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Mathematical Practices: their use across learning domains in a tertiary environment

A research report submitted to the Faculty of Science, University of the Witwatersrand, in partial fulfilment for the degree of Masters of Mathematics Education by coursework and research report.

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Date: 30 June 2009

DECLARATION

I, Lynette Manson, declare that this research report is my own, unaided work. It is being submitted for the degree of Master of Science in Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any other degree or examination at any other university.

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Lynette Manson 30 June 2009

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ABSTRACT

This research presents a case study at a South African University, involving students who had studied mathematics in a pre-undergraduate Foundation Programme (FP) and who were currently in their first year of study in Information Technology (IT) at the same institution. The study investigated a possible relationship between the teaching approach used in the FP mathematics classroom and the extent of students' abilities to use important mathematical practices, such as using procedures flexibly; using representation; understanding/explaining concepts; questioning; justifying claims; disagreeing; strategising; and generalising, in an undergraduate IT context. Focus group interviews and task-based interviews were used to answer three related questions: "To what extent are students aware of differences in teaching approaches between FP mathematics and undergraduate study?"; "To what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses?"; and "In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?" A *bricolage* of learning theories was used as a framework for understanding the possible relationships between teaching approach, development of mathematical practices and learning transfer. The students in the focus groups described the teaching approach used in the FP mathematics classes as student-centred, whereas many of the undergraduate IT lectures and tutorials were described as teacher-centred. The students felt that the approach used in the FP mathematics classroom was beneficial to further study, in that it taught them how to become responsible for their own learning and brought about deep understanding of the mathematical concepts learned in the FP. The task-based interviews showed that all students used mathematical practices to solve IT problems to a greater or lesser extent. The use of these mathematical practices was best understood as being influenced by all past cognitive, social and cultural experiences, and was therefore not a case of "transfer" in the traditional sense of the word. Instead, the use of mathematical practices could be described as an extreme case of "cognitive accommodation" from a cognitive constructivist perspective, or a case of "generality" from a situative perspective. Furthermore, an inter-relationship emerged between student-centred teaching, students' productive disposition towards mathematics, and the extent of "transfer" of mathematical practices to the IT domain. This interesting relationship warrants further investigation.

Key words: mathematical practices; student-centred teaching approach; learning transfer; cognitive perspective; situative perspective

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CHAPTER 1

CONCEPTUALISATION OF THE STUDY

1.1. Introduction and Problem Formulation

Many tertiary institutions require certain levels of mathematical proficiency in their new student cohorts – particularly those who will study undergraduate subjects such as engineering, medicine, computer science, and economics, to name a few. One of the roles of undergraduate foundation programmes in universities across South Africa is that of preparing students for undergraduate study, and either establishing or further developing important practices of critical thinking, problem-solving, argumentation and decision-making used in undergraduate study and later life. This issue was brought into focus for me at Monash South Africa, where I taught mathematics in the Foundation Programme (FP) for six years. I came to realise that as a mathematics teacher I had a potentially crucial role in helping my students to develop these practices. This brought me to question the effectiveness of how I developed mathematical practices among my students. In order to promote these practices I had attempted to teach in a way that would encourage my students to take responsibility for their own learning. I planned lessons and activities that I thought would promote the development of mathematical practices, which include using procedures flexibly; using representation; understanding/explaining concepts; questioning; justifying claims; disagreeing; strategising; and generalising. One of my goals was to help students develop these practices in a way that would enable them to use their learning in situations outside of the context in which they had originally been used. This goal was especially important – students were required to have certain mathematical knowledge for use in non-mathematics undergraduate study, such as Information Technology and Business Studies. They should be able to use the practices they had learned in the Foundation Programme mathematics classes in their undergraduate study.

I was interested to know what my FP students' experiences were in terms of the mathematical practices they learned in my mathematics class, whether or not the students thought these practices were beneficial to them, and whether or not they were maintaining or furthering the use of these practices in undergraduate study. Furthermore, I wanted to investigate if the

teaching approach used in my Foundation Programme mathematics classes enabled the further use of mathematical practices in largely traditional teacher-centred, large class undergraduate environments. Hence, my research study was guided by the following research questions:

To what extent are students aware of differences in teaching approaches between Foundation Programme mathematics and undergraduate study?

To what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses?

In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?

Cohen, Raudenbush and Ball (2003) suggest that "learning opportunities" are located where interactions exist between teachers and students; teachers and curriculum, and students and curriculum. Their proposal has potentially significant impact on understanding learning. Boaler (2002) describes an earlier rendition of Cohen et al's proposal: "… few learning occasions can be understood without consideration of the contribution made by the teacher, the students, the discipline of mathematics, and the ways that they interact within environments" (p. 244). Figure 1.1. below is a replica of Cohen et al's (2003) model.

All instruction may be modelled by representing complex interactions between these three entities and the related environments. Just as undesirable "cue-based [learning] practices" (Boaler, 1997, 1998, 2002) are developed through these interactions, so too can desirable mathematical proficiency also be developed in these interfaces (Boaler, 2002). I have found Cohen et al's model to be of value for understanding the aspects of my study which deal with student experiences of learning, and of the effects these experiences have had on further undergraduate learning. My study scrutinises the three interfaces spoken of by Cohen, Raudenbush and Ball (2003) in the tertiary education environment. I argue that the instruction my Foundation Programme students received was student-centred, and that students developed mathematical practices, through which they developed mathematical proficiency.

These practices and proficiencies were used in their first year study in information technology.

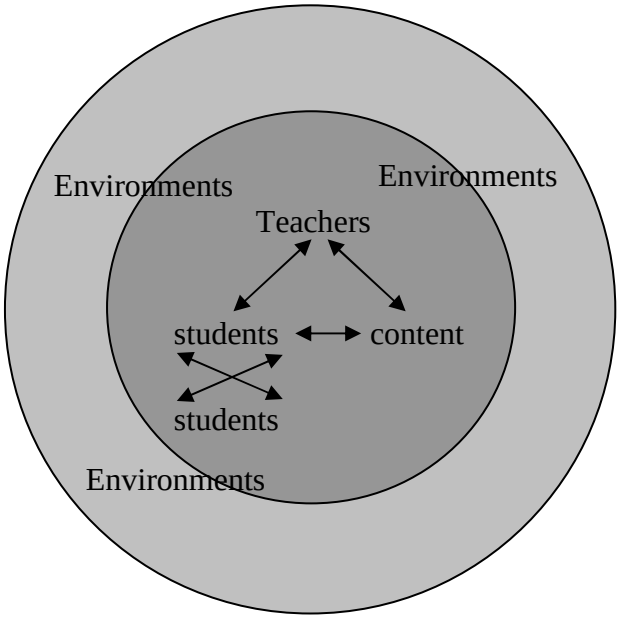

Figure 1.1. Instruction as interaction (from Cohen et al., 2003)

It is argued by many prominent mathematics education researchers that a focus on teaching and learning approaches will enable understanding of differences between effective and ineffective teaching (Boaler, 2002). While this is not my immediate focus, the first two of my research questions address students' experiences of teaching approaches used in the Foundation programme at Monash South Africa, and their beliefs about whether or not these approaches benefited them in undergraduate study. My desire is that learning within a particular tertiary environment may be better understood by educationists, and that furthering understanding of tertiary learning may subsequently positively influence teaching and learning in tertiary environments.

1.2. Rationale

This study provides insight into how students' experiences of my approach to teaching in FP mathematics, (e.g. Brodie, Lelliott, & Davis, 2002; Chung & Walsh, 2000) and an "habitual

inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and ones' own efficacy" (Kilpatrick, Swafford, & Findell, 2001 p.116) may work together to facilitate learning transfer. In addition, the study contributes to understanding the nature of learning transfer between domains – specifically between mathematics and tertiary study in information technology.

My inquiry into tertiary learning can be split into two main themes. The first investigates students' experiences of their learning in the Monash South Africa Foundation Programme in mathematics and the extent to which they believed that the approaches they learned were useful in further undergraduate study. The second theme investigates the ways in which students actually used mathematical practices such as 'using procedures flexibly'; 'using representation'; 'understanding/explaining concepts'; 'questioning'; 'justifying claims'; 'disagreeing' 'strategising'; and 'generalising' in undergraduate study in information technology. These practices were developed from those described by Ball (2003), which will be listed in section 1.3.1 and discussed in more detail in section 2.2.2.

The first theme links to the second, in that the first focuses on descriptions given by the students of their experiences in the Foundation Programme and how these experiences related to their current studies in IT whereas the second looked at what students actually did when they solved IT problems in terms of the mathematical practices they used. Gaining insight into personal experiences of students might provide a tentative explanation of the ways in which they might use their learning in the Foundation Programme in other areas of study – in this case, IT. The significance of the second theme of my study; namely, establishing the ways in which mathematical practices were used by the students to solve undergraduate information technology (IT) problems is twofold. Firstly, that the lecturers of these students may be more informed about the mathematical practices their students bring with them into first year from the FP and how the students might use them in the IT context. Such information may allow lecturers to further promote such practices in the specific IT context and unify their teaching approaches with that of the Foundation Programme teachers. Secondly, this study can contribute to the debate about transfer of mathematical practices to the domain of information technology. Currently the transfer debate is very lively (cf. 1.3.3; 2.5). Observing the mathematical practices students used when they solve IT tasks and how they used them provided interesting understandings about aspects of transfer. At the start of the study I assumed that transfer entailed the direct use of knowledge from one situation to

another. Through analysis of my data, and doing in-depth reading of transfer literature, I came to realise that this is not possible. My study thus reflects a more tentative view of transfer that developed as I progressed with my research. At the same time, however, education in a Foundation Phase Programme must be about building students' competence to participate in further study. So asking questions about how learning in one context influences learning in another is a legitimate and important question.

My study involves understanding how a number of inter-related concepts and theories can be integrated to help understand more about undergraduate student learning and transfer. They relate in terms of whether a relationship exists between student experiences of teaching approaches and the ways in which the mathematical practices that demonstrate mathematical proficiency are used in undergraduate IT study. These relationships may be understood through using particular learning theories as frameworks. In the remainder of this chapter I briefly discuss each, and describe briefly how they can be interrelated. Understanding their interrelationship is important for understanding students' experiences and how they, in turn, impact undergraduate study in IT.

1.3. Central Conceptual Issues

1.3.1. Mathematical practices

Many educational institutions which require mathematics as an entrance subject for study in certain disciplines expect and assume incoming students to be proficient in mathematics. Mathematical proficiency is a term used by Kilpatrick et al (2001) which incorporates "aspects of expertise, competence, knowledge, and facility in mathematics" (p.116), and "[captures] what [the authors] believe is necessary for anyone to learn mathematics successfully" (ibid.). Mathematical practices (Ball, 2003) are what people who are competent or proficient users of mathematics *do* when they are working with mathematics. Mathematical proficiency (Kilpatrick et al., 2001) and mathematical practices (Ball, 2003) are linked, in that the former is how adept/efficient a person is at using mathematics and the latter is what proficient people *do* when they do mathematics. The way in which mathematical proficiency

is linked with mathematical practices is summarised below, and discussed in more detail in section 2.2.2.

Mathematical proficiency (Kilpatrick et al., 2001) is characterised by five interdependent, non-hierarchical 'strands', which focus on various aspects of mathematics. They are described in more detail in section 2.2.1. The first strand is "conceptual understanding", which describes the ability to organise facts into a coherent whole (p. 118), connect facts or methods to contexts, and make connections between concepts. "Procedural fluency" is the ability to use procedures appropriately, flexibly, accurately and efficiently (p. 121). "Strategic competence" describes how the ability to formulate, represent and solve problems allows proficient mathematics users to develop "strategies for solving non routine problems" (p. 127). "Adaptive reasoning" entails justifying conclusions through logical deduction, using tools such as "representation, analogical reasoning and metaphors" (p.129); which are described by Kilpatrick et al (2001) as "aids to learning and transfer" (p. 129). The proficient mathematics user has "productive disposition" when he or she sees sense in mathematics, perceives it as useful and worthwhile, and sees him- or herself as an "effective learner of mathematics" (p. 131).

Proficient mathematics users demonstrate their proficiency by carrying out certain mathematical practices. To effectively link mathematical proficiency and practices, the RAND Mathematics Study Panel (Ball, 2003) stated the following,

> *"A major part of the knowledge teachers need for teaching relates to mathematical proficiency and how it can be developed in their students. […] A second critical priority, if teachers are to help all students attain mathematical proficiency, is the identification, analysis, and development of mathematical practices.* ***In fact, our conception of practices can be seen as another way of framing important aspects of these strands of proficiency.****" (p. 10) (emphasis mine).*

The RAND Mathematics Study Panel (Ball, 2003) has identified certain mathematical practices that demonstrate mathematical proficiency (Kilpatrick et al., 2001) in students. These practices are relevant to mathematics students of any age in any location, because they form the basis of doing mathematics. The Panel (Ball, 2003) describes "representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalising ideas and recognising patterns" (p. 32) as mathematical practices.

Certain undergraduate and post graduate disciplines outside of mathematics require some mathematical application, such as patterning, graph drawing and interpretation, argument and justification, analytical thinking and interpretation. Therefore, mathematical practices need to be "deliberately cultivated and developed" (Ball, 2003 p. 35) at all levels of learning so that students become mathematically proficient and able to use these practices when necessary to solve problems in other contexts. The Panel calls for "… support for concentrated work along three dimensions" (Ball, 2003 p. 33). This entails:

> *"…examining these mathematical practices as they are used in different settings: disciplinary practices, students' out of school practices, practices employed by adults in everyday and work life…"* (ibid.).

Even though the RAND Mathematics Study Panel's (Ball, 2003) mathematical practices provide a description of how proficient mathematicians 'do' mathematics, some aspects of these practices may be more observable than others when students are busy using mathematics to solve problems. In this study ideas from Kilpatrick et al's (2001) and the Panel's (Ball, 2003) work were used to describe what students were doing when they solved information technology (IT) problems. I found during my data analysis that I needed to design my own list of 'mathematical practices' specifically for this study because I sometimes needed more description of what the students were doing when they solved IT problems than that described by the Panel. The list of practices presented in the abstract is my own list of practices and was designed using the notions of proficiency (Kilpatrick et al., 2001) and practices (Ball, 2003). I explain the design of my list of practices in more detail in chapter 3 (cf 3.7.2.). However, I use Ball's notion of mathematical practices often in my initial discussion to set the scene, and acknowledge the Panel's work wherever I quote the practices described by Ball (2003). My set of practices were then linked back (cf Table 2.1.; section 5.5.) to the practices described by the Panel to facilitate understanding of the extent to which mathematical practices were used or not in solving IT problems.

Now that I have shown the connection between mathematical proficiency and practices, I suggest that mathematical practices leading to mathematical proficiency are crucial to the student who is not necessarily going to continue with mathematics as a subject but will need these mathematical practices in his/her IT studies. I suggest that it is of interest to those in the field of mathematics education to obtain insight into the development and use of mathematical practices in non-mathematics subjects in higher education. Part of my study

investigates if students experienced the mathematics teaching approaches in the Foundation Programme as "student-centred", and if so, the extent of the effectiveness of student-centred teaching in developing and using mathematical practices in undergraduate study.

1.3.2. Student-centred teaching

Current education theory encourages teachers to recognise the student's central role in knowledge construction and to support students to recognise their own involvement in making sense of new knowledge. One may argue that sense-making can be made possible through a "student-centred" teaching approach. A current point of contention amongst those studying "student-, learner-, or child-centredness" is the meaning of that phrase. The question may be asked: "the centre of what?" and this question has raised uncertainty, debate and widely varying definitions. According to Chung and Walsh (2000),

> *"the term has masked complex and contradictory underlying assumptions about children and their learning and development that need to be brought to the fore if the education of young children is to be adequately addressed"* (p. 229).

Chung and Walsh (2000) provide a comprehensive historical account of the development of the term "child-centred", in terms of curriculum. "Child-centred" took on three main meanings during the late 1800's and early 1900's. The first was influenced by Froebel, who, considering the "inner being" (Chung & Walsh, 2000 p. 218) of the child as "part of the continuous whole of the person" (ibid.), placed the child "in the centre of things because 'all things are seen only in relation to himself, to his life'" (Froebel, 1889 in Chung & Walsh, 2000 p. 218). In American schools this was interpreted by placing the child at the centre of his/her environment, enabling him/her to "observe, test, experiment, contemplate and use" (Hailmann, 1886 in Chung & Walsh, 2000 p. 220) everything around him/her.

By the end of the 1800's the child had *become* the centre of the curriculum, in the sense that the focus on curriculum development was informed by the child's mental, rather than natural development; through the use of testing and developmental measurement. Psychology (for example, contributions by Dewey and Thorndike) was the main influence of the swing away from the traditional understanding of "child-centred" (Chung & Walsh, 2000).

Further inroads since the 1920's have been made towards understanding how people learn, argued by theorists such as Piaget, Vygotsky, and Wenger; and these viewpoints have in turn broadened the debate about what constitutes student-centredness. I describe student-centredness in cognitive and social constructivist terms below, and then leave the rest of my discussion of student-centredness to chapter 2 (cf. 2.3).

One of the ways in which a teaching approach can be described as "student-centred" (Brodie et al., 2002) is where the teacher structures classroom activities in ways that will provide a scaffold for students to build useful and valuable knowledge in relation to their prior knowledge (ibid.). It is generally understood that a student-centred approach to teaching should improve students' conceptual understanding (Brodie et al., 2002). Although Boaler (2002) does not use the actual term, "student-centred", she provides multiple examples (e.g. Boaler, 1997, 2002, 2003) of schools in which carefully-designed interactions between teachers, students and curriculum have resulted in the development of usable and relevant mathematical knowledge in students. Boaler's work pre-dated the ground-breaking discussions of mathematical proficiency and practices by Kilpatrick, Swafford and Findell (2001) and the Rand Mathematics Study Panel (Ball, 2003) respectively, but serve to suggest a link between teacher/student/curriculum/environmental interactions and the use of mathematical practices to develop proficiency. In section 2.3. I argue how student-centredness can be defined in terms of the three cornerstones of teacher, student and curriculum.

Student-centred teaching, in one of its many manifestations, has been incorporated into the South African Outcomes-Based Education (OBE) policy (National Department of Education, 1997, 2002). As national policy it encompasses all levels of learning in mathematics education – from pre-school to post-graduate level. Student-centred teaching in the tertiary environment is desirable, but not necessarily accomplished. Therefore, part of the significance of this study is to make a link between students' perceptions of student-centred teaching, positive learning experiences, and the establishment of mathematical practices in undergraduate study.

In the next chapter I explain how, theoretically, the teaching approach used in a classroom can influence how mathematical practices are subsequently used in further study in domains that are related to – but are not part of – the mathematics domain. I also explain how, when a teacher uses a student-centred approach to teaching, s/he may encourage students to use their

knowledge in varying contexts – even those far removed from the original learning situation. I then argue that, according to the literature I have cited, the teaching approach I used to teach mathematics to the foundation programme students was student-centred.

Using knowledge from prior learning in a new learning situation is often described as "knowledge transfer". I provide a brief overview of the concept here, and then discuss it in further detail in sections 2.5.8 and 2.5.9.

1.3.3. Knowledge transfer

The concept of knowledge transfer from one situation to another has been subject to extensive debate. Transfer issues became increasingly contested between behaviourists, gestaltists and cognitivists (Carraher & Schliemann, 2002; Lobato, 2006), from the early to mid 1900's. Much of the argument revolved around exactly what knowledge was being 'transferred'. The debate livened considerably during the 1980's, where situated theorists argued vehemently that since all learning is situated in the context in which it was first learned, knowledge transfer, in its original definition of the term, cannot occur (Lave, 1988). However, a broad range of aspects of daily life assume the success of transfer of schooled mathematical knowledge (Britton, New et al., 2007a; Britton, New, Sharma, & Yardley, 2005; Hatano & Greeno, 1999). Mathematics is used while performing ordinary arithmetic computations when in the supermarket buying groceries; when using specifically-taught mathematics, such as basic financial calculations to perform one's job; when learning new mathematical concepts using prior conceptual understanding; and when applying difficult and complicated mathematical concepts in further study, such as engineering studies.

While we know that students use basic mathematical concepts in new situations (Carraher & Schliemann, 2002; Hatano & Greeno, 1999; Salomon & Perkins, 1989) much debate exists concerning the existence of transfer of complex conceptual understanding to new complex contexts. Lave (1988) questions the validity of past transfer experiments in this regard, while researchers such as Salomon and Perkins (1989) specify the different 'kinds' of transfer that can take place. Cognitivists specify exactly how transfer experiments should be run, so that any transfer observed to be taking place has been previously systematically defined – in terms

of the content expected to have been transferred, the transfer environment and the experimental design (Lobato, 2006).

What has become clear is that the phenomenon called ‘transfer’ is understood differently by situated theorists and cognitive theorists. Two important aspects of this are firstly, disagreement about *what* is transferred; and secondly, about *how* transfer should be measured. Resulting proposals about transfer include the following: (1) the word ‘transfer’ is maintained but used only when referring to the original description of the transfer of “identical elements” (Lobato, 2006) used by the behaviourists in the early 1900’s (Carraher & Schliemann, 2002); (2) the word ‘transfer’ is not used at all any more, as its original meaning has changed so much that it needs another description (and has since been defined using numerous new terms) (Carraher & Schliemann, 2002; Engle, 2006; Greeno, 2006; Lobato, 2006); (3) different ‘kinds’ of transfer need to be recognised and defined carefully; giving meaning to the way prior knowledge can be used in new situations (Perkins & Salomon, 1989; Salomon & Perkins, 1989).

Since my study considered the use of mathematical practices by IT students when solving novel IT tasks – a case of ‘transfer’ from one domain to another – the “transfer dilemma” (Carraher & Schliemann, 2002), as it was briefly summarised above, was observed in its multi-facetedness. This made the task of recognising, understanding and interpreting transfer in my study relatively complicated. In addition, it became important to ascertain whether or not the assumption that schooled mathematical knowledge is transferable is justifiable. If such an assumption is not justifiable, then it is possible that a different strategy will need to be designed to plan how to prepare people for future living and working situations.

1.3.4. Theoretical underpinnings

Underpinning the concepts of student-centredness, mathematical practices and transfer is the way that they may be observed and interpreted. This study has been theoretically framed by merging cognitive constructivist and situative perspectives of learning. I explain in the next chapter how these two perspectives contrast, and how I have combined them to gain a richer understanding of students’ learning experiences and use of mathematical practices in tertiary

study. Bearing in mind that my choice of a particular theoretical framework immediately exposes certain weakness and strengths of my research design, as well as highlighting my own personal predisposition, I discuss how I have made sense of students' opinions and actions with respect to how they used mathematical practices in undergraduate IT studies.

1.4. Background and Context of the Study

Tertiary curricula often differ significantly between institutions. Even though institutions are accountable to higher education councils within their respective countries, they remain relatively autonomous, compared with primary and secondary schooling. I situate this study, with respect to mathematics teaching and learning at Monash University South Africa (MSA), an Australian-owned university. I provide an overview of the Foundation Programme mathematics course which prepared students for undergraduate study at MSA, and of the approaches I used to teach the course.

1.4.1. Mathematics in the Monash South Africa Foundation Programme

Students who take mathematics in the Foundation Programme (FP) continue with undergraduate study in information technology (IT) and business studies. Having consulted with the IT and business lecturers we designed a mathematics programme to address the mathematical needs of students entering these courses. The resulting work programme consisted of two modules – Math A and Math B, which are summarised in Appendix A.

The teaching approach I used was generally different from the way other mathematics teachers in the Programme taught and was influenced by knowledge gained over the years from research in teaching and learning of mathematics. I approached the Math A programme by making use of whole class discussion, encouraging students to work with their peers as much as possible, and using different formative assessment techniques. For example, a volunteer would work through a problem in front of the class on the board, and we would discuss what s/he was doing, adding ideas, arguing with each other about different ways the

problem could be tackled and questioning in non-threatening ways why s/he chose to do a problem in a particular way. As the year progressed, students' voluntary participation in whole class discussion increased noticeably. In addition formative assessment such as regular compulsory homework allowed students to work on their own and to write notes to me in their homework about concepts they did not understand. I found that initially students would be reluctant to experiment with solution ideas; for fear that they might be 'wrong'. Handing in homework provided them with a non-threatening way to experiment with ideas in pencil, knowing that I would provide feedback about their conceptual understanding and correct their procedures.

Examples of other formative assessment included making mind maps to summarise whole sections of work; writing letters to an imaginary friend who had been absent for a particular lesson, and who needed a written explanation using definitions and representations in as much detail as possible to understand what had been missed; group activities; and correction of deliberately incorrectly-worked problems with explanations of where the problems were and what was wrong. Verbal and written personal and whole class feedback was given for formative assessment done. Students often commented in hindsight that the workload was "tough" but that they appreciated the ways I made them work because it helped them to "know mathematics better".

When the time came to teach Math B in the second half of the year, I had gained more insight into teaching and learning mathematics, which I wanted to try out in my own teaching. The approach I used to teach Math B was adapted from a study by Herrenkohl and Guerra (1998), in which two primary science classrooms were the subject of specific interventions designed to guide students to constructing specified aspects of scientific understanding. Based on the nature of scientific knowledge, individual students in the study were allocated 'roles' that they were to assume during small group activity. These roles required them to focus on one of the following properties of scientific knowledge: (1) making a prediction and building a theory; (2) summarising results; and (3) relating the evidence or results to the prediction and theory. The roles were designed to encourage students to "check other students' work" (p. 431).

Herrenkohl and Guerra (1998) reported that "Although theoretical perspectives clearly support the importance of student engagement, translating these ideas into viable pedagogical practices has not been as successful" (p.432). For this reason the "roles" were developed to

address three aspects of learning science that they argued were shortcomings in current science education. Summarised, they include,

> *"…(a) moving classrooms beyond traditional Initiation-Response-Evaluation (IRE; Mehan, 1979) sequences, (b) making the "rules" of scientific discourse and inquiry explicit to students by establishing intellectual roles that focus on guiding students to differentiate and subsequently coordinate theories and evidence, and (c) implementing the previous principles within a social environment that offers students opportunities to practice their cognitive skills.* (p. 434).

A significant aspect of using 'roles', in terms of the applicability of this study to my own, was the deliberate encouragement of the student to develop "deeper scientific thinking" (p. 439), through considering their own and others' ideas "worthy of reflection and response" (ibid.). Assigning 'roles' encouraged two aspects of learning which are important. First, discussion, question and argument between peers are likely to make students realise that there are aspects of the material that they do not fully understand. Second, students are exposed to many ideas which are often different from their own, but possibly equally plausible (Herrenkohl & Guerra, 1998).

While each role focused on one of the identified scientific practices, the students interchanged roles so that they could have experience using all of the scientific practices. These practices were to be promoted by explicitly teaching each student to ask questions pertinent to the 'role' s/he was currently playing. The teacher would be available to help the students to ask directed questions while they were still learning how to fill their 'roles'. Examples of questions related to each respective practice were, prediction: "What do you think is going to happen?"; summarising: "What did you find out?" and relating results to theory: "Did your results support your theory?".

I now explain how I based my teaching on this study; describing the similarities and differences between them. My objective was to help my students to become more mathematically proficient. I made a decision to teach my students how to ask questions that would focus on a particular proficiency strand (Kilpatrick et al., 2001) (cf. 2.2.1.), which I described to them as the following inter-twined, interdependent areas of doing mathematics:

1. competently using mathematical procedures and formulae,

2. understanding mathematical concepts so that they could flexibly use related formulae and procedures,

3. being able to make decisions and use effective strategies to solve problems, enabling them to strategically use their conceptual understanding and mathematical procedures flexibly,

4. being able to reason and argue through problems, using well-thought-out justifications to convince others of the validity of their claims, and

5. being able to see mathematics as a challenge, so that the above four principles could become part of a collection of tools with which problems could be solved.

My goal was to make the five proficiency strands explicit to the students. I did not focus on the mathematical practices, as described by the RAND Mathematics Study Panel because I was less familiar with them at the time. In contrast, I understood the strands of mathematical proficiency well, and saw the descriptions of what each strand entailed as detailed and helpful to me as a teacher – both in my own mathematics practice, and in helping students to be more proficient in mathematics. I set out to make the strands explicit to my students. I taught them how to question each other according to aspects of each strand, because doing this might help the students to become more proficient in mathematics.

Although I originally explicitly taught the students to ask questions based on descriptions of the proficiency strands, I came to understand better during this study that what people *do* are practices, and that the practices described by the RAND Mathematics Study Panel are the evidence of mathematical proficiency. These practices needed to be linked with how mathematical proficiency is described and how I originally used them to inform my teaching, because it would be mathematical practices, that would most likely be transferred to new contexts. In this study I looked for mathematical practices used by students who were now studying IT, but the terms used to describe the practices are coloured by my work with the students in Kilpatrick et al's proficiency.

Each 'role' in my lessons comprised a mathematical proficiency strand, which had been explained to the Math B students. I explained that they needed to use their understanding of

the particular proficiency strand while they were playing that role, by asking strand-directed questions. The students were assigned into groups of four, and sometimes five, and gave themselves their 'roles' – that is, each student took responsibility for asking questions pertaining to a particular proficiency strand. While I did not assign a role to the strand of 'productive disposition', because I was not sure how to do this, I hoped that I would observe an increase in productive disposition during the classroom sessions. The students were given a list of example questions focusing on each proficiency strand (see Appendix B) that could be asked, in order to focus predominantly on a particular strand of mathematical knowledge. For example, the conceptual understanding 'role' directed the student to focus on formulating and asking questions whenever appropriate along the lines of gaining 'conceptual understanding' of the topic as the group worked through a worksheet. The worksheets were designed by me, and consisted of questions that would lead to them gaining understanding of mathematical concepts, such as average gradient of a curve, gradient at a point and limits, and how these related to each other in differential calculus. Each time a new conceptual worksheet was given, the students would assign themselves a new role, different from any they had previously taken, so that each student had a chance to perform each role at least once.

Each group was given one A4 size exercise book in which they were to write down all the answers, comments, questions and arguments they had while they were answering the worksheet. After a worksheet had been completed one group would be assigned to present their thinking to the class. The class was encouraged to question the group, using the same kinds of questions from the strands that they had used in their groups. At this point I would also direct questions to the group and the class as a whole concerning important issues they had not thought of or had not thought through completely enough. The classroom discussion was interesting, and the students were discouraged from making disparaging comments about each others' ideas. This was not a problem, though, as students were very polite to each other and respected other groups' input. Each individual was given a copy of the worksheet for personal record.

The students initially expressed concerns about discussing the work before I had "taught" them and so possibly learning wrong information. I had quite a difficult time persuading them to trust me – that I would be walking around and listening to them all the time, that they all have different experience and knowledge and all this input would be very valuable for their learning, and that I would summarise what they had done by giving them notes and teaching

after a particular concept had been explored by them first. In this way they would understand concepts, because they were discussing ideas thoroughly with their peers, who all could contribute with different and valuable ideas. This meant that the teaching and consolidation of concepts took place after the initial exploration, and students were able to comment on and question what I was teaching them because they already understood what we were talking about.

The reason why I approached Math B in this way was because I had realised that students had probably been exposed to teaching of concepts and procedures during their school mathematics years, but had probably been deliberately encouraged very little, if at all, to solve problems, argue and justify their thinking, and work with pattern recognition and rule formulation while doing mathematics (Ball, 2003; Kilpatrick et al., 2001). I predicted that if I made the components of the strands more explicit to the students, I might encourage them to take more responsibility for their own learning, and they would come to see mathematics as an exciting, challenging subject, rather than a dry, boring, mysterious set of rules.

Traditionally, I would have provided an explanation and notes to the class, using whole class discussion extensively; after which the students would go and 'practice' as many and varied examples as possible, in order to reinforce what they had understood, or to find out what they did not understand; in which case I would explain again. I found this new approach refreshing, and it appeared to have positively affected the students' learning. I argue that it was student-centred, with respect to curriculum, pedagogy and inter-personal relationships (Brodie et al., 2002), which I discuss in more detail in section 2.3.

According to my understanding of the literature (cf 1.3.2. and 2.3.), the above description indicates that I used a student-centred teaching approach in my teaching. I structured classroom activities in ways that would provide a scaffold for students to build useful and valuable knowledge from pre-existing knowledge (Brodie et al., 2002) (cf 1.3.2.) and my students participated in discussion more frequently over the year and appeared to engage more with problems given to them. Through some students' comments and questions I observed that their knowledge had become increasingly focused on problem-solving and reasoning rather than on whether or not their answers were 'right' or 'wrong'. I became increasingly aware of students' needs, through what they expressed to me verbally and through how they answered questions.

According to my understanding of the literature cited in sections 1.3.2 and 2.3. my teaching approach was student-centred. However, my view is not sufficient. Part of this study was to understand the extent to which students saw my teaching as student centred. Although I felt that the students had benefited from this approach, I wondered if they felt the same way and if this approach would benefit the students when they moved into undergraduate study. Would they carry on using the practices that I had come to believe are an effective way of teaching and learning mathematics? This study reports the students' experiences of the approach I used when I taught Math B in 2006.

In the next section I discuss various reports of what have been described as student-centred environments in tertiary institutions in general, and then I consider some Foundation Programmes in South African tertiary institutions. These reports are a background to my study. They all take a cognitive perspective of learning. My study differs significantly from them in that it works with merging cognitive and situative approaches. Through merging these two prominent learning theories, I seek to understand firstly, the ways in which students may have developed mathematical practices in a pre-undergraduate Foundation Programme, and secondly, how they might come to use such practices when solving problems in the new undergraduate context of IT. I remind the reader here that the following research questions guided the study:

To what extent are students aware of differences in teaching approaches between Foundation Programme mathematics and undergraduate study?

To what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses?

In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?

Mathematics teaching and learning in tertiary institutions has not been documented as extensively as that in primary and secondary school. I provide an overview of recent descriptions of different tertiary situations; with respect to students' experiences, suggested teaching approaches and a study of transfer of mathematical knowledge to other learning

domains, for the reader to gain an idea of some of the current trends in teaching and learning in tertiary institutions.

1.5. Literature Review

1.5.1. Student-centred teaching in higher education

Student-centredness, discussed briefly previously, has many associated meanings. Use of a particular meaning directly or indirectly evidences affiliation with certain learning theories, psychological, historical and religious standpoints, and so on. When the term "learner-centred" or "student-centred" is used in higher education I have observed that the meaning in the context used is not always clear. Two higher education studies, by Berry et al (1999 pp. 239-240) and Walczyk and Ramsey (2003 p. 567), summarised below, do provide detailed explanations of what "student-centredness" entails, in the contexts of their studies.

According to Brodie et al (2002), the 'form' of student-centredness deals with the activities and classroom organisational setups that teachers use. The 'substance' of student-centredness involves the way that teachers select and sequence tasks in order to scaffold tasks to promote learning. A more detailed description of the form and substance of student-centredness is provided in section 2.3. Although 'form' and 'substance' with respect to student-centredness are not referred to in any of the examples discussed below, the fact that the authors are aware of, and are reporting the attempt to, or are themselves attempting to improve the quality of student learning, indicates that the their practices are directed at being student-centred. Becoming student-centred has far-reaching implications for staff and students at a higher education institution, as well as for the institution itself.

Research has suggested that it is difficult to achieve student-centredness in higher education. Reasons for this include the large volumes of content required to be mastered by tertiary students and offered by lecturers with limited pedagogical techniques (Vega & Tayler, 2005), large class sizes found in tertiary education (Bitzer, 2005; Pokorny & Pokorny, 2005), the possibility that lecturers focus strongly on content and examinations using traditional transmission-style teaching (Slabbert & Gouws, 2006; Vega & Tayler, 2005; Walczyk &

Ramsey, 2003), and the unspoken expectation lecturers have that students must 'survive' tertiary education and become independent thinkers and students without giving them much guidance as to how this can be achieved (Bitzer, 2005; Pokorny & Pokorny, 2005). Often institutional policy influenced by administrative accountability dictates how teaching and assessment programmes are designed (Cobb, 2007). Despite the difficulties, student-centred teaching is practiced by tertiary teachers. The accounts I have reported below have used cognitive and social constructivism (cf. 2.4.2.) as frameworks for their studies and conclusions.

Vega and Tayler (2005) reported that academic staff from a range of faculties at their institution had recently begun to implement student-centred approaches based on a programme where they learned student-centred pedagogical principles. Initial 'tensions' experienced by staff while working with student-centred approaches were found to ultimately help staff to become more comfortable with their new approach, once they had dealt with these tensions. Vega and Tayler reported that the student-centred approaches used were effective and applicable across many different domains; and included using variations of peer evaluation, of small group activity, and of discussion, such as questioning, arguing, criticism and socially-based reconstruction of meaning. The authors concluded that the students "participate and learn in ways that are far more enduring in their application to life situations than would be the case in the transmission of factual knowledge" (Vega & Tayler, 2005 p.86).

Walczyk and Ramsey (2003) reported that students studying undergraduate mathematics and science majors at American universities had expressed dissatisfaction with the quality of instruction they had been receiving, and the numbers of students taking mathematics and science majors had halved since twenty years ago. The predominant tertiary teaching approach used still tended to be traditional because many lecturers believed that teaching cut into their research time. Therefore students were not actively engaged with their learning. Walczyk and Ramsey argued that "learner-centered approaches to science and mathematics instruction assume that only when students are active participants will learning be deep, enduring, and enjoyable, and transfer to contexts beyond the classroom" (p. 566). Six principles promoting teaching for deep learning and transfer were suggested. In addition, the authors listed a set of seven recommendations that would promote student-centred teaching (Chickering & Gamson, 1999 in Walczyk & Ramsey, 2003 p.567). After investigating whether or not the student-centred planning of undergraduate mathematics and science classes

was actually occurring, they observed that on the occasions where student-centred instruction was used, it had been applied across all aspects of teaching. The study did not report on the effects of student-centred teaching on the students.

A major concern was raised by Slabbert and Gouws (2006) that South African accounting students are not learning authentic accounting practices during undergraduate study. They argued that tertiary preparation for accountancy was inadequate because it was typically content-driven and prescriptive, and that it produced technicians instead of real accountants. Accountancy students should be collaboratively constructing meaning through the practices of critical thinking, problem-solving and decision-making. In many ways, their description of authentic accounting practices is similar to that of the mathematical practices advocated by the RAND Mathematics Study Panel (Ball, 2003) (cf. 2.2.2.) – that is, *doing* real accountancy. Slabbert and Gouws did not use the term "student-centred" in their discussion, but promoted a kind of student-centredness in terms of authentic accounting practices, where lecturers can provide "powerful learning environments" (p. 336) to help their students to be better prepared for accountancy practice in the real world.

Berry, McIntyre & Nyman (1999) promote active, student-centred learning mathematics in higher education, where the student is "responsible for his or her own learning" (p. 238), using computer algebraic software (CAS). Such approaches were found by students on the whole to be "stimulating and enjoyable" (p. 237), although the learning of mathematics using these aids was not always efficient. The authors did not explain what they meant by 'efficient', but I understand them to mean either that the quality of learning was not always the best that it could be, purely because exciting learning aids were being used, or that learning took longer when using CAS.

Recent research has shown that widening access to tertiary study is associated with problems such as "declining student progression and retention rates" (Pokorny & Pokorny, 2005 p. 445), partly because many tertiary educators incorrectly presume that their students will rapidly become independent learners (Alexander, Bishop, Crawford, & McCartney, 2006; Bitzer, 2005; Pokorny & Pokorny, 2005). In addition, because of widened access, student support resources are more limited than they were in the past, because they have to be spread more thinly (Pokorny & Pokorny, 2005). With respect to student feedback concerning mathematics learning, smaller tutorial sessions were preferred over large lectures, and

Goldfinch (in Pokorny & Pokorny, 2005) reported significantly improved examination results when mathematics classes were smaller, and provided a chance for greater individual interaction with teachers. According to Goldfinch, mathematics learning in particular is negatively affected by traditional transmission-based learning environments because it is a structured subject: If a concept is not understood early on in a lecture, the rest of the lecture is rendered 'unintelligible'.

It is generally understood that levels of anxiety of incoming higher education students are often high (Lucas, Damianova, Burney, & Ponto, 2006) – particularly those students who did not achieve excellent marks at school, or may not know what it takes to "survive" (Bitzer, 2005) in such different environments. Pokorny & Pokorny (2005) argued that higher education institutions need to move away from traditional transmission teaching and work on ways to support new students in their study endeavours within the mass entry environment. Bitzer (2005) suggested that higher education institutions should look at ways of improving students' writing, problem-solving and self-management skills, and Smuts (2005) suggested that student feedback is helpful for reflection on teaching (by the lecturer) and learning (by the students) in higher education.

Higher education teachers in the United Kingdom have experienced difficulties, in that if they *do* use student-centred approaches where students are viewed as responsible for self investigation of topics, the students have been known to complain about not being taught properly, which is in turn monitored by education authorities (Berry et al., 1999). Since the 'lead-to-the-answer', "'spoon feeding' approach [often leads to] artificially apparently better results" (Berry et al., 1999 p. 239), higher education teachers and students may not be prepared to brave the better approach that will benefit learning in the long run. It is possible that administrative pressures influence teaching in many educational institutions.

Monash South Africa, being a private institution in South Africa, is not funded by the South African government. However, to have all courses registered with the South African Qualifications Authorities (SAQA), the university has to comply with South African educational requirements. Study at this university is more expensive than at others in South Africa. Students and parents therefore want their 'money's worth', and are afforded input with respect to the quality of teaching offered on the South African campus. Short term visible effects of teacher-centred learning, such as copious lecture notes and assessment

scores are desired by many staff and students. Thus, the problems described by Berry et al (1999) also occur at Monash South Africa.

The above examples of teaching in higher education institutions frequently refer in passing to transfer of learning to new contexts as either expected or taken for granted. While none of the above studies focus on transfer, their incidental references to transfer suggest firstly, that the ability to transfer knowledge and subject specific practices is assumed as integral to tertiary learning and essential for taking the knowledge and skills learned into the job market; and secondly, that transfer is facilitated through the implementation of student-centred teaching approaches. Both of these ideas are discussed in detail in chapter 2.

Remaining with the current focus on higher education, and because this study explored the experiences and learning of Foundation Programme students who have moved into their first year of undergraduate study in IT, it is fitting that I report on some of the research into teaching and learning that has recently taken place in other South African foundation programmes.

1.5.2. Foundation programmes in South Africa

Many universities in South Africa have instituted pre-undergraduate foundation programmes in different disciplines. One rationale for the existence of such programmes is that many students who enter into undergraduate study are under-prepared for tertiary education in a number of ways, such as low academic knowledge and skills and inability to cope with the pressures of social and academic university life (du Plessis, Janse van Rensburg, & van Staden, 2005; Pokorny & Pokorny, 2005; Wood & Lithauer, 2005). Such programmes have been the focus of research, with respect to the quality of what they provide academically, within their respective disciplines.

The foundation programme at the Nelson Mandela Metropolitan University (NMMU) in Port Elizabeth aims to “provide students with an alternative access route to tertiary education” (Wood & Lithauer, 2005), by equipping them with the necessary knowledge and skills required for undergraduate study. Students were asked to elaborate on their experiences of the foundation programme. Along with the social and emotional support that the students

received in the foundation programme was the finding that their undergraduate academic performance was the same as, and often better than, comparable students who had been directly admitted into mainstream undergraduate study (ibid.). It was found by Wood and Lithauer (2005) that 'better academic performance' could not be attributed to a single factor, such as cognitive development, but to the holistic skills which the foundation programme helped to develop in the students. Lizzio et al (2002, in Wood & Lithauer, 2005 p. 1008) agree that holistic learning, namely, academic and social development, influences student performance in undergraduate study. They suggested that students' "perceptions of their course" (p. 1008) influenced their learning within that course. "Students who [had] a negative opinion of their course tend[ed] to have a surface approach to learning, whereas those who [were] positive tend[ed] towards deep learning" (ibid.). This was stated by Wood et al to be an important consideration for foundation programme course design. Some students at the NMMU also felt that although the cognitive requirements of the foundation programme were not challenging enough, on hindsight, the student-centred approach of the foundation programme had prepared them holistically for further undergraduate study.

The foundation programme offered to information and communication technology (ICT) students at the Vaal University of Technology was developed because of the unacceptably low pass rates for the *Programming 1* course (du Plessis et al., 2005). Like Wood & Lithauer (2005), du Plessis et. al. (2005) (and all reports referred to in the previous section) raised concerns over the inadequacy of the cognitive skills shown by undergraduate students. They investigated the relevance of their foundation programme to the students' cognitive and social needs, as well as to their experience. They found that good English proficiency and introductory ICT skills obtained in the foundation programme positively influenced the Programming 1 results. However, disturbingly, the mathematics marks did not show any significant correlation with the Programming 1 results. The authors were astounded at this finding, as they reported that literature

> *'…indicates that mathematics is an important tool in developing learners' logical thinking skills and [that] it enhances reasoning, which is the basis of logical thinking skills and problem solving. It furthermore indicates that mathematical abilities play an important role in the successful learning of programming skills…* (du Plessis et al., 2005).

This finding is somewhat surprising. According to Bruce, Scot Drysdale, Kelemen and Tucker (2003) and Henderson (2003) any kind of mathematics is appropriate for

programming, as it is not the content of the maths that is needed by programming students, but rather the thinking skills that are developed for further use during the process of general programming. I suspect that the 'failure' to use/transfer mathematical knowledge in this case is an example of fragmented mathematical knowledge being of limited use in situations other than the mathematics classroom (Boaler, 1998). Also, it is possible that the environment and conditions of transfer have been dictated by the authors' interpretation of what counts as transfer, and what they expected to be transferred – a behaviourist/cognitivist understanding of transfer (cf 2.5.).

Wood & Lithauer (2005) (NMMU) and du Plessis et. al. (2005) (Vaal University of Technology) reported some of the cognitive and affective aspects of students studying in pre-undergraduate foundation programmes; the commonality between the reports being the necessity for pre-undergraduate students to perform better in their first year of undergraduate study. The Monash South Africa Foundation Programme (MSAFP) has the same goals. The difference between the two reported studies about learning in South African foundation programmes and my study is that these studies did not specifically investigate learning transfer, and mine focused on transfer of mathematical practices to undergraduate study in information technology. Mathematics is a compulsory subject in the MSAFP for those students wanting to study business or IT, and the question was: In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?

The relevance of the NMMU study to my own is the description of the holistic nature of the foundation year appearing to have a significant influence on students' undergraduate learning. This study appears to emphasise a more situative perspective on learning; which is pertinent to my study, since my study investigates transfer from both cognitive and situative perspectives. Recent literature indicates that all aspects of a student's life plays a role in knowledge transfer, and it is therefore possible that holistic learning positively influences learning transfer. The relevance of the Vaal University of Technology study to my own is the apparent lack of transfer of mathematical knowledge from the foundation year to undergraduate IT. Since IT was also the focus of my study, I was interested to find out if taking a situative perspective on learning transfer changed the interpretation of how mathematics is used in undergraduate IT study.

A large amount of literature referring to learning transfer is available. It discusses issues such as whether or not transfer occurs (e.g. Carraher & Schliemann, 2002; Lave, 1988), the theoretical perspectives from which transfer can be investigated (e.g. Anderson, Reder, & Simon, 1996; Carraher & Schliemann, 2002; Greeno, 1997), and suggestions for facilitating transfer from prior to new learning situations (e.g. Alexander & Murphy, 1999; Engle, 2006). Much of the literature discusses transfer from learning in institutions to 'everyday life' or to the business place, or transfer within the school environment. Very little appears to be reported about the transfer of mathematical knowledge to further tertiary learning situations. Following is a report of transfer research that took place in a university environment in Australia. The research is framed within a cognitive constructivist perspective (cf. 2.4.2.) and therefore has clearly defined the tasks and environment in which the extent of transfer was to be measured. Also, it was the only study that I could find in the literature that discusses transfer of mathematical knowledge to undergraduate study.

1.5.3. Transfer of mathematical practices to other undergraduate contexts

A study at the University of Sydney, Australia (Britton, New et al., 2007b; Britton et al., 2005; Roberts, Sharma, Britton, & New, 2007) investigated the ability of first year undergraduate students to transfer mathematics to science contexts. While the authors are aware of controversy surrounding the concept of transfer, they maintain that "it is a fundamental, if implicit, assumption of the modern day education system that students possess this ability" (Roberts et al., 2007 p. 1). They argue that "transfer is the 'ultimate goal of education'" (ibid.). The authors go on to describe 'far transfer' as the type of transfer "in which educators are most interested – the application of knowledge and skills to dissimilar contexts, at later times, in novel ways" (Roberts et al., 2007 p. 2) – hence my interest in their work.

These authors have attempted a novel way of measuring transfer by considering graphing skills in their calculations: a measure they have referred to as 'graphicacy'. Their rationale behind using graphicacy originated from Gill's (Britton, New et al., 2007b) suggestion that understanding of graphing concepts predicted success on a later mathematics exam. Results of their study showed that

> *"… transfer ability is positively correlated with graphicacy, although we do not know whether superior ability to interpret graphical representations of mathematical data is a cause or consequence of superior ability to transfer mathematical learning. However the strength of the correlation has encouraged some of us to change our emphases in teaching. We now devote more time to explaining the interpretation of graphs in the context of our own scientific disciplines. Further work is needed to confirm that this approach improves mathematics transfer and numeracy"* (Britton, New, Roberts, & Sharma, 2007 p. 140)

The research team subsequently postulated that this understanding of graphing may "underpin higher order mathematical concepts" (Roberts et al., 2007 p. 4), and is either a result of, or a pre-requisite for, "deep mathematical understanding" (ibid.). These findings resulted in the development of a transfer index, which measured the degree of transfer of mathematical knowledge (namely, of logarithms and exponents) to first year studies in mathematics, physics and microbiology.

There was found to be further need for study of the more qualitative aspects of transfer of mathematical knowledge, through interviews with the students – in particular, "interviews with those students who seemed able to use a piece of mathematics in a context, but did not display evidence of having that piece of mathematical knowledge on the purely mathematical questions" (Britten, personal communication, May 2007). Recent understanding about transfer indicates that this is explainable through understanding how students are able or unable to use their school learning in new contexts, and how they often use innovative ways to solve mathematical problems in non-school contexts because they cannot draw on the mathematical knowledge that they learned at school (For example, Boaler, 1997, 1998). Vice versa, students often do not relate what they do in real life to instructional situations (Boaler, 1997, 2000b). This can be because of piecemeal teaching of mathematical concepts and procedures that may takes place at school, which have never become part of personal sense-making by the student.

With the University of Sydney study in mind, my study investigates students' transfer of mathematical practices to the tertiary learning domain of IT. Unlike the other study, my study is qualitative, and examines students' experiences of tertiary study, linked with their abilities to demonstrate transfer of mathematical practices while doing first year IT tasks. Robert et al's (2007) statement that "it is a fundamental, if implicit, assumption of the modern day education system that students possess [transfer-] ability" (p. 1) is significant; because a certain level of mathematical knowledge is a requirement for undergraduate study in many

areas. Therefore it was of concern to me as a mathematics teacher preparing students to use their mathematical practices in undergraduate IT study, to understand the extent of students' abilities to make use of mathematical practices in their IT courses.

In chapter 2 I argue that to frame my study learning theories of cognitive constructivism and situated learning can be used in support, rather than in conflict, of each other, in order to gain a broader perspective of learning (Boaler, 2000a), and transfer. This possibility opened up an avenue to investigate, understand and interpret how transfer of mathematical practices to other non-mathematics undergraduate subjects might be possible. This study shows that like learning, the transfer of knowledge involves multiple aspects of a person living in the world – aspects of culture, previous experiences, prior learning, social interactions, and others; and can be better understood from social, anthropological and cognitive perspectives working in conjunction with each other.

1.6. Outline of Research Report

The study has been reported in the following format: Chapter 2 is comprised of a detailed discussion of mathematical proficiency (Kilpatrick et al., 2001) and mathematical practices (Ball, 2003), and how these may be developed in the student through a student-centred teaching approach. These components of learning mathematics have been framed by means of a '*bricolage*' (Cobb, 2007) of two learning theories, and are used together to analyse and interpret the research findings. Chapter 3 describes the research design and methodology. Chapters 4 and 5 provide detailed analyses of focus group interview and task-based interview data respectively, in terms of the students' experiences in tertiary study and their use or non-use of mathematical practices when doing information technology problems. Chapter 6 is an explanation of the manner in which student experiences in the Foundation Programme and first year study influenced their use of mathematical practices in IT, with respect to cognitive and situative perspectives of learning. Recommendations for further study are provided, as are interesting questions that arose but were not answered during the study.

1.7. Conclusion

In this chapter I situated my study within a tertiary learning environment and described some of the issues which higher education students might face during their tertiary studies. I briefly discussed the teaching and learning environments of some international South African undergraduate and foundation programmes. I alluded to the possibility that a student-centred teaching approach improves students' senses of well-being in a new learning environment, and allows for learning to be used in contexts outside the original context in which the learning occurred. I also introduced the idea that understanding knowledge transfer is key to understanding aspects of teaching and learning – even when it is not a focus of such studies. I provided a description of the mathematics learning environment in which the respondents in this study were situated.

The next chapter considers theoretical aspects of the study: the theoretical framework which underpins the study, as well as theoretical issues relating to knowledge transfer. I also explain how mathematical practices are integrally involved in the investigation into students' experiences of learning in undergraduate study and the transfer of mathematical knowledge into undergraduate studies in Information Technology (IT).

CHAPTER 2

THEORETICAL PERSPECTIVES, LEARNING TRANSFER AND THE ROLE OF STUDENT-CENTRED TEACHING: A '*BRICOLAGE*'

2.1. Introduction

The focus of this study was to investigate students' learning experiences in the pre-undergraduate Foundation Programme, and relate these to their beliefs concerning the usefulness of the approaches they learned and to the extent of transfer of mathematical practices to undergraduate study in information technology. Research has shown that students who find themselves mastering mathematical subject content and being able to use it and its associated skills and practices effectively in other walks of life will also describe their learning experiences of that subject positively (Ball, 2003). Therefore, student experiences of my teaching approaches, and their perceptions about how helpful these teaching approaches were in undergraduate study, are important aspects influencing transferability of mathematical practices to other academic domains in undergraduate study. The existence of a certain level of proficiency in the mathematics domain is necessary before it can be used in other domains. This study is dependent on an understanding of what mathematical practices are, how they are used as a whole by mathematically proficient people, and why they are important indicators of proficiency in mathematics (cf 1.3.1.).

I shall begin my theoretical discussion by first reviewing what the literature says about mathematical proficiency and the practices used by mathematically competent people to demonstrate proficiency and to show the links between practice and proficiency (cf 1.3.1). I then explain what comprises a student-centred teaching approach (cf 1.3.2.), and how development of mathematical practices in students may be brought about through using such an approach (cf 1.3.2.).

The extent of actual use of mathematical practices students demonstrated while doing IT tasks is related to my first two research questions through an understanding of how mathematical practices may be developed and used by students, and how they may be transferred between different contexts (cf 1.3.3.). It may be argued that knowledge transfer is a key indicator of

successful learning. How one understands transfer is, however, dependent on the theoretical perspectives of learning that one takes. This chapter will secondly discuss the theoretical underpinnings of the study (cf 1.3.4) and explain why a certain selection of theoretical perspectives has been used. Thirdly, transfer will be explained in terms of historical perspectives (cf 1.3.3.) and then in terms of the theoretical perspectives I have chosen to frame my study (cf 1.3.3.). Lastly, I argue how a student-centred teaching approach may positively influence students' abilities to transfer knowledge, by linking it with how transfer of mathematical practices may be encouraged through a student-centred teaching approach.

2.2. Mathematical Proficiency and Practices

2.2.1. Mathematical proficiency

Mathematical competence has recently been framed by Kilpatrick et al (2001) into the notion of *mathematical proficiency*. Mathematically proficient people demonstrate a set of skills that are mutually influential; described as the 'intertwined strands of proficiency' (cf 1.3.1.). Listed, the strands are "conceptual understanding", "procedural fluency", "strategic competence", "adaptive reasoning" and "productive disposition". A brief description of each strand follows:

Conceptual understanding involves the comprehension of concepts, and relating them to operations and other concepts. Sometimes, when one thinks about the idea of 'conceptual understanding', it seems obvious that students need to understand what they are doing. However, I have included below a quote from Kilpatrick et al (2001), showing that there is more to this strand than first meets the eye. Having conceptual understanding means that there is a certain amount of continuity to the mathematical thinking process, and that there are links between conceptual understanding and what a person then *does* with the given problem to solve it.

> '*Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know* ***more than isolated facts and methods****. They understand* ***why a mathematical idea is important*** *and the* ***kinds of contexts in which is it useful****. They have organized their knowledge* ***into a coherent whole*** *… Because facts and methods learned with*

> *understanding are connected, they are* ***easier to remember and use*** *… A significant indicator of conceptual understanding is* ***being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes****'*. (Kilpatrick et al., 2001 p. 118) (emphasis mine).

This description has helped me to understand the importance of conceptual understanding of mathematics in a subject such as IT because certain IT areas will make use of mathematical conceptual understanding as a foundation for further IT conceptual understanding. For example, a firm conceptual understanding of place value is needed when working with computer systems, because these calculations take place in a binary number system. Understanding binary place value and performing calculations, such as addition in base 2 can be very confusing, because the concept of 'carrying' a number over to the place value on the left is embedded in understanding the base 10 system. If such calculations are done in base 10 without full understanding of base 10, then when it comes to performing the same operations in base 2 the student is unable to extend their conceptual understanding from base 10 to base 2.

Not all computational situations are alike. The student needs to be able to draw from a selection of procedural tools to solve a particular problem. This will require conceptual understanding of *why* one procedure or algorithm is preferable over another, but also requires students to use that procedure flexibly, in a range of problem situations.

> *"Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently"* (Kilpatrick et al., 2001 p. 121).

In mathematics, *procedural fluency* is the proficient use of formulae, rules and procedures. For example, students need to know the rules for differentiation, and practise them in all sorts of varying procedural problems, so that competence in the procedure is achieved and they "are not handicapped in developing the other strands of proficiency" (p.122). IT procedures, algorithms and rules are often IT-specific, but also often include the use of mathematical procedures. For instance, students may be required to optimise the running of a business using programming techniques; where they have to use the principles of linear programming to perform procedures such as drawing graphs, setting up systems of inequalities and equations, and defining optimisation functions. Logarithms base 2 are used procedurally when doing calculations relating to the internal workings of computers, where the binary number system is dominant. Often mathematical procedures need to be selected appropriately – dependent on

conceptual understanding in order to apply them correctly – to solve IT problem situations. At other times mathematical procedures are used as the basis for other more complex calculations. Flexible, appropriate and efficient use of such procedures is crucial for solving many IT problems. Kilpatrick et al warn that

> *"without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematics problems"* (ibid.).

Procedures practiced without conceptual understanding lead to the incorrect use of those procedures. For example, computer modelling for business decision-making requires proficient use of linear programming in two or more dimensions in a spreadsheet environment. (Instead of having to determine the optimal number of chairs and couches in a problem the decision-maker might have to determine the optimal number of chairs, couches, tables, footstools, etc; making the problem multi-dimensional). Students often use the spreadsheet procedurally without understanding the system of inequalities they should be setting up. As in mathematics classes, they often do not understand the described problem, guess the definitions of the variables involved, and therefore guess the inequalities to be defined, which often make no sense. The result is that they insert incorrect formulae into the spreadsheet and ask the spreadsheet solver to solve the problem, which it has been programmed to do. One can "throw" almost any numbers at a solver and it will "solve" the problem – however meaningless the solution. Thus, the result of the situation is an incorrectly-solved problem, which makes no sense to the student. Furthermore, it is unlikely that s/he will be able to use these procedures in any problems other than those directly used when first taught the procedures (Kilpatrick et al., 2001); and even less so in other domains.

Strategic competence involves the problem-solving skills of the individual – s/he must be able to formulate and represent a particular problem, and then solve it. The reason why strategic competence as a component of mathematical proficiency is so important is because,

> *"[students] encounter situations in which part of the difficulty is to figure out exactly what the problem is. Then they need to formulate the problem so that they can use mathematics to solve it."* (Kilpatrick et al., 2001 p. 124).

I suggest that having strategic competence allows a student to use mathematical knowledge strategically in new domains. Being able to use mathematical problem-solving practices to

solve problems in domains such as IT, which are mathematical in many aspects, would be a competence desired by the IT student.

Representation is also key to problem-solving. Representation requires firstly, understanding the situation and knowing the steps taken in using its "core mathematical elements" (p. 124), and secondly, planning how to solve the problem. Kilpatrick et al (2001) describe in detail the role of representation recognised by the strategically competent student. Representation of a problem may be shown in a variety of ways.

> *" … the student's first step in solving [the problem] is to represent it mathematically in some fashion, whether numerically, symbolically, verbally, or graphically … Representing a problem situation requires, first, that the student build a mental image of its essential components …"* (Kilpatrick et al., 2001 p. 124).

If a student does not represent a problem during solving a task it is possible to interpret this absent visible representation as a lower problem-solving proficiency. For example, Goldin (1997) describes reduced problem-solving ability in respondents who have to be prompted while doing a task. However, a lack of a visual or even verbal representation during any task-based interview does not necessarily mean that representation is not being used, and that an individual's strategic competence is limited in some way. Representation may be mental, which indicates *better* strategic[1] competence:

> *"In contrast, **a more proficient approach** is to construct a problem model – that is, a mental model of the situation described in the problem. A problem model is not a visual picture per se; rather, it is any form of mental representation that maintains the structural relations among the variables in the problem"* (Kilpatrick et al., 2001 p. 124) (emphasis mine).

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[1] The different forms of the word 'strategy', after the word 'strategem', or 'plan', are used often in this report, and are located under the umbrella of 'strategic competence'. Strategic competence refers to being able to use strategic thinking, to solve problems. It is putting strategy into action. For example, 'strategic decision making' is referred to on page 36; 'strategic thinking' is used on page 65; 'strategies for recognizing…' on page 65; 'strategic ability' on page 66; 'strategising' is first seen on page 81 and is then used as my choice of word to describe the practice in this study (cf 2.2.2.); and 'strategic manipulation' is used on page 165.

Adaptive reasoning describes a person's ability to justify claims made through "logical thought, reflection [and] explanation" (Kilpatrick et al., 2001 p. 116). The authors continue: "…in mathematics, adaptive reasoning is the glue that holds everything together" (p. 129). Good argumentation skills demonstrate proficiency in adaptive reasoning, because valid argumentation demonstrates the "capacity to think logically about the relationships among concepts and situations [through] deductive reasoning [and] is used to settle disputes and disagreements" (ibid.).

Kilpatrick et al (2001) argue that besides the more obvious use of deductive reasoning that demonstrates adaptive reasoning, such as formal proof, there exists the often less-considered place of intuitive and inductive reasoning "based on pattern, analogy and metaphor" (p. 129). I raise this issue here because the mathematical adaptive reasoning used within the domain of IT may very well be characterised by this kind of thinking.

Students cannot justify their claims unless they understand the concepts behind their arguments. If their claims are not justified, they have no argument. While the concepts behind the arguments may be found predominantly within the IT domain, the ability to make analogies and recognise patterns that describe 'rules' may be a valuable asset transferred from mathematics to IT.

> "*Analogical reasoning, metaphors, and mental and physical representations are 'tools to think with', often serving as sources of hypotheses, sources of problem-solving operations and techniques, and* ***aids to learning and transfer****"* (Kilpatrick et al., 2001 p. 129) (emphasis mine).

Recognising analogy, or similarities between situations, is suggested by Kilpatrick et al as one of the facets of successful transfer from one situation to another. Therefore, valid reasoning, using pattern recognition, analogy and metaphor from the domain of mathematics may be important aspects of reasoning through and making convincing arguments while solving IT problems. I further discuss analogy when I discuss transfer in section 2.5.

Productive disposition, the fifth strand of mathematical proficiency, entails the

> "*habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy"* (p.116).

Repeated success in all of the other strands develops increased productive disposition; which in turn positively influences students' proficiency in the other strands. In addition, students' beliefs in their abilities to solve mathematical problems, as well as seeing problems as challenges to be conquered rather than as meaningless tests of their abilities are aspects of positive productive disposition. I suggest that having a high level of productive disposition is necessary for successful use of mathematical practices in other domains.

According to a report published by the RAND Mathematics Study Panel (Ball, 2003), it is common for mathematics teaching to focus on developing the first two strands, to the detriment of the others. The report suggests developing certain mathematical *practices* will aid students in attaining mathematical proficiency. These *practices* can best be described as what mathematically proficient people *do* to be successful users of mathematics. The way in which Ball (2003) explains how practices and proficiency relate to each other is summarised in section 1.3.1, and I provide more detail here about how practices lead to proficiency. Proficient users of mathematical knowledge need more than mathematical knowledge itself, because knowledge may be present but not used proficiently (Ball, 2003; Boaler, 1997, 1998, 2000b). Competent learning and use of mathematics includes strategic decision-making concerning how mathematical concepts, procedures and tools may be used in different circumstances. To be more specific, proficient users of mathematics "flexibly and skilfully engage" (Ball, 2003 p.33) with the practices of "representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalising ideas and recognising patterns" (p. 32) (cf 1.3.1.); whereas less proficient users do not. The extent of mathematical proficiency in students (Kilpatrick et al., 2001) can be observed when students use mathematical practices (Ball, 2003). After the next section, which discusses mathematical practices, I describe how proficiency and practices are related to each other through relating Kilpatrick et al's (2001) work with the practices listed by the RAND Mathematics Study Panel (Ball, 2003).

2.2.2. Mathematical practices – what they entail

Mathematical practices include mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement,

generalising ideas, and recognising patterns (Ball, 2003), and fall into the broad areas of representation, justification and generalisation. These practices have been found to be significantly lacking in American schools, and I suggest that this can probably be generalised to South Africa, if one takes into consideration the amount of literature available on the subject of mathematical competence in school students. I discuss each of the three broad mathematical practices below.

Firstly, representation takes on many different forms when used mathematically. It may be used to describe a physical relationship, where algebraic variables and mathematical symbols are used to model situations. Such representations would, for example, be used extensively by engineers (for example, in modelling the flow of water through a pipe), by physicists (for example, in representing the forces between particles), by economists (for example, in modelling the relationship between the repo rate and inflation), or by climatologists (for example, in using probabilistic modelling to predict the height of the 100-year floodline). Alternatively, mathematical ideas may be represented in a number of ways, where a choice needs to be made to depict the idea in the best way possible. For example, when representing data a decision may need to be made as whether to represent the data in a table or in the form of a bar graph. Lastly, mathematical symbolic language is used to precisely and elegantly describe mathematical expressions. For example, in words, one may say, "the average speed of a vehicle depends on the distance between two given points with respect to the time taken to travel between the points". In mathematical symbols, one may write $speed = \frac{s(t+h)-s(t)}{h}$, to mean the same thing, having defined *s*, *t* and *h*. It is very clear that the proficient mathematics user will need to be comfortable with using and interpreting all of the different ways of using representation.

Secondly, the Panel (Ball, 2003) refers to justification as "articulated and reasoned claims, [and] rationally negotiated disagreement" (p.32). Justification is integral to argumentation, because it is the only way to convince yourself or somebody else that what is being claimed is valid. In addition, one would need to be able to justify other people's claims as well to understand their arguments. Therefore, justification of claims, methods and solutions "certif[ies] and establish[es] knowledge" (p. 37). Different people can practice justification at different levels, depending on their comparative levels of conceptual understanding of the

material. Therefore, the degree and complexity of argument often gives a good indication of a student's mathematical proficiency.

Thirdly, generalisation is related to working with patterns, structures or relationships – ubiquitous in mathematics – where a proficient mathematics user will instinctively try to describe these using some kind of 'rule' or formula written in the form of data or mathematical symbols (Ball, 2003). This shows the interrelationship of mathematical practices – generalising patterns into a rule will require competent use of representation of some form; as well as justification of the defining rule and representation selected. From my years of experience as a practising teacher I have come to realise that working with patterns requires many slightly different aspects of practice, which are relevant to this discussion, as I will be looking for these aspects in my data analysis (cf ch.5). Firstly, students have to recognise that a pattern exists in the first place. Secondly, there must be recognition of how the pattern is propagated, which is followed thirdly by an idea of what should be done to extend it. Fourthly, the pattern is described by a rule, which is usually represented using a variety of mathematical symbols. A less proficient person may get 'stuck' at any one of these stages. For example, for the series $2 + 4 + 8 + 16 + \ldots$, generalisation entails the realisation that the series follows a pattern, that each added value is double the previous value, that the next number would be 32, and that the final representation of the pattern using a general rule is $\left(\sum_{i=1}^{n} 2^i\right)$; which also requires proficient use of relevant symbols and terms.

In summary, according to the RAND mathematics Study Panel, teaching of the practices of representation, justification and generalisation should be visible to the student, where learning by *doing mathematical practices* is key. Ball (2003) and colleagues in the RAND report focus on what is needed at the school level, in order to provide the mathematical competency needed for students to live in society, as well as that required for those teachers and students involved in "more ambitious curricula" (p. 35), such as higher levels of secondary school mathematics. The way that the RAND Mathematics Study Panel explains how mathematical practices are related to mathematical proficiency is outlined in section 1.3.1. The bulk of the report then addresses improving classroom practices for preparation of scholars for using mathematics in mathematical domains other than algebra, non-school contexts and future employment; but is sketchy where mathematical proficiency may be required for future study – particularly in non-mathematics tertiary education, which is the focus of the current study.

Drawing on their work, I suggest that students also need to be further prepared mathematically during tertiary study in non-mathematics domains, as well as for living in society. Just as promoting appropriate levels of proficiency needs to be addressed at different school levels, so the tertiary education system also needs to investigate what should be taught to develop mathematical proficiency in pre-undergraduate students, who are not necessarily going to study tertiary mathematics.

How do practices and proficiency coincide? Through examining each of the practices of "representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalising ideas and recognising patterns" (Ball, 2003 p. 32) with respect to the detailed description about mathematical proficiency (Kilpatrick et al., 2001), I provide a way to further link proficiency with practices. The first-mentioned practice of representation is clearly linked with proficiency by Kilpatrick et al (2001). It appears that representation is one of the key practices that bring about the interdependence of the proficiency strands because according to Kilpatrick et al, representation is key to more than one strand, and is partly responsible for proficiency in that strand. For conceptual understanding,

> *"[a] significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different." (p. 119).*

Representing problems is a crucial aspect of problem-solving, which is an aspect of strategic competence (Kilpatrick et al., 2001). According to these authors, the student first has to formulate the problem. Thereafter,

> *"[w]ith a formulated problem in hand, the student's first step in solving it is to represent it mathematically in some fashion, whether numerically, symbolically, verbally or graphically. […] Representing a problem situation requires, first, that the student build a mental image of its essential components. […] To represent a problem accurately, students must first understand the situation, including its key features. They then need to generate a mathematical representation of the problem that captures the core mathematical elements and ignores the irrelevant features." (p. 124).*

Formal proof and other forms of deductive reasoning form part of adaptive reasoning. According to Kilpatrick et al, representation-building experiences can help children to develop "sophisticated reasoning abilities" (p. 129). In addition,

> *"[a]nalogical reasoning, metaphors, and mental and physical representations are 'tools to think with,' often serving as sources of hypotheses, sources of problem-solving operations and techniques, and aids to learning and transfer". (p.129).*

Representation is clearly a practice that characterises the proficient mathematician through linking more than one strand. Secondly, I argue that "articulated and reasoned claims and rationally negotiated disagreement (Ball, 2003 p. 32), or *justification* (Ball, 2003 p. 37) are practices that can also clearly be linked with proficiency via the adaptive reasoning strand. Kilpatrick et al (2001) comment, "we use *justify* in the sense of provide sufficient reason for" (p. 130) (emphasis original). According to Kilpatrick et al reasoning does not have to be "confined to formal proof and other forms of deductive reasoning" (p. 129). Mathematical reasoning includes "informal explanation and justification, [and] intuitive and inductive reasoning, based on pattern, analogy and metaphor". (p.129); which can be another way of describing "articulated and reasoned claims". "Rationally negotiated disagreement" (Ball, 2003 p. 32) is a practice described thoroughly through the adaptive reasoning strand of proficiency, in that deductive reasoning "is used to settle disputes and disagreements. […] In principle, [students] need only to check that their reasoning is valid" (Kilpatrick et al., 2001 p. 129). For Kilpatrick et al "adaptive reasoning is the glue that holds everything together, the lodestar that guides learning" (p.129). Therefore, it is crucial that justification – a practice that leads to proficiency in adaptive reasoning – is present in mathematics students.

"Generalising ideas and recognising patterns" (Ball, 2003 p. 27) is a practice not as clearly described by Kilpatrick et al as an evidence of proficiency as the previous practices were. This practice overlaps multiple proficiency strands, as it is required to demonstrate different kinds of proficiencies. Firstly, students who are strategically competent "also need to see that some representations share common mathematical structures. […] More expert problem solvers focus more on the structural relationships within problems, relationships that provide the clues for how the problems might be solved" (p. 125). This statement describes the practice of pattern recognition, which I suggest is a fore-runner of generalisation. Generalisation. may be understood as 'rule' formulation. Kilpatrick et al (2001) describe a similar idea, when they state that "strategic competence involves learning to replace by more

concise and efficient procedures those cumbersome procedures that might at first have been helpful in understanding the operation" (p. 127). Other forms of pattern recognition as an aspect of proficiency include recognising "connections among concepts" (Kilpatrick et al., 2001 p. 118) (conceptual understanding strand); and algorithms as being "powerful tools" (p. 121) that are the result of generalised patterns that show the organised structure of mathematics. Kilpatrick et al do not focus on generalisation as a specific term to describe mathematical proficiency. However, because working with mathematical patterns in different ways is an aspect of so many proficiency strands, and because rule formulation or generalisation requires working with patterns, I suggest that generalisation as a practice is directly linked with overall mathematical proficiency.

Lastly, "attentive use of mathematical language and definitions" (Ball, 2003 p. 27) is not a special focus of Kilpatrick et al's discussion of the five proficiency strands. I suggest that it is essential to be used by any mathematically proficient student, because of the central role of communication in mathematics learning, using mathematical language and definitions (e.g. Ball, 2003; Pirie, 1998).

Over the last few pages I explained how different aspects of mathematical proficiency (Kilpatrick et al., 2001) may be demonstrated when students do certain practices, detailed by the RAND Mathematics Study Panel (Ball, 2003). The Panel focused on three overarching practices of 'representation', 'justification', and 'generalisation', and Table 2.1. below shows how I grouped each of the practices of "mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas, and recognizing patterns" (Ball, 2003 p. 32) into these three overarching practices (this grouping was not specifically done by the Panel).

In my analysis I use words that are slightly different (cf 1.2.) from the list of practices presented by the Panel (Ball, 2003 p. 32). Because mathematical practices and proficiency inform each other, the analysis done in this research report works with both practices and proficiency, and uses terminology from both to describe what students were doing when they were solving IT problems. The terms used for my analysis were carefully chosen for their usefulness in describing what the students were doing, and then related back to the original "practices" terms (Table 2.1.); so that the data analysis linked to a framework of mathematical practices. I explain what I did in more detail in chapter 3 (cf 3.7.2.); but for now I show in

Table 2.1. how I have related my own practices to those of the Panel (ibid.) and the three overarching practices.

Table 2.1. Relationship between Practices (Ball, 2003) and the categories used for my analysis of task-based interviews

RAND Mathematics Study Panel practices (Ball, 2003 p. 32)	Overarching practice (Ball, 2003 pp. 36-38) (my interpretation – not specifically linked by Ball)	My own categories of practices, as related to those of the Panel
representation	REPRESENTATION	using representation using procedures flexibly
attentive use of mathematical language and definitions	REPRESENTATION, JUSTIFICATION, GENERALISATION	Using representation
articulated and reasoned claims	JUSTIFICATION	understanding/explaining concepts justifying strategising
rationally negotiated disagreement	JUSTIFICATION	questioning disagreeing
generalising ideas	GENERALISATION	generalising
recognising patterns	GENERALISATION	generalising

In this section, I have linked the practices described by the RAND Mathematics Study Panel (Ball, 2003) with the strands of mathematical proficiency described by Kilpatrick et al (2001) and with my own practices I used in my data analysis. Now that I have discussed mathematical practices in so much detail, it is important that I explain why they are so important.

2.2.3. Why focus on practices?

Mathematical practices are valuable if they could be transferred to other contexts – for use in further study, in everyday life, or in the professional world. If mathematical practices are not transferable, then institutions requiring mathematics at any particular level of competence as a prerequisite for further study or future employment would need to reconsider their entrance requirements.

The mathematical practices and proficiency strands have some generic, as well as highly specific aspects. Apart from the specificity of mathematical concepts, procedures and representations, the practices of "articulated and reasoned claims, rationally negotiated disagreement, generalising ideas, and recognising patterns" (Ball, 2003 p. 32) could just as easily be used to describe knowledge in other domains. However, in other social domains, what counts as an argument is different from that of mathematics, because the objects and methods of the discipline are different. In contrast, in many areas of information technology the problems to be solved require practices, which are specifically mathematical, but are in an IT context. For example, when analysing algorithms, which is done in programme design, mathematics is the authority. In informatics, the authority for argument is experience and documented evidence, which is different from specifically mathematical argument. That is, the mathematical meaning of mathematical practices (Ball, 2003) and proficiency (Kilpatrick et al., 2001) is specific, and clearly illuminated by the authors, shown in the discussion in the previous section and section 1.3.1. The mathematical specificity of "practices" and "proficiency" is the meaning I use in my analysis. The fact that the above-mentioned practices could sometimes be seen as generic is also a strength rather than a weakness in this study, in that their aspects of non-specificity helped me to see their movement across domains more easily than if they had been purely mathematical. Areas of IT, such as using programming languages, such as Java, do not use mathematical content knowledge (cf 5.3) but do require "articulated and reasoned claims, rationally negotiated disagreement, generalising ideas, and recognising patterns" (Ball, 2003 p. 32). Depending on the IT problem to be done, I would expect to see practices in both their generic and more specific mathematical forms. More on this issue is discussed in sections 5.3. and 5.5.

To summarise, a focus on mathematical practices is valuable because, as I have previously argued (cf 1.2.), representation, justification and generalisation are mathematical practices

that are useful for problem solving in the IT domain, and if we could, through mathematics, purposefully teach such practices, their uses elsewhere can be valuable.

Having been provided with 'names' for these different practices, it may be easier to know what to look for when examining whether or not they are used in non-mathematics undergraduate contexts. Now that a description has been provided of what mathematically proficient people *do* as they successfully practise mathematics, the next questions to answer are: (1) how do students learn practices? and (2) how does the teacher promote such practices in the classroom?

2.2.4. Learning mathematical practices

Highlighting *doing* mathematical practices leads to the obvious question: *how* do students learn to *do* mathematical practices? *Doing* mathematics brings attention on the *subject* of mathematics. '*How* practices are learned' puts a focus onto theories of teaching and learning mathematics, as well as how mathematical practices may be used in other situations: the transfer of mathematical practices.

My research is framed by two learning theories. The first is the theory of cognitive constructivism, which explains how the individual, being individually responsible for his or her own learning, makes sense of new concepts through social interactions (Hatano, 1996). This sense-making takes place continuously through the process of 'equilibration' (Smith, diSessa, & Roschelle, 1993), whereby misconceptions are considered as central to construction of new learning (Hatano, 1996; Smith et al., 1993). Cognitive construction of knowledge is discussed in more detail in section 2.4.2.

The second theoretical perspective is that of situated cognition (e.g. Brown, Collins, & Duguid, 1989; Lave, 1988, 1993; Lave & Wenger, 1991), which focuses more strongly on learning as situated within a social context. Learning is equated to "active engagement" in "valued enterprises" (Wenger, 1998 p. 4). Learning may also be defined as increased participation within a community of practice, where the participants are engaged in a common enterprise with common goals, and they find meaning through their practices, as their

identities become increasingly embedded in what they do (Wenger, 1998). A more detailed explanation of situated learning, as well as the reason why the two learning theories are used to frame this research, are found in section 2.4.

2.2.5. Teaching mathematical practices

Teaching mathematical practices entails a focus on *activity*: that is, *learning* by *doing* maths (Ball, 2003), through interaction of students with other students in a student-centred environment. My last point is made because it is unlikely that a student will develop the practices of representation, generalisation, and especially justification, if he or she is watching the teacher do it and not *actively doing* it him- or herself. Hence, a student-centred learning environment would be more likely to foster such practices. What do I mean by 'student-centred'?

2.3. Student-Centred Teaching

First, I discuss some historical milestones in the development of the concept of student-centred teaching and then go on to explain what it entails in the modern classroom context. Student-centred teaching emerged in the United States of America late in the 19th century (Cuban, 1993 p. 40) and in the United Kingdom in the 1960's (Darling, 1990) Students' *needs* were emphasised through experiencing integrated content (Cuban, 1993; Darling, 1990), rather than content-driven curriculum. The student-centred concept does not dictate teaching methodology, nor is it a learning theory. Rather, it is an *ideology* (Darling, 1990), which, if it is part of the teacher's practices, can emerge as a 'student-centred teaching practice'. Student-centred teaching draws from the work of early teachers, such as Edward Sheldon and Francis Parker (Cuban, 1993), and educational psychologists, such as John Dewey and Freud (Chung & Walsh, 2000), about how children learn and think, and what motivates them. Typically, student-centred instruction encourages a range of teaching strategies, including group work, investigation, discussion, argument, and creative exercises. Ideally students are motivated and encouraged to contribute to curriculum design (Darling, 1990). According to Brodie, Lelliott

& Davis (2002) student-centred teaching has three facets. The first considers curriculum, the second, pedagogy and the third, interpersonal relations.

Brodie et al (2002) argue that curricula can be described with respect to the amount of insulation school subjects have with respect to each other. Strongly insulated subjects have strong 'boundaries', which keep them distinct and separate from other subjects (Bernstein, 1982). In contrast, student-centred teaching often implements an 'integrated' curriculum. Such curricula exhibit subjects with weakened boundaries, so that the student should be able to experience learning across a wide range of subjects. Ideally, students would be absorbed in activities which interest them because they are more relevant to their everyday lives, and the teacher designs activities so that students are directed towards optimum learning through social interaction.

However, some conceptual difficulties have arisen in relation to integrated curricula. As Brodie et al (2002) point out, if a curriculum is to address students' *needs* as described by Cuban (1993), two questions need to be asked: first, *"what needs?"*; and second, *"who gets to decide what those needs are?"* Subjects with weakened boundaries have a danger of losing their subject-specific discourse (Bernstein, 1996; Taylor, 1999 in Brodie et al., 2002). As a mathematics teacher, I find this of particular concern in mathematics, as mathematics-specific discourse is specialised (Bernstein, 1996). Many authors have reported the inability of students to transfer knowledge from an 'everyday' to a formal context, and vice versa (Lave, 1988) (cf 2.5). There is a resulting concern that integrated curricula will disadvantage students, with respect to subject specific discourses. Brodie et al (2002) comment:

> *"Therefore, a curriculum which focuses substantially on the everyday, at the expense of formal knowledge, is in danger of denying access to the disciplined knowledge to the very people who have been denied it most in the past"* (p.97).

Furthermore, weakened classification places '…additional demands on teachers…', who will require knowledge in many domains and who have to demonstrate '…increased mediational skills' in the classroom (Brodie et al., 2002).

Student-centred pedagogy considers how "teachers [act] to make links between students' current meanings and new knowledge" (Brodie et al., 2002 p. 98). This activity is often informed by Piagetian and Vygotskian learning theories because constructivism has as its core

the view that students construct new understanding socially from prior learning either by assimilation, accommodation and equilibration or through the ZPD via inter- and intra-psychological interactions. Brodie et al also emphasise that the teacher is active in his or her facilitation or mediation role. He or she needs to continually ascertain where the student currently is with respect to understanding and either set up situations that promote cognitive conflict for the student (Piagetian), or promote understanding through the notion of scaffolding (Vygotskian) (ibid.).

The third issue that Brodie et al identify is interpersonal relationships. "Relationships of respect and trust between teachers and learners" (Brodie et al., 2002 p. 96) are integral to student-centred classrooms. When students do not fear teachers they are more likely to feel free to offer suggestions and opinions during classroom interactions. Likewise, teachers would listen to and acknowledge such offerings and respond in a caring manner. In such an environment constructive argument may flourish, because teachers carefully consider what their students are saying and encourage communication skills in a respectful manner, also expecting respectful responses, during classroom conversations.

To achieve effective student-centred teaching, the teacher would need to consider all three aspects of a student-centred approach. In so doing, s/he would tailor his or her practices to suit the conceptual level and age group of the students, and facilitate knowledge development in the students' terms. Brodie et al (2002) argue that teaching can be student-centred in form or substance, or ideally, both. *Forms* are activities and classroom organisational setups that teachers use. They *can* be student-centred; although a teacher could implement student-centred forms that are not student-centred in substance. For example, a class may be given an investigation to do, which is in *form* student -centred. However, the task may be intellectually inappropriate for the students or scaffolded inappropriately, making the *substance* of the task non-student-centred. The *substance* of student-centredness entails

> *"selection and sequencing of tasks in relation to learners' current knowledge and providing for the required conceptual development in a subject area…"* (Brodie et al., 2002 p. 100)

A traditional lecture, which is not student-centred in *form* may have the *substance* of student-centredness, due to the teacher encouraging, and actually *listening to*, and appropriately responding to, questions from students.

From a pedagogical perspective, the student-centred teacher would need to practice certain *forms*, to achieve the *substance* of student-centeredness; and in so doing, would increase the likelihood of the development of mathematical practices, which are the essence of mathematical proficiency. In order to further understand my students' development and use of mathematical practices, I move to an explanation of how I theoretically frame my study.

2.4. Theoretical Framework

Theories of learning arise from the desire to describe and explain how people learn. While theories are most crucially descriptive rather than prescriptive, Cobb (2007) has described a framework for merging multiple learning theories to help design and revise learning programmes to facilitate learning. I found his discussion helpful for my study, because the framework he has described has allowed me to understand how I can "analyse students' mathematical learning that is tied to the classroom social setting in which that learning actually occur[ed]" (Cobb, 2007 p. 30). In other words, I have a theoretical justification to describe my students' individual cognitive practices within a social setting. Like Cobb, I have used more than one theory of learning to understand students' experiences of transfer.

2.4.1. Using multiple learning theories

For research purposes, when we choose a theoretical standpoint, we need to justify our choice, which will "make it open to scrutiny and discussion" (Cobb, 2007). I intend to justify the choice I have made to use multiple theoretical perspectives using the concept of pragmatic realism (ibid.).

Theories of learning arise because of the attempt to understand and explain an observed reality, such as whether and how learning takes place. To more deeply understand learning, I have found that I need to understand the philosophies of realism, and then pragmatic realism, which explains how multiple theoretical perspectives may be used to describe learning. Realism makes claims about ontology, which is the study of the existence of entities. Realism

originates from Plato, who took a stance from a world- or life-view outside of an observed situation and made deductions from that position about the situation (Gunter, 1980). For the realist, the world has a separate and independent existence "in abstraction from man" (Gunter, 1980 p. 3). Therefore, mathematical realism, like realism in general, would say that mathematical entities exist independently of the human mind. Thus humans do not invent mathematics, but rather discover it. For example, a four-sided figure exists outside of the human mind. According to Gunter, many learning theories have their origins in realism:

> *"[Realism] dominated theoretical-pedagogic thought until about the first quarter of this century, giving rise to one educational doctrine or theory after another, each the product of some closed world- and life-view or other and claiming to be scientific and valid"* (Gunter, 1980 p. 2).

Epistemology is the study of knowledge and what can be known. According to Brodie (personal communication), 'the issue … is less about whether a reality exists or not, as to how we come to know it'. Genetic epistemology, developed by Piaget,

> *"focuses on the activity of construction as the process by which the individual learns, and by which knowledge is created"* (Lerman, 1989 p.214).

For Piaget, the learning of mathematics was not concerned "with valid or invalid mathematical statements [which are ontological issues of truth or reality], but with how the individual gains that knowledge" (ibid.). Following this idea, what is important in this study is not ontology, but epistemology: not the definition of reality but how we come to understand what we observe and experience.

2.4.2. Constructivism

Constructivism is not only Piagetian. Both Piaget and Vygotsky rejected the behaviourism and empiricism that were prevalent at the time (Lerman, 1989; Moll, 2002). Vygotskian psychology is also regarded as 'constructivist' with a social and cultural emphasis. The main difference between these two types of constructivism is that Piaget describes construction of knowledge as taking place internally prior to being communicated socially; whereas Vygotsky describes knowledge construction as being the result of internalisation of social interactions (Moll, 2002).

Constructivism has associated with its broad label many different assumptions from both epistemological and ontological viewpoints (Moll, 2002). Constructivism can be understood in two broad senses. From an ontological viewpoint, constructivism can be understood from a postmodernist philosophical viewpoint as a reaction against the realist ontology; in which "everything is constructed 'all the way down'" (Moll, 2002 p. 19), and there is no absolute reality. Such an epistemological view is diametrically opposed to realism.

In contrast, if constructivism is understood as a theory of learning or development[2], it does not have to be opposed to a realist ontology, because this perspective argues that there is a reality of development, which can be described by constructivism This view can be consistent with a realist perspective, as Moll (2002) so eloquently states:

> *"…there are two different strands within constructivism as social theory. […] Some versions regard everything as socially constructed (and hence are anti-realist) while others accept that there is a non-social domain of forces which also need to be taken into account in understanding how, and to what extent, things are socially constructed (such forms of social constructivism are clearly not opposed to a realist thesis) (p. 23).*

From the viewpoint that constructivism can describe the reality of learning or development, *a* knowledge of what is experienced may be constructed, but it is neither absolute nor the only way to explain what is observed (Moll, 2002). Thus, it may be understood that the level of truth or absolutism may differ for different definitions of constructivism as a realist theory of learning. For constructivism as a learning theory knowledge is understood as fallible: "Concepts are public, as their meaning is their use, and so too is understanding" (Lerman, 1989 p. 221). In my study I view constructivism in a way that that does not devalue the significance of reality on learning, and I also consider the importance of the student's cognitive processes during the construction of knowledge.

Acknowledging the debate about whether constructivism and realism are consistent or not, I align myself, together with Cobb (2007) and Moll (2002), that they are. So I use constructivism as a theory of learning and development with a realist ontology, and also argue

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[2] Constructivism is not a theory of pedagogy, but may be used very effectively to inform pedagogy (Cobb, 2007; Moll, 2002).

that knowledge is not absolute (Lerman, 1989). This position incorporates the social constructivist viewpoint which maintains that some, but not all, behaviours are naturally and not culturally developed (Moll, 2002). It is from this constructivist position that learning transfer is explored.

I also hold that from the constructivist perspective to which I adhere there is *no single valid methodology* for understanding how knowledge is acquired; and this position has powerful implications for my argument to use a multiple theoretical framework for my study. The philosophy of pragmatic realism lends weight to why multiple theoretical perspectives may be used to understand learning and transfer.

2.4.3. Pragmatic realism

Pragmatic realism explains how multiple theoretical perspectives may be used to describe learning. Both constructivism (my understanding as described above) and pragmatic realism assert that a universal theory that describes an observed reality does not exist (Cobb, 2007). While adherents to a particular perspective might believe that their viewpoint precisely explains observed phenomena, every theoretical perspective has its limitations; and when we choose to use a particular perspective we have immediately made a choice that will "reflect particular interests and concerns" (Cobb, 2007 p. 12), but not others. Pragmatic realism is not concerned with defining a particular reality, but with fallible truths of what is observed. Peoples' activities take place in the realities "in which they actually live their lives" (Cobb, 2007 p. 10). Therefore, instead of prescribing how reasoning and interpretation of observations should take place, as is found in the field of realism, inquiry is enacted by "flesh-and-blood people and how they can use multiple explanations to explain what they observe" (Cobb, 2007 p. 11). In summary, Cobb (2007) argues that using multiple perspectives may provide valuable insight for understanding the teaching and learning of mathematics.

Before I relate the above argument to my study, I will explain Cobb's argument regarding the use of multiple theoretical perspectives. Cobb (2007) argues that understanding how mathematical reasoning develops may "feed back to inform [mathematical] instructional design and teaching" (p.29). To accomplish this he makes use of the concept of '*bricolage*', as described by Gravemeijer (1994 in Cobb, 2007 p.29):

> *"[Design] resembles the thinking process that Lawler (1985) characterises by the French word **bricolage**, a metaphor taken from Claude Levi-Strauss. A **bricoleur** is a handy man who invents pragmatic solutions in practical situations … [T]he bricoleur has become adept at using whatever is available. The bricoleur's tools and materials are very heterogeneous: Some remain from earlier jobs, others have been collected with a certain project in mind."* (emphasis original).

Developing a bricolage to understand my data will be useful because the study considers the cognitive actions of individual students, but those same students' experiences and practices are embedded in multiple social and symbolic environments. The study investigates the experiences of my students as they learned in two different study programmes: Foundation Programme mathematics and first year undergraduate information technology (IT). It also investigates the extent of transfer of mathematical practices from the mathematics domain to the IT domain, which is often explained as a set of decisions an individual must make when using pre-existing knowledge to solve novel problems. With this in mind my study needs to consider individual cognition as well as sociocultural perspectives on learning; which means that I need to be able to draw from multiple theoretical perspectives. The difference between the two perspectives I am using: cognitive and situative, relate to their emphasis on the individual and the social.

2.4.4. Individual – social relationships

Some mathematics researchers focus on distinguishing between individual and social learning environments when they attempt to explain learning (Cobb, 2007). However, this assumes that the 'individual' is fundamentally the same construct across all theoretical perspectives, which is not so. I will restrict my explanation of the individual to cognitive and sociocultural perspectives, both of which I will be using as frameworks to explain my observations. In the previous section I clarified that constructivism is associated with a range of perspectives which share fundamental concerns but have some differences. Those in which I am particularly interested are cognitive constructivism, principally developed by Piaget in the 1950's and 60's; and situated learning or situated cognition, developed by Lave (1988), Brown, Collins and Duguid (1989) and Lave and Wenger. Before I explain how these perspectives are useful for framing my research, I will explain how the 'individual' may be understood.

For the researcher describing learning from a cognitive perspective, the individual may be described as an 'epistemic' student who is the representative individual from which knowledge about how students learn is gleaned, and from whom inferences about specific students' mathematical reasoning may be made (Cobb, 2007). According to the cognitive tradition, which was principally influenced by Piaget, learning involves internal sense-making by the student of the external environment, through the processes of assimilation, accommodation and equilibration. Such cognitive restructuring, or genesis, is defined by the student's own reorganising of existing knowledge structures into more sophisticated structures (Olivier, 1989).

Social interaction is an important aspect of cognitive constructivism, because although understanding is constructed by the individual mind, the construction is based on equilibration through realisation of the presence of misconceptions[3], brought about by social interaction. Learning takes place through the processes of assimilation and accommodation, which are both co-ordinated by equilibration. Assimilation occurs when new information is similar enough to existing knowledge structures, or schemata, for it to be incorporated into the existing schemata without being changed (Olivier, 1989). Accommodation takes place when the student must adjust his or her schemata, to 'accommodate' new information (Nussbaum & Novick, 1982; Olivier, 1989). Equilibration is the process of restructuring existing knowledge into more powerful knowledge structures (Hatano, 1996; Olivier, 1989). It is possible that new knowledge may be (probably will be) yet again limited in some way, requiring further refinement. Equilibration, as a pathway to more refined and sophisticated knowledge, produces cognitive change through social interaction with peers and teachers.

Cognitive constructivism may be used to describe two aspects of the learning of mathematics. Firstly, mathematical understanding is recursive and moves between levels of sophistication (Cobb, 2007). That is, when students are novices in a particular mathematical context they re-formulate mental images of an activity specific to each context (ibid.). As they gain proficiency they are more able to generalise and formalise their knowledge to make it

[3] Misconceptions are not always 'wrong' conceptions, or errors. Often they are valid interpretations of existing knowledge applied inappropriately to new contexts.

applicable across different contexts. The proficient student is able to manipulate mathematical procedures, algorithms and truths across mathematical contexts and into novel tasks. Secondly, cognitive constructivism helps the researcher to understand how mathematical reasoning is developed in specific mathematical domains, such as 'number concept' or 'statistical reasoning' (Cobb, 2007). The quality of learning is described by the qualitative increased sophistication of mathematical reasoning in the specific domain (ibid.). If this is the case, then cognitive constructivism can help me to understand how 'mathematical practices' (what proficient mathematicians *do* when using mathematics) (Ball, 2003) are used by students in an undergraduate IT context. If mathematical practices are used when students solve IT problems then equilibration, assimilation and/or accommodation may be identifiable at this time.

Situated cognition refers to the social and context-dependent construction of knowledge, in which 'situations might be said to co-produce knowledge through activity' (Brown et al., 1989 p. 32). This argument is in direct contrast to many teaching approaches which separate knowing and doing because they argue that knowledge that is formal, abstract and de-contextualised is more transferable (Brown et al., 1989; Lave, 1988). Schools and other formal learning institutions expect that knowledge gained will automatically be transferred to new learning situations or the workplace. In contrast, situated theorists argue that learning is more closely tied to the context in which it was learned, and all learning is contextual (Anderson et al., 1996; Engle, 2006; Lave, 1988). Learning therefore cannot be directly transferred to another context, and the new situation requires significant modifications to knowledge in order to be useable.

Brown et al (1989) argue that a concept that is usable in other contexts, like the meaning of a word learned for the first time, is always "under construction" (p.33) because it evolves contextual meaning every time it is used and becomes increasingly densely textured. In addition, concepts are only fully understood through use: using them entails changing the user's view of the world; and using them entails adopting the belief system of the culture in which they are used (Brown et al., 1989). Using the equal sign is an example of such a situation (e.g. Essien & Setati, 2006; Molina & Ambrose, 2006). A school student learning about mathematical operations for the first time will understand the idea of $2+3=\square$ to mean "we must add 2 and 3 and get an answer". Since the emphasis is on the operation of addition the equal sign is understood as a symbol that signifies providing the answer to the operation.

As the child moves through school grades he or she restructures (hopefully) original conceptual understanding through use of the equal sign in new problem contexts. The concept of $\square = 2 + 3$ should be understood in terms of the relational aspect of the equal sign, where '$\square$' represents something that is equivalent to '2 + 3'. This relational understanding develops texture when used in different contexts such as showing that $(x + y)^2 = x^2 + 2xy + y^2$ or proving the identity $\tan x = \frac{\sin x}{\cos x}$, or logically deducing that if $a = b$ and $b = c$, then $a = c$.

The beginner computer programmer develops further conceptual understanding of the operational aspect of the equal sign when writing his or her first programme: to write on the screen "Hello World". The programmer uses knowledge of the equal sign in a different way from the mathematician, because although '=' is understood as 'do something', it is also understood as a symbol to be used for assigning meaning to a variable. Therefore '=' can be used to represent both numbers and text, as follows:

```
x = "Hello World";
print(x);
```

The community of computer programmers will have no problem talking about and using their operational understanding of the equal sign in this context, but a mathematician who is not a computer programmer may have difficulty participating in the conversation. Although the '=' sign is used in these different ways, it is important to realise that understanding its operational meaning is crucially important before it can evolve further contextual meaning. A sophisticated understanding of the equal sign requires flexible movement between the different meanings – depending on context. The same applies to conceptual tools, such as mathematical formulae, which are used differently by engineers and physicists (Brown et al., 1989).

According to Brown et al (1989) teachers often teach concepts as isolated fragmented facts. Alternatively, they can design 'authentic activities' that relate concepts to others, bring increased meaning to the new concepts learned, and so make them usable in other contexts. This has strong implications for knowledge transfer, which is discussed in the next section; but for now I want to emphasise that the theory of situated cognition considers knowledge to be shared and dynamic within a culture and a context.

Understanding how learning takes place in the individual and within a community in different contexts is important in my study for understanding how students' experiences of my teaching approach are related to their development of mathematical practices and whether and how these practices were transferred to a new learning context.

2.5. The Notion of 'Transfer'

The topic of transfer has engendered much debate over the years. I will discuss some of the key concepts that relate to transfer, which include: what is meant by '*transfer*', in the context that it will be used in this study; *what* is transferred?'; and the concept of transfer from *where* to *where*. Each of these key concepts opens a huge arena of research, opinion and argument. My understanding has been shaped by the comparatively small amount of literature I have read and I am aware of the vast amounts of research and documented theoretical arguments that are available, and how complicated the study of transfer is. I will discuss some of the research reported in the past ten to fifteen years and how the learning theories discussed previously have contributed to the transfer debate. Reading the literature has significantly influenced my original understanding of transfer as being behaviourist or early cognitivist in nature, to being influenced by an individual's past cognitive and social experiences, and the community in which one participates.

2.5.1. What is meant by 'transfer'?

Transfer may be broadly explained as occurring when "prior learning affect[s] new learning or performance" (Marini & Genereux, 1995 p. 2), or as "the process of using knowledge acquired in one situation in some new or novel situation" (Alexander & Murphy, 1999 p. 561). Transfer typically involves the student, the instructional tasks, the instructional context, the transfer task and the transfer context (Marini & Genereux, 1995). A list such as this could be read without much further thought because it appears to be obvious. However, the debate around transfer relates to these five elements and has been described as "lively" at times by De Corte (1999), and "spirited" by Alexander and Murphy (1999) (for example, see the

discussion between Anderson et al and Greeno in the Educational Researcher, 1996-1997, volumes 25(4) and 26(1)).

De Corte (1999), in his introduction to a series of discussions dedicated to transfer in the 31st edition of the International Journal of Educational Research, has indicated that the themes of such debate include positions such as: that transfer does not occur (e.g. Detterman, 1993 in De Corte, 1999 p. 556), that transfer only takes place between identical situations (e.g.Thorndike & Woodworth, 1901 in Cox, 1997 p. 41-42), that transfer of general skills does take place across diverse situations (e.g. Katona, 1940 and Wertheimer, 1945 in De Corte, 1999 p. 556), and that transfer of problem-solving abilities can be improved under certain teaching conditions (e.g. Mayer & Wittrock, 1996 in De Corte, 1999 p. 556). In the 1960's to 1980's transfer studies were mostly associated with an individual cognitivist interpretation of experiments (Lave, 1988 pp. 1-44). Transfer has more recently also been argued from a situative perspective, and described as 'generality' or 'productivity' by Greeno and some of his colleagues (e.g. 1997), and by Lobato (2006) and Engle (2006) with respect to how effectively situations or contexts may be related to each other by the student.

The literature suggests that there is no doubt that at least some 'kinds' of transfer exist. "When confronted with a novel problem, humans almost always try to make sense of the new situation and find a way to solve the problem with concepts and methods they have learned to use in other situations" (Hatano & Greeno, 1999 p. 647). It appears that positions taken about transfer depend largely upon the theoretical lens through which one gazes. In the following sections I discuss first, understandings about *where* certain knowledge came from and *where* it may be used in a new situation; and (2) understandings of *what* is transferred, with respect to cognitive and situative learning theories.

2.5.2. Transfer from where to where?

It is likely that a person asking questions about transfer and all that it entails will begin by wondering if what people learn at school is in any way helpful for what they do in their everyday lives or in the jobs that they do when they have finished school. Many teachers indicate that while students are in high school learning about geometry, trigonometry and

many aspects of algebra, most of them are asking exactly this question. De Corte (1999) states that industry is "strongly interested in the transfer of learning" (p. 555), and according to Simons (1999) the literature has extensively reported transfer of schooled knowledge to everyday life, as researched in instructional and organisational environments. A well-known example of such research includes Boaler's three-year investigation in two British schools, Amber Hill and Phoenix Park, with respect to how it is possible through reform-oriented teaching to facilitate the use of mathematics in non-school situations (Boaler, 1997, 1998).

While much attention has thus far been directed towards the transfer of knowledge from school to everyday life, two other types of transfer that need more investigation (Simons, 1999) are from *prior* knowledge and skills to *new knowledge in the same domain*, and from *new* knowledge and skills to *new knowledge in a new domain*. From a cognitive perspective, the question of transfer from *where* to *where* takes into account the amount of overlap between old and new contexts (Simons, 1999). A large amount of overlap between transfer settings can promote 'near transfer'; where knowledge is automatised or practised in a small range of situations and used in similar situations. Conversely, little or no overlap between two contexts can promote the occurrence of 'far transfer' (e.g. Simons, 1999), which can be best described when the student is able to decontextualise and abstract recently-learned knowledge and use it in a variety of new situations (ibid.).

As already stated, the questions, "*what is being transferred*?", and "*to what extent is* 'it' *transferred*?", has been the topic of much debate. Historically, transfer has been described by behaviourists, gestaltists, cognitive constructivists and situated cognitivists, and a few of the main proponents of each theoretical standpoint are discussed below. Thereafter, I discuss transfer from cognitive and situative perspectives in more detail, because it is from these two perspectives that I analyse the data from my study. I explain transfer from these two theoretical perspectives in terms of how it is understood, how such understanding influences learning, and how teachers can promote 'transfer ability' in their students.

2.5.3. Transfer of what?

The notion of transfer was first discussed in detail by Thorndike and Woodworth (e.g. 1901 in Carraher & Schliemann, 2002; or 1901 in Lobato, 2006) from an empiricist or behaviourist perspective. Central to this rendition of transfer was the understanding that the student passively makes use of 'identical elements' of knowledge from one situation to another (Carraher & Schliemann, 2002; Lobato, 2006). Thorndike and Woodworth concluded that transfer of knowledge took place when situations shared certain physical features, and that students should therefore be taught only what would be useful to them in the real world or work situation. For example, learning about Roman Numerals would be left out of the mathematics curriculum because it was not seen to be applicable outside of school learning (Carraher & Schliemann, 2002).

Discussion in the second half of the 1900's appears to have revolved largely around the transfer of knowledge and skills, and was generally understood first through a Gestaltist and then through a cognitive lens. The gestaltists moved away from attempting to understand how units of knowledge could be transferred from one situation to the next and focused rather on "understanding the structural features of the task" (Carraher & Schliemann, 2002 p. 2). However, experimental tasks were still fairly similar in structure to those of behaviourist experiments and instructions were given with the task in order to aid the clarification of the transfer that was to take place (ibid.).

Cognitivists investigated mental schemata that are formed and adjusted during learning. Lobato (2006) describes the altered understanding of transfer that came with the 'cognitive revolution' (p. 433). People were understood to "construct mental symbolic representations of initial learning and transfer situations" (p. 433). Instead of the transfer of identical elements as described by Thorndike and Woodworth (1901 in Lobato, 2006), transfer was seen to have occurred if the symbolic representations were identical, or overlapping, or seen to be related to each other in some way. According to Piaget, learning takes place through the processes of assimilation, accommodation and equilibration (cf. 2.4.4.). With respect to knowledge transfer any or all of these three processes would also be involved in recognising the usefulness of certain knowledge and altering understanding of the applicability of the knowledge to a new context. In so doing more powerful and richly textured internal knowledge structures are developed, which incorporate the knowledge applicable to widening contexts.

During the 1970's and 80's, transfer research was dealt a great blow because although novel problem solving experimental transfer tasks were used, it appeared that unless prompting with respect to the original tasks was given, transfer of identified skills and knowledge did not take place (Carraher & Schliemann, 2002; Lave, 1988). At this stage in transfer research the *what* of transfer was pre-defined, as an expectation of the knowledge that was to be used in the new situation; as was the experimental environment in which the transfer was being investigated.

More recently – in the last thirty years – learning has been discussed on a large scale from a more socially situated perspective, described by Lave (1988), and Lave and Wenger (1991) as 'situated in context'. Proponents of situated theories of learning have argued that one should carefully consider the implications of the question, transfer of *what?* Understanding transfer using the lens of social learning theories means that the *what* is not a skill or an algorithm or some other "unit of knowledge" (Hatano & Greeno, 1999), but rather *how* people interact with each other, or think "in interpersonal environments and with other informational systems" (p. 647). That is, the emphasis moved away from the '*what*' from a cognitive perspective to the '*how*' from a situative perspective. Hatano and Greeno argue that if transfer is being investigated from a socio cultural perspective, one has to acknowledge that units of knowledge are not "uniformly transferred from one situation to the next" (ibid.) They have also suggested that this understanding of *how* transfer takes place should perhaps encourage us to think about 'productivity' rather than 'transfer'; where productivity is understood as the "extent to which learning in some activity has effects in subsequent activities of different kinds" (ibid.). It is apparent here that the focus is not on the unit of knowledge but on participation in interpersonal relationships and situations.

Therefore, cognitive and situative viewpoints of transfer differ considerably. The cognitive theorist asks "*what?*" and the situative theorist asks "*how?*". What is it about how researchers investigate transfer that may result in these different explanations of transfer? This is discussed next.

2.5.4. Cognitive transfer research

Cognitive transfer research takes place in a very structured manner. The instruction, transfer tasks and transfer environment are all designed with specific outcomes in mind and are investigated as separate components of the transfer experiment as a whole. The idea of "*experiment*" is a remnant of behaviourist research, but it is still used to investigate transfer from a cognitive perspective (Lave, 1988; Lobato, 2006). Transfer research would be conducted as follows: An aspect of students' knowledge would be considered; after which the research environment would be designed as the medium in which the transfer of that knowledge could be measured (Simons, 1999). An example of such a research design is that followed by Roberts et al (2007) (cf 1.5.3.).

Salomon and Perkins (1989) suggest that "transfer is not at all a unitary phenomenon. Rather, transfer can occur by different routes dependent on different mechanisms and combinations of mechanisms" (p.115). Their description revolves more on the *how* than the *where* of transfer. *Low-road transfer* depends on extensive, varied practice and occurs by the automatic triggering of well-learned behaviour in a new context. *High-road transfer* occurs by the process of "mindful abstraction" of something in one context and application in a new context'. Mindful abstraction entails the purposeful de-contextualisation and re-representation in a new more general form (Salomon & Perkins, 1989 p.126).

Many theorists argue that transfer understood from a purely cognitive perspective may neglect understanding of the importance of social aspects of transfer. Situative researchers such as Lave (1988) and Lobato (2006) critique certain ways in which transfer has been described by suggesting that these descriptions have "decontextualised the transfer from the concrete experience of the learner" (Lobato, 2006 p. 434). Assuming that transfer is the application of knowledge in a new situation, (e.g. Simons, 1999), "suggests that knowledge is theoretically separable from structures in which it is developed or used" (Lobato, 2006 p. 434). Furthermore, transfer *tasks* that have been designed have been seen to be "independent of students' purposes and construction of meaning" (ibid.); while transfer *environments* are often not seen as an integral part of the transfer process, but as either supporting or interfering with the process being studied (ibid.). When particular learning is not visible to the researcher, the conclusion may be that the students could not transfer the original knowledge (Carraher &

Schliemann, 2002), or that some students exhibited better or worse learning than others (Boaler, 2000a).

Research has found that transfer is an unexpectedly rare phenomenon, when it is the subject of cognitive experiments (Hatano & Greeno, 1999; Lobato, 2006). In the first few chapters of her book Lave (1988) focuses on these problematic conclusions. Since Lave was one of the first researchers to describe in detail the inconsistencies of transfer research, as well as to argue a completely new perspective from which to understand transfer, I quote a portion of her writing which criticises traditional cognitive transfer research:

> *"All of this underscores the static quality of transfer in experimental practice: it is treated as a process of taking a* ***given*** *item and applying it somewhere else. The characterization of analogies as crystallized objects follows partly from the functional theory of transfer which treats cognition as the literal, uniform transportation of tools for thinking from one situation to the next. But its practical origins lie in the normative orientation that guides the construction of experiments. For, so long as evaluation of subjects' performances is the goal, and it is to be achieved by comparison to an ideal view of correct understanding, then the experimenter must determine what will constitute correct problem solutions (as in all the experimental studies described). The task then becomes to get the subject to match the experimenter's expectations. In this situation the target analogy* ***is*** *a preformulated, static object, and its unmodified use by the subject is the objective of the exercise. As the experiments clearly demonstrate, matching transfer expectations takes considerable effort on the part of both experimenter and subject. It may be that this matching game – rather than transfer – is the (unintended) subject of these experiments* (pp. 37-38) (emphasis original).

Thus, Lave (1988), Lobato (2006), Greeno and colleagues (e.g. 1997; 1999), and Engle (2006), amongst many other authors, argue that understanding transfer from a situative perspective incorporates much more than the transfer of various kinds of units of knowledge, as seen from the cognitive perspective. In later reports Boaler redirected her interpretation of her observations from an originally individual perspective to a situative perspective, which she claimed "has increased [her] understanding of the influence that the classroom community and the social and cultural processes that stemmed from that community had upon students' production of knowledge in different situations" (Boaler, 2000a p. 114). Transfer from a situative perspective, with respect to the student and the teacher, where learning and its application in other contexts is explained, is discussed next.

2.5.5. Situative transfer research

A cognitive understanding of transfer highlighted the predominant thinking throughout the twentieth century that transfer was, in a sense, represented by a linear progression of events, where the student in some way makes use of previously-learned knowledge to deal with problems faced in novel situations. Transfer has been progressively understood as a multi-faceted phenomenon, and thus requires more complex explanations than those previously used.

Lave, in her introductory work in the 1980's and early 1990's (e.g. Lave, 1988, 1993; Lave & Wenger, 1991) argued the multi-faceted nature of transfer. According to Lave (1988), learning is situated in a context, and is not readily transportable in its exact form to new contexts. Reasons for this are that two situations are never exactly the same, and there are many facets to consider in a transfer situation. Furthermore, students find it very difficult to "reconcile former experience with the information at hand" (Carraher & Schliemann, 2002 p. 4). Research at this time moved into investigating *how* people used previous knowledge in new contexts (not *what* knowledge), because it was apparent that they do rely on previous knowledge and experience to help them work through novel situations (Carraher & Schliemann, 2002; Hatano & Greeno, 1999).

Brown Collins and Duguid (1989) developed over the next few years a theory of situated cognition; later to become a design for classroom practice, known as cognitive apprenticeship. Explaining it briefly, it is through the individual's active participation and authentic activity in the complex social structures of different communities, that learning takes place. Through continued use of new knowledge and practices within a particular context and within a particular community the knowledge itself takes on form and further meaning particular to that community. For example, the activities of a community of software writers are usually indecipherable to those outside of the community, because the tools used by that community (for example, the computer language and the programming algorithms being used) have specific significance. "The culture and the use of [tools] act together to determine the way practitioners see the world; and the way the world appears to them determines the culture's understanding of the world and of the tools" (p. 33).

Transfer, also described as an ability to generalise learning, comes about by learning to "participate in interactions in ways that succeed over a broad range of situations" (Greeno, 1997 p. 7). Greeno's argument is entirely framed by a situative understanding of how learning takes place, with claims that "all instruction takes place in complex learning environments" (p.9). According to this perspective knowledge is shared and increasingly textured by all of the minds and resources that are participating in that situation (Brodie, personal communication). Each mind brings with it understanding, ideas, beliefs, and all of the social and educational experiences that it has. Thus, participation in a learning community results in knowledge being continually refined as it is being used by the community, and its broadening use in different situations. If this is the case, then the way(s) in which instruction is organised plays a crucial role in knowledge production, and with it, the enabling (or disabling) of transfer (or generalisation).

The phenomenon of transfer is a complex one, as has been described above, but viewing transfer from both situative and cognitive perspectives is beneficial to understanding the context of my study. Earlier on I explained why I am considering multiple theoretical perspectives (cf 2.4.3.) to frame my study and to explain different perspectives of my students' learning. Below I explain how the characteristics and the extent of any kind of transfer can be understood as individual constructs. To accomplish this I will discuss the student as the person 'doing' the transfer. Furthermore, because all learning is situated and knowledge is shared and co-constructed within a community of practice, transfer is also heavily dependent on situative aspects of both the prior learning context and the transfer context.

2.5.6. The student and transfer: A cognitive perspective

Without ignoring the debate with respect to transfer, Alexander and Murphy (1999) argue that certain "fundamental processes remain relatively constant" (p. 561) and are summarised as follows from a cognitive perspective: Firstly, transfer is more likely to be performed better by experts and highly competent students than by students "newly acclimated in their fields" (p. 562), who would be more likely to expend cognitive efforts grasping the original learning than applying it. Furthermore, the student who displays less competence in the original

material possibly lacks the strategic thinking for, and even possibly the interest in, applying the learning to novel situations. “Able performance” appears to play a significant role in the extent of the transfer that takes place.

Secondly, also from a cognitive perspective, schooled knowledge appears to be, on the whole, poorly transferred to new situations. Near transfer is more likely to occur than far transfer. Fragmented mathematical knowledge, which is procedurally, rather than conceptually understood, has reduced potential for transfer – especially far transfer – across domains. This is because the student does not recognise its applicability anywhere else other than the immediate context. Even so, it is more likely that transfer will take place if students are provided with strategies to recognise how they can use knowledge in new contexts. Thirdly, as an extension of the second point, Alexander and Murphy explain that transfer will be more likely to occur when the learning environment is “intentionally orchestrated to encourage cross-situation and cross-domain transfer and reward such efforts” (p.564). Here, we are looking at the facilitation of far transfer which, they argue, requires long term instructional commitment, as well as knowledgeable and dedicated teachers; as “instruction on this level is very demanding” (ibid.). We find such a situation in the Phoenix Park school discussed by Boaler (1997). In this situation the teachers were totally dedicated to reform teaching in the sense that it was the only approach used by all teachers in the school over a three year period. The students, although resistant to the approach at first, came to recognise its value because the mathematics they were learning at school was like the mathematics they had to use outside of school (Boaler, 1997, 2000a); and *doing* maths was socially constituted over this time as the full range of mathematical practices discussed earlier in this chapter.

Alexander and Murphy (1999) argue that transfer and an ability to reason analogically[4] are at least related processes, if transfer is not a special case of analogical reasoning. This is because students whose abilities to recognise the similarities between particular tasks or contexts are limited will be unlikely to engage in competent procedural or conceptual transfer across tasks

[4] *In analogical reasoning, an analogy for a given thing or situation is found, where the analogy is like the given thing in some way. E.g. This company is like a racehorse. It's run fast and won the race, and now it needs food and rest for a while, or ‘hat is to head as glove is to …’, written as hat:head::glove:? (http://changingminds.org/disciplines/argument/types_reasoning/analogical_reasoning.htm)*

or contexts. While Alexander and Murphy spend a great deal of time defining analogical reasoning and its relationship to their understanding of transfer, I merely wish to bring to the reader's attention the idea that generalisation involves at least a competent understanding of domain knowledge, motivation to use knowledge in more than the immediate context in which it is learned, and a strategic ability to recognise where it may be used and how it may need to be altered in order to work in the novel context in which it has been implemented (Alexander & Murphy, 1999). Refer to my earlier discussion (cf. 2.2.1.) concerning the importance of analogical reasoning as an aspect of adaptive reasoning – it appears that transfer and the practice of generalisation (part of the proficiency strand of adaptive reasoning (Kilpatrick et al., 2001) are related through at least an ability to reason analogically; which is interesting, as these two research teams made these comments independently of each other.

2.5.7. The student and transfer: A situative perspective

A situative theory of learning assumes certain understanding of how the learning environment is structured. Engle (2006) found that transfer was facilitated when teachers deliberately framed, or "fostered intercontextuality" (p. 482) in their teaching of biology to grade five children. Greeno (1997) argues that gaining skill and knowledge for its own sake should not be encouraged in the classroom. Rather, instructional activities should be focused on "students' contributions to broader social activities" (Greeno, 1997 p. 10) and the promotion of "evaluation of goals and progress of learning activities"(ibid.); which should make their learning more meaningful to them. When the student has a sense of purpose in the content to be learned, then transfer is likely to be facilitated (e.g. Perkins, 1992 in Alexander & Murphy, 1999). In addition, if an instructional programme promotes self and peer evaluation and mental skills, such as questioning, conjecturing, and arguing, then the students are taught methods of inquiry as well as skill acquisition; which are beneficial to generalisation of learned material. In situative terms, "generalisation" can be synonymous with "transfer".

In contrast, an instructional programme which consists of pre-set practiced problems aimed at the acquisition of skills, teaches students how to 'do school' but little else. This was the case in the Amber Hill school described by Boaler (1997), and is also warned against by Alexander and Murphy (1999) in their suggestions for instructionally enabling students to deal with

future transfer situations. For the Amber Hill students their mathematics was understood within the context of the school community. Through using the mathematics in the way in which they were instructed they adopted the belief system of the culture (school) in which the mathematics was used (Brown et al., 1989), which subsequently influenced how they understood the world with respect to mathematics. Using mathematics was "cue-based" (Boaler, 1997, 2000a), and the cues entailed using the mathematical content most recently-learned to solve textbook problems on the same content, and recognising the graded difficulty of textbook questions to answer problems (ibid.). Without deliberately teaching the Amber Hill students how to participate in a community of school mathematics users, through their enculturation (Brown et al., 1989) within this community, these students became experts in the community of school mathematics users (Boaler, 2000b), but did not use their knowledge outside the classroom in other real-life contexts.

The Phoenix Park students also adopted the belief system of the culture in which they learned mathematics, which was school. But in contrast, this mathematics was useable in other contexts because of the socially constituted ways in which they had used it at school. The open-ended questions that they used as the tools for learning had relevance in more than the classroom and examination contexts, and the students were able to draw from their classroom knowledge in real life contexts. Boaler (1997, 2000a) discusses the difference between *how* the students used their mathematics in different contexts from a situative perspective. It was "differences in constraints and affordances between the two settings" (p. 118) that made the Phoenix Park students able and the Amber Hill students unable to transfer their mathematical knowledge – not their lack of understanding of mathematics. Therefore, a situative perspective considers differential use of mathematical practices in different settings (Boaler, 2000a), where success and failure is not "viewed as attributes of individuals, but as specialised social and institutional arrangements" (Lave, 1993 in Boaler, 2000a p. 118).

2.5.8. Teaching for transfer

According to Hatano and Greeno (1999), who view learning from a situative perspective, transfer will occur fairly often if it is supported socioculturally. Since learning itself is heavily dependent on interaction between other people and tools, so too will transfer be dependent on

these factors. Furthermore, since learning is more successful when activities and interactions are interesting and significant, so too will transfer be more successful (ibid.). As far as the student is concerned, the kinds of interactions between his or her capabilities and the sociocultural context will influence firstly, whether or not he or she is willing to apply previous learning, and secondly, whether or not his or her performance is regarded as successful transfer (ibid.).

If one considers that constructivist notions of learning also recognise the essential role of social interaction for learning – but that the individual rather than the community is the focus of the explanation – then the above argument raised by Hatano and Greeno applies just as comfortably to the cognitive theorist as it does to the situative theorist. What significance does this knowledge have for the practising teacher?

Many authors have provided input concerning the role of the teacher in influencing the possibility of transfer in the classroom. Alexander and Murphy (cognitive) (1999) suggest that transfer will be facilitated when "helpful hints or cues [are given to] prompt students when to use trained knowledge or skills" (p. 563; 'trained' is their word), and Hatano and Greeno (situative) (1999) state that transfer is "depend[ent] on socio-cultural enablements" (p. 653). Boaler concluded after her three-year-long research in the two schools, Phoenix Park and Amber Hill, that the inquiry-based approach that the teachers at Phoenix Park used in their classroom teaching "prepared students for the real world" (Boaler, 1998 p. 60). Lobato (situative) (2006) explained the notion of "focusing phenomena", as developed by Lobato, Ellis and Muñoz (2003, in Lobato, 2006). Focusing phenomena are

> *"…the features of classroom environments that regularly direct students' attention towards certain (mathematical) properties or patterns when a variety of features compete for students' attention. They emerge not only through the instructor's actions but also through mathematical language, features of the curricular materials, and the use of artefacts"* (p. 442-443).

In investigating transfer from introductory computer programming, Salomon and Perkins (1989) commented that

> *"high-road transfer [ ] depends on an instructional style that either provokes students to abstract mindfully from the programming context principles of greater scope, or explicitly provides such principles and assures that students thoroughly understand them in their greater scope"* (p.132).

It appears that constructivists and situative theorists agree that 'transfer ability' needs to be deliberately fostered by means of dedicated problem-based teaching approaches– either in the individual or in a classroom community – even though their research methods and interpretations of transfer may be different. All these researchers argue that knowledge should not be gained for its own sake, because if it is more meaningful to the student, it is more likely to be transferred to other situations. Students should be given opportunities to experience how knowledge and skills are possible to use in other relevant contexts. Alexander and Murphy (1999) summarise this idea by stating,

> *"students should be engaged in meaningful, problem-based activities for which the knowledge and skills acquired are means for accomplishment rather than ends in themselves" (Alexander & Murphy, 1999 p. 563).*

From a multiple theoretical perspective, students who experience different teaching approaches will participate differently, and therefore develop different ways of knowing (Hatano & Greeno, 1999). The instruction that students receive can therefore be planned to prepare them for transfer. By being aware of this, the teacher can plan lessons aimed at helping students to realise when, where and how they can use something newly-learned in other situations – that is, to make knowledge accessible in other contexts. Through meaningful discourse (e.g. argument, justifying claims, suggesting and evaluating ideas) (Hatano & Greeno, 1999; Simons, 1999), engagement in meaningful problem-based activities and "mentally taxing reflection" that develops skills (Alexander & Murphy, 1999; Hatano & Greeno, 1999), and linking of fundamental concepts to other concepts and the real world (Alexander & Murphy, 1999; Boaler, 2000a), students can be shown how to link new information to what they already know and how to use their previous knowledge independently. Teachers are encouraged by these authors to model, promote and reward deep conceptual thinking to promote transfer (Alexander & Murphy, 1999), or foster intercontextuality (Engle, 2006).

In summary, although approaches to the study and subsequent confirmation of occurrences of transfer are different, pedagogic implications from the different theories are remarkably similar. Note that the outcomes of such approaches would be viewed differently by these theorists as either facilitating transfer as an individual aptitude, or as generative learning in the classroom community. Through the teaching approach that was used at Phoenix Park, the students in Boaler's (1997) study had developed a "belief that mathematics entailed active

and flexible thought [and] had developed the ability to adapt and change methods to fit new situations" (Boaler, 1998 p.57).

The opposite may also occur: if teachers practise disconnected, decontextualised, fragmented instruction, infrequently discuss relationships between concepts across domains, rarely promote intercontextual learning, and promote ritual-like practice of procedures and algorithms, transfer will probably be discouraged; manifested by the 'failure' of students to transfer knowledge (Alexander & Murphy, 1999). Boaler's (1997, 1998) description of the students' learning at Amber Hill was an example of "inflexible, school-bound" (Boaler, 1998 p.60) knowledge that was of limited use. A key concept to discuss is therefore that of student-centred teaching. In the next section I discuss how the notion of student-centred teaching may be linked with that of teaching for fostering transfer. I suggest that student-centred teaching approaches can effectively promote transfer, and I expand on this idea below.

2.5.9. Links between student-centred teaching and transfer

In the previous section I explained how transfer-directed teaching is considered by many to be essential for aiding the student to know how to compare contexts, make analogies and appropriately choose existing knowledge to use and modify where necessary in new situations. In this section I argue how a student-centred teaching approach can be conducive to making conceptual links between contexts, and therefore, contribute to knowledge transfer.

I explained in section 2.3. how student-centred teaching incorporates curriculum decisions, interpersonal relationships between teachers and students, and pedagogical decisions. It is in the area of pedagogy where I argue that links between student-centred teaching approaches and fostering transfer can be made. Teaching to foster transfer was discussed in the previous section, and the reader is reminded that the ways in which transfer may be understood depend largely on the theoretical framework through which one designs research studies and interprets the results of such studies. Although some researchers suggest that transfer is a theory and not a problem to be solved by understanding it through other theories (e.g. Carraher & Schliemann, 2002), the majority of current literature explains transfer through cognitive and socio-cultural constructivist or situative learning theories. If student-centred

pedagogy “involves teachers acting to make links between students’ current meanings and new knowledge” (Brodie et al., 2002 p. 98), then we are back to a discussion of how people learn. Therefore, if student-centred pedagogy involves teacher facilitation or mediation in order to help the student to make links between prior and new constructs, as does teaching for fostering transfer, then it is reasonable to suggest that student-centred teaching fosters transfer.

It is easier to envision student-centred teaching fostering near transfer, from a cognitive perspective, because near transfer involves prior to new learning in the same domain. However, referring to my previous discussion about making transfer possible through deliberate reference to other contexts (Alexander & Murphy, 1999), socio-cultural enablements (Hatano & Greeno, 1999), and so on, I also suggest that student-centred teaching can also foster far transfer. The following research supports this position.

Clarke et al (2004) investigated how a particular problem-based learning environment, called the Interactive Mathematics Programme (IMP) might influence secondary school children’s’ mathematical achievement and beliefs about the character of mathematics learned at school. Many exciting and significant findings emerged from this study. From an affective point of view, the IMP students viewed themselves as more mathematically able; they were more positive about their mathematics classes; and they achieved higher mathematics Scholastic Aptitude Test (SAT) results, compared with children who learned mathematics in a traditional learning environment (pp. 14, 15). A second set of findings, based on the students’ perceptions of mathematics showed that IMP students were “significantly more likely to perceive mathematics as a mental activity”, to view mathematics as “arising from individual and societal need”, to “perceive mathematics as having applications in daily use”, and to believe that “mathematical ideas can be expressed ‘in everyday words that anyone can understand’” (p. 15). In contrast, traditional stream students tended to view mathematics as “independent, absolute and unvarying” (ibid.). In addition, IMP students tended to value “interactive learning situations”, where writing and talking were viewed as helpful to learning; as opposed to traditionally instructed children, who viewed the material presented by the teacher and textbook, and ‘drill work’ as the path to mathematical knowledge (ibid.).

Clarke et al (2004) have suggested that students who have learned in an inquiry-oriented, problem-based environment “can be expected to perform more successfully on both

conventional and non-routine tasks than students lacking that experience" (ibid.); and therefore, that "a problem-based curriculum is capable of developing traditional mathematical skills at least as successfully as conventional instruction" (ibid.). Boaler's (1997) study showed similar results. The Phoenix Park students who learned mathematics in a reform student-centred environment were able to tackle open-ended mathematical tasks more adeptly than their counter-parts from Amber Hill, who were taught using a traditional teaching approach, specifically towards achieving in the GCSE standardised examination. However, the students from Phoenix Park were also able to at least equal the GCSE examination scores achieved by those students from Amber Hill. These findings indicate the ability of progressively-taught students to use their mathematical knowledge in a wider variety of contexts than do traditionally taught students.

From a pedagogical perspective, these examples show how certain *forms* of practice achieved the *substance* of student-centeredness (Brodie et al., 2002). They also indicate that students found these approaches of benefit to them in more contexts than that of school; which substantiates my argument that a student-centred teaching approach would also be likely to promote students' abilities to transfer their knowledge to new situations.

2.6. Conclusions

Drawing from multiple theoretical perspectives, from studies of student-centred teaching, and from recently-formed understanding of transfer, I have reported suggestions that near and far transfer may be facilitated through "fostering intercontextuality" (e.g. Engle, 2006); which I suggest may be best facilitated through a student-centred teaching approach. Cognitive and situative theorists understand learning from different perspectives, but the need to explicitly teach mathematics so that transfer may be made more achievable is promoted by both theoretical viewpoints.

My study investigates whether challenging mathematical content can be taught and learned in a student-centred tertiary environment, so that students can become more mathematically proficient and use mathematical practices in subsequent non-mathematics study. My research attempts to make further inroads into understanding transfer of mathematical practices into

the domain of undergraduate information technology. I refer to my research questions, listed again below for the reader's convenience. The wording and ordering of the questions alludes to my hypothesis that student-centred teaching promotes transfer, and my study investigates the accuracy of such a belief.

> To what extent are students aware of differences in teaching approaches between Foundation Programme mathematics and undergraduate study?
>
> To what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses?
>
> In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?

The first two questions investigate firstly, the students' awareness of the kinds of teaching approaches used by their teacher (me) in the Foundation Programme; and secondly, whether or not they thought that the teaching approaches that they had experienced in the Foundation Programme had prepared them for undergraduate study enough to consciously use the mathematical practices they had developed. The third question investigates actual instances of transfer and seeks to link what the students *said* they used or did to what they were *observed* to do – regardless of their own awareness thereof. If they demonstrated verbally their awareness of the student-centred approaches that I believed I had used, then the question that would be most interesting to find an answer for would be whether or not this teaching approach promoted transfer. Actual demonstration of the use of mathematical practices would be a persuasive indication that the practices had been transferred to their study in IT.

The next chapter describes the research design. Two methodologies were used; namely, focus group interviews and task-based interviews. The two sets of data are then reported and merged to provide some answers to the research questions.

CHAPTER 3

RESEARCH DESIGN

3.1. Introduction

The study is qualitative and is situated in an interpretivist paradigm. According to Merriam (1997), qualitative inquiry

> *"focuses on meaning in context, [and] requires a data collection instrument that is sensitive to underlying meaning when gathering and interpreting data"* (p.1).

Qualitative inquiry does not depend on large samples and statistical evidence on which claims are based (Opie, 2004); rather it focuses on interview and observation data (ibid.) to provide a rich description (Merriam, 1997) of a particular group of individuals. For this purpose I report a case study; whereby a case is "a single entity, a unit around which there are boundaries" (Merriam, 1997 p. 27). For this case study I have used two instruments of enquiry: focus group interviews and task-based interviews. With reference to the research questions, it is clear that the evidence gathered was subject to students' descriptions of their experiences (Babbie & Mouton, 2001) of the Foundation Mathematics Programme and of their undergraduate studies and how they solved tasks; and my subsequent interpretation of what they said and did in the interviews.

This chapter describes the research design used for the study: methodology, sample, research context, research instruments, rigour, data collection; data analysis and ethical considerations.

3.2. Methodology

3.2.1. The approach used

I made use of a case study methodology to answer my research questions. The case study has been described by Babbie and Mouton (2001) as 'the intensive investigation of a single unit'.

In this case the unit was a group of students to whom I taught Foundation Programme mathematics, and the case is a case of student experiences of different teaching approaches and transfer of mathematical practices in a tertiary environment. Many authors of qualitative research methods argue that a case study investigates a multiple set of variables, which leads to a 'thick' description of the subjects' experiences (Babbie & Mouton, 2001; Cohen, Manion, & Morrison, 2000; Gomm, 2004; Merriam, 1997), and where the reader can experience the world from someone else's point of view (Gomm, 2004). As such, the purpose of the case study was not to produce generalisations, but to "provide the reader with a rich description of social life" (Gomm, 2004 p. 13). Through focus group and task-based interviews I was able to gain some insight into the experiences and practices of the students in the sample. Defining characteristics of case studies have been outlined by authors such as Lincoln and Guba (1985), Merriam (1997) and Cohen et al (2000). Those I have discussed below are features that pertain especially to my study:

1. Case studies are concerned with a rich and vivid description of events relevant to the case.

   The data was analysed with respect to what the students in the focus groups said they experienced when I taught them and what they were doing and experiencing in their first year of undergraduate study. They were video-taped whilst doing IT tasks, in order to ascertain which mathematical practices (if any) were used to solve the tasks. Extensive notes and comments were made during analysis of the video recordings.

2. Case studies blend a description of events when they are analysed

   The focus group interviews gave me an understanding of the students' experiences while they were studying with me. Thematic analysis allowed me to identify common experiences and opinions, as well as possible outliers. The task-based interviews allowed me to observe the mathematical practices that the students were actually demonstrating, in addition to what they said they used. The video-recordings of the task-based interviews allowed me to watch and play back and note any mathematical practices used by the students as the tasks were carried out. The two sets of interviews were very useful in marrying experience with performance, and allowed me to give a rich description of the situation.

3. Case studies focus on individual actors or groups of actors, and seeks to understand their perceptions of events

   Through focused interaction with a few select individuals I was able to gain insight into the specifics (Merriam, 1997) of their experiences and practices and obtain further understanding of how they used mathematical practices in first year undergraduate IT studies.

4. Case studies highlight specific events that are relevant to the case

   As I stated at the beginning of this section, "the case is a case of student experiences of different teaching approaches and transfer of mathematical practices evidenced in a tertiary environment". Therefore, the questions and tasks given to the participants were designed to highlight these experiences and practices, so that I could gain understanding of the case as a whole.

3.2.2. Rigour concerning case studies

Rigour in research deals with '… presenting insights and conclusions that ring true to readers, educators and other researchers…' (Merriam, 1997:199). The phenomenon of *rigour* has often been explained as an element of *trustworthiness* (e.g. Lincoln & Guba, 1985; Merriam, 1997; Opie, 2004), or *validity, reliability, verisimilitude, plausibility or relevance* (Freeman et al, 2007). Qualitative research has been viewed by many researchers as untrustworthy because it is not standardised, and hence diminishes the accountability of the researcher to his or her audience (ibid.). However, those involved in qualitative research have made valuable inroads into defining rigour within this paradigm.

The well-known work of Lincoln and Guba (1985) has been cited by many authors involved in qualitative research because it describes in detail how rigour in qualitative research differs from that of quantitative research. The terms 'validity' and 'reliability' are problematic, as they originate in the quantitative paradigm. Lincoln and Guba place emphasis on aspects of the research such as whether or not the interviews were validly and reliably constructed,

whether or not the content of documents such as interview transcripts were properly analysed, and if the conclusions of the case study depend upon the data. Internal validity for the qualitative researcher looks at how well the research report represents the situation being investigated. Findings are always presented through the eyes of the interpreter, who always changes the reality of the event in some way; and in any case, reality can never be fully and accurately represented in a research study. Therefore, validity, or *trustworthiness* may be strengthened by allowing the research to be scrutinised, so as to make its interpretations believable. Interpretivist research does not usually make claims to generalisability, because the researcher wants to understand the details of a particular case, not attempting to make inferences about a general population. In addition, while the researcher is not necessarily distanced from what is observed in a traditional sense, objectivity lies with the actual data and whether or not the data are "confirmable".

Merriam (1997) also states that validity in qualitative research involves ethical considerations. Opie (2004) generalises validity more, as '… the *relationship* between a claim and the result of a data-gathering process', while Maxwell (1992) defines in detail five different types of validity that can be found within qualitative studies. Validity in my study will be measured according to the description of Maxwell (1992): particularly 'descriptive', 'interpretive' and 'theoretical' validity.

"Descriptive validity" deals with the researcher's openness in relaying the primary data obtained. For my research much of what is concluded is via analysis of interview data. I argue that what follows renders descriptive validity to my study. Firstly, the focus group interviews and task-based interviews were audio- and video-recorded respectively. All written material was used for reference purposes, and all material was kept on record. Secondly, patterns and agreement in my data were addressed by continual backwards and sideways reference to the transcribed data and notes made from the video-recordings of the focus groups and task-based interviews respectively (Babbie & Mouton, 2001; Merriam, 1997). "Interpretive validity" describes my interpretation of the raw data I received from the data collection process. Categories were designed which were an interpretation of the statements made by the focus group respondents. Interpretive validity was maintained by my use of exemplary quotes from the original data to support an observation or claim made. Maintaining "theoretical validity" was an important consideration in my study, as the research design and subsequent conclusions were made with respect to the theoretical frameworks which informed my

research: namely, that is, the development and transfer of mathematical practices was investigated with respect to a combination of cognitive constructivist and situative understandings of learning.

Lincoln and Guba (1985) suggest that the term *dependability* be used in qualitative studies, rather than reliability or repeatability, because the subjects in a case study are not likely to do or say the same things twice. Nor are two different investigators likely to follow exactly the same investigative pathway. Traditionally, reliability or repeatability was a pre-condition for validity. Guba (1981 in Lincoln & Guba, 1985 p. 316) argues that demonstration of *trustworthiness* should establish *dependability*; but that this argument on its own is weak, and dependability should be demonstrated as strongly as possible. An audit of the research process to establish dependability, as described by Lincoln and Guba (1985), was made on a continuous basis by my research project supervisor, and I list the components of the audit process briefly: (1) Detailed discussion of the proposed research took place. (2) Formal agreement of the officially-presented proposed research was made, and included ethical considerations, and proposed logistics of the project; (3) Confirmation of the trustworthiness of the data collection analysis and interpretation was made; and (4) Feedback and re-negotiation elicited a final report of the research.

In terms of this research, issues of bias were present and need to be accounted for. Lincoln and Guba (1985) describe 'credibility' in research, which maintains that research findings are not derived from the bias of the researcher. What they argue is not that the researcher does not influence, or is not influenced by, his/her respondents; but that pre-formed ideas that the researcher may have, or judgements made too early about the data have been avoided when the research is interpreted and reported. In addition, negative, as well as positive aspects of the research should be shown to have been considered. This has relevance to my research, as firstly, I was of the opinion before my data collection began, that my teaching approach was student-centred, and that of tertiary education lecturers was teacher-centred. It was necessary to ascertain through the data whether or not these assumptions were correct. Careful reference to theoretical discussions of student-centredness while interpreting students' comments brought me to the conclusion that my approach was indeed largely student-centred, but that the approach of some of the IT lecturers was also student-centred. This is discussed in more detail in chapter 4. Secondly, and related to the first point, I might have felt uncomfortable with my data if it had shown that my teaching was not student-centred, or that the students did

not learn or use mathematical practices when studying first year undergraduate IT. Continuous honest reference to theoretical and analytical frameworks, as well as rigorous thematic analysis (cf. 3.6.) during data analysis and interpretation (Hitchcock & Hughes, 1995), discouraged mis-reporting or leaving out negative findings.

Thirdly, many of the focus group respondents may have wanted to tell me what I want to hear in order to please me, because we had a good working relationship when I taught them. I had to take care that my interview questions were asked in such a way as to obtain opinions from them that were as honest as possible. To do this I discussed aspects of their experiences in different ways in a single interview, so that their experiences and views could be related in different ways, and contradictions would be more likely to be detected. Fourthly, while qualitative research is more likely to work with purposeful rather than random sampling, selecting my subjects had the potential to add bias to my research. I had to be careful not to select only those students who would give me the responses I wanted to hear. I addressed this by selecting subjects who were likely to give me interesting feedback, but who did not necessarily appreciate my teaching approach at the time. In addition, before the start of each focus group interview I stipulated to the students that I needed total honesty from them concerning the questions I asked them.

3.3. The Sample

The students who participated in the study were among the 2006 Foundation Programme mathematics cohort and were in first year undergraduate study in Information Technology, sub-majoring in Computing or Business Systems at Monash South Africa in 2007, when data were collected.

I taught two different sets of students in 2006. The first was a small group of seven, and the second, a larger group of thirty-two pre-IT students. Three students from the smaller group originally took part in interviews to provide pilot data; but the data were subsequently incorporated into the study findings. Both sets of students were exposed to the same teaching approaches in my mathematics classroom. Both groups were studying the same six first year undergraduate non-mathematics core subjects in IT in 2007, with their other two subjects

being non-mathematics ‘electives’. As will be discussed in the next section, there were two methods of data collection: focus group interviews and task-based interviews.

Purposive sampling was used to select the respondents. This is because I needed to identify “good respondents” (Merriam, 1997 p. 85), who could “express their thoughts, feelings [and] opinions – that is offer a perspective – on the topic being studied” (ibid.). Students were selected depending on how much I thought they might participate in the focus group discussions – those who I thought would readily volunteer thoughts and opinions about my questions were asked to participate. Not all the students who learned mathematics with me in the FP are still studying at MSA or doing IT, and were therefore left out of the selection process.

The focus group interviews consisted of four focus groups, of three students each. Originally it was planned that four students should take part in each interview, but it turned out that each time an interview was run, a student was not able to attend that day. According to constructivist and participative theories of learning, as described in chapter 2, learning takes place in a social environment. The theoretical framework that I used was a *bricolage* of theories based on learning through social interactions. Thus, for the task-based interviews the students worked in pairs, specifically because they had an opportunity to prompt each other. The students who participated in the focus group interviews were originally asked to take part in the task-based interviews. However, a few of them became too pressurised by study commitments and I asked others from the same large group of 2006 students to participate instead. Everyone except one person attended the task-based interviews. A student from the pilot group (Tungamirai) was asked to take the absent person’s place because he was on campus at the time and was available, and he was the only student to have done all four of the tasks.

3.4. Task Preparation

Originally the tasks were given to me by the lecturers of the first year core IT subject lecturers. I had requested that doing the tasks should require mathematical practices such as using mathematical procedures, understanding/explaining concepts; questioning; disagreeing,

using strategic thinking and decision-making, recognising patterns, generalising, justifying claims, and using graphical and symbolic representation, (cf. 1.2.; 2.2.2.), and the lecturers had considered these requirements when they gave me the tasks. Many of the tasks were very long, and the lecturers did not fully understand the mathematical practices I was trying to elicit from the tasks. I did not know enough about the tasks to work with them myself. João, a third year IT student, who had not only been one of my mathematics students previously in the Foundation Programme, but had also been a tutor-mentor of the respondents while they were in the Programme, helped me with task preparation. Once I obtained the tasks, João worked through them, and with my guidance concerning the eliciting of mathematical practices, selected, re-wrote, edited, and shortened them. He and Alain, another ex-student of mine and also a tutor-mentor of the respondents, also provided worked solutions to the finalised tasks. João and Alain spent a few hours with me discussing the mathematical practices embedded in the tasks, while I probed and argued about the intricacies of the tasks and how mathematical practices might be used in their solution. I was satisfied by the time I had finished the task preparation that after the three of us had worked through and discussed them, the tasks were usable for the interviews.

3.5. The Pilot Study

I ran one pilot focus group interview. The group included three students from the smaller 2006 Foundation Programme group. The interview ran for about ninety minutes and the students informed me that they were happy to spend that time with me because they found the discussion interesting. Thereafter, the other respondents were told that the interviews would take between sixty and ninety minutes, to which all agreed. Because the pilot focus group was successful I made no changes to the focus group interview schedule and used the pilot interview as part of the reported data.

For the task-based interviews I would have needed two pilot studies, as only two of the four tasks would be done in one interview. Only one pilot study was done, with two of the four tasks. Too few students had agreed to participate in these interviews due to increased first year work load, and I needed all of them to participate. The fact that João did the tasks with me and commented on them as a student, acted as a pseudo-pilot interview and gave me an

idea of how long the tasks would take and how difficult the first year students might find them.

The task-based pilot interview checked firstly, whether the tasks were done in a similar way to how João or Alain did them. I had to have an idea if students' thinking was at all similar to that shown by João or Alain. If this was not the case, why was it not so, how was the thinking different, and would this be a significant problem in subsequent interviews? Secondly, how important was it to pre-design the questions that I or João would ask while the tasks were being completed? It turned out that some of the mathematical thinking and task analysis shown by the participants took unexpected pathways, and that the interview questions would be more helpful if they were unstructured and addressed what was being done in that particular interview. João's presence in the interviews helped in this respect because he was able to analyse what was being done at the time and work with the respondents' current thinking, rather than what was pre-planned.

A negative aspect of the task-based interviews was the fact that doing the tasks took considerable time to complete. Potentially, doing each task well (except for the computer systems task) would require hours of work. I had to allow the respondents to do as much of the task as was possible for the interview, but had to watch the time and cut it short. Allowing thirty to forty-five minutes per task would allow me to observe whether and how mathematical practices might be used in IT. João and Alain advised me that observing students planning and explaining how the tasks could be done would provide this insight. They made this judgement because they had done the tasks and we had discussed them thoroughly. Moving into implementation of the planning would require more specific IT practices, and would not necessarily provide further information about use of mathematical practices.

3.6. Data Collection

Because a case study purposefully works with a small number of respondents, methods such as focus group interviews are reasonable ways of gathering evidence that provide rich data

(Opie, 2004). The two sources of data I used for my study were focus group interviews (e.g. Merriam, 1997) and task-based interviews (Goldin, 1997).

3.6.1. Focus group interviews

I led four focus groups interviews, and I had them audio recorded for subsequent coding. The focus groups considered areas of consensus and disagreement about what was experienced in the Foundation Programme mathematics classroom, particularly with respect to the Math B unit (see Appendix A); as well as what was being experienced currently in undergraduate study. The interview questions were semi-structured and followed guidelines suggested by researchers such as Opie (2004), Babbie and Mouton (2001), Neumann (1994) and Fraenkel and Wallen (1990) and are shown in Appendix C. Three students participated in each focus group and each interview lasted for sixty to ninety minutes.

In the discussion that follows, I briefly outline the disadvantages of using focus groups as an instrument of case study research, and address these. In addressing the disadvantages I justify the use of focus groups for data collection in this study. Focus groups have been criticised because they tend to take place within "unnatural social settings" (Babbie & Mouton, 2001 p. 292). Also, the likelihood of respondents influencing each other and themselves during an interview is strong (Gomm, 2004). However, social interaction produces negotiated meaning (Babbie & Mouton, 2001; Patton, 1990 in Merriam, 1997), that is most likely inaccessible when interviewing individuals (Babbie & Mouton, 2001). Therefore, as long as such influences become explicit in the reporting of the data (Gomm, 2004), this possibility is not necessarily disadvantageous to the research. Group discussion tends to generate ideas new to individuals, resulting in data different from that obtained from individuals (Gomm, 2004). I used focus groups to find out how my ex-students experienced their learning, as a socially negotiated activity, in my classroom, compared with that of their undergraduate courses. Using focus groups allowed me to "observe interaction on a topic" (Babbie & Mouton, 2001 p. 292). Furthermore, as Morgan (in Babbie & Mouton, 2001) states,

> *"Group discussions provide direct evidence about similarities and differences in the participants' opinions and experiences as opposed to reaching such conclusions from post hoc analyses of separate statements from each interviewee" (p. 292).*

Being aware of other disadvantages of focus groups alerted me to possible bias when interviewing the students. Focus groups are usually semi-structured, with many of the questions to be asked formulated beforehand; giving the researcher strong influence over the questions asked, the direction of the interview and the conclusions drawn (Opie, 2004). Another disadvantage of using focus groups is that of not being able to investigate the opinions and experiences of respondents to the same depth or detail as working with a single interviewee (Babbie & Mouton, 2001).

However, advantages associated with focus groups pertinent to my study counter these disadvantages. Firstly, it is possible to glean a large amount of information on a particular topic in a short period of time, because a group of respondents will provide a lot of information (Bell, 2005). Too many respondents in one group would have made the group difficult to manage, and would also have limited the amount of useful information that could be obtained from the interview (Babbie & Mouton, 2001). A smaller group of respondents allowed me to manage the interview better, while also being able to broach certain questions more deeply, and elicit a response for each question from every respondent (Merriam, 1997). Running more than one focus group allowed me to obtain rich information in order to obtain new insights from a larger number of respondents than from one group only (Babbie & Mouton, 2001).

In addition, although focus groups may take place in an open-ended conversational manner, I was still able to direct the questions towards the aim of the study (Yin, 1994). I have shown in Appendix C how the questions I asked in the focus groups specifically addressed the research questions. The use of focus groups allowed me to probe any interesting comments made by respondents that I had not originally included in my interview question schedule.

As expected, a few students arrived late at the interviews (Merriam, 1997), and each time an interview took place one student did not arrive at all for one or another reason, so reducing the possible data obtained from a focus group and possibly weakening the analysis. In addition, certain respondents were not as talkative as others.

All of the above issues are part of a decision that had to be made: a focus group should not be too large, as large groups prevent detailed information from being gleaned. However, if the group is too small, and one or two respondents do not arrive for any particular reason, the

effectiveness of the focus group is diminished. Both issues were applicable to the organisation of my focus group interviews.

3.6.2. Task-based interviews

Task-based interviews (Goldin, 1997) were used to qualitatively investigate the use of mathematical practices in non-mathematics undergraduate courses. The first year IT students all studied the compulsory 'core' courses of 'Computer Programming I', 'Computer Systems' 'Networks and Data Communications', and 'IT in Organisations'. The first three of these subjects are deemed mathematical in nature and require entering undergraduate students to have a mathematics background (this was stipulated in the course requirements booklet for new students). Mathematical content only in the form of basic algebraic concepts was required in these subjects, and included set notation, place value, simple and compound interest, and symbolic representation, to name a few. However, abstraction and problem-solving ability were also required to a great degree, and I supposed that the mathematical *practices* supposedly gained in a foundation mathematics course for IT should be at least as important for the students to master as conceptually understanding the *content* of such a course; which is one of my reasons for undertaking this study.

Task-based interviews are valuable as a research instrument because the information obtained using such an instrument allows the researcher to answer central questions, such as "what long term consequences are innovative teaching methods having for [students'] mathematical development?" (Goldin, 1997). Observing and interviewing students performing first year computing tasks was helpful for understanding and pinpointing whether and how mathematical practices were useful in these computing courses. The task-based interviews were chosen as a research instrument, because they provided information about how the students could use mathematics when doing various IT tasks. Doing tasks allowed students to actually demonstrate mathematical practices, as opposed to merely talking about them.

The IT tasks have been briefly explained for the non-IT reader, as well as the mathematical practices embedded in them in Appendix E. Because the tasks were examples of what would be given to the first year students, they were not contrived and were realistic in their

expectations of the first year IT student. Therefore, instead of the task being specifically designed to elicit demonstration of mathematical practices, these tasks were set as they would be for the first year student, and analysed according to the mathematical practices students needed in order to accomplish them. Because of this, the accompanying interviews were unstructured, as it was difficult to predict how the students would approach the questions, or what mathematical practices they would use to do the tasks.

Four IT tasks were used for the study. One came from the Networks and Data Communications 1 unit, a second semester unit, named the "networking task". Another came from the Computer Systems unit, called the "computer systems task". Through a few previous informal conversations I learned that many first year students found this course difficult. The other two tasks were set by the Java programming lecturer, based on the Java I unit. Although they were both set in Java they possessed different features (see Appendix E). The first was called the "prime numbers task", and the second was called the "banking system task". These two tasks implemented fairly high level use of Java because the students had completed the course in the first half of 2007; although all had not necessarily passed it. The students in the sample were studying mostly second semester IT courses at the time of the interviews. Those who failed Java I would be able to continue with second semester subjects (except Java II), and had to repeat Java I in 2008.

The students did the tasks in pairs and there were seven-task based interviews in total. Since the tasks were time-consuming, all four tasks were not done by all the participant groups but were divided between the groups. Three pairs of students worked on the first two tasks. Four different pairs worked on the second two tasks. The first of these was the pilot study pair, whose interview was included in the data analysis. With a few exceptions, the students who participated in the focus group interviews also participated in the task-based interviews. The data collection took place over about four weeks, as I had to find times in the day that the students did not have to attend other classes. The interviews were video-recorded, so that facial expressions, silences and gestures, as well as discussion, could be recorded. The students were requested to 'think aloud' at all times, in order to aid me in identifying the kind of mathematical thinking that was taking place. All interviews were conducted in English, although English was an additional language for all but one of them. On occasion students noticeably struggled to express themselves, but were given time to explain themselves thoroughly.

The students were mostly left to do the tasks without much interruption from me, although I asked questions and occasionally voiced issues. They were asked to discuss their thinking with each other as much as possible, so that I could clarify what mathematical practices were being used by listening very carefully to what they said. I also interjected with questions such as 'What are you doing now?'; 'Why are you doing that?; 'Why did you choose to do that?'; 'How do you know that is what you must do?'; 'What sort(s) of problem(s) are you facing here?'; and each pair was asked at the end of their interview what kinds of mathematics they thought they had needed to do the tasks.

João also asked more IT-specific questions, such as 'if you have decided to do *x*, what are the implications of that decision on your design?'. The participants sometimes spoke to João, rather than me, because he was more IT proficient than I was. This was helpful, because it a) opened up the conversation, and b) clarified content, as he was able to ask more IT-specific questions than I was.

The task-based interview is a useful research instrument that allows the researcher to observe and subsequently make inferences about learning that took place. However, if inferences were to be drawn from observed mathematical behaviour, I had to consider certain issues fundamental to many qualitative methods, and applicable also to task-based interviews.

3.6.3. Rigour relating to the focus group and task-based interviews

Focus group and task-based interviews have common features with each other, generic to many qualitative methods. Firstly, both draw on human responses which are unpredictable; requiring the pursuit of unexpected avenues of inquiry. Such flexibility allows the interview to proceed naturally, rather than pre-determining the direction of the responses (Goldin, 1997). Interviews require reproducibility, in the sense that the "same interview [needs] to be administered repeatedly […] in different contexts" (Goldin, 1997 p. 53). As the "research experience base" accumulates, repeatability becomes more possible (ibid.). An important distinction must be made at this point. The qualitative researcher will argue against the possibility of reproducibility in interviews, maintaining that no two participants can be expected to respond in the same ways. It is true that the interactions between the interviewer

and respondent cannot be expected to be reproducible, but the research instrument, (the questions) which should be directed towards investigating a particular phenomenon, can and should be reproducible in its purpose, because it is designed by the researcher, without the participant (ibid.).

Rigour in interviews considers a few key aspects. Firstly, being flexible is important. In the focus group interviews the questions to be asked were pre-designed, but as the interviewer, I needed to be flexible in my responses to students' comments, and to be able to follow up on interesting ideas that arose at any time. For the task-based interviews João explained to me that there were many ways to complete the tasks, and that the worked solutions were only an example of what could be done. Because of my insufficient knowledge in the IT areas I was investigating, either João or Alain was present in the interviews to help me conduct questioning along any unexpected avenues, as mentioned above. Their presence gave flexibility to the interviews because they were able, where I was not, to analyse what was being done at the time in a particular interview and work with the respondents' current thinking.

Secondly, the role of theory in all of the interviews needed to be constantly considered: a) Did theory inform the structuring of the interview, and if so, how? b) How did theory guide the choice of questions used in the interview, and c) What was the extent of consistency between the observations and inferences made during the interview and the theoretical framework? Theory informs not only the conclusions drawn from raw data, but the *whole* research process – the research design, task design, data collection and drawing of inferences and conclusions. In order for the researcher to maintain qualitative validity using the task-based interview he or she must distinguish between observation (made during the interview) and inference (after the interview) – "inevitably guided" (Goldin, 1997 p. 55) by theoretical underpinnings. Since qualitative data collection is a subjective process, there is uncertainty in the drawing of inferences from observations (ibid.). As described above, both the focus group (cf. 3.5.1. and Appendix C) and task-based (cf. 3.4.) research instruments were informed by theory in their original design, and then were further refined with the theoretical framework in mind. The refinement of the research instrument (p. 53) and a strong theoretical framework informing the drawing of inferences together improve validity of the process.

The thread running through Goldin's discussion about the theoretical basis of task-based interviews is crucially important to the researcher:

> *"We simply have the choice of proceeding unscientifically, choosing tasks that seem interesting and just 'seeing what happens', or trying to proceed systematically with tasks explicitly described and designed to elicit behaviours that are to some degree anticipated"* (pp. 58, 59).

The tasks and related interviews in my study were necessarily divergent from this stipulation. I could not design the tasks – they came from the school of IT and were initially designed by others, who did not necessarily fully understand what mathematical practices are, but argued that to do their subjects mathematical knowledge was required.

The third issue addresses how the social, cultural and psychological contexts of the interview influence the interview outcomes. According to Goldin (1997), contextual issues in task-based interviews, like other interviews, must be considered by the researcher; and will be unique to each interview, because each participant is a unique human being. Power relations are always present, and different individuals will respond differently to them. Also, the participants' expectations and interpretations of the significance of the interviews to their personal lives, such as their marks, or whether certain comments or answers are 'correct' or not, needs to be addressed at the beginning of the interviews. The emotional states of participants will also affect the response and explanations given when participating in both focus group and task-based interviews. Contexts associated with the interviews themselves are ascribed different meanings by different individuals, which will also affect their responses, to a greater or lesser extent. For example, when doing a task a particular question may trigger associations previously experienced that may affect the strategies chosen by an individual to solve that particular problem.

For the interviews in this study, the following situations were common to almost all of the respondents. (1) The focus group interviews were done at the beginning of the second semester of their first year, and the task-based interviews were done in the middle of the semester. By the time the task-based interviews had started the students were already under a large amount of pressure because of assignments that were due around that time. A few commented to me that they had not had much sleep recently because of this. (2) The networking task was very new to them, because it was a subject that they had only started in the second semester of 2007, and they had only covered the first half of the course. The

networking lecturer was aware of this, and for the purposes of my interviews had set a task that covered only the first few concepts taught in the course. (3) The students were relaxed in the focus groups and did not appear to be outwardly concerned about the 'correctness' of their responses in the task-based interviews. I had a good relationship with them and they were aware that they could give their honest opinions about the questions I asked them in the focus groups. I informed them at the beginning of the task-based interviews that they would need to explain everything to me, because I did not know anything about the tasks they were doing. They were very helpful in this regard, and explained throughout the interview what they were doing, as though they were teaching me.

Goldin emphasised an important consideration to make in the drawing of inferences from the task-based interviews he designed: whether or not, the participant was prompted, and if so, to what extent? For example, planning strategies used by participants for problem-solving that were not prompted indicated full use of problem-solving techniques, where planning competencies led to use of other problem-solving competencies. Prompted planning gives rise to incomplete problem-solving competencies (Goldin, 1997 p. 57). This would be true for the interviews I held. As the intervention supplied by the interviewer gains specificity, the 'less extensive' is the information obtained about the participant's full strategic ability. Therefore the interviewer is continuously required to make decisions about the level of specificity of the hint he or she needs to give the participant in order to solve a particular problem (p.57). Since I could not do the tasks myself, I was not able to prompt the participants. However, João and Alain were able to help me interrogate students' responses and build on them where necessary.

A limitation of my study that affects my research design arises on the issue of the use of mathematical practices in non-mathematics subjects. It would have been difficult to determine whether or not mathematical practices were indeed being demonstrated if the IT tasks were too difficult for the participant(s) to work through. This came to mind because I personally have very little knowledge of the IT subjects used in the tasks in my interviews, although I know that I can demonstrate mathematical practices, such as using procedures flexibly; using representation; understanding/explaining concepts; questioning; justifying claims; disagreeing; strategising; and generalising (cf 2.2.2.), in non-mathematics subjects. Because I cannot do the IT tasks, I would not be able to demonstrate mathematical practices in this area. Therefore, because many first year undergraduate IT students might be in a similar situation,

the IT tasks for the interviews were deliberately set at a level that the least knowledgeable of the participants could work through. Tasks possibly became easier when working with a peer. This also reduced the need for prompting by the interviewer, as they prompted each other.

3.7. Data Analysis

3.7.1. The focus group interviews

The focus group interviews were transcribed and the analysis of the data was by means of coding categories derived from the transcripts; also known as "pattern-matching" (Yin, 1994), or "theme analysis" (Gomm, 2004). "Categories" are concepts that have been derived through extensive and repeated study of transcripts of the data; through the "constant comparison method" (Lincoln & Guba, 1985). Category construction has an intuitive component – however, it is essential that the process is "informed by the study's purpose, the investigator's orientation and knowledge, and the meanings made explicit by the participants themselves" (Merriam, 1997 p. 179). What this means is that I designed the interview questions having in mind the issues I wanted to hear about, but described the actual categories after the transcribed data were obtained and thoroughly scrutinised. To do this, I constantly moved between the data, the notes made during analysis, and related studies and literature (Hitchcock & Hughes, 1995). Respondents' remarks were repeatedly compared with each other, giving rise to "units of potentially meaningful information" (Hitchcock & Hughes, 1995; Lincoln & Guba, 1985; Merriam, 1997); which were then grouped into themes because they had something in common.

In order to maximise validity and avoid bias I designed the categories to be both positive and negative. I then searched the data for any evidence of responses that might have made me uncomfortable to report; such as comments that might undermine pre-conceived notions I might have had about students' experiences. For example, note in Appendix D that category 3 describes the students' experiences over the period of the Math B class. The sub-category that observes a "change from negative to positive" was observed in the data, which would please me as their ex-teacher. Therefore, I added the sub-category, "change from positive to negative", so that I would report any negative experiences described, which may not have

been easy to admit as their ex-teacher but must be considered as a researcher. In this way, sub-categories were added that would ensure that I was as objective as I could be when reporting and analysing my data.

3.7.2. The task-based interviews

The IT tasks in the study were first analysed according to the mathematical practices I thought might be embedded in them; and then analysed according to the mathematical practices I saw the students demonstrating in order to solve them. Analysis of the task-based interviews took on a somewhat inductive nature.

The mathematical practices that I was looking for were listed before I watched the video recordings, and were altered as I watched the videos and listed the practices I observed the students using. The names I used to describe the practices were informed by the concepts of proficiency and practices. The RAND Mathematics Study Panel (Ball, 2003) links practices and proficiency by stating (cf 1.3.1.):

> *"… our conception of practices can be seen as another way of framing important aspects of these strands of proficiency" (p. 9).*

In sections 1.3.1. and 2.2., I highlighted additional links between proficiency and practices that are not part of the literature, but that contribute to clarifying the relationship between practices and proficiency. I also listed my own terms for the practices I saw the students using when they were doing IT problems and linked these with the existing terms for practices (Ball, 2003) and proficiency (Kilpatrick et al., 2001) (cf Table 2.1.).

My reason for using my own terms to describe practices the students used when they did IT tasks is as follows. My teaching was originally informed by the notion of proficiency, and not practices (cf 1.4.1.). It was therefore easier to work with these same proficiency strands when first working with the IT tasks. When I later understood the Panel's (Ball, 2003) practices more clearly, I realised that these mathematical practices (Ball, 2003) should be used as an analytic framework to categorise my observations, because they are what people *do* when they are proficient mathematics users. However, some of the Panel's practices did not fully

describe what I saw as the students did the IT tasks. I therefore used descriptions that were informed by my understanding of proficiency and by how I had originally taught my students to understand proficiency, because proficiency and practices are closely linked (cf 2.2.2.).

The result is that I 'broke down' some of the Panel's list of mathematical practices into more specific terms. First, "articulated and reasoned claims" and "rationally negotiated disagreement" could incorporate different aspects of *justification* (cf Table 2.1.), and I wanted to specify what my students were actually doing when they did the IT tasks, before I claimed that what they were doing was the practice of justification. To clarify what students were doing I used the terms 'understanding/explaining concepts'; 'questioning'; 'justifying'; 'disagreeing'; and 'strategising', to bring attention to the different aspects of the overarching practice of justification. Second, "attentive use of mathematical language" can also be a description of a few different aspects of mathematics. Mathematical language incorporates symbolic representation and formulae, which are forms of representation. Mathematical definitions are also aspects of mathematical language. The practices of "attentive use of mathematical language", as well as the Panel's term, "representation", were all described as 'representation' in my data analysis; and the ways in which representation was used were specified in the analysis of the tasks in chapter 5. Third, as a mathematics teacher I understand that "generalising ideas and recognising patterns", as discussed in section 2.2.2. are two distinct aspects of rule formulation in mathematics; but they are not as distinctly different when used in IT, unless the IT being done is mathematics (for example, in a subject such as mathematical modelling in IT). When I examined the tasks that the students were to do in the task-based interviews (cf chapter 5), I was not always able to distinguish between "generalising ideas and recognising patterns", and therefore made them a single category of 'generalisation'.

The terms I finally used were: "using procedures flexibly"; "using representation"; "understanding/explaining concepts"; "questioning"; "justifying"; "disagreeing"; "strategising"; and "generalising" (Table 2.1.); so that the mathematical practices being used in the IT context could be described and then re-categorised into the three mathematical practices of representation, justification and generalisation (Ball, 2003).

I made note of all the occasions in all of the interviews when a particular practice was in evidence. I watched the video recordings of the interviews and made extensive notes

regarding all the mathematical practices that I observed. Some students' comments were transcribed if I believed at the time they might be important evidence for claims I might make. Many of the practices overlapped with each other, but this was to be expected, since the strands of mathematical proficiency are inter-twined and inter-dependent (Kilpatrick et al., 2001). I tried to note the practices of individuals, but this task was almost impossible, as one person would continue an explanation where the other left off – demonstrating shared knowledge being used and developed for the purpose of solving a novel task. When this occurred, an explanation from another person sometimes referred to a practice different from the first. But then the first person would take over again from the second, remaining with that explanation of the practice. Therefore, on many occasions it would have been inaccurate to say that one person showed a particular practice.

3.8. Limitations with Respect to Research Design

The analysis of the data was done using descriptions derived from the notions of mathematical proficiency (Kilpatrick et al., 2001) and mathematical practices (Ball, 2003). A limitation of defining categories in this way is that they still needed to be linked back to the overarching practices of representation, justification and generalisation that had been used by Ball (2003) to frame proficiency (cf 2.2.). This linking back has not been done in the literature, and is therefore tentative.

Furthermore, although the mathematical practices and proficiency literature was described in terms of mathematical specificity, I commented in section 2.2.3. that they could also be applied somewhat generically to other domains. I needed to emphasise that social domains, such as the IT social informatics domain dealt with practices that might be given the same descriptive terms as 'representation', 'attentive use of language and definitions', 'articulated and reasoned claims', 'rationally negotiated disagreement', 'generalising ideas', and 'recognising patterns' (Ball, 2003 p. 32), works with different objects and methods from those of mathematics or the mathematical domain of IT; and thus did not use the mathematical practices described in this study. This is a limitation of the study, in that these differences are not obvious when first considered and need to be elucidated.

The sample selection process I used renders certain strengths to my research. Firstly, I had worked with these students for at least one hour a day while they were in the Foundation Programme, and I developed a close working relationship with them. On many occasions they participated in discussion and argument with each other and me in the mathematics classroom. I believe that this good relationship is part of the reason that they could be honest when I asked them questions about their experiences with me.

The weaknesses of my proposed methodology include the following issues, which were minimised through constant awareness of their possible pitfalls, and making every possible effort to counteract them. Firstly, all communication had to take place in English because it is the only language that we all had in common. All of the students in my case study except one were English Additional Language (EAL) students. A few of them had difficulty in expressing themselves. I had to give the respondents chances to explain themselves before moving on to new questions. I was also careful to give students a chance to enquire after any question that seemed unclear. Secondly, and significantly, there was a danger of bias creeping into my data gathering process, because I purposefully selected my sample and because I was investigating my own practice. There was the obvious need to guard against selecting students who I knew would give me the responses that I was hoping to receive, or who would confirm what I said for the purpose of pleasing me. However, due to the reasons stated in the previous paragraph, this same weakness was also a valuable asset to my research. Included in my sample were students who I know had a negative attitude towards my teaching approach, at least while they were my students. At the time of the interviews I was unaware of whether or not they had changed their attitudes since leaving the Foundation Programme.

The fact that the research was a case study rendered my results non-generalisable. Anecdotally, I am aware of many differences between cohorts of students when I teach them each year, and therefore it would be unrealistic to try to claim that my findings are generalisable to other groups of students – even those that I teach. However, a rich description of the case study may allow future research to be directed at finding out in a more generalisable way how student learning promotes transfer of mathematical practices to non-mathematics undergraduate study.

3.9. Ethical Considerations

The University of the Witwatersrand Ethics Committee ethics protocol has been followed for this research and an application for clearance on ethics issues was approved (protocol number 2007ECE53). Beyond this I realise that I am accountable to society for the integrity of my research in terms of what is reported and how I obtained my information (Babbie & Mouton, 2001). Likewise, I have a responsibility to openly disclose my research (ibid.). At the same time, I need to uphold the privacy of my respondents. With this in mind, the students I asked to participate in the study were told what it was about and they signed informed consent forms. They were assured of confidentiality and anonymity, and were told verbally and in writing that they could withdraw from the study at any time (a few did withdraw, because of study pressures). I also requested that they remain honest in their responses, even if they had something negative to say, and that they provide full explanations of their thoughts, to aid my understanding. I had a teacher-student relationship with each of my respondents, and once my research was nearing an end in 2008, I presented my findings to them. They were major participants in the research and I believed that this practice would give them a sense of continuity, as well as identity, with the research.

I am aware that power relationships always exist in researcher-subject relationships (Opie, 2004). I had power as their ex-teacher and as a member of staff at the university. I also had a different sort of power as the researcher, and was careful not to abuse it by asking embarrassing questions, or putting a particular respondent 'on the spot'. I also had to forestall a couple of complaints about the teaching approaches of other lecturers, to maintain professionalism and the integrity of the research.

Lastly, the respondents and records will remain protected once the research has been completed and documented, and destroyed after five years.

3.10. Conclusions

In summary, according to Naroll (1962 in Freeman et al., 2007 p. 28), quality qualitative research incorporates "(a) a thorough description of design and methods, (b) adequate

demonstration of the relationship of claims to data, and (c) thoughtful consideration by the researcher of the strengths and limitations of the study". In this chapter I have shown that I have considered these factors in my research design and analysis.

The study was an attempt to link what students in the sample shared about their experiences in my Foundation Programme mathematics classroom and their first year of undergraduate study in IT with whether and how mathematical practices were used in undergraduate study in IT. Chapter 4 provides a discussion and analysis of students' experiences and views regarding study in the Foundation Programme and first year undergraduate IT. Chapter 5 examines evidence of transfer of mathematical practices from Foundation Programme mathematics to the IT first year core courses of Java I, Networking and Computer Systems.

CHAPTER 4

ANALYSIS OF FOCUS GROUP DATA

4.1. Introduction

Through the focus group interviews, I was able to obtain information on the first two of my research questions, which were: 'To what extent are students aware of differences in teaching approaches between Foundation Programme mathematics and undergraduate study?' and 'To what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses?' The students who were interviewed gave feedback with respect to what they thought a teaching approach is, and described what they thought my teaching approach was and that of their lecturers in their first year of undergraduate study in Information Technology. They also volunteered their opinions about their experiences in my classroom and their current learning environments. Thereafter the students discussed whether or not they believed that the practices they had learned with me were of benefit to them, and whether or not they were currently using them. Being aware that these insights were gained from student experiences and opinions, I have discussed the mathematical practices that they actually used in the task-based interviews in the next chapter.

4.2. Developing Themes

The analysis of the data obtained in the focus groups was by means of coding categories derived from the data. The transcripts were read and re-read extensively. Students' remarks were repeatedly compared with each other, giving rise to units of potentially meaningful information (Gomm, 2004; Hitchcock & Hughes, 1995; Lincoln & Guba, 1985; Merriam, 1997; Yin, 1994), which were then grouped into themes because they had something in common. The resulting categories are shown in Appendix D. The grouping of categories in relation to how they addressed the research questions gave rise to three overarching themes.

Firstly, I discuss the awareness and views of the students, which was a theme which ran strongly through all of the interviews. Another clear theme emerging from the data was that of the opening up of mathematics for the students. A third theme was the usefulness of mathematical practices in first year study; which is key to this study; and which also links the first two research questions to the third question of whether mathematical practices were actually used in doing IT tasks.

4.3. Theme 1: Student Awareness and Views

It was important to confirm that the students could explain what a 'teaching approach' is, distinguish between 'student-centred' and 'teacher-centred' approaches, and could give examples of each, before I asked them to describe their personal experiences. I needed to determine if they shared understandings with me about these important concepts of the study. Discussions with all four focus groups began with the concept of what constitutes a teaching approach. Each interview included asking students for their own explanations of how they understood student-centred and teacher-centred teaching approaches. All twelve of the students showed understanding of what a teaching approach is. I have explained how a student-centred teaching approach may be defined in chapter 2 (cf. 2.3). Its components include curriculum decisions, interpersonal relationships between teachers and students, and pedagogical decisions. Students' comments fell within all of these three components of student-centred teaching. These observations are discussed in some detail below.

4.3.1. Curriculum decisions

Concerning *curriculum decisions*, some students referred to the relevance of their mathematics learning – both classroom organisational aspects and aspects of content to their everyday lives. Others spoke of the relevance of the mathematics they had learned in the Foundation Programme mathematics classroom to their present and future non-mathematical academic situations. Still others referred to the mathematics they had learned in terms of subject specific terms and conceptual understanding, relevance to other mathematical knowledge they had (seeing mathematics as an integrated whole), or of skills in which they

had improved through improved understanding of mathematical concepts, such as problem-solving abilities and analysis and interpretation of problem situations.

Reference to relevance of group work to a future working environment could be seen in the following exemplary comment:

> *"Yeah – at the start – with groups – I found it was not, like – good, but I came to realize that groups – its not like the school groups – they will continue even in their real life situation … I have to, like, work with my team. And then if I'm not willing to work with my team, our team will never become successful. So I attempt(ed) to start looking at this group work and start to be part of the group. Because at the time I found "Ach – it's just a waste of time – we start discussing some weird things". Then I found – ah – like – it's very important to start* [inaudible] *the communication – how to deal with people. And that was during the semester – during your, um, working with me* (Keabetswe)[5]

The students explained how learning certain content knowledge became an important knowledge base for other undergraduate subjects. For example, many appreciated having done introductory probability theory in the Foundation Programme, because it is a conceptually difficult subject, and especially difficult for those newly introduced to it. It gave them a strong foundation for at least three of the undergraduate courses they did in IT: Business Statistics, Boolean Algebra and Computer Modelling for Business Decisions.

In addition, Linear Programming, Logarithms, and basic arithmetic were listed as being essential for undergraduate study. Competence in problem-solving and logic was described by many as being a critical requirement for subjects such as Java and Modelling for Business Decisions.

> *"I did probability um…Last semester I did it in business statistics. This semester I'm doing computer models for business decisions, and I'm doing linear programming, something that I did last year. So, it's just…right now, it's a revision, it's like a continuation of what I did last year".* (Kabo)

[5] Names have been changed to ensure confidentiality and anonymity.

> *"Um…besides the fact that Java also uses things like integers, I'm using examples of Java, you also use things like integers, and binaries and…things basically…um…things we have learnt from the first week of mathematics. Um…I also think there is a thinking element to it, like in mathematics, we did a topic, we actually in almost every topic we did something called problem solving. Um…Java is a lot like problem solving…".* (Molefhi)

Seeing themselves as having improved subject specific knowledge was of significant importance in one way or another for every student interviewed. Some of the students interviewed came into the Foundation Programme with a high level of mathematical competence to begin with – such as Cambridge A levels or a grade 12 mathematics. These particular students were in the Foundation Programme not because of their immediate need to improve their mathematical knowledge, but more because of other needs, such as that of improving their English language skills. Even these students reported a positive influence of being in my Foundation Programme mathematics classroom. Students with lower mathematical qualifications, such as Cambridge O level (IGCSE), formed the majority of the Foundation Programme student body. Whatever level students entered the Programme with, they reported that their learning was useful for subsequent learning. Eric (grade 12, Democratic Republic of Congo), who entered the Programme with excellent mathematical competence, explained his experience of my teaching approach as follows:

> *"Yeah, but, um, I was a little bit skeptical …"* (Eric)
>
> ***LM:*** *I noticed!*
>
> ***E:*** *Yeah! So, you know, I was kind of like, you know, what's going on, what's happening? But, um, to see how people – because I was fine with that topic, myself – but to see how people benefited from doing what they did, because you know, you write from what um, you understood, and it stays in your head. So, to see how people took advantage of that and benefited from that, it's really… it was good.*
>
> ***LM:*** *So, so … the student comes in and says, um … "No, I just… you know, um, she's doing this stuff. I can't relate to it. Um, I know calculus already. No, I don't need help here. What are we doing this for?" What will you say to them?*
>
> ***E:*** *"You don't – you don't know everything". That's what I would say to them. You know, because I learnt a lot of things, just… because here in your head it's like: "ok, I've done this before, I know this." But you know, go and do it again; you find that I was doing this wrong. I now understand it more.*

Some students reported less about how their mathematical knowledge had improved, and more about how my approach had fine-tuned their analysis and problem-solving skills, or how working in groups to discuss problems had improved their overall abilities to communicate their knowledge.

Many students reported how they had not been "good in maths" when they were at school, but that my approach had helped them to gain confidence in solving problems, to understand concepts better, to relate concepts to each other, to relate mathematics to their lives, and to be able to discuss and argue with others. These reports came in conjunction with the actual content matter that they covered in the course, showing that specific content was seen as relevant and important for further undergraduate study, while other content was appreciated for the practices it encouraged.

> *"I feel…you…I remember you always had this thing, it is not about being able to do a problem, but being able to understand it – understand what you are doing. So…yeah…I felt…I will tell them the approach Mrs Manson gives you – it helps you to really understand what you are doing and not just being able to do it and in the long run…I mean when exam time comes, finally being able to do a certain problem."* (Tapfuma)

What did these comments show me about how choice of curriculum may be student-centred or not? The different reports about the relevance of maths to life and future careers, and to learning other academic material, as well as to further learning and understanding of mathematics itself, show that according to students' experiences, there was a balance in the Foundation Programme mathematics curriculum. In Bernstein's terms (cf. 2.3.) the subject boundaries were not so rigid as to allow no linking between mathematics and other areas of life and learning; nor were the boundaries so weak as to have lost subject specificity. Brodie et al (2002) (cf. 2.3.) question: "who gets to decide what students needs are when designing curricula?" As daunting a task as this is, (and I do not claim to have "got it right"!), the students' comments suggest that they experienced elements of student-centredness in the Foundation Programme mathematics curriculum, because of its usability in other areas of life, such as immediate and future study, and life situations.

However, in saying this, I suggest that whether a curriculum is student-centred or not, or is situated at some point in between a technocratic (top-down) or critical (socially contextual) approach (Cornbleth, 1990); the way it is presented and the teacher's classroom practice are important factors in the extent to which teaching is student or teacher-centred. Therefore, I

needed to know whether or not the students I interviewed believed that my approach had been student-centred, and if their learning in undergraduate study was any different from their learning with me; and if so, in what way. Therefore, to ascertain whether or not they found my approaches to be student-centred with respect to all three perspectives of student-centredness, I addressed the issues of inter-personal relationships and pedagogies used, in terms of how the students experienced them.

4.3.2. Interpersonal relationships

The students gave me the impression as they were discussing their understanding of student-centred teaching approaches, that interacting with each other was an important component of student-centred classrooms:

> *Um…with the teacher centred, I think um…it is more focused on the teacher, like the teacher is the one that delivers the information and provides the solutions and there is not a lot of interactions with the students or the students themselves do not interact among them or do not interact with the teacher.* (Tungamirai)

All of the students referred in various ways to the importance of *interpersonal relationships* for their learning of mathematics in the Foundation Programme. Four out of twelve students made similar references to learning in one or two of their undergraduate lectures and tutorials. Phrases such as working in "little groups", "working together", students interact[ing]", and "discussion/discussing", were common when referring to learning Foundation Programme mathematics, and also indicated that the interpersonal relationships they were referring to as significant for their learning were predominantly between themselves as students, rather than between them and me or another lecturer. For example, Kabo explained,

> *"You always let us work… um…together. For example, first semester, it was time when you, you made us do, to write a cass* [a term used for continuous assessment] *in groups and, and in second semester, you made us to create groups in which we were together to solve certain problems … Yeah … And it was very good, it also let us to know each other, how each and everyone operates."* (Kabo)

The same point was made, through some negative comments about what they experienced in undergraduate the lectures, which mostly (but not always) did not incorporate student discussion or relationships that might have facilitated learning. For example,

> *"If you want to discuss, it'll be in your own time with maybe a group of people you'd actually discuss to meet at a certain time. But in lectures and tutorials, never. You don't have room to share anything."* (Eric)

Kelebogile explained that the trust that develops between a teacher and her students is a significant contributor to student-centred teaching (cf. 2.3.). In other parts of the interview she described the trust that she had put in me helpful for her learning of a subject that tended to intimidate her.

> *"Um, ok, I think a teaching approach, isn't it how the teacher just tries to gain the confidence of the students so that they can now start, well, the teacher can start sharing the information about a particular topic. So, you try to get your students to believe in you and actually want to listen to you, then you start teaching. I think that's the approach."* (Kelebogile)

and

> *Ah, you were like, giving us questions every day, and you know, if I look at what I'm doing right now, I don't attend to anything every night, but with maths I make sure I do my work every day. You know like, you were really motivating us, and I found it very useful…* (Kelebogile)
>
> ***LM:*** *In what ways did I motivate you?*
>
> ***K:*** *Like, I could see you really were willing to help me … You know, the thing is you would, uh, every time we had, uh homework or whatever, you would, you know, mark it and give this back to us, so we'd know where we are, the progress, how we're doing, what we can do better, what we can improve on, so you know, that kept us going.*

These findings are congruent with those of Boaler (2000b pp. 13-17) in her discussions of the significance of interpersonal relationships in the learning of mathematics. Boaler states: "many of the students interviewed cited their relations with other members of the group as the single most important factor influencing their predilection towards mathematics" (p. 14). Judging by the fact that all of the students I interviewed also mentioned working together as an important dimension of their learning and understanding mathematics, shows that inter-personal relationships are considered by the students as significant for learning mathematics. This finding shows that what they experienced could be interpreted as part of a student-centred teaching approach, based on Brodie et al's (2002) description of student-centredness discussed in chapter 2 .

4.3.3. Pedagogical practices

Student-centred pedagogy considers how "teachers [act] to make links between learners' current meanings and new knowledge" (Brodie et al., 2002 p. 98) (cf. 2.3.). This can take on many forms, as I learned from my conversations with the students. When asked for an explanation in their own words for student- or teacher-centred approaches, most of the students related what they understood as student- or teacher-centred approaches with what teachers do when they teach. For example,

> *"um…I will say teaching approach, is the manner that you deliver a subject to the students. Um…do they understand it? How you deliver mathematics terms to the students. The way you deliver these terms and do you deliver it in the way that they understand? How do you get them to …let's say how do you get the students to know what is going on? …I remember in our class, there those who did A level and some Zimbabwean students who didn't. How are you going to approach these two different groups, because they have a different understanding of the subject"* (Tapfuma)

> *"[…] require us to actually discover things for ourselves rather than actually just giving us notes and uh… methods and uh…just ways of doing it, - let us actually battle it out first for ourselves and then later on actually be coming and explain …"* (Molefhi)

> *"… we started those groups and it's all about self-discovery of the actual subject. Um…I found that when you actually learn it yourself and doing it in groups, discussing it and you learn the mistakes and…because usually you actually get a lot of mistakes if you are doing it as a group, so you start learning all the mistakes, that way you know what not to do from that and then, I generally think you seem to remember more if you actually discover it yourself than when somebody tells you"* (Molefhi)

Undergraduate studies usually consist of one two-hour lecture and one two-hour tutorial per subject per week. Foundation Programme mathematics classes usually consisted of five one-hour classes in which there were between twenty and thirty students per class. They were more like the undergraduate tutorials in that the tutorial classes held a maximum of twenty-five students and were times when students generally worked on assignments and tutorial questions given to them. Nobody commented on the significant change in the learning environment between the FP and first year undergraduate study. My supposition is that they just accepted that this was how it was going to be from now on. In general, students explained their understanding of teacher- or student-centredness through descriptions and examples of how teachers or lecturers presented content to them. They had a lot to say about the approaches that their lecturers and tutors used, and indicated experiencing mostly student-

centred pedagogy in the Foundation Programme and mostly teacher-centred pedagogy in undergraduate study.

Lectures were described most of the time as being teacher-centred, and a time when lecturers read from the computerised content slides to the class, without expounding on them:

> *"Yeah, … and for lectures where* [inaudible] *you just get into the lecture and then, the lecturer would get in and then read the summaries on the slides, no elaboration, no [inaudible] um…he or she does not expand anything that isn't on the slides, just read everything that's there, something we can do ourselves…"* (Kabo)

> *"…like we had a lecturer last semester, you'd ask a question in class, he'd actually laugh first and tell you:" don't worry, you will know the answer!" And that's it. Like just asking a genuine question you really do not know he'd actually do that and um…you'll probably never know."* (Molefhi)

> *"It's like you really, like somebody can just come and stand in from of you and just read the slides, for like thirty minutes, then say: "ok, you can go". And, I don't know, in the tutorials the tutors just come in and say: 'ok, we have a tutorial, let me see the work you did'"*. (Kudzai)

Through various descriptions of their experiences, the students showed that undergraduate lecturers' pedagogies predominantly did not help them to make links between previous and new knowledge. The students gave many examples of teacher-centred pedagogy while they were explaining their views to me. They also tended to refer to pedagogy (rather than curriculum or inter-personal relationships) when describing their understanding of student-centred approaches. Not surprisingly they showed acute awareness of the need for the teacher to present material to them in a way that would facilitate their understanding.

Positive accounts of learning mathematics were described by all twelve students with respect to classroom organisation that promoted communication; such as arguing, discussing, solving problems together, and interaction. Most of them referred to the benefits of solving problems together with peers, because discussion and arguments helped them to understand the problems, as well as to find possible solutions. In addition, learning how to understand and interpret problems by using representations of the problems and possible solutions to the problem was referred to on numerous occasions. For example, Karabo found meaning in

mathematics by learning how to represent probabilistic situations by drawing tree diagrams. Tapfuma and Keabetswe repeatedly referred to the benefits of understanding the big picture of mathematics concepts through, for example, representing them in mind maps or writing letters to explain a concept to a hypothetical friend who had been "absent" when it was taught:

> *"And…a lot of people have different approaches. I think your approach was…was…was... it had a, a very…I don't know how to say this, but it impacted me in a very very nice way. … You had us drawing, like, uh, trees…what are those called again?"* (Karabo)
>
> ***LM:*** *Probability trees?*
>
> ***K:*** *Yeah! Probability trees and like diagrams to understand what you were trying to teach us, and as I drew those, I sort of like, really got to understand what maths is all about. You have to break things down in order to understand them and the much bigger picture.*
>
> *"Yeah, we even write, like, letters, which uh, was my first time to write some… a letter in maths."* (Keabetswe)
>
> ***LM****: How do you feel?*
>
> ***K:*** *Ya…. It was quite weird, but I think its helped me a lot because it makes me more…it was so interesting – how could I write a letter in maths subject… because maths is …* [inaudible] *maths is all about numbers, but I think your approach- it was quite, quite different because I got to understand that […] I was not that good in mathematics before I came in the course. I used to know I have to, like, take the formulas and start writing, not understanding the actual thing, but since… after you taught me to understand the theory behind maths I started to understand…*

Both classroom organisation and deliberate teaching of problem-solving techniques and representations were components of my pedagogy that was designed to encourage students to use previous knowledge and experience in order to learn new material, solve problems and fit together previously separated concepts of mathematics into a bigger picture or more meaningful whole.

As previously stated, students were acutely aware of pedagogy that helped them to learn. They also reported any approaches used by undergraduate lecturers that aided learning by making links to previous knowledge or to their own lives – that is, *relevance*. Although lecturers sometimes had to deal with large classes that did not easily support personal interaction between students or themselves and students, these particular lecturers frequently

held whole class discussion and debates, encouraged students to split up into discussion groups for short discussion and subsequent report-backs to the class, or took the time to explain to and listen to students during the lectures and tutorials. As far as I could determine (because I had asked the students not to name lecturers when discussing their experiences of teaching approaches), as many as four lecturers were reported to use student-centred pedagogies, and as a result were held in high regard by the students, because they were able to understand their work.

> *"Ok, Also adding to him. It's not only about the bad but also there is the good that they are doing. I mean, one of my lecturers actually I understand. Every time she comes to class there is always that … you know … that … in everything that we learn, there is always that approach that she will come and say…she will come and tell us everything and try and relate what she knows with daily activities around that."* (Kaone)

According to these findings, students were indeed aware of differences in teaching approaches between my Foundation Programme mathematics teaching and most of their undergraduate lectures. The above comments give some indication of their views on the issue; but I will discuss these views in more detail in the next section. Before I do this, I first briefly discuss possible sources of bias in the data and analysis, and then make some important conclusions about whether or not they saw my approach as "student-centred" using the way that I have described student-centredness in chapter 2.

I was aware of the potential for bias (cf 3.8.) to affect the data analysis. Bias could have appeared from different sources: Since all of the respondents were my own ex-Foundation Programme students, the possibility existed for interview questions concerning students' experiences to lead students into certain responses. A second potential problem was that of students tailoring their responses to please me because of their past relationships with me. In addition, the possibility existed that I would read more into students' responses than they intended. The first potential bias was addressed by the design of interview questions using guidelines from Merriam (1997) (see Appendix C). To address the latter two issues I searched through the data and provided categories for evidence that disproved what I might have wanted to hear when I interpreted and analysed students' comments (cf 3.7.1.).

Furthermore, I acknowledge that although rich data were obtained by using both focus group and task-based interviews, any conclusions made could not be generalised to other Monash University South Africa Foundation Programme students or students in other universities.

Their experiences, as they were related to me, pertained only to my teaching. Other teachers use different approaches, and therefore my ex-students' experiences and uses of mathematical approaches could not be generalised to other situations.

I made the statement in chapter 2 that, 'forms [of student-centredness] are activities and classroom organisational setups that teachers use. The substance of student-centredness entails "selection and sequencing of tasks in relation to students' current knowledge and providing for the required conceptual development in a subject area…" (cf. 2.3.). With reference to their learning in the Foundation Programme, I have shown that the students described their experiences of a teaching approach that was student-centred in both form and substance. In terms of form, the classroom was organised so that students could interact with each other and me, because I would walk around, listening to conversations and asking and answering questions about what the students were doing at the time. In terms of substance, the tasks that were given, for example, mind maps, letters to a friend, representational tasks, discussion tasks and scaffolded tasks, were at a level that was helpful for student learning of new material.

In this section I established that (1) the students experienced differences between teaching approaches used in the Foundation Programme mathematics unit and those of first year undergraduate IT core units; and (2) the students experienced my teaching as student-centred, with respect to curriculum, pedagogies and inter-personal relationships. In the following sections I highlight two other themes which emerged from the interview data. The first deals with how they spoke of the way mathematics was opened up for them during their year in the Foundation Programme. The second explores the students' views of the usefulness of my teaching approach for supporting future learning situations and how useful they judged their newly-developed mathematical knowledge and practices in their undergraduate studies in Information Technology. Closely related to this finding was how mathematical practices are useful for dealing with real life and future work situations.

4.4. Theme 2: Opening Up Mathematics

Although all Foundation Programme mathematics students had done mathematics in the past, many had not seen the sense in doing it and/or conceptually understood it. Therefore, it was

significant for the students in the study that through learning mathematics in the Foundation Programme mathematics was ‘opened up’ for them. Many students did not refer only to improved understanding of concepts, but also to becoming competent in different kinds of mathematical practices, which is an indication of overall increased mathematical proficiency (cf 2.2.). Although mathematical proficiency includes the productive disposition strand, I discuss the opening up of mathematics in terms of increased productive disposition separately, as it was an unexpected finding that this strand filtered through the majority of students’ comments. Hence, the discussion of the opening up of mathematics in terms of

1. Increased overall proficiency, and
2. Productive disposition

4.4.1. Increased proficiency

Many students across all of the focus groups spoke about their improved mathematical proficiency and use of mathematical practices. They also spoke of their recognition of mathematics as worthwhile and useful. According to Alexander et al (1999), proficiency in one context encourages transfer into another context. Therefore, I argue that the students’ own recognition of their improved proficiency and use of mathematical practices in mathematics is a significant finding, because this should have a positive impact on use of these practices in a non-mathematics context. This section is a brief discussion of what the students said about their mathematical proficiency in mathematics and use of mathematical practices in mathematics and undergraduate IT study. I discuss their comments about the practices that they said they used in the IT context under theme 3.

Students mostly spoke of their improved conceptual understanding of the content of mathematics, which was liberating for them. Being able to understand a problem and being able to make informed decisions about how it could be solved was also spoken of by many of them. The inter-connectedness and inter-dependency of the proficiency strands was visible in their comments, because usually when they spoke, their experiences incorporated at least two of the strands. The terminologies used concerning proficiency (Kilpatrick et al., 2001) and practices (Ball, 2003) were used interchangeably by the speakers, but it was clear that the students believed their mathematical proficiency had increased. For example, Karabo spoke of increased conceptual understanding together with an improved ability to analyse and

understand a problem, rather than attempting to do it procedurally without understanding. Underlying his comment was his belief in his improved abilities to do mathematics, both of which had changed since leaving school.

> *"In school, I used to think "Ah, maths – its either you were born with it or you are never gonna be able to do maths". But then coming here – you, you kind of like showed us, yeah, that maths is just – you just need to understand what you're doing before you do it. Instead of going to the equation and just looking at it and adding figures and coming up with an answer. You're trying to state what exactly … because it's … Just look at the question – understand it and then try to see if you really understand it by what it said: Summarising what the question wants from you, trying to find the solution because I think when you're following those steps, you can really say "Ok, now this is the answer". If it's something else, then I must have done something wrong somewhere before reaching my answer"* (Karabo)

The students' realisation that mathematics is not only procedural opened up their understanding that mathematics is more than an extremely difficult subject that did not make sense, but that had to be studied because it was an important subject to know. The strands of proficiency became accessible to them, and they observed their own development in the strands other than procedural fluency – particularly with respect to how seeing mathematics as sensible and valuable affected their conceptual understanding and problem-solving; and vice versa. Not seeing the point of doing mathematics and lacking conceptual understanding had affected their ability to solve mathematical problems.

> *"… I used to know mathematics is all about, like, the way it was … the way I was taught mathematics, I just came to class and solved problems:* $x + y = 2$ *, and then I start solving. But since I came here, every topic I did, like, an application. Because if there wasn't an application I didn't get a true picture. What this topic is all about. What are they trying to clarify? I started asking myself' "Why am I doing this topic? Because it seems like it is useless. Does it fit into being able to think logically –?" But, yeah, the logical thinking I understand, but why do I have to do something which is more technical? But if you do an association; every topic you do an association, every topic you do some … you associate with a real life situation, you take mathematics into practice. That will make students motivated to learn more about mathematics."* (Keabetswe)

> *"Will procedure also be the same as method? [**LM:** Yeah…] Ok…Yeah, you learned methods* [at school] *and then with time after doing a lot of questions, they hope that you actually realise for yourself why…actually be able to learn and explain for yourself why I do it that way and…but a lot of the time, students are not really concerned with why they are doing that way. If I have a method that works, hey great, I will just answer the question that way. I will just plot in numbers, not even know what it means. And then a day comes when you are asked a question*

> *that is similar to that one but not necessarily something you have done in class. And maybe it requires for you to change the subject of the formula for example and then you are completely lost, you have no idea of what is going on. That is when you start checking: "Wait a minute, we were not taught this!" But just…it just requires you to have an understanding and just re-apply what you have already learnt. But all you know is just your method, that's it. So…"* (Molefhi)

When I asked them why they thought they had to study mathematics in the Foundation Programme some students interviewed spoke about their fear of mathematics, their weakness in the subject, and how doing mathematics in the Foundation Programme had offered them a new way to approach mathematics. They described their changed abilities to "do maths". Change included increased confidence, increased conceptual understanding, increased problem-solving abilities, an increased appreciation of discussion and argument with peers, and increased understanding of mathematics as worthwhile and useful – an indication of the role of productive disposition in mathematical proficiency.

> *"… I think I did maths to develop an approach towards problems- to understand how to solve problems Y'know- to build… in a way for me its like building a step. It gave me confidence"* (Kaone)

> *"Ok, for me, I think I learned what I called the language of maths, rather than the equations of maths. I turned to…to…to actually erase that fear that I had of maths. …. It's like actually you know, being an African and actually taken to learn …uh ... Korean. Obviously in your first days you won't like it but it goes with interest and benefit that you want to have. Like for instance, if I had to learn what I call like language of maths, I had to speak maths, so to… to… to be sure that I understand maths, to be sure that I understand what I'm talking about. I can explain to someone in* [inaudible] *because I don't only know the formulas, I know the language"* (Kaone)

> ***LM**: What is speaking maths? What is that- What do you do when you speak maths?*

> ***K**: Its turning formulas into a language. […] I can actually tell you that this means this; this means this – I could tell someone who don't know, like… this is … what do I know about this, rather than trying to write something that I don't know. I could share my experience. I could share what I have with someone, not only actually only writing. It's not only about writing maths. It's also about telling somebody. Talking about it. Understanding the concepts.*

What came through strongly in students' comments about their increased proficiency was how increased understanding gave them the tools to solve problems better. As already stated,

the common thread running through these comments was increased productive disposition. I report on this below.

4.4.2. Productive disposition

Recall that "*productive disposition* refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001) (cf 2.2.1.). It is difficult to 'test' for productive disposition, I did not deliberately question the students about their productive disposition regarding mathematics; and the following discussion was an unexpected outcome of the focus group interviews. I have always believed that the best way to 'test for' productive disposition is to work closely with my students in the classroom. That is, I would watch what they were doing and listen carefully to what they were saying, in order to gain insight into their beliefs about mathematics. However, to my surprise, their beliefs about mathematics became very evident during the interviews, and it gave me a deeper and more meaningful insight about their experiences with me, how they saw mathematics after they left me, and how productive disposition interacts with the other proficiency strands.

Some students made it clear that studying mathematics with me had changed not only how they learned mathematics, but also how their studies in general were influenced by their new broader understanding of mathematical proficiency. Woven through Tapfuma's comment about his increased ability to understand and use his mathematics is the strand of productive disposition: his own recognition and appreciation of his increased proficiency and the manner in which he was able to use his understanding

> *"Yeah, um…the maths, as I've been saying before, it gave me a different approach, a different way to look at the subjects that I am taking. Without it, I would have still been at the same point you know, where as I said before it is not just being able to do it but understand it. I would just have been like: "ok I can do this, and this", you know and just doing what you can do. And if you don't, try and understand – if you can't, just leave it. Yeah, or you get someone else; just to show you, to have a rough idea, yeah. So, it gave me that understanding, of breaking down, what I am supposed to do and how I should do it, where is it leading to? The maths really did help…"* (Tapfuma)

The students who came into the programme with excellent content knowledge were negative about my teaching approach to begin with as they already had the content knowledge from school. However, all those interviewed realised that what I taught them was useful, in the sense that they learned how to think in a different way, and that this aspect of what they did with me was "worthwhile" to use the same word that Kilpatrick et al (2001) used to describe productive disposition. They recognised that their understanding was broader than it had been previously.

> *"Because I went through it like I said, I went through it* (the content at school), *so in the beginning it* (FP teaching approach) *was quite weird and confusing and then as we continued actually we started to gain meaning from it, I actually started to understand it and it made sense."* (Molefhi)

A few students believed that the mathematics they did with me allowed them to see its relevance, together with the idea that maths is not only learned as a set of rules 'told' by the teacher. They spoke of how mathematics had become real for them. For example, I asked them, having been in my classroom the previous year, how they would respond to a current FP student complaining about having to do mathematics in the Foundation Programme.

> **"***…this is real. Just that it is not like in high school whereby most of stuff, teachers used to do for us. So you have to learn to do things by yourself."* (Ditso)

> *"um…then…but now I feel…maths is real, maths is basically problem solving and especially as IT student, it is pretty much everything you are doing which is problem solving. So, especially the way that was brought to us, I feel yeah, definitely, it helped us."* (Tapfuma)

> *"Um, yeah, I think it was actually a good thing that we were introduced to math, because it kind of like, opens your mind to some extent from the whole – a bit from the whole high school way of learning to now a bit more grown-up way of learning where you have to, you know, you have to solve things for yourself; and* (at school) *you're just given – and you just have to plug in different numbers, and then that's*

> *it. So, it really helped because now you can use some of that knowledge from ADP*[6] *and then just try to implement it into your other things you do."* (Kudzai)

In addition, some of the students – particularly those who had entered the FP with 'O' level mathematics – spoke of the improvement in their general ability to *do* the mathematics; having come from a background where they either thought that they were 'bad in maths', or were frightened of it, or both:

> *"Jah … it was quite weird, but I think its helped me a lot because it makes me more … it was so interesting – maths is … people think that maths is all about numbers, but I think your approach – it was quite … quite different, because I get to understand that I … I was not that good in mathematics before I came in the course. I used to know I have to, like, take the formulas and start writing, not understanding the actual thing, but since … after you taught me to understand the theory behind maths I started to understand that we can be given something like "one plus one", then you say "Why are you using one plus one?" and I start to really […] think beyond mathematics – how to do the steps – the steps that I have to take in order to solve problems".* (Keabetswe)

The students recognised and described for themselves how their outlooks about mathematics had changed since doing mathematics in the Foundation Programme. This was significant because their productive disposition had also improved, evidenced by numerous descriptions of their improved mathematical understanding and recognition of the usefulness of mathematics. According to Boaler (1997, 1998, 2000b) mathematical knowledge that is more than procedural is more likely to be usable in novel problems. The fact that some of the students (Kabo, Keabetswe, Karabo, Tapfuma, Kaone, Kelebogile) reported having a broader mathematical understanding after doing the Foundation Programme mathematics course, and that their mathematical knowledge comprised more than knowledge of formulae and procedures, gives weight to my subsequent argument that their knowledge would be more likely to be usable in the context of undergraduate IT. Increased mathematical proficiency had given them greater access to use of mathematical practices when solving IT problems. Below I discuss the third theme emerging from the focus group interviews.

[6] The ADP was the name previously used for the Monash South Africa Foundation Programme

4.5. Theme 3: Usefulness of Teaching Approaches

The students in the focus groups indicated that my teaching approach in the Foundation Programme led to certain expectations for subsequent study. There were mixed opinions among students about how helpful my teaching approach was in preparing them for the teaching approach and learning experiences in first year undergraduate study.

4.5.1. Benefits of FP teaching approach in undergraduate study

As a teacher I desired that my teaching approach would encourage certain practices that students would find useful in areas of learning other than mathematics. Other techniques that I encouraged included the use of group discussion; developing questioning practices that would focus on considering Kilpatrick et al's (2001) five strands; having to explain in the form of a 'letter to a friend who was absent' the meanings of newly-learned concepts; translating mathematical representations to meanings in words, or vice versa; and breaking down complicated problems into parts that could provide some direction to solving the problem.

In this study, I wanted to test the accuracy of my belief that the approach I used while teaching FP mathematics helped my students to use mathematical practices in undergraduate study. In order to find out more about how my students used mathematical practices I first questioned them about whether they believed that the mathematical practices and other techniques they learned and used with me benefited them in undergraduate study. Subsequently, I interviewed students from the same class while they were busy with IT tasks. From these interviews I ascertained which mathematical practices each student used, which would in turn indicate whether s/he used mathematical practices in undergraduate study in IT.

Initially, I was preparing to make a simple report about the extent to which students said they used practices they had learned with me during their undergraduate studies. After coding the transcripts I realised that the report would not be as simple as I had first imagined. The students reported that learning in my FP classroom did have an influence on their individual work habits in undergraduate study, but that the transition from my classroom to that of undergraduate study was extreme for most of them. Their descriptions of their use of

mathematical practices and other useful learning techniques, such as working in groups to solve problems and using mind-mapping ranged from 'deliberately used' to 'not used'.

Certain mathematical practices were assimilated and used deliberately by some students because they worked well and became part of the student's regular practice. It also appears, judging from students' comments, that the more such practices are used in different ways and different subject areas, the more they are deemed useful by the user and subsequently, the more they are used. Also, their use becomes increasingly flexible – an important aspect of successful transfer, from the perspectives of both cognitive and situative theorists (Alexander & Murphy, 1999; Hatano & Greeno, 1999; Marini & Genereux, 1995; Simons, 1999).

The practices and other techniques that were reported by different students as helpful and used most often include group discussion and argument, mind mapping and representational and problem-solving strategies. Group discussion was reported as being helpful for solving problems, working on assignments, learning about new concepts in difficult subject matter, and clarifying understanding through argument and justification of claims. For example, Tapfuma explained his current study habits, and how they were influenced by his time spent in the Foundation Programme:

> *"Um…me…I agree with the group work, because it really helped. I guess…I remember there was a time when we used to split and we had another lecture for maths also, but now in that lecture, things were totally different. I do not know if it is…if anything …I would tell all lecturers to present subjects the way you did Maths to us. […]. Um…the thinking part, definitely. I mean just…I remember you made us do mind maps; I never thought mind maps…I usually thought it was ridiculous, not of interest. The first two, I did them in the morning, but in the end…I think yeah, they help you solve the problem. Like Rudo said, in programming, we were told um…to make a program for this company that does this…this, this, and suddenly you get your program there, your branches…I started doing it, I need to do this, I need to do that, you know, and you end up putting them down in steps, ok this first, this next, you know … And with the group work yeah, asking others. I remember, there was an assignment … actually I never thought that would happen, but we actually got to meet in groups at Tunga's house. And everyone you know…worked with their laptops doing the thing, but then when we got questions, it would not just be – . Ok, Rudo asked a question, she just passed the question to the whole floor, and you just find different ideas coming out of people. And it gives a better understanding, and I enjoyed the Maths part of it."* (Tapfuma)

Many students had developed a new way of approaching problems. Many of them explained how their problem-solving strategies had improved through using techniques and practices, such as mind mapping and representation respectively to break down the problem. Keabetswe

referred to his problem-solving approaches often, having previously described his belief that he was never 'very good in maths':

> *Even now, even right now, I am still using the approach to solving problems for my assignments because they are more technical and for you to do, to solve the technical problems you have to, like, understand what the case study first in detail before answering what you are supposed to do, but, with that approach that I've gained, I've managed to use it- and I'm still gonna use it in future* (Keabetswe).

Using newly-learned problem-solving approaches will always be easier when encouraged and aided by the teacher; because if it is a deliberate pedagogy, then the teacher, as well as the student, will be involved in thinking where and how such strategies may be used. This should promote frequency as well as flexibility of use in different situations. The students singled out a couple of lecturers who used this kind of pedagogy. I suggest that they talked about these experiences because they appreciated the fact that they could see how some of the practices learned in my mathematics classroom could be used in new situations – encouraging flexibility of their use.

> *Um…I am still using the same approach in […] computer modelling. In tutorials, we still work in groups, we come to sit together and then…um…each and every one of us would present the solution and we have got to debate, looking into the solution disagree and then argue in certain situations*…(Kabo)
>
> ***LM**: So you are arguing?*
>
> ***K**: Yes…*
>
> ***LM**: Ok. Um is that the whole tutorial or what? Is that because of you or the lecturer or both?*
>
> ***K**: Um…yes, because of the lecturer…*

A few students said that they used the practices and other techniques they had used with me, but that it was for the sake of academic survival that they tended to discuss the work with friends or do further research on their own. In terms of discussion, they had valued it as a tool for learning while they had been learning mathematics in the Foundation Programme, and had commented positively often in the interviews about being encouraged to discuss their thinking. Now that they were in first year, many teaching approaches used did not encourage similar opportunities – even in the tutorial sessions, where discussion could be used

constructively. When content became difficult, or they had assignments due, students were more likely to work together.

> *Group work, as well* (Eric)
>
> ***LM:*** *Ok, where were you using group work?*
>
> ***E:*** *Um, I don't really use it. It's kind of like just before an assignment, that's when we get together with people and work on the assignments. Because, we don't – we don't have that opportunity to do it in class or in tutorials, so yeah…*

Academic survival included increased personal research for a few of the students. Many of the students explained that first year was extremely difficult because they were left on their own. Therefore, they needed to be more pro-active in their learning. Some of them explained that it was through the way they had learned mathematics in the Foundation Programme mathematics classroom that they had been given access to knowing how to find things out for themselves, and so understand new material better.

> ***LM****: Ok, […] 'I am using the same approaches in my studies now that I used in the Foundation Program maths classes'. Is this applicable to you? Some of the time? All of the time? None of the time?*
>
> *"Um…I'd say sometimes. Sometimes optional and sometimes forced into it. Like if you have a situation…ok sometimes where…actually I think sometimes it is always forced, you do it sometimes and you are forced by the way the lecturers are teaching If a person comes like you have already said, comes into a lecture room with a 190 students and then he comes and reads slides for you, one after the other, … You do not learn anything if he is not elaborating, so that sort of forces you to actually have a hands on approach by yourself, like sort of the one where we were in groups last year – um … it forces you to do that, you have to actually go to the library, and then look for concepts […] with them if you have to, you actually have to find out things for yourself, not because we choose to like I said, it's forced."* (Molefhi)

Three of the twelve students said in their interview that they did not use many of the practices that they had learned with me. The negative responses tended to follow the theme of '*those practices we learned were good and useful, but we are not using them now because current study does not make possible that kind of practice*'. Reasons given for this included the fact that they were too busy trying to survive the work load to discuss work with each other; that they felt that sometimes they were competing with each other and therefore did not want to share their little understanding with their peers; or that they did not because their lecturers were not promoting that sort of approach.

> *"Not really. Especially when it's discussions and things like arguments, people just don't have time. You only find people discussing or arguing if they can't meet a deadline for an assignment. And, otherwise, everyone's just doing their own thing. But in ADP, we had to like, sit in groups in class, and now we can even sit in one corner and the other person in the other corner and no-one talks to anyone during the lecture or tutorial. You just stand up and leave. So now what we did in ADP is totally different, it's like a totally different school. As if we moved to a totally different place from where we were in ADP, so…"* (Kudzai)

In this particular case I was interested to see if this sort of situation was evident when the students did the task-based interviews with me. What they *said* they did and what they *actually* did was an area of interest for me – because I suspected that some students were not aware that the practices that they had learned with me were being used in their undergraduate studies. Also, some of the other comments that they made in the same interview appeared to contradict this view. For example, Kudzai said, shown above, that she did not really use the practices that she had learned in the Foundation Programme in undergraduate study; but she also said that what she had learned was useful in undergraduate study, because she did not receive much guidance from her first year lecturers:

> *"Um, like right now, in first year, I can't say there's been a lecturer who's actually taken the time to explain something so that I can understand. They just give me information, then I have to sit down and try to get the facts that I need. And, like, from second semester ADP, that's what we did in the beginning, where you were given information then you had to like, pick up stuff, and understand what you had to know. So, it actually helped me in that first year now I recall that ok, I did this before. I can sit down with that information then take out what I actually need for me to, you know, pass the unit or do well, you know. So that's how it helped me…"* (Kudzai)

Thus, it was interesting to observe their behaviours during the task-based interviews, without their interpreted explanations of what they did, to see if they were really not using mathematical practices in first year.

A number of students explained how the practices they had learned with me in the Foundation Programme were useful and relevant to them in their everyday lives. Students' beliefs about the usefulness of mathematical practices included their lives in a real world in which there are many different facets – two of which are ordinary daily life and future work situations. From a situative perspective on learning, one cannot ignore students' reference and use of mathematical practices in their daily lives, as communities of practice overlap and influence each other (Wenger, 1998). Therefore, I believe it is natural for them to refer, when they feel

it is necessary, to other facets of their lives, and because of that, I briefly report their opinions on the subject.

4.5.2. Usefulness in other facets of life

The students mentioned the usefulness of the practices, techniques and content learned in my Foundation Programme mathematics classroom to life outside of academic work. A few students spoke of the importance of learning how to work better with other people in groups, as teamwork would be an important aspect to their work in IT in future employment.

> *Um…I would say um…I think for me personally, all the aspects that I learned from you are really helping me out because um…I am not really um…a people-people and now with IT, we have to work in groups and um…personally, I do not feel like working in groups is really necessary because I feel like it undermines my ability to actually express myself, but now, since doing your course, now I understand that it not all about what I did, because in the end, I am not able to do everything and I don't have all the answers. And now you see, I understand from the experience I got from you, I actually appreciated it when we worked in groups and I found it actually much easier if I worked with other people…* (Tungamirai)

> ***LM**: What do you mean you find it easier…you find it easier to work with other people or you find it easier to do the work if you're working with other people?*

> ***T**: No, I find it easier to work with other people. I might not necessarily agree with what they say but you know I find it easier and I understand why the groups are so important.*

> ***LM**: Why? Why are they so important?*

> ***T:** Because like um…[…] when eventually I do leave Monash, in about six years or so, we want…we are going to be working in organisations and we'll be put in certain groups and we will have to perform a specific task. So, if you are not able to communicate to the next person in your group, you then…that task is going to die. Eventually, it is going to cost you a lot of money and you will not be able to utilise your capabilities to the maximum. So working with other people is really essential because IT is more of a group … it is more of a project subject, and you just have to learn…*

> *I think that […] we are all going to leave school and everything, we are probably going to be grouped up with people, whether you like it or not, you are going to be doing your job…* (Tapfuma)

Ordinary life was referred to a few times, where students were aware of where their mathematics content knowledge might serve them.

> *… you don't use, like … its like buying food in a shop. You don't buy food so that you eat it in a day. On that day. What about tomorrow? For me, I think that you don't learn everything that you learn in a day or in a year, to only use it in the coming year or tomorrow. No! You use it – it's for the future. You learn it so you can relate it to other situations of … y'know, not only about school academic, but in other real life situations.* (Kaone)

In summary, although most of the students interviewed said that they deliberately used some of the practices and other techniques they had learned with me because they had found them to be useful for learning, some of them also said that they used them because it was a means of academic survival in undergraduate study. Some others said that they did not use them at all. I propose that they said that they did not use practices because the two situations were too different for the students to be able to recognise where or how these may be used in the new situation, even if they had believed that the practices had been useful in mathematics. Many students commented on how useful the mathematical practices or other techniques learned in the Foundation Programme were in other facets of their lives, such as other academic work and real life and work situations.

Students tended to comment more deliberately about using learning techniques than about using mathematical practices. They would identify a particular technique, such as 'group work' or 'writing letters' and then describe how they used it or not in first year. Their descriptions of how they used practices in first year IT were less deliberate. The words they used were not specifically words taken from Kilpatrick et al's proficiency, or Ball's practices, but their descriptions below of the practices they used were clear nevertheless. As students began to use mathematical practices in undergraduate study, their practices were altered so that they were usable in the new context of IT. This is significant for understanding my third research question and the fifth chapter is entirely devoted to how students actually used mathematical practices in undergraduate IT study.

4.5.3. Use of mathematical practices

When I conducted my interviews, I found it difficult to separate Kilpatrick's et al (2001) proficiency and the RAND Mathematics Study Panel's (Ball, 2003) through the students' descriptions of what they did. I decided to include all the terms from both documents into what I have called 'practices', although I am aware that some of the words I have used originate with the 'proficiency' concept. I have shown that practices were defined to "frame" proficiency, because "a clearer articulation of what the strands of mathematical proficiency mean and how they relate to each other and interact over the course of a student's learning of mathematics" (Ball, 2003 p. 9) was needed (cf 1.3.1.). I therefore argue that using terms from both concepts was acceptable, although I would have to relate them back to the overarching practices of representation, justification and generalisation at the end of the study, in order to ascertain which practices had been used by the students when solving IT problems.

From what students said, it appears that they used mathematical practices in many different ways in IT. As for mathematical proficiency, these practices are intertwined and interdependent. This is shown in students' comments, where more than one practice is mentioned in a description of what the students do and think. Also, absence of a specific term when describing a practice in a student's comment does not necessarily mean that the practice is not used or recognized by the student. Firstly, the student might not know the 'name' of the practice being described, and is merely explaining what s/he does in a particular IT context. Secondly, not specifically talking about a particular practice may mean that the practice is being subconsciously used. However, it is clear from the comments made that the practices are not separate from each other, and are used in IT whenever necessary.

When the students spoke of mathematical practices they often referred to what they were currently doing, in relation to the mathematical practices established in the Foundation Programme. They were not always specific about the how a particular practice learned and used in mathematics was used now that they were in first year IT. For example, in the way that Keabetswe described how he solved computer programming problems, he showed clearly how strategic planning of a solution was linked with how well he understood the problem, and how components of the problem and solution fitted together by means of conceptual understanding of content. At the same time, he spoke of Java programming – not mathematics. His comment gives insight into how not only mathematical practices are

interwoven, but also how they may be used in non-mathematics domains (I have highlighted these key aspects of his comment).

> *"Ja, Java. You **have to understand**. 'Cos right now we're doing … what we are doing right now is completely foreign from what we did last semester, because its mainly design; **and design is concerned with understanding clearly what you have to do** – first – before … you can't just, like, **write it without understanding**. You have to **critically read and understand** what you are supposed to do **and how you associate** every, um, everything …"* (Keabetswe)

Discussion, debate, argument and justification were occasionally referred to. The students often spoke about the merits of discussion in learning mathematics. While discussion does not necessarily incorporate justification and argument, these two practices are largely dependent on discussion. Therefore, while I do not assume that any time that discussion was mentioned by a student, justification was also taking place, there were a few occasions where a student spoke about arguing aiding understanding. I suggest that where arguing occurs, justification is also likely to occur, because the person doing the arguing or questioning of another's statement would also not be likely to let it rest until satisfied with the other's explanation. The actual use of justification was seen more in the task-based interviews in chapter 5, where students did not have to understand what justification was in order to say whether or not they used it.

> *"… Yeah, I think it really did help me because for another subject last semester, we had together, um…if I did not understand something, let us say Tunga knew better than me, how else can I deal with it? **And we always used to "fight". It is not just you showing me what to do, I want to know what exactly I am supposed to do and that will give me better understanding** of the subject, which is I guess is better in the long run. **You really understand** what to do…"* (Tapfuma)

> *"…um…I am still using the same approach in data modelling and computer modelling. In tutorials, we still work in groups, we come to sit together and then…um…each and every one of us would present the solution and **we have got to debate, looking into the solution disagree and then argue in certain situations**…"* (Kabo)

> *"…Arguing and reasoning it is the base of everything"* (Tungamirai)

In the Foundation Programme Math B course (Appendix A) students were encouraged to question each other in terms of the proficiency strands (cf. 1.4.1.). A few referred to the advantage of using questioning in order to better understand content, justify claims and interpret and solve problems. Again, the inter-dependency and connection between proficiency strands is evident in these comments. A few students used questioning of their own or a peer's thinking in order to clarify a problem, justify why s/he should use a particular way to solve it, and choose a solution process best suited for the particular problem.

> *"… problem solving is actually now easier because um…I am able* ***to ask myself questions****, um…such as: "****Why*** *am I doing this?* ***How*** *am I supposed to do it? And* ***what*** *am I supposed to do?" It really helped me, with problem solving questions."* (Keabetswe)

Using the practice of representation in many aspects of IT is crucial. Students never used the term 'representation', but many described situations where use of this practice was either advantageous to solving a problem, or was a requirement before solving the rest of the problem. On the whole the students described using representations in IT, but did not mention the mathematical representations they used in the Foundation Programme, such as graphing, algebraic notation and formulae.

> *"… before you even write the program, you* ***need to have some sort of diagram that actually shows you****, ok; this element interacts with this element and, using that, you actually find out that that is how your whole program is going to be written and stuff …"* (Tungamirai)

Using representation as part of strategic planning to solve problems was described by a few students. The close relationship of representation to problem solving is shown clearly by Kilpatrick et al (2001), and the fact that students spoke about how they linked the two showed firstly, how their proficiency had increased, and secondly, how they were able to describe their use of these practices in the IT context.

> *"Yeah. Because I have, like, my website authoring I have to, like, draw mind map of some – before I attempt – even for Java. I have, like, to* ***draw a mind map of how am I going to develop a code****. How to design –* ***how am I going to approach my design****. I have to like draw first. …* (Keabetswe)
>
> ***LM:*** *Ok. And this is your decision …? Not your lecturer telling you to do it?*

> ***K: "****Ja, its my decision, because you can't like do something you end up missing out, making things become mixed up … So you have to like,* ***draw a picture*** *and just show – like – maybe I should do this and, I have to like do this … next …"*

During the focus group interviews I noticed that the students never referred to the practice of generalisation. I knew that they had learned about mathematical generalisation in the Foundation Programme, because, for example, they had needed to model information into formulae in linear programming; and generalize their knowledge about gradients, tangents and limits to produce for themselves a formula to describe gradient at a point in introductory differential calculus. After having deliberately asked one group if they used generalisation in IT one student described where this practice may be applicable in IT, but he was not sure about his analogy.

> *"For example, when we do Java, we have um…like…they say: "You have a class and then each class has an attribute and methods or behaviours." So, usually, they give us an example. They say: "um…think of a class like um…Dog, and then the dog would have a colour whether big or small," so those would be like attributes. And then the behaviours would be like um…it runs fast, what…what…what…The general concept is like, it is still a dog but, because now, each dog has different attributes…"* (Tungamirai)

However, when the students did IT tasks, they did use generalisation in certain of the tasks, when required. More is discussed on this matter in chapter 5. I have given an overview of the kinds of statements made by the students regarding their reported use of practices in first year. I conclude that although the students interviewed had mixed beliefs about whether or not the practices they learned were of benefit to them in undergraduate study, through verbal descriptions of what they did, evidence indicates that they were using mathematical practices.

4.6. Conclusion

Three main themes emerged from the focus group interviews with my students concerning their experiences of the teaching approach used in the Foundation Programme and their use of mathematical practices. They are

1. Student awareness and views,
2. Opening up mathematics, and

3. Usefulness of teaching approaches.

The students interviewed indicated that they sometimes or often recognised the need for, and used, mathematical practices and other learning techniques in undergraduate study. The next process was to determine whether the students showed *evidence* of using mathematical practices when doing non-mathematical tasks in IT subjects. This addressed issues of transfer of mathematical practices, which is an important aspect of learning mathematics: if mathematical practices cannot be used in other subjects there are a couple of important implications:

1. Whether or not a prescribed qualification in mathematics should be a pre-requisite for undergraduate study in non-mathematics subjects such as IT.

2. How useful the learning of mathematics as a subject on its own is for subsequent use in other subjects that require maths; for example, in business or biological and earth sciences. An alternative possibility would then be to learn business-specific or biology-specific or geography-specific mathematics.

Transfer of mathematical practices as demonstrated by the task-based interview respondents is discussed in the following chapter.

CHAPTER 5

DATA ANALYSIS: TASK-BASED INTERVIEWS

5.1. Introduction

Task-based interviews were set up to answer primarily the third research question, which asks, “In what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?” Put in another way it asks about the transfer, as described in chapter 2, of mathematical practices to IT.

This chapter first analyses the tasks in terms of the mathematical practices that might be needed to do the IT tasks given to the students. (Recall that the students had been originally taught to ask each other questions focusing on the mathematical proficiency strands in the Foundation Programme mathematics class (cf 1.4.1.)). Thereafter, the practices that the students actually used, (described by the terms, ‘using procedures flexibly’, ‘understanding/explaining concepts’, ‘using representations’, ‘questioning’, ‘justifying’, ‘disagreeing’, ‘strategising’, and ‘generalising’) to complete the tasks successfully are presented (Tables 5.1. to 5.4.). The practices that the students used were then categorised into the practices of representation, justification and generalisation (Table 5.5.) The results showed that the overarching practices (cf 2.2.2.) of *representation*, *justification* and *generalisation*, as defined by the RAND mathematics study Panel (Ball, 2003) were used by the students in the task-based interviews, to a greater or lesser degree. It was found that the students did use the mathematical practices that they had developed during the Foundation programme, but that in doing so, used the practices in conjunction with other content knowledge and practices from other domains and life experiences, in order to solve the problems given in the IT tasks. Therefore, from a cognitive perspective (cf 2.5.4), varying degrees of cognitive re-structuring were required to solve the novel problems. From a situative perspective (cf 2.5.5.), the students had to recognise the similarities between the prior learning and new problem contexts. In addition, they needed to recognise and select from other contexts what was useable in each new problem context. This finding is consistent with the findings of other cognitive and situative transfer research (cf 2.5.4 – 2.5.7.).

5.2. Mathematical Practices: an overview

To reiterate, mathematical *proficiency* is what people *show* when they are proficient users of mathematics. *Practices* are what proficient people *do* when they are engaged with mathematical tasks. According to the RAND Mathematics Study Panel (Ball, 2003) mathematical practices are

> *"the mathematical activities in which mathematically proficient people engage as they structure and accomplish mathematical tasks. This focus on practices calls attention to aspects of mathematical proficiency that are often left implicit in instruction, going beyond specific knowledge and skills to include the habits, tools, dispositions, and routines that support competent mathematical activity"* (p.11).

Mathematical practices are "the way in which people approach, think about, and work with mathematical tools and ideas" (p.32). Examples of such practices include "mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas, and recognizing patterns" (p. 32). I described these practices in detail as aspects of representation, justification, and generalisation (cf. 2.2.2.); and argued that they are interdependent. I also showed that practices were defined by the RAND Mathematics Study Panel (Ball, 2003), in order to frame mathematical proficiency, which, in the Panel's opinion, needed to be more clearly articulated (cf 1.3.1.). Therefore, mathematical practices and proficiency are inextricably linked.

Both mathematical practices and proficiency are mathematics specific; but their possible generic application over other domains requiring mathematics, such as IT, allow for their identification to be possible in these non-mathematics domains. However, although the context of the more technical side of IT is domain-specific, the authority with which mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas, and recognizing patterns is used is strictly mathematical (cf 2.2.3.). As such, the nature of use of these practices remains mathematical.

Although the students might incorporate mathematical practices in their approaches to solving the IT tasks, my discussion of transfer suggests that the practices would not be used in their originally-learned forms, but would be altered by their users in order to be rendered useful for solving the IT problems – hence the possible interpretation of a generic nature of these practices (cf 2.2.3.). Students would need to use other knowledge and experiences in

conjunction with the practices, in complex combinations, so that the tasks could be solved. This would make it sometimes difficult to immediately identify the mathematical practices used.

This idea is viable from both cognitive and situative perspectives. From a cognitive perspective the significance of restructuring knowledge into more general structures is that such knowledge may be used by the individual for far transfer (Perkins & Salomon, 1989) because it is no longer new and usable in only one kind of problem. Transfer theorists have discussed in depth individuals' needs to familiarise themselves with new knowledge so that it may be used over an increasingly wide set of problem situations (e.g. Alexander & Murphy, 1999). From a situated perspective no two situations or contexts are the same (Lave, 1988) and therefore people solve problems by drawing from whatever similar experiences they have had in order to come up with a solution. In Lave's opinion 'generalised' knowledge is less likely to be used to solve a problem because it is not linked with similar experiences and is therefore less accessible. These are two very contrasting ways of understanding how mathematical practices might be changed in order to solve IT tasks. I explain in the second part of the chapter how the students' solving of the tasks can be explained from both perspectives. In the first part of this chapter I discuss the mathematical practices that are embedded in the tasks and the mathematical practices the students used to do the tasks.

5.3. Task Analysis

In order to identify the practices used by the students to do the IT tasks, the tasks were first analysed in terms of the mathematical practices that were needed to do the tasks. The tasks dealt with diverse areas of computer technology as a whole and also supported diverse solution processes. The first was concerned with how important security is when managing any computer network; specifically with the design of a "lock out" setup when there are too many incorrect login attempts by a particular network user. The second dealt with the design of a simple banking system, including security issues, what the system could do and who could bank in the system. The third looked at the internal workings of a computer at the circuit level – how the components of a circuit could be switched on and off in order to switch the entire circuit on or off. The fourth was an exercise at the programming level to investigate

how a programme could be written to list all of the prime numbers from 1 to 100. These tasks have been named ‘The Network Task’, ‘The Banking System Task’, ‘The Computer Systems Task’ and ‘The Prime Numbers Task’ respectively, for the remainder of this report. How mathematical practices might be used in order to solve these IT problems is indicated in my analysis of the tasks below, in terms of the mathematical practices I expected to observe in the task solutions. More detailed information, including worked solutions and detailed task background and discussion is found in Appendix E.

5.3.1. “Using Procedures Flexibly”

Procedural fluency in computing largely entails knowing how to use the language-specific ‘syntax’ (see Appendix E for an explanation of ‘syntax’) to write a programme that has been designed to perform certain functions. Thus, I did not expect mathematical procedures and algorithms to feature prominently in the IT tasks because they are specific to mathematics, and unlikely to be used often in computing. However, certain mathematical procedures are embedded in two of the tasks.

5.3.1.1. The computer systems task

The first part of the computer systems task dealt with all the possible simultaneous combinations of ‘on’ or ‘off’ of three circuits. The number of possible combinations may be calculated using probability theory, obtaining an answer of 2^3, or 8 possible combinations. For the IT students who had done mathematics in the Foundation Programme, this could have been done by trial and error, by listing all the combinations, by using a probability tree to represent the situation and resulting sample space, or by applying the binomial distribution formula. When the students did the task they listed the combinations of on or off procedurally, without any explanation of the way that eight possible combinations for three switches could be determined using probability theory. Therefore, although listing the combinations of “on” or “off” of three circuits would have been considered as requiring conceptual understanding as it was taught in mathematics, it was done as a procedure.

For completion of the second part of the computer systems task the procedures of factorising, using the distributive law and use of identities for simplification were needed (see Appendix E); although when the students learned this IT subject they were given IT-specific names for these procedures.

5.3.1.2. The prime numbers task

The procedural aspect of writing the programme to list the prime numbers from 1 to 100 entails using the mathematical definition of a prime number to write a 'rule' or 'definition' in Java-specific syntax; so that when the programme runs, it can 'test' each number that it sees for its 'primeness'. Therefore, mathematically, the student has to know the definition of a prime number. From an IT perspective, the student has to know how programming syntax may be used correctly so that the Java language can compile the syntax to run the programme. The programmer has to write a programme that will loop through all the numbers between 1 and 100, and ask the programme to test each number for primeness, using the test or rule that was previously defined. The procedures and concepts of the task are difficult to separate: one cannot be done correctly without the other.

Writing a Java procedure for this task embodies mathematical conceptual components, which, if not considered by the students when inserting their rule/definition into the syntax of the programme, will result in the inadequate running of the programme. For example, the final list may not contain *all* of the prime numbers from 1 – 100, or it might contain some numbers which are not prime. In other words, what one does with the definition of a prime number will determine whether or not a programme will be written that fulfils the specifications of the task. Therefore, for the definition, "a prime number is a number that is divisible only by itself and one", flexible use of this definition, linked with conceptual understanding is strongly integrated with the programming syntax to be used. It is important that when planning their programme, the students question what a prime number *is*, as well as what it *is not*. Therefore, the algorithm chosen to test for prime numbers will necessarily incorporate how to test each successive number from 1 to 100, how to test if the number is prime or not and what kind of programming structures should be used that will do the job most efficiently.

5.3.1.3. The networking and banking system tasks

The networking and banking system tasks did not require flexible choice or use of any mathematical procedures; excepting for the programming into the banking system at a much later stage formulae that calculate interest accrued on the accounts. However, specifying and using the mathematical formulae was not part of the basic planning of the banking system and will therefore not be discussed.

5.3.2. "Understanding/Explaining Concepts"

Much of the conceptual understanding needed to complete the tasks was IT domain-specific. However, some specific mathematical conceptual understanding was needed by the students in order to complete the computer systems and prime numbers tasks. I suggest that if certain specific fundamental mathematical concepts had not been organised into a coherent whole by the student in the mathematics class, it is unlikely they would be recalled in appropriate ways in the new context of IT. In chapter 2 (cf. 2.5.4.) I described how transfer is understood by cognitive theorists to primarily entail the use of pre-defined units of knowledge in new problem situations in daily life or the workplace (Anderson et al., 1996; Carraher & Schliemann, 2002; Greeno, 1997; Lobato, 2006). Those who have new and unfamiliar knowledge are less likely to transfer that understanding to unfamiliar contexts than those who have '"extensive length and breadth of subject content knowledge" (Alexander & Murphy, 1999 p. 566), leading to depth of processing information, strategic use of that knowledge and generalisation of that knowledge, as demonstrated through analogical reasoning' (Alexander & Murphy, 1999; Simons, 1999) (cf. 2.5.6.). I suggest that this "extensive length and breadth of subject knowledge" is being able to explain the concepts involved; and use them in unfamiliar tasks requires the students to have familiarity with the prior knowledge as well as the new situation, in order to see similarities between the two contexts (Alexander & Murphy, 1999; Anderson et al., 1996). From a situative perspective, recognising similarities between situations requires the student to have deep conceptual understanding of the prior situation – allowing for increased flexibility (Boaler, 2000a) in their knowledge, to make it useful in other situations.

5.3.2.1. The computer systems task

The computer systems task was an example of what students learned in their introduction to computer circuit design. Designing circuits and simplifying other designs to their simplest form is important because more complicated circuits give off more heat, which is undesirable and an ongoing problem in computer hardware design. More background about the task is given in Appendix E. The task required the students to list the possible combinations of three circuits either switched on or off before the remainder of the task could be completed. The rest of the task was to simplify the components of a digital circuit using Boolean logic.

Procedures, definitions and formulae are meaningless without accompanying conceptual understanding, because they become isolated. Conceptually understanding the binomial situation of 'ON' or 'OFF', and using a probability tree to help the students represent and visualise it to solve the problem, would have been helpful for successfully completing this task, although it could have been done procedurally, and without understanding. Also required for the task were the understanding, interpretation and use of the mathematical symbols of set theory, $\cup$ and $\cap$; as well as X and its complement $\left(X \text{ and } X'\left(\overline{X} \text{ or "not } X\text{"}\right)\right)$. These symbols also exist in Boolean algebraic identities and carry similar conceptual meaning. The students needed to have sufficient conceptual understanding of the Boolean identities, which had been more recently learned that year, to manipulate and rearrange them if necessary to simplify the Boolean expression. Correct manipulation of the identities required understanding of combinations of X, Y, Z, $\overline{X}, \overline{Y}$ *and* $\overline{Z}$, with the symbols $\cup$ and $\cap$.

The mathematics of this task was well within the scope of the students' conceptual understanding, and should have been easy for those who had studied mathematics in the Foundation Programme. My focus was to determine the extent to which the students could use their understanding of these particular mathematical concepts to decide how and where their knowledge could be used in the computer systems task. I expected the students to be able to make the link between what they did in probability theory in the Foundation Programme and Boolean algebra in the computer systems course – so indicating use of their conceptual understanding in another domain.

The second part of the task required the students to conceptually understand how Boolean identities function, as well as how they can be manipulated, to simplify circuits. The identities in the text book were not written in all the possible ways in which they could be used, and a couple of the identities needed for simplifying the Boolean expression in the task required the rearranging – sometimes in a fairly sophisticated way – using understanding of Boolean logic. Therefore, the task required not only the choice of the appropriate identity, but also the understanding of how it may be used in an unfamiliar situation. In order to use them strategically and efficiently, the students had to understand the components of the identities, what they all mean, how they relate to each other, and the ways in which they can be substituted into a Boolean expression to simplify the expression.

5.3.2.2. The prime numbers task

For the prime numbers task it was very important that students understood the implications of a prime number being divisible by only 1 and itself. Therefore, in their programme they would have to apply this conceptual understanding by specifying in their programme that a 'listable' number does not have any other factors excepting for itself and 1; or that a number should *not* be listed if it is divisible by any number other than itself and 1. These two ways of saying the same thing would use differently-written definitions/rules that the programme would have to use in its test for primeness.

Computing students are usually aware that they can also often write the programme to solve such a problem by finding a formula that will calculate the next required number from the previous by using a formula. For example, if the question asked them to print all the odd numbers between 1 and 100, the best way to write the programme would be to start at the first odd number (1) and then tell the programme to add two to the previous number to calculate the next odd number. Thus they would write a loop that would repeat this formula until the nearest odd number equal to, or less than 100 is reached. In the prime numbers task the students had to understand that for prime numbers there is no formula that can be written to calculate the next prime number following the previous, and that they would have to devise another strategy. Note that strategic thinking is therefore also necessary to do this task; and I will explain the strategic thinking required for the task in the next section.

Conceptual understanding in the prime numbers task integrates understanding the mathematical definition of a prime number with understanding the different ways in which programming syntax can be used to solve the problem. Associated with this understanding is choosing the best way to use syntax to produce a programme which does what it is supposed to do.

5.3.2.3. The networking and banking system tasks

Mathematical conceptual understanding is not embedded in these two tasks. The only conceptual understanding required to do these tasks is discipline-specific. This immediately calls for a question to be asked: Does the lack of mathematical conceptual understanding and procedures mean that mathematics is not involved in these tasks? A behaviourist might say so, because transfer across settings or contexts entails the principle of 'identical elements' (Carraher & Schliemann, 2002; Greeno, 2006). I argue that this is probably not the case at all. Mathematics is not comprised of only procedures and conceptual understanding, although many individuals might understand mathematics in this way. Kilpatrick et al (2001) argued that mathematical proficiency *includes* such elements; but that in a much more complex manner, it is comprised of the interdependency of all five strands of mathematics. Therefore, just because the networking and banking system tasks do not obviously entail mathematical procedures and conceptual understanding, it does not mean that mathematical practices are not embedded or used in these areas of IT. In section 5.4. I show how different mathematical practices have indeed been used in all four IT tasks.

5.3.3. "Strategising and Using Representations"

The reason why strategic competence as a component of mathematical proficiency is so important is because

> *"[students] encounter situations in which part of the difficulty is to figure out exactly what the problem is. Then they need to formulate the problem so that they can use mathematics to solve it."* (Kilpatrick et al., 2001 p. 124).

Commonly referred to in mathematics as "problem-solving", this strand deals with the strategies[7] one uses to analyse a problem situation, represent it in some way, and then design a way to solve the problem. Problem-solving has been the subject of much study, because the possible use of problem-solving abilities in contexts other than the contexts in which they are most commonly-used, such as mathematics, is desired by future employees and teachers alike.

I surmised that recognising use of mathematical problem-solving to IT problem-solving would be difficult, because the strategies to be used to solve problems are not predictable. Different people can solve a particular problem in different ways. Transferring problem-solving strategies to another domain would be even more difficult to identify, because we would be outside the domain of mathematics. Would problem-solving skills used and observed in mathematics be evident in IT? I found Perkins and Salomon's (1989) discussion of use or transfer of problem-solving skills useful in this respect. Their discussion of use of problem-solving strategies in novel situations revolved around the hypothetical extent of a master chess player's ability to successfully strategise a battle plan. There were no similar elements between a chess *game* and a real-life battle *plan*. However, it was believed by many early cognitive theorists that the chess player should have strategising skills to competently produce a battle plan.

Perkins and Salomon (1989) use elements of both cognitive and situative understandings of transfer when they discuss generalised and context-dependent strategies. They concluded that problem-solving strategies could be both general and context-dependent. Where general problem-solving heuristics had been deliberately cultivated in students, the students showed increased abilities to use those learned heuristics in new situations. However, people were also seen to draw from more than learned heuristics, to solve problems in markedly different problem situations. This deliberation of the different levels of problem-solving abilities from familiar to unfamiliar contexts led to their explanation of a "low road" and a "high road" to transfer (Perkins & Salomon, 1989; Salomon & Perkins, 1989). Elaborating,

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[7] The practice of 'strategising' has been described in a few different ways (cf 2.2.1.); but the terms used all describe the tactical planning that one needs to do to solve a problem. Recall that the term "strategising" is the one used for the analysis of what students did when they solved the IT tasks (cf 5.4.)

> *"[the] low road" to transfer, depends on extensive and varied practice of a skill to near automaticity […]. A skill so practiced in a large variety of instances becomes applied to perceptually similar situations by way of response or stimulus generalization. For example, having driven different cars under a variety of conditions allows us to shift to driving a truck fairly easily"* (Perkins & Salomon, 1989 p. 22).

Conversely, a high road to transfer may be described as follows:

> *"People sometimes abstract principles in advance, keeping them in mind in anticipation of appropriate opportunities for application, or, in a new situation, reach back to prior experiences and abstract from them principles that might be relevant…[T]he expert chess player mobilized to save his country in our opening story would be expected to mine the context of chess for chess-bound principles such as 'get hold of the board's center,' decontextualize them, and apply them in forms like 'let's capture or destroy the enemy's command centers'"* (Perkins & Salomon, 1989 p. 22).

The notions of low road transfer; that is, "automatatising practices skills" (ibid. p. 24), and high road transfer; that is, "mindfully decontextualising principles" (ibid. p. 24), or using "deliberate mindful abstraction of a principle" (ibid. p. 22); are useful in understanding how mathematical problem-solving strategies can be used in IT. Firstly, some of the IT tasks in this study required low road transfer; which would be evidenced by students using strategies and heuristics practiced in my mathematics class, or prior to that, to solve a "perceptually similar" (p. 22) IT problem. In these cases use of mathematical strategising would be identifiable, because the mathematics would still be present. High road transfer of problem-solving strategies is much less likely to be recognised as mathematical, because it would be abstracted from mathematics and applied to the new problem in any way necessary to solve the problem. This kind of strategising in IT will be more a way of thinking than strategically using *mathematics*. For example, if a student was also a competent chess player, or played online strategic computer games, these experiences might significantly act with or without mathematical competence to influence his or her ability to problem-solve in IT.

I also suggest that the problem-solving required in some of the IT tasks would be manifested differently, because different people problem-solve in different ways, depending on their personal understanding and organisation of the concepts they are applying to solve the problem; and also depending on the experiences and input of the peers they may be working with.

Kilpatrick et al described how strategic thinking is implemented in problem solving. Their description of solving unfamiliar problems is remarkably similar to those of a low and high road to transfer. For low road transfer,

> *"… [students] are likely to need **experience and practice** in problem formulating as well as in problem solving."* (Kilpatrick et al., 2001 p. 124) (emphasis mine).

For high road transfer,

> *"A fundamental characteristic needed throughout the problem-solving process is flexibility. **Flexibility develops through the broadening of knowledge required for solving non-routine problems** rather than just routine problems …"* (Kilpatrick et al., 2001 p. 126) (emphasis mine).

In terms of the IT domain, the computer systems and networking tasks given in these interviews were routine, and the prime numbers and banking systems tasks were non-routine. However, with respect to using mathematical practices, they were all non-routine, because they existed outside of the mathematics domain. In addition, proficient strategic competence may be demonstrated in both the banking systems and computer systems tasks; although the banking system task was entirely non-numeric and written as a word problem, and the computer systems task was almost entirely symbolic. This is partly why it is complicated to define the strategising that may be transferred from the mathematics to the IT domain.

The components of strategic competence include the practices of problem understanding and formulation, representation, choosing solution strategies, problem-solving and flexible thinking. Concerning strategic competence in general, I have chosen to use the term "strategising" when describing what some of the students were doing at the time. "Strategising" is what I observed many of the respondents doing, and that term seems to be a good description of what was taking place. In my detailed discussion about the practices observed in the task-based interviews, I explain more specifically what I mean when I see them strategising, when necessary. Strategic competence has closely associated with it the essential practice of representation.

Representation, a practice identified by both the RAND Mathematics Study Panel (Ball, 2003), and Kilpatrick et al (2001), is key to problem-solving. Being able to *make* a representation may require conceptual understanding; but *using* it to help solve the problem falls into the realm of strategic competence (Kilpatrick et al., 2001). Kilpatrick et al describe

in detail the role of representation for the strategically competent student. It is interesting to observe that representation of a problem may be shown in a variety of ways.

> *" … the student's first step in solving [the problem] is to represent it mathematically in some fashion, whether numerically, symbolically, verbally, or graphically … Representing a problem situation requires, first, that the student build a mental image of its essential components …"* (Kilpatrick et al., 2001 p. 124).

Much of the representation in IT is done by means of software, designed specifically for the user to represent a system to ascertain if it works before writing the necessary programmes to make it function. Alternatively, representation may be written simply in the form of mind maps, flow charts or tree diagrams, or even less formally, for example, as a system of arrows linking ideas or a group of pictures representing aspects of the larger system.

It is important to point out that in computing, like mathematics, there will be levels of proficiency observed in individuals. One cannot assume, just because a problem has not been represented visually, that problem-solving skills are lacking, or that an individual's strategic competence is limited in some way. Representation may be mental, which may indicate better strategic competence:

> *"In contrast, **a more proficient approach** is to construct a problem model— that is, a mental model of the situation described in the problem. A problem model is not a visual picture per se; rather, it is any form of mental representation that maintains the structural relations among the variables in the problem"* (Kilpatrick et al., 2001 p. 124) (emphasis mine).

During my analysis of the task-based interviews with the computing students I looked for evidence of mental and written representation. Mental representation might be indicated by verbal descriptions of the problem, and/or possible solution strategies discussed with a peer.

I commented previously that João and Alain[8] were likely to produce better worked solutions for the tasks than the respondents were, because they had had more experience dealing with solving problems in these areas of IT. (c.f. 3.4.). Therefore, while their solutions were not the only way to solve the problems, nor necessarily the best way, their knowledge and experience made it possible for me to predict and understand some of the solution strategies that might have been used by the students. They shared the task of producing model solutions to the four tasks, some of which are shown, and others discussed in terms of their key ideas in appendix E. Each of these solutions is only one way of solving the tasks, and I did not expect the students to reproduce them. I did, however, expect to observe the students using "flexible thinking" (Kilpatrick et al., 2001) in order to do the tasks. The strategising I expected to observe in the tasks is described below:

5.3.3.1. The computer systems task

The implementation of strategic practices becomes important in the computer systems task at the point where Boolean identities must be used to simplify a Boolean expression that defines the design of a digital circuit. Designing a circuit with multiple switches requires the combination of switches/components to be in its simplest possible form, with the smallest number of components possible – a large number of components means that the size of the processor chip to be used will have to be bigger, which will increase the heating factor[9] of the chip during use.

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[8] Recall that Joao and Alain helped me in my task-based interviews. They had been my students and were in their final undergraduate year at the time of the interviews. Because they were by then subject specialists, they were valuable to me in interpreting the tasks, giving me a good idea of the practices that would be needed to do the tasks, drawing up model solutions, and interpreting students' comments and probing them further during the interviews.

[9] It is well-known that control of heating with respect to computer processors is of primary concern to computer system designers (e.g. Gomaa, Powell, & Vijaykumar, 2004).

Understanding the problem in this case involves how the student is able to recognise that the given expression is not in its simplest form. Knowing what the Boolean identities are, and understanding how they may be manipulated to facilitate their implementation in the solution is necessary for being able to subsequently choose which identity may be used at any particular time to simplify the expression.

5.3.3.2. The prime numbers task

Different kinds of strategic thinking are needed for this task. Firstly, closely tied to the need for the students to consider that a prime number is divisible only by itself and one is the ability to use this understanding strategically to include in their programme what makes a number *not prime*. This consideration would demonstrate their understanding of the problem, as well as initiate formulation of a programming strategy. Using representation to aid their thinking process may take the form of a list or flow diagram, or be mental. Secondly, decisions must be made with respect to the design of the actual programme layout. Refer to Appendix E, where I have discussed a possible programming strategy in more detail. Use of a "loop" would be the most effective way to write the programme; but depending on how the loop is implemented, different "syntax" or programming language would be used. Decisions made early on in the solution process would have ramifications throughout the rest of the programme. For example, if an 'if – then' statement is used, then the syntax to be used for the rest of the programme must be compatible with the 'if – then' statement. In computing this kind of decision is termed a 'selection construct' or 'decision construct', which also shows that it is within the realm of strategic thinking and decision-making. The objective of this task was to determine if, and to what extent, the students were using strategic thinking, in both their programming and in the content being used in the programme. They would need to work flexibly with earlier decisions made, and follow through with further related decisions in order to write a functioning programme.

Representation takes the form of Java-specific symbols and syntax. It is possible to plan the programme without the use of these symbols and syntax, but it would not be possible to write the programme without using them. Nevertheless, it would be useful, but not essential to use Java syntax during the planning stages. Syntax and symbols that should be used might include

"if", "do while" and "run" statements, and definitions of the programme variables that would be used, such as "integer", or "number". In addition, the use of representations such as flow charts or mind maps would be beneficial to the planning of such a programme.

5.3.3.3. The networking task

The banking system and networking tasks did not involve mathematical content, but required an ability to make strategic decisions. Therefore, the presence of mathematics would be obscure, although the practice of strategic thinking should have been evident when the task was attempted. The content and conceptual understanding in the networking task were new for the students. They had only started the course in the same semester as the tasks were given, and they had not done a similar course prior to this. Therefore, I did not expect them to demonstrate a variety of strategic practices, because they were still acclimating themselves in terms of content (Alexander & Murphy, 1999). Furthermore, it has been shown that students need to have a sense of purpose of use of prior knowledge in the "situations in which the knowledge can and should be transferred" (Alexander & Murphy, 1999 p. 563). From this, I suggest that they should have a certain amount of familiarity with a new context before they can recognise what prior learning may be applicable to solve problems related to it. The task difficulty was actually of a low level (refer to Appendix E), and the question required a solution approach that would have been knowledge recall and procedural for a student who was more familiar with the subject matter. However, because it was new for the students in the study, I understood it to need a certain amount of strategic thinking at the time the interviews took place. The strategising in this task entailed using the text book to aid the systematic interpretation and understanding of what each layer of a network is responsible for; understanding its role, and then making a decision about its role in network security and lockout. The text book explained and summarised all of these aspects of a network, but the task required the student to think about how the layers worked together as a whole in order to provide all aspects of the network to the system, including security. Once the role of each layer was understood, the layer(s) responsible for encryption and decryption could be identified (a more detailed description of the typical kind of thought process needed to answer this question is seen in Appendix E).

Using representation in this task is more at a conceptual level, where the names of each layer are a representation of what the layer is responsible for in the network. The strategic competence needed in this task does not require other kinds of representation, but does require conceptual understanding of the layers in a network, as well as argumentative reasoning, which will be detailed in the next section (cf. 5.4.5.).

5.3.3.4. The banking system task

The banking system task was extremely complicated. The students could have spent much longer on the task than the sixty minutes that they were given. Decision-making at many different levels was required of the student. The task required the student to be able to judge possible ramifications of a particular decision before going through with it, and accordingly accept or reject that decision; and then hypothesise new possibilities. Decisions made at the outset would significantly influence those made later.

While doing the task later realisations that an earlier decision was not the best that could have been made may also have necessitated alteration of those decisions. Alternatively, additions could be made to the design if an element had been omitted; but with the possible need to re-start the design from the beginning. The task therefore incorporated decision-making at multiple levels, together with the need for students to take into consideration other important aspects while trying to make strategic decisions about the best design to use. These other aspects included the kinds of accounts a person could open; the personal details that might be required by the bank; the different kinds of interest that could be calculated for a bank account; or the ways in which the account holders were added to the database of account holders. In Appendix E I have shown as best as I can as a non-IT specialist writing to non-IT specialists how strategic thinking and argumentation at multiple levels influence future and past decisions; how multiple strategies, decisions and a high level of flexibility are needed to design a simple banking system, and how a possible solution for the task might be reached.

Furthermore, a solution would ideally include a representation of the banking system as a whole, indicating how, for example, information about the customer, the kinds of accounts, how data is stored, and how interest is calculated, are linked to each other and how they may

be "seen and called" by the programme. The representation used and the decisions made during task progression also inform each other. That is, using representation should help the students to fine-tune their thinking; but in attempting to make a representation of the system, students first have to determine the kinds of elements that will be in their system. For example, a decision to sort different attributes of the system into classes may lead to an initial visual representation using a particular combination of class diagrams; which in turn may show that certain aspects of the big picture are missing. This would subsequently lead to the alteration of the decisions initially made. In this example, a visual representation is valuable to the system designer(s) because it shows in pictorial form where the design may need re-consideration.

5.3.4. "Questioning, Justifying, Disagreeing and Generalising"

Kilpatrick et al (2001) describe the strand of adaptive reasoning as,

> "*… the capacity to think logically about the* ***relationships among concepts and situations****. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to* ***justify*** *the conclusions … Our notion of adaptive reasoning is much broader [than formal proof and other forms of deductive* ***reasoning****], including not only informal explanation and justification but also intuitive and* ***inductive reasoning*** *based on* ***pattern, analogy, and metaphor****"*. (Kilpatrick et al., 2001 p. 129) (emphasis mine).

The strand of adaptive reasoning includes "pattern, analogy and metaphor" (generalisation) and "reasoning" (justification) in one strand, and is the reason why I discuss them under a single heading. Generalisation and justification are considered as two distinct practices by the RAND Mathematics Study Panel (Ball, 2003). According to the Panel, 'questioning, justifying and disagreeing' are practices linked with the overarching practice of 'justification', whereas 'generalising' is a practice in itself.

The Rand Mathematics Study Panel (Ball, 2003) describes justification as being able to make "**articulated** and **reasoned** claims [and participate in] **rationally negotiated** disagreement" (p. 32). All of the words in this description are important because they describe the different but essential components of justification, and I have highlighted some to bring attention to their significance. Reasoning is more effective if it is well-articulated and rational. The

‘negotiation’ that takes place during argumentation brings meaning to the discussion between two or more people. ‘Meaning’ is described by Wenger (1998) as a crucial component of learning. ‘Disagreement’ encourages people in a mathematical argument to call for justification from each other in order to obtain clarification of claims or suggestions. Use of these practices in other domains would therefore be highly valuable to the student.

I first discuss the components of justification as I use them in my analysis. I found it easier to use the terms, ‘questioning, justifying and disagreeing’ in my description, because of the ways in which the students did the tasks; before grouping them under the practice of justification. Thereafter, I discuss the practice of “generalising”. Similarly to “strategising”, I suggest that mathematical use of ‘questioning, justifying and disagreeing’ in IT is not always easy to identify. Making claims and justifying them will be domain-specific, because the justification for making those claims will use conceptual reasons from that domain. Although reasoning with justification must be used in IT, can we know that it was a mathematical skill used in the IT domain? Or is it a mixture of practices from various domains that have been implemented because they are useful to doing these kinds of IT tasks (far transfer: Perkins & Salomon, 1989)? Can we project what mathematical reasoning should look like if or when it is used in another domain?

I found it very difficult to analyse some of the tasks in terms of the mathematical questioning, justifying and disagreeing that might be used. It makes sense that the tasks containing mathematical procedures, concepts and strategies would be easier to analyse in terms of mathematical justification used. They are the prime numbers and computer systems tasks. The banking system and networking tasks, which do not involve these mathematical elements, require the use of reasoning with justification, but the presence of mathematics is obscure. I discuss each of the tasks below in terms of the need for reasoning with justification that may be embedded in the task, but I do not argue that the justification used necessarily only originates in mathematics.

My students had been encouraged to reason using rationally negotiated disagreement while they were studying differential calculus in particular. Before they studied this section in mathematics with me, they had been encouraged to use all of the mathematical practices; but the encouragement had been implicit. When I realised that some students were beginning to use practices, such as reasoning, disagreement requiring justification and strategic thinking

more frequently, but others were not, I began to encourage them more explicitly to focus specifically on the different strands of proficiency. When they started learning differential calculus I explained to them what the strands of proficiency entailed and why their knowing about the strands should improve their mathematical ability. I showed them how to ask questions of the other members of their groups, so as to purposefully focus their attention on the different strands (cf. 1.5.1.). For example, they could use a question such as '*why do you want to do it like that?'* to draw justification from a peer of a solution to a problem they were given. Alternatively, a question such as '*what is actually the problem here?*' would be framed to the whole group to focus everybody's thinking on problem identification, so that solution strategies could be initiated. Therefore, questioning each other in order to learn became a focus of doing mathematics during this time.

Being able to formulate a question served two important learning goals for the students: One was that in order to formulate the question, a student needed to think carefully about what was taking place in order to frame the question in such a way as to be understandable by those in the group. In so doing, the student would have to focus attention on the specifics of the problem. The other was to press the other group members to clarify and explain their own thinking, so as to be convincing – whether it was conceptual and/or strategic and/or demonstrating reason/justification. I suggest that although 'questioning' is not a mathematical practice described by the RAND Mathematics Study Panel (Ball, 2003), it may be a significant practice that teachers and students should be encouraged to work on, because it frames students' thinking in terms of the different aspects of mathematical proficiency. I looked for evidence of questioning during the task based interviews. Wherever I observed questioning being used I interpreted it as part of the practice of justification, as it most closely associates with 'reasoning' or 'rationally negotiated disagreement', encouraging students to think about how best to frame an answer to a question. Where students were specifically disagreeing with each other I specified the practice that they were using as 'disagreeing'. 'Disagreeing' did not necessarily have justification associated. This would imply that one person would be requiring justification from the other, but was not experiencing an attempt to be convinced by the other person.

The inclusion of "inductive reasoning based on pattern, analogy and metaphor" (Kilpatrick et al., 2001 p. 129) may be understood as part of the practice of generalisation. Concerning generalisation, it is worth investigating if generalised mathematical knowledge can be used in

another domain. I do not use the term ‘generalised’ in the transfer sense, where some cognitivist theorists argue that learning has to be generalised, or de-contextualised before it can be transferred. I refer to ‘generalisation’ as a mathematical practice: “*Generalization involves searching for patterns, structures, and relationships in data or mathematical symbols*” (Ball, 2003 p. 38). One may argue that similarities exist between the two meanings of the word, and I agree; but the difference between them is that the latter does not entail ‘decontextualisation’, but rather is used to describe a specific practice mathematically proficient people do when they are working with patterns. If used in the latter sense, I propose that rules generated while working with mathematical patterns may be useable in other domains, such as IT. If used in the former sense I propose that mathematical patterns may be generalised or abstracted into non-contextual rules or principles, for use in other contexts.

If the problem-solving process is only internal and not verbal, one cannot observe the strategising, decision-making and deliberation with or without justification taking place. One may only observe the outcome of such processes. For example, when a person thinks through a problem on his or her own, s/he might go through a number of repeated cycles of personal suggestion and rejection with or without conscious justification of either. This makes it difficult for a researcher to analyse. Although this process could be understood as part of strategising I discuss it here. When a person is thinking strategically, other practices are being used at the same time. For example, considering and rejecting strategies incorporates reasoning, which in turn incorporates justification of why a strategy should be considered further or rejected. I suggest that personal reasoning would use similar processes.

The initial planning of a system requires a choice of and subsequent decision about the design to be used, which will in turn require justification. Choice and justification thereof will require proficient conceptual understanding of the requirements of the task, and what the best way will be to do it – particularly because such tasks are usually done in teams, and there will naturally be much argument as to *how* to tackle the problems. I believe that strategic competence and adaptive reasoning are strongly represented in the tasks used for my interviews.

I have on numerous occasions in the past discussed with different IT lecturers why they think that mathematics is necessary for learning IT. Some said it is not necessary. The common response from those who have mathematical backgrounds was that being able to do many of

the IT units on offer at university requires them to think strategically and be able to think in abstraction. While this is a vague statement, I have explained that strategic thinking is related to making "articulated and reasoned claims" (Ball, 2003). I suggest that abstraction is related to generalisation – both of which have been identified by the RAND Mathematics Panel (ibid.) as mathematical practices. Therefore, it can be argued that mathematics is necessary for learning IT.

5.3.4.1. The computer systems task

The first part of the computer systems task consisted of determining how many combinations of three switches would turn a circuit on or off. I expected to observe little disagreement with respect to how this could be determined because choice was not involved in the process, although even recognising whether choice is involved or not requires some reasoning with and without justification. In the second part of this task I expected to observe claims with justification about how and when different identities could be used in simplifying the expression.

Using Boolean identities required working with abstraction and generalisation, in that the identities themselves are generalisations which need to be understood and used flexibly to simplify given expressions. Manipulating the identities required staying in the realm of abstraction in order to present the identity in a different form which would make it more useable for simplifying the expression. Simplification by substituting a term in the expression with an identity is still staying in the realm of abstraction – at no time during this process will the student use numbers or evaluate the expression. The final answer is given as an abstract set of terms which is translated afterwards to a diagram of a set of components, which represents the computer chip to be used. The whole exercise is maintained at a relatively high level of abstraction.

5.3.4.2. The prime numbers task

This task required multiple levels and types of strategic thinking, which should in turn promote incidences of suggestion with justification, or rejection of suggestions with justification as the students thought about what should be included in the programme and the ordering of programming syntax. The ways in which reasoning and justification could be used in planning the prime numbers programme are outlined in Appendix E. Details are not given, because the programme could be written correctly in many different ways. Questioning, justifying and disagreeing will probably be observed when students decide on the kind of structure the programme will take (for example, a 'do…while' loop, as opposed to a 'repeat until' loop). Which syntax to be used inside the main structure will also need to be argued and justified; as well as how the variable in the programme should be defined. I expected mathematical reasoning in this task to revolve around how to test for prime or not prime, and what would be considered a good test, from a programming perspective. Examples of what could be argued is found in Appendix E.

The task deals with abstraction at different levels. Defining a prime number symbolically requires abstraction of the definition, in that one may define a prime number in words easily, but in order to tell a computer programme how to recognise a prime number, the students must use symbolic notation as a representation of that definition, within the correct syntax of the computer language.

5.3.4.3. The networking task

For the networking task, justification of the decision made as to which network component(s), or layer(s), are involved in security revolves around understanding what all of the layers do, and justifying a claim that a particular layer *is not* involved in security. The explanation in the model solution in Appendix E shows this more clearly. Rejection of a claim that a layer is involved in security requires an explanation of how the layer functions, and what it does not do. Justification here depends on the conceptual understanding of the functions of the layers. No mathematical concepts are needed. Like the strategic thinking required in this task for new students, questioning, disagreeing and justification will be at a low level; because the task will ultimately be found in the realm of knowledge recall when the students have finished the

course, and as such, will not be a topic of much argument and discussion. No generalisation is required for this task.

5.3.4.4. The banking system task

Having already described the complexity of strategies required for planning of a good, efficient design for a banking system, I expect disagreeing, reasoning and justification to be present during the planning. So many different designs are possible and typically, with a team of designers, generally the case in IT projects, different people will come up with different, equally legitimate design strategies. Team members will have to convince others of the worth of their ideas or argue against others' strategies – both requiring justification. Furthermore, I argue that this task will not advance without disagreeing and justification taking place with each decision made, because it is extremely reliant on strategic design with accompanying argumentation prior to any plan being produced. Any justification taking place is domain-specific, and no mathematical concepts are used in this task.

The whole banking system is actually a generalisation, because it is an abstraction of the future working programme, before the programme has been written. Other generalisation required within the task is evident in the decisions that will need to be made (Appendix E) concerning the information the classes will contain, how information will be accessed from where it will be stored, and the elements of the banking system that will apply to all individuals in the system. Generalising the plan entails dealing with any variations in client information, account types, changing interest rates, and requires a great deal of abstraction at all levels of the task solution.

5.3.5. Productive Disposition

> *"If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must* ***believe*** *that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out"* (Kilpatrick et al., 2001 p. 131) (my emphasis).

I draw on Kilpatrick et al's (2001) discussion about productive disposition in mathematics, and suggest that if there is evidence that mathematical practices can be transferred to other non-mathematics contexts, then so too must productive disposition be transferred along with these practices. The reason why I claim this is because of Kilpatrick et al's claim that the strands are fully inter-related and inter-dependent. If one is missing or under-developed, the others will be affected negatively; and if all of procedural fluency, conceptual understanding, strategic competence and adaptive reasoning are present, then it is reasonable to assume that so is productive reasoning.

I will not discuss this strand in much detail in this section. It was possible that doing IT tasks was not conducive to drawing out what students believed about mathematics and their own abilities to do mathematics. However, it was possible, by listening to what they said, that I would get a sense of what they thought about the usefulness of mathematics in other domains. It was not possible for me to predict where I thought productive disposition might be used in the IT tasks because productive disposition is possibly more hidden as an indication of personal beliefs about mathematics – and I therefore waited to see if it was present in any way, and reported it when observed.

I also wondered whether productive disposition, if present, is perhaps functional in facilitating transfer if it is present. In other words, if Kilpatrick et al are correct, that if a student believes that mathematics is understandable and useful, and that this belief positively impacts the other strands, then it is possible that this belief will also positively impact how well the student can use his or her mathematical practices in new contexts. I also suggest that understanding where productive disposition 'fits' into transfer of mathematical practices in this way may aid me in the analysis of transfer from a situative perspective.

Having analysed the tasks in terms of the mathematical practices I expected to observe, I remind the reader that their use would be within the new context of IT, and it is therefore unlikely they would be used in the same way as they would have been to solve mathematical tasks. I suggest this re-contextualisation is necessary for success in using mathematical practices in a new domain. Because the students were told prior to participating in the task-based interviews that I would be looking for the kinds of mathematical thinking they would be using to solve the tasks, the possibility that they were linking what they were doing to mathematics was made more explicit that it would otherwise have been, if they had been

solving the tasks in their ordinary undergraduate coursework. The next section discusses the extent of students' uses of mathematical practices while doing first year undergraduate IT tasks.

5.4. Mathematical Practices Observed in the Task-Based Interviews

5.4.1. An Overview of the Practices Used

Each task was analysed in terms of the number of times the students used specific practices to complete the task. Average frequencies were calculated; but I emphasise that the frequencies provided are only shown to give the reader an idea of which practices were used more or less, and relative to use of the other practices. At no time do I attempt to use this data for quantitative analysis of observations. The results are summarized in Tables 5.1. to 5.4. below. The terms used for the practices listed in Tables 5.1. to 5.4. are my own (cf 1.2.), and are designed from my understanding of proficiency and practices, to aid in describing accurately what I saw the students doing. These terms were then related back to the three overarching practices of representation, justification and generalisation, as shown in chapter 2 (Table 2.1.), so that I could ascertain if transfer of mathematical practices occurred or not, and how.

5.4.1.1. An Overview of the Practices Used by Students in the Networking and Banking System tasks

Since mathematics content was not involved in the banking and networking tasks, the practices observed, shown in Tables 5.1. and 5.2. below, were associated only with the task subject matter. Tables 5.1. and 5.2. show that all of the practices are represented, excepting for representation and generalisation in the networking task which did not require any sort of written representation or "rule" formulation. Table 5.1. shows that the networking task required more explaining concepts than other practices, as expected, because the subject matter was new to the students. Use of the other practices would have been limited while the

students were still grappling with the new concepts. This observation is in line with Alexander and Murphy's (1999) argument that new subject matter requires acclimation before other knowledge may be linked with it.

Table 5.1. The Mathematical Practices Used to Solve the Networking Task

Task 1: Networking	Using procedures flexibly	Understanding/ explaining concepts	Using representation	questioning	Justifying	Disagreeing	Strategising	Generalising
Kabo and Molefhi	–	10	–	5	4	1	2	–
Ravi and Tinashe	1	8	–	–	3	6	1	–
Eric and Tungamirai	Did not do this task due to time constraints							
Mean frequency (to the nearest whole number)	1	9	–	3	4	4	2	–

Table 5.2. shows relatively high incidences of understanding/explaining concepts, representation, justification and strategising. Because none of these were used to do any kind of mathematics, the question may be asked as to the association of these practices with those learned in the Foundation Programme mathematics class. I explain towards the end of the chapter, from a situative perspective, how mathematical practices might be related to IT (cf 5.5.2).

Table 5.2. The Mathematical Practices Used to Solve the Banking System Task

Task 2: Banking System	Using procedures flexibly	Understanding/ explaining concepts	Using representation	Questioning	Justifying	Disagreeing	Strategising	Generalising
Kabo and Molefhi	2	12	10	7	31	13	28	
Ravi and Tinashe	5	13	11	1	8	3	16	–
Eric and Tungamirai	1	12	6	4	13	6	13	4
Mean frequency (to the nearest whole number)	3	13	9	4	17	7	19	1

5.4.1.2. An Overview of the Practices Used by Students in the Computer Systems and Prime Numbers Tasks

In general, the students who did the computer systems and prime numbers tasks used all of the mathematical practices listed. Mathematical content was embedded in the Computer Systems task (cf. 5.3.1.). Refer to Appendix E for an explanation why. Therefore, mathematics was embedded in any discussion that took place, even though it was within the domain of IT. Concerning the prime numbers task, where students were to write a programme to print (list) all the prime numbers between 1 and 100, the programming itself required programming syntax and conceptual understanding which was not mathematical in content. The discussion that ensued between students during this task swung between the mathematics of the task and the syntax of the programme. Therefore, when reporting the practices I observed during the completion of the Prime Numbers Task, I differentiated between those that were programming-related and those that were mathematical. Tables 5.3. and 5.4. below show the mathematical practices used in the Computer Systems and Prime Numbers Tasks respectively. The numbers of incidences that were specifically mathematical in content used in the Prime Numbers Task are shown in bold numbers in brackets after the total number of incidences observed for that practice.

Table 5.3. shows that the students tended to use more understanding/explaining concepts, representation, justifying and strategising in general to complete the Computer Systems Task. Incidences of using procedures, questioning each other, disagreeing and generalisation were observed relatively fewer times. Table 5.4. shows that strategising and justification were used by the students in the Prime Numbers task much more frequently than the other practices; although the practices of understanding/explaining concepts, representation, disagreeing and generalisation were used often during the task.

Table 5.3. The Mathematical Practices Used to Solve the Computer Systems Task

Task 3: Computer Systems	Using procedures flexibly	Understanding/ explaining concepts	Using representation	Questioning	Justifying	Disagreeing	Strategising	Generalising
Rudo and Tungamirai	3	10	4	3	11	5	11	1
Jennifer and Carol	4	6	6	4	5	–	5	–
Ditso and Keabetswe	3	13	6	–	4	–	6	3
Kaone and Yvonne	5	7	6	1	5	1	7	1
Mean frequency (to the nearest whole number)	4	9	6	2	6	2	7	1

Table 5.4. The Mathematical Practices Used to Solve the Prime Numbers Task

Task 4: prime numbers **(specifically mathematical practices in bold – others are programming specific)**	Using procedures flexibly	Understanding/explaining concepts	Using representation	Questioning	Justifying	Disagreeing	Strategising	Generalising
Rudo and Tungamirai	1**(1)**	9**(6)**	4**(3)**	1**(0)**	16**(10)**	9**(4)**	24**(17)**	3**(2)**
Jennifer and Carol	4**(0)**	7**(5)**	7**(7)**	2**(1)**	9**(6)**	6**(4)**	22**(11)**	7**(7)**
Ditso and Keabetswe	1**(1)**	9**(5)**	5**(4)**	4**(3)**	12**(6)**	2**(1)**	16**(12)**	7**(5)**
Kaone and Yvonne	1**(1)**	4**(4)**	8**(8)**	2**(2)**	7**(6)**	5**(4)**	12**(7)**	5**(5)**
Mean frequency (to the nearest whole number)	2**(1)**	7**(5)**	6**(6)**	2**(2)**	11**(7)**	6**(3)**	19**(12)**	6**(5)**

Tables 5.1. to 5.4. provide an overview of the practices that I observed the students using while they were doing the tasks. The question, however, was whether or not there was evidence of use of mathematical practices developed in the Foundation Programme. The following section relates the mathematical proficiency observed while students did the tasks, and relates this to the practices, as previously described in section 5.2.1. This section provides more detail about what the students did when they were doing the IT tasks. Thereafter, I discuss all the observed practices with respect to the overarching categories of representation, justification and generalisation (cf. 5.2.), and draw conclusions about whether or not I can argue that mathematical practices were transferred, according to constructivist and situative perspectives. In what follows I discuss each proficiency strand in relation to the four tasks.

5.4.2. "Using Procedures Flexibly"

Most of the procedures and algorithms required were IT discipline-specific and also task-specific, although mathematical procedures were sometimes required for completing the prime numbers and computer systems tasks. Only the prime numbers and computer systems

tasks were analysed according to how the students used procedures flexibly. All of the students interviewed were able to use mathematical procedures flexibly to a greater or lesser extent within the scope of the tasks given to them.

5.4.2.1. The prime numbers task

The students reminded themselves of what constitutes the set of prime numbers and then planned how to test if a number is prime or not. Included in their discussions was conversation around even and odd numbers, and how prime numbers relate to these. Yvonne and Kaone specified that a prime number is not divisible by anything except for one, but later realised their mistake through discussion and corrected themselves to mean "divisible by itself and one". The others defined a prime number as being a number divisible by itself and one. All of the students were surprised by the fact that writing the programme to list or print the primes between 1 and 100 was actually very complicated. They were not aware at the time, as they quickly defined prime numbers, that knowing what a prime number is was not the same as writing a computer programme applying their conceptual understanding of the definition. All those who did the prime numbers task decided as a matter of course that their programme needed to restrict the number to be tested to between 1 and 100, and that numbers needed to be incremented by 1 each time the loop (also unanimously decided) was implemented.

5.4.2.2. The computer systems task

The students showed mathematical procedural fluency, along with task-specific procedural fluency, for the computer systems task. All of those interviewed were able to procedurally list the eight possible combinations for variables X, Y and Z in the truth table, although it was by using a pattern taught to them, rather than by using combination theory (i.e. $2^3 = 8$) taught to them in the Foundation Programme. They may have been aware of the link, but only Rudo and Ditso verbalised the similarities between on/off and combination and binomial theory; and this was only when I asked them if there was a link. Three out of the eight people who did

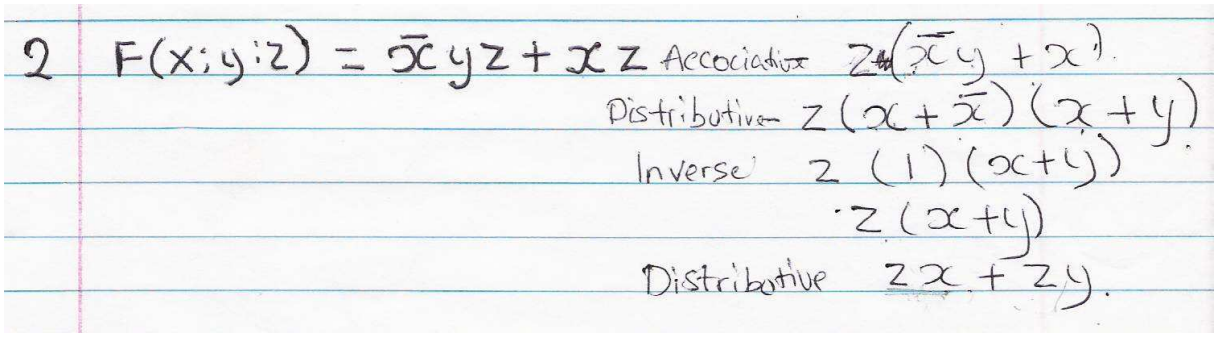
this task deliberately explained that what they were doing was a component of Boolean logic, and that it was very similar to the set theory and probability theory that they did with me.

All eight students were also able to use AND and OR in the mathematical sense, as well as to choose when to use these terms and their associated symbols and operations. ('$\cap$', '$\times$'; '$\cup$' and '+'). They were taught how to use AND and OR again when they did the computer systems course, in terms of Boolean algebra (see Appendix E); but whether or not they were taught these Boolean principles with conceptual understanding is unknown. They tended to do this part of the task procedurally, but also correctly. I observed that they did not necessarily verbally relate what they were doing in the task to what they did with me in mathematics. However, regardless of whether or not they verbalised awareness of these similarities, they all used the AND/OR operations correctly and could distinguish between them.

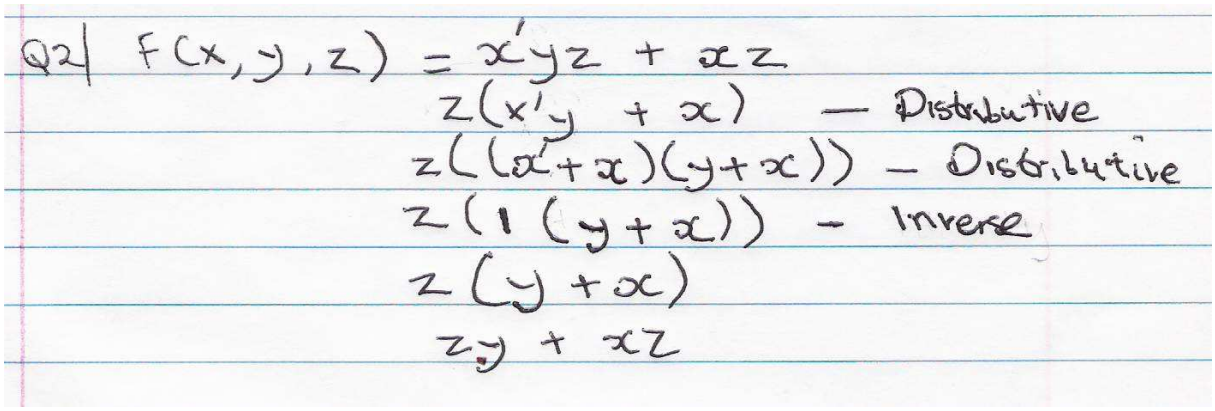
Kilpatrick et al (2001) define procedural fluency as: "… referring to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121). The identities available, from which the students needed to choose and use appropriately to simplify the Boolean expression, included the distributive, associative, commutative, inverse, complement, identity, dominance, and absorption laws. The students who were able to simplify the expression completely chose and used some of these identities appropriately and accurately, shown in Figure 5.1. below. They reached the same fully-simplified expression of ($xz + yz$). These students were able to use the identities flexibly – meaning that they were able to identify and use an appropriate variation, or variations, of the basic law(s) to complete the problem. Their lecturer expected them to write down which law they used for a particular step, and so it was clear how the identities had been used. For example, in all three of the solutions provided in Figure 5.1. below, a variation of the Boolean distributive law was identified and applied to the partially-simplified expression of $z(\overline{x}y + x)$ to result in $z(x + \overline{x})(x + y)$.

This first step required flexible use of the distributive law in order to be able to further simplify the expression[10]. In addition, all used the inverse law to substitute the value of '1' in the place of $(x+\bar{x})$.

Kaone and Yvonne[11]:



Ditso and Keabetswe



[10] This may be understood more clearly by comparing it with the trigonometric identity, $\sin^2\theta + \cos^2\theta = 1$. In my experience, students find it relatively easy to use this identity in this exact form when simplifying a trigonometric expression. However, although they can follow the procedure of manipulating it into a new form, for example, $\sin^2\theta = 1 - \cos^2\theta$, they do not find it easy to recognize when to do so, or how to strategically use it in this form to simplify the expression.

[11] Kaone and Yvonne wrote that they used the associative law, but it was actually the distributive law

Tungamirai and Rudo[12]:

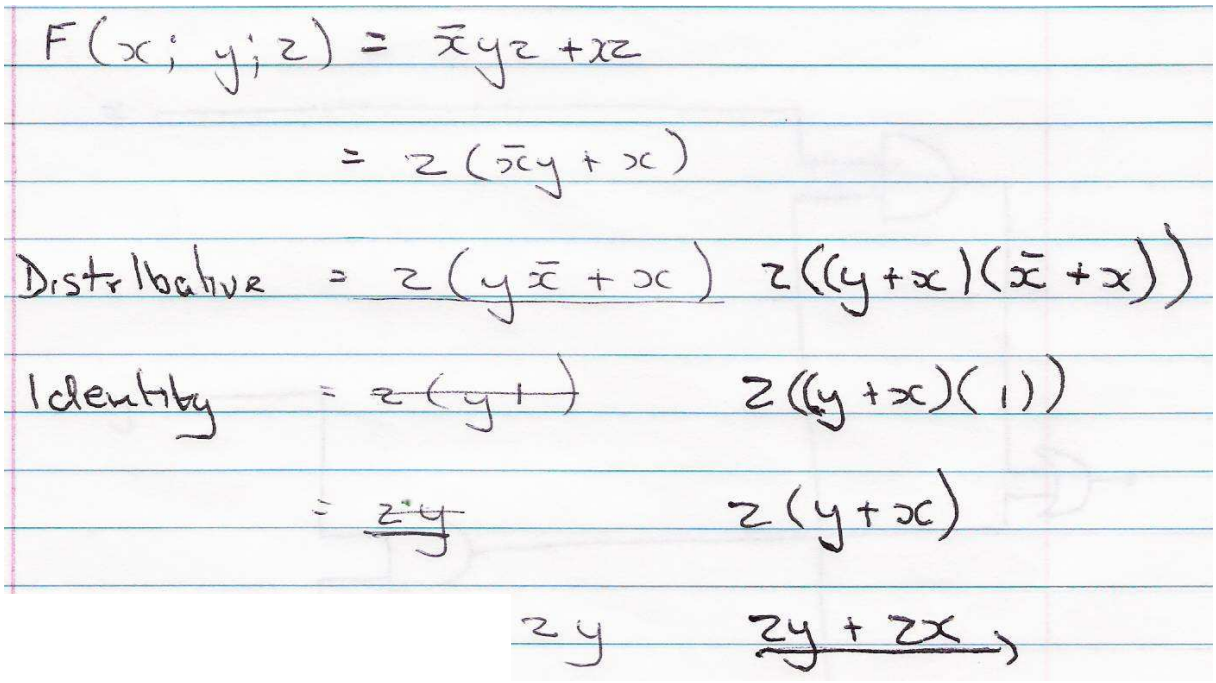


Figure 5.1. Flexible, accurate and efficient use of some Boolean identities to simplify the Boolean expression in the Computer Systems Task

Therefore, according to the explanation given by Kilpatrick et al concerning "when and how" to use procedures, and perform them "flexibly, accurately and efficiently" (ibid.), the students showed mastery of using mathematical procedures for this task, showed in Figure 5.1.

The layout of each proof was very similar to the way South African school learners are expected to lay out a Euclidean geometry 'proof', together with the reasons (theorems) for making any claims. Correctly-used laws were verified for me by João, who checked all of the solutions. Note that the first step in all three examples in Figure 5.1. is identical. Each pair of students who did this task got as far as this step, and quickly; but only the students named in Figure 5.1. finished the problem. It seemed to be very easy for them to identify and use the basic distributive identity as the first step to simplifying the expression. I suggest that it may have easy for all of the students to simplify as far as this step because use of the distributive

[12] Tungamirai and Rudo wrote that they used the identity law, but what they actually used was the inverse law.

law here is the same as a basic factorising procedure. The fact that factorising is done repeatedly in many areas of mathematics seemed to aid the students in their identification of the first identity they could use for this problem.

All students struggled thereafter to complete the problem, and even those who succeeded took quite a lot longer to finish. This is possibly because the identities used after the first step were newly-learned and more difficult to recognise (Refer to Alexander and Murphy (1999) on analogical transfer (cf. 2.5.6.)). Using identities or procedures to simplify mathematical expressions is not necessarily transferred directly to an IT problem. Analogy is made, suggest Alexander and Murphy (ibid.), when the similarities between the two situations are more evident: most likely when the prior and new learning situations are conceptually familiar enough to allow the analogy to be recognisable. The factorising in the first line of the solution was a procedure learned and competently done early in schooling. It has probably been used in a variety of different problems in mathematics, and possibly also in physical science. Hence the students' abilities to apply it with ease to the IT problem. Thereafter, new identities learned must be manipulated and applied to solving the problem; which the students found more difficult. Two out of the five pairs were not able to complete the problem

Compare this to a grade 11 mathematics student in South Africa being given the following expression to simplify:

$$\tan^2 x\left(\sin^3 x - \sin x\right)$$

It is simplified by means of knowing what the trigonometric identities are, being able to choose which, if any, can be applied to the given problem, and if possible, how they might be manipulated into an alternative form so as to be usable, to simplify the expression. Factorisation is also used in this problem, but is slightly more difficult to recognise where it is used; along with strategically using knowledge about identities:

$$\tan^2 x\left(\sin^3 x-\sin x\right) \quad (1)$$
$$=\tan^2 x\left[\sin x\left(\sin^2 x-1\right)\right] \quad (2)$$
$$=\tan^2 x\left[-\sin x\left(1-\sin^2 x\right)\right] \quad (3)$$
$$=\tan^2 x\left[-\sin x\left(\cos^2 x\right)\right] \quad (4)$$
$$=\tan^2 x\times-\sin x\times\cos^2 x \quad (5)$$
$$=\frac{\sin^2 x}{\cos^2 x}\times-\sin x\times\cos^2 x \quad (6)$$
$$=-\sin^3 x$$

Line 2 of the solution indicates a relatively simple factorisation procedure; but simultaneously a strategy needs to be designed about how factorisation initiates solution of the rest of the problem. The concepts and identities to be used as tools for simplifying a trigonometric expression have been learned more recently and are relatively unfamiliar to the learner. They are therefore more difficult to use, according to Alexander and Murphy (1999), compared with the factorisation procedure in line 2. Lines 3, 4 and 5 show strategic manipulation and use of the identity $\cos^2 x+\sin^2 x=1$; and line 6, use of the identity $\tan x=\frac{\sin x}{\cos x}$ and recognition that it is applicable to squared trigonometric ratios. It is likely that many more students would be able to finish the simplification up to the end of line 2; but fewer would complete it. The situation is similar to the way that the IT computer systems task was done.

Therefore, using procedures flexibly with respect to the Boolean expression requires "*knowledge of* ***procedures*** [factorising/use of Boolean identities], *knowledge of* ***when and how to use them appropriately*** [choosing which identities to use], *and* ***skill in performing them flexibly, accurately, and efficiently***" [factorising correctly/using variations of basic identities correctly] (Kilpatrick et al., 2001) (emphasis mine). I conclude that six out of ten students used procedures flexibly – the newer content learned being used with more difficulty.

5.4.3. "Understanding/Explaining Concepts"

I suggest that when mathematics is used in another domain, the practice of understanding mathematical concepts will be altered, so that they may be applied to solve the new problem.

This is consistent with what situative theorists argue about the use of prior knowledge in novel problems (e.g. Carraher & Schliemann, 2002; Hatano & Greeno, 1999). Alteration of knowledge in order to solve a novel problem was demonstrated when students had to use conceptual understanding of prime numbers in order to write a programme in Java; and also when they had to use their conceptual understanding of binomial probability to solve the computer systems task. The other tasks did not require the understanding of any mathematical concepts in order to do the tasks, and are not discussed under "understanding/explaining concepts".

5.4.3.1. The prime numbers task

Table 5.4. shows that 'understanding/explaining concepts' featured often while the students worked with the prime numbers task. Although it was not the most commonly-used practice, it was used frequently enough to show that they used conceptual understanding to help them to plan their Java programme.

The fact that a prime number is divisible only by itself and one is a definition that students learn and apparently understand early in their school careers. However, it was interesting to observe how the interviewed students used this understanding in the IT domain. They did not use their conceptual understanding that a prime number *cannot be divided by anything else* other than itself and one when they began to plan and write their programme. For example, one way to write the programme would be to specify in the programme what made a number *not* prime. Considering prime numbers in this way would be an example of how prior understanding can be used in different ways in order to be useable in new contexts (e.g. Carraher & Schliemann, 2002; Lobato, 2006; Perkins & Salomon, 1989).

In reminding themselves and each other what the characteristics of prime numbers are, the students all forgot to specify that prime numbers are divisible by themselves and one and nothing else. During the conversations that they had with each other while doing the task, I observed that they actually knew this but neglected to keep it foremost in their minds while trying to write the programme to list these numbers. Thus, they tried to write the programme by specifying that the variable must be divisible by itself and one. For example, if they

defined the variable that they would work with as "number" they would write in their programme: "$\frac{number}{number}=1$ *and* $\frac{number}{1}=number$". The problem with this, and none of them, excepting for Rudo, realised until I pointed it out to them, was that *every* number between 1 and 100 is divisible by itself and one. For example, even the number 12 fits into this definition. Rudo had specified this aspect of prime numbers early on in their discussion but then did not carry her understanding through to the new context of writing the programme. Jennifer, Yvonne and Tungamirai verbally acknowledged that they had not taken this fact into consideration, and that they had to include a test somewhere in their programme for numbers that were not divisible by anything else. None of the others attempted to specify that the number that the programme selects must also not be divisible by anything else, until after I had made them aware of this necessity. After realising that they needed to use a deeper understanding of prime numbers, they commented that the task was more difficult than they had first thought, because their mathematical understanding needed to be linked with Java programming syntax, which was not easy. Further discussion of writing the programme lies within the strategic competence strand of mathematical proficiency (cf. 5.4.4.).

5.4.3.2. The computer systems task

Explaining mathematical concepts was practiced more than any other mathematical practice used in this task (see Table 5.3.). The students often did not verbally explain the mathematical concepts to me or each other, but used their understanding of Boolean algebra to do the task.

The first part of the computer systems task was predominantly procedural, as already discussed. However, Ditso and Keabetswe specifically stated that one could not do this task applying only the rules of AND and OR ($\cup$ *and* $\cap$) – one had to understand what they mean and how they may be used. Rudo also explained how Boolean logic differs from probability theory: Although $X'\left(or\ \overline{X}\right)$ can be interpreted as "not X" in both mathematical systems, in probability theory $X'\left(or\ \overline{X}\right)$ is the complement of X and $X+\overline{X}=1$. In Boolean algebra, $X\ and\ \overline{X}$ mean "on" and "off" respectively, and $X+\overline{X}=0$. She was the only person who deliberately explained this difference to me, but I observed that the other students who did the

task understood how X *and* $\overline{X}$ functioned to switch a circuit on or off. All those who did this task had no problems explaining how the mathematical terms AND and OR related to the task, how and why they should be used, and how they switched the circuit on or off. None of the students verbally related their conceptual understanding of combination theory using $2^3 = 8$ to calculate the combinations of ON and OFF of three switches. They did the calculation procedurally. How do I interpret this finding in terms of use of conceptual understanding in a new domain? I will discuss this after presenting my analysis of the second part of the computer systems and prime numbers tasks.

The second part of this task, requiring use of other practices, such as the strategic manipulation of identities with accompanying reasoned argumentation, and the procedural use of Boolean identities, would predictably have been prohibited by a lack of conceptual understanding of how and why a particular identity could or could not be used. That is, there is an indication that the presence of conceptual understanding influences other practices, such as strategic thinking. This second part of the computer systems task consisted entirely of Boolean algebra, which, although it consists of logic developed specifically for the binary computer system, is mathematics and can also be investigated in terms of the mathematical proficiency shown when doing it. All of the students who did this task needed to look up the identities in the text book, as they did not remember exactly what they were. This does not imply lack of ability to use conceptual understanding in other contexts. Efficient memory-based reproduction of the identities would probably just indicate that the students had used them enough to have remembered exactly what they looked like and how they could be rearranged and substituted into an expression (Kilpatrick et al., 2001).

5.4.4. "Strategising and Using Representations"

Kilpatrick et al (2001) describe strategic competence as the ability to "*formulate mathematical problems, represent them, and solve them*" (p. 124). Furthermore, in every day life students

> "*encounter situations in which part of the difficulty is to figure out exactly what the problem is. Then they need to formulate the problem so that they can use mathematics to solve it. Consequently, they are likely to need experience and*

> *practice in problem formulating as well as in problem solving. They should know a variety of solution strategies as well as which strategies might be useful for solving a specific problem"* (ibid.).

While Kilpatrick et al specifically describe mathematical strategic competence, I reviewed Perkins and Salomon's (1989) discussion of the strategic heuristics and general problem-solving approaches through 'low road' and 'high road transfer' (cf. 5.3.3.). I also aligned what they discussed with Kilpatrick et al's description of strategic competence. Although strategic competence is integrated with the other strands of mathematical proficiency, strategic thinking is not necessarily content-dependent, but should be identifiable within other contexts if it is present.

Strategic competence incorporates the 'use of representations', and 'strategising, shown in Tables 5.1. to 5.4. An average of 25 incidences in the prime numbers task; 13 incidences in the computer systems task; 2 incidences in the networking task; and 28 incidences of strategising in the banking systems task showed that strategising was used frequently in all of the tasks excepting for the networking task. 'Strategising' incorporates decision-making, strategic problem-solving, and using prior decisions to influence later decisions. How strategising and representation was used in each task is discussed below.

5.4.4.1. The Prime Numbers Task

The students needed to be able to understand what the problem was, so that they could decide on the best strategy to use to solve that particular problem. Ditso and Keabetswe clearly showed their competence in strategising when doing the prime numbers task. Before they had even considered the contents of their programme (step 4 in Figure 5.2. below) they had written a scheme of what the task entailed (steps 1-3) – a form of representation that they would use to solve the problem. The representation was a pathway to 'problem formulation' (Kilpatrick et al., 2001).

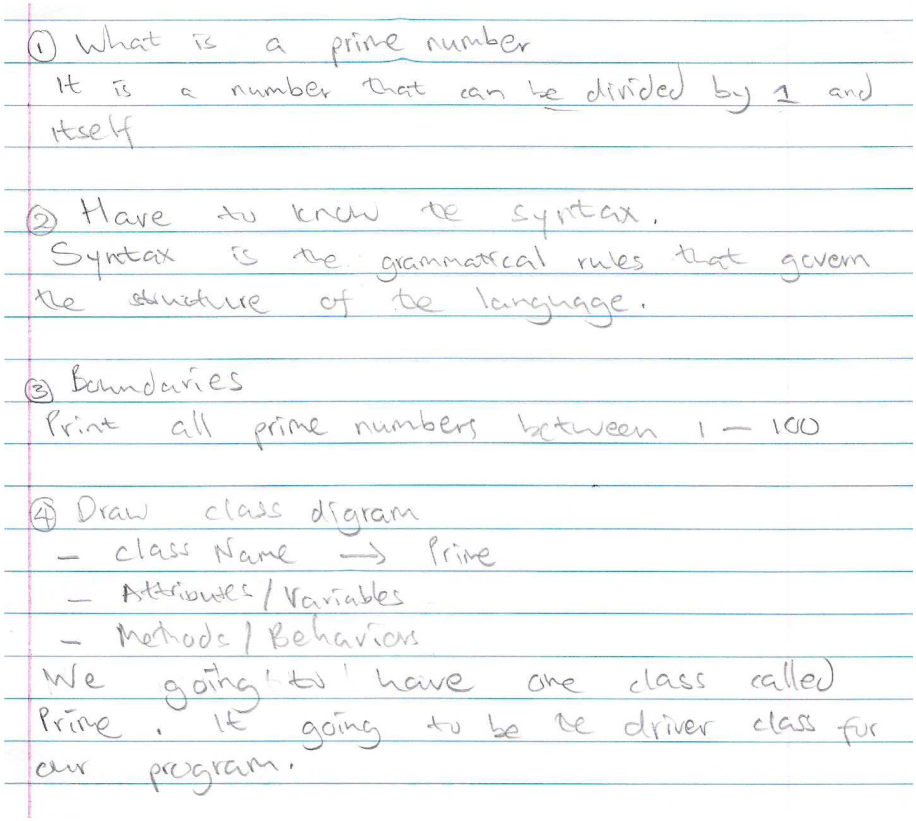


Figure 5.2. Strategic planning for the prime numbers task, as shown by Ditso and Keabetswe

Writing down their strategy demonstrates how interlinked and interdependent not only the proficiency strands, but also the mathematical practices are. Step 4 in Figure 5.2. shows Ditso's and Keabetswe's use of representation of how they intended their programme to be written. A "class diagram" is often used as part of planning, as it shows generally what should be considered in the different aspects of the programme. These two students were going to draw a class diagram that contained the class name (prime number), its attributes and variables (for example, the fact that a prime number must be an integer – which will be in the form of symbolic representation of the variable), and the behaviour(s) of the defined variable (for example, stating that a prime number is divisible by itself and one).

The others who did this task also showed what they planned to do, but their plans were less clear and logically structured than that shown in Figure 5.2. Sometimes the planning was mental and not written down. Jennifer verbalised her planning throughout the initial stages of the discussion, frequently starting with an idea and rejecting it with a reason for rejection, almost before the idea had finished being verbalised. As she verbalised Carol wrote some ideas down, but this was more like brainstorming than strategising.

Tungamirai elected not to use any of the Java II techniques he knew because Rudo had not done Java II and would not conceptually understand his suggestions. He often acted as a "sounding board" for Rudo's ideas – questioning her and commenting on her ideas, rather than offering any himself. However, after realising that *all* numbers are divisible by themselves and one, he became more active in the planning process. At this stage he suggested that not only must they reject all even numbers (except for two), but also they must reject any numbers that are divisible by 2, 3, 5 and 7; and start their programme loop at $number \geq 8$. He did not go higher than "divisible by 7", and did not explain why he stopped at that value. He did not explain any of this reasoning aloud and moved through this series of ideas extremely quickly and mentally. When he realised that the task was a lot more complicated than merely coming up with a quick strategy to list the required prime numbers, he slowed down in his mental processing, started to verbalise again, and considered Rudo more in his deliberations.

What are the implications of these findings in terms of mathematical practices? Firstly, it is clear that strategic thinking, or problem solving, is very complex and is not restricted to a set of heuristics that may be followed in order to solve a problem. Problems may be solved in many different, but correct and acceptable ways. A vast amount of research has been done with respect to aspects of problem solving, both in mathematics and in general. Since entire theses may be written regarding this topic alone, I will not be able to do it justice here; but what is clear is that the practice of strategising requires clear problem conceptualisation, visualisation/representation, solution strategies/planning, decision-making, and justification. The students demonstrated use of these practices as components of the practice of strategising. Secondly, it is possible that working together on the tasks facilitated the strategising process, more than if they had worked alone on the tasks. I make this claim because they discussed many ideas while they were trying to solve the problems, and often built on each others' ideas during their discussions. Problem-solving is probably made easier through interaction and discussion.

5.4.4.2. The computer systems task

For the computer systems task using Boolean logic the second part of the question required the strategic use of identities to simplify the expression $F(x;y;z) = \bar{x}yz + xz$. Observing the students doing this part of the task showed me clearly how closely strategy is associated with procedural fluency, conceptual understanding and justification. The students had to use their conceptual understanding of the identities to choose the correct identity to apply to the problem in an appropriate way at the appropriate step in the simplification. They also had to simplify the expression by using the identity correctly, often needing to argue for and justify a particular idea; and then explain what they did by writing down the identity used next to the simplified step.

However, reasons were not always provided for their choices of the appropriate identity. Sometimes one student would point at a particular identity in the book, and the other would nod, and they would use it. Alternatively, the other would shake his or her head with no further argument, and they would think again quietly to themselves. In these few cases, it seemed that there was no verbal disagreement because it was obvious to both students that they could or could not use that identity. Where manipulation of the identity was required before it could be used strategically (cf. 5.4.2.2.), justification and negotiated disagreement was invariably used, because the outcome of the suggestion made by one person was not immediately obvious to the other. Here it was clear that strategising was closely accompanied by justification.

All the students used the same strategy of working through the list of identities in the text book and attempting to apply the rule of each to the expression. Those who did not manage to simplify it further than the first step all tried to apply the identity in its original form directly to the expression, while those who completed it successfully also did so correctly by flexibly manipulating the identities – by working backwards and forwards with their alternative forms and checking if any form of the identity changed the expression to a form that could be further worked on (cf. 5.4.2.2.). In addition, all of the successful students except Kaone demonstrated their decision-making abilities via a degree of argumentation, resulting in subsequent rejection or choice of a particular identity.

Kaone, instead of coming up with his own arguments, usually waited for Yvonne to make a suggestion, and explain her choices before agreeing with her. If another suggestion was to be made, it would be by Yvonne, who had rejected her own suggestion and come up with something else. During this process Yvonne verbalised all her thinking. Although Kaone seemed to understand the strategies she was using, he did not often suggest strategies, decisions or arguments. This finding suggests that not all students used practices to the same extent when solving IT problems.

Keabetswe explained, as he and Ditso were thinking through the problem, that in order to simplify the expression successfully one had to approach the problem from different perspectives: if a chosen strategy does not give a successful outcome, one has to keep trying different approaches until the problem can be solved. Ditso added to this the importance of using the desired outcome as a guide to what kind of strategy to use and how to use it to solve the problem. This conversation was significant, in terms of Perkins and Salomon's two articles (Perkins & Salomon, 1989; Salomon & Perkins, 1989) explaining 'high' and 'low road' transfer. Here we have a glimpse into how these two students have used a combination of non-learned problem solving strategies (high road) with problem-solving heuristics (low road) that have been successful in the past. Using the desired outcome to give guidance on how to first approach the problem, as well as coming at the problem from different perspectives until an acceptable solution can be found demonstrates that these two students were able to use heuristics as well as 'think out of the box' in order to problem-solve in a novel task. Strategising is crucial to doing mathematics: it was also used competently and successfully in the new context of IT.

5.4.4.3. The networking task

The networking task, requiring an explanation of how "lockout" occurs, required mostly conceptual understanding of the functioning of the different components of a network. All of the students except one (who had already completed the course) were still busy with this course at the time of the interviews, and were still unfamiliar with its concepts. Strategic problem solving did not apply a great deal here, although to answer the question the students had to do what I will call "low level strategising". They all used the text book which provided

them with a list and summary of the components of a network system. What they needed to do – and they all did this – was to work through the functions of each component and use their understanding of how each functioned in order to state if that component was associated with network security or not. Depending on their conceptual understanding, different students gave different answers; but the conceptual understanding required was that of networking and not of mathematics, and therefore I am not reporting it in this study.

5.4.4.4. The banking system task

The strategising needed for the banking system task was complicated and essential to do before any ideas could be implemented. Ravi and Tinashe both wanted to move straight into coding, and were asked by João to please plan the big picture first. After this suggestion these two students treated the task similarly to everyone else. Their problem-solving as a team appeared to be slightly limited, because João had to ask a few leading questions to aid their initial thinking. I found that Ravi, who was outspoken during most of the interview, did not believe that planning was an important feature of the banking system task. He commented that planning was only important for the novice programmer. In his own words, paraphrased, he said that the '*hectic programmer' won't do this planning phase because the specs and plans are all in his head. He will go straight into programming and make changes as he goes along*. In this he was entirely incorrect, as good programmers spend a great deal of time planning before they start with the 'coding up' part of any task (IT programming lecturer, personal conversation). This was a significant finding, because his understanding about the importance of strategic planning in this task was inaccurate, and he did not appear to believe that planning was important, at least for this task. Therefore, he did not use the practice of strategising in this task until João actually asked him to; bringing to my mind questions about the extent of his use of strategising in other subjects. Therefore, I suspected that Ravi had not transferred his abilities to strategise or use heuristics in other contexts, or had not developed them adequately in the first place; possibly because he did not believe that they were necessary. His comment was one of the few made during the task-based interviews that gave me insight into how mathematical productive disposition might be visible in other contexts (cf. 4.6.). Also, his comment gave me insight into the possibility that productive disposition (personal beliefs about the usefulness of mathematical practices) is key for the use of

mathematical practices in other contexts. This latter possibility was an unexpected outcome of the study and is not part of the current discussion. I take it up again in chapter 6.

Planning the "big picture" was a significant strategy to be considered; but also embedded in the task was the need for multiple opportunities for decision-making and representation. As I already explained (cf. 5.3.3.4. and Appendix E), the complexity of strategies needed to do this task was vast, and decisions made would automatically influence how subsequent decisions would have to be made. Sometimes the students would have an initial plan – only to change it, because they had worked themselves into a situation that was unrealistic or too difficult to code. Ravi and Tinashe changed their plan three times, as they argued about which classes to have and how the classes would 'see' each other (shown by arrows drawn between classes). This made it possible to observe the problem-solving process rather well.

Representation was used extensively by all those doing the task and took the form of a series of boxes linked by arrows (for example, see Figure 5.3. below). Inside each box, which represented a "class", was a list of the "attributes" belonging to that class. For example, the "personal details" or "client" class would list attributes such as the person's name, ID number, birth date and address. The big picture plan required the designer to decide what classes to have in the system, how the classes would relate to each other and what attributes to have in which class. This form of representation was expected because it originated as a software package called 'BlueJ', designed to help the user to solve problems like the banking system task. For the purpose of the interview BlueJ was not used and what would ordinarily have been done on BlueJ was done on paper (more detail is provided in Appendix E).

Before drawing up a plan Molefhi and Kabo discussed the task at length, and made written notes containing some key words that they wanted to remember. They then drew a set of class diagrams, after which they decided together how the classes were to be linked using arrows. They only produced one plan, as shown below in Figure 5.2., where one can clearly see how they represented the system and its linked components, as well as the grouped attributes. The significance of their producing only one plan was that while some people drew diagrams *while* they were deciding what the big picture should look like, changing the representation as they discussed their ideas, these two discussed and argued at length and wrote a few key words down *before* producing a final drawing. It does not imply that they planned less, but rather that their mental strategising was more prominent, helping them to produce a

representation of their decisions after negotiation had taken place. Kilpatrick et al's (2001) description of some peoples' abilities to make mental representations as a problem-solving strategy seems to be what I observed in this case.

As Eric and Tungamirai worked on the banking system, Eric appeared to lack confidence and was quiet. He had been an excellent mathematics student, but had explained in the focus group interview that his experience with Java had been negative; he had struggled with Java I, although he passed it, and had decided to drop the computing course and take up IT in business systems instead, which would allow him specifically not to study Java II. Tungamirai, on the other hand was very confident. (He had recently completed Java II and loved this aspect of IT). Eric made a few suggestions, listening closely to counter-arguments and suggestions from Tungamirai, and together they came up with a plan for a system. They appeared to work well together, although Tungamirai seemed to play the role of tutor rather than peer. Eric's mathematical strategic competence was excellent (he excelled in every practice when he was a mathematics student in the Foundation Programme), but there was no indication that it was being used with the same ability in this task because he was fairly quiet during the interview. However, he seemed to have no problem working with counter-arguments and new ideas; even though his knowledge of the new context was possibly not proficient (Alexander & Murphy, 1999). He seemed readily able to understand the usefulness of, and the concepts and counter-arguments involved with new ideas that Tungamirai introduced to him. They planned the banking system on paper – Eric did all of the drawing and writing. The representation they made was thorough and they made one change to the plan while they discussed the specifications of the task.

To conclude, it is possible that Eric's mathematical strategic competence was being used in this task, but that it was non-verbal and therefore not easy to observe. Tungamirai showed a high level of strategic competence, but in an unusual way. Being aware that Eric was not at the same level of programming competence as himself, he tutored Eric with respect to new concepts at the same time as making decisions about better ways to plan the system. In this way Eric was never left behind or left out of the discussion and appeared to have learned a great deal during the session.

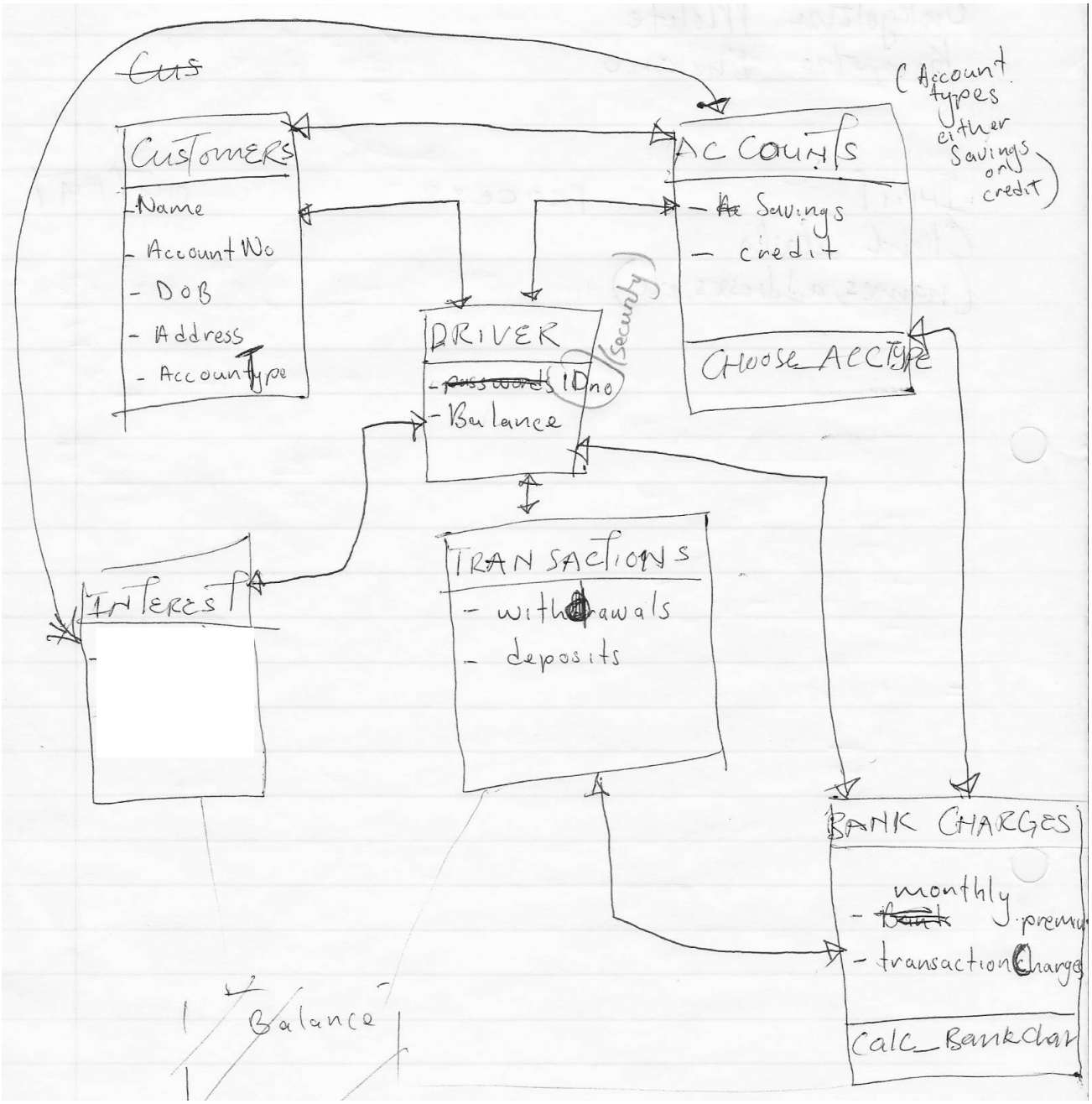


Figure 5.3. Banking system design by Molefhi and Kabo

While the banking system design could have no single correct answer, it had to include the specifications defined in the question, as well as needing to be as efficient as possible. For computer software designers efficiency is an important consideration to make when judging the value of the programme. Thus, this aspect of designing the system was foremost in the minds of the students; which is part of the reason why their big picture plans changed (on paper and mentally) as they thought about the components of the system. They all said as much to me, and explained that the way that they designed how classes communicated with

each other would positively or negatively influence their later decisions and the overall worth of the design. I came to the conclusion that all those interviewed demonstrated the problem-solving strategies of planning, representation and decision-making, to a greater or lesser extent when working with the banking system task.

5.4.5. "Questioning, Justifying, Disagreeing and Generalising"

Constructing arguments, making reasoned claims, using questioning, disagreeing and making abstractions or generalisations, were described as being ways to demonstrate the practices of justification and generalisation (cf. 5.3.4.). In some of the tasks it was very clear that if the students were to use strategic thinking, then it would be an automatic requirement for also making reasoned claims and justifying, because a suggestion for use of a particular strategy would require justification of that suggestion. This was observed in many cases, in all four of the tasks given, although in some tasks there was more scope for such practices than others. For example, the banking system task required complex strategic thinking; and because students were working together on the same task they had to convince each other of the worth of their strategy, or the shortcomings of another strategy suggested. In contrast, the networking task required a low level of strategic thinking, because the concepts were new to the students. The suggestions that were made concerning the naming of OSI layers involved in network security had to be followed up with reasons for why a suggestion was valid or another invalid. Therefore, although justification was used in both these examples, it was used for different purposes: one was to help make a set of complex decisions and the other was used in discussion for aiding understanding about how security is implemented in computer networks. The practices of constructing arguments and using justification were more linked to mathematical thinking in the computer systems and prime numbers tasks than the other two tasks. This is discussed in detail below.

Generalisation (rule formulation from recognising and working with patterns, and general use of pre-formed mathematical rules), was difficult to identify. Tables 5.1. to 5.4. show that generalisation, as it is understood in mathematics, was not observed frequently in the tasks (cf. 5.3.4.). All of the students who participated in the interviews had used disagreeing and justification when they were in the Foundation Programme and were aware of the level of

importance placed on this practice for learning mathematics. The students had all already expressed to me during the focus group interviews that their mathematical understanding benefited from discussing ideas, making suggestions, reasoning and arguing with each other in the Foundation Programme; which had helped them to better understand the significance of the different aspects of their mathematical knowledge (cf 4.3. and 4.5.).

5.4.5.1. The prime numbers task

Many incidences of justification were observed when the students did the prime numbers task. Typically, a programme to list/print all the prime numbers between 1 and 100 would have many possible solutions – many of these possibly being correct. The students all demonstrated instances of disagreeing, and justification, and to a lesser extent, questioning while they were doing this task, as shown in Table 5.4. In general, 'strategising' and 'justifying' were observed frequently during this task, which shows that students generally did not allow suggestions for strategies to write the programme to go unchallenged. As they justified their thinking to each other they demonstrated conceptual understanding of the Java, as well as the mathematics involved in the task. 'Questioning' did feature during the task, but to a lesser extent than the other two practices of justifying and disagreeing.

Kaone often tended to be satisfied with the way his partner, Yvonne, explained her suggestions to him and did not make many suggestions himself, or argue something different or justify any thoughts he had. Yvonne did not often have an opportunity to question his thinking, but sometimes argued her own points by explaining her reasoning. When they began to design a programme to list the prime numbers from one to a hundred Kaone wanted to use a "do while" statement and a loop. Yvonne wanted to run through all of the numbers between one and a hundred and tell the programme to reject any number that is divisible by anything excepting for one – she had not yet realised the flaw in her thinking and Kaone did not contradict her. They also disagreed about which syntax to use, but neither person justified his or her argument. Thus, during this interview, argumentation with justification was observed less frequently than during the others.

Jennifer and Carol, and Ditso and Keabetswe all made suggestions and justified their thinking, but they generally appeared to be in agreement with each other. Table 5.4. shows very little disagreeing taking place between Ditso and Keabetswe. Typically they would discuss one person's ideas, using justification, rather than having trying to justify a clear counter argument. Jennifer and Carol would question each other, but it was more for the purpose of clarifying something than deliberately pressing the other for justification. Alternatively, justification of an idea, when it occurred, acted to strengthen a suggestion rather than disagreeing or a counter suggestion.

Rudo and Tungamirai argued well with each other, but it was usually Tungamirai who questioned Rudo's ideas and encouraged her to justify them by producing counter arguments than the other way around. However, he deliberately kept his counter arguments at a questioning level, rather than suggesting things he knew she would not follow – he commented that he had ideas that he knew would work better that those being used, but that they were learned in Java II and she would not be aware of them. He became very uncomfortable when he realised he was not going to solve the prime numbers problem in ten minutes, as he had first supposed, but he then chose to work with her, and use his higher level of conceptual understanding to help solve the problem. Though Rudo did not use verbal justification very much, she appeared to be thinking clearly through the task problems. It was difficult to determine the extent to which she used mathematical argumentation in solving IT problems – even those which were more like mathematics than others – that is, the prime numbers and computer systems, rather than networking and banking system tasks. Perhaps much of her problem-solving and justification was internal – it was difficult to ascertain.

5.4.5.2. The computer systems task

Table 5.3. shows that excepting for Rudo and Tungamirai, the students who did the computer systems task used justifying roughly to the same extent. Ditso and Keabetswe did not question each other at all and Jennifer and Carol, and Ditso and Keabetswe did not use disagreement to press each other for justification of their suggestions. Often they would justify a suggestion, even if justification was not verbally asked for. Sometimes a raised eyebrow or other non-verbal request for explanation encouraged someone to explain why they thought a particular

identity would be valid to use to simplify the Boolean expression. The students struggled to simplify the expression after the first step had been done (cf. 5.4.4.2. and Figure 5.1.) and justified why they thought a particular identity was appropriate, rather than why they disagreed with a suggestion or asked for a better explanation.

At one stage, during the simplifying of the Boolean expression, Jennifer took on the role of questioner for a little while; to promote a way of thinking that might help solve the problem, while Carol would make suggestions as answers to her questions. At this time both would also counter-argue Carol's ideas, mostly giving reasons for their disagreements, which were usually accepted by each other.

Figure 5.1. shows some of the struggle Rudo and Tungamirai experienced to simplify the expression (a few ideas were crossed out and corrected while they were doing the task). A relatively high frequency of justification was observed concerning these two students, as well as an accompanying high level of disagreement. Therefore, it appears that disagreement promoted much of the justification in this interview.

Yvonne tended to justify her thoughts as she verbalised them – with or without Kaone's request to do so. Therefore, I concluded that because Kaone did not play a large role in questioning or disagreeing with Yvonne, she tended to see justifying her thinking as an integral part of her strategic planning of the computer systems and prime numbers tasks.

5.4.5.3. The networking task

Table 5.1. shows that although 'understanding/explaining concepts' occurred the most in the networking task, as expected, because this area of IT was new to the students, incidences of questioning, justifying and disagreeing were also observed, but to a lesser degree. Kabo and Molefhi questioned each other quite frequently, in order to understand the concepts behind the OSI layers, before coming up with an agreed-upon conclusion about which layer was responsible for security. Note that they disagreed little – after explaining to each other what the roles of the different OSI layers were, they agreed on what the answer should be. The justifying that took place between these students was mostly by explaining their conceptual

understanding while making a suggestion to the other. Conversely, Ravi and Tinashe disagreed frequently but questioned each other little. However, they were seen to frequently justify their disagreements and claims and ended the session by agreeing on an answer to the question.

As previously explained, this question required only low level justification. All students used justification to explain their understanding of concepts associated with security issues in computer networks. From what was observed while they did this and other tasks, I conclude that justification was being practiced at different levels by the students, depending on the level of complexity required by the question.

5.4.5.4. The banking system task

The practices of questioning, disagreeing and justification were observed when students were doing the banking systems task (Table 5.4.). Because there were so many different ways to do this task, all students doing the task had to justify their strategies and claims to each other – usually because one would make a suggestion which the other would disagree with or question. Kabo and Molefhi used all three practices frequently; whereas the other two pairs used them less. As soon as either one disagreed with the other, he asked for justification of his reasoning. I also observed that neither of them would let an explanation go unchallenged if it was not clear enough. The justification used while Kabo and Molefhi were doing the task was often based on justification through disagreement; whereas that used between the other two pairs was based more on justifying a suggestion to further explain an idea which was not yet understood by the other person.

In relation to questioning, questioning was encouraged as a strategy for learning in the Foundation Programme, and was used significantly by Kabo and Molefhi, and less by the other two pairs. Usually questioning was used to encourage a clearer explanation of an idea or suggestion. For example, Molefhi questioned Kabo about his suggestion to use a class diagram for planning of the banking system, as opposed to using an Input-Process-Output chart (IPO chart), which was what Molefhi wanted to use. Each argued his idea, giving reasons why his was the better strategy, until Molefhi accepted Kabo's argument and said that

it would work as well. At this point he began to question what Kabo would do next, and why and how, as though he wanted Kabo to think more completely through his plan and further convince him of its value. Molefhi kept his questioning role for quite some time, until Kabo began to struggle with naming classes and linking them to each other. Then Molefhi began to offer suggestions to finish the design, and they came up with the representation shown in Figure 5.3. They worked well together and pushed each other hard for convincing arguments.

I was able to observe during an unexpected but valuable exchange between Tungamirai, Eric and João, how Tungamirai was able to discuss, argue and question at different conceptual levels – depending on who he was talking to. During planning of the banking system Tungamirai had a long and spirited argument with João, who had questioned him concerning his decision about using an array list to store the data for the banking system. João, having had more experience than Tungamirai in programming, was able to challenge Tungamirai in a way that Eric could not do. Tungmirai was forced to provide carefully-thought through justification for his decision as he argued for the legitimacy of his claim. While Eric learned concepts which had previously been unclear to him, Tungamirai was challenged to justify at a higher conceptual level some of the understanding he had about elements of programming. As shown in Table 5.2. the incidences of justification shown by Tungamirai and Eric increased suddenly and drastically when Tungamirai had to justify his thinking to João.

Impulsive suggestions were rarely made, but when they were, they were not accepted by the other student. Either questioning or counter-justification was used on these occasions to press for clarity or re-consideration of an idea. Tinashe's arguments and counter suggestions, when he made them, tended to be quite subtle; and it was easy to miss the fact that he was disagreeing with Ravi. Mostly, Ravi did not justify his thinking very much – whether it was to strengthen a claim or a disagreement.

The two main kinds of justification observed during this task were (1) discussion and disagreement of ideas using occasional reasoning, or justification of arguments in order to strengthen them; and (2) deliberate mutual questioning throughout the interview, so as to press the other to justify claims or suggestions made. Used in the second way, argumentation practices were more explicit. Table 5.2. shows how much more Kabo and Molefhi, as opposed to the others, deliberately questioned each other, and that they had used questioning as a deliberate problem-solving tool while they did these tasks. They appeared to find

questioning useful to direct their thinking and help them to stay focused on the problem. Note also, (Table 5.2.) how many more instances of disagreement and justification they showed than the other students who did this task. The uses of these three practices appeared to reinforce each other. I suggest that teaching students how to question each other to improve and reinforce their uses of mathematical argumentation is a valuable learning tool for students to use in later learning situations.

5.4.6. Productive Disposition

Evidence of productive disposition was easier to observe in the focus group interviews, where the respondents were describing their experiences of and opinions about learning mathematics in the Foundation Programme. Verbally they described how this experience had affected their beliefs about learning mathematics and about how competence in mathematics was useful in other areas of life. I suggest that the reason why this practice was not well-observed in the task-based interviews was because they became involved in what they were doing, and were *using* the mathematics. One could perhaps conclude that if they were using the mathematics to solve IT problems, then they had to a certain extent developed the belief that mathematics is useful and worthwhile – otherwise they would not have linked the domains to do the tasks.

5.5. The Mathematical Practices of Representation, Generalisation, and Justification

The eight practices that I observed the students using to help them to solve IT tasks included using procedures flexibly; using representation; understanding/explaining concepts; questioning; justifying; disagreeing; strategising; and generalising (Tables 5.1. to 5.4.). These are the terms I used to describe what the students were doing at the time, and they were informed by my understanding of practices and proficiency, and helped me to analyse my data. They needed to be incorporated into the overarching practices of representation, justification and generalisation (Ball, 2003) as a final indication of mathematical practices

that were used to solve IT problems. Table 5.5. below shows how each of the eight observed practices I used related to the three overarching practices (cf Table 2.1.).

Table 5.5. Way in which observed practices can be incorporated into the three practices of representation, justification and generalisation (Ball, 2003), based on broad ways the three practices could be used in IT.

	Representation	**Justification**	**Generalisation**
Practices observed in the IT tasks taken from Table 2.1.	• Using procedures flexibly • Using representation	• Understanding/explaining concepts • Questioning • Justifying • Disagreeing • Strategising	• Generalising

'Using procedures flexibly' was put into the representation category. Although mathematical procedures and formulae are used in many mathematical situations, including problem-solving, reasoning and rule formulation, the IT representations including flow charts, computer programmes, programming environments such as BlueJ, models and modelling languages are more likely to use mathematical procedures than other areas of IT. I wanted to include "using procedures flexibly" in only one of the three overarching mathematical practices, and representation was the most appropriate.

A rich description of mathematical reasoning is given by Kilpatrick et al (2001); where conceptual understanding and strategic thinking are integral to judging the "legitimacy of a proposed strategy" (p. 131). Thus, 'understanding/explaining concepts' and 'strategising' have been included, along with 'questioning', justifying' and 'disagreeing' (Table 5.5.), into the overarching mathematical practice of justification. Justification is explained in section 2.2.2. as "articulated and reasoned claims, [and] rationally negotiated disagreement" (Ball, 2003 p. 32). It is clear how the practice of justification is necessary for the proficient mathematics user, and where it is crucial for solving the IT tasks in this study. Generalisation was given its own category and included incidences of pattern recognition and rule formulation in an IT context.

The total usages of practices in each task are shown in Table 5.6. Concerning the practice of representation, for the networking task representation was seen only once. As already stated, (cf 5.4.1.1.) this subject was new to the students, and those who did this task mostly used the practice of explaining concepts to each other, and questioned and justified claims made about which OSI layers were involved with lockout. For the banking system task the whole final plan was a representation on paper, in place of being represented on a computer screen using the BlueJ software package (Appendix E). So while the incidences of representation appear relatively fewer times than expected for such a representational task, much of the representation could be described as 'one continuous incident', in a way. In the computer systems and prime number tasks use of representation was observed approximately equally; which is as expected because both tasks required students to understand and use symbolic representation as key aspects of doing the tasks (cf 5.3. and Appendix E).

For the networking, banking system and computer systems tasks generalisation was used very rarely. The networking was new to the students and they were in the process of conceptually understanding how networks function. Neither of the latter two tasks required practices such as pattern recognition and rule formulation, which comprise generalisation, and the low incidence of this practice was not unexpected.

The prime numbers task was different, in that it required the students to be able to generalise the concept of prime numbers into some sort of rule, or definition, before they could write the rule into a computer programme. This is where most of the students struggled, as described in detail in section 5.4.; but more incidences of generalisation were observed in this task than the other tasks, as expected, because of the nature of the question.

Table 5.6. shows clearly how frequently justification, as an umbrella description of what takes place when people use reasoning, was used by the students while doing all four of the tasks. To solve the tasks justification was used significantly more than any other practice, which not only indicates its importance in IT, but also shows that the students were able to use this practice to do the tasks.

Table 5.6. Incidences of representation, justification and generalisation observed in each IT task.

Name of student	**Representation**	**Justification**	**Generalisation**
Networking Task			
Ravi and Tinashe	1	18	0
Kabo and Molefhi	0	22	0
Tunga and Eric	-	-	-
Total	**1**	**40**	**0**
Banking System Task			
Ravi and Tinashe	16	41	0
Kabo and Molefhi	12	91	0
Tunga and Eric	7	48	4
Total	**35**	**180**	**4**
Computer Systems Task			
Rudo and Tunga	7	40	1
Carol and Jennifer	10	20	0
Ditso and Keabetswe	9	23	3
Kaone and Yvonne	11	26	1
Total	**37**	**109**	**5**
Prime numbers Task			
Rudo and Tunga	5	59	3
Carol and Jennifer	11	46	7
Ditso and Keabetswe	6	41	7
Kaone and Yvonne	9	30	5
Total	**31**	**176**	**22**

Therefore, the mathematical practices of representation, justification and generalisation have been described in terms of a mixture of 'sub-practices', informed by the notions of mathematical practices (Ball, 2003) and mathematical proficiency (Kilpatrick et al., 2001) (cf 2.2.2.). As such they were useful for gaining some insight into how the students used mathematics to solve IT tasks. In what ways does this finding provide information about whether or not mathematical practices were transferred to the IT domain?

5.6. Did Transfer Take Place?

As stated in the introduction to this chapter the purpose of this part of the study was to identify aspects of computing subjects that require mathematical proficiency from the IT students in the form of mathematical practices that are used in the IT domain. In this section I will examine whether or not I think that transfer of mathematical practices occurred, with respect to how transfer has been defined and described by different learning theorists. I will do this first from a cognitive perspective and then from a situated perspective.

5.5.1. From a cognitive perspective:

A cognitive perspective on transfer considers what happens in the individual's mind when transfer occurs. In cognitive transfer studies the researcher traditionally has a preconceived idea of what will be transferred and sets up the experimental environment and test conditions to specifically measure the extent of transfer. There is often some kind of teaching towards the topic of transfer to be measured, after which the topic is first tested within the context it was taught, and then in a new context. Since I did not set up my transfer investigation this way, it is difficult to make cognitive claims about how mathematical practices were transferred to IT tasks. Rather, I analysed the mathematical practices that the students used in the IT tasks, suggesting more of a situated focus.

However, recent arguments from Carraher and Schliemann (2002) suggest that transfer can be understood from a cognitive perspective, even though studies were not traditionally

experimentally designed. They also argue firstly, that transfer in its original sense of the word rarely takes place, if ever. Secondly, the term ‘transfer’ should only be used in its original definition given by the behaviourists who were the first to consider such a notion: that of transfer of identical elements. Other ways in which knowledge is often observed to be used in new situations is not ‘transfer’ in the traditional behaviourist understanding of the term, but another way to understand how people learn. In this sense, transfer could be described as an extreme case of cognitive accommodation, using cognitivist terminology; where knowledge structures are altered in order to accommodate new information. Thus, along with situative theorists, some cognitivists have suggested that the term does not adequately describe what is actually taking place. Using mathematical practices in IT may be understood in this way. Cognitive re-structuring will take place to resolve cognitive conflict (the newness of the problem), and so necessary knowledge and experience will be used to solve the problem to overcome the cognitive conflict.

I suggest that the concept of cognitive accommodation, rather than transfer, can be consistent with the terms used by Salomon and Perkins (1989) and Perkins and Salomon (1989). Transfer in a behaviourist sense could be described as ‘low road transfer’, while ‘high road transfer’ is another way to describe cognitive accommodation. At this point in my discussion I have realised, and argue, that there are so many different ways in which transfer can be understood that it is time in the history of transfer research to come to an agreement about what should be called ‘transfer’ and what should be called ‘learning’.

From a cognitive perspective one might conclude that the students used mathematical practices in the IT tasks to different extents. Some of the practices were used; but in a changed way, so that sometimes the ‘mathematicalness’ of the practices was hidden. It is possible that Carraher and Schliemann’s (2002) arguments concerning transfer as cognitive accommodation are a suitable explanation for the findings in this study.

5.5.2 From a situative perspective

A situated theory of learning does not lend itself to the study of pre-specified transfer of mathematical practices. The original problem that Lave (1988) discussed at length in her book was that there had been too much emphasis in the past on investigating the transfer of

mathematical knowledge to other situations – in particular to peoples' everyday lives. One of her main reasons for trying to understand in a different way how people use mathematics in ordinary situations was that transfer studies traditionally investigated specified aspects of mathematics. In so doing, researchers would pre-define what they expected to be transferred and would investigate this transfer without taking into consideration the complex social learning and experiences of people, and how these experiences influence decisions they might make and how they might solve problems. The environments in which the transfer studies took place were contrived by the researchers, and were also separated from the ordinary experiences of their subjects. Therefore, for Lave the word "transfer" has cognitive connotations and brings to mind limitations of such a description, with respect to certain learning dimensions (Lave, 1988) (cf chapter 2). Situated theorists suggest that the word "transfer" is replaced with "generality" or "productivity" (Greeno, 1997); or "generative learning" (Engle, 2006), which is "learning that results in the flexible use of what has been learned in a wide range of relevant future situations" (ibid. p. 452).

What this meant for my data is that I did not expect mathematical practices to be transferred directly from the mathematics to the IT context, because of the differences between the two contexts. Because a situated perspective of learning describes learning through understanding how all social and cultural aspects of a person's life are parts of the learning experience, the notion of transfer of specific and separate mathematical practices does not make sense in such a perspective. If students were to draw what they needed to solve IT tasks from all of their past experiences, it means that their mathematics class was only one of those experiences and their mathematical knowledge would have been used together with all the other aspects to solve the IT problems. From this perspective it then became almost impossible to identify the mathematics that was "transferred" to the IT tasks. Rather, I needed to examine whether or not the IT problems were solved, how they were solved, and where the students made direct reference to the mathematics that they might have used. Also, I could not conclude that there was "failure to transfer" if the students did not specifically refer to mathematics knowledge during the interviews. All I can say is that they were successful or not in doing the IT tasks in the time given for the interview.

Practices were unlikely to be transferred directly and without change to the new context (Lobato, 2006) because firstly, they were being used within the entirely new context of IT, and secondly, they were being used by people who continued to learn academically and

socially since the practices were first introduced. The new experiences brought into the task situations will have changed the ways that the students interpreted and solved those particular new tasks. Examples of "successful" or "unsuccessful" transfer should be scrutinised with these situational factors in mind.

Task performance could have been influenced negatively by a number of factors, resulting in some students' inabilities to complete the tasks or obtain incorrect results. At the time the interviews were run the students were in the middle of the semester, with assignments for their current courses due (this they expressed to me) and end-of-year examinations around the corner. The fact that the students were with me being interviewed rather than working on an assignment may have made some of them anxious about what they were not doing and so not focus on the given tasks. Furthermore, it is possible that productive disposition towards the IT course material, as well as mathematics, could have influenced how tasks were done (Kilpatrick et al., 2001). A bad experience with certain course material may have resulted in a negative attitude towards the task, so negatively influencing task completion. Alternatively, if they had had good conceptual understanding of the course material and a positive attitude towards the task, it would be more likely that they would be able to complete the task successfully.

As well as the interview timing and environment playing important roles in influencing transfer of mathematical procedure, the social factors (of which I was both aware and unaware) must be considered, if I am to interpret transfer from a situated perspective (Lave, 1988). Each individual of the pair of students interviewed would have brought understanding, experiences, beliefs about both mathematics and IT, personal fears and insecurities, and so on, to the table. Once they started working on the task together, those understandings, experiences, etc, will have been distributed over the pair, so that completion of the task (or not) was performed by the pair and not the individuals. The shared knowledge present between two people meant that one person's comments and ideas could be taken up and built on by the other person. Shared knowledge goes even further than this. Un-verbalised thought processes stimulated by the question and related comments would have been simultaneous with verbalised suggestion, argument and justification. This would have been aided by their discussions during working with the task – bouncing ideas off each other, accepting or rejecting these ideas, and finally coming to consensus about the final product – either completed successfully or not completed.

It is important to acknowledge that the interview environment was not the natural learning environment of the students. It was set up by the researcher (myself) in order to observe something pre-defined by the researcher. For a true situated perspective I should have observed the students' interactions and tasks in their natural learning environments (the lectures and tutorials) and made extensive notes about the practices they were using there. However, this was not possible, given the scope of this research project. Therefore, I had to make do with the unnatural environment and data collection, and have taken it into account in my interpretations of what I observed. The students were asked before the interviews started to verbalise what they were doing as much as possible while they were doing the tasks; because the mathematical practices being used needed to be as explicit as possible in order for me to be able to recognise them in an IT context. Therefore, they had an idea of what I was investigating, which would have acted as a kind of cue to link the tasks with their mathematical knowledge.

If transfer is to be understood in this light, then all those interviewed were able to use mathematical practices to a greater or lesser extent to do the tasks, because they recognised what needed to be done and began to do it. This was true for all the tasks, but I use the following as examples to provide evidence for my claim.

For the computer systems task conceptual understanding of AND/OR linked with $\times$/+ and $\cap$/$\cup$ respectively appeared to be present, transferred to, and used within the IT context for all of the students interviewed; although they did not all verbalise recognition of the links between the probability theory and Boolean concepts. Where the second part of the task required flexible use of conceptual understanding of new identities three out of the four pairs of students interviewed successfully manipulated and used Boolean mathematical identities to simplify expressions in a new context. The others recognised what they needed to do, but were not successful in completing the problem in the limited time given. This would possibly have been interpreted as partial transfer or failure to transfer, from a cognitive perspective; but from a situative perspective, as already discussed, is a more complicated interpretation. Students also commented that they found it easier when they shared the task, which demonstrates the significance of the social interaction for learning. I discuss the interactive aspects of learning with respect to transfer of mathematical practices in more detail in chapter 6.

All students recognised their need to strategically use their understanding of prime numbers, together with Java-specific syntax to write the prime numbers programme (cf 5.4.4.). Likewise, they understood the need to strategically plan their use of Boolean identities to simplify the Boolean expression; or plan how the components of the banking system should interact to produce an efficiently working system; or work through how each of the OSI layers functioned, to help them decide which were implemented in network security. Concerning the prime numbers and banking system tasks, nobody completed the task: some got further than others. Therefore, it would not be accurate to suggest that transfer occurred for some students and not others. It is possible that the ways the students used strategic thinking demonstrates how the extent of transfer depends on the levels of competence and familiarity they had with the subject matter to be transferred (Alexander & Murphy, 1999).

In addition, one may question the legitimacy of a conclusion that any strategising observed in the solution of an IT problem is the result of transfer of mathematical practice of strategising. The same may be true for the practice of argumentation. My answer to this is: If I was to conclude that such practices are solely the result of transfer of mathematical practices, I would be guilty of trying to put mathematical practices into a box. As a mathematics education researcher, I am attempting to promote the opposite of this – mathematical practices should be part of a person's whole experience. Therefore it would be logical that all of a person's experiences, including those of mathematics would ideally be implemented in solving problems – not only IT, but also problems in other domains, the workplace and everyday life.

Mathematical argument would necessarily implement mathematical conceptual understanding, procedures, representations and justification, as well as strategies for constructing credible argument and an overall belief that mathematics as a whole made up of many aspects makes sense. This list includes all of the strands that would be shown in the proficient mathematics user. Since the networking and banking system tasks contained no mathematics it could have been concluded that mathematical argument, reasoning and justification were not used in the IT tasks. This conclusion seems a little too simple – belief exists amongst many mathematics and IT practitioners that mathematical thinking is essential for learning IT. Insight into how students might perceive the importance of mathematical practices to solve IT problems was given by Tungamirai in the following conversation after he and Eric had completed their tasks:

> **LM:** *Now I've actually asked you to do a task … but, what do you think about … I mean, if somebody comes along to you and says 'but we're not doing maths in IT – I don't know why we had to do maths". What would you say? Why?*
>
> **Tunga:** *Its funny you asked that question, 'cos I was actually thinking to myself as we were going through this, 'I'm not using maths in any way here'. But I hadn't actually realised that I was actually using all the things I had learned. But its not the same like in the practical like we did in maths. Its just that way of thinking like we used in maths, like to problem solve, to generalise, and all the supporting stuff. So in the end, like, you might not see it physically as in doing the maths, but you are given that background where you are able to have systematic thinking and systematic approach to solving a problem … solving things, y'know? And that is actually what we learned. And I think if you don't have maths you won't be able to do IT – or you don't have maths thinking. I don't think the content is really relevant – its just the thinking and the approach you take.*

The "supporting stuff", as Tungamirai described it, is shown in Tables 5.1. to 5.4. observed when the students were using mathematical practices of understanding/explaining concepts, using representations, questioning, justifying, disagreeing, strategising and generalising, for non-mathematics IT tasks. From a situative perspective, this comment explains how mathematical practices can be relevant and usable in IT.

5.7. Conclusion

The students used mathematical practices to solve IT problems to a greater or lesser extent. From a situative perspective they could be said to have transferred mathematical practices to IT; but the extent of transfer was not measurable because such a perspective of transfer takes into account all applicable contexts in relation to the students. From a cognitive perspective although the study was not designed in a cognitive experimental format, the ways that students approached the solution of the IT tasks could be interpreted as complex cognitive accommodation, in the process of further learning and use of knowledge in new contexts.

In the final chapter I return to my three research questions and attempt to answer them by drawing from the results of the study, and the theoretical and analytical frameworks used to analyse the data, and link those with my understandings of transfer from both cognitive and situative perspectives. I make a suggestion about how transfer of mathematical practices might be facilitated. In addition, I discuss the limitations of the study and make suggestions about how the study may lead to further investigation.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1. Introduction

This research study deals with students' experiences of teaching approaches used during their Foundation and first years of study, whether or not they believed the teaching approaches they experienced provided them with useful learning strategies, and the extent of their use of the mathematical practices of representation, justification and generalisation in undergraduate study in IT. Chapter 4 of this work was a discussion of what the students said they experienced, in terms of teaching approaches in the Foundation Programme and first year of undergraduate study, and their opinions about the usefulness of the mathematical practices and other techniques they were exposed to in the Foundation Programme now that they were studying first year Information Technology. Chapter 5 detailed the students' actual uses of mathematical practices while they were doing various IT tasks that came from their first year subjects. In this chapter I link two important issues that emerged from chapters 4 and 5. First I account for the similarities and differences between what the students said about the extent of use of mathematical practices and what they actually did, taking account of both cognitive and situative perspectives on learning. Second I relate the students' experiences of teaching approaches in the Foundation Programme to the transfer of mathematical practices into solving IT problems. In the last section of this chapter I make some suggestions about where this study might be used as a basis for further research in the field of transfer of mathematical practices to further post-school study in non-mathematics areas.

6.2. Transfer of Mathematical Practices

My evidence suggests that different aspects of the phenomenon of transfer, or generality, can be investigated through focus group interviews and task-based interviews. The focus group interviews helped me to understand the students' experiences of the teaching and learning

situation prior to use in first year IT. Talking about their experiences gave some insight into what they thought about mathematical practices prior to a situation where they might need those practices. Doing tasks that used some or all of the practices that they developed in the Foundation Programme gave insight into how mathematical practices might be selectively and differentially used to solve novel problems in a different knowledge domain.

6.2.1. What students said / What they experienced

The main findings concerning what students said and experienced were as follows. Firstly, they believed that what they had learned in the Foundation Programme was of benefit to them because their mathematical proficiency had increased through the use of what they experienced as student-centred teaching approaches. Bearing in mind that mathematical proficiency consists of five interwoven interdependent strands, not all the students interviewed experienced increased proficiency in the same ways. Some found that their conceptual understanding had vastly improved; positively impacting their productive disposition. Others had noticed their improved abilities to argue and justify their thinking; or spoke of their improved problem-solving abilities, also leading to improved productive disposition. Although the students may have left the Foundation Programme with differing levels of mathematical proficiency, they spoke positively of their increased proficiency, which was manifested as increased productive disposition in many different ways (cf. 4.4.2.).

Secondly, a few students specified that some practices and new-found skills that they had used in the Foundation programme were very useful and that they were continuing to use them in undergraduate study. These included mind mapping content and concepts for planning assignments and tasks and remembering large amounts of information, problem-solving strategies, and finding ways of justifying ideas to others verbally.

Thirdly, many students said that they had found certain approaches useful in mathematics, but that these approaches were not always useful in undergraduate study. They would explain this almost apologetically, because they knew that I considered them important; but the reality was that their studies were too intense, they were too stressed, and their work was too different from mathematics to use much of the mathematical practices that they learned in the

Foundation Programme. Some commented that deliberate questioning and argumentation was important for learning, and had helped them to understand mathematics better, but did not feature much at all in their learning in first year, because they did not have time. Most lecturers did not promote these practices, and passing their courses was a matter of survival for the students.

Fourthly, when asked what, if anything, they used of the mathematics they had learned in the Foundation Programme in first year, they tended to try to list all of the content that was similar between their courses and mathematics, rather than describe mathematical practices. However, after comparing what they said with what they actually did when they were doing IT tasks, it appeared that students were, in fact, using these important practices, for study in IT, although they were mostly not consciously aware of this. Some said they were not using practices, but meant teaching approaches and learning techniques, such as group discussion and mind mapping. They demonstrated when doing IT tasks that they were using all or most of the mathematical practices. These practices included using procedures flexibly; using representation; understanding/explaining concepts; questioning; justifying; disagreeing; strategising; and generalising and were aspects of the practices of representation, justification and generalisation described by the RAND mathematics Study Panel (Ball, 2003). However, they were used in different ways from their original uses, because now they were used to solve IT problems.

Further comparison of what the students said about using mathematical practices in IT, with what they actually did is made after a brief summary is given of the findings of the task-based interviews in terms of use of mathematical practices by the students.

6.2.2. What the students actually did

In chapter 5 I showed that mathematical practices (Ball, 2003) were used by all of the students interviewed, to a greater or lesser extent. The practice of generalisation was seen infrequently, while representation and justification were seen frequently in different ways.

Some mathematical content was required to do the computer systems and prime numbers tasks. In these cases the use or non use of mathematical practices was more obvious, because the IT itself was more similar to mathematics, and the practices are situated in mathematics. Where the IT being done was not mathematical, but required practices such as representation, strategising, decision making, reasoning and justification, it was more difficult to identify the mathematical practices being used, because mathematical conceptual understanding and procedures were not present. I suggest that such may be a case of high road transfer (Perkins & Salomon, 1989; Salomon & Perkins, 1989), and the mathematical practices being used would have had to go through a process of alteration in order to be usable in the context.

For the computer systems task and the prime numbers task, the mathematical practices were used in ways which were expected. These included using standard representations of binomial probabilities, factorised and simplified Boolean identities and listing the sample space for combinations of ON and OFF, and choosing and justifying the choices of Boolean identities in order to simplify Boolean circuit representations. These representations required the students to have conceptual understanding of the mathematics they used. Defining and representing prime numbers for use in a computer programme required the conceptual understanding of prime numbers to be applied in a programming situation. The application of mathematical knowledge was transferred even further than to new mathematical contexts when mathematical thinking was required to solve novel IT problems – a possible demonstration of far transfer. Students were seen to use the mathematical practices of representation, justification and generalisation even in this possible far transfer situation; although some appeared to show far transfer more clearly than others, through specific verbalisation while doing the tasks, and through their extensive use of representation. Students who verbalised their disagreements and argued and justified their cases made it easier for me to analyse the kinds of practices they were using to do the tasks. It also appears that the ability to far transfer mathematical practices was facilitated by the students' working together on the tasks.

The question may be asked: Would these practices used in IT be mathematical practices that were transferred, or are they IT practices that are very similar to mathematical practices? I argue that they are transferred mathematical practices, because these practices are so clearly mathematical, even though they might well be part of the practices necessary for proficiency in another domain. The students used them where necessary in conjunction with mathematical

content knowledge; and at other times used the practices separately from mathematical content knowledge. They were, however, identified as the same practices. Therefore, the practices used are more likely to be transferred mathematical practices, because they were most likely developed as intertwined strands in mathematics classes, rather than in anther domain. From this point of view, many IT practices are transferred/generalised mathematical practices.

Note that I do not argue that these practices necessarily originate only in mathematics. If one was to understand the use of knowledge in other domains as the transfer of identical packets of knowledge from one context to another, one might be more likely to want to identify a single origin of that knowledge. Alternatively, knowledge used in new contexts may be understood as having multiple origins, and is used however, wherever, and whenever necessary. If this is so, then the conclusion can be made that mathematical practices may have been transferred to an IT context, but were used selectively, with the aid of other experiences and knowledge, to solve the IT problems. The emphasis has been moved from "what" is transferred, discussed in cognitive research, to the "how" does the transfer occur, as discussed in situated research (Hatano & Greeno, 1999) (cf. 2.5.). Mathematical practices are therefore altered from their original mathematical use – hence the word "generalised" (Greeno, 1997) (cf. 5.5.) used as an alternative to "transferred" in the previous paragraph.

Evidence showed that the students used mathematical practices in the new context of undergraduate study in IT. Whether or not this was a case of "transfer" is discussed in more detail below.

6.3. Situative and Cognitive Explanations: Transfer or Not?

Reference to the discussion of a "transfer dilemma" (Carraher & Schliemann, 2002) in chapter 2 (cf. 2.5.) serves to remind the reader that a dilemma exists concerning what should be described as 'transfer', how to study it, and how to understand it. The concept of transfer needs clarification. Where do we draw the line between calling what we observe 'transfer' and calling it 'learning'? Or is transfer an integral part of learning? Or can transfer theory be understood as a new learning theory?

The findings of this study are in agreement with findings from the literature – knowledge learned in one context can be used in another context. Use of mathematical practices was in evidence when students were solving IT problems, but these practices were not an exact replica of what they were when they were developed and used in the mathematics class. I expand on a point I find particularly interesting. Some of us have experienced thinking of some mathematical knowledge we know from school to answer a question or work out something mathematical. Then when someone asks us how we knew we should do it that way, we say "well I don't know where I learned that, but I just know it". And then we use it in various other contexts; not being too concerned where the knowledge came from originally – it is merely useful to draw from to solve the immediate problem.

When the students in my investigation were using mathematical practices and content knowledge to solve problems in a new domain, they appeared to do so without being constantly aware of the mathematical practices being used. Being largely unaware of the mathematical practices they might have been using was a possible reason why they either did not talk about them much in the task-based interviews, or specifically said they did not use these practices during the focus group interviews. But they did use the practices, and used them appropriately, to solve the IT problems. Carraher and Schliemann (2002) spoke of the children in their study 'crafting' a solution that they were satisfied with, using whatever knowledge and experience that was available for their crafting, but not being consciously aware of it. This supports my findings. Therefore, it is not necessarily surprising that in the task-based interviews only three students deliberately referred to mathematical content that they were using while they were solving IT problems, and only one referred to how the practices learned in mathematics had influenced his thinking in IT in general. The others did not verbalise similarities between their prior learning and the new situation. What they were doing was using whatever prior learning and experience they had that was necessary to solve the problems; in conjunction with knowledge and experience of their peers as part of the solution process.

From a situative perspective, the social and cultural backgrounds of the students were an integral part of crafting a solution. They were drawing from relevant previous experiences and situations to solve the given problems. Some were able to do this more successfully than others, because some pairs worked through the tasks more competently in the time given. Alternatively, this finding could also be interpreted as students cognitively restructuring

knowledge in a very complex manner, so that it could be used to solve new problems in IT (a new domain and therefore across a very far cognitive distance). Therefore, in the light of this argument, the term 'transfer', meaning "transfer of identical elements", is misleading and unhelpful for explaining the findings of this study, and in accordance with recent arguments in this direction of research.

Furthermore, the students were outspoken about how mathematics had been made not only more understandable to them, but also more sensible and meaningful. Their increased access to mathematics allowed them to see themselves as increasingly competent doers of mathematics. Regardless of whether or not I can conclude that "transfer" or "generative learning" or "cognitive re-structuring" occurred, I can conclude that their learning experiences had become a part of the knowledge they could draw on to solve future problems in an academic context.

In conclusion, learning theorists appear to have a wide range of explanations for the same observations of how students solve novel tasks. I suggest that they are making similar points, but are using the terminology consistent with their own theoretical perspectives. I argue, therefore, that a 'bricolage' of learning theories does appear to be effective to lend a broader understanding of how students use mathematical practices to solve non-mathematics problems.

Students' responses in the focus group interviews brought my attention to another interesting but unexpected possibility with respect to use of mathematical practices in contexts other than mathematics. The role of productive disposition may be far greater than that of interdependence with the other strands involved in mathematical proficiency. I alluded to this in section 4.4.2. and here I take it up further and raise further questions.

6.4. Making New Links

In chapter 4 productive disposition was alluded to many times by all of the students throughout the course of the focus group interviews. I began to understand more deeply its role in gaining mathematical proficiency, but also realised that it has a possible important role

in high road transfer of mathematical knowledge and practices. To explain what I mean, I refer to Figure 6.1 below. My suggestion is that a student-centred teaching approach and productive disposition both contribute to transfer of mathematical practices to the IT domain; but that in a different way, student-centred teaching also contributes to improved productive disposition. I suggest that these links deserve deeper investigation.

I have already suggested a link between student-centred teaching and transfer, and I begin my discussion with this connection. Thereafter, I propose two new relationships: the first between student-centred teaching approaches and productive disposition, and the second between productive disposition and transfer. I explain each relationship below and then discuss the implications thereof.

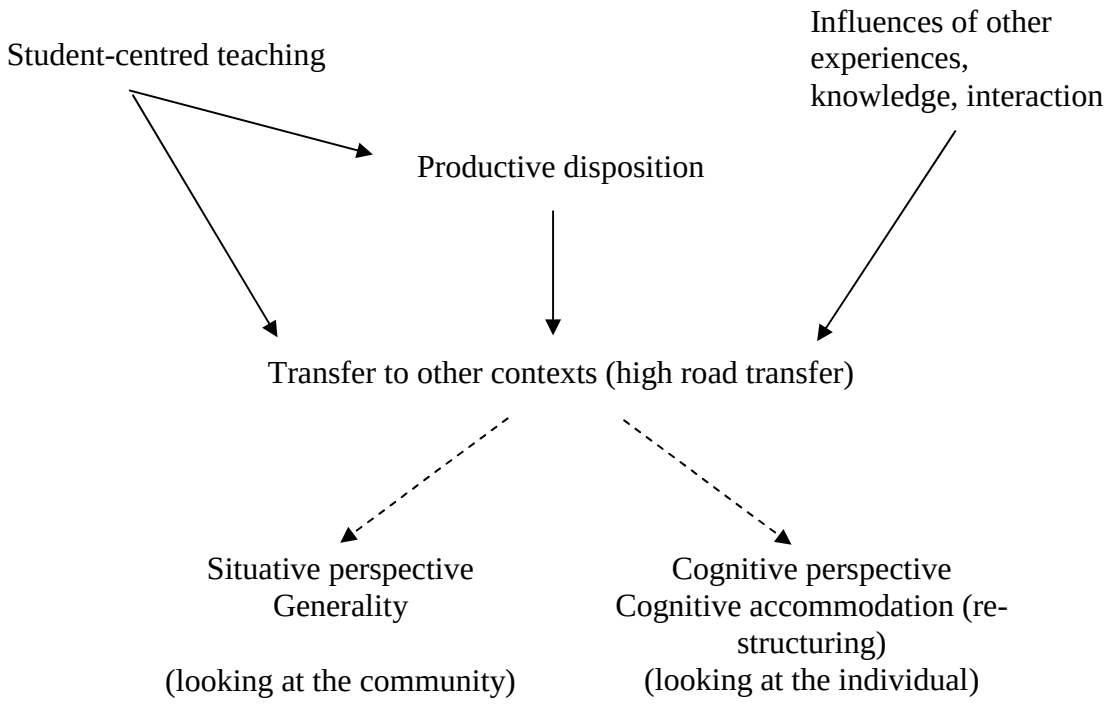


Figure 6.1. Proposed model of links between student-centred teaching approaches, productive disposition and transfer of mathematical practices

6.4.1. The link between student-centred teaching and transfer

I acknowledge that I cannot conclude that because students experienced my teaching approach as student-centred, I therefore teach using a student-centred approach. Since the students in the sample experienced my teaching approach as student-centred, in terms of the three categories of student-centredness described by Brodie et al (2002), there appears to be a connection between student-centred teaching and use of the mathematical practices encouraged in the Foundation Programme classroom in undergraduate IT.

This link has been made by a number of authors in the recent past, although the term "student-centred teaching" was not necessarily used. Engle's (2006) discussion of "framing interactions" in order to "foster generative learning" is an excellent example of how teachers can use pedagogy to foster transfer. Others, for example, Greeno (2006), Hatano and Greeno (1999), Boaler (2000a, 2000b), and Lobato (2006), have emphasised the importance of using pedagogical practices to help students to use their knowledge in different contexts. Words and phrases, such as "positioning" (Greeno, 2006), and "focusing phenomena" (Lobato, 2006) have been used to distinguish pedagogies informed by a situative perspective, from more traditional pedagogies informed by constructivism or behaviourism. Perkins and Salomon speak of "guidelines [being] available for classroom practices that can foster the transfer of knowledge and skills" (Perkins and Salomon, 1987, 1988 in Perkins & Salomon, 1989). Constructivists also emphasise the importance of helping students to recognise where prior learning may be useful in potential new situations, using concepts such as "cueing" (Alexander & Murphy, 1999), and "scaffolding" (Vygotsky, 1978); or speak of the extensive degree of cognitive accommodation needed by students to use solve novel mathematical problems, which can be facilitated by the teacher (Carraher & Schliemann, 2002; Simons, 1999). Therefore, whether cognitive or situative, theorists agree that transfer is facilitated via deliberate intervention through the teacher's use of a student-centred pedagogy.

In this study, through relating students' descriptions of their experiences and views in the Foundation Programme mathematics class, I showed that they had experienced a student-centred teaching approach. I also showed that the students were able to use the mathematical practices they had developed in the Foundation Programme to solve IT problems in first year undergraduate study. Therefore, although student-centred teaching is not necessarily the only way in which transfer can be fostered, my study shows that in accordance with other studies,

student-centred teaching may foster transfer. In the case of this study the 'transfer' that took place was across domains.

6.4.2. The link between student-centred teaching and productive disposition

In chapter 4 I showed that the students described their learning experiences in the Foundation Programme mathematics classroom as being student-centred. Without them necessarily using the term, they described the curriculum, the pedagogy and the relationship I had with them (cf. 4.3.) as being helpful for learning and understanding mathematics. Many students described how mathematics had been opened up for them – it made more sense and they were able to solve problems because the problems made more sense. Others spoke about being able to make conceptual connections that had not been apparent previously. They were given access to the choices that could be made concerning solution strategies and interpretation of their solutions, as well as the problems themselves. They were also aware, when they moved into first year IT, that some of the lecturers designed their teaching so that learning was facilitated, and others did not. Those who did not were reported as being teacher-centred. Chapter 4 also details how the students appeared to have markedly improved their productive disposition with respect to mathematics because of this. Many described how they were able to see problems as challenges, rather than as indecipherable; and how mathematics related to their lives and other learning domains. Again, it was clear that mathematics had begun to make sense to them.

Therefore, the students' "habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001 p. 116) was enhanced, through their reported experiences in the FP programme. I argue that this conclusion is important as a base for the link I will make between productive disposition and transfer of mathematical practices to the non-mathematics domain of IT.

6.4.3. The link between productive disposition and transfer

I suggest that increased productive disposition enhances the possibility for transfer – even far transfer between domains. As explained above, allusions to productive disposition occurred mostly in the focus group interviews, while mathematical practices used in first year IT were identified more clearly in the task-based interviews, and talked about infrequently in the focus groups. How then can I link productive disposition to transfer? The link I make is tentative, and more research on this issue is needed. This is my argument so far:

Because students displayed a variety of mathematical practices while they were doing the IT tasks, I was able to conclude that they were probably not aware that they were actually using practices that had originated in their mathematical knowledge. This was the reason why they either did not talk much in the task-based interviews about most of the mathematical practices that they used in first year; or even deliberately said that they did not use them much for various reasons (cf. 4.5.1.). It appears that even though I had attempted to make the proficiency strands explicit to the students, these were either not made explicit enough; or the strands and their associated practices were not used for a long enough period of time to become part of the students' identities and awareness; or the students used them, when necessary, but were not consciously aware that they were using practices that were mathematical in origin. The second and third points are more likely, for the reason stated below, to be explanations for most students' failure to verbally link mathematical practices to the IT tasks they were doing.

The strands had been made explicit to the students, and the Math B calculus course was designed solely around students' development and use of practices embedded in the proficiency strands. However, they only had a period of thirteen weeks where they were explicitly using the strands of conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning to help them to learn mathematics. While the students responded positively to this approach in my classroom (albeit, after a period of doubt and disbelief), they did not have a chance to cultivate their understanding of how the strands were central to learning mathematics over an extended period of time. They had commented that most of their first year lecturers had not used an approach that would help them learn new content or link what they had learned about the proficiency strands to new knowledge. This would explain why only one student recognised that what he was doing a lot of the time in IT

was actually part of the set of mathematical practices he had developed in my classroom; or linked the practices he was using in IT to the practices he had developed in mathematics (refer to Tungamirai's comment cf. 5.5.2.). However, not being aware of their use of mathematical practices does not imply that they did not transfer these practices to IT.

Thus, how do I make the link between productive disposition and transfer, if the students did not consciously make it? Student-centred teaching positively influences productive disposition[13] (Brodie et al., 2002), and positively influences transfer (e.g. Engle, 2006; Greeno, 2006; Lobato, 2006). The third link requires further investigation, but my study indicates a possible relationship between productive disposition and transfer. If productive disposition is the ability to see mathematics as worthwhile and a belief in one's ability to do mathematics (Kilpatrick et al., 2001), then, because of the interwoven nature and interdependence of the strands, productive disposition influences one's abilities to use mathematical practices. The students spoke about their increased productive disposition and demonstrated high road transfer. It is therefore possible that increased productive disposition was partly responsible for their abilities to use their mathematical practices in novel problems in a new domain (a case of far transfer). Thus, it is possible that productive disposition directly influences transfer. Conversely, it is also possible that being able to use one's knowledge in new situations positively influences productive disposition. This converse relationship makes sense if one considers that the proficiency strands are interwoven and inter-dependent. It also makes sense that a person's ability to see mathematics as worthwhile, and to recognise his or her increased ability to understand and do it, will also increase his or her ability to use it elsewhere in new situations – or vice-versa – even in other domains. If this is the case, it also helps to explain why the presence of strong positive productive disposition is important for development of proficiency of the other strands; especially adaptive reasoning and strategic competence, which are necessary for successfully doing many kinds of IT problems.

[13] In student-centred teaching interpersonal relationships, pedagogy and curriculum together improve learning. In mathematics learning manifests as using mathematical practices (Ball, 2003), and increased productive disposition, as an aspect of increased proficiency (Kilpatrick et al., 2001).

Continuing this line of argument, I argue that the teacher is responsible for deliberately planning to aid the development of productive disposition in the mathematics student. S/he can do this by actively cultivating in his or her classroom a student-centred approach, in the areas of both pedagogy and interpersonal relationships. Where other authors have suggested using teaching approaches such as cueing (Alexander & Murphy, 1999), framing (Engle, 2006), using focusing phenomena (Lobato, 2006), or others, in order to facilitate transfer, I propose to add 'cultivating productive disposition' to the list.

6.4.4. Summarising the model

In conclusion, use of mathematical practices to solve non-mathematics IT problems may be a case of far transfer (Perkins & Salomon, 1989) or of complex cognitive accommodation (Carraher & Schliemann, 2002), or 'generality' (Greeno, 2006). Whatever we call it, it entails the selective use of prior knowledge and experience (not only mathematical), and the 'crafting' of unique solutions. Student-centred teaching may be a key link in students' abilities to transfer knowledge, because as well as being integral to facilitating transfer directly, it also appears to be implicated in improving students' productive disposition. Productive disposition, in turn, appears to be the phenomenon that helps students to recognise the usefulness of mathematics in other contexts. Further research into these interlinking aspects of learning mathematics may be invaluable for making mathematics more useable in non-mathematics contexts.

6.5. Limitations of the Study

6.5.1. The sample

Interviewing my own students was a possible source of bias. This included the possibility of my not being objective, of reading more into students' responses than I should, asking biased and directed questions, and not knowing if or how students who had other mathematics teachers in the foundation programme had developed and used mathematical practices in

undergraduate study. With respect to the last disadvantage listed, the study would have been better informed if a wider range of ex-Foundation Programme students could have been interviewed; but the teaching approaches of the other mathematics teachers were not necessarily the same as mine, and therefore, I would not be able to draw the same kinds of conclusions from other students as I had from my own. One cannot generalise the findings of my study to all teachers in this or other foundation programmes, because it depends on how they have worked with the teacher/learner/curriculum/environment interfaces, as to the resulting mathematical practices students may or may not use once they leave their respective foundation programmes.

With respect to the first three disadvantages of interviewing my own students, these disadvantages are typical of any interview data. I suggest that the study would have been more objective if I had interviewed students from another institution, who I did not know; but then my research questions would have been different, since I would not be linking students' current experiences with those in my Foundation Programme classroom. Therefore, although interviewing my ex-students had the possibility of bias creeping into interviewing techniques and data analysis, I was careful to design interview questions to be as rigorous as possible (see chapter 3), according to suggestions by Merriam (1997). Categories were introduced that would allow for contradiction of any findings that I might have favoured and emphasised, such as students saying that they had had good experiences in my mathematics class.

6.5.2. Analytical indicators

The biggest difficulty I had in data analysis in this study was that a complete set of detailed indicators for what counts as 'mathematical practices' is not available. 'I linked mathematical practices with proficiency by searching for commonalities between the Ball (2003) and Kilpatrick et al (2001) texts, over and above those already made by Ball (2003), in order to reinforce their relationship, and to be able to describe how mathematical practices were used to solve IT tasks. A more detailed set of indicators would be very helpful for more detailed studies along the same theme as this one.

The same may be said of transfer – currently there is a lot of debate around what constitutes transfer, but very little consensus about the terms that should be used to characterise the different kinds of transfer that can take place. Part of the dilemma is that we still do not know if transfer should be described through existing learning theories, or if it is a theory of its own that needs it own definition. Transfer researchers have suggested varying ways to explain, describe and classify transfer (e.g. Barnett & Ceci, 2002; Carraher & Schliemann, 2002; Salomon & Perkins, 1989); but in order for this phenomenon to be studied, an analytical framework, through which different kinds of transfer observed can be categorised, must be provided first. It is easy to become bogged down in the debate and then become unsure how to analyse data, because there are no common terms to describe the different kinds of transfer one might observe.

6.6. Recommendations

This research study was informed by my initial curiosity about the students who I had taught. How had they experienced my teaching? Did the short period my students spent with me have a positive influence on their undergraduate learning in a subject that requires a certain extent of mathematical knowledge or accomplishment as an entrance requirement? How was mathematical knowledge used in their undergraduate studies? What would the answers mean to me in my teaching of future students in a similar environment? These questions translated into the research questions that guided this study: Firstly, to what extent are students aware of differences in teaching approaches between Foundation Programme mathematics and undergraduate study? Secondly, to what extent do students believe that their experiences of the teaching approaches in the Foundation Programme mathematics class have helped them in undergraduate study in other courses? Thirdly, in what ways are the mathematical practices taught in the Foundation Programme used in undergraduate study in IT?

This study provided some answers to these questions and resulted in a proposed model describing how student-centred teaching, productive disposition and transfer might be related to each other. Further investigation can be done in these areas in the following ways: A second case study may be done, investigating far transfer or generative learning in higher education situations, working with students from different learning backgrounds. Such a study

may throw light onto questions such as ‘Are there other ways in which to learn mathematical practices?’, ‘To what extent are students able to use mathematical practices in new contexts if the practices were not explicitly discussed and fostered with them?’, Does the extent of students’ uses of mathematical practices in mathematics correlate closely with that in other domains, and ‘What are the consequences of teacher-centred versus student-centred approaches for students’ later abilities to use mathematical knowledge and practices in other non-mathematics academic contexts?’.

The possible relationship between productive disposition and transfer hinted at in this study might be deliberately investigated. Such a study would investigate students’ beliefs and use of mathematics, as related to each other and other non-mathematics domains.

In addition, I made the assumption that developing questioning practices among students was good for the students in terms of framing aspects of mathematics that were considered important; that is, the strands of mathematical proficiency. I did this before I had thought about this particular study and I made assumptions about the usefulness of questioning as a practising teacher. It would be useful to study how deliberately asking questions to frame particular aspects of mathematics (proficiency strands) help students to develop mathematical practices. Also, does deliberately teaching students how to frame questions focusing on each of the strands make maths more useable in other domains?

6.6. Concluding Remarks

In my introductory remarks in chapter 1, I raised the question of the relevance of mathematics to study in IT. My research shows that mathematics is necessary for study in many areas of IT. However, contrary to what is commonly believed, it is not necessarily the mathematical content that is transferred, using a traditional understanding of transfer of elements of knowledge, but rather the situative transfer of mathematical content and practices by the student to the new context of IT. I observed that new knowledge or solutions to problems in a new domain may be intricately ‘crafted’ by the student, using not only the necessary mathematical content and practices, but also any necessary IT content knowledge, *plus* any social or cultural experiences necessary to do the problem. And all this can be made more

possible by using a student-centred approach to teaching, preferably in both the prior learning and in the current learning contexts.

REFERENCES

Alexander, P. A., & Murphy, P. K. (1999). Nurturing the seeds of transfer: a domain-specific perspective. *International Journal of Educational Research, 31,* 561-576.

Alexander, S., Bishop, P., Crawford, E., & McCartney, M. (2006). Mathematics for computer scientists: problems and solutions. *Int. J. Mathematical Education in Science and technology, 37*(5), 505-513.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated Learning and Education. *Educational Researcher, 25*(4), 5-11.

Babbie, E., & Mouton, J. (2001). *The Practice of Social Research*. Cape Town: Oxford University Press.

Ball, D. L., (Chr). (2003). *Proficiency for all students: Toward a strategic research and development program in mathematics education. DRU-2773-OERI, Rand Mathematics Study Panel for the Office of Educational Research and Improvement, US Department of Education* (Vol. 2008). Santa Monica, CA: RAND. Available online at: http://www.rand.org/pubs/monograph_reports/MR1643/index.htm, accessed August 9, 2008.

Barnett, S. M., & Ceci, S. J. (2002). When and Where Do We Apply What We Learn? A Taxonomy for Far Transfer. *Psychological Bulletin, 128*(4), 612-637.

Bell, J. (2005). *Doing your Research Project* (4th ed.). Berkshire: Open University Press.

Bernstein, B. (1982). On the Classification and Framing of Educational Knowledge. In T. Horton & P. Raggatt (Eds.), *Challenge and Change in the Curriculum* (pp. 157 - 176): Milton Keynes: The Open University.

Bernstein, B. (1996). Pedagogic codes and their modalities of practice. In B. Bernstein (Ed.), *Pedagogy, symbolic control and identity: Theory, research and critique* (pp. 17 - 38). London: Taylor & Francis.

Berry, J., McIntyre, P., & Nyman, M. (1999). *Student Centred Learning in Undergraduate Mathematics.* Paper presented at the Delta '99 Symposium on Undergraduate Mathematics: The Challenge of Diversity, Brisbane, Australia.

Bitzer, E. (2005). First-year students' perceptions of generic skills competence and academic performance: a case study at one university. *South African Journal of Higher Education, 19*(3), 172-187.

Boaler, J. (1997). *Experiencing School Mathematics. Teaching Styles, Sex and Settings.* Buckingham: Open University Press.

Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Boaler, J. (2000a). Exploring Situated Insights into Research and Learning. *Journal for Research in Mathematics Education, 39*(1), 113-119.

Boaler, J. (2000b). Mathematics from another World: Traditional Communities and the Alienation of Learners. *Journal of Mathematical Behaviour, 18*(4), 1-19.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239-258.

Boaler, J. (2003). When Learning no Longer Matters – standardized testing and the creation of inequality. *Phi Delta Kappan, 84*(7), 502-506.

Britton, S., New, P. B., Roberts, A. L., & Sharma, M. D. (2007). Investigating students' ability to transfer mathematics. In J. Sachs & A. Brew (Eds.), *Transforming a University: The scholarship of teaching and learning in practice*. Sydney, NSW: University of Sydney.

Britton, S., New, P. B., Roberts, A. L., Sharma, M. D., Brew, A., & Sachs, J. (2007a). Investigating students' ability to transfer mathematics. In *Transforming a university: the scholarship of teaching and learning in practice* (pp. 127-140). Sydney: Sydney University Press.

Britton, S., New, P. B., Roberts, A. L., Sharma, M. D., Brew, A., & Sachs, J. (2007b). Investigating students' ability to transfer mathematics. In A. Brew & J. Sachs (Eds.), *Transforming a university: the scholarship of teaching and learning in practice.* Sydney, NSW: Sydney University Press.

Britton, S., New, P. B., Sharma, M. D., & Yardley, D. (2005). A case study of the transfer of mathematics skills by university students. *International Journal of Mathematical Education in Science and Technology, 36*(1), 1-13.

Brodie, K., Lelliott, A., & Davis, H. (2002). Developing learner-centred practices through the FDE programme. In J. Adler & Y. Reid (Eds.), *Challenges of teacher development: an investigation of take-up in South Africa* (pp. 94-101). Pretoria: van Schaik.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.

Bruce, K. B., Scot Drysdale, R. L., Kelemen, C., & Tucker, A. (2003). Why Math? *Communications of the ACM, 46*(9), 40-44.

Carraher, D., & Schliemann, A. D. (2002). The Transfer Dilemma. *The Journal of the Learning Sciences, 11*(1), 1-24.

Chung, S., & Walsh, D. J. (2000). Unpacking child-centredness: a history of meanings. *Journal of Curriculum Studies, 32*(3), 215-234.

Clarke, D., Breed, M., & Fraser, S. (2004). The Consequences of a Problem-Based Mathematics Curriculum. *The Mathematics Educator, 14*(2), 7-16.

Cobb, P. (2007). Putting Philosophy to Work. Coping With Multiple Theoretical Perspectives. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 3-38). USA: Information Age Publishing Inc.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, Instruction, and Research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed.). London: RoutledgeFalmer.

Cornbleth, C. (1990). *Curriculum in Context*. London: Falmer Press.

Cox, B. D. (1997). The Rediscovery of the Active Learner in Adaptive Contexts: A Developmental-Historical Analysis of Transfer of Training. *Educational Psychologist, 32*(1), 41-55.

Cuban, L. (1993). *How teachers taught: constancy and change in American classrooms 1880 - 1890*. New York: Teachers' College Press.

Darling, J. (1990). Progressivism and individual needs. In N. Entwistle (Ed.), *Handbook of educational ideas and practices* (pp. 43-51). London: Routledge.

De Corte, E. (1999). On the road to transfer: an introduction. *International Journal of Educational Research, 31*, 555-559.

du Plessis, L. A., Janse van Rensburg, G., & van Staden, C. J. (2005). Realities of a foundation programme implemented as a vehicle to align learners cognitively for entry into a national diploma in information and communication technology. *South African Journal of Higher Education, 19*(5), 863-879.

Engle, R. A. (2006). Framing Interactions to Foster Generative Learning: A Situative Explanation of Transfer in a Community of Learners Classroom. *The Journal of the Learning Sciences, 15*(4), 451-498.

Essien, A., & Setati, M. (2006). Revisiting the equal sign: Some grade 8 and 9 learners' interpretations. *African Journal of Research in SMT Education, 10*(1), 47-58.

Fraenkel, J., & Wallen, N. (1990). *How to design and evaluate research in education*. New York: McGraw-Hill.

Freeman, M., deMarrais, K., Preissle, J., Roulston, K., & St. Pierre, E. A. (2007). Standards of Evidence in Qualitative Research: An Incitement to Discourse. *Educational Researcher, 36*(1), 25-32.

Goldin, G. A. (1997). Qualitative Research Methods in Mathematics Education. *Journal for Research in Mathematics Education. Monograph, 9*, 40 - 62 & 164 - 177.

Gomaa, M., Powell, M. D., & Vijaykumar, T. N. (2004). *Heat-and-run: leveraging SMT and CMP to manage power density through the operating system.* Paper presented at the Architectural Support for Programming Languages and Operating Systems archive. Proceedings of the 11th International Conference on Architectural Support for Programming Languages and Operating Systems, Boston, MA, USA.

Gomm, R. (2004). *Social Research Methodology: A Critical Introduction*. Hampshire: Palgrave Macmillan.

Greeno, J. G. (1997). On Claims That answer the Wrong Questions. *Educational Researcher, 26*(1), 5-17.

Greeno, J. G. (2006). Authoritative, Accountable Positioning and Connected, General Knowing: Progressive Themes in Understanding Transfer. *The Journal of the Learning Sciences, 15*(4), 537-547.

Gunter, C. F. G. (1980). *Aspects of Educational Theory* (D. G. v. Wageningen, Trans. 5th ed.). Stellenbosch: University Publishers and Booksellers.

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In S. P., P. Nesher, P. Cobb, G. G. & G. B. (Eds.), *Theories of mathematical learning* (pp. 197 - 217). New Jersey: Lawrence Erlbaum.

Hatano, G., & Greeno, J. G. (1999). Commentary: alternative perspectives on transfer and transfer studies. *International Journal of Educational Research, 31*, 645-654.

Henderson, P. B. (2003). Mathematical Reasoning in Software Engineering Education. *Communications of the ACM, 46*(9), 45-50.

Herrenkohl, L. R., & Guerra, R. (1998). Participant structures, scientific discourse, and students engagement in fourth grade. *Cognition and Instruction, 16*(4), 431 - 473.

Hitchcock, G., & Hughes, D. (1995). *Research and the Teacher. A qualitative introduction to school-based research*. New York: Routledge.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington DC: National Academy Press.

Lave, J. (1988). *Cognition in Practice: Mind, mathematics and Culture in Everyday Life*. Cambridge, United Kingdom: Cambridge University Press.

Lave, J. (1993). Situating Learning in Communities of Practice. In L. B. Resnick, J. M. Levine & S. D. Teasley (Eds.), *Perspectives on Socially Shared Cognition* (pp. 63-82). Washington DC: American Psychological Association.

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.

Lerman, S. (1989). Constructivism, Mathematics and Mathematics Education. *Educational Studies in Mathematics, 20*(2), 211-223.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic Inquiry*. Newbury Park, CA: Sage.

Lobato, J. (2006). Alternative Perspectives on the Transfer of Learning: History, Issues and Challenges for Future Research. *The Journal of the Learning Sciences, 15*(4), 431-449.

Lucas, M., Damianova, M., Burney, S., & Ponto, M. (2006). *Globalization and cultural specificity in stress and coping among female and male students studying in South Africa and Australia.* Paper presented at the International Education A Matter of Heart, Kuala Lumpur, Malaysia.

Marini, A., & Genereux, R. (1995). The Challenge of Teaching for Transfer. In A. McKeough, J. Lupart & A. Marini (Eds.), *Teaching for Transfer: Fostering Generalisation in Learning* (pp. 1-19). Mahway, NJ; Hove, UK: Laurence Erlbaum Associates.

Maxwell, J. A. (1992). Understanding and validity in qualitative research. *Harvard Educational Review, 62*(3), 279-300.

Merriam, S. B. (1997). *Qualitative Research and Case Study Applications in Education*. San Francisco: Jossey-Bass Publishers.

Molina, M., & Ambrose, R. C. (2006). Fostering relational thinking while negotiating the meaning of the equals sign. *Teaching Children Mathematics,*(September 2006), 111-117.

Moll, I. (2002). Clarifying Constructivism in a Context of Curriculum Change. *Journal of Education, 27*, 7-32.

National Department of Education. (1997). Curriculum 2005: Lifelong Learning for the 21st Century. A support document issued by the NDE.

National Department of Education. (2002). *National Curriculum Statement Grades 10 - 12 (General) overview* o. Document Number)

Neuman, W. L. (1994). *Social research methods: qualitative and quantitative approaches*. Boston: Allyn and Bacon.

Nussbaum, J., & Novick, S. (1982). Alternative frameworks, conceptual conflict and accommodation: towards a principled teaching strategy. *Instructional Science, 11*, 183-200.

Olivier, A. (1989, 3 - 7 July). *Handling Pupils' Misconceptions.* Paper presented at the Thirteenth National Convention on Mathematics, Physical Science and Biology Education, Pretoria.

Opie, C. (2004). *Doing Educational Research. A Guide to First Time Researchers*. London: Sage Publications.

Perkins, D. N., & Salomon, G. (1989). Are Cognitive Skills Context-Bound? *Educational Researcher, 18*(1), 16-25.

Pirie, S. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping stones. In H. Steinbring, B. B. & S. A. (Eds.), *Language and communication in the mathematics classroom* (pp. 7 - 29). Reston, Virginia: NCTM.

Pokorny, M., & Pokorny, H. (2005). Widening Participation in higher education: student quantitative skills and independent learning as impediments to progression. *International Journal of Mathematical Education in Science and Technology, 36*(5), 445 - 467.

Roberts, A. L., Sharma, M. D., Britton, S., & New, P. B. (2007). An index to measure the ability of first year science students to transfer mathematics. *International Journal of Mathematical Education in Science and Technology, 38*(4), 429 – 448.

Salomon, G., & Perkins, D. N. (1989). Rocky roads to transfer: rethinking mechanisms of a neglected phenomenon. *Educational Psychologist, 24*(2), 113-142.

Simons, P. R. J. (1999). Transfer of Learning: Paradoxes for Learners. *International Journal of Educational Research, 31*, 577-589.

Slabbert, J. A., & Gouws, D. G. (2006). The quest for powerful learning environments in tertiary education. *South African Journal of Higher Education, 20*(2), 336-351.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences, 3*(2), 115-163.

Smuts, B. (2005). Using class interviews to evaluate teaching and courses in higher education. *South African Journal of Higher Education, 19*(5), 955.

Vega, Q. C., & Tayler, M. R. (2005). Incorporating course content while fostering a more learner-centred environment. *College Teaching, 53*(2), 83-86.

Vygotsky, L. (1978). *Mind in Society*. Cambridge MA: Harvard University Press.

Walczyk, J. J., & Ramsey, L. L. (2003). Use of learner-centred instuction in college science an Mathematics classrooms. *Journal of Research in Science Teaching, 40*(6), 566-584.

Wenger, E. (1998). *Communities of Practice: Learning, meaning and identity*. New York: Cambridge University Press.

Wood, L. A., & Lithauer, P. (2005). The 'added value' of a foundation programme. *South African Journal of Higher Education, 19*(5), 1002-1019.

Yin, R. K. (1994). *Case Study Research. Design and Methods* (2nd ed.). California: SAGE Publications.

APPENDICES

Appendix A: The Mathematics Content Studied in 2006

Math A:

1. The Number System: Definitions, examples, symbols: $\mathbb{R}$; $\mathbb{N}$; $\mathbb{Q}$; $\mathbb{Z}$; domain and range; set-builder and interval notation.

2. Functions and Relations: Definitions; function notation; Inverse relation; mixed graphs; plotting to discover different shapes and what makes them that shape: straight lines, parabolas, logarithmic, exponential, hyperbolas, circles.

3. Linear Function: $y = mx + c$

   Transformations

   Concept of gradient: positive/negative/steepness

   Concept of y-intercept

   Gradient-intercept method of drawing (needed for linear programming)

   Dual intercept method of drawing

   Domain and range

   // and $\perp$ lines.

   Special lines ($y = \pm x$, $x = k$, $y = k$)

   Finding equation of line

   Word applications: introduction to modelling; includes valid questions from the disciplines of business, economics, biology, science, marketing

   Reading and interpretation of pre-drawn straight line graphs e.g. distance/time graphs

4. Simultaneous Equations: Substitution method; two and three unknowns.

5. Linear Programming: Basic inequalities on a number line

   Interpreting set-builder and interval notation by drawing on a number line; open and closed circles

   Drawing linear inequalities; shading above and below line

   Definitions within linear programming

   Number sentences from word problems; interpretation of number sentences by drawing of appropriate graphs

   Interpretation of information presented by graphs in decision-making

   Optimisation

6. Quadratic Function: Discovery of basic shape by plotting

Symmetry – what causes it to have this shape?

Domain and range

x and *y* intercepts

Transformations: $y = A(x + b)^2 + B$; what do the different letters represent? Vertical and horizontal shift; maxima/minima; increasing and decreasing functions; domain and range of transformations

Inverse relations (basic concept)

Drawing sketches

Interpreting sketches in terms of models of situations e.g. profit in business applications, distance-time curves

7. Exponents: Laws and definitions governing integral exponents

Laws and definitions governing rational exponents

Manipulation and simplification using laws

Equations: with rational exponents (where unknown is the base) and variable exponents (exponent is unknown)

As above, but with base e

Exponential graphs: x/y intercepts; asymptotes, domain/range increasing and decreasing functions

Transformations: $y = A e^{x + b} + B$

Interpretation from word problems and drawing graph; interpretation of graph, relation to disciplines of study as described above; calculations from word problems e.g. population growth, time periods; exponential curves in modelling situations

8. Logarithms: Definitions; Inverse relations; expressions and equations; base e; graphs and transformations; word applications

9. Mixed Applications: Word problems

Calculations

Graphs

Interpretation of questions and findings; multiple disciplines

Math B:

1. Probability Theory: random experiment; sample space; complementary events, mutually exclusive events; independent events; conditional probability; random variable, mean, variance, standard deviation

2. The Binomial Distribution: Binomial expansion; binomial distribution; word applications; expectation, variance and standard deviation

3. Introduction to Limits: concept: 'from above and below'; terminology: 'tends to'; calculations; simple graphical representations

4. Average Gradient: equation derived from $m = \frac{y_2 - y_1}{x_2 - x_1}$; how gradients change according to the position on a curve; average speed; concept of 'average'

5. Instantaneous Gradient: gradient at a point; tie in the concept of limits; gradient of a curve; tangents and normals; differentiation rules and chain rule

6. Curve Applications: curve sketching and interpretation

7. Word Applications: word problems; interpretation; analysis application of above theory in real life situations.

Appendix B: Example Questions for the Roles Played by Math B Students

1. **Conceptual Understanding:**

   What shall we do next?

   Why are we doing it this way?

   What is the next step?

   Where does this lead?

   What does this mean?

   What are the differences?

   How do we relate this to what we already know?

   How do we represent this graphically?

   How do we interpret this answer?

2. **Procedural Fluency:**

   How do we write this correctly?

   What is the way to represent this in maths/words?

   What formulae could we use?

   What have we left out?

3. **Strategic Competence:**

   What is the problem?

   How can we reformulate the problem?

   How can we use maths to solve this problem?

   What prior knowledge is applicable, and why?

   How can we make sense of this?

   What is relevant/irrelevant?

4. **Adaptive Reasoning:**

   What other concepts are related to what we have here?

   What other situations are related?

   What patterns can we find?

   Are you making a claim? If yes, what?

   What validates your statement/conclusion/claim?

   Provide a sufficient reason

   Justify your answer

   Convince me!

Appendix C: The Instrument Used for Focus Group Interviews

Type of Question	Refers to research question 1	Refers to research question 2	Refers to research question 3
1. 'Hypothetical' *"…what the respondent might do or what it might be like in a particular situation; usually begins with 'what if' or 'suppose'…" (Merriam, 1997:77)*	1. how would you define the concept 'teaching approach'? 2. How would you describe the approach(es) I used for teaching maths in the FP? 3. Choose a particular subject you have learned since you started undergraduate study. **Do not tell me what the subject is or who teaches it**. How would you describe the approach(es) that the lecturer used/ uses for teaching it? ↓ 4. (follow on) – can you give me examples of a typical lesson? 5. Suppose one of the current FP maths students approached you with the comment, *"Mrs Manson is doing all this stuff in maths we aren't used to …"* – How would you respond to him/her?	1. Why do you think you did maths in the FP? ↓ 2. (follow-on) What part of the maths has the most relevance to your undergraduate studies? 3. (follow-on) Why do you say this? Expand on previous answer. 4. How do you think you would have coped in undergraduate study if you had not done FP mathematics? ↓ 5. (follow-on) What aspects of study do you think might have been more difficult if you had not done FP maths?	
2. 'Devil's Advocate' *"…challenges the respondent to consider an opposing view… e.g 'some people would say' …" (Merriam, 1997:77)*	*'I am using the same approaches in my studies now that I learned to use in FP maths classes"* 1. Is this statement applicable to you? ↓ 2. (follow-on) If yes, why		1. Your friend makes the comment, *"Maths in the FP was a waste of time!"* Respond to this comment. 2. Your friend goes on to say, *"We didn't*

	do you agree with the statement? If you don't agree, what is different?		*need to do maths in the FP – we aren't even using any of it now!"* Respond to this comment.
3. 'Ideal Position' *"…describes an ideal situation… yields information and opinion" (Merriam, 1997:77)*		1. Describe some ideal ways to learn your undergraduate subjects.	1. Ideally, what things taught or learned in FP mathematics should be usable in the subjects you are studying in undergraduate IT?
4. 'Experience' *"…personal experiences, perception and interpretations…"* (my own idea)	**Lectures:** 1. Please describe a typical lecture – what does the lecturer **do** to teach? 2. In what ways do you learn new concepts in lectures? 3. Do whole-class discussions take place in lectures? **Tutorials:** 4. What does the lecturer spend his/her time doing? 5. What do <u>you</u> spend your time doing? 6. Do you ever work in groups? (is this on your – or the lecturer's initiative?) 7. Does the lecturer ask questions? ↓ (follow-on) what kinds? **(each of the following will be dealt with separately)** • Method/procedure (what) • Conceptual (why)	1. What aspects of FP maths were different from school? In terms of: *Content?* *How content was taught* *Any other aspects of the FP course?* 2. Has the FP maths course had any **positive** effects on undergraduate study? – if so, what? 3. Has the FP maths course had any **negative** effects on undergraduate study? – if so, what?	1. What subject(s) that you are currently, or have been, studying in undergraduate study use aspects of **(each of the following will be dealt with separately)** • problem-solving • generalisation from the specific to the general • logical thinking • arguing/reasoning • modelling from real situations to something like formulae, diagrams, or other?

	• Justification (justify your claim) • Problem-solving (how) 8. How much whole-class discussion takes place?		
5. 'Interpretive' *"…interviewer advances an interpretation of what the respondent has been saying and asks for a reaction…" (Merriam, 1997:77)*	I will wait for responses in the actual interview, and use this questioning technique, if necessary.		

Appendix D: The Categories Constructed for the Focus Group Interviews

1	Meaning of 'teaching approach'	**1.1.**describes generally what a teaching approach is			
		1.2. Describes 'learner-centred'			
		1.3. Describes 'teacher-centred'			
2 ***1***	Teaching approaches in the FP	**2.1.** Teacher-centred			
		2.2. Learner-centred			
3 ***1***	Learning experiences in **FP**	**3.1.** Reference to mathematical practices learned / used			
		3.2. Classroom activities	**3.2.1.** positive		
			3.2.2. negative		
			3.2.3. neutral		
			3.2.4. reference to importance in other aspects of life		
		3.3. Change from negative to **positive**			
		3.4. Change from positive to **negative**			

4 ***1***	Teaching approaches in undergrad	**4.1.** Lecturer	**4.1.1**. Teacher-centred		
			4.1.2. Learner-centred		
		4.2. Tutor	**4.2.1.** Teacher-centred		
			4.2.2. Learner-centred		
5 ***1***	Learning experiences in undergraduate study	**5.1.** Comparing FP with undergrad (perceptions)			
		5.2. Positive			
		5.3. Negative			
		5.4. Neutral			

6 ***2***	Mathematical practices used in **undergraduate study**	**6.1.** yes	Encouraged / promoted / used as a 'habit' from FP	**6.1.1.** Yes	• Procedure • Concept und. • Prob-solving • Justification • Strategy • Decision-making • Questioning • Pattern-recognition • Generalisation • Representations • Productive disposition
		6.2. No			
		6.3. Current beliefs about maths			
7 ***2***	Individual work habits /Group work / discussion / argumentation used as a learning tool / good learning	**7.1.** Deliberate learning tool			
		7.2. Survival learning tool			
		7.3. Benefits outside of studies			
		7.4. benefits for undergrad studies			
		7.5. no			
8 ***1***	Expectations of classroom behaviours learning tools	**8.1.** Learning experiences	**8.1.1.**disappointed		
			8.1.2. satisfied		
		8.2. Teaching approaches	**8.2.1.**disappointed		
			8.2.2. satisfied		
9 ***2***	Verbally demonstrating need for FP maths and purpose of FP maths	**9.1.** content	**9.1.1.** Positive / sees connection		
			9.1.2. Negative / doesn't see connection		
		9.2. practices	**9.2.1.** Positive / sees connection		
			9.2.2. Negative / doesn't see connection		

10	Reference to secondary schooling	**10.1.** Teaching approaches used			
		10.2. Learning experiences			
		10.3. content			
		10.4. Math practices used			
11	Lecturers/tutors themselves demonstrate use of math practices	**11.1.** yes			
		11.2. no			
12 ***1***	Ex-FP students compare themselves with non-FP undergrad peers	**12.1.** Favourably			
		12.2. Unfavourably			
13 ***1***	Effective uses of **my FP** teaching approaches compared with other FP learning	**13.1** postitive			
		13.2 negative			
		13.3 no opinion – just a reference to other FP experience			
14 ***2***	Reflections on FP / outcomes / roll-overs	**14.1** postitive			
		14.2 negative			
		14.3 suggestions			

Appendix E: The Instrument Used for the Task-Based Interviews

This appendix lists briefly the tasks in the research instrument and the mathematical practices required to do the IT tasks competently. It also explains briefly and simply the main concepts of the tasks – particularly for the benefit of the non-IT-proficient reader.

Many computer designs have an iterative nature. Part of the learning taking place in undergraduate study of IT is experiencing the initiation of a design, only to realise that something important was left out, resulting in the need to go back to ‘the drawing board’, so to speak. Hopefully, the task-based interviews will help to answer the question, “How are mathematical practices implemented for thinking in IT?”

The Networking Task

Explain how a Microsoft NT Server or Novell Network Server locks out an account after a certain number of failed logon attempts. Explain why this type of security is important. Explain which OSI layers play a role in helping to encrypt passwords for security. Justify all your arguments.

Analysis and projected mathematical practices

Networks and Data Communications introduces to students how computers 'talk' to each other in a network. Students learn about the hardware required, the standards that make things work together, and how to operate a network. OSI is an acronym for 'Open Systems Interconnection', which is a model that explains how computers 'talk' to each other in a network. The OSI layers describe all the components of the network. For example, the first layer defines how computers are physically joined to each other by electrical wiring. Another layer controls how information is sent to the correct computer on the network. Yet another layer will make sure that the data is correctly communicated from one machine to another; and so on. As the question states, password and logon information are security issues and are also controlled by a layer, or layers; and the question is asking which layer(s) will be involved. This is a fairly low level question, and was used in the instrument because the students had just started learning about this when they participated in my interviews.

Uses mathematical procedures:

- Identify OSI layers and defining what each layer does
- Describing the lockout procedure
- Describing the login procedure
- Basic programming syntax
- Correct use of terminology

Explains Understanding of Concepts:

- Why security is necessary – big picture
- Why certain OSI layers deal with security
- What the OSI layers do and relate to each other – not just definitions, but understanding
- The lockout process
- How the particular network functions (e.g. Novell)

Applies Concepts Flexibly

- Problem solving associated with design
- Decision making concerning choices faced and most efficient choice

Constructs Arguments

- Giving reasons for making a particular decision
- Argumentation with justification concerning decisions
- Evaluation with justification

Productive disposition:

- Appreciates where this fits into the big picture
- Appreciates the importance of security in a network

The Banking Systems Task

Design a small banking system which has clients. Each client can have a savings and credit card accounts.

Consider the questions that follow and justify all your answers.

a) What classes would you have?

b) How would these classes 'see' each other?

c) What do the classes need to remember? (i.e. what are the attributes of each class?)

d) How will each class store its information?

e) What would the behaviours (methods) of the classes be?

Analysis and projected mathematical practices

This task is very open-ended. It probably would have been even more so if it had not been broken down into sub-questions. Therefore, many designs may be submitted, all of which could be correct. This question would ordinarily be done using a software package called BlueJ. The software itself would require certain techniques to be used, but these will not be discussed, as the entire task was done on paper for the purposes of the interviews.

Efficiency of design is an important aspect of computer programming in general. With this in mind, some designs may be more 'efficient' than others, which is a significant aspect of decision-making in computer programming.

The task has been broken down into smaller sub-questions, which follow on from each other. That is, a decision made at the upper levels of the question, e.g. question (a), will influence how and why subsequent decisions are made, e.g. question (b). In general, there is a natural hierarchy that needs to be addressed when doing this task. As the respondent moves down through the sub-questions, the concepts move from the more general, 'big picture' considerations of the task to the more specific considerations – the needs of the actual system being designed. It is an interesting aside to inquire if this hierarchy would still be followed, even if the task was not encouraging this hierarchical thinking. Is the hierarchical analysis a natural part of producing a good design for this task, or did it only occur because the question was broken down in the way that it was?

The task also has evolutionary aspects. What I mean by this is that it is possible to begin a design, believing all the classes have been considered, and move through the rest of the sub-questions; only to realise that something else essential needs to be added; which in turn, may alter the initial design concept significantly. Evolution of a design is necessary for improved efficiency in most situations (IT lecturer, personal conversation). In fact, as we worked through this particular task, we realised that extending the time to be spent on it resulted in the design becoming progressively bigger, as we kept adding 'important' classes to the system. João commented that one could spend a week on this

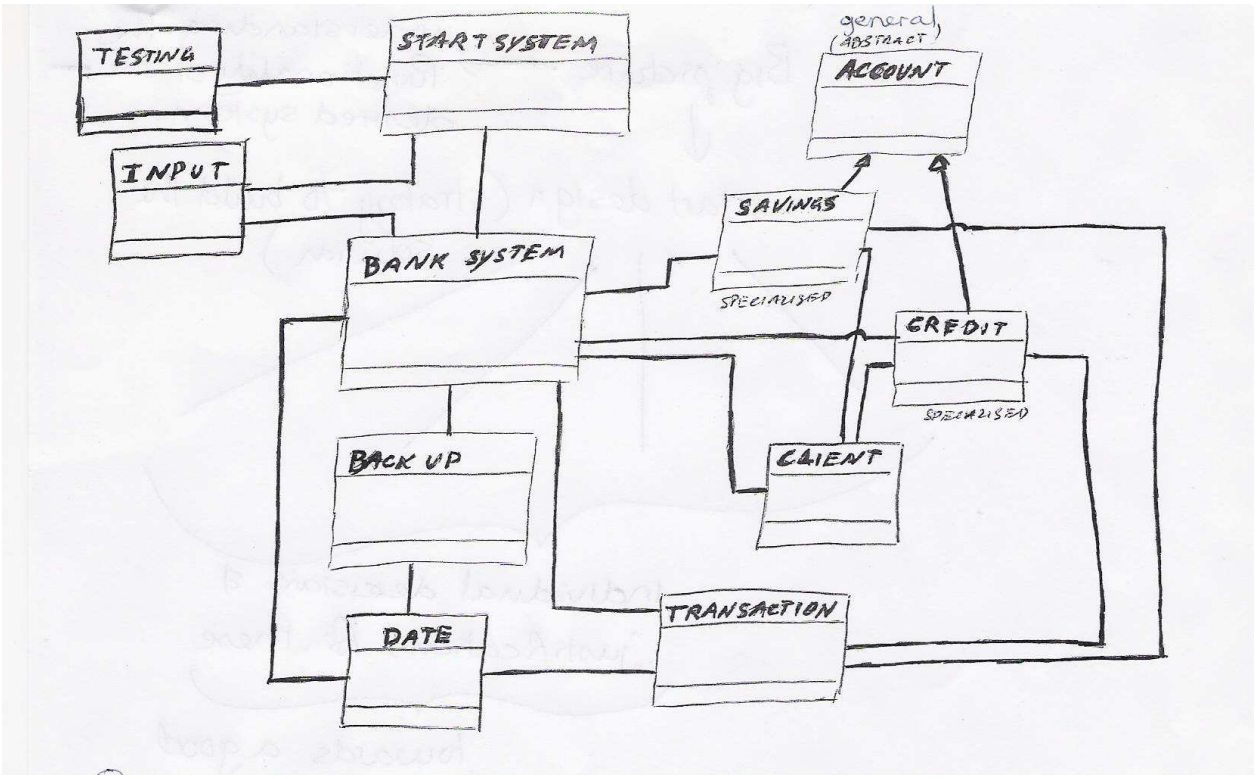
particular task, upgrading it. Therefore, we decided to allow the task interview to go on between 60 and 90 minutes, depending on how it was panning out with respect to demonstration of mathematical practices or other interesting argument. We elected not to look for the 'best' possible design – merely an indication that *critical thinking*/decision *evaluation* was taking place and that the respondents were noticing the need for evolution of their designs.

There are a few aspects to this task that make it excellent for observing certain mathematical practices. These include:

- How the design process was initiated – what sort of thinking is required? Did they observe the prior necessity for the 'big picture' conceptualisation?
- What makes a good or not-so-good design and why?
- What sorts of decisions have to be made?
- Have the students considered design efficiency; did they *evaluate* their decisions/designs; and how have they *justified* their *decisions*?
- How do the decisions made in the initial design influence subsequent decisions?
- What sorts of decisions have been made that differ from decisions made by other paired respondents who have also done task 1?
- What consequences or problems have been initiated by decisions made higher up in the design process that affect decisions and design further down in the design process?

Ultimately, the conclusion we made about the working of this task was that it required a great deal of decision-making; with a direct relationship to critical evaluation and justification of the decisions made. Also key to this task was the use of representations and symbols. These were subject-specific, and so were not mathematical in content; but were mathematical in the sense that understanding of the representations and symbols and when to use them was an important part of completing the task efficiently. Other important mathematical practices inherent to this task included analysis, problem-solving, strategising, generalisation procedural fluency and conceptual understanding. Below is an example where each of these was used by my tutor-assistant in his solution of the task.

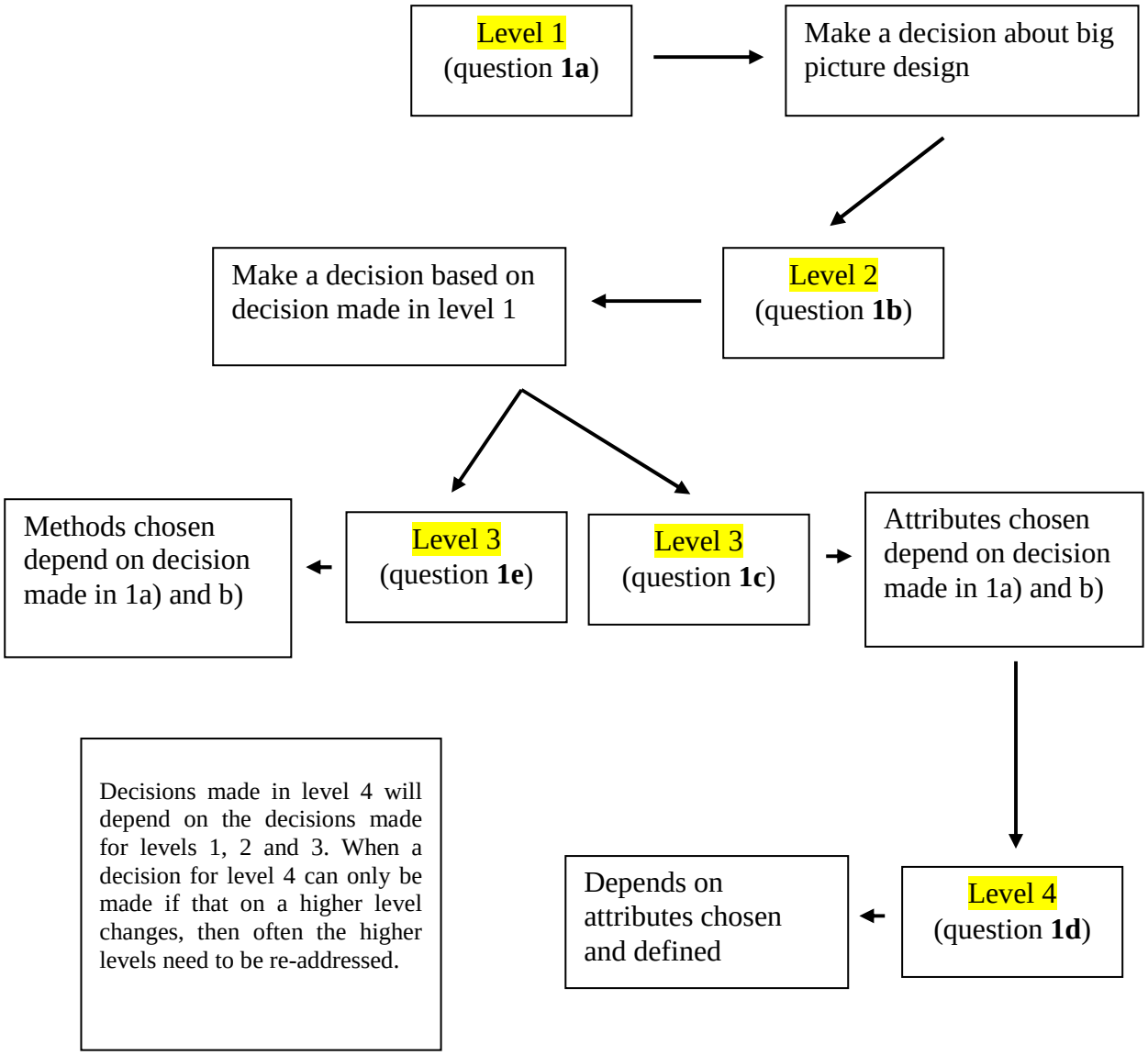
João's design is shown below, and an explanation for how such a design may be achieved is described under his solution.



A flow diagram of the logical ordering of the design is shown above, but I must emphasise that in reality the design is not linear, as it appears on the flow diagram. In reality the team tasked to design such a system would start the design by deciding what the 'big picture' would look like. From here, the design process would progressively narrow down to the designing of each component of the big picture, as shown in the flow diagram. The wording of the task indicates how this would take place, although the real system would not use questions helpfully scaffolded for the designers. The scaffolding was necessary for the task because the students are beginner programmers and would possibly become lost in the project if not helped.

The non-linearity of the task is seen in practice, where during, say, level 3, the designer(s) would realise that they have left something important out of the main design, or want to reassign an 'attribute' to another 'class'. This will have a snowballing effect on the rest of the design, and the group will generally find that they often need to re-work the main design; which in turn influences the other levels. A more proficient team will need to do this less, because their collective experience will pre-empt problems before they go too far down the process. My students, because they have significantly less experience, typically did many iterations and changes to all levels of their design at many different times. Therefore the process was not linear, although the same thinking processes and strategic and argumentative practices as those used by 'experts' are required to do the task.

As the design moves from the big picture to the specific needs of individual classes, there is a tendency towards requiring more use of procedural practices and less of strategic and argumentative practices (not that any of these are ever absent from any design level). That is, the big picture requires strategic planning with associated justification of decisions made, along with conceptual understanding of how the classes will together contribute to the whole banking system. The programming required to make each class behave in its specified way would have been defined a long way back in the design process and writing the programme requires use of syntax and terms in a defined order and specific to the programming language being used. Programming itself requires a certain amount of design – there are many different programme designs that will result in the same final output; but some programmes will be more efficient than others. Thus, even at the more procedural programming level, strategy, argumentation and justification of different strategies are necessary for better programme efficiency.



Level 1: big picture (macro):
requires:

- Listing of essential classes
- Why listed classes are needed
- Requirements of the banking system
- How classes 'see' each other
- How information may be stored
- Big picture design
- Representation of system in class diagram
- Small strategies – what goes into a class/how classes see each other
- Questioning and justifying level 1 decisions
- Evaluating level 1 decision

Specific mathematical procedures and concepts are not needed

Levels 2 and 3 (intermediate)
requires:

- Linking classes
- Defining the behaviours/methods of a class, depending on the choice of class & attribute
- Representation of system in class diagram
- Which attributes go into which classes
- How attributes, classes and storage link to each other
- Justify level 2 decision
- Evaluate levels 1 and 2 decisions

Decisions made in level 4 will depend on the decisions made for levels 1, 2 and 3. When a decision for level 4 can only be made if that on a higher level changes, then often the higher levels need to be re-addressed.

The Computer Systems Task

Use MMLogic to construct a digital logic circuit corresponding to the standard Sum-of-Products expression for the function:

$$F(x; y; z) = \bar{x}yz + xz$$

1. ***List the truth table for F.***
2. ***Simplify the expression using Boolean algebra and identities.***
3. ***List the truth table for the simplified expression.***
4. ***Construct the logic circuit diagram using MMLogic and verify that the circuit produces the same output as the truth table.***

Solution:

1.

$F(x,y,z) = \bar{x}yz + xz$

List the truth table for F.

INPUTS						OUTPUTS
x	y	z	$\bar{x}$	$\bar{x}yz$	xz	$\bar{x}yz + xz$
0	0	0	1	0	0	0
0	0	1	1	0	0	0
0	1	0	1	0	0	0
0	1	1	1	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	0	0	0	0
1	1	1	0	0	1	1

2. Simplify the expression using identities

PROOF	IDENTITY NAME
$\bar{x}yz + xz$	
$= (\bar{x}y + x)z$	Distributive law
$= [(\bar{x}+x)(y+x)]z$	Distributive law
$= 1 \cdot (y+x)z$	Inverse law
$= (x+y)z$	Commutative law

$F(x,y,z) = \bar{x}yz + xz = (x+y)z$

3. List the truth table for the simplified expression

INPUTS				OUTPUT
x	y	z	$x+y$	$(x+y)z$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

4) Construct logic circuits for both and verify that both circuits produce the same outputs.

(print version)

George Boole developed a system of logic, later known as Boolean logic, which was recognised as a base for understanding how computers function at the circuit level. Since computer logic is grounded in the 'on/off' scenario, it was reasonable to suggest that the binary number system, namely one and zero, could be used to represent 'on' and 'off'. Boolean logic was originally set within the concept of 'true' or 'false', which lends itself to the binary on/off situation. Boolean algebra is largely a system of specific identities set in the binary number system, and is used to simplify complicated circuit designs into the simplest form that may be used in the final design of circuits, such as those used in computer central processing units (CPU's) or screen cards, cell phones or modern washing machines and cars – all of which contain processing and memory functions. The identities function in a very similar way to other mathematical identities, such as trigonometric identities or Euclidean definitions, and are regarded as 'true', and subsequently used for further calculation.

I argue, therefore, that this question requires the use of the full range of mathematical practices defining the 'mathematically proficient' individual: Procedurally competent use of Boolean identities requires the conceptual understanding of how and why they are used; and their use requires their strategic manipulation, together with justification for their strategic use or non-use; which in turn requires conceptual understanding of the significance and correct procedural use of the identities. It is not clear by looking at the question where productive disposition contributes to proficiency here; but one would have to listen carefully to verbal explanations of the problem and its solution in order to identify this aspect of proficiency. Thus, all the proficiency strands of mathematics are required in their entirety for this IT problem.

The mathematical practices used by the students to do this problem could include

Uses Mathematical Procedures:

Produce a completed 'truth table' using combinations of on and off or X and X'.

Circuit diagram with correct gates' which represent AND, OR, NAND, NOR (or the combinations of ON and OFF)

Correct use of mathematical objects such as terminology, definitions and symbols

Explains Understanding of Concepts:

Correct conceptual choice of AND/OR with $\cap$ / $\cup$ symbols in Boolean logic

Choice of using X and X', $\cap$ / $\cup$, etc. symbols and their relation to ON / OFF

How groups of symbols combine to determine how the circuit is switched on or off e.g. X'YZ', etc.

Choosing appropriate Boolean identities for simplification of expression

Understand the relationship between simplified form and original form (must give identical result)

Applies Concepts Flexibly

"More expert problem solvers focus more on the structural relationships within problems, relationships that provide the clues for how problems might be solved"

"…A fundamental characteristic needed throughout the problem-solving process is flexibility. Flexibility develops through the broadening of knowledge required for solving non-routine problems rather than just routine problems" (Kilpatrick et al., 2001)

Choose correct identities for simplifying expression

Choosing ordering and when to use (or not use) identities for simplifying expression

Knowing when problem is fully-simplified

Working with problem-solving processes: given, unknown, condition, *and* solution (see above).

Given: original problem

Unknown: end solution result

Condition: Result must be simplification of original

Solution: simplified result: tested using truth table or software check programme

Constructs Arguments

Deductive reasoning used for simplifying using identities

Justify use of one identity over another

Evaluate legitimacy with justification of simplified form – did it give the same result as original form?

Productive Disposition

Appreciates where the mathematical practices fit into the big picture

The Prime Numbers Task

Write a programme in Java to print all the prime numbers from 1 – 100.

Programming tends to be very procedural in nature – especially once the programmer becomes busy with the actual task of coding. Procedure in programming includes using the correct syntax and algorithms. Syntax is specific to the programming language being used, and requires 100% correct use, or the programme will not run. This part of programming is memory-based, although prolonged use of a particular language will result in automatically correct use of syntax. Examples of syntax include 'if' and 'while' statements, which, if used, programme the computer to expect specific inputs from the programmer. The use of 'loops' is an invaluable tool for the programmer, as the computer can be told to perform a huge number of iterations of the same task until it is 'told', by some condition being reached, to end the loop.

This particular task expected the student to stop before implementing actual coding procedures (verbally communicated to the students). What mathematical practices were required from the student before the writing of the code would be done?

Uses Mathematical Procedures and Explains Understanding of Concepts:

Students needed to be able firstly to fully define a prime number, and secondly to demonstrate understanding of what a prime number is and is not in order to write a programme to list all prime numbers from 1 to 100. It is not enough to recognise a prime number when you see one. The programme needs to check every number between 1 and 100 and find which is and which is not a prime no. Therefore, one needs to understand the relationship that exists between the prime and non prime numbers.

Choice and appropriate use of mathematical symbols in order to restrict the list of numbers to between 1 and 100. The question did not specify whether 1 and 100 were to included, and so use of $1 \leq number \leq 100$ or $1 < number < 100$, or a correct variation of this notation would have been considered appropriate.

Applies Concepts Flexibly

Strategically designing a computer programme is like writing a Euclidean geometry proof. If logical sequencing of information, arguments and commands is absent, the programme will not work (the intention of the programme will not be implementable by the computer).

There is as much emphasis on what a prime is as what it is not.

Setting up a 'loop' specific to Java requires the strategic decision to use numeric values that the syntax in the loop can work with – for example, each time the loop runs it has to add 1 to the previous number it tested for prime before it can test the new number. The mathematical practice of addition of integers must be flexibly applied to the strategy chosen for the running of the programme.

given, unknown, condition, *and* solution:
given: List prime nos.
unknown: how to apply the definition of a prime number to programming.
condition: define prime and non prime
solution: systematic checking of every number by means of a loop and 'testing algorithm'.

Constructs Arguments
Justify use of one algorithm or particular syntax over another

Productive Disposition
How mathematics concepts relate to programming procedures and strategies
The importance of planning

Procedural fluency in IT is required when using Java-specific programming language syntax, for example. Use of the correct syntax ensures that the programme will know what to do. For example, every language needs to be able to set up a looping structure. Setting up a loop was necessary in this task for the programme to know to keep counting from 1 to 100 and testing if each value was a prime number or not. Computing students are expected to know how to set up a loop in Java 1. A looping structure has to have a 'test' which will tell the programme whether or not to do the loop again. This 'test' may be at the beginning or end of the loop. Correct syntax is also required to tell the programme when to 'add 1' to the previous number tested, so that the loop may run from 1 to 100, and when to stop adding 1 to the previous number. All of these procedures are defined by the language-specific 'syntax'. The syntax defines how a specific language implements a loop. Examples of Java syntax include 'if' and 'while' statements. Procedural fluency in programming entails knowing the order of commands to put into the programme, so that Java will be able to read what to do next. Added to this knowledge is a need to

continue to use particular syntax once certain syntax has been selected for use. For example, if a ‘do while’ statement has been used at the beginning of a loop, then the body of the loop must be begun with a left brace ({), and ended with a right brace (}) – otherwise the programme will not recognise the loop and not run. Another procedure that may be used by the students in the banking systems task is that of setting up an ‘array’ or ‘data base’, which are ways of storing information, for example, lists of addresses and telephone numbers which need to be extracted and used by the banking system. These principles are all procedural, and need to be used completely correctly when programming.