COMPARISON OF MULTI–CRITERIA DECISION ANALYSIS METHODS IN MINE PLANNING AND RELATED CASE STUDIES

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg, 2017

DECLARATION

I declare that this research report is my own unaided work. It is being submitted to the Degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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ABSTRACT

The mining environment is characterised by various stakeholders with unique expectations and future uncertainties. In order to make decisions in an uncertain environment that has various stakeholders with differing unique expectations, Multi-Criteria Decision Making (MCDM) methods are used. MCDM methods are sub-divided into Multi-Objective Decision Making (MODM) and Multi-Criteria Decision Making (MCDA) techniques. Given an uncertain mining environment and a multitude of MCDA techniques, it is necessary to analyse how MCDA techniques used in mine planning and related problems produce consistent results. It is also necessary to establish the ideal number of alternatives and criteria to use to increase confidence in the decision making process.

A total of 246 case studies were sourced from journal; symposia; and conference papers. The case studies were narrowed to those with numerical content, leaving 110 case studies. A total of 40 out of the 110 case studies had original decision matrices and these were chosen for analysis. Different alternatives in the case studies were ranked using eight MCDA techniques. MCDA techniques were chosen because they are used to solve problems with a finite number of alternatives.

Analytical Hierarchy Process (AHP); Elimination and Choice Expressing Reality (ELECTRE); Multi-Attribute Utility Theory (MAUT); Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE); Simple Additive Weighting Method (SAW); Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS); Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR); and Yager's method were selected. "Similarity percentages" and "average similarity percentages" were calculated for the ranked alternatives.

The 1998 economic meltdown and the 2008 Global Financial Crisis (GFC) resulted in increased use of MCDA techniques. The increased use of MCDA techniques was in response to uncertainties in response to the 1998 and 2008 events. The AHP was the most commonly used technique while Fuzzy set theory was used to address

uncertainty. Most MCDA techniques suffer from rank reversal. In order to reduce rank reversal, nine criteria are recommended for use with MCDA techniques. In addition, a maximum of four alternatives is recommended for use with MCDA techniques.

To my parents ‘Matlalane Ts’eliso Julia Mahase and Mojalefa T’sepo Allen Mahase. I love you.

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LIST OF ACRONYMS

AHP: Analytic Hierarchy Process

ANP: Analytic Network Process

CBR: Case Based Reasoning

CI: Consistency Index

CP: Compromise Programming

CR: Consistency Ratio

DEA: Data Envelopment Analysis

GFC: Global Financial Crisis

ELECTRE: Elimination and Choice Expressing Reality

MACBETH: Measuring Attractiveness by a Categorical Based Evaluation Technique

MADM: Multi-Attribute Decision Making

MCDM: Mutli-Attribute Decision Making

MODM: Multi Objective Decision Making

MCDA: Multi-Criteria Decision Making

MCQA: Multi-Criteria Q-Analysis

NAIADE: Novel Approach to Imprecise Assessment and Decision Environment

PROMETHEE: Preference Ranking Organisation Method for Enrichment Evaluation

RI: Random Index

SAW: Simple Additive Weighing

SMART: Simple Multi-Attribute Rating Technique

TOMASO: Technique for Ordinal Multi-Attribute Sorting and Ordering

TOPSIS: Technique for Order Preference Similarity to Ideal Solution

USD: United States Dollar

UTA: Utilities Additives method

WP: Weighted Product

VIKOR: Vise Kriterijumska Optimizacija I Kompromisno Resenje

1 INTRODUCTION

1.1 Background

Natural resources are present in a country to serve the needs of different groups of people within the country, wherein the different groups are known as stakeholders. To address the varying needs of the stakeholders, these natural resources have to be managed optimally. In order to manage these resources, Multi-Criteria Decision Analysis (MCDA) techniques have been used. The techniques have been used to simultaneously integrate all the different needs and preferences of stakeholders (Herath and Prato, 2006).

Mining is no exception to the above, considering the fact that minerals are exploited for the benefit of different stakeholders. The stakeholders include the immediate community; employers; employees; shareholders; and government. Figure 1-1 shows the different stakeholders in the South African mining industry. Figure 1-1 also shows impacts of the stakeholders, their expectations and expected roles.

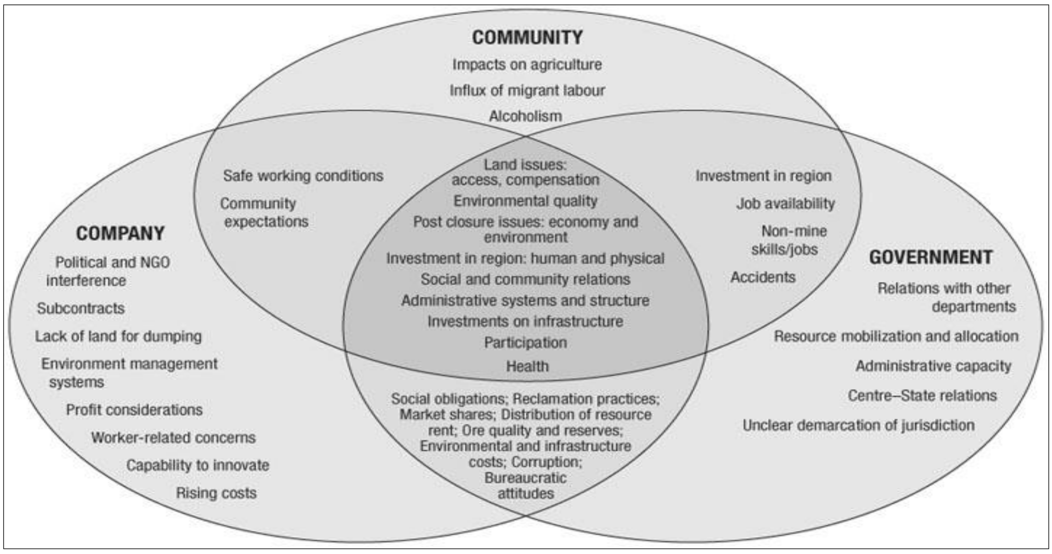

Figure 1-1 Different stakeholders in the mining industry

Source: Lebel (2003)

Resource management often involves decision making and one of the biggest challenges of decision making in mining is the one which Saaty (2007) called the "unknown future". This 'unknown future' is characterised by uncertainty of what challenges the future will present. Businessdictionary.com defines uncertainty as: *Decision making: Situation where the current state of knowledge is such that (1) the order or nature of things is unknown, (2) the consequences, extent, or magnitude of circumstances, conditions, or events is unpredictable and (3) credible probabilities to possible outcomes cannot be assigned. Although too much uncertainty is undesirable manageable uncertainty provides the freedom to make creative decisions. Information theory: Degree to which available choices or the outcomes of possible alternatives are free from constraints* (WebFinance Inc., 2017).

According to Ernst & Young (2016), the top ten risks facing mining and metals companies in South Africa, in descending order of ranking are:

- The decision whether to buy or invest to acquire competitive advantage;
- Productivity improvement;
- Access to capital and challenging fundraising conditions;
- Resource nationalism through mandated beneficiation and increased taxes;
- Social license to operate;
- Volatility of the South African currency and commodity prices;
- Capital project allocation execution risk associated with cost and schedule overruns;
- Access to energy due to rising energy prices;
- Cyber security in a digital world; and
- Lagging innovation.

In order to address the challenge of uncertainty, scenario planning is used where mine planners consider multiple possible scenarios before a decision can be taken (Petit and Fraser, 2013; Saaty, 2007). Matos (2007) further defined scenario planning as a way of modelling uncertainty, whereby the most likely future scenarios are expressed in qualitative ways.

In order to select the best scenario amongst other identified scenarios, multiple criteria and different alternatives are used together with the decision maker's judgement for the overall decision making process. This is achieved by using MCDA methods, where different criteria and alternatives are assigned numerical values to derive an optimal outcome (Petit and Fraser, 2013).

1.2 Problem statement and motivation

The increasing amount of uncertainty and competitiveness of the global market has forced companies to increase their levels of efficiency, responsiveness and flexibility (Oliveira *et al*, 2013). To achieve these increased levels, the most decisive step was turning to MCDA methods to aid in strategic decision making, as human judgement is considered as "limited" when several criteria and alternatives need to be evaluated simultaneously.

The shift is evident as there are several research studies conducted on MCDA methods which were pioneered as early as the 1970s (Oliveira *et al*, 2013). These research studies have seen the development of various MCDA methods and differentiation of these methods. These methods have been used to address everyday problems faced in the mining industry and to yield acceptable solutions.

For this research study, as many as 246 mine planning and related case studies were identified. From these case studies, between one and four MCDA techniques were used to solve the individual case studies. This gives rise to questions which the research report seeks to answer: "*How appropriate is the MCDA method used with increasing number of criteria and alternatives? What is the most suitable number of criteria and alternatives to use when using MCDA techniques?"*

Other applicable methods not used in a particular case study were utilised to validate the results yielded by the methods used. This research study also shows the applicability of these methods as the number of alternatives and criteria changes. The effectiveness of each method with increasing criteria and alternatives were then observed and accounted for.

1.3 Significance of the research study

Yavuz (2015b) defined Decision Making (DM) as a selection process for the best alternative in order to obtain a defined goal under conditions of uncertainty. DM is divided into structuring of problems and analysis of problems. In structuring of problems, a problem is defined and then the criteria and alternatives are identified and determined. In the analysis of problems, either qualitative or quantitative analysis is used to determine the appropriate decision to be taken (Yavuz, 2015b). In some instances, both qualitative and quantitative analysis may be used concurrently to solve a problem.

Both qualitative and quantitative analyses are essential in problem solving. According to Kazakidis *et al* (2004), qualitative analysis is mainly based on expert knowledge and experience and it is used if the analytical team has extensive experience to make informed judgements. Quantitative analysis is used where there is limited experience or a problem to be addressed is more complex (Kazakidis *et al*, 2004). In this analysis, facts or data collected together with a mathematical formula that includes the objectives; variables; and constraints of a problem are used. Figure 1-2 shows the relationship between problem structuring and problem analysis in problem solving.

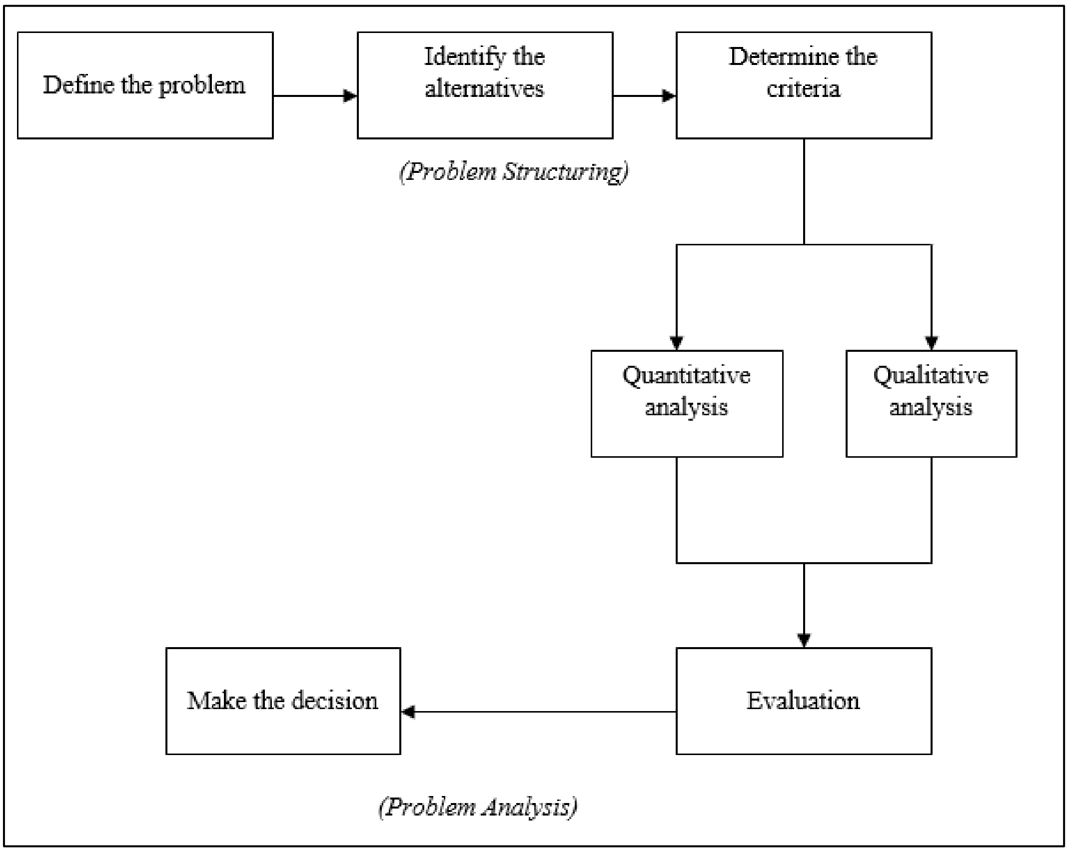

Figure 1-2: Qualitative and quantitative analysis in problem solving

Source: Kazakidis *et al* (2004)

From Figure 1-2, quantitative analysis is analogous to literature work done by authors comparing the different MCDA methods and stating their advantages; disadvantages; and applications. This research study will be used to provide results for quantitative analysis. Once quantitative and qualitative analyses of the case studies have been conducted, an evaluation and a decision will be made. Evaluation and decision making includes observing the behaviour of different methods with each case study; and comparing the ranking of different alternatives as determined by different MCDA techniques.

The similarities observed when ranking alternatives will be utilised to determine the ideal number of criteria and alternatives to use in decision making with MCDA techniques. This research study will recommend the ideal number of criteria and alternatives to use in decision making as means of reducing uncertainty. The recommended number of criteria and alternatives will in turn give the decision maker (mine planners and other practitioners) a level of confidence during the decision making process.

1.4 Objectives of the research study

The objectives of the research study are to:

- Establish the trend when different MCDA methods are used to solve the identified cases;
- Determine the appropriate method to use when the number of criteria and alternatives increases;
- Validate the results obtained with existing literature by various authors; and
- Determine the ideal number of criteria and alternatives to use when making decisions under uncertainty using MCDA techniques.

1.5 Structure of the research report

In order to achieve the objectives of the research study as stated in Section 1.4, the report is divided into five chapters. Chapter 1 has introduced background information on decision making in an uncertain mining environment and has provided the problem statement. Chapter 2 reviews the literature focusing on the MCDA methods considered in the study. The criteria used to select the MCDA methods are also discussed. Chapter 3 presents the selected case studies and the methodology used for their selection. Chapter 3 also introduces "similarity percentages" and how they are used for analysis of the case studies.

Results and analysis are presented in Chapter 4 where case studies are analysed in terms of the criteria; alternatives; and "similarity percentages". Conclusions and recommendations are made in Chapter 5. The graphical presentation of the outline of different chapters within the research report is indicated in Figure 1-3.

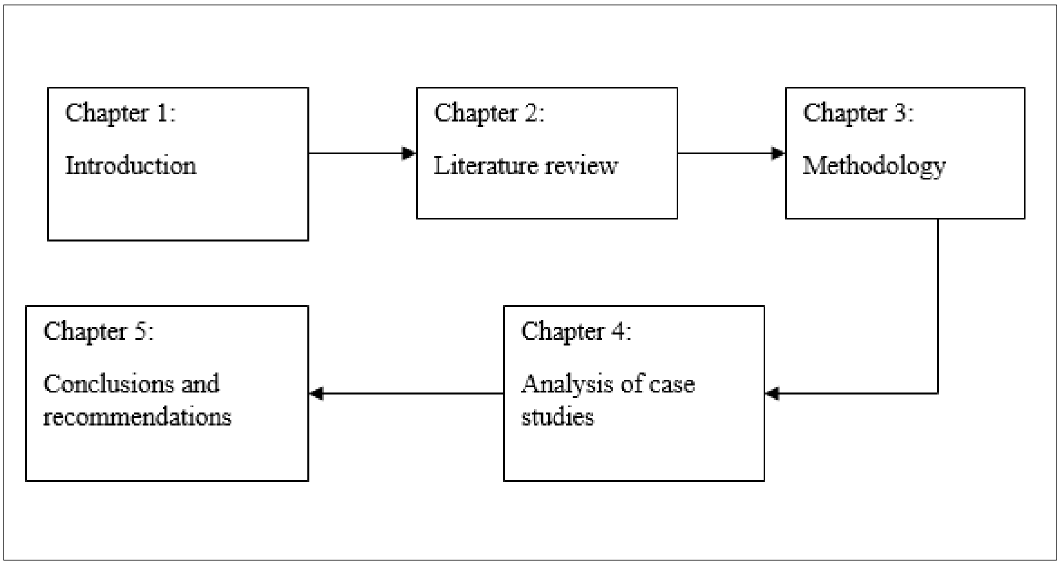

Figure 1-3 Graphical presentation of the report structure

2 LITERATURE REVIEW

2.1 Introduction

This chapter presents the literature related to the different MCDA methods. It aims to indicate the significance of this study in relation to existing literature and it discusses in detail the methods selected for analysis. Chapter 2 also explains the selection process of the methods chosen. Frequency of use of the MCDA techniques as identified by different authors were used to select the methods.

2.2 Multi-criteria decision analysis

Decision making using multiple criteria and alternatives, for instance MCDA techniques, has shown to be reliable in an uncertain environment. MCDA techniques are a class of Multi-Criteria Decision Making (MCDM). MCDM is a process used to solve problems with more than one objective (Pohekar and Ramachandran, 2003). The solution determined depends on the preference of the decision maker. MCDM is divided into Multi Objective Decision Making (MODM) and Multi-Criteria Decision Analysis (MCDA) techniques. MODM techniques are used to address problems with an infinite number of alternatives (Musingwini, 2010).

MCDA techniques are used to solve problems with a finite number of alternatives that are selected from a set of established alternatives. These techniques are also known as Multi-Attribute Decision Making (MADM) techniques (Musingwini, 2010). This research study focuses on established cases with a defined number of alternatives hence MCDA techniques will be used for analysis. MCDA implementation in problem solving is in four steps and these steps are (Musingwini, 2010):

1. Identifying objectives and representing them as criteria;
2. Assigning numerical value or weights to the criteria;

3. Measuring the efficiencies of alternatives against different criteria to yield outcomes; and
4. Making an appropriate decision from the outcome.

To yield an outcome an alternative A_i is selected from a set of alternatives $A = \{A_1, A_2, \ldots\ldots, A_m\}$. Alternative A_i is measured against a decision criterion C_j from a set of criteria $C = \{C_1, C_2, \ldots\ldots, C_n\}$. An outcome, O_{ij} is determined by measuring the efficiency of A_i against C_j. For m alternatives and n criteria, the alternatives and criteria are arranged in a (m x n) matrix to yield outcomes, as shown in Table 2-1.

Table 2-1 The structure of a generic multi-criteria decision analysis problem

		Criteria					
		C_1	C_2	…	C_j	….	C_n
Alternatives	A_1	O_{11}					
	A_2		O_{22}				
	….			…			
	A_i				O_{ij}		
	…					…	
	A_m						O_{mn}

A: Alternatives
C: Criteria
O: Outcome of measuring the efficiency of the alternative *A* in terms of criterion *C*

Source: Musingwini (2010)

2.3 Literature on MCDA methods

There are various MCDA methods that can be used to aid in decision-making. These methods are identified and categorised into different categories where classification may vary according to individual authors. Although the classification is unique for each author, there are common methods identified. The aim of this section is to review the categories and common methods, as presented by various authors. The seven authors that were considered were Fülöp (2005); Figueira *et al* (2005); Sanandaji (2006); Peniwati (2007); Velasquez and Hester (2013); De Montis *et al* (2008); and Musingwini (2010). These authors were considered as they reviewed the different methods and associated categories.

2.3.1 Classification on a study by Fülöp in 2005

Fülöp (2005) defined multi-attribute decision problems as problems that have a finite number of criteria and alternatives. To solve such problems, Multi-Attribute Decision Making (MADM) methods are used. Fülöp (2005) classified MADM methods into Cost-Benefit Analysis (CBA); Elementary methods; Multi-Attribute Utility Theory (MAUT) methods; outranking methods; group decision making methods; and sensitivity analysis.

Elementary methods were further sub-divided using pros and cons analysis; minimum and maximum methods; conjunctive and disjunctive methods; and lexicographic methods (Fülöp, 2005). These methods were considered to be simple and not data-intensive. They also needed a single decision-maker. A single decision maker is required where decisions are less complex and the method used is simple, such as selection of the best alternative between two alternatives that yield the same outcome.

The MAUT methods were methods which had alternatives measured against different criteria. Numerical values assigned to different criteria were used to determine the importance of criteria in a decision (Fülöp, 2005). They were classified into Simple Multi-Attribute Rating Technique (SMART); generalised means; and the Analytic Hierarchy Process (AHP).

Fülöp (2005) defined outranking methods as those methods which compare the performance of two alternatives under a criterion. Outranking methods were divided into Preference Ranking Organisation Method for Enrichment Evaluation

(PROMETHEE); and Elimination and Choice Expressing Reality (ELECTRE) methods (Fülöp, 2005).

2.3.2 Classification by Figueira *et al* in 2005

Figueira *et al* (2005) classified outranking based multiple criteria decision making methods based on pairwise comparison of actions. These methods were divided into outranking methods; Multi-Attribute Utility and Value Theories (MAUT/ MAVT); and non-classical MCDA approaches. The outranking methods were further divided into ELECTRE methods; PROMETHEE methods; pairwise criterion comparison approach; and one outranking method for stochastic data.

MAUT/ MAVT were further divided into MAUT method; Utilities Additives (UTA) method; Analytic Network Process (ANP); AHP; and Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH) (Figueira *et al*, 2005). Non-classical MCDA approaches were introduced to account for any uncertainties associated with a changing environment. Under non-classical MCDA approaches the Fuzzy set approach; and Technique for Ordinal Multi-Attribute Sorting and Ordering (TOMASO) tool-based software were placed (Figueira *et al*, 2005).

2.3.3 Classification by Sanandaji in 2006

Sanandaji (2006) referred to decision making methods as models and classified them into Multi-Criteria Decision Making (MCDM); Multi-Attribute Decision Making (MADM); rational decision making; irrational decision making; and non-rational decision making models. The MCDM was further divided into ELECTRE-2; PROMETHEE-2; AHP; Compromise Programming (CP); and Multi-Criteria Q-Analysis (MCQA). MADM was subdivided into Simple Additive Weighing (SAW) method; Weighted Product (WP) method; and Technique for Order Preference Similarity to Ideal Solution (TOPSIS) method. For each of the decision making classes, the advantages and disadvantages were discussed by Sanandaji (2006).

2.3.4 Classification by Peniwati in 2007

Peniwati (2007) identified fifteen MCDA methods. These methods were divided into three classes namely structuring methods; ordering and ranking methods; and structuring and measuring methods. Structural problems are explained as those problems where key words are used to establish a relationship between analogies and the problem. Analogy association; boundary examination; brainstorming/brain writing; morphological connection; and why-what's stopping methods were put under this class.

Structuring and measuring methods consisted of Bayesian analysis; MAUT; and AHP (Peniwati, 2007). In this class of methods, different alternatives were established and summed up to rank criteria. Ordering and ranking methods included voting method; nominal group technique; Delphi technique; disjointed incrementalism; matrix evaluation; goal programming; conjoint analysis; and outranking methods. These methods were considered as single criterion analysis methods (Peniwati, 2007).

2.3.5 Classification by other authors

Velasquez and Hester (2013) conducted a study with the aim of identifying MCDA methods and their applications. They used databases like Springer; ScienceDirect; IEEExplore; journal articles; and conference proceedings to identify publications on MCDA. The identified publications were narrowed to publications of the most commonly used methods. The identified case studies were grouped according to the frequency of usage of the MCDA techniques. MAUT; AHP; Fuzzy set theory; Case Based Reasoning (CBR); and Data Envelopment Analysis (DEA) were identified. Simple Multi Attribute Rating Technique (SMART); goal programming; ELECTRE; PROMETHEE; SAW; and TOPSIS methods were also identified.

MAUT was the most frequently used method amongst all of the identified methods. MAUT was also mostly used with other methods when two or more methods were utilised. Two or more methods were used so that an alternative method could make up for the shortcomings of a particular method. The advantages; disadvantages and applications of the identified methods are tabulated in Table 2-2.

Table 2-2 Summary of MCDM methods (adapted from Velasquez and Hester, 2013)

Method	Advantages	Disadvantages	Application
Multi-Attribute Utility Theory (MAUT)	Takes uncertainty into account; can incorporate preferences	Needs a lot of input; preferences need to be precise	Economics; business and finance; actuarial; water management, energy management
Analytical Hierarchy Process (AHP)	Easy to use; scalable; hierarchy structure can easily adjust to fit many sized problems; not data intensive	Problems due to interdependence between criteria and alternatives; can lead to inconsistencies between judgement and ranking criteria; rank reversal	Performance-type problems; resource management, corporate policy and strategy; public policy; political strategy; and planning
Case Based Reasoning (CBR)	Not data intensive; requires little maintenance; can improve over time; can adapt to changes over the environment	Sensitive to inconsistent data; requires many cases	Businesses and finance; vehicle insurance; healthcare; and engineering design
Data Envelopment Analysis (DEA)	Capable of handling multiple inputs and outputs; efficiency can be analysed and quantified.	Does not deal with imprecise data; assumes that all input and output are exactly known.	Economics; healthcare; utilities; road safety; agriculture; business and finance
Fuzzy set theory	Allows for imprecise input; takes into account insufficient information	Difficult to develop; can require numerous simulations before use	Engineering; economics; environmental; social; healthcare and management
Simple Multi Attribute Ranking Technique (SMART)	Simple; allows for any type of weight assessment technique; less effort by decision makers	Procedure may not be convenient to solve problems in a real-life situation	Environmental; construction; transportation and logistics; military; manufacturing and assembly problems
Goal Programming (GP)	Capable of handling large-scale problems; can produce infinite alternatives	Cannot generate weight coefficients hence it typically needs to be used in combination with other MCDM methods to yield weight coefficients	Production planning; scheduling; healthcare; portfolio selection; distribution systems; energy planning; water reservoir management; scheduling and wildlife management
Elimination and Choice Expressing Reality (ELECTRE)	Takes uncertainty and vagueness into account	Its process and outcome can be difficult to explain in layman's terms; outranking causes the strengths and weaknesses of the alternatives to not be directly identified	Energy; economics and business; environmental; water management; and transportation problems
Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)	Easy to use; does not require assumption that criteria are proportional	Does not provide a clear method by which to assign weights	Environmental; hydrology; water management; business and finance; chemistry; logistics and transportation; manufacturing and assembly; energy; agriculture
Simple Additive Weighting	Ability to compensate among criteria; intuitive to decision makers; calculation is simple and does not require complex computer programs	Estimates revealed do not always reflect the real situation; result obtained may not be logical	Water management; business and finance
Technique for Order Preference Similarity to Ideal Solution (TOPSIS)	Has a simple process; easy to use and program; the number of steps remains the same regardless of the number of attributes	Its use of Euclidean distance does not consider the correlation of attributes; difficult to weight and keep consistency of judgement	Supply chain management and logistics; engineering; manufacturing systems; business and finance; environmental; human resources; and water resources management

De Montis *et al* (2008) established that environmental problems are characterised by high risks; high stakes; urgency; and dispute. To solve environmental problems, De Montis *et al* (2008) identified and analysed the most commonly used MCDA methods. There were seven methods identified from this analysis and they were assessed in terms of their use and application. The classification of the methods was made into single criterion methods; outranking methods; and programming methods.

De Montis *et al* (2008) in their classification, defined single criterion approach methods as methods where a single attribute is generated from different criteria. The single attribute is used to compare different alternatives. Single criterion approach methods were classified into CBA; MAUT; AHP; and Evaluation matrix (Evamix).

ELECTRE III; Regime; and Novel Approach to Imprecise Assessment and Decision Environments (NAIADE) were classified under outranking methods. These were methods in which the decision maker may change preference based on the available information (De Montis *et al*, 2008). Programming methods were methods where mathematically formulated alternatives were generated as part of the solution. Programming methods were divided into goal programming and multiple objective programming.

Musingwini (2010) identified five most commonly used MCDA techniques based on the frequency of the techniques used not only in other sectors, but also in the mining industry. The frequency of usage of different MCDA techniques was narrowed down to the mining industry. The identified techniques were MAUT; AHP; ELECTRE; PROMETHEE; and TOPSIS. These techniques were compared under criteria of foundation; theoretical basis; measurement criteria; and determination of weights of criteria.

Table 2-3 shows a summary of the different methodologies compared under the different criteria. Musingwini (2010) identified AHP as the most frequently used

MCDA technique for solving mining-related problems. MCDA methods are progressively used for solving problems and their successful use has given decision makers confidence when dealing with uncertainty in the mining industry. From Tables 2-2 and 2-3, it is noticeable that different MCDA methods are used to address various problems and they all have unique properties.

Table 2-3 Comparison of the five multi-criteria decision analysis methodology categories

	Multiple Attribute Utility Theory (MAUT)	**Analytical Hierarchy Process (AHP)**	**Elimination and Choice Expressing Reality (ELECTRE)**	**Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)**	**Technique for Order Preference Similarity to Ideal Solution (TOPSIS)**
Foundation	Classical MCDA approach	Classical MCDA but hierarchical approach	Outranking procedure	Outranking procedure	Classical MCDA approach
Theoretical basis	Utility function additive model	Pair-wise comparison (weighted eigenvector evaluation)	Pair-wise comparison (concordance analysis)	Pair-wise comparison (preference function)	Dimensionless Euclidean space evaluation model
Measurement of criteria	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to a dimensionless numerical scale)
Determination of weights of criteria	Trade-off based weights (generate weights using swing, direct-ratio, or Eigenvector methods)	Trade-off (generate weights using Saaty's Eigenvector and geometric mean)	Non-trade-off (does not provide procedure to obtain weights)	Non-trade-off (does not provide procedure to obtain weights)	Trade-off based weights
Result	Relative preference order	Relative preference order	A set of non-dominated alternatives	Partial and complete ranking order	Relative preference order

Source: Geldermann and Rentz (2005) and Chen (2006), cited in Musingwini, (2010)

2.4 Approach to selecting MCDA methods

The choice of MCDA methods to be used in this research study was based on two factors:

- The extent of use of the method, as shown by the seven authors discussed previously in Section 2.3; and
- The extent of use of the method to solve the identified mine planning; and related case studies.

All the seven authors, as indicated in Section 2.3, through varying methods of classification, identified AHP; MAUT; PROMETHEE; and ELECTRE as the most commonly used methods. Figueira *et al* (2005) and Peniwati (2007) had outranking methods as a common class of methods. ELECTRE and PROMETHEE methods were found in this class. In addition, Sanandaji (2006) and Musingwini (2010) identified TOPSIS as one of the frequently used methods. Figueira *et al* (2005) introduced the fuzzy method as a way of accounting for uncertainty in problems that were addressed.

2.4.1 The extent of use for each technique in the identified case studies

Various journal and symposia papers were reviewed for choice of case studies. The journal and symposia papers presented different case studies where decisions were made with different MCDA techniques. The scope of case studies was narrowed down to case studies on mine planning and those related to mine planning. Case studies were narrowed down to mine planning in order to show the:

- Extent of use of MCDA techniques in mine planning, in terms of decision making;
- Progression of MCDA techniques over the years; and
- The different levels of planning where MCDA techniques have been applied.

Not all of the papers on mine planning and related case studies were available in the public domain. The papers that were unavailable were from the China Knowledge Resource Integrated database website (www.cnki.net) and registration request to the database led to a denied access. The available papers were sourced from different journals and portals like Researchgate.net. The identified journals and symposia papers were published in different areas of mine planning. The following areas were identified within the case studies:

- Strategic mine planning;
- Economic aspects in mine planning;
- Mining method selection;
- Location of facilities (tailings dams; processing plants; crushers);
- Mining equipment selection (road headers, trucks);
- Selection of remedial sites for post mining activities; and
- Mining layout selection.

A total of 246 papers were identified and their years of publication was used to illustrate the use of MCDA techniques over the years. The years of publication range from 1984 to 2016, as shown in Appendix 7-1. Different MCDA techniques were used to rank alternatives in each case study. Figure 2-1 shows the frequency of use in raking different alternatives of case studies in the identified papers.

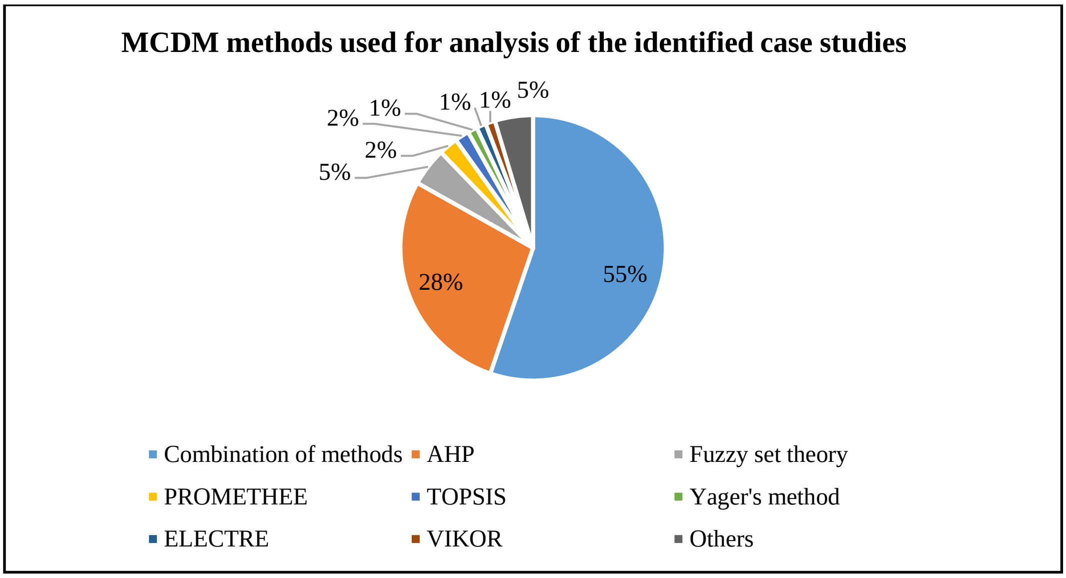

Figure 2-1 MCDA techniques used in the case study and their frequency of use

Figure 2-1 shows that the most frequently used MCDA techniques are AHP; ELECTRE; Fuzzy logic; TOPSIS; VIKOR and Yager's method. The high frequency of usage for the techniques is attributed to the simplicity and ease of use of these techniques. The combination of the techniques was analysed to demonstrate which MCDA techniques are most frequently included in the combination.

A combination of techniques is used where one technique is used to assign weights to criteria and the other technique is used to rank alternatives. For instance, Stojanovic; Bogdanovic; and Urosevic (2015) (cited in Musingwini, 2016) selected the optimal technology for surface mining using AHP and ELECTRE. The weights of different criteria were determined using AHP while the alternatives were ranked using ELECTRE. Figure 2-2 depicts the frequency of different methods combined in the 55% combination of methods, as shown in Figure 2-1.

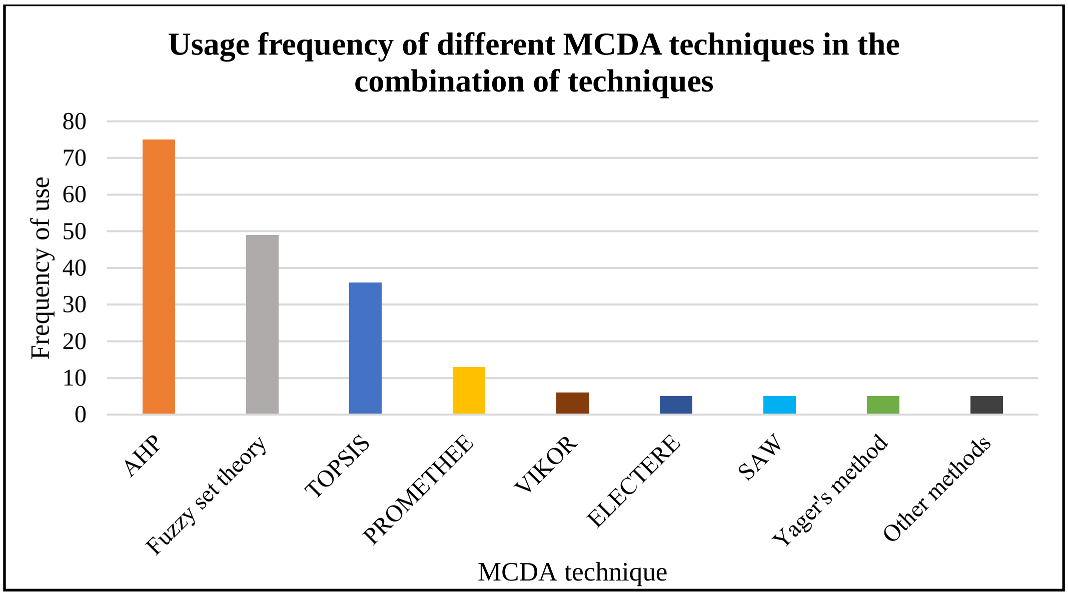

Figure 2-2 Usage frequency of different MCDA techniques in the combination of MCDA techniques

Figures 2-1 and 2-2 illustrate that the most frequently used MCDA techniques are AHP; Fuzzy logic; TOPSIS; PROMETHEE; VIKOR; ELECTRE; SAW; and Yager's method. These techniques had a high frequency of use due to their simplicity and ease of use. These MCDA techniques were chosen for analysing the case studies because they do not require complex computation methods like software and reference can be made to the existing literature, in terms of their usage. AHP is most commonly used method for it accounts for consistency of matrices hence eliminating inconsistencies (Musingwini, 2016). AHP can be used with easily accessible software like Microsoft Excel and it ranks alternatives consistently.

Broekhuizen *et al* (2015) selected case studies solved using MCDA techniques with publication years ranging from 1986 to 2013. The selected case studies were classified according to approaches used to address uncertainty. Five approaches were identified, namely; Bayesian framework; deterministic sensitivity analysis; Fuzzy set theory; Gray theory; and probabilistic sensitivity analysis. Fuzzy set theory was the most frequently identified approach, accounting for 45% of the case studies (Broekhuizen *et al*, 2015). Figure 2-3 illustrates the results from the study by Broekhuizen *et al* (2015).

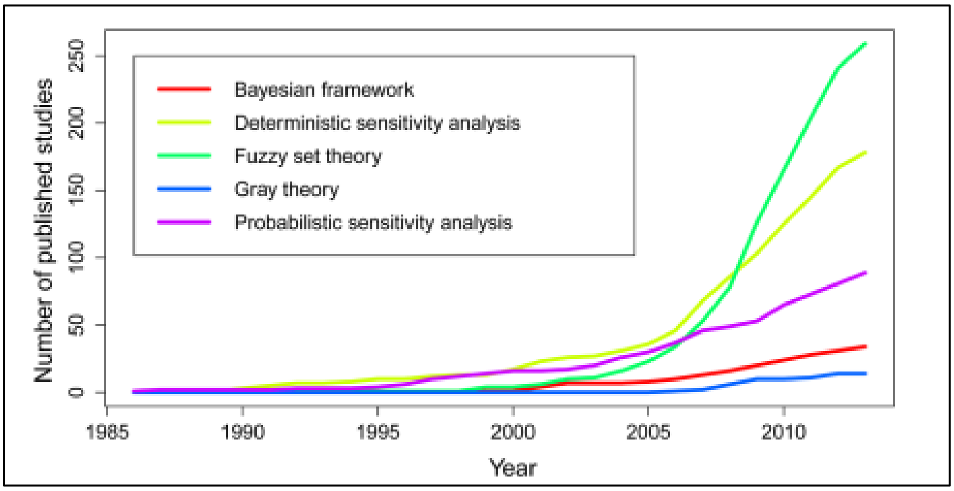

Figure 2-3 Different approaches of dealing with uncertainty in different published MCDA case studies

Source: Broekhuizen *et al* (2015)

Figure 2-3 illustrates that Fuzzy logic is most frequently used to address uncertainty in MCDA techniques, not as an independent MCDA technique. The most commonly used MCDA techniques (Section 2.3.5) included MAUT, so MAUT was substituted for fuzzy set theory in the selected techniques for analysis.

2.5 MCDA techniques selected to analyse the case studies

A total of eight MCDA techniques were selected for analysis of the case studies. SAW; PROMETHEE; MAUT; TOPSIS; VIKOR; AHP; and Yager's method are the eight selected techniques. The principle behind these techniques is discussed at length in Sections 2.5.1 to 2.5.8.

2.5.1 Simple Additive Weighting (SAW)

The Simple Additive Weighting (SAW) method is one of the most widely used technique because of its simplicity (Memariani *et al*, 2009). It is an important technique because it forms the basis of methods such as AHP and PROMETHEE in terms of calculating scores used to rank alternatives. The SAW method involves calculating normalized values for each alternative in a decision matrix. Equation 2-1 shows how normalized values for a normalized decision matrix, *R* are calculated (Memariani *et al*, 2009):

$$r_{ij} = \frac{x_{ij}}{max(x_j)} \qquad 2\text{-}1$$

Where x_{ij} is the value assigned for alternative A_i in criterion C_j in the original decision matrix *X*; *max* x_j is the maximum x_{ij} value in criterion Cj; and r_{ij} is the normalized value for alternative A_i in criterion C_j of matrix *X*. Equation 2-1 holds for criteria to be maximized while in criteria to be minimized Equation 2-2 holds (Memariani *et al*, 2009):

$$r_{ij} = \frac{min(x_i)}{x_{ij}} \qquad 2\text{-}2$$

Where x_{ij} is the value assigned for alternative A_i in criterion C_j in the original decision matrix *X*; *min* x_j is the minimum x_{ij} value in criterion C_j and r_{ij} is the normalized value for alternative A_i in criterion C_j of the original decision matrix *X*. For an alternative A_i, each normalized value r_{ij} is multiplied by weightings assigned to different criteria (W_j) and the sum of the product (W_j* r_{ij}) yields a weighted score for each alternative, V_i. Equation 2-3 illustrates how V_i is calculated (Memariani *et al*, 2009):

$$V_i = \sum(W_j * r_{ij}) \qquad 2\text{-}3$$

The weighted score, V_i, is used to rank different alternatives. The higher the value of V_i, the higher the ranking of the alternative. The advantage of the SAW method is that it is easy to use due to its simplicity (Velasquez and Hester, 2013). The disadvantage of the SAW is that the calculations seldom yield solutions that do not reflect the practical situations (Velasquez and Hester, 2013).

2.5.2 Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE)

Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE) is an MCDA method that was developed by Brans in 1982. It was further modified by Brans and Mareschal in 1994 (Kasperczyk and Knickel, n.d.). Elevli and Demicri (2004) described PROMETHEE as a method that is:

- Easy to understand, making it easy for the user or decision maker to understand the mathematics behind the method, yielding results that are transparent and can be easily scrutinised by parties other than the decision maker; and
- Easy to use since two parameters of the function are associated with the criterion to yield a single-valued outranking related to this criterion. When two alternatives cannot be compared in terms of preference or indifference then the two alternatives are said to be incomparable.

PROMETHEE is considered as an outranking method (Kasperczyk and Knickel, n.d.). A matrix V, consisting of v_{ij} elements is constructed, where a performance value of alternative A_i under criterion C_j, v_{ij} is shown, for $i = 1, \ldots\ldots, m$ and $j = 1, \ldots.. ,n$. A matrix K is constructed from matrix V, where k_{ij} shows the utility of alternative A_i, based on criteria C_j and its relationship with v_{ij} is shown in Equation 2-4 (Elevli and Demicri, 2004):

$$k_{ij} = f(v_{ij}) \tag{2-4}$$

The utility k_{ij} has values that lie between 0 and 1. A matrix G of quantitative weights of different criteria is also constructed where it represents weights (g) of different criteria, from 1 to j. When the three matrices V, K and G have been constructed, each alternative is ranked using Equation 2-5 (Elevli and Demicri, 2004):

$$N = K * G \tag{2-5}$$

Where N is a vector that shows the overall benefit of each alternative. After determining N, PROMETHEE is utilised in two stages. The first stage, PROMETHEE I is used for partial pre-order ranking of alternatives. PROMETHEE II is the last stage utilised for complete ranking of alternatives (Elevli and Demicri, 2004).

During the comparison of alternatives, a preference function, P(x_i,x_k) is used to represent the intensity of the preference of alternative x_i over alternative $x_{k,,}$ where $k \in$ (1, 2, …..n). Different types of preferences are represented thus (Elevli and Demicri, 2004):

- P(x_i,x_k) = 0 shows indifference or no preference of alternative x_i, over alternative x_k
- P(x_i,x_k) ~ 0 shows weak preference of x_i over x_k
- P(x_i,x_k) ~ 1 shows strong preference of x_i over x_k
- P(x_i,x_k) = 1 shows strict preference of x_i over x_k

The generalized criterion function of two alternatives i and k, H(d_{ik}) is used and it is represented in Equation 2-6:

$$H(d_{ik}) = \begin{cases} 0 \; if \; d_{ik} = 0 \\ 1 \; if \; d_{ik} \neq 0 \end{cases} \tag{2-6}$$

Where $d_{ik} = v(x_i) - v(x_k)$ is the deviation between two criteria $v(x_i)$ and $v(x_k)$ associated with alternatives A_i and A_k, respectively (Deshmukh, 2013). From preferences, a preference index is established to define the intensity of the decision maker's preference for x_i over x_k , $\Pi(x_i,x_k)$. The preference index is shown by Equation 2-7 as (Elevli and Demicri, 2004):

$$\Pi(x_i x_k) = \frac{\sum_{j=1}^{k} w_j * P(x_i,x_k)}{\sum_{j=1}^{k} w_j} \qquad \text{2-7}$$

Where w_j is the weighted average for criterion C_j. Similar to Equation 2-4, the preference index lies between 0 and 1, therefore the following holds (Elevli and Demicri, 2004):

- $H(d_{ik})$ ~ 0 shows weak preference of alternative x_i over x_k;
- $H(d_{ik})$ ~ 1 shows strong preference of alternative x_i over x_k for all criteria.

The preference index is used to determine the positive preference flow to measure an outranking character and negative preference flow to measure an outranked character for an alternative, as indicated by Equations 2-8 and 2-9 (Elevli and Demicri, 2004):

$$\Phi^{+}(x_i) = \sum_{k=1}^{m} \Pi(x_i x_k) \qquad \text{2-8}$$

and

$$\Phi^{-}(x_i) = \sum_{k=1}^{m} \Pi(x_k x_i) \qquad \text{2-9}$$

The difference between the positive preference flow (Equation 2-8) and negative preference flow (Equation 2-9) is called net flow (Elevli and Demicri, 2004). Equation 2-10 represents the net flow or the net preference of an alternative A_i, which is calculated as shown:

$$\Phi(x_i) = \Phi^{+}(x_i) - \Phi^{-}(x_i) \qquad \text{2-10}$$

Net flow is characteristic of PROMETHEE II, which is used for complete ranking of alternatives (Deshmukh, 2013). PROMETHEE II also ranks all the alternatives, from the best to the worst. If, for alternatives *i* and *k*, $\Phi(x_i)$ is greater than $\Phi(x_k)$ then alternative *i* is preferred. If $\Phi(x_i)$ is equal to $\Phi(x_k)$ then the alternatives are said to be indifferent from each other (Elevli & Demicri, 2004). The preference is important for it is used to evaluate the alternatives.

2.5.3 Multi-Attribute Utility Theory (MAUT)

Liu (2015) described the Multi-Attribute Utility Theory (MAUT) as one of the most widely used methodologies in multiple attribute decision making processes. MAUT is used to compare multiple attributes, by assigning numerical values (utilities) to objectives and giving them unit scales. Unit scales are assigned to allow for comparison under uniform conditions. Attributes (or criteria) being compared can either complement each other or substitute each other. MAUT provides the decision maker with qualitative and quantitative analysis to justify the chosen alternatives, given the existing criteria (Collins *et al*, 2006).

In MAUT, two extremes or conditions ranging from 0 to 1 are established, with 0 being the worst and 1 being the best. From there, utility functions of criteria lying between two extremes are established with respect to the best and worst values (or extremes) and weight assessments are made (Liu, 2015). Utilities can be determined through addition (additive utility) or multiplication (multiplicative utility). The aim of this method is to maximize the combined utility values in order to make the best decision.

The utility function, $U(x)$ for an attribute under a specific alternative is calculated using Equation 2-11:

$$U(x) = \frac{x - x_i^-}{x_i^+ - X_i^-} \tag{2-11}$$

Where x_i^- is the value of attribute x_i under the worst alternative; x_i^+ is the value of attribute x_i under the best alternative; and x is a value of attribute x_i under a specific alternative (Liu, 2015). Relative weights of attributes (k_i) are calculated by comparison of attributes and generating ratios such that $0 \leq k_i \leq 1$ and $\sum k_i = 1$ (Liu, 2015).

The relative weights are then multiplied by the individual utility functions to determine their utility, hence their ranking (Liu, 2015). The utility (U) is calculated as shown by Equation 2-12:

$$U = \sum_{i=1}^{m} k_i * U_i(x) \qquad 2\text{-}12$$

Where k_i is the relative weight of the i^{th} attribute and $U_i(x)$ is the utility function of the i^{th} attribute where $0 \leq U_i(x) \leq 1$. The alternative with the highest utility value is the one taken for consideration. The type of utility in Equation 2-12 is called the additive utility and the attributes are dependent on each other (Collins *et al*, 2006). With multiplicative utility, on the other hand, the utility is determined using Equation 2-13:

$$U = \frac{\prod_{i=1}^{m}[K * k_i * U_i(x) + 1]}{K} \qquad 2\text{-}13$$

Where k_i is the relative weight of attribute A_i and $0 \leq k_i \leq 1$; $U_i(x)$ is the utility function of the i^{th} attribute and K is a scaling constant that is determined as depicted in Equation 2-14:

$$1 + K = \prod_{i=1}^{m}(1 + (K * k_i)) \qquad 2\text{-}14$$

K must be iteratively determined (Collins *et al*, 2006). In this utility, all attributes are independent of each other and $-1 < K < 0$ indicates utility independence (Collins *et al*, 2006). MAUT has an advantage of aiding the decision maker to develop practical preference criteria. MAUT also aids in determining which assumptions are practical and in analysing the calculated utility functions (Velasquez and Hester, 2013).

2.5.4 Elimination and Choice Expressing Reality (ELECTRE)

The Elimination and Choice Expressing Reality or Elimination Et Choir Traduisant La Realite method is also known as ELECTRE method. It dates back as early as 1966, but only became widely known in 1968 (Buchanan and Sheppard, 1998). The ELECTRE is used when there are at least three criteria, in some instances from five to thirteen criteria to be compared. There are four sub-divisions of the method, namely ELECTRE I; II; III; and IV). ELECTRE usage consists of two main concepts which are threshold establishment and outranking a set of alternatives under defined criteria (Buchanan and Sheppard, 1998).

Firstly, for two alternatives *a* and *b* belonging to a set of alternatives *A* and defined criteria g_j (where j =1, 2,….r) a preference is introduced. In a preference alternative *a* (or *b*) is either preferred to; indifferent to; or cannot be compared to *b* (or *a*). A preference is described thus (Buchanan and Sheppard, 1998):

- $a\mathbf{P}b$: *a* is preferred to *b*, also presented by $g(a) > g(b)$
- $a\mathbf{I}b$: *a* is indifferent to *b*, also presented by $g(a) = g(b)$
- $a\mathbf{J}b$: *a* cannot be compared to *b*

Secondly, the indifference threshold, q is derived and its purpose is to measure weak preference. It is characteristic of the decision maker since it incorporates how the decision maker feels about the comparison (Buchanan and Sheppard, 1998). The indifference threshold is incorporated into the traditional preference modelling, as shown in Equation 2-15:

- $a\mathbf{P}b$ – *a* is preferred to *b*, where $\mathrm{g}(a) > g(b) + q$
- $a\mathbf{I}b$ – *a* is indifferent to *b*, where $|g(a) - g(b) \leq q$

- $a\mathbf{J}b$ – a is indifferent to b 2-15

Thirdly, a range or buffer between **P** (preference) and **I** (indifference) is created. This range or zone between the two extremes is known as the intermediary zone or the zone of weak preference (Buchanan and Sheppard, 1998). It is introduced to the preference model as a preference threshold, p. A "double threshold" model, is yielded where **Q** measures weak preference and **P** measures strong preference (Buchanan and Sheppard, 1998). Equation 2-16 represents the relationship between **P** and **Q** as:

- $a\mathbf{P}b$ – a is strongly preferred to b, where $g(a) - g(b) > p$
- $a\mathbf{Q}b$ – a is weakly preferred to b, where $q < g(a) - g(b) \leq p$
- $a\mathbf{I}b$ – a is indifferent to b and b to a; where $|g(a) - g(b)| \leq q$ 2-16

Once thresholds have been used to describe different models, the ELECTRE method then builds an outranking relation, **S.** For instance, $a\mathbf{S}b$ means that at least a is as good as b, or a is not worse than b. For each of the criterion under study (j), $a\mathbf{S}_jb$ means that a is good as b with respect to criterion j (Buchanan and Sheppard, 1998).

After **S,** two definitions, concordance and discordance, also known as harmony and disharmony respectively are introduced. A criterion j is in concordance with assertion $a\mathbf{S}b$ if "a is as good as b" or $g_j(a) \geq g_j(b) - q_j$ even if $g_j(a)$ is less than $g_j(b)$ with a value of q_j. A criterion j is in discordance with assertion $a\mathbf{S}b$ if b is strongly preferred to a, or $g_j(b) \geq g_j(a) + p_j$.

Concordance and discordance definitions seek to address whether, for a certain criterion j, and alternatives a and b there is a harmony or disharmony with the statement of whether a is as good as b (Buchanan and Sheppard, 1998). To quantify concordance and discordance of alternatives a and b and to measure the strength of assertion $a\mathbf{S}b$, Concordance (C) indices are introduced, see Equation 2-17:

$$C(a,b) = \frac{1}{k} {}_{j=1}^{r} k_j c_j(a,b); where\ k = {}_{j=1}^{r} k_j \tag{2-17}$$

Where $C(a,b)$ is the concordance index of alternatives $(a, b) \in A$; k_j is the importance coefficient of criterion j and A is the set of all alternatives. Lastly, a discordance matrix is derived. In the same manner, a discordance matrix is derived from the discordance index. The discordance and concordance matrices are combined to form a credibility matrix, which is used to provide the assertion between alternatives *a* and *b* (Buchanan and Sheppard, 1998). The final ranking of alternatives is done with the credibility matrix and graph theory concepts.

ELECTRE methods account for uncertainties through creating thresholds and intermediary zones between two extreme preferences (Velasquez and Hester, 2013). Conversely, the intermediary zones are created to cover the strengths and weaknesses of the alternatives. ELECTRE methods may appear to be challenging to comprehend and not user friendly (Velasquez and Hester, 2013).

2.5.5 Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS)

TOPSIS is a MCDA method that was first proposed by Hwang and Yoon in 1981 (Kabir and Hasin, 2012). TOPSIS is used to solve multiple attribute decision making problems. The assumption of TOPSIS is that each evaluation indicator has a feature that is either decreasing or increasing monotonically (Chen *et al*, 2015). TOPSIS is easy to implement and useful when the decision maker wants a simple weighting approach (Kabir and Hasin, 2012).

TOPSIS establishes a positive ideal solution composed of optimal values and a negative ideal solution consisting of the worst values for all indicators. The indicators established are compared to the two ideal solutions through the calculation of Euclean distances (Kabir and Hasin, 2012). The best solution has the shortest distance to the positive ideal solution and the longest distance to the negative ideal solution (Chen *et al*, 2015). Kabir and Hasin (2012) defined the positive ideal solution as a solution that maximises the benefit criteria, while minimising the cost criteria. The contrary is true for the negative ideal solution.

Firstly, for attributes (C_j, j = 1, 2,….., n) and alternatives (A_i, i = 1, 2,……, m), a normalised decision matrix is constructed, which allows for the comparison of attributes or criteria through giving them a comparable scale (Kabir and Hasin, 2012). Normalisation is calculated using Equation 2-18:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad \text{2-18}$$

Where r_{ij} is the normalised value of the performance attribute; and x_{ij} is an element of the matrix X indicating the performance of the i^{th} alternative A_i with respect to the j^{th} attribute C_j (Kabir and Hasin, 2012).

Secondly, a weighted normalised decision matrix is constructed for the criteria and a set of weights for the criteria C_j is assumed. Weighting is done to show the importance of a criterion relative to other criteria (Kabir and Hasin, 2012). Each column of the normalised matrix is multiplied with the corresponding weight w_j to get a matrix of order m x n. Equation 2-19 shows how a weighted normalised value (or outcome), v_{ij} is calculated (Kabir and Hasin, 2012):

$$\upsilon_{ij} = w_j r_{ij},\ for\ i = 1, 2, \ldots\ldots, m\ and\ \ j = 1, 2, \ldots\ldots\ldots, n \qquad \text{2-19}$$

Where w_j is the relative importance of each attribute to the others; and r_{ij} is the normalised value of the performance of attribute A_i with respect to criterion C_j. Thirdly, a negative ideal solution (A^-) and a positive ideal solution (A^+) are developed. The ideal solutions are defined in terms of the weighted normalised values. To calculate a positive ideal solution (A^+) and a negative ideal solution (A^-) . Equations 2-20 and 2-21 are used respectively (Kabir and Hasin, 2012):

$$A^{+} = \{\upsilon_1, \ldots\ldots, \upsilon_n\} where\ \upsilon_j = \{max(\upsilon_{ij})\ if\ j \in J; min(\upsilon_{ij})\ if\ j \in \check{J}\} \qquad 2\text{-}20$$

$$A^{-} = \{\upsilon_1^{-}, \ldots\ldots, \upsilon_n^{-}\} where\ \acute{\upsilon}_j = \{min(\upsilon_{ij})\ if\ j \in J; max(\upsilon_{ij})\ if\ j \in \check{J}\} \qquad 2\text{-}21$$

Where J and $\check{J}$ are sets of benefit attributes and cost attributes, respectively. In an ideal situation, J is supposed to be larger than $\check{J}$. Separation measures of each alternative (from the positive and negative ideal alternatives) S_i^+ and S_i^- are calculated using Equations 2-22 and 2-23:

$$S_i^{+} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{+})^2} \qquad 2\text{-}22$$

$$S_i^{-} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{-})^2} \qquad 2\text{-}23$$

For $i = 1, 2, \ldots.., m$ and the weight of each pair of alternatives A_{ii} relative to criterion C_j, given by v_{ij}. Equation 2-24 shows how the relative closeness or similarity of alternatives to the ideal solution C_i^+ is calculated:

$$C_i^{+} = \frac{S_i^{-}}{(S_i^{+} + S_i^{-})}, 0 < C_i^{+} < 1 \qquad 2\text{-}24$$

Thirdly, the alternatives are ranked by comparing C_i^+ values. The alternative with the highest relative closeness to the ideal solution is taken as the best alternative (Kabir and Hasin, 2012). TOPSIS is user friendly, making it a commonly used method. The number of steps in problem solving is not affected by the varying number of criteria (Velasquez and Hester, 2013). The Euclean distance does not consider linked criteria. Consistent weighing of attributes is challenged when the number of criteria increases (Velasquez and Hester, 2013).

2.5.6 Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR)

Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a technique that was initially introduced by Serafim Opricovic in 1998 (Bulgurcu, 2016).

Opricovic and Tzeng further developed the VIKOR method and in 2004 it was fully recognised (Bulgurcu, 2016). VIKOR is an MCDA technique that aids in making a decision by comparing solutions to the ideal solution. The best solution is considered as the compromise solution and a solution closest to the compromise solution is considered to be the preferred solution (Bulgurcu, 2016).

Firstly, when using VIKOR, a decision matrix X is developed. The decision matrix is then normalised to yield a normalised matrix R and normalisation is conducted as shown in Equation 2-25 (Hayati *et al*, 2015):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}} \qquad 2\text{-}25$$

Where x_{ij} is the value assigned to alternative A_i in criterion C_j of matrix X; and r_{ij} is the normalised value for alternative A_i in criterion C_j in matrix R. Secondly, the normalised decision matrix R is multiplied with the corresponding weighting of criterion C_j (W_j) to obtain a weighted normalised matrix F, as shown in Equation 2-26 (Hayati *et al*, 2015):

$$f_{ij} = x_{ij} * W_{ij} \qquad 2\text{-}26$$

Where f_{ij} is the weighted normalised value for matrix F. Thirdly, the ideal positive f_j^* and ideal negative f_j^- solutions are determined. For a criterion to be maximised f_j^* is the maximum value of f_{ij} and f_j^- is the minimum value of f_{ij} for criterion C_j. For a criterion to be minimised, f_j^* is the minimum value of f_{ij} and f_j^- is the maximum value of f_{ij} for criterion C_j (Hayati *et al*, 2015).

The calculated ideal solutions are used to determine the satisfaction (S_i) and rejection (R_i) indices (Hayati *et al*, 2015). S_i shows the distance of an alternative to the ideal positive value, while R_i shows the maximum distance from the ideal positive value. Equation 2-27 demonstrates how R_i and S_i are calculated (Hayati *et al*, 2015):

$$S_i = \sum W_j * \frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \qquad and\ R_i = max\left[W_j * \frac{f_j^* - f_{ij}}{f_j^* - f_j^-}\right] \tag{2-27}$$

Lastly, for each alternative A_i, the VIKOR index (Q_i) has to be calculated. Q considers the satisfaction and rejection indices, together with a ranking constant. Q is calculated according to Equation 2-28 (Hayati *et al*, 2015):

$$Q_i = \nu\left[\frac{S_i - S^-}{S^* - S^-}\right] + (1 - \nu)\left[\frac{R_i - R^-}{R^* - R^*}\right] \tag{2-28}$$

Where, for *n* alternatives, S^* and S^- are maximum and minimum *S* values while S_i is the *S* value for alternative A_i. R^* and R^- are also maximum and minimum *R* values while R_i is the *R* value for alternative A_i. ν is the ranking constant and is usually determined by the decision maker. The VIKOR index is used to rank alternatives, whereby the highest ranking alternative will have the smallest Q_i value and the lowest ranking alternative will have the Q_i highest value (Bulgurcu, 2016).

The main advantage of using the VIKOR method is that the preferred alternative is chosen through maximising the "satisfaction" or S_i while minimising the "regret" or R_i for that alternative (Gavade, 2014). The disadvantage of the VIKOR method is that it uses discrete values (not a range of values) and discrete values do not reflect practical situations.

2.5.7 Analytical Hierarchy Process (AHP)

The Analytical Hierarchy Process (AHP) is a multi-criteria decision making method that was first introduced by Saaty in 1977. This method is used to solve complex decision problems by using objectives; criteria; and alternatives (Triantaphyllou and Mann, 1995). All these are used to obtain the weights of criteria once a pairwise comparison of criteria has been conducted.

The AHP is a method that enables comparison of both qualitative and quantitative factors to aid in the selection process (Abdalla *et al*, 2013). The AHP is based on a mathematical framework of matrix and vector algebra (eigenvectors). The mathematical foundation eases calculations and enables the use of Microsoft Excel (Musingwini and Minnitt, 2008).

Firstly, a pairwise comparison of criteria is conducted to determine the weight of each criterion (Abdalla *et al*, 2013). The weight of criterion C_i relative to criterion C_j is given by v_{ij} and a pairwise comparison is consistent if conditions in Equation 2-29 hold:

$$\upsilon_{ij} = \frac{1}{\upsilon_{ji}} \; for\; i \neq j \; and \; \upsilon_{ii} = 1 \qquad 2\text{-}29$$

The determined weights are then used to form a pairwise comparison matrix *A* of order *n* x *n*. The matrix is known as a reciprocal matrix due to the reciprocal relationship that each criterion has to the next one (Abdalla *et al*, 2013). From matrix *A*, a weight vector *w* of *n* objectives is established. This is an eigenvector and it averages all possible ways of viewing alternatives. Equation 2-30 shows the relationship between the eigenvector *w* and eigenvalue (λ_A) of matrix *A* (Musingwini and Minnitt, 2008):

$$Aw = \lambda_A \qquad 2\text{-}30$$

One of the properties of the AHP is that it is able to assist the decision maker with improving inconsistency to near-consistency (Abdalla *et al*, 2013). Inconsistency may be due to human judgements that are characteristic of the decision maker. The pairwise matrix is evaluated for consistency. A matrix is said to be consistent if the relative importance is consistent (Abdalla *et al*, 2013). Equation 2-31 shows the condition of consistency, according to Musingwini and Minnitt (2008):

$$\lambda_{max} = n \qquad 2\text{-}31$$

Where n is the order of the matrix dealt with and $\lambda_{\max}$ is the corresponding eigenvector of the matrix. The Consistency Index (CI) is calculated to measure the deviation of a value from consistency (Abdalla *et al*, 2013). Equation 2-32 indicates how the CI is calculated:

$$CI = \frac{(\lambda_{max} - n)}{(n-1)} \qquad 2\text{-}32$$

The Consistency Ratio (CR) is calculated to determine the acceptable consistency of the decision maker. To calculate CRs, Random Indices (RIs) are necessary (Musingwini and Minnitt, 2008). Saaty (1980) (cited in (Musingwini and Minnitt, 2008)) simulated large samples of random matrices then increased their order, to determine their corresponding CIs.

The CIs corresponding to the random matrices generated are known as RIs. The CR is determined by dividing a CI by its corresponding RI. A CR of 0 means that the decision maker's judgements are perfectly consistent while a CR of 1 means that the decision maker's judgements are completely inconsistent and cannot be trusted (Musingwini and Minnitt, 2008). For an acceptable degree of consistency, the efficiencies of all alternatives on a criterion, $\mathbf{E}_{ij}$ are normalised (Musingwini and Minnitt, 2008). Hence for m alternatives on a criterion, the normalised values, $\boldsymbol{E}_{ij}^{N}$, are calculated as shown in Equation 2-33 (Musingwini and Minnitt, 2008):

$$\boldsymbol{E}_{ij}^{N} = \frac{\boldsymbol{E}_{ij}}{\sum_{i=1}^{m} \boldsymbol{E}_{ij}} \qquad 2\text{-}33$$

The normalised performance matrix values are multiplied by the eigenvector (w) then summed to yield a performance score. Performance scores are important for they indicate the rank of alternatives. The higher the performance score, the better the alternative (Musingwini and Minnitt, 2008). There is interdependence between criteria when using the AHP technique. This interdependence can result in inconsistencies when judging and ranking criteria (Velasquez and Hester, 2013).

2.5.8 Yager's method

Yager's method or Yager's fuzzy bags is a method introduced by Yager and it involves designing structures to view multisets for mathematical analysis. These multisets are referred to as "bags". Yager also defined these "bags" as a set linked with a count function (Biswas, 1999). This method is based on decision makers and fuzzy bags. For a problem of selecting the most suitable action out of *n* alternatives based on *K* criteria and *m* judges or decision makers in the form of fuzzy bags, Yager's method involves the following steps of analysis (Biswas, 1999):

For *n* alternatives A_i (i = 1, 2, ….., n) and *k* criteria C_j (j = 1, 2, …….., n); each of the different criteria, p_{ij} are judged by *m* experts, E_1, E_2, ….., E_m. The criteria values then form a fuzzy bag that is drawn from the set *C* of all criteria that corresponds to each alternative, A_i. From this a Matrix *M* is formed, where M = (n x k) matrix (Biswas, 1999).

From Matrix *M*, criteria values {p_{i1}, p_{i2}, ……, p_{ik}} that correspond to alternatives A_i are used to form a fuzzy bag F_i that is drawn from the criteria set, C = {C_1, C_2, ….., C_k} (Biswas, 1999). The aim is to choose the most suitable alternative. To achieve this, two fuzzy bags must have their similarity measured. Consequently, this method is termed the "Similarity Measurement Method" (Biswas, 1999).

Once the matrix *M* has been constructed, a fuzzy bag F_i that corresponds to alternative A_i (for instance a fuzzy bag A_1 will correspond to an alternative A_1, and so on) is produced through evaluation by multiple judges. For each fuzzy bag, F_i, the support fuzzy bag is given by Equation 2-34 (Biswas, 1999):

$$sf(F_i) = \left\{\frac{C_1}{\left\{\frac{1}{m}\right\}}, \frac{C_2}{\left\{\frac{1}{m}\right\}}, \frac{C_3}{\left\{\frac{1}{m}\right\}}\right. \qquad \text{2-34}$$

It is important to realise that for *i* alternatives, $sf(F_i) = F$. From Equation 2-34, the obtained fuzzy bag F_i is compared to the established fuzzy bag F to determine similarity through similarity measures. For instance, $sf(F_1)$ will be compared to F. For *n* alternatives, similarity measures S_i are then calculated using Equation 2-35 (Biswas, 1999):

$$S_i = S(F, F_i) for\ i = 1, 2, 3, \ldots, n \tag{2-35}$$

These similarity measures are arranged in descending order of magnitude as indicated in Equation 2-36:

$$S_{r_1} \geq S_{r_2} \geq S_{r_3} \geq \ldots . \geq S_{r_n} \tag{2-36}$$

Where S_{r_i} is the measure of similarity index between any fuzzy bag, F_i and the fuzzy bag F, for n alternatives. Lastly, the alternative corresponding to the highest similarity measure is taken as the best alternative, that is A_{r1} (Biswas, 1999). Yager's method has a limitation of not dealing with a high number of criteria (Yavuz, 2015a).

2.6 Chapter summary

The aim of Chapter 2 was to introduce MCDA methods and associated literature. Seven authors who reviewed different MCDA methods were selected and their work reviewed. Although each of the authors had a unique classification, they all commonly identified certain methods. In addition, the case studies identified for the research study were classified in terms of methods used to address them.

A total of 246 mine planning and related case studies were identified and grouped according to the MCDA techniques employed to rank alternatives. The frequency of use of different techniques in the 246 case studies was the basis of selecting techniques. The frequency of use was paired with work done by different authors to select techniques to be used. The methods selected for analysis were AHP; MAUT; TOPSIS; ELECTRE; VIKOR; PROMETHEE; SAW; and Yager's method.

The eight methods selected for analysis were briefly introduced and the principle behind them stated. Chapter 3 presents case studies used for analysis and the methodology used for analysis of these case studies. "Similarity percentages" and average "similarity percentages" are also introduced in Chapter 3, as means of analysing the identified case studies.

3 METHODOLOGY

This chapter presents the number of case studies reviewed and justification for their selection. It also highlights different MCDA techniques selected for analysis of the reviewed case studies as explained in Chapter 2. The methodologies used for obtaining different rankings and that adopted for comparison of alternative rankings obtained from different techniques are explained in Section 3.3. An approach of presenting results obtained is discussed in Section 3.4.

3.1 Case studies selected for the analysis

A total of 246 mine planning and related case studies were identified from the public domain, as listed in Appendix 7-1. The case studies were narrowed down to case studies with numerical information that could be analysed as data. This procedure resulted in 107 case studies being considered for the research study. In addition, 11 papers were identified as duplicate papers. Eight of the 11 case studies represented four case studies while the other three had inadequate numerical information. The eight papers were published by the same author, but in different journals and symposia.

The total number of case studies considered for the research study was 111. Case studies that could not be analysed due to absence of original decision matrix were not considered for this study. Publication dates of the identified case studies date as far back as 1984 to the most recent, 2016. Figure 3-1 illustrates the selection flow process of the 111 case studies out of the 246 identified case studies.

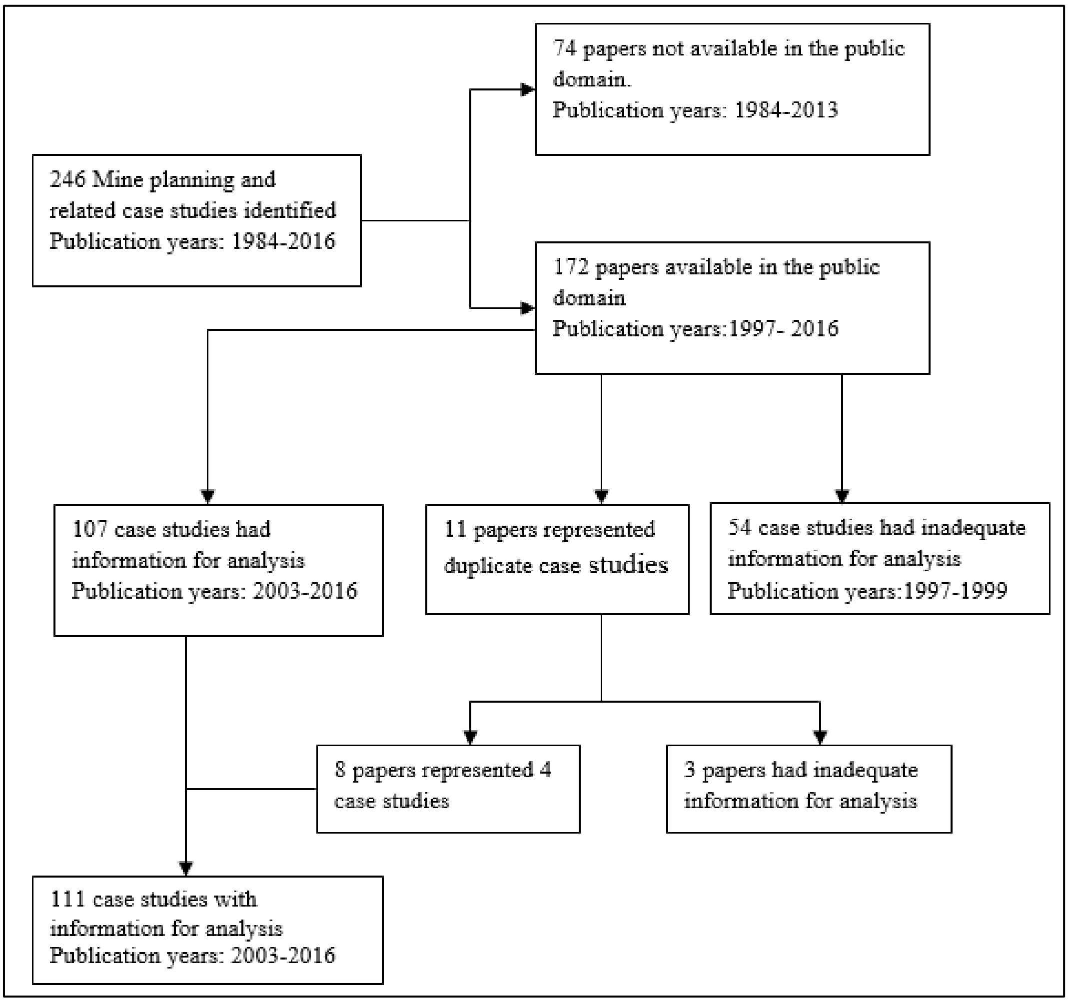

Figure 3-1 Flow process diagram for selection of case studies

3.2 MCDA techniques selected for the analysis

The selection of MCDA techniques used for the analysis of the case studies was based on the most commonly used methods as identified by different authors. The techniques were also selected based on the frequency of use of each techniques, as shown in the 246 identified case studies (Section 2.4). These two approaches resulted in AHP, ELECTRE; MAUT, PROMETHEE, SAW, TOPSIS, VIKOR; and Yager's method as the techniques selected for analysis.

3.3 The analysis procedure

The procedure for analysing the case studies was done in two steps namely; the ranking of alternatives in each case study and calculation of a “similarity percentage” for each case study. In both steps Microsoft Excel was used to present a list of case studies and for calculation purposes. Microsoft Excel was used for performing calculations due to its availability and simplicity of the computation process.

3.3.1 Ranking of alternatives

In order to normalise matrices for calculation purposes, original matrices are required. The normalisation process is unique for each MCDA technique. Once a decision matrix was identified it was normalised using a method unique to each MCDA technique. A matrix is normalised to enable comparison of alternatives under the same conditions. The units of different parameters (for instance costs; distances; and criteria) are made dimensionless in this step.

A normalised matrix consisting of dimensionless numbers is generated from the normalisation process. From the normalised matrix calculations were executed to produce a single value for each alternative. The values were used to rank the alternatives. The ranking process is unique for each selected MCDA technique. Appendices 7-3 to 7-10 illustrate the procedure of normalisation and ranking of alternatives for each technique. Appendices 7-3 to 7-11 are included in the attached memory stick and they are also available on request.

3.3.2 Comparison of rankings obtained for different alternatives

Results from each of the MCDA techniques were compared in terms of ranking of different alternatives as shown in Appendices 7-3 to 7-11. Appendices 7-3 to 7-11 are included in the attached memory stick and they are available on request. Each set of the ranked alternatives was analysed by comparing it to another set and the "similarity percentage" measured. The "similarity percentage" was measured through comparison of the obtained ranking of the alternatives through one method to the ranking obtained through another method. For instance, if one alternative has the same ranking as an alternative ranked with another technique then the "similarity percentage" is 100%.

Conversely, if there is zero similarity in terms of ranking between two alternatives then the "similarity percentage" is zero (0%). An average "similarity percentage" of a technique was then obtained by calculating the average of "similarity percentages" of alternatives within a set. The average "similarity percentage" assists in providing a level of precision in terms of ranking of different alternatives. The higher the percentage, the higher the precision (closeness of data) and high precision results in more confidence during the decision making process.

Table 3-1 serves as an illustrative example of how the "similarity percentage" and the average "similarity percentage" were calculated. Three methods (M1; M2; and M3) were used to rank four alternatives (A1; A2; A3; and A4) under a number of criteria. Results from different methods were compared in terms of "similarity percentages" and average "similarity percentages".

Table 3-1 Generic similarity percentages of alternatives ranked using three MCDA methods

Alternatives	Ranking according to different MCDA methods			Similarity Percentage (%)
	M1	M2	M3	
A1	4	4	2	66.7
A2	3	3	4	66.7
A3	1	1	1	100
A4	2	2	3	66.7
Average similarity percentage				**75.0**

The average "similarity percentage" for each case study was plotted against the number of criteria and alternatives for each case study. A trend was observed such that a relationship between the criteria; alternatives; and average "similarity percentage" was established. The relationship was used to recommend the ideal number of alternatives and criteria to use when decision making is conducted using MCDA techniques.

3.4 Presentation of results

The identified case studies are included in Appendix 7-1 while Appendices 7-3 to 7-11 (included in the memory stick attached and are available on request) illustrate calculations using different MCDA techniques. Calculations of the "average similarity percentages" for the case studies are included in Appendix 7-11 (included in the memory stick attached and is available on request). In the appendices, different colours were used to distinguish between case studies and to show the status of the case studies. The colours were also used to illustrate maximum and minimum values; minimising or maximising of criteria; weighting of criteria; and ranking of alternatives. Table 3-2 depicts different colour codes and their description.

Table 3-2 Generic colour coding used and their descriptions

Appendix	Colour coding	Description
7-1		Case studies not available in the public domain
		Case studies available in the public domain with adequate information
		Case studies available in the public domain, but with inadequate information
		Duplicate case studies
7-3 to 7-11		Weighting of criteria and whether criteria are maximised or minimised
		Minimum values in the matrix
		Maximum values in the matrix
		Ranking of alternatives according to the original MCDA technique used in the case study
		Ranking of alternatives according to the MCDA technique different from the one originally used in the case study

In some case studies, some of the MCDA techniques chosen for the analysis could not be used to rank the alternatives. Such case studies lacked an original decision matrix in them; complete information (alternatives or criteria); or weighting of criteria. In such instances, the ranking was indicated with “not applicable” (N/A) and the reason for inapplicability of the technique given. The applicability of the method was based on the information provided in the case studies and its completeness. Appendices 7-3 to 7-11 (included in the memory stick and are available on request) show such case studies and the reason for them not being applicable.

The results obtained after analysing the case studies with different MCDA methods were discussed in terms of:

- Effect of using alternative methods to those used in case studies on the final result, in terms of ranking of alternatives;
- Observed behaviour of the methods used for analysis with increasing number of criteria and alternatives;
- Closeness or deviation of ranking of alternatives with different methods used; and
- The ideal MCDA method as the number of criteria and alternatives change.

3.5 Chapter summary

Chapter 3 introduced the case studies; techniques; and procedure chosen for analysis. A total of 111 mine planning and related case studies were selected out of the 246 identified case studies. MCDA methods used for analysis were chosen based on their frequency of use to analyse the identified case studies.

A total of eight methods were selected, namely AHP; ELECTRE; MAUT; PROMETHEE; SAW; TOPSIS; VIKOR and Yager's method. The case studies and calculations were presented in Appendices 7-1 to 7-11. Appendices 7-3 to 7-11 are included in the memory stick and they are also available on request. Assumptions made; results; and analysis based on the 111 case studies are presented in Chapter 4.

4 ANALYSIS OF CASE STUDIES

Chapter 4 presents the results from the analysis of the identified case studies using the selected MCDA techniques. The chapter also states different assumptions made, concerning the techniques used for analysis. The "similarity percentage" for each case study is used to establish the relationship between the accuracy of techniques and the number of criteria and alternatives. The identified case studies are analysed in Sections 4.2 to 4.4. Section 4.5 states the possible sources of error.

4.1 Assumptions and versions of techniques selected for analysis

Some of the selected MCDA techniques have more than one version. In such cases, a specific version was chosen and its choice justified. ELECTRE I and II of the ELECTRE methods were chosen. PROMETHEE I and II of the PROMETHEE methods were also chosen. In some techniques, assumptions had to be made so that the techniques could be implemented. Assumptions were made with VIKOR and ELECTRE methods.

4.1.1 ELECTRE methods

The ELECTRE technique has four versions namely; ELECTRE I; II; III; and IV (Figueira *et al*, n.d.). ELECTRE I was established to choose a set of alternatives and ELECTRE II was established to rank a set of alternatives, from best to worst (Figueira *et al*, n.d.). ELECTRE III was invented to rank alternatives using a fuzzy relation and ELECTRE IV was built to generate a set of non-fuzzy outranking relations in instances where criteria weights cannot be established.

There is more complexity with movement from ELECTRE I to ELECTRE IV (Figueira *et al*, n.d.). For instance, ELECTRE II has two thresholds while ELECTRE III has three thresholds, in a fuzzy environment. This means that when more thresholds are introduced, it becomes less reliable to work with the thresholds, especially in instances where thresholds have to be assumed. In most of the identified case studies there were no thresholds so the thresholds had to be assumed.

The selected methods for analysis were ELECTRE I and II, due to their appropriateness to the case studies. ELECTRE I was used to generate concordance and discordance matrices, such that concordance and discordance indices could be calculated. ELECTRE II was used to rank the alternatives from best to worst based on the indices. In order to use ELECTRE II, the concordance and discordance thresholds had to be assumed. The thresholds are compared to concordance and discordance indices of different alternatives as calculated in ELECTRE I such that (Figueira *et al*, n.d.):

- If a concordance index is greater than the concordance threshold, then an alternative is ranked better; and
- If a discordance index is less than the discordance threshold, then an alternative is ranked better.

Threshold values lie between zero and one (Figueira *et al*, n.d.). There are no absolute values for the thresholds, but they are set by decision makers. The concordance threshold has to be bigger than the discordance threshold. Buchanan and Sheppard (1998) stated that in order to rank alternatives in the real world, the thresholds must have non-zero values. Milani *et al* (2006) further stipulated that high threshold values are not ideal since they make ranking of alternatives difficult.

Collette and Siarry (2003) as cited in Milani *et al* (2006) established that the normally used values for concordance and discordance thresholds are 0,7 and 0,3, respectively. The established values were then used by Milani *et al* (2006); Chatterjee *et al*. (2009) and Chatterjee *et al* (2014). For this research study, the concordance and discordance indices were also assumed to be 0,7 and 0,3, respectively.

4.1.2 PROMETHEE methods

There are two PROMETHEE methods namely; PROMETHEE I and II (Brans and Vincke, 1985). PROMETHEE I is used to partially rank alternatives while PROMETHEE II is used for complete ranking of alternatives. In the calculations, PROMETHEE I was used to partially rank alternatives and hence generate a matrix of preference. In the matrix, preference of alternatives over other alternatives was calculated for each criterion. The overall preference index was obtained by multiplying the preference of each alternative by the weighting of the corresponding criterion.

PROMETHEE II was used for complete ranking of the alternatives. Using a matrix generated from PROMETHEE I, an incoming flow and outgoing flow were calculated (Appendix 7-6, included in the memory stick and also available on request). PROMETHEE II was then used to obtain the net flow which was used to rank alternatives. The net flow is defined as the difference between the outgoing flow and incoming flow, in terms of the preference of an alternative.

4.1.3 VIKOR method

When ranking alternatives using VIKOR method, the index value (Q) of each alternative has to be calculated (Jahan *et al*, 2013). The index value is used to rank the alternatives, where the alternative with the minimum Q is considered as the best alternative. To calculate Q, the "weight of the strategy of the majority of attributes" (*v*) has to be calculated (Jahan *et al*, 2013).

The value of *v* ranges from 0 to 1, but if *v* is not given, an assumption of its value is normally 0,5. Chatterjee *et al* (2009); Wang and Pang (2011); and Jahan *et al* (2013) all assumed the value of *v* as 0,5 in their use of the VIKOR method. For calculations using the VIKOR method in the research study, *v* was also taken as 0,5.

4.1.4 MAUT method

For analysis using the MAUT method, the Additive Utility Theory (AUT) was chosen over the Multiplicative Utility Theory (MUT). The constant in the MUT must be iteratively determined and could therefore not be done in this research study due to time constraints as many case studies had to be evaluated.

Iterative calculations need repeated cycles of information. Equation 2-14 in Section 2.5.3 shows the constant K and how it is calculated. Collins *et al* (2006) preferred the AUT to the MUT because the AUT is easier to use due to easy calculations. In addition, AUT can be used in spreadsheets making it more versatile relative to the MUT.

4.2 Analysis of the 246 identified case studies

The mining industry earns the majority of Africa's revenue from exports (Southern African Resource Watch, 2009). South Africa is no exception, having the mining industry directly contributing about 7.7% and indirectly contributing 17.7% of the Gross Domestic Product (GDP) in 2015 (Chamber of Mines of South Africa, 2015). The mining sector directly and indirectly contributes to the economic growth of South Africa.

In 2015 the South African mining sector sold commodities worth R391.4 billion. A total of R271 billion out of R391.4 billion from sold commodities came from exported commodities (Chamber of Mines of South Africa, 2015). South Africa interacts with other countries in the world through activities like global trading. This means that any major negative event in other countries has a global impact, South Africa included.

4.2.1 The 1998 economic meltdown

In 1998, the Thai government dissociated their currency from the United States Dollar (USD). The banks and currency markets in Thailand collapsed, affecting the whole of Asia and the world at large (National Treasury, 1998). South Africa was also impacted by this event, whereby in 1998 there was a decrease in foreign earnings and decreased national income. The economic annual growth decreased from 3% in 1997 to 0,2% in 1998 (National Treasury, 1998).

4.2.2 The 2008 Global Financial Crisis

The Global Financial Crisis (GFC) of mid-2008 impacted the global financial atmosphere negatively (Nhleko and Musingwini, 2016). Developing countries were also impacted by the recession since there was a decrease in the mining exports (Southern African Resource Watch, 2009). The commodity prices also sharply declined due to decreased demand for commodities compounded with increased supply of the commodities (Southern African Resource Watch, 2009).

The mid-2008 GFC also resulted in restricted access to capital for various projects (Nhleko and Musingwini, 2016). This meant that mining projects had to compete with other projects for the limited capital. The restricted access to capital implied that strategic decisions made for projects had to be as accurate as possible. Any wrong decisions in the projects would have a negative impact on the already scarce capital.

The 1998 economic slowdown and the mid-2008 GFC had a negative impact on both developing and developed countries. Figure 4-1 illustrates the growth rate in USD for Chile; Australia; Peru and South Africa (Hausmann, 2014). The growth rate is shown for the period 1996 to 2014. From Figure 4-1 it can be noted that the four countries were negatively affected by the 1998 economic meltdown and the

2008 GFC. South Africa was most adversely impacted by the 2008 GFC and it failed to recover afterwards, whereas other countries started to pick up from 2013.

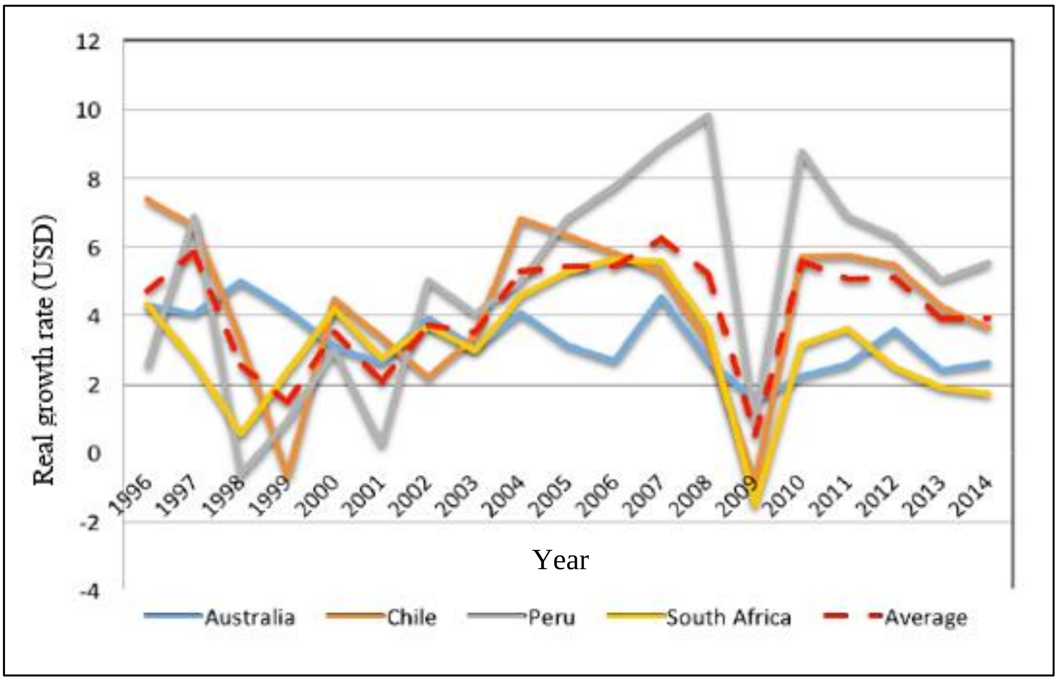

Figure 4-1 The growth rate in USD for Chile; Australia; Peru and South Africa

Source: Hausmann (2014)

4.2.3 The relationship between the 1998 slowdown; the mid-2008 GFC and application of MCDA techniques

Figure 4-2 depicts the usage of MCDA techniques in the 246 identified case studies, from 1997 to 2016. Figure 4-2 illustrates the increased use of MCDA techniques in the case studies published in 1999. The number of MCDA techniques used in case studies published rose from one in 1997 to three in 1999. Case studies published between 2008 and 2016 showed an increase in the number of MCDA techniques used. The increase was in comparison to case studies published in 2007, where only two MCDA techniques were used.

There are four case studies published as far as of March 2016. It should be noted that this number may increase, as the year has not ended. Figures 4-1 and 4-2 show a correlation between the increased use of MCDA techniques and the period post 1998 and 2008. The increased use of MCDA techniques could be attributed to the increased uncertainties in 1999 and 2008. The economic recession had more impact on the countries than the 1998 slowdown as shown in Figure 4-1. To address the decision-making uncertainty post 2008, more MCDA techniques were employed.

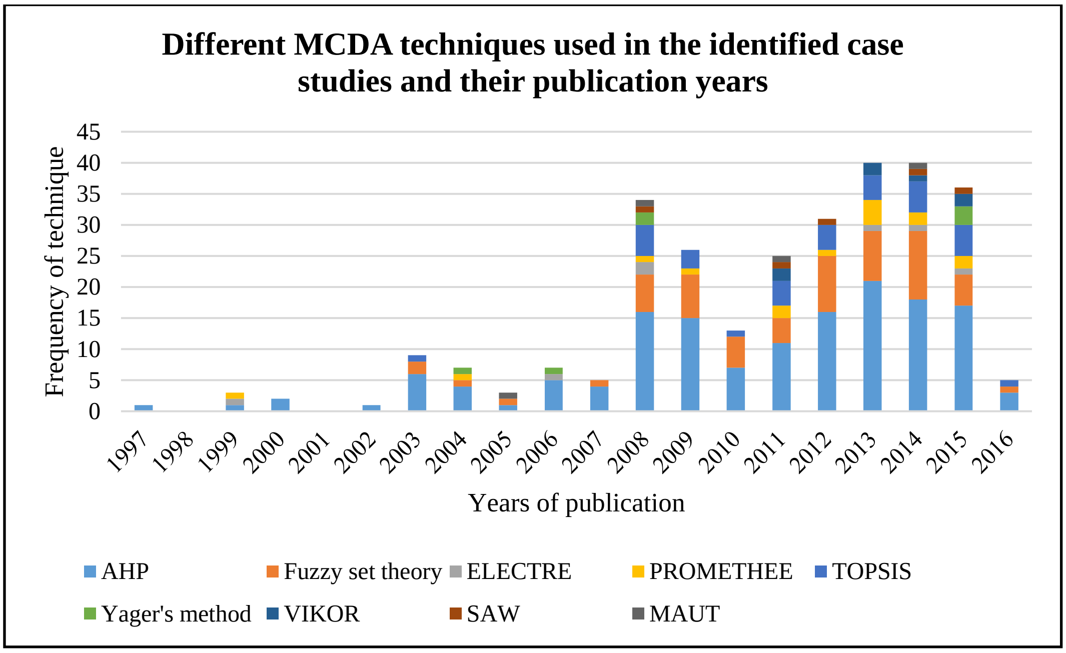

Figure 4-2 Different MCDA techniques used in the identified case studies and their publication years

Figure 4-2 illustrates three or more MCDA techniques being used in case studies published after 2008. The maximum number of MCDA techniques observed was eight, with AHP and Fuzzy set theory being prominent. AHP is considered as easy to use while showing an ordered structure of alternatives and criteria. The Fuzzy set theory was identified as one of the approaches to address uncertainty when using MCDA techniques (Broekhuizen *et al*, 2015). To deal with uncertainties in decision making, more MCDA techniques were used to solve the case studies.

4.2.4 Summary on the analysis of the 246 identified case studies

Figure 4-1 depicts the 1998 economic meltdown and the 2008 GFC. After the 1998 and 2008 crises, there was an increased risk and uncertainty associated with the global economic environment. This necessitated use of decision making techniques in an uncertain environment. Figure 4-1 also illustrates a level of recovery post 1998 compared to post 2008, where a slow recovery was exhibited. This observation correlates with the use of MCDA techniques post 1998 and 2008, as shown in Figure 4-2.

Figure 4-2 illustrates the increasing number of MCDA techniques used with progressing years. This was interpreted as representing a response to making decisions in the increasingly uncertain environment. To decrease uncertainty in decision making, more MCDA techniques were employed. Figure 4-2 shows the correlation between uncertainty and the use of MCDA techniques in solving case studies. After the 1998 and 2008 global financial crises, uncertainty around decision making resulted. The uncertainty resulted in increased usage of MCDA techniques. The more uncertain a decision is, the more the MCDA techniques should be used. MCDA techniques are preferred for decision making in an uncertain environment.

4.3 Limitations of the identified 246 case studies

A total of 246 case studies were identified. A total of 111 out of 246 case studies had matrices or numerical content. Not every case study that had numerical content had an original decision matrix. Some case studies had normalised decision matrices which could not be further used for calculations, as explained in Section 3.4. Such case studies were included in the Appendices 7-3 to 7-11(included in the memory stick and are also available on request) and given a N/A caption. A total number of 40 case studies with original decision matrices were identified.

4.4 Calculations of case studies with original decision matrix included

In total, 246 case studies were also identified and a trend established in terms of MCDA techniques used to solve them. Subsequently.40 out of the 246 identified case studies were used for further analysis, due to them having original decision matrices. Ranking of alternatives and different MCDA techniques were used to generate “similarity percentages” and average similarity percentages”. The original matrices aided in the calculations using different MCDA techniques to rank the alternatives as per technique. Appendices 7-3 to 7-10 show calculations conducted using the eight MCDA techniques, namely AHP; ELECTRE; MAUT; PROMETHEE; SAW; TOPSIS; VIKOR and Yager’s method. Ranking of alternatives from each method was presented in the calculations and in Appendix 7-11.

Appendix 7-11 also illustrates calculations of “similarity percentages” and average “similarity percentages” for the selected 40 case studies. The “similarity percentages” and average “similarity percentages” were calculated as shown in Section 3.3.2. In some instances, there were case studies with a lot of criteria and alternatives, and they could not fit the portrait design of the page. All case studies which could not fit the portrait design were grouped to and fitted into the landscape design of the page. The case studies are shown in Appendices 7-2 to 7-11.

The “similarity percentages” and “average similarity percentages” for each case study were compiled and a table constructed. Appendix 7-2 illustrates the number of criteria; alternatives and the “average similarity percentages” for each case study. The information presented in Appendix 7-2 was graphically presented in Figure 4-3, which exhibited the relationship between criteria; alternatives and the “average similarity percentages” for the case studies.

Figure 4-3 shows a relationship between the number of criteria and average "similarity percentage". The average "similarity percentage" is observed to increase with increasing number of criteria. There is also a relationship between the average "similarity percentage" and the number of alternatives. The average "similarity percentage" is observed to decrease with an increasing number of alternatives.

Relationships are visible in case studies 4; 5; 8; 12; 14; 22; 24; 34 and 40, where high positive cusps are identified. All the case studies with high positive cusps have the number of alternatives ranging between two and five; and the number of criteria ranging between five and nine. The average "similarity percentages" range between 65,63% and 100% for the high positive cusps.

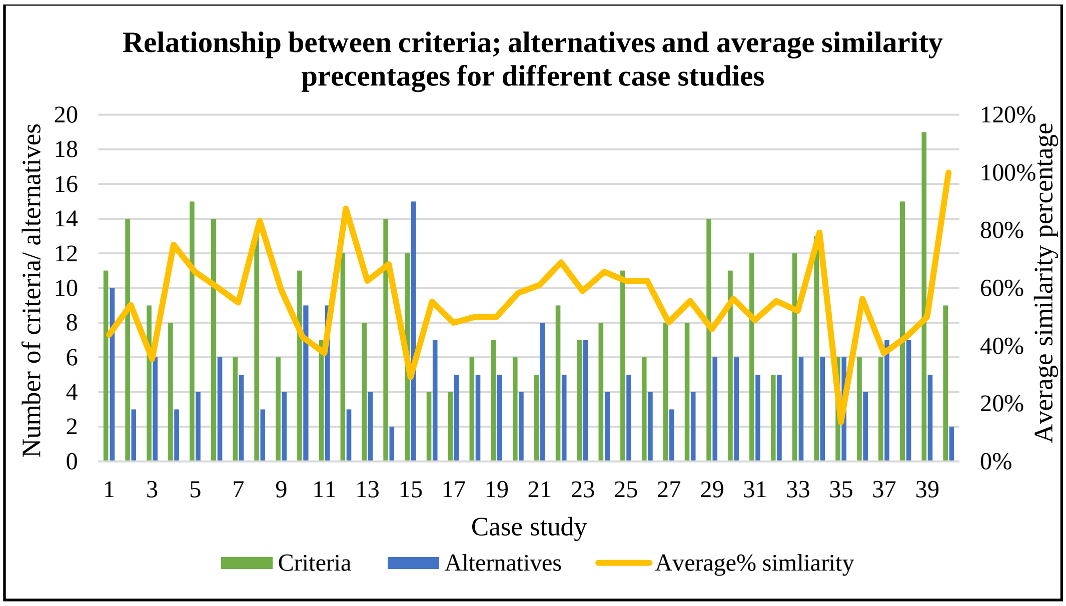

Figure 4-3 Relationship between criteria; alternatives and average similarity percentages for different case studies

Figure 4-3 also has case studies exhibiting negative cusps. The cusps are at their lowest in case studies 3; 11; 15; 35 and 37. There is also a relationship observed between the number of criteria; alternatives and the average "similarity percentage". The criteria in case studies 3; 11; 15; 35 and 37 range between six and

12, while the number of alternatives range between 6 and 15. The average "similarity percentage" range of the five case studies lies between 29,17% and 28,06%. The lowest average "similarity percentage" recorded is at 13,58%, where the number of alternatives and criteria are equal, as in case study 35, Appendix 7-2.

4.4.1 The significance of the "similarity percentages" and the average "similarity percentages"

The "similarity percentage" is a percentage computed from counting the number of times an alternative has the same ranking using different MCDA techniques. The average "similarity percentage" is an average calculated from the overall ranking of alternatives using the eight selected MCDA techniques, as shown in Section 3.3.2. Appendix 7-11 shows the computation of "similarity percentages" and average "similarity percentages". These two percentages are significant because they show similarity of rankings for each alternative from each MCDA technique.

If all or most of the MCDA methods used yield the same ranking for each alternative, then a high "similarity percentage" results. If fewer MCDA techniques show the same ranking for an alternative, then a low "similarity percentage" results. A high "similarity percentage" is desirable because it indicates that almost all the alternatives were similarly ranked by different techniques. Similarly, ranked alternatives increase the confidence level when ranking alternatives from the best to worst.

It is significant to have a high "similarity percentage" and hence a high average "similarity percentage" in terms of ranking alternatives. Alternatives ranked with high average "similarity percentage" leads to decision making with confidence. Every factor that can enhance ranking of alternatives with high average "similarity percentage" has to be maximised. In this research study the average "similarity percentage" is enhanced from a criteria and alternatives point of view.

4.4.2 Relationship between the number of criteria and the average "similarity percentages"

Figure 4-4 illustrates the relationship between the number of criteria and the average "similarity percentages" for the 40 selected case studies. There is an observed increase in the average "similarity percentages" with increasing number of criteria. There is also an increased clustering in terms of "similarity percentage" observed in the region of eight criteria. The highest "similarity percentage" is observed in a region of nine criteria, with an average similarity percentage of 100%.

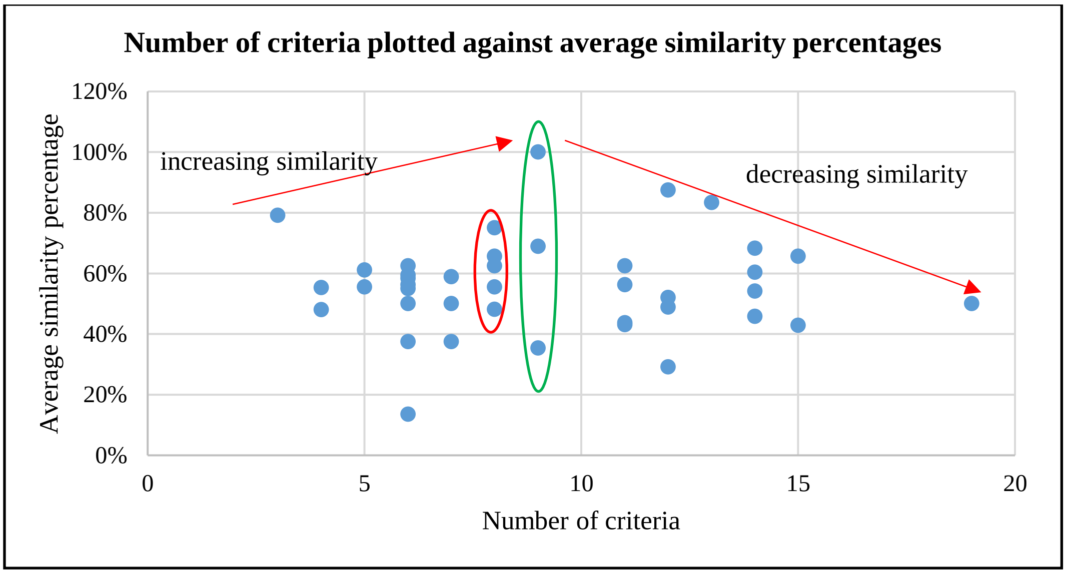

Figure 4-4 Number of criteria plotted against average similarity percentages for the case studies

The area circled in red in Figure 4-4 shows the criteria which yielded the most clustering. The highest precision in terms of average "similarity percentage" is shown as the area between the two arrows (circled in green). Criteria number of 8 yielded the highest clustering while the highest average "similarity percentage" was observed at 9 criteria (circled in green). Beyond 9 criteria, the average "similarity percentage" gradually drops. The precision observed at 9 criteria can be interpreted as the level where the highest average "similarity percentage" can be obtained.

4.4.3 Relationship between the number of alternatives and the average "similarity percentages"

Figure 4-5 illustrates the relationship between the number of alternatives and the average "similarity percentages" for the selected 40 case studies. The average "similarity percentage" decreases with increasing number of alternatives. It can be concluded that there is an inverse relationship between the average "similarity percentage" and the increasing number of alternatives. The opposite is observed in Figure 4-4, where the average "similarity percentage" is observed to increase with increasing number of criteria, up to 9 criteria.

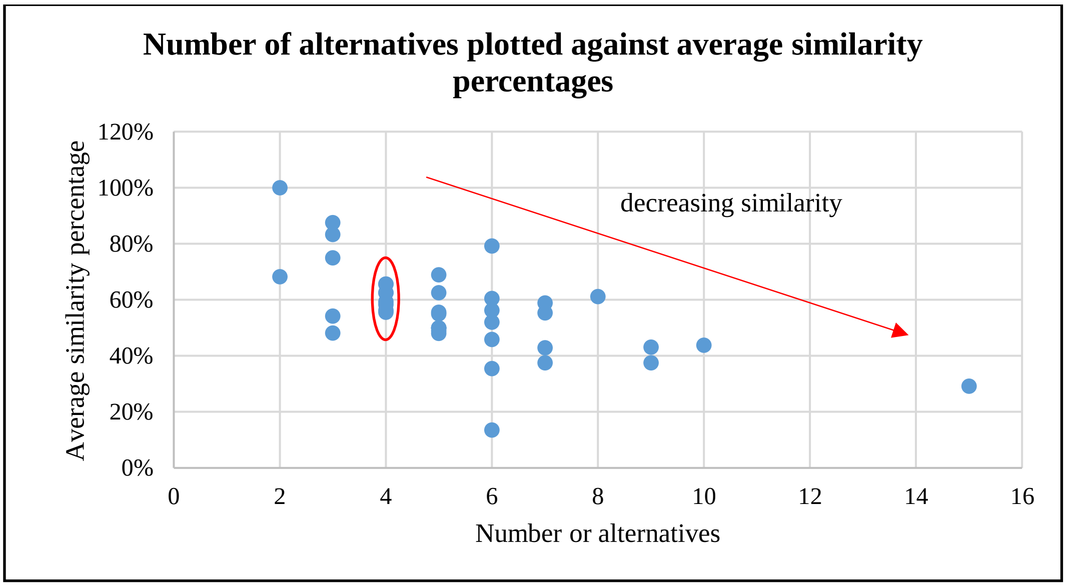

Figure 4-5 Number of alternatives plotted against average similarity percentages for the case studies

From Figure 4-5, areas circled in red show criteria with highest clustering. The area circled in red represents four alternatives, where high correlation is observed, due to high clustering. From Figure 4-5, it can be deduced that the area circled in red has higher precision due to the high clustering. Four alternatives yielded the highest precision, while the highest average “similarity percentage” was observed at two alternatives.

From four alternatives, the average “similarity percentage” starts dropping, indicating that more alternatives yield a smaller average “similarity percentage”. The dropping of the average “similarity percentage” is relative to the region of alternatives less than four. The average “similarity percentage” at 15 alternatives is 27,19% and the lowest average “similarity percentage” is recorded at 6 criteria (13,58%). The area of four criteria can be interpreted as a boundary for the number of alternatives that have a high possibility of yielding a high average “similarity percentage”.

4.4.4 The recommended number of criteria

From the number of criteria perspective, the highest accuracy is visible at 9 criteria. The advantage of having more criteria is that more criteria act as a screen to eliminate the unnecessary alternatives, while leaving the realistic alternatives. In this way, inconsistencies associated with the presence of irrelevant alternatives are eliminated. For this research study, a maximum number of criteria is considered as nine.

Conversely, too many criteria may result in repetition of some criteria. For this research study, too many criteria are considered as exceeding nine criteria. The repeated or duplicate conflicting criteria result in inconsistent choice of alternatives. According to Orrin and GuoQing (2016), it is important to cluster criteria such that a more valid ranking of alternatives is obtained.

From Figure 4-4 it is ideal to use at most nine criteria to obtain the highest accuracy. Criteria less than eight can be further divided in order to generate at least eight criteria. Criteria more than nine can be carefully clustered such that the resulting number of criteria is nine or less. A range of criteria (for instance eight to nine criteria) is preferred because it does not give a specific value but instead it gives a range of values, accounting for uncertainty.

Saaty and Ozdemir (2003) (cited in Yavuz, 2015a) stipulated that the maximum number of criteria that can be used in MCDA calculations is nine. Nine is the ideal number because the limit of the human mind to process calculations is exceeded when criteria considered is greater than nine. The exceeded limit of the mind results in inconsistencies in calculations due to attention span and memory span limitations. (Yavuz, 2015a). The maximum of "seven plus two" (nine) criteria is further emphasised by Yavuz and Pillay (2007).

The nine criteria correlate with the maximum accuracy obtained at nine criteria, as depicted in Figures 4-3 and 4-4. However, from 2003 to 2016 (Figure 4-2), there may have been developments in terms of making calculations relatively easier using a high number of criteria. Examples of such developments include software and computer programs. Due to developments to make calculations less extensive, the maximum number of criteria might be bigger than nine. Easier computation developments may possibly shift the maximum number of criteria to exceed nine.

4.4.5 The recommended number of alternatives

As more criteria are added in the decision making process, the number of alternatives is expected to decrease. This is because more criteria refine the alternatives by eliminating the irrelevant ones and leaving the relevant ones. This means a decreased possibility of choosing the irrelevant alternative. In addition, MCDA techniques involving pairwise comparison of alternatives become long when there are more alternatives to be compared. Long calculations may result in inconsistencies, which may affect ranking of alternatives. Hence fewer alternatives are preferred to a high number of alternatives.

From Figure 4-5, the maximum number of alternatives to use is four. Four was chosen because of the high precision observed with four alternatives. In addition, two criteria gave the highest accuracy as depicted in Figure 4-5. At zero and one alternatives, nothing is observed because at least two alternatives are needed to make a comparison. Hence only one boundary is existent, which is at four criteria.

4.4.6 Summary on the case studies with original decision matrix included

The 40 case studies identified were used to establish a relationship between the average “similarity percentages”; the criteria; and alternatives. The average “similarity percentages” are found to increase with the decreasing number of alternatives and increasing number of criteria. A maximum of four alternatives is recommended to give precise ranking of alternatives while two alternatives give the most accurate ranking. A maximum of 9 criteria yield an accurate ranking of alternatives.

4.5 Possible sources of error

There are different possible sources of error identified in the research study. They are errors associated with long calculations; errors associated with the presence of zero values; errors associated with weaknesses of MCDA techniques used for analysis and errors associated with limited information supplied in the case studies.

4.5.1 Errors associated with long calculations

Some of the MCDA techniques have methodologies involving extensive calculations. AHP; ELECTRE and PROMETHEE have calculations to generate a pairwise comparison of alternatives. Pairwise comparison of alternatives generates a pairwise comparison matrix; a concordance/discordance matrix or a preference matrix. The more the alternatives; the bigger the matrix generated.

Appendices 7-3; 7-4 and 7-6 (included in the attached memory stick and are available on request) show ranking of alternatives using AHP; ELECTRE and PROMETHEE, respectively. The three techniques involving a calculation during pairwise comparison of alternatives is seen to be the longest at the case study with 15 alternatives. If calculations are long they may result in inconsistencies, affecting the ranking of alternatives. Techniques like AHP further compare alternatives under individual criteria, resulting in longer calculations. It is better to have a limited number of criteria and alternatives in order to increase consistency during calculations using MCDA techniques.

4.5.2 Errors associated with zero-values in the decision matrix

Some case studies have zero values present in their original decision matrix. The formula for scoring the alternatives changes because in some division by zero is impossible. For instance, the formula of calculating scores of alternatives changes in the SAW technique due to the presence of zero. This implies that in criteria where the alternatives scored a zero, the calculation is different from where alternatives score a non-zero value.

In AHP as seen in Appendix 7-3, zero values result in an inconsistent matrix. The matrix generated has a negative consistency. A perfectly consistent method must have a consistency ratio of zero (Saaty and Tran, 2007). The presence of a zero value may result in inconsistencies when ranking alternatives, especially when using techniques like SAW and AHP.

4.5.3 Errors associated with limitations of the MCDA techniques used

Certain limitations associated with different MCDA techniques may be possible sources of error. For instance, with Yager's method, normalised minimum values are used to rank alternatives. If alternatives have the same minimum normalised scores, then it means all of them are ranked first. All alternatives ranking first are not representative of a real-life situation since different alternatives may not have the same preference.

There are MCDA techniques that suffer from rank reversal. Wang and Triantaphyllou (2006) defined rank reversal as an irregularity that results in different MCDA techniques ranking the same alternatives differently in a case study. Rank reversal is evident in Appendix 7-11, where all case studies are ranked using eight MCDA techniques, under almost the same conditions. However, the ranking of alternatives using the eight techniques yields different alternative rankings.

Wang and Triantaphyllou (2006) concluded that both real-life case studies and MCDA techniques are prone to rank reversal. They also established that alternatives with a small difference in terms of their scoring, are prone to rank reversal. The rank reversal rates were higher with increasing number of alternatives (Wang and Triantaphyllou, 2006). The rank reversal increases with an increasing number of criteria.

4.5.4 Errors associated with limited information supplied in the case studies

Case studies were collected from journal papers and proceedings of conferences, as detailed in Section 2.4.1. The content of these papers is limited in most cases. Assumptions made; constants used for calculations; and full calculation procedures are omitted in some papers. The limited content results in the limited information supplied in the paper.

Some MCDA techniques require certain information for their methodology to be used. If limited information is supplied then assumptions are made, like in Section 4.1. Assumptions do not always represent the real-life situations. For instance, when thresholds are assumed, they might not necessarily represent the actual thresholds which would be characteristic of a specific case study.

4.5.5 Summary on the possible sources of error

The possible errors identified during the analysis were errors associated with long calculations and errors associated with the presence of zero-values in the original matrix. In addition, errors associated with limitations of MCDA techniques used and errors due to assumptions made from limited information were also identified. It is important to identify the sources of error so that they are addressed to yield consistent ranking.

4.6 The MCDA techniques as assistance in decision making

It is significant for the decision maker to note that MCDA techniques do not give the right answer as stated by Belton and Steward (2002). MCDA techniques only assist in the decision making process. This implies that once alternatives have been ranked then the decision maker has to put some thought into the ranking. This would ensure that the most reasonable or ideal solution is implemented.

For instance, if the first ranked alternative is not the implemented alternative, the decision maker has to put some thought process on the solution. If the second ranked is the most feasible one, then the second alternative is chosen over the first ranked alternative. The input of the decision maker is important in the decision making process. An example would be of choosing an ideal plant location. Computation using MCDA techniques may choose an area in close proximity to, for instance, the community. If relocation of the community is expensive for the company, then the next best location is considered.

4.7 Chapter summary

Chapter 4 has presented the assumptions made in order to enable use of some of the selected MCDA techniques. A total number of 40 case studies with original decision matrices out of 111 case studies were identified and chosen for analysis, due to them having original decision matrices. Alternatives and different MCDA techniques were used to establish “similarity percentages” and average “similarity percentages”, as indicated in Section 4.4

The average “similarity percentage” was used to establish the relationship between the varying number of criteria with varying number of alternatives. From the established relationship, the ideal number of alternatives and criteria was recommended. The ideal number of criteria and alternatives would help the decision maker to rank alternatives using MCDA techniques with high accuracy and precision. The recommended number of alternatives is a maximum of four. A maximum of nine criteria is also recommended. Chapter 5 concludes on the findings of the research study and recommends future work.

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

Chapter 5 provides a summary of the findings from the research. In addition, conclusions and recommendations are made in this chapter to establish a way forward. The summary of the research report with reference to the objectives of the research report is presented in Section 5.2. Section 5.3 highlights the limitations of the research study. Section 5.4 recommends work that can be done in future, branching from this research study.

5.2 Findings of the research study

The mining industry is characterised by a lot of decision making, from every day decisions to complex strategic decisions. The mining industry is also characterised by uncertainties that seem to have escalated over the years. To make decisions under uncertainty, MCDA techniques have been used. It is essential to limit uncertainty when making decisions. One way of limiting uncertainty is to look at the behaviour of frequently used MCDA techniques with changing number of criteria and alternatives. Once a behavioural pattern is observed, the ideal number of criteria and alternatives to use efficiently with MCDA techniques would be recommended.

A total of 246 mine planning and related case studies were identified from the public domain. The case studies were sourced mainly from journals and conference proceedings. The 246 case studies were grouped according to their years of publication and the MCDA techniques used to solve them. From 1997 to 2016 there was an observed increase in the number of MCDA techniques used to solve the case studies. The increase was particularly noted after the 1998 economic slowdown and the 2008 GFC. This indicates more MCDA techniques were used in decision making, in response to the uncertainty associated with the 1998 and 2008 financial crises.

Out of the 246 identified case studies, only 40 case studies with the original decision matrices were selected. The selected case studies were solved using the eight most frequently used MCDA techniques namely: AHP; ELECTRE; MAUT; PROMETHEE; SAW; TOPSIS; VIKOR; and Yager's method. Fuzzy set theory was excluded from the list of most frequently used MCDA techniques since it is used to address uncertainty when using MCDA techniques. The identified MCDA techniques were used to rank alternatives for each case study. The ranking of alternatives was used to calculate the "similarity percentages" of the different rankings.

From the calculated "similarity percentages" a maximum of four alternatives and nine criteria were recommended to use with different MCDA techniques. The highest "similarity percentage" was observed at nine criteria and two alternatives. The recommended number of alternatives and criteria would be the one used with different MCDA techniques with increased consistency. The majority of MCDA techniques suffer from rank reversal. In Section 4.5.3 it was established that rank reversal is associated with increasing number of alternatives and criteria. To decrease rank reversal a maximum of four alternatives and nine criteria are recommended.

The research study established several findings. Firstly, from the 246 case studies, AHP was most frequently used when case studies were grouped according to MCDA techniques used to solve them. Secondly, the Fuzzy set theory was used in conjunction with other MCDA techniques to address uncertainty in decision making. The uncertainty could be associated with the aftermath of the 1998 economic meltdown and the 2008 GFC.

Thirdly, alternatives should be reduced to a maximum of four alternatives to avoid any inconsistencies that may occur. As more alternatives are eliminated, alternatives significant to the solution are left for analysis. Lastly, the number of criteria should be kept to a maximum of nine to avoid errors associated with rank reversal. Rank reversal affects MCDA techniques like AHP and ELECTRE. For criteria greater than nine, the criteria must be regrouped under key or parent criteria such that a maximum of nine criteria are considered.

Similarly, criteria less than eight should be dissociated. This research study has therefore quantitatively re-confirmed the Yavuz and Pillay (2007); and Saaty and Ozdemir (2003) (cited in Yavuz, 2015a) findings that the maximum criteria should be nine. In addition, this research has also established that alternatives should be limited to a maximum of four for MCDA analysis.

MCDA techniques provide a set of possible alternatives from which a decision maker can choose a final decision. MCDA techniques aid in the decision making process. The decision maker has to think about the solution presented by different MCDA techniques and how possible it is, before its implementation.

5.3 Limitations of the research

The following are the identified limitations associated with the research:

- The study was only limited to papers that are freely available in the public domain. These freely available papers may not fully represent the case studies solved using MCDA techniques in the mining industry;
- Some of the identified case studies had no original decision matrices. Lack of original decision matrices further limited the number of case studies to be analysed; and
- Assumptions made in some MCDA techniques did not necessarily represent the real-life situation. The assumptions made may have contributed to inconsistencies when ranking different alternatives.

5.4 Recommendations for future work

There is an increase in uncertainty, as depicted by Figures 4-1 and 4-2. The mining industry is also affected by the uncertainties. Five most frequently used approaches were recommended to deal with uncertainty when using MCDA techniques. The five approaches identified were Bayesian framework; deterministic sensitivity analysis; Fuzzy set theory; Gray theory; and probabilistic sensitivity analysis.

This study focused on establishing and recommending the ideal number of criteria and alternatives when using MCDA techniques to rank alternatives. A recommended area of research would be the application of the identified approaches to the MCDA techniques so as to rank alternatives. The "similarity percentages" would then be used to compare ranking of alternatives from MCDA techniques.

The "similarity percentages" would be used to recommend the number of alternatives and criteria to use when MCDA techniques are combined with approaches dealing with uncertainty. The "similarity percentages" and the commended number of criteria and alternatives would be compared under two scenarios and the best scenario to deal with uncertainty determined.

6 REFERENCES

Abdalla, S., Kizil, M. and Canbulat I. (2013) Development of a method for layout selection using Analytical Hierarchy Process, *13th Coal Operators' Conference*, Vol. 123, No. 37, pp. 27-37. INTERNET. http://ro.uow.edu.au/cgi/viewcontent.cgi?article=2096&context=coal, [Accessed 12 December 2015]

Belton, V. and Stewart, T. (2002) *Multiple criteria decision analysis- an integrated approach*, Kluwer Academic Publishers, Massachusetts, USA. INTERNET. https://books.google.co.za/books?isbn=079237505X, [Accessed 28 August 2016]

Biswas, R. (1999) An application of Yager's bag theory in multi criteria based decision making problems, *International Journal of Intelligent Systems*, Vol. 14, No. 12, pp. 1231-1238. INTERNET. http://onlinelibrary.wiley.com/doi/10.1002/(SICI)1098-111X(199912)14:12%3C1231::AID-INT4%3E3.0.CO;2-6/epdf, [Accessed 17 December 2015]

Brans, P. H. and Vincke, P.H. (1985) A preference ranking of alternatives organization method: The PROMETHEE method for multiple criteria decision-making, *Management Science*, Vol. 31, No. 6, pp. 647-656. INTERNET. http://www.lamsade.dauphine.fr/~mousseau/pmwiki-2.1.5/uploads/Research/Brans1985.pdf, [Accessed 19 July 2016]

Broekhuizen, H., Groothuis-Oudshoorn, C.G.M., Van Til, J.A., Hummel, J. M. and Ijzerman, M. J. (2015) A review and classification of approaches for dealing with uncertainty in multi-criteria decision analysis for healthcare decisions, *PharmacoEconomics*, Vol. 33, No. 5, pp. 445-455. INTERNET. http://www.ncbi.nlm.nih.gov/pmc/articles/PMC4544539/pdf/40273_2014_Article_251.pdf, [Accessed 15 August 2016]

Bulgurcu, B. (2016) Investment decision using the VIKOR method in European Union countries, *American Journal of Contemporary Research*, Vol. 6, No. 2, pp. 16-24. INTERNET. http://www.aijcrnet.com/journals/Vol_6_No_2_April_2016/3.pdf, [Accessed 01 August 2016]

Buchanan, J. and Sheppard, P. (1998) Ranking projects using the ELECTRE method. s.n, Hamilton, Canada. INTERNET. http://www.orsnz.org.nz/conf33/papers/p58.pdf, [Accessed 17 May 2017]

Chamber of Mines of South Africa (2015) Integrated Annual Review 2015, pp 23-25. Chamber of Mines of South Africa, Johannesburg, South Africa. INTERNET. http://www.chamberofmines.org.za/industry-news/publications/annual-reports, [Accessed 04 November 2016]

Chatterjee, P., Mondal, S. and Chakaborty, S. (2014) A comprehensive solution to automated inspection device selection problems using ELECTRE methods, *International Journal of Technology*, Vol. 2, No.1, pp. 193-208. INTERNET. www.ijtech.eng.ui.ac.id/index.php/journal/article/download/410/189, [Accessed 20 July 2016]

Chatterjee, P., Mukherjee, S. and Chakaborty, S. (2009) Supplier selection using compromise ranking and outranking methods, *Journal of Industrial Engineering International*, Vol. 7, No. 14, pp. 61-73. INTERNET. http://www.sid.ir/en/VEWSSID/J_pdf/117320111407.pdf, [Accessed 19 July 2016]

Chen, C.T., You, M.L. and Wen, K.L. (2015) The weighting analysis of TOPSIS via the grey system theory , *International Journal of Information and Education Technology*, Vol. 5, No. 2, pp. 95-99. INTERNET. http://www.ijiet.org/papers/483-S00002.pdf, [Accessed 23 February 2016]

Collins, T. R., Rossetti, M. D. and Natchmann, H.L. (2006) The use of Multi-Attribute Utility Theory to determine the overall best-in-class performer in a benchmarking study, *Benchmarking: An International Journal*, Vol. 13, No. 4, pp. 431-446. INTERNET. http://www.emeraldinsight.com/doi/pdfplus/10.1108/14635770610676281, [Accessed 20 March 2016]

De Montis, A., DeToro, P. D., Droste-Franke, B., Omann, I. and Stagl, S. (2008) *Assessing the quality of different MCDA methods*, pp. 99-133. INTERNET. http://people.unica.it/adm/files/2008/11/05_de_monti_et_al.pdf, [Accessed 17 May 2017]

Deshmukh, S.C. (2013) Preference Ranking Organization Method of Enrichment Evaluation (PROMETHEE), *Proceedings of the 1st International Conference on Waste Water Technology for Green Economy, Pune, Maharashtra, India*, Vol. 1, pp. 55-61. INTERNET. http://seeram.org/Environment%20Observer-Waste%20Water%20Technology%20For%20Green%20Economy.pdf#page=55, [Accessed 17 May 2017]

Elevli, B. and Demicri, A. (2004) Multicriteria choice of ore transport system for an underground mine: application of PROMETHEE methods, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 223, No. 38, pp. 251-256. INTERNET. http://reference.sabinet.co.za/webx/access/journal_archive/0038223X/2874.pdf, [Accessed 12 December 2015]

Ernst & Young (2016) Business risks facing mining and metals 2015-2016. INTERNET. http://www.ey.com/GL/en/Industries/Mining---Metals/Business-risks-in-mining-and-metals, [Accessed 01 March 2016]

Figueira, J., Mousseau, V. and Roy, B. (2005) ELECTRE methods. In J. Figueira, S. Greco and M. Ehrgott, (eds), *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York, USA. INTERNET. http://www.springer.com/it/book/9780387230672, [Accessed 17 May 2017]

Fülöp, J. (2005) *Introduction to decision-making methods*. Working Paper 05-6, Laboratory of Operations Research and Decision Systems, Computer and Automation Institute, Hungarian Academy of Sciences, Budapest, Hungary. INTERNET. http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf, [Accessed 17 May 2017]

Gavade, R. K. (2014) Multi-criteria decision making: an overview of different selection problems and methods, *International Journal of Computer Science and Information Technologies*, Vol. 5, No. 4, pp. 5643-5646. INTERNET. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.433.1775&rep=rep1&type=pdf, [Accessed 01 August 2016]

Hausmann, R. (2014) Understanding South Africa's poor economic performance. INTERNET. http://www.politicsweb.co.za/news-and-analysis/understanding-sas-poor-economic-performance--ricar, [Accessed 25 August 2016]

Hayati, M., Rajabzadeh, R. and Darabi, M (2014) Determination of optimal block size in Angouran mine using VIKOR method, *Journal of Materials and Environmental Science,* Vol. 6, No. 11, pp. 3236-3244. INTERNET. http://www.jmaterenvironsci.com/Document/vol6/vol6_N11/372-JMES-Hayati.pdf, [Accessed 01 August 2016]

Herath, G. and Prato, T. (2006) Role of multi-criteria decision making in natural resource management, In Herath, G. and Prato, T. eds. *Using multi-criteria decision analysis in natural resource management*, Ashgate Publishing Limited, Hampshire, England. INTERNET. https://books.google.co.za/books?isbn=0754645967, [Accessed 20 February 2016]

Jahan, A., Edwards, K.L. and Bahraminasab, M. (2013) *Multi-criteria decision analysis for supporting the selection of engineering materials in product design*, Second edition, Butterworth Heinemann, Oxford, England.

Kabir, G. and Hasin, M. (2012) Comparative analysis of TOPSIS and fuzzy TOPSIS for the evaluation of travel website service, *International Journal of Quality Research*, Vol. 6, No. 3, pp. 169-185. INTERNET. http://www.ijqr.net/journal/v6-n3/1.pdf, [Accessed 16 January 2016]

Kasperczyk, N. and Knickel, K. (undated) Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE). INTERNET. http://www.ivm.vu.nl/en/Images/MCA4_tcm234-161530.pdf, [Accessed 17 May 2017]

Kazakidis, V., Mayer, Z., and Scoble, M. J. (2004) Decision making using the Analytical Hierarchy Process in mining engineering, *Transactions of the Institute of Mining and Metallurgy (A: Mining Technology)*, Vol. 113, No. 1, pp. 30-42. INTERNET. https://www.researchgate.net/publication/233649010_Decision_making_using_the_analytic_hierarchy_process_in_mining_engineering, [Accessed 22 February 2016]

Lebel, J. (2003) In_focus Health: an ecosystem approach, pp. 32-40, *Canada: The International Development Research Centre*. INTERNET. http://idl-bnc.idrc.ca/dspace/bitstream/10625/30918/14/IDL-30918.pdf, [Accessed 29 November 2015]

Liu, Y. (2015) Multi Attribute Utility Theory (MAUT) models with Interactions, Wright State University, Ohio, USA. INTERNET. http://slideplayer.com/slide/4741033/, [Accessed 01 December 2015].

Matos, M. A. (2007) Decision under risk as a multicriteria problem, *European Journal of Operations Research*, Vol. 181, No. 3, pp. 1516-1529. INTERNET. http://ac.els-cdn.com/S0377221706002062/1-s2.0-S0377221706002062-main.pdf?_tid=17c5bf84-eedb-11e5-bb49-00000aab0f27&acdnat=1458506226_b043979ef20995657427eae31f04e681, [Accessed 16 January 2016]

Memariani, A., Amini, A. and Alinezhad, A. (2009) Sensitivity analysis of simple additive weighting method (SAW): the results of change in the weight of one attribute on the final ranking of alternatives, *Journal of Industrial Engineering*, Vol. 4, No. 4, pp. 13-18. INTERNET. http://www.qjie.ir/article_28_195a2285c846b262f048d6e8144b236a.pdf, [Accessed 01 August 2016]

Milani, A., Shanian, A. and El-Lahham, C. (2006) Using different ELECTRE methods in strategic planning in the presence of human behavioral resistance, *Journal of Applied Mathematics and Decision Sciences*, Vol. 1, No. 1, pp. 1-19. INTERNET http://www.emis.de/journals/HOA/JAMDS/Volume2006/10936.pdf [Accessed 20 July 2016]

Musingwini, C. and Minnitt, R. A. (2008) Ranking the efficiency of selected platinum mining methods using the Analytical Hierarchy Process, *Third International Platinum Conference 'Platinum in Transformation'*, pp. 319-326. Johannesburg: *The Southern African Institute of Mining and Metallurgy* INTERNET. http://www.hydrometallurgy.co.za/Pt2008/Papers/319-326_Musingwini.pdf, [Accessed 20 February 2016]

Musingwini, C. (2010) A review of the theory and application of multi-criteria decision analysis techniques in mine planning, in *Proceedings of Mine Planning and Equipment Selection (MPES) 2010*, pp. 129-140. Melbourne*: The Australasian Institute of Mining and Metallurgy*.

Musingwini, C. (2016) Keynote address: Application of Multi-Criteria Optimisation Techniques in mine planning, *6th International Conference on Computer Applications in the Minerals Industries (CAMI 2016)*, pp. 1-10. Istanbul: *T. C. Istanbul Universitesi* and *Universite Laval.*

National Treasury (1998). National treasury annual report 1998. INTERNET. http://www.treasury.gov.za/publications/annual%20reports/national%20treasury/nt%20annual%20report%201998.pdf, [Accessed 20 August 2016]

Nhleko, A.S. and Musingwini, C. (2016) Estimating cost of equity in project discount rates: comparison of the capital asset pricing model and Gordon's wealth growth model, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 116, No.3, pp. 215-220. INTERNET. http://www.scielo.org.za/pdf/jsaimm/v116n3/05.pdf, [Accessed 01 November 2016]

Oliveira, M., Fontes, D. B. M. M. and Pereira, T. (2013) Multicriteria decision making: a case study in the automobile industry. *FEP Working Papers: Research Work in Progress,* No. 483, pp. 1-17. INTERNET. http://wps.fep.up.pt/wps/wp483.pdf, [Accessed 19 December 2015]

Orrin, C. and GouQinn, L. (2016) Linking disjoint supermatrices and criteria clusters, *Journal of Multi-Criteria Decision Analysis- Optimization, Learning and Decision Support*, Vol. 23, No. 3-4, pp. 139-159. INTERNET. http://onlinelibrary.wiley.com/doi/10.1002/mcda.1570/epdf. [Accessed 20 August 2016]

Peniwati, K. (2007) Criteria for evaluating group decision making methods, *Mathematical and Computer Modelling*, Vol. 46, No. 7-8, pp. 935-947. INTERNET. http://www.sciencedirect.com/science/article/pii/S0895717707001021, [Accessed 02 March 2016]

Petit, P., and Fraser, P. (2013) What is the best energy-delivery system for hand-held stope drilling and associated equipment in narrow-reef hard rock mines?, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 113, No.3, pp. 243-249. INTERNET. http://www.saimm.co.za/publications/journal-papers/details/1/5658, [Accessed 20 January 2016]

Pokehar, S. and Ramachandran, M. (2003) Application of multi-criteria decision making to sustainable energy planning- a review, *Renewable and Sustainable Energy Reviews*, Vol. 8, No. 2004, pp. 365-381. INTERNET. https://sites.uni.edu/apetrov/wind/Fuzzy/Pokehar2003.pdf, [Accessed 17 May 2017]

Saaty, T. (2007) Multi-decisions decision-making: in addition to wheeling and dealing our natural political bodies need a formal approach to prioritization, *Mathematical and Computer Modelling*, Vol. 46, No. 7, pp. 1001-1006. INTERNET. http://www.sciencedirect.com/science/article/pii/S0895717707000817, [Accessed 23 February 2016]

Saaty, T. and Tran, L. (2007) On the invalidity of fuzzifying numerical judgements in the Analytic Hierarchy Process, *Mathematical and Computer Modelling*, Vol. 46, No.7-8, pp. 962-975. INTERNET. http://ac.els-cdn.com/S0895717707000787/1-s2.0-S0895717707000787-main.pdf?_tid=b212ca50-70db-11e6-a7d0-00000aacb35d&acdnat=1472800136_2d748cec7bf9d131bd3681f61395a187, [Accessed 20 August 2016]

Sanandaji, H. (2006) *A study of different decision-making models and their pros and cons*, Carleton University, Ottawa, Canada.

Southern African Resource Watch (2009) *Impact of the global financial crisis on mining in Southern Africa*, South African Resource Watch, Johannesburg, South Africa. INTERNET. http://www.sarwatch.org/sites/sarwatch.org/files/Publications_docs/sarw_financial_impact_mining_30jun09_report.pdf, [Accessed 17 May 2017]

Triantaphyllou, E. & Mann, S. H. (1995) Using Analytical Hierachy Process for decision making in engineering applications: some challenges, *International Journal of Industrial Engineering: Application and Practice*, Vol. 12, No. 1, pp. 35-44. INTERNET. http://bit.csc.lsu.edu/trianta/Journal_PAPERS1/AHPapls1.pdf, [Accessed 16 January 2016]

Velasquez, M. and Hester, P.T. (2013) An analysis of multi-criteria decision making methods, *International Journal of Operations Research*, Vol. 10, No. 2, pp. 56-66. INTERNET. http://www.orstw.org.tw/ijor/vol10no2/ijor_vol10_no2_p56_p66.pdf, [Accessed 12 February 2016]

Wang, C-H. and Pang, C-T. (2011) Using VIKOR method for evaluating service quality of online auction under fuzzy environment, *International Journal of Computer Science and Engineering and Technology*, Vol. 1, No. 6, pp. 307-314. INTERNET. http://www.ijcset.net/docs/Volumes/volume1issue6/ijcset2011010609.pdf, [Accessed 21 July 2016]

Wang, X. and Triantaphyllou, E. (2006) Ranking irregularities when evaluating alternatives using ELECTRE methods, *International Journal of Management Science*, Vol. 36, No. 1, pp. 45-63. INTERNET. http://ac.els-cdn.com/S0305048305001696/1-s2.0-S0305048305001696-main.pdf?_tid=d334c428-6eeb-11e6-95a6-00000aacb35f&acdnat=1472587162_511339a129c390c2fee76ea89fcbbe89, [Accessed 25 August 2016]

WebFinance Inc. (2017) BusinessDictionary: uncertainty definition. INTERNET. http://www.businessdictionary.com/definition/uncertainty.html [Accessed 16 May 2017]

Yavuz, M. (2015a) Equipment selection based on the AHP and Yager's method, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 115, No. 5, pp. 425-433. INTERNET. http://www.scielo.org.za/pdf/jsaimm/v115n5/15.pdf, [Accessed 19 February 2016]

Yavuz, M. (2015b) Making hard decisions in mining operations, *Proceedings of the 23rd International Symposium on Mine Planning and Equipment Selection (MPES)*, pp. 175-188, Sandton Convention Centre, Johannesburg, South Africa

Yavuz, M. and Pillay, S. (2007) Mining method selection by multiple criteria decision making tool by M.R. Bitarafan and M. Ataei, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 104, No. 9, pp. 137. INTERNET. https://journals.co.za/content/saimm/107/2/AJA0038223X_3298 [Accessed 9 December 2016]

6.2 **The following references do not appear in the main text but they are used as sources of data in Appendices 7-1 to 1-11**

Ataei, M. (2005) Multi-criteria selection for an alumina-cement plant location in East Azerbaijan province of Iran, *The Journal of the Southern African Institute of Mining and Metallurgy*, Vol. 105, No. 7, pp. 507-514. INTERNET. http://www.saimm.co.za/Journal/v105n07p507.pdf, [Accessed 12 December 2015]

Ataei, M., Shereski, F., Jamshidi, M. and Jalali, S. (2008) Suitable mining method for Golbini No.8 deposit in Jajarm (Iran) using TOPSIS method, *Mining Technology*, Vol. 117, No. 1, pp. 1-5. INTERNET. https://www.researchgate.net/requests/r18384312, [Accessed 20 August 2016]

Betrie, G.D. (2014) *Risk management of acid rock drainage under uncertainty* (PhD Thesis). Okanagan, The University of British Columbia, British Columbia, Canada. INTERNET. https://open.library.ubc.ca/cIRcle/collections/ubctheses/24/items/1.0074404, [Accessed 12 December 2015]

Betrie, G. D., Sadiq, R., Morin, K. A. and Tesfamariam, S. (2013) A multi-criteria decision analysis: approach for selection of remedial alternatives at mine sites, *IWWA: Reliable Water Technology*, Vol. 6, No. 1, pp. 3-8. INTERNET. https://www.imwa.info/docs/imwa_2013/IMWA2013_Betrie_515_R.pdf, [Accessed 15 December 2015]

Bitafaran, M. and Ataei, M. (2004) Mining method selection by multiple criteria decision making tools, *The Journal of the Southern African Institute of Mining and Metallurgy*, Vol. 104, No. 9, pp. 493-498. INTERNET. https://www.saimm.co.za/Journal/v104n09p493.pdf, [Accessed 17 December 2015]

Cokorilo, C. and Milicic, M. (1991) Quality comparison of mining equipment based on their technical characteristics, *Mining Science and Technology*, Vol. 13, No. 3, pp. 291-296. INTERNET. http://ac.els-cdn.com/016790319190484T/1-s2.0-016790319190484T-main.pdf?_tid=e3bd2dca-eed1-11e5-bfe1-00000aacb35f&acdnat=1458502274_204a4e194794d53530411fcd8b4868aa [Accessed 10 December 2015]

Cook, W.D., Johnston, D.A. and Kress, M. (1993) Selecting a new technology strategy using multiattribute, multilevel decision trees, *Journal of Multi-Criteria Decision Analysis*, Vol. 2, No. 3, pp. 285-290. INTERNET. http://onlinelibrary.wiley.com/doi/10.1002/mcda.4020020303/epdf, [Accessed 10 December 2015]

Hekmat, A., Osanloo, M. and Shirazi, M. K. R. (2008) New approach for selection of waste dump sites in open pit mines, *Institute of Materials, Minerals and Mining*, Vol. 117, No. 1, pp. 24-31. INTERNET. http://www.tandfonline.com/doi/pdf/10.1179/174328608X343768?needAccess=true, [Accessed 23 February 2016]

Karadogan, A., Karhiman, A. and Ozer, U. (2008) Application of fuzzy set theory in the selection of an underground mining method, *The Journal of The Southern African Institute of Mining and Metallurgy*, Vol. 108, No. 2, pp. 73-79. INTERNET. http://www.scielo.org.za/pdf/jsaimm/v108n2/02.pdf, [Accessed 01 March 2016]

Mayer, Z. and Scoble, M. J. (2002) Selection of opencast mining equipment by a multi-criteria decision making process, *Transactions of the Institute of Mining and Metallurgy (A: Mining Technology)*, Vol. 111, No. 2, pp. 136-142.

Mihai, A., Mancea, A. and Ekenberg, L. (2015) A MCDM analysis of the Rosa Montana gold mining project, *Sustainability Journal*, Vol. 7, No. 6, pp. 7264-7288. INTERNET. http://www.mdpi.com/2071-1050/7/6/7261/htm, [Accessed 19 December 2015]

Prado-Lopez, V., Steward, T., Makowski, M. and Winterfeldt, D. V. (2013) Value measurement analysis of energy tradeoffs in South Africa, *Proceedings of the International Symposium on Sustainable Systems and Technologies,* Vol. 1, pp. 1-24. INTERNET. https://figshare.com/articles/Value_Measurement_Analysis_of_Energy_Tradeoffs_in_South_Africa/805096, [Accessed 17 January 2016]

Romero, J. I. and Aldana, F. A. C. (2014) Mining method selection methodolgy by multiple criteria decision analysis - a case study in Columbian coal mining, *International Symposium of the Analytic Hierachy Process*, Vol. 3, No. 1, pp. 1-11. INTERNET. https://www.academia.edu/7314524/MINING_METHOD_SELECTION_METODOLOGY_BY_MULTIPLE_CRITERIA_DECISION_ANALYSIS_-_CASE_STUDY_IN_COLOMBIAN_COAL_MINING, [Accessed 22 January 2016]

Samanta, B., Sakar, B. and Mukherjee, S. (2002) Selection of opencast mining equipment by a multi-criteria decision making process, *Transactions of the Institute of Mining and Metallurgy (A: Mining Technology)*, Vol. 111, No. 2, pp. 136-142. INTERNET. http://www.tandfonline.com/doi/pdf/10.1179/mnt.2002.111.2.136, [Accessed 15 August 2016]

Scheffer, A., Roth, T. and Ahlf, W. (2014) Sustainable decision making under uncertainty: a case study in dredged material management, *Environmental Science Europe: A SpringerOpen Journal*, Vol. 26, No. 7, pp. 1-12. INTERNET. http://enveurope.springeropen.com/articles/10.1186/2190-4715-26-7, [Accessed 22 February 2016]

Sousa Junior, T. W., Souza, M. J. F., Cabral, I. E. and Diniz, M. E. (2014) Multi-criteria decision aid methodology applied to highway truck selection at a mining company, *Revista Escola de Minas, Ouro Preto*, Vol. 67, No. 3, pp. 285-290. INTERNET. http://www.scielo.br/pdf/rem/v67n3/a07v67n3.pdf, [Accessed 20 February 2016]

Stewart, M., Brent, G., Giurco, D. and Petrie, J. G. (2001) Decision making for sustainability: the case of mineral development in Australia, *6th World Congress of Chemical Engineering, Melbourne Australia*, 23-27 September 2001, pp. 1-10. INTERNET. http://cfsites1.uts.edu.au/find/isf/publications/giurcoetal2006sustainablemineralsdevelopment.pdf, [Accessed 22 February 2016]

Taheri, M., Ataee-Pour, M. and Irannajad, M. (2008) Selection of tailings site of Gol-E-Gohar iron ore processing plant using the MADM method, *The International Journal of Mineral Resources Engineering*, Vol. 13, No. 2, pp. 73-84. INTERNET. http://acikarsiv.atilim.edu.tr/browse/634/mineral2008_3.pdf?show, [Accessed 20 December 2015]

Wu, H., Yuan, J., Zhang, Y. and Song, S. (2007) The evaluation of the core competition of the Wugang mining cooperation using the Analytic Hierarchy Process, *The International Journal of Mineral Resources Engineering*, Vol. 12, No. 2, pp. 119-126.

Yadzi, S. F. (2014) A general framework for sustainability performance assessment of the steel industry, *International Journal of Sustainable Human Development*, Vol. 2, No. 1, pp. 8-17. INTERNET. http://ijshd.eduservgroup.com/wp/wp-content/uploads/2014/05/Vol2Iss1-p8-17-Fekri-Yazdi.pdf, [Accessed 20 January 2016]

Yazdani-Chamzini, A. and Yakhchali, S. H. (2012) Handling equipment selection in open pit mines based on group decision making, *International Journal of Industrial Engineering Computations*, Vol. 3, No. 105, pp. 907-924. INTERNET. http://www.growingscience.com/ijiec/Vol3/IJIEC_2012_42.pdf, [Accessed 30 March 2016]

7.2 Case studies and their corresponding criteria; alternatives; and average similarity percentages

Case study	Criteria	Alternatives	Average similarity %
1	11	10	43,75
2	14	3	54,17
3	9	6	35,42
4	8	3	75,00
5	15	4	65,63
6	14	6	60,42
7	6	5	55,00
8	13	3	83,33
9	6	4	59,38
10	11	9	43,06
11	7	9	37,50
12	12	3	87,50
13	8	4	62,50
14	14	2	68,25
15	12	15	29,17
16	4	7	55,36
17	4	5	48,00
18	6	5	50,00
19	7	5	50,00
20	6	4	58,33
21	5	8	61,11
22	9	5	68,89
23	7	7	58,93
24	8	4	65,63
25	11	5	62,50
26	6	4	62,50
27	8	3	48,15
28	8	4	55,56
29	14	6	45,83
30	11	6	56,25
31	12	5	48,89
32	5	5	55,56
33	12	6	52,08
34	13	6	79,17
35	6	6	13,58
36	6	4	56,25
37	6	7	37,50
38	15	7	42,86
39	19	5	50,00
40	9	2	100,00