

Predicting Exchange Rates using Taylor Rule
Fundamentals: Evidence From a Portfolio Optimisation
Framework.

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Abstract

The paper studies exchange rate predictability using Taylor rule fundamentals in an optimal portfolio framework. The study seeks to link exchange rate dynamics with capital flows. Profit-seeking economic agents are assumed to repatriate funds across borders in response to differentials in rates of return from risky assets of portfolios held. We develop a uncovered portfolio return parity (UPRP) based exchange rate model in which changes in the short-term nominal exchange rate depend on the difference of optimal portfolio returns between two economies. In a two country economy where USA is taken as the foreign country we test the model in 5 countries namely South Africa, South Korea, Brazil, Mexico and Poland. The model is benchmarked against a UIP model and a Random walk model in order to establish whether the study's extension enriches exchange rate prediction literature. We find that the main UPRP model outperforms the Random walk model in the 12 month horizon for 4 out of 5 countries using CW statistics. For the 1-month horizon the main model is outperformed by the Random walk model in 4 out of 5 countries and for the 2-month and 3-month horizons the main model beats the Random walk using CW statistics. Theil's U statistics also show that with the exception of South Korea, the main model beats the Random walk across all countries in the 3 and 12-month horizons. We conclude that out-of-sample exchange rate forecasting based on an optimal framework and Taylor rule fundamentals produces better nominal exchange rate forecasts relative to the Random walk model and UIP model.

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1 Introduction

The paper investigates exchange rate predictability using Taylor rule fundamentals in an optimal portfolio framework. Predicting exchange rate movements is important for at least three reasons. Firstly, as pointed out by Maciel, Gomide, Santos and Ballini (2014) exchange rate prediction is important because portfolio diversification ties assets to international markets. Consequently, predicting exchange rates is beneficial for strategic financial planning through asset allocation, risk management and trading strategies. Secondly, knowledge of exchange rates anchors the facilitation of finance and investment management as well as hedging strategies, as noted by Wan (2012). Thirdly, the accuracy of exchange rate prediction is important for international transactions and denomination of contracts.

Taylor (1995), Engel, Mark and West (2007) and Rogoff (2009) acknowledge that the prediction of exchange rates has proved to be no easy feat in spite of the numerous empirical studies conducted. Kilian and Taylor (2001) agreed that since the seminal work of Meese and Rogoff (1983), there is a prevalent view that models of exchange rate forecasting fail to beat the benchmark random walk model. However, in her review of recent empirical studies on exchange rate determination in advanced economies, Rossi (2013) finds that there are models that beat the random walk at short horizons. These models include the net foreign asset pricing model proposed by Gourinchas and Rey (2007) and models that use Taylor fundamentals (e.g.: Molodtsova & Papell, 2009).

The contribution the paper seeks to make is with regard to the stimulation of flow of funds across borders. The literature that is in line with Molodtsova and Papell (2009) which incorporates Taylor rule fundamentals to predict exchange rates does so on the basis of the uncovered interest rate parity relationship (hereafter UIP). The UIP assumes that funds flow across borders in response to differentials in the short-term interest rates. However, profit-seeking economic agents may move funds in response to differentials in other rates of return. For example, Hau and Rey (2006) put forward the idea that economic agents re-balance their equity holdings in response to the differential between equity returns in a relationship called uncovered equity parity (hereafter UEP).

The paper introduces Taylor rule fundamentals in a model that integrates both the UIP and UEP through the uncovered portfolio return parity (hereafter UPRP). We test whether exchange rate prediction can be improved by incorporating Taylor rule fundamentals within the framework where agents move funds across borders in response to an optimised uncovered

portfolio return differential. We assume a portfolio made up of three assets: money market instruments, bonds and equities. The economic agent forms an optimal portfolio from these assets in order to derive an optimal return. Our contribution is to introduce Taylor rule fundamentals in the context of an optimised portfolio return model. We then evaluate the performance of our model by comparing it to a UIP model and a Random walk model in order to check if our extension adds value to the exchange rate prediction literature.

The outline of this paper is as follows: Section 2 reviews literature on exchange rate determination and forecasting. Section 3 provides an overview of the theoretical framework of the study. Section 4 presents empirical analysis of the study where we describe the data, econometric estimation techniques used as well as forecast evaluation methods and finally presents estimations of the the study and their interpretation. Section 6 presents robustness checks of the main model. Finally section 7 provides concluding remarks on the main findings of the study.

2 Literature Review

The three influential theories of exchange rate determination are the Uncovered Interest Rate Parity (UIP), Purchasing Power Parity (PPP) and the monetary approach. PPP theory states that the ratio of price levels between two countries determines the nominal exchange rate between the two respective currencies. Assuming the Law of One Price, PPP is an equilibrium condition which equates exchange rates and inflation differentials across any two countries (Dornbusch, 1985). PPP remains a useful cornerstone of international economics. Dornbusch (1985) established that the PPP is a crucial tool for macroeconomic policy. Kwapong (2005) showed that the PPP helps us gain a sense of real exchange rate levels in the market. Furthermore, Haidar (2011) describes how PPP can be used to determine whether one currency is overvalued or undervalued.

PPP theory is a subject of controversy because of varying outcomes regarding its validity (Symansky et al., 1994). For example Krugman (1978) reports the empirical failure of PPP theory in the short term. Rogoff (1996, 2009), Froot and Rogoff (1995), Sarno and Taylor (2002), Taylor (1995, 2003) and Wang (2009) survey an extensive range of PPP literature and conclude with the general consensus that the real exchange rate tends towards its PPP level in the long run. However, a wide range of studies exist which criticize evidence of PPP theory and report conflicting views. These include Murray and Papell (2005), Taylor and Taylor (2004) and Wu and Wu (2001). Studies reporting empirical success of PPP are Frankel and Rose (1996) as well as Frankel (1990). In terms of forecastability Cheung, Chinn and Pascual (2005), Engel, Mark and West (2007), Rogoff and Stavrageva (2008) report marginal success of PPP as a forecaster of nominal exchange rate at long horizons.

The second important theory of exchange rate determination is the UIP. It is a key equilibrium relationship in international finance as it links interest rates across countries with the expected change in exchange rate between currencies. The UIP implies that at any given time a rational risk-neutral investor has the choice to either earn interest from holding domestic assets or equally earn foreign denominated interest from foreign assets (Isard, 2006). This means that if the difference of the foreign interest rate minus the domestic rate is positive, then the domestic currency can be expected to appreciate. The picture painted by the literature shows that evidence about the validity of the UIP is inconclusive. For example Bekaert et al. (2007) show inconsistency with UIP and Froot and Thaler (1990) characterize the UIP as an anomaly due to lack of empirical support and its ambiguity as an equilibrium condition.

In order to evaluate whether or not UIP holds, the literature tests whether the forward rate is an unbiased forecaster of the spot exchange rate. If the UIP holds the coefficient of the logarithm of the spot exchange rate regressed against the interest rate differential must be equal to 1 and the intercept must be 0. However, Froot and Thaler (1990) and Lewis (1995) show that almost all empirical studies in the literature analyzed reveal a negative coefficient for most countries and where any are positive, none is ever close to 1. Engel (1996), Baillie and Bollerslev (2000), McCallum (1994) and Mark and Wu (1998) refute the UIP and as well find negative coefficient values. However, McCallum (1994) states that negative coefficients rejects unbiasedness but is not a sufficient condition for the rejection of UIP.

Another factor which makes the UIP controversial is that it tests differently over different forecasting horizons. Chinn and Meredith (2004) find that the UIP is ineffective in predicting exchange rate movements in the short-term. However Molodtsova and Papell (2009) and Clark and West (2006) report some success at short-horizons. Alexius (2001), Chinn and Meredith (2004) and Mehl and Cappiello (2009) find support for UIP in the long term while Cheung et al. (2005) and Alquist and Chinn (2008) show that for most countries long horizon predictability of UIP is not ever marginally superior to the random walk. In addition to the fact that the validity of UIP varies with horizon, it turns out that UIP tests differ depending on frequency of the data. For example Chabound and Wright (2005) use high-frequency intraday data to test UIP theory and find supportive results with the coefficient values close to 1. However this outcome is sensitive to slightest adjustment of the size of the estimation window.

The third critical theory of exchange rate determination is the monetary approach. According to the monetary approach, exchange rates can be expressed in terms of the relative price of two currencies. Isard (1978) shows that the monetary approach is derived from equilibrium conditions namely the UIP, PPP and money demand equations. Meese and Rogoff (1983) influentially illustrated that structural models like the monetary model were outperformed by the random walk in predicting exchange rates. Neely and Sarno (2002) and Rossi (2013) review monetary approach literature and reveal that since the crucial findings of Meese and Rogoff empirical support on the monetary models is generally inconsistent. Moreover MacDonald and Taylor (1994) show that monetary models are inadequate in accounting for exchange rate deviations when used as as forecasting instruments.

There is vast literature which finds that the monetary approach holds in the long run. This includes MacDonald (1999), Husted and MacDonald (1998), Groen (2000), Rapach and Wohar (2002), Sarno, Valente and Wohar (2004), Mark and Sul (2001), Macdonald

and Taylor (1991), Kim and Mo (1995). On the other hand Mark (1995) and Chinn and Meese (1995) show that monetary approach fails to hold in the short-run. In line with the conflicting empirical results of the monetary approach to determining exchange rates, studies such as Berkowitz and Giorgianni (2001), Rapach and Wohar (2002) and Kilian (1999) find unconvincing empirical support for monetary models at both short and long horizons while Cheung et al. (2005) reject the monetary approach altogether at both short and long horizons.

Altogether exchange rate predictability remains difficult to achieve. There seems to be dissociation between exchange rate movements and macroeconomic fundamentals. The literature also reveals conflicting results across different forecasting horizons. The influential exchange rate models of the 90's offer trivial out-of-sample predictability (Cheung et. al, 2005). Even the stock markets and exchange rates relationships tested for are not robust. For example Tabak (2006) finds exchange rates and stock prices are significantly related in Brazil, Kutty (2010) finds that exchange rates and stock prices are significantly related in (Mexico and South Korea) and Abdalla and Murinde (1997) find a significant relationship between exchange rates and stock prices in Pakistan and India. All these studies use Granger causality tests. However No causal relationship between exchange rates and stock prices is found by Zia and Rahman (2011) (Pakistan) and by Kenani, Maoni, Kaunda and Nyirenda (2012) (Malawi).

Engel and West (2004) proposed the possibility of using interest rate reaction functions in models which determine exchange rates and not just as policy functions. They show that combining UIP with an interest rate reaction function is better at exchange rate determination than UIP on its own for Deutschmark-Dollar real exchange rates. Engel and West (2006) further survey the exchange rates six G7 countries and use interest rate reaction functions to state monetary policy rules for the home and foreign country. In advanced economies, Wang and Wu (2012) find that models with an interest rate reaction function beat the random walk, PPP and monetary models in semi-parametric interval forecasting for the out-of-sample exchanges rate of twelve OECD countries. Mark (2009) finds predictability in the Deutschmark - Dollar exchange rate but recognizes structural breaks within the model when interest rate reaction functions are applied.

There is budding literature spearheaded by Molodtsova and Papell (2009) which shows short-run exchange rate predictability using Taylor rules. Molodtsova and Papell (2009) and Molodtsova et al. (2008, 2011) use Taylor rules to integrate monetary policy in forecasting exchange rates. Molodtsova et al. (2008, 2011) use real-time data and find out-of-sample

predictability in the Deutschmark-Dollar nominal exchange rate. Molodtsova and Papell (2009) addresses unavailability of real-time data of most of the countries by using periodically revised data that excludes ex post data and find evidence for out-of-sample predictability in 11 out of 12 OECD countries from 1973 to 2006. Although the Taylor rule based models outperform the UIP, PPP and monetary models Rogoff and Stavrakeva (2008) argue that these results are ‘exaggerated’ due to the misunderstanding of the Clark and West (2006, 2007) out-of-sample test statistics (CW, henceforth), poor robustness checks and excessive dependence on asymptotic test statistics.

Alba, Park and Xie (2015) highlight that there is comparatively less literature on Taylor rule fundamentals as predictors of exchange rates for emerging markets. Alba et al. (2015), Kim and Ryu (2013) and Moura (2010) are part of the few studies. Alba et al. (2015) follow Molodtsova and Papell (2009) and find that Taylor rules produce moderate but not tremendous out-of-sample exchange rate predictions in thirteen inflation targeting countries in Africa, Asia, and Latin America. Kim and Ryu (2013) follow Mark (1995) and confirm out-of-sample predictability using Taylor rules on the U.S. Dollar–Korean Won exchange rate via CW tests. Moura (2010) follows Cheung et al. (2005) and runs country-by-country regressions but with forecasts derived from an error correction model for five Latin American economies from 1999 to 2007. The study concludes with strong predictability using the Taylor models.

Regarding exchange rate determination, the Taylor rule models also show inconsistencies in their results. For example Byrne Korobilis and Ribeiro (2016) show that predictability is present in some sample periods and missing in others. Also, Rossi (2013) shows that exchange rate predictability is dependent on the exact model specification and it is country specific. Nevertheless, Byrne et al. (2016) find evidence in favor of Taylor rule fundamentals using time-varying forecasting regressions for the exchange rates of 18 OECD countries. Rossi (2013) finds significant predictability of Taylor rules at short horizons via CW tests and unconvincing results at long horizons.

It is important to note that the cornerstone of the Taylor rule models of exchange rate determination is the UIP or more specifically interest rate differential. Cappiello and De Santis (2007) develop the equilibrium condition Uncovered Return Parity (URP) which links expected exchange rate change with expected return differentials on risky assets. According to URP when expected returns on a domestic security exceed expected returns on a foreign security, the domestic currency depreciates against the foreign currency. Since covariances of returns are linear operators Cappiello and De Santis (2007) use them test URP in German

and Swiss security returns relative to US returns. The tests reveal positive results for URP in that over the 15 passed years when expected equity returns are higher in the US than in Germany and Switzerland there is an associated depreciation in the US dollar in respect of the Deutschmark and the Swiss Frank. On the other hand the study reports uncertainty in the US versus UK pair with the equity and bond markets.

In light of inconsistencies in the findings we identify that UIP, PPP, the monetary models and other named models of exchange rate determination are limited. This limitation is in the sense that exchange rate dynamics do not fully compensate for margins in inflation rates, interest rates, relative monetary supply or equity returns as the models suggest. Building on concepts like the URP this study proposes that a broader approach is missing. We exploit the favorable power of Taylor rules in predictability and formulate a parity condition which links risky asset returns with exchange rates. This study extends the literature by taking portfolio return differentials as drivers of the exchange rate within an optimal portfolio construction using the Taylor rule fundamentals.

3 Theoretical Framework

Markowitz (1952) provided the mathematical method which uses the mean and variances of returns on assets to select the best portfolio. Markowitz portfolio theory (MPT hereon) uses the assumption that the investor is risk-averse and is therefore in pursuit of small risk as well as high expected return on assets. MPT can also be seen as a portfolio optimization problem so that a portfolio reaches the required rate of return with minimal volatility through an optimally selected set of weights. The instrument's variance of its rate of return is used as proxy for the instrument's volatility. In spite of MTP criticism shown for instance by Michaud (1989) and Mangram (2013) MPT remains a strong analysis tool because it provides a simple mechanism of diversification and management of risk. MPT formulation sets limits on predicted loss on assets therefore the constructing portfolios also allows for some capital allocation.

We consider a simple two-country economic environment and assume a portfolio consists of three assets $i = 1, 2, 3$ money market instruments, equities and bonds respectively. We denote the random variable r_i the rate of return for assets i for $i = 1, 2, 3$ with mean \bar{r}_i where r_1 =short rate, r_2 =stock return and r_3 =bond rate. For a 3-asset portfolio p if $w = (w_1, w_2, w_3)^T$ is a set of associated weights, the rate of return r_p is given by: $r_p = \sum_{i=1}^3 w_i r_i$ and the expected return on p , $\bar{r}_p = \sum_{i=1}^3 w_i \bar{r}_i$. It can be shown easily that portfolio variance $\sigma_p^2 = w^T \Omega w$ where Ω is the variance-covariance matrix (see appendix). The optimum portfolio minimizes portfolio variance subject to the constraints that the weights sum up to 1 and each weight is non-negative. To develop a framework which minimizes volatility of return of portfolio the specified optimization problem is:

$$\begin{aligned} \min_{w_1, w_2, w_3} \frac{\sigma_p^2}{2} &= \min_{\{w\}} w^T \Omega w / 2 \\ \text{subject to } :1 &= \sum w \\ \text{and } :w_i &\geq 0, \forall i \end{aligned}$$

The Lagrangian for the problem is set as:

$$\mathcal{L} = w^T \Omega w / 2 + \lambda_1 [1 - \sum w]$$

The problem admits a first order solution given by the matrix system:

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

or equivalently $\Omega \underline{w} = \underline{1}$ with a solution: $\underline{w}^o = \Omega^{-1} \underline{1}$. \underline{w}^o is a set of optimum weights $\{w_{1t}^o, w_{2t}^o, w_{3t}^o\}$ which are used to calculate the optimal portfolio return r_{pt}^o expected by the investor at time t and $\underline{1}$ is a unit vector.

In order to calculate the optimal solution we will need to find the best econometric model to extract the time-varying variances and covariances. Wu et al. (2010) study the performance of variations of Exponentially Weighted Moving Average (EWMA hereon) models and GARCH models to forecast volatility for stock index returns and find results favorable to EWMA. Following Wu et al. (2010) the EWMA is applied to extract time-varying variances and covariances. The formulae are given as:

$$\begin{bmatrix} \sigma_{1t}^2 \\ \sigma_{2t}^2 \\ \sigma_{3t}^2 \end{bmatrix} = (1 - \lambda) \sum_{t=1}^T \lambda^{t-1} \begin{bmatrix} (r_{1t} - \bar{r}_{1t})^2 \\ (r_{2t} - \bar{r}_{2t})^2 \\ (r_{3t} - \bar{r}_{3t})^2 \end{bmatrix}$$

for variances and

$$\begin{bmatrix} \sigma_{(12)t}^2 \\ \sigma_{(13)t}^2 \\ \sigma_{(23)t}^2 \end{bmatrix} = (1 - \lambda) \sum_{t=1}^T \lambda^{t-1} \begin{bmatrix} (r_{1t} - \bar{r}_{1t})(r_{2t} - \bar{r}_{2t}) \\ (r_{1t} - \bar{r}_{1t})(r_{3t} - \bar{r}_{3t}) \\ (r_{2t} - \bar{r}_{2t})(r_{3t} - \bar{r}_{3t}) \end{bmatrix}$$

and for covariances. λ is the decay factor parameter. RiskMetrics (1996) show that the role of λ is to specify the relative weights to be applied to the returns and past volatility

data used in the current volatility estimation. Brooks (2002) reports that in practical terms it is likely that volatility is more responsive to recent events than events in the more distant past. EWMA becomes a better measure of volatility since the recent observations have more weight and the weights related to these past observations decrease exponentially over time. Using EWMA Ω_t the time-varying variance-covariance matrix is calculated. By solving the optimization problem for the solution $\underline{w}_t^o = \Omega_t^{-1} \underline{1}$ the set of optimal weights is computed and is used to calculate the optimal portfolio return as follows:

$$r_{pt}^o = w_{1t-1}^o i_{t-1} + w_{2t-1}^o \Delta q_{t-1} + w_{3t-1}^o R_{t-1} \quad (1)$$

i_t is the monthly short-term nominal interest rate, Δq_t change in equity returns, R_t is the long-term nominal interest rate.

3.1 Taylor Rule Fundamentals

Taylor rules were proposed by Taylor (1993) as plain unsophisticated rules with which the Fed Funds rate adjusts for changes in inflation and output. They are mathematical models which are used to replicate central banks' decision-making behavior with respect to the changes in the short-term interest rate. Clarida, Gali and Gertler (1998) point out that Taylor rules and other interest rate reaction functions stem from the observation that the short-term interest rate is the central bank's effective instrument for monetary policy. The central banks achieve the targeted nominal interest rate via open market operations. Blanchard (2010) shows that a Taylor rule has described the behavior of the Federal Reserve in the USA and the European Central Bank in the euro-zone very well over the past fifteen to twenty years in spite of both banks not explicitly following a Taylor rule. As a result, the important outcome of the introduction of Taylor rule is that it has shaped the way monetary policy is viewed. That is, central banks tend to view monetary policy in terms of an interest rate rule rather than nominal money growth (Blanchard, 2010).

We assume that the short-term nominal interest rate i_t is the nominal interest rate set by central banks depending on the economic environment. We follow Molodtsova and Papell (2009) in their examination of out-of-sample predictability of exchange rates using Taylor rules and specify the short-term nominal interest rate by a backward-looking Taylor rule. We also follow Clarida et al. (1998, 2000) who suggest that since central banks have a tendency to smooth out movements in the nominal interest rate then a interest rate smoothing term must be included . We assume that both the home country and foreign country follow the same Taylor rule with the exception that for the home country the term for the log real exchange rate is included. Moura (2010) shows that when the foreign country is the United States of America we may assume that the short-term interest rate rate does not respond to movements in the real exchange rate. The Taylor rules for the home and foreign country are:

$$i_t = \phi_i i_{t-1} + \phi_\pi (\pi_t - \bar{\pi}_t) + \phi_y y_t + \phi_e e_t \quad (2)$$

$$\bar{i}_t = \bar{\phi}_i \bar{i}_{t-1} + \bar{\phi}_\pi (\bar{\pi}_t - \bar{\bar{\pi}}_t) + \bar{\phi}_y \bar{y}_t \quad (3)$$

π_t is the monthly rate of inflation, $\bar{\pi}_t$ target level of inflation, y_t output gap (deviation of actual output from potential output), e_t is the monthly log level of the real effective exchange rate.

Variables of the foreign country (USA) are analogous and are marked by $(\bar{})$

We now answer the question of what equations (2) and (3) imply with respect to movements in the exchange rate. So that by subtracting the Taylor rule of the domestic country from the Taylor rule of the foreign country we can use the analogy of Molodtsova and Papell (2009) which states that any shocks to the variables on the right-hand side of (2) and (3) will stimulate changes in the exchange rate. Therefore according to the UIP changes in the exchange rate follow the equation:

$$i_t - \bar{i}_t = \alpha_0 + \phi_e e_t + \phi_\pi (\pi_t - \bar{\pi}_t) - \bar{\phi}_\pi (\bar{\pi}_t - \bar{\bar{\pi}}_t) + \phi_i i_{t-1} - \bar{\phi}_i \bar{i}_{t-1} + \phi_y y_t - \bar{\phi}_y \bar{y}_t \quad (4)$$

Assuming UIP holds, The mechanism of change in the exchange rate may occur in the following ways:

- If inflation rises above the target target inflation in the home country, the central bank will want to cool down the heating economy by raising the short-term interest rate. Higher interest rates attract foreign capital into the home country, there is increased demand for the domestic currency and this results in immediate appreciation of the domestic currency.
- If the output gap increases in the home country the central bank will raise the short-term interest rate resulting in an appreciation of the domestic currency.
- If the real exchange rate in the home country increases the central bank will raise the short-term interest rate to cause an appreciation of the domestic currency.
- If there is smoothing of the interest rate in the home country, an increase in the lagged interest rate causes an increase current and expected future interest rates, this results in an immediate and appreciation of the domestic currency.

If UIP holds we can deduce the mathematical relationship of the UIP model which shows that predicted changes in the exchange rate are driven by an interest rate differential as follows: $E(\Delta S_{t+1}) = \alpha_0(i_t - \bar{i}_{t-1})$. S_t is the log of the nominal exchange rate, α_0 constant. The nominal exchange rate is the domestic price per unit of foreign currency therefore a rise in S_t reflects an appreciation of the domestic currency against the foreign currency. Dornbusch (1976) famously developed the overshooting model which states that this immediate appreciation of is followed by a forecasted depreciation when UIP is assumed to hold. However, the UIP has been found to be empirically invalid in the short-run as discussed earlier.

Molodtsova and Papell (2009) dispute the validity of the UIP and propose that an increase in home country's interest rate will result in an immediate appreciation followed by a sustained forecasted appreciation of the domestic currency. Therefore the signs of the coefficients of the short-term interest rate are reversed to reflect the invalidity of the UIP. Molodtsova and Papell's (2009) proposition poses the crucial link between Exchange rates and Taylor rules central to the study.

3.2 Uncovered Portfolio Return Parity (UPRP)

The work of Hau and Rey (2006) paved the way for studies in relationships between exchange rate dynamics and equity returns. In the study they offer and test the idea that when equity returns are higher in the home country relative to the foreign country then the home currency may be expected to depreciate vis-à-vis the foreign currency. This depreciation occurs via re-balanced portfolios and repatriated dividends. Hau and Rey (2006) postulate that investors are willing to sell off the foreign asset holdings in their portfolios that they forecast will be able to keep the same level of risk. When investors sell their foreign assets the exchange rate goes down. Furthermore, since higher equity returns often imply better performer of the equity market and good performance consequently implies higher dividend payouts. this higher dividends stimulate capital outflows thereby driving the exchange rate down.

It is by expansion and exploration of the hypotheses of Hau and Rey (2006) that Cappiello and De Santis (2005, 2007) coined the UEP. Much like the UIP the UEP is a no arbitrage model in which the exchange rate is equal to the difference of the returns on risky securities and bonds between two economies. Interestingly Cenedese, Payne and Sarno (2015) have recently concluded that stock market returns possess insignificant information in explaining exchange rate dynamics. The study also reveals that the change in exchange rate is not equalized by the country-level equity returns differentials. Cenedese, Payne and Sarno (2015) prove that when an investor employs a trading strategy that goes long (buying) in countries offering higher expected equity returns and goes short(selling) in those countries with lower returns produces considerable returns and Sharpe ratios (Sharpe ratios are a measure of portfolio performance).

The above supports the notion that changes in the expected exchange rate affects portfolios. Therefore we propose UPRP as the mechanism through which the short-term nominal exchange rate adjusts to the difference in optimal portfolio returns in two economies. We

set up the exchange rate forecasting relation following Lothian and Wu (2011) represented mathematically by the relationship: $\Delta S_{t+1} = \beta_0 + \beta_1(r_{pt}^o - \bar{r}_{pt}^o)$, $\beta_{0,1}$ constant. The UPRP relationship illustrates that we can expect an immediate and forecasted depreciation of the foreign currency by the factor β_1 when the domestic optimal portfolio return is greater than the foreign optimal return by one percentage point. The depreciation occurs since we make the assumption in line with Cenedese, Payne and Sarno (2015) that excess returns exist after controlling for the risk factors in the international equity markets. Moreover, the UPRP captures the idea that when returns expected from domestic risky assets are higher in the home country than returns from foreign assets the investor is only willing to diversify by investing in the foreign assets if the foreign currency will strengthen against the home currency.

We do not statistically test this relationship directly but rather we use the formulation to derive an exchange rate forecasting model and we test whether the UPRP has significant statistical power in explaining exchange rate movements. The UPRP relationship tells us that higher return currencies depreciate. When the expected portfolio return in one market is lower than the expected portfolio return that could be earned from another market, the currency in the market offering a lower portfolio return is expected to appreciate. Moreover, According to the UPRP when expected optimal portfolio return in the home country is higher than expected optimal portfolio return in a foreign the domestic currency is expected to depreciate. In this case the model reflects that the foreign investor runs a loss when he invests in the foreign country hence to maintain a balance he must be compensated for what he would have gained from the depreciation of the home currency.

The expanded forecasting equation is:

$$\Delta S_{t+1} = \beta + \phi_i w_{1t-1}^o i_{t-1} + \phi_q w_{2t-1}^o \Delta q_{t-1} + \phi_R w_{3t-1}^o R_{t-1} - \bar{\phi}_i \bar{w}_{1t-1}^o \bar{i}_{t-1} - \bar{\phi}_q \bar{w}_{2t-1}^o \Delta \bar{q}_{t-1} - \bar{\phi}_R \bar{w}_{3t-1}^o \bar{R}_{t-1} + \epsilon_t \quad (5)$$

β constant, ϵ_t error term.

We introduce Taylor rules to equation(5)by substitution of (2) and (3) into (1):

$$r_{pt}^o = w_{1t-1}^o (\phi_i \dot{i}_{t-2} + \phi_\pi (\pi_{t-1} - \hat{\pi}_{t-1}) + \phi_y y_{t-1} + \phi_e e_{t-1}) + \phi_q w_{2t-1}^o \Delta q_{t-1} + \phi_R w_{3t-1}^o R_{t-1} \quad (6)$$

$$\bar{r}_{pt}^o = \bar{w}_{1t-1}^o (\bar{\phi}_i \bar{i}_{t-2} + \bar{\phi}_\pi (\bar{\pi}_t - \bar{\pi}_t) + \bar{\phi}_y \bar{y}_{t-1}) + \bar{\phi}_q \bar{w}_{2t-1}^o \Delta \bar{q}_{t-1} + \bar{\phi}_R \bar{w}_{3t-1}^o \bar{R}_{t-1} \quad (7)$$

Following Molodtsova and Papell (2009) the signs of the coefficients of the short-term interest rate are reversed to reflect the invalidity of the UIP. Combining the equations and simplifying parameters produces a symmetric exchange rate model which is the main forecasting equation:

$$\begin{aligned} \Delta S_{t+1} = & \beta - \beta_i \dot{i}_{t-2} - \beta_\pi \pi_{t-1} - \beta_y y_{t-1} - \beta_e e_{t-1} + \beta_q \Delta q_{t-1} + \beta_R R_{t-1} \\ & + \bar{\beta}_i \bar{i}_{t-2} + \bar{\beta}_\pi \bar{\pi}_{t-1} + \bar{\beta}_y \bar{y}_{t-1} - \bar{\beta}_q \Delta \bar{q}_{t-1} - \bar{\beta}_R \bar{R}_{t-1} + \epsilon_t \end{aligned} \quad (8)$$

- is for foreign variables. $w_{1t-1}^o \phi_i = \beta_i$, $w_{1t-1}^o \phi_\pi = \beta_\pi$, $w_{1t-1}^o \phi_y = \beta_y$, $w_{1t-1}^o \phi_e = \beta_e$, $w_{2t-1}^o \phi_q = \beta_q$, $w_{3t-1}^o \phi_R = \beta_R$ with parameters $\phi_\pi \hat{\pi}_{t-1}$, $\bar{\phi}_\pi \bar{\pi}_t$, β , β_i , β_π , β_y , β_e , β_q , β_R .

4 Empirical Methodology

The study uses both macroeconomic and financial monthly data extracted from Monetary and Financial Statistics database (IMF and IFS respectively). The countries considered are Brazil, Mexico, Poland, South Korea and South Africa while the foreign reference country is the United States. The data sample is separated into two periods- the period for estimation and the rest for forecasting. For Brazil and Mexico the data sets are from January 1996 to February 2015; for Poland the data is from February 2001 to February 2015 and for South Africa and South Korea the data is from February 1992 to February 2013. The data base year is 2010. Data transformations are in line with Molodtsova and Papell (2009). For example CPI measures price level and industrial production index IPI is used as proxy for output since monthly GDP is unavailable (where IPI unavailable manufacturing production index is used). The long-term interest rate is proxied by the ten year government bond rates while the short rate is proxied by the three month Treasury bill rates. The exchange rates are from and as per the definition of Federal Reserve Bank of St. Louis (FRED) database. Share prices are proxied by the total share prices for all shares for all countries obtained from FRED. Following RiskMetrics (1996) a decay factor of $\lambda = 0.97$ is used because the data is monthly. Time varying historical averages are used to calculate the variances and covariances. The portfolio optimization problem is solved for using the Simplex method.

4.1 Rolling Window Forecasting

For a given sample data from $[1, N]$ we are interested in forecasting h steps ahead- h is the forecast horizon. Letting the forecasting period be $[N + h, \dots, N + k]$ results in $[h - k + 1]$ forecasted observations. Initially, the size of the estimation sample w is selected and it is fixed. Generally, $w \leq N$. To implement the Rolling Window estimation initializes at N where the model is first estimated over $[N - w + 1, N]$ then a h periods ahead forecast is computed for $N + h$. At $N + 1$, the first observation is discarded and the model is re-estimated over $[N - w + 2, N + 1]$ then a forecast is computed for $N + h + 1$. This data selection and model re-estimation process continues as the sample rolls onward. All the forecasts are made over a constant h so this helps in handling structural breaks. Since the model is estimated on the last w observations only w observations enter into the information of the coefficients of the model. Consideration must be taken in choosing a large enough size of w in order to avoid problems arising from choosing too small a sample size for coefficient estimates. Model evaluation is more effective over different forecast horizons.

4.2 Stationarity

Non-stationary in the variables may result in spurious regressions therefore we test variables for stationarity using the Augmented Dickey-Fuller (ADF) test. Before this, the best lags for forecasting are identified using Akaike Information Criterion (AIC). If the variable is stationary it is also referred to as integrated of order 0 denoted I(0) alternatively the variable is non-stationary. The first difference of the non-stationary must be tested using ADF procedure and if the null hypothesis of assuming of unit root is rejected, the variable is I(1) or integrated of order 1. The following table outlines results:

Table 1: Stationarity tests

STATIONARITY COEFFICIENTS	SOUTH AFRICA	SOUTH KOREA	BRAZIL	MEXICO	POLAND	USA
ΔS_t	I(1)**	I(0)*	I(1)**	I(0)+	I(0)**	-
$w_{1t-1}^o i_{t-2}$	I(0)**	I(0)**	I(0)*	I(0)**	I(0)**	-
$w_{1t-1}^o \pi_{t-1}$	I(0)**	I(0)**	I(0)**	I(0)**	I(0)**	-
$w_{1t-1}^o y_{t-1}$	I(0)**	I(0)**	I(0)**	I(0)**	I(0)**	-
$w_{1t-1}^o e_{t-1}$	I(0)**	I(0)**	I(0)**	I(0)**	I(0)**	-
$w_{3t}^o \Delta q_{t-1}$	I(0)**	I(0)**	I(0)**	I(0)**	I(0)**	-
$w_{2t}^o R_{t-1}$	I(0)**	I(0)**	I(0)**	I(0)**	I(1)**	-
$\bar{w}_{1t-1}^o \bar{i}_{t-2}$	-	-	-	-	-	I(0)*
$\bar{w}_{1t-1}^o \bar{\pi}_{t-1}$	-	-	-	-	-	I(0)**
$\bar{w}_{1t-1}^o \bar{y}_{t-1}$	-	-	-	-	-	I(0)**
$\bar{w}_{2t-1}^o \Delta \bar{q}_{t-1}$	-	-	-	-	-	I(0)**
$\bar{w}_{3t-1}^o \bar{R}_{t-1}$	-	-	-	-	-	I(0)**
\tilde{i}_{t-1}	I(1)**	I(1)**	I(0)+	I(0)**	I(0)*	-
\tilde{i}_{t-1}	-	-	-	-	-	I(1)**

NOTES: Two asterisks ** indicate significance at 1%; one asterisk * indicates significance at 5%; a plus sign + indicates significance at 10%

Table 2: Descriptive statistics

DESCRIPTIVE STATISTICS		ΔS_t	$w_{1t-1}^o i_{t-2}$	$w_{1t-1}^o \bar{\pi}_{t-1}$	$w_{1t-1}^o y_{t-1}$	$w_{1t-1}^o e_{t-1}$	$w_{2t-1}^o \Delta q_{t-1}$	$w_{3t-1}^o R_{t-1}$	i_{t-2}	i_{t-2}, \bar{i}_{t-2}
SOUTH AFRICA	Mean	-0.05	0.044	0.15	0.00009	-0.0045	0.021	-0.0024	0.1	
	Median	-0.055	0.041	0.056	0	0	0.014	-0.0019	0.1	
	Std. Dev.	0.13	0.036	0.17	0.02	0.063	0.029	0.0073	0.034	
	Kurtosis	3.1	2.3	3.3	6.8	9.9	12	5.5	2.8	
	Jarque-Bera Probability	2.7	11	46	170	570	940	90	11	
	Correlation	0.26	0	0	0	0	0	0	0	0.61
SOUTH KOREA	Mean	-0.024	0.029	0.014	0.00025	-0.003	0.015	-0.0044	0.066	
	Median	-0.0085	0.019	0.013	0	0	0.011	-0.0013	0.046	
	Std. Dev.	0.13	0.033	0.013	0.013	0.062	0.027	0.011	0.046	
	Kurtosis	6.8	3.9	3.1	9.1	13	7	12	4.7	
	Jarque-Bera Probability	240	80	29	420	1200	160	940	110	
	Correlation	0	0	0	0	0	0	0	0	0.59
BRAZIL	Mean	-0.035	0.002	0.00067	0.0000091	-0.000092	0.001	-0.0095	0.17	
	Median	-0.02	0.0002	0.000097	0	0	0.00012	-0.0025	0.16	
	Std. Dev.	0.16	0.0037	0.0012	0.00026	0.0039	0.003	0.024	0.073	
	Kurtosis	3.7	8.6	8.5	11	29	3.1	5.8	3.9	
	Jarque-Bera Probability	29	520	510	670	7100	14	130	50	
	Correlation	0	0	0	0	0	0	0	0	0.59
MEXICO	Mean	-0.037	0.037	0.024	-0.00012	0.0073	0.044	-0.013	0.11	
	Median	-0.025	0.023	0.014	0	0	0.028	-0.0034	0.075	
	Std. Dev.	0.084	0.046	0.031	0.0034	0.034	0.05	0.037	0.086	
	Kurtosis	3	4.9	6.2	5.9	6.5	3.7	29	5	
	Jarque-Bera Probability	20	130	230	100	180	28	7200	130	
	Correlation	0	0	0	0	0	0	0	0	0.73
POLAND	Mean	-0.0039	0.0035	0.0018	0.0005	0.00087	0.043	-0.0059	0.0024	
	Median	0.011	0.00003	0.000012	0	0	0	-0.0017	0	
	Std. Dev.	0.093	0.0061	0.0037	0.0041	0.014	0.063	0.012	0.0067	
	Kurtosis	3.6	4.5	8.1	24	17	2.2	4.3	20	
	Jarque-Bera Probability	13	83	320	3200	1300	27	58	2314	
	Correlation	0	0	0	0	0	0	0	0	0.37
USA	Mean		0.0054	0.0049	0.000047		0.026	-0.0026	0.014	
	Median		0	0	0		0	0	0	
	Std. Dev.		0.013	0.0087	0.0024		0.044	0.0062	0.016	
	Kurtosis		7.5	5.9	17		4.5	2.8	2.7	
	Jarque-Bera Probability		440	230	1800		120	0.76	29	
	Correlation		0	0	0		0	0.69	0	1

The descriptive statistics show that the addition of the time-varying weights to the variables make the series leptokurtic. This produces an abstraction of the data from normality in the sense that the data has longer and fatter tails and very peaky around the central peak in the distributions. Generally, the means and standard deviation of the series is close to zero therefore the series are not very volatile. It can be observed as well that the short-term interest of the US is highly correlated with that of South Africa, South Korea, Brazil and Mexico. Correlation is lower between the the US and Polish short-term interest rate

5 Results

The main model given by Equation (8) is estimated using Ordinary Least Squares (OLS). The following tables show the regression results and graphical representations of the actual series against the forecasts

Figure 1: ACTUAL AND FORECAST CHANGES IN EXCHANGE RATES

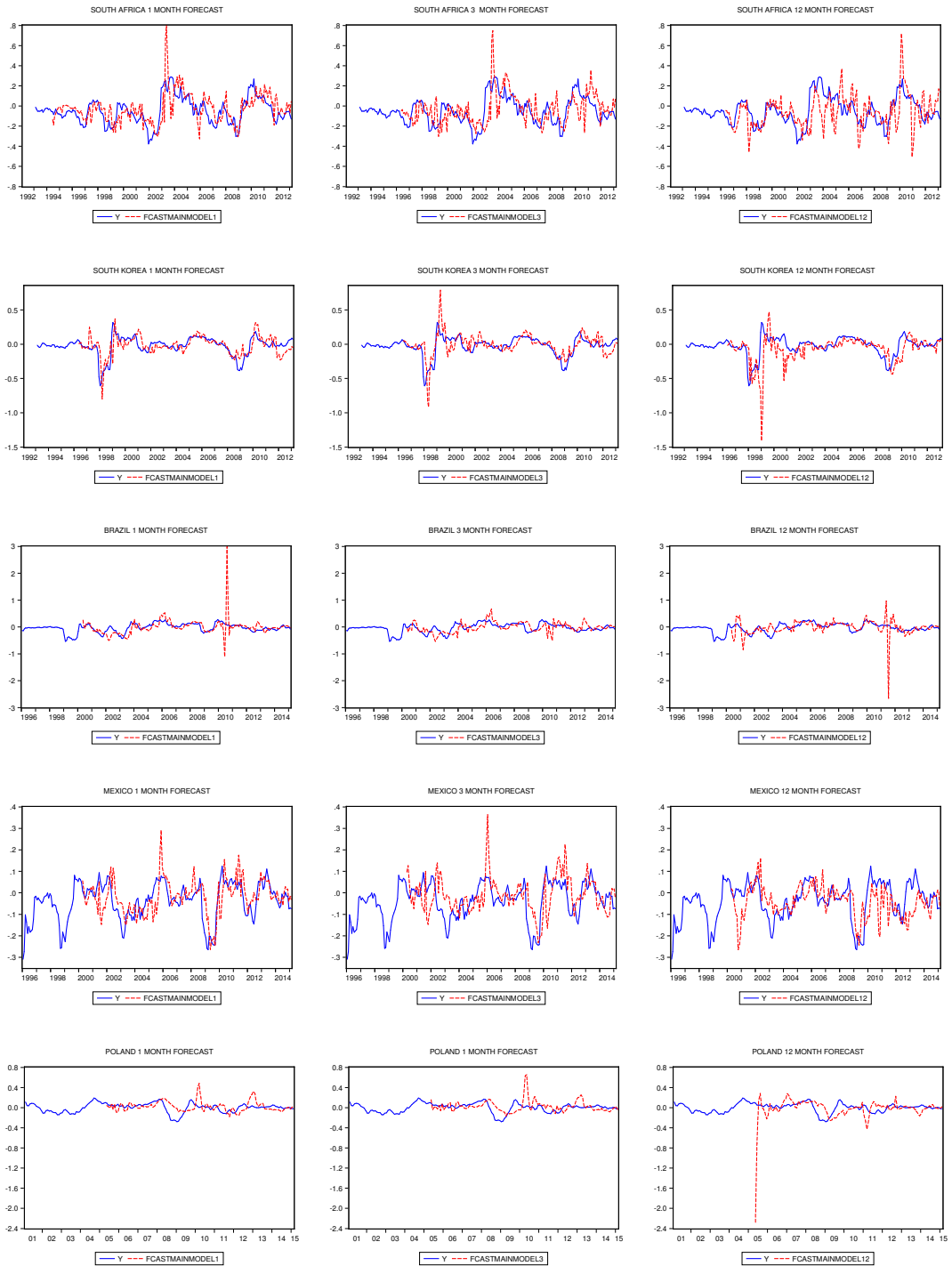


Figure (1) shows that on the whole, the estimated series appears to be tracking the actual series.

Table 3: Estimation of equation (8) regression coefficients

	SOUTH AFRICA			SOUTH KOREA			BRAZIL			MEXICO			POLAND		
	1	3	12	1	3	12	1	3	12	1	3	12	1	3	12
COEFFICIENTS	$\Delta S_{t+1} = \beta - \beta_i i_{t-2} - \beta_\pi \pi_{t-1} - \beta_y y_{t-1} - \beta_e \epsilon_t + \beta_q \Delta q_{t-1} + \beta_R R_{t-1} + \bar{\beta}_i i_{t-2} + \bar{\beta}_\pi \pi_{t-1} + \bar{\beta}_y y_{t-1} - \bar{\beta}_q \Delta \bar{q}_{t-1} - \bar{\beta}_R \bar{R}_{t-1} + \epsilon_t$														
β	0.070 (0.095)	0.108 (0.073)	0.005 (0.057)	0.012 (0.063)	-0.001 (0.063)	0.074 (0.035)*	-0.015 (0.019)	0.033 (0.031)	0.025 (0.037)	-0.010 (0.028)	-0.003 (0.029)	-0.070 (0.025)**	-0.05 (0.02)*	-0.07 (0.02)**	-0.02 (-0.02)
$-\beta_i$	-3.9 (1.665)*	-3.7 (2.7)	7 (5.9)	20 (11)	23 (9.9)*	16 (3)**	-34 (54)	-56 (61)	-150 (56)*	-4.3 (7.7)	-23 (13)	16 (8.3)	64.26 (-146.61)	51.57 (-59.38)	-6.93 (-7.45)
$-\beta_\pi$	3.6 (1.758)*	3.7 (2.5)	-8.8 (5.4)	-11 (7.3)	-11 (7.4)	-1.8 (3.3)	58 (97)	250 (110)*	280 (110)*	3.9 (8.1)	20 (12)	-17 (9.9)	-194.41 (-372.82)	-28.66 (-97.01)	5.34 (-6.21)
$-\beta_y$	1.6** (0.53)	2.9** (0.87)	0.32 (0.93)	-3.4* (1.4)	-0.82 (1.1)	2.3** (0.78)	-987* (41)	33 (29)	-36 (41)	-20* (7.5)	17 (14)	1.5 (9.6)	-8.3 (-33.87)	18.33 (-24.97)	-2.08 (-2.68)
$-\beta_e$	-1.7** (0.13)	-1.2 (0.58)	0.026 (0.64)	-1.9** (0.48)	-1.1* (0.45)	1.6** (0.56)	-43** (6)	1.7 (8.6)	-4 (13)	0.063 (0.96)	-0.19 (1.6)	0.54 (0.95)	14.88 (-29.76)	-24.94 (-19.38)	-0.35 (-0.78)
β_q	-0.41 (0.9)	-0.93 (0.66)	-1.1 (0.66)	2** (0.57)	2.6** (0.91)	1.2 (0.66)	0.31 (6.9)	5.5 (3.9)	-5.3 (11)	0.51 (0.26)	0.78 (0.44)	0.4 (0.34)	-0.02 (-0.75)	-0.23 (-0.6)	0.16 (-0.24)
β_R	-6.7 (6.7)	-9.1* (3.5)	-4.36 (5.31)	4.8 (4.9)	-7.9 (4.1)	-1.4 (4.3)	4.1 (2.1)	8.7 (5)	8.1** (2.9)	-5.3** (1)	-1.5 (1.7)	3.9* (1.6)	-2.3 (0.99)*	-2.86 (0.86)**	-1.48 (-1)
$\bar{\beta}_i$	-26 (6)	72 (55)	1.8 (4.8)	70 (64)	86 (47)	-9.6** (2.8)	-34 (42)	33 (39)	24 (44.)	38 (38)	98 (65)	-13 (63.)	43 (-61)	-41.15 (-49.5)	-41.31 (-54.82)
$\bar{\beta}_\pi$	-7.1** (2.2)	-12** (2.2)	-3.1 (1.9)	3 (3.7)	4.2 (2.7)	3.7 (2.5)	-1.3 (1.6)	-1.4 (1.7)	-2.6 (1.8)	-4.4** (0.73)	-5.8** (1.9)	1.4 (1.9)	-3.08 (-1.75)	-2.84 (-1.44)	3.91 (1.63)*
$\bar{\beta}_y$	5.7 (8.2)	0.16 (8.3)	-15 (11)	-24* (9.5)	-18* (8.2)	130** (4.45)	14** (5.3)	3.7 (5.8)	4 (6.3)	-6 (2.7)	-0.98 (5.3)	3.4 (7.5)	-0.58 (-5.52)	-3.59 (-4.44)	-6.82 (-6.75)
$-\bar{\beta}_q$	-0.058 (0.34)	0.01 (0.79)	0.38 (0.76)	0.88* (0.34)	0.81 (0.4)	-0.59 (0.3)	-0.02 (0.21)	-0.21 (0.27)	0.31 (0.38)	-0.4 (0.29)	0.12 (0.32)	0.17 (0.36)	0.66 (0.29)*	0.94 (0.24)**	-0.16 (-0.29)
$-\bar{\beta}_R$	-13.2* (6)	-17** (5.3)	0.83 (9.4)	-3.6 (5.3)	-6.3 (4.5)	-0.23 (3.6)	-15 (2.4)	2.1 (2)	-9.2* (3.7)	-8.8** (2.2)	3 (3)	-11** (2.7)	-3.93 (-2.11)	-6.01 (1.77)**	-0.8 (-1.85)
Observations:	50	50	50	50	50	50	51	51	51	50	50	50	50	50	50
R-squared:	0.72	0.68	0.38	0.61	0.69	0.84	0.76	0.71	0.66	0.73	0.55	0.55	0.36	0.59	0.53
F-statistic:	8.8	7.2	2.1	5.5	7.7	19	11	8.5	6.8	9.3	4.3	4.3	1.9	4.88	3.97
Durbin-Watson:	1.2	0.97	0.77	0.6	0.54	1.3	1.4	1.1	0.77	1.3	0.94	0.77	0.78	1.2	0.77

NOTES: Standard errors in parentheses. Two asterisks ** indicate significance at 1%; one asterisk * indicates significance at 5%

In Table (3) we focus on the significance level of the coefficients. This reveals statistically how much the dependent variable affects the independent variable. Table (3) shows statistical insignificance of i_{t-2} optimally weighted domestic short-term interest rates in most countries across all horizons with the exception of South Africa at the 1 month horizon, South Korea in the 3 and 12 month horizon and Brazil in the 12 month horizon. Similarly, $-w_{2t-1}^o \Delta q_{t-1}$ the optimally weighted domestic equity returns in all countries across all horizons with the exception of South Korea in the 3 month horizon. The foreign output gap and the real exchange rate affects changes in the South Korean nominal exchange rate at all forecast horizons. The real exchange rate is significant in explaining changes in the nominal exchange rate for at least one forecast horizon in South Africa, South Korea and Brazil this is consistent with the findings of Mohanty and Klau (2004) that central banks tend to respond to changes in the real exchange rates in emerging economies. Foreign inflation rates also explain changes

6 Robustness Checks

The main forecast model equation (8) is tested for robustness by benchmarking its performance against that of two other models. The first model is a random walk model following Moura (2010) and the second model is an unrestricted UIP model following Molodtsova and Papell (2009). The former regresses the nominal exchange rate on lagged nominal exchange rate while the latter regresses the nominal exchange rate on an interest rate differential.

$$\Delta S_t = aS_{t-1} + \epsilon_t \quad (9)$$

$$\Delta S_t = c + d(i_{t-1} - \bar{i}_{t-1}) + u_t \quad (10)$$

a , c and d are constants; ϵ_t and u_t are error terms.

Table 5: Estimations of equation (9) and (10) regression coefficients

	SOUTH AFRICA			SOUTH KOREA			BRAZIL			MEXICO			POLAND		
	1	3	12	1	3	12	1	3	12	1	3	12	1	3	12
COEFFICIENTS															
	$\Delta S_t = aS_{t-1} + \epsilon_t$														
a	0.89 (0.054)**	0.67 (0.12)**	-0.16 (0.21)**	0.89 (0.042)**	0.69 (0.13)**	-0.026 (0.21)	0.93 (0.052)**	0.72 (0.11)**	0.27 (0.098)**	0.88 (0.063)**	0.49 (0.17)**	-0.51 (0.17)**	0.87 (0.073)**	0.43 (0.19)*	-0.33 (0.12)*
Observations:	50	50	50	50	50	50	51	51	51	50	50	50	50	50	50
R-squared:	0.88	0.55	0.03	0.92	0.64	-0.03	0.82	0.33	-0.23	0.77	0.22	0.27	0.75	0.17	0.10
Durbin-Watson:	1.6	0.48	0.12	1.2	0.46	0.086	1.3	0.52	0.18	1.2	0.44	0.27	1	0.43	0.2
COEFFICIENTS															
	$\Delta S_t = c + d(i_{t-1} - \bar{i}_{t-1}) + \epsilon_t$														
c	0.01 (0.13)	-0.03 (0.12)	-0.42 (0.058)**	0.16 (0.14)	-0.11 (0.12)	0.21 (0.12)	-0.34 (0.069)**	-0.29 (0.072)**	0.03 (0.11)	-0.069 (0.079)	-0.11 (0.089)	-0.281 (0.096)**	0.03 (0.037)	0.00 (0.041)	-0.11 (0.049)*
d	-0.24 (2.1)	0.44 (1.9)	5.8 (0.74)**	-7.5 (5.6)	3.423 (4.5)	-9 (4.8)	2.9 (0.66)**	2.4 (0.71)**	-0.8 (1.1)	1.4 (2.3)	2.4 (2.5)	6.4 (2.5)*	-0.98 (1.2)	-0.29 (1.2)	2.9 (1.1)*
Observations:	50	50	50	50	50	50	51	51	51	50	50	50	50	50	50
R-squared:	0.00	0.00	0.58	0.15	0.04	0.23	0.45	0.31	0.03	0.01	0.03	0.09	0.03	0.00	0.22
F-statistic:	0.045	0.16	66	8.7	2	15	40	22	1.6	0.7	1.6	4.6	1.5	0.12	13
Durbin-Watson:	0.14	0.15	0.27	0.097	0.15	0.084	0.36	0.34	0.19	0.24	0.24	0.27	0.24	0.23	0.32

Standard errors in parentheses. Two asterisks ** indicate significance at 1%; one asterisk * indicates significance at 5%

From Table (5) we note the violation of the UIP consistent with the literature. The coefficient d is not equal to 1 and is not statistically significant in most countries and the intercept is

not equal to zero. The UIP model exhibits low R^2 while the Random walk model has high R^2 values.

6.1 Forecast Evaluation

The study determines performance of the estimated forecasts by use of Theil's U statistics and CW statistics. The forecast periods evaluated are the 1,3 and 12 months. Molodtsova and Papell (2008) and Hubrich and West (2010) review some of the methods used in determining out-of-sample predictive power of forecasting models in the literature. They find that the traditional comparisons of the mean square prediction errors (MSPEs henceforth) of the proposed model to that of a benchmark model are limited when the benchmark model is nested in the proposed exchange rate model. These findings build on Clark and West (2007) who develop the CW statistic as a comparison of adjusted MSPEs when the benchmark model is nested in the proposed exchange rate model. The CW statistic is used to establish the model with superior predictive power and this model is the one with the smallest MSPE.

We use the CW statistics adjusted MSPE as specified by Clark and West (2007) since both (9) and (10) are nested in(8). In light of CW statistics usefulness Rogoff and Stavrakeva (2008) still maintain that Theil's U statistics are better sized and superior especially when they are bootstrapped. Therefore we follow Engel, Mark and West (2015) and Moura (2010) and include Theil's U statistics. They compare the MSPE of the main exchange rate model to that of either the random walk with no drift or the UIP model. Following Alquist and Chinn (2008) we infer the the CW statistics by setting up a formal test with a null hypothesis of equal MSPEs hence equal accuracy of two models the main model and the alternative model (Random walk with no drift or UIP).

A more comprehensive CW statistics procedure is outlined in Clark and West (2007). Following Molodtsova et al. (2008) and Nikolsko-Rzhevskyy and Prodan (2012) we outline briefly how the MSPEs are calculated for the deduction of the U and CW statistics in the study. First, we assume the main model is denoted with the subscripts MM , the first alternative model is denoted with the subscript RW and the second alternative model is denoted with the subscript UIP . S_t is the actual time-series realization or log nominal exchange value at t; $\hat{S}k_{m,t}$ is the k-month forecast of the model m where $k = 1, 3, 12$ and $m = MM, RW, UIP$; P is the rolling window size. Then we define the sample forecast errors as: $\hat{e}_{m,t} = S_t - \hat{S}k_{m,t}$. The Mean square prediction error (MSPE) of model m is given by $MSPE^m = P^{-1} \sum_{t=T-P+1}^T \hat{e}_{m,t}^2$.

Theil's U-Test is given by:

Thiel's $U_{RW} = \sqrt{MSPE^{MM}/MSPE^{RW}}$ and Thiel's $U_{RW} < 1$ means MM outperforms RW .

Clark and West (2007) CW statistic:

When comparing predictability of the main model MM and the random walk RW for example:

Let $\hat{e}_{Adj}^2 = (\hat{S}k_{RW,t} - \hat{S}k_{MM,t})^2$ and $Adj = P^{-1} \sum_{t=T-P+1}^T (\hat{S}k_{RW,t} - \hat{S}k_{MM,t})^2$ so that the Adjusted MSPE for the k -month forecast is $\hat{f}k_t = MSPE^{RW} - (MSPE^{MM} - Adj)$.

We implement the hypothesis test as follows:

$$\begin{aligned} H_0 : \hat{f}k_t &= 0 \\ H_1 : \hat{f}k_t &> 0 \end{aligned}$$

To carry out the CW test we regress the difference or $\hat{f}k_t$ on a constant c and then conduct t -test for the null hypothesis that MSPEs are equal. When the value of the difference is positive, the main model beats the alternative model in predictive ability. The reported CW statistic is the t -statistics of the regression. We reject the null of c is equal to or less than zero (hence deduce that the main model outperforms the alternative model) when the CW statistic is greater than the critical value at the given significance level (i.e. greater than 1.282 at 10% level or 1.645 at 5% level).

The CW test is used twice in the study. In the first instance predictability of the main model is tested against that of a driftless random walk. In the second instance predictability of the main model is tested against that of the UIP model, by extension this also tests whether inclusion of fundamentals under an optimal framework in the main model results in better performance than the traditional UIP model. Furthermore, for improved inference of the test statistics we must use equation estimating which have been corrected for heteroskedasticity. We correct the standard errors by application of Dewey-West.

Table 6: Out of sample forecast evaluation

	SOUTH AFRICA			SOUTH KOREA			BRAZIL			MEXICO			POLAND		
HORIZON	1	3	12	1	3	12	1	3	12	1	3	12	1	3	12
	Statistics for equations (8) and (9)														
CW	1.1	0.76	6.6**	2*	2.2*	2.5*	-0.4	-0.96	4.4**	0.57	0.76	4.9**	-1.8	-1.6	0.21
<i>Thiel's U_{RW}</i>	1.5	0.85	0.8	0.72	0.93	0.39	1.2	0.72	0.49	1.1	0.76	0.78	1.6	0.71	0.72
	SOUTH AFRICA			SOUTH KOREA			BRAZIL			MEXICO			POLAND		
HORIZON	1	3	12	1	3	12	1	3	12	1	3	12	1	3	12
	Statistics for equations (8) and (10)														
CW	7.6**	6.6**	5.2**	5.6**	6.4**	2.4*	6.5**	4.7**	5.9**	6.7**	5.7**	4.7**	4.7**	5.4**	1
<i>Thiel's U_{UIP}</i>	0.53	0.57	1.1	0.68	0.64	0.4	0.67	0.71	0.55	0.53	0.68	0.7	0.83	0.64	0.77
Two asterisks ** indicate significance at 1%; one asterisk * indicates significance at 5%															

The results show evidence of exchange rate predictability within a optimal portfolio framework with Taylor rule fundamentals. By means of the CW statistic the main model beats the randomwalk model at the 5% level for 1 (South Korea) out 5 countries at all horizons. The main model beats the randomwalk model at the 1% significance level for the 12-month ahead forecast in South Africa, Brazil and Mexico. However the main model fails to beat the random walk model in all three horizons in Poland. The CW statistics show that the main model beats the UIP model at all horizons in all countries at the 1% significance level with the exception of the 12-month horizon in Poland. Theil's U statistics are mostly close to 1 with the lowest being 0.4. In the cases where the CW statistics is in favor of the Random walk model or when the two statistics conflict the Theil's U statistics are close to unity. The Theil's U statistics also show that with the exception of South Korea, the main model fails to beat the Random walk across all countries in the 1-month horizon.

7 Conclusion

The paper proposes the UPRP relationship in a framework where agents move funds across borders in response to an optimized uncovered portfolio return differential. Assuming a simple two country economy in which an investor's portfolio is made up of three risky assets namely money market instruments, bonds and equities. The economic agent derives an optimal return by means of the objective to minimize portfolio risk or variance. The paper further introduces Taylor rule fundamentals within the context of the said optimized portfolio return model. The time-varying nature of the portfolio risk is captured by exponentially weighted moving average (EWMA) variances and covariances. Predictability of the main model of the study is benchmarked against two alternative models, the driftless random walk and the UIP. A result to note from the study is that the UIP fails to hold across all the countries in that the slope is not equal to 1 and the intercept is not equal to zero at all horizons considered. We conclude that out-of-sample exchange rate forecasting based on an optimal framework and Taylor rule fundamentals produces better nominal exchange rate forecasts relative to the Random walk model and UIP model.

The CW statistics reveal that the main model outperforms the Random walk model in the 12 month horizon for 4 out of 5 countries. The CW statistics further show that for the 1-month horizon the main model is outperformed by the Random walk model in 4 out of 5 countries. Furthermore the main model is only superior in predictability at the 2-month and 3-month horizon in one country (South Korea). However, the Theil's U statistics also show that with the exception of South Korea, the main model beat the Random walk across all countries in the 3 and 12-month horizons. Perhaps it is a worthwhile extension to investigate the main model at longer horizons or in the view of a return maximizing investor. Theretofore out-of-sample exchange rate forecasting using UPRP reveals that the combination of an optimal framework and Taylor rule fundamentals produces better nominal exchange rate forecasts relative to the Random walk model and UIP model. The study further illuminates the subject of drivers of exchange rate in the context of investors within integrated international markets. It is important to develop crucial links between variables which affect exchange rate dynamics because these dynamics in turn affect portfolio and investment choices.

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8 Appendix

8.1 Markowitz Theory of Portfolio Optimization

For a 3-asset portfolio p if $w = (w_1, w_2, w_3)^T$ is a set of associated weights, then the rate of return r_p is given by: $r_p = \sum_{i=1}^3 w_i r_i$ and the expected return on p , $\bar{r}_p = \sum_{i=1}^3 w_i \bar{r}_i$. We show that the portfolio variance $\sigma_p^2 = w^T \Omega w$ where Ω is the variance-covariance matrix:

$$\text{Var}[X] = \sigma_X^2 = E[X - \bar{X}]^2$$

$$\begin{aligned}\sigma_p^2 &= E[w_1(r_1 - \bar{r}_1) + w_2(r_2 - \bar{r}_2) + w_3(r_3 - \bar{r}_3)]^2 \\ &= w_1^2 E[r_1 - \bar{r}_1]^2 + w_2^2 E[r_2 - \bar{r}_2]^2 + w_3^2 E[r_3 - \bar{r}_3]^2 \\ &\quad + 2w_1w_2 E[(r_1 - \bar{r}_1)(r_2 - \bar{r}_2)] + 2w_1w_3 E[(r_1 - \bar{r}_1)(r_3 - \bar{r}_3)] + 2w_2w_3 E[(r_2 - \bar{r}_2)(r_3 - \bar{r}_3)]\end{aligned}$$

$$\begin{aligned}\text{Var}[r_p] &= \sum_{i,j=1}^3 \text{Cov}[w_i r_i, w_j r_j] \\ &= \sum_{i,j=1}^3 w_i r_i w_j r_j - \overline{w_i r_i w_j r_j} \\ &= \sum_{i,j=1}^3 w_i r_i w_j r_j - w_i \bar{r}_i w_j \bar{r}_j \\ &= \sum_{i,j=1}^3 w_i \text{Cov}[r_i, r_j] w_j \\ &= w^T \Omega w\end{aligned}$$