthly, investors base their investment decisions on the expected return and risk. Finally, all markets are perfectly efficient (Darko, 2012).

The expected return of the portfolio is measured by the mean return, while the risk of the portfolio is measured by the variance. Levy and Markowitz (1979) performed a study of estimating the expected return of a function of mean and variance of return of 149 mutual funds. The objective of the study was to prove the validity of using mean return, and they subsequently found that ordering a portfolio by mean variance was almost identical to the order obtained by using expected return.

Varian (1993) simplifies the portfolio selection theory as, given an expected return and coefficient variance of securities, the risk of a portfolio could be minimised through Markowitz Model. Tobin (1958) included upon Markowitz work a risk-free rate, which affects the expected returns and variance of the other asset classes within a portfolio due to the combination of risky and riskless assets. Sharpe (1967) assumed Tobin and Markowitz work was in fact creating a market portfolio. By adding Beta, Sharpe included a risk measure, which varied from the market, through the development of the Capital Asset Pricing Model (CAPM).

McLeod (1998), provided limits to the Markowitz model, stating the model in the South African context fails to adequately diversify the portfolio. McLeod found three out of the seven indices were never included in the portfolio and one index only be allocated a maximum of 3% of funds. However, McLeod (1998) found using tracking error as a risk rather than variance had better results for diversification using the Markowitz model. The issue with a tracking error in our context is the lack of designated benchmark to measure against the result. McLeod (1998), suggests a good diversified portfolio using the Markowitz model will be achieved depending on good expected return forecasts of securities by analysts.

Du Plessis and Ward (2009) applied the Markowitz portfolio theory to the Johannesburg Security Exchange (JSE) using the top 40 JSE companies to determine whether an optimal portfolio can be identified. The research found using Markowitz theory, the returns outperformed the market. Albeit, the period used in the research was a short 11 years.

Biggs and Kane (2009) dealt with portfolio optimization using the Markowitz model. Their study suggests that through the Markowitz model, the classical minimum-risk problem can be unsatisfactory in practice, because it leads to unrealistically smallholding of certain assets. The solution was to introduce restrictions on each allocation, and use a combination of local and global assets to determine a satisfactory optimization.

To improve the efficiency of the stock portfolio, Xia Lau Yang (2006) used genetic algorithm. A Genetic algorithm is used to find the parameters and rules of investing. The research found by using genetic algorithms the portfolio is of higher returns and lower risk than other models.

Maharakkhaka (2011) used three classes of asset in the portfolio, namely; Security Exchange of Thailand (SET) Index, Thai investment grade corporate bond Index, and Thai government Treasury bill, to determine the performance using the mean variance efficiency. The research found the mean variance was relevant to the Thai security market and the allocations did not differ from the naïve asset allocations by investors.

As stated by Rubinstein (2002), Markowitz portfolio selection is a regular method among investors to determine their portfolios and measure alpha against their portfolio. This research paper is built upon Markowitz work, which aims to determine the portfolio selection of the South African investment market. According to Darko (2012), Markowitz mean-variance optimization is widely used around the world, being applied by financial advisory firms to determine an optimal portfolio in decision-making. Applications in asset allocation, index matching and creation and active portfolio management have been widely used.

All of the above applications to different contexts are an example of how Markowitz portfolio selection model has been applied. The examples show the results of their output, the variations used, and successes and failures of the model.

2.6 Asset allocation heuristic techniques

Traditional asset allocation theory is rooted in the basis of the mean–variance portfolio optimization framework developed by Markowitz (1952). However, the mean–variance optimization model is difficult to practically implement because of the challenges with accurately predicting the expected returns and covariance's for asset (Chaves, Hsu, Li, & Shakernia, 2011). The challenges in practically implementing Markowitz's portfolio optimization have led to a wide gap between the theory and practice (Chopra, 1993). In practice, institutional pension funds largely take on an equity/bond allocation of 60/40, with alternative asset classes attributing only modest weights. This is a heuristic technique, which is an approach to problem solving and discovery that attempts to practically use a method not guaranteed to be optimal or perfect, in a situation where attaining an optimal solution is impossible or impractical. Heuristic techniques can be used to speed up the process of finding a satisfactory solution.

2.6.1 *Traditional generic portfolio:*

The one Heuristic technique in this study is to create a generic portfolio and compare it to the Markowitz optimization models. This portfolio is based on the risk (variance) of the traditional 60/40 equity/bond portfolio variants is dominated by equity market risk because stock market volatility is significantly larger than bond market volatility (Chaves, Hsu, Li, & Shakernia, 2011). Additionally, allocations to alternative asset classes are too small to contribute meaningfully to the portfolio risk. In this sense, a 60/40 portfolio variant earns much of its return from exposure to equity risk and little from other sources of risk, making this portfolio approach under-diversified in its risk exposure (Chaves, Hsu, Li, & Shakernia, 2011).

To create the generic portfolio, this research applies an index composite used by a South African asset manager. This index is a composite of asset class indices, FTSE/JSE All share index, All bond Index, STeFI, Gold, MSCI World index and JPM Global bond index. The index starts with a simple equity, bond and cash allocation of 65%, 25% and 10%, respectively (Brooke, 2014). The composite is used as a valuable tool that provides insight into investment trends in South Africa.

2.6.2 *Equally weighted portfolio:*

The other alternative asset allocation strategy is the equal weighting. one of the most naïve portfolio heuristics is equal weighting. Investors do not need to assume any knowledge regarding the distribution of the asset class returns. The equally weighted portfolio is mean–variance optimal only if asset classes have the same expected returns and covariances. This strategy, empirically, provides superior portfolio returns when applied to U.S. and global equity portfolio construction (Chaves, Hsu, Li, & Shakernia, 2011). This research will similarly investigate its portfolio returns against the Markowitz optimized portfolios.

2.7 Strategies/variations in this research application of Markowitz model

Using the information presented in the tables above, this research presents four main strategies developed to determine the Markowitz Portfolio selection efficient frontier and portfolios:

- **Quants portfolio from strategy 1:** The first strategy attempts to minimise the risk of the portfolio given a specific target return. The target returns are chosen using a range of the minimum and maximum historic rolling 10 year returns of South African asset classes. Ten portfolios are developed using this target return range from historic minimum to maximum return over the 10 year period. Ten portfolios are created for each rolling 10 year period from December 1998. The portfolio with the highest Sharpe ratio for each year was chosen as the optimal portfolio of the year. This carries on until a portfolio is chosen in a similar method for each year, therefore creating a portfolio for the entire sample period.
- **Quants portfolios from strategy 2:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 3 percent. Optimal portfolios are created on a rolling 10 year period from December 1998.
- **Quants portfolios from strategy 3:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 4 percent. Optimal portfolios are created on a rolling 10 year period from December 1998.
- **Quants portfolios from strategy 4:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 5 percent. Optimal portfolios are created on a rolling 10 year period from December 1998.

These variations have been determined in the research, portfolios have been presented, the efficient frontier for each strategy is available and the optimal asset class allocation for each strategy has an outcome in the results chapter of this research.

2.8 Conclusion of literature review

Investors do not understand economic forces fully, therefore the surety of expected future returns are uncertain. Therefore, reducing risk should be an aim to determining a portfolio. Using Markowitz Modern Portfolio Theory, we can construct a portfolio along the efficient frontier. According to Darko (2012), Markowitz mean-variance optimization is widely used around the world, being applied by financial advisory firms to determine an optimal portfolio in decision-making. Applications in asset allocation, index matching and creation and active portfolio management have been widely used. The optimal portfolio weights will be able to solve the issue of tracking a multi-asset class index. The portfolio weights will be able to provide a platform for passive investing in the multiple asset classes without the need of an active asset manager allocation fund according to their views of expected returns. The results will attempt to reduce the cost of investment management and maintain the required investment returns.

2.8.1 *Research question 1:*

This research paper asks the question, what are the results (return, risk and risk adjusted return) of the optimal asset allocation for South African assets, by employing the Markowitz portfolio selection model?

2.8.2 *Research question 2:*

Summarised by Du Plessis and Ward (2009), Sharpe (1967) assumed Tobin (1958) and Markowitz (1952) work was in fact creating a market portfolio. By adding Beta, Sharpe included a risk measure, which varied from the market. Our research is built upon Markowitz work, which aims to determine the portfolio of the market. Therefore, using Markowitz portfolio theory how does the optimal asset allocation portfolio compare against South African asset class funds over the long-term history?

CHAPTER 3. RESEARCH METHODOLOGY

3.1 Research methodology/paradigm

The research methodology used for this research is a quantitative determination of the optimal portfolio weights, using the Markowitz efficient frontier method to determine a portfolio selection. The expected returns and covariance of assets will be determined by using historical data of each asset. Markowitz showed the variance of return could be used as a measure of portfolio risk, to then be applied to determine significant diversification through portfolio selection. By applying Markowitz portfolio theory in the South African context, we can determine the optimal portfolio weightings.

3.2 Research design

The method used to conduct this research is an application of a quantitative mathematical model, which is applying Markowitz portfolio selection theory in a South African context. After collecting data, and validating the data, Markowitz portfolio theory has been designed to create the optimal portfolio. As presented in the literature review, Markowitz portfolio selection theory has been tested and applied by research peers. Therefore, the model is respected among the investment community and its application in a South African context will be of interest.

The model has presented a few disadvantages as shown by McLeod (1998), who provided limits to the Markowitz model, stating the model in the South African context fails to adequately diversify the portfolio. McLeod found three out of the seven indices were never included in the portfolio and one index only be allocated a maximum of 3% of funds. To overcome such problems, researchers have created parameters to the allocation and used modified methods to determine the variables in the method. McLeod (1998) used a variable method that uses tracking error as a risk rather than variance,

while Biggs and Kane (2009) created restrictions on minimum and maximum allocations per asset. These variations have not been applied in this research paper, as such problems were not encountered, but these variations could be applied in future research papers using a similar data set.

The steps used to develop the portfolio weights, through the model, will be presented in the research instrument section. The output of the model is expected to show various weights according to the assets chosen and therefore the returns, variance and standard deviation of the portfolio.

3.3 Population and sample

3.3.1 *Population*

The population of the research is determined through to two main areas. The variety of assets to be considered and the amount of data points of each asset. South African investors can invest in a wide variety of assets. The main population of assets that have been used for this research are as follows:

- South African equity (JSE All share index)
- Global equity (MSCI world index)
- South African bond (JSE all bond index)
- Global Bond (JP Morgan bond index)
- South African cash
- Gold

The MSCI World Index is recognised as a composite of the world's developed economies, constituting of companies from around the world. The J.P. Morgan index covers a variety of debt denominated asset classes ranging from emerging and developed market bond indices.

Government restrictions have been imposed on the allocation investors can make on certain assets. Government restrictions on a typical multi-asset class fund are that a fund cannot invest more than 25% of the portfolio in foreign assets (Leape & Thomas, 2011). This restriction has been included in the data analysis. The ability to include global assets in a portfolio was not always possible before 1995, SA investors could not invest outside our borders because of exchange controls. In June 1995, these were relaxed and 5% of assets under management could be invested offshore via asset swaps, and in June 1996, this limit was relaxed further to 10%. The limit was again increased to 15% in March 1998. In 2000, collective investment schemes (such as unit trusts) were allowed to take 20% offshore, and in 2005 investment managers had their limits increased to 25%, but retirement funds had to wait until February 2008 to increase their global investments to 20%. Over time, the regulation moved to prudential limits and became much simpler. Finally, in December 2010, retirement funds had their offshore limit increased to 25% (plus an extra 5% in Africa) (Leape & Thomas, 2011).

The population is chosen based which assets are available for institutional investors. These assets have been sampled based on availability of data.

3.3.2 *Sample and sampling method*

The method of this research paper is best reliable with data that the dates as far back as possible. This data is hard to find for all assets, but MacroSolutions, a multi-asset class investment manager, focuses on long-term investing has been able to provide us with data for a number of asset classes dating back 1988. The data has been retrieved with monthly data points for the following asset classes:

Table 1: Start dates of asset class data

Assets	Start Date
South African Equities	December 1960
South African Bonds	December 1960
South African Cash	December 1960
Foreign Equities	December 1969
Foreign Bonds	December 1987
Gold	December 1967

Selection of the data and sampling of the entire data set used for calculating the portfolios has been determined by creating the most comparable portfolio that represents how South African balanced funds have allocated in the past, and to create a portfolio that can be used as an index for future comparisons. Industry norm depicts a typical portfolio allocation as 60%, 30% and 10% in equity, bonds and cash respectively. So the data sampled represent as significant account of a typical asset allocation universe.

The table below represents all the South African balanced funds considered in the research. The funds considered are restricted to large funds that are currently above R1billion in assets under management, according to Morningstar. This is done to encompass funds that manage the majority of assets across the country and reference the performance of funds that represent a considerable amount of pension funds. Performance bias may be created due to skills, resources and liquidity of the large funds relative to smaller funds. The performances of the funds are calculated from the date their performance data is available via the Inet BFA portal. This may also create bias, as the sample size diminishes in earlier dates due to fewer companies and the size of those companies that operated. This performance data is used to calculate the average South African balanced fund performance. The performances are then used to compare against the returns of the Markowitz optimal portfolios calculated in this research paper.

Table 2: South African balanced funds and start date of performance

Fund Manager	Start Date		Fund Manager	Start Date
ABSA Absolute Fund A	1994/05/31		Momentum Balanced Fund R	1995/06/30
Allan Gray Balanced Fund A	2000/04/30		Nedgroup Investments Managed Fund A1	2009/04/30
Allan Gray Stable Fund A	2000/09/30		Nedgroup Investments Opportunity Fund A	1999/12/31
Coronation Balanced Defensive Fund A	2010/05/31		Nedgroup Investments Stable Fund A1	2011/07/31
Coronation Balanced Plus Fund A	1996/06/30		Old Mutual Balanced R	1994/03/31
Coronation Capital Plus Fund	2001/11/30		Old Mutual Flexible R	1996/08/31
Coronation Global Managed [ZAR] Feeder Fund A	2009/12/31		Old Mutual Real Income Fund A	2007/05/31
Coronation Market Plus Fund	2001/11/30		Old Mutual Stable Growth A	2008/04/30
Coronation Optimum Growth Fund A	1999/03/31		Prudential Balanced Fund A	1999/08/31
Discovery Balanced Fund	2008/09/30		Prudential Inflation Plus Fund A	2001/06/30
Foord Balanced Fund R	2002/09/30		PSG Balanced Fund A	1999/09/30
Foord International Feeder Fund R	2006/03/31		PSG Flexible Fund	1999/07/31
Investec Cautious Managed Fund A	2008/06/30		Sanlam Investment Management Balanced Fund R	1995/02/28
Investec Cautious Managed Fund B	2008/06/30		Sanlam Investment Management Inflation Plus Fund	1999/09/30
Investec Managed Fund A	2001/01/31		STANLIB Balanced Cautious Fund A	2009/02/28
Investec Managed Fund B	2004/07/31		STANLIB Balanced Fund R	2004/05/31
Investec Managed Fund R	1995/10/31		STANLIB Multi-Manager Balanced Fund A	2001/12/31
Investec Opportunity Fund R	1998/07/31			

3.4 The research instrument

A model is created by firstly calculating the monthly returns of assets and the standard deviation. The monthly returns of each asset are calculated as follows:

$$R_{it} = \frac{P_{it}}{P_{it-1}} - 1 \quad \textit{. . . Equation 1}$$

Where Rit is the monthly return on asset i at time t, Pit is the adjusted monthly closing price of the asset i at time t, and pit-1 is the adjusted monthly closing price of the stock i at time t-1.

The standard deviation, to determine the risk of assets, is calculated as follows:

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(R_i - E(R_i))^2}{n}} \quad \textit{. . . Equation 2}$$

The expected portfolio return is calculated as follows, based on weighting of each asset class and its expected return:

$$E(R_p) = \sum_{i=1}^{n} X_i E(R_i) \quad \textit{. . . Equation 3}$$

Where “Xi” is the weighting of portfolio investment in asset class “i” and “E(Ri)” is the expected rate of return on asset class “i”.

The risk of the portfolio is calculated based on the correlation coefficient between assets:

$$\sigma_p^2 = \sqrt{\sum_{i=1}^{n}(X_i^2\sigma_i^2) + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \neq j}}^{n}(X_i X_j \sigma_{ij})} \quad \textit{. . . Equation 4}$$

Where “Pij” is the correlation coefficient between asset classes i and j. The covariance between asset classes i and j can be written as:

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \quad \ldots \quad \textit{Equation 5}$$

The optimal portfolio is found by evaluating that selection with the greatest excess return (expected return less the risk-free rate) relative to the standard deviation. The equation to find the optimal portfolio with the highest return while minimising risk is based on:

$$\theta = \frac{E(R_p) - R_f}{\sigma_p} \quad \ldots \quad \textit{Equation 6}$$

Investors may have intentions of maximising return relative to the risk, therefore may not be satisfied with investing in a minimum variance portfolio that does not have the highest Sharpe ratio. The tangency portfolio creates a portfolio with the maximum Sharpe ratio, thus:

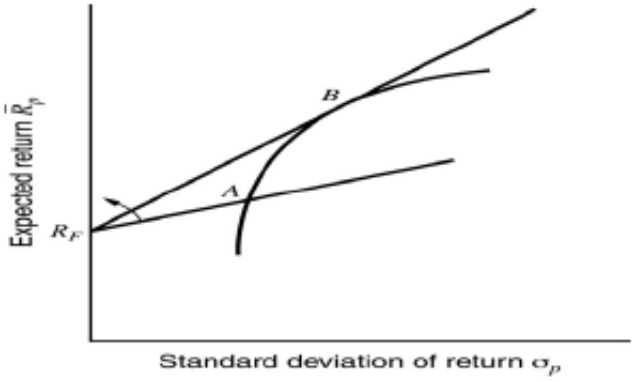


Figure 2: Efficient frontier using the risk free rate, (Tobin, 1958)

Where, “Rf” is the risk free rate and, A and B are portfolios plotted as returns relative to the standard deviation of the portfolio’s.

We provide constraints on the output in order to create a portfolio totalling 100% weighting and with no negative weighting (shorting of assets). Constrains used here:

1. $\sum_{i=1}^{n} X_i = 1$ . . . ***Equation 7***

2. $\sum_{i=1}^{n}(X_i - E(R_i)) = E(R_p)$ . . . ***Equation 8***

3. $X_i \geq 0, \quad i = 1, 2, \ldots, n$ . . . ***Equation 9***

4. $X_i \leq 25\%, \quad i = Global\ equities, Global\ bonds$ . . . ***Equation 10***

5. $X_i + X_j \leq 25\%, \quad i = Global\ equities. j = Global\ bonds$ . . . ***Equation 11***

3.5 Procedure for data collection

Market data of asset classes has been attained from Inet BFA and MacroSolutions, a multi-asset class manager. Data has been attained dating back 1988. The Inet BFA software has been used to attain all data onto excel for analysis and interpretation. The data of South African balanced fund performance has also been attained in a similar method from the Inet Bridge software.

3.6 Data analysis and interpretation

The optimal portfolios are all calculated on a 10-year rolling basis, starting with the first optimal portfolio allocations in December 1998. Portfolios have then been created yearly until December 2014. Performance returns are then calculated using back testing from the inception of the portfolio from December 1998 until the most recent data point October 2015.

The optimal portfolio requires an input on expected returns. The expected return input will be determined by using the investment benchmark South African multi-asset managers use to determine the required long-term returns, which is inflation plus an

excess return between 1 percent and 5 percent. The expected returns input will provide us with various portfolios', which are plotted as the expected results of the efficient frontier. The research presents four main strategies developed to determine the Markowitz Portfolio selection efficient frontier and portfolios, with explanation on their respective expected returns input:

- **Quants portfolio from strategy 1:** The first strategy attempts to minimise the risk of the portfolio given a specific target return. The target returns are chosen using a range of the minimum and maximum historic rolling 10-year returns of South African asset classes. Ten portfolios are developed using this target return range from historic minimum to maximum return over the 10-year period. Ten portfolios are created for each rolling 10-year period from December 1998. The portfolio with the highest Sharpe ratio for each year was chosen as the optimal portfolio of the year. This carries on until a portfolio is chosen in a similar method for each year, therefore creating a portfolio for the entire sample period.
- **Quants portfolios from strategy 2:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 3 percent. Optimal portfolios are created on a rolling 10-year period from December 1998.
- **Quants portfolios from strategy 3:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 4 percent. Optimal portfolios are created on a rolling 10-year period from December 1998.
- **Quants portfolios from strategy 4:** The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 5 percent. Optimal portfolios are created on a rolling 10-year period from December 1998.

The performance measures of optimal portfolios are calculated based on various periods. The performance measure being mainly annualised returns, standard deviation and sharpe ratio. The interpretation of the optimal portfolio will be analysed in the following periods:

- Since inception of the optimal portfolios, December 1998,
- In x number of years (past 15 years, past 10 years, past 5 years, past 3 years, past 1 year)
- In half decades and decades (1998-2000, 2000-2010, 2010-2015)

The performance measures are presented in a back tested calculation of the optimal portfolio weights. The back testing process will assume the results of the Markowitz Model are practically applied as a passive multi-asset fund. The fund will apply the weightings to the assets and track the returns, based on those weightings, for the next 5 years, 10 years, 15 years and so on. These results will allow us to determine how the Markowitz portfolio would have performed had it been applied without any hindsight bias.

The application of determining back-tested returns based on Markowitz model weights can then be compared to the performance of South African multi-asset fund returns. This will allow us to determine whether the active management returns attained by asset managers have outperformed or underperformed Markowitz Model returns.

3.7 Limitations of the study

This study attempts to establish the performance of asset classes over a period and to determine the optimal asset allocation weightings throughout that period. The period will be subject to data availability. The best data is from Inet BFA and a multi-asset class manager that can provide data on assets from 1988. Pre-1988 data may be likely unavailable or unreliable for all the asset classes. The longer the historical data, the greater the ability to use statistics to create reliable predictions.

The study focuses entirely on assets a South African asset manager could purchase on behalf of their clients. Foreign assets pre-1994 could not be purchased, therefore, will be excluded from the study pre-1994 (Leape & Thomas, 2011). Currently, the foreign asset allocation holding is limited to 25 percent for South African funds. This limitation is applied throughout the analysis period, in order to create a portfolio that can be extrapolated to indicate future expected returns.

The number of asset classes will be limited to listed assets. This is because investment managers on behalf of their clients can purchase these assets with reasonable liquidity and long-term data on these assets can be attained with accuracy. Private equity, which is an option for some multi-asset managers, will not be considered in this research.

The study will focus on major asset classes that have traditionally been accepted as the lion share of portfolios' allocation by South African managers (Brooke, 2014), which are the following: South African equities, South African bonds, South African cash, world equity index, world bonds, and gold. Platinum and palladium ETF's will be excluded, as these assets are new in terms of investment ability, accessibility and liquidity and will therefore not provide a fair result of allocation weightings.

3.8 Validity and reliability

According to Golafshani (2003), in quantitative research, we have to ask ourselves whether our research is valid and reliable through two main questions: 1) are the means of measurement accurate and do they measure what they are meant to measure, and 2) can the results be repeated? By applying the Markowitz portfolio theory the data can be measured accurately and produce answers to the research question. The research using the model is reliable as it can be repeated in various markets around the world. The reliability does depend on the availability of data.

CHAPTER 4. RESULTS & DISCUSSIONS

4.1 Introduction

This chapter presents the results attained following the utilisation of the research methodology and data analysis described in chapter 3. The final sample of the data is described in section 4.1 and the key issues that had to be dealt with in analysing the data are presented first in section 4.2. The results obtained to answer the first research question are presented through the determination of the optimal asset allocation in South Africa, having used the methodologies in chapter 3. The research question will further describe the optimal asset allocation based on the target return. Then the second research question presents the index created that can be used to compare the past and future returns of asset managers, based on the required return of an investment. The chapter is then concluded by summarising the results obtained.

4.2 Sample Profile

The data sampled from the asset returns were from 31/12/1988 until 31/10/2015. This provides a long-term analysis period, which South African funds could predominately invest in offshore assets, making the results a fair representation of practical possible asset allocations. The final asset classes chosen to be included in the analysis are South African equities, South African bonds, South African cash, MSCI global equities, JP Morgan global bonds and gold. These are indices sourced from the Johannesburg Stock Exchange, MSCI and JP Morgan through the software portal INET BFA. This sample data is representative of a typical South African balanced fund asset allocation, as noted by Brooke (2014).

4.3 Data analysis

Table five below shows the historic nominal and real returns, standard deviation and the risk/return ratio (hence forth to be mentioned as the Sharpe ratio), of the sample period mentioned above. The real returns are historic excess returns over and above inflation, which is the biggest risk to investment as it reduces spending power, therefore picking up the time value of money. The investor's decision of which asset to invest will be dependent on expectations of one of, or a combination of, the expected return, risk (standard deviation) and the Sharpe ratio. In this research, analysis and sample, SA equities had the highest return and the highest risk (standard deviation), while SA cash had the lowest return with the lowest risk. Finally, SA cash had the highest Sharpe ratio (return to risk ratio), while Gold had the lowest Sharpe ratio. Foreign asset returns do include the movements in the Rand to US Dollar exchange rate, because the assets are priced in US dollars, so the exchange rate movements may appreciate or depreciate the actual returns of the underlying assets.

Table 3: Expected returns, standard deviation and return to risk ratio of asset classes from December 1987 to October 2015.

	Historic returns		Standard deviation		Return/risk	
	Nominal Historic returns	Real Historic returns	Standard deviation of nominal returns	Standard deviation of real returns	Return/risk of nominal returns	Return/risk of real returns
SA Equity: ALSI	16.15%	7.92%	21.38%	20.73%	0.76	0.38
SA Bond: ALBI	13.54%	5.49%	8.82%	9.02%	1.54	0.61
SA Cash	11.62%	3.70%	4.58%	3.68%	2.54	1.01
GBL: Equity	15.29%	7.12%	20.06%	19.22%	0.76	0.37
Gbl: JPM GBI	13.70%	5.63%	14.66%	12.91%	0.93	0.44
Gold	10.21%	2.39%	19.12%	18.60%	0.53	0.13

Table 6 and 7 below shows the covariance and correlation of the underlying assets over the sampled period. This covariance matrix has been used to determine the allocations in the Markowitz portfolios. The interpret the results we will use the correlation coefficient; because covariance informs us if the assets have a relationship, but does not necessarily tell us how strong the relationship is. A few of the highest correlated assets are SA equity with Global equity, and also Global bonds with gold, while assets that indicate no relationship are SA equity with SA bonds and Global equity with SA cash. The largest indirect correlation relationship is between SA cash and gold.

Table 4: The covariance matrix from the period December 1987 to October 2015

	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold
SA Equity: ALSI	0.0430	0.0024	0.0006	0.0189	-0.0046	0.0106
SA Bond: ALBI	0.0024	0.0081	0.0020	-0.0001	-0.0013	-0.0007
SA Cash	0.0006	0.0020	0.0014	0.0011	-0.0003	-0.0010
GBL: Equity	0.0189	-0.0001	0.0011	0.0369	0.0103	0.0073
Gbl: JPM GBI	-0.0046	-0.0013	-0.0003	0.0103	0.0167	0.0127
Gold	0.0106	-0.0007	-0.0010	0.0073	0.0127	0.0346

Table 5: The correlation matrix from the period December 1987 to October 2015

	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold
SA Equity: ALSI	1.00	-0.03	-0.14	0.43	-0.18	0.19
SA Bond: ALBI	-0.03	1.00	0.44	-0.14	-0.07	-0.21
SA Cash	-0.14	0.44	1.00	-0.01	0.18	-0.33
GBL: Equity	0.43	-0.14	-0.01	1.00	0.39	0.13
Gbl: JPM GBI	-0.18	-0.07	0.18	0.39	1.00	0.50
Gold	0.19	-0.21	-0.33	0.13	0.50	1.00

Using the information presented in the tables above, this research presents four main strategies developed to determine the Markowitz Portfolio selection efficient frontier and portfolios (as discussed in the methodology chapter):

Quants portfolio from strategy 1: The first strategy attempts to minimise the risk of the portfolio given a specific target return. To be called "Minimising Standard deviation".

Quants portfolios from strategy 2: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 3 percent. To be called "Targeting inflation + 3%".

Quants portfolios from strategy 3: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 4 percent. To be called "Targeting inflation + 4%".

Quants portfolios from strategy 4: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 5 percent. To be called "Targeting inflation + 5%".

These strategies are presented, in the following section, with their portfolios and returns over investment periods given their respected conditions/targets.

4.4 Presenting Portfolios

4.4.1 *Quants portfolio strategy 1: Markowitz target return while minimize risk portfolios*

Applying the Markowitz target return and minimize risk strategy, we present the results of the portfolios. The Quants portfolio strategy 1, which aims to minimize risk while targeting a specific return is presented in figure 3 and table 7 below. The portfolio allocations are calculated using Markowitz portfolio theory on a 10 year rolling basis. Table 7 shows the portfolio weightings for each year given specific target returns. The target returns are determined by applying a minimum target return based on the minimum annualised return of the assets in the sample, and then the maximum target return based on the maximum annualised return of the assets in the sample. These two numbers become a range of min and max, and then used to extrapolate a few more target returns that fit within that range. This is all done on a rolling 10 year basis for each year.

The portfolios created in Table 7 provide us with an efficient frontier for each year. The optimal portfolio for each year is then chosen based on the highest Sharpe ratio to be included in the Quants portfolio strategy 1 as the investable portfolio. This is done for each year until a rolling portfolio is created from year 1998 to 2015, as presented in Figure 3 below (a summary of Table 7).

Figure 3: Portfolio allocations of the Quants strategy 1 for the 10 year rolling period of December 1998 to October 2015

Minimise STDev Portfolio							
Year	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold	Portfolio Total Weight
1998/12/31	1.4%	0.6%	86.8%	3.4%	2.9%	5.0%	100%
1999/12/31	4.0%	0.6%	84.2%	1.1%	5.6%	4.5%	100%
2000/12/31	5.1%	2.4%	83.5%	0.0%	4.9%	4.1%	100%
2001/12/31	6.6%	3.9%	78.7%	0.0%	10.3%	0.5%	100%
2002/12/31	5.2%	1.0%	86.3%	0.0%	3.9%	3.6%	100%

2003/12/31	1.3%	5.6%	88.9%	0.0%	0.0%	4.2%	100%
2004/12/31	3.7%	7.4%	85.7%	0.0%	0.0%	3.2%	100%
2005/12/31	6.0%	16.7%	72.9%	0.0%	0.0%	4.5%	100%
2006/12/31	7.6%	3.2%	84.0%	0.0%	0.0%	5.2%	100%
2007/12/31	11.9%	4.2%	82.5%	0.0%	1.4%	0.0%	100%
2008/12/31	7.0%	7.3%	80.1%	0.0%	0.0%	5.5%	100%
2009/12/31	4.0%	0.0%	93.1%	1.2%	1.0%	0.7%	100%
2010/12/31	5.9%	0.3%	92.6%	0.0%	0.0%	1.1%	100%
2011/12/31	5.8%	11.1%	79.9%	0.0%	0.0%	3.2%	100%
2012/12/31	0.7%	14.7%	74.5%	6.7%	0.0%	3.4%	100%
2013/12/31	0.0%	8.0%	86.0%	6.0%	0.0%	0.0%	100%
2014/12/31	0.0%	10.3%	82.7%	7.0%	0.0%	0.0%	100%
Average Allocation	4.5%	5.7%	83.7%	1.5%	1.8%	2.9%	
Average Allocation STDev	3.1%	5.1%	5.5%	2.6%	2.9%	2.0%	

The Markowitz portfolio model has been able to create portfolios that have allocated weights in each one of the asset classes at some point in the analysis period. Throughout the period SA cash has been the predominant investment allocation averaging 83.7%, a combination of bonds has been the second highest allocation, followed by a combination of equities. Both SA cash and SA bonds had the lowest standard deviation of annualised returns throughout the analysis period and with the highest Sharpe ratio, indicating the low risk appetite of this portfolio. Given the high allocation of cash in the portfolio, we can expect a low volatility portfolio, as will be presented in the return results section 4.5.

The portfolios created in Table 7 below provide us with an efficient frontier calculated for each year, given a certain target return. Table 7 shows, given a certain target return, the portfolio weights, variance, standard deviation, annualised returns and Sharpe ratio over the past 10 year data. The information is calculated and provided based on a rolling 10-year data. The portfolio with the highest Sharpe ratio for each year is chosen as the portfolio to be applied in the “minimising stdev portfolio” (as shown in Figure 3).

Table 6: Portfolio allocation of the Quants portfolio strategy 1 - Markowitz target return while minimizing risk over period of October 1995 to October 2015

1998/12/31	Target Return:	4.0%	5.3%	6.7%	8.0%	9.3%	10.7%	12.0%	13.3%	14.7%	16.0%
	SA Equity: ALSI	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
	SA Bond: ALBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
	SA Cash	4%	14%	24%	35%	45%	55%	66%	76%	86%	87%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%
	Gold	96%	86%	76%	65%	55%	45%	34%	24%	13%	5%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.022	0.017	0.013	0.009	0.006	0.004	0.002	0.001	0.001	0.000
	Std Dev.	14.7%	13.0%	11.3%	9.6%	7.9%	6.2%	4.6%	3.2%	2.3%	2.2%
	Returns	4.0%	5.3%	6.7%	8.0%	9.3%	10.7%	12.0%	13.3%	14.7%	16.0%
	Return/Risk Ratio	0.27	0.41	0.59	0.84	1.19	1.72	2.61	4.21	6.49	7.29
1999/12/31	Target Return:	5.0%	6.3%	7.7%	9.0%	10.3%	11.7%	13.0%	14.3%	15.7%	17.0%
	SA Equity: ALSI	0%	0%	0%	0%	0%	0%	0%	0%	4%	1%
	SA Bond: ALBI	0%	0%	0%	0%	0%	0%	0%	0%	1%	6%
	SA Cash	2%	14%	25%	37%	49%	61%	73%	84%	84%	78%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	1%	13%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	6%	1%
	Gold	98%	86%	75%	63%	51%	39%	27%	15%	4%	0%
	Portfolio Total	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

	Weight									
	Variance	0.023	0.018	0.013	0.009	0.005	0.003	0.001	0.001	0.000	0.001
	Std Dev.	15.3%	13.3%	11.3%	9.3%	7.4%	5.5%	3.7%	2.5%	2.2%	2.9%
	Returns	5.0%	6.3%	7.7%	9.0%	10.3%	11.7%	13.0%	14.3%	15.7%	17.0%
	Return/Risk Ratio	0.33	0.48	0.68	0.97	1.41	2.14	3.49	5.84	7.07	5.86
2000/12/31	Target Return:	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%	17.0%
	SA Equity: ALSI	0%	0%	0%	0%	0%	0%	1%	5%	6%	4%
	SA Bond: ALBI	0%	0%	0%	0%	0%	0%	0%	2%	5%	11%
	SA Cash	12%	24%	37%	49%	62%	74%	85%	84%	75%	62%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	4%	12%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	5%	10%	11%
	Gold	88%	76%	63%	51%	38%	26%	13%	4%	0%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.018	0.013	0.009	0.005	0.003	0.001	0.001	0.000	0.001	0.001
	Std Dev.	13.6%	11.5%	9.4%	7.4%	5.4%	3.6%	2.3%	2.2%	2.4%	3.2%
	Returns	8.0%	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%	17.0%
	Return/Risk Ratio	0.59	0.78	1.06	1.49	2.23	3.65	6.01	6.78	6.54	5.38
2001/12/31	Target Return:	13.0%	13.6%	14.1%	14.7%	15.2%	15.8%	16.3%	16.9%	17.4%	18.0%
	SA Equity: ALSI	0%	0%	2%	5%	7%	7%	7%	7%	7%	6%
	SA Bond: ALBI	0%	0%	0%	3%	4%	7%	10%	13%	16%	19%
	SA Cash	14%	52%	88%	84%	79%	70%	62%	55%	48%	41%
	GBL: Equity	0%	0%	0%	0%	0%	0%	2%	4%	7%	9%
	Gbl: JPM GBI	0%	0%	0%	5%	10%	16%	19%	21%	23%	25%

	Gold	86%	48%	10%	4%	1%	0%	0%	0%	0%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.019	0.005	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002
	Std Dev.	13.9%	7.3%	2.2%	2.1%	2.2%	2.3%	2.6%	3.0%	3.4%	3.9%
	Returns	13.0%	13.6%	14.1%	14.7%	15.2%	15.8%	16.3%	16.9%	17.4%	18.0%
	Return/Risk Ratio	0.93	1.85	6.38	6.91	7.04	6.81	6.20	5.60	5.08	4.63
2002/12/31	Target Return:	13.0%	13.3%	13.7%	14.0%	14.3%	14.7%	15.0%	15.3%	15.7%	16.0%
	SA Equity: ALSI	0%	0%	1%	5%	7%	7%	6%	6%	5%	4%
	SA Bond: ALBI	0%	0%	0%	1%	3%	7%	12%	18%	23%	29%
	SA Cash	47%	68%	88%	86%	81%	73%	64%	56%	47%	39%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%
	Gbl: JPM GBI	0%	0%	0%	4%	9%	13%	17%	21%	25%	25%
	Gold	53%	32%	11%	4%	0%	0%	0%	0%	0%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.010	0.003	0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.002
	Std Dev.	9.9%	5.7%	2.3%	2.1%	2.1%	2.3%	2.7%	3.1%	3.7%	4.3%
	Returns	13.0%	13.3%	13.7%	14.0%	14.3%	14.7%	15.0%	15.3%	15.7%	16.0%
	Return/Risk Ratio	1.31	2.35	5.89	6.78	6.75	6.36	5.65	4.92	4.29	3.76
2003/12/31	Target Return:	9.0%	9.7%	10.3%	11.0%	11.7%	12.3%	13.0%	13.7%	14.3%	15.0%
	SA Equity: ALSI	18%	15%	13%	11%	9%	6%	4%	1%	0%	0%
	SA Bond: ALBI	0%	0%	0%	0%	0%	0%	0%	6%	25%	59%
	SA Cash	5%	18%	31%	44%	57%	70%	83%	89%	66%	25%

	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	8%	14%
	Gold	77%	66%	56%	45%	34%	23%	13%	4%	0%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.031	0.023	0.016	0.010	0.006	0.003	0.001	0.001	0.001	0.004
	Std Dev.	17.7%	15.1%	12.5%	10.0%	7.4%	5.0%	3.0%	2.3%	3.3%	6.2%
	Returns	9.0%	9.7%	10.3%	11.0%	11.7%	12.3%	13.0%	13.7%	14.3%	15.0%
	Return/Risk Ratio	0.51	0.64	0.83	1.10	1.57	2.45	4.39	5.91	4.30	2.43
2004/12/31	Target Return:	8.0%	9.1%	10.2%	11.3%	12.4%	13.6%	14.7%	15.8%	16.9%	18.0%
	SA Equity: ALSI	18%	16%	13%	10%	7%	4%	0%	0%	0%	0%
	SA Bond: ALBI	0%	0%	0%	0%	0%	7%	25%	47%	70%	92%
	SA Cash	1%	20%	40%	60%	79%	86%	72%	47%	22%	0%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	3%	6%	8%	7%
	Gold	81%	64%	47%	31%	14%	3%	0%	0%	0%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.031	0.019	0.010	0.004	0.001	0.001	0.001	0.002	0.005	0.008
	Std Dev.	17.5%	13.8%	10.2%	6.6%	3.5%	2.6%	3.4%	4.9%	6.7%	8.7%
	Returns	8.0%	9.1%	10.2%	11.3%	12.4%	13.6%	14.7%	15.8%	16.9%	18.0%
	Return/Risk Ratio	0.46	0.66	1.01	1.72	3.58	5.13	4.26	3.22	2.51	2.07
2005/12/31	Target Return:	10.0%	10.7%	11.3%	12.0%	12.7%	13.3%	14.0%	14.7%	15.3%	16.0%
	SA Equity: ALSI	0%	0%	0%	0%	6%	6%	5%	4%	1%	0%

	SA Bond: ALBI	0%	0%	0%	0%	0%	17%	34%	52%	69%	86%
	SA Cash	2%	26%	50%	74%	88%	73%	57%	39%	22%	4%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	1%	5%	8%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	98%	74%	50%	26%	6%	4%	4%	4%	3%	2%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.038	0.021	0.009	0.003	0.001	0.001	0.001	0.002	0.003	0.005
	Std Dev.	19.6%	14.6%	9.7%	5.2%	2.8%	2.8%	3.5%	4.6%	5.7%	6.9%
	Returns	10.0%	10.7%	11.3%	12.0%	12.7%	13.3%	14.0%	14.7%	15.3%	16.0%
	Return/Risk Ratio	0.51	0.73	1.17	2.31	4.48	4.70	3.98	3.21	2.69	2.32
2006/12/31	Target Return:	10.4%	10.8%	11.6%	12.3%	13.1%	13.9%	14.7%	15.4%	16.2%	17.0%
	SA Equity: ALSI	0%	0%	0%	8%	9%	10%	11%	10%	13%	60%
	SA Bond: ALBI	0%	0%	0%	3%	19%	35%	50%	67%	80%	40%
	SA Cash	0%	31%	83%	84%	67%	50%	33%	14%	0%	0%
	GBL: Equity	0%	0%	0%	0%	0%	0%	1%	4%	5%	0%
	Gbl: JPM GBI	25%	25%	7%	0%	0%	0%	0%	0%	0%	0%
	Gold	75%	44%	9%	5%	5%	5%	5%	5%	2%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.040	0.017	0.002	0.001	0.001	0.001	0.002	0.003	0.005	0.018
	Std Dev.	20.1%	13.0%	3.9%	2.6%	2.8%	3.5%	4.6%	5.8%	7.0%	13.3%
	Returns	10.4%	10.8%	11.6%	12.3%	13.1%	13.9%	14.7%	15.4%	16.2%	17.0%
	Return/Risk Ratio	0.52	0.83	2.94	4.82	4.73	3.91	3.18	2.68	2.32	1.28

2007/12/31	Target Return:	10.8%	11.1%	12.2%	13.3%	14.4%	15.6%	16.7%	17.8%	18.9%	20.0%
	SA Equity: ALSI	0%	1%	12%	16%	17%	21%	41%	61%	81%	100%
	SA Bond: ALBI	0%	0%	4%	23%	44%	63%	48%	33%	18%	0%
	SA Cash	75%	96%	83%	58%	28%	0%	0%	0%	0%	0%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	25%	3%	1%	0%	0%	0%	0%	0%	0%	0%
	Gold	0%	0%	0%	3%	12%	15%	11%	6%	1%	0%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.003	0.001	0.001	0.001	0.003	0.005	0.010	0.019	0.032	0.048
	Std Dev.	5.7%	3.5%	2.6%	3.6%	5.3%	7.4%	10.2%	13.8%	17.8%	21.9%
	Returns	10.8%	11.1%	12.2%	13.3%	14.4%	15.6%	16.7%	17.8%	18.9%	20.0%
	Return/Risk Ratio	1.87	3.20	4.68	3.67	2.72	2.12	1.64	1.29	1.06	0.91
2008/12/31	Target Return:	11.0%	11.8%	12.6%	13.3%	14.1%	14.9%	15.7%	16.4%	17.2%	18.0%
	SA Equity: ALSI	5%	7%	7%	8%	8%	8%	9%	15%	30%	45%
	SA Bond: ALBI	0%	7%	19%	30%	41%	52%	64%	59%	33%	8%
	SA Cash	92%	80%	66%	51%	37%	23%	8%	0%	0%	0%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	2%	5%	8%	11%	14%	17%	20%	26%	36%	47%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.012	0.025
	Std Dev.	2.5%	2.4%	2.8%	3.4%	4.1%	4.9%	5.7%	7.0%	10.9%	15.9%
	Returns	11.0%	11.8%	12.6%	13.3%	14.1%	14.9%	15.7%	16.4%	17.2%	18.0%

	Return/Risk Ratio	4.38	4.82	4.51	3.96	3.45	3.04	2.73	2.34	1.58	1.13
2009/12/31	Target Return:	10.0%	10.7%	11.3%	12.0%	12.7%	13.3%	14.0%	14.7%	15.3%	16.0%
	SA Equity: ALSI	4%	6%	7%	9%	10%	11%	15%	21%	26%	32%
	SA Bond: ALBI	0%	7%	21%	35%	48%	62%	57%	42%	26%	11%
	SA Cash	93%	82%	63%	44%	25%	6%	0%	0%	0%	0%
	GBL: Equity	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	1%	5%	9%	13%	16%	20%	28%	38%	47%	57%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.000	0.000	0.001	0.001	0.002	0.003	0.005	0.009	0.016	0.025
	Std Dev.	1.5%	1.7%	2.6%	3.6%	4.7%	5.8%	7.2%	9.7%	12.7%	15.8%
	Returns	10.0%	10.7%	11.3%	12.0%	12.7%	13.3%	14.0%	14.7%	15.3%	16.0%
	Return/Risk Ratio	6.60	6.11	4.41	3.35	2.72	2.31	1.94	1.51	1.21	1.01
2010/12/31	Target Return:	10.0%	10.8%	11.6%	12.3%	13.1%	13.9%	14.7%	15.4%	16.2%	17.0%
	SA Equity: ALSI	6%	8%	10%	13%	15%	18%	25%	32%	39%	46%
	SA Bond: ALBI	0%	14%	27%	40%	54%	63%	50%	37%	24%	11%
	SA Cash	93%	73%	54%	35%	16%	0%	0%	0%	0%	0%
	GBL: Equity	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	1%	5%	8%	12%	15%	19%	25%	31%	37%	43%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.000	0.000	0.001	0.001	0.003	0.004	0.006	0.011	0.017	0.024

	Std Dev.	1.1%	1.7%	2.7%	3.9%	5.0%	6.2%	8.0%	10.3%	12.9%	15.6%
	Returns	10.0%	10.8%	11.6%	12.3%	13.1%	13.9%	14.7%	15.4%	16.2%	17.0%
	Return/Risk Ratio	8.71	6.35	4.25	3.20	2.61	2.23	1.83	1.49	1.26	1.09
2011/12/31	Target Return:	9.0%	9.7%	10.3%	11.0%	11.7%	12.3%	13.0%	13.7%	14.3%	15.0%
	SA Equity: ALSI	3%	6%	8%	9%	11%	18%	28%	37%	47%	69%
	SA Bond: ALBI	9%	11%	30%	49%	68%	62%	46%	30%	14%	0%
	SA Cash	83%	80%	56%	32%	7%	0%	0%	0%	0%	0%
	GBL: Equity	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	2%	3%	7%	10%	14%	20%	26%	33%	39%	31%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.000	0.000	0.000	0.001	0.002	0.003	0.007	0.013	0.022	0.035
	Std Dev.	1.4%	1.4%	2.0%	2.9%	4.0%	5.6%	8.3%	11.5%	14.8%	18.6%
	Returns	9.0%	9.7%	10.3%	11.0%	11.7%	12.3%	13.0%	13.7%	14.3%	15.0%
	Return/Risk Ratio	6.63	6.70	5.21	3.78	2.94	2.22	1.56	1.19	0.97	0.81
2012/12/31	Target Return:	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%	17.0%	18.0%
	SA Equity: ALSI	1%	6%	8%	10%	13%	22%	30%	38%	47%	56%
	SA Bond: ALBI	15%	22%	38%	55%	68%	54%	41%	27%	13%	0%
	SA Cash	74%	65%	44%	20%	0%	0%	0%	0%	0%	0%
	GBL: Equity	7%	1%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gold	3%	7%	11%	15%	19%	24%	29%	35%	40%	44%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

	Variance	0.000	0.000	0.000	0.001	0.002	0.004	0.007	0.013	0.020	0.028
	Std Dev.	1.2%	1.4%	2.1%	3.1%	4.3%	6.1%	8.6%	11.3%	14.0%	16.8%
	Returns	9.0%	10.0%	11.0%	12.0%	13.0%	14.0%	15.0%	16.0%	17.0%	18.0%
	Return/Risk Ratio	7.59	7.00	5.21	3.84	3.06	2.28	1.74	1.42	1.21	1.07
2013/12/31	Target Return:	8.0%	9.2%	10.4%	11.7%	12.9%	14.1%	15.3%	16.6%	17.8%	19.0%
	SA Equity: ALSI	0%	6%	15%	20%	26%	33%	40%	46%	54%	85%
	SA Bond: ALBI	8%	21%	30%	40%	47%	36%	25%	14%	3%	0%
	SA Cash	86%	61%	40%	18%	0%	0%	0%	0%	0%	0%
	GBL: Equity	6%	4%	0%	0%	0%	0%	0%	0%	0%	0%
	Gbl: JPM GBI	0%	3%	9%	11%	11%	8%	5%	2%	0%	0%
	Gold	0%	4%	6%	11%	16%	23%	30%	37%	44%	15%
	Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	Variance	0.000	0.000	0.001	0.002	0.003	0.006	0.009	0.013	0.018	0.031
	Std Dev.	1.2%	1.6%	2.9%	4.3%	5.8%	7.5%	9.3%	11.3%	13.3%	17.6%
	Returns	8.0%	9.2%	10.4%	11.7%	12.9%	14.1%	15.3%	16.6%	17.8%	19.0%
	Return/Risk Ratio	6.75	5.62	3.65	2.73	2.24	1.89	1.64	1.47	1.34	1.08
2014/12/31	Target Return:	8.0%	9.1%	10.2%	11.3%	12.4%	13.6%	14.7%	15.8%	16.9%	18.0%
	SA Equity: ALSI	0%	2%	9%	15%	22%	28%	34%	39%	41%	39%
	SA Bond: ALBI	10%	20%	26%	32%	39%	31%	19%	7%	0%	0%
	SA Cash	83%	63%	43%	24%	4%	0%	0%	0%	0%	0%
	GBL: Equity	7%	9%	7%	5%	2%	1%	0%	0%	2%	6%
	Gbl: JPM GBI	0%	1%	7%	14%	20%	23%	24%	24%	17%	0%
	Gold	0%	5%	8%	10%	13%	18%	24%	30%	40%	54%

Portfolio Total Weight	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Variance	0.000	0.000	0.001	0.002	0.003	0.004	0.006	0.009	0.013	0.017
Std Dev.	0.9%	1.6%	2.7%	3.9%	5.2%	6.5%	8.0%	9.5%	11.2%	13.1%
Returns	8.0%	9.1%	10.2%	11.3%	12.4%	13.6%	14.7%	15.8%	16.9%	18.0%
Return/Risk Ratio	8.71	5.77	3.75	2.88	2.41	2.10	1.84	1.65	1.51	1.38

4.4.2 *Quants portfolio strategy 2, 3 and 4: Minimising risk while targeting a return of inflation plus 3 percent, 4 percent and 5 percent.*

Applying the Markowitz target return and minimize risk strategy, we present the results of the portfolios using a 10 year rolling inflation plus 3 to 5 percent as a target return. The Quants portfolios aim to minimize risk while targeting these specific returns. The portfolio allocations are calculated using Markowitz portfolio theory on a 10 year rolling basis. These targets are used in this research because South African balanced fund managers normally use this as a benchmark to their portfolios.

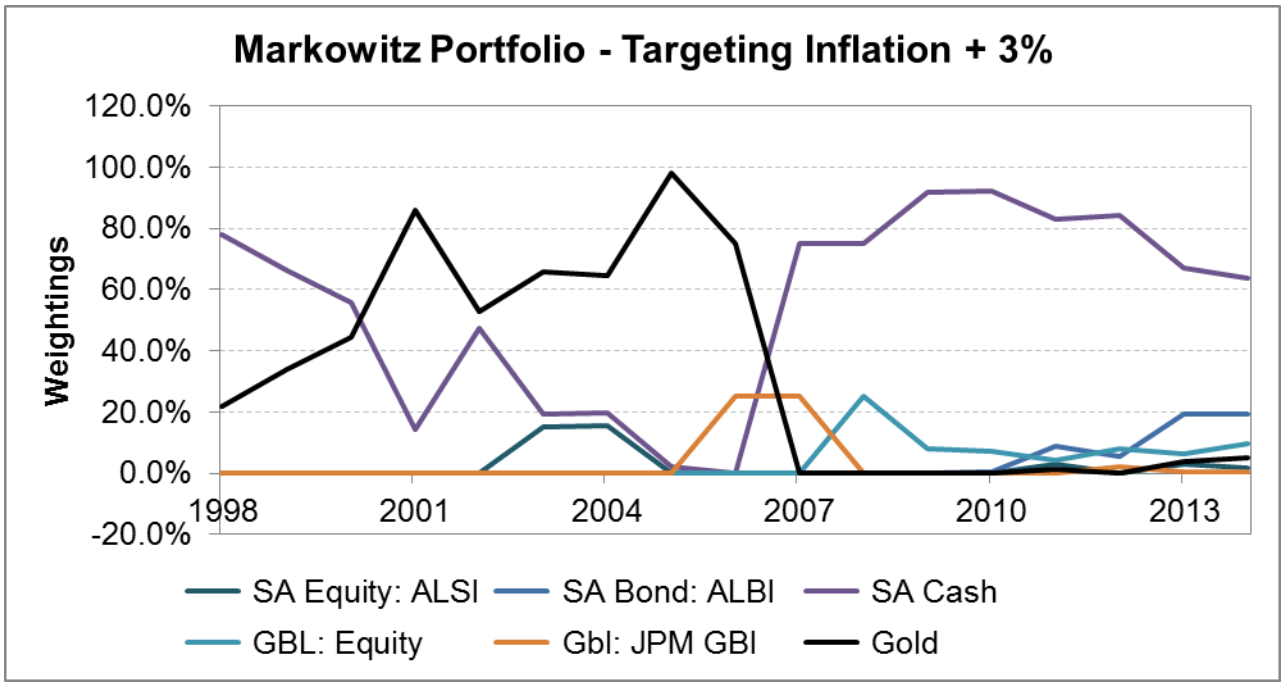
Using Markowitz portfolio theory, we are able to determine the optimal portfolio for each year, given the preceding 10 year data history. This is done for each year until a rolling portfolio is created from year 1998 to 2015, as presented in Figure 4, 6 and 8 below for each of the strategies.

Throughout the period SA cash has been the predominant investment allocation averaging 55.0%. Gold has the second highest allocation, which was mainly between year 2001 and 2006. Cash and gold both have a high standard deviation of allocation weighting. Equities and bonds average a low allocation throughout the period. The reason for gold's high allocation, albeit it's high volatility, is because during the period between year 2001 and 2006 all the other asset classes were performing well beyond or below the portfolio's target of inflation plus 3 percent. SA equities and bonds performed above 12% while global equities and bonds achieved negative returns. This meant the portfolio selected gold to achieve the target return while using cash to minimise risk.

Figure 4: Portfolio allocations of the Quants strategy 2 for the 10 year rolling period of December 1998 to October 2015

Markowitz Portfolio - Targeting Inflation + 3%							
	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold	Portfolio Total Weight
1998/12/31	0.0%	0.0%	78.0%	0.0%	0.0%	22.0%	100%
1999/12/31	0.0%	0.0%	66.2%	0.0%	0.0%	33.8%	100%
2000/12/31	0.0%	0.0%	55.7%	0.0%	0.0%	44.3%	100%
2001/12/31	0.0%	0.0%	14.1%	0.0%	0.0%	85.9%	100%
2002/12/31	0.0%	0.0%	47.3%	0.0%	0.0%	52.7%	100%
2003/12/31	15.2%	0.0%	19.2%	0.0%	0.0%	65.6%	100%
2004/12/31	15.6%	0.0%	19.8%	0.0%	0.0%	64.6%	100%
2005/12/31	0.0%	0.0%	2.1%	0.0%	0.0%	97.9%	100%
2006/12/31	0.0%	0.0%	0.0%	0.0%	25.0%	75.0%	100%
2007/12/31	0.0%	0.0%	75.0%	0.0%	25.0%	0.0%	100%
2008/12/31	0.0%	0.0%	75.0%	25.0%	0.0%	0.0%	100%
2009/12/31	0.0%	0.0%	91.8%	8.2%	0.0%	0.0%	100%
2010/12/31	0.0%	0.4%	92.3%	7.4%	0.0%	0.0%	100%
2011/12/31	2.8%	8.8%	82.8%	4.1%	0.0%	1.5%	100%
2012/12/31	0.0%	5.4%	84.1%	8.1%	2.3%	0.0%	100%
2013/12/31	2.8%	19.4%	66.9%	6.5%	0.5%	3.9%	100%
2014/12/31	1.8%	19.3%	63.8%	9.6%	0.5%	5.0%	100%
Average Allocation	2.2%	3.1%	55.0%	4.0%	3.1%	32.5%	
Average Allocation STDev	5.0%	6.6%	31.7%	6.5%	8.2%	34.9%	

Figure 5: Portfolio allocations of the Quants strategy 2 for the 10 year rolling period of December 1998 to October 2015

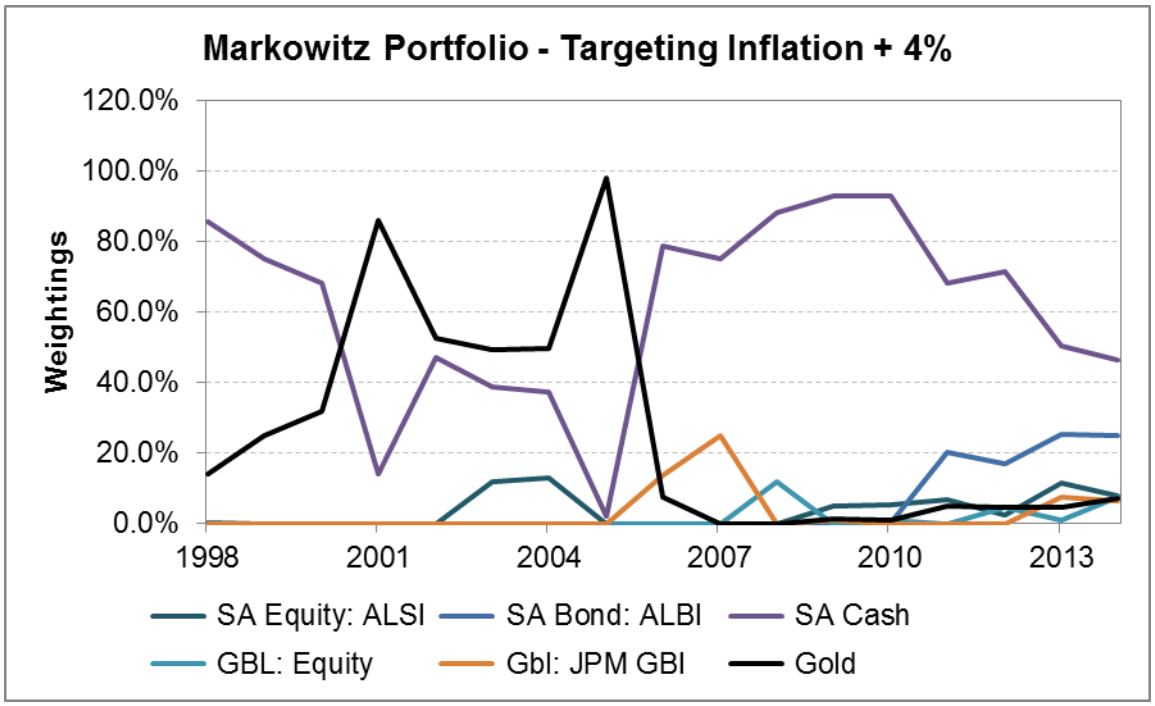


Throughout the period SA cash has been the predominant investment allocation averaging 60.7%, a higher cash allocation than portfolio strategy 2 albeit the higher target return. Gold has the second highest allocation, which was mainly between year 1998 and 2005. Cash and gold both have a high standard deviation of allocation weighting. The allocation to bonds has increased since 2011 till recent, while equities generally remained a low allocation. Allocation to bond has increased because during the most recent period all the other asset classes returns has deviated away from the portfolio's target of inflation plus 4 percent. This allocation has occurred even though SA equities and global equities had a higher sharpe ratio than bonds during 2011 until 2014 giving returns of 19.48 percent and 30.95 percent. The improvement in Sharpe ratios of equity assets more recently has led to an increase in equity allocation. This means the allocation in assets will gradually vary between years as data that is more recent is accounted in the analysis.

Figure 6: Portfolio allocations of the Quants strategy 3 for the 10 year rolling period of December 1998 to October 2015

Markowitz Portfolio - Targeting Inflation + 4%							
	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold	Portfolio Total Weight
1998/12/31	0.3%	0.0%	85.6%	0.0%	0.0%	14.2%	100%
1999/12/31	0.0%	0.0%	75.0%	0.0%	0.0%	25.0%	100%
2000/12/31	0.0%	0.0%	68.2%	0.0%	0.0%	31.8%	100%
2001/12/31	0.0%	0.0%	14.1%	0.0%	0.0%	85.9%	100%
2002/12/31	0.0%	0.0%	47.3%	0.0%	0.0%	52.7%	100%
2003/12/31	11.8%	0.0%	38.7%	0.0%	0.0%	49.5%	100%
2004/12/31	13.0%	0.0%	37.4%	0.0%	0.0%	49.5%	100%
2005/12/31	0.0%	0.0%	2.1%	0.0%	0.0%	97.9%	100%
2006/12/31	0.0%	0.0%	78.9%	0.0%	13.7%	7.4%	100%
2007/12/31	0.0%	0.0%	75.0%	0.0%	25.0%	0.0%	100%
2008/12/31	0.0%	0.0%	88.2%	11.8%	0.0%	0.0%	100%
2009/12/31	4.9%	0.0%	93.0%	0.0%	0.9%	1.2%	100%
2010/12/31	5.1%	0.3%	92.8%	1.0%	0.0%	0.8%	100%
2011/12/31	6.6%	20.1%	68.4%	0.0%	0.0%	4.9%	100%
2012/12/31	2.4%	17.1%	71.4%	4.6%	0.0%	4.6%	100%
2013/12/31	11.5%	25.3%	50.4%	0.8%	7.5%	4.5%	100%
2014/12/31	7.7%	25.1%	46.2%	7.4%	6.4%	7.3%	100%
Average Allocation	3.7%	5.2%	60.7%	1.5%	3.1%	25.7%	
Average Allocation STDev	4.8%	9.7%	26.9%	3.3%	6.8%	31.0%	

Figure 7: Portfolio allocations of the Quants strategy 3 for the 10 year rolling period of December 1998 to October 2015

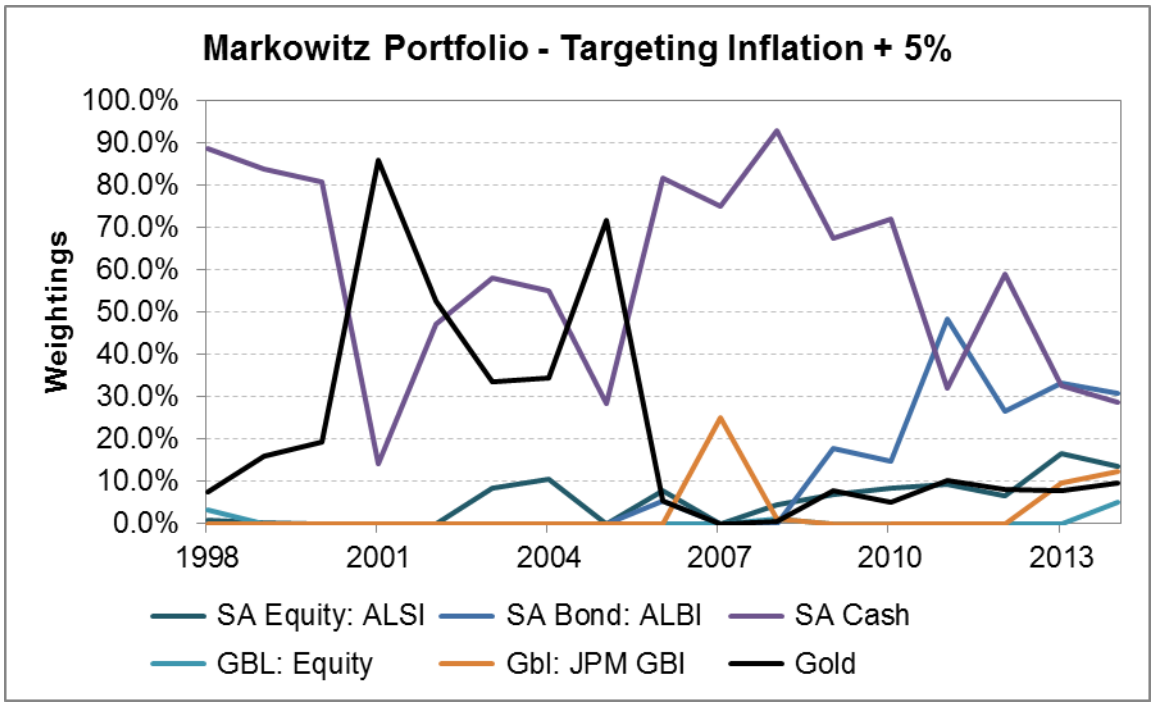


Similary with Quants strategy 4, as seen in Figure 8 and 9 below, SA cash has been the predominant investment allocation, a higher average cash allocation than portfolio strategy 2 albeit the higher target return. Gold has also had a high allocation, mainly in earlier years of the period, while the bond allocation has increased in recent years. Given the strategies higher required return, it is not a surprise to see a higher allocation in SA equities in comparison to the previous strategies. Averaging an SA equity allocation of 10.4 percent when targeting inflation plus 5 percent, versus 3.1 and 5.2 percent allocation in SA equity for portfolios targeting inflation plus 3 and 4 percent respectively.

Figure 8: Portfolio allocations of the Quants strategy 4 for the 10 year rolling period of December 1998 to October 2015

Markowitz Portfolio - Targeting Inflation + 5%							
	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold	Portfolio Total Weight
1998/12/31	0.9%	0.0%	88.5%	3.1%	0.0%	7.5%	100%
1999/12/31	0.1%	0.0%	83.8%	0.0%	0.0%	16.1%	100%
2000/12/31	0.0%	0.0%	80.7%	0.0%	0.0%	19.3%	100%
2001/12/31	0.0%	0.0%	14.1%	0.0%	0.0%	85.9%	100%
2002/12/31	0.0%	0.0%	47.3%	0.0%	0.0%	52.7%	100%
2003/12/31	8.4%	0.0%	58.2%	0.0%	0.0%	33.4%	100%
2004/12/31	10.5%	0.0%	55.1%	0.0%	0.0%	34.5%	100%
2005/12/31	0.0%	0.0%	28.4%	0.0%	0.0%	71.6%	100%
2006/12/31	7.7%	5.3%	81.7%	0.0%	0.0%	5.3%	100%
2007/12/31	0.0%	0.0%	75.0%	0.0%	25.0%	0.0%	100%
2008/12/31	4.4%	0.0%	92.9%	1.0%	1.2%	0.5%	100%
2009/12/31	7.0%	17.8%	67.4%	0.0%	0.0%	7.8%	100%
2010/12/31	8.3%	14.6%	72.1%	0.0%	0.0%	4.9%	100%
2011/12/31	9.2%	48.4%	32.1%	0.0%	0.0%	10.3%	100%
2012/12/31	6.6%	26.5%	58.9%	0.0%	0.0%	8.1%	100%
2013/12/31	16.6%	33.3%	32.6%	0.0%	9.7%	7.8%	100%
2014/12/31	13.5%	30.8%	28.6%	5.1%	12.3%	9.6%	100%
Average Allocation	5.5%	10.4%	58.7%	0.5%	2.8%	22.1%	
Average Allocation STDev	5.3%	15.5%	24.5%	1.4%	6.8%	25.5%	

Figure 9: Portfolio allocations of the Quants strategy 4 for the 10 year rolling period of December 1998 to October 2015



4.5 Research question 1

Research Question 1: What are the returns, risk and return to risk ratio of the optimal Markowitz portfolio allocations using specific targets?

4.5.1 *Back-tested annualised returns of quant portfolios*

The annualised returns in the table below depict the performance of the quantitative portfolios back-tested from 1998 until October 2015. Backtesting is used as a method to apply the historic portfolio weights of each portfolio and determine how the portfolio strategies would have predicted actual results. The results are presented in this section by comparing the results of the following strategies:

- Equally weighted portfolio (all assets are weighted equally),
- The generic portfolio (60:30:10 weighting in equity:bond:cash),
- Portfolio quants strategy 1 (minimise standard deviation)
- Portfolio quants strategy 2 (target inflation plus 3 percent)
- Portfolio quants strategy 3 (target inflation plus 4 percent)
- Portfolio quants strategy 4 (target inflation plus 5 percent)

As seen in Table 7 below, the equal weighted and generic portfolios generally outperformed throughout the period, due to their high allocation in equities, which is the asset class with the highest return performance. The portfolio with the lowest return performance is the quants strategy 1, minimising standard deviation portfolio, achieving an annualised return of 8.9 percent over 15 years. The weakest return period for portfolio strategy 1 has been between 2010 and 2015, which was a bull run period for SA and global equities. During the 2010 to 2015 period, the high performance of the generic portfolio of 14.5 percent (with the highest allocation to equities), performed favourabley versus the lowest performance of portfolio strategy 1 of 6.9 percent (holding the lowest weight in equity assets). The portfolio strategy 1 has the highest allocation to SA cash

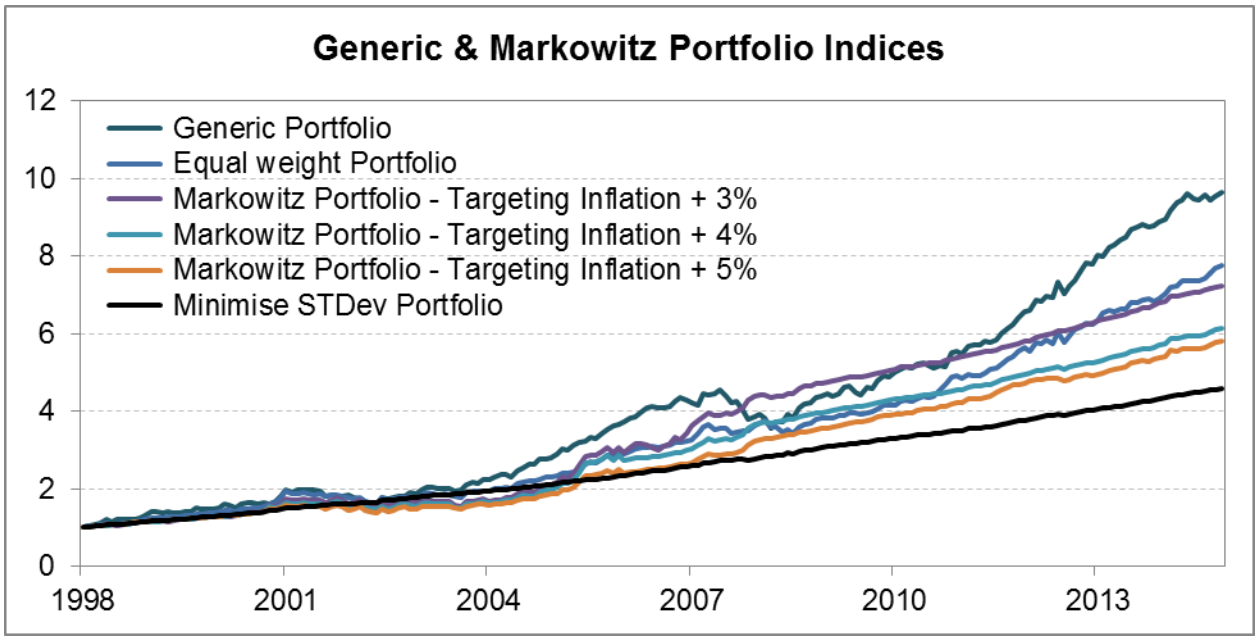
throughout the period, so retains a low volatility portfolio but with a lower return. The quants strategy portfolio 2, 3 and 4, which aim to target a return above inflation, performed well over the investment period. Majority of the higher returns were earned during the earlier years of the analysis term. The quants portfolios highest return period was from year 2000 to 2010, performing well even during the 2008 financial crisis.

Table 7: Annualised returns of generic portfolio, equal weight portfolio and the optimised quant portfolios over various periods.

Annualised Returns	Equal weight Portfolio	Generic Portfolio	Minimise STDev Portfolio	Markowitz Portfolio - Targeting Inflation + 3%	Markowitz Portfolio - Targeting Inflation + 4%	Markowitz Portfolio - Targeting Inflation + 5%
1 Years	13.4%	9.8%	6.8%	7.7%	8.2%	9.1%
3 Years	11.9%	14.6%	6.9%	8.0%	7.6%	7.5%
5 Years	13.3%	14.4%	6.9%	7.6%	7.6%	8.4%
10 Years	12.9%	13.3%	8.1%	13.3%	12.1%	12.2%
15 Years	12.2%	13.3%	8.9%	12.3%	11.1%	10.7%
1998 - 2000	18.4%	23.2%	13.9%	13.1%	13.3%	13.7%
2000 - 2010	11.5%	12.7%	9.8%	14.7%	12.9%	11.8%
2010 - 2015	13.8%	14.5%	6.9%	7.7%	7.6%	8.5%

As demonstrated in Figure 10 below, over the long term, the best performing quants portfolios based on just annualised return was the generic portfolio, the equal weighted portfolio, the inflation plus 3 percent, the inflation plus 4 percent, the inflation plus 5 percent and lastly the minimising standard deviation portfolio. The surprising result is that the lower inflation targeting quants portfolio (inflation plus 3) outperformed the higher inflation targeting portfolios. Intuitively we would expect the higher inflation-targeting portfolio to take up more risk, in equities, and therefore outperform over the long term, but the portfolio underperformed during the 10 year, 15 year and since inception. A big factor in the inflation plus 3 percent portfolio's outperformance over the other inflation targeting portfolios was dbefore the 2008 financial crisis. In between 2004 and 2007, the portfolio had the highest allocation in gold on a rolling basis, a time when gold returned 32.13 percent annualised return.

Figure 10: Back-tested indices of the optimised quant portfolios over period of December 1998 to October 2015



4.5.2 *Back-tested risk and return-risk ratio of the quant portfolios*

The performance used to present the results so far has been mainly on an annualised return basis, but suppose an investor has a view to not only invest in a portfolio based on the highest returns, but based on the portfolio that will provide the best return given the lowest possible risk. The investor will have to pay more attention to measuring the Sharpe ratio or a risk-adjusted return. The table below looks at the standard deviation and Sharpe ratio of the quants portfolios over the 3-year (short term) and 15 year (long term). Over the short term the targeting inflation plus 3 percent portfolio has had the least standard deviation, this resulted in the portfolio achieving the highest Sharpe ratio. The least performing portfolio has been the targeting-inflation plus 5 percent portfolio, which had a high standard deviation given its low 7.5% three year annualised return.

Over the long term the targeting-inflation plus 5 percent portfolio does a lot better, being second least volatile portfolio at a standard deviation of 8.2%. The quants strategy 1 portfolio, that purely aims to minimise standard deviation, has by far the least volatility over the 15 year period. The standard deviation was 2.2%, a large variance in comparison to the second least standard deviation portfolio at 8.2%. This allowed the portfolio to outperform all other portfolios when purely looking at the Sharpe ratio. Over the long term the equally weighted and generic portfolios outperformed the targeting inflation plus 3 and 4 percent portfolios on a risk adjusted basis.

Table 8: Annualised returns and Standard deviation of generic portfolio and optimized quant portfolios over the short and long term.

Annualised Returns	Equal weight Portfolio	Generic Portfolio	Minimise STDev Portfolio	Markowitz Portfolio - Targeting Inflation + 3%	Markowitz Portfolio - Targeting Inflation + 4%	Markowitz Portfolio - Targeting Inflation + 5%
3 Years Annualised Returns	11.9%	14.6%	6.9%	8.0%	7.6%	7.5%
3 Year Std Dev.	2.7%	4.8%	0.7%	0.5%	1.2%	2.6%
Annual Management Fee	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%
3 Years Annualised Returns after fees	9.4%	12.1%	4.6%	5.6%	5.3%	5.2%
Return/Risk Ratio	**3.44**	**2.50**	**6.34**	**10.55**	**4.30**	**1.99**
15 Years Annualised Returns	12.2%	13.3%	8.9%	12.3%	11.1%	10.7%
15 Year Std Dev.	8.4%	11.6%	2.2%	12.4%	10.7%	8.2%
Annual Management Fee	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%

15 Years Annualised Returns after fees	11.7%	12.8%	8.4%	11.8%	10.6%	10.2%
Return/Risk Ratio	**1.40**	**1.10**	**3.85**	**0.95**	**0.99**	**1.25**

Having an equally weighted or generic portfolio is proving to be an adequate strategy when investing over the long term, but may face higher volatility over the short term. The minimising standard deviation portfolio is great when attempting to purely reduce the investment risk, as it has one of the highest Sharpe ratios over the long and short term. But, investing using this strategy will result in significant lower returns relative to the other quant strategies in this research. The inflation targeting plus 3 percent portfolio outperformed over the short term, but was the least performing on a risk-adjusted basis over the long term. The portfolio retained higher returns than the other inflation targeting portfolios, but its high volatility reduced the risk-adjusted return. This tells us, if using the method and process chosen in this research paper, whether an investor targets inflation 3 percent or 5 percent, there is no proof that the one with the higher target return will outperform on a return basis and underperform on a risk measure.

4.6 Research question 2

Research Question 2: Does the optimal Markowitz portfolios outperform the South African balanced fund managers returns over the past 15 years?

This section of the research will determine the performance of the optimised quants portfolios over the performance of the South African active balanced fund returns. The SA balanced fund returns are calculated as an average of performing balanced fund over the same period. Inet BFA provides a list of funds and their historic performance, of which an average of the performance of these funds is used to create a comparable performance index.

The figure below is a representation of five quants/passive portfolios relative to the average SA balanced fund over the past 20 years. The first portfolio is the Generic quants portfolio using an allocation of SA equities (45 percent), SA bonds (20 percent), SA cash (7.5 percent), global equities (20 percent), global bonds (5 percent) and gold (2.5 percent), this portfolio allocation is just a proxy used by the MacroSolutions balanced fund index (Brooke, 2014). The second portfolio is an equal weighted portfolio, which equally weights all the assets without a decision or calculation to allocated or diversify against set principles or theory. The next four portfolios are generated by creating Markowitz strategies presented above, the first is referenced on the most optimal portfolio based on the highest Sharpe ratio strategy. The rest are created through targeting inflation plus 3, 4 and 5 percent to determine the optimal portfolio over the period.

Figure 11: Generic, equal weighted and Markowitz optimal portfolio indices relative to the average South African balanced funds over the period of December 1998 to October 2015

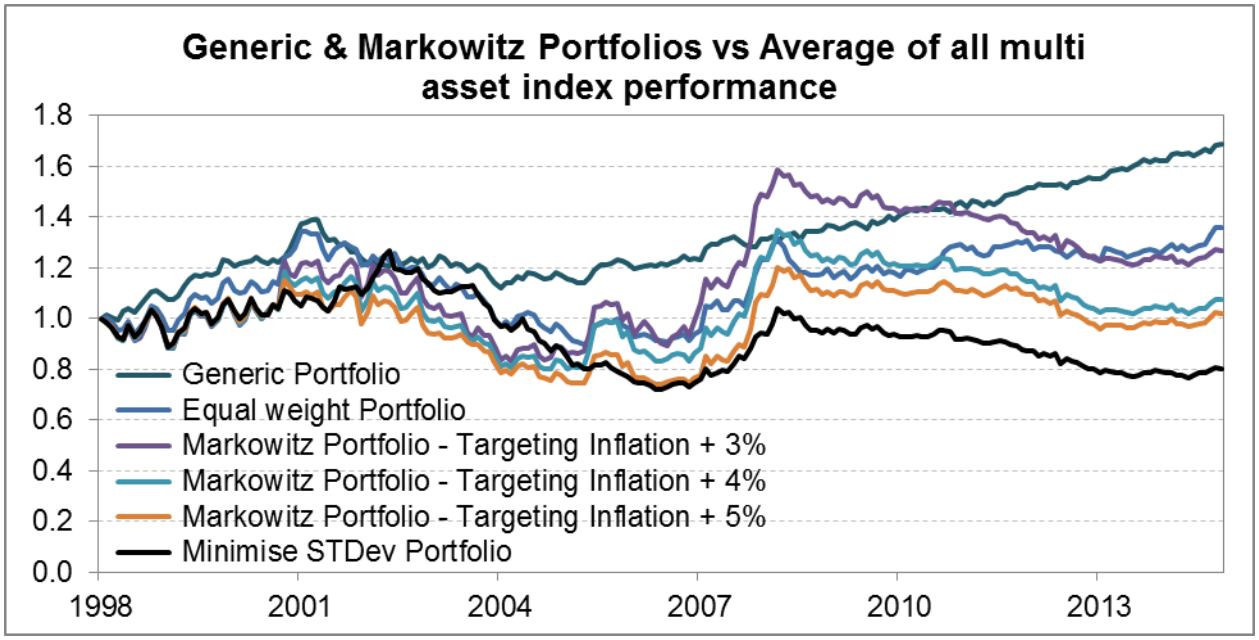


Figure 11 above is explained by noting that equally weighted, generic and all the Markowitz optimised portfolio strategies are measured relative to the average SA balanced fund determined from Inet BFA. The chart explains relative performance from

purely an annualised return basis throughout the period. If the lines in the chart are above 1 then the index has outperformed the average SA balanced funds, and if the lines are below 1 then the index has underperformed. The figure clearly shows how each of the portfolios performed against the SA balanced fund over the period. The figure reveals that all the Markowitz optimised quants portfolios strategies have outperformed the average SA balanced fund since inception (December 1998), except for the optimised portfolio that aims to minimise standard deviation. The best performing has been the generic portfolio and the equally weighted portfolio, which is expected as they have a higher allocation to the riskier SA equities and global equities. The least performing was the Markowitz optimal portfolio that aims to minimise risk. The generic portfolio varies the least relative to the SA balanced fund, even though it clearly outperforms, showing how the two portfolios have the most similar risk profiles, with the highest covariance. The 2008 financial crisis had a negative impact on the performance of the SA balanced funds, clearly showing how lower allocation equities contributed to optimised inflation targeting portfolios outperformance during that period. This highlights the risk of a high allocation in riskier assets that can result in dramatic losses over certain market events. Overall, the optimised portfolios do have periods of underperformance during year 2001 to 2006, and 2009 to 2014, but over the long term, the inflation-targeting optimised portfolios have soundly outperformed the average SA balanced fund.

Table 9 below looks at the annualised returns and standard deviation of the optimised quant portfolios calculated using Markowitz theory, and looks at the average SA balanced fund over various periods. It also shows the outperformance/underperformance of the optimised portfolios versus average SA balanced funds. Over the short term, the optimal Markowitz portfolios outperformed over the past 1 year return, but underperformed when looking at a 3-year return and 5-year return. Over the past year SA equities had a difficult period, achieving only 5.54 percent return, which explains the lower performance of SA balanced fund managers who generally have a higher allocation to SA equities. While, over the long term, the optimal Markowitz portfolios all outperformed during the 15-year period, but the optimal Markowitz minimising risk portfolio underperformed during the 5, 10 and 15 year period. This means from a purely

return basis, just investing in the portfolio with the least risk will constantly result in inferior return performance. The optimal portfolios that target inflation plus 3 and 4 percent, performed well during the 10-15 year, losing most of the relative outperformance during the equity bull years of 2009 to 2014.

On a Sharpe ratio, the optimised quants portfolios have been able outperform the fund managers over the 3 year short term and 15 year long term, as a result of the lower standard deviation/risk. The portfolios with the highest Sharpe ratio over the 15-year term is the minimising standard deviation portfolio, targeting inflation plus 5 percent, targeting inflation plus 4 percent and the targeting inflation plus 3 percent, each outperforming the SA balanced fund managers' Sharpe ratio with 3.85, 1.25, 0.99 and 0.95 respectively.

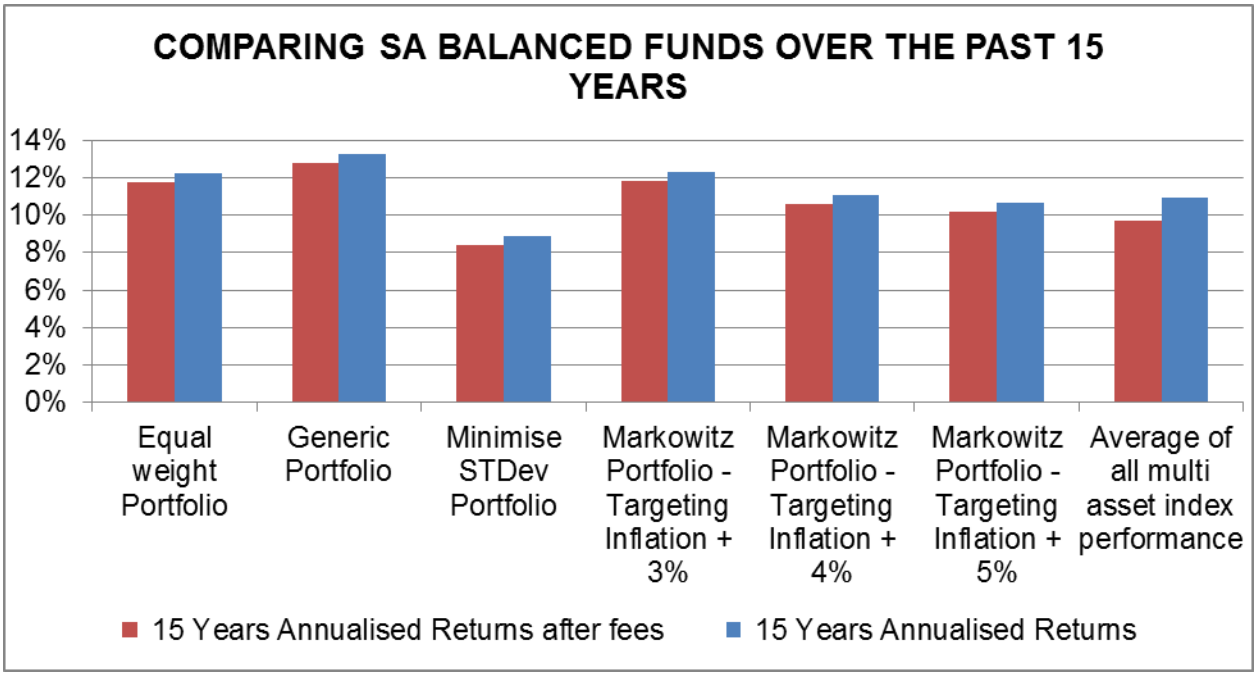
Table 9: Annualised returns and Standard deviation of optimised quant portfolios and average SA balanced fund over various periods. Showing outperformance/underperformance of optimised quant portfolios versus average SA balanced funds.

Annualised Returns	Minimise STDev Portfolio	Markowitz Portfolio - Targeting Inflation + 3%	Markowitz Portfolio - Targeting Inflation + 4%	Markowitz Portfolio - Targeting Inflation + 5%	Average of all multi asset index performance	Minimise STDev Portfolio vs Average of all multi asset index performance	Markowitz Portfolio - Targeting Inflation + 3% vs Average of all multi asset index performance	Markowitz Portfolio - Targeting Inflation + 4% vs Average of all multi asset index performance	Markowitz Portfolio - Targeting Inflation + 5% vs Average of all multi asset index performance
1 Years	6.8%	7.7%	8.2%	9.1%	5.8%	Outperformed	Outperformed	Outperformed	Outperformed
3 Years	6.9%	8.0%	7.6%	7.5%	10.2%	Underperformed	Underperformed	Underperformed	Underperformed
5 Years	6.9%	7.6%	7.6%	8.4%	10.2%	Underperformed	Underperformed	Underperformed	Underperformed
10 Years	8.1%	13.3%	12.1%	12.2%	9.3%	Underperformed	Outperformed	Outperformed	Outperformed
15 Years	8.9%	12.3%	11.1%	10.7%	10.9%	Underperformed	Outperformed	Outperformed	Underperformed
1998 - 2000	13.9%	13.1%	13.3%	13.7%	12.3%	Outperformed	Outperformed	Outperformed	Outperformed
2000 - 2010	9.8%	14.7%	12.9%	11.8%	11.0%	Underperformed	Outperformed	Outperformed	Outperformed
2010 - 2015	6.9%	7.7%	7.6%	8.5%	10.2%	Underperformed	Underperformed	Underperformed	Underperformed
3 Years Annualised Returns	6.9%	8.0%	7.6%	7.5%	10.2%	Underperformed	Underperformed	Underperformed	Underperformed
3 Year Std Dev.	0.7%	0.5%	1.2%	2.6%	4.5%	Outperformed	Outperformed	Outperformed	Outperformed
Annual Management	0.4%	0.4%	0.4%	0.4%	1.1%				

Fee									
3 Years Annualised Returns after fees	4.6%	5.6%	5.3%	5.2%	9.0%	Underperformed	Underperformed	Underperformed	Underperformed
Return/Risk Ratio	**6.34**	**10.55**	**4.30**	**1.99**	**2.00**	Outperformed	Outperformed	Outperformed	Outperformed
15 Years Annualised Returns	8.9%	12.3%	11.1%	10.7%	10.9%	Underperformed	Outperformed	Outperformed	Underperformed
15 Year Std Dev.	2.2%	12.4%	10.7%	8.2%	11.0%	Outperformed	Underperformed	Outperformed	Outperformed
Annual Management Fee	0.4%	0.4%	0.4%	0.4%	1.1%				
15 Years Annualised Returns after fees	8.4%	11.8%	10.6%	10.2%	9.7%	Underperformed	Outperformed	Outperformed	Outperformed
Return/Risk Ratio	**3.85**	**0.95**	**0.99**	**1.25**	**0.89**	Outperformed	Outperformed	Outperformed	Outperformed

Figure 12 summarises the returns and risk of each of the funds. The research presents the Markowitz optimised quant portfolios as being desirable in minimising the risk of a portfolio, while maintaining more than adequate levels of return relative to the performance of the average SA balanced fund. The performance of the inflation targeting funds clearly indicate how well they have performed in the long term relative SA balanced fund managers.

Figure 12: Annualised return and Standard deviation of generic portfolio, Markowitz optimal portfolios and average SA balanced fund between October 1995 to October 2015



4.5 SUMMARY OF RESULTS AND DISCUSSIONS

The research methodology described in chapter 3 was used to gather and analyse the data and the results were presented in this chapter. From the final sample, the results of two questions to the research were presented. From analysing the generic portfolio, the equally weighted portfolio, the optimal Markowitz minimising standard deviation portfolio and the three targeting inflation portfolios, the research firstly answered what are the returns, risk and return to risk ratio of the optimal Markowitz portfolio allocations using

specific targets? Then secondly answered, does the optimal Markowitz portfolio outperform the South African balanced fund manager's returns over the past 15 years? The research created the portfolios based on a 10 year rolling data set starting from December 1998. Then measured, analysed and back-tested the return results of six quants portfolios starting from a return date of December 1998 until October 2015. These were compared to the average South African balanced fund over a 15-year investment period.

Across the optimal quants portfolios created using Markowitz portfolio theory SA cash has been the predominant investment allocation averaging 83.7%, followed by gold mainly receiving higher allocations between 1998 and 2006. Cash and gold both have a high standard deviation of allocation weighting. Equities and bonds average a low allocation throughout the period. a combination of bonds has been the second highest allocation, followed by a combination of equities. The allocation to bonds has increased since 2011 till recent, while equities generally remained a low allocation. Given the high allocation of cash in the portfolio, the research found expected a low volatility portfolio. The higher the required return in the portfolio strategy the higher the allocation in SA equities in comparison to the previous strategies.

On just a return measure, the best performing portfolio has been the generic portfolio and the equally weighted portfolio, which is expected as they have a higher allocation to the riskier SA equities and global equities. The least performing was the Markowitz optimal portfolio that aims to minimise standard deviation over the period. Overall the optimised portfolios do have periods of underperformance during year 2001 to 2006, and 2009 to 2014, but over the long term the optimised portfolios have soundly outperformed the average SA balanced fund.

On a Sharpe ratio, the optimised quants portfolios have been able outperform the SA active fund managers over the 3 year short term and 15 year long term, as a result of the lower standard deviation/risk. The portfolios with the highest Sharpe ratio over the 15-year term is the minimising standard deviation portfolio, targeting inflation plus 5 percent, targeting inflation plus 4 percent and the targeting inflation plus 3 percent, each

outperforming the SA balanced fund managers' Sharpe ratio with 3.85, 1.25, 0.99 and 0.95 respectively.

CHAPTER 6 CONCLUSION

6.1 SUMMARY OF THE STUDY

Markowitz (1959) considers two main objectives for investors. Firstly, investors want their returns to be as high as possible, and secondly investors want the returns to be consistent, dependable and as expected, therefore uncertainty is a risk they require to be reduced. The optimal portfolio for an investor lies in the relationship between expected return and risk. The trade-off between return and risk is different for each investor, suggesting investors with a greater risk appetite can expect higher returns and conversely for investors who can't bear a lot of risk. To minimise risk, investment managers use diversification of their portfolios.

This research attempts to initially determine the performance of investment classes a typical portfolio would have been to hold over the past decades. Performance will be measured using relatively standard investment performance measures, (Antolin, 2008). By applying Markowitz (1952) portfolio selection model, the optimal portfolio selection can be determined. The research then uses the asset allocation ratios, how determine how the portfolio selection would have performed since 1995 on a risk-adjusted basis. The optimal allocation mix is encouraged to be used as a benchmark or an investment index tracker fund for the financial industry. The purpose of this research is to determine what has been the optimal asset allocation in South Africa over a long-term investment period, based on past risk-adjusted performance, using Markowitz portfolio selection model.

The data sampled from the asset returns were from 31/12/1987 until 31/10/2015. This provides a long term analysis period, which South African funds could for the majority of the period invest in offshore assets, making the results a fair representation of practical possible asset allocations. The final asset classes chosen to be included in the analysis are South African equities, South African bonds, South African cash, global equities, global bonds and gold. These are indices sourced from the Johannesburg Stock

Exchange, MSCI and JP Morgan through the software portal Inet BFA. This data sample is representative of a typical South African balanced fund asset allocation, as noted by Brooke (2014).

There are four main strategies/variations to be used in this research to determine the Markowitz Portfolio selection efficient frontier and portfolios:

- Quants portfolio from strategy 1: The first strategy attempts to minimise the risk of the portfolio given a specific target return.
- Quants portfolios from strategy 2: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 3 percent.
- Quants portfolios from strategy 3: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 4 percent.
- Quants portfolios from strategy 4: The second strategy attempts to create an optimal portfolio through minimising risk while targeting a return of inflation plus 5 percent.

Applying the Markowitz target return and minimize risk strategy, we present the results of the portfolios in the efficient frontier. Suppose an investor has other preferences than taking the least possible amount of risk (thus investing in minimum variance portfolio), but rather investing with the highest reward for risk taken. In this case, the portfolio with maximum Sharpe ratio will be the most optimal portfolio for a low risk investor. Quants portfolio strategy 1, which chooses the optimal portfolio over the period is one such portfolio. Throughout the period the portfolio had a SA cash predominantly as an investment with an allocation averaging 83.7%, a combination of bonds has been the second highest allocation, followed by equities.

Cash and gold both have a high standard deviation of allocation weighting. Equities and bonds average a low allocation throughout the period. a combination of bonds has been the second highest allocation, followed by a combination of equities. The allocation to

bonds has increased since 2011 till recent, while equities generally remained a low allocation. Given the high allocation of cash in the portfolio, the research found expected a low volatility portfolio. The higher the required return in the portfolio strategy the higher the allocation in SA equities in comparison to the previous strategies.

On just a return measure, the best performing portfolio has been the generic portfolio and the equally weighted portfolio, which is expected as they have a higher allocation to the riskier SA equities and global equities. The least performing was the Markowitz optimal portfolio that aims to minimise standard deviation over the period. Overall the optimised portfolios do have periods of underperformance during year 2001 to 2006, and 2009 to 2014, but over the long term the optimised portfolios have soundly outperformed the average SA balanced fund. The lower inflation targeting quants portfolio outperformed the higher inflation targeting portfolios. Intuitively we would expect the higher inflation-targeting portfolio to take up more risk, in equities, and therefore outperform over the long term, but the portfolio underperformed during the 10 year, 15 year and since inception. All the inflation-targeting portfolios still managed to outperform the SA balanced fund managers.

On a Sharpe ratio, the optimised quants portfolios have been able outperform the SA active fund managers over the 3 year short term and 15 year long term, as a result of the lower standard deviation/risk. The portfolios with the highest Sharpe ratio over the 15-year term is the minimising standard deviation portfolio, targeting inflation plus 5 percent, targeting inflation plus 4 percent and the targeting inflation plus 3 percent, each outperforming the SA balanced fund managers' Sharpe ratio with 3.85, 1.25, 0.99 and 0.95 respectively.

6.2 RECOMMENDATIONS

The simulations and optimisations were done using 15 years of asset class data show that a passive asset allocation investment strategy, based on Markowitz's optimal portfolio theory, outperformed the average South African balanced fund, but underperformed the risk parity and equal weighted portfolios. However, the Markowitz

strategy does not outperform consistently over multiple periods, as indicated by the short to medium term underperformance to the average South African balanced fund. The results of the optimal portfolios shows that the Markowitz optimal portfolio can be used as an investment strategy and as a benchmark to measure against active asset allocation performance.

When comparing the Markowitz optimal portfolio's to the asset allocation heuristic portfolios performance, we find the asset allocation heuristic portfolio in terms of absolute returns outperforms due to the level of risk undertaken. However, if we target the same risk as the asset allocation heuristic portfolio to create a Markowitz portfolio, we find the Markowitz portfolio outperforms the risk parity in absolute return, risk and risk adjusted return. Signifying, the Markowitz portfolio simulations can be used to compare how asset managers have performed based on their returns, risk and risk adjusted returns achieved.

From this empirical analysis, one can make use of the portfolio allocations to determine future expected returns given a targeted return, risk and risk-adjusted return. In addition, it is seen that based on this study if any investors construct their portfolio based on Markowitz model, it may offer more than the active manager return. Therefore, it may be concluded that Markowitz Model performs well in the South African asset allocation environment.

6.3 SUGGESTIONS FOR FURTHER RESEARCH

Further research using a longer period and more assets would enhance the quality of this study, given the availability of long-term data. Furthermore, the inclusion of more onerous constraints that are reflective of an asset manager's mandate from a client, might improve the practicability of the outcome. Nevertheless, these findings present strong evidence that the Markowitz optimal portfolio does provide the basis of a useful passive investment strategy and benchmark.

This study has successfully constructed an optimal portfolio applying Markowitz portfolio selection theory. Further research can focus on developing a portfolio that looks primarily on the riskier assets, therefore excluding cash from the allocation weightings. Cash can be used as the main an asset to minimise risk, but how would the portfolio look if cash was used as risk free rate. This would mean the optimal portfolio designed would have to allocate to only riskier assets, which would make the results interesting to view.

REFERENCES

Antolin, P. (2008). *Pension fund performance*. Available at SSRN 1368816.

Bartholomew-Biggs, M. C., & Kane, S. J. (2009). A global optimization problem in portfolio selection. *Computational Management Science*, *6*(3), 329-345.

Brooke, P. (2014). *Long term Perspectives*. Old Mutual Investment Group: MacroSolutions, Cape Town

Chaves, D. B., Hsu, J. C., Li, F., & Shakernia, O. (2011). Risk parity portfolio vs. other asset allocation heuristic portfolios. *Journal of Investing*,*20*(1), 108-118.

Chopra, V. K. (1993). Improving optimization. *The Journal of Investing*, *2*(3), 51-59.

Darko, S. (2012). *Constructing optimal stock portfolio with Markowitz Model* (Doctoral dissertation, Department of Mathematics (Institute of Distance Learning), Kwame Nkrumah University of Science and Technology).

Du Plessis, A. J., & Ward, M. (2009). *A note on applying the Markowitz portfolio selection model as a passive investment strategy on the JSE*. Investment Analysts Journal, (69), 39-45.

Feibel, B. J. (2003). *Investment performance measurement* (Vol. 116). John Wiley & Sons.

Golafshani, N. (2003). Understanding reliability and validity in qualitative research. *The qualitative report*, *8*(4), 597-606.

Leape, J., & Thomas, L. (2011). *Prudential regulation of foreign exposure for South African institutional investors*. Centre for Research into Economics and Finance in Southern Africa, London School of Economics and Political Science.

Levy, H., & Markowitz, H. M. (1979). Approximating expected utility by a function of mean and variance. *The American Economic Review*, 308-317.

Maharakkhaka, B. (2011). *The Performance of mean variance portfolio selection and its opportunity cost: The case of Thai Securities*. 2011 International Conference on Economics and Finance Research. IPEDR vol.4 (2011) © (2011) IACSIT Press, Singapore

Markowitz, H. (1952). Portfolio selection*. *The journal of finance*, *7*(1), 77-91.

Markowitz, H. (1959). Portfolio selection: efficient diversification of investments. *Cowies Foundation Monograph*, (16).

McLeod, W. (1998). Enhancements to the Markowitz mean-variance optimisation process of asset allocation.

Rubinstein, M. (2002). Markowitz's" portfolio selection": A fifty-year retrospective. *Journal of Finance*, 1041-1045.

Sarker, M. R. Markowitz Portfolio Model: Evidence from Dhaka Stock Exchange in Bangladesh.

Savage, L. (1954). The Foundations of Statistics Wiley & Sons. *New York*.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk*. *The journal of finance*, *19*(3), 425-442.

Sharpe, W. F. (1992). Asset allocation: Management style and performance measurement. *The Journal of Portfolio Management*, *18*(2), 7-19.

Statssa (2013). *The South African CPI Sources and Methods Manual Release v .2*

Varian, H. (1993). A portfolio of Nobel laureates: Markowitz, Miller and Sharpe. *The Journal of Economic Perspectives*, 159-169.

APPENDIX A

Annual returns of asset classes

Date	SA Equity: ALSI	SA Bond: ALBI	SA Cash	GBL: Equity	Gbl: JPM GBI	Gold
1988/12/31	14.8%	8.3%	13.4%	52.9%	29.5%	3.4%
1989/01/31	44.4%	14.9%	14.1%	51.3%	24.8%	16.8%
1989/02/28	57.8%	16.6%	14.7%	40.0%	21.7%	1.8%
1989/03/31	57.9%	13.5%	15.2%	36.9%	21.4%	0.9%
1989/04/30	69.1%	12.8%	15.8%	36.6%	22.3%	0.0%
1989/05/31	49.5%	11.2%	16.3%	42.0%	27.8%	-3.4%
1989/06/30	56.4%	9.9%	16.7%	34.9%	26.5%	-1.3%
1989/07/31	54.0%	11.9%	17.2%	33.9%	20.5%	-6.6%
1989/08/31	68.4%	12.8%	17.6%	44.2%	22.9%	-5.8%
1989/09/30	55.8%	14.9%	18.1%	36.5%	16.9%	-3.4%
1989/10/31	42.6%	15.5%	18.4%	20.6%	11.0%	-5.8%
1989/11/30	51.7%	18.3%	18.8%	29.9%	19.0%	-0.8%
1989/12/31	55.5%	21.8%	19.1%	25.6%	14.4%	0.0%
1990/01/31	53.8%	21.8%	19.4%	14.9%	13.2%	0.8%
1990/02/28	40.2%	21.6%	19.7%	6.6%	8.1%	8.9%
1990/03/31	33.5%	23.4%	19.9%	1.6%	9.2%	7.1%
1990/04/30	21.3%	22.9%	20.2%	-1.7%	7.4%	5.4%
1990/05/31	37.9%	23.6%	20.4%	2.1%	2.8%	-1.8%
1990/06/30	21.4%	20.6%	20.6%	3.1%	2.8%	-7.7%
1990/07/31	22.2%	21.5%	20.8%	-4.6%	3.9%	-6.2%
1990/08/31	11.5%	20.2%	20.9%	-15.8%	1.0%	-5.3%
1990/09/30	2.8%	18.6%	21.0%	-25.3%	2.2%	-5.2%
1990/10/31	2.6%	18.4%	21.0%	-13.8%	7.1%	-8.8%

1990/11/30	-5.6%	18.2%	21.0%	-18.4%	8.1%	-11.9%
1990/12/31	-5.1%	16.2%	21.0%	-16.3%	12.0%	-11.7%
1991/01/31	-16.9%	16.7%	20.9%	-9.9%	15.1%	-15.1%
1991/02/28	-5.6%	18.8%	20.7%	4.0%	17.9%	-20.5%
1991/03/31	-8.2%	17.7%	20.6%	10.3%	17.9%	-19.0%
1991/04/30	4.0%	19.4%	20.4%	14.5%	21.6%	-14.4%
1991/05/31	1.6%	17.8%	20.2%	7.2%	19.4%	-6.3%
1991/06/30	11.7%	17.8%	20.0%	3.9%	18.7%	-0.9%
1991/07/31	15.1%	15.5%	19.7%	9.8%	19.9%	0.9%
1991/08/31	16.2%	14.4%	19.5%	22.0%	24.5%	-2.8%
1991/09/30	24.8%	14.2%	19.3%	36.9%	25.1%	-6.4%
1991/10/31	37.2%	15.8%	19.2%	29.6%	23.8%	1.0%
1991/11/30	41.2%	16.2%	19.1%	25.7%	23.4%	3.8%
1991/12/31	31.0%	14.4%	18.9%	27.4%	23.7%	1.9%
1992/01/31	46.0%	13.4%	18.8%	24.2%	22.1%	5.0%
1992/02/29	32.7%	12.2%	18.6%	12.3%	22.2%	9.3%
1992/03/31	27.5%	16.0%	18.4%	4.8%	18.8%	6.1%
1992/04/30	17.7%	16.6%	18.3%	3.3%	15.9%	0.0%
1992/05/31	23.7%	19.7%	18.1%	2.6%	16.4%	-4.8%
1992/06/30	14.2%	23.4%	17.9%	0.6%	15.4%	-6.5%
1992/07/31	1.5%	29.9%	17.7%	-3.1%	16.1%	-4.7%
1992/08/31	-2.8%	34.7%	17.3%	-1.8%	15.3%	-4.8%
1992/09/30	0.6%	38.6%	17.0%	-0.2%	16.9%	0.0%
1992/10/31	-11.6%	33.9%	16.5%	-0.4%	17.7%	-1.0%
1992/11/30	-6.8%	29.8%	16.1%	9.4%	17.4%	-4.6%
1992/12/31	-2.0%	27.8%	15.7%	6.1%	16.4%	-3.7%
1993/01/31	-1.5%	29.0%	15.2%	6.7%	18.7%	-1.9%
1993/02/28	-1.7%	29.8%	14.8%	12.4%	22.3%	1.9%
1993/03/31	3.8%	24.5%	14.4%	24.4%	25.0%	5.8%
1993/04/30	11.8%	21.4%	14.0%	27.8%	25.6%	15.8%
1993/05/31	10.7%	20.6%	13.7%	28.5%	25.5%	26.3%
1993/06/30	15.4%	20.7%	13.4%	40.9%	30.9%	30.0%

1993/07/31	25.9%	18.6%	13.2%	46.0%	30.4%	37.3%
1993/08/31	32.3%	16.9%	13.0%	50.6%	32.1%	34.3%
1993/09/30	21.3%	17.1%	12.9%	47.7%	32.3%	39.2%
1993/10/31	34.0%	21.4%	12.8%	44.6%	25.7%	28.2%
1993/11/30	34.6%	27.2%	12.7%	31.8%	25.0%	31.1%
1993/12/31	54.8%	32.0%	12.5%	37.1%	25.0%	34.6%
1994/01/31	42.6%	26.1%	12.4%	46.0%	24.3%	35.6%
1994/02/28	45.9%	24.6%	12.2%	39.2%	19.7%	43.5%
1994/03/31	42.7%	23.9%	12.0%	25.0%	16.6%	52.7%
1994/04/30	47.5%	29.6%	11.9%	25.3%	16.3%	31.6%
1994/05/31	38.8%	21.2%	11.7%	26.0%	18.4%	20.0%
1994/06/30	36.0%	13.4%	11.6%	21.6%	14.2%	15.4%
1994/07/31	38.8%	8.3%	11.5%	20.2%	14.0%	15.7%
1994/08/31	48.3%	0.2%	11.4%	15.6%	7.9%	16.5%
1994/09/30	54.3%	-3.5%	11.3%	12.3%	5.0%	12.7%
1994/10/31	49.7%	-3.5%	11.3%	13.0%	7.2%	13.6%
1994/11/30	41.5%	-5.9%	11.3%	15.8%	7.7%	8.1%
1994/12/31	22.7%	-9.0%	11.4%	10.2%	5.7%	0.0%
1995/01/31	8.7%	-5.7%	11.6%	0.8%	5.7%	0.7%
1995/02/28	8.7%	-4.0%	11.8%	3.9%	10.0%	-7.7%
1995/03/31	9.4%	-1.1%	12.1%	13.4%	15.7%	-16.1%
1995/04/30	4.6%	-5.0%	12.4%	13.3%	17.1%	-7.1%
1995/05/31	3.8%	0.7%	12.7%	12.3%	19.0%	-3.3%
1995/06/30	2.7%	6.8%	13.0%	10.7%	16.9%	-4.0%
1995/07/31	-1.5%	10.1%	13.3%	13.0%	15.2%	-13.0%
1995/08/31	-2.7%	21.0%	13.5%	10.9%	16.1%	-8.4%
1995/09/30	2.1%	25.8%	13.8%	17.6%	18.6%	-10.0%
1995/10/31	3.6%	27.0%	14.1%	14.6%	20.1%	-5.3%
1995/11/30	6.3%	28.4%	14.4%	22.7%	21.8%	-2.1%
1995/12/31	8.8%	30.2%	14.6%	24.8%	22.7%	2.9%
1996/01/31	39.3%	33.6%	14.8%	29.5%	19.6%	5.6%
1996/02/29	33.4%	26.1%	14.9%	33.4%	20.3%	8.4%

1996/03/31	30.9%	23.5%	15.1%	33.8%	18.2%	13.5%
1996/04/30	30.4%	19.6%	15.2%	42.5%	24.9%	23.1%
1996/05/31	27.6%	20.2%	15.3%	40.4%	20.7%	22.1%
1996/06/30	29.9%	25.7%	15.4%	41.7%	21.5%	18.1%
1996/07/31	24.3%	18.3%	15.5%	36.1%	28.7%	24.1%
1996/08/31	23.5%	17.9%	15.7%	38.8%	31.1%	26.1%
1996/09/30	24.4%	14.9%	15.8%	41.9%	30.5%	25.0%
1996/10/31	23.2%	9.4%	16.0%	50.7%	36.8%	29.6%
1996/11/30	15.0%	8.0%	16.1%	49.8%	33.7%	23.8%
1996/12/31	9.3%	6.6%	16.3%	46.3%	34.0%	26.4%
1997/01/31	-0.6%	8.1%	16.5%	41.7%	28.6%	13.3%
1997/02/28	9.0%	14.9%	16.7%	32.1%	19.1%	7.1%
1997/03/31	7.6%	15.9%	17.0%	22.0%	13.2%	-1.3%
1997/04/30	4.6%	23.0%	17.2%	14.0%	4.9%	-11.9%
1997/05/31	5.4%	22.1%	17.4%	20.4%	6.7%	-9.6%
1997/06/30	10.5%	18.5%	17.4%	28.5%	9.3%	-9.4%
1997/07/31	16.0%	26.6%	17.5%	36.1%	4.5%	-14.3%
1997/08/31	11.9%	22.0%	17.5%	28.5%	6.3%	-14.5%
1997/09/30	6.1%	23.2%	17.4%	28.1%	6.3%	-13.9%
1997/10/31	-3.2%	24.4%	17.4%	20.1%	6.0%	-15.8%
1997/11/30	-3.4%	25.2%	17.3%	19.2%	6.3%	-16.9%
1997/12/31	-4.5%	29.2%	17.2%	20.9%	5.5%	-20.3%
1998/01/31	0.6%	25.4%	17.1%	27.7%	13.6%	-8.8%
1998/02/28	1.8%	24.8%	16.9%	37.4%	17.6%	-7.8%
1998/03/31	9.5%	27.2%	16.7%	50.8%	21.5%	-3.8%
1998/04/30	18.4%	26.4%	16.4%	47.3%	23.8%	3.2%
1998/05/31	11.3%	20.5%	16.1%	39.0%	23.2%	-3.1%
1998/06/30	-6.5%	10.8%	15.9%	53.5%	38.3%	16.9%
1998/07/31	-3.9%	6.3%	15.8%	49.3%	41.9%	20.0%
1998/08/31	-30.9%	-8.9%	15.9%	42.4%	49.9%	24.2%
1998/09/30	-26.6%	-1.0%	16.2%	26.5%	42.1%	14.2%
1998/10/31	-9.2%	10.4%	16.7%	35.8%	32.7%	12.9%

1998/11/30	-8.8%	7.1%	17.3%	41.1%	32.5%	14.3%
1998/12/31	-10.1%	5.0%	17.8%	50.3%	38.9%	17.2%
1999/01/31	-9.0%	5.3%	18.2%	51.9%	38.7%	16.1%
1999/02/28	-14.3%	11.3%	18.5%	41.0%	35.5%	19.0%
1999/03/31	-13.4%	8.1%	18.8%	39.0%	34.9%	16.4%
1999/04/30	-11.7%	8.4%	18.9%	40.5%	30.4%	12.5%
1999/05/31	-12.5%	11.0%	19.1%	36.9%	27.4%	11.6%
1999/06/30	7.2%	18.4%	19.3%	18.5%	5.7%	-8.3%
1999/07/31	4.1%	21.3%	19.2%	16.3%	5.9%	-11.1%
1999/08/31	45.0%	43.1%	18.9%	26.6%	-2.4%	-15.8%
1999/09/30	38.3%	31.4%	18.3%	33.1%	1.7%	10.2%
1999/10/31	26.1%	22.2%	17.4%	36.1%	5.3%	8.6%
1999/11/30	38.0%	26.8%	16.6%	31.9%	5.1%	14.3%
1999/12/31	61.5%	29.9%	15.7%	31.3%	-0.6%	12.9%
2000/01/31	49.8%	27.7%	15.0%	20.5%	-2.3%	2.8%
2000/02/29	38.4%	20.3%	14.3%	22.4%	-0.8%	9.9%
2000/03/31	27.7%	20.2%	13.7%	29.3%	4.9%	3.4%
2000/04/30	7.9%	16.7%	13.1%	25.0%	6.7%	8.3%
2000/05/31	16.2%	19.8%	12.6%	27.7%	10.4%	12.1%
2000/06/30	12.0%	20.5%	12.2%	26.2%	15.2%	20.0%
2000/07/31	11.7%	22.9%	11.8%	24.0%	11.8%	23.8%
2000/08/31	25.3%	24.0%	11.5%	30.2%	12.4%	28.1%
2000/09/30	23.6%	22.6%	11.3%	30.7%	16.1%	5.1%
2000/10/31	16.1%	19.9%	11.2%	25.1%	17.6%	10.5%
2000/11/30	5.9%	20.3%	11.1%	15.9%	23.2%	12.0%
2000/12/31	-0.1%	19.4%	11.0%	7.3%	26.1%	9.4%
2001/01/31	9.7%	21.5%	10.9%	15.8%	28.3%	13.5%
2001/02/28	15.7%	24.9%	10.8%	4.5%	27.4%	5.0%
2001/03/31	5.2%	23.6%	10.8%	-7.9%	21.4%	17.5%
2001/04/30	23.8%	26.2%	10.8%	-0.5%	20.3%	12.8%
2001/05/31	30.9%	27.1%	10.9%	-1.7%	16.2%	13.4%
2001/06/30	22.8%	31.2%	10.9%	-4.8%	16.1%	21.2%

2001/07/31	13.6%	29.7%	10.9%	-4.1%	20.0%	16.2%
2001/08/31	8.7%	27.4%	10.9%	-9.9%	27.8%	14.6%
2001/09/30	1.0%	27.9%	10.9%	-10.3%	33.3%	46.3%
2001/10/31	8.3%	30.4%	10.8%	-6.7%	36.5%	33.3%
2001/11/30	24.5%	28.4%	10.7%	12.2%	40.5%	41.9%
2001/12/31	29.1%	17.8%	10.6%	32.4%	57.4%	71.4%
2002/01/31	17.0%	12.8%	10.5%	17.5%	44.5%	66.7%
2002/02/28	23.6%	7.3%	10.4%	27.8%	45.9%	66.7%
2002/03/31	38.2%	6.9%	10.3%	36.3%	42.6%	67.4%
2002/04/30	26.5%	11.8%	10.4%	15.2%	39.2%	59.1%
2002/05/31	23.1%	12.0%	10.4%	6.9%	31.2%	59.1%
2002/06/30	19.0%	6.5%	10.4%	8.3%	44.8%	45.8%
2002/07/31	11.2%	8.6%	10.5%	-2.3%	38.7%	43.5%
2002/08/31	11.0%	7.8%	10.7%	4.0%	37.7%	48.9%
2002/09/30	20.5%	6.3%	10.8%	-5.0%	29.5%	20.0%
2002/10/31	13.6%	6.0%	11.1%	-9.8%	15.2%	25.0%
2002/11/30	4.8%	9.3%	11.3%	-23.1%	0.1%	4.9%
2002/12/31	-8.2%	16.0%	11.7%	-42.4%	-14.5%	-8.3%
2003/01/31	-11.9%	21.2%	12.0%	-40.1%	-8.4%	0.0%
2003/02/28	-19.8%	23.6%	12.3%	-43.2%	-11.6%	-11.4%
2003/03/31	-27.2%	27.3%	12.6%	-47.2%	-13.5%	-19.4%
2003/04/30	-29.1%	21.5%	12.7%	-42.8%	-18.7%	-20.0%
2003/05/31	-20.7%	22.1%	13.0%	-26.0%	1.3%	-11.4%
2003/06/30	-18.6%	24.7%	13.2%	-28.7%	-15.3%	-20.0%
2003/07/31	-0.9%	19.8%	13.3%	-21.2%	-19.5%	-12.1%
2003/08/31	-0.9%	20.4%	13.3%	-22.1%	-23.8%	-11.4%
2003/09/30	-2.2%	23.5%	13.3%	-17.1%	-25.0%	-11.1%
2003/10/31	8.0%	23.6%	13.1%	-14.1%	-21.4%	-14.3%
2003/11/30	5.5%	18.9%	12.7%	-17.5%	-20.3%	-9.4%
2003/12/31	16.1%	18.1%	12.4%	3.1%	-11.7%	-6.1%
2004/01/31	27.8%	13.8%	11.9%	15.4%	-6.7%	-8.6%
2004/02/29	34.2%	13.6%	11.5%	18.6%	-8.4%	-3.2%

2004/03/31	43.7%	12.3%	11.1%	15.5%	-9.4%	3.4%
2004/04/30	42.7%	9.9%	10.6%	26.9%	4.6%	7.1%
2004/05/31	25.5%	7.0%	10.1%	1.3%	-15.6%	-3.2%
2004/06/30	24.9%	5.7%	9.6%	3.8%	-12.2%	-3.6%
2004/07/31	20.7%	7.7%	9.2%	1.1%	-7.3%	-10.3%
2004/08/31	24.8%	11.6%	8.9%	5.1%	0.8%	-6.5%
2004/09/30	35.9%	10.1%	8.5%	9.7%	-0.3%	-9.4%
2004/10/31	23.0%	10.4%	8.2%	1.3%	-1.5%	0.0%
2004/11/30	32.2%	11.9%	8.1%	7.0%	2.0%	3.4%
2004/12/31	25.4%	15.2%	8.0%	-1.9%	-6.3%	-9.0%
2005/01/31	21.5%	18.1%	7.9%	-6.0%	-8.2%	-9.6%
2005/02/28	27.5%	19.3%	7.9%	-2.0%	-5.8%	-4.3%
2005/03/31	28.2%	15.2%	7.8%	9.7%	4.1%	1.6%
2005/04/30	24.5%	18.6%	7.7%	-2.7%	-2.0%	0.5%
2005/05/31	36.3%	18.3%	7.6%	15.9%	12.8%	6.1%
2005/06/30	44.3%	20.1%	7.6%	18.4%	15.4%	23.5%
2005/07/31	51.4%	18.9%	7.5%	23.5%	11.8%	24.4%
2005/08/31	42.1%	14.9%	7.4%	14.3%	2.7%	10.4%
2005/09/30	47.8%	13.6%	7.3%	16.8%	0.7%	17.2%
2005/10/31	45.2%	11.9%	7.3%	24.3%	7.5%	20.1%
2005/11/30	38.3%	12.0%	7.2%	24.2%	4.5%	22.0%
2005/12/31	47.3%	10.8%	7.1%	23.4%	4.8%	31.6%
2006/01/31	58.7%	10.3%	7.1%	20.2%	-2.0%	36.2%
2006/02/28	45.6%	9.0%	7.1%	20.9%	1.4%	36.0%
2006/03/31	57.4%	12.8%	7.0%	16.7%	-6.1%	34.5%
2006/04/30	73.0%	11.8%	7.0%	23.5%	-5.1%	47.7%
2006/05/31	53.3%	10.7%	7.0%	17.5%	-1.2%	53.7%
2006/06/30	53.9%	3.9%	7.0%	26.3%	6.9%	48.4%
2006/07/31	41.5%	3.6%	7.0%	20.5%	6.9%	53.4%
2006/08/31	46.2%	3.6%	7.0%	31.5%	13.0%	59.6%
2006/09/30	36.1%	5.0%	7.1%	40.4%	25.1%	55.0%
2006/10/31	45.7%	7.5%	7.2%	34.2%	15.6%	40.3%

2006/11/30	46.5%	6.0%	7.3%	34.4%	21.3%	45.0%
2006/12/31	41.2%	5.5%	7.4%	33.2%	16.9%	35.6%
2007/01/31	32.2%	5.2%	7.5%	39.0%	22.8%	35.2%
2007/02/28	38.6%	5.8%	7.6%	36.9%	24.8%	41.2%
2007/03/31	37.6%	5.6%	7.8%	37.7%	27.8%	33.7%
2007/04/30	36.7%	6.0%	8.0%	37.3%	24.4%	22.0%
2007/05/31	42.9%	6.0%	8.1%	33.0%	8.8%	8.3%
2007/06/30	36.9%	7.7%	8.3%	22.2%	1.1%	4.9%
2007/07/31	40.2%	7.3%	8.4%	23.9%	7.7%	8.5%
2007/08/31	33.9%	8.0%	8.6%	16.2%	5.1%	6.5%
2007/09/30	37.4%	9.0%	8.8%	7.9%	-3.9%	11.1%
2007/10/31	37.7%	7.4%	9.0%	7.1%	-3.5%	15.3%
2007/11/30	29.8%	4.9%	9.1%	7.3%	3.5%	13.8%
2007/12/31	19.2%	4.2%	9.4%	7.0%	8.2%	29.3%
2008/01/31	10.1%	2.9%	9.6%	3.3%	20.3%	47.0%
2008/02/29	21.9%	0.7%	9.8%	7.1%	24.8%	53.6%
2008/03/31	11.1%	0.6%	9.9%	8.1%	33.3%	56.4%
2008/04/30	11.9%	-1.6%	10.1%	5.4%	23.2%	35.8%
2008/05/31	14.1%	-3.0%	10.3%	3.4%	23.2%	44.1%
2008/06/30	10.1%	-2.7%	10.5%	-0.1%	29.8%	57.6%
2008/07/31	-0.4%	5.2%	10.8%	-7.5%	17.1%	39.7%
2008/08/31	-0.8%	5.9%	10.9%	-5.5%	17.8%	31.5%
2008/09/30	-18.0%	6.0%	11.2%	-10.7%	27.4%	41.2%
2008/10/31	-30.9%	3.8%	11.4%	-12.6%	54.1%	40.6%
2008/11/30	-27.7%	10.3%	11.5%	-15.5%	54.1%	51.7%
2008/12/31	-23.2%	17.0%	11.7%	-16.6%	56.6%	40.6%
2009/01/31	-22.1%	14.8%	11.8%	-19.9%	41.0%	34.8%
2009/02/28	-37.6%	12.5%	11.8%	-30.9%	27.5%	27.5%
2009/03/31	-28.5%	13.1%	11.8%	-31.5%	15.3%	13.9%
2009/04/30	-30.3%	15.0%	11.7%	-31.7%	12.6%	13.4%
2009/05/31	-25.9%	17.5%	11.5%	-31.6%	9.7%	15.3%
2009/06/30	-24.9%	19.3%	11.3%	-30.0%	3.3%	-2.0%

2009/07/31	-9.4%	11.3%	11.0%	-16.2%	13.0%	10.2%
2009/08/31	-6.8%	11.6%	10.7%	-15.2%	11.5%	16.1%
2009/09/30	7.7%	9.1%	10.3%	-10.3%	4.0%	2.2%
2009/10/31	29.2%	9.3%	9.9%	-4.7%	-7.6%	10.0%
2009/11/30	30.3%	4.6%	9.5%	-2.3%	-15.4%	5.6%
2009/12/31	32.1%	-1.0%	9.1%	1.6%	-20.8%	-0.2%
2010/01/31	33.2%	1.7%	8.8%	2.8%	-20.2%	-13.4%
2010/02/28	48.3%	6.8%	8.4%	18.5%	-15.7%	-11.7%
2010/03/31	44.1%	9.0%	8.1%	16.7%	-19.4%	-6.0%
2010/04/30	41.8%	9.3%	7.9%	20.5%	-7.2%	16.3%
2010/05/31	21.9%	9.4%	7.7%	10.4%	-1.1%	18.0%
2010/06/30	21.8%	9.9%	7.5%	10.1%	3.7%	31.9%
2010/07/31	19.5%	13.0%	7.4%	3.5%	-0.5%	16.3%
2010/08/31	11.7%	14.6%	7.3%	-3.2%	0.9%	21.9%
2010/09/30	21.1%	15.3%	7.2%	-0.6%	-1.6%	20.5%
2010/10/31	18.3%	16.8%	7.1%	1.4%	-3.7%	15.5%
2010/11/30	15.3%	14.4%	7.0%	2.1%	-4.6%	12.9%
2010/12/31	19.0%	15.0%	6.9%	0.5%	-4.8%	14.6%
2011/01/31	20.6%	12.2%	6.8%	12.9%	-0.4%	15.7%
2011/02/28	23.6%	10.1%	6.7%	10.7%	-4.1%	14.6%
2011/03/31	15.2%	8.3%	6.5%	5.6%	0.2%	17.8%
2011/04/30	17.8%	9.4%	6.4%	5.6%	-1.1%	15.9%
2011/05/31	23.2%	11.4%	6.3%	14.2%	-0.4%	12.7%
2011/06/30	24.6%	11.3%	6.2%	15.7%	-2.9%	7.2%
2011/07/31	13.1%	8.5%	6.1%	9.3%	0.0%	25.3%
2011/08/31	16.9%	9.0%	6.0%	9.2%	3.5%	39.2%
2011/09/30	3.6%	5.9%	5.9%	11.9%	22.3%	37.8%
2011/10/31	9.4%	7.7%	5.8%	16.4%	18.4%	43.7%
2011/11/30	11.7%	9.9%	5.8%	16.8%	23.3%	40.6%
2011/12/31	2.6%	8.8%	5.7%	15.8%	30.7%	34.8%
2012/01/31	10.8%	13.5%	5.7%	5.9%	18.0%	41.9%
2012/02/29	9.6%	13.6%	5.7%	6.1%	15.0%	33.7%

2012/03/31	7.5%	13.2%	5.7%	14.8%	19.9%	30.5%
2012/04/30	8.1%	12.7%	5.7%	13.6%	23.4%	25.3%
2012/05/31	5.1%	11.1%	5.7%	11.9%	30.0%	26.6%
2012/06/30	9.2%	14.6%	5.7%	15.1%	24.7%	32.5%
2012/07/31	14.5%	17.4%	5.7%	21.8%	26.4%	22.5%
2012/08/31	18.0%	13.5%	5.7%	30.4%	20.9%	10.4%
2012/09/30	24.4%	17.0%	5.6%	25.5%	6.1%	16.5%
2012/10/31	18.6%	13.2%	5.6%	20.1%	11.6%	10.4%
2012/11/30	19.8%	14.2%	5.6%	25.3%	13.3%	9.4%
2012/12/31	26.7%	16.0%	5.5%	21.3%	5.4%	10.8%
2013/01/31	23.7%	13.7%	5.5%	34.1%	13.3%	9.1%
2013/02/28	19.3%	14.3%	5.4%	34.6%	19.1%	7.2%
2013/03/31	22.5%	14.4%	5.4%	35.6%	19.7%	15.0%
2013/04/30	16.1%	16.9%	5.4%	35.7%	14.2%	3.0%
2013/05/31	30.7%	11.5%	5.3%	52.4%	13.2%	4.0%
2013/06/30	21.0%	6.3%	5.3%	44.6%	15.2%	-11.7%
2013/07/31	23.0%	1.6%	5.2%	47.9%	13.5%	-4.0%
2013/08/31	22.8%	0.2%	5.2%	44.8%	15.3%	1.1%
2013/09/30	27.0%	3.1%	5.2%	46.0%	14.0%	-9.9%
2013/10/31	26.2%	4.1%	5.2%	46.7%	11.9%	-11.7%
2013/11/30	21.6%	1.8%	5.2%	45.0%	8.9%	-16.9%
2013/12/31	21.4%	0.6%	5.2%	57.8%	18.2%	-10.5%
2014/01/31	14.9%	-2.7%	5.2%	46.0%	23.1%	-4.7%
2014/02/28	22.8%	-1.0%	5.2%	45.3%	19.5%	1.4%
2014/03/31	23.6%	0.6%	5.3%	37.2%	15.9%	-6.7%
2014/04/30	30.1%	-3.0%	5.3%	38.0%	18.6%	4.2%
2014/05/31	21.8%	2.9%	5.4%	23.5%	8.6%	-5.9%
2014/06/30	32.7%	5.4%	5.4%	33.7%	13.8%	15.9%
2014/07/31	28.3%	7.2%	5.5%	26.3%	13.1%	7.4%
2014/08/31	24.5%	11.7%	5.6%	26.3%	9.3%	-3.7%
2014/09/30	15.4%	5.8%	5.6%	26.5%	13.7%	3.0%
2014/10/31	12.5%	9.0%	5.7%	18.4%	8.5%	-0.2%

2014/11/30	14.4%	13.1%	5.8%	18.8%	8.6%	1.9%
2014/12/31	10.9%	10.1%	5.9%	17.4%	11.7%	9.2%
2015/01/31	17.1%	21.2%	6.0%	10.9%	2.3%	3.7%
2015/02/28	16.1%	15.1%	6.1%	16.7%	4.5%	-2.3%
2015/03/31	12.5%	12.4%	6.1%	22.4%	10.8%	5.1%
2015/04/30	14.8%	11.5%	6.2%	20.8%	7.6%	4.3%
2015/05/31	8.5%	9.5%	6.2%	23.7%	8.5%	10.1%
2015/06/30	4.8%	8.2%	6.3%	18.3%	7.3%	4.2%
2015/07/31	4.4%	8.2%	6.3%	25.3%	10.8%	-0.1%
2015/08/31	1.1%	5.4%	6.3%	20.4%	17.4%	9.3%
2015/09/30	4.8%	7.0%	6.4%	17.6%	20.2%	12.1%
2015/10/31	5.5%	4.4%	5.9%	21.6%	22.8%	19.0%

APPENDIX B

Generic, equally weighted, optimal Markowitz portfolios and the average South African balanced fund indices

Date	Equal weight Portfolio	Generic Portfolio	Minimise STDev Portfolio	Markowitz Portfolio - Targeting Inflation + 3%	Markowitz Portfolio - Targeting Inflation + 4%	Markowitz Portfolio - Targeting Inflation + 5%	Average of all multi asset index performance
1998/12/31	1.00	1.00	1.00	1.00	1.00	1.00	144.98
1999/01/31	1.04	1.05	1.02	1.03	1.02	1.02	150.64
1999/02/28	1.06	1.08	1.03	1.04	1.04	1.04	155.40
1999/03/31	1.08	1.13	1.05	1.05	1.05	1.05	164.31
1999/04/30	1.11	1.19	1.06	1.06	1.06	1.06	168.16
1999/05/31	1.09	1.14	1.07	1.06	1.07	1.07	159.59
1999/06/30	1.09	1.19	1.08	1.06	1.07	1.08	169.04
1999/07/31	1.11	1.21	1.09	1.06	1.08	1.09	166.81
1999/08/31	1.10	1.20	1.10	1.07	1.09	1.10	161.84
1999/09/30	1.14	1.20	1.12	1.13	1.13	1.13	157.82
1999/10/31	1.17	1.25	1.14	1.14	1.14	1.14	163.07
1999/11/30	1.20	1.30	1.15	1.15	1.15	1.15	172.18
1999/12/31	1.25	1.41	1.16	1.16	1.16	1.17	189.89
2000/01/31	1.24	1.40	1.17	1.15	1.16	1.17	188.42
2000/02/29	1.25	1.37	1.18	1.19	1.19	1.19	182.25
2000/03/31	1.27	1.40	1.19	1.16	1.17	1.18	179.72
2000/04/30	1.27	1.36	1.20	1.19	1.20	1.21	170.84
2000/05/31	1.28	1.36	1.21	1.20	1.20	1.21	168.66
2000/06/30	1.30	1.40	1.22	1.21	1.22	1.23	174.04

2000/07/31	1.31	1.42	1.23	1.22	1.22	1.23	174.79
2000/08/31	1.35	1.50	1.25	1.24	1.24	1.25	185.24
2000/09/30	1.36	1.48	1.26	1.25	1.25	1.26	182.14
2000/10/31	1.37	1.48	1.27	1.26	1.27	1.27	174.58
2000/11/30	1.38	1.46	1.28	1.28	1.28	1.29	172.95
2000/12/31	1.40	1.52	1.30	1.28	1.28	1.29	182.79
2001/01/31	1.45	1.61	1.31	1.29	1.29	1.30	191.22
2001/02/28	1.44	1.59	1.32	1.29	1.30	1.31	188.66
2001/03/31	1.41	1.51	1.33	1.31	1.32	1.33	178.77
2001/04/30	1.47	1.61	1.35	1.33	1.33	1.34	187.80
2001/05/31	1.48	1.65	1.36	1.34	1.34	1.35	194.52
2001/06/30	1.51	1.65	1.38	1.40	1.39	1.38	196.66
2001/07/31	1.50	1.61	1.38	1.38	1.38	1.38	189.77
2001/08/31	1.53	1.65	1.40	1.40	1.39	1.40	195.29
2001/09/30	1.60	1.59	1.43	1.58	1.53	1.48	186.46
2001/10/31	1.64	1.67	1.44	1.54	1.50	1.47	189.75
2001/11/30	1.76	1.81	1.47	1.61	1.55	1.51	199.23
2001/12/31	1.92	1.97	1.51	1.74	1.65	1.57	207.65
2002/01/31	1.86	1.92	1.50	1.70	1.61	1.53	201.40
2002/02/28	1.88	1.96	1.52	1.70	1.61	1.53	203.91
2002/03/31	1.90	1.98	1.53	1.75	1.65	1.57	206.53
2002/04/30	1.87	1.97	1.54	1.71	1.62	1.54	213.56
2002/05/31	1.85	1.96	1.54	1.71	1.62	1.54	217.36
2002/06/30	1.86	1.93	1.56	1.71	1.62	1.54	212.39
2002/07/31	1.79	1.79	1.56	1.63	1.54	1.47	201.91
2002/08/31	1.85	1.85	1.59	1.72	1.63	1.55	206.62
2002/09/30	1.83	1.80	1.60	1.76	1.67	1.59	207.34
2002/10/31	1.82	1.81	1.61	1.72	1.63	1.55	207.08
2002/11/30	1.79	1.83	1.61	1.60	1.51	1.44	213.65
2002/12/31	1.75	1.76	1.62	1.64	1.56	1.48	209.79
2003/01/31	1.76	1.72	1.64	1.70	1.61	1.53	204.14
2003/02/28	1.69	1.67	1.64	1.61	1.52	1.45	199.10
2003/03/31	1.64	1.59	1.64	1.56	1.48	1.41	190.55
2003/04/30	1.60	1.57	1.65	1.54	1.46	1.39	188.81
2003/05/31	1.77	1.76	1.69	1.64	1.55	1.47	204.85

2003/06/30	1.71	1.72	1.70	1.56	1.48	1.41	206.11
2003/07/31	1.73	1.77	1.71	1.60	1.52	1.44	210.30
2003/08/31	1.77	1.82	1.74	1.66	1.58	1.50	213.76
2003/09/30	1.77	1.79	1.75	1.70	1.61	1.53	212.75
2003/10/31	1.80	1.89	1.77	1.65	1.56	1.49	222.09
2003/11/30	1.75	1.87	1.77	1.63	1.54	1.47	225.40
2003/12/31	1.85	1.98	1.80	1.69	1.60	1.53	235.88
2004/01/31	1.92	2.05	1.82	1.74	1.64	1.55	239.09
2004/02/29	1.88	2.04	1.82	1.67	1.60	1.53	239.39
2004/03/31	1.84	2.00	1.83	1.67	1.60	1.53	240.42
2004/04/30	1.87	2.01	1.84	1.67	1.60	1.53	239.23
2004/05/31	1.85	1.99	1.85	1.67	1.60	1.54	238.31
2004/06/30	1.79	1.95	1.86	1.56	1.52	1.49	238.88
2004/07/31	1.79	1.97	1.87	1.53	1.50	1.48	238.95
2004/08/31	1.90	2.10	1.89	1.66	1.61	1.55	251.11
2004/09/30	1.92	2.16	1.91	1.68	1.62	1.57	260.75
2004/10/31	1.92	2.15	1.92	1.72	1.65	1.59	266.56
2004/11/30	1.95	2.22	1.94	1.74	1.67	1.61	282.40
2004/12/31	1.94	2.25	1.94	1.68	1.63	1.58	291.32
2005/01/31	1.99	2.29	1.96	1.71	1.65	1.60	291.82
2005/02/28	2.00	2.36	1.98	1.72	1.66	1.61	299.84
2005/03/31	2.05	2.37	1.98	1.79	1.72	1.65	295.03
2005/04/30	2.02	2.30	1.99	1.76	1.70	1.64	288.99
2005/05/31	2.15	2.48	2.01	1.86	1.77	1.70	304.33
2005/06/30	2.18	2.52	2.03	1.92	1.82	1.73	310.63
2005/07/31	2.20	2.62	2.05	1.91	1.82	1.73	325.36
2005/08/31	2.19	2.63	2.06	1.91	1.82	1.74	329.85
2005/09/30	2.25	2.76	2.08	2.01	1.90	1.80	344.89
2005/10/31	2.30	2.76	2.09	2.09	1.96	1.84	339.55
2005/11/30	2.30	2.79	2.11	2.12	1.98	1.86	346.33
2005/12/31	2.34	2.91	2.13	2.17	2.02	1.89	364.52
2006/01/31	2.40	3.04	2.16	2.29	2.14	1.97	382.82
2006/02/28	2.40	3.01	2.17	2.27	2.12	1.96	382.40
2006/03/31	2.45	3.12	2.19	2.38	2.22	2.03	396.10
2006/04/30	2.51	3.20	2.21	2.59	2.41	2.16	399.88

2006/05/31	2.63	3.23	2.23	2.84	2.65	2.32	394.41
2006/06/30	2.69	3.32	2.23	2.87	2.68	2.34	396.32
2006/07/31	2.67	3.28	2.24	2.88	2.68	2.35	392.17
2006/08/31	2.76	3.42	2.26	2.96	2.76	2.41	407.15
2006/09/30	2.87	3.54	2.28	3.05	2.85	2.46	416.93
2006/10/31	2.87	3.62	2.30	2.93	2.73	2.40	427.16
2006/11/30	2.91	3.67	2.33	3.08	2.87	2.49	437.82
2006/12/31	2.89	3.73	2.35	2.92	2.73	2.40	451.87
2007/01/31	2.97	3.83	2.37	3.07	2.76	2.43	461.08
2007/02/28	3.01	3.87	2.39	3.16	2.79	2.45	465.12
2007/03/31	3.06	4.00	2.42	3.15	2.81	2.47	477.87
2007/04/30	3.07	4.07	2.44	3.12	2.81	2.50	489.35
2007/05/31	3.09	4.13	2.45	3.05	2.82	2.51	492.00
2007/06/30	3.05	4.08	2.46	2.99	2.83	2.52	487.39
2007/07/31	3.09	4.10	2.48	3.10	2.87	2.54	486.05
2007/08/31	3.12	4.13	2.50	3.15	2.90	2.56	487.66
2007/09/30	3.20	4.26	2.54	3.31	2.92	2.60	495.39
2007/10/31	3.21	4.34	2.57	3.28	2.93	2.63	510.34
2007/11/30	3.24	4.28	2.58	3.42	2.98	2.64	501.32
2007/12/31	3.26	4.21	2.60	3.62	3.02	2.66	496.39
2008/01/31	3.43	4.17	2.61	3.77	3.14	2.77	472.64
2008/02/29	3.60	4.44	2.67	3.85	3.21	2.83	499.02
2008/03/31	3.65	4.43	2.68	3.95	3.29	2.90	495.62
2008/04/30	3.52	4.45	2.71	3.88	3.23	2.85	494.41
2008/05/31	3.57	4.53	2.74	3.90	3.25	2.87	501.08
2008/06/30	3.57	4.40	2.74	3.96	3.30	2.91	482.58
2008/07/31	3.43	4.20	2.74	3.92	3.27	2.88	465.96
2008/08/31	3.46	4.25	2.77	3.98	3.32	2.93	477.18
2008/09/30	3.50	4.01	2.75	4.07	3.39	2.99	453.35
2008/10/31	3.50	3.80	2.74	4.27	3.56	3.14	429.14
2008/11/30	3.63	3.86	2.77	4.38	3.65	3.22	426.65
2008/12/31	3.68	3.92	2.81	4.43	3.69	3.26	433.93
2009/01/31	3.74	3.83	2.84	4.43	3.71	3.28	420.89
2009/02/28	3.59	3.55	2.84	4.34	3.69	3.29	395.98
2009/03/31	3.60	3.74	2.87	4.39	3.73	3.33	407.65

2009/04/30	3.45	3.72	2.87	4.40	3.75	3.35	406.67
2009/05/31	3.54	3.91	2.91	4.45	3.78	3.39	423.61
2009/06/30	3.44	3.82	2.91	4.44	3.79	3.40	421.17
2009/07/31	3.59	4.08	2.95	4.57	3.86	3.44	440.69
2009/08/31	3.66	4.20	2.98	4.63	3.90	3.47	453.33
2009/09/30	3.68	4.21	3.00	4.66	3.92	3.49	455.05
2009/10/31	3.79	4.36	3.04	4.71	3.95	3.53	467.87
2009/11/30	3.84	4.39	3.07	4.72	3.97	3.55	465.47
2009/12/31	3.81	4.46	3.08	4.76	4.00	3.57	475.12
2010/01/31	3.81	4.39	3.10	4.78	4.02	3.58	469.64
2010/02/28	3.88	4.45	3.12	4.81	4.04	3.62	474.28
2010/03/31	3.88	4.61	3.14	4.84	4.07	3.66	486.66
2010/04/30	3.95	4.65	3.16	4.87	4.10	3.70	488.50
2010/05/31	3.94	4.50	3.17	4.88	4.12	3.71	476.58
2010/06/30	3.93	4.41	3.18	4.89	4.13	3.73	472.90
2010/07/31	3.96	4.62	3.21	4.93	4.17	3.76	484.70
2010/08/31	4.00	4.57	3.22	4.94	4.18	3.80	481.58
2010/09/30	4.07	4.78	3.25	4.98	4.22	3.85	500.54
2010/10/31	4.16	4.91	3.27	5.02	4.25	3.88	507.93
2010/11/30	4.15	4.88	3.29	5.04	4.27	3.90	509.07
2010/12/31	4.15	5.02	3.31	5.06	4.30	3.93	516.98
2011/01/31	4.26	5.08	3.32	5.13	4.32	3.92	518.48
2011/02/28	4.29	5.15	3.34	5.15	4.35	3.95	521.88
2011/03/31	4.25	5.12	3.36	5.16	4.36	3.97	521.60
2011/04/30	4.32	5.21	3.38	5.19	4.39	4.01	526.64
2011/05/31	4.40	5.25	3.39	5.21	4.41	4.04	530.14
2011/06/30	4.35	5.17	3.40	5.23	4.42	4.04	525.07
2011/07/31	4.38	5.13	3.41	5.24	4.44	4.07	520.06
2011/08/31	4.56	5.17	3.44	5.25	4.46	4.13	522.98
2011/09/30	4.67	5.16	3.44	5.30	4.48	4.12	527.55
2011/10/31	4.87	5.50	3.48	5.35	4.52	4.20	549.48
2011/11/30	4.92	5.54	3.50	5.37	4.55	4.22	551.24
2011/12/31	4.84	5.47	3.50	5.40	4.56	4.22	551.98
2012/01/31	4.95	5.66	3.54	5.44	4.62	4.31	559.76
2012/02/29	4.91	5.69	3.56	5.47	4.64	4.32	565.35

2012/03/31	4.92	5.70	3.56	5.49	4.64	4.31	571.76
2012/04/30	4.99	5.81	3.59	5.53	4.68	4.36	577.43
2012/05/31	5.08	5.76	3.60	5.55	4.70	4.38	575.30
2012/06/30	5.11	5.84	3.63	5.59	4.75	4.47	577.18
2012/07/31	5.20	6.00	3.67	5.64	4.81	4.56	584.99
2012/08/31	5.33	6.14	3.69	5.68	4.85	4.61	598.02
2012/09/30	5.42	6.23	3.72	5.71	4.89	4.67	603.11
2012/10/31	5.53	6.40	3.74	5.75	4.91	4.68	617.27
2012/11/30	5.64	6.55	3.76	5.79	4.95	4.73	626.76
2012/12/31	5.54	6.59	3.79	5.81	4.98	4.77	630.00
2013/01/31	5.77	6.88	3.84	5.90	5.04	4.81	652.04
2013/02/28	5.73	6.84	3.85	5.92	5.05	4.80	648.67
2013/03/31	5.84	6.96	3.88	5.97	5.08	4.84	661.46
2013/04/30	5.75	6.92	3.90	6.00	5.11	4.85	656.49
2013/05/31	6.05	7.33	3.93	6.08	5.14	4.86	695.18
2013/06/30	5.77	7.02	3.90	6.07	5.09	4.78	672.50
2013/07/31	5.93	7.24	3.93	6.12	5.13	4.82	684.68
2013/08/31	6.11	7.37	3.96	6.15	5.17	4.87	695.59
2013/09/30	6.16	7.62	3.99	6.20	5.22	4.92	715.58
2013/10/31	6.26	7.82	4.02	6.24	5.25	4.95	728.84
2013/11/30	6.22	7.80	4.02	6.28	5.25	4.92	730.52
2013/12/31	6.33	8.01	4.06	6.33	5.29	4.96	748.49
2014/01/31	6.54	7.99	4.07	6.35	5.31	4.98	743.28
2014/02/28	6.61	8.21	4.10	6.41	5.37	5.06	752.03
2014/03/31	6.58	8.28	4.11	6.44	5.41	5.09	757.32
2014/04/30	6.63	8.40	4.13	6.47	5.44	5.13	765.03
2014/05/31	6.62	8.48	4.16	6.50	5.47	5.15	774.50
2014/06/30	6.79	8.69	4.19	6.57	5.54	5.23	785.95
2014/07/31	6.81	8.73	4.20	6.60	5.57	5.26	785.86
2014/08/31	6.86	8.80	4.24	6.67	5.62	5.32	783.54
2014/09/30	6.88	8.73	4.26	6.68	5.61	5.29	784.55
2014/10/31	6.83	8.77	4.28	6.72	5.66	5.34	781.91
2014/11/30	6.88	8.89	4.32	6.79	5.70	5.38	793.53
2014/12/31	7.04	8.95	4.34	6.83	5.73	5.41	799.78
2015/01/31	7.21	9.18	4.38	6.95	5.87	5.58	808.25

2015/02/28	7.23	9.39	4.40	6.96	5.86	5.56	824.69
2015/03/31	7.35	9.43	4.43	7.01	5.91	5.60	829.13
2015/04/30	7.37	9.60	4.45	7.02	5.92	5.62	841.68
2015/05/31	7.37	9.48	4.47	7.06	5.93	5.60	835.87
2015/06/30	7.36	9.43	4.49	7.07	5.94	5.60	828.15
2015/07/31	7.44	9.58	4.53	7.13	5.98	5.65	833.02
2015/08/31	7.57	9.45	4.54	7.17	6.04	5.72	827.05
2015/09/30	7.69	9.53	4.56	7.21	6.09	5.78	820.34
2015/10/31	7.75	9.64	4.57	7.24	6.12	5.82	826.97